## ***Part 1***

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Unit Title: Fantastic Functions!

**Unit Strand 1: Understanding Functions (P30.8)**

Micro Concepts:

Intercepts, extremum, domain/range, function notation, evaluating values of functions, graphing functions, function mapping, relation terminology, vertical line test, horizontal line test

Generalizations:

- A function's range is all possible values in the y-axis.
- A function's domain is all possible values in the x-axis.
- Functions can be represented as either equations or graphically.
- A relation is a set of ordered pairs, where each ordered pair consists of an input value (domain) and an output value (range).
- A one-to-one function is defined as for every value in the domain, there exists a unique value in the range.
- A onto function is defined as for every value in the domain, there exists a value in the range.
- A many-to-one function is defined if there is more than one value in the domain that maps to a single value in the range.

**Unit Strand 2: Transformations (P30.7)**

Micro Concepts:

Order of operations, translations, reflections, stretching, function notation, coefficient, constant term, evaluating values of functions, graphing functions, intercepts, domain, range, slope, vertex

Generalizations:

- Functions are translated horizontally to the right when the constant value h is positive.
- Functions are translated horizontally to the left when the constant value h is negative.
- Functions are translated vertically upwards when the constant value k is positive.
- Functions are translated vertically downwards when the constant value k is negative.
- Functions are stretched horizontally by a factor of b.
- Functions are stretched vertically by a factor of a.
- Functions are reflected about the y-axis when b is negative.
- Functions are reflected about the x-axis when a is negative.
- All functions are transformed by manipulating the constants a, b, h, k in the form $y-k=af(b(x-h))$.

**Unit Strand 3: Manipulating Function Expressions (P30.6, P30.8)**

Micro Concepts:

Evaluating function values, function notation, operations on functions, function distribution, inverse relations, relation pairs, sketching functions, slope, domain, range, x-axis, y-axis, origin

Generalizations:

- Multiple functions can be combined by adding, subtracting, multiplying or dividing.
- Functions can be distributed on to other functions to create composite functions.
- Composite functions can be represented by a graph.
- Graphs can represent the changes in a function from adding, subtracting, multiplying and dividing by another function.
- The reflection of a function can be represented by a graph.
- Every one to one relation will have an inverse.
- An inverse relation is not a function.
- Relation inverses are derived from swapping each of the elements with the other element from its ordered pair.

## ***<u>Part 2</u>***

| Unit Title: Fantastic Functions! | Conceptual Lens: Relationships | Grade / Curriculum: Grade 11/12 / Pre-Calculus 30 |
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| Outcomes from the Saskatchewan mathematics curriculum |
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| P30.6 - Demonstrate an understanding of operations on, and compositions of, functions.<br>P30.7 - Extend understanding of transformations to include functions (given in equation or graph form) in general, including horizontal and vertical translations, and horizontal and vertical stretches.<br>P30.8 - Demonstrate understanding of functions, relations, inverses and their related equations resulting from reflections through the:<br>- x-axis<br>- y-axis<br>- line $y=x$. |

| Unit Overview | Concepts in the Unit: |
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| Get ready to enter the fantastic world of functions with us! We will explore how functions can be transformed to model real-world scenarios. Did you know your accountant uses composite functions to calculate your taxes? How about that Snapchat uses functions and their inverses to keep your data safe? Wow! | relations, functions, domain, range, evaluating functions, equation forms of functions, graphical form of functions, mapping terms (one-to-one, onto, many-to-one), horizontal line test, vertical line test, reflections, translations, stretching, $y-k=af(b(x-h))$, operations on functions (equation and graph form), compositions of functions, inverses of relations, transforming equations and graphs |

