A STUDY OF
THE STABILITY OF
THE MOODY-WACKER AMPLIFIER

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of
Master of Science
in the Department of Electrical Engineering
University of Saskatchewan

by

P.T. SANJIVA MURTHY

Saskatoon, Saskatchewan
June 1965

The University of Saskatchewan claims copyright in conjunction with the author. Use shall not be made of the material contained herein without proper acknowledgement.
ACKNOWLEDGEMENTS

The author wishes to express his sincere gratitude to Professor A.G. Wacker for his invaluable guidance throughout this work. He also wishes to thank Mr. A.E. Krause for his suggestions.

This work is supported by the National Research Council of Canada under Grant No. A-2193
ABSTRACT

The work presented in this thesis consists of the study of the stability of the Moody-Wacker amplifier. Generalized stability criteria based on the index principle from complex variables are used extensively in the stability investigations. Initially, it is assumed that the tunnel diode is a pure negative resistance device. The stability of two stage amplifiers incorporating two such negative resistances was investigated. The stability of both inductor input and capacitor input circuits using constant k filter sections and the influence of increasing the number of filter sections in the transmission line was studied. These investigations reveal that appreciable gain cannot be secured with a reasonable number of filter sections in the transmission line.

An attempt was made to achieve stability by making the resistive cut-off frequency of the tunnel diode nearly equal to the cut-off frequency of the transmission line. The stability of three stage amplifiers designed on this basis and which incorporate the equivalent circuit of the tunnel diode was investigated. With this arrangement there seems to be a possibility of realizing a gain of the order of 10.

The stability of two stage amplifiers using maximally flat Butterworth filters as the lumped transmission line was investigated by applying Routh's criterion. The effect of increasing the number of reactive elements in the transmission line was examined. The tunnel diode was treated to be a pure
negative resistance even in these investigations. The results obtained in this case are identical with those obtained using constant k filters as the lumped transmission line.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgements</td>
<td>i</td>
</tr>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>iv</td>
</tr>
<tr>
<td>Illustrations</td>
<td>vii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.1 General Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Scope of the work</td>
<td>3</td>
</tr>
<tr>
<td>2. BACKGROUND INFORMATION</td>
<td>5</td>
</tr>
<tr>
<td>2.1 The characteristics of the tunnel diode</td>
<td>5</td>
</tr>
<tr>
<td>2.2 The Moody-Wacker Amplifier</td>
<td>7</td>
</tr>
<tr>
<td>2.3 Stability of the tunnel diode circuits</td>
<td>10</td>
</tr>
<tr>
<td>3. STABILITY OF AMPLIFIERS USING CONSTANT K FILTER SECTIONS AND PURE NEGATIVE RESISTANCES</td>
<td>30</td>
</tr>
<tr>
<td>3.1 Frequency scaling and impedance scaling</td>
<td>30</td>
</tr>
<tr>
<td>3.2 Stability of amplifiers with capacitor input circuits</td>
<td>36</td>
</tr>
<tr>
<td>3.2.1 Two stage amplifier with one filter section per stage</td>
<td>36</td>
</tr>
<tr>
<td>3.2.2 Two stage amplifier with two filter sections per stage</td>
<td>47</td>
</tr>
<tr>
<td>3.2.3 Two stage amplifier with three filter sections per stage</td>
<td>52</td>
</tr>
<tr>
<td>3.3 Stability of amplifiers with inductor input circuits</td>
<td>53</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (CONT'D)  


4.1 Stabilization of the amplifier reducing the resistive cut-off frequency of the tunnel diode 56  
4.2 Design consideration 60  
4.3 Variables to be considered in the investigations of the stability of the amplifier 63  
4.4 Stability of three stage amplifiers with three filter sections per stage 70  
4.4.1 Stability of the amplifiers with capacitor input circuits 70  
4.4.2 Stability of the amplifiers involving inductor input circuits 72  
4.5 Stability of a hypothetical circuit which does not include the transmission line 106  

5. STABILITY OF THE AMPLIFIERS INCORPORATING BUTTERWORTH FILTERS 109  
5.1 The polynomials that determine the stability of the capacitor and the inductor input circuits 109  
5.2 Stability of the amplifiers incorporating Butterworth filters consisting of two elements 114  
5.3 The effect of increasing frequency sensitive components in the filter section on the stability of the capacitor input circuits 121  
5.4 Effect of increasing reactive elements in the filter section on the stability of the inductor input circuits 124
TABLE OF CONTENTS (CONT'D)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. CONCLUSION</td>
<td>127</td>
</tr>
<tr>
<td>7. REFERENCES</td>
<td>131</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>133</td>
</tr>
<tr>
<td>APPENDIX B</td>
<td>138</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Current versus voltage characteristic of the tunnel diode</td>
<td>6</td>
</tr>
<tr>
<td>2.2</td>
<td>Small signal equivalent circuit of the tunnel diode</td>
<td>6</td>
</tr>
<tr>
<td>2.3</td>
<td>Transmission line terminated in a negative resistance</td>
<td>9</td>
</tr>
<tr>
<td>2.4</td>
<td>An amplifier with n stages</td>
<td>9</td>
</tr>
<tr>
<td>2.5</td>
<td>Equivalent circuit for determining stability by using soldering iron type of impedance function</td>
<td>17</td>
</tr>
<tr>
<td>2.6</td>
<td>Equivalent circuit for determining stability by using pliers type of impedance function</td>
<td>19</td>
</tr>
<tr>
<td>2.7</td>
<td>Closed contour C in the S plane within which the existence of poles or zeros of F(S) have to be determined. C is the standard contour for exploring the right half plane</td>
<td>22</td>
</tr>
<tr>
<td>2.8</td>
<td>The plot of real versus imaginary parts of F(S) as S travels on C</td>
<td>22</td>
</tr>
</tbody>
</table>

### Illustration of the stability criteria for networks with one negative resistance:

2.9 By superposing the reflection of $Z_s(S)$ in the real axis                     | 24   |
2.10 By marking arrows                                                           | 24   |

### Illustrations of the stability criteria for networks with two negative resistances:

2.11 Network                                                                     | 26   |
2.12 Plot of $Z_{1s}(S)$ at $R_{N1}$ without $R_{N2}$                           | 26   |
2.13 Plot of $Z_{2s}(S)$ at $R_{N2}$ with $R_{N1}$ included                    | 26   |
**LIST OF FIGURES (CONT'D)**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Illustrations of the stability criteria for inductor input circuits</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.14</td>
<td>Network</td>
</tr>
<tr>
<td>2.15</td>
<td>Plot of impedance function $Z_{ls}(S)$</td>
</tr>
<tr>
<td>3.1</td>
<td>Typical circuit to be considered to obtain the variables required for stability investigations</td>
</tr>
<tr>
<td>3.2</td>
<td>Circuit in which the normalized impedances of all its elements are displayed</td>
</tr>
<tr>
<td>3.3</td>
<td>Impedance plot on the normalized scale</td>
</tr>
<tr>
<td>3.4</td>
<td>(a) Two stage amplifier with one filter section per stage (capacitor input)</td>
</tr>
<tr>
<td>3.4</td>
<td>(b) Two stage amplifier with two filter sections per stage (capacitor input)</td>
</tr>
<tr>
<td>3.5</td>
<td>Plot of normalized impedance at resonance points versus $R_N/Z_{01}$ of part 1</td>
</tr>
<tr>
<td>3.6</td>
<td>Experimental curves of part 1</td>
</tr>
<tr>
<td>3.7</td>
<td>Experimental curves of part 2</td>
</tr>
<tr>
<td>3.8</td>
<td>Plot of the normalized impedance at resonance points versus $R_N/Z_{01}$ of part 1</td>
</tr>
<tr>
<td>3.9</td>
<td>Plot of normalized impedance at resonance points versus $R_N/Z_{01}$ of part 2</td>
</tr>
<tr>
<td>3.10</td>
<td>Plot of normalized impedance versus $R_N/Z_{01}$ of specific cases</td>
</tr>
<tr>
<td>3.11</td>
<td>Two stage amplifier with three filter sections per stage (capacitor input)</td>
</tr>
<tr>
<td>3.12</td>
<td>Plot of normalized impedance at resonance points versus $R_N/Z_{01}$ of part 1</td>
</tr>
<tr>
<td>3.13</td>
<td>Plot of normalized impedance at resonance points versus $R_N/Z_{01}$ of part 2</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>3.14</td>
<td>Two stage amplifier with one filter section per stage (inductor input)</td>
</tr>
<tr>
<td>3.15</td>
<td>Plot of normalized impedance at resonance points versus $R_N/Z_0$</td>
</tr>
<tr>
<td>4.1</td>
<td>Two stage amplifier incorporating the equivalent circuit of the tunnel diode whose resistive cut-off frequency is equal to the cut-off frequency of the transmission line</td>
</tr>
<tr>
<td>4.2</td>
<td>Frequency response of the amplifier</td>
</tr>
<tr>
<td>4.3</td>
<td>Typical circuit to be considered to obtain the variables required for stability investigations</td>
</tr>
<tr>
<td>4.4</td>
<td>Circuit in which the normalized impedance of each element is displayed</td>
</tr>
<tr>
<td>4.5</td>
<td>Plot of $Y$ versus $x$</td>
</tr>
<tr>
<td>4.6</td>
<td>Flow chart that generates data cards</td>
</tr>
<tr>
<td>4.7</td>
<td>Three stage amplifier with three filter sections per stage (capacitor input)</td>
</tr>
</tbody>
</table>

Plots of normalized impedances at resonance points versus $L_{f_c}/Z_0$

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8</td>
<td>$G = 5; K_1 = 0.25; K_2 = 1.1$; part (1,2,3)</td>
</tr>
<tr>
<td>(a,b,c)</td>
<td></td>
</tr>
<tr>
<td>4.9</td>
<td>$G = 5; K_1 = 0.25; K_2 = 1.1$; part (1,2,3)</td>
</tr>
<tr>
<td>(a,b,c)</td>
<td></td>
</tr>
<tr>
<td>4.10</td>
<td>$G = 5; K_1 = 1.0; K_2 = 1.1$; part (1,2,3)</td>
</tr>
<tr>
<td>(a,b,c)</td>
<td></td>
</tr>
<tr>
<td>4.11</td>
<td>$G = 10; K_1 = 0.25; K_2 = 1.1$; part (1,2)</td>
</tr>
<tr>
<td>(a,b)</td>
<td></td>
</tr>
<tr>
<td>4.11c</td>
<td>$G = 10; K_1 = 0.25; K_2 = 1.1$; part 3</td>
</tr>
<tr>
<td>4.12</td>
<td>$G = 10; K_1 = 0.5; K_2 = 1.1$; part (1,2)</td>
</tr>
<tr>
<td>(a,b)</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>4.12c</td>
<td>$G = 10; K_1 = 0.5; K_2 = 1.1; $ part 3</td>
</tr>
<tr>
<td>4.13</td>
<td>$G = 10; K_1 = 0.25; K_2 = 1.4; $ part (1,2)</td>
</tr>
<tr>
<td>(a,b)</td>
<td></td>
</tr>
<tr>
<td>4.13c</td>
<td>$G = 10; K_1 = 0.25; K_2 = 1.4; $ part 3</td>
</tr>
<tr>
<td>4.14</td>
<td>$G = 10; K_1 = 0.5; K_2 = 1.4; $ part (1,2)</td>
</tr>
<tr>
<td>(a,b)</td>
<td></td>
</tr>
<tr>
<td>4.14c</td>
<td>$G = 10; K_1 = 0.5; K_2 = 1.4; $ part 3</td>
</tr>
<tr>
<td>4.15</td>
<td>$G = 10; K_1 = 1.0; K_2 = 1.4; $ part (1,2)</td>
</tr>
<tr>
<td>(a,b)</td>
<td></td>
</tr>
<tr>
<td>4.15c</td>
<td>$G = 10; K_1 = 1.0; K_2 = 1.4; $ part 3</td>
</tr>
<tr>
<td>4.16</td>
<td>$G = 15; K_1 = 0.25; K_2 = 1.1; $ part (1,2)</td>
</tr>
<tr>
<td>(a,b)</td>
<td></td>
</tr>
<tr>
<td>4.16c</td>
<td>$G = 15; K_1 = 0.25; K_2 = 1.1; $ part 3</td>
</tr>
<tr>
<td>4.17</td>
<td>$G = 15; K_1 = 0.25; K_2 = 1.56; $ part (1,2)</td>
</tr>
<tr>
<td>(a,b)</td>
<td></td>
</tr>
<tr>
<td>4.17c</td>
<td>$G = 15; K_1 = 0.25; K_2 = 1.56; $ part 3</td>
</tr>
<tr>
<td>4.18</td>
<td>Three stage amplifier with three filter sections per stage (inductor input)</td>
</tr>
<tr>
<td>4.19</td>
<td>$G = 5; K_1 = 0.25; K_2 = 1.1; $ part (1,2,3)</td>
</tr>
<tr>
<td>(a,b,c)</td>
<td></td>
</tr>
<tr>
<td>4.20</td>
<td>$G = 5; K_1 = 0.50; K_2 = 1.1; $ part (1,2,3)</td>
</tr>
<tr>
<td>(a,b,c)</td>
<td></td>
</tr>
<tr>
<td>4.21</td>
<td>$G = 5; K_1 = 1.0; K_2 = 1.1; $ part (1,2,3)</td>
</tr>
<tr>
<td>(a,b,c)</td>
<td></td>
</tr>
<tr>
<td>4.22</td>
<td>$G = 10; K_1 = 0.25; K_2 = 1.1; $ part (1,2)</td>
</tr>
<tr>
<td>(a,b)</td>
<td></td>
</tr>
<tr>
<td>4.22c</td>
<td>$G = 10; K_1 = 0.25; K_2 = 1.1; $ part 3</td>
</tr>
<tr>
<td>4.23</td>
<td>$G = 10; K_1 = 0.25; K_2 = 1.4; $ part (1,2)</td>
</tr>
<tr>
<td>(a,b)</td>
<td></td>
</tr>
<tr>
<td>4.23c</td>
<td>$G = 15; K_1 = 0.25; K_2 = 1.4; $ part 3</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>4.24</td>
<td>$G = 15; K_1 = 0.25; K_2 = 1.1; \text{ part (1,2)}$</td>
</tr>
<tr>
<td>4.24c</td>
<td>$G = 15; K_1 = 0.25; K_2 = 1.1; \text{ part 3}$</td>
</tr>
<tr>
<td>4.25</td>
<td>$G = 15; K_1 = 0.25; K_2 = 1.56; \text{ part (1,2)}$</td>
</tr>
<tr>
<td>4.25c</td>
<td>$G = 15; K_1 = 0.25; K_2 = 1.56; \text{ part 3}$</td>
</tr>
<tr>
<td>4.26</td>
<td>A hypothetical circuit which consists of three identical tunnel diodes arranged in parallel</td>
</tr>
<tr>
<td>4.27</td>
<td>Circuit consisting of a single tunnel diode which is an exact equivalent of the parallel combination of three tunnel diodes</td>
</tr>
<tr>
<td>5.1</td>
<td>Two stage amplifier incorporating Butterworth filters as the lumped transmission line (capacitor input)</td>
</tr>
<tr>
<td>5.2</td>
<td>Two stage amplifier incorporating Butterworth filter as the lumped transmission line (inductor input)</td>
</tr>
<tr>
<td>5.3</td>
<td>Two stage amplifier with two reactive elements per stage (capacitor input)</td>
</tr>
<tr>
<td>5.4</td>
<td>Amplifier with two reactive elements per stage (inductor input)</td>
</tr>
<tr>
<td>5.5</td>
<td>Reduced form of the circuit 5.3</td>
</tr>
<tr>
<td>5.6</td>
<td>Reduced form of the circuit 5.4</td>
</tr>
<tr>
<td>5.7</td>
<td>Plot of $R_C/R_Y$ against $\sqrt{L/C} R_C$</td>
</tr>
<tr>
<td>5.8</td>
<td>Two stage amplifier with 5 reactive elements per stage (capacitor input)</td>
</tr>
<tr>
<td>5.9</td>
<td>Two stage amplifier with 3 reactive elements per stage (capacitor input)</td>
</tr>
<tr>
<td>5.10</td>
<td>Two stage amplifier with 3 reactive elements per stage (inductor input)</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES (CONT'D)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.11</td>
<td>Two stage amplifier with 5 reactive elements per stage (capacitor input)</td>
<td>123</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

1.1 General Introduction

The tunnel diode is a two terminal semiconductor device which displays a negative resistance over a portion of its d.c. current-voltage characteristic. This incremental negative resistance combined with its high frequency performance has made it a useful tool in the hands of the circuit engineer. Because of the simplicity with which it performs a number of circuit functions, it has found extensive applications in the design of the electronic circuits such as oscillators, trigger circuits, and wide band amplifiers (Gentile, 1962). Amplifiers with large gains, wide bandwidths, and relatively low noise figures can be designed incorporating the tunnel diode in a passive environment (Smilen, et al., 1960).

