Improving the Spectral Efficiency of Modulation on Conjugate-Reciprocal Zeros (MOCZ) for Non-Coherent Short Packet Communications

A thesis submitted to the
College of Graduate and Postdoctoral Studies
in partial fulfillment of the requirements
for the degree of Master of Science
in the Department of Electrical and Computer Engineering
University of Saskatchewan
Saskatoon

By
Aiman Asad Siddiqui

©Aiman Asad Siddiqui, December 2022. All rights reserved.
Unless otherwise noted, copyright of the material in this thesis belongs to the author.
Permission to Use

In presenting this thesis in partial fulfillment of the requirements for a Postgraduate degree from the University of Saskatchewan, I agree that the Libraries of this University may make it freely available for inspection. I further agree that permission for copying of this thesis in any manner, in whole or in part, for scholarly purposes may be granted by the professor or professors who supervised my thesis work or, in their absence, by the Head of the Department or the Dean of the College in which my thesis work was done. It is understood that any copying or publication or use of this thesis or parts thereof for financial gain shall not be allowed without my written permission. It is also understood that due recognition shall be given to me and to the University of Saskatchewan in any scholarly use which may be made of any material in my thesis.

Disclaimer

Reference in this thesis to any specific commercial products, process, or service by trade name, trademark, manufacturer, or otherwise, does not constitute or imply its endorsement, recommendation, or favoring by the University of Saskatchewan. The views and opinions of the author expressed herein do not state or reflect those of the University of Saskatchewan, and shall not be used for advertising or product endorsement purposes.

Requests for permission to copy or to make other uses of materials in this thesis in whole or part should be addressed to:

Head of the Department of Electrical and Computer Engineering
University of Saskatchewan
57 Campus Drive
Saskatoon, Saskatchewan S7N 5A9 Canada

OR

Dean
College of Graduate and Postdoctoral Studies
University of Saskatchewan
116 Thorvaldson Building, 110 Science Place
Saskatoon, Saskatchewan S7N 5C9 Canada
Abstract

Future internet of things (IoT) applications need to meet the stringent requirements of ultra high-reliability and ultra low-latency. To meet the ultra low-latency requirements, the IoT networks will be employing the short data packets for data transmission between the devices. Employing the short data packet communications (SPCs) is not straightforward as there are several design problems related to the SPCs which still remain unsolved.

Since the block length for SPCs is finite; the channel estimation is a challenging problem. This is because the conventionally used known pilot symbols to estimate the channel will severely degrade the spectral efficiency of SPCs. Recently a novel non-coherent modulation technique named as modulation on conjugate reciprocal zeros (MOCZ) was proposed which supports the blind detection of transmitted data, i.e., detection without the knowledge of channel. It is also well known that SPCs suffers from data rate loss as compared to the channel capacity limit. Hence, in this thesis, we aim to increase the spectral efficiency of MOCZ.

We improve the spectral efficiency of MOCZ by proposing a technique named as spectrally-efficient modulation on conjugate reciprocal zeros (SE-MOCZ) which combines MOCZ with a technique named as faster-than-Nyquist (FTN) Signaling. Hence; in SE-MOCZ, we end up transmitting the coefficients of MOCZ, modulated on $T$-orthogonal pulses, at a rate faster than the Nyquist limit, i.e., $\tau T$, instead of $T$, where $0 < \tau < 1$. That said, we intentionally introduce inter symbol interference (ISI) between the received samples of SE-MOCZ. To partially remove the ISI, we introduce a discrete-time filter at the receiver. We further optimize the radius of complex zeros of SE-MOCZ in the presence of ISI. Simulation results show the gains of proposed SE-MOCZ in terms of spectral efficiency.
I would like to thank multiple people for the support they provided me during my studies, which made this dissertation a reality. Though, solely thanking them will not do justice to the constant efforts that they made in helping me towards the completion of my dissertation.

First, I would like to thank my supervisors, Prof. Ha. Nguyen and Prof. Ebrahim Bedeer Mohamed, for their constructive and meaningful feedback throughout my program. Their high expertise and proficient skills helped me in carrying out the impactful research. I am glad that I was supervised by them as it helped me in developing proficient skills for my career.

Then, I would like to thank the other members of committee, which include Prof. Brian Berscheid and Prof. Banani Roy from University of Saskatchewan, for the efforts they made in reviewing and evaluating this thesis.

Following that, I would like to extend my gratitude towards my family for their outstanding support during my studies. Last but not the least, I would like to give my thanks to my friends Atefeh, Botao, Muhammad, Khai, Son, Alireza, Ali, Nghia, Minh, in Communications Theories Research Group (CTRG), as it was my pleasure to work with them.
List of Figures

2.1 Block diagram of $M$-QAM. .................................................. 6
2.2 Mapping of the $K = 7$ group of bits to the $K = 7$ constellation points of 2-QAM/BPSK. ... 7
2.3 Mapping of the $K = 7$ group of bits to the $K = 7$ constellation points of 16-QAM. ......... 9
2.4 Block diagram of MOCZ. .................................................. 10
2.5 Mapping of $K = 7$ group of bits onto $K = 7$ complex zeros for BMOCZ message $m = (1, 0, 1, 0, 1, 1, 1)$. Black circles represent complex zeros corresponding to the message $m$, red ring represents the unit circle and the black rings represent the circles of radii $R > 1$ and $1/R$. The decision boundaries between sectors are represented by solid black lines. ............. 10
2.6 Mapping of $K = 7$ group of bits onto $K = 7$ complex zeros for 16-PMOCZ, given the message $m = (0001, 1101, 0010, 0000, 1100, 0111, 0110)$. The solid black lines represents the decision boundary between the sectors. The decision boundaries between the sub-sectors are omitted for clarity of presentation. ............................................. 11
2.7 The unique $MK$ complex zeros which belong to the complex zero codebook $\zeta$ are represented by the cyan circles for $K = 7$ and 16-PMOCZ. .................................................. 12
2.8 Complex zeros when transmitted coefficients are disturbed by the noise at $E_b/N_0 = 19$ dB. Black circles represent the complex zeros of transmitted polynomial and magenta circles represent the complex zeros of received polynomial. ............................................. 14
2.9 Complex zeros for frequency-selective channel, i.e., $L_{ch} = 4$, and no noise scenario. Black circles represent the complex zeros of $X(z)$, i.e., transmit complex zeros, magenta circles represent the complex zeros of received signal, i.e., $Y(z)$, and green circles represent the complex zeros of the channel, i.e., $H(z)$. .................................................. 17
2.10 Complex zeros for MOCZ with frequency-selective channel, i.e., $L_{ch} = 4$, and noise with $E_b/N_0 = 19$ dB. Green circles represent the complex zeros of $H(z)$. .................................................. 18
2.11 Demodulation of the message $m = (0001, 1101, 0010, 0000, 1100, 0111, 0110)$ for 16-QAM. Black circles represents the elements of the discrete-time transmit vector $x$ in IQ-plane, blue circles represents the constellation points for the 16-QAM, and magenta circles represents the elements of the received and equalized vector $y$. The dashed lines represent the boundaries between the constellation points. .................................................. 19
2.12 Detection of $K$ transmit complex zeros of MOCZ when $K = 7$ and $L_{ch} = 3$. Black circles are the complex zeros of $X(z)$, cyan circles are the $MK$ complex zeros of the codebook $\zeta$, magenta circles are the complex zeros of $Y(z)$, and green circles is the complex zeros of $H(z)$. .................................................. 21
2.13 Figure elaborating the difference between Nyquist and FTN signaling. The colored pulses, except black one, represent the RC pulses, where the received waveform representing the summation of these pulses is shown in the black color. Lastly, the vertical dashed-dotted lines represent the sampling intervals .................................................. 28
3.1 A block diagram of the proposed SE-MOCZ. .................................................. 36
3.2 Mapping of $K = 7$ groups of bits onto $K = 7$ complex zeros for BMOCZ message $m = (1, 0, 1, 0, 1, 1, 1)$ . Black circles represent the complex zeros corresponding to the message $m$. The red ring represents the unit circle and the black rings represent the circles of radius $R > 1$ and $1/R$. The solid black lines represents the decision boundary between the sectors. ............. 40
3.3 Mapping of $K = 7$ groups of bits onto $K = 7$ complex zeros with 8-PMOCZ when the message is $m = (111, 110, 001, 101, 000, 111, 110)$. The solid black lines represent the decision boundary between the sectors. The decision boundaries between the sub-sectors are omitted for clarity. 40
3.4 The unique $2KM$ complex zeros which belong to the complex zero codebook $\zeta$ are represented by cyan circles for $K = 7$ and $M = 4$. .................................................. 42
3.5 Complex zeros for BMOCZ with flat fading channel, i.e., $L_{ch} = 1$, and no noise. Black circles are the complex zeros of $X(z)$, and magenta circles are the received complex zeros, i.e., complex zeros of the polynomial $Y(z)$. .................................................. 42
3.6 Complex zeros for SE-BMOCZ with $\tau = 0.9$, flat fading channel, i.e., $L_{ch} = 1$, and no noise. Black circles are the complex zeros of $X(z)$, magenta circles are the complex zeros of $Y(z)$, and white circles are the complex zeros due to ISI resulting from FTN signalling, i.e., complex zeros of the polynomial $G(z)$.

3.7 Complex zeros for SE-BMOCZ with $\tau = 0.9$, flat fading channel, i.e., $L_{ch} = 1$, and $E_b/N_0 = 19$ dB. Black circles are the complex zeros of $X(z)$, magenta circles are the complex zeros of $Y(z)$, and white circles are the complex zeros of $G(z)$.

3.8 Complex zeros of $G(z)$ and the $2KM$ unique complex zeros of the codebook $\zeta$ with $K = 8$ and $\tau = 0.9$. White circles represent the complex zeros of $G(z)$, and cyan circles represent $2KM$ unique complex zeros of the codebook $\zeta$ for SE-BMOCZ.

3.9 Maximizing the distance between complex zero $\alpha_G$ and the closest complex zeros of the codebook $\zeta$ for even $K$. Note that the outer complex zeros of $G(z)$ are omitted for clarity of presentation.

3.10 Maximizing the distance between the complex zero $\alpha_G$ and the closest complex zeros of the codebook $\zeta$ for odd $K$. The outer complex zeros of $G(z)$ are omitted for clarity of presentation.

3.11 Complex zeros of the received signal without implementing the partial-complex-zeros-removal filter at $E_b/N_0 = 19$ dB. Black circles are the complex zeros of $X(z)$, magenta circles are the complex zeros of $Q(z)$, and white circles are the complex zeros of $G(z)$.

3.12 Complex zeros of the signal at the output of partial-complex-zeros-removal filter at $E_b/N_0 = 19$ dB. Black circles are the complex zeros of $X(z)$, magenta circles are the complex zeros of $Q(z)$, and white circles are the complex zeros of $F(z)$.

3.13 Assigning the received complex zeros to $K$ sectors for $K = 7$, $L_{ch} = 2$, and $\tau = 0.9$. Black circles are the complex zeros of $X(z)$, cyan circles are the $2KM$ complex zeros of the codebook $\zeta$, magenta circles are the complex zeros of $Q(z)$, green circle is the complex zeros of $H(z)$, and white circles are the complex zeros of $F(z)$.

3.14 BER performance of BMOCZ and SE-BMOCZ when $K = 7$, $L_{ch} = 10$, and $\tau = 0.9$. BER performance of BMOCZ and SE-BMOCZ using the ML detector when $K = 7$, $L_{ch} = 10$, and $\tau = 0.8$.

3.15 BER performance of BMOCZ and SE-BMOCZ using the RFMD detector when $K = 40$, $L_{ch} = 10$, and $\tau = 0.9$.

3.16 BER performance of SE-BMOCZ using the RFMD detector when $K = 40$, $L_{ch} = 10$, and $\tau = 0.9$.

3.17 BER performance of 4-PMOCZ and SE-4-PMOCZ using the RFMD detector when $K = 20$, $L_{ch} = 7$, and $\tau = 0.9$.
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BMOCZ</td>
<td>Binary Modulation on Conjugate Reciprocal Zeros</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Key</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>DiZeT</td>
<td>Direct Zero Testing</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FTN Signaling</td>
<td>Faster-than-Nyquist Signaling</td>
</tr>
<tr>
<td>IoT</td>
<td>Internet of Things</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter Symbol Interference</td>
</tr>
<tr>
<td>IQ</td>
<td>In-phase and Quadrature</td>
</tr>
<tr>
<td>LUT</td>
<td>Look Up Table</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
</tr>
<tr>
<td>MOCZ</td>
<td>Modulation on Conjugate Reciprocal Zeros</td>
</tr>
<tr>
<td>M-PMOCZ</td>
<td>M-ary Phase Modulation on Conjugate Reciprocal Zeros</td>
</tr>
<tr>
<td>M-QAM</td>
<td>M-ary Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>MSB</td>
<td>Most Significant Bit</td>
</tr>
<tr>
<td>RFMD</td>
<td>Root Finding Minimum Distance</td>
</tr>
<tr>
<td>SPC</td>
<td>Short Packet Communications</td>
</tr>
<tr>
<td>URLLC</td>
<td>Ultra-Reliable Low-Latency Communications</td>
</tr>
</tbody>
</table>
1 Introduction

1.1 Motivation

The emerging applications of internet of things (IoT) will be delay sensitive and will require high-reliability during the wireless data transmission, i.e., they need to fulfil the stringent requirements of ultra-reliable low-latency communications (URLLC). Depending on the application, for example tactile internet systems, the end-to-end latency requirements can go down to 1 ms or below [1]; while the reliability requirements can be as strict as achieving the error rate of $10^{-9}$ [2]. To meet the stringent requirements of low-latency, finite blocklength of the message will be employed as opposed to the conventionally used large blocklength of the message [3]. The wireless transmission which utilizes the finite blocklength of the message is termed as short packet communications (SPCs).

The SPCs poses several challenges, and one of them is the channel estimation. During the wireless transmission, the transmitted signal gets reflected, diffracted, and scattered by the objects present in the environment. As a result, multi-path components of the transmitted signal are received which interfere constructively and destructively, hence, distort the transmitted signal. Conventionally, known pilot symbols are used for the channel estimation such that various equalization techniques can be employed to cancel the channel effect. However, using pilots in SPCs will severely degrade the spectral efficiency of the transmission due the length of the message/data packet being comparable to the pilot length. Further, depending on the application, if the data transfer is sporadic, the channel may need to be estimated with every transmission which will further degrade the spectral efficiency. Hence, non-coherent modulation techniques that do not use pilots for channel estimation are needed for SPCs. One such technique which is named as modulation on conjugate reciprocal zeros (MOCZ) was proposed recently in [4].

The finite blocklength of SPCs result in degradation of the achievable data rate as compared to the channel capacity. To compensate for the data rate loss; recently, faster-than-Nyquist (FTN) signaling has been proposed as a candidate solution for SPCs [5]. The idea of FTN signaling started when J. E. Mazo in 1975, showed that the transmission of the pulses carrying the data can be accelerated by 25% beyond the Nyquist limit without degradation in the bit error rate (BER); however at the cost of increased complexity at the transmitter, receiver or both, due to the intentionally introduced inter symbol interference (ISI) between the received samples. Hence, FTN signaling can improve the spectral efficiency while maintaining the same bandwidth at the cost of increased processing complexity.
1.2 Research Objective

**Improving the spectral efficiency of MOCZ for non-coherent short packet communications.** In order to achieve this objective, we combine MOCZ with FTN signaling by accelerating the transmission of $T$-orthogonal pulses carrying the coefficients of MOCZ by $\tau T$, where $0 < \tau < 1$ is the FTN acceleration parameter. As in MOCZ, the information bits are modulated onto the complex zeros of transmitted baseband signal in the $z$-domain, hence; the ISI due to FTN signaling translates to adding more complex zeros to the received signal. To achieve the noise robustness for transmit complex zeros in the presence of these additional complex zeros, we derive the optimal radius for the modulation of transmit complex zeros in the $z$-domain. We also design the partial-complex-zeros-removal filter to remove part of the additional complex zeros which were introduced due to the FTN signaling ISI.

1.3 Organization of Thesis

The thesis is written in a manuscript format and is organized in various chapters as follows. The first Chapter discusses the motivation and the research objective for this thesis. The second Chapter reviews in details the modulator and demodulator of a conventional modulation technique, i.e., $M$-QAM, and the recently proposed MOCZ to elaborate the difference between both techniques. Further, the second Chapter presents the effect of the wireless channel on both modulation techniques, i.e., $M$-QAM and MOCZ. That said, the literature review is presented towards the end of the Chapter. The third Chapter is presented in the form of manuscript for the paper which is under review for publication in *IEEE Transaction on Wireless Communications*. The chapter discusses the integration of FTN signaling into MOCZ to improve the spectral efficiency of MOCZ, provides the derivation of optimal radius for SE-MOCZ, as well as, the design of the partial-complex-zeros removal filter. The summary and the future research problems are presented in the fourth Chapter.
References


2 Background

In wireless communications, the aim is to reliably transmit the data to the receiver. For data transmission, the incoming bits are converted to the modulation symbols. These modulation symbols are then carried by the Nyquist pulses for the generation of continuous time signal. This continuous time signal is up-converted to the higher frequency known as carrier frequency in order to produce the radio-frequency (RF) signal which will be transmitted wirelessly. During the transmission, due to the scattering, reflection, and diffraction of the transmitted signal, multiple copies of the signal are received. If the duration of all the received copies, i.e., time delay spread, exceeds the duration of modulation symbols of the transmitted signal, these copies will appear as resolvable multi-path components of the transmitted signal. As a result, the channel response can be modelled as a multi-tap filter and is referred to as frequency-selective channel. The effect of frequency-selective channel is that each frequency component in the transmitted signal gets affected by a different channel gain. To cancel the effect of the frequency-selective channel, equalization techniques are employed which require the channel state information (CSI). Conventionally, the known symbols commonly referred as pilot symbols are sent along with the transmit data to acquire the CSI. The pilots can be employed without noticeable degradation in spectral efficiency of the transmission if the packet length of the transmit data is infinite or sufficiently long. The challenge comes in when the data packet length is short.

