A TUNNEL DIODE ANALOGUE

A thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of
Master of Science
in the Department of Electrical Engineering
University of Saskatchewan

by

H.N. Mahabala

Written under the Supervision of
Professor R.S.C. Cobbold

Saskatoon, Saskatchewan

October, 1961

The University of Saskatchewan claims copyright in
conjunction with the author. Use shall not be made of
the material contained herein without proper acknowledg-
ment. Quotations are to be acknowledged and authorities
cited.
ACKNOWLEDGMENTS

The author wishes to express his sincere thanks to Professor R.S.C. Cobbold for his valuable guidance at every stage of this work. Thanks are also due to Professor H.F. Moody and Mr. A.G. Wacker for their keen interest and helpful criticism during the course of this project.

The financial support to the author from the National Research Council, Canada (Grant No. A 578) is gratefully acknowledged.
ABSTRACT

A versatile low frequency analogue capable of simulating non-linear characteristics of a tunnel diode is described. The parameters of the analogue are easily variable enabling the tolerance of circuits to variations in tunnel diode parameters to be readily ascertained. Further, the large magnitude of the parameters and the small bandwidth makes investigations into stability criteria, non-linear oscillations and switching to be carried out with ease.

Linear stability criteria are verified and demonstrated experimentally by the use of the analogue. Determination of the limit cycle and the hysteresis in boundary conditions is discussed with the aid of presently available non-linear analysis, and results demonstrated experimentally.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(1) General Introduction</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(2) Tunnel Diodes</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(3) Purpose of an Analogue</td>
<td>4</td>
</tr>
<tr>
<td>II</td>
<td>DESCRIPTION OF THE ANALOGUE</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>(1) Specification</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>(2) Circuit Description</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(3) Bandwidth</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>(4) Characteristics</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>APPLICATIONS OF THE ANALOGUE</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>(1) Stability Criteria</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>(a) Theory</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>(b) Experimental Verification</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>(2) Steady State Oscillation</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>(3) Boundary Conditions</td>
<td>33</td>
</tr>
<tr>
<td>IV</td>
<td>CONCLUSIONS</td>
<td>38</td>
</tr>
<tr>
<td>V</td>
<td>REFERENCES</td>
<td>40</td>
</tr>
<tr>
<td>VI</td>
<td>APPENDIX</td>
<td>41</td>
</tr>
<tr>
<td>FIG.</td>
<td>TITLE</td>
<td>Page</td>
</tr>
<tr>
<td>------</td>
<td>-----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>Current and Conductance vs. Voltage Characteristic of a Ge Tunnel Diode</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Equivalent Circuit for a Tunnel Diode</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Characteristics that Combine to form the Approximate Characteristic of a Tunnel Diode</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>Schematic of Circuits that Generate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) Initial Forward Resistance</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>(b) Non-linear Negative Resistance</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>Complete Circuit of the Analogue</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>Variation of Input Resistance and Capacitance with Frequency</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>Voltage-Current Characteristic of the Analogue Superimposed on that for a Ge Tunnel Diode</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>Voltage-Current Characteristic of the Analogue</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) Superposition of Component Characteristics</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>(b) Typical Characteristic</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>(c) Family of Characteristics (Ip, Vg, and Iv)</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>Equivalent Circuit Employed to Discuss the Stability Criteria</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>Stability Criteria</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) Stability Criteria Expressed in a Graphical Form</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>(b) Normalized Plot to Obtain C for a Given 2o/2n</td>
<td>22</td>
</tr>
<tr>
<td>11</td>
<td>Experimental Verification of the Stability Criteria</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) Region of Exponential Decay</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>(b) Region of Sinusoidal Decay</td>
<td>27</td>
</tr>
<tr>
<td>12</td>
<td>Experimental Verification of the Stability Criteria</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) (1) Region of Sinusoidal Growth</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>(2) Steady State Oscillation</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>(b) Growth of Oscillation Superimposed on the D.C. Characteristic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1) Sinusoidal Steady State Oscillation</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>(2) Nonsinusoidal Steady State Oscillation</td>
<td>27</td>
</tr>
<tr>
<td>FIG.</td>
<td>TITLE</td>
<td>Page</td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>13</td>
<td>Variation of the Frequency of Oscillation with Capacity</td>
<td>31</td>
</tr>
<tr>
<td>14</td>
<td>Plot of Circuit Characteristics with Capacity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) Percentage Frequency Deviation</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>(b) Percentage Harmonic Content</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>(c) D.C. Bias Point</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>(d) Voltage Excursion</td>
<td>34</td>
</tr>
<tr>
<td>15</td>
<td>Hysteresis in the Boundary Condition for</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>Starting and Stopping of Oscillation for the Analogue</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Simplified Equivalent Circuit for Obtaining the Input Impedance</td>
<td>42</td>
</tr>
</tbody>
</table>
I INTRODUCTION

