TRANSIENT EFFECTS IN THE DISCHARGE OF COMPRESSED AIR
FROM A CYLINDER THROUGH AN ORIFICE

A Thesis

Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of
Doctor of Philosophy
in the Department of Mechanical Engineering
University of Saskatchewan

by

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Saskatoon, Saskatchewan
May, 1968

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A CYLINDER THROUGH AN ORIFICE

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SYNOPSIS

The thesis describes the expansion process in a cylinder and the accompanying transient flow through an orifice when compressed air is discharged from the cylinder. The orifice which is located at one end of the cylinder is small enough to prevent the development of significant wave action.

The aim of the investigation is to study the behaviour of flows through circular orifices under transient upstream conditions and to compare such flows with those through similar orifices under steady state conditions.

A critical review of previous work is included from which it is evident that further experimental work is necessary both for a better understanding of the nature of transient flow and to obtain results of practical value. The experiments in the present investigation have been designed from dimensional considerations. The main conclusions that have been made are as follows:

The expansion of the air in the cylinder is not reversible. Gradients of temperature in the radial and axial directions are quickly established after the commencement of discharge and heat is transferred from the cylinder wall to the residual air. In practice it is impossible to achieve a truly adiabatic wall.

An experimental correlation between the pressure and weighted average temperature of the air in the cylinder during expansion has been obtained. This relationship can be used to predict the transient average temperature of the air from the instantaneous pressure in the cylinder.

The transient mass flow through a given orifice, and the corresponding discharge coefficient, have been found to be functions of the length-diameter ratio of the orifice, the pressure ratio across it and the heat transferred to the residual air. The latter involves a consideration of the temporal sequence of the discharge which makes it impossible to obtain general statements about transient flows.

From dimensional considerations, and from experiments, it has been possible to obtain hypothetical transient mass flows without heat transfer. These flows are consistently lower than those under steady state conditions when geometrical and physical similarity is preserved.

Heat transfer to the residual air in the cylinder during discharge increases the instantaneous mass flow through the orifice. Its effect on the mass flow may be quite pronounced for relatively slow discharges.

The effect on the transient mass flow of the length-diameter ratio of an orifice has been found to be similar to that for steady state conditions.

ACKNOWLEDGEMENTS

The work reported in this thesis was supervised by Dr. B.E.L. Deckker to whom the author wishes to express his thanks for his valuable advice and encouragement throughout the course of the investigation and in the preparation of the thesis. His constant interest in the work is also highly appreciated.

Thanks are also due Mr. A. Dixon for his help with some of the electronic equipment and to Mr. G.N. Rice for his assistance in the laboratory.

This project was supported by the National Research Council of Canada under Grant No. A-1618. Grateful acknowledgement is also due the National Research Council of Canada for financial support in the form of a Scholarship.

The author also wishes to record his thanks to the University of Saskatchewan for the provision of laboratory and computation facilities.

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LIST OF SYMBOLS

a, b, c, and d	Constant coefficient
C_d	Discharge coefficient ($= \dot{m}/\dot{m}_{theo}$, from same upstream conditions)
D	Diameter of cylinder
d	Diameter of orifice
G	Mass flow number ($= (\dot{m} \sqrt{kRT_0})/(p_0 d^2)$)
h	Roughness of flow passage
k	Ratio of specific heats (C_p/C_v)
L	Length of cylinder
l	Length of orifice
M	Mach number
M_a	Mass of residual air in cylinder
$\dot{m}$	Mass flow rate
N	Bulk modulus
p	Pressure
R	Characteristic gas constant
Re	Reynolds number
T	Temperature
T_s	Temperature of residual air corresponding to isentropic expansion
t	Time
t_c	Thermocouple time constant
V	Volume of cylinder
v	Mean velocity at orifice
X	Dimensionless group ($= 1/(Z - 0.04)$)
Y	Dimensionless group ($= X^2$)

LIST OF SYMBOLS (CONT'D)

Z	Dimensionless group $(= 1/(p_a/p_o)(t/\gamma)^{1/2})$
μ	Coefficient of viscosity
ρ	Density
γ	Characteristic time $(= L/\sqrt{kRT_{oo}})$

Subscripts:

o	Stagnation condition
oo	Initial condition
a	Atmospheric condition
s	Steady flow
t	Transient flow
ts	Isentropic transient flow

1. INTRODUCTION

1.1 Nature and Scope of the Work

The transient flow of a compressible fluid through an orifice when the pressure and temperature upstream of the orifice change with time occurs in many processes in mechanical engineering. The rapid and efficient evacuation of burned gas from the cylinder of an internal combustion engine, and the associated problem of port design, and the measurement of pulsating flow are two examples which appear to be of continuing interest to mechanical engineers. For over a century, the flow of a compressible fluid through an orifice, or a nozzle, under steady conditions has been studied intensively both experimentally and theoretically. The study of compressible flow under transient upstream conditions has a more recent history and has been accelerated by the development of high speed reciprocating internal combustion engines. It is only since 1949 that many details of the transient flows that occur in the air exchange process in high speed internal combustion engine have been revealed by analyses using the method of characteristics^{(1)(2)*}. However, considering the discharge process alone, agreement between the integrated mass flow obtained from the results of such an analysis and that calculated from the initial and final thermodynamic states of the gas in the cylinder has been unsatisfactory⁽²⁾. It is believed that the reason for this discrepancy is the lack of suitable values of the discharge coefficient for the exhaust ports. Unlike a

* Numbers in brackets refer to references listed on page 115.

steady flow through an orifice, or port, a transient flow depends not only on the instantaneous upstream conditions but also on the previous history of the flow. This factor has not been considered by previous investigators although it is clear that changing upstream conditions involve a thermodynamic process which is associated with the passage of time. Experimental approaches to the problem of transient flow have been mainly concerned with correctly evaluating the area of the engine ports and, since the flow conditions are always complex, such treatments can hardly be expected to clarify the physical aspects associated with transient flows.

To the knowledge of the author, it is impossible in an analytical treatment to take account of all the factors that might affect a transient flow under the conditions prevailing during a practical discharge process. Even in the relatively simple case of steady flow through an orifice, mathematical, or semi-empirical, solutions which have been obtained for either one or two dimensional compressible flows⁽³⁾⁽⁴⁾⁽⁵⁾ are applicable only to sharp edge orifices⁽⁶⁾. No analytical treatment has yet been proposed which can take into account the geometry of the orifice or the effects of entrainment downstream. The accurate measurement of steady flow, therefore, still relies on empirical formulae such as those specified in National Codes. The purpose of the investigation reported in this thesis is to systematically study the behaviour of relatively slow transient flows under controlled experimental conditions. It is believed that data obtained in this way will help towards an understanding of the more difficult problem of pulsating flow.

1.2 Statement of the Problem

The term 'slow transient' flow as used in the preceding paragraph is intended to distinguish such a flow from a transient flow in which pronounced wave action during discharge generates an appreciable bulk velocity of the gas in the cylinder towards the orifice, and which gives rise to a nonuniformity in the thermodynamic state in the cylinder. Wave action occurs when the diameter of the orifice is relatively large and when the contents of a cylinder are suddenly released. For a long cylinder, the flow, as a result of wave action, may be treated as a succession of quasi-steady flows. A slow transient flow through an orifice applies specifically to a discharge process in which the cylinder contents are suddenly released but no finite amplitude waves are generated so that the approach velocity of the gas towards the orifice is negligible. In such a case any nonuniformity in the pressure of the gas cannot be detected anywhere in the cylinder, except in a narrow region near the orifice where the flow is accelerated to comply with the boundary conditions associated with the orifice. Thus, the contents of the cylinder may be considered to be in a state of virtual stagnation. These conditions are encountered in an orifice-cylinder configuration in which the ratio of the orifice area to that of the cylinder cross-section normal to the axis of the orifice is made small enough. Therefore, the discharge of gas from the cylinder may be considered, for practical purposes, to be a free expansion and changes in the thermodynamic state of the gas in the cylinder occur in a smooth and continuous manner.

Although wave action in the cylinder must be regarded as the mechanism by which the pressure of the gas in the cylinder is reduced both in the case of a fast transient and in the case of a slow transient discharge, it is clearly incorrect to treat the latter as a quasi-steady process. However, at any instant during discharge, similarity of the stagnation pressure, the stagnation temperature and the local acceleration of the flow near the upstream face of the orifice provide a basis for comparison between the instantaneous upstream conditions in a transient flow and those found in a steady flow through a geometrically similar orifice. Although it is not certain that for given values of the stagnation pressure and temperature in the cylinder there is dynamic similarity between the two flows through the orifice, the arguments presented in reference (6) lead to the conclusion that the state of virtual stagnation and equality of the pressure are sufficient to compare transient and steady flows. It is on this basis that the transient effects may be determined.

1.3 Brief Description of the Investigation

Instead of a complex engine configuration, a cylinder is used as an expansion chamber and compressed air is discharged to the atmosphere through an orifice mounted co-axially at one end. At a given instant the actual flow through the test orifice is calculated from a mass balance of the residual air in the cylinder. For this reason, it is important to establish the thermodynamic state of the residual air at every instant during the expansion process.

In the course of the present investigation, it has been

found that the assumption of isentropic expansion which is usually made in discharge calculations is not generally valid. Although the inner surface of the cylinder was insulated from the air in the cylinder, heat transfer to the air during expansion indicates that, for a practical cylinder configuration, it would be impossible to achieve an adiabatic wall. From the measurement of the temperature of the residual air during the expansion, an empirical correlation between the weighted average temperature and the pressure of the gas in the cylinder has been obtained. The temperature predicted by this correlation at any instant during a discharge has been found to be satisfactory for all the experiments.

Using dimensional arguments, fluid dynamical effects and the effects of heat transfer to the residual air on the flow through an orifice have been examined both together and separately. For a given orifice the overall discharge coefficient has been found to be different from that of a steady flow. Further, more detailed, investigations into the effect of the starting process of the discharge and the adjustment of the flow pattern in the orifice to changes in the upstream conditions appear to be worthwhile.

2. REVIEW OF PREVIOUS WORK

2.1 Introduction

In order to place the present study of slow transient flows through orifices in its correct perspective, it is necessary to review the previous contributions which relate either directly or indirectly to it.

When a compressed gas is discharged from a cylinder through an orifice, the expansion of the gas in the cylinder is of direct interest since a precise knowledge of the thermodynamic properties of the gas in the cylinder at any instant is necessary for calculating the transient mass flow through the orifice. Although the present investigation is confined only to those discharges in which wave action in the cylinder is negligible, nevertheless, with further investigations into transient flows in view, published work dealing with expansion processes which are accompanied by the discharge of a gas from a vessel both with and without wave action has been reviewed. For the same reason, a review of the literature on transient flows through orifices includes not only transient flows associated with a relatively slow discharge but also those flows in which wave action is present upstream of the orifice.

2.2 The Expansion of Gas in a Vessel During Discharge

2.2.1 The thermodynamic nature of the expansion process

Perhaps the earliest experimental investigation into the thermodynamic behaviour of a gas in a vessel during expansion was

that of Clément and Désormes (1819)⁽⁷⁾ for the determination of the ratio of the two specific heats, C_p/C_v , or the index of isentropic expansion, k . Later, Gay-Lussac and Welter⁽⁷⁾ modified Clément's and Désormes' experiment by changing the compression process into an expansion. In the modified experiment, a small quantity of gas at room temperature and at a pressure, p_i , slightly greater than the atmospheric pressure, p_a , was allowed to expand from a vessel during the rapid opening and closing of a stopcock. Under this condition, the expansion of the gas in the vessel was regarded as reversible and adiabatic. After the expansion, the gas remaining in the vessel was allowed to attain its initial temperature at constant volume by the transfer of heat from the atmosphere through the vessel, the final pressure being p_f . The index of isentropic expansion was calculated from the relationship:

$$k = \frac{\ln(p_i/p_a)}{\ln(p_i/p_f)} .$$

Some years later, Lummer and Pringsheim⁽⁷⁾ carried out an experiment in which the change of temperature of the gas during expansion was measured. Partington⁽⁸⁾ modified their temperature measurement technique and obtained values of the specific heat ratio, k , more accurately, using the relationship:

$$\frac{T_i}{T_f} = \left(\frac{p_i}{p_f}\right)^{\frac{k-1}{k}} .$$

Both in the original experiment of Clément and Désormes and in the subsequent modifications, the important feature was the small initial pressure difference across the orifice and, therefore, the small amount of gas which flowed out of the vessel. The small

initial difference which resulted in an expansion of short duration was designed to obtain a good approximation to an adiabatic process. Also, since the kinetic energy of the gas leaving the vessel was a very small fraction of the initial internal energy of the gas, any error due to irreversibility was minimised.

There is little information on the expansion of a compressible fluid in a practical discharge process in which the initial pressure difference across the orifice, or port, is high. Weaving⁽⁹⁾ carried out experiments which simulated the discharge of gas from an engine cylinder. To determine the index of expansion, he used a method of calculation similar to that of Clément and Désormes substituting the values of three pressures obtained by measurement, thus:

$$n = \frac{\ln(p_i) - \ln(p_{\min})}{\ln(p_i) - \ln(p_f)},$$

where p_i and p_f are the initial and final pressures in the cylinder and $p_{\min}$ is the minimum pressure at the end of discharge. Weaving found that value of the index of expansion was always lower than the ratio of the two specific heats for the different gases used in his experiments. In explaining this result, he stated that the principal factors which contributed to the low values of the index of expansion were the heat transfer by forced convection from the cylinder wall to the gas during expansion, and the degeneration of energy by turbulence, particularly towards the end of the discharge.

Using Weaving's experimental data for air, Cole⁽¹⁰⁾ calculated the rate of heat transfer to the air for the duration of

the discharge. By comparing the calculated rate of heat transfer with the estimated rate for a two-stroke cycle engine of the same cylinder capacity under full load conditions, he concluded that Weaving's value of the index of expansion, equal to 1.25, was too low.

Weaving's conclusions regarding the values of the index of expansion were criticized also by Kestin and Glass⁽¹¹⁾. They pointed out that Weaving assumed the expansion of the gas in the cylinder was polytropic, which implied that the process was reversible. Therefore, his explanation that the reduction of the value of the index of expansion was due to the degeneration of kinetic energy by turbulence was a contradiction.

In an analysis of the time of discharge, Geyer⁽¹²⁾ also assumed that the expansion of the gas in the cylinder during discharge was polytropic. He assumed that heat would be transferred from the cylinder wall to the gas, since a high temperature difference would be created during the expansion. He accordingly chose a value of the index of expansion which was considerably lower than the ratio of the two specific heats. However, neither Weaving's nor Geyer's assumption has been confirmed experimentally, such as the measurement of the temperature of the gas in the cylinder during expansion.

Potter and Brown⁽¹³⁾ were the first to measure the transient temperature of a gas in a vessel during expansion. In their experiments, Argon, Nitrogen and Carbon dioxide were discharged from a spherical vessel into the atmosphere through an orifice. Using a number of thermocouples arranged in the form of a 'tree' on

the vertical axis of symmetry in the spherical vessel, they measured the transient temperature during expansion. Only average values of the temperatures measured were presented in their report. They observed that the average temperature departed from that corresponding to an isentropic expansion, and they attributed this difference to radiation heat transfer from the wall of the vessel to the thermocouples. However, when vessels with different radiative properties were used they found that the thermocouple radiation errors were very small so that their earlier explanation for the differences between the measured temperatures and the isentropic values was not correct.

Some years later, Potter and Levy⁽¹⁴⁾ measured the transient temperature at four positions on the axis of a cylinder from which air was discharged to the atmosphere through a relatively small orifice. In their experiments, only small differences in temperature at the four positions were found. However, about ten seconds after the commencement of the discharge, the temperature reached a minimum and then increased, while the pressure in the cylinder continued to fall. They believed that this behaviour was due to heat transfer by convection from the wall to the air during expansion. Differences between the measured temperatures and the isentropic values corresponding to a given expansion ratio were expressed in the form, $\Delta T / \Delta T_s$, where ΔT is the actual temperature drop at any instant after the commencement of the discharge and ΔT_s is the corresponding temperature drop for an isentropic expansion to the same pressure. Their results were in general

agreement with those found earlier by Potter and Brown⁽¹³⁾. Potter and Levy apparently did not measure the radial temperature gradient in the cylinder during expansion. They assumed the temperature distribution to be uniform, presumably because they found no appreciable differences of temperature in the four axial positions. The average value of the temperature at these four positions was, therefore, taken to be the temperature of the air in the cylinder.