| What we would like students to know | What we would like students to understand | What we would like students to be able to do |
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| 1. Know that relations are a set of ordered pairs that correspond to inputs and outputs.<br><br>2. Know that functions can be represented in various forms (equations, graphs, tabular, etc).<br><br>3. Know the various transformations to functions, including translations, reflections, and stretches.<br><br>4. Know that functions can be combined by adding, subtracting, multiplying or dividing.<br><br>5. Know that functions can be combined to form composite functions. | 1. The concept of a function as something that takes inputs and produces outputs.<br><br>2. Transformations are given by manipulating the constants a, b, h, k in $y-k=af(b(h-k))$ to produce translations, reflections, and stretches.<br><br>3. Associative and commutative properties of arithmetic apply to functions.<br><br>4. How composite functions are formed and how to distribute functions onto other functions. | 1. Determine the intercepts, extremum, range, and domain of a function.<br><br>2. Determine the type of relations using the horizontal and vertical line test.<br><br>3. Apply transformations like translations, reflections, and stretches to functions.<br><br>4. Interpret a graph and produce the corresponding equation in $y-k=af(b(h-x))$ format. |

<table>
<tr><td>6. Know that every one-to-one relation has an inverse.</td><td>5. Inverses are produced by swapping the ordered pair of a relation.</td><td>5. Sketch graphs of functions that have undergone transformations.<br><br>6. Apply basic arithmetic to functions.<br><br>7. Combine functions into composite functions.<br><br>8. Determine the inverse of a function.</td></tr>
<tr><th>Generalizations</th><th colspan="2">Guiding Questions</th></tr>
<tr><td>1. A function’s range is all possible values in the y-axis.<br><br>2. A function’s domain is all possible values in the x-axis.</td><td colspan="2">Factual Questions:<br>- What is the range and domain of a function?<br>- How do you determine the domain of a function?<br>- How do you determine the range of a function?<br><br>Conceptual Questions:<br>- What is the relationship between the domain and range of a function?<br>- Can a function have the same range as another function but a different domain? Why or why not?</td></tr>
<tr><td>3. Functions can be represented as either equations or graphically.</td><td colspan="2">Factual Questions:<br>- What are two common ways to represent a function?<br>- How is a function represented on a graph?<br>- What is the vertical line test?<br>- What is the horizontal line test?<br>- How can you graphically determine the intercepts, extremum, range, and domain of a function?<br><br>Conceptual Questions:<br>- Why is it important to consider the domain and range of a function when graphing it?<br>- What are some advantages and disadvantages of representing a function as an equation versus a graph?</td></tr>
<tr><td>4. A relation is a set of ordered pairs, where each ordered pair consists of an input value (domain) and an output value (range).</td><td colspan="2">Factual Questions:<br>- What is the difference between a function and a relation?<br>- What is the difference between a one-to-one, onto, and many-to-one function? Can you provide an example of each?<br>- What tests can you use to see if a function is one-to-one, or onto?</td></tr>
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| 5. A one-to-one function is defined as for every value in the domain, there exists a unique value in the range.<br><br>6. A onto function is defined as for every value in the domain, there exists a value in the range.<br><br>7. A many-to-one function is defined if there is more than one value in the domain that maps to a single value in the range. | Conceptual Questions:<br>- How can you determine whether a relation is a function or not?<br>- Can a function be both one-to-one and onto? Is that always the case?<br>- Why are all functions relations, but not all relations are functions?<br>- In what real-life situations might a function be onto, one-to-one, many-to-one? |
| 8. Functions are translated horizontally to the right when the constant value h is positive.<br><br>9. Functions are translated horizontally to the left when the constant value h is negative.<br><br>10. Functions are translated vertically upwards when the constant value k is positive.<br><br>11. Functions are translated vertically downwards when the constant value k is negative. | Factual Questions:<br>- What is a vertical translation?<br>- What is a horizontal translation?<br><br>Conceptual Questions:<br>- How are the intercepts impacted by translations? How is the slope impacted?<br>- How is the domain and range affected by translations?<br>- What real life situations are represented by changing the constant values h and k? |
| 12. Functions are stretched horizontally by a factor of b.<br><br>13. Functions are stretched vertically by a factor of a.<br><br>14. Functions are reflected about the x-axis when a is negative.<br><br>15. Functions are reflected about the y-axis when b is negative. | Factual Questions:<br>- How is the graph of a function affected if a is greater than 0, equal to 0, or less than 0?<br>- How is the graph of a function affected if b is greater than 0, equal to 0, or less than 0?<br>- What is a reflection?<br>- What does it mean to stretch a graph?<br><br>Conceptual Question:<br>- Can an increasing graph become a decreasing graph by altering a and b, or vice versa?<br>- Are all reflections symmetrical? Why or why not? |
| 16. All functions are transformed by manipulating the constants a, b, h, k in the form $y-k=af(b(x-h))$. | Factual Questions:<br>- What equation is used to express transformations?<br>- What information can be gathered from functions written in the form $y-k=af(b(x-h))$?<br>- What types of transformations exist? |

