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ABSORPTION OF SUNLIGHT BY ATMOSPHERIC SODIUM

A

THESIS

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ABSTRACT.

A photometer has been built which is sensitive only to light in a very narrow spectral region near the sodium D-lines. The light is scattered by sodium vapour at low density and the scattered light recorded by a photoelectric spectrometer. The latter resolves the two lines and allows the effect of stray light to be eliminated. With sunlight the radiation detected is at the bottom of the Fraunhofer lines and the residual intensity there may be measured. It is found that this intensity decreases slightly (relative to the continuum) as the sun approaches the horizon. This effect is attributed to atmospheric sodium; the amount found by this method agrees well with that deduced from twilight measurements made the same day.

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CHAPTER I.

INTRODUCTION.

1.1. General.

This thesis is devoted to a study of methods of detecting absorption of solar energy by terrestrial atmospheric sodium. The wavelengths in the solar spectrum absorbed are 5890 and 5896A - the wavelengths of the sodium D-lines. It is shown how this evaluation of absorption yields directly a value for the number of free sodium atoms present in the earth's atmosphere per vertical column of unit cross section. A significant result of the investigation is an accurate evaluation of the central intensities of the Fraunhofer D-lines. A historical resume of discoveries and measurements in connection with atmospheric sodium is traced, followed by an account of some preliminary measurements of sodium density by an indirect method made by the author.

The principal obstacle in making measurements of solar energy in the D-wavelength regions of its spectrum lies in the fact that measurements have to be made of the central intensities in the troughs of the Fraunhofer lines. These intensities are only about 5% of the nearby continuum. When it is considered that use of dispersing instruments yield central intensities, before corrections are made, from 200 to 500% too large (20,21,22), it is evident that if the

atmospheric absorption (which is of the order of 0.5 to 1.0% of the continuum) is to be detected, new techniques must be used. It is shown how use of resonance radiation enables the shortcomings of a dispersing instrument (used directly) to be overcome. The equipment used in the experiment is described and a discussion given of the interpretation of the data. The method of eliminating effects of changes in solar intensity and secondary absorption effects in the resonance tube are also dealt with. The results presented are based on 11 sets of measurements made on the days listed below. It will be shown that the corresponding number densities of sodium atoms per vertical column of unit cross section have been obtained with accuracies ranging from 5 to 50%.

29th December, 1955	26th February, 1956
30th December, 1955	27th February, 1956
10th January, 1956	28th February, 1956
31st January, 1956	2nd April, 1956
25th February, 1956	14th April, 1956

1.2. Historical Resumé.

The earliest indication of the presence of sodium in the terrestrial atmosphere came in 1929 when W.M. SLIPHER (1) observed a yellow line of estimated wavelength 5892A in spectrographic photographs of night sky emission. In 1932 J. DUFAY (2) observed the yellow line and estimated its wavelength as 5892A. CURRIE and EDWARDS (3) in the course of an auroral investigation at Chesterfield in the polar year 1932-3 detected emission in the wavelength region 5890A which exhibited noticeable enhancement at certain times. Other

estimates of the wavelength of this emission followed. These are listed below:

L.A. SOMMER (1932) (4)	5887A
J. CABANNES (1934) (5)	5888A
L. VEGARD & E. TONSBORG (1935) (6)	5892.6A
J. CABANNES & J. DUFAY (1938) (7)	5894A
R. BERNARD (1938) (8)	5893A

In 1938 the yellow line was shown by BERNARD (8) and (9) and CABANNES, DUFAY & GAUZIT (10) to be in fact a doublet; thus establishing beyond doubt the identity of the line as the sodium doublet and the existence of free sodium in the atmosphere. Estimated heights for the atmospheric sodium made at this time varied between values of 60 and 130km. BERNARD (8) and (9) observed a sudden fall-off of intensity of the emitted D-lines in twilight. Attributing this to the shadowing of a thin sodium layer by the earth at sunset, he obtained a layer height of 60km. CABANNES, DUFAY & GAUZIT (10) compared the intensity of the yellow nocturnal emission observed at the zenith with that at the horizon, and estimated a layer height of 130km. Taking a value of $1.35/\text{ergs}/\text{second}/\text{m}^2$ for the average energy radiation of the green line, in the South of France at the beginning of winter 1935-6; the latter workers estimated the yellow energy radiation to be $0.65 \text{ ergs}/\text{second}/\text{m}^2$, this being the energy released by $2 \cdot 10^{11}$ transitions per second of the type $^2P_{\frac{1}{2}} - ^2S_{\frac{1}{2}}$ and $^2P_{\frac{3}{2}} - ^2S_{\frac{1}{2}}$. Thus the first indication of the number of free sodium atoms per vertical column/ cm^2

~~density~~ was made. While CABANNES, DUFAY & GAUZIT (10) proposed a cosmic origin for the sodium, BERNARD (9) ascribed to the sodium a terrestrial origin viz. salt particles carried aloft by ascending air currents. CHAPMAN (11) listed the following as possible sources of atmospheric sodium (a) salt particles from the oceans carried up by ascending currents (b) volcanic dust shot up to heights of 10-30km (c) meteors (d) interstellar sodium swept up by the earth as it moves through space with the solar system (e) solar streams of gas such as whose ionized constituents produce geomagnetic storms. To this day the origin of the sodium is still not completely understood. CHAPMAN (11) put forward an explanation to the mystery of the continued free life of the sodium atoms of the upper atmosphere in the presence of the oxygen atoms known to exist there. The most likely reactions terminating the free life of the sodium were stated to be:

where M is a third body N_2 or O and at lower levels:

whereas it may be renewable by the two body reaction:

CHAPMAN (11) suggested that there was no necessary contradiction between BERNARD'S (9) height estimate of 60km and

the estimate by CABANNES, DUFAY & GAUZIT (10) of 130km. BERNARD'S (9) estimate was based on observations of the enhanced twilight emission and therefore the emission at the lower levels where the sodium density was greatest, whereas the night sky emission on which the measurements of CABANNES, DUFAY & GAUZIT (10) were based doubtless occurred at a much higher level. On the basis of the chemical reactions described above, having regard to the vertical distribution of O and O₃, CHAPMAN (11) suggested an exponential distribution for the free sodium with an abrupt decrease in density at the lower edge. This suggestion was substantiated by the work of ELVEY & FARNSWORTH (12) who investigated spectrophotometrically the intensity of the emitted D-lines and showed that if the log of emitted intensity is plotted against height of the earth's shadow during twilight, a linear relation is evident. Subsequent investigations aimed at locating accurately the level of and density distribution with height in the sodium layer. Measurements by VEGARD & KVIFTE (13) led to the postulation of the existence of a layer screening the solar radiation at 44km and the conclusion that this layer was the upper limit of the ozone which would absorb sunlight in the 2000-3000Å region and therefore that the twilight excitation of the sodium was due to ultra-violet radiation. However, BRICARD & KASTLER (14) (15) (16) were able to show that the sodium was excited by yellow light and that the absorption of the yellow solar

radiation should be considered in finding an effective screening due to attenuation in the lower layers of the atmosphere. DUFAY (17) thus obtained a screening height of 25km and was led to a height for the sodium layer of 80-90km. Further attention was focused on the density distribution in the layer by BARBIER (18) and HUNTEN (19). The latter found an exponential distribution of sodium with a scale height of $7.5 \pm 2\text{km}$ from $85 \pm 3\text{km}$ to 100 and probably 115km; below 85km the density dropped off quite rapidly. The density at 85km was estimated approximately as 10^4 atoms/c.c. which corresponds to 2×10^9 atoms/vertical column/cm². The latter measurement was based on estimates of the maximum scattered emission by the sodium atoms in twilight using BRICARD & KASTLER'S (14) figure of 8.10^8 quanta/cm² column/second/unit solid angle. HUNTEN (19) pointed out that the biggest uncertainty in the measurement was in the value for the flux of solar radiation incident on the atmosphere because the depth of the Fraunhofer D-lines was not accurately known. Accordingly a first approach in finding a value of sodium atomic density lay in determining accurately the residual intensities at the bottom of the Fraunhofer D-lines

1.3 Recent Work.

Recently an attempt was made in the Physics Department of the University of Saskatchewan, to obtain the Fraunhofer D-line profiles using a 21-foot concave grating and scanning with a photo-multiplier in its image plane.

This work is described in an earlier thesis (22). In spite of the high theoretical resolving power of the grating (150,000) and the precautions taken to eliminate scattered light, the profiles showed central intensities of between 20 and 25% of that of the nearby continuum. After correcting for the finite resolution of the instrument central intensities of 7.7 and 8.5% of the nearby continuum for the D2 and D1 lines respectively, were obtained. It was felt that although the profiles had been obtained by this method with good accuracy, the central intensities were considerably in error. Therefore work was begun with a view to obtaining precise central intensity values. Such data would have a twofold application:

- (a) It would enable the Fraunhofer D-line profiles to be accurately drawn at, and near, the central wavelengths.
- (b) It would make available a much better value for the flux of solar radiation incident on the earth's surface of wavelengths in the region of the sodium D-line central wavelengths. Hence a better evaluation of the amount of free sodium in the atmosphere could be made from the twilight measurements.

It will be shown in the next chapter how an absorption cell containing sodium vapour was used in conjunction with the system described in (22) to substantially

reduce the error in the central intensity measurements of the Fraunhofer D-lines. The new values for the central intensities at the bottom of the D-lines were found to be :

(a) $(5.0 \pm 0.6)\%$ for D1,

(b) $(4.8 \pm 0.6)\%$ for D2.

These values yielded a value of 1.1×10^{10} atoms of free sodium per vertical column of unit cross section, corresponding to the seasonal period of high emission intensity of the D-lines in the twilight.

In order to produce the sodium vapour densities required in the absorption cell when used in the above experiment the tube had to be maintained at temperatures in the range 80 - 200°C. Considerable experience was gained at this stage in producing absorption cells having good transmission characteristics, a resistance to attack by hot sodium vapour and having a low concentration of unwanted gases. The experience gained making these cells proved valuable in the subsequent experiments using the resonance tube and absorption cell.

In 1950 a study of high dispersion stellar spectrograms taken at the Mount Wilson Observatory was made by SANFORD (23) with a view to attempting to identify telluric sodium absorption lines. No positive identification could be made of either sodium D absorption line. The stronger D2 sodium line appeared in the region of strong absorption lines produced by terrestrial atmospheric

water vapour, so it was not possible to study this line. In stellar spectrograms, due to the large plate velocities, the stellar and inter-stellar sodium absorption lines are shifted by Doppler effect so that the terrestrial lines can, in general, be studied against the stellar continuum. SANFORD (23) found the spectrogram of α Virginis, Ce 2325, taken May 27th, 1940 using the Coudé spectrograph of the 100-inch telescope, most suitable for study, since this had a high dispersion (2.9A per mm), was taken at large Zenith distance of $Z = 68^\circ$, and utilized a fine grain emulsion. This spectrogram showed two lines at 5895.85 and 5895.94A of possible terrestrial origin having observed equivalent widths of 0.0032 and less than 0.0028A respectively. SANFORD (23) concluded that neither of these lines could be definitely attributed to telluric D1 absorption but the intensity of any such absorption, if present, did not exceed about 0.001A equivalent width. * This meant that the number of free sodium atoms per cm^2 column at night is not greater than 10^{10} .

It is important to note the date of exposure of the spectrographic plate which was given most attention by SANFORD (23) and to refer to HUNTEN'S (28) findings of a seasonal variation of terrestrial sodium density. The measurements actually made were of intensity of twilight sodium emission, taken over a two-year period. The intensity variation in terms of $Q_p(5893)$, where $Q_p(\lambda)$

* When reduced to the zenith.

represents the maximum angular surface brightness in units of 10^7 quanta/cm²/sec/sterad, is shown in Figure 14.

HUNTEN (28) shows that the number of sodium atoms per cm² column is very nearly $10^8 Q_p$. Thus Figure 14 shows that the sodium density is almost at a minimum and of the order of 9×10^8 at the time of the year at which the spectrogram studied by SANFORD (23) was taken. Hence it is not surprising that SANFORD found little or no evidence of absorption by terrestrial sodium. The author was fortunate in that he was able to make measurements in the months of December, January, February and March, corresponding to the season of maximum sodium density.

Figure 14. Seasonal variation of N_x - after Huntten.

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Figure 14. Seasonal variation of N_x - after Huntten.

CHAPTER II.

PRELIMINARY INVESTIGATIONS.

2.1. An indirect method of finding the number of free Sodium atoms per vertical column of unit cross section.

If a sensitive instrument, set to detect wavelengths in the region of 5893A, is pointed at a clear sky at night the sodium D-lines are readily detected. If the observations are continued until the end of twilight the emitted D-lines are noticed to increase in intensity after astronomical dawn. The inverse effect is also found. Shortly after sunset the sodium D-lines are strongly enhanced compared to the nocturnal intensity. A plot of intensity against time is shown in Figure 1. The twilight enhancement results from the production of resonance radiation as the sunlight illuminates the terrestrial sodium layer. Several minutes after astronomical dawn the extreme upper regions of the layer are illuminated first and as the earth's shadow falls with progressing time so more and more of the sodium layer is energized by the solar radiation. The emitted resonance radiation seen by the observer thus increases to a maximum as the whole layer is progressively illuminated. As has already been indicated the dependence of emitted resonance radiation on the time in twilight must

obviously be a function of the distribution of the free sodium atoms in the layer. Also the final maximum emitted intensity of radiation must depend on the total number of free sodium atoms in the layer; since this condition is only attained when all the sodium atoms in the viewing field of the detecting instrument are illuminated.

Figure 1. The mean variation of intensity of the D-lines emitted in twilight for the month of Dec. 1939. The zero represents the beginning of astronomical dawn. (After Elvey and Farnsworth (12)).

It would appear then, that if this maximum emitted intensity can be measured and the characteristics of the solar radiation illuminating the layer are known, an evaluation can be made of the number of free sodium atoms in a vertical column of unit cross section in the layer. The relation between the maximum resonance radiation intensity and the number of atoms in a vertical column of unit cross

section has been developed in a previous thesis (22). It is shown that if the solar radiation can be considered approximately black-body at a temperature of 6050°K, the number of emitted resonance quanta per atom per second is $18p_0$, where p_0 is the ratio of the radiation in the troughs of the Fraunhofer D-lines to the nearby continuum for zero density of atmospheric sodium. The expression giving N_x , the number of free sodium atoms per vertical, centimetre square, column is given by:

$$\begin{aligned} N_x &= \frac{\text{Observed brightness (Quanta/cm}^2\text{/sec/sterad)}}{\text{Number of quanta/atom/sec/sterad}} , \\ &= \frac{\text{Observed brightness}}{\frac{18p_0}{4\pi}} , \end{aligned}$$

and involves p_0 directly. Thus an evaluation of N_x can be made if the maximum brightness of the D-light can be measured and if an accurate value for p_0 is available.

The p_0 values obtained by determining the Fraunhofer profiles were:

$$p_{01} = 8.5\% ,$$

$$p_{02} = 7.7\% ,$$

where suffixes 1,2 refer to the D1 and D2 lines respectively (22). Values obtained by ALLEN (20) for p_0 were:

$$p_{01} = 7\% ,$$

$$p_{02} = 6\% ,$$

PRIESTER (21) found profiles for selected parts of the sun's disc. Using his central intensity values, average p_0 values were obtained for the whole of the sun's disc.

These were:

$$p_{01} = 6.6\% ,$$

$$p_{02} = 5.8\% ,$$

Using a technique similar to that of PRIESTER (21), SHANE (29) obtained the following p_0 values relating to the centre and edge of the solar disc,

$$\begin{array}{ll} p_{01} = 5.8\% & \text{for } \cos \theta = 1 , \\ p_{02} = 5.2\% & \end{array}$$

$$\begin{array}{ll} p_{01} = 8.0\% & \text{for } \cos \theta = .262 , \\ p_{02} = 7.0\% & \end{array}$$

where θ is the angle between the line of sight and the normal to the sun's surface. The large differences in these values illustrate the acute dependence which the central intensities of the Fraunhofer D-lines in the solar spectrum have on the part of the solar disc which is observed. The p_0 values obtained by PRIESTER (21) show a similar dependence on the part of the solar disc observed. This is discussed more fully at a later stage.

Our values could not be considered accurate in view of the large corrections which had to be made. However, it was felt that the central intensity values of ALLEN (20), PRIESTER (21) and SHANE (29) could still be improved upon

if a suitable correction could be applied to our profiles. Furthermore, it was important to determine the p_0 values relating to light emitted from the whole of the sun's disc since these are the values actually operative in producing the resonance emission in the terrestrial sodium layer. Undoubtedly it is possible to use the values relating to specific areas of the solar disc and to obtain a mean value for the whole disc from these. Indeed an averaging of PRIESTER'S (21) values was carried out and is described later. However, a determination of the central intensities of the Fraunhofer D-lines when observing the whole of the solar disc was obviously desirable.

2.2 A more accurate determination of the p_0 values.

If an absorption profile having zero central intensity is examined spectrophotometrically, the profile observed will rarely show zero central intensity. This is due (a) to the resolution function of the instrument which causes wavelengths to be detected other than those observed and (b) to scattered light. Thus if two lines of approximately the same form, one having zero central intensity and the other having a weak central intensity of about 5% of the nearby continuum, are examined with the same instrument and under precisely the same conditions, both will show finite central intensities. It is reasonable, however, providing the lines are of the type specified above,

to assume that the difference between the observed central intensities is equal to that of the line having the finite central intensity. One thus obtains, effectively, a zero correction at the central wavelength from the line having known zero central intensity.

The technique of finding a zero correction for the Fraunhofer profiles was based upon absorbing light from the incident solar beam. The wavelength distribution in the absorbed light had to be such that the resultant composite profile would not be greatly affected in shape but would be depressed to have a zero central intensity as shown in Figure 2. Obviously the absorption had to be produced by sodium vapour. If a broad absorption line was required high sodium vapour densities would be entailed. This would necessitate the absorption cell being maintained at high temperatures. The following requirements were thus specified for the absorption cell:

1. Good transmission characteristics.
2. Invulnerability to attack by sodium vapour.
3. Pure sodium vapour with negligible foreign gas or with a predetermined amount of inert gas.

Attention must now be given to the form of absorption lines that can be produced by an absorption cell of the type specified.

Figure 2. Ideal form of the composite absorption profile required to permit an evaluation of p_0 .

2.3. Absorption by pure sodium vapour.

The equation describing the transmission of parallel light through an absorbing medium from a source emitting a continuous spectrum is:

$$I_f = I_0 e^{-k_f x}, \quad (1)$$

where x is the thickness of the absorbing layer and k_f is the absorption coefficient. I_f , which defines the absorption line and k_f , may exhibit a dependence on frequency f as shown in Figure 3. The half breadth of the absorption line Δf is indicated in the figure. It is the total breadth of the absorption coefficient curve at the

point where k_f has fallen to one half its maximum value $k_{\max}$.

Figure 3. Dependence of (a) I_f and (b) k_f on frequency f .

In the absence of electrical and magnetic fields there are three significantly different processes that contribute to the ~~formation~~ of the absorption line of a gas. Each process has a broadening effect on the line. They are:

- (a) Natural broadening due to the finite lifetime of the excited state.
- (b) Doppler effect broadening due to the motion of the atoms.
- (c) Pressure broadening due to collisions

HFS ?

between the absorbing gas molecules
and between the absorbing gas and
foreign gas molecules.

It is shown by MITCHELL & ZEMANSKY (24) that if only
processes (a) and (b) are present the absorption coefficient
is given by the following expression:

$$k_f = k_o \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2} dy}{a^2 + (w-y)^2} \quad , \quad (2)$$

where $w = \frac{2(f-f_o)}{\Delta f_D} \sqrt{\log 2} \quad , \quad (3)$

$$a = \frac{\Delta f_N}{\Delta f_D} \sqrt{\log 2} \quad , \quad (4)$$

$$y = \frac{2\delta}{\Delta f_D} \sqrt{\log 2} \quad , \quad (5)$$

$$\Delta f_D = \text{Doppler breadth} = \frac{2\sqrt{2R\log 2}}{c} f_o \sqrt{\frac{T}{M}} \quad , \quad (6)$$

$$\Delta f_N = \text{natural breadth} = \frac{1}{2\pi c} \quad , \quad (7)$$

and $k_o = \frac{2}{\Delta f_D} \sqrt{\frac{\log 2}{\pi}} \frac{\lambda_o^2}{8\pi} \frac{g_2}{g_1} \frac{N}{\tau} \quad , \quad (8)$

Other symbols not defined above are:

R = The gas constant.

c = Velocity of light.

