ACOUSTICS OF THUNDER

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of

Doctor of Philosophy

in the Department of Physics
University of Saskatchewan

by

BHARTENDU

Saskatoon, Saskatchewan
October, 1964

The University of Saskatchewan claims copyright in conjunction with the author. Use shall not be made of the material contained herein without proper acknowledgment.
ACKNOWLEDGEMENTS

It has been a pleasure to work with Dr. B. W. Currie who supervised this study. His informal, scholarly, and stimulating advice has been very useful. His help in the beginning of the author's stay in Canada is acknowledged with deep gratitude.

I would like to thank Dr. H. M. Skarsgard for his help in discussions on positive ion oscillations. I also have Dr. J. Maybank to thank for some useful discussions.

Help rendered by Mr. Wayne Morley and Mr. Don Prickett was much appreciated.

Research scholarships from World University Service of Canada and the Institute of Upper Atmospheric Physics are gratefully acknowledged.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter/Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title Page</td>
<td>i</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>ii</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>iii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xiii</td>
</tr>
<tr>
<td><strong>CHAPTER 1</strong>                      INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Purpose and Motivation</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Synopsis of the Chapters</td>
<td>3</td>
</tr>
<tr>
<td><strong>CHAPTER 2</strong>                      ANATOMY OF THUNDER</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Some Early References</td>
<td>5</td>
</tr>
<tr>
<td>2.3 Recent References</td>
<td>11</td>
</tr>
<tr>
<td>2.4 Terminology</td>
<td>14</td>
</tr>
<tr>
<td>2.4.1 Visual characteristics of Lightning Flashes</td>
<td>14</td>
</tr>
<tr>
<td>2.4.2 Features of Thunder</td>
<td>17</td>
</tr>
<tr>
<td>2.5 Some General Considerations</td>
<td>18</td>
</tr>
<tr>
<td><strong>CHAPTER 3</strong>                      SOME THEORETICAL CONSIDERATIONS</td>
<td>24</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>24</td>
</tr>
<tr>
<td>3.2 Inaccurate Hypotheses</td>
<td>24</td>
</tr>
<tr>
<td>3.3 Shock Waves</td>
<td>25</td>
</tr>
<tr>
<td>3.4 Positive Ion Oscillations</td>
<td>39</td>
</tr>
</tbody>
</table>
CHAPTER 4 INSTRUMENTATION AND RECORDS 47
  4.1 General Scheme 47
  4.2 Microphones 50
    4.2.1 Hot Wire Microphone 50
    4.2.2 Wide Range Crystal Microphone 54
    4.2.3 Woofer 54
  4.3 Recorders and Filters 54
    4.3.1 Recorders 54
    4.3.2 Filters 57
  4.4 Records 60

CHAPTER 5 SPECTRAL DENSITY OF THUNDER 76
  5.1 Introduction 76
  5.2 Theory 77
  5.3 Autocovariance and Spectral density analysis 79
    5.3.1 Programme 79
    5.3.2 Accuracy 80
  5.4 Spectral Density Estimates 82
    5.4.1 Infrasonic spectral density estimates for 1962 records 82
    5.4.2 Infrasonic spectral density estimates for 1963 records 89
    5.4.3 Sonic spectral density estimates for 1963 records 105
    5.4.4 Reliability of the spectral density estimates 114
<table>
<thead>
<tr>
<th>CHAPTER 6</th>
<th>DIRECTION OF ARRIVAL AND NATURE OF PRESSURE VARIATIONS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Direction of arrival</td>
<td>123</td>
</tr>
<tr>
<td>6.1.1</td>
<td>Computational methods</td>
<td>123</td>
</tr>
<tr>
<td>6.1.2</td>
<td>General arrangement</td>
<td>125</td>
</tr>
<tr>
<td>6.1.3</td>
<td>Results</td>
<td>126</td>
</tr>
<tr>
<td>6.2</td>
<td>Nature of the pressure variations</td>
<td>146</td>
</tr>
<tr>
<td>6.2.1</td>
<td>Records of thunder</td>
<td>147</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER 7</th>
<th>FURTHER INVESTIGATIONS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Remillard's Invariant Quantity</td>
<td>155</td>
</tr>
<tr>
<td>7.1.1</td>
<td>Theory</td>
<td>155</td>
</tr>
<tr>
<td>7.1.2</td>
<td>Experimental Results</td>
<td>157</td>
</tr>
<tr>
<td>7.2</td>
<td>Audibility of thunder</td>
<td>162</td>
</tr>
<tr>
<td>7.2.1</td>
<td>Effect of temperature and wind</td>
<td>163</td>
</tr>
<tr>
<td>7.2.2</td>
<td>Results</td>
<td>170</td>
</tr>
<tr>
<td>7.3</td>
<td>Duration of thunder</td>
<td>172</td>
</tr>
<tr>
<td>7.4</td>
<td>Colours of Lightning</td>
<td>175</td>
</tr>
<tr>
<td>7.5</td>
<td>Cloud and Ground flashes</td>
<td>178</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER 8</th>
<th>SUMMARY</th>
<th>Page</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>APPENDIX A</th>
<th>DERIVATION OF THE EQUATION (3. 21)</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>Boltzmann Equation</td>
<td>186</td>
</tr>
<tr>
<td>A.2</td>
<td>Equation of Continuity</td>
<td>188</td>
</tr>
<tr>
<td>A.3</td>
<td>Equation of Momentum Transfer</td>
<td>189</td>
</tr>
<tr>
<td>A.4</td>
<td>Linearized Equation of Motion</td>
<td>192</td>
</tr>
<tr>
<td>A. 5</td>
<td>Equations of Continuity for Ions and Electrons</td>
<td>194</td>
</tr>
<tr>
<td>A. 6</td>
<td>Positive Ion Oscillations</td>
<td>196</td>
</tr>
<tr>
<td>A. 7</td>
<td>Significance of the Electric Field</td>
<td>199</td>
</tr>
</tbody>
</table>

**APPENDIX B** PROGRAMME OF THE SPECTRAL ANALYSIS

| B. 1 | Copy of the Spectral analysis programme for IBM-1620 computer in fortran language | 201 |

**APPENDIX C** GAUSSIAN FIT AND TEST

| C. 1 | Gaussian fit to the histogram | 204 |
| C. 2 | Normality test | 205 |
| C. 3 | Copy of the Gaussian fit and test programme for IBM-1620 computer in fortran language | 207 |

**APPENDIX D** DERIVATION OF EQUATION 7.1

**APPENDIX E** METEOROLOGICAL PARAMETERS

| E. 1 | Explanation of the symbols | 216 |

BIBLIOGRAPHY

221
## List of Figures

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Photographic reproductions of peal, clap, roll and rumble of thunder</td>
<td>19</td>
</tr>
<tr>
<td>3.1</td>
<td>A cylindrical mass of fluid, of unit cross section, passing from right to left through the shock transition zone</td>
<td>27</td>
</tr>
<tr>
<td>3.2</td>
<td>The Rankine-Hugoniot and isentropic relations</td>
<td>32</td>
</tr>
<tr>
<td>3.3</td>
<td>Entropy change across the shock wave</td>
<td>34</td>
</tr>
<tr>
<td>4.1</td>
<td>Block diagram of arrangement of instruments for recording pressure variations in 1962</td>
<td>48</td>
</tr>
<tr>
<td>4.2</td>
<td>Block diagram of arrangement of instruments for recording pressure variations in 1963</td>
<td>48</td>
</tr>
<tr>
<td>4.3</td>
<td>Circuit diagram of the hot-wire microphone for recording pressure variations</td>
<td>52</td>
</tr>
<tr>
<td>4.4</td>
<td>Plot of the resistance of the hot-wire microphone versus heating current</td>
<td>53</td>
</tr>
<tr>
<td>4.5</td>
<td>Frequency response of the wide-range crystal microphone</td>
<td>55</td>
</tr>
<tr>
<td>4.6</td>
<td>Frequency response of the woofer</td>
<td>56</td>
</tr>
<tr>
<td>4.7</td>
<td>Circuit diagrams of the 15 cps low-pass and high-pass filters</td>
<td>58</td>
</tr>
<tr>
<td>4.8</td>
<td>Frequency response of the 15 cps low-pass and high-pass filters</td>
<td>59</td>
</tr>
<tr>
<td>4.9</td>
<td>Photographic reproduction of thunder from a ground flash obtained by the visicorder</td>
<td>61</td>
</tr>
<tr>
<td>4.10</td>
<td>Photographic reproduction of thunder from a cloud flash obtained by the visicorder</td>
<td>62</td>
</tr>
<tr>
<td>4.11</td>
<td>Photographic reproduction of thunder from a cloud flash with many branches obtained by the visicorder</td>
<td>63</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.12</td>
<td>Photographic reproduction of musical thunder from a cloud flash obtained by the visicorder</td>
<td>64</td>
</tr>
<tr>
<td>4.13</td>
<td>Photographic reproduction of few loud claps of thunder obtained by the visicorder</td>
<td>65</td>
</tr>
<tr>
<td>4.14</td>
<td>Photographic reproduction of rumbling thunder obtained by the visicorder</td>
<td>66</td>
</tr>
<tr>
<td>4.15</td>
<td>Photographic reproduction of thunder from a complicated flash obtained by the visicorder</td>
<td>67</td>
</tr>
<tr>
<td>4.16</td>
<td>Photographic reproduction of roll and clap of thunder obtained by the visicorder</td>
<td>68</td>
</tr>
<tr>
<td>4.17</td>
<td>Photographic reproduction of thunder from distant flash obtained by the visicorder</td>
<td>69</td>
</tr>
<tr>
<td>4.18</td>
<td>Two typical records of thunder obtained by the Sanborn recorder</td>
<td>70</td>
</tr>
<tr>
<td>4.19</td>
<td>Two typical records of thunder obtained by the Sanborn recorder</td>
<td>71</td>
</tr>
<tr>
<td>4.20</td>
<td>Three typical records of thunder obtained by the Sanborn recorder</td>
<td>72</td>
</tr>
<tr>
<td>5.1</td>
<td>Spectral densities of two thunder records Record numbers July 1, 1962 (2.1) and July 1, 1962 (2.2)</td>
<td>83</td>
</tr>
<tr>
<td>5.2</td>
<td>Spectral densities of two thunder records Record numbers July 1, 1962 (3.1) and July 1, 1962 (3.2)</td>
<td>84</td>
</tr>
<tr>
<td>5.3</td>
<td>Spectral densities of two thunder records Record numbers July 8, 1962 (2.4) and July 8, 1962 (2.9)</td>
<td>85</td>
</tr>
<tr>
<td>5.4</td>
<td>Spectral density of the thunder record number July 21, 1962 (2.6)</td>
<td>86</td>
</tr>
<tr>
<td>5.5</td>
<td>Spectral densities of two thunder records Record numbers August 1, 1962 (2.1) and August 1, 1962 (2.2)</td>
<td>87</td>
</tr>
</tbody>
</table>
FIGURE | Page
---|---
5.6 | Spectral densities of two thunder records
Record numbers July 21, 1962 (1.18) and August 19, 1962 (1.1) 88
5.7 | Spectral density of rumbling thunder record number June 29, 1963 (4.1) 92
5.8 | Spectral density of thunder record number July 5, 1963 (1.1) 93
5.9 | Spectral density of thunder record number July 5, 1963 (1.2) 94
5.10 | Spectral density of rumbling thunder record number July 9, 1963 (1.1) 95
5.11 | Spectral density of thunder record number July 9, 1963 (4.1) from a cloud flash 96
5.12 | Spectral density of thunder record number July 9, 1963 (4.3) from a cloud flash 97
5.13 | Spectral density of few loud claps of thunder record number July 9, 1963 (4.4) 98
5.14 | Spectral density of thunder record number July 16, 1963 (1.1) from a cloud flash 99
5.15 | Spectral density of thunder record number July 16, 1963 (2.1) 100
5.16 | Spectral density of musical thunder record number July 24, 1963 (3.1) from a cloud flash 101
5.17 | Spectral density of thunder record number August 8, 1963 (3.1) from a cloud flash 102
5.18 | Spectral density of thunder record number August 8, 1963 (3.2) from a cloud flash 103
5.19 | Spectral density of thunder record number August 8, 1963 (3.3) from a ground flash 104
5.20 | Sonic spectral densities of three thunder records from ground flashes. Record numbers August 3, 1963 (1.1), August 3, 1963 (1.2) and August 8, 1963 (3.3) 110
<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.21a</td>
<td>Sonic spectral densities of two thunder records from cloud flashes. Record numbers August 8, 1963 (1, 1) and August 8, 1963 (3.2)</td>
<td>111</td>
</tr>
<tr>
<td>5.21b</td>
<td>Sonic spectral density of the thunder record number August 8, 1963 (3.4) from a cloud flash</td>
<td>112</td>
</tr>
<tr>
<td>5.22</td>
<td>Sonic spectral density of the thunder record number August 8, 1963 (2.1) from a complicated flash</td>
<td>113</td>
</tr>
<tr>
<td>5.23</td>
<td>Histogram and Gaussian curve of pressure amplitudes of the thunder record number July 5, 1963 (1.1)</td>
<td>119</td>
</tr>
<tr>
<td>5.24</td>
<td>Histogram and Gaussian curve of pressure amplitudes of the thunder record number July 9, 1963 (4.1)</td>
<td>120</td>
</tr>
<tr>
<td>5.25</td>
<td>Histogram and Gaussian curve of pressure amplitudes of the thunder record number August 8, 1963 (3.3)</td>
<td>121</td>
</tr>
<tr>
<td>5.26</td>
<td>Histogram and Gaussian curve of pressure amplitudes of the thunder record number August 8, 1963 (2.1)</td>
<td>122</td>
</tr>
<tr>
<td>6.1</td>
<td>Coordinates of the three microphones</td>
<td>124</td>
</tr>
<tr>
<td>6.2</td>
<td>Coordinates and the directions of the three hot-wire microphones used</td>
<td>124</td>
</tr>
<tr>
<td>6.3</td>
<td>Photographic reproduction of the thunder record number August 25, 1963 (1.3) from a cloud flash on three hot-wire microphones</td>
<td>128</td>
</tr>
<tr>
<td>6.4</td>
<td>Photographic reproduction of the thunder record number August 25, 1963 (1.4) from a cloud flash on three hot-wire microphones</td>
<td>129</td>
</tr>
<tr>
<td>6.5</td>
<td>Photographic reproduction of the thunder record number August 26, 1963 (2.1) from a complicated flash on three hot-wire microphones</td>
<td>130</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>6.6</td>
<td>Photographic reproduction of the thunder record number August 26, 1963 (2.3) from a cloud flash on three hot-wire microphones</td>
<td></td>
</tr>
<tr>
<td>6.7</td>
<td>Photographic reproduction of the thunder record number August 26, 1963 (2.5) from an overhead cloud flash on three hot-wire microphones</td>
<td></td>
</tr>
<tr>
<td>6.8</td>
<td>Photographic reproduction of the thunder record number August 26, 1963 (2.6) from an overhead cloud flash on three hot-wire microphones</td>
<td></td>
</tr>
<tr>
<td>6.9</td>
<td>Photographic reproduction of the thunder record number August 26, 1963 (2.7) from a ground flash on three hot-wire microphones</td>
<td></td>
</tr>
<tr>
<td>6.10</td>
<td>Photographic reproduction of the thunder record number August 26, 1963 (2.8) from two simultaneous ground flashes on three hot-wire microphones</td>
<td></td>
</tr>
<tr>
<td>6.11</td>
<td>Records of a shotgun on the hot-wire microphone and the woofer</td>
<td></td>
</tr>
<tr>
<td>6.12</td>
<td>Photographic reproduction of five compressional peals of thunder from a ground flash</td>
<td></td>
</tr>
<tr>
<td>6.13</td>
<td>Photographic reproduction of four records of thunder showing the nature of pressure variations</td>
<td></td>
</tr>
<tr>
<td>7.1</td>
<td>Plot of the most intense frequency maximum versus reciprocal time interval between the flash and the thunder</td>
<td></td>
</tr>
<tr>
<td>7.2</td>
<td>Plot of the frequency maximum versus reciprocal time interval between the flash and the thunder</td>
<td></td>
</tr>
<tr>
<td>7.3</td>
<td>Trajectory of the sound originating at a point on the lightning channel</td>
<td></td>
</tr>
<tr>
<td>7.4</td>
<td>Separate histograms of the duration of thunder recorded during 1962 and 1963</td>
<td></td>
</tr>
<tr>
<td>FIGURE</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>7.5</td>
<td>Combined histogram of the duration of thunder recorded during 1962 and 1963</td>
<td>174</td>
</tr>
<tr>
<td>D.1</td>
<td>Diagram of a sphere and a cylinder</td>
<td>212</td>
</tr>
<tr>
<td>D.2</td>
<td>Diagram of the cylindrical source intersecting and expanding sphere</td>
<td>212</td>
</tr>
</tbody>
</table>
## List of Tables

<table>
<thead>
<tr>
<th>TABLE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. 1</td>
<td>Over-Pressure on the shock wave front</td>
<td>38</td>
</tr>
<tr>
<td>4. 1</td>
<td>Classification of the records of thunder</td>
<td>74</td>
</tr>
<tr>
<td>5. 1</td>
<td>Infrasonic power spectra in 1962</td>
<td>90</td>
</tr>
<tr>
<td>5. 2</td>
<td>Infrasonic power spectra in 1963</td>
<td>106</td>
</tr>
<tr>
<td>5. 3</td>
<td>Audiofrequency power spectra in 1963</td>
<td>115</td>
</tr>
<tr>
<td>6. 1</td>
<td>Parameters for direction of arrival</td>
<td>127</td>
</tr>
<tr>
<td>6. 2</td>
<td>Direction of arrival</td>
<td>138</td>
</tr>
<tr>
<td>6. 3</td>
<td>Nature of pressure variation</td>
<td>150</td>
</tr>
<tr>
<td>7. 1</td>
<td>Invariant quantity</td>
<td>159</td>
</tr>
<tr>
<td>7. 2</td>
<td>Vertical temperature gradient and wind shear</td>
<td>170</td>
</tr>
<tr>
<td>E. 1</td>
<td>Meteorological parameters</td>
<td>218</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Purpose and Motivation

Lightning and its audible manifestation, thunder, are among the impressive phenomena of nature. While lightning has been and continues to be the subject of extensive experimental and some theoretical studies, quantitative investigations of thunder have been rare. Only two experimental studies of the spectral characteristics of thunder have been previously made; neither used modern equipment. A substantial literature on the qualitative characteristics of thunder does exist, and has been used by one investigator in an attempt to explain its audible characteristics.

Thunder differs notably from other loud sounds in the atmosphere in several aspects. Sounds from explosions start approximately from a point source and involve both heating of the air and the expansion of the gaseous products of the explosions. Sounds from gun-fire are related to the movement of the projectile through the atmosphere and to the expansion of the hot gases from the barrel of the gun. In contrast, thunder is induced by the almost instantaneous heating of the atmosphere along approximately a line source. Products of combustion and the movement of a solid object relative to the atmosphere are not involved. The sounds from thunder are likely to be complex, because the line source usually has variations of direction and has branchings. They also occur in
cloud, falling rain, and under circumstances where large gradients of temperature and wind velocity are possible.

The first actual recording of thunder was made by Wilhelm Schmidt (1914) in Germany with simple but ingenious devices as described later. He showed that most of the energy in the spectrum of thunder was contained in the infrasonic frequencies with a maximum transmission at 1.85 c.p.s. His observations indicated that the strongest pressure variations were rarefactive in nature. Audio frequencies ranged mostly from 15 to 45 and from 75 to 125 c.p.s. The second experimental study was made by Arabadji (1952) in Russia. He used essentially the same type of equipment to record the sub-audio pressure variations associated with thunder. In his case the maximum transmission of energy occurred apparently at 0.5 c.p.s., and the strongest pressure variations were compressive.

The great variations in intensity of sound that are described as peals, claps, and sometimes rolls of thunder, are due to the complex geometry of the flash and because some portions or points on the lightning channel may generate more energy than other portions (Mache, 1960). A laboratory spark gives rise to an explosion at every branch (Trowbridge, 1903b). The long continued rumbling of thunder can be due to reflection from the surrounding hills (Humphreys, 1940), or due to reflection in clouds because of evaporation and condensation of the water drops due to sound pressure and the effect of
water drops on the viscosity of the air (Zanotelli, 1951), or due to a graupel layer in the clouds (Remillard, 1960).

These contradictory results suggested that a further experimental study of the pressure variations associated with thunder was needed to explain its audibility. It also seemed worthwhile to seek further information on peals, claps, rumbling, and other commonly used expressions for describing thunder.

The region surrounding Saskatoon appeared particularly suitable for such studies. Though the number of thunderstorms experienced is not large, the country is reasonably flat. Thus, the reflection from hills, etc., should not affect the observations. The scattered bluffs of poplar might affect the amplitudes of the observed pressure changes, if they were close to the observing station and in the path of the approaching sound wave. However, any effect on the spectral characteristics and the direction of arrival of sounds of thunder would be negligible.

1.2 Synopsis of the Chapters

Chapter 2 is a historical review of the literature of thunder, referring in particular to its origin and cause. The terminology commonly used in describing thunder and lightning is summarized. A photograph of records to illustrate the terminology is included.

Chapter 3 discusses some important hypotheses related to thunder. Basic ideas of shock waves and the positive ion oscillations are considered in this chapter.
Chapter 4 includes a description of the instrumentation used in this investigation, and presents photographic reproductions of typical records of infrasonic and sonic pressure variations of air associated with thunder.

Chapters 5 and 6 present the main results. The former describes the subaudio and the audio spectral density estimates, and the latter describes the results on the direction of arrival of sounds of thunder from particular lightning flashes. The nature of large pressure variations is also examined in this chapter.

Chapter 7 presents some interesting observations obtained during the main investigation. Some of the questions discussed are:

Is the spectrum of thunder invariant as proposed by Remilard (1960)?
What is the maximum distance at which thunder is normally heard?
What is the most common duration of thunder? Are there different colours of lightning? Is there any relation between the colour of the lightning and subsequent thunder? How many lightning flashes go to earth? Does thunder from a ground flash differ from that for a cloud flash? What is the characteristic of thunder from a nearby flash and how does it differ from that due to a far flash?

The final Chapter 8 includes a summary of the results and conclusions. Some suggestions for further work are also mentioned.
2.1 Introduction

The literature on thunder is extensive, going back into the written records of early civilizations. Much of it is speculative. Not until the beginning of this century was there any real attempt to link the audible characteristics of thunder to the inducing agency (lightning) and to the effects of the atmosphere between the lightning flashes and the listener. Early accounts refer to the thunderbolt (the combined phenomena of lightning and thunder), attributing it to a mythical or religious being as a method of showing his displeasure. Later references are quasi-scientific, giving explanations without any attempt to test them by observations. Only recently have systematic observations been made on lightning and thunder with a view to finding an acceptable explanation of the observed features. A terminology has developed over the years for describing thunder and lightning. As an aid in understanding the references referred to in the following sections and chapters, definitions and explanations of terms are included.

2.2 Some Early References

Since the dawn of civilization, man has been pondering the cause of thunder. To the ancient Egyptians, the god Typhon (Seth) hurled the thunderbolt; to the early Greeks and Romans, a lightning flash was one of the chief indications of the displeasure of the father of gods (Zeus for the Greeks and Jove or Jupiter for Romans). The
people of India in vedic times were attracted to it with a feeling of awe. Rig-Veda, the oldest book of India, includes many poetic verses concerning lightning. A quotation follows: "The lightning lows like a cow, it follows as a mother follows after her young, that showers (of the Maruts) may be let loose" (Muller, 1869). In Indian mythology, Indra, who was king of Devas (angels or gods), although often identified with the Supreme Being, used the thunderbolt (vajra) to reveal his power and to annihilate the immorals. All the great religions of the World view the thunderbolt as radiating theism. The Koran, sacred scripture of Islam, is most specific when it says, "The thunder hymneth His praise and (so do) the angels for awe of Him. He launcheth the thunderbolts and smiteth with them when He will while they dispute (in doubt) concerning Allah, and He is mighty in wrath". (The translation is taken from The World's Great Religions, volume 2, page 138, Special Family Edition,)

The first known explanation of thunder that had scientific implications was by Aristotle (1961, p. 222). In about 350 B.C., he wrote, "Thunder is due to the forcible ejection of dry exhalation trapped in the clouds in the process of condensation. The ejected exhalation usually catches fire, and this produces lightning (which thus occurs, in spite of appearances, after thunder)". He continued (Aristotle, 1961, p. 225), "Different kinds of sound are produced because of the lack of uniformity in the clouds, and because hollows occur where their density is not continuous". About two hundred years later Lucretius (1929) gave many absurd causes of thunder. He said
that the thunder might be caused in the following ways: (1) when clouds clashed together face to face; (2) when clouds scraped along one another's sides, and made a noise like wind in a flapping awning or paper; (3) when wind was caught in a cloud and suddenly burst it; (4) when wind blew through the clouds, like a forest; (5) when the wind burst a cloud open; (6) when the rain waves in the clouds broke; (7) when lightning, falling from one cloud into another, hissed; (8) when the lightning burned the cloud up; and (9) when the ice and hail in the clouds crashed.

The philosophical giants had solved the mysteries of thunder and none dared to reopen the subject till 1637, when Descartes suggested that thunder was due to the higher clouds descending into the lower, and that the loudness of the sound came from the resonance of the air. He should not be judged too severely because even an eminent scientist like Benjamin Franklin, who performed his famous kite experiment in June 1752 and identified lightning as a form of electric spark, believed that thunder was caused by the colliding together of clouds (Remillard, 1960, p. 7).

Little more than a quarter of century later R. Hooke, the renowned physicist, wrote his accounts of thunder (Gunther, 1930). He was the first to note that the rumbling noise after the first clap seemed to be several echoes from distant places. He also observed, "If it rained when it thundered, it immediately after the clap poured down much faster, much as if a gale of wind had suddenly shook a tree, all whose leaves are full with drops of water."
Francois Arago (1854) condensed more factual information on thunder into one volume than has yet been done. He did not propose any new explanation, and was sceptical about the vacuum theory, common in his time. This was to the effect that lightning produced a vacuum where it passed, and the noise was a consequence of the re-entry of the air. He asked, "But by what physical cause does the lightning produce vacuum?"; and concluded, "The explanation of thunder is, therefore, yet to be found (Remillard, 1960, p. 10)".

In the late nineteenth and early twentieth centuries (1870-1920) various explanations of thunder appeared in the literature. The first challenge to the vacuum theory came from R. S. Mershon (1870), who proposed an electrolysis theory. He wrote, "The electricity in passing from one cloud to another, or to the earth decomposes the water in the cloud into its component gases; and the great heat of the electricity ignites and explodes these gases, and reforms them into water". He continued, "The violence of the explosion depends on the volume of the electricity set free, and the amount of water decomposed; or, if a volume of electricity passes through a perfectly dry medium there can be no decomposition of water, and consequently no detonation". This idea cannot be discarded entirely for J. Trowbridge, eminent American physicist, carried out laboratory experiments on electric discharges, and found that the presence of moisture enhanced the noise. He was led to conclude, "The noise of lightning discharges is doubtless enhanced in the same manner by the presence of great moisture..."
in the clouds (Trowbridge, 1903a). This theory in essence was further advanced by E. L. Bates (1903). He, while criticising the vacuum theory, even crossed the barriers of common decency and sarcastically remarked, "If this air rushing to fill a vacuum theory is still being advanced by the salaried professors of science . . . ".

The editors were compelled to add a note of their own. In this note they defended the vacuum theory but did not discard completely the electrolysis effect. The remarks of Bates prompted Prof. J. A. Lyon, one of the salaried professors, to answer the charges. He (Lyon, 1903) made the following points: (1) "It is safe to say that if enough oxygen and hydrogen to make the water of a cloudburst, such as he (Bates) imagines, were to be suddenly exploded, say, above New York city, the concussion and flame produced would lay the greater part of the city in ruins"; (2) "The appearance due to an explosion of gases in the atmosphere would be entirely different from a flash of ordinary lightning"; and (3) "The lightning discharge is evidently close akin to the spark discharge of a leyden jar or battery. This makes a loud and sharp sound under conditions where oxygen and hydrogen cannot possibly be produced". He asserted, "The probable cause of thunder is the violent concussion of the atmosphere along the track of the lightning flash, due to the intense heat (and consequent sudden expansion of the air and vapor) which we know to be produced by the discharge". Still, Lyon did not discard the vacuum theory entirely; he indicated, as an illustration, the noise produced by breaking a piece of bladder tied
over the hose of a vacuum pump. His explanation of the observational fact that heavy rain pours after a clap of thunder went on as follows: "The small water droplets are heavily charged with the same kind of electricity (say positive) and hence mutually repel each other until the discharge of lightning takes place. When this mutual repulsion ceases and the air is agitated by the thunder, the small droplets coalesce into larger ones and come down as rain."

Another theory which could be named the *steam theory* was put forward by V. R. Reynolds (1903). Reynolds suggested that the discharge caused suspended moisture in its path to change into steam at enormous pressures, and its effect was a violent detonation or blow upon the surrounding air.

M. Hirn (1888) may be commended for presenting a theory which can hardly be improved upon today. He stated, "The sound which is known as thunder is due simply to the fact that the air traversed by an electric spark, that is, a flash of lightning, is suddenly raised to a very high temperature, and has its volume, moreover, considerably increased. The column of this gas thus suddenly heated and expanded is sometimes several miles long, and as the duration of the flash is not even a millionth of a second, it follows that the noise bursts forth at once from the whole column, though for an observer in any one place it commences where the lightning is at the least distance". The theory had no experimental backing at the time, and was lost among others.
2.3 Recent References

Until approximately the beginning of the present century, explanations of thunder and its acoustic characteristics were based on little or no systematic observational data. Regular observations on the audible range of thunder were started by Professor L. C. Veenema in 1895 and continued until 1916. He (Veenema, 1917, 1918) was primarily interested in determining the range of audibility of thunder. His only piece of equipment was an ordinary watch. He had to light a match to read the watch every time he heard thunder. If a reader remains unimpressed by Veenema's enthusiasm and the difficulties under which he observed, he should read his recommendation of a stopwatch. He (Veenema, 1920) recorded 9 instances when he heard thunder at distances of 30 to 40 km; 12 instances, 40 to 50 km; 2 instances, 50 to 60 km; 2 instances, 60 to 70 km; 2 instances 70 to 80 km; 1 instance, 80 to 90 km; and 2 instances over 100 km. Many of these observations were for sound paths over ocean. He took note of factors affecting the audibility, and came to the following six conclusions: (1) Only from a very intense cloud to ground lightning flash was thunder heard over long distances; (2) The hearing ability of the observer and quietness of his environment played an important role; (3) The evening and night hours appeared to be more favourable for the propagation of sound than the day hours; (4) the wind direction, at least up to the cloud level, seemed to have no influence; (5) in late summer and autumn, the audibility conditions
were much more favourable than in spring and early summer; and (6) the audibility of thunder was diminished by irregularities and turbulence in the atmosphere. The last may explain the fact that the audibility of thunder is smaller over land than ocean.

