

A NEW KEYNESIAN APPROACH TO CONSUMPTION BASED
ASSET PRICING

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By
Christopher Wing Chi Tsang

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OR

Dean
College of Graduate and Postdoctoral Studies
University of Saskatchewan
116 Thorvaldson Building, 110 Science Place
Saskatoon, Saskatchewan
Canada
S7N 5C9

Abstract

In the decade following the global financial crisis, modern monetary theory has been forced to push the envelope via interventionist interest rate policy across geographies, and even open market asset purchases by central banks in select geographies, testing the bounds of optimal monetary policy. The unprecedented entanglement of monetary policy and asset prices alludes to a relative blind-spot in the New Keynesian literature – embedding asset pricing explicitly. The purpose of this paper is to develop an asset pricing model where monetary policy impacts real variables and thus can be analyzed – the ambition is to provide a proof-of-concept in developing a lens through which one may observe the effect monetary policy has on the real economy via symptoms observed in financial markets (e.g. risk premiums) in a paradigm consistent with modern New Keynesian theory. While asset pricing models and New Keynesian monetary models exist individually, the challenge is integrating the two concepts in an appropriate framework and interpreting the result.

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List of Abbreviations

CCAPM	Consumption Capital Asset Pricing Model
CRRA	Constant Relative Risk Aversion
DIS	Dynamic IS equation
DSGE	Dynamic Stochastic General Equilibrium
EIS	Elasticity of Intertemporal Substitution
GDP	Gross Domestic Product
NKPC	New Keynesian Phillips Curve
RBC	Real Business Cycle

1

Introduction

The efficacy and optimal role of monetary policy is a popular and constant debate in macroeconomics. Monetary economics seeks to better understand the interplay between the business cycle, inflation, and the optimal monetary policy – if any. To analyze monetary impacts, there has been a rigorous discourse concerned with developing an appropriate framework – one with satisfactory mechanics, such as defensible microeconomic foundations, which is also consistent with empirical data and past business cycle fluctuations. In recent years, global macroeconomic developments and shocks have led to an interesting interplay between monetary policy, global financial markets, and asset prices.

The global financial crisis has pushed central banks around the world to respond with non-traditional monetary policy aimed at mitigating the economic impact experienced historically as financial systems emerge from such crises. The desire to stabilize global economies and financial markets via countercyclical monetary policy has led to an unprecedented relationship between monetary policy and asset prices. The important historical consequence of the current interventionist monetary policy experiments around the world will be reflected in data subsequent to the events. In order to understand the consequences of such monetary policy, it is necessary to build economic models capable of interpreting and analyzing such data in the context of financial market implications. The purpose of this paper is to develop an asset pricing model where monetary policy impacts real variables and thus can be analyzed – the ambition is to provide a proof-of-concept in developing a lens through which one may observe the effect monetary policy has on the real economy via symptoms observed in financial market characteristics (e.g. risk premiums). While asset pricing models and monetary models exist individually, the challenge is integrating the two concepts in an appropriate framework and interpreting the result.

The classical dichotomy (Patinkin, 1965) is the belief that real variables are independent of monetary variables. Money is said to be “neutral” if changes in the money supply only have an impact on nominal variables and no impact on real variables. Any model with the ambition of connecting monetary policy and the real economy must reject the classical dichotomy and correspondingly demonstrate that money is not neutral. The New Keynesian framework combines monopolistic competition with nominal rigidity caused by price or wage stickiness, or both, resulting in short run non-neutrality of monetary policy. Though a review of the New Keynesian school of thought encounters a vast literature representing an evolution in

macroeconomic thought spanning decades, asset pricing has not been an explicit focus of the literature to date. The purpose of this paper is to develop an asset pricing model consistent with a world where monetary policy can impact real variables, and thus an asset pricing model which is developed within an explicit New Keynesian paradigm.

Modern monetary policy at many central banks is focused on the type of stabilization policy advocated by the New Keynesian school of thought. In the face of an unanticipated exogenous shock to the economy, New Keynesian analysis suggests an optimal nominal interest rate rule following what is prescribed by Taylor (1993), which responds to the level of inflation and the output gap periodically. Such a policy is thought to minimize periodic inflation and the periodic output gap, minimizing some of the distortions caused by an exogenous shock in an economy facing nominal rigidity. However, in the pursuit of stabilizing the economy via interest rates, households, firms, and central banks should be cognizant of potential unintended consequences of policy — asset prices are a key area of study for potential unintended consequence, especially at the individual household level due to the fact periodic asset prices have a significant impact on household wealth. If policy responses impact asset prices, the door opens for households to consider the current and expected future policy environments when transacting in asset markets and for firms to have a similar consideration when contemplating repurchasing or issuing equity capital.

The rest of the paper is organized as follows. Chapter 2 will review the New Keynesian school of thought, the dynamic stochastic general equilibrium (DSGE) framework that is the workhorse of the New Keynesian paradigm, and consumption based asset pricing models. Chapter 3 will work through the theoretical background for a basic New Keynesian model by following the work of Galí (2008), and a basic consumption based asset pricing model by following Cochrane (2009) and Romer (2012), and will embed the resulting relationships from the New Keynesian model into the consumption based asset pricing model. The combination is first demonstrated for a closed economy before broadening to an open economy. Due to the inability to find a closed-form solution, the resulting relationship must be linearized around a steady state using a second-order approximation. Closed economy results are presented in Section 3.1, and open economy results are presented in Section 3.2. Chapter 4 discusses the conclusions of the paper, as well as potential areas of further research.

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Prior Research

There are multiple relevant sub-disciplines discussed within this paper — the New Keynesian macroeconomic framework; dynamic stochastic general equilibrium (DSGE) modeling; and asset pricing theory – each with its own wealth of prior research. The New Keynesian framework cannot be captured by an isolated seminal work, the paradigm represents an evolution in macroeconomic theory with microeconomic foundations dating back to the 1970s. DSGE models are a class of general equilibrium models widely used in macroeconomics built from microeconomic foundations where markets clear through price and quantity adjustments, significantly overlapping with the New Keynesian literature. They can be used to describe the evolution of economic variables over time, specifically in response to exogenous shocks. Asset pricing is a broad field with a multiplicity of models, and must be narrowed specific to the use in this paper. The chapter is presented in three sections. Section 2.1 discusses the background and history of research in New Keynesian economics. Section 2.2 provides a brief introduction to DSGE models as they apply to New Keynesian monetary models. Section 2.3 reviews the partition of asset pricing literature relevant to this particular paper.

2.1 The New Keynesian School

This section will cover a brief chronology and the development of key features from the New Keynesian literature. For an alternative brief review of the New Keynesian literature see Woodford (2009); for a more inclusive review of the literature and theory see Woodford (2003), Snowdon and Vane (2005), and Galí (2008).

As the name suggests, the New Keynesian school of thought evolves from the orthodox Keynesian school which itself has its roots in the classic text *The General Theory of Employment, Interest, and Money* (Keynes, 1936). Economic instability, according to Keynes, is the product of “animal spirits” which leave investors, business confidence, and thus economies susceptible to erratic shocks. The primary determinant of aggregate output and employment is proposed to be nominal aggregate demand; with wages and prices not fully flexible, where an economy might otherwise be slow to return to full employment after a shock, stabilizing policy with a focus on aggregate demand would be desirable. Both monetary and fiscal policy could serve such a purpose, but the former is thought to have a quicker and more targeted impact on aggregate demand. Hicks (1937) brings forth the IS-LM model which illustrates the basic tenets of orthodox Keynesian thought, and thus began the development of large-scale macroeconomic models reliant on systems of structural

equations built from Keynes' *General Theory*.

In the 1970s, the orthodox Keynesian approach came under criticism. Sargent and Wallace (1975) demonstrate a simple ad hoc model asserting that the introduction of rational expectations leads to the neutrality of monetary policy – the policy-ineffectiveness proposition. The work of Lucas (1976) brings forth a need for the Keynesian paradigm to evolve, using empirical data to delineate orthodox Keynesian deficiencies. The downfall of large-scale macroeconomic models built on Keynesian fundamentals is highlighted by Lucas as the static nature of the relationships embedded within the structural equations, and the resulting inability of such models to capture potential behavioral changes which follow structural deviations such as policy or environmental change. Parameters of such models are typically estimated based on the historical data available, which can only possibly reflect environments already captured in the time series. What is referred to as the “Lucas Critique” highlights that a shift in the policy environment – a structural change – will cause behavioral responses and ultimately parameters must change to reflect the new policy variables and subsequent behavior; to accurately evaluate changes in policy, invariant parameters are required. *After Keynesian Macroeconomics* by Lucas and Sargent (1978) is titled as if to eulogize Keynesian theory. The analysis is an extension of the Lucas Critique pointed toward post-Keynes theoretical work which, over several decades, stretched Keynes's postulations into structural macroeconomic models relying on parameter stability and policy-invariant structures. The key criticism of Lucas and Sargent has never been Keynesian foundations, but rather the lack of a credible mechanism for optimizing behavior to bring forth nominal rigidities, tentatively suggesting that equilibrium business cycle models avoid many of the shortcomings highlighted.

In the face of empirical challenges, a vast body of work developed introducing the dynamics and micro-foundations the Keynesian approach was criticized for lacking, while staying true to Keynesian foundations. By disposing of the assumption that markets clear continuously, Fischer (1977) and Phelps and Taylor (1977) illustrate how nominal shocks can impact the real economy in a framework with rational expectations, a result directly contradicting the widespread belief that rational expectations begets the classical dichotomy. Fischer's work introduces the idea of embedding rigidities in nominal wage by using staggered or overlapping long-term wage contracts. Phelps and Taylor independently come to a similar and complementary conclusion using price rigidity as a source nominal stickiness necessary to extinguish the classical dichotomy in a framework with rational expectations and perfect information. The use of long-term contracts by Fischer, and prices set in advance by Phelps and Taylor, imply costs associated with the activities of price setting and wage negotiation. Sheshinski and Weiss (1977) develop the concept of “menu costs”, the explicit costs associated with changing prices, asserting that adjusting prices can be costly and thus arguing that firms will adjust prices intermittently instead of continuously. Small menu costs introduce a friction which makes prices sticky and thus can ultimately lead to an aggregate demand externality bearing a large cost to society, even if inflation is perfectly anticipated. The term can be broadened beyond the literal interpretation of

menu costs associated with printing new menus to include a multiplicity of costs business managers incur when nominal prices change.

Taylor (1979, 1980) discusses the appeal of rational expectations as evidenced by empirical and theoretical findings, and explores the microfoundations of how fixed-length staggered wages and contracts can introduce frictions which impact the real economy even within a rational expectations framework. Using one-year staggered wage contracts, Taylor connects the degree to which policy is accommodative with the resulting fluctuations in the output gap – a real effect contradicting the policy-ineffectiveness proposition in Sargent and Wallace (1975) despite assuming rational expectations. Taylor’s result suggests the existence of ideal stabilizing policies in such a framework. Extending on the concept of staggered contracts, the pricing mechanism in Calvo (1983), dubbed “Calvo Pricing”, exhibits nominal rigidity by modeling a firm’s ability to reset price as a probabilistic function of the survival rate — price-setters are thus uncertain of how long nominal prices will persist with the periodic outcome following a geometric distribution. The stochastic nature introduces complexities as firms care about their prices relative to other firms intertemporally when making their price-setting decision. Even if individual prices change frequently, the Calvo mechanism can lead to prices which adjust slowly, as price setting firms in a given period are influenced by the price level inertia of firms that cannot reset prices in the same period. Such inertia leads to persistent real effects even when prices can adjust frequently.

Within the context of bounded rationality, Akerlof and Yellen (1985) discuss how nominal price and wage inertia could be caused by firms not changing price unless there is a significant enough benefit, suggesting a threshold that must be met instead of continuous updating. While the “near-rational” behavior causes only small second-order losses at the level of the individual agent, there are relatively large resulting first-order variations in aggregate employment and output in the economy. Mankiw (1985) similarly makes the case for sticky prices, illustrating how large welfare losses can result when firms optimize with small menu costs due to private incentives. While investigating the importance of monopolistic competition as a more accurate alternative to perfect competition, Blanchard and Kiyotaki (1987) illustrate a general equilibrium model combining a monopolistically competitive model, following Dixit and Stiglitz (1977), with other imperfections to generate aggregate demand effects which perfect competition cannot. Monopolistic competition is more consistent with orthodox Keynesian beliefs; whereas perfectly competitive markets clear continuously, Keynesian agents are price-setters. A focus on monopolistic competition moves the mechanism for nominal stickiness away from previous work on wages and staggered labour contracts toward prices set by firms, as individual firms set prices more slowly than the rapid adjustments suggested by the existence of a Walrasian auctioneer in perfectly competitive markets. Through the lens of monopolistic competition, significant nominal frictions can be generated from the microfoundations of a profit maximizing firm facing a menu cost – a small but real cost of adjusting prices. Consistent with the results discussed in Akerlof and Yellen (1985) and Mankiw (1985), the model developed by Blanchard and Kiyotaki exemplifies how frictions

in the form of small second-order menu costs faced by individual agents can lead to first-order effects on output and welfare of the aggregate economy.

Ball et al. (1988) provides a comprehensive review of several of the key assumptions that began being integrated into Keynesian models over the preceding decade, and test the implications against empirical data, illustrating how far the literature developed over the period by systematically contrasting a basic New Keynesian model with the Lucas (1973) model and previous critiques of orthodox Keynesian failures. By the late 1980s, structural Keynesian models could embed imperfect competition and a price-setting mechanism with friction (e.g. Calvo pricing) to address empirical failures of orthodox Keynesian structural models, as pointed out by Lucas, while doing so in a manner adding microeconomic foundations which are more dynamic in nature. The decline of Keynesian economics in the 1970s is discussed by Ball et al. as at least partly due to the lack of an acceptable mechanism which leads to nominal rigidities from behavior consistent with optimization, and the improvement in the 1980s is explicated as developing "... models in which optimizing agents choose to create nominal rigidities" (Ball et al., 1988, p.2). After examining historical data for evidence of price stickiness, the proposed model is demonstrated to match history well both qualitatively and quantitatively, with parameters that are realistic, and illustrates a policy trade-off which corresponds to the average rate of inflation — countries with low inflation exhibit relatively flat short-run Phillips curves while countries with high inflation exhibit steeper curves. The intuition is that firms in an environment with higher average inflation will choose to adjust prices more frequently, which in turn *reduces* the effects of nominal shocks. As the magnitude of the result is substantial, not only does it violate the classical dichotomy, but it also alludes to a significant role for monetary policy in such an economy.

Mankiw (1989) addresses real business cycle (RBC) theory as a competing explanation of macroeconomic fluctuations compared to New Keynesian theory. Shortcomings of RBC theory include challenges modeling a procyclical real wage and the explanation that welfare drops in a recession only because of a decrease in technology modeled as a negative technological shock. Mankiw's main reservation with New Keynesian theory is the limited understanding of the nominal rigidities which are a cornerstone of any New Keynesian model. Ball and Romer (1990) are similarly critical of the fact that prior attempts to explain wage and price rigidity focus on real rigidity instead of nominal rigidity, asserting that real rigidity alone does not imply nominal rigidity but instead nominal rigidity requires a nominal source of friction. However, Ball and Romer illustrate that combining nominal frictions and real rigidity proves powerful in terms of explaining non-neutrality of money for empirically plausible parameters.

The confluence of contributions led to what Goodfriend and King (1997) dubbed the "New Neoclassical Synthesis" asserting that by combining the dynamic aspect of RBC with the microfoundations provided by imperfect competition and nominal rigidities, what are now known as New Keynesian models were born. Goodfriend and King discuss the four key ingredients for the new neoclassical synthesis as intertemporal

optimization, rational expectations, imperfect competition, and costly price adjustments. The first two elements build on new classical and RBC literature while the latter two elements build on the New Keynesian literature, with a shared dependence on microeconomics as common ground between the RBC and New Keynesian elements. The modeling and analysis from Goodfriend and King illustrates a resulting framework for the economy in which money is not neutral in the short-run, inflation targeting is the key policy tool, and central bank credibility is pivotal to the efficacy of policy.

Clarida et al. (1999) develop a New Keynesian model centered around three key equations in a DSGE framework, and use the model to demonstrate the importance of credibility in addressing the trade-off between inflation and output by deriving optimal policy both with and without central bank commitment. The three key equations are: i) a dynamic version of the investment-savings (DIS) curve whereby current output is a function of expected future output and interest rates; ii) a New Keynesian influenced version of the Philips curve whereby inflation depends only on current and expected future economic states; and iii) a monetary policy function — in this case a nominal interest rate rule. In the presence of nominal price rigidities using a Calvo pricing mechanism, monetary policy fluctuations impact the short-term real rate and the optimal policy is one where the nominal rate adjusts more than one-for-one with inflation, consistent with the principle motivated by Taylor (1993).

Christiano et al. (2005) and Smets and Wouters (2003, 2007) exemplify the culmination of New Keynesian progress from orthodox Keynesian roots — structural models desired by orthodox Keynesians, but modernized by nimble components built on the microeconomic foundations of monopolistic competition, rational expectations, and nominal rigidities. The work by Smets and Wouters demonstrates a micro-founded DSGE model that performs favorably compared to Bayesian Vector Autoregressions in terms of out-of-sample forecasting using U.S. and European historical data. The modern models are DSGE models suitable for policy analysis, which have been widely adopted by central banks. Galí (2008) analyzes optimal monetary policy within a basic New Keynesian model. The analysis suggests optimal monetary policy can be approximated via a Taylor interest rate rule where the central bank, under commitment, targets an optimal level for the output gap in order to stabilize the price level. The optimal policy in turn seeks to minimize the impact, or welfare losses, felt due to the dynamic series of short-term distortions caused by the rigidity of staggered price-setting in the economy.

De Paoli et al. (2010) study asset prices in a New Keynesian model using second-order numerical perturbation methods, with a focus on the relationship between the level of nominal rigidity and corresponding risk premia. De Paoli et al. provide impulse response functions of certain variables as well as numerical output of certain asset pricing characteristics such as returns and risk premia, but due to the use of numerical approximation, deriving the expression for firm prices is not the focus of the paper. Whereas De Paoli et al. is focused on numerical experiments in a New Keynesian framework and the corresponding output of asset

pricing characteristics, Chapter 3 differs by embedding a New Keynesian framework directly into the firm price equation then analytically deriving a Taylor approximation of the relationship between asset prices and various parameters in a simplified New Keynesian framework before providing an illustrative calibration of the result.

2.2 Dynamic Stochastic General Equilibrium (DSGE)

Dynamic stochastic general equilibrium (DSGE) refers to a modeling framework frequently used in contemporary macroeconomic theory that describes the evolution of economic variables over time (dynamic), where the economy is exposed to exogenous random shocks (stochastic), and all equilibrium quantities are accounted for (general equilibrium). DSGE models have become the workhorse for New Keynesian theory, thus a brief history of the DSGE literature is covered as it pertains to New Keynesian analysis. For a more comprehensive discussion see Fernández-Villaverde (2010) and Romer (2012).

Blanchard (2016) provides a succinct delineation of the three key modeling choices which distinguish the DSGE framework: the behavior of the agents present in the model is built from microfoundations; the economy is imperfectly competitive with distortions such as nominal rigidity, monopoly power, and imperfect information; and the model does not solve each equation individually but rather the model is estimated as a system. Taking the temperature of the current state of DSGE models, Blanchard provides a balanced discussion of current deficiencies and criticisms contrasted with a solid but “eminently improvable” (Blanchard, 2016, p.1) foundation. Suggestions for improvement include better integration of the work from other fields of economics on consumer behavior, and more willingness among the field to “share the scene with other types of general equilibrium models” (Blanchard, 2016, p.4).

While the literature of New Keynesian theory has become increasingly entangled with the DSGE framework, it is important to distinguish the two – modern New Keynesian models are DSGE models but not all DSGE models are New Keynesian. The roots of the DSGE framework are often credited to Kydland and Prescott (1982) for their work spearheading real business cycle (RBC) theory, another school of thought which uses DSGE modeling that was developed contemporaneous to the evolution of the New Keynesian school. Developed after Lucas (1976) demanded the field of macroeconomics to more explicitly model microfoundations, Kydland and Prescott use a calibrated DSGE framework to analyze and develop an RBC model with the desirable building blocks of rational expectations, optimizing behavior, and market clearing. Responding to the concerns of the Lucas Critique, the early combinations of the DSGE framework and RBC modeling created what seemed to be a relatively parsimonious macroeconomic model which closely resembled observed historical data. The resulting reliance on technological shocks as an explanation for much of the business cycle has since been refuted, for example in Summers (1986) and Mankiw (1989), but the use of recursive methods helped motivate DSGE models as a tool for modern quantitative macroeconomic analysis.

The general equilibrium model developed by Blanchard and Kiyotaki (1987) provides what has become a standard New Keynesian building block which injects monopolistic competition but lacks the dynamism exemplified in RBC models with intertemporal general equilibrium frameworks. Indeed, as the New Keynesian school sought to free itself from the grips of the Lucas Critique, the literature worked toward using its own version of a DSGE framework. In her comments in Goodfriend and King (1997), Ellen McGrattan concedes that work she contributed to in Chari et al. (1997) is essentially a dynamic version of the Blanchard-Kiyotaki model. DSGE models generally contain a continuum of infinitely lived agents, such as households and firms, optimizing intertemporally across an infinite-horizon. As emphasized by Woodford (2003), in theory, dynamic models based on optimizing behavior at the forward-looking individual level defend against the Lucas Critique via resulting policy-invariant parameters. A fully dynamic model with defensible microfoundations leads to a model suited for analysis of policy intervention, shocks, and various transmission mechanisms. Examples of modern DSGE models include the work by Clarida et al. (1999), Christiano et al. (2005), and Smets and Wouters (2003, 2007). Christiano et al. (2017) discusses how DSGE models have changed in response to the financial crisis.

Due to the complexity of aspiring to provide a rich macroeconomic model spanning an infinite horizon, one of the foremost problems continually encountered in the development of DSGE models is the fact that the framework rarely lends itself to closed-form solutions, especially with realistic assumptions. Fernández-Villaverde et al. (2016) contains a comprehensive review of DSGE solution methods. The commonality is that solution methods generally involve three key ingredients: a system of equations describing equilibrium in the form of Euler equations or value functions; a form of approximation, for example linearization via Taylor approximation around a non-stochastic steady state; and a form of estimation, for example calibration using empirical parameters. For the purposes of this paper, the important notion is that while modern New Keynesian models generally cannot be solved explicitly with a closed-form solution, they can be approximated and estimated locally. Though imperfect, if the model is rich enough, the lack of a closed-form solution does not necessarily preclude a model from providing useful inference as evidenced by recent literature.

