DISTRIBUTION SYSTEM RELIABILITY ASSESSMENT

A Thesis Submitted to
the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements for
the Degree of Master of Science
in the Department of
Electrical Engineering

University of Saskatchewan

by

Mohinder Singh Grover

October, 1972

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Head of the Department of Electrical Engineering
University of Saskatchewan
Saskatoon, Canada.
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UNIVERSITY OF SASKATCHEWAN

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"DISTRIBUTION SYSTEM RELIABILITY ASSESSMENT"

Student: Mohinder Singh Grover      Supervisor: R. Billinton

M.Sc. Thesis presented to the College of Graduate Studies

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ABSTRACT

An acceptable level of customer reliability is a major factor in the design and operation of an efficient and economic distribution system. Quantitative reliability assessment provides a valuable tool for predicting the performance adequacy of the system. This thesis illustrates the application of simple probability techniques to the evaluation of the generally accepted reliability measures of outage frequency and duration at various system load points. Component permanent, temporary, maintenance and overload outage modes are considered in the reliability predictions.

The results predicted by using an existing approximate method are compared with those obtained by using a Markov technique for a two state fluctuating failure environment covering normal and adverse weather periods. Simple equations are developed to represent the various load point failure modes and the results are comparable with those predicted by the Markov approach. A three state weather model is proposed which includes the effect of disaster adverse weather periods on system reliability. A technique which exists in the literature for forming an equivalent component is investigated and it is shown that such an approach is not valid when dependent effects exist in the system. A failure modes and effect analysis approach is illustrated by application to the reliability analysis of a practical configuration. This technique provides an insight into the inherent system failure processes and gives an indication of the action most likely to achieve the required improvement at minimum cost.
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1. INTRODUCTION

An adequate supply of power is an essential ingredient in any scheme to improve the standard of living in a developing country. The annual consumption of energy per head of population is usually an accurate measure of the degree of advancement or of prosperity. The economic implications associated with power development are relatively straightforward. The determination of what is an "adequate" supply, however, is not an easy problem. The increasing criticality of industrial processes as well as load concentration and customer reliance on electric energy calls for a virtually continuous uninterrupted supply. It is not possible to absolutely guarantee this requirement and any attempt to do so is impractical and uneconomical. It becomes essential to evaluate where the limited available funds should be utilised to produce the maximum returns. The incremental benefits associated with equipment and configuration changes cannot be evaluated by qualitative reliability criteria. Such measures are based on simple rules of thumb and do not adequately reflect the effect of equipment performance characteristics, network configuration, system operating conditions and in fact those elements that do influence the reliability of the system. These factors can only be incorporated in the analysis through quantitative reliability techniques which utilise probabilistic models of the system components. One objection often raised to the utilisation of probability techniques is the absence of accurate component data. It should be appreciated, however, that the results obtained are simply estimates based upon the available information. As such, they can be extremely valuable in consistently comparing alternate configurations and the relative benefits of configuration changes.
The maximum contribution to the total number of customer supply interruptions arises from the overhead distribution system. This is primarily due to the environment in which these systems operate. In adverse weather conditions, the physical stresses placed upon the system components can be very much higher than those encountered under normal weather conditions. Distribution facilities are normally concentrated over a relatively small area and therefore are more liable to be totally affected by adverse weather conditions.

The application of probability techniques in the quantitative evaluation of transmission and distribution schemes received its present impetus with the publication of two papers\(^{(1,2)}\) in 1964. A considerable amount of work has been done in the establishment of consistent techniques since that time. The approximate equations described in reference 2 included the weather conditions by assuming a two state model in which weather was classified into normal and adverse periods. The results predicted by these equations were compared with those obtained by a Markov approach in a 1967 publication\(^{(4)}\) and it was illustrated that the results obtained by these two techniques were not consistent.

This thesis extends the concepts illustrated in the development of the earlier equations\(^{(2)}\). The equations presented in this thesis give results which compare very closely with those obtained by a Markov approach and are flexible enough to consider the occurrence or nonoccurrence of repair of components during adverse weather conditions.

A technique for developing equivalent components in a two state weather environment has been suggested in the literature\(^{(10)}\). This approach was investigated. It was found to be inaccurate and did not give results which are comparable to those obtained by an exact Markov approach.
A further extension of the two state weather model has been suggested in this thesis to recognize the effect of severe hurricanes and tornadoes. The three state weather model proposed incorporates these extreme weather conditions and the results predicted compare closely with those obtained by a Markov approach.

This thesis also presents simple techniques for the quantitative evaluation of temporary load point interruptions. This aspect has not been developed in any of the previous publications. Temporary outages of components can cause many load point outages and can be a major cause of customer dissatisfaction. This aspect is extended to include the two state weather environment.

Maintenance of components is another event which can result in load point interruptions. The equations suggested in the literature(5) are not valid when the maintenance time of a component is overlapped by more than one component forced outage. This thesis presents some simple equations to represent the occurrence of this event.

A useful approach in the study of systems involving a large number of components is that of failure mode and effect analysis. This technique has been used in this thesis and requires a thorough knowledge of the operation of the system and the effect of component failures on load points. The recognition of failure modes and their cause effect relationships is a valuable tool in improving customer service continuity. If the reliability predictions do not meet a predetermined goal, a logical improvement procedure is to look for ways to mitigate the causes underlying the events that contributed most to failure.
In those cases in which the parallel facilities are not redundant, the overload outages of components can result in high unreliability values at the system load points. Overload of the components, however, does not necessarily imply an outage. The reliability indices obtained due to component overload outages will depend upon the assumptions made for those events which occur given the component overload. The reliability indices obtained due to component overload should be considered separately from those which actually do result from the complete interruption of customer supply. In some cases it may be considered desirable to accept the possible equipment damage rather than interrupt the customer.

The results obtained by using the equations developed in this thesis have been continually compared with those obtained by a more sophisticated Markov approach. Within the bounds of distributional assumptions, all the applications considered can be analysed by the Markov technique. A principal concern in this thesis has been the development of relatively simple equations and techniques which can be used in practical system studies. The techniques presented in this thesis should provide considerable assistance in developing quantitative incremental reliability costs for any point in a system. In addition, these techniques should provide efficient and economic methods to determine the most effective means by which any selected level of security of supply can be realised in a practical system.
2. ASPECTS, MEASURES AND DATA REQUIREMENTS OF THE DISTRIBUTION SYSTEM RELIABILITY PROBLEM

2.1 Introduction

An electric distribution system is that portion of the electric power system between the bulk power source or sources and the consumer's facilities. It is not possible to select a single voltage level below which all utilities consider the facilities to be in the distribution component class. In general, however, the large utilities designate higher voltage levels to distribution facilities than do the smaller utilities. Distribution systems are normally divided into six parts, namely subtransmission circuits, distribution substations, distribution or primary feeders, distribution transformers, secondary circuits and consumer service connections. Within the past few years, considerable attention has been devoted to the reliability evaluation of these portions of the system. As previously noted, the primary impetus for this work came from two 1964 IEEE papers\(^{(1,2)}\). The reliability indices obtained from the presently available techniques are very dependent on the method of defining system success and failure. This requires the identification of the different component failure modes and the effect of these on load point and system success. This task involves a thorough engineering appraisal and appreciation of the operation of the system.

An essential part of any quantitative reliability study is the definition of suitable criteria against which the performance of the system can be compared. In the distribution area the indices which seem to be
the most appropriate are frequency, average duration of customer outage and the total average annual customer outage time. The calculation of these indices requires the collection of relevant component and system data. The accuracy of the predicted results is very much dependent on the accuracy of the data used in the reliability calculations. The nature of the data required depends upon the reliability indices selected for the description of the behaviour of the system. With the general acceptance of frequency and duration indices, considerable attention is now being devoted to data collection procedures. A very limited amount of data is available for distribution system components. The first comprehensive IEEE publication\(^{15}\) on data collection for distribution schemes appeared in 1968. Other material has been published since that time. A recent publication\(^{20}\) illustrates with actual data the advances made in this area in the CEBG system in the United Kingdom. It should be recognized, however, that a great deal of work still remains to be done in this field.

### 2.2 Failure Modes Of A Component

The failure modes of a component as considered in this thesis are shown in Figure 2.1. The definitions of the various failure modes are given in Appendix 1. The permanent outage of a component requires it to be taken out of service for a period of time during which it is repaired. The actual outage time may be the replacement time for a spare component. Many of the outages on distribution schemes are of a temporary nature. If a component fault is cleared by a reclosing operation of a circuit breaker or by an automatic switching operation, a temporary outage is said to
Figure 2.1  Distinct Failure Modes of a Component
have occurred. The duration associated with such component outages is generally of the order of a few minutes. Components are also taken out for maintenance and inspection. This action is designated as preventive maintenance rather than the corrective maintenance which follows a permanent outage. From the viewpoint of reliability, this is another mode of component failure as the component is not available for performing its intended function. The final mode of component failure as illustrated in Figure 2.1 is the overload outage of a component. Under certain outage and system conditions, components may be called upon to carry loads which exceed their capability. This can result in overload outage of the components. In an actual system, depending upon the amount of overload and the system philosophy, the components may be called upon to carry the overload or be removed from service to prevent loss of life or permanent damage. In some cases, the system load may be reduced to relieve the overload.

It should be appreciated that an overload outage of a component is different from the other failure modes described above. The overload of a component may or may not result in an actual outage. The other three modes of failure illustrated in Figure 2.1 do represent actual failure of the component.

2.3 Failure Modes Of A Load Point

Component outages may or may not lead to load point interruption depending upon the configuration of the system. The various basic events
which lead to load point failure and are evaluated in this thesis are listed below using the system of Figure 2.2.

Figure 2.2 System For Illustration of Load Point Failure Modes

(i) The permanent outage of component 1 overlapping the permanent outage of component 2 or vice versa causing interruption of supply at load points A and B.

(ii) The permanent or temporary outage of component 3 causes an interruption of supply to the load point B.

(iii) The maintenance outage period of component 1 is overlapped by a permanent or a temporary outage of component 2 or a maintenance outage period of component 2 is overlapped by a permanent or a temporary outage of component 1 resulting in the interruption of supply at the load points A and B.

(iv) The permanent outage of component 1 is overlapped by a temporary outage of component 2 or the permanent outage of component 2 is overlapped by a temporary outage of component 1 causing interruption of supply at load points A and B.
Component 1 is on permanent outage and during its repair period, the load at load points A and B rises to a value which results in the overload of component 2. Under these circumstances, component 2 may be taken out of service or the load at any or both of the load points reduced to relieve the overload. In the former case both load points will be interrupted, while in the latter case a fraction of the total system load will be interrupted. A similar event can occur if components 1 and 2 are interchanged in the above description.

The probability of an event in which component temporary outages overlap is very small and, therefore, such an event is not included. The probability of component overload occurring during small durations associated with temporary outages can also be neglected. One basic assumption made in this approach is that the system is capable of satisfying its function with all components in service.

In addition to the load point failure modes described above, there are many other modes of system operation which can be regarded as failures to meet acceptable standards. These include the voltage levels at the load points outside the desirable limits, violation of transient stability limits, violation of frequency limits etc. Although the criterion of voltage levels in distribution schemes is quite important, it has not been considered in this thesis. The techniques described in the following chapters are, however, capable of considering this mode of load point failure. The transient stability, frequency and other criteria are normally considered to be exterior to the distribution system area.
2.4 The Effect Of Weather On Distribution System Performance

A particular distribution system can be composed of either overhead or underground facilities or both. Overhead schemes have to operate in a changing environment. Under severe weather conditions, the components fail quite frequently while under normal weather conditions the failures occur less frequently. It was reported in reference 20 that on the British Electricity Board, in 1968, virtually all the failures occurred in adverse weather conditions. If the components making up the system are simultaneously affected by the severe weather, several component failures can occur during the short adverse weather period. This phenomenon of component failures during this period is called "a bunching effect of adverse weather associated failures." Distribution schemes, being concentrated in small areas, are greatly affected by adverse weather conditions. The reliability predictions without weather considerations can be quite optimistic and the results can be considerably in error. This has been illustrated in Chapter 3.

2.5 Existing Reliability Indices

No single measure of distribution system reliability completely describes the ability of the system to provide satisfactory service. A number of indices have been suggested in the existing literature\(^1,2,9,14\) for the evaluation of system adequacy. These indices are as follows:

(i) Average number of service interruptions per customer served per year.
(ii) Average restoration time per interruption.
(iii) Average total interruption time per customer served per year.

(iv) Maximum expected number of interruptions experienced by any one customer.

(v) Maximum expected restoration time experienced by any one customer.

(vi) Probability that any one customer will be out of service at any one time longer than a specified period.

The first three measures give average service reliability in terms of customer satisfaction. The next two indices give the poorest service reliability provided to any customer on the system. The last measure is of value to a power company if the design criterion is set up in terms of a probability value so that no customer on the system is out of service longer than a certain specified time.

In addition to these general reliability indices, a few more existing measures which are pertinent to a particular method of calculation are:

(i) Index of Reliability... It is defined as the ratio of the total customer hours that service is available to the total customer hours served.

(ii) Average Annual Customer Interruption Rate (A.A.C.I.R)\(^{(1)}\)

It is defined as the expected number of days in a year that a specified outage condition for the load bus will occur.

(iii) Interruption Load Magnitude Index (I.L.M.I)\(^{(14)}\)... It is defined as the average magnitude of load interrupted per outage during a specified period of time. It is estimated from the operating history by dividing the total amount of load interrupted during a specified period by the number of outages resulting in customer interruptions
during that period.

(iv) The upper and lower limits of customer minutes of outage per year. These indices indicate the upper and lower limits of time the repair crews have to remain in the field for repairs.

2.6 Selection Of Reliability Indices

Satisfactory service from the viewpoint of the customers is influenced largely by the average frequency and duration of outages. It is also possible to establish acceptable standards of frequency and duration from the customer complaint records. This approach has been illustrated in Appendix 2. The indices developed in this thesis are at a particular load point. They can be easily converted to customer indices given the number of customers associated with the load point.

The reliability measures evaluated in this thesis are as follows:

(1) Average number of service interruptions per load point per year

(ii) Average restoration time at each load point

(iii) Average total interruption time per load point per year

The customer oriented indices can be obtained from the above load point measures as follows:

\[
\text{Average number of service interruptions per customer per year} = \frac{\text{Number of customer interruptions per year}}{\text{Number of customers served by the load points}}
\]
Average restoration time per interruption in hours

\[ \text{Average restoration time per interruption in hours} = \frac{\text{Number of customer interruption hours during the year}}{\text{Number of customer interruptions during the year}} \]

Average total interruption time per customer served per year

\[ \text{Average total interruption time per customer served per year} = \frac{\text{Number of customer interruption hours per year}}{\text{Number of customers served by the load points}} \]

The number of service interruptions is directly related to the restoration time. A reduction in the number of overlapping outages will facilitate more rapid coverage of all outages resulting in a reduced outage time of the components.

2.7 Component Parameters Required For Distribution System Reliability Studies

The component parameters required in the evaluation of frequency and duration indices of load point failures are listed below.

Component failure and repair rates

The component failure rates required are:

(a) Normal weather permanent outage rate. This failure rate is given by

\[ \lambda = \frac{C}{Y} \]

where \( C \) is the number of normal weather component failures during the observation period and \( Y \) is the summation of the normal weather exposure times for the component during the observation period.

(b) Adverse weather permanent outage rate. This value is given by:

\[ \lambda' = \frac{C'}{Y'} \]

where \( C' \) is the number of adverse weather component failures during the
observation period and $Y'$ is the summation of the adverse weather exposure times.

If the adverse weather is divided into different classes then the failure rate for each class is required.

(c) Temporary outage rate. If no distinction is made between the normal and adverse weather temporary outages, this value is estimated by dividing the number of component temporary outages during the observation period by the number of unit calendar years of component exposure during the observation period. If temporary outages are resolved into normal and adverse weather failures, the failure rates under different weather conditions are determined in the same manner as that described for permanent outages.

(d) Maintenance outage rate. This value is given by:

$$\lambda'' = \frac{C''}{Y''}$$

where $C''$ is the number of component maintenance outages during the observation period and $Y''$ is the summation of observation periods for the component.

Component repair times under different outage conditions are estimated as follows:

$$r = \frac{\text{Total outage time of the component for a given type of failure during the observation period}}{\text{Total number of outages of the given type during the observation period}}$$

In addition to the data described above, all the component and load data necessary for load flow studies on a system are required to evaluate overload outages of the system components. This includes component
and circuit ratings, impedances, loads at different system buses etc.

2.8 Weather Statistics Required

As noted in section 2.4, weather considerations are very important in the reliability evaluation of overhead distribution schemes. (The weather statistics required for the mathematical models studied in this thesis are:

(i) Average duration of a normal weather period.

(ii) Average duration of an adverse weather period.

The probability distributions of the weather durations are assumed to be exponential. The long term reliability predictions are, however, not dependent upon this assumption. If the adverse weather is classified into different categories, the average duration of each category is required.

The problem of data collection can be eased to a considerable extent by pooling data from different sources. The pooling of data, however, requires the following two conditions:

(i) There is no good engineering reason for the data groups to be appreciably different.

(ii) A statistical test must show that the means and variances of the underlying populations from which groups of data have been drawn are not significantly different.

In the following chapters, techniques have been developed to take into consideration all the modes of load point failure presented in this Chapter. Temporary and permanent component outages have been recognized as distinct failure modes. Temporary outages can cause considerable irritation to the customers. In some industrial processes where continuity of supply is essential, a considerable financial loss can occur in the form of spoiled
material, lost productions etc. due to temporary supply interruptions. The separation of various modes of component failure can pinpoint the components and their parameters which make a significant contribution to system failure. The ability to determine reliability indices in terms of frequency and duration of interruptions is very important. A single reliability index such as customer outage hours does not describe the true performance. A customer having five outages of one hour average duration has the same customer hours outage index (5 hours) as a customer having one outage of five hour average duration. These two widely different situations cannot be appreciated by a single reliability index. Some customers on the system may be able to stand longer but infrequent outages while the reverse may be true for other ones.
3. EVALUATION OF THE PERMANENT OUTAGE CONDITION

3.1 Introduction

The primary emphasis in the initial publications\((1,2,19)\) in the quantitative reliability evaluation of transmission and distribution schemes was on the forced outage of system components arising from permanent outages. A basic assumption in all the applications is that component forced outages occur randomly and independently. Todd's method\((1)\) which evaluates reliability in terms of an Average Annual Customer Interruption Rate involves the application of simple probability techniques. This method essentially investigates the simultaneous conditions that must prevail for power to flow in series and parallel combinations of system components. The forced outage rate in this method is defined as the ratio of the sum of the days on which outages of a minimum specified duration occur to the sum of the unit days. It further assumes that all outages occurring during one day are simultaneous and therefore gives pessimistic results. In reference 3, it was established that the assumption of complete statistical independence of forced outages was generally unrealistic and led to unreasonably low estimates of outage rates at different load points. Additional analytical work was done\((2,4,5,7)\) to extend the models to take into consideration the dependent failures occurring during adverse weather periods. As will be seen in the following chapters, component forced outages are an integral part of many other modes of load point failures and therefore, the establishment of accurate models for the evaluation of this aspect of the reliability problem is very important.
3.2 Techniques For Evaluation Of Frequency And Duration Indices

The two basic techniques which exist in the present literature for reliability evaluation of distribution schemes in terms of frequency and duration indices are:

(i) The approximate technique (2)

(ii) The Markov technique (4)

If the necessary distributional assumptions are valid, power system modelling in the form of Markov processes is the most accurate representation for reliability evaluation of these systems. The benefits of the approximate technique arise from limitations associated with the computer requirements of large scale Markov modelling. The approximate technique consists of a group of simple equations which provide numerical estimates of the rates and durations of system load point failures. This method, essentially, employs simple rules of conditional probability.

3.2.1 The approximate technique

The basic assumptions made initially in this technique are:

(1) Component up times are exponentially distributed
(2) Component down times are exponentially distributed.

It was proved later by the use of renewal theory (11) that these assumptions are not a real constraint in the calculation of frequency and duration measures.

A system of two components in series with outage rates $\lambda_1$ and $\lambda_2$ and repair time $r_1$ and $r_2$ respectively has the following reliability indices
System outage rate \( \lambda_s = \lambda_1 + \lambda_2 \) (3.1)
System average outage duration \( r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2}{\lambda_s} \) (3.2)
System total average outage time \( \frac{\lambda_s r_s}{1 + \lambda_s r_s} \) (3.3)

If the above components are connected to form a two component parallel system, the corresponding reliability indices are:

System outage rate \( \lambda_p = \lambda_1 \lambda_2 (r_1 + r_2) \) (3.4)
System average outage duration \( r_p = \frac{r_1 r_2}{r_1 + r_2} \) (3.5)
System total average outage time \( \frac{\lambda_p r_p}{1 + \lambda_p r_p} \)

These formulae can be extended for the consideration of a larger number of series or parallel components.

To illustrate the application of these formulae, consider a hypothetical system shown in Figure 3.1. The component failure and repair parameters

![Figure 3.1 A Typical System](image-url)
are given in Table 3.1

<table>
<thead>
<tr>
<th>No.</th>
<th>COMPONENT</th>
<th>Failure Rate failures/year</th>
<th>Expected Repair Time Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Line # 1</td>
<td>0.5</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>Line # 2</td>
<td>0.5</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>Line # 3</td>
<td>0.1</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>Transformer</td>
<td>0.01</td>
<td>20.0</td>
</tr>
<tr>
<td>5</td>
<td>Circuit Breaker</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

The reliability indices obtained from these parameters are given in Table 3.2

<table>
<thead>
<tr>
<th>Load Point</th>
<th>Failures/year</th>
<th>Average outage duration Hours</th>
<th>Average total outage time Hours/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.4281x10^-3</td>
<td>3.75</td>
<td>0.1605x10^-2</td>
</tr>
<tr>
<td>C</td>
<td>0.1243</td>
<td>4.10</td>
<td>0.468</td>
</tr>
</tbody>
</table>

It can be seen from this Table that the outage rate of load point C is governed by the series combination of line section 3 and the transformer.

In many distribution schemes, the outage time is not the actual repair time but the time required to switch out the failed component and to close a normally open switch to provide an alternate path. This aspect is illustrated in the simple examples(24) shown in Figure 3.2.
Figure 3.2  Sectionalized Circuits

(a) Manually Sectionalized Primary Main
(b) Loopied Primary With Manual Sectionalizing
Figure 3.2a represents a radial manually sectionalized primary main arrangement. The fault characteristics for this arrangement are as follows:

Primary Main
- 0.10 faults/circuit mile-year
- 3.0 hours, average repair time

Primary Lateral
- 0.25 faults/circuit-mile-year
- 1.0 hours, average repair time

Manual Sectionalizing time = 0.5 hours.

The interruption analysis for this system is given in Table 3.3.

**TABLE 3.3**

**INTERUPTION ANALYSIS OF SYSTEM OF FIG. 3.2a**

<table>
<thead>
<tr>
<th>Component</th>
<th>Load Point A</th>
<th></th>
<th>Load Point B</th>
<th></th>
<th>Load Point C</th>
<th></th>
<th>Load Point D</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda )</td>
<td>( r )</td>
<td>( \lambda r )</td>
<td>( r )</td>
<td>( \lambda r )</td>
<td>( r )</td>
<td>( \lambda r )</td>
<td>( r )</td>
</tr>
<tr>
<td>Primary Main</td>
<td>1/year</td>
<td>Hrs.</td>
<td>Hrs./yr.</td>
<td>Hrs.</td>
<td>Hrs./yr.</td>
<td>Hrs.</td>
<td>Hrs./yr.</td>
<td>Hrs.</td>
</tr>
<tr>
<td>Section 1</td>
<td>0.20</td>
<td>3.0</td>
<td>0.60</td>
<td>3</td>
<td>0.60</td>
<td>3</td>
<td>0.60</td>
<td>3</td>
</tr>
<tr>
<td>2 mi x 0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 2</td>
<td>0.20</td>
<td>0.5</td>
<td>0.10</td>
<td>3</td>
<td>0.60</td>
<td>3</td>
<td>0.60</td>
<td>3</td>
</tr>
<tr>
<td>2 mi x 0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 3</td>
<td>0.10</td>
<td>0.5</td>
<td>0.05</td>
<td>0.5</td>
<td>0.05</td>
<td>3</td>
<td>0.30</td>
<td>3</td>
</tr>
<tr>
<td>1 mi x 0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 4</td>
<td>0.10</td>
<td>0.5</td>
<td>0.05</td>
<td>0.5</td>
<td>0.05</td>
<td>0.5</td>
<td>0.05</td>
<td>3</td>
</tr>
<tr>
<td>1 mi x 0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary Lateral</td>
<td>0.25</td>
<td>1.0</td>
<td>0.25</td>
<td>1.0</td>
<td>0.25</td>
<td>1.0</td>
<td>0.25</td>
<td>1.0</td>
</tr>
<tr>
<td>1 mi x 0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permanent Interruption</td>
<td>0.85/yr</td>
<td>1.05 hrs./yr</td>
<td>1.55 hrs./yr</td>
<td>1.80 hrs./yr</td>
<td>2.05 h/yr</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate due to primary mains and lateral only</td>
<td>( r_A = 1.24 ) hrs.</td>
<td>( r_B = 1.82 ) hrs.</td>
<td>( r_G = 2.12 ) hrs.</td>
<td>( r_D = 2.42 ) hrs.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
where $r_j$ represents the average outage time at load point $j$.

If an alternate feeder is available, as shown in Figure 3.2b, to form a looped primary system with manual sectionalizing, the interruption analysis of Table 3.3 is modified to the following form.

<table>
<thead>
<tr>
<th>Component</th>
<th>Load Point A</th>
<th>Load Point B</th>
<th>Load Point C</th>
<th>Load Point D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$</td>
<td>$r$</td>
<td>$\lambda r$</td>
<td>$r$</td>
</tr>
<tr>
<td>Primary Main</td>
<td>1/year</td>
<td>Hrs./yr.</td>
<td>Hrs./yr.</td>
<td>Hrs./yr.</td>
</tr>
<tr>
<td>Section 1</td>
<td>0.20</td>
<td>3</td>
<td>0.60</td>
<td>0.5</td>
</tr>
<tr>
<td>Section 2</td>
<td>0.20</td>
<td>0.5</td>
<td>0.10</td>
<td>3.0</td>
</tr>
<tr>
<td>Section 3</td>
<td>0.10</td>
<td>0.5</td>
<td>0.05</td>
<td>0.5</td>
</tr>
<tr>
<td>Section 4</td>
<td>0.10</td>
<td>0.5</td>
<td>0.05</td>
<td>0.5</td>
</tr>
<tr>
<td>Primary Lateral</td>
<td>0.25</td>
<td>1.0</td>
<td>0.25</td>
<td>1.0</td>
</tr>
<tr>
<td>Permanent Interruption Rate due to primary mains and lateral only</td>
<td>$r_A = 1.24$ Hrs.</td>
<td>$r_B = 1.24$ Hrs.</td>
<td>$r_C = 0.94$ Hrs.</td>
<td>$r_D = 0.94$ Hrs.</td>
</tr>
</tbody>
</table>

In the above analysis only single contingency events have been considered and higher order contingency events, being less probable, have been neglected.
Figure 3.3 Effect of Varying Manual Sectionalizing Time on System Reliability Indices
The effect of varying the manual sectionalizing time on the outage rate and expected outage duration of the various load points for the systems of Figure 3.2 is shown in Figure 3.3. It can be seen that the variation in manual sectionalizing time does not have any effect on the load point outage rates for both the configurations in Figure 3.2. Any increase in manual sectionalizing time does, however, result in an increase in expected outage time at the various load points. The provision of an alternate path from the source, does not have any effect on the load point outage rates but reduces the expected outage time. These general conclusions are, of course, intuitively obvious but the technique permits quantitative evaluation of the load point indices and the benefits associated with additional facilities and reduced switching times.

3.2.2 The Markov technique

A sequence of events where the outcome depends on an element of chance is called a stochastic process. The Markovian process is a special class of stochastic process in which future states of the process are dependent only upon the immediate past. The power system reliability problem can be represented by Markov processes that are discrete in space and continuous in time. The use of Markovian models in power system reliability evaluation was initially proposed in a 1964 I.E.E.E publication\(^{(19)}\). The application of Markov processes to the transmission system reliability problem was first illustrated in a 1967 I.E.E.E publication\(^{(4)}\). Since then a great deal of work has been done on the application of Markov processes in this area.
Consider a simple system component for which the failure and repair rates are characterized by exponential distributions.

Let \[ \lambda = \text{The failure rate of the component} \]
\[ \mu = \text{The repair rate of the component} \]

The state space model describing this simple case is shown in Figure 3.4.

