A HIGH SPEED
DESIGN AND OPTIMIZATION
METHOD FOR MILLIMETER WAVE
E-PLANE BANDPASS FILTERS

A Thesis Submitted to
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ABSTRACT

Millimeter wave waveguide filters are widely used in modern microwave communication systems, especially in personal communication systems. Traditional approximate and empirical design techniques cannot achieve accurate results and manual tuning becomes less efficient in the high order filter design. Since the 1970's, significant progress has been made in the computer-aided design of microwave circuits. With the increase in the availability and capability of computers, research has shifted its focus to the numerical characterization and modeling of waveguide filter components. In this thesis, a high speed computer-aided design method for E-plane waveguide bandpass filters is developed. Discontinuity between an empty waveguide and the bifurcated or trifurcated waveguide is analyzed only once. The obtained scattering matrices are stored and repeatedly used throughout the synthesis and optimization procedures. A new method that uses three optimization variables regardless of the filter order is adopted to optimize the initial design. The CAD program is verified by comparing analysis results with existing results from various sources and several high order filters are realized by the CAD program with desired specifications. With the computer-aided design method developed in this thesis, E-plane waveguide bandpass filter at millimeter wave frequencies range can be designed accurately without post-production tuning. A significant amount of computer memory and analysis time will be saved due to the new synthesis and optimization procedure.
DEDICATION

This thesis is dedicated to the author's parents, Chuchu Chen and Rui Wen for their everlasting love, care and encouragement.
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LIST OF SYMBOLS

\( \alpha \) \hspace{1cm} \text{Impedance scaling parameter} \\
\( \beta_{nm} \) \hspace{1cm} \text{Phase constant of the } nm\text{-th mode} \\
\( \gamma_{nm} \) \hspace{1cm} \text{Propagation constant of the } nm\text{-th mode} \\
\( \gamma_{IL} \) \hspace{1cm} \text{Isolation bandwidth factor} \\
\( \varepsilon \) \hspace{1cm} \text{Dielectric permittivity of the medium} \\
\( \varepsilon_0 \) \hspace{1cm} \text{Permittivity of free space} \\
\( \varepsilon_r \) \hspace{1cm} \text{Relative dielectric constant} \\
\( \eta \) \hspace{1cm} \text{Waveguide characteristic impedance} \\
\( \theta_i \) \hspace{1cm} \text{Phase constant of } i\text{-th resonator} \\
\( \lambda \) \hspace{1cm} \text{Wavelength} \\
\( \lambda_g \) \hspace{1cm} \text{Guide wavelength} \\
\( \lambda_{g,nm} \) \hspace{1cm} \text{Guide wavelength of the } nm\text{-th mode} \\
\( \lambda_{g0} \) \hspace{1cm} \text{Mid-band guided wavelength} \\
\( \lambda_{gL} \) \hspace{1cm} \text{Guide wavelength in the resonator section at } f_L \\
\( \lambda_{gH} \) \hspace{1cm} \text{Guide wavelength in the resonator section at } f_H \\
\( \lambda_0 \) \hspace{1cm} \text{Free space wavelength} \\
\( \lambda_{c,nm} \) \hspace{1cm} \text{Cutoff wavelength for } nm\text{-th mode}
\( \mu \)  
Dielectric permeability of the medium

\( \mu_0 \)  
Permeability of free space

\( \mu_r \)  
Relative permeability constant

\( \phi_i^X \)  
Normalized \( i \)-th mode in region \( X \)

\( \phi \)  
Phase angle of the K-inverter

\( \omega \)  
Angular frequency

\( a \)  
Broad dimension of rectangular waveguide

\( a_i \)  
Incident power at port \( i \)

\( b \)  
Narrow dimension of rectangular waveguide

\( b_i \)  
Reflected power at port \( i \)

\( f \)  
Frequency

\( f_{c,nn} \)  
Waveguide cutoff frequency

\( f_L \)  
Lower cutoff frequency of bandpass filter

\( f_H \)  
Upper cutoff frequency of bandpass filter

\( k_{c,nn} \)  
Cutoff wave number of the \( nn \)-th mode

\( k' \)  
Impedance inverter value

\( l_i \)  
Length of \( i \)-th resonator

\( m \)  
Integer determining the propagation mode along the \( y \) axis in waveguide

\( n \)  
Integer determining the propagation mode along the \( x \) axis in waveguide

\( t \)  
Thickness of the septum
\( j = \sqrt{-1} \)

\( w_i \) Length of \( i \)-th metal septum

\( x_s \) Normalized series reactance

\( x_p \) Normalized shunt reactance

\( A^+, B^-, C^- \) Incident field coefficients

\( A^-, B^+, C^+ \) Excited field coefficients

\( BW \) Bandwidth

\( E \) Electric field intensity

\( H \) Magnetic field intensity

\( H_0 \) Amplitude constant of magnetic field

\( H_{mn}^U \) Generic element of coupling matrix \([H^U]\)

\( IL \) Isolation

\( INL \) Insertion loss

\( K_{n,n+1} \) Normalized impedance inverter value

\( L \) Length of the transmission line

\( N \) Order of the filter

\( P_A \) Power absorbed by the filter

\( P_{in} \) Incident power from the generator

\( P_L \) Power transmitted to the load

\( P_R \) Power reflected back to the generator
RL \quad \text{Return loss}

S_{mn} \quad \text{Elements of } S\text{-parameter}

TE_{mn} \quad \text{Transverse electric mode}

TM_{mn} \quad \text{Transverse magnetic mode}

V^+ \quad \text{Incident wave voltage}

V^- \quad \text{Reflected wave voltage}

X_s \quad \text{Series reactance}

X_p \quad \text{Shunt reactance}

Y_i^x \quad \text{Wave admittance of } i\text{-th mode in region } X

Z_0 \quad \text{Characteristic impedance}

Z_n \quad \text{Impedance of the distributed element}

Z_{TE,nn} \quad \text{Wave impedance of } TE_{nn} \text{ mode}

Z_{TM,nn} \quad \text{Wave impedance of } TM_{nn} \text{ mode}
Chapter 1
Introduction

1.1 A Survey of the Microwave System

Microwaves are electromagnetic waves of very short wavelength. The manner in which the microwave spectrum fits into the overall electromagnetic spectrum is displayed in Figure 1.1. The electromagnetic spectrum is broken into bands for the sake of convenience and identification. Two kinds of designations are commonly used. Table 1.1 shows the industry standard of spectrum bands. It is designated by the Institute of Electrical and Electronic Engineers (IEEE).

In the band designated by the International Radio Consultative Committee (CCIR), the microwave band extends from 300 MHz to 300 GHz. The frequency range from 300 MHz to 3 GHz is called the ultra high frequency (UHF) band, the range from 3 to 30 GHz is the super high frequency (SHF) band, and the range from 30 to 300 GHz is the extremely high frequency (EHF) band. EHF band is also called the millimeter (mm) waveband, since the wavelength of electromagnetic radiation varies from 10 mm at 30 GHz to 1 mm at 300 GHz. Millimeter waves can be classified as
microwaves since millimeter wave technology is quite similar to microwave technology.

Figure 1.1: Electromagnetic spectrum.
Table 1.1: IEEE/Industry Standards Frequency Bands.

<table>
<thead>
<tr>
<th>Band</th>
<th>Frequency Range (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HF</td>
<td>0.003–0.030</td>
</tr>
<tr>
<td>VHF</td>
<td>0.030–0.300</td>
</tr>
<tr>
<td>UHF</td>
<td>0.300–1.000</td>
</tr>
<tr>
<td>L</td>
<td>1.00–2.00</td>
</tr>
<tr>
<td>S</td>
<td>2.00–4.00</td>
</tr>
<tr>
<td>C</td>
<td>4.00–8.00</td>
</tr>
<tr>
<td>X</td>
<td>8.00–12.0</td>
</tr>
<tr>
<td>Ku</td>
<td>12.0–18.0</td>
</tr>
<tr>
<td>K</td>
<td>18.0–27.0</td>
</tr>
<tr>
<td>Ka</td>
<td>27.0–40.0</td>
</tr>
<tr>
<td>Millimeter</td>
<td>40.0–300.0</td>
</tr>
<tr>
<td>Submillimeter</td>
<td>greater than 300</td>
</tr>
</tbody>
</table>

Over the frequency range of the microwave band, special microwave transmission lines, antennas, semiconductor devices, and tubes are required.

1.1.1 Why Microwave Devices Are Needed

At the very high frequency of microwaves, conventional transistors, ICs, and wiring will not work as a result of considerable lead reactance and transit time. Therefore, special microwave devices are required. The first problem, lead reactance, is illustrated in Figure 1.2. A 10 V AC oscillator is connected to a 50 Ω resistor by a copper wire 2.5 cm long and 1 mm in diameter. It has a dc resistance of only 0.4 mΩ and an inductance of 0.027 μH, which for low-frequency electronics is negligible compared with the 50 Ω load resistor. However, the inductive reactance \( X_L \) of the wire increases with frequency \( X_L = 2πfL \). At 60 Hz \( X_L = 10^{-3} \) Ω, which is negligible,
and the full 10 V signal from the oscillator appears across the load resistor. At 6 MHz $X_L = 1 \, \Omega$, which is still small compared to the 50 \, \Omega load resistor. But at 6 GHz, a microwave frequency, $X_L = 1000 \, \Omega$. As a result, almost all of the oscillator voltage is dropped across the connecting wire and is not received by the load resistor.

![Figure 1.2: Lead reactance.](image)

The lead reactance effect gets even worse at higher frequencies. Consequently, wires or printed circuit boards cannot be used to connect microwave devices. Special microwave transmission lines are required to conduct the microwave signal from one part of the microwave system to another part.

The second problem is transit time and is illustrated with a field-effect transistor (FET) (bipolar transistors and triode tubes suffer from similar problems.). In the FET, a change in gate voltage produces a change in the electron flow from the source to the drain. As the current flows through the resistor, the drain voltage is amplified under the control of the gate voltage. In practice, a finite time is required for the electrons to move from the source to the drain. At microwave frequencies, this
finite time becomes a large fraction of the signal cycle. This will dramatically reduce the quantity of the electrons that moves from the source to the drain. The resultant current and drain voltage under these conditions are reduced in amplitude compared with the low frequency case.

Because of the lead reactance and transit time problems, special microwave devices must be used in place of the wiring, transistors, and ICs generally used in low-frequency electronics.

1.1.2 Microwave Systems

Two special features, the high frequency and short wavelength, make microwaves desirable for communications and radar applications. The high frequency provides wide bandwidth capability. A 10% bandwidth system at 10 GHz provides a bandwidth of 1 GHz. All the information in all communication systems using the lower radio frequency spectrum can be incorporated into this bandwidth, including AM and FM radio, shortwave radio, broadcast television, and mobile radio. Because of the short wavelength of microwaves, high-gain antennas with narrow beam-width used in radar application can be constructed.

Most microwave systems are located in the 300 MHz to 30 GHz range. High atmospheric absorption exists at some frequencies in the 30 to 300 GHz range, so
long-range communication and radar systems are not practical. With continuing advances in microwave devices, more and more microwave systems are being developed in the millimeter portion of the microwave band.

1.1.3 Microwave Transmission Lines

Low frequency components, like transistors and capacitors at frequencies below the microwave range, are connected by wires. The electron flow in the wire carries the electrical signal from one part to another part of a component. Microwaves, however, cannot be conducted through wire, as was explained in the last section. Inside the microwave equipment, microwaves are still considered as waves. They travel with their wavelike characteristics from one part to another part [1]. Even though the parts are only a fraction of an inch apart, the microwaves travel as waves, and the microwave power must be guided from one part to another part. The device that guides the microwave power is called a microwave transmission line. Although there are hundreds of microwave transmission lines, there are only three basic types: waveguide, coaxial cable, and microstrip [2].

The shape and materials of each type of transmission line varies according to its application. A waveguide is a hollow metal pipe. It usually has a rectangular cross section, but can have a circular or oval cross section. The microwaves travel as waves from one component to the next through the waveguide.
Coaxial cable consists of an inner conductor surrounded by an outer conductor. An insulator fills the space between the two conductors, and the microwaves travel through the insulator. Microstrip consists of a conductor, an insulator, and a flat plate called the ground plane.

Waveguide offers the advantages of high power and low loss, but large size and narrow bandwidth. Coaxial cable, on the other hand, has large bandwidth and small size, but has high attenuation and limited power-handling capability. Microstrip allows complex circuits to be fabricated easily, but it offers very high loss.

1.2 Microwave Filters

An electrical filter is a device or circuit that selectively rejects some frequencies, while favoring other frequencies. The favored frequencies are called the passband, while the rejected frequencies are called the stopband. The filter operates by providing a large attenuation for stopband frequencies and a negligible attenuation for passband frequencies. Figure 1.3 shows the general representation of a filter in a microwave network. The power is divided into several components at the input and the output plane of the filter.

\[ P_{in} \] is the incident power from the generator.

\[ P_A \] is the power admitted to the filter.
$P_R$ is the power reflected back to the generator.

$P_L$ is the power transmitted to the load.

![Diagram of a filter circuit with input and output ports, showing power flow](image)

**Figure 1.3:** General filter representation.

By conservation of energy,

$$P_{in} = P_R + P_A$$

(1.1)

If the filter is lossless and there are no reflections, $P_L = P_A$ and $P_L = P_{in}$. The insertion loss (INL) in dB at a particular frequency can be defined as

$$INL = -10 \log \left( \frac{P_L}{P_{in}} \right)$$

(1.2)

while the return loss (RL) is given by

$$RL = -10 \log \left( \frac{P_R}{P_{in}} \right)$$

(1.3)

Filters at microwave frequencies are realized using distributed elements [3] (transmission lines and waveguides) or a combination of lumped (capacitors and inductors) and distributed elements. Electric and magnetic fields have to be considered in the design of filters using distributed elements, rather than the voltage and current as
in the case of lumped elements. Designing filters using distributed elements is more involved, and the synthesis involves complicated procedures requiring empirical adjustment to experimental filters for obtaining a filter with satisfactory frequency response. Figure 1.4 shows examples of waveguide filters realized by using either septa or irises coupled resonators.

![Waveguide filters](image)

(a) Septum coupled filter. (b) Iris coupled filter.

A typical application of a microwave filter is at the input of a microwave receiver. Microwave frequencies from many systems are picked up by the receiving antenna and could enter the receiver and cause interference. The filter allows only those frequencies in the assigned operating range of the system to pass and rejects all others. Filter performance is shown in Figure 1.5. The graph shows the attenuation of a microwave signal passing through the filter as a function of frequency. At frequencies below the filter passband, the attenuation is high and most of the microwave signal is
attenuated. In a narrow frequency range (about the center frequency of 10GHz, 10MHz on either side of the center frequency), almost all of signal passes through the filter. At frequencies above the passband, most of the signal is attenuated.