Wilhelm Schmidt (1914) obtained the first actual recordings of the pressure variations associated with thunder. His ingenuity may be appreciated when one learns of the two devices which he constructed, one to record infrasonic frequencies (in his case frequencies less than 5 c.p.s.) and the other to record audio frequencies in the range of 15 c.p.s. to 200 c.p.s. For infrasonic pressure variations, he used a wooden box of about 210 liters volume and with an opening of 250 cm² area. An aluminum plate was suspended in the opening on two long threads so that it could swing in and out with the pressure variations. A lever system transferred the motion of the plate to a pen which left a trace on a moving sheet of smoked paper. For the audio pressure variations, a gramophone horn was used to direct the waves into a vertical chimney. Inside the chimney was a sensitive smokey flame which responded to the pressure changes. A tape was moved at a high speed above the flame. Each flicker of the flame, caused by an air condensation or rarefaction, resulted in an increase or decrease of the soot deposited on the tape. From the speed of the tape and the distances between deposits, the time intervals between condensations were computed. Schmidt was able to arrive at the following important conclusions: (1) Most of the energy in the spectrum of thunder was
contained in infrasonic frequencies, having a maximum at 1.85 cycles/sec; (2) most likely frequencies of thunder which could be heard were those between 15 and 40 cycles/sec., although a smaller maximum occurred between 75 and 120 cycles/sec., and (3) the strongest portion of the pressure wave from thunder was a rarefaction.

Thirty-eight years later V. I. Arabadji (1952), a Russian physicist, repeated Schmidt's observations. He was interested only in the infrasonic frequencies and used similar but more sensitive equipment. He employed a large air cavity with a tube 3 cm in length and 1.2 cm in internal diameter connecting it to the outer atmosphere. A light aluminum plate, suspended in the tube, vibrated under the influence of the pressure vibrations induced by the thunder. The vibrations were transmitted apparently by mechanical means to a paper recorder. His conclusions were as follows: (1) the maximum energy of thunder fell at frequencies in the range from 0.25 to 2.00 cycles/sec., and between these much more at 0.50 cycles/sec., and (2) the strongest blow of thunder corresponded to a compression.

Remillard (1960) approached the problem by compiling all the available reports on thunder and using these to deduce models of the lightning flash and the subsequent physical processes that explain the average or most-frequently occurring characteristics. His work was essentially statistical and theoretical. The simplest model of a lightning flash was a straight line source of thermal energy of infinite extent. The mathematics was tractable, but the model was
not very realistic. A second model assumed that the lightning channel was a finite, linear, and cylindrical source of thunder. It was a realistic model. His main conclusions were as follows: (1) the strongest portion of the direct sound wave was rarefractive; (2) the frequency spectrum of thunder was invariant; that is, the product of the frequency and the observation distance was a constant, whose value is 9.8 km-cycles/second; and (3) the pressure, observed at distance b and time t, radiated from a cylindrical source of height 2h and radius a with n branches was given by
\[ p = p(a, b, h_1, t) + p(a, b, h_2, t) + \cdots + p(a, b, h_n, t), \]
where \( h_n \) was the height from the ground along the main channel to the nth branch. His other conclusions were: (1) impulses which originated at branch points were the cause of thunder claps; and (2) a graupel layer in a thunder cloud acted as the sound reflector which produced the long roll of thunder.

2.4 Terminology

Thunder originates at a lightning flash; though this statement has not gone undisputed (Talman, 1935; Remillard, 1960, p. 9). As lightning is indeed the source of thunder it is considered worthwhile to describe the lightning forms briefly before discussing the various terms employed to describe thunder.

2.4.1 Visual characteristics of lightning flashes.

Lightning is classified according to its general appearance (Humphreys, 1920, p. 368). The usual designations are:
**Streak Lightning.** - Lightning flash which appears to the unaided eye as one or more sinuous lines or streaks. Often there is one main trunk with a number of branches, all occurring at the same time and apparently instantaneously, while at other times there are two or more simultaneous though locally disconnected streaks; the discharge frequently flickers.

Some authors (Schonland, 1950, p. 39) use this term to refer only to cloud-to-ground flashes and employ the term Air Discharge to designate a flash which does not reach the ground. In this thesis the term 'Ground flash' is used to designate a cloud-to-ground flash and the term 'Cloud flash' is used to refer to a flash which does not reach ground.

**Sheet Lightning.** - Lightning flash within the clouds which appears like a great sheet of flame that usually wanders, flickers, and glows. Often a blurred streak is seen through the thinner portions of the intervening cloud.

**Ribbon Lightning.** - Streak lightning that occurs in a very strong wind which moves the conduction channel fast enough to make the individual strokes appear separated, much like the posts in a picket fence (Kennelly, 1899; Schonland, 1950, p. 42).

**Ball Lightning.** - It is said to appear like a moving ball whose diameter varies between 0.5 inch to 6 feet. Its life time ranges from a few seconds to many minutes and often ends with a loud explosion. Its colour is variously given as white, red, yellow, and blue (Schonland,
The explanation of this rare form is still in dispute. One hypothesis is that the ball lightning is a ball of plasma in resonance with an external radio frequency electric field. Kapitza (Ritchie, 1961) postulates that the electromagnetic disturbances caused by the thunderstorm feed the energy into the plasma ball.

Various other forms of lightning have been reported (Humphreys, 1920; Brooks, 1938), but these are so rare as not to warrant description here.

A lightning discharge is composed of many separate components and streamers. From the optical and electrical investigations the following subdivisions of a lightning discharge are deduced (Hillerbrand, 1961); (1) predischarge in the cloud and development of a stepped leader between cloud and earth (Leader streamer); (2) production of the brightly-luminous return streamer; (3) discharges within the cloud as well as between cloud and earth (dart leader); (4) production of a second return streamer; (5) possible repetition of the third and fourth processes; and (6) discharges after the last return streamer.

The term stroke should be distinguished from the terms streamer and component. Each separate stroke of a flash is of dual nature (Schonland, 1956). A downward-moving weakly luminous process, the leader streamer, is followed, upon arrival near the ground, by an intensely luminous and much more rapid main or return streamer which traverses the leader channel in the reverse direction. Thus, the term stroke implies the combined process of leader and return streamers. A stroke whose luminosity decays abruptly is called a discrete
stroke. If a stroke is followed by a luminosity which persists in a channel for a longer time than 40 milliseconds, it is called a long-continuing stroke, and if a stroke is followed by continuing luminosity which lasts less than 40 milliseconds, it is called a short-continuing stroke (Kitagawa, Brooks, and Workman, 1962). It should be noted that the usual stroke interval is about 40 milliseconds.

A lightning flash which involves one or more continuing strokes is called a hybrid flash. A flash which involves only discrete strokes is called a discrete flash. Cloud-to-cloud flashes show no return streamers (Schonland, 1950, p. 79; Wang, 1963b).

2.4.2. Features of Thunder

Official definitions of thunder usually include a presumption as to its cause. As ideas about the cause have changed, so have the definitions. The current definition (Glossary of Meteorology, Huschke, 1959) defines thunder as the sound emitted by rapidly expanding gases along the channel of a lightning discharge. The cause (or causes) of thunder is far from being completely understood, and a final definition of thunder is still to be stated.

There are no consistent definitions of various terms that are used to describe the audible components of thunder, the common ones are peal, clap, roll, and rumble. These terms have been used in this thesis in the following sense:

Peal: - A sudden loud sound which repeats itself, and in general, lasts for a long time. As recorded on a hot-wire microphone, a peal exhibits a rapid change from the ambient pressure, the pressure
gradually returning to normal. Small amplitude and short period oscillatory components are superimposed. It is frequently preceded and followed by small amplitude pressure variations. Fig. 2.1a is a typical record.

Clap: A sudden isolated pressure change of varying but shorter duration than a peal. Few or no small-amplitude, short-period pressure variations are superimposed. The pressure change associated with a clap in general is smaller than that associated with a peal. On the hot-wire microphone records, claps can be easily identified by their impulsive nature. Fig. 2.1b shows a typical record. The distinction between clap and peal is not great and the expressions have often been used synonymously in the literature (Remillard, 1960, p. 3).

Roll: It is used to describe irregular variations in the pressure of sound. The variations are not as large as those associated with peals. Also no small amplitude higher frequency pressure variations are superimposed. Fig. 2.1d shows roll as recorded by a hot-wire microphone.

Rumble: Weak sounds, or murmurs, which prolong over a considerable period of time. In the literature, roll and rumble have been used synonymously (Remillard, 1960).

2.5 Some General Considerations

One characteristic of thunder which distinguishes it from an explosive sound is the large variation in intensity and the long con-
Fig. 2.1 Photographic reproductions of peal, clap, roll and rumble of thunder.
(a) Five Peals of Thunder (Speed 5"/sec)

(b) Five Claps of Thunder (Speed 1"/sec)

(c) One Clap of Thunder (Speed 5"/sec)

(d) Roll of Thunder (Speed 5"/sec)

(e) Rumbling of Thunder (Speed 1"/sec)
tinued rumblings. This can possibly be explained in several ways. Humphreys (1940) summarized some of these as follows: (1) various portions of the lightning channel are not equally distant from the observer, and hence the sound will not all reach him simultaneously, but continuously over an appreciable interval of time; (2) the lightning path is crooked and because of this, sections of the path here and there are at equal distance from the observer, at the circumference of circles of which the observer is the center, while other portions are directed more or less radially from him. This could account for loud crashes (peals and claps) accompanying thunder; and (3) if several discharges occur rapidly in time, interference and reinforcement of the compressional waves should give rise to musical notes.

Some portions or points on the lightning channel may generate more energy than other portions (Mache, 1960). Trowbridge (1903b) showed by laboratory experiments that whenever a powerful spark forks an explosion takes place. At every branching point in the lightning discharge an explosive phenomenon may take place. This could account for large peals and claps of thunder. Reflection from the reflecting surfaces like surrounding hills, etc., could accentuate and prolong sounds. According to Humphreys (1940) this is not an important factor. The rumbling is more or less the same whether over a prairie, or an ocean, or among the mountains. Zanotelli (1951) suggested that instead of reflection from hills, reflection in clouds because of evaporation and condensation of the water drops due to the
sound pressure and the effect of water drops on the viscosity of the air is responsible for the long persistence of the rumbling. Remillard (1960) proposed that a graupel layer in the thundercloud acts as a reflecting surface and helps to prolong the sound.

The second distinctive characteristic of thunder is that it can only be heard over short distances in comparison to the usual explosive sound. This may be due to the fact that temperature inversions are unlikely to occur in the lower atmosphere during thunderstorm conditions and thus all sounds are bent upward. During calm weather conditions with radiative cooling the surface temperature inversions occur in the lower atmosphere and sounds are bent downward and reach the earth at distant places. The effect of temperature lapse rate and wind shear on the propagation of sound is discussed in Chapter 7.

The energy converted into atmospheric waves from the lightning flash can be calculated in the following manner which is essentially the one followed by Remillard (1960).

The total energy \( W_t \) dissipated in a flash is given by

\[
W_t = \frac{Q^2}{2C}
\]  

(2.1)

where \( Q \) is the average net transfer of charge to the ground for each lightning flash, and \( C \) is the capacity of the cloud-ground condenser with air as the dielectric.

It is convenient to assume that the boundary of the stroke channel acts like an impervious interface, whose radial variations
correspond to those of the luminous channel. The sound energy $W_s$ (Suits, 1936) is given by

$$W_s = \frac{1}{2} \sum_{n} \text{m}_n v^2 = \frac{1}{2} M v^2$$  \hspace{1cm} (2, 2)

where $m_n$ is the mass of the air molecule, $M$ is the total mass of air moved, and $v$ is the velocity of expansion (assumed constant).

Substituting in Eq. 2.2 the value $M = \pi r^2 h \rho$, where $r$ is the instantaneous radius of the cylindrical channel, $h$ is its height, and $\rho$ is the undisturbed density of air, we obtain

$$W_s = \frac{1}{2} \pi r^2 h \rho v^2$$  \hspace{1cm} (2, 3)

Flowers (1943) found experimentally that for laboratory sparks the channel area is directly proportional to the current flowing in the channel, that is $\pi r^2 = K i$, where $K$ is a constant of proportionality and $i$ is the current. Assuming that this relationship holds for a lightning flash, and substituting for $\pi r^2$ and $v = \frac{dr}{dt}$, the Eq. 2.3 reduces to the following form

$$W_s = \frac{\rho h k^2}{8 \pi} \left( \frac{di}{dt} \right)^2$$  \hspace{1cm} (2, 4)

For a typical flash $W_t$ is found to be $4.8 \times 10^{10}$ joules. If three strokes are assumed per flash the average energy dissipated in each stroke is $1.6 \times 10^{10}$ joules (Remillard, 1960). Using Eq. 2.4, he calculated the average sound energy in the production of thunder. During the time it takes the current to reach a maximum (which is 5 microseconds) from an initial value of zero, the sound energy is $4 \times 10^7$ joules, and the sound energy during the remainder of the period is $9 \times 10^5$ joules.
This shows that most of the sound energy (98%) is produced during the first five microseconds, the time taken for current to reach a maximum. Thus, according to Remillard (1960), only 0.25 percent of the total dissipated energy of a lightning flash goes into the production of thunder. He also asserted that he performed some experiments (not reported) and found this value to be in agreement with his experimental results. Suits (1936), on the other hand, estimated that in a laboratory spark as much as 97% of the energy radiated during the initial period when the channel is expanding is in the form of sound energy. The remaining 3% goes into the production of light.

The calculations of Remillard (1960) included the period when the channel was contracting at a much slower rate and his value gave the acoustic efficiency averaged over the entire life of the stroke.
CHAPTER 3

SOME THEORETICAL CONSIDERATIONS

3.1 Introduction

A number of explanations for the production of thunder have been put forward by various observers. Chapter 2 described such explanations. This chapter discusses several important hypotheses related to thunder. The current view is that due to the sudden heating in the lightning channel compressional shock waves are generated and these give rise to sound waves, some portion of which are audible. Though the role of electrons in producing the sound waves, due to their mutual repulsion, has been considered, the effect of positive ions has not been discussed in the literature. The role of positive ions is considered in this chapter as well as some basic ideas concerning shock waves.

3.2 Inaccurate hypotheses

One interesting and plausible, but erroneous, hypothesis about the cause of the production of thunder is that it is caused by the collapse of the partial vacuum produced by the heat generated by the lightning. The cooling in this case must be rapid, especially at the instant the discharge ceases, but not so rapid as to create sound and, consequently, ever to produce peals and claps of thunder which always follow intense lightning.
Another attractive hypothesis about the origin of thunder is that it is due to the mutual repulsion of electrons along the path of the lightning discharge. There are many objections to this hypothesis (Humphreys, 1940). If such a repulsion really occurs to the extent indicated, one may, therefore, expect a rod of mercury, carrying current, to spread out. Instead, it in fact draws together, and with a strong current even pinches itself in two. Also, if mutual repulsion actually drove the electrons violently apart, one would expect the discharge instantly to dissipate, producing some kind of brush effect, rather than concentrating along the streak. Thus, it appears improbable that electron repulsion is able to explain heavy peals of thunder, though it may contribute to some extent to the expansion process. Indeed, as eminent a scientist as Schmidt (1914, p. 495) thought that electronic repulsion was the main cause of thunder.

3.3 Shock Waves

The heating in the lightning channel is so excessive that the resulting sudden expansion simulates a violent explosion and, hence, creates a shock wave. The compressional shock waves produced from spark discharges have been photographed by Topler (1867), Dvorak (1880), and Wood (1899). Wood even photographed the reflections of these shock waves. In this connection, the author wishes to describe a noteworthy incident during these investigations. On the evening of July 1, 1962, Saskatoon experienced a very severe thunder-
storm. The author was busy taking observations inside a metallic building and the hot-wire microphone was located inside a car in the field. A lightning flash reached the earth only a few feet from the microphone. The dazzling light and cacophony of sound bewildered the author, and he could not help falling on the floor. The station transformer was burnt out and the power went off permanently for the whole night. The platinum grid of the hot-wire microphone suffered a mechanical fracture. This rupture suggests the presence of large pressure change, possibly so large as to be considered as associated with a shock front. Large induced currents associated with the flash could burn out the grid, but the metallic body of the car should have provided adequate shielding.

A shock wave, in its simplest form, may be considered as a moving, stable front in a fluid, across which the fluid properties, for example pressure, temperature, etc., change discontinuously.

The basic shock wave equations, known as the Rankine and Hugoniot equations, are derived here. The treatment adopted is essentially the one outlined by Penney and Pike (1950).

Consider a cylinder of fluid (shown in Fig. 3.1) of unit cross-section and containing the shock front zone. Assume that the flow where the two regions of the fluid are separated by the transition zone, is steady and one dimensional. Also assume that inside the zone conditions are steady, and that the frame of reference is chosen so as to make this zone stay at rest. The conditions throughout each side of the zone are considered to be the same everywhere. Use a
Fig. 3.1 A cylindrical mass of fluid, of unit cross section, passing from right to left through the shock transition zone.
suffix 1 to denote properties of a fluid element on the low pressure side of the zone, and a suffix 2 to denote properties on the high pressure side. Suppose $\rho$ is the fluid density, and $u$ the velocity.

Because the fluid is not accumulating anywhere, the rate of mass flow across any section must be constant, say $m$. Thus

$$m = \rho_2 u_2 = \rho_1 u_1 \quad (3.1)$$

Since the shock compresses the fluid, $u_2$ must be less than $u_1$, and the length of the cylinder is therefore decreasing. The rate of change of momentum of the fluid in the cylinder is simply $m (u_2 - u_1)$. This must be due to the difference in pressure, $p_1 - p_2$, at the two ends. Hence the equation of momentum is

$$p_1 - p_2 = m(u_2 - u_1) \quad (3.2)$$

The work done by the cylinder of fluid in unit time is $p_2 u_2 - p_1 u_1$. This work should be equal to the internal plus kinetic energy per unit mass. Then the equation of conservation of energy is

$$p_2 u_2 - p_1 u_1 = m(E_1 + 1/2 u_1^2 - E_2 - 1/2 u_2^2) \quad (3.3)$$

From these three equations the Rankine-Hugoniot equations can be derived easily.

From Eq. 3.1 we have

$$u_2 = \frac{m}{\rho_2} = mv_2$$

$$u_1 = \frac{m}{\rho_1} = mv_1$$

where $v_1$, $v_2$ are specific volumes.
Hence, Eq. 3.3 reduces to
\[ p_2 v_2 - p_1 v_1 = (E_1 - E_2) + 1/2(u_1 - u_2)(\bar{u}_1 + u_2) \]
Substituting for \( u_1 - u_2 \) from Eq. 3.2 and for \( u_1 \) and \( u_2 \) from Eq. 3.1, the following equation is obtained.
\[ p_2 v_2 - p_1 v_1 = (E_1 - E_2) + 1/2(p_2 - p_1)(v_1 + v_2) \]
or
\[ E_2 - E_1 = 1/2(p_1 + p_2)(v_1 - v_2) \] (3.4)

The velocity \( U \) with which a shock front advances into a stationary fluid is equal to \( u_1 \). It follows from Eq. 3.2 that
\[ u_1 = \left( u_2 - \frac{p_1 - p_2}{m} \right) \]
\[ = \left( \frac{v_2 u_1}{v_1} - \frac{p_1 v_1}{u_1} + \frac{p_2 v_1}{u_1} \right) \]
or
\[ U = u_1 = v_1 \left[ \frac{p_2 - p_1}{v_1 - v_2} \right]^{1/2} \] (3.5)

It can be shown (Pike, and Penney, 1950) that as \( p_2 \rightarrow p_1 \), \( U \rightarrow C_1 \), where \( C_1 \) is the velocity of sound in the fluid at pressure \( p_1 \) and density \( \rho_1 \).

The mass velocity \( w \) behind a shock front advancing into a stationary fluid is the difference between \( u_1 \) and \( u_2 \), \( u_1 - u_2 \), and the direction of \( w \) is the same as that of \( U \).

From Eq. 3.1
\[ u_2 = \frac{\rho_1}{\rho_2} u_1 = \frac{v_2}{v_1} u_1 \]
or

\[ 1 - \frac{u_2}{u_1} = 1 - \frac{v_2}{v_1} \]

or

\[ u_1 - u_2 = \frac{u_1}{v_1} (v_1 - v_2) \]

by substituting for \( u_1 \) from Eq. 3.5 and obtain

\[ u_1 - u_2 = \left[ (p_2 - p_1) (v_1 - v_2) \right]^{1/2} \]

Hence

\[ w = u_1 - u_2 = \left[ (p_2 - p_1) (v_1 - v_2) \right]^{1/2} \]

Equations 3.4, 3.5 and 3.6 are the Rankine-Hugoniot Equations. For a perfect gas (Penney and Pike, 1950)

\[ E = \frac{pv}{(\gamma - 1)} \]

(3.7)

where \( \gamma \) is the ratio of specific heat at constant pressure to the specific heat at constant volume. By substituting for the internal energies in Eq. 3.4 we obtain the following equations.

\[ \frac{p_2 v_2}{(\gamma - 1)} - \frac{p_1 v_1}{(\gamma - 1)} = \frac{1}{2} (p_1 + p_2) (v_1 - v_2) \]

and

\[ 2(p_2 v_2 - p_1 v_1) = (\gamma - 1) (p_1 v_1 + p_2 v_2 - p_1 v_2 - p_2 v_1) \]

and

\[ v_2 \left[ 2p_2 + (\gamma - 1) p_2 + (\gamma - 1) p_1 \right] = v_1 \left[ 2p_1 + (\gamma - 1) p_1 \right] \]

\[ p_1 + (\gamma - 1) p_2 \]
and

\[ v_2 \left( \rho_2 \left( \gamma +1 \right) + \rho_1 \left( \gamma -1 \right) \right) = v_1 \left( \rho_1 \left( \gamma +1 \right) + \rho_2 \left( \gamma -1 \right) \right) \]

and

\[ \frac{v_1}{v_2} = \frac{\left( \gamma +1 \right) \rho_2 \left( \gamma -1 \right) \rho_1 + \left( \gamma +1 \right)}{\left( \gamma -1 \right) \rho_2 \left( \gamma -1 \right) \rho_1 + \left( \gamma +1 \right)} \]  

(3.8)

By writing the Eq. 3.8 in terms of densities, the following equation is obtained

\[ \frac{\rho_2}{\rho_1} = \frac{\left( \gamma +1 \right) \rho_2 \left( \gamma -1 \right) \rho_1 + \left( \gamma +1 \right)}{\left( \gamma -1 \right) \rho_2 \left( \gamma -1 \right) \rho_1 + \left( \gamma +1 \right)} \]  

(3.9)

This relation can be rewritten as

\[ \frac{\rho_2}{\rho_1} = \frac{\left( \gamma +1 \right) \rho_2 \left( \gamma -1 \right) \rho_1 + \left( \gamma +1 \right)}{\left( \gamma -1 \right) \rho_2 \left( \gamma -1 \right) \rho_1 + \left( \gamma +1 \right)} \]  

(3.10)

Eq. 3.10 is plotted in Fig. 3.2 for a diatomic gas with

\[ \gamma = 1.4 \]

together with the corresponding isentropic condition

(Pain, and Rogers, 1962)

\[ \left( \frac{\rho_2}{\rho_1} \right)^\gamma \]  

(3.11)

It appears that the limiting density ratio for the normal shock wave is 6, and this corresponds to an infinite pressure ratio. The isentropic relation is seen to be a tangent to the Rankine-Hugoniot curve at the origin, showing that weak shock waves are approximately
Fig. 3.2 The Rankine-Hugoniot and isentropic relations (following Pain and Rogers, 1962).
isentropic (constant entropy). However, a more elaborate treatment shows that the weak shock wave is accompanied by some increase in entropy (Pain and Rogers, 1962).

The entropy change across the shock wave is (Shapiro, 1953)

\[
\frac{\phi_2 - \phi_1}{R} = \frac{\gamma}{\gamma - 1} \left[ \frac{2}{(\gamma + 1)M_1^2} + \frac{(\gamma - 1)}{(\gamma + 1)} \right] + \frac{1}{\gamma - 1} \left[ \frac{2\gamma}{\gamma + 1} M_1^2 - \frac{(\gamma - 1)}{(\gamma + 1)} \right]
\]

(3.12)

where \( \phi \) are entropies per unit mass, and \( M_1 \) is the Mach number given by \( M_1 = \frac{u_1}{C_1} \). Eq. 3.12 shows that for a perfect gas with \( 1 < \gamma < 1.67 \) the entropy change is always positive when \( M_1 \) is greater than unity, and is always negative when \( M_1 \) is less than unity. The general form of Eq. 3.12 is shown in Fig. 3.3. According to the second law of thermodynamics, the entropy may not decrease during an adiabatic change (Shapiro, 1953, p. 116). It follows that the rarefaction shock, in which the flow changes discontinuously from a subsonic to a supersonic state, is impossible.

Fundamental differential equations describing shock waves are non-linear and, hence, shock reflection may differ in some respects from the familiar linear phenomena associated with sound. A simple case is the reflection of a plane shock wave impinging normally on a wall. The gas pressure near the wall, initially \( p_1 \), is raised to value \( p_2 \) by the passage of the incident shock, and to a
Fig. 3.3 Entropy change across the shock wave (following Shapiro, 1953).
Fig. 4.1 Block diagram illustrating typical arrangement of instruments for the recording of pressure variations associated with thunder in the summer of 1962.

Fig. 4.2 Block diagram illustrating typical arrangement of instruments for the recording of infrasonic and sonic pressure variations associated with thunder in the summer of 1962.
value $p_3$ as the reflected shock passes in the opposite direction.

The ratio $\alpha$ of the excess pressure behind the reflected and incident shocks is given by (Pain and Rogers, 1962)

$$\alpha = \frac{p_3 - p_1}{p_2 - p_1} = \frac{1 + \frac{\lambda^2}{p_1}}{p_2} + \frac{\lambda^2}{p_2^2}$$

(3.13)

where $\lambda^2 = \frac{\gamma - 1}{\gamma + 1}$ and the inverse pressure ratio $\frac{p_2}{p_1}$ is given by

$$\frac{p_2}{p_1} = \frac{2}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}$$

in terms of the shock Mach number $M_1$. It is found that for weak shocks $\alpha = 2$, an expected result from linear motion showing a simple doubling of pressure by the reflection, and for very strong shock $\alpha \approx 2 + \frac{1}{\lambda^2}$, which for air has a value of eight. Thus, the reflection of strong shocks gives a significant increase of pressure.

An extensive discussion of the shock reflection and diffraction is given by Pain and Rogers (1962).

The blast wave associated with an explosion is essentially a shock wave of spherical form initiated by the impulsive release of a fixed quantity of energy. The problem is discussed by Taylor (1950). The spherical shock is shown to propagate outwards with radius.

$$\rho_s = S(\gamma) t^{2/5} \left( \frac{E_v}{\rho} \right)^{1/5}$$

(3.14)

where $S(\gamma)$ is a calculated function of $\gamma$ and is approximately equal to 1 for $\gamma = 1.4$, $E_v$ is the energy per unit volume, $\rho$ is the atmospheric density, and $t$ is the time.
Lin (1954) has extended the analysis to the cylindrical shock wave and finds that the radius $r_c$ of a strong cylindrical shock wave, produced by a sudden release of energy $E_e$ per unit length, grows with time according to

$$r_c = s_0 t^{1/2} \left( \frac{E_e}{\rho} \right)^{1/4}$$  \hspace{1cm} (3.15)

In this type of wave the pressure behind the shock decays inversely with the square of the radius, so that (Lin, 1954)

$$p = 0.216 \left( \frac{E_e}{r_c^2} \right)$$  \hspace{1cm} (3.16)

Remillard (1960), using the Rankine-Hugoniot equations, predicted that the maximum pressure at the lightning stroke is 300 atmospheres. Kitagawa (1963), using Equation 3.16 and the value of the maximum pressure calculated by Remillard, and the value of energy of the flash calculated in Chapter 2, calculated the shock wave range. It is of the order of a few meters. Since the energy varies widely from stroke to stroke, the wavefronts, caused by several stroke elements involved in one lightning flash, propagate with different velocities in the vicinity of the channel. This effect is one of the factors which makes the recorded thunder waveforms complex.

The shock wave is, perhaps, responsible for the characteristic sharp crack produced by a nearby flash (Loeb, 1954). Some heat will remain, even after the luminosity has extinguished in the channel. It will cause the channel to expand to larger dimensions.
Fig. 4.3 Circuit diagram of the hot-wire microphone for the recording of pressure variations associated with thunder.
This will ultimately produce rarefaction along the channel followed by collapse and damped oscillations in the air. The oscillating air column is crooked and a few kilometers long. The atmospheric oscillations may give rise to low frequency murmurs, or rumbling as they are usually called.

Zhivlyuk and Mandel'shtam (1961), identifying the thunder with the shock wave produced by the rapid expansion of the channel, calculated the pressure on the front of the shock wave and the temperature in the lightning channel.

The radius \( r \) and the velocity \( v \) of the shock wave is given by the following formulas (Zhivlyuk and Mandel'shtam, 1961)

\[
r = 1.11 \gamma^{1/4} \rho_0^{5/56} J^{5/14} t^{13/56}
\]  
\[
v = 5.55 \times 10^5 \gamma^{1/4} \rho_0^{5/56} J^{5/14} t^{-43/56}
\]

where \( r \) is in cm, \( v \) in cm/sec, \( J \), the current, in kiloamps, \( t \), the time in microsecond, and \( \rho_0 \), the density, is in units of \( 1.29 \times 10^{-3} \) gm/cm\(^3\). The coefficient \( \gamma \) is the ratio of the energy required for the translational motion of the gas to the total energy released in the channel; for a spherical shock wave \( \gamma = 0.1 \), and for a cylindrical wave from a laboratory spark discharge \( \gamma = 0.2 \). Eqs. 3.17 and 3.18 are not derived in the referred paper and have been quoted in this paper from other sources mentioned therein.
These assume that the current in the lightning channel varies as \( t^{3/4} \) and the conductivity of the channel is practically constant.

The pressure \( p_f \) on the front of the shock wave is connected with the velocity of the front by the relation

\[
p_f = \frac{2 \rho_0 v^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1} p_0
\]

where \( p_0 \) is the atmospheric pressure, and \( \gamma \) is the ratio of specific heats.

If \( J = 30 \) kiloamps for a typical flash, and it is assumed that this value is reached within 10, 100, and 1000 microseconds respectively, the values of \( p_f \) for various values of \( r \) are as tabulated in Table 3.1

**TABLE 3.1**

(Following Zhivlyuk and Mandel'shtam)

Over-Pressure on the front of shock wave

<table>
<thead>
<tr>
<th>Current growth rate Kiloamps/microsec.</th>
<th>Relative pressure on the front of the shock wave Kgm/cm(^2)</th>
<th>( r ) (cm) = 5</th>
<th>50</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>30/10</td>
<td></td>
<td>94</td>
<td>9.2</td>
<td>0.9</td>
</tr>
<tr>
<td>30/100</td>
<td></td>
<td>9.5</td>
<td>0.25</td>
<td>0.02</td>
</tr>
<tr>
<td>* 30/1000</td>
<td></td>
<td>2.8</td>
<td>Shock wave turns into sound wave</td>
<td>Shock wave turns into sound wave</td>
</tr>
</tbody>
</table>

* In the original paper it is written 30/100, likely a printing error. Note: 1 Kgm/cm\(^2\) = 980.6 millibar.
Table 3.1 shows that at an average current growth rate of 30 kiloamps/100 microsecond, considerable pressures are developed on the front of a shock wave, and, as a consequence, the thunder can damage objects located at a distance up to several meters. To get an idea of the magnitude of the pressures involved, it should be noted that an excess pressure of 0.07 kgm/cm² acting on the entire surface of a window pane will shatter it, 0.14 kgm/cm² will damage frame buildings and telegraph poles, and 0.365 kgm/cm² produces cracks in 9 inch brick walls.

