# ALLOCATION OF TRANSMISSION LOSS IN DEREGULATED POWER SYSTEM NETWORKS

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# ABSTRACT

Deregulation in the electric power industry is having sweeping consequences for energy markets. Consumer choice and the changing role of the electricity industry in the marketplace have created new challenges that have to be addressed. New planning, operating and marketing issues related to deregulation are evolving and these issues are being addressed. Although the concept of deregulation has stemmed from the desire for a competitive wholesale power market, the implementation of deregulation means different things to different utilities depending upon the socio-economic structure of the region where the utilities reside. Full deregulation allows bilateral contracts between the suppliers and the buyers. This also leads to non-discriminatory open access to unbundled transmission and distribution networks and abolition of region based monopoly systems. The supervisory control is required for system security and reliability purpose only.

This concept, however, leads to the confusion of transmission loss sharing and generation of the reactive power. Each transaction would cause some transmission loss and thus needs some reactive power to maintain the voltage profile in the system.

Two methods have been developed to determine a generator's share of transmission loss in a fully deregulated power system. They are the Incremental Load Flow Approach (ILFA) and the Marginal Transmission Loss Approach (MTLA). The ILFA is a very simple and straightforward approach that employs an iterative load flow technique. The MTLA proposes a splitting of transmission loss starting from a generalized transmission loss expression and finds the transmission loss share of a generator by utilizing the marginal rate of transmission loss. Results obtained from both approaches agree well. The computational time required by the ILFA is more than that of the MTLA. The details of the two methods along with numerical results have been presented in this thesis.

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| | | |
|---|---|---|
| $[A_p]$ | = | coefficient of power loss equation in matrix form |
| $a_{pij}$ | = | element of matrix $[A_p]$ |
| $B_{ij}$ | = | loss coefficient |
| $[B_p]$ | = | coefficient of power loss equation in matrix form |
| $b_{pij}$ | = | element of matrix $[B_p]$ |
| $[C]$ | = | diagonal matrix with elements $\left( \cos\delta_i / \lvert V_i \rvert \right)$ |
| $c_i$ | = | element of matrix [C] |
| $[D]$ | = | diagonal matrix with elements $\left( \sin\delta_i / \lvert V_i \rvert \right)$ |
| $d_i$ | = | element of matrix [D] |
| $G_a$, $P_{G1}$ | = | real power generation of Generator A |
| $[I_B]$ | = | bus current matrix |
| $I_i$ | = | complex bus current at bus i |
| $[I_p]$ | = | matrix of real components of bus currents |
| $I_{pi}$ | = | real component of bus current at bus i |
| $[I_q]$ | = | matrix of reactive components of bus currents |
| $I_{qi}$ | = | reactive component of bus current at bus I |
| $[J]$ | = | Jacobian matrix |
| $K_{lo}$ | = | loss constant |
| $Load_a$, $P_{D5}$ | = | real power demand of customer A |
| $Loss_a$, $L_a$ | = | share of real transmission loss of Generator A |
| $M_a$ | = | share of reactive transmission loss of Generator A |
| $[P]$ | = | real power injection matrix |
| $[P_D]$ | = | real power demand matrix |
| $P_{Di}$ | = | real power demand at bus i |

# CHAPTER 1: INTRODUCTION

## 1.1 Evolution of Power Systems

Electric utilities have been in business for more than one hundred years. Over this time period power system networks and utilities have gone through various stages of evolution. Starting from very small networks, utilities have become widespread and complex in nature. Electricity has become an essential commodity and millions of pieces of equipment and accessories are in use worldwide which are solely dependent on electric power. Electric power system networks and their operations are different in different countries.

Operation of a power system is a complex phenomenon which not only depends on the state of technology but also on complex issues like economy, social advancement, environmental impact and political decisions. These factors vary from country to country and so do the power system networks and their mode of operation. Every power generating installation, in general, involves an enormous amount of money. That is why, any change in the network or its operation often raises passionate debate. But, due to the demands of social and technological advancements, the changes in power system networks and their operation are inevitable. Change or modification in a progressive way is a natural phenomenon.

The Pearl Street Power Station in New York is the pioneer in electric power generation [1]. This very company established by T. A. Edison was also responsible for power transmission and distribution to a small network compared to today's average size networks. That network was entirely isolated and showed the pathway for modern interconnected network systems which are serving billions of people all over the world today.

At the beginning, electric utilities were supplying power to neighborhoods only, as electricity was not a well-known commodity by that time. With the growing demand for electricity, the networks were required to expand. The contentious issue of right of business in a particular area was sparked off with the expansion of the networks. The monopoly system had been accepted as a solution and the utilities divided the territory among themselves for supplying electric power.

Since then the utilities have gone through different phases with the changes in technology. With the introduction of openness in technical fields In North America, Brazil, United Kingdom and many other countries, the utilities are being forced to change from monopoly systems to deregulated systems, although full deregulation is yet to be realized.

### 1.2 Power Systems in Different Countries

The original idea of a monopoly system still exists in different countries of the world whereas some countries are progressing rapidly towards deregulated systems. The nature of existing systems in different countries, their operation and modifications are very complex and can not be described with a single standard model. Every country has its own problems ranging from social to political and their network systems evolve based on these factors.

In most of the developing countries, the power networks can be categorized broadly as monopoly systems. In India, the regions have been divided among different utilities and within the allocated region the operating utility works all alone. Although after having independence in 1947 India nationalized most of its heavy and medium industries, the power sector remains outside of this nationalization scheme. Both public and private utilities have been operating within their allocated regions and they are connected by tie lines. This scenario can be termed as a decentralized monopoly system. The semi-privatized system in India is now working towards a deregulated environment.

In Bangladesh, the system has been running on a very old concept of operation [2]. The generation, transmission and most of the distribution is controlled by the state-owned Bangladesh Power Development Board (BPDB). Due to a high level of system loss in the distribution sector, two other government agencies have been assigned some of the

distribution responsibilities. Recently, to overcome the power shortage in the country, BPDB has started buying power from a private generating company for the first time in its history.

The electric power system in Brazil consists of two major interconnected systems (South-Southeastern-Central and North-Northeastern) [3] and many small isolated systems in remote regions. These interconnected systems are separated and operationally independent. A government controlled holding company, Eletrobrás, is responsible for implementing Brazil's electric power policy. The company plans, finances, coordinates and supervises programs for the construction, expansion and operation of electric power generation, transmission and distribution systems. Power generation and transmission are dominated by Eletrobrás, while distribution is predominantly the domain of other companies owned by state and municipal authorities and a few privately owned utilities. Main transmission lines in Brazil are the property of Eletrobrás subsidiaries and state companies such as Cesp, Cemig and Copel.

On the way to full deregulation, Brazil is currently in the process of creating a wholesale energy market, roughly similar to the system Argentina has had for the past 6 years. Under such a system, generation, consumption and prices would follow free market conditions, and would allow for quicker responses to the fluctuations of supply and demand.

While approaching a new millenium, Canada's power generators face the challenges of deregulation and market restructuring. Deregulation is based on the principles that all customers share both the costs and the benefits of existing, regulated generation units. The customers will benefit from competition and the generators of electricity will benefit from efficient operation.

With the inducement of Alberta's Electric Utilities Act (EUA) on January 1, 1996, Alberta became the pioneer in deregulating the power industry in Canada [4]. The act has been introduced with the goal of introducing customer choice and service and promoting more efficient and competitive pricing. This will allow generators to determine their own pricing and investment policies according to the demands of the marketplace and competitive market forces, rather than allowing regulatory authorities

to determine these policies for the generators. The customers would choose their own retailer of electricity. Edmonton Power initiated the concept of customers contracting with their utility company as the flexibility offered by deregulation.

Under the EUA, all generators of power in the province sell into the Power Pool of Alberta and distributors buy from the Pool. The provincial grid of transmission lines is supervised by an independent authority, which ensures that power generators and distributors can access the market on fair terms. Owners of transmission facilities and existing generating units must file separate tariffs (the costs and terms of service) with the Alberta Energy and Utilities Board (AEUB) for approval. The tariffs are implemented on an interim refundable basis subject to the results of a public review prior to final approval. Because distributors must pay a proportionate share of the cost of generating and transmitting electricity, they will receive a proportionate share of the total refunded amount, depending on their customer base.

Transmission and distribution systems under the EUA will remain regulated, and existing distribution utilities such as Edmonton Power will continue to provide connections to customers, and maintain their distribution lines. Starting in 1999, a limited number of large customers in Alberta are expected to have direct access to the Power Pool, with customer choice available to the remainder of customers in 2001. Also starting in 2001, the legislated financial arrangements ("hedges") established under the Electric Utilities Act, which are currently regulated by the AEUB, will be replaced by long-term arrangements which ensure fair sharing of costs and benefits. The terms of these arrangements will be set by an Independent Assessment Team to be approved by the AEUB in 1999. In future the province expects separate generation, transmission and distribution operations to accommodate new generation and retail operations.

In Saskatchewan, SaskPower is the only authorized utility to serve the consumers and the monopoly system has been in operation till date. However, there is a growing demand for power in Saskatchewan and SaskPower has evaluated options to add new power supply using the most cost-effective and feasible long-term solution available. As a part of the plan, SaskPower will purchase 210 megawatts of non-utility (NUG) power from the Meridian Cogeneration Project from December 1999 [5].

The traditional electric utility, a self-contained generation, transmission and distribution enterprise, is disappearing. With the developments in the power sectors across the United States, Canada and Europe, competition will be an unavoidable reality in Saskatchewan as early as the turn of the century. In a deregulated environment, competitors would be able to use SaskPower's transmission and distribution infrastructure.

### 1.3 Deregulation of Power Systems

Deregulation of the electricity industry is a growing trend around the world. One of the fundamental objectives of deregulation is to provide consumers with meaningful choices. Deregulation in the electric power industry is having sweeping consequences for energy markets. Consumer choice and the changing role of the electricity industry in the marketplace have created new challenges that have to be addressed. New planning, operating and marketing issues related to deregulation are evolving and these issues are being addressed. Although the concept of deregulation has stemmed from the desire for a competitive wholesale power market, the implementation of deregulation means different things to different utilities depending upon the socio-economic structure of the region where the utilities reside. Of all the ideas related to the implementation of deregulation, the following three are becoming prevalent in the electric power industry.

- An electric utility may be divided into three separate entities according to their roles in the electricity business. These entities are: generation, transmission and distribution. Each entity runs their business independent of each other.
- An independent operator (pool) may buy electricity from the generators and sell it to the customers. The independent operator acts as a broker between the buyers and sellers and also is responsible for operating the system.
- In a fully deregulated system, each consumer is free to select its source of supply.

The main objective of the first type of deregulation is to gradually withdraw the crown investment and allow citizens to buy shares of the electric industry. In the second type, an independent operator works as a middleman between the generators and the consumers. Generating units or companies offer their price to the middleman who has

the buying offer from the demand side too. The function of the middleman is to match the prices and regulate the power flow between the suppliers and buyers. In a fully deregulated system, the generators are responsible for their respective loads and their share of transmission losses. In addition, each generator must pay a combination of network access and usage fee to the network entity. Full deregulation allows direct contract between the suppliers and the buyers and restricts the function of the middleman to the supervision of the transmission network only. This concept, however, leads to the confusion of transmission loss sharing and generation of the reactive power. Each transaction would cause some transmission loss and needs some reactive power to maintain the voltage profile in the system.

The first step to full deregulation is an open access system where any utility would have access to the transmission lines. The open access system has been in operation for the last few years. The most developed deregulated system so far is an energy pool which matches the bids of buyers and sellers. The participating utilities get access to the transmission lines through this pool. The pool determines the share of the transmission losses and charges the utilities. The determination of transmission loss share is a key issue and very few methods and models have been proposed to deal with it so far. Some work has been done to determine the criterion based on which the allocation of transmission loss should be awarded.

The concept of deregulation gradually took over researchers' attention in mid 80's. Prior to that, investigation on economic optimization, spinning reserve, tie system and other power system related researches were based on conventional monopoly system. As some of the utilities have started to implement deregulation policy in mid 80's, researchers have become more and more interested in different aspects of deregulation.

Different aspects of the transmission access issue have been discussed by Vojdani et al. [6]. Reference 1 gives an overall view of the deregulation process with details of the industry structure, system operation, transmission rights, pricing models reliability under transmission open access and economic issues. This work provides a guideline for different problems arising from deregulation issue.

Basically, the allocation of transmission share problem branches out into two different viewpoints. One group is trying to establish a relation between a load and contribution of the generators in the system to the particular load. The other group is trying to solve the problem from cost analysis and its distribution. They have been trying to find a suitable model so that the total cost of the transmission facilities could be recovered with profit.

Recently Strbac et al. [7,8] proposed an algorithm for the allocation of transmission system usage. This method finds the traceable contributions of each generator and of each load to line flows instead of marginal contribution. The allocation has been proposed on the basis of maximum flows in the lines and the maximum flow condition in a line has been determined by considering different load conditions and contingency. Based on this maximum flow, the contribution of each generating unit and load in each branch has been determined according to the proportionality principle. The basic idea of these papers are to establish a link between a generating unit and a load. In other words, these papers find the contribution of a generating unit to a particular load and determine transmission usage based on this. To do this, the components of the network (bus, line etc.) have been defined in a particular fashion (commons, links etc.) and finally an acyclic state-graph of the network has been drawn based on these definitions. Once the acyclic state-graph has been drawn, the contribution of each generation to each load has been found in a progressive direction or up-stream method. This concept of contribution basically comes from the concept of proportional sharing of power at the commons. Although the contribution of a generator could be traced and found to a load – these papers do not indicate anything about bilateral contract between a generating unit and a load.

Bialek [9,10,11] has reported a method to trace generators' contribution to load based on the proportional sharing principle. Although the author has used the same principle used by Strbac et al., Bialek presents different algorithms. The two algorithms have been developed based on lossless flow. The two algorithms namely 'upstream-looking algorithm ' and ' downstream-looking algorithm ' have been used by assuming the losses either to the generation or to the load side. The upstream-looking algorithm determines the losses first and then breaks it down into components and adds the

components to nodal demands. In the downstream-looking algorithm the total transmission loss is broken down into components and subtracted from individual generators to build a virtual lossless system. Both approaches trace power flow in the transmission network either from generator to load or from load to generator depending on the up or down stream-looking algorithm. Finally, the allocation of charge is determined based on this electricity tracing method.

Macqueen and Irving [12] proposed a new method to find the loss adjustment factor for allocating losses of network to consumers. The share of the consumers has been determined by their demands. The authors have proposed the allocation of losses in three different ways – Uniform Loss Allocation, Loss Allocation on the Basis of Demand and Loss Allocation on the Basis of Demand Squared. The amount of loss has been determined by a load flow study and a directed graph of the network. The fixed and variable losses have been treated separately. The fixed losses have been considered as a part of load whereas variable losses are treated in a normal way as regular losses in the network. The concept of proportionality has been used and the losses are allocated according to proportion.

Li and David [13] have proposed an optimal solution for multi-area wheeling usage. The proposed method finds the path (wheeling utility) that should be used by a wheeling transaction to minimize the cost and determines the cost to be paid by each transaction based on marginal cost pricing. The method finds the optimal power flow (OPF) for the wheeling utility itself and then searches for the best solution for wheeling transaction. The loss has been included in the wheeling charge. Any number, say M, of utilities could be used to transfer the wheeling power, but in actual practice OPF will determine this number which could be less than M. The work does not provide the answer to what share the wheeling utility would get if it does not have to carry any power due to OPF solution but has simply been connected to the system for contingency.

Under deregulation, the pricing of electricity has become a complex issue. Lima [14] has proposed a method that describes how an agent should be charged for using a transmission network. An agent works as a middleman, and there is no direct contract

between generators and consumers with respect to monetary transactions. The paper describes how a third agent becomes affected due to the association of other agents. This sort of association actually leads to the concept of deregulation in the reverse direction and it should not be encouraged. To prevent this, some form of incentives should be provided to the agent who could gain more by forming an association. Actually, the counter-flow agents would be interested in such association and they must be compensated for not doing so. The interest of the work has been totally confined in transmission and does not recognize the source-sink relation or link.

An interesting new direction has been provided by Tsukamoto and Iyoda [15] for allocating the transmission network cost to the wheeling parties based on the MW-mile method as marginal cost pricing is incapable of solving this problem accurately. The concept of game theory has been incorporated to deal with the litigious problems arising due to deregulation. Game theory is an interesting concept and has had wide use in economics. If more than one party is involved in any situation (bidding, poll etc.) then game theory is a suitable choice to solve the dispute. In modern power systems, there would be many individual buyers, sellers and wheeling utilities (transmission network). The buyers and sellers use the same wheeling utilities. The individual seller and buyer might have their own contract but as all transaction parties have been using the same transmission network they should discuss among themselves to reach an agreement on how to share this cost. This agreement could be reached by using the cooperative game theory. If the transaction parties co-operate with each other they could save some cost. The limit of cooperation and the distribution of savings can be determined by the utilization of game theory. The method needs load flow calculation as most of the other methods need. After obtaining a load flow for the base case (the native system of the system of the wheeling utility itself), an additional load flow is done for each transaction. The application of game theory in power system is very prospective and it could be used to settle the problem of reserve under deregulated environment.

A generalized mathematical framework for energy transactions under deregulation has been defined by Galiana and Ilić [16]. In a very organized approach, the paper introduces the concept of deregulation and related outlines. From a well-defined concept of transaction, the work introduces a virtual network of transactions for better

understanding of the actual transactions in a deregulated network. It considers different existing methods of transactions under open access system and proposes a generalized transaction matrix that takes care of all these methods. The transaction matrix has been used to calculate the power injections and losses in a practical network. Based on security criterion, the method can modify and propose a new mode of transactions or rescheduling of transactions. The losses can be allocated to each transaction in some proportion determined by the nature of the network and by the type and level of all transactions. It also shows a way out for the congested conditions in networks.

### 1.4 Objectives and Scope of the Research

Full deregulation is a dynamic concept and is bringing radical changes in power system area. It does not recognize region-based monopoly systems and increases competition amongst the participating utilities. The prime objective of a fully deregulated system is to provide customers their choices of utilities. This realization would be achieved only when a customer would be allowed to enter in a bilateral contract with a generating utility. Eventually, the bilateral contract brings up the issue of transmission loss allocation among the utilities.

Allocation of transmission losses is a challenging and contentious issue in a fully deregulated system. Very little work has been reported in the literature that deals with the allocation of transmission losses. The objectives of this research are:

- To develop a method for transmission loss allocation in deregulated networks.
- To study the effects of various load conditions on transmission loss allocation based on the developed method.

Two methods have been developed to determine a generator's share of transmission loss in a fully deregulated power system. In one, a mathematical approach has been developed to determine the transmission losses that each of the sources is responsible for and the method is termed as Marginal Transmission Loss Approach (MTLA). The mathematical approach is based on Kron's transmission loss expression [17] and results in an iterative process. In the other method, a modified load flow is utilized to assess transmission losses and the method is termed as Incremental Load Flow Approach

(ILFA). At each load bus, load is increased in a discrete step while the loads at other buses are kept constant. The resulting differential transmission loss is attributed to a corresponding generator. The details of these proposed methods are presented in Chapters 3 and 4 with numerical examples. The resulting shares of transmission losses in a hypothetical system utilizing both methods have been compared to validate the methods.

### 1.5 Outline of the Thesis

The thesis is organized in six chapters. Chapters 1 and 2 deal with the basic concepts of power systems. Power systems in different countries have adopted different methods for operation and the coming wave of deregulation is changing the setup. This has been briefly discussed in Chapter 1. Chapter 2 describes power system operation, AC load flow analysis and transmission losses in power system networks. This chapter also provides the mathematical basis for the Marginal Transmission Loss Approach.

AC load flow analysis has been utilized to develop one of the proposed methods for transmission loss allocation. A test system is described which has been used for analysis in Chapters 3 and 4. The Incremental Load Flow Approach (ILFA) is presented in Chapter 3 along with numerical examples. Various load conditions have been considered and the transmission loss allocations have been shown in tables as well as plotted in graphs.

The loss allocations obtained by the ILFA have been verified by the Marginal Transmission Loss Approach (MTLA) in Chapter 4. The mathematical analysis for the MTLA is reported in this chapter. Load conditions similar to those discussed in Chapter 3 have been applied to obtain loss allocations. The loss allocations resulting from the two methods are compared. This chapter includes graphs of loss allocations for various load conditions to provide a better view of the methods.

Chapter 5 presents a comparative study of the two methods developed. It uses an example system to show the allocation of transmission losses in a more practical network. Finally, Chapter 6 reports conclusions and proposals for future work.

# CHAPTER 2: TRANSMISSION LOSS AND AC LOAD FLOW ANALYSIS

## 2.1 Power System Operation

A Power system is a composite entity. Its three functional areas are generation, transmission and distribution. The combination of these three utilities and their optimized utilization is the goal of power system operation. The generation may consists of different kinds of power plants e.g. thermal, hydro, nuclear etc. An operating authority wants to use the available generating units in an optimized and efficient way. Loads in a network vary during the day and also during various seasons. Efficient power system utilities take all those factors in consideration and operate for minimal operating cost.

Loads in a network follow some patterns and go high and low at different times of a day. Load forecast predicts the nature of load demands from load patterns and coming events with a high accuracy. The utilities use this prediction for determining the number of generating units required to meet the anticipated load, an essential activity of a power system, generally known as unit commitment.

Unit commitment dictates the number of generating units to be in spinning condition to meet load demands during the 24 hours. It also states the order of the units to be engaged in production according to the production cost of the units and starting time of the units. Production cost varies from unit to unit depending on their working principle. The production costs of hydro units are far less that those of the gas turbine and thermal units. It costs much more to produce energy in a gas turbine unit compared to that of a similar size thermal unit. The starting time of gas turbine and hydro units are much lower than conventional thermal plants. These two factors, production cost and starting

time, determine the allocation of load among various units. This activity is commonly known as load dispatch.

Once the load dispatching schedule has been prepared, the need of AC load flow analysis comes into the picture. AC load flow analysis provides bus voltages, line flows, power mismatch etc. and helps to determine the feasibility of a load dispatching schedule.

## 2.2 AC Load Flow Technique

Load flow analysis is fundamental to the operation and study of power systems. In fact, load flow forms the heart of power system analysis. Load flow analysis plays a key role in the planning of additions or expansions to transmission and generation facilities.

In general, load flow analysis solves for any unknown bus voltages and unspecified generation and finally for the complex power flow in the network components for a given power system network, with known loads and some set of specifications or restrictions on power generations and voltages. The losses in individual components and the total network as a whole are usually calculated. The system is often checked for component overloads and voltage outside allowable tolerance.

Basically, two methods are widely used for load flow analysis. They are the Gauss-Seidal and Newton-Raphson methods. Both methods need certain input parameters for performing an analysis. Y-bus or Z-bus is required to be calculated before proceeding for a solution. Buses in a network are divided into three categories: swing bus, generator or PV bus and load or PQ bus. Each bus is associated with four parameters: voltage magnitude, phase angle and real and reactive power.

Swing bus - it is a generator bus whose voltage and angle have been specified. The real and reactive power generation are to be calculated.

Generator bus - the bus voltage and real power generated have to be specified and the reactive power and the bus angle are to be determined.

Load bus - the bus with the load is connected and the real and reactive load are known. The bus voltage and angle have to be determined.

The load flow technique actually solves a set of simultaneous equations in an iterative way. Newton-Raphson method is widely used for load flow analysis for its inherent nature of fast convergence. The following simultaneous equations are required for the solution of load flow by Newton-Raphson method.

$$P_k = \sum_{n=1}^{N} |V_k V_n Y_{kn}| \cos(\theta_{kn} + \delta_n - \delta_k) \tag{2.1}$$

$$Q_k = -\sum_{n=1}^{N} |V_k V_n Y_{kn}| \sin(\theta_{kn} + \delta_n - \delta_k) \tag{2.2}$$

Where,

$P_k$ = real power at bus $k$

$Q_k$ = reactive power at bus $k$

$V_k$ = voltage at bus $k$

$V_n$ = voltage at bus $n$

$Y_{kn}$ = element of bus admittance matrix between buses $k$ and $n$

$\theta_{kn}$ = angle associated with $Y_{kn}$

$\delta_k$ = bus angle of bus $k$

The method starts with some initial values for the specified parameters, $P$ and $Q$ for every bus except the swing bus. Estimated values of $V$ and $\delta$ for each bus except the swing bus, for which they are known, are used to calculate the same parameters. The mismatch in power calculation originating from specified and calculated values are determined for each bus.

$$\Delta P_k^{(0)} = P_{ks} - P_{kc}^{(0)}$$

$$\Delta Q_k^{(0)} = Q_{ks} - Q_{kc}^{(0)}$$

where the subscripts $s$ and $c$ mean specified and calculated values respectively and the superscript represents the iteration number. In the next step the Jacobian is determined.

$$\begin{bmatrix} \Delta P_1^{(0)} \\ \cdots \\ \Delta P_{N-1}^{(0)} \\ \Delta Q_1^{(0)} \\ \cdots \\ \Delta Q_{N-1}^{(0)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial|\delta_1|} & \cdots & \frac{\partial P_1}{\partial|\delta_{N-1}|} & \frac{\partial P_1}{\partial|V_1|} & \cdots & \frac{\partial P_1}{\partial|V_{N-1}|} \\ \cdots & & & & & \\ \frac{\partial P_{N-1}}{\partial|\delta_1|} & \cdots & \frac{\partial P_{N-1}}{\partial|\delta_{N-1}|} & \frac{\partial P_{N-1}}{\partial|V_1|} & \cdots & \frac{\partial P_{N-1}}{\partial|V_{N-1}|} \\ \frac{\partial Q_1}{\partial|\delta_1|} & \cdots & \frac{\partial Q_1}{\partial|\delta_{N-1}|} & \frac{\partial Q_1}{\partial|V_1|} & \cdots & \frac{\partial Q_1}{\partial|V_{N-1}|} \\ \cdots & & & & & \\ \frac{\partial Q_{N-1}}{\partial|\delta_1|} & \cdots & \frac{\partial Q_{N-1}}{\partial|\delta_{N-1}|} & \frac{\partial Q_{N-1}}{\partial|V_1|} & \cdots & \frac{\partial Q_{N-1}}{\partial|V_{N-1}|} \end{bmatrix} \begin{bmatrix} \Delta\delta_1^{(0)} \\ \cdots \\ \Delta\delta_{N-1}^{(0)} \\ \Delta|V_1^{(0)}| \\ \cdots \\ \Delta|V_{N-1}^{(0)}| \end{bmatrix} \tag{2.3}$$

Equation (2.3) can be written as

$$\begin{bmatrix} \Delta P^k \\ \Delta Q^k \end{bmatrix} = [J] \begin{bmatrix} \Delta\delta^k \\ \Delta|V^k| \end{bmatrix}$$

or

$$\begin{bmatrix} \Delta\delta^k \\ \Delta|V^k| \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P^k \\ \Delta Q^k \end{bmatrix}$$

Equation (2.3) is solved by inverting the Jacobian ($J$) and errors in voltages and angles are calculated. New values of $V$ and $\delta$ are estimated by subtracting these errors from respective parameters. These new voltages and angles are used to calculate new bus powers using Equations (2.1) and (2.2). This process is repeated until mismatch at each bus comes down within a tolerable limit. The Kirchhoff's law, the algebraic sum of all flows at a bus must be zero, should be satisfied by the load flow and can be used as a convergence constraint.

### 2.2.1 Example of Load Flow Analysis

A small network is shown in Figure 2.1 to illustrate the load flow analysis. Bus 1, 2 and 3 are defined as swing bus, generator bus and load bus respectively. Line and generator parameters are shown in Tables 2.1 and 2.2. The base values are 100 MVA and 138 kV.

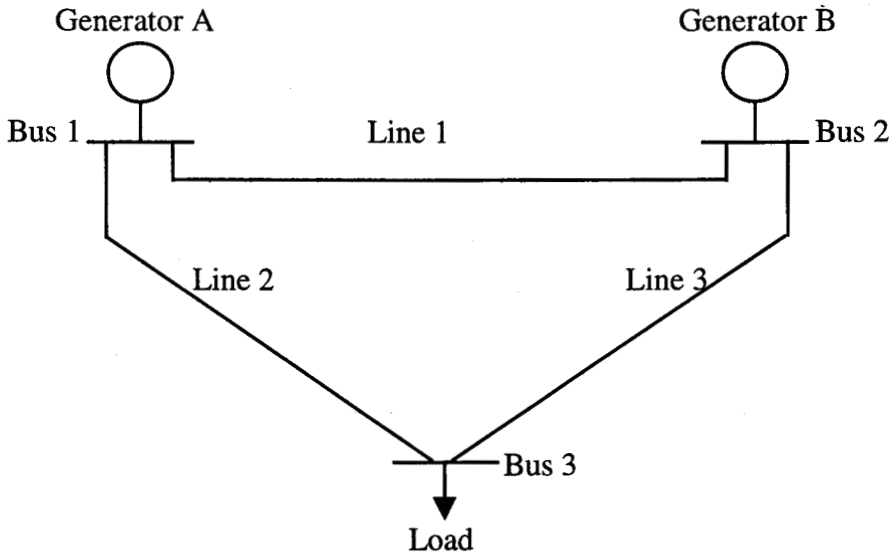


Fig.2.1 : A small power system network.

Table 2.1: Line parameters for the system shown in Figure 2.1.

| Line Number | From Bus | To Bus | Length (km) | Resistance p.u. | Reactance p.u. |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 80 | 0.01575 | 0.07876 |
| 2 | 1 | 3 | 50 | 0.0105 | 0.05251 |
| 3 | 2 | 3 | 100 | 0.02625 | 0.13127 |

Table 2.2: Generating unit characteristics for the system shown in Figure 2.1.

| Generating Unit | Cost Function ($/hr) | Maximum Output (MW) | Minimum Output (MW) |
|---|---|---|---|
| Generator A | $0.022P_1^2+12.45P_1+70$ | 500 | 90 |
| Generator B | $0.024P_2^2+13.65P_2+80$ | 400 | 40 |

The real and reactive power demands are 200 MW and 80 MVAR respectively and supplied by both units.

