TRANSMISSION LINE RELIABILITY MODELING INCORPORATING EXTREME ADVERSE WEATHER CONSIDERATIONS

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ABSTRACT

This thesis illustrates the development of a reliability model for two redundant transmission lines, which incorporates normal and bad weather conditions. The two indices of system average failure rate and system average outage duration are evaluated for this model using a Markov approach, an approximate equations method and Monte Carlo simulation. The results show that a very optimistic evaluation can be obtained if the effects of bad weather are ignored. The results obtained using the approximate method and the Monte Carlo simulation technique are compared with those obtained using the theoretically exact Markov approach. The comparison indicates that the approximate method provides, under certain conditions, a practical approach for general transmission and distribution system analysis as it can be applied directly in minimal cut applications.

The conventional two weather state model is extended in this thesis to a reliability model which incorporates normal, adverse and major adverse weather conditions. Similar analyses were conducted for the conventional two state model and the developed three state model. The effects due to a portion of the bad weather occurring in major adverse weather were evaluated. The results indicate that the two weather state model does not reflect the increasing importance of major averse weather and can underestimate the potential error. A comparison of the results obtained using the three methods of analysis places some limits on the use of the approximate equations.

A series of sensitivity studies were conducted to examine the response of the two weather state models to a specific set of weather parameter changes. In these analyses the average durations of normal and adverse weather relative to the base case, the percentage of line failures occurring in bad weather and the percentage of bad weather failures occurring in major adverse weather were varied. Model acceptability analysis was conducted by comparing the error factors obtained using the two weather models under different combinations of weather conditions. Application zones for both weather models are illustrated in the thesis for the conditions considered.

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λ_{av} = The average annual failure rate of a component.

λ = The normal weather failure rate of a component.

λ' = The adverse weather failure rate of a component.

λ^{ma} = The major adverse weather failure rate of a component.

r = The average repair time of a component.

μ = The average repair rate of a component.

N = The average duration of normal weather period.

A = The average duration of adverse weather period.

M = The average duration of a major adverse weather period.

λ_{SL} = The failure rate of a load point due to overlapping component outages.

r_{SL} = The average duration associated with load point failure

Chapter 1
Introduction

1.1 Power system reliability evaluation

The basic function of an electric power system is to supply its customers with electricity at a reasonable cost and with an acceptable level of continuity and quality. Power system reliability evaluation can be used to provide a measure of the ability of a power system to perform its intended function. The concept of reliability can be subdivided into the two main aspects of system adequacy and system security [1]. System security relates to the ability of the system to respond to disturbance arising within the system. System adequacy relates to the existence of sufficient facilities within the system to satisfy the customer demands within the system operating constraints. This includes the facilities necessary to generate sufficient energy and the transmission required to connect the generation to the actual customer load points. The research described in this thesis is in the adequacy domain.

A power system can be divided into generation, transmission and distribution facilities according to their functions. The three subsystems of generation, transmission and distribution can therefore be designated as power system functional zones. Reliability evaluation can be conducted in each of these functional zones or in the combinations that give the hierarchical levels [2] shown in Figure 1.1.

Reliability assessment at hierarchical level I (HLI) is concerned only with the generation facilities. In an HLI study, the system generation is examined to determine its adequacy to meet the total system load requirement considering random failures, and corrective and protective maintenance of the generating units. The transmission and distribution system and the ability to move the generated energy to the consumer load points are not included in this analysis. This activity is usually termed as "generating capacity reliability evaluation".

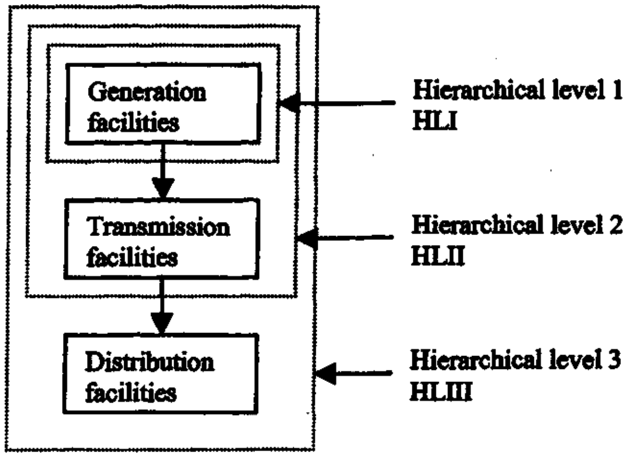


Figure1.1 Hierarchical levels

Hierarchical level II (HLII) assessment includes both generation and transmission facilities. HLII studies can be used to assess the adequacy of a system including the impact of various reinforcement alternatives at both the generation and transmission levels on the load point and overall system indices. Reliability analysis at this level is usually termed as "composite system or bulk system evaluation".

Hierarchical level III evaluation (HLIII) includes all three functional zones and starts at the generating points and terminates at the individual load points in the distribution system. A practical power system is very complex and therefore it is very difficult to evaluate the entire power system as a single entity using a completely realistic and exhaustive technique. HLIII studies are, therefore, not usually done directly. The analysis is usually performed only in the distribution functional zone and HLII load point indices are used as input values to the zone.

Distribution system reliability evaluation can be conducted within the distribution functional zone to obtain quantitative adequacy indices at the customer load points. These indices reflect the topology of the distribution network, the components used and the system operating philosophy.

Customer interruptions caused by generation and transmission system failures are normally only about 20 percent of the total load interruptions. The remaining 80 percent of customer interruptions occur within the distribution system [3]. Power system reliability assessment without considering the distribution facilities therefore recognizes only a relatively small portion of the total outages.

The reliability assessment techniques first used in practical application were deterministic in nature and some of these are still in use today. Although deterministic techniques were developed in order to combat and reduce the effects of random failures on a system, these techniques did not and cannot account for the probabilistic or stochastic nature of system behavior, of customer demands and of component failures. Probabilistic techniques that consider the stochastic nature of system behavior have been recognized since at least the 1930's[4]. These techniques were not widely used in the past due to lack of data, computer resources and realistic reliability techniques. With the development of computer techniques and the establishment of electric utility reliability data banks, probabilistic techniques have been widely developed and used by most utilities in many areas such as design, planning and maintenance [5-9].

The probability techniques used in power system reliability evaluation can be divided into the two categories of analytical methods and stochastic simulation approaches. The analytical and simulation approaches used in HLI are well developed and have been applied extensively in system planning and operation throughout the world. Considerable effort has also been expended during the last two decades on developing techniques and criteria for composite generation and transmission system assessment. The theories and techniques used to evaluate basic distribution system reliability indices are also highly developed as illustrated in the next section.

1.2 Distribution system reliability evaluation

Prior to the establishment of the hierarchical level approach, transmission system reliability evaluation was generally done in a similar way to that in distribution systems. The techniques developed initially for transmission and distribution system evaluation are still used and are being extended in the reliability assessment of distribution systems.

A distribution system can be divided into the two types of basic radial networks and parallel and meshed networks. Many systems can be connected in a meshed structure but are operated radially. The application of probability techniques in the quantitative evaluation of transmission and distribution schemes received its present impetus with the publication of two papers [10, 11] in 1964, which proposed a technique based on approximate equations for evaluating the rate and duration of outages. This technique formed the basis and starting point for most of the more modern developments. This technique was expanded and advanced in subsequent publications [12]. These papers developed models and equations for series and parallel systems which in addition to proposing indices of failure rate, average outage duration and average annual outage time included the consideration of failure bunching due to adverse weather conditions.

The application of Markov processes to transmission system evaluation is illustrated in [13]. This method is used as the primary evaluation method in certain applications and is frequently used as a means of checking approximate techniques. It is extremely useful as a standard evaluation method against which the accuracy of an approximate method can be compared. This method can, however, be very complicated and difficult to apply in large distribution networks.

An alternative approach to the Markov process is a method based on a set of approximate equations to evaluate the failure rate, outage duration and annual outage time or unavailability at a given load point. Reference [14] and [15] present a consistent set of equations for series/parallel system reduction which include adverse weather and permanent, temporary, maintenance and overload outage considerations.

A failure modes and effects analysis approach is illustrated in reference [15]. The failure modes are directly related to the minimal cut sets of the system and therefore the latter are used to identify the failure modes. The failure modes that are identified in this way represent component outages that must overlap to cause a system outage. A second order minimal cut outage event is seen as a set of parallel elements and its effect can be evaluated using the equations for parallel components. Since each of these overlapping outage events will cause system failure, these events are effectively in series from a reliability point of view. The research described in this thesis is focused on the analysis of a parallel transmission line reliability model which includes the recognition of both normal and adverse weather conditions.

1.3 Major adverse weather considerations

A major contribution to the total number of customer supply interruptions is due to failures in the overhead distribution system. This is primarily due to the weather environment in which these systems operate. The physical stresses placed upon the system components can be very much higher in bad weather than those encountered under normal weather conditions. Distribution facilities are normally concentrated in a relatively small area and therefore are liable to be affected in total by adverse weather conditions within the area. It was reported by the British Electricity Board, in 1968, that virtually all their failures occurred in adverse weather conditions, such as lighting, wind and icing [16]. Reliability predictions without incorporating weather considerations can be quite optimistic and the results can be considerably in error [17].

In addition to generally adverse weather conditions, major adverse weather conditions such as heavy storms, freezing rain and tornados, can have great impact on power system operations. The January 1998 ice storm, which occurred in Quebec, eastern Ontario and parts of the Maritimes, disrupted electric power supplies to over three million people and left more than 25 people dead [18]. Studies have shown that the effects on long term system reliability indices of extremely adverse weather are negligible if the frequency and duration of the extremely adverse weather is very small compared with that of normally encountered adverse weather [17]. Many researchers in the United States and Canada believe that extreme weather is not only becoming more frequent, but it is also getting more violent. The number of extreme low pressure systems, the barometric engines that drive the most devastating storms, could increase by more than 20 per cent by 2040. Precipitation in the form of snow or ice in the winter and rain at other times will become dramatically heavier [18]. This suggests that the effects of major adverse weather conditions may not be negligible, and that they could have considerable impact on power system reliability indices.

1.4 Research objectives

The objectives of the research work described in this thesis are to develop and examine a two parallel component transmission line reliability model considering both adverse weather and major adverse weather conditions. This reliability model should be applicable to the basic failure mode and effects analysis approach to distribution system reliability assessment. The analysis of the developed model is conducted using Markov analysis, an approximate equation approach and Monte Carlo simulation. The results obtained by the respective approaches are compared and used to illustrate the most useful approach for practical system application.

Applied techniques

The Markov approach is considered to be the most accurate method for distribution system reliability assessment given that all the assumptions are valid [13]. A sequence of events where the outcome depends on the element of chance is called a stochastic process. A Markov process is a special class of stochastic process in which future states of the process are dependent only upon the immediate past. A reliability problem can be represented by a Markov process that is discrete in space and continuous in time. The use of Markov processes in power system reliability evaluation was initially proposed in 1964 [19]. The practical application of a Markov process to transmission system reliability evaluation was first illustrated in 1967 [13]. Since then a great deal of work has been done on the application of Markov processes in a number of power system areas.

The Markov approach can be very complicated and difficult to apply when the number of states becomes large. The state space diagram becomes complicated when the adverse and major adverse weather conditions are included, even for the two transmission line reliability model. It is therefore important to develop a simpler approximate method having acceptable accuracy, which can be easily applied to the failure mode and effects analysis approach to distribution system reliability assessment. The development of a set of suitable approximate equations is described in this thesis.

Simulation techniques estimate the reliability indices by directly simulating the actual process and the random behavior of the system. The available techniques can be classified into the two categories of non-sequential or state sampling and sequential approach or state duration sampling.

In the basic state sampling approaches, it is assumed that each element has failure and success states, and that the component states are independent events. The behavior of each element is sampled using a uniform distribution between [0,1]. A system state depends on the combination of all element states [20, 21]. The advantages of this approach are:

1) The basic reliability data requirements are relatively low and only component-state probabilities are required.
2) The simulation procedure is very simple, as only uniformly distributed random numbers are generated and utilized.

A major disadvantage of this technique is that it cannot be used by itself to accurately calculate frequency and duration related indices.

The sequential method simulates the component and system behavior in chronological time. In this approach, chronological component state processes for all the components are created using random number generators and the probability distributions of the failure and repair processes [22, 23]. The chronological system states are then created by combining the individual chronological component state processes. The sequential technique can be used to simulate any state residence time distribution and to calculate actual frequency indices and the probability distributions of the relevant reliability indices. Compared to the state sampling approach, the sequential technique requires considerably more computing time and storage. The sequential technique is utilized in the research described in this thesis.

1.5 Outline of the thesis

The two weather state model for two parallel redundant transmission lines is introduced in Chapter 2. The basic approach to developing approximate equations for the solution of this system is presented in detail. The results obtained using these equations are compared with those obtained by a relatively precise Markov approach. A Monte Carlo simulation technique approach is also illustrated in this chapter and the results obtained by each method are compared.

A three weather state model is developed in Chapter 3 based on the two weather state model introduced in Chapter 2. The adverse weather state in the two weather state model is now redesignated simply as being a bad weather state and subsequently divided into two parts, adverse weather and major adverse weather, in the newly created three weather state model. The percentage of bad weather failures which occur in a major adverse weather becomes an important factor in the system reliability evaluation using the three state weather model. The approach used to develop approximate equations for the three weather state system is presented in detail. A Markov approach is also utilized to obtain a relatively more accurate result. A Monte Carlo simulation approach is also illustrated in this chapter. The results obtained using the three methods are compared, and the effects of varying the percentage of failures occurring in bad weather and the percentage of bad weather failures occurring in major adverse weather is examined using the three solution methods.

Selected sensitive analysis is described in Chapter 4. The weather conditions are varied to illustrate the effects of major adverse weather occurring more frequently. These effects are compared for the two state and three state weather models to determine which model is more appropriate under various conditions. Chapter 5 presents a summary and the conclusions of this research work.

Chapter 2
Two weather state model analysis

2.1 Introduction

A transmission and distribution system can be composed of either overhead or underground facilities or both. Overhead transmission lines are exposed to a wide range of weather conditions. Experience indicates that the failure rate of overhead transmission lines is a continuous function of the weather to which they are exposed. In some weather conditions, the failure rate can be many times greater than that found in the most favorable weather conditions. Extremely adverse weather conditions are generally infrequent and of short duration. During these periods, however, the failure rates of components increase sharply and the probability of overlapping failures is very much greater than that in favorable weather. The phenomenon of component failure during this period is called "failure bunching due to adverse weather." Distribution schemes concentrated in small areas are greatly affected by adverse weather conditions. If the adverse weather condition is neglected, the reliability indices evaluated for a load point can be over-optimistic and consequently very misleading. [15]

Failure bunching due to increased component failure rates in adverse weather should not be considered as common mode or common cause failures. This is an entirely separate phenomenon. The failure processes associated with overlapping component outages in adverse weather assume that these outages are independent events. The failure bunching occurs due to the enhanced component outage rate during the adverse weather.

Common mode failure rates during adverse weather periods are normally much greater than those in favorable weather conditions. Common cause failures can have a dominant influence on system failure probability and reliability indices. This influence can be considerably greater than that of independent failures during the common environment created by adverse weather[24]. The research work in this thesis is focused on the effects of overlapping independent component outages due to adverse and major adverse weather. Common mode failure effects are not considered in this thesis.

The basic concepts of weather state modeling are first introduced in this chapter. This is followed by a description of the Markov approach, the approximate equation method, and a Monte Carlo simulation procedure, and their application to a simple two parallel component model in a two state weather environment. The system reliability is examined in terms of the system average failure rate and the system average outage duration obtained using each method.

2.2 Weather state modeling

As noted above, the failure rate of a transmission line is directly related to the weather conditions that the line is exposed to. It is difficult if not impossible to describe the failure rate variation by a continuous function. A set of discrete states can be used to describe the component failure rate rather than a continuous function. Due to difficulties in system modeling, data collection and date validation, the data set must be restricted to a very limited number of states. This number should be sufficient to represent the failure bunching phenomenon but small enough to make the solution tractable.

According to IEEE Standard 346 [26], the weather environment can be divided into the three classes of normal, adverse and major storm disaster. These are defined as follows:

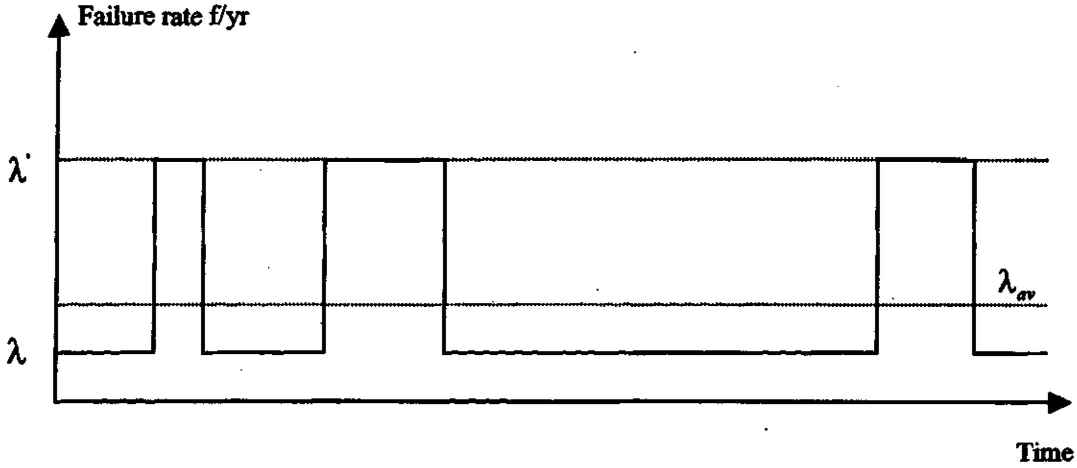
(1) Normal weather: Includes all weather not designated as adverse or major adverse.

(2) Adverse weather: Designated weather conditions which cause an abnormally high rate of forced outages for exposed components while such conditions persists, but do not qualify as major storm disasters. Adverse weather conditions can be defined for a particular system by selecting the proper values and combinations of conditions reported by the weather bureau: thunderstorms, tornadoes, wind velocities, precipitation, temperature, etc.

(3) Major storm disaster: Designates weather which exceeds the design limits of plant and which satisfies all of the following:

---- extensive mechanical damage to plant.
---- more than a specified percentage of customers out of service
---- service restoration times longer than a specified time.

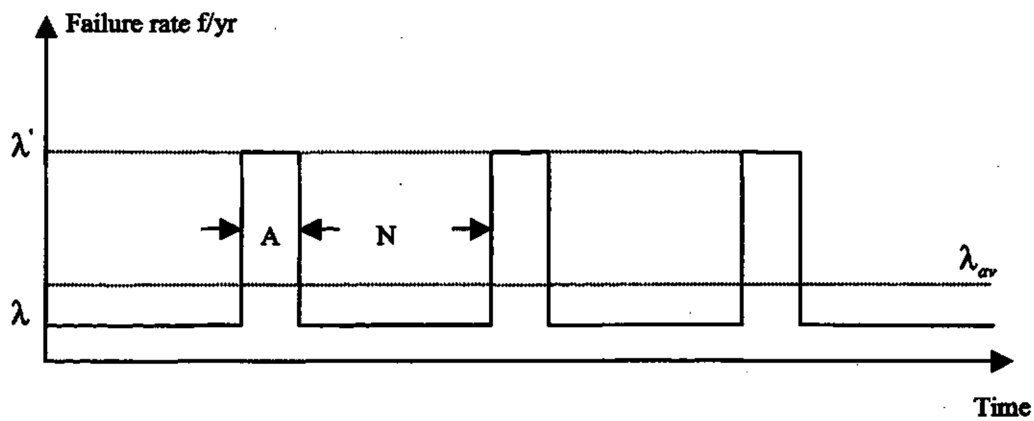
The first two weather conditions of normal and adverse weather are considered in this chapter. The major storm disaster condition designated as major adverse weather is discussed in Chapter 3. The wide range of weather conditions is therefore classified in this chapter into the two categories of normal weather and adverse weather. A simple two state fluctuating environment has been proposed in which weather duration distributions are assumed to be exponential [10]. This model is shown in Figure 2.1. The random occurrence pattern of adverse weather periods and their associated failure rate is averaged and represented by a regular pattern in Figure 2.2.



λ = Normal weather failure rate (failures per year of normal weather).

λ' = Adverse weather failure rate (failures per year of adverse weather).

Figure 2.1 Random history (two state environment)



A= Average duration of an adverse weather period.

N= Average duration of a normal weather period.

Figure 2.2 Average history (two state environment)

2.3 Adverse weather considerations

The weather conditions in Figures 2.1 and 2.2 are constrained in two states and therefore all subsequent failures should be allocated to one of these states depending on the prevailing weather at the time of the failure. The average failure rate (λ_{av}) of the component is given by Equation 2.1, and is the general statistic obtained without regard to weather conditions.

$$\lambda_{av} = \lambda(\frac{N}{N+A}) + \lambda'(\frac{A}{N+A}) \qquad (2.1)$$

In the case of a simple radial or series system, the load point failure rate is λ_{av} and recognition of λ and λ' as separate entities is not required in the reliability model.

In the case of a two component redundant supply, the conventional formula [4] for the load point failure rate λ_L is:

$$\lambda_L = \lambda_{av1}\lambda_{av2}(r_1 + r_2) \qquad (2.2)$$

where r_1 and r_2 are the average component repair times.

The values of λ and λ' can be evaluated from λ_{av} using Equation 2.1 if the values of N, A and the proportion of failures (F) occurring in adverse weather are known. This is shown in the following equations.

$$\lambda = \lambda_{av}\frac{N+A}{N}(1-F) \qquad (2.3a)$$

$$\lambda' = \lambda_{av}\frac{N+A}{A}F \qquad (2.3b)$$

2.4 Model including adverse weather conditions

The approximate and Markov approaches for the analysis of the two parallel redundant component system model are essentially those described in Reference [15] with the same basic assumptions. The probability of residing in the failed state is an important parameter in certain overall system studies. It is perhaps less important and secondary to other factors such as the configuration failure rate or failure frequency in other studies. The procedure used in [15] has been followed in this research work. Numerical results for the system failure rate and system average outage duration in a two line redundant configuration are shown later in this chapter. Approximate equations for the system failure rate are presented which include both normal and adverse weather considerations.

Several possible models including common mode outages are described and analyzed in [24]. The basic model [15] is shown in Figure 2.3. In this figure, n is the transition rate from normal weather to adverse weather and is the reciprocal of the average duration of the normal weather period. The transition rate m from adverse weather to normal weather is the reciprocal of the average duration of the adverse weather period. The transition rates λ and μ are the respective component failure and repair rates in Figure 2.3. This model assumes that there is no repair during adverse weather.

2.5 Markov approach

2.5.1 System state space probabilities

Appendix A provides a brief introduction to the Markov approach to reliability evaluation. The frequency balance approach [27] was used to obtain the set of equations designated as Equation 2.4 for determining the system steady state probabilities. These equations are written assuming that the state residence times are exponentially distributed. In Equation 2.4, P_1, P_2 …… P_8 are the steady state probabilities.

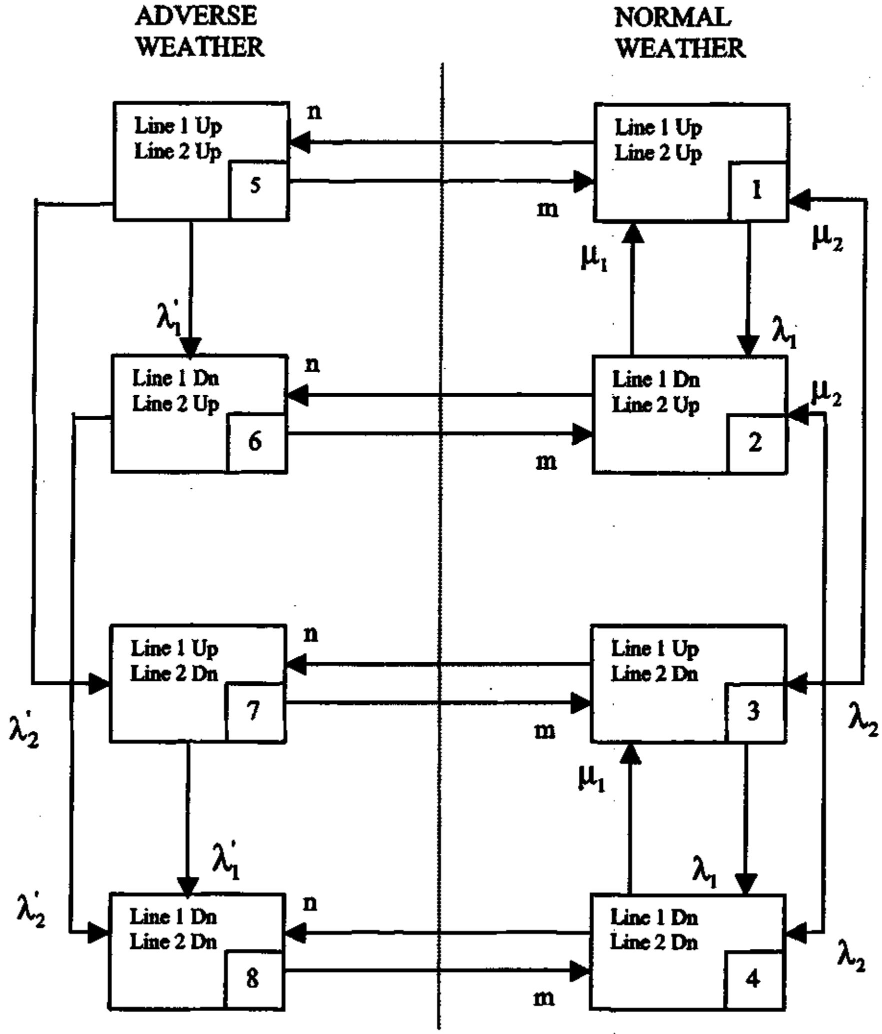


Figure 2.3 Model for two components including adverse weather effects

$$
\begin{aligned}
&(\lambda_1+\lambda_2+n)P_1-\mu_1P_2-\mu_2P_3-P_5m=0\\
&-\lambda_1P_1+(\mu_1+\lambda_2+n)P_2-\mu_2P_4-P_6m=0\\
&-\lambda_2P_1-\mu_1P_4+(\lambda_1+\mu_2+n)P_3-mP_7=0\\
&-\lambda_2P_2-\lambda_1P_3+P_4(\mu_1+\mu_2+n)-mP_8=0 \qquad (2.4a)\\
&-nP_1+(\lambda_1'+\lambda_2'+m)P_5=0\\
&-nP_2-P_5\lambda_1'+(\lambda_2'+m)P_6=0\\
&-nP_3-P_5\lambda_2'+(\lambda_1'+m)P_7=0\\
&-nP_4-\lambda_2'P_6-\lambda_1'P_7+mP_8=0
\end{aligned}
$$

The solution of the set of equations given in Equation 2.4 must satisfy the following condition:

$$\sum P_i = 1.0 \tag{2.4b}$$

i.e. the state probabilities must sum to one.

The set of simultaneous linear equations can be presented in the following matrix form

$$[0] = \begin{bmatrix} DN & -mI \\ -nI & DA \end{bmatrix} [P] \tag{2.5}$$

The matrices DN and DA correspond to transitions out of the normal and adverse weather states respectively.

[0] is a null vector.
[I] is an identity matrix.
[P] is a column vector of steady state probabilities.