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| | Conceptual Questions:<br>- Is there a maximum number of transformations that can be applied to a single function?<br>- How do different transformations simultaneously alter a graph?<br>- Can all graphs be written in the form y-k=af(b(x-h))?<br>- How does y-k=af(b(x-h)) compare to other forms of equations (slope-intercept form, point-slope form, standard form)? |
| 17. Multiple functions can be combined by adding, subtracting, multiplying or dividing.<br><br>18. Functions can be distributed on to other functions to create composite functions. | Factual Questions:<br>- How do you find the sum, difference, product or quotient of two functions?<br>- How do you substitute one function into another function?<br><br>Conceptual Questions:<br>- When would you use the skill of combining multiple functions?<br>- What can we learn by taking the function of another function? |
| 19. Composite functions can be represented by a graph.<br><br>20. Graphs can represent the changes in a function from adding, subtracting, multiplying and dividing by another function.<br><br>21. The reflection of a function can be represented by a graph. | Factual Questions:<br>- What information do you need to sketch the graph of a function?<br>- How do you sketch the graph of a function's reflection?<br>- How do you represent in a graph adding, subtracting, multiplying or dividing functions?<br><br>Conceptual Question:<br>- How can we use graphs to determine the equation of a function? |
| 22. Every one to one relation will have an inverse.<br><br>23. An inverse relation is not a function.<br><br>24. Relation inverses are derived from swapping each of the elements with the other element from its ordered pair. | Factual Questions:<br>- What is the definition of an inverse relation?<br>- What is the definition of a relation pair?<br>- What makes a relation different from a function?<br>- How do you determine the inverse of a relation?<br><br>Conceptual Questions:<br>- How do the domains and ranges of a relation pair compare to each other?<br>- What can a relations' inverse tell us about the relation?<br>- How can you determine inverses by evaluating graphs of relations? |

## Debatable Questions

1. Which is better: representing a function as an equation or as a graph? or is there another way that's better?
2. Do we need to understand the equation of a function in order to apply a transformation to it? Why or why not?
3. Is it important to be able to find the inverse of a function? Think about its importance in cryptography or engineering?

## Pre-Skills needed for this unit

Basic algebra: Students should be comfortable with solving equations, simplifying expressions, and manipulating variables. This includes applying order of operations correctly.

Graphing: Students should be able to plot points on a Cartesian plane, identify x- and y-coordinates, identify slopes, and understand the properties of a graph visually. Students will need to construct exact graphs of functions, as well as be able to roughly sketch these graphs.

Problem-solving: Students must have critical thinking skills used to analyze and interpret word problems and graphs. These critical thinking skills are derived from practice applying mathematical concepts to real-life situations.

Understanding of graphing technology: Students should have a firm understanding of how to use Desmos and graphing calculators in order to support their own learning and practice.

## Assessments for Learning (broad strokes)

Concept maps: Students can create concept maps to show their understanding of the relationships between functions, compositions of functions, and transformations. They can use arrows and labels to show how concepts are related to one another. Students can refer back to these maps to review concepts, as well as the relationships between concepts. These will be handed in at the end of each week and returned at the beginning of the next week, allowing for time to review each student's understanding and provide feedback.

Low-stakes quizzes: Short quizzes are to be given at the end of each week, assessing the material that was covered that week. These quizzes provide feedback about students' understanding and reinforce key concepts, as well as give students the opportunity to practice the same style of questions that can/will appear on the Unit Exam. These quizzes can be graded on completion or participation rather than accuracy to reduce stress and anxiety.

