

DIGITIZING ANALOG RECORDS AS APPLIED TO CARTOGRAPHIC DATA

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in partial Fulfilment of the Requirements

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University of Saskatchewan

by

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Saskatoon, Saskatchewan

June, 1968

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ABSTRACT

The application of the digital computer has been extended to include the analysis of analog records. Cartographic data, a highly sophisticated form of analog record, must be digitized by suitable devices for the development of an Automatic Cartography System. This thesis describes the results of tests performed on one such digitizer and the feasibility of a proposed improvement.

The low cost, position-sensing servomechanism of the Pencil Follower manufactured by d-mac Company Ltd., exhibits characteristics which were undesirable when digitizing maps and charts; namely lag, inconsistent resolution, and poor lock-on. The feasibility of replacing the servomotor drive with a stepper motor drive was investigated, with the necessary circuitry developed and tested. Preliminary tests were performed on a one axis model to verify the proposed melioration.

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1. INTRODUCTION

1.1 General:

With the advent of the digital computer, there has been major interest throughout the world in digitizing analog records. The advantages of having such records in a digital form are basically two fold; substantial improvements in handling large quantities of data, and the simplicity with which digital information can be mathematically manipulated and processed by a digital computer.

Graphical records produced by seismologists and the medical profession, and photographs taken in high-energy physics experiments are examples of analog records suitable for computer analysis^(1,2,3). Some of this data is available such that on-line digitizing can be accomplished without producing a hard copy record. However, there exists a multitude of hard copy records produced before these on-line techniques are perfected and economically useful. Thus, the role of digitizers becomes important in the transition stage between hard copy and completely automatic on-line digitizing.

Digitizing the extremely large amount of complex data contained on existing hard copy maps and charts is absolutely essential for the development of Automatic Cartography Systems. The majority of the time consumed in updating and drawing new maps is derived from the manual tracing and retracing of the same outline, or replotting the same position on different scales, projections, etc. This tedious and time consuming manipulation and plotting can be accomplished more efficiently under digital computer control, if the immense quantity of background map information is in digital form.

Having established the necessity for a device to digitize cartographic data, an investigation of commercially available digitizers is necessary. The quantity and complexity of map and chart information place stringent requirements on the digitizer. These general specifications can be grouped into resolution and format.

The resolution necessary for the digitizer depends upon the accuracy of the existing map information. A resolution of $\pm 0.004''$ is a generally accepted figure as the minimum width of line generally scribed by a cartographer is $0.004''$. This restriction in line width is due to the printing process which will not accurately reproduce lines finer than $0.003''$ ^(4,5).

The next consideration is format, which includes the type and quantity of information to be digitized. Almost all the data on a map can be reduced to line and point information, with the exception of names, symbols, etc. There exists a practical limitation to the density of information to maintain legibility; investigations indicate that there are approximately 20" of line, 6 point symbols, and 8 names per square inch of map⁽⁶⁾. Map and chart sizes range from 10" x 10" to 50" x 50"; to become any more specific requires more knowledge of the data displayed. Assuming a map area of approximately 1,000 square inches and line data recorded in increments of $0.004''$, there exists approximately 5×10^6 co-ordinates of line data per map, indicative of the large quantities of data to be recorded.

Thus, a high resolution ($\pm 0.004''$) must be maintained over a large area (1,000 square inches) by a cartographic digitizer. Further requirements are imposed by the central processing unit and the drawing

device used in the system. The Automatic Cartography System at the University of Saskatchewan uses a PDP-8 computer manufactured by Digital Equipment Corporation. This is a small, general purpose digital computer with rather limited storage facilities. The rate at which the computer can accept and suitably process the data depends upon these storage facilities and the complexity of the computer programs. Thus, the speed of the digitizer must not exceed the rate at which the PDP-8 can accept incoming data. Another requirement is that line information must be in a continuous digitized form, so that drawing with the Gerber Model 32 Plotting Table can be accomplished. This is an automatic drafting instrument requiring the input line data in a continuous form for efficient plotting. Also, it must be possible to "label" line and point information when digitizing, so that coastal outlines, roads, rivers, political boundaries, depth soundings, etc., can be discriminated.

These specifications and requirements provide the foundation necessary to choose the most suitable commercial digitizer for the Automatic Cartography System under study at the University of Saskatchewan. The following section reviews the types of commercially available instruments and the justification for choosing one particular device.

1.2 Digitizers for Cartographic Applications

Digitizers are of three basic types: (1) manual, (2) semi-manual, and (3) automatic. Manual devices require operator positioning and manual recording of the positional co-ordinates on punched paper tape or punched cards suitable for computer input. If we assume that

the operator can move to a new position and record the new co-ordinates once every 10 seconds, then with 5×10^6 co-ordinates of line data on a 1,000 square inch chart, it would take approximately 2.5 years, at 10 hours of digitizing per day, to completely digitize the line data on one chart. This makes the manual digitizer impractical for cartographic data and has promoted the development of semimanual and automatic techniques.

Semimanual digitizers require that the operator accurately follow the line or point to the position he wishes to digitize. The digital co-ordinates are automatically transferred to magnetic tape or digital computer store. To obtain accurate results the operator must be skilled in line following and digitizing speeds are comparable to those encountered during the scribing of cartographic data.

Solid state semimanual devices such as the "RAND Tablet",^(17, 18, 19, 20) M.H. Levin's magnetic device,⁽²¹⁾ the "LINCOLN Wand",⁽²²⁾ the "IBM Copy Write Tablet",^(23, 24) the "WESTLAND Trace Reader",^(25, 26) and the "SYLVANIA Data Tablet",⁽²⁷⁾ utilize unique properties of the operator's stylus and the tracing surface to obtain the digital position of the stylus. The limited working area (10" x 10") and insufficient resolution (± 0.01 ") limit their use to non-cartographic applications.

Numerous Electromechanical Followers are commercially available. These devices have some form of mechanical follower mechanically connected or electrically linked to the stylus such that the digital position of the stylus is available by monitoring the follower. Mechanical Carriage Record Readers^(7, 9) are not suitable for map work because of the physical loading on the operator and their limited resolution (± 0.01 ").

The slow positioning speeds of Closed Circuit T.V. Followers limit their use to basically point data. (8,10,11,14,15,16) The "Pencil Follower" and the "Universal Graphics Processor" are both suitable for high accuracy, large area digitizing with the former exhibiting superior resolution, less physical loading of the operator, and reduced initial cost. (12,13)

As distinct from semimanual methods, automatic digitizers do not require manual positioning and following with a stylus. The digital co-ordinates are obtained automatically eliminating operator error and providing accurate results at high digitizing speeds. However, these two advantages must be weighed against the disadvantages discussed below.

Automatic Scanners, such as drum scanners and flying spot C.R.T. scanners, digitize the complete map surface representing each 0.004" x 0.004" area by a binary "1" or "0". (33,34,35,36,37) This complete scan requires recording approximately ten times as much data as there is useful line information. Also, the digitized information is in a raster format with no "labels", resulting in extensive programming time and storage facilities, as well as manual "labelling", to produce continuous, "labelled", digitized line data. Thus, automatic scanners do not meet the requirements for the system.

Automatic Line Followers, can produce continuous, labelled, digitized line data. (26,28,29,30,31,32) The effective following speed is related to the complexity of information because extensive manual override is necessary to direct the device at ambiguous points, such as branches, intersections, dotted lines, etc. Since a map contains many such ambiguous points, the average following speed is much less than

the maximum according to machine specifications. Also, complete on-line digital computer control is necessary, resulting in reduced time for the processing of the digitized results. This disadvantage coupled with the high cost of such an automatic system does not justify the purchase of such a device for the Automatic Cartography Project at the University of Saskatchewan. As a result the low cost, semimanual, "Pencil Follower" manufactured by d-mac Company Limited was obtained to perform map digitizing.

The "Pencil Follower" consists of a pen-coil (stylus) which generates a magnetic field to activate the servo driven follower. Such a position-sensing servo feedback system exhibits certain characteristics which provide the machine limitations.

The purpose of this work was to establish these limitations and to investigate reducing these restrictions by using a stepper motor drive. To achieve this end, the analysis is divided into two primary divisions. The first division establishes the limitations of the servo Pencil Follower. Experiments are performed to measure the lag, step response, and resolution of this device and to indicate why these are limitations in digitizing practical cartographic data.

The second division establishes the concept of a digital Pencil Follower driven by the stepper motors, and presents preliminary results to indicate the feasibility of such a device.

2. THE PENCIL FOLLOWER

2.1 The Servo Follower

2.1.1 Basic system

The operation of the model 10000 Pencil Follower manufactured by d-mac Company Limited must be described to provide sufficient background for a detailed analysis and understanding. (12)

With reference to Figure 2.1.1, digitizing is accomplished by pointing the pen (stylus) at the point of interest or tracing it along the line to be digitized. A wire wound coil in the pen generates a symmetrical a.c. magnetic field, inducing an error voltage in coils mounted on the servo-driven follower. This voltage is amplified and energizes the servomotors to drive the follower to a position directly below the stylus.

Two detector coils, each wound on a ferrite core to improve pickup, are used for each of the X and Y axes. They are connected so that the error voltage is zero when the pen-coil is mid way between them. The induced voltage is always in phase or in antiphase (180° out of phase) with respect to the pen-coil. Thus, the magnitude of the error voltage and its phase result in the follower being propelled the required distance in the correct direction. This coil configuration is illustrated in Figure 2.1.2, for one axis, i.e. one set of pickup coils. A more concentrated magnetic field is obtained by inserting a pointed, high permeability core in the pen-coil. This high accuracy pen is available with the Pencil Follower. (38)

A block diagram of the associated electronics is indicated in Figure 2.1.3. The 400Hz a.c. field is produced by a pen-coil driven by

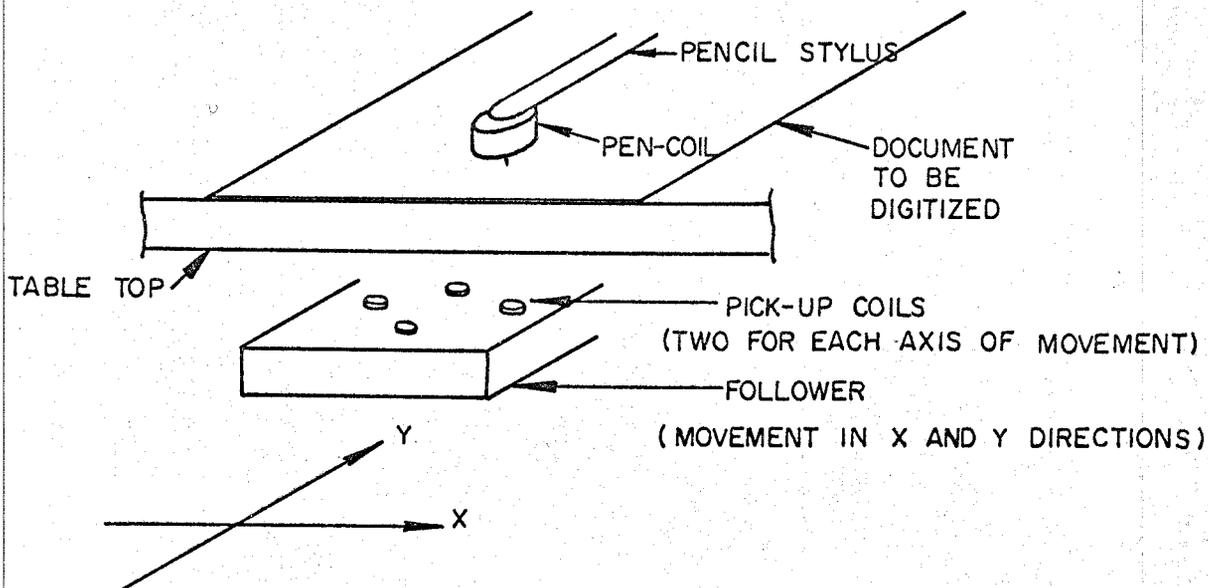


FIG.- 2.1.1 PENCIL FOLLOWER OPERATION

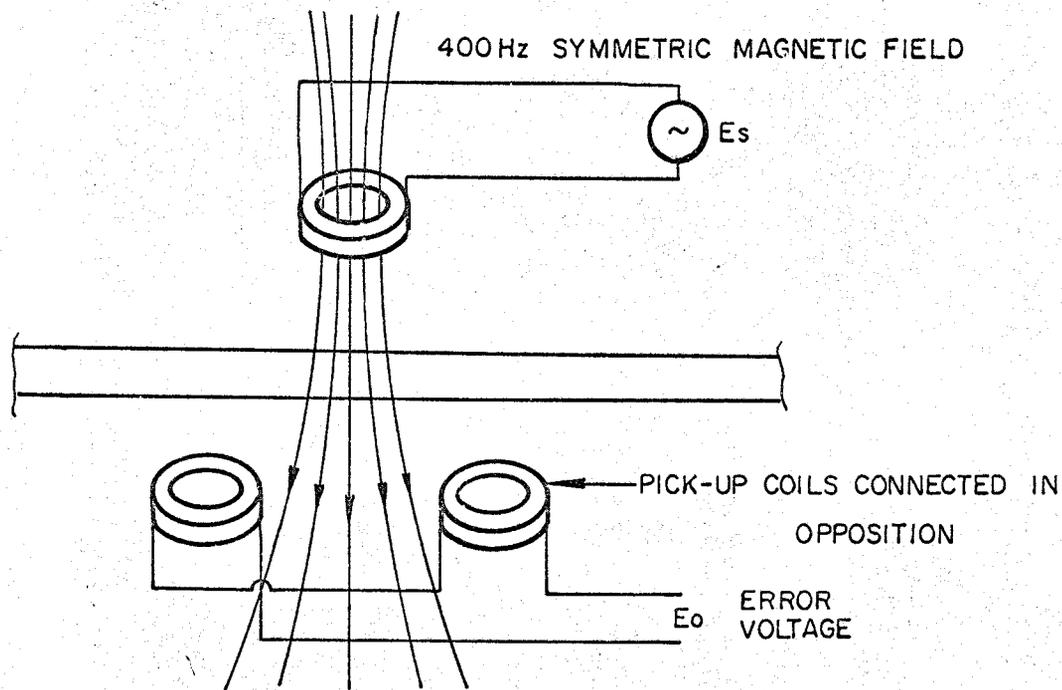


FIG.- 2.1.2 PEN AND DETECTOR COIL CONFIGURATION.

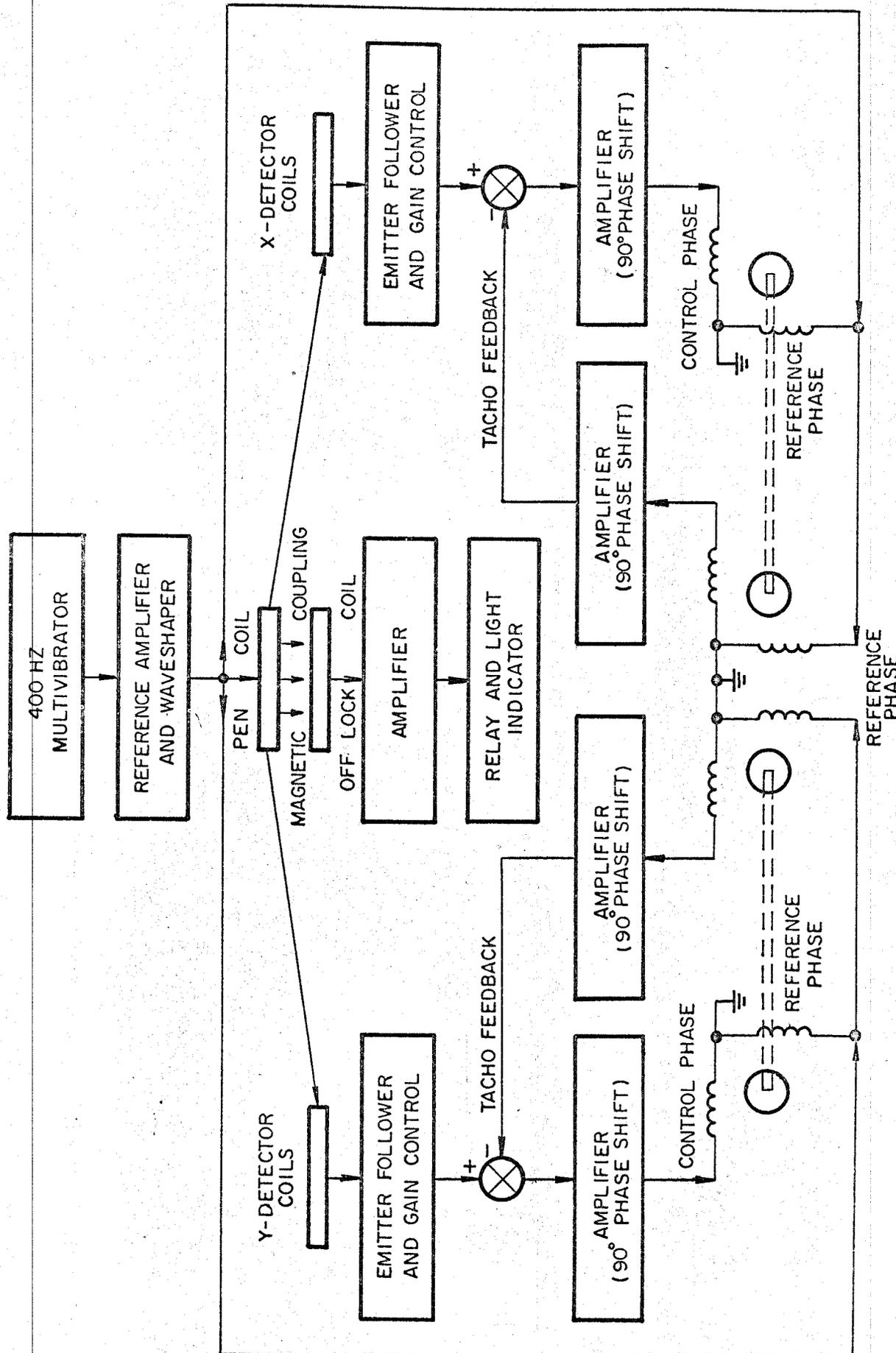


FIG-2.1.3 BLOCK DIAGRAM OF PENCIL FOLLOWER ELECTRONICS.

a 400Hz multivibrator and amplifier. The off-lock pickup coil is located midway between the two X and Y axes coils. The induced voltage approaches a maximum when the pen-coil is centered above the off-lock coil. Deviation from this central position energizes a relay, turning on an indicator light to show that the off-lock condition exists.

The output of the tachometer is reduced in magnitude and phase shifted 90° so that it can be subtracted (i.e. 180° out of phase) from the error voltage. The resultant error signal is amplified and phase shifted to be 90° out of phase with the reference voltage and used to energize the servo motors accordingly.

Carriage movement is accomplished via drive wires wound on the motor pulleys. A straight wire drive is used on the X axis and a differential pulley and drive wire arrangement is used on the Y axis. Shaft encoders are also connected to the drive wires by means of drive pulleys, and indicate the X and Y location of the carriage.

The mechanical shaft encoders consist of a disc containing sixteen tracks of conducting and non-conducting segments. This rotates over a stationary disc which contains sixteen brushes, such that each angular position corresponds to a unique combination of closed and open switches on the sixteen brush outputs. The binary "1's" and "0's" generate a Cyclic Permuting Binary Code resulting in a digitized resolution of 0.004". There is a separate 16 bit encoder for each of the X and Y axes, and positional information is obtained by sampling these binary outputs.

The Pencil Follower has two modes of operation, point and line digitizing. When digitizing in the point mode, the operator

moves the stylus to the location he wishes to digitize, and signals the computer for the transfer of the X-Y co-ordinates from the shaft encoders by pressing a footswitch. Continuous digitizing of line information requires the line mode of operation. The digital co-ordinates are transferred at a rate controlled by an external clock. Transfer is initiated by pressing the footswitch and discontinued upon its release.

The transfer of co-ordinates is accomplished via the interface. The interface couples the shaft encoder output of the Pencil Follower to the computer so that the continuous (line) and point mode of operation is available. The computer is interrupted when the footswitch is depressed or the external clock generates a pulse. The computer then goes into a subroutine to service the digitizer and the co-ordinates are transferred, via the interface, under the program control. For more details on the method used to accomplish this, see Appendix A.

2.2 Characteristics

2.2.1 A suitable mathematic model

The Pencil Follower is basically a position sensing a.c. servo system with velocity feedback. A complete block diagram representation for one axis of movement is indicated in Figure 2.2.1. This is self explanatory except for the Distance to Voltage Conversion block. This block represents the error voltage induced in the detector coils as a function of the distance between the pen-coil and the null point of the appropriate two detector coil configuration. The unity feedback is obvious from the knowledge that the detector coils are mounted

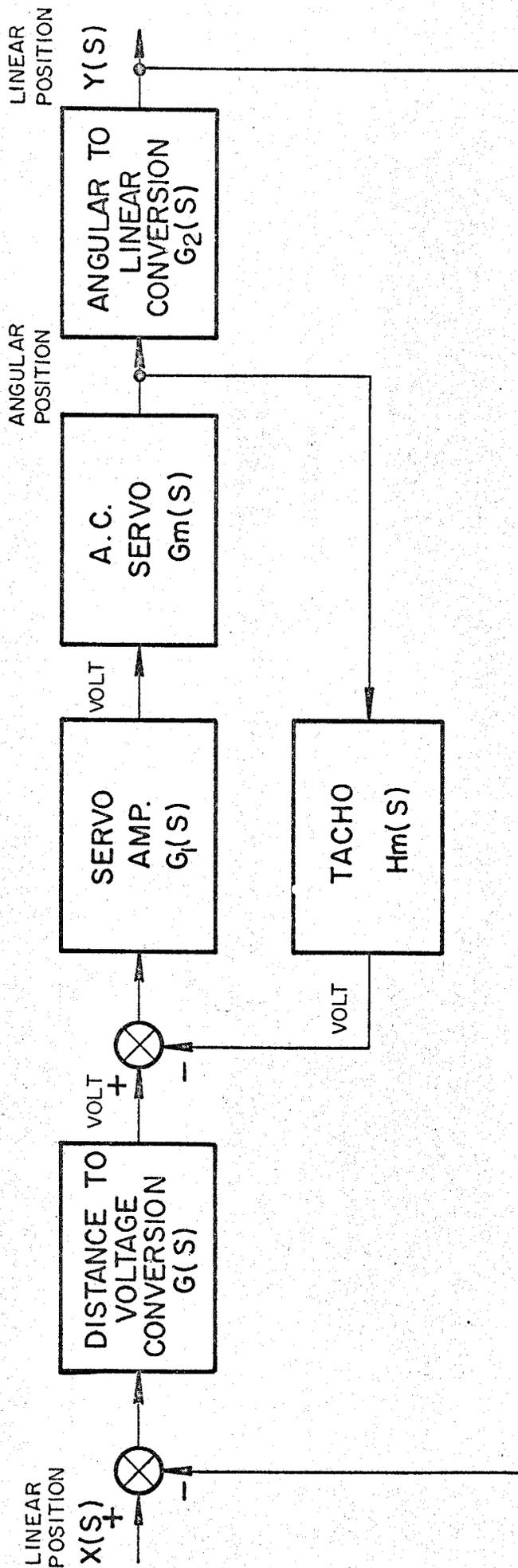


FIG-2.2.1 BLOCK DIAGRAM OF A ONE AXIS MODEL

directly on the follower, which is driven by the servo motor.

The field equations necessary to determine the transfer function $G(s)$, are complicated by the high permeability cores of the detector coils. These cores distort the magnetic field generated by the pen-coil and make theoretical calculations extremely complex. Thus, the transfer characteristics were obtained experimentally using the apparatus described in Appendix B.

The voltage induced in the detector coils is a function of both X and Y axis position. This is illustrated in the constant voltage contours shown in Figure 2.2.2 and Figure 2.2.3. Figures 2.2.4 and 2.2.5 indicate the voltage induced in the X axis coils and the Y axis coils along the X and Y Axes, respectively. To represent this system by a linear control system, the transfer function $G(s)$ was linearized as shown in Figure 2.2.4. This representation is useful over this distance because pen velocities which result in the follower lag increasing beyond 1.0", will cause the lag to increase rapidly until the follower halts.

The expressions for the transfer functions of the block in Figure 2.2.1 are:

$$G(s) = \beta_1 K_3 \dots \dots \dots (1)$$

$$G_1(s) = K_1 \dots \dots \dots (2)$$

$$G_m(s) = \frac{K_m}{s(1 + s\tau_m)} \dots \dots \dots (3)$$

$$H_m(s) = s\beta_2 K_t \dots \dots \dots (4)$$

and $G_2(s) = K_2 \dots \dots \dots (5)$

Y-AXIS IN INCHES. CONSTANT VOLTAGE CURVES FOR X-AXIS COILS.

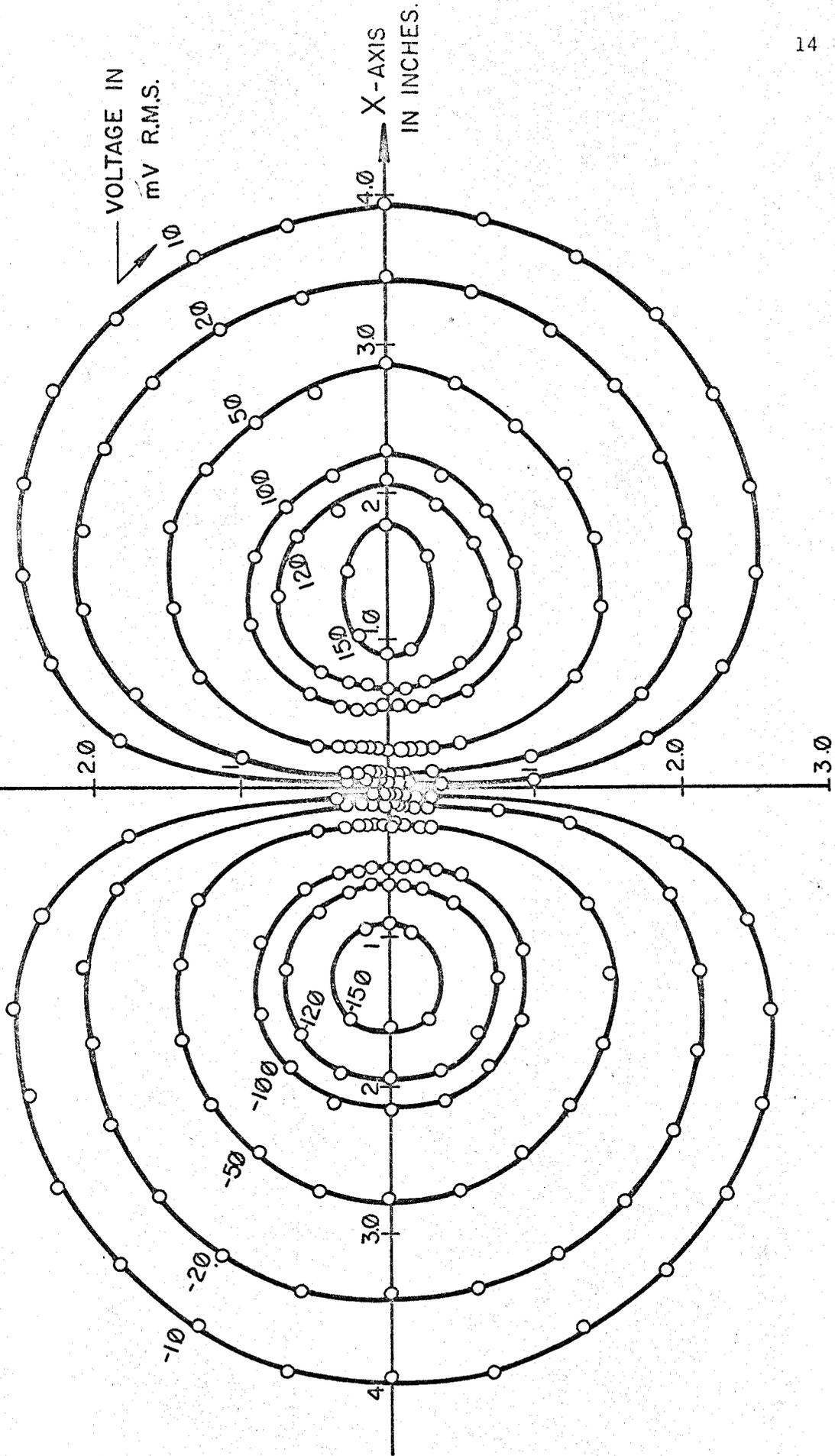


FIG.-2.2.2 CONSTANT VOLTAGE CONTOURS (X-AXIS)

CONSTANT VOLTAGE CURVES
FOR Y-AXIS COILS.

