AN OPTIMAL MULTIVARIABLE STABILIZER

FOR A

MULTIMACHINE PLANT

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By

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ABSTRACT

In recent years, intermachine oscillations in a multimachine plant have received particular attention from power system engineers due to the undesirable effects of these oscillations on the system, and also due to the fact that the individual stabilizers now in use are not effective in damping out all modes of oscillation. A multivariable stabilizer may be advantageous to stabilize a multimachine plant in which all parallel connected units in the plant are acted upon at the same time.

In this thesis, an optimal multivariable stabilizer is proposed based on the Generalized Heffron-Phillips model of a multimachine plant and on linear optimal control theory. The feedback function of the stabilizer is determined by minimizing a given quadratic cost function subject to the system dynamic constraints. The effect of different weighting matrices (Q) on the system performance is discussed. Nonlinear simulation results for different disturbance cases are reported to demonstrate the great capability of such a controller to damp out the intermachine oscillations.

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[Q]=Diag[0 0.001 0.1 0]	
[Q]=Diag[0 0.0001 0.1 0]	
(b) Terminal voltages of machines 1 to 3 with	58
[Q]=Diag[0 0.0005 0.1 0]	
[Q]=Diag[0 0.001 0.1 0]	
[Q]=Diag[0 0.0001 0.1 0]	
(c) Rotor speeds of machines 1 to 3 with	59
[Q]=Diag[0 0.0005 0.1 0]	
[Q]=Diag[0 0.0005 0.15 0]	
[Q]=Diag[0 0.0005 0.05 0]	
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PRINCIPAL SYMBOLS

System quantities (pu., except as indicated)

D_k		damping coefficient of the kth machine
E_{fk}		field voltage of the kth machine. One per unit corresponds to one per unit open circuit terminal voltage
$\bar{E}_{qdk}$	E_{qdk}	voltage behind X_q of the kth machine (phasor and magnitude)
E_{qk}'		voltage proportional to the field flux linkages of machine k
H_k		inertia constant of the kth machine (secs)
$\bar{I}_k$	I_k	kth machine armature current (phasor and magnitude)
P_{ek}		electrical power output of kth machine
P_{mk}		mechanical power on the shaft of the kth machine
T_{dok}'		field open circuit time constant of the kth machine
V_{dk}	V_{qk}	direct and quadrature axis components of the kth machine terminal voltage
$\bar{V}_{in}$	V_{in}	infinite bus voltage (phasor and magnitude)
V_{refk}		reference input to the exciter of machine k
$\bar{V}_{tk}$	V_{tk}	terminal voltage of the kth machine (phasor and magnitude)
X_{ek}		Thevenin equivalent reactance at the terminal of the kth machine
X_k		transformer reactance in series with machine k
a_{ik}		impedance factors
i_{dk}	i_{qk}	direct and quadrature axis components of the kth machine armature current
δ_{ik}		angle between the rotors of machine i and k (rad.)

δ_k	rotor angle measured between the q-axis of the kth machine and the infinite bus voltage (rad.)
ω_k	rotor speed of the kth machine (rad./sec.)
ω_o	synchronous speed (rad./sec.)
λ_i	eigenvalues of [A]
ρ_i	eigenvalues of the plant after control
[A]	plant matrix
[B]	input matrix
[C]	output matrix
[I _n]	nxn unit matrix
[Q]	weighting matrix for state variables or output variables
[R]	weighting matrix for input variables
[U]	control input vector of the plant
[X]	state vector of the plant
[Y]	output vector of the plant
$[\Delta E_f]$	mx1 vector containing the deviations of E_{fk} 's
$[\Delta E_q']$	mx1 vector containing the deviations of E_{qk} 's
$[\Delta E_{qd}]$	mx1 vector containing the deviations of E_{qdk} 's
$[\Delta P_e]$	mx1 vector containing the deviations of P_{ek} 's
$[\Delta P_m]$	mx1 vector containing the deviations of P_{mk} 's
$[\Delta V_{ref}]$	mx1 vector containing the deviations of V_{refk} 's
$[\Delta V_t]$	mx1 vector containing the deviations of V_{tk} 's
$[\Delta \omega]$	mx1 vector containing the deviations of ω_k 's

Prefix

 Δ indicates small changes Σ summation

Subscript

o indicates operating point value

Superscript

 -1 inverse of a matrix

T transpose of a matrix

' transient quantities of a synchronous machine

" subtransient quantities of a synchronous machine

Re denotes the real part of a complex quantity

[] denotes vector or matrix

| | denotes absolute value

— over a letter indicates phasor quantity

CHAPTER 1

INTRODUCTION

1.1 INTERMACHINE OSCILLATIONS IN A MULTIMACHINE PLANT

In the rapid development of electrical power systems, different types of large capacity generators, fast, high gain excitation systems and extremely high voltage transmission lines with long distances have been used. This makes the power system stability problems more severe than was the case earlier. In a multimachine plant, electromechanical oscillations which are due to the interactions of the machines have received particular attention from power system engineers because the damping of these oscillations is usually poor even for machines with damper windings and they can be inadvertently enhanced by the inappropriate action of certain control systems which may be added to the plant. In this case, a number of problems may be encountered [12][25], namely:

(1) the rise of periodic variations in the electrical quantities of voltage and current at the bus or in the tie line connecting the plant to the system;

(2) the rise of sustained hunting of machines and controllers within a plant;

(3) the initiation of torsional shaft oscillations in a particular multistage turbo-generator unit.

The most extreme of these problems is dynamic instability due to negative damping. Even in stable situations insufficient damping of these oscillations may limit the ability to transmit power out of the plant.

For an m -machine plant, if the plant is not electrically symmetrical with respect to the common plant bus due to differences between the generating units or their loadings, or from differences in the machine controllers or their settings, this system will tend to have m different electromechanical oscillating modes. One of these modes represents oscillation between the plant as a whole and the system bus, while the others represent intermachine oscillations in the plant.

How to minimize these undesirable oscillations and their effects becomes an important and practical problem in power system control studies. Since the electromechanical oscillations are usually fast compared with the governor and

the turbine time constants, stabilizing action can not be obtained through these systems. The use of fast, high gain exciters makes damping signals more effective. Also, investigations have shown that it is more convenient, more effective and cheaper in certain situations to use excitation control to improve the stability of power systems than to use the other methods, such as adding parallel transmission lines, using capacitor compensators, etc.. Because of this, considerable attention and efforts have been directed toward the excitation control systems.

1.2 INDIVIDUAL POWER SYSTEM STABILIZER

Up to now, individual power system stabilizers (simply termed PSS) have been developed and are widely used for each machine as shown in the single line diagram of Figure 1.1. Usually only one signal, rotor speed or acceleration, AC bus frequency or electrical power, is injected into each excitation control loop. Reference [5] and reference [16] are representative papers which can be used for design and analysis of a power system stabilizer used for one generator.

Figure 1.2 is the block diagram of a typical PSS in which $G(s)$ is the transfer function of the PSS. $G(s)$ is

G: Generator
EX: Excitation system
STB: Stabilizer

Figure 1.1 Individual power system stabilizer for each machine

Figure 1.2 Block diagram of a PSS

designed usually by frequency response techniques using a linear model of the generator. For any input signal, the transfer function of the stabilizer must compensate for the gain and phase characteristics of the excitation system, the generator, and the power system. It is important to realize that, although the individual power system stabilizers are widely used now and yield satisfactory results in certain situations to provide damping to some oscillations, there still exist some inherent disadvantages, namely:

(1) For a multimachine plant, each machine has its own stabilizer which is individually designed for that machine. The interactions between machines can not be considered easily and this may result in one or more modes of oscillation being left undamped. The individual stabilizer is only sensitive to a certain range of oscillation frequencies, this characteristic limits its further use in a sophisticated system which needs increased damping of both the internal plant oscillations and the oscillation of the entire plant with respect to the system to which it is connected.

(2) The individual stabilizer is based on one stabilizing signal, which may produce some undesirable effects on the system, such as the shaft torsional vibration problem. The danger of mutual excitation of one machine from another may arise or be enhanced by the inappropriate

cross coupling of excitation systems from the input of a speed stabilizing signals [12][24][25].

(3) It is important to realize that the transfer function of an individual stabilizer designed by frequency response techniques and assuming a linear generator model is strongly influenced by voltage regulator gain, generator power level, and AC system strength. Consequently any change of these factors can strongly influence the function of the stabilizer.

1.3 MULTIVARIABLE STABILIZER

In order to overcome these disadvantages of an individual stabilizer, a multivariable stabilizer for a multimachine plant has been proposed recently [24] such that all parallel connected units in a plant can be acted upon at the same time in a coordinated way. The block diagram for a multivariable stabilizer is shown in Figure 1.3.

The influence of such a stabilizer on improving system dynamic performance is discussed in detail in Chapter 4. Here only one important merit is mentioned, that is, modern control theory can be easily applied to design it. It is relatively easy to develop guidelines for the setting of such stabilizers because modern control theory has been fully developed. Experience has shown that modern control theory is a very useful tool for designing a multi-input

multi-output control system in many areas of engineering.

G: Generator

Ex: Excitation system

Figure 1.3 Multivariable stabilizer for a multi-machine plant

In Reference [24], modal control theory was used to design a multivariable stabilizer. The purpose of this thesis is to present another new type of multivariable stabilizer designed by using linear optimal control theory. The details of this approach are given in Chapter 3, while in Section 1.4, only the very basic concept of the linear optimal control theory is introduced.

1.4 OPTIMAL MULTIVARIABLE STABILIZER

This stabilizer is called an "optimal multivariable

stabilizer" since optimal control theory is used to design it. The characteristics of linear optimal control theory can be summarized as follows:

(1) It is based on the time domain state space equations of a system rather than on the frequency domain transfer function of the system, which is used in so-called traditional control theory.

(2) It is suited to a multi-input multi-output system.

(3) An analysable formula can be used from linear optimal control theory to calculate the optimal feedback function so that a specified performance cost function can be minimized.

In the early 1970's, Yu, Y.N. [26] and Anderson, J.H. [1], et al, began to introduce the optimal control techniques to excitation and/or governor control systems for turbine-generator units. The research work in this area has been developed quickly and fruitful results in this area (References from [1] to [3], [7] to [11], [13], [17], [18], [20], [27]) demonstrate that the optimal controller for a single machine can improve the system dynamic performance much more than other types of controllers both theoretically and practically. The extension of this theory to a multimachine plant has been made by this author to obtain a new type of multivariable stabilizer which shows its higher

potential for providing damping of the intermachine oscillations as reported in Chapter 4 of this thesis.

In the optimal multivariable stabilizer design, some special problems associated with such a design must be considered carefully and adequately. These are:

- the determination of a suitable reduced order model to be used to represent the multimachine plant simply and adequately. This is discussed in Chapter 2.

- the determination of a suitable weighting matrix Q to obtain the relative optimal results. This is discussed in Chapter 3.

- The dynamic performance of the optimal multivariable stabilizer must be checked using the appropriate simulation model and suitable internal and external disturbances. The results of these studies are given in Chapter 4.

CHAPTER 2

MATHEMATICAL MODEL

2.1 SELECTION OF A PLANT MODEL

In controller studies, the primary step is to select a satisfactory model for the plant which is to be controlled. The following are the main considerations for selecting the appropriate plant model for the studies reported in this thesis.

(1) Reduced model

Theoretically, the more precise a model is, the more accurate will be the results; however, if the order of the model for a synchronous machine is too high, several specific problems may be involved, namely:

- Some of the state variables of the model might not be measurable, in which case state variable observers would have to be designed in order to implement a controller which is based on state variables.

- In the multimachine case with a large number of machines, the order of the system will be very high; consequently, large storage and long computation times are required for the design, and the implementation of a multivariable stabilizer is impractical.