![Diagram showing filter performance](image)

**Figure 1.5: Performance of filters.**

Filters have four types of passbands. Figure 1.6(a) shows a low-pass filter, which passes all frequencies up to a certain frequency and then attenuates all higher frequencies. A high-pass filter illustrated in Figure 1.6(b) allows no signals to pass until a certain frequency is reached, and then passes all higher frequencies. A bandpass filter (Figure 1.6(c)) passes signals only in a specified frequency band. Above or below this band, the microwave signal is attenuated. In contrast, a bandstop filter (Figure 1.6(d)) passes microwave signals at almost all frequencies, except in a narrow range where it stops or prevents the signals from passing.
1.3 Background of CAD in Microwave Circuit Design

Since the 1970s, significant progress has been made in the computer-aided design of microwave circuits. A computer with appropriate software has become an indispensable tool in all phases of microwave circuit design procedures. Proper use of the computer in the design procedure allows better understanding of the particular
design problem, which leads to a significant decrease in the time and cost of experimental investigations. This fact is important for the design and manufacture of modern microwave networks because of the limitations for incorporating any modifications in circuits fabricated by microwave integrated circuit (MIC) technology and monolithic microwave integrated circuit (MMIC) technology.

Figure 1.7 schematically presents the sequence of steps for the CAD procedure that have to be performed to design a microwave circuit. In the initial design stage, microwave engineers use their knowledge of microwaves, experience, and intuition to determine the configuration and topology of the circuit. The performance of the initial circuit design is evaluated by computer-aided analysis methods. Computer-aided analysis allows the application of accurate and sophisticated models of passive and active circuit elements. Alternative designs may be investigated quickly and efficiently with the assistance of modern computer technology.

In the next stage of the CAD procedure, characteristics of a designed circuit computed by circuit analysis subroutines are compared with the given specifications. If the results do not satisfy the desired specifications, the circuit design parameters must be changed appropriately. The sequence of circuit analysis, comparison of circuit characteristics with design specifications, and parameter modification is performed until acceptable performance goals for the circuit are met. This kind of CAD of microwave circuits is performed by using optimization methods. The optimization
methods may also be used for parameter estimation of passive and active devices on the basis of experimental data.

![Diagram of design procedure]

**Figure 1.7:** Computer-aided design procedure.
Information on the influence of small changes in design parameters on circuit characteristics provides sensitivity analysis. Sensitivity analysis may be used in the optimization subroutines for efficient computation of the derivatives of performance functions.

1.4 Objectives of The Research

In the last decade, microcomputers have become rather inexpensive so that more and more engineers and designers use them in their microwave system designs. As a result, computational methods will become very important, because the means of applying them in microwave circuit design will continue to grow.

Millimeter-wave waveguide filters are widely used in modern communication systems, especially in personal communication systems due to their small size. Network theory and field theory are normally used in waveguide filters design and analysis in which microwave circuit elements are represented by a wide variety of matrices. One of these design examples can be found in [4] in which H-plane iris coupled filters are realized. Because the width of each iris in the filter is different, the CAD program has to analyze every discontinuity and store every obtained matrix. This will consume a large amount of time and computer resources. The problem becomes worse when the order of the filter increases.
The objective of this research is to develop a quick filter synthesis and optimization method for E-plane metal insert filters. The thickness of inserted metal septa remains constant. Therefore, discontinuities between an empty waveguide and the bifurcated or trifurcated waveguide are considered the same and will be analyzed only once. The obtained scattering matrices will be stored and used repeatedly throughout the synthesis and optimization procedures.

In an accurate analysis procedure, one has to take into account the interaction not only of the dominant mode but also of the higher order modes generated at the edge of septa. The dimension of scattering matrices will increase with the number of higher order modes. The developed method dramatically improves the synthesis and optimization speed, because it only needs to analyze one instead of multiple discontinuities just as in [4].

In this research, the work includes:

- Develop a high speed synthesis method for E-plane metal insert filters.
- Optimize the initial design data using a fixed number of optimization variables regardless the filter order.
- Use the developed CAD program to design E-plane waveguide filters.
• Compare computed result with existing results from published papers.

1.5 Organization of the Thesis

This thesis is organized into six chapters. The first chapter introduces the subject of the thesis and describes its organization. It also presents a brief background of microwave filter and a survey of CAD in microwave circuit design. Chapter two first gives an overview of a microwave waveguide that is used to develop the waveguide filter in the thesis. Then the network techniques, including scattering parameters and general scattering matrix technique, are presented as a tool to represent the waveguide. Waveguide discontinuities and analysis methods are introduced in chapter three. In chapter four, a method is developed for the fast synthesizing of E-plane bandpass filter with single or double metal insert. The complete CAD procedure is provided in this chapter. The optimization technique is also discussed in this chapter. This technique uses fewer optimization variables than that required by the existing techniques. Results from this design method are presented in chapter five, as well as the comparison with results from various resources. The thesis is concluded in chapter six. Future work is also represented in the last chapter.
Chapter 2

Literature Overview and Background

2.1 Literature Overview

Microwave filters are the most common passive components in any microwave network. Research on microwave filters commenced in 1937. Much of the work was done at various laboratories that were set up during World War II. A very good historical account of the research done in microwave filter design and development can be found in [5]. A concise summary of the development in the theory and design of microwave filters is given in [6]. N. Marcuvitz [7], R. Levy [8, 9], and many other well-known researchers contributed to the development of filter design. Much of the earlier work done in filter design was based on empirical relations and models for the waveguide discontinuities obtained directly for Marcuvitz's work [7].

In the past two decades, with the increase in the availability and capability of computers, research has shifted its focus to the numerical characterization and modeling of waveguide filter components. A good review of the computational methods used for the solution of electromagnetic fields along with an introduction to
numerical methods is given in [10]. Some numerical methods that are applied to the passive components are described in [11]. A specific numerical method is selected based on the type of structure being analyzed and on the memory requirements and CPU speed of the computer. Although complete numerical 3D analysis techniques, such as the finite element or finite difference approach, have been commercially available, the analytical mode matching method (Chapter 3) proves to be faster and more efficient for analyzing and optimizing waveguide components having well defined boundaries [12, 13].

In the early 1980s, E-plane technology was in transition between the laboratory and the commercial world. Today its applications are extensive. However, a significant amount of research is still conducted on E-plane circuits and systems. Most of the advances are relatively significant and the whole subject is nearing its maturity.

E-plane filters in the form of metal insert mounted in the E-plane of a rectangular waveguide were originally proposed in [14] as low cost mass-producible circuits for microwave frequencies. With the widespread use of integrated E-plane circuits in millimeter-wave applications, metal insert filters have been under active investigation over the past few years. This is evidenced by the considerable number of papers published on the design of this type of filter [15-25].

The reason for their suitability to mass production is that the inserts can be developed easily on metal sheets by photo-etching, stamping or pressing. Moreover, in
contrast to fin-line filters, metal insert filters have no dielectric substrates that cause additional losses. Therefore, they are well suited for narrow-band high Q application such as converters.

In practice, the filter structure consists of a number of resonators separated by inductive axial septa. Owing to the difficulties which arise in tuning filters of small dimensions in applications, it is desirable that the design procedure of these filters be based on a highly accurate analysis that includes the effect of the finite metallization thickness and the higher order mode coupling between the septa. Thus, metal insert filters are usually designed using a computer-aided design (CAD) algorithm consisting of an analysis routine and an optimization routine which in turn fine tunes the filter circuit to make its performance satisfy a set of specifications [15, 18, 22].

In this thesis a successful attempt is made to develop a CAD method for the design, analysis and optimization of E-plane bandpass filters. By efficient combination of modern network theory and field theory the speed and memory requirements of the program developed were improved without losing the accuracy of the design.

2.2 The Foundation of Waveguides

Waveguides are hollow metal pipes and may have either circular or rectangular cross sections. Figure 2.1 shows an end view of a rectangular waveguide.
The dimension $a$ is the wider dimension, and $b$ is the more narrow one. These letters are considered the standard form of notation for waveguide dimensions and will be used throughout this thesis.

![Rectangular Waveguide](image)

**Figure 2.1:** End view of the rectangular waveguide.

Figure 2.2 shows the coordinate system used to denote dimensions and directions in microwave discussions. The $a$- and $b$-dimensions of the waveguide correspond to the $x$ and $y$ axes of a Cartesian system, while the $z$ axis is the direction of wave propagation.

Although a waveguide is a transmission line, it is different from a conventional two-wire transmission line. The signal in a microwave waveguide propagates as an electromagnetic wave, not as a current.

The word MODE is used to describe a wave that travels along a waveguide. Unlike free space waves, there can be many modes travelling along a waveguide at a single frequency. Each mode has a different electric and magnetic field configuration, and in an ideal waveguide there will be no interaction between different modes.
The specific type of field found in transmission lines is a transverse electromagnetic (TEM) field. The term transverse implies fields at right angles to each other. Thus, the electric and magnetic fields are perpendicular to the direction of travel.

The TEM wave will not propagate in a waveguide because certain boundary conditions apply. While the wave in the waveguide propagates through the air in a manner similar to free-space propagation, the phenomenon is bound by the walls of the waveguide which implies that certain conditions must be met. The boundary conditions for waveguides are as follows:

- The electric field must be orthogonal to the conductor in order to exist at the surface of the conductor.
- The magnetic field must not be orthogonal to the conductive surface of the waveguide.
To satisfy these boundary conditions, the waveguide gives rise to two types of propagation modes: transverse electric mode and transverse magnetic mode. For transverse electric, all the electric field components are transverse to the direction of travel. In transverse magnetic modes, all the magnetic field components are transverse to the direction of travel. The TEM mode violates the boundary conditions because the magnetic field is not parallel to the surface and thus, it does not occur in waveguides.

When describing the various modes of propagation, a shorthand notation is used:

\[ TE_{nm} \text{ or } TM_{nm} \]

where,

\( TE \) stand for transverse electric mode,

\( TM \) for transverse magnetic mode,

\( n \) is the number of half-wavelengths along the \( x \) axis (the \( a \) dimension) and

\( m \) is the number of half-wavelengths along the \( y \) axis (the \( b \) dimension).

Some electrical parameters are used to express characteristics of microwave waveguides. These include:

- Electric and magnetic field patterns
- Cut-off frequencies of modes
- Impedance of modes
- Guide wavelength of modes
These parameters will be discussed here using the case of rectangular waveguide that is used to realize the filter structure in this thesis.

When considering the source-free interior region of a rectangular waveguide, Maxwell's equations are written as follows:

\begin{align*}
\epsilon \nabla \cdot E &= 0 \quad (2.1) \\
\mu \nabla \cdot H &= 0 \quad (2.2) \\
\nabla \times E &= -j\omega \mu H \quad (2.3) \\
\nabla \times H &= j\omega \epsilon E \quad (2.4)
\end{align*}

where,

\begin{align*}
\mu &= \mu_0 \mu_r, \quad \epsilon = \epsilon_0 \epsilon_r, \text{ and } \omega = 2\pi f, \\
\mu_0 \text{ and } \epsilon_0 \text{ are the permeability and permittivity of free space respectively,} \\
\mu_r \text{ and } \epsilon_r \text{ are the relative permeability and relative permittivity respectively of the medium filling the wave guide and} \\
f \text{ is the operating frequency.}
\end{align*}

The following wave equations are obtained by the solutions of Maxwell's equations in rectangular coordinates:

\begin{align*}
\nabla^2 E + \omega^2 \mu \epsilon E &= 0 \quad (2.5) \\
\nabla^2 H + \omega^2 \mu \epsilon H &= 0 \quad (2.6)
\end{align*}
Waveguide modes need to have a component of the field in the direction of propagation. These are called the longitudinal components of field which are the \( z \)-directed components of the field. There is an equation for each of the three components of the electric field and another three equations for components of the magnetic field.

When only the \( z \)-component is considered and it assumes a wave propagating in the \( +z \) direction with propagation constant \( \gamma \), the \( z \)-component of equation (2.5, 2.6) is obtained.

\[
\nabla_z^2 E_z(x, y) + (\omega^2 \mu \varepsilon + \gamma^2) E_z(x, y) = 0 \tag{2.7}
\]

\[
\nabla_z^2 H_z(x, y) + (\omega^2 \mu \varepsilon + \gamma^2) H_z(x, y) = 0 \tag{2.8}
\]

By applying the boundary conditions and the method of separation of variables, all field components of TE and TM modes are obtained.

For TE modes, the longitudinal magnetic component of the field is

\[
H_z(x, y) = H_0 \cos \frac{m \pi x}{a} \cos \frac{m \pi y}{b} \tag{2.9}
\]

where \( H_0 \) is the amplitude constant.

Note that \( n = 0, 1, 2, \ldots \) and \( m = 0, 1, 2, \ldots \), but \( n \) and \( m \) cannot both equal 0, because all the other field components would vanish.
The propagation constant is given by

\[ \gamma_{nn} = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2} - \omega^2 \mu \epsilon \]  

(2.10)

The cutoff wave number can be found by

\[ k_{c,nn} = \sqrt{\frac{n^2 \pi^2}{a^2} + \frac{m^2 \pi^2}{b^2}} = 2\pi f_c \sqrt{\mu \epsilon} \]  

(2.11)

The cutoff frequency is defined as the frequency that makes \( \gamma_{nn} = 0 \). Thus

\[ f_{c,nn} = \frac{1}{2\pi \sqrt{\mu \epsilon}} \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2} \]  

(2.12)

If \( f > f_{c,nn} \), the \( nm \)-th mode will propagate and \( \gamma_{nn} = j\beta_{nm} \). The phase constant \( \beta_{nm} \) is

\[ \beta_{nm} = \sqrt{\omega^2 \mu \epsilon - k_{c,nn}^2} = \frac{2\pi}{\lambda_{g,nn}} \]  

(2.13)

where \( \lambda_{g,nn} \) is guided wavelength.

If \( f < f_{c,nn} \), from Equation (2.10), \( \gamma_{nn} \) is real, the wave is attenuated and does not propagate.

In the waveguide, the distance along the \( z \) axis between two points of constant phase is called the guided wavelength. The guide wavelength is greater than the free space wavelength \( \lambda_0 \). The guide wavelength \( \lambda_g \) of \( nm \)-th mode is given by
The wave impedance is defined as the ratio of the electric field to the magnetic field. For the \(TE_{mn}\) mode, the wave impedance is given by

\[
Z_{TE_{mn}} = \frac{E_y}{H_x} = \frac{\omega \sqrt{\mu \varepsilon}}{\beta_{nm}} \eta
\]

where \(\eta = \sqrt{\frac{\mu}{\varepsilon}}\) is called the waveguide characteristic impedance.

For \(TM\) modes, a similar procedure can be followed. By solving the Equation (2.7), the longitudinal component of the electric field is obtained.

\[
E_z(x,y) = E_0 \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad n = 1, 2, 3, \ldots \quad m = 1, 2, 3, \ldots
\]

The equations for \(\gamma_{mn}\), \(\beta_{nm}\), \(f_{c,nn}\), and \(\lambda_{g,nn}\) are the same as for \(TE_{mn}\) mode.