The matrices DN and DS for the model of Figure 2.3 are shown below:

$$[DN] = \begin{bmatrix} \lambda_1 + \lambda_2 + n & -\mu_1 & -\mu_2 & 0 \\ -\lambda_1 & \lambda_2 + \mu_1 + n & 0 & -\mu_2 \\ -\lambda_2 & 0 & \lambda_1 + \mu_2 + n & -\mu_1 \\ 0 & -\lambda_2 & -\lambda_1 & \mu_1 + \mu_2 + n \end{bmatrix}$$

$$[DA] = \begin{bmatrix} \lambda_1' + \lambda_2' + m & 0 & 0 & 0 \\ -\lambda_1' & \lambda_2' + m & 0 & 0 \\ -\lambda_2' & 0 & \lambda_1' + m & 0 \\ 0 & -\lambda_2' & -\lambda_1' & m \end{bmatrix}$$

The system state probabilities can be obtained using Equation 2.5 and the condition given by Equation 2.4b. Analytical expressions can also be developed to determine the steady state probabilities using the mathematical approach given in [13].

2.5.2 System average failure rate

The procedure used to determine the system average failure rate is briefly presented as follows. The stochastic transitional probability matrix P from the state space model shown in Figure 2.3 is given below.

[P]=

$$\begin{bmatrix} 1-(\lambda_1+\lambda_2+n) & \lambda_1 & \lambda_2 & 0 & n & 0 & 0 & 0 \\ \mu_1 & 1-(\mu_1+\lambda_2+n) & 0 & \lambda_2 & 0 & n & 0 & 0 \\ \mu_2 & 0 & 1-(\lambda_1+\mu_2+n) & \lambda_1 & 0 & 0 & n & 0 \\ 0 & \mu_2 & \mu_1 & 1-(\mu_1+\mu_2+n) & 0 & 0 & 0 & n \\ m & 0 & 0 & 0 & 1-(\lambda_1'+\lambda_2'+m) & \lambda_1' & \lambda_2' & 0 \\ 0 & m & 0 & 0 & 0 & 1-(\lambda_2'+m) & 0 & \lambda_2' \\ 0 & 0 & m & 0 & 0 & 0 & 1-(\lambda_1'+m) & \lambda_1' \\ 0 & 0 & 0 & m & 0 & 0 & 0 & 1-m \end{bmatrix}$$

States 4 and 8 are system failure states and can therefore be designated as absorbing states [27]. A new matrix Q is obtained by eliminating the absorbing states. The truncated matrix is shown below:

$$Q = \begin{bmatrix} 1-(\lambda_1+\lambda_2+n) & \lambda_1 & \lambda_2 & n & 0 & 0 \\ \mu_1 & 1-(\mu_1+\lambda_2+n) & 0 & 0 & n & 0 \\ \mu_2 & 0 & 1-(\lambda_1+\mu_2+n) & 0 & 0 & n \\ m & 0 & 0 & 1-(\lambda_1'+\lambda_2'+m) & \lambda_1' & \lambda_2' \\ 0 & m & 0 & 0 & 1-(\lambda_2'+m) & 0 \\ 0 & 0 & m & 0 & 0 & 1-(\lambda_1'+m) \end{bmatrix}$$

Matrix [N] given by Equation 2.6 is the fundamental matrix, in which each element represents the time spent by the process in a given state before being absorbed.

$$[N]=[I-Q]^{-1} \tag{2.6}$$

Starting in state 1, the total expected time before entering the absorbing state of the process is $M_{1,6}$,

where $M_{1,6}=\sum_{i=1}^{6}N(1,i)$, and the system failure rate $\lambda_{SL}=1/M_{1,6}$.

2.5.3 System average outage duration

The average system outage duration r_{SL} is given by

$$r_{SL}=\frac{Cumulative\quad probability\quad of\quad the\quad failure\quad state}{Cumulative\quad frequency\quad of\quad the\quad failure\quad state}$$

The probabilities of the failure states are shown in Equation 2.4. In this case, the cumulative probability of failure is the sum of P_4 and P_8. The frequency of failure is given by

$$frequency\quad of\quad failure=P_4(\mu_1+\mu_2)$$

the system average outage duration is given by

$$r_{SL}=\frac{P_4+P_8}{P_4(\mu_1+\mu_2)} \tag{2.7}$$

2.6 Approximate approach

2.6.1 System average failure rate

The development of the system average outage rate formula for the model shown in Figure 2.3 is done in four distinct steps. The assumptions associated with the development of the approximate formula are as follows:

1. Times to failure and repair are exponentially distributed during both normal and adverse weather periods.
2. The duration of normal and adverse weather periods are exponentially distributed.
3. Repair times are typically very short compared with the times to failure and times between storms.
4. There is no repair during adverse weather.

The approximate equation development proceeds as follows:

(1) The initial failure occurs during normal weather and the second failure occurs during normal weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(failure \quad rate)_1 = \frac{N}{N+A}\{\lambda_1(\lambda_2 r_1) + \lambda_2(\lambda_1 r_2)\}$$

Where

$\dfrac{N}{N+A}$ = Long term fraction of time that the weather is normal,

$\lambda_2 r_1 \approx 1 - e^{-\lambda_2 r_1}$ i.e. The probability that component 2 fails during the repair period of component 1,

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(2) The initial failure occurs during normal weather and the second failure occurs during adverse weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(failure \quad rate)_2 = \frac{N}{N+A}\{\lambda_1 \frac{r_1}{N}\lambda_2' A + \lambda_2 \frac{r_2}{N}\lambda_1' A\}$$

Where

$\frac{r_1}{N} = 1 - e^{-r_1/N}$ i.e. The probability that the repair of component 1 is not completed within the normal weather,

$\lambda_2' A = 1 - e^{-\lambda_2' A}$ i.e. The probability that component 2 fails during the adverse weather,

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(3) The initial failure occurs during adverse weather and the second failure also occurs during adverse weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(failure \quad rate)_3 = \frac{A}{N+A}\{\lambda_1'(\lambda_2' A) + \lambda_2'(\lambda_1' A)\}$$

Where

$\frac{A}{N+A}$ = The long term fraction of time that the weather is adverse,

$A\lambda_2' \approx 1 - e^{-A\lambda_2'}$ i.e. The probability that component 2 fails during the adverse weather period,

The remaining terms in the above expression follow the same argument but with components 1 and 2 interchanged.

(4) The initial failure occurs during adverse weather and the second failure occurs in normal weather. The contribution to the system failure rate due to this mode of failure is given by:

$$(failure \quad rate)_4 = \frac{A}{N+A}\{\lambda_1'(1 - A\lambda_2')(\lambda_2 r_1) + \lambda_2'(1 - A\lambda_1')(\lambda_1 r_2)\}$$

Where

$1 - A\lambda_2' \approx e^{-A\lambda_2'}$ i.e. The probability that component 2 does not fail during adverse weather,

$\lambda_2 r_1 \approx 1 - e^{-\lambda_2 r_1}$ i.e. The probability that component 2 fails during the repair of component 1, The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

Thus the overall outage rate for a system containing two parallel redundant lines is:

$$\lambda_{SL} = \sum_{i=1}^{4} (failure \quad rate)_i \qquad (2.8)$$

Results obtained using the approximate equations are shown in Table 2.1 together with theoretically "exact" results obtained using the Markov approach. In this analysis, $\lambda_{av} = 1.0$ f/yr, N=200hours, A=2hours, and r=7.5 hours. The error factor shown in Table 2.1 is obtained by expressing the calculated load point failure rate including adverse weather effects using Equation 2.8 in per unit of the predicted value obtained using Equation 2.2. The percentage error is obtained by expressing the approximate system failure rate obtained using Equation 2.8 in per unit of the "exact" system failure rate obtained by the Markov approach.

Table 2.1 System average failure rate for a two component redundant system (Markov and approximate approaches)

% of Line Failures Occurring in Adverse Weather	System Failure Rate (f/yr) Approximate Approach.	Error Factor	System Failure Rate (f/yr) Markov Approach	Percentage Error
0	0.001665	0.972125	0.001725	3.507200
10	0.002120	1.238274	0.002186	3.016378
20	0.003462	2.021836	0.003536	2.102201
30	0.005690	3.322946	0.005772	1.413129
40	0.008804	5.141737	0.008868	0.715438
50	0.012805	7.478344	0.012809	0.027589
60	0.017693	10.33290	0.017580	0.644807
70	0.023468	13.70555	0.023157	1.346915
80	0.030131	17.59641	0.029532	2.028195
90	0.037681	22.00563	0.036682	2.721941
100	0.046119	26.93333	0.044599	3.406865

The values of each term of Equation 2.8 are shown in Table 2.2. In this table, NN represents the term in which the initial failure occurs in normal weather and the second failure also occurs in normal weather. NA represents the term in which the initial failure occurs in normal weather and the second failure occurs in adverse weather. AN indicates that the initial failure occurs in adverse weather and the second failure occurs in normal weather, AA represents the condition in which the initial failure occurs in adverse weather and the second failure occurs in adverse weather.

Table 2.2 Values of each term in Equation 2.8

% of Line Failures Occurring in Adverse Weather	NN	NA	AA	AN	System Failure Rate (f/yr) Approximate Approach.
0	0.001665	0	0	0	0.001665
10	0.001348	0.000156	0.000461	0.000155	0.002120
20	0.001065	0.000277	0.001845	0.000275	0.003462
30	0.000816	0.000363	0.004151	0.000361	0.005690
40	0.000599	0.000415	0.007379	0.000411	0.008804
50	0.000416	0.000432	0.011530	0.000427	0.012805
60	0.000266	0.000415	0.016603	0.000409	0.017693
70	0.000150	0.000363	0.022598	0.000357	0.023468
80	0.000067	0.000277	0.029516	0.000272	0.030131
90	0.000017	0.000156	0.037356	0.000152	0.037681
100	0	0	0.046119	0	0.046119

Table 2.2 clearly shows that only when the percentage of line failure occurring in adverse weather is generally lower than 20%, do most of the contributions to the system average failure rate come from the NN term. In the other cases, most of the contributions to the average system failure rate come from the AA term, for which both failures occur in adverse weather. From a practical point of view, this is the dominant term in Equation 2.8.

The variation in the error factor is shown in Figure 2.4 as a function of the percentage of failures that occur in adverse weather. It can be seen that the error factor increases rapidly as the percentage of adverse weather failures increases, and consequently a very optimistic evaluation is obtained if the weather effect is ignored.

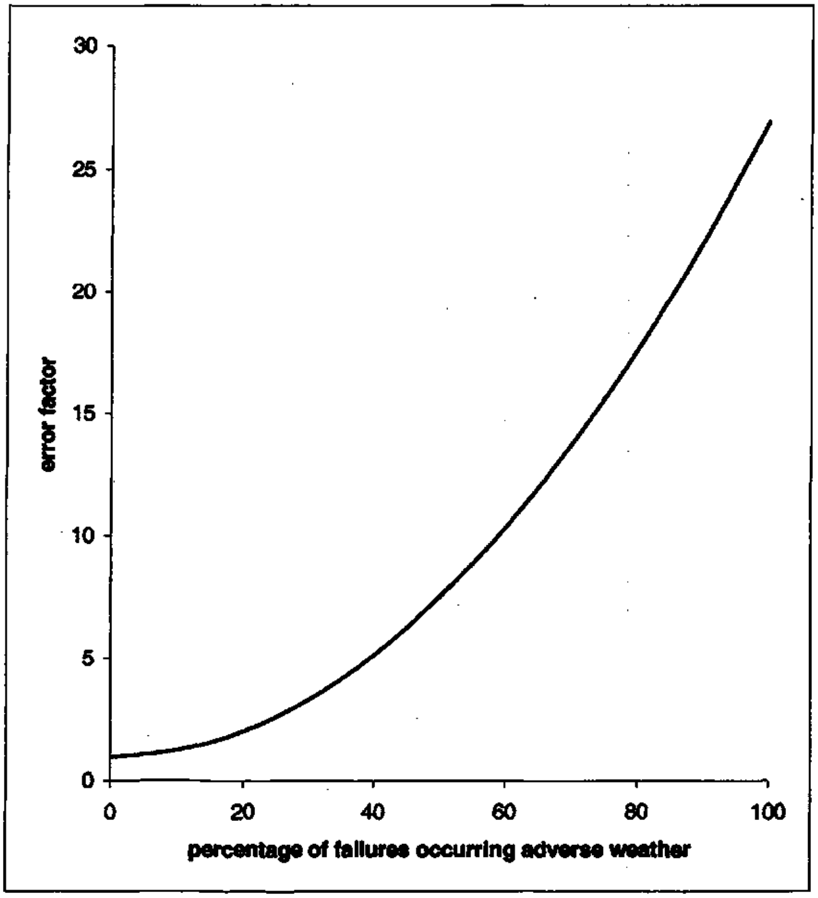


Figure 2.4 Error factor in the estimated failure rate

The error factor curve shown in Figure 2.4 was obtained for N=200 hours and A=2 hours, and is very dependent on these values. Assume that adverse weather occurs more frequently and for shorter durations such that N=100 hours and A=1 hour. The values of λ_{av} and r are unchanged at λ_{av}=1.0 f/yr and r=7.5hours. Table 2.3 presents the results in this case. Table 2.4 shows the results when the adverse weather occurs less frequently and lasts for longer periods of time. In this case, N=400 hours and A=4 hours. It can be seen from Table 2.3 and 2.4 that the percentage error associated with the approximate equation approach is not constant and varies with the percentage of failures occurring in adverse weather.

Table 2.3 System average failure rate for a two component redundant system (N = 100 hours, A= 1 hour)

% of Line Failures Occurring in Adverse Weather	System Failure Rate (f/yr) Approximate Approach.	Error Factor	System Failure Rate (f/yr) Markov Approach	Percentage Error
0	0.001600	0.934250	0.001725	7.273579
10	0.001837	1.073034	0.001959	6.208742
20	0.002499	1.459302	0.002640	5.356355
30	0.003584	2.093121	0.003768	4.873079
40	0.005093	2.974553	0.005342	4.657925
50	0.007027	4.103664	0.007368	4.633740
60	0.009384	5.480520	0.009815	4.383400
70	0.012166	7.105183	0.012709	4.272284
80	0.015373	8.977719	0.016006	3.953681
90	0.019004	11.09819	0.019743	3.746015
100	0.023059	13.46667	0.023895	3.496474

Table 2.4 System average failure rate for a two component redundant system (N =400hours, A=4 hour)

% of Line Failures Occurring in Adverse Weather	System Failure Rate (f/yr) Approximate Approach.	Error Factor	System Failure Rate (f/yr) Markov Approach	Percentage Error
0	0.001697	0.991062	0.001725	1.639504
10	0.002607	1.522737	0.002629	0.832891
20	0.005326	3.110544	0.005294	0.610328
30	0.009854	5.754758	0.009653	2.086752
40	0.016191	9.455652	0.015628	3.603755
50	0.024338	14.21350	0.023160	5.086503
60	0.034296	20.02858	0.032185	6.555992
70	0.046064	26.90116	0.042643	8.021004
80	0.059643	34.83152	0.054477	9.483102
90	0.075034	43.81994	0.067629	10.94960
100	0.092237	53.86667	0.082050	12.41599

The variation in the value of the error factor for each of the three cases is shown in Figure 2.5 as a function of the percentage of failures that occur in adverse weather. Figure 2.5 shows that when adverse weather occurs more frequently but with short durations (N= 100 hours , A=1 hour), the error factor increases slowly as the percentage of failures occurring in adverse weather increases. When adverse weather occurs less frequently but with longer durations (N = 400 hours, A = 4 hours), the error factor increases rapidly as the percentage of failures occurring in adverse weather increases.

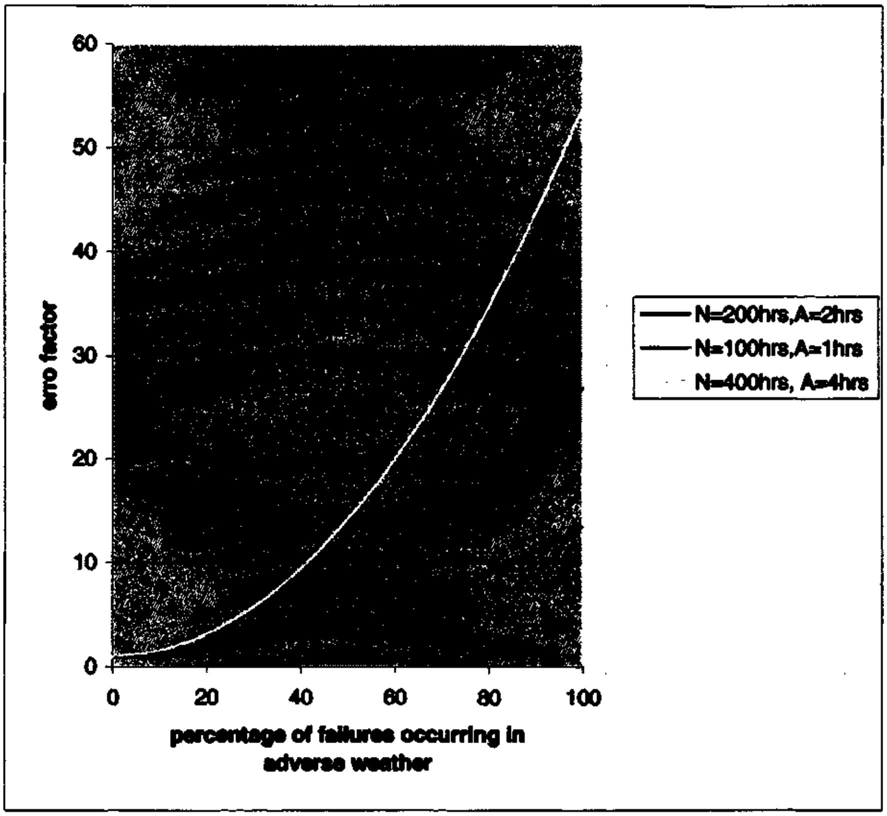


Figure 2.5 Error factor for three cases

In these examples, the ratios of N and A have been held constant. The probabilities of the adverse and normal weather state are therefore constant.

2.6.2 System average outage duration

Under the condition of no repair during adverse weather, failures which occur in adverse weather have to wait for repair for an average period equal to the average duration of the adverse weather. Referring to the model of Figure 2.3, the average outage duration for which failures occur during adverse weather periods is given by:

$$\frac{r_1 r_2}{r_1 + r_2} + A \tag{2.9}$$

If the contribution to the system failure rate due to the second failure occurring during normal weather (mode 1 and 4) and the second failure occurring during adverse weather (mode 2 and 3) are given by λ_{en} and λ_{ea} respectively, then the average outage duration is equal to the sum of the weighted average outage duration associated with each of these outage rate values as shown in Equation 2.10.

$$r_{SL} = \frac{\lambda_{en}}{\lambda_{en} + \lambda_{ea}}\left(\frac{r_1 r_2}{r_1 + r_2}\right) + \frac{\lambda_{ea}}{\lambda_{en} + \lambda_{ea}}\left(\frac{r_1 r_2}{r_1 + r_2} + A\right) \tag{2.10}$$

where

$$\lambda_{en} = (failure \quad rate)_1 + (failure \quad rate)_4$$

$$\lambda_{ea} = (failure \quad rate)_2 + (failure \quad rate)_3$$

The results using Equation 2.10 are shown in Table 2.5 together with theoretically "exact" results obtained using the Markov approach .The percentage error is defined as in the system average failure rate calculation shown in Tables 2.1 and 2.2.

Table 2.5 System average outage duration for two lines in parallel (Markov and approximate approaches)

% of Line Failures Occurring in Adverse Weather	Markov Approach (hrs)	Approximate Approach (hrs)	Percentage Error
0	3.792172	3.750000	1.112095
10	4.369792	4.331717	0.871319
20	5.006310	4.975442	0.616585
30	5.370957	5.336527	0.641048
40	5.555791	5.520455	0.636010
50	5.654495	5.618254	0.640913
60	5.710261	5.673625	0.641572
70	5.743787	5.706781	0.644278
80	5.764802	5.727552	0.646172
90	5.778469	5.741026	0.647964
100	5.787500	5.750000	0.647952

As in the case of the system failure rate, the percentage error in the system average outage duration varies with percentage of failures in adverse weather. The variation and the percentage error is, however, relatively small in this case.

2.7 Monte Carlo simulation method

In a time sequential simulation, an artificial history containing the up and down times of the system elements is generated in chronological order using random number generators and the probability distributions of the element failure and restoration parameters. A sequence of system operating-repair cycles is obtained from the generated component histories using the relationships between the element states and the system states. The system reliability indices can be obtained from the artificial history of the system.

2.7.1 Element models and parameters

The essential requirement in time sequential simulation is to generate realistic artificial operating/restoration histories of the relevant elements. These artificial histories depend on the system operating/restoration modes and the reliability parameters of the elements.

Transmission lines are normally represented by the two-state model shown in Figure 2.6 where the up state indicates that the line is in the operating state and the down state implies that the element is inoperable due to failure.

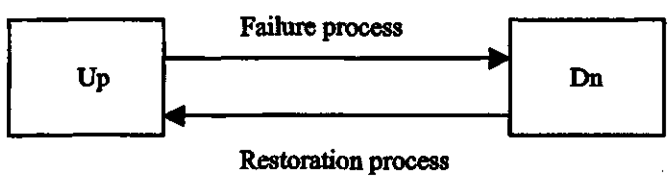


Figure 2.6 Basic two state diagram

The time for which the line remains in the up state is called the time to failure (TTF) or failure time (FT). The time in which the element is in the down state is called the restoration time or the time to repair (TTR). The process of transiting from the up state to the down state is the failure process. Figure 2.7 shows a simulated line operating/restoration history.

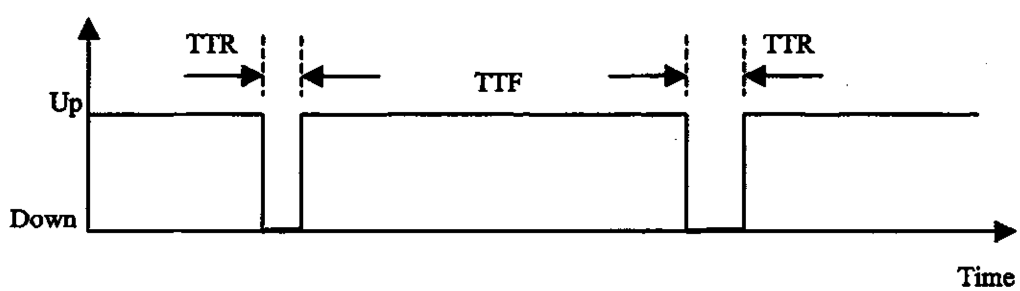


Figure 2.7 Line operating/repair history

The parameters TTF, TTR are random variables and are assumed to be exponentially distributed. The probability density function (p.d.f) of an exponential distribution is

$$f_T(t) = \begin{cases} \lambda e^{-\lambda t} & 0 < t < \infty \\ 0 & otherwise \end{cases} \qquad (2.11)$$

2.7.2 Generation of random numbers

A uniform distribution can be generated directly by a uniform random number generator. Random variables associated with other distributions are produced using the generated uniform number. The following example shows how to convert a uniform distribution into an exponential distribution using the inverse transform method.

The cumulative probability distribution function for the exponential distribution (2.11) is

$$U = F_T(t) = 1 - e^{-\lambda t} \qquad (2.12)$$

where U is a uniformly distributed random variable over the interval [0,1].
Solving for T:

$$T = -\frac{1}{\lambda}\ln(1-U) \qquad (2.13)$$

since (1-U) is distributed in the same way as U, then:

$$T = -\frac{1}{\lambda}\ln U \qquad (2.14)$$

2.7.3 Determination of the reliability indices

System failure occurs when the failure of one component overlaps that of the other component. The duration of this event is called overlapping time. Typical operating/repair sequences for a two component system are illustrated in Figure 2.8.

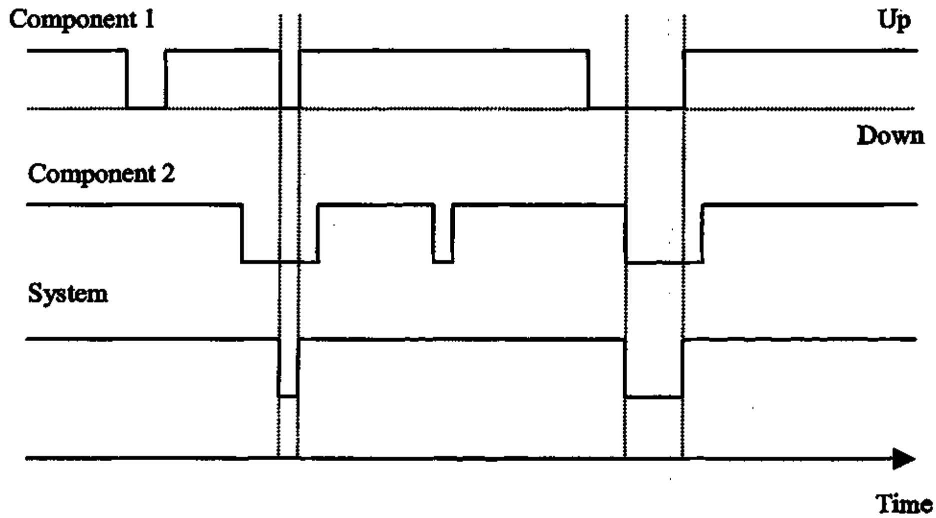


Figure 2.8 Typical operating/repair sequences for a two component system

System failure frequency $= \dfrac{\sum f}{\sum T}$ (f/yr) (2.15)

System average down time $r_{sys} = \dfrac{\sum T_d}{\sum f}$ (hours) (2.16)

System availability AV$= \dfrac{\sum T_u}{\sum T}$ (2.17)

Where

$\sum f$ = total number of system failures

$\sum T$ = total simulation time

$\sum T_u$ = total system up time

$\sum T_d$ = total system down time

The system average failure rate can be obtained using the following equation:

$$AV = 1 - \frac{\lambda_{sys}}{\lambda_{sys} + \mu_{sys}} = \frac{\mu_{sys}}{\lambda_{sys} + \mu_{sys}} = \frac{1}{\lambda_{sys} r_{sys} + 1}$$

So: $\lambda_{sys} = (\frac{1}{AV} - 1)\frac{1}{r_{sys}}$ (2.18)

2.7.4 Simulation procedure

Figure 2.3 shows the state space diagram for a two component system exposed to normal and adverse weather. The process used to evaluate the system reliability indices using time sequential simulation consists of the following steps:

Step 1: Choose state 1 as the starting state, in which both components are available and the weather is normal.

Step 2: Generate three random numbers associated with the process involving the component failure rates and the transition rate from normal weather to adverse weather. Convert these random numbers into the times to failure for both components (TTF1, TTF2) and the time to adverse weather (TTA) using λ_1, λ_2 and n respectively.

Step 3: Compare TTF1, TTF2 and TTA.
Time = Time + min{TTF1,TTF2,TTA}

At the beginning of the process, Time in the right hand side of the above equation is zero. If the time is greater than the desired years of simulation, go to step 20.

If TTF1 is the shortest time, the system will enter state 2. The TTA should be updated by deducting TTF1. Then go to step 4.

If TTF2 is the shortest time, the system will enter state 3. The TTA should be updated by deducting TTF2. Then go to step 7.

If TTA is the shortest time, the system will enter state 5. Then go to step 12.

Step 4: If the system enters state 2 from state 1 or 4, generate two random numbers associated with μ_1 and λ_2 respectively, In the case of the TTA, use the updated value in step 3. If the entry is from state 6, generate a random number associated with n.

Step 5: Convert these random numbers into the time to repair of component 1 (TTR1), the time to failure of component 2 (TTF2) and the time to adverse weather (TTA) if necessary.

Step 6: Compare TTF2, TTR1 and TTA.

Time = Time + min{TTF2,TTR1,TTA}

If the time is greater than the desired years of simulation, go to step 20.

If TTF2 is the shortest time, the system will enter state 4. The TTA and TTR1 should be updated by deducting TTF2. Then go to step 10.

If TTR1 is the shortest time, the system will go back to state 1.The TTA should be updated by deducting TTR1. Then go to step 2.

If TTA is the shortest time, the system will enter state 6. The TTR1 should be updated by deducting TTA. Then go to step 14.