The temperature \( T \) in electron volts can be estimated by the following expression, which is based on the hydrodynamical theory of development of the channel (Zhivlyuk and Mandel'shtam, 1961).

\[
T = 4.35 \rho_0^{1/4} (J t^{-3/4})^{2/7}
\]

(3.20)

where \( J \) is in kiloamps, \( t \) in microseconds, and \( \rho_0 \) is in units \( 1.29 \times 10^{-3} \text{ gm/cm}^3 \). If a typical value of \( J \) is 30 kiloamps, and if \( t \) takes values between 100 and 1000 microseconds, \( T \) ranges from 16,000 to 25,000 \( ^0 \text{K} \). This compares favourably with the values of temperature in the lightning channel as calculated by the spectroscopic method.

3.4 Positive Ion Oscillations*

The positive ions in the lightning channel should contribute to the production of thunder in some degree. The presence of the

* It has been assumed here that in the plasma of lightning channel only electrons and positive ions are present. In a more rigorous treatment negative ions should probably be considered.
large electric fields and currents in the channels suggests the possibility of positive ion electrostatic oscillations. This is considered in the following paragraphs.

A spark can be considered as an ionized column of air, which is capable of many types of oscillatory motions. If the electric field, \( \vec{E} \), and current density, \( \vec{j} \), are parallel to the direction of propagation, electrostatic restoring forces, resulting from \( \text{div} \ \vec{j} \) are present. Two types of oscillations occur: Those in which the electrons oscillate, the positive ions remaining stationary; and those in which only the positive ions oscillate. The ELECTRON ELECTROSTATIC OSCILLATIONS give rise to high frequencies and fall outside the scope of this investigation. Only POSITIVE ION ELECTROSTATIC OSCILLATIONS are likely to produce subsonic and sonic waves. These ionic sound waves in a plasma have been observed experimentally (Alexeff, and Neidigh, 1961, 1963). Ionic sound waves, like ordinary sound waves, are longitudinal compressional waves with the momentum due to the mass density and the restoring force due to thermal pressure. The positive ion oscillations are waves of relatively low frequency, in which electrical neutrality is preserved to a very high degree.

The velocity \( V \) of the acoustic wave (Spitzer, 1956) is given by

\[
V = \left( \frac{Z \gamma_e k T_e + \gamma_i k T_i}{m_i} \right)^{1/2}
\]

(3.21)
where \( Z \) is the atomic number, \( k \) is Boltzmann's constant, \( T \) is the absolute temperature, \( \gamma \) is the ratio of specific heats \( (\gamma = \frac{5}{3} + \frac{1}{p'}), \) \( p' \) is the number of degrees of freedom, and \( m \) is the mass. The subscripts \( i \) and \( e \) refer to 'ions' and 'electrons' respectively.

The derivation of the Eq. 3.21 requires a substantial amount of mathematics and is contained in the Appendix A.

\[
\text{If } T_e \gg T_i \Rightarrow \\

v^2 = \frac{Z}{3} \gamma \left( \frac{3k T_e}{m_i} \right) \quad (3.22)
\]

The expression within brackets on the righthand side of Eq. 3.22 is the square of a thermal velocity. Thus, the velocity of the ionic acoustic wave is proportional to the random velocity of the ions if they have the electron temperature.

This mechanism is based on the shielding of the positive ions by the electrons. When the wavelength of the oscillations is less than the Debye shielding distance \( h \), given by the formula

\[
h = \left( \frac{k T_e}{4 \pi \gamma_e e Z} \right)^{1/2} \quad (3.23)
\]

where \( e \) is the electron charge, and \( \gamma_e \) is the electron density, the shielding by electrons is no longer possible, and such waves cannot exist. If we assume \( T = 2.5 \times 10^4 \text{ K} \), \( \gamma_e = 10^{11} \text{ cm}^{-3} \) (Loeb,
1954, p. 355), and substitute $k = 1.38 \times 10^{-16}$ erg/degree, and $e = 4.8 \times 10^{-10}$ e.s.u., $h$ is found to be less than a millimeter. This shows that the shielding is present in the case of lightning and such oscillations are possible.

It may be noted here that, although the electric field $\mathbf{E}$ does not appear in the Eqs. 3.21 and 3.22, it plays an important role in these oscillations. The significance of the electric field is discussed in Appendix A.

If the lightning channel can be considered a plasma, the velocity of the ionic sound wave can be easily determined. The spectra of lightning show intense NII lines (Salanave, 1961), and the excitation temperatures have been calculated (Prueitt, 1963). For a typical case, take $T = 2.42 \times 10^4$ K (assuming that the electron temperature is equal to the exciting temperature), $m_1$ for NII is 7.67 $\times 10^{-24}$ gms, $k = 1.38 \times 10^{-16}$ c.g.s. units, $Z = 1$, and $\sqrt{\approx 1}$.

Substituting these values in Eq. 3.22 we have

$$v = \left( \frac{1 \times 1.38 \times 10^{-16} \times 2.42 \times 10^4}{7 \times 1.67 \times 10^{-24}} \right)^{1/2} \text{ cm/sec}$$

$$= 5.4 \times 10^5 \text{ cm/sec}$$

This value is about 16 times the velocity of sound. If we consider a vibrating string model, and take the length of the cloud to ground lightning channel as 6 km (Malan, and Schonland, 1951b) the fundamental frequency $f$ will be determined from the following formula

$$f = \frac{v}{2L}$$
This value is of the order at which a frequency maximum was observed by Arabadji (1952). The results reported in Chapter 5 also show some evidence of maxima at about this frequency. However, the spectral density estimates are not accurate below 0.75 c.p.s. and not too much confidence can be placed in this comparison.

Neutral particles will be present and should affect the oscillations. The effect of the neutral particles is considered below. The Equations of motion for ions and electrons are (Skarsgard, 1964)

\[
\begin{align*}
\frac{n_i m_i}{2} \frac{\partial \vec{V}_i}{\partial t} &= n_i Z_e \vec{E} - \nabla p_i - n_i m_i \nabla_e (\vec{V}_i - \vec{V}_e) \\
&- n_i m_i \nabla_{in} (\vec{V}_i - \vec{V}_n)
\end{align*}
\]  

(3.24)

and

\[
\begin{align*}
\frac{n_e m_e}{2} \frac{\partial \vec{V}_e}{\partial t} &= -n_e e \vec{E} - \nabla p_e - n_e m_e \nabla_e (\vec{V}_e - \vec{V}_i) \\
&- n_e m_e \nabla_{en} (\vec{V}_e - \vec{V}_n)
\end{align*}
\]  

(3.25)

where \(\nabla\)'s are collision frequencies, \(\vec{V}\)'s are average macroscopic velocities, and the subscript \(n\) designates neutral.

Adding Equations 3.24 and 3.25 and keeping in mind the assumptions made in the Appendix A, we obtain
\[
\rho_m \frac{\partial \mathbf{v}}{\partial t} = -\nabla p - n_i m_i \mathbf{v}_{\text{in}} (\mathbf{v}_i - \mathbf{v}_n) - n_e m_e \mathbf{v}_{\text{en}} (\mathbf{v}_e - \mathbf{v}_n)
\]

(3.26)

The last two terms on the right hand side of Equation 3.26 describe damping due to collisions with neutrals.

Assume that \( \mathbf{v}_n = 0 \) and provided \( \lambda_e << \lambda_i \), and \( \mathbf{v}_e = \mathbf{v}_i = \mathbf{v} \), then

\[
\rho_m \frac{\partial \mathbf{v}}{\partial t} = -\nabla p - C^1 \mathbf{v}
\]

(3.27)

where

\[
C^1 = n_i m_i \mathbf{v}_{\text{in}} + n_e m_e \mathbf{v}_{\text{en}}
\]

(3.28)

Differentiate \( \rho \frac{\partial \mathbf{v}}{\partial t} \) as Eq. A-12, and obtain

\[
\rho_m \frac{\partial^2 \mathbf{v}}{\partial t^2} = \frac{(1 + Z)}{m_i} \sqrt{k T} \rho_m \nabla (\nabla \cdot \mathbf{v}) - C^1 \frac{\partial \mathbf{v}}{\partial t} \quad (3.29)
\]

For a plane wave travelling in the x-direction

\[
\frac{\partial^2 \mathbf{v}_x}{\partial t^2} = \frac{(1 + Z)}{m_i} \sqrt{k T} \frac{\partial^2 \mathbf{v}_x}{\partial x^2} - \frac{C^1}{\rho m} \frac{\partial \mathbf{v}_x}{\partial t}
\]

(3.30a)

\[
\frac{\partial^2 \mathbf{v}_x}{\partial t^2} = \mathbf{v}_0^2 \frac{\partial^2 \mathbf{v}_x}{\partial x^2} - \mathbf{v} \frac{\partial \mathbf{v}_x}{\partial t}
\]

(3.30b)

where

\[
\frac{C^1}{\rho m} = \frac{n_i m_i \mathbf{v}_{\text{in}} + n_e m_e \mathbf{v}_{\text{en}}}{n_i m_i + n_e m_e} = \mathbf{v}
\]
and $V_0$ is the velocity for no damping.

Assume that the solution of Eq. 3.30b is of the form

$$v_x \propto e^{j(\omega t - \beta x)}$$

where $\beta$ may be complex. Substitute this in Eq. 3.30b then we have

$$-\omega^2 v_x = -\beta^2 V_0^2 v_x - j \omega v_x$$

or

$$\beta = \pm \frac{\omega}{V_0} (1 - j \frac{\sqrt{V}}{\omega})^{1/2}$$

(3.31)

If $\frac{\sqrt{V}}{\omega} << 1$, damping is unimportant. For $\frac{\sqrt{V}}{\omega} \gg 1$,

$$\beta = \pm \frac{\omega}{V_0} (-j \frac{\sqrt{V}}{\omega})^{1/2}$$

$$= \pm \frac{\omega}{V_0} \left( \frac{\sqrt{V}}{\omega} e^{j3\pi/2} \right)^{1/2}$$

$$= \pm \frac{\omega}{V_0} \sqrt{\frac{V}{\omega}} \left( -\frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right)$$

and

$$\beta = \pm \frac{\sqrt{\omega V}}{V_0} (1 - j)$$

(3.32)

Hence,

$$v_x \propto \exp j \left( \omega t - \frac{\sqrt{\omega V}}{V_0} x \right) \exp \left( -\frac{\sqrt{\omega V}}{V_0} x \right)$$

(3.33)
\[
\exp\left(-\frac{\sqrt{\frac{\omega \nu}{2}}}{V_0} x\right)
\]
describes the attenuation of the wave.

If \(\frac{\sqrt{\frac{\omega \nu}{2}}}{V_0} L \gg 1\), standing waves would not be expected.

In our case \(\omega \approx 2 \times 1 \text{ sec}^{-1}\), \(L = 6 \times 10^3\) meters,
\(V_0 = 5.4 \times 10^3\) m/sec and \(\nu \approx 10^{12} \text{ sec}^{-1}\) for \(N_2\) (Brown, 1959), if we assume the partial pressure of \(N_2\) atoms to be about 1 atmosphere, normalized to a temperature of 273\(^{\circ}\)K. Substituting these values \(\frac{\sqrt{\frac{\omega \nu}{2}}}{V_0} L \approx 10^6 \gg 1\).

Thus, the wave is attenuated due to the presence of neutrals.

It follows that if the concentration of neutral gas atoms in the lightning channel is of the order of magnitude of that assumed, positive ions will not have any significant effect on thunder production.
CHAPTER 4

INSTRUMENTATION AND RECORDS

4.1 General Scheme

During the summer of 1962, observations of the infrasonic pressure variations resulting from lightning discharges were taken. For recording the pressure variations of subaudio frequencies, a Hot-Wire Microphone (abbreviated in future HWM) was employed. The output of the HWM was fed into one channel of a two-channel Sanborn recorder, and records of pressure versus time were obtained on heat sensitive paper. Later in the summer, it was decided to determine the nature of impulses received, and because the deflection of the HWM for such frequencies is unidirectional, a 12-inch woofer, used as a microphone rather than as a speaker, was employed. The woofer had strong resonances, the diaphragm tending to oscillate with resonant frequencies after being set in motion. These oscillations often showed a modulation pattern similar to the gross features of the slower pressure change recorded by the HWM. The woofer was connected to the second channel of the Sanborn recorder. The simultaneous records on both channels made it possible to distinguish between the compressional or rarefactive nature of the beginnings of a peal of thunder. Fig. 4.1 illustrates the experimental arrangement in a block diagram. The observations were taken at a place outside Saskatoon, where local noises were at a low level.

During the summer of 1963, the observations were extended to audio frequencies. For this purpose, a Wide-Range Crystal micro-
Fig. 4.1 Block diagram illustrating typical arrangement of instruments for the recording of pressure variations associated with thunder in the summer of 1962.

Fig. 4.2 Block diagram illustrating typical arrangement of instruments for the recording of infrasonic and sonic pressure variations associated with thunder in the summer of 1962.
phone (abbreviated in future as WRM), Type 412X8, with a sound level meter, type 412, used as an audio amplifier, was employed. The output of this assembly was fed to one of the three channels of a 906C Honeywell Visicorder Oscillograph. The remaining two channels were used for the HWM and the woofer as mentioned previously. To record only subaudio pressure changes on the HWM, a 15-cycle per second low-pass filter was designed and used to filter out the high frequency components of the thunder. To record only audio pressure variations on the WRM, a 15-cycle per second high-pass filter was designed and used to filter out the infrasonic component of thunder. Fig. 4.2 shows the set-up in a block diagram.

A scheme was also developed to determine the direction of arrival of the sounds. Three HWM were used. The HWM were separated by sufficient distances to give adequate time resolution. Details of the location of the microphones are given in Chapter 6.

The site in 1963 was about 6 miles N. E. of Saskatoon, where noises due to passing cars and trains were not appreciable. The ground was level and except for two nearby houses and a windbreak of trees, had no large reflecting surfaces such as the face of hills.

The microphones were placed on the ground at some distance from the trailer in which the recording equipment was kept on occasions when there was little wind and no rain. Often wind and rain accompanied the storms when close, and the microphones were
placed inside a car. By opening and closing the proper windows and
doors the wind noise was reduced greatly. Damage to the microphones
by rain was also avoided. Records from the microphones when oper-
ated from inside the car showed no significant differences from the
ones from outside the car. For infrasonic frequencies, the wave-
length is much longer than the dimensions of doors and windows of
the car, and resonance effects should not be appreciable.

4.2 Microphones
4.2.1 Hot Wire Microphone*

In almost all cases where attempts are made to measure
or analyse sounds, microphones with some form of diaphragm which
respond to the pressure changes are used. Such microphones are,
in general, insensitive to notes of low pitch, and are, moreover,
easily disturbed by the vibrations communicated through the mounting
of the diaphragm. An HWM does not have this disadvantage since it
responds to changes in temperature of a heated wire, the temperature
changes being induced by the moving air. If the displacements in the
progressive waves are extremely small, they can be enhanced by
resonance.

The HWM as commonly used, consists essentially of a
closed vessel in the form of a Helmholtz resonator, across the open-
ing of which is a grid of thin platinum wire. The diameter of the wire

* HWM were borrowed from the Defence Research Board,
Ottawa. These were employed at one time for locating artillery.
is about 0.0006 cm. The wire is heated electrically. When sound waves impinge on the orifice of the resonator, the volume of the air inside begins to vibrate and a portion of the air flows in and out through the orifice. It thus cools and changes the resistances of the hot wire. The manner in which heat escapes from the cylindrical wire is fairly complicated. It depends on the air current and on whether or not the wire is at rest. The theoretical and experimental aspects of this problem have been dealt with extensively in the literature (King, 1914; Richards, 1923; Maxwell, 1928). The change in resistance of the hot-wire gives a measure of the pressure oscillations in the air.

For measuring the resistance changes the HWM was placed in one of the arms of Wheatstone bridge as indicated in Fig. 4.3. A constant d, c. of 25 milliampères was passed through the grid of the HWM and the bridge was balanced. Any change of resistance due to impinging of sound disturbed the balance of the bridge and produced a signal which was recorded in the recorder.

The sensitivity of the microphone depends on the heating current and is expressed in the form of a graph of resistance versus current. A plot of resistance versus heating current is given in Fig. 4.4. Another way of defining the sensitivity is to plot the ratio of change of resistance to resistance versus resistance (Tucker and Paris, 1921), and can be easily obtained from Fig. 4.4. The system was adjusted such that for a pressure change of 0.2 microbars, the HWM produced a deflection of 1 division on the Visicorder. In practice this sensitivity had to be reduced considerably. The difficulty
Fig. 4.3 Circuit diagram of the hot-wire microphone for the recording of pressure variations associated with thunder.
Fig. 4.4 Plot of the resistance of the hot-wire microphone versus heating current passing through the wire. The top most curve is for no sound, the middle curve is for a sound pressure level (SPL) of 99 db, and the bottom curve is for a sound pressure level of 112 db.
of measuring the intensity in infrasonic frequencies made it impossible

to make reliable measurements of the frequency response of the HWM.
The frequency response of the HWM for audio frequencies was deter-
mined and it showed that the microphone had a resonant frequency at
55 cycles per second. Rough measurements for lower frequencies
showed no resonant responses. It is fairly certain that none exist
for these microphones at subaudio frequencies.

4.2.2 Wide Range Crystal Microphone

The assembly of the 412X8 WRM and 412 Sound Level
Meter constitutes a complete measuring system for sound. It measures
Sound Pressure Level (SPL)* at the microphone in terms of decibels.

It provides sound pressure level measurements from 75 to 200 db
SPL over the frequency range of 10 cycles per second to 30 kilocycles
per second. The frequency response of the assembly is shown in

Fig. 4.5 (according to the information supplied by the manufacturer).

4.2.3 Woofer

A 12-inch loudspeaker was mounted on a cylindrical tin

can. Its length was 40 cm and internal diameter 27.5 cm. Three

holes of diameter 0.8 cm were drilled into it for equalizing the pressure.

The frequency response of the woofer is illustrated in Fig. 4.6.

4.3 Recorders and Filters

4.3.1 Recorders

During the course of the investigations three recorders

* SPL = 20 \log_{10} \frac{P}{P_0}, \text{ where } P_0 = 0.0002 \text{ microbars.}
Fig. 4.5 Frequency response of the wide-range crystal microphone as supplied by the manufacturer.
Fig. 4.6 Frequency response of the woofer.
S.P.L. in db

Freqency

30 50 100 500 1K 5K 10K 50K
were used; a twin-channel Sanborn recorder and an Edin oscillographic recorder during 1962, and a multi-channel Honeywell Visicorder during 1963.

The Sanborn recorder was a two channel magnetic oscillograph which produced records on a heat sensitive paper. When operated in the critically damped condition, the half-power point of a signal, whose peak to peak amplitude is 60% of the chart width, occurred at a frequency of about 55 cycles per second. The paper speeds used were 50 mm/sec and 100 mm/sec.

The Edin Oscillographic recorder was a multi-channel ink recorder, and was employed only for one storm. The paper speed used was 25 mm/sec. Though its characteristics were not known, they should not be significantly different from that of the Sanborn Recorder.

The Honeywell multi-channel 906C visicorder oscillograph with M200-120 electromagnetic damped type subminiature galvanometers was flat (=5%) from d.c. to 120 c.p.s. and the sensitivity was 1.42 millivolts per inch, though it was reduced appreciably in operation. The chart speeds used were 1 inch/second, 5 inch/second, and 25 inch/second.

4.3.2 Filters

Fifteen cycle per second low-pass and high-pass filters were designed following Fryer (1959). The circuit diagrams of these filters are given in Fig. 4.7 and the characteristics are indicated in Fig. 4.8.
Fig. 4.7 Circuit diagrams of the 15 cps low-pass and high-pass filters.
Fig. 4.8 Frequency response of the 15 cps low-pass and high pass filters.
The use of filters was terminated later on when it was realized that no significant effect could be found. The signal of thunder contains most of the power in lower infrasonic frequencies, and the spectral analysis was performed for frequencies less than 20 cycles per second. Hence the low-pass filter served no useful purpose. The frequency response of the WRM assembly drops off appreciably below 10 cycles per second and is reasonably flat for sonic frequencies, and so a high-pass filter could be dispensed with.

4.4 Records

Figures 4.9 to 4.20 inclusive are typical examples of pressure variations associated with thunder. Schmidt (1914) classified the records into two groups, group a and group b. The records of group a start with a sharp violent pressure fluctuation, while those of group b increase gradually to a maximum. Besides these two kinds, a third kind of record has been observed in our case. This has been designated group c and is from distant flashes. These records do not possess much audio frequency content and can be associated with dull murmurs, or rumblings. Table 4.1 shows the classification of these records along with the records, described in Chapter 6 for which directions of arrival of sounds have been calculated. The interpretation of this classification and various components will be discussed in the following chapters of this thesis.

To facilitate identification and review of the records the system illustrated in the following example was used. August 8, 1963
Fig. 4.9 Photographic reproduction of thunder from a ground flash. This record number August 8, 1963 (3, 3) was obtained by the visicorder. The infrasonic spectral density for this record is represented in Fig. 5.19 and the sonic spectral density is represented in Fig. 5.20. The top trace is from the hot-wire microphone, the middle from the wide-range crystal microphone and the bottom from the woofer.
Fig. 4.10 Photographic reproduction of thunder from a cloud flash. This record number August 8, 1963 (3.4) was obtained by the visicorder. The sonic spectral density for this record is represented in Fig. 5.21b. The top trace is from the hot-wire microphone, the middle from the wide-range crystal microphone and the bottom from the woofer.
Fig. 4.11 Photographic reproduction of thunder from a cloud flash with many branches. This record number August 8, 1963 (3.2) was obtained by the visicorder. The infrasonic spectral density for this record is represented in Fig. 5.18 and the sonic spectral density is represented in Fig. 5.21a. The top trace is from the hot-wire microphone, the middle from the wide-range crystal microphone and the bottom from the woofer.
Fig. 4.12 Photographic reproduction of musical thunder from a cloud flash. This record number July 24, 1963 (3.1) was obtained by the visicorder. The infrasonic spectral density for this record is represented in Fig. 5.16. The top trace is from the hot-wire microphone, the middle from the wide-range crystal microphone and the bottom from the woofer.
Fig. 4.13 Photographic reproduction of new loud claps of thunder. This record number July 9, 1963 (4.4) was obtained by the visicorder. The infrasonic spectral density for this record is represented in Fig. 5.13. The top trace is from the hot-wire microphone, the middle from the wide-range crystal microphone and the bottom from the woofer.
Fig. 4.14 Photographic reproduction of rumbling thunder. This record number July 9, 1963 (1.1) was obtained by the visicorder. The infrasonic spectral density for this record is represented in Fig. 5.10. The top trace is from the hot-wire microphone, the middle from the wide-range crystal microphone and the bottom from the woofer.
Fig. 4, 15 Photographic reproduction of thunder from a complicated (partly cloud with branches and partly ground) flash. This record number August 8, 1963 (2. 1) was obtained by the visicorder. The sonic spectral density for this record is represented in Fig. 5. 22. The top and bottom traces are from the hot wire microphones, the former being twice as sensitive as the latter, and the middle trace is from the wide-range crystal microphone.
Fig. 4.16 Photographic reproduction of thunder records obtained by the visicorder. The top trace is from the hot-wire microphone, the middle from the wide-range crystal microphone and the bottom from the woofer.

(a) One clap of thunder record number July 20, 1963 (1.1). The upward deflection on the woofer indicates a compression. (b) Roll of thunder record number July 20, 1963 (2.1). The upward deflection on the woofer indicates a compression. (c) Another example of roll of thunder record number July 20, 1963 (2.2).
Fig. 4.17 Photographic reproduction of thunder records from distant flashes obtained by the visicorder. The top trace is from the hot-wire microphone, the middle from the wide-range crystal microphone and the bottom from the woofer. No high frequency components of thunder are present as is evident from the wide-range crystal microphone and woofer traces. (a) record number July 5, 1963 (1.1). (b) record number July 5, 1963 (1.2).
Fig. 4.18 Photographic reproduction of two thunder records from a hot-wire microphone. The records were obtained by the Sanborn recorder. (a) The infrasonic spectral density of this record number July 1, 1962 (2.1) is represented in Fig. 5.1. (b) The infrasonic spectral density of this record number July 1, 1962 (2.2) is represented in Fig. 5.1.
Fig. 4.19 Photographic reproduction of two thunder records. The records were obtained by the Sanborn recorder. The top trace is from the woofer and the bottom trace is from the hot-wire microphone. 

(a) The infrasonic spectral density of the record number July 21, 1962 (2.6) is represented in Fig. 5.4. The time interval between the perception of the flash and the thunder heard was 16 seconds. (b) This thunder record number July 21, 1962 (2.4) was obtained 6 seconds after the perception of the flash.
Fig. 4.20 Photographic reproduction of three thunder records obtained by the Sanborn recorder. The top trace is from the woofer and the bottom from the hot-wire microphone. A downward deflection on the woofer indicates a compression. (a) This record number July 21, 1962 (2.1) shows the thunder originating with a large peal that is compressional in nature as is evident by the downward deflection on the woofer, and followed by smaller peals and rumbles. (b) This record number July 21, 1962 (2.2) shows that the thunder originated with a large compressional peal, as the deflection on the woofer is downward, and followed by small and large peals and rumbles. The thunder persisted for a long time. The second set of peals is as large as the first one. (c) This record number July 21, 1962 (2.3) is similar to (a) except that it is associated with more rumbles and the nature of the first peal is unidentified.
(3.4) means a thunderstorm occurring on August 8, 1963, and the fourth record of thunder on the third roll of recording paper used on this occasion. The 3.4 is enclosed in brackets to avoid confusion with the comparable numerical system used for identifying tables, figures, etc., in this thesis.

Various meteorological parameters for the dates listed in table 4.1 are included in Appendix E.
TABLE 4.1

Classification of records of thunder

<table>
<thead>
<tr>
<th>Record Number</th>
<th>Figure</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 8, 1963 (3.3)</td>
<td>4.9</td>
<td>a</td>
</tr>
<tr>
<td>August 8, 1963 (3.4)</td>
<td>4.10</td>
<td>b</td>
</tr>
<tr>
<td>August 8, 1963 (3.2)</td>
<td>4.11</td>
<td>b</td>
</tr>
<tr>
<td>July 24, 1963 (3.1)</td>
<td>4.12</td>
<td>b</td>
</tr>
<tr>
<td>July 9, 1963 (4.4)</td>
<td>4.13</td>
<td>b</td>
</tr>
<tr>
<td>July 9, 1963 (1.1)</td>
<td>4.14</td>
<td>c</td>
</tr>
<tr>
<td>August 8, 1963 (2.1)</td>
<td>4.15</td>
<td>a</td>
</tr>
<tr>
<td>July 20, 1963 (1.1)</td>
<td>4.16</td>
<td>b</td>
</tr>
<tr>
<td>July 20, 1963 (2.1)</td>
<td>4.16</td>
<td>b</td>
</tr>
<tr>
<td>July 20, 1963 (2.2)</td>
<td>4.16</td>
<td>a</td>
</tr>
<tr>
<td>July 5, 1963 (1.1)</td>
<td>4.17</td>
<td>c</td>
</tr>
<tr>
<td>July 5, 1963 (1.2)</td>
<td>4.17</td>
<td>c</td>
</tr>
<tr>
<td>July 1, 1962 (2.1)</td>
<td>4.18</td>
<td>b</td>
</tr>
<tr>
<td>July 1, 1962 (2.2)</td>
<td>4.18</td>
<td>b</td>
</tr>
<tr>
<td>July 21, 1962 (2.6)</td>
<td>4.19</td>
<td>b</td>
</tr>
<tr>
<td>July 21, 1962 (2.4)</td>
<td>4.19</td>
<td>b</td>
</tr>
<tr>
<td>July 21, 1962 (2.1)</td>
<td>4.20</td>
<td>a</td>
</tr>
<tr>
<td>July 21, 1962 (2.2)</td>
<td>4.20</td>
<td>a</td>
</tr>
<tr>
<td>July 21, 1962 (2.3)</td>
<td>4.20</td>
<td>a</td>
</tr>
<tr>
<td>August 25, 1963 (1.3)</td>
<td>6.3</td>
<td>b</td>
</tr>
<tr>
<td>August 25, 1963 (1.4)</td>
<td>6.4</td>
<td>a</td>
</tr>
</tbody>
</table>
Table 4.1 continued

<table>
<thead>
<tr>
<th>Record Number</th>
<th>Figure</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 26, 1963 (2.1)</td>
<td>6.5</td>
<td>b</td>
</tr>
<tr>
<td>August 26, 1963 (2.3)</td>
<td>6.6</td>
<td>b</td>
</tr>
<tr>
<td>August 26, 1963 (2.5)</td>
<td>6.7</td>
<td>b</td>
</tr>
<tr>
<td>August 26, 1963 (2.6)</td>
<td>6.8</td>
<td>b</td>
</tr>
<tr>
<td>August 26, 1963 (2.7)</td>
<td>6.9</td>
<td>b</td>
</tr>
<tr>
<td>August 26, 1963 (2.8)</td>
<td>6.10</td>
<td>a</td>
</tr>
</tbody>
</table>
5.1 Introduction

Frequency analysis (spectral analysis or power spectral analysis) is both a tool and blueprint, for it sometimes helps us in solving difficult problems and in presenting a descriptive and meaningful interpretation of a set of observations. It can also be used to solve problems of a statistical nature where the excitation function is random and can only be described in terms of a power spectrum or a correlation function.

A statistician might argue against a power spectrum analysis because of the time and labour required in taking Fourier transforms. He might prefer the autocorrelation function because an exact and complete knowledge of the values of any choice determines the values of other choice. However, the following arguments can be advanced in favour of choosing a power spectrum: (1) Physicists have become accustomed to thinking in terms of frequencies more clearly than in terms of time. Hence, to them, the correlation which is a function of lag (delay time), is more difficult to envisage than its Fourier transform, the power spectrum. The fact that a power spectrum is easier to interpret has been supported by Paulson (1961), who plotted the power spectrum and correlation of a series prepared by Summer (Brooks and Carruthers, 1953); (2) The data analysed in all practical situations do not represent the actual output of the random process,
but will have been modified by the transmission characteristics of the recording devices. For the correction of the estimates of the power spectrum the procedure is simply division of a frequency function by another frequency function, while the correction procedure for the autocorrelation estimate requires a Fourier transformation, division of the resulting frequency function by another frequency function, and an inverse Fourier transform; and (3) a smoothed power spectrum estimate fluctuates in an independent manner, while the estimates of autocorrelation function have fluctuations which are far from independent and could easily deceive an interpreter.

5.2 Theory

If $X_t$ is a continuous stochastic process which is both stationary and ergodic, the autocovariance function, $C_{\tau}$, at lag $\tau$ of the process is defined by

$$C_{\tau} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} X_t \cdot X_{t+\tau} dt$$  \hspace{1cm} (5.1)

The function $C_{\tau}$ is often referred to as the autocorrelation function. However, Blackmann and Tukey (1958) suggest that this name should be applied to the normalized ratio $\frac{C_{\tau}}{C(0)}$.

Equation (5.1) can be reduced (Blackmann and Tukey, 1958, p. 84-88; Paulson, 1963, p. 51) to the following form

$$C_{\tau} = \int_{-\infty}^{\infty} P(f) e^{i2\pi ft} df$$  \hspace{1cm} (5.2)
where

\[
P(f) = \lim_{T \to \infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} X(t) e^{-i2\pi ft} dt \right|^2 \quad (5.3)
\]

\(P(f)\) describes the power spectrum of the stationary random process. It can also be expressed (Blackmann and Tukey, 1958) as a Fourier transform of the autocovariance function as expressed by the following equation

\[
P(f) = \int_{-\infty}^{\infty} C(\tau) e^{-i2\pi f \tau} d\tau \quad (5.4)
\]

The above equation is sometimes used as a definition of the power spectrum. \(P(f) \, df\), to be precise, represents the contribution to the variance from frequencies between \(f\) and \(f + df\). Hence, a more correct designation of \(P(f)\) would be spectral density or power spectral density.