The wave impedance is

\[
Z_{TM_{mn}} = \frac{E_z}{H_y} = \frac{\beta_{nm}}{\omega \sqrt{\mu \varepsilon}} \eta
\]

The cutoff frequencies for the various modes of a particular waveguide can be arranged in ascending order of magnitude. It may happen that two or more modes have the same cutoff frequency. Such modes are referred to as degenerate. If the lowest
mode is not degenerate, excitation of the waveguide at a frequency above the lowest cutoff, but less than the second highest, can only produce one propagating mode. Other modes that may be initially produced will be attenuated along the waveguide, leaving only the lowest mode propagating. The lowest mode is then said to be dominant.

The $TE_{10}$ mode is the dominant mode in the rectangular waveguide and is the best mode for low-attenuation propagation in the $z$ axis. The nomenclature $TE_{10}$ indicates that there is one half-wavelength in the $a$-dimension and zero half-wavelengths in the $b$ dimension. The dominant mode exists at the lowest frequency at which the waveguide is half-wavelength. The propagation constant, guide wavelength, cutoff frequency, and cutoff wavelength for the $TE_{10}$ mode are given by

$$\beta_{10} = \sqrt{\omega^2 \mu \varepsilon - \left(\frac{\pi}{a}\right)^2}$$  \hspace{1cm} (2.18)

$$\lambda_{g,10} = \frac{2\pi}{\sqrt{\omega^2 \mu \varepsilon - \left(\frac{\pi}{a}\right)^2}}$$  \hspace{1cm} (2.19)

$$f_{c,10} = \frac{1}{2a\sqrt{\mu \varepsilon}}$$  \hspace{1cm} (2.20)

$$\lambda_{c,10} = 2a$$  \hspace{1cm} (2.21)
2.3 Scattering Parameters

Circuit elements can be characterized as having two or more terminals. Elements that are connected together in some fashion are known as networks. Network analysis is concerned with the characteristics of the complete network rather than the individual components. The pair of external terminals used to connect a termination is known as a port, and a network with \( n \) ports is called an \( n \)-port network.

Different network parameters, or coefficients, are used to represent a network. In the low frequency range, impedance parameters (\( z \)-parameters), admittance parameters (\( y \)-parameters) and chain parameters (\( ABCD \)-parameters) are used to evaluate the network.

However, the network parameters listed above require open and short circuits to evaluate the coefficients. As the frequency moves into microwave range, it becomes difficult to realize an open or short circuit. In addition, many active devices react to open and short circuit terminations by oscillating, and any measurements made under these conditions are meaningless for linear circuit design.

One particularly useful network representation, which was developed to characterize microwave circuits, is scattering or \( S \)-parameters. The \( S \)-parameters can be measured using any convenient termination and, unlike other parameter sets, exist
for most practical networks. The most important feature is the ease and accuracy with which these parameters can be measured at very high frequencies and the direct physical interpretation of the coefficients.

Figure 2.3 shows a two-port network and its S-parameters. At the input port (port 1), the incident and reflected wave voltages are \( V_1^+ \) and \( V_1^- \) respectively. At the output port (port 2), they are \( V_2^+ \) and \( V_2^- \). The S-parameters are defined by

\[
V_1^- = S_{11} V_1^+ + S_{12} V_2^+
\]

\[
V_2^- = S_{21} V_1^+ + S_{22} V_2^+
\]

S-parameters are complex variables relating the incident wave to the reflected wave. The S-parameters are given by

\[
S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} = \text{reflection coefficient at port 1 with } V_2^+ = 0,
\]

\[
S_{12} = \left. \frac{V_1^-}{V_2^+} \right|_{V_1^+ = 0} = \text{transmission coefficient from port 2 to port 1 with } V_1^+ = 0,
\]

\[
S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0} = \text{transmission coefficient from port 1 to port 2 with } V_2^+ = 0,
\]

\[
S_{22} = \left. \frac{V_2^-}{V_2^+} \right|_{V_1^+ = 0} = \text{reflection coefficient at port 2 with } V_1^+ = 0.
\]
Figure 2.3: Scattering parameters for two-port networks.

The uniform transmission line can be regarded as a two-port junction. Figure 2.4 shows a uniform transmission line of length $L$. The incident and reflected waves are $a_1$ and $b_1$ in port 1, and $a_2$ and $b_2$ in port 2. Then from the following equations,

$$b_1 = a_2 e^{-eta L} \quad (2.24)$$

$$b_2 = a_1 e^{-eta L} \quad (2.25)$$

the scattering matrix for the transmission line in the reference planes A and B is obtained as follow.

$$[S] = \begin{bmatrix} 0 & e^{-eta L} \\ e^{-eta L} & 0 \end{bmatrix} \quad (2.26)$$

where $\beta$ is the propagation constant.
The two-port S-parameters can be extended to an N-port network like shown in Figure 2.5.

In this case, the S matrix is given by the following equation:

\[
\begin{bmatrix}
 b_1 \\
 b_2 \\
 b_3 \\
 \vdots \\
 b_N \\
\end{bmatrix}
=
\begin{bmatrix}
 S_{11} & S_{12} & \cdots & S_{1N} \\
 S_{21} & S_{22} & \cdots & S_{2N} \\
 \vdots & \vdots & \ddots & \vdots \\
 S_{N1} & S_{N2} & \cdots & S_{NN} \\
\end{bmatrix}
\begin{bmatrix}
 a_1 \\
 a_2 \\
 a_3 \\
 \vdots \\
 a_N \\
\end{bmatrix}
\]

(2.27)
According to the above equation, the elements of the scattering matrix can be derived. The definition of each parameter is similar to that for a two-port network.

\[ S_{ii} = \frac{b_i}{a_i} \Bigg|_{a_n=0, n \neq i} \]  

= reflection coefficient at port \( i \) when all other ports are matched.

\[ S_{in} = \frac{b_n}{a_i} \Bigg|_{a_n=0, n \neq i} \]  

= transmission coefficient from port \( i \) to port \( n \) when all other ports are matched.

The condition \( a_n = 0, n \neq i \) means that no circuit port, except port \( i \), is excited by a signal generator and each simultaneously terminates in its reference impedance which is not displayed in Figure (2.5), so that the waves \( b_n \) from those ports are totally absorbed in these terminations.

2.4 Generalized Scattering Matrix

The scattering matrix technique discussed in the last section describes the reflection and transfer characteristics of a junction. It is useful when it deals with the signal flow from a junction in a single-moded transmission line system. The method, however, becomes unsatisfactory in two situations. First, when the transmission line system is multi-moded, all of the propagating modes interact via discontinuity and S-parameters can be defined for each of these modes. The other situation occurs when two discontinuities are located in an extreme proximity. In this case, interactions
between two discontinuities of all higher-order modes can no longer be neglected even if these modes are evanescent. The generalized scattering matrix (GSM) technique first introduced by Mittra and Pace [31] is the way to treat these kinds of problems.

GSM technique is an extension of the conventional scattering matrix technique in a single-moded transmission line system. It takes into account the dominant modes, as well as all of the high-order modes including evanescent ones. Moreover, in the case of two cascaded junctions, by using the generalized S-parameters of each junction, the overall scattering phenomena including the interactions between the two junctions can be described accurately. Since all the higher order modes are included, the interaction between junctions can be correctly described even if that the distance between them is extremely small. Furthermore, the scattering characteristics of a complicated discontinuity can be described if the generalized scattering parameters of each of the simpler junctions created by decomposition of the original junction can be found.

The definition of GSM is explained in a bifurcated waveguide problem (Figure 3.3). If the $n$-th TE mode ($TE_{n0}$) is incident upon the junction plane ($z=0$) from region $I$, fields are reflected back to region $I$ and transmitted into region $II$ and $III$. The $y$ direction component of the fields ($E_y$) is expanded in terms of the modes in each region and the coefficient of the $n$-th mode referred to the plane $z=0$ is defined as the amplitude of that mode. Assume that the amplitude of the $n$-th mode incident in region $I$ is one, then the amplitude of the $m$-th scattered mode is $S_{11}(m, n)$ and the amplitude
of the \( p \)-th and \( q \)-th scattered modes transmitted (not necessarily propagating) in region II and III are \( S_{21}(p, n) \) and \( S_{31}(q, n) \). \( S_{11}(m, n), S_{21}(p, n) \) and \( S_{31}(q, n) \) are the generalized scattering elements of matrices \([S_{11}],[S_{21}]\) and \([S_{31}]\), respectively. Other generalized scattering matrix coefficients are similarly defined. It should be noted that these generalized scattering matrices are infinite-dimensional matrices in general.

In microwave structures, the incident signal scatters into an infinite number of modes at the discontinuity. For a two-port network consisting of waveguide structure (Figure 2.3), there may exist more than one transmitted and reflected signal wave at each port. It is difficult to express such a structure using conventional scattering parameters by equations (2.22, 2.23). The problem can be solved easily by using generalized scattering matrix technique in which each scattering parameter no longer remains a single element but is expressed in the form of a matrix. The definition of general scattering parameters of a two-port network is rewritten below:

\[
[S_{11}] = \begin{bmatrix} V_{i}^{-} \\ V_{i}^{+} \end{bmatrix} \begin{bmatrix} \gamma_{1} \end{bmatrix} \quad (2.28a)
\]

\[
[S_{12}] = \begin{bmatrix} V_{i}^{-} \\ V_{i}^{+} \end{bmatrix} \begin{bmatrix} \gamma_{2} \end{bmatrix} \quad (2.28b)
\]

\[
[S_{21}] = \begin{bmatrix} V_{i}^{-} \\ V_{i}^{+} \end{bmatrix} \begin{bmatrix} \gamma_{1} \end{bmatrix} \quad (2.28c)
\]

\[
[S_{22}] = \begin{bmatrix} V_{i}^{-} \\ V_{i}^{+} \end{bmatrix} \begin{bmatrix} \gamma_{2} \end{bmatrix} \quad (2.28d)
\]

where,
\([V_1^+], [V_2^+]\) are the incident electromagnetic fields to port 1 and 2, respectively, in the form of column vectors,

\([V_1^-], [V_2^-]\) are the reflected electromagnetic fields from port 1 and 2, respectively, in the form of column vectors and

\([0]\) is a column null vector.

The equation relating to the overall scattering matrix becomes:

\[
\begin{bmatrix}
[V_1^-] \\
[V_2^-]
\end{bmatrix} =
\begin{bmatrix}
[S_{11}] & [S_{12}]
\end{bmatrix}
\begin{bmatrix}
[V_1^+]
\end{bmatrix}
\]

(2.29)

For an explanation, consider a simple two modes case. The scattering matrix is expressed as a matrix of matrices as shown below:

\[
[S] =
\begin{bmatrix}
S_{11}(1, 1) & S_{11}(1, 2) & S_{12}(1, 1) & S_{12}(1, 2) \\
S_{11}(2, 1) & S_{11}(2, 2) & S_{12}(2, 1) & S_{12}(2, 2) \\
\cdots & \cdots & \cdots & \cdots \\
S_{21}(1, 1) & S_{21}(1, 2) & S_{22}(1, 1) & S_{22}(1, 2) \\
S_{21}(2, 1) & S_{21}(2, 2) & S_{22}(2, 1) & S_{22}(2, 2)
\end{bmatrix}
\]

(2.30)

Assume that \(TE_{10}\) mode is the incident mode and \(TE_{20}\) mode propagates in addition to the incident mode. \(S_{11}, S_{12}, S_{21}, S_{22}\) have the same meaning as before. The first and second labels in the bracket represent the incident and reflected mode respectively. The value of "1" stands for \(TE_{10}\) mode, while the value of "2" stands for \(TE_{20}\) mode. Therefore, \(S_{21}(1, 2)\) represents the transmission coefficient with incident \(TE_{10}\) mode at port one resulting in a transmitted \(TE_{20}\) mode at port two.
2.5 Summary

An overview of the transmission mode of wave in waveguide has been presented in this chapter. Scattering parameters are useful to represent the single mode traveling along the waveguide. However, scattering parameters become unsatisfactory when multiple modes existing in the waveguide or when there are discontinuities in the waveguide. These situations will be encountered when waveguide filter is developed in this thesis. Thus, generalized scattering matrix technique, an extension of the conventional scattering parameter technique, is used in this design.
Chapter 3
Waveguide Discontinuities and Analysis Techniques

At low frequencies, circuit elements such as lumped resistances, lumped inductances, and lumped capacitances are combined in various ways to form circuits or networks for different purposes. As mentioned in chapter 1, such lumped elements will not work at microwave frequencies. Instead, a waveguide or any other type of transmission line is used to replace conventional forms of circuit elements.

At microwave frequencies, circuit elements are obtained by introducing discontinuities along transmission lines or waveguides. These discontinuities could be introduced in the form of certain shapes of conductors, dielectrics, or magnetic materials. They produce reflections and may change the propagation characteristics of the electromagnetic wave. For this reason, waveguide elements are far more complicated than low-frequency wire lines, and the evaluation of their characteristics is comparatively difficult. In this chapter, the waveguide discontinuities are introduced and the most efficient and accurate analysis method that can solve the discontinuity problem is discussed.
3.1 Waveguide Discontinuities

A straight, uninterrupted transmission line is considered continuous. Any
bends, shorted or open circuits, gaps, width changes, external loads and transitions are
discontinuities in the transmission line. Sometimes discontinuities are undesirable and
cause reflection and mismatch. In other cases, discontinuities can be used to
accomplish impedance matching, filtering and many other functions.

Whenever there is an abrupt change in the dimensions of the waveguide, the $E$
and $H$ fields make a transition across the region. Higher-order modes propagate at the
point of these abrupt changes or discontinuities. This effectively changes the
impedance of the guide at this point. The higher-order modes rapidly attenuate outside
this region, because the guide is physically made for the propagation of the dominant
mode only.

In a rectangular waveguide, shunt susceptances can be made in a variety of
ways and attached to the guide. Inductive susceptances can be obtained by using the
discontinuities shown in Figure 3.1 and the discontinuities giving rise to capacitive
susceptances are shown in Figure 3.2.
Figure 3.1: Inductive elements in rectangular waveguide.
(a) Asymmetrical window. (b) Symmetrical window.
(c) Double windows. (d) Triple windows.

Figure 3.2: Capacitive elements in rectangular waveguide.
(a) Asymmetrical window. (b) Symmetrical window.
(c) Double windows. (d) Ridged windows.
3.2 The Mode Matching Method

The mode matching method [11] is useful when dealing with the geometry of the structure that can be identified as a junction of two or more regions, like waveguide discontinuities. Considering the contributions from the propagating and evanescent TE and TM modes to the overall electromagnetic field, the mode matching technique has the ability to include higher order mode’s excitations and interactions. Thus, the method is more accurate for analyzing waveguide discontinuity when compared with other numerical techniques, like the Finite Element Method [11].