In spite of the conclusion reached by Weaving that the expansion in a cylinder during discharge was diabatic⁽⁹⁾, in the case of discharge from an engine cylinder it is customary to assume that the expansion of the gas in the cylinder is isentropic⁽¹⁵⁾⁽¹⁶⁾⁽¹⁷⁾⁽¹⁸⁾. However, to the knowledge of the author no measurements to confirm this assumption have been reported.

2.2.2 Prediction of pressure in the cylinder during discharge

A number of investigators have predicted the pressure-time relationship in a cylinder during a rapid discharge. The expansion of the gas in the vessel was always assumed to be isentropic.

Giffen⁽¹⁹⁾ analysed the discharge of a compressed gas from a cylinder directly to the atmosphere through an orifice mounted coaxially at one end. He applied the one-dimensional pressure-wave theory due to Earnshaw⁽²⁰⁾ to the gas in the cylinder and the quasi-steady one-dimensional equation of St. Venant and Wantzel⁽²¹⁾ to the flow through the orifice. From these equations he predicted the rate of fall of pressure of the gas in the cylinder and was also able to account for the pressure in the cylinder being less than that of the atmosphere near the end of discharge. Giffen's analysis was the

first to reveal the role of wave action in the evacuation of a cylinder. Later, a more detailed theoretical and experimental analysis of wave action and of the vacuum found in a cylinder was given by Bannister and Mucklow⁽²²⁾. In their analysis, they considered the discharge from a cylinder directly to the atmosphere, and also into a pipe of the same diameter as that of the cylinder. The predicted pressures and those measured have been found to be in good agreement. The results of Bannister and Mucklow may be considered to be exact since there was no need to make any assumption concerning the value of discharge coefficient.

Deckker⁽²⁾ predicted the pressure-time curve in the cylinder of a motored engine on the assumption that the expansion was isentropic and taking into account the effect of wave motion in the exhaust and induction pipe systems as determined by the method of characteristics. In his experiments, the release pressure was boosted to a value comparable to that in a running engine. He used the discharge coefficients derived by Wallace and Mitchell⁽¹⁵⁾ and obtained good agreement between the theoretical pressure-time curve and the indicator diagram. Benson⁽²³⁾⁽²⁴⁾ used the method of characteristics to analyse the discharge from a cylinder under conditions similar to those treated by Giffen⁽¹⁹⁾. In this investigation a constant discharge coefficient was assumed.

The prediction of the pressure in a cylinder during discharge cannot by itself be expected to reveal the nature of the expansion process. For discharge through an orifice, or port, whose area is smaller than that of the cylinder, the pressures obtained

analytically by using either a constant or a variable coefficient of discharge and those measured, would likely be in agreement with the indicator diagram for the cylinder. For example, Deckker⁽²⁾ found that, in spite of the agreement between the predicted and measured values of the pressure in the cylinder, the integrated mass discharged obtained from his analysis was substantially different from that calculated from the initial and final thermodynamic states of the gas and the volume of the cylinder. It would appear, therefore, that the pressure in the cylinder is not a sensitive enough index of the correctness of the values of the discharge coefficient used in the calculations. It should also be noted that, in view of Weaving's⁽⁹⁾ results, although these were not conclusive, the assumption of polytropic expansion in the cylinder may be open to doubt. It is certain that for a discharge occupying a longer time than that from an engine cylinder, the assumption of isentropic expansion would not be valid because of irreversible transfer of heat between the gas and the cylinder wall.

2.3 Transient Flow Through an Orifice

2.3.1 Flow through an orifice in the absence of wave action in the cylinder

There is little published work that can be said to be directly related to the aims of the present investigation of transient flow through an orifice.

In Weaving's⁽⁹⁾ experiments simulating the discharge from an engine cylinder, wave action was not observed and values of the

discharge coefficient for the port were obtained from the condition that the pressure in the cylinder decreased smoothly. It was unfortunate, for the reasons stated in the previous section, that his method of determination of the index of expansion was open to doubt. From the results of his experiments Weaving also deduced that the discharge coefficients were constants, although as he has stated (page 105 of reference 9), he was aware that in a steady flow, the discharge coefficient of an orifice varied with the pressure ratio across the orifice. While an overall average discharge coefficient may be relevant in the case of port design, it is of no value in the present study.

Wallace and Mitchell⁽¹⁵⁾ obtained values of discharge coefficients from pressure measurements in the cylinder and in the exhaust pipes during the discharge of air from the ported cylinder of an opposed-piston engine. The cylinder had a ring of rhomboidal-shaped ports which were connected symmetrically to two exhaust pipes by means of a belt. The calculation of the mass flow through the ports was based on the smoothed pressure-time curves for the expansion in the cylinder, the consideration of wave motion in the exhaust pipes, and quasi-steady flow from the cylinder to the exhaust pipes through the restriction caused by the port. The expansion of the air in the cylinder was assumed to be adiabatic and reversible. In order to extend their results to other gases at high temperatures, as would be found in a firing engine, they introduced a 'modified' Reynolds number based on the acoustic velocity in the cylinder. In this way, they were able to present their experimental

results as a function of the pressure ratio across the ports, the height of the opened ports and the temperature of the gas in the cylinder.

Considering the conventional definition of the discharge coefficient, it is not clear why Wallace and Mitchell sought to obtain a dependence of the coefficient on the temperature of the gas in the cylinder. Further, the smoothed pressure curve in the cylinder derived by them was based on a one-dimensional model in which a port was located coaxially at the end of the cylinder. The configuration of the ported cylinder used in their experiments was, however, much more complicated.

Benson⁽¹⁶⁾ applied the method of characteristics to the discharge process of an idealised engine cylinder, that is a cylinder with a port at one end, and claimed a high degree of accuracy for the prediction of the pressure in the cylinder. He also showed that the approach velocity towards the orifice was small. Therefore, in calculating the discharge under dynamic, or engine conditions, wave action in the cylinder was neglected. In Benson's experiments, dynamic discharge coefficients were obtained from the analysis of the indicator diagram of a motored engine cylinder under the assumption that the air in the cylinder expanded isentropically. The dynamic values were compared with those obtained from steady flow tests in which air was passed through the cylinder of the engine and the exhaust ports. In a communication, Deckker⁽²⁵⁾ pointed out that the accuracy of the prediction of the pressure in the cylinder during discharge depended on the use of the correct values of discharge coefficient and these were not available. In fact, as it

appeared later, in Benson's analysis the coefficient had been taken to be unity. Thus, it is clear that, within limits, the pressure-time relationship in the cylinder is insensitive to the value of the discharge coefficient, and cannot be used as a check on the correctness of the discharge coefficient. This observation applies equally to the method adopted by Wallace and Mitchell⁽¹⁵⁾ to calculate the discharge coefficient. Deckker⁽²⁵⁾ also referred to the lack of dynamical similarity in Benson's steady and dynamic tests. The importance of dynamical similarity in comparing steady and unsteady flows has been discussed by Deckker⁽²⁶⁾ in a recent paper.

It may be noted that the published work in this area is concerned usually with the average value of the discharge coefficient for the purpose of port design. No attempt has been made to analyse the behaviour of the flow or the effects of the independent variables on the transient flow through the port, or orifice.

2.3.2 Flow through an orifice due to the passage of a pressure-wave

The application of the method of characteristics to the solution of problems involving a non-steady flow in a duct is now well established. By this method one can determine either graphically or numerically the state and motion of the gas, and the propagation of the pressure waves in a duct, when the initial and boundary conditions are given. The flow behind the front of the wave is usually considered to be steady so that steady flow boundary conditions, which in many cases can easily be found analytically, are used in the method of characteristics.

The boundary condition is usually assumed to be established

immediately after the arrival of a wave front at the section considered. However, Rudinger⁽²⁷⁾ has found that at the open end of a duct steady conditions were established only some time after the arrival of the incident wave, provided the wave was of sufficiently long duration. He derived an expression for the time-lag required for a flow to become adjusted to the steady state and verified his predicted values experimentally. Later, Rudinger⁽²⁸⁾ used the expression for the time-lag to modify the boundary conditions for an orifice located at the end of a circular duct and analysed the reflection process when a pressure wave was incident at the orifice. This 'relaxation' time, which is required when conditions change rapidly, is an interesting concept. It may account for the differences that have been observed between steady and transient flows through an orifice, since in the latter, conditions upstream of the orifice are changing continuously.

Recently, Trengrouse, Imrie and Male⁽²⁹⁾ used a shock tube to examine the discharge coefficient for an orifice located at the end of the tube. They compared their results with steady flow values obtained by the method of flow measurement prescribed in the British Standard Code⁽³⁰⁾. Their steady flow tests were carried out using the same diameter ratio for the orifice-pipe configuration and for the same pressure ratios across the orifice as in the shock tube experiments. The differences found by them in the two cases may be a result of neglecting equality of the approach velocity, which, from the point of view of dynamical similarity, is more important than the diameter ratio. In fact, as had been pointed out by Rudinger⁽³¹⁾,

the losses determined by the shock tube technique are steady flow values.

2.3.3 Pulsating flow through an orifice

The problem of pulsating flow has attracted the attention of investigators for a long time. The main emphasis of much of the earlier work has been on the use of orifices, particularly a sharp-edged orifice, for the measurement of pulsating flows, since the orifice meter is a convenient and economical method of flow measurement.

An extensive review of published work on the measurement of pulsating incompressible and compressible flows prior to 1955 has been given by Oppenheim and Chilton⁽³²⁾. For the most part, the earlier work was concerned with incompressible flows.

When an orifice meter is used in the measurement of pulsating flow, the general method of approach is to treat the flow as being one-dimensional and quasi-steady. Two approaches have been made to improve the accuracy of the measurement. The first method seeks to eliminate, or to reduce, the influences due to pulsation in the measurement of flow. For example, 'critical nozzle' flow and air box methods⁽³³⁾⁽³⁴⁾ have been introduced for this purpose. The second approach which, in general, is more favoured by investigators is to obtain an overall correction factor which may be applied to the results obtained from considerations of one-dimensional steady flow. In order to do this, a limit must be found for the amplitude and frequency of pulsation for which the steady flow measuring technique can be used without serious error.

This limit is generally expressed in terms of the Strouhal number, N_s , which is a function of the ratio of the local acceleration, $\partial v / \partial t$, and the convective acceleration, $v(\partial v / \partial x)$. The Strouhal number is defined by:

$$N_s = (d \times f) / \bar{v} ,$$

where $\bar{v}$ is the time average velocity of the flow, d is the diameter of the orifice used in the measurement and f is the frequency of pulsation.

If the Strouhal number is small enough, the dynamic effect, represented by the local acceleration, may be neglected and the flow may thus be treated as being steady. Schultz-Grunow⁽³⁵⁾ suggested that as a limit the inverse of the Strouhal number, $1/N_s$, should be greater than 510. For pulsating flows greater than the limiting value, efforts have been directed to the determination of the nature of the correction factor associated with the use of steady flow measurement techniques⁽³⁶⁾⁽³⁷⁾⁽³⁸⁾⁽³⁹⁾⁽⁴⁰⁾. The point of major concern is the correction of the square root error in measuring the differential pressure across the orifice. In a one-dimensional steady flow equation, the mass flow through the orifice may be expressed as:

$$\dot{m} = K \sqrt{\Delta p}$$

where K is a constant and Δp is the pressure differential across the orifice. In the measurement of pulsating flow, it is necessary to know the average value of Δp over a time period $t_2 - t_1$

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \sqrt{\Delta p} dt \quad \dots 2.1$$

However, a damped manometer would indicate a value given by

$$\sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \Delta p \, dt} \quad \dots 2.2$$

The difference between the two averaged quantities in 2.1 and 2.2 is considered an important source of error in pulsating flow measurement. Some attention has also been given to purely physical aspects of pulsating flows, such as, the correlation of the Strouhal number and the Reynolds number of the flow, and the form and amplitude of the wave. Experimental aspects, such as the design of the manometer and its attachments, the calibration of the manometer, and the geometry and the arrangement of the metering orifice have also been considered in some detail. However, in spite of the effort in this field which is reflected in the large number of published investigations, little agreement among investigators has yet been reached⁽⁴¹⁾ and no firm conclusions can be drawn about the best method of measuring a compressible pulsating flow.

A theoretical approach to pulsating flow through an orifice has been given by Hall⁽⁴²⁾ on the basis of the application of the one-dimensional flow equations to instantaneous conditions. Baird and Bechtold⁽⁴³⁾ introduced an electrical analogy for the orifice metering problem. Earles and Zarek⁽⁴⁴⁾ have attempted to relate the results obtained from experiments on the passage of pressure waves through an orifice to the correction factor for pulsating flow measurement that had been determined by them in earlier experiments⁽³⁸⁾. Benson and El Shafie⁽⁴⁵⁾ used the one-dimensional quasi-steady flow theory to predict wave action in the pipe-orifice configuration.

It was claimed that this was a promising approach to the calculation of instantaneous mass flows through an orifice under pulsating conditions. The general validity of the statement is open to doubt as the question of dynamical similarity is involved. Some remarks on the validity of the application of the results of steady flow experiments to transient flows have already been given. It may be noted that neither a theoretical nor an experimental approach has yet been developed which enables the accurate measurement of pulsating flows to be made.

It is hoped that a more extensive study of slow transient flows through an orifice along the lines undertaken in this thesis will lead to a better understanding of the transient behaviour of pulsating flows.

3. DESIGN AND DIMENSIONAL CONSIDERATIONS

3.1 Introduction

In the present state of art, analytical methods applied to a transient flow through an orifice would yield solutions only under simplifying assumptions. The results from such an analysis would, therefore, have a limited usefulness, particularly since the aim of the present investigation is to differentiate between transient and steady flows. Because the relative importance of the independent variables in the problem is not known, the omission of any term for the sake of simplification is not justified. For this reason it is necessary to rely on the analysis of data from systematic experiments designed on the basis of dimensional analysis. Although it is recognized that this method of analysis is circumscribed by the ability of the experimenter to predict correctly the quantities that influence the problem, nevertheless, it is felt that this approach would give a better insight into the physical nature of the expansion in the cylinder and the transient flow through the orifice.

3.2 Expansion of Air in the Cylinder During Discharge

The problem of the expansion of air in a cylinder during discharge is a basic one in thermodynamics. However, as may be inferred from the literature review, little information can be found about the nature of a practical expansion process during which heat transfer between the gas and the cylinder occurs. It is impossible

to provide an adiabatic boundary for the system since there is no known material with the required insulating property which would have an energy content substantially smaller than that of the air in the cylinder. The effect of heat transfer to the air in the cylinder is particularly important if the discharge time is relatively long. Even for the short discharge times and for the small temperature differences encountered in an experiment like that of Clément and Désormes, the error due to heat transfer from the vessel to the gas within it could be appreciable if the ratio of the surface area to the volume of the vessel is relatively large⁽⁸⁾.

The assumption of polytropic expansion is unsatisfactory when the pressure difference across the orifice is high for the greater part of the discharge and when the area of the orifice is comparable with the cross sectional area of the cylinder. Since, under these conditions the kinetic energy of the mass leaving the cylinder will be appreciable, it is very unlikely that the expansion process within the cylinder would be free from irreversibilities. A polytropic process, like an isentropic process, is a limiting one which can never be realized in a practical expansion because of the relatively large kinetic energy of the leaving gas and the irreversible transfer of heat to the gas from the cylinder wall.

The only reliable evidence for the thermodynamic nature of the expansion is the simultaneous measurement of the pressure and the temperature during the process. A uniform distribution of temperature throughout the cylinder during expansion apparently has always been tacitly assumed in the previous published work. However,

preliminary measurements made within the cylinder during discharge have shown that the temperature distribution was not uniform from which it was inferred that the expansion would not be isentropic.

To evaluate the mass in the cylinder at any instant during discharge, an instantaneous average temperature, T_o , weighted for density, must be determined, corresponding to the instantaneous pressure, p_o , in the cylinder. This temperature may be regarded as that which is attained by the total mass of air in the cylinder after layers at different temperatures have been mixed completely and isentropically. At any instant, corresponding to a given pressure in the cylinder, the ratio of the average temperature, T_o , to the isentropic temperature, T_s , is taken as a measure of the departure of the actual expansion from the isentropic value.

For discharges in which air in the cylinder is initially in thermal equilibrium with the cylinder wall, the independent variables which influence the departure of the average temperature from the isentropic value are considered to be the ratio of the surface area to the volume of the cylinder, the initial thermodynamic state of the air, the instantaneous pressure in the cylinder and the time required to reach the given pressure from the commencement of discharge. The effect of changes in the diameter and in the geometry of the orifice are implicit in the above considerations. For a cylinder of given dimensions, the important aspect of the expansion is the temporal sequence, or history, of the process which may be determined from the pressure-time and the temperature-time relationships for the air in the cylinder.