Define

\[ P_0(t) = \text{Probability of the component being UP at time t.} \]
\[ P_1(t) = \text{Probability of the component being DOWN at time t.} \]

Consider an incremental time interval \( dt \) and assume that the probability of occurrence of two events in this small interval of time is negligible. The probability that the component is in state 0 at time \( t + dt \) is derived from the probability that it was in state 0 at time \( t \) and did not transit
to state 1 in time $dt$ or that it was in state 1 at time $t$ and transferred
to state 0 in time $dt$. Then

$$P_0(t+dt) = P_0(t)(1-\lambda dt) + P_1(t)\mu dt$$

(3.6)

Similarly

$$P_1(t+dt) = P_0(t)\lambda dt + P_1(t)(1-\mu dt)$$

(3.7)

These expressions can be placed into the following differential equation
form.

$$P_0(t) = -\lambda P_0(t) + \mu P_1(t)$$

$$P_1(t) = \lambda P_0(t) - \mu P_1(t)$$

In the matrix form,

$$\begin{bmatrix} P_0(t) \\ P_1(t) \end{bmatrix}' = \begin{bmatrix} -\lambda & \mu \\ \lambda & -\mu \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \end{bmatrix}$$

Or

$$P'(t) = AP$$

where $A$ is called the transitional rate matrix.

The limiting state probabilities or the so called steady state
probabilities can be obtained by setting the differential matrix equal
to zero. The system equations then become:

$$-\lambda P_0 + \mu P_1 = 0$$

$$\lambda P_0 - \mu P_1 = 0$$

$$P_0 + P_1 = 1.0$$

Solving these equations,

$$P_0 = \frac{\mu}{\lambda + \mu} \quad \text{and} \quad P_1 = \frac{\lambda}{\lambda + \mu}$$

(3.8)

$P_0$ represents the availability of the component and

$P_1$ represents the unavailability of the component.

If each component $i$ is represented by a failure rate $\lambda_i$ and
a repair rate $\mu_i$, the availabilities and unavailabilities respectively
are: \( \frac{\mu_1}{\lambda_1 + \mu_1} \) and \( \frac{\lambda_1}{\lambda_1 + \mu_1} \). Consider a total number of \( n \) components and a particular outage state \( B_j \) in which \( m \) units are on outage and the remaining \((n-m)\) components are in operation. Assuming independent behaviour, the steady state probability of existence of this state is given by:

\[
P(B_j) = \prod_{i=1}^{m} \left\{ \frac{\lambda_1}{\lambda_1 + \mu_1} \right\} \times \prod_{k=1}^{n-m} \left\{ \frac{\mu_k}{\lambda_k + \mu_k} \right\}
\]

(3.9)

The rate of departure from this state is given by

\[
D(B_j) = \sum_{i=1}^{m} \mu_1 + \sum_{k=1}^{n-m} \lambda_k
\]

Transfer from \( B_j \) will occur if another failure or repair occurs. The duration in this state is given by the reciprocal of the rate of departure from this state\(^{16}\). The duration \( R \) of the state \( B_j \) is, therefore, given by

\[
R = \frac{1}{D(B_j)}
\]

(3.10)

The frequency of occurrence of a state is equal to the product of the steady state probability of existence of the state and the rate of departure from that state. Using this principle, the probabilities, frequencies and durations for a two component system with the state space diagram shown in Figure 3.5 are given in Table 3.4.

![Two Component State Space Diagram](image-url)
TABLE 3.4

FREQUENCY AND DURATION OF THE STATES FOR A TWO COMPONENT SYSTEM

<table>
<thead>
<tr>
<th>STATE</th>
<th>COMPONENT #1</th>
<th>COMPONENT #2</th>
<th>Duration</th>
<th>Probability</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>UP</td>
<td>UP</td>
<td>$\frac{1}{(\lambda_1+\lambda_2)}$</td>
<td>$\mu_1 \mu_2 /D$</td>
<td>$\mu_1 \mu_2 (\lambda_1+\lambda_2) /D$</td>
</tr>
<tr>
<td>2</td>
<td>DOWN</td>
<td>UP</td>
<td>$\frac{1}{(\mu_1+\lambda_2)}$</td>
<td>$\lambda_1 \mu_2 /D$</td>
<td>$\lambda_1 \mu_2 (\mu_1+\lambda_2) /D$</td>
</tr>
<tr>
<td>3</td>
<td>UP</td>
<td>DOWN</td>
<td>$\frac{1}{(\lambda_1+\mu_2)}$</td>
<td>$\mu_1 \lambda_2 /D$</td>
<td>$\mu_1 \lambda_2 (\lambda_1+\mu_2) /D$</td>
</tr>
<tr>
<td>4</td>
<td>DOWN</td>
<td>DOWN</td>
<td>$\frac{1}{(\mu_1+\mu_2)}$</td>
<td>$\lambda_1 \lambda_2 /D$</td>
<td>$\lambda_1 \lambda_2 (\mu_1+\mu_2) /D$</td>
</tr>
</tbody>
</table>

where $D = (\lambda_1+\mu_1) (\lambda_2+\mu_2)$

For a system of two components in parallel with full redundancy, state #4 is the only failed state and the frequency of occurrence of this state is given by:

$$f_4 = \frac{\lambda_1 \lambda_2 (\mu_1+\mu_2)}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}$$

Considering

$\mu$'s $\gg \lambda$'s and $\mu = \frac{1}{x}$

$$f_4 \propto \lambda_1 \lambda_2 (\frac{1}{x_1} + \frac{1}{x_2}) x_1 x_2$$

$$\propto \lambda_1 \lambda_2 (x_1+x_2)$$
This result is the same as equation 3.4.

The average duration of this state is given by:

\[ R = \frac{1}{\mu_1 + \mu_2} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{r_1 r_2}{r_1 + r_2} \]

This result is the same as equation 3.5.

The mean up time of the system can also be obtained by using discrete Markov chain concepts (22). The stochastic transitional probability matrix for a two component system is given by:

\[
P = \begin{bmatrix}
1 & 2 & 3 & 4 \\
1 - (\lambda_1 + \lambda_2) & \lambda_1 & \lambda_2 & 0 \\
\mu_1 & 1 - (\lambda_2 + \mu_1) & 0 & \lambda_2 \\
\mu_2 & 0 & 1 - (\lambda_1 + \mu_2) & \lambda_1 \\
0 & \mu_2 & \mu_1 & 1 - (\mu_1 + \mu_2)
\end{bmatrix}
\]

State 4 is designated as an absorbing state and a new truncated matrix \( Q \) is obtained by eliminating the absorbing state.

\[
Q = \begin{bmatrix}
1 & 2 & 3 \\
1 - (\lambda_1 + \lambda_2) & \lambda_1 & \lambda_2 \\
\mu_1 & 1 - (\lambda_2 + \mu_1) & 0 \\
\mu_2 & 0 & 1 - (\lambda_1 + \mu_2)
\end{bmatrix}
\]

Let \( I = \text{Identity matrix} \)

\[
[I - Q] = \begin{bmatrix}
\lambda_1 + \lambda_2 & -\lambda_1 & -\lambda_2 \\
-\mu_1 & \lambda_2 + \mu_1 & 0 \\
-\mu_2 & 0 & \lambda_1 + \mu_2
\end{bmatrix}
\]
Let

\[ N = \text{Fundamental matrix, in which } n_j \text{ is the time spent by the process} \]

in state \( S_j \) before being absorbed.

\[
N = [I - Q]^{-1} = \frac{1}{D} \begin{bmatrix}
(\lambda_2 + \mu_1) (\lambda_1 + \mu_2) & \lambda_1 (\lambda_1 + \mu_2) & \lambda_2 (\lambda_2 + \mu_1) \\
\mu_1 (\lambda_1 + \mu_2) & (\lambda_1 + \lambda_2) (\lambda_1 + \mu_2) - \lambda_2 \mu_2 & \lambda_2 \mu_1 \\
\mu_2 (\lambda_2 + \mu_1) & \lambda_1 \mu_2 & (\lambda_1 + \lambda_2) (\lambda_2 + \mu_1) - \lambda_1 \mu_1 \\
\end{bmatrix}
\]

\[ D = \lambda_1 \lambda_2 (\lambda_1 + \lambda_2 + \mu_1 + \mu_2) \]

Starting in state 1 the time before entering the failed state (the absorbing state) of the process

\[ T_s = \frac{(\lambda_2 + \mu_1) (\lambda_1 + \mu_2) + \lambda_1 (\lambda_1 + \mu_2) + \lambda_2 (\lambda_2 + \mu_1)}{\lambda_1 \lambda_2 (\lambda_1 + \lambda_2 + \mu_1 + \mu_2)} \]

For \( \mu^* > \lambda^* \)

\[ T_s = \frac{\mu_1 \mu_2 + \lambda_1 \mu_2 + \lambda_2 \mu_1}{\lambda_1 \lambda_2 (\mu_1 + \mu_2)} \]

\[ = \frac{1 + \lambda_1 r_1 + \lambda_2 r_2}{\lambda_1 \lambda_2 (r_1 + r_2)} \]

Or \( \lambda_p = \lambda_1 \lambda_2 (r_1 + r_2) \)

This result is the same as equation 3.4.

The size of the stochastic transitional probability matrix increases rapidly for a large number of components. Under such circumstances, this matrix can be partitioned into lower order matrices which are easy to invert.

For the partitioned matrices, the corresponding inverses are as follows:
\[
\begin{bmatrix}
A \\
B \\
C \\
D
\end{bmatrix}^{-1} = \begin{bmatrix}
-(C-DB^{-1}A)DB^{-1} & (C-DB^{-1}A)^{-1} \\
B^{-1}+B^{-1}A(C-DB^{-1}A)DB^{-1} & -B^{-1}A(C-DB^{-1}A)^{-1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
A \\
B \\
C \\
D
\end{bmatrix}^{-1} P = \begin{bmatrix}
P \\
-PQS^{-1} \\
P(QS^{-1}FI^{-1} - CI^{-1})
\end{bmatrix}
\]

\[
\begin{bmatrix}
A \\
B \\
C \\
D
\end{bmatrix}^{-1} F = \begin{bmatrix}
-FP \\
S^{-1}(1+RPQS^{-1}) \\
-S^{-1}(FI^{-1}+RP(QS^{-1}FI^{-1} - CI^{-1}))
\end{bmatrix}
\]

\[
\begin{bmatrix}
A \\
B \\
C \\
D
\end{bmatrix}^{-1} I = \begin{bmatrix}
1 \\
(I^{-1}GP+I^{-1}HS^{-1}RP) \\
U \\
V
\end{bmatrix}
\]

Where

\[
W = (A - CI^{-1}G) \quad Q = (B - CI^{-1}H) \quad R = (D - FI^{-1}G)
\]

\[
S = (E - FI^{-1}H) \quad P = (W-QS^{-1}R)^{-1}
\]

\[
U = I^{-1}GPQS^{-1} - I^{-1}HS^{-1}(1+RPQS^{-1})
\]

\[
V = I^{-1} - I^{-1}GP(QS^{-1}FI^{-1} - CI^{-1}) + I^{-1}H(S^{-1}FI^{-1}+S^{-1}RP(QS^{-1}FI^{-1} - CI^{-1}))
\]

The results obtained for the failure rate of systems of two and three components in parallel by the two Markov approaches described above are given in Table 3.5. This table also includes the effects of normal and adverse weather conditions which are discussed in the following section. It can be seen from this table that the two Markov techniques give results which are close to each other. The matrix inversion approach requires a knowledge of the state in which the process starts. In many cases it is not possible to logically decide the state in which the process commences.
This approach is not carried any further in this thesis due to this limitation.

3.3 Weather Modelling

As pointed out in section 2.4, system reliability can be greatly affected by environmental stresses. The environment for a generating unit or any component contained indoors is generally constant. For such components, the failure rate can be considered independent of changes in the weather. Overhead distribution schemes, however, are directly exposed to a changing environment. During adverse weather conditions, the failure rate of lines and other exposed equipment increases rapidly and then reverts to its former value after the adverse weather period is over.

Since the severity of the adverse weather varies, the failure rate of the components under various adverse weather conditions will also vary. It is difficult to incorporate all types of adverse weather conditions in reliability calculations. A simple two state fluctuating environment covering normal and adverse weather has been proposed in which weather duration distributions are assumed to be exponential(2). This model is shown in Figure 3.6a. The random occurrence pattern of adverse weather periods and their associated failure rates is averaged and represented by a regular pattern as shown in Figure 3.6b. The definition of an adverse weather condition must be carefully related to the conditions within the system which actually do result in an increased rate of component failure. If these definitions are too stringent, very few adverse weather periods will be recognized. If, however, these definitions include too many mild,
Figure 3.6 Weather Modelling

(a) Random Pattern of occurrence of Adverse Weather
(b) Two State Weather Model
relatively undestructive adverse weather conditions, the failure bunching effect of more severe periods will be diluted. It should be noted that disaster adverse weather conditions such as major hurricanes and tornadoes cannot be lumped with other less violent periods. To recognize the effect of such periods on system reliability, a three state weather model is presented later in this chapter.

The effect of varying the percentage of component failures during adverse weather periods is shown in Table 3.5. The table also compares the results obtained in each case by the frequency and duration approach and the matrix inversion technique. The effects of adverse weather periods are discussed in more detail later in this chapter.

**TABLE 3.5**

**COMPARISON OF THE MARKOV TECHNIQUES**

\[ \lambda v_1 = \lambda v_2 = \lambda v_3 = 0.5 \text{ failures/yr.} \quad r_1 = r_2 = r_3 = 7.5 \text{ hours} \]

Average duration of an adverse weather period = 1.5 hours
Average duration of a normal weather period = 200.0 hours

<table>
<thead>
<tr>
<th>Percentage of component failures during adverse weather</th>
<th>Two components in parallel Failures/year</th>
<th>Three Components in parallel Failures/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq. and Duration</td>
<td>Matrix Inversion</td>
<td>Freq. and Duration</td>
</tr>
<tr>
<td>---------------------</td>
<td>-----------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>0</td>
<td>0.00043</td>
<td>0.00043</td>
</tr>
<tr>
<td>10</td>
<td>0.00047</td>
<td>0.00045</td>
</tr>
<tr>
<td>20</td>
<td>0.00072</td>
<td>0.00072</td>
</tr>
<tr>
<td>30</td>
<td>0.00120</td>
<td>0.00120</td>
</tr>
<tr>
<td>40</td>
<td>0.00188</td>
<td>0.00188</td>
</tr>
<tr>
<td>50</td>
<td>0.00267</td>
<td>0.00263</td>
</tr>
<tr>
<td>60</td>
<td>0.00371</td>
<td>0.00370</td>
</tr>
<tr>
<td>70</td>
<td>0.00493</td>
<td>0.00492</td>
</tr>
<tr>
<td>80</td>
<td>0.00627</td>
<td>0.00624</td>
</tr>
<tr>
<td>90</td>
<td>0.00782</td>
<td>0.00780</td>
</tr>
<tr>
<td>100</td>
<td>0.00958</td>
<td>0.00956</td>
</tr>
</tbody>
</table>
The amount of error introduced by neglecting the weather aspect can be seen by considering the typical distribution scheme shown in Figure 3.7. The component outage and repair statistics are given in Table 3.6.

![Diagram of a typical distribution scheme]

**Figure 3.7** A Typical Distribution Scheme

**TABLE 3.6**

<table>
<thead>
<tr>
<th>Line Sections</th>
<th>Failure rate failures/yr</th>
<th>Expected repair time hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Average duration of a normal weather period = 200.0 hours
Average duration of an adverse weather period = 1.5 hours
As the adverse weather associated component failures increase, the difference between the bus failure rate values calculated using equation 3.4 and those calculated considering adverse weather periods increase quite rapidly. The amount of error introduced is clearly depicted in Figure 3.8 in which the load point failure rate is shown in per unit of the value calculated using equation 3.4. It can further be noted that the amount of error is also governed by the duration of normal and adverse weather periods.

The inclusion of weather conditions in reliability predictions can be carried out by two techniques.

(i) The approximate technique

(ii) The Markov technique

3.3.1 The approximate technique

This technique was first described in reference 2. In this section of the thesis, mathematical expressions for calculating the various indices of reliability in simple series and parallel systems are presented. The assumptions involved which are pertinent to the mathematical modelling of the permanent outage condition are listed below:

(i) Times to failure and repair times are exponentially distributed during both normal and adverse weather periods.

(ii) The durations of normal and adverse weather periods are exponentially distributed.
Figure 3.8 Error in the System Outage Rate Predictions Due to Exclusion of Weather Conditions
(iii) The adverse weather periods are of very short duration in comparison with the component times to failure. These periods are also short compared to the mean component repair times.

(iv) Component repair rates are very large compared to their failure rates.

The following parameters are required for a reliability study:

\[ \lambda_1, \lambda_2, \lambda_3 \]

\[ \lambda_n = \text{Normal weather component failure rate; failures/year of normal weather} \]

\[ \lambda'_1, \lambda'_2, \lambda'_3 \]

\[ \lambda'_n = \text{Adverse weather component failure rate; failures/year of adverse weather} \]

\[ r_1, r_2, r_3 \]

\[ r_n = \text{Expected repair time for all permanent outages; years} \]

\[ N = \text{Expected duration of a normal weather period; years} \]

\[ S = \text{Expected duration of an adverse weather period; years} \]

The approximate overall failure rate \( \lambda_{av,i} \) for the ith component is given by:

\[ \lambda_{av,i} = \frac{N}{N+S} \lambda_1 + \frac{S}{N+S} \lambda'_1 \]

(3.11)

The overall outage rate of a series system is given by

\[ \lambda_s = \sum_{i=1}^{n} \lambda_{av,i} \text{ outages/calendar year} \]

The error incurred due to the assumption of independence in series systems is negligible because in these systems, single component failures greatly outnumber the overlapping failures. This is why, no significant error is introduced in Figure 3.8 in the outage rate of load point 2 with lines 1, 2 and 3 in service.
If the series system is in parallel with other components, it is necessary to calculate the normal and adverse weather failure rates for the equivalent series system. For the equivalent component

\[ \lambda_e = \sum_{i=1}^{n} \lambda_i \]  
failures/year of normal weather

and

\[ \lambda'_e = \sum_{i=1}^{n} \lambda'_i \]  
failures/year of adverse weather

Expected value of system down time = \[ \sum_{i=1}^{n} \frac{\lambda_{sv,i} r_i}{\lambda_e} \]  
Years

If the total system consists of a series connection between the source and the load, the load point reliability measures are as follows:

Annual outage rate = \( \lambda_e \)

Expected outage duration \( r_s = \sum_{i=1}^{n} \frac{\lambda_{sv,i} r_i}{\lambda_e} \)  
(3.11 a)

Average total outage time = \[ \frac{r_s}{r_s + \frac{1}{\lambda_e}} = \frac{r_s \lambda_e}{1 + r_s \lambda_e} \]

\[ \approx \lambda_e r_s \]  
Years/year

In a parallel system, the system components are considered in pairs and are reduced to an equivalent component for further combination. The development of average outage rate formulae for two components in parallel involves four different phases.

(1) The initial failure occurs during normal weather and the second failure also occurs during normal weather. The contribution to the system failure rate due to this mode of failure is given by:

\[ (\text{Failure Rate})_1 = \frac{N}{N+5} \left[ \lambda_1 (1 - \frac{r_1}{N}) (\lambda_2 r_1) + \lambda_2 (1 - \frac{r_2}{N}) (\lambda_1 r_2) \right] \]
If the normal and adverse weather concept is not applied, then
\[ \frac{N}{N+8} = 1 \] and
\[ \lambda_p = \lambda_1 \lambda_2 (r_1 + r_2), \text{as shown in equation 3.2.} \]
(ii) The initial failure occurs during normal weather and the second failure occurs during adverse weather. The contribution to the system failure rate due to this mode of failure is given by:

\[
(Failure \ rate)_2 = \frac{N}{N+S} \left[ \lambda_1 \frac{\tau_1}{N} + \lambda_2 \frac{\tau_2}{N} \right]
\]

Again

\[
\frac{N}{N+S} = \text{The long term fraction of time that the weather is normal.}
\]

\[
\frac{\tau_1}{N} = 1 - e^{-\lambda_1/N} = \text{The probability that repair on component 1 is not completed within the normal weather i.e. the probability that weather becomes adverse while component 1 is being repaired.}
\]

\[
\lambda_2^S \approx 1 - e^{-\lambda_2^S} = \text{The probability that component 2 fails during the adverse weather. The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.}
\]

\[
(Failure \ rate)_2 = \frac{N}{N+S} \cdot \frac{S}{N} \left[ \lambda_1 \lambda_2^S + \lambda_2 \lambda_1^S \right]
\]

(iii) The initial failure occurs during adverse weather and the second failure also occurs during adverse weather. The contribution to the system failure rate due to this mode of failure is given by:

\[
(Failure \ rate)_3 = \frac{S}{N+S} \left[ \lambda_1^S \cdot (\lambda_2^S) + \lambda_2^S(\lambda_1^S) \right]
\]
Here $\frac{S}{N+S}$ = The long term fraction of time that the weather is adverse.

$S\lambda_2' \sim 1 - e^{-S\lambda_2'}$ = The probability that component 2 fails during an adverse weather period. The remaining terms in the above expression follow the same argument but with components 1 and 2 interchanged. It has been further assumed that no repair is carried out during an adverse weather period.

$$(\text{Failure rate})_3 = \frac{N}{N+S} \left[ \frac{2S^2}{N} \lambda_1' \lambda_2' \right]$$

(iv) The initial failure occurs during adverse weather and second failure occurs in normal weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(\text{Failure rate})_4 = \frac{S}{N+S} \left[ \lambda_1'(1-S\lambda_2') \left( \lambda_2' \lambda_1' \right) + \left( \lambda_2' \right) \left( 1-S\lambda_1' \right) \lambda_1' \lambda_2' \right]$$

Here $\frac{S}{N+S}$ = The long term fraction of time that the weather is adverse.

$1 - S\lambda_2' \sim e^{-S\lambda_2'}$ = The probability that component 2 does not fail during the adverse weather.

$\lambda_2' \lambda_1' \sim 1 - e^{-\lambda_2' \lambda_1'}$ = The probability that component 2 fails during the repair of component 1. The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

Thus the overall outage rate for a system of two components in parallel
is given by
\[ \lambda_{SL} = \frac{N}{N+5} \left[ \lambda_1 \lambda_2 (r_1 + r_2) + \frac{2}{N} (\lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 r_2) + \frac{8}{N} (\lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 r_2) + \frac{28}{N} \lambda_1 \lambda_2 \right] \] (3.12)
failures/calendar year

If the parallel system operates in parallel with other components, the normal weather and adverse weather rates are required for the equivalent component representing the parallel system in further combinations. These rates are as follows.
\[ \lambda_1' = \lambda_1 \lambda_2 (r_1 + r_2) + \frac{2}{N} (\lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 r_2) \] (3.12 a)
\[ \lambda_2' = \lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 r_2 + 28 \lambda_1 \lambda_2 \] (3.12 b)
The type of weather existing when the second component fails determines whether a term contributes to the equivalent component normal or adverse weather outage rate.

Expected down time as a result of two overlapping outages is given by:
\[ r_{SL} = \frac{r_1 r_2}{r_1 + r_2} \]
The desired reliability measures for a system of two components in parallel are as follows:
The average number of interruptions at the load point = \( \lambda_{SL} \)
The average duration of interruption = \( r_{SL} \) years
The average total outage time = \( \lambda_{SL} \cdot r_{SL} \) years/year

Using this technique, the predicted failure rate and the average total outage duration for systems of two, three and four components in
Figure 3.9  Effect of Bunching of Adverse Weather Associated Component Failures on System Outage Rate
Figure 3.10  Effect of Bunching of Adverse Weather Associated Component Failures on System Total Outage Time
parallel are shown as a function of the percentage of component failures occurring during adverse weather periods in Figures 3.9 and 3.10. It can clearly be seen from these results that as the adverse weather associated component failures increase, the system outage rate and the average total outage time also increase. This increase in load point indices can be quite large.

3.3.2 The Markov technique.

The Markov technique described in section 3.2.3 can be extended to include the weather conditions. The mathematics of simple probability combinations as described in that section cannot be applied when the weather conditions are involved.

Using the same assumptions as described in section 3.3.1, the Markov approach can be applied to a single component operating in a two state fluctuating environment as follows:

Let \( \lambda, \mu = \) Normal weather failure and repair rates
\( \lambda', \mu' = \) Adverse weather failure and repair rates
\[ n = \frac{1}{N} \] where \( N \) is average duration of a normal weather
\[ m = \frac{1}{S} \] where \( S \) is average duration of an adverse weather

The state space diagram for this system is shown in Figure 3.11
The system differential equations from the state space diagram are as follows:

\[
\begin{bmatrix}
\dot{P}_0(t) \\
\dot{P}_1(t) \\
\dot{P}_2(t) \\
\dot{P}_3(t)
\end{bmatrix} =
\begin{bmatrix}
-(\lambda+n) & m & \mu & 0 \\
\ n & -(\mu+\lambda) & 0 & \mu' \\
\lambda & 0 & -(\mu+n) & m \\
0 & \lambda' & n & -(\mu'+m)
\end{bmatrix}
\begin{bmatrix}
P_0(t) \\
P_1(t) \\
P_2(t) \\
P_3(t)
\end{bmatrix}
\]

The steady state probabilities can be found by setting the differential matrix equal to zero. Assuming \( \mu = \mu' \).
\[
\begin{align*}
\frac{P_0 - \mu}{(n+m)} & = \frac{m n + m^2 + \lambda m + \mu m}{(\lambda + \mu)(\lambda^2 + \mu) + m(\mu + \lambda) + n(\lambda + \mu)} \\
\frac{P_1 - \mu}{(n+m)} & = \frac{m n + m^2 + \lambda n + \mu n}{(\lambda + \mu)(\lambda + \mu) + m(\mu + \lambda) + n(\lambda + \mu)} \\
\frac{P_2 - 1}{(n+m)} & = \frac{\lambda m^2 + \lambda \lambda m + \lambda \mu + \lambda \mu m + \lambda \mu m}{(\lambda + \mu)(\lambda + \mu) + m(\mu + \lambda) + n(\lambda + \mu)} \\
\frac{P_3 - 1}{(n+m)} & = \frac{\lambda n^2 + \lambda \mu n + \lambda m \mu + \lambda \mu m}{(\lambda + \mu)(\lambda + \mu) + m(\mu + \lambda) + n(\lambda + \mu)}
\end{align*}
\]

If no repair is carried out during the adverse weather period then \( \mu' = 0 \) and the steady state probabilities are modified as follows:

\[
\begin{align*}
\frac{P_0 - \mu}{(n+m)} & = \frac{m n + m^2 + \lambda m + \mu m}{\mu(\lambda + \lambda) + \lambda m + \lambda \lambda} \\
\frac{P_1 - \mu}{(n+m)} & = \frac{m n + m^2 + \lambda n + \mu n}{\mu(\lambda + \lambda) + \lambda m + \lambda \lambda} \\
\frac{P_2 - 1}{(n+m)} & = \frac{m(\lambda m + \lambda n) + \lambda \lambda n}{\mu(\lambda + \lambda) + \lambda m + \lambda \lambda} \\
\frac{P_3 - 1}{(n+m)} & = \frac{\lambda n^2 + \lambda \mu n + \lambda m \mu + \lambda \mu m}{\mu(\lambda + \lambda) + \lambda m + \lambda \lambda}
\end{align*}
\]

The mean up time of the component by a matrix inversion method is given by:

\[
T_s = \frac{m + \lambda^* n}{\lambda \lambda^* + \lambda m + \lambda \lambda^*}
\]
The average failure rate is given by

\[ \lambda_{av} = \frac{1}{T_s} = \frac{\lambda^1 + \lambda m + \lambda' n}{m + \lambda' + n} \]

As \( \lambda^1 < \lambda m + \lambda' n \)

and \( \lambda < m + n \)

\[ \lambda_{av} \approx \frac{\lambda m}{m+n} + \frac{\lambda' n}{m+n} \]

\[ \approx \frac{N}{N+S} \lambda + \frac{S}{N+S} \lambda' \]

This result is identical to equation 3.11

The system state space diagram becomes quite important as the number of components in the system increases. The state space diagrams for systems of two and three components are shown in Figures 3.12 and 3.13. The diagrams for four and five components are included in Appendix 4. The steady state probabilities associated with each of the system states can be found by solving a set of simultaneous linear equations of the following matrix form

\[ [0] = [DM \quad m \ I \ ] \ [P] \]

(3.21)

The matrices DM and DS correspond to transitions out of normal and adverse weather states respectively

\[ [0] \text{ is a null matrix} \]

\[ [I] \text{ is an identity matrix} \]
Figure 3.12 State Space Diagram For a Two Component System
Figure 3.13  State Space Diagram For a Three Component System
$[p]$ is a column vector of steady state probabilities.

To determine the steady state probabilities, the solution of the set of equations represented by 3.21 has to satisfy the following condition

$$\sum_{i} p_i = 1.0$$ i.e. the availabilities of the states must sum to one.