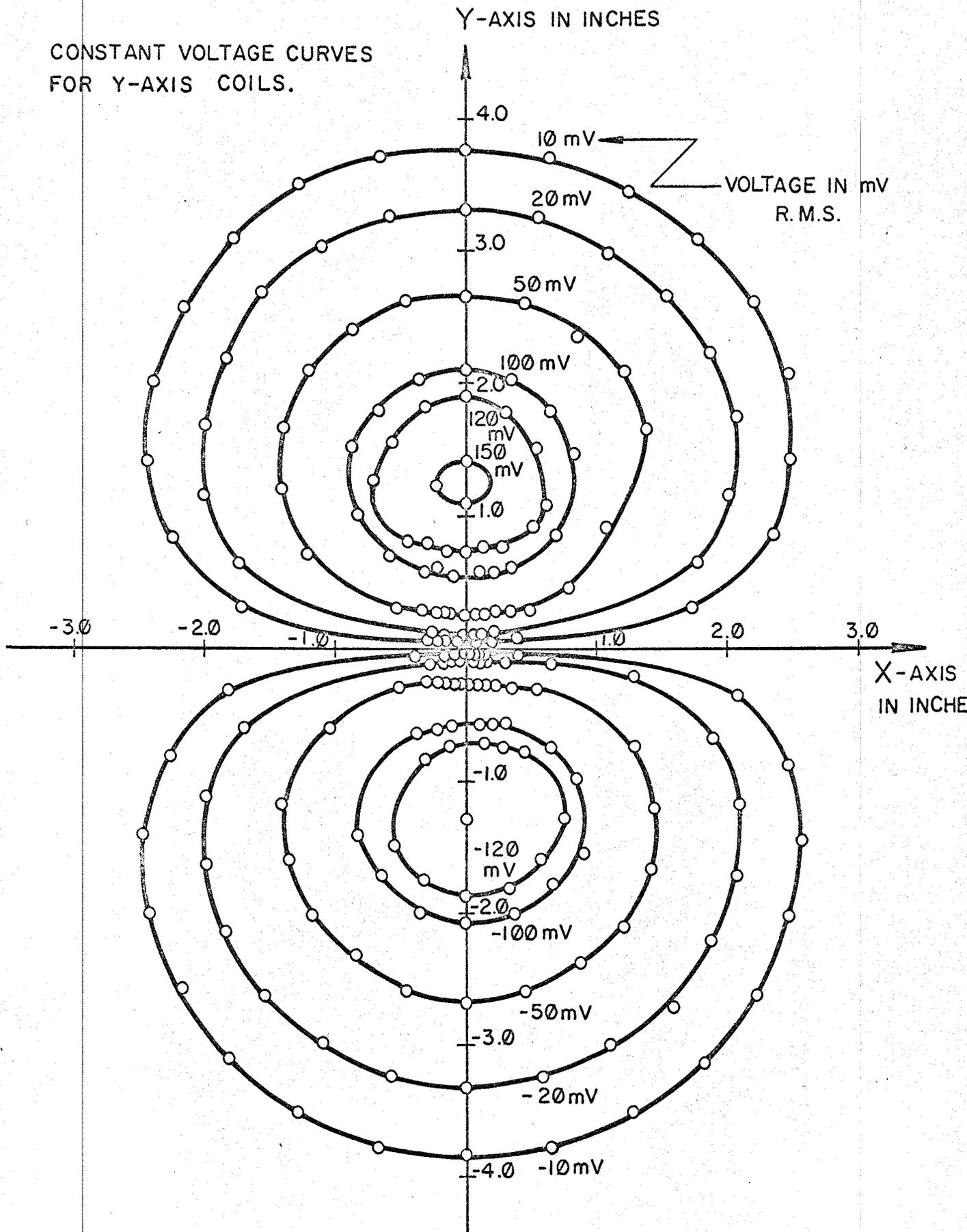


FIG.-2.23 CONSTANT VOLTAGE CONTOURS (Y-AXIS)

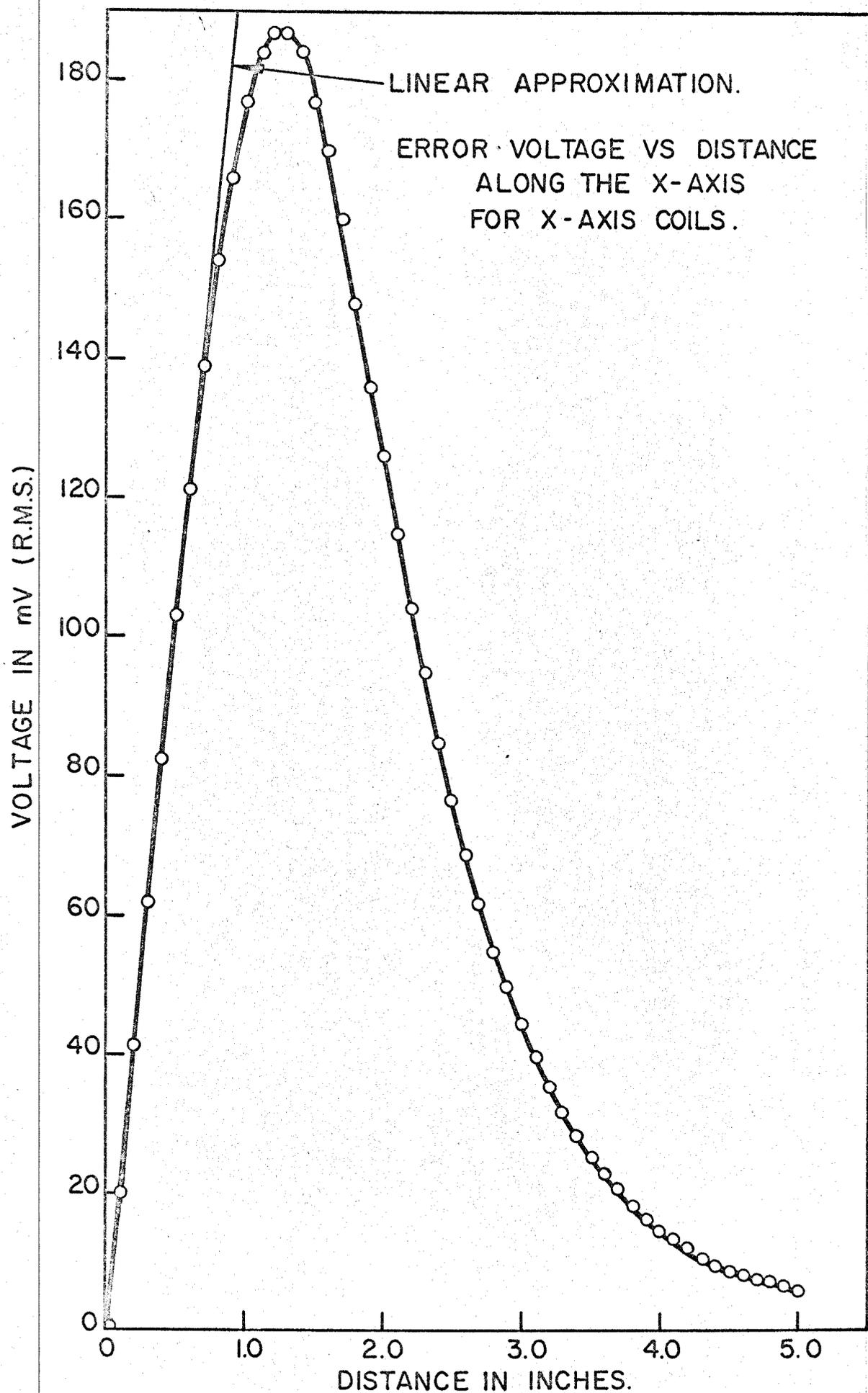


FIG-2.24 ERROR VOLTAGE CHARACTERISTICS (X-AXIS)

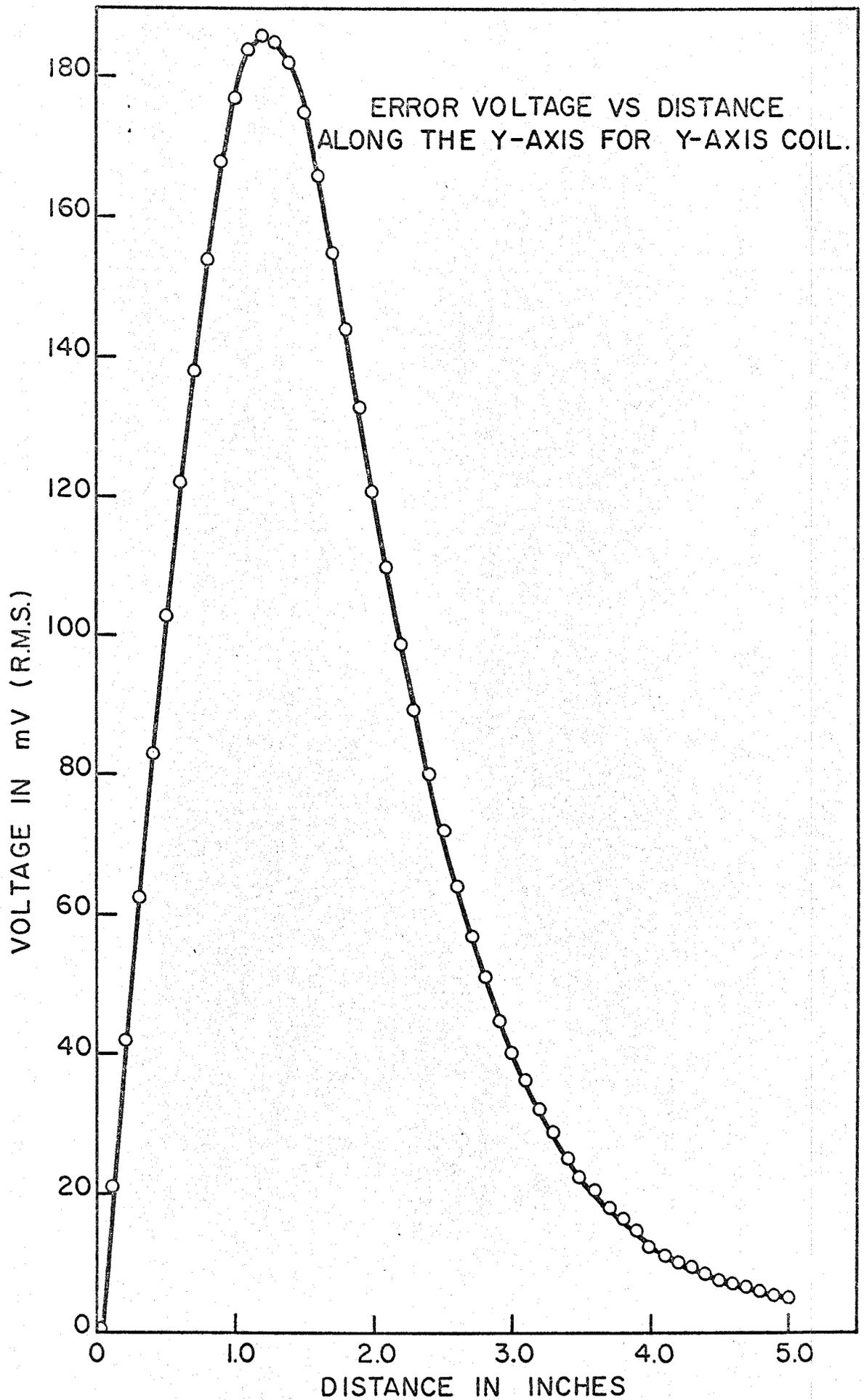


FIG.-2.2.5 ERROR VOLTAGE CHARACTERISTICS (Y-AXIS)

where:

- β_1 - detector coil amplifier gain
 - β_2 - tacho feedback amplifier gain
 - K_1 - servo amplifier gain
 - K_2 - angular to lineal conversion constant
 - K_3 - slope of error voltage vs distance (linear approximation)
 - K_m - motor gain constant
 - K_t - tacho sensitivity constant
- and τ_m - motor time constant.

The closed loop transfer function for this system becomes,
(see Appendix F).

$$\frac{Y(s)}{X(s)} = \left(\frac{\beta_1 K_1 K_2 K_3 K_m}{\tau_m} \right) \frac{1}{s^2 + \left(\frac{1 + \beta_2 K_1 K_m K_t}{\tau_m} \right) s + \frac{\beta_1 K_1 K_2 K_3 K_m}{\tau_m}} \quad \dots (6)$$

This is of the form,

$$\frac{Y(s)}{X(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2} \dots \dots \dots (7)$$

where

$$\omega_n^2 = \frac{\beta_1 K_1 K_2 K_3 K_m}{\tau_m} \dots \dots \dots (8)$$

and

$$\delta = \frac{1 + \beta_2 K_1 K_m K_t}{2 \sqrt{\beta_1 K_1 K_2 K_3 K_m \tau_m}} \dots \dots \dots (9)$$

Equation (7) represents the closed-loop transfer function of a second-order system where δ is the damping ratio and ω_n is the undamped natural frequency. (41) Measurements on the system established the following results:

$$\beta_1 = 0.2$$

$$\beta_2 = 1.51 \times 10^{-3}$$

$$K_1 = 1,000$$

$$K_2 = 3.103 \times 10^{-2} \text{ in/rad} \quad (\text{see Appendix D})$$

$$K_3 = 0.2 \text{ volt/in.} \quad (\text{see Figure 2.2.4})$$

$$K_m = 47.2 \text{ rad/volt-sec.} \quad (\text{see Appendices D and E})$$

$$K_t = 2.96 \times 10^{-2} \text{ volt/rad/sec.} \quad (\text{Reference 42})$$

and $\tau_m = 0.1075 \text{ sec.} \quad (\text{see Appendices D and E}).$

Thus, from equation (8)

$$\omega_n = 23.32 \text{ 1/sec}$$

and

$$\delta = 0.62$$

Equations (7) and (9) establish the basis for a mathematical model representing the Pencil Follower. The following section presents the experimental results of tests performed and the theoretical results using the above model.

2.2.2 Response

The specifications provided by d-mac for Pencil Follower Type PF10000 are given in Table 1. (38) These are not sufficient and must be elaborated upon to determine the resolution and the accuracy of the digitized output for the conditions encountered while digitizing cartographic data.

TABLE 1

Pencil Follower Specifications

Reading Area	- 18" x 40"
Resolution of Position Readout	- 0.1mm (0.004")
Servo Positioning Accuracy	- ± 0.1 mm (± 0.004 ")
Servo Dynamic Accuracy	- ± 0.2 mm (± 0.008 ") up to 2.5 cm/sec (1 in/sec)
	- ± 1.0 mm (± 0.04 ") up to 10 cm/sec (4 in/sec)
Maximum Speed of Movement	- 25 cm/sec (10 in/sec)
Maximum Lock-on Distance	- 10 cm (4")

Firstly, the operator's line following speed varies with the complexity of the map detail, resulting in limited confidence in the accuracy of the digitized output. This confidence level is established by the lag which exists between the stylus and the servofollower at different stylus speeds, and must be accurately determined. Secondly, the follower response to sudden, jerky stylus movements encountered when moving from point A to point B, must be determined to establish the settling time of the servofollower. This becomes important in digitizing point information where the stylus movements are not continuous.

Two other considerations that require investigation are the difference between electrical and mechanical (optical) center of the stylus, and the effect of follower stiction on the servo positioning accuracy. The difference between mechanical and electrical center be-

comes important in the rotation of the stylus for different operator positions and stiction is a problem because the error voltage, and thus the torque produced by the servo motors, is a minimum at the null point.

The follower lag, as a function of stylus velocity, was measured along the X axis using the test apparatus described in Appendix B and Program Number 1 described in Appendix C. Both axes of operation are identical except that the X axis is longer, allowing more time for the test apparatus to reach a constant velocity; thus providing superior test results. The results of these tests are indicated in Figure 2.2.6.

The expression for the steady state error (i.e. lag) of a second order system to a ramp (i.e. velocity) input is given by equation (10). (41)

$$E = \frac{2\delta}{\omega_n} V \dots \dots \dots (10)$$

Where:

E = lag

V = slope of the ramp input (i.e. velocity)

For this system, this expression becomes

$$E = 0.532V$$

which is also plotted in Figure 2.2.6.

The experimental results compare favorably with those predicted from the model, with the important results that: (1) lag increases linearly with velocity, and (2) the lag exceeds 0.004" for pen velocities greater than 0.1 inches per second. Thus, the operator

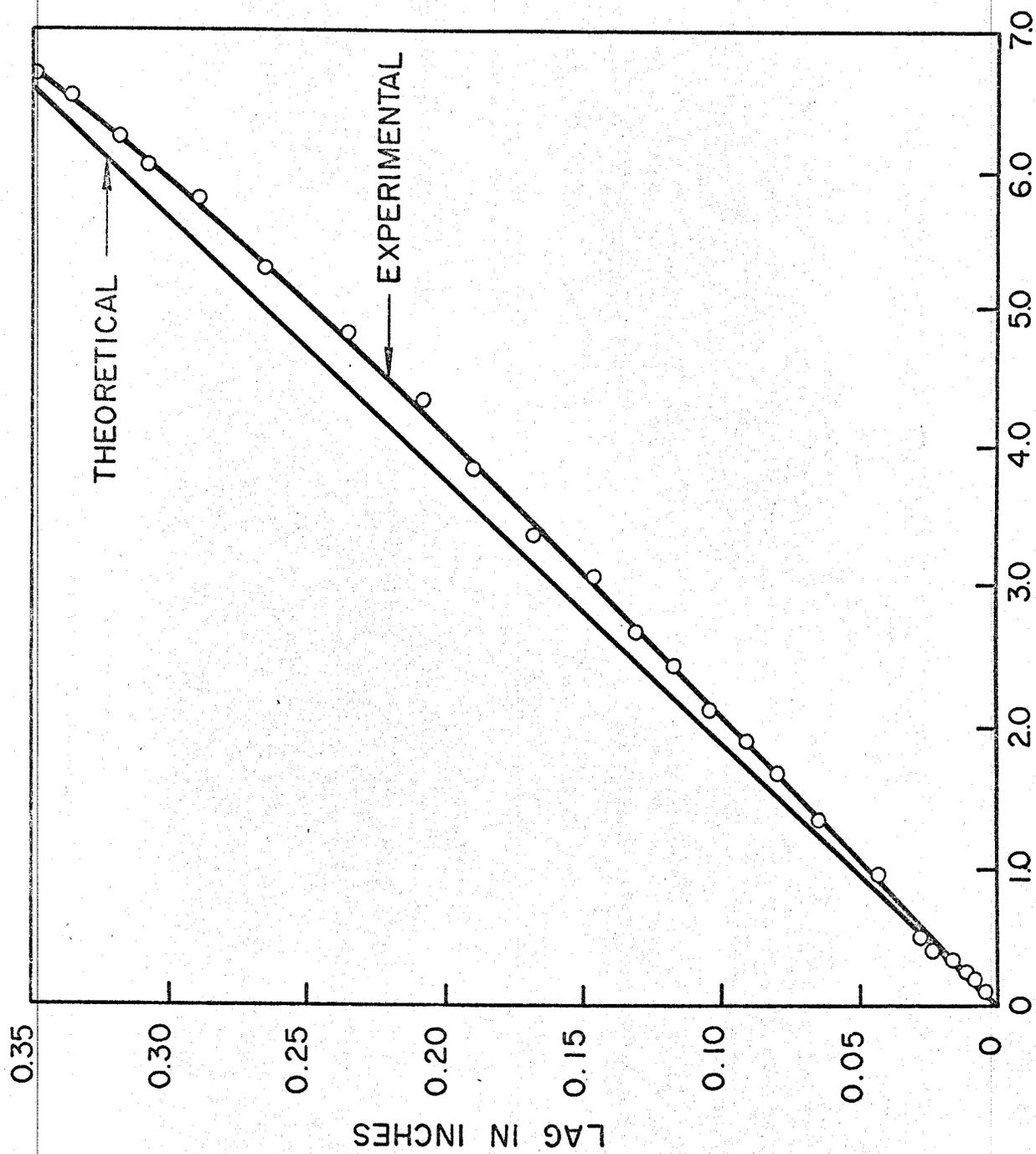


FIG.-2.2.6 LAG CHARACTERISTICS

is restricted to maximum speeds of 0.1 inches per second to obtain digital results within the desired resolution of $\pm 0.004''$.

The second test was to measure the settling time when the stylus was moved from point A to point B. Although step changes in stylus position would not occur in practice, these represent the most severe type of jerky perturbation and are a true test of system performance. A step movement in pen position was simulated by constricting the movement of the follower, moving the stylus the desired distance, and then releasing the follower. The initial movement of the follower operated a micro switch which enabled an external clock, transferring the co-ordinates at 10 millisecond intervals. Program Number 2 (Appendix C) then processed these results. The measurements were made along the X axis as the X axis trolley weighed more than the Y axis carriage, resulting in more system inertia and the worst case condition. The results are plotted on the graph of Figure 2.2.7.

The step response of such a second order system is given by equation (11). (41)

$$y(t) = A \left[1 - \frac{e^{-\delta \omega_n t}}{\sqrt{1 - \delta^2}} \sin \left(\omega_n (\sqrt{1 - \delta^2}) t + \tan^{-1} \frac{\sqrt{1 - \delta^2}}{\delta} \right) \right] \dots (11)$$

This becomes,

$$y(t) = A [1 - 1.275 e^{-14.44t} \sin [18.3t + 0.902]]$$

which is plotted for "A" equal to 1.0" in Figure 2.2.7.

The experimental results indicate that the system exhibits some higher order effects. These are due to the presence of friction, wire stretch, the non-linear characteristics of the a.c. servo motor, and the distance to voltage conversion transfer function $G(s)$.

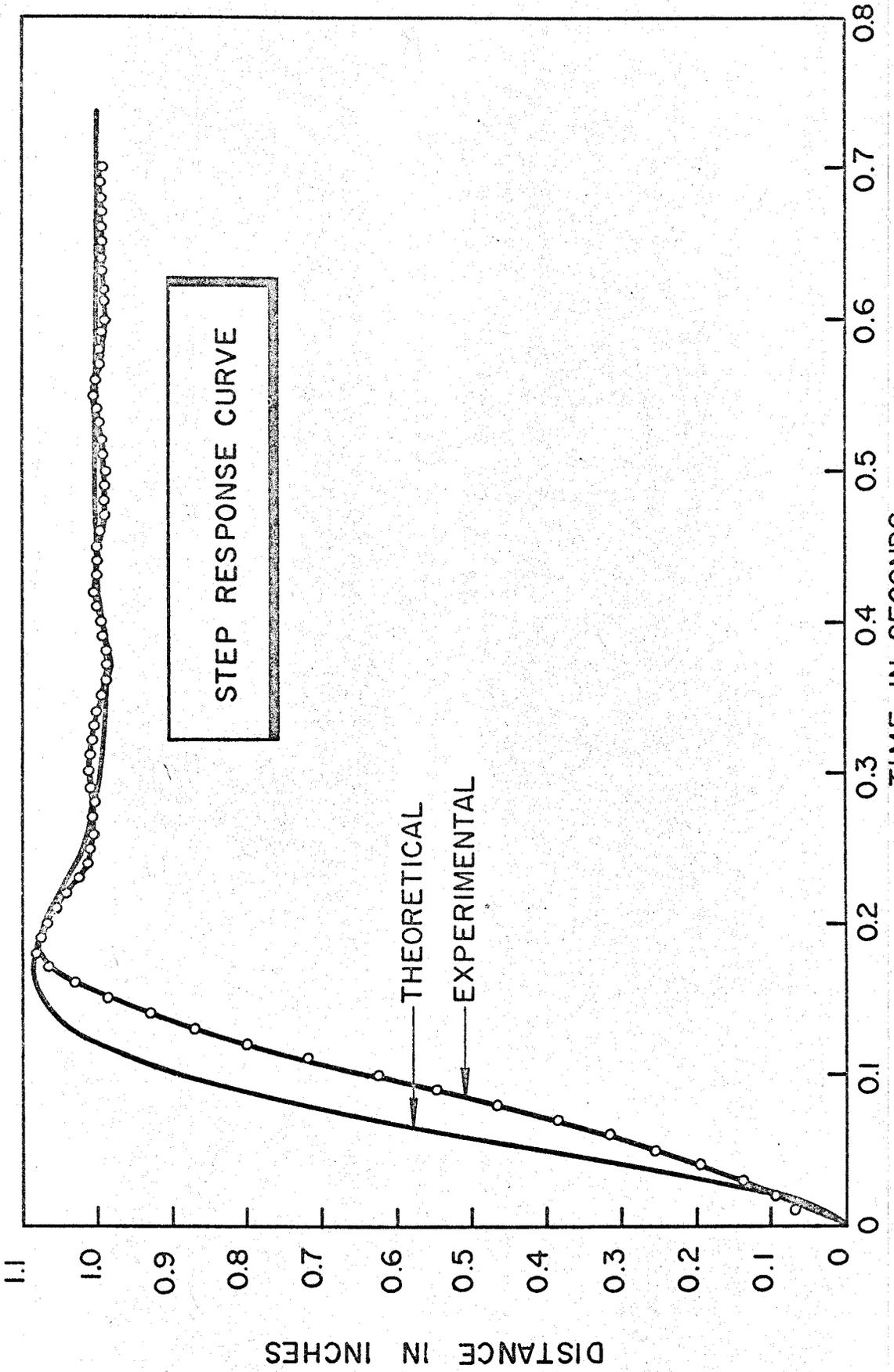


FIG-2.27 STEP RESPONSE

The important result, with regard to digitizing, is the settling time, which is the length of time before the oscillations are less than ± 0.004 ". The plot indicates that 0.7 seconds is required before the follower has stabilized to its final position for a step change of 1.0" in the pen position. This means that no co-ordinates should be transferred to the computer during this 0.7 second interval if an accurate final position is desired. For sudden stylus movements up to 1.0 inch this does not present a limitation, because the time required for the operator to be absolutely certain he is at the correct location and then depress the footswitch to initiate co-ordinate transfer would exceed 0.7 seconds. However, for distances greater than this, the non-linearities of the distance to voltage transfer function $G(s)$ becomes predominant and this increases the time required for the follower to reach the final position. Thus, for sudden changes in pen position between 1.0 inches and 4.0 inches (the maximum "lock-on" distance) the operator must exercise judgment upon the time at which he initiates co-ordinate transfer.