Indeed, as far as the field has developed, there is still significant discourse over certain deficiencies. Though widely adopted as a modeling convenience, Calvo pricing has come under criticism from empirical DSGE analysis. Through investigating *How Structural Are Structural Parameters?* Fernández-Villaverde et al. (2007) suggest that structural DSGE parameters used in modern models with Calvo pricing are misspecified due to the pricing mechanisms inability to capture the proper evolution of price. Chari et al. (2009) derives a similar result, concluding that New Keynesian models are still not developed enough to provide useful policy analysis. Charles Plosser shares a discomfort with the fact that “micro data on price behavior is not particularly consistent with the implications of the usual staggered price-setting assumptions in these models” (Plosser et al., 2012, p.5). State dependent pricing, such as the model proposed in Gertler and Leahy (2008) has been proposed as an attractive alternative, but introduces its own host of complexities and requires

certain restrictions and assumptions to obtain an analytical solution. As alluded to by Blanchard (2016), New Keynesian DSGE models still need to evolve from their nascent state, but appear to be a favorable foundation to build upon.

2.3 Consumption Based Asset Pricing

New Keynesian DSGE literature is at the forefront of modern macroeconomics and is currently evolving. Indeed, despite constant progress toward refining contributions, the core foundation of the New Keynesian framework is well established. In the last decade post-global financial crisis, modern monetary theory has been forced to push the envelope via interventionist interest rate policy across geographies and even open market asset purchases by central banks in select geographies, testing the bounds of the aforementioned developments surrounding non-neutrality and optimal monetary policy. The result is an unprecedented entanglement of monetary policy and asset prices, which therein alludes to a relative blind-spot in the New Keynesian literature – embedding asset pricing explicitly.

Rather than relitigating the long history of asset pricing literature, as it is tangential to the purpose of this paper, this section introduces a basic but well-established consumption-based asset pricing model, following Cochrane (2009) which provides a comprehensive foundation for modern asset pricing, and Romer (2012) which introduces consumption-based asset pricing within the context of macroeconomic analysis. The basic premise is that as agents make a consumption versus savings decision with a potential portfolio of assets included, the pricing equation follows from the first-order condition which relates asset price with the marginal rate of substitution between present and future consumption, accounting for investment returns. Assets, or firms, are explicitly valued using the stream of expected future dividends, or cash flows, and the stochastic discount factor which represents the marginal rate of substitution, also called the pricing kernel. This lens can be applied to any financial security or any stream of uncertain cash flows, and results in an asset pricing framework in which agents care not only about the level of asset prices but the covariance between asset prices and their periodic consumption, also called the “consumption beta”.

The foundation for consumption-based asset pricing began with the work of Rubinstein (1976) on valuing stochastic income streams. Lucas (1978) derived Euler equations for asset pricing in an exchange economy, serving as a building block for future empirical work on consumption-based asset pricing. Work in Breeden and Litzenberger (1978) and Breeden (1979) bring forth more general modeling which has come to define the consumption capital asset pricing model (CCAPM). For a full review of the CCAPM literature, including a comprehensive review of the plethora of empirical tests, see Breeden et al. (2015).

3

Theory and Model

This chapter will include two parts, Section 3.1 works through real firm price in a closed economy as an introduction to the concept in a simplified framework, Section 3.2 covers the same concept in an open economy. Despite the use of several ad hoc simplifying assumptions, opening the economy introduces significant complexity to the model due to the introduction of additional terms and notation. Each case will begin with a definition of firm price using a consumption-based approach. Standard New Keynesian assumptions are introduced and embedded in order to augment the asset pricing model in a framework consistent with the New Keynesian paradigm. The intended result is an asset pricing framework which is consistent with the assumptions of a basic New Keynesian economy.

3.1 Real Firm Price in a Closed Economy

Beginning with a consumption-based approach to asset pricing similar to what is found in Cochrane (2009) and Romer (2012), firms at the initial time period, denoted as f_t , are priced using the stream of expected future dividends, and a stochastic discount factor (also called the pricing kernel, or marginal rate of substitution):

$$f_t = \sum_{j=1}^{\infty} \beta^j E_t \left[\frac{u'(C_{t+j})}{u'(C_t)} d_{t+j} \right] \quad (3.1)$$

The consumption-based asset pricing model is a version of the discounted cash flow valuation model that is ubiquitous in finance, with the microeconomic wrinkle provided by the marginal rate of substitution between present and future consumption. By embedding the trade-off in terms of individual agent's periodic utility, a key result is that agents in a consumption-based framework care specifically about not just the level of asset prices but also the covariance between asset prices and their periodic consumption. C_t represents periodic consumption, while d_t denotes periodic dividends, and β is the standard discount rate from present value theory.

Utility is assumed to follow a separable isoelastic utility function, where σ is the elasticity of intertemporal substitution for consumption and φ is the elasticity of intertemporal substitution for labour, and its

corresponding marginal utility function and marginal rate of substitution between present and future consumption:

$$U(C_t, N_t) = \begin{cases} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1-\varphi}}{1-\varphi} & \sigma \neq 1; \varphi \neq 1 \\ \frac{C_t^{1-\sigma}}{1-\sigma} - \ln(N_t) & \sigma \neq 1; \varphi = 1 \\ \ln(C_t) - \frac{N_t^{1-\varphi}}{1-\varphi} & \sigma = 1; \varphi \neq 1 \\ \ln(C_t) - \ln(N_t) & \sigma = 1; \varphi = 1 \end{cases}$$

$$\Rightarrow u'(C_t) = \begin{cases} C_t^{-\sigma} & \sigma \neq 1 \\ \frac{1}{C_t} & \sigma = 1 \end{cases}$$

$$\Rightarrow \frac{u'(C_{t+j})}{u'(C_t)} = \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma}$$

Real dividends, d_t , are defined as firm profits, or the portion of real output Y_t that is retained by the firm after paying labour its real wages. Where W_t represents nominal wages, P_t represents the price level, and N_t is the quantity of labour, real firm profits can be defined as:

$$d_{t+j} = Y_{t+j} - \frac{W_{t+j}}{P_{t+j}} N_{t+j}$$

Substituting the results from the assumed period utility function and definition of real dividends into (3.1) results in:

$$f_t = \sum_{j=1}^{\infty} \beta^j E_t \left[\left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} \left(Y_{t+j} - \frac{W_{t+j}}{P_{t+j}} N_{t+j} \right) \right] \quad (3.2)$$

The next steps begin to follow the Basic New Keynesian Model in Galí (2008). The model introduces infinitely-lived utility maximizing households which allocate consumption expenditure across a continuum of goods. The goods market only considers a final goods sector. Periodic consumption, C_t , represents an index of consumption aggregated across the continuum of goods, within $[0, 1]$ space, and with constant elasticity of substitution between goods as follows:

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

$C_t(i)$ and $P_t(i)$ represent the quantity and price, respectively, of good i . The aggregate price level is defined similarly:

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

Assuming optimal behavior given prices results in the following relationship between the product of the price index and quantity index, and total consumption expenditure:

$$\int_0^1 P_t(i)C_t(i)di = P_t C_t$$

Firms produce differentiated goods, with the index i corresponding to the goods produced by each firm. Technology, A_t , is identical across firms. Labour, $N_t(i)$, is the input for each firm, with firms as price takers in the labour market. Firm output, $Y_t(i)$, is thus defined as follows:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

Firms face the isoelastic demand schedule of households, and set prices following a pricing mechanism as proposed in Calvo (1983). The use of differentiated goods introduces a degree of imperfect competition in the goods market. The Calvo pricing mechanism delineates that each firm has a probability $(1 - \theta)$ to have the ability to reset prices, which introduces price stickiness as price dynamics are driven by firms reoptimizing periodically. The result is a 3-equation framework as follows (as utilized in Appendix A):

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \hat{Y}_t \tag{A.1}$$

$$\hat{Y}_t = -\frac{1}{\sigma} (i_t - E_t[\pi_{t+1}] - r_t^n) + E_t[\hat{Y}_{t+1}] \tag{A.2}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{Y}_t + v_t \tag{A.3}$$

Equation (A.1) is the New Keynesian Phillips Curve (NKPC) which expresses periodic inflation, π_t , as a function of expected inflation in the next period, π_{t+1} , and the current output gap, \hat{Y}_t , with $\kappa = \lambda \left[\sigma + \frac{(\varphi+\alpha)}{1-\alpha} \right]$ and $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Equation (A.2) is the dynamic IS equation (DIS) which expresses the periodic output gap, \hat{Y}_t , as a function of the difference between the real interest rate, $r_t \equiv i_t - E_t[\pi_{t+1}]$, and the natural rate of interest, r_t^n , and next period's expected output gap. Equation (A.3) is a simple Taylor-type interest rate rule where the periodic nominal interest rate, i_t , responds to inflation and the output gap in the current period with coefficients ϕ_π and ϕ_y measuring the responsiveness of policy to each variable respectively. The policy shock v_t follows an AR(1) process with zero mean where $\rho_v \in [0, 1)$:

$$v_t = \rho_v v_{t-1} + \epsilon_t^v$$

Returning to the firm pricing equation and imposing two equilibrium conditions on (3.2):

$$\text{i) Goods Market Clearing} \Rightarrow C_t = Y_t$$

$$\text{ii) Labour Market Clearing} \Rightarrow \frac{W_{t+j}}{P_{t+j}} = \frac{\partial Y_{t+j}}{\partial N_{t+j}} = (1 - \alpha) A_{t+j} N_{t+j}^{-\alpha}$$

$$\Rightarrow f_t = \sum_{j=1}^{\infty} \beta^j E_t \left[\left(\frac{Y_{t+j}}{Y_t} \right)^{-\sigma} (Y_{t+j} - (1 - \alpha) A_{t+j} N_{t+j}^{1-\alpha}) \right]$$

$$\Rightarrow f_t = \sum_{j=1}^{\infty} \beta^j E_t [Y_{t+j}^{1-\sigma} Y_t^\sigma - (1 - \alpha) Y_{t+j}^{-\sigma} Y_t^\sigma A_{t+j} N_{t+j}^{1-\alpha}]$$

It should be noted that the labour market clearing condition introduces the output elasticity of labour, the parameter $(1 - \alpha)$. Using the approximation of the relationship between aggregate output (Y_t), employment (N_t), and technology (A_t) from Galí (2008, p.46):

$$Y_t \approx A_t N_t^{1-\alpha}$$

$$\Rightarrow Y_{t+j} \approx A_{t+j} N_{t+j}^{1-\alpha}$$

$$\Rightarrow f_t = \sum_{j=1}^{\infty} \beta^j E_t [Y_{t+j}^{1-\sigma} Y_t^\sigma - (1 - \alpha) Y_{t+j}^{-\sigma} Y_t^\sigma Y_{t+j}]$$

$$\Rightarrow f_t = \sum_{j=1}^{\infty} \beta^j E_t [Y_{t+j}^{1-\sigma} Y_t^\sigma - (1 - \alpha) Y_{t+j}^{1-\sigma} Y_t^\sigma]$$

$$\Rightarrow f_t = \sum_{j=1}^{\infty} \beta^j E_t [\alpha Y_{t+j}^{1-\sigma} Y_t^\sigma]$$

$$\Rightarrow f_t = \alpha \sum_{j=1}^{\infty} \beta^j E_t [Y_{t+j}^{1-\sigma} Y_t^\sigma] \tag{3.3}$$

The above result is an equation for real firm price which depends on the expected path of periodic output. Intuitively α acts as a scale factor, even though capital has not been modeled explicitly, α represents the output elasticity of capital which corresponds to the previously defined parameter for the output elasticity of labour. Unfortunately, this equation at time t comes without having formed a meaningful expectation for output in each discrete future period. Even a relatively simple closed economy model is abruptly confronted

with one of the downfalls of DSGE modeling — there does not seem to be a closed-form solution that would allow for a clean interpretation of the above relationship. In order to simplify the interpretation of the above result, a second-order Taylor approximation of (3.3) around the steady state value of output is necessary. A second-order approximation will preserve second-order moments which include variance terms. Where \bar{Y} is the steady state value of output and the output gap is defined as $\hat{Y}_t = \frac{Y_t - \bar{Y}}{\bar{Y}}$ (see Appendix B for full approximation):

$$\begin{aligned}
Y_{t+j}^{1-\sigma} Y_t^\sigma &\approx \bar{Y} + \sigma(Y_t - \bar{Y}) + (1 - \sigma)(Y_{t+j} - \bar{Y}) \\
&\quad + \frac{1}{2}\sigma(\sigma - 1)\bar{Y}^{-1}(Y_t - \bar{Y})^2 + \frac{1}{2}(1 - \sigma)(-\sigma)\bar{Y}^{-1}(Y_{t+j} - \bar{Y})^2 \\
&\quad + \sigma(1 - \sigma)\bar{Y}^{-1}(Y_t - \bar{Y})(Y_{t+j} - \bar{Y}) \\
\Rightarrow Y_{t+j}^{1-\sigma} Y_t^\sigma &\approx \bar{Y}[1 + \sigma\hat{Y}_t + (1 - \sigma)\hat{Y}_{t+j} + \frac{1}{2}\sigma(\sigma - 1)\hat{Y}_t^2 - \frac{1}{2}\sigma(1 - \sigma)\hat{Y}_{t+j}^2 \\
&\quad + \sigma(1 - \sigma)\hat{Y}_t\hat{Y}_{t+j}] \\
\Rightarrow f_t &\approx \alpha\bar{Y} \sum_{j=1}^{\infty} \beta^j E_t[1 + \sigma\hat{Y}_t + (1 - \sigma)\hat{Y}_{t+j} + \frac{1}{2}\sigma(\sigma - 1)\hat{Y}_t^2 \\
&\quad - \frac{1}{2}\sigma(1 - \sigma)\hat{Y}_{t+j}^2 + \sigma(1 - \sigma)\hat{Y}_t\hat{Y}_{t+j}]
\end{aligned} \tag{3.4}$$

At this stage the expression found in Galí (2008, p.51) for the output gap in terms of the economy's modeled monetary policy shock v_t can be utilized (see Appendix A for full derivation):

$$\hat{Y}_t = -(1 - \beta\rho_v)\Lambda_v v_t \tag{3.5}$$

As previously noted, v_t is exogenous and follows an AR(1) process with a zero mean. Note that $v_t = \rho_v v_{t-1} + \epsilon_t^v$ with $E_t[\epsilon_{t+i}] = 0$ and $Var_t[\epsilon_{t+i}] = \sigma_\epsilon^2 \quad \forall \quad i > 0$. By substituting in (3.5), the sum in (3.4) can be rewritten as follows (see Appendix B for full derivation):

$$\begin{aligned}
\Rightarrow f_t &= \alpha\bar{Y} \left[\frac{\beta}{1 - \beta} - \sigma(1 - \beta\rho_v)\Lambda_v v_t \left(\frac{\beta}{1 - \beta} \right) - (1 - \sigma)(1 - \beta\rho_v)\Lambda_v v_t \left(\frac{\beta\rho_v}{1 - \beta\rho_v} \right) \right. \\
&\quad + \frac{1}{2}\sigma(\sigma - 1)(1 - \beta\rho_v)^2 \Lambda_v^2 v_t^2 \left(\frac{\beta}{1 - \beta} \right) \\
&\quad \left. - \frac{1}{2}\sigma(1 - \sigma)(1 - \beta\rho_v)^2 \Lambda_v^2 \left[v_t^2 \left(\frac{\rho_v^2 \beta}{1 - \rho_v^2 \beta} \right) + \sigma_\epsilon^2 \frac{\beta}{(1 - \beta)(1 - \rho_v^2 \beta)} \right] \right. \\
&\quad \left. + \sigma(1 - \sigma)(1 - \beta\rho_v)^2 \Lambda_v^2 v_t \left(\frac{\beta\rho_v}{1 - \beta\rho_v} \right) \right]
\end{aligned} \tag{3.6}$$

Parameters can be collected, where:

$$\begin{aligned}
b_0 &= \alpha \left(\frac{\beta}{1-\beta} \right) \bar{Y} \\
b_1 &= \alpha \bar{Y} \left[-\sigma(1-\beta\rho_v)\Lambda_v \left(\frac{\beta}{1-\beta} \right) - (1-\sigma)(1-\beta\rho_v)\Lambda_v \left(\frac{\beta\rho_v}{1-\beta\rho_v} \right) \right. \\
&\quad \left. + \sigma(1-\sigma)(1-\beta\rho_v)^2\Lambda_v^2 \left(\frac{\beta\rho_v}{1-\beta\rho_v} \right) \right] \\
b_2 &= \frac{1}{2}\alpha\bar{Y}\sigma(\sigma-1)(1-\beta\rho_v)^2\Lambda_v^2 \left[\left(\frac{\beta}{1-\beta} \right) + \left(\frac{\rho_v^2\beta}{1-\rho_v^2\beta} \right) \right] \\
b_3 &= \frac{1}{2}\alpha\bar{Y}\sigma(\sigma-1)(1-\beta\rho_v)^2\Lambda_v^2 \left(\frac{\beta}{(1-\beta)(1-\rho_v^2\beta)} \right) \\
&\Rightarrow f_t = b_0 + b_1v_t + b_2v_t^2 + b_3\sigma_\epsilon^2 \tag{3.7}
\end{aligned}$$

Using a second-order Taylor approximation results in an approximated linear equation for real firm price. The real firm price in the basic closed New Keynesian economy can be approximated as a function of the monetary policy shock v_t , the square of the monetary policy shock v_t^2 , the variance of the error term from the AR(1) process for the monetary policy shock σ_ϵ^2 , and coefficients b_i which depend on various economic parameters defining the steady state of the economy. Recall that the derivation above is relevant where $\sigma \neq 1$. If $\sigma = 1$, consistent with log utility, and $u(c_t) = \log(c_t)$, it can be shown $\sigma = 1$ becomes a special case where $b_0 = \alpha\bar{Y} \left(\frac{\beta}{1-\beta} \right)$; $b_1 = \alpha\bar{Y} \left(\frac{\beta}{1-\beta} \right) (1-\beta\rho_v)\Lambda_v$; and $b_2 = b_3 = 0$ which means that variances do not impact firm prices (see Appendix F for full derivation). This is due to the constant relative risk aversion (CRRA) associated with power utility, where $\sigma = 1$ is consistent with $CRRA = 1$ and the elasticity of intertemporal substitution $EIS = \frac{1}{\sigma} = 1$. The result suggests that in an economy characterized by agents with $CRRA = \sigma = 1$ firm prices around a non-stochastic steady state in period t are a function of steady state output, the economic parameters characterizing the steady state, and the monetary policy shock realized in period t .

However, where $\sigma \neq 1$ the approximation becomes more interesting. For the ease of interpretation, prior to making any assumptions about σ , each of the coefficients b_i will be approximated by calibrating the equation with the following assumptions from Galí (2008, p.52), which are quarterly where appropriate:

Table 3.1: Summary Calibration Parameters — Closed Economy

Parameter	Assumed Value
β	0.99
α	$\frac{1}{3}$
ϵ	6
φ	1
θ	$\frac{2}{3}$
ρ_v	0.5
ϕ_π	1.5
ϕ_y	$\frac{1}{8}$

Where the calibration strays from Gali's assumptions is in examining the case where $\sigma \neq 1$ as the special case of $\sigma = 1$ has already been discussed. The direction and magnitude of the resulting b_i coefficients from (3.7) can be analyzed using the above inputs and equations for a variety of potential σ , and thus the remaining parameters λ , κ , and Λ_v , can be solved in terms of σ . As well, b_0 depends only on α and β and thus can be calibrated independent of σ .

$$\begin{aligned} \Rightarrow \lambda &= \frac{(1-\theta)(1-\beta\theta)}{\theta} = 0.17 \\ \Rightarrow \kappa &= \lambda \left[\sigma + \frac{(\varphi + \alpha)}{1-\alpha} \right] = 0.17\sigma + 0.34 \\ \Rightarrow \Lambda_v &= \frac{1}{(1-\beta\rho_v)(\sigma(1-\rho_v) + \phi_y) + \kappa(\phi_\pi - \rho_v)} = \frac{1}{0.4225\sigma + 0.403125} \end{aligned}$$

$$b_0 = \alpha \left(\frac{\beta}{1-\beta} \right) \bar{Y} = 33\bar{Y} \quad (3.8)$$

Since $\beta = 0.99$ is a quarterly estimate, consistent with an annual steady state real rate of approximately 4 percent, \bar{Y} correspondingly represents a quarterly measure of steady state output in this calibration. The calibrated parameter values and corresponding values for λ , κ , and Λ_v , can be used to solve for b_1 , b_2 , and b_3 in terms of σ :

$$b_1 = \frac{1}{3} \left(\frac{0.249975\sigma(1-\sigma)}{(0.4225\sigma + 0.403125)^2} - \frac{0.495(1-\sigma)}{0.4225\sigma + 0.403125} - \frac{49.995\sigma}{0.4225\sigma + 0.403125} \right) \bar{Y} \quad (3.9)$$

$$b_2 = \left(\frac{4.22189(\sigma-1)\sigma}{(0.4225\sigma + 0.403125)^2} \right) \bar{Y} \quad (3.10)$$

$$b_3 = \left(\frac{5.59191(\sigma - 1)\sigma}{(0.4225\sigma + 0.403125)^2} \right) \bar{Y} \quad (3.11)$$

Part of the motivation for solving other parameters and coefficients in terms of σ is the fact that σ is perhaps the most contentious parameter to calibrate empirically. In their seminal work on the equity risk premium puzzle, Mehra and Prescott (1985) review past studies estimating the parameter with resulting values ranging from 0 to 2. Indeed, Mehra and Prescott struggle to reconcile empirical measures of the coefficient of relative risk aversion with the empirical risk-free rate and equity risk premium — under conventional asset pricing models the empirical equity risk premium requires an implausibly high σ , whereas the empirical risk-free rate requires a much lower σ . Lucas (2003) uses $\sigma = 1$ or log utility, discussing how risk aversion coefficients used in literature range from 1 to 4. Using per capita consumption growth rates and the after-tax return on capital in the United States Lucas argues that σ under CRRA preferences is at most 2.5, and further suggests that a value of 2.5 would lead to much larger interest rate differentials between mature and fast-growing economies than what is observed. Using labour supply data Chetty (2006) estimates $\sigma \approx 1$, with estimates ranging from 0.15 to 1.78, and argues that $\sigma = 2$ should serve as an upper bound based on patterns of labour supply and complementarity between consumption and labour. Havránek (2013) provides a meta-analysis of the literature on the elasticity of intertemporal substitution (EIS) for consumption, the inverse of σ ($\frac{1}{\sigma}$) in the context of this model, finding that past research using micro-based data contains bias overstating estimates by an average of 0.5. Adjusting for bias, Havranek suggests corrected means of approximately 0.3 - 0.4 for the micro-based estimates, and suggests that EIS estimates above 0.8 cannot be reconciled with what is observed empirically. $\frac{1}{\sigma} = EIS < 0.8$ implies $\sigma > 1.25$ and likewise a range of approximately 0.3 - 0.4 on EIS suggests a corresponding value for σ roughly between 2.5 and 3.3. Gandelman et al. (2015) studies the relative risk aversion coefficient at the country level, across a sample of 75 countries, with individual estimates falling between 0 and 3 and median and mean estimates close to 1.