The mode matching procedure begins with the expansion of unknown fields in the individual regions in terms of their respective normal modes. Since the functional form of the normal modes is known, what is left in the procedure is to solve an infinite set of linear simultaneous equations to determine the unknown modal coefficients associated with the field expansions in various regions. Since it is impossible to achieve an exact solution for this infinite set of equations, the number of modes is truncated to a finite number in order to achieve approximated results.

The method will be demonstrated by analyzing an infinitely thin and semi-infinitely long septum in a waveguide as shown in Figure 3.3. Assume that the waveguide is lossless and there is no field or structural variation in the \( y \)-direction so that the \( y \) dependent functions are eliminated in the discussion.
The total electric and magnetic fields in Region I (z<0) are

\[ E_y^I = \sum_{i=1}^{k} [A_i^+ \phi_i^I(x) e^{-\gamma_I z} + A_i^- \phi_i^I(x) e^{\gamma_I z}] \]  

(3.1)

\[ H_x^I = \sum_{i=1}^{k} Y_i^I [A_i^+ \phi_i^I(x) e^{-\gamma_I z} - A_i^- \phi_i^I(x) e^{\gamma_I z}] \]  

(3.2)

Figure 3.3: Mode matching in bifurcated waveguide.

For Region II (z>0, 0<x<a_1)

\[ E_y^II = \sum_{i=1}^{M} [B_i^+ \phi_i^II(x) e^{-\gamma_II z} + B_i^- \phi_i^II(x) e^{\gamma_II z}] \]  

(3.3)

\[ H_x^II = \sum_{i=1}^{M} Y_i^II [B_i^+ \phi_i^II(x) e^{-\gamma_II z} - B_i^- \phi_i^II(x) e^{\gamma_II z}] \]  

(3.4)

For Region III (z>0, a_1<x<a_2)

\[ E_y^III = \sum_{i=1}^{N} (C_i^+ \phi_i^{III}(x) e^{-\gamma_III z} + C_i^- \phi_i^{III}(x) e^{\gamma_III z}) \]  

(3.5)

\[ H_x^III = \sum_{i=1}^{N} Y_i^{III} (C_i^+ \phi_i^{III}(x) e^{-\gamma_III z} - C_i^- \phi_i^{III}(x) e^{\gamma_III z}) \]  

(3.6)
where,

\[ A_i^+ , \quad B_i^- , \quad \text{and} \quad C_i^- \] are the given incident field coefficients from regions I, II, and III, respectively,

\[ A_i^- , \quad B_i^+ , \quad \text{and} \quad C_i^+ \] are the unknown excited field coefficients in regions I, II, and III, respectively,

\( \phi_i^I , \quad \phi_i^\text{II} , \quad \text{and} \quad \phi_i^\text{III} \) are normalized modes in regions I, II, and III, with propagation constants \( \gamma_i^I , \quad \gamma_i^\text{II} , \quad \text{and} \quad \gamma_i^\text{III} \), respectively. The general form of normalized modes is given in [4]. In this case, normal mode in each region is

\[ \phi_i^I (x) = \sqrt{\frac{2}{a}} \sin \left( \frac{\eta x}{a} \right) \]  
\[ \phi_i^\text{II} (x) = \sqrt{\frac{2}{a_1}} \sin \left( \frac{m \eta x}{a_1} \right) \]  
\[ \phi_i^\text{III} (x) = \sqrt{\frac{2}{a_2}} \sin \left( \frac{m \eta x}{a_2} \right) \]

\( Y_i^I , \quad Y_i^\text{II} \) and \( Y_i^\text{III} \) are corresponding wave admittance of \( i \)-th mode in Region I, II, III and are defined as follows:

\[ Y_i^q = \frac{\gamma_i^q}{j \omega \mu} \quad q = I, \ II \ or \ III \]  

Considering the fact that the field is continuous at the junction, the electric field equation can be obtained as
The corresponding magnetic field equation is

\[ \sum_{i=1}^{L} (A_i^+ + A_i^-) \phi_i^l (x) = \begin{cases} \sum_{i=1}^{M} (B_i^+ + B_i^-) \phi_i^{ll} (x) & 0 \leq x \leq a_i \\ \sum_{i=1}^{N} (C_i^+ + C_i^-) \phi_i^{lll} (x) & a_i < x \leq a \end{cases} \]  
(3.11)

Furthermore, the orthogonality property of modal functions is used to derive a set of equations involving the unknown field coefficients. This is done by multiplying both sides of equation (3.11) by \( \phi_i^l (x) \) and integrating it with respect to \( x \) from 0 to \( a \).

\[ \int_0^a \left( \sum_{i=1}^{L} (A_i^+ - A_i^-) Y_i^l \phi_i^l (x) \right) \sqrt{\frac{2}{a}} \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi x}{a} \right) \, dx \]

\[ = \sum_{m=1}^{M} (B_m^+ - B_m^-) \int_0^{a_i} \sqrt{\frac{2}{a_i}} \sin \left( \frac{m \pi x}{a_i} \right) \sin \left( \frac{n \pi x}{a_i} \right) \, dx \]

\[ + \sum_{n=1}^{N} (C_n^+ - C_n^-) \int_0^{a_i} \sqrt{\frac{2}{a_i} \frac{a_i}{a_i - a}} \sin \left( \frac{m \pi x}{a_i} \right) \sin \left( \frac{n \pi x}{a_i} \right) \, dx \]  
(3.13)

Now, from the left-hand side of equation (3.13), the following equation is obtained.
\[
\int_0^a \sin\left(\frac{b_1 x}{a}\right) \sin\left(\frac{b_2 x}{a}\right) \, dx = \begin{cases} 
0 & \text{if } r \neq l \\
\frac{a}{2} & \text{if } r = l 
\end{cases}
\] (3.14)

Thus, only \( r = l \) will be taken into account. Since \( a_2 = a - a_1 \), equation (3.13) becomes

\[
\sqrt{\frac{2}{a}} \sum_{\ell=1}^{N} \left(A_\ell^+ + A_\ell^-\right) = \sqrt{\frac{2}{a}} \sum_{m=1}^{N} \sin\left(\frac{m \pi x}{a_1}\right) \sin\left(\frac{h \xi}{a_2}\right) \, dx
\]

\[
+ \sqrt{\frac{2}{a}} \sum_{n=1}^{N} \left(C_n^+ + C_n^-\right) \sin\left(\frac{m \pi x}{a_1}\right) \sin\left(\frac{h \xi}{a_2}\right) \, dx
\] (3.15)

Following the same procedure, the magnetic field equation is obtained from (3.12).

\[
\sqrt{\frac{2}{a}} \sum_{\ell=1}^{N} \left(Y_\ell^+ - A_\ell^\prime\right) = \sqrt{\frac{2}{a}} \sum_{m=1}^{N} Y_m^\prime \left(B_m^+ + B_m^-\right) \sin\left(\frac{m \pi x}{a_1}\right) \sin\left(\frac{h \xi}{a_2}\right) \, dx
\]

\[
+ \sqrt{\frac{2}{a}} \sum_{n=1}^{N} Y_n^\prime \left(C_n^+ - C_n^-\right) \sin\left(\frac{m \pi x}{a_1}\right) \sin\left(\frac{h \xi}{a_2}\right) \, dx
\] (3.16)

Equation (3.15) and (3.16) can be expressed easily by following matrix equations that have a more straightforward look.

\[
A^+ + A^- = [H^{I,II}] (B^+ + B^-) + [H^{I,III}] (C^+ + C^-)
\] (3.17)

\[
[Y^\prime] (A^+ - A^-) = [H^{I,II}] [Y^\prime] (B^+ - B^-) + [H^{I,III}] [Y^\prime] (C^+ - C^-)
\] (3.18)

where,
The known and unknown field coefficients are defined as vector forms

\[
A^+ = \begin{bmatrix}
A_1^+ \\
A_2^+ \\
\vdots \\
A_L^+
\end{bmatrix}, \quad B^- = \begin{bmatrix}
B_1^- \\
B_2^- \\
\vdots \\
B_M^-
\end{bmatrix}, \quad C^- = \begin{bmatrix}
C_1^- \\
C_2^- \\
\vdots \\
C_N^-
\end{bmatrix}
\]

\[
A^- = \begin{bmatrix}
A_1^- \\
A_2^- \\
\vdots \\
A_L^-
\end{bmatrix}, \quad B^+ = \begin{bmatrix}
B_1^+ \\
B_2^+ \\
\vdots \\
B_M^+
\end{bmatrix}, \quad C^+ = \begin{bmatrix}
C_1^+ \\
C_2^+ \\
\vdots \\
C_N^+
\end{bmatrix}
\]

\([H^{I,II}]\) and \([H^{I,III}]\) are called the coupling matrices and have the size \(L \times M\) and \(L \times N\), respectively. \(H_{bn}^{I,II}\) is the generic element of matrix \([H^{I,II}]\), while \(H_{bn}^{I,III}\) is the element of matrix \([H^{I,III}]\). \(H_{bn}^{I,II}\) and \(H_{bn}^{I,III}\) are defined as follows.

\[
H_{bn}^{I,II} = \int_0^a \sin \left(\frac{m\pi x}{a_1}\right) \sin \left(\frac{n\pi x}{a}\right) \, dx
\]

(3.19)

\[
H_{bn}^{I,III} = \int_0^a \sin \left(\frac{m\pi x}{a_2}\right) \sin \left(\frac{n\pi x}{a}\right) \, dx
\]

(3.20)

Admittance matrices \(Y_I\), \(Y_{II}\) and \(Y_{III}\) are the diagonal matrices with the diagonal elements \(Y_{1I}\), \(Y_{II}\) and \(Y_{III}\), respectively.

To simplify the matrix operation, the matrices and field vectors are combined as follows.

\[
H = \begin{bmatrix}[H^{I,II}][H^{I,III}]\end{bmatrix}, \quad Y_d = \begin{bmatrix}[Y_{II}] & [0] \\
[0] & [Y_{III}]\end{bmatrix}
\]
\[ D^+ = \begin{bmatrix} [B^+] \\ [C^+] \end{bmatrix} \quad D^- = \begin{bmatrix} [B^-] \\ [C^-] \end{bmatrix} \quad [Z_a] = [Y^I]^{-1} \]

where \([Z_a]\) is the inverse matrix of \([Y^I]\).

After replacing the combined matrices in Equation (3.17) and (3.18), the following matrix equations are obtained.

\[
A^+ + A^- = \left[ H^{I.I} \right] \left[ H^{I,III} \right] \begin{bmatrix} B^+ + B^- \\ C^+ + C^- \end{bmatrix} = \left[ H \right] \left( D^+ + D^- \right) \quad (3.21)
\]

\[
A^+ - A^- = [Z_a] [H^{I.I}] [H^{I,III}] \begin{bmatrix} [Y^I] \\ [0] \end{bmatrix} \begin{bmatrix} B^+ - B^- \\ C^+ - C^- \end{bmatrix} = [Z_a] [H] [Y_d] (D^+ - D^-) \quad (3.22)
\]

The junction is considered to be a two-port network. Its scattering parameters can be derived from the following equations

\[
A^- = [S_{11}] A^+ + [S_{12}] D^- \quad (3.23)
\]

\[
D^+ = [S_{21}] A^+ + [S_{22}] D^- \quad (3.24)
\]

From the definition of S-parameters in equations (3.23, 3.24) and by rearranging the equations (3.21, 3.22), the multi-mode S-parameters of the junction are finally obtained by the mode matching method as follows.

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\[ [S_{22}] = ([I] + [Z_d][H'][Y_d][H])^{-1}([I] - [Z_d][H'][Y_d][H]) \] (3.25a)

\[ [S_{21}] = 2([I] + [Z_d][H'][Y_d][H])^{-1}[Z_d][H'][Y_d] \] (3.25b)

\[ [S_{12}] = [H]( [I] + [S_{21}] ) \] (3.25c)

\[ [S_{11}] = [H][S_{21}] - [I] \] (3.25d)

where,

\([I]\) is the identity matrix and

\([ H' ]\) is the transpose matrix of \([H]\).

In equation (3.25), it should be noticed that the S-parameters, \([S_{11}], [S_{12}], [S_{21}],\) and \([S_{22}],\) are matrices with the dimensions \(N\times N, N\times M, M\times N,\) and \(M\times M.\)

### 3.3 Relative Convergence Problem

As shown in the previous section, the mode matching method is based on the expansion of an electromagnetic field in terms of an infinite series of normal modes. In practical computations, however, only a finite number of normal modes can be considered in each region due to the finite calculation capacity of the computer. This means the infinite series must be truncated to obtain a solution. It has been observed that the numerical results converge to different values depending on the way the series are truncated [26]. This is called relative convergence.
The relative convergence phenomenon has been studied in several papers [27, 28] and is not the focus of this thesis. Following the known result, the number of modes in Regions II and III are chosen according to the equations below to avoid the relative convergence problem.

Suppose that $L$, $M$, and $N$ are the number of modes in Regions I, II and III, respectively (see Figure 3.3). When $L$ is fixed, the ratio between $L$, $M$ and $N$ should be maintained as follows.

\[
\frac{L}{M} = \frac{a}{a_1} \quad (3.26a)
\]

and

\[
\frac{L}{N} = \frac{a}{a_2} \quad (3.26b)
\]

3.4 Summary

Waveguide discontinuity and its analysis technique are discussed in this chapter. Application of the mode matching method to characterize E-plane discontinuities of the waveguide is given in detail. The relative convergence problem encountered in the mode match method is considered and the method for solving this problem is presented.
Chapter 4
Synthesis and Optimization

4.1 Performance Parameters of A Bandpass Filter

Before the filter synthesis, the performance parameters of the waveguide bandpass filter must be specified. The performance is represented by a set of parameters characterizing the frequency response of the filter. The characteristics of a bandpass filter as the function of frequency and relational parameters are shown in Figure 4.1.

The following terms are normally used to describe the performance of bandpass filters:

- Lower cutoff frequency, $f_L$.
- Upper cutoff frequency, $f_H$.
- Operation bandwidth is the difference between the upper and lower cutoff frequency, $BW = f_H - f_L$.
- Center frequency, $f_0$, is the mean of the lower and upper cutoff frequency.
• Passband ripple.

• Insertion loss, $\text{INL}$ and passband return loss, $\text{RL}$.

**Figure 4.1:** Characteristics of a Bandpass Filter.

Microwave power that is sent down a transmission line to the filter is called incident power. Because there is a mismatch between the transmission line and the filter, some of the power is reflected back down the transmission line. This part of power is called reflected power. The power that passes through the filter is called the transmitted power. The ratio of the transmitted power to the incident power in dB
terminology is the insertion loss. The ratio of the reflected power to the incident power in dB is the return loss.