The effect of pen orientation on the follower output co-ordinates becomes important in the process of digitizing line and point data when one considers that the operator does not always hold the stylus at the same orientation to the X and Y axis, but maintains it at positions most comfortable for his operation.

The mechanical center of the high accuracy pen (stylus) was constricted so that the stylus would rotate but not be displaced along the X and Y axes. The switch option of Program Number 1 (Appendix C) was used to read in the co-ordinates for every ten degrees of stylus

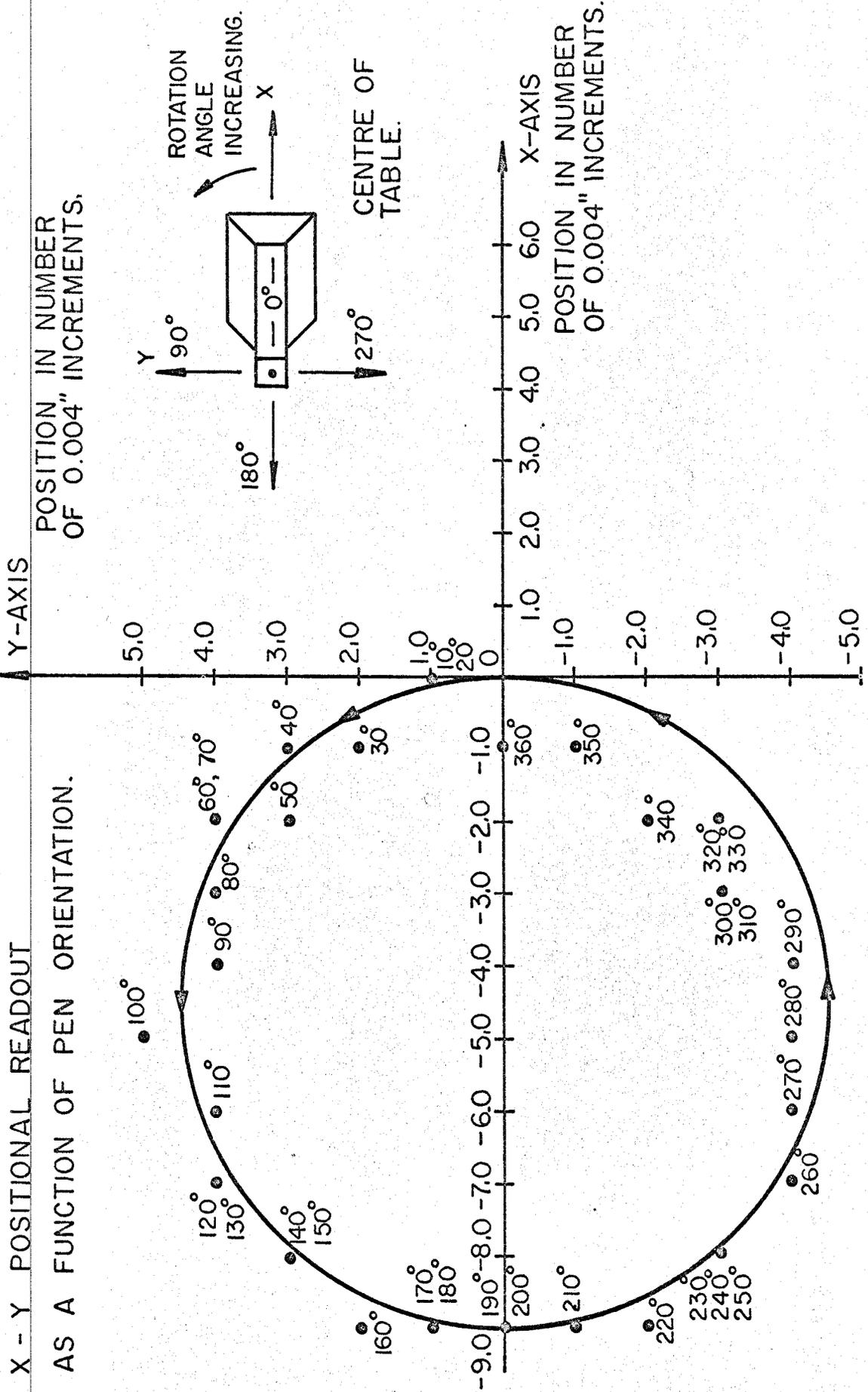


FIG- 2.2.8 RESULTS OF PEN ORIENTATION TESTS .

rotation. Typical results are shown in Figure 2.2.8. They indicate an approximate circular form of error of 0.036" (9 increments) in diameter. This would appear to indicate that the tip of the pen was bent off center, but adjusting this did not improve the results. It was experimentally verified, by tilting the plane of the pen, that the plane of the pen coil was not parallel to the operating surface, thus generating a different null point for various pen rotations. Although the high permeability tip was supposed to concentrate the magnetic field at the tip, this effect was reduced by the fact that the tip was glued into the ferrite core of the pen coil. The pen tip then distorts the field rather than generating it, reducing the effect of the magnetic field concentration.

From the digitizing point of view, this effect tends to restrict the operator to maintaining one pen angle for all situations, to obtain the desired resolution. Thus, his operation and speed are further restricted by keeping to this one pen orientation.

The fourth investigation was to measure the positioning accuracy and repeatability. This was accomplished with the test apparatus and the switch option of Program Number 1 described in Appendices B and C, respectively. A high ratio gear box was used so that the pen could be moved in very small increments of 0.001". The co-ordinates were obtained for each increment of movement and a typical set of results is shown in Figure 2.2.9 for 0.020" of movement in the positive X direction and back in the negative X direction.

The results indicate that the encoder output changed approximately every four 0.001" movements, but it was observed, on

different locations on the table, that the movement was a maximum of thirteen 0.001" movements. The repeatability, indicated by the hysteresis effect, was approximately 0.004" but also varied up to 0.012" at different parts of the table. Measurements were not taken for every square inch of the reading surface but the general result was that although a positioning accuracy of ± 0.004 " was attained over most of the table, this could approach ± 0.012 " at some specialized locations. This latter effect is the result of non uniform frictional characteristics over following surface and is explained in more detail under the next section on limitations.

The maximum "lock-on" distance was found to be approximately four inches. This is a measurement of the maximum distance the pen can be from the follower and still provide sufficient magnitude of signal to cause the follower to approach the pen. The follower would "home" in from a maximum distance of six inches if the operator was willing to wait about one minute.

These results obtained on lag, response, pen orientation, positioning accuracy and maximum lock-on distance present a better understanding of how this device will perform under the conditions encountered during the digitizing of map information. More important, the results indicate that there exist device limitations. The next section is devoted to the discussion of how these limitations affect the digitizing of complex cartographic data and provides the introduction to the next topic on the investigation of a digital pencil follower.

2.2.3 Device limitations

The experimental results indicate some limiting features of the Pencil Follower. Firstly, the lag between the stylus and the follower exceeds 0.004" for stylus velocities greater than 0.1 inches per second. This is not a limitation when digitizing highly complex cartographic data with many irregular features, as the operator must follow at slow speeds to follow the line accurately. However, when relatively regular line data is encountered the speed of digitizing can be increased beyond 0.1 inches per second, and the speed limitation becomes serious. The most critical aspect of this is that the operator is not aware of his exact digitizing speed and thus never knows when the resulting data becomes inaccurate.

The maximum lock-on distance and the response to jerky motions of the pen present a serious problem in the digitizing of point data. To digitize randomly spaced points it is necessary for the operator to move slow enough from one point to another such that the follower will maintain the lock-on condition. If he fails to do this or lifts the stylus off the paper surface while moving to the next point of interest, he must return to the last digitized point and "pick-up" the follower again. This is both an inconvenience and a waste of time, resulting in poor efficiency. Lock-on has been achieved over the whole table on more recent models of the Pencil Follower; however this particular one was not the improved version. (12)

The non uniform frictional characteristics of the mechanical follower coupled with the very low torque produced by the servo motor as the follower approaches the null point, result in irregular follower

positioning over the useful digitizing surface and inconsistent accuracy characteristics. The follower position at which such stiction problems occur is difficult to predict, thus reducing the confidence in the digitized data. Although the resolution of $\pm 0.004''$ is generally maintained, the effects of stiction can distort the digitized data.

The problem of pen orientation, although very serious, can be rectified by careful adjustment of the pen tip and the plane of the pen-coil. The experimental results obtained for pen orientation were included to indicate that the pen, directly obtained from the manufacturer, did exhibit orientation problems. However, it was rather simple to eliminate this difficulty once it was determined that a problem did exist.

3. THE INCREMENTAL PENCIL FOLLOWER

3.1 General Concept

The previous sections have indicated some limiting features of the servo driven Pencil Follower. Although the lock-on distance and positioning sensitivity can be improved by increasing the forward loop gain of the system, stability characteristics limit this. Also, some added form of feedback or compensation is required to reduce the velocity lag to a minimum. The very nature of the input and the non-linear characteristics of $G(s)$ make this difficult to synthesize and implement. The problem of stiction is difficult to predict and eliminate with the low torque characteristics near the null point.

To overcome some of the limitations, a stepper motor drive was proposed. The advantages of a stepper motor drive over the servo system are high torque characteristics, predictable lag characteristics, elimination of shaft encoders, and improved lock-on distance without stability problems. These will become more evident with the following description and discussion.

The basis for the operation of the Incremental Pencil Follower is the stepper motor. This is a digital device which converts an electrical pulse into distinct angular rotation, with a one to one correspondence between the number of input pulses and the number of angular increments. The concept of stepper motors is not new, but recent advances improving the operating speed and the torque characteristics have increased the number of applications. Their high starting and holding torque are very useful properties to reduce the effects of stiction.

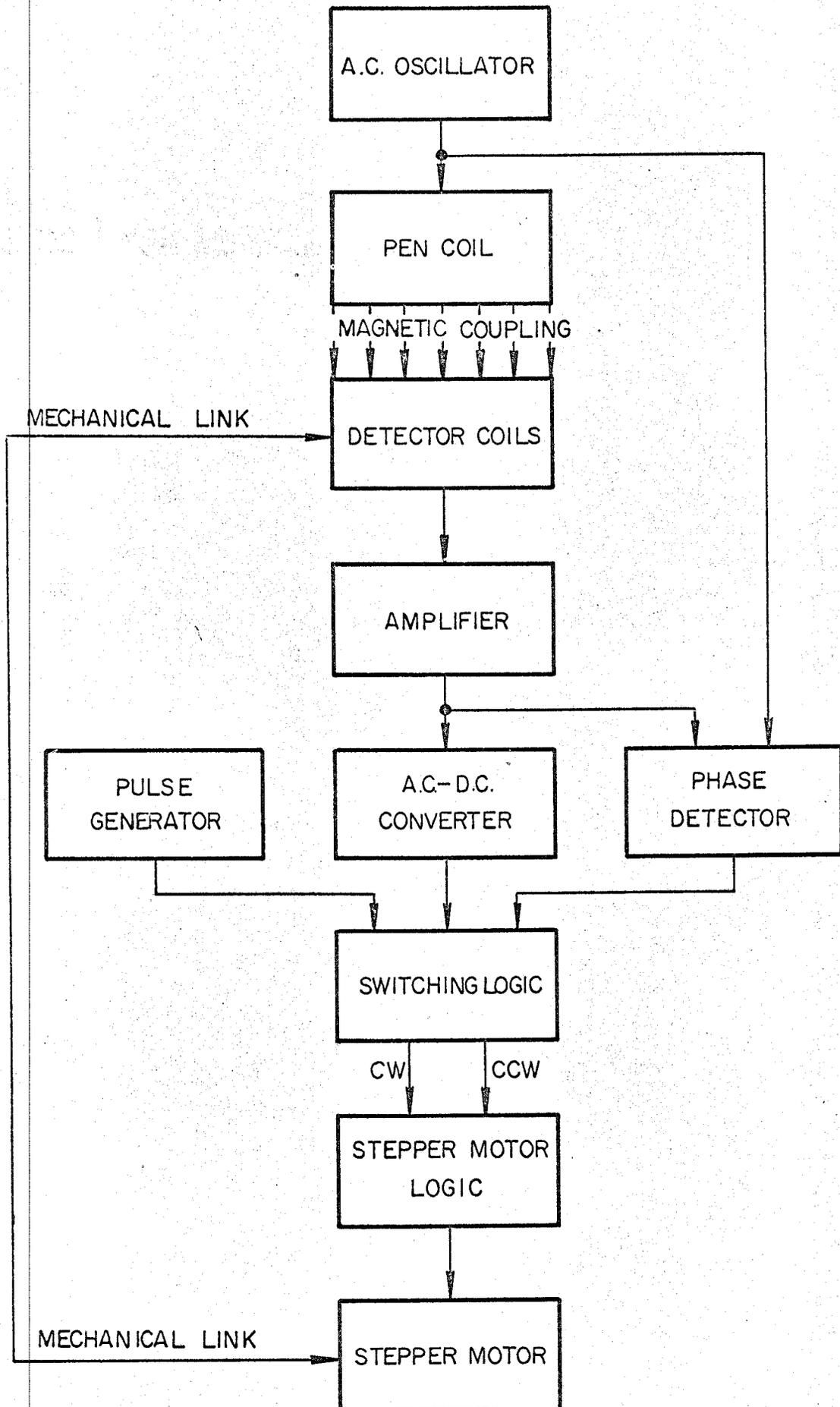


FIG.-3I.1 BLOCK DIAGRAM OF THE INCREMENTAL PENCIL FOLLOWER FOR ONE AXIS.

Figure 3.1.1 gives a block diagram of a suitable system employing stepper motors as the driving element. The operation is similar to the servo driven unit with the pen-coil generating a symmetrical a.c. magnetic field. This induces a voltage in the detector (pick-up) coils which is amplified and phase compared to the pen-coil voltage. The amplitude and phase information is used to activate the switching logic to steer the pulses to the correct terminals of the stepping motor input. The stepper motor then drives the trolley containing the detector coils to the null point at which time the output of the ac-dc converter is zero, preventing any more pulses from activating the motor. With this understanding of the basic operation, some general requirements for the various blocks in Figure 3.1.1 will be outlined.

Firstly, the pen and detector coil arrangement must have characteristics similar to those of the d-mac Pencil Follower; that is, one null point with a constant phase on either side of this null point and a distinct phase shift across it. The induced voltage must be above the noise levels to extract useful phase information up to distances of 40" between the pen and detector coils.

The amplifier must have sufficient gain to provide a useful signal to operate the phase detector over the complete operating area, thus providing lock-on over the entire digitizing surface. It must be carefully designed knowing the pen, detector coil characteristics, and the ac-dc converter sensitivity, to provide a 0.004" dead band at the null point while still providing sufficient gain to obtain a

large lock-on distance. The ac-dc converter and the phase detector must be capable of providing logic levels to switch the pulses to the correct motor inputs. These will be two state devices to indicate when and where the pulses are to be directed.

The logic switching circuitry will provide solid state switches, which are activated by the logic levels from the ac-dc converter and the phase detector, to direct the pulses from the pulse generator to the correct input terminal of the stepper motor logic. The stepper motor logic then converts the incoming pulses to switched phase sequences on the motor to obtain the angular rotation in the correct direction. The motors must be capable of moving the carriage containing the detector coils in 0.004" increments at high stepping speeds, via some drive mechanism.

With this brief explanation of the components necessary for the operation of an Incremental Pencil Follower, the advantages of this system become more evident. Firstly, the high starting torque of the stepping motor allows carriage movement even for the unpredictable frictional characteristics due to stiction. The digital nature of this motor eliminates the need for shaft encoders, as a binary up-down counter can be used to count the pulses to the stepper motor terminals. The state of this counter at any time provides the position of the follower in the number of 0.004" increments from the reference (zero increments).

Once lock-on has initially been achieved, the follower lag will be a maximum of one increment of 0.004" for pen velocities up to the maximum following speed determined by the pulse generator controlling the stepping rate of the stepper motor. Lock-on can be achieved over

the complete table surface by using only the phase information from the pick-up coils. This can be achieved without sacrificing stability or sensitivity as explained in a later section on the actual design.

3.2 Basic System

3.2.1 Stepper motor and drive

Stepper motors basically fall into two classes: phase-pulsed synchronous, and solenoid-ratchet.⁽⁴³⁾ The former are phase synchronous motors which have special pole piece arrangements so that stepping is accomplished with the progressive switching of two or more phases. Steppers in the second class use electromagnetic actuation of a solenoid core or armature to mechanically produce the shaft rotation. In general, the phased-pulsed synchronous stepper motors have fewer moving parts, longer life, and faster response. Also, their high starting and holding torque, and high stepping rates make them more suitable for this specific application.

The characteristics of the Slo-Syn HS-25 stepper motor and driving logic manufactured by Superior Electric Company (see Table I) are superior to those available from most other companies, providing high stepping rates for small step angles.⁽⁴⁴⁾ This motor can provide a maximum follower speed of two inches per second if each step corresponds to a 0.004" movement. However, it is necessary to observe the torque output of the motor for such speeds. If the velocity of the follower can be represented by

$$v = 4000t$$

during the first millisecond of a step, and then represented by

$$v = 8 - 4000t$$

TABLE I

HS-25 Characteristics

Step Angle	$1.8^{\circ} \pm 0.09^{\circ}$ noncumulative error
Maximum Stop-Start Stepping Rate	500 steps per second
Starting Torque	25 oz. in. (at 500 steps per second without error)
Holding Torque	58 oz. in.

during the second millisecond as shown in Figure 3.1.2, where v = follower velocity in inches per second and t = time in seconds, the maximum velocity is 4 inches per second at $t = 1.0$ milliseconds. This results in an average velocity of two inches per second. Since the maximum starting torque produced by the HS-25 at a stepping rate of 500 steps per second is 25 oz. in., the largest weight that can be accelerated is 0.6 lb. Although the displacement of a stepper motor is more complex with it exhibiting some higher order effects such as overshoot during a single step, this does give an indication of the maximum follower weight at this speed. This is a limitation which must be considered when designing the mechanical drive and follower assembly.

There are basically four possible methods of converting the angular rotation of the motor to lineal motion of the follower:

(i) wire drive, (ii) tape drive, (iii) lead screw drive, and (iv) rack and pinion drive. The wire drive has the advantage of keeping the follower weight down by mounting both the X and Y drive motors on the main frame instead of the follower. The disadvantage of this form of drive is the effects of wire stretch and slip on the pulleys due to high accelerations. For example, the 25 oz. in. of torque will produce approximately 0.06" of stretch in a 0.02" diameter steel wire 80" long. Although the follower motion would be complex due to the elastic characteristics of the wire, etc., this calculation does indicate a limiting feature of the wire drive.

The steel tape drive would not have the same stretch problem as the wire. However, it is not as flexible and would be more difficult to implement. Also, it requires careful design of the sprockets and

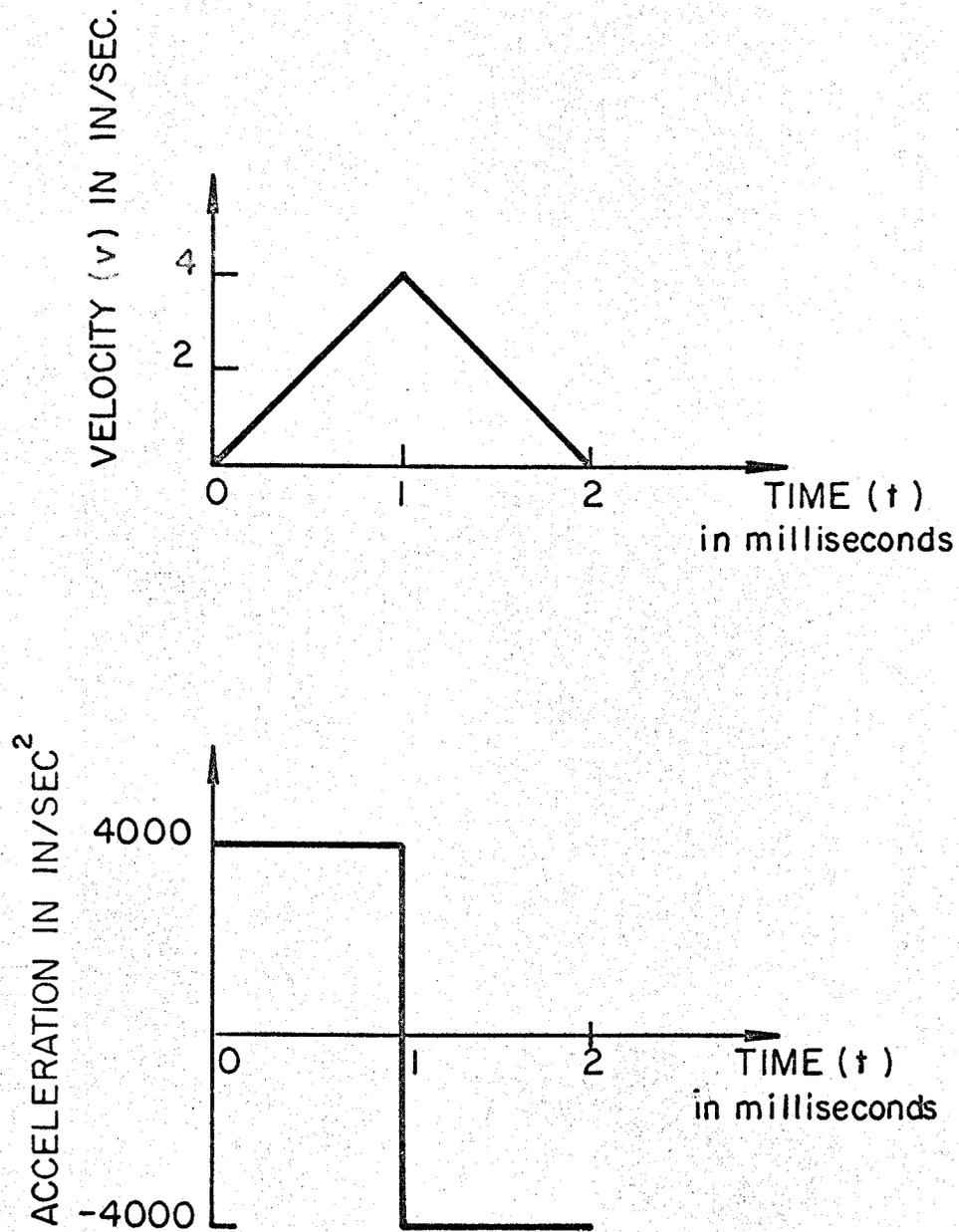


FIG.-3.1.2 FOLLOWER MOTION

the steel drive belt to minimize weight, wear, and slippage.

The leadscrew offers a very precise method of converting an angular rotation to a lineal movement. It would require mounting the Y axis motor on the trolley, resulting in an increase in trolley inertia. Also, this form of drive is somewhat more expensive than those previously mentioned.

The fourth possibility, the rack and pinion drive, could result in a very simple mechanical assembly. If the X and Y axes drive motors were mounted on the trolley, a 0.00393" lineal movement per 1.8° step could be accomplished with a 0.25" diameter pinion (16 teeth, 64 D.P.) mounted on the motor shaft and the appropriate 64 D.P. rack. This requires no reduction gear boxes, but has the disadvantage of increased follower weight because both the X and Y axes drive motors must be mounted on the trolley.

Of the four drive mechanisms described, the tape drive is probably the best for this application. Since the primary consideration in the design of this medium speed assembly is the weight of the follower, a differential tape drive on the Y axis can keep the weight to a minimum while reducing the problem of stretch encountered in wire drives. This is only a recommendation and the design of this mechanical drive requires much more research and consideration than described in this chapter.

3.2.2 Pen and detector coil configuration

As was indicated before, the coil arrangement must be such that there exists a point of zero induced voltage below the pen-coil

and a phase shift across this null. A two detector coil configuration using air core detector coils revealed some serious limiting features. Although a null point could be attained directly below the pen-coil, there exist other points of null and phase shift. This is indicated in Figure 3.2.1.

The magnetic flux lines are such that three null points exist between Regions I and II, Regions I and III, and Regions II and IV, respectively. There also exists a 180° phase shift in the induced voltage as the pickup coils pass from one region to the next. This undesirable feature was reduced on the d-mac Pencil Follower by strategically spacing the pickup coils and adding ferrite cores to distort the magnetic field so that Regions III and IV were beyond the pickup capabilities of the device. However, to attain lock-on over the complete working surface it is necessary to have a single null point

This was accomplished by the use of a single detector coil, per axis, in a plane orthogonal to that of the pen-coil, as opposed to the double coil arrangement on a parallel plane. Thus, the plane of the pen-coil is horizontal and the plane of the detector coil is vertical, with a point of zero induced voltage directly below the pen-coil as illustrated in Figure 3.2.2.

The pickup was improved by placing a ferrite core in the detector coil. The ferrite core distorts the magnetic field and increases the induced voltage. Also, tuning the pickup coil to the operating frequency provides increased output voltage.

