**TRANSMISSION LINE RELIABILITY MODELS INCLUDING COMMON
MODE AND ADVERSE WEATHER EFFECTS**

**A Thesis Submitted to
the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements for
the Degree of Master of Science
in the Department of
Electrical Engineering**

University of Saskatchewan

by

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April, 1980

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**"TRANSMISSION LINE RELIABILITY MODELS INCLUDING COMMON
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ABSTRACT

This thesis illustrates the development and analysis of a group of transmission system models which include both common mode failures and adverse weather conditions. Both of these factors have been considered as separate phenomenon in previously developed models and as such have been subjected to individual and independent analysis. The models illustrated in this thesis are used to examine the sensitivity of selected reliability indices to individual parameter variation in the composite model. Valid data collection requires an understanding of the use to be made of the data and the relative importance of the data elements in the subsequent analysis. Models such as those presented in this thesis can be used to examine the data needs and the sensitivity of the system indices to model and data approximations.

Approximate equations are developed for calculating the system failure rate and average outage time in two and three line redundant configurations including both adverse weather and common mode considerations. These approximate equations are developed to permit relatively easy utilization of the model in distribution and transmission system reliability studies. The results obtained using the approximate equations have been compared with those predicted by a theoretically accurate Markov approach. These equations are further extended including temporary and maintenance outage conditions.

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LIST OF SYMBOLS

- λ_{av} = The average annual failure rate of a component.
 λ = The normal weather failure rate of a component.
 λ' = The adverse weather failure rate of a component.
 λ_{cmv} = The average annual common mode outage rate.
 λ_c = The normal weather common mode outage rate.
 λ'_c = The adverse weather common mode outage rate.
 λ'' = The maintenance outage rate of a component.
 r = The average repair time of a component.
 r_c = The average repair time of common mode outage.
 r'' = The average maintenance time of a component.
 μ = The average repair rate of a component.
 μ_c = The average repair rate of common mode outage.
 μ'' = The average maintenance rate of a component.
 N = The average duration of normal weather period.
 S = The average duration of adverse weather period.
 n = The reciprocal of N .
 m = The reciprocal of S .
 λ_{sl} = The failure rate of a load point due to overlapping component permanent outages.
 λ''_{sl} = The maintenance outage rate of a load point.
 r_{sl} = The average duration associated with λ_{sl} .
 r''_{sl} = The average duration associated with λ''_{sl} .
 λ_{se} = The series system failure rate.
 r_{se} = The series system average outage duration.

1. INTRODUCTION

There has been a considerable increase in the past few years in the development of modelling techniques and data collection for power system and subsystem reliability evaluation. (The area of transmission system reliability evaluation has historically been somewhat slower to develop than that of generation systems in both modelling techniques and data collection.) Recent research funding, however, should provide considerable impetus to this development. The creation of models and the examination and justification of associated data are a joint and highly interrelated activity. (The application of probability techniques in the quantitative evaluation of transmission and distribution schemes received its present impetus with the publication of two papers [3,6] in 1964, and was expanded and advanced in subsequent publications [5]. These papers developed models and equations for series and parallel system considerations which in addition to proposing indices of failure rate, average outage duration and average annual outage time included consideration of failure bunching due to adverse weather conditions.) This is an important phenomenon particularly in distribution system networks. It does, however, pose considerable difficulty in regard to data collection. It is relatively simple to determine if a failure occurs during adverse weather. It is not so simple,

however, to determine appropriate data for the incidence and duration of the adverse weather when failure does not occur. The additional problem of the lack of data regarding distribution system component populations has resulted in very little data being made available on component normal and adverse weather failure rates.

Failure bunching of parallel facilities due to adverse weather is sometimes mistakenly considered as a common mode or common cause outage. This is not true in regard to the models presently available [3,5], although common mode outages can certainly occur in adverse weather. (A COMMON MODE OUTAGE IS A SITUATION IN WHICH MULTIPLE OUTAGES OCCUR DUE TO A SINGLE EVENT WHERE THESE OUTAGES ARE NOT CONSEQUENCES OF EACH OTHER. The common cause outage phenomenon has been examined in several recent publications, [4,2,8] where it was shown that the recognition of the phenomenon can significantly effect the predicted reliability indices.) (In the case of adverse weather modelling as conducted in References [3] and [5] the common factor is the environment in which the multiple components find themselves. The actual failures are then assumed to be independent events occurring with enhanced failure rates due to the common adverse weather environment.)

This thesis extends the concepts and places in focus

the existing models and equations for considering adverse weather effects and also common mode outages. The equations presented in this thesis give results which compare very closely with those obtained by a Markov approach and are flexible enough to consider the occurrence or nonoccurrence of repair of components during adverse weather conditions.

This thesis also presents simple techniques for the quantitative evaluation of temporary load point interruptions together with common cause outage considerations. Temporary outages of components can cause many load point outages and can be a major cause of customer dissatisfaction. This aspect is extended to include the two state weather environment.

Maintenance of components is another event which can result in load point interruptions. The equations given in the literature [15] are extended to take maintenance outages into account together with common cause and overlapping outages.

In this thesis it has been clearly shown that the common cause failure phenomenon has a dominant influence on the system failure probability and reliability indices. This influence is considerably greater than the common environment phenomenon created by adverse weather during

which independent failures occur. This is an important realization in that while it is extremely desirable to be able to collect all possible data regarding component failure and to utilize detailed models in subsequence analysis of alternate configurations, this is not always practical or feasible. It may be necessary to make certain reductions in modelling complexity which retain the important and dominant aspects in order to keep the problem to a manageable size. This is particularly true in composite or bulk transmission system reliability evaluation where the size of the system and the need to evaluate the complex power and voltage patterns under component outage conditions has at the present time led to the utilization of relatively simple component failure models. Failure bunching under adverse weather conditions has not been extensively utilized in composite system reliability evaluation [1]. Common mode considerations have been included in recent evaluations [8] and can be a significant factor in certain system applications. Composite system reliability evaluation at the present time uses a load flow type representation of the system in which lines simply terminate at a bus without extensive representation of the bus switching and breaker configuration. The effect of these elements are reflected into the line failure parameters in a variety of ways. The most direct way is by simply adding on a terminal component. This implies a

degree of independence between line failures which may not actually exist and which hides the possible loss of several lines due to a single terminal element failure. This would not be a common mode line failure if the resulting reliability model used in the study were sufficiently detailed to include the failed element. In this case the system response would be to remove the lines due to the active failure of the component. If however, the failed component is not represented in the reliability model but is simply part of a general element designated as a terminal than the act of removing the particular group of lines becomes a common mode failure. This situation is not well understood and is a problem in both data collection and in the subsequent creation of models for alternate configurations using the data presently and previously collected. It appears therefore to be extremely important to determine the dominant influences in system failure evaluation and make sure that they are included in the analysis and that second order effects are given an appropriate level of attention.

2. COMMON CAUSE OUTAGE IN MULTIPLE CIRCUIT TRANSMISSION LINES

2.1 Introduction

The power system network forms an integral link between primary sources of energy and the actual consumer of electrical energy. Reliability or more strictly security of adequate power supply, has become an extremely important aspect in the minds of power system engineers as they attempt to provide an uninterrupted electric supply. It is not feasible economically and technically to attempt to design a power system with one hundred percent reliability and some risk must be accepted. Both economical and environmental pressure have encouraged power supply utilities to utilize common structures to support two or more overhead transmission circuits and to construct cable and overhead routes on a common right of way. Similarly, substation and switching stations have been developed that compact the station, reduce clearance between components and minimise the use of land. All of these features increase the probability of occurrence of common mode failures.

Initial literature [5,6] on composite system reliability evaluation, have assumed that outages occur simultaneously due to independent causes. This assumption

is not true for all outage situations. The outage of two or more circuits on the same transmission tower can occur due to a single cause. The mechanical failure of the tower would result in both lines failing due to the single event. A physical disturbance such as an earthquake or tornado can result in the outage of two or more transmission circuits on separate tower structures. These types of outages should be clearly distinguished from independent outages.

The probability of occurrence of an event consisting of a set of simultaneous independent outages is the product of the individual outage probabilities. If the probabilities of individual outages are low, the product can become extremely small. The probability of common cause outage resulting in a similar event can, however, be many times larger. The effect of such outages on reliability indices can be significant as compared with second and higher order simultaneous independent outages. It is not justifiable, therefore to consider higher order outages while neglecting more significant common cause or common mode outages.

The Task Force on Common Mode Outage of Bulk Power Supply Facilities from the IEEE Subcommittee on the Application of Probability Methods (APM) in the Power Engineering Society suggested [16] a common mode outage model for two transmission lines on the same right of way or

on the same transmission tower. Billinton [2] proposed some additional possible common cause outage models. The detailed analysis and comparison of these models is shown in References 4 and 8.

2.2 Component Outages

The term outage describes the state of a component when it is not available to perform its intended function due to some event directly associated with that component. An outage may or may not cause an interruption of consumer service depending on the system configuration. Three general categories of component outages can be recognized.

(1) Independent Outage

(2) Dependent Outage

(3) Common Mode or Common Cause Outage

(1) Independent Outage

An outage is independent outage if the occurrence or nonoccurrence of any other outage or event has no effect on the probability of that outage.

(2) Dependent Outage

An outage is a dependent outage if the occurrence or nonoccurrence of another outage effect the probability of that outage.

(3) Common Mode Outage

A common mode outage is an event having a single

external cause with multiple failure effects where these effects are not consequences of each other [16].

2.3 Common Mode Outage Models

A common arrangement of two transmission lines on the same tower is shown in Figure 2.1. Two transmission lines on the same right of way is illustrated in Figure 2.2. Outage of both lines of these arrangements can occur due to single cause such as floods or aircraft interference or events initiating with the terminal facility. Several possible common cause outage models are described in References [2,15] for two lines on the same tower or on the same right of way.

Before any model is used in a reliability study however, it is essential that its validity be examined in the area for which it is being proposed. For example, although the common mode failure process itself may be the same, the repair process following such an outage varies from system to system and utility to utility due to different operational procedures and standards. To account for these differences at least three different processes have been suggested [2] which differ mainly in the type of repair performed when both components are in the down state.

(2.1)

(2.2)

Figure 2.1 Two Transmission Lines on the Same Transmission Tower.

Figure 2.2 Two Transmission Lines On the Same Right of Way.

2.3.1 Model I

The state space diagram shown in Figure 2.3 represents two cases (a) and (b) in which $\mu_c = 0$ for case (a) and $\mu_c > 0$ for case (b). The model of Figure 2.3(a) was proposed by the Task Force of the APM Subcommittee. In the models of Figure 2.3 and in subsequent models in this thesis

λ = Component failure rate . (failure/yr)

μ = Component repair rate. (repair/yr)

λ_C = Common mode outage rate

μ_C = Common mode repair rate .

The repair process is often described by average repair times rather than the repair rates. In this case

r = Average repair time = $1/\mu$.

The state space diagram illustrates a set of possible transitions from each state considering the independent and common cause failure rates. The model of Figure 2.3(a) assumes that the repair process for all failures including common cause failures is basically the same. There may, however, be some common mode failures for which a common

Figure 2.3 Models For Two Components Including Common Mode Outages.

cause repair applies. For example, the failure of a transmission structure can put both lines in the down state and repair of the transmission structure can make both lines operable simultaneously. The model of Figure 2.3(b) is now applied. The reliability analysis for both cases are shown in the following sections.

2.3.1(a) System State Space Probabilities

The frequency balancing approach can be used to solve for the state space probabilities in the models of Figure 2.3. The basic concepts associated with the frequency balancing approach to the determination of state probabilities are outlined in Appendix I. The closed form solution [7] for the system state space probabilities are as follows:

$$\begin{aligned}
 P_1 &= \{\mu_1\mu_2(\lambda_1 + \lambda_2 + \mu_1 + \mu_2) + \mu_C(\lambda_1 + \mu_2)(\mu_1 + \lambda_2)\}/D \\
 P_2 &= \{\lambda_1\mu_2(\lambda_1 + \lambda_2 + \mu_1 + \mu_2) + \lambda_C\mu_2(\lambda_1 + \mu_2) + \lambda_1\mu_C(\lambda_1 + \mu_2)\}/D \\
 P_3 &= \{\lambda_2\mu_1(\lambda_1 + \lambda_2 + \mu_1 + \mu_2) + \lambda_C\mu_1(\lambda_2 + \mu_1) + \lambda_2\mu_C(\lambda_2 + \mu_1)\}/D \\
 P_4 &= \{\lambda_1\lambda_2(\lambda_1 + \lambda_2 + \mu_1 + \mu_2) + \lambda_C(\lambda_1 + \mu_2)(\lambda_2 + \mu_1)\}/D \quad (2.1)
 \end{aligned}$$

Where:

$$\begin{aligned}
 D &= (\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_1 + \lambda_2 + \mu_1 + \mu_2) + \lambda_C^2(\mu_1 + \mu_2)^2 + \mu_C^2(\lambda_1^2 + \lambda_2^2) \\
 &\quad + (\lambda_1\mu_1 + \lambda_1\mu_2 + \lambda_2\mu_1 + \lambda_1\lambda_2 + \lambda_2\mu_2 + \mu_1\mu_2)(\lambda_C + \mu_C)
 \end{aligned}$$

If the distributions of the time to failure and time to repair of both of the lines are identical with equal mean

values, the state probabilities for this model are as follows:

$$\begin{aligned} P_1 &= (\lambda + \mu) \{2\mu^2 + \mu_C(\lambda + \mu)\} / D \\ P_2 = P_3 &= (\lambda + \mu) (2\lambda\mu + \lambda_C\mu + \mu_C\lambda) / D \\ P_4 &= (\lambda + \mu) \{2\lambda^2 + \lambda_C(\lambda + \mu)\} / D \end{aligned} \quad (2.2)$$

Where:

$$D = 2(\lambda + \mu)^3 + \lambda_C(\lambda + \mu)(\lambda + 3\mu) + \mu_C(\mu^2 + 3\lambda^2 + 4\lambda\mu)$$

The expressions for the state probabilities for the case in which $\mu_C = 0$ can be obtained by substituting this value in the Equations 2.1 and 2.2.

2.3.1(b) System Average Outage Rate

The development of the system average outage rate formula for the model of Figure 2.3 involves three different phases. It is not always necessary to utilize a Markov or state space approach to obtain the reliability parameters for each configuration. Approximate equations for system failure rate and outage time can be developed in certain cases. Appendix 2 presents the basic assumptions used in developing these approximate equations. Utilizing this approach the development of the system average outage rate formula for the models of Figure 2.3 proceeds in three steps. They are as follows:

1. Component 1 fails first and component 2 fails during the repair of component 1.

$$(\text{Failure Rate})_1 = \lambda_1(\lambda_2 r_1)$$

Where: λ_1 = Failure rate of component 1.

$\lambda_2 r_1$ = The probability that component 2 fails during the repair of component 1

$$= 1 - e^{-\lambda_2 r_1}$$

2. Component 2 fails first and component 1 fails during the repair of component 2.

$$(\text{Failure rate})_2 = \lambda_2(\lambda_1 r_2)$$

where: λ_2 = Failure rate of component 2

λ_1 = Failure rate of component 1

$\lambda_1 r_2 = 1 - e^{-\lambda_1 r_2}$ = Probability that component 1 fails during the repair of component 2.

3. Both lines fail due to the occurrence of common cause outage.

$$(\text{Failure rate})_3 = \lambda_c$$

Where: λ_c = common mode outage rate

Thus the overall outage rate for the model of Figure 2.3 is given by

$$\begin{aligned} \lambda_{SL} &= \lambda_1(\lambda_2 r_1) + \lambda_2(\lambda_1 r_2) + \lambda_c \\ &= \lambda_1 \lambda_2 (r_1 + r_2) + \lambda_c \end{aligned}$$

(2.3)

2.3.1(c) System Average Outage Duration

The expressions for system average outage duration are shown in equations 2.4 and 2.5 for the models in Figure 2.3 (a) and (b) respectively.

$$r_{SL} = \frac{1}{\mu_1 + \mu_2} = \frac{r_1 \cdot r_2}{r_1 + r_2} \quad (2.4)$$

$$r_{SL} = \frac{1}{\mu_1 + \mu_2 + \mu} = \frac{r_1 \cdot r_2 \cdot r_c}{r_1 r_2 + r_2 r_c + r_1 r_c} \quad (2.5)$$

2.3.1(d) System Annual Outage Time

The system average annual outage time can be obtained from the product of system failure rate and average outage duration.

$$U_{SL} = \lambda_{SL} r_{SL} \quad (2.6)$$

2.3.2 Model 2

The state space diagram for Model 1 shown in Figure 2.3(b) indicates that two repair processes are available when the system is in the down state due to a common cause failure. These two individual repair phenomena are the individual element repair and the joint common cause repair. If most of the common cause failure occur due to terminal equipment failures and both the lines are put into service

simultaneously even if the repair of one of the lines is completed early, it is necessary to modify the models of Figure 2.3 to eliminate the individual repair phenomena when common cause failure occurs. Such situations are suitably described by Model 2 whose state space diagram is given in Figure 2.4.

Figure 2.4 A State Space Model For Two Components Including Common Mode Outages.

2.3.2(a) System State Probabilities

The closed form solutions for the state probabilities[8] are given in Equations 2.7 and 2.8 for the model of Figure 2.4.

$$P_1 = \mu_1 \mu_2 \mu_c / D$$

$$P_2 = \lambda_1 \mu_2 \mu_c / D$$

$$P_3 = \mu_1 \lambda_2 \mu_c / D$$

(2.7)

$$P_4 = \lambda_1 \lambda_2 \mu_C / D$$

$$P_5 = \mu_1 \mu_2 \lambda_C / D$$

Where $D = \mu_C (\lambda_1 + \mu_1) (\lambda_2 + \mu_2) + \mu_1 \mu_2 \lambda_C$

For identical lines:

$$P_1 = \mu^2 \mu_C / D$$

$$P_2 = P_3 = \lambda \mu \mu_C / D$$

(2.8)

$$P_4 = \lambda^2 \mu_C / D$$

$$P_5 = \mu^2 \lambda_C / D$$

Where $D = \mu_C (\lambda + \mu)^2 + \lambda \mu^2$

2.3.2(b) System Average Outage Rate

The development of the system average outage rate formula for Model 2 involves all the three failure modes used in the case of Model 1.

$$\lambda_{SL} = \lambda_1 \lambda_2 (r_1 + r_2) + \lambda_C \quad (2.9)$$

2.3.2(c) System Average Outage Duration

There are two system failure states in the model of Figure 2.4. The expected average down time of this system can be obtained using the concepts shown in Appendix 1. The expected down time of a system is defined as follows:

$$\frac{\text{Cumulative probability of the down states}}{\text{Cumulative frequency of the down states}} = \frac{P_d}{F_d}$$

$$P_d = P_4 + P_5 = \frac{\lambda_1 \lambda_2 \mu_c + \mu_1 \mu_2 \lambda_c}{D} \quad (\text{from equation 2.7})$$

$$F_d = f_4 + f_5 = (\text{freq. of encounter between states 4 and 5})$$

$$= \frac{\lambda_1 \lambda_2 \mu_c (\mu_1 + \mu_2) + \mu_1 \mu_2 \lambda_c \mu_c}{D}$$

$$r_{SL} = \frac{P_d}{F_d} = \frac{\lambda_1 \lambda_2 \mu_c + \mu_1 \mu_2 \lambda_c}{\lambda_1 \lambda_2 \mu_c (\mu_1 + \mu_2) + \mu_1 \mu_2 \lambda_c \mu_c} \quad (2.10)$$

$$= \frac{\lambda_1 \lambda_2 r_1 r_2 + \lambda_c r_c}{\lambda_1 \lambda_2 (r_1 + r_2) + \lambda_c}$$

2.3.2(d) System Annual Outage Time

The system average annual outage time

$$U_{SL} = \lambda_1 \lambda_2 r_1 r_2 + \lambda_c r_c \quad (2.11)$$

The model shown in Figure 2.4 can be further modified. Common cause outages can occur in two different ways in a multicircuit transmission system. The failure of a transmission tower or terminal equipment can be represented and repaired as shown in Figure 2.4. The failure of both transmission lines due to a common cause event, can in some cases be repaired simultaneously. If, however, the repair of one line is completed sooner than the other, it may be put into service before the second line. The model of

Figure 2.4 does not clearly distinguish these two categories of common cause failures. An improved representation of Model 2 is shown in Figure 2.5, where λ_{c1} and λ_{c2} are two different common cause outage components. The common cause failures accounted for by λ_{c2} are repaired at a rate of μ_c . The repair of the two lines arising due to the rest of the common cause failures are carried out using the independent repair processes.

Figure 2.5 A modified Model Including Two Types of Common Mode Outages.

2.4 Analysis And Comparison of Common Cause Outage Models

The models described in this chapter are some of the possible models obtained by combining the basic independent and common cause failure models. The selection of one of

these models will be based on the configuration and data of the particular situation. This section shows the effect of the selection of the model on system reliability indices and emphasizes the importance of choosing the right model.