- Isolation is the minimum stopband insertion loss and is defined by

\[
IL = 20 \log \frac{1}{|S_{21A}|}
\]  

(4.1)

where,

\(S_{21A}\) is the maximum value of the magnitude of \(S_{21}\), the transmission coefficient in the stopband. For example, 60 dB isolation means the signal amplitudes of undesired frequencies will be attenuated by at least 60 dB with respect to those of passband.

- Isolation bandwidth factor, \(\gamma_I\).

The bandwidth with upper and lower frequency points corresponding to the specified isolation loss is called the isolation bandwidth, \(\Delta f_i\). The isolation bandwidth factor is the ratio of isolation bandwidth to operation bandwidth.

\[
\gamma_I = \frac{\Delta f_i}{f_H - f_L}
\]  

(4.2)

4.2 Synthesize of E-Plane Bandpass Filters

Waveguide bandpass filters are generally realized by cascading half-wave rectangular resonators. The coupling between adjacent resonators can be achieved by means of an inductive septum, a post or an iris. The most popular bandpass filter
configuration consists of alternating inductive septa and half-wave slot resonators. For low-loss performance, these filters invariably adopt either large-gap fin lines or pure-metal-insert E-plane configurations. Pure-metal-insert filters combine the advantages of low-cost production through batch-processing techniques, and low-loss performance of conventional waveguide circuits. In this thesis, metal insert configuration is used to discuss the design techniques and realize the bandpass filter.

The CAD program used to realize the bandpass filter consists of synthesis, analysis and optimization subroutines. The synthesis is carried out in three steps. In the first step, the generalized scattering matrix at a septum edge, which is a kind of waveguide discontinuity (see Figure 4.2), is calculated. Because each septum extends from the lower broad wall to the upper broad wall of the waveguide and is uniform, these generalized scattering parameters can be calculated exactly by the mode matching method. The second step is to calculate the scattering parameters of a septum. This is done by placing two junctions back to back, as shown in Figure 4.3, and combining the S-parameters derived in step 1 for both edges of the septum. In the last step, the generalized S-matrix technique is applied again to obtain the total scattering parameters of the composite structure to combine several septa into a filter circuit. By using the generalized S-matrix, the interactions of both the propagating mode (dominant mode) and the evanescent modes (higher order modes) are take into account. This is quite important for accurate design of E-plane filters.
4.2.1 Obtain S-parameters at the Waveguide Junction

The synthesis begins with analyzing the bifurcated waveguide in Figure 4.2, where the septum is located at the center along the E-plane. For $TE_{e0}$ fields incident from both sides, the total fields in each region are composed of the incident fields and the scattered fields due to the junction. The boundary value problem is solved by first expanding the total fields in terms of the $TE$ normal modes and then matching the tangential components of the fields at the junction. The amplitudes of the normal modes are conveniently represented by the elements of column vectors as shown in Figure 4.2, where $(A^+, A^-), (B^+, B^-), \text{ and } (C^+, C^-)$ are the amplitude vectors of the incident and scattered fields in region I, II and III, respectively. The continuity condition on the tangential components of the electric and magnetic fields is then applied across the aperture ($z=0$). The resulting equations are rearranged into the forms shown in equation (3.21, 3.22). The final solution is the scattering matrix containing the four elements given by equation (3.25). The symbols have the same meaning as in equation (3.25). $[S_{11}], [S_{12}], [S_{21}], \text{ and } [S_{22}]$ are matrices containing the scattering characteristics of the fundamental mode, as well as the higher order evanescent modes.

In this analysis case of finite thickness septum discontinuity, the same procedure of mode matching method is followed that is discussed in detail in the last chapter, and has the same generalized scattering parameters. The only exception is the procedure by which the coupling matrix $H$ is derived.
Figure 4.2: Scattering on the bifurcated waveguide junction.

The metal septum used in the former example was considered to be infinitesimally thin. This assumption is applicable only when the thickness of the septum is extremely small compared to the guide wavelength, typically less than 0.3 percent. When a thicker metal insertion is used or the operating frequency is in the range of millimeter-wave frequencies, this criterion is difficult to satisfy, and the effect of the thickness must be considered in the analysis in order to obtain a more accurate design. The elements of the coupling matrix due to the orthogonal expansion are given by

$$H_{mn}^{I,II} = \int_{x=0}^{a} \sin \frac{\alpha}{a} \sin \frac{\alpha}{a_1} \alpha dx$$

(4.3)

$$H_{ln}^{I,III} = \int_{x=a_1+t}^{a} \sin \frac{\alpha}{a} \sin \frac{\alpha}{a_2} \alpha (x-(a_1+t)) dx$$

(4.4)
4.2.2 Obtain S-Parameters of the Finite-Length Septum

With the knowledge of the scattering parameters for a single junction, the overall composite scattering matrix can be obtained by a network combination in terms of the generalized scattering matrices as shown in Figure 4.3.

![Figure 4.3: The scattering network representation and combination of one septum.](image)

Suppose that the TE-type wave from Region I is incident upon Junction A. At this junction, fields are reflected back into Region I and transmitted into Regions II. After traveling a distance \(w\), part of the wave transmitted into Region II is reflected back, and part is transmitted into Region III at Junction B. This process continues until the intensity of the reflected wave decays. The multiple-reflection phenomenon
between Junction A and B is implied in a matrix manipulation that yields the scattering matrix for a finite length septum.

Let \([S_A]\) and \([S_B]\) represent the scattering matrices for the isolated junctions A and B, respectively. In the last chapter, the element values for both matrices have been obtained in equation (3.25). The characteristics of these two junctions can be considered essentially the same except for the opposite orientation. As discussed in chapter 2, for the two-port networks, port 1 is defined to be the port on the left side and port 2 on the right side. Thus,

\[
[S_A] = [S_B] = [S_{11}]
\]
\[
[S_{12}] = [S_{21}] = [S_{12}]
\]
\[
[S_{21}] = [S_{12}] = [S_{21}]
\]
\[
[S_{22}] = [S_{11}] = [S_{22}]
\]

Now, define the \([L]\) matrix as

\[
[L] = \begin{bmatrix}
[L_1] & [0] & \cdots & [0] \\
[0] & [L_2] & \cdots & [0] \\
\vdots & \vdots & \ddots & \vdots \\
[0] & [0] & \cdots & [L_W]
\end{bmatrix} \quad (4.5)
\]

where:

- \([0]\) represents zero matrix and
- \([L_i]\), which is the element matrix of \([L]\), represents the wave propagating (for propagating modes) or attenuating (for evanescent modes) for a distance of \(w\) in
guide region $II$. $[L_i]$ is a diagonal matrix whose diagonal elements are $L_{i,nn} = e^{-\beta_{in}w}$. $\beta_{in}$ is the propagation constant of the $n$-th mode in the $i$-th guide of the bifurcated section.

The combination of matrices $[S^d]$, $[L]$ and $[S^g]$ results in the composite scattering matrix $[S^c]$ for the septum of length $w$. As a result, the expressions for the reflection and incident coefficient are

$$[S_{11}^c] = [S_{11}] + [S_{12}][L][S_{22}][I] - [L][S_{22}][L][S_{22}]^{-1}[L][S_{21}] \quad (4.6a)$$

$$[S_{12}^c] = [S_{12}][I] - [L][S_{22}][L][S_{22}]^{-1}[L][S_{21}] \quad (4.6b)$$

$$[S_{11}^c] = [S_{12}^c] \quad (4.6c)$$

$$[S_{22}^c] = [S_{11}^c] \quad (4.6d)$$

$[S_{11}]$, $[S_{12}]$, $[S_{21}]$, and $[S_{22}]$ are scattering matrices. Only the first element of each matrix is of interest, because it is the fundamental reflection coefficient caused by a fundamental incident field.

4.2.3 Synthesize Filter Structures by Connecting Septa

In the last step of synthesis, several finite length septa are cascaded with appropriate separations to form a filter circuit. This is done by combining generalized $S$ matrices of several septa.
The procedure depicted in Figure 4.4 shows the cascading of two septa represented by \([S^i]\) and \([S^j]\). The combination of \([S^i]\) and the transmission line scattering matrix \([T]\) results in matrix \([S^{iS}]\), which is afterwards combined with \([S^{jS}]\) to yield \([S]\). \([S]\) represents the total scattering matrix of two septa with a separation of \(l\). This procedure may be repeated to obtain the scattering parameters of the entire filter structure consisting of any number of septa.

![Figure 4.4: Synthesize the filter by combining septa.](image)

Using the generalized scattering matrix enables us to find the interactions between junctions due to the dominant mode and to all higher order modes. The effect of higher order mode interactions becomes more important for filters in which the septa are narrow and the resonators are coupled strongly.
4.3 Double Septa E-plane Filters

The filters discussed in the last section have only one metal insert, which is normally placed at the center of the waveguide. Single metal insert filters offer good performance when the passband is in the middle of the dominant mode \( (TE_{10}) \) waveguide band. For filters designed to operate near the higher end of the waveguide band, the stopband frequencies on the higher side exceed the dominant mode waveguide band. For frequencies exceeding the cutoff frequency of the regions between the septa and the waveguide sidewalls, waves are no longer evanescent, and power is increasingly transported by the propagating waves along the septa. The inductive coupling between the resonators is destroyed, and results in a reduction of stopband attenuation. A detail investigation with the possible solutions to this problem can be found in [16]. Basically, the solution to this problem is to reduce the coupling between the resonators by increasing the attenuation of the evanescent modes in the narrow waveguide region. This can be achieved by

- increasing the thickness of the septum,
- reducing the waveguide sidewall dimensions and
- using double metal insert instead of a single insert.

It has been demonstrated in [16] that the use of a thick septum leads to high passband insertion loss, and the reduction of the sidewall dimensions requires the use of tapered sections at the filter ends which will increase the cost and the length of the
filter. The double planar integrated filter, where two metal inserts are mounted in the E-plane of the waveguide, combines many advantages such as low costs, low passband and high stopband insertion loss. The CAD method that synthesizes two metal inserts filter will be discussed as follows.

The method of how to analyze one metal insert has been discussed in detail. This method can be easily extended to the case of two inserts. Basically, the procedure of analyzing a two-insert-filter is the same as analyzing one insert. Only the general scattering parameters at the trifurcated waveguide junction need to be modified in the new structure. The end and the top view of the new structure are shown in Figure 4.5 (a) and (b), respectively.

The total electromagnetic fields in Region I \((z<0)\)

\[
E_y^I = \sum_{i=1}^{N} \left( A_I^{+I} \phi_i^I (x)e^{-\gamma_I^I z_1} + A_I^{-I} \phi_i^I (x)e^{\gamma_I^I z_1} \right) \tag{4.7}
\]

\[
H_z^I = \sum_{i=1}^{N} \left( A_I^{+I} \phi_i^I (x)e^{\gamma_I^I z_1} - A_I^{-I} \phi_i^I (x)e^{-\gamma_I^I z_1} \right) \tag{4.8}
\]

For Region II \((z>0, 0<x<x_1)\)

\[
E_y^H = \sum_{i=1}^{N} \left( A_i^{+H} \phi_i^H (x)e^{-\gamma_H^H z} + A_i^{-H} \phi_i^H (x)e^{\gamma_H^H z} \right) \tag{4.9}
\]

\[
H_z^H = \sum_{i=1}^{N} \left( A_i^{+H} \phi_i^H (x)e^{-\gamma_H^H z} - A_i^{-H} \phi_i^H (x)e^{\gamma_H^H z} \right) \tag{4.10}
\]

For Region III \((z>0, x_2<x<x_3)\)
\[ E_y^{III} = \sum_{i=1}^{N_2} (A_i^{III+} \phi_i^{III}(x)e^{-iyz} + A_i^{III-} \phi_i^{III}(x)e^{iyz}) \] (4.11)

\[ H_x^{III} = \sum_{i=1}^{N_3} Y_i^{III} (A_i^{III+} \phi_i^{III}(x)e^{-iyz} - A_i^{III-} \phi_i^{III}(x)e^{iyz}) \] (4.12)

Figure 4.5: (a) End view of trifurcated waveguide.
(b) Scattering on the trifurcated waveguide junction.
For Region IV \((z>0, x_4<x<a)\)

\[
E_y^I = \sum_{i=1}^{K_3} (A_i^{\pi+} \phi_i^I(x)e^{-\gamma_i^I x} + A_i^{\pi-} \phi_i^I(x)e^{\gamma_i^I x})
\]

\[
H_x^I = \sum_{i=1}^{N_3} Y_i^I (A_i^{\pi+} \phi_i^I(x)e^{-\gamma_i^I x} - A_i^{\pi-} \phi_i^I(x)e^{\gamma_i^I x})
\]

where:

- \(A_i^{\pi+}, A_i^{\pi-}, A_i^{\pi-}, \) and \(A_i^{\pi+}\) are the given incident field coefficients from region I, II, III and IV respectively,
- \(A_i^{\pi-}, A_i^{\pi+}, A_i^{\pi+}, \) and \(A_i^{\pi+}\) are the unknown excited field coefficients in region I, II, III and IV respectively,
- \(Y_i^I, Y_i^II, Y_i^III,\) and \(Y_i^IV\) are the wave admittance which have the same definition as in Equation (3.10).
- \(\phi_i^I, \phi_i^II\) and \(\phi_i^III\) are normal modes in regions I, II, III and IV, with propagation constants \(\gamma_i^I, \gamma_i^II\) and \(\gamma_i^III\), respectively. In this case, normal mode in each region is

\[
\phi_i^I(x) = \sqrt{\frac{2}{a}} \sin \left(\frac{n_i \pi x}{a} \right)
\]

\[
\phi_i^II(x) = \sqrt{\frac{2}{a_1}} \sin \left(\frac{n_i \pi x}{a_1} \right)
\]

\[
\phi_i^III(x) = \sqrt{\frac{2}{a_2}} \sin \left(\frac{n_i \pi x}{a_2} \right)
\]

\[
\phi_i^IV(x) = \sqrt{\frac{2}{a_3}} \sin \left(\frac{n_i \pi x}{a_3} \right)
\]
Considering the field is continuous at the junction, the electric field equation can be obtained as

\[
\sum_{i=1}^{N} (A_i^{I+} + A_i^{I-}) \phi_i^I (x) \quad 0 \leq x < x_1
\]

\[
\sum_{i=1}^{N} (A_i^{II+} + A_i^{II-}) \phi_i^{II} (x) \quad x_2 \leq x < x_3 \quad (4.19)
\]

\[
\sum_{i=1}^{N} (A_i^{III+} + A_i^{III-}) \phi_i^{III} (x) \quad x_4 \leq x \leq a
\]

and the corresponding magnetic field equation,

\[
\sum_{i=1}^{N} (A_i^{IV+} + A_i^{IV-}) Y_i^{IV} \phi_i^{IV} (x) \quad 0 \leq x < x_1
\]

\[
\sum_{i=1}^{N} (A_i^{V+} + A_i^{V-}) Y_i^{V} \phi_i^{V} (x) \quad x_2 \leq x < x_3 \quad (4.20)
\]

\[
\sum_{i=1}^{N} (A_i^{VII+} + A_i^{VII-}) Y_i^{VII} \phi_i^{VII} (x) \quad x_4 \leq x \leq a
\]