- λ_o, f_o = Wavelength and frequency at the centre of the lines.
- T = Absolute temperature.
- M = The molecular weight of the vapour producing the absorption.
- τ = The mean life of the excited state.
- g = The statistical weight of the levels involved in the absorption process. The suffix 1 refers to the ground state and suffix 2 to the excited state.
- N = Atomic density of the absorbing vapour.

δ is an arbitrary distance from the point $f-f_o$ used in setting up the integral combining the Doppler effect broadening and natural broadening processes. The quantity 'a' is called "natural damping ratio".* The quantity k_o is an ideal quantity, the maximum absorption coefficient when Doppler broadening alone is present. For small values (< 0.01) of the natural damping ratio 'a', equation (1) can be put in the form:

$$\frac{k_f}{k_o} = e^{-w^2} - \frac{2a}{\sqrt{\pi}} \left[1 - 2wF(w) \right] \quad , \quad (9)$$

where $F(w) = e^{-w^2} \int_0^w e^{-y^2} dy \quad , \quad (10)$

A table of values of $F(w)$ has been worked out by LASH MILLER

* (The value of 'a' for sodium vapour at 160°C for the transition $3^2S_{\frac{1}{2}} - 3^2P_{\frac{1}{2}}$ is 0.0052).

& GORDON (25). The ratio $\frac{k_f}{k_0}$ is thus readily evaluated for any part or all of an absorption line. The quantity $k_0 x$, in general determines the depth and width of the absorption line. $k_0 x$ is often referred to as the central optical depth of the line. In an absorption tube 'x' is fixed and equation (8) shows that the optical depth of an absorption line is dependent on the temperature of the vapour in the coldest part of the tube. This follows from the fact that N is a function of the vapour pressure of the absorbing gas which is in turn a function of the temperature. In Figure 4 the Fraunhofer D1 profile and absorption profiles for the sodium D1 lines corresponding to various temperatures, are shown plotted. Equation (2) was used to obtain the profiles which are those which would be produced by an absorption tube of length 6cms., containing pure sodium vapour. It is evident that if these lines are compounded with the solar Fraunhofer profile to give the shape of the resultant profile which would be produced by sunlight passing through the sodium absorption tube, a smooth profile will not result. The resultant profile will be essentially unaffected in the wings and will show a discontinuous dip at central wavelengths. Undoubtedly the conditions shown in Figure 2 will not be realized. We consider now the effects of process (c), pressure broadening, on the shape of the absorption line.

Figure 4. D1 absorption profiles using pure sodium at various temperatures compared with the D1 Fraunhofer profile.

2.4. Absorption by Sodium in the presence of Helium.

As the pressure in an absorbing gas, or the pressure of a foreign non-absorbing gas, is increased, the probability of collisions between absorbing atoms or between absorbing atoms and those of the foreign gas is also increased. The effects of such collisions can vary greatly. In certain cases a collision can occur between two atoms in which no energy, or an extremely small amount of energy, is transferred. Such collisions involve a perturbation of the excited atom so that its radiation or absorption characteristic is altered in some way. Thus an absorbing gas at relatively high pressure, or in the presence of a foreign gas at high pressure, shows an increase in breadth of its absorption lines. This broadening indicates the collisions which affect the absorbing characteristic of the atoms. As the pressure of the foreign gas is increased the following characteristics are observed:

- (1) The absorption line is broadened.
- (2) The maximum of the absorption line is shifted.
- (3) The absorption line becomes asymmetrical.

If the pressure of the foreign gas is not too great (less than 0.5 Atmospheres for sodium in the presence of helium) only characteristic (1) may be noted. The absorption may then be treated in terms of the simple Lorentz Theory.

The expression for the absorption coefficient in the form given by Lorentz is:

$$k_f = \frac{\text{const}}{1 + \left[\frac{2(f-f_0)}{\Delta f_L} \right]^2}, \quad (11)$$

where Δf_L is called the Lorentz half-breadth and is given by:

$$\Delta f_L = \frac{1}{\pi} x (\text{The number of broadening collisions per second per absorbing atom.})$$

$$\text{or } \Delta f_L = \frac{Z_L}{\pi} = \frac{2}{\pi} \sigma_L^2 N \sqrt{2\pi RT \left(\frac{1}{M_1} + \frac{1}{M_2} \right)}, \quad (12)$$

where σ_L^2 is the effective cross section for Lorentz broadening,

N is the number of foreign gas molecules per c.c.

M is the molecular weights of the gases involved.

The equation may also be written:

$$\Delta f_L = P(\text{m.m.Hg.}) 1.95 \cdot 10^9 \sqrt{\frac{2}{\pi} \frac{R}{T} \left(\frac{1}{M_1} + \frac{1}{M_2} \right)} \sigma_L^2, \quad (13)$$

To find the absorption line produced by an absorbing gas in the presence of a foreign non-absorbing gas it is necessary to combine the three broadening agents viz: Lorentz broadening, Natural broadening and Doppler broadening. This has been done by REICHE (26) who obtained the following expression for the absorption coefficient:

$$k_f = \frac{2k_0}{\pi(\Delta f_N + \Delta f_L)} \int_{-\infty}^{\infty} \frac{\exp\left[-\left[\frac{2\delta}{\Delta f_D} \sqrt{\log 2}\right]^2\right] d\delta}{1 + \left[\frac{2}{\Delta f_N + \Delta f_L} (f - f_0 - \delta)\right]^2}, \quad (14)$$

Defining w and y as in equations (3) and (5) and letting:

$$a' = \frac{\Delta f_N + \Delta f_L}{\Delta f_D} \sqrt{\log 2}, \quad (15)$$

equation 14 becomes:

$$k_f = k_0 \frac{a'}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2} dy}{a'^2 + (w-y)^2}, \quad (16)$$

This equation has the same form as equation (2). However a' can no longer be regarded as small since it can be made as large as desired by increasing the pressure of the foreign gas i.e. by increasing Δf_L . Thus the $F(w)$ values applicable to equation (2) can no longer be applied and the integral in equation (16) must be evaluated by series or by numerical integration. A table of values for k_f/k_0 for several values of a' and w is given in the appendix of reference 24.

It is perhaps appropriate to point out at this stage the relation between absorption and resonance scattering, the principle of which is utilized in detecting the absorption by the terrestrial atmospheric sodium of solar radiation. The considerations given above in

sections 2.3 and 2.4 have a direct bearing on the theory of resonance radiation which is discussed in Chapter IV. The energy absorbed from a collimated beam of radiation defined by equations (2) and (16) re-appears as resonance radiation (when the absorbing vapour is viewed from a direction other than that of the incident radiation).

The latter is not seen but the resonance radiation, having a spectral distribution equivalent to that absorbed from the incident beam, is observed. Thus, in general, for small vapour densities, the resonance line has approximately the same shape as the absorption line.

With the above mathematical tools available it was possible to formulate the shape of absorption lines which would produce the desired depression in the Fraunhofer profiles. It was necessary to keep the value of a' as small as possible (i.e. to keep the pressure of the foreign gas low) so as not to produce any distortion in or displacement of the absorption line. Further broadening could still be produced by increasing the optical depth of the line (i.e. by increasing the operating temperature). Using helium as the foreign gas, it was found that suitable absorption profiles could be produced for the optimum values of 1.5 for a' and 150-180°C for the operating temperature range. A family of absorption profiles for the D1 and D2 sodium lines is shown in Figures 5 and 6, together with the appropriate solar Fraunhofer profile.

Fig.5. Absorption profiles for the pressure broadened D1 sodium lines for various values of T and a' compared with the Fraunhofer D-lines.

Fig.6. Absorption profiles for the pressure broadened D2 sodium lines for various values of T and a' compared with the Fraunhofer D-lines.

These curves were obtained using the k_f/k_o values given in the appendix of reference 24. For large values of $\lambda - \lambda_o$, k_f/k_o was found using an expression developed from equation (14). For an a' value of 1.5 it was found that the absorption cell had to be filled with helium at a pressure of 170mm Hg. (at room temperature). Consideration will now be given to the method of making the absorption cell.

2.5. Construction of the sodium vapour absorption cell.

The form and dimensions of the cell are shown in Figure 7. This tube passed the solar beam without

Figure 7. The Absorption Cell.

reducing the aperture of the dispersing system. Pyrex plate glass was chosen as material for the windows to ensure minimum attenuation of the light beam. In order to protect the walls of the tube against attack by the sodium vapour they were lined with a protective glaze. This glaze is made from a proprietary formula by the General Electric

Company and used in the manufacture of sodium vapour lighting equipment. A sample quantity of the glaze was kindly sent to us by Mr. R. Schmies of the General Electric Company. The surface to be treated was moistened with the glaze solution which rapidly dried leaving a white deposit. The glass was then fired at red heat causing the deposit to become transparent. The firing process left the glass slightly distorted with a finely pebbled transparent surface where the glaze had been applied. The window discs were placed on an iron plate upon which had previously been sprinkled a 1/8th inch layer of finely powdered carbon. After treating with the glaze solution the cold glass disc was pressed into the carbon powder, with the treated surface uppermost, to form a plane impression. Heat was then applied from below to the iron plate until the firing temperature was approached. The red firing temperature was then attained with heat applied both from above and below. The carbon powder functioned (a) to prevent the disc from buckling on softening at the firing temperature and (b) as an insulating blanket retarding the cooling of the disc thereby minimising warping.

After firing, the window discs were polished using a fine rouge (before fusing to the tube.) The side-arm AA' (Figure 8) was fused to the centre of a piece of Pyrex tubing about 12 inches long and of diameter appropriate to

Manometer

Diagram of the system for drying the specimen

the cell. The central portion of the tubing was treated with the sodium resistant glaze and on cooling was cut to the required length so that the side-arm was still central. The ends of the tube were then ground flat using a carborundum powder (180 grade) and the windows fused on. This assembly was then annealed to minimize the severe strains introduced by firing-on the sodium protective glaze. Transmission for white light proved to be about 50%. The tube was now ready for attaching at a' (Figure 8) to the pumping system which included provision for injection of the sodium and helium.

The pumping arrangement is illustrated in Figure 8. A single stage oil diffusion pump backed by a Welch Duo-Seal mechanical pump provided the vacuum. The helium was fed in at D from a gas cylinder via stopcock P2. The mercury manometer M enabled a rough estimate of pressure in the system to be made.

With a rubber stopper sealing the open tube at O the system was evacuated using the backing pump only and then flushed with helium, this process being repeated several times. (In the course of this flushing process a thin walled glass ampoule of pure sodium was embedded in a 2-inch thickness of powdered solid carbon dioxide contained in a beaker. The vacuum system was flushed with helium and immediately the gas reached atmospheric pressure, indicated by the gas bubbling out at the manometer

reservoir, the neck of the ampoule was broken under the dry ice, the stopper at O was removed and the ampoule quickly dropped open end down into the vertical tube via O. The restriction in this tube trapped the ampoule at B. The entry tube was now sealed at O with a glass blower's torch and pumping commenced immediately the glass hardened. In this way the sodium was introduced into the vacuum system without pollution. The ampoules of pure sodium were kindly provided by the General Electric Company. The diffusion pump was now used as well as the rotary backing pump to withdraw all occluded gases. The ampoule at B was then gently warmed with a bunsen flame until the sodium liquified and dropped into the cell. Some of the sodium was caught in the seal-off restriction B2 and was removed by further gentle heating at this point, the liquid sodium being forced into the cell by allowing a small increase in the helium pressure. Gases given off during the transfer of the sodium from the restriction at B to the cell were rapidly drawn off, thus further improving the purity of the sodium. Pumping was now continued for about one hour, after which helium at a pressure of 170mm Hg as measured by the manometer was introduced and the tube sealed off at B2. It was now necessary to construct an oven to house the cell and the absorption unit would be ready for use.

2.6. Construction of the absorption-cell oven.

A diagram of the oven is shown in Figure 9.

The absorption cell supports S were made from 1/4-inch asbestos sheet and served also to divide the oven into three compartments; the two end compartments were maintained at about 5°C higher temperature than the central compartments in order to prevent condensation of the sodium vapour on the cell windows. This temperature differential was achieved by having two heater windings W1, W2, supplied in parallel from a variable ac voltage source, located at the ends of the copper tube. The windings were made from nichrome wire (22 B&S, 1.02 ohms per foot) and each had a total resistance of 12 ohms. The windings were insulated from the copper tube by a single layer of asbestos paper. The latter was dampened before being rolled onto the tube. In this way the asbestos insulating layer was made to fit snugly to the copper tube. The iron bands B served to hold the first asbestos layer in position and also to provide anchors and terminals for the heater windings. Thermal insulation was achieved using the same asbestos paper. This was cut into strips as wide as the length of the copper tube. After dampening the strips were wound on to the tube, the side tube T being readily forced through the damp asbestos paper, until a 3/16th inch thickness of insulation had been applied. Asbestos string was used as a binding to hold

Figure 9. Diagram of the absorption cell oven - first model.

the insulation in position. Temperatures relating to the vapour pressure were taken at the coldest point in the oven by the thermometer X which was inserted via the insulated side tube T. The oven windows O were cut from 3/32-inch glass and held flush with the end of the copper tube by annular shaped asbestos rings C which clipped on to the oven by means of the spring clips D. Considering the simplicity of design, remarkably good control was maintained over the oven temperature during the observations.

2.7. Method of obtaining the p_0 values.

The following simple procedure was adopted in forming the composite solar D-line profiles. The sunlight was passed through the absorption cell before entering the dispersing system, which consisted of a monochromator ahead of a 21-foot concave grating having 750,000 lines in 5". A single profile took about two minutes to scan, the profile being recorded by one of the pens of a two-pen Speedomax (20mv full scale) instrument. The solar intensity was monitored by having a fixed 1P21 detector gathering the light in the first order of the spectrum and recorded using the second pen of the Speedomax recorder. Profiles of the two lines were studied separately. One of the lines was scanned with the cell at room temperature and then successive scans were made while the absorption cell temperature was gradually raised to a

maximum of about 240°C. Successive profiles were also taken as the cell gradually cooled. Temperature changes were made slowly so as to minimise any lag in the thermometer. Observations were made for each D-line on separate occasions. The minimum central intensity values for the profile were plotted against absorption cell temperature for the two D-lines. The relations are shown in Figure (1) of Appendix I. Attention was then given to finding the absorption cell temperature which would produce the correct depression in the Fraunhofer profiles. Of the solar D-line profiles available those of PRIESTER (21) had been obtained with the minimum of correction. PRIESTER (21) obtained seven profiles relating to different parts of the sun's disc. Parts of the solar disc were defined by annular rings in terms of θ where θ was the angle between the line of sight and the normal to the sun's surface. These profiles differed appreciably. However, on examination of the curves drawn on the same graph it was evident that the profile for $\sin \theta = 0.60$ would lie very close to a profile averaged for the whole of the disc. With the above considerations in mind it was decided to use the profile obtained by PRIESTER (21) for $\sin \theta = 0.60$ as a basis of comparison for obtaining the composite profile. An average was obtained however for the central intensity value. This was done by plotting the central intensity values relating to the parts of the solar disc

defined by $\sin \theta$ and weighting the curve for (a) the area of the annular ring appropriate to a given value of contributing to the total area of the disc and (b) the limb darkening. The latter was represented by an approximation suggested by MINNAERT (27) with a coefficient $\beta = 1.6$.

Figures 2 and 3 of Appendix I show the solar Fraunhofer D-line profiles obtained by PRIESTER (21) for $\sin \theta = 0.6$ and with the central intensity corrected to represent the whole of the solar disc. The same profiles are shown depressed to have a zero central intensity. Calculated profiles which would be produced by sunlight passing through the sodium vapour at different temperatures are also shown. In finding the composite profile which gave the best fit to that of PRIESTER'S (21), consideration had to be taken of the resolution function of the instrument. The width of the resolution function is also shown in Figures 2 and 3 of Appendix I. This represents the profile of a line produced by the dispersing system for virtually monochromatic radiation. The central intensity value of the Fraunhofer absorption line would be obtained with the instrument set at λ_0 , but due to the wide resolution function other nearby wavelengths would contribute to its magnitude. Therefore ordinate differences between a composite profile and that of PRIESTER'S (21) were found for a number of values of $\lambda - \lambda_0$ and multiplied by the

corresponding ordinate value of the resolution function. In this way small differences at the central wavelength were greatly weighted. The resultant product of the two ordinate values was plotted against $\lambda - \lambda_0$ and the area under the curve examined. The composite profile giving the smallest area under the product curve was selected as fitting most closely to the desired profile. The temperatures corresponding to the curves of best fit for the D1 and D2 Fraunhofer profiles were 167 and 158°C respectively. Reference to the plot of minimum central intensity against absorption cell temperature now gave the true p_{01} and p_{02} values. The change in value of p_{01} produced by the pressure broadened absorption of the sodium at 167°C was $(5.4 \pm 0.3)\%$. The change in the observed p_{02} value corresponding to 158°C was $(5.2 \pm 0.3)\%$. A subsequent correction was applied to allow for the fact that the solar intensity at 1A° from the centre of the line is only about 92% of the continuum and the error was increased to 0.6% to allow for uncertainties in the temperature measurement and uncertainties in the vapour pressure relation for sodium. The final corrected values obtained were:

$$p_{01} = (5.0 \pm 0.6)\%$$

$$p_{02} = (4.8 \pm 0.6)\%$$

These values were considerably smaller than any values found previously. As will be seen they show excellent

agreement with the values found ultimately using the resonance technique. The work described above was summarized in a paper published in the December, 1955 issue of the Canadian Journal of Physics. A reprint of this paper forms Appendix I.

CHAPTER III.

EXPERIMENTAL TECHNIQUES.

3.1. General.

It was necessary before attempting to detect the telluric sodium absorption to make some estimates, in the light of existing knowledge concerning the sodium layer, of the magnitude of the effects likely to be produced. In any event in order to cater for small sodium densities it was desirable to devise a system having as great a sensitivity as possible to the effects of absorption. In this chapter attention will be given to estimating the possible magnitude of the effects of the sodium absorption. Two techniques suitable for detecting the sodium telluric absorption will be briefly described and compared. It will be shown how one technique possesses considerable advantages over the other and was, therefore, chosen for making the measurements. The first method adopted (subsequently referred to as method I) will be outlined in this chapter. Many of the techniques in this method have a wide application. Therefore details of the apparatus will be given in the appendix. The second method (hereinafter referred to as method II) will also be outlined, details of the apparatus and theoretical

considerations being dealt with in later chapters.

3.2. Initial Considerations.

It has already been indicated how the presence of sodium atoms in the upper atmosphere is readily detected by observing (a) the scattered D-light in twilight (References 8,9,12 & 19) and (b) the nocturnal emission of the D-lines (References 10,12,49,50 & 51). In chapter II a discussion of absorption of radiant energy was given with particular reference to absorption by free sodium atoms. The equations describing absorption presented in chapter II show that no appreciable absorption occurs until a critical value of the optical depth, k_0x , is reached. As the optical depth is increased the absorption line becomes deeper and broader. After the centre of the line is completely absorbed only the broadening process is effective as k_0x values are increased. A family of absorption lines for pure sodium vapour for various values of k_0x is shown in Figure 4. Undoubtedly, any absorption of solar energy by terrestrial atmospheric sodium would not be strong. In order to produce absorption of, say 50% of the nearby continuum at the central wavelength for D2, about 10^{10} free sodium atoms per square centimetre would be required. This figure is considerably in excess of ^{most of the} estimates based on the emitted intensity of the resonance radiation from the terrestrial atmospheric sodium in twilight. Any strong absorption would probably,

in spite of the limitations already discussed, have been detected by investigators studying the solar D-line profile. As already pointed out a recent study of high dispersion stellar spectrograms by SANFORD (23) revealed no strong absorption in the region of wavelength 5896A in the continuum of the stellar spectrograms.

In order to get a clear picture of the possibility of detecting absorption by terrestrial atmospheric sodium, a family of absorption lines was drawn for various values of N_x for the D1 and D2 lines assuming Doppler broadening. These are shown in Figure 10. In drawing the line profiles a temperature of 250°K was taken as the temperature of the sodium layer. Temperatures of this order are reported by the Rocket Panel (Reference 30) for heights equivalent to that assumed for the sodium layer viz. 85km. A value for τ of 1.48×10^{-8} seconds was used. This value obtained by LADENBURG & THIELE (32) is generally considered the most accurate value available at this time. The optical depth of the profiles were then given by the expression:

$$k_0 x = 8.05 \times 10^{-12} N_x \left(\frac{g_2}{g_1} \right), \quad (1)$$

where the symbols have already been defined.

