DESIGN AND SIMULATION OF SOFTWARE FOR A HIGH SPEED DIGITAL DISTANCE RELAY

A Thesis
Submitted to the College of Graduate Studies and Research in Partial Fulfilment of the Requirements for the Degree of Master of Science in the Department of Electrical Engineering University of Saskatchewan

by

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ABSTRACT

Distance relays are used for the protection of transmission lines in electric power systems. These relays estimate an apparent impedance of the protected line from the relay location to the fault and compare it with the relay settings for making decisions. Many approaches have been suggested in the past for developing distance relaying criteria that can be implemented using digital processors. One of the approaches used modal components for developing the distance relaying criterion.

The work reported in this thesis investigates the previously suggested distance relaying criterion that used modal transformations. Depending on the type of fault, this criterion requires that some coefficients be computed in the on-line mode. An improved criterion is proposed in this thesis which eliminates the on-line computations of the coefficients. Also, the proposed criterion minimizes the number of divisions. Consequently, it reduces the computational burden on the digital processor. The derivation of the proposed criterion is described.

The proposed criterion is used for developing the relay software that is written in the assembly language of the TMS-32010 digital processor. The performance of the relay software was evaluated using the fault data generated by the Electro-Magnetic Transient Program (EMTP). A six bus representation of the Saskatchewan Power Corporation system was modelled in the EMTP. Some of the results of the simulation studies are reported in this thesis.
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LIST OF PRINCIPAL SYMBOLS

EMTP  Electro-Magnetic Transient Program.

SPC  Saskatchewan Power Corporation.

\(v_a, v_b\) and \(v_c\)  phase voltages at the fault.

\(i_a, i_b\) and \(i_c\)  phase currents in the fault.

p.u.  per unit.

\(v_{1e}\)  mode-1 voltage at the fault.

\(v_{2e}\)  mode-2 voltage at the fault.

\(v_{0e}\)  mode-0 voltage at the fault.

\(i_{1f}\)  mode-1 current in the fault.

\(i_{2f}\)  mode-2 current in the fault.

\(i_{0f}\)  mode-0 current in the fault.

\(v_1\)  mode-1 voltage at the relay location.

\(v_2\)  mode-2 voltage at the relay location.

\(v_0\)  mode-0 voltage at the relay location.

\(i_1\)  mode-1 current at the relay location.
\( i_2 \) mode-2 current at the relay location.
\( i_0 \) mode-0 current at the relay location.
\( R_1 \) mode-1 resistance of the protected line.
\( R_0 \) mode-0 resistance of the protected line.
\( L_1 \) mode-1 inductance of the protected line.
\( L_0 \) mode-0 inductance of the protected line.
\( R_f \) fault resistance represented in the modal networks.
\( x \) distance of a fault from the relay location divided by the total length of the line. This will be referred to as the p.u. distance of the fault.
\( v_{r1} \) mode-1 component of the compensated voltage.
\( v_{r2} \) mode-2 component of the compensated voltage.
\( v_{r0} \) mode-0 component of the compensated voltage.
\( v_{d1} \) ratio of \( v_1 \) and \( v_{r1} \).
\( v_{d2} \) ratio of \( v_2 \) and \( v_{r2} \).
\( v_{d0} \) ratio of \( v_0 \) and \( v_{r0} \).
\( v_{d01} \) ratio of \( v_{r0} \) and \( v_{r1} \).
\( v_{d02} \) ratio of \( v_{r0} \) and \( v_{r2} \).
\( v_{d21} \) ratio of \( v_{r2} \) and \( v_{r1} \).
\( c_1, c_2 \) and \( c_0 \) coefficients of the previously suggested criterion.
$a_1$, $a_2$ and $a_0$ coefficients of the criterion proposed in this thesis.

C.B. Circuit Breaker.
1. INTRODUCTION

1.1. General

An electric power utility generates electrical power, transmits it over transmission lines and supplies it to consumers through distribution systems. The objective of a power utility is to supply electrical power to consumers at reasonable cost and with minimum interruptions. However, abnormal conditions and faults can cause interruptions in power supply. Occurrence of a fault is generally caused by insulation failure. A fault can

- lead to unstable operation,
- cause extensive damage to equipments and
- result in loss of revenue.

To minimise these effects, equipments of a power system must be protected adequately [1]. A protection system is used for this purpose.

1.2. Power System Protection

The present practice is to divide a power system into protection zones, such as a generator protection zone, a transformer protection zone, a bus protection zone and a transmission line protection zone [2]. Combinations of relays and circuit breakers are used for protecting each zone. Relays measure electrical parameters of power systems, such as voltages, currents and frequency. These parameters change during faults.

The changes in the electrical quantities provide useful information to the relays. The relays detect the onset of a fault and take appropriate deci-
sions. If the fault is in the zone of the relay, it issues a command to trip the circuit breakers controlling the zone. The circuit breakers then disconnect the faulted section from the remaining system.

1.3. Distance Relays

Distance relays are used for protecting transmission lines. These relays estimate the impedance of the protected line from the relay location to the fault. The estimated impedance is compared with the relay settings to determine whether the fault is in the protected zone. If the fault is in the protected zone, the relay issues a command to trip the line circuit breaker.

A distance relay scheme uses six separate impedance measuring units, three for phase faults and three for ground faults. These units measure impedance continuously and compare it with the relay settings. In the event of a fault in the zone of the relay, one or more of these measurements will yield a decision to trip the line circuit breakers. However, this scheme is complex in design, costly and physically large. To overcome these shortcomings, polyphase distance relays were developed [3]. Many approaches have been suggested in the past for developing polyphase distance relaying criteria. Some of them use a single criterion, and others use two or more criteria for detecting all types of shunt faults.

1.4. Developments in Protective Relays

Relays are usually designed and built using electro-mechanical and solid-state technology. However, for the last several years research has been done in the application of computers for protective relaying. The use of digital computers in protection systems was started when minicomputers became available [4]. Initially, researchers proposed a single digital computer for implementing all relaying functions to be performed in a substation [5]. Recent developments in the integrated circuit technology have reduced the size and cost of digital processors while increasing their capabilities. There-
fore, digital processors provide a viable alternative to solid-state or electro-
mechanical relays [6].

One of the major areas of research has been digital processor based dis-
tance relays for the protection of transmission lines. Mann and Morrison
developed the software for implementing polyphase distance relaying criteria
on a digital computer [7]. The software identifies fault type and selects an
appropriate set of voltages and currents for calculating impedances. Phadke
et al. derived a distance relaying criterion using symmetrical components
transformation [8]. Later, they used the criterion for developing a microcom-
puter based distance relay [9]. Borndard and Bastide designed and built a
multiprocessor based distance relay [10]. Frendo and Kitai used bit-slice
processors for implementing a distance relay [11]. Sachdev and Kolla applied
modal transformation to develop a distance relaying criterion. This criterion
has computational advantage over the criterion used in the symmetrical com-
ponent based distance relay [12, 13]. However, some of the coefficients of
the criterion are required to be calculated in the on-line mode. Other or-
ganisations and individuals have also conducted research for developing com-
putationally efficient distance relaying criteria.

1.5. Objectives of the Project

The major objectives of the research work reported in this thesis are to

• investigate the previously suggested distance relaying criterion that
  uses modal transformation,

• improve the previously suggested criterion by reducing the com-
  putational requirements,

• design a high speed digital distance relay for shunt faults on
  transmission lines,

• write the relay software in the assembly language of the
  TMS-32010 digital processor and

• evaluate the performance of the relay using data generated by the
Electro-Magnetic Transient Program (EMTP) and a model of a transmission system.

1.6. Outline of the Thesis

This thesis is organised in six chapters and six appendices. Power system protection, distance protection, developments in distance relaying and the major objectives of the research work are briefly described in this chapter.

The previously suggested distance relaying criterion that used modal transformation is reviewed in Chapter 2. Some of the coefficients of the criterion must be calculated in the on-line mode. This factor makes the criterion computationally inefficient.

An improved criterion is derived in Chapter 3. The coefficients of the proposed criterion are constants and are not required to be calculated in the on-line mode. Also, the proposed criterion requires one division, compared to up to six divisions of the previously suggested criterion, for calculating the distance to the fault. The proposed criterion, therefore, has an advantage over the previously suggested criterion.

The procedures for generating the input signals, calculating the compensated voltages, and simulating low-pass filters and analog-to-digital converters are discussed in Chapter 4. That chapter also describes the major functions of the modules used for developing the relay software. The software was written in the assembly language of the TMS-32010 digital processor. The performance of the relay software was evaluated using the data generated from the EMTP. A six bus representation of the Saskatchewan Power Corporation (SPC) system was modelled in the EMTP. The test results of the simulation studies performed are reported in Chapter 5. The sixth chapter provides a summary of the research work and draws appropriate conclusions. This chapter is followed by the list of references.
There are six appendices in this thesis. The important properties of the modal transformation are reviewed in Appendix A. The procedure for designing low-pass filters is described in Appendix B. Appendix C provides information on the TMS-32010 digital processor. The parameters of the six bus model of the SPC transmission system are listed in Appendix D. Appendix E gives information about the Electro-Magnetic Transient Program. Some results of the simulation studies are reported in Appendix F.
2. DISTANCE RELAYING CRITERION USING MODAL TRANSFORMATION

2.1. Introduction

The subjects of power system protection, distance protection and developments in digital distance relaying were briefly presented in the previous chapter. Power systems experiencing unbalanced faults can be analysed using symmetrical, Clarke’s or modal transformations. It is also possible to use these transformations for formulating criteria for detecting shunt faults on a transmission line.

The symmetrical components transformation contains complex numbers and is suitable for calculating positive, negative and zero sequence components of voltage and current phasors [3]. This method is, however, not valid for processing instantaneous values of voltage and current signals. Clarke’s components, on the other hand, contain only real numbers and are suitable for calculating the α, β and 0 sequence components of phasors as well as instantaneous values of functions describing voltages and currents [14]. However, two elements in this transformation are irrational numbers. Like the Clarke’s transformation, modal transformation is suitable for calculating mode-1, mode-2 and mode-0 components of voltages and currents. The elements of the modal transformation are real numbers and are 0, 1 or -1. Appendix A briefly reviews important properties of this transformation.

The modal transformation has computational advantage over both the symmetrical components and the Clarke’s components transformations [13].
Therefore, Sachdev and Kolla used the modal transformation technique for developing the distance relaying criterion [12, 13]. This chapter reviews the previously suggested criterion.

2.2. Distance Relaying Criterion Using Modal Transformation

The previously suggested distance relaying criterion that used the modal transformation is briefly reviewed in this section.

Consider a three-phase power system whose single line diagram is shown in Figure 2.1. Also consider that distance relays are provided at both ends of the transmission line. In this figure, the transmission line connects buses P and Q. G1 and G2 are generators representing two power systems. The circuit breakers (C.B.'s) provided at line terminals, in conjunction with relays, protect the line.

Now assume that the system experiences a fault at location F on the transmission line. Figure 2.2 shows mode-1, mode-2 and mode-0 networks of the system when experiencing a fault at F. The transmission line is modelled as a series R-L circuit. The modal voltages $v_{1e}$, $v_{2e}$ and $v_{0e}$ at the fault, are functions of the modal voltages at bus P and the modal currents in the line and the fault. The following equations express the modal voltages in terms of other voltages, currents and the line parameters.

$$v_{1e} = v_1 - \frac{x}{\pi} \left( R_1 i_1 + L_1 \frac{d}{dt} i_1 \right) - R_f i_{1f}$$ (2.1)

$$v_{2e} = v_2 - \frac{x}{\pi} \left( R_1 i_2 + L_1 \frac{d}{dt} i_2 \right) - R_f i_{2f}$$ (2.2)

$$v_{0e} = v_0 - \frac{x}{\pi} \left( R_0 i_0 + L_0 \frac{d}{dt} i_0 \right) - R_f i_{0f}$$ (2.3)

where:
Figure 2.1: A sample power system experiencing a fault at F [13].

Figure 2.2: The modal networks of the sample power system shown in Figure 2.1 [13].
\( v_1, v_2, v_0 \) are the instantaneous modal voltages at the relay location,
\( i_1, i_2, i_0 \) are the instantaneous modal currents at the relay location,
\( R_1, R_0 \) are the mode-1 and mode-0 resistances of the protected line,
\( L_1, L_0 \) are the mode-1 and mode-0 inductances of the protected line,
\( i_{1f}, i_{2f}, i_{0f} \) are the modal currents in the fault,
\( R_f \) is the fault resistance represented in the modal networks and
\( x \) is the distance of a fault from the relay location divided by the total length of the line. This will be referred to as the per unit distance of the fault in this thesis.

Mode-1 impedance is identical to the positive sequence impedance, mode-2 impedance is identical to the negative sequence impedance and mode-0 impedance is identical to the zero sequence impedance [12]. As positive and negative sequence impedances of a balanced transmission line are equal, the mode-1 and mode-2 impedances of the line are also equal. Therefore, replica impedances can be represented by the mode-1 and mode-0 impedances of the transmission line.

Replica impedances are used in conventional relays. These impedances are equal to the sequence impedances of the transmission line. The relay currents passed through the replica impedances provide voltages that represent voltage drops across the line as if the current was flowing in the line. The modal voltage drops in the replica impedances of the transmission line are

\[
v_{r1} = R_1 i_1 + L_1 \frac{d}{dt}i_1, \quad (2.4)
\]
\[ v_{r2} = R_1 i_2 + L_1 (d/dt)i_2 \quad \text{and} \quad (2.5) \]

\[ v_{r0} = R_0 i_0 + L_0 (d/dt)i_0. \quad (2.6) \]

Substituting \( v_{r1} \) for \( R_1 i_1 + L_1 (d/dt)i_1 \) in Equation 2.1, the following equation is obtained.

\[ v_{1e} = v_1 - x v_{r1} - R_f i_{1f} \quad (2.7) \]

This equation can be rearranged to provide the following equation.

\[ v_{1e} = v_{r1} (v_{d1} - x) - R_f i_{1f} \quad (2.8) \]

where:

\[ v_{d1} = v_1 / v_{r1}. \]

Substituting \( v_{r2} \) for \( R_1 i_2 + L_1 (d/dt)i_2 \) in Equation 2.2, the following equation is obtained.

\[ v_{2e} = v_2 - x v_{r2} - R_f i_{2f} \quad (2.9) \]

This equation can be rearranged to provide the following equation.

\[ v_{2e} = v_{r2} (v_{d2} - x) - R_f i_{2f} \quad (2.10) \]

where:

\[ v_{d2} = v_2 / v_{r2}. \]

The following equation is obtained using a similar procedure.

\[ v_{0e} = v_{r0} (v_{d0} - x) - R_f i_{0f} \quad (2.11) \]

where:
\[ v_{d0} = v_0 / v_{r0}. \]

Power systems experience ten types of shunt faults, namely, a three-phase fault, three two-phase faults (b-c, a-b, c-a), three two-phase to ground faults (b-c-g, a-b-g, c-a-g) and three single-phase to ground faults (a-g, b-g, c-g). These shunt faults can be represented by arrangements illustrated in Figure 2.3. In this figure a, b, and c represent three phases of a power system. Table 2.1 describes phase voltage and phase current conditions at the fault for each fault. In this table \( v_a \), \( v_b \), and \( v_c \) are phase voltages at the fault and \( i_a \), \( i_b \), and \( i_c \) are phase currents in the fault. These conditions are transformed to equivalent modal conditions that are listed in Table 2.2 [13]. The following paragraphs use the conditions described in this table for deriving the distance to a fault from voltages and currents at the relay location.