To fulfil the requirements of ultra low-latency, future internet of things (IoT) applications will be using short packet communications (SPCs). Usage of the pilot symbols to estimate the channel will severely degrade the spectral efficiency of the transmission when the blocklength is finite, i.e., short, because of data packet length being comparable to the pilot length. Another issue for channel estimation in case of SPCs is that the highly dispersive channels cannot be estimated accurately. This is because as the number of channel taps increases, more pilot symbols are needed to correctly estimate CSI; however, we might not be able to send the required number of pilot symbols due to the limited packet length. Lastly, sporadic data transfer between the devices employing SPCs will further increase the challenge as the channel gets changed with every transmission; hence, need to be re-estimated. Therefore, sporadic data transfer will further decrease the spectral efficiency in SPCs if pilots are used to estimate the channel. That said, to overcome the aforementioned issues; non-coherent modulation techniques are required for SPCs. One such technique was recently proposed in [1] which is named as modulation on conjugate reciprocal zeros (MOCZ). As MOCZ is a novel technique, so to elaborate the difference between MOCZ and the conventional modulation scheme such as $M$-QAM modulation, this chapter presents in details the modulator and demodulator of both modulation techniques, as well as, presents the effect of the frequency-selective channel on these modulations.
This chapter is organized as follows: First, the modulators of M-QAM and MOCZ are discussed in Section 2.1. Following that, the effect of wireless channel is discussed in Section 2.2. Section 2.3 discusses the demodulators of M-QAM and MOCZ. Finally, the literature review is presented in Section 2.4.

2.1 Modulators

2.1.1 Modulator of M-QAM

The block diagram of the M-QAM modulation is shown in Fig. 2.1. In the modulation process [2], first, the incoming data bits of the information message \( m \) are grouped into \( K \) groups of bits or symbols, i.e., \( m = (m_1, m_2, \ldots, m_k, \ldots, m_K) \), where \( m_k \) is the \( k \)th group consisting of \( \log_2 M \) bits and \( M \) is the constellation size of M-QAM. These \( K \) group of bits are then mapped into \( K \) constellation points in the IQ-plane using LUTs, where I represents the in-phase axis which indicates the real component of the constellation point and Q represents the quadrature axis which indicate to the imaginary component of the constellation point. This mapping of \( K \) group of bits to the \( K \) constellation points produces the discrete-time transmit signal, i.e., \( x = [x_0, x_1, \ldots, x_k, \ldots, x_{K-1}]^T \), \( x_k \in \mathbb{C} \) which represents the set of complex numbers.

To understand the mapping, let us consider \( M = 2 \), i.e., 2-QAM which is commonly referred as binary phase shift key (BPSK) modulation. In BPSK, each group of bits \( m_k \), \( k = 1, \ldots, K \), consists of only 1 bit. The two possible values of \( m_k \), i.e., 0 and 1, are represented by the two unique constellation points in IQ-plane. Hence, one way of mapping is that if \( m_k = 0 \), it will be represented by the constellation point -1; else, if the \( m_k = 1 \), it will be represented by the constellation point 1. The LUT for axis I for 2-QAM/BPSK is shown in the Fig. 2.2a where Fig. 2.2b shows \( M = 2 \) constellation points in the IQ-plane for 2-QAM/BPSK. Note that the constellation points for 2-QAM/BPSK have Q component equals 0. Fig. 2.2c shows the mapping of the message \( m = (1, 0, 1, 0, 1, 1, 1) \) to the constellation points of 2-QAM/BPSK modulation. As \( m_1 = 1 \); using the LUT, it will be mapped into the constellation point having the value \( 1 + 0j \). The next group of bit \( m_2 = 0 \); hence, it will be mapped to the constellation point \( -1 + 0j \). Similarly, rest of the \( m_k \), \( k = 3, \ldots, 7 \), are mapped to the constellation points as shown in the Fig. 2.2c.

Now, let us discuss the mapping for higher order M-QAM. In M-QAM, \( M > 2 \), as each group of bits \( m_k \) consists of \( \log_2 M \) bits, half of the bits are mapped on the axis I and the remaining half are mapped on the axis Q. Therefore, there will be \( M/2 \) unique numerical values on each axis to uniquely map all possible combination of bits in \( m_k \). To have a clear understanding, consider \( M = 16 \). The LUTs for the axis I and Q for 16-QAM are shown in the Fig. 2.3a. The constellation points corresponding to \( M = 16 \) combinations of \( m_k \), using the gray mapping are shown in the Fig. 2.3b. In the gray mapping, the bits between the neighbouring constellation points differ by only one bit to reduce the probability of error during the detection. That said, consider the message \( m = (0001, 1101, 0010, 0000, 1100, 0111, 0110) \). As \( m_1 = 0001 \), the first two bits in \( m_1 \), i.e., 00 are mapped on the axis I, and the last 2 bits, i.e., 01 are mapped on the axis Q. Using the LUTs for axis I and Q in Fig. 2.3a, we can see that the first two bits in \( m_1 \) will be mapped
Transmitter

Group the bits into \( K \) groups
\( m = (m_1, m_2, \ldots, m_k, \ldots, m_K) \)

\((b_1, b_2, \ldots, b_n)\)

Log \( 2 \) bits

Serial-to-parallel converter

\((\log_2 M)/2\) bits

\((\log_2 M)/2\) bits

LUT for I axis

Map bits to complex number/constellation points

\( \text{Re}\{x\} = \text{Re}\{[x_0, x_1, \ldots, x_{K-1}]^T\} \)

\( \text{Im}\{x\} = \text{Im}\{[x_0, x_1, \ldots, x_{K-1}]^T\} \)

Receiver

Equalization

\( \text{Re}\{\hat{x}\} = \text{Re}\{[\hat{x}_0, \hat{x}_1, \ldots, \hat{x}_{N-1}]^T\} \)

\( \text{Im}\{\hat{x}\} = \text{Im}\{[\hat{x}_0, \hat{x}_1, \ldots, \hat{x}_{N-1}]^T\} \)

De-map complex number/constellation points to bits

De-map complex number/constellation points to bits

Decision for I axis

(\(\log_2 M)/2\) bits

Decision for Q axis

(\(\log_2 M)/2\) bits

Parallel-to-serial converter

\((\log_2 M)/2\) bits

Combine \( K \) group of bits
\( \hat{m} = (\hat{m}_1, \hat{m}_2, \ldots, \hat{m}_k, \ldots, \hat{m}_K) \)
to the sequence of bits

\((\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n)\)

Multipath channel \( h \) and additive noise \( w \)

\( y \)

\( \text{Re}\{\hat{y}\} = \text{Re}\{[\hat{y}_0, \hat{y}_1, \ldots, \hat{y}_{N-1}]^T\} \)

\( \text{Im}\{\hat{y}\} = \text{Im}\{[\hat{y}_0, \hat{y}_1, \ldots, \hat{y}_{N-1}]^T\} \)

Figure 2.1: Block diagram of \( M \)-QAM.
<table>
<thead>
<tr>
<th>Bits Value</th>
<th>Numerical value for the I axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) LUT for axis I.

(b) Constellation points corresponding to $m_k$, $k = 1, \ldots, K$.

<table>
<thead>
<tr>
<th>$m_k$, $k = 1, \ldots, K$</th>
<th>Bits in $m_k$</th>
<th>Mapped constellation points</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>1</td>
<td>$1+0j$</td>
</tr>
<tr>
<td>$m_2$</td>
<td>0</td>
<td>$-1+0j$</td>
</tr>
<tr>
<td>$m_3$</td>
<td>1</td>
<td>$1+0j$</td>
</tr>
<tr>
<td>$m_4$</td>
<td>0</td>
<td>$-1+0j$</td>
</tr>
<tr>
<td>$m_5$</td>
<td>1</td>
<td>$1+0j$</td>
</tr>
<tr>
<td>$m_6$</td>
<td>1</td>
<td>$1+0j$</td>
</tr>
<tr>
<td>$m_7$</td>
<td>1</td>
<td>$1+0j$</td>
</tr>
</tbody>
</table>

(c) Mapping of the message $m = (1, 0, 1, 0, 1, 1, 1)$ to constellation points.

Figure 2.2: Mapping of the $K = 7$ group of bits to the $K = 7$ constellation points of 2-QAM/BPSK.
to the numerical value of 1 on the axis I and last two bits in \( m_1 \) will be mapped to the numerical value of \(-1j\) on the axis Q. Hence, \( m_1 \) will be mapped to the constellation point \( 1 - 1j \) as can be seen from the Fig. 2.3c. Given that \( m_2 = 1101 \), using the LUTs, one can reach to a conclusion that it will be mapped to the constellation point \(-3 - 1j\). Similarly, rest of the \( m_k, k = 3, \ldots, 7 \), are mapped to the constellation points as shown in the Fig. 2.3c.

### 2.1.2 Modulator of MOCZ

The block diagram of MOCZ is shown in the Fig. 2.4. Similar to \( M\)-QAM, the incoming bits in MOCZ are first grouped into \( K \) group of bits, i.e., \( m = (m_1, m_2, \ldots, m_k, \ldots, m_K) \), where \( m_k, k = 1, \ldots, K \), consist of \( \log_2 M \) bits and \( M \) is the constellation size of MOCZ. These \( K \) group of bits will be mapped onto the \( K \) unique complex zeros \( \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_K]^T \) in the \( z \)-domain. Following that, a sequence of \( K + 1 \) coefficients corresponding to these \( K \) complex zeros is generated, i.e., \( x = [x_0, x_1, \ldots, x_k, \ldots, x_K]^T, x_k \in \mathbb{C} \), which represent the discrete-time transmit signal.

To understand how the \( K \) group of bits are mapped onto \( K \) complex zeros, let us consider the binary case, i.e., \( M = 2 \), which is referred as binary modulation on conjugate reciprocal zeros (BMOCZ). First, the unit circle in \( z \)-plane is divided into \( K \) sectors such that \( k \)th group of bits, i.e., \( m_k \), is mapped onto the complex zero in the \( k \)th sector. As a complex zeros can be characterized with its phase and radius, so the phase of \( k \)th complex zeros is calculated as \( 2\pi(k - 1)/K \). The radius of \( k \)th complex zero, i.e., \( |\alpha_k| \), is calculated based on the value of \( m_k \). If \( m_k = 0 \), the radius \( |\alpha_k| \) is chosen \( 1/R \), where \( R > 1 \); however, if \( m_k = 1 \), \( |\alpha_k| \) will be \( R \). Fig. 2.3c shows the mapping of \( K = 7 \) group of bits onto the \( K = 7 \) complex zeros for the message \( m = (1, 0, 1, 0, 1, 1, 1) \). As \( m_1 = 1 \), the radius of the first complex zeros, i.e., \( |\alpha_1| \) is \( R \), where the phase of \( \alpha_1 \) is calculated as \( 2\pi(k - 1)/K = 2\pi(1 - 1)/7 = 0 \). The radius \( |\alpha_2| \) is \( 1/R \) as \( m_2 = 0 \), while the corresponding phase is \( 2\pi(1)/7 \). Similarly, rest of \( m_k, k = 3, \ldots, 7 \), are mapped onto the complex zeros as shown in the Fig. 2.3c.

Now we discuss the mapping of \( K \) group of bits onto the \( K \) complex zeros for higher order MOCZ. The higher order MOCZ is referred to as \( M \)-ary phase modulation on conjugate reciprocal zeros (\( M\)-PMOCZ).

Since in \( M\)-PMOCZ, \( m_k \) contains more than 1 bit, the most significant bit (MSB) in \( m_k \) is modulated onto the radius \( |\alpha_k| \). The remaining \( \log_2 M - 1 \) bits in \( m_k \) are mapped on the phase of \( \alpha_k \). In order to do so, each of \( k \)th sector needs to be further divided into \( M'/M/2 \) sub-sectors. The phase of a sub-sector present in \( k \)th sector is calculated as \( 2\pi(k - 1)/K + 2\pi d/(M'K) \), where \( d \in \{0, \ldots, M' - 1\} \) is the decimal value of the remaining \( \log_2 M - 1 \) bits after excluding the MSB. Fig. 2.4 shows the mapping of group of bits to the complex zeros given 16-PMOCZ for message \( m = (0001, 1101, 0010, 0000, 1100, 0111, 0110) \). As \( m_1 = 0001 \) and the MSB of \( m_1 \) equals 0, the radius \( |\alpha_1| \) is \( 1/R \) as shown in the Fig. 2.4. The decimal value of remaining 3 bits in \( m_1 \), i.e., \( \text{Bi2De}(001) = 1 \), as a result, the phase of \( \alpha_1 \) is \( 2\pi(1 - 1)/7 + 2\pi(1)/((8)(7)) = 2\pi(1)/56 \) as shown in the Fig. 2.4 where \( \text{Bi2De}(.) \) represents the binary to decimal conversion operator. That said, the MSB of \( m_2 \) is 1, so \( |\alpha_2| \) is \( R \) while its phase is \( 2\pi(2 - 1)/7 + 2\pi(5)/((8)(7)) = 2\pi(13)/56 \). Similarly, rest of
<table>
<thead>
<tr>
<th>Bits Value</th>
<th>Numerical value for the I axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>-3</td>
</tr>
<tr>
<td>01</td>
<td>-1</td>
</tr>
<tr>
<td>00</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bits Value</th>
<th>Numerical value for the Q axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>-3j</td>
</tr>
<tr>
<td>01</td>
<td>-1j</td>
</tr>
<tr>
<td>00</td>
<td>1j</td>
</tr>
<tr>
<td>10</td>
<td>3j</td>
</tr>
</tbody>
</table>

(a) LUT for axis I and Q.

(b) Constellation points corresponding to $m_k$, $k = 1, \ldots, K$.

(c) Mapping of the message $m = (0001, 1101, 0010, 0000, 1100, 0111, 0110)$ to constellation points.

Figure 2.3: Mapping of the $K = 7$ group of bits to the $K = 7$ constellation points of 16-QAM.
Transmitter

Group the bits into K groups
\[ m = (m_1, m_2, \ldots, m_k) \]

Map K group of bits onto K complex zeros
\[ (\alpha_1, \alpha_2, \ldots, \alpha_K) \]

Generate polynomial having K+1 coefficients

Multipath channel \( h \) and additive noise \( w \)

Receiver

Combine the bits together to produce the sequence of received bit
\[ (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n) \]

Detect K complex Zeros
\[ (\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_K) \]

De-Map K complex zeros to K group of bits
\[ \hat{m} = (\hat{m}_1, \hat{m}_2, \ldots, \hat{m}_k) \]

**Figure 2.4:** Block diagram of MOCZ.

**Figure 2.5:** Mapping of \( K = 7 \) group of bits onto \( K = 7 \) complex zeros for BMOCZ message \( m = (1, 0, 1, 0, 1, 1, 1) \). Black circles represent complex zeros corresponding to the message \( m \), red ring represents the unit circle and the black rings represent the circles of radii \( R > 1 \) and \( 1/R \). The decision boundaries between sectors are represented by solid black lines.
Figure 2.6: Mapping of $K = 7$ group of bits onto $K = 7$ complex zeros for 16-PMOCZ, given the message $m = (0001, 1101, 0010, 0000, 1100, 0111, 0110)$. The solid black lines represents the decision boundary between the sectors. The decision boundaries between the sub-sectors are omitted for clarity of presentation.

$m_k$, $k = 3, \ldots, 7$, are mapped to the complex zeros as can be seen from Fig. 2.6.

Once the $K$ group of bits are mapped onto the complex zeros, the sequence of $K + 1$ polynomial coefficients, i.e., $x_k$, $k = 0, \ldots, K$, corresponding to the $K$ complex zeros $\alpha_k$, $k = 1, \ldots, K$, are generated which represent discrete-time transmit signal $x$ in MOCZ. A Toeplitz iterator for the polynomial coefficients generation was presented in [1]. However, as will be discussed in Section 2.2, the notation of $z$-transform used in this thesis is different than the notation used in [1]; hence, a modified Toeplitz iterator is presented below. That said, the polynomial’s coefficient vector $x$ is calculated as:

$$x = e^{j\phi_0} \frac{x^{(K)}}{\|x^{(K)}\|_2},$$

(2.1)

where $\phi_0$ is the global phase and the vector $x^{(K)}$ is generated iteratively. The $s$th vector $x^{(s)}$, which is generated in the $s$th iteration, is given as:

$$x^{(s)} = s_1^{(s-1)} - \alpha_s s_2^{(s-1)},$$

(2.2)

where $s = 2, \ldots, K$ and $\alpha_s$ represents the $s$th complex zero. The elements of the vectors $s_1^{(s-1)}$ and $s_2^{(s-1)}$ belong to the vector $x^{(s-1)}$ which was generated in the previous iteration, i.e., iteration number $s - 1$; hence, $s_1^{(s-1)} = [x_0^{(s-1)}, \ldots, x_{(s-1)}^{(s-1)}]^T$, and $s_2^{(s-1)} = [0, x_0^{(s-1)}, \ldots, x_{(s-1)}^{(s-1)}]^T$. The vector $x^{(1)}$, which is required to generate $x^{(2)}$, is given as $x^{(1)} = [1 \ - \alpha_1]$.

As in MOCZ, the unit circle in the $z$-domain is divided into $K$ sectors and the complex zero in the $k$th sector represent $\log_2 M$ bits of $m_k$; hence, in the codebook of MOCZ, there will be $M$ possible complex zeros
in every sector. That said, the complex zero codebook $\zeta$ of MOCZ is defined by the cartesian product of $K$ complex zero sets, i.e., $\zeta = \zeta_1 \times \zeta_2 \times \cdots \times \zeta_K$, where $\zeta_k$ is defined as:

$$\zeta_k = \left\{ \alpha_k^{(0)(R)}, \ldots, \alpha_k^{(M'-1)(R)}, \alpha_k^{(0)(1/R)}, \ldots, \alpha_k^{(M'-1)(1/R)} \right\}, \quad (2.3)$$

where $\alpha_k^{(d)(r)} = re^{j2\pi((k-1)M'+d)/(M'K)}$, $d \in \{0, \ldots, M'-1\}$, and $r \in \{1/R, R\}$. From each of $k$th zero set, one complex zero is selected based on $m_k$. Hence, the codebook $\zeta$ defines $M^K$ combination of the transmit complex zeros vector $\alpha$. The $MK$ unique complex zeros of codebook $\zeta$ for $K = 7$ and $M = 16$ are shown in the Fig. 2.7.