Numerous tunnel diode amplifiers have been described in the literature (Sard, 1960; Trambarulo, 1960; Harmann, 1961). A cursory examination of the trade journals also indicates that a number of amplifiers using commercially available tunnel diodes are available. The vast majority of amplifiers described in the literature are relatively narrow band amplifiers with typical bandwidths from 1% to 25% of the centre frequency. The centre frequencies typically range from 100 Mc to 10 Gc. Wideband amplifiers with bandwidths between 60% to 125% of the centre frequency have been described (Fleri, et al., 1961; MacGlashan, 1962; Locherer, 1962). King and Sharpe
(King, et al., 1961) describe a wideband amplifier designed for 5 db gain using one tunnel diode. Its bandwidth is 200% of the centre frequency; the centre frequency is about 350 Mc.

Moody-Wacker (Moody, et al., 1961) conceived the idea of designing a wideband amplifier whose bandwidth is of the order of 200% of the centre frequency on the basis of the impedance matching. In this amplifier the negative resistance property of the tunnel diode is exploited to achieve power amplification. This amplifier consists of a number of lumped transmission lines in tandem with the tunnel diode at the end of each successive stage to provide an impedance match to the lower impedance of the following stage. Tysseland (Tysseland, 1962) designed a two stage amplifier on the basis of this concept. He found it was unstable, but succeeded in stabilizing it by inserting a resistance in series with the tunnel diode nearest to the source. However, this resistance was not taken into account in the design consideration. For the logical design of the amplifier, it is very necessary to study its stability.

The method of determining stability by finding the location of the poles and zeros of impedance function is not easily applicable to the circuits like the Moody-Wacker amplifier. The degree of the numerator or the denominator polynomial of the impedance function is quite large, and factoring such polynomials to find the roots is rather formidable. Mahabala's generalized stability criteria (Mahabala, 1964)
which provides an alternative approach to the problem is used extensively in this work for the determination of stability. Mahabala's theory is based on a theorem in complex variables known as the index principle.

1.2 Scope of the work

The work to be presented in this thesis consists of the study of the stability of the Moody-Wacker amplifier. Assuming the tunnel diode to be a pure negative resistance device, the stability of the two stage amplifiers incorporating such negative resistances was investigated. One filter section of the constant k prototype was used as the lumped transmission line in each stage of these amplifiers. For this particular case, the results based on Mahabala's theory were verified experimentally and good agreement was found between the experimental and the theoretical results. The stability investigations were repeated with two and three filter sections in each stage of the amplifier. Maximum gain that could be secured with one filter section in each stage of the amplifier was of the order of 5 db. However, the gain was found to decrease with the increase in the number of filter sections.

The stability of the amplifier incorporating the equivalent circuit of the tunnel diode and using constant k filter sections as the lumped transmission line was studied. In this case a resistance of proper value was assumed to be in series with the negative resistance of the tunnel diode, which in
effect, makes the resistive cut-off frequency of the tunnel diode equal to the cut-off frequency of the transmission line. Design procedure of the amplifier was modified to allow for this resistance and the stability of three stage amplifiers consisting of inductor input and capacitor input circuits was investigated. Stable gains of the order of 10 db with bandwidths of the order of 200% of the centre frequency appear to be theoretically possible, but this was not experimentally verified.

The possibility of using maximally flat Butterworth filters as the lumped transmission lines in two stage amplifiers was also examined. For this investigation the tunnel diode was treated as a pure negative resistance device. The stability of amplifiers with capacitor input circuits involving 2, 3, and 5 reactive elements in each transmission line was investigated by applying Routh's criterion. The investigations were repeated for amplifiers with the inductor input circuits. The analysis shows that it is impossible to design a stable amplifier with inductor input circuits. Stable gains of the order of 5 db can be realized with capacitor input circuits having three reactive elements in the transmission line. However, the gain was found to decrease with the increase in the number of frequency sensitive elements in the transmission line.
2. BACKGROUND INFORMATION

2.1 The characteristics of the tunnel diode

The tunnel diode consists of an abrupt p-n junction in which the p and n sides of the junction are so heavily doped with impurities that the Fermi level lies in the conduction band in the n-type semiconductor and in the valence band in the p-type semiconductor. Such semiconductors are said to be degenerate semiconductors. Some desirable characteristics of the tunnel diode are due to quantum mechanical tunneling (L. Esaki, 1958), which arises from the degenerate nature of the semiconductors used in their construction. These characteristics are: high frequency response, incremental negative resistance, and the relative invariance of the characteristic with temperature. The current-voltage characteristic of the tunnel diode is shown in Fig. 2.1. As the forward voltage is increased from zero, the current increases from zero to a maximum value $I_p$ at a voltage $V_p$, then decreases to a minimum value $I_v$ at a voltage $V_v$ and then increases again for larger voltages. From the characteristic it is evident that the incremental negative resistance, which is equal to the slope of the characteristic, varies from point to point and is negative for voltages between $V_p$ and $V_v$. In Fig. 2.1 the conductance plot is also shown.

In Fig. 2.2 the small signal equivalent circuit of a tunnel diode biased in the negative resistance region is shown. This has been found to be adequate for many applications. $R_s$ is the series resistance which includes the bulk resistance
Fig. 2.1 Current versus voltage characteristic of the tunnel diode.

Fig. 2.2 Small signal equivalent circuit of the tunnel diode.
of the semiconductor and the resistance of the connecting leads; \( L_s \) is the inductance of the connecting leads and package. The capacitance \( C_d \) in the circuit is the sum total of junction capacitance, diffusion capacitance, and the linear stray capacitance. \( R_N \) is the negative resistance of the device.

2.2 The Moody-Wacker Amplifier

For the sake of completeness a brief description of the Moody-Wacker amplifier follows. Consider a distributed transmission line of characteristic impedance \( Z_{01} \) terminated in a negative resistance \(-R_N\) and an impedance \( Z_{02} \) connected in parallel. The following condition must be valid if this line is to be terminated in its characteristic impedance.

\[
\frac{1}{Z_{01}} = \frac{1}{Z_{02}} - \frac{1}{R_N}
\]

or

\[
\frac{1}{Z_{02}} = \frac{1}{Z_{01}} + \frac{1}{R_N}
\]

2.1

From 2.1 it is evident that the load impedance \( Z_{02} \) is less than \( Z_{01} \). This load impedance \( Z_{02} \) can be furnished by a second stage similar to one just described. The transmission line of stage 2 is matched if

\[
\frac{1}{Z_{03}} = \frac{1}{Z_{02}} + \frac{1}{R_N} = \frac{1}{Z_{01}} + \frac{2}{R_N}
\]

2.2

where \( Z_{03} \) is the characteristic impedance of stage 3. In a similar manner a number of stages can be added. For proper
termination of the \( n \)th stage, the following relation holds:

\[
\frac{1}{Z_{0n+1}} = \frac{1}{Z_{0n}} + \frac{1}{R_N} = \frac{1}{Z_{01}} + \frac{n}{R_N} \tag{2.3}
\]

In 2.3, \( Z_{0n+1} \) is the characteristic impedance of the stage following the \( n \)th stage or the load impedance if the amplifier consists of only \( n \) stages. Equation 2.3 assumes that all negative resistances are equal.

Since the lines are matched, the voltage gain is unity. A power gain, however, is realized due to the reduction in the impedance level and is given by

\[
G = \frac{Z_{01}}{Z_{0n+1}} \tag{2.4}
\]

If lumped, rather than the distributed transmission lines, are used and the series inductance and the series resistance of the tunnel diode are neglected, then the capacity of the tunnel diode can be absorbed in the capacity of the transmission lines. All the stages are arranged to have the same cut-off frequency. The cut-off frequency and the characteristic impedance of the lines are given by

\[
f_c = \frac{1}{\sqrt{\pi L_n C_n}} \tag{2.5}
\]

\[
Z_{0n} = \sqrt{\frac{I_n}{C_n}} \tag{2.6}
\]
Fig. 2.3 Transmission line terminated in a negative resistance.

Fig. 2.4 An amplifier with n stages.
where $L_n$ and $C_n$ are the inductance and capacitance of the lumped transmission line forming the $n^{th}$ stage of the amplifier (Fig. 2.3 and Fig. 2.4). By rearranging these equations the parameters of the transmission lines can be obtained in terms of $Z_{0n}$ and $f_o$.

\[
L_n = \frac{Z_{0n}}{\pi f_o} \quad 2.7
\]

\[
C_n = \frac{1}{\pi f_o Z_{0n}} \quad 2.8
\]

Tysseland (Tysseland, 1962) designed a two stage amplifier using the above technique. The cut-off frequency and the gain of this amplifier were 600 Mc and 3.9, respectively. In order to keep the losses to a minimum at these high frequencies, strip line microwave components were used in the construction of the transmission lines. A 5 ohm resistor was inserted in series with the tunnel diode nearest to the source to stabilize the amplifier. The main defect in the design procedure was that this 5 ohm resistor was not taken into account in the design equations. This means that the first stage of the amplifier was not terminated in its characteristic impedance.

2.3 Stability of the tunnel diode circuits

The stability of a network can be determined through the properties of the impedance function. In this case we have to factor the numerator (or the denominator) polynomial of the
impedance function. The roots of the numerator (or denominator) polynomial are called the zeros (or poles) of the impedance function. For the network to be stable, none of the zeros or poles of the impedance function should have positive real parts. In other words, the zeros or poles should not appear on the right hand half of the complex frequency plane.

The stability criteria discuss, in essence, the stability of a linear negative resistance under a known external impedance configuration. Hines (Hines, 1960) considered the stability of a tunnel diode with a resistance connected across its terminals. According to him the conditions for potential stability (which means all the open circuit natural frequencies can be shifted to the left hand half of the complex frequency plane) of a tunnel diode are

\[
\frac{R_s}{R_N} < 1
\]

\[
\frac{L_s}{R_N^2 C_d} < 1
\]

Moreover, the diode can be stabilized by a positive resistance \( R \) if the following inequality is satisfied.

\[
1 > \frac{R + R_s}{R_N} > \frac{L_s}{R_N^2 C_d}
\]
Smilen and Youla (Smilen, et al., 1961) derived the stability criterion for a tunnel diode terminated with a complex passive load. The necessary conditions for stability are

\[ \frac{R_s}{R_N} < 1 \]  \hspace{1cm} (2.11)

\[ \frac{L_s}{R_N^2C_d} < F(\theta) \]

where

\[ F(\theta) = \frac{\theta^3}{1+\theta^2} \frac{1}{\theta - \text{arc tan } \theta} \]  \hspace{1cm} (2.12)

In the above expression \( \theta \) is given by

\[ \theta = \sqrt{\frac{R_s}{R_N}} \sqrt{1 - \frac{R_s}{R_N}} \]

Davidson (Davidson, 1963) attempts to stabilize a tunnel diode by means of a resistance \( R \) and capacitance \( C \) placed in parallel across its terminals. The necessary and sufficient conditions for stability are:

\[ R_N C_d > R C \]

\[ R_N > R + R_s \]

\[ R \left[ 1 + \gamma (1-\beta) \right] + \alpha \beta^2 \left( \frac{R_N}{R} \right) - \beta \gamma^2 (R_N - R_s) > \alpha \beta R_N \]  \hspace{1cm} (2.13)
where \( \alpha = \frac{L_s}{R_N C_d} \); \( \beta = 1 - \frac{R_s}{R_N} \); \( \gamma = \frac{R_C}{R_N C_d} \).

By setting \( \alpha = 0 \), this reduces to Hine's criterion. Davidson obtains the stability criteria for this configuration on the assumption that \( f(R, C) \) should be maximum and \( \frac{R_s}{L_s/R_N^2 C_d} \ll 1 \).

\( f(R, C) \) is given by

\[
f(R, C) = R + \alpha \beta R_N - \gamma^2 (R_s - R_N) > \alpha \infty R_N
\]

The conditions on \( R \) and \( C \) which maximize \( f(R, C) \) are

\[
RC = \frac{L_s}{2(R_N - R_s)}
\]

The stability criteria is

\[
1 > \frac{R + R_s}{R_N} > \frac{L_s}{R_N^2 C_d} \left[ 1 - \frac{\frac{L_s}{R_N^2 C_d}}{4(1 - \frac{R_s}{R_N})} \right] + R_s
\]

The bounds on \( \alpha \) for stabilization through shunt resistance and capacitance is

\[
0 < \alpha < 2(1 - \frac{R_s}{R_N})
\]

Ivan T. Frisch (Frisch, 1964) considers the stability of a circuit which results when a tunnel diode is connected to a distorted Butterworth filter. The term distorted Butterworth filter refers to a Butterworth filter in which the reactive elements have losses associated with them. Each inductor \( L_i \)
has a resistance of value \( \frac{d}{R_N C_d (1-d)} \) in series with it and each capacitance \( C_i \) has a conductance \( \frac{d}{R_N C_d (1-d)} \) in parallel with it. The quantity \( "d" \) depends on the parameters of the tunnel diode and is given by

\[
d = \frac{1}{\frac{L_s}{R_N R_s C_d} + 1}
\]

The manner of selecting the number and value of the reactive elements is rather involved and will not be discussed here. The sufficient conditions for potential stability are

\[
\frac{L_s}{R_N^2 C_d} \leq 3(1-d^2)
\]

\[
\frac{R_s}{R_N} \leq 3d(1-d^2)
\]

\[
0 < d < 1
\]

These different criteria are restricted to the circuits involving one tunnel diode. They are not of much significance in the present situation, since the Moody-Wacker amplifier contains more than one tunnel diode.