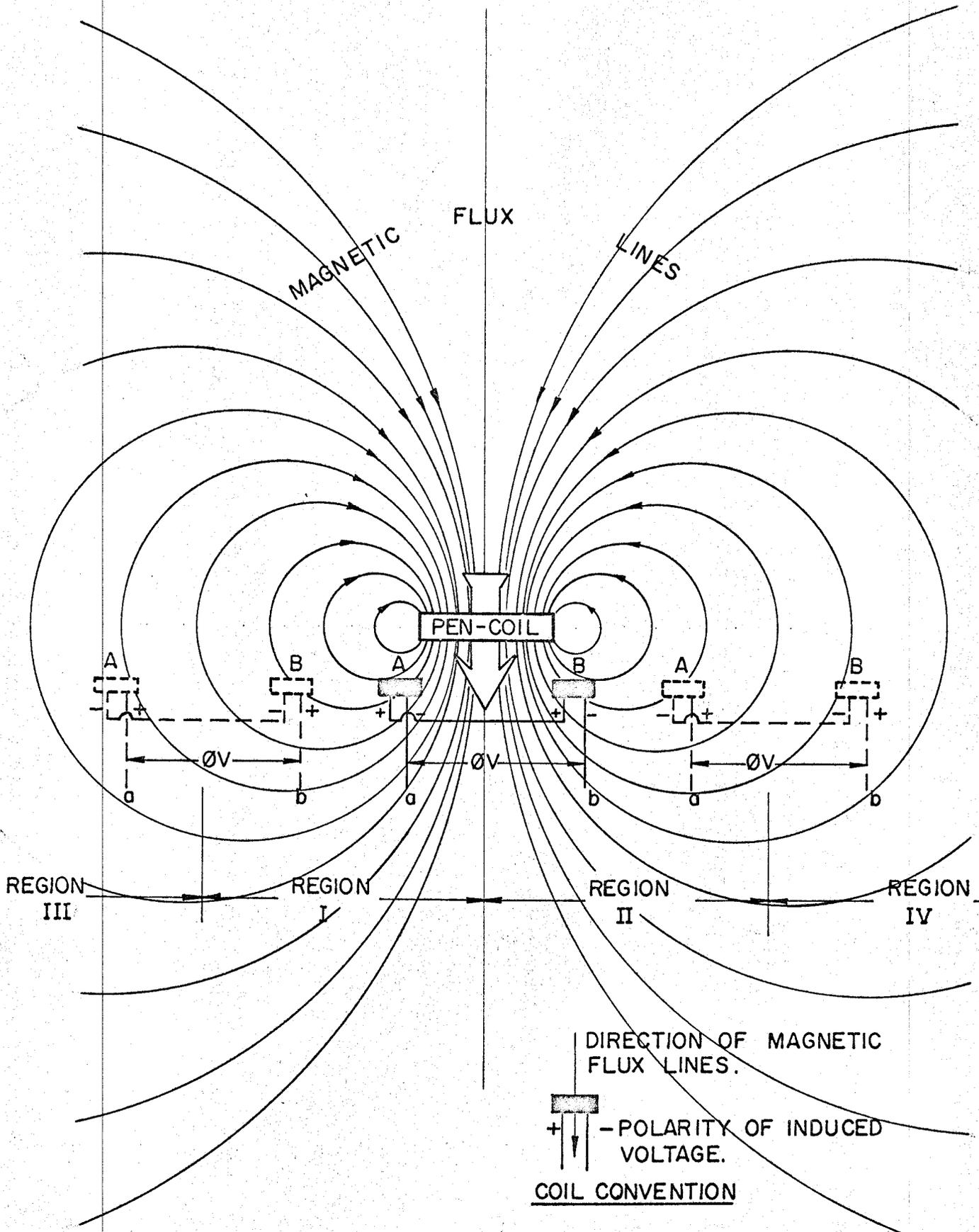


FIG-3.2.1 TWO DETECTOR COIL NULL POINTS

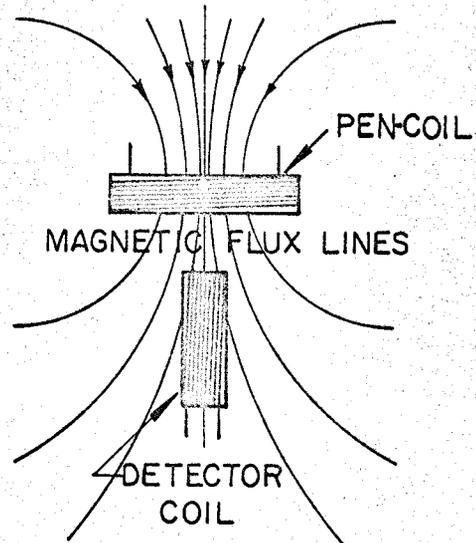
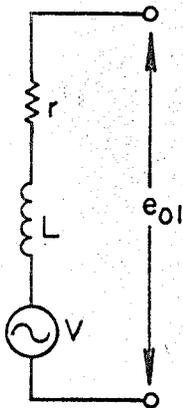
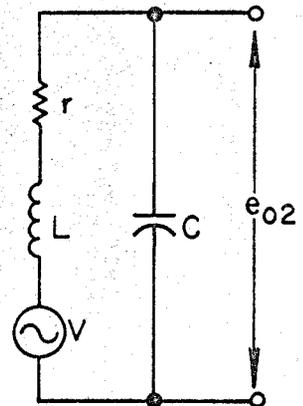


FIG.-3.2.2 IMPROVED COIL CONFIGURATION.



CASE I (SIMPLE)



CASE II (TUNED)

FIG.-3.2.3 EQUIVALENT CIRCUITS

This is easily explained with the use of Figure 3.2.3. "V" represents the voltage induced in the coil of inductance "L" and series resistance "r". Assuming that V is a sinusoidal voltage of fundamental frequency ω_1 , then $V = E_m \cos\omega_1 t$. The voltage e_{o1} is,

$$e_{o1} = E_m \cos\omega_1 t \quad \dots(1)$$

If "C" is chosen such that the equivalent circuit of Case II is series resonant, then

$$\omega_1 = \frac{1}{\sqrt{LC}} \quad \dots(2)$$

It is simple to show that,

$$e_{o2} = \frac{1}{\omega_1 r C} E_m \sin\omega_1 t \quad \dots(3)$$

Substituting equation (2) into equation (3),

$$e_{o2} = \frac{1}{r} \left(\frac{\sqrt{L}}{C}\right) E_m \sin\omega_1 t \quad \dots(4)$$

thus,

$$e_{o2} = \frac{1}{r} \left(\frac{\sqrt{L}}{C}\right) e_{o1} \angle -90^\circ \quad \text{at } \omega = \omega_1 \quad \dots(5)$$

The result of tuning this coil to the fundamental frequency of operation is to produce a 90° phase shift and an amplitude gain factor of $\frac{1}{r} \left(\frac{\sqrt{L}}{C}\right)$. Since "V" is either in phase or in antiphase (180°) with respect to the pen-coil voltage, e_{o2} will be phase shifted -90° or $+90^\circ$ with respect to the pen-coil voltage.

The choice in the size of the pen and detector coils is a trade off between bulk, position-voltage sensitivity, and ease in choosing suitable capacitors to tune the circuit. By substituting $C = \frac{1}{\omega_1^2 L}$ into equation (3) it is obvious that the higher the operating frequency, the larger the gain factor. The operating frequency was chosen to be 5KHz due to the frequency response of the low cost operation amplifiers used, as is explained more fully in a later section.

Figure 3.2.4 indicates the voltage vs distance characteristics for a 1" diameter pen coil consisting of 400 turns of #27 wire, and driven by a 15 volt (peak to peak), 5KHz sinusoidal signal, and a detector coil of 500 turns of #34 wire wound in a 1/2" diameter ferrite rod, tuned with a 1.0 μ f capacitor. The gain factor for this configuration ($\frac{1}{r} \sqrt{\frac{L}{C}}$) was approximately 10 at 5KHz.

The important characteristics of this graph are: (i) the relationship is approximately linear up to 1", (ii) the induced voltage is 1.25 mV (R.M.S.) at a distance 0.002" from null point, (iii) the peak induced voltage is 450 mV (R.M.S.), and (iv) the induced voltage drops to 0.2 mV(R.M.S.) at 40" from the null point. This last point indicates that if a dead zone of 0.004" is required the maximum lock-on distance will be approximately 18" if only the amplitude information from the pick-up coils is used. Since a lock-on distance of at least 40" is required, another coil was added to indicate a maximum voltage at the null-point of the detector coil. This zone coil is used to ensure that the phase information at voltages less than 1.25 mV, not in the dead zone, can be used to achieve the lock-on condition. This 1"

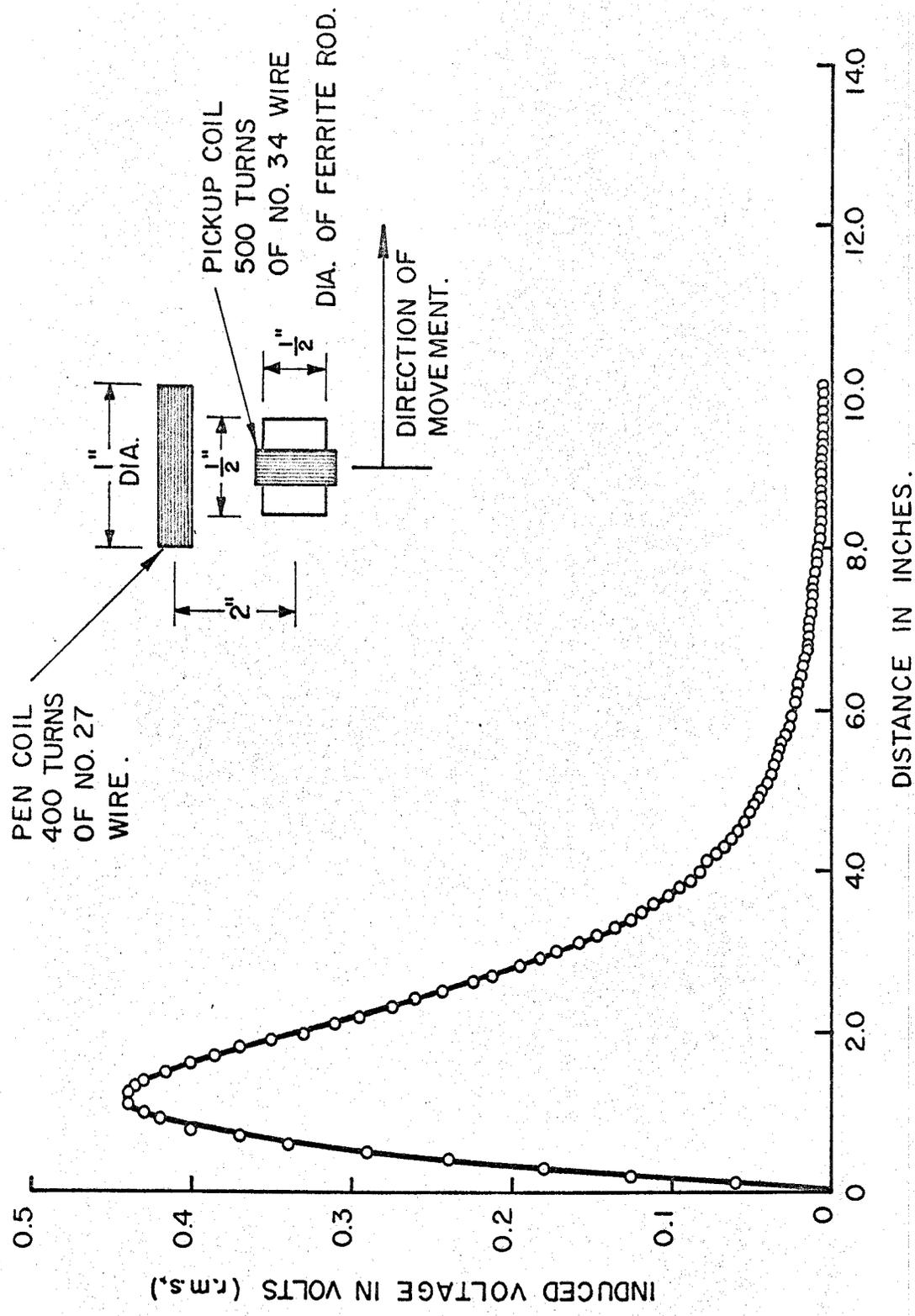


FIG.-3.2.4 VOLTAGE-DISTANCE CHARACTERISTIC.

diameter coil consisting of 100 turns of #34 wire was placed in the horizontal plane (parallel to the pen-coil) encircling the detector coil and produced a voltage of approximately 30 mV (R.M.S.) in the dead band.

3.2.3 Electronics

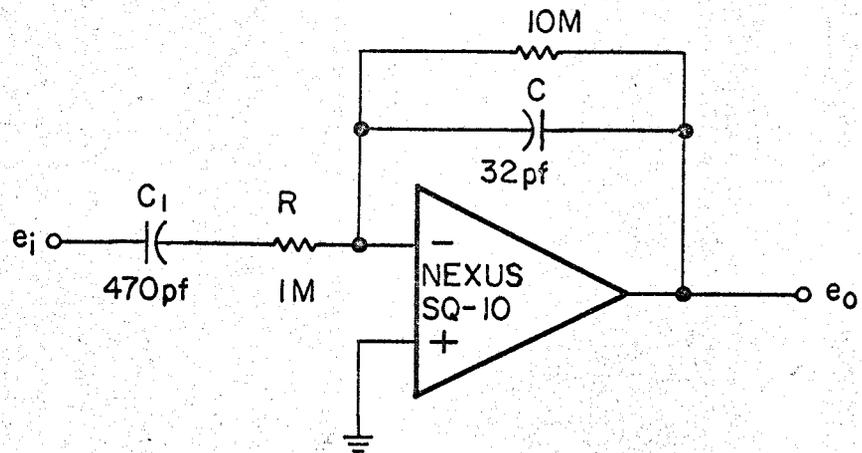
(i) Amplifier

The signal from the detector coil must be amplified to provide sufficient signal to operate the phase detector, etc., over a range of inputs from 0.2 to 450 mV (R.M.S.). Since it is more convenient to do phase detection by coincidence detection of signals in phase (0°) or in antiphase (180°), a 90° phase shift is necessary on either the pen-coil or detector coil signal. This is accomplished with a simple integrator. Thus, this section consists of two parts: (a) Integrator and (b) A.G.C. Amplifier.

(a) Integrator

The phase of the detector coil signal was shifted by -90° with the use of the integrator shown in Figure 3.2.5. This provides both the integrating or phase shifting action as well as a high input impedance (approximately $1M\Omega$) necessary to attain the gain factor of $\frac{1}{r} \sqrt{\frac{L}{C}}$ of the tuned coil arrangement.

The $10M\Omega$ feedback resistor and capacitor C1 provide d.c. stability and block low frequency pick up, respectively, without altering the characteristics of the integrator at the 5KHz operating frequency.



$$e_o \approx e_i \angle -90^\circ \text{ at } 5 \text{ KHz}$$

FIG.-3.2.5 INTEGRATOR

This aspect of blocking low frequency noise is important because 60Hz signals will be amplified by approximately 100 while 5 KHz signals are amplified by 1 due to the gain of an integrator being inversely proportional to the frequency of the sinusoidal input.

(b) A.G.C. Amplifier

The signal level from the integrator, which only performs phase shifting, varies from 0 to 450 mV (R.M.S.) with the useful range from 0.2 mV to 450 mV.

This large range of input signal must be amplified to a level suitable for activating the switching logic and operating the phase detector. The simplicity of design and the low cost of modern operational amplifiers make them a highly desirable form of linear amplifier. They can be easily adapted to amplify a.c. signals if the frequency response characteristics are carefully noted.

The Nexus SQ-10 is a low cost, general purpose operational amplifier, with moderately high open loop gain (30,000) and a unity gain crossover at 5MHz.⁽⁴⁵⁾ Since they are internally compensated at 20db per decade, the maximum closed loop gain that can be attained at a frequency of 10KHz is 50 db (gain of 320). The experimental closed loop frequency response for a voltage gain of 220 is indicated in Figure 3.2.6 with the amplifier operating in the non-inverting mode. This indicates a resonant peak at 12KHz. With a high gain system this is an undesirable feature which can be eliminated by external compensation.

AMPLITUDE BODE PLOT

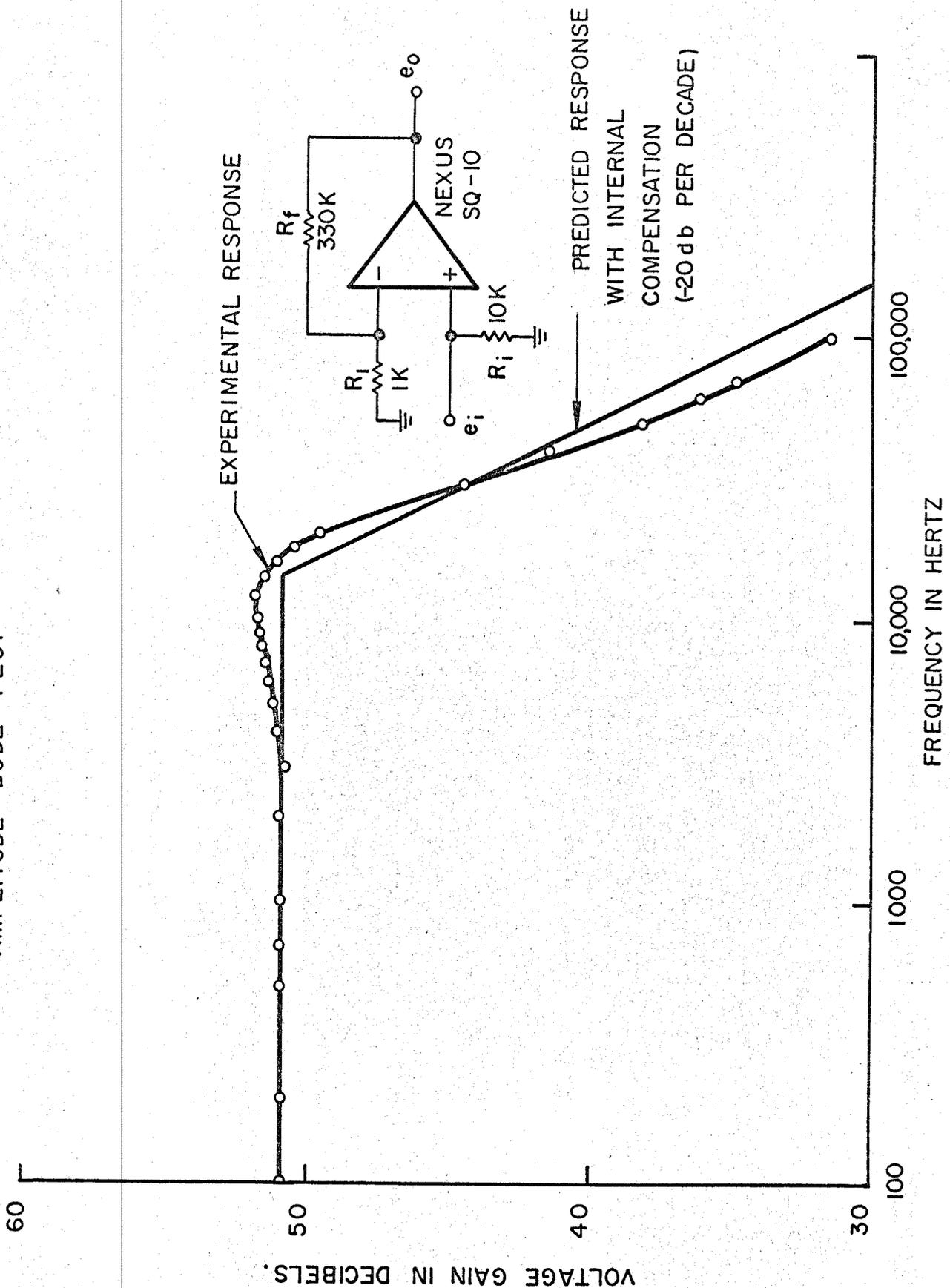


FIG.-3.2.6 FREQUENCY RESPONSE OF A NEXUS SQ-10 OPERATIONAL AMPLIFIER

A suitable band pass amplifier configuration is shown in Figure 3.2.7 with a gain expression $\frac{e_o}{e_i} = 1 + \frac{Z_f}{Z_i}$

This becomes:

$$\frac{E_o(s)}{E_i(s)} \approx \frac{s \tau_3}{(1 + s\tau_1)(1 + s\tau_2)}$$

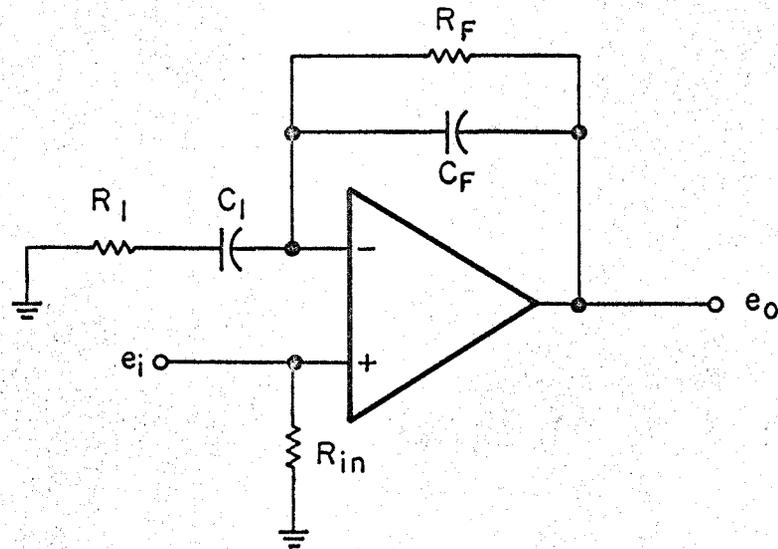
$$\tau_1 = R_1 C_1$$

$$\tau_2 = R_F C_F$$

$$\text{and } \tau_3 = C_1 R_F.$$

By setting $R_F = 330K\Omega$, $C_F = 70\text{pf}$, $R_1 = 1K\Omega$, $C_1 = 0.056\mu\text{f}$, and $R_{in} = 10 K\Omega$, the center frequency becomes 5KHz with corner frequencies of 3KHz and 7KHz. The experimental response is indicated in Figure 3.2.8. The deviation from the ideal asymptotes at frequencies greater than 10KHz is due to the internal compensation effect revealed in Figure 3.2.6. The maximum voltage gain is 250 with zero phase shift at 5KHz. The high and low frequency roll offs eliminate the 12KHz resonance and reduce the gain of 60Hz pick-up, respectively.

The wide range of input signals presents a second problem, that of saturation for large inputs. Although a square wave input to the phase detector is desirable, saturation of the SQ-10 amplifier produces phase shift and distortion at the output according to the amount of overdrive. This aspect of phase shift is undesirable because a constant phase must be maintained. Thus, some form of control must be used to prevent overdriving the amplifier.



GAIN VERSUS FREQUENCY (LOG SCALE)
AMPLITUDE BODE PLOT

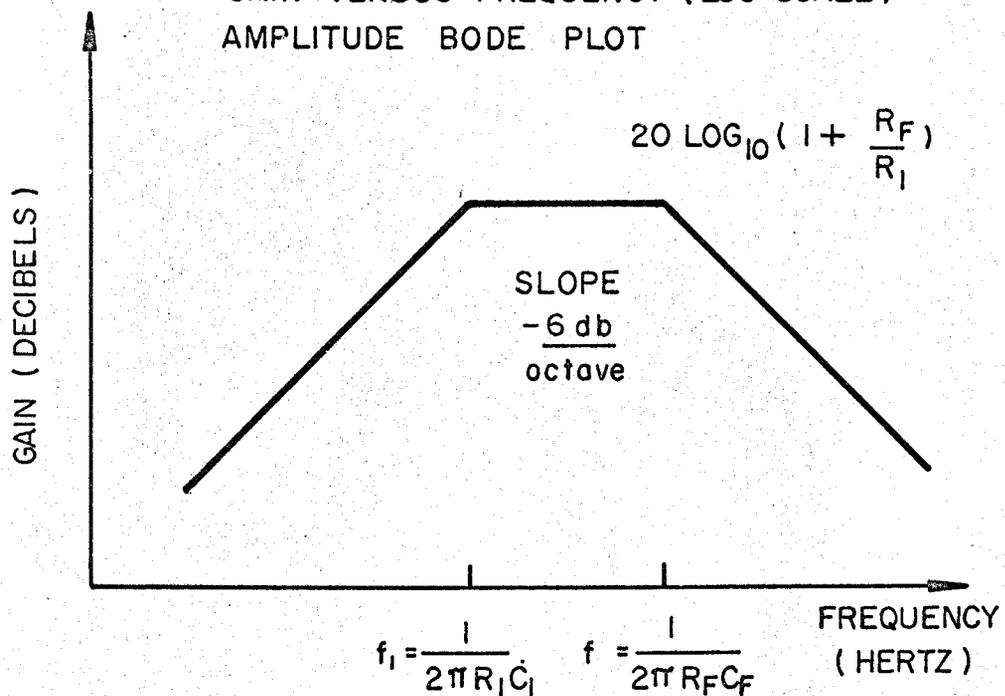


FIG.-3.2.7 NON INVERTING BAND PASS AMPLIFIER
AND CHARACTERISTIC .

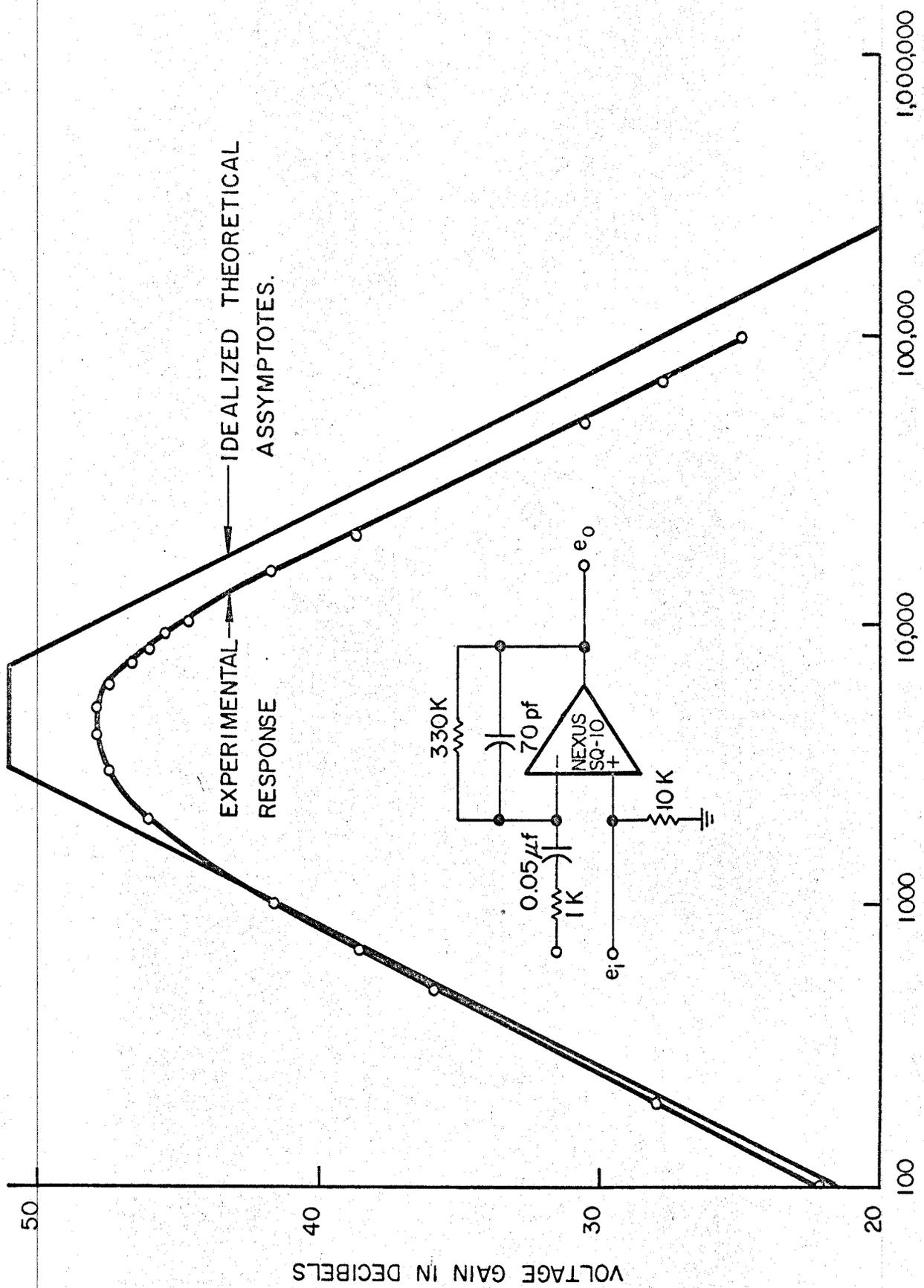


FIG.-3.2.8 FREQUENCY RESPONSE OF A BAND PASS AMPLIFIER

Various forms of diode clipping and limiting were investigated. However, the capacitive effects of these devices in the feedback loop presented some other phase shift problems. A simple method of automatic gain control (a.g.c.) was employed to prevent saturation. This consisted of varying the impedance of a semiconductor diode by passing a d.c. current through it proportional to the a.c. output amplitude of the amplifier.

