INVESTIGATIONS OF MICROSTRIP
PERIODIC STRUCTURES

A Thesis
Submitted to the College of Graduate Studies and Research
in Partial Fulfillment of the Requirements
for the Degree of
Master of Science
in the
Department of Electrical Engineering
University of Saskatchewan

by

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Saskatoon, Saskatchewan
November 1990

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Periodic structures are used as microwave filters because of their inherent passband-stopband characteristics. The structure exhibits alternative passbands and stopbands. The widths and positions of the passbands and stopbands are changed with the parameters of the structure. The analysis of non-planar periodic structure filters are well documented in the literature. Recently, investigations of these structures in the microstrip configuration have also been reported. The structures under consideration in the past investigations were microstrip lines with sinusoidally varying width and triangularly width modulated microstrip lines.

The work reported in this thesis is an investigation of microstrip periodic structures with rectangular and circular geometries. Closed form expressions for the ABCD parameters of a structure with N unit cells are derived. The input-output characteristics of the structures are very similar to a bandpass/bandstop filter. The design procedure for a periodic structure bandpass filter is presented. Bandpass filters are designed and fabricated. Theoretical results show agreement well with the measurement. The variations of the bandwidth and center frequency with the dimensions of the structures are studied.
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( [A]_T )</td>
<td>Transfer Matrix of a Uniform Line</td>
</tr>
<tr>
<td>( [A]_J )</td>
<td>Transfer Matrix of a Junction with a Shunt Susceptance</td>
</tr>
<tr>
<td>( A_u, B_u, C_u, D_u )</td>
<td>Elements of Transfer Matrix of a Unit Cell</td>
</tr>
<tr>
<td>( A_{11}, A_{12}, A_{21}, A_{22} )</td>
<td>Elements of Wave Amplitude Transmission Matrix</td>
</tr>
<tr>
<td>( B )</td>
<td>Junction Susceptance</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>Junction Capacitance</td>
</tr>
<tr>
<td>( c )</td>
<td>Velocity of Light</td>
</tr>
<tr>
<td>( C_n^+, C_n^- )</td>
<td>Incident &amp; Reflected Waves at Port n</td>
</tr>
<tr>
<td>( C_{n+1}^+, C_{n+1}^- )</td>
<td>Incident &amp; Reflected Waves at Port n+1</td>
</tr>
<tr>
<td>( \cos )</td>
<td>Cosine Function</td>
</tr>
<tr>
<td>( \csc )</td>
<td>Cosecant Function</td>
</tr>
<tr>
<td>( d )</td>
<td>Length of a Unit Cell</td>
</tr>
<tr>
<td>( \overline{E} )</td>
<td>Electric Field</td>
</tr>
<tr>
<td>( f )</td>
<td>Frequency</td>
</tr>
<tr>
<td>( h )</td>
<td>Thickness of the Substrate</td>
</tr>
<tr>
<td>( \overline{H} )</td>
<td>Magnetic Field</td>
</tr>
<tr>
<td>( I_n, I_{n+1} )</td>
<td>Currents at Ports n and n+1</td>
</tr>
<tr>
<td>( J_n(x) )</td>
<td>Bessel Function</td>
</tr>
<tr>
<td>( J'_n(x) )</td>
<td>Bessel Function Derivative</td>
</tr>
<tr>
<td>( k_0 )</td>
<td>Wave Number</td>
</tr>
<tr>
<td>( \ln )</td>
<td>Logarithm to the Base e</td>
</tr>
<tr>
<td>( L_1, L_2 )</td>
<td>Junction Inductances</td>
</tr>
<tr>
<td>( \overline{M} )</td>
<td>Magnetic Current</td>
</tr>
<tr>
<td>( S_{11} )</td>
<td>Return Loss</td>
</tr>
<tr>
<td>( S_{21} )</td>
<td>Insertion Loss</td>
</tr>
<tr>
<td>( \sin )</td>
<td>Sine Function</td>
</tr>
<tr>
<td>( [S] )</td>
<td>Eigen Vector Matrix</td>
</tr>
<tr>
<td>( [S]^{-1} )</td>
<td>Inverse of Eigen Vector Matrix</td>
</tr>
<tr>
<td>( V_n^+, V_n^- )</td>
<td>Incident &amp; Reflected Voltage Waves at Port n</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$V_{n+1}^+, V_{n+1}^-$</td>
<td>Incident &amp; Reflected Voltages Waves at Port n+1</td>
</tr>
<tr>
<td>$W_1, W_2$</td>
<td>Widths of the Uniform Lines</td>
</tr>
<tr>
<td>$w$</td>
<td>Width of Microstrip Feed Line</td>
</tr>
<tr>
<td>WAT</td>
<td>Wave Amplitude Transmission</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Axes of Cartesian Coordinates</td>
</tr>
<tr>
<td>$Y_B^+, Y_B^-$</td>
<td>Bloch-wave Admittance in $+z$ and $-z$</td>
</tr>
<tr>
<td>$Z_{11}, Z_{12}, Z_{21}, Z_{22}$</td>
<td>Elements of Z-matrix</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>Characteristic Impedance of Thinner Line</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>Characteristic Impedance of Wider Line</td>
</tr>
<tr>
<td>$Z_B^+, Z_B^-$</td>
<td>Bloch-wave Impedance in $+z$ and $-z$</td>
</tr>
<tr>
<td>$Z_{in}$</td>
<td>Input Impedance</td>
</tr>
<tr>
<td>$Z_L$</td>
<td>Load Impedance</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>Characteristic Impedance of Microstrip Input and Output Lines</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Propagation Constant</td>
</tr>
<tr>
<td>$\delta(y)$</td>
<td>Dirac Delta Function of y</td>
</tr>
<tr>
<td>$\varepsilon_0 = 10^{-9}/36\pi$</td>
<td>Permittivity of Vacuum</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Relative Dielectric Constant</td>
</tr>
<tr>
<td>$\varepsilon_{\text{eff}}$</td>
<td>Effective Dielectric Constant</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Complex Propagation Constant</td>
</tr>
<tr>
<td>$\Gamma_0, \Gamma_n$</td>
<td>Characteristic Reflection Coefficient at the at the zeroth and nth terminal</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Free Space Wavelength</td>
</tr>
<tr>
<td>$[\Lambda]$</td>
<td>Eigen Value Matrix</td>
</tr>
<tr>
<td>$\mu_0 = 4\pi 10^{-7}$</td>
<td>Permeability of Vacuum</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$3.14159 \ldots$</td>
</tr>
<tr>
<td>$\theta, \theta_1, \theta_2$</td>
<td>Electrical Length of the Uniform Lines</td>
</tr>
<tr>
<td>$\rho, \phi, z$</td>
<td>Cylindrical Coordinate System</td>
</tr>
<tr>
<td>$r, \phi, \theta$</td>
<td>Spherical Coordinate System</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular Velocity</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Cut Off Frequency</td>
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1. Introduction

1.1. General

Planar microwave circuits (PMC) are suitable as circuit elements in microwave integrated circuits because of their planar configurations. The interest in PMCs arises mainly from their advantages with respect to volume, weight, reliability, cost and reproducibility. Combined with the recent developments in the microwave solid state devices PMCs have caused a radical change in the microwave technology far beyond what was thought possible in the beginning.

The basis of the PMCs is the microstrip transmission line which consists of a strip conductor separated from a ground plane by a dielectric layer (Figure 1.1). It can be used for passive components as well as for interconnections. Although microstrip is the most popular transmission line for the microwave integrated circuits other transmission lines being used are slotline, coplanar waveguide and coplanar strip. Cross-sectional views of

![Figure 1.1: Microstrip line configuration](image-url)
these lines are shown in Figure 1.2. There are several variations of the microstrip configuration that have also been suggested for use in the microwave integrated circuits. These include inverted microstrip, suspended microstrip, microstrip with overlay, strip dielectric waveguide and inverted strip dielectric waveguide. Cross-sectional views of these structures are illustrated in Figure 1.3.

The strip line shown in Figure 1.4 is another very commonly used transmission line which resembles microstrip line. Most of the basic circuit design information available for the strip line is also applicable to the microstrip line.

Figure 1.5 illustrates the electromagnetic fields distribution of the microstrip transmission line.

A complete RF circuit can be photo-etched on a dielectric, ferromagnetic or semiconductor substrate by microstrip technology and many active and passive components can be combined to provide overall signal generation or processing. This new technology has a very important impact on advanced system designs, particularly on the low power microwave components market.

Planar microwave circuits are now available to the microwave systems as a challenging alternative to conventional waveguides or coaxial circuits. Several waveguide systems have been successfully replaced by microstrip systems and the TEM (Transverse Electro-Magnetic) equivalent of the most of the waveguide components have been worked out.

The mode of propagation of the microstrip transmission lines is quasi-TEM which allows an easy approximate analysis and yields wide band circuits. The characteristic parameters are determined by the width of the strip, thickness and the dielectric constant.
Figure 1.2: Planar transmission lines used in microwave integrated circuits
Figure 1.3: Various transmission lines derived from microstrip
In spite of all these advantages, the limitations of the planar microwave circuits are poor power handling capacity, low Q factor and the inability to adjust the circuit parameters once the circuit is fabricated. Also the dielectric-air interface changes the mode of propagation from TEM to a hybrid mode and therefore, it is much more difficult to study the high frequency performance of the circuit.

Various methods of the analysis [2] of the microstrip transmission lines are available in the literature. The TEM-mode of the propagation is considered in the quasi-static
method. In this method the characteristics of the microstrip line are calculated from the electrostatic capacitance of the structure. Numerous method are available to calculate the electrostatic capacitances, some of them are conformal mapping [3] and integral equation method [4].

1.2. Literature Review of the Microstrip Lines

A basic building block of the microwave integrated circuits is the microstrip line. The theory and design of these lines have been reported in the technical literature. In this section the past work done on the microstrip lines and components using them will be briefly reviewed.

A number of researchers have worked on the rigorous formulation of the characteristic impedance, attenuation and propagation constant of the microstrip lines with various configurations. Wheeler [5] and Gupta et al [6] presented formulas for the characteristic impedances of a strip line and its conductor loss is given by Howe [7]. The effective length for the strip open end effect and the formula for the strip line step discontinuity have been developed by Altschuler and Oliner [8]. A conformal mapping technique was used by Collin [9] for the description of the characteristic impedance of a strip of arbitrary width placed midway between the two ground planes.

A simple synthesis formula for the width of a thin microstrip line is given by Wheeler [10]. This initial value for the width may be used in the formulas given by Hammerstad and Jensen [11] to obtain a very accurate value for the width and effective dielectric constant for a conductor with nonzero thickness. The added electrical length beyond the mechanical length of an open microstrip line is given by Hammerstad and Bekkadal [12]. The formula for microstrip discontinuity is developed by Gupta et al [6]. Bahl and Garg developed accurate formulas for the characteristic impedance, attenuation
due to the conductor loss and substrate dielectric loss for the microstrip line with a finite strip thickness [13].

Microstrip antennas have been developed utilizing the radiation characteristics of the microstrip resonators. Microstrip antennas have now become a topic of considerable research, and numerous articles on the theory and design of such antennas are available in the literature [14, 15, 16, 17].

1.3. Nonuniform Lines

Nonuniform lines have varying characteristic impedance and width in the direction of propagation. In the planar circuit designs nonuniform lines have been found to be of considerable importance in recent years. Initially, they were used in broadband matching, but now nonuniform lines have been used as circuit elements and resonators in the design of filters, directional couplers, etc [18, 19].

1.4. Periodic Structures

Transmission lines loaded at periodic intervals with a reactive element are referred to as a periodic structure. Such a loaded line may be an ordinary waveguide with periodically spaced identical obstacles inserted across it as shown in Figure 1.6. It may be a guide with periodically spaced tees, each similarly terminated, or it may be a more complicated structure such as a series of identical cavities each with two waveguide outputs, one on each side. The output of one cavity forming the input to the next. A coaxial transmission line can be made periodically loaded by introducing thin circular diaphragms at regular intervals. Also, there is a class of nonuniform microstrip lines where the nonuniformity is distributed periodically along the axial direction.

The periodically loaded transmission lines have many interesting and useful
properties. These properties have been studied by many investigators in the past. The analyses of non-planar type of periodic structures are well documented in the literature [20, 21, 22, 23]. Planar periodic structures are found to be of considerable importance because of their interesting and useful properties. These properties have been studied by many researchers in the past. The structures under consideration were (i) microstrip lines with sinusoidally varying width [18] and (ii) triangularly width modulated microstrip lines [19].

