OPTIMUM ADJUSTMENT

OF

HYDRO GENERATOR GOVERNORS

A Thesis Submitted to
the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements for
the Degree of Doctor of Philosophy
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by

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ABSTRACT

An important aspect of power utility operation is effective frequency and voltage regulation of the electrical supply. Techniques for establishing quality settings of system controllers are of economic and system performance significance. In this thesis methods for determining quality frequency regulation of generators using hydraulic turbines as prime movers are developed. The quality frequency regulation process takes into account the energy demands and delivery requirements of the utility customers and the mechanical constraints of the generating installation. The methods presently employed are evaluated and the deficiencies established. System operational experience and needs are used as a basis for mathematically defining an optimum unit frequency regulation process.

A linear time invariant model is employed in the analysis. A single unit isolated load configuration adapted with several versions of frequency controllers is studied. Interconnected system phenomena are investigated and the related limitations of present day governing facilities are determined. The adjustment techniques presently used for specific system frequency controllers are extended and more flexible criteria are suggested.

A procedure is developed for selecting optimum governor settings of a hydro-electric power generator regulated by a realistic general type of governor. Multivariable optimization search techniques are employed in the selection of these adjustments. The performance criterion, a special case of Krasovskii's integral, is a function of the system state variables. The method described can be extended to the optimization of automatic voltage regulators and interacting frequency controllers.
ACKNOWLEDGEMENTS

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<tr>
<td>e</td>
<td>subscript which denotes a variable or a parameter of the equivalent unit</td>
</tr>
<tr>
<td>i</td>
<td>numerical subscripts denoting a particular unit or bus</td>
</tr>
<tr>
<td>q</td>
<td>complex variable operator</td>
</tr>
<tr>
<td>D</td>
<td>unit damping coefficient (the per unit change in unit torque/per unit change in ((\omega - \omega_n)))</td>
</tr>
<tr>
<td>D(S)</td>
<td>denominator of the system transfer function</td>
</tr>
<tr>
<td>G</td>
<td>gate position (ft)</td>
</tr>
<tr>
<td>G_o</td>
<td>base gate position (ft)</td>
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<td>G(S)</td>
<td>transfer function of forward path</td>
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<td>H(S)</td>
<td>transfer function of feedback path</td>
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<td>J</td>
<td>integral error criterion</td>
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<tr>
<td>K_{ir}</td>
<td>linearized synchronizing torque coefficient, (per unit change in power transfer per radian of electrical angular displacement between two systems)</td>
</tr>
<tr>
<td>K_c</td>
<td>electro-hydraulic three-mode governor gain</td>
</tr>
<tr>
<td>MW</td>
<td>machine base (megawatt)</td>
</tr>
<tr>
<td>N(S)</td>
<td>numerator of system transfer function</td>
</tr>
<tr>
<td>S</td>
<td>Laplace operator ((S = s' + j\omega))</td>
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<tr>
<td>T_{ac}</td>
<td>unit accelerating torque (MW seconds/radian)</td>
</tr>
<tr>
<td>T_{aco}</td>
<td>base unit accelerating torque (MW seconds/radian)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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</tr>
<tr>
<td>$T_d$</td>
<td>system retarding torque (MW seconds/radian)</td>
</tr>
<tr>
<td>$T_{do}$</td>
<td>base system retarding torque (MW seconds/radian)</td>
</tr>
<tr>
<td>$T_{t1}$</td>
<td>retarding torque, equivalent to electrical load (MW seconds/radian)</td>
</tr>
<tr>
<td>$T_{t10}$</td>
<td>base retarding torque (MW seconds/radian)</td>
</tr>
<tr>
<td>$T_m$</td>
<td>mechanical starting time of turbine, generator and load inertias (seconds)</td>
</tr>
<tr>
<td>$T_r$</td>
<td>dashpot relaxation time (seconds)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>servomotor time constant (seconds)</td>
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<tr>
<td>$T_t$</td>
<td>turbine torque (MW seconds/radian)</td>
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<tr>
<td>$T_{t0}$</td>
<td>base turbine torque (MW seconds/radian)</td>
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<tr>
<td>$T_{tl}$</td>
<td>tie-line torque (MW seconds/radian)</td>
</tr>
<tr>
<td>$T_{tl0}$</td>
<td>base tie-line torque (MW seconds/radian)</td>
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<tr>
<td>$T_W$</td>
<td>water starting time for a given gate position (seconds)</td>
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<td>$T_{11}$</td>
<td>time constant of the electro-hydraulic three-mode governor electrical amplifier (seconds)</td>
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<td>$T_{2}$</td>
<td>time constant of the electro-hydraulic transducer of the electro-hydraulic three-mode governor (seconds)</td>
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<td>$T_{3}$</td>
<td>time constant of the lead network of the general three-mode governor (seconds)</td>
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<td>$T_{4}$</td>
<td>time constant of the servomotor of the general three-mode governor (seconds)</td>
</tr>
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<td>$Z$</td>
<td>input signal of the servomotor of the general three-mode governor (ft)</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>base input signal of the servomotor of the general three-mode governor (ft)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>temporary speed droop (p.u.)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>damping ratio</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>$\lambda_1$---$\lambda_6$</td>
<td>dimensionless ratios which relate the system parameters</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>permanent speed droop (p.u.)</td>
</tr>
<tr>
<td>$\tau_j$</td>
<td>integral error criterion weighting factor</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>base unit frequency (radians/second)</td>
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<tr>
<td>$\omega_s$</td>
<td>system frequency (radians/second)</td>
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<tr>
<td>$\omega_n$</td>
<td>unit damping frequency, nominally set equal to $\omega_0$ (radians/second)</td>
</tr>
<tr>
<td>$\omega_{set}$</td>
<td>governor reference frequency (radians/second)</td>
</tr>
<tr>
<td>$\omega_{ref}$</td>
<td>reference bus frequency (radians/second)</td>
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1. INTRODUCTION

1.1 Scope of Research

The prime function of any power utility is to satisfy, most economically, the electrical demands of its customers. These demands manifest themselves in terms of the required quantity and the quality of the delivered energy. The latter is usually specified by the maximum acceptable frequency and voltage deviations and the duration of the regulating process. To achieve economy the power utility must consider the problems of generation and regulation of its product under constantly varying conditions. In this thesis techniques are developed which may be employed to achieve quality regulation of frequency of a hydro-electric power station. System operational experience suggests that the optimum quality frequency regulation is accomplished when the governors are adjusted such that, given a step change in electrical load, the frequency deviation approaches the form of a slightly under critically damped sinusoid (damping ratio $\zeta = .707$, referred to henceforth as 'optimum' damping).

Electrical power system behavior for most analyses can be adequately described by four variables; real power, reactive
power, voltage and frequency. All are interdependent; however, it has been demonstrated by Venikov\textsuperscript{2} and Kirchmayer\textsuperscript{3} that the real power and frequency phenomena can, under normal operation, be considered to be independent of those of the voltage and reactive power. Numerous successful studies have been conducted on system models which describe either the voltage or frequency control loop\textsuperscript{4}. This concept of voltage and frequency independence is exploited throughout the studies reported in this thesis.

A stable, linear, time-invariant system is usually assumed when quality regulation of either frequency or voltage is considered. This linear model can be employed when the deviations of system variables are small. Since these are the power system operating conditions during the greatest portion of time, considerable economic value could exist from a thorough investigation of such a model.

The correct adjustment of frequency controllers improves system performance and economy. Excessive frequency hunting induces a faster rate of mechanical deterioration of the controllers and thus increases down-time and maintenance costs. An excessively active system requires more spinning energy for the same loading (assuming the same maximum frequency fluctuations). This results in a higher cost of generation. A reduction of the dynamic stability limit will result if additional spinning inertia is not provided. The possibility
of another disturbance occurring while the system is imbalanced is dependent on the duration of the regulating process, this also induces larger frequency swings. In addition to the above, ineffective use of frequency controllers calls for greater attention on the part of plant and dispatching personnel. Techniques by which the optimum (fulfilling the above requirements) governor parameter settings can be selected improves the economy of electrical power generation. This consequential economy can be considered as the prime reason for conducting the analysis described in this thesis. Throughout the study standard control system analytical techniques are employed. The emphasis is on the engineering implications of the results rather than on the mathematical features of the various methods used.

1.2 Search for Quality of Regulation - Historical Review

The search for methods by which the optimum power system frequency controller settings can be selected has been concentrated in three areas. In chronological order, the first was concerned with the determination of the stability boundary of the generating unit and then the search for and selection within the stable region of the most suitable system time response. The assessment was done by a visual inspection of the sample test results. The second technique assessed the system performances in the stable region by
evaluating an error criterion. Depending on the definition of the criterion, it was either minimized or maximized. The controller settings which corresponded to the desired extreme value of the criterion were selected as the point for operation. The most recent method employs complex variable analysis to determine how the various system adjustable parameters affect the system modes of response. The parameter combination which provides a desired system damping and frequency is selected for system operation. In the subsequent sections each of these analytical approaches is reviewed and the significant features applicable to the present study are discussed.

1.2.1 Stability boundary analysis

The Routh-Hurwitz criterion for stability can be employed to determine the stability limit of a linear system. Its application requires the knowledge of the system characteristic equation. The Routh array is established and the conditions for stability are then determined. The stability boundary is determined by a set of conditions which give at least one response component with zero damping. In this section and subsequent work it may be assumed that this technique is used to determine the stability boundary of the system being considered.

I.A. Vyshnёgradskii considered the performance of a
general third order system. Similar analytical methods have been used recently by power system engineers. Before establishing the stability boundary the system characteristic equation is normalized and the Laplace operator is transformed. This technique is illustrated hereunder.

Equation 1.1 is the characteristic equation of a third order system. If the equation were divided by \(a_0\) and a new operator \(q = \sqrt[3]{\frac{a_3}{a_0}} s\) introduced, the system characteristic equation, as a function of the \(q\) operator, would become Equation 1.2,

\[
q^3 + Aq^2 + Bq + 1 = 0
\]

where \(A = \frac{a_2}{3\sqrt[3]{a_0a_3^2}}\) and \(B = \frac{a_1}{3\sqrt[3]{a_0^2a_3}}\). The system stability limit is plotted in Figure 1.1 as a function of \(A\) and \(B\). Vyshnegradskii divided the stable region of operation into three areas distinguished by the relative position of the roots on the complex \(q\) plane. The location of the roots and the boundaries demarcating the various modes of response are illustrated in Figure 1.1. Several features of this analysis are worth noting. Firstly, the ratios \(A\) and \(B\) are dimensionless, the value of this becomes obvious in the
FIG-1.1 VYSHNEGRADSKII'S HYPERBOLA FOR A THIRD ORDER SYSTEM.

FIG-1.2 PAYNTER'S STABILITY BOUNDARY OF A HYDRO POWER STATION.
material which follows. Secondly, irrespective of the magnitudes of $a_3$, $a_2$, $a_1$ and $a_0$, a specific combination of $A$ and $B$ defines a unique set of characteristic equation roots. Lastly, this analysis determines what the modes of system response are but not the actual system time response.

A similar transformational technique was utilized by Paynter\(^7\) to find the optimum governor settings for a hydroelectric power system. The characteristic equation of the linear system model which he studies was third order. In this case the dimensionless ratios are $\lambda_1$ and $\lambda_2$ which are functions of the governor and system parameters (see Chapter 2). The stability boundary as a function of $\lambda_1$ and $\lambda_2$ is given in Figure 1.2. The magnitudes of governor parameters which should be chosen such that optimum quality of frequency regulation results are selected as follows. The time response of the system frequency to a step change in load is recorded for various settings of $\lambda_1$ and $\lambda_2$. The curves obtained are visually examined and the one which most closely resembles a damped sinusoid with a damping ratio of .707 is selected as the optimum. This establishes Paynter's criterion for optimum quality of frequency regulation as $\lambda_1 = .5$ and $\lambda_2 = .2$.

The application of this method by Hovey\(^8\) to an actual power system confirms the practical value of this criterion. Hovey did however note that under certain conditions the
suggested settings had to be altered. He recognized that the amount of change needed for those defined by Paynter's criterion depended on several factors. These factors are discussed in detail in Chapter 3.

The system stability limit is the condition when one or more of the roots of the characteristic equation has a zero real part (the real parts of the remaining roots are negative). If the position of the imaginary axis of the complex plane is shifted to the left, i.e. a new complex variable \( v = q - \alpha \) is introduced, and for each incremental \( \alpha \) the conditions for stability are found, a series of contours within the Vyshnegradskii hyperbola would result. Each contour defines the system parameter magnitudes which correspond with a constant damping of the characteristic equation root nearest the imaginary axis. There exists a maximum value of \( \alpha \) beyond which stability of the system cannot be achieved irrespective of system parameter adjustment. This type of analysis may be applied to any system, Figure 1.3 illustrates the results obtained by Tsypkin and Bromberg\(^9\) for a third order process.

Maximum stability occurs when the time response component with the smallest damping exponent is located on the complex plane at its left-most position. The selection of the greatest or a predetermined degree of damping can be used as a criterion for controller settings. Point C of Figure 1.3, where all three roots are equal, represents the condition of
FIG. 1.3 TSYPKIN AND BROMBERG’S EQUAL DAMPING ANALYSIS OF A THIRD ORDER SYSTEM.

FIG. 1.4 STEIN’S OPTIMUM CRITERION FOR HYDRO-ELECTRIC POWER STATION CONTROL.

FIG. 1.5 OPTIMUM GOVERNOR SETTINGS ACCORDING TO DIFFERENT INTEGRAL ERROR CRITERIA.
maximum damping.

1.2.2 Root-locus analysis

The root-locus technique can be used to determine the variation of the roots of the system characteristic equation as a function of various system parameters. These roots are plotted on a complex variable plane and thus for a particular unit setting it is possible to determine the frequency and damping factor of all the time response components. It differs from Vyshnegradskii's analysis in that it gives also the degree of stability. In addition, it permits the systematic examination of how various unit parameters affect the degree of system stability.

Some of the more significant contributions to power system analysis by the root-locus method have been made by Van Ness\textsuperscript{10}. A procedure for determining the root-locus of a large interconnected system has been established and a method by which the rate of change of the roots as a function of a selected system parameter has been described (so-called "sensitivity analysis"). The combination of these two features make the method of Van Ness invaluable for maximizing the damping of a system. In a general control system and in particular in the frequency control of a power system the condition of maximum damping (as discussed above) is not synonymous with optimum quality of regulation. Therefore
the method as described is not used here as a criterion for quality of regulation.

Meloy has suggested a variation of Van Ness's method by which he could select governor settings to give quality regulation of generator frequency. He defined a system performance criterion which is a function of the system stability and response. He then selected the combination of governor parameter values (after examining the root-locus) which best satisfied his criterion. When this is applied to a practical power system, the quality of regulation is better than that obtained by the maximum damping criterion.

1.2.3 Quality assessing criteria

A method which resembles that of Tsypkin and Bromberg but gives an effective means of calculating the controller settings required for quality frequency regulation has been devised by Stein. The time response of the output variable (frequency deviation) given a step change in operating conditions of a third order system can be written as

\[ y(t) = C_1 e^{-\gamma t} \sin \omega t + C_2 e^{-\mu t} \] (C_1, C_2, \gamma and \mu are functions of the system parameters). Stein determined, for different operating conditions, the time required for the exponential terms of y(t) to decay to a certain amplitude. Contours denoting the time required at various values of a_1 and a_2 (a_1 and a_2 are the normalized coefficients of Stein's equation) for the
exponentials to decay to 0.1 are illustrated in Figure 1.4. The minimum time required was selected as the optimum. This minimum does not correspond to the minimum of Figure 1.3; i.e. minimum time to equilibrium does not correspond with maximum damping.