The method of determining stability by finding the location of the poles and zeros of the impedance function is not easily applicable to the circuits like the Moody-Wacker amplifier as the degree of the polynomial is quite large. To factor
such a polynomial in order to find its roots is rather formidable. Since it is sufficient to find whether the zeros or poles lie on the R.H.S. plane or not, methods such as Routh's criterion can be used. Even here we need a characteristic polynomial whose derivation may be tedious.

Davidsohn (Davidsohn, et al., 1960) has used the index principle from complex variables to determine the stability of networks involving one negative resistance. The index principle enables us to determine the difference between the zeros and poles of the impedance function lying on the R.H.S. plane, but not the exact number of zeros or poles. Mahabala's generalized stability criteria enables us to determine not only the exact number of zeros or poles of the impedance function which lie in the R.H.S. plane, but also the stability of the circuits containing more than one negative resistance by successive application of the index principle.

Since a thorough understanding of Mahabala's theory is vital to follow the present work, a brief summary of this theory is presented. Mahabala's theory depends on certain properties of the impedance function; hence, those properties will be discussed first.

There are two types of impedance functions which are of interest to us and which depend on the manner of injecting a disturbance into the circuit (Guilmin, 1957). The type of entry in which a $\mathcal{S}$ function of voltage is injected in series with a branch is called plier's type of entry. The plier's
type of impedance function is the driving point impedance between two terminals formed when a branch is opened. The type of entry by which a function of current is applied between a pair of nodes is called soldering iron type of entry. The soldering iron type of impedance function is the driving point impedance across any two nodes. Both these impedance functions are expressed as a quotient of two polynomials.

\[ Z_s(S) = \frac{N_s(S)}{D_s(S)} \quad 2.20 \]

\[ Z_p(S) = \frac{N_p(S)}{D_p(S)} \quad 2.21 \]

The subscripts p and S refer to "pliers" and "soldering iron" types of impedance functions, respectively.

Now let us consider the stability of the circuit shown in Fig. 2.5 for soldering iron type of entry. Let us determine the voltage \( v(t) \) developed across the nodes A and B for a function of current. The voltage transform \( V(S) \) is the product of the Laplace transform of \( \xi(t) \) and \( Z_s(S) \).

\[ V(S) = \mathcal{L}\left(\xi(t)\right)Z_s(S) \]

The inverse transformation presents

\[ v(t) = \mathcal{L}^{-1} Z_s(S) \]

\[ = \mathcal{L}^{-1} \frac{N_s(S)}{D_s(S)} \]

\[ = \sum_{r=1}^{n} a_r e^{S_r t} \quad 2.23 \]
Fig. 2.5 Equivalent circuit for determining stability by using soldering iron type of impedance function.
ar is a constant that depends on the initial conditions of the network. The quantities S1 to Sr are the roots of the denominator polynomial Ds(S) = 0 and are called the poles of the impedance function Zs(S). If any of the poles of Zs(S) have positive real parts, the disturbance grows with time. For the network to be stable, none of the poles of the soldering iron type of impedance function should have positive real parts. In addition, poles on the jω axis should be simple for stability. The equation 2.23 must be modified for multiple poles.

Let us derive the conditions required of Zp(S) in order that the network be stable. The response of the circuit shown in Fig. 2.6 for a function of voltage is given by

\[ i(t) = \mathcal{L}^{-1} \frac{1}{Z_p(S)} \]

\[ i(t) = \mathcal{L}^{-1} \frac{D_p(S)}{N_p(S)} \]

\[ i(t) = \sum_{r=1}^{n} c_r e^{S_r t} \]

Sr is a constant that depends on the initial conditions of the network. The quantities S1 to Sr are the roots of the polynomial Np(S) = 0 and are called the zeros of Zp(S). If any zero of Zp(S) have positive real parts, then the disturbance
Fig. 2.6 Equivalent circuit for determining stability by using pliers type of impedance function.
grows sinusoidally or exponentially depending on the imaginary part of that zero. In order for the network to be stable, the zeros of the pliers type of impedance function should not lie on the R.H.S. plane. In addition, any zeros on the \( j\omega \)-axis should be simple for the network to be stable. The equation 2.25 must be modified if any multiple zeros exist.

It can be shown that the circuit whose stability is determined on the basis that it satisfies the conditions required of \( Z_s(S) \), also satisfies the conditions required of \( Z_p(S) \). Other important properties of these impedance functions can be derived by the following relations obtained with reference to Fig. 2.6.

\[
Z_s(S) = \frac{N_s(S)}{D_s(S)} \quad 2.26
\]

\[
Z_p(S) = R_N + Z_s(S) = \frac{R_N \times D_s(S) + N_s(S)}{D_s(S)} \quad 2.27
\]

\[
Z'_s(S) = \frac{R_N \times Z_s(S)}{R_N + Z_s(S)} = \frac{R_N \times N_s(S)}{R_N D_s(S) + N_s(S)} \quad 2.28
\]

Since \( Z_s(S) \) and \( Z_p(S) \) have the same denominator, the poles of \( Z_s(S) \) are the poles of \( Z_p(S) \). The numerator and the denominator of \( Z_p(S) \) (or \( Z'_s(S) \)) do not have any factors \((S-S_\infty)\) in common. Hence, the zeros of \( Z_p(S) \) are the poles of \( Z'_s(S) \) and vice versa. The poles of \( Z'_s(S) \) are the natural frequencies of the network.
and, as such, should be the same for all pairs of nodes (Guilmin, 1957). This means that the poles of $Z_{2s}(S)$ at any other pair of nodes are the poles of $Z_s(S)$.

The essence of the foregoing analysis is that it is sufficient to study either a $Z_p(S)$ or $Z_s(S)$ at a convenient pair of nodes to determine stability. The analysis has revealed the following significant properties of the impedance function.

1. The poles of $Z_s(S)$ are the poles of $Z_p(S)$
2. The zeros of $Z_p(S)$ are the poles of $Z_s'(S)$
3. The poles of $Z_s'(S)$ are the poles of $Z_{2s}(S)$

By combining these properties it can be shown that the zeros of $Z_p(S)$ are the poles of $Z_{2p}(S)$.

These properties of the impedance function, in conjunction with the corollary of Cauchy's integral law which is also called the index principle, forms the basis of Mahabala's generalized stability criteria. The index principle states: "If $S$ travels a simple closed curve $C$ in the $S$-plane and if there are no singular points other than poles inside $C$, and on $C$, then the number of times $F(s)$ passes around the origin in the $F$ plane is equal to the weighted sum of the number of zeros inside $C$, minus the weighted sum of the number of poles inside $C$, where in each case the weighting number is the order of the zero or the pole."

Since it is the right hand plane which is of interest to us to determine the stability, the curve $C$ can be considered to be made up of the imaginary axis from $(0, \infty)$ to $(0, -\infty)$ and
Fig. 2.7 Closed Contour C in the S plane within which the existence of poles or zeros of F(S) have to be determined. C is the standard contour for exploring the right half plane.

Fig. 2.8 The plot of real versus imaginary parts of F(S) as S travels on C.
a large semicircle from \((0, \infty)\) to \((0, -\infty)\). This standard contour \(C'\), as shown in Fig. 2.7, must be traversed in the anticlockwise direction to conform with the requirements of the index principle. The impedance plot of \(Z_s(S)\) for real frequencies can be computed and the plot corresponding to negative \(j\)-axis is merely reflection in the real axis of the plot for positive frequencies. The direction of traverse of the curve in the \(F\)-plane is from d.c. to negative infinite frequency and from positive infinite frequency to d.c. as shown in Fig. 2.9.

Let the number of poles of \(Z_s(S)\) be \(P\). The plot of \(Z_s(S)\) corresponding to the standard contour \(C'\) is obtained by plotting \(Z_s(S)\) for real frequencies and adding its reflection in the real axis. The plot of \(Z_p(S)\) is the same as the plot of \(Z_s(S)\) but with the origin shifted to the point \((RN, 0)\) because \(Z_p(S) = Z_s(S) - RN\). The number of encirclements of the new origin \(X\) gives the excess of zeros over poles of \(Z_p(S)\) inside \(C'\):

\[
x_1 = N_1 - P_1
\]

\[
N_1 = P_1 + x_1
\]

For the network to be stable, the zeros of \(Z_p(S)\) should not appear on the R.H. half of the complex frequency plane.

\[
x_1 + P_1 = 0
\]
Illustration of the stability criteria for networks with one negative resistance:

Fig. 2.9 By superposing the reflection of $Z_s(S)$ in the real axis.

$x = Z - P = 2$

Fig. 2.10 By marking arrows.
Without adding the reflection of the impedance plot and without having to traverse the curve to meet the requirements of the index principle, the number of encirclements of the new origin can be found by a simple method. Arrows are marked one for infinite frequency, another for d.c., and two arrows at each zero crossing in the direction of decreasing frequency as shown in Fig. 2.10. The number of arrows on any one side of the new origin gives the index X. According to the convention to be followed, the anticlockwise arrows are considered as positive and the clockwise arrows are counted as negative. Now to determine the stability, all that we need to know is the zero crossings, and the direction of them. The digital computer programme is arranged to calculate the impedance at frequencies slightly below and above the resonant frequency to find the direction of zero crossings.

We shall apply the stability criteria for a capacitor input network with two negative resistances as shown in Fig. 2.11. When both the negative resistances are removed, we will have a passive circuit. All impedance functions of a passive circuit are positive real (Guillemin, 1957). One of the implications of positive realness is that the zeros of plier's type of impedance function and poles of soldering iron type of impedance function lie on the L.H. half of the complex frequency plane. Since it is the negative resistance which is of concern and which causes trouble, it is desirable to view the network from this active element. With both the resistances removed, the
Illustrations of the stability criteria for networks with two negative resistances:

**Fig. 2.11 Network.**

**Fig. 2.12 Plot of** $Z_{ls}(s)$ **at** $R_{N1}$ **without** $R_{N2}$. 

**Fig. 2.13 Plot of** $Z_{ls}(s)$ **at** $R_{N2}$ **with** $R_{N1}$ **included.**
soldering iron type of impedance function $Z_{1s}(S)$ at $R_{N1}$ is plotted and then the origin is shifted to $(R_{N1}, 0)$ to get the impedance plot of $Z_{1p}(S)$. (See Fig. 2.12) where $Z_{1p}(S)$ is the pliers type of impedance function at $R_{N1}$. The number of encirclements of the new origin $X_1$ gives $(N_1 - P_1)$. Since $P_1$ of $Z_{1p}(S)$ is zero for a passive circuit, the index $X_1$ gives the number of zeros of $Z_{1p}(S)$. According to the stability criteria postulated for $Z_{1p}(S)$, this subnetwork is stable provided $X_1 = 0$. This subnetwork is referred to as part I of the circuit in the future discussions.

With the resistance $R_{N1}$ included in the circuit, the soldering iron type of impedance function $Z_{2s}(S)$ is plotted at $R_{N2}$ to find the stability of the complete network. The origin is shifted to $(R_{N2}, 0)$ to get the impedance plot of $Z_{2p}(S)$ as shown in Fig. 2.13. The number of encirclements of the new origin $X_2$ gives $(N_2 - P_2)$ of $Z_{2p}(S)$. Recalling that the zeros of $Z_{1p}(S)$ are the poles of $Z_{2s}(S)$ and the poles of $Z_{2s}(S)$ are also the poles of $Z_{2p}(S)$, we can write that

$$X_2 = N_2 - P_2$$

$$X_2 = N_2 - X_1$$

$$N_2 = X_2 + X_1$$

The complete network will be stable if and only if $N_2 = 0$. The complete network is referred to as part II in our future
Illustrations of the stability criteria for inductor input circuits:

Fig. 2.14 Network.

Fig. 2.15 Plot of impedance function $Z_{ls}(s)$. 
discussions. If the subnetwork is unstable and the complete network is stable as in the present situation, then the network is said to be "conditionally stable". When both subnetwork and complete network are stable the network is considered to be "absolutely stable". A network is "absolutely stable" only when the resistance of the tunnel diode -RN is greater than the impedance at the farthest zero crossing of part I and part II.

Now we shall apply the stability criteria to an inductor input circuit which is shown in Fig. 2.14. Since the element which is next to the tunnel diode is a series inductance, the impedance plot goes to \((0,\infty)\) at infinite frequency. This circuit will be stable only when RN is less than R₀ (see Fig. 2.15). Among the networks with negative resistances, those in which the first reactive element after the negative resistance is a shunt capacitor are stable when RN is greater than the impedance at the farthest zero crossing, whereas those in which the first reactance element is a series inductor are stable provided the negative resistance value is less than the impedance at the lowest zero crossing.
3. STABILITY OF AMPLIFIERS USING CONSTANT K FILTER SECTIONS AND PURE NEGATIVE RESISTANCES

3.1 Frequency scaling and impedance scaling

In this chapter the results of the investigations of the stability of the two stage amplifier using constant K filters are presented. The series inductance $L_s$ and the series resistance $R_s$ of a tunnel diode are neglected in these initial stability investigations. Since we are using lumped transmission lines, the capacity of the tunnel diode can be absorbed in the transmission line capacity. When this is effected the tunnel diode can be looked upon as a pure negative resistance device.

It is often convenient when working with tunnel diodes to consider frequency and impedance scaled versions of the actual circuit under investigation. This is particularly true in experimental investigations as frequency and impedance ranges in which instrumentation is difficult can be avoided. The effect of frequency and impedance scaling on stability assumes paramount importance.

In any network the reactances associated with inductors and capacitors always appear in conjunction with the complex frequency $S$ in the form of $L_s$ and $C_s$. If all inductors and capacitors in a network are multiplied by the same constant, the impedance of the network will remain unchanged provided $S$ is divided by the same constant. This procedure, therefore, produces a frequency scaled network. The value of the impedance at the resonant points
also remain unchanged, although the resonant frequencies change by the scaling factor. Since the stability of the circuit depends on the resonant impedances, the stability of the original circuit and the scaled version are identical. In the case of magnitude scaling, all the inductances and resistances are multiplied and the capacitances are divided by the same constant. This in effect multiplies the impedance function by the same constant but does not affect the position of poles and zeros in the S plane. Thus, magnitude scaling also has no effect on the stability of the network.