The dynamic impedance of a semiconductor diode, neglecting capacitive effects, can be obtained from the ideal diode equation given below, (46)

$$I = I_s \left(e^{\frac{qV}{kT}} - 1 \right) \quad \dots (6)$$

where

I_s = Reverse saturation current

q = Electronic charge

k = Boltzman's constant

T = Absolute Temperature

and r_d = dynamic resistance of the diode.

$$r_d = \frac{dV}{dI} = \frac{kT}{qI_s e^{\frac{qV}{kT}}} \quad \dots (7)$$

If

$$e^{\frac{qV}{kT}} \gg 1, I \approx I_s e^{\frac{qV}{kT}} \quad \text{and} \quad r_d \approx \frac{kT}{qI} \quad \dots (8)$$

At room temperature ($T = 300^\circ\text{K}$) this becomes the well known expression

$$r_d = \frac{0.026}{I} \quad \dots (9)$$

Thus, the dynamic resistance of a semiconductor diode is inversely proportional to the d.c. current through it.

This was used in the design of the a.g.c. amplifier of Figure 3.2.9. The band pass amplifier previously described is the basis of this circuit with the ac-dc converter changing the amplitude of the a.c. output to a proportional d.c. current which varies the diode impedance " r_d ". The transfer characteristic for this configuration is shown in Figure 3.2.10. By cascading two such amplifiers the required gain can be achieved. At 0.2 mV (40" away from the null point) the output of two amplifiers would be approximately 1.5 volts (R.M.S.) and for the maximum input of 450 mV the output would be approximately 5.5 volts (R.M.S.) which is less than the maximum output before clipping occurs.

(ii) Phase Detector

The phase detector must provide two binary levels corresponding to the 0° or 180° phase difference between the pen-coil reference voltage and the a.g.c. amplifier output. These two levels must be capable of driving the switching logic.

There exists various forms of transformer phase detectors which provide an output proportional to the phase difference between two input signals. However, these generally require constant amplitude input voltages to produce a representative output. The output voltage of the a.g.c. amplifier is not of constant amplitude and since there exists only two distinct phase differences, a discrete rather than a continuous output is more desirable.

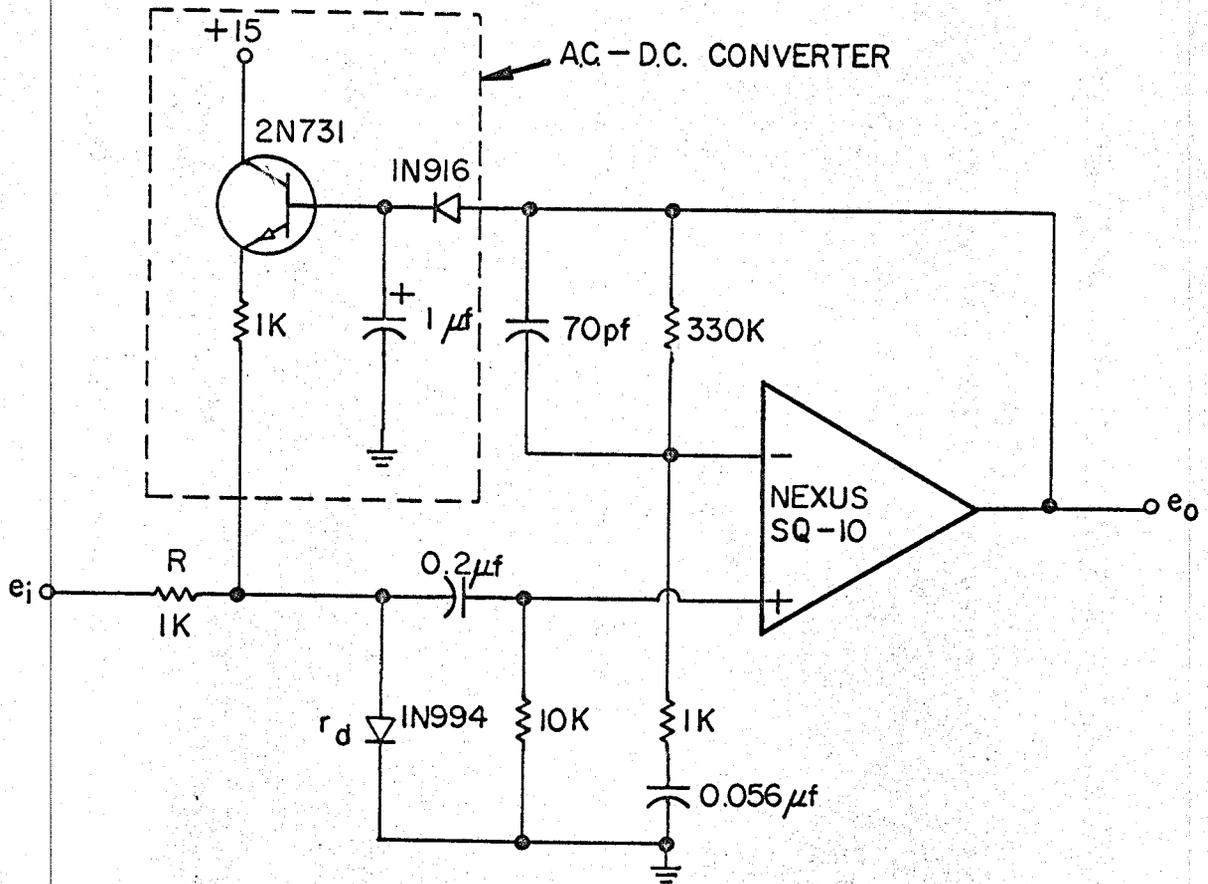


FIG.-3.2.9 A.G.C. AMPLIFIER

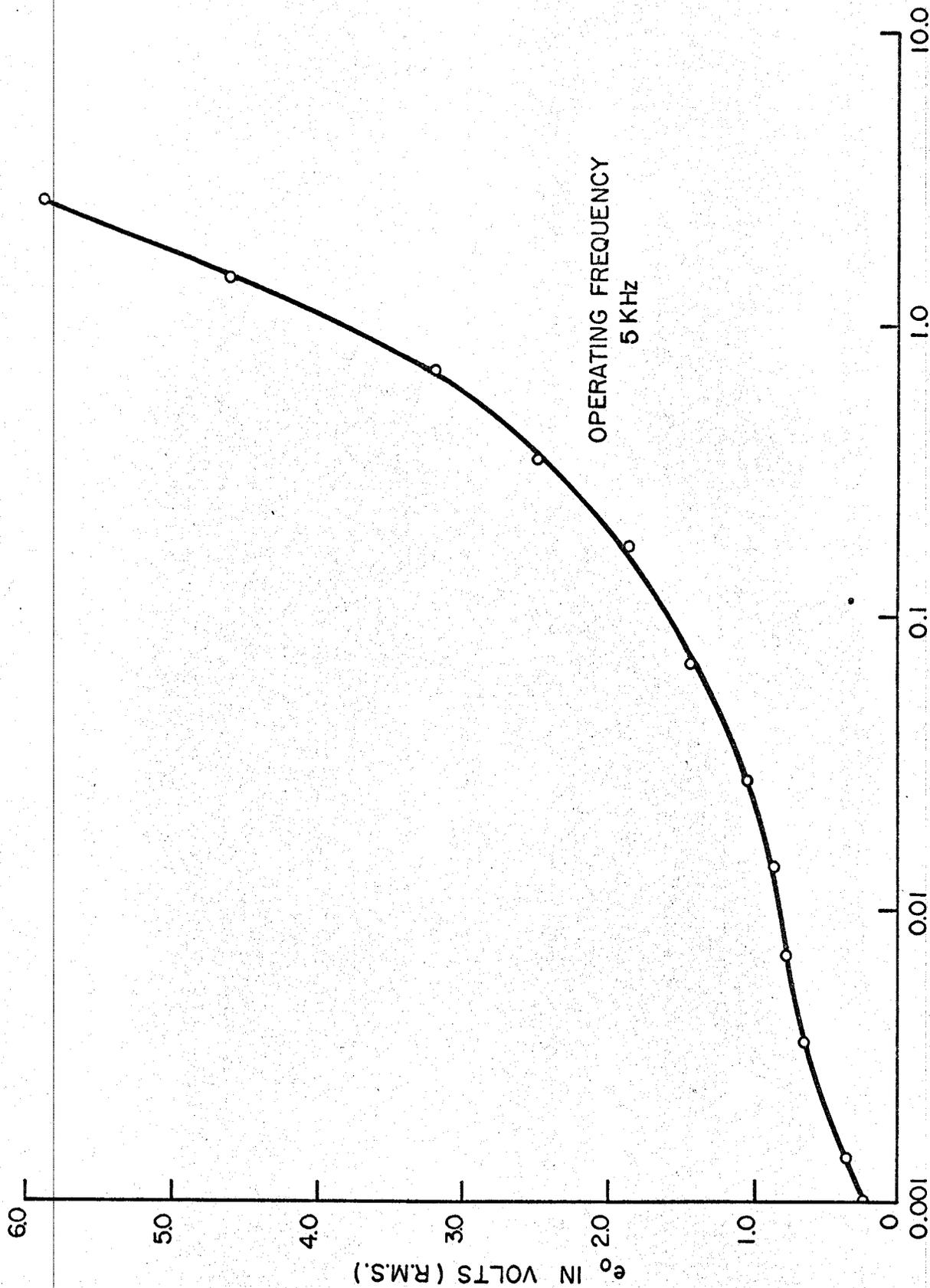


FIG-3.2.10 INPUT-OUTPUT CHARACTERISTIC OF A.G.C. AMPLIFIER.

A very simple phase detector can be constructed from an "AND" gate, by first limiting the sinusoidal input and producing a square wave. The "AND" gate then detects the coincidence of the two inputs and the average output is a measure of the phase difference.

Figure 3.2.11 indicates one such system. The output of the diode "AND" gate is filtered so that the output before level conversion is -3.5 volts for 180° phase difference and -11.5 volts for 0° phase difference. The limiters function well for inputs greater than 1.5 volts (R.M.S.) and the resulting output after level conversion is:

- (1) -15 volts for 180° phase difference
- (2) 0 volts for 0° phase difference

These represent the two binary levels available to activate the switching logic.

(iii) Switching Logic

This section consists of two parts: (a) ac-dc converter and comparator, and (b) logic. Part (a) describes the circuitry necessary to get the output of the a.g.c. amplifier and the zone coils into binary levels to activate the logic. The logic in part (b) consists of the solid state switches which are activated by the comparators to switch the pulses to the correct motor inputs.

a) A.C.-D.C. Converter and Comparator

Two of these devices are necessary, one for each of the outputs of the a.g.c. amplifier and the zone coil. The former will establish the 0.004" dead zone at the null. This is accomplished by setting the reference voltage on the comparator equal to the output of the a.g.c. amplifier at a distance of 0.002" from the null point. Simple diode rectification and filtering of the a.g.c. output yields a -3.5 volt d.c.

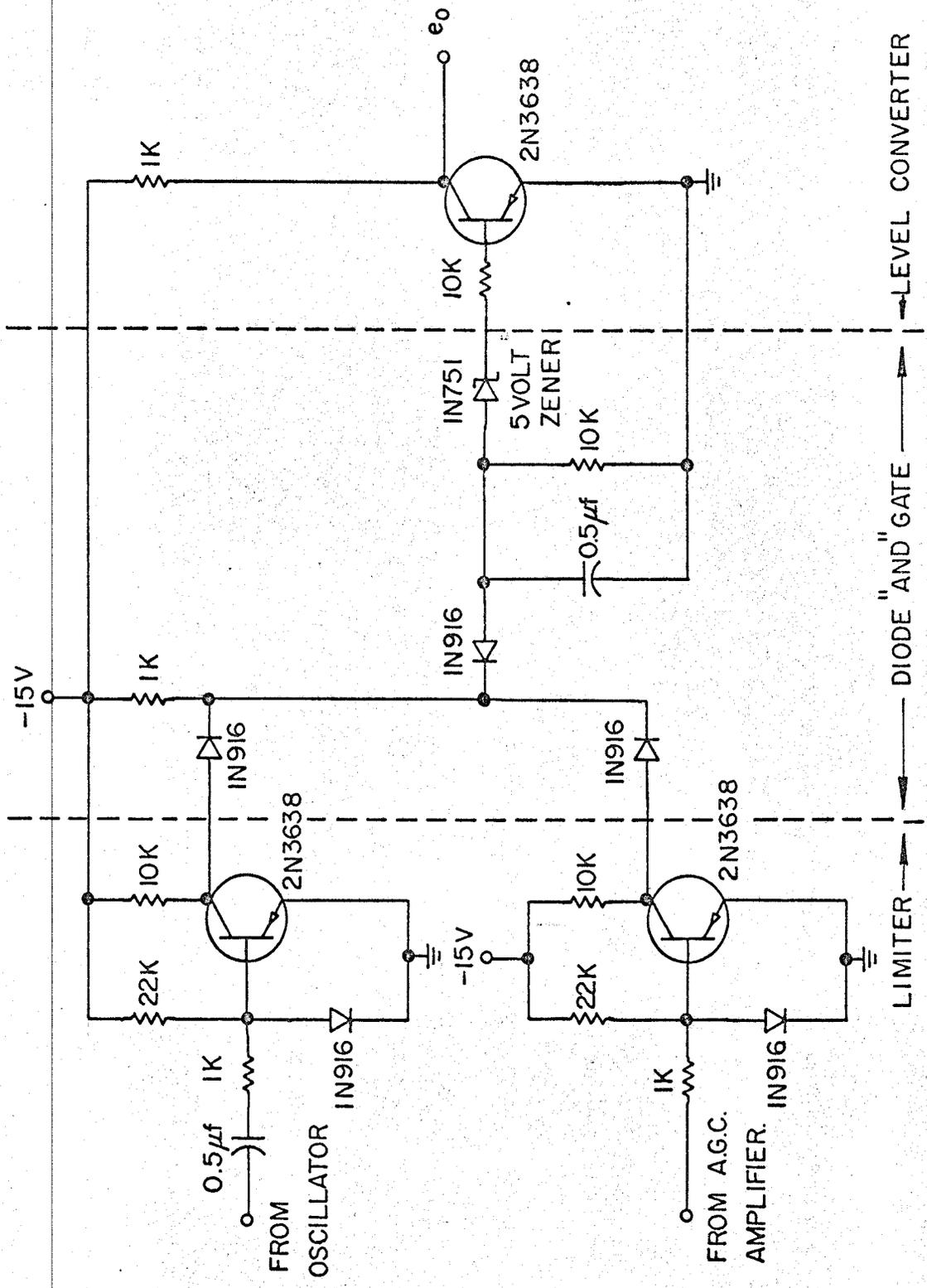


FIG.-32.11 PHASE DETECTOR

signal at this point. The comparator then generates one binary level in the dead band and another level beyond it. A similar system is necessary for the zone-coil output where maximum voltage exists in the dead band.

The circuitry shown in Figure 3.2.12 provides an output of 0 volts for the input below the reference (i.e. dead band) and -15 volts for the input above the reference. The reference voltage can be adjusted between 0 and -5 volts. The $4.8\text{K}\Omega$ and $10\text{K}\Omega$ resistor divider keep the input to a level less than -5 volts to protect the operational amplifier. Since the amplifier is operated in the open loop configuration, the hysteresis effect is minimized to less than 50 mV.

The second comparator for the zone-coils is shown in Figure 3.2.13. The operation is identical except for the addition of the amplifier stages to bring the 30 mV signal from the zone-coil to a 3 volt level. Thus, the output is -15 volts in the null zone and 0 volts beyond the null zone.

(b) Logic

The drive logic for the stepping motor (purchased with the motor) requires ground to -10 volt pulses of 50 μ sec. duration to activate the motor. An adjustable pulse generator using the simple unijunction oscillator shown in Figure 3.2.14 was constructed. The $100\text{K}\Omega$ potentiometer allows adjustment of the frequency up to 1000Hz, to obtain the maximum stepping rate to which the motor can respond.

The logic levels available from the phase detector, a.g.c., comparator, and the zone comparator are summarized in Table II.

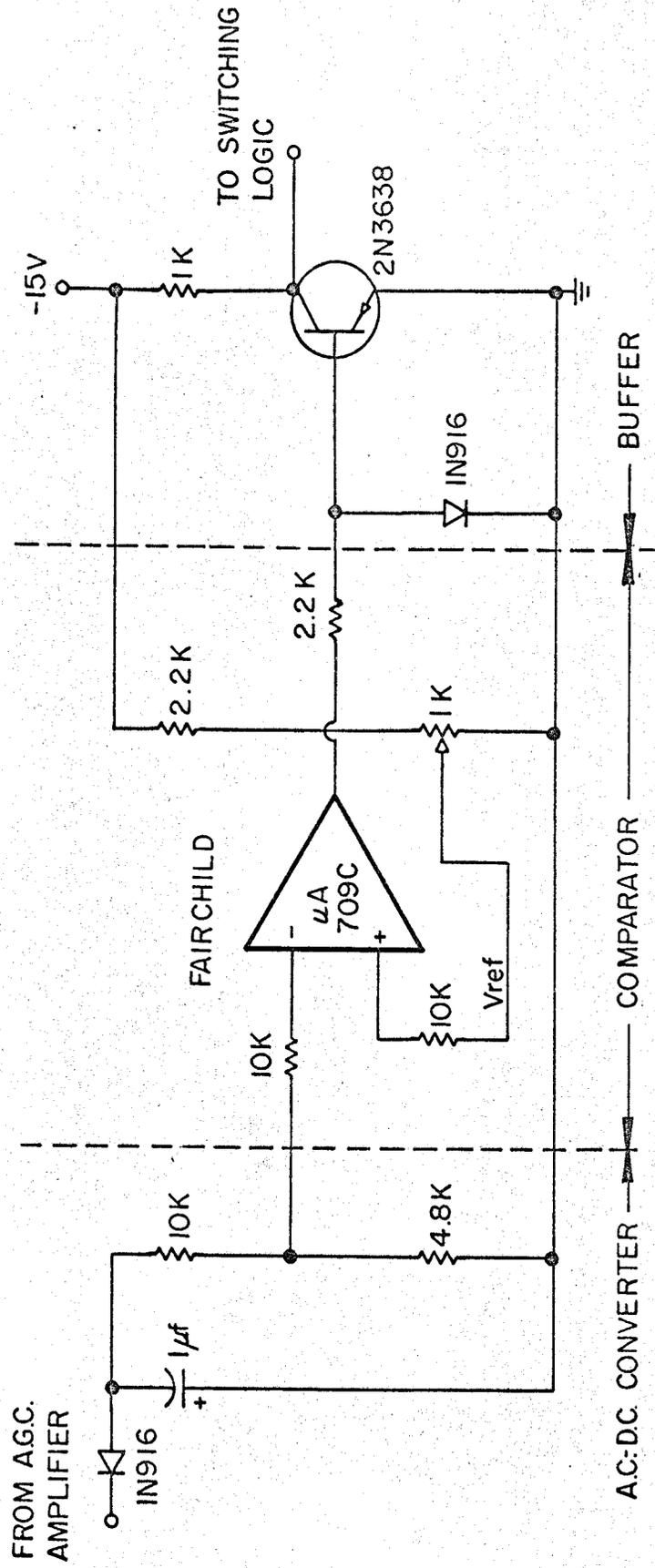


FIG.-3.2.12 A.G.C. COMPARETOR

TABLE II

Logic Levels

Device	Condition	Level (in volts)
Phase Detector	-Signals - in phase (0°)	0
	- in antiphase (180°)	-15
A.G.C. Comparator	within dead band	-15
	Outside dead band	0
Zone Comparator	around null zone	0
	outside null zone	-15

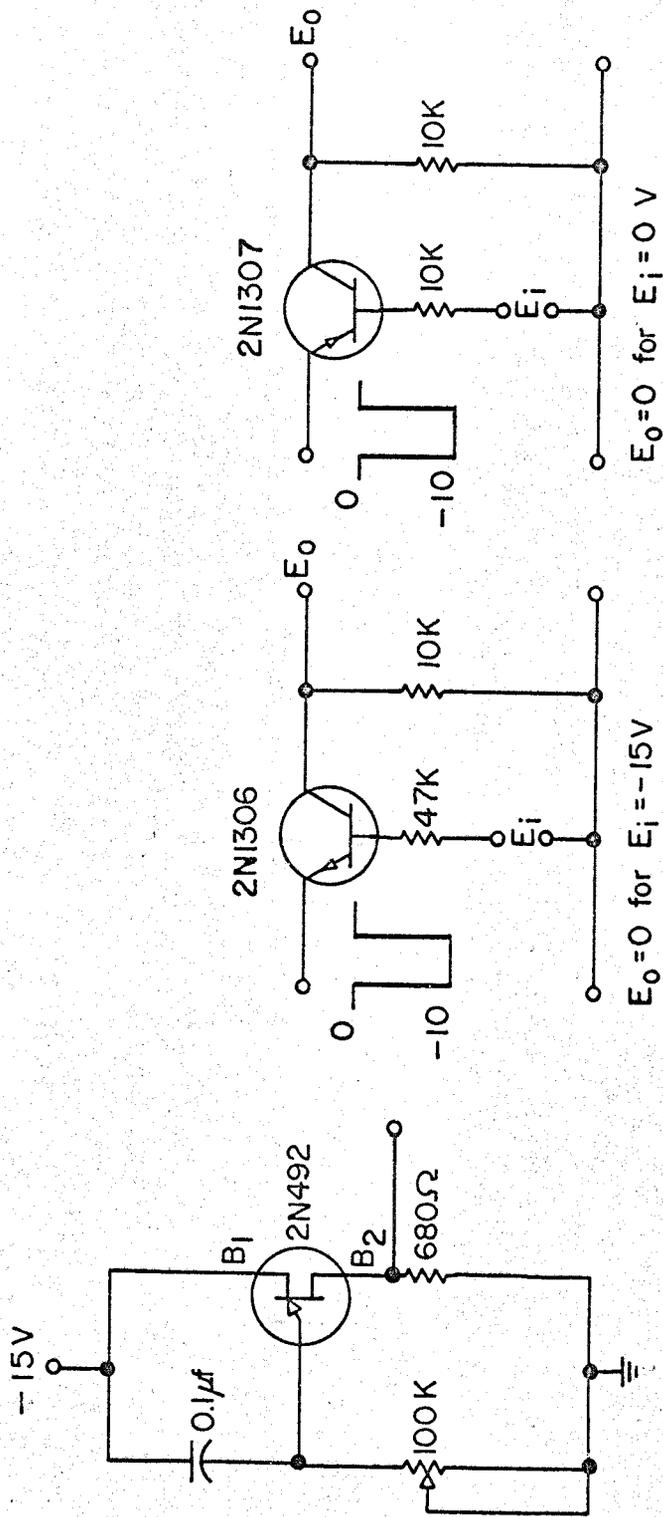


FIG-3.2.14 PULSE GENERATOR

FIG.-3.2.15 TRANSISTOR SWITCHES.

$E_o = 0$ for $E_i = 0$ V

$E_o = 0$ for $E_i = -15$ V

The output of the phase detector must be used to steer the pulses to the clockwise (c.w.) or counterclockwise (c.c.w.) input on the logic drive for the correct motor rotation. The output of the a.g.c. and zone comparators are to be used to stop any pulses reaching the motor logic only when the follower is in the dead band.

The two transistor switch arrangements shown in Figure 3.2.15 were used to provide the necessary switching of the pulse train as shown in Figure 3.2.16. The operation is straight forward with the first parallel switch arrangement ensuring that the pulse train is only interrupted when the follower is in the dead zone (i.e. output of a.g.c. comparator -15V, output of zone comparator 0 V) and the second set of switches directs the pulses to the correct terminal according to the phase information from the phase detector.

(iv) Counter

To record the position of the follower and thus obtain the digital position of the follower, a binary up-down counter was constructed. The purpose of this counter is to keep a count of the pulses which go to the terminals of the stepping motor. Since each pulse represents a 0.004" follower movement, the counter indicates the number of 0.004" movements from the reference zero count. This is a simpler and more economical method of obtaining the digital co-ordinates than the use of shaft encoders.

A 14 bit binary up-down counter using Digital Equipment Corporation (DEC) modules was constructed as shown in Figure 3.2.17. (47)* The use of 3 inverters operating in parallel was to provide sufficient current to the level inputs of the 14 Flip Flops. The down count

* DEC Symbology used in Figure 3.2.17.