The quantities on which the departure of the actual expansion from the isentropic one are considered to depend, are listed with their dimensions as follows:

L	(L)	length of the cylinder,
D	(L)	diameter of the cylinder,
p_o	$(ML^{-1}T^{-2})$	instantaneous pressure in the cylinder,
p_{oo}	$(ML^{-1}T^{-2})$	initial pressure in the cylinder,
T_{oo}	(θ)	initial temperature of the air in the cylinder,
t	(T)	duration of the expansion from the commencement of the discharge to the instant considered,
R	$(L^2T^{-2}\theta^{-1})$	gas constant of the fluid under expansion,
N	$(ML^{-1}T^{-2})$	bulk modulus of the fluid, and
p_a	$(ML^{-1}T^{-2})$	atmospheric pressure.

The functional relationship may be written:

$$T_o/T_s = f_1(L, D, p_o, p_{oo}, T_{oo}, t, R, N, p_a)$$

By applying Buckingham's Pi-Theorem (detail procedures are given in Appendix A), a relationship between the relevant dimensionless groups is obtained:

$$T_o/T_s = f_2(L/D, p_a/p_o, p_{oo}/p_a, N/p_o, t/\tau),$$

where $\tau = L/\sqrt{kRT_{oo}}$ is a characteristic time. Since L/D and $N/p_o = k$ are constant in the present experiments, the relationship is reduced to:

$$T_o/T_s = f(p_a/p_o, p_{oo}/p_a, t/\tau) \quad \dots 3.1$$

Equation 3.1 enables the effect of heat transfer from the cylinder wall on the average thermodynamic state of the air in the cylinder to be examined. The heat transferred cannot be evaluated in detail, since the modes of heat transfer, the variation of the thermal properties of the system and the transient temperature distribution in the cylinder wall are not known. In any case, an investigation involving these quantities is outside the scope of the present study. Equation 3.1 serves as a basis for the design of experiments for obtaining data required in the calculation of the residual mass of air in the cylinder during the discharge.

3.3 Transient Flow Through an Orifice

This thesis is mainly concerned with the flow through a circular orifice under the relatively slow transient conditions generated by the discharge of compressed air from a cylinder in which wave action is negligible. Although the results of earlier published work are reasonable, nevertheless, some of the assumptions concerning the expansion process in the cylinder are open to doubt and comparison between flows under steady and unsteady conditions cannot be wholly justified because of the lack of dynamical similarity. Intuitively, it is to be expected that flow through an orifice under transient conditions will be different from that in the steady state and this is also the general conclusion that may be drawn from the work of other investigators. However, from systematic experiments and by paying particular attention to the requirements for dynamical similarity, it is hoped that more definite conclusions can be drawn about the nature of

transient flows. The impossibility of achieving absolute dynamical similarity between the present experiments and those for the steady state reported earlier⁽⁶⁾ has already been the subject of some comment. However, it is believed that overall similarity of the flow pattern upstream of the orifice in the two sets of experiments is an adequate basis of comparison between them.

The transient mass flow through an orifice is considered to depend on the instantaneous conditions upstream of the orifice and the pressure differential across the orifice, except when the orifice is choked and the flow is independent of the down-stream pressure. It is also necessary to take into account the 'history' of the discharge which, in the present investigation, has been correlated by the initial thermodynamic state of the cylinder contents and by the time taken to reach the instantaneous pressure.

The independent variables that are considered to affect the transient mass flow through an orifice and their dimensions are as follows:

1. Orifice and cylinder geometry:

D	(L)	cylinder diameter,
L	(L)	cylinder length,
d	(L)	orifice diameter,
l	(L)	orifice length, and
h	(L)	roughness of the flow passage.

2. Flow conditions across and at the orifice:

p_o	$(ML^{-1}T^{-2})$	instantaneous upstream pressure,
T_o	(θ)	instantaneous upstream temperature,

p_a	$(ML^{-1}T^{-2})$	downstream pressure,
p	$(ML^{-1}T^{-2})$	instantaneous orifice local pressure,
T	(Θ)	instantaneous orifice local temperature, and
v	(LT^{-1})	average velocity at the orifice.

3. Transient effects and 'history' of the flow:

p_{oo}	$(ML^{-1}T^{-2})$	initial pressure of the discharge,
T_{oo}	(Θ)	initial temperature of the discharge, and
t	(T)	time required for discharge to pressure p_o .

4. Properties of the gas:

R	$(L^2T^{-2}\Theta^{-1})$	gas constant,
N	$(ML^{-1}T^{-2})$	bulk modulus of the gas, and
μ	$(ML^{-1}T^{-1})$	viscosity of the gas.

Thus, at any instant, the transient mass flow through the orifice may be written:

$$\dot{m}_t = \dot{m}_t(D, L, d, l, h, p_o, T_o, p_a, p, T, v, p_{oo}, T_{oo}, t, R, N, \mu)$$

Application of Buckingham's Pi-Theorem yields the following dimensionless relationship:

$$G_t = \frac{\dot{m}_t \sqrt{kRT_o}}{p_o d^2} = g_t \left(\frac{L}{D}, \frac{l}{d}, \frac{d}{D}, \frac{h}{d}, \frac{p_a}{p_o}, \frac{p_{oo}}{p_a}, \frac{p_o}{p}, \frac{T_{oo}}{T_o}, \frac{T_o}{T}, k, Re, M, \frac{t}{\tau} \right).$$

.... 3.2

where, G_t = the transient mass flow number,

$\dot{m}_t$ = the measured transient mass flow through the orifice
at any instant,

$\tau = L/\sqrt{kRT_{oo}}$, the characteristic time and

Re and M are the Reynolds number and the Mach number respectively at the upstream face of the orifice.

The transient nature of the discharge is defined explicitly

by the dimensionless time, t/τ , and by those groups which specify the chronological aspects (or the history) of such a process, namely, the initial temperature ratio, T_{oo}/T_o , and the initial pressure ratio, p_{oo}/p_a , and the ratio L/D which is proportional to the surface area for a given volume of the cylinder. In a steady flow, these groups need not be considered and Equation 3.2 reduces to:

$$G_s = \frac{\dot{m}_s \sqrt{kRT_o}}{p_o d^2} = g_s \left(\frac{1}{d}, \frac{d}{D}, \frac{h}{d}, \frac{p_a}{p_o}, \frac{T_o}{T}, \frac{p_o}{p}, k, Re, M \right), \quad \dots 3.3$$

where G_s is the steady mass flow number and $\dot{m}_s$ is the measured steady mass flow.

It is only on the basis of systematic experiments designed in accordance with Equations 3.2 and 3.3 that flows under steady and transient conditions can be compared.

To keep the experimental work within reasonable limits, it is necessary to reduce the number of variables in Equations 3.2 and 3.3. It has been established⁽⁶⁾ that in a steady flow the effect of the Reynolds number, based on the mass flow and on the orifice diameter, is constant for any flow in the range of the present investigation. From physical considerations, there does not appear to be any reason why the effect of Reynolds number would be different in the case of transient flow, so that there is no need to treat the Reynolds number as an independent quantity. Further, using the arguments presented in reference (6), the pressure ratio, p_o/p , the temperature ratio, T_o/T , and the Mach number, M , at entry to the orifice may be expressed as functions of p_a/p_o , the pressure ratio across the orifice. The effect of the velocity approaching the up-

stream face of the orifice is negligible if the value of the diameter ratio, d/D , is kept small, while the effect of roughness of the orifice wall, h/d , would be essentially constant if the procedure for the manufacture and inspection of the test orifices is adhered to rigorously. In this investigation, each orifice was examined under a low power microscope in order to maintain a uniform quality.

Since dry air only was used in this investigation, the properties of the fluid, k and R , can also be regarded as constants.

Now, the temperature ratio, T_{oo}/T_o , in Equation 3.2 may be expressed as:

$$T_{oo}/T_o = (T_{oo}/T_s)(T_s/T_o) = (T_{oo}/T_s)/(T_o/T_s) \quad \dots 3.4$$

where T_s is the temperature corresponding to a pressure, p_o , during an isentropic expansion from the initial conditions, p_{oo} and T_{oo} . Thus, the temperature ratio, T_{oo}/T_s , is a unique function of the pressure ratio, p_{oo}/p_o , or the product of the pressure ratios p_{oo}/p_a and p_a/p_o , and along an isentrope,

$$\frac{T_{oo}}{T_s} = \left(\frac{p_{oo}}{p_o}\right)^{(k-1)/k} = \left(\frac{p_{oo}}{p_a}\right)^{(k-1)/k} \cdot \left(\frac{p_a}{p_o}\right)^{(k-1)/k}.$$

The ratio, T_o/T_s is a measure of the departure of the temperature in the actual expansion from the isentropic value, as discussed in the previous section. From Equation 3.1, the temperature ratio, T_o/T_s , is a function of the pressure ratios, p_{oo}/p_a and p_a/p_o , and the dimensionless time, t/τ . Thus, Equation 3.4 may be written:

$$T_{oo}/T_o = F_1(p_a/p_o, p_{oo}/p_a, T_o/T_s) \text{ or}$$

$$T_{oo}/T_o = F_2(p_a/p_o, p_{oo}/p_a, t/\tau).$$

Using the arguments presented in the preceding paragraph, Equations 3.2 and 3.3 are reduced to the following forms:

For transient flow,

$$G_t = g_{t1}(l/d, p_a/p_o, p_{oo}/p_a, T_o/T_s), \quad \dots 3.5a$$

or

$$G_t = g_{t2}(l/d, p_a/p_o, p_{oo}/p_a, t/\gamma). \quad \dots 3.5b$$

For steady flow, the equation is,

$$G_s = g_s(l/d, p_a/p_o). \quad \dots 3.6$$

Thus, on the basis of the similarity of the flow pattern upstream of the orifice and for the same thermodynamic state of the air in the cylinder, it is possible to distinguish experimentally those independent quantities which may give rise to differences between steady and transient flows through an orifice. It is the purpose of this investigation to examine the effect of the different independent groups in Equation 3.5 on transient flows and to compare the results with those obtained for steady flows on the basis of Equation 3.6⁽⁶⁾.

4. APPARATUS AND EXPERIMENTAL PROCEDURE

4.1 General Description of Apparatus

A photograph of the apparatus is shown in Figure 4.1 and a schematic layout is given in Figure 4.2. The apparatus consists essentially of a cylinder which is used as the expansion chamber with the test orifice mounted coaxially at one end, and the associated pressure and temperature recording equipment.

Air from a reservoir is first passed through a container filled with silica-gel. The moist air is passed slowly through the gel at such a rate that sufficient water vapour is extracted to depress the dew point temperature of the air to a value lower than the minimum temperature expected to occur in the cylinder during expansion.

A calibrated Bourdon pressure gauge and a copper-constantan thermocouple, with a temperature potentiometer, are used to measure the steady initial pressure and temperature, respectively, before the commencement of the discharge. The transient pressure in the cylinder during discharge is measured by a strain gauge-type pressure transducer and a pre-amplifier. The transient temperature is measured by means of a miniature chromel-constantan thermocouple and a high-gain D.C. differential pre-amplifier. Outputs from the thermocouple and pressure transducer as displayed on an oscilloscope were recorded photographically using high speed Polaroid film.

4.2 The Cylinder and Test Orifices

The cylinder and the test orifice constitute the most

FIGURE 4.1 Photograph of Apparatus

1. compressor	7. miniature thermocouples
2. air reservoir	8. pressure transducer
3. air dryer	9. ice bath
4. cylinder	10. high-gain DC differential unit
5. orifice	11. strain-gauge pre-amplifier
6. thermocouple	12. camera

FIGURE 4.2 Schematic Layout of Apparatus

important parts of the experimental apparatus. A circular cylinder of 4 in. internal diameter and 24 in. long is mounted horizontally and used as the expansion chamber from which compressed air is discharged through the test orifice. A cross-sectional view of the cylinder and orifice configuration is given in Figure 4.3. The test orifice is mounted coaxially at one end of the cylinder by flanges of suitable design. The expansion chamber is comprised of two steel tubes each 12 in. long, joined by standard steel flanges. This enables the cylinder length to be changed if necessary although, in the present experiments, the length of the cylinder has been kept constant. In order to minimize the heat transfer from the wall to the air in the cylinder, the inner surface of the cylinder is insulated by a layer of fiberfrax 0.035 in. thick which is overlaid by a layer of aluminium foil 0.0006 in. thick. The foil is used to reduce the radiation error of the thermocouple. Tappings were provided in the cylinder wall at distances of 4.00, 14.25 and 21.50 inches from the upstream face of the orifice to allow the pressure and temperature to be measured during the discharge.

Thirteen orifices, geometrically similar to those used in the earlier investigation under conditions of steady flow⁽⁶⁾ have been used in the experiments described in this thesis. Considerable care was given to the manufacture and inspection of these orifices to ensure that the flow passages and the upstream faces of the orifice plates were smooth and square with one another, and that the edge was free from burrs. The length-diameter ratios of these orifices are 2.0, 1.5, 1.0, 0.5 and 0.05, the latter corresponding to sharp edge orifices prescribed by the British

FIG. 4.3 Cross-sectional view of the cylinder and test orifice.

Standards Code⁽³⁰⁾. The orifice diameters have been limited by the requirement that the ratio of the diameter of the orifice to that of the expansion chamber, d/D , is small enough to preclude the possibility of appreciable wave action in the cylinder. The largest orifice used in this experiment is 0.25 inch in diameter, for which the approach velocity factor calculated by the British Standards Code of flow measurement⁽³⁰⁾ is 1.000007. The geometry of these five groups of test orifices is shown in Figure 4.4 and their leading dimensions are given in Table 1.

To charge the cylinder, the orifice is closed by a laminated cellophane diaphragm which is clamped on the downstream side of the test orifice by means of a sharp-edged ring whose inside diameter is larger than the diameter of the orifice. This arrangement ensures that the diaphragm will be torn close to the ring and that 'petalling' of the cellophane will not interfere with the jet issuing from the orifice during discharge.

For discharges at initial pressures above 60 psig, rupture of the cellophane diaphragm is initiated by means of a solenoid-operated needle which may be seen in Figure 4.1 (page 33). At pressures less than 60 psig the discharge is initiated by cutting the cellophane diaphragm by means of a circular sharp-edged cutter, shown in Figure 4.5, which leaves the orifice clear. Both the solenoid-operated needle and the circular cutter form part of the triggering circuit for the oscilloscope, so that the instant of commencement of the discharge is clearly indicated in the photographic record.

Before the present apparatus was adopted, an attempt was

FIG. 4.4 Geometry of test orifices

TABLE I

Leading Dimensions of Test Orifices

DIAMETER, d (in.)		LENGTH, l (in.)	LENGTH DIAMETER RATIO (l/d)
Nominal	Exact		
1/4	0.2504	0.5000	2.0
3/16	0.1875	0.3750	2.0
1/4	0.2504	0.7500	1.5
3/16	0.1877	0.5625	1.5
1/4	0.2504	0.2500	1.0
3/16	0.1876	0.1875	1.0
1/4	0.2505	0.1250	0.5
3/16	0.1875	0.0938	0.5
9/32	0.2828	0.0141	sharp edge
1/4	0.2488	0.0125	sharp edge
7/32	0.2196	0.0110	sharp edge
3/16	0.1875	0.0094	sharp edge
1/8	0.1238	0.0062	sharp edge

FIG. 4.5 Diaphragm cutter and orifice mounting flange

made to initiate the transient flow from a steady state by using arrangement shown in Figure 4.6. This was accomplished by means of a shock-operated cut-off valve. While such a design produced very satisfactory starting conditions for the discharge, it was found that random disturbances were introduced into the transient pressure and temperature signals, particularly during the later stages of the discharge. The apparatus was designed to avoid the possibility of interference with the initiation of the transient flow by the use of a mechanical device for the opening of the orifice. However, it was established that the use of a conventional shock tube gave equally satisfactory starting conditions while the overall discharge was not disturbed by random fluctuations in the pressure.

4.3 Experimental Procedure

Prior to an actual test, the cylinder is first purged of moist air by filling and emptying it several times with dry air. The cylinder is then charged to the required initial pressure slowly enough to ensure that the air is adequately dried during its passage through the silica-gel before entering the cylinder. The temperature of the air in the cylinder is allowed to attain equilibrium with the cylinder wall before initiating the discharge at the selected initial pressure. The rupture of the cellophane diaphragm and the triggering of the two beams of the oscilloscope occur simultaneously, the transient pressure and temperature being recorded photographically. Datum lines for the temperature and pressure are superimposed on the photographic record before and after each discharge.