A considerable amount of research work has contributed to setting up various types of models to fit the control studies. It is shown in References [7] and [11] that a third order model for a synchronous machine is sufficient for many excitation system control studies at least for small signal disturbances. Also, Reference [12] recommends that for small disturbance studies it is sufficient to use a linearized version of the synchronous machine model and for this purpose, a Heffron-Phillips linearized model can be employed.

It is apparent that an optimal controller based on a reduced order linearized model remains optimal only with respect to the reduced model and not to the real system. It is proved in the literature [7], [11] and [17] that it is satisfactory to use a controller based on a reduced model to improve dynamic performance of the real system because the sensitivity of the model to the neglected factors, such as stator resistance, magnetic saturation, etc., is quite low in most situations.

(2) Linearized model and non-linear simulation

Since the intent was to use linear optimal control theory for designing a multivariable stabilizer, the plant model had also to be linear. All the nonlinear system equations must be linearized about an operating point where small signals are assumed to apply. Consequently, the optimal controller based on this linear model is optimal only with respect to the linearized model and to the particular operating point chosen. However, the nonlinear model can be used to simulate the system performance and to check whether the linear model is adequate or not. Also, the nonlinear model is used in choosing the best weighting matrix Q to obtain the relative optimal results for a real system. It is convenient to use a linear model for design and a nonlinear model for simulation and checking in the controller studies.

(3) State variables and output variables

It should be pointed out that the choice of different state variables will result in different state space models [8][10][11][17]. The Generalized Heffron-Phillips model of a multimachine plant reported by Vournas and Fleming in Reference [23] is a simple yet sufficiently accurate model which can be readily used for multivariable stabilizer design. The state variables chosen for this model are $[X] = [\Delta\delta \ \Delta\omega \ \Delta E_q' \ \Delta E_f]^T$ and the output variables are

$[Y] = [\Delta P_e \ \Delta \omega \ \Delta V_t \ \Delta E_f]^T$. The advantages of this model are:

- The physical meaning of the model is very clear and can be easily shown in a block diagram and in state space equation form.

- The degree of interactions between the speed loop and the excitation loop, as well as the interactions between the multimachines, can be expressed qualitatively and quantitatively in this model. Therefore, the multivariable stabilizer based on this model can act effectively on the electromechanical oscillations caused by these interactions.

- It is a simple and sufficiently accurate model for design and simulation.

Accordingly, this model was chosen for the optimal multivariable stabilizer design. The following sections briefly describe the derivation of the model as done by Vournas and Fleming [23]. This material is included here for completeness and convenience of readers.

2.2 THE HEFFRON-PHILLIPS MODEL FOR A SINGLE-MACHINE PLANT

A single line diagram of a single machine-infinite bus system is shown in Figure 2.1(a). Figure 2.1(b) shows the corresponding phasor diagram for this situation. The following assumptions are adopted for this system as well as for the multimachine model:

(a)

(b)

Figure 2.1 (a) Single machine-infinite bus system
 (b) Phasor diagram for system in Figure 2.1(a)

- (1) The resistances of the system components are not included,
- (2) Transformer voltages and magnetic saturation of the machine are not included,
- (3) The damping windings are not included,
- (4) The mechanical power of the turbine is assumed to remain constant during the investigation.

Under these assumptions, the basic equations, which are the stator voltage equations, the rotor winding equation, the power equation and the swing equation, become:

$$V_d = X_q i_q \quad (2.1)$$

$$V_q = E_q' - X_d' i_d \quad (2.2)$$

$$V_t = \sqrt{V_d^2 + V_q^2} \quad (2.3)$$

$$(1 + T_{d0}' P) E_q' = E_f - (X_d - X_d') i_d \quad (2.4)$$

$$E_{qd} = E_q' + (X_q - X_d') i_d \quad (2.5)$$

$$P_e = E_{qd} i_q \quad (2.6)$$

$$\frac{2H}{\omega_0} P^2 \delta + \frac{D}{\omega_0} P \delta = P_m - P_e \quad (2.7)$$

$$P \delta = \omega - \omega_0 \quad (2.8)$$

where $\omega_0 = 377$ rad./sec. is the synchronous speed.

From the phasor diagram shown in Figure 2.1,

$$i_d = \frac{E_q' - V_{in} \cos \delta}{X_d' + X_e} \quad (2.9)$$

$$i_q = \frac{V_{in} \sin \delta}{X_q + X_e} \quad (2.10)$$

Combining these equations, eliminating all variables other than δ , E_q' , P_e and V_t and linearizing around an operating point, the Heffron-Phillips model for a single

machine is obtained as follows and the block diagram is shown in Figure 2.2.

$$\Delta P_e = K_1 \Delta \delta + K_2 \Delta E_q' + K_8 \Delta V_{in} \quad (2.11)$$

$$\Delta E_q' = \left(\frac{K_3}{1 + k_3 T_{do}' s} \right) (\Delta E_f - K_4 \Delta \delta + K_9 \Delta V_{in}) \quad (2.12)$$

$$\Delta V_t = K_5 \Delta \delta + K_6 \Delta E_q' + K_7 \Delta V_{in} \quad (2.13)$$

Figure 2.2 Block diagram of Heffron-Phillips linearized model for a single machine-infinite bus system

The definition of all the coefficients and their operating point values are summarized here:

$$K_1 = \left. \frac{\Delta P_e}{\Delta \delta} \right|_{E'_q, V_{in}} = \frac{x_q - x'_d}{x_e + x'_d} i_{qo} V_{ino} \sin \delta_o - \frac{E_{qdo} V_{ino}}{x_e + x_q} \cos \delta_o$$

$$K_2 = \left. \frac{\Delta P_e}{\Delta E'_q} \right|_{\delta, V_{in}} = \frac{x_e + x_q}{x_e + x'_d} i_{qo}$$

$$K_3 = \frac{x_e + x'_d}{x_e + x_d}$$

$$K_4 = -T'_{do} \left. \frac{\Delta \dot{E}'_q}{\Delta \delta} \right|_{E'_q, V_{in}} = \frac{x_d - x'_d}{x_e + x'_d} V_{ino} \sin \delta_o$$

$$K_5 = \left. \frac{\Delta V_t}{\Delta \delta} \right|_{E'_q, V_{in}} = \frac{x_q}{x_e + x_q} \frac{v_{do}}{V_{to}} V_{ino} \cos \delta_o - \frac{x'_d}{x_e + x'_d} \frac{v_{qo}}{V_{to}} V_{ino} \sin \delta_o$$

$$K_6 = \left. \frac{\Delta V_t}{\Delta E'_q} \right|_{\delta, V_{in}} = \frac{x_e}{x_e + x'_d} \frac{v_{qo}}{V_{to}}$$

$$K_7 = \left. \frac{\Delta V_t}{\Delta V_{in}} \right|_{\delta, E'_q} = \frac{v_{do}}{V_{to}} \frac{x_q}{x_e + x_q} \sin \delta_o + \frac{v_{qo}}{V_{to}} \frac{x'_d}{x_e + x'_d} \cos \delta_o$$

$$K_8 = \left. \frac{\Delta P_e}{\Delta V_{in}} \right|_{\delta, E'_q} = - \frac{x_q - x'_d}{x_e + x'_d} i_{qo} \cos \delta_o + \frac{E_{qdo}}{x_e + x_q} \sin \delta_o$$

$$K_9 = T'_{do} \left. \frac{\Delta \dot{E}'_q}{\Delta V_{in}} \right|_{\delta, E'_q} = \frac{x_d - x'_d}{x_e + x'_d} \cos \delta_o$$

2.3 GENERALIZED HEFFRON-PHILLIPS MODEL FOR A MULTIMACHINE PLANT

Figure 2.3 (a) is the single line diagram of an m-machine plant connected to an infinite bus and Figure 2.3(b) is the phasor diagram for this system.

The Generalized Heffron-Phillips model of a multimachine plant is an extension of the single machine model.

In the phasor diagram (Figure 2.3 (b)), it should be emphasized that the machines are represented as variable voltages behind q-axis reactances (E_{qd} in series with X_q). This representation of the machines was chosen as the basis for the multimachine model derivation.

Since the voltage E_{qdk} will always lie on the q-axis of the kth machine, the effect of the kth machine on ith machine is evaluated in two ways, i.e, through a change of the angle between the machine rotor (δ_{ik}) and through a change in the magnitude of E_{qdk} . With this representation of the machines shown in Figure 2.4, the Thevenin Equivalent at the terminals of each machine can be taken. Such an equivalent for the ith machine is shown in Figure 2.5.

(a)

(b)

Figure 2.3(a) Single line diagram of an m -machine plant connected to an infinite bus
 (b) Phasor diagram for the system in Figure 2.3 (a)

Figure 2.4 Machine in the representation of E_{qd} behind q-axis reactance

Figure 2.5 Thevenin equivalent circuit for the plant as seen from the terminal of the i th machine

The Thevenin parameters are obtained as follows:

$$x_{ei} = x_i + x_{oi} \quad (2.15)$$

$$\bar{v}_{ei} = \sum_{\substack{k=1 \\ k \neq i}}^n a_{ik} \bar{E}_{qdk} \quad (2.16)$$

$$a_{ik} = \frac{x_{oi}}{x_k + x_{qk}} \quad (2.17)$$

$$\frac{1}{x_{oi}} = \sum_{\substack{k=1 \\ k \neq i}}^n \frac{1}{x_k + x_{qk}} \quad (2.18)$$

where $\bar{V}_{ei}$ and $\bar{E}_{qdk}$ are phasors.

For the infinite bus n

$$\left. \begin{aligned} x_{qn} &= 0 \\ E_{qdn} &= V_{in} \\ \delta_n &= 0 \end{aligned} \right\} \quad (2.19)$$

With the Thevenin equivalent and the phasor diagram of Figure 2.3, the current of the i th machine can be obtained by taking the d- and q-axis components of the voltages:

$$i_{di} = \frac{1}{x'_{di} + x_{ei}} \left(E'_{qi} - \sum_{\substack{k=1 \\ k \neq i}}^n a_{ik} E_{qdk} \cos \delta_{ik} \right) \quad (2.20)$$

$$i_{qi} = \frac{1}{x_{qi} + x_{ei}} \left(\sum_{\substack{k=1 \\ k \neq i}}^n a_{ik} E_{qdk} \sin \delta_{ik} \right) \quad (2.21)$$

Using the same procedure as in the single machine model, from Eqns. (2.1) to (2.8) and (2.20) to (2.21), one gets the linearized equations of the plant in matrix form

with the $[K]$ coefficients generalized and three new coefficient matrices $[R1]$ to $[R3]$ representing the variation of the E_{qd} 's for a change in the other machine variables:

$$[\dot{\Delta\omega}] = \omega_0 [T_M]^{-1} ([\Delta P_m] - [\Delta P_e]) - [D] [T_M]^{-1} [\Delta\omega] \quad (2.22)$$

$$[\Delta P_e] = [K1] [\Delta\delta] + [K2] [\Delta E_q'] + [K8] [\Delta E_{qd}] \quad (2.23)$$

$$[\dot{\Delta E_q'}] = [T_d]^{-1} ([\Delta E_f] - [K4] [\Delta\delta] + [K9] [\Delta E_{qd}] - [K3]^{-1} [E_q']) \quad (2.24)$$

$$[\Delta V_t] = [K5] [\Delta\delta] + [K6] [\Delta E_q'] + [K7] [\Delta E_{qd}] \quad (2.25)$$

$$[R1] [\Delta E_{qd}] = [R2] [\Delta\delta] + [R3] [\Delta E_q'] \quad (2.26)$$

The mxm parameter matrices are defined as follows:

mechanical starting times (inertia constants)	$[T_M] =$	$\begin{bmatrix} 2H_1 & & & \\ & 2H_2 & & \\ & & \ddots & \\ & & & 2H_m \end{bmatrix}$	} (2.27)
damping factors	$[D] =$	$\begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \ddots & \\ & & & D_m \end{bmatrix}$	

field open circuit
time constants

$$[T_d] = \begin{bmatrix} T_{doi} & & & \\ & T_{do2} & & \\ & & \ddots & \\ & & & T_{dom} \end{bmatrix}$$

