Methods of Determining the Parameters Representing the Cross-Magnetizing Effect in Saturated Synchronous Machines

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by

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Abstract

In recent studies of the saturation effect in saturated synchronous machines, it has been recognized that the magnetic coupling between the direct and quadrature axes of these synchronous machines (the cross-magnetizing phenomenon) plays an important role in their analysis using the two-axis frame models. To determine the parameters that represent this cross-magnetizing effect, the saturation characteristic curves in the various intermediate axes of the synchronous machines are needed. If a synchronous machine has both direct- and quadrature-axis excitation windings, its saturation characteristic curves in the various intermediate axes can be measured by exciting the machine from both these two windings simultaneously under open-circuited condition. However, for the industrial synchronous machines, which have no excitation winding in their quadrature axis, there has been till now no technique for measuring these saturation characteristic curves experimentally. Wu, in his Master’s thesis [1], has proposed an “Analytical” method for the determination of these saturation characteristic curves.

In the present thesis, an experimental method for the measurement of the saturation characteristic curves in the various intermediate axes of synchronous machines, which will be called the “Back-to-Back” method, is proposed. Applying this method, the saturation characteristic curves and the parameters representing the cross-magnetizing effect of a synchronous machine are experimentally obtained. By comparing these results with those obtained from Wu’s “Analytical” method, the accuracy and the validity of the assumptions of this analytical method are thus verified. Moreover, the importance of the inclusion of the cross-magnetizing effect in the two-axis frame model is demonstrated by comparing the measured and the calculated active and reactive power/load angle curves of the synchronous machine used in the investigations reported in this thesis.
Acknowledgements

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AT$</td>
<td>ampere-turns.</td>
</tr>
<tr>
<td>$AT_t$</td>
<td>total ampere-turns.</td>
</tr>
<tr>
<td>$AT_d$</td>
<td>d-axis ampere-turns.</td>
</tr>
<tr>
<td>$AT_q$</td>
<td>q-axis ampere-turns.</td>
</tr>
<tr>
<td>$B$</td>
<td>air-gap magnetic flux-density.</td>
</tr>
<tr>
<td>$B_1$</td>
<td>fundamental component of $B$.</td>
</tr>
<tr>
<td>$B_d$</td>
<td>amplitude of the d-axis component of $B_1$.</td>
</tr>
<tr>
<td>$B_q$</td>
<td>amplitude of the q-axis component of $B_1$.</td>
</tr>
<tr>
<td>$E$</td>
<td>electromotive force.</td>
</tr>
<tr>
<td>$E_s$</td>
<td>saturated value of $E$.</td>
</tr>
<tr>
<td>$E_u$</td>
<td>unsaturated value of $E$.</td>
</tr>
<tr>
<td>$E_f$</td>
<td>electromotive force due to the field current.</td>
</tr>
<tr>
<td>$E_{fs}$</td>
<td>saturated value of $E_f$.</td>
</tr>
<tr>
<td>$E_{qd}$</td>
<td>electromotive force due to the change of the flux in the d-axis, $\phi_{dq}$.</td>
</tr>
<tr>
<td>$E_{dq}$</td>
<td>electromotive force due to the change of the flux in the q-axis, $\phi_{qd}$.</td>
</tr>
<tr>
<td>$F$</td>
<td>magnetomotive force.</td>
</tr>
<tr>
<td>$I$</td>
<td>current.</td>
</tr>
<tr>
<td>$I_a$</td>
<td>armature current.</td>
</tr>
<tr>
<td>$I_{ab}$</td>
<td>base value of armature current.</td>
</tr>
<tr>
<td>$I_{ad}$</td>
<td>d-axis component of armature current.</td>
</tr>
<tr>
<td>$I_{aq}$</td>
<td>q-axis component of armature current.</td>
</tr>
<tr>
<td>$I_f$</td>
<td>field current.</td>
</tr>
</tbody>
</table>
\( I_{fb} \) base value of field current.
\( I_{f1} \) field current of the tested synchronous machine.
\( I_{f2} \) field current of the auxiliary synchronous machine.
\( P \) active power.
\( P_1 \) active power of the tested synchronous machine.
\( P_2 \) active power of the auxiliary synchronous machine.
\( P_{12} \) sum of \( P_1 \) and \( P_2 \).
\( Q \) reactive power.
\( R \) resistance.
\( R_a \) armature resistance.
\( S \) saturation factor.
\( S_b \) base value of power.
\( S_d \) d-axis saturation factor.
\( S_q \) q-axis saturation factor.
\( V \) voltage.
\( V_t \) terminal voltage.
\( V_{tb} \) base value of terminal voltage.
\( V_{t1} \) terminal voltage of the tested synchronous machine.
\( V_{t2} \) terminal voltage of the auxiliary synchronous machine.
\( V_{t12} \) vector sum of \( V_{t1} \) and \( V_{t2} \).
\( V_i \) internal voltage.
\( V_{id} \) d-axis component of the internal voltage.
\( V_{iq} \) q-axis component of the internal voltage.
\( X \) reactance.
\( X_l \) armature leakage reactance.
\( X_d \) d-axis synchronous reactance.
\( X_q \) q-axis synchronous reactance.
\( X_{ds} \) saturated value of \( X_d \).
\( X_{qs} \) saturated value of \( X_q \).
\( X_{md} \) d-axis mutual reactance.
$X_{mq}$ q-axis mutual reactance.

$X_{mdu}$ unsaturated value of $X_{md}$.

$X_{mqv}$ unsaturated value of $X_{mq}$.

$X_{mds}$ saturated value of $X_{md}$.

$X_{mqs}$ saturated value of $X_{mq}$.

$Z_b$ base value of impedance.

$\beta$ angle between the axis of the total ampere-turns and the direct axis.

$\beta_1$ angle of the relative position of the tested machine's stator with respect to the auxiliary machine's stator.

$\delta_t$ load angle.

$\delta_i$ internal load angle.

$\delta'$ angle between the axis of the air-gap magnetic flux and the direct-axis of the machine.

$\phi$ flux

$\phi_{dq}$ change of the flux in the d-axis due to the cross-magnetizing effect.

$\phi_{qd}$ change of the flux in the q-axis due to the cross-magnetizing effect.

$\theta$ the displacement angle from a point in the air gap to the pole axis.

$\tau$ half of the width of the pole arc.

$\mu$ equivalent permeability
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c.m.e.</td>
<td>cross-magnetizing effect</td>
</tr>
<tr>
<td>e.m.f.</td>
<td>electromotive force</td>
</tr>
<tr>
<td>e.m.fs.</td>
<td>electromotive forces</td>
</tr>
<tr>
<td>Eq.</td>
<td>Equation</td>
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<td>Figs.</td>
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</tr>
<tr>
<td>m.m.f.</td>
<td>magnetomotive force</td>
</tr>
<tr>
<td>m.m.fs.</td>
<td>magnetomotive forces</td>
</tr>
<tr>
<td>o.c.c.c.</td>
<td>open-circuit characteristic curve</td>
</tr>
<tr>
<td>o.c.c.cs.</td>
<td>open-circuit characteristic curves</td>
</tr>
<tr>
<td>p.u.</td>
<td>per unit</td>
</tr>
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</table>
Chapter 1

Introduction

The synchronous machine, as the source of electricity, is a very important part in the power system. In order to ensure the stability, continuity and efficiency of the power system, it is a fundamental requirement to accurately know the performance of the synchronous machines when they are connected in the power system. To achieve this, more accurate models of the synchronous machines have to be used in the power system analysis.

Among the many factors affecting the performance of a synchronous machine, saturation phenomenon is the one of the most importance. At the same time, it is almost the hardest one to handle, so it has attracted a lot of interests and concerns [1-18]. With the development of the technology to make full use of the materials, it is a tendency that more and more synchronous machines are designed to operate in saturated, sometimes, even heavily saturated state. So the accurate expression of the saturation in the models of synchronous machines is becoming more and more important.
1. 1. Models of Saturated Synchronous Machines

Many different models of the synchronous machines have been suggested and used in practice [4,8-11,19-21]. They range from the very accurate and complicated models, e.g., the finite element models [19, 21] to the simple two-axis frame model [8-10,20], which is commonly used in the power system analysis. This two-axis frame model is considered to be accurate enough for the not saturated cases. However, the presently used methods for representing saturation in this two-axis frame model do not usually give enough accurate results for the cases of saturated synchronous machines [9,16].