The variables that are to be considered during the investigations of stability can be obtained from the analysis of a simple circuit terminated in a negative resistance as shown in Fig. 3.1. A filter whose characteristic impedance and cutoff frequency are \( Z_{01} \) and \( f_c \) respectively is terminated in a resistance \(-R_N\). The values of the inductance and the capacitance in the circuit are given by 2.5 and 2.6, respectively. Combining equations 2.3 and 2.7 we obtain

\[
I_n = \frac{Z_{01}}{\pi f_c \left[ 1 + (n) \frac{Z_{01}}{R_N} \right]}
\]

and \( Z_L \) the normalized impedance of \( I_n \) with respect to \( Z_{01} \)

\[
Z_L = j \frac{2f_c}{f_c \left[ 1 + (n) \frac{Z_{01}}{R_N} \right]}
\]
Fig. 3.1 Typical circuit to be considered to obtain the variables required for stability investigations.

Fig. 3.2 Circuit showing the normalized impedances of all its elements.
Fig. 3.3 Impedance plot on the normalized scale.
In a similar manner, from equations 2.3 and 2.8, we obtain

\[ C_n = \frac{1}{\pi f_c Z_{01}} \left[ 1 + (n-\frac{Z_{01}}{R_N}) \right] \]  \hspace{1cm} 3.3

and \( Z_0 \), the normalized impedance of \( C_n \) with respect to \( Z_{01} \)

\[ Z_0 = \frac{-j}{2f/f_c \left[ 1 + (n-\frac{Z_{01}}{R_N}) \right]} \] \hspace{1cm} 3.4

It can be seen from equations 3.2 and 3.4 that the normalized input impedance \( \frac{Z}{Z_{01}} \) of the circuit shown in Fig. 3.2 is a function of \( n \), \( f/f_c \), and \( \frac{R_N}{Z_{01}} \). From this analysis it is apparent that the impedance measured at any point in the Moody-Wacker amplifier will be a function of the same three variables. Since we are restricting ourselves to the investigation of the stability of the two-stage amplifier, the only variables we have to consider are \( \frac{R_N}{Z_{01}} \) and \( f/f_c \).

Since the predominant variables in the investigation of stability are \( \frac{R_N}{Z_{01}} \) and \( f/f_c \), it is convenient to use the impedance diagram on a normalized scale as shown in Fig. 3.3 rather than on the absolute scale. The normalized frequency instead of absolute frequency \( f \) is shown as a parameter along the curve. All amplifiers having the same value of \( \frac{R_N}{Z_{01}} \) will have the same impedance plot. When normalized scales are used the origin should be shifted to \( \frac{R_N}{Z_{01}} \) to determine stability of the network. In Fig. 3.3, \( X_1 \), \( X_2 \) are the zero crossings and \( X \) is the position...
Fig. 3.4a. Two stage amplifier with one filter section per stage (capacitor input).

Fig. 3.4b. Two stage amplifier with two filter sections per stage (capacitor input).
of the new origin. Irrespective of the values of \( R_N \) and \( Z_{01} \), the new origin will be at \( X \) as long as \( \frac{R_N}{Z_{01}} \) is constant. This means that all amplifiers having the identical values of \( \frac{R_N}{Z_{01}} \) will have the same stability.

3.2 Stability of amplifiers with capacitor input circuits

3.2.1 Two stage amplifier with one filter section per stage

With one filter section in each stage of the transmission line as shown in Fig. 3.4, the stability investigations were made for different values of \( \frac{R_N}{Z_{01}} \) between 0.1 and 1.5. The normalized impedance at resonance points is plotted against \( \frac{R_N}{Z_{01}} \) as shown in Fig. 3.5. In this figure, solid curves correspond to part I and dotted curves correspond to part II. The curves are numbered in the order of increasing frequency. Here we will adopt a graphical technique which seems to be logical and also useful for determining the stability of such networks. For values of \( \frac{R_N}{Z_{01}} \) from 0.1 to 0.5 only one impedance curve corresponding to part I and none of the curves of part II lie above the \( R_N \) line. For each impedance curve lying above the \( R_N \) line, the number of encirclements of the new origin is 2. The encirclements are positive or negative depending on whether the impedance has a positive or negative reactive component respectively as the frequency tends to zero. A plus or minus sign near the d.c. impedance curve indicates the sign of the reactance for low frequencies.
Let us consider a region of $\frac{R_N}{Z_{01}}$ from 0.1 to 0.5. The impedance curve corresponding to the second zero crossing of part I lies above the $R_N$ line. The index is positive as the reactive part of the impedance is positive for very low frequency.

$$X_1 = N_1 - P_1 = 2 \quad 3.5$$

For part I the network is passive and the number of poles of $Z_p(S)$ lying on the R.H.S plane is zero.

$$X_1 = N_1 = 2 \quad 3.6$$

Since all the impedance curves of part II lie below the $R_N$ line in the region under our perusal, the index $X_2$ is zero.

$$X_2 = N_2 - P_2 = 0 \quad 3.7$$

Recalling that the zeros of $Z_{1p}(S)$ are the poles of $Z_{2s}(S)$ which are also the poles of $Z_{2p}(S)$, we can write that $P_2 = N_1 = 2$. Substituting this $P_2$ in equation 3.7, we obtain.

$$N_2 - 2 = 0$$

$$N_2 = 2 \quad 3.8$$

According to the stability criteria postulated for the pliers type of impedance function, the network is unstable.

Now we shall consider the region of $\frac{R_N}{Z_{01}}$ from 0.5 to 0.75. The impedance curves corresponding to the second zero crossing
Fig. 3.5 Plot of normalized impedance at resonance points versus $\frac{R_N}{Z_0}$ of part 1.

Solid lines - Part I
Dotted lines - Part II
of part I and part II lie above the $R_N$ line. From the above discussion it follows that the number of zeros of $Z_{lp}(S)$ and, hence, the number of poles of $Z_{2p}(S)$ is +2. Since the reactive part of the impedance is negative for very low frequencies in the case of part II, the number of encirclements of the new origin in part II is

$$N_2 - P_2 = -2$$

$$N_2 - 2 = -2$$

$$N_2 = 0$$

The analysis indicates that the network is conditionally stable in the region.

For values of $\frac{R_N}{Z_{01}}$ greater than 0.75 the network is absolutely stable, since all impedance curves corresponding to part I and part II lie below the $R_N$ line.

As a check on the correctness of the theoretical results, it was decided to investigate the stability of the amplifier (shown in Fig. 3.4a) experimentally. In order to facilitate the experimental measurements it was felt desirable to work at low frequencies. A tunnel diode analogue which has the same terminal characteristic as that of a tunnel diode (Cobbold, et al., 1963) was used in the low frequency version of the amplifier. This analog also provides facilities to alter some of the significant parameters of the tunnel diode and thus enables one
Fig. 3.6 Experimental curves of part 1.
Fig. 3.7 Experimental curves of part 2.
to vary $\frac{R_N}{Z_{01}}$. As it was strongly emphasized in the previous section that the significant ratios which are of interest to determine stability are $\frac{R_N}{Z_{01}}$ and $f/f_c$, the actual values of the cut-off frequency $f_c$ and the characteristic impedance $Z_{01}$ are unimportant. Any convenient values of $Z_{01}$ and $f_c$ can be chosen. For the particular amplifier tested, the characteristic impedance and the cut-off frequency were 235.5 ohms and 15 kc respectively. These values were dictated by the available coils and the bandwidth of the tunnel diode analogue.

The experimental verification consists of two parts: (i) to find whether the results based on Mahabalais's theory agree with those obtained experimentally or not, and (ii) to determine the agreement between the computer values and the experimental values of resonant impedances.

During the experimental verification the amplifier was found to be unstable for values of $\frac{R_N}{Z_{01}}$ between approximately 0.1 and 0.5, conditionally stable between 0.5 and 0.75 and absolutely stable beyond 0.75 (see Table 3.1). This strongly indicates that the experimental results agree with the theoretical results. A small difference between the experimental values of resonant impedances and the values obtained on the digital computer was noticed. In Fig. 3.6 and 3.7 the continuous curves and the dotted curves refer to the computer values and to the experimental values respectively. This small disparity can be attributed to the fact that the resistance of the inductors was not taken into account when the resonant impedances were
<table>
<thead>
<tr>
<th>$\frac{R_N}{Z_{01}}$</th>
<th>STABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>unstable</td>
</tr>
<tr>
<td>0.5</td>
<td>conditionally stable</td>
</tr>
<tr>
<td>0.6</td>
<td>conditionally stable</td>
</tr>
<tr>
<td>0.7</td>
<td>conditionally stable</td>
</tr>
<tr>
<td>0.8</td>
<td>absolutely stable</td>
</tr>
<tr>
<td>0.9</td>
<td>absolutely stable</td>
</tr>
<tr>
<td>1.0</td>
<td>absolutely stable</td>
</tr>
<tr>
<td>1.2</td>
<td>absolutely stable</td>
</tr>
<tr>
<td>1.5</td>
<td>absolutely stable</td>
</tr>
</tbody>
</table>
Fig. 3.8 Plot of the normalized impedance at resonance points versus $R_N/Z_0$ of part 1.
Fig. 3.9 Plot of normalized impedance at resonance points versus $R_n/Z_0$ of part 2. Normalized frequencies are displayed along the impedance lines 5 and 7. The numbers at the end of the curves indicate the order of the zero crossings.
Fig. 3.10 Plot of normalized impedance versus specific cases. Normalized frequencies are displayed along the impedance plot.
computed on the computer. Zeros could not be obtained in the unstable region during the experiment.

3.2.2 Two stage amplifier with two filter sections per stage

The stability of the amplifier with two filter sections in each stage of the transmission line as shown in Fig. 3.4b was investigated for different values of \( \frac{R_N}{Z_{01}} \) from 0.1 to 2.0. Since the graphical technique is best suited to determine the stability, the normalized impedance at resonance points obtained on the computer is plotted against \( \frac{R_N}{Z_{01}} \) (see Fig. 3.8 and Fig. 3.9). In this case we obtain 4 zero crossings in part I and 6 zero crossings in part II. The impedance curves corresponding to the second zero crossing of part I and part II intersect with the \( R_N \) line at the point where \( \frac{R_N}{Z_{01}} = 0.35 \). This point of intersection, which appears to correspond to the intersection at \( \frac{R_N}{Z_{01}} = 0.75 \) in the case of the amplifier with one filter section, has receded now to the point where \( \frac{R_N}{Z_{01}} = 0.35 \). The fourth and the fifth zero crossings are not observed for values of \( \frac{R_N}{Z_{01}} = 0.9 \) and 1.0. The possible values of zero crossings in this doubtful region are indicated by the dotted lines. The disappearance of the zero crossings at these points can be explained by reference to the Fig. 3.10 which represents a complex impedance plane in which the normalized reactance is plotted against normalized resistance. The impedance plot for the ratios of \( \frac{R_N}{Z_{01}} = 0.8, 0.9, 1.0 \) and 1.1 for frequencies in the vicinity of the frequency of the zero crossing under
consideration is shown in Fig. 3.10. Since the impedance plot corresponding to ratios of \( \frac{R_N}{Z_{01}} = 0.9 \) and 1.0 is below the horizontal axis of the impedance plane the fourth and the fifth zero crossings are not observed at these ratios. In Fig. 3.9 another peculiarity of the fourth and the fifth zero crossings of part II is that the respective frequencies at which they appear vary with \( \frac{R_N}{Z_{01}} \). In other words, \( f/f_c \) is a function of \( \frac{R_N}{Z_{01}} \) which is very conspicuous in Fig. 3.9. From the previous discussion it can be shown that the amplifier with two filter sections in each stage of the transmission line is unstable for all ratios from 0.1 to 2.0. Careful study of the graphs shown in Fig. 3.9 and Fig. 3.10 reveal that when the impedance line corresponding to part I is extrapolated, it will probably cut the \( R_N \) line where \( \frac{R_N}{Z_{01}} \) is about 4 and the amplifier may be stable for values of \( \frac{R_N}{Z_{01}} > 4 \).

Now let us examine whether this region of \( \frac{R_N}{Z_{01}} \) is a useful region to design an amplifier of appreciable gain or not. If it is required to construct an amplifier of gain 10 with \( \frac{R_N}{Z_{01}} = 4.0 \), the number of stages the amplifier should have can be obtained from equations 2.3 and 2.4.

\[
G = 1 + n \frac{Z_{01}}{R_N}
\]

\[
n = (G-1) \times \frac{R_N}{Z_{01}}
\]

\[
= (10-1) \times 4 = 36
\]
Fig. 3.11 Two stage amplifier with three filter sections per stage (capacitor input).
Fig. 3.12 Plot of normalized impedance at resonance points versus $R_N/Z_{01}$ of part 1.
Fig. 3.13 Plot of normalized impedance at resonance points versus $\frac{R_N}{Z_0}$ of part 2. Normalized frequencies are displayed along the impedance lines 5 and 7. The numbers at the end of the curves indicate the order of the zero crossings.
Since it is not advisable and also highly impracticable to build an amplifier having 36 stages, this stable region in the present situation is undoubtedly not a useful region.

3.2.3 Two stage amplifier with three filter sections per stage

Stability investigations were repeated with three filter sections in each stage of the transmission line as shown in Fig. 3.11 to examine the effect of the increase in the number of filter sections on the stability of the system. The plot of impedance at resonance points against $\frac{R_N}{Z_{01}}$ is shown in Figs. 3.12 and 3.13. In this situation 5 zero crossings in part I and 7 zero crossings in part II are noticed. The impedance at the first, the third and the sixth zero crossings of part I are very nearly equal. Impedance curves corresponding to the second and the fourth zero crossings of part II cut the $R_N$ line at 0.3 and 0.85, respectively. The sixth zero crossing of part II is not noticed in the region of $\frac{R_N}{Z_{01}}$ from 0.7 to 1.1. The dotted line in Fig. 3.13 indicates the possible value of this zero crossing in this region. From the position of the impedance curves with reference to $R_N$ line, it is evident that the amplifier is unstable throughout the region of investigation. From the trend of the curves we can anticipate that the impedance curve corresponding to the sixth zero crossing of part I may intersect with the $R_N$ line for $\frac{R_N}{Z_{01}}$ in the vicinity of 5.5.
To realize gain 10 with this configuration, the number of stages in the amplifier should be as given by equation 3.11.

\[ n = (G-1) \times \frac{R_N}{Z_{01}} \]

\[ = (10-1) \times 5.5 = 49.5 \]

Let us recall that the same gain can be secured with 36 stages in the amplifier if we have two filter sections in each stage. This strongly indicates that the increase in the number of filter sections instead of improving the situation makes it worse.

3.3 Stability of amplifiers with inductor input circuits

The stability of an inductor input amplifier (Fig. 3.14) containing one filter section in each amplifier stage can be obtained from Fig. 3.15. In this figure the plot of normalized impedance at the resonance points against \( \frac{R_N}{Z_{01}} \) is shown. It was pointed out in the previous discussion that an inductor input circuit will be stable provided \( R_N \) is less than the impedance of the lowest zero crossing. With this in view, a careful study of the graph shown in Fig. 3.15 reveals that the circuit is unstable for all ratios of \( \frac{R_N}{Z_{01}} \) between 0.1 and 1.5. As it was found that the increase in the number of filter sections in the transmission line did not improve the stability of capacity input circuits, the investigation of the stability of inductor input circuits with more than one filter section was not attempted.
Fig. 3.14 Two stage amplifier with one filter section per stage (inductor input).
Fig. 3.15 Plot of normalized impedance at resonance points versus $R_n/Z_0$. 