The generalized coefficients K and R are defined as follows:

$$K1_{ik} = \frac{\Delta P_{ei}}{\Delta \delta_k}$$

$$K2_{ii} = \frac{\Delta P_{ei}}{\Delta E'_{qi}}, \quad K2_{ik} = 0 \quad (i \neq k)$$

$$K3_{ii} = \frac{x_{ei} + x'_{di}}{x_{ei} + x_{di}}, \quad K3_{ik} = 0 \quad (i \neq k)$$

$$K4_{ik} = -T'_{doi} \frac{\Delta \dot{E}'_{qi}}{\Delta \delta_k}$$

$$K5_{ik} = \frac{\Delta V_{ti}}{\Delta \delta_k}$$

$$K6_{ii} = \frac{\Delta V_{ti}}{\Delta E'_{qi}}, \quad K6_{ik} = 0 \quad (i \neq k)$$

$$K7_{ik} = \frac{\Delta V_{ti}}{\Delta E_{qdk}} \quad (i \neq k), \quad K7_{ii} = 0$$

$$K8_{ik} = \frac{\Delta P_{ei}}{\Delta E_{qdk}} \quad (i \neq k), \quad K8_{ii} = 0$$

$$K9_{ik} = T'_{doi} \frac{\Delta \dot{E}'_{qi}}{\Delta E_{qdk}}$$

$$R1_{ik} = -\frac{\Delta E_{qdi}}{\Delta E_{qdk}} \quad (i \neq k), \quad R1_{ii} = 1$$

$$R2_{ik} = \frac{\Delta E_{qdi}}{\Delta \delta_k}$$

$$R3_{ii} = \frac{\Delta E_{qdi}}{\Delta E'_{qi}}, \quad R3_{ik} = 0 \quad (i \neq k)$$

The values of these coefficients as functions of the operating point are given in Appendix A. After eliminating the vector $[\Delta E_{qd}]$ from Eqns. (2.23) to (2.25) using equation (2.26), one obtains the Generalized Heffron-Phillips model:

$$[\Delta P_e] = [M1][\Delta \delta] + [M2][\Delta E_q'] \quad (2.28)$$

$$[\dot{\Delta E_q'}] = [T_d]^{-1} ([\Delta E_f] - [M4][\Delta \delta] + [M3][\Delta E_q']) \quad (2.29)$$

$$[\Delta V_t] = [M5][\Delta \delta] + [M6][\Delta E_q'] \quad (2.30)$$

where

$$\left. \begin{aligned} [M1] &= [K1] + [K8][R1]^{-1}[R2] \\ [M2] &= [K2] + [K8][R1]^{-1}[R3] \\ [M3] &= -[K3]^{-1} + [K9][R1]^{-1}[R3] \\ [M4] &= [K4] - [K9][R1]^{-1}[R2] \\ [M5] &= [K5] + [K7][R1]^{-1}[R2] \\ [M6] &= [K6] + [K7][R1]^{-1}[R3] \end{aligned} \right\} \quad (2.31)$$

The physical meaning of [M1] to [M6] is more general than [K1] to [K9] (shown in Figure 2.2). Since only independent variables are involved in their definition, the E_{qd} 's having been eliminated. With all other δ 's and E_q 's kept constant, the definitions of M1 to M6 are:

$M1_{ik} = \frac{\Delta P_{ei}}{\Delta \delta_k} = M1_{ki}$	synchronizing coefficient between machine i and k
$M2_{ik} = \frac{\Delta P_{ei}}{\Delta E'_{qk}}$	change in electrical power due to increased field flux linkages of the kth machine
$M3_{ik} = T'_{doi} \frac{\Delta \dot{E}'_{qi}}{\Delta E'_{qk}}$	magnetizing effect of the kth machine field flux linkages
$M4_{ik} = -T'_{doi} \frac{\Delta \dot{E}'_{qi}}{\Delta \delta_k}$	demagnetizing effect due to the angle of kth machine
$M5_{ik} = \frac{\Delta V_{ti}}{\Delta \delta_k}$	change in magnitude of the terminal voltage due to the kth machine angle
$M6_{ik} = \frac{\Delta V_{ti}}{\Delta E'_{qk}}$	change in magnitude of the terminal voltage due to the kth machine field flux linkages

The block diagram of Generalized Heffron-Phillips model is shown in Figure 2.6. It gives a complete description of the interaction relationships between m machines.

Further, the Generalized Heffron-Phillips model can be expressed in a state space form, i.e.,

$$\dot{[X]} = [A] [X] + [B] [U] \quad (2.32)$$

$$[Y] = [C] [X] \quad (2.33)$$

in order to satisfy the requirement for design using linear optimal control theory.

Let

$$\begin{aligned}
 \text{the state vector } [X]_{(3m \times 1)} &= [\Delta\delta \quad \Delta\omega \quad \Delta E_q'] \\
 \text{the input vector } [U]_{(2m \times 1)} &= [\Delta P_m \quad \Delta E_f] \\
 \text{the output vector } [Y]_{(3m \times 1)} &= [\Delta P_e \quad \Delta\omega \quad \Delta V_t]
 \end{aligned} \quad (2.34)$$

From equation (2.22) and (2.28) to (2.30),
the plant matrix is :

$$[A]_{(3m \times 3m)} = \begin{bmatrix} 0 & [I_m] & 0 \\ -\omega_o [T_M]^{-1} [M1] & -[D] [T_M]^{-1} & -\omega_o [T_M]^{-1} [M2] \\ -[T_d]^{-1} [M4] & 0 & [T_d]^{-1} [M3] \end{bmatrix}, \quad (2.35)$$

the input matrix is:

$$[B]_{(3m \times 2m)} = \begin{bmatrix} 0 & 0 \\ \omega_o [T_M]^{-1} & 0 \\ 0 & [T_d]^{-1} \end{bmatrix}, \quad (2.36)$$

and the output matrix is:

$$[C]_{(3m \times 3m)} = \begin{bmatrix} [M1] & 0 & [M2] \\ 0 & [I_m] & 0 \\ [M5] & 0 & [M6] \end{bmatrix}, \quad (2.37)$$

where $[I_m]$ is an $m \times m$ unit matrix.

Figure 2.6 The block diagram of Generalized Heffron-Phillips model

2.4 INCLUDING THE AVR'S IN THE GENERALIZED HEFFRON-PHILLIPS MODEL

So far, the machine model in the state equation form with both the field voltage and the mechanical power as independent inputs has been obtained.

According to the assumption (4) given in section 2.2, the changes in mechanical power input P_m are considered zero.

When each generator in the plant is assumed to be excited by a static AVR of the type shown in Figure 2.7, the generalized state equation for this system can be obtained from Figure 2.7 directly as follows:

Figure 2.7 Static excitation system

$$[\dot{\Delta E}_f] = \frac{1}{[T_A]} ([K_A][\Delta V_{ref}] - [K_A][\Delta V_t] - [\Delta E_f]) \quad (2.38)$$

Now the generalized block diagram shown in Figure 2.8 can be added to the Generalized Heffron-Phillips model.

Figure 2.8 The generalized block diagram of static AVR system

In Figure 2.8 the excitation gain matrix is :

$$[K_A] = \begin{bmatrix} K_{A1} & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & K_{Am} \end{bmatrix} ,$$

and the excitation time constant matrix is :

$$[T_A] = \begin{bmatrix} T_{A1} & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & T_{Am} \end{bmatrix} .$$

(2.39)

The components of the vector $[\Delta E_f]$ are now state variables not input variables as in Figure 2.6 and $[\Delta V_{ref}]$ becomes the input vector.

Then, the state vector is :

$$\begin{matrix} [X] \\ (4mx1) \end{matrix} = [\Delta\delta \quad \Delta\omega \quad \Delta E_q \quad \Delta E_f]^T$$

The input vector is :

$$\begin{matrix} [U] \\ (mx1) \end{matrix} = [\Delta V_{ref}] \quad (2.40)$$

and the output vector is :

$$\begin{matrix} [Y] \\ (4mx1) \end{matrix} = [\Delta P_e \quad \Delta\omega \quad \Delta V_t \quad \Delta E_f]^T$$

The state and output equations are in the form of

$$\dot{[X]} = [A][X] + [B][U] \quad \text{and} \quad (2.41)$$

$$[Y] = [C][X] \quad (2.42)$$

Now, the plant matrix is :

$$[A] = \begin{bmatrix} 0 & [I_m] & 0 & 0 \\ -\omega_o [T_M]^{-1} [M1] & -[D] [T_M]^{-1} & -\omega_o [T_M]^{-1} [M2] & 0 \\ -[T_d]^{-1} [M4] & 0 & [T_d]^{-1} [M3] & [T_d]^{-1} \\ -[K_A] [T_A]^{-1} [M5] & 0 & -[K_A] [T_A]^{-1} [M6] & -[T_A]^{-1} \end{bmatrix} \quad (2.43)$$

The dimension of [A] is 4mx4m.

The input matrix is :

$$\begin{matrix} [B] \\ (4mxm) \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ [K_A] [T_A]^{-1} \end{bmatrix} \quad (2.44)$$

and the output matrix is:

$$[C]_{(4 \times 4m)} = \begin{bmatrix} [M1] & 0 & [M2] & 0 \\ 0 & [I_m] & 0 & 0 \\ [M5] & 0 & [M6] & 0 \\ 0 & 0 & 0 & [I_m] \end{bmatrix} \quad (2.45)$$

2.5 THE DATA OF A THREE MACHINE PLANT AND ITS MODEL

In this thesis a three machine plant is considered. This was based on the physical elements of the Boundary Dam Power Plant of the Saskatchewan Power Corporation. The configuration of this system is shown in Figure 2.9.

Figure 2.9 A three machine plant to infinite bus system

The data and operating points of the three machines are given in Table 2.1 and Table 2.2 respectively. All pu values have been referred to a 100 MVA base. The data of the excitation system for the three machine plant are shown

in Table 2.3.

Table 2.1 Generator data for a three machine plant shown in Figure 2.9

Generator number	Rating (MVA)	Inertia constant H(secs)	Damping factor D(pu)	$X_d = X_q$ (pu)	$X'_d = X'_q$ (pu)	T'_{do} (secs)	External reactance (pu)
1	166.6	4.30	8.0	1.164	0.146	3.84	0.0194
2	166.6	4.30	8.0	1.029	0.124	3.84	0.0194
3	325.0	10.35	10.0	0.625	0.084	6.86	0.0327
Infinite bus	/	1000	/	.0001	.0001	1000	0.067

Table 2.2 Operating points for machines shown in Figure 2.9

Generator number	Active Power (MW)	Reactive power (MVAR)	Terminal voltage (pu)	Voltage angle (degree)	Power angle (degree)	E_{qd} (pu)
1	90.00	27.55	1.050	13.777	50.13	1.6831
2	90.00	27.55	1.050	13.777	47.53	1.5876
3	175.50	20.51	1.050	15.863	57.57	1.5701
Infinite bus	-345.11	16.74	1.000	0.000	0.00	1.0000

Table 2.3 The data of excitation systems for the three machine plant

Generator number	Excitation gains K_A	Time constant $T_A(\text{secs})$	Limitation gains K_p
1	135.3	0.02	5.0
2	287.4	0.05	5.0
3	324.0	0.02	5.0

Under the above conditions, the plant matrix A , the input matrix B and the output matrix C of this system can be calculated by use of the Generalized Heffron-Phillips model and the results are shown in Table 2.4, Table 2.5 and Table 2.6 respectively. These are the matrices for the equations $\dot{[X]} = [A][X] + [B][U]$ and $[Y] = [C][X]$.