The simultaneous equations for the system shown in Figure 2.1 are

$$P_2 = |V_2 V_1 Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) + |V_2^2 Y_{22}| \cos(\theta_{22}) + |V_2 V_3 Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2) \quad (2.4)$$

$$P_3 = |V_3 V_1 Y_{31}| \cos(\theta_{31} + \delta_1 - \delta_3) + |V_3 V_2 Y_{32}| \cos(\theta_{32} + \delta_2 - \delta_3) + |V_3^2 Y_{33}| \cos(\theta_{33}) \quad (2.5)$$

$$Q_2 = -|V_2 V_1 Y_{21}| \sin(\theta_{21} + \delta_1 - \delta_2) - |V_2^2 Y_{22}| \sin(\theta_{22}) - |V_2 V_3 Y_{23}| \sin(\theta_{23} + \delta_3 - \delta_2) \quad (2.6)$$

$$Q_3 = -|V_3 V_1 Y_{31}| \sin(\theta_{31} + \delta_1 - \delta_3) - |V_3 V_2 Y_{32}| \sin(\theta_{32} + \delta_2 - \delta_3) - |V_3^2 Y_{33}| \sin(\theta_{33}) \quad (2.7)$$

Since Bus 2 is a generator bus and voltage for this bus has been specified, Equations (2.4), (2.5) and (2.7) are sufficient for obtaining the solution of load flow analysis. Unspecified voltages and angles are estimated as 1 p.u. and zero radian initially.

Equation (2.3) for the current system stands as:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} 19.53 & -7.33 & -1.46 \\ -7.33 & 25.63 & 5.12 \\ 1.46 & -5.12 & 25.65 \end{bmatrix}^{-1} \begin{bmatrix} 0.798 \\ 1.998 \\ 0.7902 \end{bmatrix}$$

The errors in initial estimated voltages and angles after the first iteration are

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} 0.0785 \\ 0.0915 \\ 0.0446 \end{bmatrix}$$

The errors in voltages and angles are subtracted from their initial estimation to get a new set of estimated values. The new estimated values are:

$$\begin{bmatrix} \delta_2 \\ \delta_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} -0.0785 \\ -0.0915 \\ 0.9554 \end{bmatrix}$$

This process continues unless the errors come down within a pre-specified tolerable limit. The Solution of the AC load flow analysis is shown in Tables 2.3 and 2.4 which provides bus voltages, angles and line flows.

Table 2.3: AC load flow solution for the system shown in Figure 2.1.

| Bus | Voltage p.u. | Phase Angle degrees | Real Generation p.u. | Reactive Generation p.u. | Real Load p.u. | Reactive Load p.u. |
|---|---|---|---|---|---|---|
| 1 | 1.0 | 0.0 | 1.2391 | 0.7493 | 0.0 | 0.0 |
| 2 | 1.0 | 0.7222 | 0.8 | 0.2463 | 0.0 | 0.0 |
| 3 | 0.95 | -3.9598 | 0.0 | 0.0 | 2.0 | 0.8 |

Table 2.4: Line flows in the network shown in Figure 2.1.

| Line | From Bus | To Bus | Real Power p.u. | Reactive Power p.u. |
|---|---|---|---|---|
| 1 | 1 | 2 | -0.1537 | 0.0317 |
| | 2 | 1 | 0.1541 | -0.0298 |
| 2 | 1 | 3 | 1.3928 | 0.7175 |
| | 3 | 1 | -1.367 | -0.5886 |
| 3 | 2 | 3 | 0.6459 | 0.2761 |
| | 3 | 2 | -0.633 | -0.2114 |

The system shown in Figure 2.1 is small and Tables 2.3 and 2.4 show the typical format of load flow outputs which include bus voltages, angles, generation and line flows. Generator A connected to the swing bus is supplying (1.2391 + j0.7493 p.u.), a major portion of the demand, while generator B is providing 0.8 + j0.2463 p.u. to meet the load. The voltage of bus 3 is 0.95 p.u. because of the load connected to it. This voltage can be enhanced by connecting a tap-changing transformer to this bus. Table 2.4 shows the line flows. It may be noticed from Table 2.4 that line 2 is carrying the maximum load (1.3928 + j0.7175 p.u.) because of the system configuration. Although Buses 1 and 2 have the same voltage magnitude, a small amount of power flows from bus 2 to bus 1 due to a difference between their bus angles.

### 2.2.2 Transmission Loss Calculation from Load Flow

A load flow analysis can be utilized to determine total transmission loss in a system as well as losses in individual components i.e., in transformers or transmission lines. A load flow analysis provides real and reactive powers at different buses. Total transmission loss can be calculated easily from the algebraic sum of powers at all buses. Loss in an individual line can be determined by power calculation at two buses that are connected by the particular line. The difference between the line flows, at the two ends, of a single line indicates the transmission loss in that line. For example, 1.3928 p.u. real power flows from bus 1 to 3 and 1.367 p.u. real power flows from bus 3 to 1. The difference between these two flows, 0.0258 p.u., represents the line loss in line 1. Total real and reactive line losses are 0.0391 p.u. and 0.1956 p.u. respectively for the system shown in Figure 2.1.

The loss calculation by load flow is more accurate than any other method. A fully deregulated network requires total and individual line loss in the system. The conventional load flow gives the line loss based on few initial assumptions e.g., real powers of the generator buses have to be specified.

## 2.3 Transmission Loss in Power Systems

Transmission network is the electrical highway through which the electrical power flows. It is a network that connects the consumers to the generators or producers of electrical power. Since the transmission lines are made of physical conductors, loss of electrical power occurs in the line during its flow. Every transaction causes some transmission loss and it is quite significant in a large network. Transmission losses are unavoidable, and therefore, generating units are supposed to supply the losses along with the demand in a network.

Transmission loss can be divided into two parts: real and reactive. Real power generated by a system should be equal to the sum of its loads and line losses. It is needless to say that without providing the transmission loss, the system load could not be supplied.

On the other hand, reactive power in a system is required for system stability. Reactive power loss must be provided for maintaining the voltage profile in a network. Unlike real power, reactive power does not have any direct monetary value but it is an extremely important factor in power system operation.

Total transmission loss can be calculated in different ways. One of the popular ways is to utilize Kron's [17] transmission loss formula. This formula is based on a number of assumptions and calculates transmission loss in terms of generations. In spite of the assumptions, Kron's formula gives a fairly close result when compared to the results obtained through more accurate methods. But the advantage of this formula is that it is a function of generation only, and therefore, one can find approximate transmission loss in a system by knowing the generation of individual plants of the system.

### 2.3.1 Transmission Loss Expression

A mathematical expression for transmission loss can be derived by utilizing the standard relationship of complex powers at all buses in a network.

$$S_l = P_l + jQ_l = \sum_i S_i \tag{2.8a}$$

or,

$$S_i = V_i I_i^*$$

where,

$S_l$ = total complex power loss

$P_l$ = total real power loss

$Q_l$ = total reactive power loss

$S_i$ = complex power at bus i

$V_i$ = voltage at bus i and

$I^*_i$ = complex conjugate of bus current at bus i.

Equation (2.8) can be written as

$$S_l = [V_B]^T [I_B]^* \tag{2.8b}$$

where,

$$[V_B] = [Z_B]\,[I_B]$$
$$[Z_B] = [R] + [jX]$$
$$[I_B] = [I_p] + [jI_q]$$

$[Z_B]$ = bus impedance matrix of the system

$[R]$, $[X]$ = real and reactive components of the bus impedance matrix

$[I_B]$ = bus current matrix

$[I_p]$, $[I_q]$= real and reactive components of the bus current matrix

Replacing $V_B$ and $I_B$ by their real and imaginary parts, Equation (2.8b) becomes:

$$\begin{aligned} S_l &= [I_B]^T [Z_B][I_B]^* \\ &= ([I_p] + [jI_q])^T ([R] + [jX])([I_p] + [jI_q])^* \end{aligned}$$

After separating the real and imaginary parts

$$P_l = [I_p]^T [R][I_p] + [I_q]^T [R][I_q] \tag{2.9}$$

and

$$Q_l = [I_p]^T [X][I_p] + [I_q]^T [X][I_q] \tag{2.10}$$

At any bus i the following relationship holds

$$P_i + jQ_i = V_i I_i^* \tag{2.11}$$

where,

$$V_i = |V_i|(\cos\delta_i + j\sin\delta_i)$$

$$I_i = I_{pi} + jI_{qi}$$

$V_i$ = voltage at bus i

$I_i$ = bus current at bus i

Equation (2.11) can be written as

$$P_i + jQ_i = |V_i|(\cos\delta_i + j\sin\delta_i)(I_{pi} + jI_{qi})^* \tag{2.12}$$

Equating the real and imaginary parts of Equation (2.12)

$$P_i = |V_i| I_{pi} \cos\delta_i + |V_i| I_{qi} \sin\delta_i \tag{2.13}$$

$$Q_i = |V_i| I_{pi} \sin\delta_i - |V_i| I_{qi} \cos\delta_i \tag{2.14}$$

Solving Equations (2.13) and (2.14) for $I_{pi}$ and $I_{qi}$

$$I_{pi} = \frac{P_i \cos\delta_i + Q_i \sin\delta_i}{|V_i|}$$

$$I_{qi} = \frac{P_i \sin\delta_i - Q_i \cos\delta_i}{|V_i|}$$

or in vector form

$$[I_p] = [C][P] + [D][Q] \tag{2.15}$$

$$[I_q] = [D][P] - [C][Q] \tag{2.16}$$

where,

$$[C] = \text{diagonal matrix with elements} \left( \cos\delta_i \big/ |V_i| \right)$$

$$[D] = \text{diagonal matrix with elements} \left( \sin\delta_i \big/ |V_i| \right)$$

Now Equation (2.9), for the real part of the transmission loss, becomes

$$\begin{aligned} P_l &= [I_p]^T [R][I_p] + [I_q]^T [R][I_q] \\ &= ([C][P] + [D][Q])^T [R]([C][P] + [D][Q]) + ([D][P] - [C][Q])^T [R]([D][P] + [C][Q]) \end{aligned}$$

or,

$$\begin{aligned} P_l &= [P]^T \left([C]^T [R][C] + [D]^T [R][D]\right)[P] - [P]^T \left([D]^T [R][C] + [C]^T [R][D]\right)[Q] \\ &\quad + [Q]^T \left([D]^T [R][C] - [C]^T [R][D]\right)[P] + [Q]^T \left([C]^T [R][C] + [D]^T [R][D]\right)[Q] \end{aligned}$$

which can be written in matrix form as:

$$P_l = \begin{bmatrix} [P]^T & [Q]^T \end{bmatrix} \begin{bmatrix} [A_p] & -[B_p] \\ [B_p] & [A_p] \end{bmatrix} \begin{bmatrix} [P] \\ [Q] \end{bmatrix} \tag{2.17}$$

where,

$$[A_p] = [C]^T [R][C] + [D]^T [R][D]$$
$$[B_p] = [D]^T [R][C] - [C]^T [R][D]$$

and

$$[C]^T [R][C] = \begin{bmatrix} c_1 & 0 & 0 \ldots & 0 \\ 0 & c_2 & 0 \ldots & 0 \\ \ldots & & & \\ 0 & 0 & 0 \ldots & c_n \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & \ldots & r_{1n} \\ r_{21} & r_{22} & \ldots & r_{2n} \\ \ldots & & & \\ r_{n1} & r_{n2} & \ldots & r_{nn} \end{bmatrix} \begin{bmatrix} c_1 & 0 & 0 \ldots & 0 \\ 0 & c_2 & 0 \ldots & 0 \\ \ldots & & & \\ 0 & 0 & 0 \ldots & c_n \end{bmatrix}$$

$$= \begin{bmatrix} c_1 r_{11} c_1 & c_1 r_{12} c_2 & \ldots & c_1 r_{1n} c_n \\ c_2 r_{21} c_1 & c_2 r_{22} c_2 & \ldots & c_2 r_{2n} c_n \\ \ldots & & & \\ c_n r_{n1} c_1 & c_n r_{n2} c_2 & \ldots & c_n r_{nn} c_n \end{bmatrix}$$

The elements of $[A_p]$ are

$$\begin{aligned} a_{pij} &= c_i r_{ij} c_j + d_i r_{ij} d_j \\ &= \frac{\cos\delta_i \cos\delta_j}{|V_i||V_j|} r_{ij} + \frac{\sin\delta_i \sin\delta_j}{|V_i||V_j|} r_{ij} \\ &= \frac{r_{ij}}{|V_i||V_j|} \cos(\delta_i - \delta_j) \end{aligned}$$

Similarly,

$$b_{pij} = \frac{r_{ij}}{|V_i||V_j|} \sin(\delta_i - \delta_j)$$

Equation (2.17) becomes

$$P_l = [P]^T [A_p][P] - [P]^T [B_p][Q] + [Q]^T [B_p][P] + [Q]^T [A_p][Q] \tag{2.18}$$

Again at any bus the following can be written

$$P_i = P_{Gi} - P_{Di}$$

where,

$P_{Gi}$ = power generation at bus i

$P_{Di}$ = power demand at bus i

Similar relation also holds for the reactive part.

Using this relation Equation (2.18) can be rewritten as

$$P_l = \left([P_G]^T - [P_D]^T\right)A_p([P_G]-[P_D]) - \left([P_G]^T - [P_D]^T\right)B_p([Q_G]-[Q_D]) + \left([Q_G]^T - [Q_D]^T\right)B_p([P_G]-[P_D]) + \left([Q_G]^T - [Q_D]^T\right)A_p([Q_G]-[Q_D]) \tag{2.19}$$

After expansion Equation (2.19) becomes

$$\begin{aligned} P_l = {} & [P_G]^T[A_p][P_G] - [P_D]^T[A_p][P_G] - [P_G]^T[A_p][P_D] + [P_D]^T[A_p][P_D] - [P_G]^T[B_p][Q_G] \\ & + [P_D]^T[B_p][Q_G] + [P_G]^T[B_p][Q_D] - [P_D]^T[B_p][Q_D] + [Q_G]^T[B_p][P_G] - [Q_D]^T[B_p][P_G] \\ & - [Q_G]^T[B_p][P_D] + [Q_D]^T[B_p][P_D] + [Q_G]^T[A_p][Q_G] - [Q_D]^T[A_p][Q_G] - [Q_G]^T[A_p][Q_D] \\ & + [Q_D]^T[A_p][Q_D] \end{aligned} \tag{2.20}$$

This is the standard transmission loss equation for real power. A similar expression for reactive power can also be obtained. This expression will be used in Chapter 4 for the formulation of the Marginal Transmission Loss Approach.

## 2.4 Role of Transmission Loss in Present and Future Models of Network.

In old monopolized systems, transmission loss plays a key role in determining economic system operation. A utility runs its network for minimum operating cost. A load dispatch centre (LDC) decides the generation of committed units in a network such that the production cost is minimized. The cost associated with transmission loss is distributed and charged to the consumers.

Deregulation of electrical power market has modified the role of transmission loss. The allocation of transmission loss has become a contentious issue. Due to open access policy in some countries, utilities share a common transmission facility. Transmission losses occur due to the flow of power for all transactions and it is difficult to find the component of the loss occurring due to a particular transaction. Some work has been done to find a generator's contribution to a particular load or to a line flow [8,11]. Those works use proportional methods for separating the flows in a line and trace back to generators from loads to find a generator's contribution to load. Although these methods

identify links between sources and sinks, these do not help much in the case of a bilateral contract between a seller and a buyer.

Very few countries, that include England, Brazil, Canada and USA, are trying to implement the concept of deregulation. The existing systems are actually different combinations of old monopoly and predicted deregulated systems. In a deregulated world, however, utilities would be facing new competitors that could charge lower rates. Under deregulation, local utilities would have to allow competitive power producers to use their transmission lines. A local utility, typically the owner of a transmission facility, would charge other utilities who would use their transmission network.

In some regions, power systems are still controlled by single entities (SaskPower) [5]. They can buy power from independent generators. The controlling utility can easily determine how much power they would buy and the associated transmission loss. The transmission loss, resulting from the purchase of external power, can be summed up with the transmission loss occurring due to internal supply.

On its way to full deregulation, some systems have introduced the idea of energy pool or independent system operator (ISO). The energy pool structure is a spot market through which demands for electricity are met on the basis of hourly price and volume bids by generators and buyers. The ISO or the pool matches the sellers and the buyers based on their bidding price and supervises the overall system security and reliability. The energy pool keeps record of the transactions and allocates the transmission loss among the participating utilities. The challenge that the pool or the ISO has been facing is how to allocate the transmission loss and what should be the criterion for charging other utilities. As no unanimous approach exists, different networks have been practicing different methods. These methods are not accepted by all utilities in general, although they have been using them due to the unavailability of a proper solution. Some of the methods are

- Embedded cost pricing
- Cost Causation-based pricing
- Usage-based pricing

- Location-specific pricing
- Real-time pricing
- Congestion pricing
- Opportunity cost pricing
- Market-based pricing

In a fully deregulated environment, consumers would have the choice to buy power from any generating companies. A consumer could take a decision by comparing the rates and services provided by different utilities. For the last 100 years, utilities have operated as local monopolies, with varying degrees of price and practice regulation from governments. In a deregulated environment, consumers could choose an electric company the way they now choose a long-distance telephone service.

The challenge arises immediately for the allocation of transmission loss among the competing utilities. As an individual buyer can have a bilateral agreement with any utility, the loss related to this supply of power should be taken into consideration in preparing the agreement. The transmission loss originating from a bilateral contract should be assessed in such a way that the assessment should be viewed as fair and transparent.

Power systems typically operate under slowly changing conditions that can be analyzed using steady state analysis. Moreover, transmission systems operate under balanced or near-balanced condition allowing per phase analysis to be used with a high degree of confidence in the solution. These lead to the use of load flow technique as a more accurate way of finding transmission loss.

The proper calculation of transmission loss is very important for deregulated systems. The main objective of this calculation is to distribute the loss among the utilities in a fair and equitable manner. In a fully deregulated environment, transmission loss should be calculated in terms of loads, instead of generations.

# CHAPTER 3: INCREMENTAL LOAD FLOW APPRAOCH

## 3.1 Incremental Load Flow Approach

Transmission loss is a function of system configuration and it varies with load and generation. For a fixed system configuration loss varies with load and generation. The system configuration of a power system network usually remains unchanged unless lines are taken out of service due to a fault or maintenance. As a result, transmission loss could be easily determined with the help of a load flow analysis.

A conventional load flow analysis is performed to obtain line flows, line losses, bus voltages etc. It does not provide the share of transmission loss of a particular generator in a system. Some methods have been reported to find the individual contribution of a load or a generation in the aggregate [8,9,11]. These methods, too, are based on conventional load flow analyses. These methods, however, do not validate the concept of full deregulation. Those analyses do not take bilateral contracts between buyers and producers of electrical power into account.

In a fully deregulated environment, a particular utility would be responsible for supplying its contracted consumer along with the loss associated with the demand, and there would be a number of bilateral agreements in a network. A modified load flow analysis has been developed, named as the Incremental Load Flow Approach (ILFA), to determine the loss associated with an individual transaction.

In the ILFA, a load flow program is run for load levels from zero to their given level for each load in a system in a sequential manner. Loads are increased by a small increment in every iteration. Each Generator is assumed to have a fixed consumer or load in the system and supposed to produce the power to meet the load demand of its customer and the associated loss. When a certain load is increased by a pre-specified increment, the

corresponding increase in transmission loss is assigned to the generator that is in contract with this particular consumer. In other words, the generator (Generator X), which has a bilateral agreement with a consumer (Consumer A), would be responsible for the load demand (of Consumer A) and the loss originating from the contract.

The increased transmission loss for an incremental change in a given load is calculated and is assigned to a corresponding generator who is responsible for supplying load. During an iteration, only one load is incremented while other loads are held constant.

### 3.2 Assumptions in the ILFA

A load flow technique in general requires that the power generation for PV buses be specified. According to the ILFA, the generation of a PV bus should be the sum of its load and its share of the total line loss. This share would be calculated from the incremental transmission loss. The loss is unknown prior to the first iteration. To overcome this difficulty in the first iteration, the associated loss is neglected and the generation of a PV bus is made equal to zero. This would not affect the result as long as the increment size is kept small.

After the first iteration the generation at a PV bus can be assigned as the sum of its discrete incremental load and the respective transmission loss from the previous iteration. This means that the assigned loss is always lagging behind the actual loss by one step, the difference would be very small if the step size is kept relatively small.

It has been assumed that each load has a constant reactive to real power ratio. This reactive ratio, the ratio of reactive to real power, is defined as,

$$\mu = \frac{\text{reactive power}}{\text{real power}}$$

This ratio depends on the nature of a load and may vary from customer to customer.

### 3.3 Example System

A small hypothetical system has been considered in this section for the purpose of numerical examples related to the allocation of transmission loss.

The hypothetical system consists of six buses with two generators and two loads. The generators and loads represent bulk producers and bulk consumers. The loads and the generators are connected through five lines. Two tap-changing transformers are connected on the load sides. The system is shown in Figure 3.1.

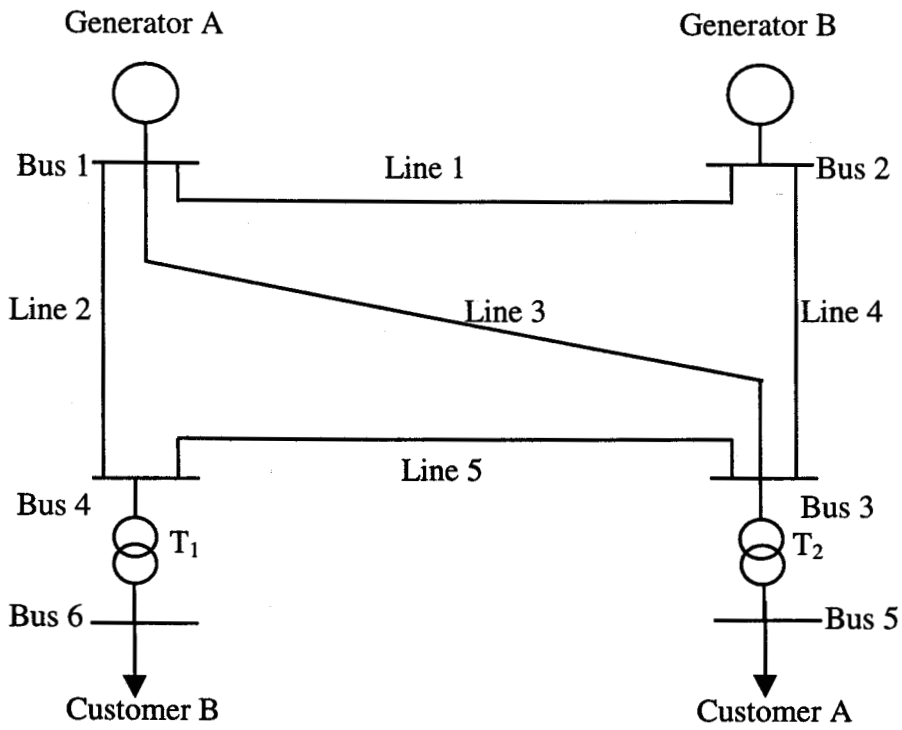


Fig. 3.1 : Test system network.

Generators A and B are connected to Buses 1 and 2 respectively. Customers A and B are connected to Buses 5 and 6 respectively. Two tap changing transformers $T_1$ and $T_2$ are connected between Buses 4 - 6 and 3 - 5 respectively. Under the existing situation, the total load could be supplied by the combined generation of Generators A and B. But the goal is to investigate the effect of the condition where each generator is tied to its own load by virtue of a contract. In a deregulated system, the concept of region does not exist and hence the customers are free to choose their own utilities. The objective is to find the share of transmission loss due to bilateral contracts between generating utilities and customers. For this purpose it has been assumed that bilateral contracts exist between Generator A and Customer A and between Generator B and Customer B. As a result of the contracts, Generator A will supply the load demand of Customer A only,

and Generator B will meet the demand of Customer B. As the generators are bound to supply certain amount of loads to their respective customers they are supposed to satisfy the associated transmission losses. The unknown at this point is the loss that an individual generator is responsible for.

The system parameters of the hypothetical system are shown in Tables 3.1 - 3.3. Bus 1 has been considered as the swing bus except in Case 1. The base values are 100 MVA and 138 kV. The tap changing transformers are set at nominal setting of 1 initially.

Table 3.1: Line parameters.

| Line Number | From Bus | To Bus | Length km | Resistance (p.u.) | Reactance (p.u.) |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 80 | 0.0157 | 0.0787 |
| 2 | 1 | 4 | 50 | 0.0105 | 0.0525 |
| 3 | 1 | 3 | 120 | 0.0367 | 0.1837 |
| 4 | 2 | 3 | 50 | 0.0105 | 0.0525 |
| 5 | 3 | 4 | 100 | 0.0262 | 0.1312 |

Table 3.2: Transformer data.

| Transformer Number | From Bus | To Bus | Resistance (p.u.) | Reactance (p.u.) |
|---|---|---|---|---|
| 1 | 4 | 6 | 0.0053 | 0.0367 |
| 2 | 3 | 5 | 0.0039 | 0.0315 |

Table 3.3: Generating unit characteristics.

| Generating Unit | Cost Function ($/hr) | Maximum Output (MW) | Minimum Output (MW) |
|---|---|---|---|
| A | $0.022P_1^2+12.45P_1+70$ | 500 | 90 |
| B | $0.024P_2^2+13.65P_2+80$ | 400 | 40 |

### 3.4 Allocation of Transmission Loss by Using the ILFA

In order to appreciate loss allocation in a deregulated system, different base cases have been studied first with the help of a conventional load flow analysis. Base cases assume

that the hypothetical system contains only one load and one generator. Base case load flow analyses help to find individual transmission loss for each generator. After the base cases, the system has been analyzed for two loads and their corresponding generators with the help of conventional load flow technique. This helps to find the effect of combined load on transmission loss and generations. Finally, the Incremental Load Flow Approach has been utilized to determine the share of transmission loss of the generators.

### 3.4.1 Case 1

At this stage, it is assumed that the hypothetical system has a generator (Generator B) and a load (Customer B). Generator B has been in contract with Customer B for supplying its demand. Two different load situations have been considered and the conventional load flow program has been used for these two load levels for finding the transmission loss. The corresponding generations and line losses are shown in Tables 3.4 and 3.5.

Table 3.4: Real and reactive power generation and line loss for Case 1.a.

| | $L_a$ (p.u.) | $L_b$ (p.u.) | $G_a$ (p.u.) | $G_b$ (p.u.) | Line loss (p.u.) |
|---|---|---|---|---|---|
| Real | 0 | 1.5 | 0 | 1.5799 | 0.0799 |
| Reactive | 0 | 0.9 | 0 | 1.3412 | 0.4412 |

Table 3.5: Real and reactive power generation and line loss for Case 1.b with reduced load.

| | $L_a$ (p.u.) | $L_b$ (p.u.) | $G_a$ (p.u.) | $G_b$ (p.u.) | Line loss (p.u.) |
|---|---|---|---|---|---|
| Real | 0 | 0.9 | 0 | 0.9243 | 0.0243 |
| Reactive | 0 | 0.54 | 0 | 0.6745 | 0.1345 |

It has been mentioned earlier that Generator B is operating only to meet the demand of Customer B. Table 3.4 shows that for a load of 1.5 + j0.9 p.u. the required generation is 1.5799 + j1.3412 p.u. The associated line loss is 0.0799 + j0.4412 p.u. The load of

Customer A is kept at zero and hence the generation of Generator A is also forced to zero. This is due to the fact that Generator A is contractually obligated to supply Customer A.

The load of Customer B has been changed to 0.9 + j0.54 p.u. The corresponding generation is 0.9243 + j0.6745 p.u. and the line loss is 0.0243 + j0.1345 p.u. (Table 3.5).

### 3.4.2 Case 2

This case has been investigated to see the effect of the bilateral contract between Generator A and Customer A on the associated transmission loss. The load of Customer B is set to zero and this allows the Generator B to produce nothing.

The generation and the line loss due to this contract is shown in Table 3.6.

Table 3.6: Real and reactive power generation and line loss in the system for Case 2.

| | Load $L_a$ (p.u.) | Load $L_b$ (p.u.) | Generation $G_a$ (p.u.) | Generation $G_b$ (p.u.) | Line loss (p.u.) |
|---|---|---|---|---|---|
| Real | 1.5 | 0 | 1.5527 | 0 | 0.0527 |
| Reactive | 0.9 | 0 | 1.2057 | 0 | 0.3057 |

For a load of 1.5 + j0.9 p.u. by Customer A, the Generator A is producing 1.5527 + j1.2057 p.u. and the calculated line loss is 0.0527 + j0.3057 p.u.

### 3.4.3 Case 3

After having the load flow solutions for individual contracts between the generators and the customers, both generators and loads have been brought into the system, which is our original system under consideration. Two different load situations have been considered - Customer A has same load in both situations while Customer B has different loads. Due to the bilateral contracts, Generator A would provide power to Customer A and Generator B would provide power to Customer B.

The generations of individual units and total line losses obtained from the conventional load flow program have been presented in Tables 3.7 - 3.10. The conventional load flow study needs the generation of PV buses to be specified and these generations are obtained from Cases 1 and 2.

Table 3.7: Real and reactive power generation and line loss in the system for Case 3.a (equal load demand) with bus 1 as the slack bus and the generation $G_b$ has been obtained from Case 1.

| | Load $L_a$ (p.u.) | Load $L_b$ (p.u.) | Generation $G_a$ (p.u.) | Generation $G_b$ (p.u.) | Line loss (p.u.) |
|---|---|---|---|---|---|
| Real | 1.5 | 1.5 | 1.5187 | 1.5799 | 0.0986 |
| Reactive | 0.9 | 0.9 | 1.4679 | 0.9036 | 0.5716 |

Table 3.8: Real and reactive power generation and line loss in the system for Case 3.a (equal load demand) with bus 2 as the slack bus and the generation $G_a$ has been obtained from Case 2.

| | Load $L_a$ (p.u.) | Load $L_b$ (p.u.) | Generation $G_a$ (p.u.) | Generation $G_b$ (p.u.) | Line loss (p.u.) |
|---|---|---|---|---|---|
| Real | 1.5 | 1.5 | 1.5527 | 1.5457 | 0.0984 |
| Reactive | 0.9 | 0.9 | 1.4615 | 0.9090 | 0.5705 |

Table 3.9: Real and reactive power generation and line loss in the system for Case 3.b (different load demand) with bus 1 as the slack bus and $G_b$ has been obtained from Case 1.

| | Load $L_a$ (p.u.) | Load $L_b$ (p.u.) | Generation $G_a$ (p.u.) | Generation $G_b$ (p.u.) | Line loss (p.u.) |
|---|---|---|---|---|---|
| Real | 1.5 | 0.9 | 1.5354 | 0.9244 | 0.0597 |
| Reactive | 0.9 | 0.54 | 0.9068 | 0.8841 | 0.3509 |

Table 3.10: Real and reactive power generation and line loss in the system for Case 3.b (different load demand) with bus 2 as the slack bus and $G_a$ has been obtained from Case 2.

| | Load $L_a$ (p.u.) | Load $L_b$ (p.u.) | Generation $G_a$ (p.u.) | Generation $G_b$ (p.u.) | Line loss (p.u.) |
|---|---|---|---|---|---|
| Real | 1.5 | 0.9 | 1.5527 | 0.9071 | 0.0598 |
| Reactive | 0.9 | 0.54 | 0.9039 | 0.8875 | 0.3514 |

In Table 3.7, the loads are equal to 1.5 + j0.9 p.u. and bus 1 has been considered as the slack bus. The generation $G_b$ has been taken from Case 1 that is 1.5799 p.u. In Table 3.8, bus 2 has been considered as the slack bus and the generation $G_a$ has been taken from Case 1 that is 1.5527 p.u. The real and reactive line losses are less than the sum of the losses found in Cases 1 and 2. But, the generation of reactive power does not drop for each generator. As for example, Table 3.8 shows that the reactive power generation of A (1.4679 p.u.) has been increased while the opposite has happened to B. Reactive generation of B has dropped to 0.9036 p.u. which is slightly higher than the reactive load of Customer B. This shows clearly that Generator B is not producing as much reactive power as it produced under Case 1. Generator B is supposed to supply the load of Customer B along with the loss originating from this bilateral transaction. This is due to the fact that transmission losses, both real and reactive are nonlinear functions of load and generation.