1.1.1. Traditional two-axis frame model

Figure 1.1 gives the phasor diagram of the traditional two-axis frame model of saturated synchronous machines. In this figure, \( R_a \) is the armature resistance, \( V_t \) is the terminal voltage and \( I_a \) is the armature current, \( I_{ad} \) and \( I_{aq} \) are the components of \( I_a \) in the direct and quadrature axes respectively, \( E_{fs} \) is the saturated electromotive force (e.m.f.) due to the magnetic field that is produced by the field winding excitation, and the angle \( \delta_1 \) between \( E_{fs} \) and \( V_t \) is the load angle which represents the operating state of the machine.
Figure 1.1: Traditional Two-Axis Frame Model of Saturated Synchronous Machines
In this model, the saturation effect is represented by the saturated direct- and quadrature-axis reactances, $X_{ds}$ and $X_{qs}$\cite{4,8,9}, which are given by the following expressions:

\begin{align*}
X_{ds} &= X_l + X_{mds} \\
X_{qs} &= X_l + X_{mqs}
\end{align*}

where $X_l$ is the leakage reactance and $X_{mds}$ and $X_{mqs}$ are the saturated d- and q-axis mutual reactances respectively. These saturated d- and q-axis mutual reactances are calculated as follows:

\begin{align*}
X_{mds} &= S_d X_{mdu} \\
X_{mqs} &= S_q X_{mqu}
\end{align*}

where $S_d$ and $S_q$ are the d- and q-axis saturation factors and $X_{mdu}$ and $X_{mqu}$ are the unsaturated d- and q-axis mutual reactances respectively. The values of the saturation factors $S_d$ and $S_q$ can be obtained from the d- and q-axis open-circuit characteristic curves respectively. In both axes, the relationship between the electromotive force (e.m.f.) $E$ and the total ampere-turns $AT$ has similar variation, which can be depicted as in Fig. 1.2. For each axis, the value of the unsaturated mutual reactance $X_{mu}$ is the slope of the air-gap line of the corresponding open-circuit characteristic curve that can be expressed as follows:

\begin{equation}
X_{mu} = \frac{E_u}{AT}
\end{equation}
Figure 1.2: Definition of the Saturation Factors Using the Open-Circuit Characteristic Curves

And its saturated value can be expressed as follows:

\[ X_{ms} = \frac{E_s}{AT} \]  \hspace{1cm} (1.6)

From Eqs. (1.5) and (1.6), the following relationship can be obtained:

\[ X_{ms} = \frac{E_s}{E_u} X_{mu} \]  \hspace{1cm} (1.7)

From Eq. (1.7), the saturation factor can thus be obtained as follows:

\[ S = \frac{E_s}{E_u} \]  \hspace{1cm} (1.8)
Several studies have shown that the use of the traditional two-axis frame model results in great discrepancies between the calculated and experimental results of the performance of saturated synchronous machines [8,9,16]. El-Serafi et. al. found, in their study of the steady-state performance of a microalternator, that there are up to 21% discrepancies between the tested and calculated results [16]. This discrepancy had been attributed to the improper representation of the saturation effect in the two-axis frame model [9].

In the traditional two-axis frame model of saturated synchronous machines, it is incorrectly assumed that there is no magnetic coupling between the direct and quadrature axes [5,10]. However, it has been recognized recently that this magnetic coupling between these two axes, which is usually called the cross-magnetizing phenomenon in the literature, plays an important role in the analysis of the saturated synchronous machines [15,17]. It has also been found that the inclusion of the cross-magnetizing effect improves the accuracy of the two-axis frame model, and makes the calculated results closer to the measured results. El-Serafi et. al. [16] have found in their investigations of the steady-state performance of a microalternator that the discrepancies between the measured and the calculated results have been reduced from 21% to less than 3%.
1.2. The objectives of this thesis

In order to accurately describe the cross-magnetizing effect (c.m.e.), two parameters have been introduced by El-Serafi et. al. [15,16]. To determine the values of these two parameters, the open-circuit characteristic curves (o.c.c.cs.) in the various axes of the synchronous machine and their corresponding curves of the angles between the resultant air-gap magnetic fluxes and the direct axis have to be known. These curves can be easily measured if the tested synchronous machine has both d- and q-axis excitation windings. However industrial synchronous machines have only one excitation winding that is located in their direct axis. Thus, for this type of machines, there is till now no experimental method to measure their saturation characteristic curves in the intermediate axes and to determine their parameters that represent the cross-magnetizing effect. To determine these saturation characteristic curves and the parameters representing the cross-magnetizing effect, Jiwei Wu has proposed, in his Master’s thesis [1], an “Analytical” method.

The main purpose of the present thesis is to present a new experimental method, which will be called the “Back-to-Back” method, for measuring the saturation characteristic curves and determining the parameters representing the cross-magnetizing effect of the industrial synchronous machines, which have no field windings in their quadrature
axis. These experimental results will be used to verify the accuracy of the "Analytical" method proposed by Wu and its assumptions by comparing the results from both methods. In this context, the main objectives of this thesis are focused on the following aspects:

1. To introduce a new experimental method for the measurement of the saturation characteristic curves in the various axes of synchronous machines that have no excitation windings in their quadrature axis;

2. To use the measured results of this new experimental method to determine the parameters representing the cross-magnetizing effect of this type of synchronous machines;

3. To verify the accuracy of the "Analytical" method, which was proposed by Wu, by comparing the results obtained from the new experimental method with those obtained from this "Analytical" method;

4. To verify the accuracy of the two-axis frame model of the synchronous machines, when it is modified by including the parameters representing the cross-magnetizing effect that have been obtained from both the new experimental and the "Analytical" methods.
Chapter 2

The Cross-Magnetizing Phenomenon

As mentioned in Chapter 1, the neglect of the magnetic coupling between the direct and quadrature axes in the traditional two-axis frame model of saturated synchronous machines is one of the main sources of errors when this model is used in power system analysis. This magnetic coupling is usually referred to in the literature as the cross-magnetizing phenomenon and results in that the magnetic flux in each axis is dependent not only on the magnetomotive force (m.m.f.) of the same axis, but also on the magnetomotive force (m.m.f.) of the other axis.

In this chapter, the cross-magnetizing phenomenon will be presented and the inclusion of its effect in the two-axis frame model will be discussed. A known "Analytical" method [1] for determining the parameters representing this cross-magnetizing effect will also be summarized.

2.1. The concept of the cross-magnetizing phenomenon

The concept of the cross-magnetizing effect in a synchronous machine can be simply explained by using Figs. 2.1 and 2.2. When the synchronous machine is only excited from the direct axis by the ampere-
Figure 2.1: Saturation Curves of a Synchronous Machine in Various Axes

Figure 2.2: Flux Vector Diagram of a Synchronous Machine
turns $AT_d$, the air-gap magnetic flux $\phi_d$ can be found from the d-axis saturation curve (Fig. 2.1). Similarly, when the machine is only excited from the quadrature axis by the ampere-turns $AT_q$, the air-gap magnetic flux $\phi_q$ can be found from the q-axis saturation curve (Fig. 2.1). If the machine is excited from both the direct and quadrature axes simultaneously with the ampere-turns $AT_d$ and $AT_q$ in the direct and quadrature axes respectively, the magnitude of the resultant air-gap magnetic flux $\phi_r$ corresponding to the total ampere-turns $AT_t$ can be found from the saturation curve corresponding to the axis of the total ampere turns ($\beta$-axis in Fig. 2.1) [13, 15]. It should be noticed that the angle $\delta'$ between the axis of the resultant air-gap magnetic flux $\phi_r$ and the direct axis of the machine is in general smaller than the angle $\beta$ between the axis of the total ampere-turns $AT_t$ and this direct axis. This is due to both the saliency and the different saturation levels in the various axes of the synchronous machines.

Resolving $\phi_r$ into its d- and q-axis components, $\phi_{rd}$ and $\phi_{rq}$, it can be found that $\phi_{rd}$ is smaller than $\phi_d$ and $\phi_{rq}$ is smaller than $\phi_q$ (Fig. 2.2). The differences of the fluxes in the direct and quadrature axes, $\phi_{dq}$ and $\phi_{qd}$, due to the magnetic coupling between the direct and quadrature axes, i.e., due to the cross-magnetizing phenomenon, can thus be used to represent the cross-magnetizing effect in the two-axis frame synchronous machine models. The magnitudes of $\phi_{dq}$ and $\phi_{qd}$ can be expressed by the following equations:
\[
\phi_{dq} = \phi_d - \phi_{ld} \quad (2.1)
\]
\[
\phi_{qd} = \phi_q - \phi_{lq} \quad (2.2)
\]

Since the fluxes are usually represented in the phasor diagrams of synchronous machines by corresponding induced e.m.fs or voltage drops, it is convenient to include the cross-magnetizing effect in the two-axis frame model also in the form of induced e.m.fs or voltage drops. In the phasor diagram of Fig. 2.3, the voltage drop concept is used. In it, the reduction in the d-axis magnetic flux due to the cross-magnetizing effect, \( \phi_{dq} \), is represented by the voltage drop \( E_{qd} \). Also, the reduction in the q-axis magnetic flux due to the cross-magnetizing effect, \( \phi_{qd} \), is represented by the voltage drop \( E_{dq} \). In a properly chosen per unit system [22], the values of these voltage drops are equal to the corresponding magnetic flux changes, i.e., \( E_{qd} = \phi_{dq} \) and \( E_{dq} = \phi_{qd} \). In this case, the values of \( E_{qd} \) and \( E_{dq} \) could be obtained from the open-circuit characteristic curves using the following equations:

\[
E_{qd} = E_q - E \cos \delta' \quad (2.3)
\]
\[
E_{dq} = E_q - E \sin \delta' \quad (2.4)
\]
Figure 2.3: Phasor Diagram of a Synchronous Machine Including the Cross-Magnetizing Effect
where $E_d$, $E_q$ and $E$ are obtained from the d-, q- and β-axis open-circuit characteristic curves of the synchronous machines respectively and $\delta'$ is obtained from the curves of the angles between the axis of the air-gap flux and the direct axis [13,15].

2.2. Determination of the parameters representing the cross-magnetizing effect

The parameters representing the cross-magnetizing phenomenon in the two-axis frame models of synchronous machines, namely $E_{qd}$ and $E_{dq}$, could be determined by Eqs. (2.3) and (2.4) if the open-circuit characteristic curves in the various axes of the machine and the corresponding curves of the angles $\delta'$ between the axis of the air-gap flux and the direct axis are known. These curves could be directly measured if the synchronous machine has two field windings: one in the direct axis and the other in the quadrature axis and the machine can thus be excited from both these two windings [13,15]. However, the industrial synchronous machines usually have no field winding in their quadrature axis and thus these saturation characteristic curves and the parameters representing the cross-magnetizing effect cannot be obtained from the measured results. In the following section, an
analytical method proposed by Wu [1] for determining these curves and parameters of this type of synchronous machines will be summarized.