Table 2.4 The plant matrix A

0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0
-122.37	18.247	28.168	-0.930	0	0	-144.09	26.197	36.829	0	0	0	0
18.231	-126.89	29.605	0	-0.930	0	26.215	-152.46	40.250	0	0	0	0
11.733	12.344	-79.217	0	0	-0.483	14.308	15.557	-91.374	0	0	0	0
-0.847	0.154	0.202	0	0	0	-1.597	0.308	0.373	0.260	0	0	0
0.137	-0.796	0.195	0	0	0	0.273	-1.587	0.378	0	0.260	0	0
0.065	0.071	-0.385	0	0	0	0.111	0.126	-0.661	0	0	0.146	0
-381.33	-184.08	24.734	0	0	0	-1461.2	-1016.9	-912.1	-50.000	0	0	0
-98.102	-403.80	28.446	0	0	0	-681.72	-1490.6	-753.4	0	-20.000	0	0
-190.19	-320.47	-1048.5	0	0	0	-1707.4	-2126.1	-5232.3	0	0	-50.000	0

Table 2.5 The input matrix B

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
6765.000	0	0
0	5748.000	0
0	0	16200.000

Table 2.6 The output matrix C

2.791	-0.416	-0.643	0	0	0	3.287	-0.598	-0.840	0	0	0
-0.416	2.895	-0.675	0	0	0	-0.598	3.478	-0.918	0	0	0
-0.644	-0.678	4.350	0	0	0	-0.786	-0.854	5.017	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0
0.056	0.027	-0.004	0	0	0	0.216	0.150	0.135	0	0	0
0.017	0.070	-0.005	0	0	0	0.119	0.259	0.131	0	0	0
0.012	0.020	0.065	0	0	0	0.105	0.131	0.323	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	1

The matrices M1 to M6 which represent the interactions between three machines are shown in Table 2.7

Table 2.7 The matrices M1 to M6

Column Row		1	2	3
M1	1	2.7915	-0.4163	-0.6426
	2	-0.4159	2.8947	-0.6753
	3	-0.6442	-0.6778	4.3496
M2	1	3.2868	-0.5976	-0.8401
	2	-0.5980	3.4778	-0.9182
	3	-0.7856	-0.8542	5.0171
M3	1	-6.1314	1.1810	1.4318
	2	1.0499	-6.0944	1.4516
	3	0.7609	0.8677	-4.5350
M4	1	3.2506	-0.5911	-0.7761
	2	-0.5245	3.0563	-0.7490
	3	-0.4427	-0.4838	2.6393
M5	1	0.0564	0.0272	-0.0037
	2	0.0171	0.0703	-0.0049
	3	0.0117	0.0198	0.0647
M6	1	0.2160	0.1503	0.1348
	2	0.1186	0.2593	0.1311
	3	0.1054	0.1312	0.3230

CHAPTER 3

OPTIMAL CONTROLLER DESIGN

3.1 INTRODUCTION

In this Chapter, the method of designing an optimal controller is presented. Section 3.2 briefly describes the basic concepts of linear optimal control theory. Section 3.3 gives one method to solve the Riccati equations to get the optimal parameters of the controller. In Section 3.4, an optimal multivariable stabilizer for a multimachine plant is developed.

3.2 OUTLINE OF LINEAR OPTIMAL CONTROL THEORY

3.2.1 The Principle Of Designing An Optimal State Variable Feedback Controller Using Parameter optimization [4]

According to linear optimal control theory, for a given system represented in the state equation form:

$$\dot{[X]} = [A] [X] + [B] [U] \quad (3.1)$$

where $[X]$ is the state vector of dimension $n \times 1$

$\dot{[X]}$ is the time derivative of $[X]$, dimension $n \times 1$

$[A]$ is the coefficient matrix of dimension $n \times n$

$[B]$ is the input matrix of dimension $n \times r$

$[U]$ is the control vector of dimension $r \times 1$,

the optimal control may be realized by use of the all state variable feedback. The block diagram of the closed-loop system is shown in Figure 3.1.

Figure 3.1 Closed-loop system using state variable feedback

The closed-loop control system which makes a given cost function minimum is defined as the optimal control system corresponding to this given cost function. From this definition, there is not any one absolute optimal control system. A system which is optimal for one cost function need not be optimal for any other cost function. Therefore, the design of an optimal controller involves finding such an

optimal feedback matrix K that a given cost function is minimized. The given cost function describes a desired system performance criterion.

In this case a quadratic cost functions denoted as

$$J = \frac{1}{2} \int_0^{\infty} ([X]^T [Q] [X] + [U]^T [R] [U]) dt \quad (3.2)$$

was used. In equation (3.2), $[Q]$ is an $n \times n$ weighting matrix and $[R]$ is an $r \times r$ weighting matrix. All the non-diagonal elements of $[Q]$ and $[R]$ are zeros. $[Q]$ and $[R]$ are positive definite (or semi-positive definite) and symmetric matrices which are chosen by the investigator. In general, $[R]$ is chosen as a unit matrix. High value elements of $[Q]$ penalize the deviations of the corresponding state variables more severely than lower value elements.

From linear optimal control theory, when $[Q]$ and $[R]$ are given, the optimal feedback function $[K]$ which leads to the minimization of the quadratic cost function is obtained by using

$$[K] = -[R]^{-1} [B]^T [P] \quad (3.3)$$

where $[P]$ is the solution of the following Riccati matrix equation

$$[A]^T [P] + [P] [A] - [P] [B] [R]^{-1} [B]^T [P] + [Q] = [0] \quad (3.4)$$

Then the optimal control vector $[U]$ is obtained where

$$[U] = [K] [X] \quad (3.5)$$

Two aspects need to be considered before obtaining $[K]$; one is how to solve equation (3.4), another is how to choose $[Q]$. The first problem is discussed in Section 3.3 and the second problem is solved in Section 3.4.

3.2.2 The Principle Of Designing An Optimal Output Variable Feedback Controller

Sometimes it is difficult to measure all the state variables or part of the state variables in a system. For example, among the state variables in the Generalized Heffron-Phillips model described in Chapter 2 $[X] = [\Delta\delta \ \Delta\omega \ \Delta E_q' \ \Delta E_f]^T$, $\Delta\delta$ and $\Delta E_q'$ can not be measured directly and accurately, but the output variables $[Y] = [\Delta P_e \ \Delta\omega \ \Delta V_t \ \Delta E_f]^T$ can be measured easily and accurately. To solve this practical engineering problem, an optimal output variable feedback controller represented as follows is used.

For a given linear time-invariant system

$$\dot{[X]} = [A] [X] + [B] [U] \quad (3.6)$$

$$[Y] = [C] [X] \quad (3.7)$$

where the state vector $[X]$ and the output vector $[Y]$ have the same dimension $n \times 1$, $[C]$ is the output matrix of dimension $n \times n$. The given quadratic cost function is

$$J = \frac{1}{2} \int_0^{\infty} ([Y]^T [Q] [Y] + [U]^T [R] [U]) dt \quad (3.8)$$

Then, substituting (3.7) into (3.8) gives

$$J = \frac{1}{2} \int_0^{\infty} ([X]^T [C]^T [Q] [C] [X] + [U]^T [R] [U]) dt \quad (3.9)$$

From linear optimal control theory, when $[Q]$ and $[R]$ are given, the optimal output feedback function K which leads to minimization of the quadratic cost function (3.9) is obtained by

$$[K_y] = -[R]^{-1} [B]^T [P] [C]^{-1} \quad (3.10)$$

where $[P]$ is the solution of the following Riccati matrix equation [4]

$$[A]^T [P] + [P] [A] - [P] [B] [R]^{-1} [B]^T [P] + [C]^T [Q] [C] = [0] \quad (3.11)$$

Therefore, the optimal control vector $[U]$ is

$$[U] = [K_y] [Y] \quad (3.12)$$

Now the closed-loop system can be shown as in Figure 3.2.

Figure 3.2 The closed-loop system using output variable feedback

3.3 NUMERICAL SOLUTION OF RICCATI EQUATION

3.3.1 Introduction

Because $[A]^T [P] + [P] [A] - [P] [B] [R]^{-1} [B]^T [P] + [Q] = [0]$ consists of a second-order item $[P] [B] [R]^{-1} [B]^T [P]$, it is a multi-dimension and second-order algebraic equation. If the given system possesses complete controllability, there must exist an unique solution for $[P]$ and the corresponding feedback matrix must lead to the minimization of the given cost function of the system.

Among several commonly used methods of solving the Riccati equation for $[P]$, such as, the iterative method [14], the eigenvector method [15], the integration method [22], etc., the iterative method is a popular one because of its high precision, fast calculation speed and simplicity. In Section 3.3.2, the iterative method is described briefly.

3.3.2 Iterative Method

To solve $[A]^T [P] + [P] [A] - [P] [B] [R]^{-1} [B]^T [P] + [Q] = [0]$ by the iterative method, first, convert it to the following form:

$$[P]([A] - [B][R]^{-1}[B]^T[P]) + ([A] - [B][R]^{-1}[B]^T[P])^T[P] + [P][B][R]^{-1}[B]^T[P] + [Q] = [0] \quad (3.13)$$

Then from (3.13) obtain the iterative equation as

follows:

$$\begin{aligned}
 & [P]^{(k+1)} ([A] - [B][R]^{-1}[B]^T [P]^{(k)}) + \\
 & ([A] - [B][R]^{-1}[B]^T [P]^{(k)}) [P]^{(k+1)} + \\
 & [P]^{(k)} [B][R]^{-1}[B]^T [P]^{(k)} + [Q] = [0]
 \end{aligned} \tag{3.14}$$

where the superscripts (k) and (k+1) represent the kth and (k+1)th iteration respectively.

If an initial $[P]^{(0)}$ is given, $[P]^{(1)}$ can be calculated from (3.14), then replace $[P]^{(1)}$ for $[P]^{(0)}$ to obtain $[P]^{(2)}$ from (3.14) and continue the iterative calculation in this way until

$$\left| p_{ij}^{(k+1)} - p_{ij}^{(k)} \right| \leq \varepsilon \quad (i=1,2,\dots,n; j=1,2,\dots,n) \tag{3.15}$$

where ε is called the permitted error or precision of iteration. Finally, $[P]^{(k)}$ is the numerical solution for (3.14).

It can be proved that if the eigenvalues of the matrix $[A] - [B][R]^{-1}[B]^T [P]^{(k)}$ all be negative real values and matrix Q is positive definite (or semi-positive definite), the solution $[P]^{(k+1)}$ must be positive definite. If the system possesses complete controllability, as long as an initial guess of $[P]^{(0)}$ is positive definite (or semi-positive definite) and enables $([A] - [B][R]^{-1}[B]^T [P]^{(k)})$ to be stable, the iterative process must converge to a unique real solution $[P]$ and the corresponding feedback matrix must lead to minimization of

the given cost function of the system.

If the coefficient matrix A is a stable matrix itself, $[P]^{(0)} = [0]$ can be chosen and $([A] - [B][R]^{-1}[B]^T[P]^{(0)})$ is stable.