Because of the difficulty in recording the transient

1. plenum chamber	6. high pressure air
2. air dryer	7. diaphragm
3. shock-operated valve	8. needle
4. cylinder	9. silica-gel
5. test orifice	10. baffle

FIGURE 4.6 Early Apparatus using Shock-Operated Cut-off Valve

temperature at more than one position in the cylinder, it is necessary to repeat the discharge. In each case the thermocouple is located in a different predetermined position in the cylinder in order to obtain a meaningful average of the temperature during discharge.

5. TRANSIENT PRESSURE AND TEMPERATURE MEASUREMENT

5.1 Pressure-Time Record

The transient pressure of the air in the cylinder during discharge is measured by a temperature compensated strain gauge-type pressure transducer of 0.25 in. diameter having a time constant of 0.01 millisecond. The transducer can be located in three different positions in the cylinder wall and at convenient radial positions inside the cylinder. The manner in which the pressure transducer is installed in the cylinder wall is shown in Figure 5.1.

Typical records of the transient pressure and the transient temperature of the air in the cylinder during discharge are shown in Figures 5.2(a) to 5.2(f).

5.2 Transient Temperature Measurement

5.2.1 Introduction

The measurement of the transient temperature of the air in the cylinder during a discharge is a difficult undertaking. In order to obtain the true value of the transient temperature at any instant, an instrument which responds very quickly to the changes of temperature is necessary. After a literature survey of the techniques of temperature measurement had been carried out, it was decided to use a thermocouple to measure the transient temperatures. It was felt that the long history of the development and application of thermocouples under similar conditions provided a reliable basis for its use. Another factor in the choice of a thermocouple is that it is

FIGURE 5.1 Installation of Pressure Transducer

FIGURE 5.2(a-c) Typical pressure and temperature records measured at radial position, r , in the cylinder

FIGURE 5.2(d-f) Typical pressure and temperature records measured at radial position, r , in the cylinder

relatively cheap, easy to construct and simple to use.

5.2.2 Choice of the thermocouple

Owing to the wide spread use of thermocouples in the laboratory and in industry, a number of combinations of materials have been developed over the years for different applications. The choice of a thermocouple depends on such factors as the range of temperature for which it is to be used, the linearity of the temperature-electromotive force relationship, the response time, the wire sizes and the chemical and mechanical properties of the available materials.

In the present investigation, the primary consideration was a rapid response of the thermocouple to a temperature change, a property associated with a small wire diameter. The second consideration was that the thermocouple output should be a maximum having regard to the performance of the available recording equipment and the temperature range that was expected during the discharge. The temperature-emf relationship of several common combinations of thermocouples are given in Figure 5.3, and in view of the conditions likely to be encountered in the present experiments, it is clear that a chromel-constantan thermocouple meets the stipulated requirements.

5.2.3 Design of thermocouple

A chromel-constantan thermocouple with wire diameter of 0.0005 inch is used for the measurement of transient temperatures in the cylinder. The design and the installation of the thermo-

FIGURE 5.3 Temperature-electromotive force relationship for common thermocouple materials

couple is shown in Figure 5.4. Thermocouple wires are carried in a two-hole ceramic tube, 1/32 in. outside diameter, which is supported in a stainless steel tube, which is 1/16 in. outside diameter and 6 inches long. The end of the steel tube is sealed with cement to prevent leakage of air from the cylinder through either the ceramic tube or the stainless steel tube. The junction of the thermocouple is left bare outside of the tube as shown in Figure 5.4.

By the use of block adaptors, the thermocouple junction can be located in the cylinder at any one of six predetermined radial positions whose distances measured from the axis of the cylinder are 0.000, 0.658, 1.187, 1.532, 1.813 and 1.938 inches. The four intermediate positions correspond to the mean radii of four co-axial annuli which divide the cylinder cross-section into four equal areas. The two remaining positions correspond to the cylinder axis and the wall, respectively. The reason for the choice of these positions are discussed in a later section.

To increase the accuracy of the measurement, a cold junction maintained at 32 deg. F. in an ice bath is used.

5.2.4 Thermocouple error - conduction and radiation

There are a number of publications⁽⁴⁶⁾⁽⁴⁷⁾⁽⁴⁸⁾ on errors in temperature measurement arising from the use of thermocouples. In general, errors in the measurement of the temperature of a gas stream are due to: (a) heat conduction away from the thermocouple junction, (b) the heat loss due to radiation, (c) the time lag due to convection and (d) the error due to stream velocity. That is, the true temperature, θ , of the gas can be expressed as:

FIGURE 5.4 Arrangement of Miniature Thermocouple

$$\Theta = \Theta_w + \left[\begin{array}{c} \text{conduction} \\ \text{error} \end{array} \right] + \left[\begin{array}{c} \text{radiation} \\ \text{error} \end{array} \right] + \left[\begin{array}{c} \text{convection} \\ \text{error} \end{array} \right] + \left[\begin{array}{c} \text{velocity} \\ \text{error} \end{array} \right],$$

where Θ_w is the wire, or indicated, temperature. The velocity contribution to the error need not be considered since, in the present investigation, the air velocity in the cylinder is zero. In this section, errors that arise in the measurement of transient temperatures due to heat conduction and radiation are discussed.

The difference between the temperature, Θ_w , indicated by the thermocouple junction and the true temperature, Θ , depends on a very complicated heat transfer process occurring at the junction. For the steady state, and for negligible radiation between the gas and the thermocouple junction, an approximate analysis based on an energy balance⁽⁴⁸⁾ gives the following results for the radiation error and the conduction error:

$$\text{Radiation error} = \frac{\sigma T_w^4 \epsilon_w}{h} \left[1 - (T_D/T_w)^4 \right]; \quad \dots \quad 5.1$$

$$\text{Conduction error} = \frac{\psi}{1 - \psi} (T_w - T_B), \quad \dots \quad 5.2$$

where, ϵ_w = thermocouple wire emissivity,

σ = Stefan-Boltzmann constant,

h = film coefficient of thermocouple wire,

T_D = absolute temperature of cylinder wall,

T_w = absolute temperature of wire,

T_B = absolute temperature of thermocouple support,

$\psi = \text{sech} \left[(hL_w^2)/(k_w D_w) \right],$

k_w = thermal conductivity of thermocouple wire,

D_w = diameter of thermocouple wire and

L_w = thermocouple wire loop length.

It is recognized that heat transfer under transient conditions may be different from that of steady state, although transient heat transfer data are not available. Therefore, in a qualitative sense, the magnitude of the errors due to radiation and conduction may be estimated for the transient case from quasi-steady considerations.

It has been found⁽⁴⁸⁾ that if the ratio of the thermocouple wire loop length to the wire diameter, L_w/D_w , is greater than 30, the error due to heat conduction becomes insignificant. The design of the thermocouple used in the present investigation fulfills this criterion, and, thus, the conduction error of the thermocouple may be neglected.

It is difficult to estimate the error due to radiation under the present experimental conditions, owing to a lack of the necessary heat transfer data, namely, the thermocouple wire emissivity, the film coefficient and the exact temperature of the cylinder wall during discharge. To estimate the maximum possible error due to radiation in the temperature measurement, the most favourable conditions for radiation heat transfer are assumed, namely, black body radiation with wire emissivity equal to unity, and the maximum temperature difference which is expected to occur between the cylinder wall and the thermocouple junction during the discharge.

To estimate the maximum temperature difference, the minimum indicated temperature, T_w , of the thermocouple is assumed to be equal to the minimum temperature of the gas in the cylinder, while the maximum temperature, T_D , is assumed to be equal to that of the

cylinder wall at the beginning of the discharge. The minimum temperature, T_w , occurs at a pressure of about 10 psig during the discharge while the temperature, T_D , which is that of the aluminium foil will, in fact, be lower than the initial temperature. Thus, under the extreme conditions, the minimum temperature is 400°R , the maximum temperature is 530°R , the absolute pressure p_0 in the cylinder is approximately 2 atmospheres and the Mach number in the cylinder is zero. Extrapolating from the nomogram given in reference (48), the maximum error due to radiation is estimated to be 0.3°F . This error is within the accuracy of the experimental observations and may be neglected.

It follows from the preceding arguments that the errors due to heat conduction and radiation in the measurement of transient temperatures in the cylinder during discharge are negligible.

It is evident that caution must be exercised in the use of correction factors in the absence of precise data on heat transfer. If inaccurate data are used to estimate errors in temperature measurement they may lead to erroneous results. For example, in the over-corrected value of the temperature obtained by Potter and Levy⁽¹⁴⁾, after corrections had been made for the radiation error, some of the air temperatures were estimated to be lower than those corresponding to isentropic expansion.

5.2.5. Determination of the time constant for the thermocouple and the error due to the convective heat transfer

The response time of the thermocouple is of considerable importance. The time lag in the response of the thermocouple

and the correction for this error may be found in the literature (47)(49)(50)(51). It may be shown that, when a thermocouple is subjected to a step change of temperature from T_1 to T_2 , the change of the indicated temperature of the thermocouple, T_w , follows an exponential law,

$$(T_1 - T_w)/(T_1 - T_2) = 1 - e^{-t/B}, \quad \dots 5.3$$

where B depends on the properties of the thermocouple, namely, the wire diameter, the density of the material of the wire, the heat capacity and film coefficient of the wire. The time constant of the thermocouple, t_c , is usually defined as the time when $t_c = t = B$ (Equation 5.3), so that t_c is the time required for the thermocouple to indicate 63.2 percent of the step change of the temperature.

Because of the lack of heat transfer data for thermocouple materials, it is often necessary to determine the time constant experimentally. The apparatus used for this purpose in the present experiments is shown in Figure 5.5. A step change of the temperature is obtained by suddenly releasing the spring, to withdraw the heating coil which initially enveloped the thermocouple junction. The variation of the indicated temperature is recorded photographically by means of an oscilloscope and camera.

From a number of records obtained from the test, the time constant of the thermocouple was determined to be 9.91 milliseconds.

For the maximum initial pressure and for the orifices used in the present experiments, the maximum rate of temperature change is about 40°F per second during the discharge. If the thermocouple is subjected to a step change of temperature of 40°F , the minimum

FIGURE 5.5 Apparatus for Determination of the Time Constant for Thermocouple

time required to indicate 98.2 percent of the temperature change within one second may be found from Equation 5.3 when $t = 4B$.

The corresponding time constant is given by,

$$t_c = (1/4)(1/40)(10)^3 = 6.25 \text{ milliseconds.}$$

Clearly, the chromel-constantan thermocouple of wire diameter 0.0005 in. is not fast enough for the rates of temperature change likely to be encountered in the temperature measurements. Since the diameter of the wire is already very small, the response of the thermocouple cannot be further improved by reducing its wire diameter. An attempt was, therefore, made to reduce the time constant by means of a phase advance circuit and an additional D.C. amplifier as suggested by Sirs⁽⁵²⁾. This attempt did not prove successful due to the noise introduced by the phase advance circuit and the preamplifier. When the preamplifier was removed, the thermocouple output was reduced by the presence of the phase advance circuit.

To correct the error due to the time lag in the response of the thermocouple, it was decided to adopt the correction formula given by Moffat⁽⁵¹⁾, namely,

$$T = T_w + t_c (dT_w/dt), \quad \dots 5.4$$

where T is the true instantaneous temperature of the air at the position measured in the cylinder. Equation 5.4 is true only if the following assumptions are fulfilled, namely:

- (a) The relationship of the temperature-emf of the thermocouple is linear.
- (b) The temperature measurement is free from errors due to

conduction, radiation and velocity.

- (c) The temperature of the gas is uniform at the junction of the thermocouple.

The first two assumptions are approximately true in the present experiments. With regard to the last assumption, it is known that the temperature distribution in the cylinder during discharge is not uniform throughout. Nevertheless, since the junction of the thermocouple is small and the radial temperature variation in the cylinder, not too large, it is reasonable to assume uniformity of temperature in the vicinity of the thermocouple junction.

5.3 Calibrations

Before and after each experiment, comprising eighteen discharges from the same conditions which were usually completed within 15 hours, the instruments were calibrated directly against standard equipment of the required accuracy. The direct calibration serves both to eliminate errors due to the nonlinear characteristics of the instrumentation and to allow the maximum beam deflection of the oscilloscope to be used. The oscilloscope-transducer system was calibrated statically by means of dead-weight gauge-tester. Calibration of thermocouple-oscilloscope system was also made statically against the millivolt output of a standard Cambridge potentiometer. The temperature-millivolt relationship of the thermocouple was obtained from the National Bureau of Standards Circular No. 561 recommended by the thermocouple manufacturer. Interpolation data have been calculated by the second degree Lagrange equation⁽⁵³⁾.

Each of the calibration records was measured at least ten times using the micrometer device described later. The mean value of these ten measurements was used to evaluate the pressure or the temperature from the photographic records obtained during the discharge.

The sweep time of the oscilloscope was also checked by means of an accurate time-mark generator and was corrected accordingly.

5.4 Measurement of Transient Pressure and Temperature Records

The micrometer device used in the calibration of the transient pressure and temperature traces is shown in Figure 5.6. The sharp-pointed scribe can be moved in two perpendicular directions on the surface of the table by the two micrometers. The transient pressure and temperature record is placed on the table and their ordinates are measured by means of the micrometers. The minimum resolution of the micrometers is 0.0005 inch. However, due to the uncertainty of locating the exact centre of a trace the last decimal place of the readings from the micrometers are not considered to be significant.

It may be seen from Figure 5.2 (page 46) that instantaneous values of the pressure have been measured with reference to the atmospheric line taken during each test. Temperatures have been measured with reference first to the initial temperature of the air in the cylinder and then to the ice point. This procedure was adopted since it is difficult to obtain the zero millivolt datum line by placing the hot junction of the thermocouple in an ice bath during the test.

FIGURE 5.6 Micrometer Device for the Measurement of Pressure and Temperature Record

6. DISCUSSION OF RESULTS

6.1 The Expansion of Air in the Cylinder During Discharge

The results presented in this chapter have been obtained from experimental measurements. Values of the recorded instantaneous pressures and temperatures have been fitted into polynomial equations of the order compatible with the required accuracy. The curve fitting, the correction of temperature, the temperature averaging procedure, and the calculations have been obtained by means of a digital computer.

Measurements of the instantaneous pressure and the instantaneous average temperature for several discharges have been incorporated into a general relationship, (Equation 3.1, page 25), which can be used for the prediction of the instantaneous value of the transient temperature in the cylinder during discharge.

6.1.1 Pressure distribution in the cylinder

It was stated earlier that in a slow transient discharge the pressure is uniform in the cylinder, except in a narrow region close to the orifice where local acceleration of the air occurs. The uniform pressure in the cylinder has been verified by locating the pressure transducer at the wall, and at the axis of the cylinder at three different positions. For discharges from a given initial pressure through the same orifice, identical pressure-time records have been obtained at these different positions in the cylinder.

No attempt has been made to measure the distribution of pressure in the region where acceleration occurs close to the orifice. Since the space occupied by this region is small compared with the volume of the cylinder, nonuniformity in the pressure and temperature can have only a very small effect on the mass in the cylinder as calculated on the uniform pressure and weighted average temperature in the cylinder.

6.1.2 Temperature distribution in the cylinder

Typical photographic records of the variation of temperature at different radial positions in the cylinder have been presented in Figure 5.2 (page 46). Measurement of the transient temperature was first made on the axis of the cylinder and, during the discharge, it was observed that the temperature decreased to a minimum and then increased, although the pressure of the air continued to decrease. This behaviour was similar to that which had been observed by Potter and Levy⁽¹⁴⁾ in their experiments except that the minimum temperature was reached after a longer interval of time from the commencement of discharge. The difference in time can be attributed to the larger volume and larger diameter of the cylinder used in their experiments. Potter and Levy concluded that during expansion of the air convective heat transfer became predominant after some time and was responsible for the increase in temperature of the air at the cylinder axis.

Measurements of the transient temperature were then made at different radial positions in the cylinder. As can be seen in Figure 5.2 (page 46), it is clear that the distribution of the

temperature in the cylinder at any instant during the discharge is not uniform. Nonuniformity of the temperature in the axial direction is much smaller.

During discharge it is not known whether the radial distribution of temperature is continuous or whether there is some form of stratification, at least in the early stages of the discharge. However, it appears reasonable that once radial and axial gradients of temperature occur convection currents will be set up which, because of asymmetry, lead to the random fluctuations of temperature that have been observed towards the end of the discharge. It may also be seen that these fluctuations occur earlier when the position of the thermocouple is nearer to the cylinder wall. The instantaneous temperatures are measured from the mean curve drawn through the fluctuating part of the temperature-time records.