Even without an exhaustive review of the literature pertaining to the coefficient of relative risk aversion, or the inversely the elasticity of intertemporal substitution, it becomes clear that σ in this context is not definitive though there is some information useful in triangulating the neighborhood of plausible empirical values of σ . Instead of embedding a prescriptive value of σ , as that would be tangential to the purpose of the paper, a range of plausible values can be calibrated and used to approximate the b_i coefficients and their behavior about that range of values. As expressed in (3.9), (3.10), and (3.11) respectively, b_1 has no positive zeros, while b_2 and b_3 have zeros at $\sigma = 0$ and $\sigma = 1$.

Recall from (3.7):

$$f_t = b_0 + b_1 v_t + b_2 v_t^2 + b_3 \sigma_\epsilon^2 \quad (3.7)$$

Table 3.2: Calibration Summary — Closed Economy

σ	$\frac{1}{\sigma} = EIS$	b_0	b_1	b_2	b_3
0	Undefined	$33\bar{Y}$	$-0.41\bar{Y}$	0	0
0.25	4.00	$33\bar{Y}$	$-8.37\bar{Y}$	$-3.06\bar{Y}$	$-4.05\bar{Y}$
0.5	2.00	$33\bar{Y}$	$-13.64\bar{Y}$	$-2.80\bar{Y}$	$-3.70\bar{Y}$
0.75	1.33	$33\bar{Y}$	$-17.39\bar{Y}$	$-1.53\bar{Y}$	$-2.02\bar{Y}$
1.0	1.00	$33\bar{Y}$	$-20.18\bar{Y}$	0	0
1.5	0.67	$33\bar{Y}$	$-24.09\bar{Y}$	$2.95\bar{Y}$	$3.90\bar{Y}$
2.0	0.50	$33\bar{Y}$	$-26.68\bar{Y}$	$5.42\bar{Y}$	$7.18\bar{Y}$
2.5	0.40	$33\bar{Y}$	$-28.53\bar{Y}$	$7.43\bar{Y}$	$9.85\bar{Y}$
3.0	0.33	$33\bar{Y}$	$-29.91\bar{Y}$	$9.08\bar{Y}$	$12.02\bar{Y}$
3.5	0.29	$33\bar{Y}$	$-30.98\bar{Y}$	$10.43\bar{Y}$	$13.82\bar{Y}$

For example, if it is believed the economy is best characterized by $\sigma = 1.5$ then the calibration for (3.7) becomes:

$$f_t = 33\bar{Y} - 24.09\bar{Y}v_t + 2.95\bar{Y}v_t^2 + 3.90\bar{Y}\sigma_\epsilon^2 \quad (3.12)$$

Or alternatively the firm price equation can be written relative to steady state output:

$$\frac{f_t}{\bar{Y}} = 33 - 24.09v_t + 2.95v_t^2 + 3.90\sigma_\epsilon^2 \quad (3.13)$$

To interpret the relationship between the policy shock and firm price for each value of σ , prospective values for v_t can be plugged into the corresponding calibrated equation. For example, if $\sigma = 1.5$ then equation (3.12) would be used. Equation (3.5) can be used to find the output gap for prospective pairs of σ and v_t . Continuing with the example of $\sigma = 1.5$, and assuming for ease of illustration that $\sigma_\epsilon \approx 0$, (3.5) suggests a shock of $v_t = 1\%$ results in an output gap of $\hat{Y}_t = -0.49\%$ which leads to a corresponding change in firm price of -0.73% ¹. Similarly for $\sigma = 1.5$ a shock of $v_t = 10\%$ is associated with an output gap of $\hat{Y}_t = -4.87\%$

¹Recall that the output gap is expressed relative to steady state output, that is $\hat{Y}_t = \frac{Y_t - \bar{Y}}{\bar{Y}}$. As well, firm price changes are relative to $\bar{f} = 33\bar{Y}$ which is the case when $v_t = 0$ and $\sigma_\epsilon = 0$. The results vary in σ as follows:
For $\sigma = 0$, $v_t = 1\%$ results in $\hat{Y}_t = -1.25\%$ and a change in firm price of -0.01% .
For $\sigma = 0.25$, $v_t = 1\%$ results in $\hat{Y}_t = -0.99\%$ and a change in firm price of -0.25% .
For $\sigma = 0.5$, $v_t = 1\%$ results in $\hat{Y}_t = -0.82\%$ and a change in firm price of -0.41% .
For $\sigma = 0.75$, $v_t = 1\%$ results in $\hat{Y}_t = -0.70\%$ and a change in firm price of -0.53% .
For $\sigma = 1$, $v_t = 1\%$ results in $\hat{Y}_t = -0.61\%$ and a change in firm price of -0.61% .
For $\sigma = 1.5$, $v_t = 1\%$ results in $\hat{Y}_t = -0.49\%$ and a change in firm price of -0.73% .
For $\sigma = 2$, $v_t = 1\%$ results in $\hat{Y}_t = -0.40\%$ and a change in firm price of -0.81% .
For $\sigma = 2.5$, $v_t = 1\%$ results in $\hat{Y}_t = -0.35\%$ and a change in firm price of -0.86% .
For $\sigma = 3$, $v_t = 1\%$ results in $\hat{Y}_t = -0.30\%$ and a change in firm price of -0.90% .
For $\sigma = 3.5$, $v_t = 1\%$ results in $\hat{Y}_t = -0.27\%$ and a change in firm price of -0.94% .
For $\sigma = 4$, $v_t = 1\%$ results in $\hat{Y}_t = -0.24\%$ and a change in firm price of -0.96% .

and a corresponding decrease in firm price of -7.21% .

In interpreting the results it is important to recall (3.5) which establishes a negative relationship between the output gap and the economy's monetary policy shock v_t . Further, recall that $\sigma_\epsilon^2 = Var_t[\epsilon_{t+i}]$.

$$\hat{Y}_t = -(1 - \beta\rho_v)\Lambda_v v_t \quad (3.5)$$

Interpreting b_0 is relatively easy as it is essentially the present value of a perpetuity, where the coupon payment or dividend is the firm's share of steady state output in the economy and is thus proportional to α and \bar{Y} , the only other factor effecting b_0 is β which corresponds to the discount rate.

The coefficient b_1 represents the first-order effect of the monetary policy shock on firm prices in the economy. It is constructive to recall that the shock v_t is introduced into the model for firm prices via the output gap, \hat{Y} . Since v_t is negatively related with the output gap as illustrated in (3.5), positive shocks decrease output in a persistent fashion due to the autoregressive nature of the shock, thus the negative relationship between firm value and v_t reflected in b_1 . The other phenomenon is that b_1 is decreasing in σ and conversely b_1 is positively related to the *EIS*. At first glance, the result may seem similar to what would be expected in a consumption and savings problem, but it is important to recognize that in this model *there is no alternative mechanism for savings* aside from investing in firms. The mechanism for fluctuations in periodic consumption is via the periodic output gap which is driven by v_t . Tracing back to the beginning, it appears that the differing impact of v_t on firm prices for various levels of σ flows from the impact of σ on the utility function and thus the stochastic discount factor — specifically the ratio of periodic marginal utilities of consumption, combined with fluctuations in C_t and C_{t+j} — the path of consumption which is distorted by the monetary policy shock's impact on the periodic output gap. Since v_t follows an AR(1) process and $\rho_v = 0.5$ in the calibration, $v_t > 0$ will have a much larger negative impact on C_t than C_{t+j} . Thus in the stochastic discount factor $\beta^j \frac{u'(C_{t+j})}{u'(C_t)} = \beta^j \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma}$ the impact of v_t will decrease the denominator by more than the numerator. Since the ratio of future and present consumption is to the power of $-\sigma$, the result is that $v_t > 0$ will decrease the stochastic discount factor and *the magnitude of the decrease* will be positively related to σ . The decrease in the stochastic discount factor decreases firm prices as the stochastic discount factor is a scale factor on dividends, as reflected in (3.1). Conversely if $v_t < 0$ the opposite occurs where C_t increases by more than C_{t+j} and thus the stochastic discount factor increases resulting in higher firm prices.

In order to interpret b_2 and b_3 , recall that the model's only mechanism to save and transfer wealth intertemporally is via capital invested in firms. Thus embedded in b_2 and b_3 are observations of two competing effects simultaneously: the risk aversion component associated with the investment decision and a prudence component associated with the savings decision. Since households are risk averse, in isolation higher volatility should reduce corresponding asset values when considering the household investment decision. Conversely,

risk averse households facing an intertemporal savings decision with stochastic future income will exhibit a level of prudence, or precautionary savings as coined by Leland (1968), with the competing effect that a higher volatility of expected future income in isolation should increase the value of savings as a household seeks to self-insure future consumption.

Due to the competing factors, as illustrated in Table 3.2, b_2 and b_3 are dependent on the level of household risk aversion, σ , both in terms of the magnitude of each coefficient as well as the direction. The second-order effect of the output change resulting from the monetary policy shock is approximated by b_2 , while the second-order effect of the policy shock's variance is approximated by b_3 . For $\sigma = 0$, the approximation of $b_2 = b_3 = 0$ reflects the fact that household utility exhibits no risk aversion and thus there is no risk aversion on the investment component and no prudence on the savings component. As soon as $\sigma > 0$, the effect of risk aversion is immediately observed via negative approximations for b_2 and b_3 which reflect lower asset values due to correspondingly higher risk premiums. However, as σ increases the effect of prudence is observed as b_2 and b_3 increase until $b_2 = b_3 = 0$ at $\sigma = 1$. For $\sigma > 1$, prudence becomes the dominating factor and b_2 and b_3 are positive and increasing in σ .

3.2 Real Firm Price in a Small Open Economy

The open economy model developed within builds on the closed economy model in Section 3.1. While the closed economy model closely follows Galí (2008), the open economy model strays due to ad hoc assumptions given that Galí's methodology is focused primarily on monetary policy, and is agnostic to firm prices. The goal is to observe how asset prices relate to monetary policy in a basic open economy framework, and thus certain simplifying assumptions are used.

The open economy uses the same initial definition of real firm price:

$$f_t = \sum_{i=1}^{\infty} \beta^i E_t \left[\frac{u'(c_{t+i})}{u'(c_t)} d_{t+i} \right] \quad (3.1)$$

And the same assumption of a period utility function is used, given by:

$$U(C_t, N_t) = \begin{cases} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1-\varphi}}{1-\varphi} & \sigma \neq 1; \varphi \neq 1 \\ \frac{C_t^{1-\sigma}}{1-\sigma} - \ln(N_t) & \sigma \neq 1; \varphi = 1 \\ \ln(C_t) - \frac{N_t^{1-\varphi}}{1-\varphi} & \sigma = 1; \varphi \neq 1 \\ \ln(C_t) - \ln(N_t) & \sigma = 1; \varphi = 1 \end{cases}$$

$$\Rightarrow u'(C_t) = \begin{cases} C_t^{-\sigma} & \sigma \neq 1 \\ \frac{1}{C_t} & \sigma = 1 \end{cases}$$

$$\Rightarrow \frac{u'(c_{t+j})}{u'(c_t)} = \left(\frac{c_{t+j}}{c_t} \right)^{-\sigma}$$

Marginal utility above is identical to Section 3.1, as is the elasticity of intertemporal substitution for both consumption and labour. Again, real dividends are defined as follows:

$$d_{t+j} = Y_{t+j} - \frac{W_{t+j}}{P_{t+j}} N_{t+j}$$

Substituting the assumed period utility function and definition of real dividends into (3.1) results in the following:

$$f_t = \sum_{i=1}^{\infty} \beta^i E_t \left[\left(\frac{c_{t+i}}{c_t} \right)^{-\sigma} \left(Y_{t+i} - \frac{W_{t+i}}{P_{t+i}} N_{t+i} \right) \right] \quad (3.2)$$

The same equilibrium condition as before is imposed on the labour market, but note that the previous goods market clearing condition from Section 3.1 is not imposed:

$$\text{i) Labour Market Clearing} \Rightarrow \frac{W_{t+j}}{P_{t+j}} = \frac{\partial Y_{t+j}}{\partial N_{t+k}} = (1 - \alpha) A_{t+j} N_{t+j}^{-\alpha}$$

$$\Rightarrow f_t = \sum_{i=1}^{\infty} \beta^i E_t \left[\left(\frac{c_{t+i}}{c_t} \right)^{-\sigma} \left(Y_{t+i} - (1 - \alpha) A_{t+i} N_{t+i}^{1-\alpha} \right) \right]$$

Due to the fact the economy is open and thus output has both a domestic component and a foreign component, the following analysis introduces notation to differentiate certain domestic and foreign variables, using superscript “h” for domestic variables and superscript “f” for foreign variables, for example:

$$Y_{t+j}^h \equiv \text{Domestic Output}$$

$$Y_{t+j}^f \equiv \text{Foreign Output}$$

Thus for pricing domestic firms the notation changes as follows:

$$\Rightarrow f_t^h = \sum_{i=1}^{\infty} \beta^i E_t \left[\left(\frac{c_{t+i}^h}{c_t^h} \right)^{-\sigma} \left(Y_{t+i}^h - (1 - \alpha) A_{t+i}^h (N_{t+i}^h)^{1-\alpha} \right) \right]$$

Instead of Galí’s 3-equation framework used in the closed economy, the open economy requires both a foreign system and a domestic system (as utilized in Appendix C). Assuming that the domestic system is small relative to the foreign system and that foreign policy is thus agnostic to domestic shocks, the foreign system can be modeled in a 3-equation framework that resembles the closed economy model:

$$\pi_t^f = \beta E_t[\pi_{t+1}^f] + \kappa \hat{Y}_t^f \quad (\text{C.1})$$

$$\hat{Y}_t^f = -\frac{1}{\sigma} (i_t^f - E_t[\pi_{t+1}^f] - r_t^n) + E_t[\hat{Y}_{t+1}^f] \quad (\text{C.2})$$

$$i_t^f = \rho + \phi_\pi \pi_t^f + \phi_y \hat{Y}_t^f + v_t^f \quad (\text{C.3})$$

The foreign policy shock follows an AR(1) process, where $\rho_v \in [0, 1)$:

$$v_t^f = \rho_v v_{t-1}^f + \epsilon_t^{vf}$$

The foreign system in the open economy is built on the same underlying assumptions and mechanics as the closed economy system. Households are infinitely-lived and utility maximizing agents allocating consumption across a continuum of goods. The goods market only considers a final goods sector, and thus only final goods are traded implicitly. Imperfect competition is introduced by the assumption that firms produce differentiated goods with identical technology, with firms facing an isoelastic demand schedule from households and setting prices following a Calvo pricing mechanism with the probability of being able to adjust prices set to $1 - \theta$. Calvo pricing introduces price stickiness as only select firms can reset prices periodically.

The 3-equation framework for the domestic system is unique in terms of the monetary policy equation:

$$\pi_t^h = \beta E_t[\pi_{t+1}^h] + \kappa \hat{Y}_t^h \quad (\text{C.6})$$

$$\hat{Y}_t^h = -\frac{1}{\sigma} (i_t^h - E_t[\pi_{t+1}^h] - r_t^n) + E_t[\hat{Y}_{t+1}^h] \quad (\text{C.7})$$

$$i_t^h = \rho + \phi_\pi^h \pi_t^h + \phi_y^h \hat{Y}_t^h + \phi_\pi^f \pi_t^f + \phi_y^f \hat{Y}_t^f + v_t^h \quad (\text{C.8})$$

The NKPC in (C.6) and the DIS in (C.7) are identical to the closed economy model as well as the aforementioned foreign system in the open economy model, and thus include the same underlying assumptions. The difference is that since the economy is now open and the foreign economy is large relative to the domestic economy, domestic policy should not be agnostic to foreign shocks and should respond in some fashion. In addition to responding to domestic inflation and the domestic output gap, in the absence of an explicit and rigorous model for trade and exchange rates, the ad hoc assumption is made that the domestic policy function in (C.8) takes the form of a Taylor-type policy function modified to respond directly to foreign inflation and the foreign output gap. The motivation is that under the assumption that the domestic economy is small and open, foreign inflation and foreign output gaps will have a meaningful impact on the domestic economy as the foreign system makes up the bulk of aggregate demand in the open economy.

For example, where $\phi_y^f > 0$, consider a scenario where actual foreign GDP is in excess of potential GDP and thus there is a foreign output gap ($\hat{Y}_t^f > 0$). Since the domestic economy is assumed to be small, the corollary is that the foreign economy determines the overwhelming majority of aggregate demand. Since the economy is open, excess aggregate demand from the foreign output gap will spill into the domestic system via trade (implicitly in this model), and will have the same effects as excess demand domestically such as pushing domestic prices higher. By having the domestic policy function in (C.8) respond directly to the foreign output gap, all else equal the domestic policymaker in this example will raise domestic nominal interest rates in a contractionary manner to counteract the excess aggregate demand, even though it is from the foreign economy.

The domestic policy shock also follows an AR(1) process, with the same $\rho_v \in [0, 1)$:

$$v_t^h = \rho_v v_{t-1}^h + \epsilon_t^{vh}$$

Domestic consumption will equal domestic income in equilibrium, but now domestic income will have three components: domestic labour income, the domestic share of domestic capital income, and the domestic share of foreign capital income — thus foreign capital income imputed into domestic consumption introduces the potential for impact of foreign assets on domestic income. For this purpose, domestic ownership of domestic capital and foreign capital are both key to representing domestic income and thus domestic consumption.

$k_t^h \equiv$ the share of domestic capital owned domestically

$k_t^f \equiv$ the share of foreign capital owned domestically

$$\begin{aligned} \Rightarrow c_t^h &= \frac{W_t^h}{P_t^h} N_t^h + k_t^h [Y_t^h - (1 - \alpha) A_t^h (N_t^h)^{1-\alpha}] \\ &\quad + k_t^f [Y_t^f - (1 - \alpha) A_t^f (N_t^f)^{1-\alpha}] \\ &= (1 - \alpha) A_t^h (N_t^h)^{1-\alpha} + k_t^h [Y_t^h - (1 - \alpha) A_t^h (N_t^h)^{1-\alpha}] \\ &\quad + k_t^f [Y_t^f - (1 - \alpha) A_t^f (N_t^f)^{1-\alpha}] \end{aligned}$$

The same approximation is used for the relationship between aggregate output, employment, and technology, as before from Galí (2008):

$$\begin{aligned} Y_t &\approx A_t N_t^{1-\alpha} \\ \Rightarrow N_t &\approx \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

However, now the definition must be partitioned to distinguish domestic and foreign contributions such that:

$$\begin{aligned}\Rightarrow c_t^h &\approx (1 - \alpha)Y_t^h + k_t^h(\alpha Y_t^h) + k_t^f(\alpha Y_t^f) \\ &= [1 - \alpha(1 - k_t^h)] Y_t^h + \alpha k_t^f Y_t^f\end{aligned}$$

Since the result will be calibrated around a steady state, assume that levels of capital ownership are constant over time, such that $k_t^h = k_{t+j}^h = k_1$ and $k_t^f = k_{t+j}^f = k_2$, while setting $\mu_1 = (1 - \alpha(1 - k_1))$ and $\mu_2 = \alpha k_2$, the result is an equation for domestic consumption which clearly delineates proportional contributions from domestic and foreign output:

$$\Rightarrow c_t^h \approx \mu_1 Y_t^h + \mu_2 Y_t^f \quad (3.14)$$

Additionally, using the approximate relationship between aggregate output, employment, and technology, the equation for firm prices simplifies to:

$$\Rightarrow f_t^h \approx \sum_{i=1}^{\infty} \beta^i E_t \left[\left(\frac{c_{t+i}^h}{c_t^h} \right)^{-\sigma} (\alpha Y_{t+i}^h) \right] \quad (3.15)$$

Combining (3.14) and (3.15):

$$\Rightarrow f_t^h \approx \alpha \sum_{i=1}^{\infty} \beta^i E_t \left[\left(\frac{\mu_1 Y_{t+i}^h + \mu_2 Y_{t+i}^f}{\mu_1 (Y_{t+i}^h) + \mu_2 Y_{t+i}^f} \right)^{\sigma} Y_{t+i}^h \right] \quad (3.16)$$

In order to linearize (3.16) around the steady state, the following assumption will be made about the steady state level of output:

$$\begin{aligned}\bar{Y}_t^h &= \bar{Y}_{t+j}^h = \bar{Y} \\ \bar{Y}_t^f &= \bar{Y}_{t+j}^f = u\bar{Y}\end{aligned}$$

Essentially, assume that the steady state value of foreign output is proportional to the steady state value for domestic output, and the coefficient “ u ” reflects such proportionality. For example, if the steady state value of foreign output is 2 times larger than the domestic economy’s output, then $u = 2$. Alternatively, one could conceptualized foreign output as representing the rest of the global economy and thus “ u ” can reflect the scale factor which depends on the size of the domestic economy relative to the global economy. If the domestic economy represents 10% of global output, then one would set $u = 9$ as the domestic economy represents $\frac{1}{1+u}$ of the global economy.