(1) General Introduction

Tunnel diodes and other two terminal non-linear negative resistance devices have provided the circuit designer with new tools which promise to be extremely valuable in producing new circuit characteristics, or which will simplify the design of present special circuitry. His familiarity with vacuum tubes and transistors, operated both in the linear and non-linear modes, offers little assistance in understanding and utilizing these devices. In fact, the combination of both non-linearity and negative resistance in a single device may appear at first a little formidable. Nevertheless, simplicity of construction, low terminal capacity, and excellent high frequency performance of tunnel diodes have much to commend themselves to the designer's attention with regard to both linear and non-linear operation. In particular, their small inherent capacity and voltage difference between the two stable states suggest their wide application in the high-speed digital computer field. Unloaded switching times of a few tenths of a nanosecond have already been demonstrated for the more recently developed tunnel diodes. However, the complete utilization of these properties in high-speed logical circuitry still remains to be achieved. Basic to this problem is the understanding of the switching phenomena in non-linear devices under various external impedance configurations. The negative resistance region of a tunnel diode can be used for wide-band amplification and
oscillation, and as such its non-linearity needs investigation. The understanding of these and other related problems can be greatly facilitated by studies on a tunnel diode analogue, which is the topic of this thesis.

(2) Tunnel Diodes

A tunnel diode consists of a very abrupt p-n junction with both the p and n regions "doped" to the extent that the Fermi level lies within the valence or conduction band. It exhibits a current-voltage characteristic, at room temperatures, similar to that shown in Fig. 1, and as such can be considered to be a voltage-controlled non-linear resistance device. The departure of the characteristic from that of a p-n junction is due to the process of quantum mechanical tunneling. Since the process of tunneling has associated with it transit times in the order of the relaxation time of the material (typically $10^{-12}$ Sec.), the operation of a tunnel diode, in many practical cases, is restricted by the parasitic impedances.

Such a device is normally characterized by a peak current $I_p$, a peak to valley current ratio $I_p/I_v$, a negative resistance or conductance $-R_n$, $-g$, and a gap voltage $V_g$. For a given material and doping, $I_p$ and $g$ are proportional to the junction area. The valley current is strongly influenced by the presence of trapping levels in the forbidden gap of the space charge region\(^1\), and decreases with decreasing concentration of trapping centres. The gap voltage is dictated by the energy gap of the intrinsic material and may vary from 300 mV for Ge to 0.9 V for GaAs.
Fig. 1. Current and Conductance vs. Voltage Characteristic of a Ge Tunnel Diode.
Fig. 2. Equivalent Circuit for a Tunnel Diode.
convenient if an analogue, which simulates the characteristics of a tunnel diode and has easily variable parameters, can be employed.

Biasing the tunnel diode in the negative resistance region is essential for amplification and oscillation, and it is of interest to study the stability of this biasing. Tunnel diodes cannot be used with ease for the above purpose, because of the difficulty in suppressing high frequency parasitic oscillations encouraged by its large bandwidth. Even if an attempt is made to derive a stable negative resistance by the use of a large external capacity, the circuit arrangement is extremely critical and has little flexibility owing to severe restrictions on the length of connecting leads. The problem can be overcome by the use of an analogue with small bandwidth.

Switching being one of the important fields of application of tunnel diodes, an investigation into the switching phenomena is very desirable, but is fraught with many difficulties. Firstly, there is no access to all the points in the equivalent circuit which makes difficult an investigation into the effects of tunnel diode capacity and lead inductance on switching. Secondly, since the capacity \( (C_b + C_d) \) is in the order of a few tens of picofarads, the use of ordinary probes for observation may disturb the circuit conditions significantly. Thirdly, the magnitudes of the voltage and current are small and are not easily and accurately measurable. Finally, highly sophisticated techniques are
required for observing the mode of switching due to the very small switching times. An analogue with access to every point on the equivalent circuit, large voltage and current magnitudes, and low speed of switching, has distinct advantages over a tunnel diode for preliminary studies on switching.
II DESCRIPTION OF THE ANALOGUE

(1) Specification

Geller and Mantek \(^2\) have used an analog computer to simulate the approximated characteristics of a tunnel diode for large signal studies. However, their technique has limited use. A versatile analogue should have the same equivalent circuit as that of a tunnel diode. Once a non-linear resistance \(R_N\) is available, capacitances, inductances, and resistances can be appended externally to obtain an analogue. Hence the problem of building an analogue reduces to that of generating \(R_N\) by a suitable means. If diodes and transistors are used to generate \(R_N\), it will have a small bandwidth because of the very nature of the components used. The variation of conductance \(g\) with voltage should be similar to the one shown in Fig. 1(b), except for a change in the voltage scale. This would imply that the V-I characteristic of the analogue fits the one for a tunnel diode drawn to an enlarged scale. Since the analogue should hold good even for D.C., the use of reactances for generation of the non-linearity is inadmissible. The range of variation for the parameters of the analogue were set as follows:

\[
I_p = 0.10 \text{mA}; \quad \frac{I_p}{I_n} = 2 - \text{a large value}
\]
\[
V_g = 0.5 - 5 \text{ volts}; \quad -R_N = -50 - -1000 \text{ ohms}.
\]

(2) Circuit

The characteristic of a tunnel diode can be compounded from four separate characteristics as shown in Fig. 3. The individual circuits which generate these four components will now be considered.
Fig. 3. Characteristics that Combine to Form the Approximate Characteristic of a Tunnel Diode.
(C1) Schematic of the circuit employed is shown in Fig. 4 (a). Diodes D1 and D2 form the two arms of a bridge. Initially the bridge is balanced by means of R2 so that the voltage across AB is zero. As A is taken positive with respect to B by an external voltage, the terminal current flows entirely through D1 and D2, since R1 and R2 are arranged to be very large. Current saturation results when D1 becomes reverse biased, and peak current Ip will be nearly equal to the initial current in D1, and hence can be controlled by R1. The slope resistance can be controlled by R5.