When \(X(t)\) is of a finite duration, it is sampled at equal intervals of time according to the sampling theorem (Shanon, 1949) in the time domain. It states, "If a function \(X(t)\) contains no frequencies higher than \(f_N\) cycles per second, it is completely determined by sampling ordinates at a sequence of points spaced \(\frac{1}{2f_N}\) seconds apart". The integrals of Eqs. 5.1 and 5.4 are replaced by their corresponding sums.
5.3 Autocovariance and Spectral density analysis

5.3.1 Programme

For the records of 1962, the autocovariance and spectral density were evaluated using the LGP-30 computer; a suitable programme had been written by Mr. H. J. Toop, modified later by Dr. G. G. Shepherd. The LGP-30 takes about 7 hours to compute 81 solutions for a series of 801 numbers. Because of the long time taken by the LGP-30, a programme was written for the IBM-1620 computer in fortran language with the help of Mr. R. H. Paulin. A copy of this programme is included in Appendix B. The IBM-1620 takes about one fourth of the time taken by LGP-30. This programme takes a maximum of 1500 numbers for 500 solutions. Records for 1963 were analysed using the new programme and the IBM-1620 computer.

The programme accepts \( q + 1 \) readings \( X_0, X_1, X_2, \ldots, X_q \), and computes the following expressions

\[
X = \frac{1}{q + 1} \sum_{i=0}^{q} X_i \quad (5.5)
\]

\[
X' = X_i - \overline{X}, \quad i = 0, 1, 2, \ldots, q \quad (5.6)
\]

\[
C(j) = \frac{1}{q - j} \sum_{i=0}^{q-j} X_i' X_{i+j}' \quad j = 0, 1, 2, \ldots, \gamma \quad (5.7)
\]

\[
P(k) = C_0 + 2 \sum_{j=1}^{\gamma-1} C_j \cos \frac{k_j \pi}{\gamma} + C_\gamma \cos k \pi \quad , \quad (5.8)
\]

\[
k = 0, 1, 2, \ldots, s.
\]
\[ P(f) = \beta P(k) + \gamma P(k) + \delta P(k+1) \quad (5.9) \]

If \( \beta = 0.54, \gamma = 0.23, \) and \( \delta = 0.23, \) the smoothing operation is called hamming (Blackman and Tukey, 1958) and is used in our case.

Equation 5.7 gives autocovariance, and Equation 5.8 and 5.9 give, respectively, the unsmoothed and smoothed spectral densities.

The actual programme is more general than outlined here.

5.3.2 Accuracy

The basic problem arises from the fact that actual records are finite instead of infinite in duration, and the frequency bandwidths \( \Delta f \) are finite rather than zero in width. For a fixed record length \( T \) one might try to improve the knowledge of \( P(f) \) by taking \( \Delta f \) smaller and smaller. However, an uncertainty principle holds between the record duration \( T \) and frequency bandwidth \( \Delta f \) such that \( T \Delta f \geq C'' \), where \( C'' \) is a fixed constant (Bendat, 1958). Thus, for a finite \( T \), too small a value of \( \Delta f \) violates this principle.

Another important error is aliasing; this arises as a result of sampling \( X(t) \) at, say, equal intervals of time \( \Delta t \), and later confusing some of the higher frequency content in the original wave with the lower frequencies. This can be reduced, though not eliminated, by making sure that the records do not contain much higher frequencies than the Nyquist frequency given by \( f_N = \frac{1}{2 \Delta t} \).

Other errors, resolution and very low frequency bandwidths,
have been discussed in detail by Bendat (1958), and Blackmann and Tukey (1958).

In order to determine the reliability of spectral density estimates, significance tests have been developed (Blackmann, and Tukey, 1958; Pierson and Marks, 1952). These tests assume that the time series has been produced by a Gaussian process. The tests of normality involve a prodigious amount of arithmetic and time. Consequently, a suitable programme for the IBM-1620 was written in fortran language with the help of Mr. R. H. Paulin. The Gaussian fit and test procedure and programme are discussed in Appendix C. Other tests (Van Isacker, 1961) assume that the distribution is Laplacian. However, the safest test is to ensure that the spectral density plot is smooth and reproducible under similar experimental conditions (Ward, 1960; Paulson, 1963, p. 55).

Lastly, the question of what should be the maximum lag has to be faced by anyone who performs this analysis. Blackmann and Tukey (1958, p. 11) state that the maximum lag for which autocovariance is estimated, should not be greater than 10 percent of the duration of the record. Jenkins (1961), on the other hand, asserts that this could be safely taken to be as large as 30 percent. Increasing the maximum lag gives useful information as resolution becomes better, but the statistical reliability is sacrificed (Bendat, 1958). The maximum lag used generally was 10 percent of the record duration; in a few cases it was extended to 15 percent.
5.4 Spectral Density Estimates

5.4.1 Infrasonic Spectral density estimates for 1962 records

Spectral density estimates were obtained for frequencies less than 9 cycles per second for 11 thunder records. These are presented in Figures 5.1 to 5.6 inclusive. An average of eight records (July 1, 1962 (2.1), (2.2), (3.1), (3.2), July 8, 1962 (2.4), 2.9), July 21, 1962 (1.18), and August 19, 1962 (1.1)) has been reported previously (Bhartendu and Currie, 1963). It showed no dominant primary maximum.

All spectra indicate that the spectral density (power) decreases with the increase in frequency. They all exhibit two kinds of maxima, primary and secondary. The primary maxima are in the range of 1.00 to 6.00 cps. The most intense primary maximum occurs between 1.00 and 2.00 cps in seven records, between 2.00 and 3.00 cps in three records, and at a frequency greater than 3.00 cps in one record. The secondary maxima are too small in amplitude to satisfy any statistical test of significance, and consequently, the actual significance of these cannot be ascertained. It should be noted, however, that the secondary maxima often occur in one spectrum at the frequencies of primary maxima of other spectra.

The spectra can be divided into three groups: Type A, type B, and type C. The spectra of type A indicate only one prominent primary maximum at a frequency in the range under consideration. The type B spectra indicate more than one primary maximum, and often suggest the presence of harmonics. These "harmonics" are not
Fig. 5.1 Spectral densities of two thunder records on hot-wire microphone. The duration of upper record number July 1, 1962 (2.2) was 20 seconds. The duration of lower record number July 1, 1962 (2.1) was 25 seconds.
Fig. 5.2 Spectral densities of two thunder records on hot-wire microphone. The duration of upper record number July 1, 1962 (3.2) was 20 seconds. The duration of lower record number July 1, 1962 (3.1) was 30 seconds.
Fig. 5.3 Spectral densities of two thunder records on hot-wire microphone. The duration of upper record number July 8, 1962 (2.9) was 20 seconds. The duration of lower record number July 8, 1962 (2.4) was 20 seconds.
Fig. 5.4 Spectral density of a thunder record on hot-wire microphone. The duration of the record number July 21, 1962 (2.6) was 17 seconds.
Fig. 5.5 Spectral densities of two thunder records on hot-wire microphone. The duration of upper record number August 1, 1962 (2.2) was 17 seconds. The duration of lower record number August 1, 1962 (2.1) was 22 seconds.
Fig. 5.6 Spectral densities of two thunder records on hot-wire microphone. The duration of upper record number August 19, 1962 (1.1) was 18 seconds. The duration of lower record number July 21, 1962 (1.18) was 20 seconds.
likely to be due entirely to statistical fluctuations. Aliasing is not substantial, for the energy associated with higher frequencies is small, and even if aliasing is significant, it should not cause a regular recurrence of maxima. The type C spectra show no intense primary maxima. Out of eleven spectra presented, two are type A, four type B, and five type C.

Table 5.1 summarizes the characteristics of these spectra.

5.4.2 Infrasonic spectral density estimates for 1963 records

Spectral density estimates for thirteen records for frequencies lower than 20.00 cps were evaluated. These are presented in Figures 5.7 to 5.19 inclusive. A 15 cps low-pass filter was used to minimize aliasing due to audio frequencies. As will be seen from the spectra, the use of the filter did not alter appreciably the results. The results are not significantly different from those for 1962.

Different lengths (durations) of the same record were selected and the analysis was performed separately for all the different portions of the record. No significant change in the frequency of dominant primary maximum (or maxima) was found. Thus, the dominant primary maximum (or maxima) in the spectral density estimate for a particular record of thunder should not be considered as governed by the duration for which it was analysed.

Individual spectra are similar in general detail. Power decreases with increase in frequency. The primary maxima occur in the range of 0.75 cps to 4.00 cps. The most intense maximum occurs between 1.00 and 2.00 cps for eight records, and between
<table>
<thead>
<tr>
<th>Record Number</th>
<th>Primary Maxima a maxima c. p. s.</th>
<th>Secondary Maxima c. p. s.</th>
<th>Type</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 1, 1962(2.1)</td>
<td>1.00</td>
<td>3.50</td>
<td>B</td>
<td>Maximum at 1 c. p. s. is most intense.</td>
</tr>
<tr>
<td></td>
<td>2.50</td>
<td>5.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.00</td>
<td>7.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 1, 1962(2.2)</td>
<td>2.25</td>
<td>4.00</td>
<td>A</td>
<td>Maximum at 2.25 is very intense</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 1, 1962(3.1)</td>
<td>1.50</td>
<td>3.25</td>
<td>C</td>
<td>There is no intense maximum</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.50-4.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 1, 1962(3.2)</td>
<td>2.00</td>
<td>2.75</td>
<td>C</td>
<td>There is no intense maximum</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 8, 1962(2.4)</td>
<td>1.00</td>
<td>2.00</td>
<td>A</td>
<td>The maximum at 1.00 c. p. s. is intense.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.25-5.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.75-7.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 8, 1962(2.9)</td>
<td>2.75</td>
<td>4.25</td>
<td>B</td>
<td>Maximum at 2.75 is most intense</td>
</tr>
<tr>
<td></td>
<td>5.50</td>
<td>7.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 21, 1962(1.18)</td>
<td>1.50</td>
<td>3.25</td>
<td>C</td>
<td>There is no intense maximum</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 21, 1962(2.6)</td>
<td>2.33</td>
<td>5.33</td>
<td>C</td>
<td>There is no intense maximum</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>August 1, 1962(2.1)</td>
<td>1.00</td>
<td>4.75</td>
<td>B</td>
<td>Maximum at 1.00 c. p. s. is most intense.</td>
</tr>
<tr>
<td></td>
<td>2.25-2.50</td>
<td>5.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table (5.1) continued

<table>
<thead>
<tr>
<th>Record Number</th>
<th>Primary Maxima (c.p.s)</th>
<th>Secondary Maxima (c.p.s)</th>
<th>Type</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 1, 1962(2,2)</td>
<td>3.33</td>
<td>7.00</td>
<td>C</td>
<td>There is no intense maximum</td>
</tr>
<tr>
<td>August 19, 1962(1,1)</td>
<td>1.25 4.00</td>
<td>2.00 3.00 5.50 6.25 7.75-8.00 9.00</td>
<td>B</td>
<td>Maximum at 1.25 is most intense, There is enough power (relatively speaking) at 3.00 c.p.s and could be considered as primary maximum.</td>
</tr>
</tbody>
</table>
Fig. 5.7 Spectral density of rumbling thunder record number June 29, 1963 (4.1) on a hot-wire microphone. The duration of the record was 20 seconds. A 15 cps low-pass filter was employed.
Fig. 5.8 Spectral density of thunder record number July 5, 1963 (1.1) on a hot-wire microphone. The duration of the record was 20 seconds. A 15 cps low-pass filter was employed.
Fig. 5.9 Spectral density of thunder record number July 5, 1963 (1.2) on a hot-wire microphone. The duration of the record was 15 seconds. A 15 cps low-pass filter was employed.
5.10 Spectral density of rumbling thunder record number July 9, 1963 (1, 1) on a hot-wire microphone. The duration of the record was 20 seconds. A 15 cps low-pass filter was employed.
Fig. 5.11 Spectral density of thunder record number July 9, 1963 (4.1) from a cloud flash on a hot-wire microphone. The duration of the record was 20 seconds. A 15 cps low-pass filter was employed.
Fig. 5.12 Spectral density of thunder record number July 9, 1963 (4.3) from a cloud flash on a hot-wire microphone. The duration of the record was 20 seconds. A 15 cps low-pass filter was employed.
Fig. 5.13 Spectral density of few loud claps of thunder record number July 9, 1963 (4.4) on a hot-wire microphone. The duration of the record was 15 seconds. A 15 cps low-pass filter was employed.
Fig. 5.14 Spectral density of thunder record number July 16, 1963 (1.1) from a cloud flash on a hot-wire microphone. The duration of the record was 17.5 seconds. A 15 cps low-pass filter was employed.
Fig. 5.15 Spectral density of thunder record number July 16, 1963 (2.1) on a hot-wire microphone. The duration of the record was 21 seconds. A cps low-pass filter was employed.
Fig. 5.16 Spectral density of musical thunder record number July 24, 1963 (3,1) from a cloud flash on a hot-wire microphone. The duration of the record was 16 seconds. A 15 cps low-pass filter was employed.
Fig. 5.17 Spectral density of thunder record number August 8, 1963 (3.1) from a cloud flash on a hot-wire microphone. The duration of the record was 20 seconds. No filter was employed.
Fig. 5.18 Spectral density of thunder record number August 8, 1963 (3.2) from a cloud flash on a hot-wire microphone. The duration of the record was 20 seconds. No filter was employed.
Fig. 5.19 Spectral density of thunder record number August 8, 1963 (3.3) from a ground flash on a hot-wire microphone. The duration of the record was 15 seconds. No filter was employed.
2.00 and 3.00 cps in five records. Three types of spectra were again noticed. Three of type A, four of type B, and six of type C spectra were observed. The characteristics of these spectra are given in Table 5.2.

It should now be emphasized that the consistency of the spectral density estimates of 1962 and 1963 when different instrumentation was employed, suggests that the dominant primary maxima are real and not accidental (for example, random). The type of spectra has no apparent connection with the type of record (types a, b, and c), type of thunder (peals, claps, rolls, and rumblings), and the distance of the flash. However, it is felt that present observations are not adequate enough to conclude this with absolute certainty. It is considered that the spectra are related to the flash structure and geometry. Present observations of only thunder are not enough to determine such a relation. Simultaneous optical observations of lightning and acoustical observations of resulting thunder are required to ascertain this.

5.4.3 Sonic spectral density estimates for 1963 records

Spectral density estimates of seven thunder records (three from cloud flash, three from ground flash, and one ultra high speed record from a complicated flash) are presented in Figures 5.20 to 5.22 inclusive.

The audio frequency spectra of thunder from cloud and ground flashes are similar. The most common frequency ranges
<table>
<thead>
<tr>
<th>Record Number</th>
<th>Description</th>
<th>Time difference between lightning and thunder, sec.</th>
<th>Primary Maxima c.p.s.</th>
<th>Secondary Maxima c.p.s.</th>
<th>Type</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 29, 1963(4.1)</td>
<td>Rumbling thunder</td>
<td>15</td>
<td>2.00</td>
<td>7.50 13.00 19.00</td>
<td>A</td>
<td>The maximum at 2.00 c.p.s. is most intense.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>There is no intense maximum.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Maximum at 2.75 c.p.s. could be considered as primary maximum.</td>
</tr>
<tr>
<td>July 5, 1963(1.1)</td>
<td>Rumbling thunder</td>
<td>10</td>
<td>2.00</td>
<td>4.00 2.75 4.75-5.50 9.75-10.00</td>
<td>C</td>
<td>There is no intense maximum.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Maximum at 3.25 c.p.s. is equally intense as that at 2.50 c.p.s.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>and could be regarded as primary maximum.</td>
</tr>
<tr>
<td>July 5, 1963(1.2)</td>
<td>Very calm atmosphere</td>
<td>11.75-12.00</td>
<td>2.50</td>
<td>3.25 4.25 7.00 11.75-12.00</td>
<td>C</td>
<td>The maximum at 2.50 c.p.s. is most intense.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rumbling thunder</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15 c.p.s. filter</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 9, 1963(1.1)</td>
<td>Rumbling thunder</td>
<td>--</td>
<td>1.00</td>
<td>2.00 3.00 7.00 7.75</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>The maximum at 1.00 c.p.s. is most intense.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table (5.2) continued

<table>
<thead>
<tr>
<th>Record Number</th>
<th>Description</th>
<th>Time difference between lightning and thunder, sec.</th>
<th>Primary Maximum c.p.s.</th>
<th>Secondary Maxima c.p.s.</th>
<th>Type</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 9, 1963(4.1)</td>
<td>cloud flash</td>
<td>12</td>
<td>1.00&lt;br&gt;2.25</td>
<td>3.00&lt;br&gt;3.75-4.00&lt;br&gt;4.50&lt;br&gt;5.50&lt;br&gt;6.50&lt;br&gt;8.00&lt;br&gt;10.50-10.75&lt;br&gt;14.25</td>
<td>B</td>
<td>The maximum at 1.00 c.p.s. is most intense.</td>
</tr>
<tr>
<td></td>
<td>15 c.p.s. filter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>July 9, 1963(4.3)</td>
<td>cloud flash</td>
<td>14</td>
<td>0.75&lt;br&gt;1.50&lt;br&gt;2.25</td>
<td>3.25&lt;br&gt;4.25&lt;br&gt;5.75&lt;br&gt;6.50&lt;br&gt;7.75&lt;br&gt;10.00&lt;br&gt;11.00-11.25&lt;br&gt;12.25&lt;br&gt;16.00&lt;br&gt;18.00</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>15 c.p.s. filter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>July 9, 1963(4.4)</td>
<td>Loud claps</td>
<td>--</td>
<td>2.00</td>
<td>1.25&lt;br&gt;2.75&lt;br&gt;4.50&lt;br&gt;6.50&lt;br&gt;8.00&lt;br&gt;13.50&lt;br&gt;16.50</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>15 c.p.s. filter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Record Number</td>
<td>Description</td>
<td>Time difference between lightning and thunder, sec.</td>
<td>Primary Maximum or Maxima c.p.s.</td>
<td>Secondary Maxima c.p.s.</td>
<td>Type</td>
<td>Remark</td>
</tr>
<tr>
<td>-------------------</td>
<td>-------------------------------</td>
<td>-----------------------------------------------------</td>
<td>----------------------------------</td>
<td>------------------------</td>
<td>------</td>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>July 16, 1963(1, 1)</td>
<td>cloud flash</td>
<td>--</td>
<td>2.75</td>
<td>4.25-4.50</td>
<td>C</td>
<td>There is no intense maximum,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.25-8.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>17.50-17.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 16, 1963(2, 1)</td>
<td>--</td>
<td>--</td>
<td>2.50</td>
<td>1.50</td>
<td>C</td>
<td>There is no intense maximum. The maximum at 4.00 c.p.s. could be considered as a primary maximum,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.00-8.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12.00-12.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.00-16.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>17.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 24, 1963(3, 1)</td>
<td>cloud flash</td>
<td>--</td>
<td>1.00</td>
<td>3.50</td>
<td>A</td>
<td>The maximum at 1.00 c.p.s. is prominent. The maxima are not intense</td>
</tr>
<tr>
<td></td>
<td>musical thunder</td>
<td></td>
<td></td>
<td>4.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>15 c.p.s. filter</td>
<td></td>
<td></td>
<td>6.25-6.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Record Number</td>
<td>Description</td>
<td>Time difference between lightning and thunder, sec.</td>
<td>Primary Maximum or Maxima c. p. s.</td>
<td>Secondary Maxima c. p. s.</td>
<td>Type</td>
<td>Remark</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-------------------</td>
<td>---------------------------------------------------</td>
<td>-----------------------------------</td>
<td>----------------------------</td>
<td>------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>August 8, 1963(3, 1)</td>
<td>cloud flash</td>
<td>12</td>
<td>2.00</td>
<td>3.25 5.75 7.25 9.75 13.00 14.50 15.00-15.75</td>
<td>C</td>
<td>There is no intense maximum.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>August 8, 1963(3, 2)</td>
<td>cloud flash</td>
<td>15</td>
<td>2.00 4.00</td>
<td>5.00 6.00 7.00 8.25 9.50 11.25 12.00-12.25 14.00 17.25 19.50</td>
<td>B</td>
<td>The maximum at 2.00 c. p. s. is most intense.</td>
</tr>
<tr>
<td></td>
<td>many branches</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>August 8, 1963(3, 3)</td>
<td>Ground flash</td>
<td>--</td>
<td>1.00</td>
<td>3.75 5.75 7.00-7.50 9.25-9.50 10.75 12.25 13.25 14.50 16.00 19.50</td>
<td>A</td>
<td>Maximum power is at 1.00 c. p. s.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 5.20 Spectral densities of three thunder records from ground flashes on a wide-range crystal microphone.

The paper speed was 5 inches per second.

(a) --- record number August 3, 1963 (1.1)

(b) — record number August 3, 1963 (1.2)

(c) --- record number August 8, 1963 (3.3)
Fig. 5.21a Spectral densities of two thunder records from cloud flashes on a wide-range crystal microphone. The paper speed was 5 inches per second.

(a) — record number August 8, 1963 (1.1)
(b) —— record number August 8, 1963 (3.2)
Fig. 5. 21b Spectral density of a thunder record number August 8, 1963 (3.4) from a cloud flash on a wide-range crystal microphone. The paper speed was 5 inches per second.
Fig. 5.22 Spectral density of a thunder record number August 8, 1963 (2.1) from a complicated flash (partly cloud and partly ground) with branches on a wide-range crystal microphone. The paper speed was 25 inches per second.
of maxima are 22-28 cps, 52-56 cps, and 66-78 cps. Spectral maxima also occur in the ranges 34-40 cps, 88-90 cps, and at 122 cps, and 202-204 cps. Sometimes harmonics are present. The resolution for Figs. 5.20 and 5.21 was not enough and, hence the Nyquist frequency had to be fixed at 100 cps. Because of the substantial power contained in frequencies greater than 100 cps, aliasing would be present. But the aliasing should not be significant in the spectrum illustrated in Fig. 5.22 in which the Nyquist frequency was 250 cps. The primary maximum in this spectrum occurs in the same general regions as those in Figs. 5.20 and Figs. 5.21. Hence, Figs. 5.20 and 5.21 should not be considered entirely inaccurate.

Incidental to these measurements it was possible to note the sound pressure level (SPL) at the WRM. The measured average sound pressure level (SPL) at the WRM was 91 db with respect to a standard pressure of 0.0002 microbars. This gives a pressure change of 71 microbars. The maximum SPL recorded was 111 db, giving a pressure change of 71 microbars.

The characteristics of these sonic spectra are summarized in Table 5.3.

5.4.4 Reliability of the spectral density estimates

The error in the frequency because of fluctuations in chart speed was within 1 per cent. To determine the frequency distribution of the pressure amplitudes, histograms were plotted, Gaussian curves were fitted, and the normality was tested, as discussed in Appendix C,
<table>
<thead>
<tr>
<th>Record Number</th>
<th>Description</th>
<th>Time difference between lightning and thunder, sec.</th>
<th>Frequency band c. p. s.</th>
<th>Primary Maxima c. p. s.</th>
<th>Secondary Maxima c. p. s.</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 3, 1963(1.1)</td>
<td>Ground flash</td>
<td>18</td>
<td>20-100</td>
<td>76-78</td>
<td>46-50</td>
<td>The maximum at 76-78 c. p. s. is sharp and intense. The power at 46-50 c. p. s. is greater than that at 66 c. p. s. The power at 100 c. p. s. range is much greater than that at low audio frequency and at 76-78 c. p. s.</td>
</tr>
<tr>
<td>August 3, 1963(1.2)</td>
<td>Ground flash</td>
<td>26</td>
<td>20-100</td>
<td>52-54</td>
<td>22</td>
<td>Primary maxima are sharp and almost equally intense. Not many secondary maxima are present. The power at 100 c. p. s. is very less (contrary to other spectra). This may well be because of distant flash.</td>
</tr>
<tr>
<td>August 8, 1963(3.3)</td>
<td>Ground flash</td>
<td>--</td>
<td>20-100</td>
<td>28</td>
<td>46</td>
<td>There are no sharp and intense maxima. The power at 100 c. p. s. region is greater than that at low audio frequencies.</td>
</tr>
</tbody>
</table>
Table 5.3 continued

<table>
<thead>
<tr>
<th>Record Number</th>
<th>Description</th>
<th>Time difference between lightning and thunder, sec.</th>
<th>Frequency band c.p.s.</th>
<th>Primary Maxima c.p.s.</th>
<th>Secondary Maxima c.p.s.</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 8, 1963(3, 2)</td>
<td>Cloud flash with many branches</td>
<td>15</td>
<td>20-100</td>
<td>26</td>
<td>54-56</td>
<td>The primary maxima are intense. The maximum at 26 c.p.s. is most intense. Maximum at 78 c.p.s. is greater than at 40 c.p.s. The power at 100 c.p.s. region is greater than at 20 c.p.s. but lower than the value at 26 c.p.s.</td>
</tr>
<tr>
<td>August 8, 1963(3, 4)</td>
<td>Cloud flash</td>
<td>--</td>
<td>20-100</td>
<td>22</td>
<td>48</td>
<td>The primary maxima are sharp and equally intense. Power at 56 c.p.s. and 62 c.p.s. is greater than that at 48 c.p.s. Power at 100 c.p.s. region is comparable to that at low audio frequencies, though it is less than 72 and 22 c.p.s. maxima.</td>
</tr>
<tr>
<td>Record Number</td>
<td>Description</td>
<td>Time difference between lightning and thunder, sec.</td>
<td>Frequency band c.p.s.</td>
<td>Primary Maxima c.p.s.</td>
<td>Secondary Maxima c.p.s.</td>
<td>Remark</td>
</tr>
<tr>
<td>---------------</td>
<td>--------------------------------------</td>
<td>-----------------------------------------------------</td>
<td>-----------------------</td>
<td>-----------------------</td>
<td>-------------------------</td>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>August 8, 1963(1.1)</td>
<td>Cloud flash with many branches</td>
<td>10</td>
<td>20-100</td>
<td>24</td>
<td>32</td>
<td>Primary maxima are prominent and stand, in order of intensity, at 24 c.p.s., 78 c.p.s., 88-90 c.p.s., 70 c.p.s. The power at 100 c.p.s. region exceeds that at any maximum or at low audio frequencies.</td>
</tr>
<tr>
<td>August 8, 1963(2.1)</td>
<td>Complicated flash, Partly cloud and partly ground with branches</td>
<td>5</td>
<td>20-250</td>
<td>56</td>
<td>92</td>
<td>Primary maxima are sharp and intense and stand, in order of power, at 56 c.p.s., 66 c.p.s., 122 c.p.s., 204-206 c.p.s. The power at 122 c.p.s. is comparable to that at low audio frequencies. The spectrum did not change with the record length.</td>
</tr>
</tbody>
</table>
for all the records of 1963 for which spectral density was evaluated. All distributions were non-normal and leptokurtic. Figs. 5.23 to 5.26 illustrate the distributions of the records whose spectral densities are represented in Figs. 5.8, 5.11, 5.19 and 5.22 respectively.

The spectra are quite smooth and the maxima occur in the same general region in all records. Thus, it is concluded that the spectral density estimates are accurate.
Fig. 5.23 Histogram and the Gaussian curve of pressure amplitudes of the thunder record number July 5, 1963 (1.1). The spectral density of this record is presented in Fig. 5.8. The distribution is leptokurtic and non-normal.
Fig. 5. 24 Histogram and the Gaussian curve of pressure amplitudes of the thunder number July 9, 1963 (4.1). The spectral density of this record is presented in Fig. 5. 11. The distribution is leptokurtic and non-normal.
Fig. 5.25 Histogram and the Gaussian curve of pressure amplitudes of the thunder record number August 8, 1963 (3, 3). The spectral density of this record is presented in Fig. 5.19. The distribution is leptokurtic and non-normal.
Fig. 5, 26 Histogram and the Gaussian curve of pressure amplitudes of thunder record number August 8, 1963 (2, 1). The spectral density of this record is presented in Fig. 5, 22. The distribution is leptokurtic and non-normal.
CHAPTER 6

DIRECTION OF ARRIVAL AND NATURE OF PRESSURE VARIATIONS

6.1 Direction of arrival

The angle of incidence and the azimuth of a down-coming sound wave can be determined (Meisser, 1927) by recording its times of arrival at three different microphones separated from each other by a few hundreds of meters. By determining the directions of arrival of peals and claps of thunder and knowing the general direction of the lightning flash, it could be decided whether or not the peals and claps arrive directly from the flash or from a combination of the flash and reflection from graupel layers in the clouds.

6.1.1 Computational Methods

$P_0$, $P_1$ and $P_2$ are the three receiving microphones as shown in Figure 6.1. Imagine a Cartesian coordinate system such that the origin is at $P_0$, the $x$-axis is along the line $P_0P_1$, and the $z$-axis is vertically upwards. Assume the ground to be horizontal, then the coordinates of $P_0$, $P_1$ and $P_2$ are respectively, $(0,0,0)$, $(x_1,0,0)$, and $(x_2,y_2,0)$. Let the times of arrival of a plane, downcoming wave at the three microphones be respectively $T_0$, $T_1$ and $T_2$.

If the speed of the sound at the ground is $C$, and the wave normal makes angles $\alpha$, $\beta$, $\gamma$, with the $x$, $y$ and $z$ axes respectively

$$x_1 \cos \alpha = C(T_1 - T_0) = \Delta_1$$ (6.1)
Fig. 6.1 Coordinates of the three microphones.

Fig. 6.2 Coordinates and directions of the three hot-wire microphones used.
and
\[ x_2 \cos \alpha + y_2 \cos \beta = C(T_2 - T_0) = \Delta_2 \quad (6.2) \]

Also
\[ \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \quad (6.3) \]

Substituting the value of \( \cos \alpha \) from Eq. 6.1 into 6.2 yields
\[ \cos \beta = \frac{\Delta_2}{y_2} - \frac{x_2}{y_2} \frac{\Delta_1}{x_1} \quad (6.4) \]

Substituting the values of \( \cos \alpha \) and \( \cos \beta \) from Eqs. 6.1 and 6.4 into 6.3 leads to
\[ \sin \gamma = \left[ \left( \frac{\Delta_1}{x_1} \right)^2 + \left( \frac{\Delta_2}{y_2} - \frac{x_2}{y_2} \frac{\Delta_1}{x_1} \right)^2 \right]^{1/2} \quad (6.5) \]

and
\[ \tan \phi = \frac{\cos \beta}{\cos \alpha} = \frac{\Delta_2}{y_2} \frac{x_2}{y_2} \frac{\Delta_1}{x_1} \quad (6.6) \]

where \( \gamma \) is the angle with the vertical, and, hence, is the angle of incidence, and \( \phi \) is the azimuthal angle with respect to the x-axis. The angle of descent, the angles the downcoming wave makes with the horizontal, is simply \( \frac{\pi}{2} - \gamma \).

6.1.2 General arrangement

Three HWM were placed as shown diagrammatically in Fig. 6.2. The distances between the microphones and their directions relative to each other are shown in the Figure. The signals from the microphones were recorded on the three channels of the visicorder.
as described in Chapter 4. Observations were taken during thunderstorms on August 25 and 26, 1963. Values of temperatures were obtained from the Airport Weather Office, and were used to compute the speed of the sound waves. Table 6.1 gives the necessary parameters for computing $\sqrt{\gamma}$ and $\phi$.