Impedance at Resonance Points $Z_n/Z_0$ vs. $R_n/Z_0$. 

- Graph shows varying lines indicating different points of resonance. 
- $R_n$ and $Z_0$ are normalized impedance values. 
- The graph includes a reference line labeled "RN LINE."

4.1 Stabilization of the amplifier reducing the resistive cut-off frequency of the tunnel diode.

A tunnel diode cannot oscillate at frequencies above its resistive cut-off frequency. An amplifier in which the resistive cut-off frequency of the tunnel diodes used is made equal to or less than the cut-off frequency of the transmission line cannot oscillate outside the pass band. Now we need confine ourselves with stabilizing the amplifier up to the cut-off frequency of the transmission line. By inserting a resistance of proper value in series with the tunnel diode, one can effect a reduction in its resistive cut-off frequency. The resistance \(r_s\) to be inserted to accomplish this can be obtained by the following equation (Gentile, 1962)

\[
\text{fg} = \frac{1}{2\pi C_d R_N} \sqrt{\frac{R_N - R_s'}{R_s}}
\]

4.1

\(R_s'\) is the sum of \(r_s\) and the series resistance \(R_s\) of the tunnel diode. Solving 4.1 for \(R_s'\) we obtain

\[
R_s' = \frac{R_N}{1 + 4\pi^2 f_g^2 C_d^2 R_N^2}
\]

4.2

where \(f_g, C_d\), and \(R_N\) are the resistive cut-off frequency, the capacitance, and the negative resistance of the tunnel diode, respectively. When \(r_s\) of proper value is inserted in series
with the tunnel diode, the impedance of the portion $R_S$ and $L_S$ becomes appreciable and it can no longer be neglected. Furthermore, $R_S'$ in series with $-R_N$ makes the low frequency effective resistance of the tunnel diode ($R_S'-R_N$) and thereby changes the characteristic impedance of the subsequent stages of the transmission line. If we desire to reduce the resistive cut-off frequency of the tunnel diode significantly we must allow for $R_S$ in the design equations of the transmission line.

To see whether the idea of reducing the resistive cut-off frequency has any merit, it was decided to investigate the stability of the circuit shown in Fig. 4.1. This circuit incorporates a resistance in series with the tunnel diode, which in effect, reduces the resistive cut-off frequency of the tunnel diode to the cut-off frequency of the transmission line. The stability of this circuit was investigated theoretically and experimentally, and good agreement was found between the theoretical and experimental results. The amplifier has a stable gain of 3.0. Recalling that an amplifier with the same number of filter sections and of the same gain but involving pure negative resistance was unstable, one can state emphatically that the reduction of the resistive cut-off frequency of the tunnel diode certainly improves the stability of the amplifier. The frequency response of the amplifier is shown in Fig. 4.2. The response is uniform up to 2 K.C. Some ripples can be noticed between 2 K.C. and 12 K.C. and the response drops to near zero at 15 K.C., which is the cut-off
Fig. 4.1 Two stage amplifier incorporating the equivalent circuit of the tunnel diode whose resistive cut-off frequency is equal to the cut-off frequency of the transmission line.
Fig. 4.2 Frequency response of the amplifier.

\[ \frac{E_o}{E_c} \text{ REFER TO FIG 4.1} \]
frequency of the amplifier. The ripples between 2 K.C. and 15 K.C. are due to the poles and zeros of the impedance function. These variables in gain would probably be less severe if more filter sections were used in each stage.

4.2 Design consideration

It was pointed out in the previous section that the design of the transmission line should be modified to allow for $R_S$ which is in series with the negative resistance $-R_N$ of the tunnel diode. Allowance can be made for this resistor by replacing $R_N$ with $R_N-R_S$ in equations 2.4 and 3.11. Design equations of the transmission line then become

$$\frac{1}{Z_{On+1}} = \frac{1}{Z_{01}} + \frac{n}{R_N-R_S} \quad 4.3$$

$$G = 1 + n \frac{Z_{01}}{R_N-R_S} \quad 4.4$$

$$\frac{Z_{01}}{R_N-R_S} = \frac{G-1}{n}$$

In the expressions 3.1 and 3.3, when $R_N$ is replaced by $(R_N-R_S)$ and $\frac{G-1}{n}$ is substituted for $\frac{Z_{01}}{R_N-R_S}$, we obtain

$$\frac{1}{\pi \sqrt{C_0} \left\{1 + \frac{(n-1)(G-1)}{n}\right\}} \quad 4.5$$

$$C_n = \frac{1}{\pi \sqrt{Z_{01}} \left\{1 + \frac{(n-1)(G-1)}{n}\right\}} \quad 4.6$$

Normally, in the design of an amplifier the gain, bandwidth, and possibly the input impedance would be specified. It is therefore desirable to obtain the tunnel diode parameters in terms of these quantities to facilitate the selection of
proper tunnel diodes for a particular amplifier. To accomplish this we shall express the capacitance $C_d$ of the tunnel diode to be a multiple $K_1$ of the total shunt capacity $\frac{C_1+C_2}{2}$ at the junction of the first and the second stage of the amplifier.

$$C_d = K_1 \frac{C_1+C_2}{2} \quad 4.7$$

From 4.6

$$C_d = \frac{K_1}{2\pi f_c Z_{01}} \left[ \frac{2n+G-1}{n} \right] \quad 4.8$$

We will assume that $f_g$ is a multiple $K_2$ of the cut-off frequency of the amplifier

$$f_g = K_2 f_c \quad 4.9$$

The equations 4.8 and 4.9 do not impose any restrictions on $K_1$ and $K_2$, respectively. The range of values of $K_1$ and $K_2$ to be considered in the stability investigations will be taken up a little later. Combining equations 4.2 and 4.4 a cubic in $R_N$ can be obtained.

$$4c^2 f_g c^2 R_N^3 - 4c^2 f_g c^2 (\frac{nZ_{01}}{G-1}) R_N^2 - \frac{nZ_{01}}{G-1} = 0 \quad 4.10$$

Combining equations 4.8, 4.9 and 4.10 we obtain

$$\frac{K_1^2 K_2^2}{Z_{01}^2} \left[ \frac{2n+G-1}{n} \right]^2 R_N^3 - \frac{K_1^2 K_2^2}{G-1} (2n+G-1)^2 R_N^2 - \frac{nZ_{01}}{G-1} = 0 \quad 4.11$$

Solving the cubic equation 4.11 yields $R_N$ in terms of $G$, $n$,.
$Z_0$, $K_1$, and $K_2$. An approximate solution of 4.11 is given by

$$R_N = \frac{n \cdot Z_0}{3(G-1)} \left[ 1 + \left\{ 2 + 3\left(\frac{x}{K_1 K_2}\right)^2\right\}^{\frac{1}{3}} + \left\{ 2 + 3\left(\frac{x}{K_1 K_2}\right)^2\right\}^{\frac{1}{3}} \right]$$

This solution yields a value of $R_N$ accurate to within 1% provided

$$1.12 K_1 K_2 < x$$

where $x = \frac{3(G-1)}{2n+G-1}$.

The complete development of the equations 4.12 and 4.13 is given in Appendix A. It is also shown in Appendix A that $x$ can never exceed 3 regardless of the number of stages in the amplifier or its gain. Combining this result with 4.13 we obtain the following inequality

$$3 \geq x > 1.12 K_1 K_2$$

From 4.14 it is evident that a necessary condition for 4.12 to be accurate to 1% is that $K_1 K_2 < 2.678$. This means that the value of $R_N$ as given by 4.12 is accurate only when the product of $K_1$ and $K_2$ is relatively small. Fortunately, the experimental investigations show that stable amplifiers tend to have a low value of $K_1 K_2$ so that 4.12 is a useful design equation.
4.3 Variables to be considered in the investigations of the stability of the amplifier.

From the analysis of a simple circuit shown in Fig. 4.3, a set of variables which can be used in the investigation of stability of the amplifier incorporating the equivalent circuit of the tunnel diode can be obtained. Since the normalization of impedances of the elements with respect to \( Z_{01} \) was useful previously, the same technique is adopted in the present situation. The normalized impedances of all these elements are shown in Fig. 4.4. The normalized input impedance with respect to \( Z_{01} \) is a function of \( G \), \( n \), \( m \), \( K_1 \), \( K_2 \), \( f/f_c \), and \( L.f_c/Z_{01} \) where \( m \) is the number of filter sections in the transmission line. Mathematically this can be expressed as,

\[
\frac{Z}{Z_{01}}(G, n, m, K_1, K_2, f/f_c, L.f_c/Z_{01}) = 4.15
\]

In order to limit the investigations to a reasonable number, the variables \( m \) and \( n \) were specified. It was decided to investigate the stability of three stage amplifiers with three filter sections in each stage as shown in Fig. 4.7. The investigations were carried out for 3 values of gain, 3 values of \( K_1 \), 2 values of \( K_2 \), and 6 values of \( L.f_c/Z_{01} \).

Now let us examine in greater detail the range of values of \( K_1 \) to be considered in the stability investigations. The factor that merits paramount importance in this situation is that though the impedance of \( C_d \) decreases with frequency, it
Fig. 4.3 Typical circuit to be considered to obtain the variables required for stability investigations.

\[ j2 \frac{\omega}{\xi_c} \frac{1}{1 + (\xi_n - 1)(\xi_e - 1)} \]

Fig. 4.4 Circuit in which the normalized impedance of each element is displayed.

\[ \frac{\omega L}{Z_{01}} = 2 \pi \frac{\xi}{\xi_c} \left( \frac{L \cdot \frac{\xi}{\xi_c}}{Z_{01}} \right) \]
must be reasonably greater than \( R_N \) at the highest frequencies of interest. Otherwise, it short circuits \( R_N \) and offsets the effect of \( R_N \) on which the amplifying action depends. The impedance of \( C_d \) at the cut-off frequency can be computed from 4.8 and it is given by

\[
|X_{cd}| = \frac{n \cdot Z_{01}}{K_1(2n+G-1)}
\]

4.16

We will assume that this reactance should be greater than \( R_N \) of the tunnel diode.

\[
|X_{cd}| > R_N
\]

4.17

Substituting the values of \( X_{cd} \) and \( R_N \) from 4.16 and 4.12, respectively, in 4.16, we obtain

\[
\frac{n \cdot Z_{01}}{K_1(2n+G-1)} > \left[ \frac{n \cdot Z_{01}}{3(G-1)} \right]^{\frac{1}{3}} + \left\{ 2 + 3\left( \frac{x}{K_1K_2} \right)^2 \right\}^{\frac{1}{3}} + \left\{ 2 + 3\left( \frac{x}{K_1K_2} \right)^2 \right\}^{\frac{1}{3}}
\]

4.18

Rearranging 4.18 and substituting \( x \) for \( \frac{3(G-1)}{2n+G-1} \), we obtain

\[
K_1 < \frac{x}{1 + \left\{ 2 + 3\left( \frac{x}{K_1K_2} \right)^2 \right\}^{\frac{1}{3}} + \left\{ 2 + 3\left( \frac{x}{K_1K_2} \right)^2 \right\}^{\frac{1}{3}}}
\]

4.19

The plot of the R.H.S. of 4.19 against \( x \) for different values of \( K_1K_2 \) is shown in Fig. 4.5. Recalling that if the equation for \( R_N \) is to be reasonably accurate, \( K_1K_2 \) should not exceed 3, it is obvious from Fig. 4.5 that 4.19 can never be satisfied
if $K_1$ is greater than 0.895. This means that for values of
$(K_1K_2)$ and $x$ for which 4.12 yields a value for $R_N$ accurate to
within 1%, $K_1$ cannot be greater than 0.895 if the reactance
of $C_d$ is to be greater than the negative resistance of the
tunnel diode within the pass band of the amplifier. The infor-
mation as to how much less $K_1$ must be than 0.895 for $X_{od}$ to
be greater than $R_N$ can be determined from Fig. 4.5. The exact
value will depend on $X$ and $(K_1K_2)$. The regions of $x$ in which
the approximation made in the evaluation of $R_N$ is valid and
the regions in which it is invalid are shown by a dotted line.
The values of $R_N$ are accurate to within 1% to the right of
this line of demarcation and they are not accurate to within
1% of the left of this line. Hence, it is necessary to confine
ourselves with regions of $x$ to the right of the demarcation
line. The dots in Fig. 4.5 depict the cases investigated.

To determine the stability of amplifiers of the form
shown in Fig. 4.7, we require the impedance plot at three
points in the circuit where we insert the negative resistances.
A programme was written to generate the data cards required
to run Mahabala's programme for obtaining the impedance at
the resonance points. The flow chart of the programme that
generates the data cards is shown in Fig. 4.6.

In the stability investigations the impedance at the
resonant frequencies was obtained for 6 values of $L/fc/Z_0$
between 0.0005 and 0.10 at the three points in the circuit
Fig. 4.5 Plot of Y versus x.
Fig. 4.6 Flow chart that generates data cards.

START

READ Z,G,R,T,B,NF,FRE,T2,

K1=1,K2=2,K3=3,K4=4,
K5=5,K6=6,K7=7

DIMENSION X(3),
C(3),D(3),L(3)

COMMON G,D,P,K1,K2,
K3,K4,K5,K6,K7,M,NF

TA = T

106

YA

YB

Q

DO 25 J=1,3,1

X(J)

C(J)

D(J)

P(J)

117

CN

CM

H

W

WI

W2

RN

GN

RS

U

RL

GL

TA - 3

> 0

KA = 1

KB = 1

< 0

KA - 2

= 0

KA = KA + 1

KA = KA + 1

TA = TA - 1

GO TO 104

SET CARDS FOR POSITION 1

SET CARDS FOR POSITION 2

SET CARDS FOR POSITION 3

GO TO 106

SET CARDS FOR POSITION 2

ARE THERE MORE CARDS?

YES

NO

GO TO 5

END
where the negative resistances are to be inserted. The resonant impedances were plotted against \( L.f_0/Z_0 \) in Figs. 4.8 to 4.17 and also in Figs. 4.19 to 4.25. The points that correspond to the same frequency are joined by solid lines. As enough points were not obtained to unambiguously define the curves in some regions, the points in those regions are joined by dotted lines. Invariably zero crossings were found at \( L.f_0/Z_0 = 0.05 \) and 0.1 which have no counterparts in the other regions of \( L.f_0/Z_0 \). Such zero crossings are shown as single points on the graphs.

These graphs are grouped as follows. Wherever it is convenient the resonant impedances corresponding to part 1, part 2 and part 3 are displayed on the same page and they are referred to as a, b, and c of the same figure, respectively. In some cases it was found rather inconvenient to plot all the curves on the same page. Under such circumstances the resonant impedances of part 1 and part 2 are shown on the same page and those of part 3 are shown on a separate page. The sequence of numbering is maintained the same.