(i) A three-phase fault:

As listed in Table 2.2, the following equalities represent a three phase fault.

\[
\begin{align*}
  v_{1e} &= 0 \\
  v_{2e} &= 0 \\
  v_{0e} &= 0
\end{align*}
\]

Using the fault condition described by Equation 2.12, Equation 2.8 converts to

\[ v_{r1} (v_{d1} - x) - R_f i_{1f} = 0. \]  

(2.15)

The distance of the fault obtained by rearranging this equation is

\[ x = v_{d1} + e_r, \]

(2.16)

where:
Figure 2.3: The types of transmission line shunt faults (a) a three phase fault (b) a phase \(b-c\) fault (c) a phase \(a-b\) fault (d) a phase \(c-a\) fault (e) a phase \(b-c\) to ground fault (f) a phase \(a-b\) to ground fault (g) a phase \(c-a\) to ground fault (h) a phase \(a\) to ground fault (i) a phase \(b\) to ground fault and (j) a phase \(c\) to ground fault.
Table 2.1: The phase voltage and current conditions representing shunt faults [12].

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Phase voltages at the fault</th>
<th>Phase currents in the fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-ph</td>
<td>$v_a = v_b = v_c$</td>
<td>$i_a + i_b + i_c = 0$</td>
</tr>
<tr>
<td>b-c</td>
<td>$v_b = v_c$</td>
<td>$i_b + i_c = 0$; $i_a = 0$</td>
</tr>
<tr>
<td>a-b</td>
<td>$v_a = v_b$</td>
<td>$i_a + i_b = 0$; $i_c = 0$</td>
</tr>
<tr>
<td>c-a</td>
<td>$v_c = v_a$</td>
<td>$i_c + i_a = 0$; $i_b = 0$</td>
</tr>
<tr>
<td>b-c-g</td>
<td>$v_b = v_c = 0$</td>
<td>$i_a = 0$</td>
</tr>
<tr>
<td>a-b-g</td>
<td>$v_a = v_b = 0$</td>
<td>$i_c = 0$</td>
</tr>
<tr>
<td>c-a-g</td>
<td>$v_c = v_a = 0$</td>
<td>$i_b = 0$</td>
</tr>
<tr>
<td>a-g</td>
<td>$v_a = 0$</td>
<td>$i_b = i_c = 0$</td>
</tr>
<tr>
<td>b-g</td>
<td>$v_b = 0$</td>
<td>$i_a = i_c = 0$</td>
</tr>
<tr>
<td>c-g</td>
<td>$v_c = 0$</td>
<td>$i_a = i_b = 0$</td>
</tr>
</tbody>
</table>

$$e_r = \frac{-R_f i_{1f}}{v_{r1}}.$$  

Similarly, the following equation is obtained by using Equations 2.10 and 2.13.

$$x = v_{d2} + e_r$$  \hspace{1cm} (2.17)

where:

$$e_r = \frac{-R_f i_{2f}}{v_{r2}}.$$
Table 2.2: The modal voltage and current conditions representing shunt faults [13].

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Modal voltages</th>
<th>Modal currents</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-ph</td>
<td>$v_{0e}=v_{1e}=v_{2e}=0$</td>
<td>$i_{0f}=0$</td>
</tr>
<tr>
<td>b-c</td>
<td>$v_{1e}=v_{2e}$</td>
<td>$i_{1f}=-i_{2f}$; $i_{0f}=0$</td>
</tr>
<tr>
<td>a-b</td>
<td>$v_{1e}=0$</td>
<td>$i_{1f}=2i_{2f}$; $i_{0f}=0$</td>
</tr>
<tr>
<td>c-a</td>
<td>$v_{2e}=0$</td>
<td>$i_{2f}=2i_{1f}$; $i_{0f}=0$</td>
</tr>
<tr>
<td>b-c-g</td>
<td>$v_{0e}=v_{1e}=v_{2e}$</td>
<td>$i_{1f}+i_{2f}+i_{0f}=0$</td>
</tr>
<tr>
<td>a-b-g</td>
<td>$v_{1e}=0$; $v_{2e}=-v_{0e}$</td>
<td>$i_{1f}2i_{2f}+i_{0f}=0$</td>
</tr>
<tr>
<td>c-a-g</td>
<td>$v_{2e}=0$; $v_{1e}=-v_{0e}$</td>
<td>$-2i_{1f}+i_{2f}+i_{0f}=0$</td>
</tr>
<tr>
<td>a-g</td>
<td>$v_{0e}+v_{1e}+v_{2e}=0$</td>
<td>$i_{1f}=i_{2f}=i_{0f}$</td>
</tr>
<tr>
<td>b-g</td>
<td>$v_{0e}-2v_{1e}+v_{2e}=0$</td>
<td>$i_{2f}=0$; $i_{1f}=-i_{0f}$</td>
</tr>
<tr>
<td>c-g</td>
<td>$v_{0e}+v_{1e}-2v_{2e}=0$</td>
<td>$i_{1f}=0$; $i_{2f}=-i_{0f}$</td>
</tr>
</tbody>
</table>

Since $v_{1e}$ and $v_{2e}$ are both zero for a three phase fault, they are also equal. This procedure provides the following condition.

$$v_{1e} = v_{2e} \quad (2.18)$$

Therefore, the right hand sides of Equations 2.8 and 2.10 can be equated to provide the following equation.

$$v_{r1} (v_{d1} - x) - R_f i_{1f} = v_{r2} (v_{d2} - x) - R_f i_{2f} \quad (2.19)$$

Rearranging this equation, Equation 2.20 is obtained.

$$x = \frac{v_{d1} - (v_{r2}/v_{r1}) v_{d2}}{1 - v_{r2}/v_{r1}} - \frac{R_f i_{1f} - R_f i_{2f}}{v_{r1} (1 - v_{r2}/v_{r1})} \quad (2.20)$$
Defining $v_{r2}/v_{r1}$ as $v_{d21}$ and substituting it in Equation 2.20, the following equation is obtained.

$$x = \frac{v_{d1} - v_{d21}}{1 - v_{d21}} + e_r$$  \hspace{1cm} (2.21)

where:

$$e_r = \frac{-R_f i_{1f} + R_f i_{2f}}{v_{r1} (1 - v_{d21})}.$$

Equation 2.14 cannot be used to determine $x$ because mode-0 currents and voltages are zero in the system. Equation 2.16, 2.17 or 2.21 can be selected as a criterion for calculating the distance of a three phase fault from the relay location.

(ii) Phase to phase faults:

Modal conditions for the phase to phase faults on a transmission line are listed in Table 2.2. These conditions can be used to derive distance relaying criterion for phase to phase faults.

A phase b-c fault:

For a phase b-c fault, the modal condition is:

$$v_{1e} = v_{2e}.$$  \hspace{1cm} (2.22)

Equation 2.22 is the same as Equation 2.18 that was used in the last section for calculating the distance of a three phase fault. Therefore, Equation 2.21 can also be used for calculating the distance $x$ for a phase b-c fault.

A phase a-b fault:

For a phase a-b fault, the modal condition is:

$$v_{1e} = 0.$$  \hspace{1cm} (2.23)
Equation 2.23 is the same as Equation 2.12. From that condition, the criterion described in Equation 2.16 was derived. Therefore, the criterion of Equation 2.16 also provides the distance $x$ in the case of a phase a-b fault.

**A phase c-a fault:**

For a phase c-a fault, the modal condition is:

$$v_{2e} = 0.$$  \hspace{1cm} (2.24)

Equation 2.24 is the same as Equation 2.13. From that condition, the criterion described in Equation 2.17 was derived. Therefore, the criterion of Equation 2.17 also provides the distance $x$ in the case of a phase c-a fault.

The discussion in this section shows that Equations 2.16, 2.17, and 2.21 provide the distance relaying criterion for phase a-b, phase c-a and phase b-c faults respectively.

**(iii) Two phase to ground faults:**

The modal conditions for the two phase to ground faults on a transmission line are listed in Table 2.2. These conditions can be used to derive the distance relaying criterion for two phase to ground faults.

**A phase b-c to ground fault:**

For a phase b-c to ground fault, the modal conditions are:

$$v_{1e} = v_{2e} = v_{0e}.$$  \hspace{1cm} (2.25)

These conditions can be rewritten as follows:

$$v_{1e} = v_{2e};$$  \hspace{1cm} (2.26)

$$v_{1e} = v_{0e} \quad \text{and}$$  \hspace{1cm} (2.27)

$$v_{2e} = v_{0e}.$$  \hspace{1cm} (2.28)

Equation 2.26 is the same as Equation 2.18. From that condition, the criterion described in Equation 2.21 was derived. Therefore, the criterion of Equation 2.21 also provides the distance $x$ in the case of a phase b-c to ground fault.
The condition described by Equations 2.27 can be used by equating the right hand sides of Equations 2.8 and 2.11. This procedure provides the following equation.

\[ v_{r1} (v_{d1} - x) - R_f i_{1f} = v_{r0} (v_{d0} - x) - R_f i_{0f} \]  (2.29)

Rearranging this equation, Equation 2.30 is obtained.

\[ z = \frac{v_{d1} - (v_{r0}/v_{r1}) v_{d0}}{(1 - v_{r0}/v_{r1})} - \frac{R_f i_{1f} - R_f i_{0f}}{v_{r1} (1 - v_{r0}/v_{r1})} \]  (2.30)

Defining \( v_{r0}/v_{r1} \) as \( v_{d01} \) and substituting it in Equation 2.30, the following equation is obtained.

\[ z = \frac{v_{d1} - v_{d01} v_{d0}}{1 - v_{d01}} + e_r \]  (2.31)

where:

\[ e_r = \frac{-R_f i_{1f} + R_f i_{0f}}{v_{r1} (1 - v_{d01})} \]

The modal condition described by Equations 2.28 can also be used for deriving the criterion in the case of a phase b-c to ground fault. If \( v_{2e} \) and \( v_{0e} \) from Equations 2.10 and 2.11 are substituted in Equation 2.28, the following equation is obtained.

\[ v_{r2} (v_{d2} - x) - R_f i_{2f} = v_{r0} (v_{d0} - x) - R_f i_{0f} \]  (2.32)

Rearranging this equation, provides Equation 2.33.

\[ z = \frac{v_{d2} - (v_{r0}/v_{r2}) v_{d0}}{(1 - v_{r0}/v_{r2})} - \frac{R_f i_{2f} - R_f i_{0f}}{v_{r2} (1 - v_{r0}/v_{r2})} \]  (2.33)
Defining \(v_{d0}/v_{r2}\) as \(v_{d02}\) and substituting it in Equation 2.33, the following equation is obtained.

\[
x = \frac{v_{d2} - v_{d02} v_{d0}}{1 - v_{d02}} + e_r
\]  

(2.34)

where:

\[
e_r = \frac{-R_f i_{2f} + R_f i_{0f}}{v_{r2} (1 - v_{d02})}.
\]

A phase a-b to ground fault:

The modal conditions for a phase a-b to ground fault are:

\[
v_{1e} = 0 \quad \text{and} \quad v_{2e} = -v_{0e}.
\]  

(2.35)  

(2.36)

The condition described in Equation 2.35 is identical to the condition of Equation 2.12 for a three phase fault. From that condition, the criterion described in Equation 2.16 was developed. Therefore, the criterion of Equation 2.16 also provides the distance \(x\) in the case of a phase a-b to ground fault.

The condition described by Equation 2.36 can be rewritten as:

\[
v_{2e} + v_{0e} = 0.
\]  

(2.37)

Therefore, the sum of right hand sides of Equations 2.10 and 2.11 can be equated to zero. This procedure provides the following equation.

\[
v_{r2}(v_{d2} - x) - R_f i_{2f} + v_{r0}(v_{d0} - x) - R_f i_{0f} = 0
\]  

(2.38)

Rearranging this equation, Equation 2.39 is obtained.
\[ x = \frac{v_{d2} + \left(\frac{v_{r0}}{v_{r2}}\right) v_{d0}}{1 + \frac{v_{r0}}{v_{r2}}} - \frac{R_f i_{2f} + R_f i_{0f}}{v_{r2} \left(1 + \frac{v_{r0}}{v_{r2}}\right)} \quad (2.39) \]

If the previously defined identity \( v_{d02} \) is substituted for \( \frac{v_{r0}}{v_{r2}} \) in Equation 2.39, the following equation is obtained.

\[ x = \frac{v_{d2} + v_{d02} v_{d0}}{1 + v_{d02}} + e_r \quad (2.40) \]

where:

\[ e_r = -\frac{R_f i_{2f} + R_f i_{0f}}{v_{r2} \left(1 + v_{d02}\right)}. \]

A phase c-a to ground fault:

The modal conditions for a phase c-a to ground fault are:

\[ v_{2e} = 0 \quad \text{and} \quad (2.41) \]

\[ v_{1e} = -v_{0e}. \quad (2.42) \]

The condition described by Equation 2.41 is identical to the condition of Equation 2.13 for a three phase fault. From that condition, the criterion described in Equation 2.17 was developed. Therefore, the criterion of Equation 2.17 also provides the distance \( x \) in the case of a phase c-a to ground fault.

The condition described by Equation 2.42 can be rewritten as:

\[ v_{1e} + v_{0e} = 0. \quad (2.43) \]

Substituting for \( v_{1e} \) and \( v_{0e} \) from Equations 2.8 and 2.11 into the above equation, the following equation is obtained.
\[ v_{r1}(v_{d1} - x) - R_f i_{1f} + v_{r0}(v_{d0} - x) - R_f i_{0f} = 0 \] (2.44)

Rearranging this equation, Equation 2.45 is obtained.

\[ x = \frac{v_{d1} + (v_{r0}/v_{r1}) v_{d0}}{(1 + v_{r0}/v_{r1})} - \frac{R_f i_{1f} + R_f i_{0f}}{v_{r1}(1 + v_{r0}/v_{r1})} \] (2.45)

If the previously defined identity \( v_{d01} \) is substituted for \( v_{r0}/v_{r1} \) in Equation 2.45, the following equation is obtained.

\[ x = \frac{v_{d1} + v_{d01} v_{d0}}{1 + v_{d01}} + e_r \] (2.46)

where:

\[ e_r = -\frac{R_f i_{1f} + R_f i_{0f}}{v_{r1}(1 + v_{d01})}. \]

Equations 2.21, 2.31, and 2.34 give the distance of a phase b-c to ground fault. Equations 2.16 and 2.40 provide the distance of a phase a-b to ground fault. Similarly, Equations 2.17 and 2.46 give the distance of a phase c-a to ground fault.