(C2) Fig. 4 (b) shows the schematic of a complementary pair transistor amplifier with positive feedback used to generate C2. The incidence of the negative resistance is controlled by the reverse bias on T1 set by R6. The magnitude of the negative resistance is determined by R4. The negative resistance region is rendered non-linear by the gradual reduction of the loop gain by a series of diodes D3, and bias on D3 determines the valley voltage. The negative resistance region is terminated once the collector injection of T1 sets in.

(C3) Characteristic (C3) is a diode characteristic with a large delay, and as such is produced by a number of diodes in series. The desired gap voltage can be adjusted by a combination of Silicon and Germanium diodes. These diodes also introduce non-linearity in (C3) by reducing the loop again.

(C4) Characteristic (C4) is generated by a diode with a resistance in series for finer adjustments of the slope
Fig. 5. Complete Circuit of the Analogue.

D₁ & D₂ Ge Diodes 1N39
D₃ Series of Diodes (2 Si + 3 Ge)
D₄ Series of Diodes (7 Si + 2 Ge) - Controls Vₖₚ
D₂₁ & D₂₂ Zener Diodes 1N1355 & M102
T₁ n.p.n. Transistor 2N358
T₂ p.n.p. Transistor 2N601

Rᵥ₁ 10K - Controls Iₚ
Rᵥ₂ & Rᵥ₃ 470
Rᵥ₄ 10K - Zeroing
Rᵥ₅ 1K - Controls Initial Slope
R₆ 12K
Rᵥ₇ 0.5M - Controls Vₚ
Rᵥ₈ 10K - Controls Iₚ
Rᵥ₉ 25K - Controls Rₙ
R₁₀ 1K
R₁₁ 22K
R₁₂ 100
R₁₃ 22K
Rᵥ₁₄ 100 - Controls Reverse Slope Resistance
Rᵥ₁₅ 10K - Overall Control of Neg. Resistance

All Resistances are in ohms.
Fig. 6. Variation of Input Negative Resistance and Capacitance with Frequency.
Negative Resistance, ohms.
finite useful bandwidth for the analogue. For purposes of the present investigation a 10% tolerance was set on $R_{in}$ and the resultant bandwidth was 65 kc. The effect of variation in $C_{in}$ is made negligible by the use of a large external capacity (0.09 µF) many times that of the inherent capacity (0.009 µF). The maximum frequency of sinusoidal oscillation was set at 50 kc. Hence, the rise and fall times in the case of switching should be greater than 10 µs. In the case of non-linear oscillation, the significant harmonic should be less than 50 kc. The input impedance in the pass band can be taken to be composed of a resistance $R_o$ and a capacity $C_o$ given by

$$R_o = -\frac{R_1}{\beta_{20}}, \quad \text{and}$$

$$C_o = \frac{\beta_{20}}{\omega N_1} \left[ 1 + C \omega (\beta_{20} R_2 + R_1) \right]. \quad ....(4)$$

(4) Characteristics

The characteristic of the analogue drawn to a reduced scale is superimposed on the characteristic of a tunnel diode in Fig. 7 and shows a good fit. The various families of characteristics obtained with the analogue are shown in Fig. 8. If one attempts to display the characteristics of the analogue on an oscilloscope by the use of a very high frequency sinusoidal sweep, a split will be observed in the characteristic. This split is due to the change in the sense of the capacitive component of the input current during the return trace. In other words $C_{in} \frac{dv}{dt}$ changes sign during the return trace. In fact, $C_{in}$ can be determined as the ratio of the
Fig. 7. Voltage-Current Characteristic of the Analogue Superimposed on that for a Ge Tunnel Diode.
Fig. 8. Voltage - Current Characteristic of the Analogue.

(a) Superposition of Component Characteristics to form the Characteristic of the Analogue. $X = 1 \text{ V/cm, } Y = 5 \text{ mA/cm.}$

(b) Typical Characteristic.