A simplified approach for analyzing a nonuniform line is modelling the structure as a number of small uniform transmission line sections of different widths, connected in cascade [18, 24, 25]. The overall wave amplitude transmission (WAT) matrix can be obtained by matrix multiplication and then from the resultant WAT-matrix various characteristics of the structure can be computed.

The Zig-Zag shaped structure has been analyzed by considering each cell of the structure as two radial lines of annular sectors, connected face to face [19]. From the solution of the wave equation, the electromagnetic fields inside the structure have been
obtained. Defining the modal voltage and modal current, the Admittance-matrix of each of the radial lines has been determined. By combining Admittance-matrices of the two radial lines, the overall Admittance-matrix and hence the WAT-matrix of the unit cell has been obtained.

1.5. Thesis Objective

From the literature survey, it is found that periodic structures with simple microstrip geometry have not yet been reported in the open literature. It is therefore, worthwhile to undertake the study of a microstrip periodic structure which can be used in the microwave integrated circuits. The microstrip periodic structures under consideration are formed with uniform lines periodically loaded by a distributed load. Such a periodically loaded microstrip line has been realized by (i) two different uniform microstrip lines periodically connected, (ii) a uniform microstrip line periodically loaded by open ended microstrip lines, (iii) a uniform microstrip line periodically loaded by circular microstrip conductors. Specific objectives for the present research work are:

1) Investigation of the filter like characteristics of the microstrip periodic structures.

2) Selection of the appropriate analytical technique which can accurately predict the performance of a given periodic structure.

3) Comparison of the performance of three different types of structures as bandpass and bandstop filters.

4) Development of a design procedure for a microwave filter using the structures under consideration.
1.6. Thesis Outline

In this chapter different types of planar structures are presented, and an updated literature review on periodically loaded microstrip lines is done.

The basic concept of the periodic structures is discussed in chapter 2. The conventional method for obtaining the propagation constant of a periodic structure is outlined, and a detailed study of the passband-stopband characteristics of the capacitively loaded coaxial transmission line is presented. The formulas for the characteristic impedance and input impedance of a terminated periodic structure are given. Also, the design procedure of a bandpass periodic structure filter is presented.

In the first part of chapter 3, a planar periodic structure formed with two different uniform lines is analyzed using the transmission line model. The effect of a step discontinuity is considered. Closed form expressions for the ABCD parameters of a symmetric structure with N cells are obtained. A bandpass periodic structure filter is designed and the theoretical results are computed and compared with the measurement.

The analysis of a uniform microstrip transmission line periodically loaded by open ended microstrip lines is also presented here. A bandpass filter is designed and fabricated and the experimental results are compared with the theory. The variations of the bandwidth and center frequency of the passband with the physical dimensions are also presented.

In chapter 4, the periodic structure discussed in the first part of chapter 2, is analyzed by field analysis. The wider line is considered as a rectangular cavity resonator and the thinner line as its feed lines at the input and output ports. The modal expansion method is invoked to obtain the electromagnetic fields inside the cavity. The equivalent
conductance for the radiation loss of the microstrip cavity is obtained. The ABCD parameters of the cavity are determined including the radiation loss. The radiation loss is also incorporated in the analysis.

A planar periodic structure formed with a uniform microstrip transmission line periodically loaded by microstrip circular patches is analyzed in chapter 5. The circular conductor is considered as a cylindrical cavity resonator with the uniform lines as its feed lines at the input and output ports. The fields inside the cavity are found by the modal expansion method, and the parameters of the ABCD-matrix of the circular cavity are obtained. A bandpass filter is designed, fabricated and the theoretical results are compared with the measured results. The effects of the various parameters of the periodic structure on the bandwidth and center frequency of the filter are studied.

Conclusions are presented in chapter 6.
2. Basic Concept of Periodic Structure

2.1. Introduction

The parameters of importance in a periodic structure are the Bloch-wave propagation constant and characteristic impedance. The Bloch-wave propagation constant is the phase shift per period, and characteristic impedance is defined as the impedance presented to the voltage and current waves at the input terminals of a unit cell. In a propagating mode, the wave travels without attenuation. The propagation constant and characteristic impedance are therefore, real quantities. In a nonpropagating mode, the above parameters are purely imaginary. In order to demonstrate the propagation characteristics of a periodic structure an example of a capacitively loaded coaxial cable is worked out. The expressions for the propagation constant and characteristic impedance are presented. The variation of propagation constant with frequency is also shown. The input impedance of a terminated periodic structure is obtained and the design procedure of a periodic structure as a bandpass filter is outlined.

2.2. Capacitively Loaded Coaxial Line

In this section, we shall consider a simple example of a capacitively loaded transmission line, in order to introduce the conventional method of analysis and typical properties of a periodic structure.

A coaxial transmission line can be made periodically loaded by introducing thin circular diaphragms at regular intervals, as shown in Figure 2.1. The diaphragm may be
Figure 2.1: Capacitive loading of a coaxial line by means of thin circular diaphragms.

machined as an integral part of the center conductor. The local storage energy is increased by the fringing electric field in the vicinity of the diaphragm. From a circuit viewpoint, this may be accounted for, by a shunt capacitance. The total field can be described in terms of the incident, reflected and transmitted dominant TEM modes and a superposition of an infinite number of higher-order modes. An approximate expression for the shunt susceptibility of the diaphragm is [20]

$$\bar{B} = \frac{B}{Y_c} = \frac{8(b-c)^2 \ln(b/a)}{\lambda_0 c[\ln(b/c)]^2} \ln \csc(\pi/2) \frac{b-c}{b-a}$$

where $Y_c = [60 \ln(b/a)]^{-1}$ 1/ohms is the characteristic admittance of an air filled coaxial line. The expression for $\bar{B}$ is accurate for $b-a < 0.1 \lambda_0$. In this region, $\bar{B}$ is directly proportional to $\omega$.

In order to analyze a periodic structure, an equivalent network for a single section or unit cell should be constructed. The voltage and current waves that may propagate along the network consisting of the cascade connection of an infinite number of the basic net-
work have to be analyzed. For the structure of Figure 2.1 an equivalent network of a basic section is a shunt normalized susceptance $\bar{B}$ with a length $d/2$ of transmission line on either side, as in Figure 2.2(a). The voltage-current relationships at the input and output of the n-th section in the infinitely long cascade connection are illustrated in Figure 2.2(b).

The ABCD-matrix of a unit cell is defined as

$$
\begin{bmatrix}
V_n \\
I_n
\end{bmatrix} =
\begin{bmatrix}
A_u & B_u \\
C_u & D_u
\end{bmatrix}
\begin{bmatrix}
V_{n+1} \\
I_{n+1}
\end{bmatrix}
$$

where the voltage and current are the total voltage and current amplitudes, i.e., the sum of the incident and reflected TEM waves at the terminal plane. The circuit for a unit cell may be considered as three circuits in cascade, a transmission line of length $d/2$, followed by a shunt susceptance $\bar{B}$, and then followed by another transmission line of length $d/2$. The ABCD-matrix for each of these networks is, respectively [20]
\[
[A]_{T1} = \begin{bmatrix}
\cos (\theta/2) & j \sin (\theta/2) \\
\sin (\theta/2) & \cos (\theta/2)
\end{bmatrix}
\]

\[
[A]_{L} = \begin{bmatrix}
1 & 0 \\
jB & 1
\end{bmatrix}
\]

\[
[A]_{T2} = \begin{bmatrix}
\cos (\theta/2) & j \sin (\theta/2) \\
\sin (\theta/2) & \cos (\theta/2)
\end{bmatrix}
\]

where \(\theta/2 = k_0 d/2\), \(k_0 = \omega (\mu_0 \varepsilon_0)^{1/2}\).

The transmission matrix for the unit cell is the product of the ABCD-matrices of these three networks, and hence we have

\[
[U] = \begin{bmatrix}
A_u & B_u \\
C_u & D_u
\end{bmatrix} = [A]_{T1}[A]_{L}[A]_{T2}
\]

The overall ABCD parameters of a unit cell are obtained as

\[
A_u = D_u = \cos \theta - 0.5B \sin \theta
\]

\[
B_u = j(0.5B \cos \theta + \sin \theta + B/2)
\]

\[
C_u = j(0.5B/2 \cos \theta + \sin \theta + B/2)
\]

Note that for a symmetrical network \(A_u = D_u\).
2.3. Propagation Constant

When a wave is propagating in a periodic structure, and if it is assumed that the structure is lossless, the absolute values of the voltage and the current at the \((n+1)\)th terminal must be equal to the voltage and current at the \(n\)th terminal. Therefore,

\[
V_{n+1}^\pm = e^{\gamma d} V_n^\pm
\]

\[
I_{n+1}^\pm = e^{\gamma d} I_n^\pm
\]

where \(\gamma = \alpha + j\beta\) is the complex propagation constant of the periodic structure.

In terms of the ABCD-matrix of the unit cell, we have

\[
\begin{bmatrix}
V_n \\
I_n
\end{bmatrix} =
\begin{bmatrix}
A_u & B_u \\
C_u & D_u
\end{bmatrix}
\begin{bmatrix}
V_{n+1} \\
I_{n+1}
\end{bmatrix} = e^{\gamma d}
\begin{bmatrix}
V_{n+1} \\
I_{n+1}
\end{bmatrix}
\]

or

\[
\begin{pmatrix}
A_u & B_u \\
C_u & D_u
\end{pmatrix}
- e^{\gamma d}
\begin{pmatrix}
0 & 0 \\
0 & e^{\gamma d}
\end{pmatrix}
\begin{bmatrix}
V_{n+1} \\
I_{n+1}
\end{bmatrix} = 0
\]

This equation is an eigenvalue matrix equation for \(\gamma\). A nontrivial solution for \(V_{n+1}, I_{n+1}\) exists only if the determinant is equal to zero. Hence

\[
\begin{vmatrix}
A_u-e^{\gamma d} & B_u \\
C_u & D_u-e^{\gamma d}
\end{vmatrix} = A_u D_u - B_u C_u + e^{2\gamma d} - e^{\gamma d}(A_u + D_u) = 0
\]

For a reciprocal network \(A_u D_u - B_u C_u = 1\), thus we obtain

\[
1 + e^{2\gamma d} - e^{\gamma d}(A_u + D_u) = 0
\]
or

\[ e^{-yd} + e^{yd} \cdot (A_u + D_u) = 2 \cosh yd \cdot (A_u + D_u) = 0 \]

Then

\[ \cosh yd = (A_u + D_u) / 2 \quad (2.1) \]

For the capacitively loaded coaxial line, we can write

\[ \cosh yd = \cos \theta - 0.5B \sin \theta \quad (2.2) \]

When the absolute value of the right hand side of equation 2.2 is less than unity, we must have \( \gamma = j\beta \) and \( \alpha = 0 \); that is

\[ \cos \beta d = \cos \theta - 0.5B \sin \theta \]

When the right hand side of equation 2.2 is greater than unity, \( \gamma = \alpha \) and \( \beta = 0 \); so

\[ \cosh \alpha d = \cos \theta - 0.5B \sin \theta \geq 1 \]

Finally, when the right hand side of equation 2.2 is less than -1, we must have \( \gamma d = j\pi + \alpha \), so that

\[ \cosh \gamma d = \cos(j\pi + \alpha) = -\cosh \alpha d \]

\[ = \cos \theta - 0.5B \sin \theta < -1 \]

Thus, there will be frequency bands for which a wave propagates unattenuated along the structure separated by frequency bands through which the wave is cut off and does
not propagate. The former is called a passband, and the latter is referred to as a stopband. Note that \(-\gamma\) is also a solution so that, propagation in both directions is possible.

The waves propagating along a periodic structure are often called Bloch-waves by analogy with the quantum-mechanical electron waves that may propagate through a periodic crystal lattice in a solid.

**2.4. Wave Analysis of a Periodic Structure**

In each unit cell of a periodic structure the forward-and backward-propagating waves exist, and the structure can alternatively be analyzed in terms of these waves.