**Huffmann Polynomial and Optimal Radius for MOCZ:** Generally, the complex zeros are unstable such that even noise having a small variance has a tendency to move the location of complex zeros in the $z$-domain. However, if the complex zeros corresponds to a special polynomial named as Huffmann Polynomial, the complex zeros can achieve noise robustness \[1\]. This can be seen from Fig. 2.8a which shows the complex zeros corresponding to the Huffmann polynomial at $E_b/N_0 = 19$ dB, where Fig. 2.8b shows the complex zeros corresponding to the non-Huffmann polynomial, also at the same $E_b/N_0$ value. It can be seen from these two figures that the received complex zeros corresponding to the non-Huffmann polynomial are severely disturbed by the noise as compared to complex zeros corresponding to the Huffmann polynomial. The Huffmann polynomial have three noticeable properties \[1\]. First, its complex zeros are present on two circles which are centered at the origin with the radius $R$ and $1/R$. Secondly, it is important to optimize the radius $R$ to achieve the noise robustness for the transmit complex zeros. The last noticeable property of the Huffmann polynomial is that its auto-correlation, i.e., $a = x^*x^-$, where $x^-$ represent the conjugate and time reversed coefficient vector, helps in channel length estimation and time synchronization due to
its impulse like appearance [1,8]. As it is important to optimize the radius of transmit complex zeros for noise robustness, authors in [1] derived the optimal radius for MOCZ by maximizing the minimum distance between the complex zeros in the codebook \( \zeta \). That said, the optimal radius \( R \) for BMOCZ is given as:

\[
R = \sqrt{1 + 2\lambda \sin(\pi/K)},
\]

(2.4)

where \( \lambda \) is approx. 1/2. For PMOCZ, the radius \( R \) is given as:

\[
R = \sqrt{1 + \left( \ln (2M) - 0.3 \right) \sin \left( \frac{\pi}{KM'} \right)},
\]

(2.5)

where \( \ln \) denotes the natural logarithm. It is worth mentioning that there can be other polynomials that can lead to good noise robustness properties but they are out of the scope of our current research.

### 2.2 Effect of the Wireless Channel

When the transmit signal is passed through the wireless channel, it experiences the fading phenomenon. This phenomenon of transmit signal fading can be categorized into two categories, i.e., large scale fading and small scale fading. The large scale fading takes place over longer distance; whereas, the small scale fading occurs over a shorter distance, i.e., comparable to the wavelength of the transmitted signal.

The large scale fading is sub-divided into two categories named as i) path-loss and ii) shadowing. The phenomenon of path-loss is defined as the loss in the transmit signal power due to the constructive and destructive interference between multi-path components of the transmit signal, generated by a fixed set of reflectors present in the channel, given that the signal travels through a long distance, i.e., 100-1000 m [4]. Hence, the exact value of path-loss is deterministic at a particular distance. The power loss that occurs due to the reflection, diffraction, scattering, and absorption of the transmitted signal by random objects such as buildings, trees, hills, etc., in the channel, given that the signal travels through a distance much greater than the wavelength of the transmitted signal, is referred as shadowing.

On the other hand, the small scale fading is categorized into i) multi-path propagation effect and ii) time variance of the channel. The random variation in the signal power over shorter distances, i.e., less than the wavelength, due to the constructive and destructive interference between the multi-path components of the transmitted signal is referred as multi-path propagation effect. The time variance of the channel is defined as the phenomenon of change in the channel state with time; either, due to the movement of objects present in the environment, or due to the movement of transmitter, receiver, or both.

Now, we will discuss the important terms related to the multi-path propagation. The first term related to the multi-path propagation is the time delay spread \( T_d \) which is defined as time duration between the arrival of first and last multi-path component of the transmitted signal. If \( T_d \) is less than the symbol duration \( T \) of the transmitted signal, then the multi-paths cannot be resolved and the channel can be estimated with a single-tap filter. The inverse of \( T_d \) is the coherence bandwidth \( B_c \) of the channel, i.e., \( B_c = 1/2T_d \), which is defined as the bandwidth over which the channel response in the frequency domain remains constant. Given
Figure 2.8: Complex zeros when transmitted coefficients are disturbed by the noise at $E_b/N_0 = 19$ dB. Black circles represent the complex zeros of transmitted polynomial and magenta circles represent the complex zeros of received polynomial.
that \( B \) is the bandwidth of the transmit signal, if \( T_d \) is less than \( T \), so \( B_c \) will greater than \( B \) because of the inverse relationship between symbol duration and bandwidth. Hence, for the single-tap channel, all the frequency components in the transmitted signal are affected by the same channel gain; therefore, such type of channel is referred as flat fading channel. If the time delay spread \( T_d \) exceeds the time duration \( T \), so there will be multiple resolvable multi-path components of the transmitted signal in the time domain and the channel can be modelled by a multi-tap filter. In the frequency domain, it results in different frequency components in the transmitted signal being affected by the different channel gains; hence, this channel is referred to as frequency-selective channel. The frequency-selective channel is modelled by the convolution operation between the transmitted signal and channel response in the time domain; whereas, in the frequency domain, it is modelled by the multiplication operation between the transmitted signal and the channel response.

That said, we will now discuss the terms related to the time variance of the channel. First, the coherence time \( T_c \) is defined as the time period over which the channel state remains the same. If \( T_c \) is much greater than the symbol duration \( T \) of the transmitted signal, it means the channel state changes slowly and more than one transmitted symbol will be affected by the same channel conditions or channel response. This phenomenon is referred as slow fading. If \( T_c \) is less than \( T \), so single transmitted symbol will be affected by the various channel responses and such a phenomenon is hard to deal with and is referred as fast fading. The inverse of \( T_c \) is the doppler spread \( B_d \), i.e., \( B_d = 1/4T_c \). The doppler spread is the frequency delay spread of the channel which occurs due to the movement of the transmitter, receiver, or both. The reason behind doppler spread is the change in the carrier frequency of the transmitted signal by the doppler frequency \( f_d \) due to the movement. If \( T_c \) is less than \( T \), so \( B_d \) will be greater than \( B \) and the received signal will experience spectral broadening.

In this thesis, we assume that: (i) the channel is frequency-selective, and (ii) the channel response remains the same over blocklength of the message. That said, the discrete-time received signal vector \( y \) in the presence of the frequency-selective channel \( h \) and the white Gaussian noise \( w \) is given as:

\[
y = x * h + w, \tag{2.6}
\]

where * represent the convolution operator which characterize the effect of the frequency-selective channel on transmit signal \( x \) in the time domain. The discrete-time frequency-selective channel \( h \) is defined as [4]:

\[
h = \sum_{l=0}^{L_{ch} - 1} a_l \delta[m - \phi_l], \tag{2.7}
\]

where \( L_{ch} \) is the total number of channel taps, \( a_l \) is the magnitude of the \( l \)th tap, and \( \phi_l \) is the \( l \)th delay.

### 2.2.1 Effect of Channel on M-QAM

In M-QAM, the elements of the received vector \( y \), i.e., \( y_n, n = 0, \ldots, N - 1 \), where \( N = K \), are corrupted from the inter symbol interference (ISI) due to the frequency-selective channel \( h \). To cancel the ISI before the detection, equalization techniques are utilized. The equalization techniques can be classified into two
categories, i.e., i) linear equalization and ii) non-linear equalization. The linear equalization techniques includes zero forcing (ZF) equalization and minimum mean-square error (MMSE) equalization [4]. The ZF equalizers performs the equalization by multiplying the inverse of a known channel response with the frequency response of received signal. As a result, if the magnitude of the channel response is very low, there will be huge noise enhancement due to the ZF equalization. That said, the MMSE equalizers minimizes the square of error between the estimated and the original data symbols for the cancellation of channel effect. In the frequency domain, it is equivalent to dividing the frequency response of the received signal with the sum of the channel response and the noise. Hence, MMSE does not lead to as much noise enhancement as compared to the ZF equalizers. The non-linear equalization results in better performance as compared to the linear equalization. That said, the equalizers which fall under the category of non-linear equalizers include decision feedback (DF) equalizers. The DF equalizers uses a feedback filter to feed the detected sequence back into the system in order to improve the detection. The downside of DF equalizers is that they suffer from the error propagation [4]. To achieve the optimal performance, the maximum likelihood sequence estimation (MLSE) equalization can be employed. The MLSE estimates the probability of transmitted data sequence based on the received sequence of data or symbols. Given the received data sequence, whichever sequence of transmit data maximizes the likelihood of transmit data, that sequence will be chosen as the transmitted data sequence. The disadvantage of MLSE is that its complexity increases with the delay spread length [4].

The linear and non-linear equalization techniques require the channel state information which is acquired usually with the help of pilot symbols. For the scenario when channel state information is not available, blind equalization such as those presented in [5], [6], and [7] can be employed.

2.2.2 Effect of the Channel on MOCZ

Similar to M-QAM, the elements of the receive vector \( y \) in MOCZ, i.e., \( y_n, n = 0, \ldots, N - 1 \), where \( N = K + L_{ch} \), are also corrupted from ISI due to the frequency-selective channel. As discussed earlier, the information bits in MOCZ are modulated onto the complex zeros in the \( z \)-domain of the transmit signal \( x \); therefore, MOCZ does not require equalization techniques to cancel the ISI introduced by the frequency-selective channel. To understand this, let us re-write (2.6) in the \( z \)-domain as:

\[
Y(z) = X(z)H(z) + W(z),
\]

where \( Y(z), X(z), H(z), \) and \( W(z) \) are the \( z \)-domain polynomials of \( y, x, h, \) and \( w \), respectively. The polynomial \( X(z) \) corresponding to the transmit coefficients vector \( x \) is defined as:

\[
X(z) = \sum_{k=0}^{K} x_k z^{-k},
\]

where \( x_k \) is the \( k \)th coefficient in the vector \( x \) and the degree of \( X(z) \) is \( K \) if \( x_K \neq 0 \). Similarly, the degrees of \( Y(z), H(z), \) and \( W(z) \) are \( N - 1, L_{ch} - 1, \) and \( N - 1, \) respectively [7]. Since a polynomial can be represented

\(^1\) Note that the notation of the \( z \)-transform in (2.9) is slightly different than the notation used in [1].
as the product of its complex zeros if it has a degree greater than 1; therefore, \( X(z) \) can be represented as:

\[
X(z) = x_0 \prod_{k=1}^{K} (z - \alpha_k),
\]

(2.10)

where \( x_0 \) is the constant term in (2.9) and \( \alpha_k \) is the \( k \)th complex zero of \( X(z) \), i.e., \( X(\alpha_k) = 0, \ k = 1, \ldots, K \). In the same manner, the polynomials \( H(z) \), \( W(z) \), and \( Y(z) \) can be expressed as the products of their complex zeros. Hence, it is evident from (2.8) that the frequency-selective channel will introduce \( L_{ch} - 1 \) extra complex zeros in the received signal. This is the reason that the length of received vector \( y \) in case of MOCZ is \( N > K \), as the received vector \( y \) represent the coefficients corresponding to the complex zeros of \( X(z) \) and \( H(z) \). If the magnitude of \( X(z)H(z) \) in (2.8), i.e., \( |X(z)H(z)| \), is greater than the magnitude of the noise, i.e., \( |W(z)| \), the noise \( W(z) \) will not introduce the extra complex zeros in the received signal [8]; however, this noise will disturb the positions of complex zeros of \( X(z) \) and \( H(z) \) from their original locations in the \( z \)-domain. Fig. 2.9 shows the complex zeros of transmit signal \( X(z) \) and received signal \( Y(z) \), in the presence of frequency-selective channel \( H(z) \) with \( L_{ch} = 4 \) and no noise. It can be seen from the figure that the number of received complex zeros is greater than the number of transmit complex due to the extra complex zeros introduced by the channel \( H(z) \). Fig. 2.10 shows the complex zeros of \( X(z) \) and \( Y(z) \), at \( E_b/N_0 = 19 \) dB. It can be seen from the figure that the received complex zeros are disturbed from their actual locations due to the noise \( W(z) \). That said one can detect the \( K \) transmitted zeros non-coherently, i.e., without the knowledge of the complex zeros introduced by the channel, using the MOCZ codebook as will be explained in the next section.
Figure 2.10: Complex zeros for MOCZ with frequency-selective channel, i.e., $L_{ch} = 4$, and noise with $E_b/N_0 = 19$ dB. Green circles represent the complex zeros of $H(z)$.

2.3 Demodulator

2.3.1 Demodulator of $M$-QAM

To detect the transmitted signal $\hat{x}$ based on the equalized signal $\hat{y}$ in $M$-QAM, the minimum distance rule is used. Hence, the value of each $n$th received and equalized sample, i.e., $\hat{y}_n$, $n = 0, ..., N - 1$, where $N = K$, is compared with all possible $M$ constellation points in the IQ-plane, whichever constellation points is found closer to $\hat{y}_n$, that constellation points is chosen as the $n$th transmitted constellation point. Fig. 2.11 shows the demodulation of the message $m = (0001, 1101, 0010, 0000, 1100, 0111, 0110)$ for 16-QAM. It can be seen that the first received and equalized sample $\hat{y}_0$ fall within the boundary of the constellation point $1 - 1j$, i.e., it is found closer to the constellation point $1 - 1j$ as compared to the other constellation points; hence, $\hat{x}_0$ is estimated as $1 - 1j$. The second received and equalized sample $\hat{y}_1$ is found closer to the constellation point $-3 - 1j$; therefore, $\hat{x}_1$ is estimated as $-3 - 1j$. Similarly, the rest of $\hat{x}_k$, $k = 2, ..., 6$, can be determined by comparing the minimum distance between the received and equalized sample $\hat{y}_n$ and the constellation points. Once $\hat{x} = [\hat{x}_0, \hat{x}_1, ..., \hat{x}_{K-1}]^T$ is determined, it will be demodulated to the group of bits $\hat{m}_k$.

2.3.2 Demodulator of MOCZ

For the non-coherent detection of MOCZ, three detectors were proposed in [1] namely the root finding minimum distance (RFMD) detector, maximum likelihood (ML) detector, and an efficient detector named as direct zero-testing (DiZeT) detector.
Figure 2.11: Demodulation of the message \( \mathbf{m} = (0001, 1101, 0010, 0000, 1100, 0111, 0110) \) for 16-QAM. Black circles represents the elements of the discrete-time transmit vector \( \mathbf{x} \) in IQ-plane, blue circles represents the constellation points for the 16-QAM, and magenta circles represents the elements of the received and equalized vector \( \hat{\mathbf{y}} \). The dashed lines represent the boundaries between the constellation points.

i) Root Finding Minimum Distance (RFMD) detector:

The RFMD detector detects \( K \) transmitted complex zeros based on the minimum distance between the received complex zeros and the \( MK \) complex zeros of the codebook \( \mathbf{\zeta} \). First, the received complex zeros corresponding to the received coefficient vector \( \mathbf{y} \) are determined by using a root finding technique. Then, each of the received complex zero \( \alpha_n, \, n = 1, \ldots, N - 1 \), is classified into one of the \( K \) sectors using the following criterion [1]:

\[
\Xi_k = \left\{ n \mid \forall k' \neq k : \min_{d \in \{0, \ldots, M' - 1\}} \min_{r \in \{1/R, R\}} d(\alpha_n, \alpha_k^{(d)(r)}) \leq \min_{d \in \{0, \ldots, M' - 1\}} \min_{r \in \{1/R, R\}} d(\alpha_n, \alpha_k^{(d)(r)}) \right\},
\]

(2.11)

where \( \Xi_k \) defines the set of the indices \( n \) of received complex zeros, which are classified in the \( k \)th sector, and \( d(\alpha_n, \alpha_k^{(d)(r)}) \) represents the Euclidean distance between the \( n \)th received complex zero \( \alpha_n \) and the codebook complex zero \( \alpha_k^{(d)(r)} \) belonging to the \( k \)th sector; therefore, \( \alpha_k^{(d)(r)} = r e^{j2\pi((k-1)M'+d)/M'K} \), \( d \in \{0, \ldots, M' - 1\} \), and \( r \in \{1/R, R\} \). Hence, the indices of received complex zeros, which are found closer to one of the codebook zero corresponding to the \( k \)th sector as compared to the codebook zeros of the other sectors, are assigned to \( \Xi_k \). This implies that more than one received complex zero can be assigned to \( k \)th sector. Therefore, one needs to eliminate the extra complex zeros classified in the \( k \)th sector; hence, pick only one received complex zero from each sector as the transmitted complex zero based on the minimum distance of received complex zero from the codebook zeros. This method of eliminating the extra complex zeros from
the kth sector; hence, finding the radius $\hat{r}_k$ and phase $\hat{d}_k$ corresponding to the kth transmitted complex zeros can be mathematically described as [1]:

$$[\hat{r}_k, \hat{d}_k] = \arg \min_{d \in \{0, \ldots, M' - 1\}} \min_{r \in \{1/R, R\}} d(\alpha_k^{(d)}(r), \alpha_n).$$  \quad (2.12)$$

If MSB of $\hat{m}_k$ is denoted by $\hat{c}_k$; then, $\hat{c}_k$ is defined as:

$$\hat{c}_k = \begin{cases} 
0, & \hat{r}_k = 1/R \\
1, & \hat{r}_k = R 
\end{cases}. $$  \quad (2.13)$$

That said, the least significant bits $\hat{b}_k$ in $\hat{m}_k$ are represented by the binary equivalent value of $\hat{d}_k$, i.e., $\hat{b}_k = \text{De2Bi}(\hat{d}_k)$.

Fig 2.12 shows the detection process of MOCZ using the RFMD detector. The Figure shows that the received complex zero denoted as $\hat{\alpha}_1$ is assigned to the first sector. This is because the distance of $\hat{\alpha}_1$ from the codebook zeros at the phase 0 and radius $R$ was found to be small as compared to the distance of $\hat{\alpha}_1$ from codebook zeros corresponding to the rest of sectors. Following the same criterion, the rest of the received complex zeros are assigned to one of the $K$ sectors. After the received complex zeros are assigned to the $K$ sectors, the extra complex zeros (if there are any) from each sector are eliminated. Fig. 2.12 shows that two received complex zeros are classified in the first sector; one of the received complex zero is close to the codebook zero having the radius $R$, and the other received complex zero is close to the codebook complex zero having the radius $1/R$. However the distance of $\hat{\alpha}_1$ from the codebook zero having the radius $R$ is less as compared to the distance of the other received complex zero from the codebook zero having the radius $1/R$. Therefore, the received complex zero denoted as $\hat{\alpha}_1$ is decided as the first transmitted complex zero having the phase 0 and radius $R$. That said, the second sector have only one received complex zero $\hat{\alpha}_2$, which is close to the codebook zero having the radius $1/R$; therefore, $\hat{\alpha}_2$ is decided to be the second transmitted zero having the radius $1/R$ and phase $(2\pi)/7$. Similarly, the rest of the transmitted complex zeros are determined.