Stein's method for selecting governor settings is based on the direct measurement of the time required for the transient signal to reduce to a predetermined value. An indirect scheme of performance evaluation can be achieved by integrating the deviation of the output signal or some function of the error with respect to time. Some criteria which have been applied to process control\textsuperscript{13} are $\int c \, dt$, $\int |c| \, dt$, $\int c^2 \, dt$ and $\int c^{2n} \, dt$, $n$ and $m$ are integers. The adoption of governor settings to give a minimum for any one of these criteria results in performance near the stability limit. Consequently for power system analysis, these criteria are not suitable for the determination of controller settings which would give quality regulation. Figure 1.5 illustrates the location and the associated time responses of the suggested setting as determined by three different criteria.

The aforementioned error criteria are functions of only the error and time. If the derivatives of the error are available, various other criteria can be suggested. Of these the most common employs the error and its first derivative, $\int (c^2 + c_2(c')^2) \, dt$.\textsuperscript{14} Considering a specific case the
correct choice of the weighting factor $C_3$ would result in improved quality; i.e. nearer the results of Stein. However no technique exists at the present time by which a value of $"C_3"$ can be selected for the general case.

Krasovskii\textsuperscript{15} has shown that if the desired process response can be described by a differential equation, then a criterion can be established which will lead to this response. He has described a general error criterion which permits a process to be optimized such that, given a specific disturbance, the output time response will be of a certain form. It is shown in Chapter 5 of this thesis that a criterion can be established which is a minimum when the system time response is a critically damped sinusoid.

In summary, past researchers have concentrated their efforts in three areas. The results though applicable to certain cases demonstrate short-comings insofar as the general problem of quality regulation is concerned. The advantages and disadvantages of each are discussed in the following section.

1.3 Discussion of Quality Criteria

The Vyshnegradskii's hyperbola provides a convenient means of evaluating the stability of hydro stations which can be represented by third order models. The performance of such a station can be represented as a function of two variables.
The point where the three roots are equal (point C in Figure 1.3) can be chosen for governor settings, this gives near-optimum quality regulation.

When applying a method similar to that used by Vyshnegradskii the number of variables is equal to n-1 where n is the order of the characteristic equation. Often a hydro-electric generating station cannot be adequately represented as a third order system, therefore more variables must be introduced. It is cumbersome to represent the system stability on a plane for more than two variables. The ratios as determined by the Vyshnegradskii method are normally a function of system parameters raised to some fractional power. This makes the manipulation of the ratios and interpretation of the results difficult. The analyses indicate regions of stability and instability and not the degree of stability. The damping factors of the roots of the characteristic equation are not given for various system operating conditions.

Another disadvantage of this analysis is that it considers only the characteristic equation, which in itself cannot give complete information about the system time response. Any criterion which determines quality settings of the system controllers must be a function of the system time response.

The work of Paynter, though similar to that of Vyshnegradskii solved the problem of the awkward forms of the dimensionless ratios. He transformed the system character-
istic equation (which is third order) in such a fashion that \( \lambda_1 \) and \( \lambda_2 \) become functions of governor and system parameters raised to the first power. In Chapter 3 of this thesis Paynter's method is extended to the analysis of higher order systems. The disadvantage of this technique is that it is also an analysis of the characteristic equation and does not account for the total description of the time response.

The Vyshnegradskii hyperbola does not give the degree of system stability at various controller settings. The analysis of Tsypkin and Bromberg permits the determination of the damping exponent of the least damped root at various system settings. For determining quality regulation of a system, this method is better than Vyshnegradskii's since a certain degree of stability can be selected. Its usefulness for determining quality regulation is restricted since it also does not contain complete time response information.

The curves obtained by Tsypkin and Bromberg specify the damping exponent of the least damped root or roots at various system conditions. The root-locus method of analysis provides a facility to determine the frequency and damping of all the roots for any system condition. Since this analysis determines the behavior of all the roots, it is possible under certain conditions to use this information to make a better estimate of system performance.

For large interconnected systems, Van Ness has developed
a procedure for determining the root-locus. However, as the complexity of the model increases the root-locus becomes a maze of curves and it becomes extremely difficult from the examination of the loci to predict what system conditions correspond to optimum regulation. Meloy has described a means of assigning varying degrees of significance to the various loci. Based on his interpretation a particular setting is selected. The frequency regulation process achieved by this method is characterized by a lightly damped sinusoidal response. Another method employing root-locus techniques is described in Chapter 4.

The quality criteria which are functions of the system transient response are the most useful. The criteria define figures of merit for various responses. By proper selection of the criterion it is possible to segregate a particular response (the optimum) from amongst the total. Thus, for the determination of optimum quality of frequency regulation a criterion is to be calculated which is an extremum for an optimumly damped sinusoid.

The dilemma in applying this technique is the selection of an appropriate criterion. Two different types have been described, that as employed by Stein and those which are integrals of some function of the error signal.

Stein's calculation which defines the time required for the transient to reduce to a certain percentage of its maximum amplitude is a direct time measurement of the
regulating process. The results obtained are of engineering value, however, a considerable amount of calculating effort is required. For fourth and higher order systems, the analytical work associated with the necessary calculations becomes formidable. Stein's method though exacting in its results is cumbersome in its application.

The integral of some function of the error is another means of measuring the system performance since the final result is dependent on signal deviation and duration of the regulating process. The problem of controller parameter setting selection reduces to a multivariable optimization search. The criteria which are functions of only the error signal are a minimum in a region near the stability boundary of the system. This corresponds to system operation with an overly active governor and thus undesirable for field application. The merit of using this type of technique is its ease of implementation.

The theorem of Krasovskii permits the establishment of an error criterion that is a minimum when the system response is of a predetermined form. The disadvantage in applying this theorem is that the desired final solution must be known beforehand. From this information (the form of the final solution) it is then possible to establish the criterion. Often in the problem of power system frequency regulation, the engineer does not know the exact form of the response.
sought. The derivatives of the error signal must also be available.

The research reported in this thesis is a model analysis and for this case the required derivatives are available without differentiations. Chapter 5 describes how an error criterion which minimizes for an optimumly damped sinusoid can be selected. This method provides a facility by which quality regulation of a hydro-electric power station can be obtained.

1.4 Outline of Research and Presentation in Thesis

The purpose of the author's research is to improve the system performance under normal operating conditions. This state of operation is characterized by the occurrence of only small load changes. The variation in system state variables is relatively small, consequently a linear time invariant model can be used to represent the system.

In brief, the objective of the investigation discussed in this thesis is to develop and demonstrate methods by which optimum quality regulation of frequency of a single hydro-electric generating station can be achieved. This is accomplished by the selection of parameter settings of frequency controllers which are in use at the present time.

The technical content of this thesis is divided into four areas, each constitutes a single chapter. Chapter 2 develops
the model used in the investigation. Chapter 3 is devoted to the calculation of the stability limits as influenced by operating conditions and parameter magnitudes. The following chapter discusses the application of root-locus techniques and the next chapter illustrates the adaptation of Krasovskii's theorem for the determination of quality regulation. An assessment of the suggested quality criteria is given. Limitations of present frequency controllers as applied to the present problem are discussed. Lastly, comments pertaining to future controller design are made.
2. THE SYSTEM MODEL

2.1 General

In the introduction it was stated that the objective of this research was to determine methods for selecting the optimum settings of present power system frequency controllers. This study is therefore one of analysis. Fundamental to any effective analysis is the need for an accurate model on which the investigation can be conducted. If the description of such a model is not known, then the actual system must be used in the experimentation. By using the actual system as part of the experiment, the analyst avoids the errors associated with modelling. In the use of the real system, the benefits of accuracy are offset by difficulties in measurement, inter-facing, and ease of testing a broad range of conditions. The experimentation on hydro station equipment, because of cost and possible consequences, requires cautious and planned procedures, consequently it is more convenient to employ a model of the actual system. The analyst conducts the tests on a model, solves for the optimum and then transfers the results to the actual system.

The use of analogue and digital computers in the analysis of the model is advantageous for several reasons.
This increases the speed of tests, flexibility of test conditions, repetitiveness of tests, testing of adverse conditions and number of situations examined. In addition, a relatively accurate representation of the system is known. Due to the above reasons, the author employed a model representation for analysis.

In this chapter the mathematical representation of the system is presented. The model characterizes the considerations necessary to determine the frequency of a hydro-electric power station. In this model no voltage control effects are included, even though it is appreciated that there is interaction between the voltage and frequency in a power generation scheme. Experience has shown that significant engineering results are still obtained if interaction is neglected.

The schematic diagram of the system defining the frequency of a single machine is illustrated in Figure 2.1. The main components are the frequency controller (governor), the prime-mover (penstock and turbine), the inertia of the generator and prime-mover, the transmission network and the electrical load. In each component only the factors which significantly affect the frequency are examined (i.e. torque, inertia, damping, power transfer, etc.). In several portions of this thesis, the schematic is modified according to the considerations being made in that particular section.

Before the transfer functions of the components in
FIG. 2. BLOCK DIAGRAM OF THE FREQUENCY DEFINING COMPONENTS OF A HYDRO-ELECTRIC POWER STATION.

UNIT FREQUENCY

ENERGY SOURCE

UNIT TORQUE ANGLE

INERTIA OF GENERATOR AND PRIME MOVER

ACCELERATING TORQUE

PRIME MOVER

INPUT TORQUE

GATE OR VALVE POSITION

GOVERNOR

REFERENCE FREQUENCY

TRANSMISSION LINE

TORQUE AT OTHER END OF LINE
Figure 2.1 are given, it is of considerable interest and value to examine the performance of a single machine in an interconnected power system. An appreciation of the interaction of a single unit with the remainder of the power grid permits an assessment to be made of what factors should be considered in single unit studies. At this point it is adequate to state that the tie-line influences the final optimum governor setting. The tie-line phenomena also illustrate the need for new design concepts in frequency controller synthesis.

2.2 Frequency Characteristics of an Interconnected System

For illustrative purposes a rudimentary analogue can be used to describe the frequency characteristics of an interconnected power system. Refer to Figure 2.2. The system electrical angle is represented by a horizontal sheet of elastic material. The frequency is given by a constant base frequency \( f_0 \) plus the rate of change in the vertical position of the sheet. Loads are shown as forces pushing downward on the sheet while generation centres are upward forces. The load and generation characteristics effecting frequency are represented by a mass, dashpot and spring combination. The governors are similarly expressed.

The first derivative of torque angle is transmitted as
FIG-2.2 MECHANICAL ANALOG OF AN INTERCONNECTED POWER SYSTEM.
an error signal to the governors. Because of the energy storage and time lag characteristics of the loads, generators and governors and the coupling features of the elastic sheet an oscillation in torque angle will occur. The electrical torque angle deviations at one load centre affect not only the nearest generator but are radiated to other load and generating points. Depending on the characteristics of the participating system components, varying degrees of oscillation within the entire system will result. The power system frequency control problem as is discussed in this thesis is analogous to the selection of mass, spring and dashpot magnitudes of the frequency controllers such that the torque angle variation is damped in an optimum fashion. In the real system the omnidirectional energy coupling between the loads and the generators of the above model is replaced by discrete paths via transmission lines. A better analogue is one where the sheet is replaced by elastic strings and the elasticity of the links is made proportional to the degree of electrical coupling which exists in the real system.

In the present investigation the area in the immediate vicinity of a generating point and the nearest load centre is isolated from the remainder of the system. The study considers the procedure of adjusting the particular governor such that frequency oscillations are damped in quality fashion. Another consideration is the adjustment of the controllers
given a disturbance from outside the area. The coupling to
the generator with the outside disturbance is by a trans-
mission line.

The remainder of this chapter presents the representation
of a single machine feeding an isolated load with and with-
out coupling to the remainder of the system.

2.3 Transfer Functions of a Single Unit

In this section the transfer functions of the machine
components are given. The assumptions which are inherent in
this type of representation are also stated.

One of the objectives of the author's research was that
the techniques developed are to be applicable to present-day
facilities. Most of the electrical power generated in Canada
is produced using hydraulic prime movers, thermal prime movers
are the second most important. Of the two schemes of genera-
tion, the control of frequency of a hydro station is more
difficult than that of a thermal station. The penstock, in
addition to causing a time lag, introduces an initial response
opposite to system demand. It has been assumed that if tech-
niques can be developed which give good quality regulation
for hydro systems, similar schemes can be extended to thermal
units. In Chapter 5, on the method of Krasovskii, a general
method for obtaining optimum settings for frequency controllers
is described.
The block diagram of the frequency control loop of a hydro station is illustrated in Figure 2.3. The transfer functions of each block and the list of related assumptions is given below. The transfer functions, expressed in per unit form for each block are given hereunder.

1) Mechanical-hydraulic temporary droop type governor\(^16\).

\[
\frac{G}{G_o}(S) \left/ \frac{\Delta \omega}{\omega_o}(S) \right. = \frac{(T_r S + 1)}{T_r T_s S^2 + (T_s + \delta T_r + \sigma T_r)S + \sigma} \quad 2.1
\]

2) Electro-hydraulic three-mode (P-I-D) without the permanent droop feedback loop included\(^17\).

\[
\frac{G}{G_o}(S) \left/ \frac{\Delta \omega}{\omega_o}(S) - \sigma \left( \frac{G}{G_o}(S) \right) \right. = \frac{K'(T_s S + 1)(T_2 S + 1)}{S T_2(.05S + 1)^2(.018 + 0.2\xi S + 1)} \quad 2.2
\]

3) Penstock and hydraulic turbine\(^3\).

\[
\frac{T_t}{T_{to}}(S) \left/ \frac{G}{G_o}(S) \right. = \frac{1 - T_w S}{1 + T_w S} \quad 2.3
\]

4) Spinning inertia\(^3\).

\[
\frac{\omega}{\omega_o}(S) \left/ \frac{T_{ac}}{T_{aco}}(S) \right. = \frac{1}{T_m S} \quad 2.4
\]
FIG.-2.3 BLOCK DIAGRAM OF THE FREQUENCY CONTROL LOOP OF A HYDRO-ELECTRIC POWER STATION.
5) Tie-line coupling\(^3\).

\[
\frac{T_{tl}}{T_{tlo}}(S) \left/ \left( \frac{\omega}{\omega_0}(S) - \frac{\omega^*}{\omega_0}(S) \right) \right. = \frac{K}{S} \quad 2.5
\]

6) System damping\(^3\).

\[
\frac{T_d}{T_{do}}(S) \left/ \left( \frac{\omega}{\omega_0}(S) - \frac{\omega^*}{\omega_0}(S) \right) \right. = D \quad 2.6
\]

The assumptions inherent in this type of representation are:

1. a linear system representation is valid; this implies that only small signal disturbances are to be considered,
2. turbine torque is proportional to gate position,
3. water and conduit elasticity have negligible effect on the transfer function of the turbine and penstock,
4. all damping torques due to prime mover, generator and system vary linearly with frequency,
5. the speed control system is independent of the voltage control system, and
6. a lossless transmission network is employed.
Throughout this study, the system as illustrated in figure 2.3 or slight variations of it will be used as the model for analysis. For representation of a thermal generating station the reader is referred to Kirchmayer.\(^3\)

2.4 Establishment of a Single Machine Equivalent

Once the performance of the individual units of a interconnected power system is thoroughly appreciated and techniques have been developed by which quality regulation of frequency can be achieved, it will then be necessary to study the behavior of the interconnected system. For the moment assume that the performances of the participating units has been optimized on an individual operating basis, the units are interconnected and the characteristics of the overall system are to be studied. Because of analytical resource limitations, it is often necessary to reduce the complexity of a large system. One technique commonly utilized for the reduction of the complexity in system dynamics studies is the replacement of parallel operating units by a single machine equivalent. Appendix A describes a method for the selection of the "most significant" single machine equivalent for a multi-machine bank. The "most significant" is defined as that which satisfies a specific comparison criterion.