Tables 3.9 and 3.10 shows the generation and loss for different demands of Customers A and B. Customer B's load ($L_b$) has been changed to 0.9 + j0.54 p.u. while Customer A's ($L_a$) remains unchanged at 1.5 + j0.9 p.u. The study has been done again taking bus 1 and 2 as the slack bus and corresponding PV bus generations are obtained from Cases 1 and 2. Comparing the results shown in Table 3.6 with Tables 3.9 and 3.10, Generator A is producing much less reactive power than it produces under Case 2. It is clear that, from Table 3.9, more burden of producing reactive power has been taken by Generator B (0.8841 p.u.) while Generator A (0.9068 p.u.) is producing less than that produced in Case 2 which is just higher than the reactive demand of Customer A. Again the study has been made under this case is to show how the generations vary with the selection of slack bus. In Tables 3.11 and 3.12, the generation, total transmission loss and contribution of the generators to transmission loss have been summarized.

The situations considered in Case 3 leads to a set of questions. What should be the appropriate share of each generator in producing real and reactive power? What would happen if one of the generators is unable to produce its due share and then to whom would this generator be liable for this support? The incremental load flow approach can be utilized to find the appropriate share of each generator in terms of real and reactive power.

Table 3.11: Comparison of real loss contribution for the three different cases.

| | Load (p.u.) | | Slack Bus | System Loss (p.u.) | Generation to mitigate loss (p.u.) | |
|---|---|---|---|---|---|---|
| | $L_a$ | $L_b$ | | | $G_a$-$L_a$ | $G_b$-$L_b$ |
| Case 1.a | 0 | 1.5+j0.9 | 2 | 0.0799 | 0 | 0.0799 |
| Case 2 | 1.5+j0.9 | 0 | 1 | 0.0527 | 0.0527 | 0 |
| Case 3.a | 1.5+j0.9 | 1.5+j0.9 | 1 | 0.0986 | 0.0187 | 0.0799 |
| | | | 2 | 0.0984 | 0.0527 | 0.0457 |
| Case 1.b | 0 | 0.9+j0.54 | 2 | 0.0243 | 0 | 0.0243 |
| Case 3.b | 1.5+j0.9 | 0.9+j0.54 | 1 | 0.0597 | 0.0354 | 0.0244 |
| | | | 2 | 0.0598 | 0.0527 | 0.0071 |

Table 3.12: Comparison of real loss contribution for the three different cases.

| | Load (p.u.) | | Slack Bus | System Loss (p.u.) | Generation to mitigate loss (p.u.) | |
|---|---|---|---|---|---|---|
| | $L_a$ | $L_b$ | | | $G_a$-$L_a$ | $G_b$-$L_b$ |
| Case 1.a | 0 | 1.5+j0.9 | 2 | 0.4412 | 0 | 0.4412 |
| Case 2 | 1.5+j0.9 | 0 | 1 | 0.3057 | 0.3057 | 0 |
| Case 3.a | 1.5+j0.9 | 1.5+j0.9 | 1 | 0.5716 | 0.5679 | 0.0036 |
| | | | 2 | 0.5705 | 0.5615 | 0.0090 |
| Case 1.b | 0 | 0.9+j0.54 | 2 | 0.1345 | 0 | 0.1345 |
| Case 3.b | 1.5+j0.9 | 0.9+j0.54 | 1 | 0.3509 | 0.0068 | 0.3441 |
| | | | 2 | 0.3514 | 0.0039 | 0.3475 |

### 3.4.4 Case 4

In the ILFA, the loads are increased in incremental steps in a sequential manner. In any iteration, only one customer load is increased while the other loads are held fixed. Let us assume that load $L_a$ is increased by a step of $\Delta L_a$ while $L_b$ stays at its previous level. A load flow program has been used to find the generation and transmission loss for this condition. Since load demand of Customer B ($L_b$) is unchanged, the resulting incremental transmission loss becomes the obligation of Generator A to produce it in order to support the incremental load demand of customer A. In the next iteration, $L_b$ has been increased by a step size of $\Delta L_b$ while $L_a$ remains fixed. Again, generations and transmission losses are calculated and the incremental loss is assigned to Generator B. This is done as only $L_b$ has been incremented in this iteration and Generator B is providing power to Customer B.

In order to obtain a load flow solution, the buses connected with generating units have to be declared as voltage controlled (PV) bus. The power generations at these buses are required to be specified prior to the load flow. It is a tricky situation where generation of the PV bus should be the sum of load and its share of the total line loss before the calculation of the line loss.

Generator B is connected to Bus 2, which has been declared as PV bus. The generation at Bus 2 has been calculated on the basis of current load ($L_b$) and transmission loss calculated from the previous load. Since the transmission loss for the first increment of load $L_b$ is unknown, the output of Generation B ($G_b$) has been specified equal to $\Delta L_b$ during the first increment of Customer B's load. In the next increment of Customer B's load (iteration number 4), $G_b$ has been updated as $L_b$ plus the corresponding share of the line loss from the previous iteration. This approximation works well as long as the step size remains small which is 1MW in this case.

Using this approximation and reactive ratio of 0.6, the incremental load flow program has been utilized to obtain generations and transmission losses. Individual generations and transmission losses are shown in Tables 3.13 and 3.14.

Table 3.13: Individual generations and total transmission loss obtained from ILFA for equal load condition.

| | $L_a$ (p.u.) | $L_b$ (p.u.) | $G_a$ (p.u.) | $G_b$ (p.u.) | Line loss (p.u.) |
|---|---|---|---|---|---|
| Real | 1.5 | 1.5 | 1.5414 | 1.557 | 0.0984 |
| Reactive | 0.9 | 0.9 | 1.4635 | 0.9072 | 0.5707 |

Table 3.14: Individual generations and total transmission loss obtained from ILFA for different load condition.

| | $L_a$ ( p.u. ) | $L_b$ ( p.u. ) | $G_a$ ( p.u. ) | $G_b$ ( p.u. ) | Line loss ( p.u. ) |
|---|---|---|---|---|---|
| Real | 1.5 | 0.9 | 1.5414 | 0.9182 | 0.0597 |
| Reactive | 0.9 | 0.54 | 0.9057 | 0.8853 | 0.3511 |

The main goal of the ILFA is to allocate transmission losses among the generating utilities in a deregulated power system network. Six different situations have been considered for this purpose and the corresponding transmission loss allocations have been assessed.

### 3.4.4.1 Equal Load and Equal Reactive Ratio

The loads of Customers A and B have been kept fixed and their reactive power ratios have been varied from 0.6 to 0.3. Allocations of transmission loss have been found for the corresponding variations in reactive power. Table 3.15 shows the calculated allocation of transmission loss for Generators A and B. The losses, both real and reactive, decrease with a decrease in reactive ratio ($\mu$) i.e., reactive load.

The loss allocation curves, with load varied from zero to respective demand of Customers A and B, are shown in Figures 3.2 and 3.3 for a reactive ratio of 0.6.

Table 3.15: Calculated share of transmission loss of generator A and B for equal load and equal reactive ratio.

| Real $L_a$ (p.u.) | Real $L_b$ (p.u.) | Reactive ratio μ | Calculated Loss Share of A (p.u.) | | Calculated Loss Share of B (p.u.) | |
|---|---|---|---|---|---|---|
| | | | real | Reactive | Real | Reactive |
| 1.5 | 1.5 | 0.6 | 0.0405 | 0.243 | 0.0579 | 0.3277 |
| 1.5 | 1.5 | 0.5 | 0.036 | 0.2158 | 0.0518 | 0.2934 |
| 1.5 | 1.5 | 0.4 | 0.0322 | 0.194 | 0.0469 | 0.2653 |
| 1.5 | 1.5 | 0.3 | 0.0293 | 0.1769 | 0.0432 | 0.2432 |

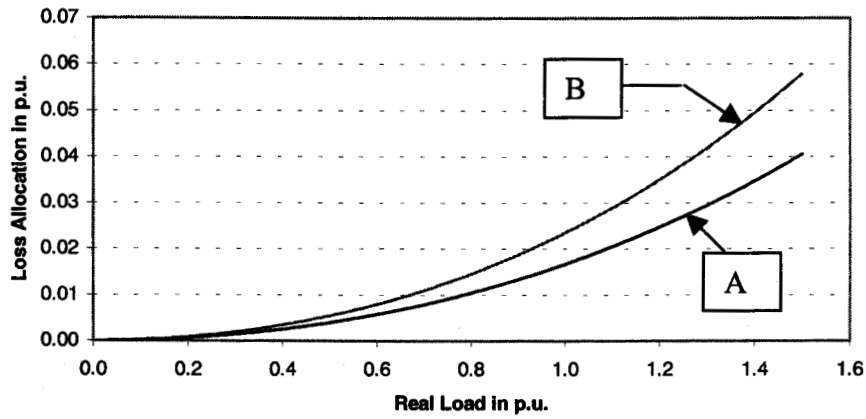


Fig. 3.2: Loss allocation of real power for Generators A and B (μ=0.6).

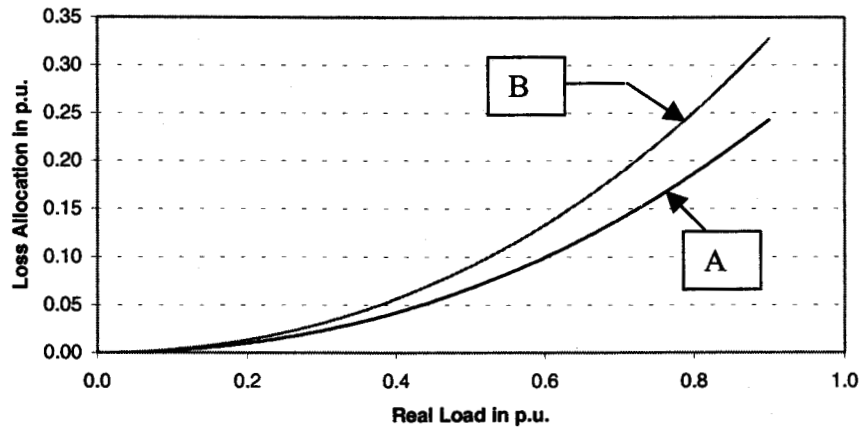


Fig. 3.3: Loss allocation of reactive power for Generators A and B (μ=0.6).

Figures 3.2 and 3.3 show the allocation of real and reactive components of transmission loss for Generators A and B. Loss allocations, both real and reactive, are higher for Generator B. Generator A is getting advantage of line 3 in the network and hence gets a lesser share of transmission loss.

### 3.4.4.2 Equal Load and Different Reactive Ratio

The real loads of Customers A and B are equal with unequal reactive ratio. The reactive ratio is varied for Customer B while a constant reactive ratio is maintained for Customer A. The allocations of transmission losses are shown in Table 3.16.

Table 3.16: Calculated share of transmission loss of generator A and B for equal load and unequal reactive ratio.

| Real $L_a$ (p.u.) | Real $L_b$ (p.u.) | Reactive ratio $\mu$ | Reactive ratio $\mu$ | Calculated Loss Share of A (p.u.) | | Calculated Loss Share of B (p.u.) | |
|---|---|---|---|---|---|---|---|
| | | | | real | Reactive | Real | Reactive |
| 1.5 | 1.5 | 0.6 | 0.3 | 0.039 | 0.2353 | 0.0441 | 0.2488 |
| 1.5 | 1.5 | 0.6 | 0.4 | 0.0395 | 0.2376 | 0.0478 | 0.2698 |
| 1.5 | 1.5 | 0.6 | 0.5 | 0.0400 | 0.2402 | 0.0523 | 0.296 |
| 1.5 | 1.5 | 0.5 | 0.3 | 0.0351 | 0.2116 | 0.0438 | 0.2469 |
| 1.5 | 1.5 | 0.5 | 0.4 | 0.0356 | 0.2136 | 0.0473 | 0.2676 |
| 1.5 | 1.5 | 0.4 | 0.3 | 0.0319 | 0.1923 | 0.0435 | 0.245 |

A change in the reactive load of Customer B affects the loss allocation of Generator A by a small magnitude. The major change in loss allocation goes to generator B's account for obvious reason.

Transmission loss allocations are shown in Figures 3.4 - 3.7. Graphs are plotted (Figures 3.4 - 4.7) for the two combinations of reactive and real loads and all graphs exhibit similar trend. Real loads are equal for both customers whereas reactive loads are different. This condition has been reflected in Figures 3.4 - 3.7 where reactive loss allocation for Generator B rises very sharply. This is due to the fact that relative position of Customer B with respect to Generator B in the network requires more reactive power to be produced by Generator B to maintain the voltage profile.

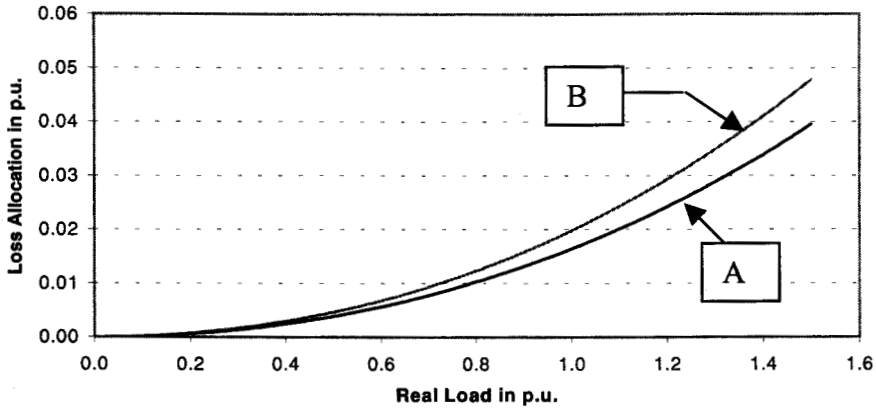


Fig. 3.4: Loss allocation of real power for Generators A and B ($\mu_a$=0.6, $\mu_b$=0.4).

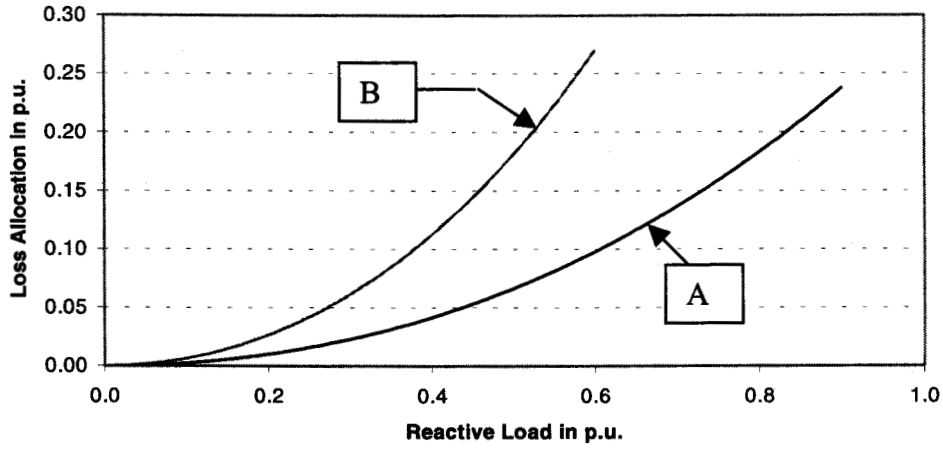


Fig. 3.5: Loss allocation of reactive power for Generators A and B ($\mu_a$=0.6, $\mu_b$=0.4).

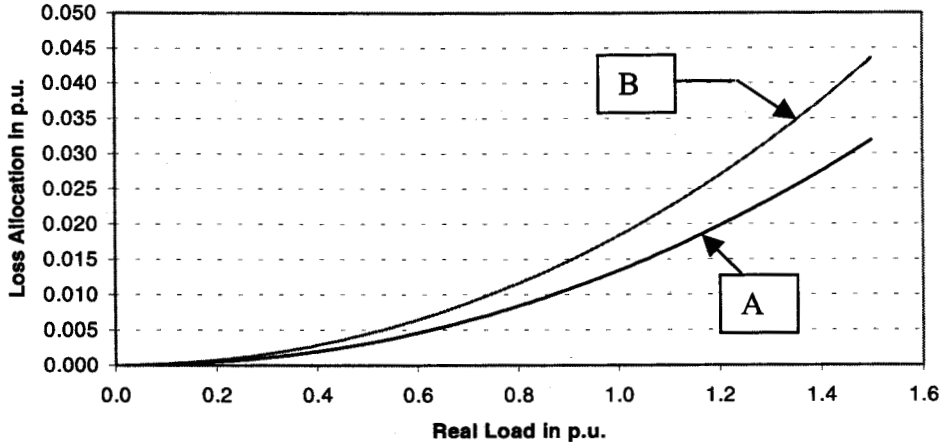


Fig. 3.6: Loss allocation of real power for Generators A and B ($\mu_a$=0.4, $\mu_b$=0.3).

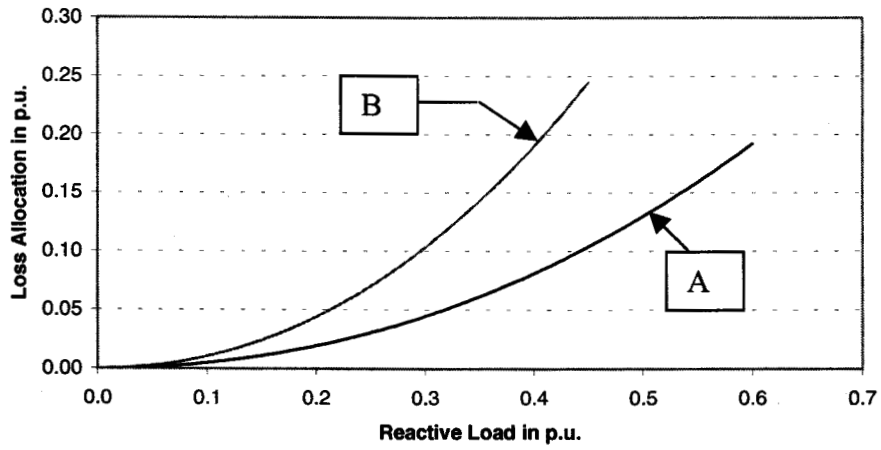


Fig. 3.7: Loss allocation of reactive power for Generators A and B ($\mu_a$=0.4, $\mu_b$=0.3).

### 3.4.4.3 Unequal Load and Equal Reactive Ratio

The real load demands of Customers A and B have been kept different while keeping their reactive ratios identical. The reactive ratio has been changed from 0.6 to 0.3. Although the reactive ratio is same for both loads but load demands are different, the trend in loss allocation remains same. The calculated loss allocation is shown in Table 3.17.

Table 3.17: Calculated share of transmission loss of generator A and B for unequal load and equal reactive ratio.

| Real $L_a$ (p.u.) | Real $L_b$ (p.u.) | Reactive ratio $\mu$ | Calculated Loss Share of A (p.u.) | | Calculated Loss Share of B (p.u.) | |
|---|---|---|---|---|---|---|
| | | | Real | reactive | Real | reactive |
| 1.5 | 0.9 | 0.6 | 0.0409 | 0.2449 | 0.0187 | 0.1064 |
| 1.5 | 0.9 | 0.5 | 0.0367 | 0.2187 | 0.0169 | 0.0968 |
| 1.5 | 0.9 | 0.4 | 0.0331 | 0.1977 | 0.0156 | 0.0886 |
| 1.5 | 0.9 | 0.3 | 0.0302 | 0.1814 | 0.0147 | 0.0819 |

Due to a smaller demand by Customer B, the loss allocation of Generator B also becomes smaller than that of Generator A. From Table 3.15 it is evident that loss allocation decreases with lower reactive ratio.

Figures 3.8 and 3.9 show the loss allocation under the situation described for a reactive ratio of 0.5.

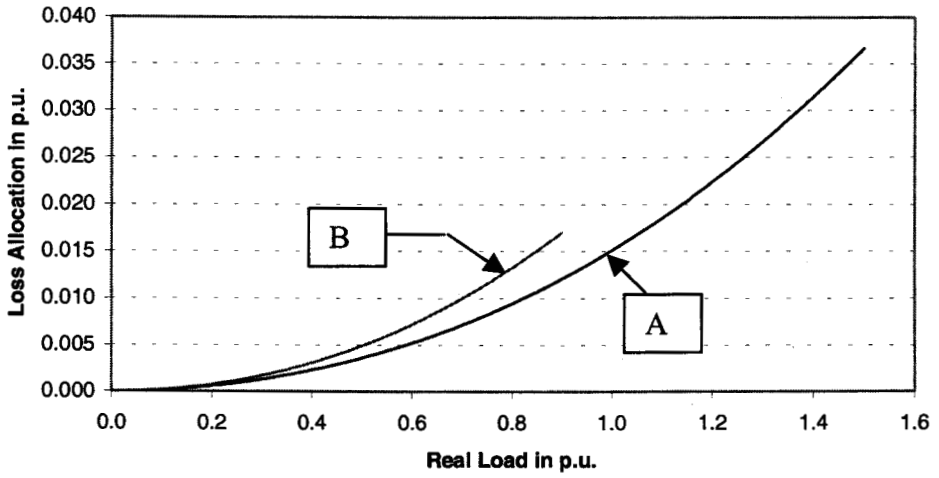


Fig. 3.8: Loss allocation of real power for Generators A and B for unequal load demands ($\mu$=0.5).

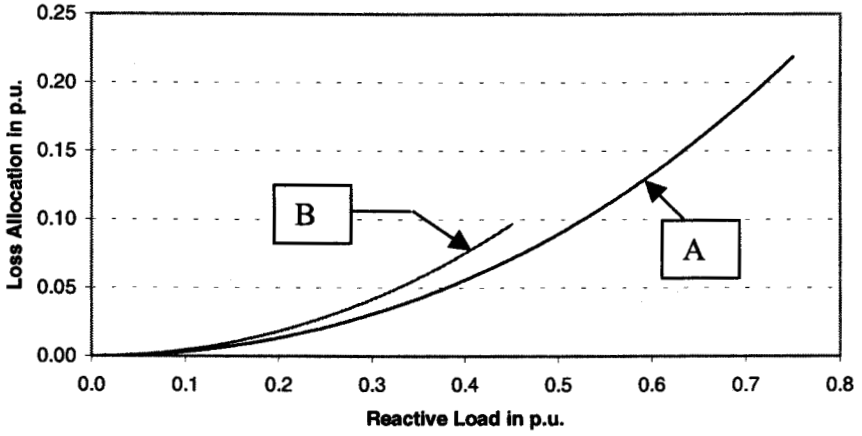


Fig. 3.9: Loss allocation of reactive power for Generators A and B for unequal load demands ($\mu$=0.5).

The load demands of Customers A and B are different while having equal reactive ratio. The curves show that the loss allocations of Generator B is steeper than that of Generator A even though Customer B has a lower demand of power compared to Customer A.

### 3.4.4.4 Unequal Load and Different Reactive Ratio

This is a combination of load situations described in sections 3.4.4.2 and 3.4.43. The reactive ratios as well as the real loads, of Customers A and B are different. The reactive ratio of Customer B has been varied while maintaining a constant reactive ratio for customer A e.g. 0.6. Transmission losses are shown in Table 3.18.

Table 3.18: Calculated share of transmission loss of generator A and B for unequal load and unequal reactive ratio.

| Real $L_a$ (p.u.) | Real $L_b$ (p.u.) | Reactive ratio $\mu$ | Reactive ratio $\mu$ | Calculated Loss Share of A (p.u.) | | Calculated Loss Share of B (p.u.) | |
|---|---|---|---|---|---|---|---|
| | | | | Real | Reactive | real | Reactive |
| 1.5 | 0.9 | 0.6 | 0.3 | 0.0397 | 0.239 | 0.0148 | 0.0836 |
| 1.5 | 0.9 | 0.6 | 0.4 | 0.0401 | 0.2407 | 0.0157 | 0.0899 |
| 1.5 | 0.9 | 0.6 | 0.5 | 0.0405 | 0.2427 | 0.0171 | 0.0974 |
| 1.5 | 0.9 | 0.5 | 0.3 | 0.0359 | 0.2156 | 0.0147 | 0.0832 |
| 1.5 | 0.9 | 0.5 | 0.4 | 0.0363 | 0.217 | 0.0156 | 0.0894 |
| 1.5 | 0.9 | 0.4 | 0.3 | 0.0328 | 0.1965 | 0.0147 | 0.0825 |

Figures 3.10 - 3.13 show loss allocations for different combinations of real loads and reactive ratios.

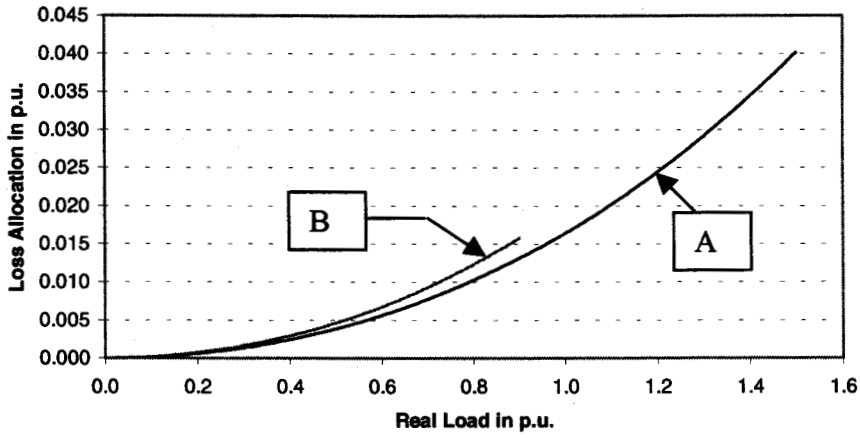


Fig. 3.10: Loss allocation of real power for Generators A and B for unequal demands ($\mu_a$=0.6, $\mu_b$=0.4).

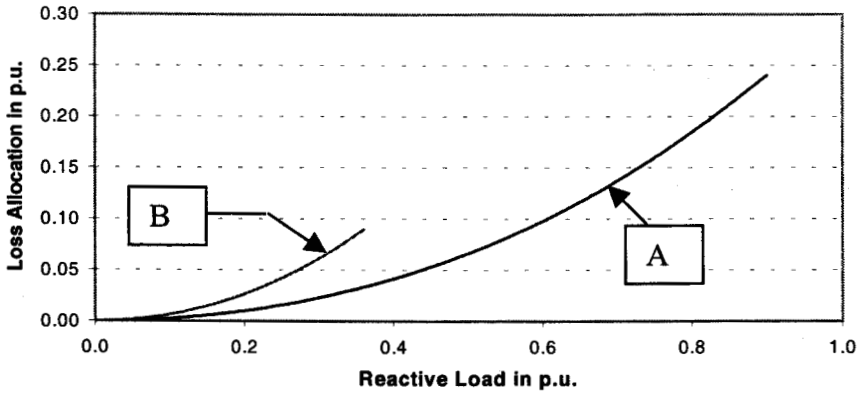


Fig. 3.11: Loss allocation of reactive power for Generators A and B for unequal demands ($\mu_a$=0.6, $\mu_b$=0.4).

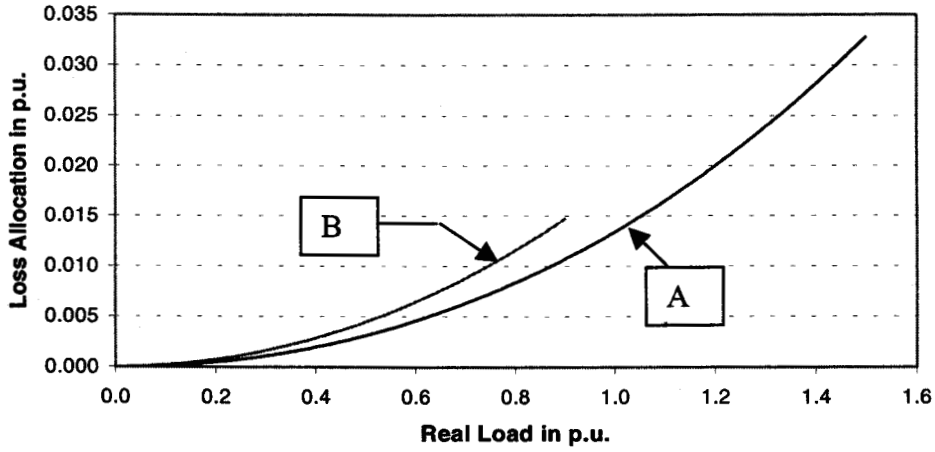


Fig. 3.12: Loss allocation of real power for Generators A and B for unequal demands ($\mu_a$=0.4, $\mu_b$=0.3).

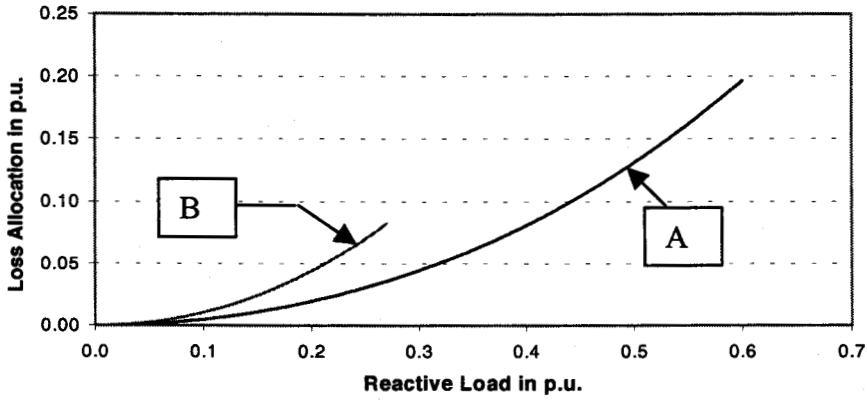


Fig. 3.13: Loss allocation of reactive power for Generators A and B for unequal demands ($\mu_a$=0.4, $\mu_b$=0.3).