With reference to Figure 2.3, the amplitudes of the forward- and backward-propagating waves at the nth terminal plane \(C_n^+\) and \(C_n^-\) are related to the forward- and backward-propagating waves at the \((n+1)\)th terminal plane by the wave amplitude transmission (WAT) matrix as follows:

\[
\begin{bmatrix}
    C_{n+1}^- \\
    \vdots \\
    jB \\
    \vdots \\
    C_{n+1}^+
\end{bmatrix} = \begin{bmatrix}
    C_n^+ \\
    \vdots \\
    \vdots \\
    C_n^-
\end{bmatrix}
\]

**Figure 2.3:** Wave amplitudes in a periodic structure.
On the other hand, we must have

\[ c_{n+1}^+ = e^{-\gamma d} c_n^+ \]

and

\[ c_{n+1}^- = e^{+\gamma d} c_n^- \]

Hence equation 2.3 becomes

\[
\begin{bmatrix}
    A_{11} - e^{\gamma d} & A_{12} \\
    A_{21} & A_{22} - e^{\gamma d}
\end{bmatrix}
\begin{bmatrix}
    c_{n+1}^+ \\
    c_{n+1}^-
\end{bmatrix} = 0
\]

A nontrivial solution for \( c_{n+1}^+, c_{n+1}^- \) is obtained only if the determinant equals zero. Thus, the eigenvalue equation for \( \gamma \) is

\[ A_{11}A_{22} - A_{12}A_{21} + e^{2\gamma d} - e^{\gamma d}(A_{11} + A_{22}) = 0 \]

Since the determinant of the transmission matrix \( A_{11}A_{22} - A_{12}A_{21} \) is equal to unity when normalized wave amplitudes are used we obtain

\[ \cosh \gamma d = (A_{11} + A_{22})/2 \]  

(2.4)

For the capacitively loaded coaxial line discussed earlier, the wave-amplitude transmission matrices for the three sections of the unit cell are
The WAT-matrix for the unit cell is obtained by multiplying the three component matrices together. The elements of this matrix are obtained as

\[ [A]_{T1} = \begin{bmatrix} e^{jk_0d/2} & 0 \\ 0 & e^{jk_0d/2} \end{bmatrix} \]

\[ [A]_L = \begin{bmatrix} \frac{2+j\overline{B}}{2} & j\overline{B}/2 \\ -j\overline{B}/2 & \frac{4+\overline{B}^2}{2(2+j\overline{B})} \end{bmatrix} \]

\[ [A]_{T2} = \begin{bmatrix} e^{jk_0d/2} & 0 \\ 0 & e^{jk_0d/2} \end{bmatrix} \]

The elements of this matrix are obtained as

\[ A_{11} = \frac{1}{2}(2+j\overline{B})e^{j\theta} \]

\[ A_{12} = j\overline{B}/2 \]

\[ A_{21} = -j\overline{B}/2 \]

\[ A_{22} = \frac{4+\overline{B}^2}{2(2+j\overline{B})} e^{-j\theta} \]

where \( \theta = k_0d \). Using equation 2.4, it is found that
which is similar to equation 2.2 obtained earlier.

2.5. $K_0$-$\beta$ Diagram

In this section a detailed study of the passband-stopband characteristics of the capacitively loaded coaxial transmission line is discussed. The equation for the propagation constant $\beta$ in a periodic structure is usually plotted on a $k_0$-$\beta$ plane. The variations of $\beta$ versus $k_0$ show immediately the frequency bands for which unattenuated propagation can take place and also the frequency bands in which the wave is attenuated. The resultant plot is called the $k_0$-$\beta$ diagram.

For the capacitively loaded coaxial line, equation 2.2 gives

\[
\cos \beta_d = \cos k_0 d - 0.5 \overline{B} \sin k_0 d = \cos k_0 d - K k_0 d \sin k_0 d
\]  

(2.5)

where $\overline{B}/2$ has been expressed as $K k_0 d$. The variation of $k_0 d$ versus $\beta_d$ is plotted in Figure 2.4 with $K=2$ (ohms Rad.)$^{-1}$. The real value of $\beta_d$ is plotted in Figure 2.4. The first low frequency passband is for $k_0 d=0$ to $0.416\pi$ Rad.. This passband is followed by a stopband and further alternating passbands and stopbands. As $k_0 d$ increases, the widths of the passbands in terms of the frequency decrease, since the rate of change of equation 2.5 increases. The corners of the bands occur when the magnitude of the right hand side of equation 2.5 is equal to unity. The lower corner of the first passband occurs when $\beta_d=\pi$, and $k_0 d$ is obtained by equation

\[
\cos k_0 d - K k_0 d \sin k_0 d = -1
\]
Figure 2.4: $k_0d-\beta d$ diagram for a capacitively loaded coaxial line, $K=2$ (ohms Rad.)\(^{-1}\).

The higher corner occurs when $\beta d=0$, and $k_0d$ is obtained by equation

$$\cos k_0d-Kk_0dsin k_0d=1$$

For this structure one corner of the passband always occurs when the length of $d$ equals one half-wavelength, and in the present case when $k_0d$ is a multiple of $\pi$. 
The $k_0-\beta$ characteristics of the other types of periodic structures are similar to those in Figure 2.4. For example if inductance is used instead of capacitance, the relative locations of the passbands and stopbands are interchanged. The low frequency passband will be a stopband since the shunt inductors will short-circuit the line at low frequency.

2.6. Characteristic Impedance

The characteristic impedance is another important parameter in connection with periodic structure, and it is defined as the impedance $Z_B$ presented to the voltage and current waves at the input terminals of a periodic cell. An expression for it in terms of the parameters of the ABCD-matrix may be obtained from

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A_u & B_u \\ C_u & D_u \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = e^{\gamma d} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}$$

which may be written as,

$$(A_u - e^{\pm \gamma d}) V_{n+1}^\pm = -B_u I_{n+1}^\pm$$

Hence

$$Z_B^\pm = V_{n+1}^\pm / I_{n+1}^\pm = -\frac{B_u}{A_u - e^{\pm \gamma d}}$$

Replacing $e^{\pm \gamma d}$, one obtains

$$Z_B^\pm = \frac{2B_u}{D_u - A_u \pm [(A_u + D_u)^2 - 4]^{1/2}}$$

where the positive and negative signs are for propagation in the $+z$, and $-z$ directions, respectively. If the network is symmetric, $A_u = D_u$. In this case the characteristic impedance will be
In order to eliminate the Bloch-wave reflection in the -z direction, the periodic structure must be terminated to a load \( Z_L = Z_B^+ \). Similarly, a series load \(-Z_B^-\) with the source will eliminate the reflection from the source end.

### 2.7. Input Impedance of Terminated Periodic Structure

Consider a periodic structure of length \( d \) and having \( N \) cells terminated in a load impedance \( Z_L \) at the \( N \)th terminal. The voltage at the \( N \)th terminal plane is

\[
V_L = V_{BN} = Z_L I_{BN}
\]

Then

\[
V_{BN}^+ + V_{BN}^- = Z_L (I_{BN}^+ + I_{BN}^-)
\]

\[
= Z_L (Y_B^+ V_{BN}^+ + Y_B^- V_{BN}^-)
\]

where \( Y_B^+ \) and \( Y_B^- \) are the Bloch-wave admittances of the waves in the +z and -z directions, respectively. The reflection coefficient \( \Gamma_N \) at the \( N \)th terminal plane is

\[
\Gamma_N = \frac{V_{BN}^-}{V_{BN}^+} = \frac{1 - Z_L Y_B^+}{Z_L Y_B^- - 1}
\]

\[
= -\frac{Z_B^- Z_L - Z_B^+}{Z_B^+ Z_L - Z_B^-}
\]

For a symmetrical structure \( Z_B^+ = -Z_B^- = Z_B \) (Equation 2.6), and the expression of \( \Gamma_N \) is
\[ \Gamma_N = \frac{Z_L - Z_B}{Z_L + Z_B} \]

At the 0th terminal plane the Bloch-wave reflection coefficient is

\[ \Gamma_N = \frac{V_{B0}^-}{V_{B0}^+} = \frac{V_{BN}^- e^{-jN\beta d}}{V_{BN}^+ e^{+jN\beta d}} = \Gamma_N e^{-j2N\beta d} \]

where \( V_{B0}^+ \) and \( V_{B0}^- \) are forward and backward travelling waves of the periodic structure, which are different from \( V^+ \) and \( V^- \) of the input line, therefore, \( \Gamma_0 \) is not the input reflection coefficient.

The input impedance of the periodic structure is

\[ Z_{in} = \frac{V_{B0}^+ + V_{B0}^-}{I_{B0}^+ + I_{B0}^-} = \frac{V_{B0}^+(1 + \Gamma_0)}{V_{B0}^+/Z_B - V_{B0}^-/Z_B} \]

\[ = Z_B \frac{1 + \Gamma_0}{1 - \Gamma_0} \]

Putting the values of \( \Gamma_0 \) and \( \Gamma_N \)

\[ Z_{in} = Z_B + jZ_L \tan N\beta d \]

The input reflection coefficient \( \rho \) may be obtained by

\[ \rho = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} \]

where \( Z_o \) is the characteristic impedance of the input transmission line.

From the equation of the input impedance for the periodic structure, it can be seen
that when \( N\beta d = n\pi, n=1,2,\ldots \) and \( \beta \) is real (in a passband), then \( Z_{in} = Z_L \). This means that the periodic structure is at resonance, and the frequency of resonance may be obtained from the relation \( \beta d = n\pi/N \).

2.8. Periodic Structure as a Filter

The ideal filter is a network which provides perfect transmission for all frequencies in certain passband regions and infinite attenuation in the stopband regions. Such ideal characteristics are not possible to obtain, and the purpose of filter design is to approximate the ideal requirements with an acceptable tolerance. There are three types of filters, namely: (1) low-pass filters, that transmit all signals from zero frequency to a cutoff frequency \( \omega_c \) and attenuate all signals with frequencies above the cutoff value; (2) high-pass filters, that reject all frequencies below the cutoff value \( \omega_c \) and pass all frequencies above \( \omega_c \); (3) bandpass filters, that transmit signals with frequencies in the range between \( \omega_1 \) to \( \omega_2 \) and attenuate signals with frequencies outside this range. The stopband filter is the complement to the passband filter, which rejects frequencies in the range from \( \omega_1 \) to \( \omega_2 \).

At low frequencies the filter elements are inductors and capacitors. These elements have very simple frequency characteristics, and a very general and complete synthesis procedure has been developed for the design of filters using them. The filter design problem at microwave frequencies where distributed parameter elements must be used is much more complicated.

Many microwave elements have frequency characteristics essentially like those of an ideal inductive or capacitive reactance over a limited frequency range. In this case a low frequency prototype filter may be used as a model. Then, the microwave filter will be realized by replacing all inductors and capacitors by suitable microwave circuit elements that have similar frequency characteristics over the range of interest.
The periodic structure with passband-stopband characteristics can be used as a filter. The advantages of using periodic structure filters over conventional ones are as follows:

(i) Unlike conventional filter structures, periodically loaded lines exhibit alternating bands of propagation and attenuation, making it possible to realize a bandpass filter with an infinite number of passbands.

(ii) The design procedure of filters is better controlled by the parameters of the periodic structure. For the Capacitively loaded coaxial line susceptance B and periodicity d, adjust respectively the width and the position of a passband.

(iii) Because of the smooth variation of the geometry of the structure, better compensation effects are automatically realized.

The following conditions must be satisfied to design a periodic structure bandpass filter:

i) The absolute value of equation \((A_u + D_u)/2\) should be equal to unity at the desired corner frequencies of the passband.

ii) The characteristic impedance of the unit cell should be equal 50 ohms at the center frequency.

The parameters of interest for a filter are the return loss and insertion loss. For an ideal filter, the magnitude of these quantities should be zero and one, respectively, in the passband. The return loss, and insertion loss can be found from the elements of the scattering matrix of the structure, and are
\[ Return\ Loss = |S_{11}| = \left| \frac{\text{Reflected\ voltage\ at\ port\ 1}}{\text{Incident\ voltage\ at\ port\ 1}} \right| \]

\[ Insertion\ Loss = |S_{21}| = \left| \frac{\text{Incident\ voltage\ at\ port\ 2}}{\text{Incident\ voltage\ at\ port\ 1}} \right| \]

where \( S_{11} \) and \( S_{21} \), in terms of the ABCD parameters of the structure are obtained as

\[ S_{11} = \frac{A-B/Z_o+CZ_o-D}{A+B/Z_o+CZ_o+D} \]

and

\[ S_{21} = \frac{2}{A+B/Z_o+CZ_o+D} \]

where \( Z_o \) is the characteristic impedance of the input and output transmission lines.