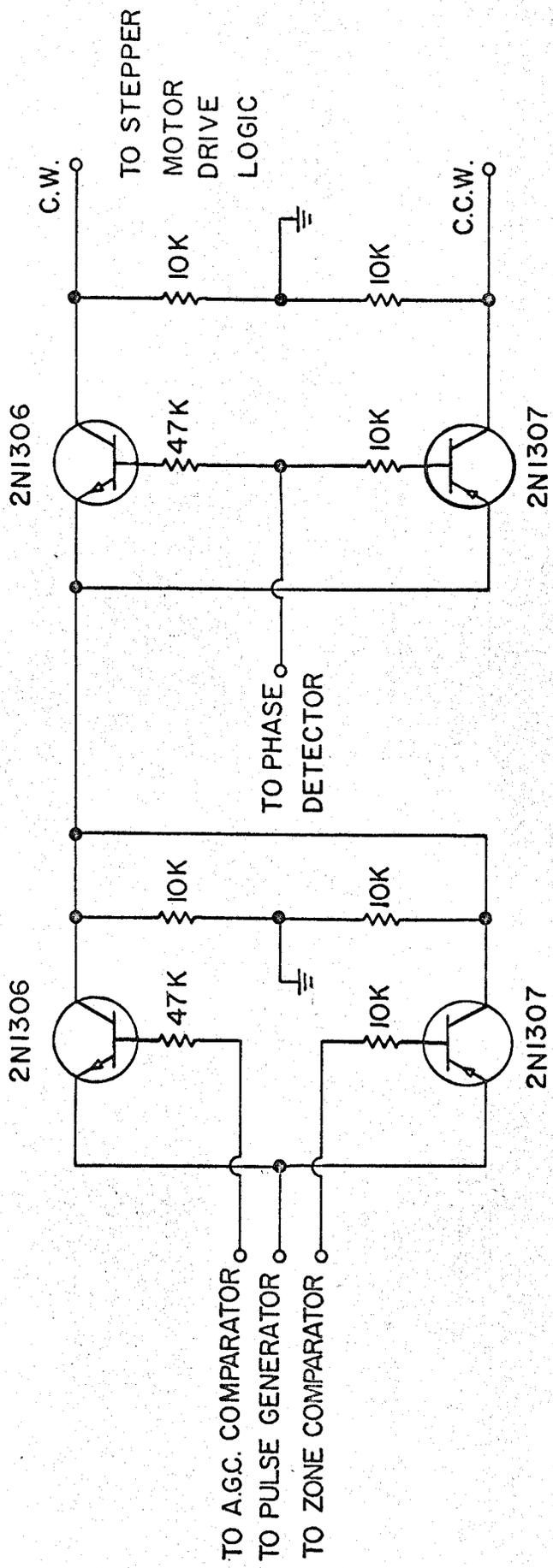


FIG.-3.2.16 SWITCHING LOGIC

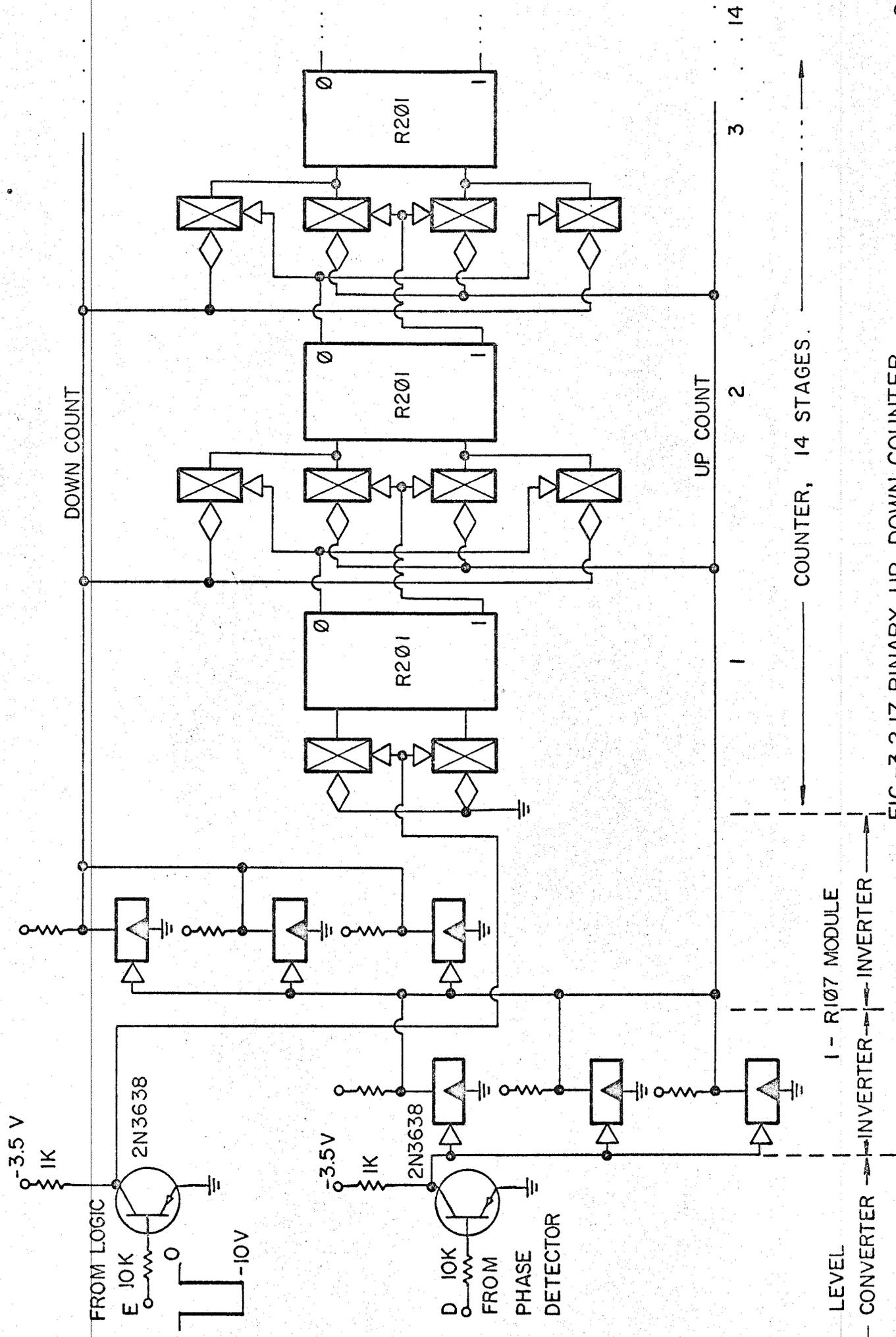


FIG-3.2.17 BINARY UP DOWN COUNTER.

corresponds to clockwise motor movement and the up count to counter clockwise rotation; the 14 stages are required to handle 10,000 increments of 0.004" in the 40" span.

(v) Oscillator

The oscillator used to drive the pen coil with a 5KHz, 15 volt (peak to peak) sinusoidal voltage is shown in Figure 3.2.18. The pen-coil is used as a reactive, frequency determining element in the tuned circuit of a Colpitts Oscillator. The disadvantage of this configuration is that if the pen coil is changed, the coil inductance may be different and the frequency would also be altered. Thus, it is probably desirable to place a small, variable inductor in series with the pen-coil to help tune the circuit to the desired frequency if more than one form of pen-coil is required.

The advantage of this circuit is again simplicity. This arrangement eliminates using a conventional oscillator in conjunction with a power amplifier to drive the low impedance pen coil. Both the oscillator and the power amplifier are combined in this single transistor configuration.

3.3 Discussion

The complete electronics as shown in Figures 3.3.1 and 3.3.2 was constructed for a one axis model of the Incremental Pencil Follower. A test jig using a Slo-Syn HS-25 Stepping Motor and a rack and pinion drive was constructed for test purposes. The basic construction was similar to that described in Appendix B with the wire drive replaced by

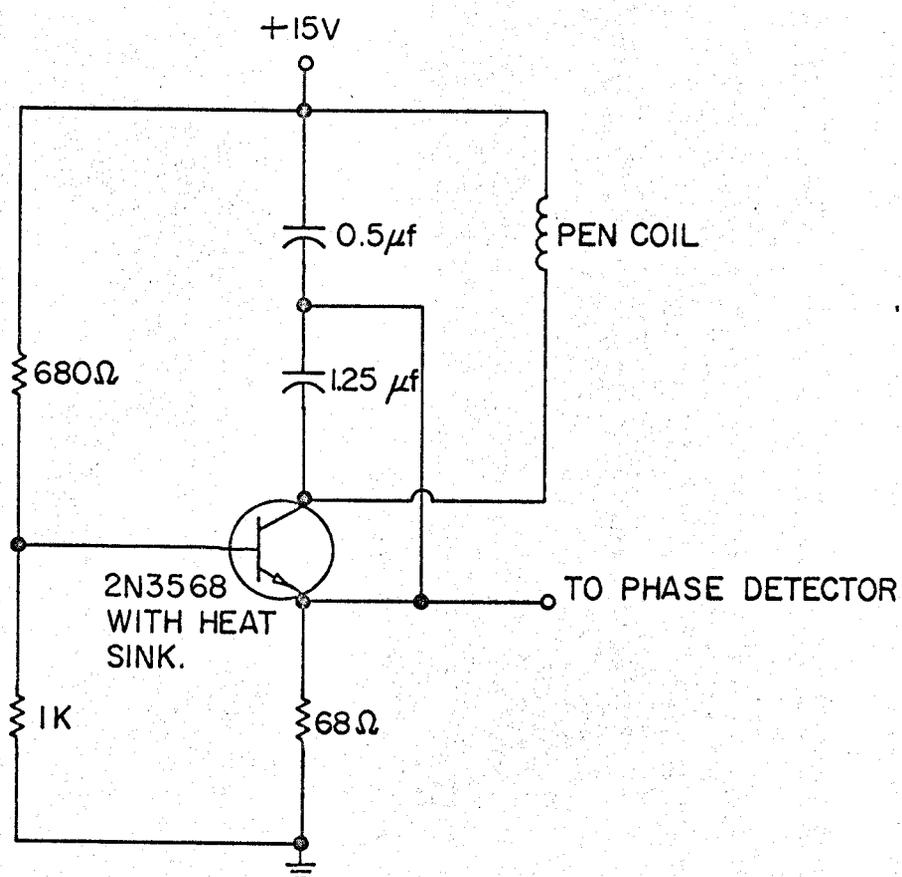
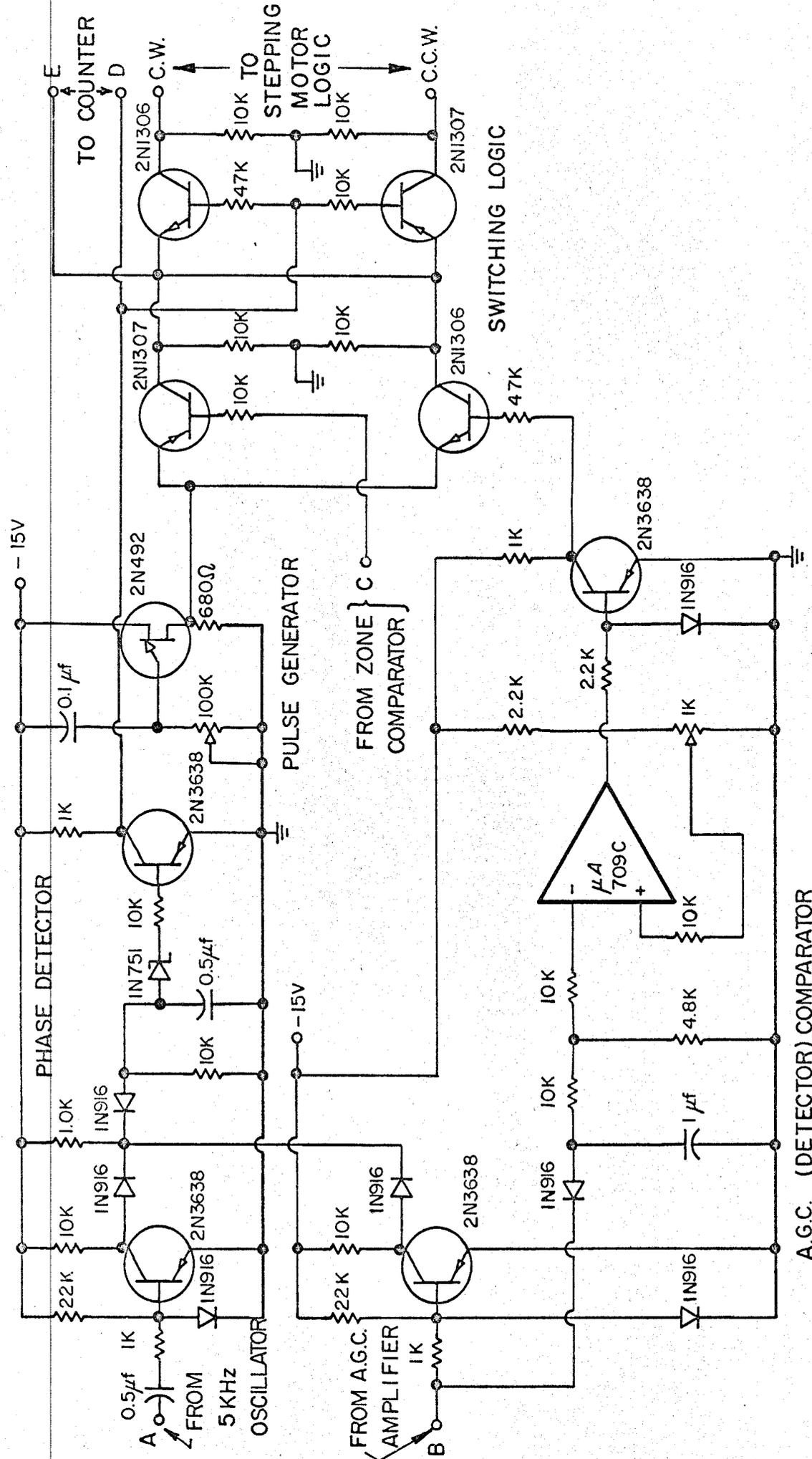


FIG.-3.2.18 5 KHz COLPITTS OSCILLATOR



A.G.C. (DETECTOR) COMPARATOR

FIG.-3.3.2 PART II OF THE ELECTRONICS

the rack and pinion to accomplish a 0.00393" lineal movement per step of the stepper motor. The driving motor and zone and detector coils were mounted on the moveable block to provide one axis of movement.

Some difficulty was encountered with the magnetic pick-up due to the transients in the stepper motor when it was phase switched. Magnetic shielding of the motor and the use of co-axial cable from the detector and zone coils to the associated electronics reduced this effect. Also, it was necessary to place the electronics in a metal box to shield it.

The reference voltage on the detector comparator was adjusted until the follower oscillations ceased. However, it was necessary to add some damping in the form of friction to the follower to reduce stepper motor overshoot, and attain a 0.004" dead band (i.e. 0.002" on either side of the null point). The high gain a.g.c. amplifier stages performed satisfactorily with the follower capable of homing in from a distance of 40".

The precision of this device was measured to be ± 0.002 ". This can be simply explained by referring to Figure 3.3.3. Since the follower can only be located in discrete positions determined by the stepping motor, the pen-coil can only lie anywhere within that approximate 0.004" interval (0.002" on either side of follower position) without the follower moving. The Figure indicates that if the pen-coil starts from point "A" and is moved to point "B" and then back to "A" again, the position of the follower, and thus the value in the up-down counter, would be the same. However, if the pen-coil was returned to

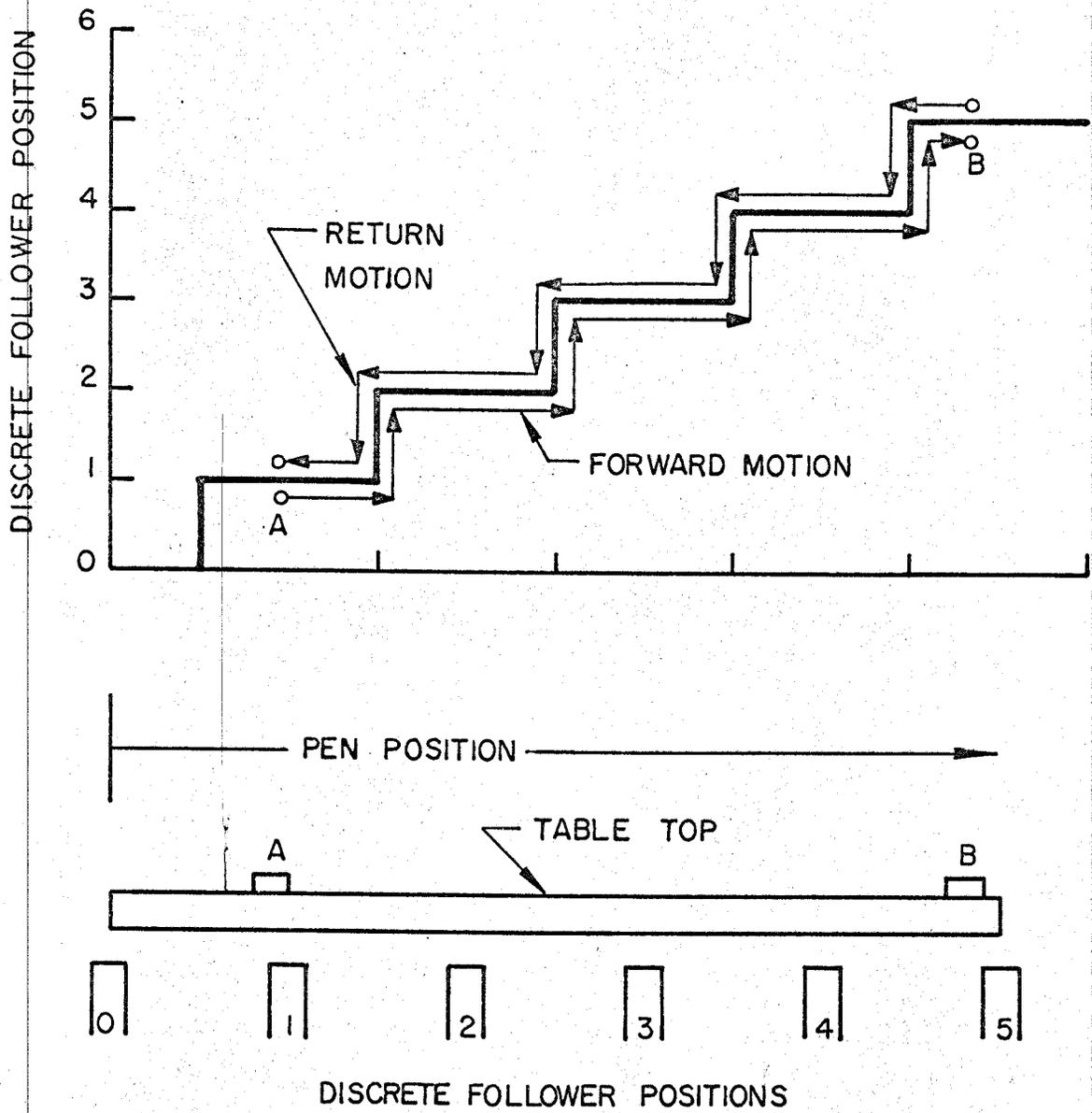


FIG.-3.3.3 INCREMENTAL PENCIL FOLLOWER REPEATABILITY.

any location within that 0.004" dead band the follower position would also be the same as the starting position. Thus, the resolution of this device is 0.00393", i.e. if the pen-coil is located some distance from the starting point, the digital co-ordinate read from the counter will be in error a maximum of 0.00393". The advantage of the stepper motor drive is that the high starting and holding torque prevent the effects of stiction from altering this resolution.

The lag of this system is a maximum of 0.00393" up to pen velocities of 1.0 inch per second. The motor could not be operated at the maximum stepping rate due to follower inertia. However, with improved mechanical design, this system has the capability of following at speeds up to 2.0 inches per second. Velocities greater than this would cause the follower lag to increase indefinitely, as the follower cannot "catch-up" to the stylus.

4. CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

The complexity of map and chart information place stringent requirements on a device suitable for digitizing such data. Although there exists many commercially available digitizers, the Pencil Follower manufactured by d-mac Company was chosen as the best device for the Automatic System of Cartography at the University of Saskatchewan.

Numerous tests were performed on this device to determine its response to conditions encountered while digitizing cartographic data. The results indicated some device limitations; namely lag, non uniform resolution, and poor lock-on distance. These restrict the operators' movements and reduce the reliability of the digitized output.

A stepper motor drive was proposed to reduce these limitations. The necessary electronics was constructed and tested, with a simple one axis test jig used to perform preliminary tests on the system. The use of the stepping motor as a digital form of actuator eliminated the problem of stiction and provided a simpler method for obtaining follower position without using shaft encoders. The use of a binary up-down counter to record motor position eliminated the mechanical loading and alignment problems of shaft encoders, while allowing the reference zero position to be chosen as desired. Improvements in coil configurations coupled with the use of high gain, automatic gain control amplifiers, allowed lock-on to be achieved anywhere over the 40" distance. A positioning resolution of 0.00393" was attained and the lag was limited to 0.00393" for pen velocities up to 1.0 in/sec. Although the motors were capable of moving a mass at 2 in/sec., the actual torque, speed characteristics of the motor and the weight of the test jig prevented the use of the

maximum stepping rates.

Although a two axes system was not constructed, the results of the preliminary tests indicated that such a system is feasible. The electronics necessary for the second axis is identical to that developed. There does exist some problems in constructing a two axes follower for minimum weight while maintaining sufficient positioning accuracy. The mechanical design was not completed in this dissertation, but some of the problems were exposed and a recommendation made. The recent addition of the HS-50 and HS-50L into the Slo-Syn line of stepper motors, and the new high torque Fujitsu stepping motors (Models 110 and 111) available from Icon Corporation, 237 Binney Street, Cambridge, Mass. 02142, U. S. A., provide a better alternative than the HS-25 used for these experimental purposes because improved torque outputs would ease the weight limitation in the mechanical design.

4.2 Recommendations for Future Research

The limiting feature on all semi-manual digitizing devices is the actual digitizing speed. In areas of complex line data, the operator is restricted to following speeds of approximately 0.1 inches per second to maintain close proximity with the actual line he is attempting to digitize. This is not a machine limitation, but rather a limitation originating from the digitizing technique employed. There are two basic methods by which this technique, and thus the digitizing speed, can be improved.

Firstly, an optical error detector could be mounted on the follower to compensate for human errors. The operator would visually approximate to the line while allowing the error detector to measure his deviation from the line. Thus, the operator's position and the error would both be conveyed to the computer to provide the correct digitized co-ordinate. This error detector could be a television camera, a rotating aperture, or an array of photocells which are mounted directly on the follower or optically linked to the follower via a fiber optic bundle.

This system would also permit character recognition which would be very useful in digitizing depth soundings. Digitizing of the depth sounding location would be accomplished by pointing with the stylus in the normal manner. Recording the actual depth of water at that location (i.e. the depth sounding) would then be performed by the optical detector scanning the numerals and the computer performing character recognition. Since the numerals involved are well defined and of a standard size and format, the character recognition process would be reasonably simple and reliable.

The second method is to go to an automatic system. If the immense software sorting of the data from automatic scanners could be solved, this device does hold promise. However, the automatic line follower appears to be the best automatic digitizer at present, as it is capable of producing "labelled", continuous, digitized line data. However, there exists some problems when this device reaches ambiguous

intersections, dotted lines, etc. There are three possible methods of directing the line follower at these ambiguous locations:

(i) complete software control, (ii) operator intervention and control, and (iii) directer control. Previous research and development in the first two areas has met with limited success. Although a system employing both software and operator control would be an improvement, the use of approximate (rough) digitizing performed on a semimanual device to direct an automatic line follower at the points of ambiguity appears promising. Although the digitizing would be performed in two steps, this directer control would eliminate much software and operator control. However, a detailed study of all types of control and some improved definitions of what will be encountered while digitizing map data, would result in a more complete understanding so necessary for the development of such an automatic system.

The problems in digitizing cartographic data are just beginning to be defined as indicated by the broad areas for further research discussed here. These problems must be defined and solved so that existing map and chart information can be digitized and stored in a digital data bank, to provide the background data for such future developments as automatic plotting and controlling of the routes travelled by ships, planes, and spacecraft. The importance of solving these massive problems and digitizing existing data cannot be over-emphasized when one considers such future developments.

5. REFERENCES

1. Bordner, C.A.; Brenner, A.E.; deBruyne, P.; "A Computer Controlled Precision Film Scanner". The Radio and Electronic Engineer, Vol. 33, No. 5, March, 1967, pp. 171-176.
2. Ledley, R.S.; "Optical Processing in Medical Sciences", Computers and Automation, July, 1966, pp. 14-18.
3. Manual of Photogrammetry, Third Edition, 1966, Vol. II, pp. 759-800.
4. Bickmore, D.P.; "Cartographic Data Banks", A paper presented to the Congress of the International Federation of Surveyors (Commission V) Rome, May, 1965.
5. Bickmore, D.P.; Boyle, A.R.; "An Automatic System of Cartography", A paper presented at the Technical Symposium of the International Cartographic Association, Edinburgh, 1964.
6. Richardson, M. Anne; "The Oxford Cartographic Bank, A feasibility study of accuracies, store sizes and operation time", Paper presented at the Third International Conference on Cartography, Amsterdam, April 17-22, 1967.
7. Calma Co., Model 302, 303 and 480 Digitizer Data Sheet.
8. Auto-trol Corp., Model 3929-XY Recorder, Graphic Digitizing Data Sheet.
9. Benson-Lehner Corp., Model LARR-VH, Record Reading Systems Data Sheet.
10. Siegel, L.; "Digitizing Graphic Records for Computer Analysis", I.E.E.E. Trans. on Biomedical Engineering, Vol. BME-14, No. 1 Jan., 1967, pp. 7-10.
11. Coradi Corp. Digimeter - Description Note, Zurick, Switzerland.
12. d-mac Ltd., The Pencil Follower Data Sheet.
13. Concord Ltd., Mark 8, Universal Graphics Processor Data Sheet.
14. Benson-Leher Corp., Computer Graphics, Vol. 3, No. 11, April, 1966.
15. Universal Drafting Machine Corp., Orthomat Mark II Digitizing Machine Data Sheet.
16. Gerber Scientific Instrument Co., Data Sheet.

17. Davis, M.R.; Ellis, T.O.; "The RAND Tablet: A Man-Machine Graphical Communication Device" Memorandum RM-4122-ARPA, August, 1964.
18. Davis, M.R.; Ellis, T.O.; "The RAND Tablet: A Man-Machine Graphical Communication Device". 1964 Proc. F.J.C.C., Vol. 26, pp. 325-331.
19. Davis, M.R.; Ellis, T.O.; "The RAND Tablet: A Man-Machine Graphical Communication Device". Instruments and Control Systems, Vol. 38, Dec., 1965, pp. 101-103.
20. Data Equipment Co., Data Sheet on the Grafacon.
21. Lewin, M.H.; "A Magnetic Device For Computer Graphic Input". 1965 Proc. F.J.C.C., Vol. 27, pp. 831-838.
22. Roberts, L.G.; "The LINCOLN Wand", 1966 Proc. F.J.C.C., Vol. 28, pp. 223-227.
23. Simek, J.G.; Tunis, C.J.; "Handprinting-Input Device For Computer Systems", IBM SDD Technical Report TR01.961, May 31, 1966.
24. Simek, J.G.; Tunis, C.J.; "Handprinting-Input Device For Computer Systems", I.E.E.E. Spectrum, Vol. 4, No. 7, July 1967, pp. 72-81.
25. Normalair Ltd., Westland Trace Reader Data Sheet.
26. Hargreaves, B.; Joyce, J.D.; Cole, G.L.; Foss, E.D.; Gray, R.G.; Sharp, E.M.; Sippel, R.J.; Spellman, T.M.; Thorpe, R.A.; "Image Processing Hardware For A Man-Machine Graphical Communication System", 1964 Proc. F.J.C.C., Vol. 26, pp. 363-386.
27. Sylvania Corp., Model DT-1 Sylvania Data Table Data Sheet.
28. Gerber Scientific Instrument Co., Graphic to Digital Data Conversion Data Sheet.
29. California Computer Products Inc., Bulletin No. 175D, June, 1967, on Automatic Trace Digitizer.
30. Hewlett Packard/Moseley Division, Model F3B and 7500 Series Line Followers Data Sheets.