In Appendix A it is shown that a single machine equivalent of a multi-machine bank can be chosen such that the
steady-state operating conditions are the same as that of
the original bank of machines and the transient response of
the model approximates that of the original. The procedure
followed is to calculate the magnitudes of the model para-
eters which cause a duplication of the steady-state con-
ditions; the remainder are then adjusted such that the
transient response of the model follows as "close as possible"
the transient of the real. Formulae are given by which the
steady-state defining parameters are determined. A systematic
scheme by which the values of the remaining parameters are
chosen is also given.

It is shown that an exact single machine equivalent is
possible only in certain circumstances (i.e. given any load
disturbance, the time response of the equivalent is the same
as that of the original). In a bank of n parallel operating
machines mutual energy exchanges can occur amongst the units
in addition to the energy exchange with the remainder of the
system. However, when a single machine is substituted for
the bank, only one mode of energy exchange can exist (i.e.
between it and the remainder of the system). These energy
oscillations cause acceleration and deceleration of the respect-
ive inertias and thus introduce frequency variations. The
replacement of a bank by a single equivalent results in the
filtering of inter-unit oscillations.

In Appendix A it is indicated when units can be lumped
without significantly altering the system performance. The converse is also illustrated, that is, it is shown when the behavior is materially affected and thus a single machine equivalent should not be used. Engineering judgement is required to determine when this simplifying procedure can be of benefit.

It is emphasized that the concept of employing a single machine equivalent is not new. The values of the parameters of the equivalent are often assumed to be equal to the averages of the parameter values of the actual machines operating in parallel. The advantages of using the optimizing method in lieu of the averaging method is given in Appendix A. It is concluded that this technique does provide a means of establishing a significant single machine equivalent. It illustrates the conceptual limitations of such a single machine equivalent.
3. SINGLE MACHINE STABILITY ANALYSIS

3.1 Reasons for Stability Analysis

All the possible system modes of operation can be defined by points in an n-dimensional space, where n is the number of adjustable system parameters. Before searching for the particular point in this hyperspace which corresponds to quality regulation of frequency, it is advantageous to first consider the description of the surface in the n-dimensional region which circumscribes stable system operation. Knowledge of stability limit information permits a reduction of the region in which the search for optimum settings is to be concentrated, it indicates whether such a quality setting might exist and it illustrates how various parameters influence system performance and thus indirectly indicates how the quality of regulation is affected. This analysis is conducted only for a system controlled by a mechanical-hydraulic temporary droop governor. The analysis of a system employing a different controller is analogous.

For a generator controlled by a mechanical-hydraulic temporary droop governor a two-dimensional hyperspace is adequate to describe system performance since temporary droop \( (6) \) and dashpot relaxation time \( (T_p) \) are the only readily
adjustable parameters. The method used in this thesis to determine the regions of stability is the same as that employed by Paynter.

The work of Paynter and Hovey consider the stability limits of a single machine feeding an isolated load and having no system damping, no permanent droop and no servomotor lag. The author's research extends this analysis and investigates the effects on the single unit stability limit when these factors are introduced in the system description. In this chapter the characteristic equation is determined for the system illustrated in Figure 2.3. This model includes the following which Paynter's model did not; system damping, coupling to an infinite bus, permanent droop feedback and a finite servomotor time. Various forms (i.e. including some or all of these) of the model are examined. In each case the stability limit as a function of the governor settings is evaluated.

3.2 Characteristic Equation Normalization

In determining the stability limits of his model, Paynter first manipulated the characteristic equation coefficients such that they became functions of two dimensionless ratios rather than the system parameters. His model was a function of four parameters. He then transformed the Laplace operator \( s \). This normalization permitted a reduction in the number of
system variables, relative ease of comparison with different units and the specification of optimum governor settings in terms of a constant set of ratios.

A similar approach is taken in this study. With the introduction of each new parameter in the system description, another dimensionless ratio is defined. The ratios are selected in such a fashion that each is a function of only one adjustable governor parameter (this simplifies the establishment of the unit stability boundary). The equations for these ratios are given by Equation 3.3.

Equation 3.1 is the characteristic equation of a single machine as considered by Paynter. The unit is feeding an isolated load and has zero system damping (D), permanent droop (σ) and servomotor time (Tₛ).

\[ \frac{\delta T_m T_r T_w}{2} S^3 + (\delta T_r T_m T_w) S^2 + (T_r T_w) S + 1 = 0 \quad 3.1 \]

λ₁ and λ₂, the previously mentioned dimensionless ratios, are defined as \( \lambda_1 = T_w / \delta T_m \) and \( \lambda_2 = T_w / T_r \). Substituting these into Equation 3.1 and replacing the Laplace operator by \( S = q / T_w \), the characteristic equation takes on the form given by Equation 3.2.

\[ \frac{q^3}{2\lambda_1\lambda_2} + \left( \frac{1}{\lambda_1\lambda_2} - \frac{1}{\lambda_2} \right) q^2 + \left( \frac{1}{\lambda_2} - 1 \right) q + 1 = 0 \quad 3.2 \]
The system stability limit can be plotted as a function of $\lambda_1$ and $\lambda_2$. Note that $\lambda_1$ is a function of one of the governor parameters while $\lambda_2$ is a function of the other.

A different set of ratios and $S$ transformation could have been used. Another possible set is $\beta_1 = T_r/T_m$, $\beta_2 = T_w/T_m$, $\beta_3 = \delta T_r/T_m$ and $S = P/T_m$, where $P$ is a complex variable. The disadvantages of this form of normalization is that three variables result of which one is a function of both $\delta$ and $T_r$. In fact, numerous combinations of ratios and transformations can be suggested, however, the one initially chosen by Paynter will be used as a basis in this study.

In passing, it should be pointed out that if, in the derivation of the system transfer functions, a transformation of the form $f(q) = \int_0^\infty f(t)e^{-\frac{q}{T_w}t} dt$ were used instead of the standard Laplace transform and dimensionless ratios $\lambda_1$ and $\lambda_2$ were substituted, then the resulting system characteristic equation would be Equation 3.2.

Equation 3.4 is the characteristic equation of the model illustrated in Figure 2.3 in terms of the Laplace operator $S$ and the system parameters. Various forms of this equation expressed in terms of $\lambda_n$, $n = 1, 2, \ldots, 6$ and the operator $q$ where $S = q/T_w$ are also listed. The equations of $\lambda_n$ are given in Equation 3.3. Note that each is a function of only one variable governor parameter. The definition of $\lambda_1$ and $\lambda_2$ is left as defined by Paynter, thus permitting
comparison of these results with those previously obtained.

\[ \lambda_1 = \frac{T_w}{\delta T_m} \quad \lambda_2 = \frac{T_w}{T_r} \quad \lambda_3 = 2\delta T_w/T_s \quad \lambda_4 = \sigma/\delta \quad \lambda_5 = 1/\delta D \quad \lambda_6 = 1/\delta KT_w \]

The characteristic equation of the model as illustrated in Figure 2.3 in terms of the Laplace operator \( S \) and system parameters is given by Equation 3.4.

\[
\frac{T_s T_r T_m T_w}{2} S^5 + \left( \frac{T_s T_r (T_w D + T_m)}{2} + T_m \right) + \frac{T_m T_w}{2} (T_s + \delta T_r + \sigma T_r) S^4 \\
+ (T_s T_r (T_w K + D) + \frac{T_w D + T_m}{2})(T_s + \delta T_r + \sigma T_r) + \delta T_m T_w - T_w T_r) S^3 \\
+ (T_s T_r K + (T_w K + D)(T_s + \delta T_r + \sigma T_r) + \sigma(T_w D + T_m) + (T_r - T_w)) S^2 \\
+ (K(T_s + \delta T_r + \sigma T_r) + \sigma(T_w K + D) + 1) S \\
+(\sigma K) = 0
\]

To determine the characteristic equation of a single machine for any combination of the following, isolated load operation, no system damping, no permanent droop and/or zero servomotor time, the appropriate parameters are set to zero. Next the ratios for \( \lambda_n \) and \( S = q/T_w \) are substituted into the modified form of Equation 3.4.

The normalized characteristic equation of a single machine feeding an isolated load with \( T_s = 0, D = 0 \) and \( \sigma = 0 \) (Hovey and Fayniter's model) as obtained by operating on
Equation 3.4 is given by Equation 3.5.

\[ q^3 + (2-2\lambda_1)q^2 + (2\lambda_1 - 2\lambda_1\lambda_2)q + 2\lambda_1\lambda_2 = 0 \]  

Equation 3.5.

The normalized characteristic equation for a single machine feeding an isolated load with finite servomotor time \( T_s \) and permanent droop \( \sigma \) and zero system damping \( D \) is given by Equation 3.6.

\[ q^4 + (\lambda_2 + \lambda_3 + \frac{\lambda_3\lambda_4}{2} + 2)q^3 + (2\lambda_2 + \lambda_3 + \lambda_4 + \frac{\lambda_2\lambda_3\lambda_4}{2} - \lambda_1\lambda_3)q^2 + (\lambda_1\lambda_3 + \lambda_2\lambda_3\lambda_4 - \lambda_1\lambda_2\lambda_3)q + \lambda_1\lambda_2\lambda_3 = 0 \]

Equation 3.6.

The normalized characteristic equation of a unit as described by Equation 3.4 is given by Equation 3.7.

\[ \frac{q^5}{\lambda_1\lambda_2\lambda_3} + \left(\frac{1}{\lambda_2\lambda_3\lambda_5} + \frac{2}{\lambda_1\lambda_2\lambda_3} + \frac{1}{\lambda_1\lambda_3} + \frac{1}{2\lambda_1\lambda_2} + \frac{\lambda_4}{2\lambda_1\lambda_2}\right)q^4 + \left(\frac{1}{\lambda_2\lambda_3\lambda_6} + \frac{2}{\lambda_2\lambda_5} + \frac{1}{\lambda_3\lambda_5} + \frac{\lambda_4}{2\lambda_2\lambda_5} + \frac{1}{\lambda_1\lambda_2} + \frac{\lambda_3\lambda_5}{\lambda_3\lambda_5} + \frac{\lambda_4}{\lambda_1\lambda_2} + \frac{\lambda_4}{\lambda_1\lambda_2} - \frac{1}{\lambda_2}\right)q^3 + \left(\frac{2}{\lambda_2\lambda_3\lambda_6} + \frac{1}{\lambda_2\lambda_6} + \frac{\lambda_4}{\lambda_2\lambda_6} + \frac{\lambda_4}{2\lambda_5} + \frac{\lambda_4}{\lambda_1\lambda_2} + \frac{1}{\lambda_2} - \lambda_1\lambda_2\lambda_3\lambda_4\right)q^2 + \left(\frac{2}{\lambda_3\lambda_6} + \frac{1}{\lambda_2\lambda_6} + \frac{\lambda_4}{\lambda_2\lambda_6} + \frac{\lambda_4}{2\lambda_6} + \frac{\lambda_4}{\lambda_5} + 1\right)q + \frac{\lambda_4}{\lambda_6} = 0 \]

Equation 3.7.

In this investigation the stability limits of Equations 3.6 and 3.7 are determined as functions of \( \lambda_1 \) to \( \lambda_4 \) and \( \lambda_1 \) to \( \lambda_6 \).
respectively.

The normalized characteristic equations (Equations 3.5, 3.6 and 3.7) are given for specific models, namely various forms of a generating station using hydro as its prime mover and employing a temporary droop type of governor. This analysis could as readily be applied to a steam driven station or a hydro station controlled by an electro-hydraulic type of governor (or any other realistic system). The resultant normalized equation depends on the model studied and the transformation used. The work of Vyshnegradskii illustrates one means of normalization; the method of conformal mapping is another.

3.3 Determination of the Stability Limits

The stability limits of a linear process can be established by operating on the system characteristic equation using the Routh-Hurwitz criterion. The stability boundary in Paynter's analysis and in this thesis is plotted as a function of \( \lambda_1 \) and \( \lambda_2 \) (this being sufficient to define the setting of a temporary droop hydraulic governor).

The stability limit as a function of \( \lambda_1 \) and \( \lambda_2 \) can be calculated for any combination of constant \( \lambda_3, \lambda_4, \lambda_5, \lambda_6 \) or it can be determined for a particular machine. In the analysis of a specific machine, parameters other than \( \delta \) and \( T_r \) (\( \sigma \), \( T_\delta \), \( D \) and \( K \)) are fixed and consequently as \( \lambda_1 \) varies...
so do \( \lambda_3, \lambda_4, \lambda_5, \) and \( \lambda_6 \).

In the case where \( \lambda_3, \lambda_4, \lambda_5 \) and \( \lambda_6 \) are constants the magnitude of \( \lambda_1 \) at the stability limit can be calculated as \( \lambda_2 \) is incremented in the range of interest. It will be found that \( \lambda_1 \) varies as a function of \( \lambda_2 \) and if \( \lambda_3 \) to \( \lambda_6 \) were held constant then this means that the results are not applicable to a specific machine. In the second case, for a specific machine, the value of \( \lambda_1 \) determines the magnitude of \( \lambda_3 \) to \( \lambda_6 \). The information available from each method is illustrated in the subsequent sections.

For details as to how the Routh-Hurwitz criterion was employed the reader is referred to Appendix B. Here formulae are given by which \( \lambda_1 \) is calculated for constant \( \lambda_2 \) to \( \lambda_6 \). An iterative solution of \( \lambda_1 \) for a specific machine is also described.

3.3.1 Stability limit variations with constant \( \lambda_3 \) and \( \lambda_4 \)

The stability effects of a finite servomotor time \((T_s)\) and permanent droop \((\sigma)\) was studied by substituting numerical values for \( \lambda_3 \) and \( \lambda_4 \) into the system characteristic equation (Equation 3.6). Due to the large number of calculations involved, a digital computer programme was used and the resultant stability boundary was shown on a \( \lambda_1 \lambda_2 \) plane (refer to Figure 3.1).
FIG. 3.1 STABILITY LIMIT VARIATIONS DUE TO SERVOMOTOR OPERATING TIME, A FINITE $\lambda_3$.

FIG. 3.2 STABILITY LIMIT VARIATIONS DUE TO PERMANENT DROOP, A FINITE $\lambda_4$.