2.9. Conclusion

The basic concept of the periodic structures was discussed with an example of a capacitively loaded coaxial transmission line. The expressions for the complex propagation constant and characteristic impedance of the periodic structures were presented. The condition for propagation was also specified. It was noted that a periodically loaded transmission line exhibits alternating propagation and attenuation bands. The width and position of the passband can be controlled by the parameters of the structure.
3. Microstrip Periodic Structure with Uniform Lines

3.1. Introduction

The analyses of two different planar periodic structures formed with uniform microstrip transmission line sections connected in a periodic fashion are presented in this chapter. The ABCD-matrix of a period (a unit cell) is determined first, from which the propagation constant and the characteristic impedance are obtained. The above matrix is diagonalized in order to obtain a simplified form of the ABCD matrix of a periodic structure with N unit cells. The scattering parameters are then determined from the ABCD matrix of the structure. Bandpass filters are designed utilizing the design procedure presented in the preceding chapter. Two different periodic structure filters are fabricated and their performances are evaluated. Numerical results are compared with the experimental data. The effect of the number of sections in a structure on the frequency response are shown. Various filter characteristics of a terminated periodic structure are presented.

3.2. Periodic Structure with Different Transmission Lines in Series

The periodic structure under consideration is illustrated in Fig. 3.1(a). It consists of a number of unit cells connected in cascade. A unit cell is a combination of three microstrip lines as shown in the Fig. 3.1(b). The unit cell can be characterized by its equivalent ABCD-matrix. The ABCD matrix of the unit cell is obtained by multiplying the ABCD matrices of the transmission line sections, and the step discontinuities arising from the junctions of the two microstrip lines.
Fig. 3.1: (a) Geometry of a planar periodic structure; (b) A unit cell of the structure

Fig.3.2 represents the T equivalent circuit for the step discontinuity consisting of two lumped inductances and a lumped capacitance. The effect of the junction inductances can be accounted for by shortening the line lengths by $\Delta l_1$ and $\Delta l_2$ (Fig 3.3). The formulas for inductances, capacitance and lengths $\Delta l_1$ and $\Delta l_2$ are given in the literature [11] and will be presented later. Thus, the junction of two uniform microstrip lines is equivalent to a shunt capacitance, and can be represented by the ABCD-matrix given below

$$
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_j =
\begin{bmatrix}
1 & 0 \\
jB & 1
\end{bmatrix}
$$
Figure 3.2: Approximate T equivalent circuit for a step discontinuity

Figure 3.3: Approximate compensation for a step discontinuity

In the foregoing $B=\omega C_0$, $\omega$ is the angular frequency and $C_0$ is the shunt capacitance.

For a uniform line having an electric length $\theta$ the ABCD-matrix is given by [20]

$$
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_{T,L} = 
\begin{bmatrix}
\cos \theta & jZ_0 \sin \theta \\
j \sin \theta / Z_0 & \cos \theta
\end{bmatrix}
$$
where $Z_0$ is the characteristic impedance of the line, $\theta = kd$, $k = \omega (\varepsilon_0 \varepsilon_r \mu_0)^{1/2}$, $\varepsilon_e$ is the effective dielectric constant of the substrate on which the structure is etched and can be obtained from the equation

$$
\varepsilon_e = \frac{1}{2} (\varepsilon_r + 1) + \frac{1}{2} (\varepsilon_r - 1) (1 + 10h/w)^{-1/2}
$$

where $\varepsilon_r$ is the relative dielectric constant, $h$ is the thickness of the substrate and $w$ is the width of the uniform microstrip transmission line.

The overall ABCD-matrix of the unit cell is given by

$$
[U] = \begin{bmatrix}
A_u & B_u \\
C_u & D_u
\end{bmatrix} = [A_{T1}][A_j][A_{T2}][A_j][A_{T3}]
$$

where

$$
[A_{T1}] = [A_{T3}] = \begin{bmatrix}
\cos (\theta_1/2) & jZ_1 \sin (\theta_1/2) \\
j1/Z_1 \sin (\theta_1/2) & \cos (\theta_1/2)
\end{bmatrix}
$$

and

$$
[A_{T2}] = \begin{bmatrix}
\cos \theta_2 & jZ_2 \sin \theta_2 \\
j1/Z_2 \sin \theta_2 & \cos \theta_2
\end{bmatrix}
$$

$\theta_1$ and $\theta_2$ are the electrical lengths of the transmission line sections (including the effect of junction inductances) and $Z_1$, $Z_2$ are the characteristic impedances.

The overall ABCD parameters of a unit cell are obtained as
\[
A_u = D_u = \cos(\theta_1) \cos(\theta_2) - 0.5 \sin(\theta_1) \sin(\theta_2) (Z_1/Z_2 + Z_2/Z_1) \\
+ 0.5 Z_1 Z_2 B^2 \sin(\theta_1) \sin(\theta_2) - Z_1 B \sin(\theta_1) \cos(\theta_2) - Z_2 B \sin(\theta_2) \cos(\theta_1)
\]  
(3.1)

\[
B_u = j \{ Z_1 \sin(\theta_1) \cos(\theta_2) + 0.5 \sin(\theta_2)/Z_2 [Z_2^2 - Z_1^2 + \cos(\theta_1)(Z_1^2 + Z_2^2)] \\
- Z_1 Z_2 B \sin(\theta_1) \sin(\theta_2) - Z_1^2 B(1 - \cos(\theta_1)) \cos(\theta_2) + 0.5 Z_1^2 Z_2^2 \sin(\theta_2)(1 - \cos(\theta_1)) \}
\]  
(3.2)

\[
C_u = j \{ 1/Z_1 \sin(\theta_1) \cos(\theta_2) + 0.5 Z_2 \sin(\theta_2) [1/Z_2^2 - 1/Z_1^2 + \cos(\theta_1)(1/Z_1^2 + 1/Z_2^2)] \\
+ B \cos(\theta_2)(1 + \cos(\theta_1)) - Z_2/Z_1 B \sin(\theta_1) \sin(\theta_2) - 0.5 Z_2 B^2 \sin(\theta_2)(1 + \cos(\theta_1)) \}
\]  
(3.3)

The junction capacitance, inductances, and the value of \( \Delta l_1, \Delta l_2 \) are obtained from the following expressions [6]:

\[
\frac{C_0}{(W_1 W_2)^{1/2}} (pF/m) = [10.1 \log(\varepsilon_r) + 2.33] W_2 W_1 - 12.6 \log(\varepsilon_r) - 3.17
\]

This expression is in error by less than 10% when \( \varepsilon_r \leq 10 \), and \( 1.5 \leq W_2/W_1 \leq 3.5 \). The junction capacitance has been calculated using the above equation.

The values of the inductances in the equivalent circuit are obtained from the relations

\[
L/h (nH/m) = 40.5 [W_2/W_1 - 1.0] - 75 \log(W_2/W_1) + 0.2 [W_2/W_1 - 1.0]^2
\]

The inductance per unit length of a microstrip is

\[
L_{W1} = \frac{Z_0(W_2)(\varepsilon_{eff}(W_2))^{1/2}}{c}
\]

\[
L_{W2} = \frac{Z_0(W_1)(\varepsilon_{eff}(W_1))^{1/2}}{c}
\]

so that
\[ L_1 = \frac{L_{W1}}{L_{W1} + L_{W2}} - L \]

\[ L_2 = \frac{L_{W2}}{L_{W1} + L_{W2}} - L \]

and

\[ \Delta l_1 = \Delta l_2 = \frac{L}{(L_{W1} + L_{W2})^{1/2}} \]

where \( W_1 \) and \( W_2 \) are the widths of the lines, \( c = 3 \times 10^8 \) m/s, \( h \) is the substrate thickness and \( Z_0(W) \) and \( \varepsilon_{\text{eff}}(W) \) are given in [26].

**3.2.1. Overall ABCD-matrix of the Structure**

In the diagonalized form, the ABCD-matrix of the unit cell is represented as

\[ [U] = [S][\Lambda][S]^{-1} \]

where \([S]\) represents the eigen vector matrix of the unit cell, \([\Lambda]\) is the eigen value matrix (diagonal) and \([S]^{-1}\) is the inverse of \([S]\). For a symmetric matrix they are obtained as

\[
[S] = \begin{bmatrix}
-B_u & -B_u \\
A_u \cdot \lambda^+ & A_u \cdot \lambda^-
\end{bmatrix}
\]

\[
[\Lambda] = \begin{bmatrix}
\lambda^+ & 0 \\
0 & \lambda^-
\end{bmatrix}
\]
\[
[S]^{-1} = \frac{1}{\Delta} \begin{bmatrix}
-B_u & -B_u \\
\lambda^+ - A_u & A_u - \lambda^-
\end{bmatrix}
\]

\(\lambda^+\) and \(\lambda^-\) are eigen values of the unit cell matrix, and they are obtained as

\[
\lambda^+ = \sqrt{(A_u^2 - 1)}
\]

\(\lambda^- = \sqrt{(A_u^2 - 1)}\)

The value of \(\Delta\) is derived as

\[
\Delta = B_u (\lambda^- - \lambda^+)
\]

When \(N\)-such cells are connected in cascade, the overall ABCD-matrix of \(N\) unit cells is

\[
[U_N] = \begin{bmatrix}
A_N & B_N \\
C_N & D_N
\end{bmatrix}
\]

\[=[U]^N = [S][\Lambda]^N[S]^{-1}
\]

Where

\[
[\Lambda]^N = \begin{bmatrix}
(\lambda^+)^N & 0 \\
0 & (\lambda^-)^N
\end{bmatrix}
\]

From 3.4, the ABCD parameters of the periodic structure are obtained as

\[
A_N = D_N = [(\lambda^+)^N + (\lambda^-)^N]/2
\]

\[
B_N = B_u [(\lambda^+)^N - (\lambda^-)^N]/(\lambda^+ - \lambda^-)
\]

\[
C_N = C_u [(\lambda^+)^N - (\lambda^-)^N]/(\lambda^+ - \lambda^-)
\]
The above formulation, therefore, yields the closed form expressions for the ABCD parameters of a structure with any number of cells.

3.2.2. Results and Discussion

A three-section periodic structure bandpass filter was designed and fabricated with center frequency of 6 GHz, a bandwidth of 2 GHz and characteristic impedance of 50 ohms at the center frequency (Fig. 3.4). The characteristic impedance of the narrow microstrip transmission line was chosen as $Z_1=50$ ohms. The electrical lengths of the microstrip lines and the characteristic impedance of the wider line $Z_2$ were obtained utilizing equations 2.1, 2.6, 3.5 and filter design criteria presented in chapter 1. The value of $Z_2$ was found to be 26.5 ohms, and the physical dimensions were obtained as $d_1=0.9$ cm and $d_2=1.68$ cm. The substrate had a dielectric constant of 2.2 and a thickness of 0.159 cm.

The insertion loss and return loss of the structure were measured by the HP8510B Network Analyzer and compared with the theoretical results. The measurement had an error of 0.1 dB. The results are shown in Figures 3.5 and 3.6. The slight difference in the passband can be attributed to the radiation loss, dielectric loss and the copper loss. These losses have not been considered in the computation. The radiation loss can be reduced by making the width of the microstrip lines as low as possible or by reducing the dielectric thickness. It can be eliminated completely by using a strip line configuration. The dielectric loss will also be reduced considerably by reducing the thickness of the dielectric substrate.

The expressions for the propagation constant $\beta$ and characteristic impedance $Z_o$, have been computed for various circuit parameters. Figures 3.7 and 3.8 show the variations of $\beta$ and $Z_o$ versus frequency with $Z_1$ as a parameter. It is seen that in the passband
Figure 3.4: Experimental periodic structure filter with $d_1=0.9$ cm, $d_2=1.68$ cm, $h=0.159$ cm, $\varepsilon_r=2.2$, $Z_1=50$ ohms, $Z_2=26.5$ ohms.
Figure 3.5: Variation of the insertion loss with frequency for the periodic structure.
Figure 3.6: Variation of the return loss with frequency for the periodic structure.
Figure 3.7: Propagation constant $\beta$ versus frequency.

Figure 3.8: Characteristic impedance (normalized) versus frequency.
Figure 3.9: Plot of the bandwidth vs. the patch length $d_2$. $Z_1=50$ ohms, $Z_2=26.5$ ohms and $d_1=0.9$ cm.