2.2.1. "Analytical" method [1]

In this method, the intermediate axis (β-axis) open-circuit characteristic curves and their corresponding curves of the angle $\delta$, which are required to determine the parameters representing the cross-magnetizing effect, are calculated analytically using the measurable d- and q-axis open-circuit characteristic curves. The d-axis open-circuit characteristic curve could be easily determined by the conventional open-circuit tests [23-25], while the q-axis open-circuit characteristic curve could be determined from the "Maximum Lagging Current" method [25]. For the calculation of the intermediate-axis open-circuit characteristic curves, an equivalent permeability is assumed for the magnetic circuit of the synchronous machine. For the salient-pole synchronous machine which will be used in the investigations of this thesis, it is assumed that, in the unsaturated condition, the value of this equivalent permeability along the periphery of the rotor has the form shown in Fig. 2.4, where $\mu$ is the value of the equivalent permeability under the pole arc and $2\tau$ is the width of the pole arc. The permeability under one pole pitch can thus be expressed by the following equation:
In the saturated condition, the equivalent permeability at a point \( \theta \) away from the pole axis can be obtained by modifying its unsaturated value using an appropriate saturation function. Such a saturation function depends on the value of the magnetomotive force (m.m.f.) at this point, namely \( F(\theta) = AT \cos(\theta - \beta) \), and can be expressed as a polynomial function of \( F(\theta) \). Thus, this saturation function can be represented in the following form:
\[ S(\theta) = 1.0 - a_1 |F(\theta)| - a_2 |F(\theta)|^2 \]  \hfill (2.6)

where \( a_1 \) and \( a_2 \) are machine dependent coefficients, \(|F(\theta)|\) is the magnitude of the magnetomotive force (m.m.f.) at the point \( \theta \), and \( S(\theta) \) is the saturation function at this point.

The air-gap magnetic flux density at point \( \theta \) can thus be determined from the following equation:

\[ B(\theta) = k_1 F(\theta) \mu(\theta) S(\theta) \]  \hfill (2.7)

where \( k_1 \) is a constant depending on the machine dimensions. The fundamental component of this air-gap magnetic flux density, \( B_1(\theta) \), can be represented by the following equation:

\[ B_1(\theta) = B_d \cos \theta + B_q \sin \theta \]  \hfill (2.8)

where \( B_d \) and \( B_q \) are the amplitudes of the d- and q-axis components of the air-gap magnetic flux density. These d- and q-axis components can be calculated by applying the Fourier series analysis to Eq. (2.7) as follows:

\[ B_d = \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} B(\theta) \cos \theta \, d\theta \]

\[ = \frac{2}{\pi} \int_{-\pi}^{\pi} k_1 \mu F(\theta) (1.0 - a_1 |F(\theta)| - a_2 |F(\theta)|^2) \cos \theta \, d\theta \]  \hfill (2.9)

\[ B_q = \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} B(\theta) \sin \theta \, d\theta \]
The d-axis component of the air-gap magnetic flux $\phi_d$ can be obtained by integrating the d-axis magnetic flux density over one pole pitch and can be expressed as follows:

$$\phi_d = k_2 B_d$$

(2.11)

where $k_2$ is a constant which depends on the dimensions of the machine. Substituting Eq. (2.9) in Eq. (2.11), $\phi_d$ can be given by the following expression:

$$\phi_d = \frac{2}{\pi} \int_{-\pi}^{\pi} k_2 k_1 \mu F(\theta)(1.0 - a_1 F(\theta) - a_2 |F(\theta)|^2) \cos \theta d\theta$$

$$= \frac{2}{\pi} \int_{-\pi}^{\pi} k F(\theta)(1.0 - a_1 F(\theta) - a_2 |F(\theta)|^2) \cos \theta d\theta$$

(2.12)

where $k$ is a constant which is machine dependent. Similarly, the q-axis component of the air-gap magnetic flux $\phi_q$ can be obtained by integrating the q-axis component of the air-gap magnetic flux density over one pitch and can be expressed as follows:

$$\phi_q = \frac{2}{\pi} \int_{-\pi}^{\pi} k_2 k_1 \mu F(\theta)(1.0 - a_1 F(\theta) - a_2 |F(\theta)|^2) \sin \theta d\theta$$

$$= \frac{2}{\pi} \int_{-\pi}^{\pi} k F(\theta)(1.0 - a_1 F(\theta) - a_2 |F(\theta)|^2) \sin \theta d\theta$$

(2.13)
The constant $k$ and the pole arc width $2\tau$ can be determined from the measurable unsaturated d- and q-axis mutual reactances of the synchronous machine, $X_{md}$ and $X_{mq}$ respectively. These mutual reactances in per unit are given by the following formula:

$$X_m = \frac{\phi}{A\tau}$$ (2.14)

If the synchronous machine is unsaturated and excited in the direct axis, the air-gap magnetic flux can be calculated from Eq. (2.12) as follows:

$$\phi_d = \frac{2}{\pi} \int_{-\tau}^{\tau} kF(\theta) \cos \theta d\theta$$

$$= \frac{k}{\pi} A\tau (2\tau + \sin 2\tau)$$ (2.15)

Substituting Eq. (2.15) in Eq. (2.14), the unsaturated d-axis mutual reactance in per unit can be represented as follows:

$$X_{md} = k(2\tau + \sin 2\tau)/\pi$$ (2.16)

Similarly, if the machine is excited from the quadrature axis and the machine is unsaturated, the air-gap magnetic flux can be calculated from Eq. (2.13) as follows:

$$\phi_q = \frac{2}{\pi} \int_{-\tau}^{\tau} kF(\theta) \sin \theta d\theta$$

$$= \frac{k}{\pi} A\tau (2\tau - \sin 2\tau)$$ (2.17)
and the unsaturated q-axis mutual reactance in per unit can be represented as follows:

\[ X_{mq} = k(2\tau - \sin 2\tau)/\pi \] \hspace{1cm} (2.18)

From Eqs. (2.16) and (2.18), the following relationships can be obtained:

\[
\begin{align*}
\frac{2\tau}{\sin 2\tau} &= \frac{X_{md} + X_{mq}}{X_{md} - X_{mq}} \\
k &= \frac{\pi}{4\tau}(X_{md} + X_{mq})
\end{align*}
\hspace{1cm} (2.19) \hspace{1cm} (2.20)

Knowing the values of the d- and q-axis mutual unsaturated reactances \(X_{md}\) and \(X_{mq}\), the pole arc width \(2\tau\) and the constant \(k\) of Eqs. (2.12) and (2.13) could thus be obtained by solving Eqs. (2.19) and (2.20).

Using the d- and q-axis open-circuit characteristic curves, the coefficients \(a_1\) and \(a_2\) of Eq.(2.6) could be determined from Eqs. (2.12) and (2.13) by one of the known fitting techniques [1]. Appendix 1 gives the details of the FORTRAN program that used to calculate the coefficients \(a_1\) and \(a_2\) for the synchronous machine used in the investigations of this thesis.

To determine the open-circuit characteristic curve in an intermediate axis (β-axis), Eqs. (2.12) and (2.13) could be used to calculate the d- and q-axis components of the air-gap magnetic flux, namely \(\phi_{id}\) and \(\phi_{iq}\). Knowing \(\phi_{id}\) and \(\phi_{iq}\), the total air-gap flux \(\phi_t\) can then be calculated as follows:
The angle between the axis of the resultant air-gap magnetic flux and the
direct axis, namely $\delta'$, can be calculated as follows:

$$\delta' = \tan^{-1} \left( \frac{\theta_{tq}}{\phi_{td}} \right)$$

(2.22)

Knowing $\phi_{td}$, $\theta_{tq}$, $\phi_{d}$ and $\phi_{q}$, the parameters representing the cross-
magnetizing effect can thus be calculated by Eqs. (2.1) and (2.2). From Eqs.
(2.1), (2.2), (2.12) and (2.13), the magnitudes of $E_{qd}$ ($= \phi_{dq}$) and $E_{dq}$ ($= \phi_{qd}$)
can be expressed as follows:

$$E_{qd} = \begin{cases} 
(a_{1}d_{1} + 3a_{2}d_{2}AT_{d})AT_{q}^{2} & (\beta \leq \frac{\pi}{2} - \tau) \\
(a_{1}(b_{1} - c_{d1})AT_{d}^{2} + 2a_{1}b_{2}AT_{d}AT_{q} + (a_{1}b_{3} + 3a_{2}d_{2}AT_{d})AT_{q}^{2} & (\beta > \frac{\pi}{2} - \tau)
\end{cases}$$

(2.23)

$$E_{dq} = \begin{cases} 
((2a_{1}d_{1} + 3a_{2}d_{2}AT_{d})AT_{q} - a_{1}c_{q1}AT_{q})AT_{q} & (\beta \leq \frac{\pi}{2} - \tau) \\
(a_{1}b_{2} + 3a_{2}d_{2}AT_{q})AT_{d}^{2} + 2a_{1}b_{3}AT_{d}AT_{q} + a_{1}(b_{4} - c_{q1})AT_{q}^{2} & (\beta > \frac{\pi}{2} - \tau)
\end{cases}$$