The continuum reference for absorption lines produced by atmospheric sodium is the solar intensity at the bottom of the Fraunhofer lines (Figure 11). This is not accurately known. Therefore, the change in the amount of absorption

Figure 10. Absorption profiles produced by atmospheric sodium for various values of N_x as observed at noon and sunset.

corresponding to a change in the sun's elevation must be studied. The path length of the sun's rays through the sodium layer, assumed of finite thickness and at a height of 85km as found by HUNTEN & SHEPHERD (31) is proportional to $\sec \alpha$ where α is the angle between the sun's rays and the normal to the layer. At noon during the spring and early summer months the value of $\sec \alpha$ is approximately 2.

Figure 11. Intensity distribution at the bottom of one of the Fraunhofer D-lines showing the terrestrial sodium absorption.

The minimum value of $\sec \alpha$ which can be obtained is somewhat less than this. At sunset $\sec \alpha$ is about 6. The path length can thus be changed by a factor of about 3 in the summer months. In winter the path length is only twice as great at sunset as at noon. The change in absorption corresponding to $\sec \alpha$ increasing from 2 to 6

is shown in Figure 10. It can clearly be seen from the curves that a value of $N_x = 5 \times 10^9$, will produce appreciable change in absorption between noon and sunset in summer. For $N_x = 10^9$, the absorption is never greater than 10% and for N_x less than 5×10^8 , virtually no absorption at all is present. Estimates for N_x values based primarily on measurements of twilight sodium emission varied between 10^8 and 10^{10} . Assuming the upper value to be correct the absorption should be readily detected; if the smaller value is correct, detection of the absorption would be impossible.

3.3. Methods for detecting terrestrial sodium absorption.

The first method tried represented, to some extent, a refinement and extension of the techniques developed to obtain accurate measurements of solar intensity at the bottom of the Fraunhofer D-lines as described in chapter II. The measurement was made in the following way: After producing a narrow band width of solar radiation about the D-lines the solar beam was split (Figure 12). The split beams were passed alternately so as to fall ultimately on to the cathode of a 1P21 photomultiplier, which, together with amplifier and phase-sensitive-detector, formed the detecting unit. One of the two resultant beams passed through a sodium absorption cell at such temperature that an absorption line of depth about 50% of the nearby continuum was produced. The other beam was attenuated

Figure 12. Schematic diagram of Method I.

using a comb-type variable light attenuator so as to compensate for the 50% transmission of the sodium absorption cell and the uneven dividing by the beam splitter. Using this attenuator the two beams could be made to fall with equal intensity on the photo-cathode of the detector and so produce a balance. The spectral nature of the light in the two beams falling on the photomultiplier at noon and sunset is shown in Figure 13. Beam B having

Figure 13. Spectral distribution of energy in the two light beams falling on the detector.

no energy absorbed in the detecting system, shows a net change in energy content indicated by the shaded portion between the limiting absorption lines formed at noon and sunset. Thus, after balancing the two beams at noon, a progressive unbalance should develop as sunset approaches.

This unbalance appeared as an approximately triangular ac wave form at the output of the detector and amplification was provided in order to attain the greatest possible sensitivity. This ac signal was then fed to a phase sensitive detector and the resultant dc output represented the degree of unbalance of the two light beams. The dc output was fed to an Esterline Angus recording meter. The oven temperature was controlled to within 0.1°C using a thermistor as control element in an ac bridge. Power was supplied via a thyatron regulated by the bridge output. Considerable refinement was necessary in the electronics associated with the detector in order to achieve a satisfactory response and undistorted output wave forms. Appendix II contains details of the techniques used in this scheme.

About the time that Method I was being made ready for taking observations, work was begun on Method II. Immediately the possibilities of the alternative techniques became apparent it was necessary to press ahead with construction of the apparatus for this scheme in order that both methods be available, if necessary, at the time of maximum sodium density, i.e. in the period January to March (Reference 28). Method II was based on the principle of observing the resonance radiation produced by radiation incident on a cell containing sodium vapour. This scheme reproduced in the laboratory effectively the phenomenon of

the twilight emission.

This method achieved very high resolution since only those wavelengths absorbed by sodium are scattered by resonance; indeed, as has already been indicated in chapter II, the two processes are essentially the same observed in different ways. The resonance tube was illuminated with sunlight and placed in front of the slit of a spectrometer. The latter scanned the region of the sodium D-lines and its output showed the two resonance lines well resolved and of easily measurable magnitude above a continuum background produced by scattered white light. The emitted intensities of the resonance radiation lines were proportional to the residual intensity at the bottom of the Fraunhofer lines including the small additional absorption due to the terrestrial sodium i.e. p in Figure 11. If the terrestrial sodium were sufficiently dense to produce a change in p between noon and sunset, this change should affect the resonance emission in two ways (a) the magnitude of the emission lines should decrease as the solar elevation decreases (b) the line intensity ratio should change slightly. It was also possible, by using a projection lamp as a standard source to provide an intense illumination having a continuous spectrum for the resonance tube. This was used to make accurate measurements of the p values in the solar illumination. The ratio of the resonance emission produced by sunlight to that produced by the lamp,

corrected for the relative intensity of the two illuminating beams, gives directly the value of p . (A discussion of the quantitative aspects of the measurements is given below).

The principal difficulty with this scheme lay in reducing the scattered white light which at first produced a background considerably greater than the emission. This masking of the emission by the background was particularly severe when using sunlight since the wavelengths producing the resonance radiation are only about $1/20$ th the intensity of the nearby continuum which produces the background.

However, by careful masking of the resonance tube it was possible, ultimately, to get emission to background ratios using sunlight, of greater than 1:1. At this stage it was decided to adopt the resonance tube method in preference to Method I for making the final measurements.

The reasons for such a choice are as follows:

The resonance emission observed in Method II was virtually a direct observation of unabsorbed radiation at the central wavelengths of the Fraunhofer D-lines and measurements were easily made. Undoubtedly the resolution achieved in Method I was equal to that of Method II but interpretation of the magnitude of the effects observed would be difficult. This difficulty arises from the fact that absorption changes produced by the terrestrial atmospheric sodium are proportional to the shaded area shown in Figure 13. This area is not known in relation to the total area under

the curve which represents the total energy content of the beam. A careful calibration probably making the measurement fully dependent on the absorption tube temperature and temperature-density relation for sodium would thus be necessary. Perhaps the primary objection to Method I was the possibility of sodium condensing on the windows of the absorption cell. Precautions were taken to prevent such effects as described in Appendix II. However, any minute condensation on the windows of the absorption cell would immediately affect the transmission of the cell, causing an unbalance in the two beams. Thus changes in the transmission of the cell due to sodium condensation would not be readily distinguished from real absorption effects produced by the terrestrial sodium.

Using Method I a continuous curve, (which is desirable), representing the amount of absorption by the terrestrial sodium would be shown on the output recording meter in the course of a noon to sunset run. This curve would be independent of small fluctuations in solar intensity since only the difference in intensity between the reference beam and that showing the terrestrial sodium absorption, is amplified in the detector system. On the other hand, observations using Method II would have to be made at intervals of about 20 to 30 minutes in order to permit calibration measurements to be made. Only about nine points on the absorption versus time plot

could thus be obtained in the course of a noon to sunset run in winter. However, since only one of the sodium absorption lines could conveniently be studied at once, using Method I, it was not possible to examine the different degree of absorption which must occur in the two lines as the path length of the sunlight through the sodium layer increases. Undoubtedly, two plotted curves, relating to the D1 and D2 lines, obtained simultaneously, and representing the terrestrial sodium absorption would be more valuable than a continuous curve representing absorption in one D-line.

With these considerations in mind all emphasis was placed on Method II in preparing to detect the terrestrial atmospheric absorption.

CHAPTER IV.

RESONANCE RADIATION AND ITS APPLICATION IN MEASURING TERRESTRIAL SODIUM ABSORPTION.

4.1. General.

We consider now Method II in greater detail. This chapter falls logically into two parts. In Part I a development of the theory of resonance radiation and its applications in this investigation is introduced and a discussion of the method of analysing the data is presented. Part 2 contains a description of the equipment used.

Part 1.

THEORY AND ANALYSIS.

Theory of Resonance Radiation.

4.2. Certain solids and liquids, when illuminated by monochromatic radiation of frequency f , will themselves emit light, usually of frequencies different from f . This process is termed fluorescence. The fluorescence of sodium vapour was studied in the last decade of the 19th Century. Although spectral bands, now known to be due to the Na_2 molecule, were detected, it was not possible to detect the sodium D-lines when exciting sodium vapour with white light, or with light from a flame

containing a sodium chloride solution. WOOD (33 & 34) was first to excite the D-line fluorescence in sodium vapour by the action of D-lines themselves. This type of fluorescence was termed resonance radiation by WOOD (33 & 34) since it was predicted by the classical theory of a light wave vibrating at the same frequency as the dipole oscillations of the medium. The reason why earlier attempts failed was due to the fact that too high a sodium density was used. The emitted resonance radiation was re-absorbed by other unexcited atoms in the resonance tube and the radiation was thus imprisoned within the vapour. This tendency to imprison the resonance radiation affects the sensitivity of response of the emitted radiation to changes in the intensity of the exciting radiation. This effect is important when interpreting the terrestrial sodium absorption curves and is discussed later.

The phenomena of resonance radiation can be explained for sodium, in terms of the energy level diagram shown in Figure 15. Assuming white light to be incident on the sodium, only lines of the principal series are absorbed since most of the atoms are in the ground ($3^2S_{1/2}$) state. The atoms can re-emit the lines appropriate to the transition $3^2P_{3/2}-3^2S_{1/2}$, $3^2P_{1/2}-3^2S_{1/2}$, $4^2P_{3/2}-3^2S_{1/2}$, $4^2P_{1/2}-3^2S_{1/2}$, $5^2P_{3/2}-3^2S_{1/2}$ etc. The emitted

Figure 15. Energy level diagram for sodium.

radiation is resonance radiation since all the wavelengths emitted are contained in the exciting radiation. If the excited sodium vapour is contained in a glass vessel, only the D-lines are observed in emission as other resonance lines are of wavelengths short enough to be absorbed by the glass. Resonance radiation can thus be defined as follows: If a group of atoms in the normal state absorb light of a certain frequency, and subsequently re-emit light of the same frequency, the emitted light is termed resonance radiation.

In order to distinguish between resonance radiation and fluorescence, the lowest energy levels of thallium are shown in Figure 16. The atom can absorb

Figure 16. Lower energy levels of thallium.

the two lines of wavelengths 3776 and 2768A by the transition $6^2P_{\frac{1}{2}}-7^2S_{\frac{1}{2}}$, and $6^2P_{\frac{1}{2}}-6^2D_{3/2}$. If the atom absorbs wavelength 3776A, it reaches the $7^2S_{\frac{1}{2}}$ state from which it may revert to the normal state emitting the resonance line or may return to the metastable $6^2P_{3/2}$ state with the emission of the fluorescent line 5350. Similarly two lines - the resonance line 2768 and the fluorescent line 3530 - result if light of wavelength 2768A is absorbed. Of the four lines emitted on absorption of the wavelengths 3776 and 2768A, two are resonance lines and two are fluorescent lines.

4.3. The Resonance Lamp.

The theoretical considerations presented above enable specifications to be laid down for the production of an efficient resonance tube or lamp. This may be defined as a vessel containing an unexcited gas or vapour which can absorb a beam of radiation from an outside source (an exciting source) and which, as a result of this optical excitation, emits resonance radiation in all directions. Evidently a crude form of resonance lamp is readily made by distilling sodium into a glass or quartz bulb under vacuum. Such an arrangement can indeed be used to demonstrate the phenomenon of resonance radiation, as was originally done by WOOD (33 & 34). To perform such a demonstration the bulb is illuminated

by an intense white light source or by light from a sodium vapour Pirani-type lamp. With the bulb at room temperature only stray light due to reflection is observed. As the tube temperature is slowly raised by gentle heating with a bunsen burner or electrical element, the vapour pressure of the sodium increases and the characteristic yellow colour of the sodium resonance radiation appears in the path of the exciting beam. As the temperature, and hence the vapour pressure inside the bulb is raised further, the whole bulb glows with resonance radiation. On increasing the vapour pressure still further, only the portion of the vapour lying near the window where the exciting beam enters the tube is luminous. Further increase of vapour pressure results in no atomic resonance radiation being emitted at all. It is important to note that the resonance radiation observed only from the region of the path of the optical beam is the result of a single atomic absorption and emission. The glowing of the whole volume of the bulb indicates secondary (and tertiary), etc. absorption and emission by unexcited, closely packed, atoms about the exciting beam. When the resonance radiation is extinguished the atomic density is so high that the resonance radiation is imprisoned within a small volume by the multiple scattering of the resonance radiation. Two disadvantages of the simple resonance

lamp described are:-

- (1) the presence of stray light due to reflection, and
- (2) the presence of a layer of unexcited vapour between the exciting beam and the window from which the beam emerges.

These disadvantages are minimized in the lamp illustrated

Figure 17. Resonance Lamp with baffling to minimize reflected light.

The screens S and S' and the entrance window projection enable the exciting beam to be confined in the region of the exit window so that the resonance radiation

does not traverse an unexcited layer of vapour. By drawing-off the rear of the tube in the manner shown, the internally reflected light is trapped and little stray light appears at the exit window. The vapour pressure in the lamp is determined by controlling the temperature of the cell C containing the sodium. The lamp is maintained at a temperature greater than that of C to prevent condensation of the sodium on the lamp windows.

The design of the resonance lamp used in Method II is shown in Figure 18. An image of the solar

Figure 18. Form and dimensions of the resonance lamp used in Method II.

disc or lamp filament was produced at O by the exciting radiation I_f , and the emitted resonance radiation I_{rf} was observed through the side of the small diameter tube at Ex.

The diameter of the tube was increased at the shoulder S so that the diverging light beam first struck the tube walls in a region remote from the exit window and near the light trap. The end of the tube containing the windows was maintained at a higher temperature than the drawn-off end so that condensation was confined to the tail of the tube. Extensive baffling was necessary - in spite of the light trap - to cut down the reflected stray light before the lamp could be used to produce prominent emission lines using sunlight. Once efficient operation of the lamp had been achieved, observations of the effects of terrestrial sodium on the solar spectrum were commenced.

4.4. Determination of an optimum operating temperature for the Resonance Lamp.

The resonance lamp was first illuminated with light from the standard source and the operating temperature of the tube adjusted until satisfactory emission of the D-lines was observed by the spectrometer. (This scanned a 20A band of wavelengths centred at 5893A). In the course of this adjustment the emission lines were absent until a lamp temperature of about 120°C was attained. The ratio of the D2 line intensity to that of the D1 line at this temperature was about 1.8:1, indicating that even with the correspondingly low vapour pressure, some of the resonance radiation was being lost in the lamp

due to absorption by unexcited atoms. The ideal intensity ratio of the two lines is 2:1 but the D2 line is affected more strongly in absorption than the D1 line. As the temperature was increased the intensity of the resonance emission increased and the D2:D1 line ratio decreased. In the region of 160°C the maximum D-line intensities were observed and the ratio fell to about 1:1. As temperatures were increased above 160°C the emitted D-line intensities decreased rapidly. When using the resonance lamp it was desirable to obtain substantial resonance emission in order to facilitate measurements and it was also desirable to preserve a low vapour pressure to minimize absorption of the resonance radiation. The temperature corresponding to the smallest accurately measurable emission lines was used to operate the tube. Thus a temperature in the range 145 - 150°C was maintained at the rear end of the tube with the window end always several degrees higher.

Once the optimum operating temperature for the tube was determined the lamp was ready for use. An actual Varian recorder trace showing the D-line emission produced by illuminating the resonance lamp with sunlight is shown in Figure 20. The importance of the extensive and elaborate baffling provided to suppress the reflected radiation can be seen from this trace since the background produced by reflected light is as great as the emission. The background intensity is proportional to the intensity of the solar beam

illuminating the resonance lamp, while the emitted D-lines are proportional to p in Figure 11. By plotting the ratio of the intensity of the emitted D-lines to the background intensity it was possible to observe changes in the value of p as the path length of the solar radiation through the terrestrial sodium layer varied. If no absorption is produced in the solar spectrum by the terrestrial atmospheric sodium (p is constant) and the above ratio is constant. A device will subsequently be described, the use of which enabled absolute values of p to be determined and also the p_0 values in the Fraunhofer D-lines.

4.5. Preliminary Analysis of the Resonance Emission, using sunlight.

For a fixed operating temperature of the resonance lamp the intensity of the resonance emission produced by white light will in general be proportional to the intensity of the incident radiation. The width of a resonance line is of the same order as that of a Doppler broadened line produced at the same temperature as the resonance line. The lines emitted from the lamp are thus sensitive to a very small range of wavelengths in the exciting radiation (this accounts for the high resolution achieved in Method II). When sunlight is used to excite the sodium vapour in the lamp the effective exciting continuum for the resonance radiation (assuming for the moment negligible terrestrial sodium absorption) is p_0 times the intensity I_s of the

continuum at a few Angstroms on either side of the Fraunhofer lines; whereas the continuum intensity for the reflected radiation producing the background is proportional to I_s . Consider the D2 emission lines and call the intensity of the line produced by sunlight D_{s2} , the intensity of the line produced by the standard white light source (of intensity I_L), D_{L2} , and the observed intensity of the respective backgrounds B_s and B_L , we have:

$$\begin{aligned} D_{s2} &= S P_{o2} I_s , \\ D_{L2} &= S I_L , \end{aligned} \quad (1)$$

where S is a scattering function for the sodium vapour, but

$$\begin{aligned} I_s &= k B_s , \\ I_L &= k B_L , \end{aligned} \quad (2)$$

where k is the same for both equations and represents the degree of reflection by the resonance lamp.

Hence:

$$\frac{D_{s2}}{D_{L2}} = \frac{P_{o2} B_{s2}}{B_L} ,$$

and

$$P_{o2} = \frac{D_{s2}}{D_{L2}} \times \frac{B_L}{B_{s2}} , \quad (3)$$

Any terrestrial atmospheric absorption will reduce the value of p_o given in equation 3. Strictly speaking, if the

terrestrial sodium absorption is assumed to be effective, p_0 is replaced by p in equation 3. Measurement of the quantities given in equation 3 at several instants between noon and sunset, i.e. for different values of $\sec \alpha$, should reveal the amount of absorption by the terrestrial atmospheric sodium present in the solar spectrum. This measurement should also yield a value for p_0 which assumes zero terrestrial sodium absorption. The same equation applies to the D1 line - the suffix 2 being replaced by suffix 1 to the quantities defined above.

Using the most intense source available to illuminate the resonance lamp for standardizing the measurements, the deflection on the recorder equivalent to B_L was barely detectable for half scale deflection produced by D_{L_2} (or D_{L_1}). The accuracy in the measurement of B_L was thus very low. Little improvement in accuracy was achieved by measuring B_L using the high gain output of the spectrometer control unit to give a substantial deflection on the Varian recorder since the recorder was driven off scale for most of the scan by the relatively large resonance lines. Values of about 8 and 10% were obtained for p_2 and p_1 respectively from measurements of the above type. A second disadvantage lay in the fact that when using sunlight to illuminate the resonance lamp the depth of the Fraunhofer lines in the background B_g could not readily be measured. The observed depth of the Fraunhofer line in the continuum background must be known if the true value of the D_g emission

is to be obtained, since this emission must be measured from the trough of the observed Fraunhofer line and not from the level of the continuum (Figure 19). It is, of

Figure 19. Diagram showing the quantities measured to yield D_s .

course, possible to obtain the apparent depth of the Fraunhofer line before the observations are started, with the tube cold. In the absence of the resonance emission the Fraunhofer lines appear in the background and can be measured as a fraction of the background continuum. When making measurements of the resonance emission produced by sunlight D_s is found by measuring d_s and B_s . Whence:

$$D_s = d_s + qB_s;$$

where q is the observed fractional depth of the Fraunhofer

line. Evidently more confidence should be placed in the measurements if the value of q were obtainable at regular intervals in the course of a noon to sunset run.