The nature of the loss allocation curves remains similar for variations in load demands and reactive ratios. In both cases of reactive ratios, real loss allocation of Generator A follows that of B closely whereas reactive loss allocations differ significantly.

### 3.4.4.5 Different Load and Equal Reactive Ratio

The load of Customer A has been kept constant while that of customer B has been varied from 90 to 40 MW. Reactive ratios are assumed to be equal and constant for both customers. The corresponding transmission loss allocations are shown in Table 3.19.

Table 3.19: Calculated share of transmission loss of generator A and B for different load and equal reactive ratio.

| Real $L_a$ (p.u.) | Real $L_b$ (p.u.) | Reactive ratio $\mu$ | Calculated Loss Share of A (p.u.) | | Calculated Loss Share of B (p.u.) | |
|---|---|---|---|---|---|---|
| | | | real | Reactive | real | Reactive |
| 1.5 | 0.9 | 0.6 | 0.0409 | 0.2449 | 0.0187 | 0.1064 |
| 1.5 | 0.8 | 0.6 | 0.0411 | 0.2457 | 0.0145 | 0.0828 |
| 1.5 | 0.7 | 0.6 | 0.0414 | 0.2470 | 0.0109 | 0.0624 |
| 1.5 | 0.6 | 0.6 | 0.0417 | 0.2484 | 0.0079 | 0.0452 |
| 1.5 | 0.5 | 0.6 | 0.0421 | 0.2504 | 0.0054 | 0.0309 |
| 1.5 | 0.4 | 0.6 | 0.0426 | 0.2523 | 0.0034 | 0.0195 |

It can be noticed from Table 3.19 that the loss allocation of A varies very little with the load variation of customer B. This is due to the fact that the load of Customer A is held constant.

The loss allocations for different load combinations under this situation have been shown in Figures 3.14 and 3.15. The load demand of Customer B has been varied while maintaining a constant load for Customer A. Figures 3.14 and 3.15 show the loss allocations when real load demand of Customer B is 80 MW or 0.8 p.u. and that of Customer A remains at 150 MW or 1.5 p.u. Both real and reactive loss allocations of Generator B is smaller than that of Generator A. This is due to the obvious fact that Generator B is required to provide a smaller load than that of Generator A.

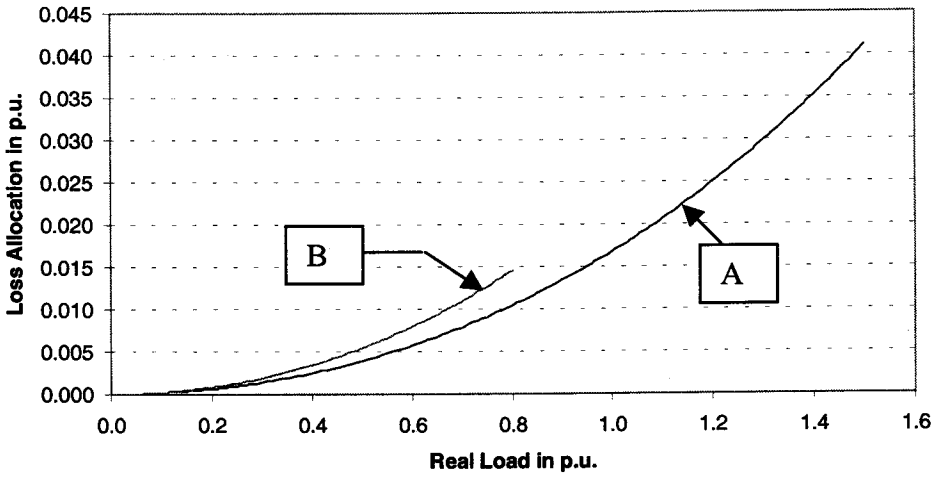


Fig. 3.14: Loss allocation of real power for Generators A and B for different demands ($\mu$=0.6).

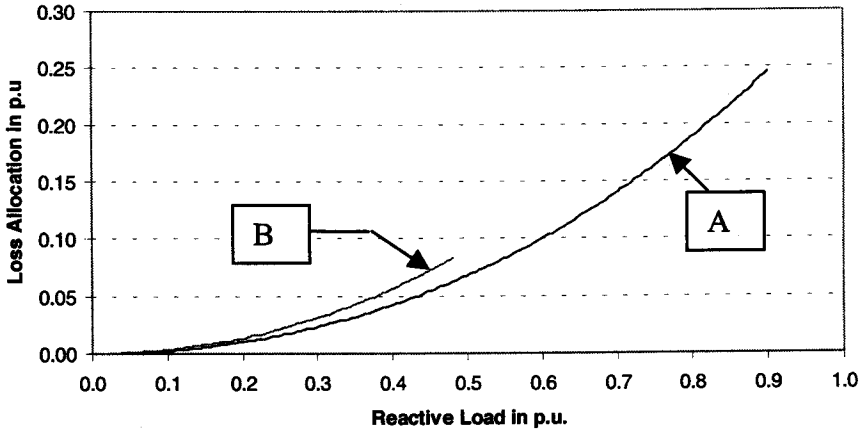


Fig. 3.15: Loss allocation of reactive power for Generators A and B for different demands ($\mu$=0.6).

### 3.4.4.6 Modified System

Transmission loss is a function of network configuration. If a line is taken out of the network or added to the network, the load flow pattern would change. A slightly modified system network has been used to determine its effect on loss allocation. Line number 3 has been omitted from the system shown in Figure 3.1. The load has been maintained same for both customers while two different reactive ratios have been used. The loss allocation for the modified system network has been shown in Table 3.20.

Table 3.20: Calculated share of transmission loss of generator A and B for modified system.

| Real $L_a$ (p.u.) | Real $L_b$ (p.u.) | Reactive ratio μ | Calculated Loss Share of A (p.u.) | | Calculated Loss Share of B (p.u.) | |
|---|---|---|---|---|---|---|
| | | | Real | Reactive | real | Reactive |
| 1.5 | 1.5 | 0.6 | 0.0514 | 0.2993 | 0.0569 | 0.3228 |
| 1.5 | 1.5 | 0.4 | 0.0412 | 0.2398 | 0.0453 | 0.2572 |

Due to the omission of one of the lines the other lines have been forced to carry more load and hence the total loss in the system has increased. Comparing the losses in Table 3.9 with Table 3.18 it can be noticed that Generator B is benefited at the cost of Generator A. This happened because line number 3 was connected directly between Generator A and Customer A. When this line was taken out of the network, Customer A was directly affected. Line 3 did not provide much support to the power flow between Generator B and Customer B and this has been reflected in the loss allocation.

Figures 3.16 and 3.17 show the loss allocations for the modified system network. The omission of line 3 has affected Generator A directly. Generator A's as its share of transmission loss has been increased as shown in Figures 3.16 and 3.17. Comparing Figures 3.2 and 3.3 with Figures 3.16 and 3.17 respectively, it can be observed that the loss allocations of Generators A and B increase in a similar way. The loss allocations of Generator A and B would be exactly equal if the impedances of line 1 and 5 were equal.

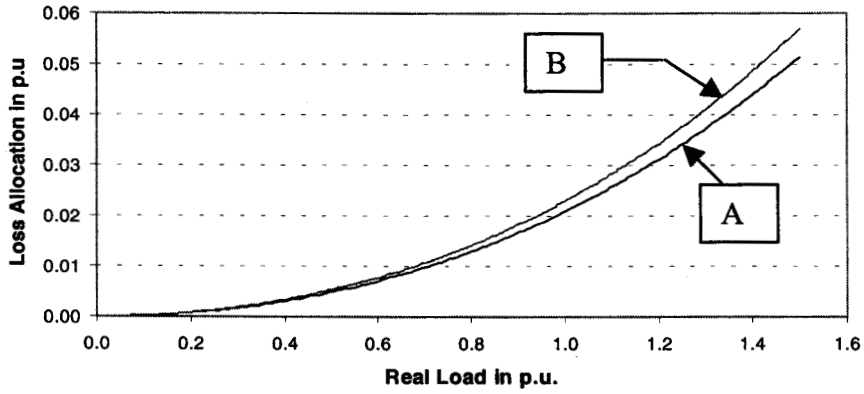


Fig. 3.16: Loss allocation of real power for Generators A and B for modified system (μ=0.6).

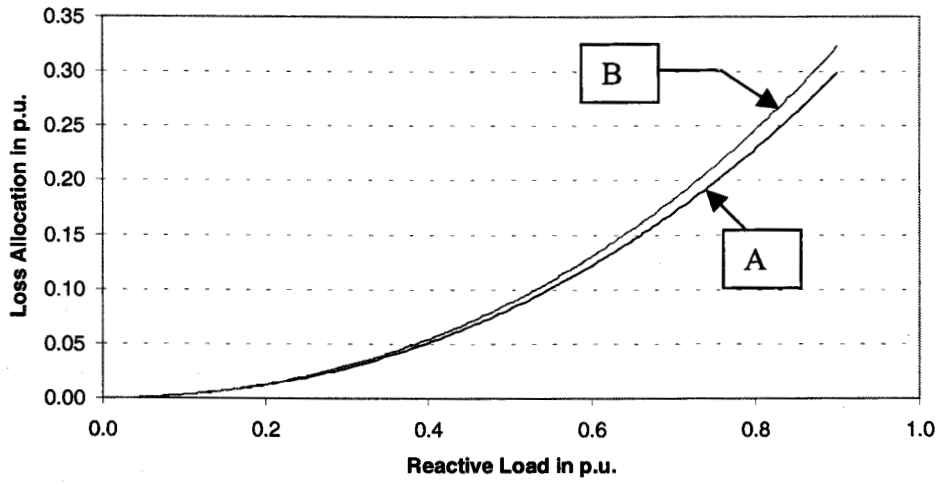


Fig. 3.17: Loss allocation of reactive power for Generators A and B for modified system ($\mu$=0.6).

### 3.4.5 System without Voltage Constraints

Let us consider one of the load situations described in Section 3.4.4.1, Customers A and B have equal demand of 150 MW with equal reactive ratio of 0.6. Table 3.21 shows generations, calculated shares of loss and contributions towards the mitigation of loss of each generator. The two generators' contributions towards the reactive component of the transmission loss as obtained by the ILFA are different from their calculated shares of loss. This difference occurs due to the nature of the load flow technique. This can be attributed to the load flow method where the generation of reactive power is determined by restricting the voltage levels.

Table 3.21: Generation and share of loss allocation for a load condition of Section 3.4.4.1 (load = 1.5+j0.9 p.u.).

| | | Generation (p.u.) | Share of Loss (p.u.) | Generation to mitigate loss (p.u.) |
|---|---|---|---|---|
| Generator A | Real | 1.5414 | 0.0405 | 0.0414 |
| | Reactive | 1.4635 | 0.243 | 0.5635 |
| Generator B | Real | 1.557 | 0.0579 | 0.057 |
| | Reactive | 0.9072 | 0.3277 | 0.0072 |

When a generator bus is declared as the PV bus, the load flow determines the reactive power that has to be produced by each generator in order to maintain the bus voltage. A slight modification has been done to investigate what happens if the PV buses are declared as PQ buses. By relaxing the voltage restrictions, this modification allows us to calculate a generator's share of reactive power loss. The corresponding results have been shown in Table 3.22. Table 3.23 shows the bus voltages with and without voltage restriction.

Table 3.22: Modified generation in the system without voltage restriction.

| | $G_a$ ( p.u. ) | $G_b$ ( p.u. ) | Line loss ( p.u. ) | Generation to mitigate loss (p.u.) | | Calculated share of loss (p.u.) | |
|---|---|---|---|---|---|---|---|
| | | | | $G_a$ -$L_a$ | $G_b$ - $L_b$ | $L_a$ | $L_b$ |
| Real | 1.5407 | 1.5568 | 0.0976 | 0.0407 | 0.0568 | 0.0399 | 0.0577 |
| Reactive | 1.1455 | 1.2197 | 0.5653 | 0.2455 | 0.3197 | 0.2407 | 0.3246 |

Table 3.23: Bus voltages with and without voltage restrictions in p.u.

| | Bus 1 | Bus 2 | Bus 3 | Bus 4 | Bus 5 | Bus 6 |
|---|---|---|---|---|---|---|
| With voltage restriction | 1.04 | 1.04 | 0.9803 | 0.9698 | 0.9617 | 0.9609 |
| Without voltage restriction | 1.04 | 1.055 | 0.9907 | 0.973 | 0.9633 | 0.9644 |

This study has to be performed in each system to ascertain the acceptability of the voltage levels. In a system, a generator may not be able to produce its share of reactive power loss due to its voltage restriction. However, the ILFA can be used to calculate the share of the reactive power loss by relaxing voltage restrictions. This will indicate how much a generator would have to depend on others in terms of reactive power and may lead to a suitable financial compensation. The assessment of the interdependence and financial compensation is beyond the scope of this thesis.

The modification confirms the line loss share of the generators calculated by the ILFA. Comparing loss allocation and generation to mitigate loss of each generator from Tables 3.21 and 3.22, it can be said that the loss share calculated by the ILFA is quite fair to both generators. Due to this modification, the voltages at different buses have been

changed which is obvious. But this result indicates the fair share of the transmission loss if the generators could run with relaxed voltage constraints.

### 3.5 Overview of Loss Allocations

An extensive study has been done to find transmission loss allocation in a fully deregulated power system network. Different load combinations have been taken into consideration and loss allocations have been calculated using the ILFA. The ILFA can be utilized to assess the allocation of real and reactive transmission losses in a deregulated system with bilateral contracts.

A generator in a system may not be able to supply its due share of real and reactive power. Results obtained from the ILFA can be utilized to assess the shortfall of the generator. Real component of transmission loss is directly related to monetary issues and hence should be resolved in a fair way. The reactive loss distribution is also of importance as it must be provided to maintain the system voltage level.

# CHAPTER 4: MARGINAL TRANSMISSION LOSS APPROACH

## 4.1 General Transmission Loss Formula

In a monopoly system, generating units are committed and dispatched in a way that the total operating cost is minimized. Transmission loss is viewed as a part of the overall operating cost and included in the optimization process with the help of Lagrange multipliers. In a deregulated environment, generating entities are responsible for supplying power to their own customers. Generating entities will use a common transmission facility to transport power to their respective customers. Transmission facilities would provide equal access to all participating (competing) generating entities. Each generating entity should produce enough power to meet its load and its share of transmission loss.

Transmission loss can be calculated accurately by AC load flow analysis. Due to the computational complexities of AC load flows, other methods have been developed to calculate transmission loss in a simpler way. The use of a generalized transmission loss formula that does not depend on network configuration is a convenient way for calculating transmission loss.

Due to its application in economic optimization, transmission loss, in general, is expressed in terms of active power generations only. The George's formula [18] is the simplest one that is given by

$$P_l = \sum_{i=1}^{m} \sum_{j=1}^{m} (P_i B_{ij} P_j) \tag{4.1}$$

Where,

$P_l$ = total transmission loss

$P_i$ = active power injection at bus $i$.

The coefficients $B_{ij}$ are commonly referred to as loss coefficients. A more generalized formula is given by Kron. Kron's Loss formula [17] is

$$P_l = K_{lo} + \sum_{i=1}^{m} B_{i0} P_i + \sum_{j=1}^{m} (P_i B_{ij} P_j) \tag{4.2}$$

In both formulae, active generations are used as the only variables to reduce the computational complexities. The main advantage of the generalized loss formulae is that they are easy to use and do not require iterations like load flow.

It is clear from George's and Kron's formula that neither of them reflect the load situations in the network. Although the generations used in these formulae are based on load demand in the network, they do not provide sufficient information with respect to the transmission loss allocation of an individual generator in a deregulated network. In order to allocate transmission loss, the contribution of individual loads to loss in a system has to be determined. Any change in a load should be taken into consideration and its effect should be reflected in transmission loss calculation. This would indicate the particular generator that is supposed to change its output according to the change in its load.

In order to allocate transmission loss among the generators in a fair way, a modified transmission loss formula is necessary which would allow us to find an individual generator's responsibility in a deregulated network.

### 4.2 Marginal Transmission Loss

The basic principle of the generalized loss formulae can be used in a modified form for the purpose of transmission loss allocation. The modified transmission loss formula should be a function of individual loads in a system and should be able to respond to changes in loads. As generation in a deregulated system should depend upon contracted load and its share of transmission loss, the modified loss formula would include generation in an indirect manner.

The example system described in Chapter 3 has been used to develop the modified loss formula. In that system Generator A and B are supplying load $L_a$ and $L_b$ respectively. It

is assumed that Generator A should produce enough power to supply Customer A and satisfy its share of transmission loss. Mathematically this can be written as

$$G_a = Load_a + Loss_a \tag{4.3}$$

Where,

$G_a$ = active power generation of Generator A in MW,

$Load_a$ = active power demand of a bulk consumer in MW that Generator A is obliged to supply and

$Loss_a$ = Generator A's share of transmission loss in MW.

In the same way, Generator B should produce enough power to meet the demand of Customer B along with the associated transmission loss. Equation (4.3) implies that the generation is a function of load. A conventional loss formula uses the amount of generation that includes loss too. But, in a deregulated system it is required to separate the loss that is originated due to a particular load. In a monopoly system, the losses are distributed as a consequence of economic operation. In a deregulated network, it is important and necessary to keep track of the losses arising from individual loads.

A modified loss equation has been derived which is a function of loads. The modified loss equation can be used to calculate loss associated with an individual load in a deregulated system with bilateral contracts.

### 4.3 Mathematical Model

Transmission loss in a network can be expressed as [19]:

$$\begin{aligned} P_l = {} & [P_G]^T[A_p][P_G] - [P_D]^T[A_p][P_G] - [P_G]^T[A_p][P_D] + [P_D]^T[A_p][P_D] - [P_G]^T[B_p][Q_G] \\ & + [P_D]^T[B_p][Q_G] + [P_G]^T[B_p][Q_D] - [P_D]^T[B_p][Q_D] + [Q_G]^T[B_p][P_G] - [Q_D]^T[B_p][P_G] \\ & - [Q_G]^T[B_p][P_D] + [Q_D]^T[B_p][P_D] + [Q_G]^T[A_p][Q_G] - [Q_D]^T[A_p][Q_G] - [Q_G]^T[A_p][Q_D] \\ & + [Q_D]^T[A_p][Q_D] \end{aligned} \tag{4.4}$$

Equation (4.4) is dependent on real and reactive power generations and demands in a network. For the purpose of transmission loss allocation, Equation (4.2) can be modified as a function of power demands. The required modifications depend on

system configuration and the example system used in Chapter 3 has been used here to perform the necessary modifications.

The traditional generalized loss formulae have been based on some assumptions. In order to derive the modified form of loss equation few assumptions have been considered as well. These assumptions help to keep the challenge of transmission loss allocation manageable and do not affect the results significantly. They are:

1. Z-bus remains constant over the range of load variation. Although a Z-bus changes with a change in the setting of tap-changing transformers, its overall effect can be neglected.
2. Bus angles remain constant for a part of the load variation. In this case, three different sets of angles have been used for the entire load range. More sets may be used for better accuracy.
3. For the first step of the iteration, the loss has been assumed to be zero. This makes the required generation equal to the load at the beginning.
4. Reactive to real power ratio remains constant. This reduces the number of variables in transmission loss equation.
5. Bus voltages remain constant at 1 p.u. for the entire range of the load.

The formula for allocation of individual generator's transmission loss is based on the assumptions mentioned above. The next section describes the development of the required modified transmission loss formula.

### 4.3.1 Simplified Loss Equation for the Example System

The system that we are considering for the purpose of allocating transmission loss consists of six buses. The loss equation, Equation (4.4), has been simplified and expressed in terms of the system parameters. Bus 3 has been taken as the reference. The parameters in Equation (4.4) can be written as:

$$P_G = \begin{bmatrix} P_{G1} \\ P_{G2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad Q_G = \begin{bmatrix} Q_{G1} \\ Q_{G2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad P_D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ P_{D5} \\ P_{D6} \end{bmatrix}, \qquad Q_D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Q_{D5} \\ Q_{D6} \end{bmatrix}$$

$$A_p = \begin{bmatrix} a_{p11} & a_{p12} & a_{p14} & a_{p15} & a_{p16} \\ a_{p21} & a_{p22} & a_{p24} & a_{p25} & a_{p26} \\ a_{p41} & a_{p42} & a_{p44} & a_{p45} & a_{p46} \\ a_{p51} & a_{p52} & a_{p54} & a_{p55} & a_{p56} \\ a_{p61} & a_{p62} & a_{p64} & a_{p65} & a_{p66} \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 & b_{p12} & b_{p14} & b_{p15} & b_{p16} \\ b_{p21} & 0 & b_{p24} & b_{p25} & b_{p26} \\ b_{p41} & b_{p42} & 0 & b_{p45} & b_{p46} \\ b_{p51} & b_{p52} & b_{p54} & 0 & b_{p56} \\ b_{p61} & b_{p62} & b_{p64} & b_{p65} & 0 \end{bmatrix}$$

The diagonal elements of $B_p$ are zero because of the zero angle of the sine term in the expression of $b_{pij}$. The subscripts used in the matrices are in coordination with bus numbers. Every element of Equation (4.4) is a matrix and there are sixteen terms in the equation and each term consists of three elements. It would be a huge equation obviously. Each term has been expanded separately to avoid unnecessary complexities and finally they are gathered in one single equation.

The terms of Equation (4.4) have been expanded into algebraic form as follows:

$$P_G^T A_p P_G = a_{p11} P_{G1}^2 + a_{p22} P_{G2}^2 + (a_{p12} + a_{p21}) P_{G1} P_{G2} \tag{4.5a}$$

$$P_D^T A_p P_G = a_{p51} P_{G1} P_{D5} + a_{p61} P_{G1} P_{D6} + a_{p52} P_{G2} P_{D5} + a_{p62} P_{G2} P_{D6} \tag{4.5b}$$

$$P_G^T A_p P_D = a_{p15} P_{G1} P_{D5} + a_{p16} P_{G1} P_{D6} + a_{p25} P_{G2} P_{D5} + a_{p26} P_{G2} P_{D6} \tag{4.5c}$$

$$P_D^T A_p P_D = a_{p55} P_{D5}^2 + a_{P66} P_{D6}^2 + (a_{p56} + a_{p65}) P_{D5} P_{D6} \tag{4.5d}$$

$$P_G^T B_p Q_G = b_{p12} P_{G1} Q_{G2} + b_{p21} P_{G2} Q_{G1} \tag{4.5e}$$

$$P_D{}^T B_p Q_G = b_{p51} Q_{G1} P_{D5} + b_{p61} Q_{G1} P_{D6} + b_{p52} Q_{G2} P_{D5} + b_{p62} Q_{G2} P_{D6} \tag{4.5f}$$

$$P_G{}^T B_p Q_D = b_{p15} P_{G1} Q_{D5} + b_{p16} P_{G1} Q_{D6} + b_{p25} P_{G2} Q_{D5} + b_{p26} P_{G2} Q_{D6} \tag{4.5g}$$

$$P_D{}^T B_p Q_D = b_{p56} P_{D5} Q_{D6} + b_{p65} P_{D6} Q_{D5} \tag{4.5h}$$

$$Q_G{}^T B_p P_G = b_{p21} P_{G1} Q_{G2} + b_{p12} P_{G2} Q_{G1} \tag{4.5i}$$

$$Q_D{}^T B_p P_G = b_{p51} P_{G1} Q_{D5} + b_{p61} P_{G1} Q_{D6} + b_{p52} P_{G2} Q_{D5} + b_{p62} P_{G2} Q_{D6} \tag{4.5j}$$

$$Q_G{}^T B_p P_D = b_{p15} Q_{G1} P_{D5} + b_{p16} Q_{G1} P_{D6} + b_{p25} Q_{G2} P_{D5} + b_{p26} Q_{G2} P_{D6} \tag{4.5k}$$

$$Q_D{}^T B_p P_D = b_{p65} P_{D5} Q_{D6} + b_{p56} P_{D6} Q_{D5} \tag{4.5l}$$

$$Q_G{}^T A_p Q_G = a_{p11} Q_{G1}{}^2 + a_{p22} Q_{G2}{}^2 + (a_{p12} + a_{p21}) Q_{G1} Q_{G2} \tag{4.5m}$$

$$Q_D{}^T A_p Q_G = a_{P51} Q_{G1} Q_{D5} + a_{p61} Q_{G1} Q_{D6} + a_{p52} Q_{G2} Q_{D5} + a_{p62} Q_{G2} Q_{D6} \tag{4.5n}$$

$$Q_G{}^T A_p Q_D = a_{p15} Q_{G1} Q_{D5} + a_{p16} Q_{G1} Q_{D6} + a_{p25} Q_{G2} Q_{D5} + a_{p26} Q_{G2} Q_{D6} \tag{4.5o}$$

$$Q_D{}^T A_p Q_D = a_{p55} Q_{D5}{}^2 + a_{p66} Q_{D6}{}^2 + (a_{p56} + a_{p65}) Q_{D5} Q_{D6} \tag{4.5p}$$

Using the properties of $A_p$ and $B_p$ and taking advantage of the system configuration, the above expressions can be reduced in size. The following relationships have been used to reduce the size of the expressions.

$$a_{pij} = a_{pji}$$

$$b_{pij} = -b_{pji}$$

$$a_{p5i} = 0, \quad i \neq 5$$

$$a_{pi5} = 0, \quad i \neq 5$$

$$b_{p5i} = 0$$

$$b_{pi5} = 0$$

From assumption number (6), the reactive load can be expressed in terms of real load which eventually reduces the number of variables.

$$Q_{Di} = \mu \times P_{Di}$$

where, $\mu$ = reactive ratio

The transmission loss equation has to be separated into respective parts for the purpose of allocating transmission loss to each generator in the system.

### 4.3.2 Solution Approach

In our example system, there are two generators that are in separate bilateral contracts with two customers. Hence it is required to break up the total transmission loss into two parts so that each generator would know its share of generation. The total transmission loss can be written as

$$P_l = L_a + L_b \tag{4.6}$$

$$Q_l = M_a + M_b \tag{4.7}$$

Where,

$L_a$ = share of real transmission loss of Generator A

$L_b$ = share of real transmission loss of Generator B

$M_a$ = share of reactive transmission loss of Generator A

$M_b$ = share of reactive transmission loss of Generator B

With respect to the network shown in Figure 3.1,

$P_{G1}$ = $G_a$ = active power generation of A,

$Q_{G1}$ = reactive power generation of A,

$P_{G2}$ = $G_b$ = active power generation of B,

$Q_{G2}$ = reactive power generation of B,

$P_{D5}$ = $Load_a$ = active power demand of customer A,

$Q_{D5}$ = reactive power demand of customer A,

$P_{D6}$ = $Load_b$ = active power demand of customer B,

$Q_{D6}$ = reactive power demand of customer B,

Generator A is supplying the load of Customer A, $L_a$, which is ($P_{D5}$ + $jQ_{D5}$) in the expression and Generator B is supplying the load of Customer B, $L_b$, which is ($P_{D6}$ + $jQ_{D6}$). Since the reactive load has been associated with the real load by the factor $\mu$, Equation (4.4) can be expressed in terms of real load only.