(2.24)

where

$$c_{d1} = \frac{k}{3\pi} (9 \sin \tau + \sin 3\tau),$$

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\[ c_{dl} = \frac{k}{3\pi} (9 \sin \tau + \sin 3\tau), \]

\[ c_{q1} = \frac{k}{3\pi} (8 - 9 \cos \tau + \cos 3\tau), \]

\[ d_1 = \frac{k}{3\pi} (3 \sin \tau - \sin 3\tau), \]

\[ d_2 = \frac{k}{8\pi} (4\tau - \sin 4\tau), \]

\[ b_1 = \frac{k}{3\pi} (9 \cos \beta - \cos 3\beta), \]

\[ b_2 = \frac{k}{3\pi} (3 \sin \beta - \sin 3\beta - 3 \cos \tau - \cos 3\tau), \]

\[ b_3 = \frac{k}{3\pi} (3 \cos \beta + \cos 3\beta), \]

and \[ b_4 = \frac{k}{3\pi} (9 \sin \beta + \sin 3\beta - 9 \cos \tau + \cos 3\tau). \]

Appendix 2 gives the MATLAB program for the calculation of the parameters \( E_{qd} \) and \( E_{dq} \) of the synchronous machine used in the investigations throughout this thesis.
Chapter 3

The “Back-to-Back” Method for Determining the Parameters Representing the Cross-Magnetizing Effect

As discussed in Chapter 2, there is till now no experimental method for measuring the saturation characteristic curves in the intermediate axes of synchronous machines that do not have field windings in their quadrature axis. In this chapter, an experimental method for measuring these saturation characteristic curves in the intermediate axes of such type of synchronous machines will be presented. The principle of this proposed experimental method, which will be called the “Back-to-Back” method, will be first introduced. To demonstrate this method, a set of a salient-pole synchronous machine coupled to a similar specially designed machine, which is available in the Department’s Laboratory, is used. The saturation characteristic curves of this salient-pole synchronous machine in the various axes are determined by applying this method. Using these saturation characteristic curves, the parameters representing the cross-magnetizing effect of this machine are calculated. Moreover, these experimental results are compared with those obtained from the “Analytical” method described in Chapter 2.
3.1. Principle of the “Back-to-Back” method

As mentioned in Chapter 2, the saturation characteristic curves in the intermediate axes of synchronous machines, which are needed to determine the parameters representing the cross-magnetizing effect, can only be obtained if the synchronous machine has excitation windings in both its direct and quadrature axes. In the proposed new method, namely the “Back-to-Back” method, the armature three-phase winding is to be used to generate the magnetic flux needed to determine the saturation characteristic curves in the various axes of the synchronous machines that have no excitation windings in their quadrature axis, e.g., the industrial synchronous machines. By exciting this three-phase symmetrical armature winding with three-phase symmetrical currents of the rated frequency, a rotating air-gap magnetic flux similar to the one produced by exciting the rotor field winding will be produced in the synchronous machine. If the axis of this produced air-gap flux is adjusted to locate in the direct, quadrature or any intermediate axis (β-axis), the saturation characteristic curves in the direct, quadrature or intermediate axes can then be obtained.

If the tested synchronous machine, which is denoted in Fig. 3.1 by SM1, is coupled mechanically to an auxiliary synchronous machine SM2 (Fig.3.1) of the same number of poles and of the same ratings as SM1 through an adjustable coupling, the relative position of the rotors of these
two machines could be adjusted by this coupling. Thus, with this adjustable coupling, the angle between the direct axes of these two machines, $\beta_1$, can be adjusted to any angle from 0 to 90 electrical degrees. When the two direct axes of these two machines are aligned, $\beta_1$ is equal to zero, while $\beta_1$ is equal to $90^\circ$ when the direct axis of the tested machine SM1 is aligned with the quadrature axis of the auxiliary machine SM2. For the experiments, which will be carried out in this thesis, a set of three machines: two similar salient-pole synchronous machines and a DC machine, will be used. In this set, one of the salient-pole synchronous machine will be used as the tested
machine, SM1, while the other, which is specially designed to have its stator position adjustable with respect to the fixed stator of the tested machine, will be used as the required auxiliary machine, SM2. Thus, the adjustment of the stator of this auxiliary machine will result in the change of the relative position of the stators of the two machines with respect to their pole structures as it is in the case of adjusting the relative position of the rotors by an adjustable coupling.

In carrying out the "Back-to-Back" test, the tested machine is kept unexcited while the field current of the auxiliary machine is adjusted to a suitable value and the DC machine is connected to a DC supply. If the stators of these tested and auxiliary machines are aligned together and their three-phase armature windings are connected in series to a power system, the rotating magnetomotive forces (m.m.fs.) of these two machines will have the same speed and the same relative positions with respect to their stators and pole structures.

Keeping the output power of the auxiliary machine \( P_2 \) to be zero (through adjusting the power of the DC machine DM), the axis of the produced rotating magnetic flux of the auxiliary machine will be located very close to its direct axis. This relationship can be illustrated by the diagram of Fig. 3.2, if the armature resistance of SM2 is neglected. If the position of the stator of the tested synchronous machine SM1 is adjusted at an angle \( \beta \) with respect to the stator of the auxiliary machine SM2, the axis
Figure 3.2: The Phasor Diagram of SM2 for the “Back-to-Back” Method
(Neglecting Resistances)
of the produced rotating magnetic flux in the tested machine SM1 will be closely located at an angle $\beta$ with respect to its direct axis. Figure 3.3 illustrates the phasor diagram of the tested synchronous machine SM1 for this case (armature resistance is neglected).

From the phasor diagram of Fig. 3.3, the d- and q-axis components of the internal voltage of the tested machine, $V_{id1}$ and $V_{iq1}$, could be expressed as follows:

$$V_{id1} = V_{il} \sin \delta_{i1} - I_a X_{i1} \sin \beta$$

$$V_{iq1} = V_{il} \cos \delta_{i1} - I_a X_{i1} \cos \beta$$

where $V_{il}$ is the measured terminal voltage of the tested machine, $I_a$ is the measured armature current, $\delta_{i1}$ is the load angle of the tested machine, and $X_{i1}$ is the leakage reactance of the tested synchronous machine.

The load angle $\delta_{i1}$ could be calculated from the following relationship:

$$\delta_{i1} = \beta - (90^\circ - \varphi_1)$$

where $\varphi_1$ is the power factor angle of SM1, which could be calculated from the measured input power $P_1$, terminal voltage $V_{il}$ and armature current $I_a$ as follows:

$$\varphi_1 = \cos^{-1} \left( \frac{P_1}{V_{il}/I_a} \right)$$
Figure 3.3: The Phasor Diagram of SM1 for the “Back-to-Back” Method
(Neglecting Resistances)
It should be noted that, in the case of neglecting the resistances of the tested and the auxiliary synchronous machines, the angle between the axis of the magnetomotive force (m.m.f.) of the tested synchronous machine and its direct axis, i.e. the angle $\beta$, is equal to the angle of the relative position of the stator of the tested machine with respect to the stator of the auxiliary machine, $\beta_1$.

The internal voltage of the tested machine SM1, $V_{i1}$, which is the induced electromotive force (e.m.f.) in the machine due to the magnetomotive force of the machine's stator winding when $I_a$ flows in it, can be calculated as follows:

$$V_{i1} = \sqrt{V_{id1}^2 + V_{iq1}^2}$$  \hspace{1cm} (3.5)

The angle of this internal voltage $V_{i1}$ with respect to the q-axis of the phasor diagram of Fig. 3.3, which is equal to the angle between the resultant air-gap magnetic flux and the direct axis of the tested machine, $\delta$, can be calculated as follows:

$$\delta_{i1} = \tan^{-1}(V_{id1} / V_{iq1})$$  \hspace{1cm} (3.6)

By changing the supply voltage $V_{t12}$, the armature current $I_a$ can thus be adjusted. For each value of the armature current $I_a$, the terminal voltage of the tested machine $V_{t1}$ is measured. From these measured values, the corresponding internal voltage of the tested machine $V_{i1}$, i.e., the induced electromotive force $E$ of the open-circuit characteristic curve of the tested machine, and the angle $\delta_{i1}$, i.e., the angle between the resultant air-gap
magnetic flux and the direct axis of the tested machine, δ', could thus be calculated using Eqs. (3.5) and (3.6). Knowing the values of these induced electromotive forces E and the angles δ for a reasonable number of armature currents I_a, the intermediate axis (β-axis) open-circuit characteristic curve and the corresponding curve of the angle between the resultant air-gap magnetic flux and the direct axis of the tested machine could be obtained. Adjusting the relative position of the stators of the tested and the auxiliary synchronous machines at different angles and repeating the "Back-to-Back" test, a family of the open-circuit characteristic curves in various intermediate axes (β-axes) and their corresponding curves of the angle δ' could thus be obtained.