FIG. 3.3 STABILITY LIMITS VARIATIONS DUE TO SERVOMOTOR OPERATING TIME AND PERMANENT DROOP.
The effects of a finite servomotor time constant and permanent droop is represented by Figures 3.1, 3.2 and 3.3. Figure 3.1 illustrates the effect of a finite servomotor time (permanent droop is zero), the unstabilizing effects of this is obvious. The curve labelled PSC is the stability curve of Paynter's model. In Figure 3.2, the effects of a finite permanent droop are illustrated. As would be predicted, the permanent droop contributes to the unit stability (the area of stable operation increases). Figure 3.3 is a combination of the results of Figure 3.1 and 3.2 (i.e., a finite permanent droop and servomotor time). In all figures, the stability limit curve obtained by Paynter (PSC) is included for comparison purposes. These results illustrate that the servomotor time constant and permanent droop have a pronounced effect on the unit stability limit and therefore cannot be disregarded in studies of quality regulation of hydro-electric power generator frequency.

While these curves (Figure 3.1, 3.2 and 3.3) show a significantly altered area of stable operation, it must be realized that these results are for constant $\lambda_3$ and $\lambda_4$ and not for a particular unit. To clarify, given a unit with the following parameter values $T_m = 10.0$ sec., $T_w = 1.0$ sec., $\sigma = .05$ p.u., and $T_s = .05$ sec. the value of $\lambda_3$ varies from 4 to 40 and $\lambda_4$ varies from .5 to .05 as $\delta$ is changed from .1 p.u. to 1 p.u. Thus as $\lambda_1$ is decreased by increasing the
temporary droop) the magnitude of \( \lambda_3 \) is increased and \( \lambda_4 \) is decreased. The net result if considering a specific unit would be a smaller region of stable operation than that given in Figure 3.3. The stability boundary of a particular unit is illustrated in section 3.3.2.

Three different response curves are given in Figure 3.4. For each, the magnitudes of \( T_m, T_w, T_r, T_s, \delta \) and \( \sigma \) were chosen such that \( \lambda_1 = 0.7, \lambda_2 = 0.2, \lambda_3 = 20.0 \) and \( \lambda_4 = 0.1 \). It is to be noted that the amplitudes and frequencies of the time responses are different but the damping ratios are approximately the same.

Fixing the values of \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) defines a unique set of system characteristic equation roots (in terms of the complex variable \( q \)). For the system with a \( T_w = 2.0 \), the frequency of the time response is one half of that for the system with \( T_w = 1.0 \). This is because in the normalization of the characteristic equation the complex variable \( q = ST_w \), thus for a constant \( q \), \( S \) (which defines the frequency of the time response) must vary inversely as \( T_w \).

The difference in the amplitude of the transient response of the second and third case (\( T_m = 2.38 \) sec. and \( T_m = 3.57 \) sec.) can be accounted for as follows. The transfer function describing changes in system frequency given retarding torque variations in a generating unit controlled by a temporary droop governor (Figure 2.3) is given by Equation 3.8.
FIG. 3-4 VARIATION OF SYSTEM TRANSIENT RESPONSE AT A GIVEN $\lambda_1$ $\lambda_2$ $\lambda_3$ $\lambda_4$. 

- $\lambda_1 = 0.7$
- $\lambda_2 = 0.2$
- $\lambda_3 = 20$
- $\lambda_4 = 0.1$

- $\sigma = 0.06$
- $\delta = 0.6$
- $T_s = 0.12$
- $T_w = 2.0$
- $T_m = 4.77$

- $\sigma = 0.06$
- $\delta = 0.6$
- $T_s = 0.06$
- $T_w = 1.0$
- $T_m = 2.38$

- $\sigma = 0.04$
- $\delta = 0.4$
- $T_s = 0.04$
- $T_w = 1.0$
- $T_m = 3.57$
\[
\frac{\omega}{\omega_0}(S) = \frac{T_1}{T_{10}} = \frac{N(S)}{D(S)} = \frac{(1 + \frac{T_w}{2} S)(T_w T_r S^2 + (T_w + \delta T_r + \sigma T_r) S + \sigma)}{T_m T_s T_r T_w S^4 + \frac{T_w T_m (T_w + \delta T_r + \sigma T_r) + T_w T_r T_m}{2} S^3}
\]

+ ((T_w + \delta T_r + \sigma T_r + \frac{T_w}{2}) T_m - T_w T_r) S^2

+ (T_r + \sigma T_m - T_w) S + 1

3.8

If \( \lambda_1 \) to \( \lambda_4 \) and \( q = T_w S \) were substituted into \( N(S) \), the numerator, it would take on the form of Equation 3.9.

\[
N(q) = \frac{\delta q^3}{\lambda_2 \lambda_3} + \frac{(2 \delta + \frac{\delta}{\lambda_3} + \frac{\delta}{2 \lambda_2} + \frac{\sigma}{2 \lambda_2}) q^2}{\lambda_2 \lambda_3}
\]

\[
+ (\frac{\sigma}{2} + \frac{2 \delta}{\lambda_3} + \frac{\delta}{\lambda_2} + \frac{\sigma}{\lambda_2}) q + \frac{\delta}{\lambda_4}
\]

3.9

\( N(q) \) is not only a function of \( \lambda_1 \) to \( \lambda_4 \) and \( q \) but also a function of \( \delta \) and \( \sigma \). Stated differently, the characteristic equation is normalized (by substitution of \( \lambda_1 \) to \( \lambda_4 \) and \( q \)) but the operational part, \( N(S) \), of the transfer function is not. Thus for the example with \( \delta = .4 \), the amplitude of the transient should be approximately \( 2/3 \) of the case with \( \delta = .6 \).

This is verified by examining the respective time responses. It is to be noted that it is not exactly \( 2/3 \) since \( N(q) \) is also a function of \( \sigma \), which contributes a small change.

The significance of this is that if a hydro-electric generator is controlled by temporary droop governor adjusted according to some combination of \( \lambda_1 \) to \( \lambda_4 \), then the exact form of the transient response cannot be predicted, only the
approximate damping ratio can. In Paynter's analysis, the damping ratio remained constant for fixed $\lambda_1$ and $\lambda_2$. This is illustrated by Equation 3.10 which is the normalized transfer function of system frequency for given load changes.

$$\frac{N(q)}{D(q)} = \frac{\delta \left( \frac{1}{2\lambda_2} q^2 + \frac{\lambda_1}{\lambda_2} \right)}{\frac{1}{2\lambda_1\lambda_2} q^3 + \left( \frac{1-\lambda_1}{\lambda_1\lambda_2} \right) q^2 + \left( \frac{1-\lambda_2}{\lambda_2} \right) q + 1}$$

3.3.2 Stability limit variations of a single machine

The stability limit of a particular unit is generally required rather than information which describes the performance of various units, (i.e. constant $\lambda_3$ and $\lambda_4$ as given in the preceding section). This section describes stability limit variations of a single unit feeding an isolated load with finite servomotor operating time and permanent droop. The condition of a single unit connected to an infinite bus is also investigated. These results indicate how the settings as suggested by Paynter's criteria ($\lambda_1 = .5$ and $\lambda_2 = .2$) should be altered when the effects of these parameters are pronounced.

Figure 3.5 is a plot of the stability limit curves of a single unit with a finite servomotor time and variable permanent droop. As was previously shown (Figure 3.2) the permanent droop has a stabilizing effect. For illustrative purposes the stability boundary variation of one machine is
**FIG.-3.5** STABILITY LIMIT VARIATIONS OF A SINGLE UNIT AS A FUNCTION OF PERMANENT DROOP.

**FIG.-3.6** STABILITY LIMIT VARIATIONS OF A SINGLE UNIT COUPLED TO AN INFINITE BUS AS A FUNCTION OF SYNCHRONIZING TORQUE COEFFICIENT.
given. A digital computer programme was written to calculate the stability limit curves given the description of the machine (i.e., $T_m$, $T_w$, $T_s$ and $\sigma$ are defined or adjustable as desired). The method used for determining $\lambda_1$ and $\lambda_2$ is given in Appendix B.

For quality regulation of frequency to be realized when the permanent droop is large the values of $\lambda_1$ and $\lambda_2$ suggested by Paynter are increased. This essentially means that more of the system stabilization effort is provided by the permanent droop feedback loop and the temporary droop feedback ($\delta$ and $T_2$) features are less significant as far as stability is concerned. It must not be inferred that it is desirable to remove the temporary droop loop and compensate this by a high permanent droop. If this were done, the system would be stable but quality regulation of frequency would not be achieved.

Similar results (Figure 3.6) are given for a single unit feeding an infinite bus. As in the case of a single machine feeding an isolated load, a digital computer programme was written to calculate the stability boundary for a particular set of machine parameters. The results indicate that as the tie-line synchronizing torque coefficient ($K$) is increased the region of stable operation is reduced.

These results were predicted by Messerle\textsuperscript{19}, who suggested that in the case of a unit feeding an infinite bus, the
governor, set for isolated load operation, causes hunting rather than damping. With the temporary droop set low, the penstock-spinning inertia oscillation augments the tie-line hunting. At low values of $\lambda_1$ (or large $\delta$) the governor is made insensitive and thus the infinite bus determines the response of the machine. It is to be noted that at low values of $\lambda_1$, $\lambda_2$ has little effect on the unit stability.

3.3.3 Discussion of results

The results obtained for a single machine feeding an isolated load (Figure 3.1, 3.2 and 3.3) explain why the performances of a unit cannot always be predicted when its governor settings are selected according to Paynter's criterion. The reason for system performance deterioration with a large servomotor time constant as experienced by Schleif and Wilbor$^{20}$ is explained by the aforementioned results. The distance on a $\lambda_1\lambda_2$ plane from the point of governor setting to the stability boundary is a crude indication of the degree of system damping (refer to Tsypkin and Bromberg). Therefore these results dictate in what direction the governor settings are to be modified (i.e. from that suggested by Paynter and Hovey) to compensate for the relocation of the stability boundary due to the effects of $T_s$ and $\sigma$.

In the case of a single machine feeding an infinite bus,
the governor reduces the region of stable system operations. The characteristics of the temporary droop hydraulic governor and the penstock are such that they cannot contribute to the damping of the high frequency tie-line oscillation. In practice a tie-line frequency oscillation which is a factor of ten higher than the penstock-spinning inertia natural frequency is not uncommon. It is standard practice in such cases to desensitize the governor and use it only for load sharing purposes (i.e. by the permanent droop setting). The frequency regulation is provided by system damping.

To reiterate, the stability analyses as described in this chapter provide an appreciation for the effects which influence the performance of a single machine. Paynter's method is extended to include a finite servomotor time constant and permanent droop. For interconnected systems the influence of the tie-line is illustrated. It is obvious that if tie-line oscillations are to be effectively damped that new types of frequency controllers must be designed.

3.4 Limitations of the Stability Analysis

The limitation of stability analysis is that it only defines regions of stable and unstable operation. For a particular setting no measure of the quality of frequency regulation is obtained (except for the trivial case of operation on the stability boundary). If optimum quality
regulation of frequency is to be obtained, the entire stable region must be searched and a particular performance selected. It is to be recalled that the system performance cannot be predicted on the basis of a constant combination of \( \lambda_1 \lambda_2 \lambda_3 \lambda_4 \).

Therefore in the determination of controller settings for various machines, the stable region in each case must be searched since the final results are not transferable from unit to unit (as was the case with Paynter's model). The basis of Paynter's criterion is that a fixed combination of \( \lambda_1 \) and \( \lambda_2 \) defines a unique system response; with the introduction of \( \lambda_3 \) and \( \lambda_4 \) into the model, this condition no longer exists.

Another restriction is that the results are displayed on a \( \lambda_1 \lambda_2 \) plane, thus the solution is normally given as a function of two variables. By drawing a series of curves and interpolating, the information can be illustrated as a function of three variables. In the general case it is necessary to assess system performance using any type of governor, be it one with two or \( n \) adjustable parameters.

This analysis though valuable in indicating the limits of stability does not describe an effective means of determining the governor settings sought. The subsequent chapters of this thesis are devoted to the determination of a quality criterion.
4.1 General

In Chapter 3 stability analyses were carried out for various configurations of single machine operation. Since this data contains little information regarding the degree of system damping it is of limited value for indicating quality governor settings. It is to be recalled that the governor settings which cause an optimum damping of frequency deviations are being sought.

For the particular model used in this investigation (employing a mechanical-hydraulic temporary droop governor) the frequency deviation caused by a step variation in retarding torque is, in the region of normal operation, comprised of a damped sinusoid and two exponential components. For the same system regulated by an electro-hydraulic three-mode governor (P-I-D) the frequency deviation, in the region of normal operation, is comprised of three exponentials and two damped sinusoids. Since the output frequency deviation is some combination of these individual components, it is instructive to know how the damping factor and frequency of each is affected by changes in governor settings. Under
certain circumstances, as is shown later, it is possible from
the frequency and damping information (S-plane) to determine
the output time response. If the time response for the
various controller settings is known, then the one which "best"
satisfies the quality criterion can be chosen for system
operation.

Since the ultimate goal of this research project was to
define some technique or techniques by which the frequency
controllers could be adjusted such that quality regulation of
frequency would result, the performance of the system as a
function of its parameters had to be surveyed.

The root-locus method is a formalized technique of
plotting the loci of the characteristic equation roots as a
function of a designated system parameter. These roots
define the damping and modes of oscillation of the various
response components. Since the equation is usually in the
form of a polynomial some means of polynomial factoring must
be used. Because of the numerous calculations required, this
can be best done by means of a digital computer programme.

The root-locus method is briefly reviewed in this chapter
and its adaptation to the speed control problem is given. A
technique for selecting governor settings which results in
quality regulation of frequency is described. Lastly, the
limitations of this method and the need for further analysis
is discussed.
4.2 Root-Locus Analysis

The transfer function of a single loop feedback control system is given in Equation 4.1 (refer to Figure 4.1). For damping factor and frequency information of the system output, it is necessary to examine only the system characteristic equation, \( D(S) = 0 \). The values of \( d_j \) in Equation 4.1 are the system roots. \( N(S) \) affects the amplitude of the various response components \( (C_i) \). \( a_i \) and \( b_j \) are constants defined by the system characteristics.

![Block Diagram of a Feedback Control System](image-url)
\[
\frac{C(S)}{R(S)} = \frac{G(S)}{1+H(S)G(S)} = \frac{N(S)}{D(S)}
\]

\[
\begin{align*}
N(S) &= \frac{a_n S^n + a_{n-1} S^{n-1} + \cdots + a_1 S + a_0}{b_m S^m + b_{m-1} S^{m-1} + \cdots + b_1 S + b_0} \\
D(S) &= S^{m+n} + \frac{C_1}{S+d_1} + \frac{C_2}{S+d_2} + \cdots + \frac{C_m}{S+d_m}
\end{align*}
\]

To determine the effect of a given parameter on the system response, the characteristic equation is divided into two parts, one which has as a common factor the parameter in question and the other which does not. Refer to Equation 4.1. \(A\) is the parameter to be varied, normally referred to as the "root-locus gain" and \(Z_j\) and \(P_j\) are constants.

\[
D(S) = A(Z_i S^i + Z_{i-1} S^{i-1} + \cdots + Z_1 S + Z_0) + (P_j S^j + P_{j-1} S^{j-1} + \cdots + P_1 S + P_0) = 0
\]

or

\[
\frac{A(Z_i S^i + Z_{i-1} S^{i-1} + \cdots + Z_1 S + Z_0)}{(P_j S^j + P_{j-1} S^{j-1} + \cdots + P_1 S + P_0)} = -1
\]

The characteristic equation can be arranged with any parameter as the root-locus gain. \(A\) is normally incremented in some predefined manner, and for each value the roots of \(D(S)\) are calculated. The roots are then plotted on a complex variable plane as a function of \(A\).