1. $X = 0.5 \text{ V/cm, } Y = 2 \text{ mA/cm.}$

2. Same as (a)

(c) Family of Characteristics

1. Varying $I_p = 6, 8, 10, \text{ mA.}$

2. Varying $V_g = 2, 5, 3.25, 4 \text{ Volts.}$

3. Varying $I_v = 1, 2, 3 \text{ mA.}$

Scale same as (a).
Fig. 8.
displacement of the characteristic along the current axis from the D.C. characteristic to the slope of the input voltage, when the analogue is swept by a sawtooth.
III APPLICATIONS OF THE ANALOGUE

(1) Stability Criteria

(a) Theory

The stability criteria for tunnel diodes discuss in essence the stability of a linear negative resistance under known external impedance configuration. The small signal equivalent circuit employed for this analysis is shown in Fig. 9 and an expression for the input current \( i_1(t) \), when the input is a positive \( \delta \) function can be easily derived as

\[
i_1(t) = \frac{1}{L(\alpha - \gamma)} \left[ (\alpha - \beta) e^{\alpha t} - (\gamma - \beta) e^{\gamma t} \right]. \quad \ldots \ldots (5)
\]

where \( \alpha \) and \( \gamma \) are given by

\[
\alpha = -\frac{1}{2} \left( \frac{R_p}{L} - \frac{R_t}{L} \right) \sqrt{\frac{1}{4} \left( \frac{R_p}{L} - \frac{R_t}{L} \right)^2 - \left( 1 - \frac{R_t}{L} \right)},
\]

\[
\gamma = -A \pm \sqrt{A^2 - B}
\]

and

\[
A = \frac{1}{2} \left( \frac{R_p}{L} - \frac{R_t}{L} \right), \quad B = \left( 1 - \frac{R_t}{L} \right).
\]

The expression for the current \( i_2(t) \) through the negative resistance is given by

\[
i_2(t) = \frac{\frac{\beta}{L(\alpha - \gamma)}}{\text{LO}(\alpha - \gamma)} (e^{\alpha t} - e^{\gamma t}). \quad \ldots \ldots (6)
\]

Stability is usually discussed by using (5), but it is very much easier if one uses (5) instead, since it removes the ambiguity about the final state of \( i_2(t) \) in certain cases. From (6) it is clear that the current response \( i_2(t) \) will be oscillatory or otherwise according as \( \sqrt{A^2 - B} \) is imaginary or real. In both the cases the response grows or decays
Fig. 9. Equivalent Circuit Employed to Discuss the Stability Criteria.
depending on the real part of \( \alpha \) or \( \beta \) being positive or both. Thus \( i_2(t) \) can assume any one of the various possible modes, namely, exponential growth or decay, sinusoidal growth or decay, depending on the impedance conditions.

The stability criteria has been discussed and results expressed in an algebraic form by many authors\(^{(3,4,5)}\). However, \( \text{Hines}^{(5)} \) has expressed the criteria in a graphical form, and a slightly different version of the same will be used for our discussion. In Fig. 10 (a) both the axes are normalized and a given set of parameter condition will determine a point in one of the regions from which one can find the nature of the response. Each region will now be discussed in detail.

**Region I** Since both \( \alpha \) and \( \beta \) are real and negative, the response decays exponentially. The current \( i_2(t) \) increases by a step and decays sinusoidally, whereas \( i_1(t) \) increases initially starting from zero and finally decays.

**Region II** In this case both \( \alpha \) and \( \beta \) are complex and have negative real parts making the response oscillatory and decaying. Current \( i_1(t) \) starts as a sinusoid of maximum amplitude and continues to decay exponentially.

**Region III** Both \( \alpha \) and \( \beta \) are complex and have positive real parts making the response grow in an oscillatory fashion. The frequency and amplitude during growth can be determined accurately from the linear analysis. However, those under steady state conditions are determined by the non-linearity. The steady state response in this region will be considered in...
Fig. 10. (a) Stability Criteria Expressed in a Graphical form. (1) - Plot of \( 2 \frac{Z_0}{R_N} - \left( \frac{Z_0}{R_N} \right)^2 \) vs \( \frac{Z_0}{R_N} \).

(2) - Plot of \( \frac{Z_0}{R_N} \) vs \( \frac{Z_0}{R_N} \), where \( Z_0 = \sqrt{\frac{L}{C}} \).

ABDE corresponds to \( R_N = -100 \) ohms, \( R_T = 63.4 \) ohms, \( L = 470 \mu H \).

(b) Normalized Plot of \( \left( \frac{R_N}{Z_0} \right)^2 \) vs \( \frac{Z_0}{R_N} \) for determining \( C \) for a given \( \frac{Z_0}{R_N} \).
taking a tunnel diode beyond the point of intersection of the load line with the characteristic to effect switching.

The regions I and II are stable, whereas III, IV, and V are unstable. The above analysis holds good only for linear negative resistance, and as such the boundary between the several regions are unique and present no hysteresis. The influence of non-linearity on the above analysis will be discussed in due course.

(b) Experimental Verification

For verifying and demonstrating the various regions, it is necessary to set up a typical parameter condition and compare the actual response with the one predicted by the theory. It is also necessary to make the response repetitive to facilitate observation on an oscilloscope. Since the region I and II are stable, the parameter conditions can be set permanently and the response excited by an external disturbance periodically. Whereas in regions III and IV, even the parametric conditions have to be set up intermittently. Hence different experimental procedures are necessary for the two cases. Region V needs no special verification.