(iv) Single phase to ground faults:

The modal conditions for single phase to ground faults on a transmission line are listed in Table 2.2. These conditions can be used to derive the distance relaying criterion for single phase to ground faults.

A phase a to ground fault:

The modal condition for a phase a to ground fault is:

\[ v_{1e} + v_{2e} + v_{0e} = 0. \] (2.47)

Substituting for \( v_{1e} \), \( v_{2e} \), and \( v_{0e} \) from Equations 2.8, 2.10, and 2.11 in the above equation, Equation 2.48 is obtained.
\[
v_{r1} (v_{d1} - x) - R_f i_{1f} + v_{r2} (v_{d2} - x) - R_f i_{2f} + v_{r0} (v_{d0} - x) - R_f i_{0f} = 0
\]

(2.48)

Rearranging this equation, the following equation can be obtained.

\[
x = \frac{v_{d1} + (v_{r2}/v_{r1}) v_{d2} + (v_{r0}/v_{r1}) v_{d0}}{1 + (v_{r2}/v_{r1}) + (v_{r0}/v_{r1})}
\]

\[
- \frac{R_f i_{1f} + R_f i_{2f} + R_f i_{0f}}{v_{r1} (1 + (v_{r2}/v_{r1}) + (v_{r0}/v_{r1}))}
\]

(2.49)

If the previously defined identities \( v_{d21} \) for \( v_{r2}/v_{r1} \) and \( v_{d01} \) for \( v_{r0}/v_{r1} \) are substituted in Equation 2.49, the following equation is obtained.

\[
x = \frac{v_{d1} + v_{d21} v_{d2} + v_{d01} v_{d0}}{1 + v_{d21} + v_{d01}} + e_r
\]

(2.50)

where:

\[
e_r = - \frac{R_f i_{1f} + R_f i_{2f} + R_f i_{0f}}{v_{r1} (1 + v_{d21} + v_{d01})}
\]

(2.51)

A phase b to ground fault:

The modal condition for a phase b to ground fault is:

\[-2 v_{1e} + v_{2e} + v_{0e} = 0.
\]

Substituting for \( v_{1e} \), \( v_{2e} \), and \( v_{0e} \) from Equations 2.8, 2.10, and 2.11 in the above equation, Equation 2.52 is obtained.
Rearranging this equation, the following equation can be obtained.

\[ x = \frac{v_{d1} - \left(v_{r2}/2v_{r1}\right) v_{d2} - \left(v_{r0}/2v_{r1}\right) v_{d0}}{1 - \left(v_{r2}/2v_{r1}\right) - \left(v_{r0}/2v_{r1}\right)} \]

\[ + \frac{-2R_f i_{1f} + R_f i_{2f} + R_f i_{0f}}{2v_{r1} \left(1 - \left(v_{r2}/2v_{r1}\right) - \left(v_{r0}/2v_{r1}\right)\right)} \]  

(2.53)

If the previously defined identities \( v_{d21} \) for \( v_{r2}/v_{r1} \) and \( v_{d01} \) for \( v_{r0}/v_{r1} \) are substituted in Equation 2.49, the following equation is obtained.

\[ x = \frac{v_{d1} - \left(v_{d21}/2\right) v_{d2} - \left(v_{d01}/2\right) v_{d0}}{1 - \left(v_{d21}/2\right) - \left(v_{d01}/2\right)} + \epsilon_r \]

(2.54)

where:

\[ \epsilon_r = \frac{-2R_f i_{1f} + R_f i_{2f} + R_f i_{0f}}{2v_{r1} \left(1 - \left(v_{d21}/2\right) - \left(v_{d01}/2\right)\right)} \]

A phase c to ground fault:

The modal condition for a phase c to ground fault is:

\[ v_{1e} - 2v_{2e} + v_{0e} = 0. \]

(2.55)

Substituting for \( v_{1e}, v_{2e}, \) and \( v_{0e} \) from Equations 2.8, 2.10, and 2.11 in the above equation, Equation 2.56 is obtained.
\[ v_{r1} (v_{d1} - x) - R_f i_{1f} - 2[v_{r2} (v_{d2} - x) - \]
\[ R_f i_{2f}] + v_{r0} (v_{d0} - x) - R_f i_{0f} = 0 \tag{2.56} \]

Rearranging this equation, the following equation can be obtained.

\[ x = \frac{v_{d1} - 2(v_{r2}/v_{r1}) v_{d2} + (v_{r0}/v_{r1}) v_{d0}}{1 - 2(v_{r2}/v_{r1}) + (v_{r0}/v_{r1})} \]

\[ - \frac{R_f i_{1f} - 2R_f i_{2f} + R_f i_{0f}}{v_{r1} (1 - 2(v_{r2}/v_{r1}) + (v_{r0}/v_{r1}))} \tag{2.57} \]

If the previously defined identities \( v_{d21} \) for \( v_{r2}/v_{r1} \) and \( v_{d01} \) for \( v_{r0}/v_{r1} \) are substituted in Equation 2.49, the following equation is obtained.

\[ x = \frac{v_{d1} - 2 v_{d21} v_{d2} + v_{d01} v_{d0}}{1 - 2 v_{d21} + v_{d01}} + e_r \tag{2.58} \]

where:

\[ e_r = - \frac{R_f i_{1f} - 2 R_f i_{2f} + R_f i_{0f}}{v_{r1} (1 - 2 v_{d21} + v_{d01})}. \]

Equations 2.50, 2.54 and 2.58 provide the distances of a phase a to ground, phase b to ground, and phase c to ground faults respectively.

A distance relaying criterion can be obtained by combining the above criteria into a single equation as follows.

\[ x = \frac{c_1 v_{d1} + c_2 v_{d2} + c_0 v_{d0}}{c_1 + c_2 + c_0} + e_r \tag{2.59} \]
In the above equation \( e_r \) is the error term in the measurement of the distance due to the fault resistance. The coefficients \( c_1, c_2, \) and \( c_0 \) are fault dependent and can be selected after determining the type of fault. The values of the coefficients, \( c_1, c_2, \) and \( c_0 \), given in Table 2.3 should be used for calculating the distance of a fault. Some of the coefficients of this criterion are required to be calculated in the on-line mode. This factor makes the criterion computationally inefficient.

### Table 2.3: A list of values of coefficients \( c_1, c_2, c_0 \) used in distance relaying criterion [13].

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Criterion given by Equation</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-ph</td>
<td>2.16 ( 1 ) ( 0 ) ( 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.17 ( 0 ) ( 1 ) ( 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.21 ( 1 ) (-v_{d21}) ( 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b-c</td>
<td>2.21 ( 1 ) (-v_{d21}) ( 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a-b</td>
<td>2.16 ( 1 ) ( 0 ) ( 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c-a</td>
<td>2.17 ( 0 ) ( 1 ) ( 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b-c-g</td>
<td>2.21 ( 1 ) (-v_{d21}) ( 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.31 ( 1 ) ( 0 ) (-v_{d01})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.34 ( 0 ) ( 1 ) (-v_{d02})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a-b-g</td>
<td>2.16 ( 1 ) ( 0 ) ( 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.40 ( 0 ) ( 1 ) ( v_{d02})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c-a-g</td>
<td>2.17 ( 0 ) ( 1 ) ( 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.46 ( 1 ) ( 0 ) ( v_{d01})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a-g</td>
<td>2.50 ( 1 ) ( v_{d21}) ( v_{d01})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b-g</td>
<td>2.54 ( 1 ) (-v_{d21}/2) (-v_{d01}/2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c-g</td>
<td>2.58 ( 1 ) (-2v_{d21}) ( v_{d01})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.4 lists the number of divisions required for estimating the distance, \( x \), for the ten shunt faults. In this table, the number of divisions required for calculating voltage ratios \( (v_{d1}, v_{d2} \) and \( v_{d0}\)), coefficients \( (c_1, c_2 \)
and \( c_0 \) and distance \( x \) for various types of shunt faults are given. This criterion requires minimum of one division and maximum of six divisions. Divisions usually put considerable computational burden on the digital processor.

**Table 2.4:** A list of the number of divisions required for estimating the distance \( x \).

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Criterion given by Equation</th>
<th>Number of Divisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-ph</td>
<td>2.16</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2.17</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2.21</td>
<td>4</td>
</tr>
<tr>
<td>b-c</td>
<td>2.21</td>
<td>4</td>
</tr>
<tr>
<td>a-b</td>
<td>2.16</td>
<td>1</td>
</tr>
<tr>
<td>c-a</td>
<td>2.17</td>
<td>1</td>
</tr>
<tr>
<td>b-c-g</td>
<td>2.21</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2.31</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2.34</td>
<td>4</td>
</tr>
<tr>
<td>a-b-g</td>
<td>2.16</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2.40</td>
<td>4</td>
</tr>
<tr>
<td>c-a-g</td>
<td>2.17</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2.46</td>
<td>4</td>
</tr>
<tr>
<td>a-g</td>
<td>2.50</td>
<td>6</td>
</tr>
<tr>
<td>b-g</td>
<td>2.54</td>
<td>6</td>
</tr>
<tr>
<td>c-g</td>
<td>2.58</td>
<td>6</td>
</tr>
</tbody>
</table>

It is desirable that on-line computations of the digital processor should be kept to the minimum. This aspect should be examined and an attempt should be made to improve the distance relaying criterion.
2.3. Summary

The modal transformation offers computational advantage over the symmetrical and the Clarke’s transformations. The application of modal transformation for deriving the distance relaying criterion is reviewed in this chapter. The number of divisions required to evaluate the distance of faults is identified. An attempt is made in the next chapter to improve the distance relaying criterion.
3. DERIVATION OF THE IMPROVED DISTANCE RELAYING CRITERION

3.1. Introduction

The application of modal transformation for distance relaying was described in the previous chapter. The distance relaying criterion reviewed in that chapter requires that some coefficients be calculated in the on-line mode. This factor makes the criterion computationally inefficient. This aspect was examined and an improved criterion was developed. The improved criterion is presented in this chapter.

3.2. The Improved Distance Relaying Criterion

Consider the power system used in Chapter 2 for deriving the previously suggested distance relaying criterion. The single line diagram of the power system and its modal networks are shown in Figures 2.1 and 2.2. As described in that chapter, the modal voltages, $v_{1e}$, $v_{2e}$ and $v_{0e}$, at the fault are functions of the modal voltages at bus P and the modal currents in the line and the fault. The modal voltages at the fault are defined by Equations 2.1, 2.2 and 2.3. These equations are used in this section for deriving the improved distance relaying criterion.

Substituting $v_{r1}$ for $R_1 i_1 + L_1 (d/dt)i_1$ in Equation 2.1, $v_{1e}$ is defined as

$$v_{1e} = v_1 - x v_{r1} - R_f i_{1f}.$$  \hspace{1cm} (3.1)

Substituting $v_{r2}$ for $R_1 i_2 + L_1 (d/dt)i_2$ in Equation 2.2, $v_{2e}$ is expressed as
\[ v_{2e} = v_2 - x v_{r2} - R_f i_{2f}. \]  \hspace{1cm} (3.2)

Substituting \( v_{r0} \) for \( R_0 i_0 + L_0 (d/dt)i_0 \) in Equations 2.3, \( v_{0e} \) is expressed as

\[ v_{0e} = v_0 - x v_{r0} - R_f i_{0f}. \]  \hspace{1cm} (3.3)

Equations 3.1, 3.2 and 3.3, and the modal conditions of Table 2.2 can be used for developing the improved distance relaying criterion. The procedure used for deriving the improved criterion for the ten types of shunt faults is described in this section.

(i) A three phase fault:

The modal conditions for a three phase fault are given by Equations 2.12, 2.13 and 2.14. Using the condition described by Equation 2.12, Equation 3.1 becomes

\[ v_1 - x v_{r1} - R_f i_{1f} = 0. \]  \hspace{1cm} (3.4)

Re-arranging this equation, the following equation is obtained.

\[ x = \frac{v_1}{v_{r1}} - \frac{R_f i_{1f}}{v_{r1}} \]  \hspace{1cm} (3.5)

The above equation can be rewritten as:

\[ x = \frac{v_1}{v_{r1}} + e_r. \]  \hspace{1cm} (3.6)

where:

\[ e_r = \frac{-R_f i_{1f}}{v_{r1}}. \]

Similarly, the following equation is obtained by using Equation 3.2 and Equation 2.13.
\[ x = \frac{v_2}{v_{r2}} + e_r \]  

(3.7)

where:

\[ e_r = \frac{-R_f i_{1f}}{v_{r2}}. \]

Since \( v_{1e} \) and \( v_{2e} \) are both zero for a three phase fault, they are also equal. This procedure provided Equation 2.18. Therefore, the right hand sides of Equations 3.1 and 3.2 can be equated to provide the following equation.

\[ v_1 - x v_{r1} - R_f i_{1f} = v_2 - x v_{r2} - R_f i_{2f} \]  

(3.8)

Rearranging this equation, Equation 3.9 is derived.

\[ x = \frac{v_1 - v_2}{v_{r1} - v_{r2}} + e_r \]  

(3.9)

where:

\[ e_r = \frac{-R_f i_{1f} + R_f i_{2f}}{v_{r1} - v_{r2}}. \]

Equation 3.3 cannot be used to determine \( x \) because mode-0 currents and voltages are zero in the system. Equation 3.6, 3.7 or 3.9 can be used to determine the distance of the fault from the relay location during a three phase fault.

(ii) Phase to phase faults:

The modal conditions for the phase to phase faults on a transmission line are listed in Table 2.2. The improved criterion for calculating the distance of a two-phase fault is derived in this section.
A phase b-c fault:

Equation 2.22 provides the modal condition for a phase b-c fault. When the condition described by Equation 2.22 was used in the case of a three phase fault, it provided Equation 3.9. Therefore, this equation is also valid for calculating the distance of a phase b-c fault.

A phase a-b fault:

Equation 2.23 provides the modal condition for a phase a-b fault. When the condition described by Equation 2.23 was used in the case of a three phase fault, it provided Equation 3.6. Therefore, this equation is also valid for calculating the distance of a phase a-b fault.

A phase c-a fault:

Equation 2.24 provides the modal condition for a phase c-a fault. When the condition described by Equation 2.24 was used in the case of a three phase fault, it provided Equation 3.7. Therefore, this equation is also valid for calculating the distance of a phase c-a fault.

