LOAD FLOW ANALYSIS IN COMPOSITE
SYSTEM RELIABILITY EVALUATION

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of
Master of Science
in the Department of Electrical Engineering

by

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Saskatoon, Saskatchewan
May 1971

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ACKNOWLEDGEMENTS

The author wishes to express his indebtedness to Dr. R. Billinton for his guidance and encouragement during the course of this work. Appreciation is also extended to Professor K. Bollinger for his helpful advice and assistance during the course of this research work.

The author also wishes to acknowledge the Saskatchewan Power Corporation for providing the data used in some parts of this thesis.

Acknowledgement of the financial support given by National Research Council of Canada under Grant No. A-2711 is made.
University of Saskatchewan

Electrical Engineering Abstract 71A136

"Load Flow Analysis in Composite System Reliability Evaluation"

Student: Sudhendu B. Dhar  Supervisor: Dr. R. Billinton

M.Sc. Thesis Presented to the College of Graduate Studies
May 1971

Abstract

Composite system reliability evaluation is important in the
determination of critical outage situations. This involves
numerous load-flow studies for different system component outage
conditions.

The object of this research project is to examine the existing
methods suitable for repeated load-flow analysis in composite system
reliability evaluation. A new method and a few modifications have
been developed as an extension of the basic Newton's method for
load-flow solutions. The main emphasis in the new methods is on the
computational time and rate of convergence in comparison with other
existing methods for load-flow analysis. The numerical examples
illustrate that successful load-flow solutions and considerable
computing time can be saved by the proposed methods.
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1. INTRODUCTION

1.1. General

In the past decade, high speed digital computers have been successfully employed in carrying out load-flow studies for the evaluation of power system operating performance. A load-flow study consists of an evaluation of the power flows, voltages and currents at various points in an electrical network for specified terminal conditions. A load-flow programme provides a mathematical model of the steady-state system conditions. The solution of this model for a given set of input parameters lead to a better appreciation of the system operating conditions. This insight is invaluable when considering system modifications, design parameters, operating economics and quality of service indices.

In the planning of a power system, provisions are normally made which enable the system to operate satisfactorily with the outage of a critical line or transformer\(^1,2\). To test this capability, it is necessary to determine which line or system component will produce critical outage conditions. A series of load-flow cases must be scheduled for the assumed outage of various network components i.e. lines, transformers, generators, etc. With the numerical techniques available to date, this problem is a difficult and time consuming one even with a high speed computer.

1.2 Statement of the Problem

A considerable amount of work has been done in the electrical engineering department of this University in the field of power system reliability evaluation. A method for evaluating the reliability at any
point in a composite system including both generation and transmission facilities using a conditional probability approach was proposed by Billinton. A digital computer programme has been developed using this technique for evaluating the reliability levels in a practical system. This programme creates the possible system component outage conditions and performs load-flow analysis at selected load-levels to determine the reliability indices. A reliability study of the Saskatchewan Power Corporation 138-238 KV network with 21 transmission lines and 4 transformers for three load levels and single outage conditions requires approximately 7 minutes. The computer time required for double outage conditions of the same system is about 15 times higher.

The object of this research work is to study various existing numerical methods for load-flow analysis and to develop some numerical techniques for repeated power-flow studies which will require less computer time to evaluate the system reliability.

1.3. Present Numerical Methods for Load-flow Studies

Various numerical techniques have been suggested for efficient solution of the a.c. power-flow problem. In a recent survey, these methods are classified as being either direct or iterative. All methods are actually iterative in one sense because the basic problem involves the solution of a system of nonlinear equations. The so-called direct methods, however, employ the direct solution of a related linear system in the iterative algorithm, whereas the iterative methods use a scheme of successive displacement. The programmes developed so far for single load-flow studies are extremely efficient as everything
possible has been done to limit the storage requirements and computation time. In repeated load-flow calculations by a direct method, further reduction in computation time is possible by applying the information already obtained to the succeeding cases. There is relatively little published material in this area of application.

1.4 Main Contributions

In recent years, much interest has been shown in the Newton method\textsuperscript{7,10,11,12} for load-flow solution. Some of the practical problems involved with the actual computer implementation of the method have also been solved\textsuperscript{13,14,15}. The available literature indicates that very little work has been done to modify the Newton method for repeated load-flow calculations.

This thesis presents a new numerical technique and some modifications of the present methods for load-flow analysis. The proposed modifications are simple and can be readily implemented into a standard Newton approach. The new concepts developed for load-flow analysis were first applied to a hypothetical system and then to a practical system to verify the efficiency of the new methods. All the major existing numerical methods for load-flow analysis have also been studied and investigated thoroughly.

The last part of the thesis concentrates on system reliability applications of the suggested numerical methods for repeated load-flow calculations. Some theoretical investigations for further reduction in the computer time and for better convergence of the load-flow solution using a function minimization technique are also proposed for
future work in this field.

1.5 Facilities for Studies

All the digital computer programmes used for the power system load-flow studies were developed in Fortran IV, on the IBM 360 digital computer at the University of Saskatchewan Computer Centre. The details of various existing numerical techniques, the new developed methods and the attendant results are presented in the following chapters.
2. EXISTING LOAD FLOW SOLUTION TECHNIQUES

2.1. General

A basic problem in a practical power system is the determination of current and power flow and the distribution of voltage levels throughout the system for any given set of generation, load and transmission conditions. The current, power and voltage can be determined by the application of a load-flow technique which mathematically models the actual operation of the system. Considerable work has been done in this area and many papers and computer programs have been written to solve this problem.

A load-flow study normally involves the calculation of power flows and voltages in the network for specified bus conditions. A single phase representation is adequate since power systems are usually balanced. Four quantities are associated with each node, the real and reactive power, the voltage magnitude and the voltage phase angle. Three types of buses are usually represented in the load-flow calculation and at a bus, two of the four quantities are specified. It is also necessary to select one bus, designated the swing bus, to provide the additional real and reactive power to supply the transmission losses, which are unknown until the final solution is obtained. The remaining buses of the system are designated either as voltage controlled buses or load buses. The real power and voltage magnitude are specified at a voltage controlled bus. The real and reactive powers are specified at a load bus.

The infeed data for load-flow can be classified in two groups
Figure 2.1

General Power System Network
as shown in Figure 2.1.

(i) Data for generator nodes.

(ii) Data for load nodes.

2.2 Power System Equations

An electric network such as a power system can be represented by a system of node equations as follows:

\[ I_k = \sum_{m=1}^{N} Y_{km} V_m \]  \hspace{1cm} (2.1)

where

- \( Y_{km} \) is an element of the system admittance matrix.
- \( V_m \) represents the complex voltage at the node \( m \).
- \( I_k \) is the complex current flowing into the node

Complex power \( S_k \) at node \( k \) is given by equation (2.2).

\[ S_k = V_k \sum_{m=1}^{N} Y_{km}^* V_m^* \]

\[ S_k = P_k + jQ_k = |V_k| \sum_{m=1}^{N} |V_m^*| |Y_{km}^*| e^{j(\delta_k - \delta_m - \theta_{km})} \]  \hspace{1cm} (2.2)

where \( P_k \) and \( Q_k \) are the real and reactive powers respectively entering node \( k \) and the asterisk indicates the complex conjugate.

The voltage phase angles are designated by \( \delta \) and the admittance angle by \( \theta \). As noted earlier, in the normal load-flow study, three types of nodes are considered and at each node there are four variables \( P_k, Q_k, V_k \) and \( \delta_k \). The swing node or bus has both the magnitude and angle of its voltage specified i.e. \( V_k \) and \( \delta_k \). In a power flow study it is therefore necessary to solve a system of \( N-1 \) simultaneous equations similar in form to equation 2.2 under
certain terminal conditions.

Node Equations:

The total assumed complex power at any bus \( k \) is

\[
P_k - jQ_k = V_k^* I_k \quad (2.3)
\]

and the current is

\[
I_k = (P_k - jQ_k)/V_k^* \quad (2.4)
\]

In a transmission network, the node current could either be coming from a generator or flowing to a load. If there were no generation or load at a node point, the current \( I_k \) for that node would be zero.

In the formulation of the network equation, if the line charging is included in the system admittance matrix, then equation 2.4 represents the total current at the bus. If the shunt elements are not included in the parameter matrix, the total current at a bus \( k \) is

\[
I_k = (P_k - jQ_k)/V_k^* - b_k V_k \quad (2.5)
\]

where \( b_k V_k \) is the total shunt current flowing from bus \( k \) to ground.

Line Flow Equations:

If line flows are desired in addition to the voltage solution, then the current at bus \( k \) in the line connecting bus \( k \) to bus \( m \) is given by:

\[
i_{km} = (V_k - V_m) Y_{km} + V_k \frac{b_{km}}{2} \quad (2.6)
\]

where

\( Y_{km} \) is the transfer admittance between \( k \) and \( m \)

\( b_{km} \) is the total line charging admittance
\[ V_k \frac{b_{km}}{2} \] is the current contribution at bus \( k \) due to the line charging. Line flow equation is given by

\[ P_{km} - jQ_{km} = V_k^* i_{km} \] \hspace{1cm} (2.7)

or

\[ P_{km} - jQ_{km} = V_k^* (V_k - V_m)Y_{km} + V_k^* V_k \frac{b_{km}}{2} \] \hspace{1cm} (2.8)

\[ P_{mk} - jQ_{mk} = V_m^* (V_m - V_k)Y_{mk} + V_m^* V_m \frac{b_{km}}{2} \]

where \( P_{km} \) and \( Q_{km} \) are the real and reactive power flow from \( k \) to \( m \) respectively. Equations (2.8) are also applicable to the case in which \( k \) and \( m \) are connected by a transformer having a nominal turns ratio. If the turns ratio is not nominal and the tap appears at the node \( k \) end of the branch then equation (2.8) is modified as follows:

\[ P_{km} - jQ_{km} = V_k^* (nV_k - V_m)Y_{km} + V_k^* V_k \frac{b_{km}}{2} \] \hspace{1cm} (2.9)

If the tap appears at the node \( m \) end of the branch then:

\[ P_{km} - jQ_{km} = V_k^* (1/nV_k - V_m)Y_{km} + V_k^* V_k \frac{b_{km}}{2} \] \hspace{1cm} (2.10)

The power loss in any branch is the algebraic sum of \( (P_{km} - jQ_{km}) \) and \( (P_{mk} - jQ_{mk}) \).

2.3 Solution Techniques:

In the load flow problem, the network equations are usually expressed in one of the following forms:

\[ YV = I \]
\[ or \hspace{1cm} ZI = V \]
\[ YZ = U, \]

where \( U \) is the unit matrix.

The solution techniques for solving systems of equations of

In equations 2.9 and 2.10 \( Y_{km} \) includes the factor \( n \).
the form shown in equation (2.11) can be generally divided into two catagories, Direct methods and Iterative methods.

Direct Methods:

In the direct methods, \( Y \) is inverted explicitly or equivalently and the equations are solved directly from

\[
V = Y^{-1}I
\]

The above equation can also be written as

\[
V = Y^{-1}[S^*]_{V/V}
\]

or

\[
V = Z[S^*]_{V/V}
\]

The various direct methods differ only in the way in which the right hand side of (2.12) is formed. The problem requires that after each estimate of \( V \) has been obtained, the right hand side \([S^*]_{V/V}\) is recomputed with the latest values of \( V \) and the process is repeated until convergence is reached. All the direct methods contain some iterative aspects but for simplicity the overall method is still referred to as a direct method. Most books on numerical analysis contain some of these methods, e.g., Gaussian elimination, triangular decomposition, partitioning and various inverse assembly techniques.

Iterative Methods:

Iterative methods produce answers in an unspecified number of steps and the desired value \( V \) is obtained by a process of successive approximation. The steps may be considered as follows:

(i) The equations are rearranged such that any estimate of \( V \) gives a new estimate of \( V \).
(ii) The process of obtaining a new and better estimate of V is accelerated if possible.

(iii) Suitable tests of convergence are applied to assess the adequacy of the solutions obtained.

2.4. Existing Methods Studied

Gauss-Seidel Iterative Method

The Gauss-Seidel process is a practical iterative scheme for finding the solution of a set of linear equations in certain circumstances.

Let \( A = (a_{ij}) \), \( b = (b_1, b_2 \ldots b_N) \) and define a sequence of vectors,

\[
x_i = [x_1^r, x_2^r \ldots x_N^r]
\]

\( r \geq 0 \)

\( i = 1, 2 \ldots N \)

\( x^0 \) is arbitrary and \( x^{r+1} \) is obtained from \( x^r \) by finding the solution of the triangular system:

\[
a_{11}x_1^{r+1} + a_{12}x_2^r + \ldots + a_{1N}x_N^r = b_1 \\
a_{21}x_1^{r+1} + a_{22}x_2^{r+1} + \ldots + a_{2N}x_N^r = b_2 \\
a_{N1}x_1^{r+1} + a_{N2}x_2^{r+1} + \ldots + a_{NN}x_N^{r+1} = b_N
\]

The solution of the load-flow problem is initiated by specifying the voltages at certain buses and assuming voltage values at the remaining buses.

Consider the 3-node system of Figure 2.2
Figure 2.2

Single Line Diagram of a 3-node System

The Gauss-Seidel form of equations is as follows:

\[ v_1^{p+1} = -\frac{Y_{12}}{Y_{11}} v_2^p - \frac{Y_{13}}{Y_{11}} v_3^p + \frac{S_1}{Y_{11}} \left[ \frac{1}{v_1^p} \right] \]  
\[ v_2^{p+1} = \frac{Y_{21}}{Y_{22}} v_1^{p+1} + \frac{S_2}{Y_{22}} \left[ \frac{1}{v_2^{p+1}} \right] - \frac{Y_{23}}{Y_{22}} v_3^p \]  
\[ v_3^{p+1} = -\frac{Y_{31}}{Y_{33}} v_1^{p+1} - \frac{Y_{32}}{Y_{33}} v_2^{p+1} + \frac{S_3}{Y_{33}} \left[ \frac{1}{v_3^{p+1}} \right] \]

(2.13)  
(2.14)  
(2.15)

No equation is required for the swing bus as the voltage and phase angle are specified. If \( v_1 \) is the swing bus voltage, then only equations in \( v_2 \) and \( v_3 \) would be treated in each iteration cycle.

At a generating node \( k \), where \( |V_k| \) and \( P_k \) are specified, the general equation can be written as

\[ v_k^{p+1} = (v_k^{p+1})' + \left[ \frac{P_k - Q_k}{Y_{kk}} \right] \frac{1}{v_k^{p+1}} \]

(2.16)

where \( Q_k \) is unknown and \( (v_k^{p+1})' \) is the voltage component of \( v_k^{p+1} \) calculated from the known parameter. The exact value of
Q_k may be found by solving the equation

\[ Q_k = \sum_{m=1}^{N} \sum_{m \neq k}^{N} y_{km} V_m V_m^* \]  

(2.17)

where \( J_m \) represents the imaginary part of complex variable.

The real and imaginary magnitudes obtained from equation (2.16) are adjusted such that the polar magnitude is equal to the specified value. An acceleration factor \( \alpha \) can be introduced during the iteration process in the following manner.

\[ V_{P+1}^c = \alpha(V_{P+1}^c - V_P) + V_P \]  

(2.18)

where \( V_{P+1}^c \) is the latest calculated value of voltage. A considerable number of studies have been done using this method on both hypothetical and practical systems and detailed results are given later in this thesis.

Newton-Raphson Method

The load-flow problem can be solved by the Newton-Raphson method using a set of non-linear equations to express the specified real and reactive powers in terms of bus voltages. The technique is an iterative process for solving a set of general non-linear equations of the following type:

\[ f_1 (x_1, x_2, \ldots, x_N) = b_1 \]
\[ f_2 (x_1, x_2, \ldots, x_N) = b_2 \]
\[ f_N (x_1, x_2, \ldots, x_N) = b_N \]

If \( x_N^0 \) is an approximation to the solution of the equation, with error \( \Delta x_N \) at any stage \( p \) of the iteration cycle,

\[ f_1 (x_1^0 + \Delta x_1, x_2^0 + \Delta x_2, \ldots, x_N^0 + \Delta x_N) = b_1 \]
\[ f_2 \left( x_1^0 + \Delta x_1, x_2^0 + \Delta x_2, \ldots x_N^0 + \Delta x_N \right) = b_2 \]
\[ f_N \left( x_1^0 + \Delta x_1, x_2^0 + \Delta x_2, \ldots x_N^0 + \Delta x_N \right) = b_N \]

By Taylor's theorem

\[ f_1 \left( x_1^0 + \Delta x_1, x_2^0 + \Delta x_2, \ldots x_N^0 + \Delta x_N \right) = f_1(0) + \Delta x_1 \frac{\partial f_1}{\partial x_1} \bigg|_0 + \Delta x_2 \frac{\partial f_1}{\partial x_2} \bigg|_0 + \cdots + \Delta x_N \frac{\partial f_1}{\partial x_N} \bigg|_0 + \phi_1 = b_1 \]

Where \( \phi_1 \) is a function of higher power of \( \Delta x_1, \Delta x_2 \ldots \Delta x_N \) and second, third, etc. derivative of the function of \( f \).

If the initial estimate for \( x_N \) is near the solution value, the \( \Delta x_N \) will be relatively small and all terms of higher power can be neglected. Therefore, the resulting linear set of equations are

\[ f_1(0) + \Delta x_1 \frac{\partial f_1}{\partial x_1} \bigg|_0 + \Delta x_2 \frac{\partial f_1}{\partial x_2} \bigg|_0 + \cdots + \Delta x_N \frac{\partial f_1}{\partial x_N} \bigg|_0 = b_1 \]
\[ f_2(0) + \Delta x_1 \frac{\partial f_2}{\partial x_1} \bigg|_0 + \Delta x_2 \frac{\partial f_2}{\partial x_2} \bigg|_0 + \cdots + \Delta x_N \frac{\partial f_2}{\partial x_N} \bigg|_0 = b_2 \]
\[ f_N(0) + \Delta x_1 \frac{\partial f_N}{\partial x_1} \bigg|_0 + \Delta x_2 \frac{\partial f_N}{\partial x_2} \bigg|_0 + \cdots + \Delta x_N \frac{\partial f_N}{\partial x_N} \bigg|_0 = b_N \]

The equations in matrix form are as follows:

\[
\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} \bigg|_0 & \frac{\partial f_1}{\partial x_2} \bigg|_0 & \cdots & \frac{\partial f_1}{\partial x_N} \bigg|_0 \\
\frac{\partial f_2}{\partial x_1} \bigg|_0 & \frac{\partial f_2}{\partial x_2} \bigg|_0 & \cdots & \frac{\partial f_2}{\partial x_N} \bigg|_0 \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_N}{\partial x_1} \bigg|_0 & \frac{\partial f_N}{\partial x_2} \bigg|_0 & \cdots & \frac{\partial f_N}{\partial x_N} \bigg|_0 
\end{bmatrix}
\begin{bmatrix}
\Delta x_1 \\
\Delta x_2 \\
\vdots \\
\Delta x_N 
\end{bmatrix}
= 
\begin{bmatrix}
b_1 - f_1(0) \\
b_2 - f_2(0) \\
\vdots \\
b_N - f_N(0)
\end{bmatrix}
\]
This is of the form \( Ax = b \), where \( A \) is the coefficient matrix which is to be evaluated at every iteration, \( x \) is the unknown vector, and \( b \) is the residual vector.

The new values for \( x_N \) are calculated from

\[
x_N = x_N + \Delta x_N \tag{2.21}
\]

The process is repeated until the two successive values for each \( x_N \) differ only by a specified tolerance. If the coefficient matrix changes slowly then it can be re-evaluated after every \( k \)th iteration.

From relation (2.2) the complex power at bus \( k \) is:

\[
P_k + jQ_k = \sum_{m=1}^{N} Y_{km} V_m^* V_k
\]

or

\[
P_k + jQ_k = |V_k|^* \sum_{m=1}^{N} |Y_{km}| |V_m|^* e^{j(\delta_k - \delta_m - \theta_{km})} \tag{2.22}
\]

Separating real and imaginary parts

\[
P_k = |V_k|^* \sum_{m=1}^{N} |Y_{km}| |V_m|^* \cos(\delta_k - \delta_m - \theta_{km}) \tag{2.23}
\]

\[
Q_k = |V_k|^* \sum_{m=1}^{N} |Y_{km}| |V_m|^* \sin(\delta_k - \delta_m - \theta_{km}) \tag{2.24}
\]

Equations (2.23) and (2.24) result in a set of non-linear simultaneous equations, one for voltage controlled buses and two for load buses of the system. Thus there are \( 2(N-1)-k \) equations to be solved for a load-flow solution by this method, where \( k \) is the number of voltage controlled nodes other than the swing bus. In the rectangular co-ordinate form two equations are required for all nodes and \( 2(N-1) \) non-linear simultaneous equations must be solved.

As the complex power equation is defined as a function of
voltage and phase angle, at any node \( k \) where \( P_k \) and \( Q_k \) are specified then the mismatch \( W_k \) is given by

\[
W_k = S_k \text{ specified } - F(V, \delta) = S_k \text{ specified } - V_k \sum_{m=1}^{N} Y_{km} V_m
\]

and this must be zero. In the generating node where only \( P_k \) is specified,

\[
W'_k = P_k \text{ specified } - |V_k| \sum_{m=1}^{N} |Y_{km}| |V_m| \cos(\delta_k - \delta_m - \theta_{km})
\]

must be equal to zero.