It would be convenient if it is possible to set up conditions for the various regions by a change of single parameter. Obviously L and C are the two convenient choices, in which case the locus of the point will be a straight line parallel to the x-axis such as ABDK. However, varying of C has to be preferred to varying L, in as much as the parameter $R_T$ can be easily left unaltered. This method has the additional
advantage of the bias arrangement requiring no adjustment during the transition. Fig. 10(b) gives a normalized plot of \( 2c/R_N \) against \( R_n^2/2c^2 \) from which the value of \( C \) for any given point for the values of \( R_N \), \( R_n \) and \( L \) shown can be easily read off.

**Regions I and II** The negative resistance region of the analogue was made linear to conform with the requirements of the theory and a suitable bias point was set up. It is preferable to bias through a large resistance than through an inductance. Capacity \( C \) was adjusted so that the analogue stayed at any desired point between \( A \) and \( B \). The response was excited from a narrow high amplitude voltage pulse and the resulting current wave forms \( i_1(t) \) and \( i_2(t) \) were displayed on an oscilloscope. The various wave forms obtained are shown in Fig. 11, and good agreement between the observed frequency and rate of decay with that from calculation was obtained within limits of experimental accuracy.

**Region III and IV** The analogue was biased as before and the capacity \( C \) adjusted to make the analogue stay in region I. Capacity \( C \) can be reduced periodically by a known value by the use of a mechanical relay or a transistor switch, and the second method reduces the jitter in the wave form. If the capacity is reduced in such a manner that no charge is transferred to or from the circuit, the resulting response will be incoherent, as it can be initiated only by noise. However, if the switching removes or injects charge greater than that due to random fluctuations, which is always the case in practice due to stray capacities, the response will be coherent enabling
 steadier waveform to be obtained. The nature of the response corresponds to the residual value of capacity. For example, as the capacity is reduced from 0.4 \( \mu F \) to 0.06 \( \mu F \), the response grows sinusoidally. The calculated frequency and rate of growth can be verified experimentally and wave forms are shown in Fig. 12 (a). The steady state waveform can be sinusoidal or otherwise. Region IV cannot be easily demonstrated by reducing capacity. However, switching \( R_T \) or \( R_H \) can be used to demonstrate region IV.

It is of interest to examine the growth in region III on an X-Y display, superimposed on the D.C. characteristic, since it enables one to determine the region of the D.C. characteristic traversed during the growth of the response. In Fig. 12 (b) only the growth of the oscillation was displayed by selective blanking of the scope, and it should be noted that the few cycles of growth are very nearly sinusoidal. If desired, the time during the growth can be indicated by additional marker blanking pulses. However, the usual method of displaying the response using a time base has to be preferred for greater accuracy in measurement of time.

The stability criteria discussed above does not take non-linearity into consideration, and as such cannot be used to determine the frequency under steady-state conditions. Further, the linear theory predicts the boundary between the regions to be single valued, which is however not true when non-linearities are present. The above two factors will now be discussed in detail.
Fig. 11. Experimental verification of Stability Criteria.
(a) Region of Exponential Decay
   (1) $i_1(t)$  (2) $i_2(t)$
(b) Region of Sinusoidal Decay
   (1) $i_1(t)$  (2) $i_2(t)$
Scale: $X - 50 \mu S/cm$, $Y - 2 mA/cm$.

Fig. 12. Experimental verification of Stability Criteria.
(a) (1) Region of Sinusoidal Growth $X - 5 mS/cm$.
    (2) Steady State Oscillations $X - 50 \mu S/cm$.
Scale: $Y - 2 mA/cm$.
(b) Growth of Oscillation Superimposed on the
    D.C. Characteristic.
   (1) Sinusoidal Steady State Oscillation.
   (2) Non-Sinusoidal Steady State Oscillation.
Scale: $X - 0.5 V/cm$, $Y - 2 mA/cm$.
All Currents are $i_2(t)$.
Fig. 11.

Fig. 12. (a)

Fig. 12. (b)
(2) **Steady State Oscillations**

The problem of an exact analytical determination of the limit cycle for oscillation in a non-linear device is in general very difficult. A reasonably exact analytical solution can be obtained in general by numerical methods using a computer. Analytical solutions have been obtained in cases of special types of non-linearity and circuit conditions. However, it is desirable to be able to predict the limit cycle with reasonable accuracy by an easier method.

Brunetti(6) has suggested a graphical method for evaluating the amplitude of the limit cycle based on the concept of an average negative resistance. The average negative resistance is the equivalent resistance, that would deliver the same power as the non-linear resistance would for a given amplitude of oscillation about the bias point. It can be expressed mathematically as the ratio of the square of the r.m.s. amplitude of an applied pure sinusoidal voltage to the power delivered. The average negative resistance $R_{av}$ can be determined analytically, if the non-linearity can be expressed as a power series. However, one can always determine $R_{av}$ experimentally as the ratio of an applied pure sinusoidal voltage to the fundamental component of current for a given amplitude of oscillation. By constructing a graph of $R_{av}$ against the amplitude of oscillation and assuming the average negative resistance to be equal to $1/CR_T$, he can determine the amplitude of oscillation. He is also able to derive the condition for starting of oscillations in a non-linear device from the above graph.
Schuller and Gärnter\(^{(7)}\) have derived an expression for the frequency of oscillation for small values of \(gR_T\) and assuming the non-linearity about the bias point to be symmetric and follow a cubic law. The differential equation for a general system reduces to the van-der Pol's equation under their simplifying assumption, for which analytical solutions are available. They derive an expression for the frequency of small oscillations in the form,

\[
f = \frac{1}{2\pi\sqrt{LC}} \cdot \frac{1}{1 + \frac{1}{16LC(R_TC - LC)^2}}.
\]  