(iii) Two-phase to ground faults:

The modal conditions for two-phase to ground faults on a transmission line are listed in Table 2.2. The improved criterion for calculating the distance of two-phase to ground faults is derived in this section.

A phase b-c to ground fault:

Equations 2.26, 2.27 and 2.28 describe the modal conditions for a phase b-c to ground fault.

When the condition described by Equation 2.26 was used in the case of a three phase fault, it provided Equation 3.9. Therefore, this equation is also valid for calculating the distance of a phase b-c to ground fault.

The condition described by Equation 2.27 states that $v_{1c}$ is equal to $v_{0c}$. Therefore, the right hand sides of Equations 3.1 and 3.3 can be equated. This procedure gives the following equation.
\[ v_1 - z\ v_{r1} - R_f \ i_{1f} = v_0 - z\ v_{r0} - R_f \ i_{0f} \]  \hspace{1cm} (3.10)

Rearranging this equation, Equation 3.11 is derived.

\[ z = \frac{v_1 - v_0}{v_{r1} - v_{r0}} + e_r \]  \hspace{1cm} (3.11)

where:

\[ e_r = \frac{-R_f \ i_{1f} + R_f \ i_{0f}}{v_{r1} - v_{r0}}. \]

The condition described by Equation 2.28 states that \( v_{2e} \) is equal to \( v_{0e} \). Therefore, the right hand sides of Equations 3.2 and 3.3 can be equated. This procedure gives the following equation.

\[ v_2 - z\ v_{r2} - R_f \ i_{2f} = v_0 - z\ v_{r0} - R_f \ i_{0f} \]  \hspace{1cm} (3.12)

Rearranging this equation, provides Equation 3.13.

\[ z = \frac{v_2 - v_0}{v_{r2} - v_{r0}} + e_r \]  \hspace{1cm} (3.13)

where:

\[ e_r = \frac{-R_f \ i_{2f} + R_f \ i_{0f}}{v_{r2} - v_{r0}}. \]

Equations 3.9, 3.11 and 3.13 provide an estimate of the distance of a phase b-c to ground fault.

**A phase a-b to ground fault:**

The modal conditions for this fault are given by Equations 2.35 and 2.36. When the condition described by Equation 2.35 was used in the case of a three phase fault, it provided Equation 3.6. Therefore, this equation is also valid for calculating the distance of a phase a-b to ground fault.
The condition described by Equation 2.36 states that \( v_{2e} \) is equal to \(- v_{0e} \). Therefore, the sum of the right hand sides of Equations 3.2 and 3.3 can be equated to zero. This procedure provides the following equation.

\[
v_2 - x v_{r2} - R_f i_{2f} + v_0 - x v_{r0} - R_f i_{0f} = 0 \tag{3.14}
\]

Rearranging this equation, Equation 3.15 is obtained.

\[
x = \frac{v_2 + v_0}{v_{r2} + v_{r0}} + e_r \tag{3.15}
\]

where:

\[
e_r = -\frac{R_f i_{2f} + R_f i_{0f}}{v_{r2} + v_{r0}}.
\]

Equations 3.6 and 3.15 provide an estimate of the distance of a phase a-b to ground fault.

A phase c-a to ground fault:

The modal conditions for this fault are given by Equations 2.41 and 2.42. When the condition described by Equation 2.41 was used in the case of a three phase fault, it provided Equation 3.7. Therefore, this equation also provides the distance of a phase b-c to ground fault.

The condition described by Equation 2.42 states that \( v_{1e} \) is equal to \(- v_{0e} \). Therefore, the sum of the right hand sides of Equations 3.1 and 3.3 can be equated to zero. This procedure provides the following equation.

\[
v_1 - x v_{r1} - R_f i_{1f} + v_0 - x v_{r0} - R_f i_{0f} = 0 \tag{3.16}
\]

Rearranging this equation, Equation 3.17 is derived.
\[ x = \frac{v_1 + v_0}{v_{r1} + v_{r0}} + e_r \]  \hspace{1cm} (3.17)

where:

\[ e_r = - \frac{R_f i_1 f + R_f i_0 f}{v_{r1} + v_{r0}}. \]

Equations 3.7 and 3.17 provide distance of a phase c-a to ground fault.

Equations 3.9, 3.11, and 3.13 provide the distance of a phase b-c to ground fault. Equations 3.6 and 3.15 give the distance of a phase a-b to ground fault. Equations 3.7 and 3.17 describe the distance of a phase c-a to ground fault.

(iv) Single phase to ground faults:

The modal conditions for single phase to ground faults on a transmission line have been listed in Table 2.2. The improved criterion for calculating distance of single phase to ground faults is derived in this section.

A phase a to ground fault:

The modal condition for phase a to ground fault is given by Equation 2.47. This equation states that

\[ v_{1e} + v_{2e} + v_{0e} = 0. \]  \hspace{1cm} (3.18)

Substituting for \( v_{1e} \), \( v_{2e} \) and \( v_{0e} \) from Equations 3.1, 3.2 and 3.3 in the above equation, the following equation is obtained.

\[ v_1 - x v_{r1} - R_f i_1 f + v_2 - x v_{r2} - R_f i_2 f + v_0 - x v_{r0} - R_f i_0 f = 0 \]  \hspace{1cm} (3.19)

Rearranging this equation, provides
where:

\[ e_r = - \frac{R_f i_{1f} + R_f i_{2f} + R_f i_{0f}}{v_{r1} + v_{r2} + v_{r0}}. \]

**A phase b to ground fault:**

The modal condition for phase b to ground fault is given by Equation 2.51. This equation states that

\[ -2v_{1e} + v_{2e} + v_{0e} = 0. \]  

(3.21)

Substituting for \( v_{1e}, v_{2e} \) and \( v_{0e} \) from Equations 3.1, 3.2, and 3.3 in the above equation, the following equation is obtained.

\[ -2(v_1 - z v_{r1} - R_f i_{1f}) + v_2 - z v_{r2} - R_f i_{2f} + v_0 - z v_{r0} - R_f i_{0f} = 0 \]

(3.22)

Rearranging this equation, gives

\[ x = \frac{-2v_1 + v_2 + v_0}{-2v_{r1} + v_{r2} + v_{r0}} + e_r \]

(3.23)

where:

\[ e_r = - \frac{-2 R_f i_{1f} + R_f i_{2f} + R_f i_{0f}}{-2 v_{r1} + v_{r2} + v_{r0}}. \]
A phase c to ground fault:

The modal condition for phase c to ground fault is given by Equation 2.55. This equation states that

\[ v_{1e} - 2v_{2e} + v_{0e} = 0. \]  \hspace{1cm} (3.24)

Substituting for \( v_{1e}, v_{2e}, \) and \( v_{0e} \) from Equations 3.1, 3.2, and 3.3 in the above equation, the following equation is obtained.

\[
v_1 - xv_{r1} - R_f i_{1f} - 2(v_2 - xv_{r2} - R_f i_{2f}) + v_0 - xv_{r0} - R_f i_{0f} = 0 \]  \hspace{1cm} (3.25)

Rearranging this equation, provides the following equation.

\[
x = \frac{v_1 - 2v_2 + v_0}{v_{r1} - 2v_{r2} + v_{r0}} + e_r \] \hspace{1cm} (3.26)

where:

\[
e_r = -\frac{R_f i_{1f} - 2R_f i_{2f} + R_f i_{0f}}{v_{r1} - 2v_{r2} + v_{r0}}.\]

Equation 3.20 provides the distance of phase a to ground fault. Equation 3.23 gives the distance of a phase b to ground fault. Equation 3.26 describes the distance of a phase c to ground fault.

The following equation is valid for all types of shunt faults if suitable values are assigned to the coefficients \( a_1, a_2 \) and \( a_0 \) depending upon the type of the fault.
\[ x = \frac{a_1 v_1 + a_2 v_2 + a_0 v_0}{a_1 v_{r1} + a_2 v_{r2} + a_0 v_{r0}} + e_r \]  

The term \( e_r \) is the error in the estimate of the distance due to the resistance of the fault. Table 3.1 gives a list of the values that the coefficients should be assigned for each shunt fault. The coefficients \( a_1, a_2 \) and \( a_0 \) are either 0, ±1, or ±2 and are not required to be calculated in the on-line mode. Compared to this, the coefficients \( c_1, c_2 \) and \( c_0 \) used in the previous chapter are required to be calculated in the on-line mode for five types of shunt faults.

The number of divisions required for evaluating the criterion of Equation 3.27 is listed in Table 3.2. The improved criterion requires only one division for each shunt fault, whereas the previously suggested criterion required one to six divisions depending on the type of fault. Since divisions take considerable computation time of the digital processor, the proposed criterion has an advantage over the criterion described in the previous chapter.

### 3.3. Assigning Values to \( a_1, a_2 \) and \( a_0 \)

The improved criterion for calculating the distance of a shunt fault from the relay location has been developed in the previous section. This criterion requires that a fault be classified and an appropriate value be assigned to the coefficients. The shunt faults on a transmission line can be classified by considering the phase currents in the pre and post fault states [6]. Sachdev and Baribeau used phase currents for identifying the type of a shunt fault [15]. Later, Sachdev and Kolla calculated the product of the phase currents and the positive sequence impedance of the protected line, and then applied a similar criterion for identifying the fault type [13]. The same procedure is used in this work for selecting values of the coefficients \( a_1, a_2, \) and \( a_0 \). The procedure is briefly reviewed.
Table 3.1: A list of values of coefficients used in the improved distance relaying criterion.

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Criterion given by Equation</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-ph</td>
<td>3.6</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3.7</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3.9</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>b-c</td>
<td>3.9</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>a-b</td>
<td>3.6</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c-a</td>
<td>3.7</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>b-c-g</td>
<td>3.9</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3.11</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>3.13</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>a-b-g</td>
<td>3.6</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3.15</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>c-a-g</td>
<td>3.7</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3.17</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>a-g</td>
<td>3.20</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>b-g</td>
<td>3.23</td>
<td>-2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>c-g</td>
<td>3.26</td>
<td>1</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 3.2: A list of number of divisions required for evaluating an improved distance relaying criterion.

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Criterion given by Equation</th>
<th>Number of Divisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-ph</td>
<td>3.6</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3.7</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3.9</td>
<td>1</td>
</tr>
<tr>
<td>b-c</td>
<td>3.9</td>
<td>1</td>
</tr>
<tr>
<td>a-b</td>
<td>3.6</td>
<td>1</td>
</tr>
<tr>
<td>c-a</td>
<td>3.7</td>
<td>1</td>
</tr>
<tr>
<td>b-c-g</td>
<td>3.9</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3.11</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3.13</td>
<td>1</td>
</tr>
<tr>
<td>a-b-g</td>
<td>3.6</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3.15</td>
<td>1</td>
</tr>
<tr>
<td>c-a-g</td>
<td>3.7</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3.17</td>
<td>1</td>
</tr>
<tr>
<td>a-g</td>
<td>3.20</td>
<td>1</td>
</tr>
<tr>
<td>b-g</td>
<td>3.23</td>
<td>1</td>
</tr>
<tr>
<td>c-g</td>
<td>3.26</td>
<td>1</td>
</tr>
</tbody>
</table>
Let $v_{ra}$, $v_{rb}$, and $v_{rc}$ define the products of phase a, b and c currents and the positive sequence impedance of the protected line. The voltages $\Delta v_{ra}$, $\Delta v_{rb}$ and $\Delta v_{rc}$ are calculated by subtracting the pre-fault values of $v_{ra}$, $v_{rb}$ and $v_{rc}$ from that of post-fault values and if these voltages exceed a threshold value, $\epsilon$, it is concluded that system is experiencing a fault. The phase current conditions of Table 2.1 are applied to the calculated voltages for identifying the type of shunt fault and selecting appropriate values of the coefficients. Figure 3.1 shows the flow chart that can be used for selecting the coefficients. The procedure is as follows.

1. If $|\Delta v_{ra}|$ is greater than the threshold value, proceed to the next step; otherwise go to step 7.

2. If $|\Delta v_{rb}|$ is greater than the threshold value, proceed to the next step; otherwise go to step 4.

3. The system is experiencing a three-phase, a phase a-b or a phase a-b to ground fault. Assign the following values to the coefficients.

   $$a_1 = 1$$
   $$a_2 = 0$$
   $$a_0 = 0$$

Note that a fault is a three-phase fault if $|\Delta v_{rc}|$ is greater than the threshold value. Block 31 in the flow chart is redundant because the criterion 1 for a three-phase fault, criterion for a phase a-b fault and criterion 1 for a phase a-b to ground fault are the same.

4. If $|\Delta v_{re}|$ is greater than the threshold value, continue to the next step; otherwise proceed to step 6.

5. The system is experiencing a phase c-a fault or phase c-a to ground fault. Assign the following values to the coefficients.

   $$a_1 = 0$$
   $$a_2 = 1$$
   $$a_0 = 0$$
Figure 3.1: Flow chart for selection of coefficients $a_1$, $a_2$, and $a_0$. 
6. The system is experiencing a phase a to ground fault. Assign the following values to the coefficients.

\[ a_1 = 1 \]
\[ a_2 = 1 \]
\[ a_0 = 1 \]

7. If \(|\Delta v_{rb}|\) is greater than the threshold value, proceed to the next step; otherwise go to step 11.

8. If \(|\Delta v_{rc}|\) is greater than the threshold value, continue to the next step; otherwise proceed to step 10.

9. The system is experiencing a phase b-c fault or a phase b-c to ground fault. Assign the following values to the coefficients.

\[ a_1 = 1 \]
\[ a_2 = -1 \]
\[ a_0 = 0 \]

10. The system is experiencing a phase b to ground fault. Assign the following values to the coefficients.

\[ a_1 = -2 \]
\[ a_2 = 1 \]
\[ a_0 = 1 \]

11. If \(|\Delta v_{rc}|\) is greater than the threshold value, continue to the next step; otherwise proceed to step 13.

12. The system is experiencing a phase c to ground fault. Assign the following values to the coefficients.

\[ a_1 = 1 \]
\[ a_2 = -2 \]
\[ a_0 = 1 \]

13. Concludes that the system is not experiencing a fault.
3.4. Summary

The improved criterion for estimating the distance of a shunt fault from the relay location has been developed using the modal transformation. The number of divisions required for evaluating the improved distance relaying criterion is identified. A procedure for selecting the appropriate values of the coefficients, $a_1$, $a_2$ and $a_0$, for different types of faults has been described. The coefficients take a value of 0, ±1 or ±2 depending on the fault type and are not required to be calculated in the on-line mode. The improved criterion requires only one division for estimating the distance of a shunt fault and is computationally efficient compared to the previously suggested criterion.
4. DESIGN OF A DIGITAL DISTANCE RELAY

4.1. Introduction

The improved distance relaying criterion has been described in the previous chapter. This criterion was used for developing software for a digital distance relay. The software was written in the assembly language of a TMS-32010 digital processor. The developed software was tested using a simulator on a VAX 11/780 digital computer that was available at the University of Saskatchewan. To facilitate testing of the relay software, the input signals were generated and low-pass filters and analog to digital converter were simulated on the digital computer. This chapter describes the major functional blocks of a digital distance relay. Also presented are the procedures for generating input signals and the compensated voltages. The procedures for simulating low-pass filters and analog to digital converters are then discussed. Finally, the functions of the modules used for developing the relay software are described.