In order to solve the sum in (3.16), a second-order Taylor approximation around the steady state value of output will be used. Let $\mathbf{Y} = (Y_t^h, Y_{t+k}^h, Y_t^f, Y_{t+j}^f)$ such that:

$$\left(\frac{\mu_1 Y_t^h + \mu_2 Y_t^f}{\mu_1 (Y_{t+j}^h) + \mu_2 Y_{t+j}^f} \right)^\sigma Y_{t+j}^h = F(Y_t^h, Y_{t+k}^h, Y_t^f, Y_{t+j}^f) = F(\mathbf{Y})$$

Then write (3.16) in terms of $F(\mathbf{Y})$:

$$\Rightarrow f_t^h \approx \alpha \sum_{i=1}^{\infty} \beta^i E_t [F(\mathbf{Y})] \quad (3.17)$$

Thus the approximation becomes (see Appendix D for detailed approximation):

$$\begin{aligned} F(\mathbf{Y}) &\approx \bar{Y} + \frac{\sigma \mu_1}{\mu_1 + \mu_2 u} (Y_t^h - \bar{Y}) + \frac{\sigma \mu_2}{\mu_1 + \mu_2 u} (Y_t^f - u \bar{Y}) \\ &+ \left(\frac{\mu_1 + \mu_2 u - \sigma \mu_1}{\mu_1 + \mu_2 u} \right) (Y_{t+j}^h - \bar{Y}) - \frac{\sigma \mu_2}{\mu_1 + \mu_2 u} (Y_{t+j}^f - u \bar{Y}) \\ &+ \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})^2 \\ &+ \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_2^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^f - u \bar{Y})^2 \\ &+ \frac{1}{2} \left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_{t+j}^h - \bar{Y})^2 \\ &+ \frac{1}{2} \left(\frac{\sigma(\sigma+1)\mu_2^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_{t+j}^f - u \bar{Y})^2 \\ &+ \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1 \mu_2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})(Y_t^f - u \bar{Y}) \\ &+ \frac{1}{2} \left(\frac{\sigma \mu_1 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})(Y_{t+j}^h - \bar{Y}) \\ &- \frac{1}{2} \left(\frac{\sigma^2 \mu_1 \mu_2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})(Y_{t+j}^f - u \bar{Y}) \\ &+ \frac{1}{2} \left(\frac{\sigma \mu_2 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^f - u \bar{Y})(Y_{t+j}^h - \bar{Y}) \\ &- \frac{1}{2} \left(\frac{\sigma^2 \mu_2^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^f - u \bar{Y})(Y_{t+j}^f - u \bar{Y}) \\ &+ \frac{1}{2} \left(\frac{\sigma \mu_2 (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_{t+j}^h - \bar{Y})(Y_{t+j}^f - u \bar{Y}) \end{aligned} \quad (3.18)$$

Where \bar{Y} is the steady state value of domestic output one can express the domestic output gap as $\hat{Y}_t^h = \frac{Y_t^h - \bar{Y}}{\bar{Y}}$ and where $u\bar{Y}$ is the steady state value of foreign output the foreign output gap can be expressed as $\hat{Y}_t^f = \frac{Y_t^f - u\bar{Y}}{(u\bar{Y})}$. Combining (3.17) and (3.18) and writing in terms of respective output gaps results in a second-order approximation for firm price in terms of the macroeconomic parameters and the domestic and foreign output gaps:

$$\begin{aligned}
f_t \approx & \alpha \bar{Y} \sum_{j=1}^{\infty} \beta^j E_t \left[1 + \frac{\sigma \mu_1}{\mu_1 + \mu_2 u} \hat{Y}_t^h + \frac{\sigma \mu_2 u}{\mu_1 + \mu_2 u} \hat{Y}_t^f \right. \\
& + \left(\frac{\mu_1 + \mu_2 u - \sigma \mu_1}{\mu_1 + \mu_2 u} \right) \hat{Y}_{t+j}^h - \frac{\sigma \mu_2 u}{\mu_1 + \mu_2 u} \hat{Y}_{t+j}^f + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_t^h)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_t^f)^2 + \frac{1}{2} \left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_{t+j}^h)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma+1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_{t+j}^f)^2 + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^h \hat{Y}_t^f \\
& + \frac{1}{2} \left(\frac{\sigma \mu_1 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^h \hat{Y}_{t+j}^h - \frac{1}{2} \left(\frac{\sigma^2 \mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^h \hat{Y}_{t+j}^f \\
& + \frac{1}{2} \left(\frac{\sigma \mu_2 (\mu_1 + \mu_2 u - \sigma \mu_1) u}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^f \hat{Y}_{t+j}^h - \frac{1}{2} \left(\frac{\sigma^2 \mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^f \hat{Y}_{t+j}^f \\
& \left. + \frac{1}{2} \left(\frac{\sigma \mu_2 u (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_{t+j}^h \hat{Y}_{t+j}^f \right] \tag{3.19}
\end{aligned}$$

Next using the results from Appendix C which expresses the output gaps, \hat{Y}_t^f and \hat{Y}_t^h , in terms of their respective monetary policy shocks:

$$\hat{Y}_t^f = \psi_{yv} v_t^f = -(1 - \beta \rho_v) \Lambda^f v_t^f \tag{C.4}$$

$$\hat{Y}_t^h = \psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f = -(1 - \beta \rho_v) \Lambda^h v_t^h + (1 - \beta \rho_v) \Lambda^h \Lambda^f (\phi_\pi^f \kappa + \phi_y^f (1 - \beta \rho_v)) v_t^f \tag{C.17}$$

The equation (3.19) can be written in terms of the domestic and foreign policy shocks which are left in terms of ψ_{yv} , ψ_{yv}^h , and ψ_{yv}^f to keep the notation clearer for collecting terms:

$$\begin{aligned}
f_t \approx & \alpha \bar{Y} \sum_{j=1}^{\infty} \beta^j E_t \left[1 + \frac{\sigma \mu_1}{\mu_1 + \mu_2 u} (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f) + \frac{\sigma \mu_2 u}{\mu_1 + \mu_2 u} \psi_{yv} v_t^f \right. \\
& + \left(\frac{\mu_1 + \mu_2 u - \sigma \mu_1}{\mu_1 + \mu_2 u} \right) (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f) - \frac{\sigma \mu_2 u}{\mu_1 + \mu_2 u} \psi_{yv} v_{t+j}^f \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv} v_t^f)^2 \\
& + \frac{1}{2} \left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma+1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv} v_{t+j}^f)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} v_t^f (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f) \\
& + \frac{1}{2} \left(\frac{\sigma \mu_1 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f) (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f) \\
& - \frac{1}{2} \left(\frac{\sigma^2 \mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f) \psi_{yv} v_{t+j}^f \\
& + \frac{1}{2} \left(\frac{\sigma \mu_2 (\mu_1 + \mu_2 u - \sigma \mu_1) u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} v_t^f (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f) \\
& - \frac{1}{2} \left(\frac{\sigma^2 \mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv})^2 v_t^f v_{t+j}^f \\
& \left. + \frac{1}{2} \left(\frac{\sigma \mu_2 u (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f) \psi_{yv} v_{t+j}^f \right]
\end{aligned} \tag{3.20}$$

As shown in detail in Appendix E, (3.20) can be expressed in terms of first-order and second-order terms for each shock, where the z_i below represent coefficients that are a collection of the parameters that come from grouping terms in (3.20):

$$\begin{aligned}
f_t \approx & \alpha \bar{Y} \sum_{j=1}^{\infty} \beta^j E_t \left[1 + z_1 v_t^h + z_2 v_t^f + z_3 v_{t+j}^h + z_4 v_{t+j}^f + z_5 (v_t^h)^2 \right. \\
& + z_6 (v_t^f)^2 + z_7 (v_{t+j}^h)^2 + z_8 (v_{t+j}^f)^2 + z_9 v_t^h v_t^f + z_{10} v_{t+j}^h v_{t+j}^f \\
& \left. + z_{11} v_t^f v_{t+j}^h + z_{12} v_t^f v_{t+j}^f + z_{13} v_t^h v_{t+j}^h + z_{14} v_t^h v_{t+j}^f \right]
\end{aligned} \tag{3.21}$$

Working through the summation results in the following linear equation for firm prices (see Appendix E for more detail):

$$\begin{aligned}
\Rightarrow f_t \approx & \alpha \bar{Y} \left[\frac{\beta}{1-\beta} + \frac{\beta}{1-\beta} z_1 v_t^h + \frac{\beta}{1-\beta} z_2 v_t^f \right. \\
& + \frac{\beta \rho_v}{1-\beta \rho_v} z_3 v_t^h + \frac{\beta \rho_v}{1-\beta \rho_v} z_4 v_t^f + \frac{\beta}{1-\beta} z_5 (v_t^h)^2 + \frac{\beta}{1-\beta} z_6 (v_t^f)^2 \\
& + z_7 \left[(v_t^h)^2 \left(\frac{\beta \rho_v^2}{1-\beta \rho_v^2} \right) + (\sigma_\epsilon^h)^2 \left(\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} \right) \right] \\
& + z_8 \left[(v_t^f)^2 \left(\frac{\beta \rho_v^2}{1-\beta \rho_v^2} \right) + (\sigma_\epsilon^f)^2 \left(\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} \right) \right] + \frac{\beta}{1-\beta} z_9 v_t^h v_t^f \\
& + z_{10} \left[\left(\frac{\beta \rho_v^2}{1-\beta \rho_v^2} \right) v_t^h v_t^f + \frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} \sigma_{\epsilon h, \epsilon f} \right] + \left(\frac{\beta \rho_v}{1-\beta \rho_v} \right) z_{11} v_t^f v_t^h \\
& + \left(\frac{\beta \rho_v}{1-\beta \rho_v} \right) z_{12} (v_t^f)^2 + \left(\frac{\beta \rho_v}{1-\beta \rho_v} \right) z_{13} (v_t^h)^2 + \left(\frac{\beta \rho_v}{1-\beta \rho_v} \right) z_{14} v_t^f v_t^h \left. \right]
\end{aligned} \tag{3.22}$$

In order to more clearly illustrate the linear equation, terms can be grouped with respect to first-order and second-order effects of the various shocks, including interactions, and thus the open economy result can be written in the same format as the closed economy result, but with more terms given the influence of foreign monetary policy shocks and corresponding interactions. Additionally, most of the b_i terms are now much more complex with significantly more parameters embedded, though the first four terms are optically similar to Section 3.1 (see Appendix E for more detail, including each b_i):

$$\begin{aligned}
\Rightarrow f_t \approx & b_0 + b_1 v_t^h + b_2 (v_t^h)^2 + b_3 (\sigma_\epsilon^h)^2 + b_4 v_t^f + b_5 (v_t^f)^2 \\
& + b_6 (\sigma_\epsilon^f)^2 + b_7 v_t^h v_t^f + b_8 \sigma_{\epsilon h, \epsilon f}
\end{aligned} \tag{3.23}$$

The result is a linear equation approximating real firm price as a function of the domestic monetary policy shock v_t^h , the square of the domestic monetary policy shock $(v_t^h)^2$, the variance of the error term from the domestic monetary policy shock $(\sigma_\epsilon^h)^2$, the foreign monetary policy shock v_t^f , the square of the foreign monetary policy shock $(v_t^f)^2$, the variance of the error term from the foreign monetary policy shock $(\sigma_\epsilon^f)^2$, the interaction between the domestic and foreign monetary policy shock $v_t^h v_t^f$, and correlation of the error term in the domestic and foreign monetary policy functions denoted $\sigma_{\epsilon h, \epsilon f}$.

The next step is to calibrate the model, using several of the same assumptions as Section 3.1:

Table 3.3: Summary Calibration Parameters — Open Economy

Parameter	Assumed Value
β	0.99
α	$\frac{1}{3}$
ϵ	6
φ	1
θ	$\frac{2}{3}$
ρ_v	0.5
$\phi_\pi = \phi_\pi^f = \phi_y^h$	1.5
$\phi_y = \phi_y^f = \phi_y^h$	$\frac{1}{8}$
λ	0.17
κ	$0.17\sigma + 0.34$
$\Lambda^h = \Lambda^f$	$\frac{1}{0.4225\sigma + 0.403125}$

Notably the simplifying assumption has been made that the interest rate rule coefficients are the same in the domestic and foreign economy, that is $\phi_\pi = \phi_\pi^f = \phi_y^h$ and $\phi_y = \phi_y^f = \phi_y^h$. Despite the simplifying assumptions, this is a point of departure compared with the closed economy model because now in addition to σ there is also k_1 and k_2 as choice variables — capital ownership now matters. In theory, an attempt could be made to solve for the asset allocation of domestic and foreign households from Euler equations describing utility maximization, but such a solution would require assumptions about forward-looking return expectations from investing in firms and would thus require a formulation of expectations of future firm prices.

In practice, the equity “home bias” is a well documented anomaly whereby actual empirical asset allocations of households cannot be reconciled with optimizing household behavior in an open economy. The equity home bias was initially discussed in French and Poterba (1991) which identified that investors hold the overwhelming majority of their wealth domestically, consistent across all five of the largest stock markets in the world at the time (United States; Japan; United Kingdom; Germany; and France) despite generally well known and accepted benefits of international diversification. Coval and Moskowitz (1999) reinforce the result, finding that home bias occurs even within the United States as domestic investment managers empirically prefer firms with local headquarters — illustrating that the home bias anomaly occurs not only between national borders but also within national borders. Obstfeld and Rogoff (2000) delineates equity home bias as one of *The Six Major Puzzles in International Macroeconomics*. While there are proposed explanations such as the information immobility discussed in Van Nieuwerburgh and Veldkamp (2009), a satisfactory solution has not been widely accepted and the anomaly is still not well understood. For a more thorough discussion of the home bias anomaly and associated literature, see Lewis (1999) and Karolyi and Stulz (2003).

Instead of attempting to prove or disprove home bias, assumptions can be made in order to exemplify a few distinct asset allocations for domestic and foreign households, including a hypothetical asset allocation consistent with home bias and one contradictory to home bias, and each asset allocation can be calibrated. Assuming that the amount of domestic and foreign capital is distributed proportional to output and that the combined amount of capital owned domestically — which consists of domestic capital owned domestically and foreign capital owned domestically — will be similarly proportional to the domestic economy’s size relative to global output such that:

$$1 = k_1 + uk_2 \tag{3.24}$$

For the purpose of illustration, assume $u = 9$, and thus the small open domestic economy is 10% of global output, then a few pairs (k_1, k_2) can be chosen that satisfy (3.24) in order to illustrate three separate hypothetical scenarios:

$$k_1 = 0.10, k_2 = 0.10 \tag{3.25}$$

$$k_1 = 0.95, k_2 = \frac{1 - 0.95}{9} = 0.005 \tag{3.26}$$

$$k_1 = 0.01, k_2 = \frac{1 - 0.01}{9} = 0.11 \tag{3.27}$$

(3.25) is a baseline scenario with balanced capital ownership (such that $k_1 = k_2 = \frac{1}{1+u}$); (3.26) is a scenario illustrating relatively high domestic ownership of domestic capital and correspondingly low domestic ownership of foreign capital, consistent with a degree of home bias; and (3.27) illustrates relatively low domestic ownership of domestic capital which is compensated by higher domestic ownership of foreign capital — this could be conceptualized as a “hedging” economy where domestic agents seek to diversify away from exposure to the domestic economy via disproportionate allocations of capital to foreign assets, which is essentially the opposite of home bias and is used to illustrate the contrast. The following three tables illustrate the results of the assumptions in (3.25), (3.26), and (3.27) respectively:

Table 3.4: Calibration Summary — Open Economy ($k_1 = 0.1, k_2 = 0.1$)

σ	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
0	$33\bar{Y}$	$0.33\bar{Y}$	0	0	$0.58\bar{Y}$	0	0	0	0
0.25	$33\bar{Y}$	$-5.46\bar{Y}$	$-1.48\bar{Y}$	$-4.25\bar{Y}$	$5.22\bar{Y}$	$-1.81\bar{Y}$	$-5.80\bar{Y}$	$3.08\bar{Y}$	$10.45\bar{Y}$
0.5	$33\bar{Y}$	$-9.28\bar{Y}$	$-1.35\bar{Y}$	$-4.93\bar{Y}$	$7.21\bar{Y}$	$-1.35\bar{Y}$	$-5.53\bar{Y}$	$2.52\bar{Y}$	$11.35\bar{Y}$
0.75	$33\bar{Y}$	$-12.00\bar{Y}$	$-0.74\bar{Y}$	$-4.39\bar{Y}$	$8.12\bar{Y}$	$-0.63\bar{Y}$	$-4.25\bar{Y}$	$-1.26\bar{Y}$	$9.86\bar{Y}$
1.0	$33\bar{Y}$	$-14.03\bar{Y}$	0	$-3.45\bar{Y}$	$8.52\bar{Y}$	0	$-2.98\bar{Y}$	$0.01\bar{Y}$	$7.90\bar{Y}$
1.5	$33\bar{Y}$	$-16.89\bar{Y}$	$1.43\bar{Y}$	$-1.37\bar{Y}$	$8.69\bar{Y}$	$0.92\bar{Y}$	$-1.02\bar{Y}$	$-2.00\bar{Y}$	$4.27\bar{Y}$
2.0	$33\bar{Y}$	$-18.82\bar{Y}$	$2.63\bar{Y}$	$0.50\bar{Y}$	$8.57\bar{Y}$	$1.50\bar{Y}$	$0.26\bar{Y}$	$-3.42\bar{Y}$	$1.50\bar{Y}$
2.5	$33\bar{Y}$	$-20.23\bar{Y}$	$3.61\bar{Y}$	$2.07\bar{Y}$	$8.36\bar{Y}$	$1.88\bar{Y}$	$1.12\bar{Y}$	$-4.42\bar{Y}$	$-0.58\bar{Y}$
3.0	$33\bar{Y}$	$-21.31\bar{Y}$	$4.41\bar{Y}$	$3.37\bar{Y}$	$8.14\bar{Y}$	$2.14\bar{Y}$	$1.72\bar{Y}$	$-5.14\bar{Y}$	$-2.15\bar{Y}$
3.5	$33\bar{Y}$	$-22.17\bar{Y}$	$5.06\bar{Y}$	$4.45\bar{Y}$	$7.93\bar{Y}$	$2.33\bar{Y}$	$2.16\bar{Y}$	$-5.69\bar{Y}$	$-3.36\bar{Y}$

Table 3.5: Calibration Summary — Open Economy ($k_1 = 0.95, k_2 = 0.005$)

σ	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
0	$33\bar{Y}$	$0.33\bar{Y}$	0	0	$0.58\bar{Y}$	0	0	0	0
0.25	$33\bar{Y}$	$-7.81\bar{Y}$	$-2.93\bar{Y}$	$-4.09\bar{Y}$	$10.45\bar{Y}$	$-4.53\bar{Y}$	$-6.44\bar{Y}$	$7.28\bar{Y}$	$10.27\bar{Y}$
0.5	$33\bar{Y}$	$-13.17\bar{Y}$	$-2.68\bar{Y}$	$-3.82\bar{Y}$	$15.44\bar{Y}$	$-3.43\bar{Y}$	$-5.04\bar{Y}$	$6.06\bar{Y}$	$8.78\bar{Y}$
0.75	$33\bar{Y}$	$-16.98\bar{Y}$	$-1.46\bar{Y}$	$-2.22\bar{Y}$	$18.26\bar{Y}$	$-1.62\bar{Y}$	$-2.60\bar{Y}$	$3.08\bar{Y}$	$4.81\bar{Y}$
1.0	$33\bar{Y}$	$-19.84\bar{Y}$	0	$-0.27\bar{Y}$	$19.97\bar{Y}$	0	$-0.40\bar{Y}$	0	$0.67\bar{Y}$
1.5	$33\bar{Y}$	$-23.86\bar{Y}$	$2.82\bar{Y}$	$3.52\bar{Y}$	$21.81\bar{Y}$	$2.35\bar{Y}$	$2.82\bar{Y}$	$-5.15\bar{Y}$	$-6.29\bar{Y}$
2.0	$33\bar{Y}$	$-26.57\bar{Y}$	$5.19\bar{Y}$	$6.71\bar{Y}$	$22.69\bar{Y}$	$3.83\bar{Y}$	$4.85\bar{Y}$	$-8.92\bar{Y}$	$-11.41\bar{Y}$
2.5	$33\bar{Y}$	$-28.55\bar{Y}$	$7.12\bar{Y}$	$9.31\bar{Y}$	$23.14\bar{Y}$	$4.80\bar{Y}$	$6.19\bar{Y}$	$-11.69\bar{Y}$	$-15.17\bar{Y}$
3.0	$33\bar{Y}$	$-30.06\bar{Y}$	$8.69\bar{Y}$	$11.43\bar{Y}$	$23.39\bar{Y}$	$5.46\bar{Y}$	$7.10\bar{Y}$	$-13.77\bar{Y}$	$-18.01\bar{Y}$
3.5	$33\bar{Y}$	$-31.28\bar{Y}$	$9.99\bar{Y}$	$13.18\bar{Y}$	$23.53\bar{Y}$	$5.93\bar{Y}$	$7.75\bar{Y}$	$-15.38\bar{Y}$	$-20.21\bar{Y}$

Table 3.6: Calibration Summary — Open Economy ($k_1 = 0.01$, $k_2 = 0.11$)

σ	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
0	$33\bar{Y}$	$0.33\bar{Y}$	0	0	$0.58\bar{Y}$	0	0	0	0
0.25	$33\bar{Y}$	$-5.21\bar{Y}$	$-1.36\bar{Y}$	$-4.21\bar{Y}$	$4.67\bar{Y}$	$-1.63\bar{Y}$	$-5.49\bar{Y}$	$2.74\bar{Y}$	$10.24\bar{Y}$
0.5	$33\bar{Y}$	$-8.87\bar{Y}$	$-1.24\bar{Y}$	$-4.94\bar{Y}$	$6.34\bar{Y}$	$-1.22\bar{Y}$	$-5.23\bar{Y}$	$2.23\bar{Y}$	$11.27\bar{Y}$
0.75	$33\bar{Y}$	$-11.47\bar{Y}$	$-0.67\bar{Y}$	$-4.49\bar{Y}$	$7.04\bar{Y}$	$-0.58\bar{Y}$	$-4.00\bar{Y}$	$1.12\bar{Y}$	$9.98\bar{Y}$
1.0	$33\bar{Y}$	$-13.42\bar{Y}$	0	$-3.63\bar{Y}$	$7.30\bar{Y}$	0	$-2.79\bar{Y}$	$0.01\bar{Y}$	$8.20\bar{Y}$
1.5	$33\bar{Y}$	$-16.15\bar{Y}$	$1.31\bar{Y}$	$-1.70\bar{Y}$	$7.30\bar{Y}$	$0.84\bar{Y}$	$-0.92\bar{Y}$	$-1.75\bar{Y}$	$-4.87\bar{Y}$
2.0	$33\bar{Y}$	$-18.00\bar{Y}$	$2.41\bar{Y}$	$0.05\bar{Y}$	$7.07\bar{Y}$	$1.38\bar{Y}$	$0.31\bar{Y}$	$-2.97\bar{Y}$	$2.31\bar{Y}$
2.5	$33\bar{Y}$	$-19.35\bar{Y}$	$3.31\bar{Y}$	$1.52\bar{Y}$	$6.79\bar{Y}$	$1.74\bar{Y}$	$1.13\bar{Y}$	$-3.83\bar{Y}$	$0.40\bar{Y}$
3.0	$33\bar{Y}$	$-20.38\bar{Y}$	$4.04\bar{Y}$	$2.74\bar{Y}$	$6.52\bar{Y}$	$2.00\bar{Y}$	$1.71\bar{Y}$	$-4.45\bar{Y}$	$-1.05\bar{Y}$
3.5	$33\bar{Y}$	$-21.21\bar{Y}$	$4.64\bar{Y}$	$3.76\bar{Y}$	$6.27\bar{Y}$	$2.18\bar{Y}$	$2.13\bar{Y}$	$-4.91\bar{Y}$	$-2.18\bar{Y}$

Before interpreting the results, it is imperative to recall the assumptions made in Appendix C relating to the monetary policy shocks: foreign monetary policy illustrated in (C.3) is agnostic to domestic shocks because the domestic economy is small relative to the foreign economy, while domestic monetary policy illustrated in (C.8) takes into account foreign shocks:

$$i_t^f = \rho + \phi_\pi \pi_t^f + \phi_y \hat{Y}_t^f + v_t^f \quad (\text{C.3})$$

$$i_t^h = \rho + \phi_\pi^h \pi_t^h + \phi_y^h \hat{Y}_t^h + \phi_\pi^f \pi_t^f + \phi_y^f \hat{Y}_t^f + v_t^h \quad (\text{C.8})$$

The first four coefficients, b_0 to b_3 , can be compared directly to the results found in the closed economy in Section 3.1. Clearly, b_0 has not changed as it constitutes parameters that are identical in both the closed economy and open economy models — it is still the present value of a perpetuity where the coupon or dividend payment is the firm's share of steady state output in the economy, proportional to \bar{Y} , and is only a function of β which is unchanged. It should be noted that the scenario with relatively high domestic ownership of domestic capital in Table 3.5 closely resembles the approximated results for b_0 to b_3 in Section 3.1.