The cases that are investigated and the summary of the results corresponding to the absolute stability of the amplifiers are furnished in Tables 4.1 and 4.2. The reader can get all the pertinent information regarding the absolute stability of the amplifiers investigated from these tables as they depict the regions wherein the amplifiers are absolutely stable and the regions in which they are not absolutely stable.
If the reader is curious about the detailed behavior of the zero crossings in the various regions of $L.f_0/Z_0l$ and their relative position with respect to $R_N$ line he should refer to Figs. 4.8 to 4.25. A critical examination of these figures and the successive application of the index principle will make it possible to explore the regions in which the amplifiers are conditionally stable.

4.4 Stability of three stage amplifiers with three filter sections per stage.

4.4.1 Stability of the amplifiers with capacitor input circuits.

The amplifier consisting of three stages with capacitor input circuits is shown in Fig. 4.7. The impedance at the resonant frequencies at all the three points where we incorporate the negative resistances are plotted against $L.f_0/Z_0l$ in Fig. 4.8 (a,b,c). The study of Fig. 4.8 reveals that the amplifier of gain 5 for which $K_1 = 0.25$ is absolutely stable for all values of $L.f_0/Z_0l$. However, the zero crossings at $L.f_c/Z_0l = 0.1$ in all the three parts are very close to the $R_N$ line. It is not desirable to choose this value of $L.f_c/Z_0l$ to design the amplifier. As the circuit will be extremely critical, any small variation in any one of its parameters may shift the position of the zero crossings and thereby render the amplifier unstable.
The stability of the amplifiers of gain 5 for which $K_1 = 0.5$ and $K_1 = 1.0$ can be studied with reference to the Fig. 4.9 (a,b,c) and Fig. 4.10 (a,b,c), respectively. It can be seen from these figures that for these higher values of $K_1$, the amplifier is unstable in the range of $Lf_c/Z_{01}$ greater than 0.05.

In view of all these facts, it is desirable to choose smaller values of $K_1$ and to work in the lower regions of $Lf_c/Z_{01}$. In other words, the series inductance $L$ and the junction capacitance $C_d$ of the tunnel diodes used in the design of the amplifier should be small.

The stability of amplifiers of gain 10 for which $K_2 = 1.1$ and $K_1 = 0.25$ can be studied with reference to Fig. 4.11 (a,b,c). The stability of amplifiers of the same gain but for which $K_2 = 1.1$ and $K_1 = 0.50$ can be determined with reference to Fig. 4.12 (a,b,c). Fig. 4.13 (a,b,c), Fig. 4.14 (a,b,c), and Fig. 4.15 (a,b,c) reveal the stability of amplifiers of gain 10 for which $K_2 = 1.4$ and $K_1 = 0.25$, 0.50 and 1.0, respectively. The examination of all these figures indicates that all these amplifiers are not absolutely stable.

The stability of the amplifier of gain 15 for which $K_2 = 1.1$ and $K_1 = 0.25$ can be determined with reference to Fig. 4.16 (a,b,c). The stability of the amplifiers of the same gain but for which $K_2 = 1.56$ and $K_1 = 0.25$ can be found from Fig. 4.17 (a,b,c). It is evident from the study of these figures that the amplifiers of gain 15 are not absolutely stable.
A cursory examination of all the figures 4.11 (a,b,c) to 4.17 (a,b,c) indicates that the amplifiers of gain 10 and 15 are not even conditionally stable in most regions of $L/f_c/Z_0$. 

During the investigation of the stability of the amplifiers of gain 5 and 10 it was noticed that there is a tendency toward instability for higher values of $K_1$. With this in view the investigation of the stability of the amplifier of gain 15 for values of $K_1 = 0.5$ and $K_1 = 1.0$ was not attempted.

4.4.2 Stability of the amplifiers involving inductor input circuits.

The stability of the amplifiers involving inductor input circuits as shown in Fig. 4.18 was investigated. It can be seen from Fig. 4.19 (a,b,c) and Fig. 4.20 (a,b,c) that the amplifiers of gain 5 for which $K_1 = 0.25$ and $K_1 = 0.50$ are absolutely stable in all the investigated regions of $L/f_c/Z_0$, except at $L/f_c/Z_0 = 0.10$. From Fig. 4.21 (a,b,c) it is apparent that the amplifier of the same gain but for which $K_1 = 1.0$ is stable for values of $L/f_c/Z_0$ between 0.000316 to 0.0316 and unstable for values of $L/f_c/Z_0$ greater than 0.0316.

Stability of the amplifiers of gain 10 for which $K_1 = 0.25$ and $K_2 = 1.1$ can be determined with reference to Fig. 4.22 (a,b,c). Surprisingly enough the amplifier is found to be stable for values of $L/f_c/Z_0$ between 0.000316 and 0.10. However, it is unstable at $L/f_c/Z_0 = 0.10$. Fig. 4.23 (a,b,c) reveals the stability of the amplifiers of the same gain but
for which $K_1 = 0.25$ and $K_2 = 1.1$. The amplifier is absolutely stable for all values of $L\cdot f_c/Z_0$ from 0.000316 to 0.0316 and unstable in the regions where $L\cdot f_c/Z_0$ is greater than 0.0316. It is interesting to note that in this situation the second zero crossings of part 1 and part 2 are very close to the $R_N$ line. One should be vitally concerned about this crucial zero crossing during the design stage, because the amplifier will become unstable if there is a shift in the position of this zero crossing. The comparison of the Figs. 4.22 (a,b,c) and 4.23 (a,b,c) strongly indicates the tendency of the amplifier to become unstable with an increase in the value of $K_2$.

Stability of the amplifiers of gain 15 for which $K_1 = 0.25$ and $K_2 = 1.1$ can be determined with reference to Fig. 4.24 (a,b,c). From Fig. 4.25 (a,b,c) the stability of the amplifier of the same gain but for which $K_1$ and $K_2$ are 0.25 and 1.56, respectively, can be found. The examination of all these figures points out that all these amplifiers are not absolutely stable.
TABLE 4.1
CAPACITOR INPUT CIRCUITS

GAIN 5

<table>
<thead>
<tr>
<th>Figure</th>
<th>K1</th>
<th>K2</th>
<th>Range</th>
<th>Unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8(a,b,c)</td>
<td>0.25</td>
<td>1.1</td>
<td>From 0.0005 to 0.1</td>
<td>Beyond 0.05</td>
</tr>
<tr>
<td>4.9(a,b,c)</td>
<td>0.50</td>
<td>1.1</td>
<td>From 0.0005 to 0.05</td>
<td>Beyond 0.05</td>
</tr>
<tr>
<td>4.10(a,b,c)</td>
<td>1.00</td>
<td>1.1</td>
<td>From 0.0005 to 0.05</td>
<td>Beyond 0.05</td>
</tr>
</tbody>
</table>

GAIN 10

<table>
<thead>
<tr>
<th>Figure</th>
<th>K1</th>
<th>K2</th>
<th>Range</th>
<th>Unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.11(a,b,c)</td>
<td>0.25</td>
<td>1.1</td>
<td>-</td>
<td>In all the regions investigated</td>
</tr>
<tr>
<td>4.12(a,b,c)</td>
<td>0.50</td>
<td>1.1</td>
<td>-</td>
<td>&quot;</td>
</tr>
<tr>
<td>4.13(a,b,c)</td>
<td>1.00</td>
<td>1.1</td>
<td>-</td>
<td>&quot;</td>
</tr>
<tr>
<td>4.14(a,b,c)</td>
<td>0.25</td>
<td>1.4</td>
<td>-</td>
<td>&quot;</td>
</tr>
<tr>
<td>4.15(a,b,c)</td>
<td>0.50</td>
<td>1.4</td>
<td>-</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

GAIN 15

<table>
<thead>
<tr>
<th>Figure</th>
<th>K1</th>
<th>K2</th>
<th>Range</th>
<th>Unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.16(a,b,c)</td>
<td>0.25</td>
<td>1.1</td>
<td>-</td>
<td>In all the regions investigated</td>
</tr>
<tr>
<td>4.17(a,b,c)</td>
<td>0.25</td>
<td>1.56</td>
<td>-</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
### TABLE 4.2
**INDUCTOR INPUT CIRCUITS**

<table>
<thead>
<tr>
<th>GAIN 5</th>
<th>Figure</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>The Region of $L\cdot f_c/Z_{01}$ in which the amplifier is &quot;Absolutely Stable&quot; is Unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.19(a,b,c)</td>
<td>0.25</td>
<td>1.1</td>
<td>From 0.000316 to 0.1 At 0.1</td>
</tr>
<tr>
<td></td>
<td>4.20(a,b,c)</td>
<td>0.50</td>
<td>1.1</td>
<td>From 0.000316 to 0.1 At 0.1</td>
</tr>
<tr>
<td></td>
<td>4.21(a,b,c)</td>
<td>1.00</td>
<td>1.1</td>
<td>From 0.000316 - 0.0316 Beyond 0.0316</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GAIN 10</th>
<th>Figure</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>The Region of $L\cdot f_c/Z_{01}$ in which the amplifier is &quot;Absolutely Stable&quot; is Unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.22(a,b,c)</td>
<td>0.25</td>
<td>1.1</td>
<td>From 0.000316 to 0.1 At 0.1</td>
</tr>
<tr>
<td></td>
<td>4.23(a,b,c)</td>
<td>0.25</td>
<td>1.4</td>
<td>From 0.000316 - 0.0316 Beyond 0.0316</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GAIN 15</th>
<th>Figure</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>The Region of $L\cdot f_c/Z_{01}$ in which the amplifier is &quot;Absolutely Stable&quot; is Unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.24(a,b,c)</td>
<td>0.25</td>
<td>1.1</td>
<td>- In all the regions investigated</td>
</tr>
<tr>
<td></td>
<td>4.25(a,b,c)</td>
<td>0.25</td>
<td>1.56</td>
<td>-</td>
</tr>
</tbody>
</table>

*Figure K1 K2 is "Absolutely Stable"*
Fig. 4.7 Three stage amplifier with three filter sections per stage (capacitor input).
G = 5; K1 = 0.25; K2 = 1.1; part 3

Fig. 4.8c

IMPEDEANCE AT RESONANCE POINTS $Z/\zeta_{01}$

G = 5; K1 = 0.25; K2 = 1.1; part 2

Fig. 4.8b

G = 5; K1 = 0.25; K2 = 1.1; part 1

Fig. 4.8a
G = 5; K₁ = 0.5; K₂ = 1.1; part 3

Fig. 4.9c

IMPEDANCE AT RESONANCE POINTS $\frac{Z}{Z₀}$

---

G = 5; K₁ = 0.5; K₂ = 1.1; part 2

Fig. 4.9b

---

G = 5; K₁ = 0.5; K₂ = 1.1; part 1

Fig. 4.9a
G = 10; \( K_1 = 0.25 \); \( K_2 = 1.1 \); part 3

Fig. 4.11c
G = 10; K₁ = 0.5; K₂ = 1.1; part 2

Fig. 4.12b

G = 10; K₁ = 0.5; K₂ = 1.1; part 1

Fig. 4.12a
G=10; K_1=0.5; K_2=1.1; part 3

Fig. 4.12c
G=10; K1=0.25; K2=1.4; part 2

Fig. 4.13b

Impedance at Resonance Points $\frac{Z}{Z_0}$

G=10; K1=0.25; K2=1.4; part 1

Fig. 4.13a
Fig. 4.13o

IMPEDEANCE AT RESONANCE POINTS $z/z_0$.

G=10; $K_1=0.25$; $K_2=1.4$; part 3

G.0; K_1=0.25; K_2=1.4; part 3

Fig. 4.13o
G = 10; K₁ = 0.5; K₂ = 1.4; part 2

Fig. 4.14b

G = 10; K₁ = 0.5; K₂ = 1.4; part 1

Fig. 4.14a

Impedance at Resonance Points $\frac{z}{z_0}$
IMPEDEANCE AT RESONANCE POINTS \( \frac{Z}{Z_0} \)

\[ G = 10; K_1 = 0.5; K_2 = 1.4; \]

Fig. 4.14e
G = 10; \( K_1 = 1.0; \) \( K_2 = 1.4; \) part 2

Fig. 4.15b

G = 10; \( K_1 = 1.0; \) \( K_2 = 1.4; \) part 1

Fig. 4.15a
IMPEDEANCE AT RESONANCE POINTS $\frac{z}{z_0}$

$G=10; K_1=1; K_2=1.4; \text{Part } 3$

Fig. 4.150
G = 15; $K_1 = 0.25; K_2 = 1.1$; part 2

Fig. 4.16b

G = 15; $K_1 = 0.25; K_2 = 1.1$; part 1

Fig. 4.16a
\( G=15; \ k_1=0.25; \ k_2=1.1; \) part 3

**Fig. 4.16c**

**Impedance at Resonance Points \( Z/Z_0 \)**

- **Y-axis:** Values ranging from -2 to 2
- **X-axis:** \( L \times F_0 / Z_0 \)

Graph showing changes in impedance with varying values on the x-axis and y-axis.
G=15; K₁=0.25; K₂=1.56; part 2

Fig. 4.17b

G=15; K₁=0.25; K₂=1.56; part 1

Fig. 4.17a
G = 15; \( K_1 = 0.25 \); \( K_2 = 1.56 \); part 3

Fig. 4.17c
Fig. 4.18 Three stage amplifier with three filter sections per stage (inductor input).
\[ G = 5; \, K_1 = 0.25; \, K_2 = 1.1; \, \text{part 1} \quad \text{Fig. 4.19a} \]

\[ G = 5; \, K_1 = 0.25; \, K_2 = 1.1; \, \text{part 2} \quad \text{Fig. 4.19b} \]

\[ G = 5; \, K_1 = 0.25; \, K_2 = 1.1; \, \text{part 3} \quad \text{Fig. 4.19c} \]
\[ G = 5; K_1 = 0.5; K_2 = 1.1; \text{ part 3} \]

**Fig. 4.20c**

\[ G = 5; K_1 = 0.5; K_2 = 1.1; \text{ part 2} \]

**Fig. 4.20b**

\[ G = 5; K_1 = 0.5; K_2 = 1.1; \text{ part 1} \]

**Fig. 4.20a**
G=10; K_1=0.25; K_2=1.1; part 2

Fig. 4.22b

G=10; K_1=0.25; K_2=1.1; part 1

Fig. 4.22a
$G=10; \; K_1=0.25; \; K_2=1.1; \; \text{part 3}$

**Fig. 4.22c**

**Graph: Impedance at Resonance Points**

- **RN Line**
- **Impedance**
  - $\frac{Z}{Z_{01}}$

**Axes:**
- X-axis: $0.000316 \rightarrow 0.01$
- Y-axis: $-2.0 \rightarrow 2.0$
G = 10; K₁ = 0.25; K₂ = 1.4; part 2

G = 10; K₁ = 0.25; K₂ = 1.4; part 1
G=10; K_1=0.25; K_2=1.4; part 3

Fig. 4.23c
G=15; K_1=0.25; K_2=1.1; part 2

Fig. 4.24b

G=15; K_1=0.25; K_2=1.1; part 1

Fig. 4.24a
G=15; K_1=0.25; K_2=1.1; part 3

Fig. 4.24c
G = 15; K₁ = 0.25; K₂ = 1.56; part 1

Fig. 4.25a

G = 15; K₂ = 1.56; part 2

Fig. 4.25b
$G=15; K_1=0.25; K_2=1.56$; part 3

Fig. 4.25c
1.5 Stability of a hypothetical circuit which does not include the transmission line.