For a discharge from a given initial pressure and for a given orifice, typical relationships between transient pressures and the temperatures at six radial positions in the cylinder are reproduced in Figure 6.1 in the form of the logarithmic values of the pressure and temperature ratios. The six radial positions measured from the cylinder axis are 0.000, 0.685, 1.187, 1.532, 1.813 and 1.938 inches. Since the pressure-time relationships for the six curves shown in Figure 6.1 are identical, the pressure ratio ordinate is also a measure of time as given by the scale included in the Figure. For any pressure ratio in the cylinder, differences of temperature among the six radial positions give the radial temper-

FIGURE 6.1 Typical Pressure-Temperature Relationships of the Air at Six Radial Positions in Cylinder during Discharge

ature distribution at that instant.

In Figure 6.1, the straight line corresponds to isentropic expansion. Along a line of constant pressure ratio, corresponding to an instant during the discharge, the horizontal intercept between the curve for each of the six positions in the cylinder and the straight line is a measure of the departure of the temperature of the air in the cylinder from the isentropic value. The departure becomes greater as the cylinder wall is approached. Near the axis of the cylinder, the transient temperatures follow closely the isentropic values for the greater part of the discharge; at the wall, after a small decrease initially, the temperature is almost constant.

As at the axis of the cylinder, the temperature at each of the other five radial positions decreases to a minimum and then increases as the pressure continues to fall during the discharge. The time at which the minimum temperature is reached occurs earlier at positions closer to the wall. This behaviour indicates that the main reason for the departure of the expansion from the isentropic law is the transfer of heat to the air from the cylinder wall.

For a given radial position on the three axial planes, differences of temperature are not as pronounced as those in the radial direction. The distances of these three planes from the upstream face of the orifice are 4.00, 14.25 and 21.50 inches. At each plane the temperatures measured at the six different radial positions have been averaged on the basis of the weighted density. Details of the temperature averaging procedure are discussed in the following section. The typical relationship between the average

temperature and the corresponding pressure for each of the three axial planes is shown in Figure 6.2 as the three broken lines drawn through the experimental points. The solid line represents the weighted average temperature of the air in the cylinder.

6.1.3 Weighted average temperature of the air in the cylinder during expansion

The relationship between the instantaneous pressure, p_o , and the weighted average instantaneous temperature, T_o , of the air in the cylinder during a discharge is shown as the solid line in Figure 6.2. The average temperatures of the air in the cylinder are obtained by a double averaging procedure in the radial and axial directions weighted on the basis of the density of the air. It is with the object of weighting for the density that the radial positions of the thermocouple junction have been located at the mean radii of the concentric annuli which divide the cylinder cross-sectional area into four equal parts. The temperature corresponding to the arithmetic mean of the densities obtained at the different radial positions is, therefore, the average temperature of a plane. The average temperature of the air in the cylinder has also been obtained on the basis of the weighted density which is dependent on the axial temperature distribution at the three sections. The average temperature obtained in this way represents the uniform temperature that would be attained throughout the cylinder at any instant during the discharge if the air is mixed completely and isentropically.

By using a relationship between the instantaneous pressure and the instantaneous average temperature such as that in Figure 6.2,

FIGURE 6.2 Average Pressure-Temperature Relationships of the Air in Three Axial Planes and of the Air in the Cylinder

the average thermodynamic state of the air at any instant during the expansion may be obtained from the indicator diagram in the cylinder. At any instant, the residual mass of the air in the cylinder is calculated from the equation of state using instantaneous values of the pressure and the average temperature in the cylinder. The transient mass flow through the orifice is found from the equation of the conservation of mass flow.

It may be seen from Figure 6.2 that departure of the weighted average temperature of the air from the corresponding isentropic value begins a very short time after the commencement of discharge. The extent of the departure gradually increases as the discharge proceeds and reaches a point when the change of temperature with respect to the change of pressure is zero, that is,

$$d(\ln(T_{oo}/T_o))/d(\ln(p_o/p_a)) = 0,$$

corresponding to a turning point in the curve of the pressure-temperature relationship. Further decrease of the pressure is then accompanied by an increase in the average temperature of the air in the cylinder.

6.1.4 Pressure-temperature correlation and the prediction of the temperature for the air in the cylinder during discharge

Relationships between the transient pressure and the average temperature of the air in the cylinder, similar to those in Figures 6.1 and 6.2, have been obtained for discharges from initial pressures, p_{oo} , of 18, 30, 60, 80 and 100 psig. and for a constant initial temperature, T_{oo} , (equal to room temperature) using sharp edge orifices of diameters 0.2828, 0.2488, 0.2196, 0.1875 and 0.1238 inches. For example, the relationship between the pressure and the

average temperature for an initial pressure of 60 psig. using three different orifices is shown in Figure 6.3 in which it is seen that the extent of the departure from the isentropic law is dependent on the orifice diameter.

The magnitude of the departure from the isentropic law may be expressed by the following relationship:

$$\begin{aligned}\ln\left(\frac{T_{oo}}{T_o}\right) &= \ln\left(\frac{T_{oo}}{T_s} \cdot \frac{T_s}{T_o}\right), \\ &= \ln\left(\frac{T_{oo}}{T_s}\right) - \ln\left(\frac{T_o}{T_s}\right), \\ &= \frac{k-1}{k} \ln\left(\frac{p_{oo}}{p_o}\right) - \ln\left(\frac{T_o}{T_s}\right)\end{aligned}$$

The first term on the right hand side of the above equation is the isentropic temperature ratio corresponding to the expansion ratio, p_{oo}/p_o , and is represented by the straight line in Figure 6.3. The second term is a measure of the departure from the isentropic value and is equal to the distance between the actual curve and the isentrope.

It is clearly desirable to derive a simple expression which would correlate the transient pressures and temperatures obtained experimentally and which would also be valid for any discharge under the conditions of the present investigation.

Equation 3.1 of the dimensional analysis, shows that the temperature ratio, T_o/T_s , is a function of p_{oo}/p_a , p_a/p_o and t/γ . It has been found by trial and error that, for a given initial pressure ratio, p_{oo}/p_a , the temperature ratio, T_o/T_s , can be correlated with p_a/p_o and with t/γ by a functional expression of the form:

FIGURE 6.3 Pressure and Weighted Average Temperature of the Residual Air in Cylinder During Discharge for a Given Initial Pressure

$$T_o/T_s = f(Z),$$

where $Z = 1/(p_a/p_o)(t/\tau)^{\frac{1}{2}}$.

It follows that, in general,

$$T_o/T_s = F(Z, p_{oo}/p_a) \quad \dots 6.1$$

The results of this correlation for different initial pressures and for orifices of different diameters are shown in Figure 6.4 where T_o/T_s is plotted against Z with the initial pressure ratio as parameter. Each curve has been obtained from the pressure-time and temperature-time records for discharges using orifices of different diameters.

The accuracy of this correlation has been analysed by the method of the least square polynomial regression and the standard error of the temperature ratio, T_o/T_s , has been found to be about 0.0057, which is considered to be satisfactory.

For practical purposes, the curves shown in Figure 6.4 may be used to calculate the rate of change of the mass of air in the cylinder. However, it is generally difficult to obtain the slope of any one of these curves with sufficient accuracy, particularly in those regions where the curves are relatively flat. The determination of the slope is simplified by a suitable transformation of the coordinates. The transformation is carried out in two steps in order to achieve the maximum accuracy. In the first step, when the Z -coordinate is transformed into

$$X = 1/(Z - 0.04),$$

a major segment of each curve becomes a straight line whose equation is:

FIGURE 6.4 Average Pressure and Temperature Correlation of the Air in Cylinder During Discharge

$$T_o/T_s = aX + b, \quad \dots 6.2$$

where, a and b are constant coefficients whose values depend on the initial pressure of the discharge. The results of the transformation are shown plotted in Figure 6.5. The loci of the terminal points of straight lines is given by the equation:

$$T_o/T_s = -0.1543 X + 1.080.$$

The lines represented by equation 6.2 corresponds to the later part of the discharge. In the second step of the transformation, those segments of the curves which correspond to the earlier stage of a discharge became straight lines when the X-coordinate is transformed by

$$Y = X^2,$$

as in Figure 6.6. The equation of this family of lines is given by

$$\begin{aligned} T_o/T_s &= c Y + d, \\ &= c X^2 + d, \end{aligned} \quad \dots 6.3$$

where, c and d are constant coefficients.

The coefficients in the equations 6.2 and 6.3 have been established by means of a least square fit through the experimental points, and their values are given in Table 2.

The relationship between the transient pressure and the average temperature established in this section not only serves for the calculation of the mass of the air in the cylinder but also gives an insight into the thermodynamics of the expansion process during a relatively slow discharge. The measurement of the transient pressure in the cylinder during discharge is clearly an easier experimental task than the measurement of a meaningful temperature.

FIGURE 6.5 Straight Line Transformation of the Temperature Correlation for Discharges

FIGURE 6.6 Straight Line Transformation of the Temperature Correlation for Early Stages of Discharge

TABLE 2

Coefficients for Equations of Transient
Pressure-Temperature Correlation

$$T_o/T_s = a X + b \quad \dots 6.2$$

$$T_o/T_s = c Y + d \quad \dots 6.3$$

Initial pressure (psig)	Coefficients			
	a	b	c	d
100	0.1079	0.9126	0.0365	0.9951
80	0.0905	0.9290	0.0312	0.9944
60	0.0631	0.9295	0.0149	0.9949
30	0.0418	0.9446	0.00898	0.9979
18	0.0352	0.9352	0.00562	0.9975

The use of the empirical relationship, Equations 6.2 and 6.3, to predict the average temperature of the air from values of the instantaneous pressure obtained from an indicator diagram is illustrated in Figure 6.7 in which the predicted and measured values of the temperature have been compared. In that figure, the solid lines represent the calculated values obtained from the pressure-time records using Equations 6.2 and 6.3, while the experimental values obtained by the double averaging procedure outlined earlier are given by the points. It is evident that agreement between the measured and the predicted values is very good, the maximum error being less than one percent.

6.1.5 Index of expansion

In the review of literature presented in Chapter 2, the limiting nature of a polytropic process has been discussed. It is doubtful whether the use of a constant index of expansion to relate the pressure and temperature in a practical discharge process is meaningful, particularly under the conditions associated with a relatively slow discharge.

At a given instant during a discharge, the thermodynamic state of the air in the cylinder defined by p_o and T_o can be related to the initial state by an equation of the form:

$$T_{oo}/T_o = (p_{oo}/p_o)^{(n'-1)/n'}, \quad \dots 6.4$$

where n' is a numerical value.

It follows that:

$$(n' - 1)/n' = \ln(T_{oo}/T_o)/\ln(p_{oo}/p_o),$$

FIGURE 6.7 Comparison of Predicted Temperature in Cylinder during Discharge with Experimental Values

$$\begin{aligned}
&= \ln \left[(T_{oo}/T_s)(T_s/T_o) \right] / \ln(p_{oo}/p_o), \\
&= \ln(T_{oo}/T_s) / \ln(p_{oo}/p_o) - \ln(T_o/T_s) / \ln(p_{oo}/p_o), \\
&= (k - 1)/k - \ln(T_o/T_s) / \ln(p_{oo}/p_o),
\end{aligned}$$

or,

$$1/n' = 1/k + \ln(T_o/T_s) / \ln(p_{oo}/p_o). \quad \dots 6.5$$

Clearly n' is not constant throughout the expansion process since the term T_o/T_s is a function of the initial pressure ratio, p_{oo}/p_a , and the dimensionless group, $Z = 1/(p_a/p_o)(t/\tau)^{\frac{1}{4}}$, (Figure 6.4). Thus, n' will also be a function of the two groups, p_{oo}/p_a and Z . In a practical expansion process, n' is always less than k since the last term of Equation 6.5 is always positive. In the limiting case of an isentropic expansion, $T_o = T_s$, and n' becomes constant and equal to the ratio of the specific heats, k .

The index n' in Equations 6.4 and 6.5 is different from the index n associated with a polytropic process. Although the values of n' found from Equation 6.5 may be substituted into Equation 6.4 to relate the instantaneous pressure, p_o , and temperature, T_o , to their initial values, nevertheless, it cannot be regarded as an index of expansion. It must be noted that n' as calculated from Equation 6.5 can neither be used to relate the instantaneous conditions at different points during a process to the initial state, nor can it be used to calculate any quantity which depends on the path of the process.

A number of values of the index, n' have been calculated by means of Equation 6.4, from pressure and temperature measurements

obtained in the course of the present investigation and the results have been plotted against $Z = 1/(p_a/p_o)(t/\gamma)^{\frac{1}{\gamma}}$ as shown in Figure 6.8. In this Figure, it is seen that values of n' for the different discharges considered in the analysis fall approximately on a single curve. The dependence of n' on the initial pressure, p_{oo}/p_a , appears to be insignificant. In the earlier stages of the expansion, corresponding to large values of Z , n' approaches the value of the ratio of the two specific heats and, towards the end of the discharge, corresponding to small values of Z , n' decreases to a value less than unity. In a very rapid discharge, however, it is not known whether the assumption of isentropic expansion is a valid one although the value of Z for the whole of the expansion will be large. The amount of heat transferred to the air in the cylinder during an expansion process evidently depends both on the duration of the expansion and on the temperature difference between the air and the cylinder wall. In Figure 6.8, conditions associated with large values of Z correspond to short discharge times and small temperature differences. In a rapid discharge, however, although the duration of the expansion is small, nevertheless, during the later stages the temperature difference between the cylinder wall and the air is large. At any rate, if heat is transferred to the air in the cylinder, the index n' in Equations 6.4 and 6.5 will no longer be constant. Therefore, it would appear that the assumption made by Geyer⁽¹²⁾ and by Weaving⁽⁹⁾ that the expansion is polytropic, is clearly incorrect.

Values of the temperature obtained from the present

FIGURE 6.8 Variation of the Index of Expansion

experiments and those calculated from the initial conditions by means of a constant index of expansion are compared in Figure 6.9. The broken lines through the points represent the locus of the quasi-static states obtained from pressure and temperature measurements while the solid lines correspond to calculations using the constant value of n found from the conditions at the points marked. Differences between the two temperatures indicate the errors when a constant exponent is used. It is clear that the results calculated in this way are not satisfactory.

6.1.6 Effect of heat transfer on the pressure-time relationship of air in the cylinder during discharge

The effect of heat transfer on the transient temperature of the air in the cylinder during expansion has already been discussed in previous sections. The question may be raised whether the transfer of heat affects the pressure in the cylinder during the expansion process. Under the conditions of the present investigation, heat is transferred spontaneously from the cylinder wall to the air when the temperature difference between them is increasing during expansion. It is believed that the magnitude and the rate of heat transfer do not have an appreciable effect on the pressure-time relationship for the air in the cylinder. Heat transferred to the air tends to increase the pressure but, at the same time, the increase in pressure will be balanced by an increase in the discharge rate, so that the pressure-time record in the cylinder remains unchanged. To verify this hypothesis, an experiment has been carried out with different rates of heat transfer to the air

FIGURE 6.9 Comparison of Temperatures Calculated using Constant Indices of Expansion with Experimental Values

using an electric heating coil.

An electrical heater of 700 watts maximum output was installed inside the cylinder to provide different rates of heat transfer during the expansion process. The pressure-time records obtained from such experiments are given in Figure 6.10. Figure 6.10a is for a discharge under normal conditions, that is without an additional supply of heat. Figure 6.10b shows the result obtained when the addition of heat starts simultaneously with the commencement of discharge. Figure 6.10c shows the result of an experiment in which heat is added to the air in the cylinder prior to the commencement of the discharge. The discharge is initiated when an increase of pressure due to the addition of heat is first observed. When these records are superimposed on one another, no differences in the rate of change of pressure can be observed. Although the results of this simple experiment can only be regarded as being qualitative, nevertheless, it is reasonable to conclude that when heat is transferred spontaneously between the air and the wall of the cylinder, the rate of change of pressure in the cylinder is unaffected.

6.2 Transient Flow Through Orifices

6.2.1 Introduction

To obtain the transient mass flow through an orifice during the discharge of air from a cylinder, the instantaneous residual mass of the air in the cylinder is first determined in terms of known values of the pressure and the average temperature from the

FIGURE 6.10 Pressure-time Relationships for Discharges with Different Rates of Heat Transfer to the Residual Air in the Cylinder

$$p_{oo} = 60 \text{ psig}, d = 0.1875 \text{ in.}, t = 0.49784 \text{ sec./in.}$$

- (a) No heat during discharge
- (b) Heat added from the commencement of discharge
- (c) Heat added for some time before discharge

equation of state.

