The Design of a

REINFORCED CONCRETE BOW STRING ARCH

over the

North Saskatchewan River, Ceepee, Sask.

A thesis presented to the Faculty of Engineering, University of Saskatchewan, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering

by

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April, 1936.

Acknowledgements

The writer wishes to acknowledge with many thanks the assistance given by Mr. A. P. Linton, Bridge Engineer of the Saskatchewan Department of Highways, in the furnishing of the river profile and foundation data at the bridge site.

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Preface

In selecting a bridge design as a thesis, the writer had in mind a problem which exists in the prairie provinces. The building of our highway system necessitates the erection of suitable structures to cross rivers and creeks. Many ferries serve at river crossings now, but these provide slow transportation and do not operate the full year. Increase in speeds and volume of automobile and motor truck traffic have brought about a demand for highway bridges at these points.

Economy, permanence and appearance are the factors to be considered in selecting the type of bridge. Our natural materials of construction are very limited in this area, wood and materials for making concrete being the only building material available. Timber structures are not satisfactory from the standpoint of permanence or aesthetics and are not feasible for large spans. Our choice of material is then between imported steel and concrete. These competitive materials must be considered in the design of bridges of ordinary span length. Concrete is the more desirable from the aspect of permanence, maintenance and aesthetics. The difference in economy between steel and concrete depends chiefly on the physical conditions, but generally the costs of these structures are about the same.

One of the objects of present construction is to provide labor for relief of unemployment. Therefore, in choosing materials for construction this must be kept in mind. A cost analysis of concrete and steel structures shows that labor is a much greater item in the former. For this reason concrete structures should be erected in preference to steel wherever practical, providing the comparative costs are close.

A highway bridge has been contemplated for several years at a location on the North Saskatchewan River known as Ceepee. The feasibility of a concrete structure there has been doubted by engineers and others interested in the proposed project. This thesis is the design of a reinforced concrete bridge which the writer believes is practical.

General

The difficulty in the erection of a concrete bridge at this site arises on account of poor foundation conditions and the necessity of long spans for economical design. Good foundations are important for concrete structures, as excessive settlement will cause cracking and probably failure. Few long span concrete bridges have been built in this country on account of uncertainties and difficulties met in design. The writer proposes to overcome these obstacles by choosing a type of structure adapted to these conditions. The other difficulties will be eliminated by special features in design and construction.

In this thesis the writer will not enter into the detailed design of parts of the structure which do not present any problem of particular interest. Consequently the thesis deals chiefly with the design of the concrete arches and hinges.

Technical

The type of structure chosen is a multiple bow string arch. Yielding foundations, temperature changes and stress adjustments in the concrete will not imperil the safety of this structure, providing the end reactions can be taken properly at all times. This requires supports which will vary in span and rotate to accommodate changes in the arch and any displacement of the supports. Such are provided in this design. Uncertainties in stress conditions exist when a structure does not act as assumed in the theory of design. A bow string arch may be designed as a two hinged arch having a tie to carry the horizontal thrust. The same arch may be designed as a continuous arch and tie. Small spans are usually designed as two hinged arches and constructed with the arch and tie continuous. In the case of long spans, 130 ft. or over this cannot be done, as the secondary stresses introduced by arch shortening, concrete shrinkage, dead load elastic extension of the tie become of such magnitude that the structure cannot be designed. These secondary stresses will be eliminated by placing hinges at the springing and crown of the arch ring and allowing the ring to deflect as a three hinged arch, the horizontal tie being hinged also. The hinge at the crown will be a temporary as this is not required once the dead load deflections and rotations have taken place, the live load stresses being very small. The springing hinges will be permanent ones, since these must provide a normal reaction to the support in the case of foundation settlement continuing after the structure is completed.

Rigid frame action between the arch and deck must be eliminated in order that the structure may act as a three hinged arch. For this reason a portion of the road slab, the centre panel of the sway bracing and the concrete encasing of the hangers will not be poured until the arch ring is carrying the full dead load of the bridge. This will also allow the full dead load extension of the hangers to take place without the opening of tension cracks in the concrete encasing.

The effect of rigidity between the ring and the deck would be to shift the line of thrust (which is calculated to pass through the arch axis for dead loads) to a lower position, thus setting up moments that must be taken by the arch ring and floorsystem.

Rigid frame action under permanent conditions will be minimized by making the hanger section flexible. This will be done by designing the hanger with little resistance to bending.

Economies of Design

From a preliminary study of the costs of varying the number of spans for the bridge site, the writer concluded that the cost of the structure did not change much for three, four or five spans. As the design of long spans presented more difficulty and some interesting problems worth studying, the writer chose the triple span structure.

In the preliminary study, two octagonal boom sections connected with a web and a rectangular section were considered. The former resulted in a saving of concrete of 9%, but the extra cost of steel spiralling, forming and pouring a boom section would offset this saving. From all considerations it seemed that the rectangular section was the best. The arch ring is made of constant section. The maximum stress conditions which occur at the quarter point and at the crown are of about the same magnitude. The dead load does not vary greatly throughout and is more significant than the live load stresses. Variation in the ring cross section is therefore not warranted. A rise of one-sixth of the span was chosen, as this is the most economical ratio and is also desirable from the aesthetic viewpoint.

In determining the panel span for the floor system a length had to be chosen which fitted the structure architecturally and was in the economical range.

The foundation problem for the river piers was studied from the standpoint of economy. Three designs were investigated. The first type considered was a pier composed of two circular shafts carrying the load directly to the hard pan. This idea was abandoned, as a suitable foundation pressure could not be obtained at a reasonable cost. The second pier design considered was similar to the one finally adopted, but the foundation was carried to proper bearing material. The elevation of this was uncertain and the cofferdam construction could not be planned economically and might be impracticable. For this reason a pier supported on foundation piles was adopted. The piers are carried deep enough below river bed to prevent danger from erosion.

Aesthetic Considerations

As the arches are the outstanding structural elements of the bridge, it is natural that the appearance of the structure depends on them. Therefore it is essential that these be pleasingly proportioned in regard to each span and its relation to the structure as a whole. This was considered in choosing the arch span, rise and ring shape. It was recognized that three similar spans would not give the best appearance, as the centre span appeared flattened. This was overcome by making all the arches similar but increasing the length of the centre span. This gave a pleasing structure, the upper lines of the bridge presenting a curved effect harmonizing with the arches. It was also found the same effect could be had by making all the spans of equal length and increasing the rise of the centre arch to give a slight curved effect. As this arrangement was more economical, it was adopted for design.

The aesthetic treatment of the rest of the structure consists mainly in trying to secure good effect by simple, well proportioned parts. No intricate or ornamental treatment is used, as this adds considerably to the cost of the structure and such expenditure would not be justified for a bridge in this location.

The piers are built several feet higher than the requirements for high water, but this seemed necessary to give the best appearance, the bridge standing out in view much better with a greater clearance above the ordinary water level. The upstream and downstream faces of the piers are triangular. Circular faces would harmonize with the structure better but would add considerably to the pier cost. This did not seem worth while.

D E S I G N

GENERAL DESIGN CONSIDERATIONS

Location of Bridge

This bridge is on the Jasper Highway between Saskatoon and Battleford. The highway is constructed to the present ferry landings on the banks of the North Saskatchewan River. For this reason no study was made to determine the most economical bridge site, as the constructed highway marked the most logical location.

Bridge Dimensions

The Jasper Highway is a link in the Trans-Canada Highway System which crosses the park area of the western provinces. Tourist traffic is fairly heavy at present and will increase greatly with the development of our northern national parks. The cities of Saskatoon and North Battleford will provide a fair volume of truck and bus traffic over this bridge. From the above traffic expectations it seems that a bridge width of 22 feet clear will be suitable. This will provide two lanes of traffic with ample clearance for large vehicles and fast traffic.

Foundation conditions make the construction of approach spans impractical, so the concrete arches must cover the entire distance to be spanned. Three spans of 300 feet each are chosen to accomplish this.

The North Saskatchewan River is not navigable for boats of any size, so navigation clearances do not have to be considered, although a fair clearance is provided.

GENERAL DESIGN DATA

Type of Structure

Reinforced Concrete Multiple Bow String Arch.

Total spanned length is 920' - 3 spans.

Total bridge width is 29'

Maximum arch clear rise is 56'

Arch Axis Spans

2 spans - 288' long : rise 48'

1 span - 288' long : rise 50.5'

Panel Spans - 15 @ 16' 6"

2 @ 18' 0"

Design Specifications

Canadian Engineering Standards Association.

(a) Standard Specifications for Concrete and Reinforced Concrete.

(b) Standard Specifications for Steel Highway Bridges.

Live Loads -	Motor Trucks	15 ton (2 lanes)
	Uniform load	60 lbs./sq.ft. (Arches)
Stresses -	Reinforcing Steel	20,000 lbs. in.^{2}
	Concrete - Arches	1,200 lbs. in.^{2}
	Other stresses are governed by specifications.	

DESIGN OF FLOOR SYSTEM

Floor Slab Design

Floor beam spacing = 16.5' c. to c.

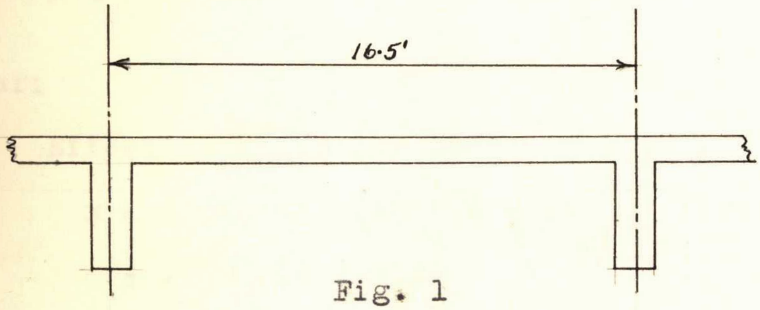


Fig. 1

Design for concentrated wheel loads - 15 ton Truck.

C. E. S. A. Specifications:

15 Ton Motor Truck Loading

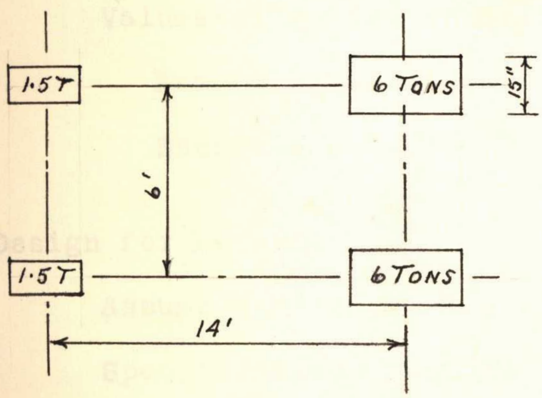


Fig. 2
Truck Loading Diagram

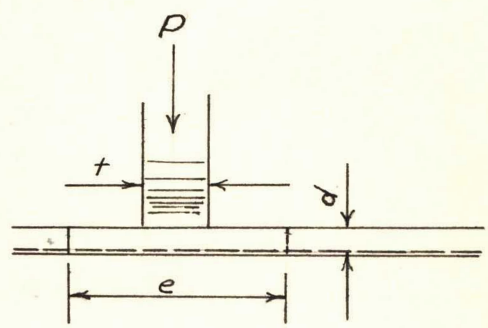


Fig. 3
Distribution of Wheel Load

Moment:

For slabs supported by transverse floor beams only, having a wheel midway between two consecutive floor beams, being loaded for maximum moment

$$e = \frac{2\,l}{3} + t \quad \text{but not greater than gauge of wheels}$$

$$e = \frac{2 \times 16.5}{3} \quad 1.25 = 12.25$$

Floor Slab Design (continued)

Ketchum's Specifications for distribution are the same as the above.

Urquhart and O'Rourke suggest 5'6" as the limit.

Shear:

Effective width for shear $e = \frac{4x}{3} + t$

$x =$ depth of slab + ½ width of floor beam

$= .75 + .54 = 1.29$

$$e = \frac{4 \times 1.29}{3} + 1.25 = 2.97$$

Ketchum gives values of e for shear as 3.0' minimum and 6.0' maximum.

Values of e used in this design will be

Moment $e = 5.5'$

Shear $e = 3.0'$

Design for Moment:

Assume a 9" slab $w = 112.5$ lbs./sq.ft.

Specifications require both positive and negative moments to be taken as 8/12 of simple support moments.

$$\text{D.L. B.M.} = \frac{8}{12} \times \frac{112.5 \times 16.5^2}{8} = 2550 \text{ ft. lbs.}$$

$$\text{L.L. B.M.} = \frac{8}{12} \times \frac{12,000 \times 16.5}{4 \times 5.5} = 6000 \text{ ft. lbs.}$$

Impact 30% $= 1800$ ft. lbs.

Design B.M. $= 10,350$ ft. lbs.

$M = K bd^2$

$$\text{Required } d = \sqrt{\frac{10,350 \times 12}{197 \times 12}} = \sqrt{52.6} = 7.25''$$

Using 5/8 ∅ steel, slab depth will be $7.25 + \frac{.625}{2} + 1.0 = 8.56''$

Floor Slab Design (continued)

Shear:

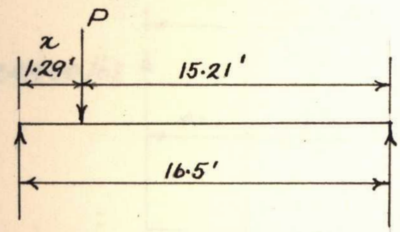


Fig. 4

Width of slab under shear is 3.0'

Shear per ft. width due to L.L.

$$= \frac{15.21}{16.5} \times \frac{12{,}000}{3.0} = 3690 \text{ lbs.}$$

$$\text{D.L. Shear} = \frac{16.5 \times 112.5}{2}$$

$$= 927 \text{ lbs.}$$

Max. Shear = L.L. = 3690
D.L. = 927
Impact = 981
5598 lbs.

Allowable shear for diagonal tension $= .02\, f'_c = 60$ lbs. in.^{2}

Using an $8\frac{1}{2}$" slab the existing shear will be

$$v = \frac{V}{j\,b\,d} = \frac{5598}{.875 \times 12 \times 7.2} = 74.1 \text{ lbs.in.}^2$$

Since 3.0 ft. is the minimum width for shear according to several specifications, this value is O.K. Also Ketchum says shear over a 3.0' width is punching shear and that diagonal tension should not be considered under a 4.5' width of slab.

If special anchorage is provided for the slab steel the allowable shear will be $.03\, f'c = 90$ lbs.in.^{2}

Slab Reinforcing:

For balanced design $p = .0112$ $\quad d = 8.5 - \frac{.625}{2} - 1.0 = 7.2$

As required $= .0112 \times 12 \times 7.2" = .97$ sq.in. per ft. width slab.

Spacing = $3\frac{3}{4}$" O.C. for 5/8"Ø

$5\frac{1}{2}$" O.C. for 3/4"Ø

Design of Floor Beams

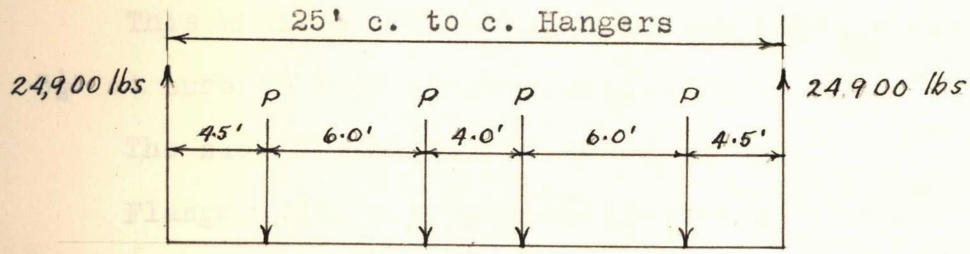


Fig. 5

Moment Design - Assuming average thickness of $9\frac{1}{2}$" for slab.

Assume width of beam 13"

D.W. of stem as 445 lbs./ft.

D.L. per ft. on beam = 16.5 x 118.5 + 445 = 2400

D.L. B.M. = $\frac{w\ l^2}{8} = \frac{2400 \times 25^2 \times 12}{8}$

= 2,250,000 in lbs.

Live Load - Max. Moment will be for 2 trucks with rear wheels on floor beam. Fig. 5.

Load P will be 12,000 + $\frac{2.5}{16.5}$ x 3000 = 12,450 lbs.

Max. L.L. B.M. = 24,900 (12.5 - 5.0) x 12

= 2,240,000 in lbs.

Total B.M. (positive)	D.L. =	2,250,000
	L.L. =	2,240,000
	Impact 30% =	672,000
	Design B.M. =	5,162,000 in lbs.

Shear Design.

D.L. = .5 x 2400 x 25	= 30,000
L.L. = 2 x 12450	= 24,900
Impact 30%	= 7,470
Design Shear (End)	= 62,370 lbs.

Design of Floor Beam (continued)

This will be a T-Beam. The slab depth will vary from $8\frac{1}{2}$" at curb to $10\frac{1}{2}$" at centre of beam.

The slab depth will be taken as $9\frac{1}{2}$"

Flange width - from specifications

$= \frac{1}{4}l = .25 \times 25 \times 12 = 75"$ (governs)

Or $= 2 \times 8t = 16 \times 9.5 = 152"$

Allowable shear in web $= v = .06f'c = 180$ lbs.in.^{2}

Web area required from $v = \dfrac{V}{jb'd}$

$$b'd = \frac{62,400}{180 \times .875} = 396 \text{ sq.in.}$$

Assuming b' as 13" d = 31". Using 2 layers of steel the total depth will be 34". This will be increased to 42" to compensate for the area deducted by the tie opening in the floor beam at the supports. Extra depth is also provided here by the 9" curb.