The coefficient b_1 still represents the first-order effect of the domestic monetary policy shock, now denoted v_t^h , on firm prices in the economy. The shock v_t^h is still introduced into the model via the domestic output gap, \hat{Y}_t^h , and is negatively related with the output gap while autoregressive in nature, thus positive shocks decrease output in a persistent fashion. The same phenomenon is observed where b_1 is decreasing in σ and positively related to the *EIS*. However, universally higher b_1 coefficients are observed — that is, less negative values — in the first and third specifications, Table 3.4 with balanced capital ownership and Table 3.6 with relatively low domestic capital ownership and high foreign capital ownership. Recall that in

Section 3.1 the negative b_1 coefficient was driven by distortions in the path of consumption caused by the monetary policy shock's impact on the periodic output gap. The same mechanism describes b_1 in the open economy context, but now the consumption path is also driven by foreign capital income which is proportional to domestic ownership of foreign capital, k_2 , and foreign capital income is independent of domestic shocks thus it dampens the impact of domestic shocks — the independent foreign capital income stream stabilizes domestic consumption. Since k_1 and k_2 are inversely related, the specification in Table 3.5 with high domestic capital ownership produces b_1 coefficients relatively close in value to the closed economy results in Section 3.1.

Despite the fact the open economy model has introduced foreign assets via foreign capital income, the same two competing effects are still simultaneously observed: the risk aversion component associated with the investment decision and a prudence component associated with the savings decision — there are still no risk-free assets and though there is an alternative foreign asset, domestic ownership of foreign assets is held at a constant level in each scenario (e.g. k_2). The open economy b_2 , the second-order effect of the domestic monetary policy shock, still exhibits the same zeros at $\sigma = 0$ and $\sigma = 1$, while b_3 , the second-order effect of the domestic policy shock's variance, exhibits a zero at $\sigma = 0$ but not at $\sigma = 1$.

For b_2 , the coefficient on $(v_t^h)^2$: at $\sigma = 0$ no risk aversion on the investment component and no prudence on the savings component are observed; from $0 < \sigma < 1$ the effect of risk aversion is observed, which is strongest at high values of k_1 (correspondingly low values for k_2) — in other words, there is a diversification benefit from higher levels of domestic ownership of foreign capital (higher k_2) which does not exist in Section 3.1; and the open economy b_2 is persistently lower (a *less positive* effect) for $\sigma > 1$ than the closed economy, especially for low k_1 . In Section 3.1, prudence becomes the dominating factor for $\sigma > 1$ which is similar here as b_2 is still increasing in σ but at a slower rate than in 3.1 — the strength of the prudence effect on b_2 appears to be lowered by high foreign capital ownership.

The second-order effect of the error variance of the domestic monetary policy shock, $(\sigma_\epsilon^h)^2$, as measured by b_3 , is universally lower for $\sigma > 0$ than in Section 3.1. The high domestic capital scenario produces b_3 coefficients relatively close to in 3.1 across all σ , while the high foreign capital scenario produces generally lower values of b_3 . The explanation for lower b_3 is likely a combination of the investment component, as foreign capital introduces diversification, and differences in prudence as foreign capital income reduces the demand for self-insurance via ownership of domestic assets. Consistently, relatively high b_3 is observed where k_1 is relatively high (*less* foreign capital income) and low b_3 where k_1 is low (*more* foreign capital income).

The remaining coefficients, b_4 through b_8 , are unique to the open economy model and thus cannot be directly compared to the results in Section 3.1. The coefficient b_4 measures the first-order effect of the foreign monetary policy shock denoted v_t^f . Notably, domestic assets respond differently to a foreign monetary policy

shock than to a domestic monetary policy shock, as illustrated by the stark contrast between universally positive values for b_4 and predominantly negative values of b_1 . In order to understand the mechanism for a seemingly anomalous result on the surface, one must trace the way foreign monetary policy shocks feed into the domestic system using relationships in Appendix C:

$$\hat{Y}_t^f = \psi_{yv}^f v_t^f = -(1 - \beta\rho_v)\Lambda^f v_t^f \quad (\text{C.4})$$

Similar to the domestic shock, the foreign shock is negatively related to the foreign output gap \hat{Y}_t^f , thus a positive shock implies a negative *foreign* output gap. Unlike the foreign policy function which is agnostic to domestic shocks, domestic policy actually responds to foreign shocks as denoted in (C.8):

$$i_t^h = \rho + \phi_\pi^h \pi_t^h + \phi_y^h \hat{Y}_t^h + \phi_\pi^f \pi_t^f + \phi_y^f \hat{Y}_t^f + v_t^h \quad (\text{C.8})$$

Thus, all else equal, *because of the fact domestic policy responds to foreign policy shocks, a positive foreign policy shock actually leads to a decrease in domestic interest rates*, which then feeds into (C.7):

$$\hat{Y}_t^h = -\frac{1}{\sigma}(i_t^h - E_t[\pi_{t+1}^h] - r_t^n) + E_t[\hat{Y}_{t+1}^h] \quad (\text{C.7})$$

The decrease in domestic interest rates in turn begets a *positive* domestic output gap, and as observed in Section 3.1, the periodic output gap leads to distortions in the stochastic discount factor. Instead of a negative output gap decreasing the stochastic discount factor as in 3.1 (the denominator, C_t , drops more than the numerator, C_{t+j}), b_4 illustrates the fact a foreign policy shock leads to a positive domestic output gap which in turn *increases* the stochastic discount factor in a manner where the magnitude of the increase is positively related to σ . However, since the economy is open there is also a secondary competing effect which comes from the fact that the path of domestic consumption is in part a function of foreign capital income — in isolation, the foreign policy shock which resulted in a negative foreign output gap, \hat{Y}_t^f , will result in persistently lower domestic foreign capital income due to the autoregressive nature of the shock, and this effect will partially offset the effect of the aforementioned positive domestic output gap in a manner which depends on the levels of domestic and foreign capital ownership — this effect in isolation is similar to the result for the first-order effect of the domestic policy shock measured by b_1 in Section 3.1, except for the fact the effect represents only a fraction of domestic income. Correspondingly, relatively large b_4 coefficients are observed which increase in σ for the scenario with low domestic ownership of foreign capital specified in Table 3.5, as the consumption impact from foreign capital income is relatively low. In the specifications in Table 3.4 and Table 3.6, it is clearer to see the competing effect of the consumption impact from foreign capital income, as the specification in 3.4 results in a b_4 coefficient that begins decreasing after $\sigma > 1.5$, and the specification in Table 3.6 results in a b_4 coefficient which peaks between $1 < \sigma < 1.5$ (b_4 is approximately equal at $\sigma = 1$ and $\sigma = 1.5$). Ultimately b_4 suggests that if the domestic policy function responds to negative

foreign output gaps with domestic monetary stimulus, the observation is a first-order effect of the foreign policy shock whereby domestic asset prices are *inflated* by the stimulus. For a positive foreign output gap and corresponding monetary tightening, domestic asset prices would deflate.

The second-order effect of the foreign monetary policy shock, $(v_t^f)^2$, behaves similarly to the domestic policy shock with competing effects from the investment decision and savings decision. For $\sigma = 0$ there is no risk aversion on the investment component and no prudence on the saving component, thus $b_5 = 0$; for $0 < \sigma < 1$, b_5 is negative but increasing due to increasing importance of prudence as σ increases; finally for $\sigma > 1$ prudence begins to dominate as b_5 is both positive for $\sigma > 1$ and increasing in σ . Notably b_5 is largest in magnitude for the scenario with low foreign capital ownership in Table 3.5 which can be thought of as higher risk premiums on domestic assets where the investment decision dominates from $0 < \sigma < 1$, and stronger prudence effects for $\sigma > 1$ where the savings decision dominates. The result is intuitive given the framework, as low foreign capital ownership implies less diversification of capital income which in turn is consistent with asset concentration leading to higher risk premiums or a higher risk aversion effect on the investment decision for domestic assets as well as a higher value of prudence as less diversified capital income increases the motive for agents to self-insure.

The second-order effect of the foreign policy shock's error variance, $(\sigma_\epsilon^f)^2$, is captured in the coefficient b_6 , and follows a similar pattern as the coefficient b_3 . At $\sigma = 0$, $b_6 = 0$ as there is no risk aversion on the investment component and no prudence on the saving component, and b_6 increases in σ becoming positive at higher values of σ . Similar to the findings with b_5 , b_6 is largest in magnitude for the scenario with low foreign capital ownership for similar reasons mentioned above.

The last two coefficients, b_7 and b_8 , measure interaction effects of the two policy functions. The second-order interaction effect between the domestic policy shock and the foreign policy shock is measured by b_7 , while b_8 captures the second-order effect of the covariance between the errors of the two policy functions. The fact that the domestic policy function has been set up to respond countercyclically to foreign shocks is likely playing a role in the interaction terms. Directionally, the behavior of b_7 is roughly the opposite pattern of the second-order effect of each policy shock individually, as $b_7 = 0$ for $\sigma = 0$, but then b_7 is positive but decreasing from $0 < \sigma < 1$, reaching $b_7 \approx 0$ at $\sigma = 1$ and then continues to decrease in σ for $\sigma > 1$. The result for b_8 is interesting as it suggests that positive correlations between the policy functions increase domestic firm prices for lower values of σ but decrease domestic firm prices for higher values of σ , or conversely that negative correlations between the policy functions increase domestic firm prices for higher values of σ but decrease domestic firm prices for lower values of σ . Notably, this effect varies based on the mix of domestic capital ownership as b_8 is negative at $\sigma = 1.5$ for the relatively high domestic ownership of domestic capital scenario in Table 3.5 whereas b_8 is first negative at $\sigma = 3$ for the relatively low domestic ownership of domestic capital scenario in Table 3.6.

4

Conclusion

The first-order observations from the calibration of the ad hoc open economy model are logical and consistent with modern macroeconomic thought. Positive monetary policy shocks defined as $v_t^h > 0$ are consistent with higher nominal interest rates, or monetary tightening, as expressed in (C.8) which all-else-equal cools the domestic economy by decreasing the domestic output gap and in turn leads to lower domestic firm prices — conversely negative policy shocks ($v_t^h < 0$) are stimulative, all-else-equal increase the domestic output gap, and in turn lead to higher domestic firm prices. The observed negative relationship between domestic firm prices and domestic nominal interest rates and the positive relationship between domestic firm prices and the domestic output gap both seem to reconcile with what is intuitively expected. Domestic inflation should have the same positive relationship with firm prices, similar to the domestic output gap, as domestic inflation feeds into the policy function in (C.8) in the same manner as the output gap. The negative relationship between domestic firm prices and the foreign output gap is intuitive once it is understood that the mechanism driving the process is the domestic policy function which reacts countercyclically to foreign output gaps and foreign inflation by design. The suggestion of this mechanism is that when faced with a global recession, a policymaker acting countercyclically in a small open economy will end up propping up domestic firm prices via such countercyclical policy intervention — an intriguing result that is worth additional investigation in the future given the actual policy interventions over the past decade, especially since the appreciation in firm prices is driven by household preferences as reflected in the stochastic discount factor instead of macroeconomic fundamentals such as higher productivity and higher dividends.

The competing risk aversion and prudence components picked up in second-order effects are harder to interpret but also seem to reconcile with macroeconomic theory, though a richer model would allow for cleaner interpretation. Households in both the closed economy and open economy models are observed to exhibit risk aversion which varies with σ and is counterbalanced by the fact second-order effects conflate prudence and risk aversion. Prudence also varies with σ , as the degree to which households seek self-insurance depends on their risk aversion and the volatility of expected future income. Adding alternative methods of savings might help tease out prudence more distinctly.

While the preceding section suggests that it is possible to combine New Keynesian macroeconomics with consumption based asset pricing, and approximate and calibrate the resulting model around a non-stochastic

steady state, even a basic model quickly runs into limitations of complexity in the derivation, estimation, and interpretation. As a proof-of-concept in embedding a New Keynesian model for the economy within a consumption based asset pricing model, there is clearly substantial room for improvement and further study across a variety of dimensions. Importantly, a second-order Taylor approximation (or potentially higher-order) allows for a “brute force” locally approximated solution regardless of how complex the model — this means a similar type of approximation is possible by following similar steps, even for a much richer model with significantly more terms. Potential dimensions for extension include the inclusion of another mechanism for intertemporal savings such as money or a bond market, or alternatively allowing levels of domestic and foreign capital to vary at the choice of individual agents making a dynamic asset allocation decision, adding habit formation to the utility function as studied in Campbell and Cochrane (1999), adding an explicit model for trade with components such as exchange rates and intermediate goods, or conducting a higher-order approximation and calibration. Further, more rigor could be added by calibrating based on empirical parameters of an actual economy instead of a hypothetical small open economy.

In the New Keynesian DSGE framework, it is imperative to accurately model decision-making at the individual agent level in order to provide rich microfoundations. Adding an alternative method of saving to complement investing in risky-assets, or firms, could help parse second-order effects between risk premiums and prudence. Habit formation could improve the richness of the utility function, and ultimately the results derived from the utility function. A higher-order approximation would provide results closer to the actual functional form that is being approximated. The trade-off of all of the aforementioned factors would be the exponentially increasing complexity of any such approximation and estimation. An explicit model for trade would similarly improve the foundations of the model while also allowing for a more realistic understanding of how shocks and policy transmit between open economies.

The concept of combining a New Keynesian DSGE model with conventional consumption-based asset pricing continues to tease a compelling possibility. Indeed, as the proof-of-concept the opportunity for extension is the most interesting part of the ad hoc open economy model — though the model is far from a complete model, it seems to have a solid foundation and should prove an important direction for further study. As illustrated in Section 3.2, the potentially complex relationships between domestic and foreign monetary policy could have profound impacts on asset prices in a New Keynesian paradigm. A lack of proper understanding of the potential asset market distortions that follow from monetary policy shocks could lead to unintended consequences in asset markets. Similarly, more developed asset pricing models built to be consistent with a New Keynesian paradigm could allow for complex ex-post analysis of monetary policy, monetary regime changes, and structural macroeconomic shifts.

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Appendix A

A Closed Economy New Keynesian Output Gap

In this Appendix we will derive the New Keynesian output gap in terms of the model's monetary policy shock and parameters in a closed economy.

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \hat{Y}_t \quad (\text{A.1})$$

$$\hat{Y}_t = -\frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n) + E_t[\hat{Y}_{t+1}] \quad (\text{A.2})$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{Y}_t + v_t \quad (\text{A.3})$$

Equation (A.1) is the New Keynesian Phillips Curve (NKPC), equation (A.2) is the dynamic IS equation (DIS), and equation (A.3) is a simple interest rate rule. Combining (A.1), (A.2), and (A.3) we can form a system of difference equations that represents the equilibrium conditions of our model, similar to what is found in Gali (2008):

$$\begin{bmatrix} \hat{Y}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t[\hat{Y}_{t+1}] \\ E_t[\pi_{t+1}] \end{bmatrix} + B_T(\hat{r}_t^n - v_t) \quad (\text{A.4})$$

Where $\hat{r}_t^n \equiv r_t^n - \rho$, and

$$A_T \equiv \Omega \begin{bmatrix} \rho & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}; B_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}; \Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$$

Here we will assume that both ϕ_π and ϕ_y are non-negative and the following holds:

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \quad (\text{A.5})$$

The above condition holds if and only if the solution to (A.4) is locally unique. This result follows the work of Blanchard and Kahn (1980) and Bullard and Mitra (2002).

Now we assume that the policy shock v_t follows an AR(1) process:

$$v_t = \rho_v v_{t-1} + \epsilon_t^v$$

We set $(\hat{r})_t^n = 0$ and guess that the solution to (A.4) takes the form of $\hat{Y}_t = \psi_{yv} v_t$ and $\pi_t = \psi_{\pi v} v_t$, where ψ_{yv} and $\psi_{\pi v}$ are coefficients we must determine. Using the NKPC from (A.1) and the guessed solutions:

$$\begin{aligned} \Rightarrow \psi_{\pi v} v_t &= \beta E_t[\pi_{t+1}] + \kappa \hat{Y}_t \\ \Rightarrow \psi_{\pi v} v_t &= \beta E_t[\psi_{\pi v} v_{t+1}] + \kappa \psi_{yv} v_t \\ \Rightarrow \psi_{\pi v} v_t &= \beta E_t[\psi_{\pi v} (\rho_v v_t + \epsilon_{t+1}^v)] + \kappa \psi_{yv} v_t \\ \Rightarrow \psi_{\pi v} v_t &= \beta \psi_{\pi v} (\rho_v v_t) + \kappa \psi_{yv} v_t \\ \Rightarrow \psi_{\pi v} &= \beta \psi_{\pi v} \rho_v + \kappa \psi_{yv} \\ \Rightarrow (1 - \beta \rho_v) \psi_{\pi v} &= \kappa \psi_{yv} \\ \Rightarrow \psi_{yv} &= \frac{(1 - \beta \rho_v)}{\kappa} \psi_{\pi v} \end{aligned} \quad (\text{A.6})$$

Using the interest rate rule in (A.3) and the DIS from (A.2) it follows:

$$\Rightarrow \hat{Y}_t = E_t[\hat{Y}_{t+1}] - \frac{1}{\sigma} \left((\rho + \phi_\pi \pi_t + \phi_y \hat{Y}_t + v_t) - E_t[\pi_{t+1}] - r_t^n \right)$$

We assume no technological shocks, i.e. $r_t^n - \rho = \sigma \psi_{ya}^n E_t[\Delta a_{t+1}] = 0$

$$\Rightarrow \hat{Y}_t = E_t[\hat{Y}_{t+1}] - \frac{1}{\sigma} \left(\phi_\pi \pi_t + \phi_y \hat{Y}_t + v_t - E_t[\pi_{t+1}] \right)$$

We can use the above results to solve for $\psi_{\pi v}$ by substituting in our guess solution $\hat{Y}_t = \psi_{yv} v_t$ and $\pi_t = \psi_{\pi v} v_t$

$$\begin{aligned} \Rightarrow \psi_{yv} v_t &= E_t[\psi_{yv} v_{t+1}] - \frac{1}{\sigma} (\phi_\pi \psi_{\pi v} v_t + \phi_y \psi_{yv} v_t + v_t - E_t[\psi_{\pi v} v_{t+1}]) \\ \Rightarrow \psi_{yv} v_t &= \psi_{yv} \rho v_t - \frac{1}{\sigma} (\phi_\pi \psi_{\pi v} v_t + \phi_y \psi_{yv} v_t + v_t - \psi_{\pi v} \rho v_t) \\ \Rightarrow \psi_{yv} v_t &= \frac{\sigma \rho v - \phi_y}{\sigma} \psi_{yv} v_t - \frac{\phi_\pi - \rho v}{\sigma} \psi_{\pi v} v_t - \frac{1}{\sigma} v_t \\ \Rightarrow \sigma \psi_{yv} &= (\sigma \rho v - \phi_y) \psi_{yv} - (\phi_\pi - \rho v) \psi_{\pi v} - 1 \end{aligned}$$

Now we can substitute in our result from (A.6):

$$\begin{aligned} \Rightarrow (\sigma(1 - \rho v) + \phi_y) \left(\frac{1 - \beta \rho v}{\kappa} \right) \psi_{\pi v} + (\phi_\pi - \rho v) \psi_{\pi v} &= -1 \\ \Rightarrow \frac{(1 - \beta \rho v)(\sigma(1 - \rho v) + \phi_y) + \kappa(\phi_\pi - \rho v)}{\kappa} \psi_{\pi v} &= -1 \\ \Rightarrow \psi_{\pi v} &= \frac{-\kappa}{(1 - \beta \rho v)(\sigma(1 - \rho v) + \phi_y) + \kappa(\phi_\pi - \rho v)} \end{aligned}$$

Letting $\Lambda_v \equiv \frac{1}{(1 - \beta \rho v)(\sigma(1 - \rho v) + \phi_y) + \kappa(\phi_\pi - \rho v)}$ it follows:

$$\psi_{\pi v} = -\kappa \Lambda_v \tag{A.7}$$

We can now substitute (A.7) into our solution in (A.6) to get an expression for ψ_{yv} solely in terms of our model's parameters:

$$\psi_{yv} = \left(\frac{(1 - \beta \rho v)}{\kappa} \right) (-\kappa \Lambda_v) = -(1 - \beta \rho v) \Lambda_v$$

Since we have now found values of our coefficients (ψ_{yv} and $\psi_{\pi v}$) that depend only on the models parameters, we can plug substitute our solutions for these coefficients back into our guess solutions for (A.4):

$$\begin{aligned} \hat{Y}_t &= \psi_{yv} v_t = -(1 - \beta \rho v) \Lambda_v v_t \\ \pi_t &= \psi_{\pi v} v_t = -\kappa \Lambda_v v_t \end{aligned}$$

It should be noted that as long as (A.5) holds, $\Lambda_v > 0$.