If a kth sector is found to be empty, i.e., $\Xi_k = \{\}$, the extra complex zeros in the neighbouring sectors can be searched to classify the closest complex zero from the neighbouring sector into the empty sector. Similar performance was observed in our simulation results when instead of picking the extra closest complex zeros from the neighbouring sector, the bits for $\hat{m}_k$ were assigned randomly for a scenario when a sector was found to be empty.

ii) Maximum Likelihood (ML) Detector:

The authors in [1], derived the maximum likelihood ML detector for MOCZ, which is given as:

$$\arg \max_{\alpha \in \zeta} p(y|\mathbf{x}(\alpha)) = \arg \min_{\alpha \in \zeta} \| (\mathbf{V}_\alpha^H \mathbf{V}_\alpha)^{-1/2} \mathbf{V}_\alpha^H y \|_2^2, $$  \quad (2.14)$$

where $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_K]^T$, $\mathbf{V}_\alpha^H$ is the generalized Vandermonde matrix, and $\mathbf{V}_\alpha$ is the Hermitian of $\mathbf{V}_\alpha^H$. The generalized Vandermonde matrix is defined below, however, in a slightly different manner than in [1].
Figure 2.12: Detection of $K$ transmit complex zeros of MOCZ when $K = 7$ and $L_{ch} = 3$. Black circles are the complex zeros of $X(z)$, cyan circles are the $MK$ complex zeros of the codebook $\zeta$, magenta circles are the complex zeros of $Y(z)$, and green circles is the complex zeros of $H(z)$.

due to different notation of $z$-transform being used in my research:

$$V_{\alpha}^H = \begin{pmatrix} 1 & \alpha_1^{-1} & \alpha_1^{-2} & \ldots & \alpha_1^{-(N-1)} \\ 1 & \alpha_2^{-1} & \alpha_2^{-2} & \ldots & \alpha_2^{-(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_K^{-1} & \alpha_K^{-2} & \ldots & \alpha_K^{-(N-1)} \end{pmatrix}. \quad (2.15)$$

That said, (2.14) will be evaluated for all the possible combinations of $K$ complex zeros from the codebook $\zeta$. Whichever combination of the $K$ complex zeros, i.e., $\alpha$, minimizes the objective function in (2.14), that combination will be selected as the combination of transmitted complex zeros. The transmitted message $\hat{m}$ will then be determined based on the sequence of the detected complex zeros.

iii) Direct Zero-Testing (DiZeT) Detector:

The simplification of ML detector (2.14) leads to an efficient detector named as direct zero testing (DiZeT) detector \[1]. As the $k$th complex zero for BMOCZ is defined as $\alpha_k = r_k e^{2\pi(k-1)/K}$, $k = 1, \ldots, K$, where $r_k$ is the radius of $\alpha_k$, therefore, if one defines $V_{\alpha}^H V_{\alpha}$ in (2.14) with its equivalent matrix $C_{\alpha}$, i.e., $C_{\alpha} = V_{\alpha}^H V_{\alpha}$,
the elements of matrix $C_{\alpha}$ are given as:

$$c_{k,m}(\alpha_k, \alpha_m) = \sum_{n=0}^{N-1} (\alpha_k^{-1} \alpha_m^{-1})^n,$$

$$= \sum_{n=0}^{N-1} (r_k^{-1} r_m^{-1})^n e^{-j2\pi n(k-m)/K}. \quad (2.16)$$

For the diagonal elements of matrix $C_{\alpha}$, (2.16) simplifies as $c_{k,k}(\alpha_k) = 1 - r_k^{-2N}/1 - r_k$. As for the off diagonal elements, if the transmitted bits are equally likely, the expected value of $(r_k r_m)^{-1}$ in (2.16) will be 1, i.e., $\mathbb{E}[(r_k r_m)^{-1}] = (R(1/R))^{-1} = 1$. Therefore, for independent and identically distributed bits, the off diagonal elements of matrix $C_{\alpha}$ will be approximated by zero because $\sum_{n=0}^{N/K-1} e^{-j2\pi n(k-m)/K} \sum_{n=0}^{N-1} e^{-j2\pi n(k-m)/K} = 0$, given that $N/K \in \mathbb{N}$, where $\mathbb{N}$ is the natural number. Hence, $(\mathbf{V}_\alpha^H \mathbf{V}_\alpha)^{-1/2}$ in (2.14) can be approximated with the diagonal matrix $\mathbf{W}_{|\alpha|}$ having the $k$th diagonal element defined as $w(|\alpha_k|) = c_{k,k}^{-1/2}(\alpha_k)$. Moreover, from (2.9) and (2.15), it can be concluded that $\mathbf{V}_\alpha^H \mathbf{y}$ in (2.14) represents the $z$-domain polynomial of the received coefficient vector $\mathbf{y}$ when evaluated at the transmitted complex zeros, i.e., $\mathbf{V}_\alpha^H \mathbf{y} = [Y(\alpha_1), Y(\alpha_2), \ldots, Y(\alpha_K)]^T$. It implies that the detection of $k$th transmitted complex zero can be performed independently from the other $K - 1$ complex zeros. As a result, ML detector in (2.14) simplifies as follows for BMOCZ [1]:

$$\hat{\alpha}_k = \arg \min_{\alpha_k \in \{R, R^{-1}\} e^j2\pi n(k-1)/K} |w(|\alpha_k|)Y(\alpha_k)|. \quad (2.17)$$

That said $\hat{\alpha}_k$ is calculated as:

$$\hat{\alpha}_k = \frac{\left| \frac{1}{2} \left[ \left| \frac{1 - R^{-2N}}{1 - R^{-2}} \frac{1}{\frac{1 - R^{-2N}}{1 - R^{-2}}^{1/2}} \mathbf{Y}(R^{-1} e^{j\pi \alpha k}) \right| \mathbf{Y}(R e^{j\pi \alpha k}) \right] \right|}{\left( \left( \frac{1 - R^{-2N}}{1 - R^{-2}} \right)^{1/2} \mathbf{Y}(R^{-1} e^{j\pi \alpha k}) \right) - \left( \left( \frac{1 - R^{-2N}}{1 - R^{-2}} \right)^{1/2} \mathbf{Y}(R e^{j\pi \alpha k}) \right)^2}, \quad (2.18)$$

By ignoring the common term, i.e., $(1 - R^{-2N})^{-1/2}$ and replacing the term $\sqrt{(1 - R^{-2N})}$ with its equivalent, i.e., $\sqrt{\frac{(1 - R^{-2})(1 - R^{-2N})}{(1 - R^{-2})^2}} = R^{-N+1}$, (2.18) can be written as:

$$\hat{\alpha}_k = \frac{1}{2} \left( \frac{1}{2} \left[ \left| \frac{1 - R^{-2N}}{1 - R^{-2}} \frac{1}{\frac{1 - R^{-2N}}{1 - R^{-2}}^{1/2}} \mathbf{Y}(R^{-1} e^{j\pi \alpha k}) \right| \mathbf{Y}(R e^{j\pi \alpha k}) \right] \right), \quad (2.19)$$

The same approximation considered above will be used for M-PMOCZ; hence, $\hat{\alpha}_k$ and $\hat{d}_k$ for M-PMOCZ can be calculated, respectively, as:

$$\hat{d}_k = \arg \min_{d \in \{1, \ldots, M'\}} \min \left\{ R^{-N+1} \left| Y \left( R^{-1} \omega_{M'} M' - M' + d \right) \right|, \left| Y \left( R \omega_{M' K} M' - M' + d \right) \right| \right\}. \quad (2.20)$$
\[ \hat{c}_k = 1 + \text{sign} \left( R^{-N + 1} \left| Y \left( R^{-1} \omega_{M'}^k M' + d_k \right) - Y \left( R\omega_{M'}^k M' + d_k \right) \right| \right), \] (2.21)

where \( \omega_{M'} = e^{j2\pi/M'} \). That said, \( \hat{b}_k \) can be estimated from \( \hat{d}_k \), i.e., \( \hat{b}_k = \text{De}2\text{Bi}(\hat{d}_k) \).

**FFT Implementation of DiZeT:** The DiZeT detector can be implemented using the fast fourier transform (FFT). To do so, let us define the discrete fourier transform (DFT) matrix \( S \) of dimensions \( N \times N \). The elements of \( S \) are given as \( S_{u,v} = e^{-j2\pi uv/N} \), where \( u = 1, 2, \ldots, N \) and \( v = 1, 2, \ldots, N \). Also, let \( D_R \) and \( D_{R-1} \) be \( N \times N \) diagonal matrices with their respective diagonal vectors given as \([1, R^1, R^2, \ldots, R^{N-1}]^T \) and \([1, R^{-1}, R^{-2}, \ldots, R^{-(N-1)}]^T \). Finally, let \( \hat{D}_R \) and \( \hat{D}_{R-1} \) be the matrices of the dimensions \((N + N') \times N \), i.e., \( \hat{D}_R = (D_R 0_{N \times N'})^T \) and \( \hat{D}_{R-1} = (D_{R-1} 0_{N \times N'})^T \), where \( N' = QKM' - N \) and \( Q = \text{ceil}(N/K) \). Having said that, the vector of \( Y(z) \) evaluated at the complex zeros having the radius \( R \), i.e., \( [Y(Re^{j2\pi(0)/QKM'}), \ldots, Y(Re^{j2\pi(QKM' - 1)/QKM'})] \), can be calculated as:

\[
Y(\alpha_Q^R) = S\hat{D}_{R-1}y = \begin{pmatrix}
\sum_{n=0}^{N-1} y_n R^{-n} e^{-j2\pi n \frac{0}{QKM'}} \\
\vdots \\
\sum_{n=0}^{N-1} y_n R^{-n} e^{-j2\pi n \frac{QKM' - 1}{QKM'}}
\end{pmatrix} = \text{FFT}(\hat{D}_{R-1})y. \] (2.22)

For BMOCZ, picking every \( Q \)th sample from (2.22) will result in \( Y(Re^{j2\pi(k-1)/K}) \), \( k = 1, \ldots, K \), which is required in (2.19) to calculate \( \hat{c}_k \). Similarly, for \( M \)-PMOCZ, upsampling of (2.22) by a factor of \( Q \) will produce \( Y \left( R\omega_{M'}^k M' + d \right) \) which is required in (2.20) and (2.21) to estimate \( \hat{d}_k \) and \( \hat{c}_k \), respectively, where \( k = 1, \ldots, K \) and \( d = 1, \ldots, M' \). Hence, after upsampling of (2.22), first \( M' \) samples corresponds to \( k = 1 \), the following \( M' \) samples corresponds to \( k = 2 \), and so on. In the same manner, \( Y(\alpha_Q^{R-1}) \) is defined as:

\[
Y(\alpha_Q^{R-1}) = S\hat{D}_Ry = \begin{pmatrix}
\sum_{n=0}^{N-1} y_n R^n e^{-j2\pi n \frac{0}{QKM'}} \\
\vdots \\
\sum_{n=0}^{N-1} y_n R^n e^{-j2\pi n \frac{QKM' - 1}{QKM'}}
\end{pmatrix} = \text{FFT}(\hat{D}_R)y, \] (2.23)

to obtain \( Y(R^{-1}e^{j2\pi(k-1)/K}) \), \( k = 1, \ldots, K \), for BMOCZ and \( Y \left( R^{-1}\omega_{M'}^k M' + d \right) \), \( k = 1, \ldots, K \), and \( d = 1, \ldots, M' \), for PMOCZ. Simulation results in [1] showed that DiZeT detector is best among the three detectors, namely RFMD, ML, and DiZeT detector, in terms of balancing between the low-complexity and performance.
2.4 Literature Review

2.4.1 Non-Coherent Communication for SPCs

Some studies have been conducted which proposed the non-coherent transmission/detection schemes for SPCs to avoid the pilot overhead. For example; in [9], a hybrid composite hypothesis test (HCHT) and generalized likelihood ratio test based non-coherent receiver was proposed for the scenario when the transmission is made by utilizing scatter radio frequency shift key. Following that, a blind index modulation technique for non-coherent detection of orthogonal frequency division multiplexing index modulation (OFDM-IM) scheme was proposed in [10]. In blind OFDM-IM, the data is transmitted via $K$ active sub-carriers out of total $N$ sub-carriers; hence, the information bits are uniquely represented by the indices of active sub-carriers. The detection can simply be performed by finding the combination of $K$ sub-carriers (valid according to the transmit signal codebook) which have maximum power. The authors studied the scheme for single input multiple output (SIMO) scenario and showed through the simulation results that the received data reliability increases with the number of antennas. That said, a novel blind transmission technique named as pilot-less one-shot (PLOS) transmission, which is based on sparse vector coding, was proposed in [11]. The mapping was achieved by encoding the control information into the non-zero blocks of sparse vector; whereas, the non-zero position of the chosen blocks encoded the transmit data. For the blind detection, deep neural network (DNN) was employed to achieve the purpose. Recently, the authors in [12] proposed the blind receiver design for uplink MIMO single-carrier interleaved frequency division multiple access (SC-IFDMA) technique to perform the blind data detection in the scenario of SPCs. The receiver first perform the blind channel shortening; and then, blindly estimate the channel frequency offset and compensate for it. Following that, the blind signal-to-noise (SNR) maximization is performed; and then, the transmit data is detected blindly by using the fractional low-order statistics constant modulus algorithm (FLOS-CMA).

2.4.2 Faster-than-Nyquist (FTN) Signaling

The idea of faster-than-Nyquist (FTN) signaling started when J. E. Mazo realized and showed in his work [13] that the transmission of Nyquist pulses can be accelerated by a factor of $0.802T$ without increasing the minimum Euclidean distance between the pulses and without degradation in BER while maintaining the same bandwidth. The violation of Nyquist criterion introduces the inter symbol interference (ISI) between the received samples; hence, requires complex processing at the receiver for proper detection of transmitted data. Since, FTN signaling helps in achieving the higher data rate as compared to the Nyquist signaling, FTN signaling has been proposed as a candidate solution to compensate for the data rate loss in SPC when compared to the channel capacity.

In the Nyquist signaling, we transmit and sample the $T$-orthogonal pulses at the intervals of $T$ such that at the sampling intervals, the contribution of the neighboring pulses is zero. On the other hand, in FTN
signaling we perform the transmission and sampling of the received signal at the intervals of $\tau T$ instead of $T$, where $0 < \tau < 1$, is the acceleration parameter. Hence, the contribution from the neighboring pulse, in case of FTN signaling, will not be zero at the sampling intervals. The difference between Nyquist signaling and FTN signaling is explained in Fig. 2.13 where, Fig. 2.13a shows the raised cosine (RC) pulses at the intervals of $T$ and Fig. 2.13b demonstrate the RC pulses at the intervals of $\tau T$. It can be seen in Fig. 2.13a that the received waveform representing the summation of these pulses exactly coincides with the peak values of the RC pulses as the contribution from the neighbouring pulses is 0 at each sampling instant. Hence, at every sampling interval $T$, we will be able to sample the exact peak values of the transmit pulses. Fig. 2.13b shows the RC pulses at the intervals $\tau T$, where $\tau = 0.7$, representing the FTN signaling. One can see from the figure that the contribution from the neighbouring pulses at the sampling intervals is non-zero. Hence, the waveform which represent the summation of these pulses does not coincides with the peak of the RC pulses at the sampling interval. Therefore, every received sample which is picked at the interval of $\tau T$ is corrupted from the contributions of the neighbouring pulses. We refer to this corruption of the received samples from the neighboring pulses as inter symbol interference (ISI). Due to the intentionally introduced ISI in FTN signaling, the transmit symbols cannot be estimated independently from each other; rather, the joint estimation of transmit symbols need to be performed since each received symbol in FTN signaling is dependant on the neighbouring symbols.

Several studies have been performed for the detection of FTN signaling in the presence of additive white gaussian noise (AWGN) channel. For example; in [14], the detection was performed by using turbo equalization when high-rate single parity-check codes were employed. That said, the performance of FTN signaling was studied using the truncated Viterbi algorithm in [15]. The authors in [15] further studied the performance of FTN signaling when convolutional encoder and interleaver is implemented to cancel the ISI during turbo equalization. Following that, the authors in [16] proposed two novel successive interference cancellation based methods, named as symbol-by-symbol sequence estimation (SSSSE) and symbol-by-symbol with go-back-$K$ sequence estimation (SSSgbKSE), for the detection of FTN signaling which are suitable for the scenario when the high acceleration parameter is employed. In [17], the performance of SSSSgbKSE detector was investigated in the presence of polar codes based channel coding. The polar coded FTN signaling does result in the improved performance of SSSSgbKSE detector. A novel Ungerboeck observation model based reduced-complexity maximum-A-posteriori (MAP) equalization technique was presented in [18]. For the scenario when FTN signaling employed $M$-ary phase shift key (PSK) symbols, a semidefinite relaxation (SDR) and Gaussian randomization based detection algorithm was presented in [19]. Following that, the authors in [20] proposed the convex quadratic relax-and-quantize sequence estimation (CQRAQSE) method for detection, which was based on convex relaxation, primal-dual predictor-corrector interior point method, and quantization. The FTN signaling was employed for SPCs in [21] to compensate for the data rate loss in comparison to the channel capacity. The authors employed non-binary low-density parity-check (NB-LDPC) codes and perform the detection by adopting the SSSSgbKSE detector. In [22], to reduce the detection
complexity of FTN signaling, the authors introduced the deep learning based list sphere decoding (DL-LSD) method, contrary to the conventional list sphere decoding (LSD) algorithm. More specifically, to reduce the complexity during soft detection, an optimal radius of LSD hypersphere is determined based on the neural network. The optimized hypersphere include significantly less number of lattice points in comparison to the original LSD. Recently, in [23], an eigenvalue decomposition based precoding technique was proposed for the FTN based index modulation technique.