The value of $\dfrac{t}{d}$ is .244. The beam falls under Case II and is not balanced design. Using the approximate formula $A_s = \dfrac{M}{f_s jd}$ the area of steel required = 7.57 sq.in.

8 - 1"☐ will be used placed in 2 rows of 4.
4 bars will run end to end and 4 will be bent up.

Bond Stress - $u = \dfrac{V}{\Sigma ojd} = \dfrac{62,400}{4 \times 4 \times .875 \times 39}$

$= 114$ lbs.in.^{2}

Allowable bond stress $= .05\ f'c = 150$ lbs.in.^{2}

(Ordinary anchorage)

Design of Floor Beam (continued)

Points to bend up horizontal steel, assuming a parabolic bending moment diagram

$$x = \frac{\ell}{2}\left(1 - \sqrt{\frac{m_2}{m}}\right)$$

For m_2 values of 2 and 4, $x = 6.25'$ and $3.75'$

Shear Design: Maximum end shear = 62,400 lbs.

Allowable shear taken by concrete = .03 f'c = 90 lbs.in.^{2}

Existing shear at support = $\frac{62,400}{.875 \times 13 \times 39}$ = 141 lbs.in.^{2}

Stirrups will be required to care for the shear. The bent up bars provide ample area to take this shear but the spacing will not do.

End shear taken by concrete = $v_c jbd$ = 40,000 lbs.

End shear taken by stirrups = 22,400 lbs.

Design shear at centre = 15,600 lbs.

Decrease in shear per ft. is $\frac{65,400 - 15,600}{12.5}$ = 3750 lbs.

Stirrups will not be required after $x = \frac{22,400}{3,750} = 5.96'$

Total shear to be taken by stirrups is

$$\frac{22,400}{.875 \times 39} \times \frac{5.96 \times 12}{2} = 23,500 \text{ lbs.}$$

3/8" Ø stirrups will be used. A_s = .22 sq.in.

Stirrup spacing = $\frac{\text{jd x allowable stress per stirrup}}{\text{shear to be carried by stirrup}}$

$$= \frac{.875 \times 39 \times .22 \times 16,000}{V} = \frac{120,000}{V}$$

The following spacing from the support is used: 1 - 3", 4 - 6", 2 - 9", 1 - 12" and the minimum spacing of 17" thereon.

Design of Hangers

The hangers must be designed to carry the maximum panel point load and a wind load.

The maximum stress in the hanger will be tension due to the combination of the direct tension load and bending due to the wind load. The deflection of the hanger will induce a second moment due to the axial load. If the hanger is hinged at the ends, the moments due to axial eccentricity from deflection and cross bending will counteract each other and only axial stress will exist at the hinge. If the hanger has fixed ends this condition cannot be obtained and the design will be uneconomical and result in a stiffer member which is an undesirable feature in this structure.

Design Data:

Dead Load at hanger	=	48,200 lbs.
Dead Load in hanger	=	8,400 lbs. (longest one)
		56,600 lbs.
Max. Live Load per hanger	=	24,900 lbs.
Impact 30%		7,500 lbs.
		32,400 lbs.
Total.....		89,000 lbs.

Wind load will be taken as 30 lbs. sq.ft.

Hanger section will be 9" x 18" for long members.

Hinges will be provided at ends.

Design of Hangers (continued)

The wind bending moment in the longest hanger (43.5')

will be $M = \frac{wl^2}{8} = \frac{45 \times 43.5^2 \times 12}{8} = 127,800$ in lbs.

The deflection at the centre calculated by Maney's Method. See Concrete Engineers Handbook - Hool & Johnson, Page 305.

$$D = c\,\frac{l^2}{d}\,(e_c + e_s)$$

For working stresses of $f_c = 1200$ lbs.in.^{2}, $f_s = 20,000$ lbs.in.^{2}

$$e_c = \frac{1200}{3 \times 10^6} = .0004 \qquad e_s = \frac{20,000}{30 \times 10^6} = .000666$$

$$D = \frac{.1041 \times 43.5^2 \times 144}{7} \times (.0004 + .000666) = 4.3''$$

The axial dead load of 56,600 lbs. will produce a moment equal and opposite to the wind moment if the hanger deflects an amount equal to $127,800 \div 56,600 = 2.26''$

Using a section with 6 - 1"□ as shown in sketch, Fig. 6.

Fig. 6

$A_s = 6.0 \qquad \frac{d'}{d} = .286$

$p = p' = .0238$

$K = 238$ (approx. value)

R.M. $= Kbd^2$

R.M. $= 238 \times 18 \times 7^2$

$= 210,000$ in.lbs.

Direct steel stress $= \frac{56,600}{6} = 9,450$ lbs.in.^{2}

Allowable total stress for combined load conditions

$= 130\% \times 20,000 = 26,000$ lbs.in.^{2}

Allowable bending stress $= 16,550$ lbs.in.^{2}

Design of Hangers (continued)

This corresponds to a moment of 106,000 in lbs. at the centre of hanger, which is 83% of the simple beam wind moment.

From the above approximate calculations it seems that the hanger section will be suitable as the wind moment at the centre will be reduced by the axial load and eccentricity.

THEORY OF TWO HINGED TIED ARCHES

General formulae for the Arch Rib Reactions will be developed from the well known equations for deflection of curved beams.

Notation:

ρ = radius of curvature

α = inclination of arch axis at any point

α_1 = inclination of arch axis at springing

L = length of arch axis

M = bending moment at any section due to given loads and the true reactions

M' = bending moment at any section due to given loads and vertical reactions only

t_a = temperature of arch ring

t_t = temperature of arch tie

H = horizontal thrust in tie

A_t = area of arch tie

A = area of rib

w = coefficient of expansion for concrete = .0000065 - ^{o}F

k = coefficient of concrete shrinkage

The arch reactions are illustrated in Fig. 7. The vertical reactions V_1 and V_2 are determined by moments about A and B. These are simple beam reactions.

$$V_1 = \Sigma P(1 - k)$$

$$V_2 = \Sigma Pk$$

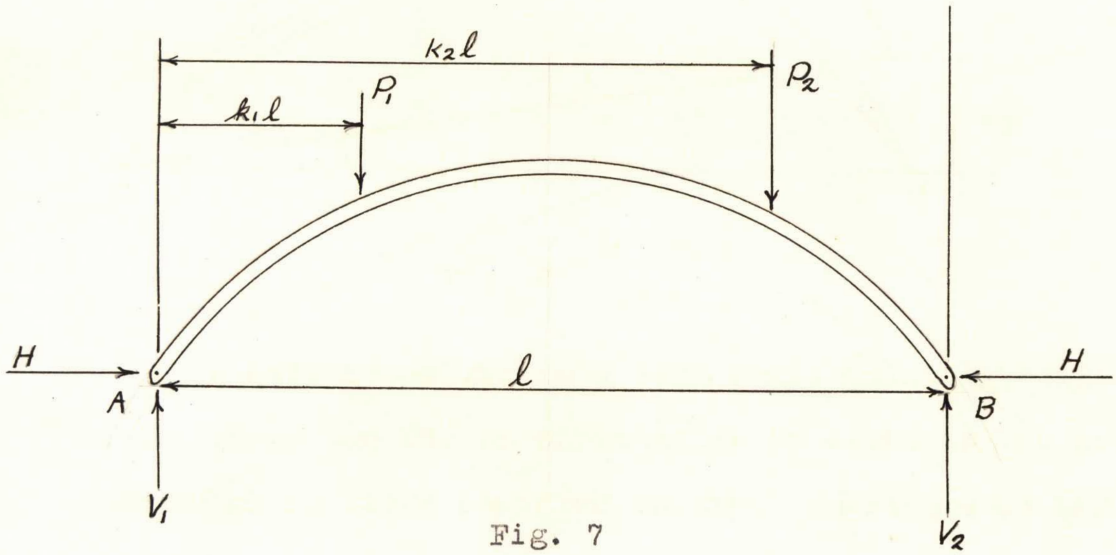


Fig. 7

The value of H can only be determined by considering the elastic deflection of one end of the arch with respect to the tangent at the other end.

Fig 8

Let ACB be a two hinged arch rib which is subjected to bending moments and axial thrust due to loads. The arch will be distorted and take up a new position AC'B, the end tangents changing angles. Now if the distorted arch is rotated about A so that the tangent at A coincides with the original tangent, then the movement of B will represent the true elastic deflection of B. This is illustrated in Fig. 8.

The elastic deflections are Δx , Δy and $\Delta \phi$ as usually designated in curved beam theory.

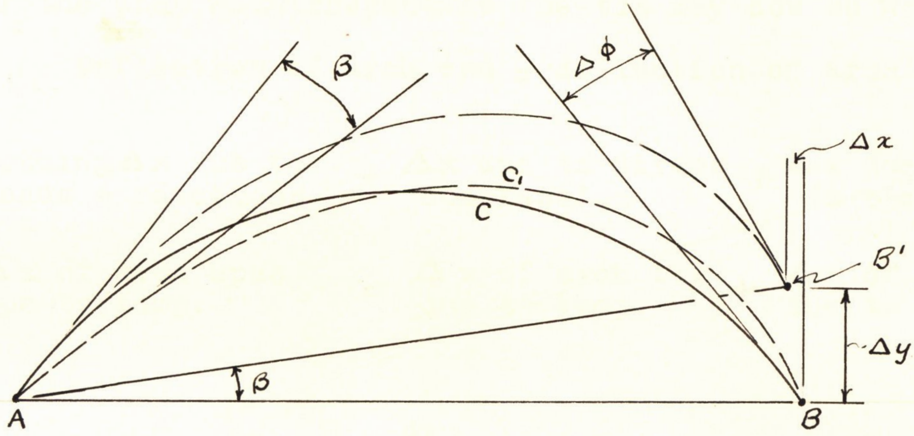


Fig. 8

The deflection Δx is a very small quantity compared to Δy since the radius of rotation is equal to the arch span which is large compared to Δy. Therefore we may place $\Delta x = 0$

The equation for the deflection Δx of a curved beam under load is $\Delta x = \int \frac{Myds}{EI}$

The deflection of a curved beam due to axial compression is $\Delta x = -\int \frac{fdx}{E} + \int \frac{fyds}{Ee}$

For change in span for temperature rise

$$\Delta x = wtl$$

For elastic extension of the tie

$$\Delta x = \frac{Hl}{AE}$$

The general expression for horizontal deflection of the arch with respect to the tie may now be written.

Deflection of arch end = deflection of arch tie.

Bending Δx due to loads & reactions $-$ Δx due to direct compression $+$ Δx due to bending in compression

$+$ Δx of arch span due to temp. $=$ Δx of arch tie due to temp. $+$ Δx of arch tie due to elasticity

or

$$\int \frac{Myds}{EI} - \int \frac{fdx}{E} + \int \frac{fyds}{E\rho} + wt_a l = wt_t l + \frac{Hl}{A_t E} \text{ ....(1)}$$

Since $M = M' - Hy$

$$\int \frac{Myds}{EI} = \int \frac{M'yds}{EI} - \int \frac{Hy^2 ds}{EI} \text{ ....(2)}$$

The moment M' is the simple beam moment for vertical loads.

Before equation (1) is solved f must be expressed in terms of H.

Determination of f: Consider the forces acting on any section of the arch as in Fig. 9 and Fig. 10

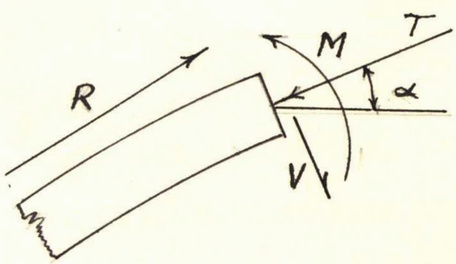


Fig. 9

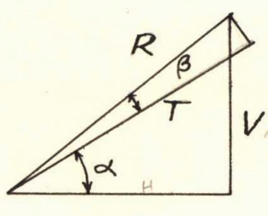


Fig. 10

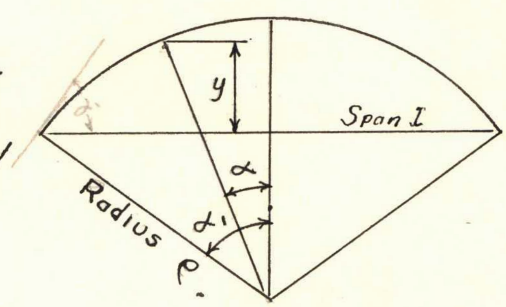


Fig. 11

Since the angle β is small $T = H \sec\alpha$

$$f = \frac{T}{A} = \frac{H \sec\alpha}{A}$$

Then $$\int \frac{fyds}{E\rho} - \int \frac{fdx}{E} = \frac{H}{E} \int \frac{\sec\alpha}{A} \left(\frac{yds}{\rho} - dx\right) \qquad \text{........(3)}$$

Now if the curvature is uniform we have from Fig. 11

$$y = \rho\cos\alpha - \rho\cos\alpha_1$$

and $dx = ds \cos\alpha$

Therefore $\frac{yds}{\rho} - dx = (\cos\alpha - \cos\alpha_1)ds - dx = -ds \cos\alpha_1$

Equation (3) becomes

$$-\frac{H}{E} \cos\alpha_1 \int \frac{ds}{A \cos\alpha}$$

From (2) and (3) equation (1) now becomes

$$\int \frac{M'yds}{EI} - H\int \frac{y^2ds}{EI} - \frac{H}{E} \cos\alpha_1 \int \frac{ds}{A\cos\alpha} + w(t_a - t_t)l - \frac{Hl}{A_tE_s} = 0$$

Solving for H

$$H = \frac{\int \frac{M'yds}{EI} + w(t_a - t_t)l}{\int \frac{y^2ds}{EI} + \frac{1}{E}\cos\alpha_1 \int \frac{ds}{A\cos\alpha} + \frac{1}{A_tE_s}} \qquad \text{............(4)}$$

Omitting the effect of temperature we have

$$H = \frac{\int \frac{M'yds}{I}}{\int \frac{y^2ds}{I} + \cos\alpha_1 \int \frac{ds}{A\cos\alpha} + \frac{1}{A_t}\frac{E_c}{E_s}} \qquad \text{..................(5)}$$

The evaluation of this equation for H, the horizontal thrust, will enable the determination of moments, thrusts and stresses for any section in the arch rib.

General Equation for Moment at any section

$$M_x = V_1x - Hy - P(x - kl)$$

General Equation for Thrust at any section

$$N = H \sec\alpha$$

Solution of Integral Expression for H

This equation may be integrated mathematically provided the equation for the curve of the arch axis is known. For uniform load this equation is a parabola. For uniform varying load the curve is a hyperbola. (See Whitney's Method of Arch Analysis.) Either of these curves gives a complicated expression for integration. The equation may be solved very easily by other methods. The first members in the numerator and in the denominator may be evaluated graphically and the remaining members integrated by the summation method.

Graphical Determination of $\int \frac{M'yds}{I}$ and $\int \frac{y^2ds}{I}$ from Eq.(5)

The arch is divided into sections of equal length. As I is constant then $\frac{ds}{I}$ is a constant. The value of $\frac{yds}{I}$ is now calculated for each section. Let Q represent this value generally for each section. The above integrals may now be written

$$\sum M'Q \quad \text{and} \quad \sum yQ$$

Now the quantity M' for a single load P on the arch at a distance kl from the support is given by these equations:

$$\text{for } x < kl \qquad M' = P(l - k)x$$

$$\text{for } x > kl \qquad M' = Pk(l - x)$$

From this we have

$$\sum M'Q = P\sum_{0}^{kl} (1 - k)xQ + P\sum_{kl}^{l} k(1 - x)Q$$

Note that this expression is similar to the moment at a point in a simple beam for the following conditions:

(1) beam span equals arch span

(2) beam supports a series of loads equal to Q

(3) the value of P is 1.

From this it is seen that the value of $\sum M'Q$ may be determined by calculating the bending moment in a straight beam under the arch load point due to a series of loads Q. Hence all the values of $\sum M'Q$ or $\sum \frac{M'yds}{I}$ may be obtained by drawing a bending moment diagram for a simple beam under loads Q. The ordinate at the load point will represent the value of $\sum M'Q$. The bending moment diagram can be constructed graphically by drawing an equilibrium polygon for the beam loads.

The quantity $\sum yQ$ can be determined graphically by a similar method. The values of Q being horizontal loads acting on a vertical cantilever of height equal to the arch axis rise. This may also be calculated by the summation method as the various terms required for summation are previously calculated.

Temperature Effects

For a variation in temperature of the structure we have considering deflections

Δx of arch rib end $=$ Δx of tie end, or

$$H \int \frac{y^2 ds}{EI} - \frac{H}{E} \cos\alpha_1 \int_0^L \frac{ds}{A \cos\alpha} + w t_a l = \frac{Hl}{A_t E_s} + w t_t l.$$

Solving for H

$$H = \frac{w(t_a - t_t) l E_c}{\int_0^L \frac{y^2 ds}{I} + \cos\alpha_1 \int_0^L \frac{ds}{A \cos\alpha} + \frac{l}{A_t} \frac{E_c}{E_s}} \quad \text{........ Eq. (6)}$$

Note that for a uniform temperature throughout the structure no thrust will exist and the temperature stresses will be zero.