There is very little documented material available regarding common cause outage data in practical systems. The Task Force on the Application of Probability Methods Subcommittee of the IEEE Power Engineering Society gave an estimate of the common cause permanent outage as 4.5% of the total permanent outage on a common right of way. This was based however, on a very small amount of data. Outage data of some double circuit lines in the Ontario Hydro system has recently indicated a percentage ratio λ_C/λ of 25.4%. After removing the three bad lines having high common cause outage, the previous ratio was calculated as 14.3%. The details of the Ontario Hydro data are discussed in Chapter 4 of this thesis. Some of these data are used in this chapter to compare the common cause outage models previously presented.

It was therefore, assumed that the common cause failure rate is less than 15% of the independent failure rate of a line. In order to examine the effect, the ratio λ_C/λ was varied from 0 to 15% in steps of 2.5%. The effect of higher common cause outages on system reliability indices is shown in Figure 2.7 where the ratio λ_C/λ is changed from 0 to 25%

in steps of 2.5%.

2.4.1 Effect of Common Cause Failure Rate on System Reliability Indices

Reliability indices for the models of Figures 2.3(a), 2.3(b), and 2.4 are shown in Table 2.1. The indices were obtained using the equations given in this chapter. The percentage error was obtained by calculating the indices using a complete Markov approach as shown in Appendix 1. These probabilities have been calculated for a range of λ_c/λ ratio of 0 to 15%. The results shown in Table 2.1 indicates that the probability of the system being in the down state is much higher for Model 2 than Model 1. The system failure rate was also calculated for each model and it can be seen from Table 2.1 that the system failure rate is very sensitive to the relative magnitude of common cause failure rate. Table 2.1 also indicates the variation of system average outage duration with the variation of common cause outage rate. The effect of r_c in Model 1 depends on the relative magnitude of r_1 and r_2 . Using Model 2 in which the common mode phenomenon produces an isolated state, the bulk of system failures occur due to common cause events as indicated in the model of Figure 2.4, the average system outage time is therefore very close to the time required to repair a common mode failure. All the indices shown in

Table 2.1

Variation in System Reliability Indices With common mode
Outage Rate For a Two Component Redundant System

Independent outage rate of each line = 0.5 f/yr
 Average repair time of each line = 7.5 hours
 Common mode repair time = 10.0 hours

λ_c / λ	Model of Figure 2.3(a)		Model of Figure 2.3(b)		Model of Figure 2.4	
	Using Equations	%Error Introduced	Using Equations	%Error Introduced	Using Equations	%Error Introduced
Pr(Both lines Down) $\times 10000$						
0.0	0.0018	0.0856	0.0013	0.0973	0.0018	0.0856
2.5	0.0553	0.0458	0.0402	0.0571	0.1445	0.0856
5.0	0.1089	0.0467	0.0792	0.0575	0.2872	0.0856
7.5	0.1624	0.0481	0.1181	0.0585	0.4299	0.0856
10.0	0.2159	0.0496	0.1570	0.0595	0.5726	0.0856
12.5	0.2694	0.0511	0.1959	0.0606	0.7151	0.0856
15.0	0.3229	0.0527	0.2348	0.0617	0.8579	0.0856
System Average Failure Rate (f/yr)						
0.0	0.0004	0.1284	0.0004	0.1284	0.0004	0.1284
2.5	0.0129	0.0869	0.0129	0.0869	0.0129	0.0869
5.0	0.0254	0.0863	0.0254	0.0863	0.0254	0.0863
7.5	0.0379	0.0860	0.0379	0.0860	0.0379	0.0860
10.0	0.0504	0.0859	0.0504	0.0859	0.0504	0.0859
12.5	0.0629	0.0858	0.0629	0.0858	0.0629	0.0858
15.0	0.0754	0.0858	0.0754	0.0858	0.0754	0.0858
System Average Outage Duration (hours)						
0.0	3.7500	0.0000	2.7273	0.0000	3.7500	0.0000
2.5	3.7500	0.0000	2.7273	0.0000	9.7730	0.0000
5.0	3.7500	0.0000	2.7273	0.0000	9.8948	0.0000
7.5	3.7500	0.0000	2.7273	0.0000	9.9294	0.0000
10.0	3.7500	0.0000	2.7273	0.0000	9.9469	0.0000
12.5	3.7500	0.0000	2.7273	0.0000	9.9575	0.0000
15.0	3.7500	0.0000	2.7273	0.0000	9.9645	0.0000

Table 2.1 are calculated using the approximate equations developed in this chapter, and compared to those obtained by an exact Markov approach. The percentage error shown in Table 2.1 clearly indicates that the use of approximate formulas introduces an error of only a fraction of a percent. The models developed in this chapter are also compared for the data provided by Ontario Hydro. These results are shown in Table 2.2.

Figure 2.6 is a graphical presentation of the effect of common cause failures on the probability of the system being down for the models of Figures 2.3 and 2.4. The probabilities and average outage durations obtained by Models (a) and (b) of Figure 2.3 are relatively close to each other. Figure 2.7 also shows that the probability of failure for Model 2 is higher than that of Model 1. Figure 2.9 shows that the average outage duration for Model 2 is very close to the time required to repair a common mode failure.

2.5 Higher Order Common Cause Outage Models

The use of more than two transmission circuits on the same transmission tower or on the same right of way is not very common but a possible arrangement of three transmission circuits on the same transmission tower is illustrated in

Table 2.2

Variation in System Reliability Indices With Common Mode
Outage Rate Using Ontario Hydro Data

Independent outage rate of each line = 0.0341 f/yr/mile
 Average repair time for each line = 7.5 hours
 (assumed)
 Average common cause repair time = 10.0 hours
 (assumed)

λ/λ	Model Of Figure 2.3(a)	Model Of Figure 2.3(b)	Model of Figure 2.4	Model Of Figure 2.5
Pr(Both Lines Down)X10000				
0.0	0.0000	0.0000	0.0098	0.0000
2.5	0.0037	0.0027	0.0196	0.0067
5.0	0.0074	0.0053	0.0294	0.0135
7.5	0.0110	0.0080	0.0392	0.0202
10.0	0.0147	0.0107	0.0489	0.0269
12.5	0.0184	0.0134	0.0587	0.0337
15.0	0.0220	0.0160	0.0685	0.0404
(System Failure rate)x100 (f/yr)				
0.0	0.0002	0.0002	0.0002	0.0002
2.5	0.0859	0.0859	0.0859	0.0859
5.0	0.1717	0.1717	0.1717	0.1717
7.5	0.2574	0.2574	0.2574	0.2574
10.0	0.3432	0.3432	0.3432	0.3432
12.5	0.4289	0.4289	0.4289	0.4289
15.0	0.5147	0.5147	0.5147	0.5147
System Average Outage Duration (hours)				
0.0	3.7500	2.7273	3.7500	3.7500
2.5	3.7500	2.7273	9.9854	6.8676
5.0	3.7500	2.7273	9.9927	6.8713
7.5	3.7500	2.7273	9.9951	6.8725
10.0	3.7500	2.7273	9.9963	6.8731
12.5	3.7500	2.7273	9.9971	6.8735
15.0	3.7500	2.7273	9.9976	6.8737

* Line length = 1 mile

Figure 2.6 Probability of System Failure As A Function of λ_c For the Models 1 and 2.

Figure 2.7 Variation in the Probability of System Failure With Common Mode Outage Rate

Figure 2.8 Variation in the System Outage Time With Common Mode Outage Rate.

Figure 2.9. A common cause outage model for three lines is shown in Figure 2.10 containing a total of eight states. In this diagram, only the common cause outage of all three lines is included though other lower order common cause events could be added if justified by the available data. A state space model is constructed including lower order common cause outages and the results were compared to those obtained by the model of Figure 2.10 for state probabilities. Table 2.3 clearly indicates that the lower order common cause outages do not dominate the system reliability indices. Therefore lower order common cause outages can in certain cases be neglected to make the state space model and analysis simpler.

Table 2.3

Effect of Lower Order Common Mode Outage Considerations
In a Three Line Redundant System

$$\lambda = 0.5 \text{ f/yr}$$

$$r = 2.5 \text{ hours}$$

λ_c/λ	Neglecting Lower Order Common Cause outages		Including Lower Order Common Cause Outages	
	Pr(All Lines in Up State)	Pr(All Lines in Down State)	Pr(All Lines in Up State)	Pr(All Lines in Down State)
0.0	0.998717	0.000000	0.998717	0.000000
2.5	0.998697	0.000004	0.998697	0.000004
5.0	0.998678	0.000007	0.998678	0.000007
7.5	0.998658	0.000011	0.998658	0.000011
10.0	0.998639	0.000014	0.998638	0.000014
12.5	0.998619	0.000018	0.998618	0.000018
15.0	0.998599	0.000021	0.998599	0.000021

The closed form expressions are developed in Reference 4

Figure 2.9 Three Transmission Lines on the Same Transmission Tower

Figure 2.10 A State Space Diagram Including Common Mode And Independent Outages For A Three Line Redundant System.

to determine the state probabilities for the model of Figure 2.10 assuming that the system contains three identical lines. A similar approach can be used to create and solve other higher order models depending upon the data availability. The state probabilities for the model of Figure 2.10 are as follows.

$$\begin{aligned}
 P_1 &= 6\mu^3/D \\
 P_2 = P_3 = P_4 &= 2\mu^2(3\lambda + \lambda_c)/D \\
 P_5 = P_6 = P_7 &= \mu(6\lambda^2 + \lambda_c(2\lambda + \mu))/D \\
 P_8 &= 6\lambda^3 + \lambda_c(2\lambda^2 + \mu\lambda + 2\mu^2)/D
 \end{aligned} \tag{2.12}$$

Where:

$$D = 6(\lambda + \mu)^3 + \lambda_c(2\lambda^2 + 7\lambda\mu + 11\mu^2)$$

Approximate formulas can also be developed to find the system failure rate and system average outage duration using the same approach as is used in the case of the two component system. In order to examine the effect of the common cause failure rate on the system failure rate in a three line case, the system failure rate is shown in Table 2.4. The Table 2.4 also indicates that neglecting the lower order common cause outages does not significantly effect the system failure rate. Figure 2.11 shows that the probability of finding all the three lines down increases sharply with a 0 to 5% variation in the common cause failure rate and to lesser degree from 5 to 15%. This observation is similar to

Three Lines In Parallel

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Figure 2.11 Variation in The Probability of the Three Lines Down State With Common Mode Outage Rate.

that for the two line case.

Table 2.4

Effect of Lower Order Common Mode Outage Considerations on
System Failure Rate for a Three Line Redundant System

$$\lambda = 0.5 \text{ f/yr}$$

$$r = 7.5 \text{ hours}$$

% System Failure Rate (f/yr)		

λ/λ	Neglecting lower order Common cause outages	Including lower order Common cause outages

0.0	0.00000027	0.00000027
2.5	0.012484	0.012500
5.0	0.024968	0.025000
7.5	0.037452	0.037500
10.0	0.049936	0.050000
12.5	0.062420	0.062500
15.0	0.074904	0.075000

2.6 Summary

A few selected common cause outage models for two and three lines on the same transmission tower or on the same right of way are discussed in this chapter. Closed form expressions for calculating steady state probabilities, system failure rate and system average outage duration of these models are also developed. Such expressions are useful for evaluating transmission line reliability indices.

The models examined in this chapter were selected from References [2,4,7]. These models must be further modified because of the existence of a changing weather environment.

Several common cause outages occur in adverse weather. The concept of adverse weather failures is introduced and discussed in detail in the next chapter of this thesis. The results shown in this chapter illustrates the importance of recognizing the effect of common cause failures, and suggest a great need to include common cause outages in the reliability evaluation of composite generation and transmission systems.

3. EFFECT OF ADVERSE WEATHER ON SYSTEM RELIABILITY INDICES

3.1 Introduction

A transmission and distribution system is composed of either overhead or underground facilities or both. Overhead schemes have to operate in a highly variable weather environment. Reliability predictions without weather considerations can be quite optimistic and the results can be considerably in error. This has been illustrated in Reference [5]. Under adverse weather conditions, the components fail quite frequently while under normal weather conditions the failures occur less frequently. Distribution schemes, being concentrated in small areas, are greatly effected by adverse weather conditions. As the severity of the adverse weather varies, the failure rate of the components under these conditions will also vary. It is, however, very difficult to incorporate all possible kinds of adverse weather conditions in reliability calculations. This chapter examines the concepts associated with adverse weather modelling and its effects on system reliability indices. This phenomenon is examined further in Chapter 4 in conjunction with common mode outage considerations.

3.2 Weather Modelling

Weather conditions can be divided into the following three categories [23]

- (1) Normal Weather
- (2) Adverse Weather
- (3) Major Storm Disaster.

A simple two state fluctuating environment covering normal and adverse weather has been proposed in which weather duration distributions are assumed to be exponential[3]. This model is shown in Figure 3.1. The random occurrence pattern of adverse weather periods and their associated failure rates is averaged and represented by a regular pattern as shown in Figure 3.2.

Figure 3.1 Random History - Two State Environment.

Figure 3.2 Average History - Two State Environment.

The definition of adverse weather must be well defined as it must include all the adverse weather which cause an increase in the failure rate of a component compared to its normal weather failure rate. It should be noted that disaster adverse weather conditions as major hurricanes and tornadoes cannot be lumped with other less violent periods. A three state weather model was presented in reference [17] to recognize the effect of such periods on system reliability and it was concluded that the system failure rate and system average outage duration increases with an increase in the average duration of disaster adverse weather periods. The effect of disaster adverse weather on the system reliability indices is negligible if the duration of disaster adverse weather is very small compared to adverse weather.

3.3 Adverse Weather Considerations

The basic approach to adverse weather modelling assumes that the fluctuating environment associated with an outdoor component such as a transmission line can be reduced to a two or three state model. The most common representation is the two state model described by the normal and adverse weather. The overall average failure rate λ_{av} for a component is divided into two components λ and λ' where

λ = Normal weather failure rate.

λ' = Adverse weather failure rate.

S = Average duration of an adverse weather period. (hrs.)

N = Average duration of normal weather period. (hrs.)

and

$$\lambda_{av} = \lambda \left(\frac{N}{N+S} \right) + \lambda' \left(\frac{S}{N+S} \right) \quad (3.1)$$

In the case of a simple radial or series system, the load point failure rate is λ_{av} and recognition of λ and λ' as separate entities is not required in the reliability model. In the case of a two component redundant supply, the conventional formula for the load point failure rate λ_L is:

$$\lambda_L = \lambda_{av_1} \lambda_{av_2} (r_1 + r_2) \quad (3.2)$$

This predicted value can prove to be extremely optimistic if the two component configuration can exist in a joint adverse weather environment. This effect is shown in

Table 3.1 for a two and three component redundant configuration. The effect of varying the actual percentage of component failures which occur during adverse weather is also illustrated. The Error Factor shown in Table 3.1 is obtained by expressing the calculated load point failure rate including adverse weather effects in per unit of the predicted value obtained using Equation 3.2.

Table 3.1

Effect of Adverse Weather On System Failure Rate

Independent outage rate of each component = 0.5 f/yr.
Average repair time of each component = 7.5 hours

$M = 200.0$ hours , $S = 1.5$ hours

Two line redundant system failure rate without considering adverse weather = 0.0004281 f/yr.

Three line redundant system failure rate without considering adverse weather = 0.000000274 f/yr.

Percentage of Independent Outages occur in Adverse W.	Two Lines in Parallel Failure/y.	Error Factor	Three Lines in Parallel Failure/y.	Error Factor
0.0	0.000431	1.0068	0.000000274	1.0000
10.0	0.000542	1.2660	0.000000871	3.1788
20.0	0.000874	2.0416	0.000004148	15.1386
30.0	0.001427	3.3333	0.000012357	45.0985
40.0	0.002202	5.1436	0.000027747	101.2664
50.0	0.00320	7.4749	0.000052568	191.8540
60.0	0.004416	10.3153	0.000089068	325.0656
70.0	0.005855	13.6767	0.000139495	509.1058
80.0	0.007516	17.5366	0.000206098	752.1825
90.0	0.009397	21.9505	0.000291130	1062.5182
100.0	0.011500	26.8629	0.00039682	1448.2482

The system failure rates shown in Table 3.1 were obtained using the equations given in Reference [5] and for the case in which no repair is performed during the adverse weather period. These equations are again listed in Appendix 3 for both the situations in which repair is performed and the

case in which repair is not performed during the adverse weather. The Error Factor for two, three and four element fully redundant configurations are shown in Figure 3.3. Similar curves are shown in Reference [5]. Figure 3.4 and 3.5 shows state space models of a two component system with and without adverse weather considerations. Adverse weather effects have also been examined using Markov models[5]. The model of Figure 3.5 recognizes repair during adverse weather by the adverse weather repair rate μ' . The normal and adverse weather transition rates are represented by n and m respectively where $n=1/N$ and $m=1/S$. Markov models were used to check the results obtained by approximate formulae in References [2] and [5] and are used later in the Chapter 4 for the same purpose. The Markov approach is discussed in Appendix 1. Figure 3.3 clearly shows the effect of adverse weather considerations on the predicted failure rate of fully redundant configurations.

3.4 Effect of Adverse Weather on System Failure Rate

Using the approximate equations (Appendix 3), the effect of variation in component parameters and weather parameters are shown in Figure 3.6 for a simple parallel redundant system of two and three components for various percentage of component failures during adverse weather periods. Figure 3.6 clearly shows the effect of varying the

Figure 3.3 Error Factor For Parallel Redundant Configurations.

Figure 3.4 State Space Model For a Two Component Redundant System.

Figure 3.5 State Space Model For a Two Component Redundant System Including Adverse Weather Effects.

Figure 3.6 Variation in System Failure Rate With Changing Environment Conditions.

adverse weather duration as well as the effect of occurrence or nonoccurrence of repair during adverse weather on the system outage rate. It can also be seen that occurrence and nonoccurrence of repair during adverse weather has no significant effect on the system outage rate but it can be easily seen that the adverse weather duration has a significant effect on the system failure rate. It is obvious that longer adverse weather periods are liable to cause more damage to the system from a reliability viewpoint than that caused by short ones.

3.5 Effect of Adverse Weather on System Outage Time

Figure 3.7 shows the effect of adverse weather duration on system outage time with the percentage change of component failures during adverse weather periods. The system average outage times were obtained for various situations using the equations given in Reference [5]. These equations are listed in Appendix 3 and the assumptions involved in these equations are given in Appendix 2. The duration of an adverse weather period does not have any effect on the system average outage time where repair is performed during adverse weather. Nonoccurrence of component repair during adverse weather introduces significant variation in system average outage time. The reason for the change in system average outage time under this condition is that the

every failure which occurs in adverse weather has to wait for repair for a period equal to the average duration of the adverse weather period.

Figure 3.7 Variation in the System Outage Time With Changing Environment Conditions.

3.6 Conclusion

The effect of adverse weather considerations on the predicted system reliability indices of a fully redundant configuration is illustrated in this chapter. The system weather considerations can have undesirable effect on predicted reliability indices in redundant configurations.

4. COMMON MODE AND ADVERSE WEATHER CONSIDERATIONS.

4.1 Introduction

This chapter illustrates the development and analysis of a group of transmission system models which include both common mode failures and adverse weather considerations. The effect of common mode failures and adverse weather in parallel redundant facilities has already been discussed individually in Chapters 2 and 3 respectively. The concept of transmission system reliability modelling including both these factors is presented in this chapter and has been published in Reference [25]. (At the present time, data collection in transmission and distribution systems is receiving considerable attention. It therefore becomes very important to determine the relative contribution to predicted reliability indices of the many factors that are present in the analysis. (These types of studies may also be helpful in data collection as well as in valid reductions in modelling complexity which retain the important and dominant aspects and also keep the problem to manageable size).

(Several possible common cause outage models including adverse weather conditions are described and analyzed in the following sections to examine the sensitivity of the

calculated reliability indices to individual parameter variation in the composite model. Approximate equations are developed to permit easy utilization of reliability models in transmission and distribution system studies.

4.2 Models Including Common Mode And Adverse Weather Conditions.

Several possible models including common mode outages were described and analyzed in Chapter 2. These models are extended in this section by including adverse weather effects. System studies are presented to show the influence of common cause outages and adverse weather on system reliability indices. Common cause outage models are described in this section for two lines on the same tower or on the same right of way.

The approximate and Markov approaches for the analysis of system models are basically those described in Reference [5] with the same basic assumptions. The probability of residing in the failed state is an important parameter in certain overall system studies. It is perhaps less important and secondary to other factors such as the configuration failure rate or failure frequency in other studies. The procedure used in Reference [5] has been followed in this section. Numerical results for the system

state probabilities, system failure rate and system average outage duration in a two line and three line redundant configurations are shown to compare the different kind of proposed models. Approximate equations for the system failure rate are developed which include both normal and adverse weather considerations and common cause outages.

4.2.1 Model A

The model of Figure 2.3 is extended in this section to include adverse weather effects. The model is shown in Figure 4.1 and illustrates a set of possible transitions from each state considering the weather fluctuations and independent and common cause failure rates. The analysis for the model of Figure 4.1 is shown in the following sections.