\(\ldots(8)\)

The above equation suggests that the frequency of oscillation falls below \(\frac{1}{2\pi\sqrt{LC}}\), as the amplitude of oscillation increases due to decrease of \(L\) or \(C\). Their analysis cannot be easily verified experimentally on the analogue, because of the difficulty in realizing the condition of small \(gR_T\). A rigorous analytical analysis for a more general case with large \(gR_T\) and asymmetry in the non-linearity remains to be done.

At any point on the boundary between regions II and III the oscillation is small in amplitude and very nearly sinusoidal. The frequency \(f_0\) during the growth of the oscillation can be derived from (7) by observing that \(A\) is zero at the boundary as

\[
f_0 = \frac{\sqrt{1 - gR_T}}{2\pi\sqrt{LC_0}},
\]  

\(\ldots(9)\)

where \(C_0\) corresponds to the capacity at the boundary. As the capacity is reduced below \(C_0\), the frequency during the growth can be derived from the linear theory, and is given by
\[ f' = \frac{\sqrt{(1 - gR_T)}}{2\pi \sqrt{LC}} \cdot \frac{1}{1 + \frac{1}{3LC} \left( \frac{R_T C}{L} - L \right)^2} \quad \quad \ldots (10) \]

Frequency \( f' \) has been plotted as a function of \( C \) in Fig. 13.

It can be seen that \( f' \) reduces to a form similar to the frequency derived by Schuller and Gartner in equation 8, when \( gR_T \) is assumed to be very small. The slight difference in the denominator can be attributed to the non-linearity being taken into consideration in (8). Let a frequency \( f'' \) be defined as

\[ f'' = \frac{\sqrt{(1 - gR_T)}}{2\pi \sqrt{LC}} \quad \quad \ldots (11) \]

and \( f'' \) is plotted in Fig. 13.

In general, as the amplitude of oscillation is increased by decreasing the capacity (or increasing \( L \)), two distinct phenomena are observed. Firstly, the harmonic content of the steady state response increases, and secondly, there may be a shift in bias point depending on the asymmetry of the non-linearity about the initial bias point. The influence of these on the frequency of oscillation will now be considered.

Grozkowski(3) has shown that in general, for voltage controlled non-linear resistance oscillating systems the presence of harmonics in the output wave form reduces the fundamental frequency to a value below that which would exist in the case of pure sinusoidal oscillation. He recognizes the fact that a negative resistance oscillatory system is unable to store energy. In other words, the net energy over a closed cycle is zero, which can be expressed mathematically in the form
above \( f' \). However, for larger amplitudes the effect of increasing harmonic contents predominates in bringing the frequency below \( f'' \).

The amplitude of oscillation was increased gradually by reducing the capacity \( C \). The frequency, harmonic content shift in the bias point, and peak amplitudes were measured for various \( C \), and the results are plotted in Figs. (13) and (14). The actual frequency at the start of oscillations agreed with \( f_0 \) within the experimental accuracy of 1%. It can be seen from Fig. (13) that for low amplitudes the measured frequency is above \( f'' \) and falls off below \( f'' \) for large amplitudes. It is always below \( \frac{1}{2\pi \sqrt{LC}} \) and above \( f' \) for appreciable value of \( gR_T \). The frequency \( f' \) departs very much from the measured frequency as can be expected from the linear approximations made in deriving (10). It appears that \( f'' \) given by (11) gives a reasonable engineering approximation for the actual frequency for moderate amplitudes. The insert in Fig. (14) shows the limits of oscillation for a few values capacity, and it is interesting to note that for small values of capacity, the negative peak extends into the reverse bias region of the characteristic.

(3) Boundary Conditions

The influence of non-linearity on the boundary between the various regions in Fig. (10) will now be considered. As both the region I and II are stable, the characteristic in the neighbourhood of the bias point can be replaced by a linear characteristic, namely the tangent at the bias point. The nature of the response is determined by this linear approximation.
Fig. 14. Variation of Percentage Frequency Deviation, Percentage Harmonic Content, D.C. Bias Point, and Voltage Excursion with Capacity. Insert shows the Region of the D.C. Characteristic Traversed for each Capacity.
and as such the boundary between I and II is that determined by the linear theory, and hence presents no hysteresis. The boundary between region V and any other region is exact, in as much as the device cannot be biased in the negative resistance region, if $R_T$ exceeds $R_N$. The boundary between regions II and III and III and IV need more elaborate consideration.