After a root-locus plot for a particular system is obtained, the analyst examines it and determines from the loci what combination of roots results in the "best" system performance.
One method of selecting a governor adjustment is to choose parameter values so that maximum damping of the roots occurs. This implies that all the roots are to be located (by selection of parameter magnitudes) as far to the left in the complex $S$-plane as possible. It has been shown that this, though giving the maximum rate of decay of the individual response components does not mean a minimum time of frequency imbalance. The reason for this is that the characteristic equation defines only the rates of damping and frequency of the components and not the amplitudes. For a hydro station, controlled by a mechanical-hydraulic temporary droop governor, the conditions for highly damped roots is also the condition when the roots are grouped (point C on the Vyshnegradskii hyperbola, Figure 1.1). Thus the component amplitudes are large.

Another method suggested in this study, defines a criterion which is a function of the location of the characteristic equation roots and the numerator $N(S)$ of the system transfer function. If skill is used in selecting the criterion (i.e. the consideration of $N(S)$ in its formulation) it is possible in certain system configurations to achieve quality regulation by examining only the root-locus. Section 4.2.1 describes the application of such a criterion for determining optimum quality adjustment of a mechanical-hydraulic temporary droop governor. Another criterion which resembles it is also
described. It is employed in a generating station controlled by an electro-hydraulic three-mode type of governor.

4.2.1 Root-locus criterion for a mechanical-hydraulic temporary droop governor

The transfer function for a hydro station (Figure 2.3) controlled by a mechanical-hydraulic temporary droop governor is given by Equation 4.3.

\[
\frac{\omega}{\omega_0} = \frac{N(S)}{D(S)} \quad 4.3
\]

where

\[
N(S) = (T_s T_r S^2 + (T_s + \delta T_r + \sigma T_r) S + \sigma)(1 + \frac{T_w}{2} S)
\]

and

\[
D(S) = \frac{T_m T_s T_r T_w}{2} S^4 + \left(\frac{T_w T_m (T_s + \delta T_r + \sigma T_r) + T_s T_r T_m}{2}\right) S^3
\]

\[
+ ((T_s + \delta T_r + \sigma T_r - \frac{T_w}{2}) T_m - T_w T_r) S^2
\]

\[
+ (T_r + \sigma T_m - T_w) S + 1
\]

In practice \(\delta\) and \(T_r\) are the most readily adjustable governor parameters so these are selected as the root-locus gains for study. The two root-locus form equations used to investigate the effects of varying these governor parameters are given in Equations 4.4 and 4.5.

\[
\frac{\delta (aS^3 + bS^2)}{cS^4 + dS^3 + eS^2 + fS + 1} = -1 \quad 4.4
\]
where

\[ a = \frac{T_m T_w T_r}{2} \]

\[ b = T_m T_r \]

\[ c = \frac{T_m T_w T_s T_r}{2} \]

\[ d = \frac{T_m T_w}{2} (T_s + \omega T_r) + T_m T_s T_r \]

\[ e = (T_m (T_s + \omega T_r) + \frac{T_m T_w}{2} - T_r T_w) \]

\[ f = (T_r - T_w) + \omega T_m \]

and

\[
\frac{1}{T_r} \left( \frac{T_m T_w T_s}{2} \right)^3 + \left( \frac{T_m T_w}{2} \right)^2 + \left( \frac{T_m T_w}{2} \right) S + (\omega T_m - T_w) S + 1 \]

\[
\frac{T_m T_w T_s}{2} S + (T_m T_w (\omega + \delta) + T_m T_s) S^3 + (T_m (\omega + \delta) - T_w) S^2 + S \]

= -1

4.5

Root-locus plots are shown for these two adjustable parameters in Figure 4.2 and 4.3. Also illustrated are traces of frequency variations as functions of time following a step change in unit load. It is to be noted that the forms of the time responses are directly related to the location of the roots of \( D(S) = 0 \) on the S-plane, and thus to the values of \( \delta \) and \( T_r \).

In the introduction it was stated that a desirable criterion for governor setting would be one which returned the system after a disturbance to normal operation as soon as possible, with little overshoot. In practice additional con-
FIG.-4.2 ROOT-LOCUS OF A HYDRO-ELECTRIC POWER
GENERATOR CONTROLLED BY A MECHANICAL-
HYDRAULIC TEMPORARY DROOP GOVERNOR
WITH $T_s = 0.05$ AND $\sigma = 0.05$.  

PATH OF EQUAL DAMPING

1. PAYNTER $\lambda_1 = 0.4$ $\delta = 0.25$
   $\lambda_2 = 0.2$ $T_r = 5.0$

2. HOVEY $\lambda_1 = 0.5$ $\delta = 0.2$
   $\lambda_2 = 0.25$ $T_r = 4.0$

3. EQUAL DAMPING AND MINIMUM $T_r$

4. EQUAL AND MAXIMUM DAMPING

$T_m = 10.0$ $T_w = 1.0$
$\sigma = 0.05$ $T_s = 0.05$
straints are imposed in the form of mechanical limits, saturation, etc.; however, in this analysis an output frequency deviation which resembles a slightly under critically damped sinusoid was selected as the criterion with no mechanical constraints imposed.

For this configuration, in the region of normal operation, the frequency deviation consists of two exponential components plus a damped sinusoidal component. Of the two exponentials, the one due to the real root far to the left on the negative real axis, due to its very short time constant, is assumed to be zero in the overall response. In Figures 4.2 and 4.3 only the upper half of the S-plane is shown since the curves below the real axis are mirror images of those above. The root on the negative real axis near the origin is plotted as a function of $T_r$, $\delta$ and $\sigma^-$, the real part of $S$.

Note that if the real root is moved far to the left by variations of $T_r$ and $\delta$ the complex root tends to move toward the imaginary axis and thus gives rise to a more lightly damped response. Conversely, by varying $T_r$ and $\delta$ to move the complex root far to the left, the real root moves in and gives rise to a slower decaying exponential. A compromise is obviously necessary. The curves labelled "path of equal damping" in Figures 4.2 and 4.3 define the values of $\delta$ and $T_r$ for which the negative real part ($-\sigma^-$) is the same for the real and complex roots. This means that following a disturb-
ance both the sinusoidal and exponential components of the
time response will decay at the same rate. At the extreme
left of the "path of equal damping" the damping is a maximum
and the time of restoration to steady-state would be a minimum
if the amplitudes of the sinusoidal and exponential components
were equal and remained unchanged for variations in $T_r$ and $\delta$.
The suggested point of governor setting would therefore be
the extreme left point on the path of equal damping (called
the point of "equal and maximum damping").

It can be shown that for a given disturbance the ampli-
tudes of the response components vary as functions of $\delta$ and
$T_r$ (see $N(S)$ of Equation 4.3). Rather than the extreme left
point in the "path of equal damping" the point where this
curve cuts a minimum $T_r$ curve is suggested as a "good"
governor setting (called the point of "equal damping and
minimum $T_r$" ). It is not to be inferred that the point of
equal damping and minimum $T_r$ is mathematically the optimum
point of operation, although it is very close to it. In
addition, this governor setting is uniquely defined on the
root-locus plot and thus can be readily selected.

In Figure 4.2, four time response curves are shown, which
represent the variation of system frequency following a step
change in load. The four curves illustrate the time response
obtained by selecting the various governor adjustment criteria.
The curves are for the Paynter, Hovey, equal and maximum
FIG.-43 ROOT-LOCUS OF A HYDRO-ELECTRIC POWER GENERATOR CONTROLLED BY A MECHANICAL-HYDRAULIC TEMPORARY DROOP GOVERNOR WITH $T_s = 0.5$ AND $\sigma = 0.035$. 

<table>
<thead>
<tr>
<th>Case</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\lambda_1$</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>$\delta$</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>$\lambda_2$</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>5.0</td>
</tr>
<tr>
<td>3</td>
<td>$\lambda_1$</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>$\delta$</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>$\lambda_2$</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>4.0</td>
</tr>
<tr>
<td>5</td>
<td>$\delta$</td>
<td>0.33</td>
</tr>
<tr>
<td>6</td>
<td>$T_r$</td>
<td>10.5</td>
</tr>
<tr>
<td>7</td>
<td>$\delta$</td>
<td>0.43</td>
</tr>
<tr>
<td>8</td>
<td>$T_r$</td>
<td>13.5</td>
</tr>
</tbody>
</table>

$T_w = 1.0$, $T_m = 10.0$, $T_s = 0.5$, $\sigma = 0.035$. 

PATH OF EQUAL DAMPING.
damping and equal damping and minimum $T_r$ governor adjustment criteria. The results obtained for these cases are similar. If the servomotor time ($T_s$) and the permanent droop ($\sigma$) are changed appreciably as in Figure 4.3 (in this example for illustrative purposes, they are varied excessively) the responses obtained by application of these criteria differ significantly.

It is to be recalled that the governor transfer function includes $T_s$ and $\sigma$, thus this method permits the determination of "quality" governor setting even if the effects of these parameters become pronounced. The other criteria cited do not permit the analyst to do this. The consideration of $T_r$ in the formulation of the criterion permits quality regulation settings to be determined using only characteristic equation information.

4.2.2 Root-locus criterion for an electro-hydraulic three-mode governor

The characteristic equation for the electro-hydraulic governor controlled system is of the form of Equation 4.6.

$$a_7s^7 + a_6s^6 + a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 = 0 \quad 4.6$$

where $a_7$ to $a_0$ are functions of the system parameters illustrated in Figure 2.3. The generator is feeding an isolated load. An electro-hydraulic governor is used and the system damping is zero. Equation 4.6 can be arranged in root-locus form with
any parameter selected as the root-locus gain. The adjustable parameters in this case are $T_1$, $T_2$ and $K'$. 

Figure 4.4 illustrates loci for a one parameter variation rather than two as in Figures 4.2 and 4.3 for the mechanical-hydraulic system. However, if two were varied a similar "path of equal damping" could be obtained for this system. In this case the point of operation was selected as the point of equal damping of the two dominant response components, which are damped sinusoidal terms. Since there are three variables a three dimensional plot is required, the extreme left point on the surface of equal damping is the point of operation referred to. Analogue computer results indicate that this gives satisfactory results (Figure 4.4). Once again it must not be inferred that the results are theoretically the best. The criterion described provides a convenient means of obtaining "near optimum" governor settings.

Figure 4.5 illustrates the improvement which can be achieved by adding derivative action to a proportional plus integral controller. In Figure 4.5 three curves are drawn. Curve B is for $T_1 = 0.3$, $T_2 = 5.9$ and $K' = 3.24$; Curve C is for $T_1 = 1.0$, $T_2 = 2.7$ and $K' = 5.5$; while Curve A is for an optimized mechanical-hydraulic governor controlled system. In all cases, $T_m$ was equal to 9.85 and $T_w$ equal to 1.24. This illustrates that Curve B is equivalent to P-I control since it corresponds to an optimized mechanical-hydraulic (Curve A)
**FIG. 4.4** ROOT-LOCUS OF AN ELECTRO-HYDRAULIC THREE-MODE GOVERNOR FREQUENCY CONTROL SYSTEM.

\[
\begin{align*}
T_m &= 9.81 & T_w &= 1.24 \\
T_1 &= 1.0 \\
T_2 &= 2.7 \\
K' &= 5.5 \\
\sigma &= 0.04
\end{align*}
\]
\[ \delta = 0.22 \]
\[ T_1 = 0.3 \]
\[ T_2 = 0.9 \]
\[ T'_1 = 3.24 \]
\[ T'_2 = 5.5 \]
\[ T_m = 9.85 \]
\[ T_w = 1.24 \]
\[ \sigma^* = 0.04 \]

\textbf{FIG.-4.5} TRANSIENT FREQUENCY RESPONSE COMPARISON BETWEEN A MECHANICAL-HYDRAULIC TEMPORARY DROOP GOVERNOR AND AN ELECTRO-HYDRAULIC THREE-MODE GOVERNOR WITH P-I AND P-I-D CONTROL.

\textbf{FIG.-4.6} LOCATION OF THE ROOTS FOR THE CURVE FOLLOWING PROGRAMME.
response and Curve C gives P-I-D correction. To obtain Curve C the criterion of equal and maximum damping was applied, while in Curve B the derivative correction was reduced.

4.3 Curve Following Programme

Section 4.2.2 illustrates that quality adjustments are obtained when the criterion of equal damping and minimum $T_r$ is employed for a system controlled by a mechanical-hydraulic temporary droop governor even when the magnitudes of $T_s$ and $\sigma$ become abnormal. An important disadvantage of this scheme is that for each system to be optimized, a root-locus diagram must be plotted, then the "path of equal damping" graphed and the point of minimum $T_r$ selected. This constitutes a considerable amount of labour in the form of analysis and graphical construction.

It is possible to circumvent both of these drawbacks. The criterion for optimum governor setting is defined as selecting, on the "path of equal damping", the unique point of minimum $T_r$. If this curve is specified in terms of the characteristic equation roots and a method of determining $T_r$ is available, then it would be possible to use a curve following programme to establish the optimum governor settings without plotting the root-locus diagram. A digital computer programme has been written which performs this operation. The path of equal damping defines a particular
combination of roots $A$, $A \pm jB$ and $C$ then $(S+A)(S+A+jB)(S+A-jB)(S+C) = 0$ is the characteristic equation. If this product is equated to the characteristic equation as defined by the system parameters, it is possible to define the "path of equal damping" in terms of $\delta$ and $T_r$. Refer to Figure 4.6. Equating the coefficients of the characteristic equation as given by the system parameters (Equations 3.4 with $D=K=0$) with that obtained from the product of $(S+A)(S+C)(S+A-jB)(S+A+jB) = 0$ the equalities given in Equations 4.7 to 4.10 result.

$$AC(A^2+B^2) = \frac{2}{T_m T_s T_r T_w}$$  

$$3A+C = \frac{1}{T_r} + \frac{\delta}{T_s} + \frac{\sigma}{T_s} + 2/T_w$$  

$$3A^2+B^2+3AC = \frac{2}{T_r T_w} + \frac{2\delta}{T_s T_w} + \frac{2\sigma}{T_r T_s} + \frac{\sigma}{T_r T_s} - \frac{2}{T_s T_m}$$  

$$A^3+AB^2+3A^2C+B^2C = \frac{2}{T_m T_s T_w} + \frac{2\sigma}{T_s T_r T_w} - \frac{2}{T_m T_s T_r}$$  

There are five unknowns and four equations, thus either $T_r$ or $\delta$ must be held constant, then the corresponding values of $A$, $B$, $C$, $\delta$ or $T_r$ are found by an iterative process.

The digital computer programme as written has as input information the numerical values of $T_m$, $T_w$, $T_s$ and $\sigma$ and an initial guess of $A$, $B$, $C$ and $T_r$. The output is the optimum governor setting in terms of $T_r$ and $\delta$ or $\lambda_1$ and $\lambda_2$. 
This type of analysis was not carried out for a generator controlled by an electro-hydraulic three-mode governor. The reason for this is that the root-locus method exhibits limitations as far as obtaining optimum quality setting for a general governor. Consequently it was decided to discontinue the research in this area and concentrate on a more general technique.