Concrete Shrinkage Effects

Concrete shrinkage has the same effect on the structure as a negative temperature difference between the arch ring and tie. The horizontal thrust will be calculated by substituting the amount of the span shortening for $w(t_a - t_t)l$ in equation (6).

DESIGN OF ARCH RINGS

General Procedure

The preliminary dead load stresses and approximate arch axis curve are determined by Whitney's Method. Then the dead load reactions and true shape of the arch axis are determined by graphical methods. Following this an elastic analysis of the arch ring is made, and the true horizontal thrust for dead load, live load and temperature changes determined. Arch ring stresses are then calculated at critical sections for various conditions of loading.

Design Data

Arch axis span l is 288'

Arch axis rise r is 48' for end arches and 51.5' for centre arch.

Panel distance for floor system = 16.5'

Arch ring dimensions 3' x 5'. Per cent steel = .01

Arch ring concrete f'c = 3000 lbs. sq. in. n = 10

This design is made for the end arches.

w_c = 5910 lbs.
Determined on page 28
w_s = 5640 lbs.

The value of w_s will be assumed the same as w_c since the difference is caused by the sudden ending of the sway bracing.

Dead Load Calculations

Determination of w_c:

Slab load per panel =	3670 x 16.5	=	60,600
Floorbeam		=	15,000
Hangers	2 x 4850	=	9,700
Ties	2 x 5620	=	11,240
Hand rail		=	600
Arch Sway Bracing	2 x 11,700	=	23,400
Arch Ring	2 x 16.5 x 2250	=	74,300
	Total Panel Load		194,840

$$\text{D.L. per ft. of ring} = \frac{194,840}{2 \times 16.5} = 5910 \text{ lbs.}$$

$$w_c = 5910 \text{ lbs.}$$

Determination of w_s:

At springing the hanger and sway bracing loads will not exist but the ring load will be increased. Correcting the above panel load for these we have

$$w_s = 5640 \text{ lbs.}$$

Dead Load Analysis by Whitney's Method

Reference: Transactions of the American Society of Civil Engineers, 1925.

From Whitney's Tables for Analysis the following constants are taken:

$$g = \frac{w_s}{w_c} = 1 \qquad N = .25 \qquad \frac{1}{r} \tan \phi_s = 4.0$$

$$C_d = \frac{r\,H_d}{w_c l^2} = .1250 \qquad \frac{V_d}{w_c l} = .5$$

Since $l = 288$ $r = 48$ $w_c = 5910$

$\tan \phi_s = .667$ arc tan $= 33^{o}40'$ $\cos \phi_s = .8323$

$$H_d = \frac{.1250 \times 5910 \times 288^2}{48} = 1,278,000 \text{ lbs.}$$

$$V_d = .5 \times 288 \times 5910 = 852,000 \text{ lbs.}$$

$$T_s = H_d \div \cos \phi_s = 1,535,000 \text{ lbs.}$$

Approximate dead load stress at the springing will be $\frac{1,535,000}{3 \times 5 \times 144} = 710$ lbs.in.^{2}

From this it seems that the maximum arch stress for combined stress conditions will not exceed the allowable of 1200 lbs. in.^{2} A more thorough analysis will now be made for dead load, live load and temperature stresses.

Elastic Analysis of Arch Ring - Procedure

The approximate arch axis is first plotted, assuming the curve is a parabola. The true dead loads on the structure are then calculated and the equilibrium polygon drawn for these loads through the crown and springing points. The location of the resultant of the loads on the half-span is determined in order that the pole distance for the equilibrium polygon be determined directly. The dead load calculations are given on page 31.

The dead load equilibrium polygon was found to coincide very closely with the parabolic curve. A smooth curve drawn to follow the equilibrium polygon is the arch axis used in the design.

The arch semi-axis is now divided into ten equal parts or voussirs and the centre of each voussir located. Table No. 1 is then constructed and the various quantities required for analysis tabulated for convenience. The equations for horizontal thrust are then solved and the maximum moments and thrusts for stress calculations tabulated in Table 2. The graphical work in connection with the analysis is shown on Drawing 5.

TABLE NO.1
ARCH ANALYSIS DATA

SECTION	y	y^2	$Q = \frac{y\,ds}{I}$	Qy	α	$\cos\alpha$	$\frac{ds}{A\cos\alpha}$
1	4.0	16.0	1.63	6.5	32° 0'	.8480	1.112
2	12.3	151.2	5.03	61.8	30° 0'	.8660	1.089
3	19.7	388	8.05	158.5	26° 30'	.8949	1.054
4	26.2	687.	10.70	280.0	24° 0'	.9135	1.032
5	32.2	1047.	13.15	423.0	21° 0'	.9336	1.010
6	37.4	1400	15.30	572.0	17° 30'	.9537	.989
7	41.6	1730	17.00	707.0	14° 0'	.9703	.973
8	44.8	2015	18.30	820.0	9° 30'	.9863	.957
9	46.8	2195	19.15	898.0	5° 0'	.9962	.947
10	47.7	2275	19.50	930.0	2° 0'	.9994	.943
	312.7		127.81	4856.8		9.3619	10.106

Dead Loads for Arch Ring Analysis

Dead Load per panel in floor system

Slab	60,600 lbs.
Floorbeam	15,000 lbs.
Arch Tie	9,200 lbs.
Hand Rail	600 lbs.
Sway Bracing	23,400 lbs.
	104,800 lbs.

or 54,400 lbs. per panel point.

Dead Load Totals

Hanger No.	Floor System	Arch Ring	Hanger	Total
8	54,400	37,200	7275	98,875
7	54,400	37,400	7000	98,800
6	54,400	37,800	6600	98,800
5	54,400	38,300	5960	98,660
4	54,400	39,200	5070	98,670
3	54,400	40,300	4020	98,720
2	54,400	41,700	1820	97,920
1	54,400	43,500	790	98,690
1d		20,300		20,300
			Total	809,435

By taking moments of the hanger loads about the crown it is found that the resultant of the dead loads acts 67' from the arch centre.

Arch Analysis - Data

Scaled length of arch semi-axis $= \frac{L}{2} = 153.8'$

Length of arch segments - $ds = 15.38'$

Moment of inertia of arch section

$$I = \frac{bd^3}{12} + (n - 1)\, A_s k^2$$

$$= 31.25 + 6.48 = 37.73$$

$A_s = .01 \times 15.0 = .15$ sq.ft. $= 21.6$ sq.in.

$$\frac{ds}{I} = \frac{15.38}{37.73} = .408$$

Transformed area $A_T = 15.0 + 1.34 = 16.34$ sq.ft.

$$\frac{ds}{A_T} = \frac{15.38}{16.34} = .943$$

Area of steel tie $= A_t = 80$ sq.in.

$$\frac{l}{A_t n} = \frac{2 \times 144.0 \times 144}{80 \times 10} = 51.8$$

$\alpha_1 = 33°40'$ Calculated and checked graphically.

$\cos\alpha_1 = .8323$

Evaluation of Equation for H Due to Vertical Loads

Values for members of equation (5) are

$\int_0^L \frac{M'yds}{I}$ = Ordinates in equilibrium polygon for loads of $\frac{yds}{I}$. Ordinates corrected for scale and pole distance. See Drawing No. 5

$$\int_0^L \frac{y^2 ds}{I} = 2 \times 4856.8 = 9713.6 \quad \text{(Table No. 1)}$$

$$\cos\alpha_1 \int_0^L \frac{ds}{A \cos\alpha} = .8323 \times 2 \times 10.106 = 16.8$$

(Table No. 1)

$$\frac{l}{A_t n} = 51.8$$

Numerator of equation (5) is then

$$9713.6 + 16.8 + \underset{51.8}{443} = \underset{9782.2}{10173.4} \quad \text{(correction)}$$

The value of H, the horizontal thrust for a load at any point, can now be found by dividing the ordinates in the equilibrium polygon by 10173 and correcting for scale and pole distance. The latter correction is 300 for pole distance.

Example: Ordinate for unit load at centre ($k = .5\ l$) is 37.0

$$\text{Value of } H = \frac{37.0 \times 300}{10{,}173} = 1.09 \text{ lbs.}$$

The corrected link polygon is drawn to form an influence line for horizontal thrust. These values are used in calculating moments and thrusts from vertical arch loads.

<u>Horizontal Thrust Due to Temperature Variations</u>

$$H = \frac{w(t_a - t_t)l\,E}{\int_0^L \frac{y^2 ds}{I} + \cos\alpha_1 \int_0^L \frac{ds}{A \cos\alpha} + \frac{l}{A_t n}} \quad \ldots\ldots\ldots \text{Eq.}(6)$$

Assuming a temperature difference of 20° between the tie and arch rib the value of the horizontal thrust is

$$H = \frac{.0000065 \times 20 \times 288 \times 3 \times 10^6 \times 144}{9713.6 + 16.8 + 443} = 1590 \text{ lbs.}$$

Moments in Rib Due to Temperature Thrust

Moment at any point is given by $M_t = H_t y$

Moment at 1/4 Point

$M = 1590 \times 36 = 57{,}300$ ft. lbs.

Moment at Crown Point

$M = 1590 \times 48 = 76{,}500$ ft. lbs.

Horizontal Thrust Due to Concrete Shrinkage

$$H_s = \frac{k\ l\ E}{\int_0^L \frac{y^2 ds}{I} + \cos\alpha_1 \int_0^L \frac{ds}{A \cos\alpha} + \frac{l}{A_t n}}$$

From the "Gold Beach Tests" the following coefficients of shrinkage were obtained.

For 1 week $k = .00007$

3 weeks $k = .00010$

1 year $k = .00018$

Total assumed $k = .00025$

For this design a shrinkage coefficient of .00018 will be assumed. This is intended to care for shrinkage that takes place after closing the temporary crown hinge.

$$H_s = \frac{288 \times .00018 \times 3 \times 10^6 \times 144}{10173} = -2205 \text{ lbs.}$$

Moments in Rib Due to Concrete Shrinkage

Moment at 1/4 Point

$M = 2205 \times 36 = 79{,}400$ ft. lbs.

Moment at Crown Point

$M = 2205 \times 48 = 106{,}000$ ft. lbs.

Live Load Moments and Thrusts at Crown and 1/4 Point

These are tabulated in Tables Nos. 2 and 3 for the conditions giving maximum positive and negative moment.

Calculation of Arch Ring Stresses

The stresses in the arch ring will be due to the following:

1. Dead Load
2. Live Loads
3. Temperature variations in the structure
4. Concrete shrinkage
5. Wind

The dead load thrust in the arch tie has been determined graphically only, Whitney's Method of Arch Analysis being used as a check. This could be determined in the same manner as the live load thrust and the dead load moment and normal thrust found for any section in the ring. If the arch axis follows the dead load equilibrium polygon closely, the dead load moments will be very small. This perfect condition cannot be obtained in the completed structure. For this reason a dead load eccentricity of .25' has been assumed to exist, at all sections, on the side of the arch axis giving the largest stress.

Maximum stresses are calculated for the critical sections of the ring. These are the quarter point and the crown. The maximum moments and thrusts for the

TABLE NO. 2.

LIVE LOAD – MAXIMUM MOMENTS AND THRUSTS AT CROWN.

Load Point	$\frac{l}{2}$ - kl.	Live Load	Unit H	Unit V_1	Unit Moment.	Moment.	Thrust.	Max. Positive Moment		Max. Negative Moment.	
								M	N	M	N
H-1	123.75	10.0	.248	.930	-1.65	-16.50	2.48			16.50	2.48
H-2	107.25	10.0	.442	.874	-2.55	-25.50	4.42			25.50	4.42
H-3	90.75	10.0	.620	.816	-3.02	-30.20	6.20			30.20	6.20
H-4	74.25	10.0	.767	.759	-1.80	-18.00	7.67			18.00	7.67
H-5	57.75	10.0	.897	.700	+0.95	+9.50	8.97	9.50	8.97		
H-6	41.25	10.0	.998	.644	+3.60	+36.00	9.98	36.00	9.98		
H-7	24.75	10.0	1.062	.586	+8.60	+86.00	10.62	86.00	10.62		
H-8	8.25	10.0	1.090	.528	+15.65	+156.50	10.90	156.50	10.90		
One half of arch loaded							61.24	288.00	40.47	90.20	20.77
Full arch loaded.							122.48	576.00	80.94	180.40	41.54

Note:- Loads in kips. Live load 600 lbs. per lin. foot. $M_c = V_1 \frac{l}{2} - Hr - P(\frac{l}{2} - kl)$.

TABLE NO. 3

MAXIMUM LIVE LOAD MOMENTS AND THRUSTS AT QUARTER POINT.

Load Point.	Live Load	Unit H	Unit Moment	Unit Normal T	Moment	Thrust.	Max. Positive Moment		Max. Negative Moment	
							M	N	M	N
C-1	10.0	.250	+ 6.05	.264	60.5	2.64	60.5	2.64		
C-2	10.0	.445	+ 11.55	.470	115.5	4.70	115.5	4.70		
C-3	10.0	.620	+ 17.62	.656	176.2	6.56	176.2	6.56		
C-4	10.0	.775	+ 24.45	.820	244.5	8.20	244.5	8.20		
C-5	10.0	.900	+ 18.0	.952	180.0	9.52	180.0	9.52		
C-6	10.0	.995	+ 10.5	1.050	105.0	10.50	105.0	10.50		
C-7	10.0	1.058	+ 4.1	1.190	41.0	11.90	41.0	11.90		
C-8	10.0	1.095	- 1.35	1.157	- 13.5	11.57			13.5	11.57
C-9	10.0	1.095	- 5.55	1.157	- 55.5	11.57			55.5	11.57
C-10	10.0	1.058	- 8.30	1.190	- 83.0	11.90			83.0	11.90
C-11	10.0	.995	- 10.20	1.050	- 102.0	10.50			102.0	10.50
C-12	10.0	.900	- 10.90	.952	- 109.0	9.52			109.0	9.52
C-13	10.0	.775	- 10.48	.820	- 104.8	8.20			104.8	8.20
C-14	10.0	.620	- 9.01	.656	- 90.1	6.56			90.1	6.56
C-15	10.0	.445	- 6.85	.470	- 68.5	4.70			68.5	4.70
C-16	10.0	.250	- 4.03	.264	- 40.3	2.64			40.3	2.64
Maximum Moments and Corresponding Thrusts.							922.7	54.02	666.7	77.16
Maximum Live Load Horizontal Thrust $= \frac{54.02 + 77.16}{1.057} = 124.5$										

Load in kips. sec. ϕ at quarter point 1.057 (19°) Live Load 600 lbs. per lineal foot of bridge.

TABLE NO. 4.

MAXIMUM MOMENT THRUSTS AND STRESSES

	Loading	Value in kips N	Value in kips M	$\frac{M}{N} = e'$	Direct Stress $\frac{N}{A_T}$	Bending Stress $\frac{Nec}{I}$
MAXIMUM POSITIVE MOMENT AT QUARTER POINT						
1	Dead Load	-1373.0	+343.0	0.25'	-585	-158
2	Live Load	- 54.0	+922.7	17.20'	- 22.9	-424
3	Temp. 20° Var.	+ 1.6	+ 57.3	36.0'	+ 0.7	- 26
4	Con. Shrinkage	+ 2.2	+ 79.4	36.0'	+ 0.9	- 36
	Total	-1423.2	+1402.4	0.987	- 606.3	- 644.
MAXIMUM NEGATIVE MOMENT AT QUARTER POINT.						
1	Dead Load	-1373.0	-343.0	0.25	-585	-158
2	Live Load.	- 77.2	-666.7	8.63	- 32.8	-307
3	Temp. 20° Var.	- 1.6	- 57.3	36.0	- 0.7	- 26
4	Con. Shrinage	+ 2.2	+ 79.4	36.0	+ 0.9	- 36
	Total	-1449.6	-1146.4	0.792	- 617.6	-527.
MAXIMUM POSITIVE MOMENT AT CROWN						
1	Dead Load	-1300.0	+325.0	0.25	-553	-150
2	Live Load	- 81.0	+576.0	7.11	- 34.4	- 265
3	Temp. 20° Var.	+ 1.6	+ 76.5	48.0	+ 0.7	- 35
4	Con. Shrinkage	+ 2.2	+106.0	48.0	+ 0.9	- 49
	Total	-1377.2	+1083.5	0.787	- 585.8	-499
MAXIMUM NEGATIVE MOMENT AT CROWN						
1	Dead Load	-1300.0	-325.0	0.25	-553.0	-150.
2	Live Load	- 41.5	-180.4	4.34	- 17.6	- 83.0
3	Temp. 20° Var.	- 1.6	- 76.5	48.0	- 0.7	- 35.0
4	Con. Shrinkage	+ 2.2	+106.0	48.0	+ 0.9	-49.0
	Total.	-1340.9	-475.9	0.355	-560.4	-317.0

Note:- Dead load eccentricity of ·25' assumed.

TABLE NO. 5

SUMMARY — MAXIMUM STRESSES AT QUARTER POINT.

MAXIMUM POSITIVE MOMENT

	Loading	Direct Stress	Bending Stress.	Max. Stress Concrete	Max. Stress. Steel.
1	Dead Load	- 585	158	743	
2	Live Load	- 22.9	424	447	
3	Temperature 20° Var.	+ 0.7	26	25	
4	Shrinkage	+ 0.9	36	35	
		- 606.3	644	1250	11990

MAXIMUM NEGATIVE MOMENT.