3.2. Effect of the armature resistance on the analysis of the "Back-to-Back" test

In Section 3.1, the armature resistances of both the tested and the auxiliary synchronous machines have been neglected to simplify the basic explanation of the principle of the "Back-to-Back" test. However, the values of these armature resistances could affect to a certain extent the accuracy of the calculation, particularly the determination of δ. Thus, it is necessary to include the effect of these armature resistances in the calculation of V_{O1} and δ'. In this case, the phasor diagrams of the auxiliary synchronous machine
SM2 and of the tested synchronous machine SM1 will be as shown in Figs. 3.4 and 3.5 respectively. From Fig. 3.5, the d- and q-axis components of the internal voltage of the tested machine could be expressed as follows:

\[ V_{id1} = V_{t1} \sin \delta_{t1} + I_a R_{a1} \cos \beta - I_a X_{f1} \sin \beta \]  \hspace{1cm} (3.7)

\[ V_{iq1} = V_{t1} \cos \delta_{t1} - I_a R_{a1} \sin \beta - I_a X_{f1} \cos \beta \]  \hspace{1cm} (3.8)

where \( R_{a1} \) is the armature resistance of the tested machine, and \( \beta \) is the angle between the armature current \( I_a \) and the \( d_1 \)-axis (Figs. 3.4 and 3.5). It should be noticed that, in this case of including the armature resistances in the analysis, the angle \( \beta \) is not equal to the angle of the relative position of the tested machine's stator with respect to the auxiliary machine's stator, i.e. \( \beta_1 \). This angle could be calculated as follows:

\[ \beta = \beta_1 - \delta_{t2} \]  \hspace{1cm} (3.9)

where \( \delta_{t2} \) is the load angle of the auxiliary synchronous machine (Fig. 3.4). The angle \( \delta_{t2} \) could be calculated as follows:

\[ \delta_{t2} = \tan^{-1} \left( \frac{I_a R_{a2}}{V_{t2} - I_a X_{q2}} \right) \]  \hspace{1cm} (3.10)

where \( V_{t2} \) is the measured terminal voltage of the auxiliary machine, \( R_{a2} \) is the armature resistance of the auxiliary machine, and \( X_{q2} \) is the q-axis synchronous reactance of the auxiliary machine. The load angle, \( \delta_{t1} \), and the
Figure 3.4: The Phasor Diagram of SM2 for the "Back-to-Back" Method
(Including Armature Resistance)
Figure 3.5: The Phasor Diagram of SM1 for the "Back-to-Back" Method
(Including Armature Resistance)
power factor angle, $\varphi_1$, of Eqs. (3.7) and (3.8) are to be calculated in the same way as given in Section 3.1.

Using Eqs. (3.7) and (3.8), the family of the open-circuit characteristics curves of the tested machine in various intermediate axes and their corresponding curves of the angle $\delta'$ could be obtained in the same manner as explained in Section 3.1.

3.3. Experimental results

To verify the accuracy of the "Analytical" method for calculating the saturation characteristic curves and the parameters representing the cross-magnetizing effect of saturated synchronous machines (Chapter 2), the "Back-to-Back" method was applied to obtain experimentally the intermediate-axis open-circuit characteristic curves and the angle $\delta'$ curves of the salient-pole machine described in Section 3.1. As mentioned before, this machine has no field winding in the quadrature axis. The nameplates and the data of both the tested and auxiliary machines are given in Appendices 3 and 4 respectively. Using these saturation characteristic curves, the parameters representing the cross-magnetizing effect were determined and compared with the results obtained from the "Analytical" method.
3.3.1. The d- and q-axis open-circuit characteristic curves

Applying the "Back-to-Back" method with the angle $\beta_1 = 0^\circ$, the d-axis open-circuit characteristic curve of the tested machine SM1 was obtained and is shown in Fig. 3.6. In this figure, the d-axis open-circuit characteristic curve that is obtained by the open-circuit test [23-25] is also given. In Fig. 3.6, $AT_d$ is the ampere-turns in the direct axis, which is related to the armature current $I_a$ in the case of the "Back-to-Back" method and to the field current $I_f$ in the case of the open-circuit test, and $E_q$ is the electromotive force induced by $AT_d$. $E_q$ is equal to the calculated internal voltage $V_{i1}$ in the case of the "Back-to-Back" method while, in the case of the open-circuit test, $E_q$ is the measured open-circuit terminal voltage. As shown in Fig. 3.6, the two d-axis open-circuit characteristic curves agree well with each other. However, there is some discrepancy between the two curves when the tested machine becomes saturated.

Similarly, the q-axis open-circuit characteristic curve of the tested synchronous machine was obtained from the "Back-to-Back" method with The angle $\beta_1 = 90^\circ$ and is shown in Fig. 3.7. In this figure, the q-axis open-circuit characteristic curve measured by the "Maximum Lagging Current" method [25] is also shown. As Fig. 3.7 shows, the two q-axis open-circuit characteristic curves obtained from these two methods agree reasonably with each other.
Figure 3.6: The D-axis Open-Circuit Characteristic Curves of the Tested Machine
Figure 3.7: The Q-axis Open-Circuit Characteristic Curves of the Tested Machine
The small discrepancies between the curves in Figs. 3.6 and 3.7 could be attributed to the following factors:

1. The value of the armature leakage reactance $X_{11}$ of the tested machine, which is used to calculate $V_{il}$ in the "Back-to-Back" method, decreases with the increase of $I_a$ due to the saturation in the leakage flux paths [26-28]. In the determination of the d- and q-axis open-circuit characteristic curves from the "Back-to-Back" method, the armature leakage reactance of the tested machine $X_{11}$ has been treated as constant and its unsaturated value has been used.

2. The effect of the iron-loss has, as usual, been ignored in the calculations of the "Back-to-Back" method.

3. The inaccuracy of the "Maximum Lagging Current" method in reaching exactly the boundary of the instability of the tested machine. This factor affects the accuracy of the q-axis open-circuit characteristic curve of the tested machine obtained by this method.
3.3.2. The $\beta$ - axis saturation characteristic curves

Applying the "Back-to-Back" method with $\beta = 15^\circ, 30^\circ, 45^\circ, 60^\circ$ and $75^\circ$, a family of the saturation characteristic curves, namely the open-circuit characteristic curves and the curves of the angle $\delta'$ between the axis of the resultant air-gap magnetic flux and the direct axis, of the tested machine in the intermediate axes were obtained. These results together with the $d$- and $q$-axis open-circuit characteristic curves of the tested machine are shown in Figs. 3.8 and 3.9. In these figures, the calculated results applying the "Analytical" method for the intermediate-axis curves are also shown. It should be mentioned that the calculation applying the "Analytical" method is based on the $d$-axis open-circuit characteristic curve obtained from the open-circuit test (Fig. 3.6) and the $q$-axis open-circuit characteristic curve obtained from the "Maximum Lagging Current" method (Fig. 3.7). From these figures, it can be found that the intermediate-axis open-circuit characteristic curves obtained from the "Back-to-Back" method and those obtained from the "Analytical" method agree well with each other. However, there are appreciable discrepancies between the results of the angle $\delta'$ curves obtained from these two methods. These discrepancies could be attributed to the following factors:
Figure 3.8: The Open-Circuit Characteristic Curves in Various Axes of the Tested Synchronous Machine
Figure 3.9: The Angles between the Axis of the Air-Gap Flux and the
Direct Axis of the Tested Synchronous Machine
1. In the "Back-to-Back" method, the angle of the relative position of the tested machine's stator with respect to the auxiliary machine's stator, $\beta_1$, is kept constant, while the angle between the axis of the magnetomotive force and the direct axis of the tested machine, $\beta$, is changing with the armature current. On the other hand, in the calculation of the "Analytical" method, the angle $\beta$ is kept constant (equal to $\beta_1$).

2. In the "Back-to-Back" method, the iron-loss has been neglected.

3. The leakage reactance of the tested machine, $X_{\parallel}$, is treated as constant and its unsaturated value is used in the "Back-to-Back" method.

4. The errors in the measured d- and q-axis open-circuit characteristic curves, especially in the case of the q-axis open-circuit characteristic curve, affect the accuracy of the calculation of the intermediate-axis saturation characteristic curves obtained from the "Analytical" method.
3.3.3. Parameters representing the cross-magnetizing effect

In order to determine the parameters representing the cross-magnetizing effect of the tested synchronous machine from the experimental results obtained from the "Back-to-Back" method, it is more convenient to express the saturation characteristic curves of the tested machine by polynomial equations as functions of the magnetomotive force $AT$ and the angle $\beta_1$, i.e., the angle of the relative position of the stator of the tested machine with respect to the stator of the auxiliary machine. The coefficients of these polynomials could be determined using any of the known fitting techniques.

In the case of the $d$- and $q$-axis open-circuit characteristic curves, the angle $\beta_1$ is equal to zero and $90^\circ$ respectively and, thus, the polynomial equations representing these curves will be only functions of the magnetomotive force $AT$ and can be written in the following forms:

$$E_q = (C_0 + C_1 AT_d + \ldots + C_m AT_d^m) AT_d$$

$$E_d = (D_0 + D_1 AT_q + \ldots + D_n AT_q^n) AT_q$$

where $m$ and $n$ are the orders of the polynomial equations representing the $d$- and $q$-axis open-circuit characteristic curves in terms of $AT_d$ and $AT_q$ respectively, and $C_0$, $C_1$, ..., $C_m$ and $D_0$, $D_1$, ..., $D_m$ are the coefficients of these polynomial equations. For the tested machine, it has been found that

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the polynomial orders \( m = n = 2 \) give a good approximation for the representation of the d- and q-axis open-circuit characteristic curves. For this case, the coefficients of these polynomials have been found to be as follows:

\[
\begin{bmatrix}
C_0 \\
C_1 \\
C_2 \\
\end{bmatrix} = \begin{bmatrix}
6.715 \times 10^{-1} \\
5.134 \times 10^{-2} \\
-4.718 \times 10^{-2} \\
\end{bmatrix} \tag{3.13}
\]

\[
\begin{bmatrix}
D_0 \\
D_1 \\
D_2 \\
\end{bmatrix} = \begin{bmatrix}
3.352 \times 10^{-1} \\
9.900 \times 10^{-3} \\
-6.200 \times 10^{-3} \\
\end{bmatrix} \tag{3.14}
\]