The matrices $DN$ and $DS$ for a two component system are shown in Table 3.7

**Table 3.7**

PARTITIONED MATRICES FOR A TWO COMPONENT SYSTEM

$$[DN] = \begin{bmatrix}
-\lambda_1-\lambda_2-n & \mu_2 & \mu_1 & 0 \\
\lambda_2 & -\lambda_1-\mu_2-n & 0 & \mu_1 \\
\lambda_1 & 0 & -\mu_1-\lambda_2-n & \mu_2 \\
0 & \lambda_1 & \lambda_2 & -\mu_1-\mu_2-n
\end{bmatrix}$$

$$[DS] = \begin{bmatrix}
-\lambda_1-\lambda_2-m & \mu'_2 & \mu'_1 & 0 \\
\lambda_2 & -\lambda_1-\mu'_2-m & 0 & \mu'_1 \\
\lambda'_1 & 0 & -\mu'_1-\lambda_2-m & \mu'_2 \\
0 & \lambda'_1 & \lambda'_2 & -\mu'_1-\mu'_2-m
\end{bmatrix}$$

The matrices $DN$ and $DS$ can be written in a similar manner for larger numbers of components. The state transition diagrams provide the most direct means for easy and accurate accounting of the permissible transfers between states in complicated systems.
The complete Markov approach can be easily applied to relatively small systems. The number of simultaneous linear equations to be solved in order to determine the steady state probabilities is $2 \times 2^n$ for an $n$ component system operating in a two state fluctuating environment. This number increases very rapidly as $n$ increases. In a five component system 64 simultaneous linear equations are necessary to determine the reliability indices. The number of components that can be dealt with by a Markov approach is, therefore, limited by the storage requirements and the rounding errors incurred in the computer solution.

It was stated in reference 2 that the approximate technique of section 3.3.1 would give results which are within a few percent of those obtained by the Markov technique. The results obtained by the above two techniques were compared in reference 4 and it was concluded that the approximate method responds quite differently to changes in system parameters. The magnitudes of the errors involved in the prediction of system failure rate for two, three, four and five component parallel systems are shown in Figure 3.14 for different component repair durations. The average outage durations obtained for two, three, four and five component parallel systems by the Markov approach are quite close to those obtained by the approximate technique.

The predicted outage rates for a two component parallel system are quite close to those obtained by the approximate technique if no repairs are performed during adverse weather periods. This is shown in
Figure 3.14 Comparison of System Outage Rates Using the Approximate and Markov Techniques, Variable Repair Durations.
Table 3.8 for different percentages of component failure during adverse weather periods.

The average outage duration predicted by the approximate technique can be quite different from that obtained by a Markov approach when no repair is performed during the adverse weather period. The magnitude of possible difference is shown in Figure 3.15 in which the load bus average outage duration as obtained by the approximate technique is shown in per unit of the value obtained by the Markov approach. The average outage duration as obtained from the approximate method does not vary with the percentage of adverse weather associated component failures. In contrast, when calculated by the Markov technique, the average outage duration increases with an increasing percentage of adverse weather component failures. This is, perhaps, intuitively obvious, because as the percentage of component failures during adverse weather increases and no repair is carried out in this period, more and more component failures are waiting repairs resulting in an increased average outage duration of the system. The approximate equation 3.12 for the failure rate assumes no repair during adverse weather whereas equation 3.5 for the average outage duration assumes that the repair process is started as soon as a failure occurs. These two equations are therefore valid under two different repair conditions. Equation 3.12 is applicable when no repair occurs during adverse weather and equation 3.5 is applicable when repair does occur.
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<th>Duration of repair, $x$ Hours</th>
<th>Failure rate/year Markov</th>
<th>Approximate</th>
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Figure 3.15 Comparison of System Average Outage Durations Using the Approximate and Markov Techniques. No Repair During Adverse Weather
3.4 Equivalent Component Formation

The equations for two components in parallel are presented in section 3.3.1. As noted previously, for three or more components in parallel, the basic approach is to consider them two at a time until the system is subsequently reduced to a single equivalent component. This simplification results in an increased error as the number of parallel components increase as shown in Figure 3.14. Unlike the two component parallel system, the results for systems of more than two parallel components differ from those obtained by the Markov approach for both repair conditions.

In reference 10, a Markov process concept is used to derive expressions for an equivalent component representation of two components in series and parallel considering two state fluctuating normal and adverse weather periods. It is also stated that this method of equivalent component development takes into consideration the dependency of transitional probabilities and the results are reasonably comparable to those obtained by a complete Markov technique. The derivation of the equivalent component formation was done using a matrix inversion technique.

The following section presents a method of forming an equivalent component using a frequency and duration technique.

In order to form an equivalent component for a two component system, the reduced system must be equivalent to the original in certain respects.
The probability of all components being down in normal weather \( \equiv \) The probability of the equivalent component being down in normal weather \[ \text{(3.22)} \]

If repair occurs in adverse weather, then the probability of the equivalent component being down in normal weather is given by equation 3.15.

In general, \( \mu_{eq} \gg \lambda_{eq} \) so that equation 3.15 can be reduced to the following form.

\[ f_{NW} = \frac{1}{(n+m)} \left[ \frac{\lambda_{eq} n^2 + \lambda_{eq} m n + \lambda_{eq} \mu_{eq} n}{\mu_{eq}^2 + m \mu_{eq} + n \mu_{eq}} \right] \quad \text{(3.23)} \]

Similarly,

The probability of all components being down in adverse weather \( \equiv \) The probability of the equivalent component being down in adverse weather \[ \text{(3.24)} \]

From equation 3.16 and using the above approximation

\[ f_{ES} = \frac{1}{(n+m)} \left[ \frac{\lambda_{eq} n^2 + \lambda_{eq} m n + \lambda_{eq} \mu_{eq} n}{\mu_{eq}^2 + m \mu_{eq} + n \mu_{eq}} \right] \quad \text{(3.25)} \]

Let

The frequency of all components being down in normal weather = \( FN \)
The probability of all components being down in normal weather = \( PN \)
The frequency of all components being down in adverse weather = \( FS \)
The probability of all components being down in adverse weather = \( PS \)
Then \( \mu_{eq} = \frac{FN + FS}{PN + FS} - \text{Frequency of transitions amongst these states} \quad \text{(3.26)} \)
From equations 3.22, 3.23, 3.24 and 3.25
\[
\frac{(n^2 + \mu_{eq}n)}{A} \lambda_{eq} + \frac{mn}{A} \lambda_{eq}' = FN \tag{3.27}
\]
\[
\frac{mn}{A} \lambda_{eq} + \frac{n^2 + \mu_{eq}n}{A} \lambda_{eq}' = PS \tag{3.28}
\]
where 
\[A = (n+m) \left( \mu_{eq}^2 + m \mu_{eq} + n \mu_{eq} \right)\]

Equations 3.27 and 3.28 can be solved for \(\lambda_{eq}\), the normal weather failure rate and \(\lambda_{eq}'\), the adverse weather failure rate of the equivalent component. The repair rate \(\mu_{eq}\), of the equivalent component is known from equation 3.26.

If no repair is performed during the adverse weather period, the following two equations can be written on the same basis as equations 3.27 and 3.28 from equations 3.19 and 3.20.
\[
(n^2 - n^2FN - mn FN) \lambda_{eq} + (mn - n(n+m)FN - (n+m)\mu_{eq}FN) \lambda_{eq}' = (n+m) \mu_{eq}n FN \tag{3.29}
\]
\[
(mn - m(n+m)PS) \lambda_{eq} + (n^2 - n(n+m)PS - (n+m)\mu_{eq}PS + n \mu_{eq}) \lambda_{eq}' = (n+m) \mu_{eq}n PS \tag{3.30}
\]

Equations 3.26, 3.29 and 3.30 give the reliability parameters of the equivalent component. This approach has been called the Block Reduction Method. The results obtained for the outage rate of a system with three components in parallel for various percentages of component failures during adverse weather periods are shown in Tables 3.9 and 3.10 and have been compared with those obtained by the approximate technique and the
exact Markov technique. Table 3.9 gives the results when repair occurs during adverse weather and Table 3.10 gives the results when no repair occurs during adverse weather periods.

It can be seen from Tables 3.9 and 3.10 that the results of the Block reduction method are quite different from the results obtained by the exact Markov approach. The results of the Block Reduction technique are instead quite close to those obtained by the approximate technique. This is, perhaps, to be expected. As noted in section 3.3.2, the approximate equation 3.12 gives results for two overlapping outages which are quite close to those obtained by the Markov approach when no repair occurs during adverse weather. The results predicted for the normal and adverse weather outage rates by the approximate equations 3.12 a and 3.12 b are close to those predicted by equation 3.29 and 3.30. If these values are again used in equation 3.12 and in the Block Reduction method, the resultant outage rate predictions should again be quite close to each other.

In reference 10, the outage rate predictions for a system of three components in parallel have been presented and are reproduced in Table 3.11 for the purpose of comparison.

A comparison of Tables 3.10 and 3.11 clearly shows that the results of the Markov technique and the approximate technique have been interchanged in Table 3.11. The results predicted by the Block Reduction technique (10) are no better than those obtained by the approximate
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### TABLE 3.10

**OUTAGE RATE FOR A THREE COMPONENT PARALLEL SYSTEM**

*NO REPAIR DURING ADVERSE WEATHER*

\[ \lambda_{av1} = \lambda_{av2} = \lambda_{av3} = 0.5 \quad S = 1.5 \text{ Hours} \quad N = 200.0 \text{ Hours} \]

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<tr>
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<td>12.5</td>
<td>1952.80</td>
<td>2094.75</td>
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### Table 3.11
OUTAGE RATE FOR A THREE COMPONENT PARALLEL SYSTEM (10)
NO REPAIR DURING ADVERSE WEATHER

\[ \lambda_{av1} = \lambda_{av2} = \lambda_{av3} = 0.5 \text{ f/year} \quad S = 1.5 \text{ Hours} \quad N = 200.0 \text{ Hours} \]

<table>
<thead>
<tr>
<th>Percentage of component failures during adverse weather</th>
<th>Repair Time Hours</th>
<th>Approx. method failures/10^7 years</th>
<th>Block reduction failures/10^7 years</th>
<th>Exact Markov</th>
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<td>2675.10</td>
<td>2645.60</td>
</tr>
</tbody>
</table>

The authors of reference 10 seem to have misinterpreted their results and as such the conclusions made in this reference are not valid.

#### 3.5 The Modified Approximate Technique

As has been noted in the earlier sections, the reliability indices obtained from the approximate technique can be quite different from those obtained by a Markov approach. Equation 3.12 gives results for two overlapping outages which are close to those obtained by a Markov technique when no repairs are performed during adverse weather periods. The results predicted by the associated equation 3.5, for the average outage duration...
do not agree with those obtained by the Markov approach. The technique

described in section 3.4 for the formation of an equivalent component
does not give results which are better than those obtained by the app-
proximate technique. The following presents a modified version of the
approximate technique for the two different situations of repair i.e

(i) When repair occurs during adverse weather periods

(ii) When repair does not occur during adverse weather periods

3.5.1 Repair during adverse weather

For a series system of two components,

System failure rate \( \lambda_s = \lambda_{av1} + \lambda_{av2} \)

System average outage duration, \( x_s = \frac{N(\lambda_1+\lambda_2 + \lambda_{av1}^2) + S(\lambda_1+\lambda_{av1}^2 \lambda_{av2})}{N(\lambda_1+\lambda_2) + S(\lambda_1+\lambda_{av2})} \)

\[ = \frac{x_1(N(\lambda_1+\lambda_{av1}^2) + x_2(N(\lambda_2+\lambda_{av2}^2))}{(N(\lambda_1+\lambda_{av1}^2) + (N(\lambda_2+\lambda_{av2}^2))} \]

\[ = \frac{x_1 \lambda_{av1}^2 + x_2 \lambda_{av2}}{\lambda_{av1} + \lambda_{av2}} \]

This result is the same as equation 3.11 a

The formation of equations for two overlapping outages in a parallel system

is basically the same as that presented in section 3.3.1.

The contribution to the system failure rate due to the first mode of

system failure is given by:

\[ \text{(Failure rate)}_1 = \frac{N}{N+S} \left( \lambda_1 \lambda_2 (x_1 + x_2) \right) \]

This is the same expression as in section 3.3.1
The contribution to the system failure rate due to the second mode of system failure is given by:

\[
(Failure \ rate)_2 = \frac{N}{N+S} \left[ \lambda_1 \left( \frac{r_1}{N} \right) \lambda_2 \left( \frac{r_2}{x_1} \right) + \lambda_2 \left( \frac{r_2}{x_2} \right) \lambda_1 \left( \frac{r_2}{x_2} \right) \right]
\]

where

\[
\frac{r_1}{N} = \text{The long term fraction of time that the weather is normal}
\]

\[
\frac{r_1}{N} \approx 1 - e^{-r_1/N} = \text{The probability that the weather becomes adverse during the repair of component 1}
\]

and

\[
\lambda_2 \left( \frac{r_2}{x_1} \right) \approx \text{The probability that component 2 fails during the overlapping period of adverse weather and the repair of component 1.}
\]

In the approximate technique, this term was written as \( \lambda_2 S \).

A similar approach is used to obtain the second term in the above expression.

The contribution to the system failure rate due to the third mode of system failure is given by:

\[
(Failure \ rate)_3 = \frac{S}{N+S} \left[ \lambda_1 \left( \lambda_2 \left( \frac{r_1}{x_1} \right) \right) + \lambda_2 \left( \lambda_1 \left( \frac{r_2}{x_2} \right) \right) \right]
\]

where

\[
\frac{S}{N+S} = \text{The long term fraction of time that the weather is adverse.}
\]

\[
\lambda_2 \left( \frac{r_2}{x_1} \right) \approx \text{The probability that component 2 fails}
\]
during the overlapping period of adverse weather and the repair of component 1. In the approximate technique, this term was written as $\lambda_2^2 S$.

Similar reasoning is applicable to the second term in the above expression.

The contribution to the system failure rate due to the fourth mode of system failure is given by the same expression as given in section 3.3.1 i.e.,

$$(\text{Failure rate})_4 = \frac{S}{N+S} \left[ \lambda_1^2 \lambda_2 r_1 + \lambda_2^2 \lambda_1 r_2 \right]$$

Thus the overall failure rate for two overlapping outages is given by:

$$\lambda_{SL} = \frac{N}{N+S} \left[ \lambda_1 \lambda_2 (r_1 + r_2) + \frac{S}{N} (\lambda_1 \lambda_2^2 \frac{r_1^2}{S+r_1} + \lambda_2 \lambda_1^2 \frac{r_2^2}{S+r_2}) \right]$$

$$+ \frac{S}{N+S} \left[ \lambda_1^2 \lambda_2 r_1 + \lambda_2^2 \lambda_1 r_2 + \lambda_1 \lambda_2 S \left( \frac{r_1}{S+r_1} + \frac{r_2}{S+r_2} \right) \right] \quad (3.31)$$

The average outage duration under this condition of repair is given by equation 3.5.

3.5.2 No repair during adverse weather

For a series system of two components,

$$\text{System failure rate, } \lambda_S = \lambda_{av1} + \lambda_{av2}$$

$$\text{System average outage duration, } r_S = \frac{N(\lambda_1 r_1 + \lambda_2 r_2) + S(\lambda_1^2 (S+r_1) + \lambda_2^2 (S+r_2))}{N(\lambda_1 + \lambda_2) + S(\lambda_1^2 + \lambda_2^2)}$$

Equation 3.12 gives the failure rate value due to two overlapping
outages when no repair occurs during adverse weather periods. Under this condition of repair every failure which occurs in adverse weather has to wait for repairs for a period equal to the average duration of the adverse weather period. (refer to Figure 3.12 and equation 3.10). The average outage duration associated with failures which occur during the adverse weather periods is given by:

$$\left(\frac{r_1r_2}{r_1+r_2}\right) + \frac{r_1}{r_1+r_2}$$

If the contributions to the system failure rate due to normal weather failures (modes (i) and (iv)) and adverse weather failures (modes (ii) and (iii)) are represented by $\lambda_{en}$ and $\lambda_{es}$ respectively, then the average outage duration is equal to the sum of the weighted durations associated with each of these outage rate values i.e.

$$r_{SL} = \frac{\lambda_{en}}{\lambda_{en}+\lambda_{es}} \left[ \frac{r_1r_2}{r_1+r_2} \right] + \frac{\lambda_{es}}{\lambda_{en}+\lambda_{es}} \left[ \frac{r_1r_2}{r_1+r_2} + s \right] \quad (3.32)$$

The results obtained by equations 3.31 and 3.32 are compared with those obtained by a Markov approach for two overlapping outages in Tables 3.12 and 3.13. It can be seen that the results obtained by these equations are quite close to those predicted by the Markov technique.
<table>
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<tr>
<th>Percentage of component failures during adverse weather</th>
<th>Repair duration Hours</th>
<th>Approx. failure rate/year</th>
<th>Markov technique failure rate/year</th>
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<td>0.000143</td>
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<td>Average Outage duration, Years</td>
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\( \lambda_{av1} = \lambda_{av2} = 0.5 \text{ f/year} \quad N = 200.0 \text{ Hours} \quad S = 1.5 \text{ Hours} \)
Two sets of equations have, therefore, been obtained. Equations 3.12 and 3.32 represent the average outage rate and the average outage duration respectively when no repair is performed during adverse weather periods. Equations 3.31 and 3.5 represent the same measures when repair occurs during the adverse weather period.

It is interesting to note that equations 3.12 and 3.31 can be derived easily by the application of renewal process theory (11). A renewal process is a process which repeats itself after every run-fail-repair cycle. The application of renewal processes illustrates that the probability distributions of times to failure and repair can be quite general. All that is required is that the distributions remain the same for each successive run-fail-repair cycle. This is shown below under the approximation that the component failure rates are much smaller than the repair rates.

Let

\[ n = \text{Number of weather change cycles in a year} \]

\[ \lambda_{av1}, \lambda_{av2} = \text{The average annual failure rates of components 1 and 2.} \]

\[ W = \text{The fraction of component failures in the adverse weather periods.} \]
(1) No repair during adverse weather

Using the same terminology as in section 3.3.1,
the number of failures of component 1 during adverse weather periods =
\[ \lambda_{av1} \bar{W} \times n (N+S) \]

The total down time of component 1 in adverse weather periods =
\[ \lambda_{av1} \bar{W} \times n (N+S) S \]

The cycle time of component 2 in adverse weather = \[ \frac{1}{\lambda_2} \]

The fraction of outages of component 2 in adverse weather which overlap
the outages of component 1 in adverse weather = \[ \frac{\lambda_{av1} \bar{W} \times n (N+S) S}{\frac{1}{\lambda_2}} \]

The number of outages of component 1 in normal weather = \[ \lambda_{av1} (N+S) n (1-W) \]

The probability of an outage of component 1 which occurs in normal
weather, extending into adverse weather = \[ \frac{\Sigma_1}{N} \]

The expected number of outages of component 1 which occur in normal
weather and extend into an adverse weather period = \[ \lambda_{av1} (N+S) n (1-W) \left( \frac{\Sigma_1}{N} \right) \]

The total outage duration associated with such outages =
\[ \lambda_{av1} (N+S) n (1-W) \left( \frac{\Sigma_1}{N} \right) S \]

The fraction of outages of component 2 in adverse weather which overlap
the outages of component 1 in normal weather and extended into adverse
weather
\[ = \frac{\lambda_{av1} (N+S) n (1-W) \left( \frac{\Sigma_1}{N} \right) S}{\frac{1}{\lambda_2}} \]
The total number of outages in adverse weather in which the outages of component 2 overlap the outages of component 1

\[
\frac{1}{\lambda_2} \left( \lambda_{av1} \frac{W n (N+S)_1}{S} + \lambda_{av1} (N+S) \frac{r_1}{N} \right) \left( \frac{1}{\lambda_2} \right) S
\]

Similarly, the total number of outages in adverse weather in which the outages of component 1 overlap the outages of component 2

\[
\frac{1}{\lambda_1} \left( \lambda_{av2} \frac{W n (N+S)_2}{S} + \lambda_{av2} (N+S) \frac{r_2}{N} \right) \left( \frac{1}{\lambda_1} \right) S
\]

The total number of overlapping outages of components 1 and 2 in adverse weather periods

\[
\lambda = \lambda_{av1} \frac{W n (N+S)_1}{S} + \lambda_{av1} \frac{r_1}{N} \frac{S}{S} + \lambda_{av2} \frac{W n (N+S)_2}{S} + \lambda_{av2} \frac{r_2}{N} \frac{S}{S}
\]

But \( \lambda_{av1} \frac{W (N+S)}{S} = \lambda_1' \) and \( \lambda_{av2} \frac{W (N+S)}{S} = \lambda_2' \)

\[
\lambda = \lambda_1' S \lambda_2' S n + \lambda_1' \frac{r_1}{S} + \lambda_2' S \lambda_1' S n + \lambda_2' \frac{r_2}{S} + \lambda_1' \frac{r_1}{S} + \lambda_2' \frac{r_2}{S}
\]

The adverse weather outage rate is therefore

\[
\lambda'_o = \frac{\lambda}{S_n} = \lambda_1' \frac{r_1}{S} + \lambda_2' \frac{r_2}{S} + \lambda_1 \frac{r_1}{S} + \lambda_2 \frac{r_2}{S}
\]

This equation is same as equation 3.12 b
The number of outages of component 1 in normal weather =
\( \lambda_{av1} (1-W) (N+S) n \)
The duration associated with these outages = \( \lambda_{av1} (1-W) (N+S) n \cdot r_1 \)
The cycle time of component 2 in normal weather = \( \frac{1}{\lambda_2} \)
The fraction of outages of component 2 in normal weather which overlap the outages of component 1 in normal weather =
\( \frac{\lambda_{av1} (1-W) (N+S) n \cdot r_1}{\frac{1}{\lambda_2}} \)
All the outages which occur in adverse weather extend into normal weather.
The number of outages of component 1 in adverse weather = \( \lambda_{av1} W n (N+S) \)
The duration associated with these outages in normal weather =
\( \lambda_{av1} W n (N+S) r_1 \)
The fraction of outages of component 2 in normal weather which overlap
the outages of component 1 which occurred in adverse weather and
extended into normal weather =
\( \frac{\lambda_{av1} W n (N+S) r_1}{\frac{1}{\lambda_2}} \)
The total number of outages in normal weather in which the outages of
compartment 2 overlap the outages of component 1.
\( \frac{\lambda_{av1} (1-W) (N+S) n \cdot r_1}{\frac{1}{\lambda_2}} + \frac{\lambda_{av1} W n (N+S) r_1}{\frac{1}{\lambda_2}} \)
Similarly, the total number of outages in normal weather in which
the outages of component 1 overlap the outages of component 2
\[
\frac{\lambda_{av2} (1-W) (N+S) n r_2}{\lambda_1} + \frac{\lambda_{av2} W n (N+S) r_2}{\lambda_1}
\]
The total number of overlapping outages of components 1 and 2 in
normal weather,
\[
B = \lambda_{av1} (1-W) (N+S) n r_1 \lambda_2 + \lambda_{av1} W n (N+S) r_1 \lambda_2
\]
\[
+ \lambda_{av2} (1-W) (N+S) n r_2 \lambda_1 + \lambda_{av2} W (N+S) n r_2 \lambda_1
\]
The normal weather outage rate is therefore,
\[
\lambda_o = \frac{B}{n N} = \lambda_1 \lambda_2 r_1 + \lambda_1 \lambda_2 r_2 + \frac{S}{N} \left[ \lambda_1' \lambda_2 r_1 + \lambda_2' \lambda_1 r_2 \right]
\]
This expression is the same as equation 3.12 a.

(ii) Repair during adverse weather

The total outage time of component 1 in adverse weather due to its outages
in adverse weather
\[
= \lambda_{av1} W (N+S) n \left( \frac{Sr_1}{S+r_1} \right)
\]
where \( \frac{Sr_1}{S+r_1} \) is the overlapping duration of the repair period of
component 1 and the adverse weather period.
The cycle time of component 2 in adverse weather
\[
= \frac{1}{\lambda_2}
\]
The fraction of outages of component 2 in adverse weather which
overlap the outages of component 1 in adverse weather =

\[ \frac{S_{r_1}}{\lambda_{av1} W (N+S) n \frac{\alpha^2}{S + x_1}} \]

The expected duration in adverse weather associated with outages of component 1 which occurred in normal weather and extended into adverse weather

\[ \frac{\lambda_{av1} (1-W) n (N+S) \frac{\alpha^2}{W} \frac{S_{r_1}}{S + x_1}}{\lambda_2} \]

where \( \frac{\alpha^2}{W} \) is the probability that the repair of component 1 is not completed in normal weather.

The fraction of outages of component 2 in adverse weather which overlap the outages of component 1 which occurred in normal weather and extended into adverse weather

\[ \frac{\lambda_{av1} (1-W) n (N+S) \frac{\alpha^2}{W} \frac{S_{r_1}}{S + x_1}}{\lambda_2} \]

The total number of outages in adverse weather in which outages of component 2 overlap the outages of component 1

\[ \frac{\lambda_{av1} W (N+S) n \frac{S_{r_1}}{S + x_1} + \lambda_{av1} (1-W) n (N+S) \frac{\alpha^2}{W} \frac{S_{r_1}}{S + x_1}}{\lambda_2} \]

Similarly, the total number of outages in adverse weather in which outages of component 1 overlap the outages of component 2

\[ \frac{\lambda_{av2} W (N+S) n \frac{S_{r_2}}{S + x_2} + \lambda_{av2} (1-W) n (N+S) \frac{\alpha^2}{W} \frac{S_{r_2}}{S + x_2}}{\lambda_1} \]
The total number of overlapping outages of component 1 and 2 in adverse weather
\[
\Lambda = \lambda_{av1} \times (N+S) \times \left( \frac{Sr_1}{S+x_1} \times \lambda'_2 + \lambda_{av1} \times (1-N) \times \frac{r_1}{N} \times \frac{Sr_1}{S+x_1} \times \lambda'_2 \right)
\]
\[
+ \lambda_{av2} \times (N+S) \times \left( \frac{Sr_2}{S+x_2} \times \lambda'_1 + \lambda_{av2} \times (1-N) \times \frac{r_2}{N} \times \frac{Sr_2}{S+x_2} \times \lambda'_1 \right)
\]
\[
= \lambda'_1 \times \lambda'_2 \times \frac{Sr_1}{S+x_1} \times \frac{Sr_2}{S+x_2} \times \left( \frac{Sr_1}{S+x_1} + \frac{Sr_2}{S+x_2} \right)
+ \lambda'_1 \times \lambda'_2 \times \left( \frac{r_1}{S+x_1} + \frac{r_2}{S+x_2} \right)
\]

The adverse weather outage rate is therefore
\[
\lambda'_e = \Lambda = \lambda'_1 \times \lambda'_2 \times \left( \frac{Sr_1}{S+x_1} + \frac{Sr_2}{S+x_2} \right)
+ \lambda'_1 \times \lambda'_2 \times \frac{r_1}{S+x_1} + \lambda'_1 \times \lambda'_2 \times \frac{r_2}{S+x_2}
\]

The fraction of outages of component 2 in normal weather that overlap the outages of component 1 which occurred in adverse weather and extended into normal weather
\[
\frac{\lambda_{av1} \times (N+S) \times \frac{kr_1}{N+x_1}}{1}
\]

where \( \frac{kr_1}{N+x_1} \) = The overlapping duration of normal weather and the repair period of component 1 = \( r_1 \) as \( N \gg r \)

The assumption implied herein is that all outages in adverse weather extend into normal weather.

The total number of outages in normal weather in which outages of component 2 overlap the outages of component 1...
\[
- \frac{\lambda_{av1} (1-W) (N+S) n r_1}{1 + \lambda_2} + \frac{\lambda_{av1} W n (N+S) r_1}{\lambda_2}
\]

Similarly the total number of outages in normal weather in which outages of component 1 overlap the outages of component 2

\[
- \frac{\lambda_{av2} (1-W) (N+S) n r_2}{1 + \lambda_1} + \frac{\lambda_{av2} W n (N+S) r_2}{\lambda_1}
\]

The total number of overlapping outages of component 1 and 2 in normal weather,

\[
B = \frac{B}{n N} = \lambda_1 \lambda_2 x_1 + \lambda_1 \lambda_2 x_2 + \frac{S}{N} \left( \lambda_1 \lambda_2 x_1 + \lambda_1 \lambda_2 x_2 \right)
\]

The normal weather outage rate is therefore,

\[
\lambda = \frac{B}{n N} = \lambda_1 \lambda_2 x_1 + \lambda_1 \lambda_2 x_2 + \frac{S}{N} \left( \lambda_1 \lambda_2 x_1 + \lambda_1 \lambda_2 x_2 \right)
\]

The overall annual outage rate from equation 3.11 is thus given by:

\[
\lambda_{SL} = \frac{N}{N+S} \left( \lambda_1 \lambda_2 (x_1+x_2) + \frac{S}{N} (\lambda_1 \lambda_2 x_1 + \lambda_1 \lambda_2 x_2) \right)
\]

\[+ \frac{S}{N+S} \left( \lambda_1 \lambda_2 \left( \frac{x_1}{N} + \frac{x_2}{N} \right) + \lambda_1 \lambda_2 \left( \frac{x_1^2}{N} + \lambda_1 \lambda_2 \left( \frac{x_2^2}{N} \right) \right) + \lambda_2 \lambda_1 \frac{r_2}{S+x_2} \right)
\]

Rearranging,

\[
\lambda_{SL} = \frac{N}{N+S} \left[ \lambda_1 \lambda_2 (x_1+x_2) + \frac{S}{N} \left( \lambda_1 \lambda_2 \frac{x_1^2}{S+x_1} + \lambda_2 \lambda_1 \frac{x_2^2}{S+x_2} \right) \right]
\]
\[
+ \frac{S}{N+S} \left[ \lambda_1^r \lambda_2^r x_1 + \lambda_2^r \lambda_2^r x_2 + \lambda_1^r \lambda_2^r S \left( \frac{r_1}{S+x_1} + \frac{r_2}{S+x_2} \right) \right]
\]

This expression is same as equation 3.31

This clearly shows that the assumptions regarding exponential distributions of component up and down times are not very stringent and the results are valid for any general distribution.