The conservation of mass flow gives the following relation:

$$\dot{m}_t = C_{dt} \rho v a = - dM_a/dt \quad \dots 6.6$$

where, $\dot{m}_t$ is the transient mass flow through the orifice, C_{dt} is the transient coefficient of discharge and M_a is the residual mass of the air in the cylinder at the instant considered.

From the equation of state, the mass of the air, M_a , in the cylinder is given by:

$$M_a = \rho_o V = p_o V / RT_o \quad \dots 6.7$$

where V is the volume of the cylinder, which is constant, p_o and T_o are respectively the instantaneous pressure and the weighted average temperature of the air in the cylinder. Substituting 6.7 into 6.6, the transient mass flow, $\dot{m}_t$, through the orifice becomes:

$$\dot{m}_t = \frac{-p_o V}{RT_o} \left(\frac{1}{p_o} \frac{dp_o}{dt} - \frac{1}{T_o} \frac{dT_o}{dt} \right) \quad \dots 6.8$$

The pressure, p_o , in the cylinder can be measured accurately from the pressure-time record of the discharge. The temperature, T_o , of the air may be calculated if the nature of the expansion process is known. For isentropic expansion, the transient temperature is related to the instantaneous pressure and the initial conditions by:

$$T_o = T_{oo} (p_o / p_{oo})^{(k-1)/k}.$$

For an actual expansion process under the conditions specified in the present investigation, the average temperatures of the residual air in the cylinder can be predicted satisfactorily from the pressure-time record for the discharge by means of the relationship

derived in Section 6.1 (page 73). The only limitation in the accuracy of the method of estimating the instantaneous mass flow rate from the equation of conservation of mass flow lies in the evaluation of the slope, dp_o/dt , from the pressure-time record, (Equation 6.8).

Although the polynomial equation obtained by curve fitting can accurately reproduce the pressure at any instant, it does not give the corresponding gradient to the same accuracy. Difficulties inherent in the curve fitting process are encountered at the beginning and towards the end of the discharge. However, in spite of the limitations of this approach it appears to be the most practical method of calculating the transient mass flow through an orifice during discharge. The accuracy of the calculated results are discussed in a later section.

6.2.2 Experimental considerations

Before discussing the transient mass flow through an orifice, it is desirable, for practical purposes, to separate the effects on the flow due to orifice geometry, to the transfer of heat and to the transient fluid dynamic factors.

The effect of orifice geometry can be eliminated by keeping constant the appropriate group which is characteristic of the orifice. The relevant geometrical group which has to be taken into account in the present experiments is the length-diameter ratio of the orifice.

The effect of heat transfer to the air in the cylinder during expansion cannot actually be isolated from the other independent variables since it is not possible in practice to achieve a

truly adiabatic cylinder wall. However, it has been found that, within limits, the rate of heat transfer to the residual air in the cylinder during expansion does not affect, in any way, the relationship between the pressure of the air and the time during discharge. It may be argued, therefore, that the same relationship between pressure and time will hold in the absence of heat transfer. Since it is difficult to postulate a significant dissipative mechanism during the expansion of the residual air in the cylinder, such an expansion may be regarded as being isentropic. Thus, using the isentropic relationship between the pressure at any instant, as given by the pressure-time record, and the temperature, the equation of conservation of mass flow, Equation 6.8, gives a hypothetical mass flow rate, $\dot{m}_{ts}$, through the orifice.

For an isentropic expansion, $T_o = T_s$, and the relationship between the mass flow number, G_{ts} , and the other independent variables, Equation 3.5a, is reduced to:

$$G_{ts} = g_{ts}(l/d, p_a/p_o, p_{oo}/p_a) \quad \dots 6.9$$

The functional dependence of G_{ts} on these independent variables will be discussed in later sections. For a given discharge, the effect of orifice geometry and of fluid dynamical variables on G_{ts} will be the same as those on the actual mass flow number, G_t .

Two sets of experiments may be carried out on the basis of the results of dimensional analysis, namely, Equation 6.9 for isentropic conditions and Equation 3.5a for diabatic conditions, namely,

$$G_t = g_t(l/d, p_a/p_o, p_{oo}/p_a, T_o/T_s) \quad \dots 3.5a$$

Values of G_{ts} and G_t may be obtained from systematic experiments in which each of the first three groups in Equation 3.5a and Equation 6.9 are always made equal. It is evident that differences in the values of G obtained from such experiments must reflect the effect of heat transfer to the residual air in the cylinder. Thus, it is possible to compare the effects of heat transfer by taking the ratio of G_t and G_{ts} which is a function of the ratio T_o/T_s , or a function of Z and the ratio p_{oo}/p_a (Equation 6.1). Thus, one may write:

$$G_t/G_{ts} = g_h'(T_o/T_s),$$

or

$$G_t/G_{ts} = g_h(p_{oo}/p_a, Z) \quad \dots 6.10$$

The discharge coefficient is defined as the ratio of the actual mass flow through an orifice to the theoretical mass flow through an ideal nozzle of equal minimum geometrical cross-sectional area, for the same upstream conditions. These conditions are specified by the pressure, temperature, and approach velocity of the air and the pressure difference across the flow passage. It may be shown that the theoretical mass flow number,

$$G_{theo} = (\dot{m}_{theo} \sqrt{kRT_o}) / (p_o d^2),$$

is a function of the pressure ratio, p_a/p_o only, and it follows that, for a transient flow,

$$\begin{aligned} C_{dt} &= \dot{m}_t / \dot{m}_{theo} = G_t / G_{theo}, \\ &= c_{dt}'(l/d, p_a/p_o, p_{oo}/p_a, T_o/T_s), \quad \dots 6.11a \end{aligned}$$

or

$$C_{dt} = c_{dt}(l/d, p_a/p_o, p_{oo}/p_a, Z). \quad \dots 6.11b$$

The corresponding expression for transient flow associated with

isentropic expansion is:

$$\begin{aligned} C_{dts} &= \dot{m}_{ts}/\dot{m}_{theo} = G_{ts}/G_{theo}, \\ &= c_{dts}(l/d, p_a/p_o, p_{oo}/p_a), \end{aligned} \quad \dots 6.12$$

For steady flow the expression is:

$$\begin{aligned} C_{ds} &= \dot{m}_s/\dot{m}_{theo} = G_s/G_{theo}, \\ &= c_{ds}(l/d, p_a/p_o) \end{aligned} \quad \dots 6.13$$

By definition, the discharge coefficient of an orifice is independent of the upstream temperature. When the expansion of the air in the cylinder is affected by the transfer of heat from the cylinder wall, the discharge coefficient depends not only on the characteristic for the orifice and the pressure ratios but also on the departure of the temperature from the isentropic value, namely, the ratio T_o/T_s .

6.2.3 Dependence of transient mass flow number on the pressure ratio

For orifice length-diameter ratios, l/d , equal to 2.0, 1.5, 1.0, 0.5, and for sharp edge orifices, variations of the transient isentropic mass flow number, G_{ts} , and of the corresponding discharge coefficient, C_{dts} , with the pressure ratio, p_a/p_o , across the orifice are shown in Figures 6.11, 6.12, 6.13, 6.14 and 6.15. The results shown in these figures are for different initial pressures but, within the limits of the overall experimental error, the initial pressure does not appear to affect the transient mass flow number, and the coefficient of discharge. For purposes of comparison, curves of the mass flow number for steady flow, G_s , and of the discharge coefficient, C_{ds} , for orifices with the same length-diameter ratios have been superimposed in these Figures.

FIGURE 6.11 Effect of Pressure Ratio, ($1/d = 2.0$)

FIGURE 6.12 Effect of Pressure Ratio ($1/d = 1.5$)

FIGURE 6.13 Effect of Pressure Ratio, ($1/d = 1.0$)

FIGURE 6.14 Effect of Pressure Ratio, ($l/d = 0.5$)

FIGURE 6.15 Effect of Pressure Ratio (sharp-edge orifices)

It is clear from Figures 6.11 to 6.15 that the mass flow numbers, G_s and G_{ts} , and the corresponding coefficients of discharge, C_{ds} and C_{dts} , are functions of the length-diameter ratio, l/d , and the pressure ratio, p_a/p_o , across the orifice. Their functional dependence on the pressure ratio, p_a/p_o , represented by g_s and g_{ts} in Equations 3.6 and 6.9 on the one hand, and by c_{ds} and c_{dts} in Equations 6.12 and 6.13 on the other, reveal the transient effects.

In Figures 6.11, 6.13, 6.14 and 6.15, the mass flow number for transient flow, G_{ts} , and the corresponding discharge coefficient, C_{dts} , are always smaller than those for steady flows (about 10 to 15 percent in the case of mass flow number), except at the end of the discharge when the flow approaches the steady state. It may also be seen that at the beginning of the discharge, especially at high initial pressures, some of the experimental points lie above the mean curve. These points are considered to be associated with the process of starting the transient flow through the orifice but may well be the effect of the particular experimental technique employed. In this technique, before the discharge is initiated, the orifice and a small region downstream of it, due to the stretching of the diaphragm material, is filled with air at the initial pressure. If this explanation is correct, the anomalous points cannot be regarded as describing the transient flow behaviour which is of interest in the present investigation. In order to understand the mechanism by which a transient flow is adjusted to changing upstream conditions, a more detailed study of the starting process alone may be worthwhile.

For a given length-diameter ratio, the pressure ratio

across the orifice, p_a/p_o , is considered to represent the inertia and compressibility forces which govern the size of the vena contracta and the development of the jet. Differences between isentropic transient flows and steady flows from the same upstream conditions are thought to be due to the inability of the transient jet formed by the orifice to adjust itself in phase with changes in upstream conditions. The phenomenon is similar to the time lag observed by Rudinger⁽²⁷⁾ before a plane wave reflected at the open end of a pipe became fully established. Although steady and transient flows have been compared on the basis of an overall physical similarity between the upstream conditions, it must be borne in mind that in a transient flow, the upstream conditions are changing continuously during discharge, while they are constant in steady flow.

6.2.4 Effect on the transient mass flow of heat transfer to the air in the cylinder

To assess the effect of heat transfer on the transient flow through an orifice, the transient mass flow number, G_t , has been compared with the corresponding isentropic transient mass flow number, G_{ts} . The experimental results are shown in Figure 6.16 in which the ratio of the mass flow numbers, G_t/G_{ts} , is plotted as a function of $Z = 1/(p_a/p_o)(t/\tau)^{\frac{1}{2}}$ for constant initial pressures. As may be deduced from Figure 6.16, increasing values of the ratio of the transient mass flow numbers, G_t/G_{ts} , are associated with the smaller values of Z . These values of Z correspond to slower discharges, and hence to larger transfers of heat to the residual air in the cylinder. In the early stages of a discharge, corresponding

FIGURE 6.16 Effect of Heat Transfer to the Residual Air on Discharge

to large values of Z , heat transfer increases the mass flow number by about five percent above the corresponding isentropic values.

For the present, only those results after 0.1 second from the beginning of the discharge can be used with confidence owing to the difficulties inherent in the curve fitting process. However, in the very early stages of a discharge the numerical value of Z will be very large since both p_a/p_o and t/τ are small. In Figure 6.16, from the trend of the curves it appears that the very early stages of the discharge may be considered to be isentropic. For discharges longer than 0.1 second the heat transfer effect becomes increasingly important and cannot be neglected, even when the inner surface of the cylinder is insulated thermally as in the experiments described in this thesis. However, it cannot be deduced with any certainty from the results obtained in this investigation that in a fast discharge occupying much less than 0.1 second flow through the orifice is free from the effects of heat transfer to the residual air in the cylinder. Clearly, in the early stages of such a discharge it would be valid to assume that the expansion is approximately isentropic. In the later stages, when the residual pressure will be small, a relatively large temperature difference will exist between the air and the cylinder wall although t/τ would be small. If the results in Figure 6.16 can be extrapolated to fast transient discharges without wave action, in the later stages of the discharge both p_a/p_o and t/τ would approach a value of unity. Therefore, a five percent increase of the mass flow number from the corresponding isentropic value may be expected to occur on the basis of the

weighted average temperature. However, no firm conclusion can be drawn at present concerning the effects of heat transfer in very rapid discharges.

The points on each curve of Figure 6.16 were obtained from orifices of different diameters and of different length-diameter ratios. It appears that the effect of heat transfer to the air in the cylinder on transient mass flow through an orifice, therefore, is independent of the geometry of the orifice. The discrepancy between the experimental points and the mean curve drawn through them, and which is particularly marked in the case of an initial pressure of 80 psig., is due to a limitation of the straight line correlation between the transient pressure and average temperature (Equations 6.2 and 6.3, page 73). While the temperature is correctly predicted by the correlation, it is evident that the rate of change of temperature in the region of the intersection of the two straight lines (Figure 6.5, page 74) cannot be determined with the same degree of accuracy.

6.2.5 Effect of length-diameter ratio of orifice on the transient mass flow

It has been established experimentally that the length-diameter ratio, l/d , of the orifice plays an important role in steady flow through orifices⁽⁶⁾. Other geometrical characteristics of an orifice, such as the shape and the direction of the flow passage⁽⁵⁴⁾, also affect steady flows. Geometrical factors may also be expected to influence the transient flow through an orifice. In the present investigation only circular square-edge orifices have

been examined. Apart from the orifice diameter the only geometrical variable is the length-diameter ratio of the orifice.

For a discharge from given initial conditions and for the same orifice diameter, the effect of the length-diameter ratio on the flow through the orifice would be reflected in the pressure-time relationship for the discharge. The most convenient method of examining this effect is to compare the transient isentropic mass flow numbers, calculated from the experimental pressure-time curves for different length-diameter ratios. Five length-diameter ratios, namely, 2.0, 1.5, 1.0, 0.5 and sharp edge orifices have been used for this purpose. The results are shown in Figure 6.17. As anticipated, the effect of the length-diameter ratio on transient mass flow is similar to that observed in steady flow⁽⁶⁾. No appreciable difference in the mass flow numbers, or in the discharge coefficients, can be found for thick orifices when the pressure ratios across the orifice are less than 0.5283. When the pressure ratio is greater than the critical value, the flow rate appears to be higher for thicker orifices at a given pressure ratio, p_a/p_o . The behaviour of sharp edge orifices is also similar to that observed in steady flows. While sharp edge orifices consistently give the lowest values of the mass flow number for pressure ratios, p_a/p_o , greater than 0.5283, the mass flow number does not become constant*

*The use of the term 'area choking' for orifices is of doubtful significance. When the pressure ratio, p_a/p_o , is equal to, or less than, the critical value the acoustic velocity is reached at the vena-contracta. However, in sharp edge orifices the area of the vena-contracta, and the mass flow, increases as the pressure ratio is decreased. The analogy to choking is valid only when the acoustic velocity is reached at the geometrical throat.

FIGURE 6.17 Effect of Length-Diameter Ratio of Orifice on the Transient Mass Flow Through an Orifice

when the pressure ratios are less than the critical value, but continuously increase as the pressure ratio is decreased.

6.2.6 The transient mass flow number

The effect on the transient mass flow of the combination of the independent dimensionless groups given by the functional relationship, Equation 3.5b, can be obtained by cross plotting the results shown in Figures 6.11 to 6.16. The results have been presented in three dimensions in the form of a number of "carpets"⁽⁵⁵⁾ for a single length-diameter ratio and for different initial pressures in Figure 6.18. In this Figure the two families of intersecting lines represent constant values of pressure ratios, p_a/p_o , on the one hand, and the dimensionless group $Z = 1/(p_a/p_o)(t/\tau)^{\frac{1}{2}}$ on the other. A similar plot for several length-diameter ratios but for one initial pressure is shown in Figure 6.19. This method of presentation of the results emphasizes the effect of initial pressure (Figure 6.18) and the effect of the length-diameter ratio of the orifice (Figure 6.19). However, for most practical purposes, the information found in Figures 6.11 to 6.16 is adequate for the determination of either the transient mass flow number, or the coefficient of discharge, for a given orifice when the pressure-time relationship of the air in the cylinder during discharge is known.