	Loading	Direct Stress	Bending Stress.	Max. Stress Concrete	Max. Steel Stress
1	Dead Load	- 585	158	743	
2	Live Load	- 32.8	307	340	
3	Temperature 20° Var.	- 0.7	26	25	
4	Shrinkage	+ 0.9	36	35	
		- 617.6	527	1143	11025.

Note:- Stresses in lbs. per sq.in.
Dead load bending stresses due to an assumed eccentricity of ·25 feet.
Value of "n" for stress calculations is 10.

TABLE NO. 6

SUMMARY – MAXIMUM STRESSES AT THE CROWN.

MAXIMUM POSITIVE MOMENT.

	Loading	Direct Stress	Bending Stress	Max. Stress Concrete	Max. Stress Steel.
1	Dead Load	- 553.	150	703	
2	Live Load	- 34.4	265	299	
3	Temperature 20° Var.	+ 0.7	35	34	
4	Shrinkage	+ 0.9	49	48	
		-585.8	499	1084	10 450

MAXIMUM NEGATIVE MOMENT.

	Loading	Direct Stress	Bending Stress	Max. Stress Concrete	Max. Stress Steel.
1	Dead Load.	-553.	150	703	
2	Live Load	-17.6	83	100	
3	Temperature 20° Var.	- 0.7	35	34	
4	Shrinkage	+ 0.9	49	48	
		-560.4	317	885	8525.

Note:- Stresses in lbs. per sq. in.
Dead load bending stresses due to an assumed eccentricity of .25 feet.
Value of "n" for stress calculations is 10.

various stress conditions are tabulated in Table No. 4. The maximum stresses are shown in Table No. 5 and Table No. 6. The stresses are shown separately so that the significance of the loading will be obvious. Also the direct stress and bending stress are both shown so that the stress distribution at the section is apparent.

Arch Ring Stresses Due to Wind

Stresses at Crown due to Truss Action of Arches: A wind load of 30 lbs. per sq.ft. will be considered as acting on both arches and hangers.

Total load will be 2(46140 + 10590) = 113,460 lbs.

The maximum moment (at the centre) considering this load to be uniformly distributed along a beam

$$M = \frac{Wl}{8} = \frac{113,460 \times 288}{8} = 4,080,000 \text{ ft.lbs.}$$

$$\text{Direct stress in arch ring} = \frac{4,080,000}{25} = 163,000 \text{ lbs.}$$

$$\text{Unit stress in arch ring} = \frac{163,000}{2355} = 70 \text{ lbs.in.}^2$$

This will exist as tension in the leeward ring and compression in the windward ring.

Note: This analysis is only approximate, as the beam is curved and torsion will also be developed.

Stresses at Springing due to Vertical Reaction: The overturning moment causes a vertical reaction of 83,000 lbs. The resultant shear across the ring will be

$$\frac{82,000 \times .8323}{3 \times 5 \times 144} = 31.6 \text{ lbs.in.}^2$$

The normal dead load stress is 710 lbs.in.^{2}
The resultant combined stress will be from

$$\frac{S_1}{2} \pm \sqrt{\left(\frac{S_1}{2}\right)^2 + S_s^{\,2}} \;, = -715 \text{ lbs.in.}^2 \text{ or } +5 \text{ lbs.in.}^2$$

From the preceding investigation of wind stresses it is seen that none of them are of importance. Also the working stresses may be increased 30% for combined loading where wind stresses are considered.

Design of Arch Sway Bracing

The purpose of the sway bracing is to transmit the shear due to wind loads to the arch ring and to give rigidity to the structure. The sway bracing must also carry shear due to loads placed unsymmetrically between the arches. The latter shear is of no significance from the design viewpoint, as the live load arch deflections are very small.

Maximum Wind Shear

This is due to a moving wind load of 30 lbs.sq.ft. The maximum shear will be taken equal to one half the wind load on the arch and hangers. This shear amounts to 28,400 lbs. (Page 36). The steel area required to take this is $\frac{28,400}{18,000}$ = 1.58 sq.in.

Rigidity

From the standpoint of rigidity the crossbracing section must be 12" x 36". Four $1\frac{1}{4}$ in.sq. bars will be used in this section. Stirrups will be placed 12" o;c.

to tie the steel together and provide a strong member. This section will meet the requirements for wind and loading shears.

Portal Design

A portal analysis should be made for the portion of the arch where there is no sway bracing. This will not be made, however, as the arch ring members are certainly large enough to care for any lateral moments due to lateral forces.

Arch Reactions due to Wind Loads

Design wind load is 30 lbs. per sq.ft.

Projected area of arch = 153.8 x 5.0

= 769.0 sq.ft. (1/2 Arch)

Centre of gravity of arch area using co-ordinates on page and taking moment areas $y_0 = \frac{\Sigma\, ydA}{\Sigma\, A} = \frac{dA\, \Sigma y}{\Sigma A}$

$$y_0 = \frac{5 \times 15.38 \times 312.7}{769.0} = 31.2' \text{ above X axis.}$$

Area = 2 x 769 = 1538 sq.ft.

Projected area of columns

Column	Axis Length		Column Length	Column y^0	Moment Area
C 1	12.8	-5.5	7.2	3.6	25.9
C 2	21.7		16.2	8.1	131.0
C 3	29.0		23.5	11.7	274.5
C 4	35.5		30.0	15.0	450.0
C 5	40.7		35.2	17.6	620.0
C 6	44.3		38.8	19.4	753.0
C 7	46.8		41.3	20.6	851.0
C 8	48.0		42.5	21.3	905.0
			234.7	117.3	4010.4

y_0 = 4010.4 ÷ 234.7 = 17.1' above floor

Area = 2 x 234.7 x .75 = 353 sq.ft.

Projected area of floor slab, arch tie and floor beams

Floor	288 x .75	= 216
Beams	16 x 1.16 x 5.0	= 93
Tie	288 x 1.67	= 480
Hand Rail	288 x .5	= 144
		933 sq.ft.

y_0 is about 1 foot above X axis.

Total overturning moment about pier tops

Arch - 1538 x 30 = 46140 x (31.2 + 5.0) = 1,670,000 ft.lbs.

Columns - 353 x 30 = 10590 x (17.1 + 3.0) = 213,000 " "

Slab, Tie - 933 x 30 = 27990 x (1 + 5.0) = 168,000 " "

$\Sigma F = 84720$ $\qquad \Sigma M = 2,051,000$ ft.lbs.

Arch Reaction on leeward side $= \dfrac{2,051,000}{25 \times 2} = 41,000$ lbs. per pier.

Some specifications require the projected area to be doubled. This would give a reaction of 82,000 lbs.

The effect of the wind load on the foundation will be considered later.

DESIGN OF ARCH SUPPORTS AND PERMANENT HINGES

The arch support will consist of a steel shoe connected by a pin to the arch and to the horizontal tie. The pin connection will serve as a permanent hinge, the necessity of which has been explained in the general discussion. The reactions from the arch rib will be transmitted to the pin connection by using a structural steel connection at the ends of the arch rib.

The writer considered using a rocker column to care for the expansion of the structure, but the idea did not seem feasible, the vertical reaction being too large to be taken in this manner when horizontal displacement was considered. Crushing of the bearing surface was inevitable.

Expansion will be provided for by using a set of caged steel rollers under one of the rib reactions. This expansion detail will be connected to the arch rib and tie by the pin.

Design of Tie Pin and Anchorage

The arch will be connected to the pin by means of 2 I_s - 33" - 132 lb. spaced 18" o.c. Web thickness = 5/8" Flange width = $11\frac{1}{2}$". A number of angles sufficient to carry the arch thrust will be riveted to the I beams.

The Maximum Rib Thrust = $H_{max} \div \cos \phi_s$ = 1,715,000 lbs.

Required bearing area on pin = 1,715,000 ÷ 24,000

= 71.5 sq.in.

Using a pin 11" in diameter the required width of bearing will be 6.5 in.

Thickness of additional plates required on webs of I's will be $\frac{6.5 - 2 \times .625}{4} = 1.31"$. Use 1-3/8" plates.

Number of rivets required per plate =

$$\frac{1.31 \times 11 \times 24{,}000}{8120} = 42$$

Design of Expansion Rollers

Design Data:

Vertical Reactions -	Dead Load =	810,000 lbs.
	Live Load =	95,000 lbs.
	Wihd Load =	82,000 lbs.
	Maximum Vertical Reaction =	987,000 lbs.

Horizontal Movement of Arch

A temperature variation of 130° (-40°F to 90°F) will be used in this design. This range is in accordance with experiments conducted for similar structures in this country.

Temperature movement will be wtl

$= .0000065 \times 288 \times 12 \times 130 = 2.9"$

Horizontal Movement of Arch Support

Foundation settlement may cause the supports to be displaced horizontally. If a difference in settlement of 1" occurs across the pier width, then the displacement of the support will be 2", since the ratio of pier width to height is 1:2. This amount will be considered in the design.

Total horizontal displacement to be provided for in the design $= 2.9 + 2.0 = 4.9"$

Bearing Requirements

The allowable bearing on steel rollers per inch of length is 600 d. (A.R.E.A. Steel Bridge Specifications). Using rollers 9" in diameter the required roller length will be $\frac{987,000}{600 \times 9} = 183"$

This will be provided by 5 rollers 36" long.

Design of Horizontal Arch Tie

Horizontal Reactions -	Dead Load	1,300,000 lbs.
	Live Load	124,500 lbs.
	Temperature	1,600 lbs.
Maximum Horizontal Reaction		1,426,100 lbs.

A working stress of 18,000 lbs. will be used in the tie.

Required section of steel tie $= \frac{1,426,000}{18,000} = 79.2$ sq.in.

Tie section will be 4 - $1\frac{1}{4}"$ x 16" plates.

Gross area $= 4 \times 1.25 \times 16 = 80$ sq.in.

Number of 7/8" ⌀ rivets required in batten splice (double shear) $= \frac{1,426,000}{16240 \times 4} = 21$ (See Drawing 4)

Deduction for net section (3 rivets) $= 4 \times 3 \times 1.25 \times 1$

$= 15.0$ sq.in.

Net section $= 80 - 15 = 65$ sq.in.

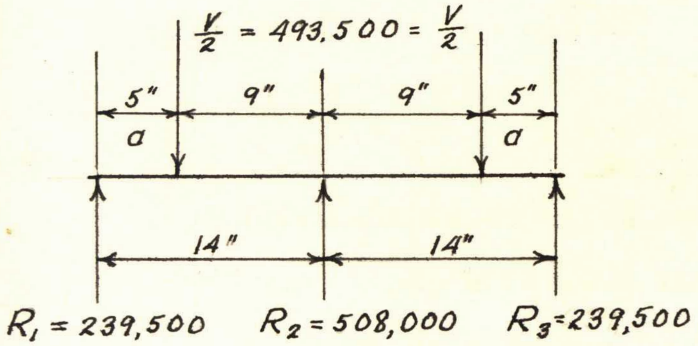
Required section here is $\frac{17}{21} \times \frac{1,428,300}{18000} = 64$ sq.in.

Dead load per ft. of tie $= \frac{80 \times 490}{144} = 272$ lbs.

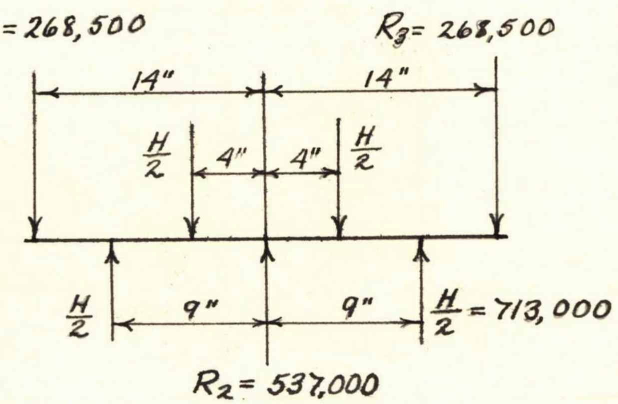
Design of Pin

Bending Moments in Pin.

Forces Acting on Pin



Elevation



Plan

The forces acting on the pin are

$H = 1,426,000$ lbs.

$V = 987,000$ lbs.

Since the pin has a redundant support the reactions will have to be determined by considering pin deflections. The centre support will be taken as redundant. Its value is given by $R = \dfrac{\int \frac{Mmdx}{EI}}{\int \frac{m^2dx}{EI}}$

Considering the vertical forces

$$R_2 = \frac{\frac{Wa}{12\ EI}\left(\frac{3}{4}\ l^2 - a^2\right)}{\frac{l^3}{48\ EI}} = \frac{\text{Deflection at centre of span due to 2 symmetrical loads of W/2 on span}}{\text{Deflection at centre of span due to unit load at centre of span}}$$

Solving the above equation, $R_2 = 508,000$ lbs.

$R_1 = R_3 = 239,500$ lbs.

Similarly considering the horizontal forces on the pin

$$R_2 = \frac{\frac{Wa_1}{12\ EI}(\frac{3}{4}\ l^2 - a_1^2) - \frac{Wa_2}{12\ EI}(\frac{3}{4}\ l^2 - a_2^2)}{\frac{l^3}{48\ EI}}$$

$$W = \frac{H}{2} \qquad a_1 = 10" \qquad a_2 = 5" \qquad l = 28"$$

Solving this equation we have

$$R_2 = 537{,}000 \text{ lbs.}$$

$$R_1 = R_2 = -268{,}500 \text{ lbs.}$$

These reactions are shown on the sketch of the pin forces.

The maximum moment in the pin will occur 5" from either end. The value of this moment is

$$M_V = 239{,}500 \times 5 = 1{,}197{,}500$$

$$M_H = 268{,}500 \times 5 = 1{,}342{,}500$$

$$M_{max} = \sqrt{M_V^2 + M_H^2} = 1{,}795{,}000 \text{ in.lbs.}$$

The allowable bending moment for a 11" dia. pin for a working stress of 18,000 lbs.in.^{2} is

$$M = \frac{f\ I}{c} = \frac{18{,}000 \times \pi\ d^3}{32} = 2{,}352{,}100 \text{ in lbs.}$$

This pin is larger than necessary for bending requirements. A pin 10" in diameter would carry the bending moment, but the larger pin is more desirable from the bearing standpoint.

Shear in Pin

The maximum shear in the pin will be between the I-Beam and the tie. Its value is

$$V_V = 493,500 - 239,500 = 254,000 \text{ lbs.}$$

$$V_H = 713,000 - 268,500 = 444,500 \text{ lbs.}$$

$$V_{max} = \sqrt{V_V^2 + V_H^2} = 512,000 \text{ lbs.}$$

$$\text{Unit shear } v = \frac{512,000}{95.03} = 5400 \text{ lbs.in.}^2$$

Bearing on Pin

The maximum pin reaction $= \sqrt{R_{2V}^2 + R_{2H}^2}$

$= 738,000$ lbs.

Bearing area required is $\frac{738,000}{24,000} = 30.7$ sq.in.

Bearing width must be $\frac{30.7}{11} = 2.8$ in.

The maximum tie thrust $= 1,426,000$ lbs.

Bearing width per eye bar must be

$$\frac{1,426,000}{4 \times 24,000 \times 11} = 1.35''$$

Combined Stresses

Since the bending and shear stresses are low, the combined stress will not be investigated. The combined stress will probably be the greatest over the centre support. The determination of this involves a great deal of work and will not be attempted here.

DESIGN OF TEMPORARY HINGES AT CROWN OF ARCH RINGS

Theory and Experimental Work

In the general discussion on this design the necessity of placing temporary hinges in the arch was outlined. The purpose of the hinge is to make the arch act as a three hinged arch until all dead load deflections and rotations have taken place, thus relieving any stresses caused by bending moments that would otherwise be set up in the ring by dead load arch shortening, concrete shrinkage and elastic extension of the arch tie.

A flexural hinge will be used in this design, as it is more economical than other types using steel seats and pins. Two types of flexural hinge are in common use, the Mesnager hinge and the Considere hinge. Flexural hinge action is obtained in both designs by reducing the sectional area at the hinge point to form a section which will give slight resistance to rotation. The Mesnager type is made by crossing steel at the hinge axis, the steel area being sufficient to carry the thrust and shear at the hinge. Articulation is provided by the slight bending of the steel.

The Considere hinge is made by reducing the sectional concrete area. The reduced area is highly reinforced and is under a large working stress. Under this condition the hinge material is sufficiently plastic to allow slight rotations to take place without much restraint.

Experiments conducted on actual hinge sections have been the basis for design. Recently the National Bureau of Standards conducted a series of experiments on the Mesnager type of hinge. A theoretical stress analysis was made also. The results of the work are summarized here.

Reference: "Articulations for Concrete Structures - The Mesnager Hinge", by B. Moreell. Journal of the American Concrete Institute, March-April 1935.

"Mesnager advances the following recommendations for the design of his hinges.

1. With a rotation of .02 radians or less, bare hinge bars can develop their full elastic strength in direct stress.

2. If the l/r ratio for the bars is between 20 and 40, no account need be taken of the bending stress for rotations under .02 radians.

3. Plain bars should have an imbedment of 45 diameters on each side of hinge.

4. The angle of inclination between the bars should be such that the line of thrust at the hinge will be well within the angle for all conditions of loading.

5. The ratio of the l/r value of the bars across the hinge should not exceed 40."

The experiments conducted by the National Bureau of Standards bring out the following points:

1. For rotations as large as .026 radians the bare bars in the hinge opening will safely take an ultimate direct thrust equal to 90% of their tensile yield point with an l/r ratio of 30 to 40.