4.2.1(a) System State Space Probabilities

The frequency balancing approach is used to obtain the set of equations designated as 4.1 for determining the system steady state probabilities. These equations are written assuming the state residence time are exponentially distributed.

$$\begin{aligned}
 (\lambda_1 + \lambda_2 + \lambda_C + n)P_1 - \mu_1 P_2 - \mu_2 P_3 - \mu_C P_4 - P_5^m &= 0 \\
 -\lambda_1 P_1 + (\mu_1 + \lambda_2 + n)P_2 - \mu_2 P_4 - P_6^m &= 0
 \end{aligned}$$

$$\begin{aligned}
\lambda_2 P_1 - \mu_1 P_4 + P_3(\lambda_1 + \mu_2 + n) - m P_7 &= 0 \\
-\lambda_C P_1 - \lambda_2 P_2 - \lambda_3 P_3 + P_4(\mu_1 + \mu_2 + \mu_C + n) - m P_8 &= 0 \\
(\lambda_1' + \lambda_2' + \lambda_C' + m) P_5 - \mu_1' P_6 - \mu_2' P_7 - \mu_C' P_8 - P_1 n &= 0 \\
-\lambda_1' P_5 + (\mu_1' + \lambda_2' + m) P_6 - \mu_2' P_8 - P_2 n &= 0 \\
-\lambda_2' P_5 - P_8 \mu_1' + P_3(\lambda_1' + \mu_2' + m) - n P_3 &= 0 \\
-\lambda_C' P_5 - \lambda_2' P_6 - \lambda_1' P_7 + P_8(\mu_2' + \mu_1' + \mu_C' + m) - n P_4 &= 0
\end{aligned} \tag{4.1}$$

To determine the steady state probabilities, the solution of the set of equations given in 4.1 has to satisfy the following condition.

$$\sum P_i = 1.0$$

i.e. the availabilities of all the states must sum to one.

The set of simultaneous linear equations can be presented in the following matrix form

$$\begin{bmatrix} 0 \end{bmatrix} = \begin{bmatrix} DN & mI \\ nI & DS \end{bmatrix} \begin{bmatrix} P \end{bmatrix} \tag{4.2}$$

The matrices DN and DS correspond to transitions out of normal and adverse weather states respectively.

[0] is a null vector.

[I] is an identity matrix.

[P] is a column vector of steady state probabilities.

The matrices DN and DS for the model of Figure 4.1

are shown in Table 4.1.

The stochastic transitional probability matrix designated 4.2 can be solved for the system state probabilities using any matrix inversion approach for the given values of λ 's, μ 's, m and n . The analytical expressions can also be developed to determine the steady state probabilities using the mathematical approach given in Appendix 1.

4.2.1(b) System Average Outage Rate

The development of system average outage rate formula for the model of Figure 4.1 is done in six distinct steps. The assumptions associated with the development of the approximate formula are given in Appendix 2. The following shows the development of the approximate technique for the two different situations of repair i.e.

- (i) When repair does not occur during adverse weather.
- (ii) When repair occurs during adverse weather.

(i) No Repair During Adverse Weather

The equations for the situation in which repair is not performed during adverse weather are given as follows:

Table 4.1 Matrices DN And DS For Model A

$$[DN] = \begin{bmatrix} \lambda_1 + \lambda_2 + \lambda_C + \mu_1 & -\mu_1 & -\mu_2 & -\mu_C \\ -\lambda_1 & \lambda_2 + \mu_1 + \mu_2 & 0 & -\mu_2 \\ -\lambda_2 & 0 & \lambda_1 + \mu_2 + \mu_1 & -\mu_1 \\ -\lambda_C & -\lambda_2 & -\lambda_1 & \mu_1 + \mu_2 + \mu_C \end{bmatrix}$$

$$[DS] = \begin{bmatrix} \lambda_1 + \lambda_2 + \lambda_C + \mu_1 & -\mu_1 & -\mu_2 & -\mu_C \\ -\lambda_1 & \lambda_2 + \mu_1 + \mu_2 & 0 & -\mu_2 \\ -\lambda_2 & 0 & \lambda_1 + \mu_2 + \mu_1 & -\mu_1 \\ -\lambda_C & -\lambda_2 & -\lambda_1 & \mu_1 + \mu_2 + \mu_C \end{bmatrix}$$

Figure 4.1 Model A For Two Components Including Common Mode And Adverse Weather Effects.

1. The common mode outage occurs in adverse weather. The contribution to the system failure rate due to this mode of system failure is given by:

$$(\text{Failure rate})_1 = \frac{N}{N+S} \{\lambda_c\}$$

Where:

$N/(N+S)$ = Long term fraction of time that the weather is normal.

λ_c = Normal weather common mode outage rate.

2. The initial failure occurs during normal weather and the second failure also occurs during normal weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(\text{Failure rate})_2 = \frac{N}{N+S} \left\{ \lambda_1 \left(1 - \frac{r_1}{N}\right) (\lambda_2 r_1) + \lambda_2 \left(1 - \frac{r_2}{N}\right) \lambda_1 r_2 \right\}$$

Where

$1 - \frac{r_1}{N} = e^{-r_1/N}$ i.e. The probability that adverse weather does not occur while component 1 is being repaired.

$\lambda_2 r_1 = 1 - e^{-\lambda_2 r_1}$ i.e. The probability that component 2 fails during the repair period of component 1.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged. The expression can be simplified to the following form:

$$(\text{Failure rate})_2 = \frac{N}{N+S} \{\lambda_1 \lambda_2 (r_1 + r_2)\}$$

3. The initial failure occurs during normal weather and the second failure occurs during adverse weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(\text{Failure rate})_3 = \frac{N}{N+S} \{ \lambda_{1N}^{r_1} \lambda_2^1 S + \lambda_{2N}^{r_2} \lambda_1^1 S \}$$

where:

$\frac{r_1}{N} = 1 - e^{-r_1/N}$ = The probability that repair of component 1 is not completed within the normal weather.

$\lambda_2^1 S = 1 - e^{-\lambda_2^1 S}$ = The probability that component 2 fails during the adverse weather. The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

4. The common mode outage occurs in adverse weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(\text{Failure rate})_4 = \frac{S}{N+S} \{ \lambda_C^1 \}$$

Where:

$S/(N+S)$ = The long term fraction of time that the weather is adverse.

λ_C^1 = Adverse weather common mode outage rate.

5. The initial failure occurs during adverse weather and the second failure also occurs during adverse

weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(\text{Failure rate})_5 = \frac{S}{N+S} \{ \lambda_1' (\lambda_2' S) + \lambda_2' (S \lambda_1') \}$$

Where:

$S \lambda_2' = 1 - e^{-S \lambda_2'}$ = The probability that component 2 fails during the adverse weather period. The remaining terms in the above expression follow the same argument but with components 1 and 2 interchanged.

6. The initial failure occurs during adverse weather and the second failure occurs in normal weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(\text{Failure rate})_6 = \frac{N}{N+S} \{ \lambda_1' (1 - S \lambda_2') (\lambda_2 r_1) + \lambda_2 (1 - S \lambda_1') (\lambda_1 r_2) \}$$

Where:

$1 - S \lambda_2' = e^{-S \lambda_2'}$ = The probability that component 2 does not fail during adverse weather.

$\lambda_2 r_1 = 1 - e^{-\lambda_2 r_1}$ = The probability that component 2 fails during the repair of component 1. The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

Thus the overall outage rate for a system of two lines

is given as follows:

$$\lambda_{SL} = \frac{N}{N+S} \{ \lambda_C + \lambda_1 \lambda_2 (r_1 + r_2) + \lambda_1 \frac{r_1}{N} \lambda_2 S + \lambda_2 \frac{r_2}{N} \lambda_1 S \} + \frac{S}{N+S} \{ \lambda'_C + \lambda_1 (\lambda_2 S) + \lambda_2 (S \lambda_1) + \lambda_1 (1 - S \lambda_2) (\lambda_2 r_1) + \lambda_2 (1 - S \lambda_1) (\lambda_1 r_2) \} \quad (4.3)$$

if the parallel system operates in parallel or in series with other components, the normal and adverse weather failure rates are required to form the equivalent component representing the parallel system in further contributions. These rates are as follows:

$$\begin{aligned} \lambda_e &= \{ \lambda_1 \lambda_2 (r_1 + r_2) + \frac{S}{N} (\lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 r_2) + \lambda_C \} \\ \lambda'_e &= \{ 2S \lambda_1 \lambda_2 + \lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 r_2 + \lambda'_C \} \quad (4.4) \end{aligned}$$

(ii) Repair During Adverse Weather

The formation of approximate equations in this case for two overlapping outages including common cause in a parallel redundant system is basically the same as that presented in previous case (No repair during adverse weather) except of mode 3 and mode 5.

Mode 3: The contribution to the system failure rate due to the third mode of failure is given by:

$$(\text{Failure rate})_3 = \frac{N}{N+S} \left\{ \lambda_1 \left(\frac{r_1}{N} \right) \lambda'_2 \left(\frac{Sr_1}{S+r_1} \right) + \lambda_2 \left(\frac{r_2}{N} \right) \lambda'_1 \left(\frac{Sr_2}{S+r_2} \right) \right\}$$

Where:

$\frac{r_1}{N} = 1 - e^{-r_1/N}$ = The probability that weather becomes adverse during the repair of component 1.

$\lambda'_2 \frac{Sr_1}{S+r_1}$ = The probability that component 2 fails

during the overlapping period of adverse weather and the repair of component 1. The similar approach is used to obtain the second term of the above expression.

Mode 5: The contribution to the system failure rate due to the fifth mode of system failure is given by:

$$(\text{Failure rate})_5 = \frac{S}{N+S} \left\{ \lambda_1 \left(\lambda_2 \frac{S r_1}{S+r_1} \right) + \lambda_2 \left(\lambda_1 \frac{S r_2}{S+r_2} \right) \right\}$$

Where:

$\lambda_2 \frac{S r_1}{S+r_1}$ The probability that the component 2 fails during the overlapping period of adverse weather and the repair of component 1. Similar reasoning is applicable to the second term of the above expression.

Thus the overall system failure rate for the condition in which repair is available during adverse weather, is given by the following equation.

$$\lambda_{sl} = \frac{N}{N+S} \left\{ \lambda_c + \lambda_1 \lambda_2 (r_1 + r_2) + \frac{S}{N} \left(\lambda_1 \lambda_2 \frac{r_1^2}{S+r_1} + \lambda_2 \lambda_1 \frac{r_2^2}{S+r_2} \right) \right\} +$$

$$\frac{N}{N+S} \left\{ \lambda_c + \lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 r_2 + \lambda_1 \lambda_2 S \left(\frac{r_1}{S+r_1} + \frac{r_2}{S+r_2} \right) \right\} \quad (4.5)$$

The normal and adverse weather failure rates for the equivalent component representing the parallel system are as follows:

$$\lambda_e = \frac{N}{N+S} \left\{ \lambda_c + \lambda_1 \lambda_2 (r_1 + r_2) + \frac{S}{N} \left(\lambda_1 \lambda_2 \frac{r_1^2}{S+r_1} + \lambda_2 \lambda_1 \frac{r_2^2}{S+r_2} \right) \right\}$$

$$\lambda'_e = \{\lambda'_c + \lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 r_2 + \lambda_1 \lambda_2 S (\frac{r_1}{S+r_1} + \frac{r_2}{S+r_2})\} \quad (4.6)$$

Results using these equations are shown in Tables 4.2 and 4.3 together with theoretically "Exact" results obtained using the Markov approach with $\lambda_{av} = 0.5$, $\lambda_{cav} = 0.05$, $N=200$, $S=1.5$, and 50% of the common mode outages occur during adverse weather. The procedure in this case is illustrated in Reference [5] and in Appendix 1. The stochastic transitional probability matrix P from the state space model of Figure 4.1 is shown below.

[P] =

$$\begin{bmatrix} 1-(\lambda_1+\lambda_2+\lambda'_c+n) & \lambda_1 & \lambda_2 & \lambda'_c & n & 0 & 0 & 0 \\ \mu_1 & 1-(\mu_1+\lambda_2+n) & 0 & \lambda_2 & 0 & n & 0 & 0 \\ \mu_2 & 0 & 1-(\lambda_1+\mu_2+n) & \lambda_1 & 0 & 0 & n & 0 \\ 0 & \mu_2 & \mu_1 & 1-(\mu_1+\mu_2+n) & 0 & 0 & 0 & n \\ m & 0 & 0 & 0 & 1-(\lambda_1+\lambda_2+\lambda'_c+m) & \lambda'_1 & \lambda'_2 & \lambda'_c \\ 0 & m & 0 & 0 & \mu_1 & 1-(\mu_1+\lambda'_2+m) & 0 & \lambda'_2 \\ 0 & 0 & m & 0 & \mu'_2 & 0 & 1-(\lambda_1+\mu'_2+m) & \lambda_1 \\ 0 & 0 & 0 & m & 0 & \mu'_2 & \mu_1 & 1-(\mu_1+\mu'_2+m) \end{bmatrix}$$

In obtaining the system mean time to failure, both states 4 and 8 can be considered as absorbing states. It is not important in which state the system failed in, only that it did fail. The truncated Q matrix is formed by deleting states 4 and 8.

TABLE 4.2
Outage Rate For a Two Component Redundant System
No Repair During Adverse Weather

Percentage Of Components Failure During Adverse Weather	Repair Time (hours)	Failure Rate λ_{SL} f/yr.		Percentage Error
		Approx.	Exact	
0.0	2.5	0.05014	0.05011	0.057
	7.5	0.05043	0.05040	0.115
	12.5	0.05072	0.05063	0.172
10.0	2.5	0.05026	0.05017	0.169
	7.5	0.05054	0.05043	0.221
	12.5	0.05083	0.05070	0.273
20.0	2.5	0.05059	0.05046	0.279
	7.5	0.05087	0.05072	0.314
	12.5	0.05115	0.05097	0.349
30.0	2.5	0.05117	0.05097	0.390
	7.5	0.05142	0.05122	0.398
	12.5	0.05169	0.05148	0.406
40.0	2.5	0.05196	0.05170	0.505
	7.5	0.05220	0.05196	0.475
	12.5	0.05244	0.05221	0.447
50.0	2.5	0.05298	0.05265	0.625
	7.5	0.05319	0.05291	0.550
	12.5	0.05341	0.05316	0.477
60.0	2.5	0.05423	0.05383	0.755
	7.5	0.05441	0.05408	0.626
	12.5	0.05460	0.05433	0.499
70.0	2.5	0.05570	0.05521	0.896
	7.5	0.05586	0.05546	0.706
	12.5	0.05600	0.05571	0.519
80.0	2.5	0.05741	0.05682	1.049
	7.5	0.05752	0.05706	0.794
	12.5	0.05762	0.05731	0.541
90.0	2.5	0.05934	0.05863	1.216
	7.5	0.05939	0.05887	0.890
	12.5	0.05945	0.05912	0.507
100.0	2.5	0.06150	0.06065	1.398
	7.5	0.06150	0.06089	0.997
	12.5	0.06150	0.06113	0.600

Table 4.3
Outage Rate For a Two Component Parallel System
Repair During Adverse Weather

Percentage Of Components Fail During Adverse Weather	Repair Time (hours)	Failure Rate λ_{SL} f/yr		Percentage Error
		Approx.	Exact	
0.0	2.5	0.05014	0.05012	0.053
	7.5	0.05043	0.05037	0.112
	12.5	0.05072	0.05063	0.173
10.0	2.5	0.05021	0.05015	0.132
	7.5	0.05052	0.05041	0.213
	12.5	0.05081	0.05067	0.276
20.0	2.5	0.05042	0.05032	0.209
	7.5	0.05078	0.05063	0.302
	12.5	0.05109	0.05091	0.357
30.0	2.5	0.05076	0.05062	0.283
	7.5	0.05124	0.05105	0.381
	12.5	0.05156	0.05135	0.418
40.0	2.5	0.05126	0.05106	0.357
	7.5	0.05188	0.05165	0.453
	12.5	0.05223	0.05199	0.464
50.0	2.5	0.05189	0.05167	0.432
	7.5	0.05270	0.05243	0.521
	12.5	0.05309	0.05283	0.497
60.0	2.5	0.05267	0.05240	0.508
	7.5	0.05371	0.05340	0.585
	12.5	0.05414	0.05386	0.521
70.0	2.5	0.05358	0.05327	0.586
	7.5	0.05490	0.05455	0.649
	12.5	0.05538	0.05508	0.539
80.0	2.5	0.05464	0.05428	0.668
	7.5	0.05628	0.05588	0.716
	12.5	0.05682	0.05650	0.555
90.0	2.5	0.05584	0.05543	0.755
	7.5	0.05784	0.05739	0.788
	12.5	0.05845	0.05812	0.573
100.0	2.5	0.05719	0.05671	0.846
	7.5	0.05958	0.05907	0.864
	12.5	0.06027	0.05991	0.593

[Q] =

$$\begin{bmatrix}
 1-(\lambda_1+\lambda_2+\lambda_c+n) & \lambda_1 & \lambda_2 & n & 0 & 0 \\
 \mu_1 & 1-(\mu_1+\lambda_2+n) & 0 & 0 & n & 0 \\
 \mu_2 & 0 & 1-(\lambda_1+\mu_2+n) & 0 & 0 & n \\
 m & 0 & 0 & 1-(\lambda_1+\lambda_2+\lambda_c+m) & \lambda_1' & \lambda_2' \\
 0 & m & 0 & \mu_1' & 1-(\mu_1+\lambda_2+m) & 0 \\
 0 & 0 & m & \mu_2' & 0 & 1-(\lambda_1+\mu_2+m)
 \end{bmatrix}$$

The fundamental matrix N is

$$[N] = [I - Q]^{-1}$$

$$\begin{bmatrix}
 \lambda_1+\lambda_2+\lambda_c+n & -\lambda_1 & -\lambda_2 & -n & 0 & 0 \\
 -\mu_1 & \mu_1+\lambda_2+n & 0 & 0 & -n & 0 \\
 -\mu_2 & 0 & \lambda_1+\mu_2+n & 0 & 0 & -n \\
 -m & 0 & 0 & \lambda_1+\lambda_2+\lambda_c+m & -\lambda_1' & -\lambda_2' \\
 0 & -m & 0 & -\mu_1' & \mu_1+\lambda_2+m & 0 \\
 0 & 0 & -m & -\mu_2' & 0 & \lambda_1+\mu_2+m
 \end{bmatrix}^{-1}$$

Starting in State 1 (i.e the system is operating in normal weather), the M.T.T.F is

$$M_{1,4\&8} = \sum_{i=1}^6 N(1,i) \quad (4.8)$$

$$\lambda_{SL} = 1/M_{1,4\&8}$$

If there is no repair during adverse weather μ_1 and μ_2 are zero. The state space diagram for a larger number of components can be drawn in a similar manner as shown in Figure 4.2 and the exact system failure rate associated with

Figure 4.2 A State Space Model For Three Components Similar to Model A.

that system can be obtained by using the above approach.

The overall approximate average outage rate resulting from three overlapping outages is given by the summation of appropriate terms of Table 4.4 i.e

$$\lambda_{SL} = \sum_{i=1}^{10} (\text{Failure rate})_i \quad (4.9)$$

The first five modes of failures shown in Table 4.4 involve the system failure in normal weather and the last five modes involve the system failure in adverse weather.

The approximate equations developed in this section for a two and three component redundant system, are quite adequate for most applications. The percentage error in the approximate system outage rates as compared to the results obtained by the Markov approach are shown in Figures 4.3(a) and 4.3(b) where $\lambda_{av} = 0.5$ f/yr, $\lambda_{cav} = 0.05$ f/yr, $N=200$, $S=1.5$ hr. and 50% of the common mode outages occur during adverse weather.

4.2.1(c) System Average Outage Duration

The development of average outage duration formula for the model of Figure 4.1 is shown below for two different situations.

- (1) When repair does not occur during adverse weather.