A load has been increased from zero to the respective customer's demand in a successive process and therefore, the change in the transmission loss for each increment can be expressed as:

$$\begin{aligned}\Delta P_l &= \Delta L_a + \Delta L_b \\ &= \frac{\partial P_l}{\partial P_{D5}}\Delta P_{D5} + \frac{\partial P_l}{\partial P_{D6}}\Delta P_{D6} \\ &= U_1 \Delta P_{D5} + U_2 \Delta P_{D6}\end{aligned} \tag{4.8}$$

$$\begin{aligned}\Delta Q_l &= \Delta M_a + \Delta M_b \\ &= \frac{\partial Q_l}{\partial P_{D5}}\Delta P_{D5} + \frac{\partial Q_l}{\partial P_{D6}}\Delta P_{D6} \\ &= W_1 \Delta P_{D5} + W_2 \Delta P_{D6}\end{aligned} \tag{4.9}$$

Where,

$$\Delta L_a = \frac{\partial P_l}{\partial P_{D5}}\Delta P_{D5}$$

$$\text{or, } \frac{\partial P_l}{\partial P_{D5}} = \frac{\Delta L_a}{\Delta P_{D5}}$$

If $\Delta L_a$ and $\Delta P_{D5}$ are small then

$$\frac{\partial P_l}{\partial P_{D5}} \approx \frac{\partial L_a}{\partial P_{D5}} = U_1$$

Similar arguments could be applied to the followings

$$\frac{\partial P_l}{\partial P_{D6}} \approx \frac{\partial L_b}{\partial P_{D6}} = U_2$$

$$\frac{\partial Q_l}{\partial P_{D5}} \approx \frac{\partial M_a}{\partial P_{D5}} = W_1$$

$$\frac{\partial Q_l}{\partial P_{D6}} \approx \frac{\partial M_b}{\partial P_{D6}} = W_2$$

Generation is dependent on load and loss, and can be expressed as:

$$P_{G1} = P_{D5} + L_a \tag{4.10}$$

$$P_{G2} = P_{D6} + L_b \tag{4.11}$$

$$Q_{G1} = \mu_a P_{D5} + M_a \tag{4.12}$$

$$Q_{G2} = \mu_b P_{D6} + M_b \tag{4.13}$$

Using Equations (4.10) - (4.13) and replacing reactive loads by real loads, the expressions (4.5a) – (4.5p) can be rewritten as:

$$P_G{}^T A_p P_G = a_{p11}(P_{D5}{}^2 + 2P_{D5}L_a + L_a{}^2) + a_{p22}(P_{D6}{}^2 + 2P_{D6}L_b + L_b{}^2) + 2a_{p12}(P_{D5}P_{D6} + P_{D5}L_b + P_{D6}L_b + L_aL_b) \tag{4.14a}$$

$$P_D{}^T A_p P_G = a_{p61}P_{D6}(P_{D5} + L_a) + a_{p62}P_{D6}(P_{D6} + L_b) \tag{4.14b}$$

$$P_G{}^T A_p P_D = a_{p61}P_{D6}(P_{D5} + L_a) + a_{p62}P_{D6}(P_{D6} + L_b) \tag{4.14c}$$

$$P_D{}^T A_p P_D = a_{p55}P_{D5}{}^2 + a_{p66}P_{D6}{}^2 \tag{4.14d}$$

$$P_G{}^T B_p Q_G = b_{p12}P_{D5}M_b + b_{p12}L_a(\mu_b P_{D6} + M_b) - b_{p12}P_{D6}M_a - b_{p12}L_b(\mu_a P_{D5} + M_a) + (\mu_a - \mu_b)b_{p12}P_{D5}P_{D6} \tag{4.14e}$$

$$P_D{}^T B_p Q_G = b_{p61}P_{D6}(\mu_a P_{D5} + M_a) + b_{p62}P_{D6}(\mu_b P_{D6} + M_b) \tag{4.14f}$$

$$P_G{}^T B_p Q_D = -\mu_b b_{p61}P_{D6}(P_{D5} + M_a) - \mu_b b_{p62}P_{D6}(P_{D6} + M_b) \tag{4.14g}$$

$$P_D{}^T B_p Q_D = 0 \tag{4.14h}$$

$$Q_G{}^T B_p P_G = -b_{p12} P_{D5} M_b - b_{p12} L_s (\mu_b P_{D6} + M_b) + b_{p12} P_{D6} M_s + b_{p12} L_b (\mu_s P_{D5} + M_s) + (\mu_a - \mu_b) b_{p12} P_{D5} P_{D6} \tag{4.14i}$$

$$Q_D{}^T B_p P_G = \mu_b b_{p61} P_{D6} (P_{D5} + L_a) + \mu_b P_{D6} (P_{D6} + L_b) \tag{4.14j}$$

$$Q_G{}^T B_p P_D = -b_{p61} P_{D6} (\mu_a P_{D5} + M_a) - b_{p62} P_{D6} (\mu_b P_{D6} + M_b) \tag{4.14k}$$

$$Q_D{}^T B_p P_D = 0 \tag{4.14l}$$

$$Q_G{}^T A_p Q_G = a_{p11} (\mu_a{}^2 P_{D5}{}^2 + 2\mu_a P_{D5} M_a + M_a{}^2) + a_{p22} (\mu_b{}^2 P_{D6}{}^2 + 2\mu_b P_{D6} L_b + L_b{}^2) + 2a_{p12} (\mu_a \mu_b P_{D5} P_{D6} + \mu_a P_{D5} M_b + \mu_b P_{D6} M_a + M_a M_b) \tag{4.14m}$$

$$Q_D{}^T A_p Q_G = \mu_b a_{p61} P_{D6} (\mu_a P_{D5} + M_a) + \mu_b a_{p62} P_{D6} (\mu_b P_{D6} + M_b) \tag{4.14n}$$

$$Q_G{}^T A_p Q_D = \mu_b a_{p61} P_{D6} (\mu_a P_{D5} + M_a) + \mu_b a_{p62} P_{D6} (\mu_b P_{D6} + M_b) \tag{4.14o}$$

$$Q_D{}^T A_p Q_D = \mu_a{}^2 a_{p55} P_{D5}{}^2 + \mu_b{}^2 a_{p66} P_{D6}{}^2 \tag{4.14p}$$

Combining the simplified expressions (4.14a) – (4.14p) and putting them in Equation (4.2) the final form of the transmission loss equation stands as:

$$\begin{aligned} P_l = {} & P_{D5}{}^2 (1+\mu_a{}^2)(a_{p11} + a_{p55}) + P_{D6}{}^2 (1+\mu_b{}^2)(a_{p22} + a_{p66} - 2a_{p62}) + 2a_{p11} P_{D5} L_a + a_{p11} L_a{}^2 \\ & + 2P_{D6} L_b (a_{p22} - a_{p62} - \mu_b b_{p62}) + a_{p22} L_b{}^2 + 2P_{D5} P_{D6} (1+\mu_a \mu_b)(a_{p12} - a_{p61}) + \\ & 2P_{D5} L_b (a_{p12} + \mu_a b_{p12}) + 2P_{D6} L_a (a_{p12} - a_{p61} - \mu_b b_{p12} - \mu_b b_{p61}) + 2a_{p12} L_a L_b + \\ & 2P_{D5} M_b (\mu_a a_{p12} - b_{p12}) + 2P_{D6} M_a (b_{p12} + b_{p61} + \mu_b a_{p12} - \mu_b a_{p61}) + 2P_{D6} M_b (b_{p62} + \\ & \mu_b a_{p22} - \mu_b a_{p62}) + 2\mu_a a_{p11} P_{D5} M_a - 2b_{p12} L_a M_b + 2b_{p12} L_b M_a + a_{p11} M_a{}^2 + a_{p22} M_b{}^2 \\ & + 2a_{p12} M_a M_b + 2(\mu_a - \mu_b)(b_{p12} + b_{p61}) P_{D5} P_{D6} \end{aligned} \tag{4.15}$$

Differentiating Equation (4.15) with respect to $P_{D5}$ we get,

$$\frac{\partial P_l}{\partial P_{D5}} = 2(1+\mu_a{}^2)P_{D5}(a_{p11}+a_{p55}) + 2a_{p11}L_a + 2a_{p11}P_{D5}\frac{\partial L_a}{\partial P_{D5}} + 2a_{p11}L_a\frac{\partial L_a}{\partial P_{D5}} +$$
$$2P_{D6}(1+\mu_a\mu_b)(a_{p12}-a_{p61}) + 2L_b(a_{p12}+\mu_a b_{p12}) + 2P_{D6}(a_{p12}-a_{p61}-\mu_b b_{p12} -$$
$$\mu_b b_{p61})\frac{\partial L_a}{\partial P_{D5}} + 2a_{p12}L_b\frac{\partial L_a}{\partial P_{D5}} + 2M_b(\mu_a a_{p12}-b_{p12}) + 2P_{D6}(b_{p12}+b_{p61}-\mu_b a_{p12} -$$
$$\mu_b a_{p61})\frac{\partial M_a}{\partial X_5} + 2\mu_a a_{p11}M_a + 2\mu_a a_{p11}P_{D5}\frac{\partial M_a}{\partial P_{D5}} - 2b_{p12}M_b\frac{\partial L_a}{\partial P_{D5}} + 2b_{p12}L_b\frac{\partial M_a}{\partial P_{D5}} +$$
$$2a_{p11}M_a\frac{\partial M_a}{\partial P_{D5}} + 2a_{p12}M_b\frac{\partial M_a}{\partial P_{D5}} + 2(\mu_a-\mu_b)(b_{p12}+b_{p61})P_{D6}$$

or,

$$U_1 - 2a_{p11}P_{D5}U_1 - 2a_{p11}L_aU_1 - 2P_{D6}(a_{p12}-a_{p61}-\mu_b b_{p12}-\mu_b b_{p61})U_1 - 2a_{p12}L_bU_1 +$$
$$2b_{p12}M_bU_1 - 2P_{D6}(b_{p12}+b_{p61}+\mu_b a_{p12}-\mu_b a_{p61})W_1 - 2\mu_a a_{p11}P_{D5}W_1 - 2b_{p12}L_bW_1 -$$
$$2a_{p11}M_aW_1 - 2a_{p12}M_bW_1 = 2(1+\mu_a{}^2)P_{D5}(a_{p11}+a_{p55}) + 2a_{p11}L_a + 2(1+\mu_a\mu_b)P_{D6}(a_{p12}$$
$$-a_{p61}) + 2L_b(a_{p12}+\mu_a b_{p12}) + 2M_b(\mu_a a_{p12}-b_{p12}) + 2\mu_a a_{p11}M_a +$$
$$2(\mu_a-\mu_b)(b_{p12}+b_{p61})P_{D6} \tag{4.16}$$

After reorganizing, Equation (4.16) can be written as:

$$\alpha_1 U_1 + \beta_1 W_1 = \gamma_1 \tag{4.17}$$

where,

$$\alpha_1 = 1 - 2a_{p11}P_{D5} - 2a_{p11}L_a - 2P_{D6}(a_{p12}-a_{p61}-\mu_b b_{p12}-\mu_b b_{p61}) - 2a_{p12}L_b + 2b_{p12}M_b$$

$$\beta_1 = -2P_{D6}(b_{p12}+b_{p61}+\mu_b a_{p12}-\mu_b a_{p61}) - 2\mu_a a_{p11}P_{D5} - 2b_{p12}L_b - 2a_{p11}M_a - 2a_{p12}M_b$$

$$\gamma_1 = 2P_{D5}(1+\mu_a{}^2)(a_{p11}+a_{p55}) + 2a_{p11}L_a + 2P_{D6}(1+\mu_a\mu_b)(a_{p12}-a_{p61}) + 2L_b(a_{p12}$$
$$+\mu_1 b_{p12}) + 2M_b(\mu_a a_{p12}-b_{p12}) + 2\mu_a a_{p11}M_a + 2(\mu_a-\mu_b)(b_{p12}+b_{p61})P_{D6}$$

Now a second equation is required for the solution of $U_1$ and $W_1$ which would be obtained from Equation (2.3). By examining Equations (2.2) and (2.3), it becomes clear that expression for $Q_l$ will be similar to that of $P_l$. The only difference will be there in $a_{pij}$ and $b_{pij}$ elements where $r$'s will be replaced by $x$'s. The expression for $Q_l$ will be

$$Q_l = P_{D5}^{\ 2}(1+\mu_a^{\ 2})(a_{p11}+a_{p55})+P_{D6}^{\ 2}(1+\mu_b^{\ 2})(a_{p22}+a_{p66}-2a_{p62})+2a_{p11}P_{D5}L_a + a_{p11}L_a^{\ 2}+2P_{D6}L_b(a_{p22}-a_{p62}-\mu_b b_{p62})+a_{p22}L_b^{\ 2}+2P_{D5}P_{D6}(1+\mu_a\mu_b)(a_{p12}-a_{p61})+2P_{D5}L_b(a_{p12}+\mu_a b_{p12})+2P_{D6}L_a(a_{p12}-a_{p61}-\mu_b b_{p12}-\mu_b b_{p61})+2a_{p12}L_aL_b+2P_{D5}M_b(\mu_a a_{p12}-b_{p12})+2P_{D6}M_a(b_{p12}+b_{p61}+\mu_b a_{p12}-\mu_b a_{p61})+2P_{D6}M_b(b_{p62}+\mu_b a_{p22}-\mu_b a_{p62})+2\mu_a a_{p11}P_{D5}M_a-2b_{p12}L_aM_b+2b_{p12}L_bM_a+a_{p11}M_a^{\ 2}+a_{p22}M_b^{\ 2}+2a_{p12}M_aM_b+2(\mu_a-\mu_b)(b_{p12}+b_{p61})P_{D5}P_{D6} \tag{4.18}$$

where,

$$a_{pij} = \frac{x_{ij}}{|V_i||V_j|}\cos(\delta_i-\delta_j)$$
$$b_{pij} = \frac{x_{ij}}{|V_i||V_j|}\sin(\delta_i-\delta_j) \tag{4.19}$$

After differentiating Equation (4.18) with respect $P_{D5}$ and performing the mathematical operations similar to that done for Equation (4.15), the following can be written.

$$\alpha_2 U_1 + \beta_2 W_1 = \gamma_2 \tag{4.20}$$

where,

$$\alpha_2 = -2a_{p11}P_{D5}-2a_{p11}L_a-2P_{D6}(a_{p12}-a_{p61}-\mu_b b_{p12}-\mu_b b_{p61})-2a_{p12}L_b+2b_{12}M_b$$

$$\beta_2 = 1-2P_{D6}(b_{p12}+b_{p61}+\mu_b a_{p12}-\mu_b a_{p61})-2\mu_a a_{p11}P_{D5}-2b_{p12}L_b-2a_{p11}M_a-2a_{p12}M_b$$

$$\gamma_2 = 2P_{D5}(1+\mu_a^{\ 2})(a_{p11}+a_{p55})+2a_{p11}L_a+2P_{D6}(1+\mu_a\mu_b)(a_{p12}-a_{p61})+2L_b(a_{p12}+\mu_a b_{p12})+2M_b(\mu_a a_{p12}-b_{p12})+2\mu_a a_{p11}M_a+2(\mu_a-\mu_b)(b_{p12}+b_{p61})P_{D6}$$

Solving Equations (4.17) and (4.20) simultaneously for $U_1$ and $W_1$,

$$U_1 = \frac{\gamma_1\beta_2-\gamma_2\beta_1}{\alpha_1\beta_2-\alpha_2\beta_1}$$
$$W_1 = \frac{\gamma_2\alpha_1-\gamma_1\alpha_2}{\alpha_1\beta_2-\alpha_2\beta_1}$$

Differentiating Equation (4.15) with respect to $P_{D6}$,

$$\frac{\partial P_l}{\partial P_{D6}} = 2(1+\mu_b^{\ 2})P_{D6}(a_{p22}+a_{p66}-2a_{p62})+2L_b(a_{p22}-a_{p62}-\mu_b b_{p62})+2P_{D6}(a_{p22}-a_{p62}$$

$$-\mu_b b_{p62})\frac{\partial L_2}{\partial P_{D6}}+2a_{p22}L_b\frac{\partial L_b}{\partial P_{D6}}+2(1+\mu_a\mu_b)P_{D5}(a_{p12}-a_{p61})+2P_{D5}(a_{p12}+$$

$$\mu_a b_{p12})\frac{\partial L_2}{\partial P_{D6}}+2L_a(a_{p12}-a_{p61}-\mu_b b_{p12}-\mu_b b_{p61})+2a_{12}L_a\frac{\partial L_b}{\partial P_{D6}}+2P_{D5}(\mu_a a_{p12}$$

$$-b_{p12})\frac{\partial M_b}{\partial P_{D6}}+2M_a(b_{p12}+b_{p61}+\mu_b a_{p12}-\mu_b a_{p61})+2M_b(b_{p62}+\mu_b a_{p22}-\mu_b a_{p62})$$

$$+2P_{D6}(b_{p62}+\mu_b a_{p22}-\mu_b a_{p62})\frac{\partial M_b}{\partial P_{D6}}-2b_{p12}L_a\frac{\partial M_b}{\partial P_{D6}}+2b_{p12}M_a\frac{\partial L_b}{\partial P_{D6}}+$$

$$2a_{p22}M_b\frac{\partial M_b}{\partial P_{D6}}+2a_{p12}M_a\frac{\partial M_b}{\partial P_{D6}}+2(\mu_a-\mu_b)(b_{p12}+b_{p61})P_{D5}$$

or,

$$U_2-2P_{D6}(a_{p22}-a_{p62}-\mu_b b_{p62})U_2-2a_{p22}L_bU_2-2P_{D5}(a_{p12}+\mu_a b_{p12})U_2-2a_{p12}L_aU_2-$$
$$2b_{p12}M_aU_2-2P_{D5}(\mu_a a_{p12}-b_{p12})W_2-2P_{D6}(b_{p62}+\mu_b a_{p22}-\mu_b a_{p62})W_2-2b_{p12}L_aW_2-$$
$$2a_{p22}M_bW_2-2a_{p12}M_aW_2=2(1+\mu_b^{\ 2})P_{D6}(a_{p22}+a_{p66}-2a_{p62})+2L_b(a_{p22}-a_{p62}-\mu_b b_{p62})$$
$$+2(1+\mu_a\mu_b)P_{D5}(a_{p12}-a_{p61})+2L_a(a_{p12}-a_{p61}-\mu_b b_{p12}-\mu_b b_{p61})+2M_a(b_{p12}+b_{p61}+$$
$$\mu_b a_{p12}-\mu_b a_{p61})+2M_b(b_{p62}+\mu_b a_{p22}-\mu_b a_{p62})+2(\mu_a-\mu_b)(b_{p12}+b_{p61})P_{D5} \tag{4.21}$$

Equation (4.21) can be rewritten as:

$$\alpha_1' U_2+\beta_1' W_2=\gamma_1' \tag{4.22}$$

where,

$$\alpha_1'=1-2P_{D6}(a_{p22}-a_{p62}-\mu_b b_{p62})-2a_{p22}L_b-2P_{D5}(a_{p12}+\mu_a b_{p12})-2a_{p12}L_a-2b_{p12}M_a$$

$$\beta_1'=2P_{D5}(\mu_a a_{p12}-b_{p12})-2P_{D6}(b_{p62}+\mu_b a_{p22}-\mu_b a_{p62})-2b_{p12}L_a-2a_{p22}M_b-2a_{p12}M_a$$

$$\gamma_1'=2P_{D6}(1+\mu_b^{\ 2})(a_{p22}+a_{p66}-2a_{p62})+2L_b(a_{p22}-a_{p62}-\mu_b b_{p62})+2P_{D5}(1+\mu_a\mu_b)$$
$$(a_{p12}-a_{p61})+2L_a(a_{p12}-a_{p61}-\mu_b b_{p12}-\mu_b b_{p61})+2M_a(b_{p12}+b_{p61}+\mu_b a_{p12}-\mu_b a_{p61})$$
$$+2M_b(b_{p62}+\mu_b a_{p22}-\mu_b a_{p62})+2(\mu_a-\mu_b)(b_{p12}+b_{p61})P_{D5}$$

Similarly, taking derivative of Equation (4.18) with respect to $P_{D6}$ the second equation can be written as:

$$\alpha_2' U_2+\beta_2' W_2=\gamma_2' \tag{4.23}$$

Where,

$$\alpha_2' = -2P_{D6}(a_{p22} - a_{p62} - \mu_b b_{p62}) - 2a_{p22}L_b - 2P_{D5}(a_{p12} + \mu_a b_{p12}) - 2a_{p12}L_a - 2b_{p12}M_a$$

$$\beta_2' = 1 - 2P_{D5}(\mu_a a_{p12} - b_{p12}) - 2P_{D6}(b_{p62} + \mu_b a_{p22} - \mu_b a_{p62}) - 2b_{p12}L_a - 2a_{p22}M_b - 2a_{p12}M_a$$

$$\gamma_2' = 2P_{D6}(1+\mu_b^2)(a_{p22} + a_{p66} - 2a_{p62}) + 2L_b(a_{p22} - a_{p62} - \mu_b b_{p62}) + 2P_{D5}(1+\mu_a\mu_b)$$
$$(a_{p12} - a_{p61}) + 2L_a(a_{p12} - a_{p61} - \mu_b b_{p12} - \mu_b b_{p61}) + 2M_a(b_{p12} + b_{p61} + \mu_b a_{p12} -$$
$$\mu_b a_{p61}) + 2M_b(b_{p62} + \mu_b a_{p22} - \mu_b a_{p62}) + 2(\mu_a - \mu_b)(b_{p12} + b_{p61})P_{D5}$$

It is important to note that $a_{pij}$ and $b_{pij}$ parameters for the reactive part have to be calculated by the relationship given in Equation (4.19).

Solving Equations (4.22) and (4.23) simultaneously for $U_2$ and $W_2$,

$$U_2 = \frac{\gamma_1'\beta_2' - \gamma_2'\beta_1'}{\alpha_1'\beta_2' - \alpha_2'\beta_1'}$$
$$W_2 = \frac{\gamma_2'\alpha_1' - \gamma_1'\alpha_2'}{\alpha_1'\beta_2' - \alpha_2'\beta_1'}$$

After evaluating $U_1$, $U_2$, $W_1$ and $W_2$, and the share of transmission loss of each generator can be evaluated using Equations (4.8) and (4.9).

### 4.4 Allocation of Transmission Loss by Using the MTLA

A computer program has been developed to solve Equations (4.8) and (4.9) for the determination of the share of transmission losses. Different load conditions have been taken into consideration to find the transmission loss shares. The results have been compared with the shares obtained in the incremental load flow technique.

To start with the allocation process, the loads are varied from zero to their respective load demands. In the MTLA, the loads are increased step by step in a sequential manner. In any iteration, only one customer load is increased while the other loads are held fixed. Let us assume that load $L_a$ is increased by a step of $\Delta L_a$ while $L_b$ stays at its previous level. Since the load demand of Customer B ($L_b$) is unchanged, the resulting incremental transmission loss becomes the liability of Generator A to produce it in order to support the incremental load demand of Customer A. In the next iteration, $L_b$ is

increased by a step size of $\Delta L_b$ while $L_a$ remains fixed. Again, generations and transmission losses are calculated and the incremental transmission loss is assigned to Generator B. This has been done as only $L_b$ has been incremented in this iteration and Generator B is providing power to Customer B.

A variety of load combinations have been utilized to find the transmission loss allocation by the MTLA. Both real load demand and reactive ratio of both customers have been varied and the transmission loss shares have been determined. The results obtained from the MTLA have been compared with those obtained from the ILFA.

### 4.4.1 Equal Load and Equal Reactive Ratio

The demand of Customers A and B have been kept fixed and their reactive power ratio has been varied from 0.6 to 0.3. Allocations of transmission loss have been calculated for the corresponding variations in reactive power. Tables 4.1 and 4.2 show the calculated allocation of transmission loss for Generators A and B. The real load demand of Customer A and B is 1.5 p.u.

It can be noticed from Tables 4.1 and 4.2 that for both real and reactive powers, difference in loss allocation for Generator A, obtained from two different methods, increases with a decrease of reactive ratio of Customer A while the opposite happens to Generator B.

Table 4.1: Real Loss Allocation for equal load and equal reactive ratio.

| Reactive Ratio | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|
| $\mu$ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.0405 | 0.0412 | 0.0579 | 0.0609 |
| 0.5 | 0.0360 | 0.0370 | 0.0518 | 0.0544 |
| 0.4 | 0.0322 | 0.0337 | 0.0469 | 0.0484 |
| 0.3 | 0.0293 | 0.0310 | 0.0432 | 0.0443 |

Table 4.2: Reactive Loss Allocation for equal load and equal reactive ratio.

| Reactive Ratio | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|
| μ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.2430 | 0.2441 | 0.3277 | 0.3398 |
| 0.5 | 0.2158 | 0.2191 | 0.2934 | 0.3038 |
| 0.4 | 0.1940 | 0.2000 | 0.2653 | 0.2705 |
| 0.3 | 0.1769 | 0.1838 | 0.2432 | 0.2475 |

Figures 4.1 to 4.4 show the real and reactive loss allocation for both generators for a reactive ratio 0.6. Each figure shows two curves, one from the MTLA and the other from the ILFA.

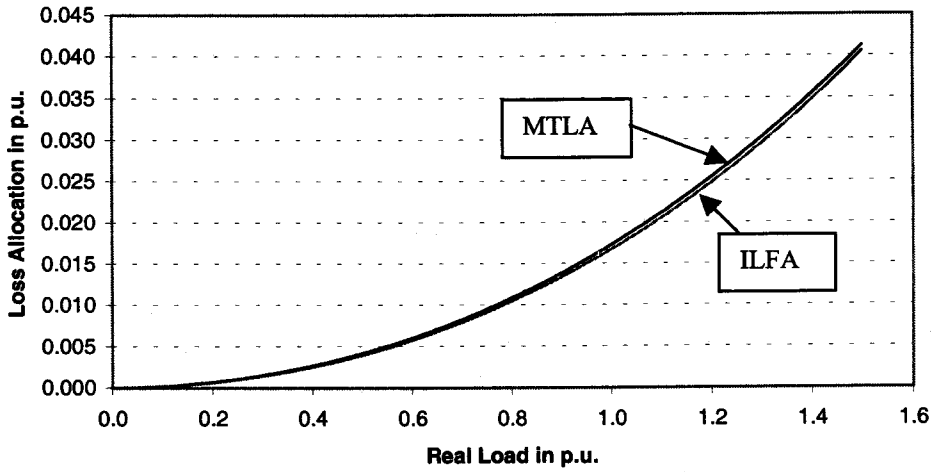


Fig. 4.1: Allocation of real transmission loss for Generator A (μ=0.6).

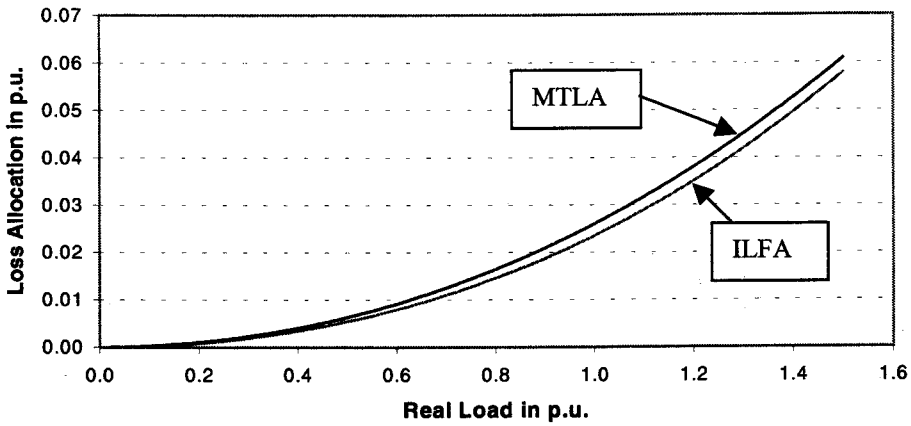


Fig. 4.2: Allocation of real transmission loss for Generator B (μ=0.6).

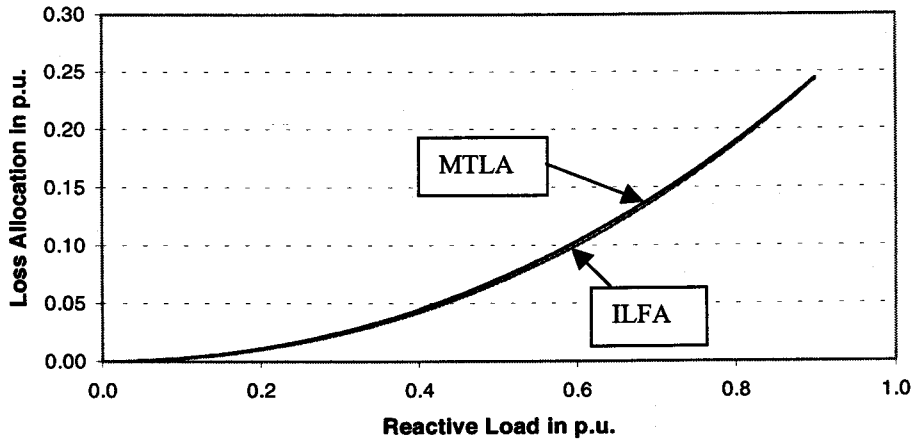


Fig. 4.3: Allocation of reactive transmission loss for Generator A (μ=0.6).

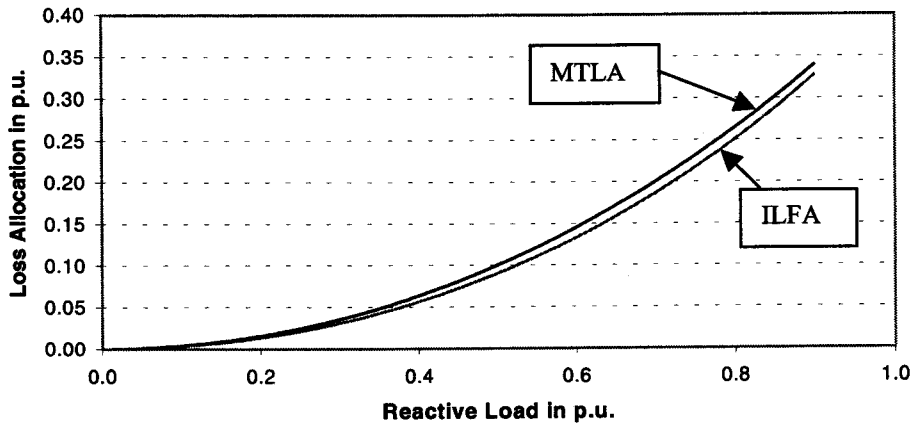


Fig. 4.4: Allocation of reactive transmission loss for Generator B (μ=0.6).

These figures represent a comparative view of transmission loss allocations obtained from the two different methods. Loss allocations for Generator A by the ILFA and the MTLA are very close for both real and reactive powers whereas those for Generator B are little different. For Generator B, losses allocated by the MTLA are slightly higher than losses allocated by the ILFA for both real and reactive powers. The trend of the curves in each case is similar and differences exist because of the assumptions made for model simplification [Section 4.3].

### 4.4.2 Equal Load and Different Reactive Ratio

The real loads of Customers A and B are considered equal with unequal reactive ratio. The reactive ratio is varied for Customer B while a constant reactive ratio is maintained for Customer A. Transmission loss allocations are shown in Tables 4.3 and 4.4. The real load demand of Customer A and B is 1.5 p.u.

Loss allocations for Generator A by the ILFA does not change appreciably for a change in the reactive ratio ($\mu_b$) of Customer B. But $\mu_b$ seems to have a significant effect on loss allocations for Generator A as obtained by the MTLA. The loss allocations for Generator B, both real and reactive, increase with an increase in reactive ratio. This rate of change in loss allocations obtained by the MTLA is higher than that obtained by the ILFA.

Table 4.3: Real Loss Allocation for equal load and unequal reactive ratio.

| Reactive Ratio | | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|---|
| $\mu_a$ | $\mu_b$ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.3 | 0.0390 | 0.0436 | 0.0441 | 0.0437 |
| 0.6 | 0.4 | 0.0395 | 0.0429 | 0.0478 | 0.0479 |
| 0.6 | 0.5 | 0.0400 | 0.0419 | 0.0523 | 0.054 |
| 0.5 | 0.3 | 0.0351 | 0.0384 | 0.0438 | 0.0439 |
| 0.5 | 0.4 | 0.0356 | 0.0378 | 0.0473 | 0.0482 |
| 0.4 | 0.3 | 0.0319 | 0.0340 | 0.0435 | 0.0441 |

Table 4.4: Reactive Loss Allocation for equal load and unequal reactive ratio.

| Reactive Ratio | | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|---|
| $\mu_a$ | $\mu_b$ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.3 | 0.2353 | 0.2561 | 0.2488 | 0.2446 |
| 0.6 | 0.4 | 0.2376 | 0.2525 | 0.2698 | 0.2678 |
| 0.6 | 0.5 | 0.2402 | 0.2476 | 0.2960 | 0.3020 |
| 0.5 | 0.3 | 0.2116 | 0.2263 | 0.2469 | 0.2456 |
| 0.5 | 0.4 | 0.2136 | 0.2234 | 0.2676 | 0.2692 |
| 0.4 | 0.3 | 0.1923 | 0.2006 | 0.2450 | 0.2466 |

Figures 4.5 - 4.12 show the real and reactive loss allocations for both generators. In each case, the two loads are considered equal with different reactive ratios.