4.2. Functional Blocks of a Digital Distance Relay

Figure 4.1 shows the major functional blocks of a typical digital distance relay. The analog input subsystem receives low level signals from potential transformers (pt’s) and current transformers (ct’s) while it protects the relay from transients. The levels of signals received from pt’s and ct’s are further reduced to avoid saturation of the analog to digital converters. The inputs are band-limited by low-pass filters to avoid aliasing. The analog input subsystem converts input currents into equivalent voltages because analog to digital converters accept voltages only.
Figure 4.1: Block diagram of a typical digital distance relay [16].
The outputs of the analog input subsystem are applied to the analog interface subsystem. This subsystem contains hardware for sample and hold, analog to digital conversion and multiplexing. The outputs of the analog input subsystem are sampled at a selected rate and are held as voltages across capacitors. The voltages are applied to an analog to digital converter that digitizes the sampled voltages.

The digitized values are stored in Random Access Memory (RAM). The relay software, that resides in Read Only Memory (ROM), uses the digitized values for analysing and making decisions. The central processing unit (CPU) performs arithmetic and logical operations. The software determines whether the fault is in the protected zone. If a fault is within the zone of protection, the relay issues a command to trip the circuit breaker.

The digital input subsystem conveys information concerning the status of the line circuit breaker and switches to the relay. The digital output subsystem conveys the decisions of the relay to the power system. Power supply to the digital relay is provided by a battery and a battery charger. This provision is made to insure that uninterrupted power supply is available to the relay.

4.3. Auxiliary Software

In this project, the relay software was developed using the improved distance relaying criterion described in the previous chapter. The relay software modules were written in the assembly language of the TMS-32010. The developed software was debugged and tested using a simulator on the VAX 11/780 digital computer. To facilitate testing of the software, auxiliary software was developed. The auxiliary and the relay software use the input signals generated by the simulation program that uses the Electro-Magnetic Transient Program (EMTP) [17]. The procedures for generating the input signals and the compensated voltages are briefly described in this section. Also described are the procedures for simulating low-pass filters and analog
to digital converters. The functions of the modules and the major blocks of the flow chart used for writing the relay software are discussed in the next section.

4.3.1. The generation of input signals

The input signals are needed for evaluating the performance of the relay. The EMTP was used to generate the required signals. A six bus representation of the Saskatchewan Power Corporation (SPC) system was modelled. A single line diagram of the SPC system is shown in Figure D.1 of Appendix D. Faults were applied on the P2C line and waveforms sampled at 23040 Hz were obtained. Appendix D lists the parameters of the SPC system. Appendix E briefly describes the important features of the EMTP. Three-phase, two-phase, two-phase to ground and single phase to ground faults were simulated at different locations on the P2C line. This line is 174.4 km long and it connects the Poplar River (PR) and Condie (CON) substations of the SPC system. The voltages and currents at the PR terminal of the line were recorded in data files.

4.3.2. Generating the compensated voltages

The improved distance relaying criterion, developed in the previous chapter, uses modal components of compensated voltages. These are voltage drops across the modal impedances of the protected transmission line. They can be generated using replica impedances in a digital relay [2, 8]. As mentioned in Chapter 2, the mode-1 and mode-0 impedances of the transmission line can be used for this purpose. If the phase currents are passed through the mode-1 impedances of the protected transmission line, they provide compensated voltages. These voltages can be used for obtaining mode-1 and mode-2 components of the compensated voltages. If the zero sequence current is passed through mode-0 impedance, it provides the mode-0 component of the compensated voltages.
The mode-1, mode-2 and mode-0 components of compensated voltages, \( v_{r1}, v_{r2}, \) and \( v_{r0} \), are as follows.

\[
v_{r1}(t) = \frac{1}{3} \left\{ R_1 i_a(t) + L_1 (d/dt)i_a(t) \right. \\
- \left. [R_1 i_b(t) + L_1 (d/dt)i_b(t)] \right\} \tag{4.1}
\]

\[
v_{r2}(t) = \frac{1}{3} \left\{ R_1 i_a(t) + L_1 (d/dt)i_a(t) \right. \\
- \left. [R_1 i_c(t) + L_1 (d/dt)i_c(t)] \right\} \tag{4.2}
\]

\[
v_{r0}(t) = \frac{1}{3} \left\{ R_0 [i_a(t) + i_b(t) + i_c(t)] \right. \\
+ L_0 [(d/dt)i_a(t) + (d/dt)i_b(t)] \\
+ (d/dt)i_c(t) \right\} \tag{4.3}
\]

where:

- \( R_1 \) and \( L_1 \) are the mode-1 resistance and inductance of the transmission line,
- \( R_0 \) and \( L_0 \) are the mode-0 resistance and inductance of the transmission line and
- \( i_a, i_b \) and \( i_c \) are instantaneous values of the phase currents in the transmission line.

The differential operator in these equations can be replaced by the central difference equation [18]. The first derivative of \( i_a \), \( (d/dt)i_a \), can be estimated from three consecutive samples of \( i_a \) [19]. If three samples of a current \( i_a \) are taken at \((k-1)\Delta T, k\Delta T\) and \((k+1)\Delta T\) seconds, the estimate of \( (d/dt)i_a \) at time \( k\Delta T \) is
\[
\frac{d}{dt}i_a(k) = \frac{i_a(k+1) - i_a(k-1)}{2 \Delta T} \quad (4.4)
\]

The derivatives of \(i_b\) and \(i_c\) can be obtained using a similar procedure.

### 4.3.3. Simulation of low-pass filters

Low-pass filters are used in digital distance relays to band limit the input voltages and currents. The cut-off frequency of the low-pass filter should be less than one half of the sampling frequency so that the effect of aliasing is minimized [20, 21]. Since a sampling frequency of 720 Hz was used for the relay, 300 Hz was chosen as the cut-off frequency of the low-pass filter. A fourth-order low-pass filter was designed for this purpose. Appendix B reviews the procedure for designing the fourth-order low-pass analog filter and transforming it to an equivalent digital filter using the bilinear transformation method [20]. This procedure provided the following transfer function:

\[
H(z) = \frac{5.4712 \times 10^{-5}(1+4z^{-1}+6z^{-2}+4z^{-3}+z^{-4})}{1-3.31196z^{-1}+4.1134z^{-2}-2.270572z^{-3}+0.470003z^{-4}} \quad (4.5)
\]

Since \(H(z)\) is \(Y(z)/X(z)\), Equation 4.5 can be rearranged as:

\[
Y(z) \{1 - 3.31196z^{-1} + 4.1134z^{-2} - 2.270572z^{-3} + 0.470003z^{-4}\} = 5.4712 \times 10^{-5} \{1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4}\} X(z). \quad (4.6)
\]

This equation rewritten in terms of the time delay of \(n\) sampling intervals is
This equation was programmed on the VAX 11/780 digital computer to simulate low-pass filters. The input phase voltages and compensated voltages were processed by filters. The output of the filters were then applied to the analog to digital converter.

4.3.4. Simulation of analog to digital converters

An analog to digital converter converts an instantaneous voltage to an equivalent number. Raw data is first processed by the low-pass filter. The filtered data is then sampled, scaled and applied to the analog to digital converter. The procedure for simulating analog to digital converters on the digital computer is reviewed in this section.

The levels of input signals applied to analog to digital converters are up to ±10 V. An analog to digital converter of (b+1) bits can be modelled as follows [22].

\[
(Q)_{10}^0 = \text{INT or RON} \left\{ \frac{P (2^b - 1)}{Y} \right\} \tag{4.8}
\]

\[
(Q)_{10}^0 = \text{INT or RON} \left\{ \frac{(2^b - |P|) 2^b}{Y} \right\} \tag{4.9}
\]

where:

\[-Y \text{ to } +Y\] is the range of the input signals,
is input voltage applied to an analog to digital converter,

\( (Q)_{10} \) is digitized value to the base 10,

\( INT \) represents the truncation of the digitized value and

\( RON \) represents the rounding of the digitized value.

Equation 4.8 represents digitized value of an input of +P volts. Equation 4.9 represents digitized value of an input of -P volts in two's complements.

4.4. The Relay Software

This section describes the software used in the design of the digital distance relay. The software was written in the assembly language of the TMS-32010 digital processor. The software consists of the following modules:

- Module-A : for detecting the onset of faults,
- Module-B : for computing modal components,
- Module-C : for selecting coefficients,
- Module-D : for estimating fault distance,
- Module-E : for making decisions.

The module A receives digitized samples of three phase voltages and compensated voltages. It then compares the latest samples of the compensated phase voltages with the corresponding samples taken one cycle earlier and checks the difference against a predetermined threshold for determining the onset of the fault.

The module B calculates the modal components of the sampled voltages. The module C implements the logic of the flow chart illustrated in Figure 3.1 for selecting coefficients. This logic identifies the fault type and assigns appropriate values to the coefficients \( a_1, a_2 \) and \( a_0 \).
The module D estimates the distance to the fault. It uses Equation 3.27 for this purpose. The module E compares the estimated distance with the relay setting for determining whether the fault is in the zone of protection or not. This module uses two counters (Tripc and Notripc) for arriving at a decision to issue a trip command or not. If the estimated distance is less than the relay setting, the counter Tripc is incremented by one and the counter Notripc is decremented by one. When the counter Tripc reaches a set threshold, a decision to trip is taken. If the estimated distance is greater than the relay setting, the counter Notripc is incremented by one and the counter Tripc is decremented by one.

The flow chart of the developed relay software is illustrated in Figure 4.2. The details of this flow chart are explained below:

1. Receive and store digitized values of phase voltages, compensated voltages.
2. Set counter for out-of-zone fault (Notripc) and counter for in-zone fault (Tripc) to zero.
3. Calculate the changes in the compensated phase voltages by subtracting the values obtained one cycle earlier from the present values.
4. If the changes in the compensated phase voltages exceed a threshold, proceed to the step 6; otherwise continue to the next step.
5. Get the next set of samples and go to step 3.
6. The system is experiencing a fault. Compute modal components of the sampled voltages.
7. Identify the fault type and select the coefficients $a_1$, $a_2$, and $a_0$ using the flow chart illustrated in Figure 3.1.
8. Calculate the apparent fault distance using Equation 3.27.
9. If the estimated fault distance is less than the setting of the distance relay, proceed to step 16; otherwise continue to the next step.
Figure 4.2: Flow chart of the developed relay software.
10. Increment the counter Notripc by one.

11. If the counter Tripc is greater than zero, decrement it by one.

12. If the fault is estimated to be outside the protected zone of the relay for twelve consecutive samples (i.e., if the counter Notripc is equal to 12), continue to the next step; otherwise proceed to step 15.

13. Get the next set of samples.

14. Reset the counters, Tripc and Notripc, to zero and go to step 3.

15. Get the next set of samples and go to step 5.

16. Increment the counter Tripc by one.

17. If the counter Notripc is greater than zero, decrement it by one.

18. If the counter Tripc is equal to a threshold, proceed to step 20; otherwise continue to the next step.

19. Get the next set of samples and revert to step 5.

20. Issue a tripping signal to the line circuit breaker.

4.5. Summary

Functional blocks of a digital distance relay have been described in this chapter. The procedure for generating input signals and compensated voltages has been discussed. Also described are procedures for simulating low-pass filters and analog to digital converter. The modules used for writing the relay software are described. The functional blocks of the flow chart of the developed software have been explained.
5. THE SIMULATION STUDIES

5.1. Introduction

The procedure for generating input signals and compensated voltages and simulating low-pass filters and analog to digital converters were described in the previous chapter. That chapter also described the functions of the modules used for developing the relay software. The performance of the software was evaluated by conducting the simulation studies on the VAX 11/780 digital computer. This chapter describes the procedure used for performing the simulation studies. Some of the results of the studies are also included in this chapter.

5.2. Testing of the Developed Software

The performance of the developed software was evaluated using transient voltages and currents that might be expected in the power systems. The simulation program, EMTP, was used for this purpose. This program uses a six bus representation of the Saskatchewan Power Corporation (SPC) system and provides the instantaneous values of voltages and currents at the relay location. These values were recorded in data files. The recorded data were used for testing the relay software. This section discusses the procedure used for evaluating the performance of the low-pass filters, the improved distance relaying criterion and the relay software.
5.2.1. Testing of low-pass filters

The procedure for simulating the low-pass filters was described in the previous chapter. This filter was tested using the fault voltages and currents generated using the EMTP. One of the voltage waveforms that was used for testing the low-pass filter is shown in Figure 5.1. The waveform contains high frequency components. This voltage was processed by the low-pass filter. Figure 5.2 shows the output of the low-pass filter. It can be observed from Figures 5.1 and 5.2 that the high frequency components have been attenuated. Figure 5.3 shows the input and the output of the low-pass filter on a common time base. This figure shows that the filter delay is of the order of one millisecond.

5.2.2. Testing the improved distance relaying criterion

The improved criterion for estimating the distance to the fault was developed in Chapter 3. The criterion was tested using the transient voltages and currents that might be expected in the power system. Line P2C of a six bus representation of the SPC system was selected for simulation studies. This line is 174.4 km long. The proposed relay was assumed to have been provided at the bus 2 end of the P2C line. Five different locations were chosen for simulating the shunt faults on the line. The onset of the faults were simulated at 0.017 seconds in the EMTP, and fault data were obtained. The instantaneous values of voltages and currents obtained from the EMTP are referred to as fault data in this chapter. Fault data were processed by the models of the low-pass filters. The output of the low-pass filters were then converted to samples taken at 720 Hz and 1440 Hz. The sampled values were used for calculating the distance to the fault.

The results of the simulation studies for shunt faults at 87.2 km (0.5 p.u.) of the P2C line are reported in this section. The results of the simulation studies performed for the shunt faults at 43.6 km (0.25 p.u.), 130.8 km (0.75 p.u.) and 174.4 km (1.0 p.u.) away from the relay location and 17.4 km (0.10 p.u.) behind the relay are included in Appendix F.
Figure 5.1: The voltage waveform applied to the low-pass filter.

Figure 5.2: Output voltage of the low-pass filter.
Figure 5.3: The input and the output voltage waveforms of the low-pass filter on a common time base.