If matrix A is unstable itself, $[P]^{(0)}$ can not be chosen as $[0]$, in this case. $[P]$ can first be obtained from a selected operating point at which matrix A is stable and therefore $[P]^{(0)}$ can be chosen as $[0]$. Then use this $[P]$ as the initial $[P]^{(0)}$ but change matrix A towards the actually chosen operating point step-by-step. A subroutine for obtaining the initial matrix $[P]^{(0)}$ has to be added in this case and this makes the program more complicated.

In order to solve (3.14) on a digital computer easily, it is necessary to convert it in some way. Briefly, equation (3.14) is of the form of equation (3.16). An algebraic matrix equation of this form

$$[F]^T[P] + [P][F] = [G] \quad (3.16)$$

can be converted to the form of

$$[F1][P1] = [G1] \quad (3.17)$$

Where $[P]$, $[F]$ and $[G]$ are $n \times n$ matrices,

$[F1]$ is a $(n(n+1)/2) \times (n(n+1)/2)$ matrix,

$[P1]$ and $[G1]$ are $n(n+1)/2 \times 1$ vectors.

Equation (3.17) is a linear algebraic equation which can be solved easily; therefore, the key to solve (3.14) is to obtain the equation in the form of (3.17). The following procedure demonstrates the five steps of the technique of obtaining (3.17). This is based on the work reported in Reference [6].

(1) Convert $[F]^T[P]$ to the following form:

$$\begin{matrix} [C1] & [P2] \\ (n^2 \times n^2) & (n^2 \times 1) \end{matrix} :$$

$$\left[\begin{array}{ccc} \text{The first row of } F^T & \text{The first row of } F^T & 0 \\ & & \vdots \\ 0 & \text{The first row of } F^T & 0 \\ \text{The second row of } F^T & \text{The second row of } F^T & \vdots \\ & & \vdots \\ 0 & \text{The second row of } F^T & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \text{The nth row of } F^T & \text{The nth row of } F^T & 0 \\ & & \vdots \\ 0 & \text{The nth row of } F^T & \vdots \end{array} \right] \left[\begin{array}{c} p_{11} \\ p_{12} \\ \vdots \\ p_{1n} \\ p_{21} \\ p_{22} \\ \vdots \\ p_{2n} \\ \vdots \\ p_{n1} \\ p_{n2} \\ \vdots \\ p_{nn} \end{array} \right]$$

(3.18)

(5) Eliminate the symmetrical elements

Since $[P]$ is symmetrical, it is only necessary to obtain the upper triangular part of $[P]$ (including diagonal elements); the number of these elements is $n(n+1)/2$ in all. Therefore, there are only $n(n+1)/2$ equations which are independent in (3.21), whereas the others $((n^2 - n(n+1)/2)$ in all) which are dependent should be eliminated. For example, P_{12} and P_{21} are the duplicated elements in $[G2]$, P_{21} should be eliminated from $[G2]$: firstly, by adding the corresponding column of P_{21} in matrix $[F2]$ (that is the $(n+1)$ th column) to the corresponding column of P_{12} in matrix $[F2]$ (that is the second column), then secondly, eliminating the $(n+1)$ th column in $[F2]$, the $(n+1)$ th row in $[F2]$, $[P2]$ and $[G2]$ at the same time. The lower triangular elements in matrix $[P]$ are all eliminated in such a way that the equations remaining in (3.21) are the independent equations in the form of $[F1][P1]=[G1]$. Solving $[F1][P1]=[G1]$ for every iteration, the final solution $[P]$ is obtained.

It should be pointed out that from the results of the studies reported in this thesis using a VAX 11-780 computer, i.e., when the iterative precision was set at 10^{-5} , about 5 iterations and 300 seconds cpu time were needed for the three machine plant system, and about 15 seconds cpu time for a single machine system using this iterative method. The storage and cpu time of a computer will increase much

more with increasing of system order, and this is the disadvantage of this method. The block diagram for optimal output feedback controller design is as follows:

Figure 3.3 Block diagram for optimal output controller design

3.4 APPLICATION OF OPTIMAL CONTROL THEORY TO A MULTIMACHINE PLANT

For a three machine plant, an optimal output multivariable stabilizer can be designed in the form shown in figure 3.4.

So far, all problems in designing this optimal output multivariable stabilizer, except how to choose the weighting matrix Q , have been solved and described in Section 3.2 and 3.3. Now the choice of the weighting matrix Q is considered.

Figure 3.4 Optimal output multivariable stabilizer for a three machine plant

3.4.1 Weighting Matrix Q

It should be emphasized again that an optimal control system is defined as that particular system which makes a given cost function of this system minimal. Obviously, if the weighting matrix Q of a quadratic cost function is changed, the corresponding optimal control system must be changed. Because the weighting matrix Q is chosen by the investigator, it can, in principle, be any arbitrary value; however, from an engineering point of view, the system must be practical and investigators must choose $[Q]$ to improve the system response in some way. $[Q]$ can not be chosen as any arbitrary value, rather $[Q]$ must be chosen to satisfy the requirements of engineering. For a multimachine plant system the system small disturbance dynamic stability is the main criterion used for checking whether a chosen $[Q]$ is suitable or not. In Reference [27], the authors explored $[Q]$ based on their experience, Moussa and Yu [17] developed a method to determine $[Q]$ in conjunction with a left shift of the dominant eigenvalues as far as the practical controller permitted in a single machine system. This author investigated an orthogonal design method to determine $[Q]$ for a single machine system [21], although it is not entirely free from the need to use experience, it is a simple and effective way to choose $[Q]$ by analyzing the trial results. The multimachine plant is more sophisticated than a single machine system, but the experience gained from

the orthogonal design method in a single machine case is useful and can greatly simplify the procedure of choosing $[Q]$ in the multimachine case. A trial and error approach as termed by this author was used for choosing a satisfactory weighting matrix $[Q]$ in the multimachine case. The procedure is as follows:

In a three machine case, $[Q]$ is a 12th order diagonal matrix expressed as

$$[Q] = \text{Diag}[q_{pe1} \ q_{pe2} \ q_{pe3} \ q_{\omega 1} \ q_{\omega 2} \ q_{\omega 3} \ q_{vt1} \ q_{vt2} \ q_{vt3} \ q_{Ef1} \ q_{Ef2} \ q_{Ef3}] \quad (3.22)$$

the subscripts of q correspond to the output variables of each machine respectively. Here, the same value of q is chosen for the same kind of variables, such as, $q_{pe1} = q_{pe2} = q_{pe3}$, etc., then the expression of $[Q]$ can be simply termed as

$$[Q] = \text{Diag}[q_{pe} \ q_{\omega} \ q_{vt} \ q_{Ef}] \quad (3.23)$$

instead of (3.22).

The experience gained from the orthogonal design method indicates to this author that among these four weighting elements, q_{ω} and q_{vt} can be defined as dominant elements which play an important role in improving system stability, while q_{pe} and q_{Ef} are the secondary elements which can be fixed firstly when q_{ω} or q_{vt} is changed to check the system performance. It is better to choose small values of q_{pe} and

q_{Ef} , therefore, q_{Pe} and q_{Ef} can be firstly fixed as zeros, then q_{ω} and q_{Vt} are changed and their influence on the dominant eigenvalues of the system are determined. Results for this determination are given in Table 3.1.

Table 3.1 The influence of q_{ω} and q_{Vt} on the dominant eigenvalues of the three machine plant system with $q_{Pe} = q_{Ef} = 0$

Trial number	q_{Vt}	q_{ω}	The dominant eigenvalues of the closed-loop system		
1	0.5	0.001	$-1.805 \pm j6.947$,	-1.810 ,	-3.334
2	0.1	0.001	$-2.072 \pm j6.868$,	-1.627 ,	-2.798
3	0.05	0.001	$-2.116 \pm j6.854$,	-1.603 ,	-2.731
4	0.5	0.0005	$-1.368 \pm j7.044$,	-2.333 ,	-5.050
5	0.1	0.0005	$-1.554 \pm j7.004$,	-2.095 ,	-4.015
6	0.05	0.0005	$-1.585 \pm j6.996$	-2.065 ,	-3.896
7	0.5	0.0001	$-0.834 \pm j7.130$,	$-3.867 \pm j11.443$	
8	0.1	0.0001	$-0.916 \pm j7.122$,	-2.952 ,	-11.061
9	0.05	0.0001	$-0.930 \pm j7.120$,	-2.909 ,	-10.010

From Table 3.1, it is to be noted that there is a counteraction between q_{ω} and q_{Vt} to shift the dominant eigenvalues of the closed-loop system; that is, one eigenvalue is shifted to the left but the other one will be shifted to the right. It should be pointed out that in order to choose the better values of q_{Vt} and q_{ω} , it is not

sufficient to consider only the dominant eigenvalues because these eigenvalues are obtained from a reduced and linearized model. A more satisfactory way is to check the system performance in response to certain disturbances using a nonlinear model. The disturbance used here is shown in Figure 3.5; it is a 10MW mechanical power step disturbance on machine 1 and which could occur in a multimachine plant.

Figure 3.5 Mechanical power step disturbance on machine 1

Figure 3.6(a)-(d) are the time responses of rotor speeds and terminal voltages determined for the machines as simulated on a nonlinear model for a disturbance as shown in Figure 3.5, and with different q_{Vt} and q_w . The nonlinear model used for this is described in detail in Chapter 4.

From the responses, it was found that when q_w was increased (q_{Vt} fixed, Figure 3.6(a)-(b)), the system damping became better, but the static deviations of the terminal

voltages increased. If q_{vt} was increased (q_{ω} fixed, Figure 3.6(c)-(d)), the tendency was reversed; i.e, system damping decreased, but the static deviations of V_t decreased also. It also appears that there exists a counteraction between system damping and static deviation of ΔV_t in choosing q_{vt} and q_{ω} .

It is to be noted also that from an eigenvalue point of view, the farther the eigenvalues are shifted to the left in the complex plane, the better the system damping will be; however, it is important to realize that the nonlinear model simulations indicate that only a limited range of q_{vt} and q_{ω} produce better system performance. From a visual examination of the time responses of the system for the tests run, the best value of $[Q]$ seems to be $[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$.

Because the optimal control improves system stability through the field voltages which change with all the output variables, the deviations of field voltages can not be penalized severely, i.e, it is better to choose $q_{Ef} = 0$.

The question of a choice of a suitable value of q_{pe} arises. q_{pe} was chosen arbitrarily to be 0.0 for this test shown in Figure 3.6(a)-(d). Figure 3.6(e)-(f) are the responses with different values of q_{pe} (q_{ω} , q_{vt} and q_{Ef} are fixed). From these responses, it can be concluded that $q_{pe} = 0$ is adequate.

In summary then, to this point the best value of $[Q]$ appears to be $\text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$.

Figure 3.6 Responses in the case of 10MW mechanical power step disturbance on machine 1 with different weighting matrix Q

(a) Rotor speeds of machines 1 to 3

- $[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$
- $[Q] = \text{Diag}[0 \ 0.001 \ 0.1 \ 0]$
- $[Q] = \text{Diag}[0 \ 0.0001 \ 0.1 \ 0]$

Figure 3.6 (b) Terminal voltages of machines 1 to 3

——— [Q]=Diag[0 0.0005 0.1 0]
 —▲— [Q]=Diag[0 0.001 0.1 0]
 —●— [Q]=Diag[0 0.0001 0.1 0]

Figure 3.6 (c) Rotor speeds of machines 1 to 3

— $[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$
 —•— $[Q] = \text{Diag}[0 \ 0.0005 \ 0.15 \ 0]$
 —●— $[Q] = \text{Diag}[0 \ 0.0005 \ 0.05 \ 0]$

Figure 3.6 (d) Terminal voltages of machines 1 to 3

Figure 3.6 (e) Rotor speeds of machines 1 to 3

— $[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$
 —•— $[Q] = \text{Diag}[0.001 \ 0.0005 \ 0.1 \ 0]$
 —•— $[Q] = \text{Diag}[0.01 \ 0.0005 \ 0.1 \ 0]$

Figure 3.6 (f) Terminal voltages of machines 1 to 3

$[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$
 $[Q] = \text{Diag}[0.001 \ 0.0005 \ 0.1 \ 0]$
 $[Q] = \text{Diag}[0.01 \ 0.0005 \ 0.1 \ 0]$

3.4.2 Optimal Output Multivariable Stabilizer

From Section 3.4.1, the weighting matrix Q has been chosen to be $[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$. Applying the optimal output feedback technique described in Section 3.2.2, the feedback matrix K_y can be obtained. Its values are shown in Table 3.2 and used in Chapter 4 for simulating the performance of the optimal controller.