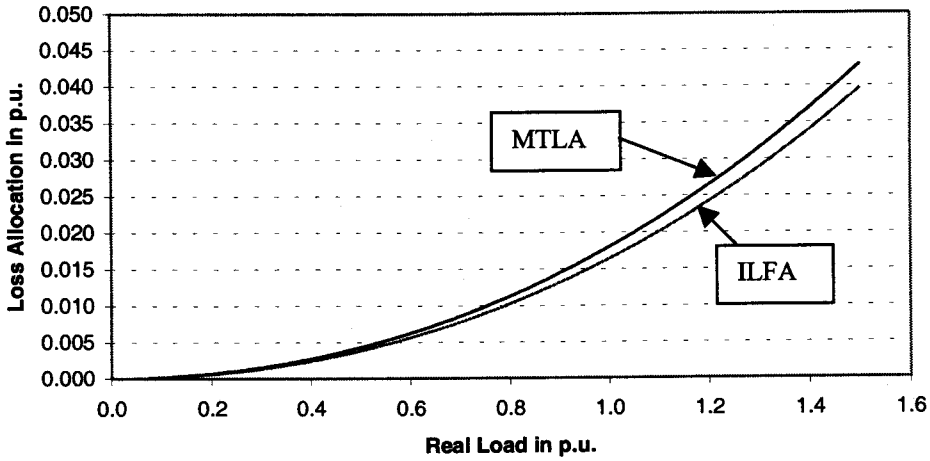


Fig. 4.5: Allocation of real transmission loss for Generator A ($\mu_a$=0.6, $\mu_b$=0.4).

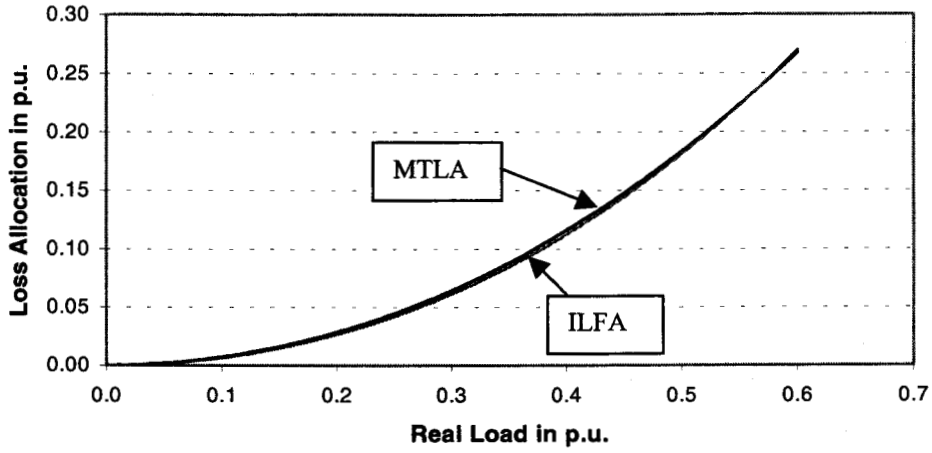


Fig. 4.6: Allocation of real transmission loss for Generator B ($\mu_a$=0.6, $\mu_b$=0.4).

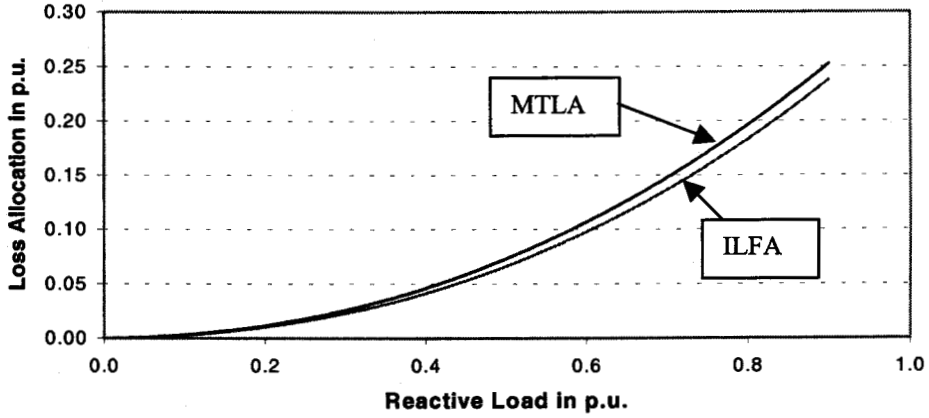


Fig. 4.7: Allocation of reactive transmission loss for Generator A ($\mu_a$=0.6, $\mu_b$=0.4).

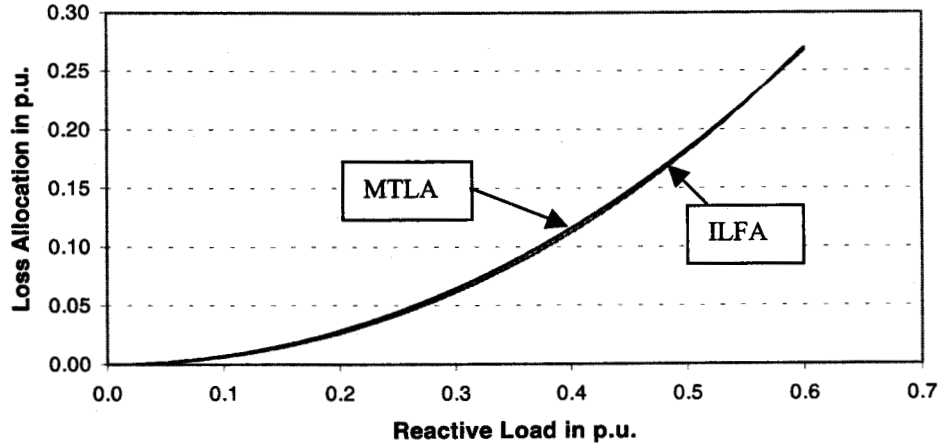


Fig. 4.8: Allocation of reactive transmission loss for Generator B ($\mu_a$=0.6, $\mu_b$=0.4).

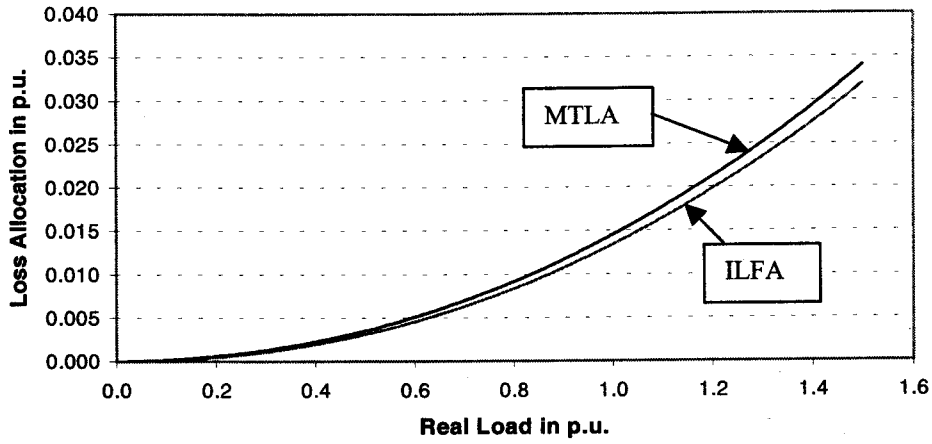


Fig. 4.9: Allocation of real transmission loss for Generator A ($\mu_a$=0.4, $\mu_b$=0.3).

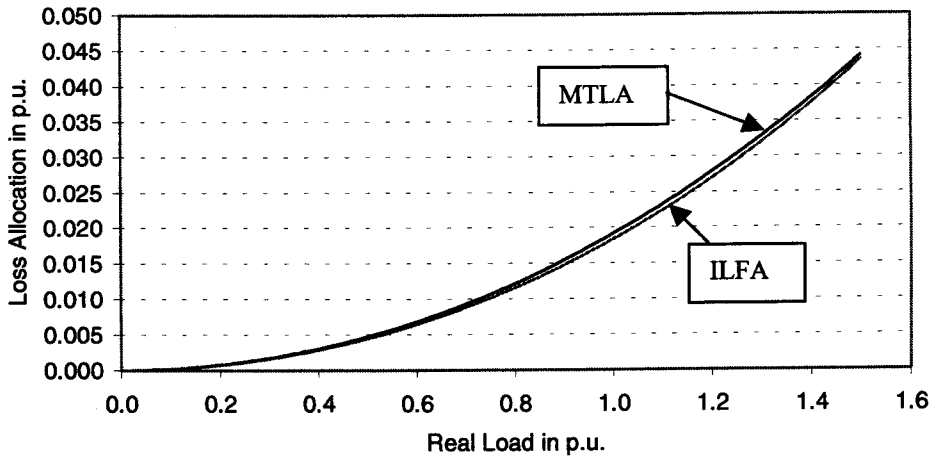


Fig. 4.10: Allocation of real transmission loss for Generator B ($\mu_a$=0.4, $\mu_b$=0.3).

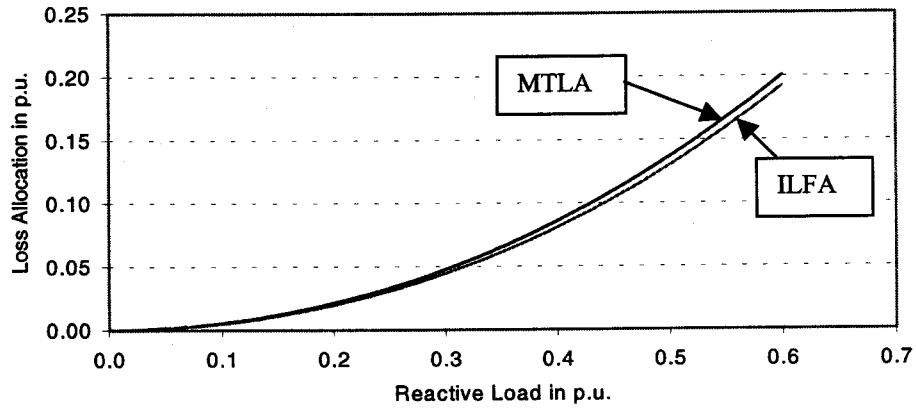


Fig. 4.11: Allocation of reactive transmission loss for Generator A ($\mu_a$=0.4, $\mu_b$=0.3).

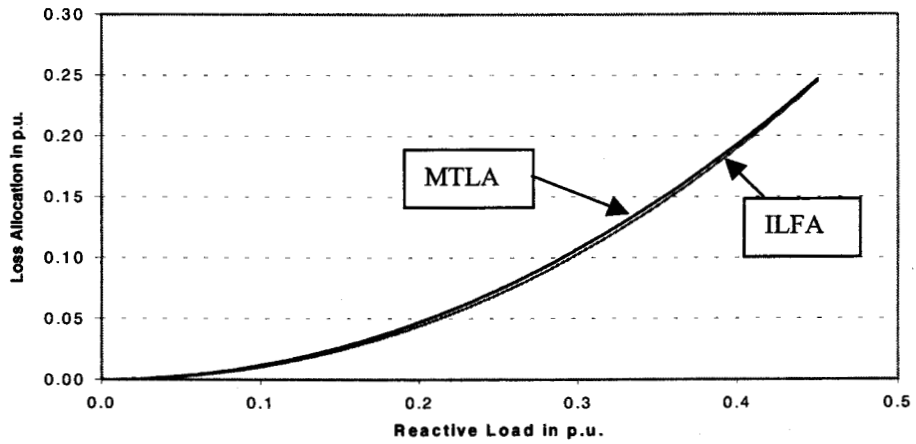


Fig. 4.12: Allocation of reactive transmission loss for Generator B ($\mu_a$=0.4, $\mu_b$=0.3).

Two sets of loss allocations have been shown in the Figures 4.5 - 4.12. It can be noticed from the graphs that the loss allocations for Generator A, obtained from two methods, differ by a small margin while those for Generator B follow each other closely.

### 4.4.3 Unequal Load and Equal Reactive Ratio

The real load demands of Customers A and B have been kept different while their reactive ratios have been maintained same. The reactive ratio has been changed from 0.6 to 0.3. The calculated loss allocations are shown in Tables 4.5 and 4.6. The real load demands of Customer A and B are 1.5 and 0.9 p.u. respectively.

Table 4.5: Real Loss Allocation for unequal load and equal reactive ratio.

| Reactive Ratio | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|
| $\mu$ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.0409 | 0.0440 | 0.0187 | 0.0198 |
| 0.5 | 0.0367 | 0.0392 | 0.0169 | 0.0178 |
| 0.4 | 0.0331 | 0.0354 | 0.0156 | 0.0164 |
| 0.3 | 0.0302 | 0.0325 | 0.0147 | 0.0153 |

Table 4.6: Reactive Loss Allocation for unequal load and equal reactive ratio.

| Reactive Ratio | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|
| μ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.2449 | 0.2582 | 0.1064 | 0.1108 |
| 0.5 | 0.2187 | 0.2298 | 0.0968 | 0.0997 |
| 0.4 | 0.1977 | 0.2075 | 0.0886 | 0.0919 |
| 0.3 | 0.1814 | 0.1915 | 0.0819 | 0.0857 |

Table 4.5 shows that Generator A's shares of real transmission losses as obtained by the ILFA and the MTLA differ significantly when the reactive ratio increases. Reactive loss allocations for Generator A and both real and reactive loss allocations for Generator B as obtained by the ILFA and the MTLA are very close.

Figures 4.13 - 4.16 show the loss allocation for equal loads with a reactive ratio 0.5.

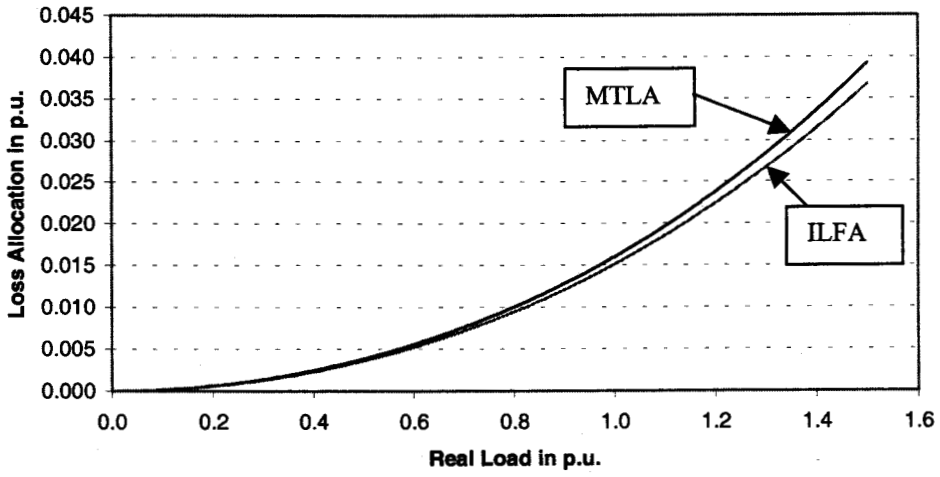


Fig. 4.13: Allocation of real transmission loss for Generator A (μ =0.5).

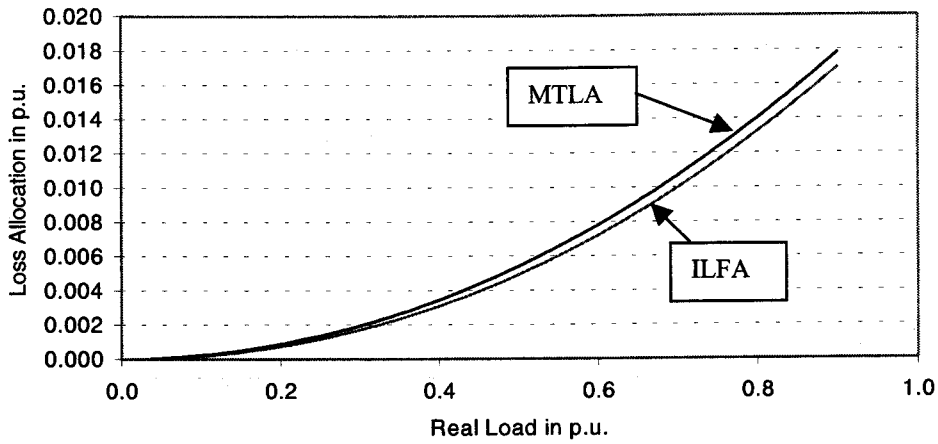


Fig. 4.14: Allocation of real transmission loss for Generator B ($\mu$ =0.5).

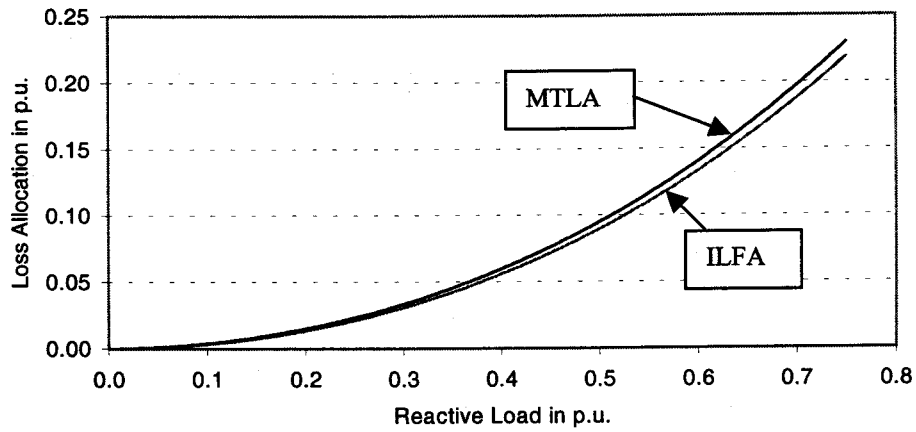


Fig. 4.15: Allocation of reactive transmission loss for Generator A ($\mu$ =0.5).

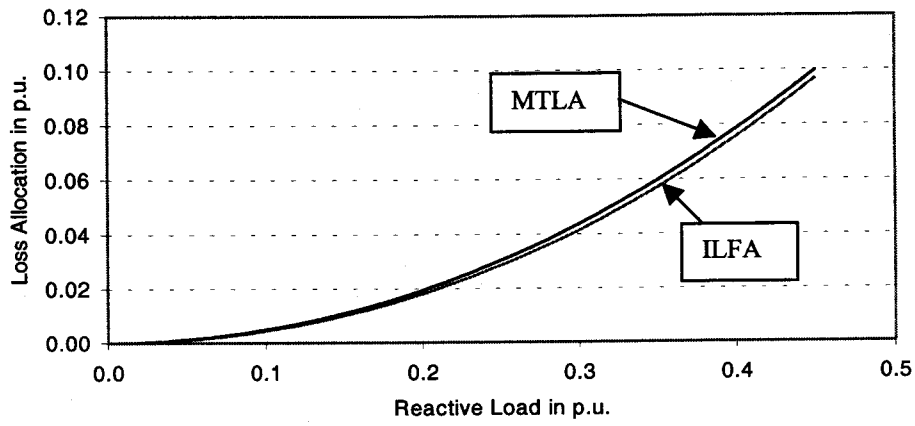


Fig. 4.16: Allocation of reactive transmission loss for Generator B ($\mu$ =0.5).

It can be noticed from Figure 4.13 that the real transmission loss allocations of Generator A as obtained from the two methods have noticeable differences whereas Figures 4.14 - 4.16 show a closer match between the loss allocations derived from the two methods.

### 4.4.4 Unequal Load and Unequal Reactive Ratio

In this section, the reactive ratios as well as the real loads of Customers A and B are considered different. The reactive ratio for Customer B has been varied while maintaining a constant reactive ratio for Customer A. Three different constant reactive ratios for Customer A have been considered in this evaluation. Transmission loss allocations are shown in Tables 4.7 and 4.8. The real load demands of Customer A and B are 1.5 and 0.9 p.u. respectively.

The real loss allocations for Generator A as obtained from the two methods have the maximum difference when two loads have the maximum difference in their reactive ratios. But the loss allocations for Generator B as obtained by the two methods are very close. The reactive loss allocations as obtained by ILFA and MTLA are very close too.

Table 4.7: Real Loss Allocation for unequal load and unequal reactive ratio.

| Reactive Ratio | | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|---|
| $\mu_a$ | $\mu_b$ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.3 | 0.0397 | 0.0452 | 0.0148 | 0.0150 |
| 0.6 | 0.4 | 0.0401 | 0.0448 | 0.0157 | 0.0162 |
| 0.6 | 0.5 | 0.0405 | 0.0444 | 0.0171 | 0.0177 |
| 0.5 | 0.3 | 0.0359 | 0.0399 | 0.0147 | 0.0151 |
| 0.5 | 0.4 | 0.0363 | 0.0396 | 0.0156 | 0.0163 |
| 0.4 | 0.3 | 0.0328 | 0.0356 | 0.0147 | 0.0152 |

Table 4.8: Reactive Loss Allocation for unequal load and unequal reactive ratio.

| Reactive Ratio | | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|---|
| $\mu_a$ | $\mu_b$ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.3 | 0.2390 | 0.2633 | 0.0836 | 0.0846 |
| 0.6 | 0.4 | 0.2407 | 0.2612 | 0.0899 | 0.0908 |
| 0.6 | 0.5 | 0.2427 | 0.2591 | 0.0974 | 0.0991 |
| 0.5 | 0.3 | 0.2156 | 0.2331 | 0.0832 | 0.0850 |
| 0.5 | 0.4 | 0.2170 | 0.2314 | 0.0894 | 0.0914 |
| 0.4 | 0.3 | 0.1965 | 0.2088 | 0.0825 | 0.0853 |

Figures 4.17 - 4.24 show loss allocations for different combinations of real loads and reactive ratios.

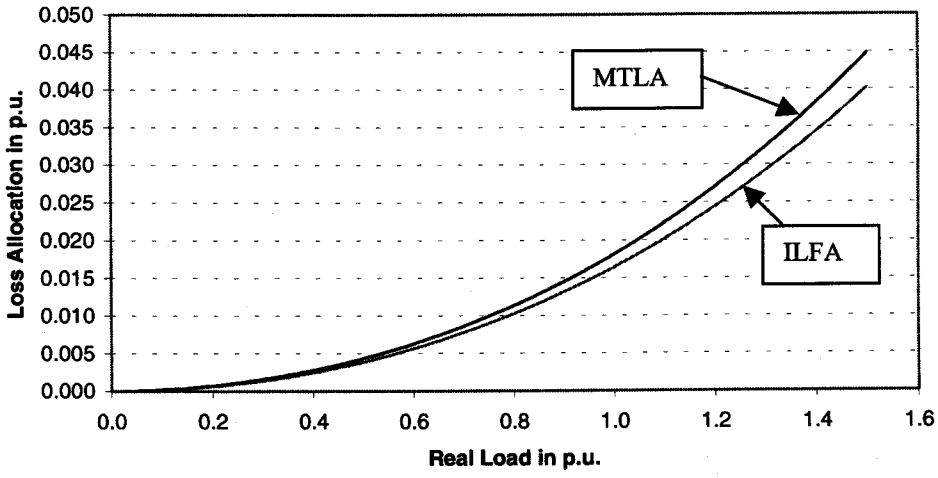


Fig. 4.17: Allocation of real transmission loss for Generator A ($\mu_a$=0.6, $\mu_b$=0.4).

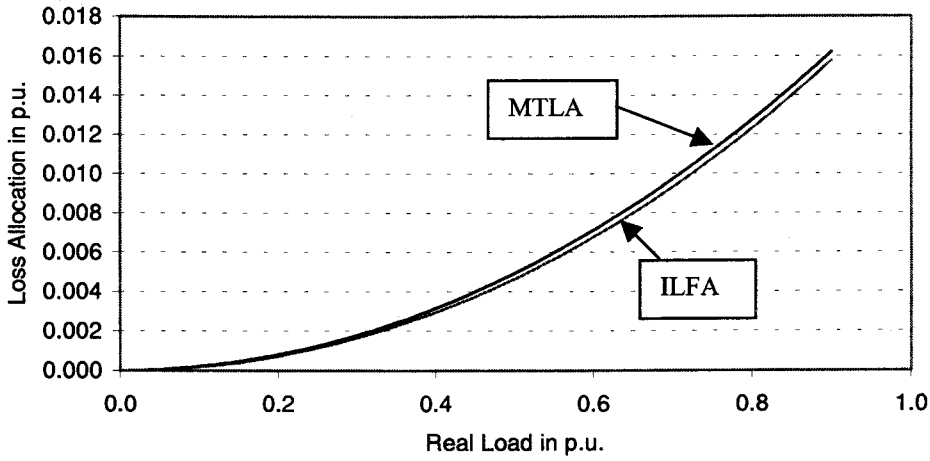


Fig. 4.18: Allocation of real transmission loss for Generator B ($\mu_a$=0.6, $\mu_b$=0.4).

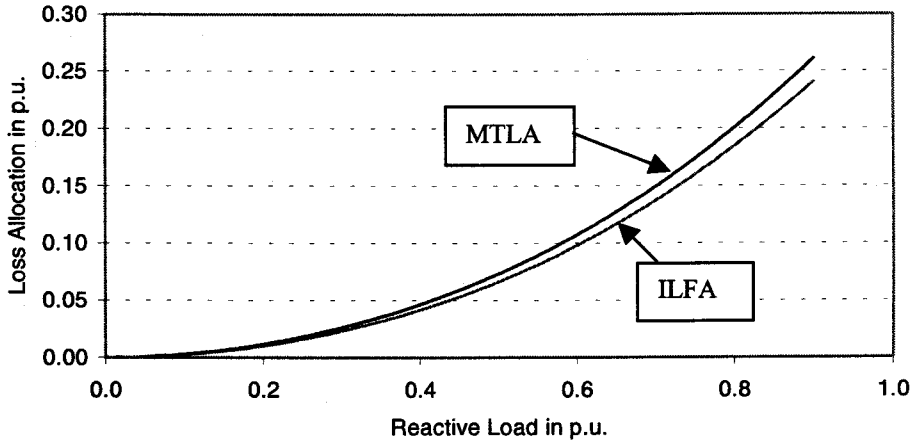


Fig. 4.19: Allocation of reactive transmission loss for Generator A ($\mu_a$=0.6, $\mu_b$=0.4).

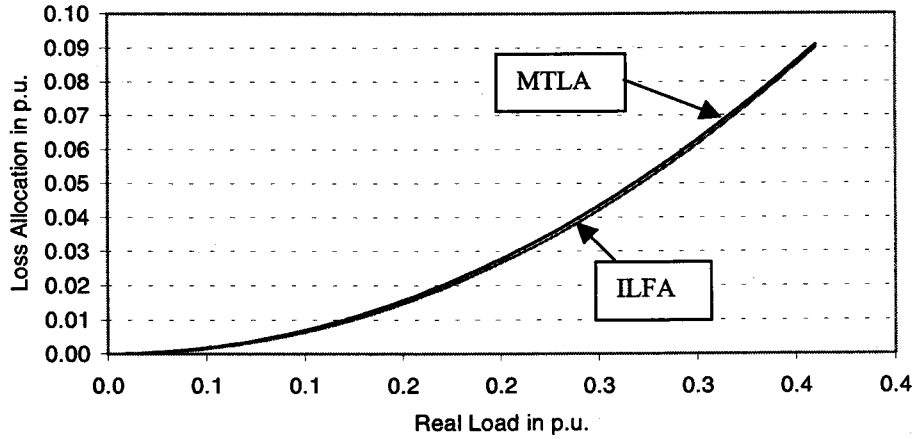


Fig. 4.20: Allocation of reactive transmission loss for Generator B ($\mu_a$=0.6, $\mu_b$=0.4).

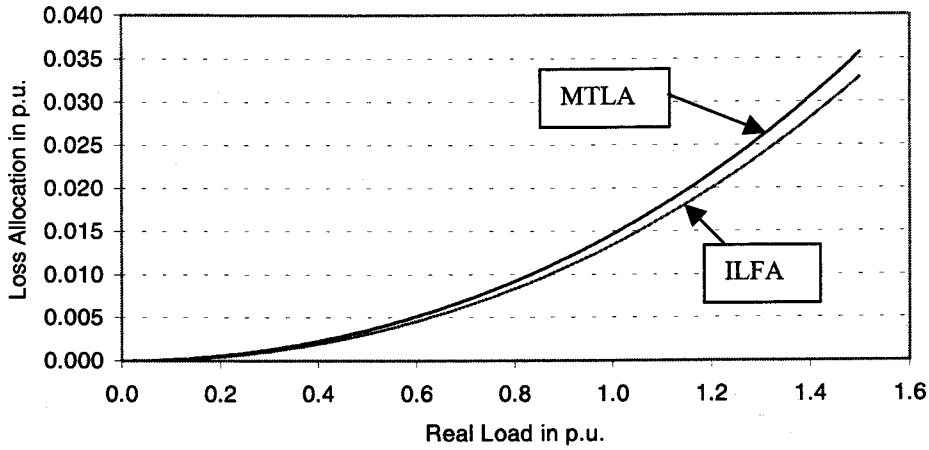


Fig. 4.21 Allocation of real transmission loss for Generator A ($\mu_a$=0.4, $\mu_b$=0.3).

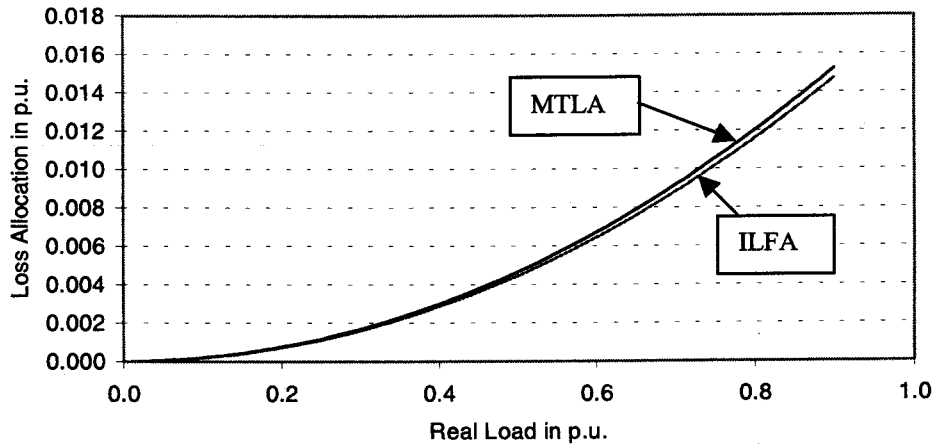


Fig. 4.22: Allocation of real transmission loss for Generator B ($\mu_a$=0.4, $\mu_b$=0.3).