(i) A three phase fault

As mentioned earlier, a six bus representation of the SPC system was modelled in the EMTP for simulating a three phase fault. Figure 5.4 shows the instantaneous values of voltages and currents at the relay location for a fault 87.2 km from the relay. Data obtained at the relay location were processed by models of low-pass filters. The output of the filters were recorded in a data file. The recorded data were then sampled at 720 Hz. The sampled values were used for calculating the distance to the fault using the procedure proposed in this thesis. Figure 5.5(a) shows the calculated distance to the fault as a function of time. An examination of this figure reveals that the calculated values of distance to the fault are in the neighbourhood of 0.5 p.u. The mean and the standard deviation of the calculated
values of the distance to the fault are 0.5006 p.u. and 0.0299 p.u. respectively.

The recorded data were also sampled at 1440 Hz, and the criterion proposed in this thesis was used for calculating the distance to the fault. Figure 5.5(b) shows the calculated distance to the fault as a function of time. An examination of this figure also reveals that estimates of the calculated distance to the fault are in the neighbourhood of 0.5 p.u. The mean and the standard deviation of the calculated values of the distance to the fault are 0.4941 p.u. and 0.0625 p.u. respectively.

(ii) A phase a-b fault

The procedure described for the three phase fault was also used for simulating a phase a-b fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure 5.6 shows the instantaneous values of voltages and currents at the relay location. Figure 5.7(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5011 p.u. and 0.0293 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure 5.7(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.4945 p.u. and 0.0619 p.u. respectively.

(iii) A phase b-c fault

The procedure described for the three phase fault was also used for simulating a phase b-c fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure 5.8 shows the instantaneous values of voltages and currents at the relay location. Figure 5.9(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5024 p.u. and 0.0087 p.u. respectively.
Figure 5.4: The instantaneous values of (a) three phase voltages and (b) three phase currents for a three phase fault at 87.2 km of the P2C line.
Figure 5.5: The calculated distance as a function of time for a three phase fault condition of Figure 5.4: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
Figure 5.6: The instantaneous values of (a) three phase voltages and (b) three phase currents for a phase a-b fault at 87.2 km of the P2C line.
Figure 5.7: The calculated distance as a function of time for a phase a-b fault condition of Figure 5.6: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure 5.9(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5024 p.u. and 0.0063 p.u. respectively.

(iv) A phase c-a fault

The procedure described for the three phase fault was also used for simulating a phase c-a fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure 5.10 shows the instantaneous values of voltages and currents at the relay location. Figure 5.11(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5038 p.u. and 0.0196 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure 5.11(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5080 p.u. and 0.0538 p.u. respectively.

(v) A phase a-b to ground fault

The procedure described for the three phase fault was also used for simulating a phase a-b to ground fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure 5.12 shows the instantaneous values of voltages and currents at the relay location. Figure 5.13(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5006 p.u. and 0.0294 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure 5.13(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.4941 p.u. and 0.0619 p.u. respectively.
Figure 5.8: The instantaneous values of (a) three phase voltages and (b) three phase currents for a phase b-c fault at 87.2 km of the P2C line.
Figure 5.9: The calculated distance as a function of time for a phase b-c fault condition of Figure 5.8: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
Figure 5.10: The instantaneous values of (a) three phase voltages and (b) three phase currents for a phase c-a fault at 87.2 km of the P2C line.
Figure 5.11: The calculated distance as a function of time for a phase c-a fault condition of Figure 5.10: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
(vi) A phase b-c to ground fault

The procedure described for the three phase fault was also used for simulating a phase b-c to ground fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure 5.14 shows the instantaneous values of voltages and currents at the relay location. Figure 5.15(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5022 p.u. and 0.0085 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure 5.15(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5026 p.u. and 0.0065 p.u. respectively.

(vii) A phase c-a to ground fault

The procedure described for the three phase fault was also used for simulating a phase c-a to ground fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure 5.16 shows the instantaneous values of voltages and currents at the relay location. Figure 5.17(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5035 p.u. and 0.0204 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure 5.17(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5019 p.u. and 0.0192 p.u. respectively.

(viii) A phase a to ground fault

The procedure described for the three phase fault was also used for simulating a phase a to ground fault. The samples were taken at 720 Hz
Figure 5.12: The instantaneous values of (a) three phase voltages and (b) three phase currents for a phase a-b to ground fault at 87.2 km of the P2C line.
Figure 5.13: The calculated distance as a function of time for a phase a-b to ground fault condition of Figure 5.12: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
Figure 5.14: The instantaneous values of (a) three phase voltages and (b) three phase currents for a phase b-c to ground fault at 87.2 km of the P2C line.
Figure 5.15: The calculated distance as a function of time for a phase b-c to ground fault condition of Figure 5.14: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
Figure 5.16: The instantaneous values of (a) three phase voltages and (b) three phase currents for a phase $c$-a to ground fault at 87.2 km of the P2C line.
Figure 5.17: The calculated distance as a function of time for a phase c-a to ground fault condition of Figure 5.16: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
and were used for calculating the distance to the fault. Figure 5.18 shows the instantaneous values of voltages and currents at the relay location. Figure 5.19(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.4897 p.u. and 0.0331 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure 5.19(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.4929 p.u. and 0.0325 p.u. respectively.

(ix) A phase b to ground fault

The procedure described for the three phase fault was also used for simulating a phase b to ground fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure 5.20 shows the instantaneous values of voltages and currents at the relay location. Figure 5.21(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5090 p.u. and 0.0787 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure 5.21(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.5035 p.u. and 0.0582 p.u. respectively.

(x) A phase c to ground fault

The procedure described for the three phase fault was also used for simulating a phase c to ground fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure 5.22 shows the instantaneous values of voltages and currents at the relay location. Figure 5.23(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.4942 p.u. and 0.0102 p.u. respectively.
Figure 5.18: The instantaneous values of (a) three phase voltages and (b) three phase currents for a phase a to ground fault at 87.2 km of the P2C line.
Figure 5.19: The calculated distance as a function of time for a phase a to ground fault condition of Figure 5.18: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
Figure 5.20: The instantaneous values of (a) three phase voltages and (b) three phase currents for a phase b to ground fault at 87.2 km of the P2C line.
Figure 5.21: The calculated distance as a function of time for a phase b to ground fault condition of Figure 5.20: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure 5.23(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.4939 p.u. and 0.0086 p.u. respectively.

(xi) Remarks

The test results of the simulation studies reported in this section indicate that the estimates of the calculated distance to the fault are in the neighbourhood of 0.5 p.u. A list of the mean and the standard deviation of the estimated distances to the fault is given in Tables 5.1 and 5.2. The results of the ten cases presented in this section have shown that satisfactory estimates of the distance to the fault can be obtained using the criterion proposed in the thesis.

When the sampling frequency of 1440 Hz was used, more information in the form of distance estimates is provided to the decision-making module of the relay. As a result a faster decision can be arrived at. This aspect is discussed in the next section.

5.2.3. Testing of the decision making module of the relay software

The results of the simulation studies reported in the previous section have indicated that the estimates of the calculated distance of the fault are in the neighbourhood of 0.5 p.u. The module-E of the relay software uses these estimates for making decisions.

As mentioned in the previous chapter, Module-E of the relay software uses a counter, Tripc, to decide whether to issue a command to trip the line circuit breakers or not. Consider that the initial value of Tripc is zero. If the estimated distance to the fault is less than the reach of the relay (0.8 p.u.), Tripc is incremented by one. The value of Tripc is compared with a threshold. If the value of Tripc is equal to the threshold, a trip command
Figure 5.22: The instantaneous values of (a) three phase voltages and (b) three phase currents for a phase c to ground fault at 87.2 km of the P2C line.
Figure 5.23: The calculated distance as a function of time for a phase c to ground fault condition of Figure 5.22: (a) using a sampling frequency of 720 Hz and (b) using a sampling frequency of 1440 Hz.
Table 5.1: A list of the mean and the standard deviation of the estimated fault distances. A sampling frequency of 720 Hz was used in the design.

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-ph</td>
<td>0.5006</td>
<td>0.0299</td>
</tr>
<tr>
<td>a-b</td>
<td>0.5011</td>
<td>0.0293</td>
</tr>
<tr>
<td>b-c</td>
<td>0.5024</td>
<td>0.0087</td>
</tr>
<tr>
<td>c-a</td>
<td>0.5038</td>
<td>0.0196</td>
</tr>
<tr>
<td>a-b-g</td>
<td>0.5006</td>
<td>0.0294</td>
</tr>
<tr>
<td>b-c-g</td>
<td>0.5022</td>
<td>0.0085</td>
</tr>
<tr>
<td>c-a-g</td>
<td>0.5035</td>
<td>0.0204</td>
</tr>
<tr>
<td>a-g</td>
<td>0.4897</td>
<td>0.0331</td>
</tr>
<tr>
<td>b-g</td>
<td>0.5090</td>
<td>0.0787</td>
</tr>
<tr>
<td>c-g</td>
<td>0.4942</td>
<td>0.0102</td>
</tr>
</tbody>
</table>
Table 5.2: A list of the mean and the standard deviation of the estimated fault distances. A sampling frequency of 1440 Hz was used in the design.

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-ph</td>
<td>0.4941</td>
<td>0.0625</td>
</tr>
<tr>
<td>a-b</td>
<td>0.4945</td>
<td>0.0619</td>
</tr>
<tr>
<td>b-c</td>
<td>0.5024</td>
<td>0.0063</td>
</tr>
<tr>
<td>c-a</td>
<td>0.5080</td>
<td>0.0538</td>
</tr>
<tr>
<td>a-b-g</td>
<td>0.4941</td>
<td>0.0619</td>
</tr>
<tr>
<td>b-c-g</td>
<td>0.5026</td>
<td>0.0065</td>
</tr>
<tr>
<td>c-a-g</td>
<td>0.5019</td>
<td>0.0192</td>
</tr>
<tr>
<td>a-g</td>
<td>0.4929</td>
<td>0.0325</td>
</tr>
<tr>
<td>b-g</td>
<td>0.5035</td>
<td>0.0582</td>
</tr>
<tr>
<td>c-g</td>
<td>0.4939</td>
<td>0.0086</td>
</tr>
</tbody>
</table>
is issued. If the estimated distance to a fault is beyond the reach of the relay and Tripc is greater than zero, Tripc is decremented by one. This process is repeated for every time a new estimate of the distance to the fault is calculated. Once Tripc reaches the threshold, it is frozen at that value. A threshold of six to twelve can be chosen so that security of the system is not affected. A threshold of nine was used for the work reported in the thesis.

The decision-making module of the relay software was tested for five different locations of shunt faults. The test results for shunt faults at 87.2 km of the P2C line are reported this section. The results for shunt faults at other locations are reported in Appendix F.

(i) A three phase fault

The decision-making module of the relay software was tested for the ten types of shunt faults. One case for a three phase fault is reported in this section. Figure 5.24(a) shows the profile of the trip counter when a sampling frequency of 720 Hz was used. An examination of this figure reveals that the trip counter is incremented up to the threshold, because the simulated fault is in the zone of the relay. The relay issues a command to trip the line circuit breakers in 13.6 ms after the inception of the fault. The operating time of the relay is less than one cycle.

Figure 5.24(b) shows the profile of the trip counter when a sampling frequency of 1440 Hz was used. As the estimated distances to the fault are less than the reach of the relay, the trip counter is incremented up to the threshold. The relay issues a command to trip the line circuit breakers in 8.0 ms after the inception of the fault. The operating time of the relay is less than one half of a cycle. An examination of Figures 5.24(a) and (b) shows that a faster decision can be arrived at when the sampling frequency of 1440 Hz is used.
Figure 5.24: The profile of the trip counter for shunt faults at 87.2 km of the P2C line: (a) when a sampling frequency of 720 Hz was used and (b) when a sampling frequency of 1440 Hz was used.
5.3. Summary and Discussion

The simulation studies used for evaluating the performance of the relay have been reported in this chapter. A six bus representation of SPC power system was used in the simulation studies. The EMTP software was used to obtain fault data for various fault conditions. Fault data were then used for estimating the distance to the fault. The decision making module of the relay software was tested and the profile of the trip counter was studied. The simulation studies demonstrated that the relay software performed satisfactorily.

The results of the simulation studies reported in this chapter have shown that the improved criterion provides satisfactory estimates of the distance to the fault. The profiles of the trip counter show that the decision making module of the relay software performed satisfactorily. When the sampling frequency of 720 Hz was used and the simulated fault was in the zone of the relay, the relay operating time is 13.6 ms after the inception of the fault. Whereas the relay operating time is 8.0 ms for a similar fault condition and the sampling frequency of 1440 Hz. The studies reported in this chapter shows that a faster decision can be arrived at when the sampling frequency of 1440 Hz is used.
6. SUMMARY AND CONCLUSIONS

6.1. Summary and Conclusions

A power system experiences an occurrence of a fault generally caused by insulation failure. A fault can lead to unstable operation and can cause extensive damage to equipment in a power system. It can also result in loss of revenue. To minimise these effects, the equipment of a power system must be protected adequately. A protection system is used for this purpose. The first chapter of the thesis has described the philosophy of power system protection, distance protection and developments in digital distance relaying. That chapter has also described the major objectives of the research work and the outline of the thesis. The major objectives of the research work reported in the thesis were to design and simulate a high speed digital distance relay for the protection of transmission lines.

Many approaches have been suggested in the past for developing distance relaying criteria that can be implemented using digital processors. The previously suggested distance relaying criterion that used modal transformations has been reviewed in Chapter 2. Some of the coefficients of the criterion are required to be calculated in the on-line mode. This factor makes the criterion computationally inefficient.

The improved criterion has been proposed and described in Chapter 3. The coefficients of the proposed criterion are constants. Also, the proposed criterion requires only one division for estimating the distance to a shunt fault. Compared to this, the previously suggested criterion required up to six divisions.
The criterion proposed in the third chapter was used for designing a high speed digital distance relay. Chapter 4 outlined the procedure used for designing the distance relay. The functional blocks of the digital distance relay have been described. A procedure for simulating low-pass filters and analog to digital converters has been reviewed. The organization of the relay software is also discussed. The relay software was written in the assembly language of the TMS-32010 digital processor.

The fifth chapter has discussed the simulation procedure used for testing the proposed criterion. A six bus representation of the SPC transmission system was modelled in the EMTP and fault data were generated. The model of low-pass filters were used for attenuating the high frequency components present in fault data. The filtered data were sampled and digitized using the simulation program for analog to digital converters. The output of analog to digital converters was used for testing the proposed criterion. Test results of the simulation studies have also been reported in the fifth chapter.