Therefore at any point \( p \) in the iteration cycle

\[
\Delta W^p = \Delta S_k^p = \Delta P_k^p + j \Delta Q_k^p = \left( \frac{\partial P_k}{\partial \delta_k} + j \frac{\partial Q_k}{\partial \delta_k} \right) \Delta \delta_k^p + \left( \frac{\partial P_k}{\partial V_k} + j \frac{\partial Q_k}{\partial V_k} \right) \Delta V_k^p
\]

Equating real and imaginary parts:

\[
\begin{align*}
\Delta P_k &= \sum_{m=1}^{N} H_{km} \Delta \delta_m + \sum_{m=1}^{N} N_{km} \Delta V_m/V_m \\
\Delta Q_k &= \sum_{m=1}^{N} J_{km} \Delta \delta_m + \sum_{m=1}^{N} L_{km} \Delta V_m/V_m
\end{align*}
\]

Where \( H_{km}, N_{km}, J_{km} \) and \( L_{km} \) are the coefficients of the Jacobian matrix, calculation of which is given in the Appendix II. These equations can be written in matrix form for the 5-bus system of Figure 2.3 as follows:
\[
\begin{bmatrix}
\Delta P_2 \\
\Delta P_3 \\
\Delta P_4 \\
\Delta P_5 \\
\Delta Q_3 \\
\Delta Q_4 \\
\Delta Q_5
\end{bmatrix} =
\begin{bmatrix}
H_{22} & H_{23} & H_{24} & H_{25} \\
H_{32} & H_{33} & H_{34} & H_{35} \\
H_{42} & H_{43} & H_{44} & H_{45} \\
H_{52} & H_{53} & H_{54} & H_{55} \\
J_{32} & J_{33} & J_{34} & J_{35} \\
J_{42} & J_{43} & J_{44} & J_{45} \\
J_{52} & J_{53} & J_{54} & J_{55}
\end{bmatrix}
\begin{bmatrix}
\Delta N_2 \\
\Delta N_3 \\
\Delta N_4 \\
\Delta N_5 \\
\Delta L_3 \\
\Delta L_4 \\
\Delta L_5
\end{bmatrix} =
\begin{bmatrix}
N_{23} & N_{24} & N_{25} \\
N_{33} & N_{34} & N_{35} \\
N_{43} & N_{44} & N_{45} \\
N_{53} & N_{54} & N_{55} \\
L_{33} & L_{34} & L_{35} \\
L_{43} & L_{44} & L_{45} \\
L_{53} & L_{54} & L_{55}
\end{bmatrix} \tag{2.27}
\]

In the above equations, node 1 has not been included as it was taken as the swing bus. With \(|V_1|\) and \(\delta_1\) fixed, \(\Delta V_1\) and \(\Delta \delta_1\) are therefore zero. At the voltage controlled bus, \(\Delta V_2\) is also zero as \(|V_2|\) is fixed.

The coefficient in equation (2.27) can be expressed as partial derivatives and are given below:

For \(k \neq m\)

\[
H_{km} = \frac{\partial P_k}{\partial \delta_m} = a_m e_k - b_m f_k
\]

\[
J_{km} = \frac{\partial Q_k}{\partial \delta_m} = -(a_m e_k + b_m f_k)
\]

\[
L_{km} = \frac{\partial L_k}{\partial \delta_m} = -(a_m e_k + b_m f_k)
\]

For \(k = m\)

\[
H_{kk} = \frac{\partial P_k}{\partial \delta_k} = -Q_k - R_k V_k^2
\]
Figure 2.3

A Simple Hypothetical Power System
\[ N_{kk} = \frac{\partial P_k}{\partial V_k} = P_k + G_{kk}V_k^2 \]

\[ J_{kk} = \frac{\partial Q_k}{\partial \delta_k} = P_k - G_{kk}V_k^2 \]

\[ L_{kk} = \frac{\partial Q_k}{\partial \delta_k} = Q_k - B_{kk}V_k^2 \]

where

\[ I_k = |I_k| e^{j\alpha_k} = a_k + jb_k \]

\[ V_m = |V_m| e^{j\delta_m} = e_m + j\phi_m \]

\[ V_{km} = |V_{km}| e^{j\theta_{km}} = G_{km} + jB_{km} \]

One feature of this approach is that these partial derivatives as given in equation (2.28) and (2.29) are easily evaluated in terms of quantities that have already been found. The load-flow equations can be formed in terms of the rectangular co-ordinate values. The results and comparisons of this method with other load-flow solution techniques are given in later part of this thesis.

Approximate Newton's method

It has been proved\textsuperscript{12} that for a small change in the magnitude of bus voltage, the real power at a bus does not change very much. Similarly, for a small change in the phase angle, the reactive power also does not change appreciably. Using the polar-co-ordinate form, the load-flow problem can therefore be simplified by assuming the elements of the sub-matrices \( N \) and \( J \) shown in equation (2.27) to be zero.
Considerable computing time for successful load-flow solutions can be saved by this method. The results of this approximate method are given in detail and discussed in the following chapters.

2.5 Review of Other Methods

This section presents a brief survey of several other developments that have recently occurred in the digital computer solution of the load-flow problem. The basic theory and the important problems of solution, acceleration criteria for convergence and the inherent computational difficulties are briefly presented.

Fletcher-Powell method:

The load-flow problem formulated by this method for a typical node \( k \) is:

\[
A_{k1} = g_1(e,f) - |V_k| = 0 \\
A_{k2} = g_2(e,f) - P_k = 0 \\
A_{k3} = g_3(e,f) - Q_k = 0
\]
where \((e_k + jf_k)\) is the complex bus voltage or bus current vector at node \(k\) and \(g(e,f)\) is a function formed with these variables.

At the generating nodes, equation (2.31) and (2.32) are selected. Equations (2.32) and (2.33) are utilized at a load node.

Therefore,

\[
F(e,f) = \sum_{Gen} A_{k1}^2 + \sum_{Gen, Loads} A_{k2}^2 + \sum_{Loads} A_{k3}^2
\]  
\[(2.34)\]

where

\[
\sum_{Gen} A_{k1}^2 = \sum_{G} \left( |e_k^2 + f_k^2|^2 - |V_k|^2 \right)
\]

\[
\sum_{Gen, Loads} A_{k2}^2 = \sum_{G,L} \left\{ \text{Re}[V_k^* (Y_{k1}V_1 + \ldots Y_{kN}V_N)] - P_k \right\}^2
\]

\[
\sum_{Loads} A_{k3}^2 = \sum_{L} \left\{ \text{Im}[V_k^* (Y_{k1}V_1 + \ldots Y_{kN}V_N)] - Q_k \right\}^2
\]

The summation is taken over all appropriate nodes. For the swing node \(e = |V|\) and \(f = 0\).

The load-flow solution can be obtained by finding the minimum of the function \(F(e,f)\) given by equation (2.34) which in this case would be zero, i.e.

\[
\min \{F(e,f)\} = 0
\]

The sum of the squares of the mismatches at each node is zero. The minimization would be stopped, however, when the function \(F(e,f)\) is smaller than a certain prescribed tolerance. This method is not competitive with standard Gauss-Seidel or Newton's method unless ill-conditioned networks are being solved. It is useful, however, if an approximate solution is required or a near solution is known in which
case very few minimization steps are necessary.

Relaxation method

This method was suggested by Jordan and uses an orderly sequence of operations on the residuals of each of the nodes as in the Gauss-Seidel method. The steps may be summarized as follows:

(a) from the voltages $V_k$, obtained from the previous iteration, calculate the current at each bus, except the swing bus from

$$I_k^* = \frac{S_k}{V_k} \quad (2.35)$$

$$I_k^* = \sum_{m=1}^{N} V_m^* V_m \quad (2.36)$$

(b) The swing bus current is

$$I_1^* = \sum_{k=1}^{N} \sum_{m=1}^{N} V_k^* V_m - \frac{S_k}{V_1} \quad (2.37)$$

where $S_k$ is the specified generation at bus k.

(c) The current residual at bus k can be found from:

$$R_k = I_k^* - I_k$$

This results from the assumed voltage solution. The voltage residual is

$$\Delta V_k = R_k / Y_{kk} \quad \text{at bus } k.$$ (2.38)

(d) A voltage correction is obtained for the bus at which the residual $R_k$ is a maximum. If the current at a bus remained constant, the residual $R_k$ would be reduced to zero.

(e) An improved estimate of voltage for bus k is then

$$v_{k}^{p+1} = v_{k}^{p} + \Delta v_{k}^{p}$$
and the new current is

\[ I_k^{p+1} = (P_k - jQ_k)/(V_k^{p+1})^* \]

As a result of the change in the current the new residual at bus \( k \) is then

\[ R_k^{p+1} = I_k^p - I_k^{p+1} + R_k^p \]

with the improved value of bus voltage \( V_k^{p+1} \) the new residuals other than bus \( k \) are calculated from

\[ R_k^{p+1} = R_k^p + Y_{km} \Delta V_k \]

where

- \( m = 1, 2 \ldots N \)
- \( m \neq k \quad k \neq s \)
- \( s \) is the swing bus.

This process is repeated, until the residual is less than a specified tolerance.

In comparison with other iterative methods, the relaxation technique suffers from being more difficult to programme. Experience has shown that the relaxation approach is not as effective as the unaccelerated Gauss-Seidel method.

Voltage Vector method:

This is a new approach to the solution of the load-flow problem and has been suggested as being faster than any present method. The mathematical model of the method is very simple. It is defined by two systems of simultaneous linear equations which can be solved using the optimally ordered Gaussian elimination technique.
The following two systems of linear equations are obtained by designating the reference node as 0 and $\theta_0 = 0$ and $V_j = V_0$

$$\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & \ldots & S_{1n} \\
S_{21} & S_{22} & \ldots & S_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
S_{n1} & S_{n2} & \ldots & S_{nn}
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\vdots \\
\theta_n
\end{bmatrix} =
\begin{bmatrix}
J_1 \\
J_2 \\
\vdots \\
J_n
\end{bmatrix} =
\begin{bmatrix}
T_{11} & T_{12} & \ldots & T_{1n} \\
T_{21} & T_{22} & \ldots & T_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
T_{n1} & T_{n2} & \ldots & T_{nn}
\end{bmatrix}
\begin{bmatrix}
V_1 - V_0 \\
V_2 - V_0 \\
\vdots \\
V_n - V_0
\end{bmatrix}$$

where

$$S_{jj} = \sum_{k=0}^{n} S_{jk} ; \quad S_{jk} = S_{kj} = \frac{V_j V_k}{Z_{jk}^2} X_{jk}$$

$$T_{jj} = \sum_{k=0}^{n} T_{jk} ; \quad T_{jk} = T_{kj} = -\frac{1}{Z_{jk}^2} X_{jk}$$

and

$V_k$ = Voltage magnitude at node $k$

$\theta_k$ = Voltage angle at node $k$

$Z_{jk}$ = branch impedance between node $j$ and $k$

$X_{jk}$ = branch reactance between nodes $j$ and $k$.

The load flow problem is formulated by the equation 2.37 for unknown voltage angles and voltage magnitudes. This method provides a reduction in the computational time and the storage requirements. The convergence is always assured for small and large practical load flow problems.

Hybrid methods:

These methods 18, 19 are mixtures of direct and iterative techniques. If the nodal admittance equation for a network is written as
\[
\begin{bmatrix}
I_k \\
I_m
\end{bmatrix} = \begin{bmatrix}
Y_{kk} & Y_{km} \\
Y_{mk} & Y_{mm}
\end{bmatrix} \begin{bmatrix}
V_k \\
V_m
\end{bmatrix}
\]

The above equations can be manipulated to
\[
\begin{bmatrix}
I_k \\
V_m
\end{bmatrix} = \begin{bmatrix}
Y_{kk} - Y_{km} Y_{mm}^{-1} Y_{mk} & Y_{km} Y_{mm}^{-1} \\
- Y_{mm}^{-1} Y_{mk} & Y_{mm}^{-1}
\end{bmatrix} \begin{bmatrix}
V_k \\
I_m
\end{bmatrix}
\]

The matrix relating currents and voltages in this is a mixed matrix of impedance elements, admittance elements, etc. The nodal requirements, in terms of power and voltage may be substituted for the currents and the system of equation solved by any iterative scheme such as the Gauss-Seidel method.

Several additional direct and iterative methods have been suggested in recent years for load-flow solution. Some of these are given in the References.6,11,14,17,21,22.

2.6 Conclusion

As can be seen from the techniques described, there are numerous methods available for the solution of the load-flow problem. Unfortunately there does not seem to be one particular method which can be considered to be the best under all possible circumstances. The effectiveness of any particular technique can not be divorced from the problem size, the system structure, the computer storage available and the coding and computation efficiency of the facility.

Several modifications have been made to the existing techniques which result in a decrease in the system solution time without increasing the storage requirements. The theoretical development and the practical application of these modifications are discussed in subsequent chapters.
3. A NEW ACCELERATED NEWTON'S METHOD FOR LOAD-FLOW SOLUTION

3.1 General

This chapter presents a new numerical method\textsuperscript{41,42} for the solution of the load-flow problem. In recent years, many excellent reports\textsuperscript{14,21,22} have been published on various aspects of load-flow studies. Several important techniques of problem formulation and solution have been proposed. The main purpose of this new method is to reduce the computational time and increase the rate of convergence of the solution in comparison with existing methods.

This new method incorporates Gaussian elimination in such a manner that the most recent information is always used at each step of the iteration cycle. The method is roughly quadratically\textsuperscript{41,42} convergent and requires only \( (N^2/2 + 3N/2) \) function evaluations per iterative step as compared to \( (N^2 + N) \) function evaluations in Newton's method\textsuperscript{41}. The detailed description of the proposed method and load-flow formulation is given below.

3.2 Newton's Method

The load-flow problem is the solution of \( N \) non-linear equations in \( N \) unknowns as given by

\[
\begin{align*}
    f_1(x) &= f_1(x_1, x_2, \ldots, x_N) = b_1 \\
    f_2(x) &= f_2(x_1, x_2, \ldots, x_N) = b_2 \\
    & \vdots \\
    f_N(x) &= f_N(x_1, x_2, \ldots, x_N) = b_N
\end{align*}
\] (3.1)

or simply in the vector form
f(x) - b = 0 \quad \text{(3.2)}

A system of non-linear equations of the form (3.1), where \( f \) and \( x \) are \( N \)-dimensional column vectors and may perhaps be complex. If \( x^* \) is a solution of (3.1) it is required to generate a sequence of approximation, \( x_k \), such that this sequence converges to \( x^* \). Newton's method applied to (3.1) generates a sequence of vectors \( x_k \) by the formula

\[
x_{k+1} = x_k - J_k^{-1} f_k
\]

where \( J_k \) is the Jacobian matrix

\[
J = \frac{\partial f}{\partial x}
\]

evaluated at \( x_k \), and \( f_k \) is the residual vector. The following local convergence theorem is well known for Newton's method\(^{19}\).

This theorem states that:

(i) in the closed region \( S \) whose interior contains a root \( x = S \) (where \( S = S_1, S_2 \ldots S_N \)) i.e. solution of (3.1), each \( f_i \) is twice continuously differentiable for \( i = 1, 2, \ldots N \).

(ii) \( J(f) \) is nonsingular at \( x = S \), and

(iii) \( x_0^0 \) is chosen in \( S \) sufficiently close to \( x = S \) then the iteration of (3.1) is convergent to \( S \).

3.3. New Accelerated Newton's Method

The technique basically consists of expanding the first equation in a Taylor series about the starting guess, retaining only linear terms, equating to zero and solving for one variable, say \( x_N \), as a linear combination of the remaining \( N-1 \) variables. In the second equation, \( x_N \) is eliminated by replacing it with its linear representation
found above, and again the process of expanding by linear terms, equating to zero and solving for one variable in terms of the remaining $N-2$ variables. The process continues in this fashion, eliminating one variable per equation, until the $N$th equation remains with only one unknown. This is shown below in matrix form:

$$
\begin{bmatrix}
  x & x & x & x & x & x \\
  x & x & x & x & x & x \\
  x & x & x & x & x & x \\
  x & x & x & x & x & x \\
  x & x & x & x & x & x \\
  x & x & x & x & x & x \\
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4 \\
  x_5 \\
  x_6 \\
\end{bmatrix}
= 
\begin{bmatrix}
  b_1 \\
  b_2 \\
  b_3 \\
  b_4 \\
  b_5 \\
  b_6 \\
\end{bmatrix}
$$

(3.3)

The unknown $x_N$ vector can be calculated by back-substitution in the generated triangularized linear system. In elimination, the pivoting effect is achieved by choosing at any step that variable having the largest absolute partial derivative. The pivoting is done without physical interchange of rows or columns. The forward triangularization of the full Jacobian is obtained in this method by working with one row at a time.

The algorithm of this method consists of the following steps:

Step I:

The first equation of (3.1) is expanded by Taylor's theorem about the point $x^n$:

$$
f_1(x) = f_1(x^n) + (x_1^{n+1} - x_1^n) \frac{\partial f_1}{\partial x_1^n} + (x_2^{n+1} - x_2^n) \frac{\partial f_1}{\partial x_2^n} + \ldots + (x_N^{n+1} - x_N^n) \frac{\partial f_1}{\partial x_N^n} + \phi_1 - b_1 = 0
$$

(3.4)

where $\phi_1$ is a function of higher powers of $x_1, x_2 \ldots x_N$ and second,
third, etc. derivatives of the function \( f_1 \). If the initial estimate for \( x_N \) is near the solution value, then all the higher power terms can be neglected. The residual vector is given by

\[
\{ f_1(x^n) - b_1 \} = f_1^n
\]

The right hand side of (3.4) is equated to zero and solved for one variable which has the greatest absolute partial derivative.

From (3.4):

\[
x_N^{n+1} = x_N^n - \sum_{j=1}^{N-1} \left( \frac{f_1^n}{1x_j^n} \right) (x_j^{n+1} - x_j^n) - \frac{f_1^n}{1x_N^n} \tag{3.5}
\]

where

\[
\frac{f_1^n}{1x_j^n} = \frac{\partial f_1}{\partial x_j}
\]

\[
\frac{f_1^n}{1x_N^n} = f_1(x^n)
\]

The constant terms \((-\frac{f_1^n}{1x_j^n} / f_1^n)\) and \((f_1^n / f_1^n)\) are stored for future use. Equation (3.5) now contains (N-1) variables and is redesignated as \( a_N \).

Therefore,

\[
a_N(x_1^{n+1}, x_2^{n+1}, \ldots, x_{N-1}^{n+1}) = \text{R.H.S. of (3.5)} \tag{3.6}
\]

Step II:

Define a new function \( g_2 \) for the second equation of (3.1) in terms of the (N-1) variables and \( a_N \) which is also a function of the (N-1) variables.

\[
g_2(x_1^{n+1}, x_2^{n+1}, \ldots, x_{N-1}^{n+1}) = f_2(x_1^{n+1}, x_2^{n+1}, \ldots, x_{N-1}^{n+1})
\]

\[
a_N(x_1^{n+1}, x_2^{n+1}, \ldots, x_{N-1}^{n+1}) \tag{3.7}
\]

The function is now expanded in Taylor series about the point
\((x_1^n, x_2^n, \ldots, x_{N-1}^n)\), linearized and solved for the variable which
has the greatest absolute partial derivative as before. Consider
\(x_{N-1}^n\) as the variable whose corresponding partial derivatives is largest
in magnitude.

\[
f_2(x_1^n, x_2^n, \ldots, x_{N-1}^n, a_N(x_1^n, \ldots, x_{N-1}^n))
\]
\[
= g_2^n + (x_2^{n+1} - x_2^n) \frac{\partial g_2}{\partial x_2^n} + (x_2^{n+1} - x_2^n) \frac{\partial g_2}{\partial x_2^n}
\]
\[
+ \ldots + (x_{N-1}^{n+1} - x_{N-1}^n) \frac{\partial g_2}{\partial x_{N-1}^n} = 0
\]  

(3.8)

All partial derivatives of (3.8) are obtained from (3.7). Since \(x_N\)
is eliminated from the first equation, therefore, from (3.7)

\[
\frac{\partial g_2}{\partial x_N^n} = 0
\]

\[
\frac{\partial g_2}{\partial x_{N-1}^n} = \frac{\partial f_2}{\partial x_{N-1}^n} + \frac{\partial f_2}{\partial a_N} \frac{\partial a_N}{\partial x_{N-1}^n}
\]

.......................  

.......................  

\[
\frac{\partial g_2}{\partial x_2^n} = \frac{\partial f_2}{\partial x_2^n} + \frac{\partial f_2}{\partial a_N} \frac{\partial a_N}{\partial x_2^n}
\]

\[
\frac{\partial g_2}{\partial x_{N-1}^n} = \frac{\partial f_2}{\partial x_{N-1}^n} + \frac{\partial f_2}{\partial a_N} \frac{\partial a_N}{\partial x_{N-1}^n}
\]

(3.9)

The derivatives of (3.8) and (3.9) are obtained from (3.7) by chain
rule differentiation. Therefore, from (3.8)

\[
x_{N-1}^n = x_{N-1}^n - \sum_{j=1}^{N-2} \left( g_2(x_j^n / g_2 x_{N-1}^n) (x_j^{n+1} - x_j^n) \right)
\]

\[
- g_2^n / (g_2 x_{N-1}^n)
\]

(3.10)
where \( g_2^n \) = is the residual vector

\[
g_2^n = (g_2^n (x)^n - b_2)
\]

\[
g_{2x_j}^n = \frac{\partial g_2^n}{\partial x_j^n}
\]

(3.11)

The left hand side of (3.10) is designated as \( a_{N-1} \) and is a function of \( N-2 \) variables.

\[
a_{N-1} (x_1^{n+1}, x_2^{n+1} \ldots x_{N-2}^{n+1}) = \text{R.H.S. of (3.10)}
\]

The ratio formed by \( (g_2^n / g_{2x_j}^n) \) and \( (g_2^n / g_{2x_{N-1}}^n) \) are stored for future use.