4.4 Summary

The root-locus method, by definition, is a characteristic equation analysis and in general time response information is not provided. A root-locus plot can readily illustrate the effect which two parameters (T_r and δ) have on the location of the characteristic equation roots, thus the modes of system oscillation and damping. For frequency controllers with more than two adjustable parameters (electro-hydraulic three-mode governor) a series of plots are necessary and therefore this technique becomes cumbersome.

One other restriction is that the "path of equal damping and minimum T_r" serves as a valuable criterion only because of the type of transfer function which a mechanical-hydraulic temporary droop governor possesses. It is possible to define a governor which would not perform optimally when this type of criterion is used, thus in general, for each type of controller an appropriate criterion must be defined.
interconnected systems the characteristic equation and a root-locus plot can be readily found. It is difficult from the examination of the resultant loci to select the particular combination of roots which is synonymous with quality operation. Furthermore, if \( N(S) \) is to be included in the criterion formulation, it must be established. For a multi-loop interconnected system this constitutes a considerable amount of algebraic manipulation. The quality criterion should therefore be based on a per machine basis. The above reasons preclude the general use of root-locus techniques in quality control analysis.

It is to be appreciated that these limitations are associated with the use of the root-locus technique in defining quality adjustments of hydro-electric station frequency controllers, not with the usefulness of this analytical tool for stability or other applications. Bearing this in mind, a more suitable method of obtaining the settings sought is needed. Such a method is described in the next chapter.
5. ADAPTATION OF KRASOVSKII'S THEOREM FOR QUALITY REGULATION

5.1 Environment of an Isolated Hydro-Electric Generator

In Chapter 4 an optimizing technique based on root-locus methods has been described by which certain types of hydro-electric power station governors can be adjusted for quality regulation of frequency. This procedure is limited since it is not general in its application, that is if another type of controller were considered the basis of the criterion might no longer be valid. A new criterion might be required and new concepts on which to base it might have to be established.

The criterion for selecting governor parameters must accommodate systems controlled by regulators with a general form of transfer function and n adjustable parameters. The criterion should also be of a form which would not restrict its application in interconnected systems. In this chapter a criterion is described for selecting governor settings of a generator which operates in an environment with the following constraints: the disturbances are step changes in load, the resultant frequency deviation in the region of final operation approaches a damped sinusoid, the maximum frequency
deviation for a fixed disturbance remains relatively constant irrespective of the damping ratios, the frequency offset after a change in load is small relative to the maximum deviation, the output signal (frequency deviation) and its first and second derivatives are available by means other than differentiation and a linear system representation is valid.

The constraints listed above do not restrict the usefulness of the proposed criterion, since these are the conditions which exist during normal generator operation. A single unit feeding an isolated load is usually subjected to step changes in load. The transient response is of the form of a damped sinusoid. It is possible to obtain the necessary signals (first and second derivatives of frequency) from the model representation.

In this chapter it is shown that a sufficiently general optimizing procedure can be defined which provides a means of determining the governor adjustments for hydro-electric generators so quality regulation of frequency results. The contents of this chapter are a brief statement of Krasovskii's theorem, its adaptation to the present problem, the optimization search used and the results.

5.2 Krasovskii's Theorem

In servomechanism design it is often necessary to
synthesize a system with the shortest possible "rise time" commensurate with an acceptable amount of overshoot. In the limit the ideal system is one which responds instantaneously. If this type of response or one which approaches it were required then the design could be based on the minimization of the integral of some positive function of the error. Two integral criteria which can be used for this purpose are \( \int_0^\infty \epsilon^2 dt \) and \( \int \left| \epsilon \right| dt \).

In the control of power system frequency a short "rise time" does not constitute quality regulation. When the governor controls are adjusted to make the integral of the frequency error squared a minimum, the rate of change of the frequency deviation is undesirably high and oscillatory. In order to achieve quality regulation settings by an optimization search some function other than \( \int \epsilon^2 dt \) must be minimized.

A.A. Krasovskii developed a method of establishing integral error criteria which are minima for different system transient responses. Stated differently if the final form of the system response sought is known, then a criterion which is a minimum for this response can be determined. The theorem is given below, this is based directly upon reference 15.

The general form of the criterion is given by Equation 5.1

\[
J = \int_0^\infty \left( \delta^2 + \tau_1^2 (\delta^1)^2 + \tau_2^4 (\delta^{11})^2 + \cdots + \tau_n^{2n}(\delta^n)^2 \right) dt \quad 5.1
\]
where $\delta$ is the regulated system variable and $\tau_j$ are weighting factors.

The minimization of the integral is equivalent to seeking a regulation process which is the nearest approximation to the process described by Equation 5.2.

$$a_n \delta^n + a_{n-1} \delta^{n-1} + \ldots + a_1 \delta^1 + \delta^0 = 0 \quad 5.2$$

where $a_j$ are the polynomial coefficients and are used to define the above weighting factors as given by Equation 5.3.

$$\tau_1^2 = a_1^2 - 2a_2$$
$$\tau_2^4 = a_2^2 - 2a_1 a_3 + 2a_4$$
$$\ldots$$
$$\tau_{n-1}^{2n-1} = a_{n-1}^2 - 2a_{n-1}a_n$$
$$\tau_n^{2n} = a_n^2 \quad 5.3$$

It can be shown that for the power system frequency control problem a criterion of the form of Equation 5.4 can be minimized to give a damped sinusoidal response.

$$J = \int \left(\Delta \omega^2 + \tau_2^4 (\omega^{11})^2\right) dt \quad 5.4$$

In Equation 5.4 $\Delta \omega$ is the generator bus frequency deviation, $\omega^{11}$ is its second derivative and $\tau_2^4$ is an appropriate weighting factor.

Before this criterion can be utilized, $\tau_2^4$ must be calculated, therefore the solution sought must be known or estimated.
In practice the complete description of this solution is not known beforehand, except that it should approach a sinusoidal with a damping ratio of .707. A method is described below which permits the determination of \( \tau^4 \) and thus the optimum regulator settings for hydro-electric frequency control systems.

5.3 Mathematical Approach

To illustrate the application of Krasovskii's theorem in the solution of the hydro-electric generator frequency regulation problem, a brief outline of the necessary mathematical considerations is given. For a more detailed development the reader is referred to Appendix C. In Figure 5.1 \( t_o \) is the time of the load disturbance \( U(t-t_o) \). The system variables at time before \( t_o \) are zero.

If the frequency deviation caused by a step disturbance is to be optimally damped an error criterion as defined by Equation 5.4 must be minimized by appropriate governor settings. To give a response with a damping ratio of .707, \( \tau^4 \) must be equal to the fourth power of the reciprocal of the natural frequency, i.e. \( \tau^4 = \left(\frac{1}{\omega^2}\right)^2 \). The natural frequency can be calculated from the integral of the frequency deviation squared and the integrals of the first and second derivatives squared.

For the system described by Figure 5.1, these integrals are given by Equation 5.5, 5.6 and 5.7.

\[
\int_0^\infty (\Delta \omega)^2 dt = \frac{A^2}{4\omega} \left( \frac{1}{\xi} + \sin(-\psi) \right) = Q_1
\]
FIG. 5.1 SCHEMATIC DIAGRAM OF THE GENERAL HYDRO-ELECTRIC POWER GENERATOR FREQUENCY CONTROL PROBLEM.
In Equation 5.5, 5.6 and 5.7 \( \psi = \tan^{-1} \frac{1}{(1-\xi^2)^{1/2}} \) and \( A \) is the amplitude of the frequency deviation. Then \( \tau_2^\psi = (Q_1/Q_2)^2 \). Similarly \( (Q_2/Q_3)^2 = \tau_2^\psi \). An advantage of the model analysis is that these integrals can be obtained by system measurement.

For the actual hydro-electric generator case, the output signal though resembling a damped sinusoid is in fact defined by a higher order polynomial (see Figure 2.3). As a result the magnitude of \( \tau_2^\psi \) as determined from system model measurements is larger than that required in Equation 5.4. Better results, in terms of final controller settings, are achieved if the approximate value of \( \tau_2^\psi \) obtained from Equations 5.6 and 5.7 is substituted for \( \tau_2^\psi \) (see Appendix C).

Equations 5.8 and 5.9 define the derivatives of the frequency deviation for a mechanical-hydraulic temporary droop governor controlled hydro-electric generator. The relationships are for isolated load and no system damping conditions. It is to be noted that the first and second derivatives must be re-defined for each new system representation studied.

\[
\int (\omega^1)^2 \, dt = \frac{A^2 \omega}{4} \left( \frac{1}{\xi} - \sin (\psi) \right) = Q_2 \quad 5.6
\]

\[
\int (\omega^1)^2 \, dt = \frac{A^2 \omega^3}{4} \left( \frac{1}{\xi} + \sin (3\psi) \right) = Q_3 \quad 5.7
\]
If the natural frequency of the system model remained constant, with variation of the controller parameters, the minimization of the criterion defined by Equation 5.4 employing a weighting factor $T_2^4$ as determined above would result in an optimumly damped system response. However, the system natural frequency will normally change with variations in controller settings. The steps necessary in the optimizing procedure are: a search in the vicinity of the initial operating point to establish the slopes of the error criterion response surface (Equation 5.4 defined in an n dimensional hyperspace where n is the number of adjustable parameters) and the adjustment of the controller parameters in the direction of a minimum. Upon establishing a new operating point $T_2^4$ is recalculated and the sequence is repeated. This procedure is repeated until the governor parameter magnitudes converge. The final controller settings are the optimum adjustments as defined by the quality regulation requirements.

In this study to establish a minimum of the error criterion, an optimization search technique based on the Taylor series expansion was employed. Any other optimization method could as readily have been used.
5.4 Results

The error criterion described in this chapter was used to establish quality settings for power generators controlled by a mechanical-hydraulic temporary droop governor, an electro-hydraulic three-mode governor and a general three-mode governor. Figures 5.2, 5.3, 5.4 and 5.5 illustrate the results achieved.

Two cases of a hydro-electric generator controlled by a mechanical-hydraulic temporary droop governor were studied. Figure 5.2 is for a short servomotor time constant \(T_s = 0.05\) seconds) while Figure 5.3 is for \(T_s = 0.5\). These values were chosen so a direct comparison could be made with the results obtained by the root-locus technique as described in Chapter 4. In each case a time response of the frequency deviation is given with the governor set according to those suggested by the optimization procedure. It is to be noted that the final results compare favorably with those obtained by the root-locus method and do meet the quality requirements as specified.

Figure 5.4 illustrates the frequency deviation time response of the hydro-electric generator at various stages of the electro-hydraulic governor optimization procedure. Note that the final response curve is similar to that achieved by means of the root-locus technique (Figure 4.4).

The curves of Figure 5.5 are for a system controlled by a general three-mode governor. The transfer function in per unit form (without the permanent droop feedback loop) is
FIG.-5.2 OPTIMIZATION SEARCH PATTERN OF A SINGLE
UNIT CONTROLLED BY A MECHANICAL-
HYDRAULIC TEMPORARY DROOP GOVERNOR
WITH $T_s = 0.05$ SEC.

$T_m = 10.00$
$T_w = 1.00$
$T_s = 0.05$
$\sigma = 0.05$
$\Delta T_1 = 0.01$
FIG.-5.3 OPTIMIZATION SEARCH PATTERN OF A SINGLE
UNIT CONTROLLED BY A MECHANICAL-
HYDRAULIC TEMPORARY DROOP GOVERNOR
WITH \( T_s = 0.5 \) SEC.
FIG. 5.4 OPTIMIZATION SEARCH OF A SINGLE UNIT FEEDING AN ISOLATED LOAD AND CONTROLLED BY AN ELECTRO-HYDRAULIC P-I-D GOVERNOR.

FIG. 5.5 OPTIMIZATION SEARCH OF A SINGLE UNIT FEEDING AN ISOLATED LOAD AND CONTROLLED BY A GENERAL THREE-MODE GOVERNOR.
As would be predicted the final parameter settings \((G_1, G_2\) and \(G_3\)) chosen by the optimization procedures results in a system response similar in "time-to-steady-state" and maximum deviation to that obtained by the electro-hydraulic three-mode system (Figure 5.4).

In all the above cases the optimization procedure developed, based on the adaptation of Krasovskii's theorem, guides the search for quality governor settings to operating conditions which result in an optimumly damped system frequency deviation. Given any other type of hydro station frequency controller, quality adjustments for it could be established using this technique.
6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

In this thesis the methods used for selecting optimum governor settings for present-day hydro-electric power generators has been reviewed. The advantages and disadvantages of the various techniques are discussed. While the Paynter, Stein, Meloy criteria have merit in specific cases, these are not suitable for general use. These suffer in terms of ease of application and/or the quality of the results obtained when used in a general system.

The most significant contribution of this research is the adaptation of Krasovskii's theorem for the optimization of the frequency response of a hydro-electric generator controlled by a realistic general type of governor. This is realized by first defining mathematically what an optimum frequency deviation response is; secondly the appropriate error criterion based on Krasovskii's theorem is selected and lastly the error criterion, which is updated during the optimization search, is minimized. This method is applicable to a general hydro-electric generating station since the description of the penstock, generator, governor and associated system components need not be of the form described in this thesis. The only constraints are that it is a linearized
model of a realistic system and that the required signals are available through measurement.

In addition to the major results arising from the application of Krasovskii's theorem, a number of additional conclusions of a more minor nature related to this general field of study were drawn. These are discussed hereunder under topical headings.

6.1.1 Single machine equivalence

A procedure is developed which can be used in lieu of the commonly employed averaging method for determining a single machine equivalent of a bank of n parallel operating units. The degree of time response similarity of any bank of n machines and the optimized equivalent is equal to or greater than that which exists between the time responses of the original bank and the equivalent obtained by simple averaging methods. Whenever a multi-machine interconnected system is simplified by lumping of machine banks, errors are introduced into the description of the variables (i.e. the voltage, frequency, et cetera). The technique developed in this thesis minimizes the magnitude of such errors.

The mathematical description of the single machine equivalent permits an appraisal of the approximations associated with lumping of machines. This provides for a better appreciation of the significance of a single machine equivalent.
For example, the frequency variations caused by intra-bank unit energy exchanges cannot be duplicated in a single machine equivalent representation. This analysis aids in deciding whether the behavior of a particular bank (or part of the bank) of machines can be adequately reproduced by a single equivalent or whether lumping is, in a particular case, to be avoided.

6.1.2 Stability limit analysis

This analysis confirms that the frequency regulation of a hydro-electric generator feeding an isolated load controlled by a mechanical-hydraulic temporary droop governor set according to Paynter's criterion cannot always be predicted (i.e. when the servomotor time ($T_s$) and/or the permanent droop ($\sigma$) influence the system response). It is demonstrated that a fixed combination of the dimensionless parameters $\lambda_1$, $\lambda_2$, $\lambda_3$ and $\lambda_4$ is not sufficient to specify a unique system performance, therefore a need exists for some other method by which the frequency controllers of such systems can be adjusted to achieve quality frequency regulation.