The studies presented in this thesis show that the proposed criterion has computational advantage over the previously suggested criterion. The test results of the simulation studies demonstrate that the proposed criterion can determine an apparent distance to the shunt faults satisfactorily. The developed relay software has also provided satisfactory results. Therefore, the proposed criterion can be used for designing a high speed digital distance relay for the protection of transmission lines. The relay operating time of the order of three-quarters of a cycle can be achieved by using 720 Hz sampling frequency. Faster relay operating times, of the order of one-half cycle, can be achieved by increasing the sampling frequency to 1440 Hz. However, the higher sampling frequency would also result in additional overhead on the data acquisition system and memory storage.
6.2. Suggestions for Future Work

The work reported in the thesis can be further extended for designing microprocessor-based relays for transmission line protection. Microprocessor-based relays use algorithms such as the Least Error Square, Kalman filter or Fourier transformation for estimating voltage and current phasors, and criteria for making decisions. It is possible that a chosen relay algorithm may provide incorrect decisions in a particular situation. Therefore, it would be desirable to use two algorithms in a relay design. The decisions of the two algorithms can be provided to the relay logic which can issue a command for opening line circuit breakers if at least one out of two algorithms yields a trip decision. This scheme will increase the reliability of the protection system but decrease its security.

If higher security is desired, the relay logic could be changed to issue trip command if both algorithms lead to a trip decision. Alternatively, more than two algorithms can be used in the relay design; their outputs can be provided to a voting logic for making final decisions.

The relay should make decisions in shorter times when a fault is close to the relay location. Also, the calculated impedances should be more accurate if the fault is near the end of the protection zone. These features can be provided by incorporating in the relay logic for switching between a short window algorithm and a long window algorithm.

In a real system, the design should also include instantaneous overcurrent, directional and multi-zone distance protection functions. These functions can be implemented using different algorithms. Also, relay hardware can be designed. The developed software and designed hardware can then be tested on the development system. If results are satisfactory, a prototype of relay can be built and tested. Finally, field tests can be carried out for proving the suitability of the design.
REFERENCES


A. MODAL TRANSFORMATION

The modal transformation is one of the techniques that can be used for analysing power systems experiencing faults. This transformation can be used for expressing the phase voltages in terms of modal voltages [13]. The following equations define the relationship between phase voltages and modal voltages.

\[
\begin{align*}
v_a &= v_0 + v_1 + v_2 \\
v_b &= v_0 - 2v_1 + v_2 \\
v_c &= v_0 + v_1 - 2v_2
\end{align*}
\] (A.1, A.2, A.3)

where:

- \(v_0, v_1, v_2\) are modal voltages at the relay location and
- \(v_a, v_b, v_c\) are phase voltages at the relay location.

From Equations A.1, A.2 and A.3 the following equations that define modal voltages in terms of phase voltages can be obtained.

\[
\begin{align*}
v_0 &= \frac{1}{3} (v_a + v_b + v_c) \\
v_1 &= \frac{1}{3} (v_a - v_b) \\
v_2 &= \frac{1}{3} (v_a - v_c)
\end{align*}
\] (A.4, A.5, A.6)

Similar equations define the relationship between the phase currents and modal currents. The elements of the modal transformation are real numbers [23]. Figure A.1, reproduced from reference [13], represents modal network of a sample power system.
Figure A.1: A sample power system and its modal network [13].
If the three phase system is considered to be balanced, the application of the modal transformation shows that:

1. the mode-0, mode-1, and mode-2 impedances are identically equal to zero, positive and negative sequence impedances respectively.

2. the currents of a mode cause voltage drops of that mode only.

The modal transformation can be applied to the instantaneous values because the elements of this transformation are real numbers. There is no phase shifting involved. Further, modal transformation offers computational advantage over symmetrical component method and Clark’s component method [13].
B. DESIGN OF LOW-PASS FILTERS

Low-pass filters are used in a digital distance relay to band limit the input voltages and currents. The cut-off frequency of the low-pass filter should be less than one half of the sampling frequency so that effect of aliasing is minimized [20, 21]. Since a sampling frequency of 720 Hz was used for the relay, the input signals are required to be band-limited to frequencies below 360 Hz. A fourth-order low-pass filter was designed for this purpose [12]. A cut-off frequency of 300 Hz was used in the design. The procedure for designing a fourth-order low-pass filter is briefly reviewed in this section. A fourth-order filter can be obtained by cascading four first order filters. The transfer function of a fourth-order filter is given by the following equation.

\[
H(s) = \left[ \frac{k}{s + k} \right]^4
\]  

(B.1)

where:

\[ k \]

is a constant.

To satisfy output of -3 db at cut-off frequency, substitute \( j\omega_c \) for \( s \) in Equation B.1 and equate it to \( 1/\sqrt{2} \). This procedure provides

\[
\left| \frac{k}{j\omega_c + k} \right|^4 = \frac{1}{\sqrt{2}}.
\]

(B.2)

From Equation B.2, following equation is obtained.
The selected cut-off frequency 300 Hz is equivalent to \( \omega_c = 600\pi \) radians per second. Putting this value of \( \omega_c \) into the Equation B.3, the value of \( k \) becomes 4333. Substituting the value of \( k \) into Equation B.1, the transfer function of the analog low-pass filter is given by the following equation.

\[
H(s) = \left[ \frac{4333}{s + 4333} \right]^4
\]  

(B.4)

The group delay introduced due to the filter is defined as

\[
T_d = -\frac{d\phi(\omega)}{d\omega}
\]  

(B.5)

where:

\( \phi \) is the argument of the transfer function.

The argument of the transfer function of the filter is

\[
\phi = -4 \tan^{-1}\left(\frac{\omega}{4333}\right).
\]  

(B.6)

Substituting Equation B.6 into Equation B.5 and performing differentiation, the following equation is obtained.

\[
T_d = \frac{4}{4333} \frac{1}{1 + (\omega/4333)^2}
\]  

(B.7)

The group delays of the designed filter are 0.9161 ms and 0.7762 ms at 60 Hz and 300 Hz respectively.

In a distance relay, low-pass filters are implemented using analog devices. In this project, low-pass filters were simulated on the VAX 11/780
digital computer. The analog filter can be transformed into an equivalent digital filter using the bilinear transformation method [20]. The procedure is briefly explained below.

The cut-off frequency of the analog filter is prewarped in terms of the cut-off frequency of the digital filter using following equation.

\[
\omega_c = \frac{2}{\Delta T} \tan \left\{ \frac{\Omega_c \Delta T}{2} \right\}
\]

(B.8)

where:

\(\Delta T\) is the inter-sampling time and

\(\Omega_c\) is the cut-off frequency of the digital filter.

The inter-sampling time was selected as 4.34 E-5 seconds. The cut-off frequency of the analog filter can be obtained by the following equation.

\[
\omega_c = \left[ \frac{2}{4.34 \times 10^{-5}} \right] \tan \left\{ \frac{600 \pi \times 4.34 \times 10^{-5}}{2} \right\}.
\]

\[= 1886.0 \text{ r/s}.\]

Substituting this value of \(\omega_c\) into Equation B.3, the value of \(k\) becomes 4336. The transfer function of the analog filter for the modified value of \(k\) can be rewritten as

\[
H(s) = \frac{4336}{s + 4336^4}
\]

(B.9)

Now \(s\) can be replaced by following equation.
\[ s = \frac{2}{\Delta T} \frac{1 - z^{-1}}{1 + z^{-1}} \]  

\[ = \frac{2}{4.34 \times 10^{-5}} \frac{1 - z^{-1}}{1 + z^{-1}} \]  

\[ = \frac{46080}{1 + z^{-1}} \]  

Substituting Equation B.12 into Equation B.9, the following equation is obtained.

\[ H(Z) = \left[ \frac{4336}{46080 \left[ (1 - z^{-1})/(1 + z^{-1}) \right] + 4336} \right]^4 \]  

(B.13)

The above equation can be rearranged as

\[ H(Z) = \left[ \frac{4336 (1 + z^{-1})}{46080 (1 - z^{-1}) + 4336 (1 + z^{-1})} \right]^4 \]  

(B.14)

This equation can be rewritten as follows.

\[ H(z) = \frac{5.4712 \times 10^{-5} (1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})}{1 - 3.31196z^{-1} + 4.1134z^{-2} - 2.270572z^{-3} + 0.470003z^{-4}} \]  

(B.15)

The frequency response of the fourth order filter is shown in Figure B.1.

Since \[ H(z) = \frac{Y(z)}{X(z)} \]  
Equation B.15 can be rewritten as:
**Figure B.1:** Frequency response of the fourth order low-pass filter.

\[
\frac{Y(z)}{X(z)} = \frac{5.4712 \times 10^{-5} (1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})}{1 - 3.31196z^{-1} + 4.1134z^{-2} - 2.270572z^{-3} + 0.470003z^{-4}} \quad (B.16)
\]

Rearranging this equation, the following equation can be obtained.

\[
Y(z)(1 - 3.31196z^{-1} + 4.1134z^{-2} - 2.270572z^{-3} + 0.470003z^{-4}) = 5.4712 \times 10^{-5} (1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})X(z).
\]  

\[ (B.17) \]

Equation B.17 can be rewritten in terms of the time delay of \( n \) sampling intervals as:
\[ y(n) = 5.4712 \times 10^{-5} (x(n) + 4x(n-1) + 6x(n-2) + 4x(n-3) \\
+ x(n-4)) + 3.31196 \ y(n-1) - 4.1134 \ y(n-2) \\
+ 2.270572 \ y(n-3) - 0.470003 \ y(n-4). \] \tag{B.18}

Equation B.18 was used in Chapter 4 to simulate low-pass filters on the digital computer.
C. AN OVERVIEW OF TMS32010 MICROCOMPUTER

The TMS32010 is 16/32-bit single-chip microcomputer [24]. It has high speed controller and an array processor. The TMS320 family utilises a modified Harvard architecture for speed and flexibility. Figure C.1 shows the block diagram of the TMS32010 digital signal processor (DSP), reproduced from the TMS320 User's Manual. The TMS32010 contains a hardware multiplier to perform a multiplication in a single 200-ns cycle.

The TMS32010 microprocessor combines the following elements onto a single chip [24]:

- Volatile 144 x 16-word read/write data memory.
- Double-precision 32-bit ALU/accumulator.
- Fast 200-ns multiplier.
- Shifter that shifts the accumulator into the data RAM.
- 16-bit data bus for fetching instruction words from off-chip at full speed.
- 4 x 12-bit stack.
- Autoincrementing/decrementing registers for indirect data addressing and loop counting.
- Single interrupt.

There are four basic arithmetic elements: the ALU, the accumulator, the multiplier, and the shifters. All arithmetic operations are performed using two's complement arithmetic.
Figure C.1: Block schematic of the TMS32010 [24].
Program memory consists of up to 4 x 1024 words of 16-bit width. Program memory mode of operation is controlled by the $MC/\overline{MP}$ pin. There are two modes of operation: microcomputer mode and the microprocessor mode. In microprocessor mode all 4 x 1024 words of memory are off-chip.

There are two 16-bit auxiliary registers. These auxiliary registers can be used for three functions: temporary storage, indirect addressing of data memory, and loop control. The program counter (PC) is a 12-bit register. The PC contains the program memory address of the next instruction to be executed. The stack is 12-bit wide and four layers deep. The status register consists of five status bits: accumulator Overflow (OV), Overflow Mode (OVM), Interrupt Mask (INTM), Auxiliary Register Pointer (ARP), Data memory page Pointer (DP).

Input and output data to and from a peripheral is accomplished by the IN and OUT instructions. IN and OUT instructions take two cycles to execute. The BIO pin is used for monitoring peripheral device status. It is an alternative to using an interrupt when we do not want to disturb time-critical loops.

The TMS32010's instruction set supports arithmetic operations, logical operations, data movements, unconditional and conditional branching, calling subroutines, and control instructions. The most of the instructions of the TMS32010 instruction set are single-cycle single word. Only branch and I/O instructions are multicycle. The hardware multiplier of the TMS32010 executes the MPY instruction in a single cycle. The TMS32010 uses three addressing modes: direct, indirect and immediate addressing.
D. ELECTRICAL PARAMETERS OF THE SPC SYSTEM

For testing of the digital distance relay a six bus representation of the Saskatchewan Power Corporation (SPC) transmission system [13] was used in Chapter 5. Figure D.1 depicts a single line diagram of the SPC system. This system was used for testing of the distance relaying scheme in Reference [13]. The same system was used for simulation studies in the Chapter 5. The system description of the SPC system is reproduced here from Reference [12]. There are three generating stations and six transmission lines in the SPC system. Out of these three generators, two, Poplar River (PR) and Boundary Dam (BD), act as 230 kV voltage sources. The third generator, the Queen Elizabeth (QE) station, acts as 138 kV source. Transformer and generator impedances are represented by an equivalent impedance. Loads are placed at the three generating stations and the Condie (CON), Regina (RG) and Wolverine (WOL) substations. Base capacity of 100 MVA and base voltage of 230 kV were used in this work.

Table D.1 lists transmission line data and Table D.2 provides generator data. Data for loads is listed in Table D.3. The pre-fault load flow data is given in Table D.4.
Figure D.1: Single line diagram of the SPC power system [13].
Table D.1: Transmission line data [13].

<table>
<thead>
<tr>
<th>Line Name</th>
<th>Length</th>
<th>Positive sequence Impedances (p.u.)</th>
<th>Zero sequence Impedance (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line Name</td>
<td>R</td>
<td>X</td>
<td>B</td>
</tr>
<tr>
<td>B2R</td>
<td>182.8</td>
<td>0.0210</td>
<td>0.1214</td>
</tr>
<tr>
<td>B3R</td>
<td>182.8</td>
<td>0.0210</td>
<td>0.1214</td>
</tr>
<tr>
<td>C1W</td>
<td>158.5</td>
<td>0.0172</td>
<td>0.1428</td>
</tr>
<tr>
<td>P2C</td>
<td>174.4</td>
<td>0.0151</td>
<td>0.1134</td>
</tr>
<tr>
<td>R4C</td>
<td>21.40</td>
<td>0.0023</td>
<td>0.0181</td>
</tr>
<tr>
<td>Q1W</td>
<td>104.6</td>
<td>0.0913</td>
<td>0.2651</td>
</tr>
</tbody>
</table>

Table D.2: Generator data [13].

<table>
<thead>
<tr>
<th>Gen. No.</th>
<th>Connected Bus Name</th>
<th>Positive sequence Impedances (p.u.)</th>
<th>Zero sequence Impedance (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>R</td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>BD</td>
<td>0.000603</td>
<td>0.037343</td>
</tr>
<tr>
<td>2</td>
<td>PR</td>
<td>0.000897</td>
<td>0.054236</td>
</tr>
<tr>
<td>3</td>
<td>QE</td>
<td>0.002160</td>
<td>0.096514</td>
</tr>
</tbody>
</table>

Table D.3: Load data [13].