31. St.Clair, J.M.; "Digitalizing Line Tracing by Line Following Using a Computer Controlled Scanner", M.Sc. Thesis, University of Maryland, 1965.
32. Krull, F.N.; Foote, J.E.; "A Line Scanning System Controlled From an On-Line Console", 1964 Proc. F.J.C.C., Vol, 26, pp. 397-410.
33. Bordner, C.A.; Brenner, A.E.; deBruyne, P.; Reuter, B.J.; Rudnick, D.; "High Precision C.R.T. Scanning System". Paper presented at the DEC Computer Society Spring Convention; DECUS Proceedings, Spring, 1967.
34. Thompson, D.R.; "An IBM Special Cartographic Scanner", Paper presented at the ASP-AC SM Annual Convention, Washington, D.C., March 6-10, 1967.
35. Gilman, W.L.; "Drum Scanning Techniques For Digitizing and Recording Image Data", 1966 I.E.E.E. International Convention Record, Part III Computers, pp. 29-38.
36. Shepherd, W.H.; "Automatic Contour Digitizer", Paper as a result of a project at the Advanced Mapping Division, USAE GIMRADA, Fort Belvoir, Virginia.
37. Shepherd, W.H.; "Automatic Contour Digitizer". Photogrammetric Engineering, Jan., 1968, pp. 75-82.
38. Reference Manual For Pencil Follower Type PF1000, d-mac Ltd., Queen Elizabeth Avenue, Glasgow, S.W. 2.
39. Small Computer Handbook - Digital Equipment Corporation.
40. Krishna, R.; "Interface Unit - Pencil Follower/PDP-8", Interim Report, Cartographic Project, University of Saskatchewan, 1967.
41. Kuo, B.C.; Automatic Control Systems. Prentice-Hall Inc., Publishers, 1965.
42. Muirhead and Company Ltd.; Size 18, Model 18 M10D2 Servomotor-tachometer generator data sheet.
43. Bailey, S.J.; "Incremental Servos, Part I", Control Engineering, Vol. 7, No. 11, Nov., 1960, pp. 123-127.
44. Superior Electric, Data Sheets on Slo-Syn Stepping Motors.

45. Nexus Research Laboratories, Inc., Data Sheets on the SQ-10, SA-10, SK-10, all silicon, solid state operational amplifiers.
46. Cochrun, B.L.; Transistor Circuit Engineering, The MacMillan Company, 1967.
47. Digital Logic Handbook - 1967, Digital Equipment Corporation.

APPENDICES

Appendix A

Interfacing the d-mac Pencil Follower

to

the PDP-8 Computer

The interface is the device responsible for transferring binary information from the 16 switch contacts of each X and Y shaft encoder into the computer. To describe its operation a brief description of the PDP-8 (Programmed Data Transfer) computer manufactured by Digital Equipment Corporation (DEC) is necessary.

The PDP-8 is a small scale, general purpose digital computer using 12 bit, two's complement arithmetic. It has a 4096 word, random-address magnetic core memory with a 1.5 microsecond cycle time. The word length is fixed at 12 bits. Standard features of the system include indirect addressing and facilities for instruction skipping and program interruption as functions of input-output device conditions. Addition is performed in 3.0 microseconds, subtraction in 6.0 microseconds.

The PDP-8 has a Teletype Model 33ASR (Automatic Send Receive) which can type in or print out at a rate of up to ten characters per second, or read in or punch out perforated tape at a ten character per second rate. Besides this standard equipment, the PDP-8 at the University of Saskatchewan has Type PC02 High Speed Perforated Tape Reader, a Type PC03 High Speed Tape Punch Control, an Extended Arithmetic Unit (EAU), and two magnetic DECTape units. These options provide improved input-output transfer rates, faster multiplication and division, and increased storage facilities.

The accumulator of the digital computer is used as the connecting link between the peripheral equipment interface and the computer. All data must be transferred through this 12 bit register, if

Programmed Data Transfers are used. Three function commands are available to generate IOP1, IOP2, and IOP4 pulses under computer control.* These pulses can be used to sense and initiate the data transfer to or from a specified external device.

The Pencil Follower has two modes of operation, point and line digitizing. When digitizing in the point mode, the operator moves the stylus to the location he wishes to digitize, and signals the computer for the transfer of the X-Y co-ordinates from the shaft encoders by pressing a footswitch. Continuous digitizing of line information requires the line mode of operation. The digital co-ordinates are transferred at a rate controlled by an external clock. Transfer is initiated by pressing the footswitch and discontinued upon its release. These two modes of operation are accomplished with the interface to be described here in.

The X and Y co-ordinates are available from the 16 bit shaft encoders, one for each axis. These switch contact combinations are transferred to the accumulator by footswitch operation in the point or line mode. Since the accumulator is a 12 bit register and the shaft encoders consist of 16 bits, complete transfer is executed in two phases. In the first phase, the least significant 12 bits are transferred directly into the accumulator and the most significant 4 bits held in a buffer register. The second phase transfers these 4 digits into the accumulator, completing the transfer for one axis. The transfer for the other axis is performed in an identical manner.

* Digital "Small Computer Handbook" - Digital Equipment Corporation.

A block diagram of the Pencil Follower Interface is shown in Figure A1. (See reference No. 40.) A brief explanation of its operation is necessary to provide sufficient background for its use when testing the Pencil Follower.

The FLAG is set by the footswitch in the point mode and by the external clock when the footswitch is depressed in the continuous (line) mode. The output of the FLAG is connected to the Program Interrupt Request Bus and the Input/Output Skip Bus. The IOT1 pulse is used to check the status of the FLAG, and the IOT4 pulse is used to clear it.

Since the same buffer registers and diode gates are used for both the X and Y axis transfer, the X-Y control Flip Flop allows for the correct axis transformation. It is always cleared initially by power clear pulses so that the X co-ordinate is transferred first. The IOT4 pulse complements this Flip Flop so that the Y co-ordinate is read in after the X. The next IOT4 pulse clears the Flip Flop to read X again, and the cycle is complete.

The Pencil Follower was assigned device number 13 and the device selector identifies the address and generates the IOT pulses from the IOP pulse issued under program control. The functions of these pulses are indicated in the table on page 89.

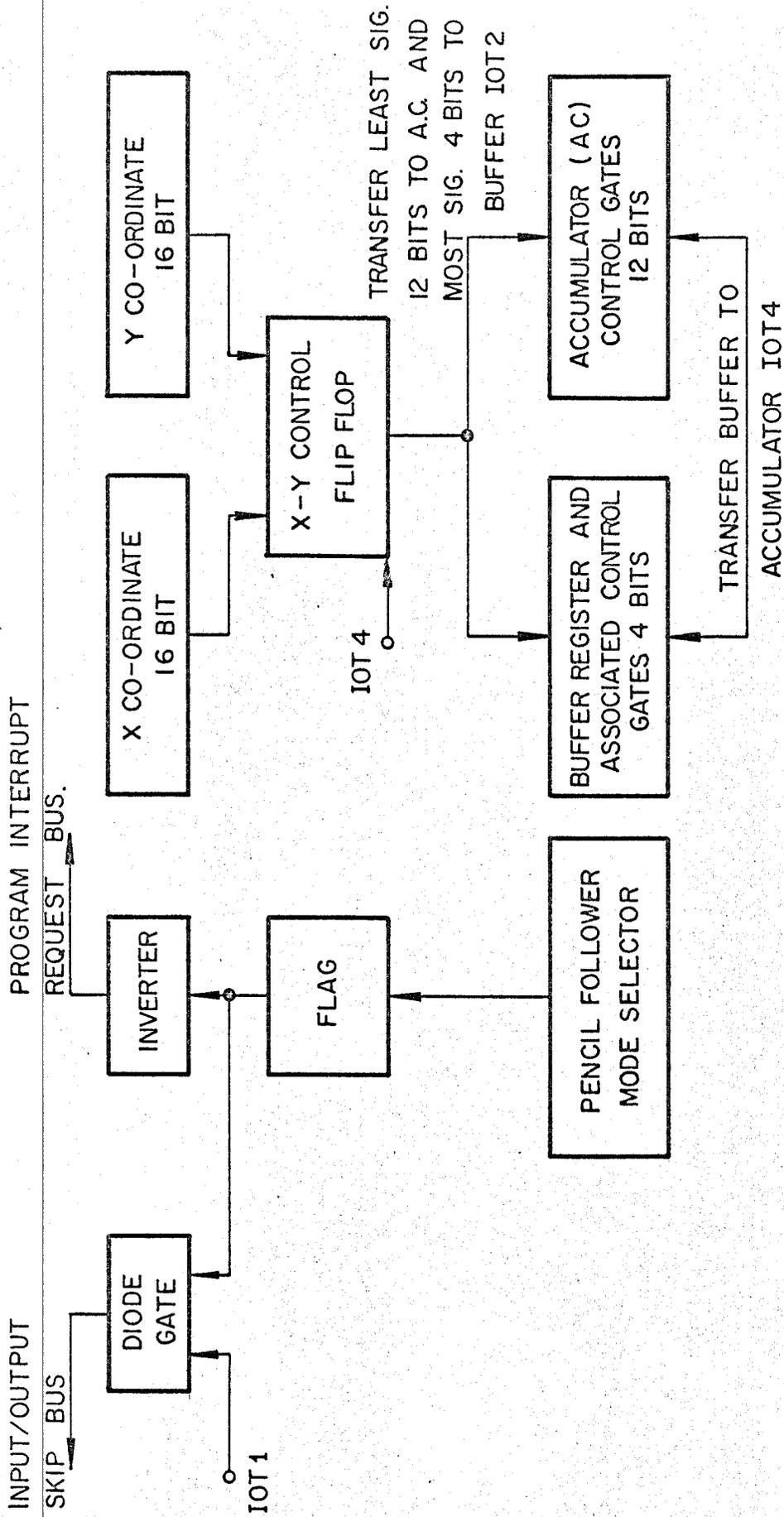


FIG-A1 BLOCK DIAGRAM OF THE PENCIL FOLLOWER INTERFACE.

TABLE 1

IOT Pulse Functions

IOT Pulse	Octal Code	Mnemonic Code	Operation
IOT 1	6131	SPF	Skip the next instruction if the Pencil Follower Flag is set. (check PF flag).
IOT 2	6132	RPL	Read Least Significant 12 bits into the accumulator and 4 most significant bits into the buffer register.
IOT 4	6134	RPM	Read most significant 4 bits from the buffer register into the accumulator Complement the control Flip Flop Clear Pencil Follower Flag

Thus, to read in both X and Y Co-ordinates two RPL and RPM instructions are generally used, so that a typical program might be:

Clear accumulator

SPF

Jump back one instruction

RPL

Store X (least sig. 12 bits)

RPM

Store X (most sig. 4 bits)

RPL

Store Y (least sig. 12 bits)

RPM

Store Y (most sig. 4 bits)

Appendix B

Test Apparatus

A mechanical apparatus shown in Figure B1, was constructed to move the pen (stylus) along the 40" length of table in a straight line by manual means or under motor control. Aluminum was used wherever possible, to prevent distortion of the magnetic field. All magnetic material was placed a sufficient distance from the pen to have negligible effect on the magnetic field.

The block carrying the stylus is moved by the drive wire with the lineal ball bushings providing smooth, precision lineal motion. Different motors and gear boxes were used with this apparatus to obtain the desired form of stylus movement. A description of the test procedures for each of the experiments performed will now be presented.

(i) Voltage position tests:

The purpose of this test was to obtain the constant voltage contours for the X and Y axes detector coils. To accomplish this a grid of radial lines were constructed (spaced 10°) with the origin at the center of the table. The test apparatus was placed so that the stylus moved along these lines in 0.05" increments. A stepping motor which moved in 1.8° steps and a 3.18" diameter drive pulley were used to perform this movement. The signals from the detector coils were disconnected from the electronics so that the follower remained stationary. Thus, the voltage induced in the detector coils was measured as a function of X and Y displacement from the null point. The null point was placed in the center of the table by allowing the follower to follow the pen to the origin of the grid of radial lines. After this the detector

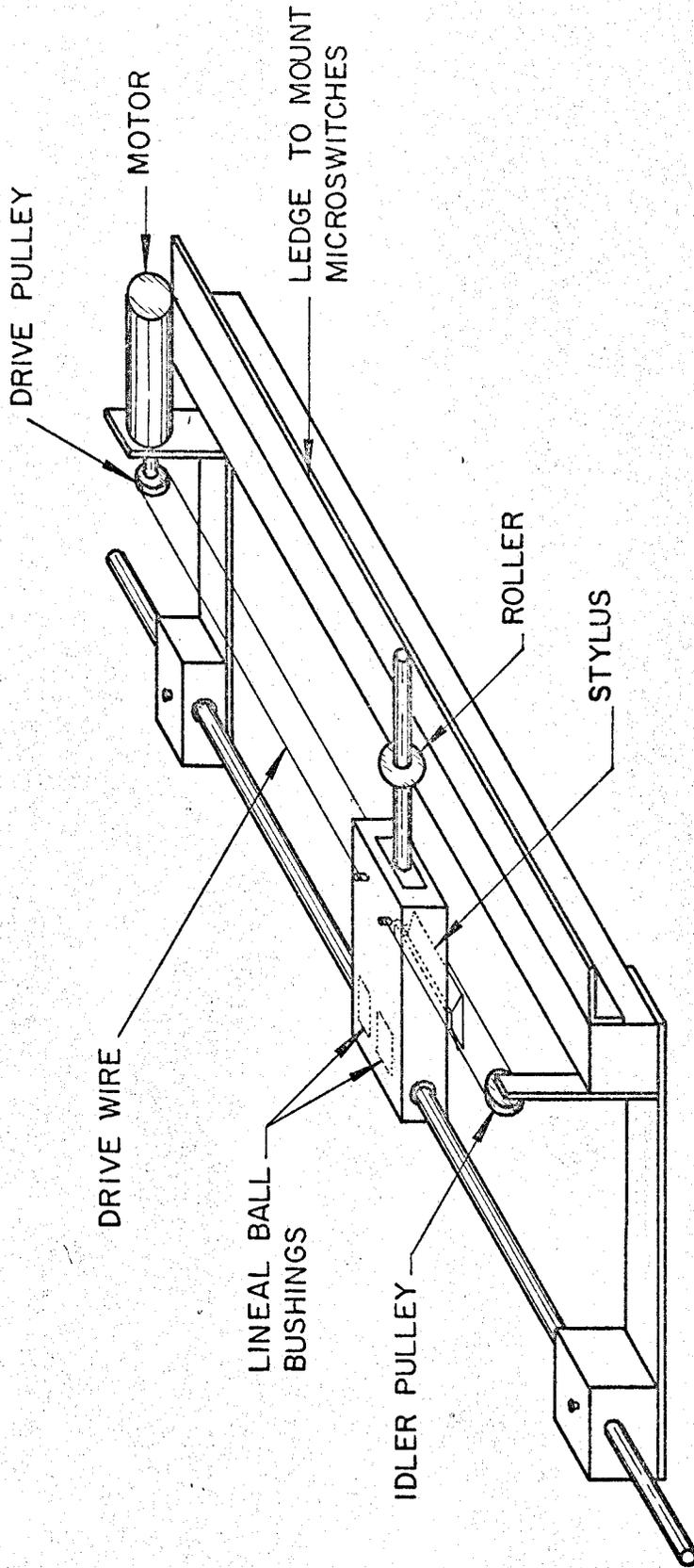


FIG.-BI TEST APPARATUS

coil signal was disconnected from the follower electronics to perform the measurements.

(ii) Lag test

The purpose of this test was to measure the lag between the follower and the stylus for different pen velocities. This was accomplished with a motor driving the block, containing the pen, at a constant velocity and reading the X-Y co-ordinates into the computer to determine the lag.

The basic test philosophy will now be described. Eight microswitches connected in series were mounted on the ledge indicated in Figure B1. These were used to replace the footswitch when the interface was operating in the point mode. Each time the block containing the pen passed a microswitch the X-Y co-ordinates of the follower were read into the computer. The first pass was run at a very low velocity (0.05in/sec), where the lag was less than 0.004", to initialize the microswitch positions. The following passes were run at the desired pen velocities and the co-ordinates of the follower were again read in as the pen passed the microswitches. These co-ordinates were then subtracted from the initializing ones to obtain the lag. The results of the eight lag measurements of the eight microswitches were averaged to obtain the average lag.

Measurements were taken along the X axis as this allowed more length for the drive motor to reach a constant speed. Pen velocities up to 7 in/sec were accomplished before the lag started to increase indefinitely. The eight microswitches, spaced at approximately 2" intervals, were used to indicate the reliability of these precision micro-

switches and to reveal the consistency of the lag over the entire table length. Program No. 1 of Appendix C was used extensively for this test.

(iii) Accuracy test

The mechanical assembly shown in Figure B1 was again used, with the motor replaced by a handwheel and a large reduction gear box. With this apparatus it was possible to move the pen in 0.001" movements monitored by a dial gauge.

The switch option of Program No. 1 Appendix C, was used in conjunction with the footswitch and the interface in the point mode to transfer the co-ordinates each time a 0.001" pen movement was completed. The accuracy and the repeatability were checked by moving a total length of 0.02" with the co-ordinates read in each 0.001" and then moving back to the starting point.

(iv) Orientation test

To measure the difference between the electrical and mechanical center, the mechanical center of the pen was constricted as shown in Figure B2 and the pen rotated. The follower co-ordinates were read in for each 10° of rotation by depressing the footswitch and using the switch option of Program No. 1, Appendix C.

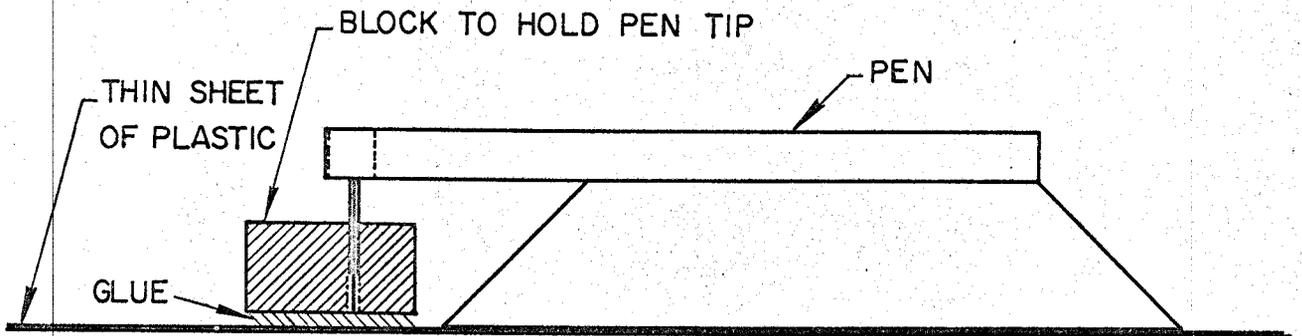
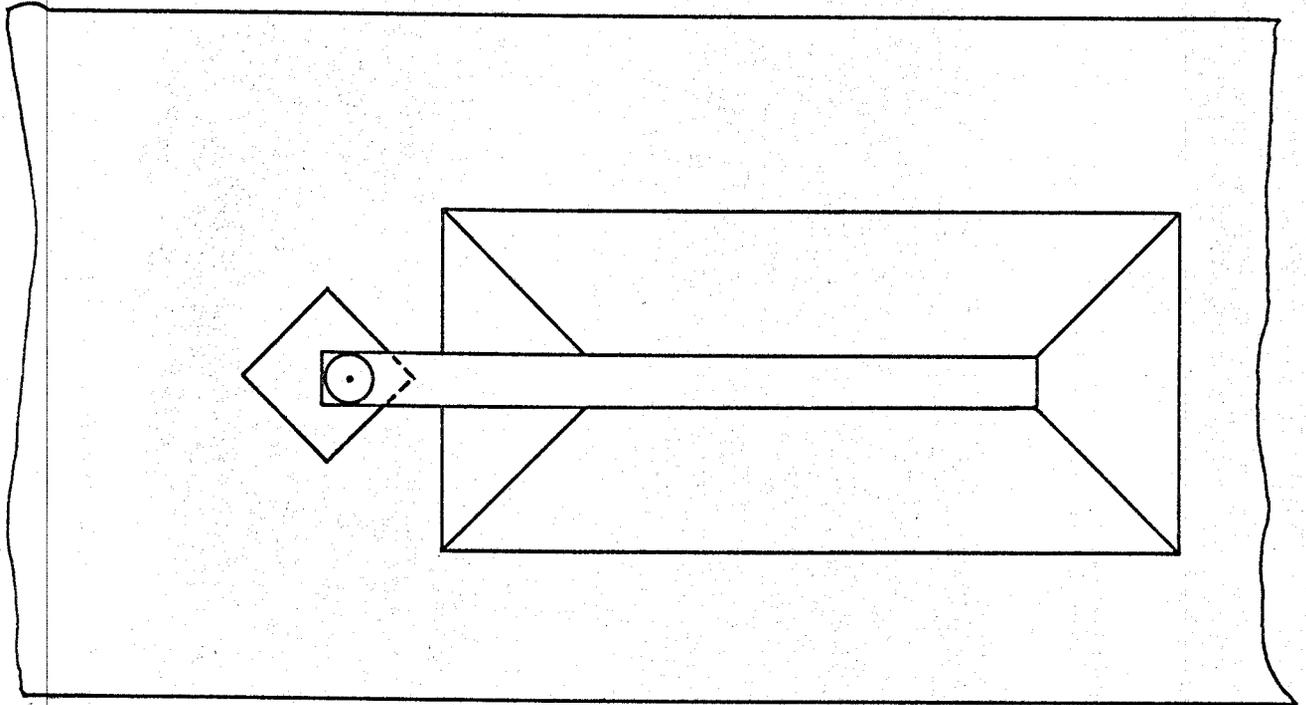


FIG.-B2 STYLUS AND APPARATUS FOR ORIENTATION TEST.

Appendix C

Computer Programs (PDP-8)

Two computer programs were written for the PDP-8 to accomplish the lag and response tests on the d-mac Pencil Follower. Both programs contain a switch option which allows type - out of the follower co-ordinates when the footswitch on the interface was pressed. This is useful for checking and also performing the accuracy and orientation tests.

(i) Program Number 1 - Program for lag tests.

The lag is to be measured at each of eight microswitches for various pen velocities. The first pass is to establish the initial position of the microswitches by moving the pen at a very low velocity (0.05 in/sec) where the lag is less than 0.004". After initializing the switch location, the tests can be performed at the desired pen velocities.

The flow chart for this program is shown in Figure C1. Two subroutines, RGTOB and SDPRNT were used to convert the Pencil Follower Code to binary and type out the lag in double precision seven place decimal format, respectively. The page allocations in the computer memory (See Computer Handbook for PDP-8 Programming) for this program were:

Page 1 - Main Program,
Page 2 - SDPRNT subroutine,
Page 3 - RGTOB subroutine,
and Page 4 - Data Store.

The main program is tabulated on page 100.

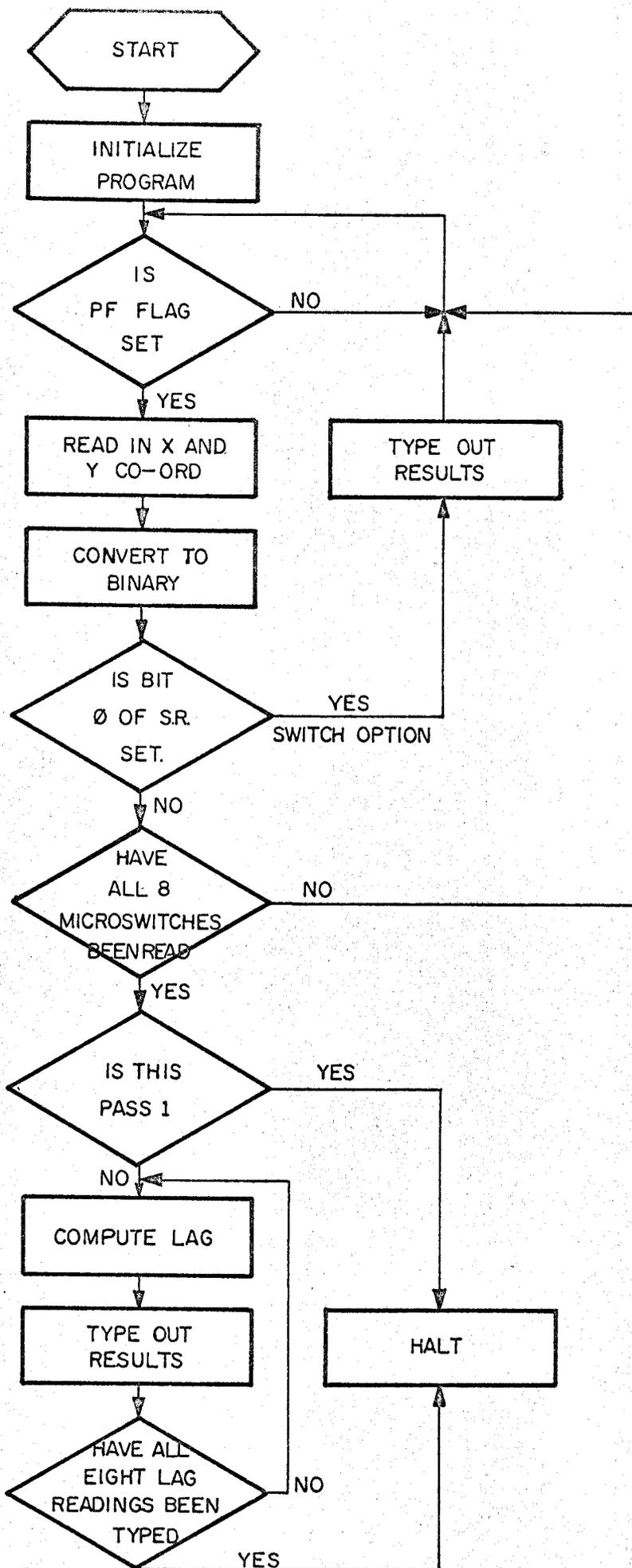


FIG.-C1 FLOW CHART FOR PROGRAM NO. 1

Program Number 1

/P. DENNIS PENCIL FOLLOWER LAG TESTS

PAGE 1

IOF

S=20

BEGN1,

CLA CLL

Initialize
Program

CMA

DCA PSNO

TAD K777

DCA Z 10

BEGN2,

CLA

TAD K7760

DCA TYCT

TAD K777

DCA Z 11

TAD K1037

DCA Z 12

IAC

CMA

DCA PRNO

READ,

SPF

Read in X and
Y Co-ordinates.

JMP .-1

RPL

DCA S+1

RPM

DCA S

RPL

DCA S+3

RPM

DCA S+2

JMS I RGTB

OSR

SPA CLA

JMP TYPE

TAD S+1

DCA I Z 10

TAD S

DCA I Z 10

TAD S+3

DCA I Z 10

TAD S+2

DCA I Z 10

ISZ SWCT

JMP READ

ISZ PSO

JMP .+3

HLT

Convert to Binary.
Check if S.R. bit 0
is a "1".
Type Out Results.
Restore Binary Equivalent
of the Co-ordinates.Check if all Co-ord.
Read.
Is this Pass No. 1?