Figure 3.10: Plot of the center frequency vs. the patch length $d_2$. $Z_1=50$ ohms, $Z_2=26.5$ ohms and $d_1=0.9$ cm.
**Figure 3.11:** Variation of the bandwidth with the characteristic impedance ($Z_1$) of the thinner line. $Z_2 = 26.5$ ohms, $d_1 = 0.9$ cm and $d_2 = 1.68$ cm.

**Figure 3.12:** Variation of the center frequency with the characteristic impedance ($Z_1$) of the thinner line. $Z_2 = 26.5$ ohms, $d_1 = 0.9$ cm and $d_2 = 1.68$ cm.
$\beta$ and $Z_B$ are real, and in the stopband they are imaginary. However, the bandwidths of the passband and stopband depend on the value of $Z_1$.

It is seen that by increasing the number of sections, the power loss ratio (ratio between the output power and input power) in the stopband decreases, and the response of the filter is improved. However, a number of ripples are produced in the passband. The number of ripples increases with $N$. This is due to the multiple reflection of the incident wave occurring at the input of each cell. The reflected fields come back to the input end of the structure with various phases and the magnitude of the resultant reflected field oscillates. Matching is perfect when $Z_B = Z_0$, which is true only at the center frequency of the passband and the resonant frequencies of the structure. Away from the center frequency, $Z_B$ differs considerably from $Z_0$ and the input impedance becomes oscillatory. This yields a mismatch to the input with subsequent ripple in the input-output characteristic of the structure.

Figures 3.9 and 3.10 show the variations of the center frequency and bandwidth of the passband with $d_2$ (the length of the wider microstrip line) as a parameter. As expected, the center frequency and bandwidth decrease with $d_2$.

In Figures 3.11, the bandwidth of the passband and adjacent stopband have been plotted against $Z_1$. It is found that the bandwidth of the passband decreases, however, the bandwidth of the adjacent stopband increases with $Z_1$, resulting a wide separation between adjacent passbands. This would be useful in designing a bandpass filter when the separation between two passbands is specified. The variation of the center frequency with $Z_1$ is shown in figure 3.12. It is seen that the rate of change of the center frequency with $Z_1$ is low.
3.3. Analysis of Uniform Line Loaded by Open Ended Microstrip Line

A periodic structure, illustrated in Figure 3.13(a) is considered in this section. It is formed with a uniform microstrip transmission line periodically loaded by open ended microstrip lines. A unit cell of the structure is shown in figure 3.13(b). The impedance of an open ended transmission line is reactive, and is equal to

\[ Z_{in} = -j \frac{Z_2}{\tan \theta} \]

In the above equation, \( \theta = k d_2 \), \( d_2 \) is the length of the transmission line with characteristic impedance of \( Z_2 \). When \( \theta = n \pi, n=1,2,\ldots \), the line is at resonance, and the corresponding frequency of resonance is obtained as

\[ f_r = \frac{n}{2 d_2 (\mu \epsilon)^{1/2}} \]

This result can be utilized in designing a periodic structure filter.

As it has been shown in figure 3.13(b) the impedance of the two open ended transmission lines are in parallel, and the effect of these impedances can be represented by ABCD-matrix as

\[ [A_Z] = \begin{bmatrix} 1 & 0 \\ 2/Z_{in} & 1 \end{bmatrix} \]

The overall ABCD-matrix of the unit cell is obtained by matrix multiplication as follows:

\[ [U] = [A_{T1}][A_Z][A_{T3}] \]

where
Figure 3.13: (a) Geometry of a planar periodic structure; (b) A unit cell of the structure

\[
[A_{T1}] = [A_{T2}] = 
\begin{bmatrix}
\cos (\theta_1/2) & jZ_1 \sin (\theta_1/2) \\
-j1/Z_1 \sin (\theta_1/2) & \cos (\theta_1/2)
\end{bmatrix}
\]

In the above, \( \theta_1 \) and \( Z_1 \) are the electrical length and characteristic impedance of the uniform line, respectively. The elements of the resultant matrix of a unit cell are
\[ A_u = D_u = \cos \theta_1 + jZ_1/Z_{in}\sin \theta_1 \]

\[ B_u = j(Z_1\sin \theta_1) - Z_1^2/Z_{in}(1-\cos \theta_1) \]

\[ C_u = j(1/Z_1\sin \theta_1) + 1/Z_{in}(1+\cos \theta_1) \]

Note that, the lower corner of the passband occurs when \( \theta_1 = \pi \). This makes the design of the filter simple.

### 3.3.1. Results and Discussion

A structure was designed and fabricated to operate as a bandpass filter with a center frequency of 6 GHz and a bandwidth of 2 GHz (Fig 3.14). The lengths and characteristic impedances obtained for the microstrip transmission lines were \( d_1 = 2.5 \text{ cm}, d_2 = 1.68 \text{ cm}, Z_1 = 50 \text{ ohms} \) and \( Z_2 = 70 \text{ ohms} \). The number of sections was taken as \( N = 3 \). The length of the open ended microstrip line \( d_2 \) is equal to half the wavelength at the center frequency, which means at center frequency the line is at resonance.

The insertion loss, and return loss of the periodic structure filter were measured and compared with the computed results. The results are illustrated in Figures 3.15 and 3.16, respectively. The measurement shows about 2 dB power loss in the passband. The losses have not been considered in the computation. The theoretical results of the insertion loss and return loss of a six-section periodic structure are also shown. The radiation loss of the structure discussed in the first part of this chapter is lower than the structure with open ended microstrip lines.

The variation of the propagation constant and characteristic impedance of the structure are plotted against frequency with \( Z_1 \) as parameter in Figures 3.17 and 3.18, respec-
Figure 3.14: Experimental periodic structure filter with $d_1=2.5$ cm, $d_2=1.68$ cm, $h=0.159$ cm, $\varepsilon_r=2.2$, $Z_1=50$ ohms, $Z_2=70$ ohms.
Figure 3.15: Variation of the insertion loss with frequency for the periodic structure.
Figure 3.16: Variation of the return loss with frequency for the periodic structure.
Figure 3.17: Propagation constant $\beta$ versus frequency.

Figure 3.18: Characteristic impedance (normalized) versus frequency.
Figure 3.19: Plot of the bandwidth vs. the length of the open ended line $d_2$. $Z_1=50$ ohms, $Z_2=70$ ohms and $d_1=2.5$ cm.

Figure 3.20: Plot of the center frequency vs. the length of the open ended line $d_2$. $Z_1=50$ ohms, $Z_2=70$ ohms and $d_1=2.5$ cm.
Figure 3.21: Variations of the bandwidth with the characteristic impedance ($Z_1$) of the uniform line. $d_1 = 2.5$ cm, $d_2 = 1.68$ cm and $Z_2 = 70$ ohms.

Figure 3.22: Variation of the center frequency with the characteristic impedance ($Z_1$) of the uniform line. $d_1 = 2.5$ cm, $d_2 = 1.68$ cm and $Z_2 = 70$ ohms.
itively. It is seen that the propagation constant and characteristic impedance are real in the passband and imaginary in the stopband and the bandwidths of the passband and stopband change with $Z_1$.

In Figure 3.19, the center frequency is plotted against $d_2$, the length of the open ended microstrip transmission lines. As expected, the center frequency decreases with the length $d_2$, because an open ended transmission line acts as a resonator, and the frequency of resonance decreases by increasing the length of the line. In Figure 3.20, it can be seen that the bandwidth of the passband also decreases with $d_2$.

Figures 3.21 and 3.22 show the variation of the bandwidth, and center frequency of the passband and adjacent stopband with $Z_1$. It is found that the bandwidth of the passband decreases, and the bandwidth of the adjacent stopband increases with $Z_1$, and the center frequency of the passband remains almost constant.

3.4. Conclusion

In this chapter two different microstrip periodic structures were analyzed by the transmission line method. The method is very simple and computationally efficient. However, there are a few limitations such as

i) If the ratio of the widths of the wider microstrip transmission line and the thinner line (structure discussed in the first part of the chapter) is large the expression for the junction capacitance is not accurate,

ii) The losses can not be incorporated in the analysis. However, for a structure in strip line version, the radiation loss will be eliminated and the present method can be applied with a reasonable accuracy.
iii) Applicable for uniform microstrip line structure only.

Two microwave periodic structure filters were designed and fabricated, and the results of the measurement were found to be close to the theoretical data. It was seen that by increasing the number of sections the frequency response of the filter in the passband oscillates and the number of peaks is related to the number of sections. The dimension of the structure determines the bandwidth and center frequency of the filter.
4. Cavity Model Analysis of a Unit Cell
with Rectangular Patch

4.1. Introduction

In the preceding chapter a planar periodic structure formed with two uniform microstrip transmission lines of different characteristic impedances was analyzed using the transmission line method. It was mentioned before that, if the ratio of the line widths is large, the expression for the junction capacitance becomes inaccurate. In such cases a more accurate method should be invoked to analyze the microstrip periodic structure.

In this chapter a method based on the cavity model is developed to analyze the periodic structure. In this method, a unit cell of the periodic structure discussed in chapter 3 is modelled as a rectangular microstrip cavity resonator with two microstrip transmission feed lines connected at the two ends. The electromagnetic fields inside the cavity are obtained using the modal expansion technique [16]. The Z-matrix of the equivalent circuit of the microstrip cavity is found from the field expressions. The equivalent conductance, $G_r$, for the microstrip cavity is computed. From the Z-matrix and the equivalent conductance for the radiation loss, the ABCD parameters of the microstrip cavity are obtained. The elements of the ABCD-matrix of the unit cell are obtained by matrix multiplication.

The performance of a periodic structure bandpass filter is examined theoretically. The variations of the bandwidth and center frequency with the physical parameters of the periodic structure are studied.
4.2. Modal Expansion Method

In this method a patch is considered to be a rectangular cavity resonator. In order to derive the expressions for the fields inside the resonator, the following assumptions are made.

i) The small thickness of the dielectric substrate between the microstrip conductor and the ground plane suggests that only the $z$ component of the electric field $\vec{E}$, and the $x$ and $y$ components of the magnetic field $\vec{H}$ (Fig. 4.1) exist in the region bounded by the microstrip and the ground plane.

ii) The field in this region does not depend on the $z$-coordinate for all frequencies of interest.

iii) The electric current in the microstrip conductor does not have components normal to the edge at any point on the edge, which implies a negligible tangential component of $\vec{H}$ along the edge.

Thus, the region between the microstrip conductor and the ground plane may be considered as a cavity bounded by a magnetic wall along the edge, and by electric walls at the top and the bottom (Fig. 4.1). In order to account for the fringing field, the magnetic wall is placed slightly away from the periphery.

4.2.1. The Wave Equation of the Electric Field

The cavity is assumed to be lossless with a perfect magnetic wall in its periphery. For microstrip feed lines, the feed source may be modelled using Huygen’s principle, by a strip of $z$-directed electric current backed by a magnetic wall [27]. When the microstrip line is thin, this current is negligible at any point on the edge of the conductor except in the immediate vicinity of the feeds. In the ideal case, the feed source can be assumed to
Figure 4.1: Geometry of a rectangular microstrip patch with dimensions $a$ along x-axis and $b$ along y axis, fed by two microstrip lines be a uniform current ribbon of constant current. Fringing of the feed line fields indicates that the width of the current ribbon is the effective width of the feed line. Thus, it is assumed that the cavity is excited by two surface current sources located at $y=0$, and $y=b$, respectively. The above excitation can be expressed as:

$$J_z = \begin{cases} I_1 \delta(y)/w + I_2 \delta(y-b)/w & \text{if } a/2 - w/2 < x < a/2 + w/2 \\ 0 & \text{Otherwise} \end{cases}$$

(4.1)

where $I_1$ and $I_2$ are the line currents, and $w$ is the width of the transmission feed lines. $\delta(y)$ represents the Dirac-delta function of $y$.