Care was taken to ensure that such sets of records were due to the thunder from a single flash, and not from two or more flashes in quick succession. The recorder was equipped with a device for placing time marks on the records. This was used to determine the intervals between times of arrival at the different microphones. Time intervals, if necessary, could also be determined from the speeds of the recording paper. Two speeds were used -- 1 in/sec and 5 in/sec.

Estimates of the heights of the sources producing peals and claps were made by noting the time interval between a flash and the arrival of the audible thunder. Assuming that this time interval is the same as that between the flash and first large disturbance on the record, and a straightline path, and knowing the angle of arrival, the height can be determined.

6.1.3 Results

Only eight records were considered free from over-lapping effects of sound from two or more flashes occurring close to each other in time. Photographic reproductions of these records are presented in Figs. 6.3 to 6.10 inclusive. The records are marked for various peals and claps for which directions were determined. The
### TABLE (6.1)

**PARAMETERS FOR DIRECTION OF ARRIVAL**

<table>
<thead>
<tr>
<th>Date</th>
<th>Temp. °C</th>
<th>Speed of sound meters/sec.</th>
<th>$x_1$ meters</th>
<th>$x_2$ meters</th>
<th>$y_2$ meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 25, 1963 (1:00 a.m.)</td>
<td>15°</td>
<td>340.4</td>
<td>413.9</td>
<td>418.8</td>
<td>179.9</td>
</tr>
<tr>
<td>August 26, 1963 (3:30 p.m.)</td>
<td>18.9°</td>
<td>342.7</td>
<td>413.9</td>
<td>418.8</td>
<td>179.9</td>
</tr>
</tbody>
</table>
Fig. 6.3 Photographic reproduction of thunder record number August 25, 1963 (1.3) from a cloud flash on three hot-wire microphones. The top trace was on mic. $P_2$, middle on mic. $P_1$ and bottom on mic. $P_0$. The time interval between the lightning flash and the thunder heard was 5 seconds. The angles of incidence and azimuth for marked points are tabulated in Table 6.2.
Fig. 6.4 Photographic reproduction of thunder record number August 25, 1963 (1.4) from a cloud flash on three hot-wire microphones. The top trace was on mic. P₂, middle on mic. P₁ and bottom on mic. P₀. The time interval between the lightning flash and the thunder heard was 2 seconds. The angles of incidence and azimuth for marked points are tabulated in Table 6.2.
Fig. 6.5 Photographic reproduction of thunder record number August 26, 1963 (2.1) from a complicated flash (partly ground and partly cloud) on three hot-wire microphones. The top trace was on mic. $P_2$, middle on mic. $P_1$ and bottom on mic. $P_0$. The time interval between the lightning flash and the thunder heard was 5 seconds. The angles of incidence and azimuth for marked points are tabulated in Table 6.2.
Fig. 6.6 Photographic reproduction of thunder record number August 26, 1963 (2,3) from a cloud flash on three hot-wire microphones. The top trace was on mic. $P_2$, middle on $P_1$ and bottom on mic $P_0$. The time interval between the lightning flash and the thunder heard was 6 seconds. The angles of incidence and azimuth for marked points are tabulated in Table 6.2.
Fig. 6.7 Photographic reproduction of thunder record number August 26, 1963 (2.5) cloud flash on three hot-wire microphones. The top trace was on mic. P₂, middle on mic. P₁ and bottom on mic. P₀. The time interval between the lightning flash and the thunder heard was 8 seconds. The angles of incidence and azimuth for marked points are tabulated in Table 6.2.
Fig. 6.8 Photographic reproduction of thunder record number August 26, 1963 (2.6) from an overhead cloud flash on three hot-wire microphones. The top trace was on mic. P₂, middle on mic. P₁ and bottom on mic. P₀. The time interval between the lightning flash and the thunder heard was 6 seconds. The angles of incidence and azimuth for marked points are tabulated in Table 6.2.
Fig. 6.8 continued
Fig. 6.9 Photographic reproduction of thunder record number August 26, 1963 (2.7) from a ground flash on three hot-wire microphones. The top trace was on mic. $P_2$, middle on mic. $P_1$ and bottom on mic. $P_0$. The time interval between the lightning flash and the thunder heard was 7 seconds. The angles of incidence and azimuth for marked points are tabulated in Table 6.2.
Fig. 6.10 Photographic reproduction of thunder record number August 26, 1963 (2,8) from two simultaneous ground flashes on three hot-wire microphones. The top trace was on $P_2$, middle on mic. $P_1$, and bottom on mic. $P_0$. The time interval between the lightning flash and the thunder heard was 5 seconds. The angles of incidence and azimuth for marked points are tabulated in Table 6.2.
angles of incidence and azimuth, description of sounds and lightning flashes, times of arrival, and the heights are tabulated in Table 6.2.

The angles of incidence vary in the range of $1^\circ 49' \text{ to } 79^\circ 14'$ (measured as indicated previously from the vertical). The polar plot of the HWM shows that the response falls off rapidly for sounds impinging at angles greater than $75^\circ$. This accounts for the absence of waves incident at higher angles, or from distant flashes. As discussed later in Chapter 7, sounds produced close to the ground are likely to be refracted upward, and thus miss the microphones.

The accuracy in the determination of angles of incidence and azimuth depends largely on the time resolution. An error may also arise from the values of the speed of sound which are used. The temperatures used were for the Saskatoon Air Terminal, a distance of about ten miles from the site. Temperature also varies with height. The humidity of the air also affects the speed, but this is small compared to the other errors. The error in speed of sound should not affect the angles appreciably and are likely to be in the same sense. Maximum possible errors are estimated to be 2 per cent in the angle of incidence, and 5 per cent in the azimuthal angle. Other errors may arise due to the fact that the ground was not horizontal and that the polar plots of all the three microphones were not exactly the same.

The errors in height determinations are large. The first assumption of a straight-line path is obviously invalidated whenever
## TABLE (6, 2)

**DIRECTION OF ARRIVAL**

<table>
<thead>
<tr>
<th>Record Number</th>
<th>Number</th>
<th>Angle of incidence</th>
<th>Azimuth</th>
<th>Time taken for sound to arrive (sec)</th>
<th>Height (km)</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 25, 1963 (1, 3) Cloud flash</td>
<td>1</td>
<td>39° 58'</td>
<td>71° 20'</td>
<td>5.0</td>
<td>1.30</td>
<td>1 is the beginning of a large peal.</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>67° 50'</td>
<td>37° 46'</td>
<td>17.0</td>
<td>2.18</td>
<td>2 is the beginning of another peal. The thunder builds up gradually.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>August 25, 1963 (1, 4) Cloud flash</td>
<td>1</td>
<td>50° 56'</td>
<td>81° 52'</td>
<td>2.0</td>
<td>0.43</td>
<td>1 is the beginning of first large peal.</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>43° 39'</td>
<td>36° 27'</td>
<td>5.8</td>
<td>1.42</td>
<td>2 is the beginning of second large peal.</td>
</tr>
<tr>
<td></td>
<td>2(a)</td>
<td>29° 49'</td>
<td>52° 30'</td>
<td>6.9</td>
<td>2.04</td>
<td>2(a) is the end of second large peal.</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>66° 30'</td>
<td>102° 57'</td>
<td>42.5</td>
<td>5.77</td>
<td>3 is the beginning of a clap obtained in the last part of record.</td>
</tr>
<tr>
<td>Number</td>
<td>Angle of incidence °</td>
<td>Azimuth °</td>
<td>Height km.</td>
<td>Time taken to arrive P₀ sec.</td>
<td>Time of arrival of sound</td>
<td>Remark</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------</td>
<td>-----------</td>
<td>------------</td>
<td>------------------------------</td>
<td>------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>1</td>
<td>37° 42'</td>
<td>57° 12'</td>
<td>1.35</td>
<td>1</td>
<td>1 is the arrival of disturbances on mics.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>66° 52'</td>
<td>319° 56'</td>
<td>2.82</td>
<td>2</td>
<td>2 is the beginning of first peal.</td>
<td></td>
</tr>
<tr>
<td>2(a)</td>
<td>74° 18'</td>
<td>318° 35'</td>
<td>2.82</td>
<td>2(a), 2(b), 2(c) are the fine structures of first peal.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2(b)</td>
<td>79° 14'</td>
<td>315° 45'</td>
<td>2.86</td>
<td>2(b), 2(c) are the fine structures of first peal.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2(c)</td>
<td>67° 16'</td>
<td>321° 45'</td>
<td>2.74</td>
<td>2(c) are the fine structures of first peal.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>69° 9'</td>
<td>316° 58'</td>
<td>2.74</td>
<td>2(c) are the fine structures of first peal.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>50° 39'</td>
<td>298° 50'</td>
<td>7.41</td>
<td>4, 5, 6, 7, and 8 are beginnings of second large peal after second peal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>46° 6'</td>
<td>299° 15'</td>
<td>7.41</td>
<td>4, 5, 6, 7, and 8 are beginnings of second large peal after second peal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>50° 39'</td>
<td>298° 50'</td>
<td>7.41</td>
<td>4, 5, 6, 7, and 8 are beginnings of second large peal after second peal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>51° 36'</td>
<td>322° 0'</td>
<td>7.75</td>
<td>4, 5, 6, 7, and 8 are beginnings of second large peal after second peal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>77° 21'</td>
<td>311° 41'</td>
<td>7.90</td>
<td>4, 5, 6, 7, and 8 are beginnings of second large peal after second peal</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**August 26, 1963 (2,1)**  
Complicated flash; partly cloud and partly ground

<table>
<thead>
<tr>
<th>Number</th>
<th>Angle of incidence °</th>
<th>Azimuth °</th>
<th>Height km.</th>
<th>Time taken to arrive P₀ sec.</th>
<th>Time of arrival of sound</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29° 16'</td>
<td>25° 17'</td>
<td>6.0</td>
<td>1</td>
<td>1 is the arrival of first large peal.</td>
<td></td>
</tr>
<tr>
<td>1(a)</td>
<td>40° 10'</td>
<td>24° 0°</td>
<td>1.79</td>
<td>1(a) is the fine structure of first large peal.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1(b)</td>
<td>55° 55'</td>
<td>81° 43'</td>
<td>1.67</td>
<td>1(b) is the fine structure of first large peal.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>45° 59'</td>
<td>81° 43'</td>
<td>1.80</td>
<td>2</td>
<td>2 is the arrival of first large peal.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>50° 10'</td>
<td>89° 24'</td>
<td>1.80</td>
<td>3</td>
<td>another large peal.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>52° 13'</td>
<td>328° 58'</td>
<td>5.15</td>
<td>4</td>
<td>and 5 are claps received later on.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>75° 41'</td>
<td>27° 35'</td>
<td>5.32</td>
<td>5</td>
<td>and 5 are claps received later on.</td>
<td></td>
</tr>
</tbody>
</table>
Table (6.2) continued

<table>
<thead>
<tr>
<th>Record Number</th>
<th>Number</th>
<th>Angle of incidence</th>
<th>Azimuth</th>
<th>Time taken for sound to arrive P₀ sec.</th>
<th>Height km.</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 26, 1963 (2.5) Cloud flash overhead</td>
<td>1</td>
<td>16° 8'</td>
<td>81° 26'</td>
<td>8.0'</td>
<td>2.62</td>
<td>1 is the beginning of a peal, easily identified on all the records.</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>50° 23'</td>
<td>33° 2'</td>
<td>9.97</td>
<td>2.18</td>
<td>2 is the beginning of largest peal.</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>20° 41'</td>
<td>241° 16'</td>
<td>12.47</td>
<td>3.99</td>
<td>3 is another peal.</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>11° 48'</td>
<td>130° 21'</td>
<td>18.42</td>
<td>6.11</td>
<td>4 and 5 are claps received later on.</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>44° 52'</td>
<td>91° 21'</td>
<td>19.20</td>
<td>4.65</td>
<td>The record is complex.</td>
</tr>
<tr>
<td>August 26, 1963 (2.6) Cloud flash overhead</td>
<td>1</td>
<td>4° 28'</td>
<td>254° 38'</td>
<td>6.0</td>
<td>2.04</td>
<td>1 is the beginning of first peal.</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1° 49'</td>
<td>218° 27'</td>
<td>6.48</td>
<td>2.22</td>
<td>2 is the beginning of second, larger than the first in amplitude, peal.</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5° 19'</td>
<td>190° 23'</td>
<td>7.36</td>
<td>2.51</td>
<td>3 and 4 are later claps.</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2° 25'</td>
<td>191° 24'</td>
<td>7.61</td>
<td>2.60</td>
<td>5 is another large peal.</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>69° 18'</td>
<td>211° 48'</td>
<td>24.28</td>
<td>2.94</td>
<td>6 is another large peal.</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>49° 43'</td>
<td>183° 0'</td>
<td>25.38</td>
<td>5.61</td>
<td>7 and 8 are claps received later on.</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>32° 53'</td>
<td>241° 47'</td>
<td>26.08</td>
<td>7.48</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>24° 31'</td>
<td>79° 4'</td>
<td>44.18</td>
<td>13.60</td>
<td></td>
</tr>
<tr>
<td>Record Number</td>
<td>Number</td>
<td>Angle of incidence</td>
<td>Azimuth</td>
<td>Time taken for sound to arrive PO·sec.</td>
<td>Height km.</td>
<td>Remark</td>
</tr>
<tr>
<td>---------------</td>
<td>--------</td>
<td>--------------------</td>
<td>---------</td>
<td>--------------------------------------</td>
<td>------------</td>
<td>--------</td>
</tr>
<tr>
<td>August 26, 1963 (2.7) Ground flash</td>
<td>1</td>
<td>42° 36'</td>
<td>265° 26'</td>
<td>7.0</td>
<td>1.76</td>
<td>1 is the first large peal.</td>
</tr>
<tr>
<td></td>
<td>1(a)</td>
<td>3° 11'</td>
<td>40° 26'</td>
<td>7.64</td>
<td>2.62</td>
<td>1(a) is the end of first peal.</td>
</tr>
<tr>
<td></td>
<td>1(b)</td>
<td>7° 38'</td>
<td>15° 8'</td>
<td>7.80</td>
<td>2.65</td>
<td>1(b) is the beginning of another small peal.</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>19° 39'</td>
<td>80° 47'</td>
<td>8.11</td>
<td>2.62</td>
<td>2 is the beginning of second large, not as large as first one, peal.</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>33° 36'</td>
<td>290° 8'</td>
<td>15.55</td>
<td>4.43</td>
<td>3 is the beginning of another peal received long after 2.</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>20° 16'</td>
<td>276° 52'</td>
<td>17.77</td>
<td>5.70</td>
<td>4 is the beginning of claps received much later.</td>
</tr>
<tr>
<td>August 26, 1963 (2.8) Two simultaneous ground flashes</td>
<td>1</td>
<td>70° 0'</td>
<td>308° 6'</td>
<td>5.0</td>
<td>0.59</td>
<td>1 is the arrival of first large peal,</td>
</tr>
<tr>
<td></td>
<td>1(a)</td>
<td>73° 52'</td>
<td>309° 37'</td>
<td>5.03</td>
<td>0.48</td>
<td>1(a) is fine structure of first large peal.</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>53° 53'</td>
<td>302° 18'</td>
<td>10.21</td>
<td>2.30</td>
<td>2 is the arrival of second large peal.</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8° 23'</td>
<td>76° 52'</td>
<td>15.53</td>
<td>5.25</td>
<td>3 is the arrival of a clap received later on.</td>
</tr>
</tbody>
</table>
there is any reflection and refraction. Also, the assumption that the time elapsed between the flash and thunder is same as that between flash and the first analysed disturbance is violated when smaller amplitude disturbances come before the larger disturbances. Hence, the height measurements should be viewed as approximate.

The record number August 25, 1963 (1.3), as illustrated in Fig. 6.3, indicates that the sounds from nearest part of the flash arrived earlier than those originating at distant parts of the flash. The first peal is from the nearest part of the flash and the second peal, received after 12 seconds, is from a higher part of the flash.

The record number August 25, 1963 (1.4), as represented in Fig. 6.4, shows that different peals of thunder arrive from different parts of the flash, the earlier peal originated from a nearer part of the flash. The height corresponding to the beginning of second peal (no. 2) is less than that of its end (no. 2a), and this suggests that the beginning of a peal is from the nearest point at the lightning channel and the end determines the farthest point on it. The angles of azimuth corresponding to clap no. 3 (this impulse is much smaller in amplitude and is associated with rumblings) is appreciably different from the others, and this could be due either to a reflection from a graupel layer or to a long branch hidden in the clouds. Its height is quite large, and could well be from a hidden branch in the cloud.

The record number August 26, 1963 (2.1), as illustrated in Fig. 6.5 is complicated. This record of thunder is from a complex
flash. It suggests that sounds received at an observer originate at
different parts of the lightning channel, and hence, thunder is due to
the superimposition of acoustic waves produced at different parts of
the lightning flash. Various peals arrive from different directions.
The small-amplitude short period waves superimposed on the larger
amplitude long period waves (called peals) arrive from the same
general directions. This is evident from the values of angles of
peals numbers 2, 2a, 2b, and 2c. This indicates that lower and
higher frequency pressure variations originate from the same source.
All the claps which are received later and are associated with rum-
blings arrive from higher parts of the flash. The exception is the
last clap (no. 8) which is not from a higher part of the flash but
from a more distant part of the flash. Its azimuth is not distinct
and hence any reflection from a graupel layer is improbable. There
is no clear evidence of a reflection in this record.

The record number August 26, 1963 (2, 3), reproduced
in Fig. 6.6 shows as in others that various peals and claps arrive
from different directions and originate at different parts of the flash.
All the peals and claps, except the last one, have arrived directly
from the flash. The last clap, no. 5, has arrived from a different
direction, its azimuth being significantly different from the others.
This can be only explained as due to a reflection from a graupel layer
in the clouds (Remillard, 1960) provided no obscure flash or a long
branch of the same flash occurred in that direction. The probability
of an obscure flash is small as its height is only 0.6 km higher than
the first flash and should have been observed by eye. The possibility of a long branch cannot be ruled out, though it seems unlikely.

The record number August 26, 1963 (2, 5), as shown in Fig. 6.7, is from an overhead cloud flash. It indicates that various peals and claps originate at various parts of the flash and arrive directly. There is no evidence of a reflection.

The record number August 26, 1963 (2, 6), as represented in Fig. 6.8, is from an overhead flash. It also shows that different peals and claps arrive from different parts of the flash. The difference between the first and second peal is only 0.48 seconds. The most frequent time interval between strokes is 40 milliseconds (Schonland, 1956) and this suggests that small-amplitude, short-period waves could be due to different strokes. In Fig. 6.10 the difference between peals 1 and 1a is only 0.03 seconds, and hence, 1a, which has arrived from the same direction as 1, could be attributed to a different stroke. An extremely rare value of 450 milliseconds time interval between strokes has been observed (Schonland, 1956) and this compares favourably with 0.48 seconds. Thus, the peals 1 and 2 could possibly be due to different strokes occurring at the same place. If two sound radiating channels were only 300 meters apart, the difference between the peals due to these two channels would be about 1 second. The spatial geometry of the flash gives such several varying distances, and hence the time interval between the peals may vary. The claps 3 and 4 arrive from the same general
direction as 1 and 2. About 16.7 sec later than clap no. 4, large peaks number 5 and 6, arrive, but their azimuths are not significantly different, and therefore any reflection is unlikely. A possibility of an obscure flash is still present. The last clap, no. 8, arrived from a different direction, its azimuth being significantly different from the others. This could only be due to a reflection, if higher branches or obscure flashes are ruled out. The height corresponding to this clap is 13.6 km, a value higher than the average height of cumulus thunderstorm clouds (Mason, 1957). Only rarely have intense thunderstorm clouds of greater heights been observed (Ludlam and Macklin, 1959). Furthermore, flashes from the top of clouds to the upper air have also been detected (Malan and Schonland, 1951a). Thus this large value of height may not be greatly in error.

The record number August 26, 1963, (2, 7) as reproduced in Fig. 6.9, is the only one from a ground flash. The first peal (no. 1) arrived directly from the flash. The height corresponding to this peal is 1.7 km. The critical height, discussed in Chapter 7, is only 17 meters. It is calculated using Eq. 7.18 and assuming $T = 292^\circ$K, $\alpha = 7.5^\circ$C/km, and $x = 2.39 \sin 42^\circ 36' = 1.62$ km. Thus, we see that the height corresponding to peal no. 1 is much greater than the critical height. Before peal no. 1 has ended, reflected waves seem to have arrived, the azimuth of 1a and 1b being different, and the second peal is due to reflection. The peal no. 3 and clap no. 4 have arrived directly from the flash. Assuming that the flash was straight and peal no. 1 is from the nearest part and clap 4 is from
the farthest part of the flash, the calculated height of the stroke is 5.7 km, a value in close agreement to 6 km obtained by Malan and Schonland (1951b). Recent measurements (Brook, Kitagawa and Workman, 1962) indicate that the weighted average height for the hybrid flashes is 5.1 km and for discrete flashes, 3.9 km.

The record number August 26, 1963 (2.8), as represented in Fig. 6.10, is of thunder resulting from two simultaneous, widely-separated (about 1 or 2 km), ground flashes that occurred in the same general direction. Two distinct sets of peals, no. 1 and 2, have arrived directly from these two flashes, the difference in azimuth being only 6 degrees. The height of the source of the first peal is much smaller than for the second peal, but is much greater than the critical height which is 16 meters in this case. The small-amplitude, short-period wave, represented by no. 1a and received after 0.03 sec from the peal number 1, arrived directly from the flash, its azimuth not being significantly different, and is conceivably due to a separate stroke. The azimuth of clap no. 3 is appreciably different from the others and should be attributed to a reflection. Again, the possibility of an obscure flash cannot be denied.

6.2 Nature of the Pressure variations

One way of probing into the mysteries of thunder is to examine the nature of the pressure variations of thunder and compare them with those of explosions. Schmidt (1914) found that the strongest
deflection in the records of thunder is a rarefaction. Arabadji (1952), on the other hand, challenging the result of Schmidt, claimed this to be a compression.

A twelve-inch woofer, used as a microphone was employed to investigate the nature of pressure variations. The characteristics of the woofer and the details of the set up are discussed in Chapter 4. In spite of strong resonances, use of the woofer made it possible to distinguish between a compression and a rarefaction. Fig. 6.11 shows four records, two on a woofer and the other two on a HWM, of the impulsive pressure wave from a shot gun. It shows clearly the sharp compressional impulse preceeding the longer lasting rarefactive change on the woofer records. On the HWM the deflection is unidirectional.

6.2.1 Records of thunder

During 1962 only 52 records were considered free from overlapping effects of the sounds from two or more flashes occurring close to each other in time. The initial pressure impulse from 90 per cent of these corresponded to a compression. In cases where the flashes were overhead and roughly horizontal, the impulses always corresponded to a compression. Figure 4.20 shows records where the initial impulse is a compression.

In 1963 besides the first peal, all the large peals and claps of thunder in the records were analysed. In many cases, particularly when the pressure variation is gradual and not large, it was extremely
Fig. 6.11 Records of a shot gun. (a) record on a hot-wire microphone with maximum sensitivity. (b) record on the hot-wire microphone with a sensitivity 1/20 of (a). (c) and (d) records on the woofer with maximum sensitivity. A downward deflection indicates a compression and an upward deflection indicates a rarefaction. The arrows on the woofer record point out the compressional impulse preceeding the longer lasting rarefaction.
difficult to know whether or not a peal or clap was compressive. But a good number of peals and claps were worth analysing. The compressional pressure variations were observed in the records of thunder from near and distant flashes, but the rarefactive pressure variations were observed only in the records of thunder from distant flashes. The rarefactive pressure variations were small in amplitude. Table 6.3 tabulates the number of compressional and rarefactive pressure variations, and the percentage of compression on various dates. It shows that 81.3 per cent of the large pressure variations are compressional in nature.

Fig. 6.12 indicates five large compressional peals obtained from a near ground flash. This figure is an enlarged photograph of a part of the whole record represented in Fig. 4.9. Fig. 6.13 shows two rarefactive peals, one compressional peal, and one compressional peal followed by a rarefactive peal; the latter types are rare.

Steep compressional shock waves produced at the flash travel with supersonic speeds. The following rarefaction, in due course, overtakes the front (Duvall, 1962) and then the amplitude and velocity begin to decrease and continue to do so until the pulse is reduced to acoustic amplitude. These compressional acoustic waves would give rise to compressional pressure variations.

It has been suggested that the pinching effect, which is the contraction of the channel carrying current due to magnetic field of the current itself, may give rise to rarefactive pressure variations.
### TABLE (6.3)

**NATURE OF PRESSURE VARIATION**

<table>
<thead>
<tr>
<th>Date</th>
<th>Number of Compressional pressure variations</th>
<th>Number of rarefactive pressure variations</th>
<th>Percentage of Compressional pressure variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962</td>
<td>47</td>
<td>5</td>
<td>90.0</td>
</tr>
<tr>
<td>June 24, 1963</td>
<td>8</td>
<td>4</td>
<td>66.7</td>
</tr>
<tr>
<td>June 29, 1963</td>
<td>72</td>
<td>14</td>
<td>84.2</td>
</tr>
<tr>
<td>July 5, 1963</td>
<td>169</td>
<td>47</td>
<td>78.3</td>
</tr>
<tr>
<td>July 9, 1963</td>
<td>50</td>
<td>10</td>
<td>83.3</td>
</tr>
<tr>
<td>July 15, 1963</td>
<td>5</td>
<td>1</td>
<td>83.3</td>
</tr>
<tr>
<td>July 16, 1963</td>
<td>36</td>
<td>6</td>
<td>85.7</td>
</tr>
<tr>
<td>July 20, 1963</td>
<td>37</td>
<td>5</td>
<td>88.1</td>
</tr>
<tr>
<td>July 24, 1963</td>
<td>43</td>
<td>9</td>
<td>82.7</td>
</tr>
<tr>
<td>August 8, 1963</td>
<td>15</td>
<td>3</td>
<td>83.3</td>
</tr>
<tr>
<td><strong>Total during 1963</strong></td>
<td><strong>435</strong></td>
<td><strong>99</strong></td>
<td><strong>81.3</strong></td>
</tr>
</tbody>
</table>
Fig. 6.12 Photographic reproduction of five compressional peals of thunder from a ground flash. The top trace was on a hot-wire microphone, middle on a wide-range crystal microphone and the bottom on a woofer. The upward deflection on the woofer record indicates a compression and a downward deflection indicates a rarefaction. The initial deflection of the peals is upward as shown by arrows.
Fig. 6.13 Reproduction of four records of thunder showing the nature of pressure variations. The top trace was on a hot-wire microphone, middle on a wide-range crystal microphone and bottom on a woofer. The upward deflection on the woofer record indicates a compression and downward deflection indicates a rarefaction. (a) Compressional pressure variation. (b) Rarefactive pressure variation. (c) Rarefactive pressure variation. (d) A compressional pressure variation followed by a rarefactive pressure variation.
Theoretical considerations (Remillard, 1960) as outlined below show that such an effect will not contribute significantly to thunder.

The mechanical force per unit volume is \( \mu jH \), where \( \mu \) is the permeability of the medium, \( j \) is the conduction current density, and \( H \) is the magnetic intensity. It is directed inwards toward the axis of the lightning channel (Cowling, 1957). Cylindrical coordinates \((R, \phi, Z)\) are used such that the axis of the channel is along the \( Z \) axis. Thus the \( j \) and \( H \) at \( R \) are functions of \( R \) only. The equilibrium equation, using mks system of units, is

\[
\frac{dP}{dR} + \mu jH = 0 \quad (6.7)
\]

where \( P \) is the pressure at \( R \). The \( j \) is given by

\[
j = \frac{1}{R} \frac{d}{dR} (RH) \quad (6.8)
\]

Substituting for \( j \) in Eq. 6.7 we have

\[
\frac{dP}{dR} + \frac{\mu}{R} \frac{d}{dR} (RH) = 0 \quad (6.9)
\]

If \( j \) equals a constant \( J \) inside \( R = a \), and vanishes outside

\[
H = \frac{RJ}{2}, \quad R \ll a \quad (6.10)
\]

Thus

\[
\frac{dP}{dR} = -\frac{\mu RJ^2}{2}, \quad R \ll a \quad (6.11)
\]

and

\[
\frac{dP}{dR} = 0, \quad R > a \quad (6.12)
\]

Hence the pressure outside \( R = a \) is a constant \( P_0 \), and
the pressure inside $R = a$ is given by

$$P = P_0 + \frac{\mu J^2}{4} (a^2 - R^2) \quad (6.13)$$

The difference in pressure between the inside and the outside of the channel is

$$\Delta P = P - P_0 = \frac{\mu J^2}{4} (a^2 - R^2) \quad (6.14)$$

Remillard (1960) estimated this to be about 0.022 atmospheres while the pressure provided by the thermal pressure due to shock wave is about 300 atmospheres as said in Chapter 3. Thus it appears that the pinch effect is of minor importance. However, to get an accurate picture the actual radial distribution of current in the channel would have to be known.

Our results agree with those of Arabadji but disagree with those of Schmidt. If the results of Schmidt are correct, which most probably they are, then different conditions of discharges must occur. At Saskatoon, most of the discharges are of the same nature as that observed in the USSR.
CHAPTER 7

FURTHER INVESTIGATIONS

7.1 Remillard's Invariant Quantity

An interesting theoretical conclusion about the frequency spectrum of thunder, suggested by Remillard (1960), is that the product of frequency maximum and the observation distance is constant. This section deals with this problem.

7.1.1 Theory*

It is assumed that the frequency spectrum corresponding to the largest peal by itself should closely resemble the frequency spectrum of thunder as a whole. The pressure variation for the largest peal, which is rarefactive according to Remillard's theoretical analysis, is given by

\[ p(b, t) = \frac{a^2 C_t P/2}{[(C_t)^2 - b^2]^{3/2}} \]  

(7.1)

where \( P \) is the atmospheric pressure, \( C \) is the velocity of sound, \( a \) is the radius of cylindrical channel of lightning radiation sound, \( b \) is the observing distance, and \( t \) is the time. Eq. 7.1 is derived in Appendix D. It shows

\[ p(b, t) \propto \frac{C_t}{[(C_t)^2 - b^2]^{3/2}} = \frac{C(t/b)}{b^2 [C^2(\frac{t}{b})^2 - 1]^{3/2}} \]

or

\[ b p(b, t) \propto \frac{1}{b} D(t/b) \]  

(7.2)

* The treatment adopted here is the same as given by Remillard (1960).
where
\[ D \left( \frac{t}{b} \right) = \frac{C(t/b)}{\left[ C^2(t/b)^2 - 1 \right]^{3/2}} \]  
(7.3)

Eq. 7.2 indicates that it is possible to describe the product of the direct pressure wave with the observation distance \( b \) as some function \( D \) of the ratio \( \frac{t}{b} \) divided by the observation distance \( b \).

The Fourier transform of Eq. 7.2 with respect to time is
\[ b \int_{-\infty}^{\infty} P(b, t) e^{j\omega t} dt \propto \frac{1}{b} \int_{-\infty}^{\infty} D(t/b) e^{-j\omega t} dt \]  
(7.4)

Substituting \( \xi = t/b \) on right-hand-side we have
\[ b \overline{\rho(b, \omega)} \propto \int_{-\infty}^{\infty} \overline{D(\xi)} e^{-j\omega b \xi} d\xi \]  
(7.5)

where the bar signifies the transformed quantity. The right-hand-side of Eq. 7.5 is \( D(b\omega) \) by definition of the Fourier transform. Hence
\[ b \overline{\rho(b, \omega)} \propto D(b\omega) \]  
(7.6)

The significance of Eq. 7.6 can be illustrated as follows:
Assume that the transform \( \rho(b, \omega) \) of the pressure radiated from a cylindrical source has been measured at a distance \( b \) from the source, and plot \( \rho(b, \omega) \) on \( Y \)-axis versus \( \omega \) on \( X \)-axis. Now, if both the abscissa and ordinate of the coordinate system are multiplied by \( b \), the curve will maintain its shape and the ordinate can be relabeled as \( D(b\omega) \) according to Eq. 7.6 so that the shape of the curve is indepen-
dent of observation distance. In other words, the product of the Fourier transform of the sound pressure wave with the observation distance is a function of the product of the observation distance with frequency $\omega$, and this function is the same irrespective of the distance of the recording instrument from the lightning channel. The theoretical value of the invariant quantity, product of frequency maximum and observation distance, is $9.8 \text{ km-cycles per second}$.