Outage Rate Associated With Different Modes of Failures Of A Three Component Parallel System

Failure Modes			Contribution to the System Failure Rate	
No.	1st	2nd	3rd	No. of Repair During Adverse Weather
	Common Mode			Repair During Adverse Weather
1	Normal Weather			$\frac{N}{N+S} [\lambda_c]$
2	N	N	N	$\frac{N}{N+S} [\lambda_1 (\lambda_2 r_3) (\lambda_3 R_1) + \lambda_1 (\lambda_3 r_1) (\lambda_2 R_2)$ + Similar terms for component 2 & 3]
3	N	A	N	$\frac{N}{N+S} [\lambda_1 (\frac{r_1}{N}) (\lambda_2' S) (\lambda_3 R_1) + \lambda_1 (\frac{r_1}{N}) (\lambda_2' S) (\lambda_2 R_2)$ + Similar terms for component 2 & 3]
4	A	N	N	$\frac{S}{N+S} [\lambda_1' (\lambda_2 r_1) (\lambda_3 R_1) + \lambda_1' (\lambda_3 r_1) (\lambda_2 R_2)$ + Similar terms for component 2 & 3]
5	A	A	N	$\frac{S}{N+S} [\lambda_1' (\lambda_2' S) (\lambda_3 R_1) + \lambda_1' (\lambda_2' S) (\lambda_2 R_2)$ + Similar terms for component 2 & 3]
6	Common Mode			$\frac{S}{N+S} [\lambda_c']$
7	A	A	A	$\frac{S}{N+S} [\lambda_1' (\lambda_2' S) (\lambda_3' S) + \lambda_1' (\lambda_3' S) (\lambda_2' S)$ + Similar terms for component 2 & 3]
8	A	N	A	$\frac{S}{N+S} [\lambda_1' (\lambda_2 r_1) (\frac{R_1}{N}) (\lambda_3' S) + \lambda_1' (\lambda_3 r_1) (\frac{R_2}{N}) (\lambda_3' S)$ + Similar terms for component 2 & 3]
9	N	A	A	$\frac{N}{N+S} [\lambda_1 (\frac{r_1}{N}) (\lambda_2' S) (\lambda_3' S) + \lambda_1 (\frac{r_1}{N}) (\lambda_3' S) (\lambda_2' S)$ + Similar terms for component 2 & 3]
10	N	N	A	$\frac{N}{N+S} [\lambda_1 (\lambda_2 r_1) (\frac{R_1}{N}) (\lambda_3' S) + \lambda_1 (\lambda_3 r_1) (\frac{R_2}{N}) (\lambda_2' S)$ + Similar terms for component 2 & 3]
$R_1 = \frac{r_1 r_2}{r_1 + r_2}, R_2 = \frac{r_1 r_3}{r_1 + r_3}, R_3 = \frac{r_1}{S + r_1}, R_5 = \frac{N r_1 r_2}{N r_1 + N r_2 + r_1 r_2}, R_6 = \frac{N r_1 r_3}{N r_1 + N r_3 + r_1 r_3}, R_7 = \frac{S r_1 r_2}{S r_1 + S r_2 + r_1 r_2}, R_8 = \frac{S r_1 r_2}{S r_1 + S r_2 + r_1 r_2}, R_9 = \frac{S r_1 r_2}{S r_1 + S r_2 + r_1 r_3}$				
IN failure mode column 'N' represents components failure in normal weather 'A' represents components failure in adverse weather				

$$R_7 = \frac{N r_1}{N + r_1}$$

No Repair During Adverse Weather
50% Common Mode Outages Occur During
Adverse Weather.

----- Two Lines in Parallel
- - - Three Lines in Parallel

$$\lambda_{av} = 0.5 \text{ f/yr.}$$

$$\lambda_{cav} = 0.05 \text{ f/yr.}$$

$$N = 200.0 \text{ h}$$

$$S = 1.5$$

Percentage Error Introduced Using Approximate Equations

Figure 4.3(b) Variation of % Error Associated With Approximate System Failure Rate

(ii) When repair occurs during adverse weather.

(i) No Repair During Adverse Weather

Under this condition of repair, failures which occur in adverse weather have to wait for repair for an average period equal to the average duration of the adverse weather. Referring to the model of Figure 4.1, the average outage duration for which failure occurs during the adverse weather period is given by:

$$\frac{r_1 r_2 r_c}{r_1 r_2 + r_2 r_c + r_c r_1} + S \quad (4.10)$$

If the contribution to the system failure rate due to the normal weather failures (modes 1,2 and 3) and adverse weather failures (modes 4,5 and 6) are given by λ_{en} and λ_{es} respectively, then the average outage duration is equal to the sum of the weighted average outage durations associated with each of these outage rate values i.e

$$r_{SL} = \frac{\lambda_{es}}{\lambda_{en} + \lambda_{es}} \left\{ \frac{r_1 r_2 r_c}{r_1 r_2 + r_2 r_c + r_c r_1} \right\} + \frac{\lambda_{en}}{\lambda_{en} + \lambda_{es}} \left\{ \frac{r_1 r_2 r_c}{r_1 r_2 + r_2 r_c + r_c r_1} + S \right\} \quad (4.11)$$

(ii) Repair During Adverse Weather

Under this condition, every failure which occurs either in normal weather or in adverse weather, can be repaired immediately. The system average outage duration is given by

Equation 4.12.

$$r_{SL} = \frac{r_1 r_2 r_c}{r_1 r_2 + r_c r_2 + r_c r_1} \quad (4.12)$$

The results using Equations 4.11 and 4.12 are shown in Table 4.5 together with theoretically "Exact" results obtained using a Markov approach. The method is discussed in Section 4.2.1(a) for determining the exact system steady state probabilities. The system average outage duration can be calculated as follow:

$$r_{SL} = \frac{\text{Cumulative probability of the down states}}{\text{Cumulative frequency of the down states}} = \frac{P_d}{F_d}$$

$$r_{SL} = \frac{P_3 + P_6}{P_3(\mu_1 + \mu_2 + \mu_c) + P_6(\mu_1' + \mu_2' + \mu_c')} \quad (4.13)$$

if there is no repair available during adverse weather

$$\mu_1' = \mu_2' = \mu_c' = 0.0$$

$$r_{SL} = \frac{P_3 + P_6}{P_3(\mu_1 + \mu_2 + \mu_c)} \quad (4.14)$$

If repair is available during adverse weather with the same rate as in normal weather therefore,

$$r_{SL} = \frac{1}{\mu_1 + \mu_2 + \mu_c} \quad (4.15)$$

The above approach can be extended to determine the system average outage duration in a three component

Table 4.5

System Outage Durations For Multiple Circuit Lines

$\lambda_{av} = 0.5 \text{ f/yr}$

,

$\lambda_{cav} = 0.05 \text{ f/yr}$

Percentage of independent outages occur in adverse Weather
r=7.5 hrs., N=200, S=1.5,
Two Lines in Parallel Sysyem Av. Outage Du.
Three Lines in Parallel System Av. Outage Du.

Exact Approx. % Error Exact Approx. % Error

No repair during adverse weather

0.0	3.491	3.471	0.573	2.765	2.750	0.530
10.0	3.493	3.474	0.551	2.7634	2.7500	0.484
20.0	3.499	3.480	0.532	2.7622	2.7500	0.439
30.0	3.507	3.489	0.516	2.7610	2.7501	0.394
40.0	3.519	3.501	0.501	2.7600	2.7504	0.349
50.0	3.533	3.516	0.486	2.7592	2.7507	0.305
60.0	3.533	3.516	0.486	2.7585	2.7513	0.260
70.0	3.569	3.552	0.459	2.7580	2.7521	0.216
80.0	3.589	3.573	0.445	2.7578	2.7530	0.172
90.0	3.610	3.595	0.430	2.7579	2.7543	0.128
100.0	3.633	3.618	0.415	2.7582	2.7559	0.084

Repair available during adverse weather

0.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
10.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
20.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
30.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
40.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
50.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
60.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
70.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
80.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
90.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000
100.0	2.7273	2.7273	0.000	2.0000	2.0000	0.000

redundant system. Those equations are shown in Table 4.4. The results obtained by these approximate equations are compared with those obtained by a Markov approach for two and three overlapping outages in Table 4.5. It is clear from Table 4.5 that the results obtained by these equations are quite close to those predicted by the Markov technique.

4.2.2 Model B

The state space diagram proposed in Reference [4] is shown in Figure 2.3. This model can be extended by including adverse weather conditions as shown in Figure 4.4 for a two non-identical component system. The state space diagram of Figure 4.1 indicates that there are two repair phenomena available when the system is in the down state. These two individual repair phenomena are the individual element repair and joint common cause repair. If in a given system, most of the common mode failures occur due to terminal equipment failures or due to tower failures and both the lines are put back into service simultaneously even if the repair of one of the lines is completed earlier, it is necessary to further modify the model of Figure 4.1 to eliminate the individual repair phenomenon from the down state. This situation is suitably described by the model of Figure 4.4. The system analysis is presented in three different parts.

4.2.2(a) System State Space Probabilities

Using the same assumptions and approach as described in Appendix 1, the Markov approach can be applied to the model of Figure 4.4, operating in a two state fluctuating environment. The state space diagram for the system of three components can be constructed. The steady state probabilities associated with each of the system states can be determined by solving the set of simultaneous linear equations of the following matrix form:

$$\begin{bmatrix} 0 \end{bmatrix} = \begin{bmatrix} DN & nI \\ nI & DS \end{bmatrix} \begin{bmatrix} P \end{bmatrix} \quad (4.12)$$

The matrices DN and DS correspond to transitions out of the normal and adverse weather states respectively. The matrices DN, DS, nI, mI, and P are defined as in Section 4.2.1(a). In order to determine the steady state probabilities, the solution of the simultaneous linear equations designated as 4.12, must satisfy the condition that the availabilities of all the states must sum to one. The matrices DN and DS for the two component system are shown in Table 4.6. The matrices DN and DS for a larger number of components can be written in a similar manner.

Table 4.6 Matrices DN And DS For Model B

$$[DN] = \begin{bmatrix} \lambda_1 + \lambda_2 + \lambda_C + n & -\mu_1 & -\mu_2 & 0 & -\mu_C \\ -\lambda_1 & \lambda_2 + \mu_1 + n & 0 & -\mu_2 & 0 \\ -\lambda_2 & 0 & \lambda_1 + \mu_2 + n & -\mu_1 & 0 \\ 0 & -\lambda_2 & -\lambda_1 & \mu_1 + \mu_2 + n & 0 \\ -\lambda_C & 0 & 0 & 0 & \mu_C + n \end{bmatrix}$$

$$[DS] = \begin{bmatrix} \lambda_1' + \lambda_2' + \lambda_C' + m & -\mu_1' & -\mu_2' & 0 & -\mu_C' \\ -\lambda_1' & \lambda_2' + \mu_1' + m & 0 & -\mu_2' & 0 \\ -\lambda_2' & 0 & \lambda_1' + \mu_2' + m & -\mu_1' & 0 \\ 0 & -\lambda_2' & -\lambda_1' & \mu_1' + \mu_2' + m & 0 \\ -\lambda_C' & 0 & 0 & 0 & \mu_C' + m \end{bmatrix}$$

Figure 4.4 Model B For Two Components Including Common Mode And Adverse Weather Effects.

4.2.2(b) System Average Failure Rate

The development of the average outage rate formula for the model of Figure 4.4 is exactly the same as that for the model of Figure 4.1. The normal and adverse weather components of system failure rate are given in Equation 4.17 and are the same as those in Equation 4.3 for the condition of no repair during adverse weather.

$$\lambda_e = \{\lambda_2\lambda_1(r_1 + r_2) + \frac{S}{N}(\lambda_1^i\lambda_2r_1 + \lambda_2^i\lambda_1r_2) + \lambda_c\} \quad (4.17)$$

$$\lambda_e^i = \{2S\lambda_1^i\lambda_2^i + \lambda_1\lambda_2^i r_1 + \lambda_2\lambda_1^i r_2 + \lambda_c^i\}$$

For the condition of repair during adverse weather, the normal and adverse weather components of system failure rate are given in Equation 4.18.

$$\lambda_e = \{\lambda_c + \lambda_1\lambda_2(r_1 + r_2) + \frac{S}{N}(\lambda_1\lambda_2^i \frac{r_1^2}{S + r_1} + \lambda_2\lambda_1^i \frac{r_2^2}{S + r_2})\} \quad (4.18)$$

$$\lambda_e^i = \{\lambda_c^i + \lambda_1^i\lambda_2r_1 + \lambda_2^i\lambda_1r_2 + \lambda_1^i\lambda_2^i S(\frac{r_1}{S + r_1} + \frac{r_2}{S + r_2})\}$$

The Fundamental matrix N used to determine the exact system outage rate by the Markov approach is the same as that derived in Section 4.1.1(b).

4.2.2(c) System Average Outage Duration

The average outage duration of a system is given by the ratio of the cumulative probability of the down states to

the cumulative frequency of the down states.

$$P_d = P_4 + P_5 + P_9 + P_{10}$$

P_d = Cumulative frequency of all down states

$$= P_4(\mu_1 + \mu_2) + P_5(\mu_c) + P_9(\mu_1' + \mu_2') + P_{10}(\mu_c')$$

For the model of Figure 4.4 P_4 , P_5 , P_9 , P_{10} are the probabilities of being in the failed states 4, 5, 9 and 10 respectively. The state space probabilities can be obtained by solving the Probability Matrix 4.12. For the condition of no repair during adverse weather, μ' and μ_c' can be taken as zero.

The average outage duration can be obtained using an approximate formula using the concept described earlier. If the contribution to the system failure rate due to the normal weather outages are given as λ_{enl} and λ_{cn} and due to adverse weather outages are given as λ_{esl} and λ_{cs} then the system average outage duration is equal to the weighted sum of the average outage durations associated with each of these outage rate values. For the condition of no repair during adverse weather

$$r_{SL} = \frac{1}{\lambda_{SL}} \left\{ \lambda_{enl} \left(\frac{r_1 r_2}{r_1 + r_2} \right) + \lambda_{cn} r_c + \lambda_{esl} \left(\frac{r_1 r_2}{r_1 + r_2} + S \right) + \lambda_{cs} (r_c' + S) \right\}$$

For the condition of repair during adverse weather

$$r_{SL} = \frac{1}{\lambda_{SL}} \left\{ \lambda_{enl} \left(\frac{r_1 r_2}{r_1 + r_2} \right) + \lambda_{cn} r_c + \lambda_{esl} \left(\frac{r_1 r_2}{r_1 + r_2} \right) + \lambda_{cs} r_c' \right\}$$

4.2.3 Model C

The model shown in Figure 4.5 is a combination of models A and B. The common cause outages occurring in normal weather, can be repaired by either the individual element repair process or by common cause repair action. The common cause outages which occur in adverse weather are repaired by common cause repair action. This recognizes that the common mode outages which occur in normal weather are different to the common cause outages which occur in adverse weather. The outage duration associated with an adverse weather common mode outage can be much shorter than that associated with a normal weather common mode outage.

4.2.3(a) System Steady State Probabilities

The steady state probabilities associated with each state of model C can be determined by solving the set of simultaneous linear equations of Matrix 4.19.

$$\begin{bmatrix} 0 \end{bmatrix} = \begin{bmatrix} DN & mI \\ nI & DS \end{bmatrix} \begin{bmatrix} P \end{bmatrix} \quad (4.19)$$

The matrices DN, DS, nI, mI, and P are defined as in Section 4.2.1(a). The matrices DN and DS for the model of Figure 4.5 are given in Table 4.7. Similar matrices can be written for higher order systems.

Table 4.7 Matrices DN And DS For Model C.

$$[DN] = \begin{bmatrix} \lambda_1 + \lambda_2 + \lambda_C + n & -\mu_1 & -\mu_2 & -\mu_C & -\mu'_C \\ -\lambda_1 & \lambda_2 + \mu_1 + n & 0 & -\mu_2 & 0 \\ -\lambda_2 & 0 & \lambda_1 + \mu_2 + n & -\mu_1 & 0 \\ -\lambda_C & -\lambda_2 & -\lambda_1 & \mu_1 + \mu_2 + n & 0 \\ 0 & 0 & 0 & 0 & \mu'_C + n \end{bmatrix}$$

$$[DS] = \begin{bmatrix} \lambda_1 + \lambda_2 + \lambda'_C + m & -\mu'_1 & -\mu'_2 & 0 & -\mu'_C \\ -\lambda'_1 & \lambda'_2 + \mu'_1 + m & 0 & -\mu'_2 & 0 \\ -\lambda'_2 & 0 & \lambda'_1 + \mu'_2 + m & -\mu'_1 & 0 \\ 0 & -\lambda'_2 & -\lambda'_1 & \mu'_1 + \mu'_2 + m & 0 \\ -\lambda'_C & 0 & 0 & 0 & \mu'_C + m \end{bmatrix}$$

Figure 4.5 Model C For Two Components Including Common Mode And Adverse Weather Effects.

4.2.3(b) System Average Outage Rate

The development of an average outage rate formula for the model of Figure 4.5 is the same as that for Models A and B of this chapter. The normal and adverse weather components of the system failure rate are also the same as that of previous models. The truncated stochastic transitional probability matrix Q is the same as that given in Section 4.2.1(a) for determining the exact system outage rate.

4.2.3(c) System Average Outage Duration

The average outage duration is given by the ratio of the cumulative probability and cumulative frequency of all the system down states.

$$P_d = P_4 + P_5 + P_9 + P_{10}$$

$$F_d = \text{Cumulative frequency of down states.}$$

$$= P_4(\mu_1 + \mu_2 + \mu_c) + P_5(\mu'_c) + P_9(\mu'_1 + \mu'_2) + P_{10}(\mu'_c)$$

The system steady state probabilities are obtained from the solution of Matrix 4.19 given in Section 4.2.3(a). The average outage duration can also be obtained using the approximate equations given below. These equations are developed using the concept described earlier. For the condition of repair during adverse weather,

$$r_{SL} = \frac{1}{\lambda_{SL}} \left\{ (\lambda_{en1} + \lambda_{cn}) \left(\frac{r_1 r_2 r_c}{r_1 r_2 + r_c r_1 + r_c r_2} \right) + \lambda_{es1} \left(\frac{r_1 r_2}{r_1 + r_2} \right) + \lambda_{cs} (r'_c) \right\}$$

and for the condition of no repair during adverse weather,

$$r_{SL} = \frac{1}{\lambda_{SL}} \{ (\lambda_{en1} + \lambda_{cn}) \left(\frac{r_1 r_2 r_c}{r_1 r_2 + r_1 r_c + r_2 r_c} \right) + \lambda_{es1} \left(\frac{r_1 r_2}{r_1 + r_2} + S \right) + \lambda_{cs} (r'_c + S) \}$$

where λ_{en1} , λ_{cn} , λ_{es1} and λ_{cs} are defined as in Section 4.2.2(c).

4.2.4 Model D

The model shown in Figure 4.6 for a two component system, is a further modification of models A, B and C. In the previous model, common cause outages were divided into two categories with one category on the two different weather sides. In practice, both types of common mode outages can occur during both normal and adverse weather periods. Therefore, normal and adverse weather common cause outages must be subdivided in two parts λ_{C1} and λ_{C2} in the normal weather and λ'_{C1} and λ'_{C2} during adverse weather. The common cause failures accounted for by λ_{C2} and λ'_{C2} are repaired at the rate of μ_C and μ'_C respectively. The repairs arising due to the remainder of the common cause failures are carried out using the normal independent repair processes. Practical application of this model requires more detailed data collection.

The system steady state probabilities, average outage rate and system average outage duration can be obtained for the model of Figure 4.6 using the concepts described

earlier. A approximate formula is given below for calculating the system average outage duration. For the condition of repair during adverse weather

$$r_{sL} = \frac{1}{\lambda_{sL}} \{ \lambda_{enl} \left(\frac{r_1 r_2}{r_1 + r_2} \right) + \lambda_{esl} \left(\frac{r_1 r_2}{r_1 + r_2} \right) + \lambda_{cn} r_c + \lambda_{cs} r'_c \}$$

and for the condition of no repair during adverse weather

$$r_{sL} = \frac{1}{\lambda_{sL}} \{ \lambda_{enl} \left(\frac{r_1 r_2}{r_1 + r_2} \right) + \lambda_{esl} \left(\frac{r_1 r_2}{r_1 + r_2} + S \right) + \lambda_{cn} r_c + \lambda_{cs} (r'_c + S) \}$$

where λ_{enl} , λ_{cn} , λ_{esl} and λ_{cs} are defined as in Section 4.2.2(c).

The results obtained by these approximate equations are compared with those obtained by a Markov approach. Figure 4.7 shows the percentage error in the approximate average outage durations for the models B, C. and D.

4.3 Analysis And Comparison of Common Mode Outage Models Including Adverse Weather Effects.

The models described in this chapter were obtained by adding the adverse weather effects to the basic independent and common cause outage models of Chapter 2. The selection of a particular model in any given case must be based on the configuration and data of the particular situation. This section shows the effect of the selection of the model on system state probabilities and on system reliability indices and emphasizes the importance of choosing the right model.

Figure 4.7 Percentage Error Associated With Approximate Outage Time.

This analysis also illustrates the effect of the variation in common cause failure rate and repair rate on the system reliability indices of various models. A meaningful analysis and comparison of the models can be achieved by using numerical results based on practical system data. The studies of this section are therefore, conducted using the outage data provided by Ontario Hydro. These data are shown in Table 4.8.

The method of calculating λ , the independent failure rate in outage/mile/year and λ_c , the common mode outage rate in outage/mile of multiple circuit length/year is as follows.

λ in failure/mile/year

$$\begin{aligned} &= (f_1 + f_2 + f_3 + \dots + f_j + \dots + f_n) / \Sigma L_j Y_j \\ &= (f_j) / (\Sigma L_j Y_j) \end{aligned} \quad (4.20)$$

λ_c in failure/mile of double circuit line/year

$$\begin{aligned} &= (c_1 + c_2 + c_3 + \dots + c_j + \dots + c_n) / \Sigma L_j Y_j \\ &= (c_j) / (\Sigma L_j Y_j) \end{aligned} \quad (4.21)$$

Where $\Sigma L_j Y_j = (L_1 Y_1 + L_2 Y_2 + \dots + L_j Y_j + \dots + L_n Y_n)$

L_j = Length of the j^{th} line.