When a microstrip patch is fed by a microstrip line, many modal waves are produced in general. The wave equation for $E$ in the cavity with excitation current $J$ is (a time dependence of $e^{j\omega t}$ is assumed) given by [27]
\[ \nabla^2 \vec{E} + k^2 \vec{E} = j\omega \mu \vec{J} - \frac{\nabla (\nabla \cdot \vec{J})}{j\omega \varepsilon} \]  

(4.2)

where \( k^2 = \omega^2 \mu_0 \varepsilon_0 \). As mentioned before, \( \vec{E} \) and \( \vec{J} \) are assumed to have only the \( z \)-component, with no variation along \( z \)-axis implying that

\[ \nabla \cdot \vec{J} = \frac{\partial J_z}{\partial z} = 0 \]  

(4.3)

Thus, equation 4.2 becomes

\[ \nabla^2 E_z + k^2 E_z = j\omega \mu J_z \]  

(4.4)

4.2.2. The Cavity Mode Eigenfunction

The electric field inside the cavity must satisfy the wave equation 4.4 and the following boundary conditions:

\[ \frac{\partial E_z}{\partial x} (x=0) = \frac{\partial E_z}{\partial x} (x=a) = 0 \]

or

\[ H_y (x=0) = H_y (x=a) = 0 \]

and

\[ \frac{\partial E_z}{\partial y} (y=0) = \frac{\partial E_z}{\partial y} (y=b) = 0 \]

or

\[ H_x (y=0) = H_x (y=b) = 0 \]

The fields inside the cavity for a given TM\(_{mn}\) mode can be obtained from the following set of equations:
\[
\vec{E}_{mn} = \psi_{mn} \hat{z} \quad (4.5)
\]

\[
\nabla \times \vec{E}_{mn} = -j\omega \mu \vec{H}_{mn} \quad (4.6)
\]

\[
(\nabla^2 + k_{mn}^2)\psi_{mn} = 0, \text{ with } \frac{\partial \psi_{mn}}{\partial x} = \frac{\partial \psi_{mn}}{\partial y} = 0 \text{ on the magnetic wall} \quad (4.7)
\]

where \( \nabla_t \) is a transverse del operator with respect to the z-axis and \( \psi_{mn} \) is the field solution for the rectangular microstrip patch resonator with resonant wave number

\[
k_{mn}^2 = \frac{\omega^2}{\varepsilon}
\]

The others EM field components are obtained from \( E_z \) using the Maxwell equations [27].

If the origin is taken at one of the patch corners of Fig. 4.1, the solution of the homogeneous wave equation of 4.7 can be written, subject to the above boundary conditions, as

\[
E_z = A_{mn} \cos \left( \frac{m\pi}{a} x \right) \cos \left( \frac{n\pi}{b} y \right) \quad (4.8)
\]

In the presence of a z-directed source at the feed, the total \( E_z \) must satisfy the wave equation 4.4. The general solution is comprised of an arbitrary linear sum of a complete set of orthogonal functions each of the form of 4.8, and can be written as

\[
E_z = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \cos \left( \frac{m\pi}{a} x \right) \cos \left( \frac{n\pi}{b} y \right) \quad (4.9)
\]

Each term of 4.9 satisfies the boundary conditions, and the unknown constants \( A_{mn} \) are obtained by putting \( E_z \) in equation 4.4 as follows:
\[ \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} (k^2 - k_{mn}^2) \cos \left( \frac{m\pi}{a}x \right) \cos \left( \frac{n\pi}{b} y \right) = j \omega \mu \mathcal{J}_z \]  

(4.10)

Note that this is in the form of a double cosine Fourier series in the x and y. It is thus evident that \( A_{mn} (k^2 - k_{mn}^2) \) are the Fourier coefficients of the right hand side of the equation 4.10 or,

\[ A_{mn} (k^2 - k_{mn}^2) = \frac{2 \varepsilon_m \varepsilon_n}{ab} \int_0^a \int_0^b j \omega \mu J_z \cos \left( \frac{m\pi}{a}x \right) \cos \left( \frac{n\pi}{b} y \right) \, dx \, dy \]

where

\[ \varepsilon_m \text{ and } \varepsilon_n = \begin{cases} 1 & \text{if } m \text{ and } n = 0 \\ 2 & \text{Otherwise} \end{cases} \]

Performing the integration, the expressions for \( A_{mn} \) are obtained as

\[ A_{0n} = \frac{2}{ab(k^2 - k_{0n}^2)} [I_1 + \cos (n\pi) I_2] \]

\[ A_{m0} = \frac{4 \sin \left[ m\pi \omega / (2a) \right] \cos (m\pi/2)}{m\pi bw(k^2 - k_{m0}^2)} [I_1 + I_2] \]

\[ A_{mn} = \frac{8 \sin \left[ m\pi \omega / (2a) \right] \cos (m\pi/2)}{m\pi bw(k^2 - k_{mn}^2)} [I_1 + \cos (n\pi) I_2] \]

\[ k_{mn}^2 = \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \]

The field inside the cavity will be of the TM type, so that all other field components can be determined from \( \mathcal{E}_z \).
Let $E_1$ and $E_2$ be the average fields at the input port and output port, respectively.

Then:

$$E_1 = \frac{1}{w} \int_{a/2-w/2}^{a/2+w/2} (E_z)_y=0 \, dx$$

$$= \sum_{n=0}^{\infty} A_{0n} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} 2A_{mn}a/m\pi\cos (m\pi/2)\sin [m\pi w/(2a)]$$

$$E_2 = \frac{1}{w} \int_{a/2-w/2}^{a/2+w/2} (E_z)_y=b \, dx$$

$$= -\sum_{n=0}^{\infty} A_{0n} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} 2A_{mn}a/(m\pi)\cos (m\pi/2)\cos (n\pi)\sin [m\pi w/(2a)]$$

In order to obtain the ABCD parameters of the microstrip cavity resonator, the $Z$-parameters should be known first. The $Z$-matrix is defined as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where $V_1$ and $V_2$ are the line voltages at the input and output ports, respectively. The port voltages are related to the electric fields and are obtained as

$$V_1 = -E_1 \, h$$

$$V_2 = -E_2 \, h$$

Comparing the $Z$-matrix definition and the equations for $V_1$ and $V_2$, which are related to $I_1$ and $I_2$, the elements of the $Z$-matrix are determined as

$$Z_{11} = Z_{22} = j\omega \mu h \left( \frac{1}{abk^2} + \sum_{n=1}^{\infty} \frac{2}{ab(k_0 n^2 - k^2)} \right)$$
It can be seen that the input impedance of the rectangular microstrip patch conductor is equivalent to infinite parallel LC resonators, for which the frequencies of resonance are given by

\[ f_r = \frac{1}{2\pi(\mu/e)^{1/2}} [(m\pi/a)^2 + (n\pi/b)^2]^{1/2} \] (4.11)

For the TM_{01} mode the equivalent L and C of the resonator are obtained as

\[ L = \frac{2\mu hb}{\pi^2 a} \]

and

\[ C = \frac{abe}{2h} \]

The ABCD parameters of the microstrip patch in terms of the Z-parameters are obtained as
\[ A_p = D_p = \frac{Z_{11}}{Z_{21}} \]
\[ B_p = Z_{12} + \frac{(Z_{11}Z_{22})}{Z_{21}} \]
\[ C_p = \frac{1}{Z_{21}} \]

It is evident that, in the passband the microstrip conductor should be close to a resonance. Thus, the dimensions of the rectangular conductor should be about \( n\lambda_0/2 \), \( n=1, 2, \ldots \) (\( \lambda_0 \) is the wave length at the center frequency).

If the characteristic impedance of the thinner line is chosen as 50 ohms, the dimension of the rectangular conductor patch (wider line) should be such that for the center frequency the cavity is at resonance. This would be useful in the design of a periodic structure filter.

In this section the ABCD parameters are derived using the modal expansion method. The parameters do not include the radiation loss. The radiation loss is discussed in the following section.

4.4. Radiation Loss

The radiated field can be found by considering the microstrip cavity as two slots spaced at a distance \( b \) in the x-y plane (Fig. 4.2). Assuming that in the passband the dominant mode is \( \text{TM}_{01} \), the variation of \( E_z \) along x-axis is constant, and along y is sinusoidal. The radiated fields in the y-z plane will vanish, because the equivalent magnetic currents are in the opposite directions (Huygen’s principle). The magnetic currents on the slots in x-y plane are:

\[
\bar{M} = \begin{cases} 
-2\hat{n} \times \hat{y} E_z = -a_2 E_0 & \text{if } -w/2 \leq x \leq w/2, \text{ and } -h/2 \leq z \leq h/2 \\
0 & \text{Otherwise}
\end{cases}
\]

where the factor of 2 accounts for the presence of the ground plane, \( \hat{n} \) is the outward
Figure 4.2: Coordinate system for slot (aperture)

normal to the magnetic wall, and $E_0 = E_1 = E_2$. Thus, if $a = \lambda_0$, and $b = \lambda_0/2$, in the passband the dominant mode is $TM_{21}$, and no power will be radiated by the magnetic walls of the microstrip patch conductor.

Since the magnetic current is the source of the outside radiation, the fields can be obtained from the electric vector potential [28]. The electric vector potential is given by [28]

$$F(r) = \varepsilon \int_{s'} \frac{M(r')}{4\pi |r-r'|} e^{-jk_0 |r-r'|} d{s'}$$

where $r'$ is the location of the source point, $r$ is the location of the observation point, $k_0$ is the wave number and $s'$ is the aperture area.

The total far-zone fields radiated by each slot of the microstrip patch are [28]
\[ E_r = E_\theta = 0 \]
\[ E_\phi = \frac{hak_0 e^{-j k_0 r}}{2\pi r} \left( \sin \theta \left( \frac{\sin X \sin Z}{XZ} \cos \left( \frac{X_0 h}{2 \sin \theta \sin \phi} \right) \right) \right) \]

where
\[ X = k_0 h/2 \sin \theta \cos \phi \]
\[ Z = k_0 a/2 \cos \theta \]
\[ k_0 = \sqrt{\frac{\omega \epsilon_0}{c}} \]

The total power loss radiated by each slot is
\[ P_r = \text{Re} \left\{ \int_0^{2\pi} \int_0^{\pi/2} -E_\phi H_\theta^* r^2 \sin \theta \, d\phi \, d\theta \right\} \]

where
\[ H_\theta = \frac{E_\phi}{\eta} \]

The conductance due to radiation for each slot is
\[ G_r = \frac{P_r}{|V|^2} \]

where \( V = V_1 = -V_2 \).

Considering radiation loss, the effective ABCD-matrix of the microstrip patch is obtained from the equation
\[
\begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ G_r & 1 \end{bmatrix} \begin{bmatrix} A_p & B_p \\ C_p & D_p \end{bmatrix} \begin{bmatrix} 1 & 0 \\ G_r & 1 \end{bmatrix}
\]

yielding
Multiplying the ABCD-matrix of the transmission lines connected to the input and output ports, the parameters of the unit cell matrix are obtained as

\[
\begin{align*}
A_u &= D_u = A_T \cos \theta + j \{0.5 \sin \theta (Z_1 C_T + B_T / Z_0)\} \\
B_u &= j \{0.5 [Z_1 A_T \sin \theta]\} + 0.5 B_T (1 + \cos \theta) - Z_1^2 (1 - \cos \theta) \\
C_u &= j \{1 / Z_1 A_T \sin \theta\} - 0.5 (1 - \cos \theta) B_T / Z_1^2 + 0.5 C_T (1 + \cos \theta)
\end{align*}
\]

where \( \theta = kd \), \( d \) and \( Z_1 \) are the length and characteristic impedance of the microstrip transmission feed lines connected to the microstrip patch conductor. The overall ABCD-matrix of the periodic structure can be obtained using equation 3.5 in chapter 3.