Gamification: Using the platform Kahoot students will participate in a quiz-style game where they will encounter both application and recall questions. The element of fun associated with competition, as well as the anonymity of results given on the platform, allow students a way to demonstrate their understanding without the stress or anxiety associated with grades. The results that Kahoot provides are given in a timely manner as statistics regarding which questions students got correct, and these results provide information as to which concepts should be reviewed and student understanding of concepts.

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| Exit Slip Concept Review: Regular exit slips for students to complete regarding individual concepts in order to indicate their readiness to be able to move to the next concept. These exit slips give students the opportunity to share specifically which concepts they feel confident in, and which they do not feel as confident in. With this information in mind, the upcoming lessons can be modified to allow for more practice/review of concepts or accelerated to keep the lessons at the appropriate level of challenge.<br><br>Mini-whiteboards: Students have the chance to practice during lessons. All students would attempt the same question simultaneously. By simply walking around and looking over students' shoulders, teachers can obtain feedback regarding which aspects of concepts students are understanding, and where exactly (which steps) students may be misunderstanding or struggling. Whiteboards allow for this feedback during independent practice as well as collaborative work.<br><br>Independent Practice Questions: Students will have practice questions assigned for each lesson that is completed in class. They will not be graded but will be available for students to practice their skills. The questions will also be anonymously peer-reviewed so that students are given the opportunity to identify errors in completing the type of questions they are working on. These practice questions will also be used for students to put in their learning journals and portfolio to show understanding. |
| Assessments as Learning (broad strokes) |
| Self-assessment checklists: Students will use a rating scale to assess their confidence about the concepts covered which will help them in visualizing their understanding of key concepts and skills related to the unit. This will include topics from function mapping, transformation of functions, compositions of functions, and inverses. This will be done a few days before the unit test so that students can reflect and identify topics they are confident in, and what areas they need to practice before the exam. (also could be used FOR learning)<br>*Ex) Stretching functions: Not confident ←----------------------x---->confident*<br><br>Peer review: Students can review and provide feedback on each other's work, such as algebra, graphs of functions, and compositions of functions. This can help students develop mindfulness and strategies to identify errors in their peers' work, as well as their own work. Students have the opportunity to learn from their peers' structure, organization, and thought processes depicted in their practice, as they can discover new perspectives and alternative means of calculation and graphing that they may choose to input into their future practice.<br><br>Learning journals: Students can keep a journal throughout the unit where they reflect on their learning, ask questions, and make connections between different concepts. The students can also use this as a place to compile all of the necessary equations and "rules" that they will learn throughout the unit. These journals give students the opportunity to personalize their notes and draw on personal connections to the material (such as prior learning) as they develop their understanding. The teacher can review these journals and provide feedback and guidance as needed. |

| Assessments of Learning (broad strokes) |
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| Unit test: A comprehensive test could be given at the end of the unit to assess students' understanding of all the concepts and skills covered. This could include a mix of multiple-choice, short-answer, and problem-solving questions. To differentiate this test for all students, it would include a variety of questions that pertain to each topic, but require students to choose a set number of questions per topic to complete. In this way, students experience autonomy and have the opportunity to demonstrate their understanding using an example in which they are most comfortable. The questions would also have a small range in difficulty, such that students who prefer a challenge can demonstrate their ability, and students who prefer to demonstrate their understanding at a more basic level can also successfully do so.<br><br>Portfolio: Students would create a portfolio of their work throughout the unit, including problem sets, graphs of functions, and compositions of functions. This will include examples from practice work that they have done to show their understanding as well as corrections to questions they had originally completed incorrectly to show how they have improved their understanding. The students will also include some reflections from the unit, such as those written in their learning journal as well as new reflections, showing their understanding of the material which will give the instructor the opportunity to see that students have reached the expected generalizations about the topic. |
| Materials needed |
| - Textbook: Pre-Calculus 12 (Student edition, Teacher edition)<br>- Whiteboard, mini white boards, dry erase markers, dry erase erasers<br>- Practice worksheets<br>- Real-world case studies of examples of functions found in science, economics, or engineering and their application<br>- Low-stake quizzes<br>- Unit exams<br>- Self-assessment checklist<br>- Learning journals<br>- iPads with speech-to-text and text-to-speech capabilities<br>- Laptops<br>- Calculators (standard and graphing)<br>- PDFs of class notes (to upload to Google Classroom)<br>- Youtube videos<br>- Voice-overs of lessons |
| Technology Use |
| - Internet<br>- Laptops<br>- Screen recording software for instructor<br>- Desmos Graphing<br>- Kahoot<br>- Graphing calculators<br>- Standard calculators<br>- Google Classroom<br>- iPads with speech-to-text and text-to-speech capabilities, as well as capability to download other assistive technology programs |