3.6 Extension Of The Equations For More Than Two Overlapping Outages

As seen in section 3.4, the results predicted for the average outage rate of more than two parallel components by the approximate technique and the Block Reduction technique do not agree with those obtained by a complete Markov approach. Both combining techniques form an equivalent component from two of the components and then combine the equivalent component with the third one. The formation of an equivalent component may not be accurate in cases where dependent effects exist in the system. In this section, the equations of two overlapping outages are expanded to three overlapping outages. The equations for four overlapping outages are given in Appendix 3.

There are eight possible modes of occurrence of three overlapping outages. They are as follows:

(1) The initial failure occurs in normal weather, the second failure occurs in normal weather and the third failure also occurs in normal weather.

(ii) The initial failure occurs in normal weather, the second failure occurs in adverse weather and the third failure occurs in normal weather.
(iii) The initial failure occurs in adverse weather, the second failure occurs in normal weather and the third failure also occurs in normal weather.

(iv) The initial failure occurs in adverse weather, the second failure occurs in adverse weather and the third failure occurs in normal weather.

(v) The initial failure occurs in adverse weather, the second failure occurs in adverse weather and the third failure also occurs in adverse weather.

(vi) The initial failure occurs in adverse weather, the second failure occurs in normal weather and the third failure occurs in adverse weather.

(vii) The initial failure occurs in normal weather, the second failure occurs in adverse weather and the third failure also occurs in adverse weather.

(viii) The initial failure occurs in normal weather, the second failure occurs in normal weather and the third failure occurs in adverse weather.

The contribution to the system failure rate due to each of these modes of failures is given below. The equations are formulated to consider the occurrence or nonoccurrence of repairs during adverse weather periods.

3.6.1 No repair during adverse weather

Node 1

\[
(\text{Failure rate})_1 = \frac{N}{N+3} \left( \lambda_1 \left( \lambda_2 r_1 \right) \left( \lambda_3 \frac{r_2}{r_1 + r_2} \right) + \lambda_1 \left( \lambda_3 r_1 \right) \left( \lambda_2 \frac{r_3}{r_1 + r_3} \right) \right)
\]

Similar terms for components 2 and 3 \hspace{1cm} (3.33 a)
where

\[ \lambda_2 r_1 = \text{The probability that component 2 fails during the repair period of component 1.} \]

\[ \frac{r_1 r_2}{r_1 + r_2} = \text{The overlapping outage duration of components 1 and 2} \]

\[ \lambda_3 \frac{r_1 r_2}{r_1 + r_2} = \text{The probability that component 3 fails during the overlapping outage duration of components 1 and 2} \]

Similar reasoning applies to the second term but with components 2 and 3 interchanged.

In the first two terms component 1 is assumed to fail first. Similar terms can be written for components 2 and 3 failing first. It is further assumed that the probability of a weather change during the repair of a component is zero. The above expression can be simplified to the following form

\[ (\text{Failure rate})_1 = \frac{N}{N+S} \left( \lambda_1 \lambda_2 \lambda_3 (r_1 r_2 + r_2 r_3 + r_3 r_1) \right) \]

Node 2

\[ (\text{Failure rate})_2 = \frac{N}{N+S} \left( \lambda_1 \left( \frac{r_1}{N} \right) \left( \lambda'_2 S \right) \left( \lambda_3 \frac{r_1 r_2}{r_1 + r_2} \right) + \lambda_1 \left( \frac{r_1}{N} \right) \left( \lambda'_3 S \right) \lambda_2 \frac{r_1 r_3}{r_1 + r_3} + \right. \]

\[ \left. \text{Similar terms for components 2 and 3} \right) \]

(3.33 b)
where

\[ \frac{L}{N} \] = The probability that the weather changes from normal to adverse during the repair of component 1.

\[ \lambda_2^S \] = The probability that component 2 fails during the adverse weather period.

\[ \lambda_3 \frac{r_1 r_2}{r_1 + r_2} \] = The probability that component 3 fails in normal weather during the repair of components 1 and 2.

Similar reasoning applies to the second term in the above expression but with components 2 and 3 interchanged.

In the first two terms component 1 is assumed to fail first. Similar terms can be written for components 2 and 3 failing first.

Mode 3

\[
(Failure\ rate)_3 = \frac{S}{N+S} \left( \lambda_1' (\lambda_2 r_1) \left( \lambda_3 \frac{r_1 r_2}{r_1 + r_2} + \lambda_1' (\lambda_2 r_1) \left( \lambda_2' \frac{r_1 r_2}{r_1 + r_2} \right) + \right) \right)
\]

where

\[ \frac{S}{N+S} \] = The long term fraction of time that the weather is adverse

\[ \lambda_1' \] = The adverse weather failure rate of component 1.

\[ \lambda_2 r_1 \] = The probability that component 2 fails during the repair of component 1.
\[ \frac{\lambda_2 \frac{x_1 x_2}{x_1 + x_2}}{r_1 + r_2} \Rightarrow \text{The probability that component 3 fails during the repair of components 1 and 2.} \]

Similar reasoning applies to the second term in the above expression but with components 2 and 3 interchanged.

The first two terms in the above expression assume component 1 failing first. It is assumed in these terms that the probability of components 2 and 3 not failing in adverse weather is one. Similar terms can be written for components 2 and 3 failing first.

\[
(Failure \ rate)_4 = \frac{8}{N+S} \left( \lambda_1' (\lambda_2 S) \left( \frac{x_1 x_2}{r_1 + r_2} \right) + \lambda_2' (\lambda_3 S) \left( \frac{x_1 x_3}{r_1 + r_3} \right) + \right)
\]

\[
\text{Similar terms for components 2 and 3} \quad (3.33 \text{d})
\]

where:

\[ \lambda_1' = \text{The adverse weather failure rate of component 1.} \]
\[ \lambda_2 S = \text{The probability that component 2 fails during the adverse weather period.} \]
\[ \lambda_3 \frac{x_1 x_2}{r_1 + r_2} = \text{The probability that component 3 fails during the repair of components 1 and 2.} \]

Similar reasoning applies to the second term in the above expression but with components 2 and 3 interchanged.

The first two terms in the above expression assume component 1 failing first. Similar terms can be written for components 2 and 3 failing first.
Mode 5

\[
\text{(Failure rate)}_5 = \frac{S}{N+S} \left( \lambda'_1 (\lambda'_2 S) (\lambda'_3 S) + \lambda'_1 (\lambda'_3 S) (\lambda'_2 S) + \right.
\]

\[
\left( \text{Similar terms for components 2 and 3} \right) \quad (3.33 \ a)
\]

where

\[
\lambda'_2 S \approx \text{The probability that component 2 fails during the adverse period.}
\]

\[
\lambda'_3 S \approx \text{The probability that component 3 fails during the adverse weather period.}
\]

In the first term, the failure of component 1 is followed by the failure of component 2 and then component 3. In the second term, the failure of component 1 is followed by the failure of component 3 and then component 2. Similar terms can be written for components 2 and 3 failing first.

Mode 6

\[
\text{(Failure rate)}_6 = \frac{S}{N+S} \left( \lambda'_1 (\lambda_2 r_1) \left( \frac{r_1 r_2}{r_1 + r_2} \right)_N (\lambda'_3 S) + \lambda'_1 (\lambda_3 r_1) \left( \frac{r_1 r_3}{r_1 + r_3} \right)_N \lambda'_2 S \right.
\]

\[
\left( \text{Similar terms for components 2 and 3} \right) \quad (3.33 \ f)
\]

where

\[
\lambda'_2 r_1 \approx \text{The probability that component 2 fails during the repair of component 1.}
\]

\[
\frac{r_1 r_2}{(r_1 + r_2)_N} \approx \text{The probability that the weather changes from normal to adverse during the repair of components 1 and 2.}
\]

\[
\lambda'_3 S \approx \text{The probability that component 3 fails during the adverse weather.}
\]
Similar reasoning applies to the second term in the above expression but with components 2 and 3 interchanged.

In the above expression the first two terms involved component 1 failing first. Similar terms can be written for components 2 and 3 failing first.

Node 7

\[
(Failure \ rate)_7 = \frac{N}{N+S} \left( \lambda_1 \left( \frac{1}{N} \right) (\lambda_2 S) (\lambda_3 S) + \lambda_1 \left( \frac{F_1}{N} \right) (\lambda_3 S) (\lambda_2 S) + \right.

\]

\[
\left. \text{Similar terms for components 2 and 3} \right) \quad (3.33 \text{ h})
\]

where

\[
\frac{F_1}{N} \approx \text{The probability that the weather changes from normal to adverse during the repair of component 1.}
\]

\[
\lambda_2 S \approx \text{The probability that component 2 fails during the adverse weather.}
\]

\[
\lambda_3 S \approx \text{The probability that component 3 fails during the adverse weather.}
\]

In the first term the failure of component 1 is followed by failure of component 2 and then component 3. In the second term, the failure of component 1 is followed by component 3 and then component 2. Similar terms can be written for components 2 and 3 failing first.

Node 8

\[
(Failure \ rate)_8 = \frac{N}{N+S} \left( \lambda_1 \left( \frac{1}{N} \right) (\lambda_2^{\text{r2}}) \left( \frac{F_1 \lambda_2}{(r_1 + r_2) N} \right) (\lambda_3 S) + \lambda_1 \left( \frac{F_1 \lambda_3}{(r_1 + r_3) N} \right) (\lambda_2 S) \right.

\]

\[
\left. + \left( \frac{F_1 \lambda_3}{(r_1 + r_3) N} \right) (\lambda_2 S) \right) \quad (3.33 \text{ h})
\]
where
\[ \lambda_2 r_1 \approx \text{The probability that component 2 fails during the repair of component 1.} \]
\[ \frac{r_1 r_2}{(r_1 + r_2)^N} \approx \text{The probability that the weather changes from normal to adverse during the repair of components 1 and 2.} \]

Similar reasoning applies to the second term in the above expression but with components 2 and 3 interchanged.

The first two terms in the above expression involved component 1 failing first. Similar terms can be written for components 2 and 3 failing first.

The overall annual outage rate resulting from three overlapping outages is therefore given by:
\[
\lambda_{SL} = \sum_{i=1}^{8} (\text{Failure rate})_i \tag{3.34}
\]

The first four modes of failure involved the system failure in normal weather and the last four modes involved the system failure in adverse weather.

All the failures which occur in an adverse weather have to wait for repairs for a period equal to the average duration of an adverse weather period.

The average outage duration associated with three overlapping outages is therefore given by:
\[
r_{SL} = \left[ \frac{\frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1}}{r_1 r_2 + r_2 r_3 + r_3 r_1} \right] \times \sum_{i=1}^{4} \frac{(\text{Failure rate})_i}{\lambda_{SL}} + \left[ \frac{\frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1} + s}{r_1 r_2 + r_2 r_3 + r_3 r_1} \right] \times \sum_{i=5}^{8} \frac{(\text{Failure rate})_i}{\lambda_{SL}} \tag{3.35}
\]
The average total outage duration $\approx \lambda_{SL} \cdot T_{SL}$ \hspace{1cm} (3.36)

3.6.2 Repair during adverse weather

**Node 1**

$$(\text{Failure rate})_1 = \frac{N}{N+S} \left( \lambda_1 \left( \lambda_2 \frac{r_1 r_2}{r_1 + r_2} \right) + \lambda_1 \left( \lambda_3 \frac{r_3 r_1}{r_1 + r_3} \right) + \lambda_2 \left( \lambda_3 \frac{r_1 r_3}{r_1 + r_3} \right) \right)$$

Similar terms for components 2 and 3)

This expression is similar to equation 3.33 a.

In simplified form,

$$(\text{Failure rate})_1 = \frac{N}{N+S} \left( \lambda_1 \lambda_2 \lambda_3 \left( r_1 r_2 + r_2 r_3 + r_3 r_1 \right) \right)$$

**Node 2**

$$(\text{Failure rate})_2 = \frac{N}{N+S} \left( \lambda_1 \left( \frac{r_1}{N} \right) \left( \lambda_2 \frac{Sr_1}{N} \right) \left( \lambda_3 \frac{N r_1 r_2}{N r_1 + N r_2 + r_1} \right) \right) + \lambda_1 \left( \frac{r_1}{N} \right) \left( \lambda_2 \frac{Sr_1}{N} \right) \left( \lambda_3 \frac{N r_1 r_3}{N r_1 + N r_3 + r_1} \right)$$

Similar terms for components 2 and 3)

where

$\frac{r_1}{N} \approx \text{The probability that the weather changes from normal to adverse during the repair of component 1.}$

$\lambda_2 \frac{Sr_1}{N} \approx \text{The probability that component 2 fails during an overlapping period of adverse weather and the repair of component 1.}$
The probability that component 3 fails during the overlapping period of normal weather and the repair of components 1 and 2.

\[ \frac{\lambda_3 \frac{N \tau_1 \tau_2}{N r_1 + N r_2 + \tau_1 \tau_2}}{N r_1 + N r_2 + \tau_1 \tau_2} \]

\[ \approx \lambda_3 \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \]

Similar reasoning is applicable to the second term in the above expression but with components 2 and 3 interchanged.

The first two terms in the above expression considered component 1 failing first. Similar terms can be written for components 2 and 3 failing first.

Mode 3

\[ \text{(Failure rate)}_3 = \frac{S}{N+8} \left( \lambda_1 \lambda_2 \frac{N r_1}{N+\tau_1} \right) \left( \lambda_3 \frac{N r_1 \tau_2}{N r_1 + N r_2 + \tau_1 \tau_2} \right) \]

\[ \left( \lambda_1 \left( \frac{N r_1}{N+\tau_1} \right) \left( \lambda_3 \frac{N r_1 \tau_2}{N r_1 + N r_2 + \tau_1 \tau_2} \right) \right) \]

\[ \left( \lambda_1 \left( \frac{N r_1}{N+\tau_1} \right) \left( \lambda_3 \frac{N r_1 \tau_2}{N r_1 + N r_2 + \tau_1 \tau_2} \right) \right) \]

where

\[ \lambda_2 \frac{N r_1}{N+\tau_1} \approx \text{The probability that component 2 fails during the overlapping period of normal weather and the repair of component 1.} \]

\[ \lambda_3 \frac{N r_1 \tau_2}{N r_1 + N r_2 + \tau_1 \tau_2} \approx \text{The probability that component 3 fails during the overlapping period of normal weather and the repair of components 1 and 2.} \]

\[ \lambda_3 \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \]
Similar reasoning is applicable to the second term in the above expression but with components 2 and 3 interchanged.

The first two terms in the above expression involved component 1 failing first. Similar terms can be written for components 2 and 3 failing first.

Mode 4

\[
\text{(Failure rate)}_4 = \frac{S}{N+S} \left( \lambda_1' \left( \lambda_2' \frac{S r_1}{S+r_1} \right) \left( \lambda_3 \frac{N r_1 r_2}{N r_1 + N r_2 + r_1 r_2} \right) + \lambda_1' \left( \lambda_3' \frac{S r_1}{S+r_1} \right) \left( \frac{N r_1 r_3}{N r_1 + N r_2 + r_1 r_2} \right) + \text{Similar terms for components 2 and 3} \right)
\]

where

\[
\lambda_2' = \frac{S r_1}{S+r_1} \approx \text{The probability that component 2 fails during the overlapping period of adverse weather and the repair of component 1.}
\]

\[
\lambda_3 \frac{N r_1 r_2}{N r_1 + N r_2 + r_1 r_2} \approx \text{The probability that component 3 fails during the overlapping period of normal weather and repair of components 1 and 2.}
\]

Similar reasoning applies to the second term in the above expression but with components 2 and 3 interchanged.

The first two terms in the above expression involved component 1 failing first. Similar terms can be written for components 2 and 3 failing first.
Mode 5

\[
\text{(Failure rate)}_5 = \frac{S}{N+S} \left( \lambda_1 \left( \lambda_2 \frac{S r_1 r_2}{S + r_1} \right) + \lambda_1 \left( \lambda_3 \frac{S r_1 r_2}{S + r_2 + r_1 r_2} \right) \right) + \text{Similar terms for components 2 and 3}
\]

where

\[
\lambda_2 \frac{S r_1}{S + r_1} = \text{The probability that component 2 fails during the overlapping period of adverse weather and the repair of component 1.}
\]

\[
\lambda_3 \frac{S r_1 r_2}{S + r_2 + r_1 r_2} = \text{The probability that component 3 fails during the overlapping period of adverse weather and the repairs of components 1 and 2.}
\]

Similar reasoning applies to the second term in the above expression but with components 2 and 3 interchanged.

The first two terms in the above expression involved component 1 failing first. Similar terms can written for components 2 and 3 failing first.

Mode 6

\[
\text{(Failure rate)}_6 = \frac{S}{N+S} \left( \lambda_1 \left( \lambda_2 \frac{r_1 N}{r_1 + N} \right) + \lambda_1 \left( \lambda_3 \frac{N r_1 r_2}{r_1 + N} \right) \right) + \text{Similar terms for components 2 and 3}
\]

where

\[
\lambda_2 \frac{r_1 N}{r_1 + N} = \text{The probability that component 2 fails during the}
\]
overlapping period of normal weather and the repair of component 1.

\[
\frac{N \cdot r_1 \cdot r_2}{(N \cdot r_1 + N \cdot r_2 + r_1 \cdot r_2) \cdot N}
\]

\( \lambda_3 \cdot \frac{S \cdot r_1 \cdot r_2}{S \cdot r_1 + S \cdot r_2 + r_1 \cdot r_2} \)

The probability that the weather changes from normal to adverse during the overlapping period of normal weather and the repair of components 1 and 2.

The probability that component 3 fails during the overlapping period of adverse weather and the repair of components 1 and 2.

Similar reasoning applies to the second term in the above expression but with components 2 and 3 interchanged.

The first two terms in the above expression involved component 1 failing first. Similar terms can be written for components 2 and 3 failing first.

**Mode 7**

\[
(Failure \ rate)_7 = \frac{N}{N+S} \left( \lambda_1 \left( \frac{r_1}{N} \right) (\lambda_2' \cdot \frac{Sr_1}{S+r_1}) (\lambda_3' \cdot \frac{S \cdot r_1 \cdot r_2}{S \cdot r_1 + S \cdot r_2 + r_1 \cdot r_2}) + \lambda_1 \left( \frac{r_1}{N} \right) \right.
\]

\[
(\lambda_3' \cdot \frac{Sr_1}{S+r_1}) (\lambda_2' \cdot \frac{S \cdot r_1 \cdot r_3}{S \cdot r_1 + S \cdot r_3 + r_1 \cdot r_3} ) +
\]

Similar terms for components 2 and 3

where

\[
\frac{r_1}{N} \Rightarrow \text{The probability that the weather changes from normal to adverse during the repair of component 1.}
\]

\[
\frac{Sr_1}{S+r_1} \Rightarrow \text{The probability that component 2 fails during the overlapping}
\]
period of adverse weather and the repair of component 1

\[ \lambda_3 \frac{S \, r_1 \, r_2}{r_1 + r_2 + r_1 \, r_2} \approx \]

The probability that component 3 fails during the overlapping period of adverse weather and the repair of components 1 and 2.

Similar reasoning applies to the second term in the above expression but with components 2 and 3 interchanged.

In the above expression, the first two terms involved component 1 failing first. Similar terms can be written for components 2 and 3 failing first.

Mode 8

\[
(Failure \ rate)_8 = \frac{N}{N+S} \left( \lambda_1 \left( \lambda_2 r_1 \right) \left( \frac{r_1 r_2}{r_1 + r_2} \right) \frac{S \, r_1 \, r_2}{r_1 + r_2 + r_1 \, r_2} + \lambda_2 \left( \lambda_3 r_1 \right) \left( \frac{r_1 r_3}{r_1 + r_3} \right) \frac{S \, r_1 \, r_3}{r_1 + r_2 + r_1 \, r_3} + \lambda_3 \left( \lambda_1 r_1 \right) \left( \frac{r_1 r_2}{r_1 + r_2} \right) \frac{S \, r_1 \, r_2}{r_1 + r_2 + r_1 \, r_2} \right)
\]

where

\[ \lambda_2 r_1 \approx \] The probability that component 2 fails in normal weather during the repair of component 1.

\[ \frac{r_1 r_2}{(r_1 + r_2) \, N} \approx \] The probability that the weather changes from normal to adverse during the overlapping period of repair of components 1 and 2.

\[ \lambda_3 \frac{S \, r_1 \, r_2}{r_1 + r_2 + r_1 \, r_2} \approx \] The probability that component 3 fails during the overlapping period of adverse weather and the repair of components 1 and 2.
Similar argument applies to the second term in the above expression but with components 2 and 3 interchanged.

The first two terms in the above expression involved component 1 failing first. Similar terms can be written for components 2 and 3 failing first.

The overall annual outage rate resulting from three overlapping outages is therefore

$$\lambda_{SL} = \sum_{1=1}^{3} (\text{Failure rate})_1$$

(3.37)

The average outage duration is given by

$$r_{SL} = \frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1}$$

(3.38)

The average total outage duration $$\simeq \lambda_{SL} \cdot r_{SL}$$

(3.39)

The results obtained from the equations given in this section have been compared with those obtained by a Markov approach. There is very little difference in the results obtained by the two techniques.

Using these equations, the effects of variation of component parameters are shown in Figures 3.16, 3.17, 3.18 and 3.19 for simple parallel system configurations of two, three and four components for various percentages of component failures during adverse weather periods. Figure 3.16 shows the effect of variation of component failure rates. The system failure rate increases with the increase in component failure rate and the percentage of component failures during adverse weather. Figure 3.17 shows the effect of variation of component repair time on the system.
failure rate when no repairs are performed during the adverse weather period. It can be seen from this figure that the component repair time does not have any effect on the system failure rate if all the failures occur during adverse weather periods. This is, perhaps, obvious as all the failures occur during adverse weather when no repairs are performed. Under these circumstances, the component repair time does not affect the system failure rate. Figures 3.18 and 3.19, respectively, show the effect of occurrence and nonoccurrence of repairs during adverse weather periods on the system failure rate and average outage duration. The system failure rate increases when no repair occurs during adverse weather. Similarly, the average outage duration of the system also increases with no repair during adverse weather. These results are intuitively obvious.

Figure 3.20 shows the effect of variation of the weather parameters on the system failure rate when repair occurs during adverse weather periods. It can be seen that for the same relative duration of normal and adverse weather periods, the longer adverse weather periods are liable to cause more damage to the system from a reliability viewpoint than that caused by shorter ones.

Figure 3.21 shows the importance of a series component in determining the failure rate of a system. In this figure, the effect of variation in the series component failure rate on the system failure rate is shown. It can be seen that the series component failure rate dominates the configuration value even for relatively small series component values.
Figure 3.16  Effect of Varying Component Failure Rates on System Failure Rate, No Repair During Adverse Weather
Figure 3.17  Effect of Varying Component Repair Times On System Outage Rate, No Repair During Adverse Weather
Figure 3.18 Effect of Occurrence and Nonoccurrence of Repair During Adverse Weather on System Outage Rate. Variable Repair Durations
Figure 3.19  Effect of Occurrence and Nonoccurrence of Repair During Adverse Weather on System Average Outage Duration, Variable Repair Times
Figure 3.20  Effect of Longer Adverse Weather Periods on System Outage Rate
Figure 3.21 Effect of Series Component Failure Rate in a Series Parallel System, No Repair During Adverse Weather
3.7 Error in an Equivalent Component Formation

The equivalent component formation, as seen in section 3.5, does not give correct results. This is because the equivalent component formation misses some of the transitions in which component 1 fails first followed by failures of components 2 and 3 respectively and component 2 fails first followed by failures of components 3 and 1 respectively. This is shown below for the adverse weather failure rate of the equivalent component when no repair occurs during the adverse weather period. Using equations 3.12a and 3.12:

\[
\lambda_{eq} = 2 \lambda_3 \left\{ \lambda_1 \lambda_2 S + \lambda_1 \lambda_2 r_2 + \lambda_1 \lambda_2 \right\} S + \lambda_3 \left\{ 2 \lambda_1 \lambda_2 S + \lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 \right\} r_3
\]

\[
+ \lambda_3 \left\{ \lambda_1 \lambda_2 \left( r_1 + r_2 \right) + \frac{S}{N} \left( \lambda_1 \lambda_2 r_1 + \lambda_2 \lambda_1 r_2 \right) \right\} \left( \frac{r_1 r_2}{r_1 + r_2} \right)
\]

(3.40)

The adverse weather failure rate of a three component system from the set of equations of 3.33 is given by:

\[
\lambda = 6 \lambda_1 \lambda_2 \lambda_3 S^2 + 2 \lambda_1 r_1 \lambda_3 \lambda_2 S + 2 \lambda_2 r_2 \lambda_1 \lambda_3 S + 2 \lambda_3 r_3 \lambda_1 \lambda_2 S
\]

\[
+ \lambda_1 \lambda_2 r_1 \left( \frac{r_1 r_2}{r_1 + r_2} \right) \lambda_3 S + \lambda_1 \lambda_3 r_1 \left( \frac{r_1 r_3}{r_1 + r_3} \right) \lambda_2 S + \lambda_2 \lambda_1 r_2 \left( \frac{r_2 r_1}{r_1 + r_2} \right) \lambda_3 S
\]

\[
+ \lambda_2 \lambda_3 r_2 \left( \frac{r_2 r_3}{r_2 + r_3} \right) \lambda_1 S + \lambda_3 \lambda_1 r_3 \left( \frac{r_3 r_1}{r_1 + r_3} \right) \lambda_2 S + \lambda_3 \lambda_2 r_3 \left( \frac{r_3 r_2}{r_2 + r_3} \right) \lambda_1 S
\]

\[
+ \lambda_1 \lambda_2 \lambda_3 \left( r_1 + r_2 + r_3 \right) + \lambda_1 \lambda_3 \lambda_2 r_1 + r_3 + \lambda_1 \lambda_2 \lambda_3 r_2 + \lambda_2 \lambda_3 r_1
\]

(3.41)

Comparing expressions 3.40 and 3.41, it can be seen that the following terms are missing in equation 3.40.
\[2 \lambda_1 \lambda_2 \lambda_3 S + \lambda_1 \lambda_2 \frac{r_1 r_3}{(r_1 + r_3)} \lambda_3 s + \lambda_1 \lambda_3 \frac{r_1 r_3}{(r_1 + r_3)} \lambda_2 s + \lambda_2 \lambda_3 \frac{r_2 r_3}{(r_2 + r_3)} \lambda_1 s\]

These terms arise from the transitions noted earlier. Similarly, it can be shown that there are some terms missing in the equivalent component normal weather failure rate. It can, however, be noted that with zero percentage of component failures during adverse weather, there is no error in the formation of an equivalent component. With 100% component failures during adverse weather, there is 50% error in the prediction of the annual outage rate of the system. Therefore, not quite as much bunching due to adverse weather failures is recognised in the formation of an equivalent component.

3.6 Star - Delta Conversion

The techniques described in the previous sections dealt with series and parallel components. In practice, however, many systems may be encountered in which none of the components is either in series or in parallel. An example of such a system is shown in Figure 3.22 a. For such a system, the concept of star - delta conversion was applied\(^{(10)}\) and equivalent formulae were developed to convert a star configuration of components into a delta configuration and vice versa. This conversion process is shown in Figure 3.22 b. The star - delta conversion concept is quite complicated and the results predicted are inaccurate due to the
Figure 3.22  A Star-Delta Conversion Technique  

(a) A Complex System  
(b) Star-Delta Conversion  
(c) System Cut Sets
formation of equivalent components. In order to deal with complex systems of this type, the technique of failure mode and effect analysis has been applied. The system failure states are determined by minimal tie sets or cut sets(7). A tie set is a set of components which form a path from input to output. A tie set is minimal if no node is traversed more than once when tracing the tie set. A cut set is a set of components which when removed divides the system into two parts, one of which contains the input, the other the output node. Minimal cut sets are those for which a subset cannot be another cut set.