Step III:

Define another function \( g_3 \) for the third equation in terms of the variables \( x_1^{n+1}, x_2^{n+1} \ldots x_{N-2}^{n+1}, a_{N-1} \) and \( a_N \). The process of expansions, linearization and elimination of the variable with the largest absolute derivative is repeated and the ratios formed by derivatives are stored as before for future use for the third equation.

\[
f_3(x_1^{n+1}, \ldots x_{N-2}^{n+1}, a_{N-1} (x_1^{n+1} \ldots x_{N-2}^{n+1}), a_N(x_1^{n+1} \ldots x_{N-2}^{n+1}))
\]

\[
= g_3 (x_1^{n+1}, x_2^{n+1} \ldots x_{N-2}^{n+1}, a_{N-1}, a_N)
\]

(3.12)

In this manner, one variable is replaced at a time from each equation and each \( g_k \) is expanded about the point \( (x_1^n, x_2^n \ldots \)

\( x_{N-k+1}^n \).
In the last step \((N-1)\) variables have been eliminated and the last equation has the form:

\[ g_N = f_N(x_1^{n+1}, a_2, a_3 \ldots a_N) \]

In this equation \(g_N\) is a function of only \(x_1^{n+1}\). Expanding, linearizing and solving for \(x_1\), gives

\[ x_1^{n+1} = x_1^n - \frac{g_N^n}{g_N x_1} \tag{3.13} \]

Where \(x_1^{n+1}\) is the improved approximation of \(x_1\). By back-substitution, the improved approximation of all other variables i.e. \(x_2^{n+1} \ldots x_N^{n+1}\) are obtained. The most recent information available is immediately used in the construction of the next variables, i.e. similar to that done in the Gauss-Seidel process for linear system of equations.

### 3.4 Simple Numerical Example

Consider the non-linear system of equations

\[ f(x_1, x_2) = x_1^2 - 2x_2 + 1 = 0 \]

\[ g(x_1, x_2) = x_1 + 2x_2^2 - 3 = 0 \]

Let the initial approximation be \(x^0 = (0,0)^T\).

Therefore,

\[ f(x_1^0, x_2^0) = 1 \]

\[ f(x_1^{1}, x_2^{1}) = f(x^0) + (x_1^{1} - x_1^0) \frac{\partial f}{\partial x_1^0} + (x_2^{1} - x_2^0) \frac{\partial f}{\partial x_2^0} \tag{3.14} \]

\[ \frac{\partial f}{\partial x_1^0} = 0 \]

\[ \frac{\partial f}{\partial x_2^0} = -2 \]
Since $|\frac{\partial f}{\partial x_2^0}| > |\frac{\partial f}{\partial x_1^0}|$, therefore $x_2$ is eliminated from (3.14)

\[
x_2^1 = x_2^0 - (x_1^1 - x_1^0)(\frac{\partial f}{\partial x_1^0} / \frac{\partial f}{\partial x_2^0} - f(x^0) / \frac{\partial f}{\partial x_2^0})
\]

\[
a_1 = x_2^0 - (x_1^1 - x_1^0)^* 0 + \frac{1}{2} = \frac{1}{2}
\]

Similarly for the second equation

\[
g(x_1^0, x_2^1) = -\frac{5}{2}
\]

\[
g(x_1, a_1) = g(x_1, a_1 (x_1))
\]

\[
\frac{\partial g}{\partial x_1} = \frac{\partial g}{\partial x_1} + \frac{\partial g}{\partial a_1} \cdot \frac{\partial a_1}{\partial x_1} = 1
\]

\[
g(x_1, a_1) = g(x^0) + (x_1^1 - x_1^0) \frac{\partial g}{\partial x_1^0}
\]

\[
= -\frac{5}{2} + (x_1^1 - x_1^0)^1
\]

\[
\therefore x_1^1 = \frac{5}{2}
\]

Therefore,

\[
(x_1^1, x_2^1) = (2.5, 0.5)^T
\]

is the solution after the first iteration by the developed algorithm.

Using the standard Newton's approach the solution vector is $(3.0, 0.5)^T$ at this point. The given algorithm does not give the same solution vector as the Newton approach at each point of the iterative cycle but does converge to the same final solution $(1.0, 1.0)^T$. This shows that mathematically the new algorithm is not equivalent to Newton's method. The rate of convergence and the computational time is discussed in more detail later in the chapter in connection with the load-flow problem.
3.5 Load-flow Formulation

The main features of this method are that equations (3.15) and (3.16) shown below are used to find new values of the unknown voltages as functions of the variables that are known or specified. The method utilizes algebraic manipulation of these equations, and does not require any matrix inversion.

\[ I_k = \sum_{m=1}^{N} Y_{km} V_m \]  \hspace{1cm} (3.15)

\[ S_k = V_k I_k^* \]  \hspace{1cm} (3.16)

where

- \( Y_{km} \) - for \( k = m \), the self admittance at node \( k \) or \( m \).
- for \( k \neq m \) the transfer admittance between \( k \) and \( m \).
- \( N \) - number of independent node equations.
- \( I_k \) - sum of the fixed currents entering node \( k \).
- \( V_k \) - voltage at node \( k \).
- \( S_k \) - complex power entering node \( k \).

From (3.16)

\[ P_k + jQ_k = \sum_{m=1}^{N} |Y_{km}| |V_m| |V_k| e^{j(\delta_m - \delta_k - \theta_{km})} \]

or

\[ P_k = \sum_{m=1}^{N} |Y_{km}| |V_m| |V_k| \cos(\delta_m - \delta_k - \theta_{km}) \]  \hspace{1cm} (3.17)

\[ Q_k = \sum_{m=1}^{N} |Y_{km}| |V_m| |V_k| \sin(\delta_m - \delta_k - \theta_{km}) \]  \hspace{1cm} (3.18)

Equations (3.17) and (3.18) are function of \( \delta \) and \( |V| \). Using as an example the simple three-node system shown in Figure 2.2, the system
equations are as follows:
\[ f_1(\delta_2, \delta_3, |V_3|) = P_2 - |Y_{21}||V_2||V_1| \cos(\delta_2 - \delta_1 - \theta_{21}) + |Y_{22}||V_2||V_2| \cos(\delta_2 - \delta_3 - \theta_{23}) \]  \hspace{1cm} (3.19)
\[ f_2(\delta_2, \delta_3, |V_3|) = P_3 - |Y_{31}||V_3||V_1| \cos(\delta_3 - \delta_1 - \theta_{31}) - |Y_{32}||V_3| \]  \hspace{1cm} (3.20)
\[ f_3(\delta_2, \delta_3, |V_3|) = Q_3 - |Y_{31}||V_3||V_1| \sin(\delta_3 - \delta_1 - \theta_{31}) - |Y_{32}||V_3| \]  \hspace{1cm} (3.21)

In the above system of equations node number 1 has not been included, since it was taken as the swing bus and thus \(|V_1|\) and \(\delta_1\) are specified. Likewise for the generating node i.e. node number 2 the variation of voltage magnitude would be zero since \(|V_2|\) is fixed and as \(P_2\) is given, only \(\delta_2\) will change. Node number 3 is the load bus where \(P_3\) and \(Q_3\) are specified and both \(|V_3|\) and \(\delta_3\) are unknown. The above three non-linear equations are a function of the three unknowns, \(\delta_2, \delta_3\) and \(|V_3|\).

The basic steps are as follows:

Step A:

Equation (3.19) is expanded in Taylor's series about the point \((\delta_2^0, \delta_3^0, |V_3^0|)\) and only the first order terms are retained to obtain the linear approximation
\[ f_1(\delta_2, \delta_3, |V_3|) = f_1^0 + (\delta_2^1 - \delta_2^0) \frac{\partial f_1}{\partial \delta_2^0} + (\delta_3^1 - \delta_3^0) \frac{\partial f_1}{\partial \delta_3^0} \]  \hspace{1cm} (3.22)
\[ + (|V_3^1| - |V_3^0|) \frac{\partial f_1}{\partial |V_3^0|} \]
where
\[ f_1^0 = P_{2c} - P_{2s} \]

\( P_{2c} \) is the calculated real power at node #2 for the initial guess of \( \delta \) and \( |V| \).

\( P_{2s} \) is the specified real power at node #2.

If the absolute value of the partial derivative of \( \frac{\partial f_1}{\partial \delta_2} \) is the largest, then \( \delta_2 \) is eliminated from equation (3.22).

\[
\delta_2^1 = \delta_2^0 - (\delta_3^1 - \delta_3^0) \frac{\partial f_1}{\partial \delta_2^0} - (|V_3^1| - |V_3^0|) \]

or

\[
a_1 = \delta_2^0 - (\delta_3^1 - \delta_3^0) A_1 - (|V_3^1| - |V_3^0|) B_1 - C_1 \tag{3.23}
\]

\( A_1, B_1 \) and \( C_1 \) of equation (3.23) are stored for future use.

Step B:

Define a function \( g_2 \) in terms of \( \delta_3 \) and \( |V_3| \).

\[
g_2 (\delta_3, |V_3|, a_1) = f_2 (\delta_3, |V_3|, a_1(\delta_3, |V_3|)) \tag{3.24}
\]

Now expand the second equation about the points \( \delta_3^0 \) and \( V_3^0 \).

\[
g_2 (\delta_3, V_3, a_1) = f_2^0 + (\delta_3^1 - \delta_3^0) \frac{\partial g_2}{\partial \delta_3^0} + (|V_3^1| - |V_3^0|) \frac{\partial g_2}{\partial |V_3^0|} \tag{3.25}
\]

The partial derivatives of equation (3.25) are obtained from (3.24) by chain rule differentiation.

\[
\frac{\partial g_2}{\partial \delta_2} = 0
\]

\[
\frac{\partial g_2}{\partial \delta_3} = \frac{\partial f_2}{\partial \delta_3^0} + \frac{\partial f_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial \delta_3^0}
\]
\[ \frac{\partial g_2}{\partial |V_3^0|} = \frac{\partial f_2}{\partial |V_3^0|} + \frac{\partial f_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial |V_3^0|} \]

Assuming that the magnitude of the partial derivative of \( \frac{\partial g_2}{\partial \delta_3} \) is greater than \( \frac{\partial g_2}{\partial |V_3^0|} \), \( \delta_3 \) is eliminated from equation (3.25).

\[ \delta_3 = \delta_3^0 - (|V_3^1| - |V_3^0|)(\frac{\partial g_2}{\partial |V_3^0|}/(\frac{\partial g_2}{\partial \delta_3}) - \frac{\partial g_2}{\partial |V_3^0|} - (|V_3^1| - |V_3^0|) A_2 - B_2 \] (3.26)

\[ a_2 = \frac{\delta_3^0}{|V_3^0|} \]

\[ a_2 = \frac{\delta_3^0}{|V_3^0|} \cdot (|V_3^1| - |V_3^0|) A_2 - B_2 \]

\[ a_2 = \frac{\delta_3^0}{|V_3^0|} \cdot (|V_3^1| - |V_3^0|) \]

\[ A_2 \text{ and } B_2 \text{ are stored for future use.} \]

Step C:

The variables \( \delta_2 \) and \( \delta_3 \) are eliminated from the first and second equations and another function \( g_3 \) is defined in terms of \( |V_3^1| \) only.

The process of expansion in Taylor's series and linearization is then repeated:

\[ g_3 (|V_3^1|, a_2, a_1) = f_3 (|V_3^0|, a_2 (|V_3^0|), a_1 (|V_3^0|, a_2 (|V_3^0|))) \]

\[ g_3 (|V_3^1|, a_2, a_1) = f_3 (|V_3^0|) + (|V_3^1| - |V_3^0|) \frac{\partial g_3}{\partial |V_3^0|} \] (3.27)

\[ \frac{\partial g_3}{\partial \delta_2} = 0 \quad \frac{\partial g_3}{\partial \delta_3} = 0 \]

\[ \frac{\partial g_3}{\partial |V_3^0|} = \frac{\partial f_3}{\partial a_2} + \frac{\partial f_3}{\partial |V_3^0|} \cdot \frac{\partial a_2}{\partial |V_3^0|} + \frac{\partial f_3}{\partial |V_3^0|} \cdot \frac{\partial a_1}{\partial a_2} \cdot \frac{\partial a_2}{\partial |V_3^0|} \]

Now from equation (3.27)

\[ V_3^1 = V_3^0 - f_3 (|V_3^0|) \cdot (\frac{\partial g_3}{\partial |V_3^0|}) \] (3.28)

Equations (3.22), (3.25) and (3.28) can be written in the following matrix form:
\[
\begin{bmatrix}
\frac{\partial f_1}{\partial \delta_2} & \frac{\partial f_1}{\partial \delta_3} & \frac{\partial f_1}{\partial V_3} \\
\frac{\partial g_2}{\partial \delta_3} & \frac{\partial g_2}{\partial |V_3|} \\
\frac{\partial g_3}{\partial V_3}
\end{bmatrix}
\begin{bmatrix}
\delta_2^{n+1} - \delta_2^n \\
\delta_3^{n+1} - \delta_3^n \\
|V_3|^{n+1} - |V_3|^n
\end{bmatrix}
= \begin{bmatrix}
f_1^o \\
f_2^o \\
f_3^o
\end{bmatrix}
\]

The improved approximation of \(\delta_2^{n+1}, \delta_3^{n+1}\) and \(|V_3|^{n+1}\) are obtained from equation (3.29) by back-substitution. It should be noted here that the most recent value is immediately used in the calculation of the next function argument in a similar way to that used in the Gauss-Seidel method.

The process is repeated from step A to step C until the calculated real and reactive powers at every bus is satisfied with a desired accuracy or the two successive values for each \(\delta\) and \(|V|\) differ only by a specified tolerance. For \(N\) linearized equations, there will be \(N\) steps. If the value of \(\Delta \delta\) and \(|\Delta V|\) for the given system varies very slowly then as in Newton's method, the triangularized Jacobian matrix may be evaluated at every \(k\)th iteration. This will give a further reduction in computation time.

### 3.6 Triangularized Jacobian

One feature of the new method is that the forward triangularization of the full Jacobian matrix is approximated by working with one row at a time and eliminating one variable from each row treated. The system of equations can be shown by the following matrix representation.
\[ J(x^{n+1} - x^n) = -f \]

where

\[ J = \text{Triangularized Jacobian} \]
\[ x = \text{unknown vector} \]
\[ f = \text{residual vector} \]

The triangularized Jacobian is given by

\[
J = \begin{bmatrix}
    f_{1j} & f_{1,N-i+2} & \ldots & f_{1,N-1} & f_{1N} \\
    f_{2j} & f_{2,N-i+2} & \ldots & f_{2,N-1} & f_{2N} \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    f_{i,j} & f_{i,N-i+2} & \ldots & f_{i,N-1} & f_{iN} \\
    f_{i-1,i} & f_{i-1,N-i+2} & \ldots & f_{i-1,N-1} & f_{i-1,N} \\
\end{bmatrix}
\]

If the assumption is made that the variables are eliminated in the order of \( x_N, x_{N-1}, \ldots, x_2 \), then the matrix formulation for 3 x 3 system is

\[
J = \begin{bmatrix}
    \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\
    \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & 0 \\
    \frac{\partial g_3}{\partial x_1} & 0 & 0
\end{bmatrix}
\]
where
\[
\frac{\partial g_2}{\partial x_1} = \frac{\partial f_2}{\partial x_1} + \frac{\partial f_2}{\partial x_3} \cdot \frac{\partial x_3}{\partial x_1}
\]
\[
\frac{\partial g_2}{\partial x_2} = \frac{\partial f_2}{\partial x_2} + \frac{\partial f_2}{\partial x_3} \cdot \frac{\partial x_3}{\partial x_2}
\]

and
\[
x_3^1 = x_3^0 - (x_1^1 - x_1^0) \left( \frac{\partial f_1}{\partial x_1} / \frac{\partial f_1}{\partial x_3} \right) - (x_2^1 - x_2^0) \left( \frac{\partial f_1}{\partial x_2} / \frac{\partial f_1}{\partial x_3} \right)
\]

\[
\frac{\partial f_1}{\partial x_3} - f_1(0) / \left( \frac{\partial f_1}{\partial x_3} \right)
\]

Therefore,
\[
\frac{\partial x_3}{\partial x_1} = - \left( \frac{\partial f_1}{\partial x_1} / \frac{\partial f_1}{\partial x_3} \right)
\]
\[
\frac{\partial x_3}{\partial x_2} = - \left( \frac{\partial f_1}{\partial x_2} / \frac{\partial f_1}{\partial x_3} \right)
\]

Now, from equation (3.32)
\[
\frac{\partial g_2}{\partial x_1} = \frac{\partial f_2}{\partial x_1} + \frac{\partial f_2}{\partial x_3} \left( - \frac{\partial f_1}{\partial x_1} / \frac{\partial f_1}{\partial x_3} \right)
\]
\[
= (f_{21} f_{13} - f_{11} f_{23}) / f_{13}
\]
\[
= \begin{vmatrix} f_{11} & f_{13} \\ f_{21} & f_{23} \end{vmatrix}
\]
\[
f_{13}
\]

Similarly,
\[
\frac{\partial g_2}{\partial x_2} = \frac{\partial f_2}{\partial x_2} + \frac{\partial f_2}{\partial x_3} \left( - \frac{\partial f_1}{\partial x_2} / \frac{\partial f_1}{\partial x_3} \right)
\]
Similarly, the derivative of the last equation is

\[
\frac{\partial g_3}{\partial x_1} =
\begin{vmatrix}
  f_{11} & f_{12} & f_{13} \\
  f_{21} & f_{22} & f_{23} \\
  f_{31} & f_{32} & f_{33} \\
\end{vmatrix}
\]

Therefore, the triangularized Jacobian is

\[
J = 
\begin{vmatrix}
  f_{11} & f_{12} & f_{13} \\
  f_{21} & f_{22} & f_{23} \\
  f_{31} & f_{32} & f_{33} \\
\end{vmatrix}
\begin{vmatrix}
  f_{11} & f_{13} \\
  f_{21} & f_{23} \\
  f_{31} & f_{33} \\
\end{vmatrix}
\begin{vmatrix}
  f_{12} & f_{13} \\
  f_{22} & f_{23} \\
  f_{32} & f_{33} \\
\end{vmatrix}
\begin{vmatrix}
  f_{13} \\
  f_{23} \\
  f_{33} \\
\end{vmatrix}
\]

\[
= (f_{22} f_{13} - f_{23} f_{12}) / f_{13}
\]

\[
= \begin{vmatrix}
  f_{12} & f_{13} \\
  (-1) f_{22} & f_{23} \\
\end{vmatrix}
\]

\[
f_{13}
\]
where the arguments are given by the following $x^n$ for $f_{ij}$

$$(x_1^n, x_2^n, a_3(x_1^n, x_2^n))$$ for $f_{2j}$ and

$$(x_1^n, a_2(x_1^n), a_3(x_1^n, a_2(x_1^n)))$$ for $f_{3j}$

where $j = 1, 2 \ldots 3$

The results obtained by this method are most encouraging in comparison with conventional Gauss-Seidel and Newton's methods. The computer programme was developed using a hypothetical test system$^3$.

The programme was later applied to a practical system to confirm the efficiency of the technique. The method is roughly quadratically convergent$^{41,42}$ and as mentioned earlier it requires only $(N^2/2 + 3N/2)$ function evaluations in this approach. There is no particular "best" method for overall load-flow analysis and rigorous convergence results have yet to be obtained.

3.7 Numerical Examples

The sample problem used for the load-flow study is that given in reference$^3$. Some of the data and the system one-line diagram (Figure 2.3) are repeated here for convenience.

In order to compare the merits of the proposed methods under as nearly identical conditions as possible they were used to solve the power system of Figure 2.3 and the simplified 20-bus SPC system, Figure 5.2, with each method starting from the same initial conditions. It is not possible to terminate the calculations at the same exact point for each method. The accuracy of the final solutions are measured by a prespecified tolerance of 0.0001. The criteria by which the
Table 3.1. 5-Bus System

Line data for the sample system shown in Figure 2.3.
Lines are assumed to be 795 ACSR 54/7
Current carrying capacity = 900 amp = 1.71 p.u.
Base MVA = 100
Base KV = 110

<table>
<thead>
<tr>
<th>Line #</th>
<th>Length Miles</th>
<th>Impedence p.u.</th>
<th>Susceptance (b/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 6</td>
<td>30</td>
<td>0.0342 + j 0.1800</td>
<td>0.0106</td>
</tr>
<tr>
<td>2, 7</td>
<td>100</td>
<td>0.1140 + j 0.6000</td>
<td>0.0352</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>0.0912 + j 0.4800</td>
<td>0.0282</td>
</tr>
<tr>
<td>4, 5, 8</td>
<td>20</td>
<td>0.0228 + j 0.1200</td>
<td>0.0071</td>
</tr>
</tbody>
</table>

Table 3.2.

Generation Data 5-Bus System

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>No. of Units</th>
<th>Capacity of each unit in MW</th>
<th>Total Bus Capacity MW</th>
<th>Voltage p.u.</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>20</td>
<td>80</td>
<td>1.05</td>
<td>0°</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>5</td>
<td>130</td>
<td>1.05</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Load Data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-</td>
<td>Swing Bus</td>
<td>1.05</td>
<td>0°</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>1.0</td>
<td>110</td>
<td>1.05</td>
<td>0°</td>
</tr>
<tr>
<td>3</td>
<td>85</td>
<td>1.0</td>
<td>0</td>
<td>1.0</td>
<td>0°</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>1.0</td>
<td>0</td>
<td>1.0</td>
<td>0°</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1.0</td>
<td>0</td>
<td>1.0</td>
<td>0°</td>
</tr>
</tbody>
</table>

All data for the 20 Bus Saskatchewan Power Corporation System is given in Chapter 5.
merit of any load-flow technique may be judged are the accuracy of the solution obtained, the time taken to reach this solution, and the amount of computer storage space used in the computation.