It is well known that the boundary conditions necessary to start and stop oscillations in a non-linear system are different. In other words, the boundary presents a phenomena of hysteresis. Appelton and van der Pol\(^9,10\) have discussed this phenomena for a simple triode oscillator. By assuming that the characteristic around the bias point can be expanded as a Maclurin's series. They predict the existence of two stable amplitudes one of which is zero, if the fifth differential coefficient is positive and the third differential coefficient is negative. Further they prove the existence of two non zero amplitudes in a small region before the start of large amplitude oscillations, and the reversibility of the smaller amplitude, by taking into consideration two more terms in the Maclurin's expansion, and hence the value of the seventh differential coefficient. Brunetti has qualitatively explained this phenomena of hysteresis by using his plot of average negative resistance against amplitude. Schuller and Gartner\(^7\) have shown by numerical analysis with the aid of a computer, the existence of hysteresis in tunnel diode oscillators. They draw attention in particular, to the existence of two stable non-zero amplitudes for the same set of circuit conditions,
when the tunnel diode is not biased at the point of maximum negative conductance.

The above phenomena of hysteresis necessarily affects the nature of the boundary between regions II and III, and III and IV. The boundary between the regions II and III consists of two branches, one for each direction of transition. The boundary for transition from II to III is same as that predicted by the linear theory, since the region II is stable and is independent of the non-linearity, whereas that for III to II is highly dependent on the non-linearity. Fig. 15 shows the phenomena of hysteresis as observed experimentally on the analogue. The region of hysteresis was accentuated by the choice of the bias point (1.7 V) in the region of high non-linearity. The analogue follows the path ABDE as the capacity is decreased and it is set into oscillation. The insert shows the small reversible region of small stable amplitude. Once the analogue is oscillating at a point beyond D, the reduction of the capacity returns it to the region of no oscillation along the path EDFA. The point of abrupt decay of oscillation F is subject to fluctuations due to noise.

The boundary between the regions III and IV as observed experimentally will have no definite relation with the one predicted by theory, because of both III and IV being unstable and the characteristic being non-linear. The actual boundary can be expected to present hysteresis. Experimental determination of this boundary could not be done due to the difficulty in determining the exact point of transition.
Fig. 15. Hysteresis in the boundary condition for starting and stopping of oscillation for the analogue.

\[ R_N = 600 \Omega. \]
\[ R_T = 53.4 \Omega. \]
\[ L = 1.47 \text{ mH}. \]
IV CONCLUSIONS

The analogue described has distinct advantages over a tunnel diode for studies regarding the various modes of operation of a tunnel diode, in that, it has flexibility in characteristics and a small bandwidth. The results obtained on the analogue can be used to explain qualitatively and sometimes quantitatively the working of a tunnel diode in a given circuit, to a first approximation. However, any phenomena that depends on the exact nature of the non-linearity may not be predicted accurately by the analogue. This cannot be considered to be a serious disadvantage in as much as every non-linear problem needs individual consideration. The role of the analogue in throwing light on the operation of a tunnel diode can best be explained from the fact that the frequency deviation and hysteresis was so prominent in the case of the analogue, that it could not have been missed by accident.

Facility to examine the current and voltage in the various elements of the equivalent circuit and the advantage of reducing the speed of switching by increasing the capacity, have proved advantageous during preliminary studies on switching. The negative valley current could be used to study switching under biasing conditions from a perfect current source, since the device will have two stable states even in the case of zero terminal current.

The analogue can be used as an educational tool in acquainting the student with the properties of non-linear resistances, negative resistances, and tunnel diodes without
the aid of sophisticated circuit layout and instruments.

The hysteresis phenomena observed suggests a possibility of building an A.C. logic using a tunnel diode with the zero amplitude and finite amplitude states as the two stable states. The optimization of the type of non-linearity and biasing point for the above purpose can be greatly facilitated by studies on the analogue.
REFERENCES


VI APPENDIX

The theory of operation of a negative resistance generator of a form used in the analogue has been discussed by a number of authors \((9, 10)\). However, the effects of collector to base capacity on the input impedance appears to be ignored. An expression for the input impedance for a more general case under sinusoidal excitation will now be derived.

On the schematic shown in Fig. 4(b) resistance \(R_5, R_7, R_8,\) and \(R_9\) can be neglected and the equivalent circuit for derivation of the input impedance reduces to the one shown in Fig. 16. From Kirchoff's laws

\[
i_{b1} = i_c + i_{c2} + i_1 \quad \cdots (1) \quad V_1 = i_{cl}R_1 \quad \cdots (4)
i_{b2} = i_c + i_{cl} \quad \cdots (2) \quad V_2 = -i_{e2}R_2 \quad \cdots (5)
i_c = j\omega C(V_2-V_1) \quad \cdots (3)
\]

In addition

\[
\alpha_1i_{e1} = i_{cl} = \beta_1i_{b1}, \quad \cdots (6)
\]

and

\[
\alpha_2i_{e2} = i_{c2} = \beta_2i_{b2}, \quad \cdots (7)
\]

under usual notation.