Appendix B

A Closed Economy Asset Pricing Model

In this Appendix we will derive solutions for various components in the closed economy asset pricing model we have developed.

$$\begin{aligned}
 f_t &\approx \alpha \bar{Y} \sum_{i=1}^{\infty} \beta^i E_t [1 + \sigma \hat{Y}_t + (1 - \sigma) \hat{Y}_{t+i} + \frac{1}{2} \sigma (\sigma - 1) \hat{Y}_t^2 \\
 &\quad - \frac{1}{2} \sigma (1 - \sigma) \hat{Y}_{t+i}^2 + \sigma (1 - \sigma) \hat{Y}_t \hat{Y}_{t+i}]
 \end{aligned} \tag{B.1}$$

In order to simplify the right hand side of (B.1), we must first simplify six sums:

$$i) \sum_{i=1}^{\infty} \beta^i E_t [1] = \frac{1}{1 - \beta} - 1 = \frac{1}{1 - \beta} - \left(\frac{1 - \beta}{1 - \beta} \right) = \frac{\beta}{1 - \beta} \tag{B.2}$$

$$\begin{aligned}
 ii) \sum_{i=1}^{\infty} \beta^i E_t [\sigma \hat{Y}_t] &= \sigma \sum_{i=1}^{\infty} \beta^i E_t [-(1 - \beta \rho_v) \Lambda_v v_t] \\
 \Rightarrow \sum_{i=1}^{\infty} \beta^i E_t [\sigma \hat{Y}_t] &= -\sigma (1 - \beta \rho_v) \Lambda_v v_t \left(\frac{\beta}{1 - \beta} \right)
 \end{aligned} \tag{B.3}$$

$$\begin{aligned}
 iii) \sum_{i=1}^{\infty} \beta^i E_t [(1 - \sigma) \hat{Y}_{t+i}] &= (1 - \sigma) \sum_{i=1}^{\infty} \beta^i E_t [-(1 - \beta \rho_v) \Lambda_v v_{t+i}] \\
 &= -(1 - \sigma) (1 - \beta \rho_v) \Lambda_v \sum_{i=1}^{\infty} \beta^i E_t [v_{t+i}] \\
 &= -(1 - \sigma) (1 - \beta \rho_v) \Lambda_v v_t \sum_{i=1}^{\infty} (\beta \rho_v)^i
 \end{aligned}$$

Since v_t is AR(1) $\Rightarrow E_t[v_{t+i}] = \rho_v^i v_t$

$$\Rightarrow \sum_{i=1}^{\infty} \beta^i E_t [(1 - \sigma) \hat{Y}_{t+i}] = -(1 - \sigma) (1 - \beta \rho_v) \Lambda_v v_t \left(\frac{\beta \rho_v}{1 - \beta \rho_v} \right) \tag{B.4}$$

$$\begin{aligned}
 iv) \sum_{i=1}^{\infty} \beta^i E_t \left[\frac{1}{2} \sigma (\sigma - 1) \hat{Y}_t^2 \right] &= \frac{1}{2} \sigma (\sigma - 1) \sum_{i=1}^{\infty} \beta^i E_t [(1 - \beta \rho_v)^2 \Lambda_v^2 v_t^2] \\
 \Rightarrow \sum_{i=1}^{\infty} \beta^i E_t \left[\frac{1}{2} \sigma (\sigma - 1) \hat{Y}_t^2 \right] &= \frac{1}{2} \sigma (\sigma - 1) (1 - \beta \rho_v)^2 \Lambda_v^2 v_t^2 \left(\frac{\beta}{1 - \beta} \right)
 \end{aligned} \tag{B.5}$$

$$v) \sum_{i=1}^{\infty} \beta^i E_t \left[-\frac{1}{2} \sigma (1 - \sigma) \hat{Y}_{t+i}^2 \right] = -\frac{1}{2} \sigma (1 - \sigma) (1 - \beta \rho_v)^2 \Lambda_v^2 \sum_{i=1}^{\infty} \beta^i E_t [v_{t+i}^2]$$

$$v_t = \rho_v v_{t-1} + \epsilon_t^v$$

$$\Rightarrow v_{t+j} = \rho_v v_{t+j-1} + \epsilon_{t+j}$$

$$v_{t+j} = \rho_v (\rho_v v_{t+j-2} + \epsilon_{t+j-1}) + \epsilon_{t+j}$$

Iterating forward j periods:

$$v_{t+j} = \rho_v^j v_t + \rho_v^{j-1} \epsilon_{t+1} + \rho_v^{j-2} \epsilon_{t+2} + \dots + \rho_v^{j-(j-1)} \epsilon_{t+j-1} + \epsilon_{t+j}$$

Note that $E_t[\epsilon_{t+i} \epsilon_{t+j}] = 0 \quad \forall \quad i \neq j$ this follows because ϵ is assumed to be i.i.d. and $E_t[v_t \epsilon_{t+i}] = 0 \quad \forall \quad i > 0$ because $E_t[\epsilon_{t+1}] = 0 \quad \forall \quad i > 0$. The meaningful result is that all cross products will equal zero when we take the expectation of $(v_{t+j})^2$.

$$\begin{aligned} \Rightarrow E_t[(v_{t+j})^2] &= E_t[(\rho_v^j v_t)^2 + (\rho_v^{j-1} \epsilon_{t+1})^2 + (\rho_v^{j-2} \epsilon_{t+2})^2 + \dots \\ &\quad \dots + (\rho_v \epsilon_{t+j-1})^2 + \epsilon_{t+j}^2] \\ \Rightarrow E_t[(v_{t+j})^2] &= \rho_v^{2j} E_t[v_t^2] + \rho_v^{2(j-1)} E_t[\epsilon_{t+1}^2] + \rho_v^{2(j-2)} E_t[\epsilon_{t+2}^2] + \dots \\ &\quad \dots + \rho_v^2 E_t[\epsilon_{t+j-1}^2] + E_t[\epsilon_{t+j}^2] \end{aligned}$$

Here we can use the fact that $E_t[v_t^2] = v_t^2$ since it is the current shock and can be measured. As well, we note that $E_t[\epsilon_{t+i}^2] = \text{Var}_t[\epsilon_{t+i}] = \sigma_\epsilon^2 \quad \forall \quad i > 0$ which follows from the fact that $E_t[\epsilon_{t+i}] = 0$

$$\Rightarrow E_t[(v_{t+j})^2] = \rho_v^{2j} v_t^2 + \sigma_\epsilon^2 \sum_{i=1}^j \rho_v^{2(j-i)} = \rho_v^{2j} v_t^2 + \sigma_\epsilon^2 \sum_{i=0}^{j-1} \rho_v^{2i}$$

We can use the generalized solution: $\sum_{n=0}^p m^n = \frac{m^o - m^{p+1}}{1 - m}$ for $m \neq 1$

$$\begin{aligned} \Rightarrow \sum_{i=0}^{j-1} \rho_v^{2i} &= \frac{1 - \rho_v^{2j}}{1 - \rho_v^2} \\ \Rightarrow E_t[(v_{t+j})^2] &= \rho_v^{2j} v_t^2 + \sigma_\epsilon^2 \left(\frac{1 - \rho_v^{2j}}{1 - \rho_v^2} \right) \\ \Rightarrow \sum_{i=1}^{\infty} \beta^j E_t[v_{t+j}^2] &= \sum_{i=1}^{\infty} \beta^j E_t[\rho_v^{2j} v_t^2 + \sigma_\epsilon^2 \left(\frac{1 - \rho_v^{2j}}{1 - \rho_v^2} \right)] \\ &= v_t^2 \sum_{i=1}^{\infty} (\beta \rho_v^2)^j + \sigma_\epsilon^2 \left(\frac{1}{1 - \rho_v^2} \right) \left(\sum_{i=1}^{\infty} \beta^j - \sum_{i=1}^{\infty} (\beta \rho_v^2)^j \right) \\ &= v_t^2 \left(\frac{\rho_v^2 \beta}{1 - \rho_v^2 \beta} \right) + \sigma_\epsilon^2 \left(\frac{1}{1 - \rho_v^2} \right) \left[\frac{\beta}{1 - \beta} - \frac{\rho_v^2 \beta}{1 - \rho_v^2 \beta} \right] \\ &= v_t^2 \left(\frac{\rho_v^2 \beta}{1 - \rho_v^2 \beta} \right) + \sigma_\epsilon^2 \left(\frac{1}{1 - \rho_v^2} \right) \left[\frac{\beta - \rho_v^2 \beta^2 - \rho_v^2 \beta + \rho_v^2 \beta^2}{(1 - \beta)(1 - \rho_v^2 \beta)} \right] \\ &= v_t^2 \left(\frac{\rho_v^2 \beta}{1 - \rho_v^2 \beta} \right) + \sigma_\epsilon^2 \left(\frac{1}{1 - \rho_v^2} \right) \left[\frac{\beta(1 - \rho_v^2)}{(1 - \beta)(1 - \rho_v^2 \beta)} \right] \\ \Rightarrow \sum_{i=1}^{\infty} \beta^j E_t[v_{t+j}^2] &= v_t^2 \left(\frac{\rho_v^2 \beta}{1 - \rho_v^2 \beta} \right) + \sigma_\epsilon^2 \frac{\beta}{(1 - \beta)(1 - \rho_v^2 \beta)} \\ \Rightarrow \sum_{i=1}^{\infty} \beta^j E_t[-\frac{1}{2} \sigma(1 - \sigma) \hat{Y}_{t+j}^2] & \\ &= -\frac{1}{2} \sigma(1 - \sigma)(1 - \beta \rho_v)^2 \Lambda_v^2 \left[v_t^2 \left(\frac{\rho_v^2 \beta}{1 - \rho_v^2 \beta} \right) + \sigma_\epsilon^2 \frac{\beta}{(1 - \beta)(1 - \rho_v^2 \beta)} \right] \end{aligned} \tag{B.6}$$

$$\begin{aligned}
vi) \sum_{i=1}^{\infty} \beta^j E_t[\sigma(1-\sigma)\hat{Y}_t\hat{Y}_{t+j}] &= \sigma(1-\sigma) \sum_{i=1}^{\infty} \beta^j E_t[(1-\beta\rho_v)^2\Lambda_v^2 v_t v_{t+j}] \\
&= \sigma(1-\sigma)(1-\beta\rho_v)^2\Lambda_v^2 v_t^2 \sum_{i=1}^{\infty} (\beta\rho_v)^j \\
\Rightarrow \sum_{i=1}^{\infty} \beta^j E_t[\sigma(1-\sigma)\hat{Y}_t\hat{Y}_{t+j}] &= \sigma(1-\sigma)(1-\beta\rho_v)^2\Lambda_v^2 v_t^2 \left(\frac{\beta\rho_v}{1-\beta\rho_v} \right) \tag{B.7}
\end{aligned}$$

Note that v_t is AR(1) $\Rightarrow E_t[v_t v_{t+j}] = v_t E_t[v_{t+j}] = v_t^2 \rho_v^j$. Now we can return to the second-order approximation for real firm prices:

$$\begin{aligned}
f_t \approx \alpha\bar{Y} \sum_{i=1}^{\infty} \beta^j E_t[1 + \sigma\hat{Y}_t + (1-\sigma)\hat{Y}_{t+j} + \frac{1}{2}\sigma(\sigma-1)\hat{Y}_t^2 \\
- \frac{1}{2}\sigma(1-\sigma)\hat{Y}_{t+j}^2 + \sigma(1-\sigma)\hat{Y}_t\hat{Y}_{t+j}] \tag{B.8}
\end{aligned}$$

Using our results from (B.2), (B.3), (B.4), (B.5), (B.6), and (B.7):

$$\begin{aligned}
\Rightarrow f_t = \alpha\bar{Y} \left[\frac{\beta}{1-\beta} - \sigma(1-\beta\rho_v)\Lambda_v v_t \left(\frac{\beta}{1-\beta} \right) - (1-\sigma)(1-\beta\rho_v)\Lambda_v v_t \left(\frac{\beta\rho_v}{1-\beta\rho_v} \right) \right. \\
+ \frac{1}{2}\sigma(\sigma-1)(1-\beta\rho_v)^2\Lambda_v^2 v_t^2 \left(\frac{\beta}{1-\beta} \right) \\
- \frac{1}{2}\sigma(1-\sigma)(1-\beta\rho_v)^2\Lambda_v^2 \left[v_t^2 \left(\frac{\rho_v^2\beta}{1-\rho_v^2\beta} \right) + \sigma_\epsilon^2 \frac{\beta}{(1-\beta)(1-\rho_v^2\beta)} \right] \\
\left. + \sigma(1-\sigma)(1-\beta\rho_v)^2\Lambda_v^2 v_t \left(\frac{\beta\rho_v}{1-\beta\rho_v} \right) \right] \tag{B.9}
\end{aligned}$$

Appendix C

An Open Economy New Keynesian Output Gap

Here we take an approach similar to Appendix A and extend it for the open economy. Now, there will be two separate systems: the foreign system and the domestic system. First, we will analyze the foreign system which will behave exactly like the closed economy in Appendix A because we assume the domestic system is small relative to the foreign system and thus foreign policy is agnostic to domestic shocks.

$$\pi_t^f = \beta E_t[\pi_{t+1}^f] + \kappa \hat{Y}_t^f \quad (\text{C.1})$$

$$\hat{Y}_t^f = -\frac{1}{\sigma} (i_t^f - E_t[\pi_{t+1}^f] - r_t^n) + E_t[\hat{Y}_{t+1}^f] \quad (\text{C.2})$$

$$i_t^f = \rho + \phi_\pi \pi_t^f + \phi_y \hat{Y}_t^f + v_t^f \quad (\text{C.3})$$

$$v_t^f = \rho_v v_{t-1}^f + \epsilon_t^{vf}$$

Aside from the superscripts specifying that the variables we are dealing with are the foreign variables, we notice that these equations are identical to Appendix A, and thus we will get a nearly identical result by duplicating the same procedure. Letting $\Lambda^f \equiv \frac{1}{(1-\beta\rho_v)(\sigma(1-\rho_v)+\phi_y)+\kappa(\phi_\pi-\rho_v)}$ it follows:

$$\hat{Y}_t^f = \psi_{yv}^f v_t^f = -(1-\beta\rho_v)\Lambda^f v_t^f \quad (\text{C.4})$$

$$\pi_t^f = \psi_{\pi v}^f v_t^f = -\kappa\Lambda^f v_t^f \quad (\text{C.5})$$

Again, the solution is identical aside from the superscripts, and the restrictions are the same as Appendix A. $\Lambda^f > 0$ as long as the Blanchard-Kahn condition is satisfied. Now we are prepared to analyze the domestic system.

$$\pi_t^h = \beta E_t[\pi_{t+1}^h] + \kappa \hat{Y}_t^h \quad (\text{C.6})$$

$$\hat{Y}_t^h = -\frac{1}{\sigma} (i_t^h - E_t[\pi_{t+1}^h] - r_t^n) + E_t[\hat{Y}_{t+1}^h] \quad (\text{C.7})$$

$$i_t^h = \rho + \phi_\pi^h \pi_t^h + \phi_y^h \hat{Y}_t^h + \phi_\pi^f \pi_t^f + \phi_y^f \hat{Y}_t^f + v_t^h \quad (\text{C.8})$$

$$v_t^h = \rho_v v_{t-1}^h + \epsilon_t^{vh}$$

The important distinction comes in (C.8) where the domestic policy function now reacts not only to domestic inflation and the domestic output gap, but also foreign inflation and the foreign output gap. The domestic policy shock v_t^h is seen directly, and the foreign policy shock v_t^f will be part of the equation through its effect on foreign output and inflation.

Similar to Appendix A we will proceed by forming a guess solution. The difference now is that we will partition our guess solution to consider both domestic and foreign policy shocks as follows:

$$\hat{Y}_t^h = \psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f \quad (\text{C.9})$$

$$\pi_t^h = \psi_{\pi v}^h v_t^h + \psi_{\pi v}^f v_t^f \quad (\text{C.10})$$

Combining (C.10) with (C.6) and then substituting in (C.9):

$$\begin{aligned}
&\implies \psi_{\pi v}^h v_t^h + \psi_{\pi v}^f v_t^f = \beta E_t[\pi_{t+1}^h] + \kappa \hat{Y}_t^h \\
&\implies \psi_{\pi v}^h v_t^h + \psi_{\pi v}^f v_t^f = \beta E_t[\psi_{\pi v}^h v_{t+1}^h + \psi_{\pi v}^f v_{t+1}^f] + \kappa(\psi_{y v}^h v_t^h + \psi_{y v}^f v_t^f) \\
&\implies \psi_{\pi v}^h v_t^h + \psi_{\pi v}^f v_t^f = \beta[\psi_{\pi v}^h \rho_v v_t^h + \psi_{\pi v}^f \rho_v v_t^f] + \kappa(\psi_{y v}^h v_t^h + \psi_{y v}^f v_t^f) \\
&\implies (1 - \beta \rho_v)[\psi_{\pi v}^h v_t^h + \psi_{\pi v}^f v_t^f] = \kappa(\psi_{y v}^h v_t^h + \psi_{y v}^f v_t^f) \\
&\implies [\psi_{\pi v}^h v_t^h + \psi_{\pi v}^f v_t^f] = \frac{\kappa}{(1 - \beta \rho_v)}[\psi_{y v}^h v_t^h + \psi_{y v}^f v_t^f]
\end{aligned}$$

Here we suggest the following solution for the pair of coefficients:

$$\psi_{y v}^h = \frac{(1 - \beta \rho_v)}{\kappa} \psi_{\pi v}^h \quad (\text{C.11})$$

$$\psi_{y v}^f = \frac{(1 - \beta \rho_v)}{\kappa} \psi_{\pi v}^f \quad (\text{C.12})$$

Next we will substitute (C.8) into (C.7) and rearrange as follows:

$$\hat{Y}_t^h = E_t[\hat{Y}_{t+1}^h] - \frac{1}{\sigma}(\rho + \phi_{\pi}^h \pi_t^h + \phi_y^h \hat{Y}_t^h + \phi_{\pi}^f \pi_t^f + \phi_y^f \hat{Y}_t^f + v_t^h - E_t[\pi_{t+1}^h] - r_t^n)$$

We assume no technological shocks, i.e. $r_t^n - \rho = \sigma \psi_{y a}^n E_t[\Delta a_{t+1}] = 0$

$$\implies \hat{Y}_t^h = E_t[\hat{Y}_{t+1}^h] - \frac{1}{\sigma}(\phi_{\pi}^h \pi_t^h + \phi_y^h \hat{Y}_t^h + \phi_{\pi}^f \pi_t^f + \phi_y^f \hat{Y}_t^f + v_t^h - E_t[\pi_{t+1}^h])$$

Using (C.9) and (C.10) to substitute for the domestic output gap and inflation, and (C.4) and (C.5) to substitute for the foreign output gap and inflation:

$$\begin{aligned}
&\implies \psi_{y v}^h v_t^h + \psi_{y v}^f v_t^f = E_t[\psi_{y v}^h v_{t+1}^h + \psi_{y v}^f v_{t+1}^f] \\
&\quad - \frac{1}{\sigma} \left(\phi_{\pi}^h (\psi_{\pi v}^h v_t^h + \psi_{\pi v}^f v_t^f) + \phi_y^h (\psi_{y v}^h v_t^h + \psi_{y v}^f v_t^f) \right. \\
&\quad + \phi_{\pi}^f (-\kappa \Lambda^f v_t^f) + \phi_y^f (-(1 - \beta \rho_v) \Lambda^f v_t^f) \\
&\quad \left. + v_t^h - E_t[\psi_{\pi v}^h v_{t+1}^h + \psi_{\pi v}^f v_{t+1}^f] \right)
\end{aligned}$$

We can collect terms with respect to the domestic policy shock v_t^h and the foreign policy shock v_t^f as follows:

$$\begin{aligned}
&\implies \sigma(\psi_{y v}^h v_t^h + \psi_{y v}^f v_t^f) = (\sigma \rho_v \psi_{y v}^h - \phi_{\pi}^h \psi_{\pi v}^h - \phi_y^h \psi_{y v}^h - 1 + \rho_v \psi_{\pi v}^h) v_t^h \\
&\quad + (\sigma \rho_v \psi_{y v}^f - \phi_{\pi}^f \psi_{\pi v}^f - \phi_y^f \psi_{y v}^f + \phi_{\pi}^f \kappa \Lambda^f + \phi_y^f (1 - \beta \rho_v) \Lambda^f + \rho_v \psi_{\pi v}^f) v_t^f
\end{aligned}$$

First we will focus on the portion of the equation related to the domestic policy shock:

$$\begin{aligned}
&\sigma \psi_{y v}^h v_t^h = (\sigma \rho_v \psi_{y v}^h - \phi_{\pi}^h \psi_{\pi v}^h - \phi_y^h \psi_{y v}^h - 1 + \rho_v \psi_{\pi v}^h) v_t^h \\
&\implies \sigma \psi_{y v}^h = (\sigma \rho_v - \phi_y^h) \psi_{y v}^h - (\phi_{\pi}^h - \rho_v) \psi_{\pi v}^h - 1 \\
&\implies (\sigma(1 - \rho_v) + \phi_y^h) \psi_{y v}^h + (\phi_{\pi}^h - \rho_v) \psi_{\pi v}^h = -1 \\
&\implies (\sigma(1 - \rho_v) + \phi_y^h) \left(\frac{1 - \beta \rho_v}{\kappa} \right) \psi_{\pi v}^h + (\phi_{\pi}^h - \rho_v) \psi_{\pi v}^h = -1
\end{aligned}$$

$$\implies \psi_{\pi v}^h = \frac{-\kappa}{(1 - \beta\rho_v)(\sigma(1 - \rho_v) + \phi_y^h) + \kappa(\phi_\pi^h - \rho_v)} = -\kappa\Lambda^h \quad (\text{C.13})$$

Where $\Lambda^h = \frac{1}{(1 - \beta\rho_v)(\sigma(1 - \rho_v) + \phi_y^h) + \kappa(\phi_\pi^h - \rho_v)}$

We now proceed to the portion of the equation related to the foreign policy shock:

$$\begin{aligned} \sigma\psi_{yv}^f v_t^f &= (\sigma\rho_v\psi_{yv}^f - \phi_\pi^h\psi_{\pi v}^f - \phi_y^h\psi_{yv}^f + \phi_\pi^f\kappa\Lambda^f + \phi_y^f(1 - \beta\rho_v)\Lambda^f + \rho_v\psi_{\pi v}^f)v_t^f \\ \implies \sigma\psi_{yv}^f &= (\sigma\rho_v\psi_{yv}^f - \phi_\pi^h\psi_{\pi v}^f - \phi_y^h\psi_{yv}^f + \phi_\pi^f\kappa\Lambda^f + \phi_y^f(1 - \beta\rho_v)\Lambda^f + \rho_v\psi_{\pi v}^f) \\ \implies (\sigma(1 - \rho_v) + \phi_y^h)\psi_{yv}^f + (\phi_\pi^h - \rho_v)\psi_{\pi v}^f &= \phi_\pi^f\kappa\Lambda^f + \phi_y^f(1 - \beta\rho_v)\Lambda^f \\ \implies (\sigma(1 - \rho_v) + \phi_y^h)\psi_{yv}^f + (\phi_\pi^h - \rho_v)\psi_{\pi v}^f &= \Lambda^f(\phi_\pi^f\kappa + \phi_y^f(1 - \beta\rho_v)) \\ \implies \psi_{\pi v}^f \left[(\sigma(1 - \rho_v) + \phi_y^h) \left(\frac{1 - \beta\rho_v}{\kappa} \right) + \phi_\pi^h - \rho_v \right] &= \Lambda^f(\phi_\pi^f\kappa + \phi_y^f(1 - \beta\rho_v)) \\ \implies \psi_{\pi v}^f &= \frac{\Lambda^f(\phi_\pi^f\kappa + \phi_y^f(1 - \beta\rho_v))\kappa}{(1 - \beta\rho_v)(\sigma(1 - \rho_v) + \phi_y^h) + \kappa(\phi_\pi^h - \rho_v)} \\ \implies \psi_{\pi v}^f &= \Lambda^h\Lambda^f(\phi_\pi^f\kappa + \phi_y^f(1 - \beta\rho_v))\kappa \end{aligned} \quad (\text{C.14})$$