Let us consider the effect on the stability of the amplifier of decreasing the transmission line filter sections. This decrease in the number of filter sections, when taken to the limit, yields a circuit of the form shown in Fig. 4.26. Some extremely unstable situations of the previous configurations of both the capacitor input and the inductor input circuits were chosen. The parameters of the tunnel diode and the load of each such situation were taken to include them in the circuit 4.26 and to investigate its stability. Since the parallel combination of three identical tunnel diodes of Fig. 4.26 is equivalent to a single tunnel diode of Fig. 4.27, the circuit shown in Fig. 4.26 can be reduced to the form as shown in Fig. 4.27. The stability of the circuit shown in Fig. 4.27 was investigated by applying Hine's (Hine, 1960) criterion. Curiously enough, it was found that all the unstable situations chosen turn out to be stable. From this it is conceivable that the decrease in the number of filter sections definitely improves the stability of the amplifier. Recall that the same conclusion was reached when the stability of the amplifier involving pure negative resistances was investigated.

Although the circuit shown in Fig. 4.26 is theoretically stable, anyone who has attempted to connect several tunnel diodes in parallel will question the validity of this theory
Fig. 4.26 A hypothetical circuit which consists of three identical tunnel diodes arranged in parallel.

Fig. 4.27 Circuit consisting of a single tunnel diode which is an exact equivalent of the parallel combination of three tunnel diodes.
as such an arrangement invariably oscillates. This instability can only be explained by assuming that Fig. 4.26 is not an adequate representation for several tunnel diodes in parallel. Strictly speaking, the circuit 4.26 is not realizable since there will at least be some stray capacity shunting the tunnel diodes. Several other factors which may contribute to the instability of the circuit are the slight difference that normally exists between "identical" tunnel diodes, and the mutual coupling between the tunnel diodes which may be rather appreciable due to the close proximity of the tunnel diodes. None of these factors are known accurately and, hence, it is difficult to make allowances for them. The inclusion of one or more filter sections between the tunnel diodes results in a circuit which is more nearly realizable as the stray shunt capacity can be absorbed in the filters. In addition, the tunnel diodes are physically removed from each other decreasing the mutual coupling that may exist. Thus, from the practical point of view there still appears to be value in the use of filter sections between the tunnel diodes to achieve stability. Coupling this with the theoretical consideration suggests that from the viewpoint of stability the number of filter sections used between the tunnel diodes should be kept to a minimum.
5. STABILITY OF THE AMPLIFIERS INCORPORATING BUTTERWORTH FILTERS

5.1 The polynomials that determine the stability of the capacitor and the inductor input circuits

The expressions for driving point impedance and transfer function of constant k filters which are terminated in their image impedance can be obtained analytically and are relatively simple. These expressions for the same filters terminated in resistances are very complicated. Furthermore, to determine the nature of poles and zeros of driving point impedances by solving the numerator or the denominator polynomial is formidable though not impossible. Since the amplifier which we propose to design is always terminated in a resistance load, it is logical that we should consider filters which incorporate such resistive terminations. As the driving point impedance and the transfer function of Butterworth filters are based on resistive loads, it was felt desirable to make an investigation of the amplifiers using such filters. This does not involve the concept of characteristic impedance as before. However, the loads will still be chosen on the D.C. basis.

The transfer function describing maximally flat steady state response ($S = j\omega$) for the Butterworth low pass prototype is (WEB, 1963)

$$t_B(j\omega)^2 = \frac{1}{1 + \omega^{2n}}$$

5.1
Fig. 5.1 Two stage amplifier incorporating Butterworth filters as the lumped transmission line (capacitor input).

Fig. 5.2 Two stage amplifier incorporating Butterworth filter as the lumped transmission line (inductor input).
where \( n \) is the number of frequency sensitive elements in the filter.

The driving point impedance of the filter is:

\[
Z_{11} = \frac{\prod_{k=1}^{k=n} (S - S_{pk}) - S^n}{\prod_{k=1}^{k=n} (S - S_{pk}) + S^n}
\]

We will restrict the stability investigations to two-stage amplifiers incorporating tunnel diodes which are treated to be pure negative resistance devices. Let us consider the capacitor input circuit shown in Fig. 5.1. The driving point impedance of the filter section 1 is \( kZ_{11} \) and that of the filter section 2 is \( Z_{11} \). If the terminating resistance of the filter section 1 is \( k \) ohms, the terminating resistance of the filter section 2 must necessarily be 1 ohm. In order to conform with the requirement that the terminating resistance should be 1 ohm, it is obvious that the parallel combination of \(-R_N\) and \(R_L\) should yield 1 ohm.

\[
\frac{R_LX - R_N}{R_L - R_N} = 1
\]

Further the combination of \(-R_N\) in parallel with 1 ohm must yield \( k \) for the correct termination of the first filter section.

\[
\frac{IX - R_N}{1 - R_N} = k
\]
\[-R_N = \frac{k}{1-k}\] 5.5

Combining equations 5.3 and 5.5 we obtain
\[\frac{k}{R_L} = 2k - 1\] 5.6

which is evidently the gain of the amplifier.

The impedance of the parallel combination of \(Z_{11}\) and \(kZ_{11}\) is
\[Z' = \frac{k}{k+1} Z_{11}\] 5.7

where \(Z_{11}\) is given by 5.2.

The total impedance of \(Z\) in parallel with \(-R_N\) is
\[Z'' = \frac{Z' X - R_N}{Z' - R_N}\] 5.8

Substituting the values of \(Z'\) and \(R_N\) from 5.5 and 5.7 into 5.8 and simplifying, we obtain
\[Z'' = \frac{kNZ_{11}}{-k(NZ_{11}-D_{Z_{11}}) + NZ_{11} + D_{Z_{11}}}\] 5.9

where \(NZ_{11}\) and \(D_{Z_{11}}\) are the numerator and the denominator of \(Z_{11}\), respectively.

The roots of the denominator of the impedance function \(Z''\) determine the stability of the network. Substituting for \(NZ_{11}\) and \(D_{Z_{11}}\) from 5.2 the denominator of \(Z''\) reduces to the form
The roots of 5.10 determine the stability of the capacitor input circuits as shown in Fig. 5.1.

It is convenient to work with admittance functions to derive the stability conditions of inductor input circuits. Let us determine the conditions of stability of inductor input circuit as shown in Fig. 5.2. \( \frac{Y_{ll}}{k} \) is the input admittance of the filter section 1 and \( Y_{ll} \) is the input admittance of the filter section 2. The admittance of the parallel combination of \( Y_{ll} \) and \( \frac{Y_{ll}}{k} \) is

\[
Y' = \frac{k+1}{k} Y_{ll}
\]

When this admittance is in parallel with a conductance \( \frac{1}{R_N} \), the total admittance would be

\[
Y'' = Y' - \frac{1}{R_N}
\]

Substituting for \( Y' \) and \( R_N \) from 5.5 and 5.11 in 5.12 we obtain

\[
Y'' = \frac{(k+1)N_{ll} - (k-1)D_{ll}}{kD_{ll}}
\]

where \( N_{ll} \) and \( D_{ll} \) are the numerator and denominator of the input admittance \( Y_{ll} \), respectively. The roots of the numerator of the admittance function determine the stability of inductor input circuits. Surprisingly enough the expressions for
driving point impedance of the capacitor input circuits and those for the driving point admittance of inductor input circuits for Butterworth filters containing the same number of reactive elements are identical. The driving point admittance for the filter consisting of "n" reactive elements is

\[ Y_{11} = \frac{\sum_{k=1}^{k=n} (S-S_{pk}) - S_n}{\sum_{k=1}^{k=n} (S-S_{pk}) + S_n} \]  

5.14

Substituting for \( N_{Y_{11}} \) and \( D_{Y_{11}} \) from 5.14 in the numerator of 5.13 we obtain

\[ \prod_{k=1}^{k=n} (S-S_{pk}) - kS^n = 0 \]  

5.15

The roots of 5.15 determine the stability of the inductor input circuit shown in Fig. 5.2.

5.2 Stability of the amplifiers incorporating Butterworth filters consisting of two elements

Let us determine the stability for the particular case of only two elements in each filter section as shown in Fig. 5.3. This is of special interest to us as its solution is already known from Mahabala's unified stability criteria. We shall examine whether our solutions reconcile with the solutions of Mahabala's theory. For a circuit having two elements in the filter section it can be shown that
\[ \prod_{k=1}^{n} (s - s_{pk}) = s^2 + \sqrt{2} s + 1 \] 5.16

Substituting 5.16 in 5.10, it can be seen that the polynomial which determines the stability of the capacitor input circuit is

\[(1+k)s^2 + \sqrt{2} s + 1 \] 5.17

By applying Routh's criterion as given in the Appendix B, it can be shown that the roots of the polynomial 5.17 will have no positive real parts provided \((k+1) > 0\). In other words, the network shown in Fig. 5.3 is stable provided \(k > -1\).

Let us recall that the polynomial which determines the stability of the inductor input circuits is given by the equation 5.15. For a circuit with two elements in the filter section, as shown in Fig. 5.4, the characteristic polynomial that determines the stability can be determined by combining the equations 5.15 and 5.16. It is given by

\[(1-k)s^2 + \sqrt{2} s + 1 = 0 \] 5.18

It can be shown that the roots of this polynomial will have no positive real parts and hence the stability of the circuit is assured provided \((1-k) > 0\). In other words, the network will be stable if \(k < 1\).

The circuit shown in Fig. 5.4 can be reduced to the form as shown in Fig. 5.5. Since a function of voltage disturbance
can be introduced in series with any branch of the circuit to
determine its stability, in the present situation it can be
injected in series with the negative resistance \(-R_N\) of Fig.
5.4. The voltages at points A and B will be the same because
the filter section 1 is the impedance scaled version of filter
section 2. The capacitor \(C/k\), the inductor \(kL\), and the resistor
\(k\) of the filter section 1 are effectively in parallel with the
capacitor \(C\), the inductor \(L\), and the resistance of 1 ohm of the
filter section 2, respectively. The circuit shown in 5.5 will
become either the capacitor input circuit or inductor input
circuit, depending on the polarities of \(R_C\) and \(R_V\). If \(R_V\) is
negative and \(R_C\) is positive, the circuit will be capacitor
input. The components of this capacitor input circuit are
given below.

\[
R_C = \frac{k}{1+k} \quad \text{and} \quad \frac{R_C}{R_V} = \frac{1-k}{1+k} \quad 5.19
\]

\[
C_1 = \frac{k+1}{k} C \quad \text{and} \quad L_1 = \frac{k}{k+1} L
\]

\[
\sqrt{\frac{C_1}{L_1}} \cdot R_C = \frac{k+1}{k} \sqrt{\frac{C}{L}} \cdot \frac{k}{k+1} = \sqrt{\frac{C}{L}} \quad 5.20
\]

In the case of inductor input circuits, \(R_V\) is positive
and \(R_C\) is negative. The parameters of this circuit shown in
Fig. 5.6 are given below.

\[
R_C = \frac{k}{1-k} \quad \text{and} \quad \frac{R_C}{R_V} = \frac{1+k}{1-k} \quad 5.21
\]
Fig. 5.3 Two stage amplifier with two reactive elements per stage (capacitor input).
Fig. 5.4 Amplifier with two reactive elements per stage (inductor input).

Fig. 5.5 Reduced form of the circuit 5.3.

Fig. 5.6 Reduced form of the circuit 5.4.
The values of $K$ are shown along the Inductor input and Capacitor input axes.

Fig. 5.7 Plot of $\frac{R_C}{R_V}$ against $\sqrt{\frac{C}{L}} \times R_C$. 
\[ C_2 = \frac{k+1}{k} C \; \; ; \; \; L_2 = \frac{k}{k+1} L \]

\[ \sqrt{\frac{C_2}{L_2}} \cdot R_c = \frac{k+1}{k} \sqrt{\frac{C}{L}} \cdot \frac{k}{1-k} = \frac{k+1}{1-k} \sqrt{\frac{C}{L}} \quad 5.22 \]

The plot of \( \frac{R_c}{R_V} \) against \( \sqrt{\frac{C}{L}} R_c \) for both capacitor input and inductor input circuits is shown in Fig. 5.7. This is superposed on the graphical representation of Mahabala's unified stability criteria. In Fig. 5.7 the different regions are classified as follows:

I - Region of Exponential decay
II - Region of Sinusoidal decay
III - Region of Sinusoidal growth
IV - Region of Exponential growth
V - Region of Exponential growth but referred to commonly as the region of switching.

From the graph, it is evident that the regions in which the capacitor and the inductor input circuits are stable and the regions in which they are unstable obtained on the basis of Mahabala's theory and as defined by the above theory are identical. We are interested in the positive values of \( k \) which are greater than unity to build an amplifier of appreciable gain. It can be seen in Fig. 5.7 that the inductor input circuits are unstable for values of \( k \geq 1 \). This means that with the inductor input circuits it is not possible to build a stable amplifier. From Fig. 5.7 it is apparent that the amplifiers with the capacitor input circuits are stable for
some values of $k > 1$. However, this useful region of $k$ is very limited as it extends from $k = 1$ to $k = 3$, as it can be seen in Fig. 5.7.

5.3 The effect of increasing frequency sensitive components in the filter section on the stability of the capacitor input circuits.

It was pointed out in the previous section that the roots of the polynomial given by 5.10 determines the stability of the capacitor input circuits. The effect of increasing the number of frequency sensitive elements will be studied by subjecting the polynomial to Routh’s criteria.

If the filter section consists of only three elements, as shown in Fig. 5.9, the polynomial that determines the stability will be

$$(l+k)s^3 + 2s^2 + 2s + 1$$  \hspace{1cm} 5.23

According to Routh’s criterion, the circuit will be stable only when $k < 3$; $k > -1$.

Since the maximum permissible value that $k$ can assume is 3, the gain of the two stage amplifier from 5.6 cannot be greater than 5. Interestingly enough, the constant $k$-filter consisting of 3 elements is exactly identical with the 3 stage maximally flat Butterworth low pass prototype. Recall that the maximum gain that can be secured with a two stage amplifier having constant $k$ filter sections consisting of 3 elements
Fig. 5.8 Two stage amplifier with 5 reactive elements per stage (capacitor input).

Fig. 5.9 Two stage amplifier with 3 reactive elements per stage (capacitor input).
Fig. 5.10 Two stage amplifier with 3 reactive elements per stage (inductor input).

Fig. 5.11 Two stage amplifier with 5 reactive elements per stage (inductor input).
given by 3.11 is 5.