The above radiation loss can also be incorporated in the transmission line method by assuming a conductance \( G_r \) due to the radiation loss parallel to the junction capacitance. The ABCD-matrix of the wider microstrip transmission line, including the junction capacitance and the conductance \( G_r \) for the radiation loss, is derived as

\[
\begin{bmatrix}
A_T & B_T \\
C_T & D_T
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
Y & 1
\end{bmatrix}
\begin{bmatrix}
\cos (\theta_1 / 2) & j Z_1 \sin (\theta_1 / 2) \\
j 1 / Z_1 \sin (\theta_1 / 2) & \cos (\theta_1 / 2)
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
Y & 1
\end{bmatrix}
\]

where

\[
Y = j B + G_r
\]
The elements of the ABCD-matrix of the unit cell are

\[ A_u = D_u = \cos \theta_1 \cos \theta_2 - 0.5 \sin \theta_1 \sin \theta_2 (Z_1/Z_2 + Z_2/Z_1) \]
\[ -0.5 Z_1 Z_2 Y^2 \sin \theta_1 \sin \theta_2 + j \left( Z_1 Y \sin \theta_1 \cos \theta_2 + Z_2 Y \sin \theta_2 \cos \theta_1 \right) \]
\[ B_u = j \left( Z_1 \sin \theta_1 \cos \theta_2 + 0.5 \sin \theta_2/Z_2 \left( Z_2^2 - Z_1^2 + \cos \theta_1 (Z_1^2 + Z_2^2) \right) \right) \]
\[ -0.5 Z_1^2 Z_2 Y^2 \sin \theta_2 (1 - \cos \theta_1) \cdot Z_1 Z_2 Y \sin \theta_1 \sin \theta_2 - Z_1^2 Y (1 - \cos \theta_1) \cos \theta_2 \]
\[ C_u = j \left( 1/Z_1 \sin \theta_1 \cos \theta_2 + 0.5 Z_2 Y \sin \theta_2 \left( 1/Z_2^2 - 1/Z_1^2 + \cos \theta_1 (1/Z_1^2 + 1/Z_2^2) \right) \right) \]
\[ + 0.5 Z_2 Y^2 \sin \theta_2 (1 + \cos \theta_1) + Y \cos \theta_2 (1 + \cos \theta_1) - Z_2/Z_1 Y \sin \theta_1 \sin \theta_2 \]

We note that \( A_u \) is complex, resulting a complex propagation constant, the real part of which is due to the radiation loss.

4.5. Results and Discussion

The insertion loss of the filter in chapter 3 was computed, and compared with the measurement. For the theoretical results the radiation loss was taken into the consideration. The comparison is illustrated in Figures 4.3. It can be seen that the theoretical results are close to the measurement. This confirms the validity of the method used to incorporate the radiation loss.

In order to examine the validity of the modal expansion method, the performance of the structure designed in chapter 3 was computed. Figure 4.3 shows the comparison between the results obtained from the modal expansion method and the transmission line method. Within the passband, the agreement is satisfactory. However, near the lower cut-off region, a discrepancy between the two methods is observed. The modal expansion method is found to differ from the experimental data in this region. This is due to the assumption of magnetic wall, which is valid only when the feed lines joining two
patches are sufficiently thin. In such a case, the field inside the patch is more like a standing wave which supports the magnetic wall assumption. In the present case, the feed lines are not thin, as compared to the width of the patch and the results deviate from the experimental data. However, for a thin feed line, the modal expansion method would yield a more accurate result as we shall see in the following chapter.

Figures 4.4 and 4.5 illustrate values of the insertion loss and return loss of a filter with \( d_1 = 0.9 \) cm, \( a = 1.16 \) cm, \( b = 1.68 \) cm and \( Z_1 = 70 \) ohms. The DC mode (\( m \) and \( n \) zero) and two higher order modes have been considered in the computation. The solid lines show the results of the insertion loss and return loss for a periodic structure with three sections. The dotted lines indicate the insertion loss and return loss for a six-section periodic structure. As expected, the loss increases with the number of sections due to the radiation. Also, at the center frequency, \( b \) is equal to \( \lambda_0 / 2 \).

The variations of the center frequency and bandwidth with the width of the rectangular microstrip conductor are plotted in Figures 4.6 and 4.7, respectively. As expected, the bandwidth decreases with the width of the patch. However, the variation of the center frequency with the width of the patch is low. In the passband the dominant mode for the microstrip cavity is \( \text{TM}_{0n} \), and for \( \text{TM}_{0n} \) mode the resonance frequency of the microstrip conductor is not a function of \( a \). Thus, if a periodic structure filter is designed such that in the passband the \( \text{TM}_{01} \) mode is dominant in the microstrip cavity then, the center frequency of the filter will not vary with the width of the cavity. However, the bandwidth of the filter is a function of the width of the cavity, and it increases with the width of the cavity. The power loss of the structure is reduced by decreasing the width of the patch.

In Figure 4.8 the bandwidth of the passband and adjacent stopband are plotted against the characteristic impedance of the transmission feed line, \( Z_1 \). It is seen that by
Figure 4.3: Variation of the insertion loss with frequency for the periodic structure designed chapter 3. 
(——) Transmission line method, (- - -) Modal expansion method
Figure 4.4: Variation of the insertion loss with frequency for the periodic structure.
Figure 4.5: Variation of the return loss with frequency for the periodic structure.
Figure 4.6: Plot of the bandwidth vs. the width of the rectangular patch. $Z_1=70$ ohms, $d_1=0.9$ cm and $b=1.68$ cm.

Figure 4.7: Plot of the center frequency vs. the width of the rectangular patch. $Z_1=70$ ohms, $d_1=0.9$ cm and $b=1.68$ cm.
Figure 4.8: Variation of the bandwidth with the characteristic impedance ($Z_1$) of the feed line. $d_1=0.9$ cm, $a=1.16$ cm and $b=1.68$ cm.

Figure 4.9: Variation of the center frequency with the characteristic impedance ($Z_1$) of the feed line. $d_1=0.9$ cm, $a=1.16$ cm and $b=1.68$ cm.
increasing the value of $Z_1$ the bandwidth of the passband decreases, and the bandwidth of the adjacent stopband increases. The center frequency of the passband is also plotted against $Z_1$ in Figure 4.9. It is noted that the center frequency is almost constant with $Z_1$.

4.6. Conclusion

The analysis of a planar periodic structure with two different uniform lines was presented employing the cavity model theory. It was noted that, from a circuit viewpoint, the rectangular microstrip patch with open output is equivalent to infinite parallel LC resonators. The resonant frequency of each resonator is a function of the width and length of the patch conductor. Thus the dimension of the rectangular patch should be such that in the passband the cavity is close to resonance. If in a passband the dominant mode for the microstrip cavity is $\text{TM}_{0m}$, the bandwidth is changed with the width of the cavity. However, the center frequency of the passband is constant with the width of the cavity. The center frequency is also constant with the characteristic impedance of the transmission feed line.

The radiation loss for the rectangular patch conductor was considered in the filter designed by transmission line method (chapter 3), and the theoretical results were found to agree closely with the experimental data. In the following chapter a periodic structure with circular patch will be considered.
5. Periodic Structure with Circular Patches

5.1. Introduction

It was shown in the previous chapter how a periodic structure with rectangular patch elements can be analyzed both by the transmission line theory and by the cavity modal theory. However, the transmission line theory is not applicable to circular patches. The cavity modal analysis of a circular patch will be presented in this chapter.

The periodic structure under consideration is illustrated in Figure 5.1(a). The geometry of a unit cell is shown in Figure 5.2(b), which consists of a circular patch and two uniform microstrip transmission lines connected at the two diametrically opposite edges of the patch. The circular conductor is considered as a cylindrical cavity resonator.

The fields inside the cavity are obtained utilizing the modal expansion method discussed in the previous chapter. The Z-matrix of the equivalent circuit of the microstrip circular conductor is determined from the fields inside the cavity. From the Z-matrix the ABCD-parameters are obtained. The ABCD-matrix of the unit cell can be obtained by multiplying the individual ABCD matrices of the cavity resonator and microstrip feed lines.

A periodic structure bandpass filter formed with uniform microstrip transmission lines and circular conductors is designed and fabricated. Numerical results for various filter characteristics are compared with the experimental data. The effects of various parameters of the periodic structure on the bandwidth and center frequency of the filter are studied.
Figure 5.1: (a) Geometry of a planar periodic structure with circular patches; (b) A unit cell of the structure.

5.2. Model Expansion of the Inside Field

The thickness of the substrate being much less than the wave length of interest, the electric field within the substrate has the z-component only. The magnetic field has essentially x and y components, and the fields do not vary along the z-direction (Fig 5.2). The component of the current normal to the edge of the microstrip conductor approaches zero at the edge, implying that the tangential component of the magnetic field at the edge is negligible. With these assumptions, the region between the microstrip disk and the ground plane can be modelled as a cylindrical cavity with electrical walls at the top and the bottom of the cavity, and with a magnetic wall along its periphery. Thus, the fields in
Figure 5.2: Geometry of a circular microstrip patch, fed by two microstrip lines.

This region corresponding to the $\text{TM}_{mn}$ modes may be determined by solving a cavity problem.

The source which excites the cavity is assumed to be (Huygen's principle) [27]

$$\bar{J} = \hat{z} J_z = \hat{z} J_z(\theta) \delta(\rho-a)/a$$

where

$$J_z(\phi) = \begin{cases} 
  I_1/w & \text{if } -\delta/2 < \phi < \delta/2 \\
  I_2/w & \text{if } \pi-\delta/2 < \phi < \pi+\delta/2 \\
  0 & \text{Otherwise}
\end{cases}$$

where $I_1$, and $I_2$ are the line currents, $w$ is the width of the feed lines, $a$ is the radius of the microstrip disk, and $\delta$ is given by the following equations:
\[
\sin \frac{\delta}{2} = \frac{w/2}{a}
\]

which yields
\[
\delta = 2\sin^{-1} \frac{w}{2a}
\]

When a microstrip patch is excited by a current source, various cavity modes are produced. The modal expansion model is used to find the electromagnetic fields inside the cylindrical cavity. The wave equation for \( \bar{E} \) inside the cavity in the presence of a current source, \( \bar{J} \), is

\[
\nabla^2 \bar{E} + k^2 \bar{E} = j\omega \mu \bar{J} - \nabla \left( \frac{\nabla \cdot \bar{J}}{j\omega \epsilon} \right)
\]

(5.1)

where \( \bar{J} \) and \( \bar{E} \) have only the z-component, with no variation along the z-direction, which implies that

\[
\nabla \cdot \bar{J} = \frac{\partial J_z}{\partial z} = 0
\]

(5.2)

Thus the wave equation becomes

\[
\nabla^2 E_z + k^2 E_z = j\omega \mu J_z
\]

(5.3)

where \( k = \omega(\mu \epsilon)^{1/2} \). The electric field must satisfy this equation and the following magnetic wall boundary condition

\[
\hat{n} \times H_\phi = 0 \text{ at } \rho = a
\]

where \( \hat{n} \) is the normal to the microstrip patch. The magnetic field component \( H_\phi \) is obtained from the Maxwell equation:
\[ \nabla \times \mathbf{E} = j\omega \mu \mathbf{H} \]

yielding

\[ H_\phi = -\frac{j}{\omega \mu} \frac{\partial E_z}{\partial \rho} \]

Thus, the magnetic wall boundary condition at \( \rho = a \) is equivalent to

\[ \frac{\partial E_z}{\partial \rho} = 0 \text{ at } \rho = a \]

For the cavity model, the electric and magnetic fields for a given mode (TM\(_{mn}\) mode) may be obtained from the following set of equations:

\[ \overline{E}_{mn} = \psi_{mn} \hat{z} \]  \hspace{1cm} (5.4)

\[ \nabla \times \overline{E}_{mn} = j\omega \mu \overline{H}_{mn} \]  \hspace{1cm} (5.5)

\[ (\nabla + k_{mn}^2)\psi_{mn} = 0, \text{ with } \frac{\partial \psi_{mn}}{\partial \rho} = 0 \text{ at } \rho = a \]  \hspace{1cm} (5.6)

where

\[ k_{mn}^2 = \omega_{mn}^2 \mu \epsilon \]

The solutions of the homogeneous wave equation 5.6 consist of an infinite set of orthogonal functions each of the form

\[ \psi_{mn} = J_n(k_{mn}\rho)\cos n\phi \]

where \( J_n(k_{mn}\rho) \) is the Bessel function of order n, and

\[ k_{mn} = \lambda_{mn}/a \]
In the above, $X_{mn}$ is the mth zero of the function $J'_n(k_{mn}\rho)$. $J'_n(k_{mn}\rho)$ represents the derivative of $J_n(x)$ with respect to $x$.