Step 7: If the system enters state 3 from state 1 or state 4, generate two random numbers associated with λ_1 and μ_2 respectively. In the case of the TTA, use the updated value in step 3. If the entry is from state 7, generate a random number associated with n.

Step 8: Convert these random numbers into the time to repair of component 2 (TTR2), the time to failure of component 1(TTF1) and the time to adverse weather (TTA) if necessary.

Step 9: Compare TTF1, TTR2 and TTA.

Time = Time + min{TTF1,TTR2,TTA}

If the time is greater than the desired years of simulation, go to step 20.

If TTF1 is the shortest time, the system will enter state 4, the TTA and TTR2 should be updated by deducting TTF1. Then go to step 10.

If TTR2 is the shortest time, the system will enter state 1, the TTA should be updated by deducting TTR2, then go to step 2.

If TTA is the shortest time, the system will enter states 7, the TTR2 should be updated by deducting TTA, then go to step 16.

Step 10: If the system enters state 4 from state 2, generate a random number associated with μ_2. Convert this number into the time to repair of component 2 (TTR2). In the case of the TTA and TTR1, use the updated value in step 6.

If the system enters state 4 from state 3, generate a random number associated with μ_1. Convert this number into the time to repair of component 1 (TTR1). In the case of the TTA and TTR2, use the updated value in step 9.

If the system enters state 4 from state 8, generate a random number associated with n. Convert this number into the time to adverse weather (TTA). In the case of the TTR1 and TTR2, use the updated values in step 18 if they are not null.

If TTR1 is zero in step 18, generate a random number associated with μ_1, and convert it into the time to repair of component 1 (TTR1). If TTR2 is zero in step 18, generate a random number associated with μ_2, and convert it into the time to repair of component 2 (TTR2). If both TTR1 and ATTR2 are zero in step 18, generate two random numbers associated with μ_1 and μ_2, and convert them into the time to repair of component 1 (TTR1) and the time to repair of component 2 (TTR2).

Step 11: Compare TTR1, TTR2 and TTA.

$\text{Time} = \text{Time} + \min\{\text{TTR1},\text{TTR2},\text{TTA}\}$

If the time is greater than the desired years of simulation, go to step 20.

If TTR1 is the shortest time, the system will go back to state 3, TTR2 and TTA should be updated by deducting TTR1. Then go to step 7.

If TTR2 is the shortest time, the system will go back to state 2, TTR1 and TTA should be updated by deducting TTR2. Then go to step 4.

If TTA is the shortest time, the system will enter state 8. TTR1 and TTR2 should be updated by deducting TTA. Then go to step 18.

Step 12: Generate three random numbers associated with the process involving the component failure rates and the transition rate from adverse weather to normal weather. Convert these numbers into the time to failure of both component 1 and 2 (TTF1 and TTF2) and time to normal (TTN) using λ_1', λ_2' and m respectively

Step 13: CompareTTF1, TTF2 and TTN.

$\text{Time} = \text{Time} + \min\{\text{TTF1},\text{TTF2},\text{TTN}\}$

If the time is greater than the desired years of simulation, go to step 20.

If TTF1 is the shortest time, the system will enter state 6.TTN should be updated by deducting TTF1. Then go to step 14.

If TTF2 is the shortest time, the system will enter state 7, TTN should be updated by deducting TTF2. Then go to step 16.

If TTN is the shortest time, the system will enter state 1. Then go to step 2.

Step 14: If the system enters state 6 from state 5, generate a random number associated with $\lambda_2^{'}$, convert it into the time to failure of component 2 (TTF2). In the case of the TTN, use the updated value in step 13.

If the system enters state 6 from state 2, generate two random numbers associated with $\lambda_2^{'}$ and m respectively. Convert these numbers into the time to failure of component 2 (TTF2) and the time to normal weather (TTN) respectively.

In the case of the TTR1, keep its value unchanged if it is not zero.

Step 15: Compare TTF2 and TTN.

Time = Time + min{TTF2, TTN}

If the time is greater than the desired years of simulation, go to step 20.

If TTF2 is the shorter time, the system will enter state 8. TTN should be updated by deducting TTF2. Then go to step 18.

If TTN is the shorter time, the system will enter state 2. Then go to step 4.

Step 16: If the system enters state 7 from state 5, generate a random number associated with $\lambda_1^{'}$ and convert it into the time to failure of component 1(TTF1). In the case of the TTN, use the value updated in step 13.

If the system enters state 7 from state 3, generate two random numbers associated with λ_1' and m respectively, convert these numbers into the time to failure of component 1(TTF1) and the time to normal weather (TTN).

In the case of the TTR2, keep its value unchanged if it is not zero.

Step 17: Compare TTF1 and TTN.

Time = Time + min{TTF1, TTN}

If the time is greater than the desired years of simulation, go to step 20.

If TTF1 is the shorter time, the system will enter state 8. TTN should be update by deducting TTF1. Then go to step 18.

If TTN is the shorter time, the system will enter state 3. Then go to step 7.

Step 18: If the system enters state 8 from state 4, generate one random number associated with m, convert it into the time to normal weather (TTN). If the system enters state 8 from state 5 or state 7, use the updated value of TTN in step 15 or step 17.

In the case of the TTR1 or TTR2, keep these value unchanged if they are not zero.

Step 19: Time = Time + TTN

If the time is greater than the desired years of simulation, go to step 20.

If the time is less than the desired years of simulation, go to step 10.

Step 20: Determine the total system down time and system failure frequency.

The total system down time is obtained by accumulating the duration of the system residence times in states 4 and 8.

The system failure frequency is obtained by accumulating the number of times that the system enters state 4 from states 2 or 3 and the number of times of the system enters state 8 from states 6 or 7.

Step 21: Obtain the system reliability indices, system failure frequency, system average downtime and system availability using Equations 2.14–2.16.

2.7.5 Simulation results

The system average failure rates obtained using the Monte Carlo simulation approach are shown in Table 2.6 together with the values obtained using the Markov approach. The system average outage time values are shown in Table 2.7.

Table 2.6 System average failure rate for a two component redundant system (MCS and Markov approaches)

% of Line Failures Occurring in Adverse Weather	System Failure Rate (f/yr) MCS Approach.	Error Factor	System Failure Rate (f/yr) Markov Approach	Percentage Error
0	0.001850	1.080400	0.001725	7.245217
10	0.002300	1.343200	0.002186	5.214090
20	0.003650	2.131600	0.003536	3.223133
30	0.005690	3.322960	0.005772	1.421344
40	0.008050	4.701200	0.008868	9.224628
50	0.012520	7.311680	0.012809	2.256382
60	0.016870	9.852080	0.017580	4.038567
70	0.023600	13.78240	0.023157	1.913547
80	0.029620	17.29808	0.029532	0.298930
90	0.037221	21.73706	0.036682	1.468159
100	0.044921	26.23386	0.044599	0.721675

Table 2.7 System average outage duration for a two component redundant system (MCS and Markov approaches)

% of Line Failures Occurring in Adverse Weather	Markov Approach (hrs)	MCS Approach (hrs)	Percentage Error
0	3.792172	3.546907	6.467665
10	4.369792	4.170287	4.565549
20	5.006310	4.829863	3.524492
30	5.370957	5.216334	2.878872
40	5.555791	5.470712	1.531357
50	5.654495	5.735687	1.435884
60	5.710261	5.687376	0.400770
70	5.743787	5.649268	1.645587
80	5.764802	5.714376	0.874722
90	5.778469	5.845143	1.153835
100	5.787500	5.693608	1.622324

The Monte Carlo approach takes considerable more solution time than either the Markov or the approximate methods. If the simulation time is 100,000 years, a solution will take 70 seconds on a Pentium II PC. The running time is less than 0.1 second for the Markov approach and the approximate approach,

The convergence associated with the Monte Carlo solution is illustrated in Figure 2.9. In this example, 50% of the line failures are assumed to occur in adverse weather. Figure 2.9 shows the system average failure rate as a function of the simulation time. It can be seen that the system average failure rate continues to fluctuate even though the simulation time is quite long. This fluctuation makes it difficult to directly compare the results with the theoretically exact values obtained using the Markov approach.

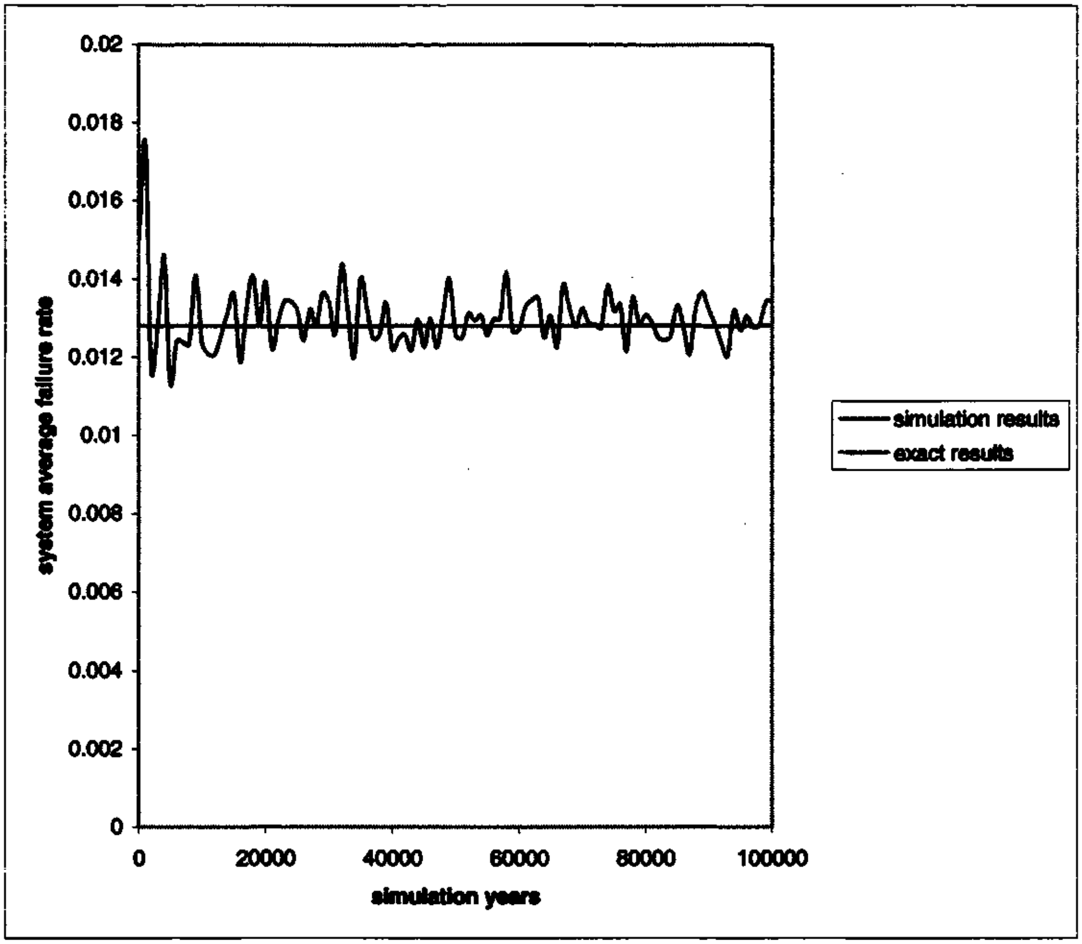


Figure 2.9 Simulation convergence (two weather state model)

2.8 Summary

This chapter introduces the basic two redundant component transmission line reliability model incorporating normal and adverse weather conditions. Two system indices, the average failure rate and the average outage duration were evaluated using the Markov approach, the approximate method and the Monte Carlo simulation technique.

The three methods all give reasonably similar results. The Markov approach can be considered to provide theoretically exact values. Under these conditions, the percentage errors in the approximate approach and the Monte Carlo simulation method are quite small. All three methods can be considered acceptable from an accuracy point of view.

The Monte Carlo Simulation approach has the problem that it does not converge even though the simulation time is very long. This could result in relatively large errors. The Monte Carlo method is also very time consuming. Both the Markov approach and the approximate method involve much less solution time. The approximate approach, however, is much more direct and simple than the Markov method. The approximate method is the most practical approach for general transmission and distribution system analysis as it can be applied directly in minimal cut applications [15].

In this chapter, the weather environment is divided into normal weather and adverse weather. The analyses conducted show that the error factor increases rapidly as the percentage of failures occurring in adverse weather increases. This clearly indicates that a very optimistic evaluation would be obtained if the effects of weather were ignored. As previously noted, IEEE Standard 346 [26] classifies the weather into normal, adverse and major storm. This chapter illustrates the effects of recognizing the increased failure rate associated with adverse weather. Chapter 3 extends the analysis by including further increases in component failure rate due to major adverse weather. The solution techniques introduced in Chapter 2 are utilized in Chapter 3 to analyze a two component redundant configuration including normal, adverse and major adverse weather conditions.

Chapter 3
Three weather state model analysis

3.1 Introduction

IEEE Standard 346 [26] subdivides the weather environment into the three categories of normal, adverse and major storm disaster. As noted in Chapter 2, only normal weather and adverse weather conditions are generally considered due to difficulties in modeling and data collection. Chapter 1 notes that major adverse weather conditions appear to be becoming more frequent and that they have a significant effect on the system reliability. It is therefore reasonable to consider that these periods should not be aggregated with other less violent periods of adverse weather.

An initial discussion of a three weather state model is presented in [17]. This model assumes that overlapping outages cannot occur in the two different types of adverse weather. Possible transitions between adverse weather and major adverse weather do, however, exist and overlapping outages could occur in these two types of adverse weather. Reference [17] briefly discusses the three state model but does not conduct any detailed analysis.

A new model which incorporating the three weather states has been developed in this research work. This model incorporates all the possible overlapping outage conditions. The three solution techniques introduced in Chapter 2 have been applied to this model. This chapter presents the results of the analysis in terms of the system average failure rate and average outage duration using the three methods.

3.2 Major adverse weather considerations

3.2.1 Parameter assumptions

In Chapter 2, the weather environment is classified into the two categories of normal and adverse weather. The two weather state space diagram is shown in Figure 3.1. In this diagram, the normal and adverse weather state transition rates are labeled as follows in order to easily extend them to the three weather state model.

na = transition rate from normal weather to adverse weather

an = transition rate from adverse weather to normal weather

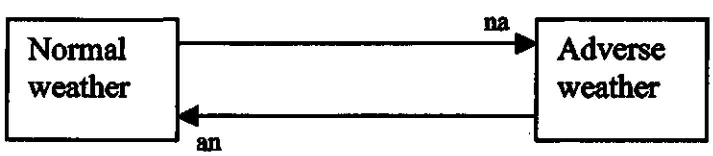


Figure 3.1 Two weather state model

In this chapter, the weather environment is classified into the three categories of normal, adverse and major adverse weather. The bad weather condition in the two weather state model is split into two parts: adverse weather and major adverse weather. The three weather state space diagram is shown in Figure 3.2.

Where:

na = transition rate from normal weather to adverse weather

an = transition rate from adverse weather to normal weather

am = transition rate from adverse weather to major adverse weather

ma = transition rate from major adverse weather to adverse weather

mn = transition rate from major adverse weather to normal weather

nm = transition rate from normal weather to major adverse weather.

Component failure is dependant on the weather state in which the component resides. The weather dependant component failure rates are defined as follow.

λ = normal weather failure rate (failures per year in normal weather).

λ' = adverse weather failure rate (failures per year in adverse weather).

λ^{ma} = major adverse weather failure rate (failures per year in major adverse weather).

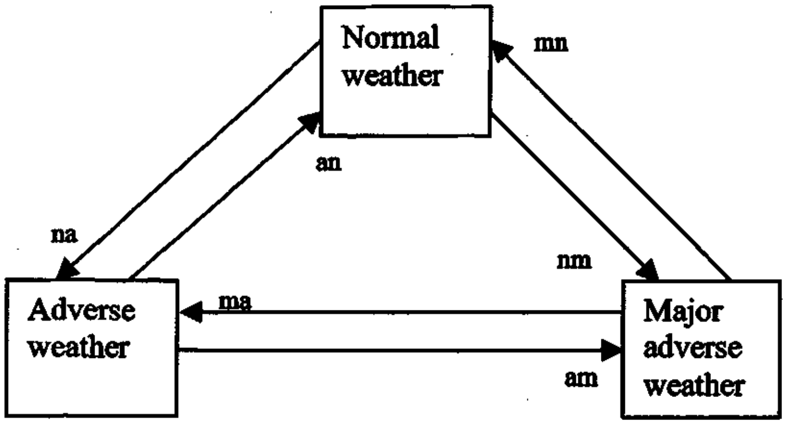


Figure 3.2 Three weather state model

It is extremely difficult to collect data on normal, adverse and major adverse weather and particularly on the component failures which occur in these periods. The following assumptions were made in order to create a realistic and practical three state weather model. It was assumed that the parameters na and an retained the same values used in the two weather model applied in Section 2.6. i.e. na = 1/200 occ/hour and an = 1/2 occ/hour. It was then assumed that major adverse weather occurs once per year and that the transition rates mn and ma are 1/2 occ/hour.

The state probabilities in Figure 3.2 are as follows:

P_N = steady state probability of normal weather

P_A = steady state probability of adverse weather

P_{MA} = steady state probability of major adverse weather

The state probabilities P_N, P_A, and P_{MA} can be obtained using the following equations:

$$\begin{cases} P_N(na+nm) = P_A an + P_{MA} mn \\ P_A(an+am) = P_N na + P_{MA} ma \\ P_N + P_A + P_{MA} = 1 \end{cases}$$

$$P_N = \frac{an(na\cdot mn+na\cdot ma+nm\cdot ma)+mn(na\cdot am+an\cdot nm+am\cdot nm)}{(na\cdot mn+na\cdot ma+nm\cdot ma)(na+an+nm)+(na\cdot am+an\cdot nm+am\cdot nm)(na+nm+mn)} \tag{3.1}$$

$$P_A = \frac{(na+nm)(na\cdot mn+na\cdot ma+nm\cdot ma)}{(na\cdot mn+na\cdot ma+nm\cdot ma)(na+an+nm)+(na\cdot am+an\cdot nm+am\cdot nm)(na+nm+mn)} \tag{3.2}$$

$$P_{MA} = \frac{(na+nm)(na\cdot am+an\cdot nm+am\cdot nm)}{(na\cdot mn+na\cdot ma+nm\cdot ma)(na+an+nm)+(na\cdot am+an\cdot nm+am\cdot nm)(na+nm+mn)} \tag{3.3}$$

The steady state probability and the frequency of encountering each state are as follows:

$P_N = 0.98987525$ $\quad P_A = 0.01001061$ $\quad P_{MA} = 0.00011414$

$f_N = 44.3464$occ/yr $\quad f_A = 43.8565$occ/yr $\quad f_{MA} = 0.9999$occ/yr

The average duration of the normal weather period (N), the average duration of the adverse weather period (A) and the average duration of the major adverse weather period (M) in this case are as follows:

$$N = \frac{1}{na+nm} = \frac{1}{\frac{1}{200}+\frac{1}{8760}} = 195.5357 \text{ hours}$$

$$A = \frac{1}{an + am} = \frac{1}{\frac{1}{2} + \frac{1}{8760}} = 1.9995 \text{ hours} \approx 2.0 \text{ hours}$$

$$M = \frac{1}{mn + ma} = \frac{1}{\frac{1}{2} + \frac{1}{2}} = 1 \text{ hour}$$

The parameters presented above are used as the basic data set in the following calculations.

3.2.2 Component average failure rate incorporating major adverse weather

The component average failure rate is given by Equation 3.4.

$$\lambda_{av} = P_N \lambda + P_A \lambda' + P_{MA} \lambda^{ma} \tag{3.4}$$

Assume that the component average failure rate λ_{av}=1.0 f/yr, 50% of the outages occur in normal weather, 45% of the outages occur in adverse weather and 5% of the outages occur in major adverse weather. The component failure rate in normal weather λ, the component failure rate in adverse weather λ' and the component failure rate in major adverse weather λ^{ma} obtained using Equation 3.4. are as follows,

$\lambda = 0.5051$ occ/yr,

$\lambda' = 44.9530$ occ/yr,

$\lambda^{ma} = 438.6465$ occ/yr.

It is important to note that in the two state weather model, the probability of normal weather is given by $\frac{N}{N+A}$ and the probability of adverse weather is given by $\frac{A}{N+A}$.

It is not correct to extend Equation 2.1 to incorporate all three weather states as shown in Equation 3.5.

$$\lambda_{av} = \frac{N}{N+A+M}\lambda + \frac{A}{N+A+M}\lambda' + \frac{M}{N+A+M}\lambda^{ma} \tag{3.5}$$

Under this condition,

$\lambda = 0.5077$ occ/yr,

$\lambda' = 44.6816$ occ/yr,

$\lambda^{ma} = 9.9268$ occ/yr.

These values are obviously incorrect.

3.3 Model including major adverse weather

A three state model including major adverse weather is shown in Figure 3.3. This is a direct extension of the two weather state model.

3.4 Markov approach

3.4.1 System state space probabilities

The frequency balance approach for determining the system steady state probabilities was used to obtain the relationships shown as Equation 3.6. These equations are written assuming the state residence times are exponentially distributed. In these equations, P_1, P_2 ...... P_{12} are the steady state probabilities.

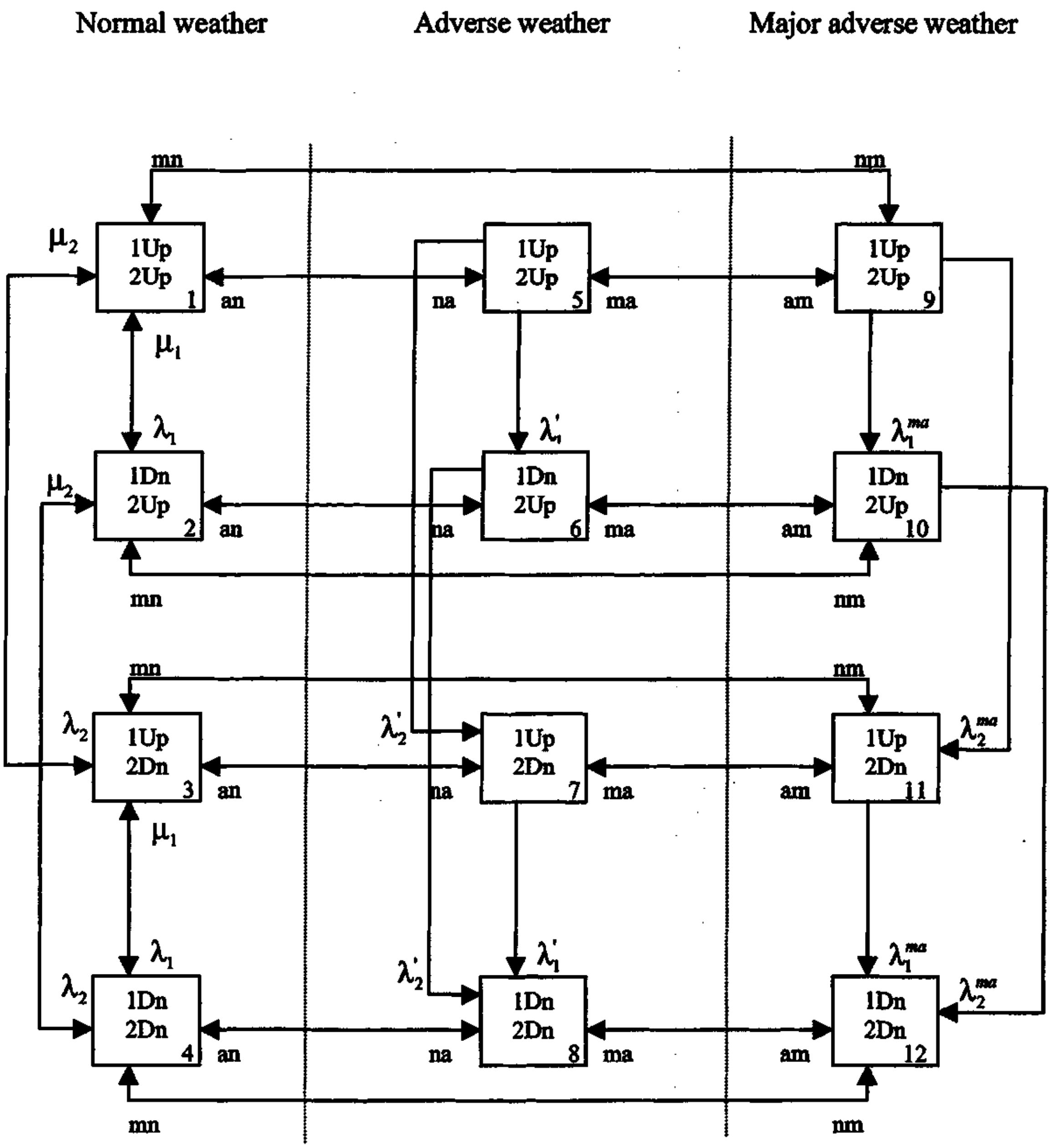


Figure 3.3 Model for two components including major adverse weather effects

$$P_1(\lambda_1+\lambda_2+na+nm)-P_2\mu_1-P_3\mu_2-P_5an-P_9mn=0$$
$$-P_1\lambda_1+P_2(\lambda_2+\mu_1+na+nm)-P_4\mu_2-P_6an-P_{10}mn=0$$
$$-P_1\lambda_2+P_3(\lambda_1+\mu_2+na+nm)-P_4\mu_1-P_7an-P_{11}mn=0$$
$$-P_2\lambda_2-P_3\lambda_1+P_4(\mu_1+\mu_2+na+nm)-P_8an-P_{12}mn=0$$
$$-P_1na+P_5(\lambda_1'+\lambda_2'+an+am)-P_9ma=0$$
$$-P_2na-P_5\lambda_1'+P_6(\lambda_2'+an+am)-P_{10}ma=0 \tag{3.6}$$
$$-P_3na-P_5\lambda_2'+P_7(\lambda_1'+an+am)-P_{11}ma=0$$
$$-P_4na-P_6\lambda_2'-P_7\lambda_1'+P_8(an+am)-P_{12}ma=0$$
$$-P_1nm-P_5am+P_9(\lambda_1^{ma}+\lambda_2^{ma}+ma+mn)=0$$
$$-P_2nm-P_6am-P_9\lambda_1^{ma}+P_{10}(\lambda_2^{ma}+ma+mn)=0$$
$$-P_3nm-P_7am-P_9\lambda_2^{ma}+P_{11}(\lambda_1^{ma}+ma+mn)=0$$
$$-P_4nm-P_8am-P_{10}\lambda_2^{ma}-P_{11}\lambda_1^{ma}+P_{12}(ma+mn)=0$$

In order to determine the steady state probabilities, the solution of the relationships given in Equation 3.6 has to satisfy the following condition:

$$\sum P_i=1.0$$

i.e. the availability of all the states must sum to one.

The set of simultaneous linear equations can be presented in the following matrix form

$$\begin{bmatrix} \\ 0 \\ \\ \end{bmatrix}=\begin{bmatrix} DN & -an\cdot I & -mn\cdot I \\ -na\cdot I & DA & -ma\cdot I \\ -nm\cdot I & -am\cdot I & DM \end{bmatrix}\begin{bmatrix} \\ P \\ \\ \end{bmatrix} \tag{3.7}$$

The matrix DN, DA and DM correspond to transitions out of the normal, adverse and major adverse weather states respectively.

[0] is a null vector.

[I] is an identity matrix.

[P] is a column vector of steady state probabilities.