2. The lateral steel reinforcement in the hinge blocks is very important. This should be designed to carry the vertical component of the bar stresses, and should be in contact with the hinge bars.

3. The hinge bars should not be covered with concrete to a depth greater than .75 the hinge span or at least not more than the hinge span. For the latter case the stiffness of the hinge and the thrust eccentricity will increase from 7.5 to 11.5 times that of the bare bars.

4. Mortar hinge covering increases the ultimate strength of the hinge from 2 to 3 times.

In the theoretical stress analysis the hinge bars are treated as a rigid frame cross braced structure acted upon by a thrust, shear and moment. The equation for stress is derived by considering the deflections of the structure under load. Other factors influencing the stress are the angle θ of the crossed bars and the l/r ratio. Charts for determining stresses are prepared with the variables θ and l/r and the ratio of shear to thrust.

Calculation of Arch Hinge Rotations

Rotation Due to Tie Extension - Dead Load Only.

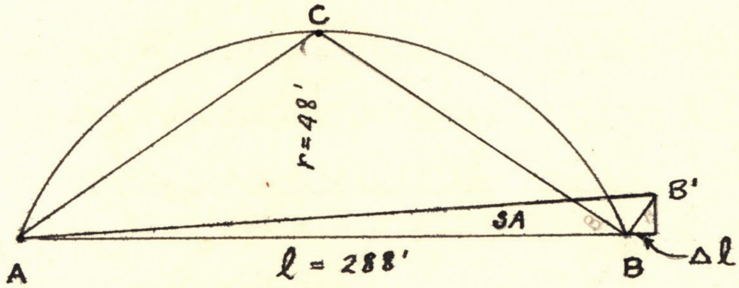
Full Dead Load Horizontal Reaction = 1,260,000 lbs.

Average working stress in steel tie is (using gross area 88 sq.in.) 1,260,000 ÷ 88 = 14,300 lbs.

Maximum working stress = 1,260,000 ÷ 71.5 = 17,650 lbs.

Dead load elastic extension of tie = e = $\frac{\text{unit stress x length}}{E}$

$$= \frac{14,300 \times 288 \times 12}{30,000,000} = 1.65 \text{ in.} = \Delta l$$



∠A

Determination of angles; tan LA = $\frac{48}{144}$ = .3333 = 18°26' = ~~3.2 [illegible] rad~~

∠C

LC = 143°08'

$$\delta A = \frac{\Delta l \times \cot B}{l} \text{ radians}$$

$$\delta A = \frac{1.65 \times 3.00028}{288 \times 12} = -.00143 \text{ radians}$$

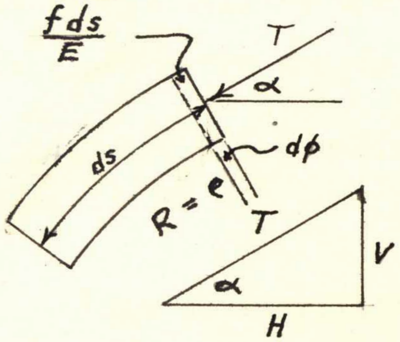
Also δB = -.00143 radians from symmetry

δC will = 2 x .00143 = .00286 radians

?

Rotations Due to Arch Shortening.

Consider a section of the arch of length ds carrying an axial thrust T.



The average compressive force

$$f = \frac{H \sec\alpha}{A}$$

$$d\phi = \frac{fds}{E} = \frac{H \sec\alpha \, ds}{AE}$$

The total angular change in one half the arch axis will be $\Delta\phi = \int_0^{\frac{L}{2}} \frac{H \sec\alpha \, ds}{AE\rho}$ Using the average value of H secα as 1,372,000 lbs. and assuming a circular arch axis with a radius of 240' we will have an approximate value for the above integral expression

H sec α Length arc $\int_0^{} ds$

$$= \frac{1,372,000 \times 153.8 \times 12}{16.34 \times 144 \times 3 \times \underset{E}{10^6} \times \underset{\rho}{240 \times 12}}$$

S = ?

$$= .000125 \text{ radians}$$

The change in the chord length of the semi-arch axis will be calculated by neglecting the deflection Δy at the crown, as this is negligible. The deflection Δx will be approximately equal to $\Delta\phi\,\rho$ since the radius of curvature is very large. The true change in chord length could be determined by locating the deflected position of the semi-arch crown by evaluating the integrals

$$\Delta y = \int_0^{\frac{L}{2}} \frac{H \sec\alpha \, ds \, x}{AE\rho}$$

$$\Delta x = \int_0^{\frac{L}{2}} \frac{H \sec\alpha \, ds \, y}{AE\rho}$$

The approximate $\Delta x = \Delta\phi e$ = .000125 x 240 = .030'
The corresponding shortening of the chord of the semi-axis will be $\Delta x \cos A$ = .03 x .94869 = .0284
Original chord length = $\sqrt{144^2 + 48^2}$ = 182'

$$\delta A = \frac{\Delta l \cot C}{l} \text{ radians} = \frac{.0284 \times .93688}{182}$$

$$= .0001462 \text{ radians}$$

The total rotation of A will be that equal to the rotation caused by the change in chord length plus one half the total change $d\phi$ of the arch axis central angle. The rotation at C will be the rotation at C caused by the change in chord length minus one half the total change $d\phi$ of the arch axis central angle. The above reasoning is illustrated diagramatically in Fig.
Note that the rotations are the result of the arch shortening in length and the fall of the crown.

The total rotations due to arch shortening will be

Rotation at A = B $.000146 + \frac{.000125}{2}$ = .000208 radians

Rotation at C $2(.000146 - \frac{.000125}{2})$ = .000168 radians

Rotations Due to Concrete Shrinkage and Temperature Changes. The arch axis does not undergo rotation due to concrete shrinkage and temperature changes. The arch span will change in length an amount equal to Kl for concrete shrinkage and wtl for temperature change.

Where K = shrinkage coefficient

w = temperature coefficient .0000065

t = temperature change

In order that temperature change adds to the rotation of the arch at the hinges, the tie must be at a higher temperature than the arch ring. This condition will not likely exist during construction. However, a difference in temperature of 15° will be assumed.

The temporary hinge will not be closed for at least one month. From the Gold Beach Tests the coefficient of shrinkage for this period will be around .00012.

The total change in span length for shrinkage and temperature will be $\ell(K+wt) = .0625'$

The angular changes will be

$$\delta A = \frac{\Delta \ell \cot B}{\ell} = \frac{.0625 \times 3.00}{288} = .000651 \text{ radians}$$

$$\delta B = \delta A = .000651 \text{ radians}$$

$$\delta C = \delta A + \delta B = .001302 \text{ radians}$$

Summary of Rotations.

	Values in Radians	
	Crown Hinge	Springing Hinge
Dead Load Arch Shortening	.000208	.000168
D.L. Extension of Tie	.002860	.001430
Concrete Shrinkage	.000720	.000360
Temperature Changes	.000582	.000291
	.004370	.002249

These values will be used in the design of the temporary flexural hinge at the crown of the arch ribs.

Design of Temporary Crown Hinge

Design Data:

Dead Load Thrust to be taken by hinge = 1,260,000 lbs.

Dead Load Shear is zero.

Maximum calculated rotation of hinge = .0044 radians = ϕ

Arch ring width = 36".

The arch ring is not sufficiently wide to accommodate the required steel area for a Mesnager hinge, so a hinge of the Considere type will be used, although the former is more desirable. The concrete in the reduced hinge section will be reinforced with interlocking spirals. Light longitudinal steel will be provided to support the spirals. Heavy longitudinal steel will be placed at the hinge axis to give resistance to any shear and to assist in carrying the hinge thrust.

With this arrangement the spiralled concrete and steel can both be counted on to carry load. The writer believes this hinge will also allow the structure to articulate without inducing moments of any significance at the hinge.

The reduced hinge section chosen is 14" x 48". The reinforcement provided is 6 - $11\frac{1}{2}$" diameter interlocking spirals, 24 - 5/8"Ø and 11 - $1\frac{1}{4}$ □sq.in. The hinge length is 18". The dimensions of this hinge are in accordance with those recommended. It is also to be noted that the expected rotation is only 1/5 of that given to experimental models.

Stress Calculations:

Transformed area of spiralled core (Scaled) is

$$A_c + (n - 1)A_s = 10.5 \times 42 \quad 5 \times 24.5 = 563.5 \text{ sq.in.}$$

Maximum direct stress = 2240 lbs.in.^{2}

Transformed Moment of Inertia of total section is

$$\frac{bd^3}{12} + (n - 1) \sum A_s k^2$$

$$= 10980 + 5 \times 6.7 \times 4.5^2 = 11,660 \text{ in.}^4$$

Rotation of hinge due to any moment is

$$d\phi = \frac{M}{EI} dl \quad \text{(Fundamental Principles)}$$

For a rotation of .0044 radians as expected

$$M = \frac{.0044 \times 3 \times 10^6 \times 11,660}{18} = 8,550,000 \text{ in lbs.}$$

Eccentricity of hinge thrust to produce this moment will be $\frac{8,550,000}{1,260,000} = 6.7"$

Stress adjustments, plastic flow and the elasticity of the arch ring will reduce the resisting moment of the hinge to a much smaller value. For an assumed moment in the hinge of 4,000,000 in lbs. the maximum bending stress would be

$$S = \frac{Mc}{I} = \frac{4{,}000{,}000 \times 5.75}{11{,}660} = 2{,}100 \text{ lbs.in.}^2 \text{ in the core.}$$

The maximum total stress would then be

$$2240 + 2100 = 4340 \text{ lbs.in.}^2$$

The writer believes that the hinge concrete will not be stressed over 4000 lbs.in.^{2}

Effect of Hinge Resisting Moments on Arch

Temporary Crown Hinge:

The maximum possible resisting moment of this hinge does not produce a thrust eccentricity greater than 6.7". In the calculation of arch ring stresses a dead load thrust eccentricity of 3" was assumed. This caused a bending stress of 150 lbs.in.^{2} at the crown and 158 lbs.in.^{2} at the 1/4 point. The resisting moment that will exist at the hinge may therefore be of importance.

Permanent Springing Pin Hinge:

The resisting moment of this hinge will be due to the pin friction. The value of this moment will be given by

$$M = \frac{PkD}{2}$$

P = direct stress on pin = 1,260,000 lbs.

k = coefficient of friction = .2 (no lubrication)

D = diameter of pin = 11"

$$M = \frac{1,260,000 \times .2 \times 11}{2} = 1,260,000 \text{ in lbs.}$$

(This moment is about ¼ of the magnitude of the maximum crown hinge moment.

DESIGN OF PIERS

The following elevations were estimated from a study of hydrographic records on this river at Edmonton and at Prince Albert.

High Water Level	1475.0
Low Water Level	1450.0

After investigating several existing structures over the Saskatchewan River, the writer decided that the water piers should be carried 16 feet below the river bed in order to avoid the danger from erosion. The foundation material at Pier No. 1 is fine sand which would erode very easily. Pier No. 2 penetrates a strata of gravel and soft yellow clay. Both of these piers will be rip-rapped to lessen the erosion danger.

The pier will be designed so that the combined dead load of the pier, superstructure and surcharge will be centric about the pier base. This is important as the foundation pressure will be uniform and any settlement will not throw the structure out of the vertical position. Unequal settlement would be very objectionable on this bridge, as the total height is about twice the pier length.

As the piers cannot be economically carried to a suitable foundation, bearing piles will be driven to the hard pan.

Dead Load Foundation Pressures

Dead load in pier - 1215 cu.yds. = 4,920,000 lbs.

Dead load from superstructure = 3,236,000 lbs.

Total dead load = 8,156,000 lbs.

Surcharge 1328 sq.ft. x 10 x 100

(Earth) = 1,328,000

9,484,000 lbs.

Water load will not be considered, as the uplift effect will more than offset this. (Uncertain).

Base area of pier = 26 x 70 = 1820 sq.ft.

Dead Load Foundation Pressure = 2.6 tons/sq.ft.

Live Load Foundation Pressures

They are not of any importance as they are of short duration and light.

Maximum live load pier reaction is

294 x 22 x 60 = 389,000 lbs.

Live Load Foundation Pressure = 0.11 tons/sq.ft.

Wind Load Foundation Pressures

See Calculation on page

Overturning moment about pier top = 4,102,000 ft.lbs.

Moment about pier base will be

$$M + \text{Force} \times \text{Pier height}$$

$$4{,}102{,}000 + 84{,}720 \times 52 = 8{,}502{,}000 \text{ ft.lbs.}$$

Moment of Inertia of pier base about

$$= \frac{bd^3}{12} = \frac{26 \times 70^3}{12} = 741{,}000 \text{ ft.}^4$$

Increase in foundation pressure on the leeward side of the bridge will be

$$f = \frac{Mc}{I} = \frac{8{,}502{,}000 \times 35}{741{,}000} = 402 \text{ lbs./sq.ft.}$$

$$= .20 \text{ tons per sq.ft.}$$

Current Pressure

For a water depth of 25' the pressure on the pier, assuming a velocity of 7 ft. per sec., will be

$$P = wK\frac{v^2}{2g} \times \text{Area} \qquad K = \text{about } .8$$

$$= 11{,}500 \text{ lbs.}$$

Moment about pier base = 328,000 ft.lbs.

Increase in foundation pressure = 16 lbs. sq.ft.

This is negligible.

Total Maximum Foundation Pressure

Dead Load	2.60 tons/sq.ft.
Live Load	.11 " " "
Wind Load	.20 " " "
	2.91 tons/sq.ft.

Bearing Piles

Number of piles required using a load of 20 tons per pile is

$$\frac{2.6 \times 1820}{20} = 237$$

225 are provided spaced 2'9" o.c.

Pier Reinforcing

The wall and slab sections of the pier will require reinforcement. Cross pier reinforcement will also be provided under the arch supports to prevent any danger of the pier splitting from lateral forces of the expansion supports.

Ice Break Protection

No steel protection will be used. The pier noses are suitably rounded and will be strengthened with additional cement.

DESIGN OF ABUTMENTS

These abutments carry the vertical reaction of the arch and must also support the terminals and the approach floor system. The latter will be cantilevered.

D.L. of Concrete in Abutment

Wt.		About Right Edge.Mo.Arm	Moment 1000 ft.lbs
Lower lift 18 x 45 x 3 x 150	=	365,000 x 9.0	3280.0
Second lift 14 x 40 x 2 x 150	=	168,000 x 8.5	1427.0
Third lift 10 x 35 x 2 x 150	=	105,000 x 8.0	840.0
Shaft 13 x 8 x 31 x 150	=	484,000 x 7.0	3388.0
Side Walls 2 x 2.5 x 17 x 9.5 x 150	=	121,000 x 19.0	2300.0
Cross Walls 26 x 10 x 1.5 x 150	=	58,500 x 27.5	1610.0
26 x 14 x 1.0 x 150	=	54,600 x 19.0	1037.0
Slab 22 x 19 x .8 x 150	=	50,800 x 18.5	940.0
Pillars 2 x 6 x 3.5 x 19 x 150	=	120,000 x 18.5	2220.0
2 x 4.0 x 1.5 x 19 x 150	=	34,200 x 18.5	633.0
		1,561,100	17,675.0
Less hollow 11 x 10 x 7 x 150	=	115,700 x 7.5	- 868.0
		1,445,400	16,807.0

$\bar{x} = 11.6'$

D.L. of Earth Surcharge

		Wt.		Mo.Arm about Front Edge	Moment 1000 ft.lbs.
Front	3.0 x 45 x 13 x 100 =	175,500	x	1.5	263.2
Back	7.0 x 40 x 13 x 100 =	366,000	x	14.5	5300.0
Sides	7.0 x 8.0 x 13 x 100 =	72,800	x	7.0	510.0
		614,300			6073.2

Resultant of all dead loads

	Wt.		Mo.
Concrete	1,445,400		16,807,000
Earth	614,300		6,073,000
Arch	1,400,000	x 6.0	8,400,000
	3,459,700		31,280,000

$\bar{x} = 9.05'$

Therefore the resultant reaction for all dead loads falls on the centre of the footing.

Dead Load Foundation Pressure

Base area = 45 x 18 = 810 sq.ft.   R = 3,460,000 lbs.

Foundation Pressure = 4270 lbs/sq.ft. or 2.13 tons/sq.ft.

Number of bearing piles required to carry load using a pile loading of 20 tons per pile = 87

The footing has ample area to take this number.

CONCLUSIONS

From the preceding work the writer concludes that the structure designed is practicable. The design stresses indicate that the critical parts are the arch supports and hinges. The remainder of the structure does not present any difficult problems.

The analysis of the arch would be simplified if the equation for the arch axis could be integrated mathematically instead of by the summation method. For bow string arches of constant section this equation approximates a parabola. The error which would be involved by assuming a parabolic curve for the determination of horizontal thrusts due to live loads, temperature variations, arch shortening and concrete shrinkage would be negligible. This is true since the horizontal arch thrust does not vary appreciably with slight changes in the shape of the curve but depends chiefly on the ratio of the arch rise to span length.

After making this design it occurred to the writer that the main advantages of the bow string arch could be attained by fixing one end of the supports and providing a support for the other end, which would allow rotation and sliding to take place. Temperature changes would have no effect on a structure supported in this way. This would result in considerable economy and would simplify the construction work.

The cost of the structure would be reduced greatly if the arch supports could be designed so that the pier width could be decreased. The pier shaft in this design and on bridges constructed is made larger than necessary from a bearing standpoint, in order to provide width for the arch supports. A great deal of unnecessary dead weight is added to the pier which results in a larger and more expensive foundation. The pier load in this design is 60% of the total dead load on the foundation.