Y_j = Years of outage recorded for the j^{th} line.

f_j = Number of Independent outages of j^{th} line in Y_j years.

c_j = Number of common mode outages of j^{th} double circuit line.

$$\lambda_c / \lambda = (\Sigma c_j) / (\Sigma f_j) \quad (4.22)$$

Table 4.8
Common Mode Outage Data of Some Double Circuit Lines
of Ontario Hydro

Designation	Common-mode outages		Total Common mode outages (3) = (1+2)	Total (*) correct trips (4)	Length of line (miles) (5)	Years of Record (6)	Mile x year (5) x (6)	λ	λ_c	$\frac{\lambda_c}{\lambda}$
	(1) NORMAL	(2) ABNORMAL						(4) 2 (5) (6)	(3) (5) (6)	(3)/2 (4)
A21L-A22L	2	5	7	5	119.6	5.0 years	598.0	0.004180	0.011706	2.8
BP76-PA27	2	0	2	1	4.7	8.0	37.6	0.013298	0.05319	4.0
B15C-B16C	0	0	0	1	2.9	3.67	10.6	0.046980	0	0.0
B18H-B20H	0	0	0	5	7.1	6.17	43.8	0.057078	0	0.0
B20P-B24P	0	0	0	2	1.2	2.08	2.5	0.40000	0	0.0
C16L-C17L	0	0	0	1	15.8	3.08	48.7	0.010267	0	0.0
C18R-C20R	0	0	0	21	29.0	5.92	171.7	0.061160	0	0.0
C4R-C5R	3	0	3	27	27.8	7.92	220.2	0.061310	0.013624	0.22
D1M-D2M	2	1	3	5	44.0	7.92	744.5	0.00336	0.00403	1.2
D22S-D23S	2	2	4	12	51.4	3.83	196.9	0.030472	0.020315	0.667
D4M-D5M	0	0	0	14	55.9	5.0	279.5	0.025050	0	0.0
E8V-E9V	0	1	1	7	35.4	6.0	212.4	0.01648	0.004708	0.29
H1Q-H2Q	0	0	0	2	16.3	4.0	65.2	0.015337	0	0.0
I23N-I25N	0	0	0	12	13.6	7.92	107.7	0.055710	0	0.0
I26N-I26N	0	0	0	23	68.8	6.58	452.7	0.023403	0	0.0
I28C-I29C	1	0	1	13	36.5	5.67	206.9	0.031416	0.004833	0.15
N20D-N21D	0	1	1	10	45.4	4.17	189.32	0.026410	0.00528	0.2
N23L-N24L	9	10	19	12	142.6	6.25	891.25	0.006730	0.021318	3.167
N27B-N28B	0	0	0	1	21.4	5.01	107.0	0.004672	0	0.0
N23M-N23M	0	2	2	67	60.5	5.42	327.91	0.102162	0.006099	0.059
N28R-N28R	0	1	1	20	112.0	8.0	896.0	0.111605	0.001116	0.0
N1M-N2M	0	0	0	17	27.6	5.17	142.7	0.039565	0	0
N21J-N22J	0	0	0	0	5.1	1.75	8.92	0	0	0
N21M-N22M	1	5	6	94	65.7	8.0	525.6	0.089421	0.011415	0.128
N3M-N4M	0	0	0	0	27.3	1.08	29.5	0	0	0
N3M-N3M	0	0	0	5	27.8	4.58	127.32	0.019635	0	0
P6C-P7C	0	0	0	0	4.4	5.17	22.75	0	0	0
P8C-P9C	0	0	0	1	4.4	6.08	26.7	0.018730	0	0
Q21P-Q22P	1	0	1	5	0.6	8.0	4.8	0.520835	0.20833	0.4
R1K-R2K	1	0	1	15	4.1	8.0	32.8	0.22866	0.030488	0.133
R13K-R15K	0	0	0	11	4.1	8.0	32.8	0.16768	0	0
R14B-R17B	0	0	0	14	28.3	8.0	226.4	0.030918	0	0
R19B-R21B	1	0	1	33	39.0	8.0	312.0	0.052885	0.003205	0.06
W1Q-W2C	0	0	0	14	60.3	6.08	366.6	0.019094	0	0
W21M-W22M	0	8	8	10	104.6	6.08	635.9	0.007863	0.012581	1.6
<hr/>										
	25	36	61	480	1365.2		8305.17	0.028898	0.007345	0.254

* Include all incidents where one of the circuits was faulted.

Does not include common cause outages

Using the Ontario Hydro data, the independent outage rate λ , common cause outage rate λ_c and the ratio λ_c/λ were calculated using Equations 4.20, 4.21, 4.22 respectively and are given in Table 4.9.

TABLE 4.9

Total independent outages per line = $480/2 = 240$

Total common cause outages/double ckt. line = 61

$$\lambda = \frac{\text{Total independent outages}}{\text{Mile.Year}}$$

$$= 240/8350.17 = 0.02890 \text{ f/mile/year}$$

$$\lambda_c = \frac{\text{Total number of common mode outages}}{\text{Mile.year}}$$

$$= 61/8305.17 = 0.00734 \text{ f/mile/year}$$

$$\lambda_c/\lambda = 61/240 = 0.254$$

i.e The common cause outage failure rate is 25.4% of the independent failure rate.

Percentage occurrences of common mode outages in adverse weather = $36.(100)/61 = 59.01$ percent.

Percentage occurrences of common mode outages in normal weather = $21.(100)/61 = 40.99$ percent.

An examination of Table 4.8 shows that three lines have a relatively very high number of common cause outages. The effect of these data on the overall failure rates can be observed by calculating the ratio λ_c/λ after these data have been removed from the population. Such a calculation yields a λ_c/λ ratio 0.143 and λ_c of 0.0341 f/year/mile and 0.00487 f/year respectively. The data provided by Ontario

Hydro were not sufficient to calculate the normal and adverse weather components of λ and λ_c . Component average down and up time data are also necessary for complete system modelling. Therefore, the rest of the necessary data has been assumed in the studies conducted. The system overall common cause outage rate is approximately 15% of the independent outage rate after removing the three bad lines. In order to examine the effect, the ratio λ_c/λ was varied from 0 to 20% in steps of 2.5%. A system composed of two identical transmission lines (on the same structure or on the same right of way) was examined in detail using the test data of Table 4.8 and varying the normal and adverse weather components of λ and λ_c . The results of these studies are presented in the following sections.

4.3.1 Effect Of Adverse Weather on System

Down State Probability

The probability of the all lines down state has been calculated using Models A, B, C and D. These probabilities have been calculated for a 0 to 20 % range of λ_c/λ ratio and fixed r_c/r ratio. Table 4.10 shows the variation in the both line down state probability using Models A, B and D. The probability of the system failure increases drastically as the common mode component increases. The effect of the environment in terms of the percentage of independent

failures occurring during adverse weather is also shown in Table 4.10. The probability of system failure is much higher for Model B than that of Models A and D for the same data and conditions. Table 4.11 shows the effect associated with neglecting adverse weather considerations when calculating the probability of the failure state. The Error Factor is shown for a 100 mile line considering Models A, B and D. The error decreases as λ_c increases. In the case of Model A, neglecting adverse weather produces at least 50% error with $\lambda_c=0.1$ and above. The Error Factor increases for large values of λ and λ_c . This effect is shown in Figure 4.8 where Error Factor is shown for lines of 20 and 40 miles in length. Figure 4.9 shows that shorter lines situations have less error in both cases. In the case of Model B the Error Factor is small even for a line of 40 miles long and therefore consideration of adverse weather could possibly be neglected. In the case of Model A neglecting the adverse weather causes an error of 30 to 40% for the same length of line. The average length of a double circuit transmission line for the Ontario Hydro data is 40 miles. Tables 4.10 and 4.11 are based on a 100 miles long transmission line. Figure 4.9 shows the Error Factor for the same conditions used in Figure 4.8 but with the percentage of common mode outages occurring during adverse weather varied from 0 to 100 % in a 40 mile line. The results shown in Figure 4.8 suggest that the percentage of common mode outages which

Table 4.10
Variation In The System Down State Probability
With Common Mode Outage Rate

Independent failure rate of each line = 0.0341 f/yr/mile
 Average repair time of each line = 7.5 hours
 Common Mode repair Time = 10.0 hours
 Length of the line = 100.0 Miles

%						
(Probability of both lines down)x10000						

λ_c/λ	0.0%	20.0%	40.0%	60.0%	80.0%	100.0% (*)

Model A

0.0	0.1143	0.2442	0.5940	1.1348	1.8412	2.6918
2.5	0.5667	0.6872	1.0280	1.5599	2.2680	3.1004
5.0	1.0183	1.1294	1.4612	1.9844	2.6741	3.5084
7.5	1.4690	1.5709	1.8936	2.4082	3.0896	3.9159
10.0	1.9189	2.0116	2.3254	2.8314	3.5044	4.3227
12.5	2.3680	2.4515	2.7565	3.2538	3.9186	4.7289
15.0	2.8164	2.8907	3.1868	3.6756	4.3321	5.1345
17.5	3.2639	3.3291	3.6164	4.0967	4.7450	5.5395
20.0	3.7106	3.7668	4.0453	4.5172	5.1573	5.9440

MODEL B

0.0	0.1526	0.3114	0.7427	1.4107	2.2842	3.3364
2.5	1.6974	1.8268	2.2302	2.8717	3.7201	4.7484
5.0	3.2394	3.3396	3.7153	4.3305	5.1538	6.1583
7.5	4.7788	4.8499	5.1980	5.7870	6.5854	7.5661
10.0	6.3155	6.3577	6.6784	7.2412	8.0148	8.9719
12.5	7.8495	7.8629	8.1563	8.6932	9.4421	10.1037
15.0	9.3808	9.3656	9.6318	10.1429	10.8673	11.7776
17.5	10.9095	10.8659	11.1051	11.5904	12.2904	13.1774
20.0	12.4355	12.3636	12.5759	13.0357	13.7113	14.5752

MODEL D

0.0	0.1526	0.3114	0.7427	1.4107	2.2842	3.3364
2.5	2.2702	2.3880	2.7803	3.4110	4.2490	5.2672
5.0	3.3261	3.4234	3.7966	4.4088	5.2291	6.2305
7.5	4.3802	4.4576	4.8113	5.4051	6.2078	7.1924
10.0	5.4324	5.4894	5.8243	6.3998	7.1850	8.1528
12.5	6.4828	6.5202	6.8356	7.3929	8.1607	9.1119
15.0	7.5314	7.5488	7.8453	8.3844	9.1349	10.0696
17.5	8.5780	8.5758	8.8534	9.3745	10.1077	11.0259
20.0	10.6659	10.6245	10.8646	11.3499	12.0489	12.9344

 (*) Percentage variation of adverse weather independent outages.

Table 4.11

Variation Of Error Excluding Adverse Weather Effects
For The System Down State Probability

(Probability of both lines down)x10000						
λ/λ	No repair during adverse w.			Repair during adverse w.		
%	Including Adverse Weather	Excluding Adverse Weather	Error Factor	Including Adverse Weather	Excluding Adverse Weather	Error Factor
<u>Model A</u>						
0.0	1.1348	0.1127	10.0692	0.7281	0.1127	6.4605
2.5	1.5599	0.4758	3.2785	1.0759	0.4758	2.2612
5.0	1.9844	0.8388	2.3657	1.4233	0.8388	1.6968
7.5	2.4082	1.2018	2.0038	1.7702	1.2018	1.4729
10.0	2.8314	1.5647	1.8095	2.1167	1.5647	1.3528
12.5	3.2538	1.9275	1.6881	2.4626	1.9275	1.2776
15.0	3.6756	2.2902	1.6049	2.8082	2.2902	1.2262
17.5	4.0967	2.6529	1.5442	3.1534	2.6529	1.1887
20.0	4.5172	3.0154	1.4980	3.4980	3.0154	1.1600
<u>MODEL B</u>						
0.0	1.4101	0.1504	9.3797	0.9719	0.1504	6.4621
2.5	2.8717	1.5986	1.7964	2.3554	1.5986	1.4734
5.0	4.3305	3.0464	1.4215	3.7346	3.0464	1.2259
7.5	5.7869	4.4938	1.2877	5.1131	4.4938	1.1378
10.0	7.2412	5.9408	1.2189	6.4897	5.9408	1.0924
12.5	8.6932	7.3873	1.1768	7.8643	7.3873	1.0646
15.0	10.1429	8.8335	1.1482	9.2371	8.8335	1.0457
17.5	11.5904	10.2792	1.1276	10.6079	10.2792	1.0319
20.0	13.0357	11.7245	1.1118	11.9769	11.7245	1.0215
<u>MODEL D</u>						
0.0	1.4107	0.1501	9.3984	0.9719	0.1501	6.4750
2.5	3.4110	1.1168	3.0542	1.8952	1.1168	1.6969
5.0	4.4088	2.0829	2.1167	2.8171	2.0829	1.3525
7.5	5.4051	3.0488	1.7729	3.7377	3.0488	1.2259
10.0	6.3998	4.0144	1.5942	4.6569	4.0144	1.1600
12.5	7.3929	4.9797	1.4846	5.5749	4.9797	1.1195
15.0	8.3844	5.9447	1.4104	6.4916	5.9447	1.0919
17.5	9.3745	6.9094	1.3568	7.4069	6.9094	1.0720
20.0	11.3499	7.8739	1.4415	8.3211	7.8739	1.0566

Figure 4.8 Variation of Error Factor With Line Length and λ_c/λ Ratio.

Figure 4.9 Variation of Error Factor With Percentage λ_c/λ Ratio and λ_c'

Error Factor = $\frac{\text{Probability of system failure including adverse w.}}{\text{Probability of system failure excluding adverse w.}}$

occur in adverse weather is a factor which should be included in the system model.

The variation in the probability of being in the two line out state as a function of the relative magnitude of the common mode failure rate is shown in Figure 4.10. The results shown in Figure 4.10 are very similar to those of Figure 2.6. The common cause failure phenomenon has a dominant influence on the system failure probability. This influence is considerably greater than the common environment phenomenon created by adverse weather during which independent failures occur. Figure 4.10 utilizes $\lambda=0.5$ f/yr, $r=10$, $N=200.0$, $S=1.5$ hours for Models A and B. The probability of the system down state is very sensitive to the value of λ_c from 0 to 5% of the independent outage rate.

A comparison study was also made for Model C, where it was assumed that the common cause outages which occur in adverse weather are of short duration compared to the outages which occur during normal weather. A model is shown in Figure 4.11 similar to model C but neglecting adverse weather considerations. The adverse weather common mode outages and repair rates are included as λ'_{cs} and r'_c respectively. Model C which has a total of ten states is now represented by a model containing 5 states. Table 4.12

Figure 4.10 Probability of System Failure As A Function of λ_c For the Models A and B.

Table 4.12

Model C. Analysis And Error Factor Neglecting Adverse Weather

Percentage of Independent Outages occur in Adverse W.	Probability of Both Lines Up	Error Factor	Probability of Both Lines Down	Error Factor
0.0	0.991802	1.00002	0.000260	0.9059
10.0	0.991706	0.9999	0.000261	0.9094
20.0	0.991635	0.9998	0.000268	0.9338
30.0	0.991587	0.9998	0.000280	0.9756
40.0	0.991563	0.9997	0.000298	1.0383
50.0	0.991560	0.9997	0.000320	1.1149
60.0	0.991578	0.9998	0.000347	1.2090
70.0	0.991616	0.9998	0.000378	1.3171
80.0	0.991674	0.9998	0.000413	1.4390
90.0	0.991750	0.9999	0.000452	1.5749
100.0	0.991843	1.00002	0.000494	1.7212

Figure 4.11 Reduced Version of Model C Neglecting Adverse Weather Effects.

shows the probability values for the both line up and both line down states together with the Error Factor neglecting adverse weather for Model C. The probabilities for both lines up and both lines down for the model of Figure 4.11 are as follows:

Probability that both lines are in the up state = 0.991779

Probability that both lines are in the Dn state = 0.000287

The above probabilities were calculated using $\lambda_{av}=3.41$, $\lambda_{cav}=0.487$, $r=10$, $r_c=15$, $r'_c=5$, and λ_c is 60 % of λ . Table 4.12 shows the probabilities of both lines Up or Down with variation in the adverse weather independent outages and 60% common mode outages in adverse weather. Table 4.12 clearly shows that the Model C can be reduced to the model of Figure 4.9. The Error Factor for the Up and Down state probabilities is close to unity, if 50% or less, independent outages occur in adverse weather. The results in Table 4.12 are for a line of 100 miles long and therefore it is obvious that adverse weather considerations can be neglected in the analysis of shorter lines.

4.3.2 Effect of Adverse Weather on System

Failure Rate

In Section 4.2.1(a), an approximate and an exact

approach was described for determining the failure rate of a multi-circuit redundant transmission system. It was also noted that the approximate failure rate formula is the same for Models A, B, C and D. The percentage Error Factor associated with the approximate system failure rate is shown in Table 4.9. The system failure rate for a two and three component system without considering adverse weather considerations but including common mode outages are as follows:

Repair time hours	Double ckt. line $\lambda_{sLf/yr/100mile}$	Triple ckt. line $\lambda_{sLf/yr/100mile}$
5.0	0.49831	0.48421
10.0	0.50949	0.48151
15.0	0.52055	0.47897

In each case $\lambda_c = 0.487$ f/yr/100 miles.

As expected, the system failure rate is dominated by the common mode failure rate. Equations have already been given in the previous sections for the system of three component in parallel with and without considering repair during adverse weather. These equations are approximate and are quite adequate for most applications. Tables 4.13 and 4.14 present results for a system containing two transmission lines on an independent tower or on same right of way with two different conditions of repair, i.e repair during adverse weather and no repair during adverse weather respectively. The system failure rate for a system of three

Table 4.13

Outage Rate For a Two Component System
Repair During Adverse Weather

Percentage of Independent Outages occur in Adverse W.	Repair Time (hours)	Exact	Failure rate λ , f/yr.	Approximate	Error Factor
0.0	5.0	0.49667	0.50037	0.50037	0.9967
	10.0	0.50789	0.51375	0.51375	0.9968
	15.0	0.51897	0.52712	0.52712	0.9969
20.0	5.0	0.50439	0.51580	0.51580	1.0122
	10.0	0.51642	0.53073	0.53073	1.0136
	15.0	0.52770	0.54438	0.54438	1.0137
40.0	5.0	0.54160	0.56333	0.56333	1.0869
	10.0	0.55753	0.58306	0.58306	1.0943
	15.0	0.56998	0.59763	0.59763	1.0949
60.0	5.0	0.60531	0.64295	0.64295	1.2147
	10.0	0.62757	0.67074	0.67074	1.2317
	15.0	0.64192	0.68687	0.68687	1.2332
80.0	5.0	0.69281	0.75467	0.75467	1.3903
	10.0	0.72329	0.79378	0.79378	1.4196
	15.0	0.74008	0.81210	0.81210	1.4217
100.0	5.0	0.80171	0.89849	0.89849	1.6088
	10.0	0.84185	0.95217	0.95217	1.6523
	15.0	0.86148	0.97331	0.97331	1.6549

Table 4.14

Outage Rate For a Two Component System
No Repair During Adverse Weather

Percentage of Independent outages occur in Adverse W.	Repair Time (hours)	Exact	Failure rate λ , f/yr/100 miles	Approximate	Error Factor
0.0	5.0	0.49647	0.50037	0.50037	0.9963
	10.0	0.50775	0.51375	0.51375	0.9966
	15.0	0.51889	0.52712	0.52712	0.9968
20.0	5.0	0.50779	0.52124	0.52124	0.0190
	10.0	0.51887	0.53407	0.53407	1.0184
	15.0	0.52982	0.54691	0.54691	1.0178
40.0	5.0	0.53624	0.58383	0.58383	1.1163
	10.0	0.56672	0.59506	0.59506	1.1123
	15.0	0.57708	0.60629	0.60629	1.1086
60.0	5.0	0.63701	0.68814	0.68814	1.2783
	10.0	0.64657	0.69669	0.69669	1.2691
	15.0	0.65603	0.70525	0.70525	1.2602
80.0	5.0	0.74595	0.83418	0.83418	1.4969
	10.0	0.75434	0.83899	0.83899	1.4806
	15.0	0.76264	0.84381	0.84381	1.4651
100.0	5.0	0.87944	1.02194	1.02194	1.7648
	10.0	0.88649	1.02194	1.02194	1.7399
	15.0	0.89348	1.02194	1.02194	1.7164

redundant lines is shown in Tables 4.15 and 4.16. The Error Factor associated with the system failure rate clearly indicates that neglecting adverse weather introduces a significant error in the overall system failure rate in the case of a two line redundant system. This effect is dependent on the common cause failure rate for the three line case and the detail in the three line state space model. This error is much smaller in the case of a three line redundant system. The system failure rate shown in these tables is calculated for the condition that 60 % common mode outages occur in adverse weather. These tables also indicate that if a majority of the independent outages occur during normal weather then a single state weather model can be used even for long double circuit lines to calculate the overall system failure rate with only a small amount of error.