The matrices DN, DA and DM for the model of Figure 3.3 are as follows:

$$[DN]=\begin{bmatrix} \lambda_1+\lambda_2+na+nm & -\mu_1 & -\mu_2 & 0 \\ -\lambda_1 & \lambda_2+\mu_1+na+nm & 0 & -\mu_2 \\ -\lambda_2 & 0 & \lambda_1+\mu_2+na+nm & -\mu_1 \\ 0 & -\lambda_2 & -\lambda_1 & \mu_1+\mu_2+na+nm \end{bmatrix}$$

$$[DA]=\begin{bmatrix} \lambda_1'+\lambda_2'+an+am & 0 & 0 & 0 \\ -\lambda_1' & \lambda_2'+an+am & 0 & 0 \\ -\lambda_2' & 0 & \lambda_1'+an+am & 0 \\ 0 & -\lambda_2' & -\lambda_1' & an+am \end{bmatrix}$$

$$[DM]=\begin{bmatrix} \lambda_1^{ma}+\lambda_2^{ma}+ma+mn & 0 & 0 & 0 \\ -\lambda_1^{ma} & \lambda_2^{ma}+ma+mn & 0 & 0 \\ -\lambda_2^{ma} & 0 & \lambda_1^{ma}+ma+mn & 0 \\ 0 & -\lambda_2^{ma} & -\lambda_1^{ma} & ma+mn \end{bmatrix}$$

The stochastic transitional probability matrix designated as Equation 3.7 can be solved for the system state probabilities using a matrix inversion approach for the given values of λ, μ, na, nm, an, am, ma and mn.

3.4.2 System average failure rate

The stochastic transitional probability matrix for the three weather states model shown in Figure 3.3 is as follows:

$$
\begin{bmatrix}
1-(\lambda_1+\lambda_2+na+nm) & \lambda_1 & \lambda_2 & 0 & na & 0 & 0 & 0 & nm & 0 & 0 & 0 \\
\mu_1 & 1-(\lambda_2+\mu_1+na+nm) & 0 & \lambda_2 & 0 & na & 0 & 0 & 0 & nm & 0 & 0 \\
\mu_2 & 0 & 1-(\lambda_1+\mu_2+na+nm) & \lambda_1 & 0 & 0 & na & 0 & 0 & 0 & nm & 0 \\
0 & \mu_2 & \mu_1 & 1-(\mu_1+\mu_2+na+nm) & 0 & 0 & 0 & na & 0 & 0 & 0 & nm \\
an & 0 & 0 & 0 & 1-(\lambda_1'+\lambda_2'+an+am) & \lambda_1' & \lambda_2' & 0 & am & 0 & 0 & 0 \\
0 & an & 0 & 0 & 0 & 1-(\lambda_2'+an+am) & 0 & \lambda_2' & 0 & am & 0 & 0 \\
0 & 0 & an & 0 & 0 & 0 & 1-(\lambda_1'+an+am) & \lambda_1' & 0 & 0 & am & 0 \\
0 & 0 & 0 & an & 0 & 0 & 0 & 1-(an+am) & 0 & 0 & 0 & am \\
mn & 0 & 0 & 0 & ma & 0 & 0 & 0 & 1-(\lambda_1^m+\lambda_2^m+ma+mn) & \lambda_1^m & \lambda_2^m & 0 \\
0 & mn & 0 & 0 & 0 & ma & 0 & 0 & 0 & 1-(\lambda_2^m+ma+mn) & 0 & \lambda_2^m \\
0 & 0 & mn & 0 & 0 & 0 & ma & 0 & 0 & 0 & 1-(\lambda_1^m+ma+mn) & \lambda_1^m \\
0 & 0 & 0 & mn & 0 & 0 & 0 & ma & 0 & 0 & 0 & 1-(ma+mn)
\end{bmatrix}
$$

States 4, 8 and 12 are designated as absorbing states and a new matrix Q is obtained by eliminating the absorbing states. The truncated matrix is shown below.

$$\begin{bmatrix}
1-(\lambda_1+\lambda_2+na+nm) & \lambda_1 & \lambda_2 & na & 0 & 0 & nm & 0 & 0 \\
\mu_1 & 1-(\lambda_2+\mu_1+na+nm) & 0 & 0 & na & 0 & 0 & nm & 0 \\
\mu_2 & 0 & 1-(\lambda_1+\mu_2+na+nm) & 0 & 0 & na & 0 & 0 & nm \\
an & 0 & 0 & 1-(\lambda_1'+\lambda_2'+an+am) & \lambda_1' & \lambda_2' & am & 0 & 0 \\
0 & an & 0 & 0 & 1-(\lambda_2'+ +an+am) & 0 & 0 & am & 0 \\
0 & 0 & an & 0 & 0 & 1-(\lambda_1'+ +an+am) & 0 & 0 & am \\
mn & 0 & 0 & ma & 0 & 0 & 1-(\lambda_1^m+\lambda_2^m+ma+mn) & \lambda_1^m & \lambda_2^m \\
0 & mn & 0 & 0 & ma & 0 & 0 & 1-(\lambda_2^m+ +ma+mn) & 0 \\
0 & 0 & mn & 0 & 0 & ma & 0 & 0 & 1-(\lambda_1^m+ +ma+mn)
\end{bmatrix}$$

As shown in Chapter 2, matrix [N] is the fundamental matrix, where n_j is the time spent by the process in state S_j before being absorbed.

$$[N]=[I-Q]^{-1}$$

Starting in state 1, the time before entering the absorbing state of the process is $M_{1,9}$,

where $M_{1,9}=\sum_{i=1}^{9} N(1,i)$,

and the system failure rate $\lambda_{SL}=1/M_{1,9}$. (3.8)

3.4.3 System average outage duration

The system average outage duration is given by

$$r_{SL} = \frac{Cumulative \quad probability \quad of \quad the \quad failure \quad state}{Cumulative \quad frequency \quad of \quad failure}.$$

The probability of the failure states are shown in Equation 3.6. In this case, the probability of failure is the sum of P_4 , P_8 and P_{12}, The frequency of failure is given by

$$frequency \quad of \quad failure = P_4(\mu_1 + \mu_2),$$

the system average outage duration is given by

$$r_{SL} = \frac{P_4 + P_8 + P_{12}}{P_4(\mu_1 + \mu_2)}. \tag{3.9}$$

3.5 Approximate approach

3.5.1 System average failure rate

The development of the system average outage rate formula using the approximate approach illustrated in Chapter 2 for the model of Figure 3.3 is done in nine distinct steps.

The following development assumes that repair does not occur during adverse and major adverse weather. In these equations, N is the average duration of normal weather. A is the average duration of adverse weather and M is the average duration of major adverse weather.

(1) the initial failure occurs during normal weather and the second failure also occurs during normal weather

$$(failure\quad rate)_1 = P_N\{\lambda_1(\lambda_2 r_1) + \lambda_2(\lambda_1 r_2)\}$$

Where

P_N = long term fraction of time that the weather is normal,

$\lambda_2 r_1 \approx 1 - e^{-\lambda_2 r_1}$ = The probability that the component 2 fails during the repair period of component 1.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(2) the initial failure occurs during normal weather and the second failure occurs during adverse weather

$$(failure\quad rate)_2 = P_N\{\lambda_1(r_1 na)(1 - r_1 nm)(1 - \lambda_2 r_1)(\lambda_2' A)$$
$$+ \lambda_2(r_2 na)(1 - r_2 nm)(1 - \lambda_1 r_2)(\lambda_1' A)\}$$

Where

$r_1 na \approx 1 - e^{-r_1 na}$ = The probability that adverse weather occurs while component 1 is being repaired,

$1 - r_1 nm \approx e^{-r_1 nm}$ = The probability that major adverse weather does not occur while component 1 is being repaired,

$1 - \lambda_2 r_1 \approx e^{-\lambda_2 r_1}$ = The probability that the component 2 does not fail while component 1 is being repaired,

$\lambda_2' A \approx 1 - e^{-\lambda_2' A}$ = The probability that the component 2 fails in the period of adverse weather.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(3) the initial failure occurs during normal weather and the second failure occurs during major adverse weather

$$(failure \quad rate)_3 = P_N\{\lambda_1(1-r_1na)(r_1nm)(1-\lambda_2r_1)(\lambda_2^{ma}M) + \lambda_2(1-r_2na)(r_2nm)(1-\lambda_1r_2)(\lambda_1^{ma}M)\}$$

Where

$1-r_1na \approx e^{-r_1na}$ = The probability that adverse weather does not occur while component 1 is being repaired,

$r_1nm \approx 1-e^{-r_1nm}$ = The probability that major adverse weather occurs while component 1 is being repaired,

$1-\lambda_2r_1 \approx e^{-\lambda_2r_1}$ = The probability that component 2 does not fail while component 1 is being repaired,

$\lambda_2^{ma}M = 1-e^{-\lambda_2^{ma}M}$ = The probability that component 2 fails in the period of major adverse weather.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(4) initial failure occurs during adverse weather and the second failure also occurs during adverse weather

$$(failure \quad rate)_4 = P_A\{\lambda_1'(\lambda_2'A) + \lambda_2'(\lambda_1'A)\}$$

Where

P_A = long term fraction of time that the weather is adverse weather,

$\lambda_2'A \approx 1-e^{-\lambda_2'A}$ = The probability that component 2 also fails during adverse weather.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(5) initial failure occurs during adverse weather and the second failure occurs during normal weather

$$(failure \quad rate)_5 = P_A\{\lambda_1'(1-\lambda_2' A)e^{-A\cdot am}(1-e^{-A\cdot an})(\lambda_2 r_1)$$

$$+\lambda_2'(1-\lambda_1' A)e^{-A\cdot am}(1-e^{-A\cdot an})(\lambda_1 r_2)\}$$

Where

$1-\lambda_2' A \approx e^{-\lambda_2' A}$ = The probability that component 2 does not fail during the adverse weather,

$e^{-A\cdot am}$ = The probability that the adverse weather does not change into major adverse weather,

$1-e^{-A\cdot an}$ = The probability that the adverse weather changes into normal weather,

$\lambda_2 r_1 \approx 1-e^{-\lambda_2 r_1}$ = The probability that component 2 fails while component 1 is being repaired.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(6) the initial failure occurs during adverse weather and the second failure occurs during major adverse weather

$$(failure \quad rate)_6 = P_A\{\lambda_1'(1-\lambda_2' A)e^{-A\cdot an}(1-e^{-A\cdot am})(\lambda_2^{ma} M)$$

$$+\lambda_2'(1-\lambda_1' A)e^{-A\cdot an}(1-e^{-A\cdot am})(\lambda_1^{ma} M)\}$$

Where

$1-\lambda_2' A \approx e^{-\lambda_2' A}$ = The probability that component 2 does not fail during adverse weather,

$e^{-A\cdot an}$ = The probability that the adverse weather does not change into normal weather,

$1-e^{-A\cdot am}$ = The probability that the adverse weather changes into major adverse weather,

$\lambda_2^{ma} M \approx 1 - e^{-\lambda_2^{ma} M}$ = The probability that component 2 fails during major adverse weather.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(7) the initial failure occurs during major adverse weather and the second failure also occurs during major adverse weather

$$(failure \quad rate)_7 = P_{MA}\{\lambda_1^{ma}(\lambda_2^{ma} M) + \lambda_2^{ma}(\lambda_1^{ma} M)\ \}$$

Where

P_{MA} = long term fraction of time that the weather is adverse,

$\lambda_2^{ma} M \approx 1 - e^{-\lambda_2^{ma} M}$ = The probability that component 2 also fails during major adverse weather.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(8) the initial failure occurs during major adverse weather and the second failure occurs during normal weather

$$(failure \quad rate)_8 = P_{MA}\{\lambda_1^{ma}(1 - \lambda_2^{ma} M)e^{-M \cdot ma}(1 - e^{-M \cdot mn})(\lambda_2 r_1)$$

$$+ \lambda_2^{ma}(1 - \lambda_1^{ma} M)e^{-M \cdot ma}(1 - e^{-M \cdot mn})(\lambda_1 r_2)\}$$

Where

$1 - \lambda_2^{ma} M \approx e^{-\lambda_2^{ma} M}$ = The probability that component 2 does not fail in the period of major adverse weather,

$e^{-M \cdot ma}$ = The probability that the major adverse weather does not change into adverse weather,

$1 - e^{M \cdot mn}$ = The probability that the major adverse weather changes into normal weather,

$\lambda_2 r_1 \approx 1 - e^{\lambda_2 r_1}$ = The probability that component 2 fails while component 1 is being repaired.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

(9) the initial failure occurs during the major adverse weather and the second failure occurs during adverse weather

$$(failure \quad rate)_9 = P_{MA}\{\lambda_1^{ma}(1-\lambda_2^{ma}M)e^{-M\cdot mn}(1-e^{-M\cdot ma})(\lambda_2^{'}A)$$

$$+\lambda_2^{ma}(1-\lambda_1^{ma}M)e^{-M\cdot mn}(1-e^{-M\cdot ma})(\lambda_1^{'}A)\}$$

Where

$1-\lambda_2^{ma}M \approx e^{-\lambda_2^{ma}M}$ = The probability that component 2 does not fail in the period of major adverse weather,

$e^{-M\cdot mn}$ = The probability that the major adverse weather does not change into normal weather,

$1-e^{-M\cdot ma}$ = The probability that the major adverse weather changes into adverse weather,

$\lambda_2^{'}A = 1-e^{-\lambda_2^{'}A}$ = The probability that component 2 also fails during adverse weather.

The remaining terms in this mode of failure follow the same reasoning but with components 1 and 2 interchanged.

The system average outage rate is:

$$\lambda_{SL} = \sum_{i=1}^{9}(failure \quad rate)_i \qquad (3.10)$$

Results obtained using these equations are shown in Tables 3.4 to 3.15 together with theoretically exact results obtained using the Markov approach. In these analyses,

λ_{av} = 1.0 f/yr, N = 195.5357 hours, A = 1.9995 hours, M = 1.0hour, and the repair time r =7.5 hours. Adverse and major adverse weather are collectively designated as bad weather and the percentage of failures occurring in bad weather was varied from 0% to 100% in each study. Table 3.1 shows the results for the situation in which 95% of the bad weather failures occurred in adverse weather and 5% of the bad weather failures occurred in major adverse weather. The percentage of bad weather failures in adverse weather was varied from 95% to 50%, and the bad weather failures in major adverse weather was varied from 5% to 50% in subsequent studies. The error factors shown in Table 3.1 were obtained by expressing the calculated load point failure rate including adverse weather and major adverse weather effects using Equation 3.10 in per unit of the predicted value obtained using Equation 2.2. The percentage error was obtained by comparing the system failure rate obtained using Equation 3.10 with the exact system failure rate obtained using the Markov approach.

Table 3.1 System average failure rate for the two lines in parallel system (95% of bad weather failures occur in adverse weather, 5% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	λ (f/yr)	λ' (f/yr)	λ^{ma} (f/yr)	Markov Method (f/yr)	Approx. Method (f/yr)	Error Factor	Percentage Error
0	1.010228	0	0	0.001724	0.001730	1.010228	0.360995
10	0.909205	9.490074	43.86465	0.002195	0.002114	1.234448	3.705970
20	0.808182	18.98015	87.72930	0.003579	0.003411	1.991938	4.705959
30	0.707160	28.47022	131.5940	0.005861	0.005621	3.282751	4.085882
40	0.606137	37.96030	175.4586	0.009028	0.008745	5.106943	3.138450
50	0.505114	47.45037	219.3233	0.013042	0.012782	7.464565	1.996073
60	0.404091	56.94044	263.1879	0.017897	0.017732	10.35567	0.920646
70	0.303068	66.43052	307.0526	0.023583	0.023596	13.78032	0.054963
80	0.202046	75.92059	350.9172	0.030059	0.030374	17.73856	1.047187
90	0.101023	85.41067	394.7819	0.037320	0.038066	22.23045	1.998693
100	0	94.90073	438.6465	0.045350	0.046671	27.25602	2.914236

The numerical values of each term in Equation 3.10 are shown in Table 3.2. In this table, NN represents the term in which the initial failure occurs in normal weather and the second failure also occurs in normal weather. NA represents the term in which the initial failure occurs in normal weather and the second failure occurs in adverse weather. NM represents the term in which the initial failure occurs in normal weather and the second failure occurs in major adverse weather. AN represents the term in which the initial failure occurs in adverse weather and the second failure occurs in normal weather, AA represents the term in which the initial failure occurs in adverse weather and the second failure occurs in adverse weather. AM represents the term in which the initial failure occurs in adverse weather and the second failure occurs in major adverse weather. MN represents the term in which the initial failure occurs in major adverse weather and the second failure occurs in normal weather. MA represents the term in which the initial failure occurs in major adverse weather and the second failure occurs in adverse weather. MM represents the term in which both failures occur in major adverse weather.

It can be seen from Table 3.2 that the bulk of the contribution to the system average failure rate comes from the AA term when a significant number of failures occur in bad weather. This is to be expected in this case as 95% of the bad weather failures occur in adverse weather. The second largest contribution comes from the MM term.

Tables 3.3 to 3.7 show the results for situations in which the percentage of bad weather failures in adverse weather and in major adverse weather are varied. It can be seen from these tables that the predicted system average failure rate increases as the percentage of bad weather failures in major adverse weather increases. Figure 3.4 shows the error factor as a function of the percentage of failures in bad weather. This figure shows the error factor variation for the different percentages of bad weather failures in major adverse weather presented numerically in Table 3.1 and Tables 3.3 to 3.7. The figure also shows the error factor associated with the two weather state model. It can clearly be seen that the two state weather model does not reflect the increasing importance of major adverse weather and underestimates the potential error.

Table 3.2 Values of each term in Equation 3.10 (5% of the bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	NN	NA	NM	AA	AN	AM	MM	MN	MA	System Average Failure Rate (f/yr)
0	0.001730	0	0	0	0	0	0	0	0	0.001665
10	0.001401	0.000143	0.000007	0.000412	0.000093	0	0.000050	0.000002	0.000005	0.002120
20	0.001107	0.000254	0.000013	0.001647	0.000165	0	0.000200	0.000003	0.000020	0.003462
30	0.000848	0.000333	0.000017	0.003705	0.000217	0.000001	0.000451	0.000004	0.000046	0.005690
40	0.000623	0.000381	0.000020	0.006586	0.000247	0.000001	0.000801	0.000005	0.000081	0.008804
50	0.000432	0.000396	0.000020	0.010291	0.000257	0.000002	0.001252	0.000005	0.000126	0.012805
60	0.000277	0.000380	0.000019	0.014819	0.000246	0.000003	0.001803	0.000005	0.000181	0.017693
70	0.000156	0.000332	0.000017	0.020171	0.000215	0.000004	0.002454	0.000004	0.000244	0.023468
80	0.000069	0.000253	0.000013	0.026346	0.000163	0.000005	0.003205	0.000003	0.000318	0.030131
90	0.000017	0.000142	0.000007	0.033344	0.000092	0.000006	0.004056	0.000002	0.000400	0.037681
100	0	0	0	0.041165	0	0.000008	0.005007	0	0.000491	0.046119

Table 3.1 and Tables 3.3 to 3.7 also show the percentage error associated with the approximate equations. This error is relatively small when the percentage of bad weather failures in major adverse weather is less than 20%. The error increases considerably as the percentage increases. The error factor shown in Figure 3.4 is overestimated when the percentage of bad weather failures in major adverse weather increases above the 20% level. Table 3.1 and Tables 3.3 to 3.7 also show the failure rates in normal, adverse and major adverse weather for each percentage of line failures in bad weather and for each considered percentage of bad weather failures in major adverse weather. As the percentage of bad weather failures in major adverse weather increases, the component failure rate in major adverse weather becomes very large and therefore the MM term dominates the calculation of the overall average system failure rate.

Under these conditions, the approximate equations considerable overestimate the overall annual system failure rate when compared with the theoretically exact values obtained using the Markov approach. This therefore place some limits on the use of the approximate equations. The following comments pertain to the approximate equation values shown in Table 3.2 and Tables 3.8 to 3.12. As previously noted in connecting with Table 3.2, the contribution from the AA term dominates the overall system average failure rate value when the percentage of bad weather failures occurring in major adverse weather is 5%. This situation changes as the percentage of bad weather failures in major adverse weather increases. Table 3.9 shows the results for 20% of bad weather failures in major adverse weather. The MM term now dominates the overall system average failure rate value. This contribution increases significantly in Tables 3.10 to 3.12 when the percentage of bad weather failures in major adverse weather increases from 30% to 50%. It can be seen from Table 3.12, in which the percentage of bad weather failures in major adverse weather is 50%, that the MM term provides the largest contribution to the overall system average failure rate value even when the percentage of line failure occurring in bad weather is as low as 10%.

Table 3.3 System average failure rate for the two lines in parallel system (90% of bad weather failures occur in adverse weather, 10% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	λ (f/yr)	λ' (f/yr)	λ^{ma} (f/yr)	Markov Method (f/yr)	Approx. Method (f/yr)	Error Factor	Percentage Error
0	1.010228	0	0	0.001724	0.001730	1.010228	0.360995
10	0.909205	8.990596	87.72930	0.002307	0.002223	1.298352	3.612612
20	0.808182	17.98119	175.4586	0.004007	0.003855	2.251528	3.773482
30	0.707160	26.97179	263.1879	0.006786	0.006626	3.869577	2.353953
40	0.606137	35.96238	350.9172	0.010593	0.010535	6.152317	0.550799
50	0.505114	44.95298	438.6465	0.015387	0.015581	9.099568	1.263415
60	0.404091	53.94358	526.3759	0.021127	0.021766	12.71115	3.020587
70	0.303068	62.93417	614.1051	0.027791	0.029087	16.98687	4.663512
80	0.202046	71.92477	701.8344	0.035321	0.037545	21.92655	6.299219
90	0.101023	80.91537	789.5638	0.043701	0.047140	27.53002	7.871083
100	0	89.90596	877.2930	0.052889	0.057872	33.79707	9.421272

Table 3.4 System average failure rate for the two lines in parallel system (80% of bad weather failures occur in adverse weather, 20% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	λ (f/yr)	λ' (f/yr)	λ^{ma} (f/yr)	Markov Method (f/yr)	Approx. Method (f/yr)	Error Factor	Percentage Error
0	1.010228	0	0	0.001724	0.001730	1.010228	0.360995
10	0.909205	7.991641	175.4586	0.002801	0.002747	1.604483	1.909127
20	0.808182	15.98328	350.9172	0.005845	0.005965	3.483804	2.059474
30	0.707160	23.97492	526.3759	0.010616	0.011382	6.647237	7.221705
40	0.606137	31.96656	701.8344	0.016910	0.018996	11.09383	12.33623
50	0.505114	39.95821	877.2930	0.024558	0.028806	16.82261	17.29509
60	0.404091	47.94985	1052.752	0.033418	0.040809	23.83264	22.11833
70	0.303068	55.94149	1228.210	0.043370	0.055005	32.12293	26.82873
80	0.202046	63.93313	1403.669	0.054288	0.071391	41.69254	31.50460
90	0.101023	71.92477	1579.128	0.066100	0.089967	52.54047	36.10756
100	0	79.91641	1754.586	0.078717	0.110729	64.66576	40.66790

Table 3.5 System average failure rate for the two lines in parallel system (70% of bad weather failures occur in adverse weather, 30% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	λ (f/yr)	λ' (f/yr)	λ^{ma} (f/yr)	Markov Method (f/yr)	Approx. Method (f/yr)	Error Factor	Percentage Error
0	1.010228	0	0	0.001724	0.001730	1.010228	0.360995
10	0.909205	6.992686	263.1879	0.003633	0.003679	2.148412	1.271776
20	0.808182	13.98537	526.3759	0.008808	0.009704	5.667130	10.17140
30	0.707160	20.97806	789.5638	0.016574	0.019802	11.56445	19.47899
40	0.606137	27.97074	1052.752	0.026412	0.033970	19.83842	28.61713
50	0.505114	34.96343	1315.940	0.037907	0.052204	30.48708	37.71729
60	0.404091	41.95612	1579.128	0.050748	0.074501	43.50848	46.80570
70	0.303068	48.94880	1842.315	0.064690	0.100857	58.90062	55.90921
80	0.202046	55.94149	2105.503	0.079522	0.131270	76.66154	65.07262
90	0.101023	62.93418	2368.691	0.095086	0.165735	96.78925	74.30043
100	0	69.92686	2631.879	0.111254	0.204249	119.2817	83.58821

Table 3.6 System average failure rate for the two lines in parallel system (60% of bad weather failures occur in adverse weather, 40% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	λ (f/yr)	λ' (f/yr)	λ^{ma} (f/yr)	Markov Method (f/yr)	Approx. Method (f/yr)	Error Factor	Percentage Error
0	1.010228	0	0	0.001724	0.001730	1.010228	0.360995
10	0.909205	5.993731	350.9172	0.004778	0.005017	2.930176	5.011924
20	0.808182	11.98746	701.8344	0.012726	0.015072	8.801817	18.43616
30	0.707160	17.98119	1052.752	0.024179	0.031887	18.62225	31.88260
40	0.606137	23.97492	1403.669	0.038157	0.055460	32.38855	45.34752
50	0.505114	29.96865	1754.586	0.053928	0.085784	50.09778	59.07162
60	0.404091	35.96239	2105.503	0.070987	0.122854	71.74697	73.06590
70	0.303068	41.95612	2456.420	0.088958	0.166666	97.33309	87.35286
80	0.202046	47.94985	2807.338	0.107552	0.217214	126.8532	101.9615
90	0.101023	53.94358	3158.255	0.126560	0.274494	160.3042	116.8887
100	0	59.93731	3509.172	0.145828	0.338498	197.6831	132.1215

Table 3.7 System average failure rate for the two lines in parallel system (50% of bad weather failures occur in adverse weather, 50% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	λ (f/yr)	λ' (f/yr)	λ^{ma} (f/yr)	Markov Method (f/yr)	Approx. Method (f/yr)	Error Factor	Percentage Error
0	1.010228	0	0	0.001724	0.001730	1.010228	0.360995
10	0.909205	4.994776	438.6465	0.006202	0.006763	3.949816	9.056653
20	0.808182	9.989552	877.2930	0.017436	0.022069	12.88818	26.57243
30	0.707160	14.98433	1315.940	0.033044	0.047640	27.82170	44.16956
40	0.606137	19.97910	1754.586	0.051456	0.083470	48.74673	62.21691
50	0.505114	24.97388	2193.233	0.071607	0.129554	75.65955	80.92493
60	0.404091	29.96866	2631.879	0.092803	0.185884	108.5564	100.2997
70	0.303068	34.96343	3070.526	0.114556	0.252455	147.4336	120.3761
80	0.202046	39.95821	3509.172	0.136530	0.329259	192.2871	141.1629
90	0.101023	44.95298	3947.819	0.158501	0.416290	243.1133	162.6425
100	0	49.94776	4386.465	0.180308	0.513541	299.9081	184.8131