| Adaptations for students |
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| Collaborative learning: Encourage collaborative learning by allowing students to work in pairs or groups, which can help to build social skills and encourage peer support. Collaborative learning also gives students the unique opportunity to learn from their peers, as all students will have different perspectives, thoughts, and opinions to contribute to discussions. Additionally, by approaching relatively challenging topics with the support of peers, students who have math anxiety may feel more comfortable participating.<br><br>Online resources: Provide materials (e.g., helpful youtube videos, notes, lecture-style voice-over of the taught lessons) on Google Classroom allowing students access to materials and lessons that they may have missed or are interested in reviewing independently.<br><br>Open-ended tasks: Provide open-ended tasks to meet the needs of individual students. For example, some students may benefit from simply solving the task to start with, while others may try more challenging approaches to problems to keep them engaged.<br><br>Parallel tasks: On quizzes and tests students will be provided with parallel questions such that they can pick the question they feel most comfortable with, while still demonstrating their understanding of the concept.<br><br>Assistive technology: Use assistive technology such as text-to-speech or speech-to-text software, screen readers, or enlarged text, to support students with disabilities or learning differences.<br><br>Visual aids: Provide visual aids such as diagrams, graphs, and charts to help students who may struggle with reading or understanding written instructions. These aids will be provided in the textbook, online, and during class via demonstrations and hand-outs. |
| Next steps for this unit |
| This unit prepares students for the following outcomes in the remainder of Pre-Calculus 30:<br>- P30.9 Demonstrate an understanding of logarithms including:<br>  ● evaluating logarithms<br>  ● relating logarithms to exponents<br>  ● deriving laws of logarithms<br>  ● solving equations<br>  ● graphing.<br>- P30.10 Demonstrate understanding of polynomials and polynomial functions of a degree greater than 2 (limited to polynomials of degree $\leq 5$ with integral coefficients).<br>- P30.11 Demonstrate understanding of radical and rational functions with restrictions on the domain.<br><br>This unit prepares students for the following outcomes in Calculus 30:<br>- C30.1 Extend understanding of functions including:<br>  ● algebraic functions (polynomial, rational, power)<br>  ● transcendental functions (exponential, logarithmic, trigonometric)<br>  ● piecewise functions, including absolute value<br>  ([C, CN, ME, R, T, V])<br>- C30.3 Demonstrate understanding of limits and continuity ([C, CN, ME, PS, R, T, V])<br>- C30.5 Extend understanding of curve sketching by applying differentiation and limits. [C, CN, V]<br>- C30.7 Demonstrate understanding of transcendental function derivatives and their applications. |

**References**


Wathall, J. T. H., (2016). *Concept-based mathematics: Teaching for deep understanding in secondary classrooms.* Corwin.

https://www.edonline.sk.ca/webapps/moe-curriculum-BB5f208b6da4613/CurriculumHome?id=159 (Pre-Calculus 30 SK Curriculum, accessed March 2023)

https://www.edonline.sk.ca/webapps/moe-curriculum-BB5f208b6da4613/CurriculumHome?id=163 (Calculus 30 SK Curriculum, accessed March 2023)