Table 4.17 shows the variation in system failure rate with change in the normal and adverse weather durations. The results in this table were obtained for $r=10.0$, $\lambda_{av}=3.41$ f/yr/100miles, $\lambda_{cav}=0.487$ and 60 % common mode outages occurring in adverse weather. A study of Table 4.17 shows that the two state weather environment must be included when the adverse weather duration increases even in cases when these storms occur less frequently.

Table 4.13

Outage Rate For a Three Component System
Repair During Adverse Weather

Percentage of Independent Outages occur in Adverse W.	Repair Time (hours)	Exact	Approximate	Error Factor
0.0	5.0 10.0 15.0	0.48267 0.48004 0.47747	0.48704 0.48716 0.48735	0.9968 0.9969 0.9970
20.0	5.0 10.0 15.0	0.47323 0.46975 0.46701	0.48756 0.48791 0.48822	0.9774 0.9756 0.9751
40.0	5.0 10.0 15.0	0.46704 0.46402 0.46185	0.49104 0.49279 0.49387	0.9646 0.9637 0.9644
60.0	5.0 10.0 15.0	0.46569 0.46519 0.46448	0.50030 0.50569 0.50841	0.9623 0.9661 0.9699
80.0	5.0 10.0 15.0	0.47144 0.47488 0.47653	0.51818 0.53053 0.53648	0.9736 0.9862 0.9950
100.0	5.0 10.0 15.0	0.48449 0.49417 0.49903	0.54752 0.57120 0.58236	1.0006 1.0263 1.0420

Table 4.16

Outage Rate For a Three Component System
No Repair During Adverse Weather

Percentage of Independent Outages occur in Adverse W.	Repair Time (hours)	Exact	Approximate	Error Factor
0.0	5.0 10.0 15.0	0.48227 0.47956 0.47696	0.48704 0.48715 0.48735	0.9960 0.9960 0.9935
20.0	5.0 10.0 15.0	0.47048 0.46804 0.46596	0.48819 0.48845 0.48879	0.9717 0.9720 0.9724
40.0	5.0 10.0 15.0	0.46495 0.46305 0.46123	0.49554 0.49607 0.49668	0.9602 0.9617 0.9631
60.0	5.0 10.0 15.0	0.46881 0.46763 0.46653	0.51489 0.51563 0.51640	0.9682 0.9712 0.9741
80.0	5.0 10.0 15.0	0.48403 0.48367 0.48336	0.55206 0.55269 0.55334	0.9996 1.0045 1.0093
100.0	5.0 10.0 15.0	0.51174 0.51219 0.51268	0.61288 0.61287 0.61288	1.0569 1.0637 1.0705

Table 4.17

Variation In System Outage Rate With The Change In
Normal And Adverse Weather Durations
No Repair During Adverse Weather

	N=100, S=1.0	N=200, S=2.0	N=300, S=1.5
Percentage of Independent Outages occur in Adverse W.	λ_{SL} f/yr Error	λ_{SL} f/yr Error	λ_{SL} f/yr Error

Two Lines in Parallel

0.0	0.50876	0.9986	0.50781	0.9967	0.50686	0.9948
10.0	0.50909	0.9992	0.50846	0.9979	0.50782	0.9967
20.0	0.51450	1.0098	0.51896	1.0186	0.52310	1.0267
30.0	0.52428	1.0301	0.53856	1.0572	0.55130	1.0821
40.0	0.53988	1.0596	0.56691	1.1127	0.59115	1.1603
50.0	0.55952	1.0982	0.60318	1.1839	0.64151	1.2591
60.0	0.58359	1.1454	0.64693	1.2697	0.70135	1.3766
70.0	0.61192	1.2010	0.69765	1.3693	0.76976	1.5108
80.0	0.64439	1.2647	0.75489	1.4817	0.84591	1.6603
90.0	0.68085	1.3363	0.81824	1.6056	0.92905	1.8235
100.0	0.72117	1.4154	0.88729	1.7415	1.01851	1.9990

Three Lines in Parallel

0.0	0.48050	0.9979	0.47955	0.9959	0.47861	0.9940
10.0	0.47724	0.9911	0.47321	0.9828	0.46934	0.9747
20.0	0.47433	0.9851	0.46800	0.9720	0.46239	0.9603
30.0	0.47192	0.9801	0.46446	0.9646	0.45873	0.9527
40.0	0.47016	0.9764	0.46302	0.9616	0.45911	0.9535
50.0	0.46916	0.9744	0.46399	0.9636	0.46395	0.9635
60.0	0.46904	0.9741	0.46765	0.9712	0.47359	0.9836
70.0	0.46989	0.9759	0.47419	0.9848	0.48820	1.0139
80.0	0.47182	0.9799	0.48377	1.0047	0.50786	1.0547
90.0	0.47489	0.9863	0.49649	1.0311	0.53255	1.1060
100.0	0.47917	0.9952	0.51242	1.0642	0.56220	1.1676

Figure 4.12 shows the variation in the Error Factor with change in length of the transmission line. The Error Factor is smaller for short lines than for longer lines. The variation in Error Factor with the change in the relative magnitude of λ_c from 0 to 20 % of the independent outage rate is shown in Figure 4.13 for a double ckt. line, 40 miles long. Neglecting the adverse weather effects results in relatively small error in the calculation of system failure rate if the data of that system suggest that the ratio λ_c/λ is 15 % or more.

4.3.3 Effect of Adverse Weather on System Average

Outage Duration.

The average outage duration for a two component redundant system without common mode and adverse weather effect is

$$r_{SL} = \frac{1}{\mu_1 + \mu_2} = \frac{r_1 r_2}{r_1 + r_2}$$

This equation is also valid when adverse weather is included and repair occurs during adverse weather at the same rate as in normal weather. When no repair occurs during adverse weather, the average outage duration is increased by a factor of S for those failures associated with adverse weather activity (5). When common mode events are included the average system outage duration is very

Figure 4.12-Variation in Error Factor With Line Length.

dependent on the model used. Using the model of Figure 2.3

$$r_{SL} = \frac{1.0}{\mu_1 + \mu_2 + \mu_C}$$

if $r_1 = r_2 = 7.5$ hours,

$$\lambda_{av1} = \lambda_{av2} = 3.410 \text{ f/yr/100 mile and } cav = 0.487$$

When $r_C = 10.0$ hours, $r_S = 2.7273$ hours

$r_C = 20.0$ hours, $r_S = 3.1579$ hours

The effect of r_C depends on its relative magnitude to r_1 and r_2 .

Using the model of Figure 2.4 in which the common cause phenomenon produces an isolated state

$$r_{SL} = \frac{\lambda_1 \lambda_2 r_1 r_2 + \lambda_C r_C}{\lambda_1 \lambda_2 (r_1 + r_2) + \lambda_C}$$

When $r_C = 10.0$ hours, $r_S = 9.9745$ hours.

$r_C = 20.0$ hours, $r_S = 19.3617$ hours.

Using the model of Figure 2.5 in which common cause outages are divided into two parts λ_{c1} and λ_{c2}

$$r_{SL} = \frac{\lambda_1 \lambda_2 (r_1 + r_2) + \lambda_{c1}}{\lambda_1 \lambda_2 (r_1 + r_2) + \lambda_C} \left(\frac{r_1 r_2}{r_1 + r_2} \right) + \frac{\lambda_{c2}}{\lambda_1 \lambda_2 (r_1 + r_2) + \lambda_C} (r_C)$$

When $r_C = 10.0$ hours, $r_S = 6.7530$ hours.

$r_C = 20.0$ hours, $r_S = 11.5470$ hours.

The results for the models A, B, C and D presented in this chapter can be compared to those from the corresponding models of Chapter 2. In the case of Model B, the bulk of system failures occur due to common cause events and the average system outage time is therefore very close to the

time required to repair a common mode failure. In the case of Model D, the system average outage duration is partially dominated by the common mode repair rate as the common mode outages have been divided into two parts. The Error Factor is calculated and shown in Tables 4.18 and 4.19 for Models A, B, and D. The results in Tables 4.18 and 4.19 show that the error in the predicted system average outage duration is higher in the case where most of the independent outages occur in adverse weather. Table 4.20 indicates the variation in system average outage duration with the variation of adverse weather independent outage rate in a two component redundant configuration considering Models A, B and D. The results in this table are based upon $r = 10.0$, $r_c = 15.0$ and 60 % common mode outages occur during adverse weather with no repair during adverse weather. Figure 4.14 shows the change in system average outage duration for the system of two and three components, with the inclusion of common mode outages considering Model A.

The average outage duration using Model C can be compared to the average outage duration for the model shown in Figure 4.11. It can be seen from Table 4.21 that after neglecting the adverse weather condition, the average outage duration is very close to the average outage duration of Model C which includes adverse weather considerations. Neglecting the adverse weather effects in this case

Table 4.18

Variation in System Average Outage Duration
With Common Mode Outage Rate
No Repair During Adverse Weather

Percentage	System average outage duration in hours					
λ_c/λ	0 % *	20 % *	30 % *	40 % *	50 % *	60 % *

MODEL A

0.0	3.7781	4.5988	5.0525	5.1997	5.2548	5.2781
2.5	4.4614	4.6411	4.8764	5.0353	5.1277	5.1812
5.0	4.5533	4.6499	4.8081	4.9459	5.0438	5.0192
7.5	4.5895	4.6536	4.7716	4.8895	4.9841	5.0536
10.0	4.6087	4.6553	4.7489	4.8507	4.9394	5.0093
12.5	4.6205	4.6566	4.7333	4.8224	4.9048	4.9733
15.0	4.6285	4.6572	4.7219	4.8007	4.8770	4.9433
17.5	4.6341	4.6575	4.7132	4.7835	4.8543	4.9179
20.0	4.6383	4.6577	4.7062	4.7696	4.8354	4.8962

MODEL B

0.0	5.0375	5.8577	6.3112	6.4586	6.5139	6.5375
2.5	13.3897	12.3566	10.5882	9.2724	8.4481	7.9340
7.5	14.5217	13.7792	12.2240	10.8032	9.7261	8.9707
7.5	14.9702	14.4026	13.1230	11.7655	10.6334	9.7707
10.0	15.2106	14.7524	13.6679	12.4263	11.3108	10.4068
12.5	15.3603	14.9763	14.0386	12.9081	11.8359	10.9246
15.0	15.4625	15.1317	14.3072	13.2749	12.2547	11.3542
17.5	15.5366	15.2460	14.5106	13.5635	12.5966	11.7165
20.0	15.5928	15.3335	14.6701	13.7964	12.8809	12.0261

MODEL D

0.0	5.0375	5.8577	6.3112	6.4586	6.5139	6.5375
2.5	10.1595	9.8380	9.1519	8.5021	8.0135	7.6691
5.0	10.4011	10.1505	9.5720	8.9541	8.4367	8.0409
7.5	10.5305	10.3258	9.8323	9.2645	8.7524	8.3363
10.0	10.6109	10.4379	10.0093	9.4505	8.9971	8.5768
12.5	10.6659	10.5156	10.1374	9.6625	9.1921	8.7762
15.0	10.7056	10.5727	10.2344	9.7978	9.3513	8.9443
17.5	10.7357	10.6141	10.3104	9.9069	9.4837	9.0879
20.0	10.7781	10.6787	10.4217	10.0722	9.6910	9.3203

(*) Percentage of independent outages occur in adverse weather.

Table 4.19

System Average Outage Duration And Error Factor
Excluding Adverse Weather
Repair During Adverse Weather

Percentage of Independent Outages occur in Adverse W.	Model	$r_1=10.0, r=7.5$	$r_2=20.0, r=7.5$	$r_3=30.0, r=7.5$
0.0	A	2.7273	1.0000	3.1579
	B	9.7535	0.9778	19.3590
	C	6.7477	0.9992	11.5439
20.0	A	2.7273	1.0000	3.1579
	B	9.5664	0.9591	18.8727
	D	6.6515	0.9849	11.2939
40.0	A	2.7273	1.0000	3.1579
	B	9.0622	0.9085	17.5620
	D	6.3982	0.9475	10.6334
60.0	A	2.7273	1.0000	3.1579
	B	8.4034	0.8425	15.8464
	D	6.0689	0.8987	9.7795
80.0	A	2.7273	1.0000	3.1579
	B	7.7296	0.7749	14.0974
	D	5.7329	0.8489	8.9059
100.0	A	2.7273	1.0000	3.1579
	B	7.1187	0.7137	12.5091
	D	5.4287	0.8039	8.1147

Table 4.20

System Average Outage Duration And Error Factor
Excluding Adverse Weather
No Repair During Adverse Weather

Percentage of Independent Outages occur in Adverse W.	Model	$r_1=10.0, r=10.0$	$r_2=20.0, r=10.0$	$r_3=30.0, r=10.0$
0.0	A	3.6079	1.3229	4.0415
	B	10.5868	1.0714	20.3617
	D	7.6975	1.1339	12.4886
20.0	A	3.6345	1.3326	4.0682
	B	10.4734	1.0500	19.7652
	C	7.5650	1.1202	12.2046
40.0	A	3.6988	1.3562	4.1325
	B	9.9465	0.9972	18.2934
	D	7.3346	1.0861	11.5029
60.0	A	3.7774	1.3850	4.2112
	B	9.2958	0.9319	16.4763
	D	7.0495	1.0439	10.6361
80.0	A	3.8534	1.4129	4.2873
	B	8.6622	0.8684	14.7077
	C	6.7714	1.0027	9.7916
100.0	A	3.9189	1.4396	4.3529
	B	8.1103	0.8131	13.1686
	D	6.5289	0.9668	9.0563

introduces an error of about 10% in the predicted system average outage duration. Therefore it can be concluded from the above comparison study that in some cases single state weather models can be used instead of using more complex two state weather models.

Table 4.21

Model C. Variation in Error Excluding Adverse Weather
No Repair During Adverse Weather

Percentage of Independent outages occur in Adverse W.	r=7.5, $r_c=10$, $r_c'=5.0$			r=7.5, $r_c=20.0$, $r_c'=5.0$		
	Sy. Av. Outage Duration (hours) (*)	Error Factor (**)		Sy. Av. Outage Duration (hours) (*)	Error Factor (**)	
0.0	4.0414	4.0363	1.0013	4.2262	4.2188	1.0017
10.0	4.0416	4.0363	1.0013	4.2302	4.2188	1.0027
20.0	4.0457	4.0363	1.0023	4.2430	4.2188	1.0057
30.0	4.0533	4.0363	1.0042	4.2628	4.2188	1.0104
40.0	4.0634	4.0363	1.0067	4.2874	4.2188	1.0162
50.0	4.0750	4.0363	1.0096	4.3146	4.2188	1.0227
60.0	4.0872	4.0363	1.0126	4.3427	4.2188	1.0293
70.0	4.0994	4.0363	1.0156	4.3703	4.2188	1.0359
80.0	4.1110	4.0363	1.0185	4.3965	4.2188	1.0421
90.0	4.1219	4.0363	1.0212	4.4208	4.2188	1.0479
100.0	4.1318	4.0363	1.0237	4.4431	4.2188	1.0532

(*) System av. outage duration including adverse weather effects.

(**) System av. outage duration excluding adverse weather effects.

4.4 Predominant Effects

The results shown clearly indicate that it may be possible to neglect the fluctuating environment effects with little error in the calculated indices in situations where common cause failures occur. It should be clearly understood however, that this may apply only in certain

cases and is not a blanket statement. From the experience and study of this chapter it is evident that under certain conditions, the existence of adverse weather can be omitted from the analysis. Some of these conditions are as follows.

1. Short transmission lines.
2. Low component failure rates.
3. Short adverse weather durations.
4. System composed of more than two redundant lines.

This phenomenon is shown in Figure 4.15 where a double circuit line is considered. The Error Factor in this case is obtained by dividing the system failure rate obtained including adverse weather by the value obtained neglecting it. In this case $\lambda = 0.5$ f/yr, $\lambda_c = 0.05$ f/yr, $r = 2.5$ hours, $N = 200$, $S = 1.5$ and 50 % of the average common mode failures are assumed to occur during adverse weather. The Error Factor can be compared to that shown in Figure 3.3 in which common mode outages are not included. Table 4.14 shows that neglecting adverse weather causes an error of about 70% in the predicted system failure rate in a two component configuration for the condition where most of the independent outages occur in adverse weather. This error reduces to less than 20% when less than 50% of the independent and common mode outages occur in adverse weather. Figure 4.15 also shows that a lower component

Error Factor = System outage rate with/without Adverse Weather

% of Lines Annual Failures in Adverse Weather

Figure 4.15: Error Factor For Multiple Line Configuration.

Figure 4.14 Variation in System Outage Time Considering Model A.

failure rate also results in a lower Error Factor. Figure 4.15 also shows that the variation in the Error Factor for two, three, four and five line redundant configurations as a function of the percentage of the failures occurring in adverse weather. The two and three line cases are the most important as complete redundancy is not normally provided in a four and five line configuration. The ordinate scales are reversed for the two line and the three, four and five line configurations.

Figure 4.16 shows that the error in this case relative to the difficulty associated with collecting and utilizing adverse weather data may be negligible. The statement that "it may be possible to neglect the fluctuating environment" includes both common mode and independent outages during adverse weather. The Ontario Hydro data appears to indicate that 60 % of the common mode outages occur in adverse weather. Figure 4.17 shows the results for a three component system with the percentage of common mode outages which occur in adverse weather varied from 0 to 100 %. The effect is minimal in these cases and indicates that the use of an overall annual common mode outage rate rather than two separate and quite distinct weather related rates may give acceptable results. This is an important point as noted in the introduction to this thesis. Calculation of adverse weather failure rates for both common mode and independent

ERROR FACTOR =

Av. outage rate considering adverse weather

Av. outage rate without considering adverse weather

Figure 4.16 Error Factor Due to Neglecting Adverse Weather.

Figure 4.17 Error Factor Due to Neglecting Adverse Weather For Variable λ_c in a Three Component System.

events requires a detailed knowledge of the environment for those situations in which failures did occur and also failure did not occur. Overall annual failure rates for both common mode and independent events are much easier to obtain than weather related rates.

Figure 4.18 shows the variation in Error Factor with respect to variation in common mode outage rate. The Error Factor increases as the relative magnitude of common mode outage rate decreases. Figure 4.18 indicates that adverse weather conditions can be neglected for those situations in which the ratio λ_c/λ is greater than 10.0%.

4.5 Conclusion

Several common cause outage models which include adverse weather effects for two and three transmission lines on the same transmission tower or on same right of way have been presented and examined in this chapter. A method of calculating system failure rate has been developed to permit relatively easy utilization of the models in distribution and transmission system reliability studies. The models proposed in this chapter are applicable to different circumstances. The selection of a particular model depends entirely on the data of the situation.

Figure 4.18 Error Factor Due to Neglecting Adverse Weather For Variable λ'_{c} .

Figure 4.19 Error Factor Due to Neglecting Adverse Weather For Variable λ_{cav} in a Three Component System.

The studies presented in this chapter clearly indicates that the fluctuating environment effects can be neglected for the situation where the common mode outage rate is higher than 10 % of the independent outage rate. In these cases neglecting the adverse weather phenomenon introduces a relatively small error in the predicted system reliability indices. The decision depends on the magnitude of λ_c which increases with line length. In the case of a long line configuration it may be necessary to incorporate the adverse weather effects in the analysis. The influence of the common cause outage phenomenon is considerably greater than the common environment phenomenon created by adverse weather during which independent failures occur. This is an important realization in that while it is extremely desirable to be able to collect all possible data regarding component failure and to utilize detailed models in subsequence analysis of alternate configurations, it may be necessary to make certain reductions in modelling complexity which retain the important and dominant aspects in order to keep the problem to manageable size. This is particularly true in composite or bulk transmission system reliability evaluation.

This chapter has clearly shown that fluctuating environmental effects become quite secondary in evaluating redundant configurations when common mode outage can occur.

This chapter focusses attention on the distinct difference between weather related independent component outages and actual common mode outage events. This distinction is an important factor in valid and useable data collection.

5. Evaluation of Maintenance and Temporary Outage Conditions

5.1 Introduction

The consideration of maintenance outages in transmission and distribution schemes is very important because these types of temporary outages can cause a considerable number of interruptions at the customer load point. The nature of maintenance outages is somewhat different from that of forced outages. It is essential to take out power system components and equipment for periodic inspection. Lines are not normally removed from service in redundant configurations when their removal would cause an interruption, excessive voltage drops or another outage due to overload. The occurrence of maintenance is therefore dependent on whether or not forced outages exist and on the load level. A maintenance outage can not therefore be described [17] as a statistically independent outage or one that is independent of the load level. The system operating experience indicates that one cause of double contingency outages is the occurrence of an outage at a time when another component has been taken out for maintenance.

Special attention has been given to maintenance outages in References [18,6,15]. In this chapter these concepts are extended by including common cause and adverse weather

effects. In actual practice, the frequency of common mode outages overlapping maintenance outages may be very low due to the condition imposed that maintenance is only performed during normal weather periods. A number of approximate equations are developed in this chapter which include common mode and adverse weather effects. Results obtained using these equations are compared to those obtained by a Markov approach.