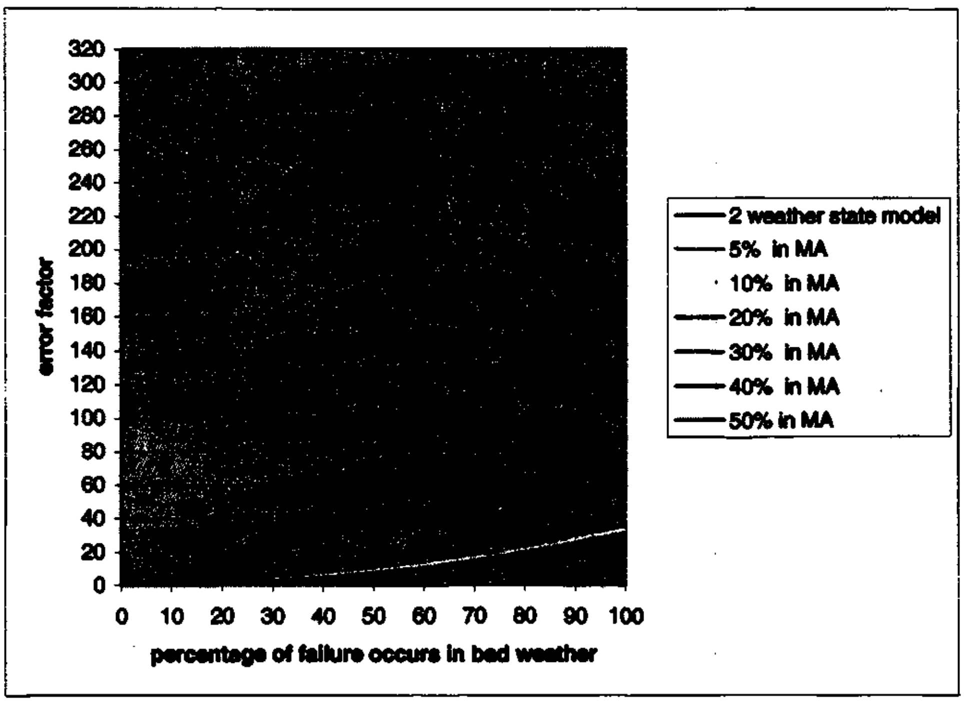


Figure 3.4 Error factor analyses

Table 3.8 Values of each term in Equation 3.10 (10% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	NN	NA	NM	AA	AN	AM	MM	MN	MA	System Average Failure Rate (f/yr)
0	0.001730	0	0	0	0	0	0	0	0	0.001730
10	0.001401	0.000136	0.000015	0.000369	0.000088	0	0.000200	0.000004	0.000010	0.002223
20	0.001107	0.000241	0.000026	0.001478	0.000157	0.000001	0.000801	0.000006	0.000038	0.003855
30	0.000848	0.000316	0.000034	0.003325	0.000205	0.000001	0.001803	0.000008	0.000086	0.006626
40	0.000623	0.000361	0.000039	0.005911	0.000234	0.000002	0.003205	0.000010	0.000150	0.010535
50	0.000432	0.000375	0.000040	0.009236	0.000243	0.000004	0.005007	0.000010	0.000233	0.015581
60	0.000277	0.000360	0.000038	0.013300	0.000233	0.000005	0.007211	0.000009	0.000331	0.021766
70	0.000156	0.000315	0.000033	0.018103	0.000204	0.000007	0.009814	0.000008	0.000446	0.029087
80	0.000069	0.000240	0.000025	0.023645	0.000155	0.000010	0.012819	0.000006	0.000577	0.037545
90	0.000017	0.000135	0.000014	0.029926	0.000087	0.000012	0.016224	0.000003	0.000722	0.047140
100	0	0	0	0.036946	0	0.000015	0.020030	0	0.000882	0.057872

Table 3.9 Values of each term in Equation 3.10 (20% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	NN	NA	NM	AA	AN	AM	MM	MN	MA	System Average Failure Rate (f/yr)
0	0.001730	0	0	0	0	0	0	0	0	0.001730
10	0.001401	0.000121	0.000029	0.000292	0.000079	0	0.000801	0.000007	0.000017	0.002747
20	0.001107	0.000214	0.000052	0.001168	0.000139	0.000001	0.003205	0.000013	0.000067	0.005965
30	0.000848	0.000281	0.000067	0.002627	0.000183	0.000002	0.007211	0.000016	0.000147	0.011382
40	0.000623	0.000321	0.000076	0.004671	0.000208	0.000004	0.012819	0.000018	0.000256	0.018996
50	0.000432	0.000334	0.000078	0.007298	0.000217	0.000007	0.020030	0.000019	0.000392	0.028806
60	0.000277	0.000320	0.000075	0.010509	0.000208	0.000010	0.028843	0.000017	0.000552	0.040809
70	0.000156	0.000280	0.000065	0.014304	0.000181	0.000013	0.039258	0.000015	0.000734	0.055005
80	0.000069	0.000213	0.000049	0.018683	0.000138	0.000017	0.051276	0.000011	0.000936	0.071391
90	0.000017	0.000120	0.000027	0.023645	0.000077	0.000021	0.064896	0.000006	0.001156	0.089967
100	0	0	0	0.029192	0	0.000026	0.080118	0	0.001393	0.110729

Table 3.10 Values of each term in Equation 3.10 (30% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	NN	NA	NM	AA	AN	AM	MM	MN	MA	System Average Failure Rate (f/yr)
0	0.001730	0	0	0	0	0	0	0	0	0.001730
10	0.001401	0.000106	0.000044	0.000223	0.000069	0	0.001803	0.000011	0.000022	0.003679
20	0.001107	0.000187	0.000077	0.000894	0.000122	0.000001	0.007211	0.000019	0.000086	0.009704
30	0.000848	0.000246	0.000099	0.002011	0.000160	0.000003	0.016224	0.000024	0.000187	0.019802
40	0.000623	0.000281	0.000112	0.003576	0.000182	0.000006	0.028843	0.000026	0.000322	0.033970
50	0.000432	0.000292	0.000115	0.005587	0.000190	0.000009	0.045066	0.000026	0.000486	0.052204
60	0.000277	0.000280	0.000109	0.008046	0.000182	0.000013	0.064896	0.000024	0.000675	0.074501
70	0.000156	0.000245	0.000094	0.010951	0.000159	0.000017	0.088330	0.000021	0.000885	0.100857
80	0.000069	0.000187	0.000070	0.014304	0.000121	0.000022	0.115370	0.000015	0.001111	0.131270
90	0.000017	0.000105	0.000039	0.018103	0.000068	0.000028	0.146015	0.000008	0.001351	0.165735
100	0	0	0	0.02235	0	0.000035	0.180266	0	0.001599	0.204249

Table 3.11 Values of each term in Equation 3.10 (40% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	NN	NA	NM	AA	AN	AM	MM	MN	MA	System Average Failure Rate (f/yr)
0	0.001730	0	0	0	0	0	0	0	0	0.001730
10	0.001401	0.000090	0.000058	0.000164	0.000059	0	0.003205	0.000014	0.000025	0.005017
20	0.001107	0.000161	0.000101	0.000657	0.000105	0.000002	0.012819	0.000024	0.000096	0.015072
30	0.000848	0.000211	0.000130	0.001478	0.000137	0.000004	0.028843	0.000031	0.000207	0.031887
40	0.000623	0.000241	0.000146	0.002627	0.000157	0.000006	0.051276	0.000033	0.000351	0.055460
50	0.000432	0.000251	0.000149	0.004105	0.000163	0.000010	0.080118	0.000033	0.000522	0.085784
60	0.000277	0.000240	0.000141	0.005911	0.000156	0.000014	0.115370	0.000030	0.000714	0.122854
70	0.000156	0.000210	0.000121	0.008046	0.000136	0.000020	0.157031	0.000025	0.000921	0.166666
80	0.000069	0.000160	0.000090	0.010509	0.000104	0.000026	0.205102	0.000018	0.001136	0.217214
90	0.000017	0.000090	0.000050	0.013300	0.000058	0.000032	0.259583	0.000010	0.001353	0.274494
100	0	0	0	0.016420	0	0.000040	0.320472	0	0.001566	0.338498

Table 3.12 Values of each term in Equation 3.10 (50% of bad weather failures occur in major adverse weather)

% of Line Failures in Bad Weather	NN	NA	NM	AA	AN	AM	MM	MN	MA	System Average Failure Rate (f/yr)
0	0.001730	0	0	0	0	0	0	0	0	0.001730
10	0.001401	0.000075	0.000072	0.000114	0.000049	0	0.005007	0.000018	0.000026	0.006763
20	0.001107	0.000134	0.000126	0.000456	0.000087	0.000002	0.020030	0.000030	0.000098	0.022069
30	0.000848	0.000176	0.000161	0.001026	0.000114	0.000004	0.045066	0.000037	0.000208	0.047640
40	0.000623	0.000201	0.000179	0.001824	0.000131	0.000007	0.080118	0.000040	0.000348	0.083470
50	0.000432	0.000209	0.000182	0.002851	0.000136	0.000010	0.125185	0.000039	0.000510	0.129554
60	0.000277	0.000201	0.000171	0.004105	0.000130	0.000015	0.180266	0.000035	0.000685	0.185884
70	0.000156	0.000175	0.000146	0.005587	0.000114	0.000020	0.245362	0.000028	0.000866	0.252455
80	0.000069	0.000134	0.000109	0.007298	0.000087	0.000027	0.320472	0.000020	0.001044	0.329259
90	0.000017	0.000075	0.000060	0.009236	0.000049	0.000034	0.405598	0.000010	0.001211	0.416290
100	0	0	0	0.011403	0	0.000042	0.500738	0	0.001359	0.513541

3.5.2 System average outage duration

The previous analyses were conducted under the condition the no repair is performed during adverse weather and therefore failures which occur in adverse weather have to wait for repair for an period equal to the average duration of adverse weather. The average outage duration for a system failure occurring during normal weather is given by Equation 3.11.

$$r_{en} = \frac{r_1 r_2}{r_1 + r_2} \tag{3.11}$$

The average outage duration due to failure during adverse weather is given by:

$$r_{ea} = \frac{r_1 r_2}{r_1 + r_2} + A \tag{3.12}$$

Following the same reasoning, failures which occur in major adverse weather have to wait for repair for an average period equal to the average duration of major adverse weather. Referring to the model of Figure 3.3, the average outage duration for which failure occurs during a major adverse weather period is given by:

$$r_{em} = \frac{r_1 r_2}{r_1 + r_2} + M \tag{3.13}$$

The annual outage duration is given by:

$$\begin{aligned} U &= \lambda_{en} r_{en} + \lambda_{ea} r_{ea} + \lambda_{em} r_{em} \\ &= \lambda_{en} (\frac{r_1 r_1}{r_1 + r_2}) + \lambda_{ea} (\frac{r_1 r_2}{r_1 + r_2} + A) + \lambda_{em} (\frac{r_1 r_2}{r_1 + r_2} + M) \end{aligned} \tag{3.14}$$

λ_{en} is the contribution to the system failure rate due to the second failure occurring during normal weather. $\lambda_{en} = \lambda_1 + \lambda_5 + \lambda_8$

λ_{ea} is the contribution to the system failure rate due to the second failure occurring during adverse weather. $\lambda_{ea} = \lambda_2 + \lambda_4 + \lambda_9$

λ_{em} is the contribution to the system failure rate due to the second failure occurring during major adverse weather. $\lambda_{em} = \lambda_3 + \lambda_6 + \lambda_7$.

The system average outage duration is given by:

$$r_{SL} = \frac{U}{\lambda_{SL}}$$

$$= \frac{r_1 r_2}{r_1 + r_2} + \frac{\lambda_2 + \lambda_4 + \lambda_9}{\lambda_{SL}} A + \frac{\lambda_3 + \lambda_6 + \lambda_7}{\lambda_{SL}} M \qquad (3.15)$$

The results obtained using Equation 3.15 are shown in Table 3.13 to Table 3.18 together with theoretically exact results obtained using the Markov approach.

Table 3.13 System average outage duration for the two lines in parallel system (95% of bad weather failures occur in adverse weather, 5% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	Approximate Approach (hrs)	Percentage Error
0	3.788567	3.750000	1.017976
10	4.374110	4.307002	1.534225
20	5.018518	4.939189	1.580722
30	5.378576	5.286376	1.714203
40	5.560292	5.455869	1.878021
50	5.657665	5.541588	2.051671
60	5.712614	5.587489	2.190327
70	5.745357	5.613291	2.298649
80	5.766003	5.628296	2.388247
90	5.779346	5.637185	2.459819
100	5.788392	5.642453	2.521232

Table 3.14 System average outage duration for the two lines in parallel system (90% of bad weather failures occur in adverse weather, 10% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	Approximate Approach (hrs)	Percentage Error
0	3.788567	3.750000	1.017976
10	4.443211	4.309873	3.000930
20	5.101665	4.876211	4.419234
30	5.434835	5.152167	5.201052
40	5.593967	5.277334	5.660273
50	5.677983	5.337732	5.992455
60	5.724478	5.368921	6.211164
70	5.751966	5.385857	6.364943
80	5.769472	5.395326	6.484931
90	5.780800	5.400653	6.576028
100	5.788279	5.403577	6.646224

Table 3.15 System average outage duration for the two lines in parallel system (80% of bad weather failures occur in adverse weather, 20% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	Approximate Approach (hrs)	Percentage Error
0	3.788567	3.750000	1.017976
10	4.680212	4.365086	6.733156
20	5.318715	4.781744	10.09589
30	5.562959	4.926465	11.44164
40	5.667303	4.981513	12.10082
50	5.719913	5.005332	12.49287
60	5.748560	5.016555	12.73372
70	5.765421	5.022052	12.89358
80	5.776358	5.024707	13.01253
90	5.783543	5.025861	13.10065
100	5.788388	5.026183	13.16783

Table 3.16 System average outage duration for the two lines in parallel system (70% of bad weather failures occur in adverse weather, 30% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	Approximate Approach (hrs)	Percentage Error
0	3.788567	3.750000	1.017976
10	4.934537	4.442942	9.962343
20	5.477658	4.741703	13.43557
30	5.645072	4.821359	14.59173
40	5.711853	4.848521	15.11474
50	5.744834	4.859497	15.41101
60	5.762683	4.864321	15.58931
70	5.773433	4.866450	15.70961
80	5.780447	4.867280	15.79753
90	5.785089	4.867441	15.86230
100	5.788363	4.867242	15.91333

Table 3.17 System average outage duration for the two lines in parallel system (60% of bad weather failures occur in adverse weather, 40% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	Approximate Approach (hrs)	Percentage Error
0	3.788567	3.750000	1.017976
10	5.139900	4.511891	12.21831
20	5.574522	4.728601	15.17478
30	5.690957	4.777596	16.04933
40	5.736212	4.793385	16.43640
50	5.758323	4.799534	16.65049
60	5.770531	4.802115	16.78209
70	5.777894	4.803157	16.87011
80	5.782815	4.803464	16.93554
90	5.786092	4.803397	16.98375
100	5.788354	4.803130	17.02080

Table 3.18 System average outage duration for the two lines in parallel system (50% of bad weather failures occur in adverse weather, 50% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	Approximate Approach (hrs)	Percentage Error
0	3.788567	3.750000	1.017976
10	5.290289	4.564779	13.71400
20	5.632962	4.725707	16.10619
30	5.717875	4.758630	16.77626
40	5.750461	4.768931	17.06871
50	5.766381	4.772866	17.22944
60	5.775203	4.774471	17.32809
70	5.780664	4.775076	17.39573
80	5.784223	4.775205	17.44431
90	5.786658	4.775091	17.48102
100	5.788369	4.774848	17.50961

It can be seen from Table 3.13 to 3.18 that system average outage duration obtained using the approximate equations are again different from the values obtained using the Markov approach. The percentage error, however, is considerable smaller than the previously noted error in the system average failure rate. As in the case of the system average failure rate, the use of the approximate equations can be considered a practical tool when the percentage of bad weather failures occurring in major adverse weather is less than 20%.

3.6 Monte Carlo simulation method

The basic concepts of time sequential Monte Carlo simulation are introduced in Chapter 2. The time sequential simulation process for the three weather state model follows the same general philosophy as for the two weather state model but with a much more complex procedure. The detailed simulation procedure is shown in Appendix B.

The system average failure rate values obtained using the sequential Monte Carlo simulation are shown in Tables 3.19 to 3.24. The system average outage duration values are shown in Table 3.25 to 3.30. The percentage of bad weather failures in major adverse weather varies from 5% to 50% in these studies.

Table 3.19 System average failure rate for the two lines in parallel system (MCS approach, 95% of bad weather failures occur in adverse weather, 5% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (f/yr)	MCS Approach (f/yr)	Error Factor	Percentage Error
0	0.001724	0.001610	0.940240	6.612529
10	0.002195	0.002310	1.349040	5.239180
20	0.003579	0.003520	2.055680	1.648505
30	0.005861	0.006090	3.556560	3.907183
40	0.009028	0.009010	5.261840	0.199379
50	0.013042	0.013310	7.773040	2.054899
60	0.017897	0.018150	10.59960	1.413645
70	0.023583	0.024370	14.23208	3.337150
80	0.030059	0.030911	18.05202	2.834426
90	0.037320	0.038421	22.43786	2.942277
100	0.045350	0.045141	26.36234	0.460860

Table 3.20 System average failure rate for the two lines in parallel system (MCS approach, 90% of bad weather failures occur in adverse weather, 10% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (f/yr)	MCS Approach (f/yr)	Error Factor	Percentage Error
0	0.001724	0.001610	0.940240	6.612529
10	0.002307	0.002420	1.413280	4.898136
20	0.004007	0.004040	2.359360	0.823559
30	0.006786	0.006880	4.017920	1.385205
40	0.010593	0.010460	6.108640	1.255546
50	0.015387	0.015290	8.929360	0.630402
60	0.021127	0.020690	12.08296	2.068443
70	0.027791	0.028661	16.73802	3.130510
80	0.035321	0.036671	21.41586	3.822089
90	0.043701	0.043781	25.56810	0.183062
100	0.052889	0.054572	31.87005	3.180245

Table 3.21 System average failure rate for the two lines in parallel system (MCS approach, 80% of bad weather failures occur in adverse weather, 20% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (f/yr)	MCS Approach (f/yr)	Error Factor	Percentage Error
0	0.001724	0.001610	0.940240	6.612529
10	0.002801	0.002870	1.676080	2.463406
20	0.005845	0.005680	3.317120	2.822926
30	0.010616	0.010960	6.400640	3.240392
40	0.016910	0.017110	9.992240	1.182732
50	0.024558	0.025100	14.65840	2.207020
60	0.033418	0.033391	19.50034	0.080793
70	0.043370	0.044281	25.86010	2.100530
80	0.054288	0.054642	31.91093	0.652078
90	0.066100	0.066393	38.77351	0.443268
100	0.078717	0.080684	47.11946	2.498825

Table 3.22 System average failure rate for the two lines in parallel system (MCS approach, 70% of bad weather failures occur in adverse weather, 30% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (f/yr)	MCS Approach (f/yr)	Error Factor	Percentage Error
0	0.001724	0.001610	0.940240	6.612529
10	0.003633	0.003640	2.125760	0.192678
20	0.008808	0.009040	5.279360	2.633969
30	0.016574	0.016460	9.612640	0.687824
40	0.026412	0.027560	16.09504	4.346509
50	0.037907	0.037661	21.99402	0.648957
60	0.050748	0.051052	29.81437	0.599038
70	0.064690	0.066053	38.57495	2.106972
80	0.079522	0.079734	46.56466	0.266593
90	0.095086	0.095196	55.59446	0.115685
100	0.111254	0.110718	64.65931	0.481780

Table 3.23 System average failure rate for the two lines in parallel system (MCS approach, 60% of bad weather failures occur in adverse weather, 40% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (f/yr)	MCS Approach (f/yr)	Error Factor	Percentage Error
0	0.001724	0.001610	0.940240	6.612529
10	0.004778	0.004940	2.884960	3.390540
20	0.012726	0.012840	7.498560	0.895804
30	0.024179	0.024690	14.41896	2.113404
40	0.038157	0.039301	22.95178	2.998140
50	0.053928	0.053072	30.99405	1.587302
60	0.070987	0.070553	41.20295	0.611380
70	0.088958	0.089835	52.46364	0.985858
80	0.107552	0.108288	63.20515	0.684320
90	0.126560	0.127051	74.19778	0.387958
100	0.145828	0.144974	84.66482	0.585621

Table 3.24 System average failure rate for the two lines in parallel system (MCS approach, 50% of bad weather failures occur in adverse weather, 50% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (f/yr)	MCS Approach (f/yr)	Error Factor	Percentage Error
0	0.001724	0.001610	0.940240	6.612529
10	0.006202	0.005970	3.486480	3.740729
20	0.017436	0.017130	10.00392	1.754990
30	0.033044	0.032301	18.86378	2.248517
40	0.051456	0.051492	30.07133	0.069964
50	0.071607	0.070813	41.35479	1.108830
60	0.092803	0.091926	53.68478	0.945013
70	0.114556	0.114989	67.15358	0.377981
80	0.136530	0.136362	79.63541	0.123050
90	0.158501	0.159067	92.89513	0.357096
100	0.180308	0.180752	105.5592	0.246245

Table 3.25 System average outage duration for the two lines in parallel system (MCS approach, 95% of bad weather failures occur in adverse weather, 5% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	MCS Approach (hrs)	Percentage Error
0	3.788567	3.766927	0.571192
10	4.374110	4.591191	4.962861
20	5.018518	5.120584	2.033788
30	5.378576	5.461182	1.535834
40	5.560292	5.521577	0.696276
50	5.657665	5.681969	0.429577
60	5.712614	5.653548	1.033957
70	5.745357	5.662470	1.442678
80	5.766003	5.744404	0.374592
90	5.779346	5.798767	0.336041
100	5.788392	5.828570	0.694113

Table 3.26 System average outage duration for the two lines in parallel system (MCS approach, 90% of bad weather failures occur in adverse weather, 10% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	MCS Approach (hrs)	Percentage Error
0	3.788567	3.766927	0.571192
10	4.443211	4.543587	2.259087
20	5.101665	5.095952	0.111983
30	5.434835	5.419739	0.277764
40	5.593967	5.594866	0.016071
50	5.677983	5.673773	0.074154
60	5.724478	5.786336	1.080588
70	5.751966	5.787574	0.619058
80	5.769472	5.775910	0.111587
90	5.780800	5.770106	0.184992
100	5.788279	5.747612	0.702575

Table 3.27 System average outage duration for the two lines in parallel system (MCS approach, 80% of bad weather failures occur in adverse weather, 20% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	MCS Approach (hrs)	Percentage Error
0	3.788567	3.766927	0.571192
10	4.680212	4.682379	0.046300
20	5.318715	5.035581	5.323353
30	5.562959	5.691283	2.306760
40	5.667303	5.583396	1.480546
50	5.719913	5.765797	0.802180
60	5.748560	5.795223	0.811730
70	5.765421	5.754769	0.184757
80	5.776358	5.738750	0.651068
90	5.783543	5.797737	0.245420
100	5.788388	5.782240	0.106213

Table 3.28 System average outage duration for the two lines in parallel system (MCS approach, 70% of bad weather failures occur in adverse weather, 30% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	MCS Approach (hrs)	Percentage Error
0	3.788567	3.766927	0.571192
10	4.934537	5.074757	2.841600
20	5.477658	5.547063	1.267060
30	5.645072	5.413431	4.103420
40	5.711853	5.718825	0.122060
50	5.744834	5.710639	0.595230
60	5.762683	5.736889	0.447604
70	5.773433	5.891255	2.040760
80	5.780447	5.777257	0.055186
90	5.785089	5.762619	0.388412
100	5.788363	5.804645	0.281290

Table 3.29 System average outage duration for the two lines in parallel system (MCS approach, 60% of bad weather failures occur in adverse weather, 40% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	MCS Approach (hrs)	Percentage Error
0	3.788567	3.766927	0.571192
10	5.139900	5.025401	2.227650
20	5.574522	5.579345	0.086520
30	5.690957	5.605793	1.496479
40	5.736212	5.674251	1.080173
50	5.758323	5.796598	0.664690
60	5.770531	5.782182	0.201910
70	5.777894	5.856770	1.365130
80	5.782815	5.839573	0.981490
90	5.786092	5.825379	0.678990
100	5.788354	5.789012	0.011370

Table 3.30 System average outage duration for the two lines in parallel system (MCS approach, 50% of bad weather failures occur in adverse weather, 50% of bad weather failures occur in major adverse weather)

% of Line Failures Occurring in Bad Weather	Markov Approach (hrs)	MCS Approach (hrs)	Percentage Error
0	3.788567	3.766927	0.571192
10	5.290289	5.144128	2.762817
20	5.632962	5.662511	0.524570
30	5.717875	5.695006	0.399956
40	5.750461	5.874181	2.151480
50	5.766381	5.706967	1.030352
60	5.775203	5.778284	0.053350
70	5.780664	5.751933	0.497019
80	5.784223	5.774462	0.168752
90	5.786658	5.797722	0.191200
100	5.788369	5.781073	0.126046

The results show that the Monte Carlo simulation approach gives values which are relatively close to the results obtained using the Markov approach for all the weather conditions studied. From an accuracy point of view, the Monte Carlo simulation approach is much more acceptable than the approximate equation method.

Compared with the Markov approach and the approximate method, Monte Carlo simulation, however, requires considerably more solution time. The simulation time for one solution of 100,000 years requires 70 seconds running time on a Pentium II PC. The running time is less than 0.1 second for the Markov approach and the approximate equation method.

Figure 3.5 shows the convergence of the system average failure rate when 50% of the failures occur in bad weather and 10% of the bad weather failures occur in major adverse weather. The simulation time was varied from 1 year to 100,000 years in steps of 1,000 years. Figure 3.5 shows that the simulation results continue to fluctuate even though the simulation time is very long. This fluctuation provides the bulk of the error when comparing the results with the exact values obtained using the Markov approach.

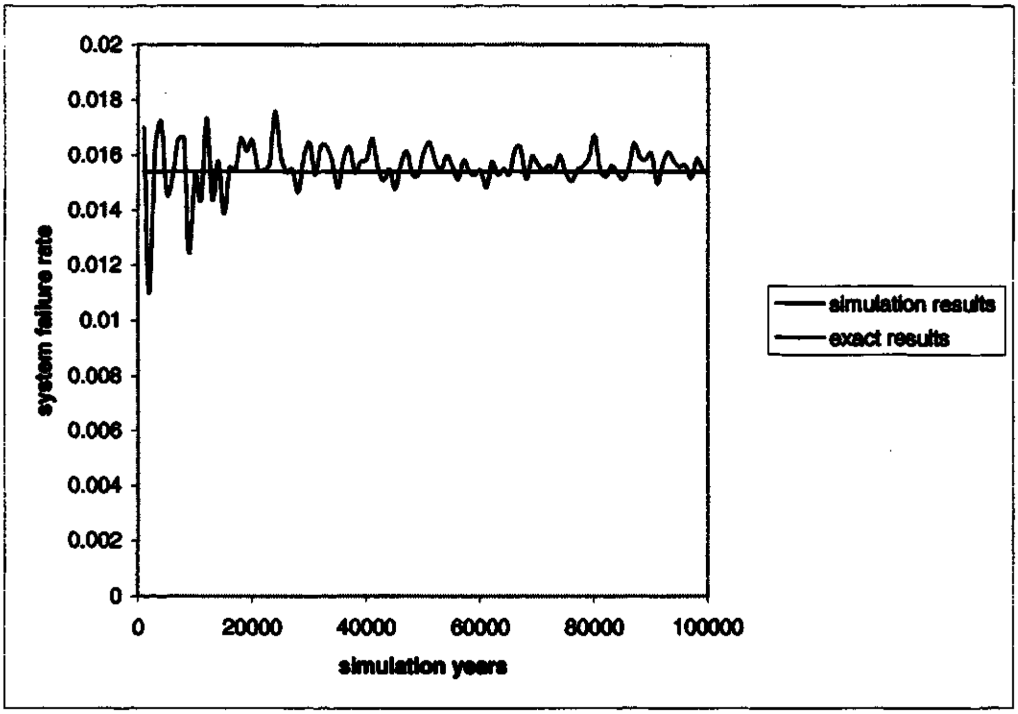


Figure 3.5 Simulation convergence (three weather state model)

3.7 Summary

This chapter illustrates the extension of the two redundant component transmission line reliability model to incorporate normal, adverse and major adverse weather conditions. The two indices of system average failure rate and system average outage duration were evaluated for this model using the Markov approach, the approximate method and the Monte Carlo simulation technique. The results show that the predicted system average failure rate increases as the percentage of bad weather failures in major adverse weather increases. Figure 3.4 shows the error factor variation for different percentages of bad weather failures in major adverse weather associated with the three weather state model and the error factor associated with the two weather state model. It can be clearly seen that the two weather state model does not reflect the increasing importance of major adverse weather and underestimates the potential error.

The results associated with the approximate equation approach show that the bulk of the contribution to the system average failure rate comes from the term in which both failures occur in adverse weather when 95% of the bad weather failures occur in adverse weather. The second largest contribution comes from the term in which both failures occur in major adverse weather. As the percentage of bad weather failures in major adverse weather increases, the component failure rate in major adverse weather becomes very large. When the percentage of bad weather failures in major adverse weather is equal or greater than 20%, the term in which both failures occur in major adverse weather dominates the calculation of the overall average system failure rate.