The eigenvalues of the system without control, and also with the optimal output control are shown in Table 3.3. The values of the dominant eigenvalues are analyzed in Section 4.3.1.

Table 3.3 The eigenvalues of the system without control and with optimal output control

Eigenvalue's number	Without control	With optimal output control
1	-44.152	-44.247
2-3	-16.829±j20.708	-17.846±j20.621
4-5	-16.550±j1.476	-22.779, * -4.014
6-7	* -2.311±j11.493	-11.027±j15.939
8-9	-3.094±j9.673	-6.163±j12.396
10-11	* -0.611±j7.190	* -1.555±j7.004
12	-3.245	* -2.095

" * " represents the dominant eigenvalues

Table 3.2 The optimal output feedback matrix K_y for the three machine plant

Feedback variables	Power $[K_{Pe}]$			Rotor speed $[K_{\omega}]$			Terminal voltage $[K_{Vt}]$			Field voltage $[K_{Ef}]$		
Machine	1	2	3	1	2	3	1	2	3	1	2	3
1	-0.0426	0.0091	0.0055	0.0080	0.0022	0.0035	-0.0013	-0.0676	-0.0530	-0.0008	0.0003	0.00008
2	0.0133	-0.0406	0.0067	0.0014	0.0108	0.0037	0.0062	0.0087	-0.0376	0.0003	-0.0016	0.00012
3	0.0069	0.0048	-0.0205	-0.0014	-0.0008	0.0118	0.0485	0.0062	-0.0407	0.0002	0.0003	-0.00032

3.5 CONCLUSIONS

In this Chapter, after the principle of designing an optimal multivariable stabilizer is described, the methods of solving two key problems involved in this design are discussed in detail; one is the iterative method to solve the Riccati equation and the other is the method of solving for the choice of matrix Q . The counteractions between improving system damping and decreasing static deviations of terminal voltages for the case of mechanical power step disturbance are discussed in detail related to the choice of suitable values of q_{vt} and q_w . The methods reported in this thesis for solving these two problems were shown to be quite satisfactory for a three machine plant.

At the end of this Chapter, the description of an optimal multivariable stabilizer which was designed for a given three machine plant is given. The next step described is the simulation of its performance to check whether it is satisfactory for damping intermachine oscillations or not. This study is reported in Chapter 4 of this thesis.

CHAPTER 4

PERFORMANCE ANALYSIS

4.1 INTRODUCTION

In this chapter the results of simulation studies are given to show how the performance of the optimal output multivariable stabilizer compares with that of the conventional controller of the form used in practice in the Boundary Dam Power Plant of the saskatchewan Power Corporation. The results demonstrate the advantages and the suitability of using such an optimal multivariable stabilizer for a wide range of operating conditions.

In Chapter 3, the design of the optimal multivariable stabilizer based on a reduced linearized model and linear optimal control theory is described. A power system, however, is a nonlinear system; can an optimal linear multivariable stabilizer designed in this way work well for a nonlinear system in which both internal and external disturbances occur? To answer this question the system must

be checked using a nonlinear simulation program. For the studies reported in this thesis the nonlinear model was simulated with the Power System Dynamic Simulation Program developed by Podmore [19] and briefly described in Section 4.2.

4.2 POWER SYSTEM DYNAMIC SIMULATION PROGRAM (PSDS)

This PSDS program was developed as a special program to study the effects of generators, excitation systems and turbine-generators on power system transient and dynamic stabilities. The main features of the program are:

(1) Inclusion of up to ten interconnected synchronous machines with a particularly high degree of flexibility in the representations of the machines, in that

- the resistances of the system components can be included,
- saturation can be represented in both axes as a function of the total air gap flux for a round rotor machine, while for a salient pole machine, saturation is represented only in the d-axis,
- one damping winding in the q-axis can be included,
- the round rotor machine representation includes transient saliency.

Comparing these representations with the assumptions listed in Section 2.2, only the transformer voltages of the machines are still not included and the mechanical input powers to the machines are kept constant in the investigation. The other assumptions are all removed.

(2) It is based on a step-by-step integration of the machine and controller differential equations. The network equations are solved at each step. A time step of 0.001 second which was used in all the simulation procedures, gave numerically stable and accurate responses with a reasonable computation time.

(3) It allows for a flexibility of choice of the type of disturbances which can be used in the study of power system transient and dynamic stabilities.

(4) It allows for flexibility in the choice of various types of controllers.

Figure 4.1 illustrates a block diagram of the synchronous generator model used in the program, the related generator equations for this model are listed in Appendix B. Note that it provides for a rotor winding in each of the d- and q-axis, magnetic saturation and coupling with the network equations, with the turbine and with the exciter. The model of the conventional excitation controller was given in Figure 2.7 and the values of its parameters for the three

machine plant were given in Table 2.3. The model of the optimal output controller which was added to the simulation program by this author is shown in Figure 4.2.

Figure 4.1 Synchronous generator model used in the simulation program

Figure 4.2 The model of the optimal output controller used in the simulation program

$[K_{pe}]$, $[K_{\omega}]$, $[K_{vt}]$ and $[K_{Ef}]$ are given in Table 3.2. This includes all the deviations of output variables of all machines; i.e, 12 variables in all for a three machine plant, must be injected into every excitation control loop through the corresponding elements of feedback matrix $[K_y]$. This is an interactive feedback between machines and between all the output variables, rather than a single variable feedback to an individual machine as in the conventional controller as shown in Figure 2.7.

4.3 COMPARISON OF SYSTEM RESPONSES WITH CONVENTIONAL AND OPTIMAL CONTROLLERS

4.3.1 Comparison Of System Eigenvalues

From the eigenvalue point of view, the comparison of performance of the optimal controller versus the conventional controller is shown in Table 4.1 which lists the dominant eigenvalues of the system found without controllers and with different controllers.

Table 4.1 The dominant eigenvalues of the system with different controllers

Eigenvalue number	Without controller	Optimal controller	Conventional controller
1-2	$-2.311 \pm j11.493$	-2.095, -4.014	$-1.765 \pm j8.282$
3-4	$-0.611 \pm j7.190$	$-1.555 \pm j7.004$	$-0.571 \pm j7.101$

From the results given in Table 4.1 it is apparent that the dominant eigenvalues were shifted farther to the left in the complex plane by the optimal control. This made the system more stable. On the contrary, the conventional controller shifted the dominant eigenvalues to the right, and this means that, instead of improving system stability, the conventional controller supplied negative damping to the system and made the system stability worse. The original

oscillation frequencies, calculated from the dominant eigenvalues, were about 1.15 Hz and 1.87 Hz. The dominant oscillation frequencies using conventional control were about 1.13 Hz and 1.35 Hz.

In addition, the settling time of the system following a disturbance may be estimated approximately from the dominant eigenvalues. This is approximately $4/|\operatorname{Re} \rho_i|_{\min}$ for a $\pm 2\%$ steady-state value band. For the optimal control the settling time was 2.57 seconds, while for the conventional control the settling time was 7.00 seconds. This was checked also on the time response curves (see Section 4.3.1 and Figure 4.4-4.5). From these results it can be concluded that for the system studied the optimal control can stabilize the system more quickly than the conventional control can.

The above comparison only considers the dominant eigenvalues which are obtained from the linearized model. Using a nonlinear model simulation (PSDS), further comparisons were completed for two typical cases which were small disturbance and large disturbance cases; one an internal disturbance and the other an external disturbance. They are discussed in Section 4.3.1 and Section 4.3.2.

4.3.2 In The Case Of Small Disturbances

The small disturbances used for the studies reported in

this thesis, were two step disturbances; one was a mechanical power step disturbance on machine 1 as shown in Figure 3.5, the other was a terminal voltage reference step disturbance on machines 1 to 3 as shown in Figure 4.3.

Figure 4.3 0.02 pu terminal voltage reference step disturbances on machines 1 to 3

The dynamic responses of the simulated system with the optimal output control and with the conventional control for the case of a 10 MW mechanical power step disturbance on machine 1 are shown in Figure 4.4 (a)-(d). The responses for the case of a 0.02 pu terminal voltage reference step disturbances are shown in Figure 4.5 (a)-(d). From these responses, the following facts are demonstrated clearly:

(1) Although the 10 MW mechanical power step disturbance was applied to machine 1 only, intermachine oscillations are present in all three machines due to the interaction between machines. In addition, oscillations

Figure 4.4 responses for the case of 10 MW mechanical power step disturbance on machine with the optimal control and the conventional control
(a) Electrical powers of machines 1 to 3

— the optimal control
 ([Q]=Diag[0 0.0005 0.1 0])
 —•— The conventional control

Figure 4.4 (b) Rotor speeds of machines 1 to 3

— the optimal control
 ($[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$)
 —•— the conventional control

Figure 4.4 (c) Terminal voltages of machines 1 to 3

— the optimal control
 ([Q]=Diag[0 0.0005 0.1 0])
 —•— the conventional control

Figure 4.4 (d) Field voltages of machines 1 to 3

—— the optimal control
 ([Q]=Diag[0 0.0005 0.1 0])
 —●— the conventional control

Figure 4.5 Responses for the case of 0.02 pu terminal voltage reference step disturbances on machines 1 to 3

(a) Electrical powers of machines 1 to 3

- The optimal control
 $([Q]=\text{Diag}[0 \ 0.0005 \ 0.1 \ 0])$
- The conventional control

Figure 4.5 (b) Rotor speeds of machines 1 to 3

— The optimal control
 ([Q]=Diag[0 0.0005 0.1 0])
 —•— The conventional control

Figure 4.5 (c) Terminal voltages of machines 1 to 3

— The optimal control
 ($[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$)
 —•— The conventional control

Figure 4.5 (d) Field voltages of machines 1 to 3

— The optimal control
 ([Q]=Diag[0 0.0005 0.1 0])
 —•— The conventional control

occur not only in active power but also in all the machine variables due to the coupling between the variables. This phenomenon can be observed also in the responses for the case of a 0.02 pu terminal voltage reference step disturbance. The interactions between machines and between variables would be more complex if the number of machines were increased. Fortunately, the Generalized Heffron-Phillips model accounts for these interactions qualitatively and quantitatively through the $[M1]$ - $[M6]$ matrices as defined in Section 2.3. The numerical values of the $[M1]$ - $[M6]$ matrices for operation around a given operating point were given in Table 2.7. The absolute values of the elements of matrices $M1$ - $M6$ indicate the degrees of these interactions and the signs of the elements indicate positive or negative interactions.

(2) In this three machine plant system, because machines 1 and 2 are almost symmetrical electromechanically with respect to the common plant bus in these small disturbance cases, this system will tend to have two dominant electromechanical oscillation modes; firstly, between machine 3 and machines 1 and 2 moving together as a pair, and secondly, the three machines moving together as a group with respect to the large system.