Using this technique, the minimal cut sets of the configuration in Figure 3.22 a are shown in series in Figure 3.22 c. Each cut set represents a distinct mode of system failure. Only series and parallel combinations are therefore required to determine the reliability indices. The results obtained by this technique for the system shown in Figure 3.22 a have been compared with those obtained by an exact Markov technique and are shown in Table 3.14.

It can be seen from this table that the results obtained by the cut set technique are very close to those obtained by a complete Markov approach for both the reliability measures i.e failure rate of the system and the associated outage duration.
<table>
<thead>
<tr>
<th>Percentage of component failures during adverse weather</th>
<th>Approx. Failure Rate $/\text{year}$</th>
<th>Approx. Average Duration $\text{hours}$</th>
<th>Markov Failure Rate $/\text{year}$</th>
<th>Markov Average Duration $\text{hours}$</th>
<th>Approx. Failure Rate $/\text{year}$</th>
<th>Approx. Average Duration $\text{hours}$</th>
<th>Markov Failure Rate $/\text{year}$</th>
<th>Markov Average Duration $\text{hours}$</th>
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<td>0.00670</td>
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<td>0.00777</td>
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</table>
3.9 Three State Weather Model

It was pointed out in section 3.3 that the disaster adverse weather periods such as severe hurricanes and tornadoes can have a significant effect on the system reliability, although their occurrence is very rare. These periods, however, cannot be lumped with other less violent periods of adverse weather. (In order to represent the effect of disaster adverse weather periods, a three state weather model is proposed. With this model, the component failure rate fluctuates between three levels corresponding to normal, adverse and disaster adverse weather periods. It is assumed that the overlapping outages cannot occur in two different types of adverse weather periods. This, in fact, implies that there is enough time between adverse weather and disaster adverse weather periods to perform repairs. It is further assumed that the repair is possible during adverse weather periods and no repair can be done during disaster adverse weather periods.)

Let

The average duration of a period between adverse weather periods = N
The average duration of a period between disaster adverse weather periods = P
The average duration of an adverse weather period = S
The average duration of a disaster adverse weather period = T
The failure rate of component 1 during normal weather = \( \lambda_1 \)
The failure rate of component 1 during an adverse weather = \( \lambda_1' \)
The failure rate of component 1 during a disaster adverse weather = \( \lambda_1'' \)
The average repair time of component $i = r_i$

The reasoning for various terms in the following modes of failure follows from equations 3.12 and 3.31. For a two component parallel system, there are seven distinct modes of system failure.

Mode 1  The initial failure occurs in normal weather and the second failure also occurs in normal weather

$$(\text{Failure Rate})_1 = \frac{N}{N+S} \left( \lambda_1 \lambda_2 r_1 + \lambda_2 r_2 \right)$$

Mode 2  The initial failure occurs in normal weather and the second failure occurs in adverse weather

$$(\text{Failure Rate})_2 = \frac{N}{N+S} \left( \lambda_1 \frac{r_1}{N} \left( \lambda_2 \frac{Sr_1}{S+r_1} \right) + \lambda_2 \frac{r_2}{N} \left( \lambda_1 \frac{Sr_2}{S+r_2} \right) \right)$$

Mode 3  The initial failure occurs in disaster adverse weather and the second failure occurs in normal weather

$$(\text{Failure Rate})_3 = \frac{T}{P+T} \left( \lambda_1' \left( 1 - \lambda_2'' \right) \lambda_2 r_1 + \lambda_2'' \left( 1 - \lambda_1' \right) \lambda_1 r_2 \right)$$

Mode 5  The initial failure occurs in adverse weather and the second failure also occurs in adverse weather

$$(\text{Failure Rate})_5 = \frac{S}{N+S} \left( \lambda_1' \left( \lambda_2' \frac{Sr_1}{S+r_1} \right) + \lambda_2' \left( \lambda_1' \frac{Sr_2}{S+r_2} \right) \right)$$

Mode 6  The initial failure occurs in normal weather and the second failure occurs in disaster adverse weather

$$(\text{Failure Rate})_6 = \frac{P}{P+S} \left( \lambda_1 \frac{r_1}{P} \lambda_2'' r + \lambda_2 \frac{r_2}{P} \lambda_1'' r \right)$$
Mode 7 The initial failure occurs in disaster adverse weather and the second failure also occurs during disaster adverse weather.

\[
(Failure \ Rate)_7 = \frac{r}{P+T} \left( \lambda_1^{11} \lambda_2^{11} T + \lambda_2^{11} \lambda_1^{11} T \right)
\]

The system failure rate due to two overlapping outages is therefore, given by

\[\lambda_{\text{SL}} = \sum_{i=1}^{2} (Failure \ Rate)_i\]

and

\[r_{\text{SL}} = \left( \frac{r_1 r_2}{r_1 + r_2} \right) \left( \frac{\sum_{i=1}^{2} (Failure \ Rate)_i}{\lambda_{\text{SL}}} \right) \left( \frac{r_1 r_2}{r_1 + r_2} + s \right) \left( \frac{\sum_{i=6}^{7} (Failure \ Rate)_i}{\lambda_{\text{SL}}} \right)\]

Similar equations can be written for three or more overlapping outages of components.

Using these equations, the effects of variation of component disaster adverse weather failure rate and disaster adverse weather period duration are shown in Figures 3.23 and 3.24. Figure 3.23 shows the effect of variation in the disaster adverse weather component failure rate and the average duration of the disaster adverse weather period on the failure rate of a two component parallel system. As is obvious, the system failure rate increases with an increase in the average duration of the disaster adverse weather period and the component disaster adverse weather failure rate.
Figure 3.23  Effects of Average Duration of Disaster Adverse Weather and Associated Component Failure Rate on Two Component Parallel System Outage Rate
Figure 3.24  Effects of Average Duration of Disaster Adverse Weather and Associated Component Failure Rate on Two Component Parallel System Outage Duration.
rate. Figure 3.24 shows the effect of the same two parameters on the average outage duration of a system of two parallel components. The average outage duration also increases with an increase in the average duration of disaster adverse weather and the associated component failure rate during this period.

The Markov approach can also be applied in the representation of a three state weather model. There are 12 possible states, under the assumptions noted above, for a system of two components. The results obtained by the approximate equations have been compared with those obtained by the Markov technique. The results are very close in all the cases studied.

3.10 Effect of Circuit Breaker Performance on Reliability

A circuit breaker is a relatively common element in a power system. Its primary purpose is to protect the system against overload in the case of a fault or other abnormal system operation. The failures of relays, instrument transformers and other associated auxiliary equipment are normally assigned to the circuit breaker. A breaker as such can fail in a number of possible ways e.g. it fails to operate on a fault, it fails to reclose, it gives a false operation, it itself becomes faulty etc. It is quite difficult to incorporate all possible circuit breaker failures in system reliability evaluation. In addition, it is also difficult to collect data for these failure modes.
Figure 3.25 Typical System Configurations

(a) A Two Component Parallel System
(b) A Three Component Parallel System
(c) A Two Load Point System
Figure 3.26 Effect of Circuit Breaker Ground Fault Rate on System Outage Rate
If a circuit breaker faults to ground, the supply to customers in the immediate vicinity of this breaker is interrupted until this breaker is switched out of the circuit. Under these conditions, a circuit breaker essentially acts as a series element in a parallel configuration of components. The effect of the circuit breaker ground fault rate on the systems given in Figure 3.25 is shown in Figure 3.26. It can be seen that as the circuit breaker ground fault rate increases, the system failure rate increases very rapidly and with very large ground fault rate values, the effect of bunching due to adverse weather failures may become insignificant. The inclusion of circuit breaker ground fault failures in reliability calculations will reduce the average outage durations experienced at various load points. This is because, in many cases, the supply can be restored to the customers by switching out the faulted circuit breaker and the switch out time is very small in comparison with the breaker repair time.

In this chapter, accurate equations have been developed for evaluating the permanent outage aspect of distribution schemes.Reliability predictions obtained without consideration of weather conditions can be considerably in error. The concept of forming an equivalent component as presented in the literature\(^{10}\) has been investigated and it has been concluded that the results obtained by this technique are not comparable with those obtained by a complete Markov approach. To avoid the formation of an equivalent component, a further extension of overlapping outage
formulae has been presented and the results are comparable with those obtained by a Markov approach. The failure mode and effect analysis technique is a very powerful method in determining system reliability indices. The equations given in this chapter apply not only to two or three component systems but to a system of any number of components. The primary consideration is the effect of the failure of a component or components on the load point. It has also been established that the assumption of exponentially distributed up and down times is not a constraint on the reliability prediction of long term system behaviour.
4. EVALUATION OF MAINTENANCE AND TEMPORARY OUTAGE CONDITIONS

4.1 Introduction

The nature of maintenance outages is basically different from that of component forced outages. Power system components must be taken out of service for periodic inspection and maintenance. Such an action is desirable to forestall by preventive methods, the occurrence of a future failure or malfunction and therefore maintenance is a dependent activity. The actual removal of a component is dependent upon the load level in the system and whether or not an outage already exists. It has been established from the operating experience that a major cause of double contingency outages is the occurrence of an outage at a time when another device has been taken out for maintenance.

The consideration of temporary outages in distribution schemes is very important because temporary interruptions can cause considerable irritation to the customers. In many industries involving continuous processes, a momentary outage can cause complete wastage of the product in process resulting in a heavy economic loss. In many overhead distribution schemes, the intensity of component temporary outages is governed by the environment. There is practically no data published on temporary outages in the existing literature and no techniques have been suggested for the evaluation of temporary interruptions. It should be noted that the distinction between temporary outages and permanent outages is not based on the duration of the outage but on the type of the disturbance. If a fault can be cleared
by the operation of a circuit breaker without it locking out or by automatic
reclosers or by repeater fuses, the resulting outage is regarded as a
temporary one. Such outages are generally of 2 to 3 minutes duration.)

4.2 A Technique For Maintenance Representation

In addition to the assumptions made for the evaluation of the
permanent outage condition, the following assumptions are required for
evaluating maintenance outages.

(1) No maintenance is carried out if there is some outage already existing
in a related portion of the system.

(ii) Maintenance occurs in normal weather and no maintenance is carried out
if adverse weather is expected to occur before the maintenance can be
completed.

(iii) Maintenance up-and-down times are exponentially-distributed.

The last assumption is not necessary for long term reliability
predictions. A different mathematical treatment of these outages is required
because of the dependent nature of the maintenance activity.

4.2.1 Component Permanent Outages Overlapping Component Maintenance Outages

(When a component has been taken out for maintenance, another component
or components may suffer a forced outage or outages causing interruption to
the system load points.) Two approaches can again be applied to evaluate this
mode of failure.

(i) The Markov approach

(ii) The approximate approach.
The Markov technique simply involves the extension of the state space diagram on the normal weather side to include the maintenance outage states.

Let

\[ \lambda_i = \text{The normal weather failure rate of component } i \]
\[ \lambda''_i = \text{The maintenance outage rate of component } i \]
\[ \mu_i = \text{The average repair rate of component } i = \frac{1}{r_i} \]
\[ \mu''_i = \text{The average maintenance rate of component } i = \frac{1}{r_i} \]

The state space diagrams including the maintenance states are shown in Figures 4.1 and 4.2 for systems of two and three components in parallel respectively. The state space diagram of two components involves 12 states and that of three components 28 states. The state space diagram for a four component parallel system has 64 states.

If, however, only normal weather is considered throughout, the state space diagram for a two component system has eight states and is shown in Figure 4.3. The probabilities of being in the various states are given by:

\[ P_1 = \frac{1}{A} \quad P_2 = \frac{\lambda_1}{A\mu_1} \quad P_3 = \frac{\lambda_2}{A\mu_2} \quad P_4 = \frac{\lambda''_1}{A\mu''_1} \]

\[ P_5 = \frac{\lambda''_2}{A\mu''_2} \quad P_6 = \frac{\lambda_1\lambda_2}{\mu_1\mu_2 A} \quad P_7 = \frac{\lambda_2\lambda''_1}{A(\mu''_1+\mu_2) \mu''_1} \]

\[ P_8 = \frac{\lambda_1\lambda''_2}{A(\mu''_1+\mu''_2) \mu''_2} \]
Figure 4.1 State Space Diagram For a Two Component Parallel System Including Maintenance States
WEATHER: NORMAL WEATHER

Parallel System
Figure 4.3 State Space Diagram For a Two Component Parallel System Excluding Weather Considerations
where
\[
\lambda = 1 + \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \frac{\lambda_1}{\mu_1} + \frac{\lambda_1^\prime}{\mu_2} + \frac{\lambda_1^\prime}{\mu_1^\prime \mu_2} + \frac{\lambda_2^\prime}{\mu_1^\prime \mu_2^\prime} + \frac{\lambda_1^\prime \lambda_2^\prime}{\mu_1^\prime \mu_2^\prime} \frac{1}{\mu_2^\prime} \approx 1
\]

and \( P_j \) = The probability of being in \( j \)th state in the system of Figure 4.3.

Thus the frequency of encountering a state in which component 1 has been taken out for maintenance and a permanent outage of component 2 overlaps it, is given by:

\[
P_7 (\mu_1^\prime + \mu_2) \]

\[
= \lambda_2 \lambda_1^\prime r_1^\prime
\]

Similarly, the frequency of encountering a state in which component 2 has been taken out for maintenance and a permanent outage of component 1 overlaps it, is given by:

\[
P_8 (\mu_1 + \mu_2^\prime)
\]

\[
= \lambda_1 \lambda_2^\prime r_2^\prime
\]

Therefore, the maintenance outage rate of a system of two parallel components is:

\[
\lambda_{ML}^\prime = \lambda_1 \lambda_2 r_1^\prime + \lambda_2 \lambda_1^\prime r_2^\prime
\]

The state space diagram grows very rapidly as the number of system components increase and it is very difficult to apply the complete Markov approach to larger systems because of computer storage and rounding error limitations. This, therefore, necessitates the formation of some approximate formulae to evaluate the reliability indices.
According to the approximate approach, the maintenance outage rate of a system of \( n \) series components is given by:

\[
\lambda_{\text{SE}}^{\text{''}} = \sum_{1=1}^{n} \lambda_1^{\text{''}}
\]

and the expected value of series system down time due to maintenance outages is

\[
x_{\text{SE}}^{\text{''}} = \sum_{1=1}^{n} \frac{\lambda_1^{\text{''}} x_1^{\text{''}}}{\lambda_{\text{SE}}^{\text{''}}}
\]

The failure rate resulting from the permanent outage of a component overlapping the maintenance outage of a component is

\[
\lambda_{\text{ML}}^{\text{''}} = \lambda_1^{\text{''}} (\lambda_2 x_1^{\text{''}})
\]

where \( \lambda_1^{\text{''}} \) = The maintenance outage rate of component 1

and \( \lambda_2 x_1^{\text{''}} = 1 - e^{-\lambda_2 x_1^{\text{''}}} \) = The probability that the second component fails during the maintenance period of component 1.

Therefore, for a system of two parallel components, the maintenance outage rate is:

\[
\lambda_{\text{ML}}^{\text{''}} = \lambda_1^{\text{''}} \lambda_2 x_1^{\text{''}} + \lambda_2^{\text{''}} \lambda_1 x_2^{\text{''}}
\]

This is the same expression as equation (4.1)

The expected value of down time for a system of two parallel components due to maintenance outages is given by:

\[
x_{\text{ML}}^{\text{''}} = \frac{\lambda_1^{\text{''}} \lambda_2 x_1^{\text{''}}}{\lambda_{\text{ML}}^{\text{''}}} \left( \frac{x_2^{\text{''}}}{r_2^{\text{''}} x_1^{\text{''}}} \right) + \frac{\lambda_2^{\text{''}} \lambda_1 x_2^{\text{''}}}{\lambda_{\text{ML}}^{\text{''}}} \left( \frac{x_1^{\text{''}} x_2^{\text{''}}}{r_1^{\text{''}} x_2^{\text{''}}} \right) \tag{4.2}
\]

The first factor of the first term is the fraction of system outages involving component maintenance outages in which component 2 failed while
component 1 was out for maintenance. The second factor of this term gives the expected value of the system down time given that component 2 fails while component 1 is out for maintenance. The second term follows the same argument with components 1 and 2 interchanged.

The results predicted by equations 4.1 and 4.2 have been compared with those obtained by a Markov approach in Tables 4.1 and 4.2. Table 4.1 indicates the results when only the normal weather is considered and Table 4.2 depicts the results for normal and adverse weather considerations.

TABLE 4.1

RELIABILITY INDICES DUE TO COMPONENT MAINTENANCE FOR A TWO COMPONENT PARALLEL SYSTEM (NORMAL WEATHER ONLY)

<table>
<thead>
<tr>
<th>Maintenance parameters</th>
<th>Approximate</th>
<th>Markov</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Failure rate</td>
<td>Average Duration</td>
</tr>
<tr>
<td></td>
<td>f/yr</td>
<td>Hours</td>
</tr>
<tr>
<td>$\lambda_1 = \lambda_2 = 0.5$ failures/year</td>
<td>$r_1 = r_2 = 7.5$ Hours</td>
<td></td>
</tr>
<tr>
<td>$\lambda_1 = 3.0$ 0/yr. $\alpha$ 5.84 Hrs.</td>
<td>0.00201</td>
<td>3.276</td>
</tr>
<tr>
<td>$\lambda_1 = 5.0$ 0/yr. $\alpha$ 5.84 Hrs.</td>
<td>0.00331</td>
<td>3.276</td>
</tr>
<tr>
<td>$\lambda_1 = 3.0$ 0/yr. $\alpha$ 8.76 Hrs.</td>
<td>0.00300</td>
<td>4.038</td>
</tr>
</tbody>
</table>
TABLE 4.2

RELIABILITY INDICES DUE TO COMPONENT MAINTENANCE
FOR A TWO COMPONENT PARALLEL SYSTEM
(TWO WEATHER STATES)

\[
\begin{align*}
\lambda_{av1} & = \lambda_{av2} = 0.5 \, \text{f/yr} \\
\lambda_1'' & = \lambda_2'' = 3.0 \, \text{f/yr} \\
N & = 200.0 \text{ Hours} \\
S & = 1.5 \text{ Hours}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Percentage of component failures adverse weather</th>
<th>Approximate</th>
<th>Markov</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Failure rate</td>
<td>Av. duration</td>
</tr>
<tr>
<td></td>
<td>failure/year</td>
<td>Hours</td>
</tr>
<tr>
<td>0</td>
<td>0.00201</td>
<td>3.280</td>
</tr>
<tr>
<td>10</td>
<td>0.00181</td>
<td>3.280</td>
</tr>
<tr>
<td>20</td>
<td>0.00161</td>
<td>3.280</td>
</tr>
<tr>
<td>30</td>
<td>0.00143</td>
<td>3.280</td>
</tr>
<tr>
<td>40</td>
<td>0.00121</td>
<td>3.280</td>
</tr>
<tr>
<td>50</td>
<td>0.00100</td>
<td>3.280</td>
</tr>
<tr>
<td>60</td>
<td>0.00080</td>
<td>3.280</td>
</tr>
<tr>
<td>70</td>
<td>0.00060</td>
<td>3.280</td>
</tr>
<tr>
<td>80</td>
<td>0.00040</td>
<td>3.280</td>
</tr>
<tr>
<td>90</td>
<td>0.00020</td>
<td>3.280</td>
</tr>
<tr>
<td>100</td>
<td>0.00000</td>
<td>—</td>
</tr>
</tbody>
</table>

It can be seen from Tables 4.1 and 4.2 that the results obtained by equations 4.1 and 4.2 are quite close to those obtained by the Markov approach.

For consideration of more than two overlapping outages in which one component is out for maintenance and two or more components are forced out, the technique of equivalent component formation has been suggested in the literature(5). According to this approach, the contribution to the system failure rate due to the maintenance of a component and two component forced outages overlapping it for a three component parallel system is given by:
\[ \lambda''_\text{ML} = \lambda''_1 \lambda_{2-3} r''_1 + \lambda''_2 \lambda_{1-3} r''_2 + \lambda''_3 \lambda_{1-2} r''_3 \]  
(4.3)

where \( \lambda''_1 \) = The maintenance outage rate of component 1
\( \lambda_{2-3} \) = The overall failure rate of components 2 and 3
\( \lambda_{2-3} r''_1 \) = The probability that components 2 and 3 are forced out during the maintenance of component 1.

Similar arguments apply to the remaining terms in equation 4.3.

And the average outage duration is given by:

\[ r''_\text{ML} = \frac{\lambda''_1 \lambda_{2-3} r''_1}{\lambda''_\text{ML}} + \frac{\lambda''_2 \lambda_{1-3} r''_2}{\lambda''_\text{ML}} + \frac{\lambda''_3 \lambda_{1-2} r''_3}{\lambda''_\text{ML}} \]  
(4.4)

where \( r_{i-j} \) = The expected overlapping outage duration of components i and j.

The reasoning explained in the formulation of equation 4.3 also applies to equation 4.4.

The results obtained by equations 4.3 and 4.4 are compared in Tables 4.3 and 4.4 with those obtained by the Markov approach. Table 4.3 gives the results when only normal weather is considered. Table 4.4 shows the results for normal and adverse weather considerations.

### TABLE 4.3

**RELIABILITY INDICES DUE TO COMPONENT MAINTENANCE**

**FOR A THREE COMPONENT PARALLEL SYSTEM**

(NORMAL WEATHER ONLY)

\( \lambda_1 = \lambda_2 = \lambda_3 = 0.5 \text{ f/year} \quad \tau_1 = \tau_2 = \tau_3 = 7.5 \text{ Hours} \)

<table>
<thead>
<tr>
<th>Maintenance Parameters</th>
<th>Approximate Failure rate F/year</th>
<th>Approximate Av. Duration Hours</th>
<th>Markov Failure rate F/year</th>
<th>Markov Av. Duration Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda'^{''}_1 = 3.0 \text{ 0/yr.} ), ( \tau = 5.84 \text{ Hrs.} )</td>
<td>0.000000256</td>
<td>2.280</td>
<td>0.000000111</td>
<td>2.280</td>
</tr>
<tr>
<td>( \lambda'^{''}_2 = 5.0 \text{ 0/yr.} ), ( \tau = 5.84 \text{ Hrs.} )</td>
<td>0.00000428</td>
<td>2.280</td>
<td>0.00000185</td>
<td>2.280</td>
</tr>
<tr>
<td>( \lambda'^{''}_3 = 3.0 \text{ 0/yr.} ), ( \tau = 8.76 \text{ Hrs.} )</td>
<td>0.00000385</td>
<td>2.63</td>
<td>0.000000205</td>
<td>2.63</td>
</tr>
</tbody>
</table>
TABLE 4.4

RELIABILITY INDICES DUE TO COMPONENT MAINTENANCE
FOR A THREE COMPONENT PARALLEL SYSTEM
(TWO WEATHER STATES)

\[
\lambda_{av1} = \lambda_{av2} = \lambda_{av3} = 0.5 \text{ f/yr} \quad r_1 = r_2 = r_3 = 7.5 \text{ Hours}
\]
\[
\lambda'' = \lambda_2 = \lambda_3 = 3.0 \text{ f/yr} \quad r''_1 = r''_2 = r''_3 = 5.84 \text{ Hours}
\]

\[N = 200,0 \text{ Hours} \quad S = 1.5 \text{ Hours}\]

<table>
<thead>
<tr>
<th>Percentage of component failures adverse weather</th>
<th>Approximate failure rate failure/10⁶year</th>
<th>Approximate Av. Duration Failure rate 10⁶year</th>
<th>Markov Failure rate 10⁶year</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.600</td>
<td>2.280</td>
<td>1.114</td>
</tr>
<tr>
<td>10</td>
<td>2.400</td>
<td>2.280</td>
<td>0.924</td>
</tr>
<tr>
<td>20</td>
<td>2.086</td>
<td>2.280</td>
<td>0.730</td>
</tr>
<tr>
<td>30</td>
<td>1.820</td>
<td>2.280</td>
<td>0.559</td>
</tr>
<tr>
<td>40</td>
<td>1.570</td>
<td>2.280</td>
<td>0.411</td>
</tr>
<tr>
<td>50</td>
<td>1.300</td>
<td>2.280</td>
<td>0.285</td>
</tr>
<tr>
<td>60</td>
<td>1.040</td>
<td>2.280</td>
<td>0.183</td>
</tr>
<tr>
<td>70</td>
<td>0.720</td>
<td>2.280</td>
<td>0.103</td>
</tr>
<tr>
<td>80</td>
<td>0.520</td>
<td>2.280</td>
<td>0.046</td>
</tr>
<tr>
<td>90</td>
<td>0.200</td>
<td>2.280</td>
<td>0.011</td>
</tr>
<tr>
<td>100</td>
<td>0.000</td>
<td>2.280</td>
<td>0.000</td>
</tr>
</tbody>
</table>

It is evident from Tables 4.3 and 4.4 that the results obtained by the use of equations 4.3 and 4.4 are considerably different from those obtained by the Markov approach in both cases.

The development of some modified equations for the representation of maintenance which give more accurate results is presented in the following pages.

The outage rate resulting from the maintenance period of a component being overlapped by two permanent outages is given by:

\[
\]
\[ \lambda''_{NL} = \lambda''_1 (\lambda_2 r''_1) \left( \lambda_3 \frac{r''_1 r''_2}{r''_1 + r''_2} \right) + \lambda''_1 (\lambda_3 r''_1) \left( \lambda_2 \frac{r''_1 r''_3}{r''_1 + r''_3} \right) \]  \hspace{1cm} (4.5)

\[ \lambda''_1 = \text{The maintenance outage rate of the component 1.} \]

\[ \lambda''_2 \approx \text{The probability that component 2 suffers a forced outage during} \]

\[ \text{the maintenance period of component 1} \]

\[ \lambda''_3 \approx \text{The probability that component 3 suffers a forced outage during} \]

\[ \text{the maintenance of component 1 and repair of component 2.} \]

The second term in equation 4.5 follows the same argument but with components 2 and 3 interchanged.

The expected outage duration associated with this outage rate is

\[ r''_{NL} = \frac{r''_1 r''_2 r''_3}{r''_1 r''_2 + r''_1 r''_3 + r''_2 r''_3} \]  \hspace{1cm} (4.6)

The contribution to the system failure rate due to the maintenance of a component and two forced outages overlapping it for a three component parallel system is therefore given by:

\[ \lambda''''_{NL} = \lambda''''_1 \lambda_2 r''''_1 \lambda_3 \frac{r''''_1 r''''_2}{r''''_1 + r''''_2} + \lambda''''_1 \lambda_3 r''''_1 \lambda_2 \frac{r''''_1 r''''_3}{r''''_1 + r''''_3} \]

\[ + \lambda''''_2 \lambda_1 r''''_2 \lambda_3 \frac{r''''_2 r''''_1}{r''''_2 + r''''_1} + \lambda''''_2 \lambda_3 r''''_2 \lambda_1 \frac{r''''_2 r''''_3}{r''''_2 + r''''_3} \]

\[ + \lambda''''_3 \lambda_1 r''''_3 \lambda_2 \frac{r''''_3 r''''_1}{r''''_3 + r''''_1} + \lambda''''_3 \lambda_2 r''''_3 \lambda_1 \frac{r''''_3 r''''_2}{r''''_3 + r''''_2} \]

The expected outage duration of \( \lambda''''_{NL} \) is equal to the weighted outage duration of each of the terms in the above equation. If \( \lambda_1 \) represents the frequency of outage of one of the above terms and \( r_1 \) its associated expected outage duration, then the expected outage duration of \( \lambda''''_{NL} \) is given by:
Using equations 4.1, 4.2, 4.5 and 4.6, the reliability indices of simple configurations of two and three components in parallel have been obtained and the results have been compared with those obtained by a Markov approach. There is very good agreement between the two sets of results. The contributions to the system failure rate due to the component maintenance for these simple configurations are plotted against the percentage of component failures during adverse weather periods in Figure 4.4. The effect of variation of component maintenance parameters has also been illustrated. It is clear that as the percentage of component failures during adverse weather increases, the contribution of maintenance to the system failure rate decreases. This is quite obvious as the component failures during adverse weather periods increase, the likelihood of component permanent outages overlapping the component maintenance outages, which occur in normal weather, decreases.