7.1.2 Experimental Results

Thirteen thunder records of suitable length, for which the distance of the lightning flash could be determined, were available. The spectral density estimates were obtained using the analysis described in Chapter 5. The flash distances were computed by noting the time intervals between flash and thunder, and multiplying these times by the speed of sound (taken to be 340 meters per second).

Objections are raised against this method of determining the distance of the flash, especially when the lightning channel is close by (Montigny, 1860; Schonland, 1950; Battan, 1964). The first objection (Schonland, 1950; Battan, 1964) arises because of the large spatial geometry of the flash. If a flash is close to the observer and a long branch is just overhead, the first sound of the thunder reaching the observer will be due to the branch. Hence, even though the main part of the flash may be far, the observer would consider it to be near confusing it with the branch. The second objection (Montigny, 1860) is that the loud sound travels faster than the ordinary sound, hence,
the distance computed using the velocity of sound will be wrong. However, an error arising because of the supersonic speed of the shock wave will be insignificant for the range of shock wave is only a few meters, as discussed in Chapter 3.

The observations and results are tabulated in Table 7.1. If an invariant characteristic exists, then a plot of frequency maximum versus reciprocal distance or reciprocal time interval between flash and the thunder should be a straight line. Fig. 7.1 illustrates the plot of the most intense frequency maximum versus reciprocal time interval between flash and thunder. The best statistical line for the observations and the theoretical straight line are plotted. The invariant quantity was determined from the slope of the curve. The computations were done on the LGP-30 computer, for which a suitable programme was available. The slope (frequency/time\(^{-1}\)) was 39.0 cycles. Using 340 meters per second as the speed of sound, the invariant quantity was calculated. It was 13.3 km-cycles per second. Fig. 7.2 shows the plot of all primary frequency maxima found in the spectral density estimate versus reciprocal of time interval between flash and thunder. The invariant quantity, calculated from the slope of this plot was 12.2 km-cycles per second.

The experimental values of the invariant quantity and the nature of the plots do not conform to the theoretical deductions as suggested by Remillard. Hence, we conclude that no such invariant characteristic exists as far as thunder in the Saskatoon region is concerned.
<table>
<thead>
<tr>
<th>Date</th>
<th>Time difference between lightning and thunder, sec.</th>
<th>Frequency as calculated from spectral analysis</th>
<th>Reciprocal of time</th>
<th>distance km</th>
<th>Remillard Invariant Quantity, Freq. x distance (Primary max. is taken, km c/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 8, 1963</td>
<td>12</td>
<td>2.00 c/s</td>
<td>.0833</td>
<td>4.08</td>
<td>8.16</td>
</tr>
<tr>
<td>August 8, 1963</td>
<td>15</td>
<td>2.00 c/s</td>
<td>.0667</td>
<td>5.10</td>
<td>10.20</td>
</tr>
<tr>
<td>July 5, 1963</td>
<td>10</td>
<td>2.00 c/s 2.75 c/s</td>
<td>.1000</td>
<td>3.40</td>
<td>6.80</td>
</tr>
<tr>
<td>July 5, 1963</td>
<td>14</td>
<td>2.00 c/s 1.00 c/s</td>
<td>.0714</td>
<td>4.76</td>
<td>9.52</td>
</tr>
<tr>
<td>July 9, 1963</td>
<td>12</td>
<td>1.00 c/s 2.25 c/s</td>
<td>.0833</td>
<td>4.08</td>
<td>4.08</td>
</tr>
<tr>
<td>July 9, 1963</td>
<td>14</td>
<td>2.25 c/s 1.5 c/s 0.75 c/s</td>
<td>.0714</td>
<td>4.76</td>
<td>10.72</td>
</tr>
<tr>
<td>July 9, 1963</td>
<td>35</td>
<td>5 c/s 4 c/s</td>
<td>.0286</td>
<td>11.9</td>
<td>59.50</td>
</tr>
<tr>
<td>June 29, 1963</td>
<td>12</td>
<td>1 c/s</td>
<td>.0833</td>
<td>4.08</td>
<td>4.08</td>
</tr>
<tr>
<td>June 29, 1963</td>
<td>15</td>
<td>2 c/s</td>
<td>.0667</td>
<td>5.10</td>
<td>10.20</td>
</tr>
<tr>
<td>June 29, 1963</td>
<td>19</td>
<td>4 c/s 2.25 c/s</td>
<td>.0526</td>
<td>6.46</td>
<td>25.84</td>
</tr>
<tr>
<td>June 29, 1963</td>
<td>13</td>
<td>1.75 c/s 0.75 c/s</td>
<td>.0769</td>
<td>4.42</td>
<td>7.74</td>
</tr>
<tr>
<td>July 21, 1962</td>
<td>16</td>
<td>2.33 c/s</td>
<td>.0625</td>
<td>5.44</td>
<td>12.68</td>
</tr>
<tr>
<td>August 19, 1962</td>
<td>23</td>
<td>1.25 c/s 4 c/s</td>
<td>.0435</td>
<td>7.82</td>
<td>9.75</td>
</tr>
</tbody>
</table>
Fig. 7.1 Plot of the most intense frequency maximum versus reciprocal time interval between flash and the thunder.
Fig. 7.2 Plot of the frequency maximum versus reciprocal time interval between flash and the thunder.
7.2 Audibility of Thunder

Infrasonic acoustic waves from large atmospheric nuclear explosions travel around the globe and have been observed by Yamamoto (1954, 1956, 1957), Donn and Ewing (1962a, 1962b), and Bhartendu and Currie (1964). The so-called anomalous propagation of the sound waves from artificial explosions over hundreds of kilometers (Meisser, 1930; Whipple, 1935; Gutenberg, 1951; Cox, 1957; Reinelt, 1964) due to their refraction and reflection at heights between 20 and 50 km in the atmosphere is well known. On the other hand, though distances as great as 100-120 km have been reported (Veenema, 1917, 1918, 1920; Breton, 1928; Taljaard, 1952), the normal distance to which thunder can be heard is seldom greater than about 25 km (Humphreys, 1940; Albright, 1947). A 16-inch gun liberates about $7.2 \times 10^9$ joules* of energy in a concentrated explosion which can be heard for about 48 km (Loeb, 1954). The energy liberated in a lightning flash is about $4.8 \times 10^{10}$ joules (Remillard, 1960) and in a stroke $1.6 \times 10^{10}$ joules, as discussed in Chapter 2. The smaller range of audibility of thunder in comparison with the sound waves produced from artificial explosions arises mainly because the source of sound and the medium of sound propagation are appreciably different in the two cases. In the case of artificial explosion sound starts from the same place, and the energy is concentrated. In the case of thunder, on the other hand, it is stretched out over the entire length of the lightning flash, and the energy is diffused through an extensive volume.

* 1 metric ton of TNT (Trinitrotoluene) releases about $4.5 \times 10^9$ joules of energy (Cox, 1958).
This difference in concentration may very well be one of the reasons to limit the audibility of thunder.

The medium is also important and should not be overlooked. The sound from an explosion is heard farthest when the air is still, and when there are temperature inversions. Conversely, sound is heard to the least distance when the atmosphere is irregular with respect to temperature, and, or, moisture distribution (Humphreys, 1940). These conditions favour the production of internal sound reflections and dissipation of energy. In the case of thunder, these latter conditions occur and limit the audibility.

The attenuation of sound for the frequencies under consideration, due to humidity, fog, rain, ordinary temperature and wind refractions, irregularities in the wind structure (gustiness), and ground propagation has been studied (Ingard, 1953; Parkin, and Scholes, 1954). The most common factors which affect the propagation of sound waves are temperature and wind and will be discussed in the next section. Detailed discussions are reported elsewhere (Mitra, 1952; Reinelt, 1964).

7.2.1 Effect of Temperature and Wind*

The treatment in this section will be from the point of view of geometrical acoustics, with the sound waves regarded as rays.

First, consider a stratified medium with no wind shear but with a linear lapse rate given by

$$T = T_0 - \alpha z$$  \hspace{1cm} (7.7)

* The treatment adopted here is essentially the same as outlined by Fleagle (1949).
where the temperature gradient $\alpha = \frac{\partial T}{\partial z}$ is greater than zero, $T_o$ is the absolute temperature at ground, and $T$ is the absolute temperature at height $z$.

The well known Laplace's Equation for the speed of sound of an ideal gas is

$$C = \left(\frac{\gamma P}{\rho}\right)^{1/2} \quad (7.8)$$

where $\gamma$ is the ratio of the specific heat of the gas at constant pressure to its specific heat at constant volume, $P$ and $\rho$ are respectively the pressure and density in the undisturbed medium. This equation assumes that the pressure changes produced by the sound rays are adiabatic.

Substitution from the equation of state for an ideal gas

$$\frac{P}{\rho} = \frac{RT}{M}$$

where $R = 8.314 \times 10^7$ ergs per °C per mole is the universal gas constant and $M$ is the molecular weight of the gas, permits Laplace's equation to be written in the form

$$C = \left(\frac{\gamma RT}{M}\right)^{1/2} \quad (7.9)$$

Substituting the values for $M = 28.97$ gm per mole, and $\gamma = 1.403$ for dry air, we have (Cox, 1957)

$$C = 20.07 \, T^{1/2} \, \text{meters per sec.} \quad (7.10)$$

The speed of sound in moist air (Gutenberg, 1942) will be greater than the value obtained from Eq. 7.10. This increase in

* In meteorological work this is generally defined as negative. In that case Eq. 7.7 would be given by $T = T_o + \alpha \cdot z$. 
speed is given by $C = \frac{1}{4} \frac{C \cdot e}{P}$, where $e$ is the partial pressure of the water vapour at $T \ O K$ and $P$ is the total pressure.

Snell's law yields,

$$\frac{\sin i}{\sin i_0} = \frac{C}{C_0} = \left( \frac{T}{T_0} \right)^{1/2} \quad (7.11)$$

where $i$ is the angle between the incident sound ray and the vertical and the subscript $0$ signifies "at the ground level".

For the sound ray that is parallel to the earth at the earth's surface, $i_0 = \frac{\pi}{2}$. Thus

$$\sin i = \frac{C}{C_0} = \left( \frac{T}{T_0} \right)^{1/2} \quad (7.12)$$

or

$$\tan i = \left( \frac{T}{T_0 - T} \right)^{1/2} \quad (7.13)$$

Hence, the slope of the ray is given by

$$\frac{dx}{dz} = \tan i = \left( \frac{T}{T_0 - T} \right)^{1/2} \quad (7.14)$$

where $x$ and $z$ represent respectively the horizontal and vertical coordinates. Substitution for $T$ from Eq. 7.7 into Eq. 7.14 yields

$$\frac{dx}{dz} = \left[ \left( \frac{T_0}{\alpha z} \right) - 1 \right]^{1/2} \quad (7.15)$$

Expanding the right hand side by the binomial theorem we have

$$\frac{dx}{dz} = \left( \frac{T_0}{\alpha z} \right)^{1/2} - \frac{1}{2} \left( \frac{T_0}{\alpha z} \right)^{-1/2} + \ldots \quad (7.16)$$
After integration the above equation becomes

\[ x = 2 \left( \frac{T_0}{\alpha} \right)^{1/2} z^{1/2} - \frac{1}{3} \left( \frac{T_0}{\alpha} \right)^{-1/2} z^{3/2} \quad \cdots \]  

(7.17)

The ratio test (Fleagle, 1949) shows that the series given by Eq. 7.17 converges rapidly within the troposphere \((z < 10 \text{ km})\) for the observed ranges of \(\alpha\) and \(T_0\). Since only the first term makes an important contribution, the sound ray which is parallel to the earth at the earth's surface can be approximated to by the first term of Eq. 7.17. Thus, we find

\[ x = 2 \left( \frac{T_0}{\alpha} z \right)^{1/2} \]  

(7.18)

This is an equation of a parabola as shown in Fig. 7.3. It is easily seen from the Fig. 7.3 that for a given distance \(b\) of the lightning channel from the observer, the limiting ray path determines a specific point \(H\) on the channel. All sounds originating at portions of the lightning channel below this point will not be able to reach an observer situated at a distance \(x \geq b\). An observer hears only sounds originating at the lightning channel above a height \(z = H\) which is a function of observation distance \(b\) and the ratio of ground temperature \(T_0\) to the temperature gradient \(\alpha\), as given by Eq. 7.18. When \(b\) and \(\frac{T_0}{\alpha}\) are such that this critical height is equal to or greater than the length of the channel, the observer will be in a shadow zone and will not hear any thunder. For an average lapse rate of 7.5 degree centigrade per kilometer (Fleagle, 1949), \(T_0 = 300^0\text{K}\), and \(z = 4\ \text{km}\), the maximum range of audibility is 25 km; for \(z = 6\ \text{km}\),
Fig. 7.3 Trajectory of the sound originating at point H on the lightning channel. The observer is at O.
$$x = 2 \sqrt{\frac{Io \gamma}{\alpha}}$$
average height of a ground flash (Malan, and Schonland, 1951b), the maximum range is 31 km.

Now, consider the path of a ray propagated horizontally in the positive x-direction at the earth's surface in an isothermal atmosphere with negative wind shear. This ray is refracted away from the earth, and is analogous to the critical ray discussed above.

If the wind-modified speeds of sound are given by

\[ U_0 = C + V_0 \quad (7.19) \]

and

\[ U = C + V \sin i \quad (7.20) \]

where

\[ V = V_0 - \beta z \quad (7.21) \]

then for the particular case in question, \( i_0 = \frac{\pi}{2} \), Snell's Law can be written

\[ \sin i = \frac{U}{U_0} = \frac{C + (V_0 - \beta z) \sin i}{C + V_0} \quad (7.22) \]

where \( V \) and \( U \) are respectively the speed of wind and the modified speed of sound, \( V_0 \) is the horizontal component of wind velocity parallel to the sound ray at \( z = 0 \), and \( \beta \) represents the horizontal component of the vertical shear lying in the plane of the ray. Usually, \( \beta \) depends on \( z \), but an estimate of the maximum curvature due to shear may be made by assuming \( \beta \) constant. Then from Eq. 7.22 we have

\[ \sin i = \frac{C}{C + \beta z} \quad (7.23) \]
or
\[ \tan i = \frac{C}{(2C_\beta z + \beta^2 z^2)^{1/2}} \]  
(7.24)

Hence the slope of the ray is
\[ \frac{dx}{dz} = \frac{C}{\beta} \left[ \frac{1}{(\frac{2Cz}{\beta} + z^2)^{1/2}} \right] \]  
(7.25)

and
\[ x = \frac{C}{\beta} \int_0^z \frac{dz}{\left[ \left( \frac{2Cz}{\beta} + z^2 \right)^{1/2} \right]} \]

For values of \( z \ll \frac{2C}{\beta} \), a quantity which is about 300 km for normal thunderstorm conditions (Remillard, 1960, p. 99), we obtain
\[ x = \frac{2}{\beta} \left( \frac{Cz}{2} \right)^{1/2} \]  
(7.27)

The trajectories given by Eq. 7.27 are parabolas. These may be compared with those obtained from Eq. 7.18 by comparing \( \frac{4T_0}{\alpha} \) and \( \frac{2C}{\beta} \). If \( T_0 = 300^\circ \text{K} \) and \( C = 330 \text{ meters per second} \), the paths of the critical rays resulting from temperature gradient and wind shear are identical when \( \frac{\beta}{\alpha} = 0.55 \times 10^{-2} \text{ meter}^2 \text{ per sec per} \ 0^\circ \text{K} \). Table 7.2 tabulates some simultaneous values of vertical temperature gradient and vertical shear \( \beta \) which give identical paths of the critical rays.
TABLE 7.2
VERTICAL TEMPERATURE GRADIENT AND WIND SHEAR

(After Fleagle, 1949)

<table>
<thead>
<tr>
<th>( \alpha ) °C/km</th>
<th>( \beta ) m/sec/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.8</td>
<td>6</td>
</tr>
<tr>
<td>7.5</td>
<td>4</td>
</tr>
<tr>
<td>5.5</td>
<td>3</td>
</tr>
</tbody>
</table>

It should be noted that \( \alpha = 9.8 \) °C/km is close to the adiabatic lapse rate of \( 10 \) °C/km for dry air, and \( 5.5 \) °C/km close to the adiabatic lapse rate of \( 6 \) °C/km for saturated air.

7.2.2 Results

On only two occasions was the time interval between the flash and the thunder more than one minute. In both cases, the thunder was a very feeble rumbling and could have easily been missed. In both instances a very moderate wind was blowing from the lightning flash towards the observer. On July 9, 1963, at about 3:30 a.m., thunder was heard 75 seconds after the flash. If we assume the speed of sound as 340 meters per second, then the distance of the flash was 25.5 km. At about 2:30 p.m. on July 20, 1963, thunder was heard from a ground flash after 70 seconds. Again, if we take a value of 340 meters per second for the sound speed, then the distance of the flash was 23.8 km. The values of both of these observed distances compare favourably with the maximum range of audibility of thunder discussed in Section 7.2.1.
It was noted occasionally that the thunder was apparently louder when the wind was blowing from the flash to the observer. This observation is in disagreement with the observations of Veenema (1918, p. 63), although in agreement with the anticipated result according to ray theory. But until the windspeeds and structure in both observations can be compared, they should not be considered as contradictory. The wind speeds in the Saskatoon observation were moderate but not gusty. The wind during a thunderstorm is often gusty, and the gustiness usually increases with wind velocity. A gusty wind produces turbulence and the resulting inhomogeneities cause considerable attenuation of sound in the atmosphere. The attenuation due to eddies is essentially the same for all directions of the wind (Ingard, 1953). This often masks the geometric effects discussed earlier. If, then, the winds during Veenema's observations were generally gusty, the direction of wind would not affect the audibility of thunder.

At times, it was noted that even with moderate winds and close lightning flashes the thunder heard was feeble. This was verified during a night storm on June 20, 1963. The lightning flashes were mostly within the clouds. They were bright and white in colour. Only when the lightning flashes were near the zenith, was the thunder faintly audible. Though lightning without audible thunder has been observed (McEachron, 1939), it seems unlikely that the storm of June 20, 1963, contained such flashes. Perhaps the explanation lies
in meteorological factors, especially the lapse rate. Observations of various meteorological parameters coincident with the thunder are necessary to determine the cause of such events.

7.3 Duration of Thunder

Two easily observed facts about thunder are the time interval between the perception of the lightning flash and the first sound of the accompanying thunder, and the duration of thunder. The former gives the distance between the observer and the nearest portion of the flash as discussed in Section 7.1.2 and 7.2. The measurements of the duration of thunder allows one to estimate the physical length of the lightning channel (Carpenter, 1907; Sourdillon, 1952). In principle, if one assumes that the thunder received is radiated from the flash and arrived directly from it, it is possible to estimate the length of the channel. However, in practice the complex spatial geometry of the flash, its orientation relative to the observer, possible reflections from graupel layers, effect of temperature gradient and wind shear on the propagation, and the presence of turbulence all make the computations extremely difficult.

The durations of thunder records on the HWM obtained in 1962 and 1963 were measured. Histograms from these measurements are plotted in Fig. 7.4 and 7.5. About 61 per cent of thunder at Saskatoon last in the range of 5 to 20 seconds. These observations indeed include the overlapping effects from two or more lightning flashes occurring rapidly in succession. Records of more than 1
Fig. 7.4 Separate histograms of the duration of thunder recorded during 1962 and 1963.
Fig. 7.5 Combined histogram of the duration of thunder recorded during 1962 and 1963.
minute duration are rare. Three records (not shown in the histogram) of duration of about two minutes were noted; one record, in 1962, of 129 seconds, and two records, 118 seconds and 125 seconds respectively in 1963. The sensitivity and location of the microphone were not the same for these observations. Hence, no quantitative deductions can be made. But, it should be said that if the sensitivity and the location of a microphone remained unchanged, and if an average value of the duration of thunder at one location is found from observation; such histograms should give the number of occurrence of lightning flashes.

It has been observed also that the duration of thunder, in general, from a cloud flash is greater than that from a ground flash. Branching lengthens the duration. The length of the path of a cloud flash is greater than that of a ground flash (Humphreys, 1920). If we assume that the direction and the distance of the ground and cloud flash were more or less the same, then it is conceivable that the cloud flash may give a longer lasting thunder.

7.4 Colours of Lightning

Russell (1908) from visual investigations of five year observed that the colours of lightning (streak and sheet combined) in order of decreasing occurrence are: red, white, blue, golden, yellow, violet, orange, and green. He also found some relation between the colour of the lightning and the subsequent thunder. He noted that red is followed by thunder of a long rolling nature, blue by thunder in peals varying both in intensity and duration of sound, white by thunder as nearly as possible resembling the rapid discharge
of ordnance, and violet and green are followed by most intense thunder.

The colours observed at Saskatoon in order of decreasing occurrence are: pink (a mixture of red and violet), violet, yellow, white, red, and blue. The pink colour is a mixed colour and could vary from violet to reddish blue. The order of these colours in an individual storm could be different. Often only a few colours are present in an individual storm and though no actual count of the occurrence of different colours was kept, qualitative observations indicate that the above is the rough order of occurrence. Another observed phenomenon, consistent with the Russell's observations, was that the flashes were generally pink or red as the storm approached, and yellow and white as the storm receded.

Though difficulty is very often encountered in distinguishing a colour, particularly among pink, violet and red, the consistency of these observations irrespective of day and night hours suggests that the colours are not optical illusions. Red colour is generally observed in far storms, but it has also been observed in overhead storms. The flashes of red colour, when examined in the spectroscope, show several lines due to hydrogen which is furnished by the decomposition of some of the water along the lightning path (Humphreys, 1940). The white flashes show no hydrogen lines. The yellow flashes are those which have lost their shorter wavelength components through absorption due to distance or intervening clouds. The spectrum of lightning and that of ordinary electric spark in the air are found to be similar (Fox, 1903). Recent measurements (Salanave, 1961) show that the
lightning spectrum resembles that of an arc in air rather than that of the spark. It is also found (Wallace, 1960; Salanave, Orville, and Richards, 1962) that singly and doubly ionized atomic nitrogen and oxygen lines are more prominent than the molecular bands attributed to CN and N₂ that appear infrequently and with variable intensity. An attempt should be made to find a correlation between visual colour and spectrum of lightning.

The most common colour observed at Saskatoon in the streak lightning is pink, and in the sheet lightning white. This is in agreement with the observations by Symons (Russell, 1908). Any suggestion that it is because of the flash or response of the eye is ruled out for it has been experimentally shown (Land, and Daw, 1962) that the colours do not vary with the duration of stimulus, provided the quantity of light is above threshold, and the field phenomena which produce colour and determine colour are activated instantaneously and do not depend on fatigue and adaptation. To see a particular colour, a certain number of photons must be received by a visual receptor (rod or cone). The average number of photons is proportional to the intensity, provided the eye does not move (Akerman, 1962), and the constant of proportionality depends on several factors, one of which is the size of patch. This physiological factor may well be responsible for producing this effect, but spectroscopic observations are needed before any physical cause can be completely discarded.

No particular attempt was made to find any relation between the colour of lightning and the resulting thunder. It was neverthe-
less. observed that often white ground flashes gave louder thunder than the pink ones, provided the wind direction and the distance of the lightning flashes were unchanged. This was due possibly to higher temperatures and to a greater degree of ionization in the white flashes.

7.5 Cloud and Ground flashes

Various optical and electrical devices have been built (Hagenguth, 1940a, 1940b; Malan, and Schonland, 1950; Clarence, and Malan, 1951; Kitagawa, and Brook, 1960; Sonde, 1963) to study the mechanism of lightning discharges. From the point of view of communication engineers, electrical engineers, physicists, economists, and medical men the most important study is to know the relative frequencies of occurrence of cloud and ground flashes. Visual observations (Golde, 1945) electrostatic field measurements (Brook, and Kitagawa, 1960; Wang, 1963a) and spheric studies (Aiya, and Sonde, 1963) have been employed to determine the ratio of cloud to ground discharges in different countries. The value of this ratio depends upon the heights and dimensions of cumulus thunderstorm clouds, orographic characteristics, and the geological formation of the soil such as resistivity (Simpson, 1927). It varies from storm to storm, and between the beginning and the end of a storm. The occurrence of cloud flashes is relatively much greater than that of ground flashes. At New Mexico, the ratio is 1.5 (Brook, and Kitagawa, 1960), at Singapore, it is 5 (Wang, 1963a), at South Africa, it is 10
(Schonland, 1950, p. 38), and 9 at Bangalore, India, (Aiya, and Sonde, 1963). The various reported estimates around the globe range between 50 and 0.7 (Hagenguth, 1951). At Saskatoon, though no record of actual counting was kept, it was quite evident that cloud flashes outnumbered the ground flashes.

As regards the characteristics of thunder due to these two kinds of discharges, it is often reported (Golde, 1945) that a close ground flash produces a sharp crack that is absent in cloud flashes. Even the term proper thunder (Remillard, 1960) has been used to imply the thunder from ground flashes. This crack or click has been interpreted (Battan, 1964) as a sign that the stepped leader has made a contact with the return streamer. At Saskatoon, no such sound was ever heard by the author in spite of consistent attempts to detect it.

Closeby flashes irrespective of their nature were characterized by a loud burst of sound; distant flashes by a rumbling which developed into a loud burst. The thunder from very distant flashes is a dull murmur or rumbling. These observations are supported by the records presented in Chapters 4 and 6. Fig. 6.4, an example of a type a record, is from a close cloud flash; the time interval between the flash and subsequent thunder being 2 seconds. Fig. 6.10, an example of a type a record, is from a ground flash, the time difference between the perception of flash and subsequent thunder being 5 seconds. Fig. 6.9 an example of a type b record, is from a ground flash, the time difference is 7 seconds. Examples of type b records from cloud flashes are numerous. For type b and c records reference
should be made to the table 4. 1 and the records presented in Chapters 4 and 6.

This observation is in agreement with that of Schmidt (1914, p. 493). However, his explanation that the main vibrations get separated from the beginning of thunder after travelling a considerable distance in the atmosphere is inexact. The accurate explanation has come from Remillard (1960) who considered the refraction of sound in the atmosphere. If we assume a temperature lapse rate of 7.5°C per km, then an observer situated at 5 km from the flash would hear the thunder originating at the channel at heights greater than 200 meters from the ground, and hence the first loud clap due to the main channel will not be heard. When the lightning flash is fairly distant, only sounds from the highest parts will be heard, producing only a rumbling. Also, the intensity of sound is inversely proportional to the square of observing distance (Carpenter, 1907), and hence, when the distance is large enough, the changes in intensity of the sound due to flash geometry will not be great to the ear. The observer will only hear feeble sounds, that is, a rumbling. This explains the type C records mentioned in Chapter 4.
A historical review of the literature on thunder, a discussion on the terminology used for describing various components of thunder, and typical records of pressure variations of thunder with time have been presented.

Various important hypotheses regarding the physical cause of the production of thunder have been examined. Some theoretical considerations of the role of positive ions in the plasma of lightning channel have been discussed. It has been found that, because of neutral molecules present in the channel, the contribution of positive ion electrostatic oscillations towards the production of thunder is negligible. The decisive cause for the production of thunder seems to be the compressional shock waves produced by the sudden heating, and perhaps enhanced by the abundant ionization, in the channel.

The analysis of pressure fluctuations in time show that most of the power in the spectrum of thunder is contained in the infrasonic frequencies. Infrasonic spectral density estimates show the following conclusions:

(a) Primary maxima (most of the energy) fall in the frequency range of 0.75 to 6.00 cps with the most intense maximum occurring in the range of 1.00 to 3.00 cps.

(b) The spectra can be classified into three groups. Type A, which contain only one prominent primary maximum, Type B, which contain more than one prominent primary maximum, and Type C, which contain no intense primary maximum.
The audio-frequency Spectral density estimates lead to the following conclusions:

(c) The spectra of thunder for cloud and ground flashes are similar.

(d) Most frequent ranges of intense spectral maxima are 22-28, 52-56, 66-78 cps. Less powerful maxima also occur in the frequency ranges 34-40, 88-90, at about 122, and 202-204 cps.

The measurements of the direction of arrival of sound of thunder lead to the following general conclusions:

(e) Peals of thunder arrive directly from the lightning flash. Successive peals of thunder originate at different parts of the flash, and rarely from different strokes.

(f) The claps (impulses which often contribute to rumbling in the last phases of thunder), in general, arrive from different directions than peals, and could either be due to reflection or due to higher and more distant parts of the flash.

(g) A reflection from a graupel layer, as suggested by Remillard, may occasionally take place.

(h) The amplitude of reflected waves, be they claps or peals, is less than that of direct waves. The reflected wave, in general, gives rise to claps and the direct wave to peals of thunder. The reflected wave may protract the sound.

(i) Short-period and small-amplitude, and long-period and large-amplitude pressure variations arrive from the same general direction. High frequency small amplitude variations may possibly be due to different strokes.
(j) Complexity of the lightning flash makes the thunder record complex.

Other results obtained in this study are listed as below:

(k) The initial impulse from most of the large pressure variations is compressional in nature.

(l) The product of frequency maximum and the observation distance is not a constant, contrary to Remillard's prediction.

(m) The normal maximum distance at which thunder can be heard is about 25 km. This is in agreement with the theoretical considerations.

(n) Histograms of thunder durations suggest that most occurrences at Saskatoon last in the range from 5.0 to 20.0 seconds.

(o) The colours of lightning observed at Saskatoon in order of decreasing occurrence are: pink, violet, yellow, white, red, and blue. The most common colour in the streak lightning is pink, and in the sheet lightning white. A white streak lightning flash, in general, gives louder thunder than a pink flash.

(p) Most of the lightning flashes at Saskatoon are cloud flashes. There is no discernible difference between the thunder heard from a cloud flash to that from a ground flash.

(q) The close flashes irrespective of their nature are characterized by a burst of sound and the distant flashes by a rumbling which develops into a loud burst. Very distant flashes give only rumbling thunder.

Thus we see that the sounds which we hear following a lightning flash are only a portion of all the sounds generated at the
flash. Most intense sounds are in the infrasonic region which we
cannot hear as our ears do not respond to such frequencies. How-
ever, the energy contained in the audio frequencies is great enough.
The loud sounds originate at the flash and arrive directly from it.
The repetition of loud sounds, which we generally call peals of thunder,
occur because they originate at different parts of the flash. Rarely,
different peals may be due to different strokes occurring in the channel
at the same place. The rumbling that follows the large sounds (Peals)
is due to sounds originating at higher and distant parts of the flash,
and likely reflections of sounds from a graupel layer (a layer of
soft ice pellets) in the clouds. The distinct claps of thunder which
follow a nearby flash contribute to rumbling in a distant flash.

The large pressure variations (sounds) of thunder are
produced by thermal expansion of the atmospheric constituent gases
in the lightning channel (this is suggested by the compressional nature
of the loud sounds).

Adequate explanation for the distinct frequency maxima
obtained in the spectral density estimates is obscure at this time.
Though it is conceivable that if a vibrating string model is assumed,
and certain lengths of the lightning path are common, infrasonic
maxima may occur. But consistent peaks in the audio frequency
range cannot be explained in this way.