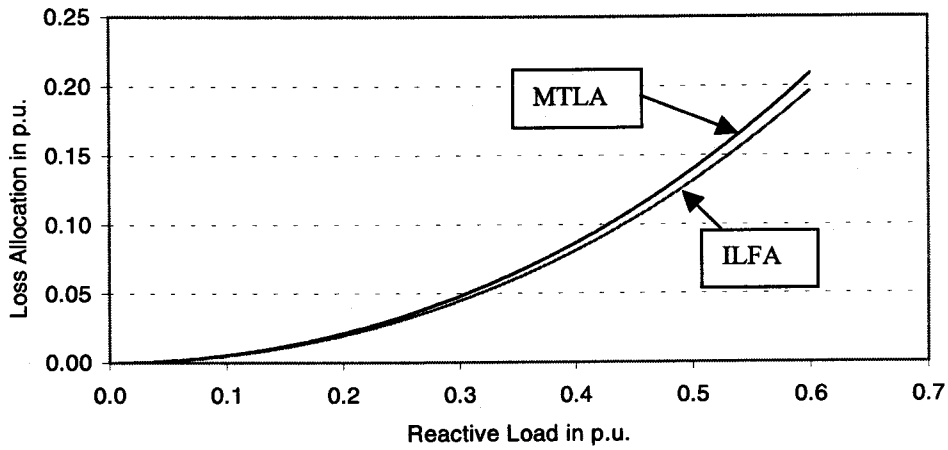


Fig. 4.23: Allocation of reactive transmission loss for Generator A ($\mu_a$=0.4, $\mu_b$=0.3).

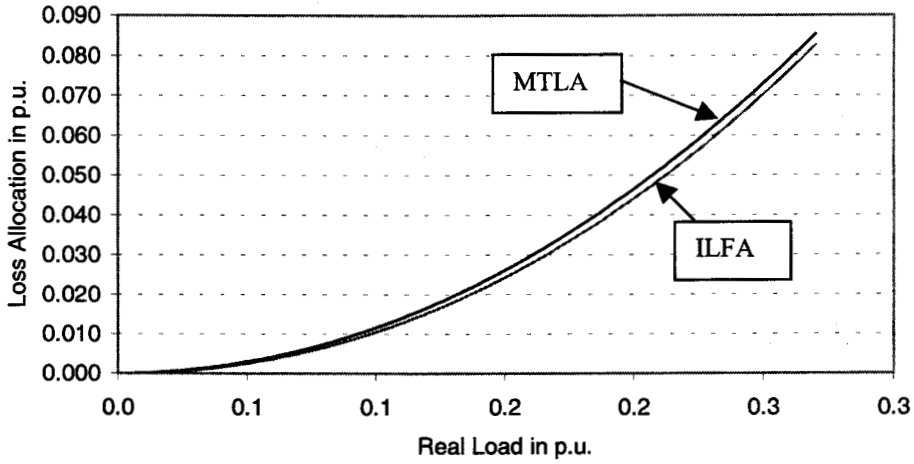


Fig. 4.24: Allocation of reactive transmission loss for Generator B ($\mu_a$=0.4, $\mu_b$=0.3).

From Figures 4.17 - 4.24, it can be seen that the loss allocations for Generator A as obtained by the two methods have higher differences than those of Generator B. Loss allocations for Generator A and B for two different reactive ratios have been plotted (Figures 4.17 - 4.24) and it can be noticed that Generator B's loss allocations as obtained by the two methods are very close to each other.

### 4.4.5 Unequal Load with Fixed Equal Reactive Ratio

The real load of Customer A has been kept constant while that of Customer B has been varied from 0.9 to 0.4 p.u. Reactive ratio is assumed to be equal for both customers which is 0.6 in this case. The loss allocations obtained from the ILFA and the MTLA are shown in Tables 4.9 and 4.10.

Table 4.9 represents the worst case scenario for Generator A. The absolute difference between the loss allocations for Generator A as obtained by the two methods does not differ by a significant amount but the relative difference (percentage error) is noticeable. This has been discussed in Section 4.5. Although the load of Customer A is fixed at 1.5 p.u., it is evident from Table 4.9 that the loss allocation for Generator A has been affected by a change in Customer B's load. With a decrease in the real power demand of Customer B, the difference in loss allocations for Generator A as obtained by the two methods increases. This is true for real as well as reactive transmission losses.

Table 4.9: Real Loss Allocation for unequal load with fixed equal reactive ratio.

| Real Demand (p.u.) | | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|---|
| $L_a$ | $L_b$ | ILFA | MTLA | ILFA | MTLA |
| 1.5 | 0.9 | 0.0409 | 0.0440 | 0.0187 | 0.0198 |
| 1.5 | 0.8 | 0.0411 | 0.0446 | 0.0145 | 0.0153 |
| 1.5 | 0.7 | 0.0414 | 0.0454 | 0.0109 | 0.0117 |
| 1.5 | 0.6 | 0.0417 | 0.0464 | 0.0079 | 0.0082 |
| 1.5 | 0.5 | 0.0421 | 0.0473 | 0.0054 | 0.0059 |
| 1.5 | 0.4 | 0.0426 | 0.0486 | 0.0032 | 0.0035 |

Table 4.10: Reactive Loss Allocation for unequal load with fixed equal reactive ratio.

| Real Demand (p.u.) | | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|---|
| $L_a$ | $L_b$ | ILFA | MTLA | ILFA | MTLA |
| 1.5 | 0.9 | 0.2449 | 0.2582 | 0.1064 | 0.1108 |
| 1.5 | 0.8 | 0.2457 | 0.2604 | 0.0828 | 0.0859 |
| 1.5 | 0.7 | 0.2470 | 0.2642 | 0.0624 | 0.0654 |
| 1.5 | 0.6 | 0.2484 | 0.2692 | 0.0452 | 0.0462 |
| 1.5 | 0.5 | 0.2504 | 0.2738 | 0.0309 | 0.0331 |
| 1.5 | 0.4 | 0.2523 | 0.2802 | 0.0195 | 0.0201 |

The loss allocations for different load combinations under this situation have been shown in Figures 4.25 - 4.28.

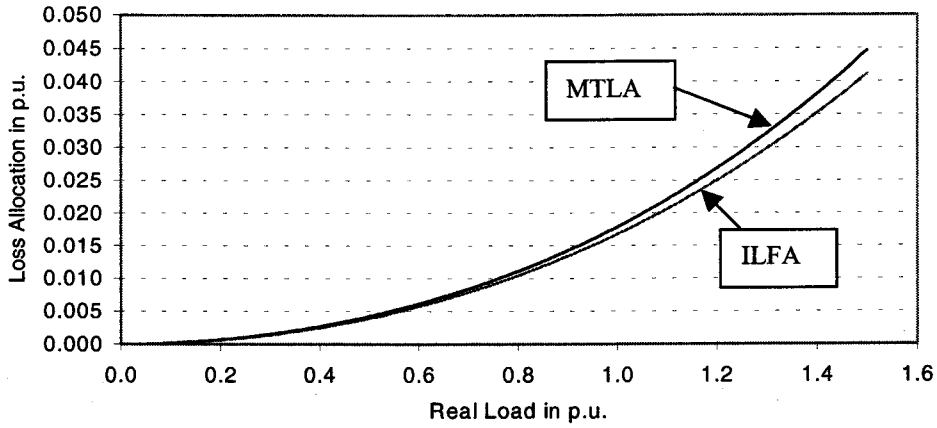


Fig. 4.25: Allocation of real transmission loss for Generator A ($L_b$=0.8 p.u., $\mu$ =0.6).

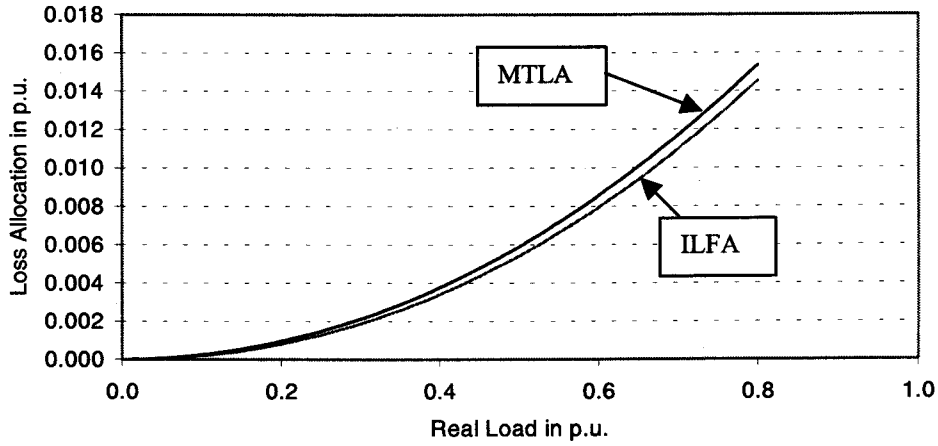


Fig. 4.26: Allocation of real transmission loss for Generator B ($L_b$=0.8 p.u., $\mu$ =0.6).

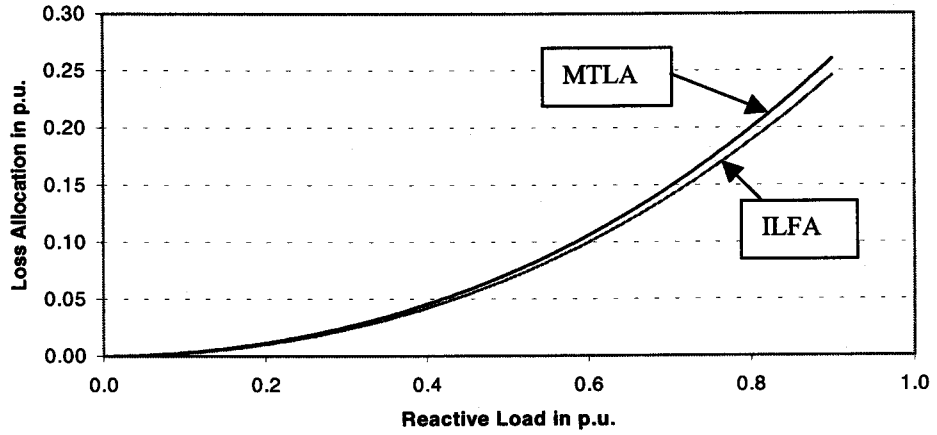


Fig. 4.27: Allocation of reactive transmission loss for Generator A ($L_b$=0.8 p.u., $\mu$ =0.6).

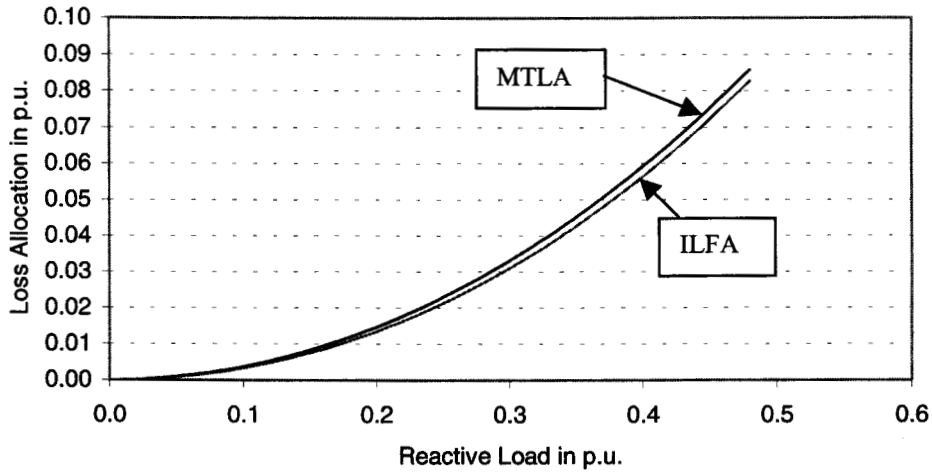


Fig. 4.28: Allocation of reactive transmission loss for Generator B ($L_b$=0.8 p.u., $\mu$ =0.6).

Figures 4.25 - 4.28 reflect the effect of load demand of Customer B on the loss allocations. Loss allocations by the MTLA are higher than those obtained by the ILFA for both generators and the difference is higher for Generator A.

### 4.4.6 Modified System

Transmission loss and its allocation will be affected by a change in the network configuration. Line number 3, connected between buses 1and 3, has been omitted from the system to investigate its effect on the loss allocation. The load has been maintained same for both customers while two different reactive ratios have been considered. The loss allocations for the modified system network are shown in Tables 4.11 and 4.12.

Similar trend of loss allocations continues for the modified system. Difference in loss allocations for Generator A as obtained by the two methods is higher than that of Generator B.

Table 4.11: Real Loss Allocation for modified system.

| Reactive Ratio | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|
| $\mu$ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.0514 | 0.0559 | 0.0569 | 0.0582 |
| 0.4 | 0.0412 | 0.0453 | 0.0453 | 0.0457 |

Table 4.12: Reactive Loss Allocation for modified system.

| Reactive Ratio | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|
| μ | ILFA | MTLA | ILFA | MTLA |
| 0.6 | 0.2993 | 0.3192 | 0.3228 | 0.3271 |
| 0.4 | 0.2398 | 0.2585 | 0.2572 | 0.2567 |

Figures 4.29 - 4.32 show the loss allocation for the modified system network.

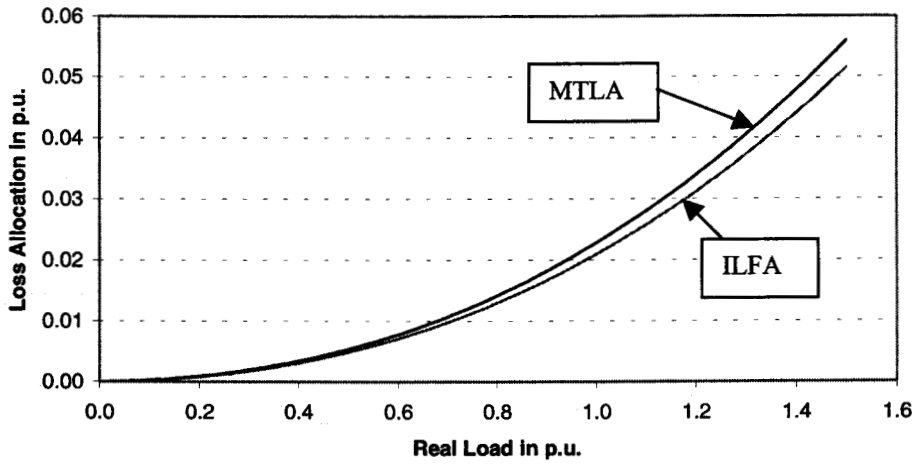


Fig. 4.29: Allocation of real transmission loss for Generator A in the modified network (μ =0.6).

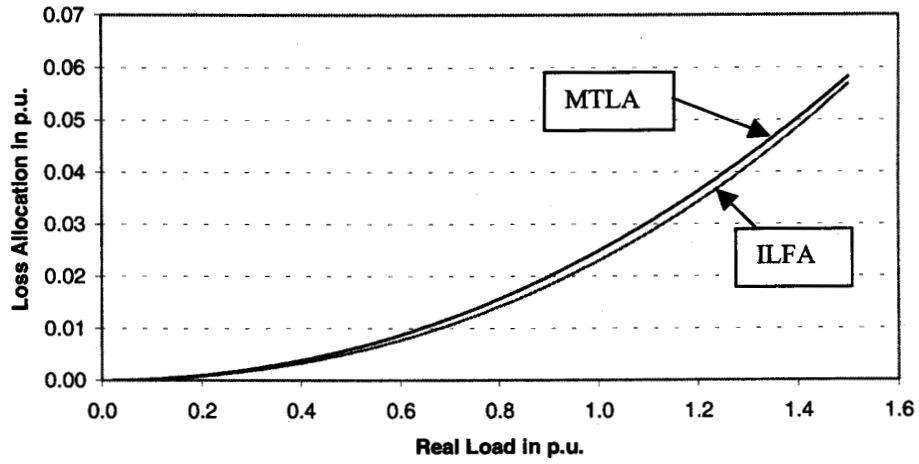


Fig. 4.30: Allocation of real transmission loss for Generator B in the modified network (μ =0.6).

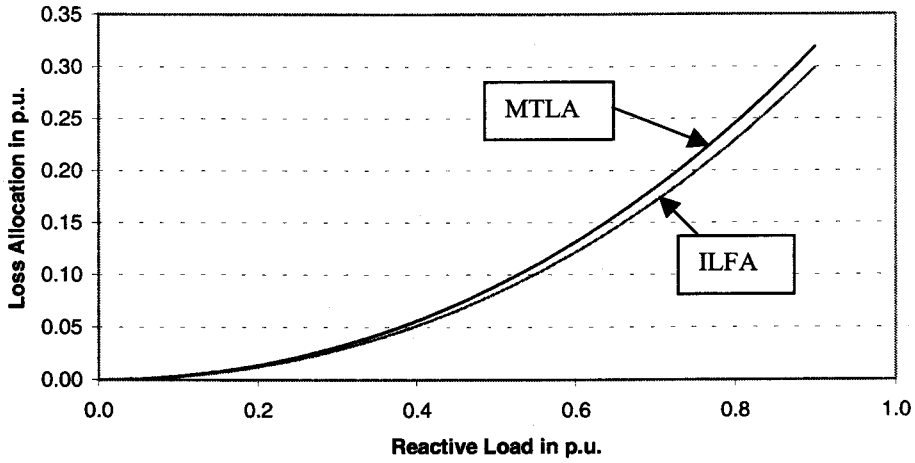


Fig. 4.31: Allocation of reactive transmission loss for Generator A in the modified network ($\mu$ =0.6).

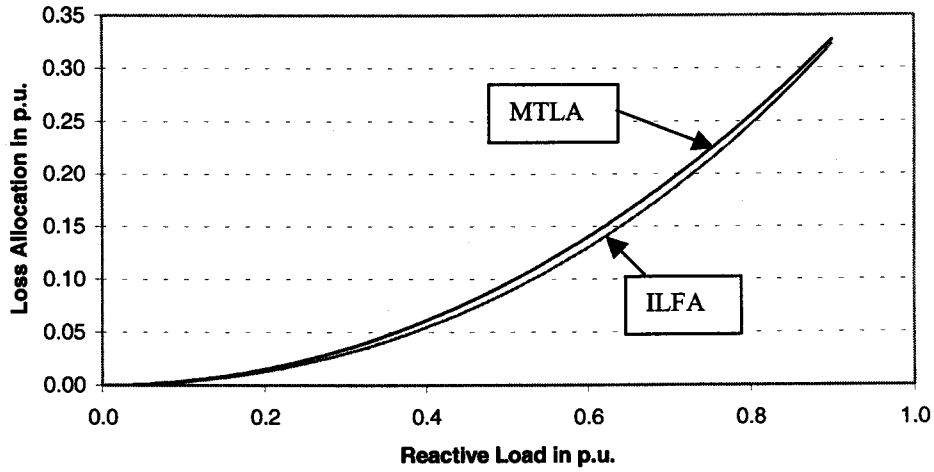


Fig. 4.32: Allocation of reactive transmission loss for Generator B in the modified network ($\mu$ =0.6).

Graphs for the modified system show that significant difference in loss allocations by the MTLA and the ILFA exists for Generator A whereas Generator B's loss allocations obtained by the two methods are very close.

### 4.5 Comparison and Discussion

The loss allocations obtained by the two different methods, the ILFA and the MTLA, differ as the real and/or reactive load changes. It is interesting to note the effect of

reactive ratio and different load demands on loss allocations. Tables 4.1 - 4.12 and Figures 4.1 - 4.32 present the loss allocations obtained by the MTLA as well as by the ILFA which clearly indicate that the differences between the loss allocations obtained by the two methods vary from nominal to significant magnitudes. The assessment of loss allocation by the MTLA is based on a number of assumptions. The assumptions have been adopted to keep the MTLA relatively simple and manageable.

In order to have a better view of the loss allocations obtained by the two methods, percentage of errors have been calculated for all load combinations. The loss allocations obtained by the ILFA have been taken as base values for error calculation.

Table 4.13 shows the percentage of errors for equal load and equal reactive ratio, for both customers. The real load for both customers is considered to be 1.5 p.u. This load condition has been previously considered in Section 4.4.1.

Table 4.13: Error table for equal load and equal reactive ratio.

| Reactive Ratio $\mu$ | Percentage of Error of Real Loss Allocation | | Percentage of Error of Reactive Loss Allocation | |
|---|---|---|---|---|
| | A | B | A | B |
| 0.6 | -1.72 | -5.21 | -0.43 | -3.67 |
| 0.5 | -2.69 | -5.05 | -1.56 | -3.55 |
| 0.4 | -4.64 | -3.19 | -3.09 | -1.96 |
| 0.3 | -5.93 | -2.59 | -3.92 | -1.78 |

The errors in loss allocation, both real and reactive, for Generator A increase with a decrease in reactive ratio while the opposite happens for Generator B. The errors in this situation, which varies between -0.43 percent to -5.93 percent, are not significant.

Table 4.14 presents the errors obtained for equal load and different reactive ratios for Customers A and B. The real load for both customers is considered to be 1.5 p.u. This load condition has been previously considered in Section 4.4.2.

Table 4.14: Error table for equal load and unequal reactive ratio.

| Reactive Ratio | | Percentage of Error of Real Loss Allocation | | Percentage of Error of Reactive Loss Allocation | |
|---|---|---|---|---|---|
| $\mu_a$ | $\mu_b$ | A | B | A | B |
| 0.6 | 0.3 | -11.83 | 1.01 | -8.86 | 1.69 |
| 0.6 | 0.4 | -8.51 | -0.25 | -6.26 | 0.76 |
| 0.6 | 0.5 | -4.69 | -3.32 | -3.10 | -2.01 |
| 0.5 | 0.3 | -9.38 | -0.25 | -6.93 | 0.54 |
| 0.5 | 0.4 | -6.29 | -1.79 | -4.56 | -0.58 |
| 0.4 | 0.3 | -6.70 | -1.42 | -4.28 | -0.64 |

The errors for the loss allocation of Generator A depends on the reactive ratio of Customer B. The errors in transmission loss allocations for Generator A subside as the difference between the two reactive ratios decreases. The error is significant when the two reactive ratios have the maximum difference. The errors obtained for Generator B's transmission loss allocations vary from positive to negative as its reactive ratio goes up. The differences between the loss allocations for Generator B as obtained by the two methods are negligible.

Table 4.15 shows the errors calculated for unequal loads and equal reactive ratio for Customers A and B. The real load demands of Customers A and B are 1.5 and 0.9 p.u. respectively. This load condition has been previously considered in Section 4.4.3.

When Customers A and B have different load demands but same reactive ratio, the errors decrease with a decrease in the reactive ratio. The errors shown in Table 4.15 vary from -3 to -7.6 percent.

Table 4.15: Error table for unequal load and equal reactive ratio.

| Reactive Ratio $\mu$ | Percentage of Error of Real Loss Allocation | | Percentage of Error of Reactive Loss Allocation | |
|---|---|---|---|---|
| | A | B | A | B |
| 0.6 | -7.67 | -5.92 | -5.43 | -4.20 |
| 0.5 | -7.00 | -5.06 | -5.06 | -3.02 |
| 0.4 | -6.71 | -4.78 | -4.95 | -3.68 |
| 0.3 | -7.63 | -3.93 | -5.52 | -4.61 |

Table 4.16 indicates the error obtained for unequal load and unequal reactive ratio. The real load demands of Customers A and B are 1.5 and 0.9 p.u. respectively. This load condition has been previously considered in section 4.4.4.

Table 4.16: Error table for unequal load and unequal reactive ratio.

| Reactive Ratio | | Percentage of Error of Real Loss Allocation | | Percentage of Error of Reactive Loss Allocation | |
|---|---|---|---|---|---|
| $\mu_a$ | $\mu_b$ | A | B | A | B |
| 0.6 | 0.3 | -13.92 | -1.75 | -10.15 | -1.11 |
| 0.6 | 0.4 | -11.52 | -2.72 | -8.48 | -1.01 |
| 0.6 | 0.5 | -9.43 | -3.56 | -6.75 | -1.66 |
| 0.5 | 0.3 | -11.03 | -2.77 | -8.11 | -2.11 |
| 0.5 | 0.4 | -8.85 | -4.01 | -6.62 | -2.21 |
| 0.4 | 0.3 | -8.61 | -3.39 | -6.23 | -3.39 |

The maximum error occurs when the two different load demands have the maximum difference in their reactive ratios. Although the errors in loss allocations for Generator B are nominal, the errors for Generator A in some cases are significantly high.

Table 4.17 presents the error calculation for different loads with equal reactive ratio for Customers A and B. The real load of Customer B has been varied from 0.9 to 0.4 p.u. while maintaining a constant load of 1.5 p.u. for Customer A. The reactive ratio is 0.6 which applies to both customers in this case. This load situation has been mentioned in section 4.4.5.

Table 4.17: Error table for unequal load with fixed equal reactive ratio.

| Real Demand (p.u.) | | Percentage of Error of Real Loss Allocation | | Percentage of Error of Reactive Loss Allocation | |
|---|---|---|---|---|---|
| $L_a$ | $L_b$ | A | B | A | B |
| 1.5 | 0.9 | -7.67 | -5.92 | -5.43 | -4.20 |
| 1.5 | 0.8 | -8.58 | -5.63 | -5.95 | -3.70 |
| 1.5 | 0.7 | -9.62 | -6.75 | -6.95 | -4.70 |
| 1.5 | 0.6 | -11.15 | -4.16 | -8.38 | -2.20 |
| 1.5 | 0.5 | -12.23 | -8.83 | -9.36 | -6.97 |
| 1.5 | 0.4 | -14.07 | -10.80 | -11.07 | -2.75 |

The errors shown in Table 4.17 are the highest among all other load combinations and this is the worst case scenario. The errors follow a rising trend in case of Generator A's loss allocations and become the maximum when the two loads have the maximum difference between them. The errors for the modified system are shown in Table 4.18.

Table 4.18: Error table for modified system.

| Reactive Ratio $\mu$ | Percentage of Error of Real Loss Allocation | | Percentage of Error of Reactive Loss Allocation | |
|---|---|---|---|---|
| | A | B | A | B |
| 0.6 | -8.74 | -2.35 | -6.66 | -1.32 |
| 0.4 | -10.01 | -0.72 | -7.79 | 0.20 |

The errors in real loss allocations for Generator A are comparatively higher than others in the table. Errors obtained for reactive loss allocations, for both generators, are quite nominal.

Figures 4.1 - 4.32 show that the outputs of the MTLA closely follow the curves obtained from the ILFA. The difference in results obtained from the two methods is small in most cases and this difference exists due to the assumptions made in both methods. The MTLA can be utilized to check the validity of the ILFA and vice versa. The Tables and the graphs clearly confirm the validity of both methods.

### 4.5.1 Modified Loss Allocations by The MTLA

It can be noticed from Tables 4.13 - 4.18 that the loss allocation errors vary from negligible to significant level depending on the load levels. It has been mentioned before that these errors occur due to the assumptions made in the MTLA. The $A_p$ and $B_p$ parameters depend on the Z-bus matrix which has been assumed to be constant in the MTLA for the sake of simplicity. The bus angles have been assumed to be constant for a certain range of load too. The bus voltages are considered 1 p.u. for all load conditions. Although these assumptions make the MTLA simpler, they are the major sources of error. An attempt has been made to recalculate the loss allocations using exact bus voltages and Z-bus for each iteration of the MTLA. This essentially makes the components of $A_p$ and $B_p$ having different values in successive iteration.

The load situation in Section 4.4.5 (Tables 4.9 and 4.10, Figures 4.25 - 4.28) results (Table 4.17) the highest error in loss allocation and, therefore, has been taken as the test case. The computer program that implements the MTLA has been modified to use the current values of $A_p$ and $B_p$ parameters and bus voltages and angles. The loss allocations from this modified program have been shown in Tables 4.19 and 4.20.

Figures 4.33 - 4.36 present loss allocations obtained by the ILFA and the modified MTLA.

Table 4.19: Modified real Loss Allocation for different load and equal reactive ratio.

| Real Demand (p.u.) | | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|---|
| $L_a$ | $L_b$ | ILFA | MTLA | ILFA | MTLA |
| 1.5 | 0.9 | 0.0409 | 0.0399 | 0.0187 | 0.0193 |
| 1.5 | 0.8 | 0.0411 | 0.0407 | 0.0145 | 0.0150 |
| 1.5 | 0.7 | 0.0414 | 0.0416 | 0.0109 | 0.0114 |
| 1.5 | 0.6 | 0.0417 | 0.0427 | 0.0079 | 0.0080 |
| 1.5 | 0.5 | 0.0421 | 0.0438 | 0.0054 | 0.0057 |
| 1.5 | 0.4 | 0.0426 | 0.0451 | 0.0032 | 0.0034 |

Table 4.20: Modified reactive Loss Allocation for different load and equal reactive ratio.

| Real Demand (p.u.) | | A's Share of Loss (p.u.) | | B's Share of Loss (p.u.) | |
|---|---|---|---|---|---|
| $L_a$ | $L_b$ | ILFA | MTLA | ILFA | MTLA |
| 1.5 | 0.9 | 0.2449 | 0.2378 | 0.1064 | 0.1082 |
| 1.5 | 0.8 | 0.2457 | 0.2417 | 0.0828 | 0.0845 |
| 1.5 | 0.7 | 0.247 | 0.2462 | 0.0624 | 0.0639 |
| 1.5 | 0.6 | 0.2484 | 0.2516 | 0.0452 | 0.0449 |
| 1.5 | 0.5 | 0.2504 | 0.2566 | 0.0309 | 0.0320 |
| 1.5 | 0.4 | 0.2523 | 0.2633 | 0.0195 | 0.0193 |

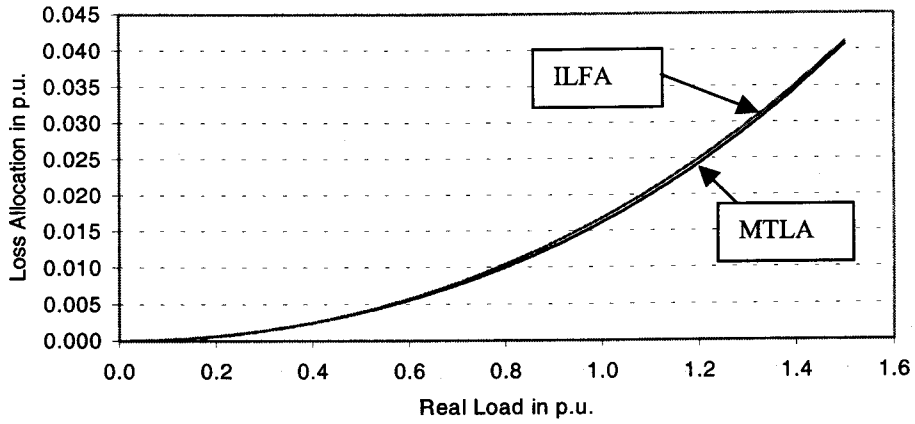


Fig. 4.33: Modified allocations of real transmission loss for Generator A ($L_b$=0.8 p.u., $\mu$ =0.6).