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2. Ketchum - Design of Highway Bridges.

3. Urquhart and O'Rourke - Design of Concrete Structures.

4. Johnson, Bryan and Turneaure - Statically Indeterminate Structures and Secondary Stresses.

5. Hool and Whitney - Concrete Designer's Manual

6. Hool and Johnson - Concrete Engineer's Handbook.

7. Journal of American Concrete Institute, March and April Numbers, 1935.

Subjects for Further Thesis Study
Suggested by this Work

1. A study of flexural hinges and supports for reinforced concrete structures.

2. Determination of deflections and stiffness of reinforced concrete members.

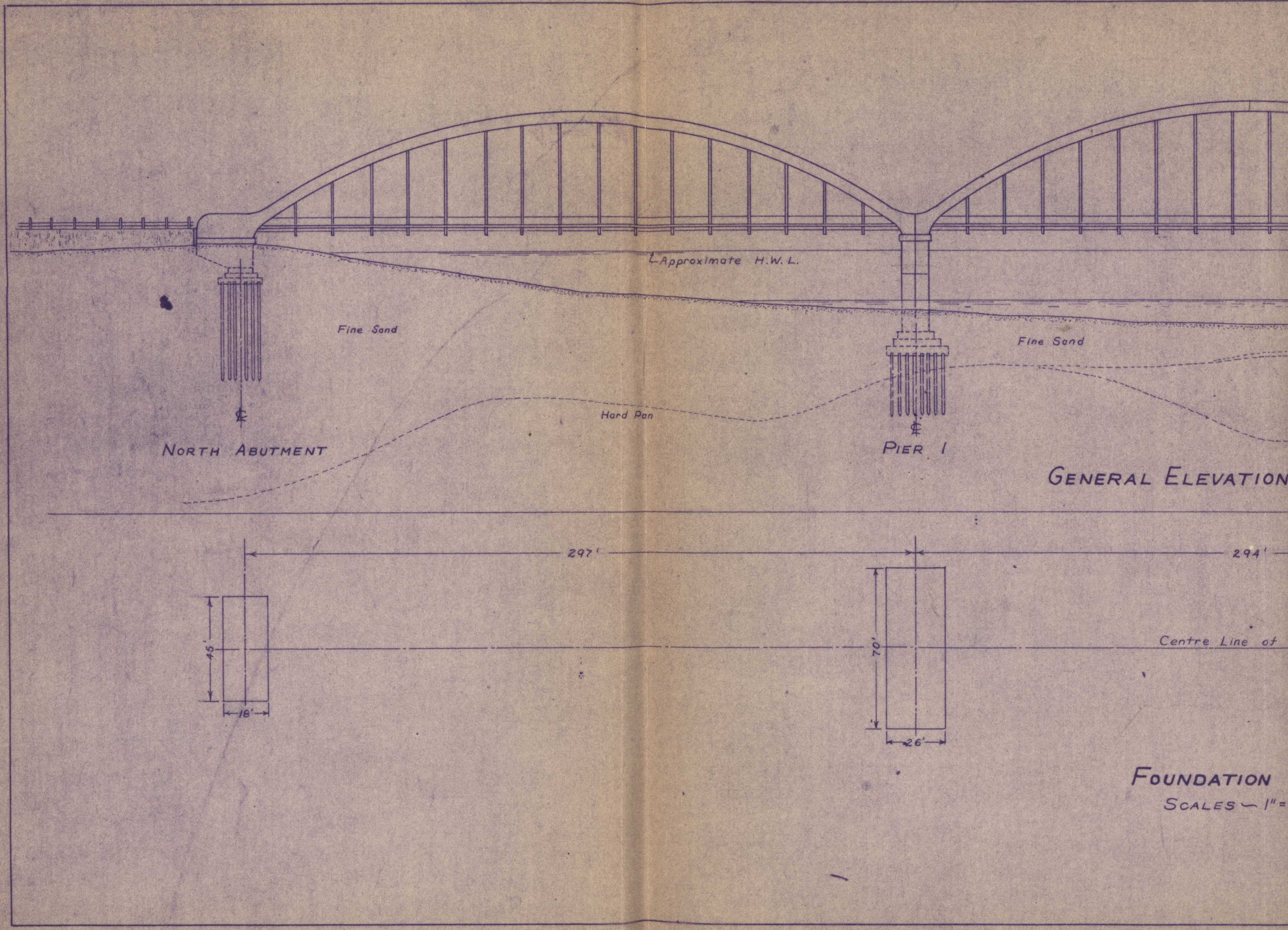
3. Photoelastic determination of stresses in a bow string arch.

   Object: (1) To find the effect of the floor system and hangers on the arch ring.

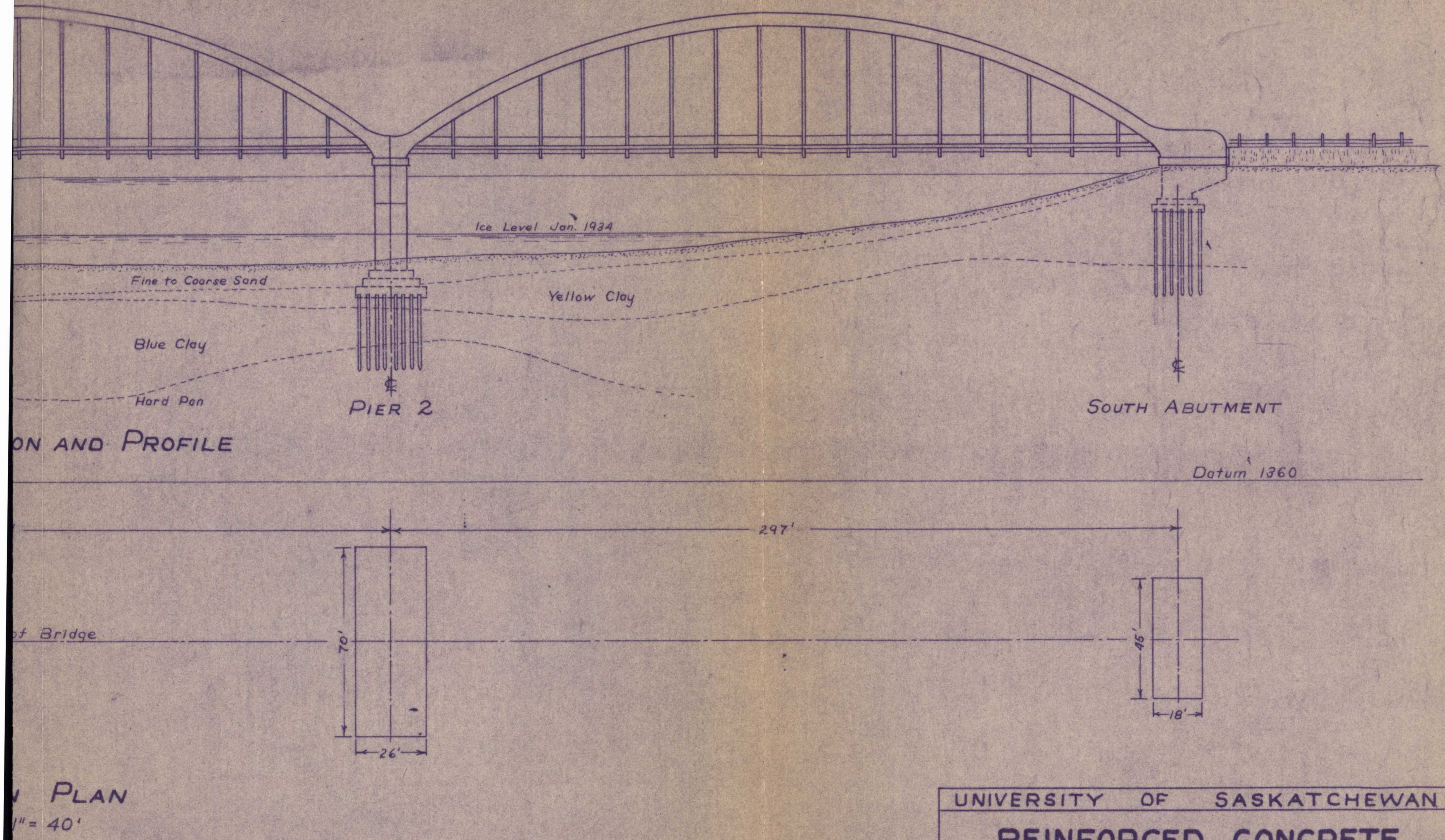
   (2) To determine the stresses at the ends of the arch ring when the tie is constructed continuous with the arch.

4. A study of piers and arch supports for reinforced concrete bow string arches from the standpoint of economics.

5. The design of a reinforced concrete bow string arch having one end fixed to the pier or constructed continuous with an adjacent span.



Approximate H.W.L.
Fine Sand
Fine Sand
Hard Pan
NORTH ABUTMENT
PIER 1
GENERAL ELEVATION
297'
294'
45'
18'
70'
26'
Centre Line of
FOUNDATION
SCALES — 1"=



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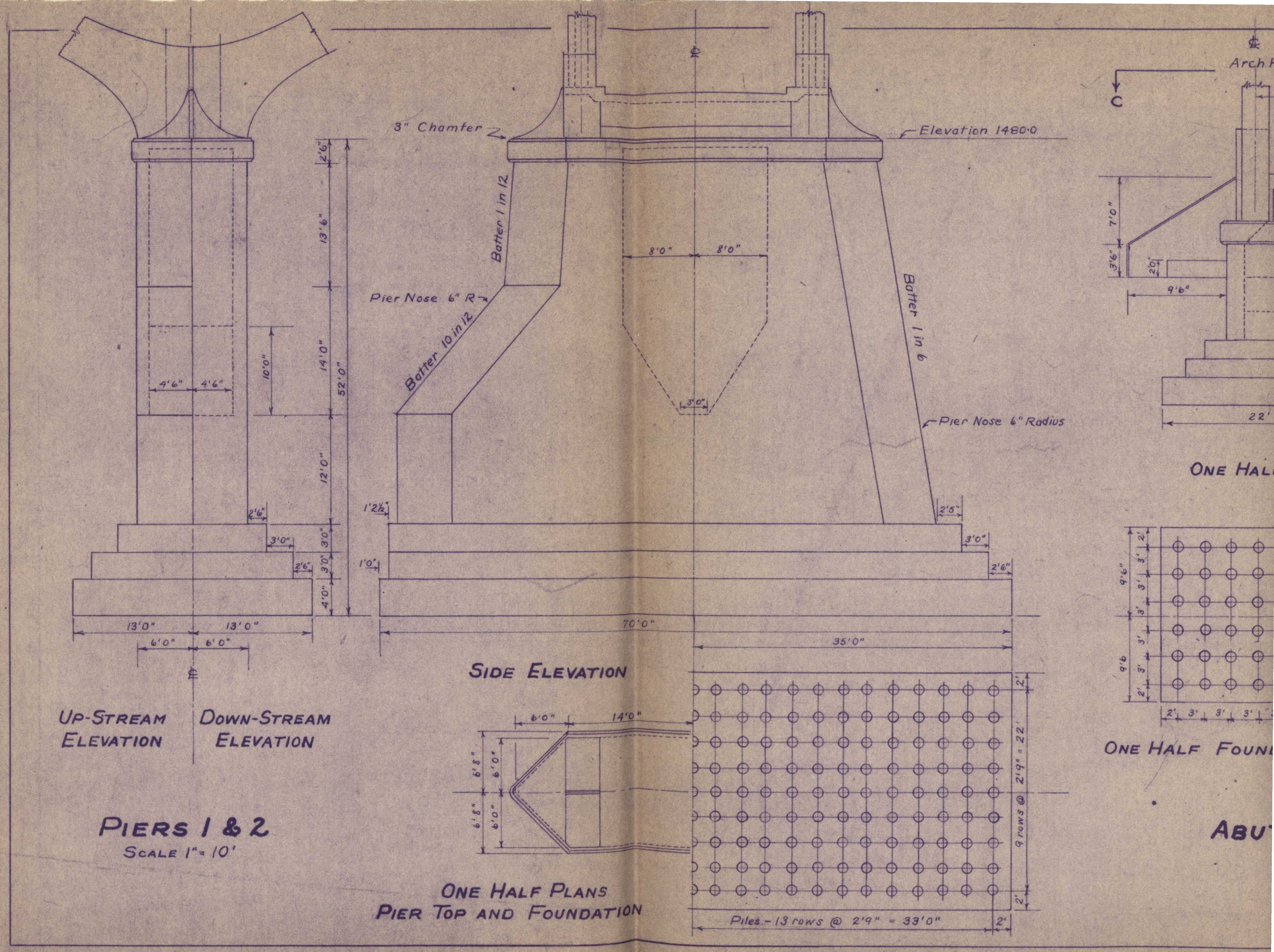
REINFORCED CONCRETE BOWSTRING ARCH BRIDGE

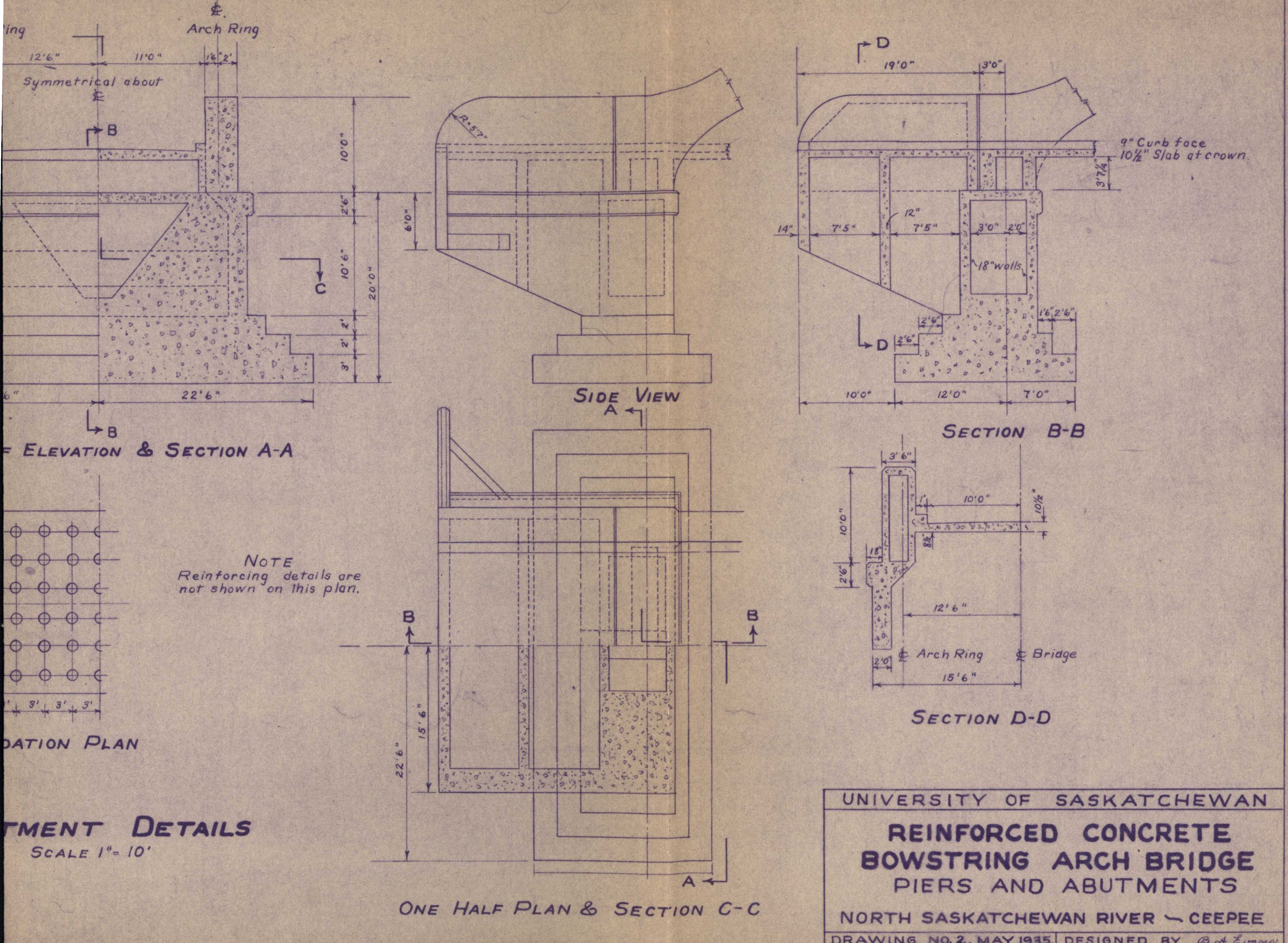
NORTH SASKATCHEWAN RIVER - CEEPEE

DRAWING NO. 1 MAY 1935	DESIGNED BY B. A. Evans

Piers 1 & 2

Scale 1" = 10'