Carrying out the expansion of the previous equations for a case in which the maintenance outage of a component is overlapped by three forced outages, the contribution to the system failure rate is given by:

\[
\lambda''_{ML} = \lambda_1'(\lambda_2'\lambda_3') \left( \lambda_4' \frac{r_1' r_2' r_3'}{r_1' r_2' + r_1' r_3' + r_2' r_3'} \right) + \lambda_1' \lambda_3' \left( \lambda_2' r_3' \frac{r_1' r_2'}{r_1' r_2' + r_1' r_3' + r_2' r_3'} \right) + \lambda_1' \lambda_2' \lambda_3' \left( \lambda_4' \frac{r_1' r_2' r_3'}{r_1' r_2' + r_1' r_3' + r_2' r_3'} \right)
\]
\[ + \lambda_1^n (\gamma_2 r_2^n) \left( \lambda_3 \frac{r_1 r_2 r_4}{r_1 + r_3} \right) + \lambda_1^n (\lambda_4 r_2^n) \left( \lambda_3 \frac{r_1 r_2 r_4}{r_1 + r_4} \right) + \lambda_1^n (\lambda_4 r_1^n) \left( \lambda_2 \frac{r_1 r_3 r_4}{r_1 + r_2} \right) + \lambda_1^n (\lambda_2 r_1^n) \left( \lambda_3 \frac{r_1 r_2 r_4}{r_1 + r_3} \right) \]

\[ \lambda_1^n (\lambda_2 r_1^n) \left( \lambda_3 \frac{r_1 r_2 r_4}{r_1 + r_3} \right) + \lambda_1^n (\lambda_4 r_2^n) \left( \lambda_3 \frac{r_1 r_2 r_4}{r_1 + r_4} \right) + \lambda_1^n (\lambda_2 r_1^n) \left( \lambda_3 \frac{r_1 r_2 r_4}{r_1 + r_3} \right) \]

\[ \lambda_2 r_1^n \sim \text{The probability that component 2 is forced out during the maintenance period of component 1.} \]

\[ \lambda_3 \frac{r_1 r_2}{r_1 + r_2} \sim \text{The probability that component 3 is forced out during the overlapping period of maintenance of component 1 and the repair of component 2.} \]

\[ \lambda_4 \frac{r_1 r_2 r_3}{r_1 r_2 + r_1 r_3 + r_2 r_3} \sim \text{The probability that component 4 is forced out during the overlapping period of maintenance of component 1 and the repair of components 2 and 3.} \]

The remaining terms in equation 4.7 follow similar reasoning. The average outage duration associated with \( x_{ML} ^n \) in equation 4.7 is given by:

\[ x_{ML} ^n = \frac{r_1 r_2 r_3 r_4}{r_1 r_2 + r_1 r_3 + r_2 r_3 + r_1 r_2 r_3 + r_2 r_3 r_4} \]  

(4.8)

The results obtained for a four component parallel system have been compared with those obtained by a Markov approach in Table 4.5. It can be seen that the results predicted by the above equations agree closely with the results obtained by the Markov approach (which requires the solution of 64 simultaneous linear equations).
Figure 4.4 Contribution to System Failure Rate Due to Component Maintenance
<table>
<thead>
<tr>
<th>P</th>
<th>Maint. rate r/year</th>
<th>Approx. Failure rate f/10⁹ yr.</th>
<th>Avg. duration Hrs.</th>
<th>Markov Failure rate f/10⁹ yr.</th>
<th>Avg. duration Hrs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1000</td>
<td>0.927</td>
<td>1.945</td>
<td>0.910</td>
<td>1.945</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>0.437</td>
<td>1.750</td>
<td>0.430</td>
<td>1.750</td>
</tr>
<tr>
<td>30</td>
<td>1000</td>
<td>0.436</td>
<td>1.945</td>
<td>0.430</td>
<td>1.945</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>0.205</td>
<td>1.750</td>
<td>0.200</td>
<td>1.750</td>
</tr>
<tr>
<td>50</td>
<td>1000</td>
<td>0.159</td>
<td>1.945</td>
<td>0.160</td>
<td>1.945</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>0.075</td>
<td>1.750</td>
<td>0.070</td>
<td>1.750</td>
</tr>
<tr>
<td>70</td>
<td>1000</td>
<td>0.081</td>
<td>1.945</td>
<td>0.080</td>
<td>1.945</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>0.018</td>
<td>1.750</td>
<td>0.020</td>
<td>1.750</td>
</tr>
<tr>
<td>90</td>
<td>1000</td>
<td>0.010</td>
<td>1.945</td>
<td>0.010</td>
<td>1.945</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>0.0005</td>
<td>1.750</td>
<td>0.0004</td>
<td>1.750</td>
</tr>
</tbody>
</table>

P = Percentage of component failures during adverse weather
This clearly shows that the equations described for maintenance can be extended to any number of components with very little loss of accuracy.

The error in the formation of an equivalent component can be seen from a consideration of equations 4.3 and 4.5. From equation 4.3, the contribution to the system failure rate due to an event in which component 1 is on maintenance and components 2 and 3 suffer forced outages during the maintenance period is given by:

$$\lambda^{\prime\prime}_{\text{ML}} = \lambda_1^{\prime\prime} \lambda_2 \lambda_3 r_1^{\prime\prime}$$

$$= \lambda_1^{\prime\prime} \lambda_2 \lambda_3 (r_2 + r_3) r_1^{\prime\prime}$$

If

$$r_1 = r_2 = r_1^{\prime\prime} = r_2^{\prime\prime} = r$$

$$\lambda^{\prime\prime}_{\text{ML}} = 2 \lambda_1^{\prime\prime} \lambda_2 \lambda_3 r^2$$

From equation 4.5

$$\lambda^{\prime\prime}_{\text{ML}} = \lambda_1^{\prime\prime} \lambda_2 \lambda_3 r \frac{r}{2} + \lambda_1^{\prime\prime} \lambda_2 \lambda_3 r \frac{r}{2}$$

$$= \lambda_1^{\prime\prime} \lambda_2 \lambda_3 r^2$$

(4.9b)

It is therefore noted that equation 4.9a gives double the value given by the correct equation 4.9b. This arises because the equivalent component does not represent the true performance in cases where component dependent behaviour is present.

To deal with complex configurations, the technique of failure modes and effect analysis is applied. Consider the distribution system of Figure 3.7. The various failure modes at load points 1 and 2 are as follows:
Load point 1

- Line sections 1, 2 and 3 out
- Line sections 1, 2 and 4 out
- Line sections 1, 2, 3 and 4 out

Load point 2

- Line sections 3 and 4 out
- Line sections 1, 2 and 4 out
- Line sections 1, 2, 3 and 4 out

The failure mode in which all the components are out makes very little contribution to the system failure rate because it is a fourth order contingency and can be ignored.

Using the above failure modes, the permanent outage and maintenance outage contribution to the system failure rate has been evaluated by the equations described in this Chapter and in Chapter 3. The results have been compared with a Markov approach involving 64 states. These results are given in Table 4.6. It, further, shows that the failure mode and effect analysis technique has a very important role in evaluating the system performance using the approximate equations.

4.2.2 Component Temporary Outages Overlapping Component Maintenance Outages

As previously noted in section 4.1, the evaluation of temporary interruptions in distribution schemes is very important. (Temporary outages are quite liable to occur during component maintenance periods due to increased activity in the component vicinity. The mathematical formulation of temporary outages proceeds as follows:
Let

\[ \lambda_{IT} = \text{The temporary outage rate of component } i. \]

The contribution to the system temporary outage rate due to a temporary outage of component 1 overlapping the maintenance outage period of component 2 is given by:

\[ \lambda_{iL} = \lambda_2 \lambda_{IT} r_2 \]  \hspace{1cm} (4.10)

where

\[ \lambda_{IT} = \text{The probability that component 1 suffers a temporary outage during the maintenance period of component 2.} \]

For a system of two parallel components, therefore

\[ \lambda_{iL} = \lambda_2 \lambda_{IT} r_1 + \lambda_2 \lambda_{IT} r_2 \]  \hspace{1cm} (4.10a)

Similarly for a three component parallel system,

\[ \lambda_{iL} = \lambda_1 \lambda_2 r_1 + \lambda_3 \lambda_2 r_1 + \lambda_2 \lambda_3 r_2 + \lambda_1 \lambda_3 r_3 \]

\[ + \lambda_2 \lambda_1 r_2 + \lambda_3 \lambda_2 r_2 + \lambda_1 \lambda_2 r_3 \]

\[ + \lambda_3 \lambda_1 r_3 + \lambda_2 \lambda_3 r_3 + \lambda_1 \lambda_3 r_2 \]

If temporary outages are separated into normal and adverse weather failures, then the \( \lambda_T \) values in the above equations represent the normal weather temporary outage rate of the components.
<table>
<thead>
<tr>
<th>Percentage of component failures during adverse weather</th>
<th>Approximate Technique</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F.R. ( f/10^6) yr</td>
<td>A.D. Hrs.</td>
<td>F.R. ( f/10^6) yr</td>
<td>A.D. Hrs.</td>
<td>F.R. ( f/10^6) yr</td>
<td>A.D. Hrs.</td>
<td>F.R. ( f/10^6) yr</td>
<td>A.D. Hrs.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>L.P. 1</td>
<td>0.558</td>
<td>2.50</td>
<td>2.186</td>
<td>2.30</td>
<td>0.555</td>
<td>2.52</td>
<td>2.140</td>
<td>2.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L.P. 2</td>
<td>432.0</td>
<td>2.08</td>
<td>2018.3</td>
<td>2.86</td>
<td>429.0</td>
<td>2.10</td>
<td>1982.0</td>
<td>2.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>L.P. 1</td>
<td>8.257</td>
<td>3.581</td>
<td>1.399</td>
<td>2.30</td>
<td>8.429</td>
<td>3.611</td>
<td>1.374</td>
<td>2.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L.P. 2</td>
<td>886.47</td>
<td>2.996</td>
<td>1614.12</td>
<td>2.86</td>
<td>885.8</td>
<td>3.021</td>
<td>1585.0</td>
<td>2.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>L.P. 1</td>
<td>55.38</td>
<td>3.663</td>
<td>0.787</td>
<td>2.30</td>
<td>55.759</td>
<td>3.695</td>
<td>0.773</td>
<td>2.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L.P. 2</td>
<td>2285.7</td>
<td>3.410</td>
<td>1210.19</td>
<td>2.86</td>
<td>2294.3</td>
<td>3.438</td>
<td>1198.0</td>
<td>2.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>L.P. 1</td>
<td>177.96</td>
<td>3.680</td>
<td>0.350</td>
<td>2.30</td>
<td>176.142</td>
<td>3.717</td>
<td>0.343</td>
<td>2.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L.P. 2</td>
<td>4683.7</td>
<td>3.524</td>
<td>806.53</td>
<td>2.86</td>
<td>4533.15</td>
<td>3.558</td>
<td>792.596</td>
<td>2.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>L.P. 1</td>
<td>412.04</td>
<td>3.687</td>
<td>0.0870</td>
<td>2.30</td>
<td>400.69</td>
<td>3.728</td>
<td>0.0859</td>
<td>2.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L.P. 2</td>
<td>8134.3</td>
<td>3.563</td>
<td>403.13</td>
<td>2.86</td>
<td>7719.23</td>
<td>3.604</td>
<td>396.242</td>
<td>2.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>L.P. 1</td>
<td>752.25</td>
<td>3.690</td>
<td>0.0000</td>
<td>—</td>
<td>758.21</td>
<td>3.736</td>
<td>0.0000</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L.P. 2</td>
<td>11812.2</td>
<td>3.582</td>
<td>0.0000</td>
<td>—</td>
<td>11808.42</td>
<td>3.626</td>
<td>0.0000</td>
<td>—</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**P.O.C.** = Permanent outage Contribution

**L.P.** = Load Point

**TABLE 4.6**

RELIABILITY INDICES FOR THE SYSTEM IN FIG. 3.5

\[
\begin{align*}
\lambda_{av1} &= \lambda_{av2} = \lambda_{av3} = \lambda_{av4} = 0.5 \text{ f/yr.} \\
\tau_1 &= \tau_2 = 7.5 \text{ Hours} \quad \tau_3 = 2.5 \text{ Hours} \quad \tau_4 = 12.5 \text{ Hours} \\
\lambda &= 3.0 \text{ f/yr.} \\
\tau &= 5.84 \text{ Hours}
\end{align*}
\]

No Repair During Adverse Weather
4.3 Component Temporary Outages Overlapping Component Permanent Outages

There is a possibility of a temporary outage occurring during the period when another component has been permanently forced out. Such an event can result in a service interruption to certain load points in the system. The probability of occurrence of a permanent outage during the period when another component is temporarily out is virtually negligible as the duration associated with such an outage is very small. Such an event is, therefore, discounted and is not considered in the reliability calculations.

For a series system of $n$ components, the temporary outage rate of the system is given by:

$$\lambda_{TS} = \sum_{i=1}^{n} \lambda_{iT}$$

If component temporary outages are not separated into normal and adverse weather failures, the outage rate of a system of two components in parallel assuming repair occurs in adverse weather is given by:

$$\lambda_{TL} = \frac{N}{N+S} \left\{ \lambda_1 \lambda_2 T \cdot \tau_1 + \lambda_2 \lambda_{1T} \cdot \tau_2 \right\} + \frac{S}{N+S} \left\{ \lambda_1^* \lambda_2 T \cdot \tau_1 + \lambda_2^* \lambda_{1T} \cdot \tau_2 \right\}$$

$$= \left\{ \lambda_1 \frac{N}{N+S} + \lambda_1^* \frac{S}{N+S} \right\} \lambda_2 T \cdot \tau_1 + \left\{ \lambda_2 \frac{N}{N+S} + \lambda_2^* \frac{S}{N+S} \right\} \lambda_{1T} \cdot \tau_2$$

$$= \lambda_{av1} \lambda_2 T \cdot \tau_1 + \lambda_{av2} \lambda_{1T} \cdot \tau_2$$

(4.11)

For a system of three components in parallel, the failure rate of the system due to component temporary outages is given by:

$$\lambda_{TL} = \lambda_{1-2} \lambda_3 T \cdot \tau_{1-2} + \lambda_{1-3} \lambda_2 T \cdot \tau_{1-3} + \lambda_{2-3} \lambda_{1T} \cdot \tau_{2-3}$$

(4.12)
where

\[ \lambda_{1-j} = \text{The overlapping outage rate of components } i \text{ and } j \]
\[ r_{1-j} = \text{The overlapping outage duration of components } i \text{ and } j \]
\[ \lambda_3 T r_{1-2} = \text{The probability that component } 3 \text{ is temporarily out during} \]
\[ \text{the overlapping outage period of components } 1 \text{ and } 2 \]

The results obtained from equations 4.11 and 4.12 are illustrated in Figure 4.5. For a two component parallel system, the contribution to the system failure rate due to component temporary outages does not vary with the percentage of component failures during adverse weather. This is evident from equation 4.11 where average annual forced outage rates of components are involved. For a three component parallel system, however, the terms \( \lambda_{1-j} \) are functions of component failure rates during adverse weather periods and therefore, the temporary outage rate of such a system varies with component failures during adverse weather periods.

Sometimes it becomes necessary to separate out the temporary outages into normal and adverse weather failures. (In many cases all component temporary outages occur during adverse weather periods). To illustrate this aspect, equations of the form described in sections 3.4 and 3.5 can be written. For a two component parallel system:

\[
\lambda_{TL} = \frac{N}{N_s} \left\{ \lambda_1 \lambda_2 T r_1 + \lambda_2 \lambda_1 T r_2 + \frac{S}{N} \left( \lambda_1' \lambda_2 T r_1 + \lambda_2' \lambda_1 T r_2 \right) \right. \\
\left. + \frac{S}{N} \left( \lambda_1 \lambda_2' T r_1 + \lambda_2 \lambda_1' T r_2 \right) + \frac{S}{N} \left( \lambda_1' \lambda_2 T s + \lambda_2' \lambda_1 T s \right) \right\} \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \heticse
Figure 4.5 System Temporary Outage Rate Due to Component Temporary Outages
Figure 4.6: 22-State Model For a Two Component Parallel System
where
\[ \lambda_{1T} = \text{The normal weather outage rate of component 1} \]
\[ \lambda_{1T} = \text{The adverse weather outage rate of component 1} \]

Equation 4.13 assumes no repair during adverse weather. Similar equations can be written for larger number of components. The accuracy of equations 4.10a and 4.13 have been tested by constructing a 22 state model for a two component system as shown in Figure 4.6. The results obtained are presented in Table 4.7.

<table>
<thead>
<tr>
<th>Percentage of component failures during adverse weather</th>
<th>Approx. A</th>
<th>Approx. B</th>
<th>Markov A</th>
<th>Markov B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00258</td>
<td>0.01210</td>
<td>0.00259</td>
<td>0.01200</td>
</tr>
<tr>
<td>10</td>
<td>0.00325</td>
<td>0.01088</td>
<td>0.00326</td>
<td>0.01074</td>
</tr>
<tr>
<td>20</td>
<td>0.00524</td>
<td>0.00967</td>
<td>0.00530</td>
<td>0.00955</td>
</tr>
<tr>
<td>30</td>
<td>0.00856</td>
<td>0.00846</td>
<td>0.00868</td>
<td>0.00836</td>
</tr>
<tr>
<td>40</td>
<td>0.01321</td>
<td>0.00725</td>
<td>0.01340</td>
<td>0.00716</td>
</tr>
<tr>
<td>50</td>
<td>0.01919</td>
<td>0.00604</td>
<td>0.01943</td>
<td>0.00597</td>
</tr>
<tr>
<td>60</td>
<td>0.02649</td>
<td>0.00483</td>
<td>0.02676</td>
<td>0.00477</td>
</tr>
<tr>
<td>70</td>
<td>0.03513</td>
<td>0.00362</td>
<td>0.03538</td>
<td>0.00358</td>
</tr>
<tr>
<td>80</td>
<td>0.04509</td>
<td>0.00241</td>
<td>0.04528</td>
<td>0.00238</td>
</tr>
<tr>
<td>90</td>
<td>0.05638</td>
<td>0.00121</td>
<td>0.05644</td>
<td>0.00119</td>
</tr>
<tr>
<td>100</td>
<td>0.06875</td>
<td>0.00000</td>
<td>0.06885</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

A - Temporary outages overlapping the permanent outages. failures/year
B - Temporary outages overlapping the maintenance outages. failures/year
It is seen from the results in Table 4.7 that the approximate equations 4.10a and 4.13 give results which are fairly close to those obtained by the state space diagramming technique.

4.4 System Studies

Using the equations presented in this chapter and those in Chapter 3, any system can be studied from a reliability viewpoint in terms of the frequency and duration of outages at the various load points. This approach is based on the recognition of failure modes and their cause effect relationships. The simple hypothetical distribution system shown in Figure 4.7 is utilized as an example. The various component and weather parameters are listed in Table 4.8.

The events that can result in load point interruptions are listed in Table 4.9 for all the load points in the system. The contributions to the load point outage rate and average outage duration due to each of the failure events have also been indicated. As can be seen, this method of analysis can clearly point out various weak links in the system and assist in carrying out further improvements in reliability.

Table 4.10 lists the contribution to the load point failure rates and outage durations if the system were underground with the same component parameters. No maintenance and temporary outages are considered for the underground system.
Figure 4.7  A Hypothetical Distribution Scheme
<table>
<thead>
<tr>
<th>Component</th>
<th>Normal weather outage rate f/yr</th>
<th>Adverse weather outage rate f/yr</th>
<th>Outage Duration Hours</th>
<th>Maintenance outage rate o/yr</th>
<th>Maintenance outage Duration Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Permanent Temporary</td>
<td>Permanent Temporary</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transformer</td>
<td>0.02</td>
<td>0.02</td>
<td>62.0</td>
<td>1.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Distributor #1</td>
<td>0.50</td>
<td>1.0</td>
<td>29.98</td>
<td>10.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Distributor #2</td>
<td>0.50</td>
<td>2.0</td>
<td>25.0</td>
<td>20.0</td>
<td>8.0</td>
</tr>
<tr>
<td>Distributor #3</td>
<td>0.50</td>
<td>2.0</td>
<td>25.0</td>
<td>20.0</td>
<td>8.0</td>
</tr>
<tr>
<td>Distributor #4</td>
<td>1.0</td>
<td>2.0</td>
<td>50.0</td>
<td>20.0</td>
<td>12.0</td>
</tr>
<tr>
<td>Distributor #5</td>
<td>0.60</td>
<td>2.0</td>
<td>30.0</td>
<td>20.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Distributor #6</td>
<td>0.30</td>
<td>1.0</td>
<td>15.0</td>
<td>10.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Distributor #7</td>
<td>0.58</td>
<td>1.0</td>
<td>29.98</td>
<td>10.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Distributor #8</td>
<td>0.58</td>
<td>1.0</td>
<td>29.98</td>
<td>10.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Average duration of a normal weather period = 200.0 Hours
Average duration of an adverse weather period = 1.50 Hours
No repair during adverse weather periods.
Load points 1, 7 and 8 are 100% reliable.
### Table 4.9

**RELIABILITY ANALYSIS OF SYSTEM OF FIG. 4.7**

<table>
<thead>
<tr>
<th>Reliability Indices at load point 2</th>
<th>Continuity</th>
<th>Permanent Outages $f/yr$</th>
<th>Temporary Outages $f/yr$</th>
<th>Duration of Permanent Outage $10^{-3}$ Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent and temporary interruptions resulting from component permanent and temporary outages.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line sections 1 and 4 out.</td>
<td>0.0037200</td>
<td>0.0011600</td>
<td></td>
<td>0.469</td>
</tr>
<tr>
<td>Line sections 1 and 5 out.</td>
<td>0.0026600</td>
<td>0.0012300</td>
<td></td>
<td>0.372</td>
</tr>
<tr>
<td>Line sections 1, 2 and 3 out.</td>
<td>0.0000283</td>
<td>0.0000256</td>
<td></td>
<td>0.391</td>
</tr>
<tr>
<td>Line sections 1, 6 and 8 out.</td>
<td>0.00001935</td>
<td>0.0000076</td>
<td></td>
<td>0.356</td>
</tr>
<tr>
<td>Line sections 1, 7 and 8 out.</td>
<td>0.00003464</td>
<td>0.0000144</td>
<td></td>
<td>0.338</td>
</tr>
</tbody>
</table>

Permanent outage rate due to overlapping component permanent outages = 0.00067 $f/yr$.
Average outage duration due to overlapping component permanent outages = 3.018 Hours.
Temporary outage rate due to component temporary outages overlapping component permanent outages = 0.00653 $f/yr$.

<table>
<thead>
<tr>
<th>Component temporary and permanent outages overlapping component maintenance outages</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Line section 1 maint., Line section 4 out.</td>
<td>0.0013600</td>
<td>0.002770</td>
<td></td>
<td>0.456</td>
</tr>
<tr>
<td>Line section 4 maint., Line section 1 out.</td>
<td>0.0004100</td>
<td>0.0006844</td>
<td></td>
<td>0.273</td>
</tr>
<tr>
<td>Line section 1 maint., Line section 5 out.</td>
<td>0.0009200</td>
<td>0.0007770</td>
<td></td>
<td>0.273</td>
</tr>
<tr>
<td>Line section 5 maint., Line section 1 out.</td>
<td>0.0005630</td>
<td>0.0009190</td>
<td></td>
<td>0.304</td>
</tr>
<tr>
<td>Line section 1 maint., Line sections 2 &amp; 3 out.</td>
<td>0.0000035</td>
<td>0.0000016</td>
<td></td>
<td>0.273</td>
</tr>
<tr>
<td>Line section 2 maint., Line sections 1 &amp; 3 out.</td>
<td>0.0000065</td>
<td>0.0000016</td>
<td></td>
<td>0.228</td>
</tr>
<tr>
<td>Line section 3 maint., Line sections 1 &amp; 2 out.</td>
<td>0.0000065</td>
<td>0.0000016</td>
<td></td>
<td>0.228</td>
</tr>
<tr>
<td>Line section 1 maint., Line sections 6 &amp; 8 out.</td>
<td>0.000000002</td>
<td>0.0000000016</td>
<td></td>
<td>0.195</td>
</tr>
<tr>
<td>Line section 6 maint., Line sections 1 &amp; 8 out.</td>
<td>0.00000002</td>
<td>0.00000003</td>
<td></td>
<td>0.182</td>
</tr>
<tr>
<td>Line section 8 maint., Line sections 1 &amp; 6 out.</td>
<td>0.00000002</td>
<td>0.00000004</td>
<td></td>
<td>0.195</td>
</tr>
<tr>
<td>Line section 1 maint., Line sections 7 &amp; 8 out.</td>
<td>0.00000003</td>
<td>0.00000005</td>
<td></td>
<td>0.171</td>
</tr>
<tr>
<td>Line section 7 maint., Line sections 1 &amp; 7 out.</td>
<td>0.00000003</td>
<td>0.00000005</td>
<td></td>
<td>0.171</td>
</tr>
</tbody>
</table>

Outage rate due to component permanent outages overlapping component maintenance outages = 0.00315 $f/yr$.
Average duration of outages due to component permanent outages overlapping component maintenance outages = 3.130 Hours.
Outage rate due to component temporary outages overlapping component maintenance and permanent outages = 0.00708 $f/yr$.  


### Reliability Indices at Load Point 1

<table>
<thead>
<tr>
<th>Component</th>
<th>Permanent Outages 1/yr</th>
<th>Temporary Outages 1/yr</th>
<th>Duration of Permanent Outage 10^-3 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent and temporary interruptions resulting from component permanent and temporary outages.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line sections 1 and 4 out.</td>
<td>0.0007200</td>
<td>0.0041610</td>
<td>0.469</td>
</tr>
<tr>
<td>Line sections 1 and 5 out.</td>
<td>0.0023600</td>
<td>0.0003290</td>
<td>0.032</td>
</tr>
<tr>
<td>Line sections 2, 3 and 4 out.</td>
<td>0.0000300</td>
<td>0.0000534</td>
<td>0.500</td>
</tr>
<tr>
<td>Line sections 2, 3 and 5 out.</td>
<td>0.0000283</td>
<td>0.0000273</td>
<td>0.290</td>
</tr>
<tr>
<td>Line sections 1, 6 and 8 out.</td>
<td>0.00002195</td>
<td>0.0000022</td>
<td>0.022</td>
</tr>
<tr>
<td>Line sections 1, 7 and 8 out.</td>
<td>0.0000134</td>
<td>0.0000144</td>
<td>0.518</td>
</tr>
</tbody>
</table>

Permanent outage rate due to overlapping component permanent outages = 0.00872 1/yr.
Average outage duration due to overlapping component permanent outages = 3.820 Hours
Temporary outage rate due to component temporary outages overlapping component permanent outages = 0.0057 1/yr.

### Component temporary and permanent outages overlapping component maintenance outages.

<table>
<thead>
<tr>
<th>Component</th>
<th>Permanent Outages 1/yr</th>
<th>Temporary Outages 1/yr</th>
<th>Duration of Permanent Outage 10^-3 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line section 1 maint., Line section 1 out.</td>
<td>0.0013680</td>
<td>0.0027300</td>
<td>0.455</td>
</tr>
<tr>
<td>Line section 2 maint., Line section 1 out.</td>
<td>0.0001406</td>
<td>0.0005890</td>
<td>0.273</td>
</tr>
<tr>
<td>Line section 3 maint., Line section 5 out.</td>
<td>0.0006232</td>
<td>0.0002730</td>
<td>0.273</td>
</tr>
<tr>
<td>Line section 4 maint., Line section 1 out.</td>
<td>0.0004479</td>
<td>0.0009130</td>
<td>0.904</td>
</tr>
<tr>
<td>Line section 5 maint., Line sections 2 &amp; 4 out.</td>
<td>0.000032</td>
<td>0.0000062</td>
<td>0.062</td>
</tr>
<tr>
<td>Line section 6 maint., Line section 3 out.</td>
<td>0.000021</td>
<td>0.0000005</td>
<td>0.005</td>
</tr>
<tr>
<td>Line section 7 maint., Line sections 2 &amp; 3 out.</td>
<td>0.000006</td>
<td>0.0000022</td>
<td>0.022</td>
</tr>
<tr>
<td>Line section 8 maint., Line sections 2 &amp; 5 out.</td>
<td>0.000003</td>
<td>0.0000002</td>
<td>0.002</td>
</tr>
<tr>
<td>Line section 9 maint., Line sections 2 &amp; 3 out.</td>
<td>0.000002</td>
<td>0.0000008</td>
<td>0.008</td>
</tr>
<tr>
<td>Line section 10 maint., Line sections 6 &amp; 8 out.</td>
<td>0.000002</td>
<td>0.0000006</td>
<td>0.006</td>
</tr>
<tr>
<td>Line section 11 maint., Line sections 1 &amp; 6 out.</td>
<td>0.000002</td>
<td>0.0000003</td>
<td>0.003</td>
</tr>
<tr>
<td>Line section 12 maint., Line sections 7 &amp; 8 out.</td>
<td>0.0000003</td>
<td>0.0000005</td>
<td>0.005</td>
</tr>
<tr>
<td>Line section 13 maint., Line sections 1 &amp; 8 out.</td>
<td>0.0000003</td>
<td>0.0000005</td>
<td>0.005</td>
</tr>
<tr>
<td>Line section 14 maint., Line sections 1 &amp; 7 out.</td>
<td>0.0000003</td>
<td>0.0000005</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Outage rate due to component permanent outages overlapping component maintenance outages = 0.00313 1/yr.
Average duration of outages due to component permanent outages overlapping component maintenance outages = 3.139 Hours.
Outage rate due to component temporary outages overlapping component maintenance and permanent outages = 0.0057 1/yr.
### Reliability Indices at Load Point A

<table>
<thead>
<tr>
<th>Component and permanent interruptions resulting from component permanent and temporary outages.</th>
<th>Permanent Outages 1/yr</th>
<th>Temporary Outages 1/yr</th>
<th>Duration of Permanent Outage 10(^{-3}) Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line sections 4 and 5 out.</td>
<td>0.00572</td>
<td>0.00876</td>
<td>0.0000195</td>
</tr>
<tr>
<td>Line sections 1 and 5 out.</td>
<td>0.00356</td>
<td>0.00252</td>
<td>0.0000317</td>
</tr>
<tr>
<td>Line sections 4, 6 and 8 out.</td>
<td>0.00091</td>
<td>0.00092</td>
<td>0.0000317</td>
</tr>
<tr>
<td>Line sections 4, 7 and 8 out.</td>
<td>0.00009</td>
<td>0.00009</td>
<td>0.0000317</td>
</tr>
<tr>
<td>Line sections 1, 6 and 8 out.</td>
<td>0.00003</td>
<td>0.00003</td>
<td>0.0000317</td>
</tr>
<tr>
<td>Line sections 2, 3 and 5 out.</td>
<td>0.00003</td>
<td>0.00003</td>
<td>0.0000317</td>
</tr>
</tbody>
</table>

Permanent outage rate due to overlapping component permanent outages = 0.00877 1/yr.
Average outage duration due to overlapping component permanent outages = 3.810 hours.
Temporary outage rate due to component temporary outages overlapping component permanent outages = 0.00912 1/yr.