Indeed, coincident observations of optical, electrical, and
acoustical aspects of lightning may provide answers to these questions.
Every attempt should be made to take such observations. Fast respond-
ing microphones for infrasonic frequencies will give more accurate pictures of the nature of pressure variations. Theoretical studies are needed to obtain a clear understanding of the physical cause or causes of the production of thunder. A study should be made to find if turbulence created by the shock waves would produce such frequencies and give such peaks.
APPENDIX A

DERIVATION OF THE EQUATION (3.21)

For the derivation of Eq. (3.21) several involved equations are needed. Their derivations are as follows:

A.1 Boltzmann Equation

Consider a gas of particles of one kind; each constituent particle is subjected to an external force, \( \vec{F} \) (\( \vec{F} \) may vary with position, \( \vec{r} \), and time, \( t \), but not with velocity, \( \vec{v} \)).

Suppose, \( \vec{v} \), is the velocity of a particle at time \( t \) and position \( \vec{r} \). Then, the velocity of the particle at time \( t + dt \) and position \( \vec{r} + \vec{v} dt \), will be \( \vec{v} + \frac{\vec{F}}{m} dt \), if it suffers no collisions.

Assume that there are \( f(\vec{r}, \vec{v}, t) \) \( dx \, dy \, dz \, d\omega_x \, d\omega_y \, d\omega_z \) particles at time \( t \), which lie in a volume element \( dx \, dy \, dz \), centered at \( \vec{r} \) and in the velocity interval \( d\omega_x \, d\omega_y \, d\omega_z \) centered at \( \vec{v} \).

After time \( dt \), neglecting collisions, the same particles from the set occupying volume \( dx \, dy \, dz \), will be centered at \( \vec{r} + \vec{v} dt \), and in the velocity interval \( d\omega_x \, d\omega_y \, d\omega_z \), will be centered at \( \vec{v} + \frac{\vec{F}}{m} dt \). The number of particles in this set is, then,

\[ f(\vec{r} + \vec{v} dt, \vec{v} + \frac{\vec{F}}{m} dt, t + dt) \, dx \, dy \, dz \, d\omega_x \, d\omega_y \, d\omega_z. \]

However, due to collisions the number in the second set will differ, in general, from the number in the first set. The net change in number of particles is proportional to \( dx \, dy \, dz \, d\omega_x \, d\omega_y \, d\omega_z \) dt. Denote this gain by \( \left( -\frac{\partial f}{\partial t} \right)_{\text{Coll}} \) \( dx \, dy \, dz \, d\omega_x \, d\omega_y \, d\omega_z \) dt, \([f(\vec{r} + \vec{v} dt, \vec{v} + \frac{\vec{F}}{m} dt, t + dt) - f(\vec{r}, \vec{v}, t)] \, dx \, dy \, dz \, d\omega_x \, d\omega_y \, d\omega_z \, dt\).
\[ \frac{d\omega_y}{dt} + \frac{d\omega_z}{dt} = \left( \frac{\partial f}{\partial t} \right)_{\text{Coll}} \frac{dx}{dt} \frac{dy}{dt} \frac{dz}{dt} \]

Divide by \( dx \ dy \ dz \ d\omega_x \ d\omega_y \ d\omega_z \) \( dt \) and in the limit as \( dt \to 0 \),

\[ \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \omega_x + \frac{\partial f}{\partial y} \omega_y + \frac{\partial f}{\partial z} \omega_z + \frac{\partial f}{\partial \omega_x} \frac{F_x}{m} + \frac{\partial f}{\partial \omega_y} \frac{F_y}{m} + \frac{\partial f}{\partial \omega_z} \frac{F_z}{m} = \left( \frac{\partial f}{\partial t} \right)_{\text{Coll}} \]

or

\[ \frac{\partial f}{\partial t} + \sum_{e} \omega_e \frac{\partial f}{\partial x_e} + \sum_{e} \frac{F_e}{m} \frac{\partial f}{\partial \omega_e} = \left( \frac{\partial f}{\partial t} \right)_{\text{Coll}} \]  \hspace{1cm} (A.1)

This is the Boltzmann equation, which states that along the trajectory of a system of particles, \( f \) is changed only by collisions.

Macroscopic particle density, \( n(\mathbf{\vec{r}}, t) \), macroscopic average velocity, \( \mathbf{\vec{v}}(\mathbf{\vec{r}}, t) \), and any general quantity, \( Q(\mathbf{\vec{w}}) \), are defined as below

\[ n(\mathbf{\vec{r}}, t) = \iiint_{-\infty}^{\infty} f(\mathbf{\vec{r}}, \mathbf{\vec{w}}, t) \ d\omega_x \ d\omega_y \ d\omega_z \]  \hspace{1cm} (A.2)

and

\[ \mathbf{\vec{v}}(\mathbf{\vec{r}}, t) = \frac{1}{n(\mathbf{\vec{r}}, t)} \iiint_{-\infty}^{\infty} \mathbf{\vec{w}} f(\mathbf{\vec{r}}, \mathbf{\vec{w}}, t) \ d\omega_x \ d\omega_y \ d\omega_z \]  \hspace{1cm} (A.3)

and

\[ Q(\mathbf{\vec{r}}, t) = \frac{1}{n(\mathbf{\vec{r}}, t)} \iiint_{-\infty}^{\infty} Q(\mathbf{\vec{w}}) f(\mathbf{\vec{r}}, \mathbf{\vec{w}}, t) \ d\omega_x \ d\omega_y \ d\omega_z \]  \hspace{1cm} (A.4)

Multiply the Boltzmann equation by \( Q(\mathbf{\vec{w}}) \) \( d\omega_x \ d\omega_y \ d\omega_z \) and integrate over all \( \mathbf{\vec{w}} \). The first term is

\[ \iiint_{-\infty}^{\infty} Q(\mathbf{\vec{w}}) \frac{\partial f}{\partial t} \ d\omega_x \ d\omega_y \ d\omega_z \]

\[ = \frac{\partial}{\partial t} \iiint_{-\infty}^{\infty} Q(\mathbf{\vec{w}}) f \ d\omega_x \ d\omega_y \ d\omega_z \]

\[ = \frac{\partial}{\partial t} (n \mathbf{\vec{v}}) \]
and the second term is
\[
\iiint_{-\infty}^{\infty} Q(\vec{\omega}) \, \omega_e \, \frac{\partial f}{\partial x_e} \, d\omega_x \, d\omega_y \, d\omega_z
\]
\[
= \frac{\partial}{\partial x_e} \iiint_{-\infty}^{\infty} Q(\vec{\omega}) \, \omega_e \, d\omega_x \, d\omega_y \, d\omega_z
\]
\[
= \frac{\partial}{\partial x_e} (n \, \omega_e Q)
\]

and the third term is
\[
\iiint_{-\infty}^{\infty} Q(\vec{\omega}) \, \frac{F_e}{m} (\vec{\eta}, \vec{\omega}) \, \frac{\partial f}{\partial \omega_e} \, d\omega_x \, d\omega_y \, d\omega_z
\]
remembering \(\int udv = uv - \int vdu\), we have
\[
= \left[ \int \left\{ Q(\vec{\omega}) \, \frac{F_e}{m} (\vec{\eta}, \vec{\omega}) \right\} \right]_{-\infty}^{\infty} - \iiint_{-\infty}^{\infty} f \, \frac{\partial}{\partial \omega_e} \left( \frac{F_e}{m} (\vec{\eta}, \vec{\omega}) \right) \, Q(\vec{\omega}) \]
\[
= - n \frac{\partial}{\partial \omega_e} \left( \frac{F_e}{m} Q \right)
\]
since the first term vanishes because \(f = 0\) at \(t = \infty\).

The final result is
\[
\frac{\partial}{\partial t} (nQ) + \frac{\partial}{\partial x_e} (n \, \frac{\partial}{\partial \omega_e} Q) - n \frac{\partial}{\partial \omega_e} \left( \frac{F_e}{m} Q \right)
\]
\[
= \iiint_{-\infty}^{\infty} Q(\vec{\omega}) \left( \frac{\partial f}{\partial t} \right)_{\text{Coll}} \, d\omega_x \, d\omega_y \, d\omega_z \quad (A.5)
\]

A.2 Equation of Continuity

This is obtained by putting \(Q = 1\) in the above equation.

The right hand side gives
\[
\iiint_{-\infty}^{\infty} \left( \frac{\partial f}{\partial t} \right)_{\text{Coll}} \, d\omega_x \, d\omega_y \, d\omega_z
\]
\[
\frac{\partial}{\partial t} \left( \begin{array}{c}
\frac{d\mathbf{\omega}}{d\mathbf{t}}
\end{array} \right) \text{Coll} \int_{-\infty}^{\infty} f \, d\mathbf{\omega}_x \, d\mathbf{\omega}_y \, d\mathbf{\omega}_z
\]

= 0

since \( \int_{-\infty}^{\infty} f \, d\mathbf{\omega}_x \, d\mathbf{\omega}_y \, d\mathbf{\omega}_z \) is the particle density and will remain unchanged due to collisions.

Also, the third term on the left hand side will vanish as seen below:

For a magnetic force

\[
\vec{F} = q \vec{\mathbf{\omega}} \times \vec{B}
\]

so, y component, say,

\[
F_y = q \left( \mathbf{\omega} \cdot \mathbf{B} \times \mathbf{B} \right)
\]

The force in one direction depends on the velocity in the transverse direction and so \( \frac{\partial F_y}{\partial \mathbf{\omega}_y} = 0 \) and in this way \( \frac{\partial F_e}{\partial \mathbf{\omega}_e} \) will be zero.

Hence we have

\[
\frac{\partial}{\partial t} n + \sum e \frac{\partial}{\partial x_e} (n \mathbf{\omega}_e) = 0
\]

or

\[
\frac{\partial}{\partial t} n + \bigtriangledown \cdot (n \mathbf{\omega}) = 0 \tag{A. 6}
\]

This is the equation of continuity.

A. 3  Equation of Momentum Transfer

This is obtained by assuming \( Q = m \mathbf{\omega} \) in the Eq. (A. 5).

The result is
\[
\frac{\partial}{\partial t} (nm \vec{u}) + \nabla \cdot (nm \vec{v} \vec{u}) - nF
\]

\[
= \iiint_{-\infty}^{\infty} m \vec{\omega} \left( \frac{\partial f}{\partial t} \right) \text{Coll.} \, d\omega_x \, d\omega_y \, d\omega_z
\]

where \( \vec{u} \) = average velocity.

From the first term

\[
\frac{\partial}{\partial t} (nQ) = \frac{\partial}{\partial t} (nm \vec{u}) ,
\]

but \( Q = m \vec{\omega} \),

hence \( \vec{u} = \vec{\omega} \)

The first term can be reduced as follows:

\[
\frac{\partial}{\partial t} (nm \vec{u})
= nm \frac{\partial}{\partial t} \vec{u} + \vec{u} \frac{\partial}{\partial t} (nm)
= nm \frac{\partial}{\partial t} \vec{u} - \vec{u} \nabla \cdot (nm \vec{u})
\]

The above follows from Equation of Continuity (A.6).

The second term is treated in the following manner

\[
\nabla \cdot (nm \vec{\omega} \vec{u})
= \frac{\partial}{\partial x} (nm \omega_x \vec{u}_x \omega_x) + \frac{\partial}{\partial y} (nm \omega_y \vec{u}_y \omega_y) + \frac{\partial}{\partial z} (nm \omega_z \vec{u}_z \omega_z)
= \frac{\partial}{\partial x} \left[ nm(\hat{i} \omega_x \omega_x + \hat{j} \omega_x \omega_y + \hat{k} \omega_x \omega_z) \right]
+ \frac{\partial}{\partial y} \left[ nm(\hat{i} \omega_y \omega_x + \hat{j} \omega_y \omega_y + \hat{k} \omega_y \omega_z) \right]
+ \frac{\partial}{\partial z} \left[ nm(\hat{i} \omega_z \omega_x + \hat{j} \omega_z \omega_y + \hat{k} \omega_z \omega_z) \right]
\]

Put \( \vec{\omega} = \vec{u} + \vec{\omega} \)
where $\vec{\omega}$ = actual velocity

$\vec{v}$ = mean velocity (= $\vec{\omega}$)

$\vec{u}$ = random velocity relative to $\vec{v}$

now,

$\omega_x = v_x + u_x$

and

$\omega_x \omega_y = v_x v_y + u_x u_y + v_x v_y + u_x u_y$

thus, we have

$\nabla \cdot (\text{nm } \vec{\omega} \vec{\omega}) = \nabla \cdot (\text{nm } \vec{v} \vec{v}) + \nabla \cdot (\text{nm } \vec{u} \vec{u})$

put $\text{nm } \vec{u} \vec{u} = \psi$ = stress tensor (Dyadic)

and expand

$\nabla \cdot (\text{nm } \vec{\omega} \vec{\omega}) = \text{nm } \vec{v} \cdot \nabla \vec{v} + \vec{u} \cdot \nabla \cdot (\text{nm } \vec{v})$ then

the second term is reduced to

$\nabla \cdot (\text{nm } \vec{\omega} \vec{\omega}) = \text{nm } \vec{v} \cdot \nabla \vec{v} + \vec{u} \cdot \nabla \cdot (\text{nm } \vec{v}) + \nabla \cdot \psi$

The third term is treated in the following way

$\vec{F}$ is given by

$\vec{F} = q \vec{E} + q \vec{\omega} \times \vec{B} - m \nabla \phi$

where $\vec{E}$ = electric field

$\vec{B}$ = magnetic field

$\vec{\omega} = \vec{v}$

$\nabla \phi$ = gravitational potential

$= -g$ at the surface of the earth.

Hence,

$\vec{nF} = n(q \vec{E} + q \vec{\omega} \times \vec{B}) - \text{nm } \nabla \phi$

$= n q (\vec{E} + \vec{\omega} \times \vec{B}) - \text{nm } \nabla \phi$
The fourth term (R.H.S.) is reduced as below.

This is the momentum gained as a result of the collisions. The collisions between identical particles produce no net momentum change, but the collisions between different kinds of particles yield a net momentum gain; call it \( \vec{P} \). Therefore the fourth term is equal to \( \vec{P} \). Thus

\[
\text{nm} \left( \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{U} \right) = n q \left( \vec{E} + \vec{V} \times \vec{B} \right) - \nabla \cdot \vec{\psi} - \text{nm} \nabla \phi + \vec{P} \quad (A.6) \]

Now,

The following macroscopic quantities are defined as indicated.

- velocity: \( \vec{U} = \frac{1}{\rho} \left( n_i m_i \vec{U}_i + n_e m_e \vec{U}_e \right) \) \quad (A.7)
- current density: \( \vec{j} = e(n_i Z \vec{U}_i - n_e \vec{U}_e) \) \quad (A.8)
- and mass density: \( \rho_m = n_i m_i + n_e m_e \) \quad (A.9)

where \( Z = \frac{n_e}{n_i} \)

A.4 Linearized Equation of Motion

Approximations: (1) Neglect second order terms in \( \vec{U}, \vec{j} \) and their derivatives.

(2) Assume electrical neutrality i.e., \( Z n_i = n_e \).

(3) Ignore \( \frac{m_e}{m_i} \) as compared to 1

and (4) Substitute \( \nabla \cdot \vec{\psi} = \nabla p \)

where \( p = \) scalar pressure.
The equations of momentum transfer for ions and electrons are as below:

The equation for ions is

\[ n_i m_i \left( \frac{\partial \vec{u}_i}{\partial t} + \vec{u}_i \cdot \nabla \vec{u}_i \right) = n_i Z e (\vec{E} + \vec{u}_i \times \vec{B}) - \nabla p_i - n_i m_i \nabla \phi + \vec{P}_{ie} \quad (A. 10) \]

and the equation for electrons is

\[ n_e m_e \left( \frac{\partial \vec{u}_e}{\partial t} + \vec{u}_e \cdot \nabla \vec{u}_e \right) = - n_e e(\vec{E} + \vec{u}_e \times \vec{B}) - \nabla p_e - n_e m_e \nabla \phi + \vec{P}_{ei} \quad (A. 11) \]

The quantities \( \vec{u}_i \cdot \nabla \vec{u}_i \) and \( \vec{u}_e \cdot \nabla \vec{u}_e \) are second order quantities and are neglected. Also from Newton's third law

\[ \vec{P}_{ie} = - \vec{P}_{ei} \]

Add the above two equations and obtain

\[ n_i m_i \frac{\partial \vec{u}_i}{\partial t} + n_e m_e \frac{\partial \vec{u}_e}{\partial t} = e(n_i Z \vec{u}_i - n_e \vec{u}_e) \times \vec{B} - \nabla p - \vec{P}_m \nabla \phi \]

or

\[ \vec{P}_m \frac{\partial \vec{u}}{\partial t} = j \times \vec{B} - \nabla p - \vec{P}_m \nabla \phi \quad (A. 12) \]

Eq. (A. 12) is the linearized equation of motion.

Eq. (A. 12) implies that,

\[ \vec{P}_m \frac{\partial \vec{u}}{\partial t} = n_i m_i \frac{\partial \vec{u}_i}{\partial t} + n_e m_e \frac{\partial \vec{u}_e}{\partial t} \]

which can be easily shown to be true as below:

Rewrite equations (A. 7) and (A. 8)

\[ \vec{u} = \frac{1}{\vec{P}_m} (n_i m_i \vec{u}_i + n_e m_e \vec{u}_e) \quad (A. 7) \]
and

\[ \vec{j} = e(n_i Z \vec{U}_i - n_e \vec{U}_e) \]  \hspace{1cm} (A.8)

Differentiate Eq. (A.7) and obtain

\[
\frac{\partial \vec{U}}{\partial t} = -\frac{1}{\rho_m} \frac{\partial \rho_m}{\partial t} (n_i m_i \vec{U}_i + n_e m_e \vec{U}_e) + \frac{1}{\rho_m} n_i m_i \frac{\partial \vec{U}_i}{\partial t} \\
+ \frac{\vec{U}_i}{\rho_m} \frac{\partial (n_i m_i)}{\partial t} + \frac{1}{\rho_m} n_e m_e \frac{\partial \vec{U}_e}{\partial t} + \frac{\vec{U}_e}{\rho_m} \frac{\partial (n_e m_e)}{\partial t}
\]

From Eq. (A.6)

\[
\frac{\partial (n_e m_e)}{\partial t} = -m_e \nabla \cdot (n_e \vec{U}_e)
\]

and

\[
\frac{\partial (n_i m_i)}{\partial t} = -m_i \nabla \cdot (n_i \vec{U}_i)
\]

Hence first, third and fifth terms on the right hand side of the above equation contain second order quantities and are neglected. This gives

\[
\frac{\partial \vec{U}}{\partial t} = \frac{1}{\rho_m} n_i m_i \frac{\partial \vec{U}_i}{\partial t} + \frac{1}{\rho_m} n_e m_e \frac{\partial \vec{U}_e}{\partial t}
\]

Hence

\[
\rho_m \frac{\partial \vec{U}}{\partial t} = n_i m_i \frac{\partial \vec{U}_i}{\partial t} + n_e m_e \frac{\partial \vec{U}_e}{\partial t}
\]

is true.

A.5 Equations of Continuity for Ions and Electrons

The Equation of Continuity (A.6) for the ion is

\[
\frac{\partial n_i}{\partial t} = -\nabla \cdot (n_i \vec{U}_i)
\]
Eq. (A.7) states
\[ \rho_m \mathbf{\nabla} \cdot n_i m_i \mathbf{\nabla} \mathbf{\nabla}_i + n_e m_e \mathbf{\nabla} \mathbf{\nabla}_e = n_i m_i \mathbf{\nabla} \mathbf{\nabla}_i + Z n_i m_e \mathbf{\nabla} \mathbf{\nabla}_e = n_i m_i \mathbf{\nabla} \mathbf{\nabla}_i \left(1 + \frac{m_e}{m_i} \mathbf{\nabla} \mathbf{\nabla}_i \right) \]

Therefore \[ n_i \mathbf{\nabla} \mathbf{\nabla}_i = \frac{1}{m_i} \left( \rho_m \mathbf{\nabla} \right) \]

Hence the Equation of Continuity for ions can be written as
\[ \frac{\partial n_i}{\partial t} = \frac{1}{m_i} \mathbf{\nabla} \cdot (\rho_m \mathbf{\nabla}) \] (A.13)

Similarly Equation of Continuity for electrons, is given by
\[ \frac{\partial n_e}{\partial t} = -\mathbf{\nabla} \cdot (n_e \mathbf{\nabla} \mathbf{\nabla}_e) \]

Equation (A.8) gives
\[ \mathbf{j} = e \left( n_i Z \mathbf{\nabla} \mathbf{\nabla}_i - n_e \mathbf{\nabla} \mathbf{\nabla}_e \right) \]
or
\[ n_e \mathbf{\nabla} \mathbf{\nabla}_e = -\frac{\mathbf{j}}{e} + n_i Z \mathbf{\nabla} \mathbf{\nabla}_i \]

Substituting for \( n_i \mathbf{\nabla}_i \) we obtain
\[ n_e \mathbf{\nabla} \mathbf{\nabla}_e = -\frac{\mathbf{j}}{e} + Z \frac{n_i}{m_i} \left( \rho_m \mathbf{\nabla} \right) \]

Hence,
\[ \frac{\partial n_e}{\partial t} = -\mathbf{\nabla} \cdot \left( \frac{\mathbf{j}}{e} + Z \frac{n_i}{m_i} \left( \rho_m \mathbf{\nabla} \right) \right) \]
or
\[ \frac{\partial n_e}{\partial t} = \frac{1}{e} \mathbf{\nabla} \cdot \mathbf{j} - Z \frac{n_i}{m_i} \mathbf{\nabla} \cdot (\rho_m \mathbf{\nabla}) \] (A.14)
Eq. (A. 14) is the Equation of Continuity for the electrons.

Now we can derive the equation (3. 21)

A. 6 Positive Ion Oscillations

Since positive ions are moving, \( \vec{U} \) oscillates, and we should use the linearized equation of motion. Eq. (A. 12) is

\[
\rho_m \frac{\partial \vec{U}}{\partial t} = j \times \vec{B} - \nabla p - \rho_m \nabla \phi
\]

(A. 12)

Assume, also, that (a) magnetic field and gravity = 0

ie \( \vec{B} = 0 = \nabla \phi \)

and (b) \( \gamma = \gamma_e = \gamma_i \)

and \( T = T_e = T_i \)

where \( \gamma \) is the ratio of specific heats and is given by, \( \gamma = \frac{2 + P'}{P} \),

where \( P' \) is the number of degrees of freedom, and \( T \) is the absolute temperature.

Now, introduce these assumptions into Eq. (A. 12) and differentiate and obtain

\[
\rho_m \frac{\partial^2 \vec{U}}{\partial t^2} + \frac{\partial \rho_m}{\partial t} \frac{\partial \vec{U}}{\partial t} = - \nabla \frac{\partial p}{\partial t}
\]

The second term involves the product of two first order terms and, hence, is neglected. It follows that

\[
\rho_m \frac{\partial^2 \vec{U}}{\partial t^2} = - \nabla \frac{\partial p}{\partial t}
\]

(A. 15)

Now,

\[
p = p_i + p_e
\]

\[
= (n_i + n_e)kT
\]
where $k$ = Boltzmann constant.

Hence,

$$\frac{\partial p}{\partial t} = kT \frac{\partial (n_i+n_e)}{\partial t} + k(n_i + n_e) \frac{\partial T}{\partial t}$$

Now, if we assume adiabatic process

$$T = C'' n^\gamma - 1$$

where $C''$ is a constant of proportionality

$$= C'' (n_i+n_e)^\gamma - 1$$

Hence,

$$\frac{\partial T}{\partial t} = C'' (\gamma - 1) (n_i + n_e)^\gamma - 2 \frac{\partial (n_i+n_e)}{\partial t}$$

Substituting for $\frac{\partial T}{\partial t}$ in above equation and obtain

$$\frac{\partial p}{\partial t} = kT \frac{\partial (n_i+n_e)}{\partial t} + k(n_i+n_e) C'' (\gamma - 1) (n_i+n_e)^\gamma - 2 \frac{\partial (n_i+n_e)}{\partial t}$$

$$= kT \frac{\partial (n_i+n_e)}{\partial t} + kT (\gamma - 1) \frac{\partial (n_i+n_e)}{\partial t}$$

$$= \left\{ \frac{\partial}{\partial t} (n_i+n_e) \right\} kT \gamma$$

Hence,

$$\frac{\partial p}{\partial t} = \sqrt{\gamma} kT \frac{\partial (n_i+n_e)}{\partial t} \quad (A. 16)$$

Rewrite Eqs. (A. 13) + (A. 14), the equations of continuity

for ions and electrons,

$$\frac{\partial n_i}{\partial t} = - \frac{1}{m_i} \nabla \cdot (\rho_m \vec{v}) \quad (A. 13)$$

and

$$\frac{\partial n_e}{\partial t} = - \frac{Z}{m_i} \nabla \cdot (\rho_m \vec{v}) + \frac{1}{e} \nabla \cdot \vec{j} \quad (A. 14)$$

but, for low frequencies, $\nabla \cdot \vec{j} = 0$. Hence

$$\frac{\partial n_e}{\partial t} = - \frac{Z}{m_i} \nabla \cdot (\rho_m \vec{v}) \quad (A. 14, 1)$$
Add Eq. (A. 13) + Eq. (A. 14, 1)
\[
\frac{\partial n_i}{\partial t} + \frac{\partial n_e}{\partial t} = - \frac{(1+Z)}{m_i} \bigtriangledown \cdot (\rho_m \vec{u})
\]
or
\[
\frac{\partial (n_i+n_e)}{\partial t} = - \frac{(1+Z)}{m_i} \bigtriangledown \cdot (\rho_m \vec{u})
\]
but from vector formula,
\[
\bigtriangledown \cdot (\rho_m \vec{u}) = \rho_m \bigtriangledown \cdot \vec{u} + \bigtriangledown \rho_m \cdot \vec{u}
\]
\[
= \rho_m \bigtriangledown \cdot \vec{u} \quad \text{since the second term is second order term.}
\]
Hence,
\[
\frac{\partial (n_i+n_e)}{\partial t} = - \frac{(1+Z)}{m_i} \rho_m \bigtriangledown \cdot \vec{u}
\]
Substituting this in Eq. (A. 16), we get
\[
\frac{\partial p}{\partial t} = - \sqrt{kT} \frac{(1+Z)}{m_i} \rho_m \bigtriangledown \cdot \vec{u}
\]
Hence
\[
\bigtriangledown \cdot \frac{\partial p}{\partial t} = - \frac{(1+Z)}{m_i} \sqrt{kT} \rho_m \bigtriangledown \cdot \vec{u}
\]
\[
- \frac{(1+Z)}{m_i} \sqrt{k} (\bigtriangledown T) \rho_m (\bigtriangledown \cdot \vec{u})
\]
or
\[
\bigtriangledown \cdot \frac{\partial p}{\partial t} = - \frac{(1+Z)}{m_i} \sqrt{kT} \rho_m \bigtriangledown \cdot \vec{u}
\]
since the second term is second order term and is neglected.
Substituting this in Eq. A.15 we get
\[
\rho_m \frac{\partial^2 \vec{u}}{\partial t^2} = \frac{(1+Z)}{m_i} \sqrt{kT} \rho_m \bigtriangledown \cdot \vec{u}
\]
or
\[
\frac{\partial^2 \vec{u}}{\partial t^2} = \frac{(1+Z)}{m_i} \sqrt{kT} \bigtriangledown \cdot \vec{u} \quad \text{(A. 17)}
\]
This Eq. (A.17) is the general equation of motion.

For a wave travelling in the x direction

\[
\frac{\partial^2 \nu_x}{\partial t^2} = \frac{(1 + Z)}{m_i} \sqrt{kT} \frac{\partial^2 \nu_x}{\partial x^2} \quad \text{(A. 18)}
\]

Evidently, this is the equation of motion of an acoustic wave of velocity, \( V \) given by

\[
V^2 = \frac{(1 + Z)}{m_i} \sqrt{kT} \quad \text{(A. 19)}
\]

If the treatment is carried out for different temperatures of ions and electrons, \( \sqrt{kT} \) will have to be replaced by the more detailed average over electrons and positive ions. The velocity of this acoustic wave is then given by

\[
V^2 = Z \frac{\sqrt{e kT_e} + \sqrt{i kT_i}}{m_i}
\]

It is the same as Eq. (3.21)

A. 7 Significance of The Electric Field

The linearized equation of motion, Eq. (A.12)

\[
\rho_m \frac{\partial \vec{v}}{\partial t} = \vec{j} \times \vec{B} - \nabla p - \rho_m \nabla \phi
\]

contains the gradient of total pressure \( p \), and also \( T_i \) and \( T_e \) appear in the Eq. 3.21 for \( V \). So the importance of \( T_i \) and of the positive ion is obvious but it is not clear how the electrons affect the wave. It has been assumed that \( n_e \) varies with \( n_i \) and that \( j \) vanishes. Hence the right hand side of the equation of Generalized Ohms Law (not derived here) (Spitzer, 1956)
\[
\frac{m_e}{n_e e^2} \frac{\partial \vec{j}}{\partial t} = \vec{E} + \vec{v} \times \vec{B} - \frac{1}{en_e} \vec{j} \times \vec{B} + \frac{1}{n_e e} \nabla p_e - \gamma \vec{j}
\]

where

\[
\gamma = \frac{1}{n_e e} \frac{Pe_i}{\vec{j}}
\]

must vanish. Also, it is assumed that \( \vec{B} = 0 \), so that

\[
0 = \vec{E} + \frac{1}{n_e e} \nabla p_e
\]

Hence, an electric field \( \vec{E} \) must be present which just cancels the term in \( \nabla p_e \). It is this electrical field which transmits to the positive ions the force associated with the electron pressure gradient. As a consequence, if \( T_e \) much exceeds \( T_i \), the velocity of these acoustic waves is much greater than the thermal velocity of the positive ions, and is about equal to the random velocity the positive ions would have if \( T_i \) were as great as \( T_e \).