Consider the responses in Figure 4.4 and Figure 4.5, a mode with a frequency of about 1.5 Hz and a slower mode of about 1.1 Hz oscillations appear to be present in the system with conventional control, but not in that with the optimal output control. This phenomenon indicates that the conventional control has little effect on damping these electromechanical oscillations, whereas, the optimal control can damp various modes of intermachine oscillations. This is because the optimal control is based on an optimal combination for all the output variable feedbacks, consequently, it can make use of the interactions sufficiently to give the best effect for the damping of all modes of oscillations.

(3) To compare the responses of the system for the two kinds of control more clearly and quantitatively, Figure 4.6 shows a typical time response of the output of a system for a step increase in input to the system. Aside from the initial and final steady-state values, three other quantities are of interest:

- (a) rise time T_r
- (b) maximal overshoot M_p
- (c) settling time T_s

In addition, the static deviation of the steady-state value is also of interest.

These quantities of interest for the responses in Figures 4.4 and 4.5 are summarized in Table 4.2. For the sake of convenience, the heading "opt." denotes the quantities with optimal control and the heading "con." indicates the quantities with conventional control.

Figure 4.6 Typical time response of system to a step increase in input

From Table 4.2, the following important conclusions can be drawn. There is no doubt that for variables P_e and ω the responses using optimal control are much better than those using conventional control including all the quantities of T_r , T_s , M_p and S_d . For the variable V_t , the responses with the optimal control are not as good as those of the conventional control. It appears that a counteraction

Table 4.2 Compared quantities of the responses in Figure 4.4 and Figure 4.5

Quantities Variables		Rise time		Maximum overshoot		Settling time		Static deviation	
		$T_r(\text{secs})$		M_p		$T_s(\text{secs})$		S_d	
		Opt.	Con.	Opt.	Con.	Opt.	Con.	Opt.	Con.
In the case of $\Delta P_{m1}=10 \text{ MW}$	P_{e1} (pu)	0.075	0.087	0.026	0.056	1.075 ($\pm 2\%$)	3.325 ($\pm 2\%$)	0.00034	0.00038
	ω_1 (rad/sec)	/	/	0.2465	0.2787	3.700 ($\pm 2\%$)	7.375 ($\pm 2\%$)	0.0010	0.0015
	V_{t1} (pu)	/	/	0.0024	0.0006	1.225 ($\pm 2\%$)	0.0 ($\pm 2\%$)	0.0030	0.0004
In the case of $\Delta V_{ref}=0.02(\text{pu})$	P_{e1} (pu)	/	/	0.0125	0.0413	0.3 ($\pm 5\%$)	5.875 ($\pm 5\%$)	/	/
	ω_1 (rad/sec)	/	/	-0.0806	-0.2567	1.925 ($\pm 5\%$)	8.275 ($\pm 5\%$)	/	/
	V_{t1} (pu)	0.7	0.075	0.0	0.00125	0.925 ($\pm 5\%$)	0.1 ($\pm 5\%$)	0.0055	0.0006

exists between improving the performance of P_e , ω and of V_t . In addition, because these two kinds of control are both proportional feedback control, consequently, there must be a static deviation of V_t in the step disturbance cases. The static deviation of V_t using optimal control is larger than that for conventional control, because smaller feedback gains are used for the optimal control.

(4) From Figure 4.4(d) and Figure 4.5(d), it is concluded that in the first period after the disturbances, the field voltages change much more in the optimal control than in the conventional control in order to provide more damping to the system. This is because in the optimal control the field voltages have direct interaction with all the output variables; whereas, in the conventional control the field voltages have direct interaction with only the terminal voltage of the individual machine. That is why the deviations of the field voltages can not be penalized severely and $[q_{E_f}] = [0]$ (as shown in Section 3.4.1) is reasonable for optimal control design.

4.3.3 In The Case Of A Large Disturbance

In Section 4.3.1, the simulation results shown that the optimal control is the better one for improving system stability in the case of the small disturbances. To ascertain whether this would still be the case for a large

disturbance, a simulation was carried out to determine the time responses for a three phase short circuit fault of 0.1 second duration on the infinite bus. The resulting responses are shown in Figure 4.7(a)-(d).

Because the three phase short circuit fault is the most severe type of fault, in this case immediately following the fault instant, the field voltages for both control systems rise almost to the same maximal values within the same time duration in response to the disturbance. Following the initial responses these field voltages differ since they are responses to different control laws.

For the short circuit duration (0.1 second), the active electric power output of the plant is reduced while the mechanical power input of all machines are kept constant; hence, there appears a very large unbalance between the mechanical power and the active electric power output of each machine, which makes the rotor speeds of the three machines rise quickly. After the disappearance of the fault at 0.1 second, the intermachine oscillations remain much longer than in the case of small disturbances. The same modes of frequency response, about 1.5 Hz and 1.1 Hz oscillations as in the small disturbance cases are observed from the responses of the conventional control but not from that of the optimal control. Furthermore, in the large disturbance case, the optimal control still has more effect

Figure 4.7 Responses for the case of a three phase short circuit of 0.1 second duration on infinite bus

(a) Electrical powers of machines 1 to 3

- The optimal control
($[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$)
- The conventional control

Figure 4.7 (b) Rotor speeds of machines 1 to 3

— The optimal control
 ([Q]=Diag[0 0.0005 0.1 0])
 —•— The conventional control

Figure 4.7 (c) Terminal voltages of machines 1 to 3

— The optimal control
 ([Q]=Diag[0 0.0005 0.1 0])
 —•— The conventional control

Figure 4.7 (d) Field voltages of machines 1 to 3

— The optimal control
 ([Q]=Diag[0 0.0005 0.1 0])
 —•— The conventional control

on stabilizing the system than does the conventional control.

The settling time T_r for variable P_e and ω is about 2.0 seconds with the optimal control, but about 7.0 seconds with the conventional control; the overshoots are also much less than those in the conventional control.

The settling times of the terminal voltages are almost the same for the two control schemes, the overall performance of terminal voltages with optimal control is not quite as good as that with the conventional control.

From all these simulation results for both the small disturbance cases and the large disturbance case, it is concluded that the overall performance with the optimal control is much better than that with the conventional control.

4.4 RATIONALIZATION OF USING THE LINEAR OPTIMAL CONTROLLER

Theoretically, the performance of a linear optimal controller designed from the linearized model can be optimal only at the designed operating point, but not at all the operating points; therefore, if optimal performance is required over the whole operating range, adaptive control is needed and an on-line micro-computer must be used to change the feedback parameters as the operating point changes, such

an on-line adaptive control may be more complicated and difficult to implement than either a fixed optimal or conventional control system.

The question arises: "can we use the fixed feedback parameters for a multimachine plant over a wide range of operating conditions and what is the result if we do so ?" It is certain that if it can be done the controller will be much simpler and more easily implemented. In this section, it is shown that the optimal controller can work satisfactorily over a wide range of operating conditions, and this is another important advantage for the optimal control. The following information is useful for establishing this advantage.

(1) From the eigenvalue point of view, the variation of the smallest real part of the dominant eigenvalue with feedback matrix K_y (refer to Table 3.2) fixed and with different operating points from 0.0 to 3.0 pu. active power on machine 3 are shown in Figure 4.8. This Figure shows that for very large power level operating conditions, the smallest real part of the eigenvalue varies very little, especially beyond the designed operating point (real power from 1.755 to 3.0 pu.).

(2) When $[Q]$ is fixed as $[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$, different $[K_y]$'s are obtained with different operating points, the values of the smallest real part of the eigenvalue in this case compared with the values of fixed $[K_y]$ are shown in Figure 4.9.

Figure 4.8 Values of the smallest real part of dominant eigenvalue at different operating points

- without control
- optimal control with K_y fixed
- ▲— conventional control

Figure 4.9 Values of the smallest real part of dominant eigenvalue at different operating points

1. — without control
2. —●— optimal control with K_y fixed
3. —▲— optimal control with K_y varying with operating points ($[Q]$ fixed as $[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$)

These two curves are very close to each other and cross at the designed operating point. Before this crossing point, the values shown by curve 2 are a little better than those by curve 3 (the real part of the eigenvalues of curve 2 are more negative), while beyond this point, the values shown by curve 2 are not as good as those shown by curve 3. If the optimal controller were chosen as the base of an adaptive control, from curve 3 it would seem that it is better to change the weighting matrix Q instead of fixing it

with different operating points in order to improve the system stability performance.

(3) With $[Q]$ fixed as $[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$, the values of different $[K_y]$'s for the optimal control with different operating points are shown in Figure 4.10 (a)-(i). From these curves, the interesting thing is that the parameters change little under a wide range of operating conditions (active power on machine 3 from 0.5 pu. to 3.0 pu.). Only $K_{y[1,3]}, K_{y[2,3]}, K_{y[3,3]}, K_{y[1,9]}, K_{y[2,9]}$ change very much under the lower power conditions (active power on machine 3 from 0.0 pu. to 0.5 pu.).

Figure 4.10 Values of different K_y at different operating points with $[Q]$ fixed as $[Q] = \text{Diag}[0 \ 0.0005 \ 0.1 \ 0]$

(a) $K_{y[1,1]} - K_{y[1,6]}$

— $K_{y[1,1]}$	—●— $K_{y[1,2]}$	—▲— $K_{y[1,3]}$
—*— $K_{y[1,4]}$	—◆— $K_{y[1,5]}$	—*— $K_{y[1,6]}$

Figure 4.10 (b) $Ky[2,1]-Ky[2,6]$

$Ky[2,1]$ $Ky[2,2]$ $Ky[2,3]$
 $Ky[2,4]$ $Ky[2,5]$ $Ky[2,6]$

Figure 4.10 (c) $Ky[3,1]-Ky[3,6]$

$Ky[3,1]$ $Ky[3,2]$ $Ky[3,3]$
 $Ky[3,4]$ $Ky[3,5]$ $Ky[3,6]$

Figure 4.10 (d) $Ky[1,7]$ - $Ky[1,9]$
 — $Ky[1,7]$ —●— $Ky[1,8]$ —▲— $Ky[1,9]$

Figure 4.10 (e) $Ky[2,7]$ - $Ky[2,9]$
 — $Ky[2,7]$ —●— $Ky[2,8]$ —▲— $Ky[2,9]$

Figure 4.10 (f) $Ky[3,7]$ - $Ky[3,9]$
 — $Ky[3,7]$ —●— $Ky[3,8]$ —▲— $Ky[3,9]$

Figure 4.10 (g) $Ky[1,10]$ - $Ky[1,12]$
 — $Ky[1,10]$ —●— $Ky[1,11]$ —▲— $Ky[1,12]$

Figure 4.10 (h) $Ky[2,10]$ - $Ky[2,12]$
 — $Ky[2,10]$ — $\bullet$ $Ky[2,11]$ — $\bullet$ $Ky[2,13]$

Figure 4.10 (i) $Ky[3,10]$ - $Ky[3,12]$
 — $Ky[3,10]$ — $\bullet$ $Ky[3,11]$ — $\bullet$ $Ky[3,12]$

(4) The responses of the optimal control with fixed $[K_y]$ and with different operating points (0.8 pu.-1.755 pu.-3.0 pu. active power on machine 3) for the cases of small disturbances and large disturbance are shown in Figure 4.11, Figure 4.12 and Figure 4.13 respectively. Only the responses on machine 1 are presented to illustrate the effects; the tendency of the responses on the other machines were found to be similar to those on machine 1.

From these Figures, it was determined that for operation at power levels below the designed operating point, the responses are not quite as good as in the case of ΔV_{ref} step disturbance, while above the designed operating point, the responses are not as good as in the case of ΔP_m step disturbance and short circuit disturbance. However the actual change in response characteristics was found to be small. The performance is satisfactory using a fixed feedback controller under a wide range of operating conditions.