Table 3.3 presents the voltage vectors obtained by the different methods for a tolerance of $10^{-4}$ in bus voltages and bus powers. The results obtained by the new method for the test system compare quite closely with more conventional load-flow techniques.

**Table 3.3.**

Voltage solution for the sample problem

<table>
<thead>
<tr>
<th>Bus</th>
<th>Gauss-Seidel</th>
<th>Newton's</th>
<th>New Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Voltage</td>
<td>Angle</td>
<td>Voltage</td>
</tr>
<tr>
<td>1</td>
<td>1.0500</td>
<td>0.0</td>
<td>1.0500</td>
</tr>
<tr>
<td>2</td>
<td>1.0500</td>
<td>7.09</td>
<td>1.0500</td>
</tr>
<tr>
<td>3</td>
<td>1.0371</td>
<td>-3.54</td>
<td>1.0371</td>
</tr>
<tr>
<td>4</td>
<td>1.0372</td>
<td>-2.86</td>
<td>1.0372</td>
</tr>
<tr>
<td>5</td>
<td>1.0369</td>
<td>-3.53</td>
<td>1.0369</td>
</tr>
</tbody>
</table>

A considerable reduction in the overall computation time is obtained by the proposed technique in comparison to the Newton and Gauss-Seidel methods \(^5, 7, 10\). The comparative results for the 5-Bus system and the 20-Bus system are given in Table 3.4 and Table 3.5. It can be seen from these Tables that the total reduction
in solution time by the new method is about 40% to 50%.

A tolerance of $10^{-4}$ was used in each case in order to obtain a more equitable comparison. The number of iterations required to achieve the desired accuracy at different nodes by the new method is less than by Newton's method. The reason is obvious, as at each step of the calculations, the latest calculated value is immediately used to calculate the following unknown variables. In this connection it should be noted, however, that the criteria for rapid convergence of the load-flow problem depends not only upon the numerical technique but also on the system size, the effect of the number of generating and load buses and the choice of the swing bus, etc.

Table 3.4 and Table 3.5 clearly show the difference in solution time between the new technique and Gauss-Seidel and Newton's method.

Table 3.4.

Comparative Results for 5-Bus System

<table>
<thead>
<tr>
<th>Methods</th>
<th>Tolerance Used</th>
<th>Time (Sec)</th>
<th>No. of Iterations</th>
<th>Maximum Mismatch (MVA)</th>
<th>Average Mismatch (MVA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss-Seidel</td>
<td>$10^{-4}$</td>
<td>2.72</td>
<td>17</td>
<td>0.001</td>
<td>0.0004</td>
</tr>
<tr>
<td>Newton's</td>
<td>$10^{-4}$</td>
<td>2.00</td>
<td>5</td>
<td>0.021</td>
<td>0.0130</td>
</tr>
<tr>
<td>New Method</td>
<td>$10^{-4}$</td>
<td>1.57</td>
<td>3</td>
<td>0.005</td>
<td>0.0010</td>
</tr>
</tbody>
</table>
Table 3.5.
Comparative Results for 20-Bus S.P.C. System

<table>
<thead>
<tr>
<th>Methods</th>
<th>Tolerance</th>
<th>Time (Sec)</th>
<th>No. of Iterations</th>
<th>Maximum Mismatch (MVA)</th>
<th>Average Mismatch (MVA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss-Seidel</td>
<td>$10^{-4}$</td>
<td>68.0</td>
<td>49</td>
<td>0.009</td>
<td>0.0010</td>
</tr>
<tr>
<td>Newton's</td>
<td>$10^{-4}$</td>
<td>56.0</td>
<td>7</td>
<td>0.158</td>
<td>0.0210</td>
</tr>
<tr>
<td>New Method</td>
<td>$10^{-4}$</td>
<td>32.0</td>
<td>5</td>
<td>0.026</td>
<td>0.0070</td>
</tr>
</tbody>
</table>

The accuracy of the power flow to a bus depends on the choice of the tolerance factor. As can be seen from the average mismatch, a tolerance of $10^{-4}$ gives reasonably accurate bus power values. The time given in Table 3.4 and Table 3.5 also includes the data preparation time for the iterative process. The program developed can also handle any change in the system network i.e. outage of lines or generators.

The triangularized Jacobian matrix in this approach is obtained directly and therefore requires somewhat less storage than a triangularized equivalent obtained from the full Jacobian by the conventional approach. The storage space saving will be quite significant in large practical system studies. Experience gained from a large number of study cases has indicated that the new method is definitely competitive with the conventional methods for load-flow...
analysis. The gain in computation time is well worth the extra computation involved in directly evaluating the triangularized Jacobian matrix.

It is too early to offer a definitive answer as to the convergence properties and other merits of the new method. It is believed that more studies should be carried out to determine the exact merits and efficiency of the proposed method over the other existing methods.

3.8. Conclusions

In this chapter a new modified Newton's method for finding the load-flow solution is investigated. The numerical examples illustrate the power of the new method. The results obtained by the new method have also been compared with those from the conventional load-flow techniques, namely the Gauss-Seidel and Newton's method. The method is similar to Newton's method and its principal objective is to reduce the computing time and number of iterations required for load-flow solution. From the test cases it has been found that:

(a) The overall computing time is 40% to 50% lower than that of Newton's and Gauss-Seidel methods.

(b) The number of iterations is less than that required for the Newton method.

Further reduction in computing time may be achieved by introducing an acceleration factor in the iteration process as reported in a recent paper \(^{16}\) by Sasson. It would be most desirable, however, for further studies to be done on the convergence behavior of the proposed method and acceleration techniques. The
latter is, of course, an interesting, useful and largely unexplored
domain of research for load-flow study by Newton's method. Further
experimentation and exploitation of the numerical techniques will
undoubtedly result in substantial improvements to the method proposed.
4. MODIFIED NEWTON METHODS

4.1 General

This chapter describes three modified algorithms for rapid load-flow solution. The first two methods were developed as a part of this research project and the last one by Broyden. Although the discussion is limited to Newton's method, the developed algorithms are general and can be used in conjunction with any of the iterative techniques. The preceding chapter illustrated the development of a modified approach for load-flow solution which appears to have considerable benefits in conventional load-flow analysis. The decrease in solution time using this approach relative to conventional Gauss-Seidel and Newton solutions are shown for a hypothetical 5 bus system. The load-flow requirement in composite reliability studies is in the form of a series of system solutions under selected outage conditions. It was therefore considered desirable to further investigate the basic Newton method for application to repeated load-flow analysis keeping in mind that the accuracy of the solution in this case need not be as high as that normally utilized in single case studies.

4.2 Method No. 1 - Newton with successive overrelaxation

In load-flow analysis by the Newton method, the problem is to solve a set of simultaneous linearized algebraic equations of the form:

\[ Jx_i = b_i \quad (4.1) \]

where \( J \) is the coefficient matrix which is to be evaluated every
iteration cycle, $x_i$ is the column matrix of unknown variables, and $B_i$ is the residual vector. These equations are usually real number equations but can also be complex number equations.

A system of equations is said to be sparse if the associated coefficient matrix is sparse. Consider a rectangular array $J$ of dimensions $N \times M$, where $k$ is the number of non-zero elements in the matrix. The figure of merit by which the sparsity of $J$ is judged is $k/N \times M$. For admittance matrices arising in electric power networks, this figure of merit is typically less than 5% for a large practical system. Using the 5% figure, on the average only 10 elements need be processed in each 200 elements row of a 200 x 200 coefficient matrix. The Jacobian matrix is also a sparse matrix as it is derived from the nodal admittance matrix. The usual procedure to solve equation (4.1) is:

(a) Calculate the coefficient matrix at each iteration.
(b) Solve the unknown vector by the Gaussian elimination.

The process is repeated until the two successive values for each $x_i$ differ only by a specified tolerance or the residual vector is less than a certain specified tolerance.

The proposed modified method utilized the following concepts:

(a) The values of $x_i$ are changing relatively slowly in the load-flow analysis by the Newton method and the elements of the Jacobian matrix can be re-evaluated every $k$th iteration rather than at each iteration. This will save considerable computation time for a practical system.
(b) Equation (4.1) can be solved by the successive overrelaxation method with an optimum relaxation factor and considerable computational time can be saved by taking advantage of zeros in the coefficient matrix. The sparsity of the coefficient matrix was not fully utilized by the ordered triangularization and back-substitution method as the zero elements are replaced by non-zero values. The convergence rate can also be increased by introducing a relaxation factor in the solution of equation (4.1).

The procedure used for the overrelaxation method and the determination of the optimum relaxation factor are described in the following section.

The Successive Overrelaxation Method:

This approach is only one of a class of methods that have been developed to accelerate the convergence of an iterative process. A physical system in the steady-state can be represented by a set of simultaneous equations of the form:

\[
\sum_{j=1}^{N} a_{i,j} x_j = b_i \quad 1 \leq i \leq N
\]

or in the vector form by:

\[
f_i (x_i) = b_i \quad (4.2)
\]

where \( f_i \) are functions relating the unknown variables \( x_i \) with the known parameters of the system. A linear system can be represented in the matrix form by

\[
Ax = B \quad (4.3)
\]

where \( A \) is a constant coefficient matrix. In solving a system of
linearized equations, the coefficient matrix is re-evaluated at every iteration or at every kth iteration. In the latter case, the assumption is made that the coefficient matrix remains almost constant for k iterations. The rate of convergence of a stationary iterative process depends on the spectral radius of the associated block Jacobi matrix. Any modification of this matrix that will reduce the spectral radius will increase the rate of convergence. The coefficient matrix is highly sparse, and therefore considerable computational time can be saved by operating only on the non-zero elements.

The system of equations is rearranged so that each diagonal element of $A$ is 1. This can be done by dividing each equation by its diagonal element for the Gauss-Seidel iterative method:

$$a_{i,i} x_i^{(m+1)} = \sum_{j=1}^{N} a_{i,j} x_j^{(m+1)} - \sum_{j=i+1}^{N} a_{i,j} x_j^{(m)} + b_i$$

where $1 \leq i \leq N$, $m \geq 0$

or, in the matrix notation

$$(D-E)x^{m+1} = Fx^m + b \quad m \geq 0$$

where $D = \text{diag} \{a_{11}, a_{22}, \ldots, a_{NN}\}$ and $E$ and $F$ are strictly lower and upper triangular $NxN$ matrices respectively. This iterative approach is the point Gauss-Seidel or point single step iterative method.

Starting with equation (4.4) the components of the unknown vector, $\tilde{x}$, are given by:

$$\tilde{x}^{(m+1)} = \sum_{j=1}^{i-1} a_{i,j} x_j^{m+1} - \sum_{j=i+1}^{N} a_{i,j} x_j^{(m)} + b_i$$

Since $a_{i,i} = 1$ and $1 \leq i \leq N$, $m \geq 0$
The actual components $x_i^{(m+1)}$ of this iterative method are then defined from

$$x_i^{m+1} = x_i^m + \omega (\tilde{x}_i^{m+1} - x_i^m)$$

$$= (1 - \omega) x_i^m + \omega \tilde{x}_i^{m+1}$$

(4.6)

where the quantity $\omega$ is called the overrelaxation factor. This relaxation factor can be easily calculated by the Power method as described in the latter part of this chapter. If the true relaxation factor $\omega$ changes significantly from step to step then the use of a single unchanged value may not lead to convergence. When the elements of the coefficient matrix change considerably then this type of iterative scheme would not be successful. The Jacobian matrix formed from the nodal admittance matrix fortunately becomes almost constant after a few iterations in the Newton method. Ideally, for every $k$th iteration a new relaxation factor should be calculated to obtain the maximum rate of convergence. As the Jacobian becomes virtually constant after a few iterative cycles, however, only one relaxation factor is normally required to give a good convergence rate.

It has been found from studies on small systems that the relaxation factor usually lies between 1.5 and 1.0. In large practical networks the optimum overrelaxation factor normally lies between 1.5 and 2.0. Numerical methods for the calculation of an optimum relaxation factor are given below.

Estimation of the Optimum Relaxation Factor:

The optimum relaxation factor can be easily obtained if the largest eigenvalue of the associated block Jacobi matrix is known.
The method of successive overrelaxation will converge for all ω such that 0 < ω < 2. The most rapid convergence occurs for a value ω_{opt}, where 1 < ω_{opt} < 2. It is important, in practical system studies to use a relaxation factor as close to ω_{opt} as possible. The problem therefore is to determine the value of ω_{opt} without adding substantially to the computer time. The value of ω_{opt} should be obtained in less time then it would take to solve the problem with any reasonable guess for ω. The optimal overrelaxation method is approximately 30 times faster than the Liebmann process (ω=1).

If the final Jacobian matrix is known, from the previous results, then the principal eigenvalue of the associated Jacobian matrix can be estimated by the Power method. If not, the Jacobian matrix of the first iteration can be used to calculate the largest eigenvalue. As the coefficient matrix does not change, the will not vary very much.

If λ_1 is the dominant eigenvalue of the associated block Jacobi matrix, then ω_{opt} can be computed by the formula^23, 25.

\[ ω_{opt} = \frac{2}{1 + (1-λ_1^2)^{1/2}} \] (4.7)

Power method for determining the largest Eigenvalue

The Power method, in its basic form is conceptually the simplest iterative procedure for approximating the dominant or principal eigenvalue of the matrix J. The literature contains several methods for determining the optimum relaxation factor. Two methods have been examined in detail and the salient points...
are included in this section.

Let \( x_k \) denote the unknown vector of equation (4.1) after the
k th sweep in the iteration process and \( E_{k-1} = x_k - x_{k-1} \) be the
error vector of \( x_k \). The error vectors \( E_k \) are easily accessible in
the relaxation program at each iteration. The method of successive
displacement is then carried out for a number of iterations with \( \omega = 1 \).
The direction of the error vector \( E_k \) will approach the eigenvector
of the block Jacobi matrix that belongs to the dominant eigenvalue
\( \lambda_1 \). It follows that, for any norm function

\[
\lambda_1 = \lim_{k \to \infty} \frac{||E_{k+1}||}{||E_k||} \quad (4.8)
\]

When the ratio of the error vectors approaches the limiting value
its square root is the dominant eigenvalue of the associated block
Jacobi matrix. The optimum relaxation factor is then given by:

\[
\omega_{\text{opt}} = \frac{2}{1 + \sqrt{1 - \lambda_1^2}}^{1/2}
\]

The second approach to approximately locating \( \omega_{\text{opt}} \) is to carry
out the process of successive overrelaxation with various values of
\( \omega \) which are believed near \( \omega_{\text{opt}} \) and to compare the results in terms of
the required number of iterations. The optimum value will not change
significantly over a wide range of studies on a given network
configuration and the approximate optimum value can be used in these
cases. Curves relating the number of iterations with the relaxation
factor are shown in Figure 5.4 of the next chapter for a
5-bus and a 20-bus system.
Convergence:

Several authors\textsuperscript{25, 26, 27} have recently examined the Gauss-Seidel type of iterative procedure as applied to systems of non-linear equations. The convergence rate of these techniques has been found to be asymptotic\textsuperscript{24}. The convergence rate of the Newton method with successive overrelaxation is dependent upon the following factors:

(a) A good starting approximation.

(b) A reasonably fast method to determine the optimum relaxation factor, $\omega_{\text{opt}}$.

Work in this area has not been sufficiently extended to completely analyze the convergence properties of this technique. This may prove to be a fruitful area for future research.

4.3 Method Number 2. Constant Jacobian Matrix

In composite system reliability studies of the type described in the literature\textsuperscript{3} and later in this thesis it is necessary to perform a base case load-flow study and then repeated load-flow studies for a list of assumed outage conditions e.g., lines, transformers, generators, etc. The admittance matrix is usually modified for each outage condition and the Jacobian is evaluated at each point in the iterative cycle. The modified technique developed in this thesis, retains the final Jacobian matrix obtained in the base case and stores it for use in the subsequent cases. The final Jacobian matrix of the base case is modified only for changes in the system network. In modifying the Jacobian of the base case only those elements which have been effected by the outage of system components are recalculated. The inverse of this Jacobian is computed only once.
The off-diagonal elements of the Jacobian matrix are a function of $|V|$, $\delta$, $R$ and $X$ as shown in equation (2.28). The diagonal elements of the Jacobian are a function of $P$, $Q$, $|V|$, $\delta$, $R$ and $X$ as shown in equation (2.29).

The Jacobian matrix will therefore be modified for change in the system network configuration and in the system load levels. The matrix obtained from the base case will be an approximation if applied in subsequent cases involving system changes. It has been found from studies on a 5-bus hypothetical system and on a 20-bus practical system that the voltage vector solution time using the approximate Jacobian matrix is about 5 to 10 times lower than that required by the standard approach. The results obtained do not appear to differ very much from those of the standard Newton method. However, by modifying the inverse of the final Jacobian matrix from the inverse of the final Jacobian matrix from the base case for any system component outage, the results obtained are virtually identical to those of the basic approach. A method for modifying the inverse of the Jacobian matrix is developed in the later part of this chapter.

Geometrical Interpretation for Convergence

Of all the methods for the numerical solution of equations, Newton's method seems to be the most generally satisfactory. The real roots of the algebraic equations are conveniently evaluated by the Newton method of tangents. The use of a constant Jacobian matrix throughout the iterative cycle can be interpreted as a geometric modification of the tangent method.
Figure 3.1

Geometrical Interpretation for Convergence
Consider Figure 3.1 to represent a magnified view of the graph of \( y = f(x) \). If a tangent is drawn from the point P whose abscissa is \( a \), this tangent will intersect the x-axis at some point T. Another line drawn from \( P_1 \) and parallel to the original tangent will cut the x-axis at a point \( T_2 \) between \( T_1 \) and S. If the curvature of the graph does not change sign between P and S, the points T, \( T_1 \), \( T_2 \) etc. etc. will approach the point S as a limit. The value of OS represents the real root of the equation shown by the successive approximations to the desired root. The use of the final Jacobian from the base case provides the constant slope from which the successive approximations are obtained. The time required for convergence to the desired roots will be minimum. The procedure illustrated in figure 3.1 shows the geometrical significance of the constant Jacobian matrix method.

**Jacobian Matrix:**

The system conditions which may result in the Jacobian matrix being altered are as follows:

(a) the addition or outage of a bus from the existing system.

(b) a new connection or removal of a line or link between two existing buses.

(c) changes in the system load levels.

The addition of a branch to a new node and the addition or removal of branches between existing nodes can easily be incorporated by modification to the Jacobian matrix.

It is believed that better results and considerable saving in computational time can be achieved by this method if the inverse of the Jacobian matrix can be evaluated by the 'row by row' method.
The rows of the inverse Jacobian matrix obtained from the final step of the base case study can be modified for the outage of system components and changes in load steps. This modified Jacobian matrix then can be used directly for further load-flow studies. The whole procedure is illustrated by the Flow-chart in the Appendix IV. By avoiding the necessity of calculating the full Jacobian matrix and its inverse at each iteration in the repeated load-flow studies, a considerable amount of computational time can be saved. The next section presents a numerical method for inverting the Jacobian matrix using the 'row by row' technique.

4.4 Matrix Inversion by the 'Row by Row' Method

The elements of the coefficient matrix are complex for practical power systems. When the system matrices are too large, the storage requirements and computation becomes difficult if not impossible. There are many advantages to inverting a matrix using the 'row by row' technique. The outage of any system component will cause a change in the Jacobian matrix as noted in the previous section. Instead of recalculating the whole Jacobian matrix and its inverse, the inverse of only those particular rows in which the elements have been changed can be calculated.

General Method:

The inversion process is based on a nodal iterative solution that satisfies Kirchhoff's first law. Only two equations are required, one for the diagonal element and one for the off-diagonal elements.

By Kirchhoff's first law
\[
N
I_{pp} = \sum_{q=1}^{\infty} I_{pq}
\]  
(4.9)

where

\[I_{pp} = \text{total input current at node } p\]

\[I_{pq} = \text{the line current between node } p \text{ and } q\]

\[N = \text{total number of lines connected to node } p\]

The line current can be expressed in terms of node voltages and transfer admittances as follows:

\[I_{pq} = (V_p - V_q) Y_{pq}\]
(4.10)

where

\[V_p = \text{Voltage at node } p\]

\[V_q = \text{Voltage at node } q\]

\[Y_{pq} = \text{The transfer or mutual admittance between node } p \text{ and } q.\]

Combining equations (4.9) and (4.10):

\[I_{pp} = Y_{pp} V_p + \sum_{q=1}^{N} Y_{pq} V_q\]

(4.11)

where

\[N Y_{pp} = \sum_{q=1}^{N} Y_{pq} = \text{Self admittance of node}\]

Solving equation (4.11) for voltage \(V_p\) at node \(p\):

\[
V_p = \frac{\sum_{q=1}^{N} Y_{pq} V_q}{Y_{pp}} - \frac{I_{pp}}{Y_{pp}}
\]
(4.12)

Consider node \(p\) to be the driving point node and apply 1.0 p.u. current at this point, then

\[
V_p^D = \frac{1.0 - \sum_{q=1}^{N} Y_{pq} V_q}{Y_{pp}}
\]
(4.13)
where
\[ V_p^D \] is the voltage at diagonal element
\[ V_q \] is the voltage at adjacent nodes

The expression for the off-diagonal node voltages is
\[
V_{OD}^p = \sum_{q=1}^{N} Y_{pq} V_q \quad (4.14)
\]

The computation is initiated by selecting one node as the driving point and by using a series of initial estimates. The calculation then proceeds from one node to another. The approximate solutions are stored and re-used for the next iteration until the change in the node voltage is less than a specified tolerance. One row of the matrix has now been inverted. When an admittance matrix is inverted the diagonal terms represent the driving point impedences and the off-diagonal elements are the transfer impedences.