4.6. The Scattering Monitor and Measuring Routine.

The disadvantages discussed above were removed by using a scattering device to monitor the solar and standard source light beams. This facilitated accurate determinations of the ratio I_S/I_L and, incidentally, showed clearly the amount of Fraunhofer absorption in the background radiation B_S . The scatterer was designed so as to effectively reproduce the backgrounds B_S , B_L , in a manner equivalent to that produced by imperfections in the resonance lamp assembly, but with greater and more easily measurable intensity. In this way the solar and standard source light beams were monitored at the point where they produced the resonance radiation. The physical arrangement of the scatterer and resonance lamp was such that they were easily interchanged in position in front of the spectrometer slit. A routine for making the measurement was now necessary. This was as follows:

1. The monitor was positioned in front of the spectrometer slit and illuminated with sunlight. The output scans showed the solar intensity I_S in the region of the sodium D-line wavelengths and also the Fraunhofer lines. (Figure 20(b)). Several

Fig. 20. (a) Resonance emission of the D-line using sunlight.
(b) Spectrum of scattered sunlight.

Fig. 21. (a) Resonance emission of the D-lines from the standard source.
(b) Continuum of scattered light from the standard source.

scans were observed to ensure that I_s was not fluctuating due to atmospheric haze or atmospheric pollution.

2. The monitor was retracted and replaced by the resonance lamp. Five scans of the resonance emission of the sodium D-lines produced by the sunlight were obtained and the time of observation of the central scan noted. (Figure 20(a)).
3. The resonance lamp and monitor were interchanged and an additional trace showing the intensity of the sunlight was obtained. This trace was required to check that no severe change in solar intensity occurred during the period of production of the scans showing the solar resonance emission.
4. The resonance lamp and monitor were interchanged so that the resonance lamp was positioned in front of the spectrometer slit and the solar illumination was replaced by the light from the standard source. (This was done using a pivoted mirror which could be introduced in such a way that the solar beam was cut off by the mirror-mount and the light beam from the standard source was deflected by the mirror along the optical axis of the baffle into the resonance lamp). Five calibration traces were then

obtained showing resonance emission produced by the light from the standard source. (Figure 21(a)).

5. The resonance lamp was removed and the scattering monitor replaced in position in front of the spectrometer slit. Five traces produced by light from the standard source showing I_L were then obtained (Figure 21(b)).

One spectrometer scan of the 20A band centred at 5893A, took about 50 seconds. This was followed by an "off" period of 10 seconds during which the scanning mechanism reset the grating for the subsequent scan. After some practice it was found possible to make the adjustment described above during the 10 second "off" period of the spectrometer between scans, so that no time delay was incurred in making the adjustments. The whole sequence of operations corresponding to one observation at a particular time of the day took about 20 minutes.

4.7. Analysis of the Data.

Using the scattering monitor, p values of the order of 5% were obtained. Since these were obviously of the correct order, observations aimed at detecting the terrestrial sodium absorption were commenced.

The traces produced by the scattering monitor, when illuminated by sunlight, yielded values of I_s and q (the percentage depth of the observed Fraunhofer lines in

relation to the continuum). The traces taken immediately before and after the period of production of the solar resonance emission of the resonance lamp provided a check on the constancy of the solar intensity.

The group of five traces taken with sunlight incident on the entrance window of the resonance lamp yielded values of d_s and B_s , the corrected emission intensity being:

$$D_s = d_s + \eta B_s ,$$

a mean D_s , $\bar{D}_s$ was then found from the five traces.

The five traces produced by illuminating the resonance lamp with the beam from the standard source yielded mean values of D_L , while a mean I_L was obtained from the five traces produced by the scattering monitor and standard source combination. The value of p at the time of observation was thus given by:

$$p_2 = \frac{\bar{D}_{s2}}{\bar{D}_{L2}} \times \frac{\bar{I}_L}{\bar{I}_s} , \quad (4)$$

where $(-)$ indicates mean values and the suffix refers to the particular line.

The log of the p values was now plotted against the values of $\sec \alpha$ appropriate to the time of observation. As shown in chapter II, the dependence of p on the path length of the solar radiation through the terrestrial sodium

layer is given by:

$$I_f = I_0 e^{-k_f x}, \quad (5)$$

the general relation for an absorption line, where I_f and k_f are the transmitted intensity and absorption coefficient at frequency f respectively. I_0 is the intensity of the incident radiation and x is the path length in the absorbing medium. Considering for the moment only the central wavelengths; since p and p_0 are proportional to the transmitted and incident intensities at the central wavelength we have:

$$p = p_0 e^{-k_{f_0} x \sec \alpha}, \quad (6)$$

where x is the effective vertical thickness of the layer,

α is the angle between the observed sun's rays and the normal to the layer,

k_{f_0} is the absorption coefficient at the central wavelengths and

$$\log p = \log p_0 - k_{f_0} x \sec \alpha. \quad (7)$$

Assuming a uniform horizontal distribution of sodium atoms in the terrestrial layer the plot of $\log p$ versus $\sec \alpha$ should thus be a straight line of negative slope $k_{f_0} x$. Assuming the resolution of the system to be such as to detect only those wavelengths near the line centre the

value of the sodium number density is obtained by measuring the slope of the above plots to yield $k_{f_0} x$.

The relation:

$$k_0 x = 8.05 \cdot 10^{-12} \frac{g_2}{g_1} N_x ,$$

as derived previously gives the value of N_x - the number density of sodium atoms per cm^2 vertical column - since k_{f_0} is nearly equal to k_0 for a Doppler broadened sodium line.

It will be noted that the above treatment is somewhat artificial in that the detecting system is assumed to be sensitive to only the central wavelengths near f_0 in the terrestrial sodium line. The response of the resonance emission line to all the absorbed wavelengths must be considered. When this is done an effective absorption coefficient k' which is less than k_0 is obtained. In the next chapter the method of determining the value of k' will be described in the course of a discussion dealing with the results obtained.

Part 2.

EXPERIMENTAL EQUIPMENT USED IN METHOD II.

4.8. Layout of the Equipment.

The arrangement of the units used in Method II is shown in Figure 22. These were housed in the Auroral Observatory of the Physics Building. The observatory was

Figure 22. Layout of the equipment used in Method II.

built with a 30 by 30 inch aperture cut in the roof to facilitate solar studies. By placing a coelostat on the observatory roof above this aperture it was possible to make unobstructed observation of the setting sun during the winter months. The coelostat was mounted above the hole in the observatory roof so as to direct a steady vertical beam of sunlight on to mirror M1. Protection for the coelostat was provided by an octagonal dome.

The mirror M1 deflected the solar beam from the coelostat along a horizontal path so that it entered either the resonance tube (which was contained in the oven O,) or the light scatterer S. The latter units were so mounted that the scatterer could be withdrawn from a pre-set position adjacent to the light baffle and retracted. The absorption cell oven could then be moved into a position equivalent to that occupied previously by the light scatterer. The lens L, of diameter and focal length $4 \frac{1}{8}$ and 23 inches respectively, produced an image of the solar disc in the resonance tube. In order to minimise heating effects produced by the solar beam in the resonance tube the infra-red absorption cell I was placed in the light beam. The resonance tube control unit was located on a shelf beneath the bench housing the equipment.

The spectrometer detected the emission from the resonance tube or the white radiation from the light

scatterer, depending on which of these units was in its pre-set position adjacent to the baffle and aligned with the spectrometer slit S_2 . The spectrometer output was fed to the spectrometer control unit and thence to a Varian recorder V.

Sunlight was replaced by light from the standard lamp source by swinging the mirror M2 into a pre-set position M2'. The mount for the latter was arranged to revolve on a vertical shaft and provision was made for setting the mirror in either of the positions M2 or M2'. This mirror could be adjusted so that the axis of the beam from the lamp coincided with the axis of the solar beam.

The individual units are described below in the order in which they are encountered by the solar light beam.

4.9. The Coelostat and Dome.

A drawing of the coelostat is shown in Figure 23. The design of this unit was based on that described by STRONG (35). Figure 24 shows the coelostat and dome mounting with the dome removed.

The coelostat functions as follows:

Mirror M1 is mounted on a rotating shaft. The latter, driven by a Telechrom motor, rotates once every 48 hours and is aligned parallel with the earth's axis. This alignment is achieved if the vertical plane including the shaft is in the North-South direction and if the

Fig. 52. The pumping system.

inclination of the shaft to the horizontal is equal to the latitude. The mirror M1 is adjusted on the mounting shaft so that the parallel solar beam which it intercepts is thrown on to the mirror M2. The rotation of mirror M1 causes its reflected beam to remain stationary as the sun traverses the heavens in the course of the day. The mounting position of the rotating mirror assembly on the base board is adjustable in the North-South direction defined by the guide rails R1, R2. The variation in the sun's declination can thus be compensated for. In winter, the reflected beam from the rotating mirror is arranged to fall on the directing mirror M2 by having the former near the South end of the guide rail. As the sun's declination increases, the movable assembly is positioned at progressively greater distance from the South end of the coelostat base, permitting mirror M2 to intercept its reflected beam. Two guide rails are provided since the rotating mirror is liable to be in the shadow of the directing mirror M2 at certain times of the day. If the East guide rail is used in the forenoon and the West guide used in the afternoon, this does not occur. The steady beam intercepted by M2 is directed as desired.

In this application M2 was adjusted to direct the beam downwards through the observatory roof aperture via a hole cut in the coelostat base. The directing mirror M2 was mounted so that it could be rotated about the two axes

X and Y, the desired position being maintained by friction in the mounting bearings.

The dome was constructed using 3/16 inch plywood panels for the body. The annular base was made from a double thickness of $\frac{3}{4}$ inch board. The "slit" was covered with 1/8 inch thick lucite panels. These were removed if the observations were made on calm days. The dome was mounted on a circular array of twelve rubber castors (Figure 24). Six of these were mounted perpendicular to the observatory roof and supported the dome. Six were positioned radially in the same plane as the roof so as to bear against the circular edge of the dome base and to hold the dome in position. These castors allowed the dome to be readily rotated. No difficulty was thus experienced in manually adjusting the orientation of the dome to pass sunlight to the coelostat. Four guy-ropes secured the dome to the roof when not in use.

4.10. The Infra-red Filter.

A 3½% solution of cupric chloride was used to filter out the infra-red radiation in the solar beam. This was contained in a cell made from plate glass, the internal dimensions of which were $4\frac{1}{2} \times 4\frac{1}{2} \times \frac{1}{4}$ inches. The cell was clamped on to the mount used for L_2 (Figure 22), so that the path length of the solar beam in the absorbing solution was $\frac{1}{4}$ inch. The transmission characteristic of the cupric

chloride in the infra and near infra-red is shown in Figure 25 (see STRONG (35)). By plotting the mean monochromatic intensity of the solar disc as influenced by the Fraunhofer lines (reference 36) and comparing this with the same plot corrected for the transmission of the cell, it was estimated that inclusion of the absorption cell reduced the energy in the beam by about 60%.

Figure 25. Transmission Characteristics for Cupric Chloride Solution in the infra and near infra-red.

4.11. The Resonance Lamp, Resonance Lamp Oven and Oven Temperature Control Unit.

A diagram of the resonance lamp is shown in Figure 18 and the resonance lamp oven is shown in Figure 26. Little need be said about the resonance lamp, since the techniques for evacuating the tube and filling with sodium are the same as those described in Appendix II for the sodium

Figure 26. The Resonance Lamp Oven.

absorption cell. The pressure in the resonance lamp before "sealing-off" was 5×10^{-7} mm.

The oven construction and mounting assembly for the glass tube was essentially the same as that of the sodium absorption cell oven described in Appendix II. An 80 ohm winding W, of 4 ohms per foot nichrome wire, was used to heat the oven. The winding was concentrated at the end of the oven containing the entrance and exit windows E_x and E_n , several turns being wound on the window projection P_i . This ensured a high temperature in the oven compartment containing the windows. An annular well was formed by the entrance window projection P_i and an extension of the outer insulation wall. This was filled with a moistened fire-brick powder which baked out to form the firm thermally insulating wall X. The entrance window was mounted at the end of the projecting tube P_1 . A diaphragm r mounted at the oven end of this tube formed the final light baffle screening the incident radiation. The exit window was mounted in the end of the tube P_2 and the two baffles b_1, b_2 , screened the exit window from unwanted light scattered inside the oven. To reduce such scattered light inside the oven the lamp was liberally coated with black paint over all its outer surface except for the viewing port and entrance window. The thermistor T was mounted in the transite partition as shown, the leads being brought out through the end wall of the oven via insulating bushes to

the terminal board B. The heater windings were also brought out to this board so that all the leads from the heater control unit were anchored conveniently at this point. The internal assembly was mounted on two threaded 4BA rods secured to the detachable end-wall of the oven so that the lamp could be readily withdrawn to permit inspection or modification. The baffles b1 and b2 were cut away near the exit window mounting tube P2 facilitating the withdrawal of the lamp mounting assembly. The temperature of the oven compartment containing the lamp tail was measured by thermometer A and once an operating temperature had been determined, this served merely to check the constancy of the oven temperature.

The circuit of the oven temperature control unit is shown in Figure 51 and the operation is explained in Appendix II.

4.12. The Adjustable Mirror and Standard Light Source.

Details of the adjustable mirror are shown in Figure 27. The mirror was held out of the solar beam, causing at the same time the radiation from the standard lamp to be reflected back along its own axis, by the thumb screw S engaging in the groove A. By disengaging S the mirror could rotate about the shaft B under the action of the spring O, causing D to come into contact with the adjustable stop K. In this position light from the standard source entered the detecting system via the light baffle and the solar radiation

Figure 27. The adjustable mirror.

was interrupted by the mirror mount. Alignment of the reflected beam from the standard source was achieved using screws s1, s2, s3. The mirror was produced by vapourizing aluminum under vacuum so as to form a highly reflective surface on the $\frac{1}{4}$ inch plate glass (c.f. STRONG (35)).

An optical bench high intensity illuminator was used for the standard source. This utilized a coil element projection lamp which operates at 6v 18 amps. The use of the large operating current in this type of lamp results in the elimination of ac fluctuations in the emitted beam intensity.

4.13. The Scattering Monitor.

Preliminary attempts to obtain the relative intensities of the two light beams entering the resonance lamp utilized an unsilvered piece of plate glass placed at an angle to the beam axis in front of the baffle tube (Figure 22) so as to reflect a fraction of the incident radiation into a separate detector. These attempts gave inconsistent results presumably due to (a) the fact that the light content of the beams at the point monitored was different from that at the point in the system where the resonance radiation was produced, (b) the different intensity distribution in the spectra of the two beams.

The scattering monitor shown in Figure 28 was designed to permit examination of the light intensity pro-

duced by the solar, or the standard source, radiation immediately in front of the spectrometer slit after the beam had passed the anti-scattering baffles. In this way, the ratio of intensities of the solar radiation and the radiation from the standard source which produced the resonance emission was determined independently of the path (or geometry) of the beams. This unit enabled the spectrometer to be used as a detector which was sensitive only to a 20Å wave band in the incident radiation centred about the wavelengths producing the resonance radiation. The monitor in conjunction with the spectrometer also showed the depth of the Fraunhofer D-lines. The scattering was produced by a stack of lightly ground glass plates P inclined to the incident radiation so that reflected light from the plates was directed away from the exit window E_x . It was essential that the light be deflected into the spectrometer by the monitor by a process other than direct reflection since this might show a critical dependence on the beam alignment. The stack of ground glass plates was equivalent in many respects to a vapour or colloidal solution in its scattering action and was thus insensitive to small changes in the direction of the incident beam. The plates were mounted on a removable frame F and housed in a light-tight brass box B. This had a small tube in one side which functioned as an exit port. The exit and entrance

windows were made to correspond geometrically to those of the resonance lamp. The emergent scattered radiation

Figure 28. Diagram of the Scattering Monitor.

was appropriately attenuated by a disc of exposed photographic negative held over the exit window by the rubber ring r ; the latter also serving to prevent stray light entering the spectrometer slit when the monitor was in operation. The short length of tubing T_1 fitted snugly in to the baffle tube and ensured correct positioning of the unit with respect to the light beams and spectrometer

slit. The tube T_2 formed a sliding bearing on a horizontal shaft attached to the mounting plate for the baffle tube, the resonance lamp oven and the scattering monitor. This enabled the latter to be readily withdrawn from, and replaced in position, before the spectrometer slit.

4.14. Mounting Arrangement for the Resonance Tube Oven, Scattering Monitor and Light Baffle.

The arrangement for mounting the interchangeable resonance tube oven and scattering monitor units in relation to the fixed light baffle tube and spectrometer slit is shown in Figure 29. The mounting plate was a sheet of $\frac{1}{4}$ inch Dural which could be attached to the front of the spectrometer by means of wing bolts located in the holes h_1 , h_2 , and the slots s_1 , s_2 , so that the port O was positioned over the spectrometer slit. The resonance tube oven was cradled in the sliding assembly A. With the scattering monitor M retracted the oven assembly could be slid along the shaft S until checked by the adjustable stop P. In this position the exit window of the oven was aligned with the aperture O and hence with the spectrometer slit; and the oven was engaged about $\frac{3}{8}$ inch in the baffle tube B so as to bring the entrance window of the oven adjacent to the aperture in the baffle plate b_2 . By sliding back the oven cradle assembly to the extreme left hand end of the shaft the scattering monitor could be slid into position so that its exit window was centred with respect to the aperture in the mounting plate, and its entrance window

Diagram of the pump and buffer
 The pump and buffer are shown in the
 figure. The pump is shown in the
 figure. The buffer is shown in the
 figure.

(which corresponded to the resonance lamp entrance window) was correctly located in relation to the aperture in the baffle b2. Corresponding to the oven engagement, the tube T formed a snug fit in the baffle tube, a stop being provided by the arm D. So far as could be judged from the results obtained the positions of the oven and scattering monitor in front of the spectrometer slit were exactly reproducible.

4.15. The Spectrometer and Recorder.

The spectrometer was one designed and built by Dr. D.M. HUNTEN (37) of the University of Saskatchewan and applied originally to auroral studies. This spectrometer utilizes the following optical components:

A high quality $1\frac{1}{2}$ inch long entrance slit; a 6-inch off-axis paraboloid collimating mirror of 34 inches focal length; a 4 x 4 inch aluminum on glass, 6,000 lines per inch, plane grating with a blaze angle of 17.5° ; a 6-inch on-axis paraboloid focusing mirror of 22 inches focal length; a small 45° plane mirror; a 1-inch long exit slit with electrically operated shutter, and a 1P21 photo-multiplier tube. The 50 second scan is produced by oscillating the grating through a small angle. A cam rotating once every 60 seconds imparts a saw tooth motion to a short lever (4 inches long). A long lever of adjustable length (about 20 inches) is fixed to the grating. By making the end of the long lever bare on the side of the short lever at various points, various

amplitudes of oscillation can be given to the grating and the spectral range can be adjusted. A second cam rotating with the first operates a micro-switch which controls the clamping relay in the amplifier and a shutter in front of the exit slit. This shutter closes and the input grid of the amplifier is grounded by the clamp, during the 10-second fly-back period. When using the spectrometer to study the sodium D-lines a spectral range of approximately 20 Å was used.

The signal from the 1P21 photo-multiplier detector is fed via a pre-amplifier (used to match ^{approximately the} output and line impedances) to the spectrometer control unit (Figure 22) which houses the main amplifiers and power supplies (c.f. Reference 37). The first amplifier provides a gain of 2 and uses a 6V6 in the output stage to develop the necessary power to drive one channel of a Brush two channel magnetic oscillograph. The second amplifier provides a further gain of 3 or 10 as desired, and drives the other channel of the recorder.

The circuit shown in Figure 30 was recently incorporated by Mr. G.F. Lyon (under the supervision of Dr. D.M. Huntten) to enable the output to be presented by a Varian recorder. The latter requires only 100mv. for full scale deflection thus obviating the necessity for the amplifiers used to provide the driving power for the Brush oscillograph. The signal from the pre-amplifier and clamp

Figure 30. Circuit modification to the spectrometer control to permit use of the Varian Recorder.

circuit is fed to the switch SW1 which selects the circuits associated with either the Varian or the Brush recorder. When using the Varian recorder the signal appears on the grid of V1 which is simply a cathode follower having a tapped cathode load permitting outputs in the ratio of 1,3,10,30,100 times the voltage developed across the bottom 1k resistor to be fed to the gain selector switch SW2 and thence to the Varian recorder. The variable resistance P1, together with the -150v supply voltage acts to control the residual voltage appearing at A and therefore as a set-zero control for the Varian. With the signal clamped the set-zero control on the Varian is used initially to give zero pen deflection with the gain selector set at x1. The gain selector is then set at x100 and the control P1 used to re-establish the zero pen deflection. The gain selector is again set at x1 and if any deflection is evident the adjusting cycle is repeated until no pen deflection occurs. The Varian circuit is then ready for operation.