For the condition of a single machine electrically coupled to an infinite bus or a comparatively large generation system the stability region as determined by the governor settings is appreciably reduced. This explains why new types of frequency controllers are necessary to damp frequency oscil-
lations caused by the electrical characteristics of the tie-line and generator inertia combination. Mechanical limitations of the temporary droop hydraulic governor and the frequency response characteristics of the governor penstock combination preclude the use of this controller for tie-line oscillation control.

As does Paynter's analysis, the transformation of the unit characteristic equation by means of dimensionless ratios $\lambda_1$ to $\lambda_6$ and the q operator simplifies the determination of the unit stability boundary and the comparison of it with that established for other machines (the ratios $\lambda_1$ to $\lambda_6$ remain relatively constant, in the region of operation, irrespective of unit parameters). Associated with this, the analyst can more readily assess the effect which a particular unit parameter has on the location of the stability boundary.

6.1.3 Root-locus analysis

The establishment of two governor adjustment criteria is the most significant product of this portion of the research. These criteria are referred to as the "equal damping and minimum $T_r$" and the "equal and maximum damping" criteria respectively. By employing these, the mechanical-hydraulic temporary droop and the electro-hydraulic three-mode governors can be adjusted for quality frequency regulation.

A digital computer curve following programme which deter-
mines the optimum quality settings of a mechanical-hydraulic temporary droop governor was developed. It saves the long computation time and graphical construction required by the conventional root-locus analysis method.

The disadvantage of using root-locus analysis to establish optimum controller settings is that, for governors with more than two adjustable parameters, the technique is cumbersome. In addition the criteria can be defined only for particular types of controllers. Given a different version (in terms of the electrical circuit and mechanical construction) a new criterion would have to be determined. However it may not be possible to define a suitable root-locus criterion for every type of controller.

6.1.4 Analysis based on Krasovskii's criterion

The most meaningful contribution which emerged from this research was the development of a governor adjustment criterion by which the optimum setting of a general frequency controller of a hydro-electric generator can be selected. This method is based on an adaptation of Krasovskii's theorem. It was tested on a generator employing a mechanical-hydraulic temporary droop governor, an electro-hydraulic three-mode governor and a general three-mode governor. In each case the search converges to governor settings which result in quality regulation of frequency.
The most important feature of this governor optimizing technique is that it directs the search toward parameter settings which cause the system time response to be of a certain form. The examination of the system frequency deviation and its derivatives provides information as to how the controllers should be adjusted for a desired time response. As was shown in Appendix C, the output frequency variation for step changes in load consists of a damped sinusoid plus other components, this causes the waveform of the final solution to resemble a sinusoid with a damping ratio slightly less than .707.

Throughout this study, only the frequency control loop of the generation plant was considered, however a similar analysis could be applied to establish the optimum settings of an automatic voltage regulator (AVR). The definition of the voltage criterion must consider what constitutes quality voltage regulation and the mechanical and electrical limitations of the AVR, thus its mathematical description would probably be different than that of the frequency criterion (Equation 5.4).

6.2 Recommendations for Further Research in this Area

A number of problems have arisen from this research which warrant further detailed study and could form the basis for continued research in this area. The most signifi-
cant of these are listed hereunder.

1) An investigation is required to establish the theoretical limits, irrespective of the type of controllers employed, beyond which the system performance cannot be improved. A frequency deviation must exist before corrective action can be initiated and a finite time is required before the necessary energy can be extracted from the energy reservoir. During this time mechanical spinning energy must be converted into electrical form to meet the electrical demands. Therefore a frequency error must always occur. Other considerations such as controller mechanical constraints and penstock pressure limits can be included in the analysis. By determining the theoretical minimum frequency regulation area which must exist, the design of controllers can be approached more realistically. Rather than trying to continually improve the system performance, it may become obvious that a further improvement at best is marginal and that concentration in this area should be discontinued.

2) A study of the synthesis of controllers to damp tie-line oscillations is needed. Since in a hydroelectric station it is difficult if not impossible (due to mechanical and frequency response considerations) to operate the wicket gates in such a fashion that tie-line oscillations are damped, it is necessary to employ a controller with speed and voltage error inputs to
control the AVR and wicket gates. At present this is being pursued, however, with the aid of optimization techniques this analysis can be accelerated.

3) The study of interconnected system controller optimization is required. Before the optimization of frequency controllers in a large interconnected system is undertaken, a better understanding of how to model the distributed system load is needed. For example, when establishing the optimum frequency adjustment of a power grid which consists of two or more generation centers and more than one load, it is no longer adequate to assume equal deviations of each load.

4) There is a need to continue the studies reported in this thesis to establish quality settings of governors (and/or automatic voltage regulation) in an interconnected power grid. In the introduction it was suggested that this is the next logical step upon completion of the single unit analysis.

5) The development of a criterion which can be used to select controller settings such that overall system quality regulation results is required. This criterion must be of a form other than the summation of the individual error integrals. It is suggested that system damping be used as a measure of performance, that is, the overall system damping is measured and maximized.
This can be done by several means, the direct consists of determining a damping effectiveness of each unit and then maximizing a quadratic function which includes a measure of all the individual unit damping. An indirect means would be to minimize the energy exchange between the participating units of the system. The above can be extended to include the voltage regulation and supplementary controls loops of the individual generating plants.
7. REFERENCES


APPENDIX A - SINGLE MACHINE EQUIVALENTS FOR MULTI-MACHINE HYDRO PLANTS

A.1 Machine Equivalence

A single machine equivalent is a unit which is substituted for two or more units. The measure of the degree of equivalence is assessed in terms of the similarity between the performance of the simplified and actual systems. The configuration of the units being grouped will determine whether an exact or an approximate equivalent exists. Given any disturbance, with an exact equivalent substituted for the bank of n machines, the overall system dynamics are exactly duplicated; with an approximate representation substituted the response differs by some degree from the actual response. Engineering judgment is required to establish what deviation from the actual response can be tolerated before the analysis becomes invalid. The final equivalent selected is a compromise between an attempt to lump a maximum number of units and to retain similarity between the actual and the simplified system performances.

Figure A.1 illustrates the application of these principles in finding a single machine equivalent for a bank of n parallel operating generators. The actual system and the model are
FIG. A.1 SCHEMATIC OF A SINGLE MACHINE EQUIVALENT SUBSTITUTED FOR $n$ PARALLEL OPERATING MACHINES.
compared on the basis of the correlation in time of the frequencies or torque angles of the models of the actual system and the equivalent system. In this study, the frequencies are compared.

A.2 System Representation

The analysis presented in this Appendix considers a power grid with hydro turbines as the prime movers and mechanical-hydraulic temporary droop governors as frequency controllers. The block diagram representation of the system discussed is given in Figure A.2. It illustrates the transfer function of one unit; n such units are assumed to be present.

In Figure A.2 the reference bus frequency is determined by the block labelled "\(\omega_{\text{ref}}\) generator", the equation for \(\omega_{\text{ref}}\) as a function of \(\omega_1, \ldots, \omega_n\) is given by Equation A.7.

For this type of system (Figure A.2) the steady-state values of \(\omega_i, G_i\) and \(T_{\text{tli}}\) (where \(i\) denotes the \(i^{\text{th}}\) machine), can be readily determined; therefore the magnitude of the parameters of the equivalent unit can be calculated to give the same steady-state conditions. For this particular system the variables \(\omega_{\text{seti}}, \omega_{\text{ni}}, MW_i, \sigma_i, K_{ir}\) and \(D_i\) determine the steady-state values of frequency, gate position and the tie-line torque, and these must be defined for the equivalent.

For convenience, the same value of \(\omega_{\text{set}}\) and \(\omega_n\) are selected for all units, \(\omega_{\text{set}}\) and \(\omega_n\) are thus defined. MW_e.
FIG. A.2 BLOCK DIAGRAM FOR FREQUENCY CONTROL OF AN INTERCONNECTED HYDRO STATION.
the megawatt base, is equal to the sum of the individual megawatt bases of the units being replaced. The equivalent megawatt base, $MW_e$, is given by Equation A.1.

$$MW_e = MW_i + \ldots + MW_j$$  \hspace{1cm} \text{A.1}

where $i \ldots j$ refer to generators represented by the single machine equivalent.

$T_{le}$, the equivalent generator load torque, is given by Equation A.2.

$$T_{le} = \frac{T_{li}.MW_i + \ldots + T_{lj}.MW_j}{MW_i + \ldots + MW_j}$$  \hspace{1cm} \text{A.2}

If the frequency base for all generators is the same, the permanent droop of the equivalent generating unit is given by Equation A.3.

$$\sigma_e = \frac{\sum_{p=i}^{j} MW_P}{\sum_{p=i}^{j} \sigma_p}$$  \hspace{1cm} \text{A.3}

If the megawatt base is other than $MW_e = MW_i + \ldots + MW_j$, the permanent droop, $\sigma_e$, must be recalculated using Equation A.4. $\sigma_{old}$ is the permanent droop given by Equation A.3 and a megawatt base given by Equation A.1.
\[ \sigma_{\text{new}} = \sigma_{\text{old}} \frac{\text{MW}_{\text{new}}}{\text{MW}_{\text{old}}} \]  

**A.4**

The synchronizing torque coefficient of the line linking the reference bus and the equivalent generator is determined by

\[ K_{\text{er}} = \sum_{p=i}^{j} \frac{K_{\text{pr}} \cdot \text{MW}_p}{\sum_{p=i}^{j} \text{MW}_p} \]  

**A.5**

If a different megawatt base is used, \( K_{\text{er}} \) is recalculated using Equation **A.6**.

\[ K_{\text{new}} = K_{\text{old}} \frac{\text{MW}_{\text{old}}}{\text{MW}_{\text{new}}} \]  

**A.6**

The reference bus frequency is given by Equation **A.7**.

\[ \omega_{\text{ref}} = \frac{\sum_{q=i}^{n} K_{q\text{r}} \cdot \text{MW}_q \cdot \omega_q}{\sum_{q=i}^{n} K_{q\text{r}} \cdot \text{MW}_q} \]  

**A.7**

where \( i, \ldots, n \) refer to buses coupled by a direct tie to the reference bus. \( \text{MW}_q \) is the effective megawatt base of the unit behind the \( q \) bus.

Equation **A.8** defines the relationship for the system steady-state frequency.
In Equation A.8 $j$ is equal to the number of simulated units.

With $\omega$, $\sigma_e$ and $K_{er}$ determined, $D_e$ can be calculated by means of Equation A.9.

\[
\frac{\omega}{\omega_0} = \frac{\sum_{p=1}^{j} \left( \frac{\omega_{setp}}{\sigma_p \cdot \omega_0} - \frac{T_{lp}}{T_{Io}} + \frac{\omega_{np}}{\omega_0} \cdot D_p \right) MW_p}{\sum_{p=1}^{j} \left( \frac{1}{\sigma_p} + D_p \right) MW_p}
\]

In Equation A.9 $m, \ldots, t$ are the generators not replaced by the single machine equivalent.

In digital and system analyzer transient stability and load flow studies a common megawatt base for all units is normally selected. In analog computer analyses where generators and their controllers are simulated together, it may be more convenient to use the individual machine ratings as the bases with different base values used for different parts of the system. Irrespective of the type of investigation being conducted, Equations A.4, A.5, and A.9 permit $\sigma_e$, $K_{er}$, and $D_e$
to be found for any megawatt base.

A.3 Mathematical Implications

When the characteristic equation of the units which are replaced is compared to the characteristic equation of the equivalent machine, it is found that the orders are different. Due to this, it may be impossible to find an exact equivalent.

Figure A.3 illustrates an actual two machine plant and a single machine equivalent plant. The transfer function $\omega_{ref}/\omega_s$ in terms of the actual system parameters is given by Equation A.10. $\omega_{ref}/\omega_s$ for the simplified representation is given by Equation A.11.

$$\omega_{ref}/\omega_s = \frac{K_{sref} \cdot MW_s \cdot F_1 \cdot F_2}{(F_1 \cdot F_2 \cdot F_s - F_2 \cdot K_{1r}^2 \cdot MW_s - F_1 \cdot K_{2r}^2 \cdot MW_s)} \quad A.10$$

$$\omega_{ref}/\omega_s = \frac{K_{sref} \cdot MW_s \cdot F_e}{(F_e \cdot F_s - K_{er}^2 \cdot MW_e)} \quad A.11$$

In Equation A.10, $F_1 = T_{m1} S^2 + D_1 S + K_{1r}$

$F_2 = T_{m2} S^2 + D_2 S + K_{2r}$

and $F_s = K_{1r} \cdot MW_1 + K_{2r} \cdot MW_2 + K_{sref} \cdot MW_s$
FIG.-A,3 SINGLE MACHINE EQUIVALENT OF TWO PARALLEL-OPERATING UNITS.
In Equation A.11, \( F_e = T_{me} S^2 + D_e S + K_{er} \)

and \( F_s = K_{er} \cdot M_{we} + K_{sr} \cdot M_{ws} \)

To determine what \( F_e \) must be for identical system performance Equations A.10 and A.11 are equated. The resultant \( F_e \) is given by Equation A.12. \( M_{we} = M_{w1} + M_{w2} \) and \( K_{er} = \left( K_{1r} \cdot M_{w1} + K_{2r} \cdot M_{w2} \right) / \left( M_{w1} + M_{w2} \right) \).

\[
F_e = \frac{A \cdot F_1 \cdot F_2}{F_2 + B \cdot F_1} \quad \text{A.12}
\]

In Equation A.12, \( A = \frac{K_{er} \cdot M_{we}}{K_{1r} \cdot M_{w1}} \) and \( B = \frac{K_{2r} \cdot M_{w2}}{K_{1r} \cdot M_{w1}} \). \( F_e \) defined by Equation A.12 and employed in Equation A.11 are not normally identical.

The approximate value of \( F_e \) obtained by the commonly used averaging method is given by Equation A.13. In this relationship \( M_{w1} = M_{w2} \) and \( M_{we} = M_{w1} + M_{w2} \).

\[
F_e = \left( \frac{T_{m1} + T_{m2}}{2} \right) S^2 + \frac{D_1 + D_2}{2} S + \frac{K_{1r} + K_{er}}{2} \quad \text{A.13}
\]

Figure A.4 illustrates frequency response curves for various machine combinations which result in the same approximate equivalent (Equation A.13). The frequency response of the system with the one machine equivalent (Curve 1) exhibits a single peak. For the actual system with two units
FIG. A.4 FREQUENCY RESPONSE OF TWO PARALLEL OPERATING UNITS.

**Approximate Response**
- $T_m = 10.0$
- $D_e = 1.0$
- $K_{fr} = 100.0$

**Exact Response 1**
- $T_m = 8.0$
- $D_1 = 1.0$, $D_2 = 1.0$
- $K_{fr} = 100.0$, $K_{fr} = 100.0$

**Exact Response 2**
- $T_m = 8.0$, $T_m = 12.0$
- $D_1 = 1.0$, $D_2 = 1.0$
- $K_{fr} = 100.0$, $K_{fr} = 100.0$

**Exact Response 3**
- $T_m = 5.5$, $T_m = 14.5$
- $D_1 = 1.0$, $D_2 = 1.0$
- $K_{fr} = 100.0$, $K_{fr} = 100.0$

**Exact Response 4**
- $T_m = 10.0$, $T_m = 10.0$
- $D_1 = 1.2$, $D_2 = 0.8$
- $K_{fr} = 75.0$, $K_{fr} = 125.0$

**Exact Response 5**
- $T_m = 5.5$, $T_m = 14.5$
- $D_1 = 1.2$, $D_2 = 0.8$
- $K_{fr} = 75.0$, $K_{fr} = 125.0$

$M_{e} = M_{1} + M_{2}$

$K_{er} = \frac{K_{fr} \times M_{1} + K_{fr} \times M_{2}}{M_{1} + M_{2}}$
operating in parallel, mutual oscillatory energy exchanges can occur between Unit 1, Unit 2 and the system; thus for parallel operation (Curves 2-5) a double peaked frequency response occurs. Equations A.10 and A.11 and Figure A.4 illustrate why in a general case it may be impossible to determine an exact single machine equivalent.