The orthogonality property of modal functions is applied to derive a set of equations involving the unknown field coefficients. This is done by multiplying both sides of equation (4.19) by \( e^I (x) \) and integrating with respect to \( x \) from 0 to \( a \).
From the left-hand side of equation (4.21), the following equation is obtained.

\[
\int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{r\pi x}{a}\right) \, dx = \begin{cases} 
0 & \text{if } r \neq n \\
\frac{a}{2} & \text{if } r = n
\end{cases}
\]  (4.22)

Thus, only \( r = n \) will be taken into account. Equation (4.21) becomes

\[
\sqrt{\frac{2}{a}} \sum_{n=1}^N (A_n^{ll} + A_n^{rl}) = \sqrt{\frac{2}{a_1}} \sum_{n=1}^{N_1} (A_n^{ll} + A_n^{rl}) \int_0^{x_1} \sin\left(\frac{n_1\pi x}{a_1}\right) \sin\left(\frac{r\pi x}{a_1}\right) \, dx
\]

\[
+ \sqrt{\frac{2}{a_2}} \sum_{n=1}^{N_2} (A_n^{ll} + A_n^{rl}) \int_{x_2}^{x_3} \sin\left(\frac{n_2\pi x}{a_2}\right) \sin\left(\frac{r\pi x}{a_2}\right) \, dx
\]

\[
+ \sqrt{\frac{2}{a_3}} \sum_{n=1}^{N_3} (A_n^{ll} + A_n^{rl}) \int_{x_4}^{x_5} \sin\left(\frac{n_3\pi x}{a_3}\right) \sin\left(\frac{r\pi x}{a_3}\right) \, dx
\]  (4.23)

Using the same method, the magnetic field equation is deduced as follows.
Equation (4.23) and (4.24) are reorganized in the form of matrices:

\[
A^{-} + A^{+} = [H^{+,}] (A^{-}) + [H^{--}] (A^{+}) + [H^{++}] (A^{--}) \]

\[
+ [H^{+,+}] (A^{++}) + [H^{--}] (A^{--}) \]

\[
Y_{l}^{-} (A^{-}) - A^{-} = [H^{,+,}][Y_{l}^{-}](A^{+}) - A^{-} + [H^{,--}][Y_{l}^{--}](A^{++}) - A^{-} \]

\[
+ [H^{+,+}][Y_{l}^{++}](A^{+++}) - A^{-} \]

where the known and unknown field coefficient are defined as vector forms

\[
A^{-} = \begin{bmatrix} A^{-}_{1} \\ A^{-}_{2} \\ \vdots \\ A^{-}_{N} \end{bmatrix} ; \quad A^{+} = \begin{bmatrix} A^{+}_{1} \\ A^{+}_{2} \\ \vdots \\ A^{+}_{N} \end{bmatrix} ; \quad A^{++} = \begin{bmatrix} A^{++}_{1} \\ A^{++}_{2} \\ \vdots \\ A^{++}_{N} \end{bmatrix} ; \quad A^{++} = \begin{bmatrix} A^{++}_{1} \\ A^{++}_{2} \\ \vdots \\ A^{++}_{N} \end{bmatrix}
\]

The elements of the coupling matrices are defined by the following equations.
\[ H_{n,n_1}^{1,II} = \int_{x=0}^{x_1} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n_1\pi x}{a_1}\right) dx \]  \hspace{1cm} (4.27)

\[ H_{n,n_2}^{1,III} = \int_{x=x_2}^{x_3} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n_2\pi (x-x_2)}{a_2}\right) dx \]  \hspace{1cm} (4.28)

\[ H_{n,n_3}^{1,IV} = \int_{x=x_3}^{x_4} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n_3\pi (x-x_3)}{a_3}\right) dx \]  \hspace{1cm} (4.29)

\( [Y^I], [Y^II], [Y^III], \) and \( [Y^IV] \) are the corresponding diagonal admittance matrices in region I, II, III, and IV with the same diagonal elements defined in equation (3.10).

The following combinations make the matrix operation simpler:

\[
[H] = \begin{bmatrix}
H^{1,II} & H^{1,III} & H^{1,IV}
\end{bmatrix}
\]

\[
[Y_d] = \begin{bmatrix}
Y^II & [0] & [0] \\
[0] & Y^III & [0] \\
[0] & [0] & Y^IV
\end{bmatrix}
\]

\[
[D^+] = \begin{bmatrix}
[A^{II+}] & [A^{II-}]
\end{bmatrix} \hspace{1cm} [D^-] = \begin{bmatrix}
[A^{III-}] & [A^{IV-}]
\end{bmatrix}
\]

Then following matrix equations are obtained after the replacement.
The junction can also be considered as a 2-port network whose scattering matrices are defined in equations (3.23, 3.24). The generalized scattering parameters of the double metal insert have the same expressions as in equation (3.25).

4.4 The Synthesis Procedure by CAD

4.4.1 Impedance Inverters and Initial Synthesis Based on Network Theory

Figure 4.6 shows the construction of the waveguide bandpass filter with inductive septa (metal inserts). The equivalent circuit of the septum can be expressed with the inductive reactance T-network [3], as shown in Figure 4.7. Thus the filter construction in Figure 4.6 can be treated as several T-networks connected by sections of transmission lines shown in Figure 4.8. It has been proved in [3] that a symmetrical T-network connected to a section of transmission line operates as an impedance
inverter (K-inverter), as shown in Figure 4.9. Therefore, the equivalent circuit in Figure 4.8 is transformed to the equivalent impedance inverter network in Figure 4.10. With these equivalencies, the well-known filter design formulas in [3] can be directly utilized for designing waveguide bandpass filters.

![Figure 4.6: Inductive septa in the waveguide bandpass filter.](image)

![Figure 4.7: Equivalent circuit of the inductive septum.](image)
Figure 4.8: Equivalent network of the waveguide bandpass filter.

Figure 4.9: K-inverter consisting of reactance T network and transmission lines.

Figure 4.10: Equivalent filter network using impedance K-inverter.
4.4.2 CAD Procedures

The synthesis of bandpass filters is finally realized by a CAD program. The synthesis procedure determines the width of each septum and the distance between two consecutive septa in the waveguide filter structure.

The synthesis method is based on a formulation proposed by Rhodes for a distributed step-impedance bandpass prototype. The procedure is as follows:

1. The order of the filter, \( N \), will give the number of resonators, and \( N+1 \) is the number of septa needed in the circuit structure to meet the design specifications. \( N \) is determined from the ripple bandwidth \( \Delta f_r \), the isolation bandwidth \( \Delta f_i \), the passband return loss \( RL \) and the stop band isolation \( IL \) of the filter (Figure 4.1) using the following equation for a Chebyshev type response. The filter order is given by:

\[
N \geq \frac{RL + IL + 6}{20 \log \left( \frac{\gamma + \sqrt{\gamma^2 - 1}}{\gamma} \right)}
\]  

(4.32)

where \( \gamma = \Delta f_i / \Delta f_r \).

2. The mid-band guide wavelength \( \lambda_{gl} \) is determined by solving

\[
\lambda_{gl} \sin \left( \frac{\pi \lambda_{ge}}{\lambda_{gl}} \right) + \lambda_{g'} \sin \left( \frac{\pi \lambda_{ge}}{\lambda_{g'}} \right) = 0
\]  

(4.33)
where:

\( \lambda_{GL} \) and \( \lambda_{GH} \) are the guide wavelengths in the resonator section at the lower
cutoff frequency, \( f_L \) and upper cutoff frequency, \( f_H \).

For a narrow-band case,

\[
\lambda_{GO} = \frac{\lambda_{GL} + \lambda_{GH}}{2}
\]  

(4.34)

A suitable numerical method is applied for solving the equation (4.33).

3. Scaling parameter \( \alpha \) is obtained from

\[
\alpha = \frac{\lambda_{GO}}{\lambda_{GL} \sin\left(\frac{\pi \lambda_{GO}}{\lambda_{GL}}\right)}
\]  

(4.35)

4. The impedance of the distributed element \( Z \) and impedance inverter values, \( k'_{n,n+1} \),

are determined by,

\[
Z_n = \frac{2\alpha \sin\left[\frac{(2n-1)\pi}{2N}\right]}{y} - \frac{1}{4\alpha^2 y} \left\{ \frac{y^2 + \sin^2\left(\frac{m\pi}{N}\right)}{\sin\left(\frac{(2n+1)\pi}{2N}\right)} \right\} - \frac{1}{4\alpha^2 \sin\left(\frac{(2n-3)\pi}{2N}\right)} \left\{ \frac{y^2 + \sin^2\left(\frac{(n-1)\pi}{N}\right)}{\sin\left(\frac{(2n-3)\pi}{2N}\right)} \right\}
\]  

(4.36)

\( n=1, 2, ..., N \)

and
\[ k'_{n,n+1} = \sqrt{y^2 + \sin^2 \left( \frac{m\pi}{N} \right)} \], \quad n = 0, \ldots, N \quad (4.37) \]

where:

\[ y = \sinh \left[ \frac{1}{N} \sinh^{-1} \frac{1}{\varepsilon} \right] \quad (4.38) \]

5. In uniform waveguide filters, the characteristic impedance of the resonator sections are identical. Hence, by scaling the impedance \( Z_n \) to unity, the K-inverters are normalized as follows:

\[ Z_0 = Z_{n+1} = 1 \quad (4.39) \]

and

\[ K_{n,n+1} = -\frac{k'_{n,n+1}}{\sqrt{Z_n Z_{n+1}}} \], \quad n = 0, \ldots, N \quad (4.40) \]

Once the K-inverter values have been calculated from equation (4.40), they can be physically realized in terms of discontinuities in a rectangular waveguide.

6. Determine the widths of E-plane septa so that the required K-inverter is realized.

The following equations are used to obtain the K-inverter values and the associated phase angle \( \phi \):

\[ K = \left| \tan \left( \frac{\phi}{2} + \tan^{-1} x_r \right) \right| \quad (4.41) \]

\[ \phi = -\tan^{-1} (2x_r + x_s) - \tan^{-1} x_s \quad (4.42) \]
\[ jx_s = \frac{1-S_{12} + S_{11}}{1-S_{11} + S_{12}} \]  

(4.43)

\[ jx_p = \frac{2S_{12}}{(1-S_{11})^2 - S_{12}^2} \]  

(4.44)

where,

\[ x_s = \frac{X_s}{Z_0} \quad \text{and} \quad x_p = \frac{X_p}{Z_0} \]

are normalized reactances and

\( S_{11} \) and \( S_{12} \), the dominant mode scattering parameters, are computed scattering parameters of the septum in the waveguide as discussed in the previous chapter.

Notice that \( x_p \) and \( x_s \) in equations (4.41, 4.42) are functions of the septum width. A root-seeking subroutine is used to find the value of septum width to satisfy the required \( K \) value and the angle \( \phi \) for each impedance inverter. A rational functional interpolation method [4] can also be used.

7. Using the result from equations (4.43, 4.44), \( \theta_j \) in Figure 4.10 is given by

\[ \theta_j = \pi + \frac{1}{2}(\phi_j + \phi_{j+1}) \]  

(4.45)

The length of the \( j \)-th resonator formed by the \( j \)-th and \((j+1)\)-th septa is given by

\[ l_j = \frac{\lambda_e}{2\pi} \theta_j \]  

(4.46)
4.5 Optimization

4.5.1 Basic Concepts

The above initial synthesis method based on network theory is approximate, because it neglects the effects of higher order mode interactions between adjacent septa. As a result, the computed as well as the measured response of the filter deviates from the desired response. It is very important to adjust circuit parameters to minimize the deviation between the circuit performance achieved and the desired response. The process of finding the set of element values that provides a circuit response as close as possible to the specified target response is known as optimization.

The deviation error is often due to differences between design assumptions and the actual circuit. It means little improvement can be gained from another design iteration. However, it is possible to identify parameters in the circuit model that can be controlled and then vary them to search for the best combination.

Traditionally, E-plane bandpass filters were designed empirically using approximate discontinuity models from the waveguide handbook [7]. Because the approximate and empirical design techniques cannot consider the higher order mode interaction between any two adjacent discontinuities, constructed filters must be tuned to achieve required specification, like bandwidth, centre frequency and return loss.
When only a few variables in a design require adjustment, manual tuning of the filter dimensions can be a feasible option. As the number of variables increases, visualization of the multidimensional variable space becomes difficult, and manual tuning becomes less efficient.

Optimization approach in CAD is the mathematical equivalent of circuit alignment using test equipment. The CAD software replaces the test equipment, and changing element values correspond to adjusting components in the circuit. The obvious advantages of using a computer model are speed and cost. It is not necessary to actually build and test the hardware. When waveguide filters are created by a CAD program, it is possible to use a more exact multi-mode field theoretical approach in analysing waveguide discontinuities.

There are two important steps in optimization: the determination of a search direction and the search for the minimum in that direction. There are two different ways of carrying out the determination of the search direction: the gradient method and the direct search method. Gradient method uses information about the slope of the performance function to dictate a direction of search where the minimum exists. The direct search algorithm does not use gradient information, and parameter modification is carried out by searching for the optimum in a systematic manner. The direct search method is used in this CAD program.
4.5.2 Error Functions

An error function is usually created before applying optimization to the design problems. The purpose of optimization is to minimize a given error function that describes the difference between computed and target responses. The smaller the value of the error function, the closer the achieved response is to the desired response.

There are various approaches used to define the error function. For example, the error function can be defined in such a way as to cause the optimization routine to converge as close as possible to a specified value, or it can be defined to converge to a design in which the system tolerances have been exceeded. In E-plane waveguide filter design, the least squared optimization function is the most widely used error function. This error function is created by summing all applicable error components at each analysis frequency. It is given by

\[
E(\bar{x}) = \sum_{i=1}^{N_{\text{err}}} \left( \frac{L_i(\text{min})}{IL(f_i)} \right)^2 + \sum_{j=1}^{N_{\text{err}}} \left( \frac{IL(f_j)}{L_j(\text{max})} \right)^2 = \text{Minimum} \quad (4.47)
\]

where,

\[
IL = 20\log\left(\frac{1}{|S_{21}|}\right),
\]

\(\bar{x}\) are parameters that will be optimized to yield a minimum error,

\(f_i\) are the frequency sample points,
\( N_{\text{stop}} \) and \( N_{\text{pass}} \) are the number of sample points in stopband and passband respectively. A number of 20–30 frequency sample points, both in passband and stopband, is sufficient.

\( L_{s(\text{min})} \) is the given minimum stopband attenuation and

\( L_{p(\text{max})} \) is the given maximum passband attenuation.

The frequency characteristics of the transmission loss are obtained by analysing the filter structure at specific frequency sample points (Figure 4.11). One will note from equation (4.47) that in order to minimize the error function, it is desirable to make insertion loss as low as possible in the passband and as high as possible in the stopband.