CHAPTER V.

RESULTS.

5.1. General.

Successful observations were made on ten different days in the period December, 1955 to April, 1956. The data obtained yielded average values of p_{01} and p_{02} which showed excellent agreement with those determined by the method described in chapter II, and also a mean value for N_x , the number of absorbing atoms per vertical column of unit cross section, for the day on which the observations were made. In this chapter the method of reducing the experimental data to obtain the plot of $\log p$ versus $\sec \alpha$ is described. The resultant values of N_x for the various days on which observations were made and the average values of p_{01} and p_{02} are presented. The method of calibrating the apparatus to obtain an effective absorption coefficient is described. A discussion of the results is given in the next chapter.

5.2. Evaluation of $k'x$.

The data available from the Varian Recorder chart for an observation made effectively at a given time on a given day are (1) the relative values of $\bar{D}_s$, $\bar{D}_L$, $\bar{I}_L$, and $\bar{I}_s$ as defined previously and (2) the time of observation as

shown by the clock (i.e. Mountain Standard Time). From the latter the value of $\sec \alpha$ is obtained in the following way. The elevation of the sun h , above the horizon is found in terms of the local apparent time of the particular observation. $\sec \alpha$ is then found from a functional relationship between h and $\sec \alpha$. The local apparent time is given by "the equation of time" and a correction of -6min. 30sec. for the longitude of Saskatoon, i.e.

$$\text{L.A.T.} = (\text{observed clock time} - \text{clock error}) - 6\text{m } 30\text{s} - \text{correction to sundial.}^* \quad \times$$

The elevation h , of the sun above the horizon is a function of the L.A.T. and the time of year as shown in Appendix III. A nomogram was used to obtain h values for the runs made between 29th December, 1955 and 31st January, 1956. This nomogram was not suitable for use after this date and the values of h corresponding to L.A.T. were calculated for the runs made 25-28th February, 1956, using the FERUT digital computer at the University of Toronto. This calculation is described in Appendix III. A second more versatile nomogram drawn under the supervision of Dr. D.M. Hunten, suitable for all months of the year was available, and was used, for all subsequent runs. The relation between h and $\sec \alpha$ for any assumed height of the sodium layer is developed in Appendix IV. Two curves relating h and $\sec \alpha$ were calculated, using the Toronto digital computer FERUT for sodium layer heights of 80 and

* This correction applies to longitude 105° West.

90km in order to examine the degree of dependence of $\sec \alpha$ on layer height. The curves lie close together showing that the $\sec \alpha$ values are not critically dependent on the layer height. A height for the sodium layer of 85km was assumed in accordance with the measurements made by HUNTEN & SHEPHERD (31). The relationship between $\sec \alpha$ and h corresponding to this height was readily available by drawing a smooth curve between those calculated for heights of 80 and 90km. This curve constituted the last link in the chain of information necessary to convert the time values into their corresponding values of $\sec \alpha$.

Before plotting the points of $\log p$ against $\sec \alpha$ a further refinement was introduced. Referring to equation (4) of chapter III, p is given by:

$$p = \frac{\bar{D}_s}{I_s} + \frac{\bar{D}_L}{I_L}, \quad (1)$$

in which $\frac{\bar{D}_L}{I_L}$ should be constant

The values of $\frac{\bar{D}_L}{I_L}$, plotted against time of day showed a tendency to vary and appreciable spread due to experimental error was evident. The primary cause of this error was the shot noise in the photo-current resulting from the smallness of the available light intensity. To reduce the latter

a smooth curve was drawn through the points and the correction factor $\frac{D_L}{I_L}$ was taken from this curve instead of using the actual value obtained at a given time. A typical plot of the correction factor against time is shown in Figure 31. Figures 32 to 42 show $\log p$ plotted against $\sec \alpha$ for the 11 sets of observations made. These points should represent a linear relationship between $\log p$ and $\sec \alpha$. In view of the considerable spread in several plots it was necessary to find the most probable straight line through the points in as objective a way as possible. To do this the method of least squares, which is described in Appendix V, was used. This method yields the constants of the most probable straight line through the points and also the probable error in the resultant slope. The values of p given by the derived straight lines for the imaginary abscissa value of $\sec \alpha = 0$ are those corresponding to zero terrestrial sodium density, i.e. p_0 . The values of p_{01} and p_{02} for each run and the mean values of these are given in Table I. As already indicated the slope of the straight line drawn through the points of $\log p$ plotted against $\sec \alpha$ gives a value of $k'x$ which, once the effective absorption coefficient k' is known, yields a value of the number of absorbing atoms in a vertical column of unit cross sectional area, i.e. Nx . The values of $k'x$ obtained from the curves shown in Figures 32 to 42 are given in Table I. The method of finding k' must now be considered.

Figure 31. Plot of the correction factor D_1/D_2 against time of day.

M.S.T.

Figures 32 to 42 incl.

Plot of $\log p$ against relative path length $\sec \alpha$. The points indicated by the dotted circles were not used in deriving the slopes from the mean-squares analysis. The significance of the dotted curves shown in Figures 34 and 40 is discussed in Chapter VI.

Figure 33.
Noon to sunset 29.12.55.

Figure 33.
Neon to sunset 30.12.55.

Figure 34.
Neon to sunset 10-1-56.

Figure 35.
Noon to sunset 31.1.56.

Figure 36.
 Noon to sunset 26.2.56.

Figure 27
Root to Street 26.2-56.

Figure 3A.
NOON TO SUNSET 27.2.56.

Figure 39.
Sunrise to noon 28.2.56.

Figure 10.
Moon to sunset 28.2.56.

Figure 11.
Moon to sunset 2.4.56.

Figure 42.
Moon to sunset 14.7.56.

Date	$k'_1 x_0$	% error.	$k'_2 x_0$	% error.	Slope Ratio k'_1/k'_2	$P_{01}\%$	$P_{02}\%$	$Nx_0 \div 10^9$	% error.
29.12.55	-.063	12%	-.067	17%	.91	6.99	5.26	7.04	6.9%
30.12.55	.000	17% *	-.033	47%	.000	4.35	4.43	2.46	20%
10.1.56	-.0487	49%	-.092	15%	.523	5.81	5.47	8.27	17%
31.1.56	-.035	13%	-.020	31%	1.75	6.34	4.88	3.17	11.2%
25.2.56	-.042	12%	-.051	9%	.820	5.66	5.00	5.45	5%
26.2.56	-.052	12%	-.045	9%	1.15	6.38	5.00	5.63	5%
27.2.56	-.009	137%	-.035	45%	.257	5.64	4.86	2.64	48%
28.2.56 (forenoon)	-.018	91%	-.041	25%	.440	5.96	5.10	3.5	31%)
28.2.56 (afternoon)	-.058	34%	-.084	19%	.69	6.49	5.74	8.72	13%)
2.4.56	+.007	111%	-.019	46%	.000	5.15	4.61	1.06	40%)
14.4.56	+.014	77%	-.016	39%	.000	6.17	5.33	0.88	29%
					<u>.000</u> Mean 0.59				

) Daily
Mean
5.88.

Note: (*) Probable Error = $\pm .026$

Mean

$$P_{01} = (5.9 \pm 0.46)\%$$

$$P_{02} = (5.06 \pm 0.24)\%$$

Table 1. Measurements and results obtained from absorption observations.

5.3. Determination of the effective absorption coefficient.

The effective absorption coefficient k' may be readily obtained by observing a known amount of absorption in a continuous light source with the apparatus. To do this the sodium absorption cell and oven described in Appendix II, pages 145 to 165, was used. The absorption cell was mounted with its axis coincident with that of the light beam from the standard source, and placed between the standard source and the mirror M_2 - Figure 22. By varying the temperature of the absorption cell, sodium absorption lines of different depths could be produced and the effect of this absorption on the resonance emission observed.

Suppose the intensity of the radiation emerging from the absorption cell at room temperature (i.e. having negligible sodium vapour density) is I_0 . With the absorption cell at temperature T the radiation incident on the resonance lamp is:

$$I_f = I_0 e^{-k_f x}, \quad (1)$$

where k_f is the absorption coefficient at frequency f and appropriate to the cell temperature T and x is the length of the cell.

At the centre of the absorption line at frequency f_0 , the transmitted light intensity is:

$$I_{f_0} = I_0 e^{-k_{f_0} x} = I_0 e^{-k_0 x}, \quad (2)$$

since $k_{f_0} = k_0$ for small values of Nx for a Doppler-broadened line.

If the scattering function of the resonance lamp was such that the emitted resonance radiation was sensitive only to frequencies near f_0 , then D_L , the relative intensity of the resonance line, would be proportional to I_{f_0} , i.e.

$$D_L = Ce^{-k_0 x}, \quad (3)$$

However, the scattering function of the resonance lamp is such that the whole of the absorption line influences the resonance radiation and D_L must be defined in terms of an effective absorption coefficient k' , i.e.

$$D_L = Ce^{-k' x}, \quad (4)$$

It is shown by HUNTEN (38) that, for small values of $k_0 x$ (1), the effective absorption coefficient is related to k_0 by:

$$k' = \frac{k_0}{\sqrt{2}}, \quad (5)$$

In order to test the validity of this relation we proceed as follows. A relation between sodium vapour density and temperature is shown in Figure 43. This was obtained from the International Critical Tables (Reference 39). It was

Figure 43. Number density of sodium atoms plotted against vapour temperature.

obviously necessary to work with N_x values equivalent to those estimated for the atmospheric sodium. Assuming N_x is approximately equal to 5×10^9 for the atmospheric sodium layer, absorption tube temperatures in the region of 80°C are, evidently, required. Consider now the expression for k_o given by equation (8), chapter II viz:

$$k_o = \frac{2}{\Delta f_D} \frac{\sqrt{\log 2}}{\pi} \frac{\lambda_o^2}{8\pi} \frac{g_2}{g_1} \frac{N}{T}, \quad (6).$$

With T defined in the region of 80°C $\Delta f_D = 1.43 \times 10^9$,
Thus k_o may be written in terms of N , i.e.

$$k_o = 6.13 \times 10^{-12} \frac{g_2}{g_1} N, \quad (7)$$

$$\text{and} \quad k_o x = 6.13 \times 10^{-12} \frac{g_2}{g_1} N_x, \quad (8)$$

where x is the path length through the absorbing medium and in the calibration experiment equal to 10cm, the length of the absorption cell.

In performing the calibration the detecting system was set up exactly as for a solar run with the absorption cell in the path of the light from the standard source. The resonance lines from the resonance lamp were observed on the Varian recorder with the absorption tube at room temperature and at various controlled temperatures up to 100°C . Between 10 and 20 scans of the emission lines were obtained for each

control-temperature of the absorption cell and an average intensity value of the emission lines was used in the analysis. As the absorption cell temperature was raised the emitted D-lines were observed to diminish in intensity. The average D_L values were expressed as a percentage of the emission values with the absorption cell cold (i.e. when no absorption is operative). If equation (3) is expressed in logarithmic form a linear relation between $\log (D_L)$ and Nx results.

Thus $\log (D_L) = A - k_0 x$

$$= A - 6.13 \cdot 10^{-12} \frac{g_2}{g_1} Nx, \quad (9)$$

The log of the D values obtained experimentally were plotted against Nx . By plotting the theoretical relation $\frac{k_0}{\sqrt{2}}$ on the same graph the validity of equation (6) was immediately apparent. Table 2 shows the relative emission intensities of the D1 and D2 lines for two runs made on different occasions. The Nx values corresponding to absorption cell temperature were obtained from the plot of sodium vapour density against temperature (Figure 43) and from the fact that $x = 10\text{cm}$ for the absorption cell.

The data for the theoretical curves corresponding to $k_0/\sqrt{2}$ is given in Table 3. Figure 44 illustrates the close correspondence between the observed points and calculated line. In view of this the value for k' given by

Absorption cell temperature °C	Nx ÷ 10 ⁹	Observed intensity of emission lines.		log D _{L2}	log D _{L1}
		D _{L2} %	D _{L1} %		
101	32.5	78.4	76.5	4.362	4.337
94.5	18.5	91.5	99	4.516	4.595
84	7.2	97.5	95.5	4.579	4.559
50	0	98	100	4.585	4.605
20	0	100	98.5	4.605	4.59
93	16.3	85	93	4.443	4.532
82.75	5.4	91	96.5	4.511	4.569
70.75	1.95	99	100	4.595	4.605
COLD	0	100	96	4.605	4.564

Table 2. Data obtained from two calibration experiments to find k' in terms of k₀ using the sodium absorption cell.

Figure 44. Plot of $\log D_1$ and $\log D_2$ against Nx compared with a theoretical relationship assuming an effective absorption coefficient of $k_0/\sqrt{2}$.

$Nx \div 10^9$	k_0		$k_0/\sqrt{2}$	
	$D_{L1}\%$	$D_{L2}\%$	$D_{L1}\%$	$D_{L2}\%$
0.5	100	100	100	100
1.0	100	99	100	100
5.0	97	94	98	96
10	94	88	95.5	91.5
20	88.7	78	92	84
30	83	69	88	77

Table 3. Theoretical values of D_L for absorption coefficients of k_0 and $k_0/\sqrt{2}$.

equation (5) was used in determining N_x for the terrestrial sodium layer.

5.4. Evaluation of N_x .

In Figures 32 to 42 the slope for $\log p_2$ plotted against $\sec \alpha$ should be twice as great as that for the plot of $\log p_1$ against $\sec \alpha$. In order to obtain a representative slope for both lines the slope values were added and divided by 3, the resultant value corresponding to p_1 , this gives the D2 measurement twice the weight, as is reasonable. Using the averaged slopes the corresponding N_x values were obtained at once from equation (1) derived in chapter III, i.e.

$$k_0 x = 8.5 \cdot 10^{-12} \frac{g_2}{g_1} N_x,$$

$$\text{or } N_{x_0} = \frac{\sqrt{2} k' x \cdot 10^{12}}{8.5}, \quad (10)$$

since $\frac{g_2}{g_1} = 1$, for the slope defined above and

$$k_0 = \sqrt{2} k'.$$

The N_{x_0} values obtained and the dates of observations are listed in Table I.

CHAPTER VI.

DISCUSSION.

6.1. General.

The results obtained in the last chapter are now subjected to criticism. The absorption effects of terrestrial atmospheric water vapour are considered as a possible agent influencing the wavelengths 5890 and 5896 studied. The ratios of the slopes associated with the D1 and D2 sodium absorption are examined in detail. The unusual distribution of the points evident in certain of the $\log p$, $\sec \alpha$ plots is considered and shown to be consistent with an uneven geographical sodium distribution in the terrestrial layer as proposed by ROACH & PETTIT (49,50 & 51). Finally the dependence of the density of free sodium atoms in the earth's atmosphere as obtained by this method, on the time of year, is examined and a comparison made with measurements made from observations of the twilight emission of the D-lines.

6.2. Water Vapour Absorption.

That absorption of the wavelengths 5890 and 5896A by the earth's atmosphere has been observed in solar

radiation, there can be little doubt. The question that must be carefully considered, however, is - "can this absorption be definitely attributed to the atmospheric sodium?" In view of the great number and complexity of spectral distribution of the absorption lines produced by atmospheric water vapour, the possibility exists that the absorption observed is due to atmospheric water vapour. Several water vapour absorption lines are known to be present in the solar spectrum near the wavelengths studied in this investigation. These lines do not need to have their central wavelengths coincident with the centre of the sodium absorption lines. The absorption in the wings of even a weak water vapour line near the centre of one of the Fraunhofer D-lines could conceivably reduce the residual intensity in the solar spectrum at the bottom of the Fraunhofer line and produce the effect observed. The observations made between 29th December, 1955 and 28th February, 1956 were made on cold winter days at temperatures in the range -30° to $+10^{\circ}$ F. The subsequent run was planned with a view to checking on the absorption effects of terrestrial water vapour and was made on April 2nd on a warm day with temperature above the freezing point. Using the relation between water-vapour-pressure and temperature given in the International Critical Tables (Reference 39), it was estimated that the water vapour

density on this day was greater than that present in the atmosphere on the days when previous observations were made, by a factor of 25. In view of this large increase in water vapour density in the atmosphere an increased slope was to be expected in the plot of $\log p$ against $\sec \alpha$ if water vapour were responsible for the absorption previously observed. Reference to Figure 41 indicates, however, that less absorption was present for the observation made on April 2nd than in the observations made previously, thus ruling out water vapour as a possible absorbing agent. The following observation made April 14th, the results of which are shown in Figure 42, confirms the above conclusion. It may thus be assumed that the absorption of the wavelengths 5890 and 5896A in the solar spectrum by the terrestrial atmosphere is not attributable to water vapour.

6.3. The D1, D2 Ratios.

An important relation which must be established before the observed absorption can be confidently attributed to atmospheric sodium is the 2:1 ratio in the slope values k'_2x , k'_1x of the $\log p$, $\sec \alpha$ curves. Table 1 shows the ratio k'_1/k'_2 for each observation made.

The positive slopes found for the D1 absorption curves obtained on April 2nd and 14th deserve special attention. It should perhaps first be borne in mind that

the intensity of the D1 resonance emission line is only about 0.6 of that of the D2 emission line with the resonance lamp at the 145°C operating temperature and illuminated with continuous radiation. Thus measurements involving the D1 resonance line are likely to be less accurate than those obtained from the D2 resonance line. This is verified by the larger probable error attached to most of the values of k'_1x as compared to those associated with k'_2x values. It is also not inconsistent to observe zero slope for the D1 absorption and yet a measurable slope for the D2 absorption since a threshold of sensitivity for the apparatus must exist below which small amounts of absorption are not detected. This would simply mean that for small atmospheric sodium densities the apparatus would detect the larger D2 absorption and not the D1. A zero value can be assigned within the probable error to the k'_1x value found on April 2nd, and may be considered normal on the basis of the above reasoning. The positive value of k'_1x shows a large probable error, the minimum value still, however, being slightly positive. An examination of the Varian chart record for this observation shows that the last four points were obtained with low values of solar intensity. This necessitates the use of high gain in the detector amplifier of the spectrometer to produce easily measurable deflection corresponding to the resonance lines, on the Varian recorder. Employment of the higher

gain results in a decrease of signal to noise ratio in the amplifier output. The recorder deflections due to the sodium resonance radiation obtained from low solar intensities and observed using high gain, represent less accurately the emitted intensity than those obtained from brighter sunlight and observed with low gain. This is due to the relatively large noise fluctuations which appear when observing the weaker emission. These noise fluctuations, generally, increase the amplifier output when observing an emission line and introduce a tendency, in spite of precautions taken when making the measurements, to overestimate the intensity of weak emission lines. Such an over estimation would, of course, show up more noticeably in the weaker D1 resonance emission line, and may be responsible for the high log p values made near sunset and resulting in the positive slopes, for the days in question. On the other hand the possibility of a systematic error introducing a positive error into the log p, sec α slopes can not be ruled out. It is encouraging to note, however, that such an error would cause the observed absorption to appear less than that actually operative and cannot, therefore, be confused with the absorption attributed to the atmospheric sodium.

In an examination of the slope ratios shown in Table 1, the D1 slope for the last two observations was

assumed to be zero. In this case three small values of k'_2x are associated with zero values for k'_1x . This agrees with the suggestion made above that small absorption in the D1 line may be below the sensitivity threshold for the apparatus. The ratio k'_1/k'_2 for the observations made 10th January, 1956 and 28th February, 1956 (forenoon) agree very closely with the theoretical value of 0.5, while the ratios for runs made 27th February, 1956 and 28th February, 1956 (afternoon) agree within the probable errors associated with the slope values. The average value of all the ratios obtained is 0.59. The 2:1 ratio in the absorption coefficients associated with the D2 and D1 sodium lines is thus established.