We can now substitute our result from (C.13) into (C.11):

$$\psi_{yv}^h = \frac{(1 - \beta\rho_v)}{\kappa}\psi_{\pi v}^h = \left(\frac{1 - \beta\rho_v}{\kappa} \right) (-\kappa\Lambda^h) = -(1 - \beta\rho_v)\Lambda^h \quad (\text{C.15})$$

And similarly we can substitute out result from (C.14) into (C.12):

$$\psi_{yv}^f = \frac{(1 - \beta\rho_v)}{\kappa}\psi_{\pi v}^f = (1 - \beta\rho_v)\Lambda^h\Lambda^f(\phi_\pi^f\kappa + \phi_y^f(1 - \beta\rho_v)) \quad (\text{C.16})$$

And finally we can use (C.15) and (C.16) to solve our original guess solutions:

$$\hat{Y}_t^h = \psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f$$

$$\hat{Y}_t^h = -(1 - \beta\rho_v)\Lambda^h v_t^h + (1 - \beta\rho_v)\Lambda^h\Lambda^f(\phi_\pi^f\kappa + \phi_y^f(1 - \beta\rho_v))v_t^f \quad (\text{C.17})$$

$$\pi_t^h = \psi_{\pi v}^h v_t^h + \psi_{\pi v}^f v_t^f$$

$$\pi_t^h = -\kappa\Lambda^h v_t^h + \Lambda^h\Lambda^f(\phi_\pi^f\kappa + \phi_y^f(1 - \beta\rho_v))v_t^f \quad (\text{C.18})$$

Appendix D

Open Economy Linearization

Linearizing this function will require evaluating several terms at the steady state. In order to show calculations in a straightforward manner we will calculate each term individually before combining.

$$\left(\frac{\mu_1 Y_t^h + \mu_2 Y_t^f}{\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f} \right)^\sigma Y_{t+j}^h = F(Y_t^h, Y_{t+k}^h, Y_t^f, Y_{t+j}^f) = F(\mathbf{Y})$$

$$F(\bar{Y}) = \left(\frac{\mu_1 \bar{Y} + \mu_2 u \bar{Y}}{\mu_1 \bar{Y} + \mu_2 u \bar{Y}} \right)^\sigma \bar{Y} = \bar{Y} \quad (\text{D.1})$$

$$\frac{\partial F}{\partial Y_t^h} = \frac{\sigma \mu_1 (\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-1}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^\sigma} Y_{t+j}^h$$

$$\Rightarrow (Y_t^h - \bar{Y}) \frac{\partial F}{\partial Y_t^h} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} = \frac{\sigma \mu_1}{\mu_1 + \mu_2 u} (Y_t^h - \bar{Y}) \quad (\text{D.2})$$

$$\frac{\partial F}{\partial Y_t^f} = \frac{\sigma \mu_2 (\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-1}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^\sigma} Y_{t+j}^h$$

$$\Rightarrow (Y_t^f - u\bar{Y}) \frac{\partial F}{\partial Y_t^f} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} = \frac{\sigma \mu_2}{\mu_1 + \mu_2 u} (Y_t^f - u\bar{Y}) \quad (\text{D.3})$$

$$\frac{\partial F}{\partial Y_{t+j}^h} = \left(\frac{\mu_1 Y_t^h + \mu_2 Y_t^f}{\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f} \right)^\sigma + \frac{(\mu_1 Y_t^h + \mu_2 Y_t^f)^\sigma}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{\sigma+1}} (-\sigma) (\mu_1) Y_{t+j}^h$$

$$\Rightarrow (Y_{t+j}^h - \bar{Y}) \frac{\partial F}{\partial Y_{t+j}^h} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} = \left(\frac{\mu_1 + \mu_2 u - \sigma \mu_1}{\mu_1 + \mu_2 u} \right) (Y_{t+j}^h - \bar{Y}) \quad (\text{D.4})$$

$$\frac{\partial F}{\partial Y_{t+j}^f} = (\mu_1 Y_t^h + \mu_2 Y_t^f)^\sigma (-\sigma) (\mu_2) (\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{-\sigma-1} Y_{t+j}^h$$

$$\Rightarrow (Y_{t+j}^f - u\bar{Y}) \frac{\partial F}{\partial Y_{t+j}^f} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} = \frac{-\sigma \mu_2}{\mu_1 + \mu_2 u} (Y_{t+j}^f - u\bar{Y}) \quad (\text{D.5})$$

$$\frac{\partial F}{\partial Y_t^h \partial Y_t^h} = \frac{\sigma(\sigma-1) \mu_1^2 (\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-2}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^\sigma} Y_{t+j}^h$$

$$\Rightarrow \frac{1}{2} (Y_t^h - \bar{Y})^2 \frac{\partial F}{\partial Y_t^h \partial Y_t^h} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}}$$

$$= \frac{1}{2} \left(\frac{\sigma(\sigma-1) \mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})^2 \quad (\text{D.6})$$

$$\begin{aligned}
\frac{\partial F}{\partial Y_t^f \partial Y_t^f} &= \frac{\sigma(\sigma-1)\mu_2^2(\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-2}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^\sigma} Y_{t+j}^h \\
&\Rightarrow \frac{1}{2}(Y_t^f - u\bar{Y})^2 \frac{\partial F}{\partial Y_t^f \partial Y_t^f} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} \\
&= \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_2^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^f - u\bar{Y})^2
\end{aligned} \tag{D.7}$$

$$\begin{aligned}
\frac{\partial F}{\partial Y_{t+j}^h \partial Y_{t+j}^h} &= (\mu_1 Y_t^h + \mu_2 Y_t^f)^\sigma (-\sigma)(\mu_1)(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{-\sigma-1} \\
&\quad + (\mu_1 Y_t^h + \mu_2 Y_t^f)^\sigma (-\sigma)(\mu_1)(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{-\sigma-1} \\
&\quad + \frac{(\mu_1 Y_t^h + \mu_2 Y_t^f)^\sigma}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{\sigma+2}} (\sigma)(\sigma+1)(\mu_1)^2 Y_{t+j}^h \\
&\Rightarrow \frac{1}{2}(Y_{t+j}^h - \bar{Y})^2 \frac{\partial F}{\partial Y_{t+j}^h \partial Y_{t+j}^h} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} \\
&= \frac{1}{2} \left(\frac{\sigma\mu_1(\sigma\mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_{t+j}^h - \bar{Y})^2
\end{aligned} \tag{D.8}$$

$$\begin{aligned}
\frac{\partial F}{\partial Y_{t+j}^f \partial Y_{t+j}^f} &= \frac{(\mu_1 Y_t^h + \mu_2 Y_t^f)^\sigma}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{\sigma+2}} (\sigma)(\sigma+1)(\mu_2)^2 Y_{t+j}^h \\
&\Rightarrow \frac{1}{2}(Y_{t+j}^f - u\bar{Y})^2 \frac{\partial F}{\partial Y_{t+j}^f \partial Y_{t+j}^f} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} \\
&= \frac{1}{2} \left(\frac{\sigma(\sigma+1)\mu_2^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_{t+j}^f - u\bar{Y})^2
\end{aligned} \tag{D.9}$$

$$\begin{aligned}
\frac{\partial F}{\partial Y_t^h \partial Y_t^f} &= \frac{\sigma(\sigma-1)\mu_1\mu_2(\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-2}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^\sigma} Y_{t+j}^h \\
&\Rightarrow \frac{1}{2}(Y_t^h - \bar{Y})(Y_t^f - u\bar{Y}) \frac{\partial F}{\partial Y_t^h \partial Y_t^f} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} \\
&= \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1\mu_2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})(Y_t^f - u\bar{Y})
\end{aligned} \tag{D.10}$$

$$\begin{aligned}
\frac{\partial F}{\partial Y_t^h \partial Y_{t+j}^h} &= \frac{\sigma\mu_1(\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-1}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^\sigma} - \sigma^2 \mu_1^2 \frac{(\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-1}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{\sigma+1}} Y_{t+j}^h \\
&\Rightarrow \frac{1}{2}(Y_t^h - \bar{Y})(Y_{t+j}^h - \bar{Y}) \frac{\partial F}{\partial Y_t^h \partial Y_{t+j}^h} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} \\
&= \frac{1}{2} \left(\frac{\sigma\mu_1(\mu_1 + \mu_2 u - \sigma\mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})(Y_{t+j}^h - \bar{Y})
\end{aligned} \tag{D.11}$$

$$\begin{aligned}
\frac{\partial F}{\partial Y_t^h \partial Y_{t+j}^f} &= -\sigma^2 \mu_1 \mu_2 \frac{(\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-1}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{\sigma+1}} Y_{t+j}^h \\
&\Rightarrow \frac{1}{2}(Y_t^h - \bar{Y})(Y_{t+j}^f - u\bar{Y}) \frac{\partial F}{\partial Y_t^h \partial Y_{t+j}^f} \Big|_{Y_t^h=Y_{t+j}^h=\bar{Y}, Y_t^f=Y_{t+j}^f=u\bar{Y}} \\
&= -\frac{1}{2} \left(\frac{\sigma^2 \mu_1 \mu_2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})(Y_{t+j}^f - u\bar{Y})
\end{aligned} \tag{D.12}$$

$$\begin{aligned}
\frac{\partial F}{\partial Y_t^f \partial Y_{t+j}^h} &= \frac{\sigma \mu_2 (\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-1}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^\sigma} - \frac{\sigma \mu_1 \mu_2 (\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-1}}{\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{\sigma+1}} Y_{t+j}^h \\
&\Rightarrow \frac{1}{2} (Y_t^f - u \bar{Y}) (Y_{t+j}^h - \bar{Y}) \frac{\partial F}{\partial Y_t^f \partial Y_{t+j}^h} \Big|_{Y_t^h = Y_{t+j}^h = \bar{Y}, Y_t^f = Y_{t+j}^f = u \bar{Y}} \\
&= \frac{1}{2} \left(\frac{\sigma \mu_2 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^f - u \bar{Y}) (Y_{t+j}^h - \bar{Y})
\end{aligned} \tag{D.13}$$

$$\begin{aligned}
\frac{\partial F}{\partial Y_t^f \partial Y_{t+j}^f} &= -\sigma^2 \mu_2^2 \frac{(\mu_1 Y_t^h + \mu_2 Y_t^f)^{\sigma-1}}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{\sigma+1}} Y_{t+j}^h \\
&\Rightarrow \frac{1}{2} (Y_t^f - u \bar{Y}) (Y_{t+j}^f - u \bar{Y}) \frac{\partial F}{\partial Y_t^f \partial Y_{t+j}^f} \Big|_{Y_t^h = Y_{t+j}^h = \bar{Y}, Y_t^f = Y_{t+j}^f = u \bar{Y}} \\
&= -\frac{1}{2} \left(\frac{\sigma^2 \mu_2^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^f - u \bar{Y}) (Y_{t+j}^f - u \bar{Y})
\end{aligned} \tag{D.14}$$

$$\begin{aligned}
\frac{\partial F}{\partial Y_{t+j}^h \partial Y_{t+j}^f} &= -\sigma \mu_2 \frac{(\mu_1 Y_t^h + \mu_2 Y_t^f)^\sigma}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{\sigma+1}} \\
&\quad + \frac{\sigma(\sigma+1) \mu_1 \mu_2 (\mu_1 Y_t^h + \mu_2 Y_t^f)^\sigma}{(\mu_1 Y_{t+j}^h + \mu_2 Y_{t+j}^f)^{\sigma+2}} Y_{t+j}^h \\
&\Rightarrow \frac{1}{2} (Y_{t+j}^h - \bar{Y}) (Y_{t+j}^f - u \bar{Y}) \frac{\partial F}{\partial Y_{t+j}^h \partial Y_{t+j}^f} \Big|_{Y_t^h = Y_{t+j}^h = \bar{Y}, Y_t^f = Y_{t+j}^f = u \bar{Y}} \\
&= \frac{1}{2} \left(\frac{\sigma \mu_2 (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_{t+j}^h - \bar{Y}) (Y_{t+j}^f - u \bar{Y})
\end{aligned} \tag{D.15}$$

Now we can combine our elements from (D.1) to (D.15) to form:

$$\begin{aligned}
F(\mathbf{Y}) \approx & \bar{Y} + \frac{\sigma\mu_1}{\mu_1 + \mu_2 u} (Y_t^h - \bar{Y}) + \frac{\sigma\mu_2}{\mu_1 + \mu_2 u} (Y_t^f - u\bar{Y}) \\
& + \left(\frac{\mu_1 + \mu_2 u - \sigma\mu_1}{\mu_1 + \mu_2 u} \right) (Y_{t+j}^h - \bar{Y}) - \frac{\sigma\mu_2}{\mu_1 + \mu_2 u} (Y_{t+j}^f - u\bar{Y}) \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_2^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^f - u\bar{Y})^2 \\
& + \frac{1}{2} \left(\frac{\sigma\mu_1(\sigma\mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_{t+j}^h - \bar{Y})^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma+1)\mu_2^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_{t+j}^f - u\bar{Y})^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1\mu_2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})(Y_t^f - u\bar{Y}) \\
& + \frac{1}{2} \left(\frac{\sigma\mu_1(\mu_1 + \mu_2 u - \sigma\mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})(Y_{t+j}^h - \bar{Y}) \\
& - \frac{1}{2} \left(\frac{\sigma^2\mu_1\mu_2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^h - \bar{Y})(Y_{t+j}^f - u\bar{Y}) \\
& + \frac{1}{2} \left(\frac{\sigma\mu_2(\mu_1 + \mu_2 u - \sigma\mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^f - u\bar{Y})(Y_{t+j}^h - \bar{Y}) \\
& - \frac{1}{2} \left(\frac{\sigma^2\mu_2^2}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_t^f - u\bar{Y})(Y_{t+j}^f - u\bar{Y}) \\
& + \frac{1}{2} \left(\frac{\sigma\mu_2(\sigma\mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \bar{Y}^{-1} (Y_{t+j}^h - \bar{Y})(Y_{t+j}^f - u\bar{Y})
\end{aligned} \tag{D.16}$$

Where \bar{Y} is the steady state value of domestic output we can express the domestic output gap as $\hat{Y}_t^h = \frac{Y_t^h - \bar{Y}}{\bar{Y}}$ and where $u\bar{Y}$ is the steady state value of foreign output we can express the foreign output gap as $\hat{Y}_t^f = \frac{Y_t^f - u\bar{Y}}{u\bar{Y}}$

$$\begin{aligned}
f_t \approx & \alpha\bar{Y} \sum_{j=1}^{\infty} \beta^j E_t \left[1 + \frac{\sigma\mu_1}{\mu_1 + \mu_2 u} \hat{Y}_t^h + \frac{\sigma\mu_2 u}{\mu_1 + \mu_2 u} \hat{Y}_t^f \right. \\
& + \left(\frac{\mu_1 + \mu_2 u - \sigma\mu_1}{\mu_1 + \mu_2 u} \right) \hat{Y}_{t+j}^h - \frac{\sigma\mu_2 u}{\mu_1 + \mu_2 u} \hat{Y}_{t+j}^f + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_t^h)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_t^f)^2 + \frac{1}{2} \left(\frac{\sigma\mu_1(\sigma\mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_{t+j}^h)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma+1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_{t+j}^f)^2 + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1\mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^h \hat{Y}_t^f \\
& + \frac{1}{2} \left(\frac{\sigma\mu_1(\mu_1 + \mu_2 u - \sigma\mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^h \hat{Y}_{t+j}^h - \frac{1}{2} \left(\frac{\sigma^2\mu_1\mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^h \hat{Y}_{t+j}^f \\
& + \frac{1}{2} \left(\frac{\sigma\mu_2(\mu_1 + \mu_2 u - \sigma\mu_1)u}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^f \hat{Y}_{t+j}^h - \frac{1}{2} \left(\frac{\sigma^2\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^f \hat{Y}_{t+j}^f \\
& \left. + \frac{1}{2} \left(\frac{\sigma\mu_2 u(\sigma\mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_{t+j}^h \hat{Y}_{t+j}^f \right]
\end{aligned} \tag{D.17}$$

Appendix E

Open Economy Solution (Approximation)

Where \bar{Y} is the steady state value of domestic output we can express the domestic output gap as $\hat{Y}_t^h = \frac{Y_t^h - \bar{Y}}{\bar{Y}}$ and where $u\bar{Y}$ is the steady state value of foreign output we can express the foreign output gap as $\hat{Y}_t^f = \frac{Y_t^f - u\bar{Y}}{(u\bar{Y})}$ as shown in Appendix D, we will start with the equation derived in (D.17):

$$\begin{aligned}
 f_t \approx & \alpha \bar{Y} \sum_{j=1}^{\infty} \beta^j E_t \left[1 + \frac{\sigma \mu_1}{\mu_1 + \mu_2 u} \hat{Y}_t^h + \frac{\sigma \mu_2 u}{\mu_1 + \mu_2 u} \hat{Y}_t^f \right. \\
 & + \left(\frac{\mu_1 + \mu_2 u - \sigma \mu_1}{\mu_1 + \mu_2 u} \right) \hat{Y}_{t+j}^h - \frac{\sigma \mu_2 u}{\mu_1 + \mu_2 u} \hat{Y}_{t+j}^f + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_t^h)^2 \\
 & + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_t^f)^2 + \frac{1}{2} \left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_{t+j}^h)^2 \\
 & + \frac{1}{2} \left(\frac{\sigma(\sigma+1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\hat{Y}_{t+j}^f)^2 + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^h \hat{Y}_t^f \\
 & + \frac{1}{2} \left(\frac{\sigma \mu_1 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^h \hat{Y}_{t+j}^h - \frac{1}{2} \left(\frac{\sigma^2 \mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^h \hat{Y}_{t+j}^f \\
 & + \frac{1}{2} \left(\frac{\sigma \mu_2 (\mu_1 + \mu_2 u - \sigma \mu_1) u}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^f \hat{Y}_{t+j}^h - \frac{1}{2} \left(\frac{\sigma^2 \mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_t^f \hat{Y}_{t+j}^f \\
 & \left. + \frac{1}{2} \left(\frac{\sigma \mu_2 u (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \hat{Y}_{t+j}^h \hat{Y}_{t+j}^f \right]
 \end{aligned} \tag{E.1}$$

Now we make use of our results from Appendix C:

$$\hat{Y}_t^f = \psi_{yv} v_t^f = -(1 - \beta \rho_v) \Lambda^f v_t^f \tag{C.4}$$

$$\hat{Y}_t^h = \psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f = -(1 - \beta \rho_v) \Lambda^h v_t^h + (1 - \beta \rho_v) \Lambda^h \Lambda^f (\phi_{\pi}^f \kappa + \phi_y^f (1 - \beta \rho_v)) v_t^f \tag{C.17}$$

Using the above equations (C.4) and (C.17) for the foreign and domestic output gaps respectively, we can rewrite (E.1) in terms of the policy shocks:

$$\begin{aligned}
f_t \approx & \alpha \bar{Y} \sum_{j=1}^{\infty} \beta^j E_t \left[1 + \frac{\sigma \mu_1}{\mu_1 + \mu_2 u} (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f) + \frac{\sigma \mu_2 u}{\mu_1 + \mu_2 u} \psi_{yv} v_t^f \right. \\
& + \left(\frac{\mu_1 + \mu_2 u - \sigma \mu_1}{\mu_1 + \mu_2 u} \right) (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f) - \frac{\sigma \mu_2 u}{\mu_1 + \mu_2 u} \psi_{yv} v_{t+j}^f \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv} v_t^f)^2 \\
& + \frac{1}{2} \left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma+1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv} v_{t+j}^f)^2 \\
& + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} v_t^f (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f) \\
& + \frac{1}{2} \left(\frac{\sigma \mu_1 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f) (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f) \\
& - \frac{1}{2} \left(\frac{\sigma^2 \mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_t^h + \psi_{yv}^f v_t^f) \psi_{yv} v_{t+j}^f \\
& + \frac{1}{2} \left(\frac{\sigma \mu_2 (\mu_1 + \mu_2 u - \sigma \mu_1) u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} v_t^f (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f) \\
& - \frac{1}{2} \left(\frac{\sigma^2 \mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv})^2 v_t^f v_{t+j}^f \\
& \left. + \frac{1}{2} \left(\frac{\sigma \mu_2 u (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h v_{t+j}^h + \psi_{yv}^f v_{t+j}^f) \psi_{yv} v_{t+j}^f \right]
\end{aligned} \tag{E.2}$$

We can rewrite the above in terms of first-order and second-order terms for each shock, where the z_i below represent coefficients that are a collection of parameters from grouping terms in (E.2):

$$\begin{aligned}
f_t \approx & \alpha \bar{Y} \sum_{j=1}^{\infty} \beta^j E_t \left[1 + z_1 v_t^h + z_2 v_t^f + z_3 v_{t+j}^h + z_4 v_{t+j}^f + z_5 (v_t^h)^2 \right. \\
& + z_6 (v_t^f)^2 + z_7 (v_{t+j}^h)^2 + z_8 (v_{t+j}^f)^2 + z_9 v_t^h v_t^f + z_{10} v_{t+j}^h v_{t+j}^f \\
& \left. + z_{11} v_t^f v_{t+j}^h + z_{12} v_t^f v_{t+j}^f + z_{13} v_t^h v_{t+j}^h + z_{14} v_t^h v_{t+j}^f \right]
\end{aligned} \tag{E.3}$$