In the case of the intermediate axes, the open-circuit characteristic curves have to be expressed by polynomial equations as functions of both the magnetomotive force \( AT \) and the angle \( \beta_1 \) of the relative position of the stator of the tested machine with respect to the stator of the auxiliary machine. Thus, the polynomial equation describing the open-circuit characteristic curves in the intermediate axes could be written in the following form:

\[
E = ( H_0 + H_1 AT + \ldots + H_i AT^i ) AT \tag{3.15}
\]

where \( i \) is the order of the polynomial equation representing the intermediate-axis open-circuit characteristic curve in terms of \( AT \), and \( H_0, H_1, \ldots, H_i \) are the polynomial equation coefficients which are functions of
the parameter $\beta_1$. These coefficients $H_0, H_1, \ldots, H_i$ could be expressed in the following matrix formula:

\[
\begin{bmatrix}
H_0 \\
H_1 \\
\vdots \\
H_i
\end{bmatrix} = K \begin{bmatrix}
1 \\
\beta_1 \\
\vdots \\
\beta_i
\end{bmatrix}
\]

where $j$ is the order of the polynomial equations representing the polynomial coefficients of the open-circuit characteristic curve in terms of $\beta_1$ and $K$ is a $(i+1) \times (j+1)$ dimension matrix with constant element values. For the tested synchronous machine, it has been found that the polynomial orders $i=2$ and $j=3$ give a good approximation for the representation of the intermediate-axis open-circuit characteristic curves. For the tested machine, the element values of the matrix $K$ have been found to be as follows:

\[
K = \begin{bmatrix}
5.698 \times 10^{-1} & 1.483 \times 10^{-2} & -5.198 \times 10^{-4} & 3.770 \times 10^{-6} \\
2.018 \times 10^{-1} & -2.114 \times 10^{-2} & 4.987 \times 10^{-4} & -3.368 \times 10^{-6} \\
-1.064 \times 10^{-1} & 7.144 \times 10^{-3} & -1.494 \times 10^{-4} & 9.567 \times 10^{-7}
\end{bmatrix}
\]

(3.17)

In the same way, the angle $\delta'$ curves for the intermediate axes (Fig. 3.9) could also be written in the following polynomial form:

\[
\delta' = F_0 + F_1 AT + \ldots + F_p A T^p
\]

(3.18)

where $p$ is the order of the polynomial equation representing the angle $\delta'$ curve in terms of $AT$ and $F_0, F_1, \ldots, F_p$ are the polynomial equation
coefficients that are functions of the parameter $\beta_1$. These coefficients $F_0, F_1, \ldots, F_p$ could be expressed in the following matrix formula:

$$
\begin{bmatrix}
F_0 \\
F_1 \\
\vdots \\
F_p
\end{bmatrix} = G
\begin{bmatrix}
1 \\
\beta_1 \\
\vdots \\
\beta_r
\end{bmatrix}
$$

(3.19)

where $r$ is the order of the polynomial equations representing the polynomial coefficients of the angle $\delta'$ curves in terms of $\beta_1$ and $G$ is a $(p+1) \times (r+1)$ dimension matrix with constant element values. For the tested synchronous machine, it has been found that the polynomial orders $p=2$ and $r = 3$ give a good approximation for the representation of the angle $\delta'$. The element values of the matrix $G$ have been found to be as follows:

$$
G =
\begin{bmatrix}
9.066 & -0.4854 & 2.620 \times 10^{-2} & -1.121 \times 10^{-4} \\
13.19 & 1.475 & -3.663 \times 10^{-2} & 2.403 \times 10^{-4} \\
6.249 & -0.6927 & 1.730 \times 10^{-2} & 1.179 \times 10^{-4}
\end{bmatrix}
$$

(3.20)

Using Eqs. (3.11) to (3.20), the parameters $E_{dq}$ and $E_{qd}$ could be calculated as follows:

$$
E_{qd} = E_q - E \cos \delta'
$$

(3.21)

$$
E_{dq} = E_d - E \sin \delta'
$$

(3.22)

In the calculation of $E$ and $\delta'$ using Eqs. (3.15) and (3.18), it should be noticed that these equations of the “Back-to-Back” method are functions of...
$AT$ and $\beta_1$. As mentioned before, the angle between the magnetomotive force and the direct axis ($\beta$) is a little smaller than the angle of the relative position of the tested machine's stator with respect to the auxiliary machine's stator ($\beta_1$), due to the effect of the armature resistance of the auxiliary synchronous machine. The difference between the two angles is dependent on the value of the magnetomotive force $AT$. If the calculation of the parameters representing the cross-magnetizing effect, i.e., $E_{dq}$ and $E_{qd}$ is to be determined as a function of a constant value of $\beta$, the corresponding value of $\beta_1$ for such a value has thus to be used in Eqs. (3.15) and (3.18). For the tested machine, the relationship between $\beta_1$ and $\beta$ in electrical degree has been found to be approximately as follows:

$$\beta_1 = \beta + 0.295AT$$  \hspace{1cm} (3.23)

Figures 3.10 and 3.11 show the results of the parameters $E_{dq}$ and $E_{qd}$ of the tested machine obtained from the "Back-to-Back" method. In these figures, the parameters $E_{dq}$ and $E_{qd}$ are given as functions of the d- and q-axis components of the total ampere-turns, i.e., $AT_d$ and $AT_q$.

The "Analytical" method has also been used to calculate the parameters representing the cross-magnetizing effect $E_{dq}$ and $E_{qd}$ of the tested machine in the manner explained in Chapter 2. These analytical results and those obtained from the experimental "Back-to-Back" method are shown in Figs. 3.12 to 3.15. As shown from these figures, there is a
good agreement between them. This verifies further the validity of the "Analytical" method and the assumptions used in it.
Figure 3.10: The Parameter $E_{dq}$ Obtained from the "Back-to-Back" Method
Figure 3.11: The Parameter $E_{qd}$ Obtained from the "Back-to-Back" Method
Figure 3.12: Comparison between the Values of $E_{dq}$ Obtained from the “Analytical” and “Back-to-Back” Methods, with (a) $AT_q = 0.50$ p.u. and (b) $AT_q = 1.00$ p.u.
Figure 3.13: Comparison between the Values of $E_dq$ Obtained from the "Analytical" and "Back-to-Back" Methods, with
(a) $AT_q = 1.50$ p.u. and (b) $AT_q = 2.00$ p.u.
Figure 3.14: Comparison between the Values of $E_{qd}$ Obtained from the “Analytical” and “Back-to-Back” Methods, with (a) $AT_d=0.50$ p.u. and (b) $AT_d=1.00$ p.u.
Figure 3.15: Comparison between the Values of $E_{qd}$ Obtained from the “Analytical” and “Back-to-Back” Methods, with (a) $AT_d=1.50$ p.u. and (b) $AT_d=2.00$ p.u.
Chapter 4

Verification of the Accuracy of the Modified Two-Axis Frame Model

To verify the accuracy of the modified two-axis frame model introduced in Chapter 2, experimental investigations have been carried out on the tested synchronous machine when it is connected to infinite bus system. The active and reactive power/load angle curves of this synchronous machine at different saturation levels (at different terminal voltages) are obtained experimentally. The measured curves are compared with those calculated using the modified two-axis frame model that includes the cross-magnetizing effect using the parameters obtained from the “Analytical” and “Back-to-Back” methods.

4.1. Calculation of the steady-state power/load angle curves including the cross-magnetizing effect

From the phasor diagram of Fig. 2.3, the following equations can be written for the direct and quadrature axes:

\[ V_i \sin \delta + I_{ad} R_a + E_{dq} - I_{aq} X_{qs} = 0 \]  \hspace{1cm} (4.1)

\[ V_i \cos \delta + I_{aq} R_a + E_{qd} + I_{ad} X_{ds} = E_{fs} \]  \hspace{1cm} (4.2)
where

\[ X_{ds} = X_{mds} + X_L \]  \hspace{2cm} (4.3)

\[ X_{qs} = X_{mq} + X_L \]  \hspace{2cm} (4.4)

From Eqs. (4.1) and (4.2), the d- and q-components of the armature current can be expressed as follows:

\[ I_{ad} = \frac{1}{X_{ds}} (E_f - V_t \cos \delta - I_{aq} R_a - E_{qf}) \]  \hspace{2cm} (4.5)

\[ I_{aq} = \frac{1}{X_{qs}} (V_t \sin \delta + I_{ad} R_a + E_{df}) \]  \hspace{2cm} (4.6)

Then, the output active and reactive powers can be calculated by the following well-known equations [23]:

\[ P = I_{ad} V_{td} + I_{aq} V_{tq} \]

\[ = I_{ad} V_t \sin \delta + I_{aq} V_t \cos \delta \]  \hspace{2cm} (4.7)

\[ Q = I_{ad} V_{tq} - I_{aq} V_{td} \]

\[ = I_{ad} V_t \cos \delta - I_{aq} V_t \sin \delta \]  \hspace{2cm} (4.8)

Substituting Eqs. (4.5) and (4.6) into Eqs. (4.7) and (4.8), the output active and reactive powers can be expressed as follows:

\[ P = P_o + P_{c.m.} \]  \hspace{2cm} (4.9)

\[ Q = Q_o - Q_{c.m.} \]  \hspace{2cm} (4.10)
where

\[
P_a = \frac{V_E f_0}{X_{ds}} \sin \delta + \frac{V_t^2}{2} \left( \frac{1}{X_{qs}} - \frac{1}{X_{ds}} \right) \sin 2\delta + V_t R_s \left( \frac{I_{ds}}{X_{qs}} \cos \delta - \frac{I_{ad}}{X_{ds}} \sin \delta \right),
\]

\[
Q_a = \frac{V_E f_0}{X_{ds}} \cos \delta - \frac{V_t^2}{2} \left( \frac{1}{X_{qs}} + \frac{1}{X_{ds}} \right) + \frac{V_t^2}{2} \left( \frac{1}{X_{qs}} - \frac{1}{X_{ds}} \right) \cos 2\delta,
\]

\[
P_{c.m.} = V_t \left( \frac{E_{dq}}{X_{qs}} \cos \delta - \frac{E_{qf}}{X_{ds}} \sin \delta \right),
\]

and \( Q_{c.m.} = V_t \left( \frac{E_{dq}}{X_{qs}} \sin \delta + \frac{E_{qf}}{X_{ds}} \cos \delta \right). \)

It can be seen from Eqs. (4.9) and (4.10) that the \( P_{c.m.} \) and \( Q_{c.m.} \) terms of the active and reactive power equations are functions of the cross-magnetizing effect and become zero if the cross-magnetizing effect is neglected.