<table>
<thead>
<tr>
<th>Component temporary and permanent outages overlapping component maintenance outages.</th>
<th>Permanent Outages 1/yr</th>
<th>Temporary Outages 1/yr</th>
<th>Duration of Permanent Outage 10(^{-3}) Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line section 4 maint., line section 5 out.</td>
<td>0.00041</td>
<td>0.00136</td>
<td>0.273</td>
</tr>
<tr>
<td>Line section 5 maint., line section 4 out.</td>
<td>0.00009</td>
<td>0.00012</td>
<td>0.046</td>
</tr>
<tr>
<td>Line section 1 maint., line section 5 out.</td>
<td>0.00082</td>
<td>0.00277</td>
<td>0.273</td>
</tr>
<tr>
<td>Line section 5 maint., line section 1 out.</td>
<td>0.00051</td>
<td>0.00091</td>
<td>0.054</td>
</tr>
<tr>
<td>Line section 4 maint., line sections 6 &amp; 8 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 6 maint., line sections 4 &amp; 6 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 8 maint., line sections 4 &amp; 6 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 4 maint., line sections 7 &amp; 8 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 7 maint., line sections 4 &amp; 8 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 8 maint., line sections 4 &amp; 7 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 1 maint., line sections 6 &amp; 8 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 6 maint., line sections 1 &amp; 8 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 8 maint., line sections 1 &amp; 6 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 1 maint., line sections 7 &amp; 8 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 7 maint., line sections 1 &amp; 8 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 8 maint., line sections 1 &amp; 6 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 2 maint., line sections 3 &amp; 5 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 3 maint., line sections 2 &amp; 5 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>Line section 5 maint., line sections 2 &amp; 3 out.</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Outage rate due to component permanent outages overlapping component maintenance outages = 0.00269 1/yr.
Average duration of outage due to component permanent outages overlapping component maintenance outages = 3.265 hours.
Outage rate due to component temporary outages overlapping component maintenance and permanent outages = 0.00665 1/yr.
<table>
<thead>
<tr>
<th>Continuities</th>
<th>Permanent Outages f/yr</th>
<th>Temporary Outages f/yr</th>
<th>Duration of permanent outages 10⁻³ Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent and temporary interruptions resulting from component permanent and temporary outages.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line sections 5, 6 and 8 out.</td>
<td>0.0000395</td>
<td>0.00000143</td>
<td>0.036</td>
</tr>
<tr>
<td>Line sections 5, 7 and 8 out.</td>
<td>0.0000380</td>
<td>0.00000151</td>
<td>0.038</td>
</tr>
<tr>
<td>Line sections 4, 6 and 8 out.</td>
<td>0.0000310</td>
<td>0.00000125</td>
<td>0.032</td>
</tr>
<tr>
<td>Line sections 4, 7 and 8 out.</td>
<td>0.0000374</td>
<td>0.00000257</td>
<td>0.359</td>
</tr>
<tr>
<td>Line sections 1, 6 and 8 out.</td>
<td>0.0000395</td>
<td>0.00000138</td>
<td>0.336</td>
</tr>
<tr>
<td>Line sections 1, 7 and 8 out.</td>
<td>0.0000384</td>
<td>0.00000144</td>
<td>0.338</td>
</tr>
</tbody>
</table>

Permanent outage rate due to overlapping component permanent outages = 0.00002 f/yr.  
Average outage duration due to overlapping component permanent outages = 3.000 Hours.  
Temporary outage rate due to component temporary outages overlapping component permanent outages = 0.00001 f/yr.

<table>
<thead>
<tr>
<th>Outages overlapping component maintenance outages.</th>
<th>f/yr</th>
<th>f/yr</th>
<th>10⁻³ Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line section 5 maint., line sections 6 &amp; 8 out.</td>
<td>0.0000001</td>
<td>0.0000003</td>
<td>0.230</td>
</tr>
<tr>
<td>Line section 6 maint., line sections 5 &amp; 8 out.</td>
<td>0.0000001</td>
<td>0.0000003</td>
<td>0.182</td>
</tr>
<tr>
<td>Line section 7 maint., line sections 5 &amp; 6 out.</td>
<td>0.0000002</td>
<td>0.0000005</td>
<td>0.195</td>
</tr>
<tr>
<td>Line section 5 maint., line sections 7 &amp; 8 out.</td>
<td>0.0000002</td>
<td>0.0000004</td>
<td>0.196</td>
</tr>
<tr>
<td>Line section 7 maint., line sections 5 &amp; 6 out.</td>
<td>0.0000003</td>
<td>0.0000007</td>
<td>0.171</td>
</tr>
<tr>
<td>Line section 8 maint., line sections 5 &amp; 7 out.</td>
<td>0.0000003</td>
<td>0.0000007</td>
<td>0.171</td>
</tr>
<tr>
<td>Line section 6 maint., line sections 5 &amp; 8 out.</td>
<td>0.0000001</td>
<td>0.0000003</td>
<td>0.182</td>
</tr>
<tr>
<td>Line section 7 maint., line sections 5 &amp; 6 out.</td>
<td>0.0000003</td>
<td>0.0000008</td>
<td>0.249</td>
</tr>
<tr>
<td>Line section 8 maint., line sections 4 &amp; 6 out.</td>
<td>0.0000003</td>
<td>0.0000009</td>
<td>0.273</td>
</tr>
<tr>
<td>Line section 4 maint., line sections 7 &amp; 8 out.</td>
<td>0.0000001</td>
<td>0.0000002</td>
<td>0.171</td>
</tr>
<tr>
<td>Line section 7 maint., line sections 4 &amp; 8 out.</td>
<td>0.0000006</td>
<td>0.0000011</td>
<td>0.228</td>
</tr>
<tr>
<td>Line section 8 maint., line sections 4 &amp; 7 out.</td>
<td>0.0000006</td>
<td>0.0000011</td>
<td>0.228</td>
</tr>
<tr>
<td>Line section 1 maint., line sections 6 &amp; 8 out.</td>
<td>0.0000002</td>
<td>0.0000004</td>
<td>0.195</td>
</tr>
<tr>
<td>Line section 6 maint., line sections 1 &amp; 8 out.</td>
<td>0.0000002</td>
<td>0.0000003</td>
<td>0.182</td>
</tr>
<tr>
<td>Line section 8 maint., line sections 1 &amp; 6 out.</td>
<td>0.0000002</td>
<td>0.0000003</td>
<td>0.182</td>
</tr>
<tr>
<td>Line section 1 maint., line sections 7 &amp; 8 out.</td>
<td>0.0000003</td>
<td>0.0000005</td>
<td>0.171</td>
</tr>
<tr>
<td>Line section 7 maint., line sections 1 &amp; 8 out.</td>
<td>0.0000003</td>
<td>0.0000005</td>
<td>0.171</td>
</tr>
<tr>
<td>Line section 8 maint., line sections 1 &amp; 7 out.</td>
<td>0.0000003</td>
<td>0.0000005</td>
<td>0.171</td>
</tr>
</tbody>
</table>

Outage rate due to component permanent outages overlapping component maintenance outages = 0.00000067 f/yr.  
Average duration of outages due to component permanent outages overlapping component maintenance outages = 1.80 Hours.  
Outage rate due to component temporary outages overlapping component maintenance and permanent outages = 0.00000067 f/yr.
### Reliability Indices at load point A

<table>
<thead>
<tr>
<th>Continuosity</th>
<th>Permanent Outages $f/yr$</th>
<th>Temporary Outages $f/yr$</th>
<th>Duration of Permanent Outage $10^{-3}$ Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent and temporary interruptions resulting from component permanent and temporary outages...</td>
<td>0.0015070</td>
<td>0.00018620</td>
<td>0.413</td>
</tr>
<tr>
<td>Line sections 6 and 7 out.</td>
<td>0.0000364</td>
<td>0.0000184</td>
<td>0.318</td>
</tr>
<tr>
<td>Line sections 1, 7 and 8 out.</td>
<td>0.0000674</td>
<td>0.0000217</td>
<td>0.357</td>
</tr>
<tr>
<td>Line sections 4, 7 and 8 out.</td>
<td>0.0000384</td>
<td>0.0000122</td>
<td>0.318</td>
</tr>
</tbody>
</table>

**Permanent outage rate due to overlapping component permanent outages = 0.00165 $f/yr$.**

**Average outage duration due to overlapping component permanent outages = 3.562 Hours.**

**Temporary outage rate due to component temporary outages overlapping component permanent outages = 0.00132 $f/yr$.**

---

### Component temporary and permanent outages overlapping component maintenance outages.

| Line section 6 maint., Line section 7 out. | 0.0005479 | 0.0009130 | 0.306 |
| Line section 7 maint., Line section 6 out. | 0.0001106 | 0.0003680 | 0.342 |
| Line section 1 maint., Line sections 7 & 8 out. | 0.0000003 | 0.0000005 | 0.171 |
| Line section 7 maint., Line sections 1 & 8 out. | 0.0000003 | 0.0000005 | 0.171 |
| Line section 8 maint., Line sections 1 & 7 out. | 0.0000003 | 0.0000005 | 0.171 |
| Line section 1 maint., Line sections 7 & 8 out. | 0.0000001 | 0.0000002 | 0.171 |
| Line section 7 maint., Line sections 1 & 8 out. | 0.0000006 | 0.0000010 | 0.228 |
| Line section 8 maint., Line sections 1 & 7 out. | 0.0000006 | 0.0000010 | 0.228 |
| Line section 7 maint., Line sections 5 & 7 out. | 0.0000002 | 0.0000003 | 0.182 |
| Line section 8 maint., Line sections 5 & 7 out. | 0.0000003 | 0.0000007 | 0.171 |

**Outage rate due to component permanent outages overlapping component maintenance outages = 0.00096 $f/yr$.**

**Average duration of outages due to component permanent outages overlapping component maintenance outages = 2.900 Hours.**

**Outage rate due to component temporary outages overlapping component maintenance and permanent outages = 0.00229 $f/yr$.**
<table>
<thead>
<tr>
<th>Contingency</th>
<th>Frequency (f/yr)</th>
<th>Average outage duration (10^{-3} years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line sections 1 and 4 out.</td>
<td>0.0010950</td>
<td>0.342</td>
</tr>
<tr>
<td>Line sections 1 and 5 out.</td>
<td>0.00032800</td>
<td>0.228</td>
</tr>
<tr>
<td>Line sections 1, 2 and 3 out.</td>
<td>0.0000003</td>
<td>0.171</td>
</tr>
<tr>
<td>Line sections 1, 6 and 9 out.</td>
<td>0.0000001</td>
<td>0.152</td>
</tr>
<tr>
<td>Line sections 1, 7 and 8 out.</td>
<td>0.0000001</td>
<td></td>
</tr>
</tbody>
</table>

Outage rate = 0.001425 f/yr.
Average outage duration = 2.768 Hours

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Frequency (f/yr)</th>
<th>Average outage duration (10^{-3} years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line sections 1 and 4 out.</td>
<td>0.0010950</td>
<td>0.342</td>
</tr>
<tr>
<td>Line sections 1 and 5 out.</td>
<td>0.00032800</td>
<td>0.228</td>
</tr>
<tr>
<td>Line sections 2, 3 and 4 out.</td>
<td>0.0000001</td>
<td>0.171</td>
</tr>
<tr>
<td>Line sections 2, 3 and 5 out.</td>
<td>0.0000003</td>
<td>0.152</td>
</tr>
<tr>
<td>Line sections 1, 6 and 8 out.</td>
<td>0.0000001</td>
<td></td>
</tr>
<tr>
<td>Line sections 1, 7 and 8 out.</td>
<td>0.0000001</td>
<td></td>
</tr>
</tbody>
</table>

Outage rate = 0.001426 f/yr.
Average outage duration = 2.768 Hours

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Frequency (f/yr)</th>
<th>Average outage duration (10^{-3} years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line sections 4 and 5 out.</td>
<td>0.0010950</td>
<td>0.342</td>
</tr>
<tr>
<td>Line sections 1 and 5 out.</td>
<td>0.00032800</td>
<td>0.228</td>
</tr>
<tr>
<td>Line sections 4, 6 and 8 out.</td>
<td>0.0000003</td>
<td>0.171</td>
</tr>
<tr>
<td>Line sections 4, 7 and 6 out.</td>
<td>0.0000001</td>
<td>0.152</td>
</tr>
<tr>
<td>Line sections 1, 6 and 8 out.</td>
<td>0.0000001</td>
<td></td>
</tr>
<tr>
<td>Line sections 1, 7 and 8 out.</td>
<td>0.0000001</td>
<td></td>
</tr>
<tr>
<td>Line sections 2, 3 and 5 out.</td>
<td>0.0000003</td>
<td></td>
</tr>
</tbody>
</table>

Outage rate = 0.001426 f/yr.
Average outage duration = 2.768 Hours
### Reliability Indices at load point 5

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Frequency f/yr</th>
<th>Average outage duration 10^-3 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line sections 5, 6 and 8 out.</td>
<td>0.0000001</td>
<td>0.173</td>
</tr>
<tr>
<td>Line sections 5, 7 and 8 out.</td>
<td>0.0000001</td>
<td>0.152</td>
</tr>
<tr>
<td>Line sections 8, 6 and 8 out.</td>
<td>0.0000003</td>
<td>0.228</td>
</tr>
<tr>
<td>Line sections 6, 7 and 8 out.</td>
<td>0.0000003</td>
<td>0.228</td>
</tr>
<tr>
<td>Line sections 1, 6 and 8 out.</td>
<td>0.0000001</td>
<td>0.171</td>
</tr>
<tr>
<td>Line sections 1, 7 and 8 out.</td>
<td>0.0000002</td>
<td>0.152</td>
</tr>
</tbody>
</table>

Outage rate = 0.0000011 f/yr.  
Average outage duration = 1,760 Hours

### Reliability Indices at load point 6

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Frequency f/yr</th>
<th>Average outage duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line sections 6 and 7 out.</td>
<td>0.0002050</td>
<td>0.273</td>
</tr>
<tr>
<td>Line sections 1, 7 and 8 out.</td>
<td>0.0000001</td>
<td>0.152</td>
</tr>
<tr>
<td>Line sections 4, 7 and 8 out.</td>
<td>0.0000005</td>
<td>0.195</td>
</tr>
<tr>
<td>Line sections 5, 7 and 8 out.</td>
<td>0.0000001</td>
<td>0.152</td>
</tr>
</tbody>
</table>

Outage rate = 0.000205 f/yr.  
Average outage duration = 2,356 Hours
5. EVALUATION OF THE OVERLOAD OUTAGE CONDITION

5.1 Introduction

In the previous chapters, continuity of supply to the load point has been recognized as the only criterion of success. This, therefore, assumes that every component in a parallel system is capable of carrying the highest load to which it may be subjected in any contingency. Such a system is termed fully redundant. A fully redundant system while quite reliable is also relatively expensive because of high component capacities. A simple means of reducing the system cost is to reduce the capacities of the components connected in parallel relative to the highest loads which they may be called upon to carry under contingency conditions. This reduction in system cost, however, will be at the expense of system reliability because with such systems even though there is continuity between the load and the source points under an outage state, there is a finite conditional probability of loss of load at the load point. This probability is a function of the system load level, configuration of the system, capability of the remaining components in the system and the duration of the contingency. It has already been pointed out that during system outages, if the load exceeds the emergency capability of a component, several situations can exist. The component may be called upon to carry the overload, it may be tripped out of service, or some load may be cut off to relieve the overload. Each of these conditions will result in a different value of load point reliability.
The first step in the estimation of overload outages is the determination of the probability that a component will not be able to carry a given contingency load. A technique of contingency curves has been described in reference 2 in which curves representing the probability distribution for the contingency load on each line in the network are precalculated. Another technique used in generating capacity reliability studies\(^{(23)}\) is to represent the system load model as a stationary Markov process. In this method it is assumed that the system daily load consists of a peak load period which exists for some time and a low load level which exists for the remainder of the day. Equations have been developed using this load model to evaluate the frequency and duration of negative capacity margins\(^{(23)}\).

5.2 Two State Model

The conditional probability approach\(^{(6)}\) does not take into account the duration of the contingency in determining the probability of carrying the contingency load. The contingency curve technique\(^{(2)}\) is accurate but cannot be easily applied to complicated networks. The load model based on stationary Markov processes\(^{(23)}\) recognises only a few load levels in the system and does not reflect the hourly load data.

The two state load model\(^{(12)}\) recognises how long a load exceeds a particular value and the time between those periods and therefore, assigns frequency and duration to any specified load level.
Figure 5.1a shows a typical hourly chronological load curve of a system over a certain period of time. The value 'L' designates the load level at which a particular contingency load exceeds the configuration capability. As can be seen from Figure 5.1b, the variations above and below 'L' can be represented as a two state renewal process. The two parameters which describe this renewal process are the average duration of the load state \( l(t) > L \) and its rate of occurrence. Here \( l(t) \) is the load level at time \( t \). By collecting hourly load data for a period of one year, these parameters can be estimated for a given load level as follows:

For each value of \( L \), two quantities are evaluated. They are

(i) \( n_p \), the number of hours for which \( l(t) < L \) and \( l(t+1) \geq L \) where \( l(t) \) is the load for hour \( t \), \( t=1, 2 \ldots 8760 \). This is equal to the number of transitions from less than to greater than load level \( L \).

(ii) \( n_L \), the number of hours that \( l(t) > L \)

From these two counts, the estimate of the frequency and duration parameters associated with the load level \( L \) can be obtained. The rate of occurrence of the high load period \( l(t) > L \) can be interpreted as the reciprocal of the mean time between high loads. The mean time between high loads is equal to the average duration of the low load period, \( l(t) < L \)

Let

\[
\lambda_L = \text{Rate of occurrence of load level } l(t) > L \quad \text{occurrences/year}
\]

\[
r_L = \text{Average duration of load level } l(t) > L \quad \text{years}
\]
Figure 5.1 Two State Load Model
(a) System Chronological Load Curve
(b) Two State Model
The probability of \( l(t) \) being greater than \( L \)

\[
P(l(t) > L) = \frac{\lambda_L}{\lambda_L + \frac{1}{r_L}}
\]

Also

The cycle time of occurrence of load level \( l(t) > L = \frac{1}{\lambda_L} + r_L \)

Therefore, \( \frac{1}{\lambda_L} + r_L = \frac{1}{n_T} \)

It follows from the above two equations that

\[
\lambda_L = \frac{n_T}{1 - n_L/8760} \quad (5.1)
\]

and

\[
r_L = \frac{n_L}{8760 n_T} \quad (5.2)
\]

These parameters for one week of hourly load data for an S.P.C. system
substation with chronological curve as given in Appendix 5 are illustrated
in Table 5.1. It can be seen from this table that the rates and durations
are well behaved functions of load level and these parameters can be
determined for any intermediate load level by interpolation.

It is clear from the above method of derivation of the frequency
and duration parameters that these reflect not only the daily, weekly and
seasonal variations of load but also include any random variations which
might occur by weather changes and other natural factors.
<table>
<thead>
<tr>
<th>Load level -% of Annual Peak</th>
<th>Rate No./week</th>
<th>Average Duration - Hours</th>
<th>Availability - Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>—</td>
<td>168.0</td>
<td>100.00</td>
</tr>
<tr>
<td>45</td>
<td>—</td>
<td>168.0</td>
<td>100.00</td>
</tr>
<tr>
<td>50</td>
<td>33.6</td>
<td>163.0</td>
<td>97.02</td>
</tr>
<tr>
<td>55</td>
<td>32.3</td>
<td>28.0</td>
<td>84.52</td>
</tr>
<tr>
<td>60</td>
<td>25.0</td>
<td>17.0</td>
<td>70.83</td>
</tr>
<tr>
<td>65</td>
<td>22.0</td>
<td>10.0</td>
<td>60.12</td>
</tr>
<tr>
<td>70</td>
<td>16.8</td>
<td>8.0</td>
<td>46.42</td>
</tr>
<tr>
<td>75</td>
<td>14.9</td>
<td>7.0</td>
<td>39.88</td>
</tr>
<tr>
<td>80</td>
<td>14.1</td>
<td>3.0</td>
<td>22.02</td>
</tr>
<tr>
<td>85</td>
<td>10.1</td>
<td>2.0</td>
<td>10.71</td>
</tr>
<tr>
<td>90</td>
<td>8.6</td>
<td>1.0</td>
<td>7.14</td>
</tr>
<tr>
<td>95</td>
<td>1.0</td>
<td>1.0</td>
<td>0.59</td>
</tr>
<tr>
<td>100</td>
<td>1.0</td>
<td>1.0</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Using these parameters of load, equations can be derived for the average interruption rate and the average duration of interruptions. The state space diagram for a two component parallel system with no maintenance and weather considerations is shown in Figure 5.2. In this diagram,

\[ \lambda_L = \text{The rate of occurrence of load level } l(t) > L \]

\[ \mu_L = \text{The reciprocal of the average duration of load level } l(t) > L \]
Figure 5.2 State Space Diagram For Permanent Outages and Overload Outages
State 6, here, represents the overload on component 1 while component 2 is on permanent outage. The frequency of occurrence of such a state, using equations 3.9 and 3.10, is given by:
\[ \lambda_{OL} = P(2) \lambda_L + P(5) \lambda_2 + P(8) \mu_1 \]
where
\[ P(j) \] represents the probability of occurrence of jth state.

\[ \lambda_{OL} = \lambda_L P_T \] (Component 2 forced out). \( P_T \) (Component 1 not forced out)

\[ = (1 - P_T (1(t) > L)) \lambda_2 P_T \] (Component 2 not forced out).

\[ P_T \] (Component 1 not forced out). \( P_T \) (1(t) > L)

\[ + \mu_1 P_T \] (Component 2 forced out). \( P_T \) (Component 1 forced out).

\[ = \lambda_L P_T \] (Component 2 forced out). \( P_T \) (Component 1 not forced out).

\[ (1 - P_T (1(t)) > L) + \lambda_2 P_T \] (1(t) > L) \( \left\{ \frac{\mu_2}{(\lambda_1 + \mu_1)} \right\} \frac{\mu_1}{(\lambda_2 + \mu_2)} \)

\[ = \lambda_L P_T \] (Component 2 forced out). \( P_T \) (Component 1 not forced out).

\[ (1 - P_T (1(t) > L)) + \lambda_2 P_T \] (1(t) > L) \( (1 - P_T \) (Component 2 forced out)) \( \left( \frac{\mu_2}{\mu_2 + \lambda_1} \right) \frac{\mu_1}{\lambda_1 + \lambda_2} \)

Since \( \mu^4 \)s \( \gg \lambda^4 \)s, the last term in the parenthesis is nearly equal to the probability that component 1 is not forced out. In that case,
\[ \lambda_{OL} = \lambda_L \Pr (\text{Component 2 forced out}) \Pr (\text{Component 1 not forced out}) \cdot (1 - \Pr (l(t) > L)) \]

\[ + \lambda_2 \Pr (l(t) > L) (1 - \Pr (\text{Component 2 forced out})) \cdot \Pr (\text{Component 1 not forced out}) \]

Equation 5.3 can also be derived from renewal process considerations.

The system load can be represented by a fictitious component with outage rate \( \lambda_L \) and expected outage duration of \( r_L \) in parallel with the two actual parallel components. Consider a situation in which component 2 is forced out and component 1 may become overloaded. For an operating time of \( T \) units, the total outage time of component 2 is given by:

\[ \frac{\lambda_2}{\lambda_2 + \mu_2} T \]

The cycle time of the load component = \( \frac{1}{\lambda_L} + \frac{1}{\mu_L} \).

The expected number of outages of component 2 which overlap the overload condition for an operation time \( T \) units is therefore,

\[ A = \frac{\lambda_2 T}{\lambda_2 + \mu_2} / (\frac{1}{\lambda_L} + \frac{1}{\mu_L}) \]

In an operation time of \( T \) units, the load condition \( l(t) > L \) exists for

\[ a \text{ time} = \frac{\lambda_L T}{\lambda_L + \mu_L} \]

The cycle time of component 2 = \( \frac{1}{\lambda_2} + \frac{1}{\mu_2} \).

Therefore, the expected number of outages of the fictitious load component which overlap the outage of component 2 for an operation time of \( T \) units
The frequency of component 1 becoming overloaded when component 2 is permanently out is therefore, given by:

\[ \lambda_{OL} = \Pr (\text{Component 1 is not forced out}) \times \frac{\Delta t P}{T} \]

\[ = \Pr (\text{Component 1 is not forced out}) \left[ \frac{\lambda_2 \mu_L}{(\lambda_2 + \mu_2)} \frac{\lambda_L}{(\lambda_1 + \mu_L)} \right] + \frac{\lambda_1}{(\lambda_1 + \mu_L)} \frac{\lambda_2 \mu_2}{(\lambda_2 + \mu_2)} \]

\[ = \Pr (\text{Component 1 is not forced out}) \left\{ (\lambda_L \Pr (1 - 1(t) > L)) \right\} \]

This equation is similar to equation 5.3. This derivation further confirms that the probability distributions of the load can be of any general shape. It can further be noted that this equation includes the durations of the outage states which are unaccounted for in a conditional probability approach.

The expected outage duration associated with outage rate \( \lambda_{OL} \) is given by:

\[ r_{OL} = \frac{r_2 \lambda_L}{r_2 + r_L} \quad (5.4) \]

Using equations 5.3 and 5.4 the average outage rate and the associated outage duration due to component overload outages can be evaluated at the various system load points.
If the overload of the remaining components results from more than one component permanent outage, equation 5.3 is modified as follows:

\[ \lambda_{oL} = \text{Pr} \ (\text{The remaining components are not forced out}) \cdot \left\{ (\lambda_L \cdot \text{Pr} (1 - 1(t) > L) \cdot \text{Pr} \ (\text{The components under consideration are forced out}) \right\} + (\text{The overlapping outage rate of the components under consideration}) \cdot \text{Pr} (1(t) > L) \cdot (1 - \text{Pr} \ (\text{The components under consideration are forced out})) \}

Since,

\[ \text{Pr} \ (\text{The remaining components are not forced out}) = 1 \]

and

\[ \text{Pr} \ (\text{The components under consideration are forced out}) = \lambda_{SL} \cdot r_{SL} \]

Therefore,

\[ \lambda_{oL} = \lambda_L \cdot \text{Pr} (1 - 1(t) > L) \cdot (\lambda_{SL} \cdot r_{SL}) + \lambda_{SL} \cdot \text{Pr} (1(t) > L) \cdot (1 - \lambda_{SL} \cdot r_{SL}) \]

\[ (5.5) \]

Using equations 5.3, 5.4 and 5.5 results have been obtained for two component and three component parallel systems. These results are shown in Figures 5.3 and 5.4 for various percentage of component failures during the adverse weather period when no repair is performed using the S.P.C system substation hourly load values. Figure 5.3 gives the system outage rate and Figure 5.4 gives the expected system outage duration. It can be seen from these curves that for a two component parallel system, there is no variation in the system outage rate and expected outage.
TWO COMPONENTS IN PARALLEL

COMPONENT CAPABILITY = 10 MW.