The hatched area of each carpet in Figures 6.18 and 6.19 is a region which cannot be attained under the heat transfer conditions of the present experiments. Small values of the pressure ratio and the small values of Z are associated with large departures of the temperature of the air expanding in the cylinder, T_o , from

FIGURE 6.18 Carpet Diagram Showing the Variation of Transient Mass Flow Number through Sharp Edge Orifices with p_a/p_0 and Z for Different Initial Pressures

FIGURE 6.19 Carpet Diagram Showing the Variation of Transient Mass Flow Number through Thick Orifices with P_a/P_0 and Z for an Initial Pressure of 60 psig.

the corresponding isentropic values, T_s , in the early stages of a discharge. Such conditions are not likely to be encountered in the relatively slow discharges examined in this thesis.

6.2.7 Remark on "hysteresis" phenomenon in transient flow through an orifice

In steady flows through orifices having a length-diameter ratio of 0.5⁽⁶⁾, a "hysteresis" loop, shown superimposed in Figure 6.14, was observed. At the time, it was suspected that this behaviour might be due to the effect of jet reattachment associated with the particular geometrical configuration of the orifice. However, a similar effect was observed when the downstream geometry of the orifice was changed. In transient flow experiments through orifices with a length-diameter ratio of 0.5, there was no definite evidence for the existence of hysteresis. Although, as may be seen in Figure 6.14 a few experimental points obtained for a discharge from an initial pressure of 80 psig. have been found to give higher values than the mean curve, near the critical pressure ratio, it is not certain whether the higher values are an effect of hysteresis or whether they are due to inaccuracies in the gradients of the pressure-time records. Such a behaviour has not been found in discharges from initial pressures of 60 psig. or less.

6.2.8 Discussion of the error of the calculated results

The accuracy of the instrumentation and some of the limitations of the experiments have already been discussed in the appropriate sections. The error which will be discussed in this

section is, perhaps, the most serious because of the uncertainty of its source. In analysing the experimental results, a polynomial equation for the pressure variation was obtained by a curve fitting process using a number of points measured from the pressure-time record. Although the curve generated by the polynomial equation passes very nearly through the selected points, it is uncertain whether the generated curve will coincide with the actual pressure-time record with the same degree of accuracy. The approximation can be improved by increasing the order of the polynomial equation, but coincidence of every point can never be achieved. For this reason, while the pressure calculated by the polynomial equation may be regarded as satisfactory, a relatively larger error in the gradient may occur randomly along the curve.

Since the errors incurred in this manner are randomly distributed along the generated pressure-time curve, an estimate of the overall error in the results may be made by means of statistical methods. A hundred points were taken randomly along the curve in Figure 6.15 and analysed by means of a polynomial least square regression. The standard error for the isentropic mass flow number, G_{ts} , has been found to be 0.0124. It follows that the limits of the 95 percent confidence level is ± 0.0183 along the mean curve. These limits are considered to be satisfactory.

7. CONCLUSIONS

A systematic experimental investigation of slow transient flows through circular square-edge orifices has been carried out. Transient flow is initiated by the sudden release of dry compressed air from a cylinder whose inner surface is insulated thermally. The test orifice is mounted coaxially at one end of the cylinder. The following conclusions have been drawn from the results of the investigation.

1. It is impossible to provide a containing vessel with an adiabatic wall for the air which undergoes expansion during discharge. During the expansion of the air, temperature gradients in both the radial and the axial directions in the cylinder are established due to the transfer of heat from the cylinder wall. The primary mode of the heat transfer is convection. Clearly, under these conditions, the expansion process of the residual air in the cylinder is not isentropic.
2. In order to determine the mass of the residual air in the cylinder, it is necessary to define a weighted average temperature of the air. At any instant during discharge the average temperature is obtained, on the basis of weighted density, by a double averaging procedure from the instantaneous temperature distributions in the radial and axial directions in the cylinder. This weighted average temperature is the temperature when the residual air is mixed completely and isentropically. From the same initial conditions, the ratio of the weighted average

temperature, T_o , to the temperature, T_s , corresponding to an isentropic expansion to the same instantaneous pressure, p_o , at a given instant, is taken to represent the departure of the actual expansion process from the isentropic law. This departure has been found to depend on the instantaneous pressure of the air and history of the process. As far as the expansion process is concerned, the geometry of the orifice appears to be unimportant. From a given initial pressure and temperature of the air, the instantaneous temperature ratio, T_o/T_s , has been found to be a function of the instantaneous pressure and the duration of the discharge to this pressure, combined in the dimensionless form $Z = 1/(p_a/p_o)(t/\tau)^{\frac{1}{4}}$. This empirical relationship may be used to predict, to an accuracy within one percent, the instantaneous average temperature of the air in the cylinder from the pressure-time record of the discharge.

3. The nature of the empirical relationship between the instantaneous pressure and temperature of the air in the cylinder during discharge invalidates the assumption of a polytropic expansion and a constant index of expansion. A polytropic expansion, as in the isentropic case, is a limiting one which can never be achieved in a practical discharge process.
4. Moderate heat transfer from the wall to the residual air in the cylinder, as in this investigation, does not affect the pressure-time relationship of the air in the cylinder during discharge. However, it is not certain whether this relation-

ship will remain unchanged under high rates of heat transfer, as in the heat release due to chemical reaction in the cylinder of an internal combustion engine.

5. When the ratio of the diameter of the orifice to that of the cylinder, d/D , is small, wave action and non-uniformity in the properties of the air in the cylinder during the discharge are negligible. Except in a narrow region near the orifice there is no bulk motion of the air which is, therefore, in a state of virtual stagnation. The decrease of pressure in the cylinder during discharge is consistent with a free expansion. Similarity in the conditions upstream of a given orifice at any instant provides a suitable basis for the comparison of transient and steady flows.
6. The transient mass flow through an orifice is affected by the following:
 - a. The transfer of heat to the residual air in the cylinder,
 - b. The pressure ratio across the orifice, and
 - c. The geometry of the orifice.
7. From dimensional considerations, inertia and compressibility forces which govern the development of the jet issuing from an orifice may be represented by the pressure ratio across the orifice, p_a/p_o . In the absence of heat transfer these forces alone determine the transient flow through a given orifice. Thus, under isentropic conditions, the mass flow number, G_{ts} , or the coefficient of discharge, C_{dts} , is a function only of the pressure ratio, p_a/p_o . The mass flow number, or the co-

efficient of discharge, under these conditions is lower than that for steady flow through geometrically similar orifices, at the same pressure ratio. The difference is attributed to the effect of the transient upstream pressure and temperature. Under continuously changing upstream conditions, the jet cannot become adjusted to the instantaneous upstream pressure so that it is never fully developed, and the minimum area of the jet is, therefore, smaller than that under steady conditions.

8. The effect of heat transfer on transient flow is represented by the ratio of the two mass flow numbers, G_t/G_{ts} . This is dependent on the initial conditions of the discharge and the dimensionless group, Z , which correlates the history of the process. For discharges through a given orifice from the same initial conditions, the mass discharged in a given time depends on the heat transferred to the residual air.
9. As in the case of steady flows, the length-diameter ratio of an orifice affects the mass flow under transient conditions, particularly when the pressure ratio is less than the critical value. The flow is higher the greater the length-diameter ratio, l/d , up to a limiting value of 2.0. For pressure ratios less than the critical value, the transient mass flow number through thick orifices approaches a constant value, while through sharp edge orifices it increases continuously as the pressure ratio is decreased. For orifices of length-diameter ratio equal to 0.5 there is no firm evidence to indicate that there is a 'hysteresis' in transient flow, as has been observed in the steady state.

10. A flow through an orifice under continuously changing upstream conditions is different from the flow through an orifice due to the passage of a flat-topped shock wave. Discharge coefficients derived from quasi-steady conditions established after the reflection of the shock wave are, in fact, steady state values. Differences between these discharge coefficients and steady state values may be due to the lack of dynamical similarity in the conditions upstream of the orifice. It is thought that similarity of the geometrical configuration only is insufficient as a basis of comparison between steady and non-steady flows.

8. SUGGESTIONS OF FURTHER INVESTIGATION

The present investigation has mainly been concerned with the following problems:

- (a) The nature of the expansion process of the air in a cylinder during a relatively slow discharge.
- (b) The behaviour of a slow transient flow through an orifice, generated by the discharge of compressed air in a cylinder.
- (c) Differences between transient and steady flows through an orifice on the basis of upstream dynamical similarity.

The study of the expansion process of the air in a cylinder during discharge has been carried out with initial temperatures which are nearly constant. The results obtained may also be valid for other initial temperatures provided that the air in the cylinder is initially in thermal equilibrium with the cylinder wall. An extension of the present work to different initial temperatures, therefore, appears to be desirable.

The correlation between the pressure and temperature of the air in the cylinder during discharge needs to be examined systematically for different conditions of heat transfer and for cylinders of different surface/volume ratios in order to generalize the equations already obtained. The form of the correlation is expected to be similar to that derived in the present investigation.

Although the pressure-temperature correlation proposed is adequate to determine the residual mass of the air in the cylinder during discharge, it is of considerable practical interest to examine in detail, the nature of the heat transfer.

While flows through an orifice both under transient and steady conditions have been examined, certain details which have not been treated in the present investigation appear to be worthy of further consideration. These are:

(a) The starting process of a jet

In order to fully understand the behaviour of transient flow through an orifice and the mechanism of the readjustment of the flow to continuously changing of upstream conditions, a study of the process of development of the jet issuing from the orifice is necessary.

(b) Flow with a finite approach velocity towards the orifice

The two investigations into steady and transient flows through an orifice have been limited to stagnant upstream conditions. It is likely that transient and steady flows through an orifice will be strongly affected by the approach velocity of the flow. In these cases, particular care must be given to satisfying the requirements of dynamical similarity.

(c) Orientation and grouping of orifices

The effects of orifice geometry, including their grouping and the direction of the meridional axis, on transient flow are also interesting subjects for more detailed studies.

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APPENDIX A: DIMENSIONAL ANALYSIS

1. Transient Flow Through an Orifice

Consider the transient flow through an orifice, initiated from the discharge of compressed gas in a cylinder to atmosphere with an initial pressure p_{oo} and temperature T_{oo} and with the flow configuration as shown above, the instantaneous mass flow through the orifice depends on the following variables:

1. The geometrical configuration
 - (a) the diameter of the cylinder, D ,
 - (b) the length of the cylinder, L ,
 - (c) the diameter of the orifice, d ,
 - (d) the length of the orifice, l , and
 - (e) the roughness of the flow passage, h .
2. Instantaneous conditions of flow
 - (a) the upstream pressure, p_o , and temperature, T_o ,
 - (b) the downstream pressure, p_a , and
 - (c) the pressure, p , the temperature, T , and the mean velocity, v , at the orifice.

3. Properties of the gas

- (a) characteristic gas constant, R ,
- (b) the bulk modulus of the gas, N (compressibility), and
- (c) the coefficient of viscosity, μ .

4. The flow history

- (a) the initial pressure, p_{oo} , and temperature, T_{oo} , and
- (b) the time, t , from the commencement of the discharge.

The functional relationship for the mass flow is

$$\dot{m}_t = F(D, L, d, l, h, p_o, T_o, p_a, p, T, v, \mu, R, N, p_{oo}, T_{oo}, t)$$

There are eighteen variables in the above equation. If the dimensions chosen are the mass, length, time, and temperature (MLT Θ), then, according to the Pi-Theorem, there are at least $18 - 4 = 14$ dimensionless groups. The variables and their dimensions are given below:

$\dot{m}_t$	... (MT ⁻¹)
D, L, d, l and h	... (L)
p_o, p_a, p_{oo}, p and N	... (ML ⁻¹ T ⁻²)
T_o, T_{oo} and T	... (Θ)
μ	... (ML ⁻¹ T ⁻¹)
R	... (L ² T ⁻² Θ ⁻¹)
v	... (LT ⁻¹)
t	... (T)

Some of the required groups may be obtained by observation, since they are the simple ratios of the variables with the same dimensions, thus:

$$\begin{aligned}
\pi_2 &= L/D, \\
\pi_3 &= 1/d, \\
\pi_4 &= d/D, \\
\pi_5 &= h/d, \\
\pi_6 &= p_a/p_o, \\
\pi_7 &= p_{oo}/p_a, \\
\pi_8 &= p_o/p, \\
\pi_9 &= T_{oo}/T_o, \\
\pi_{10} &= T_o/T, \\
\pi_{11} &= N/p = k.
\end{aligned}$$

If the primary quantities p_a , d , R , and T_{oo} are chosen, the remaining dimensionless groups may be obtained in the following manner:

$$\begin{aligned}
\pi_1 &= \dot{m}_t (p_a)^a (d)^b (R)^c (T_{oo})^d \\
&= (MT^{-1})(ML^{-1}T^{-2})^a (L)^b (L^2T^{-2}\Theta^{-1})^c (\Theta)^d \\
&= 1
\end{aligned}$$

The exponents a , b , c and d are solved from:

$$\begin{aligned}
M: \quad & 1 + a = 0 \\
L: \quad & -a + b + 2c = 0 \\
T: \quad & -1 - 2a - 2c = 0 \text{ and} \\
\Theta: \quad & -c + d = 0 \text{ and their solutions are:} \\
& a = -1, \\
& b = -2, \\
& c = 1/2, \\
& d = 1/2.
\end{aligned}$$

Therefore,

$$\pi_1 = \frac{\dot{m}_t \sqrt{RT_{oo}}}{p_a d^2}.$$

Similarly from,

$$\pi_{12} = \mu (p_a)^a (d)^b (R)^c (T_{oo})^d = 1,$$

$$\pi_{13} = v (p_a)^a (d)^b (R)^c (T_{oo})^d = 1, \text{ and}$$

$$\pi_{14} = t (p_a)^a (d)^b (R)^c (T_{oo})^d = 1,$$

it follows that:

$$\pi_{12} = \frac{\mu \sqrt{RT_{oo}}}{p_a d},$$

$$\pi_{13} = \frac{v}{\sqrt{RT_{oo}}} \text{ and}$$

$$\pi_{14} = \frac{t \sqrt{RT_{oo}}}{d}.$$

By manipulating some of these groups, other well known dimensionless groups are obtained:

$$\pi_1 = \pi_1 \pi_6 (\pi_{11})^{\frac{1}{2}} (\pi_9)^{-\frac{1}{2}} = \frac{\dot{m}_t \sqrt{kRT_o}}{p_o d^2} = G_t, \text{ the transient mass flow}$$

number;

$$\pi_{12} = \pi_{13} \pi_{10} \pi_9 (\pi_8)^{-1} (\pi_6)^{-1} (\pi_{12})^{-1} = \frac{(\frac{p}{RT}) v d}{\mu} = \frac{\rho v d}{\mu} = Re, \text{ the}$$

Reynolds number at the orifice;

$$\pi_{13} = \pi_{13} (\pi_{10})^{\frac{1}{2}} (\pi_9)^{\frac{1}{2}} (\pi_{11})^{-\frac{1}{2}} = \frac{v}{\sqrt{kRT}} = M, \text{ the Mach number at the}$$

orifice, and

$$\pi_{14} = \pi_{14} \pi_4 (\pi_{11})^{\frac{1}{2}} (\pi_2)^{-1} = \frac{t}{L/\sqrt{kRT_{oo}}} = t/\tau, \text{ dimensionless time.}$$

The functional relationship between the dependent group G_t and the other dimensionless quantities thus becomes,

$$G_t = g_t \left(\frac{L}{D}, \frac{1}{d}, \frac{d}{D}, \frac{h}{d}, \frac{p_a}{p_o}, \frac{p_{oo}}{p_a}, \frac{p_o}{p}, \frac{T_{oo}}{T_o}, \frac{T_o}{T}, k, Re, M, \frac{t}{\tau} \right) \text{ (Eqn. 3.2,}$$

page 28).

For steady flow through orifices from similar upstream conditions in a cylinder of fixed dimensions, the quantities, L/D , p_{oo}/p_a , T_{oo}/T_o and t/τ do not appear in the relevant dimensional equation, and the above relationship reduces to:

$$G_s = g_s \left(\frac{1}{d}, \frac{d}{D}, \frac{h}{d}, \frac{p_a}{p_o}, \frac{p_o}{p}, \frac{T_o}{T}, k, Re, M \right) \text{ (Eqn. 3.3, page 29)}$$

2. The expansion of the air in the cylinder during discharge

Considering the equilibrium expansion of air in a cylinder from initial pressure, p_{oo} , and temperature, T_{oo} , to the instantaneous conditions, p_o and T_o , at a time t from the commencement of the expansion, the quantities that determine the temperature ratio, T_o/T_s , are:

1. The dimensions of the cylinder: length, L , and diameter, D ,
2. instantaneous pressure of the residual air, p_o ,
3. the "history" of the expansion: initial pressure, p_{oo} , initial temperature, T_{oo} , and the duration, t ,
4. the ambient condition: atmospheric pressure, p_a , and
5. properties of the fluid: gas constant, R , and bulk modulus, N .