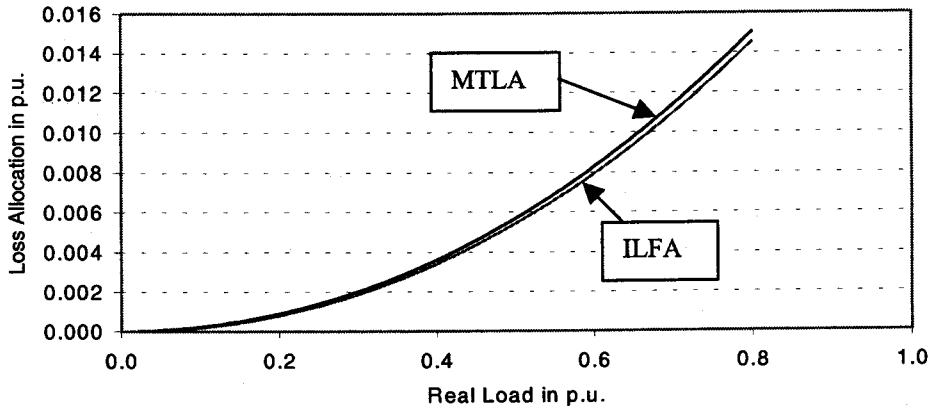


Fig. 4.34: Modified allocations of real transmission loss for Generator B ($L_b$=0.8 p.u., $\mu$ =0.6).

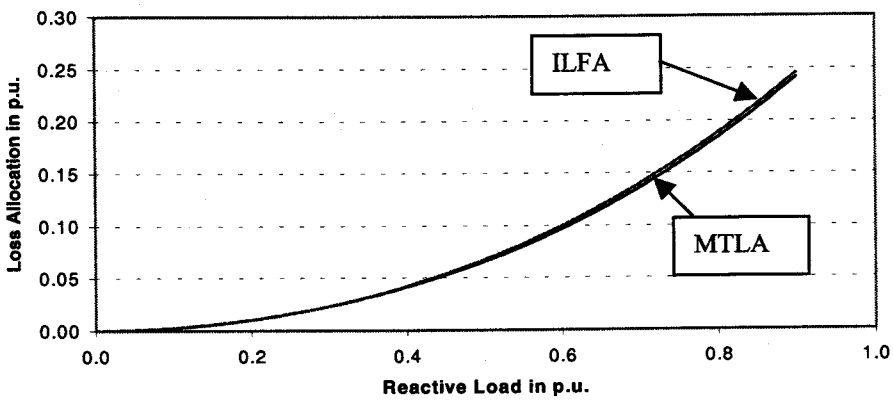


Fig. 4.35: Modified allocations of reactive transmission loss for Generator A ($L_b$=0.8 p.u., $\mu$ =0.6).

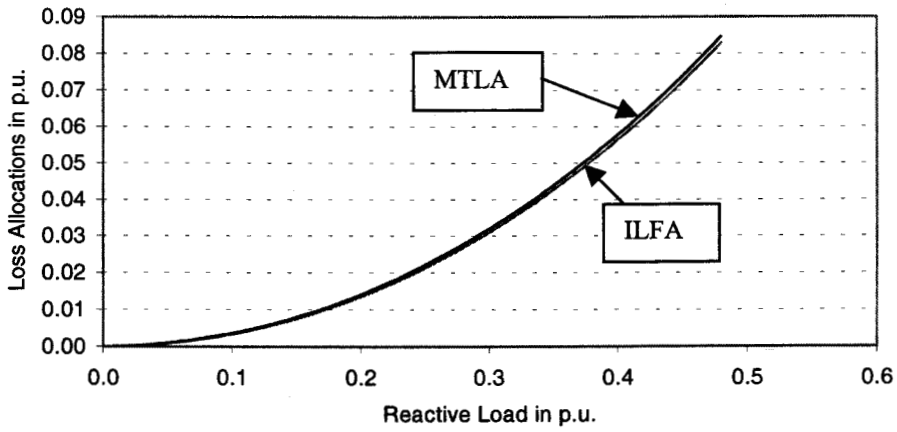


Fig. 4.36: Modified allocations of reactive transmission loss for Generator B ($L_b$=0.8 p.u., $\mu$ =0.6).

The effect of the modifications made in the MTLA is clearly evident in Figures 4.33 - 4.36 as compared to the graphs shown in Figures 4.25 - 4.28. The modifications in the MTLA have made the loss allocations obtained by the two methods closer to each other. Both real and reactive loss allocations for Generators A and B as obtained by the MTLA and the ILFA shown in Figures 4.33 - 4.36 are very close.

The percentage of errors of loss allocations for Generators A and B have been calculated for the present case and shown in Table 4.21.

The errors have been reduced by a significant margin when compared to the errors found before the modification of the MTLA. With the modifications made in the program, the errors now all reside within a reasonable range. The errors obtained for loss allocation for Generator A vary from positive to negative with a decrease in the load of Customer B. The worst case in Section 4.4.5 occurs when the difference between the load demands of the two customers becomes the maximum. The percentage error has been changed to -5.91 from a value of -14.07 after the modifications are made. Although this is a notable achievement, the modified MTLA requires an operation of AC load flow in each iteration. This is needed to have the current values of Z-bus, bus angles and voltages. The modification in the MTLA approach requires

additional computational time. In many systems the modification would not be necessary if they can live with the errors. This modification, however, validates the two methods with respect to the allocation of transmission loss.

Table 4.21: Modified error table for different maximum load and equal reactive ratio.

| Real Demand (p.u.) | | Percentage of Error of Real Loss Allocation | | Percentage of Error of Reactive Loss Allocation | |
|---|---|---|---|---|---|
| $L_a$ | $L_b$ | A | B | A | B |
| 1.5 | 0.9 | 2.29 | -3.15 | 2.87 | -1.77 |
| 1.5 | 0.8 | 0.87 | -3.57 | 1.65 | -2.04 |
| 1.5 | 0.7 | -0.57 | -4.01 | 0.36 | -2.36 |
| 1.5 | 0.6 | -2.44 | -0.88 | -1.30 | 0.77 |
| 1.5 | 0.5 | -3.81 | -4.97 | -2.50 | -3.29 |
| 1.5 | 0.4 | -5.91 | -6.17 | -4.36 | 1.42 |

# CHAPTER 5: TRANSMISSION LOSS IN A DEREGULATED EXAMPLE SYSTEM

## 5.1 ILFA vs. MTLA

The Incremental Load Flow Approach (ILFA) and the Marginal Transmission Loss Approach (MTLA) are two different methods that can be utilized to assess transmission loss shares in a deregulated network. The methods have been presented with results of transmission loss allocations in Chapters 3 and 4. A number of different load situations have been considered to find shares of transmission losses for the generators involved in the network shown in Figure 3.1. Loss allocations obtained from the two methods have been compared in the form of various tables and graphs in the previous chapter. Moreover, errors have been calculated based on the results obtained by the ILFA and discussed. It has been found that with some modifications in the assumptions made for the MTLA, the transmission loss allocations, both real and reactive, as obtained by two methods came out to be very close.

The MTLA, as described in Chapter 4, involves extensive mathematical manipulations and the final form of transmission loss equation is also quite complex in nature. The solution approach of the MTLA is dependent on network configuration. The mathematical complexity will increase with the size of the network. But the mathematical expressions are required to be evaluated only once for a network. Once the mathematical model for transmission loss allocations of a network is obtained, the rest of the calculations for allocating the losses among the generators is simple and straightforward. Even in the case of a change made in the network configuration, the mathematical expressions will not have to be evaluated from scratch. Minor modifications in the original transmission loss expression will accommodate the changes in the network.

The MTLA provides mathematical models for the shares of transmission losses and therefore provides a way of recognizing the roles that various parameters play in the transmission loss sharing. In absence of other alternatives, the MTLA can be utilized to verify the results obtained by the ILFA.

In comparison, the ILFA is simple and straightforward to use. The ILFA can be viewed as an extended and modified application of conventional AC load flow technique. The ILFA can be applied readily to a network of any practical size.

An extended network has been considered for numerical examples of transmission loss shares in a deregulated network in the following section. Due to its simplicity, the ILFA has been utilized to assess the shares of transmission losses in the following example.

### 5.2 Deregulated Example System

The deregulated system network for the example system is shown in Figure 5.1. It consists of three generators and three bulk customers or loads. According to the bilateral contracts assumed, Generators A, B and C will supply the power of Customers A, B and C respectively. Each Generator will have to generate enough power to meet its load and the associated transmission loss.

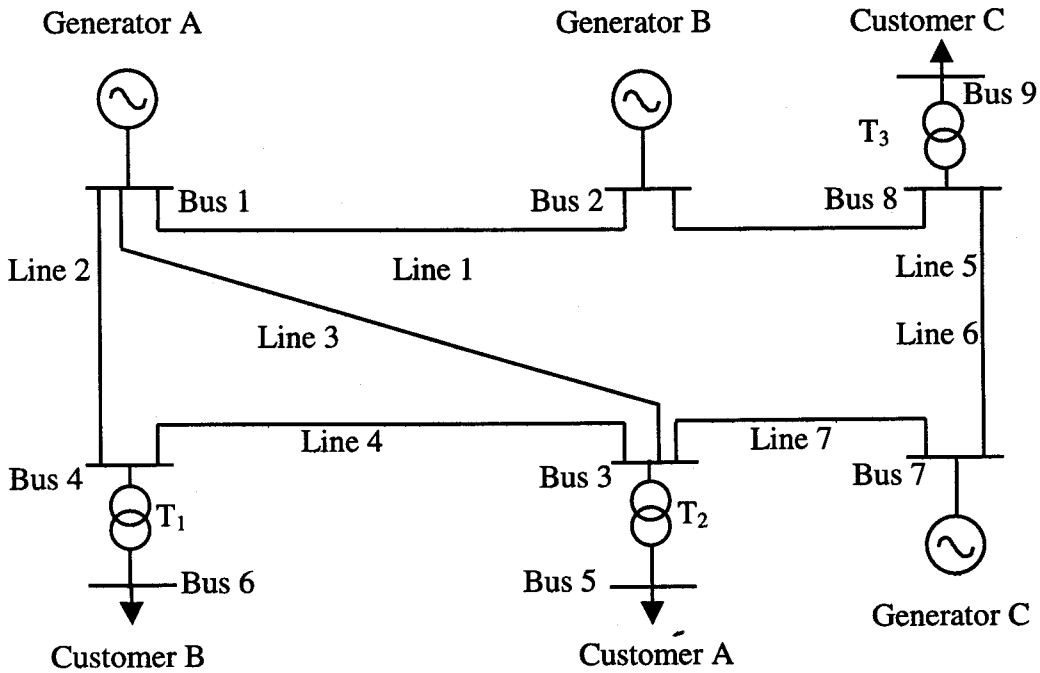


Fig.5.1. Example system.

Generators A, B and C are connected to buses 1, 2 and 7 respectively. Customers A, B and C are connected to buses 5, 6 and 9 respectively. These load buses are connected to buses 3, 5 and 8 through tap changing transformers $T_1$, $T_2$ and $T_3$ respectively. The system parameters have been included in Tables 5.1-5.3. The base values are 100 MVA and 138 kV and the tap changing transformers are set at nominal setting of 1 initially.

Table 5.1: Line parameters.

| Line Number | From Bus | To Bus | Length (km) | Resistance (p.u.) | Reactance (p.u.) |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 90 | 0.0157 | 0.0787 |
| 2 | 1 | 4 | 60 | 0.0105 | 0.0525 |
| 3 | 1 | 3 | 220 | 0.0367 | 0.1837 |
| 4 | 3 | 4 | 60 | 0.0105 | 0.0525 |
| 5 | 2 | 8 | 60 | 0.0105 | 0.0525 |
| 6 | 7 | 8 | 120 | 0.0210 | 0.1050 |
| 7 | 3 | 7 | 90 | 0.0157 | 0.0787 |

Table 5.2: Transformer data.

| Transformer Number | From Bus | To Bus | Resistance (p.u.) | Reactance (p.u.) |
|---|---|---|---|---|
| 1 | 4 | 6 | 0.0053 | 0.0367 |
| 2 | 3 | 5 | 0.0039 | 0.0315 |
| 3 | 8 | 9 | 0.0039 | 0.0315 |

Table 5.3: Generating unit characteristics.

| Generating Unit | Cost Function ($/hr) | Maximum Output (MW) | Minimum Output (MW) |
|---|---|---|---|
| A | $0.022P_1^2+12.45P_1+70$ | 500 | 90 |
| B | $0.024P_2^2+13.65P_2+80$ | 350 | 40 |
| C | $0.021P_3^2+13.25P_3+70$ | 300 | 25 |

### 5.2.1 Allocation of Transmission Loss in the Example System

Some assumptions have been made to find the transmission loss shares for Generators A, B and C. They are:

- Fixed reactive to real power ratio ($\mu$) for all loads.
- Transmission lines are considered as medium length lines and corresponding changes have been incorporated in the ILFA program.

Real power demand of each individual customer is considered as 1.6 p.u. with a reactive ratio of 0.6. Three different buses have been utilized as swing buses in three different cases. The real and reactive loss allocations using different buses as swing bus is shown in Tables 5.4 and 5.5 respectively.

Table 5.4: Real power generation and assigned loss of the generators.

| Swing Bus | Generations (p.u.) | | | Calculated Share of Loss (p.u.) | | |
|---|---|---|---|---|---|---|
| | $G_a$ | $G_b$ | $G_c$ | A | B | C |
| 1 | 1.6583 | 1.6738 | 1.6354 | 0.0567 | 0.0749 | 0.0359 |
| 2 | 1.6555 | 1.6762 | 1.6359 | 0.0563 | 0.0748 | 0.0364 |
| 7 | 1.6552 | 1.6731 | 1.6391 | 0.0561 | 0.0742 | 0.0372 |

Table 5.5: Reactive power generation and assigned loss of the generators.

| Swing Bus | Generations (p.u.) | | | Calculated Share of Loss (p.u.) | | |
|---|---|---|---|---|---|---|
| | $G_a$ | $G_b$ | $G_c$ | A | B | C |
| 1 | 1.5624 | 0.5948 | 1.1298 | 0.058 | 0.3152 | 0.0338 |
| 2 | 1.5630 | 0.5944 | 1.1298 | 0.0679 | 0.3046 | 0.0346 |
| 7 | 1.5628 | 0.5948 | 1.1294 | 0.0614 | 0.3178 | 0.0279 |

The generations and loss shares do not change appreciably for the change of reference or swing bus. Different swing buses are tried so as to find whether it affects loss allocations by ILFA or not. The real loss allocation of Generator B is the highest among the three generators because, electrically it situated farthest from its customer.

The reactive power generation and real loss allocation depend on the system configuration. In Tables 5.4 and 5.5 Generator C has been allocated the minimum real loss share because of Customer C's position in the network. Again, Generator B's reactive loss allocation is the highest.

The reactive power generation does not follow the trend exhibited by the real loss allocation. The generators' contributions, difference between generation and reactive power demand, towards the reactive component of the transmission loss as obtained by the ILFA are different from their calculated shares of loss. As for example, Generator B is supposed to produce 1.27 p.u. reactive power when bus 1 is considered as the swing bus. This is the sum of reactive power demand, 0.96 p.u, and the allocated transmission loss, 0.31 p.u. But the ILFA shows it has been producing only 0.59 p.u. and a deficit of 0.68 p.u. exists for Generator B in reactive power generation.

This difference occurs due to the nature of load flow technique. This can be attributed to the load flow method where the generation of the reactive power is determined by restricting the voltage levels. When a generator bus is declared as the PV bus, the load flow determines the reactive power that has to be produced by each generator in order to maintain the bus voltage. In a system, a generator may not be able to produce its share of reactive power loss due to its voltage restriction. However, the ILFA can be

used to calculate the share of the reactive power loss by relaxing voltage restriction. This will indicate how much a generator would have to depend on others in terms of reactive power and may lead to a suitable financial compensation [Section 3.4].

In a similar way, as mentioned for Generator B, the surplus generations of reactive power of Generators A and C can be calculated which are 0.54 p.u. and 0.14 p.u. respectively. It appears that Generators A and C are taking the burden of Generator B in terms of reactive power generation. A suitable costing method is desirable for the solution of financial compensation for Generators A and C. However, this is beyond the scope of this work.

It has been mentioned that transmission loss and its allocation among the generators vary with network configuration. In the following section, a slight modification has been made in the network in order to observe its effect on the loss allocation.

### 5.2.2 Allocation of Transmission Loss in the Modified Example System

Transmission loss depends on network configuration and so do the loss allocations. A change has been made in the example system shown in Figure 5.1 in order to observe the effect of network configuration on the loss allocations. Line 3 has been removed and reconnected between buses 2 and 4. The rest of the network remained unchanged. The modified network is shown in Figure 5.2.

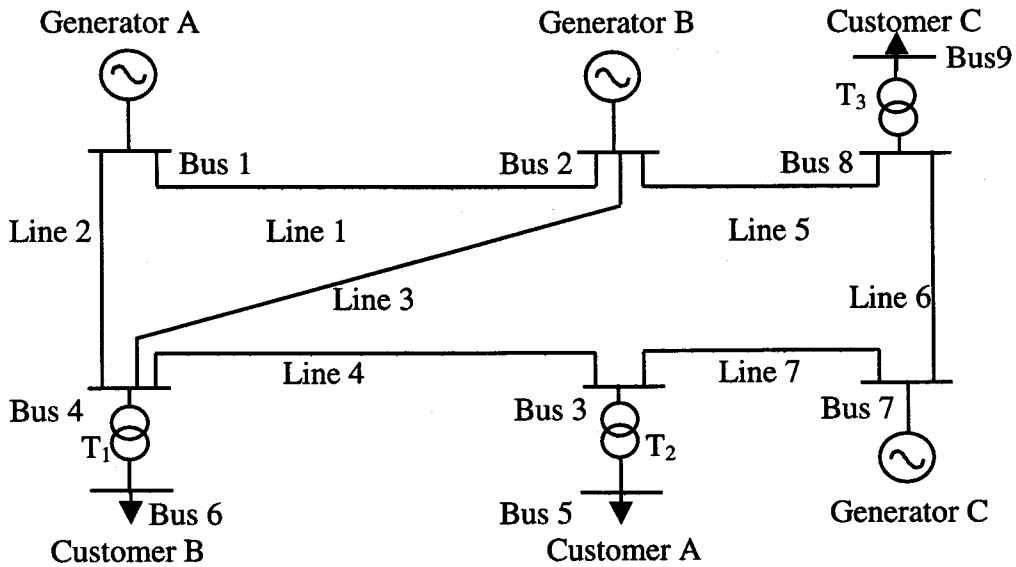


Fig.5.2. Modified example system.

Real power demand of each individual customer is considered as 1.6 p.u. with a reactive ratio of 0.6 which is the same as considered in the previous section. Three different buses have been utilized as swing buses in three different cases. The real and reactive loss allocations using different buses as swing bus is shown in Tables 5.6 and 5.7 respectively.

Table 5.6: Real power generation and assigned loss of the generators in the modified network.

| Swing Bus | Generations (p.u.) | | | Calculated Share of Loss (p.u.) | | |
|---|---|---|---|---|---|---|
| | $G_a$ | $G_b$ | $G_c$ | A | B | C |
| 1 | 1.6711 | 1.661 | 1.6365 | 0.0696 | 0.0619 | 0.037 |
| 2 | 1.6683 | 1.6636 | 1.6366 | 0.0693 | 0.0621 | 0.0371 |
| 7 | 1.6682 | 1.6605 | 1.6398 | 0.0692 | 0.0614 | 0.0379 |

Table 5.7: Reactive power generation and assigned loss of the generators in the modified network.

| Swing Bus | Generations (p.u.) | | | Calculated Share of Loss (p.u.) | | |
|---|---|---|---|---|---|---|
| | $G_a$ | $G_b$ | $G_c$ | A | B | C |
| 1 | 1.0831 | 0.9157 | 1.2942 | 0.2313 | 0.1273 | 0.0545 |
| 2 | 1.0836 | 0.9152 | 1.2942 | 0.2414 | 0.1175 | 0.054 |
| 7 | 1.0835 | 0.9157 | 1.2937 | 0.2364 | 0.1294 | 0.0471 |

It has been anticipated that Generator A's loss allocation would increase and Generator B's loss allocation would decrease because of the change made in the network. This seems logical as Customer B is now directly connected to its supplier, Generator B by Line 3. This advantage was previously enjoyed by Customer A and Generator A. Tables 5.6 and 5.7 show the generations and loss allocations for the changed system which agree well with the anticipation. It can be noticed from the tables that loss allocations,

both real and reactive, for Generator A have been increased from its original loss allocations based on the network shown in Figure 5.1. Real loss allocation for Generator A rises to 0.069 p.u. from its previous allocation of 0.056 p.u., when bus 1 is considered as the swing bus. Similarly, reactive loss allocation is now 0.23 p.u. instead of 0.058 p.u. Generator B's real loss allocation drops to 0.061 p.u. from its initial allocation of 0.074 p.u. for the original network shown in Figure 5.1. The reactive loss allocation for Generator B drops to 0.12 p.u. from a previous value of 0.31 p.u. The loss allocations for Generator C, both real and reactive, remain the minimum as they were before but they increase by a small margin due to the change made in the network.

The loss allocations for Generators A, B and C, from both network, indicate clearly what power they should produce and what one individual generator owe to the other generators in terms of reactive power generations. Since the loss allocation is already known from the ILFA, the supervising authority can easily calculate each generator's share of transmission loss and charge the generators accordingly.

# CHAPTER 6: CONCLUSIONS

## 6.1 Conclusions

There are many challenges associated with the deregulation of power systems. Some of them are technical which include transmission loss allocations, transmission access usage etc. and the others are non-technical. The non-technical challenges originate from political, socio-economical, environmental issues. Despite the potential for huge economic benefit, deregulation of power systems is basically a political decision in most countries of the world. In other parts of the world where it has not been a political issue, some groups are questioning the moral grounds of deregulation [20]. Although these are non-technical issues that are beyond the scope of this research, they influence the policies with respect to deregulation of power systems.

Of all the technical issues related to deregulation, allocation of transmission losses and transmission access are the most contentious ones. This research work dealt with the issue of allocation of transmission losses in a deregulated system with bilateral contracts. The methods used to allocate transmission losses should be viewed by the generating entities as transparent and fair.

In the research work reported in this thesis, methods have been developed to allocate transmission losses in a deregulated system where generators are tied to the loads through bilateral contracts. For a given configuration, a discrete change in a given load will be accompanied by a discrete change in transmission loss. The discrete change in the transmission loss can be attributed to the generator that is contracted to the load provided all other factors remain same. In a deregulated network with bilateral contracts, a discrete change in the transmission loss can be evaluated by increasing a load by a discrete amount while keeping all other loads at their previous levels. All loads can be changed in a sequential manner and the corresponding discrete change in

transmission loss can be aggregated. Each generator will be responsible for supplying its load as well as its share of transmission loss. In short, this is the working principle that has been used in the work reported in this thesis. This working principle has been realized by two different methods.

The Incremental Load Flow Approach is a relatively simple technique. It utilizes conventional AC load flow technique in a successive manner. Some modifications in an AC load flow are required to evaluate transmission loss allocation. The modifications are very simple and can be implemented with little difficulty. The second method, the Marginal Transmission Loss Approach is a direct consequence of mathematical reasoning. If a transmission loss can be expressed as a function of loads then its marginal rates can be evaluated with respect to the loads. Traditionally, transmission losses have been expressed as functions of generation. This is useful for the purpose of economic optimization. In order to allocate transmission loss, we need transmission loss to be expressed as a function of loads. Starting from the expression of bus injection in a network, a convenient and useful expression for transmission loss has been derived which is a function of loads. The MTLA has provided a means to check the results obtained by the ILFA. Although the MTLA uses a different approach, it provides similar loss allocations as obtained by the ILFA.

The MTLA is a complex mathematical and network dependent method. The mathematical complexity increases with the size of the network. Once the mathematical formulation of loss expression are accomplished for a network, it would be a simple task to accommodate any changes made in the network. Although the formulation of mathematical expression for the MTLA is a complex process, its execution time is much faster than that of the ILFA. The ILFA requires load flow technique that needs iterations to obtain the solution for one load level and this process has to be repeated for every incremental load level. Whereas in the MTLA, a set of simultaneous equations are solved to calculate loss allocations and no iteration is required during its execution. The ILFA requires 10 to 15 seconds for its execution for the system used as the test network whereas the MTLA requires 5 to 10 seconds.

The execution time for the ILFA would increase with the size of the network. It has been mentioned that mathematical complexity would increase with the network size and more time would be required for the formulation of the MTLA. But the execution time would not increase in the same ratio for the MTLA.

The formulation of the MTLA is based on some assumptions. Those assumptions have been made to keep the challenge of transmission loss allocation manageable. Although they help to keep the formulation of the MTLA relatively simple, it has been found that the loss allocations obtained by the MTLA differ noticeably in some load situations than those obtained by the ILFA because of the assumptions. The assumptions of constant Z-bus, bus voltages and angles (only three different values used) over the full range of load demand are responsible for the deviations in results to a large extent. In an ideal case, current bus impedance matrix, bus voltages and angles should be used instead of assuming them to be constant. But it is cumbersome to store bus impedance matrix, bus voltages and angles for every incremental load level used in the MTLA and moreover, a load flow program has to be utilized to update those parameters. The use of updated bus impedance matrix, bus voltages and angles in every load level in the MTLA, however, gives fairly accurate and close results to those obtained by the ILFA.

The methods reported in this work consider bilateral contracts between the generators and the customers in a deregulated network for the allocation of transmission losses. Full deregulation would allow bilateral contracts between the generators and the buyers and it is expected that there would be a number of such contracts in a fully deregulated system. A generator in the bilateral contract with its customer is supposed to meet the customer's load along with the associated transmission loss. In some cases, a generator might not be able to produce its share of loss. In such cases, the ILFA or the MTLA would be very useful to determine how much this particular generator would owe to other generators in the system and the liable generator would pay accordingly. In some cases, some generators may have to produce more reactive power than their allocations in order to maintain the minimum voltage level in the system. In such cases, these generators should be compensated by other generators and the ILFA or the MTLA would be a useful tool to determine other generators' liabilities.

## 6.2 Future Work

The proposed methods in this thesis considered bulk loads only. Transmission losses are allocated among the generators that are responsible for supplying loads to their respective bulk customers.

The mathematical expressions developed for the MTLA are based on the network shown in Chapter 3. These expression are not generalized and would vary from network to network. Development of a generalized model for the MTLA would be a challenging work in this regard. The generalized form would greatly simplify the implementation of the MTLA.

The following works would be quite significant in the context of the implementation of full deregulation:

- Methods to determine the payment structure when certain generators would not be able to produce its share of allocated real loss.
- Methods to determine the payment structure when certain generators would not be able to produce its share of allocated reactive loss.
- Method to find the effect of voltage profile on the allocation of real and reactive losses.

# REFERENCES


[1] Available at URL: http://www.gms.ocps.k12.fl.us/biopage/a-g/edison5.html.

[2] Available at URL: http://shakti.hypermart.net/conventional.html.

[3] Available at URL: http://www.ing.puc.cl/power/southamerica/southamerica.htm.

[4] Available at URL: http://www.energy.gov.ab.ca/electric/restruct/overview.htm.

[5] Available at URL: http://www.saskpower.com/html/message.html.

[6] A.F.Vojdani et al., "Transmission Access Issue", IEEE Transactions on Power Systems, Vol. 11, No. 1, February 1996, pp. 41-51.

[7] G. Strbac, D. Kirschen, S. Ahmed, "Allocating Transmission System Usage on the Basis of Traceable Contributions of Generators and Loads to Flows", IEEE Transactions on Power Systems, Vol. 13, No. 2, May 1998, pp. 527-534.

[8] R. Allan, D. Kirschen, G. Strbac, "Contributions of Individual Generators to Loads and Flows", IEEE Transactions on Power Systems, Vol. 12, No. 1, February 1997, pp. 52-60.

[9] J. Bialek, "Identification of Source-Sink Connections in Transmission Network", IEE Fourth Conference on Power System Control and Management, London, 16-18 April 1996, pp. 200-204.

[10] J. Bialek, "Topological Generation and Load Distribution Factors for Suppliment Charge Allocation in Transmission Open Access", IEEE Transactions on Power Systems, Vol. 12, No. 3, August 1997, pp. 1185-1193.

[11] J. Bialek, D.B.Tam "Tracing the Generators' Output", IEE International Converence on Opportunites and Advances in Electric Power Generations, Durham, UK, 18-20 March 1996, pp. 133-136.

[12] C.N.Macqueen, M.R.Irving, "An Algorithm for the Allocation of Distribution System Demand and Energy Losses", IEEE Transactions on Power Systems, Vol. 11, No. 1, February 1996, pp. 338-343.

[13] Y.Z.Li, A.K.David, "Optimal Multi-Area Wheeling", IEEE Transactions on Power Systems, Vol. 9, No. 1, February 1994, pp. 288-294.

[14] J.W.M.Lima, "Allocation of Transmission Fixed Charges: an Overview", IEEE Transactions on Power Systems, Vol. 11, No. 3, August 1996, pp. 1409-1418.

[15] Y. Tsukamoto, I. Iyoda, "Allocation of Fixed Transmission Cost to Wheeling Transactions by Cooperative Game Theory", IEEE Transactions on Power Systems, Vol. 11, No. 2, May 1996, pp. 620-627.

[16] F. D. Galiana, M. Ilic′, "A Mathematical Framework for the Analysis and Management of Power Transactions under Open Access", IEEE Transactions on Power Systems, Vol. 13, No. 2, May 1998, pp. 681-687.

[17] G.Kron, "Tensorial Analysis of Integrated Transmission System -Part II: Off-nominal Turns Ratios", AIEE Transactions, Vol. 71, January 1952, pp. 505-512.

[18] E.E.George, "Intrasystem Transmission Losses", AIEE Transactions, Vol. 62, March 1943, pp. 153-158.

[19] M.E. El-Hawary, G.S. Christensen, "Optimal Operation of Electric Power Systems", Academic Press Inc., 1979.

[20] D.R.Wooley, A.J.Cavallo, "Real Planning, Sham Competition, and State Regulation", Regulating Regional Power Systems, IEEE Press, 1995,pp.41-51.