The study also shows that the percentage error associated with the approximate equations is relatively small when the percentage of bad weather failures in major adverse weather is less than 20%. The error increases considerably as the percentage increases. Under these conditions, the approximate equations considerably overestimated the overall annual system failure rate when compared with the theoretically exact results obtained using the Markov approach. This therefore places some limits on the use of the approximate equations.

Unlike the approximate equation approach, the Monte Carlo simulation method gives values which are relatively close to the results obtained using the Markov approach for all the weather conditions studied. From an accuracy point of view, the Monte Carlo approach is much more suitable than the approximate equation method. The Monte Carlo simulation method, however, has the problem that it does not convergence even when the simulation time is very long. The resulting fluctuations in the calculated index could result in relatively large errors. The Monte Carlo simulation method is also very time consuming. The Markov approach and the approximate equation methods require much less solution time.

The studies described in this chapter are based on the condition that the major adverse weather condition occurs on average once per year. This is a general assumption which could change considerably in the future. This condition is illustrated in Chapter 4. In these studies the Markov approach is utilized to obtain fast and accurate results.

Chapter 4
Sensitive analysis

4.1 Introduction

A two parallel line reliability evaluation model incorporating two weather states is presented in Chapter 2. This is extended in Chapter 3 to a model which includes three weather states. It was found that when the percentage of failures occurring in bad weather increases, the difference in the reliability indices given by the two models also increases. When the percentage of bad weather failures occurring in major adverse weather increases, the difference in the results given by the two models further increases. These phenomena suggest that the system failure rate is related to both factors: the percentage of failures occurring in bad weather and the percentage of bad weather failures occurring in major adverse weather. An important practical question is under what weather conditions, should a particular model be used. Under certain weather conditions, the two weather state model could be considered acceptable, but under other conditions the three weather state model may be superior. This chapter presents a series of sensitivity studies conducted to examine the response of the two models to a specific set of weather parameter changes.

4.2 Establishment of a basis for comparison

The three weather state representation results in a much different reliability model than the two weather state representation. This can be seen by comparing Figures 3.3 and 2.3. There are eight states in the two weather state reliability model. The weather conditions are divided into the two categories of normal weather and bad weather. In the three weather state reliability model, there are twelve states. The weather conditions are divided into the three categories of normal weather, adverse weather and major adverse weather. It is difficult to directly compare the two models. It is therefore necessary to establish a basis upon which the two models can be evaluated. The probability of residing in each state and the frequency of entering each state are two important indices. Ideally, the two models could be considered comparable if the two indices are the same for the two models.

Under these conditions, the probability of residing in normal weather in the two state model would be equal to that in the three state model, and the frequency of entering the normal weather state in the two state model would be equal to that in the three state model. The single bad weather state in the two state model is divided into adverse weather and major adverse weather in the three state model. The probability of residing in bad weather in the two state model should be equal to the sum of the probabilities of residing in adverse weather and major adverse weather in the three state model. The frequency of entering the bad weather state in the two state model should be equal to the sum of the frequencies of entering the adverse weather state and the major adverse weather state in the three state model.

The basic data for the 3 weather state model are:

na = 1/200 occ/hour, an = 1/2 occ/hour, nm = 1/8760 occ/hour,
am = 1/8760 occ/hour, mn = 1/2 occ/hour, ma = 1/2 occ/hour.

The probability and frequency indices are:

$P_N = 0.98987525$ $P_A = 0.01001061$ $P_{MA} = 0.00011414$

$f_N = 44.3464$ occ/yr $f_A = 43.8565$ occ/yr $f_{MA} = 0.9999$ occ/yr

The basic data for the 2 weather state model are:

na = 1/200 occ/hour, an = 1/2 occ/hour

The probability and frequency indices are:

$P_N = 0.99009901$ $P_A = 0.00990099$

$f_N = 43.3663$ occ/yr $f_A = 43.3663$ occ/yr

The sum of the probabilities of residing in adverse weather and major adverse weather in the three state model is 0.01012475. This is greater than the probability of residing in the bad weather state in the two state model. The two state model data can be modified slightly to make the two probability indices comparable. The revised data for the two weather state model is:

na = 1/200 occ/hour, an = 1/2.04566 occ/hours

Using these data, the probability and frequency indices are:

$P_N = 0.98987525$ $P_A = 0.01012475$

$f_N = 43.3565$ occ/yr $f_A = 43.3565$ occ/yr

It is not possible to make both the probability and frequency indices exactly the same in the two models. It was decided therefore to make the probability indices the same and accept the small differences which result in the frequency indices.

The conditions under which the sensitivity analysis was conducted can be illustrated by considering the several sets of weather related parameters. The base case conditions are established above. Consider the situation in which the average duration of normal weather and adverse weather are five times greater than the base case value in the two state model. Under these conditions, N = 200*5 hours , A = 2.04566*5 hours

$P_N = 0.9899$ $P_A = 0.0101$

$f_N = 8.6713$ occ/yr $f_A = 8.6713$ occ/yr

The comparable conditions for the three state model are as follows:

na = 1/(5*200) occ/hour an = 1/(5*2) occ/hour nm = 1/(5*8760) occ/hour

mn = 1/(5*2) occ/hour am = 1/(5*8760) occ/hour ma = 1/(5*2) occ/hour

$P_N = 0.9899$ $P_A = 0.0100$ $P_{MA} = 0.0001$

$f_N = 8.8693$ occ/yr $f_A = 8.7713$ occ/yr $f_{MA} = 0.2000$ occ/yr

N = 977.6786 hours A = 9.9977 hours M = 5 hour

The two sets of parameters for the condition in which the average durations of normal weather and adverse weather are ten times greater than the base case conditions are as follows:

Two state model

N = 200*10 hours , A = 2.04566*10 hours

$P_N = 0.9899$ $P_A = 0.0101$

$f_N = 4.3356$ occ/yr $f_A = 4.3356$ occ/yr

Three state model

na = 1/(10*200) occ/hour an = 1/(10*2) occ/hour nm = 1/(10*8760) occ/hour

mn = 1/(10*2) occ/hour am = 1/(10*8760) occ/hour ma =1/(10*2) occ/hour

$P_N = 0.9899$ $P_A = 0.0100$ $P_{MA} = 0.0001$

$f_N = 4.4346$ occ/yr $f_A = 4.3856$ occ/yr $f_{MA} = 0.1000$ occ/yr

N = 1955.4 hours A = 19.9954 hours M = 10 hour

The two sets of parameters for the condition in which the average durations of normal weather and adverse weather are half the base case conditions are as follows:

Two state model

N = 200/2 hours, A = 2.04566/2 hours

$P_N = 0.9899$ $P_A = 0.0101$

$f_N = 86.7131$ occ/yr $f_A = 86.7131$ occ/yr

Three state model

na = 2/200 occ/hour	an = 2/2 occ/hour	nm = 2/8760 occ/hour
mn = 2/2 occ/hour	am = 2/8760 occ/hour	ma = 2/2 occ/hour
$P_N = 0.9899$	$P_A = 0.0100$	$P_{MA} = 0.0001$
$f_N = 88.6928$occ/yr	$f_A = 87.7130$occ/yr	$f_{MA} = 1.9998$occ/yr
N = 97.7679 hours	A = 0.9998 hours	M = 0.5 hour

It should be emphasized that the weather state probabilities have been held constant in the limited set of sensitive studies. There is obviously an infinity of possible combinations in which the durations of normal and bad weather can be varied.

All the conditions under which the sensitive analyses were conducted are presented in Appendix C. The average durations of normal weather and adverse weather are varied from half the base case value to ten times greater than the base case value in the two state model.

4.3 Sensitive analysis

Sensitive analyses were conducted based on the conditions presented above. The detailed results are presented in Appendix C. The error factors in the system failure rate are plotted as a function of the percentage of failures occurring in bad weather and are shown in Figures 4.1 to 4.11.

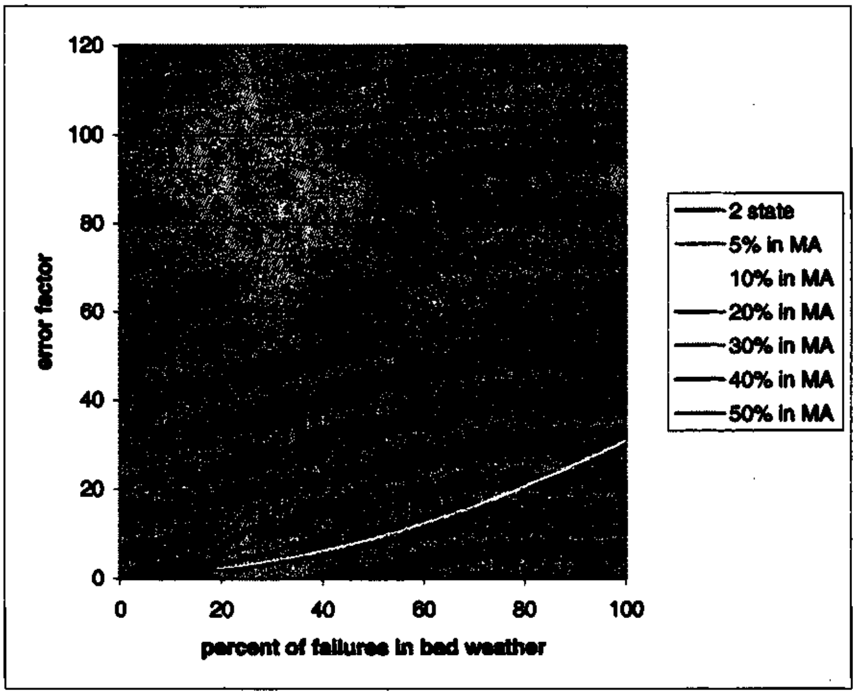


Figure 4.1 The base case analysis

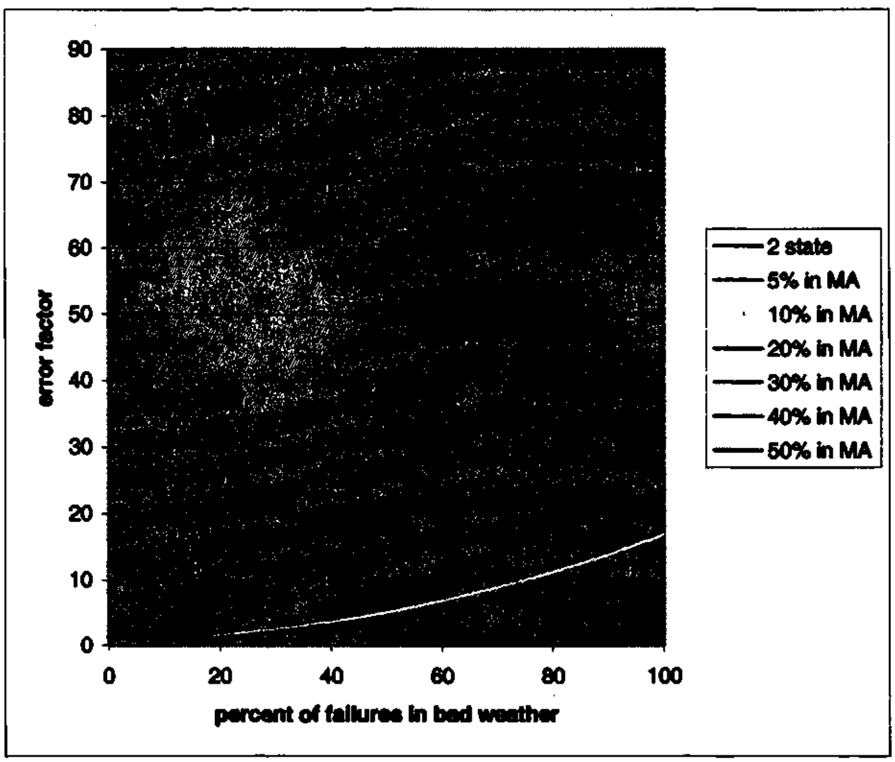


Figure 4.2 The average durations of normal and adverse weather are half the base case values

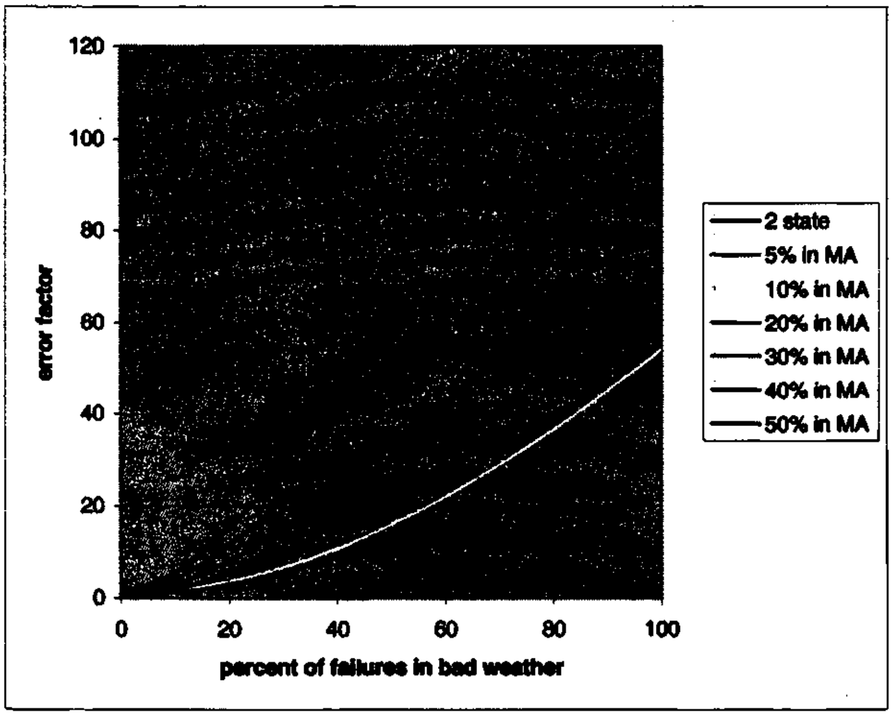


Figure 4.3 The average durations of normal and adverse weather are two times greater than the base case values

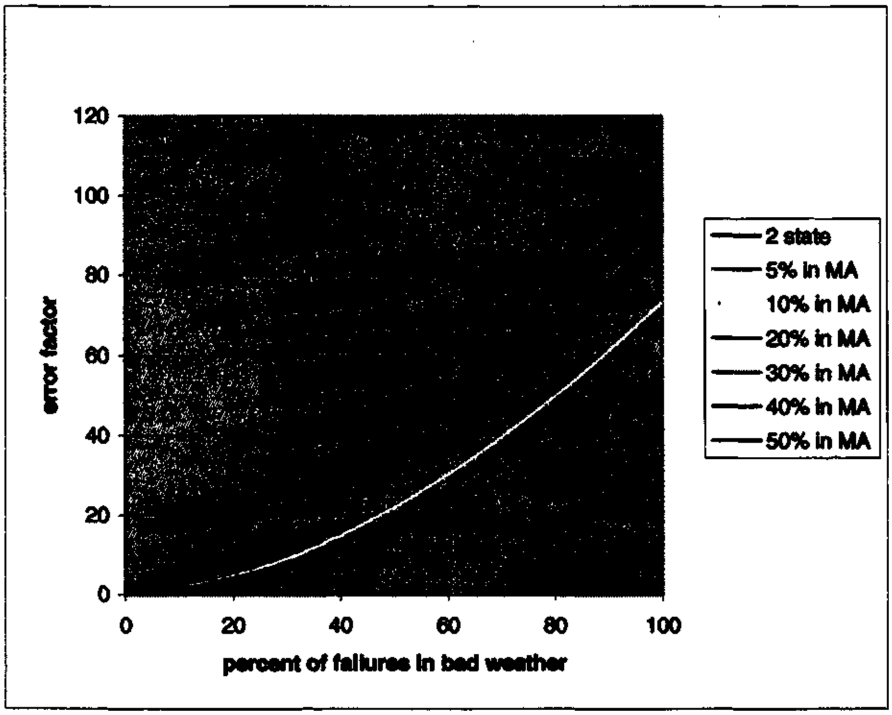


Figure 4.4 The average durations of normal and adverse weather are three times greater than the base case values

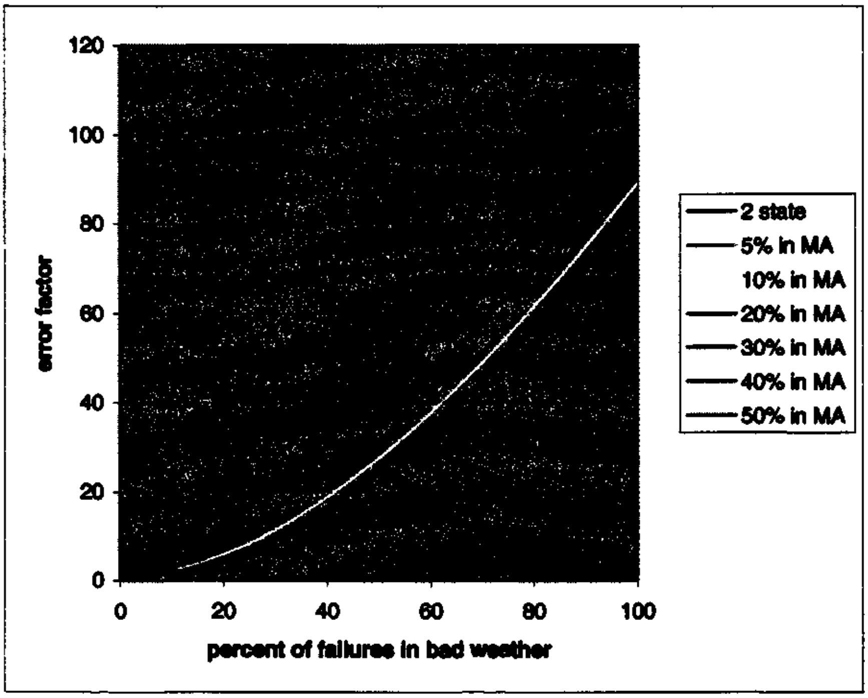


Figure 4.5 The average durations of normal and adverse weather are four times greater than the base case values

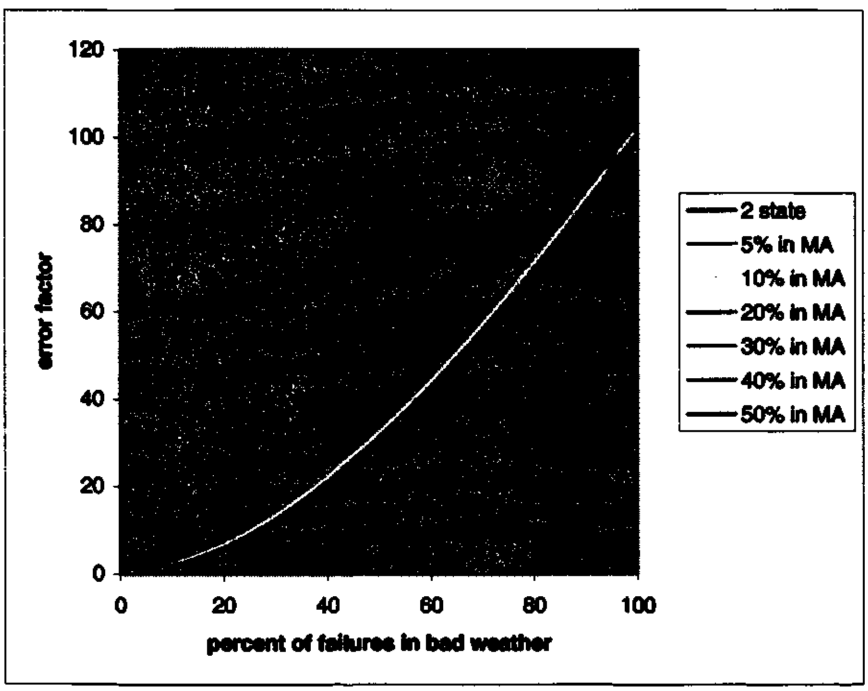


Figure 4.6 The average durations of normal and adverse weather are five times greater than the base case values

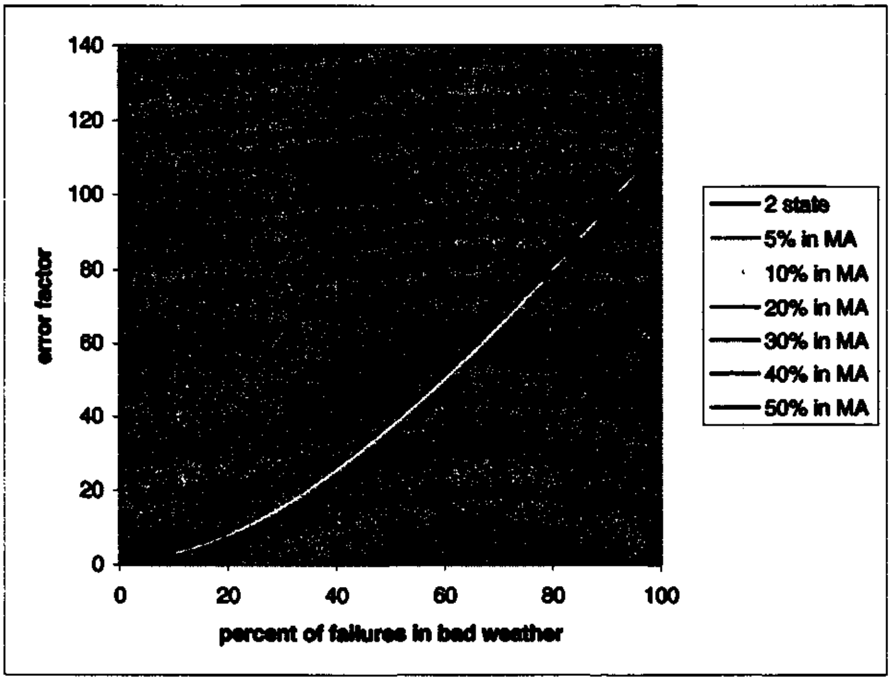


Figure 4.7 The average durations of normal and adverse weather are six times greater than the base case values

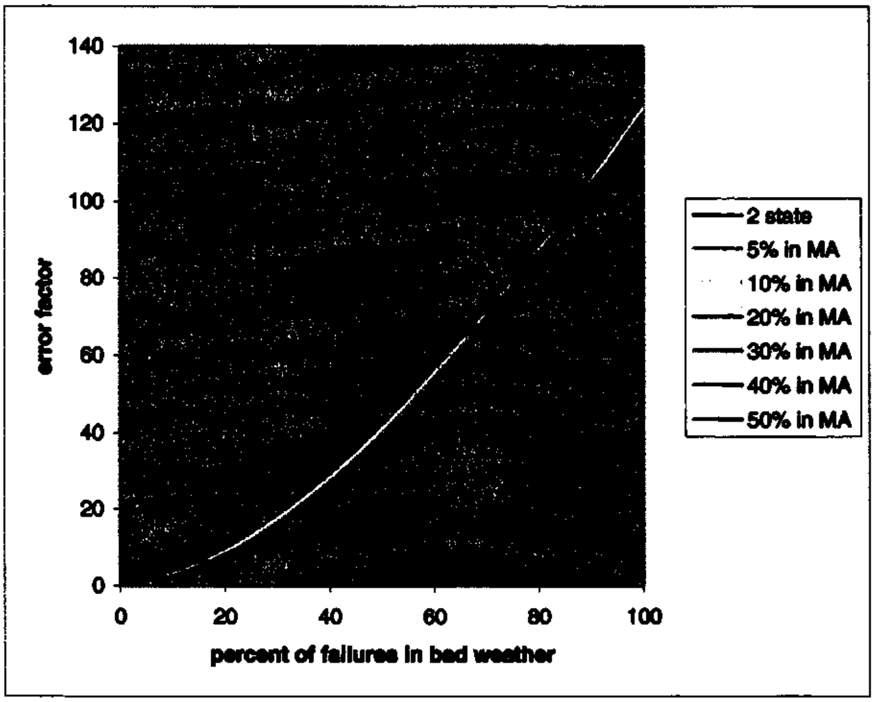


Figure 4.8 The average durations of normal and adverse weather are seven times greater than the base case values

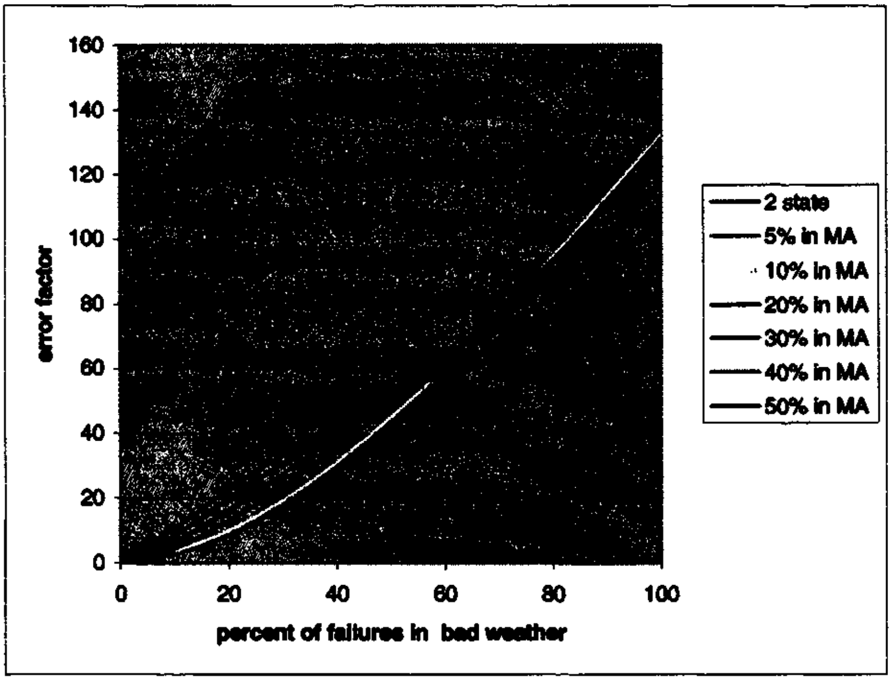


Figure 4.9 The average durations of normal and adverse weather are eight times greater than the base case values

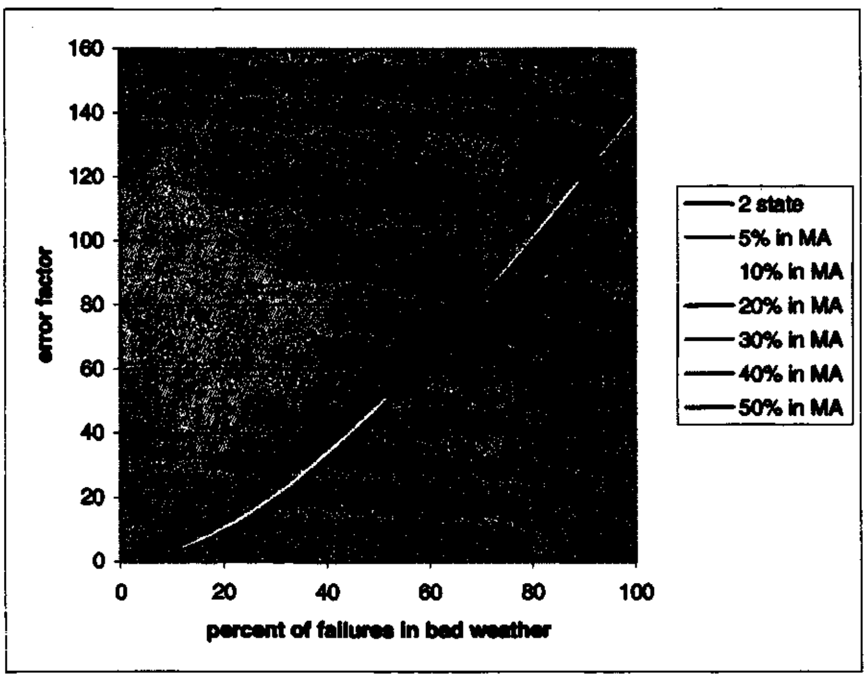


Figure 4.10 The average durations of normal and adverse weather are nine times greater than the base case values

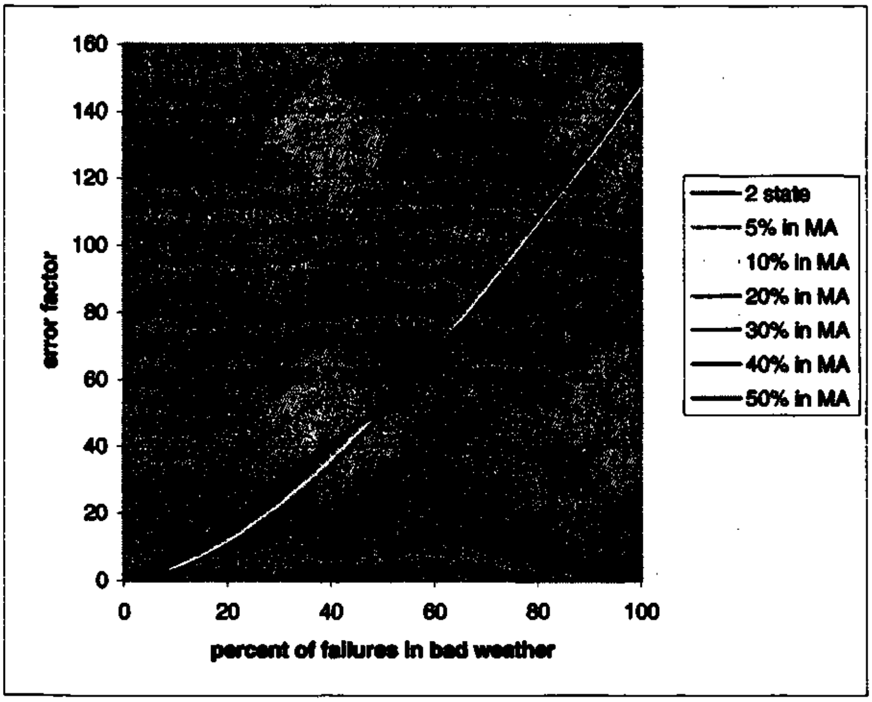


Figure 4.11 The average durations of normal and adverse weather are ten times greater than the base case values

It should be appreciated that the two state weather model is much easier to use than the three state model. It is also much easier to collect the required weather and failure rate data for the two state model. There are virtually no published practical data on weather related failure rates for either the two state model or the three state model. If the two state model can provide a reasonable estimate of the system reliability behavior then there are obvious and practical reasons why it should be employed rather than the three state representation. As shown in the previous set of figures, the two state model in some cases underestimates the error factors associated with the system failure rate prediction. Under these conditions, it may be necessary to use the three state representation. In order to provide a relatively simple basis for comparison, the following criteria are used. If the error factor obtained using the two state model is greater than that obtained using the three state model, the two weather state model is considered to be acceptable. If this is not the case, the three state model should be used.