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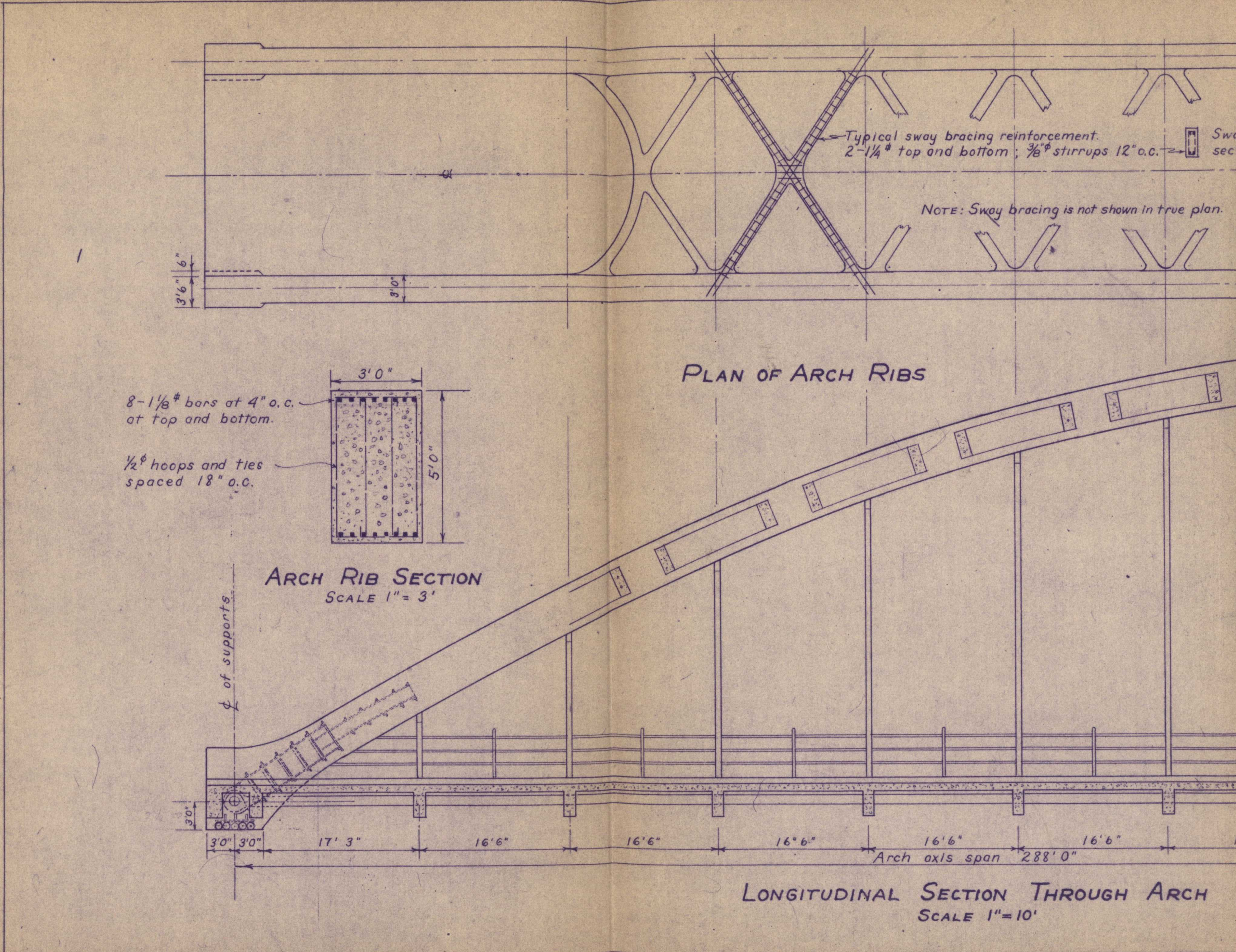
REINFORCED CONCRETE BOWSTRING ARCH BRIDGE

PIERS AND ABUTMENTS

NORTH SASKATCHEWAN RIVER — CEEPEE

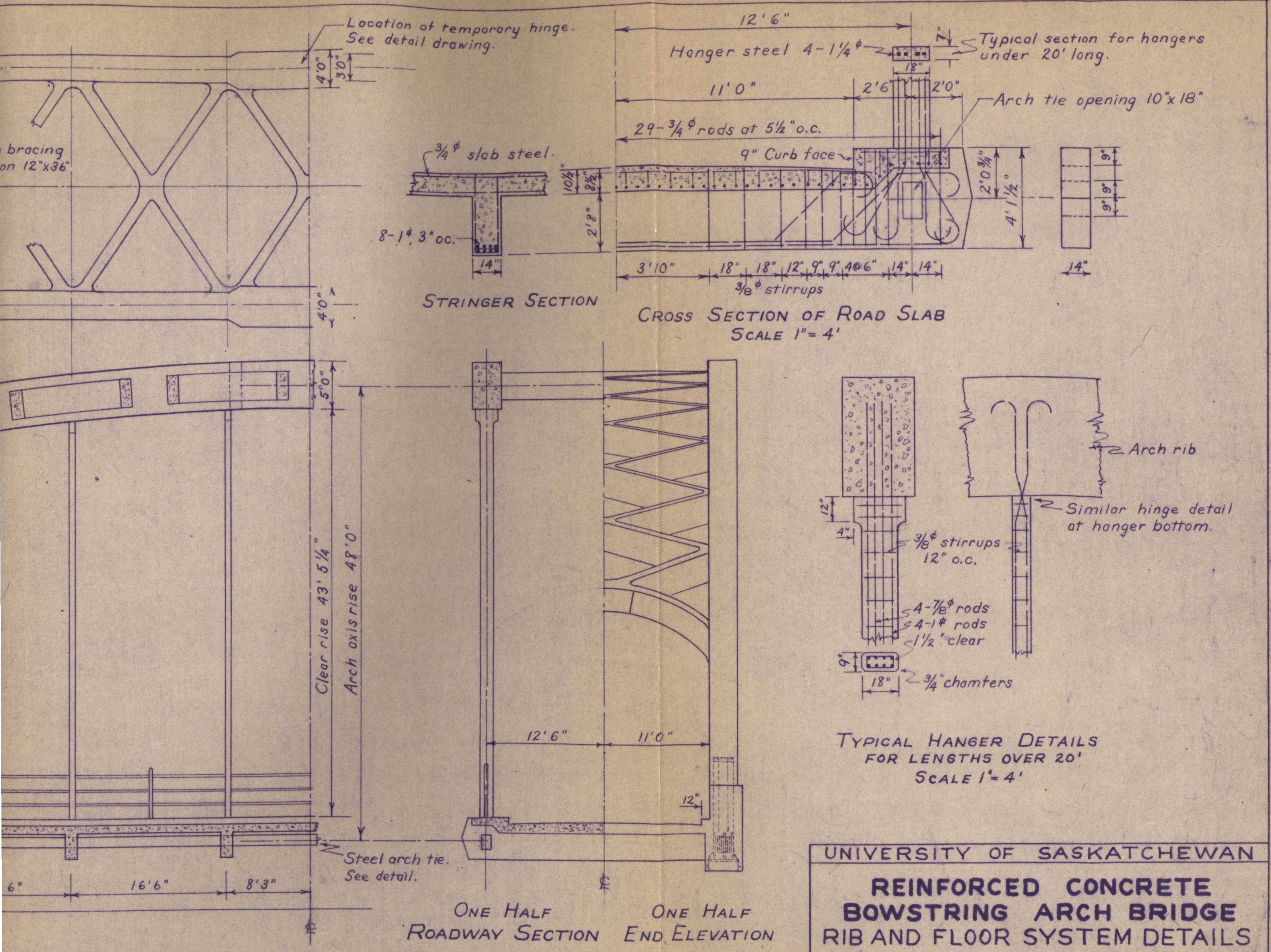
DRAWING NO. 2. MAY, 1935. | DESIGNED BY B. A. Evans



Plan of Arch Ribs

Arch Rib Section
Scale 1" = 3'

Longitudinal Section Through Arch
Scale 1"=10'



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REINFORCED CONCRETE BOWSTRING ARCH BRIDGE

RIB AND FLOOR SYSTEM DETAILS

NORTH SASKATCHEWAN RIVER — CEEPEE

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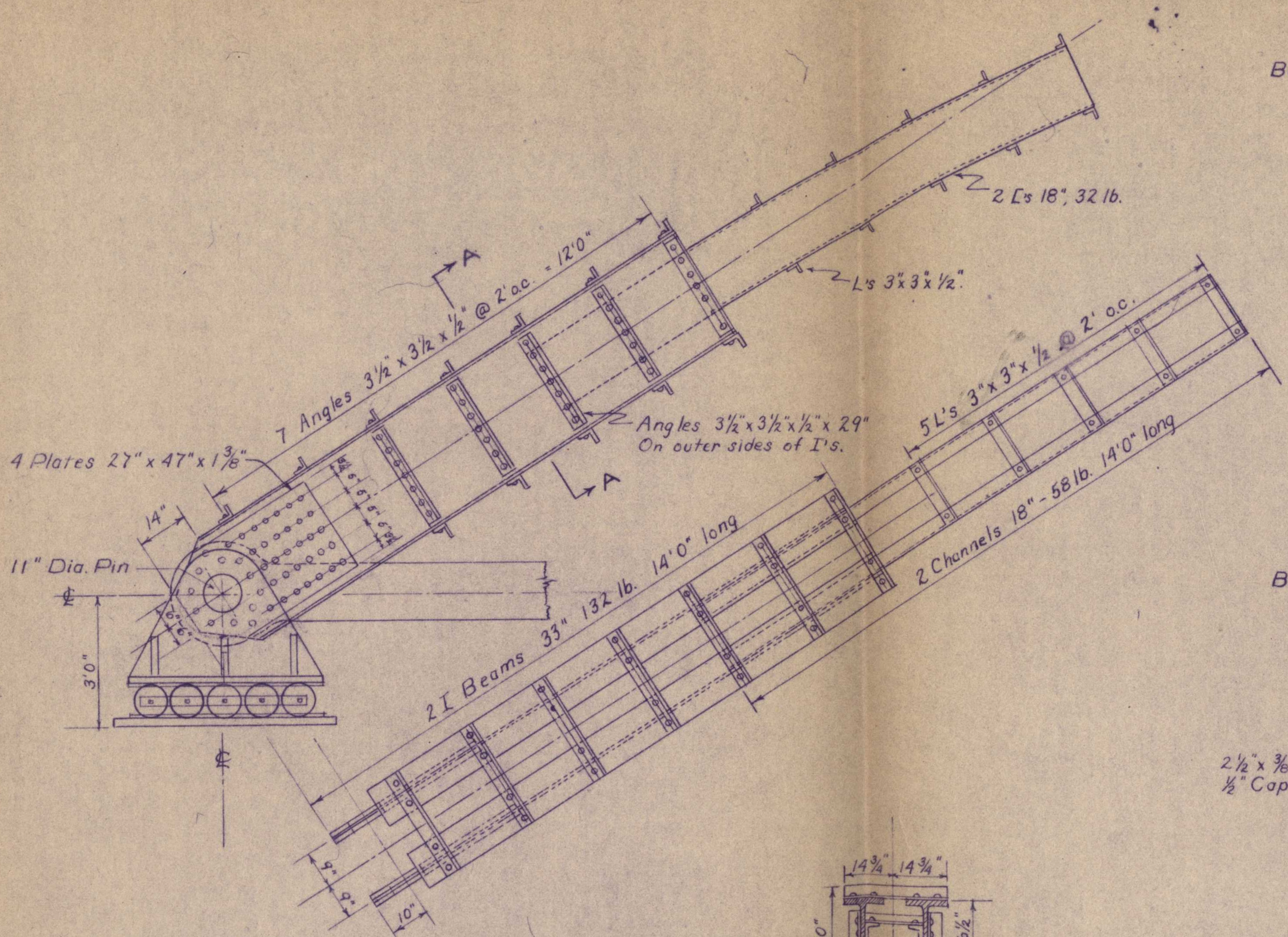
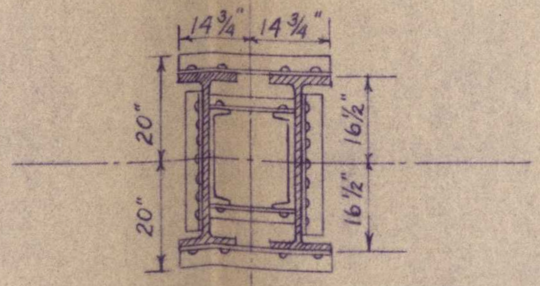
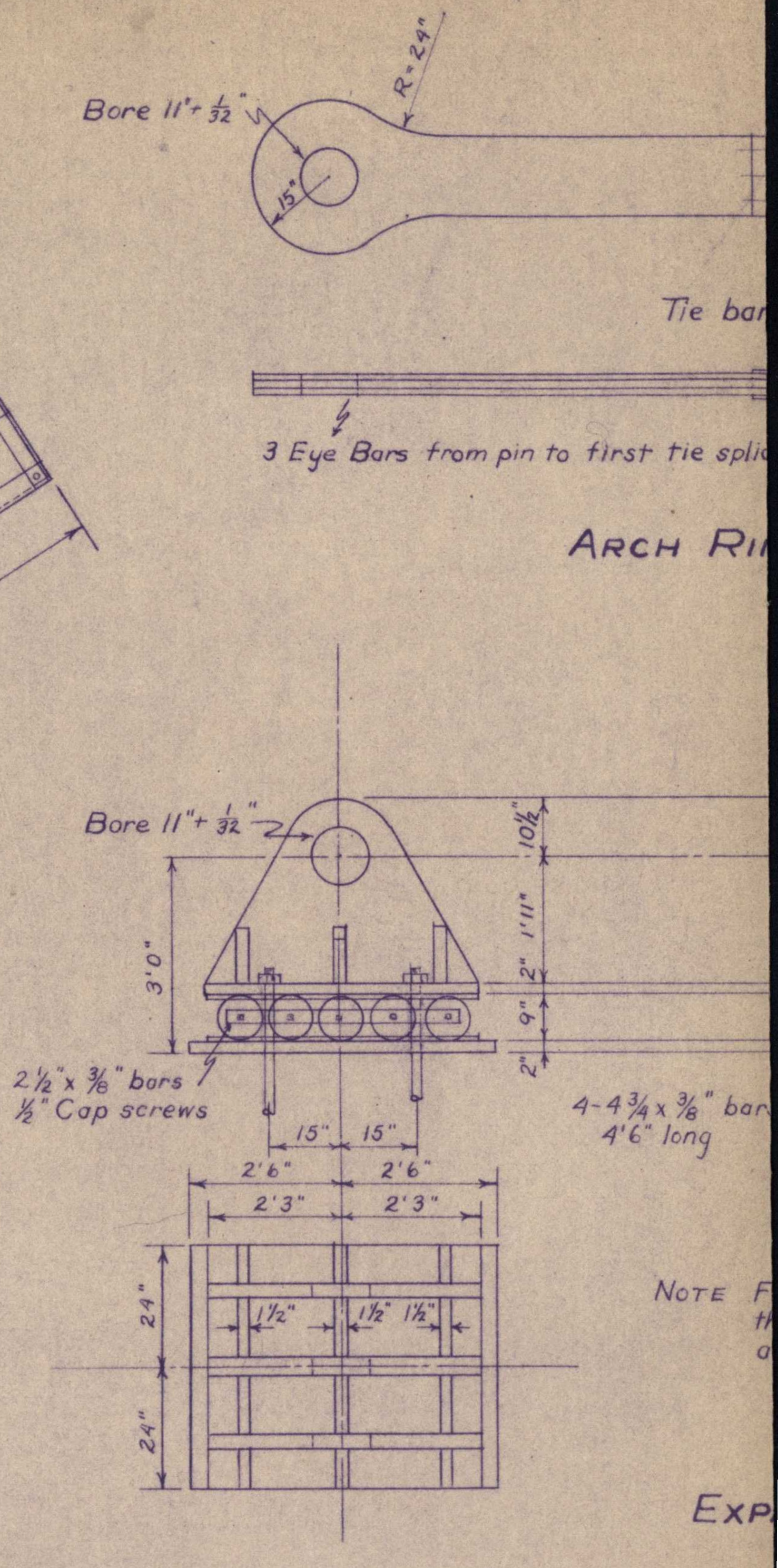


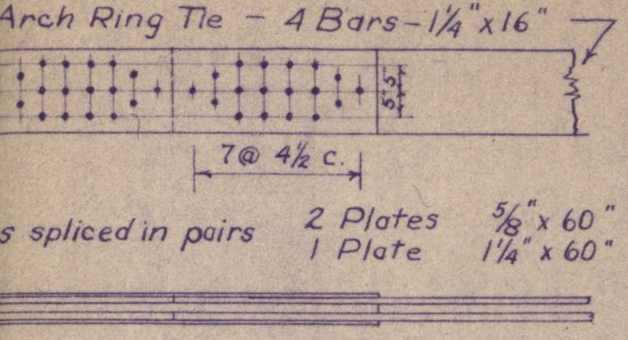
Structural Connection Between Arch Ribs and Tie

Scale 1" = 3'0"