To obtain the complete inverse matrix, the next bus or any other bus is then selected as the driving node and the next row is computed. The inversion of the matrix is then built up 'row by row'.

Application:

As noted previously, in a load-flow analysis by the Newton method it is necessary to solve a system of linearized equations of the form:
\[
Ax = B \quad (4.15)
\]

where
\[ A \] is the Jacobian matrix.
\[ x \] is the unknown vector.
\[ B \] is the residual vector.
The coefficient matrix $A$ is diagonally dominant and also very sparse. This is due to the fact that the Jacobian matrix is derived from the system admittance matrix. Considering only those elements in the row with non-zero values i.e. actual network connections, a substantial saving in input requirement can be achieved. The inverse of a diagonally dominant but non-symmetric matrix obtained by the row by row method is not very accurate. The method is however quite accurate when applied to a diagonally dominant symmetric matrix. Any unsymmetrical coefficient matrix can be made symmetrical and more diagonally dominant if both sides of the equation (4.15) are multiplied by the transpose of the coefficient matrix i.e.

$$A^T A \ x = A^T b$$

The matrix $A^T A$ is symmetrical and more diagonally dominant than $A$.

The speed of convergence can be increased by suitable acceleration factors which can be automatically selected by the program from the computational behavior. The maximum acceleration factor used is 1.8 and the minimum 1.0. A comparison of some results obtained by applying a conventional matrix inversion by Gauss-Jordan method and those obtained by using the 'row by row' matrix inversion are shown as follows:
Table 4.1

\[
<table>
<thead>
<tr>
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Diagonally dominant symmetrical matrix

Table 4.2

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<td>-0.00020</td>
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Inverse matrix obtained with 'row by row' matrix inversion. Tolerance = 10^{-7}
Table 4.3

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</table>

*Inversion obtained for tolerance $10^{-3}$*

Table 4.4

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<tbody>
<tr>
<td>1</td>
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<td>0.00117</td>
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<td>-0.00221</td>
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<td>-0.00053</td>
<td>-0.00416</td>
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<td>-0.03640</td>
<td>0.03640</td>
<td>0.03714</td>
<td>0.07755</td>
</tr>
</tbody>
</table>

*Inverse matrix obtained by conventional Gauss-Jordan method, SSP subroutine MINV.*

Substantial computing time can be saved in repeated load-flow analysis by modifying the inverse Jacobian matrix in the manner described. A matrix inversion process using known methods becomes time consuming for practical system studies because the original matrix is difficult
to store wholly in the memory. The 'row by row' technique will permit
the handling of larger size systems than is possible with the standard
available methods. The matrix is inverted by an iteration process
and therefore some control over speed and accuracy of the computation
can also be obtained.

4.5. Method No. 3. Quasi Newton Approach

Many algorithms have been proposed for solving systems of non-
linear equations. The best known method of solving a general set of
simultaneous non-linear equations in which the corrections are
computed as linear combinations of the residuals is the Newton-Raphson
method. The principal disadvantage of the Newton method is that it
requires the evaluation of the Jacobian of the system of equations
for each iteration. Recently a number of methods have been\textsuperscript{16, 30, 31, 39} reported in which the inverse Jacobian matrix is replaced by an
approximation which is modified in some simple manner at each iteration.
At each stage of the calculation the approximate Jacobian is refined
until, at convergence, it hopefully becomes the Jacobian corresponding
to the solution of the system of equations. Calculations of the
Quasi-Newton type could be quite important for two reasons: First,
the analytical expressions for the system of equations could be
sufficiently complex that their differentiation might not be practical.
This is becoming less important because of computer availability.
Secondly, the algorithm can be used in conjunction with the standard
Newton method to reduce computational time. This modification
however is not unique and has quite a few drawbacks from a programming
point of view.

Iteration Technique:

Consider a set of \( n \) non-linear equations

\[
f_j(x_1, x_2 \ldots x_n) = 0
\]

\( j = 1, 2 \ldots n \)

These may be written more concisely as

\[
f(x) = 0
\]  \hspace{1cm} (4.16)

where \( f \) and \( x \) are \( n \) dimensional column vectors and may perhaps be complex.

If \( x^* \) is a solution of equation (4.16), it is necessary to generate a sequence of approximations, \( x^k \), such that this sequence converges to \( x^* \). If \( x^k \) is the \( k \)th approximation to the solution of equation (4.16) and \( f^k \) is written for \( f(x^k) \), then Newton's method is defined by

\[
x^{k+1} = x^k - [J^k]^{-1} f^k
\]  \hspace{1cm} (4.17)

where \( J \) is the Jacobian matrix \( [\frac{\partial f^k}{\partial x^k}] \) evaluated at \( x^k \).

In the Quasi-Newton iteration the sequence of approximation to \( x^* \) is given by a formula analogous to equation (4.17):

\[
x^{k+1} = x^k + \alpha^k
\]  \hspace{1cm} (4.18)

The displacement \( \alpha^k \) is given by

\[
\alpha^k = \alpha^k S^k
\]  \hspace{1cm} (4.19)

where \( \alpha^k \) is a scalar and the vector \( S^k \) is expressed in terms of the residuals by

\[
S^k = -H^k f^k
\]  \hspace{1cm} (4.20)

The matrix \( H \) represents the \( k \)th approximation to \( J^{-1} (x^*) \). It is clear from these equations that the inclusion of \( \alpha^k \) is not strictly
necessary as it could well be absorbed in the matrix $H^k$. It is convenient to treat it as a separate entity, however, since in practice its value is often not known until the correction vector $S^k$ has been computed.

It is necessary to develop a procedure for generating a sequence of approximations to $J^{-1}(x)^*$ starting with some initial estimate $H^1$. The equation can be written

$$H^{k+1} = H^k + C^k$$

(4.21)

The matrix $C^k$ is determined by the requirement that for the case of linear equations $H^{k+1}$ should be effectively $J^{-1}$:

$$f(x) = Jx - a$$

(4.22)

Since for linear system of equations

$$J^{-1}\Delta f^i = \sigma^i$$

(4.23)

Where

$$\Delta f^i = f^{i+1} - f^i$$

And

$$H^{k+1} \Delta f^i = \sigma^i$$

(4.24)

$$i = 1, 2, \ldots, k$$

Eliminating $H^{k+1}$ from equations (4.21) and (4.24)

$$C^k \Delta f^i = \sigma^i - H^k \Delta f^i$$

$$k = 1, 2 \ldots i$$

Setting $i = k$ we have

$$C = \left[ \frac{\sigma^k [u^k]^T}{(u^k, \Delta f^k)} - \frac{H^k \Delta f^k [u^k]^T}{(u^k, \Delta f^k)} \right] M$$

(4.25)

The proof of equation (4.25) is given in detail in 29, 30, 31. The
vectors $u^k$ and $v^k$ are still undetermined and the matrix $M$ is independent of $x$, Hermitian and either positive or negative definite. Because $H_i \Delta \xi_i = \sigma_i$, this requires that
\[ C^k \Delta \xi_i = 0 \quad i = 1, 2 \ldots k-1 \] (4.26)
This requirement can be satisfied by the form of equation (4.25) if the vectors $u$ and $v$ are chosen so that they are orthogonal to the subspace spanned by $\Delta \xi_i$ (i<k), i.e.:
\[ (u^k, \Delta \xi_i) = (v^k, \Delta \xi_i) = 0 \quad (i<k) \] (4.27)
The convergence of this iteration process can easily be determined. Since the space of the residuals is an n dimensional space, the index $k$ in equation (4.27) can not exceed n because there are at most n linearly independent vectors in the space.

Determination of the $u^k$ and $v^k$ vectors:
The fact that the vectors $u^k$ and $v^k$ need only be orthogonal to the (k-1) dimensional space spanned by $\Delta \xi_i$ (i<k) clearly gives a great deal of flexibility to the algorithm. Widely different iterations could be obtained for different selections of $u^k$ and $v^k$. One possible simplification that could be made to take $u^k$ equal to $v^k$. This choice reduces equation (4.25) to
\[ C^k = (\sigma^k - H^k \Delta \xi_k) \frac{(u^k)^T M}{(u^k, \Delta \xi_k)} \] (4.28)
Broyden's iteration uses (4.28) to calculate $C_k$, but with a choice $u^k = \Delta \xi^k$ to get
\[ C^k = (\sigma^k - H^k \Delta \xi_k) \frac{(\Delta \xi_k)^T M}{(\Delta \xi_k, \Delta \xi_k)} \] (4.29)
This iteration suffers from the problem that $\Delta x^k$ will not in
general be orthogonal to $\Delta f^i$ (i<k). This difficulty can be partially
circumvented by not using $\Delta x^k$ for $u^k$ but rather that part of $\Delta x^k$
which is orthogonal to the subspace. This is only successful when
$\Delta x^k$ itself does not lie entirely within the subspace.

Prevention of Divergence:

Although it is not possible to guarantee convergence, divergence
may be prevented by ensuring that some norm of the residual vector
is nonincreasing. This may be achieved either by norm minimization
or by norm reduction strategy. Norm minimization is perhaps the
most obvious choice to make, since it gives the greatest immediate
reduction of the norm. However, in order to do this the vector function
$f$ needs to be evaluated a number of times and this means an increase
in the amount of computation required compared with the alternative
strategy of choosing the value of $\alpha^i$ which merely reduces the norm.
The algorithm for minimization of the Euclidean norm of the residual
vector as a function of the single variable $\alpha$ is given in detail by
Broyden.\textsuperscript{31}

Summary of the Algorithm:

The following calculation steps are required for solving a system
of non-linear equations by the Quasi-Newton method:

1. Obtain an initial estimate $x^0$ of the solution.

2. Obtain an initial value of $H^0$ of the iteration matrix. This
may be obtained either by computing and inverting the Jacobian matrix,
or it is possible that a sufficiently good approximation may be
available from some other source, e.g. the final value of $H^i$ from the
previous calculations.

3. Compute \( f^i = f(x^i) \).

4. Compute \( a^i = H^i f^i \).

5. Select a value of \( a_i \) of \( a \) such that the norm of \( f(x^i + a_i \delta^i) \) is less than the norm of \( f(x^i) \). This is done iteratively. The procedure is given in the Reference 31. In the course of this calculation the following will have to be computed.

\[
x^{i+1} = x^i + a_i \delta^i
\]

\[
f^{i+1} = f(x^{i+1})
\]

6. Test \( f^{i+1} \) for convergence. If the iteration has not converged, then

7. Compute \( \Delta f^i = f^{i+1} - f^i \).

8. Compute \( H^{i+1} = H^i + C^i \).

9. Repeat from step 4 until \( ||f^{i+1}|| < \varepsilon \) where \( \varepsilon \) is the tolerance used. The main two drawbacks observed in this method are

(a) Computer storage is much higher than any other existing method for load-flow solutions.

(b) It is not possible to write a general programme to calculate the vectors \( u \) and \( v \) orthogonal to the subspace spanned by the residual vector.

It was found that for small systems the total execution time was slightly less than the standard Newton method, but because of the above drawbacks the application to a practical power system network does not appear too profitable.

4.6. Conclusions

In this chapter three modified forms of Newton's method have
been investigated. The main features of the modified techniques can be summarized as follows:

(a) Method No. 1 incorporates a successive overrelaxation method instead of the Gaussian elimination approach.

(b) Method No. 2 retains the coefficient matrix from the base load-flow study for the normal network and uses it for the repeated load-flow studies.

(c) Method No. 3 presents a technique for eliminating the necessity of inverting the Jacobian matrix at each iteration.

A method for modifying the inverse of the Jacobian matrix is also presented. The advantages of the 'row by row' matrix inversion process are many from the practical applications point of view. The modifications noted above are relatively slight and can be easily incorporated into present standard Newton programmes for load-flow study. Some comparisons are made in the next chapter between different techniques for load-flow analysis as a result of numerical experiments.
5. RESULTS AND COMPARISONS

5.1. General

The criteria by which the merit of any algorithm may be assessed are the accuracy of the solution obtained, the time taken to reach this solution and the amount of computer storage space used in the computation. Frequently, these criteria are in conflict with each other and therefore some assumptions must be made before it is possible to give an overall comparison of different algorithms. In the comparisons made in this chapter the storage space requirements are omitted completely, computation time, the number of iterations required and the accuracy of different methods only are considered.

In order to compare the performance of the various methods under as nearly identical conditions as possible they were used to analyze two given power systems starting each method from the same initial conditions. It was not possible to terminate the calculations at the same point in each case as the computational procedure varied with each method. The accuracy of the final solutions are measured by a prespecified tolerance from $10^{-3}$ to $10^{-6}$. When load-flow analysis is used as a part of composite system reliability studies then the comparison of the different methods must include the following points:

(a) Iterative solution time

(b) The computing time required to process the system input data in order to obtain the parameters for the iterative calculation.

(c) The computing time required to modify the network data and to effect system operating changes.
The computer time required to perform the iterative solution also depends on the
(a) rate of convergence of the solution technique.
(b) number of logical and arithmetic operations required to complete an iteration.
(c) size and characteristics of the power system.

5.2. Power Systems

Various power systems were studied using all the methods discussed in the previous chapters. All the programmes for load-flow analysis were first developed and tested on the five bus hypothetical system shown in Figure 5.1. The developed load-flow programmes were then used to perform power-flow studies on the Saskatchewan Power Corporation 20-bus simplified system shown in figure 5.2. The data for load-flow studies are given in Tables 5.1, 5.2, 5.3 and 5.4. The Queen Elizabeth plant or bus number 1 was taken as the swing bus. The optimum acceleration factors for the real and imaginary voltage components were taken as 1.6 and 1.8 respectively. As shown in figure 5.2 there are 8 generating buses and 12 load buses in this system.

In the studies of this type, the convergence rate can be increased by neglecting the shunt capacitive susceptance components at the various buses. This tends to increase the diagonal dominance of the nodal admittance matrix. Neglecting the shunt capacitive susceptance may, however, not give sufficiently accurate results.

5.3. Accuracy of Solutions

A considerable reduction in the computer time was obtained by
using the developed techniques. One characteristic of the Newton method is that the rate of convergence increases as the solution is approached. Using a tolerance of $10^{-4}$ it is quite possible in the final iteration to reduce the norm of the residual vector by as much as a factor of $10^{-3}$. Although the iteration is terminated immediately, the residual vector is less than $10^{-4}$ and the final mismatch may be as low as $10^{-6}$. In order to check the accuracy of the solutions obtained by different techniques, the mismatch of the real and reactive bus powers for a particular tolerance were compared. This comparison gives the relative accuracy of the solutions obtained.

5.4. Acceleration for Convergence

There are many ways of increasing the rate of convergence. In the latter part of this thesis some parameter selections for load-flow study using function minimization techniques are discussed. These parameters may be used to accelerate an iterative scheme. Two types of acceleration factors have been previously noted in this thesis. In summary, they are as follows:

(i) In the Gauss-Seidel method each voltage is accelerated at the end of each iteration by a linear acceleration factor $\alpha$. This factor may be different for the real part and for the imaginary part of the voltage. In this method the difference between the value of the voltage component after each iteration and that value before iteration is multiplied by $\alpha$ and then added to the value of the component before iteration to get the modified value. When $\alpha = 1$, there is no acceleration.
(ii) To accelerate the successive overrelaxation method, the optimum relaxation factor is calculated using the Power method. Here only one acceleration factor is used to increase the convergence rate of the solution. The optimum relaxation factor found for the 5-bus hypothetical system is 1.3 and for the 20-bus Saskatchewan Power Corporation System is 1.73.

The development of acceleration techniques is a subject of much research in the field of numerical analysis, and several comprehensive treatments of iteration methods and convergence theorems are available. Several of these techniques have been used for load-flow studies. The most successful technique with a Gauss-Seidel programme is to accelerate each voltage, at each iteration, by linear acceleration factors. In the successive overrelaxation approach the acceleration factors are modified abruptly or gradually during the iteration cycle.

The acceleration technique appears to be most widely used in load-flow studies today. No complete studies are documented in the literature however on the convergence characteristics for different acceleration factors and techniques. A subsequent part of this thesis illustrates that the Newton method for load-flow solution can be accelerated using certain parameters in the iteration process.

5.5. System Results

Load-flow studies were conducted on the system shown in figure 5.1 and 5.2 using the various techniques previously illustrated. The results of these studies are shown on the following pages.
5-bus system of figure 5.1 is a slightly modified version of the system shown in Figure 2.3 in chapter 2. Bus number 1 was taken as the swing bus in these studies. In the simplified 20-bus representation of the Saskatchewan Power Corporation System shown in Figure 5.2, bus numbers 1 through 8 were used as generating nodes with bus number 1 as the swing node. The results obtained using the conventional Gauss-Seidel techniques are shown in tables 5.6A and B. The data used for load-flow analysis are given in tables 5.1, 5.2, 5.3 and 5.4.

Figure 5.1

A 5-Bus Hypothetical System
20-BUS SASKATCHEWAN POWER CORPORATION SYSTEM

Figure S.2
TABLE 5.1

20-Bus System

Line data for the simplified Saskatchewan Power Corporation System shown in figure 5.2. Lines are assumed to be 795 ACSR 54/7

Base MVA = 100

Base KV = 110

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<tr>
<th>Line #</th>
<th>Length Miles</th>
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<th>Susceptance (b/2)</th>
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<td>0.0 + j0.1150</td>
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### TABLE 5.2
**Generation Data**

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<th>Capacity of each unit MW</th>
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<th>Voltage p.u.</th>
<th>Angle</th>
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<td>1.05</td>
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*Initial estimate for generator bus voltage phase angle = 0°.*
### TABLE 5.3

Load Data

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<th>Bus No.</th>
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<th>Generation Allocated Under Peak Load</th>
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<th>Angle</th>
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<td>101.5</td>
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<td>0</td>
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Initial estimate for bus voltage phase angle = 0°.
TABLE 5.4

Transformers and Bus Reactors:

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<tr>
<th>Transformers</th>
<th>From Bus</th>
<th>To Bus</th>
<th>Tap Ratio</th>
<th>Tap Side Bus No.</th>
<th>Bus No.</th>
<th>Reactors $Y_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1, T_2$</td>
<td>1</td>
<td>11</td>
<td>1.025</td>
<td>11</td>
<td>1</td>
<td>0.033</td>
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<tr>
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<td>1</td>
<td>5</td>
<td>0.080</td>
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<tr>
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<td>2</td>
<td>1.010</td>
<td>9</td>
<td>10</td>
<td>0.043</td>
</tr>
<tr>
<td>$T_6$</td>
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<td>2</td>
<td>1.015</td>
<td>1</td>
<td>19</td>
<td>0.043</td>
</tr>
<tr>
<td>$T_{22}, T_{23}$</td>
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<td>10</td>
<td>1.015</td>
<td>10</td>
<td></td>
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</tr>
<tr>
<td>$T_{24}, T_{25}$</td>
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<td>10</td>
<td>1.025</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{12}, T_{30}$</td>
<td>11</td>
<td>19</td>
<td>1.025</td>
<td>11</td>
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<td></td>
</tr>
<tr>
<td>$T_{21}$</td>
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<td>14</td>
<td>1.015</td>
<td>12</td>
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<td>$T_{29}$</td>
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<td>5</td>
<td>1.010</td>
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</table>

TABLE 5.5

Summary of the methods of formulation and solution of the load-flow problem

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<tr>
<th>Power System</th>
<th>5-Bus System</th>
<th>20-Bus S.P.C. System</th>
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<tbody>
<tr>
<td>Solution Technique</td>
<td>Gauss-Seidel</td>
<td>Gauss-Seidel</td>
</tr>
<tr>
<td>*Newton with S.O. method</td>
<td>Newton-Raphson</td>
<td>New Acctd. Newton</td>
</tr>
<tr>
<td>Quasi-Newton</td>
<td>*Newton with S.O. method</td>
<td>-</td>
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<tr>
<td>Approx. Newton</td>
<td>Approx. Newton</td>
<td>Constant Jacobi method</td>
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<tr>
<td>Constant Jacobi method</td>
<td>Constant Jacobi method</td>
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</tr>
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* S.O. - Successive Overrelaxation
### TABLE 5.6A

Load-flow solution by the conventional Gauss-Seidel method.

5-Bus System

<table>
<thead>
<tr>
<th>Bus No</th>
<th>Voltage (p.u.)</th>
<th>Angle</th>
<th>Generator Watts</th>
<th>Generator Vars</th>
<th>Load Watts</th>
<th>Load Vars</th>
<th>Static Vars</th>
</tr>
</thead>
<tbody>
<tr>
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Losses - 0.051147 p.u.

Tolerance used - \(10^{-4}\)

Number of iterations 19

### TABLE 5.6B

Load-flow solution by the conventional Gauss-Seidel method

20-Bus System

<table>
<thead>
<tr>
<th>Bus No</th>
<th>Voltage (p.u.)</th>
<th>Angle</th>
<th>Generator Watts</th>
<th>Generator Vars</th>
<th>Load Watts</th>
<th>Load Vars</th>
<th>Static Vars</th>
</tr>
</thead>
<tbody>
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Continued Table 5.6B

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<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
<th>Generator Watts</th>
<th>Generator Vars</th>
<th>Load Watts</th>
<th>Load Vars</th>
<th>Static Vars</th>
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<td>1.11</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.542</td>
<td>-0.115</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Total Losses - 0.4283 p.u.
Tolerance used - 10^{-4}
Number of iterations - 49

The great importance of load-flow studies in planning future power system expansion as well as in determining the best operation of existing systems can not be overestimated. One new method and some modifications have been proposed as an extension of the Newton-Raphson method for use in composite system reliability studies. These techniques reduce the computation time and in some cases also reduce the memory requirement, yielding maximum computer efficiency and minimum labor in data handling. The results obtained using the different techniques for the 5-bus hypothetical system and the 20-bus practical system are given in Tables 5.7A to 5.12B.