Consider Figure 4.1. In this case, the error factors obtained using the three state model with the percentage of bad weather failures in major adverse weather varying from 5% to 50% are all greater than those obtained using the two state model. The three state model should therefore be used based on the criterion noted earlier. The difference between the two models increases as the percentage of bad weather failures in major adverse weather increases. Under these situations, the two state model underestimates the potential error caused by neglecting the major adverse weather. Model acceptability for the base case described by Figure 4.1 considering the different combinations of the percentage of line failures in bad weather and the percentage of bad weather failures in major adverse weather is shown in Table 4.1. In this table, the notation II indicates that the two state model is considered to be acceptable and the notation III indicates that the three state model is acceptable.

Table 4.1 Base case model acceptability analysis

% of Line Failures in Bad Weather	5% in Major Adverse Weather	10% in Major Adverse Weather	20% in Major Adverse Weather	30% in Major Adverse Weather	40% in Major Adverse Weather	50% in Major Adverse Weather
10	III	III	III	III	III	III
20	III	III	III	III	III	III
30	III	III	III	III	III	III
40	III	III	III	III	III	III
50	III	III	III	III	III	III
60	III	III	III	III	III	III
70	III	III	III	III	III	III
80	III	III	III	III	III	III
90	III	III	III	III	III	III
100	III	III	III	III	III	III

The profiles in Figures 4.2 to 4.5 are in general similar to those in Figure 4.1. In these figures, the average durations of normal and adverse weather vary from half the base case values to four times greater than in the base case. The three state model is the most appropriate representation under these conditions. The model acceptability considering the different combinations of the percentage of line failures in bad weather and the percentage of bad weather failures in major adverse weather under these conditions is shown in Table 4.1.

The error factors shown in Figure 4.6 were obtained under the condition that the average durations of normal and adverse weather are five times greater than in the base case. When 100% of the line failures occurs in bad weather and the percentage of bad weather failures occurring in major adverse weather is 5%, the error factor obtained using the three state model is less than that obtained using the two state model. In this situation, the two state model could be used. For other combinations of the percentage of line failures occurring in bad weather and the percentage of bad weather failures occurring in major adverse weather, the error factors obtained using the three state model are greater than those obtained using the two state model. The model acceptability in this case is shown in Table 4.2

Table 4.2 Model acceptability analysis (the average durations of normal and adverse weather are five times greater than in the base case)

% of Line Failures in Bad Weather	5% in Major Adverse Weather	10% in Major Adverse Weather	20% in Major Adverse Weather	30% in Major Adverse Weather	40% in Major Adverse Weather	50% in Major Adverse Weather
10	III	III	III	III	III	III
20	III	III	III	III	III	III
30	III	III	III	III	III	III
40	III	III	III	III	III	III
50	III	III	III	III	III	III
60	III	III	III	III	III	III
70	III	III	III	III	III	III
80	III	III	III	III	III	III
90	III	III	III	III	III	III
100	II	III	III	III	III	III

The error factors shown in Figure 4.7 were obtained under the condition that the average durations of normal and adverse weather are six times greater than in the base case. When the percentage of bad weather failures occurring in major weather is 5%, 40% or 50% and the percentage of line failures occurring in bad weather is greater than 80%, the error factors obtained using the two state model are greater than those obtained using the three state model. In these cases, the two state model should be used. When the percentage of bad weather failures in major adverse weather is 10% and 20%, the error factors obtained using the three state model are greater than those obtained using the two state model for all the percentages of line failures occurring in bad weather. The three state model should be used in these cases. When the percentage of bad weather failures occurring in bad weather is 30% and the percentage of line failures occurring in bad weather is 100%, the error factor obtained using the two state model is greater than that obtained using the three state model. In this case, the two state model should be used. The model acceptability in this case is shown in Table 4.3.

Table 4.3 Model acceptability analysis (the average durations of normal and adverse weather are six times greater than in the base case)

% of Line Failures in Bad Weather	5% in Major Adverse Weather	10% in Major Adverse Weather	20% in Major Adverse Weather	30% in Major Adverse Weather	40% in Major Adverse Weather	50% in Major Adverse Weather
10	III	III	III	III	III	III
20	III	III	III	III	III	III
30	III	III	III	III	III	III
40	III	III	III	III	III	III
50	III	III	III	III	III	III
60	III	III	III	III	III	III
70	III	III	III	III	III	III
80	III	III	III	III	III	III
90	II	III	III	III	II	II
100	II	III	III	II	II	II

The error factors shown in Figure 4.8 were obtained under the condition that the average durations of normal and adverse weather are seven times greater than in the base case. In this figure, the error factors obtained using the three state model are less than those obtained using the two state model for a number of combinations of the percentage of bad weather failures occurring in major adverse weather and the percentage of line failures occurring in bad weather. These combinations are shown in Table 4.4. The two state model should be used in these situations. Table 4.4 can be compared with Table 4.3 to note the changing utilization of the two and three state weather models.

Table 4.4 Model acceptability analysis (the average durations of normal and adverse weather are seven times greater than in the base case)

% of Line Failures in Bad Weather	5% in Major Adverse Weather	10% in Major Adverse Weather	20% in Major Adverse Weather	30% in Major Adverse Weather	40% in Major Adverse Weather	50% in Major Adverse Weather
10	III	III	III	III	III	III
20	III	III	III	III	III	III
30	III	III	III	III	III	III
40	III	III	III	III	III	III
50	III	III	III	III	III	III
60	III	III	III	III	III	III
70	III	III	III	III	III	III
80	II	III	III	III	II	II
90	II	III	III	II	II	II
100	II	III	II	II	II	II

The model acceptability examined under the condition that the average durations of normal and adverse weather are eight times greater than in the base case are shown in Table 4.5. The number of situations in which the two state model should be used is increased from that shown in Table 4.4.

Table 4.5 Model acceptability analysis (the average durations of normal and adverse weather are eight times greater than in the base case)

% of Line Failures in Bad Weather	5% in Major Adverse Weather	10% in Major Adverse Weather	20% in Major Adverse Weather	30% in Major Adverse Weather	40% in Major Adverse Weather	50% in Major Adverse Weather
10	III	III	III	III	III	III
20	III	III	III	III	III	III
30	III	III	III	III	III	III
40	III	III	III	III	III	III
50	III	III	III	III	III	III
60	III	III	III	III	III	III
70	II	III	III	III	II	II
80	II	III	II	II	II	II
90	II	III	II	II	II	II
100	II	II	II	II	II	II

The model acceptability examined under the condition that the average durations of normal and adverse weather are nine times greater than in the base case is shown in Table 4.6. The number of situations in which the two state model should be used further increases.

Table 4.7 shows the model acceptability results under the condition that the average durations of normal and adverse weather are ten times greater than in the base case.

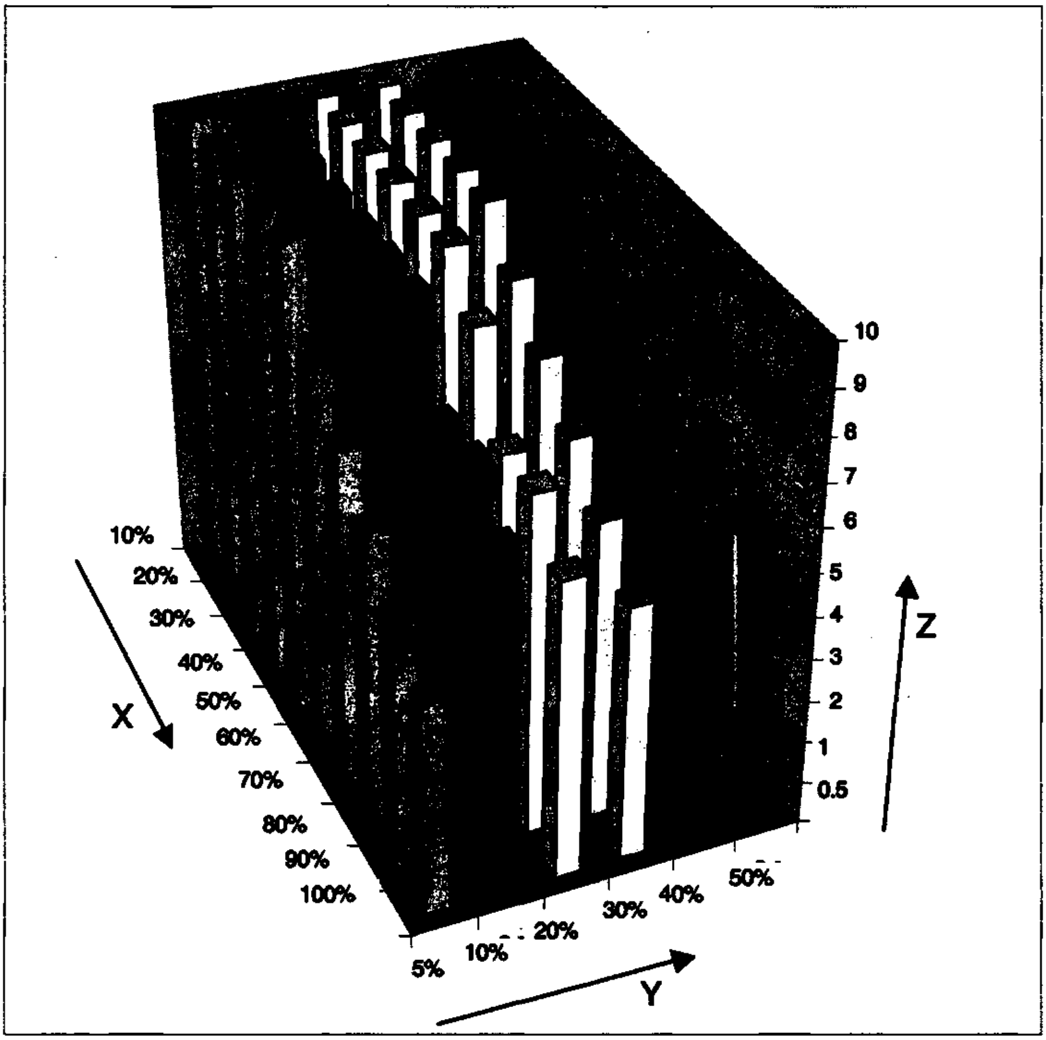
Table 4.6 Model acceptability analysis (the average durations of normal and adverse weather are nine times greater than in the base case)

% of Line Failures in Bad Weather	5% in Major Adverse Weather	10% in Major Adverse Weather	20% in Major Adverse Weather	30% in Major Adverse Weather	40% in Major Adverse Weather	50% in Major Adverse Weather
10	III	III	III	III	III	III
20	III	III	III	III	III	III
30	III	III	III	III	III	III
40	III	III	III	III	III	III
50	III	III	III	III	III	III
60	II	III	III	III	II	II
70	II	III	III	II	II	II
80	II	II	II	II	II	II
90	II	II	II	II	II	II
100	II	II	II	II	II	II

Table 4.7 Model acceptability analysis (the average durations of normal and adverse weather are ten times greater than in the base case)

% of Line Failures in Bad Weather	5% in Major Adverse Weather	10% in Major Adverse Weather	20% in Major Adverse Weather	30% in Major Adverse Weather	40% in Major Adverse Weather	50% in Major Adverse Weather
10	III	III	III	III	III	II
20	III	III	III	III	III	II
30	III	III	III	III	III	II
40	III	III	III	III	III	II
50	III	III	III	III	III	II
60	II	III	III	II	II	II
70	II	III	II	II	II	II
80	II	II	II	II	II	II
90	II	II	II	II	II	II
100	II	II	II	II	II	II

The analysis results in Tables 4.1 to 4.7 are plotted as a three-dimensional image in Figure 4.12. The application zone for the three weather state model is contained within the indicated surface. The application zone for the two weather state model is outside the indicated surface.



Axis X = the percentage of line failures occurring in bad weather

Axis Y = the percentage of bad weather failures occurring in major adverse weather

Axis Z = the base case multiplication factors for the average durations of normal and adverse weather

Figure 4.12 Application zone for the two state model and the three state model

Figure 4.12 shows that when the average normal and adverse weather durations are equal or less than five times the base case values, then the three state model should be used. The application zone of the three state model decreases as the average durations of normal and adverse weather become larger. The application zone decreases as the percentage of line failures occurring in bad weather increases.

4.4 Summary

The sensitive analysis described in this chapter was conducted utilizing the Markov approach for both the two and the three weather state models. Three factors were involved in the analysis: the average durations of normal and adverse weather relative to those in the base case, the percentage of line failures occurring in bad weather and the percentage of bad weather failures occurring in major adverse weather. The error factors obtained using the two weather state model and the three weather state model were plotted as a function of the percentage of line failures occurring in bad weather. The error factors obtained using the two models were used to judge under what weather conditions, a particular model should be used. The application zone for each model was illustrated using a three-dimensional format. Figure 4.12 shows that when the average normal and adverse weather durations are equal or less than five times the base case values, then the three state model should be used. The application zone of the three state model decreases as the average durations of normal and adverse weather become larger. The application zone decrease occurs as the percentage of line failures occurring in bad weather increases. These conclusions are based on a simple error factor criterion and a restricted set of sensitivity studies. There is obviously a very wide range of conditions which could be examined and more complex applicability criteria utilized. The studies described in this chapter do, however, illustrate the general difference between the application zones of the two weather models.

Chapter 5
Summary and conclusions

A transmission and distribution system is composed of either overhead or underground facilities or both. Overhead transmission lines are exposed to varying weather conditions. During adverse weather conditions, the physical stresses placed upon the system components can be very much higher than those encountered under normal weather conditions. Therefore the stress related component failure rates are much higher. If the weather factor is not considered in system reliability evaluation, the reliability models are not physically compatible with the actual situation and the predicted indices could be much more optimistic than the actual system performance.

The failure rate of a component is a continuous function of the weather, which suggests that it should be described either by a continuous function or by a large set of discrete states. Due to difficulties in system modeling, data collection and data validation, only normal and adverse weather are generally considered. Although data collection in transmission and distribution systems has received considerable attention and a range of data has been established, normal, adverse and major adverse weather failure rates are still extremely difficult to obtain and very few utilities can provide these data. This situation may change with the increased attention being devoted to data collection. It is important, however, to focus attention on the most significant data requirements and to ensure that maximum effort is made to obtain this information.

The basic two redundant component transmission line reliability model incorporating normal and adverse weather conditions is introduced in Chapter 2. Two system indices, the average failure rate and the average outage duration are evaluated using the Markov approach, the approximate method and the Monte Carlo simulation technique. The three methods all give reasonably similar results under certain conditions. The Markov approach can be considered to provide theoretically exact values. Under these conditions, the percentage errors in the approximate approach and the Monte Carlo simulation method are quite small. All three methods can be considered to be acceptable from an accuracy point of view for a wide range of practical application.

The Monte Carlo Simulation approach has the problem that the solution does not converge even though the simulation time is very long. This could result in relatively large errors. The Monte Carlo method is also very time consuming. Both the Markov approach and the approximate method involve much less solution time. The approximate approach, however, is much more direct and simple than the Markov method. The approximate method is the most practical approach for general transmission and distribution system analysis as it can be applied directly in minimal cut applications [15].

A two redundant component transmission line reliability model which incorporates normal, adverse and major adverse weather conditions is illustrated in Chapter 3. This is a direct extension of the two weather state model. The two indices of system average failure rate and system average outage duration are evaluated for this model using the Markov approach, the approximate method and the Monte Carlo simulation technique. The results show that the predicted system average failure rate increases as the percentage of bad weather failures in major adverse weather increases. Figure 3.4 shows the error factor variation for different percentages of bad weather failures in major adverse weather associated with the three weather state model and the error factor associated with the two weather state model. It can be clearly seen that the two weather state model does not reflect the increasing importance of major adverse weather and underestimates the potential error.

The results associated with the approximate equation approach show that the bulk of the contribution to the system average failure rate comes from the term in which both failures occur in adverse weather when 95% of the bad weather failures occur in adverse weather. The second largest contribution comes from the term in which both failures occur in major adverse weather. As the percentage of bad weather failures in major adverse weather increases, the component failure rate in major adverse weather becomes very large. When the percentage of bad weather failures in major adverse weather is equal or greater than 20%, the term in which both failures occur in major adverse weather dominates the calculation of the overall average system failure rate.

The studies also show that the percentage error associated with the approximate equations is relatively small when the percentage of bad weather failures in major adverse weather is less than 20%. The error increases considerably as the percentage increases. Under these conditions, the approximate equations considerably overestimated the overall annual system failure rate when compared with the theoretical exact results obtained using the Markov approach. This therefore places some limits on the use of the approximate equations.

Unlike the approximate equation approach, the Monte Carlo simulation method gives values which are relatively close to the results obtained using the Markov approach for all the weather conditions studied. From an accuracy point of view, the Monte Carlo approach is much more suitable than the approximate equation method. As noted earlier. the Monte Carlo simulation method, however, has the problem that it does not convergence even when the simulation time is very long. The resulting fluctuations in the calculated index could result in relatively large errors. As noted in connection with the two state model, the Monte Carlo simulation method is also very time consuming. The Markov approach and the approximate equation method require much less solution time.

Sensitive analysis was conducted utilizing the Markov approach for both the two and the three weather state models. Three primary factors involved in the analysis are the average durations of normal and adverse weather relative to those in the base case, the percentage of line failures occurring in bad weather and the percentage of bad weather failures occurring in major adverse weather. The error factors obtained using the two weather state model and the three weather state model were plotted as a function of the percentage of line failures occurring in bad weather. The error factors obtained using the two models were used to judge under what weather conditions a particular model should be used. The application zone for each model is illustrated using a three-dimensional format. Figure 4.12 shows that when the average normal and adverse weather durations are equal to or less than five times the base case values, then the three state model should be used. The application zone of the three state model decreases as the average durations of normal and adverse weather become larger. The application zone decrease occurs as the percentage of line failures occurring in bad weather increases. These conclusions are based on a simple error factor criterion and a restricted set of sensitivity studies. There is obviously a very wide range of conditions which could be examined and more complex applicability criteria utilized. The studies described in Chapter 4 do however illustrate the general difference between the practical application zones of the two weather models.

The studies described in this thesis clearly illustrate that bad weather should be divided into at least two categories and that transmission line failure data should be collected for each weather state. The use of a single overall average failure rate in predictive reliability studies can lead to optimistic appraisals of the benefits associated with adding redundant transmission lines. The models presented in this thesis form the basis for further research in this area and for extended development of data collection techniques.

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Appendix A
The Markov approach

A sequence of events where the outcome depends on the element of chance is called a stochastic procedure. The Markovian process is a special class of stochastic process in which future states of the process are dependent only upon the immediate past. The reliability problem normally deals with systems that are discrete in space and continuous in time. The use of Markovian models in power system reliability evaluation was initially proposed in a 1964 IEEE publication [19] and application to the transmission system reliability problem was first illustrated in [13]. Since then a great deal of work has been done on the application of Markov processes in this area.

Consider a simple case of a single repairable component for which the failure and repair rates are characterized by exponential distributions. The state space model describing this simple case is shown in Figure A-1.

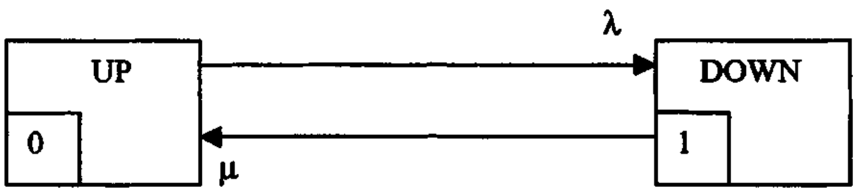

Figure A-1 Two state model of a component

Define:

$P_0(t)$ = Probability of the component being UP at time t.

$P_1(t)$ = Probability of the component being DOWN at time t.

Consider an incremental time interval dt and assume that the probability of the occurrence of two events in this small interval of time is negligible. The probability that the component is in the state 0 at time t+dt is derived from the probability that it was in state 0 at time t and did not transit to state 1 in time dt or that it was in state 1 at time t and transferred to state 0 in time dt. Then

$$P_0(t+dt) = P_0(t)(1-\lambda dt) + P_1(t)\mu dt$$

similarly $P_1(t+dt) = P_0(t)\lambda dt + P_1(t)(1-\mu dt)$

These expressions can be placed in the following differential equation form:

$$P_0'(t) = -\lambda P_0(t) + \mu P_1(t)$$

$$P_1'(t) = \lambda P_0(t) - \mu P_1(t)$$

The matrix form:

$$\begin{bmatrix} P_0'(t) \\ P_1'(t) \end{bmatrix} = \begin{bmatrix} -\lambda & \mu \\ \lambda & -\mu \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \end{bmatrix}$$

or $[P'(t)] = [A][P]$

Where A is called the transitional matrix.

The limiting state probabilities or the so called steady state probabilities can be obtained by setting the differential matrix equal to zero. The system equations then become:

$$-\lambda P_0 + \mu P_1 = 0$$

$$\lambda P_0 - \mu P_1 = 0$$

$$P_0 + P_1 = 1$$

solving these equations,

$$P_0 = \frac{\mu}{\lambda + \mu} \quad \text{and} \quad P_1 = \frac{\lambda}{\lambda + \mu}$$

Where: P_0 represents the availability of the component and

P_1 represents the unavailability of the component.

The frequency of the occurrence of a state is equal to the product of the steady state probability of existence of the state and the rate of departure from that state. Using this principle, the probabilities, frequencies and durations for a two component system with the state space diagram shown in Figure A-2 are given in Table A-1.

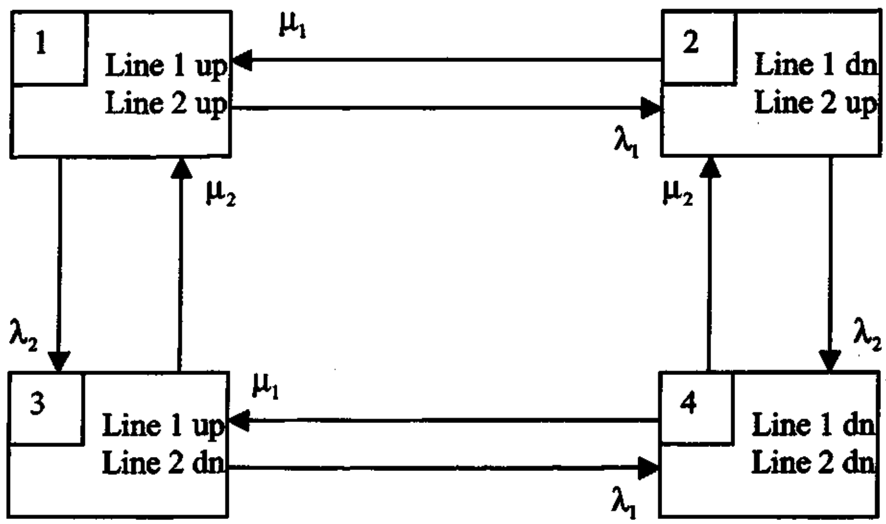

Figure A-2 Two component state space diagram

Table A-1 Frequency and duration of the states for a two component system

State	Component1	Component2	Duration	Probability	Frequency
1	up	up	$1/(\lambda_1+\lambda_2)$	$\mu_1\mu_2/D$	$\mu_1\mu_2(\lambda_1+\lambda_2)/D$
2	down	up	$1/(\mu_1+\lambda_2)$	$\lambda_1\mu_2/D$	$\lambda_1\mu_2(\mu_1+\lambda_2)/D$
3	up	down	$1/(\lambda_1+\mu_2)$	$\mu_1\lambda_2/D$	$\mu_1\lambda_2(\lambda_1+\mu_2)/D$
4	down	down	$1/(\mu_1+\mu_2)$	$\lambda_1\lambda_2/D$	$\lambda_1\lambda_2(\mu_1+\mu_2)/D$

Where $D=(\lambda_1+\mu_1)(\lambda_2+\mu_2)$

For the system of two component in parallel with full redundancy, state 4 is the only failed state and the frequency of occurrence of this state is given by:

$$f_4 = \frac{\lambda_1 \lambda_2 (\mu_1 + \mu_2)}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}$$

The average duration associated with this state is given by:

$$Duration = \frac{\text{Probability} \quad \text{of} \quad \text{the} \quad \text{state}}{\text{Frequency} \quad \text{of} \quad \text{the} \quad \text{state}}$$

$$= \frac{P_4}{P_4(\mu_1 + \mu_2)} = \frac{r_1 r_2}{r_1 + r_2}$$

The average duration associated with a cumulative group of state is given by:

$$Duration = \frac{\text{Cumulative} \quad \text{probability} \quad \text{of} \quad \text{the} \quad \text{states}}{\text{Cumulative} \quad \text{frequency} \quad \text{of} \quad \text{the} \quad \text{states}}$$

The term cumulative probability is defined as the summation of all state probabilities of that cumulative group. The term cumulative frequency of a group of states is equal to the summation of the individual frequencies reduced by the sum of the frequencies of encountering each other.