	JMP	BEGN 2	Reset Storage
	TAD	K1037	Location Counter.
	DCA	Z 10	
SUB,	TAD I Z	12	Subtract from
	CLL	NEG	Initial Co-ord.
	TAD I Z	11	
	DCA	SBR+1	
	CML	RAL	
	TAD I Z	12	
	NEG		
	TAD I Z	11	
	DCA	SBR	
	JMS I	SDNT	Type Out Results.
	SBR		
	ISZ	PRNO	Print two spaces.
	JMP	.+5	
	JMS	CRLF	Do Carriage Return
	CLA	IAC	Set PRNO to -2
	CMA		
	DCA	PRNO	
	JMS	SPACE	
	ISZ	TYCT	Check if all Co-ord.
	JMP	SUB	Typed Out.
	JMP	SUB-4	
TYPE,	JMS	CRLF	Type Out Sub-
	JMS I	SDNT	routine
	S		
	JMS	SPACE	
	JMS I	SDNT	
	S+2		
	JMS	CRLF	
	JMP	READ	
K777,	777		
K7760,	7760		
K7770,	7770		
K1037,	1037		
PSNO,	0		
TYCT,	0		
SWCT,	0		
RGTB,	RGTOB		
SBR,	0		
	0		
SDNT,	SDPRNT		
PRNO,	0		
CRLF,	0		
	CLA		
	TAD	(215)	
	JMS	PRINT	
	TAD	(212)	
	JMS	PRINT	
	JMP I	CRLF	Space and Carriage Return Subroutine

PRINT,

Ø
TLS
TSF
JMP .-1
CLA
JMP I PRINT

Type out Subroutine

SPACE,

Ø
CLA
TAD (24Ø)
JMS PRINT
TAD (24Ø)
JMS PRINT
JMP I SPACE
\$

Space Print Out
Subroutine

The procedures to be used with this program are:

A. Switch Option For Type Out of Single Co-ordinates

- (1) Enter the necessary programs into the computer memory.
- (2) Load address 2000, and set bit 0 of the Switch Register (S.R.) to a 1 for the switch option.
- (3) Start program.
- (4) Move the stylus to the desired position and depress the foot-switch to initiate co-ordinate transfer and type out results. Once the type out has been completed the program is waiting for another co-ordinate to be transferred.

B. Procedure for Lag Tests

(i) Pass 1 to initialize switch positions

- (1) Load in the appropriate programs.
- (2) Move the assembly to the starting position.
- (3) Load address 2000 and leave bit 0 at 0 (bypassing switch option).
- (4) Start the Program.
- (5) Move assembly at 0.05 in/sec to initialize switch positions. After initializing all eight positions the computer will halt.
- (6) The compute is now ready for the lag passes.

(ii) Lag Passes

- (1) Move the assembly to the starting position.
- (2) Press continue to start the program running and run the test at the desired pen velocity. After the eight microswitches have been activated the results will be typed out and the computer will HALT.

- (3) To perform another test move the assembly back to the starting position and go to step 2.

NOTE: The interface was operated in Point Mode for Program Number 1.

Program Number 2 - Program for Response Tests

The purpose of this program is to measure the response of the follower to step changes in pen position. This was accomplished by reading in the co-ordinates at finite time intervals specified by an external clock available on the interface. The frequency of the clock was set at 100 Hz so the co-ordinates were transferred in every 10 milliseconds. A microswitch, used in place of the footswitch, was mounted such that the clock was enabled when the follower moved from its starting position. The step movement in position of the stylus was simulated by constraining the follower movement, then moving the pen the desired distance, and then releasing the follower. Upon the release of the follower, the microswitch activated the clock so that a 1000 co-ordinates were read in at 10 millisecond intervals.

The flow chart for this program is shown in Figure C2. The two subroutines RGTOB and SDPRNT were again used (See Program No. 1). The program has the switch option for typing individual co-ordinates. This has the capability of reading in 1000 co-ordinates and typing out the difference between these and the initial ones. Only the X co-ordinates were stored because of limited storage in the computer memory and the test being performed only along the X axis.

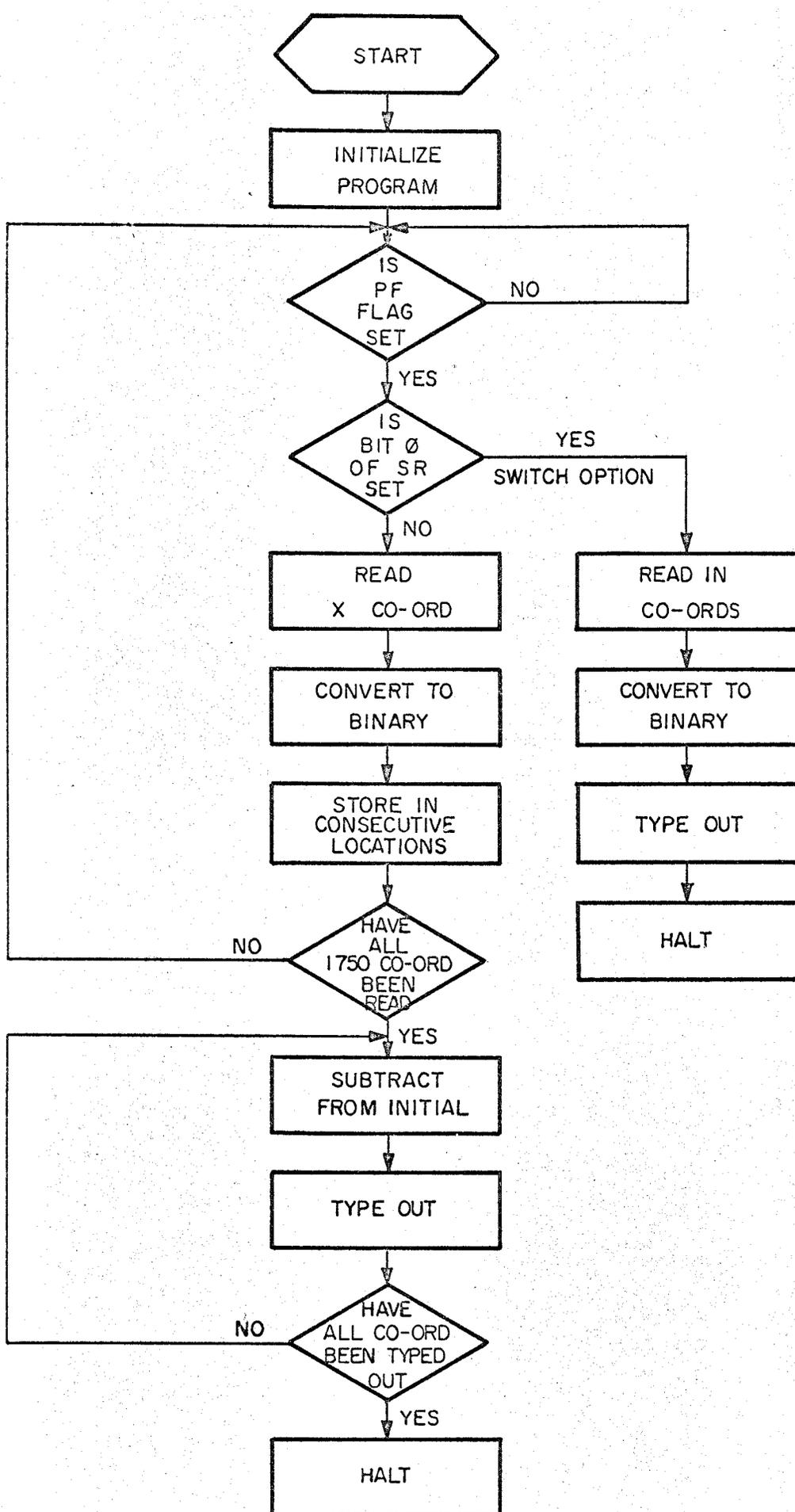


FIG.-C2 FLOW CHART FOR PROGRAM NO. 2

The page allocations are:

Page 1 - Main Program,

Page 2 - SDPRNT Subroutine,

Page 3 - RGTOB Subroutine,

and Pages 4 - 36 - Data Storage.

The main program is tabulated on page 107.

Program Number 2

/P. DENNIS

RESPONSE TESTS

*200

IOF

S=20

T=30

BEGN,

CLA		Initialize
TAD	K777	Program

DCA Z 10

TAD KN1750

DCA CRCNT

TAD KN1750

DCA TYCNT

READ,

CLA Check P.F. Flag

SPF

JMP .-1

OSR

SPA CLA

JMP HT+2

RPL

DCA Z S+1

RPM

DCA Z S

RPL

RPM

CLA

JMS I RGTB

TAD Z S+1

DCA I Z 10

TAD Z S

DCA I Z 10

ISZ CRCNT

JMP READ

TAD K777

DCA Z 10

SUB,

TAD I Z 10

CLL NEG

TAD Z T

DCA DIFF+1

CML RAL

TAD I Z 10

NEG

TAD Z T+1

DCA DIFF

JMS I SDNT

DIFF

JMS CRLF

ISZ TYCNT

JMP SUB

HT,

HLT

Initialize
Program

Check P.F. Flag

Check Bit 0 of S.R.

Read in X and Y
Co-ordinates, But
Store only X.Convert the X Co-ord.
to Binary.
Restore X Co-ord.Check if all Co-ord.
Read In.
Reset Storage
Location Counter.
Subtract From
Initial Co-ord.

Type Out Results.

Type Out Line
Feed and Carriage
Return.Check if All
Co-ord. Typed.

	JMP	BEGN	
	RPL		Read in X and Y Co-ord.
	DCA Z	S+1	
	RPM		
	DCA Z	S	
	RPL		
	DCA Z	S+3	
	RPM		
	DCA Z	S+2	
	JMS I	RGTB	Convert to Binary. Restore Co-ord.
	TAD Z	S+1	
	DCA Z	T	
	TAD Z	S	
	DCA Z	T+1	
	JMS	TYPE	Type Out
	HLT		
	JMP	READ	
TYPE,	Ø		Type Out Sub- Routine
	JMS	CRLF	
	JMS I	SDNT	
	S		
	JMS	SPACE	
	JMS I	SDNT	
	S+2		
	JMS	CRLF	
	JMP I	TYPE	
CRLF,	Ø		Carriage Return and Line Feed Subroutine
	CLA		
	TAD	(215)	
	JMS	PRINT	
	TAD	(212)	
	JMS	PRINT	
	JMP I	CRLF	
PRINT,	Ø		Type out Sub- Routine
	TLS		
	TSF		
	JMP	.-1	
	CLA		
	JMP I	PRINT	
SPACE,	Ø		Space Print out Subroutine
	CLA		
	TAD	(24Ø)	
	JMS	PRINT	
	TAD	(24Ø)	
	JMS	PRINT	
	JMP I	SPACE	
K777,	777		
KN175Ø,	-175Ø		
RGTB,	RGTOB		
SDNT,	SDPRNT		
DIFF,	Ø		

CRCNT,
TYCNT,
\$

Ø
Ø
Ø

Procedure For Response Tests

- (1) Load the necessary programs into the computer.
- (2) Load address 200, set bit 0 of the S.R. to "1", and start the program.
- (3) Put the follower in the initial position and transfer the digital co-ordinate into the computer. The X, Y co-ordinate will be read in, typed out and the program will HALT.
- (4) Set bit 0 of S.R. to 0 and press continue.
- (5) Release the follower.
- (6) After 10 sec. (1000 co-ordinates read in), type out will begin. This can be terminated after the co-ordinate output ceases to change.
- (7) To perform test again go to step 2.

Appendix D

Equivalent Motor Inertia

To calculate the motor time constant (See Appendix E), it is necessary to obtain the equivalent rotational inertia reflected to the motor shaft. The drive assembly can be represented by Figure D1. The following assumptions are made to simplify this derivation:

- (1) The weight of the drive wire is negligible,
- (2) The drive wire does not stretch,
- (3) All frictional effects are negligible, and
- (4) The drive wire does not slip on drive pulley or the idler pulleys.

The symbols used in this development are:

- J_D - drive pulley inertia,
- J_{P2} - idler pulley (radius r_2) inertia
- J_{P3} - idler pulley (radius r_3) inertia
- r_1 - drive pulley radius
- r_2 - idler pulley (inertia J_{P2}) radius
- r_3 - idler pulley (inertia J_{P3}) radius
- θ - rotation angle of drive pulley
- θ_2 - rotation angle of idler pulley (radius r_2)
- θ_3 - rotation angle of idler pulley (radius r_3)
- n_2 - number of idler pulleys (radius r_2)
- n_3 - number of idler pulleys (radius r_3)
- x - lineal motion of mass M ,
- M - Mass of the gantry
- and N - gear ratio of the gear box.

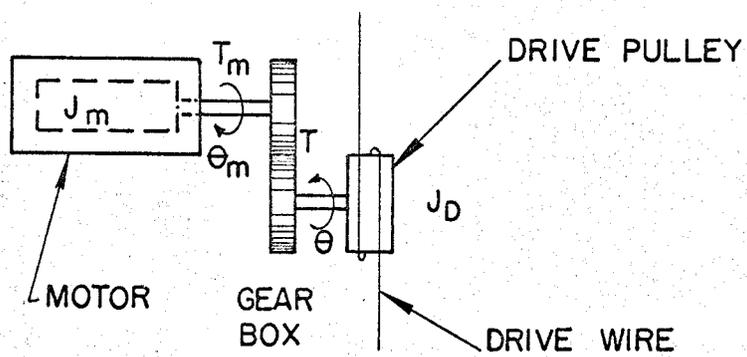
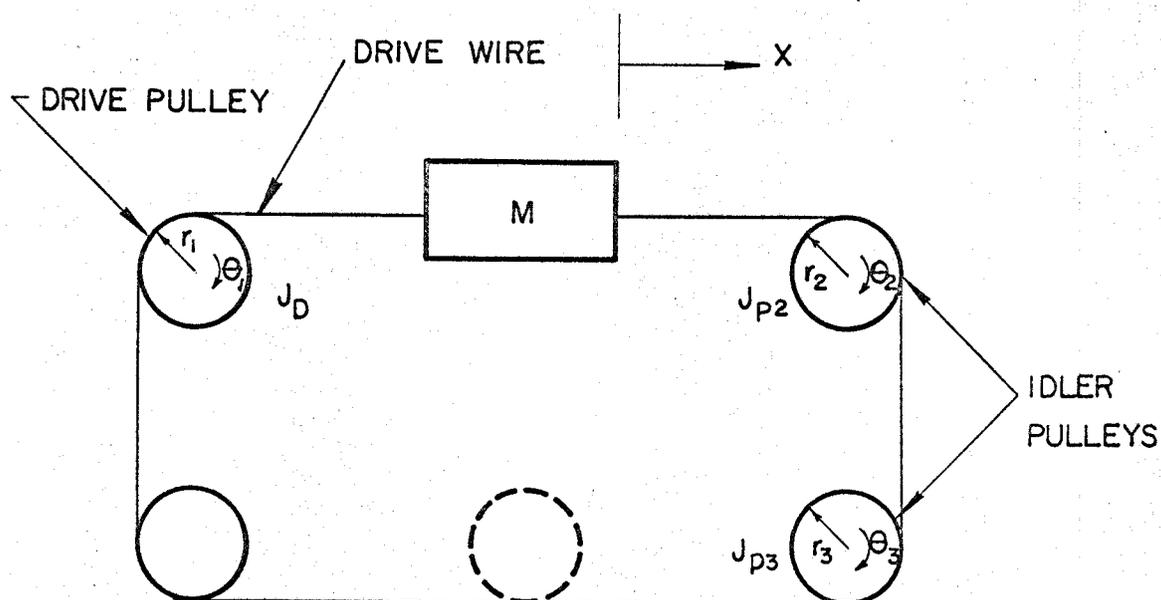


FIG.- D1 DRIVE ASSEMBLY.

The Force (F) on the drive wire due to the motion of mass M and the rotation of the idler pulleys is given by:

$$F = M \frac{d^2x}{dt^2} + F_2 + F_3 \quad \dots(1)$$

where

$$F_2 = n_2 T_2/r_2 = n_2 \frac{J_{P2}}{r_2} \frac{d^2\theta_2}{dt^2} \quad \dots(2)$$

and

$$F_3 = n_3 T_3/r_3 = n_3 \frac{J_{P3}}{r_3} \frac{d^2\theta_3}{dt^2} \quad \dots(3)$$

Since $\theta_2 r_2 = \theta_3 r_3 = x$, equation (1) becomes

$$F = \left(M + n_2 \frac{J_{P2}}{r_2} + n_3 \frac{J_{P3}}{r_3} \right) \frac{d^2x}{dt^2} \quad \dots(4)$$

This Force produces a torque on the drive pulley,

$$T = Fr_1 = \left(M + \frac{n_2}{r_2} J_{P2} + \frac{n_3}{r_3} J_{P3} \right) r_1 \frac{d^2x}{dt^2} \quad \dots(5)$$

Since $\theta r_1 = x$, this becomes,

$$T = r_1^2 \left(M + \frac{n_2 J_{P2}}{r_2} + \frac{n_3 J_{P3}}{r_3} \right) \frac{d^2\theta}{dt^2} \quad \dots(6)$$

Thus the torque on the drive shaft is give by:

$$T_t = [J_D + r_1^2 \left(M + \frac{n_2 J_{P2}}{r_2} + \frac{n_3 J_{P3}}{r_3} \right)] \frac{d^2\theta}{dt^2} \quad \dots(7)$$

To obtain the equivalent motor inertia this must be transferred through the gear box. Assuming an ideal gear box of gear ratio N , the reflected retarding torque on the motor shaft becomes,

$$T_r = N^2 \left[J_D + r_1^2 \left(M + \frac{n_2 J_{P2}}{r_2} + \frac{n_3 J_{P3}}{r_3} \right) \right] \frac{d^2 \theta_m}{dt^2} \quad \dots (8)$$

Thus, the equivalent motor inertia becomes,

$$J_{meq} = N^2 \left[J_D + r_1^2 \left(M + \frac{n_2 J_{P2}}{r_2} + \frac{n_3 J_{P3}}{r_3} \right) \right] + J_m \quad \dots (9)$$

For this system along the X axis of movement

$$M = 84.8 \text{ oz.}$$

$$J_D = 0.526 \text{ oz in}^2$$

$$J_{P2} = 0.0442 \text{ oz. in}^2$$

$$J_{P3} = 0.1303 \text{ oz. in}^2$$

$$n_2 = 4$$

$$n_3 = 4$$

$$r_2 = 0.938 \text{ in.}$$

$$r_3 = 1.0 \text{ in.}$$

$$r_1 = 0.6205 \text{ in.}$$

$$J_m = 0.029 \text{ oz in}^2$$

$$N = 1/20$$

$$\text{and thus, } J_{meq} = 0.1126 \text{ oz in}^2 \quad \dots (10)$$

Appendix E

A.C. Servo Motor Transfer Function

The torque-speed curves for the Model 18 M10D2 servomotor used in the d-mac Pencil Follower are given in Figure E1. These are linearized for the speeds encountered while digitizing data (0 - 1000 r.p.m.). The slope of the parallel, equally spaced approximation to the curves is

$$m = 2.71 \times 10^{-3} \frac{\text{oz. in. sec.}}{\text{rad.}}$$

If V = rated control voltage and k is the ratio of the torque intercept and the control voltage (i.e. $k = \frac{1.28 \text{ oz. in.}}{1/2 V}$), then for any torque T_m , the family of straight lines is represented by

$$T_m = kV + m \frac{d\theta_m}{dt} \quad \dots(1)$$

Equation (1) becomes

$$kV + m \frac{d\theta_m}{dt} = J_m \frac{d^2\theta_m}{dt^2} + f_m \frac{d\theta_m}{dt} \quad \dots(2)$$

where

J_m = motor inertia

f_m = motor viscous friction

and θ_m = motor angular rotation.

In Laplace transform notation this becomes

$$\theta_m(s) [sm - s^2 J_m - s f_m] = -kV_2(s) \quad \dots(3)$$

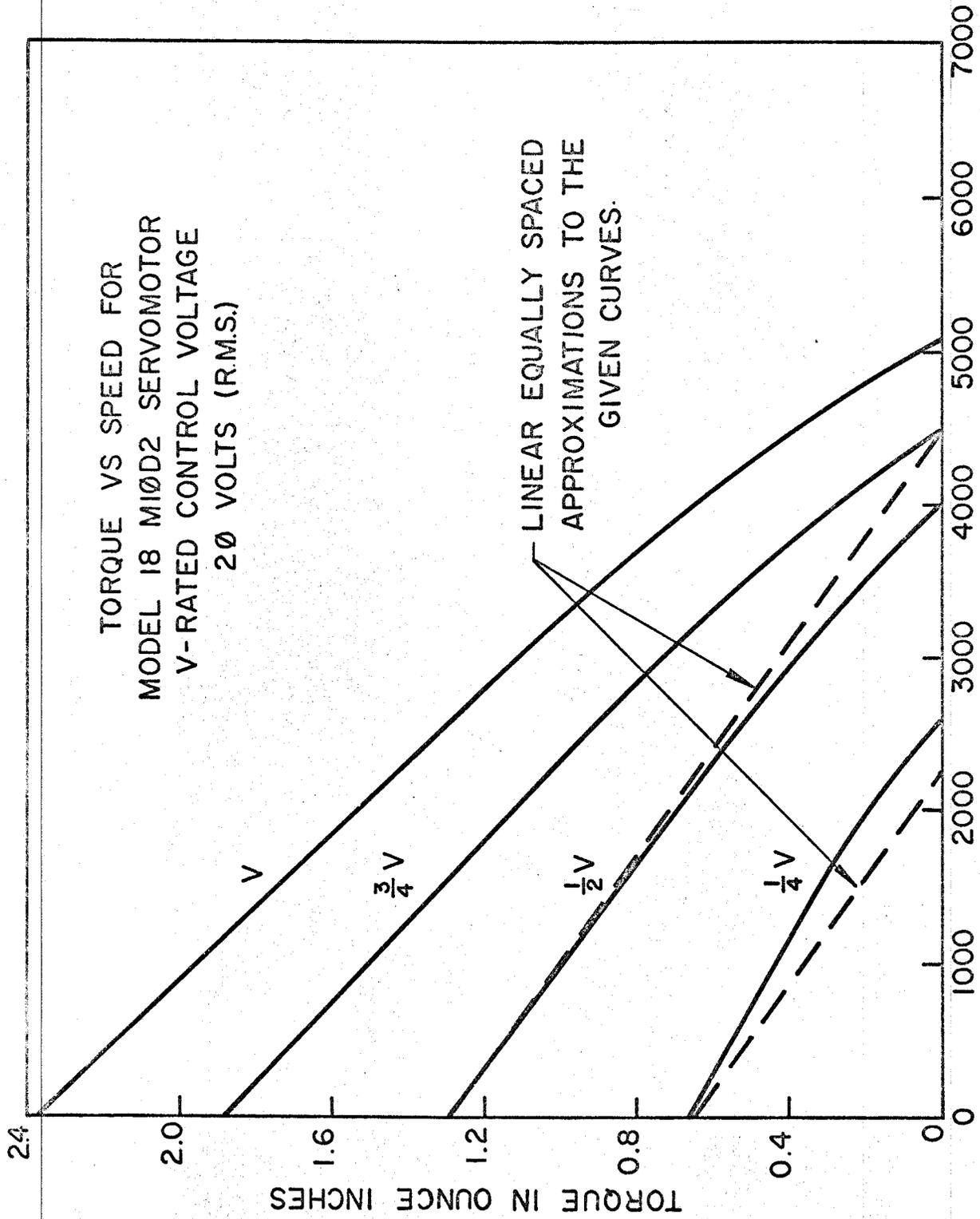


FIG.-E1 TORQUE SPEED CURVES

Thus,

$$\frac{\theta_m(s)}{V(s)} = \frac{k}{s[(f_m - m) + sJ_m]} \quad \dots (4)$$

which can be written as:

$$\frac{\theta_m(s)}{V(s)} = \frac{K_m}{s(1 + s\tau_m)} \quad \dots (5)$$

where

$$K_m = \frac{k}{f_m - m} = \text{motor gain constant,}$$

and

$$\tau_m = \frac{J_m}{f_m - m} = \text{motor time constant.}$$

If the effects of viscous friction are neglected, $f_m = 0$ and equation (5) is reduced to:

$$\frac{\theta_m(s)}{V(s)} = \frac{47.2}{s(1 + 0.1075s)} \frac{\text{rad.}}{\text{volt}} \quad \dots (6)$$

as $m = 2.71 \times 10^{-3} \frac{\text{oz.in.sec.}}{\text{rad.}}$

$$k = 0.128 \frac{\text{oz.in}}{\text{volt}}$$

$$K_m = 47.2 \text{ rad/volt sec., and}$$

$$\tau_m = 0.1075 \text{ sec.}$$

References:

- (1) Kuo - Automatic Control Systems, Prentice-Hall Inc., 1962, pp. 106-112.
- (2) Fitzgerald and Kingsley - Electric Machinery, McGraw-Hill Inc., 1961 pp. 312-315.

Appendix F

System Transfer Function

A block diagram representation for one axis of movement is indicated in Figure F1. This can be reduced as shown in Figure F2,

where

$$G_3(s) = \frac{G_1(s) G_m(s)}{1 + H_m(s) G_1(s) G_m(s)},$$

and

$$G_4(s) = G(s) G_2(s) G_3(s)$$

Now the closed loop transfer function becomes

$$\begin{aligned} \frac{Y(s)}{X(s)} &= \frac{G_4(s)}{1 + G_4(s)} \\ &= \frac{G_1(s) G_2(s) G(s) G_m(s)}{1 + H_m(s) G_1(s) G_m(s)} \cdot \frac{1}{1 + \frac{G_1(s) G_2(s) G(s) G_m(s)}{1 + H_m(s) G_1(s) G_m(s)}} \end{aligned}$$

or

$$\frac{Y(s)}{X(s)} = \frac{G_1(s) G_2(s) G(s) G_m(s)}{1 + G_1(s) G_m(s) [H_m(s) + G_2(s) G(s)]} \dots (1)$$

Substituting:

$$G_1(s) = K_1$$

$$G_m(s) = \frac{K_m}{s(1 + s\tau_m)}$$

$$H_m(s) = s\beta_2 K_t$$

$$G_2(s) = K_2$$

and $G(s) = \beta_1 K_3$

equation (1) becomes

$$\begin{aligned} \frac{Y(s)}{X(s)} &= \frac{\beta_1 K_1 K_2 K_3 K_m}{s(1+s\tau_m)} \cdot \frac{1}{1 + \frac{K_1 K_m}{s(1+s\tau_m)} (s\beta_2 K_t + \beta_1 K_2 K_3)} \\ &= \frac{\beta_1 K_1 K_2 K_3 K_m}{s(1+s\tau_m)} \frac{s(1+s\tau_m)}{s(1+s\tau_m) + K_1 K_m (s\beta_2 K_t + \beta_1 K_2 K_3)} \end{aligned}$$

therefore

$$\frac{Y(s)}{X(s)} = \frac{\beta_1 K_1 K_2 K_3 K_m}{\tau_m} \cdot \frac{1}{s^2 + \left(\frac{1 + \beta_2 K_1 K_m K_t}{\tau_m}\right)s + \frac{\beta_1 K_1 K_2 K_3 K_m}{\tau_m}}$$

This is of the form

$$\frac{Y(s)}{X(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

where

$$\omega_n^2 = \frac{\beta_1 K_1 K_2 K_3 K_m}{\tau_m} \quad \dots (2)$$

and

$$\delta = \frac{1 + \beta_1 K_1 K_m K_t}{2 \sqrt{\beta_1 K_1 K_2 K_3 K_m \tau_m}}$$