A technique was developed in Reference [15] to include temporary outages in system reliability analysis together with adverse weather effects. It should be noted that the distinction between temporary and permanent outages is not based on the duration of the outages but on the type of disturbance. A temporary outage can be understood as the clearing of a fault by the operation of a circuit breaker without locking it out or by automatic reclosers or by repeater fuses. Such outages are generally of two to three minutes durations. A scheduled outage is a situation in which an outage results when a component is deliberately taken out of service at a selected time, usually for the purpose of construction, preventive maintenance, or repair. There are some additional assumptions needed to evaluate maintenance outages.

1. Maintenance up and down times are exponentially

distributed.

2. No maintenance is carried out if adverse weather is expected to occur before the maintenance can be completed.
3. The components are not taken out for maintenance if such action cause the remaining components in a redundant system to become overloaded.
4. No maintenance is carried out if there is some outage already existing in the related portion of the system.

5.2 Component Permanent Outages Overlapping Maintenance Outages.

Power systems are composed of groups of elements or components that act in series or parallel or both with each other to carry power from generation sources to load buses. Double circuit or triple circuit lines are often used to transmit bulk power using high voltages. These lines are constructed either on the same right of way or on the same transmission tower. When a line has been taken out for maintenance, another line may suffer a forced outage or outages causing interruption to the system load bus. The

probability of residing in the failed state is an important parameter but perhaps less important and secondary to other factors such as the system failure rate and the contribution to this failure rate due to maintenance related activities. Two approaches can again be applied to evaluate this mode of failure.

- (1) The Markov approach (Appendix 1).
- (2) The Approximate approach (Appendix 2).

The Markov approach utilizes an extension of the state space diagram on the normal weather side to include the maintenance outage states as maintenance outages are assumed to occur only in adverse weather.

Let λ_1'' = The maintenance outage rate of component 1.

r_1'' = The maintenance repair time of component 1.

A simple state space model including maintenance outages [19] and excluding common mode and adverse weather effects is shown in Figure 5.1. This model has only eight states and the probabilities of being in the various states are given by:

$$\begin{aligned}
 P_1 &= \frac{1}{A} , & P_2 &= \frac{\lambda_1}{A\mu_1} , & P_3 &= \frac{\lambda_2}{A\mu_2} , & P_4 &= \frac{\lambda_1''}{A\mu_1''} \\
 P_5 &= \frac{\lambda_2''}{A\mu_2''} , & P_6 &= \frac{\lambda_1\lambda_2}{A\mu_1\mu_2} , & P_7 &= \frac{\lambda_2\lambda_1''}{A(\mu_1'' + \mu_2) + \mu_1''} \\
 P_8 &= \frac{\lambda_1\lambda_2''}{A(\mu_1 + \mu_2'')\mu_2''}
 \end{aligned}$$

Figure 5.1 State Space Model For A Two Component System Including Maintenance Outages.

$$A = 1 + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_1^n}{\mu_1^n} + \frac{\lambda_2^n}{\mu_2^n} + \frac{\lambda_1 \lambda_2}{\mu_1 \mu_2} + \frac{\lambda_2 \lambda_1^n}{(\mu_1 + \mu_2^n) \mu_1^n} + \frac{\lambda_1 \lambda_2^n}{(\mu_1^n + \mu_2) \mu_2^n} = 1$$

and P_j = The probability of being in j th state.

The maintenance outage rate of the system can be determined as the summation of frequencies of encountering the states in which independent outage of a component overlap the maintenance outage of the other component. This is given by:

$$= P_7(\mu_1^n + \mu_2) + P_8(\mu_1 + \mu_2^n) \quad (5.1)$$

$$\lambda_{ml}^n = \lambda_2 \lambda_1^n r_1^n + \lambda_1 \lambda_2^n r_2^n$$

The expected down time for a system of two parallel components due to maintenance outages is given by:

$$r_{ml}^n = \frac{\lambda_1^n \lambda_2 r_1^n}{\lambda_{ml}^n} \left\{ \frac{r_2 r_1^n}{r_2 + r_1^n} \right\} + \frac{\lambda_2^n \lambda_1 r_2^n}{\lambda_{ml}^n} \left\{ \frac{r_1 r_2^n}{r_1 + r_2^n} \right\} \quad (5.2)$$

The first term of the above expression represents the system outage time due to the independent outage of component 2 overlapping the maintenance outage of component 1. The reasoning for the second term is the same with components 1 and 2 interchanged.

The system indices obtained using Equations 5.1 and 5.2 have been compared with those predicted by the Markov

approach. The system failure rate and average outage duration with $\lambda=1.364$, $r=10.0$, $\lambda''=4.0$, $r''=5.0$ are as follows:

	Approximate	Markov
System Maintenance Outage Duration(hrs.)	3.33333	3.3333
System Maintenance Outage rate (f/yr)	0.00623	0.00618

Table 5.1 shows the comparison between the approximate equations and the Markov approach when adverse weather considerations are included. It can be seen from Table 5.1 that the results obtained using Equations 5.1 and 5.2 are quite close to those obtained by the Markov approach.

Table 5.1

Reliability Indices Due To Component Maintenance
For a Two Component Parallel System
(Normal Weather Only)

$\lambda = 1.364$ f/yr , $r = 10.0$ hours
 $\lambda'' = 4.0$ m/yr , $r'' = 5.0$ hours
 $S = 1.5$ hours , $N = 200.0$ hours

Percentage of Independent Outages occur in Adverse W.	Using Equations		Markov Approach	
	Outage Rate	Outage Time	Outage Rate	Outage Time
0.0	0.00628	3.3333	0.00618	3.3333
10.0	0.00565	3.3333	0.00556	3.3333
20.0	0.00502	3.3333	0.00494	3.3333
30.0	0.00439	3.3333	0.00433	3.3333
40.0	0.00376	3.3333	0.00371	3.3333
50.0	0.00314	3.3333	0.00309	3.3333
60.0	0.00251	3.3333	0.00249	3.3333
70.0	0.00188	3.3333	0.00185	3.3333
80.0	0.00126	3.3333	0.00124	3.3333
90.0	0.00063	3.3333	0.00062	3.3333
100.0	0.00000	3.3333	0.00000	3.3333

Figure 5.2 State Space Diagram For A Two Component Redundant System Including Maintenance And Adverse Weather Effects.

The results in Table 5.1 show that the maintenance related outage effect decreases as the adverse weather failure rate increases and the normal weather failure rate decreases. The state space diagram for the system of three components is shown in Figure 5.3 including maintenance outages and adverse weather phenomena. The outage rate resulting from the maintenance period of a component being overlapped by two independent outages is given by:

$$\lambda_{ml}'' = \lambda_1''(\lambda_2 r_1'') \lambda_3 \left(\frac{r_1'' r_2}{r_1'' + r_2} \right) + \lambda_1''(\lambda_3 r_1'') \left(\lambda_2 \frac{r_1'' r_3}{r_1'' + r_3} \right) \quad (5.3)$$

Where:

$\lambda_2'' r_1'$ = The probability that component 2 suffers a forced outage during the maintenance period of component 1.

$\lambda_3 \frac{r_1'' r_2}{r_1'' + r_2}$ = The probability that component 3 suffers a forced outage during the maintenance of component 1 and repair of component 2. The second term in the above equation follows the same reasoning but with components 2 and 3 interchanged.

The expected outage duration associated with this outage rate is:

$$r_{ml}'' = \frac{r_1'' r_2 r_3}{r_1'' r_2 + r_1'' r_3 + r_2 r_3} \quad (5.4)$$

The contribution to the system failure rate due to the maintenance of a component and two forced outages overlapping it for a three component parallel system is

Figure 5.3 State Space Diagram For A Three Component Redundant System Including Maintenance States.

therefore given by:

$$\begin{aligned} \lambda_{ml}'' = & \lambda_1'' \lambda_2 r_1'' \lambda_3 \left(\frac{r_1'' r_2}{r_1'' + r_2} \right) + \lambda_1'' \lambda_3 r_1'' \lambda_2 \left(\frac{r_1'' r_3}{r_1'' + r_3} \right) + \\ & \lambda_2'' \lambda_1 r_2'' \lambda_3 \left(\frac{r_2'' r_1}{r_2'' + r_1} \right) + \lambda_2'' \lambda_3 r_2'' \lambda_1 \left(\frac{r_2'' r_3}{r_2'' + r_3} \right) + \\ & \lambda_3'' \lambda_1 r_3'' \lambda_2 \left(\frac{r_3'' r_1}{r_3'' + r_1} \right) + \lambda_3'' \lambda_2 r_3'' \lambda_1 \left(\frac{r_3'' r_2}{r_3'' + r_2} \right) \end{aligned}$$

The expected outage duration due to maintenance outage is equal to the weighted outage duration of each of the terms in the above expression. If λ_i represents the frequency of outage of one of the above terms and r_i its associated expected outage duration of λ_{ml}'' is given by:

$$r_{ml}'' = \sum_{i=1}^6 \frac{\lambda_i r_i}{\lambda_{ml}''}$$

The above equations were given in Reference [15]. These equations are further extended in this chapter to include common mode and adverse weather effects.

The state space diagrams of Figures 5.4 and 5.5 are basically identical to Models A and B of Chapter 4 after including maintenance outages. In the model of Figure 5.4, common mode outages put both lines in the down state and repair is followed with the two separate repair processes. The model of Figure 5.5 shows that the common mode outages put the system in a separate failure state and a common mode

Figure 5.4 State Space Diagram Including Maintenance And Common Mode Outages Identical To Model A For Two Identical Lines.

Figure 5.5 State Space Diagram Including Maintenance And Common Mode Outages Identical To Model B For Two Identical Lines.

repair process is provided to bring the system back in the UP state. The average maintenance outage rate formula for the models of Figures 5.4 and 5.5 is as follows:

$$\begin{aligned}\lambda_{ml}'' &= \lambda_1''\lambda_2r_1'' + \lambda_2''\lambda_1r_2'' + \lambda_1''\lambda_c r_1'' + \lambda_2''\lambda_c r_2'' \\ &= \lambda_1''\lambda_2r_1'' + \lambda_2''\lambda_1r_2'' + \lambda_c (\lambda_1''r_1'' + \lambda_2''r_2'')\end{aligned}$$

The models in Figures 5.4 and 5.5 contain 12 and 15 states respectively. It can be seen from the model of Figure 5.3 that the state space diagram grows very rapidly as the number of system components increases. It therefore becomes difficult to apply the complete Markov approach to large systems because of computer storage and rounding error limitations. The use of approximate equations results in a small error in the predicted indices but permits a relatively easy utilization of reliability models in transmission system reliability studies.

The expected value of maintenance down time for the model of Figure 5.4 due to the maintenance outage is given by:

$$r_{ml}'' = \frac{\lambda_1''r_1''(\lambda_1 + \lambda_2)}{\lambda_{ml}''} \left(\frac{r_2r_1''r_c}{r_2r_1'' + r_2r_c + r_1''r_c} \right) + \frac{\lambda_2''r_2''(\lambda_1 + \lambda_2)}{\lambda_{ml}''} \left(\frac{r_1r_2''r_c}{r_1r_2'' + r_1r_c + r_2''r_c} \right)$$

The first factor of the first term is the fraction of system outages involving component maintenance outages in which component 2 failed or common mode outage occurs while component 1 was out for maintenance. The second factor of

this term gives the expected value of the system down time due to the outages occurring while component 1 is out for maintenance. The second term follows the same argument with the components 1 and 2 interchanged.

The expected down time in the case of the model in Figure 5.5 due to the maintenance outage is as follows:

$$r_{ml}'' = \frac{\lambda_1'' \lambda_2 r_1''}{\lambda_{ml}''} \left(\frac{r_2 \cdot r_1''}{r_2 + r_1''} \right) + \frac{\lambda_2'' \lambda_1 r_2''}{\lambda_{ml}''} \left(\frac{r_1 \cdot r_2''}{r_1 + r_2''} \right) + \frac{\lambda_c (\lambda_1'' r_1'' + \lambda_2'' r_2'')}{\lambda_{ml}''} (r_c) \quad (5.9)$$

The first and second terms of the above expression follow the same reasoning as in Equation 5.2. The third term in the above expression gives the expected system down time due to the occurrence of the common mode outage during the maintenance time of either component. These equations can be extended for a three line redundant system. It has been shown in the previous chapter that the adverse weather effects can be quite secondary compared to common mode outage effects in redundant facilities of more than two components. It has therefore not been considered of great benefit to develop state space models including adverse weather effects for a system having more than two components in parallel. This conclusion may change in the future as better data collection procedures should provide additional transition rate data for multicircuit configurations.

The results predicted using Equations 5.7, 5.8 and 5.9 have been compared with those obtained by a Markov approach. The system average maintenance outage rate and maintenance outage duration neglecting adverse weather are as follows.

	Maintenance outage rate (f/yr)		Maintenance outage Time (hours)	
	Exact	Markov	Exact	Markov
Model of Figure 5.4	0.0071	0.0072	2.7272	2.7272
Model of Figure 5.5	0.0071	0.0072	4.8551	4.8551

The system average outage rate formula is the same for both models in Figures 5.4 and 5.5. Table 5.2 shows the system maintenance outage rates together with the percentage variation of independent outages occurring in adverse weather periods. The Error Factor associated with the system average maintenance outage rate is obtained by dividing the calculated system failure rate obtained including adverse weather by the value obtained neglecting it.

Table 5.2 clearly shows the effect of neglecting adverse weather. The Error Factor increases with the relative decrease in normal weather independent outages. Table 5.3 shows the variations of system average maintenance outage time and Error Factors together with the percentage variation of adverse weather independent outages for the

models of Figures 5.4 and 5.5. It is evident from the Tables 5.2 and 5.3 that the results obtained using approximate equations are very close to those obtained by the Markov approach in the cases of both models.

Table 5.2

System Outage Rate Due To Component Maintenance
For a Two Component Parallel System
(Two Weather States)

Percentage of Independent Outages occur In Adverse W.	System Average Maintenance Outage Rate (f/yr)		Error Factor
	Using Markov	Using Equations	
0.0	0.00654	0.00665	0.9211
10.0	0.00593	0.00602	0.8352
20.0	0.00531	0.00539	0.7479
30.0	0.00469	0.00477	0.6606
40.0	0.00408	0.00414	0.5746
50.0	0.00346	0.00351	0.4873
60.0	0.00284	0.00289	0.4000
70.0	0.00223	0.00225	0.3141
80.0	0.00161	0.00163	0.2267
90.0	0.00099	0.00100	0.1394
100.0	0.00037	0.00038	0.0521

The Error Factors shown in Tables 5.2 and 5.3 indicate that adverse weather considerations have a dominant influence on the system indices. A comparison study of Table 5.1 where system maintenance outage rate is shown neglecting common mode outages, with Table 5.2, clearly shows that the common mode outages overlapping maintenance outages have a relatively small effect on the system maintenance outage rate. The system maintenance outage duration is also affected with the percentage variation of

Table 5.3

System Average Outage Time Due To Component Maintenance
Outages For a Two Component Parallel System
(Two Weather States)

Percentage of Independent Outages occur In Adverse W.	System Average Outage Time (hours) Using Equations	System Average Outage Time (hours) Using Markov	Error Factor
--	--	---	-----------------

Model of Figure 5.4

0.0	2.7272	2.7272	1.0000
10.0	2.7272	2.7272	1.0000
20.0	2.7272	2.7272	1.0000
30.0	2.7272	2.7272	1.0000
40.0	2.7272	2.7272	1.0000
50.0	2.7272	2.7272	1.0000
60.0	2.7272	2.7272	1.0000
70.0	2.7272	2.7272	1.0000
80.0	2.7272	2.7272	1.0000
90.0	2.7272	2.7272	1.0000
100.0	2.7272	2.7272	1.0000

Model of Figure 5.5

0.0	3.9937	3.9937	0.8224
10.0	4.0625	4.0625	0.8367
20.0	4.1473	4.1473	0.8542
30.0	4.2544	4.2544	0.8763
40.0	4.3939	4.3939	0.9050
50.0	4.5833	4.5833	0.9440
60.0	4.8551	4.8551	1.0000
70.0	5.2778	5.2778	1.0871
80.0	5.0256	5.0256	1.2411
90.0	7.7083	7.7083	1.5877
100.0	15.0000	15.0000	3.0895

normal weather independent outages as well as with the selection of state space models. The system average maintenance outage duration has completely different values in the two models of Figures 5.4 and 5.5. Table 5.3 suggests that adverse weather consideration could be omitted in the case of both these models if 60% or less permanent outages occur during adverse weather. This is a very important realization in the case of maintenance outage modelling which leads to reduction of the state space models.

5.3 Component Temporary Outages Overlapping

Component Maintenance Outages.

The contribution to the system failure rate due to the maintenance outages also suggests the possible importance of temporary interruptions in transmission and distribution schemes. Temporary outages are quite liable to occur during component maintenance periods due to increased activity in the component vicinity. The mathematical formulation of temporary outages proceeds as follows:

Let λ_{1T} = The temporary outage of component 1.

The state space model including maintenance and temporary outages is shown in Figure 5.6. The contribution to the system temporary outage rate due to the temporary

outage of component 1 overlapping a maintenance outage period of component 2, is given by:

$$\lambda_{TL} = \lambda_2^n \lambda_{1T} r_2^n$$

where: $\lambda_{1T} r_2^n$ = The probability that component 1 suffers a temporary outage during the maintenance period of component 2.

For the system of two parallel components:

$$\lambda_{TL} = \lambda_1^n \lambda_{2T} r_1^n + \lambda_2^n \lambda_{1T} r_2^n$$

Similarly for a three component parallel system:

$$\begin{aligned} \lambda_{TL} = & \lambda_1^n \lambda_2^n \lambda_{3T} \frac{r_1^n r_2^n}{r_1^n + r_2^n} + \lambda_1^n \lambda_3^n \lambda_{2T} \frac{r_1^n r_3^n}{r_1^n + r_3^n} \\ & + \lambda_2^n \lambda_1^n \lambda_{3T} \frac{r_2^n r_1^n}{r_2^n + r_1^n} + \lambda_2^n \lambda_3^n \lambda_{1T} \frac{r_2^n r_3^n}{r_2^n + r_3^n} \\ & + \lambda_3^n \lambda_1^n \lambda_{2T} \frac{r_3^n r_1^n}{r_3^n + r_1^n} + \lambda_3^n \lambda_2^n \lambda_{1T} \frac{r_3^n r_2^n}{r_3^n + r_2^n} \end{aligned}$$

If the temporary outages are separated into normal and adverse weather failures, then the λ_T values in the above equations represent the normal weather temporary outage rate of the components. The above equations are based on the assumption that the component temporary outages occur only when no independent permanent outages are expected to occur. Figure 5.7 shows the variation of system average temporary outage rate overlapping maintenance outages together with the percentage variation of adverse weather permanent annual outages.

Figure 5.6 State Space Diagram Including Maintenance And Temporary Outages.

Figure 5.7 System Temporary Outage Overlapping Maintenance Outage Rate Due to Component Temporary Outages.

5.4 Conclusion

This chapter briefly shows the contribution of temporary outages to the system reliability indices. The consideration of common mode outages overlapping maintenance outages has a relatively small contribution to the system reliability indices. This effect is very small in the case of redundant configurations of more than two components. Therefore, consideration of these types of outages can be omitted from the analysis. It should also be noted that the magnitude of maintenance outage rate is small compared to the system permanent outage rate. The existence and therefore inclusion of common mode outages in the analysis, will result in the system overall outage rate being much less effected by neglecting maintenance outages. The magnitude of the normal weather independent component outage rate produces a significant variation in the system maintenance outage rate. The recognition of the adverse and normal weather phenomenon is therefore important and must be included in the evaluation of system maintenance outage indices.

6. CONCLUSIONS

System reliability predictions in redundant transmission and distribution configurations are normally done assuming that component failure are independent events. A comparison of actual system performance and the calculated indices often indicate that the predictions are quite optimistic and therefore the models are not physically compatible with the actual situation. The development of the two state weather model was an attempt to recognize the variable stress level imposed upon outdoor components. It assumed dependence in regard to the environment in which the components exist and therefore dependence in regard to the stress related failure rates but also assumed independence between actual component failure events. This point is not clearly understood in many of the discussions taking place in industry and research organizations. This has become more evident with the increased attention paid to the common mode outage events. These events can physically occur in both normal and non-normal weather but multiple failures during non-normal weather are not automatically common mode or common cause events.

Data collection in transmission and distribution systems is presently receiving considerable attention and hierarchies of desirable data are being established. It

therefore becomes important to determine the relative contribution to the predicted indices of the many factors that are obviously present in the analysis. It is uneconomical and possibly impossible to include all these factors. An early and clear example of this is the use of a two level discrete weather model for what is obviously a continuous distribution of component stress and therefore failure rate. Normal and adverse weather failure rates are extremely difficult to obtain and very few utilities can provide this data. This situation may change with the increased attention being devoted to data collection. It is important, however, to focus attention on the most significant data requirements and to ensure that maximum effort is made to obtain this information.

This thesis has clearly shown that when common mode outages can occur then the fluctuating weather related effects may become quite secondary in evaluating redundant configurations. This is particularly true in the case of more than two line redundant configurations where when common mode outages can occur, the weather related effects are ~~almost~~ completely negligible and therefore, a single state weather model can be used to evaluate the system reliability indices with little error.