![Diagram](image)

**Figure 4.11:** Scheme for the computer optimization.
4.5.3 Conventional Optimization Approaches

The brute force optimization flow chart shown in Figure 4.12 presents the most common method used in situations in which an optimization is needed in the waveguide filter design [15, 18]. Optimization is accomplished by repeatedly analysing the filter response and by directly changing the septum widths and resonator lengths. The optimization variables in equation (4.47) are both the septum widths and resonator lengths and can be expressed as a vector:

$$\bar{x} = (w_1, w_2, \ldots, w_{N+1}, l_1, l_2, \ldots, l_N)$$  \hspace{1cm} (4.48)

For a filter with order $N$, the optimization variables will reach $2N+1$. Although the symmetric structure is widely used in E-plane filters that in turn reduce the optimization variables to $N + 1$, this optimization approach is still time consuming, especially when dealing with higher order filters. The brute force method also has a tendency to converge to local minimum, thus forcing the designer to begin the design process over again.

Another optimization approach is to optimize the K-inverters. For an $N$-pole filter of symmetrical structure, there are $N + 1$ K-inverters, out of which $\frac{N}{2} + 1$ have different values. In other words, every set of septum widths and resonator lengths corresponds to a set of K-inverters. When the K-inverter values are optimized, the septum widths and resonator lengths can be found by back calculation. Therefore,
using the optimization of K-inverters rather than directly optimizing the filter dimensions will reduce the number of optimization variables to $N/2 + 1$. The set of optimization variables is

$$\bar{x} = (k_{01}, k_{12}, \ldots, k_{N/2, N/2+1})$$

(4.49)

Figure 4.12: Brute force optimization flowchart.
Because of fewer variables that must be handled, this method is faster than the brute force method. However, this method is still slow when one must deal with larger order filters, and it also has the same problem as the brute force method with convergence to local minima.

4.5.4 Optimization Approach with A Fixed Number of Variables

From the above discussions, it is found that the number of optimization variables is proportional to the order of the filter. This is the reason that brute force optimization and the K-inverter optimization are not applicable in the higher order filter design. Therefore, an optimization approach that is independent of the order of the filter is needed [4, 29].

In this thesis, the return loss and the upper and lower cutoff frequencies will be used as the optimization variables instead of using the physical dimensions or the K-inverter. In this way, the number of optimization variables will always remain three, regardless of the order of the filter.

Figure 4.13 shows the general flowchart for this method of optimization. The design program using this method will create a filter, analyse the structure and check that the response matches the desired one. If they do not match, the optimization variables are changed and the design process is started all over again. This will continue until the correct filter response is finally reached. It should be noted that the
order of the filter remains the same throughout the entire design process, even though the return loss is being changed.

The idea behind this method is that every time the filter specifications are entered into the synthesis program, the program will design a filter with a frequency response that is different from the desired one. Therefore, there must exist another set of filter specifications that will produce the desired frequency response when entered into the synthesis program. The object of this technique is to find the wrong filter specifications that will in turn produce the correct response when used by the filter synthesis algorithm.

In the optimization, the more variables used in error function, the more local minimum will be produced. Once the error function converges to the local minimum, the optimization must be started over again. The optimization time is wasted. The situation may become worse in higher order filter design in which more optimization variables exist.

In this new method, the number of optimization variables is independent of the order of filters. This means that the probability of converging to a local minimum is reduced for higher order filter optimization. Table 4.1 shows the number of variables in different optimization methods discussed in this thesis.
Figure 4.13: The flowchart of optimization method with fixed number of variables.

Furthermore, a new approach in the analysis block is adopted to speed up the optimization procedure. In the analysis step, a single discontinuity between an empty
waveguide and a bifurcated or a trifurcated waveguide is analysed only once. The resulting scattering matrix is stored and repeatedly used. This will save a great amount of analysis time, as well as computer resources.

**Table 4.1: The number of variables in different optimization methods.**

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Order of the filter</th>
<th>Number of optimization variables in asymmetrical structure</th>
<th>Number of optimization variables in symmetrical structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brute force</td>
<td>$N$</td>
<td>$2N + 1$ (filter dimensions)</td>
<td>$N + 1$ (filter dimensions)</td>
</tr>
<tr>
<td>K-inverters</td>
<td>$N$</td>
<td>$N + 1$ (K-inverter values)</td>
<td>$N/2 + 1$ (K-inverter values)</td>
</tr>
<tr>
<td>This thesis</td>
<td>$N$</td>
<td>$3 (RL, f_L, and f_H)$</td>
<td>$3 (RL, f_L, and f_H)$</td>
</tr>
</tbody>
</table>

The simplex method is used for searching the minimum of error function. Suppose $n$ is the number of optimization variables. A simplex in $n$-dimensional space is characterized by the $n+1$ distinct vectors that are its vertices. In two-dimensional space, a simplex is a triangle. In three-dimensional space, which is adopted in this thesis, it is a pyramid. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the function's values at the vertices of the simplex. Usually, one of the vertices is replaced by the new point which is a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance.

**4.6 Summary**

In this chapter, the filter structures with single or double metal insert have been analyzed. A complete fast computation method has been developed. The
inductive septa along the waveguide are converted to an equivalent impedance K-inverter network. The frequency behavior of the K-inverters can be accurately predicted by using generalized scattering matrix technique. However, there is no guarantee that it will produce an accurate prediction of the frequency response of the filter, because the higher order mode interactions between adjacent K-inverters are still not taken into account. Hence, an optimization procedure to tune the filter structure is necessary.

In the optimization method developed, the number of optimization variables does not increase with increasing the filter order. Therefore, this method is faster than a conventional optimization method such as brute force method when dealing with higher order filters. In addition, the chance of convergence to a local optimization minimum is greatly reduced. As it can be found from the flowchart in Figure (4.13), the physical dimensions of the filter need to be determined in every iteration of the optimization. Thus, a fast synthesis method will improve the speed of optimization. It will be shown in the next chapter that the proposed optimization produces the desired results.
Chapter 5

Results and Discussion

In this chapter, the results of E-plane waveguide filter designs using the approach proposed in the previous chapters are presented. The synthesis, analysis, and optimization of bandpass filters are realized by a developed CAD program based on the method that is discussed in the previous chapters. It is costly and impractical to verify the design by experimentally making and testing all the filters. Therefore, the analysis subroutine is used to consider a filter structure consisting of any number of inductive septa. The analysis program takes into account the correct behavior of each septum, as well as the higher order mode coupling between the septa, and accurately simulates the characteristic response of a filter. Thus, it is important to verify the analysis method first.

Transmission characteristics of various filters, which are synthesized by either experiment or other CAD methods, are compared with the result obtained from this analysis method.
5.1 Verification of Analysis Method

To verify the reliability of the analysis method, filters with different numbers of resonators in various frequency bands will be considered first. Figures 5.1 and 5.2 show the configurations of E-plane filters and metal inserts. The dimensions of filters 1, 2 and 3 are given in Tables 5.1 and 5.2.

![Diagram of filter configurations](image)

(a) Configuration of single insert.
(b) Configuration of double inserts.
Table 5.2: Dimensions of E-plane metal insert for filter 1,2 and 3  
(All Dimensions are in millimeters).

<table>
<thead>
<tr>
<th>No.</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$w_5$</th>
<th>$l_1$</th>
<th>$l_2$</th>
<th>$l_3$</th>
<th>$l_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7721</td>
<td>3.4254</td>
<td>3.4254</td>
<td>0.7721</td>
<td>–</td>
<td>3.9527</td>
<td>3.9539</td>
<td>3.9527</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>0.5990</td>
<td>1.8110</td>
<td>2.0080</td>
<td>1.8110</td>
<td>0.5990</td>
<td>1.4390</td>
<td>1.4400</td>
<td>1.4400</td>
<td>1.4390</td>
</tr>
<tr>
<td>3</td>
<td>0.6170</td>
<td>1.9200</td>
<td>2.1000</td>
<td>1.9200</td>
<td>0.6170</td>
<td>1.9200</td>
<td>1.9270</td>
<td>1.9270</td>
<td>1.9200</td>
</tr>
</tbody>
</table>

Figure 5.2: E-plane metal insert.

Table 5.1: Configurations of Ka-, W-, and E-band filters  
(All Dimensions are in millimeters).

<table>
<thead>
<tr>
<th>No.</th>
<th>Band</th>
<th>Number of resonators</th>
<th>Waveguide Dimensions</th>
<th>Insert Thickness</th>
<th>Design Type</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ka</td>
<td>3</td>
<td>$a = 7.1120$</td>
<td>$t = 1.2500$</td>
<td>single</td>
<td>Fig. 5.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 3.55560$</td>
<td></td>
<td>insert</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>W</td>
<td>4</td>
<td>$a = 2.5400$</td>
<td>$t = 0.0500$</td>
<td>single</td>
<td>Fig. 5.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 1.2700$</td>
<td></td>
<td>insert</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>E</td>
<td>4</td>
<td>$a = 3.1000$</td>
<td>$t = 0.1000$</td>
<td>single</td>
<td>Fig. 5.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 1.5500$</td>
<td></td>
<td>insert</td>
<td></td>
</tr>
</tbody>
</table>
Filter 1 is realized in [19] according to the method presented in [30]. This method can accurately predict the response of filters up to moderate bandwidths. The transmission characteristic of Filter 1 is plotted in Figure 5.3 and is compared to the filter response calculated by the analysis program developed in this thesis.

Filter 2 was originally described in [17] and optimized by the brute force method. A prototype was fabricated [17] according to dimensions listed in Tables 5.1 and 5.2. Insertion loss of the filter measured from the prototype is plotted in Figure 5.4. Filter 3 was proposed in [18]. Brute force optimization is applied in the design again. Measured insertion loss as a function of frequency is compared with analysis results produced by CAD program developed in Figures 5.4 and 5.5.

From Figures 5.3, 5.4, and 5.5, it can be found that the transmission characteristics, either from calculation formula [19] or from the measurement [17, 18], agree excellently with the results from the analysis program developed in this thesis.

Tables 5.3 and 5.4 give the dimensions of filters 4, 5 and 6. Filters 4 and 5 are two seven resonators Ka-band filters proposed in [19]. The response of each filter is analyzed by developed analysis subroutine and then compared with the response given by [19] in Figures 5.6, 5.7, 5.8 and 5.9. From these figures, it can be noticed that the design has achieved a relatively wide bandwidth. The band edge frequencies and the stopband attenuation are in good agreement. The analysis routine produces a better passband ripple level, as shown in figures 5.7 and 5.9. The errors are due to the
approximated design used in [19], which tends to produce a slightly higher ripple level.

**Table 5.3:** Configurations of three Ka-band filters (All Dimensions are in millimeters).

<table>
<thead>
<tr>
<th>No.</th>
<th>Band</th>
<th>Number of resonators</th>
<th>Waveguide Dimensions</th>
<th>Insert Thickness</th>
<th>Design Type</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Ka</td>
<td>7</td>
<td>$a = 7.1120$</td>
<td>$t = 0.0250$</td>
<td>single insert</td>
<td>Fig. 5.6 &amp; 5.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 3.5560$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Ka</td>
<td>7</td>
<td>$a = 7.1120$</td>
<td>$t = 0.0250$</td>
<td>single insert</td>
<td>Fig. 5.8 &amp; 5.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 3.5560$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Ka</td>
<td>4</td>
<td>$a = 7.1120$</td>
<td>$t = 0.1500$</td>
<td>double insert</td>
<td>Fig. 5.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 3.5560$</td>
<td>$d = 1.8000$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.4:** Dimensions of E-plane metal insert for filter 4, 5 and 6 (All Dimensions are in millimeters).

<table>
<thead>
<tr>
<th>No.</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$w_5$</th>
<th>$w_6$</th>
<th>$w_7$</th>
<th>$w_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.1081</td>
<td>0.6668</td>
<td>0.9718</td>
<td>1.0510</td>
<td>1.0510</td>
<td>0.9718</td>
<td>0.6668</td>
<td>0.1081</td>
</tr>
<tr>
<td>5</td>
<td>0.1541</td>
<td>0.7753</td>
<td>1.1652</td>
<td>1.2802</td>
<td>1.2802</td>
<td>1.1652</td>
<td>0.7753</td>
<td>0.1541</td>
</tr>
<tr>
<td>6</td>
<td>0.4030</td>
<td>2.4480</td>
<td>3.6460</td>
<td>2.4480</td>
<td>0.4030</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No.</th>
<th>$l_1$</th>
<th>$l_2$</th>
<th>$l_3$</th>
<th>$l_4$</th>
<th>$l_5$</th>
<th>$l_6$</th>
<th>$l_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4.8556</td>
<td>5.0978</td>
<td>5.1369</td>
<td>5.1427</td>
<td>5.1369</td>
<td>5.0978</td>
<td>4.8556</td>
</tr>
<tr>
<td>5</td>
<td>3.2116</td>
<td>3.2314</td>
<td>3.2196</td>
<td>3.2171</td>
<td>3.2196</td>
<td>3.2314</td>
<td>3.2116</td>
</tr>
<tr>
<td>6</td>
<td>3.6370</td>
<td>3.6890</td>
<td>3.6890</td>
<td>3.6370</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Filter 6 is an E-plane bandpass filter with double planar insertion reported in [20]. The response given by this analysis routine does not correspond with exactly, but nevertheless very accurately matches the measured response.
From the above discussion, it can be concluded that the analysis routine used is very accurate and has a wide applicable range in analyzing E-plane bandpass filters. Narrow or wide band filters with single or double inserts can be analyzed with confidence using this subroutine.

Figure 5.3: Insertion loss of Ka-band 3-resonator filter.
Figure 5.4: Insertion loss of W-band 4-resonator filter.
Figure 5.5: Insertion loss of E-band 4-resonator filter.
Figure 5.6: Insertion loss of Ka-band 7-resonator filter (No. 4).
Figure 5.7: Passband ripple of Ka-band 7-resonator filter (No. 4).
Figure 5.8: Insertion loss of Ka-band 7-resonator filter (No. 5).
Figure 5.9: Passband ripple of Ka-band 7-resonator filter (No. 5).
Figure 5.10: Insertion loss of Ka-band double planar filter.
5.2 Synthesis and Optimization of E-plane Bandpass Filters

In this section, an X-band and two Ka-band E-plane bandpass filters will be synthesized by this CAD program. The frequency response of the initial and the optimized designs will be analyzed to show the effectiveness of the CAD program.

Table 5.5 provides the specifications for the initial design. Table 5.6 shows the initial and optimized dimensions of E-plane metal inserts. Figures 5.11 and 5.12 show the transmission characteristics of initial and optimized design. It can be found that the optimized filter has wider bandwidth and better return loss. The optimized design gives an excellent return loss on the lower and upper cutoff frequencies, while the initial design gives a much higher return loss on the cutoff frequency. The return losses given by the initial design are only about -5dB and -10dB on the lower and upper cutoff frequencies, respectively. They do not satisfy the required -16dB return loss in the passband.