6.4. Sodium Clouds.

An examination of the distribution of the points in Figures 32 to 42 reveals large differences in the spread. While the majority of the figures show a more or less progressive fall-off in the value of $\log p$ as $\sec \alpha$ increases, Figures 34 and 40 show, as indicated by the dotted line, sets of points which seem to lie along a 'U' shaped curve rather than a straight line. The results shown in Figure 40 are particularly suitable for study since, together with those shown in Figure 39, they represent observations taken on a single day, February 28th; Figure 39 being a plot of the forenoon and Figure 40a plot

of the afternoon observations. The observations taken before noon show a conventional form and yield a small N_x value; the afternoon values of $\log p$ fall off suddenly in the early part of the run and then remain fairly constant over $\sec \alpha$ values from 3 to 5. The average slope is large and a relatively large value for N_x is obtained. These characteristics are also to be found in Figure 34, and are readily explained if an uneven geographical distribution of sodium is assumed.

Considerable evidence is available to support the suggestion that the terrestrial atmospheric sodium is not uniformly distributed in the geographical sense. ROACH & PETTIT (49 & 50) have examined the geographical sodium distribution of excitation regions in the night sky and observe localized regions of maximum sodium D excitation. They find in general that during the months of minimum atmospheric sodium density that the sodium D emission is more intense in the South than in the North. During the months of high atmospheric sodium density, October to February, the sodium D emission is more intense in the North. In particular ROACH (51) has suggested a southward migration of a sodium cloud, from 45° lat. N to 5° lat. S in the months January to June and the equivalent northward movement in the months July to December. In any case, in view of the high wind velocities which have been observed in the upper atmosphere at the level of the sodium layer a

uniform geographical distribution of sodium is less likely than a non-uniform one.

Returning to Figures 34 and 40 it must be remembered that the $\sec \alpha$ values correspond to widely differing azimuth angles of the sun and hence of the region of the sodium layer producing the absorption. If as the sun moves to the West in the course of an afternoon observation, the observed sun rays pass through increasingly dense regions of the layer there will be two factors tending to increase the amount of absorption (a) the increase of path length, (b) the increase in density of the absorbing vapour. If on the other hand the observed sun rays progressively intersect the layer in regions of diminishing density the effect on absorption of the increasing path length are opposed by the tendency to a reduction in observed number density. The net effect is that the observed p values remain fairly constant as the sun sets. If one assumes a localized region in the sodium layer of high sodium density in the direction of the sun equivalent to $\sec \alpha = 3$ in Figure 40 and $\sec \alpha = 4$ in Figure 34, the distribution of the $\log p$ values is explained. The p values obtained in the course of the morning observations of February 28th (Figure 39) show a linear $\log p - \sec \alpha$ relation indicating an even sodium distribution to the East of the observer. The afternoon observations suggest higher sodium densities to the West with a highly dense

localized region in the approximate South-West.

The foregoing remarks are made with a view to explaining the unusual distribution of points in Figures 34 and 40. It is not possible from the scanty data of this type available to make inferences regarding the geographical distribution of sodium in the terrestrial atmospheric layer.

6.4. Seasonal Variation of N_x .

It is of interest to compare the values of N_x obtained from measurements of terrestrial sodium absorption with the seasonal variations of number density obtained from observations of the twilight D-line emission. Figure 45 shows the measurements of HUNTEN (28) made prior to this investigation and the measurements made by the author plotted against time of year. The seasonal variation shows up very well in the measurements of HUNTEN. Five of the measurements made by the author agree well with those made by the above investigator; five measurements - one made in late December, 1955, one made in January, 1956 and three made in late February, 1956, appear to yield excessively high N_x values. However, consideration of the self-absorption effects operative when the twilight emission is observed during periods of high terrestrial sodium density shows that these large values of N_x are not inconsistent with those predicted theoretically. The resonance radiation

Figure 45. Seasonal variation of N_x - after Hunten. (•)
 N_x values obtained from absorption measurements (+).
 N_x values obtained from twilight measurements on the same day as absorption observation. (⊗).

emitted in the terrestrial sodium layer is, in part, re-absorbed by unexcited atoms in the layer. For small sodium densities this effect is not great. As the sodium density increases the intensity of the emitted D-light increases but the absorption by the more numerous unexcited atoms in the path of the emitted light also increases; the net effect being that the emitted intensity does not increase proportionately with the increase of sodium atomic density. The theoretical aspects of the secondary absorption of the emitted D-light in twilight have been discussed by HUNTEN (28) and CHAMBERLAIN (53) and the conclusion is reached that higher sodium number densities than those calculated from the twilight measurements probably prevail during the periods of high atmospheric sodium density.

6.5. Comparison of N_x values obtained from absorption, and twilight measurements made on the same day.

Three measurements of N_x were obtained from twilight observations made on or near the same day as observations of the terrestrial sodium absorption. The results of a twilight observation made on January 9th can be associated with the absorption measurements made on January 10th. Twilight observations made February 28th and April 14th were made on the same days as absorption observations. The results obtained from these twilight observations are given below in Table 4.

Date.	Observed Nx.	Recommended Nx.
Jan. 9th	4.2×10^9	2.5×10^9
Feb. 28th	3.10×10^9	3.1×10^9
Apr. 14th	2.5×10^9	1.5×10^9

Table 4. Nx values obtained from twilight observations made on the same days as absorption observations.

The uncertainty in the Nx values arises from the fact that little time was available for setting up the spectrometer outside for the twilight observations, the instrument being in use for the absorption measurements inside the observatory until sunset. The D-lines were observed in the second order for the absorption experiment; the first order was always used for the twilight measurements. The anomalously high Nx values associated with two of the runs shown in Table 4 may possibly be attributed to the twilight emission being observed in the second-order instead of the first, as a consequence of the hasty setting up of the spectrometer. The observed values, assumed to correspond to the second-order, are corrected to correspond to first-order observations

by multiplying by the factor 0.6, and appear in the column headed Recommended Value. Comparison of the recommended Nx values with those previously obtained by HUNTEN (28) from twilight observations, Figure 14, shows close agreement. The Nx values obtained from absorption and twilight observation on April 14th show close agreement. The Nx values obtained from absorption measurements are about 2 and 3 times greater than those obtained from twilight emission on February 28th and January 9th and 10th respectively. For small sodium densities the self-absorption in the twilight emission is small and the two methods should yield comparable results. For higher sodium densities with self-absorption reducing the observed intensity of the twilight emission, higher Nx values are to be expected from absorption measurements than from twilight measurements. The agreement between the two sets of results is thus satisfactory.

CHAPTER VII.

CONCLUSION.

The preliminary measurements described in chapter II were made in the winter months of 1954-55. Work on the first method used to detect the telluric sodium absorption was commenced in May, 1955. The experience gained in building the equipment for Method I proved valuable. The techniques of detecting small intensity changes in two related light beams are of general utility and may possibly find application in other fields of study. Method II first began to bear fruit in December, 1955 and was profitably used in the course of the following four months.

The success of the techniques finally used may be attributed primarily to three factors

- (a) the extremely high resolution achieved by the resonance scattering principle,
- (b) the excellent performance of the rapid scanning spectrometer available for the investigations, in the face of extended periods of operation, and
- (c) the high efficiency of the temperature control units.

A considerable disadvantage lay in the fact that the investigator is always dependent to a large extent upon ideal weather conditions. Observations can only be made with the atmosphere perfectly clear and such conditions must persist for the duration of the noon to sunset period. The number of days, at least in the winter months, when such conditions prevail, are very few.

The results indicate, positively, absorption of the wavelengths of the sodium D-lines in the solar spectrum by the atmosphere of the earth. A critical examination of the results shows that the observed absorption can safely be attributed to atmospheric sodium. Three principal factors emerging from the analysis of the results support the conclusion that absorption by atmospheric sodium is observed. These are:

- (1) The exclusion of water vapour as a possible absorbing agent;
- (2) The fact that the D2 wavelengths are absorbed more strongly than the D1;
- (3) The agreement between the observed seasonal dependence of sodium density and that reported by other investigators.

The unusual form of some of the plots of $\log p$ against $\sec \alpha$, suggests localized regions of relatively high density in the atmospheric sodium layer. This 'lumpiness'

is reported by ROACH & PETTIT (49 & 50) and ROACH (51).

The investigation yields values of the residual intensities at the bottom of the Fraunhofer lines of $p_{01} = 5.9 \pm .46\%$, $p_{02} = 5.06 \pm .24\%$. These values agree well with previous measurements made by the author and are believed to be the most accurate yet made for the whole of the solar disc.

An improved type resonance tube, using re-entrants at the entrance and exit windows to minimize reflected light was designed. Because of the satisfactory performance of the original resonance tube made, this improved type resonance lamp was not constructed. However, it would perhaps be interesting to experiment with a lamp of this type.

An improvement to the techniques of Method II, suggested by Dr. D.M. Hunten, would be the incorporation of ratio-recording for the resonance emission. This would permit the emission from the lamp to be recorded as a fraction of the exciting radiation and automatically compensate for fluctuations in the latter.

APPENDIX I.

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THE INTENSITY AT THE BOTTOM OF THE FRAUNHOFER D LINES¹

By J. A. SCRIMGER AND D. M. HUNTEN

ABSTRACT

The line profiles were measured by scanning a photomultiplier and exit slit in the second order of a 21-ft. grating. The large distortions resulting from finite resolution were evaluated by means of an absorption cell containing sodium vapor with helium at a pressure of 170 mm. This produced an artificial line whose central intensity was known to be zero. By matching its profile carefully to that of the solar line the central intensities of D_1 and D_2 were found to be $(5.0 \pm 0.6)\%$ and $(4.8 \pm 0.6)\%$ in terms of the nearby continuum.

1. INTRODUCTION

One of the most valuable results coming from studies of the twilight sodium emission is an estimate of the number of free sodium atoms per square centimeter column in the upper atmosphere (1, 5). The data required are the scattered intensity, the scattering power of a sodium atom, and the intensity of the incident sunlight. The first two are known with reasonable accuracy, but the last is very uncertain. The intensity of the continuous spectrum is well known (6), but the radiation active in exciting the twilight scattering is at the bottom of the Fraunhofer D lines. The ratio of the central intensity to the continuous intensity (which we will call p) is not known with any accuracy. The best and most recent measurement is that of Priester (8) who used a Lummer plate crossed with a high dispersion spectrograph to observe the line profiles. The recording was done photographically, and separate measurements were made for various parts of the sun's disk. Substantial corrections had to be made for finite resolution and scattered light.

It was felt that an independent measurement by a different method, using light from the whole sun, would be desirable. This consisted of a measurement of the line profiles using a 21-ft. concave grating, and a second measurement under similar conditions but with the central intensity reduced to zero by a sodium vapor absorption tube. The difference then gave the central intensity directly. Direct photoelectric photometry was used in order to eliminate uncertainties in the standardization of photographic plates. The agreement of our results with Priester's is highly satisfactory.

2. PRELIMINARY MEASUREMENTS

It was thought at first that direct photometry with the grating would yield the required accuracy. The spectrograph is an Eagle type using a 15,000 lines per inch grating 5 in. wide. A photomultiplier and exit slit were mounted so that they could be scanned for a short distance tangent to the Rowland circle in the second order. A coelostat was arranged to bring in the sunlight without forming an image, so that the results were for the whole sun.

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A simple double monochromator was set up ahead of the grating instrument and adjusted to pass about a 20 Å band. Errors caused by scattered light were thus greatly reduced. The detector was a 1P21 photomultiplier followed by a simple but stable and linear direct-current amplifier. A similar system at a fixed position in the first order served as a monitor for changes in the incident intensity. The circuit of the amplifier was similar to one already published (4), but with the electrometer stage omitted since the photomultiplier load resistance was only 50 megohms. A two-pen recording potentiometer allowed both signals to be recorded on one chart.

The theoretical resolving power of the grating in the second order is 180,000; in practice we were able to achieve 100,000. The resulting distortion of the *D* line profiles should be small; however the values of p were about 20% rather than about 6%. Because of the double monochromator this could not be attributed to scattered light. The instrumental line profile was therefore measured using the 5790 Å line of mercury 198. (The loan of the source was kindly arranged by Dr. A. E. Douglas of the National Research Council and Dr. A. Vallance Jones of this Department.) The width halfway down was about 0.06 Å, giving the resolving power already quoted, but there were faint wings extending nearly 0.4 Å out from the center. These would have very little effect on the sharpness of lines in emission spectroscopy, but would result in a serious filling-in of the absorption lines, as was observed.

An attempt was made to correct the observed profiles by a method resembling that of Hardy and Young (3). For D_1 and D_2 the resulting values of p were 8.5% and 7.7%. These cannot be considered at all accurate, since the correction is about twice the final result.

3. THE SODIUM VAPOR TUBE

The correction at the center of a line was evaluated directly by forming an absorption line whose shape matched that of the solar line but whose central intensity was known to be zero. Any observed intensity was spurious and was of course equal to the required correction. The artificial line was produced by passing sunlight through a tube containing sodium vapor and helium at the correct pressures. The helium was required to make the line broad enough and smooth enough, since pure sodium vapor gives a very narrow line in the pressure range that was used.

Since glass is attacked by hot sodium vapor, the tube was lined with a glaze used for vapor lamps by the General Electric Company (generously given by Mr. R. W. Schmies). This is normally used with a special glass of matching thermal expansion, but was found satisfactory with Pyrex. The glaze must be fired at a temperature in the softening range of the glass, and it is difficult to keep the flat glass for the windows from distorting. This problem was solved by making a layer about 1 mm. thick of powdered graphite on an iron plate and packing it by pressure with the window blank. Heat could then be applied from above and below. The uncoated surface remained adequately smooth and kept its polish. The coated surface was finely pebbled and had to be polished with rouge on a felt pad. The two windows were found to remove about half the light from a collimated beam.

The diameter of the tube was 34 mm. and the length 92 mm. Many of the first attempts failed when the tubes cracked after the windows were sealed on, because of strains introduced when the glaze was fired. This difficulty was removed by annealing the parts in an oven before assembly. The completed tube was also annealed. Because a considerable pressure of helium was to be used, the tube was not carefully outgassed. It was evacuated by a single-stage oil diffusion pump, flushed with helium, evacuated again, and filled to the final pressure of 170 mm. Sodium purified by vacuum melting and distillation was introduced from a thin-walled ampoule.

The profile of the absorption line at different temperatures was calculated from equation (97) of MZ (book by Mitchell and Zemansky, Reference 7). This equation assumes Lorentz broadening, but should give a good approximation at the pressure used. For the pressure of helium used and for a temperature near 180°C., equations (104) and (96) of MZ give a Lorentz damping ratio $a' = 1.5$. Some lines were also calculated for $a' = 0.5$ and 1.0 to find the effect of a' on the line shape. To a very good approximation a temperature giving the same shape could be found for each value. Since there is no optimum value of a' the largest was used, so that the temperature could be kept relatively low. Calculations were made with the aid of Appendix IX in MZ. The vapor pressures used were those of Edmondson and Egerton (2).

4. MEASUREMENTS AND INTERPRETATION

The observing procedure was very simple once the spectrograph had been adjusted to give the sharpest possible lines. The two D lines were observed in different runs, since too much time would otherwise have been lost in scanning between them. The line was scanned several times as the sodium tube slowly cooled. Another measurement was made with the tube removed from the optical path. The position of the continuum was estimated by drawing a line through the trace about 1 Å from the center. The observed

FIG. 1. The observed central intensities for D_1 and D_2 as a function of the temperature of the sodium vapor tube. The temperatures found from Figs. 2 and 3 are marked on the curves.

residual intensity was the ratio of the minimum deflection to the deflection for the continuum. The results are plotted in Fig. 1 as a function of temperature of the tube. Since there is no break in the curves indicating the desired point, it is necessary to estimate the point with the aid of the calculated absorption line profiles.

The line desired has the same shape as the solar line but is displaced downward so that its central intensity is essentially zero. The shape used was that found by Priester (8) for $\sin \theta = 0.5$, θ is the angle between the line of sight and the normal to the sun's surface. An average shape for the whole disk would be preferable, but did not seem worth the considerable labor of calculat-

FIG. 2. Profiles of the D_1 line 5896 Å. The upper solid curve with points is Priester's measurement. The lower is the same displaced down. The three dotted curves are calculated for sunlight passing through the sodium vapor tube at 160, 170, 180°C. (left to right).

FIG. 3. Profiles of the D_2 line 5890 Å. The upper solid curve with points is Priester's measurement. The lower is the same displaced down. The three dotted curves are calculated for sunlight passing through the sodium vapor tube at 150, 160, 180°C. (left to right).

ing it. However, the average central intensity was calculated, allowing for the contribution of each region to the total area and for limb darkening. The latter was represented by the approximation suggested by Minnaert (6), with a coefficient $\beta = 1.6$. For D_1 and D_2 the results were $p_1 = 6.6\%$ and $p_2 = 5.8\%$. As would be expected, these are close to the values observed at $\sin \theta = 0.60$.

The line shapes actually achievable are shown by the dotted curves in Figs. 2 and 3. Also shown are the shape of the solar line as found by Priester and the same curve displaced down until it touches the horizontal axis. The dotted curve most nearly matching the desired curve in the region of the instrumental profile is the one desired. The best temperatures are 167°C. for D_1 and 158° for D_2 , with uncertainty of about $\pm 2^\circ$ in both cases. Reference to Fig. 1 then gives $p_1 = (5.4 \pm 0.3)\%$ and $p_2 = (5.2 \pm 0.3)\%$. It would seem reasonable to double the error figures to allow for uncertainties in temperature measurement, the vapor pressure relation for sodium, and the curves of Fig. 1. Account must also be taken of the fact* that the intensity $1 \text{ \AA}$ from the line center is only about 92% of the continuum (9). The final result is therefore

$$p_1 = (5.0 \pm 0.6)\%,$$

$$p_2 = (4.8 \pm 0.6)\%.$$

5. DISCUSSION

There is little doubt that p_2 should be less than p_1 ; it is satisfactory that our result is in agreement with this. The results of Priester already quoted ($p_1 = 6.6\%$ and $p_2 = 5.8\%$) differ somewhat more and are a little higher. Our values may be taken as giving a general confirmation of Priester's, but as indicating that his may be slightly too high. This is not impossible, since the corrections for instrumental effects, though far smaller than ours, were still appreciable.

It is possible that slightly better accuracy might be achieved by averaging several measurements of this type, but far greater improvement could be made by starting with a spectrograph of higher resolution. Another possibility is to use a double-beam system with one beam passing through the sodium cell and with rapid alternation of the two beams on the phototube. A measurement by this method is now in progress.

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*We are grateful to the referee for pointing out this correction.

APPENDIX II.

II.1. General.

This Appendix deals with the equipment designed and built for detecting the terrestrial atmospheric sodium absorption in solar radiation using Method I. Some of the equipment, e.g. the absorption tube and oven, found application in Method II but is described only in this Appendix in order to avoid repetition.

II.2. The Monochromator and beam comparator system.

A diagram of the arrangement of the components which comprised the optics of Method I is shown in Figure 46. These components were housed in a light-tight enclosure measuring $42\frac{1}{2}$ by 19 by 10 inches with a $\frac{3}{4}$ inch reinforced plywood base and a partition AA' dividing the monochromator from the rest of the system so as to reduce pick-up of stray light by the photo-detector P. Sunlight was focused on the entrance slit S1 by the cylindrical lens L1. The entrance slit S1 was at the focal point of a $3\frac{1}{2}$ inch diameter lens L2 of focal length $38\frac{1}{2}$ inches. Light from the entrance slit S1 was thus made parallel by L2 before striking the grating G. The latter was a 4 by 4 inch replica grating having 15,000 lines per inch. An image produced by the grating and lens L2 was deflected by a mirror M1 so as to fall in the plane of the exit slit

S2. The second order of the grating was used so that the D-lines produced by illuminating S1 with sodium light were separated by approximately 1 mm. When making observations using sunlight the grating was adjusted so that one of the Fraunhofer lines fell on the exit slit. The band width of the monochromated beam emerging from the monochromator at S2 was approximately 2A.

Light from S2 was collimated by lens L3, the collimated beam passing to the beam splitter Bs. The transmitted beam A was deflected by mirror M2 so as to be aligned parallel to the reflected beam and in such direction as to fall ultimately on the photo-cell. The baffle BB' was placed directly in front of the chopper k. The former had two apertures situated at equal heights from the base equivalent to two diametrically opposite segments on the chopper and was included to make the two beams of uniform cross section. The chopper was driven at 1800 r.p.m. by an electric motor and contained five symmetrically arranged apertures, each bounded by a pair of concentric circles and two radii subtending an angle of 36° at the centre of the chopper disc. Each beam, and the light source in the reference signal generator, was thus interrupted five times for each revolution of the chopper, producing a modulation of the beam, and a reference signal, of frequency 150 cps. The modulated beam A was passed through

Figure 46. Optical layout of Method I.

the absorption cell X and was focused by lens L4 on to the cathode of the photo-multiplier P. Beam B was deflected by mirrors M3 and M4 and also focused by lens L4 to fall at approximately the same point on the photo-cathode as beam A.