The more practically encountered channel type during wireless transmission is the frequency selective channel. The presence of frequency selective channel further increases the challenge of detection in FTN signaling. In conventional Nyquist signaling, the known pilots symbols are usually sent along with the data symbols to learn and estimate the channel. However, if the same method is employed in case of FTN signaling, pilot symbols will encounter self-interference, i.e., interference from the neighbouring pilots; hence, more involved signal processing techniques should be utilized. The authors in [24] proposed a Gaussian message passing (GMP) algorithm to perform the estimation of channel and the decoding of the message jointly. In particular, their proposed GMP algorithm was the combination of belief propagation (BP) algorithm, expectation propagation (EP), and variational message passing (VMP) algorithm. A low-complexity, semi-blind minimum mean square error based algorithm was proposed in [25] for the joint detection of data and the estimation of the channel. The authors further proposed the guidelines to design the pilot sequence for the channel estimation. Recently, in [26], the authors employed the combination of linear pre-equalization (LPE), spectral precoding, Tomlinson-Harashima precoding (THP), and conventional frequency domain channel estimation technique, to cancel the FTN signaling ISI and estimate the channel. In the aforementioned technique, LPE cancels the FTN signaling ISI; however, results in signal spectral brodening. Hence, Spectral precoding was employed to deal with the problem of signal spectral brodening. That said, spectral precoding introduces intentional ISI to counter the effect of spectral brodening of the signal. Therefore, Tomlinson-Harashima precoding (THP) was employed to cancel the ISI introduced by the spectral precoding. Following this process, as the only remaining ISI is the ISI introduced by frequency selective channel, authors employed the conventional frequency domain channel estimation techniques used for Nyquist signaling to cancel this ISI.

Some studies have been performed to enable the non-coherent detection of FTN signaling. For examples, the authors in [27] employed differentially encoded phase-shift key symbols (DPSK) for non-coherent detection of FTN signaling. Under the assumption of flat-fading channel, the FTN signaling ISI was equalized using frequency domain equalization as the complete knowledge of FTN signaling ISI is available at the receiver. Following this process, the equalized DPSK symbols were non-coherently decoded to PSK symbols using conventional differential demodulation technique. Simulation results demonstrated that the proposed scheme has a potential to outperforms the conventional coherent FTN scheme in fast fading channel at high SNR. After that, the authors in [28] employed 16-point double star quadrature amplitude modulation (QAM), instead of PSK modulation as it was the case in [27], for differential FTN (DFTN) signaling. The results did
show the benefits of employing the star-QAM aided DFTN signaling over PSK in terms of BER performance. Recently, the authors in [29] proposed the non-coherent detection of differential phase modulated FTN signaling based on the decision feedback algorithm. The proposed algorithm for non-coherent detection in the presence of unknown phase shift was proved to be of low-complexity and energy efficient; however, the authors studied the performance under the AWGN channel with constant transmission ratio.

That said, to the best of our knowledge, there are no work related to non-coherent detection of FTN signaling under the frequency-selective channel. The proposed technique in this research will easily allow the non-coherent detection of FTN signaling in the presence of frequency-selective channel when we employ the MOCZ technique. This is possible because MOCZ is a unique technique in which the information is modulated onto the complex zeros in the $z$-domain of transmit baseband signal.

### 2.5 Summary

This chapter elaborated the difference between conventional modulation technique $M$-QAM and the novel modulation technique MOCZ by discussing in details their modulators/demodulators and the effect of channel. It was shown that contrary to the conventional modulation scheme like $M$-QAM in which information bits are directly modulated onto the discrete-time transmit signal, in MOCZ, the information bits are modulated onto the complex zeros of the $z$-domain polynomial corresponding to the discrete-time transmit signal. When the transmit signal is passed through the frequency-selective channel, the extra random complex zeros from the channel are added to the signal. To perform the detection, the MOCZ receiver does not require information regarding the channel state or channel zeros. Whereas in the $M$-QAM, when the transmit signal passes through the frequency-selective channel, the $M$-QAM receiver may need the channel information to perform the non-blind channel equalization before the detection. Hence, MOCZ is a suitable technique for the transmissions which cannot acquire channel state information during transmission such as SPCs.
Figure 2.13: Figure elaborating the difference between Nyquist and FTN signaling. The colored pulses, except black one, represent the RC pulses, where the received waveform representing the summation of these pulses is shown in the black color. Lastly, the vertical dashed-dotted lines represent the sampling intervals.
References


3 Spectrally-Efficient Modulation on Conjugate-Reciprocal Zeros (SE-MOCZ) for Non-Coherent Short Packet Communications


This chapter contains the manuscript of the paper which is under review for publication in the journal IEEE Transactions on Wireless Communications. In this paper, we improved the spectral efficiency of a recently-proposed modulation scheme named as modulation on conjugate reciprocal zeros (MOCZ) by combining MOCZ with a technique named as faster-than-Nyquist (FTN) signaling. The combination of MOCZ with FTN signalling introduced inter symbol interference (ISI) between the received samples of MOCZ, which translates to the extra complex zeros in the $z$-domain. We optimized the radius of transmit complex zeros to increase the noise robustness; hence, to improve the bit error rate (BER) performance of MOCZ, in the presence of extra complex zeros (due to the FTN signaling ISI). We also designed a partial-complex-zeros-removal filter at the receiver to remove part of the complex zeros due to the FTN signaling. Lastly, we derived the maximum likelihood (ML) detector to estimate the transmitted sequence of coefficients; hence, the transmitted complex zeros in the presence of induced complex zeros corresponding the FTN signaling.
Spectrally-Efficient Modulation on
Conjugate-Reciprocal Zeros (SE-MOCZ) for
Non-Coherent Short Packet Communications

Aiman Asad Siddiqui, Ebrahim Bedeer, Ha H. Nguyen, and Robert Barton

Abstract

This paper proposes a non-coherent communication scheme for short packet communications (SPCs), called spectrally-efficient modulation on conjugate reciprocal zeros (SE-MOCZ). The proposed SE-MOCZ scheme is realized by combining the recently-proposed MOCZ and faster-than-Nyquist (FTN) signaling. Specifically, the pulses carrying the polynomial coefficients of MOCZ are accelerated beyond the Nyquist limit to enhance the spectral efficiency. The spectral efficiency enhancement, however, comes at the expense of inter-symbol interference (ISI), which increases the number of received complex zeros in the $z$-domain of the polynomial representing the received signal. We design a partial-complex-zeros-removal filter at the receiver to partially remove the extra complex zeros caused by FTN signaling. Furthermore, we also optimize the radius of the transmit complex zeros in SE-MOCZ to improve the bit error rate (BER) performance. We propose a maximum likelihood (ML) detector for the proposed SE-MOCZ scheme. To strike a balance between the BER performance and computational complexity, we adopt a root-finding minimum distance (RFMD)-based detector. Simulation results clearly show the performance advantage of the proposed SE-MOCZ when compared to MOCZ for a wide range of operating parameters.

Index Terms

Faster-than-Nyquist signaling, modulation on zeros, non-coherent detection, Nyquist criterion, short packet communications, ultra-reliable and low-latency communications.

3.1 Introduction

Future wireless communication will encompass diverse Internet of things (IoT) applications, such as telemedicine, autonomous vehicles, and automated factories, that need to fulfill the stringent requirements of ultra-reliable and low-latency communication (URLLC) [1]. For example, wireless telesurgery requires a packet error rate
of $10^{-7}$ and a latency of less than 100 ms, 25 ms, and 5 ms for audio/video feedback, haptic feedback, and interactive live holographic feedback, respectively \[2\]. Likewise, IoT applications in automated factories require an end-to-end latency in the range of 1 to 50 ms along with packet error rates from $10^{-6}$ to $10^{-9}$ \[3\]. To avoid collision in autonomous vehicles, the latency needs to be in the order of milliseconds while maintaining high reliability \[4\]. To meet such stringent latency requirements, URLLC necessitates the use of short data packets for data transmission \[1,5\].

Short packet communications (SPCs) pose several design challenges when compared to the conventional large-blocklength communications. For instance, large-blocklength communications can use pilots to efficiently estimate the wireless channel. However, using pilots to estimate the channel in SPCs will severely degrade the spectral efficiency since the pilot length is comparable to the length of short data packets \[5\]. Another design challenge for channel estimation in SPCs is that the traffic is mostly sporadic, i.e., the wireless channel changes from one transmission to the next, which requires re-estimation of the channel for every transmission \[6\]. Therefore, there is a need to design efficient transmission techniques, e.g., modulation and coding, for SPCs.

Recently, a novel modulation technique, known as modulation on conjugate-reciprocal zeros (MOCZ), was proposed for non-coherent SPCs \[7\]. In MOCZ, the information bits are modulated onto the complex zeros of the $z$-domain polynomial corresponding to samples of the transmit discrete-time signal. When the transmit signal propagates through the frequency-selective channel, the number of complex zeros of the polynomial corresponding to the received signal increases due to the insertion of the complex zeros associated with the polynomial of the channel response. At the receiver, based on the codebook of the transmit complex zeros that is known to the receiver, the transmit complex zeros can be separated from the channel complex zeros without knowledge of channel state information (CSI) \[7\].

Each of the transmit complex zeros can either carry a single information bit, which is known as binary modulation on conjugate reciprocal zeros (BMOCZ), or more than one information bit, which is known as phase modulation of conjugate reciprocal zeros (PMOCZ). Different detectors of MOCZ were proposed in \[7\] to balance the tradeoff between performance and the computational complexity. In \[8\], the authors studied performance of MOCZ under timing offset and carrier frequency offset and proposed an efficient detector that can jointly estimate the carrier frequency offset and decode the transmit information. The performance of MOCZ in multi-user transmission was recently investigated in \[9\], where the authors studied two transmission techniques, namely, MOCZ single-carrier time division multiple access (MOCZ-TDMA) and MOCZ single-carrier frequency division multiple access (MOCZ-FDMA). The authors demonstrated performance gains, in terms of block error rate, of the proposed techniques over the orthogonal frequency-division multiple access (OFDMA) technique for highly-mobile and frequency-selective channels.

Because of the use of finite (usually short) blocklength, the achievable rate of SPCs is less than the channel capacity \[10\]. Recently, faster-than-Nyquist (FTN) signaling was proposed as a promising solution to compensate for the rate loss of SPCs \[11\]. The history of FTN signaling dates back to 1975 when J. E.
Mazo showed in [12] that the uncoded transmission of binary sinc pulses can be accelerated up to 25% beyond the Nyquist limit without deteriorating the asymptotic error rate. This is remarkable considering that FTN violates the orthogonality condition and causes intersymbol interference (ISI). In summary, FTN signaling makes a better use of transmission bandwidth to increase the spectral efficiency, but at the cost of additional complexity at the transmitter and/or receiver to compensate for the introduced ISI [13].

Although the ISI generated from FTN signaling has a trellis structure [14], standard trellis-based detection techniques such as the Viterbi and BCJR algorithms are not directly applicable to detect FTN signaling due to the excessive processing complexity, especially at small acceleration parameters. Reduced-trellis or reduced-search variants of the Viterbi and BCJR algorithms can be used to detect low-order modulation FTN signaling [15, 16], but it becomes more challenging for high-order modulation FTN signaling. On the other hand, precoding techniques at the transmitter [17, 18] or estimation techniques at the receiver based on concepts from non-linear optimization theory [19, 20] can strike a balance between performance and detection complexity. For high acceleration parameters, techniques based on frequency-domain equalization [21] or symbol-by-symbol estimation [22] can achieve satisfactory detection performance. We refer the interested reader to [13] for more detailed discussion on FTN signaling.

Inspired by the advantages of MOCZ and FTN signalling, in this paper we propose and investigate a new non-coherent communication scheme, called spectrally-efficient modulation on conjugate-reciprocal zeros (SE-MOCZ) to improve the spectral efficiency of MOCZ. In a nutshell, the proposed SE-MOCZ combines MOCZ and FTN signaling in such a way that the transmission of Nyquist pulses carrying the polynomial coefficients of MOCZ is accelerated beyond the Nyquist rate. As a consequence of intentionally violating the Nyquist limit, the ISI between the received samples occurs. As explained before, in MOCZ the information bits are modulated onto the complex zeros of the transmit polynomial in the z-domain. The violation of the Nyquist limit leads to additional complex zeros in the received polynomial, complicating the detection of the information bits. To deal with these additional complex zeros (caused by ISI of FTN signaling), we design a partial-complex-zeros-removal filter at the receiver. Further, we optimize the radius of the transmit complex zeros so that they are robust against noise in the presence of the additional ISI-induced complex zeros. We develop the maximum likelihood (ML) detection rule for the proposed SE-MOCZ. Since the ML detection has high computational complexity, a low-complexity root-finding minimum distance (RFMD) detector similar to that in MOCZ can also be applied for SE-MOCZ to strike a balance between computational complexity and performance. Simulation results show that spectral efficiency gains of up to 25% can be achieved with the proposed SE-MOCZ and without any penalty in terms of average energy per bit when the ML detector is employed. When the low-complexity RFMD detector is employed, the spectral efficiency gain reduces to about 11% and at the expense of about 1 dB average energy per bit.

The remainder of this paper is organized as follows. Section 3.2 describes the system model of the proposed SE-MOCZ technique. Then, a brief review of MOCZ is presented in Section 3.3. Section 3.4 discusses the effect of ISI due to FTN signaling on the received complex zeros and optimal radius design for SE-MOCZ. In
Section 3.5, the design of a partial-complex-zeros-removal filter and different detectors, namely the ML and RFMD detectors, are presented. Simulation results are presented and discussed in Section 3.6. Section 3.7 concludes the paper.

Notations: Throughout the paper, lower-case and upper-case boldface letters denote vectors and matrices, respectively. The complex conjugate and time reversal of vector $\mathbf{a}$ is denoted as $\overline{\mathbf{a}}^T$, the transpose of vector $\mathbf{a}$ is denoted as $\mathbf{a}^T$, and the Hermitian of matrix $\mathbf{A}$ is denoted as $\mathbf{A}^H$. We use $\mathbf{I}_N$ to represent the identity matrix of size $N \times N$, $\mathbb{C}$ to represent the set of complex numbers, and $\|\mathbf{a}\|_2$ to represent the norm of vector $\mathbf{a}$. We denote a complex Gaussian random variable with mean $\mu$ and variance $\sigma^2$ as $\mathcal{CN}(\mu, \sigma^2)$. Further, $\mathbb{E}[\cdot]$, $\text{De2Bi}(\cdot)$, $\ln(\cdot)$, and $*$ represent the expectation, the decimal to binary conversion, natural logarithm, and the convolution operator, respectively.
3.2 System Model

A block diagram of the system model describing the proposed SE-MOCZ is shown in Fig. 3.1. The incoming data bits of the information message \( m \) are organized into \( K \) groups of bits as \( m = (m_1, m_2, \ldots, m_k, \ldots, m_K) \), where \( m_k \) is the \( k \)th group consisting of \( \log_2 2M \) bits and \( 2M \) is the constellation size of SE-MOCZ. The bits in \( m_k \) are mapped onto a unique complex zero \( \alpha_k \in \mathbb{C}, k = 1, \ldots, K \), in the \( z \)-domain, where the mapping criterion is explained later in Section 3.3. It follows that the information message \( m \) can be represented by a set of \( K \) unique complex zeros. For transmission, a sequence \( x = [x_0, x_1, \ldots, x_k, \ldots, x_K]^T, x_k \in \mathbb{C} \), containing \( K + 1 \) polynomial coefficients corresponding to the \( K \) unique complex zeros is generated.

In MOCZ, the coefficients in \( x \) are transmitted over the channel by modulating the basic \( T \)-orthogonal transmit pulse \( p(t) \) as \( \sum_{k=0}^{K} x_k p(t - kT) \), where \( T \) is the symbol duration. In the proposed SE-MOCZ, the coefficients in \( x \) are transmitted using FTN signaling, i.e., every \( \tau T \) seconds, where \( 0 < \tau \leq 1 \) is the acceleration parameter. Hence, the transmit signal in the proposed SE-MOCZ is given as:

\[
x(t) = \sum_{k=0}^{K} x_k p(t - k\tau T). \tag{3.1}
\]

Considering baseband transmission and let \( h(t) \) represent the impulse response of the multipath fading channel, then the received baseband signal \( r(t) \) is given as:

\[
r(t) = \int h(t') x(t - t') dt' + n(t), \tag{3.2}
\]

where \( n(t) \) is additive white Gaussian noise (AWGN) with zero mean and one-sided power spectral density (PSD) of \( N_0 \). Moreover, the multipath fading channel \( h(t) \) can be modeled as:

\[
h(t) = \sum_{l=0}^{L_{ch} - 1} a_l \delta(t - \phi_l), \tag{3.3}
\]

where \( L_{ch} \) is the number of channel taps, \( a_l \) and \( \phi_l \) are the gain and delay of the \( l \)th tap, respectively. Using (3.3), the received signal \( r(t) \) can be re-written as:

\[
r(t) = \sum_{l=0}^{L_{ch} - 1} a_l x(t - \phi_l) + n(t),
\]

\[
= \sum_{k=0}^{K} x_k \sum_{l=0}^{L_{ch} - 1} a_l p(t - k\tau T - \phi_l) + n(t). \tag{3.4}
\]

At the receiver, the received signal is passed through a filter \( p^*(-t) \) which is matched to the transmit pulse \( p(t) \). For a real transmit pulse \( p(t) \), we have \( p^*(-t) = p(-t) \). The signal after the matched filter is written as:

\[
y(t) = \int p(t') r(t - t') dt',
\]

\[
= \sum_{k=0}^{K} x_k \sum_{l=0}^{L_{ch} - 1} a_l g(t - k\tau T - \phi_l) + w(t), \tag{3.5}
\]
where $g(t)$ and $w(t)$ are given, respectively, as:

$$g(t) = \int p(t')p(t-t')dt',$$

(3.6)

$$w(t) = \int p(t')n(t-t')dt'.$$

(3.7)

The signal $y(t)$ at the output of the matched filter is then sampled at every $\tau T$ to obtain the following sampled signal:

$$y[n] = \sum_{k=0}^{K} x_k \sum_{l=0}^{L_{ch}-1} a_l g[n - k - \phi_l] + w[n].$$

(3.8)

For convenience, define the effective channel response $h_{\text{eff}}[m]$ as:

$$h_{\text{eff}}[m] = \sum_{l=0}^{L_{ch}-1} a_l g[m - \phi_l].$$

(3.9)

Then (3.8) can be equivalently written as:

$$y[n] = \sum_{k=0}^{K} x_k h_{\text{eff}}[n - k] + w[n].$$

(3.10)

Note that for $0 < \tau < 1$, the noise samples $w[n]$ are still zero-mean Gaussian random variables, but they are correlated.