The functional dependence is:

$$T_o/T_s = f_1(L, D, p_o, p_{oo}, T_{oo}, t, R, N, p_a)$$

In mass-length-time-temperature (MLT θ) system, there are, according to the Pi-Theorem, at least $10 - 4 = 6$ dimensionless groups. These

quantities and their dimensions are:

L and D	... (L)
p_o , p_{oo} , p_a and N	... ($ML^{-1}T^{-2}$)
T_{oo}	... (θ)
t	... (T)
R	... ($L^2T^{-2}\theta^{-1}$)

By observation:

$$\begin{aligned}\pi_1 &= T_o/T_s, \\ \pi_2 &= p_{oo}/p_a, \\ \pi_3 &= p_a/p_o, \\ \pi_4 &= N/p_o = k \text{ and} \\ \pi_5 &= L/D\end{aligned}$$

Choosing L, p_a , T_{oo} and t as the primary quantities the other group is

$$\begin{aligned}\pi_6 &= R(L)^a (p_a)^b (T_{oo})^c (t)^d \\ &= (L^2T^{-2}\theta^{-1}) (L)^a (ML^{-1}T^{-2})^b (\theta)^c (T)^d = 1\end{aligned}$$

The exponents a, b, c and d, are found from:

$$M: \quad b = 0$$

$$L: \quad 2 + a - b = 0$$

$$T: \quad -2 - 2b + d = 0$$

$$\theta: \quad -1 + c = 0$$

It follows that $a = -2$, $b = 0$, $c = 1$ and $d = 2$, and

$$\pi_6 = RT_{oo} t^2 / L^2$$

or,

$$\pi_6 = \sqrt{\pi_6 \pi_4} = t/\tau, \text{ where } \tau = L / \sqrt{kRT_{oo}}.$$

Therefore, the relevant dimensional equation is:

$$T_o/T_s = f_2(L/D, p_{oo}/p_a, p_a/p_o, k, t/\tau).$$

APPENDIX B: THE MASS FLOW NUMBERS THROUGH AN ORIFICE

For flows through an orifice with a configuration as shown in the above figure, the mass flow number, G , through the orifice is defined as:

$$G = (\dot{m} \sqrt{kRT_o}) / (p_o d^2)$$

where,

G = the mass flow number,

$\dot{m}$ = mass flow rate through the orifice,

k = ratio of specific heats,

T_o = upstream stagnation temperature,

p_o = upstream stagnation pressure,

d = diameter of the orifice, and

R = characteristic gas constant

The mass flow numbers under theoretical, and steady and transient conditions are obtained in the following manner:

1. Theoretical Mass Flow Number, G_{theo}

The theoretical mass flow, $\dot{m}_{theo}$, through an orifice is

calculated from the equation for isentropic flow through an ideal convergent nozzle with a throat section. It is assumed that the air in the cylinder is a perfect gas and the coefficient of discharge of the nozzle is unity. Since the approach velocity of the air towards the nozzle is zero, it can be shown that,

$$\begin{aligned}\dot{m}_{\text{theo}} &= \frac{\pi}{4} d^2 \sqrt{\frac{2k}{k-1} p_o \rho_o \left(\frac{p}{p_o}\right)^{2/k} \left[1 - \left(\frac{p}{p_o}\right)^{(k-1)/k}\right]} \\ &= \frac{p_o d^2}{\sqrt{kRT_o}} \frac{\pi}{4} \sqrt{\frac{2k^2}{k-1} \left(\frac{p}{p_o}\right)^{2/k} \left[1 - \left(\frac{p}{p_o}\right)^{(k-1)/k}\right]}\end{aligned}$$

It follows that the theoretical mass flow number is,

$$G_{\text{theo}} = \frac{\dot{m}_{\text{theo}} \sqrt{kRT_o}}{p_o d^2} = \frac{\pi}{4} k \left(\frac{p}{p_o}\right)^{1/k} \sqrt{\frac{2}{k-1} \left[1 - \left(\frac{p}{p_o}\right)^{(k-1)/k}\right]} \dots \text{B-1}$$

When the pressure ratio, p/p_o is less than 0.52828, the nozzle is choked and the flow is independent of the downstream pressure. The pressure at the throat of the nozzle is constant, and is given by:

$$p/p_o = \left(\frac{2}{k-1}\right)^{k/(k-1)} = 0.52828$$

Thus, from Equation B-1,

$$G_{\text{theo}} = \frac{\pi}{4} k \sqrt{\left(\frac{2}{k-1}\right)^{(k+1)/(k-1)}} = 0.63543$$

For flow in which the pressure ratio across the nozzle is greater than the critical value, the pressure at the exit of the nozzle is equal to the ambient pressure, p_a . Equation B-1 becomes:

$$G_{\text{theo}} = \frac{\pi}{4} k \left(\frac{p_a}{p_o}\right)^{1/k} \sqrt{\frac{2}{k-1} \left[1 - \left(\frac{p_a}{p_o}\right)^{(k-1)/k}\right]}$$

Values of G_{theo} as function of pressure ratio, p_a/p_o , are given in Table A-1.

2. The Steady Flow Mass Flow Number, G_s

When flow through an orifice from upstream stagnant conditions is steady, the mass flow number, G_s is, by definition,

$$G_s = (\dot{m}_s \sqrt{kRT_o}) / (p_o d^2),$$

where $\dot{m}_s$ is the measured mass flow through the orifice. It has been found that the dependence of the mass flow number on the pressure ratio across the orifice is a function of the length-diameter ratio of the orifice. Values reproduced from reference (6) are given in Table A-2.

3. The Transient Mass Flow Numbers

Transient mass flow through an orifice during the discharge is calculated on the principle of the conservation of mass flow from the instantaneous mass of the residual air in the cylinder. At a given instant the residual mass in the cylinder is,

$$M_a = \rho_o V = p_o V / RT_o,$$

where, V is the volume of the cylinder, ρ_o is the instantaneous average density of the air in the cylinder. The rate of change of the mass is,

$$\begin{aligned} \frac{dM_a}{dt} &= M_a \left(\frac{1}{p_o} \frac{dp_o}{dt} - \frac{1}{T_o} \frac{dT_o}{dt} \right) \\ &= \frac{p_o V}{RT_o} \left(\frac{1}{p_o} \frac{dp_o}{dt} - \frac{1}{T_o} \frac{dT_o}{dt} \right). \end{aligned}$$

The conservation of mass flow gives,

$$\dot{m}_t = - \frac{dM_a}{dt} = - \frac{p_o V}{RT_o} \left(\frac{1}{p_o} \frac{dp_o}{dt} - \frac{1}{T_o} \frac{dT_o}{dt} \right). \quad \dots B-2$$

The instantaneous weighted average temperature, T_o , and its rate of change are obtained from equations correlating the pressure and temperature during discharge (Equations 6.2, 6.3). For isentropic expansion, T_o is equal to T_s , the latter being a function only of the instantaneous pressure and the initial state of the air in the cylinder, viz.,

$$T_s = T_{oo} (p_o/p_{oo})^{(k-1)/k}.$$

If this relation is substituted into Equation B-2, the transient isentropic mass flow is,

$$\begin{aligned} \dot{m}_{ts} &= - \frac{p_o V}{RT_s} \left(\frac{1}{p_o} \frac{dp_o}{dt} - \frac{1}{T_s} \frac{dT_s}{dt} \right) \\ &= - \frac{1}{k} \frac{V}{RT_{oo}} (p_{oo}/p_o)^{(k-1)/k} \frac{dp_o}{dt} \end{aligned}$$

Two transient mass flow numbers may be defined:

$$G_t = (\dot{m}_t \sqrt{kRT_o})/(p_o d^2),$$

corresponding to conditions under which heat is transferred to the residual air, and

$$G_{ts} = (\dot{m}_{ts} \sqrt{kRT_s})/(p_o d^2),$$

in the absence of heat transfer. It should be stressed that G_{ts} is a hypothetical mass flow number which is calculated from the pressure-time records obtained in the cylinder. The extrapolation to isentropic conditions may be justified on the basis of the control experiments described in the text.

Values of G_{ts} are given in Table A-3 as function of the instantaneous pressure ratio, p_a/p_o , and the length-diameter ratio, l/d . Values of the ratio, G_t/G_{ts} , are given in Table A-4 as a function of the initial pressure, p_{oo} , and Z ($Z = 1/(p_a/p_o)(t/\tau)^{\frac{1}{k}}$).

TABLE A-1

Theoretical Mass Flow Number

Pressure ratio p_a/p_o	Theoretical mass flow number $G_{theo} = \frac{\dot{m}_{theo} \sqrt{kRT_o}}{p_o d^2}$
0.000 - 0.52828	0.63543
0.52828	0.63543
0.550	0.63469
0.575	0.63325
0.600	0.62905
0.625	0.62302
0.650	0.61509
0.675	0.60518
0.700	0.59319
0.725	0.57896
0.750	0.56237
0.775	0.54314
0.800	0.52101
0.825	0.49559
0.850	0.46632
0.875	0.43242
0.900	0.39270
0.925	0.34515
0.950	0.28589
0.975	0.20499
1.000	0.00000

TABLE A-2

Mass Flow Numbers from Steady Flow Experiments⁽⁶⁾

Pressure ratio p_a/p_o	Steady mass flow number, $G_s = \dot{m}_s \sqrt{kRT_o} / (p_o d^2)$			
	1/d=2.0	1/d=1.0	1/d=0.5	Sharp-edge
0.100	0.54564	0.54853	0.57140	0.56073
0.125	0.54564	0.54853	0.57140	0.56058
0.150	0.54564	0.54853	0.57140	0.55920
0.175	0.54564	0.54853	0.57140	0.55768
0.200	0.54564	0.54794	0.57140	0.55479
0.225	0.54564	0.54778	0.57140	0.55158
0.250	0.54564	0.54762	0.57140	0.54853
0.275	0.54564	0.54732	0.57140	0.54473
0.300	0.54564	0.54702	0.57140	0.54032
0.325	0.54548	0.54686	0.57078	0.53589
0.350	0.54548	0.54672	0.57048	0.53147
0.375	0.54534	0.54656	0.57018	0.52690
0.400	0.54534	0.54640	0.56987	0.52112
0.425	0.54518	0.54626	0.56987	0.51608
0.450	0.54489	0.54594	0.56987	0.51014
0.475	0.54459	0.54564	0.56987	0.50359
0.500	0.54397	0.54548	0.56987	0.49674
0.525	0.54259	0.54427	0.56987	0.48942
0.550	0.54184	0.54168	0.56987	0.48240
0.575	0.53864	0.53741	0.56179	0.47357
0.600	0.53544	0.53270	0.54154	0.46474
0.625	0.53011	0.52477	0.52339	0.45499
0.650	0.52355	0.51417	0.50740	0.44416
0.675	0.51472	0.50405	0.49369	0.43198
0.700	0.50451	0.49034	0.47921	0.41902
0.725	0.49018	0.47525	0.46320	0.40455
0.750	0.47540	0.45864	0.44508	0.38856
0.775	0.45682	0.44005	0.42664	0.37103
0.800	0.43609	0.41932	0.40531	0.35183
0.825	0.41292	0.39631	0.38291	0.33172
0.850	0.38810	0.37392	0.35351	0.30887
0.875	0.36037	0.34390	0.31997	0.28341
0.900	0.32456	0.31176	0.28341	0.25522
0.925	0.28646	0.27579	0.24228	0.22079
0.950	0.23618	0.22719	0.19427	0.17674
0.975	0.16791	0.16000	0.13332	0.12038
1.000	0.00000	0.00000	0.00000	0.00000

TABLE A-3

Transient Isentropic Mass Flow Number

Pressure ratio (p_a/p_o)	Transient isentropic mass flow number $G_{ts} = (\dot{m}_{ts} \sqrt{kRT_s}) / (p_o d^2)$				
	1/d=2.0	1/d=1.5	1/d=1.0	1/d=0.5	Sharp-edge
0.100	0.4825	0.4848	0.4804	0.4813	0.5236
0.125	0.4825	0.4848	0.4804	0.4813	0.5219
0.150	0.4825	0.4831	0.4804	0.4813	0.5201
0.175	0.4815	0.4831	0.4804	0.4813	0.5175
0.200	0.4804	0.4831	0.4795	0.4813	0.5148
0.225	0.4799	0.4831	0.4795	0.4808	0.5120
0.250	0.4795	0.4831	0.4795	0.4804	0.5083
0.275	0.4795	0.4831	0.4791	0.4801	0.5042
0.300	0.4795	0.4831	0.4788	0.4795	0.5004
0.325	0.4795	0.4831	0.4785	0.4791	0.4963
0.350	0.4795	0.4830	0.4778	0.4787	0.4887
0.375	0.4795	0.4830	0.4774	0.4778	0.4813
0.400	0.4787	0.4830	0.4764	0.4760	0.4755
0.425	0.4776	0.4823	0.4760	0.4751	0.4681
0.450	0.4774	0.4816	0.4751	0.4734	0.4628
0.475	0.4771	0.4775	0.4739	0.4725	0.4549
0.500	0.4769	0.4758	0.4728	0.4707	0.4478
0.525	0.4767	0.4743	0.4721	0.4681	0.4399
0.550	0.4760	0.4725	0.4707	0.4646	0.4319
0.575	0.4728	0.4699	0.4681	0.4610	0.4233
0.600	0.4716	0.4654	0.4637	0.4545	0.4138
0.625	0.4690	0.4619	0.4593	0.4460	0.4036
0.650	0.4654	0.4557	0.4540	0.4372	0.3940
0.675	0.4584	0.4495	0.4448	0.4252	0.3843
0.700	0.4513	0.4404	0.4355	0.4117	0.3720
0.725	0.4399	0.4314	0.4249	0.3981	0.3588
0.750	0.4291	0.4192	0.4161	0.3826	0.3456
0.775	0.4134	0.4034	0.3981	0.3664	0.3297
0.800	0.3949	0.3861	0.3791	0.3484	0.3156
0.825	0.3720	0.3685	0.3600	0.3262	0.2978
0.850	0.3491	0.3445	0.3367	0.3015	0.2768
0.875	0.3203	0.3173	0.3129	0.2750	0.2551
0.900	0.2909	0.2856	0.2830	0.2442	0.2292
0.925	0.2574	0.2500	0.2459	0.2089	0.2022
0.950	0.2186	0.2063	0.2019	0.1675	0.1657
0.975	0.1587	0.1499	0.1446	0.1199	0.1164
1.000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE A-4

Ratio of Transient Mass Flow Number to
Transient Isentropic Mass Flow Number

Z $\left[\frac{1}{\left(\frac{p_a}{p_o} \right) \left(\frac{t}{\tau} \right)^{\frac{1}{4}}} \right]$	Initial pressure, p_{oo} (psig)			
	80	60	30	18
	G_t/G_{ts}			
0.18	1.700	1.700	1.700	1.700
0.20	1.485	1.475	1.460	1.455
0.30	1.358	1.321	1.297	1.273
0.40	1.296	1.251	1.223	1.192
0.50	1.253	1.203	1.175	1.140
0.60	1.217	1.163	1.136	1.106
0.70	1.187	1.132	1.105	1.083
0.80	1.162	1.108	1.082	1.058
0.90	1.141	1.091	1.066	1.055
1.00	1.124	1.078	1.057	-
1.10	1.108	1.169	1.051	-
1.20	1.096	1.063	-	-
1.30	1.086	1.058	-	-
1.40	1.078	1.055	-	-
1.50	1.071	1.052	-	-
1.60	1.067	1.049	-	-
1.70	1.063	1.045	-	-
1.80	1.060	-	-	-
1.90	1.058	-	-	-
2.00	1.056	-	-	-
2.10	1.055	-	-	-
2.20	1.054	-	-	-

APPENDIX C

SOME TYPICAL PRESSURE-TIME AND
TEMPERATURE-TIME RECORDS DURING
DISCHARGE FROM CYLINDER

(a) PRESSURE CALIBRATION 5 PSI/DIV.

(b) TEMPERATURE CALIBRATION -2.0 M.V.

(c) $r = 0.0$ INCHES (AXIS) (d) $r = 0.685$ INCHES (e) $r = 1.187$ INCHES (f) $r = 1.532$ INCHES (g) $r = 1.813$ INCHES (h) $r = 1.938$ INCHES (WALL)

0 ——— 1
SCALE — 1 SECOND.

UPPER CURVE - TEMPERATURE
LOWER CURVE - PRESSURE

Figure A.1 PRESSURE AND TEMPERATURE RECORDS.