The mirrors M1, M2, M3, and M4 were similar and made from $\frac{1}{4}$ inch plate glass. Two-inch squares of the glass were cleaned and mounted in a vacuum chamber and on evacuation of the chamber were coated with aluminum by vapourizing the metal on a hot tungsten filament. This technique is described by STRONG (36). The mirrors were then cemented to brass plates using Javabond cement and then readily supported in the adjustable mirror mounts. The comb light attenuator, illustrated in Figures 46 and 13 functioned simply by advancing or withdrawing the comb edge 3 into or from the light beam, using a thumb screw to control the degree of attenuation.

II.3. The Absorption Cell Oven.

The absorption cell oven was designed to meet the following requirements:

- a. An operating temperature range of 80 - 180°C.
- b. Temperature control within $\frac{1}{4}$ °C.
- c. As good a transmission to light as possible, preferably of the order of 80 - 95%.

- d. A temperature distribution inside the tube such that the ends of the oven be about 5° warmer than the central region.

A drawing of the oven is shown in Figure 47.

The oven body was simply a $6\frac{1}{4}$ inch length of 3-inch diameter, $\frac{3}{16}$ inch walled, copper pipe. The oven ends were $\frac{1}{8}$ inch thick brass discs with $1\frac{3}{8}$ inch holes cut centrally for the oven windows. End B was set into the recessed copper pipe and held permanently by radially located screws. The other end of the tube, designed for quick removal, was fitted flush with the end of the copper pipe and was held by means of the easily accessible screws S2. The cell supporting cradles C, auxiliary heating coils h and partitions P were attached to the oven end A so that the internal assembly of the oven could be readily extracted. The glass discs forming the oven windows were recessed into the brass end plates and held by means of annular rings O. Heat was supplied to the oven by 22 B&S Nichrome wire forming a heating element wound on the outside wall of the copper pipe and also by two small auxiliary heaters h which could if necessary be switched off. Approximately 26 turns of wire were required for the main heater. These were distributed so as to give small spacing between turns at the oven ends and so provide for a differential heating - tending to maintain the ends of the oven at a higher temperature than the centre. The total

Figure 17. Diagram of the absorption cell over.

resistance of the winding was $23\frac{1}{2}$ ohms. A single layer of asbestos paper wrapped around the outer wall served to insulate the heater winding electrically from the copper pipe. Thermal insulation was then provided by rolling on several layers of asbestos paper in the manner described in chapter II. The heater winding was anchored by forming the end windings into closed loops; by tightening these loops on to the pipe the wire was held securely. The Nichrome wire was then brought out from the insulation to a terminal board T1 strapped to the oven.

The auxiliary heating coils were connected electrically as shown in Figure 48 (design details are given in

Figure 48. Control circuit for the auxiliary heaters.

the section describing the temperature control unit). These elements were made from fine Nichrome wire wound on to a $\frac{1}{4}$ inch former. The resultant elements were sufficiently

rigid to enable them to be mounted without the former in the oven by attaching each end of the coil to the support rods r by two 4BA nuts. The asbestos paper which was wrapped around the tube to form a heat insulation was extended about $\frac{3}{4}$ inch over each end of the tube. This provided a wall at the tube ends facilitating insulation of the brass end discs. End B was packed with a moistened fire brick powder which hardened to produce an excellent insulating block. Insulation at the removable end consisted of a $\frac{1}{4}$ inch asbestos annular ring G, additional insulation being provided by a rock-wool packing H. The cell cradle C and circular dividing walls P were cut from $\frac{1}{4}$ inch asbestos sheet and held in position by nuts on the 4BA threaded support rods r. By insulating the latter from the end wall A they were made to carry the electrical supply to the auxiliary heating coils h. The thermistor temperature control element R_T was located near the oven wall in one of the end sections of the oven, the leads being led out via asbestos insulating bushes in end wall A. The thermistor leads were then taken to a terminal board T2.

This oven fulfilled the requirements set out above. The partition P, the local heating coils h and the insulation provided at the end walls, enabled the end compartments to be maintained at two or three degrees higher temperature than the central section. Temperatures in the specified range were readily attainable. Little or no

light attenuation was produced by the windows. The method of obtaining the temperature control is described below.

II.4. The Oven Temperature Control Unit.

Two temperature control units were built of slightly different design, both based on the same principles. The first unit built was designed for use with the above oven and will be fully described in this appendix. The second unit was built for controlling the temperature of the resonance tube oven and contained certain modifications and improvements. This unit is mentioned in Chapter IV. The circuit diagram of the latter together with some notes on the modifications made are given in this appendix. The general principles described below are applicable to both units.

The first control unit built is shown in Figure 49. This design was based on a circuit described by BENEDICT (41). The control system utilized a temperature sensitive element which formed one arm of an ac bridge (Reference 40). The unbalance output of the bridge was used to regulate the temperature in the system. A type 14A thermistor, produced by the Western Electric Company, was used for the temperature sensitive element. This is a resistance having a large negative temperature coefficient, a description and design data being available in the pamphlet "New Horizons Using Design Components" (43). Many temperature control

systems have been described, e.g. BURWELL & PETERSON (42), which have much greater sensitivity than that of BENEDICT (41). However, these are relatively complex, whereas the simple circuit shown in Figure 49 provided ample control. The large sensitivity was provided by the thermistor R_T and the high gain of the 6SN7's in the two stage amplifier V1 and V2.

The thermistor control element R_T formed one arm of the ac bridge R_1, R_2, R_3, R_T . The resistance of this type of thermistor varies from 5k to about 500 ohms in the temperature range 95 - 180°C. A three-turn variable resistance helipot was used for R_3 . When the helipot resistance was adjusted to a value of 5k, the bridge was balanced with the thermistor at approximately 95°C. This position of the helipot thus corresponded to a control temperature of 95°C. Smaller values of the helipot resistance corresponded to higher control temperatures. Helipot settings thus corresponded to control temperatures. Resistances R_1 and R_2 were 3k precision resistors. The 5v ac output from the transformer T1 supplied power for the thyatron filaments and energized the bridge. The unbalance output, together with an out-of-phase component from R_5, C_1 and C_2 of the bridge was amplified by tubes V1 and V2 to form the control voltage which was fed to the grid of the thyatron V3. The live side of the mains 110v supply was fed via the heating element R_H and switch SW1

to the plate of the thyatron. The thermistor control element regulated the temperature in the following way.

A permanent voltage E_1 of amplitude and phase determined by the setting of potentiometer P_1 is added vectorially to the unbalance voltage E_2 of the bridge at X as shown in Figure 50. The resistance of the control element R_T is highly dependent on the oven temperature. Any change in the latter thus affects the balance of the bridge. Unbalancing the bridge causes a change in phase of the ac control voltage on the thyatron grid relative to its plate voltage. The phases are arranged so that when the oven temperature falls below the operating temperature, the control voltage moves into phase with the plate voltage of the thyatron, causing it to conduct sooner in the ac cycle and thus increase the heater current. The operating temperature is thus restored. If the oven temperature rises above the desired value, the control voltage moves out of phase with the thyatron plate voltage causing its plate current to be reduced or cut off, depending on the degree of unbalance of the bridge. The control action is illustrated in Figure 50. Figure 50a shows three resultant voltages E_{g1} , E_{g2} , E_{g3} from the bridge corresponding to the oven temperature T less than, equal to and greater than the desired temperature T_c respectively. These voltages are produced by the vectorial

Figure 50. Phase relations in the temperature control unit.

(a)

(b)

addition of the unbalance voltage E_2 and the preset out of phase voltage E_1 . Figure 50b shows these voltages plotted in relation to the thyatron plate voltage E_p . In case (1) plate and grid voltages are approximately in phase permitting pulses of plate current I_{p1} of relatively long duration. This corresponds to an excessive supply of energy causing the oven temperature to rise. In case (2), the control temperature T_c is reached. The phase of the control voltage is such as to allow the thyatron to conduct for shorter periods corresponding to a reduction in the net dc. The current I_c in this case is, ideally, just sufficient to produce heating to compensate for the heat losses from the tube. Case (3) illustrates the formation of the approximately anti-phased control voltage which holds the thyatron cut-off. In the absence of the heater current the oven temperature falls until T approaches T_c and the thyatron again conducts as the control voltage moves into phase with the thyatron plate voltage E_p .

Obviously the bridge output has to be such that excessive oven temperatures cause the heater supply current to decrease and vice versa. This condition is readily achieved by choosing the appropriate phase for E_1 . If voltage E_1 shown in Figure 50a does not produce such control, P_1 is adjusted to give the anti phased voltage E_1' . This produces an approximate reversal of phase of the control

voltages. The potentiometer P1 also serves as a sensitivity control. With P1 set to give large amplitude values of E1 the unbalance voltage has a relatively small effect on the phase of the sum voltage $\overline{E1} + \overline{E2}$. With P1 set for a small amplitude output of E1, variations in the unbalance voltage amplitude cause large changes in phase together with a variation of amplitude for the sum voltage $\overline{E1} + \overline{E2}$. The system is, therefore more sensitive to temperature changes. The amplitude value of E1 must, however, be held sufficiently high to maintain adequate control of the thyatron.

In practice excellent control of the temperature was achieved. When the control temperature was reached after first switching on the heater current, large current oscillations between zero and 1.2A were apparent. This 'hunting' gradually diminished and the current settled to an almost steady value I_c (within the range 0.65 - 0.75A at 150°C), after about 10 minutes. The optimum value of plate load for the FG27A type thyatron proved to be 23 ohms. This defined the resistance for the heater winding. With the auxiliary heaters h shorted out, about 11 watts ($I_p = 0.7A$), maintained an oven temperature of 150°C.

Assuming a value of 20 ohms for the resistance of each of the auxiliary heater windings, the net thyatron

plate load with the auxiliary heaters in circuit is 33 ohms.

An approximate plate current given by:

$$I_p = \frac{23}{33} \times 0.7 = 0.5A ,$$

was to be expected when the auxiliary heaters were switched on with the oven controlled at 150°C. The wattage consumed by the main heater winding is then 6 watts, with 0.6 watts consumed in each auxiliary element. In practice a slight rise in the value of heater current must occur to restore the input power to about 11 watts. Power was then distributed so that 10% of the total power was dissipated in each of the auxiliary heaters. This proved ideal for producing the temperature differential required in the oven, and the auxiliary heaters were made having a resistance of 20 ohms.

In thermally regulated systems, care must be taken to ensure that temperature changes produced by the heater windings are transmitted with as little delay as possible to the control element. In heating systems of the oven type there must always be a lag between the instant of changing the power in the heater elements and the instant that the control element senses a change in temperature. If this lag is appreciable the oven temperature will oscillate about the control valve, the amplitude of the oscillation depending upon the tightness of the coupling between the sources of heat and the thermo-sensitive control

thyatron was "warming-up".

- (b) It enabled a larger initial heater current to be passed. This reduced the warm-up time of the oven.
- (c) It served to prolong the thyatron life, since near the control temperature the thyatron was only called on to deliver 0.4 to 0.8 amps.

The warm-up time of the oven was generally greater than that specified for the thyatron so that once the desired oven temperature was reached, switch SW1 was changed from the "warm-up" to "control" position.

Amplification of the control voltage was achieved using a conventional two stage amplifier V1, V2. Condenser C3 was incorporated to filter out mains interference from the input stage of the amplifier. This was particularly important when two units of this type were used simultaneously. A large pulse was fed into the mains supply each time the thyatron fired. This pulse was picked up by the other unit, disturbing its normally smooth control action. The 300v H.T. and 6.3 filament voltage for the control voltage amplifier were obtained from a Lambda power supply.

The regulation produced by this control unit was estimated as $\pm 0.1\%$ at 150°C . It proved simple to operate

and required no attention while running. Its main disadvantage lay in the fact that the mains voltage was fed directly to the circuit via the small resistance oven heater. This provided a considerable hazard for the user. A second, self-contained unit, the circuit of which is shown in Figure 51, was built for supplying power to the resonance lamp oven. Some notes will be given here to explain the modifications apparent in this circuit. The unit was made self-contained by the inclusion of T1 which provided (in conjunction with the rectifier V1 and smoothing network R1, C1, C2) 100v HT for the amplifier V2, V3 and a 6.3v filament voltage. The auto transformer T3 made available a 220v ac plate voltage for the 5559 thyratron control tube V4, the increased plate voltage being provided to improve control of this tube. A simple 2:1 step-up transformer would be desirable here to achieve an isolation of the control unit from the line supply. (Such transformers were ordered for this purpose but the auto transformers were received in error). In view of the short warm-up time of the oven, the warm-up control switch was not used.

II.5. The Absorption Tube.

In Chapter II an account is given of the construction of a sodium absorption cell required for the production of pressure broadened sodium absorption lines. A temperature

Figure 51. Temperature control unit for the resonance lamp oven (improved version of the circuit shown in Fig. 49).

range of 100 - 200°C was specified for the cell. This cell was therefore coated with a sodium resistant glaze to protect the glass from attack by hot sodium vapour. The temperature range necessary for the absorption cell used in Method I was 90 - 140°C. In view of this low operating temperature the second cell was constructed without the use of the protective glaze. The technique used to introduce the sodium and evacuate the tube were similar to those described in Chapter II. However, it was important in this case to obtain as low a foreign gas pressure as possible, as a weak absorption line produced by pure sodium vapour was required. Several refinements, aimed at producing a very high vacuum in the cell, were therefore introduced. These were:

- (1) The use of a baking-out oven for degassing the cell.
- (2) Use of a three stage diffusion pump.
- (3) Pressure measurement using an electronically controlled ionization gauge fused into the pumping system.

Some notes on this equipment are given below: The reader is referred to chapter II for a description of the method of introducing the sodium.

The baking out oven is shown in Figure 52. This was constructed using Silocell bricks strapped by iron bands. The lid consisted of two bricks fitting flush with

the top, each having a semi-circular groove cut in the side to permit entry of the tube attaching the cell to the pump system. The heating element was a commercial "hot plate" type designed to produce 800 watts at 110v. The element consisted of a pair of $\frac{1}{4}$ inch diameter heating coils. These were wired in parallel, mounted in recesses cut in the inner walls of the oven and supplied with power from a Powerstat variable ac voltage source. A Fe-Constantan thermocouple was used to measure the oven temperature (Reference 44), the thermocouple current being measured by a Weston meter having a full scale deflection equivalent to 1.2 milliamperes. The oven worked satisfactorily up to 500°C (the maximum temperature measurable by the thermocouple) but was doubtless good for much higher temperatures.

The three stage diffusion pump used was a type GF 25 A made by the Consolidated Vacuum Corporation. Pressure was measured using an Edwards IG2 series ionization manometer with a Model 3 Ionization Gauge control unit.

The cell was degassed using a baking out temperature of 300°C. A pressure of 5×10^{-6} mm was attained in the pumping system before introducing the sodium. Subsequently, the cell described in Chapter II, was broken open at the seal-off and the sodium cleaned out. This was refilled with sodium using the improved pumping equipment. For this cell the pressure in the pumping system

Fig. 52. The pumping system.

before sealing off the absorption cell was 2×10^{-6} mm. Two absorption cells were thus made in order to have one as a spare.

II.6. Electronics of the signal detector system.

A block diagram of the electronics associated with Method I is shown in Figure 53. The signal produced by the two light beams falling on the 1P21 photomultiplier was fed via a cathode follower to an electronic switch. The latter functioned to eliminate spurious pulses in the signal voltage. The signal voltage was then amplified and fed via the phase sensitive detector to the dc amplifier, the output of which was connected to the input terminals of the recording meter. A reference signal was generated at the light chopper and fed to an amplifier and squaring stage. The latter generated a push-pull square wave for the 6AR8 used in the phase sensitive detector and a square wave for the electronic switch. Details of the units shown in the block diagram are outlined below. Power was supplied to the system from the following power supplies:

1. One high voltage power supply type PS-22 made by Scientific Specialities Corporation - output 500 - 1500v at 0-1 milliamperes.
 - (a) This was used for the photomultiplier accelerator voltage.

Figure 53. Block diagram of the electronics used in Method I.

2. One Lambda Power Supply Type 29. Output 6.3v, 5 amps, centre tapped, 225-325v, regulated. This was set at 300v.
 - (a) A 100v supply was obtained from this unit by stepping down the 300v output using a resistance network and smoothing condensers.
3. One Lambda Power Supply type 29 output 100-200v at 100ma max. 6.3v at 3 amps.
 - (a) A positive 100v and negative 100v was obtained from this unit and used in the electronic switch.

II.7. The Detector.

The circuit diagram for this unit is shown in Figure 54. The output of the LP21 photo-multiplier was connected directly to the grid of the 6C4 cathode follower. Voltage appearing at the cathode of the 6C4 was fed to the amplifier via the electronic switch.

II.8. The Amplifier.

The circuit for this unit is shown in Figure 55. This is a highly stable circuit utilizing feed back via R5, R3 and R4. R4 controls the amount of feed-back and thus acts as a gain control.

Figure 54. Circuit diagram of the detector.

Figure 55. Circuit diagram of the amplifier.

II.9. The Reference Signal Generator.

The electrical circuit is shown in Figure 56.

Figure 56. Circuit diagram of the reference signal generator.

The periodically interrupted light from the 6v bulb was detected by the type 923 photo-cell which produced the reference signal. Output was taken from the cathode of the 6C4.

II.10. The Reference Signal Amplifier, Phase Sensitive Detector and dc output circuit.

These units are closely related and are shown in Figure 57. The approximately sinusoidal output from the reference signal generator was applied to the grid of V1

(6SN7)

and amplified before passing to the squaring tubes V2 and V3. These functioned to cut-off the peaks of the wave form from the reference signal generator so that the amplitude of the output switching voltage was independent of that of the input. V4 served as a phase splitter providing the push-pull output for the 6AR8. The output to the electronic switch was taken from the plate of V3.

The 6AR8 tube, V5, was used instead of the two pentodes common to the more conventional type phase sensitive circuits based on that originally suggested by DICKE (45). This tube resembles in some respects a miniature cathode-ray tube. The deflectors YY' can be made by the deflector voltage to cause the focused electron beam, emerging from the cathode, suppressor grid (g_2) and accelerator grid (g_3) assembly, to fall on either of the plates P_1 , P_2 , as desired. The electron beam is modulated by a signal on the control grid (g_1) and the form of the plate currents I_{p1} and I_{p2} depends on the nature of both grid and deflector voltages. In this application the phase of the reference (deflector) voltage was such that the electron beam was directed at one plate when one light beam in the detector unit was passed by the chopper to illuminate the 1P21 photo-multiplier; and on to the other plate when the second light beam was passed by the chopper to fall on the 1P21. The idealized wave forms are illustrated in Figure 58. The current pulses in the plate circuits are different,

generally of opposite polarity, in the event of an unbalance in energy content of the two light beams producing the signal at the photo-multiplier cathode. Suppose a positive pulse is produced at P1 and a negative pulse at P2. The large time constant of R21, R22, C7 causes an approximately constant potential difference to appear across C7. The grids of the output tubes V6 and V7 are thus held at different potentials. The grid of V6 is more positive than the grid of V7. The tubes, therefore, conduct unequal plate currents and a voltage appears at the output terminals O1, O2. This voltage is ultimately dependent on the balance of the two beams in the detector unit. The voltage output at O1, O2 thus corresponds to the degree of unbalance of these beams.

In practice it was not possible to produce a smooth signal wave form; spurious pulses were produced in the wave form as shown in Figure 58. These were produced, in spite of considerable precautions, by a slight mis-alignment of the light beams in relation to the chopper. To eliminate these unwanted pulses by improving the mechanics of the chopper assembly would have necessitated a piece of high precision engineering and with no guarantee of success. An electronic switch was, therefore, designed to short out the signal voltage during the period that the undesired pulses were produced. This circuit will

now be considered.

II.11. The Electronic Switch.