For a detailed discussion, reference may be made to Spitzer (1956).
PROGRAMME OF THE SPECTRAL ANALYSIS

B. 1 Copy of the spectral analysis programme for IBM-1620 computer in fortran language.

DIMENSION X(1500), SOL(500)
100 SUMX=0.0
READ 301
301 FORMAT(5SH)
      PUNCH 301
      READ 1,ALPHA,BETA,GAMMA,DELTA,EPS,ZETA,C,NN,NR,NS
      1 FORMAT(7F7.4,314)
      READ 2,(X(I),I=1,NN)
      2 FORMAT(10F8.4)
      DO 3 I=1,NN
      3 SUMX=SUMX+X(I)
      ANN=NN
      XSAR=SUMX/ANN
      PUNCH 99
      99 FORMAT(15X.4HSUMX.15X.4HXBAR)
      PUNCH 4,SUMX,XBAR
      4 FORMAT(8X.F11.3,12X.F7.3)
      SUMX=0.0
      DO 5 I=1,NN
      5 X(I)=X(I)-XSAR
      TYPE 200
      200 FORMAT(43HSW1 ON FOR DEVIATIONS FROM MEAN,PRESS START)
      PAUSE
      IF (SENSE SWITCH 1) 75,202
      75 PUNCH 77
      77 FORMAT(/19X,12HX=*X(I)-XBAR)
      PUNCH 78,(X(I),I=1,NN)
      78 FORMAT(5(2X.F14.7))
      202 J1=NN-1
      DO 6 I=1,J1
      SAVE=X(I)-
      6 X(I)=X(I+1)-ALPHA*SAVE
      TYPE 205
      205 FORMAT(41HSW1 ON FOR WHITENED DEVIATION,PRESS START)
      PAUSE
      IF (SENSE SWITCH 1) 203,204
      203 PUNCH 79
      79 FORMAT(/19X,4HY(I))
      PUNCH 80,(X(I),I=1,J1)
      80 FORMAT(5(2X,F14.7))
      204 J2=NR+1
      DO 8 I=1,J2
      J3=NN-1
      DO 9 J=1,J3
      K=J+I-1
      9 SUMX=SUMX+X(J)*X(K)
      DEN=NN-1
      SOL(I)=SUMX/DEN
      IF (SENSE SWITCH 2) 206,8
      206 TYPE 207,1
207 FORMAT(2HC(*13*13H ) CALCULATED)
8 SUMX=0.0
81 PUNCH 83
83 FORMAT(/19X+14H AUTOCOVARIANCE)
   PUNCH 84*(SOL(I)*I=1,J2)
84 FORMAT(5(2X,F14.7))
   X1=SOL(I)
   ANR=NR
   J1=NS+1
   DO 50 I=1,J1
   J2=NR-1
   AK1=I-1
   SUM=0.0
   DO 51 J=1,J2
   AJ=J
   SUM=SUM+SOL(J+1)*COSF(AK1*AJ*3.1416/ANR)
50 X(I)=X(I)+2.*SUM+SOL(NR+1)*COSF(AK1*3.1416)
   PUNCH 85
85 FORMAT(/19X+14H POWER SPECTRUM)
   PUNCH 86*(X(I)*I=1,J1)
86 FORMAT(5(2X,F14.7))
   DO 500 I=1,J1
500 SOL(I)=0.0
   SOL(I)=BETA*X(I)+GAMMA*X(NS+1)+DELTA*X(NS)
   DO 56 I=2,NS
   SOL(I)=BETA*X(I)+GAMMA*X(I-1)+DELTA*X(I+1)
   PUNCH 89
89 FORMAT(/19X+13H SMOOTHED POWER SPECTRUM)
   PUNCH 90*(SOL(I)*I=1,J1)
90 FORMAT(5(2X,F14.7))
   TYPE 208
208 FORMAT(44H SMOOTHED POWER SPECTRUM COMPUTED-PRESS START)
   PAUSE
   DO 501 I=1,J1
501 X(I)=SOL(I)
   J1=NS+1
   DO 57 I=1,J1
   AI=I-1
   AS=NS
   X(I)=X(I)/(EPS-ZETA*COSF(AI*3.1416/AS))
   PUNCH 91
91 FORMAT(/19X+7H PRIME)
   PUNCH 92*(X(I)*I=1,J1)
92 FORMAT(5(2X,F14.7))
   DO 58 I=2,J1
   AI=I-1
   X(I)=X(I)/((SINF(AI*3.1416/(2.*AS)))/(AI*3.1416/(2.*AS))**2)
   PUNCH 93
93 FORMAT(/19X+14H DOUBLE PRIME)
   PUNCH 94*(X(I)*I=1,J1)
94 FORMAT(5(2X,F14.7))
   DO 59 I=1,J1
59 X(I)=X(I)*C
   PUNCH 95
95 FORMAT(/19X*16HC*U DOUBLE PRIME)
    PUNCH 96,((X(I)+I=1,J1))
96 FORMAT(5(2X,F14.7))
    DO 98 I=1,NN
98 X(I)=0.0
    DO 300 I=1,250
300 SOL(I)=0.0
    TYPE 108
108 FORMAT(25HENTER NEW PARAMETERS, DATA)
    PAUSE
    GO TO 100
END
APPENDIX C

GAUSSIAN FIT AND TEST

C. 1 Gaussian Fit to the Histogram*

Suppose the frequency of occurrence of amplitudes

\[ x_1, x_2, x_3, \ldots, x_n \]

is \( f_1, f_2, f_3, \ldots, f_n \).

Then the mean is defined by

\[
\bar{x} = \frac{\sum f_n x_n}{\sum f_n}
\]  

(C. 1)

The "modulus of precision", \( h \), is given by

\[
\frac{1}{h} = \left( \frac{2 \sum (\bar{x} - x_n)^2 f_n}{\sum f_n} \right)^{1/2}
\]  

(C. 2)

and the normalized equation of the law of normal frequency distribution is

\[
y = \frac{h}{\sqrt{\pi}} e^{-h^2(\bar{x} - x_n)^2}
\]  

(C. 3)

the magnitude of \( y \) at \( x = x_n \) is

\[
y_0 = \frac{h}{\sqrt{\pi}}
\]  

(C. 4)

The programme calculates the above quantities and computes the \( y \)'s for different \( x \)'s. It also gives normalized values of \( y \) which are obtained by dividing the values by \( \frac{h}{\sqrt{\pi}} \).

After a normal curve is fitted to the histogram, the next logical step is to test quantitatively whether or not the distribution is normal. There are many tests described in the literature, (Worthing and Geffner, 1960; Young, 1962). The test described here (Chambes, 1958) depends upon the first four moments about the mean of the distribution.

* The mathematics contained in this section is outlined by Worthing and Geffner (1960).
C. 2 Normality Test*

Compute expressions \( f_n(\bar{x} - x_n) \), \( f_n(\bar{x} - x_n)^2 \), \( f_n(\bar{x} - x_n)^3 \) and \( f_n(\bar{x} - x_n)^4 \).

Also calculate

\[
\mathcal{U}_1 = \frac{\sum f_n(\bar{x} - x_n)}{\sum f_n} \quad (C. 5. 1)
\]
\[
\mathcal{U}_2 = \frac{\sum f_n(\bar{x} - x_n)^2}{\sum f_n} \quad (C. 5. 2)
\]
\[
\mathcal{U}_3 = \frac{\sum f_n(\bar{x} - x_n)^3}{\sum f_n} \quad (C. 5. 3)
\]

and

\[
\mathcal{U}_4 = \frac{\sum f_n(x - x_n)^4}{\sum f_n} \quad (C. 5. 4)
\]

The four moments about the mean of the frequency distribution are obtained from the following expressions:

\[
\mu_1 = 0 \quad (C. 6. 1)
\]
\[
\mu_2 = \mathcal{U}_2 - \mathcal{U}_1^2 - \frac{1}{12} \quad (C. 6. 2)
\]
\[
\mu_3 = \mathcal{U}_3 - 3 \mathcal{U}_1 \mathcal{U}_2 + 2 \mathcal{U}_1^3 \quad (C. 6. 3)
\]

and

\[
\mu_4 = \mathcal{U}_4 - 4 \mathcal{U}_1 \mathcal{U}_3 + 6 \mathcal{U}_1^2 \mathcal{U}_2 - 3 \mathcal{U}_1^4 - \frac{1}{2} \mu_2 - \frac{1}{80} \quad (C. 6. 4)
\]

After this calculate two constants

\[
\beta_1 = \frac{\mu_3^2}{\mu_2^3} \quad (C. 7. 1)
\]

and

\[
\beta_2 = \frac{\mu_4}{\mu_2^2} \quad (C. 7. 2)
\]

* The mathematics contained in this section is exactly the same as that given by Chambers (1958).
Further, compute the quantities
\[ \gamma_1 = (\beta_1)^{1/2} \]  
(C. 8. 1)

and
\[ \gamma_2 = \beta_2 - 3 \]  
(C. 8. 2)

\( \gamma_1 \) is a measure of the symmetry of the distribution. 
\( \gamma_2 \) is said to measure "Kurtosis", departures of a symmetrical nature from normality.

Finally, determine the standard errors of \( \gamma_1 \) and \( \gamma_2 \)

\[ (S. E.) \gamma_1 = \left( \frac{6}{N} \right)^{1/2} \]  
(C. 9. 11)

and
\[ (S. E.) \gamma_2 = \left( \frac{24}{N} \right)^{1/2} \]  
(C. 9. 2)

If \( \gamma_1 \ll 2(S. E.) \gamma_1 \), and \( \gamma_2 \ll 2(S. E.) \gamma_2 \) the distribution is not significantly different from the normal form; however, if \( \gamma_1 \) and \( \gamma_2 \) are greater than twice their standard errors, the distribution is not normal. The curves which are flat-topped and short-tailed compared with the normal curve are called Platykurtic; for these \( \beta_2 \) is less than 3. The curves which are sharply peaked and long-tailed, and for which \( \beta_2 \) is greater than 3, are called Leptokurtic.

The programme is written such that the computer performs all the above steps and states whether or not the distribution is normal, and whether it is Leptokurtic or Platykurtic. A copy of the programme for IBM - 1620 Computer, (in Fortran language) is attached here.
C. 3 Copy of the Gaussian fit and test programme for IBM-1620 computer in fortran language

```fortran
DIMENSION F(50), Y(50), STORE(50), OBS(1000), FX1(50), FX2(50), FX3(50)
DIMENSION FX4(50)

DO 100 I = 1, 50
   100 F(I) = 0.0
   SUMF1 = 0.0
   SUMF2 = 0.0
   SUMF3 = 0.0
   SUMF4 = 0.0
   SUMO = 0.0
   READ 907

FORMAT (55H)
PUNCH 907
READ 90, N, NN
FORMAT (I2, I4)
10 READ 1, (OBS(I), I = 1, NN)
1 FORMAT (10F8.4)
   OMAX = OBS(1)
   DO 900 I = 2, NN
   IF(OMAX - OBS(I)) 901, 900, 900
   OMAX = OBS(I)
   CONTINUE
   OMIN = OBS(1)
   DO 902 I = 2, NN
   IF(OMIN - OBS(I)) 902, 902, 903
   OMIN = OBS(I)
   CONTINUE
   PUNCH 904, OMAX, OMIN, N
904 FORMAT (12HUPPER LIMIT =,FS.4,14HLOWER LIMIT =,FS.4,21HINTERVALS
   1 IN RANGE =,IL//)
   DO 11 I = 1, NN
   SUMO = SUMO + OBS(I)
   AN = N
   DEL = (OMAX-OMIN)/AN
   DO 7 I = 1, N
   A = I
   Ai = OMIN + A*DEL
   DO 6 J = 2, NN
   IF(OBS(J)-AI) 5,5,6
   F(I) = F(I) + 1.0
   CONTINUE
   STORE(I) = F(I)
   IF(I-1) 7,7,8
   F(I) = F(I)-STORE(I-1)
   CONTINUE
   SUMF = NN
   XBAR = SUMO/SUMF
   X = OMIN + DEL/2.
   DO 13 I = 1, N
   FX1(I) = (XBAR-X)*F(I)
   FX2(I) = FX1(I)* (XBAR-X)
   FX3(I) = FX2(I)* (XBAR-X)
   FX4(I) = FX3(I)* (XBAR-X)
   13 X = X + DEL
   DO 775 I = 1, N
   SUMFX1 = SUMFX1 + FX1(I)
   SUMFX2 = SUMFX2 + FX2(I)
   SUMFX3 = SUMFX3 + FX3(I)
   SUMFX4 = SUMFX4 + FX4(I)
```

---

The program above defines a Gaussian fit and test programme for an IBM-1620 computer, written in Fortran language. It includes definitions for arrays such as `F(50)`, `Y(50)`, `STORE(50)`, `OBS(1000)`, `FX1(50)`, `FX2(50)`, and `FX3(50)` among others. The program reads data from a file, calculates various intermediate values, and then formats and prints the results. The code snippet shows typical Fortran 77 syntax with loops, conditional statements, and arithmetic operations.
H = SQRTF ((SUMF - 1.)/(2.*SUMFX2))
HINV = 1.0/H
X = OMN + DEL/2.
DO 16 I = 1, N
Y(I) = (H/SQRTF(3.1416))*EXPF(-(H*(XBAR-X))**2))
X = X + DEL
ANU1 = SUMFX1/SUMF
ANU2 = SUMFX2/SUMF
ANU3 = SUMFX3/SUMF
ANU4 = SUMFX4/SUMF
AMU1 = 0.0
AMU2 = ANU2 - ANU1**2 - .08333
AMU3 = AMU3 - 3.*ANU1*ANU2 + 2.*ANU1**3
AMU4 = ANU4 - 4.*ANU1*ANU3 + 6.*ANU1**2*ANU2 - 3.*ANU1**4 - .5*AMU2-.O125
B1 = AMU3**2/AMU2**3
B2 = AMU4/AMU2**2
G1 = SQRTF(ABS(B1))
G2 = B2 - 3.0
SDL = SQRTF(6./SUMF)
SD2 = 2.*SDL
PUNCH 21, XBAR, HINV
21 FORMAT(7HXBAR=_,FX, 6H1/H=, F10.3//)
IF(G1 - SD2) 776, 777, 778
776 IF(G2 - 2.*SD2) 779, 780, 781
779 PUNCH 782
782 FORMAT(29H DISTRIBUTION NORMAL, SYMMETRIC)
GO TO 802
777 PUNCH 783
783 FORMAT(13HGAMMA 1 = 2 (SE 1))
GO TO 776
778 PUNCH 784
784 FORMAT (35H DISTRIBUTION NOT NORMAL, UNSYMМETRIC)
785 IF(B2 - 3.0) 786, 787, 788
786 PUNCH 789
789 FORMAT(17H CURVE PLATYKURTIC).
GO TO 802
787 PUNCH 790
790 FORMAT(8H BETA 2 = 3.)
GO TO 802
788 PUNCH 791
791 FORMAT(17H CURVE LEPTOKURTIC)
GO TO 802
780 PUNCH 792
792 FORMAT(13HGAMMA 2 = 2(SE2))
GO TO 785
781 PUNCH 793
793 FORMAT(33H DISTRIBUTION NOT NORMAL, SYMMETRIC)
GO TO 785
802 PUNCH 794
794 FORMAT(71H BETA 1 BETA 2 GAMMA 1 ST ERR 1 GAMMA
12 ST ERR 2 )
PUNCH 795, B1, B2, G1, SDL, G2, SD2
795 FORMAT(6F12.6)
PUNCH 102
102 FORMAT(22H DISTRIBUTION CURVE---)
PUNCH 22, (Y(I), I = 1, N)
DO 204 I = 1, N
204 Y(I) = Y(I)/(H/SQRTF(3.1416))
PUNCH 796
796 FORMAT(29HNORMALIZED DISTRIBUTION CURVE)
PUNCH 22, (Y(I), I = 1, N)
22 FORMAT(5(6X,F10.3))
PUNCH 101
101 FORMAT(18HHISTOGRAM DATA——)
PUNCH 23, (F(I), I = 1, N)
23 FORMAT(5(6X,F10.0))
BIG = F(I)
DO 1001 I = 2, N
IF(BIG-F(I)) 1000, 1000, 1001
1000 BIG = F(I)
1001 CONTINUE
DO 705 I = 1, N
705 F(I) = F(I)/BIG
PUNCH 797
797 FORMAT(20HNORMALIZED HISTOGRAM)
PUNCH 22, (F(I), I = 1, N)
TYPE 905
905 FORMAT(37HENTER NEW PARAMETERS, DATA—PRESS START)
PAUSE
GO TO 906
END
APPENDIX D

DERIVATION OF EQUATION 7.1

The treatment adopted here is the same as outlined by Remillard (1960).

According to Poisson's integral equation any function \( p \) which satisfies the differential equation

\[
\nabla^2 p = \frac{1}{C^2} \frac{\partial^2 p}{\partial t^2}
\]

(D. 1)

where \( C \) is a constant, can be expressed in terms of its value and its derivative with respect to \( t \), both evaluated at \( t = 0 \).

The expression for \( p \) which satisfies Eq. (D. 1) is

\[
p = \frac{\partial}{\partial t} \left( t \bar{p}_0 \right) + t \left( \bar{\frac{\partial p}{\partial t}} \right)_0
\]

(D. 2)

where the subscript 0 indicates evaluation at \( t = 0 \), and the bar signifies an average over the surface of a sphere whose radius is \( ct \).

In the Small Signal Wave Equation (D. 1) \( p \), \( c \) and \( t \) are taken respectively as the excess pressure, the speed of sound, and time. The initial conditions are; (a) the initial pressure is constant throughout the stroke channel (\( p_0 = P \)) and zero elsewhere (\( p_0 = 0 \)), and (b) it is stationary everywhere \( \left[ \left( \frac{\partial p}{\partial t} \right)_0 = 0 \right] \). Applying the later condition, the Eq. (D. 2) is reduced to

\[
p = \frac{\partial}{\partial t} \left( t \bar{p}_0 \right)
\]

(D. 3)

Consider an observer at point \( Q \) (Fig. D. 1) located in a plane perpendicular to the axis of the source and at a distance \( b \)
from it. A cylinder of length $2h$ and radius $a$ represents the acoustic source (in our case $h$ is the height of the lightning column and the bottom half is the image produced by reflection in the ground plane). A sphere of radius $Ct$ centered at $Q$, where the pressure is sought, cuts into the cylinder when $Ct > b$. Now if it were possible to calculate the area on the surface of the sphere which is intersected by the cylinder, the average value $p_0$ could be found simply by dividing this area by $4\pi (Ct)^2$, the surface area of the entire sphere, and multiplying the result by $p$. The value of pressure $p$ can then be calculated from Eq. (D.3).

As shown in figure (D.2) a cylindrical source of infinite length is considered. If we assume the origin of the cartesian coordinates at point $0$, nearest point to $Q$, the position of observer on the surface of the source, then the equations of sphere is given by

$$(x + b)^2 + y^2 + z^2 = (Ct)^2$$

and that of cylinder is $(x - a)^2 + y^2 = a^2$.

The surface area $S$ of the sphere which is intersected by the cylinder is found by projecting $dS$ onto the $xy$-plane and integrating over the area of circle which is formed by the intersection of the cylinder with the $x$-$y$ plane.

The unit vector, $\vec{n}$, normal to the surface of the sphere is given by

$$\vec{n} = \frac{1}{Ct} (\hat{i} x + \hat{j} y + \hat{k} z)$$

(D.4)
Fig. D. 1 Diagram showing a sphere of radius $C_t$ centered at $Q$ and a cylinder of height $2h$ and radius $a$. The cylinder, which represents the source, is shown intersecting the sphere.

Fig. D. 2 Cylindrical source shown intersecting a total area $S$ (shaded) of an expanding sphere centered at $Q$. 
The projection of $ds$ onto the xy-plane is given by
\[ dx
dy = \mathbf{k} \cdot \mathbf{n}
ds = \frac{z}{Ct}
ds \quad (D. 5) \]

Hence
\[
S = Ct \iint \frac{dy
dx}{z} = Ct \iint \frac{dy
dx}{[(Ct)^2 - (x + b)^2 - y^2]^{1/2}} \quad (D. 6)
\]

Consider the cylinder as infinite in length and confine the analysis to times when $Ct > (b + 2a)$. With the conditions, Eq. (D. 6) becomes
\[
S = 4Ct \iint_{0}^{2a} \frac{[x(2a - x)]^{1/2}}{[(Ct)^2 - (x + b)^2 - y^2]^{1/2}} \text{ dy dx, } Ct > b + 2a \quad (D. 7)
\]

Integrating the above integral w. r. t. t (Peirce, 1910, p. 20), one obtains
\[
S = 4Ct \int_{0}^{2a} \sin^{-1} \left[ \frac{x(2a - x)}{(Ct)^2 - (x + b)^2} \right]^{1/2} \text{ dx, } Ct > b + 2a \quad (D. 8)
\]

The initial pressure $P_0$, average over the surface of the sphere, is
\[
\bar{P}_0 = \frac{\int S \cdot P}{4 \pi \left( Ct \right)^2} \quad (D. 9)
\]
or
\[
\bar{P}_0 = \frac{P}{\pi \left( Ct \right)} \int_{0}^{2a} \sin^{-1} \left[ \frac{x(2a - x)}{(Ct)^2 - (x + b)^2} \right]^{1/2} \text{ dx} \quad (D. 10)
\]

Substituting the value of $\bar{P}_0$ from Eq. (D. 10) into Eq. (D. 3) we get
\[
p = \frac{P}{\pi C} \int_{0}^{2a} \frac{\partial}{\partial t} \sin^{-1} \left[ \frac{x(2a - x)}{(Ct)^2 - (b + x)^2} \right]^{1/2} \text{ dx} \quad (D. 11)
\]
Remembering $D \sin^{-1} u = (1 - u^2)^{1/2}$ du we get

\[ p = \frac{-PCt}{\pi} \int_0^{2a} \sqrt{\frac{x(2a - x)}{[(Ct)^2 - b^2 - 2x(a + b)]}} \cdot \frac{dx}{(Ct)^2 - (b + x)^2}, \quad (D, 12) \]

\[ Ct > b + 2a \]

The approximate solution of Eq. (D, 12) can be obtained by the use of the theorem of mean (Wilson, 1958, p. 29). It states that if $m$ and $M$ are the minimum and maximum of $h(x)$ between $\alpha$ and $\beta$ and if $f(x)$ is always positive in the interval, then

\[ m \int_\alpha^\beta f(x) \, dx < \int_\alpha^\beta f(x) \, h(x) \, dx < M \int_\alpha^\beta f(x) \, dx \quad (D, 13) \]

Now if we assume

\[ h(x) = \frac{1}{[(Ct)^2 - b^2 - 2x(a + b)]^{1/2} [(Ct)^2 - (b + x)^2]} \]

\[ 0 \leq x \leq 2a \]

and

\[ f(x) = \left[ x(2a - x) \right]^{1/2}, \quad 0 \leq x \leq 2a \quad (D, 15) \]

and $\alpha = 0, \beta = 2a \quad (D, 16)$

it is seen by inspection that the maximum value of $h(x) = M$ in the interval $0 \leq x \leq 2a$ occurs when $x = 2a$, and the minimum value, $m$, occurs when $x = 0$. Thus we have

\[ M = \frac{1}{[(Ct)^2 - (b + 2a)^2]^{3/2}} \quad (D, 17) \]

and

\[ m = \frac{1}{[(Ct)^2 - b^2]^{3/2}} \quad (D, 18) \]

It can also be seen that (Peirce, 1910, p. 31)
\[
\int_{\alpha}^{\beta} f(x) \, dx = \int_{0}^{2a} \left[ x(2a - x) \right]^{1/2} dx
\]

\[
= \left[ \frac{x - a}{2} \left\{ x(2a - x) \right\}^{1/2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x - a}{a} \right) \right]_{a}^{2a} \tag{D. 19}
\]

\[
= \frac{\pi a^2}{2}
\]

Substituting the values for \( h(x), f(x), m, M, \alpha, \) and \( \beta, \)

from equations (D. 14) to (D. 15) in Equation (D. 13), and comparing

the result with Equation (D. 12) we obtain

\[
\frac{\pi a^2/2}{\left[ (Ct)^2 - b^2 \right]^{3/2}} < -\frac{p P}{P(Ct)} < \frac{\pi a^2/2}{\left[ (Ct)^2 - (b + 2a)^2 \right]^{3/2}}, \quad Ct > b + 2a \tag{D. 20}
\]

or

\[
\frac{-a^2 Ct P/2}{\left[ (Ct)^2 - b^2 \right]^{3/2}} > p > \frac{-a^2 Ct P/2}{\left[ (Ct)^2 - (b + 2a)^2 \right]^{3/2}}, \quad Ct > b + 2a \tag{D. 21}
\]

For normal observation distance, \( b \gg 2a, \) both extremes

of Eq. (D. 21) approach each other. Hence

\[
p \approx \frac{-a^2 Ct P/2}{\left[ (Ct)^2 - b^2 \right]^{3/2}}, \quad Ct > b, \quad b \gg 2a \tag{D. 22}
\]

The equation (D. 22) gives the sound pressure from an

infinitely long source of uniform initial distribution of condensation.

The negative sign indicates that it is rarefactive pressure.
APPENDIX E

METEOROLOGICAL PARAMETERS

E. 1 Explanation of the symbols

Information about various meteorological parameters was obtained from the local Airport Weather Office and is tabulated in table E. 1. The symbols used are explained as below:

- **Cb** means cumulonimbus cloud
- **Cu** means cumulus cloud
- **Cu+** means heavy cumulus cloud
- **Ac** means altocumulus cloud
- **Sc** means stratocumulus cloud
- **Ci** means cirrus cloud
- **(i)** means scattered (that is when 5/10 or less of sky is covered)
- **(ii)** means broken (that is when 6/10 to 9/10 of sky is covered)
- **(c)** means overcast (that is when 10/10 of sky is covered)
- **A** means hail
- **T** means thunder
- **R** means continuous rain from stratus clouds
- **RW-** means light rain showers (greater than 0.10 inch)
- **RW--** means very light showers (occasional drops)
- **RW+** means rain showers with more than 0.30 inch per hour
- **↓** means north wind
- **↑** means south wind
- ← means east wind
- → means west wind
The arrows in the table show the direction of wind. The time increases as one goes down the page.

E means estimated

To understand the table an example is given. On July 9, 1963, a thunderstorm was observed between 04:00 and 06:00 a.m. C.S.T. Various layers of clouds were noted. First at an height of 4000 ft, scattered clouds and at estimated height of 6000 ft. broken clouds were noted. Second observation showed that at an estimated height of 1500 ft, scattered clouds and at 5000 ft. broken clouds were present. The third column shows that light thunder showers and occasional very light thunder showers were observed during the storm. The fourth column shows that the distribution of clouds was variable. Stratocumulus cloud covered 4/10 of the sky and cumulonimbus cloud covered 5/10 of the sky in the first observation. In the later observation stratocumulus cloud covered 7/10 of the sky and the cumulonimbus cloud covered 2/10 of the sky. The storm was south to east and then came roughly overhead. The wind speeds varied between 16 to 25 mph. First the winds were variable between north and northwest, then between west and northwest, and finally north and northeast. The temperature varied between 67° to 66° F, the relative humidity varied between 85% to 96%, and the barometric pressure at the station varied between 953.8 to 953.0 millibars.
<table>
<thead>
<tr>
<th>Date and Time</th>
<th>Cloud height x 100 ft.</th>
<th>Precipitation</th>
<th>Cloud types</th>
<th>Location</th>
<th>Wind Speeds, Direction</th>
<th>Temperature, °F</th>
<th>Relative Humidity, %</th>
<th>Pressure, mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 26, 1963</td>
<td>25050°</td>
<td>TRW-</td>
<td>Sc(2/10)Cb(8/10)</td>
<td>overhead</td>
<td>8 to 14</td>
<td>64-60</td>
<td></td>
<td>945.1 - 946.8</td>
</tr>
<tr>
<td>03:00 to 04:30 p.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>August 25, 1963</td>
<td>50090°</td>
<td>TRW--</td>
<td>Cb(3/10)Ac(7/10)</td>
<td>North-west to North</td>
<td>9 to 14</td>
<td>59-61</td>
<td></td>
<td>954.9 - 956.5</td>
</tr>
<tr>
<td>00:30 to 03:00 a.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>August 8, 1963</td>
<td>45°</td>
<td>TRW-</td>
<td>Cb(10/10)</td>
<td>overhead</td>
<td>15 to 25</td>
<td>68-65</td>
<td></td>
<td>956.6 - 960.0</td>
</tr>
<tr>
<td>01:00 to 03:00 a.m.</td>
<td>40°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 24, 1963</td>
<td>60°100°</td>
<td>T</td>
<td>Cb(7/10)Ac(3/10)</td>
<td>overhead</td>
<td>12 to 24</td>
<td>69-62</td>
<td></td>
<td>949.9 - 954.9</td>
</tr>
<tr>
<td>10:55 p.m. to 01:30 a.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE E. 1**

**METEOROLOGICAL PARAMETERS**
<table>
<thead>
<tr>
<th>Date and Time</th>
<th>Cloud height x 100 ft</th>
<th>Precipitation</th>
<th>Cloud types</th>
<th>Location</th>
<th>Wind Speeds, Direction</th>
<th>Temperature, °F</th>
<th>Relative Humidity, %</th>
<th>Pressure, mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 20, 1963</td>
<td>60°E65°</td>
<td>T</td>
<td>Cu+(3/10)Cb(5/10)</td>
<td>West</td>
<td>6 to 8</td>
<td>73-75</td>
<td>54-57</td>
<td>950.5 - 950.9</td>
</tr>
<tr>
<td>03:00 to</td>
<td>50°E65°</td>
<td>T</td>
<td>Cu+(3/10)Cb(6/10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>04:00 p.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 9, 1963</td>
<td>40°E60°</td>
<td>TRW-</td>
<td>Sc(4/10)Cb(5/10)</td>
<td>South to East overhead</td>
<td>16 to 25</td>
<td>67-66</td>
<td>85-96</td>
<td>953.8 - 953.0</td>
</tr>
<tr>
<td>04:00 to</td>
<td>E15050°</td>
<td>TRW-</td>
<td>Sc(7/10)Cb(2/10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>06:00 a.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 5, 1963</td>
<td>65°120°300°</td>
<td>T</td>
<td>Cb(4/10)Ac(1/10)Ci(4/10)</td>
<td>East to south overhead</td>
<td>9 to 25</td>
<td>82-64</td>
<td>59-94</td>
<td>950.9 - 954.4</td>
</tr>
<tr>
<td>(Frontal passage)</td>
<td>E60°</td>
<td>T+RW+A</td>
<td>Cb(10/10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>heavy storm</td>
<td>E50°60©120©</td>
<td>TRW-</td>
<td>Sc(6/10)Cb(2/10)Ac(2/10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>07:00 to</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>09:00 p.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table E. 1 continued

<table>
<thead>
<tr>
<th>Date and Time</th>
<th>Cloud height x 100 ft.</th>
<th>Precipitation</th>
<th>Cloud types</th>
<th>Location</th>
<th>Wind Speeds, Direction</th>
<th>Temperature, ° F</th>
<th>Relative Humidity, %</th>
<th>Pressure, mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 21, 1962, 02:00 p.m. to 06:00 p.m.</td>
<td>55°E60'00&quot;280'00&quot;</td>
<td>TRW -- T</td>
<td>Cb(3/10) Cu+(4/10) Ci(1/10)</td>
<td>came from NE to E</td>
<td>5 to 14</td>
<td>Temp. 68-65</td>
<td>Rel. H. 61-74</td>
<td>Pres. 956.6 - 956.0</td>
</tr>
<tr>
<td></td>
<td>45°E60'00&quot;250'00&quot;</td>
<td>TRW -</td>
<td>Cb(4/10) Cu+(2/10) Ci(1/10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cb(1/10) Cu+(3/10) Ci(2/10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cu(4/10) Cb(4/10) Ci(1/10)</td>
<td>went to SE to S to SW</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sc(4/10) Cb(4/10) Ci(1/10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 1, 1962, 08:00 to 10:00 p.m.</td>
<td>E20°</td>
<td>TRW +</td>
<td>Cb(10/10)</td>
<td>overhead</td>
<td>17 to 21</td>
<td>Temp. 55-60</td>
<td>Rel. H. 91-97</td>
<td>Pres. 945.1 - 946.8</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


Battan, L. J. 1964. The Thunderstorm (New York American Library of Canada Ltd., 156 Front Street W., Toronto 1)


Humphreys, W. J. 1920. Physics of the air, The Franklin Institute of the State of Pennsylvania (J. B. Lippincot Co.).


Jenkins, G. M. 1961. Technometrics, 3, 133.


Meisser, O. 1927. Z. Geophys. 3 Jahrgang, 285.


Muller, F. Max. 1869. Rig-Veda Sanhita (Trubner and Co., 60 Paternoster Row, London), 65.


Richards, R. C. 1923. Phil. Mag., 45, 926.


Veenema, L. G. 1917(June). Das Wetter, 127.


Veenema, L. G. 1917(Dec.) Das Wetter, 258.

Veenema, L. G. 1918(March-April) Das Wetter, 56.