The results shown in Tables 5.6A and B for the Gauss-Seidel technique can be considered as a standard of comparison in terms of
## COMPARISON OF DIFFERENT LOAD-FLOW TECHNIQUES

*System: 5-bus Hypothetical System*

### TABLE 5.7A

**Standard Newton-Raphson Method in Polar Co-ordinate Form**

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
<th>Generator</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Watts</td>
<td>Vars</td>
</tr>
<tr>
<td>1</td>
<td>1.050</td>
<td>0.0</td>
<td>0.5019</td>
<td>0.1511</td>
</tr>
<tr>
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<td>1.050</td>
<td>11.07</td>
<td>1.10</td>
<td>0.0978</td>
</tr>
<tr>
<td>3</td>
<td>1.0297</td>
<td>-0.30</td>
<td>0.0</td>
<td>-0.85</td>
</tr>
<tr>
<td>4</td>
<td>1.0217</td>
<td>-3.92</td>
<td>0.0</td>
<td>-0.40</td>
</tr>
<tr>
<td>5</td>
<td>1.0283</td>
<td>-4.92</td>
<td>0.0</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

### TABLE 5.8A

**Load-Flow solution by the Developed Accelerated Newton's Method in Polar Co-ordinate Form**

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
<th>Generator</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Watts</td>
<td>Vars</td>
</tr>
<tr>
<td>1</td>
<td>1.050</td>
<td>0.0</td>
<td>0.5017</td>
<td>0.1491</td>
</tr>
<tr>
<td>2</td>
<td>1.050</td>
<td>11.04</td>
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<td>-0.0967</td>
</tr>
<tr>
<td>3</td>
<td>1.0300</td>
<td>-4.31</td>
<td>0.0</td>
<td>-0.85</td>
</tr>
<tr>
<td>4</td>
<td>1.0269</td>
<td>-3.96</td>
<td>0.0</td>
<td>-0.40</td>
</tr>
<tr>
<td>5</td>
<td>1.0287</td>
<td>-4.95</td>
<td>0.0</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

### TABLE 5.9A

**Approximate Newton-Raphson Method**

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
<th>Generator</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Watts</td>
<td>Vars</td>
</tr>
<tr>
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<td>0.5012</td>
<td>0.1520</td>
</tr>
<tr>
<td>2</td>
<td>1.050</td>
<td>10.99</td>
<td>1.10</td>
<td>-0.1065</td>
</tr>
<tr>
<td>3</td>
<td>1.037</td>
<td>-4.33</td>
<td>0.0</td>
<td>-0.85</td>
</tr>
<tr>
<td>4</td>
<td>1.034</td>
<td>-4.00</td>
<td>0.0</td>
<td>-0.40</td>
</tr>
<tr>
<td>5</td>
<td>1.034</td>
<td>-4.97</td>
<td>0.0</td>
<td>-0.10</td>
</tr>
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</table>

### TABLE 5.10A

**Newton Method with Successive Overrelaxation**

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
<th>Generator</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Watts</td>
<td>Vars</td>
</tr>
<tr>
<td>1</td>
<td>1.050</td>
<td>0.0</td>
<td>0.5008</td>
<td>0.1521</td>
</tr>
<tr>
<td>2</td>
<td>1.050</td>
<td>11.06</td>
<td>1.10</td>
<td>-0.0982</td>
</tr>
<tr>
<td>3</td>
<td>1.0307</td>
<td>-4.30</td>
<td>0.0</td>
<td>-0.85</td>
</tr>
<tr>
<td>4</td>
<td>1.025</td>
<td>-3.94</td>
<td>0.0</td>
<td>-0.40</td>
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<tr>
<td>5</td>
<td>1.034</td>
<td>-4.96</td>
<td>0.0</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

### TABLE 5.11A

**Quasi-Newton Method**

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
<th>Generator</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Watts</td>
<td>Vars</td>
</tr>
<tr>
<td>1</td>
<td>1.050</td>
<td>0.0</td>
<td>0.5016</td>
<td>0.1513</td>
</tr>
<tr>
<td>2</td>
<td>1.050</td>
<td>11.05</td>
<td>1.10</td>
<td>0.0982</td>
</tr>
<tr>
<td>3</td>
<td>1.0308</td>
<td>-4.31</td>
<td>0.0</td>
<td>-0.85</td>
</tr>
<tr>
<td>4</td>
<td>1.0249</td>
<td>-3.94</td>
<td>0.0</td>
<td>-0.40</td>
</tr>
<tr>
<td>5</td>
<td>1.0293</td>
<td>-4.96</td>
<td>0.0</td>
<td>-0.100</td>
</tr>
</tbody>
</table>

### TABLE 5.12A

**Constant Jacobian Matrix Method**

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
<th>Generator</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Watts</td>
<td>Vars</td>
</tr>
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<td>1</td>
<td>1.050</td>
<td>0.0</td>
<td>0.5014</td>
<td>0.1562</td>
</tr>
<tr>
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<td>1.050</td>
<td>11.06</td>
<td>1.10</td>
<td>-0.0993</td>
</tr>
<tr>
<td>3</td>
<td>1.0359</td>
<td>-4.31</td>
<td>0.0</td>
<td>-0.85</td>
</tr>
<tr>
<td>4</td>
<td>1.0266</td>
<td>-3.96</td>
<td>0.0</td>
<td>-0.40</td>
</tr>
<tr>
<td>5</td>
<td>1.0362</td>
<td>-4.95</td>
<td>0.0</td>
<td>-0.20</td>
</tr>
</tbody>
</table>
### TABLE 5.7B

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>3.63</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>10.18</td>
</tr>
<tr>
<td>4</td>
<td>1.05</td>
<td>18.13</td>
</tr>
<tr>
<td>5</td>
<td>1.05</td>
<td>31.26</td>
</tr>
<tr>
<td>6</td>
<td>1.05</td>
<td>33.08</td>
</tr>
<tr>
<td>7</td>
<td>1.05</td>
<td>1.23</td>
</tr>
<tr>
<td>8</td>
<td>1.05</td>
<td>33.01</td>
</tr>
<tr>
<td>9</td>
<td>1.016</td>
<td>-3.44</td>
</tr>
<tr>
<td>10</td>
<td>1.053</td>
<td>21.88</td>
</tr>
<tr>
<td>11</td>
<td>1.092</td>
<td>-1.27</td>
</tr>
<tr>
<td>12</td>
<td>1.044</td>
<td>19.60</td>
</tr>
<tr>
<td>13</td>
<td>1.035</td>
<td>23.94</td>
</tr>
<tr>
<td>14</td>
<td>1.038</td>
<td>18.69</td>
</tr>
<tr>
<td>15</td>
<td>1.034</td>
<td>-2.49</td>
</tr>
<tr>
<td>16</td>
<td>1.030</td>
<td>14.94</td>
</tr>
<tr>
<td>17</td>
<td>1.204</td>
<td>31.31</td>
</tr>
<tr>
<td>18</td>
<td>0.989</td>
<td>-6.14</td>
</tr>
<tr>
<td>19</td>
<td>1.043</td>
<td>-1.99</td>
</tr>
<tr>
<td>20</td>
<td>0.953</td>
<td>1.13</td>
</tr>
</tbody>
</table>

### TABLE 5.8B

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>3.63</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>10.16</td>
</tr>
<tr>
<td>4</td>
<td>1.05</td>
<td>18.12</td>
</tr>
<tr>
<td>5</td>
<td>1.05</td>
<td>31.07</td>
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</table>

### TABLE 5.9B

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>3.63</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>10.18</td>
</tr>
<tr>
<td>4</td>
<td>1.05</td>
<td>18.17</td>
</tr>
<tr>
<td>5</td>
<td>1.05</td>
<td>31.04</td>
</tr>
<tr>
<td>6</td>
<td>1.05</td>
<td>33.12</td>
</tr>
<tr>
<td>7</td>
<td>1.05</td>
<td>1.23</td>
</tr>
<tr>
<td>8</td>
<td>1.05</td>
<td>33.08</td>
</tr>
<tr>
<td>9</td>
<td>1.05</td>
<td>-3.44</td>
</tr>
<tr>
<td>10</td>
<td>1.05</td>
<td>22.0</td>
</tr>
<tr>
<td>11</td>
<td>1.095</td>
<td>-1.31</td>
</tr>
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<td>12</td>
<td>1.051</td>
<td>19.35</td>
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<td>1.039</td>
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<td>1.043</td>
<td>18.54</td>
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<td>15</td>
<td>1.034</td>
<td>-2.45</td>
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<td>16</td>
<td>1.036</td>
<td>14.85</td>
</tr>
<tr>
<td>17</td>
<td>1.204</td>
<td>31.33</td>
</tr>
<tr>
<td>18</td>
<td>0.994</td>
<td>-6.02</td>
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<tr>
<td>19</td>
<td>1.052</td>
<td>-2.05</td>
</tr>
<tr>
<td>20</td>
<td>0.958</td>
<td>1.12</td>
</tr>
</tbody>
</table>
COMPARISON OF DIFFERENT LOAD-FLOW TECHNIQUES

System: Saskatchewan Power Corporation

**TABLE 5.10B**

Newton with Successive Overrelaxation

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>3.63</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>10.18</td>
</tr>
<tr>
<td>4</td>
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<td>18.19</td>
</tr>
<tr>
<td>5</td>
<td>1.05</td>
<td>31.06</td>
</tr>
<tr>
<td>6</td>
<td>1.05</td>
<td>33.08</td>
</tr>
<tr>
<td>7</td>
<td>1.05</td>
<td>-1.20</td>
</tr>
<tr>
<td>8</td>
<td>1.05</td>
<td>33.09</td>
</tr>
<tr>
<td>9</td>
<td>1.016</td>
<td>-3.49</td>
</tr>
<tr>
<td>10</td>
<td>1.049</td>
<td>22.02</td>
</tr>
<tr>
<td>11</td>
<td>1.094</td>
<td>-1.22</td>
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<td>19.70</td>
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<td>1.038</td>
<td>24.05</td>
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<td>1.042</td>
<td>18.94</td>
</tr>
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<td>-2.50</td>
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<td>15.01</td>
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<td>31.12</td>
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<td>1.060</td>
<td>-1.99</td>
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<tr>
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</table>

**TABLE 5.12B**

Constant Jacobi Matrix

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>3.63</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>10.17</td>
</tr>
<tr>
<td>4</td>
<td>1.05</td>
<td>18.29</td>
</tr>
<tr>
<td>5</td>
<td>1.05</td>
<td>31.26</td>
</tr>
<tr>
<td>6</td>
<td>1.05</td>
<td>33.08</td>
</tr>
<tr>
<td>7</td>
<td>1.05</td>
<td>1.23</td>
</tr>
<tr>
<td>8</td>
<td>1.05</td>
<td>33.14</td>
</tr>
<tr>
<td>9</td>
<td>1.016</td>
<td>-3.44</td>
</tr>
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<td>1.053</td>
<td>21.88</td>
</tr>
<tr>
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<td>1.098</td>
<td>-1.17</td>
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<td>1.043</td>
<td>19.60</td>
</tr>
<tr>
<td>13</td>
<td>1.035</td>
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<td>31.57</td>
</tr>
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<td>18</td>
<td>0.985</td>
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</tr>
<tr>
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<td>1.93</td>
</tr>
<tr>
<td>20</td>
<td>0.948</td>
<td>1.13</td>
</tr>
</tbody>
</table>
solution accuracy. It can be seen that the results are basically identical in the Tables 5.7A and B obtained using the standard Newton-Raphson method. The results shown in each of the other cases are not vastly different in terms of the bus voltages and phase angles. As these are the principal parameters required in composite reliability evaluation the other small discrepancies are not important. The approximate solutions are shown in Tables 5.7A to 5.12B.

The results were obtained using optimum acceleration factors and a tolerance of $10^{-4}$ in the bus voltage and bus powers in the Gauss-Seidel and Newton method respectively. The effect of the voltage tolerance value on the maximum bus power mismatch was also studied and the effect is illustrated in Figure 5.3. From a practical point of view the bus voltage and power values are sufficiently accurate at a tolerance of $10^{-4}$.

The solution time and the number of iterations required in each case are summarized in Tables 5.15 and 5.14. The time indicated is the execution time in each case. The optimum acceleration factors are also shown. It can be seen that both the number of iterations and the time per iteration is lower in the modified Newton and the constant Jacobi method than in the standard Newton-Raphson technique. The effect on the number of iterations of the varying the acceleration factors in the Gauss-Seidel and Newton with successive overrelaxation are illustrated in Figures 5.4 and 5.5. It can clearly be seen that the selection of an optimum acceleration factor is an important element in minimizing the problem solution time.
### TABLE 5.13
Comparative Results of 5-Bus System

<table>
<thead>
<tr>
<th>Methods</th>
<th>Tolerance</th>
<th>Time (Sec.)</th>
<th>No. of Iter.</th>
<th>Optimum Accln. factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss-Seidel</td>
<td>$10^{-4}$</td>
<td>2.72</td>
<td>19</td>
<td>1.2, 1.5</td>
</tr>
<tr>
<td>Newton-Raphson</td>
<td>$10^{-4}$</td>
<td>2.00</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>Approx. Newton</td>
<td>$10^{-4}$</td>
<td>2.58</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>New Accl. Newton</td>
<td>$10^{-4}$</td>
<td>1.57</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>Constant Jacobian</td>
<td>$10^{-4}$</td>
<td>0.98</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Newton with S. Overrelaxation</td>
<td>$10^{-4}$</td>
<td>1.92</td>
<td>16</td>
<td>1.3</td>
</tr>
<tr>
<td>Quasi-Newton</td>
<td>$10^{-4}$</td>
<td>2.45</td>
<td>14</td>
<td>-</td>
</tr>
</tbody>
</table>

### TABLE 5.14
Comparative Results of 20-Bus System

<table>
<thead>
<tr>
<th>Methods</th>
<th>Tolerance</th>
<th>Time (Sec.)</th>
<th>No. of Iter.</th>
<th>Accln. factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss-Seidel</td>
<td>$10^{-4}$</td>
<td>68.0</td>
<td>49</td>
<td>1.6, 1.8</td>
</tr>
<tr>
<td>Newton-Raphson</td>
<td>$10^{-4}$</td>
<td>56.0</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>Approx. Newton</td>
<td>$10^{-4}$</td>
<td>97.2</td>
<td>37</td>
<td>-</td>
</tr>
<tr>
<td>New Accl. Newton</td>
<td>$10^{-4}$</td>
<td>32.28</td>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>Constant Jacobian</td>
<td>$10^{-4}$</td>
<td>13.56</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>Newton with S. Overrelaxation</td>
<td>$10^{-4}$</td>
<td>46</td>
<td>50</td>
<td>1.73</td>
</tr>
<tr>
<td>Quasi-Newton</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 5.3

Effect of Tolerance on Bus Power
Figure 5.4

Convergence characteristics: Gauss-Seidel Method
20-Bus System

5-Bus System

Total number of Iterations

Overrelaxation factor $\omega$

Figure 5.5

Convergence characteristics: Successive Overrelaxation Method
The major portion of the work has been numerical experimental in nature. The application to only two different systems can not be considered to draw definite conclusions regarding the best technique. Additional analytical work is required on the convergence characteristics of each method under the possible system types. On the average there are about three lines per bus in the 20-bus system of Figure 5.2. Three complex multiplications are therefore required, together with the current calculation to obtain each new estimate of bus voltage using the admittance matrix in the Newton method. The convergence of the basic Newton-Raphson method can be further accelerated using optimally ordered elimination as proposed by Tinney.15

Further effort and experience will definitely result in substantial improvement in the techniques developed. These methods will result in considerable reduction in the computation time required for repeated load-flow studies. It may be necessary, however, to use the programmes developed on an actual production basis in order to obtain the experience necessary for the development of further improvements.

5.6. Conclusions

The power-flow analysis problem contains two basic aspects; the solution of the network equations and the satisfaction of certain nodal constraints governing the power and voltage requirements at each bus. The solution which simultaneously satisfies these two aspects can only be obtained by an iterative procedure. The main computing effort is required in the solution of the network equations.
The form of this part of the calculation distinguishes the various approaches.

The following conclusions were drawn from the studies conducted.

(a) In the case of the Gauss-Seidel and overrelaxation techniques, every system has an optimum acceleration factor. This value depends mainly on the system size, the ratio of the number of generator buses to load buses, the stabilizing effect of generator buses, the choice of the swing bus and the effect of shunt capacitances present in the system.

(b) The convergence of the iteration leading to the final value depends upon factors such as the choice of the swing bus, the coding of buses and the choice of initial estimates.

(c) The effect of the acceleration factor on the convergence rate is quite pronounced. The rate for a given acceleration factor will vary with each iteration. The variability in convergence rate as a function of the number of iterations is dependent upon the solution technique used.

(d) The effect of different tolerance values on the solution accuracy for bus and line flows is illustrated in Figure 5.3. A tolerance factor of $10^{-4}$ provides desired bus and line flow accuracy in conventional load flow studies. This may, however, not be sufficient for detailed loss studies. It must be appreciated that a reduction in the number of iterations is not the complete answer to the problem of reducing computing time. Two other factors must be considered; the time per iteration and the data preparation time. The time per iteration is directly dependent upon the numerical technique but it is also influenced
considerably by the quality of programming. This is particularly true when the program or technique can take full advantage of any zeros in the coefficient matrix or the system admittance matrix.

The principal object of the load-flow development described in this thesis has been towards their application to repeated load-flow calculations rather than single case studies. Many of the conclusions, however, are applicable to all forms of load-flow problems. The application of the concepts to repeated load-flow requirements of composite reliability studies are illustrated in detail in the next chapter.
6. COMPOSITE SYSTEM RELIABILITY STUDIES

6.1. General

During the past decade there has been considerable emphasis placed upon the development of analytical techniques for evaluating reliable and economic system operation. There are many publications available in the power system reliability area and in certain aspects of system planning and operating some of the techniques are becoming standard utility practice. This is particularly true in the generation area where the "Loss of load Approach" has become a basic technique. It is only in recent years, however, that those methods have been extended into the transmission network. A numerical method for evaluating composite system reliability was proposed by Billinton\textsuperscript{3}. This technique requires multiple load-flow studies for the different system component outage conditions. In this chapter, the load-flow techniques previously presented in this thesis are illustrated in terms of their application to the composite reliability evaluation problem.

6.2 Composite System Reliability Concept

The use of probability methods in generating capacity and transmission system reliability evaluation provides quantitative indices for planning future system requirements. It has been observed\textsuperscript{33} that there are many more publications dealing with generation capacity reliability evaluation than with transmission system reliability evaluation. The published material available in the area of composite system reliability studies is very limited.
The main object of such a study is to provide an overall reliability assessment of a composite power system. The use of the term composite indicates that both generation and transmission facilities are included in the analysis. Reliability indices are obtained at various points in the network and for different load levels.

If the transmission system is completely redundant then the load point reliability can be considered strictly in terms of continuity and success and failure is relatively easy to define. Desieno and Stine suggested Boolean algebraic approach for determining power system reliability with respect to a load point. The method however becomes very complicated for a system with several generating nodes. The probability of outage at a load point is not an acceptable single criterion. The concepts developed in connection with the Frequency and Duration Approach to generating capacity reliability evaluation have been used in the composite system technique to provide an additional frequency index at the load points. This provides a more physical index of load point reliability and one that is more acceptable to the utility industry.

Service quality criterion:

In order to obtain meaningful indices of load point reliability both the transmission and generation parameters must be introduced into the analysis. The technique which has been developed uses a conditional probability approach to evaluate the load point reliability. This approach also introduces the idea of a service quality standard as the reliability criterion rather than simple continuity between a generating source and the load point. If due to the outage of a system component,
the bus voltage at any load point falls below a specified minimum value than this is defined as a breach of continuity. Any transient voltage departure is not included in the breach of quality criterion unless it remains in the unacceptable region for a long time.

Before proceeding with the determination of load point reliability indices it is important to consider the required service quality at the various load points. This allocation is determined by considering the quality requirements necessary to achieve customer satisfaction. This is particularly important when considering system expansion planning using a consistent quantitative reliability index. It is obvious that composite system reliability evaluation using a service quality criterion involves more effort and analysis than simply breach of continuity as proposed by other authors.

Conditional Probability Approach:

In developing the composite system reliability approach, the experience and ideas gained from generation capacity and transmission reliability have been utilized. The reliability of both generation and transmission systems based on an analysis of possible failure modes and failure rates are detailed in a recent publication. The failure modes of a composite power system are an extremely important part of any reliability evaluation.

In virtually all power system reliability studies, the component failures i.e. the outage of transmission lines, transformers and generators are assumed to be independent events. It has been shown that under each outage condition there is a maximum load that can be supplied at each load point without violating the service quality
criterion. If the load at any bus can be considered as a random variable then the probability of the load exceeding some value which will violate the service quality is given by the load probability distribution curve for that particular bus. This type of failure is, in a conditional sense, the probability of the load exceeding an acceptable value dependent upon the system generation and transmission conditions. It has been shown\(^{36}\) that the probability of the load at a bus exceeding the limiting value is obtained by combining the generation system reliability with the transmission system reliability. Both the generation and transmission outage conditions are considered to be independent events which may cause a bus failure. The composite system outage probability is given in equation (6.1).