From (1), (4), (5), (6), and (7)

\[
i_1 = \frac{V_1\alpha_1}{\beta_1R_1} + \frac{V_2\alpha_2}{R_2} = (V_2-V_1)j\omega C.
\]

\[
= V_1 \left[ \frac{\alpha_1}{\beta_1R_1} + j\omega C \right] + V_2 \left[ \frac{\alpha_2}{R_2} - j\omega C \right]. \quad \cdots (8)
\]

From (2), (4), (5), (6), and (7)

\[
V_2 = \frac{-R_2\beta_2}{\alpha_2} (i_c + i_{cl}) = \frac{-R_2\beta_2}{\alpha_2} \left[ \frac{\alpha_1V_1}{\beta_1R_1} + j\omega C(V_2-V_1) \right]
\]
Fig. 16. Simplified Equivalent Circuit for Obtaining the Input Impedance.
\[ v_2 = -v_1 \cdot \frac{\alpha_1 R_1 - j\omega C}{\beta_2 R_2 + j\omega C} \] \hspace{1cm} \ldots(9)

From (8) and (9)

\[ i_1 = v_1 \left( \frac{\alpha_1 R_1 + j\omega C}{\beta_1 R_1} - \frac{\alpha_2 R_2 - j\omega C}{\beta_2 R_2 + j\omega C} \right) \]

\[ z_{in} = \frac{v_1}{i_1} = \frac{\frac{\alpha_2 R_2}{\beta_2 R_2} + j\omega C}{(\frac{\alpha_2 R_2}{\beta_2 R_2} + j\omega C)(\frac{\alpha_1 R_1}{\beta_1 R_1} + j\omega C) - (\frac{\alpha_2 R_2}{\beta_2 R_2} - j\omega C)(\frac{\alpha_1 R_1}{\beta_1 R_1} - j\omega C)} \]

\[ = \frac{\frac{\alpha_2 R_2}{\beta_2 R_2} + j\omega C}{\frac{\beta_1 \beta_2}{R_1 R_2} \left( \frac{1}{\beta_1 \beta_2} - 1 \right) + j\omega C \frac{\alpha_1 R_1}{\beta_1 R_1} + \frac{\alpha_2 R_2}{\beta_2 R_2} + \alpha_1 R_1 + \alpha_2 R_2} \]

In general following approximations regarding the transistor parameters can be made

\[ \alpha_1, \alpha_2 \ll 1 \ ; \beta_1, \beta_2 \gg 1 \]

Hence,

\[ z_{in} = -\frac{R_1 + j\omega C R_1 R_2}{\beta_2 (1 - j\omega C (R_1 + R_2))} \] \hspace{1cm} \ldots(10)

If \( \beta_2 \) is now assumed to be complex and of the form

\[ \beta_2 = \frac{\beta_2 \omega}{1 + \frac{1}{\omega \beta}} \]
where $\omega_\beta$ is the angular cut-off frequency and $\beta_{20}$ is the low frequency $\beta$ of transistor $T_2$.

Substituting for $\beta_2$ in (10) and noting that $R_2$ is very small compared to $R_1$ in practice, we get

$$Z_{in} = -\frac{\frac{R_1}{\beta_{20}} \left( 1 + \frac{i\omega}{\omega_\beta} \right) + i\omega CR_1 R_2}{1 - j\omega CR_1}$$

$$= -\frac{\frac{R_1}{\beta_{20}} \left( 1 + CR_2 \omega_\beta \beta_{20} \right)}{1 - j\omega CR_1}$$

$$= -\frac{-\frac{R_1}{\beta_{20}} \left( \omega_\beta^2 + \omega_\beta^2 \left( 1 + CR_2 \omega_\beta \beta_{20} \right)^2 \right)}{\omega_\beta - \omega_\beta^2 CR_1 \left( 1 + CR_2 \omega_\beta \beta_{20} \right) - j\omega \left( 1 + C \omega_\beta \left( R_1 + R_2 \beta_{20} \right) \right)}$$

Dividing the numerator and denominator by

$$\omega_\beta - \omega_\beta^2 CR_1 \left( 1 + CR_2 \omega_\beta \beta_{20} \right)$$

and rearranging, we obtain $Z_{in}$ in the form

$$Z_{in} = \frac{R_{in}}{1 + j\omega C_{in} R_{in}}$$

where

$$R_{in} = -\frac{R_1}{\beta_{20} \omega_\beta} \frac{\omega_\beta^2 + \omega_\beta^2 \left( 1 + CR_2 \omega_\beta \beta_{20} \right)^2}{\omega_\beta - \omega_\beta^2 CR_1 \left( 1 + CR_2 \omega_\beta \beta_{20} \right)}$$

...(11)

and

$$C_{in} = \frac{\omega_\beta \beta_{20}}{R_1} \frac{1 + C \omega_\beta \left( R_1 + \beta_{20} R_2 \right)}{\omega_\beta^2 + \omega_\beta^2 \left( 1 + CR_2 \omega_\beta \beta_{20} \right)^2}$$

...(12)

It is evident that $Z_{in}$ is a parallel combination of a resistance $R_{in}$ and a capacity $C_{in}$ given by (11) and (12). From (11) and
(12) low frequency values for $R_{in}$ and $C_{in}$ can be derived by
neglecting terms involving $\omega$ as

$$R_o = \frac{-R_1}{\beta_{20}}$$

$$C_o = \frac{\beta_{20}}{\omega R_1} \left(1 + \frac{\omega \beta_{20}}{R_1 + \beta_{20} R_2}\right).$$