S.P.C SYSTEM SUBSTATION LOAD

\( \lambda_{ov} = 0.5 /yr \)

\( \mu = 1000 /yr \)

Figure 5.3 System Outage Rate Due to Component Overload Outages
Figure 5.4 System Average Outage Duration Due to Component Overload Outages
duration with component failures during the adverse weather period. This is due to the absence of a bunching effect of adverse weather associated failures. If both the components are out, no overloading is possible. In the three component parallel system, however, two components must be on overlapping outage and the remaining component overloaded for an overload failure to occur. The probability of occurrence of overlapping outages of two components increases as the component failures during adverse weather periods increase.

Some question may arise about the validity of these equations under the condition that all the component failures occur during the adverse weather period. Under these circumstances, the system load cycle analysis should be confined to adverse weather periods and the small periods following the adverse weather in which repairs are performed. Since adverse weather periods are assumed to occur randomly, the annual load cycle of the system is used.

The effect of load growth on the system reliability can be obtained by the use of equations 5.4 and 5.5. Increasing load is equivalent to an overall decrease in component capability. This is illustrated in Figure 5.5 for systems of two and three parallel components where system steady state failure rate is plotted against the component capability in percentage of annual peak load. It can be seen that as the component capability decreases, the system outage rate increases very rapidly. In many cases the decrease in overall system reliability can be traced to one or two components which have ratings slightly below single contingency loads at the time of the annual peak.
Figure 5.5  Effect of Load Growth on System Outage Rate
In the above presentation, a single load point case has been considered. In the case of system configurations involving many load points, the evaluation of overload outages becomes quite involved. A load flow analysis must be performed to determine the load values at which overload outages of components commence. Under such conditions, 8760 load flows are required for one year of load data for every outage configuration of the system to determine the overload condition. This, therefore, requires far too much calculation.

One simplifying assumption, which is quite valid in systems in which all the load points serve similar types of loads, is that the individual loads are directly proportional to the system load. This implies that the normalized system load model is applicable at all the load points in the system. With this assumption, the load flow analysis is performed at different load levels (expressed in percentage of the peak load at each load point) to establish the load level at which the overload on the system components commences.

To illustrate this, a system configuration of Figure 5.6 with two load points has been studied. The capabilities of the lines are indicated. Two of the many possible load curtailment situations arising from overload outages have been studied and are as follows:

(i) if a line becomes overloaded, the nearest load is curtailed.
(ii) if a line becomes overloaded, the load well removed from this line is curtailed (if possible).
Table 5.2 lists the various double contingency outage events which can result in the overload of the remaining components under the situation described as (i) in the above. Table 5.3 lists the various double contingency outage events which can result in the overload of the remaining components under the situation described as (ii) in the above. The load levels at which overload outages occur are obtained by a load flow analysis. A computer program has been written on the basis of the
iterative Gauss-Seidel technique to carry out the load flow analysis at different load levels.

**TABLE 5.2**

CONTINGENCIES CONSIDERED FOR THE SYSTEM IN FIG. 5.6 POLICY (I)

<table>
<thead>
<tr>
<th>No.</th>
<th>Contingency</th>
<th>Load Point 1</th>
<th>Load Point 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 - 2 out</td>
<td></td>
<td>If load &gt; 77%</td>
</tr>
<tr>
<td>2</td>
<td>1 - 3 out</td>
<td>If load &gt; 77%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1 - 4 out</td>
<td>If load &gt; 39%</td>
<td>If load &gt; 77%</td>
</tr>
<tr>
<td>4</td>
<td>2 - 3 out</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2 - 4 out</td>
<td>If load &gt; 77%</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3 - 4 out</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 5.3**

CONTINGENCIES CONSIDERED FOR THE SYSTEM IN FIG. 5.6 POLICY (II)

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Load Point 1</th>
<th>Load Point 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 - 2 out</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1 - 3 out</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1 - 4 out</td>
<td>If load &gt; 39%</td>
</tr>
<tr>
<td>4</td>
<td>2 - 3 out</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2 - 4 out</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3 - 4 out</td>
<td></td>
</tr>
</tbody>
</table>

The results obtained by equations 5.3, 5.4 and 5.5 for this configuration are shown in Tables 5.4 and 5.5 for both the above described load curtailment policies. These tables clearly illustrate that the
reliability indices at the different load points will vary with changes in the system load curtailment policy. An approach such as this can be used to logically decide the required capabilities of the components to serve a given load. Not all loads are equally important and therefore if desirable, the reliability levels may be variable.

It must be recognized, however, that a condition in which the probability of overload occurrence is one is different from that in which there is complete discontinuity of supply to the load point. In the former case, the load at a load point or points can be curtailed or reduced to relieve the overload whereas in the latter case, such an action is not possible. Therefore, a complete disconnection of the load point implies a complete interruption until the components which caused the discontinuity are restored to service whereas an overload outage may imply a partial outage at the load point until the load falls below the capability of the remaining components or the components capable of carrying the load are restored to service. As such, the reliability indices which result from the complete interruption of service to the load point cannot be combined with those obtained from the overload outages of the components.

It is assumed, while calculating the reliability indices, in Tables 5.4 and 5.5 that the component maintenance outages cannot result in the overload outages of the remaining system components. The maintenance outages are not independent of the load level in the system and, therefore, the risk of maintenance outages unintentionally overlapping a peak load period is negligible. This risk is further reduced with the highly
predictable nature of the load and controlled nature of maintenance activity. If, however, such an event is possible, the load parameters covering the time interval during which maintenance is scheduled must be developed for use in the equations described in this Chapter to find the corresponding reliability indices.
### Table 5.4

**Reliability Indices Due to Component Overloads for the System in Fig. 5.6**

**Load Point 1: Policy (1)**

<table>
<thead>
<tr>
<th>Contingencies</th>
<th>Percentage of Component Failures During Adverse Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>Overall Values</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
</tbody>
</table>

A = Failure Rate f/yr

B = Average Outage Duration Hours
<table>
<thead>
<tr>
<th>Contingencies</th>
<th>Percentage of Component Failures During Adverse Weather</th>
<th></th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>A   0.130 x 10^{-3}</td>
<td>0.242 x 10^{-3}</td>
<td>0.586 x 10^{-3}</td>
<td>0.116 x 10^{-2}</td>
<td>0.197 x 10^{-2}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B   3.70</td>
<td>3.70</td>
<td>3.70</td>
<td>3.70</td>
<td>3.70</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A   0.205 x 10^{-3}</td>
<td>0.345 x 10^{-3}</td>
<td>0.769 x 10^{-3}</td>
<td>0.148 x 10^{-2}</td>
<td>0.247 x 10^{-2}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B   4.60</td>
<td>4.60</td>
<td>4.60</td>
<td>4.60</td>
<td>4.60</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>A   --</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B   --</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>A   --</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B   --</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>A   --</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B   --</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Overall Values</td>
<td>A  0.336 x 10^{-3}</td>
<td>0.587 x 10^{-3}</td>
<td>0.135 x 10^{-2}</td>
<td>0.264 x 10^{-2}</td>
<td>0.444 x 10^{-2}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B   4.25</td>
<td>4.23</td>
<td>4.21</td>
<td>4.206</td>
<td>4.203</td>
<td></td>
</tr>
</tbody>
</table>
**TABLE 5.5**

**RELIABILITY INDICES DUE TO COMPONENT OVERLOADS FOR THE SYSTEM IN FIG. 5.6**

<table>
<thead>
<tr>
<th>Contingencies</th>
<th>Percentage of Component Failures During Adverse Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1 A</td>
<td>0.138 \times 10^{-3}</td>
</tr>
<tr>
<td>B</td>
<td>3.70</td>
</tr>
<tr>
<td>2 A</td>
<td>0.173 \times 10^{-4}</td>
</tr>
<tr>
<td>B</td>
<td>1.86</td>
</tr>
<tr>
<td>3 A</td>
<td>0.205 \times 10^{-3}</td>
</tr>
<tr>
<td>B</td>
<td>4.60</td>
</tr>
<tr>
<td>4 A</td>
<td>--</td>
</tr>
<tr>
<td>B</td>
<td>--</td>
</tr>
<tr>
<td>5 A</td>
<td>--</td>
</tr>
<tr>
<td>B</td>
<td>--</td>
</tr>
<tr>
<td>Overall Values</td>
<td>A</td>
</tr>
<tr>
<td>B</td>
<td>4.137</td>
</tr>
</tbody>
</table>

A = Failure Rate \$/yr  
B = Average Outage Duration Hours
### TABLE 5.5 (Contd.)

**RELIABILITY INDICES DUE TO COMPONENT OVERLOADS FOR THE SYSTEM IN FIG. 5.6**

**LOAD POINT 2**

<table>
<thead>
<tr>
<th>Contingencies</th>
<th>Percentage of Component Failures During Adverse Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>Overall Values</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
</tbody>
</table>

*A = Failure Rate f/yr  
B = Average Outage Duration Hours*
6. CONCLUSIONS

The application of probability techniques in the evaluation of distribution system reliability has been investigated in this thesis. A single reliability measure such as customer minutes of outage is not entirely adequate because it does not represent a unique situation of load point unreliability. The recognition of load point reliability indices in terms of outage frequency and duration gives a physical significance to the distribution system reliability problem.

An approximate technique based on the conditional probability approach has been developed to determine the frequency and duration of interruptions at various load points of a system. A method has been suggested for determining the adequate level of load point reliability in terms of frequency and duration of interruptions. Reliability predictions, in the case of overhead systems, can be quite in error if the effect of environmental conditions in which these facilities are operating is not considered. A two state weather model has been presented in which the component failure rate fluctuates between two levels. The two levels utilised correspond to normal and adverse weather periods. The equations formulated to include weather considerations represent two distinct situations of component repair i.e.

(i) Repair occurs during adverse weather periods

(ii) Repair does not occur during adverse weather periods
The results obtained by the approximate equations have been compared with those predicted by a theoretically accurate Markov technique for simple configurations of two, three and four components in parallel. It has been seen that the approximate equations give results which are very close to those obtained by the Markov approach. The interaction of component and weather parameters on the load point reliability has been investigated for various percentages of component failures during the adverse weather period.

The concept of forming an equivalent component on the basis of the approximate technique (2) and the Markov technique (10) has been investigated. It was found that the results presented in reference 10 are incorrect and the formation of an equivalent component on the basis of a Markov technique gives results which are no better than those obtained by the approximate equations. To deal with a system with a large number of components, equations for three and four overlapping outages have been developed. A very powerful reliability technique known as failure modes and effect analysis has been applied to determine the various load point failure modes. This requires a detailed knowledge of the behaviour of the system and all the inherent system processes that lead to a load point failure. The contribution to the system unreliability due to various load point failure modes can be evaluated using the equations described in this thesis. The relative severity of certain failure categories pinpoints the areas where maximum reliability improvements can be achieved with minimum expenditures. The star-delta conversion (10)
approach to reduce the system to a simple configuration is not required if
the failure modes and effect analysis technique is utilized.

A three state weather model has been presented to recognize the
effect of very severe weather periods. This thesis illustrates that the
disaster adverse weather condition can have a significant effect on the
system reliability indices.

When the normal and adverse weather failure aspect is not considered,
component maintenance outages make a significant contribution to the load
point unreliability. The risk of component permanent and temporary
outages overlapping the component maintenance periods, however, decreases
as more and more component failures occur during adverse weather periods.
The results obtained by the approximate equations compare very closely
with those obtained by the Markov approach in which the state space
diagram is extended on the normal weather side. It is quite important
in a distribution system to consider the component temporary outages
as this form of interruption can result in considerable irritation to
the customer. The equations described in this thesis can adequately
consider temporary load point outages even when the component temporary
outages are separated into normal and adverse weather failures.

In the case of facilities which are not redundant, component
overloads can make a large contribution to the load point failures. A
load flow analysis is required to determine the load level at which the
overload outages commence. The results predicted for the outage rate
and the average outage duration will be different depending upon the
assumptions made. The approach described in this thesis assumes that
the normalized system chronological load curve is representative of the
loads at various load points in the system. Interruptions due to overloads
have been recognized separately in this thesis as component overloads
do not necessarily imply an outage. Facilities may, sometimes, be called
upon to carry the overload even at the risk of permanent damage.

It has also been proved that the assumption of exponentially
distributed component up and down times is not very stringent. The results
predicted are valid for any general probability distribution if the
distributions remain the same for every run-failure-repair cycle.

Using the techniques described in this thesis, the management
problem to decide whether to consider all points of supply as being of
equal importance, whether to do the greatest good to the greatest number,
or whether to give special consideration to particular points can be
evaluated in quantitative terms.

It must be noted that the application of the techniques described
in this thesis depends upon having collected sufficient information
regarding the component failure rates and the associated expected outage
durations. Unfortunately, a limited amount of data is available on the
reliability performance of distribution system components. The importance
of data collection with the objective of reliability evaluation of various
design alternatives should be realised and effort expended to consistently
collect the required data.
7. References


There are several existing publications which contain some extremely important definitions. The latest of these is an IEEE Committee Report\(^{(14)}\) giving definitions of some useful terms used in reliability studies of transmission and distribution facilities. The definitions of some of the basic terms used in this thesis are as follows\(^{(14)}\):

**Outage**

An outage describes the state of a component when it is not available to perform its intended function due to some event directly associated with that component.

**Types Of Outages**

1. **Forced Outage** - A forced outage is an outage that results from emergency conditions directly associated with a component requiring it to be taken out of service immediately either automatically or as soon as switching operations can be performed or an outage caused by improper operation of equipment or human error.

2. **Scheduled or Maintenance Outage** - A scheduled outage is an outage that results when a component is deliberately taken out of service at a selected time, usually for the purpose of construction, preventive maintenance or repair.
Types Of Forced Outage

(i) Persistent Cause forced outage or Permanent Outage - A persistent cause forced outage is a component outage whose cause is not immediately self clearing but must be corrected by eliminating the hazard or by repairing or replacing the affected component before it can be restored to service.

(ii) Transient Cause Forced Outage or Temporary Outage

A transient cause forced outage is a component outage whose cause is immediately self clearing so that the affected component can be restored to service either automatically or as soon as switch or circuit breaker can be reclosed.

Interruption Terms

Interruption

An interruption is the loss of service to one or more customers or other facilities and is the result of one or more component outages, depending upon the system configuration. It can further be classified into scheduled interruption which is caused by scheduled outages and forced interruption which is caused by forced outages.

Interruption Duration

Interruption duration is the period from the initiation of an interruption to a customer or other facility until service has been restored to that customer or facility.
Switching time

Switching time is the period from the time switching operation is required due to a forced outage until that switching operation is performed. Switching operations include reclosing a circuit breaker, opening of a sectionalising switch or circuit breaker or replacing a fuse.
APPENDIX 2

A METHOD FOR DETERMINING ADEQUATE RELIABILITY INDICES

It is, in general, said that it is the responsibility of the management to determine the adequate level of electric supply reliability to the customers. The procedure for the management to make such a decision in setting up a goal of reliability has been suggested below(21). Limits for frequency of interruption

The limits for the interruption frequency can be obtained from an analysis of the recorded customer complaints. The data required for this analysis are:

(1) The average period between interruptions.

(ii) The cumulative number of complaints as a function of the average period between interruptions.

A plot, as shown in Figure A.1a of the cumulative number of customer complaints as a function of the average period between interruptions is obtained. This curve can be interpreted as statistical distribution of complaints. If this curve is extrapolated, it meets the abscissa at the point X. This point represents zero complaint at an average period of X months between interruptions. It theoretically indicates that no complaints about the frequency of interruption will be lodged if all customers have an average period of X months between interruptions. Thus the acceptable value of frequency of interruption is given by:

\[ f_L = \frac{12}{X} \text{ failure/year} \]
This frequency value can be regarded as an upper limit of the acceptable value because many customers may have interruptions more than this value. A value slightly less than $z_1$ can be chosen as an acceptable average value of interruption frequency.

This method of estimation takes into consideration only those interruptions which motivate a formal complaint. The interruptions which cause substantial irritation are unaccounted for in this method.

Limits for Interruption durations

Customer irritation can be classified into two levels. First, a customer will become irritated in a relatively short time. Second, if the interruption persists for a long time, the customer becomes militant.

A method of evaluating the threshold limits of these two levels of irritation is suggested below:

The number of complaints lodged by the customers during a certain period of outage duration are plotted. A typical plot is shown in figure A.1b. It can be inferred that the high volume of complaints in the first few minutes of interruption is just to make sure that power company personnel know of the trouble. A threshold of irritation is reached at point A and a high level of irritation is reached at point B.

The threshold of militancy can be determined by an analysis of the formal complaints about interruption durations. A typical curve for the number of complaints versus the duration of outages is shown in Figure A.1c. Extrapolation of this curve to zero indicates that the customer will not, probably, complain about durations of less than 6 hours.
Thus, it can be established that all the faulted sections should be disconnected and service restored to all other customers within a duration of A minutes of the start of the interruption in order to minimize the customer irritation. The faulted sections should be restored to service within a duration of B minutes in order to avoid a higher level of irritation. In extreme cases, the service should be restored in C hours to avoid problems of customer relations.
Figure A.1  A Graphical Method To Determine Acceptable Reliability Indices

(a) Complaints About Interruption Frequencies
(b) Customer Irritation Due to Interruption Duration
(c) Complaints About Interruption Durations
APPENDIX 3

OUTAGE RATE AND AVERAGE OUTAGE DURATION FOR FOUR OVERLAPPING OUTAGES

The formulation of equations for the outage rate and average outage duration of four component parallel system follows the same basic procedure as described in Chapter 3 of this thesis. In this case there are sixteen possible modes of failure. They are:

(i) The initial failure occurs during normal weather, the second failure occurs during normal weather, the third failure occurs during normal weather and the fourth failure also occurs during normal weather.

(ii) The initial failure occurs during normal weather, the second failure occurs during normal weather, the third failure occurs during adverse weather and the fourth failure occurs during normal weather.

(iii) The initial failure occurs during normal weather, the second failure occurs during adverse weather, the third failure occurs in normal weather and the fourth failure occurs during normal weather.

(iv) The initial failure occurs in adverse weather, the second failure occurs in normal weather, the third failure occurs during normal weather and the fourth failure occurs in normal weather.

(v) The initial failure occurs in normal weather, the second failure occurs in adverse weather, the third failure occurs in adverse weather and the fourth failure occurs in normal weather.

(vi) The initial failure occurs in adverse weather, the second failure
occurs in normal weather, the third failure occurs in adverse weather and the fourth failure occurs in normal weather.

(vii) The initial failure occurs in adverse weather, the second failure occurs in adverse weather, the third failure occurs in normal weather and the fourth failure occurs in normal weather.

(viii) The initial failure occurs in adverse weather, the second failure occurs in adverse weather, the third failure occurs in adverse weather and the fourth failure occurs in normal weather.

The remaining eight modes of failure are the same as described above but with normal and adverse weather conditions interchanged.

Let \( R_1 = \frac{1}{N} \left( \frac{1}{r(j)} + \frac{1}{r(k)} + \frac{1}{r(L)} \right) \)

\[ R_2 = \frac{1}{S} \left( \frac{1}{r(j)} + \frac{1}{r(k)} + \frac{1}{r(L)} \right) \]

\[ R_3 = \frac{N \ r(j) \ r(k)}{(N \ r(j) + N \ r(k) + r(j) \ r(k))} \]

\[ R_4 = \frac{S \ r(j) \ r(k)}{(S \ r(j) + S \ r(k) + r(j) \ r(k))} \]

\[ R_5 = \frac{N \ r(j)}{(N + r(j))} \]

\[ R_6 = \frac{S \ r(j)}{(S + r(j))} \]

where \( N = \) The average duration of a normal weather
\( S = \) The average duration of an adverse weather
\( r(j) = \) The expected repair time of \( j \)th component
\( \lambda(j) = \) Normal weather failure rate of \( j \)th component
The contribution to the system failure rate due to each of the above modes of system failure for four components in parallel when repair is performed during adverse weather periods is as follows.

**Mode 1**

\[
(Failure \ rate)_1 = \frac{N}{N+S} \left\{ \lambda(j) \left( \lambda(k)R5 \right) \left( \lambda(jj)R3 \right) \left( \lambda(kk)R1 \right) + \right\} \\
\text{Similar terms}
\]

The reasoning for various terms follows from Chapter 3.

**Mode 2**

\[
(Failure \ Rate)_2 = \frac{N}{N+S} \left\{ \lambda(j) \left( \lambda(k)R5 \right) x \frac{R3}{N} \left( \lambda^*jjR2 \right) \left( \lambda(kk)R1 \right) + \right\} \\
\text{Similar terms}
\]

**Mode 3**

\[
(Failure \ Rate)_3 = \frac{N}{N+S} \left\{ \lambda(j) \cdot R5 \cdot \frac{R5}{N} \left( \lambda^*(k)R6 \right) \left( \lambda(jj)R3 \right) \left( \lambda(kk)R1 \right) + \right\} \\
\text{Similar terms}
\]

**Mode 4**

\[
(Failure \ rate)_4 = \frac{S}{N+S} \left\{ \lambda'(j) \left( \lambda(k)R5 \right) \left( \lambda(jj)R3 \right) \left( \lambda(kk)R1 \right) + \right\} \\
\text{Similar terms}
\]

**Mode 5**

\[
(Failure \ rate)_5 = \frac{N}{N+S} \left\{ \lambda(j) \left( \lambda(k)R5 \right) \left( \lambda^*(k)R6 \right) \left( \lambda(jj)R4 \right) \left( \lambda(kk)R1 \right) + \right\} \\
\text{Similar terms}
\]

**Mode 6**

\[
(Failure \ rate)_6 = \frac{S}{N+S} \left\{ \lambda'(j) \left( \lambda(k)R5 \right) \left( \lambda(jj)R4 \right) \left( \lambda(kk)R1 \right) + \right\} \\
\text{Similar terms}
\]
Mode 7

\[
(Failure \ Rate)_7 = \frac{S}{N+S} \left\{ \lambda'(j) \left( \lambda(k)R6 \right) \left( \lambda'(jj)R9 \right) \left( \lambda(kk)R1 \right) + \right\}
\]
Similar terms

Mode 8

\[
(Failure \ Rate)_8 = \frac{S}{N+S} \left\{ \lambda'(j) \left( \lambda(k)R6 \right) \left( \lambda'(jj)R4 \right) \left( \lambda(kk)R1 \right) + \right\}
\]
Similar terms

Mode 9

\[
(Failure \ Rate)_9 = \frac{S}{N+S} \left\{ \lambda'(j) \left( \lambda'(k)R6 \right) \left( \lambda'(jj)R4 \right) \left( \lambda'(kk)R2 \right) + \right\}
\]
Similar terms

Mode 10

\[
(Failure \ Rate)_{10} = \frac{S}{N+S} \left\{ \lambda'(j) \left( \lambda'(k)R6 \right) \left( \lambda(jj)R3 \right) \frac{R1}{N} \left( \lambda'(kk)R2 \right) + \right\}
\]
Similar terms

Mode 11

\[
(Failure \ Rate)_{11} = \frac{S}{N+S} \left\{ \lambda'(j) \left( \lambda(k)R5 \right) \frac{R3}{N} \left( \lambda'(jj)R4 \right) \left( \lambda'(kk)R2 \right) + \right\}
\]
Similar terms

Mode 12

\[
(Failure \ Rate)_{12} = \frac{N}{N+S} \left\{ \lambda(j) \left( \frac{R5}{N} \right) \left( \lambda(k)R6 \right) \left( \lambda(jj)R4 \right) \left( \lambda(kk)R2 \right) + \right\}
\]
Similar terms

Mode 13

\[
(Failure \ Rate)_{13} = \frac{S}{N+S} \left\{ \lambda'(j) \left( \lambda(k)R5 \right) \left( \lambda(jj)R3 \right) \frac{R1}{N} \left( \lambda'(kk)R2 \right) + \right\}
\]
Similar terms
Mode 14

\[
(Failure\ Rate)_{14} = \frac{N}{N+S} \left\{ \lambda(j) \frac{R_5}{N} \left( \lambda(k)R_6 \right) \left( \lambda(jj)R_3 \right) \frac{R_1}{N} \left( \lambda(kk)R_2 \right) \right\}
\]

Similar terms

Mode 15

\[
(Failure\ Rate)_{15} = \frac{N}{N+S} \left\{ \lambda(j) \left( \lambda(k)R_5 \right) \frac{R_3}{N} \left( \lambda(jj)R_4 \right) \left( \lambda(kk)R_2 \right) \right\}
\]

Similar terms

Mode 16

\[
(Failure\ Rate)_{16} = \frac{N}{N+S} \left\{ \lambda(j) \left( \lambda(k)R_5 \right) \left( \lambda(jj)R_3 \right) \frac{R_1}{N} \left( \lambda(kk)R_2 \right) \right\}
\]

Similar terms

The similar terms described above are obtained as follows.

Let \( j, k, jj \) and \( kk \) in the above terms represent 1, 2, 3 and 4. The following sequence of changes is made and each change represents one term in the above expressions.

<table>
<thead>
<tr>
<th>j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>jj</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>kk</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

These changes have been made assuming component 1 fails first. Similar changes can be made for components 2, 3 and 4 failing first.

Thus the system failure rate, \( \lambda_{SL} \), is

\[
\lambda_{SL} = \sum_{i=1}^{16} (Failure\ Rate)_{i}
\]
The average outage duration, $r_{SL}$

$$
\frac{r(1) r(2) r(3) r(4)}{r(1) r(2) r(3) + r(1) r(2) r(4) + r(2) r(3) r(4) + r(1) r(3) r(4)}
$$

If repair is not carried out during adverse weather periods, then $R_2$, $R_4$ and $R_6$ values in the above expressions are replaced by $S$. In that case, the expected outage time of the system is given by:

$$
r_{SL} = \sum_{i=1}^{8} \frac{(\text{Failure rate})_i}{\lambda_{SL}} + \sum_{i=9}^{16} \frac{(\text{Failure rate})_i}{\lambda_{SL}}
$$

\begin{align*}
&= \left[ \frac{r(1) r(2) r(3) r(4)}{r(1) r(2) r(3) + r(1) r(2) r(4) + r(2) r(3) r(4) + r(1) r(3) r(4)} \right] \\
&\quad + \left[ \frac{r(1) r(2) r(3) r(4)}{r(1) r(2) r(3) + r(1) r(2) r(4) + r(1) r(3) r(4) + r(2) r(3) r(4)} + S \right]
\end{align*}
APPENDIX 4

STATE SPACE MODEL FOR FOUR AND FIVE COMPONENT SYSTEMS

Figure A.2 State Space Model For a Four Component System
THIS BLOCK CONTAINS COMPONENT OUTAGE STATES 33 TO 64 SIMILAR TO STATES 1 TO 32, USING ADVERSE WEATHER FAILURE RATES.

Figure A.3 State Space Model For a Five Component System
Figure A.4  Chronological Load Curve of S.P.C. System For One Week
(Line WI7 going to Watrons and Hatfield from Wolverine)
January 21, to January 27, 1972
**APPENDIX 6**

**LIST OF SYMBOLS**

\[ \lambda_{av} = \text{The average annual failure rate of a component.} \]
\[ \lambda = \text{The normal weather failure rate of a component.} \]
\[ \lambda' = \text{The adverse weather failure rate of a component.} \]
\[ \lambda'' = \text{The maintenance outage rate of a component.} \]
\[ \lambda''' = \text{The failure rate of a component in disaster adverse weather periods.} \]
\[ \lambda_{eq} = \text{The normal weather failure rate of an equivalent component.} \]
\[ \lambda'_{eq} = \text{The adverse weather failure rate of an equivalent component.} \]
\[ r = \text{The average repair time of a component.} \]
\[ r'' = \text{The average maintenance time of a component.} \]
\[ r_L = \text{The average duration of a load level L.} \]
\[ \mu = \text{The average repair rate of a component.} \]
\[ \mu'' = \text{The average maintenance rate of a component.} \]
\[ \mu_{eq} = \text{The average repair rate of an equivalent component.} \]
\[ N = \text{The average duration of a normal weather period.} \]
\[ S = \text{The average duration of an adverse weather period.} \]
\[ T = \text{The average duration of a disaster adverse weather period.} \]
\[ n = \text{The reciprocal of the average duration of a normal weather period.} \]
\[ m = \text{The reciprocal of the average duration of an adverse weather period.} \]
\[ \lambda_{SL} = \text{The failure rate of a load point due to overlapping component permanent outages.} \]
\[ \lambda''_{ML} = \text{The maintenance outage rate of a load point.} \]
\[ \lambda_{TL} = \text{The temporary outage rate of a load point.} \]
\[ \lambda_{OL} = \text{The overload outage rate of a load point.} \]
\[ r_{SL} = \text{The average duration associated with } \lambda_{SL}. \]
\[ r''_{ML} = \text{The average duration associated with } \lambda''_{ML}. \]
\( r_{oL} \) = The average duration associated with \( \lambda_{oL} \).

\( \lambda_s \) = The series system failure rate.

\( r_s \) = The series system average outage duration.

\( W \) = The fraction of component failures in adverse weather periods.