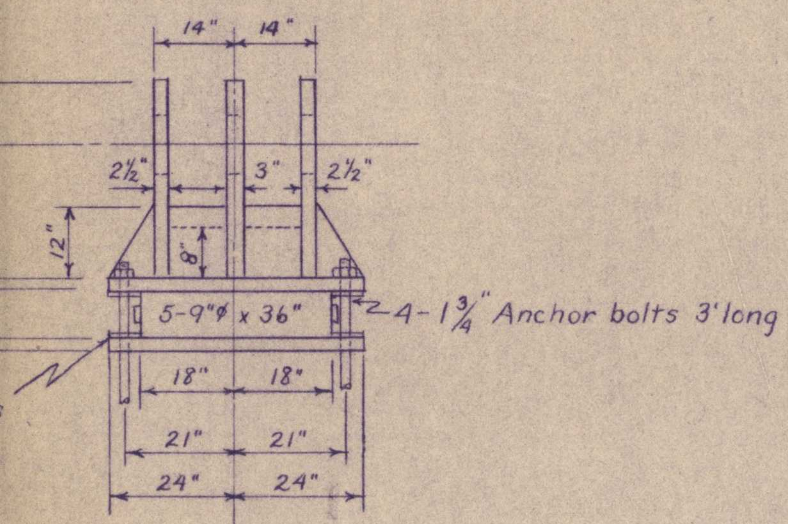


Section A-A



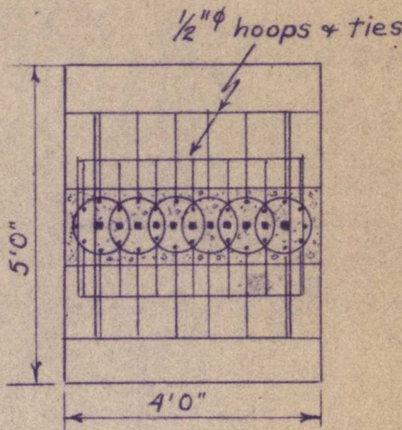


...ng Tie Details

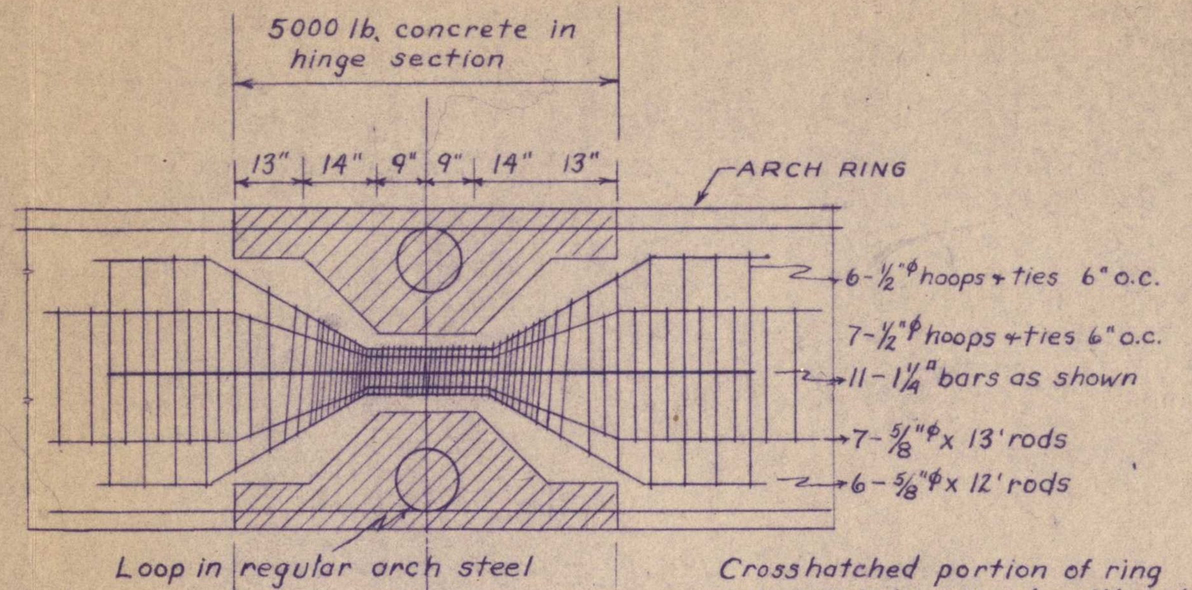


...ixed shoe details are similar to
...e above except the shoe is 9" deeper
...nd the rollers are eliminated.

...ansion Shoe Details



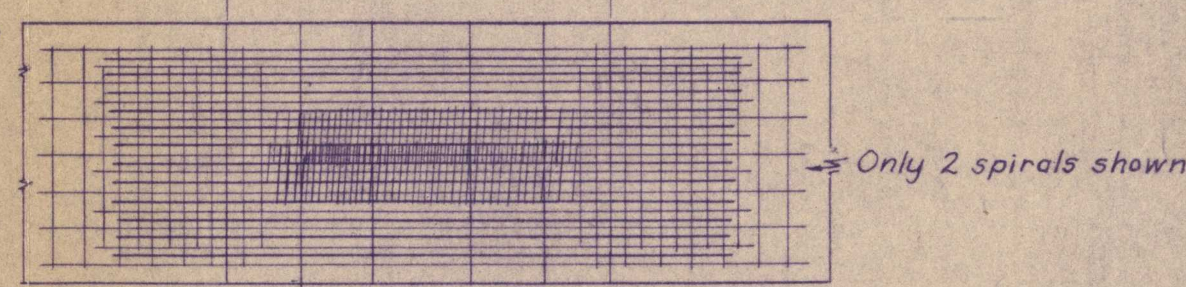
Section at Hinge



Crosshatched portion of ring must not be poured until arch carries full bridge dead load.

Elevation

Hinge consists of six columns of ¼"ϕ spiral, 1⅜" pitch, 11½" outside diameter. Columns are spaced 6½" o.c.



Plan

Temporary Arch Crown Hinge Details

Note
All Scales 1" = 3'

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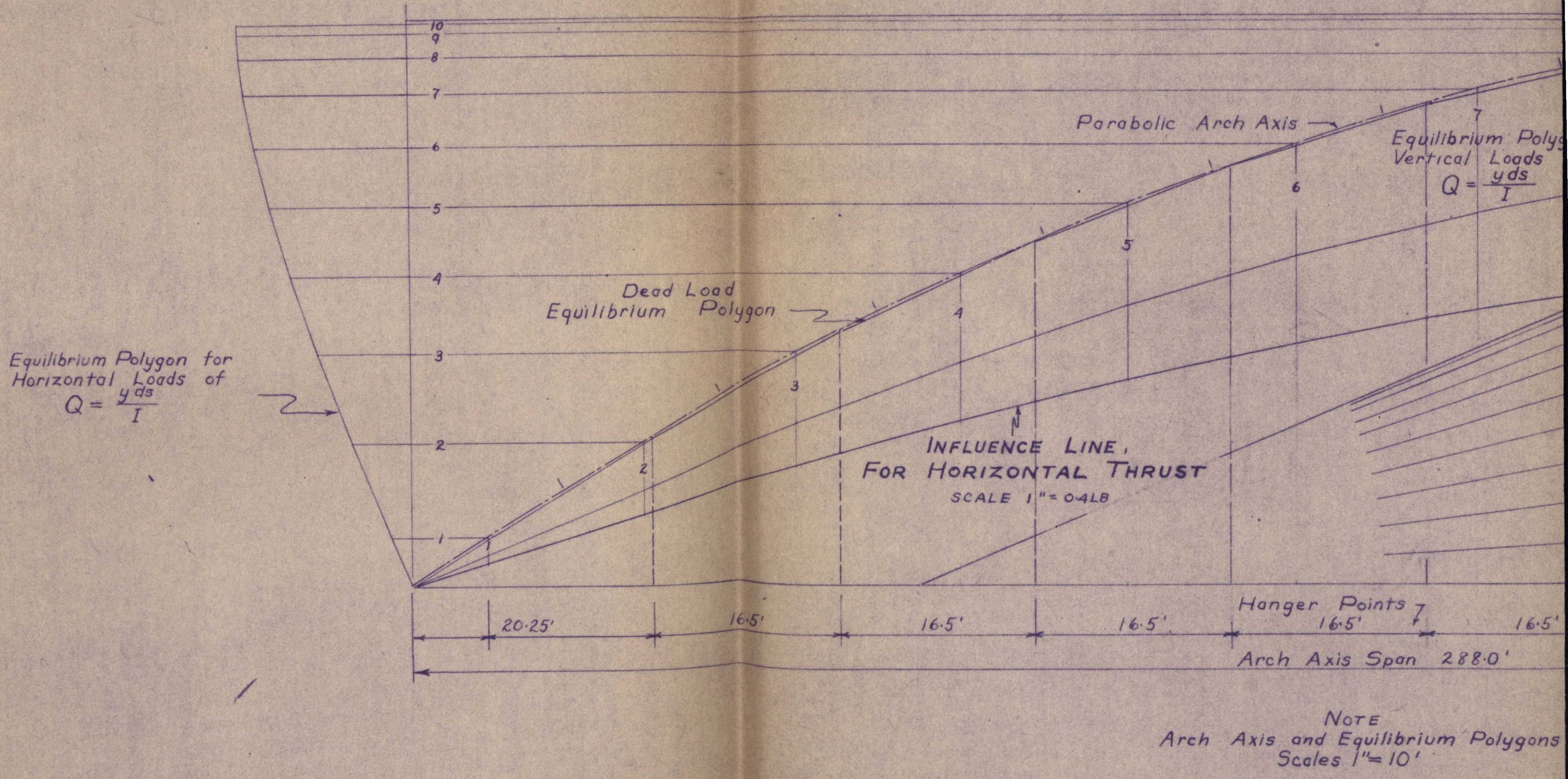
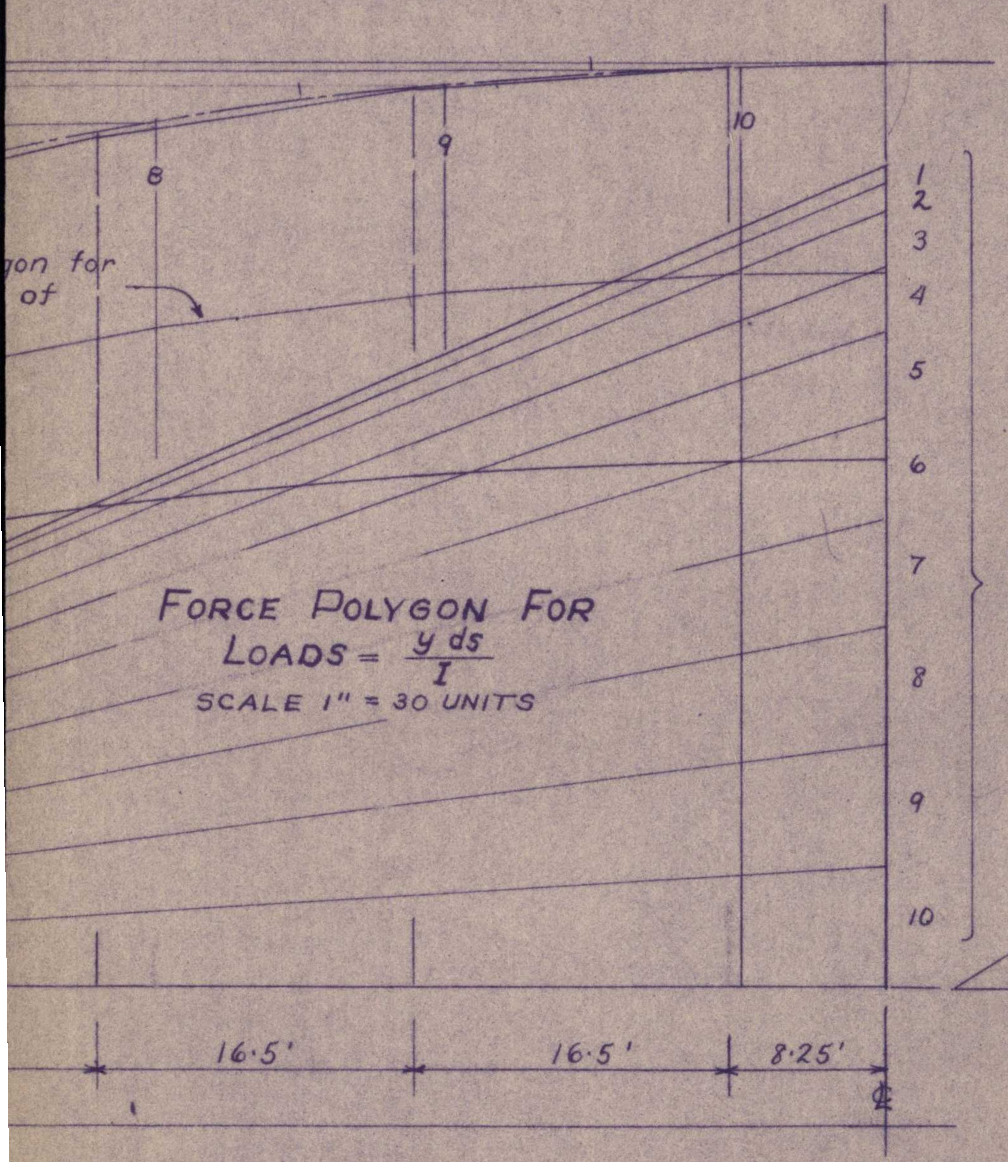
REINFORCED CONCRETE BOWSTRING ARCH BRIDGE

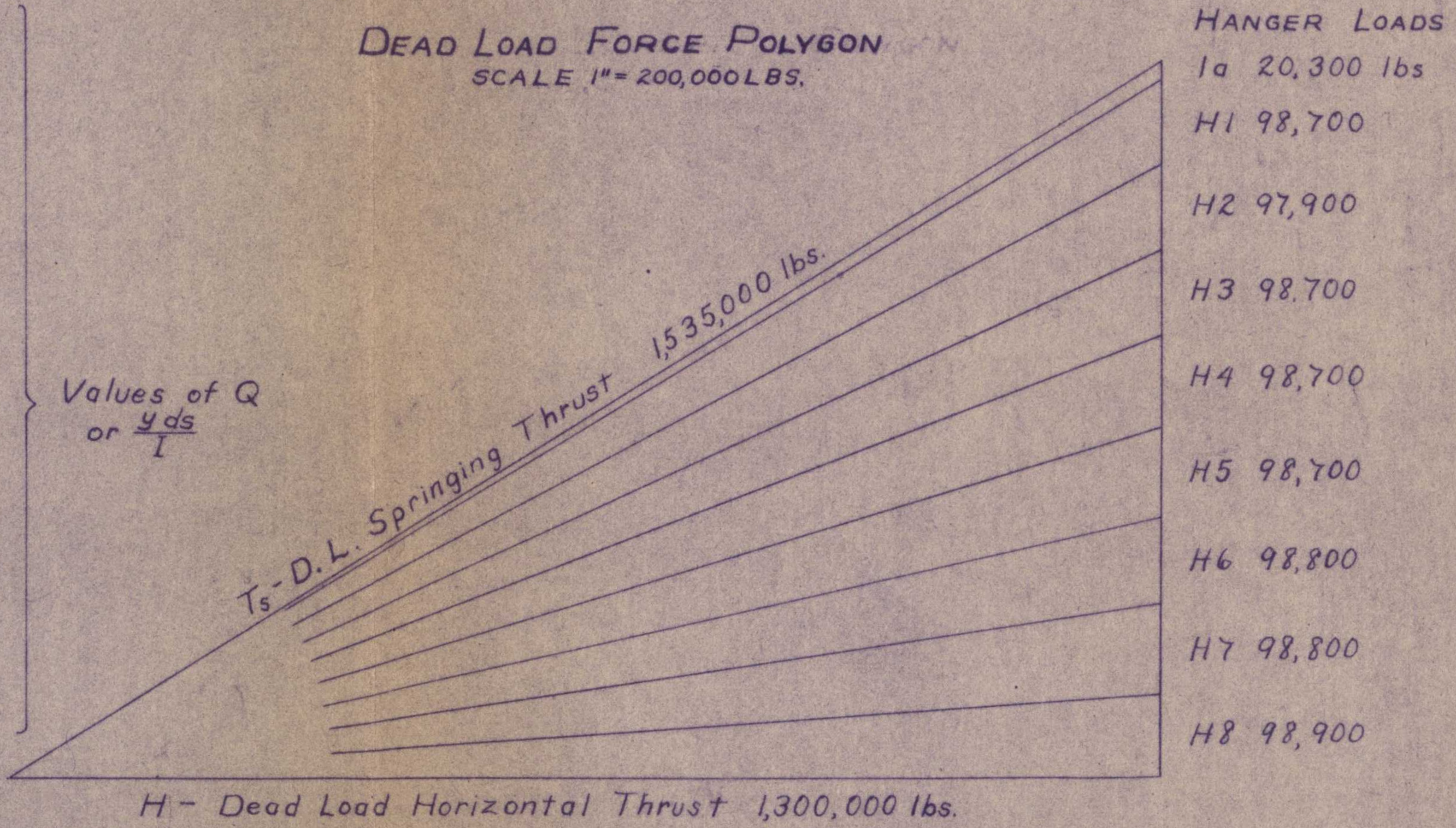
ARCH RING HINGE DETAILS

NORTH SASKATCHEWAN RIVER — CEEPEE

DRAWING No. 4. MAY 1935 | DESIGNED BY B. A. Evans


10
9
8
7
6
5
4
3
2
1
Parabolic Arch Axis
Equilibrium Polyg
Vertical Loads
$Q = \frac{y\,ds}{I}$
Dead Load
Equilibrium Polygon
Equilibrium Polygon for
Horizontal Loads of
$Q = \frac{y\,ds}{I}$
INFLUENCE LINE,
FOR HORIZONTAL THRUST
SCALE 1" = 0.4LB
Hanger Points
20.25'
16.5'
16.5'
16.5'
16.5'
16.5'
Arch Axis Span 288.0'
NOTE
Arch Axis and Equilibrium Polygons
Scales 1" = 10'





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REINFORCED CONCRETE BOWSTRING ARCH BRIDGE

GRAPHICAL ARCH ANALYSIS

NORTH SASKATCHEWAN RIVER - CEEPEE

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