To study further the effectiveness of optimization, the passband insertion loss is enlarged in Figure 5.13 to show the performance of ripple bandwidth. From Figure 5.13, it can be found that the initial design does not meet the desired 0.1 dB ripple bandwidth response. Instead of giving a required 9.5 GHz in lower cutoff frequency and 10 GHz in upper cutoff frequency, the filter response is around 9.52 GHz in lower cutoff and 9.98 GHz in upper cutoff frequency. This means that 8% or 40 MHz in
bandwidth is lost. The filter response obtained from the optimized design agrees with the requirement in ripple bandwidth. Passband ripple level is also improved in this optimized design. Therefore, it proves that this CAD program produces desired results.

**Table 5.5:** Input specifications for the X-band E-plane filter design.

<table>
<thead>
<tr>
<th>Optimization Variables</th>
<th>Initial &amp; Desired</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower cutoff frequency ($f_l$)</td>
<td>9.5 GHz</td>
<td>9.4796 GHz</td>
</tr>
<tr>
<td>Upper cutoff frequency ($f_H$)</td>
<td>10 GHz</td>
<td>10.0064 GHz</td>
</tr>
<tr>
<td>Passband return loss ($RL$)</td>
<td>16 dB</td>
<td>19.3153 dB</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Non-optimization Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolation bandwidth ($\Delta f$)</td>
</tr>
<tr>
<td>Isolation loss ($IL$)</td>
</tr>
<tr>
<td>Waveguide width ($a$)</td>
</tr>
<tr>
<td>Waveguide height ($b$)</td>
</tr>
<tr>
<td>Septum thickness ($t$)</td>
</tr>
</tbody>
</table>

**Table 5.6:** Initial and optimized dimensions of E-plane metal inserts of X-band filter (All dimensions are in millimeters).

<table>
<thead>
<tr>
<th>Septum width</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$w_5$</th>
<th>$w_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>1.7095</td>
<td>7.1322</td>
<td>8.5000</td>
<td>8.5000</td>
<td>7.1322</td>
<td>1.7095</td>
</tr>
<tr>
<td>Optimized</td>
<td>1.2799</td>
<td>6.4498</td>
<td>7.9776</td>
<td>7.9776</td>
<td>6.4498</td>
<td>1.2799</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Resonator length</th>
<th>$l_1$</th>
<th>$l_2$</th>
<th>$l_3$</th>
<th>$l_4$</th>
<th>$l_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>15.1388</td>
<td>15.3636</td>
<td>15.3726</td>
<td>15.3636</td>
<td>15.1388</td>
</tr>
<tr>
<td>Optimized</td>
<td>15.0993</td>
<td>15.3900</td>
<td>15.4030</td>
<td>15.3900</td>
<td>15.0993</td>
</tr>
</tbody>
</table>
Figure 5.11: Insertion loss of initial and optimized designs for X-band filter.
Figure 5.12: Return loss of initial and optimized designs for X-band filter.
Figure 5.13: Ripple bandwidth of initial and optimized designs for X-band filter.
Similarly, two high order Ka-band E-plane bandpass filters, with the orders of 10 and 11, are presented as follows. Tables 5.7 to 5.10 provide the specifications and dimensions of initial and optimized design. Figures 5.14 to 5.19 show the transmission characteristics of initial and optimized design. Just as in the low order filter design, this CAD program produces the desired results (lower and upper cutoff frequency, and passband return loss) in high order E-plane bandpass filter design.

Since the number of optimization variables is independent of filter order in the design procedure, this CAD program did not make extra effort to reach the final optimized results. The program finished the optimization of 10th and 11th order filters in 102 and 122 iterations, respectively. It can be concluded that the workload is the same as comparing 111 iterations in 5th order filter optimization. It proves that this CAD program is time saving and efficient, as well as accurate. In addition, the optimization never hits a local minimum.

Table 5.7: Input specifications for the Ka-band 10th order filter design.

<table>
<thead>
<tr>
<th>Optimization Variables</th>
<th>Initial &amp; Desired</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower cutoff frequency ($f_L$)</td>
<td>28 GHz</td>
<td>27.9375 GHz</td>
</tr>
<tr>
<td>Upper cutoff frequency ($f_H$)</td>
<td>30 GHz</td>
<td>30.0321 GHz</td>
</tr>
<tr>
<td>Passband return loss ($RL$)</td>
<td>20 dB</td>
<td>20.5579 dB</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Non-optimization Variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolation bandwidth ($\Delta f$)</td>
<td>4 GHz</td>
</tr>
<tr>
<td>Isolation loss ($IL$)</td>
<td>80 dB</td>
</tr>
<tr>
<td>Waveguide width ($a$)</td>
<td>7.112 mm</td>
</tr>
<tr>
<td>Waveguide height ($b$)</td>
<td>3.556 mm</td>
</tr>
<tr>
<td>Septum thickness ($t$)</td>
<td>0.025 mm</td>
</tr>
</tbody>
</table>
Table 5.8: Initial and optimized dimensions of E-plane metal inserts of Ka-band 10th order filter (All dimensions are in millimeters).

<table>
<thead>
<tr>
<th>Septum width</th>
<th>$w_1 = w_{11}$</th>
<th>$w_2 = w_{10}$</th>
<th>$w_3 = w_9$</th>
<th>$w_4 = w_8$</th>
<th>$w_5 = w_7$</th>
<th>$w_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>0.1696</td>
<td>1.2810</td>
<td>1.7444</td>
<td>1.8565</td>
<td>1.8918</td>
<td>1.9007</td>
</tr>
<tr>
<td>Optimized</td>
<td>0.1409</td>
<td>1.1944</td>
<td>1.6607</td>
<td>1.7763</td>
<td>1.8125</td>
<td>1.8217</td>
</tr>
<tr>
<td>Resonator length</td>
<td>$l_1 = l_{10}$</td>
<td>$l_2 = l_9$</td>
<td>$l_3 = l_8$</td>
<td>$l_4 = l_7$</td>
<td>$l_5 = l_6$</td>
<td></td>
</tr>
<tr>
<td>Initial</td>
<td>5.5450</td>
<td>5.7965</td>
<td>5.8192</td>
<td>5.8231</td>
<td>5.8242</td>
<td></td>
</tr>
<tr>
<td>Optimized</td>
<td>5.5325</td>
<td>5.8026</td>
<td>5.8287</td>
<td>5.8333</td>
<td>5.8345</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.9: Input specifications for the Ka-band 11th order filter design.

<table>
<thead>
<tr>
<th>Optimization Variables</th>
<th>Initial &amp; Desired</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower cutoff frequency ($f_1$)</td>
<td>28 GHz</td>
<td>27.9434 GHz</td>
</tr>
<tr>
<td>Upper cutoff frequency ($f_H$)</td>
<td>30 GHz</td>
<td>30.0256 GHz</td>
</tr>
<tr>
<td>Passband return loss ($RL$)</td>
<td>20 dB</td>
<td>20.6683 dB</td>
</tr>
<tr>
<td>Non-optimization Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isolation bandwidth ($\Delta f$)</td>
<td>3 GHz</td>
<td></td>
</tr>
<tr>
<td>Isolation loss ($IL$)</td>
<td>65 dB</td>
<td></td>
</tr>
<tr>
<td>Waveguide width ($a$)</td>
<td></td>
<td>7.112 mm</td>
</tr>
<tr>
<td>Waveguide height ($b$)</td>
<td></td>
<td>3.556 mm</td>
</tr>
<tr>
<td>Septum thickness ($t$)</td>
<td></td>
<td>0.025 mm</td>
</tr>
</tbody>
</table>

Table 5.10: Initial and optimized dimensions of E-plane metal inserts of Ka-band 11th order filter (All dimensions are in millimeters).

<table>
<thead>
<tr>
<th>Septum width</th>
<th>$w_1 = w_{12}$</th>
<th>$w_2 = w_{11}$</th>
<th>$w_3 = w_{10}$</th>
<th>$w_4 = w_9$</th>
<th>$w_5 = w_8$</th>
<th>$w_6 = w_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>0.1709</td>
<td>1.2860</td>
<td>1.7502</td>
<td>1.8636</td>
<td>1.9010</td>
<td>1.9142</td>
</tr>
<tr>
<td>Optimized</td>
<td>0.1429</td>
<td>1.2050</td>
<td>1.6743</td>
<td>1.7917</td>
<td>1.8305</td>
<td>1.8442</td>
</tr>
<tr>
<td>Resonator length</td>
<td>$l_1 = l_{11}$</td>
<td>$l_2 = l_{10}$</td>
<td>$l_3 = l_9$</td>
<td>$l_4 = l_8$</td>
<td>$l_5 = l_7$</td>
<td>$l_6$</td>
</tr>
<tr>
<td>Initial</td>
<td>5.5462</td>
<td>5.7970</td>
<td>5.8196</td>
<td>5.8235</td>
<td>5.8247</td>
<td>5.8250</td>
</tr>
<tr>
<td>Optimized</td>
<td>5.5342</td>
<td>5.8032</td>
<td>5.8291</td>
<td>5.8337</td>
<td>5.8351</td>
<td>5.8354</td>
</tr>
</tbody>
</table>
Figure 5.14: Insertion loss of initial and optimized designs for Ka-band 10th order filter.
Figure 5.15: Return loss of initial and optimized designs for Ka-band 10th order filter.
Figure 5.16: Ripple bandwidth of initial and optimized designs for Ka-band 10th order filter.
Figure 5.17: Insertion loss of initial and optimized designs for Ka-band 11th order filter.
Figure 5.18: Return loss of initial and optimized designs for Ka-band 11th order filter.
Figure 5.19: Ripple bandwidth of initial and optimized designs for Ka-band 11th order filter.
5.3 Summary

In this chapter, the results obtained from the CAD methods developed in this thesis are presented. The CAD program based on the analysis method developed is verified first. Obtained results are in good agreement with experimental and published results. The verifications of synthesis and optimization method are done by creating several waveguide bandpass filters. E-plane bandpass filters are designed and the frequency response characteristics of these filters are presented. Two high order filters are introduced in this thesis for the first time. It is proved that the CAD method and the computer program developed are effective in E-plane bandpass filter design, especially in high order filter designs. The design process in the mass production of filters can be greatly simplified with the use of the computer-aided design method.
Chapter 6

Conclusions

6.1 Contributions of the Thesis

E-plane filters with all-metal inserts are studied in this thesis. A complete computer-aided design method is developed for creating the filter structure. The synthesis subroutine gives the initial parameters of the circuit structure and the optimization subroutine fine tunes the filter circuit to make the filter performance satisfy a set of specifications. The fine tune feature is desirable in practical application because circuits used in microwave bands (especially in millimeter-wave band) are very difficult to tune physically due to their extremely small size.

By employing the accurate analysis procedure, fast computation method, and fewer optimization variables, this method is better than most existing methods, both in speed and in accuracy.

As stated in chapter 1, the research works of the thesis include:

- Develop a high speed synthesis method for E-plane metal insert filters.
• Optimize the initial design data using a fixed number of optimization variables regardless the filter order.
• Use the developed CAD program to design E-plane waveguide filters.
• Compare computed result with existing results from published papers.

In finishing these works and fulfilling our objectives, the following contributions are made.

1. In any existing waveguide filter design method, all waveguide discontinuities were analyzed one by one. The obtained scattering matrices of discontinuities should be stored before the design was finished. Therefore, the analysis of these discontinuities consumes a significant amount of computer memory and analysis time due to the large dimension of matrices. In this thesis, discontinuity between an empty waveguide and the bifurcated or trifurcated waveguide is analyzed only once. The obtained scattering matrix is stored and repeatedly used throughout the synthesis and optimization procedures. This method has never been previously reported in the literature.

2. Use of three variables makes the optimization very fast and it is independent of the order of the filter. This will greatly benefit high order filter design. This optimization method was first introduced in [33] to
realize H-plane iris coupled waveguide bandpass filter. However, it is used for the first time to realize E-plane waveguide filter by this thesis.

3. Using only three optimization variables greatly reduces the chance of convergence to a local minimum. This also speeds up the optimization procedure.

6.2 Summary

E-plane circuits for microwave applications were first introduced in 1974. Since then, a constant stream of publications on E-plane circuits has appeared in the literature. In the 1980s, commercial E-plane products appeared. Today, due to its extensive applications in military and industrial communication systems, a significant amount of research is still conducted on E-plane circuits and systems.

The relatively complicated structure and small size of E-plane circuits make them difficult to design and tune experimentally. As a result, CAD method was introduced to complete the task of design, analysis and optimization. Obviously, computational methods become important because the means of applying them in microwave design will continue to grow.
The chapter 1 of the thesis has briefly described microwave systems, microwave filter and microwave circuit CAD. General procedure of CAD is introduced.

The microwave waveguide, which is used to develop the filter, is discussed in chapter 2. The concept of scattering parameters is used extensively in microwave design because they are easier to measure and use than other kinds of parameters. The generalized scattering matrix technique allows analysis of circuit of any topology. Both of these methods are discussed in this chapter.

Chapter 3 deals with the waveguide discontinuities and the techniques analyzing them. Discontinuities are usually undesirable in the transmission line. However, proper combination of discontinuities can be used to accomplish many circuit functions such as filtering. Mode matching method is the most commonly used semi-analytical technique for analysis of waveguide discontinuities. It provides highly accurate values for the propagation constants of hybrid-guided modes as well as the description of the field distributions.

One of objectives of this research is to develop a fast CAD method to design E-plane bandpass filters. The procedure is provided in chapter 4 along with the optimization method. The optimization method is better than most existing methods when it deals with higher order filters, because a fixed number of optimized variables
is used in the associated error function. Also, it eliminates the possibility of convergence to a local minimum.

In chapter five, the results obtained from the CAD methods developed in this thesis are presented. The CAD program based on the analysis method developed is verified first. Obtained results are in good agreement with experimental and published results. The verifications of synthesis and optimization method are done by realizing several waveguide bandpass filters. E-plane bandpass filters are designed and the frequency response characteristics of these filters are presented. Two high order filters are introduced by this thesis for the first time. It is proved that the CAD method and the computer program developed are effective in E-plane bandpass filters design, especially in high order filter designs. The design process in the mass production of filters can be greatly simplified with the use of the computer-aided design method.

6.3 Limitations

Some of the limitations of the CAD method developed in this thesis are as follows:

- The filters are considered to be designed under the condition of losslessness.
- The inserts are metallic only.
- Only symmetrical filter structures are considered.
6.4 Future Work

- Waveguide filters with different dielectric filling can be studied.
- More complex fin-line structure inserts can be designed by following similar procedures to those described in this thesis.
- Asymmetric filter structures can be designed by slight modification of the developed method in this thesis.
- The method developed in this thesis can be extensively applied in other waveguide circuits design, like diplexers, multiplexers, and couplers.
REFERENCES