This thesis focusses attention on the distinct

difference between weather related independent component outages and actual common mode outage events. This distinction is an important factor in valid and useable data collection.

Approximate equations have been developed and presented for calculating the system failure rate and average outage time in two and three line redundant configurations including both adverse weather and common mode considerations. The equations formulated in this thesis to include weather considerations model two distinct situations for component repair i.e.

- (1) Repair occurs during adverse weather periods.
- (2) Repair does not occur during adverse weather periods.

The results obtained using the approximate equations have been compared with those predicted by a theoretically accurate Markov approach for simple configurations of two and three components in parallel. It has been shown that the approximate equations give results which are very close to those obtained by the Markov approach. The interaction of component and weather parameters on the load point reliability has been investigated for various percentages of component failures occurring during the adverse weather periods. These equations extend those presently existing in the published literature.

When the normal and adverse weather failure aspect is not considered, component maintenance outages make a significant contribution to the calculated system reliability indices. The risk associated with component permanent and temporary outages overlapping the component maintenance periods, however, decreases as more and more component failures occur during adverse weather periods. The results obtained using approximate equations are found to be very close to those obtained using a Markov approach. The studies conducted also show that the common mode outages overlapping maintenance outages can have a very small contribution to the calculated system reliability indices.

The effect of common mode outages in parallel redundant facilities has been illustrated in a number of publications and has been extended in this thesis. There are a number of possible models which can be used to include adverse weather and common mode effects in reliability evaluation and data is needed not only for use in the model but to actually select and define the model for a particular application.

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APPENDIX 1
THE MARKOV APPROACH

A sequence of events where the outcome depends on an element of chance is called a stochastic process. The Markovian process is a special class of stochastic process in which future states of the process are dependent only upon the immediate past. The reliability problem normally deals with systems that are discrete in space and continuous in time. The use of Markovian models in power system reliability evaluation was initially proposed in a 1964 IEEE Publication [24] and application of it to the transmission system reliability problem was first illustrated in Reference [12]. Since then a great deal of work has been done on the application of Markov processes in this area.

Consider a simple case of a single repairable component for which the failure and repair rates are characterized by exponential distribution. The state space model describing this simple case is shown in Figure A-1.

Figure A.1 Two State Model of A Component.

Define:

$P_0(t)$ = Probability of the component being UP at time t .

$P_1(t)$ = Probability of the component being DOWN at time t .

Consider an incremental time interval dt and assume that the probability of occurrence of two events in this small interval of time is negligible. The probability that the component is in the state 0 at time $t+dt$ is derived from the probability that it was in state 0 at time t and did not transit to state 1 in time dt or that it was in state 1 at time t and transferred to state 0 in time dt . then

$$P_0(t+dt) = P_0(t)(1 - \lambda dt) + P_1(t)\mu dt$$

Similarly
$$P_1(t+dt) = P_0(t)\lambda dt + P_1(t)(1 - \mu dt)$$

These expressions can be placed into the following differential equation form.

$$P'_0(t) = -\lambda P_0(t) + \mu P_1(t)$$

$$P'_1(t) = \lambda P_0(t) - \mu P_1(t)$$

The matrix form,

$$\begin{bmatrix} P'_0(t) \\ P'_1(t) \end{bmatrix} = \begin{bmatrix} -\lambda & \mu \\ \lambda & -\mu \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \end{bmatrix}$$

$$\text{or } [P'(t)] = [A][P]$$

Where A is called the transitional matrix. The limiting state probabilities or the so called steady state probabilities can be obtained by setting the differential matrix equal to zero. The system equations then become:

$$-\lambda P_0 + \mu P_1 = 0$$

$$\lambda P_0 - \mu P_1 = 0$$

$$P_0 + P_1 = 1$$

Solving these equations,...

$$P_0 = \frac{\mu}{\lambda + \mu} \quad \text{and} \quad P_1 = \frac{\lambda}{\lambda + \mu}$$

Where: P_0 represents the availability of the component
and P_1 represents the unavailability of the component.

The frequency of the occurrence of a state is equal to the product of the steady state probability of existence of the state and the rate of departure from that state. Using this principle, the probabilities, frequencies and durations for a two component system with the state space diagram shown in Figure A-2 are given in Table A-1.

Figure A-2 Two Component State Space Diagram.

Table A-1
Frequency And Duration Of The States
For A Two Component System

STATE	COMPONENT#1	COMPONENT#2	DURATION	PROBABILITY	FREQUENCY
1	UP	Down	$1/(\lambda_1 + \lambda_2)$	$\mu_1 \mu_2 / D$	$\mu_1 \mu_2 (\lambda_1 + \lambda_2) / D$
2	DOWN	UP	$1/(\mu_1 + \lambda_2)$	$\lambda_1 \mu_2 / D$	$\lambda_1 \mu_2 (\mu_1 + \lambda_2) / D$
3	UP	DOWN	$1/(\lambda_1 + \mu_2)$	$\mu_1 \lambda_2 / D$	$\mu_1 \lambda_2 (\lambda_1 + \mu_2) / D$
4	DOWN	DOWN	$1/(\mu_1 + \mu_2)$	$\lambda_1 \lambda_2 / D$	$\lambda_1 \lambda_2 (\mu_1 + \mu_2) / D$

Where $D = (\lambda_1 + \mu_2)(\lambda_2 + \mu_1)$

For the system of two components in parallel with full redundancy, state 4 is the only failed state and the frequency of occurrence of this state is given by:

$$f_4 = \frac{\lambda_1 \lambda_2 (\mu_1 + \mu_2)}{(\lambda_1 + \mu_1)(\lambda_1 + \mu_2)}$$

The average duration associated with this state is given by:

$$\begin{aligned} & \frac{\text{Probability of the state}}{\text{Frequency of the state}} \\ & = \frac{P_4}{P_4(\mu_1 + \mu_2)} = \frac{r_1 r_2}{r_1 + r_2} \end{aligned}$$

The average duration associated with a cumulative group of states is given by:

$$\frac{\text{Cumulative probability of the states}}{\text{Cumulative frequency of the states}}$$

The term cumulative probability is defined as the summation of all state probabilities of that cumulative group. The term cumulative frequency of a group of states is equal to

the summation of the individual frequencies reduced by the sum of the frequency of encounter from each other.

The mean up time of the system can also be obtained by using discrete Markov chain concepts [18]. The stochastic transitional probability matrix for a two component system is given by:

$$[P] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1-(\lambda_1+\lambda_2) & \lambda_1 & \lambda_2 & 0 \\ \mu_1 & 1-(\lambda_2+\mu_1) & 0 & \lambda_2 \\ \mu_2 & 0 & 1-(\lambda_1+\mu_2) & \lambda_1 \\ 0 & \mu_2 & \mu_1 & 1-(\mu_1+\mu_2) \end{bmatrix} \end{matrix}$$

State 4 is designated as an absorbing state and a new truncated Matrix Q is obtained by eliminating the absorbing state

$$Q = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1-(\lambda_1+\lambda_2) & \lambda_1 & \lambda_2 \\ \mu_1 & 1-(\lambda_2+\mu_1) & 0 \\ \mu_2 & 0 & 1-(\lambda_1+\mu_2) \end{bmatrix} \end{matrix}$$

Let I = Identity matrix

$$[N] = [I-Q]^{-1} = \begin{bmatrix} \lambda_1+\lambda_2 & -\lambda_1 & -\lambda_2 \\ -\mu_1 & \lambda_2+\mu_1 & 0 \\ -\mu_2 & 0 & \lambda_1+\mu_2 \end{bmatrix}^{-1}$$

Matrix [N] is Fundamental matrix, where n_j is the time spent by the process in state S_j before being absorbed.

Starting in state 1, the MTTF (The time before entering the

absorbed state) of the process is $M_{1,4}$

$$M_{1,4} = \sum_{i=1}^4 N(1,i)$$

$$\lambda_{SL} = 1/M_{1,4}$$

Where λ_{SL} is the system failure rate.

APPENDIX 2

ASSUMPTIONS IN RELIABILITY CALCULATIONS

Some of the assumptions which were made in determining the reliability calculation methods of the thesis have been mentioned in preceding sections. These assumptions are listed as follows for ease of reference:

1. Times to failure and repair times are exponentially distributed during both normal and adverse weather periods.
2. The durations of periods of normal and adverse weather are exponentially distributed.
3. Repair times are typically very short compared with times to failure and times between storms.
4. Common mode down times are exponentially distributed.
5. Maintenance down times are exponentially distributed.
6. Maintenance outages occur at random during normal weather periods except that components are not

taken out for maintenance if the maintenance could not be completed before a storm struck.

7. Storms are very short in duration compared with times to failure for components. Storms are also short compared with typical repair times.

APPENDIX 3

OUTAGE RATE AND AVERAGE OUTAGE DURATION FOR TWO AND THREE OVERLAPPING OUTAGES.

The formulation of approximate equations for the outage rate and average outage duration follows the same basic procedure as described in Chapter 4 of this thesis. These equations were given in Reference [5] for two different situations of repair i.e.

- (i) When repair occurs during adverse weather.
- (ii) When repair does not occur during adverse weather.

(i) No Repair During Adverse Weather

The development of average outage rate formula for the configuration of two components in parallel involves four different phases.

1. The initial failure occurs during normal weather and the second failure also occurs during normal weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(\text{Failure rate}) = \frac{N}{N+S} \{ \lambda_1 (1-r_1/N) (\lambda_2 r_1) + \lambda_2 (1-r_2/N) (\lambda_1 r_2) \}$$

Where:

$N/(N+S)$ = The long term fraction of time that weather is normal.

$(1-r_1/N)$ = The probability that adverse weather does

not occur while component 1 is being repaired.

$\lambda_2 r_1 = 1 - e^{-\lambda_2 r_1}$ = The probability that component 2 fails during the repair of component 1.

Since r_1 and r_2 are very less compared with N ,

$$(\text{Failure rate})_1 = \frac{N}{N+S} \{ \lambda_1 \lambda_2 (r_1 + r_2) \}$$

2. The initial failure occurs during normal weather and the second failure occurs during adverse weather.

$$(\text{Failure rate})_2 = \frac{N}{N+S} \{ \lambda_1 (r_1/N) \lambda_2 S + \lambda_2 (r_2/N) \lambda_1 S \}$$

Where:

$r_1/N = 1 - e^{-r_1/N}$ = The probability that repair of component 1 is not completed within the normal weather.

$\lambda_2 S = 1 - e^{-\lambda_2 S}$ = The probability that component 2 fails during the adverse weather. The remaining terms in this mode of failure follows the same reasoning but with component 1 and 2 interchanged.

3. The initial failure occurs during adverse weather and the second failure also occurs during adverse weather.

$$(\text{Failure rate})_3 = \frac{N}{N+S} \{ \lambda_1 (\lambda_2 S) + \lambda_2 (S \lambda_1) \}$$

Where:

$S/(N+S)$ = The long term fraction of time that weather is adverse.

$S\lambda_2' = 1 - e^{-S\lambda_2'}$ = The probability that component 2 fails during adverse weather period. The remaining terms in the above expression follow the same argument but with component 1 and 2 interchanged.

4. The initial failure occurs during adverse weather and second failure occurs in normal weather.

$$(\text{Failure rate})_4 = \frac{S}{N+S} \{ \lambda_1 (1 - S\lambda_2') (\lambda_2 r_1) + \lambda_2' (1 - S\lambda_1') (\lambda_1 r_2) \}$$

Where:

$1 - S\lambda_2' = e^{-\lambda_2 S}$ = The probability that component 2 does not fail during adverse weather.

$\lambda_2 r_1 = 1 - e^{-\lambda_2 r_1}$ = The probability that component 2 fails during the repair of component 1. The remaining terms of the above expression follow the same reasoning but with component 1 and 2 interchanged.

Thus the overall outage rate for a system of two components in parallel is given by:

$$\lambda_{SL} = \frac{N}{N+S} \{ \lambda_1 \lambda_2 (r_1 + r_2) + \frac{S}{N} (\lambda_1 \lambda_2' r_1 + \lambda_2 \lambda_1' r_2) + \frac{S}{N} (\lambda_1' \lambda_2 r_1 + \lambda_2' \lambda_1 r_2) + \frac{2S^2}{N} \lambda_1' \lambda_2' \}$$

Under the condition of no repair during adverse weather every failure which occurs in adverse weather has to wait for repair for a period equal to the average duration of

adverse weather period. If the contribution to the failure rate due to normal weather failures (Modes (1) and (4)) and adverse weather failures (Modes (2) and (3)) are represented by λ_{en} and λ_{es} respectively, then the average outage duration is equal to the sum of the weighted durations associated with each of these outage rate values i.e.

$$r_{SL} = \frac{\lambda_{en}}{\lambda_{en} + \lambda_{es}} \left\{ \frac{r_1 \cdot r_2}{r_1 + r_2} \right\} + \frac{\lambda_{es}}{\lambda_{en} + \lambda_{es}} \left\{ \frac{r_1 \cdot r_2}{r_1 + r_2} + S \right\}$$

(ii) Repair During Adverse Weather

The formation of equations for two overlapping outages in a parallel configuration is basically the same as that presented in previous case except mode 2 and Mode 4

1. Mode 2: The contribution to the system failure rate due to the third mode of system failure is given by:

$$(\text{Failure rate})_2 = \frac{N}{N+S} \left\{ \lambda_1 (r_1/N) \lambda_2^1 \left(\frac{Sr_1}{S+r_1} \right) + \lambda_2 (r_2/N) \left(\lambda_1^1 \frac{Sr_2}{S+r_2} \right) \right\}$$

where:

$\frac{r_1}{N} = 1 - e^{-r_1/N}$ = Probability that weather becomes adverse during the repair of component 1.

$\lambda_2^1 \frac{Sr_1}{S+r_1}$ = The probability that component 2 fails during the overlapping periods of adverse weather and the repair of component 1. The similar approach is used to obtain the second term of the above expression with component 1 and 2 interchanged.

2. Mode 4: The contribution to the system failure rate due to the fourth mode of system failure is given by:

$$(\text{Failure rate})_4 = \frac{S}{N+S} \left\{ \lambda_1^1 (\lambda_2^1 \frac{Sr_1}{S+r_1}) + \lambda_2^1 (\lambda_1^1 \frac{Sr_2}{S+r_2}) \right\}$$

Where:

$\lambda_2^1 \frac{Sr_1}{S+r_1}$ = the probability that the component 2 fails during the overlapping period of adverse weather and repair of component 1. The similar reasoning is applicable to the second term in the above expression with component 1 and 2 interchanged

The expected down time as a result of two overlapping outages with the condition that repair available during adverse weather is given by:

$$r_{SL} = \frac{r_1 \cdot r_2}{r_1 + r_2}$$

The overall annual outage rates resulting from three overlapping outages is given by the summation of appropriate terms of Table A-2.

$$\lambda_{SL} = \sum_{i=1}^8 (\text{Failure Rate})_i$$

The first four modes of system failure shown in Table A-2 involve the system failure in normal weather and last four modes involve the system failure in adverse weather.

Table A.2
Outage Rate Associated With Different Modes of System Failures
of A Three Component Redundant System.

Failure Modes				Contribution to the System Failure Rate	
No.	1st	2nd	3rd	No Repair During Adverse Weather	Repair During Adverse Weather
1	N	N	N	$\frac{N}{N+S} [\lambda_1(\lambda_2 r_1)(\lambda_3 R_1) + \lambda_1(\lambda_3 r_1)(\lambda_2 R_2)$ + Similar terms for components 2 & 3]	$\frac{N}{N+S} [\lambda_1(\lambda_2 r_1)(\lambda_3 R_1) + \lambda_1(\lambda_3 r_1)(\lambda_2 R_2)$ + Similar terms for components 2 & 3]
2	N	A	N	$\frac{N}{N+S} [\lambda_1(\frac{r_1}{N})(\lambda_2^2 S)(\lambda_3 R_1) + \lambda_1(\frac{r_1}{N})(\lambda_2^2 S)(\lambda_2 R_2)$ + Similar terms for components 2 & 3]	$\frac{N}{N+S} [\lambda_1(\frac{r_1}{N})(\lambda_2^2 R_3)(\lambda_3 R_5) + \lambda_1(\frac{r_1}{N})(\lambda_2^2 R_3)(\lambda_2 R_6)$ + Similar terms for components 2 & 3]
3	A	N	N	$\frac{S}{N+S} [\lambda_1^2(\lambda_2 r_1)(\lambda_3 R_1) + \lambda_1^2(\lambda_3 r_1)(\lambda_2 R_2)$ + Similar terms for components 2 & 3]	$\frac{S}{N+S} [\lambda_1^2(\lambda_2 R_7)(\lambda_3 R_5) + \lambda_1^2(\lambda_3 R_7)(\lambda_2 R_6)$ + Similar terms for components 2 & 3]
4	A	A	N	$\frac{S}{N+S} [\lambda_1^2(\lambda_2^2 S)(\lambda_3 R_1) + \lambda_1^2(\lambda_3^2 S)(\lambda_2 R_2)$ + Similar terms for components 2 & 3]	$\frac{S}{N+S} [\lambda_1^2(\lambda_2^2 R_3)(\lambda_3 R_5) + \lambda_1^2(\lambda_3^2 R_3)(\lambda_2 R_6)$ + Similar terms for components 2 & 3]
5	A	A	A	$\frac{S}{N+S} [\lambda_1^2(\lambda_2^2 S)(\lambda_3^2 S) + \lambda_1^2(\lambda_3^2 S)(\lambda_2^2 S)$ + Similar terms for components 2 & 3]	$\frac{S}{N+S} [\lambda_1^2(\lambda_2^2 R_3)(\lambda_3^2 R_8) + \lambda_1^2(\lambda_3^2 R_3)(\lambda_2^2 R_9)$ + Similar terms for components 2 & 3]
6	A	N	A	$\frac{S}{N+S} [\lambda_1^2(\lambda_2 r_1)(\frac{R_1}{N})(\lambda_3^2 S) + \lambda_1^2(\lambda_3 r_1)(\frac{R_2}{N})(\lambda_2^2 S)$ + Similar terms for components 2 & 3]	$\frac{S}{N+S} [\lambda_1^2(\lambda_2 R_7)(\frac{R_5}{N})(\lambda_3^2 R_8) + \lambda_1^2(\lambda_3 R_7)(\frac{R_5}{N})(\lambda_2^2 R_9)$ + Similar terms for components 2 & 3]
7	N	A	A	$\frac{N}{N+S} [\lambda_1(\frac{r_1}{N})(\lambda_2^2 S)(\lambda_3^2 S) + \lambda_1(\frac{r_1}{N})(\lambda_3^2 S)(\lambda_2^2 S)$ + Similar terms for components 2 & 3]	$\frac{N}{N+S} [\lambda_1(\frac{r_1}{N})(\lambda_2^2 R_3)(\lambda_3^2 R_8) + \lambda_1(\frac{r_1}{N})(\lambda_3^2 R_3)(\lambda_2^2 R_9)$ + Similar terms for components 2 & 3]
8	N	N	A	$\frac{N}{N+S} [\lambda_1(\lambda_2 r_1)(\frac{R_1}{N})(\lambda_3^2 S) + \lambda_1(\lambda_3 r_1)(\frac{R_2}{N})(\lambda_2^2 S)$ + Similar terms for components 2 & 3]	$\frac{N}{N+S} [\lambda_1(\lambda_2 r_1)(\frac{R_1}{N})(\lambda_3^2 R_8) + \lambda_1(\lambda_3 r_1)(\frac{R_2}{N})(\lambda_2^2 R_9)$ + Similar terms for components 2 & 3]
$R_1 = \frac{r_1 r_2}{r_1 + r_2}; R_2 = \frac{r_1 r_3}{r_1 + r_3}; R_3 = \frac{S r_1}{S + r_1}; R_5 = \frac{N r_1 r_2}{N r_1 + N r_2 + r_1 r_2}; R_6 = \frac{N r_1 r_3}{N r_1 + N r_3 + r_1 r_3}; R_7 = \frac{N r_1}{N + r_1};$ $R_8 = \frac{S r_1 r_2}{S r_1 + S r_2 + r_1 r_2}; R_9 = \frac{S r_1 r_3}{S r_1 + S r_3 + r_1 r_3}$					
In failure mode column: N represents component failure in normal weather A represents component failure in adverse weather					

For the situation in which repair is not provided in adverse weather, all the outages which occur in adverse weather period has to wait for the repair for a period equal to the average duration of an adverse weather period. The average outage duration associated with three component redundant system is given by:

$$r_{SL} = \left\{ \frac{1}{\mu_1 + \mu_2 + \mu_3} \right\} \left\{ \frac{\sum_1^4 (\text{Failure Rate})_1}{\lambda_{SL}} \right\} +$$

$$\left(\frac{1}{\mu_1 + \mu_2 + \mu_3} + S \right) \left\{ \frac{\sum_4^8 (\text{Failure Rate})_1}{\lambda_{SL}} \right\}$$

When repair is performed during adverse weather the expected outage time is given by:

$$r_{SL} = \frac{r_1 \cdot r_2 \cdot r_3}{r_1 r_2 + r_1 r_3 + r_2 r_3}$$