<table>
<thead>
<tr>
<th>Load No.</th>
<th>Connected Bus Name</th>
<th>Real Power P (p.u.)</th>
<th>Reactive Power Q (p.u.)</th>
<th>Equivalent Impedance (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BD</td>
<td>0.798</td>
<td>0.177</td>
<td>1.3158+j0.2919</td>
</tr>
<tr>
<td>2</td>
<td>PR</td>
<td>0.250</td>
<td>0.100</td>
<td>3.7996+j1.5198</td>
</tr>
<tr>
<td>3</td>
<td>QE</td>
<td>2.709</td>
<td>0.680</td>
<td>0.3549+j0.0891</td>
</tr>
<tr>
<td>4</td>
<td>CON</td>
<td>0.608</td>
<td>0.0545</td>
<td>1.6859+j0.1511</td>
</tr>
<tr>
<td>5</td>
<td>WOL</td>
<td>0.419</td>
<td>0.075</td>
<td>2.4277+j0.4346</td>
</tr>
<tr>
<td>6</td>
<td>RG</td>
<td>2.925</td>
<td>0.735</td>
<td>0.3280+j0.0824</td>
</tr>
</tbody>
</table>
Table D.4: Pre-fault load flow data [13].

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Bus Name</th>
<th>Load (p.u.)</th>
<th>Generation (p.u.)</th>
<th>Voltage (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BD</td>
<td>0.798+j0.177</td>
<td>2.784+j0.129</td>
<td>1.0496+j0.00</td>
</tr>
<tr>
<td>2</td>
<td>PR</td>
<td>0.250+j0.100</td>
<td>2.910+j0.184</td>
<td>1.0275+j0.2148</td>
</tr>
<tr>
<td>3</td>
<td>QE</td>
<td>2.709+j0.680</td>
<td>2.200+j0.775</td>
<td>0.9476-j0.3524</td>
</tr>
<tr>
<td>4</td>
<td>CON</td>
<td>0.608+j0.0545</td>
<td>0.000+j0.000</td>
<td>1.0136-j0.0767</td>
</tr>
<tr>
<td>5</td>
<td>WOL</td>
<td>0.419+j0.075</td>
<td>0.000+j0.000</td>
<td>1.002-j0.2142</td>
</tr>
<tr>
<td>6</td>
<td>RG</td>
<td>2.925+j0.735</td>
<td>0.000+j0.000</td>
<td>1.0038-j0.1103</td>
</tr>
</tbody>
</table>
E. ELECTRO-MAGNETIC TRANSIENT PROGRAM

The Electro Magnetic Transient Program (EMTP) [17] is documented by the Bonneville Power Administration, Portland, Oregon. This program is widely used in power utilities and research institutions for carrying out simulation studies [25]. Power system networks can be modelled to represent practical systems using the EMTP. There are a large number of preprogrammed models available in the EMTP for this purpose. These models can be used for simulating steady state and transient state phenomena in power networks.

The power networks can be modelled using voltage or current sources, multiphase equivalent circuits, distributed parameter circuits, lumped parameter circuits, and switches. The mathematical equations can then be used for representing the specified power system model. There are various subroutines in the EMTP that solve the mathematical equations and provide solutions to these models. Another feature of this program is that it can handle very large power system networks. The mathematical equations and algorithms used in the program are considered to be very accurate.

The models used in the EMTP are reasonably accurate and proven with practical systems [25]. Therefore, there are many users of this program all over the world. The EMTP has been used by the Power System Research Group, University of Saskatchewan for generating fault data and carrying out simulation studies.

Reference 13 used the EMTP for obtaining fault data. That reference
also reported the test results using fault data generated by EMTP. The procedure described in Reference 13 was followed for using the EMTP in this project. The same procedure is briefly described in this appendix. A six bus representation of the SPC transmission system, reviewed in the last appendix, was modelled in the EMTP.

The generators of the SPC system were modelled as voltage sources. The impedances of the generators and transformers were represented by equivalent impedances. The transmission lines were modelled using multi-phase distributed parameters. The equivalent impedances of the transmission line models and loads were represented in the EMTP. The switches provided in the EMTP were used for simulating various shunt faults on the transmission line models.

The inter-sampling time of 4.34 E-5 was used for obtaining the output of the EMTP. The instantaneous values of three phase voltages and three phase currents at the above specified inter-sampling time were stored in a data file. The developed relay software was tested using the fault data recorded in files. The fifth chapter has reported the test results of the studies.
F. TESTING OF THE DEVELOPED RELAY SOFTWARE

F.1. Introduction

The results of the simulation studies performed in Chapter 5 have provided satisfactory estimates of the distance to the fault at 87.2 km of the P2C line using the improved criterion. The criterion was also tested for various types of shunt faults at four different locations. One case for each different location is reported in this appendix. The test results in the case of a three-phase fault at 43.6 km, a two-phase at 130.8 km and a two-phase to ground fault at 174.4 km of the P2C line are presented. For determining the performance of the relay for the faults behind the relay location, it was assumed that the relay was provided at bus 5 end of the Q1W line. The faults were applied at 17.4 km behind the relay. The test results for a single phase to ground fault behind the relay are also reported in this appendix. In all the cases the onset of faults were simulated at 0.017 seconds in the EMTP.

(i) A three phase fault at 43.6 km of the P2C line

The procedure described in Chapter 5 was also used for simulating a three phase fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure F.1 shows the instantaneous values of voltages and currents at the relay location. Figure F.2(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.2505 p.u. and 0.0201 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for cal-
culating the distance to the fault. Figure F.2(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.2476 p.u. and 0.0244 p.u. respectively.

(ii) A two phase fault at 130.8 km of the P2C line

The procedure described in Chapter 5 was also used for simulating a three phase fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure F.3 shows the instantaneous values of voltages and currents at the relay location. Figure F.4(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.7588 p.u. and 0.0128 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure F.4(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 0.7599 p.u. and 0.0116 p.u. respectively.

(iii) A two phase to ground fault at 174.4 km of the P2C line

The procedure described in Chapter 5 was also used for simulating a three phase fault. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure F.5 shows the instantaneous values of voltages and currents at the relay location. Figure F.6(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 1.0159 p.u. and 0.0211 p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure F.6(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are 1.0240 p.u. and 0.0244 p.u. respectively. If the relay setting is chosen as 0.8 p.u., the estimated fault distances are greater than the setting.
Figure F.1: The instantaneous values of (a) three phase voltages and (b) three phase currents for a three phase fault at 43.6 km of the P2C line.
Figure F.2: The calculated distance to a three phase fault condition of Figure F.1: (a) when a sampling frequency of 720 Hz was used and (b) when a sampling frequency of 1440 Hz was used.
Figure F.3: The instantaneous values of (a) three phase voltages and (b) three phase currents for a two phase fault at 130.8 km of the P2C line.
Figure F.4: The calculated distance to a two phase fault condition of Figure F.3: (a) when a sampling frequency of 720 Hz was used and (b) when a sampling frequency of 1440 Hz was used.
(iv) A single phase to ground fault behind the relay

The procedure described in Chapter 5 was also used for simulating a single phase to ground fault behind the relay. The samples were taken at 720 Hz and were used for calculating the distance to the fault. Figure F.7 shows the instantaneous values of voltages and currents at the relay location. Figure F.8(a) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are -0.1129 p.u. and 0.0399 p.u. p.u. respectively.

Similarly, the samples were taken at 1440 Hz and were used for calculating the distance to the fault. Figure F.8(b) shows the calculated distance to the fault as a function of time. The mean and the standard deviation of the calculated values of the distance to the fault are -0.1121 p.u. and 0.0425 p.u. respectively. As faults were simulated behind the the relay location, estimated fault distances are negative.

Remarks

The test results reported in this section show that the proposed criterion can determine the distance to the fault. A summary of the mean and the standard deviation of the calculated distance to the fault is illustrated in Table F.1 and F.2.

F.1.1. Testing the decision making module of the relay software

The procedure described in Chapter 5 was used for testing the decision making module of the relay software. The results for shunt faults reported in the previous section are used for evaluating the performance of the logic of the decision making module of the relay software. This section presents the results of these tests.

(ii) Shunt faults at 43.6 km of the P2C line

The procedure described for the shunt faults at 87.2 km of the P2C line in Chapter 5 was used in this case. One case for a three phase fault is reported in this section. Figure F.9(a) shows the profile of the trip counter
Figure F.5: The instantaneous values of (a) three phase voltages and (b) three phase currents for a two phase to ground fault at 174.4 km of the P2C line.
Figure F.6: The calculated distance to a two phase to ground fault condition of Figure F.5: (a) when a sampling frequency of 720 Hz was used and (b) when a sampling frequency of 1440 Hz was used.
Figure F.7: The instantaneous values of (a) three phase voltages and (b) three phase currents for a single phase to ground fault behind relay location.
Figure F.8: The calculated distance to a single phase to ground fault condition of Figure F.7: (a) when a sampling frequency of 720 Hz was used and (b) when a sampling frequency of 1440 Hz was used.
Table F.1: A list of the mean and the standard deviation of the estimated fault distances when the sampling frequency of 720 Hz was used in the design.

<table>
<thead>
<tr>
<th>Fault Simulated at (p.u.)</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.2505</td>
<td>0.0201</td>
</tr>
<tr>
<td>0.75</td>
<td>0.7588</td>
<td>0.0128</td>
</tr>
<tr>
<td>1.00</td>
<td>1.0159</td>
<td>0.0211</td>
</tr>
<tr>
<td>-0.10</td>
<td>-0.1129</td>
<td>0.0399</td>
</tr>
</tbody>
</table>

Table F.2: A list of the mean and the standard deviation of the estimated fault distances when the sampling frequency of 1440 Hz was used in the design.

<table>
<thead>
<tr>
<th>Fault Simulated at (p.u.)</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.2476</td>
<td>0.0244</td>
</tr>
<tr>
<td>0.75</td>
<td>0.7599</td>
<td>0.0116</td>
</tr>
<tr>
<td>1.00</td>
<td>1.0240</td>
<td>0.0244</td>
</tr>
<tr>
<td>-0.10</td>
<td>-0.1121</td>
<td>0.0425</td>
</tr>
</tbody>
</table>

for this fault when the samples taken at 720 Hz were used. As the simulated fault was in the zone of the relay, the value of Tripc is reached the predetermined threshold. Therefore, the relay issues a command to trip a line circuit breaker at 13.6 ms after the inception of fault.
Figure F.9(b) shows the profile of the trip counter for this fault when a sampling frequency of 1440 Hz was used. As the fault was in the zone of the relay, a trip counter reached the predetermined threshold. Therefore, the relay issues a command to trip a line circuit breaker at 8.0 ms after the inception of fault. An examination of Figures F.9(a) and (b) shows that faster decision can be taken when the sampling frequency of 1440 Hz is used.

(iii) Shunt faults at 130.8 km of the P2C line

The procedure described for the shunt faults at 87.2 km of the P2C line in Chapter 5 was used in this case. One case for a two phase fault is reported in this section. Figure F.10(a) shows the profile of the trip counter for this fault when the samples taken at 720 Hz were used. As the simulated fault was in the zone of the relay, the value of Tripc is reached the predetermined threshold. Therefore, the relay issues a command to trip a line circuit breaker at 15.0 ms after the inception of fault.

Figure F.10(b) shows the profile of the trip counter for this fault when a sampling frequency of 1440 Hz was used. As the fault was in the zone of the relay, a trip counter reached the predetermined threshold. Therefore, the relay issues a command to trip a line circuit breaker at 8.7 ms after the inception of fault. An examination of Figures F.9(a) and (b) shows that faster decision can be taken when the sampling frequency of 1440 Hz is used.

(iv) Shunt faults at 174.4 km of the P2C line

The procedure described for the shunt faults at 87.2 km of the P2C line in Chapter 5 was used in this case. The values of the trip counter for this location of shunt faults were studied. One case for a two phase to ground fault is reported in this section. Figure F.11(a) shows the profile of the trip counter for this fault when the samples taken at 720 Hz were used. As the simulated fault was out of the zone of the relay, the value of Tripc did not reach the predetermined threshold. Therefore, the relay does not issue a command to trip a line circuit breaker.

Figure F.11(b) shows the profile of the trip counter for this fault when
Figure F.9: The profile of trip counter for shunt faults at 43.6 km of the P2C line: (a) when a sampling frequency of 720 Hz was used and (b) when a sampling frequency of 1440 Hz was used.
Figure F.10: The profile of the trip counter for shunt faults at 130.8 km of the P2C line: (a) when a sampling frequency of 720 Hz was used and (b) when a sampling frequency of 1440 Hz was used.
a sampling frequency of 1440 Hz was used. As the simulated fault was out of the zone of the relay, the value of Tripc did not reach the predetermined threshold. Therefore, the relay does not issue a command to trip a line circuit breaker.

(v) **Shunt faults behind the relay**

The procedure described for the shunt faults at 87.2 km of the P2C line in Chapter 5 was used in this case. One case for a single phase to ground fault is reported in this section. Figure F.12(a) shows the profile of the trip counter for this fault when the samples taken at 720 Hz were used. As the simulated fault was out of the zone of the relay, the value of Tripc did not reach the predetermined threshold. Therefore, the relay does not issue a command to trip a line circuit breaker.

Figure F.12(b) shows the profile of the trip counter for this fault when a sampling frequency of 1440 Hz was used. As the simulated fault was out of the zone of the relay, the value of Tripc did not reach the predetermined threshold. Therefore, the relay does not issue a command to trip a line circuit breaker.

**F.1.2. Remarks**

The improved criterion was also tested for the shunt faults at four different locations. The results of the simulation studies reported in this appendix have shown that the improved criterion provides satisfactory estimates of the distance to the fault.

The logic of decision making module of the relay software was tested. The profile of the trip counter was also studied and results have shown that relay software performed satisfactorily. For the faults in the zone of the relay, the relay takes 15.0 ms for making decision when the sampling frequency of 720 Hz is used whereas it takes 8.7 ms when the sampling rate of 1440 Hz was used.
Figure F.11: The profile of the trip counter for shunt faults at 174.4 km of the P2C line: (a) when a sampling frequency of 720 Hz was used and (b) when a sampling frequency of 1440 Hz was used.
Figure F.12: The profile of the trip counter for shunt faults behind the relay: (a) when a sampling frequency of 720 Hz was used and (b) when a sampling frequency of 1440 Hz was used.
The test results presented in this appendix have demonstrated that the relay performed satisfactorily. The advantage of using the sampling rate of 1440 Hz is that the more information in the form of fault distance estimates is available to the relay. Therefore, faster decisions can be arrived at compared to the case of 720 Hz sampling rate.

F.2. Summary

The simulation studies used for evaluating the performance of the relay have been reported in this appendix. The simulation studies demonstrated that relay software performed satisfactorily.