Since the values of the saturated reactances \( X_{ds} \) and \( X_{qs} \) and the parameters representing the cross-magnetizing effect depend on the saturation level determined by the loading conditions of the machine, the active and the reactive powers as functions of the load angle have to be calculated using iterative techniques. The flow chart of the program used for such calculations is given in Fig. 4.1.

### 4.2. Comparison of the Power/Load Angle Curves

To verify the accuracy of the modified two-axis frame model, experimental investigations are carried out on the tested synchronous
Calculate: $E_{fo}$, $I_{do}$, and $I_{qo}$

Calculate: $AT_d = I_f - I_{do}$, $AT_q = I_{qo}$

Determine: $X_{ds}$ and $X_{qs}$

Calculate: $E_{dq}$ and $E_{qd}$

Calculate: $E_r$, $I_d$ and $I_q$

Calculate: $P$ and $Q$

Figure 4.1: The Flow-Chart of the Program Used for the Calculation of the Active and Reactive Power of the Saturated Synchronous Machine
machine. The active and reactive power/load angle curves of the machine at different saturation levels (at different terminal voltages) are obtained experimentally. The measured curves are compared with those calculated using the modified two-axis frame model including the cross-magnetizing voltages obtained from both the "Analytical" and the "Back-to-Back" methods. Figures 4.2 to 4.4 show these results together with those calculated using the traditional two-axis frame model in which the cross-magnetizing effect is neglected. From these figures, the following observations could be made:

1. From Fig. 4.2, it is evident that, when the machine is unsaturated, there are very little or no practical differences among all the methods, and the active and reactive power/load angle curves from all the methods agree well with the measured results. With the increase of the saturation level of the tested machine, the discrepancies between the calculated active and reactive power/load angle curves neglecting the cross-magnetizing effect and the measured results increase.

2. When the tested machine is operating in the saturated region, the modified two-axis model reduces the discrepancies between the calculated active and reactive power/load angle curves and the measured results.
Figure 4.2: The (a) Active and (b) Reactive Power/Load Angle Curves of the Tested Machine, $V_t = 0.62$ p.u., $I_f = 1.775$ p.u.

1. Traditional model  
2. “Analytical” Method  
4. Measurement
Figure 4.3: The (a) Active and (b) Reactive Power/Load Angle Curves of the Tested Machine, $V_i = 1$ p.u., $I_f = 1.775$ p.u.

1. Traditional model
2. "Analytical" Method
4. Measurement
Figure 4.4: The (a) Active and (b) Reactive Power/Load Angle Curves of the Tested Machine, $V_t = 1.14$ p.u., $I_f = 1.775$ p.u.

1. Traditional model  2. "Analytical" Method  
3. In general, both the "Analytical" and "Back-to-Back" methods give almost the same results over the normal operating range. However, when the machine is operating at large load angles, the discrepancies in the active and reactive power/ load angle curves obtained from the two methods differ a little.

4. From these figures, it can be seen that the accuracy of the calculated reactive powers are more influenced by the neglect of the cross-magnetizing effect.
5.1. Summary

This thesis presents an experimental method for the measurement of the saturation characteristic curves in the various intermediate axes of synchronous machines. These curves are needed for the determination of the parameters representing the effect of the magnetic coupling between the direct and quadrature axes of saturated synchronous machines (the cross-magnetizing phenomenon). Using a laboratory synchronous machine, the results obtained from this method, which has been called the “Back-to-Back” method, have been compared with those obtained from the “Analytical” method, which was proposed earlier by Wu [1]. The main aspects of this thesis can be summarized as follows:

Chapter 1 gives an introduction to the traditional two-axis frame model of synchronous machines and the conventional approach of representing saturation in this model.

In Chapter 2, the concept of the cross-magnetizing phenomenon is presented and the inclusion of its effect in the two-axis frame model is
discussed. The “Analytical” method, which was proposed earlier by Wu [1] to calculate the parameters representing this cross-magnetizing effect, is summarized.

Chapter 3 introduces the principle of measuring the intermediate-axis saturation characteristic curves by the proposed “Back-to-Back” method. Applying this method, the saturation characteristic curves in the various intermediate axes of a laboratory synchronous machine are obtained experimentally, and the parameters representing the cross-magnetizing effect of this machine are calculated using these results. The analytical method, which was proposed by Wu [1], has also been used to calculate the same results as those obtained from the experimental “Back-to-Back” method and both results are compared.

In Chapter 4, the inclusion of the parameters that represent the cross-magnetizing effect in the two-axis frame model of synchronous machines is introduced. Using this modified two-axis frame model and the parameters obtained from both the “Back-to-Back” and “Analytical” methods, a comparison between the measured and the calculated values of the active and reactive powers for different load angles and saturation levels is made.

5.2. Conclusions

The investigations conducted in this thesis yield the following conclusions:
1. The proposed experimental “Back-to-Back” method provides an effective means for the measurement of the saturation characteristic curves in the various axes of industrial synchronous machines, which have no field winding in their quadrature axis. For such type of machines, there was no earlier technique to measure these curves experimentally.

2. In comparison with the known methods for measuring the q-axis open-circuit characteristic curves (o.c.c.cs.) of synchronous machines, the “Back-to-Back” method provides an alternative means for the accurate measurement of these curves.

3. In general, the “Analytical” method proposed by Wu for the determination of the saturation characteristic curves in the intermediate axes of a synchronous machine [1] is an accurate method. The saturation characteristic curves obtained from this “Analytical” method agree well with those obtained from the “Back-to-Back” method. However, there are some discrepancies between the results of the angle $\delta'$ curves obtained from these two methods. These discrepancies could be attributed to several factors discussed in detail in Chapter 3. It is believed that these discrepancies could be reduced if these factors could be eliminated.

4. For the unsaturated synchronous machines, the traditional two-axis frame model (neglecting the cross-magnetizing effect) is accurate enough.
However, for the saturated synchronous machines, the modified two-axis frame model, which includes the cross-magnetizing effect, is of much higher accuracy than the traditional model.

5. In comparison to the calculated active powers, the accuracy of the calculated reactive powers is influenced more by the neglect of the cross-magnetizing effect in the two-axis frame model.

6. In general, the results of the active power and reactive power / load angle curves obtained from both the “Analytical” and “Back-to-Back” methods have the same accuracy over the normal operating range. However, when the machine is operating at large load angles, the results obtained from both these methods differ a little.
References


[22] Rankin, A.W., “Per Unit Impedance of Synchronous Machines”, AIEE Transactions, Vol.64, 1945, pp.839-841.


Appendix 1

FORTRAN Program for the Calculation of the Parameters $a_1$ and $a_2$ in the "Analytical" Method

C Program for the Calculation of the Parameters, $a_1$, $a_2$, $C_{d1}$, $C_{d2}$, $C_{q1}$, $C_{q2}$, $d_1$, $d_2$, Used in the Analytical Method

by Yumin Xu, Nov., 1995

C

DIMENSION ATD(20),ATQ(20),YD(20),YQ(20)
DIMENSION BB(40),AA(40,2),AAT(2,40),TEM1(2,2),TEM2(2,2)
DIMENSION TEM3(2),XX(2)
REAL K
COMMON/RECl/TEM1,TEM2

XMDU=.6715
XMQU=.3352

DO 10 I=1,20
  ATD(I)=.12*(I-1)
  ATQ(I)=.12*(I-1)
  YD(I)=XMDU-(-.6715+.05134*ATD(I)-.04718*ATD(I)*ATD(I))
  YQ(I)=XMQU-(-.3352+.0099*ATQ(I)-.0062*ATQ(I)*ATQ(I))
10 CONTINUE

PI=6.*ASIN(.5)
T=.6715+.05134*ATD(I)+.04718*ATD(1)*ATD(I)
K=.25*PI*(XMDU+XMQU)/T
CD1=K/PI/3*(9*SIN(T)+SIN(3*T))
CD2=K/PI/8*(12*T+8*SIN(2*T)+SIN(4*T))
CQ1 = K/PI/3*(8-9*COS(T)+COS(3*T))
CQ2 = K/PI/8*(12*T-8*SIN(2*T)+SIN(4*T))
D1 = K/PI/3*(3*SIN(T)-SIN(3*T))
D2 = K/PI/8*(4*T-SIN(4*T))

DO 20 I=1,20
BB(I) = YD(I)
BB(I+20) = YQ(I)
AA(I,1) = CD1*ATD(I)
AA(I,2) = CD2*ATD(I)*ATD(I)
AA(I+20,1) = CQ1*ATQ(I)
20 AA(I+20,2) = CQ2*ATQ(I)*ATQ(I)

CALL TRA(40,2,AA,AAT)
DO 30 I=1,2
DO 30 J=1,2
TEM1(I,J) = 0.
DO 30 L=1,40
30 TEM1(I,J) = AAT(I,L)*AA(L,J) + TEM1(I,J)
CALL REC(2)

DO 40 I=1,2
TEM3(I) = 0.
DO 40 L=1,40
40 TEM3(I) = AAT(I,L)*BB(L) + TEM3(I)

DO 50 I=1,2
XX(I) = 0.
DO 50 L=1,2
50 XX(I) = XX(I) + TEM2(I,L)*TEM3(L)