The switching circuit and associated wave forms are shown in Figures 59 and 58. It was necessary to provide, first of all, a switching voltage which would operate to short out the signal voltage during the period of production of the spurious pulses. This circuit is best described by considering the formation of square waves which serve as a gate to permit the desired part of the signal voltage to pass to the phase sensitive detector. Once the gating voltage is formed, one of several types of diode switch (reference 46) may be used to pass the desired part of the wave form. Such switches generally fall into two groups: unidirectional and bidirectional. The former pass only one polarity of the wave form, the latter pass both polarities. Reference to Figure 58 shows that a bidirectional switch was required in this case. The switch shown in Figure 59 was finally incorporated after several preliminary tests with other types of switch. This switch has the advantage that in the absence of the selector pulse the output potential is set at ground by the diodes V11 and V12. It was important, in view of the long duration ($2,000\mu s$) of the selector pulse, that the input impedance of the switch to the selector pulse be high. The circuit of Figure 59 fulfills

Figure 57. Wave forms associated with the electronic switch.

Figure 59. Circuit diagram of the electronic switch

this requirement and little distortion of the gating wave form was apparent for the values of C11, C12, R34 and R35 shown.

The action of the gating circuit was as follows: Condenser C1 and resistance R1 differentiate the square wave reference signal voltage input at A. V1 serves as an amplifier and phase splitter. The positive pulses passed by the diodes V1a and V1b of the two output wave forms appear across R7. These pulses trigger the univibrator comprised of V3 and V4, V2 acting as an isolating tube and trigger pulse amplifier. The negative square pulses fed from the plate of V4 are differentiated by C6 and R21. The negative pulses have no effect on the second univibrator V5 and V6. The positive pulses, however, have a triggering action. The negative square wave output of univibrator V5 and V6 is fed to the grid of V7. This tube was incorporated to improve the quality of the square wave. Any unwanted positive pulses are removed by the grid stopper R25. The negative part of the square wave drives the tube to cut-off at a voltage determined by R27. This results in the production of a clean, flat top for the output square wave at the plate of the tube. The desired amplitude of the selector square wave is then applied via R28 to the grid of the phase splitter V8 and the two antiphased waves from this tube are fed to the switch. The signal voltage is applied via resistances

R32 and R33 to the switch. On receipt of the gating pulse, signal voltages are passed by the switch. When the gating pulse is removed the signal voltage is effectively grounded at X.

A 1000cps audio signal was used to test this switch. In the absence of the selector pulse only 2% of the applied signal voltage was passed. R11 controlled the length of the square pulse produced by the first univibrator. The differentiated back edge of this pulse triggered the second univibrator. Thus R11 served, in effect, as a delay control for the selector pulse. The length of the latter was controlled by R18. R11 and R18 thus enabled the position and length of the selector pulse, in relation to the signal voltage wave form, to be readily controlled. R36 served to balance the action of the selector pulses fed from the phase splitter V8. When R34 and R35 were of equal value the selector pulse appeared as a pedestal in the output signal wave form due to the different output impedances at the plate and cathode of V8. By placing R36 in parallel with R34 the value of the combination could be adjusted until the unwanted pedestal in the signal output was eliminated.

APPENDIX III.

A relation between elevation of the sun above the horizon and time of day (local apparent time) was obtained for the days February 24th and February 27th, 1956. The position of the sun at time given by the hour angle t is shown on the celestial sphere ZHAH' (Figure 60). With celestial poles at PP' and Zenith at Z, arc PZ is the co-latitude of the position of observation. The sun describes small-circle OS'O'S" having angular distance $(90 - \delta)$ from P, (where δ is the sun's declination at the time of observation). The horizon is the great circle HS'H'S" and S' and S" show the position of the sun respectively at sunset and sunrise for t increasing as indicated by the arrow. Calling the elevation of the sun above the horizon h the arc ZS is the zenith distance of the sun equal to $(90 - h)$. Thus at any instant t the solar elevation h is found by solving the spherical triangle PZS, using the following general relation for a spherical triangle:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

where a, b and c are the sides of the triangle and A, B, C the angles opposite these. Thus for the spherical triangle PZS:

$$\cos(90 - h) = \cos(90 - \delta) \cos(90 - \ell) + \sin(90 - \delta) \sin(90 - \ell) \cos t,$$

$$\text{i.e.} \quad \sin h = \sin \delta \sin \ell + \cos \delta \cos \ell \cos t,$$

Values of h for $2\frac{1}{2}^\circ$ intervals of t (equivalent to 0.04363 radians) were obtained for the two values of δ corresponding to February 24th and February 27th using FERUT, the Ferranti MkI electronic digital computer, housed at the University of Toronto. At the time that the evaluation was required a telegraphic link between the University of Saskatchewan Department of Physics and the Computation Centre of the University of Toronto was available one evening per week for transmitting calculation programmes and receiving results. Using Transcode techniques (Reference 47) the programme was punched on teletape and transmitted to the Computation Centre at Toronto using the telegraphic link.

For the days February 24th and February 27th δ values were $-9^\circ 54.8'$ and $-8^\circ 48.2'$ respectively (Reference 48). For Saskatoon $\ell = 52^\circ 10'$ giving the two constants $\cos \ell = 0.61384$ and $\sin \ell = 0.78943$. The programme used (together with some explanatory notes) is given below. Table 5 contains the resultant values of h in degrees and t in hours and minutes. h is shown plotted against t in Figure 61.

CNST 78943 + 1- (C01 Sin ℓ)
 61384 + 1- (C02 Cos ℓ)
 15366 - 1- (C03 δ_1 rad)
 17307 - 1- (C04 δ_2 rad)
 4363 + 2- (C05 2.5° in rad)
 ++ (C06 0)
 1++ (C07 1)
 5729578+2+ (C08 radn. $\rightarrow$ deg.)

INST 028

001	FNTN	009.0	C03.0	x01.0	Sin δ_1
002	MULT	x01.0	C01.0	x02.0	Sin δ_1 Sin ℓ
003	FNTN	002.0	C03.0	x03.0	Cos δ_1
004	MULT	x03.0	C02.0	x04.0	Cos δ_1 Cos ℓ
005	FNTN	009.0	C04.0	x05.0	Sin δ_2
006	FNTN	002.0	C04.0	x06.0	Cos δ_2
007	MULT	x06.0	C02.0	x07.0	Cos δ_2 Cos ℓ
008	MULT	x05.0	C01.0	x08.0	Sin δ_2 Sin ℓ
009	LOOP	002.0	000.5	000.0	
010	ZERO	Z01.0	000.0	000.0	t = 0
011	FNTN	002.0	Z01.0	Y02.0	Cos t
012	MULT	Y02.0	X04.0	Y03.0	Cos t Cos δ_1 Cos ℓ
013	ADDN	Y03.0	X02.0	Y04.0	Cos t Cos δ_1 Cos ℓ + Sin δ_1 Sin ℓ = N
014	MULT	Y04.0	Y04.0	Y05.0	N ²
015	SUBT	C07.0	Y05.0	Y06.0	1 - N ²
016	$\frac{1}{2}$ QRT	Y06.0	000.0	Y07.0	(1 - N ²) ^{$\frac{1}{2}$}
017	DIVD	Y04.0	Y07.0	Y08.0	N/(1 - N ²) ^{$\frac{1}{2}$}
018	FNTN	005.0	Y08.0	Y09.0	h (radns.)
019	MULT	Y09.0	C08.0	Z02.0	h ⁰
020	PRNT	002.2	004.0	Z01.0	print Z01, Z02 viz. h ⁰ t.
021	ADDN	005.0	Z01.0	Z01.0	(t + 2.5 ⁰)
022	TRNS	011.0	000.0	Z02.0	loop for h ⁰ $\geq$ 0.
024	OVER	X07.0	000.0	X04.0	replace Cos δ_1 Cos ℓ by Cos δ_2 Cos ℓ
025	OVER	X08.0	000.0	X02.0	replace Sin δ_1 Sin ℓ by Sin δ_2 Sin ℓ
026	TRNS	010.0	000.5	000.0	loop back to 10.
027	HALT	000.0	000.0	000.0	
028	QUIT	000.0	000.0	000.0	

hrs. t min.	h°	
	February 24th δ = -9°54.8'	February 27th δ = -8°48.2'
0.00	27.95	29.06
0.10	27.91	29.03
0.20	27.80	28.91
0.30	27.62	28.72
0.40	27.36	28.46
0.50	27.03	28.13
1.00	26.62	27.72
1.10	26.15	27.24
1.20	25.61	26.69
1.30	25.01	26.08
1.40	24.34	25.40
1.50	23.60	24.66
2.00	22.81	23.86
2.10	21.97	23.00
2.20	21.06	22.09
2.30	20.11	21.13
2.40	19.1	20.11
2.50	18.05	19.05
3.00	16.96	17.95
3.10	15.82	16.80
3.20	14.64	15.61
3.30	13.43	14.39
3.40	12.18	13.13
3.50	10.89	11.84
4.00	9.580	10.52
4.10	8.239	×
4.20	6.872	7.792
4.30	5.480	6.394
4.40	4.067	4.974
4.50	2.633	3.534
5.00	1.181	2.076
5.10	-0.287	-0.603
		× no value received

Table 5. Elevation of the sun above the horizon for February 24th and 27th, 1956.

Figure 60. Diagram of the celestial sphere illustrating the elevation of the sun in relation to the time of day.

Figure 61. Plot of solar elevation against local apparent time for February 24th and 27th, 1956, using values obtained from FERUT.

APPENDIX IV.

An expression is developed in this appendix giving the relative path length through a thin layer at height H , of a ray of solar radiation, observed at a point on the earth's surface, in terms of h (the angle of elevation of the sun above the horizon), R (the radius of the earth) and H . Values of relative path lengths were obtained for H values of 80 and 90km for h varying from 0 to 60° in 2° steps. These values were obtained using the electronic digital computer at the University of Toronto, the programme used being shown below. The relative path length values are listed in Table 6 and are shown plotted against h in Figure 62.

Assuming unit path length through the layer for the sun directly overhead (i.e. for $h = 90^\circ$), the relative path length at any h is $\sec \alpha$; where α is the angle between the solar ray and the normal to the layer at the point of intersection. Referring to Figure 63 one obtains from the sine law

$$\frac{R}{\sin \alpha} = \frac{R+H}{\sin(90+\alpha)} ,$$

whence $\sin \alpha = \frac{R}{R+H} \cos h ,$

$$\cos \alpha = \left(1 - \frac{R}{R+H} \cos^2 h \right)^{\frac{1}{2}} ,$$

$$\sec \alpha = \left(1 - \frac{R}{R+H} \cos^2 h \right)^{-\frac{1}{2}} ,$$

To solve this for $H_1 = 80\text{km}$ and $H_2 = 90\text{km}$ and for $h = 0, 2, 4, \dots, 60^\circ$
the following programme (with brief explanatory notes) for
FERUT may be used.

CNST	6367 +3 +	(C01 R = 6,367km)
	8 +1 +	(C02 $H_1 = 80\text{km}$)
	9 +1 +	(C03 $H_2 = 90\text{km}$)
	1 + +	(C04 =1)
	2 + +	(C05 =2)
	5729578+1+	(C06 radn. $\rightarrow$ deg.)

INST 023

001	OVER	C06.0	000.0	X01.0	Deg. radns. in X01
002	DIVD	C05.0	X01.0	X02.0	2° in radns.
003	ADDN	C01.0	C02.0	X04.0	$R+H_1$
004	ADDN	C01.0	C03.0	X05.0	$R+H_2$
005	DIVD	C01.0	X04.0	X06.0	$R/R+H_1$
006	DIVD	C01.0	X05.0	X07.0	$R/R+H_2$
007	LOOP	002.0	000.5	000.0	
008	ZERO	X08.0	000.0	000.0	Zero X08 $h = 0^\circ$
009	LOOP	032.0	000.4	000.0	
010	FNTN	002.0	X08.0	X09.0	$\cos h$
011	MULT	X09.0	X06.0	X10.0	$R/R+H_1 \cos h$
012	MULT	X10.0	X10.0	X11.0	$(\quad)^2$
013	SUBT	C04.0	X11.0	X12.0	$1-(\quad)^2$
014	($\frac{1}{2}$ QRT)	X12.0	000.0	X13.0	$(1-(R/R+H_1)\cos h)^{\frac{1}{2}}$
015	DIVD	C04.0	X13.0	Z01.0	$\sec \alpha$
016	MULT	X08.0	X01.0	Z02.0	h in deg.
017	PRNT	002.2	005.0	Z01.0	Print h and $\sec \alpha$
018	ADDN	X02.0	X08.0	X08.0	Add 2° in radn. to h
019	TRNS	010.0	000.4	000.0	
020	OVER	X07.0	000.0	X06.0	Change $R/R+H_1$ to $R/R+H_2$
021	TRNS	008.0	000.5	000.0	
022	HALT	000.0	000.0	000.0	
023	QUIT	000.0	000.0	000.0	

h°	Sec α	
	$H_1 = 80\text{km}$	$H_2 = 90\text{km}$
0.00	6.3675	6.0103
2.00	6.2195	5.8857
4.00	5.8312	5.5544
6.00	5.3209	5.1093
8.00	4.7916	4.6366
10.00	4.3004	
12.00	3.8684	3.7874
14.00	3.4976	3.4383
16.00	3.1820	3.1380
18.00	2.9136	2.8804
20.00	2.6846	2.6591
22.00	2.4881	2.4684
24.00	2.3186	2.3030
26.00	2.1714	2.1590
28.00	2.0428	2.0329
30.00	1.9299	1.9218
32.00	1.8302	1.8235
34.00	1.7417	1.7362
36.00	1.6629	1.6584
38.00	1.5924	1.5886
40.00	1.5292	1.5260
42.00	1.4723	1.4696
44.00	1.4209	1.4187
46.00	1.3744	1.3726
48.00	1.3324	1.3308
50.00	1.2942	1.2929
52.00	1.2596	1.2584
54.00	1.2281	1.2271
56.00	1.1995	1.1987
58.00	1.1735	1.1729
60.00	1.1500	1.1494
62.00	1.1286	1.1282

Table 6. Relative path length through the sodium layer for assumed heights of 80 and 90km, for various solar elevation angles (h).

Figure 63. Diagram showing the solar ray in relation to the sodium layer and the surface of the earth.

Figure 62. The dependence of relative path length through the sodium layer of assumed heights 80-90km, on solar elevation h .

APPENDIX V .

The method of obtaining the most probable values for the constants of a linear equation representing given data is described in several texts. (Reference 54). In this case the treatment (subject to certain modification) given by WORTHING and GEFNER (52) was followed. In Figures 32-42 the values of $x = \sec$ can be obtained to a very high degree of accuracy and may be considered to be free from error. It is necessary to find an equation of the form

$$y = a + bx \quad , \quad (1)$$

which represents the most probable straight line which can be drawn through the points obtained experimentally. Since errors are assumed only in the y values (i.e. in $\log p$) the constants a and b are given by:

$$a = \frac{\sum x^2 \sum y_0 - \sum x \sum xy_0}{n \sum x^2 - (\sum x)^2} \quad , \quad (2)$$

$$b = \frac{n \sum xy_0 - \sum x \sum y_0}{n \sum x^2 - (\sum x)^2} \quad , \quad (3)$$

where n represents the number of observed points and y_0 the observed y values.

Considerable labour is saved if (i) a straight line, say

$$y' = a' + b'x \quad , \quad (4)$$

is drawn "by eye" through the points to represent approximately the relation sought and (ii) the origin is shifted by making the substitution:

$$x' = x - c \quad , \quad (5)$$

so that the points are distributed approximately evenly either side of the ordinate $x' = 0$. One then measures the deviation y_0 of the points from the assumed line and corrections a' and b' to the constants a and b are obtained from the expressions:

$$\Delta a' = \frac{\sum x'^2 \sum \Delta y_0^2 - \sum x' \sum x' \Delta y_0}{n \sum x'^2 - (\sum x')^2} \quad , \quad (6)$$

$$\Delta b' = \frac{n \sum x' \Delta y_0 - \sum x' \sum \Delta y_0}{n \sum x'^2 - (\sum x')^2} \quad , \quad (7)$$

With weights assigned, w_1 to point 1, w_2 to point 2 etc., the constants of the linear equation are given by:

$$a = \frac{\sum wx^2 \sum wy_0 - \sum wx \sum wxy_0}{D} \quad , \quad (8)$$

$$b = \frac{\sum w \sum wxy_0 - \sum wx \sum wy_0}{D} \quad , \quad (9)$$

where $D = \sum w \sum wx^2 - (\sum wx)^2$,

and the other symbols have the meaning defined above.

In order to find the probable error in a or b we require the law of propagation of probable errors.

In general if:

$$\bar{u} = f(\bar{x}, \bar{y}, \bar{z}) \quad , \quad (10)$$

where $\bar{x}, \bar{y}, \bar{z}$ are the means of three separate sets of readings the probable error of $\bar{u}$, $P_{\bar{u}}$, may be obtained from the equation:

$$P_{\bar{u}}^2 = \left(\frac{\partial u}{\partial x}\right)^2 P_{\bar{x}}^2 + \left(\frac{\partial u}{\partial y}\right)^2 P_{\bar{y}}^2 + \left(\frac{\partial u}{\partial z}\right)^2 P_{\bar{z}}^2 \dots, \quad (11)$$

Equations (8) and (9) may be re-written as:

$$a = A \sum wy_0 - A' \sum wxy_0 = \sum [(A - A'x)wy_0] \quad , \quad (12)$$

$$b = B' \sum wxy_0 - B \sum wy_0 = \sum [(B'x - B)wy_0] \quad , \quad (13)$$

where $A = \frac{\sum wx^2}{\sum w \sum wx^2 - (\sum wx)^2}$ $A' = \frac{\sum wx}{\sum w \sum wx^2 - (\sum wx)^2}$,

$$B' = \frac{\sum w}{\sum w \sum wx^2 - (\sum wx)^2} \quad B = \frac{\sum wx}{\sum w \sum wx^2 - (\sum wx)^2} \quad ,$$

Consider the expression for b.

The different values of y_0 are regarded as independent variables, all with the same probable error P_{y_0}

i.e. $b = f(y_{01}, y_{02}, y_{03} \dots y_{0k}) \quad k \leq n \quad , \quad (14)$

Applying the law for the propagation of probable error:

$$P_b^2 = \left(\frac{\partial b}{\partial y_{01}} \right)^2 P_{y_{01}}^2 + \left(\frac{\partial b}{\partial y_{02}} \right)^2 P_{y_{02}}^2 + \left(\frac{\partial b}{\partial y_{0k}} \right)^2 P_{y_{0k}}^2, \quad (15)$$

$\left(\frac{\partial b}{\partial y_{0k}} \right)$ is found from the general expression for b, viz:

$$b = B' \sum_{i=1}^k w_i x_i y_{0i} - B \sum_{i=1}^k w_i y_{0i}, \quad (16)$$

$$\frac{\partial b}{\partial y_{0i}} = B' (w_i x_i) - B(w_i), \quad (17)$$

whence

$$P_b^2 = \left[\sum_{i=1}^k (B' w_i x_i - B w_i)^2 \right] P_{y_0}^2, \quad (18)$$

and

$$P_b = \sqrt{w^2 (B' x - B)^2} P_{y_0}, \quad (19)$$

a similar relation holds for P_a .

The expression obtained above for P_b differs from that given by WORTHING and GEFNER (52) which is demonstrably incorrect. In applying equation (19) to find the probable error in the slope of Figures 32 - 42 the points were assumed to have equal weights so that the w 's drop out of the equation. One then obtains for the probable error in b:

$$P_b = P_{y_0} \frac{1}{\sum x^2 - \frac{(\sum x)^2}{n}} \sqrt{\sum \left(x - \frac{\sum x}{n} \right)^2}, \quad (20)$$

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1

ABSORPTION OF SUNLIGHT BY ATMOSPHERIC SODIUM

A

THESIS

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ABSTRACT.

A photometer has been built which is sensitive only to light in a very narrow spectral region near the sodium D-lines. The light is scattered by sodium vapour at low density and the scattered light recorded by a photoelectric spectrometer. The latter resolves the two lines and allows the effect of stray light to be eliminated. With sunlight the radiation detected is at the bottom of the Fraunhofer lines and the residual intensity there may be measured. It is found that this intensity decreases slightly (relative to the continuum) as the sun approaches the horizon. This effect is attributed to atmospheric sodium; the amount found by this method agrees well with that deduced from twilight measurements made the same day.

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