The convolutions in (3.10) and (3.9) can be, respectively, represented in the vector form as:

$$\mathbf{y} = \mathbf{x} * \mathbf{h}_{\text{eff}} + \mathbf{w},$$

(3.11)

$$\mathbf{h}_{\text{eff}} = \mathbf{g} * \mathbf{h},$$

(3.12)

where $\mathbf{g}$, $\mathbf{h}$, $\mathbf{w}$ are vectors of samples obtained by sampling $g(t)$, $h(t)$, and $w(t)$, respectively, by the sampling period $\tau T$. The length of $\mathbf{g}$ is theoretically infinite. However, for practical purposes, we set it to $2L_g + 1$ by properly truncating the coefficients of the shaping pulse. The lengths of $\mathbf{h}$ and $\mathbf{h}_{\text{eff}}$ are $L_{ch}$ and $L_{\text{eff}} = 2L_g + L_{ch}$, respectively, and the length of $\mathbf{w}$ is $N = K + L_{\text{eff}}$. Note that, for the conventional Nyquist signalling, $\tau = 1$, hence $\mathbf{h}_{\text{eff}} = \mathbf{h}$ and $L_{\text{eff}} = L_{ch}$. From (3.11) and (3.12), it is clear that (3.8) can be written in vector form as:

$$\mathbf{y} = \mathbf{x} * \mathbf{g} * \mathbf{h} + \mathbf{w},$$

(3.13)

where the length of the vector $\mathbf{y}$ is also $N$.

The sampled signal $\mathbf{y}$ is then passed through a discrete-time filter designed to partially remove the complex-zeros introduced by FTN signaling. This filter shall be named partial-complex-zeros-removal filter and it is presented in detail in Section 3.5 Next, the filtered signal is processed by the detector to recover the information message as $\hat{\mathbf{m}} = (\hat{m}_1, \hat{m}_2, \ldots, \hat{m}_k, \ldots, \hat{m}_K)$, where $\hat{m}_k$ is the estimate (detected) version of the $k$th group of bits.
3.3 Review of MOCZ

As discussed earlier, the information message \( m \) in MOCZ consists of \( K \) groups of data bits, each having \( \log_2 2M \) bits, where \( 2M \) is the size of MOCZ constellation. Each of the \( K \) groups of data bits is mapped onto a unique complex zero in the \( z \)-domain. This means that each information message \( m \) is mapped onto a set of \( K \) complex zeros for transmission. In general, when the coefficients of the transmit polynomial are disturbed by noise, the receive complex zeros become unstable and can be far away from their original locations (i.e., the locations of the transmit complex zeros) in the \( z \)-plane. However, it has been shown that the complex zeros corresponding to the coefficients of a special polynomial called the *Huffman polynomial* can be robust to noise.

The Huffman polynomials have the following properties: (i) The complex zeros of the Huffman polynomial lie on two circles centered at the origin, where the radii of these circles are the reciprocal of each other, i.e., \( R > 1 \) and \( 1/R \), (ii) Optimizing the radius \( R \) is crucial in achieving noise robustness, and (ii) The auto-correlation of a Huffman polynomial, i.e., \( a = x \star x^\ast \), has an impulse like shape, and hence, is helpful for channel length estimation and time synchronization.

To illustrate how the modulator of MOCZ works, we first discuss the mapping from information bits to complex zeros when \( M = 1 \), i.e., for the case of BMOCZ. Then, we discuss the mapping when \( M > 1 \), i.e., \( 2M \)-PMOCZ.

In BMOCZ, each group \( m_k, k = 1, \ldots, K \), has only one bit that is mapped to a unique complex zero. The mapping is achieved by dividing the \( z \)-plane into \( K \) sectors, where each sector \( k, k = 1, \ldots, K \), contains one complex zero corresponding to one bit of the information message \( m \). The location of a complex zero in the \( z \)-plane is determined by its radius and phase. In particular, the \( k \)th complex zero \( \alpha_k \) of BMOCZ has a phase value of \( 2\pi(k - 1)/K \), whereas its radius is determined by the bit value of \( m_k \) as follows: if \( m_k = 0 \), \( |\alpha_k| = 1/R \), and if \( m_k = 1 \), \( |\alpha_k| = R > 1 \).

To further illustrate the idea of BMOCZ, consider an example of message \( m = (1, 0, 1, 0, 1, 1, 1) \) that consists of seven groups of bits, i.e., \( K = 7 \), each group having only one bit. The first bit \( m_1 = 1 \) is mapped to a complex zero located at phase \( 2\pi(k - 1)/K = 2\pi(1 - 1)/7 = 0 \) and radius \( R > 1 \) as shown in Fig. 3.2. The second bit \( m_2 = 0 \) is mapped to a complex zero at radius \( 1/R \) and phase \( 2\pi(k - 1)/K = 2\pi(1)/7 \). The other 5 bits in \( m_k, k = 3, \ldots, 7 \), are mapped similarly as shown in Fig. 3.2.

Similar to BMOCZ, \( 2M \)-PMOCZ divides the \( z \)-plane into \( K \) sectors and maps each group of bits \( m_k, k = 1, \ldots, K \), into a given sector. In particular, in \( 2M \)-PMOCZ, every group of bits \( m_k \) consists of \( \log_2 2M \) bits and the mapping is performed as follows. The most significant bit (MSB) in \( m_k \) is mapped to the radius of \( \alpha_k \), just like in BMOCZ. The remaining \( \log_2 M \) bits are mapped to the phase of \( \alpha_k \). Therefore, each of the sectors in the \( z \)-plane is further divided into \( M \) sub-sectors, where the phase of a given sub-sector within the \( k \)th sector is \( 2\pi(k - 1)/K + 2\pi d/(MK) \). Here \( d \in \{0, \ldots, M - 1\} \) is the decimal value of the remaining \( \log_2 M \) bits after excluding the MSB.
Figure 3.2: Mapping of $K = 7$ groups of bits onto $K = 7$ complex zeros for BMOCZ message $m = (1, 0, 1, 0, 1, 1, 1)$. Black circles represent the complex zeros corresponding to the message $m$. The red ring represents the unit circle and the black rings represent the circles of radius $R > 1$ and $1/R$. The solid black lines represent the decision boundary between the sectors.

Figure 3.3: Mapping of $K = 7$ groups of bits onto $K = 7$ complex zeros with 8-PMOCZ when the message is $m = (111, 110, 001, 101, 000, 111, 110)$. The solid black lines represent the decision boundary between the sectors. The decision boundaries between the sub-sectors are omitted for clarity.
To illustrate the idea of 2M-PMOCZ, consider using 8-PMOCZ (i.e., \( M = 4 \)) for the message \( m = (111, 110, 001, 101, 000, 111, 110) \) that consists of \( K = 7 \) groups of bits, each having 3 bits. The mapping of this message to complex zeros is shown in Fig. 3.3. Since the MSB of \( m_1 = 111 \) is 1, the radius of \( \alpha_1 \) is \( R \). The remaining \( \log_2 M = \log_2 4 = 2 \) bits in \( m_1 \), namely bits 11, have an equivalent decimal value \( d = 3 \) and accordingly the phase of \( \alpha_1 \) is \( 2\pi(3)/28 \). Similarly, for \( m_2 = 110 \), since the MSB is 1, the radius of \( \alpha_2 \) is set to \( R \). The decimal value of the remaining 2 bits in \( m_2 \) is 2, so the phase of \( \alpha_2 \) is \( 2\pi(6)/28 \). In a similar manner, other groups of bits \( m_k, k = 3, \ldots, 7 \), are mapped to the complex zeros as shown in Fig. 3.3.

Two different codebooks for MOCZ can be defined: one is for the complex zeros, i.e., complex zero codebook \( \zeta \); while the other is for the complex coefficients i.e., coefficient codebook \( \rho \). The MOCZ complex zero codebook \( \zeta \) is the Cartesian product of the \( K \) complex zero sets, i.e., \( \zeta = \zeta_1 \times \zeta_2 \times \ldots \times \zeta_K \) [7]. Each of the \( k \)th complex zero set \( \zeta_k \) consists of all possible \( 2^M \) complex zeros within the \( k \)th sector. Therefore, the \( k \)th complex zero set \( \zeta_k \) can be defined as:

\[
\zeta_k = \left\{ \alpha_k^{(0)(R)}, \ldots, \alpha_k^{(M-1)(R)}, \alpha_k^{(0)(1/R)}, \ldots, \alpha_k^{(M-1)(1/R)} \right\},
\]

where \( \alpha_k^{(d)(r)} = r e^{j2\pi((k-1)M+d)/(MK)} \), \( d \in \{0, \ldots, M-1\} \), and \( r \in \{1/R, R\} \).

Based on the bits in \( m_k \), only one of the complex zeros is selected from the \( k \)th complex zero set \( \zeta_k \) for transmission. Hence, the complex zero codebook of MOCZ consists of the \( 2^{MK} \) unique complex zeros, which defines the \( (2M)^K \) combinations of the complex zeros that can be transmitted based on the message \( m \). Fig. 3.4 shows all the \( 2KM \) unique complex zeros for \( K = 7 \) and \( M = 4 \).

The coefficient codebook \( \rho \) consists of all \( (2M)^K \) combinations of the complex coefficients corresponding to the \( (2M)^K \) combinations of the complex zeros defined by the complex zero codebook \( \zeta \).

Finally, the \( K + 1 \) polynomial coefficients \( x_k, k = 0, \ldots, K \), corresponding to the \( K \) complex zeros \( \alpha_k \), \( k = 1, \ldots, K \), are generated using the Toeplitz iterator presented in [7].

### 3.4 The Proposed SE-MOCZ

Similar to MOCZ, the proposed SE-MOCZ maps \( K \) groups of information bits onto \( K \) complex zeros and generates polynomial coefficients corresponding to these complex zeros for transmission. The SE-MOCZ is different from MOCZ in the way the polynomial coefficients are carried by the transmit pulses: In SE-MOCZ the pulses \( p(t) \) are transmitted every \( \tau T \), \( 0 < \tau \leq 1 \), as opposed to every \( T \) in MOCZ. Because the Nyquist criterion is intentionally violated in SE-MOCZ, ISI is introduced between the received samples, which gives rise to a larger number of received complex zeros in the \( z \)-domain. This is discussed in more detail in the following.
Figure 3.4: The unique $2KM$ complex zeros which belong to the complex zero codebook $\zeta$ are represented by cyan circles for $K = 7$ and $M = 4$.

Figure 3.5: Complex zeros for BMOCZ with flat fading channel, i.e., $L_{ch} = 1$, and no noise. Black circles are the complex zeros of $X(z)$, and magenta circles are the received complex zeros, i.e., complex zeros of the polynomial $Y(z)$. 
3.4.1 Effect of ISI due to FTN signaling on received complex zeros

Using the convolution property of the \( z \)-transform, the received signal \( y \) in (3.13) can be expressed in the \( z \)-domain as:

\[
Y(z) = X(z)G(z)H(z) + W(z),
\]

where \( Y(z), X(z), G(z), H(z), \) and \( W(z) \) are the \( z \)-domain polynomials of \( y, x, g, h, \) and \( w \), respectively. The polynomial \( X(z) \) corresponds to the transmit coefficients vector \( x \) and is defined as:

\[
X(z) = \sum_{k=0}^{K} x_k z^{-k},
\]

where \( x_k \) is the \( k \)th element of \( x \) and the degree of \( X(z) \) is \( K \) if \( x_K \neq 0 \). Similarly, the degrees of \( Y(z), H(z), G(z), \) and \( W(z) \) are \( N - 1, L_{ch} - 1, 2L_{g}, \) and \( N - 1 \), respectively. The effect of ISI due to FTN signaling is through \( G(z) \) in (3.15) and can be seen with the help of Fig. 3.5 and Fig. 3.6. These two figures show the complex zeros of \( X(z) \) and \( Y(z) \) for BMOCZ and binary SE-MOCZ (SE-BMOCZ), respectively, for the scenario of no noise and flat fading channel, i.e., \( L_{ch} = 1 \). Obviously, in both figures the transmit signals have the same number of complex zeros. However, the received signal in the case of SE-MOCZ has more complex zeros that are introduced by ISI due to FTN signaling.

As discussed earlier, the complex zeros representing the information bits should be carefully designed to be located at the optimal radii of \( R > 1 \) and \( 1/R \) to be robust against noise. In particular, the transmit coefficients should be selected to be a Huffman polynomial. However, the ISI due to FTN signaling is
determined by the transmit and receiver pulse shapes, and hence, we have no control over the complex zeros of $G(z)$ once the filters are selected. In general, the complex zeros due to $G(z)$ are sensitive to noise. This can be shown with the help of Fig. 3.7 which plots the complex zeros of the received signal for SE-BMOCZ at $E_b/N_0 = 19$ dB and flat fading channel, i.e., $L_{ch} = 1$. As can be seen from Fig. 3.7 for the same noise power; the complex zeros corresponding to $G(z)$ are distributed much more when compared to the complex zeros corresponding to $X(z)$.

### 3.4.2 Optimal radius design for the proposed SE-MOCZ

To have a reliable communication with SE-MOCZ, it is intuitively desirable to maximize the Euclidean distance between the complex zeros of $G(z)$ and the unique $2KM$ complex zeros of the codebook $\zeta$. As can be seen from Fig. 3.8, the complex zeros of $G(z)$ are, in general, far from the $2KM$ complex zeros of the codebook $\zeta$, except for two complex zeros denoted as $\alpha_G$ and $1/\alpha_G$, which are conjugate reciprocals of each other. Among these two complex zeros, $\alpha_G$ is closer to the complex zeros of the codebook $\zeta$. Therefore we aim to maximize the distance between $\alpha_G$ and the closest complex zero of the codebook. Since we have no control over the complex zeros of $\alpha_G$, we shall control the positions of the codebook zeros by jointly maximizing (i) the Euclidean distance $d_g$, and (ii) the minimum distance between $d_{\text{ph}}$ and $d_{\text{rad}}$, where $d_g$ is the distance between $\alpha_G$ and the closest codebook zero having radius $1/R$, $d_{\text{ph}}$ represents the distance between the neighboring complex zeros having radius $1/R$, and $d_{\text{rad}}$ is the distance between the conjugate reciprocal pairs of codebook complex zeros, i.e., $d_{\text{rad}} = R - 1/R$. These distance measures are illustrated in Fig. 3.9 (for even values of $K$) and Fig. 3.10 (for odd values of $K$). Observe that such joint maximization...
can be achieved by setting $\lambda d_{\text{rad}} = d_g$ because $d_{\text{ph}} > d_{\text{rad}}$, where the factor $\lambda$ is introduced to take into account that $\alpha_G$ is unstable in the presence of noise as compared to the codebook zeros. For SE-BMOCZ, the value of $d_g$ depends on whether the number of the groups of bits $K$ is odd or even, and both cases will be discussed separately.

For $K$ even, as shown in Fig. 3.9, $\alpha_G$ lies at a phase of 180 degrees and the closest codebook complex zeros are also present at the same phase, therefore $d_g = (1/R - d)$, where $d$ is the location of $\alpha_G$. It is pointed out that the value of $d$ depends on $\tau$ and the pulse shape. Furthermore, the number of complex zeros introduced by ISI changes with $\tau$ as well. It is clear that the optimal radius $R$ should satisfy

$$\lambda \left( R - \frac{1}{R} \right) = \frac{1}{R} - d, \quad \lambda > 0. \tag{3.17}$$

Note that (3.17) can be used to find the optimal radius $R$ for higher-order SE-MOCZ, i.e., SE-PMOCZ for both even and odd values of $K$. This is because in SE-PMOCZ, there are always complex zeros in the codebook that lie at the phase of 180 degrees, irrespective of the value of $K$.

For $K$ odd, as shown in Fig. 3.10, no codebook zeros are present at the phase of 180 degrees. Therefore $d_g$ can be found as:

$$d_g = \frac{\sin(\pi/K) d}{\sin(\gamma)}, \tag{3.18}$$

where $\gamma$ can be found as:

$$\frac{\sin(\gamma)}{d} = \frac{\sin(\pi - \frac{\pi}{K} - \gamma)}{1/R}, \tag{3.19}$$
Figure 3.9: Maximizing the distance between complex zero $\alpha_G$ and the closest complex zeros of the codebook $\zeta$ for even $K$. Note that the outer complex zeros of $G(z)$ are omitted for clarity of presentation.

Figure 3.10: Maximizing the distance between the complex zero $\alpha_G$ and the closest complex zeros of the codebook $\zeta$ for odd $K$. The outer complex zeros of $G(z)$ are omitted for clarity of presentation.
which simplifies to:

\[
\gamma = \arctan \left( \frac{Rd \sin \left( \pi - \frac{\pi}{K} \right)}{1 + Rd \cos \left( \pi - \frac{\pi}{K} \right)} \right),
\]

(3.20)

In summary, the optimal radius \( R \) for \( K \) odd satisfies

\[
\lambda \left( R - \frac{1}{R} \right) = \frac{\sin \left( \frac{\pi}{R} \right) d}{\sin \left( \arctan \left( \frac{Rd \sin \left( \pi - \frac{\pi}{K} \right)}{1 + Rd \cos \left( \pi - \frac{\pi}{K} \right)} \right) \right)}, \quad \lambda > 0.
\]

(3.21)

Note that the value of \( \lambda \) in both (3.17) and (3.21) can be calculated numerically to achieve the best BER performance.