WRITE(*,60)XX(1),XX(2)
WRITE(*,61)CD1,CD2
WRITE(*,62)CQ1,CQ2
WRITE(*,63)D1,D2
WRITE(*,64)T,K

60 FORMAT(1X,'a1=',E15.5,5X,'a2=',E15.5)
61 FORMAT(1X,'Cd1=',E15.5,4X,'Cd2=',E15.5)
62 FORMAT(1X,'Cq1=',E15.5,4X,'Cq2=',E15.5)
63 FORMAT(1X,'d1=',E15.5,5X,'d2=',E15.5)
64 FORMAT(1X,'T=',E15.5,6X,'k=',E15.5)

STOP
END
C----CAL TRANSFORMATION OF MATRIX, INPUT X(N,M), OUTPUT Y(M,N)----

SUBROUTINE TRA(N,M,X,Y)
DIMENSION X(40,40), Y(40,40)
DO 5 I=1,N
DO 5 J=1,M
5 Y(I,J)=X(J,I)
RETURN
END

C------CAL RECIPROCAL OF MATRIX, INPUT N & X, OUTPUT Y------

SUBROUTINE REC(N)
COMMON/RECI/X,Y
DIMENSION W(4,4), W1(4,4), X(2,2), Y(2,2)
DO 5 I=1,N
DO 10 J=1,N
10 W(I,J)=X(I,J)
DO 5 J=N+1,2*N
W(I,J)=0.
IF(I.EQ.(J-N)) W(I,J)=1.
5 CONTINUE
DO 15 K=1,N
IF(ABS(W(K,K)).LT.1.E-6) GOTO 40
DO 20 I=1,N
DO 20 J=1,2*N
IF(I.EQ.K)W1(I,J)=W(K,J)/W(K,K)
20 CONTINUE
DO 25 I=1,N
DO 25 J=1,2*N
25 W(I,J)=W1(I,J)
15 CONTINUE
DO 30 I=1,N
DO 30 J=N+1,2*N
30 Y(I,J-N)=W(I,J)
GOTO 35
40 WRITE(*,45)
45 FORMAT(1X,'Main element is zero, STOP!')
STOP 2
35 RETURN
END
Appendix 2

MATLAB Program for the Calculation of $E_{dq}$ and $E_{qd}$ in the “Analytical” Method

\begin{verbatim}
% MATLAB Program for the Calculation of Edq and Eqd in the “Analytical” Method

atd=[.008444 .281265 .351615 .5232 .694786 .866371 1.037956 1.209542];
atd=[atd 1.381127 1.552713 1.724298 2.067469 2.41064 2.839604]';

vqo=[.006143 .201883 .25134 .369835 .48487 .595902 .702388 .803784];
vqo=[vqo .899547 .989134 1.072002 1.215408 1.325417 1.409344]';

atq=[.208699 .229699 .4174 .570599 .647098 .956492 .977491];
atq=[atq 1.340778 1.347579 1.72216 1.741659 1.887352 2.017945
    2.232634]';

vdo=[.07034 .077453 .141219 .193392 .219441 .324367 .331446];
vdo=[vdo .452445 .454677 .575113 .581228 .626343 .665864 .728717]';

xmdu=.6715;
xmqu=.3352;

yd=xmdu-vqo./atd;
yq=xmqu-vdo./atq;
vqo=[0;vqo];
vdo=[0;vdo];
xd=[0;atd];
yd=[0;yd];
xq=[0;atq];
yq=[0;yq];

t=1.1386;
k=pi*(xmdu+xmqu)/4/t;

cd1=k/pi/3*(9*sin(t)+sin(3*t));
cd2=k/pi/8*(12*t+8*sin(2*t)+sin(4*t));
\end{verbatim}
cq1 = \frac{k}{\pi} \times 3 \times (8 - 9 \cos(t) + \cos(3t));

cq2 = \frac{k}{\pi} \times 8 \times (12t - 8 \sin(2t) + \sin(4t));

d1 = \frac{k}{\pi} \times 3 \times (3 \sin(t) - \sin(3t));

d2 = \frac{k}{\pi} \times 8 \times (4t - \sin(4t));

b = [yd; yq];

a = [cd1 * xd, cd2 * xd, * xd; cq1 * xq, cq2 * xq, * xq];

a1a2 = inv(a') * (a' * b);

a1 = a1a2(1);

a2 = a1a2(2);

a1, a2, cd1, cd2, cq1, cq2, d1, d2

eqo = xmdu * xd - a1 * cd1 * xd * xd - a2 * cd2 * xd * (xd * xd);

edo = xmqv * xq - a1 * cq1 * xq * xq - a2 * cq2 * xq * (xq * xq);

eqd = zeros(length(xq), 5);

for i = 1:3

  id = i * .5;

  for j = 1:length(xq)

    bet = atan(xq(j) / id);

    if bet <= (pi/2 - t)

      eqd(j, i) = xq(j) * xq(j) * (a1 * d1 + 3 * a2 * d2 * id);

    else

      b1 = k / pi / 3 * (9 * cos(bet) - cos(3 * bet));

      b2 = k / pi / 3 * (3 * sin(bet) - sin(3 * bet) - 3 * cos(t) - cos(3 * t));

      b3 = k / pi / 3 * (3 * cos(bet) + cos(3 * bet));

      eqd(j, i) = a1 * (b1 - cd1) * id * id + 2 * a1 * b2 * id * xq(j) + (a1 * b3 + 3 * a2 * d2 * id) * xq(j)^2;

    end

  end

end

edq = zeros(length(xd), 5);

for i = 1:3

  iq = i * .5;

  for j = 1:length(xd)

    if xd(j) == 0

      bet = atan(iq / xd(j));

    else

      bet = pi / 2;

    end

    if bet <= (pi / 2 - t)

      edq(j, i) = 3 * a2 * d2 * iq * xd(j)^2 + 2 * a1 * d1 * iq * xd(j) - a1 * cq1 * iq^2;

    else

      edq(j, i) = 3 * a2 * d2 * iq * xd(j)^2 + 2 * a1 * d1 * iq * xd(j) - a1 * cq1 * iq^2;

    end

  end

end
\[ b_4 = \frac{k}{\pi} 3^*(9\sin(b_\theta) + \sin(3b_\theta) - 9\cos(t) + \cos(3t)) \]
\[ b_2 = \frac{k}{\pi} 3^*(3\sin(b_\theta) + \sin(3b_\theta) - 3\cos(t) - \cos(3t)) \]
\[ b_3 = \frac{k}{\pi} 3^*(3\cos(b_\theta) + \cos(3b_\theta)) \]

\[ \text{edq}(j,i) = (a_{1b2} + 3a_{2b}^2i_q)x_d(j)^2 + 2a_{1b3}i_qx_d(j) + a_1(b_{4c}q1)i_q^2; \]

\begin{align*}
&\text{plot}(x_d,v_qo,'*'), x_d,e_qo,'-', x_q,v_do,'*'), x_q,e_do,'-');
&\text{xlabel('ATd,ATq [p.u.] \{Test: "*"; Cal: "-"\}'), ylabel('Vqo,Vdo [p.u.]');}
&\% \text{plot}(x_q,e_{qd}(;1),x_q,e_{qd}(;2),x_q,e_{qd}(;3));
&\% \text{xlabel('ATq [p.u.'], ylabel('Eqd [p.u.] for ATd=.5,1.5[p.u.']);}
&\% \text{plot}(x_d,e_{dq}(;1),x_d,e_{dq}(;2),x_d,e_{dq}(;3));
&\% \text{xlabel('ATd [p.u.'], ylabel('Edq [p.u.] for ATq=.5,1.5[p.u.']);}
\end{align*}
Appendix 3

The Name-Plates of the Tested and Auxiliary Synchronous Machines

H. BUNGART K.G. MADE IN GERMANY VDE 0530
Typ DM-SN/DG 22400-4B3 IP 21/00 I.C.I. B
No. 60 018 2GA 1320-2C JEC
Schaltg V A KVA cos 1/Min Erreg
connect. Excit.
GEN 208 11.1 4. 0.8 1800 110 V
Y IND HZ A
Schaltg V A KVA cos 1/Min Erreg
connect. Excit.
MOT 208 14.6 4. 0.9 1800 110 V
Y KAP 60 HZ 1.5A

Note: The name-plate of the auxiliary synchronous machine is the same as that of the tested machine as shown above, except that its serial no. is 60017, instead of 60018.
Appendix 4

The Per Unit Values of the Tested and Auxiliary Synchronous Machines

In the per unit system used in this thesis, the base values of the terminal voltage and the armature current of the tested synchronous machine are chosen to be their rated voltage and rated current respectively. The base value of the field current is determined using the "X_{md}- base" per unit system [22]. The per unit base values are listed in Table A4.1, and the per unit values of the parameters of the tested machine, which are needed for the analysis, are listed in Table A4.2.

Table A4.1: The Per Unit Base Values of the Tested and Auxiliary Synchronous Machines

<table>
<thead>
<tr>
<th>$V_{ib}$ (V)</th>
<th>$I_{ab}$ (A)</th>
<th>$S_b$ (KVA)</th>
<th>$Z_b$ (Ohm)</th>
<th>$I_{fb}$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>208</td>
<td>11.1</td>
<td>4.0</td>
<td>10.816</td>
<td>0.5828</td>
</tr>
</tbody>
</table>

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Table A4.2: The Per Unit Values of the Tested and Auxiliary Synchronous Machines

<table>
<thead>
<tr>
<th>$X_{mdu}$</th>
<th>$X_{mqu}$</th>
<th>$X_I$</th>
<th>$R_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6715</td>
<td>0.3352</td>
<td>0.1014</td>
<td>0.003744</td>
</tr>
</tbody>
</table>