The total field $E_z$ is a linear sum of these orthogonal functions, which must satisfy the wave equation 5.3. Mathematically, $E_z$ can be written as

$$E_z = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} J_n(k_{mn}\rho) \cos n\phi$$  \hspace{1cm} (5.7)

The field in the cavity is of the TM type, and all other field components can be found from $E_z$. The unknown constants $A_{mn}$ can be obtained by putting $E_z$ in equation 5.3. This gives

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} [k^2 - (X_{mn}/a)^2] J_n(X_{mn}\rho/a) \cos n\phi = j\omega \mu E_z$$  \hspace{1cm} (5.8)

The Bessel functions $J_n(X_{mn}\rho/a)$ are orthogonal in the interval $\rho=0$, to $\rho=a$. Therefore, we can write

$$\int_0^a \int_0^{2\pi} A_{mn} [k^2 - (X_{mn}/a)^2] \rho^2 J_n(X_{mn}\rho/a) \cos n\phi d\rho d\phi =$$  \hspace{1cm} (5.9)

$$j\omega \mu \int_0^a \int_0^{2\pi} \rho J_n(X_{mn}\rho/a) \cos n\phi d\rho d\phi$$

$$= j\omega \mu \left[ \int_0^a \int_\delta^{\delta/2} I_1/\omega \delta(\rho-a)/a \rho J_n(X_{mn}\rho/a) \cos n\phi d\rho d\phi + \int_0^{\pi+\delta/2} \int_{\pi-\delta/2} I_2/\omega \delta(\rho-a)/a \rho J_n(X_{mn}\rho/a) \cos n\phi d\rho d\phi \right]$$

Performing the integration with respect to $\phi$, one obtains
\[ A_{mn} [k^2 - (X_{mn}/a)^2] \int_0^a \rho J_n^2 (X_{mn}/\rho) d\rho = \]

\[ j \frac{2\omega \mu}{nw\sigma_m} J_n (X_{mn}/\rho) \sin n\delta/2[I_1 + \cos n\pi I_2] \]

where

\[ \sigma_m = \begin{cases} 2 & \text{if } m=0 \\ 1 & \text{Otherwise} \end{cases} \]

In particular,

\[ A_{00} = j \frac{\omega \mu \delta}{wk^2 \pi a} [I_1 + I_2] \]

\[ A_{m0} = j \frac{\omega \mu \delta}{w\pi a [k^2 - (X_{m0}/a)^2] J_0 (X_{m0})} [I_1 + I_2] \]

\[ A_{mn} = j \frac{4\omega \mu \mu \sin n\delta/2}{nw\pi [k^2 - (X_{m0}/a)^2][1 - (n/X_{mn})^2] J_n (X_{mn})} [I_1 + \cos n\pi I_2] \]

The above expressions for \( A_{mn} \) can be substituted in 5.7 to obtain the electromagnetic fields inside the cavity resonator. The ABCD parameters of the two ports circular patch will be derived in the section that follows.

5.3. Equivalent Circuit

The input voltage \( V_1 \), and the output voltage \( V_2 \) are related to the electric fields, and are given by

\[ V_1 = -E_1 h \]

\[ V_2 = -E_2 h \]
where \( h \) is the thickness of the substrate, and \( E_1 \) and \( E_2 \) are the average electric field at the input and output ports, respectively. The expressions for \( E_1 \) and \( E_2 \) are obtained as

\[
E_1 = 1/\delta \int_{-\delta/2}^{\delta/2} (E_2)_{\rho=a_e} d\phi
\]

\[
= \sum_{m=0}^{\infty} A_m J_0(X_m a_e/a) + \frac{1}{n \delta} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} 2A_{mn} J_n(X_m a_e/a) \sin n \delta/2
\]

\[
E_2 = 1/\delta a \int_{-\delta/2}^{\delta/2} (E_2)_{\rho=a_e} d\phi
\]

\[
= \sum_{m=0}^{\infty} A_m J_0(X_m a_e/a) + \frac{1}{n \delta} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} 2A_{mn} J_n(X_m a_e/a) \sin n \delta/2 \cos n \pi
\]

Note that an effective radius \( a_e \) has been introduced to account for the fringe fields along the edge of the resonator. The relation between the physical radius and the effective radius is given by [17]

\[
a_e = a \left[ 1 + \frac{2h}{\pi a \epsilon_r} (\ln \frac{\pi a}{2h} + 1.7726) \right]^{1/2}
\]

In the above equation \( \epsilon_r \) is the dielectric constant of the substrate. The elements of \( Z \)-matrix are obtained as

\[
Z_{11} = Z_{22} = -j \omega \mu h \left( \frac{\delta}{w k^2 \pi a} + \sum_{m=2}^{\infty} \frac{\delta}{w \pi a [k^2 - (X_m a)^2]} J_0(X_m a) \right)
\]

\[
+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{8 \sin^2 n \delta/2}{n^2 \delta w \pi [k^2 - (2m a)^2]} J_n(X_m a)
\]

\[
Z_{12} = Z_{21} = -j \omega \mu h \left( \frac{\delta}{w k^2 \pi a} + \sum_{m=2}^{\infty} \frac{\delta}{w \pi a [k^2 - (X_m a)^2]} J_0(X_m a) \right)
\]
It can be seen that, similar to the rectangular microstrip patch, the input impedance of a circular microstrip conductor is equivalent to infinite parallel LC resonators. The frequency of resonance of a resonator is

\[ f_r = \frac{1}{2\pi(\mu\epsilon)(X_{mn}/a_e)^{1/2}} \]

For the TM\textsubscript{11} mode the equivalent L and C of the resonator are obtained as

\[ L = \frac{8\mu ha \sin^2 \delta/2 J_1(X_{11}a_e/a)}{\delta w\pi(X_{11}/a)^2[1-(1/X_{11})^2]J_1(X_{11})} \]

\[ C = \frac{8ha \sin^2 \delta/2 J_1(X_{11}a_e/a)}{\delta\epsilon w\pi[1-(1/X_{11})^2]J_1(X_{11})} \]

The radius of the microstrip circular conductor should be such that in the passband the cylindrical cavity is near resonance. If the characteristic impedance of the uniform line is taken as 50 ohms, it is evident that at the center frequency the characteristic impedance of the periodic structure would be 50 ohms which is desirable to minimize the return loss of a terminated periodic structure.

The elements in the ABCD-matrix of the circular microstrip patch are obtained from the Z-parameters. The ABCD parameters of a unit cell are obtained by multiplying the ABCD-matrix of the microstrip feed lines connected to the input and output ports. The overall ABCD-matrix of the periodic structure is obtained using equations 3.5 in chapter 3.
5.4. Results and Discussion

Using the filter design procedure a periodic structure bandpass filter with a bandwidth of 3.8 GHz and center frequency of 7 GHz was designed and fabricated (Fig. 5.3). The characteristic impedance of the microstrip transmission feed lines was taken as $Z_1=50$ ohms. The length of the feed line was found as $d_1=0.9$ cm, and the radius of the circular microstrip patch conductor was obtained as $a=0.99$ cm. The substrate thickness was 0.159 cm with a dielectric constant of 2.2. The number of sections was $N=3$.

The theoretical results for the insertion loss and return loss are shown in Figures 5.4 and 5.5, respectively, which include the DC mode and three higher order modes. Measurement taken from a three-section periodic structure are also shown. Note that, for the structure with circular patches the ripples are higher than those in the structures discussed before. This is due to the higher rate of change of the characteristic impedance of the structure with frequency.

The variations of the propagation constant and characteristic impedance of the structure were computed and are plotted in Figures 5.6 and 5.7, respectively. Note that, similar to the periodic structures discussed in the preceding chapters, the propagation constant and characteristic impedance are real in the passband and imaginary in the stopband. Also, the bandwidth of the passband decreases with $Z_1$, and the center frequency is almost constant. However, the bandwidth of the adjacent stopband increases with $Z_1$.

The variations of the bandwidth and center frequency with the radius of the circular patch are illustrated in Figures 5.8 and 5.9, respectively. It can be seen that, the bandwidth and center frequency decrease with the radius of the patch.

The variations of the bandwidths of the passband and adjacent stopband and the
Figure 5.3: Experimental periodic structure filter with $d_1=0.9$ cm, $a=0.99$ cm, $h=0.159$ cm, $\varepsilon_r=2.2$, $Z_1=50$ ohms.
Figure 5.4: Variation of the insertion loss with frequency for the periodic structure with circular patches.
Figure 5.5: Variation of the return loss with frequency for the periodic structure with circular patches.
Figure 5.6: Propagation constant $\beta$ versus frequency.

Figure 5.7: Characteristic impedance (normalized) versus frequency.
Figure 5.8: Plot of the bandwidth vs. the radius $a$ of the circular patch. $Z_1=110$ ohms and $d_1=0.9$ cm.

Figure 5.9: Plot of the center frequency vs. the radius of the circular patch. $Z_1=110$ ohms and $d_1=0.9$ cm.
Figure 5.10: Variation of the bandwidth with the characteristic impedance ($Z_1$) of the feed line. $d_1=0.9$ cm and $a=1.4$ cm.

Figure 5.11: Variation of the center frequency with the characteristic impedance ($Z_1$) of the feed line. $d_1=0.9$ cm and $a=1.4$ cm.
tion of the center frequency of the passband are shown in the Figures 5.10, and 5.11, respectively. It is seen that the bandwidth of the passband decreases with $Z_1$. However, the bandwidth of the adjacent stopband increases with $Z_1$. The variation of the center frequency of the passband with $Z_1$ is small. Similar characteristics were also noted in the case of rectangular patches.

5.5. Conclusion

A planar periodic structure with uniform transmission line loaded with circular patches was analyzed. It was seen that, the input impedance of a circular patch is equivalent to parallel LC resonators. The frequencies of resonance are determined by the radius of the patch conductor. It was also noted that this structure has a narrow adjacent stopband in comparison with the structures discussed earlier. The theoretical results were compared with measurements and the agreement was satisfactory. The bandwidth and center frequency of the filter are determined by the parameters of the structure.
6. Conclusion

The investigations reported in the thesis can be broadly categorized as follows:

i) Analysis of planar periodic structures with different patch geometries such as rectangular, circular, etc.

ii) Performance study for periodic-structure filters with the above geometrical shapes, which provides a guideline for the design of a filter using these shapes.

iii) Experimental verification of the theory

The important conclusions and scope of the studies can be grouped into two classes.

a) Conclusions arising from the methods used to analyze the structures.

b) Performance of various periodic structures as bandpass filters.

a) CONCLUSIONS ARISING FROM THE METHODS USED:

In the thesis, two different methods were employed to analyze various periodic structures. They are the transmission line method and the modal expansion method. Numerical computations were very simple for the transmission line method, and it provided accurate results for rectangular and comb-line periodic structure filters. However, we observed that the method has few a limitations. These are listed below:
i) It is applicable for rectangular geometries only, i.e., when the patch shapes are rectangular.

ii) It does not provide very accurate results, when the feed lines joining two patches are very thin.

The modal expansion method on the other hand, is more versatile than the transmission line method. It can be used for rectangular, circular, elliptic and other geometries (except the comb-line structure). Our studies were confined to two different periodic structures, which are formed with rectangular and circular patches. Numerical results show agreement with the results obtained experimentally.

Although the modal expansion method has several advantages over the transmission line method, it has some limitations too. These are:

i) It is more numerically complex than the transmission line method.

ii) If the intermediate feed lines joining two patches are not very thin (for example greater than \( \lambda_0/4 \)), the method yields inaccurate results. This is due to the assumption of a magnetic wall which is not valid for wider feed lines.

Therefore, when the intermediate feed lines are thin (width < \( \lambda_0/4 \)), the modal expansion method provides accurate results, however, in all other cases, the transmission line model yields better accuracy. This is an important criterion for selecting the appropriate method for a structure where both methods, in principle, are applicable.

b) PERFORMANCE OF VARIOUS PERIODIC STRUCTURES AS FILTERS:

We have studied three different structures with (1) rectangular patches (2) open microstrip lines (3) circular patches. The important conclusions were as follow:
i) The theoretical results had agreement with measurements.

ii) For all structures the center frequency and bandwidth of the bandpass filters decreased with the physical parameters of the patches. These parameters were: a) the length of the rectangular patch, b) the length of the open microstrip line and c) the radius of the circular patch.

iii) The bandwidth of the bandpass filters were found to decrease with the characteristic impedance of the uniform line $Z_1$. However, the center frequency of the passband remained constant with $Z_1$.

iv) The structure formed with different uniform lines were found to have better filter performance in comparison with the others structures.

Therefore, the analytical methods presented in this thesis have been shown to provide accurate predictions of the experimental results. Also, it was demonstrated that the microstrip periodic structure can be used as a bandpass or bandstop filter. Filter response can be improved by reducing the power loss of the structure. This can be accomplished by using a stripline configuration, or by reducing the thickness of the substrate.
REFERENCES