Now let us consider the stability of the amplifier involving 5 elements in each filter section as shown in Fig. 5.8. The characteristic polynomial that determines the stability of this circuit will be

\[(1+k)S^5 + 3.236S^4 + 5.236S^3 + 5.236S^2 + 3.236S + 1 = 0 \quad 5.24\]

Applying Routh's criterion to this polynomial, it can be shown that the circuit is stable if and only if \( k < 1.147; k > -1. \)

From 5.6 it is obvious that the maximum gain that can be secured with this configuration cannot be more than 1.3. The inference that one can draw from the above discussion is that in the case of capacitor input circuits the increase in the number of frequency sensitive elements drastically reduces the gain of the amplifier. We reached the same conclusion with constant \( k \) filters.

5.4 Effect of increasing reactive elements in the filter section on the stability of the inductor input circuits.

Consider an inductor input circuit consisting of three elements in each filter section as shown in Fig. 5.10. Recalling that the polynomial that determines the stability of such circuits is given by 5.15, we can write the characteristic polynomial that determines the stability of this configuration.

\[(1-k)S^3 + 2S^2 + 2S + 1 = 0 \quad 5.25\]
Application of Routh's criterion to this polynomial yields the following conditions of stability:

\[ k > -3 \]
\[ k < 1 \]

The same technique when applied to a network having five elements in each filter section, as shown in Fig. 5.11, reveals that such a system will be stable under the conditions that \( k > -2.24 \) and \( k < 1 \). The results of the investigations are furnished in Table 5.1.

The above analysis strongly indicates that with inductor input circuits there is no useful region to build a stable amplifier of significant gain. This is evident from equation 5.6. The same difficulty was encountered when we were dealing with the constant \( k \) filters. A significant point that has emerged out of this study is that it is not possible to build a stable amplifier with inductor input circuits incorporating tunnel diode treated as a pure negative resistance device.
## TABLE 5.1

**CAPACITOR INPUT CIRCUITS**

<table>
<thead>
<tr>
<th>Number of Reactive Elements in the Filter</th>
<th>The Polynomial that Determines the Stability of the Amplifier</th>
<th>The Restrictions on K which Assure the Stability of the Amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>((1+K)S^2 + \sqrt{2}S + 1)</td>
<td>(K &gt; -1)</td>
</tr>
<tr>
<td>3</td>
<td>((1+K)S^3 + 2S^2 + 2S + 1)</td>
<td>(K &lt; 3; K &gt; -1)</td>
</tr>
<tr>
<td>5</td>
<td>((1+K)S^5 + 3.236S^4 + 5.236S^3 + 3.236S + 1)</td>
<td>(K &lt; 1.14; K &gt; -1)</td>
</tr>
</tbody>
</table>

**INDUCTOR INPUT CIRCUITS**

<table>
<thead>
<tr>
<th>Number of Reactive Elements in the Filter</th>
<th>The Polynomial that Determines the Stability of the Amplifier</th>
<th>The Restrictions on K which Assure the Stability of the Amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>((1-K)S^2 + \sqrt{2}S + 1)</td>
<td>(K &lt; 1)</td>
</tr>
<tr>
<td>3</td>
<td>((1-K)S^3 + 2S^2 + 2S + 1)</td>
<td>(K &lt; 1; K &gt; -3)</td>
</tr>
<tr>
<td>5</td>
<td>((1-K)S^5 + 3.236S^4 + 5.236S^3 + 3.236S + 1)</td>
<td>(K &lt; 1; K &gt; -2.24)</td>
</tr>
</tbody>
</table>
6. CONCLUSION

When the idea of designing the Moody-Wacker amplifier was conceived, it was thought appropriate to absorb the tunnel diode capacitance in the transmission line. To effect this incorporation the series inductance and the series resistance of the tunnel diode had to be neglected. It was presumed that the lumped transmission line could serve two useful purposes. Firstly, it permits the tunnel diode capacitance to be treated as a part of the transmission line. Secondly, at high frequencies it provides some isolation between the tunnel diodes and thus decreases the mutual interactions that are likely to occur. This could conceivably assist in stabilizing the circuit. To examine these suppositions the stability of two stage amplifiers with capacitor input circuits involving pure negative resistances was investigated. The investigations indicate the possibility of designing an amplifier of maximum gain of 5 with one filter section as the lumped transmission line. Any increase in the number of filter sections in the transmission line reduces the maximum gain that can be achieved. In other words, the decrease in the number of filter sections increases the gain achievable. If this is taken to the limit by removing all the filter sections, we will be left with pure resistances in the circuit. Theoretically, such a circuit would be stable and any gain can be obtained with such configuration. In practice, it is impossible to dispense with
all the reactive elements of the circuit as the negative resistance has some reactive elements associated with it. It is not at all clear as to how the stability of the amplifier will be influenced by the change in the number of filter sections, when these parasitic elements are included. With this in view, it was felt desirable to investigate the stability of the amplifier incorporating the actual equivalent circuit of the tunnel diode. At this stage it was felt necessary to reduce the resistive cut-off frequency of the tunnel diode nearly equal to the cut-off frequency of the transmission line to ensure that the system does not break off into oscillations outside the pass band. The investigations of the stability of three stage amplifiers with three filter sections per stage indicate that it is possible to design an amplifier of stable gain 10 with inductor input circuits. To find how the stability of the system would be affected when the number of filter sections varies, some extreme situations of instability were chosen. The stabilities of simple circuits each consisting of a single tunnel diode which is an exact equivalent of the parallel combination of three tunnel diodes of each unstable situation were considered. Surprisingly enough these hypothetical circuits were found to be theoretically stable. It should be remembered that such circuits are not practically realizable. Anyway, from this we can suspect the increase in the number of filter sections impairs the stability of the amplifier incorporating the equivalent circuit of the tunnel
diode. To make this point very clear and more convincing some additional work should be done. It is very necessary to investigate the stability of three stage amplifiers involving one filter section per stage and also of those involving two filter sections per stage, to establish the above mentioned fact.

Another significant point that has emerged out of these investigations is that for smaller values of the series inductance and the junction capacitance of the tunnel diode, the amplifier tends to become stable.

Since the Moody-Wacker amplifier is always terminated in a resistive load, it was felt logical to investigate the stability of the amplifiers involving maximally flat Butterworth filters as the lumped transmission line, as these filters are designed on the basis of resistive loads. It is possible to determine the stability of two stage amplifiers of this type through the properties of impedance functions because the relevant expression for driving point impedance and transfer function of Butterworth filters are known. The study of the results of the stability investigations indicates that with this mode of design it is possible to build a two stage amplifier of maximum gain $5$ with three reactive elements in each Butterworth filter. Any increase in the number of frequency sensitive elements decreases the maximum gain achievable.

The results of the stability investigations of the various situations indicate that from the theoretical point of
view, the inclusion of transmission line which was supposed to assist us in stabilizing the amplifier is rather disputable. By experience we know that a circuit which has no filter section between the tunnel diodes invariably oscillates. Further, such a circuit is not physically realizable. From a practical point of view there still seems to be valid reason in the use of filter section between the tunnel diodes to achieve stability. However, the number of filter sections used between the tunnel diodes should be a minimum.


APPENDIX A

In Chapter 4 the desirability of expressing the tunnel diode parameters in terms of gain, input impedance, and bandwidth of the amplifier was pointed out. The following cubic equation is obtained from the consideration

\[ 4 \pi f_g^2 c_d^2 R_N^3 - 4 \pi f_g^2 c_d^2 \left( \frac{n \cdot Z_0}{G - 1} \right) R_N^2 - \frac{n \cdot Z_0}{G - 1} = 0 \]

Setting \( f_g = K_2 f_0 \), the equation 1 can be rewritten as

\[ 4 \pi K_2^2 f_0^2 c_d^2 R_N^3 - 4 \pi K_2^2 f_0^2 c_d^2 \left( \frac{n \cdot Z_0}{G - 1} \right) R_N^2 - \frac{n \cdot Z_0}{G - 1} = 0 \]

From 4.6 we know that

\[ c_d = \frac{K_1}{2 f_0 Z_0} \left( \frac{2n+G-1}{n} \right) \]

Substituting 3 in 2, we obtain

\[ K_1^2 K_2^2 \left( \frac{2n+G-1}{n \cdot Z_0} \right)^2 R_N^3 - \frac{K_1^2 K_2^2}{G - 1} \left( \frac{2n+G-1}{n \cdot Z_0} \right)^2 R_N^2 - \frac{n \cdot Z_0}{G - 1} = 0 \]

The equation 4 can be expressed in the form

\[ pJ^3 + 3qJ^2 + r = 0 \]

where \( J = R_N; p = K_1^2 K_2^2 \left( \frac{2n+G-1}{n \cdot Z_0} \right)^2; r = \frac{-n \cdot Z_0}{G - 1}; 3q = -p \cdot r \).

Equation 5 has one real root and two complex roots. The real root is

\[ J = \left( \frac{Q-H}{Q} \right)^\frac{H}{p} - q \]
where \( Q = \left(\frac{1}{2}B\right)^{\frac{1}{3}} \sqrt[3]{1 + \frac{4H}{B^2} - 1} \); \( H = -q^2 \); and \( B = p^2 + 2q^3 \).

It will be shown shortly that the error incurred in \( Q \) is small if the \( 1 + \frac{4H}{B^2} \) term is expanded by the binomial theorem and only the first two terms retained.

\[
Q = \left(\frac{1}{2}B\right)^{\frac{1}{3}} \sqrt[3]{1 + \frac{4H}{B^2} - 1} = \frac{H}{B^3}
\]

Substituting 7 in 6 we obtain

\[
J = \left(\frac{H}{B^{1/3}} - B^{1/3}\right) - q
\]

Substitution of the values of \( H \) and \( B \) in 8 yields

\[
J = -\frac{q}{p} \left[ 1 + \left\{ 2 + \frac{27p^2r}{q^3}\right\}^{\frac{1}{3}} + \left\{ 2 + \frac{27p^2r}{q^3}\right\}^{\frac{1}{3}} \right]
\]

Substituting the values of \( S, p, q, \) and \( r \), we obtain

\[
R_N = \frac{n.Z_0}{3(G-1)} \left[ 1 + \left\{ 2 + \frac{27(G-1)^2}{K_1 K_2^2(2n+G-1)^2}\right\}^{\frac{1}{3}} + \left\{ 2 + \frac{27(G-1)^2}{K_1 K_2^2(2n+G-1)^2}\right\}^{\frac{1}{3}} \right]
\]

Let

\[
\frac{3(G-1)}{2n+G-1} = x
\]
By the method of induction it can be established that \( x \) of equation 11 cannot be greater than 3.

Let us suppose that \( x = \frac{3(G-1)}{2n+G-1} \geq 3 \)

\[ G-1 = 2n+G-1 \]

\[ 2n < 0 \]

The inadmissibility of the inequality 12 rules out the possibility of \( x \) having values greater than 3. Hence the upper bound on \( x \) is dictated by the following inequality

\[ 3 > x \]

Substituting \( x \) for \( \frac{3(G-1)}{2n+G-1} \) in 10, we obtain

\[ R_N = \frac{n \cdot Z_0}{3(G-1)} \left[ 1 + \left\{ 2 + 3(\frac{x}{K_1K_2})^2 \right\}^{\frac{1}{3}} + \left\{ 2 + 3(\frac{x}{K_1K_2})^2 \right\}^{-\frac{1}{3}} \right] \]

While deriving the above solution for \( R_N \) we made the approximation that

\[ \sqrt{1 + \frac{4H^3}{B^2}} = 1 + \frac{2H^3}{B^2} \]

Now we shall examine the error introduced by this assumption. Writing the exact expression for \( Q \) and expanding it by the binomial theorem we obtain
The series is a convergent alternating series as we know that
\[
\frac{4H^3}{B^2} = \frac{4(-q^2)^3}{p^2r + 2q^3} < 1
\]

Let us assume that we desire a value of \( Q \) accurate to within 1%. The permissible error in the quantity under the cube root sign can then be 3%. Since the error in terminating a convergent alternating series at any term does not exceed numerically the value of the first of the terms discarded the fractional error in the quantity under the cube root sign will not exceed 3% if
\[
\left| \frac{1}{5} \left( \frac{4H^3}{B^2} \right)^2 \right| < 0.03
\]

which reduces to
\[
\frac{4H^3}{B^2} < 0.12
\]

Expressing \( H \) and \( B \) in terms of \( p, q, \) and \( r \)
\[
\frac{1}{1 + \frac{p^2r}{2q^3}} < \sqrt{0.12}
\]
The values of \( p, q, \) and \( r \) of equation 4 when substituted in 19 yield

\[
\frac{K_1^2 K_2^2}{(2n+G-1)^2} \leq \frac{27}{3.774}
\]

Substituting \( x \) for \( \frac{3(G-1)}{2n+G-1} \) in 20 and simplifying, we obtain

\[
1.12 \ K_1 K_2 < x
\]

Combining the inequalities 13 and 21, we obtain

\[
3 > x > 1.12 \ K_1 K_2
\]

It is essential to realize that the restriction which this inequality imposes on \( x \) is not fundamental. It exists because of the approximation made in seeking a solution for \( R_N \). It should be noted in the region where the inequality breaks down our approximation which limits the error to less than 1% is not valid.
APPENDIX B

Routh test is a mechanistic procedure whereby a polynomial can be tested to determine if it has any zeros in the R.H. S plane. The procedure is demonstrated in the polynomial

\[ a_6s^6 + a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 \]

The coefficients of the polynomials are placed in two rows as follows

\[
\begin{array}{cccc}
  a_6 & a_4 & a_2 & a_0 \\
  a_5 & a_3 & a_1 \\
\end{array}
\]

The coefficients of the next row are formed from these two rows

\[
b_4 = \frac{a_5a_4 - a_3a_6}{a_5}
\]

\[
b_2 = \frac{a_5a_2 - a_1a_6}{a_5}
\]

\[
b_0 = \frac{a_5a_0 - 0.6a_6}{a_5}
\]

When these two rows are arranged below the first two rows, the fourth, the fifth rows are similarly calculated in succession from the previous two rows.

* (James G. Halsebrook, 1959)
The polynomial that determines the stability of amplifiers involving inductor input circuits is given by 5.15

\[ \prod_{k=1}^{n} (s - S_{pk}) - kS^n = 0 \]

By applying Routh's criterion the terms of the first column can be found and thereby the restrictions on \( k \) which assures the stability of the amplifier can be determined.

Let us recall that it is the denominator polynomial of the driving point impedance of the capacitor input circuits which determines the stability of the network. This characteristic polynomial is given by 5.10

\[ \prod_{k=1}^{n} (s - S_{pk}) + kS^n = 0 \]

Subjecting this polynomial to Routh's test the restrictions on \( k \) that assures the stability of the system can be obtained.
The coefficients $c_3$ and $d_2$ are

$$c_3 = \frac{b_4a_3 - a_5b_2}{b_4}$$

$$d_2 = \frac{c_3b_2 - b_4c_1}{c_3}$$

Routh showed that the number of zeros of polynomial in the right half $S$ plane is equal to the number of changes of sign in the elements in the first column. Thus, by forming the array and observing the number of changes of sign in the first column, the number of zeros on the R.H.S. plane is determined. If there is no change of sign in the first column, the polynomial is Hurwitz, since all the zeros lie on the left half $S$-plane.