As will be explained in Sub-section 3.5.1, a partial-complex-zeros-removal filter can be employed to cancel the inner complex zeros introduced by the ISI due to FTN signaling. Using such a filter, the complex zero \( \alpha_G \) will also be removed, making \( 1/\alpha_G \) the closest complex zero. In other words, to find the optimal radius \( R \) when taking into account the ISI in SE-MOCZ and the partial-complex-zeros-removal filter, one needs to consider the complex zero \( 1/\alpha_G \) instead of \( \alpha_G \). Performing similar analysis as before, the optimal radius \( R \) when SE-BMOCZ with even \( K \) is used, or when SE-PMOCZ (even or odd \( K \)) is used satisfies:

\[
\lambda \left( R - \frac{1}{R} \right) = (d_{filt} - R), \quad \lambda > 0,
\]

(3.22)

where \( d_{filt} = 1/d \), is the location of complex zero \( 1/\alpha_G \). Likewise, the optimal radius \( R \) when SE-BMOCZ with odd \( K \) is used can be found from:

\[
\lambda \left( R - \frac{1}{R} \right) = \frac{\sin \left( \frac{\pi}{R} \right) d_{filt}}{\sin \left( \arctan \left( \frac{d_{filt} \sin \left( \pi - \frac{\pi}{K} \right)}{R + d_{filt} \cos \left( \pi - \frac{\pi}{K} \right)} \right) \right)}, \quad \lambda > 0.
\]

(3.23)

For convenience, in the rest of the paper, we shall use the notation \( R_{opt1} \) for the optimal radius obtained from (3.17) and (3.21), and \( R_{opt2} \) for the optimal radius calculated from (3.22) and (3.23).

Before closing this section, it is pointed out that the definition of the \( z \)-transform in (3.16) is slightly different from that used in [7]. For completeness and to avoid confusion, we present next the coefficient generation technique for the proposed SE-MOCZ. The coefficient vector \( x \) is calculated as:

\[
x = x_0 x^{(K)},
\]

(3.24)

where \( x^{(K)} \) is generated iteratively and \( x_0 \) is defined as:

\[
x_0 = e^{j\phi_0} \| x^{(K)} \|^{-1}_2,
\]

(3.25)

where \( \phi_0 \) is the phase of the first complex zero, i.e., the global phase, which was chosen to be 0 in this paper.

The iterative method to generate \( x^{(K)} \) consists of \( K - 1 \) iterations. The \( s \)th iteration generates the following
vector:

\[
\mathbf{x}^{(s)} = \begin{bmatrix}
    x_0^{(s-1)} & 0 \\
    x_1^{(s-1)} & x_0^{(s-1)} \\
    x_2^{(s-1)} & x_1^{(s-1)} \\
    \vdots & \vdots \\
    x_{s-1}^{(s-1)} & x_{s-2}^{(s-1)} \\
    0 & x_{s-1}^{(s-1)}
\end{bmatrix}
\begin{bmatrix}
    1 \\
    -\alpha_s
\end{bmatrix},
\]

(3.26)

where \( s = 2, \ldots, K \), \( \alpha_s \) is the \( s \)th complex zero and \( x_i^{(s-1)}, i = 0, \ldots, s-1 \), is the \( i \)th element of vector \( \mathbf{x}^{(s-1)} \) generated in the previous iteration. The vector \( \mathbf{x}^{(1)} \), which is required in the first iteration (corresponding to \( s = 2 \)), is given as \([1 \ - \alpha_1]^T\). The final iteration (corresponding to \( s = K \)) results in \( \mathbf{x}^{(K)} \), which is required in (3.24).

### 3.5 Demodulation of SE-MOCZ

In this section, we first discuss the partial-complex-zeros-removal filter that partially removes the complex zeros introduced by ISI due to FTN signaling. We then present the ML and RFMD detectors for the proposed SE-MOCZ.

#### 3.5.1 Partial-complex-zeros-removal filter

To facilitate the detection of the complex zeros representing the information message, the sampled received signal \( \mathbf{y} \) is passed through a filter in an attempt to partially remove the complex zeros introduced by the ISI due to FTN signaling. In other words, such a filter tries to remove the complex zeros of \( G(z) \) that lie inside the unit circle. The partial-complex-zeros-removal filter is designed such that its poles are placed at the locations of the complex zeros introduced by the ISI inside the unit circle. This is achieved with the help of spectral factorization. Specifically, the vector \( \mathbf{g} \) responsible for the ISI is factorized as \( \mathbf{g} = \mathbf{f} * \overline{\mathbf{f}} \), where \( \mathbf{f} \) and \( \overline{\mathbf{f}} = \mathbf{f}^- \) are the coefficient vectors corresponding to the outer and inner complex zeros of ISI, respectively. Note that the exact spectral factorization of the square-root raised cosine pulse \( p(t) \) considered in this paper is only valid for \( \tau \geq 1/(1 + \beta) \), where \( \beta \) is the roll-off factor [23].

In the \( z \)-domain, the spectral factorization can be written as \( G(z) = F(z)F(1/z) \). By designing the partial-complex-zeros-removal filter in such a way that the \( z \)-transform of its impulse response is exactly
Figure 3.11: Complex zeros of the received signal without implementing the partial-complex-zeros-removal filter at $E_b/N_0 = 19$ dB. Black circles are the complex zeros of $X(z)$, magenta circles are the complex zeros of $Y(z)$, and white circles are the complex zeros of $G(z)$.

As can be seen from (3.27), the inner complex zeros of $G(z)$ are removed and we are left with only the outer complex zeros of $G(z)$, the channel complex zeros, and the transmitted complex zeros. All these remaining zeros are disturbed by the filtered noise. Fig. 3.11 and Fig. 3.12 show the complex zeros of $Y(z)$ and $Q(z)$, respectively, at $E_b/N_0 = 19$ dB. It can be seen from Fig. 3.12 that the inner complex zeros of $G(z)$ have been removed.

The time-domain signal $q$ at the output of the partial-complex-zeros-removal filter can be written as:

$$q = x \ast f \ast h + w_{\text{wh}},$$  \hspace{1cm} (3.28)

where length of $q$ and $w_{\text{wh}}$ is $L_{\text{filt}} = L_g + L_{\text{ch}} + K$ and $w_{\text{wh}}$ is the time-domain vector corresponding to $W_{\text{wh}}(z) = W(z)/F(1/Z)$. The PSD of $W(z)$ is $N_0 G(z)$, where $N_0$ is the one-sided PSD of $n(t)$. Therefore, the PSD of the noise after the partial-complex-zeros-removal filter is $N_0 [F(1/Z)]^2/[F(1/Z)]^2 = N_0$, which means that $W_{\text{wh}}(z)$ is white noise. Hence, the partial-complex-zeros-removal filter is also the whitening filter. However, we call it the partial-complex-zeros-removal filter to emphasize its main purpose. Note that placing the poles of the filter to cancel the complex zeros outside the unit circle will make the filter unstable. Therefore, we can only partially remove the inner complex zeros of $G(z)$ introduced by ISI due to FTN signaling.
Figure 3.12: Complex zeros of the signal at the output of partial-complex-zeros-removal filter at $E_b/N_0 = 19$ dB. Black circles are the complex zeros of $X(z)$, magenta circles are the complex zeros of $Q(z)$, and white circles are the complex zeros of $F(z)$.

3.5.2 Maximum likelihood (ML) detector

Rewrite the signal at the output of the partial-complex-zeros-removal filter in (3.28) as:

$$ q = XFh + w_{wh}, \quad (3.29) $$

where $X$ is a Toeplitz matrix of size $L_{\text{filt}} \times (L_g + L_{\text{ch}})$ whose first column is $[x_0, x_1, \ldots, x_K, 0, \ldots, 0]^T$. Similarly, the Toeplitz matrix $F$ of size $(L_g + L_{\text{ch}}) \times L_{\text{ch}}$ is defined based on the vector $f$. Based on (3.29), the maximum likelihood detection of the transmitted coefficients is given as:

$$ \hat{x} = \arg \max_{x \in \rho} p(q|x), \quad (3.30) $$

where $p(q|x) \sim \mathcal{CN}(u_q, \Sigma_q)$ and is expressed as

$$ p(q|x) = \frac{1}{(2\pi)^{L_{\text{filt}}/2} |\Sigma_q|^{1/2}} \exp \left( -\frac{1}{2} (q - u_q)^{\text{H}} \Sigma_q^{-1} (q - u_q) \right). \quad (3.31) $$

Under the assumption that the channel $h$ and noise $w_{wh}$ are zero mean complex Gaussian random vectors, it is clear that $u_q = 0$. On the other hand, the covariance matrix $\Sigma_q = \mathbb{E}[qq^\text{H}]$ is computed as:

$$ \Sigma_q = \mathbb{E}[(XFh + w_{wh})(XFh + w_{wh})^\text{H}], $$

$$ = XFE[hh^\text{H}]F^\text{H}X^\text{H} + XFE[hw_{wh}^\text{H}] + \mathbb{E}[w_{wh}h^\text{H}]F^\text{H}X^\text{H} + \mathbb{E}[w_{wh}w_{wh}^\text{H}], $$

$$ = XFD_p F^\text{H}X^\text{H} + \sigma^2 I_{L_{\text{filt}}}, \quad (3.32) $$

where $D_p = \mathbb{E}[hh^\text{H}]$ is a diagonal matrix of size $L_{\text{ch}} \times L_{\text{ch}}$ whose diagonal elements are the power delay profile (PDP) vector $p$ of the channel $h$. To be specific, we define the PDP of $h$ as $p = (p^0, p^1, \ldots, p^{L_{\text{ch}}})$. Note that
the covariance of white noise $\mathbf{w}_{wh}$ is $E[\mathbf{w}_{wh}\mathbf{w}_{wh}^H] = \sigma^2 \mathbf{I}_{\text{filt}}$, where $\sigma^2 = N_0(1 + \beta)/2T$. Furthermore, since both $\mathbf{w}_{wh}$ and $\mathbf{h}$ are zero-mean independent random vectors, we have $E[\mathbf{h}\mathbf{w}_{wh}^H] = E[\mathbf{w}_{wh}\mathbf{h}^H] = 0$. Hence, the ML rule in (3.30) can be re-written as:

$$
\hat{x} = \arg \min_{\mathbf{x} \in \mathcal{P}} \left( \frac{1}{2} \mathbf{q}^H \left( \mathbf{XFDPF}^H \mathbf{X}^H + \sigma^2 \mathbf{I}_{\text{filt}} \right)^{-1} \mathbf{q} + \log \left( (2\pi)^{L_{\text{filt}}/2} \left| \mathbf{XFDPF}^H \mathbf{X}^H + \sigma^2 \mathbf{I}_{\text{filt}} \right|^{1/2} \right) \right).
$$

(3.33)

Using the Sylvester’s determinant identity (also known as the Weinstein-Aronszajn identity) [24], namely $[\mathbf{I}_S + \mathbf{VC}] = [\mathbf{I}_O + \mathbf{CV}]$, where $\mathbf{V}$ and $\mathbf{C}$ are matrices of dimensions $S \times O$ and $O \times S$, respectively, one can re-write the second term in (3.33) as

$$
\left| \mathbf{XFDPF}^H \mathbf{X}^H + \sigma^2 \mathbf{I}_{\text{filt}} \right|^{1/2} = \left| (\mathbf{D}_p^{1/2})^H \mathbf{F}^H \mathbf{AFD}_p^{1/2} + \sigma^2 \mathbf{I}_{\text{ch}} \right|^{1/2},
$$

(3.34)

where $\mathbf{A} = \mathbf{X}^H \mathbf{X}$ is the autocorrelation Toeplitz matrix constructed from the autocorrelation vector $\mathbf{a}$.

For the proposed SE-BMOCZ, the autocorrelation vector $\mathbf{a}$ corresponding to the Huffman polynomial is constant, i.e., for all possible combinations of the coefficient vector $\mathbf{x}$, the autocorrelation vector $\mathbf{a}$ remains the same, and is given as:

$$
\mathbf{a} = \mathbf{x} \ast \overline{\mathbf{x}},
$$

$$
\mathbf{a} = [a_0, 0, \ldots, 0, a_K, 0, \ldots, 0, a_{2K}]^T,
$$

(3.35)

where the magnitudes of $a_0$ and $a_{2K}$ are much less than the magnitude of $a_K$, making $a_K$ the peak of the autocorrelation vector $\mathbf{a}$. Since the matrix $\mathbf{A}$ is based on the autocorrelation vector $\mathbf{a}$, it is constant as well, i.e., it remains the same for every possible matrix $\mathbf{X}$. It follows that the term $\log \left( (2\pi)^{L_{\text{filt}}/2} \left| \mathbf{XFDPF}^H \mathbf{X}^H + \sigma^2 \mathbf{I}_{\text{filt}} \right|^{1/2} \right)$ in (3.33) is a constant and hence can be dropped from the optimization problem. Thus, the ML rule in (3.33) can be simplified as:

$$
\hat{x} = \arg \min_{\mathbf{x} \in \mathcal{P}} \left( \mathbf{q}^H \left( \mathbf{XFDPF}^H \mathbf{X}^H + \sigma^2 \mathbf{I}_{\text{filt}} \right)^{-1} \mathbf{q} \right),
$$

$$
= \arg \min_{\mathbf{x} \in \mathcal{P}} \left( \sigma^2 \mathbf{q}^H \left( \mathbf{XFDPF}^{1/2} \mathbf{D}_p^{-1/2} \mathbf{I}_{\text{ch}} \mathbf{D}_p^{-1/2} \mathbf{XFDPF}^{1/2} \mathbf{D}_p^{-1/2} \right)^{-1} \mathbf{q} \right),
$$

(3.36)

where $(\mathbf{D}_p^{1/2})^H = \mathbf{D}_p^{1/2}$ as $\mathbf{D}_p$ is real.

To further simplify the ML detection rule, we shall make use of the Woodbury matrix identity, which is given as [25] $(\mathbf{Z} + \mathbf{ECV})^{-1} = \mathbf{Z}^{-1} - \mathbf{Z}^{-1} \mathbf{E}(\mathbf{C}^{-1} + \mathbf{VZ}^{-1} \mathbf{E})^{-1} \mathbf{VZ}^{-1}$, where the sizes of the matrices $\mathbf{Z}$, $\mathbf{E}$, $\mathbf{C}$, and $\mathbf{V}$ are, respectively, $O \times O$, $O \times S$, $S \times S$, and $S \times O$. Applying this identity results in the following:

$$
\hat{x} = \arg \min_{\mathbf{x} \in \mathcal{P}} \left( \sigma^2 \mathbf{q}^H \left( \mathbf{I}_{\text{filt}}^{-1} - \mathbf{I}_{\text{filt}}^{-1} \mathbf{XFDPF}^{1/2} \left( (\sigma^2 \mathbf{I}_{\text{ch}})^{-1} + (\mathbf{D}_p^{1/2} \mathbf{F}^H \mathbf{X}^H + \sigma^2 \mathbf{I}_{\text{filt}})^{-1} (\mathbf{D}_p^{1/2} \mathbf{F}^H \mathbf{X}^H) \mathbf{I}_{\text{ch}}^{-1} \mathbf{D}_p^{1/2} \right) \right)^{-1} \mathbf{q} \right),
$$

$$
= \arg \min_{\mathbf{x} \in \mathcal{P}} \left( \sigma^2 \left\| \mathbf{q} \right\|_2^2 - \sigma^2 \mathbf{q}^H \left( \mathbf{XFDPF}^{1/2} \mathbf{D}_p^{-1/2} \mathbf{I}_{\text{ch}} \mathbf{D}_p^{-1/2} \mathbf{XFDPF}^{1/2} \mathbf{D}_p^{-1/2} \right)^{-1} \mathbf{D}_p^{1/2} \mathbf{F}^H \mathbf{X}^H \mathbf{q} \right).$$

(3.37)
Figure 3.13: Assigning the received complex zeros to $K$ sectors for $K = 7$, $L_{ch} = 2$, and $\tau = 0.9$. Black circles are the complex zeros of $X(z)$, cyan circles are the $2KM$ complex zeros of the codebook $\zeta$, magenta circles are the complex zeros of $Q(z)$, green circle is the complex zeros of $H(z)$, and white circles are the complex zeros of $F(z)$.

Ignoring the constant term $\sigma^2\|q\|_2^2$ simplifies (3.37) to:

$$
\hat{x} = \arg\max_{x \in \rho} \left( q^H XFD_1^{1/2} (\sigma^2 I_{L_{ch}} + D_{p}^{1/2} F^H X^H XFD_{p}^{1/2})^{-1} D_{p}^{1/2} F^H X^H q \right),
$$

$$
= \arg\max_{x \in \rho} \left( q^H X F \left( D_{p}^{-1/2} (\sigma^2 I_{L_{ch}} + D_{p}^{1/2} F^H X^H XFD_{p}^{1/2}) D_{p}^{-1/2} \right)^{-1} F^H X^H q \right),
$$

$$
= \arg\max_{x \in \rho} \left( q^H X F B^{-1/2} F^H X^H q \right),
$$

(3.38)

where $B = \sigma^2 D_{p}^{-1} + F^H X^H X F$. Finally, the ML detection of the transmitted coefficients in the proposed SE-MOCZ is expressed as:

$$
\hat{x} = \arg\max_{x \in \rho} \| B^{-1/2} F^H X^H q \|.
$$

(3.39)

Since there are $(2M)^K$ possible candidates for $x$, the complexity of the ML detector increases exponentially with $K$. Also, the complexity further increases by a factor $M$ for SE-PMOCZ. Hence, low-complexity detectors are needed for employing SE-BMOCZ with a large value of $K$, as well as for SE-PMOCZ.