Probability of failure at a bus:
\[
= \sum_j P(B_j)[P_{Gj} + P_{Lj} - P_{Gj}P_{Lj}]
\] (6.1)

where:

- \(P(B_j)\) is the probability of the existence of failure \(B_j\).
- \(B_j\) is the outage condition for the transmission system i.e. the lines and transformer.
- \(P_{Gj}\) is the probability of generating capacity exceeding the reserve capacity. For this a cumulative probability for the generating unit outage condition exceeding certain capacity levels is obtained from the capacity outage probability table.
- \(P_{Lj}\) is the probability of the load at a bus exceeding the maximum load that can be supplied at that bus without failure.

The outage \(B_j\) for the transmission network is obtained by assuming that the outage condition of each component of the system network is
independent. Environmental fluctuations may cause changes in the failure rates of transmission system components. If the transmission system reliability is evaluated over a large area then the effect of abnormal weather or other environmental changes become negligible.

The composite system reliability index at any node in the system can also be defined in terms of expected or average frequency of failure. It has been shown\(^3\) that the frequency of occurrence of an outage condition is equal to the product of the probability of existance of the outage and the rate of departure from that condition. Therefore, the expected frequency of failure at any bus j is given by:

\[
\text{Expected frequency of failure} = \sum_{j} F(B_j)[P_{G_j} + P_{L_j} - P_{G_j}P_{L_j}] \quad (6.2)
\]

where

\[F(B_j)\] is the frequency of occurrence of outage \(B_j\).

It has also been shown in a recent paper\(^{37}\) that the rates of departure associated with the load model states can be included in the analysis. This becomes very complicated for anything but a very simple system. If the individual generating units are considered together with the transmission lines and transformer to determine the outage condition \(B_j\) then the generation schedule used in the load-flow analysis does not require any extra modification. The individual outage conditions as described in equation (6.1) may increase considerably depending upon the number of generating units in the system. The assumption made in the previous case is that any breach of quality is due to a line or transformer outage or to the system load exceeding the total available generation. The equations for the probability of a bus
failure and expected frequency of failure including individual generating unit failures are:

Probability of a bus failure:

\[ P(B_j) = \sum_j P(B_j) \cdot PL_j \quad (6.3) \]

Expected frequency of failure at a bus:

\[ F(B_j) = \sum_j F(B_j) \cdot PL_j \quad (6.4) \]

where

\[ B_j \] is the outage condition including generating units, lines and transformers.

### 6.3 Digital Computer Program

Over the past few years, the application of digital computer techniques for power system reliability evaluation have been well established. In the following section, the main philosophy behind a digital computer programme for composite system reliability is discussed. Following this, some results obtained by using this programme with the different load-flow techniques developed are described. The original composite system reliability program was developed using the Gauss-Seidel technique for multiple load-flow solutions.

In applying equation (6.1) to (6.4) the most important requirement is the determination of the maximum load that can be satisfied at each load point. Several assumptions were made to simplify the computer program. The load variation at each bus was represented by a normalized load duration curve approximated by a single straight line as shown in Figure 6.1.
Load Probability Distribution at a Bus

Probability of the Load Exceeding the Indicated Value

Figure 6.1

Bus Failure at Load Levels

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>PL in Equations 6.1 to 6.4</td>
<td>1.0</td>
<td>0.25</td>
<td>0.0</td>
</tr>
</tbody>
</table>
A load bus is assumed to be failed under the following conditions:

(a) If a line or transformer supplying power to a load point is overloaded.
(b) The voltage at a bus is less than a specified minimum level.
(c) The available generating capacity is less than the total system load demand.

Some buses in the system may not satisfy the reliability requirement even though their voltage level is well within the acceptable limits under normal conditions. This situation arises when the components feeding the bus have relatively high outage rates. This criterion is discussed in more detail in a recent paper on transmission planning using a reliability criterion by Billinton and Bhavaraju.

The composite system reliability study programme creates the possible system component outage conditions and performs the load-flow analysis at selected load levels to test for bus failure. If for any outage condition $B_j$, a load bus fails at any load levels, $PL$ in equation (6.1) is taken as the average of the probability value of the load level at which the load bus has failed and of the previous lower level. $PL$ is assumed to be zero if the load bus does not fail even at the peak load level as shown in Figure 6.1. The value of $PG$ in equation (6.1) is the cumulative probability of the generating capacity exceeding the available static reserve. In all the studies conducted a maximum of two simultaneous outage conditions were considered. It has been found that the computational time for more than two simultaneous outages is much higher and the corresponding risk contribution for these higher outage conditions is normally negligible.
The flow-chart for the composite system reliability programme is given in the Appendix IV. If due to some outage condition, the load-flow solution does not converge within a specified number of iterations then the system is divided into different sub-systems with each generating bus supplying loads in the sub-system. The risk contribution is then calculated for each bus using the capacity outage probability table at the generating bus and a combined load duration model for the loads being supplied. If the isolated generating bus is the system swing bus then another swing bus is selected for load-flow analysis.

In the load-flow analysis if it is found that some lines or transformers are overloaded at any particular load level then they are assumed to be tripped automatically by the protective devices. The risk contribution is evaluated only if the base case load-flow solution for all the components in service is satisfactory. To perform the complete system reliability study it is necessary to call the load-flow subroutine several times. The load-flow solutions are obtained by the conventional Gauss-Seidel, Newton-Raphson and the newly developed techniques such as accelerated Newton's method, Constant Jacobi method and the successive overrelaxation method. The program developed for the different load-flow solution techniques are not very sophisticated and there is no provision for automatic tap changing. It has been found, however, that by accelerating the load-flow solution, the total computer time required for composite system reliability studies can be reduced considerably. Each load-flow solution is obtained with the generating bus voltages set to the
maximum possible limit, without violating the VAR limits.

In the reliability programme, the system loads are represented either as constant power or constant admittance models. A constant admittance system increases the diagonal terms of the nodal admittance matrix and a significant saving in the total solution time is obtained. The admittance for this model is computed from the constant power load as specified in the data using the bus voltages obtained from the base case load-flow study. The loads can also be represented as constant power models for voltage above a specified minimum level and as constant admittance loads if the voltage falls below the specified minimum. Detailed study results are given in the following sections.

6.4. Data Requirements

The data required for a composite system reliability study is as follows:

(a) All the necessary data for a normal load flow solution.
(b) Line and transformer failure, repair and overload parameters.
(c) At each bus, the normalized load duration curve approximated to a straight line, the per unit constant power load representation, the number and capacity of the available generating units, the generation schedule at the different levels of system load with the variation approximated to a straight line, the static capacitance, the estimated voltages and voltage limits, and the reactive power generation limits.
(d) Other data:

The bus voltage limit below which the load should be modified
to a constant admittance representation, the increments for rounding the capacity outage probability tables; the probability steps by which the system load level should be increased from the minimum level to the peak load level; the data for dividing the system into subsystems when the load-flow solution does not converge for an outage condition; and the maximum number of outages to be considered.

The failure and repair parameters of the individual components are extremely important and should be obtained from the system operating records. The mechanism for obtaining this data must be consistent and the data must be analysed and interpreted in terms of its future use. Basic definitions for failure and repair parameters are available in the published literature.

6.5. Study of a Hypothetical System

The simple hypothetical system shown in Figure 5.1 was studied using the different load-flow techniques to evaluate the load point risk levels. Values were obtained for the two indices represented by equations (6.1) and (6.2). The reliability indices of this system were examined for changes in the bus voltage limits, addition of selected line elements, load model variation and the effect of representing the generating units as individual elements. The system data for the composite reliability study is given in Appendix III.

The risk levels obtained using the different load-flow techniques are shown in Table 6.1. The load-flow techniques used were the conventional Gauss-Seidel approach, the Newton-Raphson method and all the modified techniques developed as noted earlier in this thesis.
In the first study case, the load model was approximated by a straight line from the 100% to the 40% load by 10 equal probability steps. The maximum acceptable bus voltage in this case was 1.05 p.u. and the minimum acceptable limit 0.97 p.u. The load model and the lower voltage limits were varied in some of the subsequent studies as shown in Table 6.1 with the exception of case 2 all the studies shown in Table 6.1 were conducted for a maximum of two simultaneous independent outages.

The reliability indices were also obtained using equations (6.3) and (6.4) as shown in case 8 of Table 6.1. The risk level at bus number 2 is extremely low in case 8. This is partially due to using a maximum of two simultaneous independent outages and also to the fact that this is a large source of system generation.

A significant saving in computer time occurs when only single outage conditions are considered. The risk levels obtained at the different load points in this case are almost equal to the values obtained by considering generation adequacy only. A single outage approach may not be a valid indicator of system reliability if the system is designed using a single contingency criterion. Case 9 of Table 6.1 shows the effect on the reliability indices of reducing the number of steps in the load probability model. Any possible reduction in load model step will result in savings in computer solution time but the accuracy of the results will also decrease. These studies shown in Table 6.1 were conducted using the load-flow technique previously described. It was found that the reliability indices were basically the same for each load-flow technique when a relatively
tight tolerance value was used. As noted in chapter 5, a tolerance of 0.0001 in both Gauss-Seidel and Newton based methods appears to be adequate for this purpose.

6.6. Results

Risk levels for loads in the system shown in Figure 5.1.

Normal case: Line 1 to 6 in service, maximum of two simultaneous independent outages are considered; load distribution is approximated to 10 steps; load represented as constant admittance, minimum acceptable voltage at all buses: 0.97 per unit; maximum acceptable voltage at generating buses is 1.05 per unit, equation (6.1) and (6.2) used.

TABLE 6.1

Load-flow techniques used: Gauss-Seidel, Newton-Raphson, Accelerated Newton, Const. Jacobi and Successive Overrelaxation

<table>
<thead>
<tr>
<th>Changes</th>
<th>Bus #2</th>
<th>Bus #3</th>
<th>Bus #4</th>
<th>Bus #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.000062</td>
<td>0.001351</td>
<td>0.000937</td>
<td>0.8060</td>
</tr>
<tr>
<td>Single Outage</td>
<td>0.000062</td>
<td>0.0013</td>
<td>0.000903</td>
<td>0.7470</td>
</tr>
<tr>
<td>Load as Constant Power</td>
<td>0.000063</td>
<td>0.0025</td>
<td>0.00150</td>
<td>1.3067</td>
</tr>
<tr>
<td>Minimum Voltage Level</td>
<td>0.000062</td>
<td>0.0016</td>
<td>0.00415</td>
<td>3.670</td>
</tr>
<tr>
<td>1.02 p.u.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 6.1 continued

<table>
<thead>
<tr>
<th>Changes</th>
<th>Bus #2</th>
<th>Bus #3</th>
<th>Bus #4</th>
<th>Bus #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>From the Case normal</td>
<td>Proba-bility</td>
<td>Expect</td>
<td>Proba-bility</td>
<td>Expect</td>
</tr>
<tr>
<td></td>
<td>freq.</td>
<td>freq.</td>
<td>freq.</td>
<td>freq.</td>
</tr>
<tr>
<td>3(b) (</td>
<td>V</td>
<td>=1.01\text{pu})</td>
<td>0.000062</td>
<td>0.0015</td>
</tr>
<tr>
<td>3(c) (</td>
<td>V</td>
<td>=1.0\text{pu})</td>
<td>0.000062</td>
<td>0.0015</td>
</tr>
<tr>
<td>3(d) (</td>
<td>V</td>
<td>=0.99\text{pu})</td>
<td>0.000062</td>
<td>0.0013</td>
</tr>
<tr>
<td>3(e) (</td>
<td>V</td>
<td>=0.95\text{pu})</td>
<td>0.000062</td>
<td>0.0013</td>
</tr>
<tr>
<td>3(f) (</td>
<td>V</td>
<td>=0.93\text{pu})</td>
<td>0.000062</td>
<td>0.0013</td>
</tr>
<tr>
<td>3(g) (</td>
<td>V</td>
<td>=0.92\text{pu})</td>
<td>0.000062</td>
<td>0.0013</td>
</tr>
<tr>
<td>4 Line 6 removed</td>
<td>0.000064</td>
<td>0.0043</td>
<td>0.005108</td>
<td>4.5116</td>
</tr>
<tr>
<td>5 Generation 100% Reliable</td>
<td>0.0</td>
<td>0.0</td>
<td>0.000875</td>
<td>0.804</td>
</tr>
<tr>
<td>6 Line 7 Added</td>
<td>0.000061</td>
<td>0.0022</td>
<td>0.000069</td>
<td>0.0155</td>
</tr>
<tr>
<td>7 Line 7&amp;8 Added</td>
<td>0.000061</td>
<td>0.0024</td>
<td>0.000067</td>
<td>0.0126</td>
</tr>
<tr>
<td>8 Generating unit outage</td>
<td>0.0</td>
<td>0.0</td>
<td>0.000866</td>
<td>0.8114</td>
</tr>
<tr>
<td>unit outage considered separately</td>
<td>6.3, 6.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equa.</td>
<td>6.3, 6.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 Load Distribution</td>
<td>0.000062</td>
<td>0.0013</td>
<td>0.000657</td>
<td>0.5866</td>
</tr>
</tbody>
</table>

The total computer time required to evaluate the system reliability is quite different in each case. The total computer times for the normal case of Table 6.1 are given in Table 6.2. The savings in computer time using the developed load-flow techniques are further emphasized in the reliability studies of reasonably large practical power systems. The values shown in Table 6.2 include both the compilation and execution
times. A composite system reliability study of the 20-bus Saskatchewan Power Corporation System was also conducted. The results are shown in Table 6.3.

**TABLE 6.2**

Solution Time

<table>
<thead>
<tr>
<th>Load-flow Techniques</th>
<th>Computer Time for Composite System Reliability Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss-Seidel</td>
<td>8.54 min.</td>
</tr>
<tr>
<td>Newton-Raphson</td>
<td>7.53 min.</td>
</tr>
<tr>
<td>Acctld. Newton</td>
<td>6.80 min.</td>
</tr>
<tr>
<td>Constant Jacobi</td>
<td>5.78 min.</td>
</tr>
<tr>
<td>Successive Overrelaxation</td>
<td>7.04 min.</td>
</tr>
</tbody>
</table>
TABLE 6.3

Bus Risk Levels
Minimum Acceptable Voltage = 0.90 p.u.
No. of Probability Steps = 5
Single outage condition

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>10</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>11</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>12</td>
<td>0.000234</td>
<td>0.229856</td>
<td>0.112028</td>
</tr>
<tr>
<td>13</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>14</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>15</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>16</td>
<td>0.0</td>
<td>0.0</td>
<td>0.111600</td>
</tr>
<tr>
<td>17</td>
<td>0.009057</td>
<td>8.63478</td>
<td>0.119637</td>
</tr>
<tr>
<td>18</td>
<td>0.000001</td>
<td>0.001217</td>
<td>6.111601</td>
</tr>
<tr>
<td>19</td>
<td>0.0000015</td>
<td>0.001947</td>
<td>0.111602</td>
</tr>
<tr>
<td>20</td>
<td>0.003427</td>
<td>3.2725</td>
<td>0.114113</td>
</tr>
</tbody>
</table>

Computer Time
Gauss-Seidel = 52 minutes
Constant Jacobi = 36 minutes

6.8. Conclusions

In this chapter the background and concepts of the composite system reliability study program have been discussed and studied.
The composite power system reliability procedure is extremely useful in analysing a complex system and provides a valuable insight into the system operating characteristics. This approach provides a consistent quantitative yardstick which can be used by engineering personnel to illustrate to management the effect of alternate proposals and design configurations.

Considerable reduction in reliability evaluation time was experienced by accelerating the load flow solutions. All the studies show that the bus risk levels are virtually identical for the range of load-flow techniques used. The total time required to obtain the reliability indices shown in Table 6.4 was 52 minutes using the standard Gauss-Seidel load-flow technique for 20 bus SPC system. The time required when this load-flow subroutine was replaced by one based upon the constant Jacobian approach was 36 minutes. This is a saving of approximately 30%. The results in each case were virtually identical. This saving is quite significant particularly if the analysis is done repeatedly in transmission planning studies. The selection of the number of load steps, the number of independent outages and the system representation are dependent upon the system size and the basic design philosophy. In a practical system these can be determined by conducting an exhaustive series of test cases.
7. CONCLUSIONS

The great importance of load-flow studies in planning future expansion of power systems as well as in operating existing systems cannot be overestimated. In this thesis a new method and a few modifications for load-flow analysis have been proposed as an extension of the basic Newton's method for application in power system reliability studies. There are numerous publications describing the digital load-flow problem. The available techniques can be classified as being either direct or iterative.

The conventional Gauss-Seidel and Newton-Raphson methods were first investigated for power system load-flow application. From the studies of these techniques it was recognized that the Newton-Raphson method could be further exploited to reduce the computational time. The convergence characteristics of the Newton method in comparison to the conventional Gauss-Seidel method is 2 to 3 times faster. A brief review of recent significant development in numerical methods for the load-flow problem is also presented in the thesis.

A new method for power system load-flow calculation has been developed in this thesis. The new accelerated Newton method incorporates Gaussian elimination in evaluating the system Jacobian matrix. The new method reduces the computer time and memory requirements for load-flow solutions. A principle feature of the technique lies in the direct evaluation of the triangularized Jacobian. At each step of the iteration cycle the new accelerated method always uses the latest calculated value of the unknown variables. The overall computing time is 40% to 50% lower than that of Newton's and Gauss-Seidel methods.
The number of iterations are also less than that required for the Newton method. Although the studies that were conducted did not exhaust all possibilities, sufficient evidence was gathered in favour of the proposed method. It is also recognized that a reduction in the number of iterations is not the complete answer to the load-flow problem of reducing the computing time. The factor by which the convergence of any iterative scheme can be judged is the time per iteration. Further acceleration of the proposed method may be achieved by introducing an acceleration factor in the iteration process.

The main computing difficulty with the Newton method is in storing and calculating the inverse of the Jacobian matrix. Three modifications of the basic Newton method have been suggested in this thesis to overcome these difficulties. The first method incorporates the successive overrelaxation approach with an optimum relaxation factor. The necessity of evaluating the inverse of the Jacobian matrix is therefore eliminated. Sparsity of the Jacobian matrix is exploited to reduce the computational time. A new method for solving the load-flow problem has also been suggested by keeping the inverse of the Jacobian matrix constant. For the normal load-flow study this technique requires about 5 to 10 times less computing time in comparison with the conventional Newton method. As an extension of this technique a new procedure for modifying the inverse of any diagonally dominant symmetrical matrix was studied. This modification is required when the system Jacobian matrix or its inverse changes for the outage of any system component. The technique for modifying the inverse uses the 'row by row' method for evaluating
the inverse of a matrix. It is believed that the 'row by row' matrix inversion technique will give tremendous saving in computer storage for large practical systems. Another new technique for modifying the Newton method was also investigated. In this Quasi-Newton approach, the inverse of the coefficient matrix is not directly calculated but its approximate one is generated. It has been found that this technique requires much higher computer storage than all the other methods studied. The convergence characteristics are not as good as those of the other methods investigated.

Some comparisons are given for the different load-flow techniques developed and discussed. The main points of comparison are the accuracy of the solutions obtained, the number of iterations required to converge and the solution time. It was found that a tolerance of 0.0001 gives a reasonably accurate solution in all the techniques investigated. It was also found that the size and characteristics of the power system greatly influences the solution of any load-flow technique.

The developed load-flow techniques were applied in the composite system reliability studies. Calculations of the composite system reliability provides useful information for system planning and operation requirements in future years. The failure of a load bus in the reliability studies has been defined in terms of quality of service indices rather than only continuity of supply. The importance of a fast load flow technique in studies of this type is clearly illustrated by the 30% reduction in solution time obtained for the practical system shown in Figure 5.2.
Recommendations for future work:

In recent years, several rapidly convergent methods have been proposed. Among these are the conjugate gradient method\(^3\)\(^{39}\) and Fletcher-Reeves\(^4\)\(^{40}\). In both the methods the direction of the search is determined by the gradient of the function. The stability property of the method is analogous to that of the steepest descent method which has a very simple algorithm and convergence is assured from any initial approximation. The convergence rate however is very slow. On the other hand, the Newton-Raphson method as described in chapter 2 has the quadratic convergence property but the method may not converge at all. In Appendix I, one numerical algorithm is developed for load-flow solution by a non-linear programming approach. The main features of the method is that the convergence is always assured. In the Appendix, modifications of the Fletcher-Reeves method are given and then the modified forms for function minimization are applied in the Newton-Raphson method. The load-flow solution through a function minimization technique appears to be a promising avenue for further research work. The use of the load-flow solution in system expansion studies, in optimizing existing system operation or in the analysis of system failure, involving a transient network performance should be a fruitful area for further work.
8. REFERENCES


APPENDIX I

LOAD FLOW THROUGH A FUNCTION MINIMIZATION TECHNIQUE

The Nonlinear Programming Approach

As shown in chapter 3, if the equations for bus power can be expressed in terms of $\delta$ and $|V|$ then another function could be formed by the residuals of the bus powers as shown below:

$$f_1(\delta_2, \delta_3, |V_3|) = P_2 - V_2 \sum_{m=1}^{3} Y_{2m} V_m \cos(\delta_2 - \delta_m - \delta_{2m})$$  \hspace{1cm} (1)

$$f_2(\delta_2, \delta_3, |V_3|) = P_3 - V_3 \sum_{m=1}^{3} Y_{3m} V_m \cos(\delta_3 - \delta_m - \delta_{3m})$$  \hspace{1cm} (2)

$$f_3(\delta_2, \delta_3, |V_3|) = Q_3 - V_3 \sum_{m=1}^{3} Y_{3m} V_m \sin(\delta_3 - \delta_m - \delta_{3m})$$  \hspace{1cm} (3)

The new function formed:

$$F(\delta_2, \delta_3, |V_3|) = f_1^2 + f_2^2 + f_3^2$$  \hspace{1cm} (4)

or

$$F(x) = \sum_i f_i^2 (x)$$  \hspace{1cm} (5)

where in vector form

$$(x)^T = (\delta, |V|)^T$$

The non-linear programming approach utilizes the Fletcher-Reeves minimization scheme to minimize the function (5) until it approaches zero with a desired degree of accuracy. By Taylor's series:

$$F(x) = F(o) + \sum_{i=1}^{N} A_i \Delta x_i + \frac{1}{2} \sum_{i,j}^{NN} H_{ij} \Delta x_i \Delta x_j$$  \hspace{1cm} (6)

where
Differentiating equation (6) w.r.t. \( x_1, x_2 \ldots \) etc. we have the following:

\[
A_i = \frac{\partial F}{\partial x_i} \quad \text{and} \quad H_{ij} = \frac{\partial^2 F}{\partial x_i \partial x_j}
\]

where

\[
A_i + H_{ij} \Delta x_j = 0
\]

\[A_i = g(x) = \frac{\partial F}{\partial x} = \text{gradient}\]

\[
\Delta x_i = x_i^{k+1} - x_i^k
\]

\[
x_i^{k+1} = x_i^k + \Delta x_i
\]  

(7)

In a recent paper by Sasson\(^{16}\) for load-flow solution proved that the minimum of \( F(x_0) \) is found by determining \( \alpha \) which minimizes \( F(x_0 + \alpha \Delta x) \). Instead of equation (7) the new point \( x_i \) is computed from:

\[
x_i^{k+1} = x_i^k + \alpha \Delta x_i
\]  

(8)

If the \( H \) matrix is explicitly not available then it can be taken as a Hermitian matrix and can be updated by a formula\(^{21}\) that contains information on \( x_i \) and the gradient computed at \( x_i \). The process is repeated until desired accuracy at all the buses are obtained. However, in the paper by Sasson \( \Delta x_i \) is obtained from \(-J^{-1}f\) and \( \alpha \) is calculated by cubic interpolation technique which reduces the norm of \( F(x) \) at each iteration.