The mean up time of the system can also be obtained by using discrete Markov chain concepts [28]. The stochastic transitional probability matrix for a two component system is given by:

$$[P] = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 1-(\lambda_1+\lambda_2) & \lambda_1 & \lambda_2 & 0 \\ \mu_1 & 1-(\lambda_2+\mu_1) & 0 & \lambda_2 \\ \mu_2 & 0 & 1-(\lambda_1+\mu_2) & \lambda_1 \\ 0 & \mu_2 & \mu_1 & 1-(\mu_1+\mu_2) \end{bmatrix}$$

State 4 is designated as an absorbing state and a new truncated Matrix Q is obtained by eliminating the absorbing state.

$$\begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 1-(\lambda_1+\lambda_2) & \lambda_1 & \lambda_2 \\ \mu_1 & 1-(\lambda_2+\mu_1) & 0 \\ \mu_2 & 0 & 1-(\mu_1+\mu_2) \end{bmatrix}$$

Let I = Identity matrix

$$[N]=[I-Q]^{-1}=\begin{bmatrix} \lambda_1+\lambda_2 & -\lambda_1 & -\lambda_2 \\ -\mu_1 & \lambda_2+\mu_1 & 0 \\ -\mu_2 & 0 & \lambda_1+\mu_2 \end{bmatrix}^{-1}$$

Matrix [N] is the Fundamental matrix, where n_j is the time spent by the process in state S_j before being absorbed.

Starting in state 1, the MTTF (the time before entering the absorbed state) of the process is $M_{1,4}$

$$M_{1,4}=\sum_{i=1}^{4} N(1,i)$$

$$\lambda_{SL}=1/M_{1,4}$$

Where λ_{SL} is the system failure rate.

Appendix B

Detail Monte Carlo simulation procedure for three weather state model

Step 1: Choose state 1 as the starting state, in which both components are available and the weather is normal.

Step 2: Generate four random numbers associated with the process involving the component failure rates and the transition rate from normal weather to adverse weather and from normal weather to major adverse weather. Convert these random numbers into the times to failure for both components (TTF1, TTF2), the time to adverse weather from normal weather (TTAFN) and the time to major adverse weather from normal weather (TTMFN) using λ_1, λ_2 NA and NM respectively.

Step 3: Compare TTF1, TTF2, TTAFN and TTMFN.

Time = Time + min {TTF1, TTF2, TTAFN, TTMFN}

At the beginning of the process, Time in the right hand side of the above equation is zero.

If the time is greater than the desired years of simulation, go to step 30.

If TTF1 is the shortest time, the system will enter state 2. The TTAFN and TTMFN should be updated by deducting TTF1. Then go to step 4.

If TTF2 is the shortest time, the system will enter state 3. The TTAFN and TTMFN should be updated by deducting TTF2. Then go to step 7.

IF TTAFN is the shortest time, the system will enter state 5. The TTMFN should be updated by deducting TTAFN. Then go to step 12.

If TTMFN is the shortest time, the system will enter state 9. Then go to step 21.

Step 4: The system is in state 2. Generate two random numbers associated with μ_1 and λ_2 respectively.

If the system enters state 2 from state 1, in the case of TTAFN and TTMFN, use the updated value in step 3.

If the system enters state 2 from state 4, in the case of TTAFN and TTMFN, use the updated value in step 11.

If the system enters state 2 from state 6 or state 10, generate two random numbers associated with NA and NM respectively.

Step 5: Convert these random numbers into the time to repair of component 1 (TTR1), the time to failure of component 2 (TTF2), the time to adverse weather (TTAFN) or the time to major adverse weather (TTMFN) if necessary.

Step 6: Compare TTF2, TTR1, TTAFN and TTMFN.
Time = Time + min {TTF2, TTR1, TTAFN, TTMFN}

If the time is greater than the desired years of simulation, go to step 30.

If TTF2 is the shortest time, the system will enter state 4. The TTR1, TTAFN and TTMFN should be updated by deducting TTF2. Then go to step 10.

If TTR1 is the shortest time, the system will go back to state 1. The TTAFN and TTMFN should be updated by deducting TTR1. Then go to step 2.

If TTAFN is the shortest time, the system will enter state 6. The TTR1 should be updated by deducting TTAFN. Then go to step 15.

If TTMFN is the shortest time, the system will enter state 10. The TTR1 should be updated by deducting TTMFN. Then go to step 24.

Step 7: System in state 3. Generate two random numbers associated with λ_1 and μ_2 respectively.

If the system enters state 3 from state 1, in the case of TTAFN and TTMFN, use the updated value in step 3.

If the system enters state 3 from state 4, in the case of TTAFN and TTMFN, use the updated value in step 11.

If the system enters state 3 from state 7 or state 11, generate two random numbers associated with NA and NM respectively.

Step 8: Convert these random numbers into the time to repair of component 2 (TTR2), the time to failure of component 1(TTF1), the time to adverse weather (TTAFN) and the time to major adverse weather (TTMFN) if necessary.

Step 9: Compare TTF1, TTR2, TTAFN and TTMFN.
Time = Time + min {TTF1, TTR2, TTAFN, TTMFN}

If the time is greater than the desired years of simulation, go to step 30.

If TTF1 is the shortest time, the system will enter state 4, the TTR2, TTAFN, and TTMFN should be updated by deducting TTF1. Then go to step 10.

If TTR2 is the shortest time, the system will enter state 1, the TTAFN and TTMFN should be updated by deducting TTR2, then go to step 2.

If TTAFN is the shortest time, the system will enter state 7, the TTR2 should be updated by deducting TTAFN, then go to step 17.

If TTMFN is the shortest time, the system will enter states 11, the TTR2 should be updated by deducting TTMFN, then go to step 26.

Step 10: The system is in state 4.

If the system enters state 4 from state 2, in the case of TTAFN, TTMFN and TTR1, use the updated value in step 6. Generate a random number associated with μ_2, convert this random number into the time to repair of component 2 (TTR2).

If the system enters state 4 from state 3, in the case of TTA, TTM and TTR2, use the updated value in step 9. Generate a random number associated with μ_1. Convert this number into the time to repair of component 1 (TTR1).

If the system enters state 4 from state 8, generate a random number associated with NA. Convert this number into the time to adverse weather from normal weather (TTAFN). In the case of TTR1 and TTR2, use the updated values in step 20 if they are not null.

If TTR1 is zero in step 20, generate a random number associated with μ_1, and convert it into the time to repair of component 1 (TTR1). If TTR2 is zero in step 20, generate a random number associated with μ_2, and convert it into the time to repair of component 2 (TTR2). If both TTR1 and ATTR2 are zero in step 20, generate two random numbers associated with μ_1 and μ_2, and convert them into the time to repair of component 1 (TTR1) and the time to repair of component 2 (TTR2).

If the system enters state 4 from state 12, generate a random number associated with NM. Convert this number into the time to adverse weather from normal weather (TTMFN). In the case of TTR1 and TTR2, use the updated values in step 29 if they are not null.

If TTR1 is zero in step 29, generate a random number associated with μ_1, and convert it into the time to repair of component 1 (TTR1). If TTR2 is zero in step 29, generate a random number associated with μ_2, and convert it into the time to repair of component 2 (TTR2). If both TTR1 and ATTR2 are zero in step 29, generate two random numbers associated with μ_1 and μ_2, and convert them into the time to repair of component 1 (TTR1) and the time to repair of component 2 (TTR2).

Step 11: Compare TTR1, TTR2, TTAFN and TTMFN.
Time = Time + min {TTR1, TTR2, TTAFN, TTMFN}

If the time is greater than the desired years of simulation, go to step 30.

If TTR1 is the shortest time, the system will go back to state 3. TTR2, TTAFN and TTMFN should be updated by deducting TTR1. Then go to step 7.

If TTR2 is the shortest time, the system will go back to state 2. TTR1, TTAFN and TTMFN should be updated by deducting TTR2. Then go to step 4.

If TTAFN is the shortest time, the system will enter state 8. TTR1, TTR2 should be updated by deducting TTAFN. Then go to step 19.

If TTMFN is the shortest time, the system will enter state 12. TTR1, TTR2 should be updated by deducting TTMFN. Then go to step 28.

Step 12: system in state 5. Generate four random numbers associated with λ_1', λ_2', AN and AM respectively.

Step 13: Convert these numbers into the times to failures for both components (TTF1 and TTF2), the time to normal weather from adverse weather (TTNFA) and the time to major adverse weather from adverse weather (TTMFA).

Step 14: CompareTTF1, TTF2, TTNFA and TTMFA.
Time = Time + min {TTF1, TTF2, TTNFA, TTMFA}

If the time is greater than the desired years of simulation, go to step 30.

If TTF1 is the shortest time, the system will enter state 6.TTNFA and TTMFA should be updated by deducting TTF1. Then go to step 15.

If TTF2 is the shortest time, the system will enter state 7. TTNFA and TTMFA should be updated by deducting TTF2. Then go to step 17.

If TTNFA is the shortest time, the system will enter state 1.Then go to step 2.

If TTMFA is the shortest time, the system will enter state 9. Then go to step 21.

Step 15: The system is in state 6.
If the system enters state 6 from state 5, generate a random number associated with λ_2', convert it into the time to failure of component 2 (TTF2). In the case of TTNFA and TTMFA, use the updated values in step 14.

If the system enters state 6 from state 2 or state 10, generate three random number associated with λ_2', AN and AM respectively. Convert these numbers into the time to failure of component 2 (TTF2), the time to normal weather from adverse weather

(TTNFA) and the time to major adverse weather from adverse weather (TTMFA) respectively.

If the value of TTR1 existed, keep the value unchanged.

Step 16: Compare TTF2, TTNFA and TTMFA.
Time = Time + min {TTF2, TTNFA, TTMFA}

If the time is greater than the desired years of simulation, go to step 30.

If TTF2 is the shortest time, the system will enter state 8. TTNFA and TTMFA should be updated by deducting TTF2. Then go to step 19.

If TTNFA is the shortest time, the system will enter state 2. Then go to step 4.

If TTMFA is the shortest time, the system will enter state 10. Then go to step 24.

Step 17: The system is in state 7.
If the system enters state 7 from state 5, generate a random number associated with λ_1' and convert it into the time to failure of component 1(TTF1). In the case of TTNFA and TTMFA, use the values updated in step 14.

If the system enter state 7 from state 3 or state 11, generate three random numbers associated with λ_1', AN and AM respectively. Convert these numbers into the time to failure of component 1(TTF1), the time to normal weather from adverse weather (TTNFA) and the time to major adverse weather from adverse weather (TTMFA) respectively.

If the value of TTR2 existed, keep the value unchanged.

Step 18: Compare TTF1 and TTNFA and TTMFA.

Time = Time + min {TTF1, TTNFA, TTMFA}

If the time is greater than the desired years of simulation, go to step 30.

If TTF1 is the shortest time, the system will enter state 8. TTNFA and TTMFA should be update by deducting TTF1. Then go to step 19.

If TTNFA is the shortest time, the system will enter state 3. Then go to step 7.

If TTMFA is the shortest time, the system will enter state 11. Then go to step 26.

Step 19: The system is in state 8.

If the system enters state 8 from state 4 or 12, generate two random numbers associated with AN and AM, convert them into the time to normal weather from adverse weather (TTNFA) and the time to major adverse weather from adverse weather (TTMFA).

If the entry is from state 6 or 7, in the case of TTNFA and TTMFA, use the updated values in step 16 or step 18.

If the values of TTR1 or TTR2 exist, keep these value unchanged.

Step 20: Compare TTNFA and TTMFA

Time = Time + min {TTNFA, TTMFA}

If the time is greater than the desired years of simulation, go to step 30.

If TTNFA is shorter, system will enter state 4. Then go to step 10.

If TTMFA is shorter, system will enter state 12. Then go to step 28.

Step 21: The system is in state 9. Generate four random numbers associated with λ_1^{ma}, λ_2^{ma}, MN and MA respectively.

Step 22: Convert these numbers into the times to failure for both components (TTF1 and TTF2), the time to normal weather from major adverse weather (TTNFM) and the time to adverse weather from major adverse weather (TTAFM).

Step 23: CompareTTF1, TTF2, TTNFM and TTAFM.
Time = Time + min {TTF1, TTF2, TTNFM, TTAFM}
If the time is greater than the desired years of simulation, go to step 30.

If TTF1 is the shortest time, the system will enter state 10. TTNFM and TTAFM should be updated by deducting TTF1. Then go to step 24.

If TTF2 is the shortest time, the system will enter state 11. TTNFM and TTAFM should be updated by deducting TTF2. Then go to step 26.

If TTNFM is the shortest time, the system will enter state 1.Then go to step 2.

If TTAFM is the shortest time, the system will enter state 5. Then go to step 12.

Step 24: The system is in state 10.
If the system enters state 10 from state 9, generate a random number associated with λ_2^{ma}, convert it into the time to failure of component 2 (TTF2). In the case of TTNFM and TTAFM, use the updated values in step 23.

If the system enters state 10 from state 2 or state 6, generate three random numbers associated with λ_2^{ma}, MN and MA respectively. Convert these numbers into the time to failure of component 2 (TTF2), the time to normal weather from major adverse weather (TTNFM) and the time to adverse weather from major adverse weather (TTAFM) respectively.

If the value of TTR1 exists, keep this value unchanged.

Step 25: Compare TTF2, TTNFM and TTAFM.
Time = Time + min {TTF2, TTNFM, TTAFM}

If the time is greater than the desired years of simulation, go to step 30.

If TTF2 is the shortest time, the system will enter state 12. TTNFM and TTAFM should be updated by deducting TTF2. Then go to step 28.

If TTNFM is the shortest time, the system will enter state 2. Then go to step 4.

If TTAFM is the shortest time, the system will enter state 6. Then go to step 15.

Step 26: The system is in state 11.
If the system enters state 11 from state 9, generate a random number associated with λ_1^{ma} and convert it into the time to failure of component 1(TTF1). In the case of TTNFM and TTAFM, use the values updated in step 23.

If the system enters state 11 from state 3 or state 7, generate three random numbers for λ_1^{ma}, MN and MA respectively. Convert these numbers into the time to failure of component 1(TTF1), the time to normal weather from major adverse weather (TTNFM) and the time to adverse weather from major adverse weather (TTAFM) respectively.

If the value of TTR2 exists, keep this value unchanged.

Step 27: Compare TTF1 and TTNFM and TTAFM.
Time = Time + min {TTF1, TTNFM, TTAFM}

If the time is greater than the desired years of simulation, go to step 30.

If TTF1 is the shortest time, the system will enter state 12. TTNFM and TTAFM should be update by deducting TTF1. Then go to step 28.

If TTNFM is the shortest time, the system will enter state 3. Then go to step 7.

If TTAFM is the shortest time, the system will enter state 7. Then go to step 17.

Step 28: The system is in state 12.

If the system enters state 12 from state 4 or 8, generate two random numbers associated with MN and MA, convert them into the time to normal weather from major adverse weather (TTNFM) and the time to adverse weather from major adverse weather (TTAFM).

If the system enters state 12 from state 10 or state 11, in the case of TTNFM and TTAFM, use the updated values in step 23 or step 25.

If the values of TTR1 or TTR2 exist, keep these value unchanged.

Step 29: Compare TTNFM and TTAFM.

Time = Time + min {TTNFM, TTAFM}

If the time is greater than the desired years of simulation, go to step 30.

If TTNFM is shorter, system will enter state 4. Then go to step 10.

If TTAFM is shorter, system will enter state 8. Then go to step 19.

Step 30: Determine the total system down time and system failure frequency.

The total system down time is obtained by accumulating the duration of system residence times in state 4, 8 and 12.

The system failure frequency is obtained by accumulating the number of times that the system enters state 4 from states 2 or 3, the number of times that the system enters state 8 from states 6 or 7 and the number of times that the system enters state 12 from states 10 or 11.

Step 32: Obtain the system reliability indices, system failure frequency, system average downtime and system availability using the Equations from 2.14--2.16.

Appendix C
Sensitive analysis results

Table C-1 Error factors (base case)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.007564	1.006594	1.006594	1.006594	1.006594	1.006594	1.006594
10	1.276667	1.281957	1.347014	1.635711	2.121432	2.790327	3.621802
20	2.065169	2.090307	2.339821	3.413550	5.143921	7.431697	10.18245
30	3.369041	3.422594	3.962861	6.199526	9.679065	14.12033	19.29790
40	5.177301	5.272415	6.186298	9.875555	15.42440	22.28353	30.05033
50	7.480804	7.616555	8.986037	14.34213	22.13744	31.49390	41.81821
60	10.26558	10.45190	12.33836	19.51603	29.63677	41.45644	54.19701
70	13.52505	13.77275	16.22998	25.32780	37.77880	51.95175	66.90087
80	17.25115	17.55473	20.62711	31.70428	46.44110	62.81057	79.73328
90	21.42613	21.79484	25.52122	38.60218	55.53013	73.91084	92.56432
100	26.05079	26.48412	30.88712	45.97052	64.97241	85.16369	105.3000

Base case parameters: na = 1/200 occ/hour, an = 1/2 occ/hour, nm = 1/8760 occ/hour,
am = 1/8760 occ/hour, mn = 1/2 occ/hour, ma = 1/2 occ/hour.

Table C-2 Error factors (the average durations of normal and adverse weather are half the base case values)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.006669	1.005637	1.005637	1.005637	1.005637	1.005637	1.005637
10	1.140167	1.144060	1.177016	1.327174	1.584461	1.944866	2.409435
20	1.541500	1.552877	1.683150	2.260912	3.232277	4.570122	6.233159
30	2.204112	2.230351	2.517059	3.774145	5.836464	8.615765	12.01372
40	3.117773	3.176496	3.675531	5.822762	9.284026	13.85994	19.36193
50	4.300261	4.377129	5.140039	8.367269	13.47780	20.11346	27.97089
60	5.733984	5.839544	6.919940	11.39345	18.35334	27.23767	37.59569
70	7.420181	7.555608	8.986481	14.85048	23.82301	35.08377	48.02096
80	9.350101	9.536042	11.36403	18.74335	29.84312	43.56539	59.10628
90	11.52870	11.75172	14.02069	23.02143	36.33794	52.55606	70.68257
100	13.95863	14.22612	16.96549	27.67895	43.27428	61.99591	82.65891

Table C-3 Error factors (the average durations of normal and adverse weather are two times the base case value)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.007653	1.007478	1.007478	1.007478	1.007478	1.007478	1.007478
10	1.536279	1.548213	1.672923	2.209538	3.074937	4.218644	5.593957
20	3.093203	3.138752	3.595796	5.440330	8.214600	11.64346	15.52605
30	5.638008	5.730430	6.673483	10.26201	15.32134	21.22921	27.59635
40	9.128537	9.280344	10.81640	16.35382	23.71969	31.89959	40.35389
50	13.52936	13.74465	15.94579	23.48499	32.98063	43.06741	53.12220
60	18.80025	19.08413	21.99459	31.48084	42.83588	54.41727	65.59503
70	24.90936	25.26166	28.89975	40.21109	53.10741	65.77338	77.64478
80	31.82032	32.23796	36.60419	49.56966	63.67812	77.04192	89.22984
90	39.50401	39.98224	45.06345	59.48627	74.47759	88.18848	100.3655
100	47.92722	48.45924	54.22717	69.89115	85.45192	99.19251	111.0772

Table C-4 Error factors (the average durations of normal and adverse weather are three times the base case value)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.007858	1.007303	1.007303	1.007303	1.007303	1.007303	1.007303
10	1.795851	1.812441	1.991894	2.737719	3.898193	5.378031	7.104009
20	4.094515	4.155602	4.784803	7.176441	10.55004	14.48760	18.73168
30	7.814971	7.935159	9.176113	13.53560	19.17481	25.29653	31.50524
40	12.87096	13.05579	14.99611	21.31986	28.88745	36.60558	44.05322
50	19.18215	19.43237	22.10117	30.20739	39.23330	47.93507	55.97627
60	26.67430	26.98372	30.37017	39.98291	49.97163	59.10368	67.20904
70	35.27658	35.63507	39.69962	50.49639	60.97577	70.06042	77.80754
80	44.92083	45.31638	49.99856	61.64219	72.18084	80.81434	87.86163
90	55.54819	55.96352	61.18886	73.34437	83.55515	91.39660	97.46815
100	67.09776	67.51638	73.20218	85.54511	95.08544	101.8475	106.7138

Table C-5 Error factors (the average durations of normal and adverse weather are four times the base case value)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.007446	1.007601	1.007601	1.007601	1.007601	1.007601	1.007601
10	2.050166	2.075533	2.309049	3.244793	4.648260	6.377777	8.332415
20	5.067789	5.150446	5.932238	8.732382	12.43730	16.53672	20.76011
30	9.903393	10.05989	11.53784	16.31781	22.00244	27.77369	33.32227
40	16.41261	16.64365	18.85654	25.36800	32.39976	39.02330	45.03618
50	24.46340	24.75972	27.67657	35.51767	43.23797	50.00874	55.83245
60	33.93409	34.28117	37.82598	46.54447	54.35648	60.70624	65.85607
70	44.71341	45.08959	49.16171	58.30446	65.69299	71.17046	75.28071
80	56.69615	57.07698	61.55851	70.69949	77.22729	81.47184	84.25982
90	69.78825	70.14791	74.91495	83.66078	88.96233	91.68260	92.92350
100	83.90198	84.21282	89.13992	97.13643	100.9031	101.8627	101.3728

Table C-6 Error factors (the average durations of normal and adverse weather are five times the base case value)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.007690	1.007544	1.007544	1.007544	1.007544	1.007544	1.007544
10	2.302481	2.327970	2.599018	3.662764	5.213570	7.077244	9.136001
20	6.015526	6.097840	6.971008	9.971674	13.76079	17.79377	21.81918
30	11.90981	12.05601	13.64545	18.49435	23.91084	29.14495	33.98917
40	19.77110	19.97269	22.26465	28.51941	34.75361	40.27034	45.04664
50	29.40771	29.64420	32.55216	39.66410	45.97512	51.05677	55.14587
60	40.64674	40.89013	44.28703	51.70610	57.47198	61.57590	64.53842
70	53.32966	53.55090	57.29019	64.50729	69.22204	71.93517	73.44716
80	67.31406	67.48136	71.41069	77.97183	81.22668	82.22881	82.04298
90	82.47300	82.55403	86.52242	92.03162	93.49646	92.53602	90.45620
100	98.68990	98.65051	102.5136	106.6306	106.0387	102.9142	98.78101

Table C-7 Error factors (the average durations of normal and adverse weather are six times the base case value)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.007260	1.007345	1.007345	1.007345	1.007345	1.007345	1.007345
10	2.550390	2.582523	2.896756	4.092694	5.779021	7.747706	9.868969
20	6.935734	7.035100	8.004551	11.16551	14.94784	18.80498	22.52588
30	13.83230	14.00067	15.69288	20.49647	25.48650	30.04698	34.09291
40	22.94752	23.16925	25.50838	31.32462	36.58573	40.89186	44.40228
50	34.02494	34.26858	37.10808	43.26980	48.02651	51.38997	53.80241
60	46.83506	47.06492	50.22738	56.12204	59.76748	61.69387	62.62133
70	61.17440	61.35265	64.65382	69.74995	71.81570	71.94094	71.10146
80	76.86456	76.95139	80.21164	84.06184	84.18529	82.23495	79.41320
90	93.74409	93.70163	96.75326	98.98566	96.88661	92.64882	87.67433
100	111.6707	111.4641	114.1540	114.4650	109.9255	103.2350	95.96818

Table C-8 Error factors (the average durations of normal and adverse weather are seven times the base case value)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.007869	1.007133	1.007133	1.007133	1.007133	1.007133	1.007133
10	2.795411	2.834127	3.191705	4.506732	6.298514	8.329650	10.46506
20	7.832674	7.948851	9.003045	12.25353	15.93371	19.52878	22.88339
30	15.68320	15.87187	17.63252	22.26054	26.71206	30.54398	33.79285
40	25.96865	26.20083	28.52625	33.74659	37.96174	41.08358	43.42218
50	38.36094	38.59674	41.28437	46.35029	49.56218	51.33520	52.25998
60	52.57659	52.76934	55.59975	59.87444	61.51550	61.49842	60.66844
70	68.36675	68.46929	71.23064	74.19205	73.84447	71.72110	68.88784
80	85.51508	85.48241	87.97990	89.21151	86.56493	82.10164	77.07462
90	103.8327	103.6232	105.6837	104.8559	99.68077	92.70071	85.32884
100	123.1533	122.7301	124.2030	121.0601	113.1859	103.5550	93.71535

Table C-9 Error factors (the average durations of normal and adverse weather are eight times the base case value)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.007686	1.007504	1.007504	1.007504	1.007504	1.007504	1.007504
10	3.037632	3.079246	3.470325	4.870231	6.722568	8.771934	10.88324
20	8.709504	8.824917	9.931216	13.18634	16.70086	20.01082	23.01435
30	17.46843	17.64234	19.41415	23.77107	27.67336	30.84304	33.41112
40	28.84633	29.03646	31.27582	35.84129	39.09784	41.21026	42.59196
50	42.44162	42.59717	45.05797	49.04750	50.91855	51.38280	51.11867
60	57.91509	57.98065	60.41680	63.19674	63.15935	61.57762	59.35823
70	74.97449	74.89841	77.08401	78.16162	75.84493	71.93769	67.53642
80	93.37040	93.10378	94.84077	93.23917	88.98009	82.54520	75.78877
90	112.8870	112.3888	113.5093	110.1454	102.5571	93.44663	84.19808
100	133.3394	132.5745	132.9382	127.0052	116.5570	104.6614	92.81168

Table C-10 Error factors (the average durations of normal and adverse weather are nine times the base case value)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.008157	1.007678	1.007678	1.007678	1.007678	1.007678	1.007678
10	3.274085	3.313662	3.716731	5.146807	7.011180	9.046689	11.12019
20	9.552015	9.656280	10.76263	13.94710	17.29275	20.37474	23.12552
30	19.16164	19.30834	21.02520	25.08840	28.55283	31.24399	33.34468
40	31.53790	31.68133	33.77454	37.76706	40.30985	41.71084	42.43522
50	46.20926	46.29588	48.49767	51.62478	52.52705	52.07316	50.98248
60	62.78117	62.75746	64.81178	66.45553	65.22627	62.54469	59.34244
70	80.92056	80.73859	82.41914	82.11552	78.41820	73.25285	67.72506
80	100.3485	99.96535	101.0801	98.48936	92.09386	84.26443	76.24938
90	120.8267	120.2078	120.6000	115.4782	106.2305	95.60998	84.98215
100	142.1553	141.2731	140.8164	132.9954	120.7957	107.2957	93.95879

Table C-11 Error factors (the average durations of normal and adverse weather are ten times the base case value)

Percentage of failures in bad weather	2 state model	3 state model					
		5% in MA	10% in MA	20% in MA	30% in MA	40 % in MA	50% in MA
0	1.007455	1.007340	1.007340	1.007340	1.007340	1.007340	1 1.006462
10	3.511640	3.562976	4.018084	5.554645	7.467610	9.482187	11 2.457379
20	10.38893	10.52032	11.70050	14.84056	17.90477	20.57422	23 6.420686
30	20.82417	20.99771	22.72509	26.37393	29.11449	31.00223	32 12.41996
40	34.15302	34.30474	36.27541	39.40608	40.81541	41.10160	41 20.06656
50	49.83022	49.89011	51.79295	53.61384	53.04270	51.23313	49 29.04699
60	67.40865	67.30980	68.86835	68.80476	65.83138	61.60757	58 39.11226
70	86.51732	86.20194	87.18627	84.83278	79.18291	72.33065	66 50.05092
80	106.8498	106.2689	106.4953	101.5742	93.07246	83.44529	75 61.70173
90	128.1494	127.2660	126.5923	118.9220	107.4601	94.95982	84 73.91337
100	150.2042	148.9899	147.3091	136.7786	122.2983	106.8620	93 86.57859