Figure 4.11 Responses of machine 1 for the case of a 10 MW mechanical power step disturbance on machine 1 at different operating points

- 0.8 pu real power on machine 3
- - - 1.755 pu real power on machine 3
- ... 3.0 pu real power on machine 3

Figure 4.12 Responses of machine 1 for the case of 0.02 pu. terminal voltage reference step disturbances on machines 1 to 3 at different operating points

- 0.8 pu real power on machine 3
- 1.755 pu real power on machine 3
- △— 3.0 pu real power on machine 3

Figure 4.13 Responses of machine 1 for the case of a three phase short circuit of 0.1 second duration on infinite bus at different operating points

- 0.8 pu real power on machine 3
- 1.755 pu real power on machine 3
- △— 3.0 pu real power on machine 3

4.5 CONCLUSIONS

In this Chapter, the system performances obtained from simulation studies on a nonlinear generator model are reported. The time responses with conventional and optimal controllers for the case of small disturbances as well as for the case of a large disturbance showed that the optimal controller possessed higher capability to damp out all the intermachine oscillations and also to improve system dynamic stability more than the conventional controller did. It is also shown in this Chapter that the optimal controller can adapt to a wide range of operating conditions; therefore, the linear optimal controller is suitable for multimachine plant operation.

CHAPTER 5

CONCLUSIONS

The conclusions arrived at from the studies reported in this thesis fall into several categories; those related to the modelling, to the control strategy and to the simulation results.

(1) Modelling

In controller studies, the primary step is to choose a satisfactory model of the plant. The main considerations by this author are given in Section 2.1, i.e, why a reduced and linearized model with measurable output variables is suitable for a multivariable controller studies and why the Generalized Heffron-Phillips model is chosen. The Generalized Heffron-Phillips model is an extension of the well known Heffron-Phillips model of a single synchronous generator to a multimachine plant. The merits of this model are demonstrated once again in this investigation of the optimal controller, i.e, the model provides more physical

insight into the interactions between the multimachines than other models. Its lower order, still with sufficient accuracy for control studies, as well as its simple state space expressions, prove it to be a very practical and effective model. This model deals with the interactions between machines, not only qualitatively but also quantitatively since parameters can be calculated by generalized formulae such that this model can be applied to any arbitrary multimachine plant.

(2) Control strategy

In this thesis, it is shown how linear optimal control theory could be applied to a multimachine plant to develop a new kind of multivariable stabilizer which is different from others [24] in terms of control strategies. Guidelines for the setting of such a stabilizer were established and all the feedback variables of the optimal controller are the measurable output variables, which make the implementation of this optimal controller possible.

The main problems involved in using optimal control theory, which are the methods of solving Riccati equations and the determination of the suitable weighting matrix $[Q]$, are discussed in detail in this thesis. The results show that the iterative method is an accurate and simple method for solving Riccati equations. It could be used for a three machine multivariable stabilizer design. In order to choose

the best Weighting matrix $[Q]$ so as to damp the electromechanical oscillations optimally, the trial and error approach proposed by this author is an easy and effective method. By using this method, the elements q_{ω} and q_{Vt} of $[Q]$ are the primary elements which play an important role in influencing system performances, whereas, q_{Pe} and q_{Ef} are secondary elements which can be fixed at the smaller values for the trials. It was demonstrated by the system eigenvalues and performance characteristics, that in the choice of q_{Vt} and q_{ω} , counteractions exist between improving system stability and decreasing the static deviations of terminal voltages in the case of step disturbances. The larger elements q_{ω} appear to shift the dominant eigenvalue far to the left in the complex plane and increase the static deviations of terminal voltages in the case of step disturbances. The larger elements q_{Vt} , on the other hand, decrease the static deviations of terminal voltages and make the system stability worse. Therefore, considerable care and synthesis should be exercised in choosing a suitable $[Q]$. In addition, from an eigenvalue point of view, larger absolute values of the negative real parts of the dominant eigenvalues should indicate better damping of system oscillations; however, eigenvalues are obtained from linearized models. As a matter of fact, the multimachine plant is a nonlinear system, and a further check on the nonlinear simulation model reveals that the actual allowable

range of the weighting matrix Q for getting better performance of the system is limited. From this limited range of $[Q]$, the best $[Q]$ can be chosen with a few tries (trial and error approach). It should be pointed out that using a linear model for design and using a nonlinear model for checking is a practical and effective method for these kinds of control studies. It can simplify the design procedure and get the satisfactory results for the real system.

(3) Simulation results

Simulation tests have been run on a nonlinear model to compare the performance of the optimal control with that of the conventional control in three cases, i.e., the 10 MW mechanical power step disturbance on machine 1, 0.02 pu. terminal voltage reference step disturbance simultaneously on all three machines and a three phase short circuit fault of 0.1 second duration on the infinite bus. From the simulation results, it can be concluded that because the optimal control is based on the optimal interactive feedbacks between machines and between all the output variables, rather than a single variable feedback to an individual machine as shown in the conventional control, it can damp the various intermachine oscillations quickly and optimally even for a large disturbance; whereas, the conventional control can not affect the system optimally in

this way.

In addition, the responses of variables P_e and ω using optimal control are much better than those using conventional control including all the quantities of rise time T_r , settling time T_s , maximal overshoot M_p and static deviation S_d . For the variable V_t , the responses with the optimal control are not as good as those of the conventional control. It appears that a counteraction exists between improving the performances of P_e , ω and of V_t . The optimal control causes the static deviations of the terminal voltages to be larger than the conventional control also because the optimal control involves the proportional interactive feedbacks of all the output variables and the feedback gains are small. Still the static deviations in these cases are small and within acceptable bounds.

(4) Suggestions for further work

The investigation for proving the rationalization of using the fixed optimal controller was carried out in this thesis and shows that the fixed optimal controller can give satisfactory results under a wide range of operating conditions. This investigation has paved the way for further work on the fixed suboptimal controller, simple optimal on-line computer control and adaptive control based upon optimal control theory.

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7. APPENDIX A

The generalized coefficient matrices of Section 2.3 can be calculated as functions of the operating point quantities of the machines in the plant, the elements of these matrices are:

K1:

$$K1_{ii} = - \sum_{\substack{k=1 \\ k \neq i}}^n K1_{ik}$$

$$K1_{ik} = - \frac{x_{qi} - x'_{di}}{x_{ei} + x'_{di}} i_{qio} a_{ik} E_{qdko} \sin \delta_{iko} - a_{ik} \frac{E_{qdio} E_{qdko} \cos \delta_{iko}}{x_{ei} + x_{qi}} \quad (\text{for } i \neq k)$$

K2:

$$K2_{ii} = \frac{x_{ei} + x_{qi}}{x_{ei} + x'_{di}} i_{qio}$$

$$K2_{ik} = 0 \quad (\text{for } i \neq k)$$

K3:

$$K3_{ii} = \frac{x_{ei} + x'_{di}}{x_{ei} + x_{di}}$$

$$K3_{ik} = 0 \quad (\text{for } i \neq k)$$

K4:

$$K4_{ii} = - \sum_{\substack{k=1 \\ k \neq i}}^n K4_{ik}$$

$$K4_{ik} = - \frac{x_{di} - x'_{di}}{x_{ei} + x'_{di}} a_{ik} E_{qdko} \sin \delta_{iko} \quad (\text{for } i \neq k)$$

K5:

$$K5_{ii} = - \sum_{\substack{k=1 \\ k \neq i}}^n K5_{ik}$$

$$K5_{ik} = - \frac{x_{qi}}{x_{ei} + x_{qi}} \frac{v_{dio}}{V_{tio}} a_{ik} E_{qdko} \cos \delta_{iko} + \frac{x'_{di}}{x_{ei} + x'_{di}} a_{ik} E_{qdko} \frac{v_{qio}}{V_{tio}} \sin \delta_{iko} \quad (\text{for } i \neq k)$$

K6:

$$K6_{ii} = \frac{x_{ei}}{x_{ei} + x'_{di}} \frac{v_{qio}}{V_{tio}}$$

$$K6_{ik} = 0 \quad (\text{for } i \neq k)$$

K7:

$$K7_{ii} = 0$$

$$K7_{ik} = \frac{v_{dio}}{V_{tio}} \frac{x_{qi}}{x_{ei} + x_{qi}} a_{ik} \sin \delta_{iko} + \frac{v_{qio}}{V_{tio}} \frac{x'_{di}}{x'_{di} + x_{ei}} a_{ik} \cos \delta_{iko} \quad (\text{for } i \neq k)$$

K8:

$$K8_{ii} = 0$$

$$K8_{ik} = - \frac{x_{qi} - x'_{di}}{x_{ei} + x'_{di}} a_{ik} i_{qio} \cos \delta_{iko} + \frac{a_{ik}}{x_{qi} + x_{ei}} E_{qdio} \sin \delta_{iko} \quad (\text{for } i \neq k)$$

K9:

$$K9_{ii} = 0$$

$$K9_{ik} = \frac{x_{di} - x'_{di}}{x_{ei} + x'_{di}} a_{ik} \cos \delta_{iko} \quad (\text{for } i \neq k)$$

R1:

$$R1_{ii} = 1$$

$$R1_{ik} = \frac{x_{qi} - x'_{di}}{x_{ei} + x'_{di}} a_{ik} \cos \delta_{iko} \quad (\text{for } i \neq k)$$

R2:

$$R2_{ii} = - \sum_{\substack{k=1 \\ k \neq i}}^n R2_{ik}$$

$$R2_{ik} = - \frac{x_{qi} - x'_{di}}{x_{ei} + x'_{di}} a_{ik} E_{qdko} \sin \delta_{iko} \quad (\text{for } i \neq k)$$

R3:

$$R3_{ii} = \frac{x_{qi} + x_{ei}}{x'_{di} + x_{ei}}$$

$$R3_{ik} = 0 \quad (\text{for } i \neq k)$$

APPENDIX B

The equations related to the synchronous generator model shown in Figure 4.1 are summarized below:

q-axis stator equation,

$$V_q = E_q' - X_d' I_d - R_a I_q$$

d-axis stator equation,

$$V_d = E_d' + X_q' I_q - R_a I_d$$

d-axis field equation,

$$\dot{E}_q' = (K E_f - E_q' - (X_d - X_d') I_d) / T_{d0}'$$

q-axis damper equation,

$$\dot{E}_d' = (-E_d' + (X_q - X_q') I_q) / T_{q0}'$$

The rotor mechanical equations,

$$\dot{\omega} = (P_m - P_e - P_l - D\omega) / 2H$$

$$\dot{\delta} = 2\pi f_0 \omega$$

The machine saturation is represented by varying the reactances and time constants as a function of a saturation factor k , which is in turn a function of the air gap flux.

The generator mutual reactances saturate:

$$X_{ad} = K X_{ad}^0$$

$$X_{aq} = K X_{aq}^0$$

The saturated synchronous reactance are given by

$$X_d = K X_d^0 + (1-K) X_l$$

$$X_q = K X_q^0 + (1-K) X_l$$

The saturated open circuit transient time constants are:

$$T_{d0}' = T_{d0}^0 [1 - (1-K)(X_d^0 - X_d') / (X_d^0 - X_l)]$$

$$T_{q0}' = T_{q0}^0 [1 - (1-K)(X_q^0 - X_q') / (X_q^0 - X_1)]$$

The transient reactances X_d' and X_q' are not sensitive to typical variations in and are assumed to be independent of the generator saturation level.

To determine the machine saturation level, the air gap voltage is calculated using:

$$E_{aq} = E_q' - (X_d' - X_1) I_d$$

$$E_{ad} = E_d' + (X_q' - X_1) I_q$$

$$E_{at} = \sqrt{E_{aq}^2 + E_{ad}^2}$$