The Proposed Method

Memory Newton Method:

A new accelerated Newton method by finding the minimum of a function \( F(x) \) as given in equation (7) is developed. The new algorithm can be stated as follows:
\[ x^{k+1} = x^k + \delta x \] (9)

where

\[ \delta x = a \Delta x + \beta \Delta \hat{x} \] (10)
\[ \Delta x = -J^{-1} f(x_0) \]

\( J \), the Jacobian matrix is the matrix of the first partial derivatives evaluated at the point \( x^k \).

\( J = \frac{\partial F}{\partial x} \)

\( \Delta \hat{x} \) is same as \( \Delta x \) for the iteration preceding that under consideration.

\( a \) and \( \beta \) scalars are calculated at each step so as to yield the greatest decrease in the function.

The basic idea is to construct corrections \( \delta x \) leading from a nominal point \( x^k \) to a varied point \( x^{k+1} \) such that

\[ ||F(x^{k+1})|| < ||F(x^k)|| \]

In the function minimization technique as given in 16, 39, 40, one dimensional search is made to reduce the function. By introducing the second factor \( \beta \) the convergence rate increases. Since in the proposed method the constraint involves not only the correction of \( \Delta x \) of the iteration under consideration but also the correction \( \Delta \hat{x} \) of the previous iteration, the resulting algorithm is therefore named Memory Newton Method.

For any iteration except the first, the complete algorithm can be summarized as follows:

(a) For a given nominal point \( x \), the correction \( \Delta x \) is known and the vector \( \Delta \hat{x} \) is known from the previous iteration.

(b) The optimum values of the multipliers \( a \) and \( \beta \) must be determined so that the function \( F(x) \) reduces at each step.
(c) The correction $\delta x$ to the position vector $x$ is determined by using equation (10) and

(d) The new position vector $x^{k+1}$ is computed through equation (9).

Of course, operation (a) through (d) imply that $\Delta x$ is known from the previous iteration. Since this is not the case for the first iteration, some assumption concerning $\Delta x$ must be made in order to start the algorithm. The simplest assumption is $\Delta x = 0$, equivalent to starting the first step as described by Sasson. If the value of $\alpha$ in first iteration is assumed as unity then the starting iteration will be just like ordinary Newton's method.
APPENDIX II

Elements of the Jacobian Matrix:

In the Newton method the Jacobian matrix gives the linearized relationship between small changes in voltage angle and magnitude $\Delta \delta$ and $\Delta V/V$ and the incremental change in bus real and reactive power, $\Delta P$ and $\Delta Q$. The example of matrix equation is shown in equation (2.27) of chapter 2. The elements of the Jacobian matrix $H_{km}$, $N_{km}$, $J_{km}$ and $L_{km}$ can be evaluated by taking partial derivatives of the real and reactive power as follows:

$$H_{km} = \frac{\partial P_k}{\partial \delta_k}, \quad N_{km} = \frac{\partial P_k}{\partial V_m} V_m$$

(1)

$$J_{km} = \frac{\partial Q_k}{\partial \delta_k}, \quad L_{km} = \frac{\partial Q_k}{\partial V_m} V_m$$

(2)

Although the partial derivatives defined in the equation (1) based on a polar formulation of the load-flow problem, these could be computed by rectangular arithmetic. The rectangular form for formulating the load-flow problem by the Newton method has not been studied in this thesis as it required more memory. The equation for complex power at any node $k$ is:

$$P_k + jQ_k = (V_k e^{-j\delta_k}) \sum_{m=1}^{N} (V_m e^{-j\delta_m})(Y_{km} e^{-j\theta_{km}})$$

(3)

The partial derivative of equation (3) with respect to one value of $\delta_m$ other than $\delta_k$:

$$\frac{\partial P_k}{\partial \delta_m} + j \frac{\partial Q_k}{\partial \delta_m} = -j(V_k e^{j\delta_k})(V_m e^{-j\delta_m})(Y_{km} e^{-j\theta_{km}})$$

(4)
The rectangular expression for the voltage, admittance and current are:

\[ V_m = (e_m + jf_m) \]
\[ Y_{km} = (G_{km} + jB_{km}) \]
\[ I_m = (a_m + jb_m) \]

Therefore, the last two terms of equation (4) can be interpreted as a current and equating real and imaginary parts of (4) gives the following values of \( H_{km} \) and \( J_{km} \) when \( k \) is not equal to \( m \):

\[ H_{km} = a_m \bar{f}_k - b_m e_k \]
\[ J_{km} = -(a_m e_k + b_m f_k) \quad (6) \]

Similarly, for \( L_{km} \) and \( M_{km} \), the partial derivative of equation (1) is taken w.r.t. one value of \( V_m \) other than \( V_k \). The expressions are

\[ L_{km} = a_m \bar{f}_k - b_m e_k \]
\[ N_{km} = a_m e_k + b_m f_k \quad (7) \]

To evaluate the elements of the Jacobian matrix when \( m \) equals \( k \) a similar method can be used. The partial derivatives of equation (3) is taken w.r.t. \( \delta_k \):

\[
\frac{\partial P_k}{\partial \delta_k} + j\frac{\partial Q_k}{\partial \delta_k} = j(V_k e^{j\delta_k}) \sum_{m=1}^{N} (V_m e^{-j\delta_m})(Y_{km} e^{-j\theta_{km}}) - j(V_k e^{j\delta_k}) (V_k e^{-j\delta_k}) (Y_{kk} e^{-j\theta_{kk}}) \]

(8)

This can be simplified to:
\[
\frac{\partial P_k}{\partial \delta_k} + j \frac{\partial Q_k}{\partial \delta_k} = j(P_k + jQ_k) - jV_k^2 (G_{kk} - jB_{kk}) \tag{9}
\]

Equating real and imaginary parts to get \( H_{kk} \) and \( J_{kk} \):
\[
H_{kk} = -Q_k - B_{kk} V_k^2 \tag{10}
\]
\[
J_{kk} = P_k - G_{kk} V_k^2
\]

Similar manner the partial derivatives of equation (3) can be taken w.r.t. \( V_k \) as follows:
\[
\frac{\partial P_k}{\partial V_k} + j \frac{\partial Q_k}{\partial V_k} = (P_k + jQ_k) + V_k^2 (G_{kk} - jB_{kk}) \tag{11}
\]

Equating real and imaginary parts give \( N_{kk} \) and \( L_{kk} \):
\[
N_{kk} = P_k + G_{kk} V_k^2 \tag{12}
\]
\[
L_{kk} = Q_k - B_{kk} V_k^2
\]

The derivatives of above equations are given by Van Ness\(^{10} \) and Tinney\(^{15} \). The residual powers at a node are defined as
\[
\Delta P_k = P_k \text{ (scheduled)} - P_k \text{ (calculated)}
\]
\[
\Delta Q_k = Q_k \text{ (scheduled)} - Q_k \text{ (calculated)}
\]
APPENDIX III

TABLE 1

S.P.C. Major Transmission Network Reliability Load and Generation Data:

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Peak Load</th>
<th>Total Load</th>
<th>Generation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MW</td>
<td>MVAR</td>
<td>MW</td>
</tr>
<tr>
<td>1</td>
<td>72.0</td>
<td>19.2</td>
<td>220.0</td>
</tr>
<tr>
<td>2</td>
<td>48.2</td>
<td>11.3</td>
<td>105.0</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td>60.0</td>
</tr>
<tr>
<td>4</td>
<td>157.9</td>
<td>55.9</td>
<td>85.0</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>0.0</td>
<td>120.0</td>
</tr>
<tr>
<td>6</td>
<td>47.0</td>
<td>18.5</td>
<td>70.0</td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>0.0</td>
<td>70.0</td>
</tr>
<tr>
<td>8</td>
<td>0.0</td>
<td>0.0</td>
<td>280.0</td>
</tr>
<tr>
<td>9</td>
<td>125.3</td>
<td>18.8</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>11</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>12</td>
<td>30.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>13</td>
<td>42.2</td>
<td>11.6</td>
<td>0.0</td>
</tr>
<tr>
<td>14</td>
<td>10.6</td>
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<tr>
<td>15</td>
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<td>16</td>
<td>77.3</td>
<td>26.7</td>
<td>0.0</td>
</tr>
<tr>
<td>17</td>
<td>25.0</td>
<td>10.0</td>
<td>0.0</td>
</tr>
<tr>
<td>18</td>
<td>84.4</td>
<td>21.4</td>
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</tr>
<tr>
<td>19</td>
<td>77.3</td>
<td>15.4</td>
<td>0.0</td>
</tr>
<tr>
<td>20</td>
<td>54.2</td>
<td>11.5</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Lines are assumed to be 795 ACSR 54/7
Current carrying capacity = 900 amps.
Failure rate = 0.05 failures/year/mile.
Expected repair duration = 10 hours.

Load Probability Steps: 1.00 0.80 0.60 0.40 0.20 0.0 (5 step)

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>No. of Units</th>
<th>Capacity of each Unit MW</th>
<th>Total Bus Capacity MW</th>
<th>Probability of outage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>60.0</td>
<td>220.0</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
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<td>100.0</td>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>25.0</td>
<td>105.0</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10.0</td>
<td></td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>30.0</td>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>15.0</td>
<td></td>
<td>0.005</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>20.0</td>
<td>60.0</td>
<td>0.015</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>5.0</td>
<td>85.0</td>
<td>0.005</td>
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<tr>
<td></td>
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<td>20.0</td>
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<td>0.015</td>
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<td>25.0</td>
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<td>0.015</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>30.0</td>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>60.0</td>
<td>120.0</td>
<td>0.025</td>
</tr>
<tr>
<td>6</td>
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<td>5.0</td>
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</tr>
<tr>
<td></td>
<td>1</td>
<td>15.0</td>
<td></td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>30.0</td>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>20.0</td>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>35.0</td>
<td>70.0</td>
<td>0.015</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>140.0</td>
<td>280.0</td>
<td>0.05</td>
</tr>
</tbody>
</table>
APPENDIX IV

PROGRAM FLOW CHART FOR LOAD FLOW ANALYSIS

METHOD: GAUSS-SEIDEL

START

Form bus admittance matrix Y Bus

Set iteration count $k=0$

Form parameter of voltage equation
\[
\frac{P_i - jQ_i}{Y_{ii}} = x
\]
\[
y_{ij}/y_{ii} = y
\]
i = 1, 2, ..., n; j = 1, 2, ..., n if $i \neq j$

Set max$^m$ voltage change max $\Delta E = 0$ and bus count $i=1$

Test for swing bus $i \neq s$

\[
E_{i}^{k+1} = \frac{x}{(E_i^k)^*} - \frac{1}{2}yE_{j}^{k+1} - \sum_{j=i+1}^{n} yE_{j}^{k}
\]

Solve voltage eqn$^n$ for bus $i$

Calculate change in voltage of bus $i$

$\Delta E_{i}^{k} = E_{i}^{k+1} - E_{i}^{k}$
Test for max change in voltage
\[ |\Delta E_i^k|: \max \Delta E^k \]

Greater

Equal or Less

Set \( \max \Delta E^k = |\Delta E_i^k| \)

Replace \( E_i^k \) by \( E_i^{k+1} \)

Advance bus count \( i+1+i \)

Equal or Less

Test for end of one iteration \( i:n \)

Advance iteration count

Greater

Test for Convergence max
\[ \Delta E^k: \epsilon \]

Equal or Less

Test for generator bus reactive power generation limits.

Not satisfied

Change the bus voltage by \( \pm 0.01 \text{ p.u.} \)

Satisfied

Calculate the line flows and power at swing bus.

END
FLOW-CHART FOR LOAD LOW ANALYSIS

METHOD: STANDARD NEWTON-RAPHSON

START

FORM BUS ADMITTANCE MATRIX

SET ITERATION COUNT $k=0$

CALCULATE REAL AND REACTIVE BUS POWERS

CALCULATE RESIDUAL COMPLEX POWER AT DIFFERENT BUSES

$\Delta P^k = P$ specified - $P$ calculated
$\Delta Q^k = Q$ specified - $Q$ calculated

Test for convergence

$|\Delta P^k| \leq \epsilon$
$|\Delta Q^k| \leq \epsilon$

CALCULATE JACOBIAN MATRIX

$\Delta P\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \Delta \delta$

$\Delta Q\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \Delta V$

CALCULATE NEW BUS VOLTAGES

ADVANCE ITERATION COUNT

END

CALCULATE LINE FLOWS AND GENERATION AT SWING BUS
FLOW CHART FOR LOAD FLOW ANALYSIS

METHOD: ACCELERATED NEWTON'S

METHOD: CONSTANT JACOBIAN MATRIX
FLOW CHART FOR LOAD FLOW ANALYSIS

METHOD: NEWTON WITH SOR

1. SET RELAXATION FACTOR \( \omega = 1.0 \)
2. EVALUATE THE JACOBIAN MATRIX \( J \)
3. SOLVE THE LINEARIZED EQUATIONS BY SOR METHOD
4. IS \( \omega = \omega_{opt} \) ?
   - NO: ADVANCE ITERATION COUNT
   - YES: CALCULATE NEW BUS VOLTAGES
START

READ ALL THE DATA REQUIRED TO LOAD FLOW & CSR STUDY

FORM BUS ADMITTANCE MATRIX

CALL

STOP

EVALUATE CAPACITY OUTAGE TABLE

PERFORM LOAD FLOW STUDY

EVALUATE BUS RISK LEVELS

RLBTY STUDY UNDER LOAD FLOW NO. CONVERGENCE

MODIFY CAPACITY OUTAGE TABLE

PERFORM LOAD FLOW ANALYSIS

EVALUATE RLBTY FOR ISOLATED BUSES

MODIFY ADMITTANCE MATRIX
APPENDIX V

ELEMENTS OF TRIANGULARIZED JACOBIAN MATRIX

The developed programme evaluates a triangularized Jacobian matrix at each iteration as described in Chapter 3. The elements of the Jacobian matrix of equation (3.29) are calculated as described in this Appendix. The partial derivatives of $f_1$, $f_2$ and $f_3$ of equations (3.19), (3.20) and (3.21) are as follows:

\[
\frac{\partial f_1}{\partial \delta_2} |_{o} = \sum_{m=1, m \neq 2}^{3} \frac{Y_{2m} V_2 V_m}{|V_2| |V_m|} \sin(\delta_2 - \delta_m - \theta_{2m})
\]

(1)

\[
\frac{\partial f_1}{\partial \delta_3} |_{o} = - \frac{Y_{23} V_3 V_2}{|V_2| |V_3|} \sin(\delta_2 - \delta_3 - \theta_{23})
\]

(2)

\[
\frac{\partial f_1}{\partial V_3} |_{o} = - \frac{Y_{23} V_3 V_2}{|V_3| |V_2|} \cos(\delta_2 - \delta_3 - \theta_{23})
\]

(3)

\[
\frac{\partial f_2}{\partial \delta_2} |_{o} = - \frac{Y_{23} V_3 V_2}{|V_2| |V_3|} \sin(\delta_3 - \delta_2 - \theta_{32})
\]

(4)

\[
\frac{\partial f_2}{\partial \delta_3} |_{o} = \sum_{m=1, m \neq 3}^{3} \frac{Y_{3m} V_3 V_m}{|V_3| |V_m|} \sin(\delta_3 - \delta_m - \theta_{3m})
\]

(5)

\[
\frac{\partial f_2}{\partial V_3} |_{o} = - \sum_{m=1, m \neq 3}^{3} \frac{Y_{3m} V_3 V_m}{|V_3| |V_m|} \cos(\delta_3 - \delta_m - \theta_{3m}) + \frac{Y_{33} V_3 V_3}{|V_3|^2} \cos(-\theta_{33})
\]

(6)

\[
\frac{\partial f_3}{\partial \delta_2} |_{o} = \frac{Y_{32} V_2 V_3}{|V_2| |V_3|} \cos(\delta_3 - \delta_2 - \theta_{32})
\]

(7)
The first row of the Jacobian is established by equation (1), (2) and (3) respectively. The elements of the second row are defined as follows:

\[
\frac{\partial f_3}{\partial \delta_3} |_o = - \sum_{m=1 \atop m \neq 3}^3 \frac{|Y_{3m}| |V_3| |V_m| \cos(\delta_3 - \delta_m - \theta_{3m})}{m \neq 3} \tag{8}
\]

\[
\frac{\partial f_3}{\partial |V_3|} |_o = - \sum_{m=1 \atop m \neq 3}^3 \frac{|Y_{3m}| |V_m| \sin (\delta_3 - \delta_m - \theta_{3m}) + |Y_{33}| |V_3| \sin(-\theta_{33})}{m \neq 3} \tag{9}
\]

By definition, \( \frac{\partial f_2}{\partial a_1} \) is approximately equal to \( \frac{\partial f_2}{\partial \delta_2} \) which is obtained from equation (3.20)

and their values are given by equation (4), (5), and (6).

\[
\frac{\partial a_1}{\partial \delta_3} \quad \text{and} \quad \frac{\partial a_1}{\partial |V_3|}
\]

are obtained by differentiating equation (3.23) as follows:

\[
\frac{\partial a_1}{\partial \delta_3} = \frac{\partial a_1}{\partial \delta_3} = -A = - \left( \frac{\partial f_1}{\partial \delta_3} \right) |_o + \frac{\partial f_1}{\partial \delta_2} |_o
\]

\[
= -|Y_{23}| |V_3| |V_2| \sin(\delta_2 - \delta_3 - \theta_{23}) \sum_{m=1 \atop m \neq 2}^3 \frac{|Y_{2m}| |V_2|}{m \neq 2}
\]

\[
|V_m| \sin(\delta_2 - \delta_m - \theta_{2m})
\]
The element in the third row of the Jacobian matrix is given by
\[
\frac{\partial a_1}{\partial a_2} \bigg|_0 - \frac{\partial f_1}{\partial V_3} \bigg|_0 = -B_1 = - \left( \frac{\partial f_1}{\partial V_3} \bigg|_0 / \frac{\partial f_2}{\partial \delta_2} \bigg|_0 \right)
\]
\[
= -|V_{23}| V_2 \cos(\delta_2 - \delta_3 - \theta_23) / \left\{ \sum_{m=1, m\neq 2}^3 |V_{2m}| |V_2| \right\} \left\{ \sin(\delta_2 - \delta_3 - \theta_2m) \right\}
\]

(13)

The element in the third row of the Jacobian matrix is given by
\[
\frac{\partial g_3}{\partial V_3} \bigg|_0 = \frac{\partial f_3}{\partial V_3} + \frac{\partial f_2}{\partial \delta_2} \cdot \frac{\partial a_2}{\partial V_3} + \frac{\partial g_3}{\partial \delta_2} \left( \frac{\partial a_1}{\partial V_3} + \frac{\partial a_1}{\partial \delta_2} \right)
\]
where \(\frac{\partial f_3}{\partial a_1}\) and \(\frac{\partial f_3}{\partial \delta_2}\) are equal to \(\frac{\partial f_3}{\partial a_1} \bigg|_0\) and \(\frac{\partial f_3}{\partial \delta_2} \bigg|_0\) respectively.

\(\frac{\partial a_1}{\partial a_2}\) is given by equation (12) and \(\frac{\partial a_2}{\partial V_3}\) is given as follows
\[
\frac{\partial a_2}{\partial V_3} = \frac{\partial a_2}{\partial V_3} = -A_2 = - \left( \frac{\partial g_2}{\partial V_3} \bigg|_0 / \frac{\partial g_2}{\partial \delta_3} \bigg|_0 \right)
\]
where the values of \(\frac{\partial g_2}{\partial V_3} \bigg|_0\) and \(\frac{\partial g_2}{\partial \delta_3} \bigg|_0\) are defined by equations (10) and (11). These equations establish all the terms of the Jacobian matrix of equation (3.29) which leads to the solution of the improved values of \(\delta\)'s and \(V\)'s.