Where:

$$z_1 = \left(\frac{\sigma\mu_1}{\mu_1 + \mu_2 u} \right) \psi_{yv}^h$$

$$z_2 = \left(\frac{\sigma\mu_1}{\mu_1 + \mu_2 u} \right) \psi_{yv}^f + \left(\frac{\sigma\mu_2 u}{\mu_1 + \mu_2 u} \right) \psi_{yv}$$

$$z_3 = \left(\frac{\mu_1 + \mu_2 u - \sigma\mu_1}{\mu_1 + \mu_2 u} \right) \psi_{yv}^h$$

$$z_4 = \left(\frac{\mu_1 + \mu_2 u + \sigma\mu_1}{\mu_1 + \mu_2 u} \right) \psi_{yv}^f - \left(\frac{\sigma\mu_2 u}{\mu_1 + \mu_2 u} \right) \psi_{yv}$$

$$z_5 = \frac{1}{2} \left(\frac{\sigma(\sigma - 1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h)^2$$

$$z_6 = \frac{1}{2} \left(\frac{\sigma(\sigma - 1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^f)^2 + \frac{1}{2} \left(\frac{\sigma(\sigma - 1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv})^2 + \frac{1}{2} \left(\frac{\sigma(\sigma - 1)\mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^f$$

$$z_7 = \frac{1}{2} \left(\frac{\sigma\mu_1(\sigma\mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h)^2$$

$$z_8 = \frac{1}{2} \left(\frac{\sigma\mu_1(\sigma\mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^f)^2 + \frac{1}{2} \left(\frac{\sigma(\sigma + 1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv})^2 + \frac{1}{2} \left(\frac{\sigma\mu_2 u(\sigma\mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^f \psi_{yv}$$

$$z_9 = \left(\frac{\sigma(\sigma - 1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^h \psi_{yv}^f + \frac{1}{2} \left(\frac{\sigma(\sigma - 1)\mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^h$$

$$z_{10} = \left(\frac{\sigma\mu_1(\sigma\mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^h \psi_{yv}^f + \frac{1}{2} \left(\frac{\sigma\mu_2 u(\sigma\mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^h$$

$$z_{11} = \frac{1}{2} \left(\frac{\sigma\mu_1(\mu_1 + \mu_2 u - \sigma\mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^h \psi_{yv}^f + \frac{1}{2} \left(\frac{\sigma\mu_2(\mu_1 + \mu_2 u - \sigma\mu_1)u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^h$$

$$z_{12} = \frac{1}{2} \left(\frac{\sigma\mu_1(\mu_1 + \mu_2 u - \sigma\mu_1) - \sigma^2 \mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^f)^2 + \frac{1}{2} \left(\frac{\sigma\mu_2 u(\mu_1 - 2\sigma\mu_1 + \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^f$$

$$z_{13} = \frac{1}{2} \left(\frac{\sigma\mu_1(\mu_1 + \mu_2 u - \sigma\mu_1)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h)^2$$

$$z_{14} = \frac{1}{2} \left(\frac{\sigma\mu_1(\mu_1 + \mu_2 u - \sigma\mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^h \psi_{yv}^f - \frac{1}{2} \left(\frac{\sigma^2 \mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^f$$

We can simplify (E.3) as illustrated below on a term-by-term basis:

$$\sum_{j=1}^{\infty} \beta^j E_t[1] = \frac{\beta}{1-\beta} \quad (\text{E.4})$$

$$\sum_{j=1}^{\infty} \beta^j E_t[z_1 v_t^h] = \frac{\beta}{1-\beta} z_1 v_t^h \quad (\text{E.5})$$

$$\sum_{j=1}^{\infty} \beta^j E_t[z_2 v_t^f] = \frac{\beta}{1-\beta} z_2 v_t^f \quad (\text{E.6})$$

$$\sum_{j=1}^{\infty} \beta^j E_t[z_3 v_{t+j}^h] = \frac{\beta \rho_v}{1-\beta \rho_v} z_3 v_t^h \quad (\text{E.7})$$

$$\sum_{j=1}^{\infty} \beta^j E_t[z_4 v_{t+j}^f] = \frac{\beta \rho_v}{1-\beta \rho_v} z_4 v_t^f \quad (\text{E.8})$$

$$\sum_{j=1}^{\infty} \beta^j E_t[z_5 (v_t^h)^2] = \frac{\beta}{1-\beta} z_5 (v_t^h)^2 \quad (\text{E.9})$$

$$\sum_{j=1}^{\infty} \beta^j E_t[z_6 (v_t^f)^2] = \frac{\beta}{1-\beta} z_6 (v_t^f)^2 \quad (\text{E.10})$$

$$\sum_{j=1}^{\infty} \beta^j E_t[z_7 (v_{t+j}^h)^2] = z_7 \left[(v_t^h)^2 \left(\frac{\beta \rho_v^2}{1-\beta \rho_v^2} \right) + (\sigma_\epsilon^h)^2 \left(\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} \right) \right] \quad (\text{E.11})$$

$$\sum_{j=1}^{\infty} \beta^j E_t[z_8 (v_{t+j}^f)^2] = z_8 \left[(v_t^f)^2 \left(\frac{\beta \rho_v^2}{1-\beta \rho_v^2} \right) + (\sigma_\epsilon^f)^2 \left(\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} \right) \right] \quad (\text{E.12})$$

$$\sum_{j=1}^{\infty} \beta^j E_t[z_9 v_t^h v_t^f] = \frac{\beta}{1-\beta} z_9 v_t^h v_t^f \quad (\text{E.13})$$

For the following we will make use of the fact $E_t[v_t \epsilon_{t+i}] = 0$ for $i > 0$; $E_t[\epsilon_{t+i}^h \epsilon_{t+j}^f] = 0 \quad \forall \quad i \neq j$; and $E_t[\epsilon_{t+i}^h \epsilon_{t+i}^f] = \text{cov}(\epsilon_{t+i}^h, \epsilon_{t+i}^f) = \sigma_{\epsilon^h, \epsilon^f} \quad \forall \quad i > 0$. Additionally, many steps are analogous to the methods used in Appendix B.

$$\begin{aligned}
& \sum_{j=1}^{\infty} \beta^j E_t [z_{10} v_{t+j}^h v_{t+j}^f] \tag{E.14} \\
&= z_{10} \sum_{j=1}^{\infty} \beta^j E_t \left[(\rho_v^j v_t^h + \rho_v^{j-1} \epsilon_{t+1}^h + \dots + \rho_v^{j-(j-1)} \epsilon_{t+j-1}^h + \epsilon_{t+j}^h) \right. \\
&\quad \left. (\rho_v^j v_t^f + \rho_v^{j-1} \epsilon_{t+1}^f + \dots + \rho_v^{j-(j-1)} \epsilon_{t+j-1}^f + \epsilon_{t+j}^f) \right] \\
&= z_{10} \sum_{j=1}^{\infty} \beta^j E_t \left[\rho_v^{2j} v_t^h v_t^f + \rho_v^{2(j-1)} \epsilon_{t+1}^h \epsilon_{t+1}^f + \dots + \epsilon_{t+j}^h \epsilon_{t+j}^f \right] \\
&= z_{10} \sum_{j=1}^{\infty} \beta^j \left[\rho_v^{2j} v_t^h v_t^f + \text{cov}(\epsilon_{t+i}^h, \epsilon_{t+i}^f) \sum_{i=0}^{j-1} \rho_v^{2i} \right] \\
&= z_{10} \sum_{j=1}^{\infty} \beta^j \left[\rho_v^{2j} v_t^h v_t^f + \text{cov}(\epsilon_{t+i}^h, \epsilon_{t+i}^f) \left(\frac{1 - \rho_v^{2j}}{1 - \rho_v^2} \right) \right] \\
&= z_{10} \left[\left(\frac{\rho_v^2 \beta}{1 - \rho_v^2 \beta} \right) v_t^h v_t^f + \frac{\beta}{(1 - \beta)(1 - \rho_v^2 \beta)} \text{cov}(\epsilon_{t+i}^h, \epsilon_{t+i}^f) \right] \\
&= z_{10} \left[\left(\frac{\rho_v^2 \beta}{1 - \rho_v^2 \beta} \right) v_t^h v_t^f + \frac{\beta}{(1 - \beta)(1 - \rho_v^2 \beta)} \sigma_{\epsilon^h, \epsilon^f} \right]
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^{\infty} \beta^j E_t [z_{11} v_t^f v_{t+j}^h] = z_{11} v_t^f \sum_{j=1}^{\infty} \beta^j E_t [v_{t+j}^h] \tag{E.15} \\
&= z_{11} \left(\frac{\beta \rho_v}{1 - \beta \rho_v} \right) v_t^f v_t^h
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^{\infty} \beta^j E_t [z_{12} v_t^f v_{t+j}^f] = z_{12} v_t^f \sum_{j=1}^{\infty} \beta^j E_t [v_{t+j}^f] \tag{E.16} \\
&= z_{12} \left(\frac{\beta \rho_v}{1 - \beta \rho_v} \right) (v_t^f)^2
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^{\infty} \beta^j E_t [z_{13} v_t^h v_{t+j}^h] = z_{13} v_t^h \sum_{j=1}^{\infty} \beta^j E_t [v_{t+j}^h] \tag{E.17} \\
&= z_{13} \left(\frac{\beta \rho_v}{1 - \beta \rho_v} \right) (v_t^h)^2
\end{aligned}$$

$$\begin{aligned}
& \sum_{j=1}^{\infty} \beta^j E_t [z_{14} v_t^h v_{t+j}^f] = z_{14} v_t^h \sum_{j=1}^{\infty} \beta^j E_t [v_{t+j}^f] \tag{E.18} \\
&= z_{14} \left(\frac{\beta \rho_v}{1 - \beta \rho_v} \right) v_t^h v_t^f
\end{aligned}$$

By moving through the sum in (E.3) as depicted in (E.4) to (E.18), we come to:

$$\begin{aligned}
\Rightarrow f_t \approx & \alpha \bar{Y} \left[\frac{\beta}{1-\beta} + \frac{\beta}{1-\beta} z_1 v_t^h + \frac{\beta}{1-\beta} z_2 v_t^f \right. \\
& + \frac{\beta \rho_v}{1-\beta \rho_v} z_3 v_t^h + \frac{\beta \rho_v}{1-\beta \rho_v} z_4 v_t^f + \frac{\beta}{1-\beta} z_5 (v_t^h)^2 + \frac{\beta}{1-\beta} z_6 (v_t^f)^2 \\
& + z_7 \left[(v_t^h)^2 \left(\frac{\beta \rho_v^2}{1-\beta \rho_v^2} \right) + (\sigma_\epsilon^h)^2 \left(\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} \right) \right] \\
& + z_8 \left[(v_t^f)^2 \left(\frac{\beta \rho_v^2}{1-\beta \rho_v^2} \right) + (\sigma_\epsilon^f)^2 \left(\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} \right) \right] + \frac{\beta}{1-\beta} z_9 v_t^h v_t^f \\
& + z_{10} \left[\left(\frac{\beta \rho_v^2}{1-\beta \rho_v^2} \right) v_t^h v_t^f + \frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} \sigma_{\epsilon h, \epsilon f} \right] + \left(\frac{\beta \rho_v}{1-\beta \rho_v} \right) z_{11} v_t^f v_t^h \\
& + \left(\frac{\beta \rho_v}{1-\beta \rho_v} \right) z_{12} (v_t^f)^2 + \left(\frac{\beta \rho_v}{1-\beta \rho_v} \right) z_{13} (v_t^h)^2 + \left(\frac{\beta \rho_v}{1-\beta \rho_v} \right) z_{14} v_t^f v_t^h \left. \right]
\end{aligned} \tag{E.19}$$

By grouping terms above with respect to first-order and second-order effects of the various shocks, including interactions, we can write our open economy result in the same format as the closed economy result, but with significantly more terms given the influence of foreign monetary policy shocks and corresponding interactions. As well, most of the b_i terms are now much more complex with significantly more parameters embedded.

$$\begin{aligned}
\Rightarrow f_t \approx & b_0 + b_1 v_t^h + b_2 (v_t^h)^2 + b_3 (\sigma_\epsilon^h)^2 + b_4 v_t^f + b_5 (v_t^f)^2 \\
& + b_6 (\sigma_\epsilon^f)^2 + b_7 v_t^h v_t^f + b_8 \sigma_{\epsilon h, \epsilon f}
\end{aligned} \tag{E.20}$$

With the various b_i as follows:

$$b_0 = \alpha \left(\frac{\beta}{1-\beta} \right) \bar{Y}$$

$$b_1 = \alpha \left(\frac{\beta}{1-\beta} z_1 + \frac{\beta \rho_v}{1-\beta \rho_v} z_3 \right) \bar{Y} = \alpha \left(\frac{\beta}{1-\beta} \left(\frac{\sigma \mu_1}{\mu_1 + \mu_2 u} \right) \psi_{yv}^h + \frac{\beta \rho_v}{1-\beta \rho_v} \left(\frac{\mu_1 + \mu_2 u - \sigma \mu_1}{\mu_1 + \mu_2 u} \right) \psi_{yv}^h \right) \bar{Y}$$

$$\begin{aligned}
b_2 &= \alpha \left(\frac{\beta}{1-\beta} z_5 + \frac{\beta \rho_v^2}{1-\beta \rho_v^2} z_7 + \frac{\beta \rho_v}{1-\beta \rho_v} z_{13} \right) \bar{Y} \\
&= \alpha \left(\frac{\beta}{2(1-\beta)} \left[\left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h)^2 \right] + \frac{\beta \rho_v^2}{2(1-\beta \rho_v^2)} \left[\left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h)^2 \right] \right. \\
&\quad \left. + \frac{\beta \rho_v}{2(1-\beta \rho_v)} \left[\left(\frac{\sigma \mu_1 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h)^2 \right] \right) \bar{Y}
\end{aligned}$$

$$b_3 = \alpha \left[\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} z_7 \right] \bar{Y} = \alpha \left(\frac{\beta}{2(1-\beta)(1-\rho_v^2 \beta)} \right) \left[\left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^h)^2 \right] \bar{Y}$$

$$\begin{aligned}
b_4 &= \alpha \left(\frac{\beta}{1-\beta} z_2 + \frac{\beta \rho_v}{1-\beta \rho_v} z_4 \right) \bar{Y} \\
&= \alpha \left(\frac{\beta}{1-\beta} \left(\frac{\sigma \mu_1}{\mu_1 + \mu_2 u} \right) \psi_{yv}^f + \frac{\beta \rho_v}{1-\beta \rho_v} \left(\frac{\mu_1 + \mu_2 u + \sigma \mu_1}{\mu_1 + \mu_2 u} \right) \psi_{yv}^f \right. \\
&\quad \left. + \left(\frac{\beta}{1-\beta} - \frac{\beta \rho_v}{1-\beta \rho_v} \right) \left(\frac{\sigma \mu_2 u}{\mu_1 + \mu_2 u} \right) \psi_{yv} \right) \bar{Y}
\end{aligned}$$

$$\begin{aligned}
b_5 &= \alpha \left(\frac{\beta}{1-\beta} z_6 + \frac{\beta \rho_v^2}{1-\beta \rho_v^2} z_8 + \frac{\beta \rho_v}{1-\beta \rho_v} z_{12} \right) \bar{Y} \\
&= \alpha \left(\frac{\beta}{2(1-\beta)} \left[\left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^f)^2 + \left(\frac{\sigma(\sigma-1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv})^2 + \left(\frac{\sigma(\sigma-1)\mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^f \right] \right. \\
&\quad \left. + \frac{\beta \rho_v^2}{2(1-\beta \rho_v^2)} \left[\left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^f)^2 + \left(\frac{\sigma(\sigma+1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} + \left(\frac{\sigma \mu_2 u (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^f \psi_{yv} \right] \right. \\
&\quad \left. + \frac{\beta \rho_v}{2(1-\beta \rho_v)} \left[\left(\frac{\sigma \mu_1 (\mu_1 + \mu_2 u - \sigma \mu_1) - \sigma^2 \mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^f)^2 + \left(\frac{\sigma \mu_2 u (\mu_1 - 2\sigma \mu_1 + \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^f \right] \right) \bar{Y}
\end{aligned}$$

$$\begin{aligned}
b_6 &= \alpha \left(\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} z_8 \right) \bar{Y} \\
&= \alpha \left(\frac{\beta}{2(1-\beta)(1-\rho_v^2 \beta)} \right) \left[\left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) (\psi_{yv}^f)^2 + \left(\frac{\sigma(\sigma+1)\mu_2^2 u^2}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \right. \\
&\quad \left. + \left(\frac{\sigma \mu_2 u (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^f \psi_{yv} \right] \bar{Y}
\end{aligned}$$

$$\begin{aligned}
b_7 &= \alpha \left(\frac{\beta}{1-\beta} z_9 + \frac{\beta \rho_v^2}{1-\beta \rho_v^2} z_{10} + \frac{\beta \rho_v}{1-\beta \rho_v} z_{11} + \frac{\beta \rho_v}{1-\beta \rho_v} z_{14} \right) \bar{Y} \\
&= \alpha \left(\frac{\beta}{1-\beta} \left[\left(\frac{\sigma(\sigma-1)\mu_1^2}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^h \psi_{yv}^f + \frac{1}{2} \left(\frac{\sigma(\sigma-1)\mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^h \right] \right. \\
&\quad \left. + \frac{\beta \rho_v^2}{1-\beta \rho_v^2} \left[\left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^h \psi_{yv}^f + \frac{1}{2} \left(\frac{\sigma \mu_2 u (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^h \right] \right. \\
&\quad \left. + \frac{\beta \rho_v}{2(1-\beta \rho_v)} \left[\left(\frac{\sigma \mu_1 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^h \psi_{yv}^f + \left(\frac{\sigma \mu_2 (\mu_1 + \mu_2 u - \sigma \mu_1) u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^h \right] \right. \\
&\quad \left. + \frac{\beta \rho_v}{2(1-\beta \rho_v)} \left[\left(\frac{\sigma \mu_1 (\mu_1 + \mu_2 u - \sigma \mu_1)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^h \psi_{yv}^f - \left(\frac{\sigma^2 \mu_1 \mu_2 u}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^f \right] \right) \bar{Y}
\end{aligned}$$

$$\begin{aligned}
b_8 &= \alpha \left(\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} z_{10} \right) \bar{Y} \\
&= \alpha \left(\frac{\beta}{(1-\beta)(1-\rho_v^2 \beta)} \right) \left[\left(\frac{\sigma \mu_1 (\sigma \mu_1 - \mu_1 - 2\mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv}^h \psi_{yv}^f + \frac{1}{2} \left(\frac{\sigma \mu_2 u (\sigma \mu_1 - \mu_2 u)}{(\mu_1 + \mu_2 u)^2} \right) \psi_{yv} \psi_{yv}^h \right] \bar{Y}
\end{aligned}$$

Appendix F

A Special Case Closed Economy Model

In this Appendix we will derive the results for the special case of a closed economy where $\sigma = 1$ and the utility function takes the form of log utility with $U(C_t, N_t) = \ln(C_t) + \ln(N_t)$ and $u'(C_t) = \frac{1}{C_t}$. Here we can begin with our original definition of firm price in (3.1):

$$f_t = \sum_{j=1}^{\infty} \beta^j E_t \left[\frac{u'(C_{t+j})}{u'(C_t)} d_{t+j} \right] \quad (\text{F.1})$$

$$u'(C_t) = \frac{1}{C_t}$$

$$\Rightarrow \frac{u'(C_{t+j})}{u'(C_t)} = \frac{C_t}{C_{t+j}}$$

$$\Rightarrow f_t = \sum_{j=1}^{\infty} \beta^j E_t \left[\frac{C_t}{C_{t+j}} d_{t+j} \right] \quad (\text{F.2})$$

Substituting our definition of dividends into (F.2):

$$d_{t+j} = Y_{t+j} - \frac{W_{t+j}}{P_{t+j}} N_{t+j}$$

$$\Rightarrow f_t = \sum_{j=1}^{\infty} \beta^j E_t \left[\frac{C_t}{C_{t+j}} \left(Y_{t+j} - \frac{W_{t+j}}{P_{t+j}} N_{t+j} \right) \right] \quad (\text{F.3})$$

Imposing our two market clearing conditions:

i) Goods Market Clearing $\Rightarrow C_t = Y_t$

ii) Labour Market Clearing $\Rightarrow \frac{W_{t+j}}{P_{t+j}} = \frac{\partial Y_{t+j}}{\partial N_{t+j}} = (1 - \alpha) A_{t+j} N_{t+j}^{-\alpha}$

$$\Rightarrow f_t = \sum_{j=1}^{\infty} \beta^j E_t \left[\frac{Y_t}{Y_{t+j}} \left(Y_{t+j} - (1 - \alpha) A_{t+j} N_{t+j}^{1-\alpha} \right) \right] \quad (\text{F.4})$$

Here we use our approximation from Gali (2008):

$$Y_t \approx A_t N_t^{1-\alpha}$$

$$\Rightarrow Y_{t+j} \approx A_{t+j} N_{t+j}^{1-\alpha}$$

Which yields the following result:

$$\Rightarrow f_t = \sum_{j=1}^{\infty} \beta^j E_t [Y_t - (1 - \alpha) Y_t]$$

$$\Rightarrow f_t = \alpha \sum_{j=1}^{\infty} \beta^j E_t [Y_t] \quad (\text{F.5})$$

We can approximate (F.5) around the steady state value of output where \bar{Y} is the steady state value of output and the output gap is defined as $\hat{Y}_t = \frac{Y_t - \bar{Y}}{\bar{Y}}$:

$$\begin{aligned} Y_t &\approx \bar{Y} + (Y_t - \bar{Y}) \\ &\Rightarrow Y_t \approx \bar{Y}(1 + \hat{Y}) \end{aligned} \tag{F.6}$$

(F.6) looks much different than our results where $\sigma < 1$ due to the fact we're only dealing with one linear term Y_t . Now substitution (F.6) into (F.5) yields:

$$f_t \approx \alpha \sum_{j=1}^{\infty} \beta^j E_t \left[\bar{Y}(1 + \hat{Y}) \right] \tag{F.7}$$

$$f_t \approx \alpha \bar{Y} \sum_{j=1}^{\infty} \beta^j E_t \left[1 + \hat{Y} \right]$$

$$f_t \approx \alpha \bar{Y} \left[\frac{\beta}{1 - \beta} - (1 - \beta \rho_v) \Lambda_v v_t \frac{\beta}{1 - \beta} \right]$$

$$f_t \approx \alpha \bar{Y} \left(\frac{\beta}{1 - \beta} \right) [1 - (1 - \beta \rho_v) \Lambda_v v_t] \tag{F.8}$$

Now (F.8) becomes a special case of (3.7) where $b_0 = \alpha \bar{Y} \left(\frac{\beta}{1 - \beta} \right)$ and $b_1 = \alpha \bar{Y} \left(\frac{\beta}{1 - \beta} \right) (1 - \beta \rho_v) \Lambda_v$ with $b_2 = b_3 = 0$. The constant relative risk aversion associated with this special case of log utility where $\sigma = 1$ results in $b_2 = b_3 = 0$ which means that variances do not impact firm prices. Rather, our approximation for firm price collapses to a present value of steady state output plus a present value which depends on v_t , the periodic monetary policy shock in t .