3.5.3 Root-finding minimum distance (RFMD) detector

As originally described in [7], the basic idea of the RFMD detector is to identify the transmit complex zeros based on the minimum distance between the received complex zeros and the complex zeros belonging to the transmit codebook $\zeta$. In the RFMD detector, the $L_{filt} - 1$ complex zeros of the received polynomial $Q(z)$ are determined by using the received coefficient vector $q$ and a root-finding technique. To detect the $K$ transmit complex zeros, and hence, the $K$ groups of bits $\hat{m}_k$, $k = 1, \ldots, K$, first all the $L_{filt} - 1$ received complex zeros
are assigned to one of the \( K \) sectors based on the minimum distance between the received complex zeros and all the possible complex zeros within the codebook \( \zeta \). For example, the received complex zero denoted by \( \hat{\alpha}_1 \) in Fig. 3.13 is assigned to the first sector, i.e., the sector having phase 0. This is because \( \hat{\alpha}_1 \) is closest to one of the complex zeros within the first sector of the codebook \( \zeta \) as compared to the complex zeros in other adjacent sectors of the codebook. This criteria of assigning the \( j \)th received complex zero to the \( k \)th sector, when \( K > 1 \), is mathematically described as [7]:

\[
\Xi_k = \left\{ j \mid \forall k' \neq k : \min_{d \in \{0, \ldots, M-1\}} \min_{r \in \{1/R, R\}} d(\alpha_j, \alpha_k^{(d)(r)}) \leq \min_{d \in \{0, \ldots, M-1\}} \min_{r \in \{1/R, R\}} d(\alpha_j, \alpha_{k'}^{(d)(r)}) \right\}.
\] (3.40)

In the above expression \( \Xi_k \) contains the indices of the received complex zeros which are classified in the \( k \)th sector, \( j \) is the index of the received complex zero assigned to the \( k \)th sector, \( \alpha_j \) is the \( j \)th received complex zero, \( d(a, b) \) is the Euclidean distance between \( a \) and \( b \), i.e., \( d(a, b) = |a - b| \), and \( \alpha_k^{(d)(r)} \) is defined as:

\[
\alpha_k^{(d)(r)} = r e^{j 2 \pi ((k - 1)M + d)/MK},
\] (3.41)

where \( d \in \{0, \ldots, M - 1\} \) and \( r \in \{1/R, R\} \). Note that for \( K = 1 \), \( \Xi_1 = \{1, \ldots, L_{\text{ch}}\} \).

Thus, in general more than one complex zero can be assigned to the \( k \)th sector. Amongst the received complex zeros that are classified in the \( k \)th sector, the one that has the minimum distance from one of \( 2M \) possible complex zeros in the \( k \)th sector is decided as the \( k \)th transmitted complex zero and estimated by the closest complex zero of the codebook \( \zeta \). Fig. 3.13 shows that the last sector, i.e., the seventh sector, contains two received complex zeros and both are close to the codebook zero having the phase \((2\pi/6)/7\) and radius \( R \) as opposed to radius \( 1/R \). However, the received complex zero denoted by \( \hat{\alpha}_7 \) is decided as the transmitted complex zero. This is because the distance of \( \hat{\alpha}_7 \) from the codebook zero, having the phase \((2\pi/6)/7\) and radius \( R \), was smaller as compared to the distance of other received complex zeros from the same complex zero of the codebook.

The above approach of finding the \( k \)th transmitted complex zero, i.e., the radius \( \hat{r}_k \) and phase \( \hat{d}_k \) within the \( k \)th sector, can be mathematically described as [7]:

\[
[\hat{r}_k, \hat{d}_k] = \arg \min_{d \in \{0, \ldots, M-1\}, j \in \Xi_k} \min_{r \in \{1/R, R\}} d(\alpha_k^{(d)(r)}, \alpha_j).
\] (3.42)

Let the MSB and the least significant bits of \( \hat{\mathbf{m}}_k \) be denoted by \( \hat{c}_k \) and \( \hat{b}_k \), respectively, i.e., \( \hat{\mathbf{m}}_k = [\hat{c}_k, \hat{b}_k] \).

Then \( \hat{c}_k \) is given as:

\[
\hat{c}_k = \begin{cases} 
0, & \hat{r}_k = 1/R \\
1, & \hat{r}_k = R
\end{cases}
\] (3.43)

and \( \hat{b}_k = \text{De}2\text{Bi}(\hat{d}_k) \).

For the case that \( \Xi_k \) is empty for a particular \( k \)th sector, one can look for extra complex zeros in the neighboring sectors and then classify the closest complex zero from a neighboring sector into this \( k \)th sector.

53
Another option is to randomly assign the bits for that $\hat{m}_k$. The performance in both cases was observed to be the same in our simulation results, but the transmission in these cases is unreliable anyway as the noise severely disturbs the location of the $k$th complex zero. To have acceptable reliability in such a scenario, re-transmission of the packet can be requested.

Before closing this section, recall that the locations of the complex zeros introduced by the ISI due to FTN signaling are known. However, they are very sensitive to noise disturbance as discussed in Section 3.3. It appears that eliminating the complex zeros of the ISI due to FTN signaling before detecting the complex zeros of $X(z)$ does not improve the detection performance of the RFMD detector.

### 3.6 Simulation Results

In this section, we evaluate the BER performance of the proposed SE-MOCZ and compare it with the BER of MOCZ to quantify the spectral efficiency gain. In our simulations, we use a square-root raised cosine pulse with a roll-off factor $\beta = 0.3$. The span of the pulse is set by truncating the coefficients whose magnitudes are less than 0.1% of the peak magnitude. We consider sporadic transmission, hence consecutive packets experience different channel realizations.

The channel is normalized to have an average energy of one with flat PDP. To have a fair comparison with MOCZ, the energy of the transmit signal of SE-MOCZ is set to equal its counterpart of MOCZ as $K + L_{ch}$ joules so that both techniques have the same $E_b/N_0$. Therefore, the energy per bit $E_b$ in the transmit signal is $E_b = (K + L_{ch})/B$, where $B = K \log_2 2M$ is the total number of transmitted bits per packet. It follows that $E_b/N_0 = (K + L_{ch})/N_0 B$.

The spectral efficiencies of MOCZ and SE-MOCZ are calculated as $K \log_2 2M/((K + L_{ch}) (1 + \beta))$ and $K \log_2 2M/(\tau (K + L_{ch}) (1 + \beta))$, respectively.

The ML detector for MOCZ is implemented according to [7, Eqn. (45)] to have a fair comparison with the ML detector for our proposed SE-MOCZ. Note that the ML detector in [7, Eqn. (19)] is an approximate version and may result in a minor degradation as compared to the exact ML performance of MOCZ. Further, the radius $R$ of MOCZ for the binary case, i.e., BMOCZ is given as [7]:

$$R_{MOCZ} = \sqrt{1 + \sin(\pi/K)}, \quad (3.44)$$

whereas for higher-order modulation, i.e., PMOCZ, it is given as [7]:

$$R_{MOCZ} = \sqrt{1 + \left(\ln (2M) - 0.3\right) \sin \left(\frac{\pi}{KM}\right)} . \quad (3.45)$$

Fig. 3.14 plots the BER curves of BMOCZ and SE-BMOCZ versus $E_b/N_0$ obtained with ML and RFMD detectors for $K = 7$, $L_{ch} = 10$ and $\tau = 0.9$. For BMOCZ, $R_{MOCZ} = 1.1974$. For SE-BMOCZ, the parameters are $R_{opt2} = 1.1959$ and $\lambda = 2$ when the ML detector is used, whereas $R_{opt2} = 1.1232$ and $\lambda = 3.2$ for the RFMD detector. Note that the values of $R_{opt2}$ are different for the ML and the RFMD detectors because the optimal radius is found based on the BER performance of SE-MOCZ, which depends on the type of
detector. It can be seen from Fig. 3.14 that, when the ML detector is used, the proposed SE-BMOCZ achieves practically the same performance as BMOCZ. On the other hand, when the RFMD detector is used, the performance gap between SE-BMOCZ and BMOCZ is about 3 dB at the BER of $10^{-3}$. This comparison means that, using the ML detector the proposed SE-BMOCZ provides a spectral efficiency gain of $0.3519 - 0.3167 = 11.11\%$ without any penalty in transmit power or BER performance. However, when the low-complexity RFMD detector is used, in order to have the same spectral efficiency gain, SE-BMOCZ incurs about 3 dB penalty in the transmit power as compared to BMOCZ.

In Fig. 3.15, we compare the ML performance of SE-BMOCZ and BMOCZ, with and without implementing the partial-complex-zeros-removal filter when $K = 7$, $L_{ch} = 10$, and $\tau = 0.8$. It can be seen that when the partial-complex-zeros-removal filter is not implemented and the radius $R$ in SE-BMOCZ is set to be the same as that in MOCZ, i.e., $R = R_{MOCZ} = 1.1974$, the performance gap between SE-BMOCZ and BMOCZ at the BER of $10^{-3}$ is about 2.8 dB. This gap reduces to about 1.8 dB when $R_{opt1} = 1.3524$ and $\lambda = 0.6$ is used for SE-BMOCZ. Finally, when the partial-complex-zeros-removal filter is implemented with $R_{opt2} = 1.3581$ and $\lambda = 0.95$, the performance of SE-BMOCZ approaches that of BMOCZ, with only about 0.18 dB gap. This leads to $\frac{0.3959 - 0.3167}{0.3167} = 25\%$ gain in the spectral efficiency of SE-MOCZ over MOCZ. Note that when the partial-complex-zeros-removal filter is implemented, the optimal radius $R_{opt2}$ is used instead of $R_{opt1}$ as discussed in Sub-section 3.4.2. One can conclude from Fig. 3.15 that, for small values of $K$ such as $K = 7$, optimizing the radius without implementing the partial-complex-zeros-removal filter (i.e., using $R_{opt1}$) result in about 1 dB saving in transmit power, whereas implementing the partial-complex-zeros-removal filter along with the optimal radius $R_{opt2}$ for the proposed SE-BMOCZ results in about 1.8 dB saving in the transmit power.
Because of the high computational complexity of the ML detector at high values of $K$, we use the RFMD technique for the proposed SE-BMOCZ at higher values of $K$. Fig. 3.19 plots the BER performance of SE-BMOCZ with the RFMD detector when $K = 40$, $L_{ch} = 10$, and $\tau = 0.9$. The SE-BMOCZ when implemented without the partial-complex-zeros-removal filter and with $R = R_{\text{MOCZ}} = 1.0385$ needs about 2 dB more in transmit power as compared to BMOCZ in order to achieve the BER of $10^{-3}$. The error performance remains the same when $R_{\text{opt1}} = 1.0373$ (with $\lambda = 4.5$) is used in place of $R_{\text{MOCZ}}$. This indicates that, for higher values of $K$, the performance of SE-MOCZ is insensitive to optimizing the radius only. This is in contrast to the observation of Fig. 3.15 that the BER performance is quite sensitive to radius optimization at small values of $K$. Finally, when the partial-complex-zeros-removal filter is implemented along with $R_{\text{opt2}}$, the performance gap between SE-BMOCZ and BMOCZ reduces to about 1 dB, where $R_{\text{opt2}}$ and the corresponding $\lambda$ were found to be 1.0419 and 6.5, respectively. Hence, 11.11% spectral efficiency gain can be achieved by the proposed SE-BMOCZ at $K = 40$ using the RFMD detector, albeit at the expense of 1 dB penalty in the transmit power. This finding suggests that the RFMD detector is more effective and attractive for the proposed SE-MOCZ at high values of $K$.

In Fig. 3.17 we investigate the effect the channel length $L_{ch}$ on the BER performance of SE-MOCZ using the RFMD detector when $K = 40$ and $\tau = 0.9$. As expected, the BER performance degrades as $L_{ch}$ increases. The values of $R_{\text{opt2}}$ for $L_{ch} = 10, 20, 30,$ and 40 are found to be 1.0419, 1.0381, 1.0325, and 1.0278 with the values of $\lambda = 6.5, 7.2, 8.5$, and 10, respectively. The value of $R_{\text{opt2}}$ changes with $L_{ch}$ because we find $R_{\text{opt2}}$ based on the BER performance, which degrades with increasing $L_{ch}$.

The BER performance for higher-order SE-MOCZ, namely SE-4-PMOCZ is shown in Fig. 3.18 when $K = 20$, $L_{ch} = 7$, and $\tau = 0.9$. At the BER of $10^{-3}$, the performance gap between 4-PMOCZ and SE-
Figure 3.16: BER performance of BMOCZ and SE-BMOCZ using the RFMD detector when $K = 40$, $L_{ch} = 10$, $\tau = 0.9$.

Figure 3.17: BER performance of SE-BMOCZ using the RFMD detector when $K = 40$ and $\tau = 0.9$. 
4-PMOCZ is about 2 dB when no partial-complex-zeros-removal filter is included and the MOCZ radius $R_{\text{MOCZ}} = 1.0417$ is used. When the radius is optimized to be $R_{\text{opt1}} = 1.0538$ and $\lambda = 3$, the performance gap slightly reduces to about 1.7 dB. Hence, it can be concluded that for higher-order SE-MOCZ, i.e., SE-PMOCZ, the BER performance is not too sensitive to the optimization of the radius alone. When the partial-complex-zeros-removal filter is implemented along with $R_{\text{opt2}} = 1.0630$ and $\lambda = 4.2$, the performance of 4-SE-PMOCZ approaches that of 4-PMOCZ, with a gap of about 0.7 dB. This means that there is about $1.2662 - 1.1396 = 11.11\%$ spectral efficiency gain at the expense of 0.7 dB increase in the transmit power.

3.7 Conclusions

We proposed and investigated a non-coherent communication scheme for short packet communications (SPCs), called spectrally-efficient modulation on conjugate reciprocal zeros (SE-MOCZ). SE-MOCZ aims to improve the spectral efficiency of the recently-proposed MOCZ by combining MOCZ with faster-than-Nyquist (FTN) signaling. Accelerating the transmission of the pulses carrying the polynomial coefficients of MOCZ beyond the Nyquist limit introduces ISI. The ISI due to FTN signaling increases the number of received complex zeros in the $z$-domain. To partially remove these extra complex zeros, we designed a partial-complex-zeros-removal filter at the receiver. We also optimized the radius of the transmit complex zeros for SE-MOCZ to achieve better robustness against noise and ISI due FTN signaling. The maximum likelihood (ML) detector as well as the low-complexity root-finding minimum distance (RFMD) detector were presented for the proposed SE-MOCZ scheme. Compared to MOCZ, simulation results showed a spectral efficiency gain of up to 25% by the proposed SE-MOCZ when the ML detector is used with small values of

![Figure 3.18](image-url)
$K$. For high values of $K$, a spectral efficiency gain of up to 11\% can be achieved with the low-complexity RFMD detector, but at the expense of about 1.0 dB in the transmit power. Simulation results also showed that optimizing the radius of the transmit complex zeros in SE-MOCZ is beneficial at low values of $K$. For high values of $K$ or higher-order SE-MOCZ, where the RFMD detector is used, there is little gain in radius optimization. Lastly, implementing the partial-complex-zeros-removal filter along with the radius of transmit complex zeros optimally designed by taking into account such a filter improves the BER performance for both low and high values of $K$. 
References


4 Summary and Suggestions for Future Studies

4.1 Summary

Short packet communications (SPCs) is deemed to be necessary for future internet of things (IoT) applications which need to meet the stringent ultra low-latency requirements. There are several challenges associated with SPCs and one of the major challenges is the channel estimation. Conventionally, known pilot symbols are sent along with the data, which will help in determining the channel state information (CSI) based on how the known pilots get affected by the channel. As the blocklength in SPC is finite, so sending the known pilots in case of SPCs will greatly decrease the spectral efficiency of the transmission. Therefore, SPCs require the non-coherent modulation techniques, i.e., the modulation techniques which do not require CSI for reliable detection of the transmitted data.

A novel non-coherent modulation technique named as modulation on conjugate reciprocal zeros (MOCZ), which is suitable for SPC, has been discussed in this dissertation. In MOCZ, the information bits are modulated onto the complex zeros in the $z$-domain, and the coefficients corresponding to these complex zeros are transmitted through the channel. Due to the transmission via frequency-selective channel, the random complex zeros of the channel are also added to the signal which increases the number of received complex zeros. At the receiver, the transmitted complex zeros are detected without the knowledge of the channel, using one of the following three detectors, namely, root finding minimum distance (RFMD) detector, maximum likelihood (ML) detector, or direct zero-testing (DiZeT) detector.

The finite blocklength of the message in SPCs result in data rate loss as compared to the channel capacity. To recover this loss of data rate without any increase in the bandwidth, faster-than-Nyquist (FTN) signaling is deemed as the candidate solution. This recovery of data rate loss is achieved by accelerating the transmission of Nyquist pulses, carrying the data, beyond the Nyquist limit. Hence, FTN signaling improves the spectral efficiency of the transmission at the cost of introducing inter symbol interference (ISI) between the received samples.

In our research, we improved the spectral efficiency of MOCZ by proposing a novel modulation technique named as Spectrally-Efficient Modulation on Conjugate-Reciprocal Zeros (SE-MOCZ). This technique combines MOCZ with FTN signaling, i.e., in this technique the transmission of Nyquist pulses, carrying the coefficients of MOCZ, is accelerated beyond the Nyquist limit. Accelerating the transmission of pulses increases the number of complex zeros at the receiver due to the ISI introduced by the FTN signaling. The extra complex zeros corresponding to the ISI, negatively impact the BER performance. Hence, to achieve
better BER performance, we optimized the radius of the transmit complex zeros in the presence of extra complex zeros. We further designed a partial-complex-zeros-removal filter at the receiver to partially remove the complex zeros coming from the ISI. Lastly, we derived the maximal likelihood (ML) detector to optimally perform the detection in the presence ISI. To achieve a balance in terms of the high performance and low complexity, root finding minimum distance (RFMD) detector was employed at high values of $K$ and higher order SE-MOCZ. Simulation results showed that with ML detector, at small values of $K$, SE-MOCZ results in 25% improvement in the spectral efficiency. At the high values of $K$, the spectral efficiency gain of 11% can be achieved using the RFMD detector; however, at the expense of about 1 dB increase in the transmit power. The results of optimizing the radius for SE-MOCZ were not prominent for high values of $K$ and higher order SE-MOCZ when RFMD detector was employed. Further, the simulation results showed that the BER performance is improved at high and low values of $K$ by implementing the partial-complex-zeros-removal filter at the receiver.

4.2 Suggestions for Future Studies

- Due to the complexity of the ML detector, one cannot employ the ML detector for high values of $K$, as well as, for higher order SE-MOCZ, whereas using the RFMD detector results in non-optimal performance. A possible solution is to utilize a neural network consisting of several layers which takes the complex received zeros as input and decide on the transmitted complex zeros. This may result in improved BER performance, i.e., performance close to the ML detector, while maintaining the low-complexity in comparison to the ML detector.

- Other potential research problem is to employ precoding technique at the transmitter to remove the extra complex zeros due to the FTN signaling.

- Lastly, as the transmit complex zeros in SE-MOCZ are modulated onto two radii, $1/R$ and $R$, instead one can use four radii, i.e., $1/R_1$, $R_1$, $1/R_2$, and $R_2$, for the modulation such that the distance of transmit complex zeros can be further maximized from the ISI complex zeros according to the location of ISI complex zeros.