A.4 Mathematical Analysis

The analytical approach used in finding a single machine equivalent is divided into two parts. The magnitude of the parameters which affect the steady-state operation are calculated. The remaining coefficients are then chosen to make the transient response of the model approximate that of the actual system. Formulae have been given (Equations A.3, A.5 and A.9) by which $\sigma_e$, $K_{er}$ and $D_e$, the steady-state defining parameters, are calculated. This section describes how one selects the optimum values of the dynamic response describing coefficients $T_{me}$, $T_{we}$, $T_{re}$, $T_{se}$ and $\delta_e$.

The difference between the response of the actual system and the model is measured, squared and integrated with respect to time. By varying the parameter values of the model it is possible to minimize this integral of the squared error $ISE^2$. The smaller the magnitude of $ISE = \int \varepsilon^2 dt$ the better is the similarity of the actual system and the model. The exact representation is achieved when $ISE$ is zero. Since it is
desired to duplicate the dynamics of the actual system by a
simplified version, the values of $T_{ms}$, $T_{we}$, $T_{re}$, $T_{se}$ and $\delta_e$
for which ISE is a minimum are sought. The implementation
of this principle is illustrated in Figure A.5.

In this investigation, the ISE was utilized to compare
the model with the actual system. Various other criteria
such as $\int |\epsilon| dt$, $\int \epsilon^{2n} dt$ or $\int \epsilon^{2n} e^{m} dt$ where $n$ and $m$ are
integers, could have been used. The application of different
criteria will usually result in different single machine
equivalents.

The limits of integration of ISE are zero to infinity
and thus it is a function of both the transient and steady-
state response. The values of the steady-state defining
parameters, $\sigma_e$, $K_{er}$ and $D_e$ can also be selected from the
minimization of ISE, but due to computational considerations
these parameters are precalculated and the integration interval
is made finite.

Since the minimization of ISE is a five variable optimization
search problem, the solution need not be unique since
several combinations of $T_{ms}$, $T_{we}$, $T_{re}$, $T_{se}$ and $\delta_e$ may result
in a minimum. In general, the convergence to a minimum of
the ISE does not necessarily mean that the absolute minimum
value has been reached. If, after repeating the search, a
number of times with different initial conditions, the search
in each case converges to the same combination of parameter
FIG-A.5 DETERMINATION OF AN OPTIMUM SINGLE MACHINE EQUIVALENT.
magnitudes one may assume with confidence that the best solution has been found.

Reference bus frequency variation of the actual system for a step change in load is usually a damped sinusoid or a damped sinusoid plus an exponential. To give a damped sinusoidal response at least two coupled energy storage elements and an energy dissipater are required, consequently to get a "good" single machine equivalent it may not be necessary to vary all five parameters ($T_{me}$, $T_{we}$, $T_{re}$, $T_{se}$ and $\delta_e$).

In this system $T_s$, the servomotor time constant, has no appreciable effect on the system performance unless it is abnormally large. Thus it may be fixed at some representative value. Often for comparison purposes it is convenient to constrain $T_w$. By the imposition of constraints on the equivalent (i.e. fixing $T_s$ and $T_w$) the complexity of the optimization problem is reduced. It must be realized that the final model obtained with constraints will at best be only as "good as" or "poorer than" that obtained without constraints.

The conditions for a minimum ISE may be found using any of the numerous optimization techniques available. A Taylor series approximation is used in this study. The first and second derivatives of ISE as a function of each parameter are determined, then the derivatives are used to predict the magnitude of the parameters for a minimum of the ISE. This is a repetitive operation since the location of the predicted
minimum is usually only in the vicinity of the actual minimum. Optimum search patterns characterized by this search technique normally approach the final solution asymptotically. The analyst must terminate the search when an improvement in solution accuracy is not sufficient to offset the cost of another iteration.

A.5 Experimental Results

Several configurations of two machines operating in parallel were selected and an equivalent for each was established. The purpose of the experiments was to assess how various factors affected the solution. Three cases were studied, the first consisted of two units strongly coupled to an infinite bus, in the second the infinite bus was replaced by a large system, in the third the strength of the tie between the two machines and the system was reduced.

In Figure A.6 the representation and time response of the two units tightly coupled to an infinite bus are given. Machine and system characteristics were selected to give a bimodal reference bus frequency oscillation for a step change in infinite bus frequency. A single machine feeding an infinite bus exhibits a unimodal oscillation, therefore in this particular case an exact single machine equivalent is not obtainable.

In Figure A.7 the infinite bus of Figure A.6 is replaced
FIG.-A.6 TIME RESPONSE VARIATIONS BETWEEN A SINGLE MACHINE EQUIVALENT AND TWO PARALLEL MACHINES FEEDING AN INFINITE BUS.
by a relatively large system. The frequency deviation time response of the reference bus is given for the actual system, for an approximate equivalent and for an optimized equivalent. There is no discernible difference in the response of the actual and optimized representation. For the approximate equivalent case there is good agreement for the first three seconds.

The purpose of the next set of tests was to illustrate the effect that different disturbances have on the equivalent found. The model used in this set of tests consisted of two similar units connected to a large system by a comparatively weak tie. This is the arrangement which most often occurs in practice. The simulation was perturbed by three types of load disturbances, a step, a sinusoidal and a ramp. The single machine equivalents were found by either the integral of the error squared (ISE) or integral of absolute error criteria. In each case a different equivalent was obtained. Refer to Figure A.8 for the conditions of the test, Test #1 will be taken as the reference for comparison.

In Test #2 the values of $T_{we}$ and $T_{se}$ were fixed at 1.0 sec. and 0.05 sec. respectively. A reduction in $T_{we}$ would tend to increase the governor-penstock characteristic frequency; therefore, to maintain the same frequency as the actual system, the dashpot time constant, $T_{re}$, would have to be increased. The rate of response of a mechanical-hydraulic temporary droop
FIG.-A.7 TIME RESPONSE VARIATIONS OF THE APPROXIMATE AND OPTIMIZED SINGLE MACHINE EQUIVALENTS.
<table>
<thead>
<tr>
<th>TEST NO</th>
<th>CRITERIA</th>
<th>DISTURBANCE</th>
<th>EQUIVALENT UNIT</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ISE</td>
<td>$\frac{\Delta T_{1i}}{T_{1o}}$ = .05</td>
<td>$T_{me} = 4.54$  $T_{we} = 1.12$  $T_{te} = 9.92$  $T_{se} = .03$  $\delta_e = .81$</td>
<td>NO</td>
</tr>
<tr>
<td>2</td>
<td>ISE</td>
<td>$\frac{\Delta T_{1i}}{T_{1o}}$ = .05</td>
<td>$T_{me} = 4.56$  $T_{we} = 11.12$  $T_{te} = .03$  $T_{se} = .05$</td>
<td>CONSTRAINTS</td>
</tr>
<tr>
<td>3</td>
<td>ISE</td>
<td>$\frac{\Delta T_{1i}}{T_{1o}}$ = .05</td>
<td>$T_{me} = 4.57$  $T_{we} = 11.18$  $T_{te} = .03$  $T_{se} = .05$  $T_{m2} = 6.0$  $T_{m3} = 3.0$  $\delta_2 = .52$  $\delta_3 = .133$</td>
<td>IMPOSED</td>
</tr>
<tr>
<td>4</td>
<td>$\int</td>
<td>c</td>
<td>, dt$</td>
<td>$\frac{\Delta T_{1i}}{T_{1o}}$ = .05</td>
</tr>
<tr>
<td>5</td>
<td>ISE</td>
<td>$\frac{\Delta T_{1i}}{T_{1o}}$ = .01</td>
<td>$T_{me} = 4.48$  $T_{we} = 8.44$  $T_{te} = .03$  $T_{se} = .05$</td>
<td>CONSTRAINTS</td>
</tr>
<tr>
<td>6</td>
<td>ISE</td>
<td>$\frac{\Delta T_{1i}}{T_{1o}}$ = .05 $\sin 3.1 t$</td>
<td>$T_{me} = 4.50$  $T_{we} = 8.78$  $T_{te} = .03$  $T_{se} = .05$</td>
<td>CONSTRAINTS</td>
</tr>
<tr>
<td>7</td>
<td>ISE</td>
<td>$\frac{\Delta T_{1i}}{T_{1o}}$ = .05 $\sin 3.1 t$</td>
<td>$T_{me} = 4.57$  $T_{we} = 8.51$  $T_{te} = .03$  $T_{se} = .05$</td>
<td>CONSTRAINTS</td>
</tr>
</tbody>
</table>

FOR ALL TEST $\sigma_e = .05$  
$K_{er} = 329.7$  
$D_e = 2.0$  
$MW_e = 200000$

FIG.- A.8 SINGLE MACHINE EQUIVALENTS FOR VARIOUS TEST CONDITIONS.
governor is dominated by the magnitude of \( \lambda_1 \), where
\[ \lambda_1 = \frac{T_w}{\delta T_m}. \]
With a decrease in \( T_{we} \), \( \delta_e \) must be reduced if \( \lambda_1 \) is to remain constant.

In Test #3, \( T_{me} \) was slightly greater than the average value. One possible reason is that the governor on the larger unit (\( T_m = 6.0 \) sec.) was set at \( \lambda_1 = 0.32 \) while for the smaller it was set at \( \lambda_1 = 0.25 \), thus causing the system dynamics to be dominated by the larger unit.

The results of Tests #2 and #4 do not differ significantly. Thus the error criterion used, in this particular case, has a marginal influence.

Tests #5, 6 and 7 were conducted for time varying load disturbances. The significant differences in the resulting equivalents are the reduced values of \( T_{re} \). One possible reason for this is that the two units in the actual system tend to hunt more with time varying loads, thus to duplicate the performance a more active governor is required (small \( T_{re} \)) on the equivalent.

A.6 Discussions and Conclusions

A systematic method has been described to determine the single machine equivalent of a bank of \( n \) machines. The method is illustrated for the case of \( n=2 \) but this method can be extended to provide a facility by which all machine parameter values can be found for \( n>2 \). The equivalent obtained is a
function of numerous conditions such as the degree of machine similarity, the type of disturbance, the constraints imposed, the error criterion used and the degree of system coupling.

The main advantage of the technique is accuracy while its cost and time of computation are its disadvantages. Before employing this method in lieu of simpler approximation methods, engineering judgement is necessary to decide whether the refinement obtained is sufficient to offset the additional cost.
APPENDIX B - STABILITY LIMIT ANALYSIS

B.1 Description of Solution

The stability limit of a linear system can be determined by operating on the system characteristic equation (Equation B.1) using the Routh-Hurwitz stability criterion. The criterion states that for stability the determinants $D_1$ to $D_{n-1}$ must be equal to or greater than zero.

$$a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 = 0 \quad \text{B.1}$$

$$D_1 = |a_1| \quad \text{B.2}$$

$$D_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} \quad \text{B.3}$$

$$D_{n-1} = \begin{vmatrix} a_1 & a_0 & \cdots \\ a_3 & a_2 & \cdots & a_{n-3} \\ \vdots \end{vmatrix} \quad \text{B.4}$$

In Chapter 3 three different conditions were studied. The first assumed constant $\lambda_3$ and $\lambda_4$, the second considered a single unit feeding an isolated load while the third considered a single machine coupled to an infinite bus. Each situation is discussed below.
For a single machine with constant $\lambda_3$ and $\lambda_4$ and feeding an isolated load it is sufficient to examine only $D_3$. Setting $D_3 = 0$ and arranging the coefficients a quadratic equation (Equation B.5) with $\lambda_1$ as the dependent variable is obtained.

$$A_3\lambda_1^2 + B_3\lambda_1 + C_3 = 0$$  \hspace{1cm} (B.5)

where

$$A_3 = (\lambda_2 + \lambda_3/2 + \lambda_3\lambda_4/2 + 2)A_2 - (\lambda_3^2 - 2\lambda_2\lambda_3 + 2\lambda_3^2)$$  \hspace{1cm} (B.6)

$$B_3 = (\lambda_2 + \lambda_3/2 + \lambda_3\lambda_4/2 + 2)B_2 - (2\lambda_2\lambda_3 - 2\lambda_2\lambda_3^2 + 2\lambda_3^2)$$  \hspace{1cm} (B.7)

$$C_3 = (\lambda_2 + \lambda_3/2 + \lambda_3\lambda_4/2 + 2)C_2 - \lambda_2\lambda_3^2\lambda_4$$  \hspace{1cm} (B.8)

$$A_2 = \lambda_3^2(\lambda_2 - 1)$$  \hspace{1cm} (B.9)

$$B_2 = \lambda_3^2(1 + \lambda_4 - 2\lambda_2\lambda_4 - 3\lambda_2/2 - \lambda_2\lambda_4/2 - 3\lambda_2\lambda_3)$$  \hspace{1cm} (B.10)

$$C_2 = \lambda_3^2(\lambda_2\lambda_4 + \lambda_2\lambda_4^2 + \lambda_2\lambda_4^2 + \lambda_2\lambda_3\lambda_4)$$  \hspace{1cm} (B.11)

To determine the stability boundary as a function of $\lambda_1$, $\lambda_2$ and constant $\lambda_3$ and $\lambda_4$, $\lambda_1$ is calculated by means of the quadratic formula as $\lambda_2$ is incremented through the range of interest.

For a specific machine, with a constant permanent droop and servomotor time, the above procedure must be modified. For each value of $\lambda_2$ that the stability limit is to be calculated, an initial value of $\lambda_1$ is selected. This defines $\lambda_3$ and $\lambda_4$. Substituting $\lambda_2$, $\lambda_3$ and $\lambda_4$ into Equation B.5, $\lambda_1$ is calculated as before. Based on the initial guess and on the calculated guess
a new value of $\lambda_1$ is determined which in turn defines $\lambda_3$ and $\lambda_4$. This process is repeated until $\lambda_1$ converges. This sequence is conducted for all desired values of $\lambda_2$.

For a single machine feeding an infinite bus, $D_4 = 0$ defines the limiting conditions for stable operation (smallest region on the $\lambda_1\lambda_2$ plane). Since $D_4$ is a quartic function of $\lambda_1$, an analytical solution of $\lambda_1$ is cumbersome. An iterative solution is preferable. The rate of change of $D_4$ with respect to $\lambda_1$, $\frac{\Delta D_4}{\Delta \lambda_1}$, is numerically determined at some initial value of $\lambda_1$. Using this slope information to predict a new $\lambda_1$ and by repeating the sequence, the smallest positive value of $\lambda_1$ which corresponds to $D_4 = 0$ may be found. This procedure is executed for all desired values of $\lambda_2$. 
APPENDIX C - SELECTION OF THE ERROR CRITERION

C.1 Description of the Error Criterion

If a stable second order system is at steady state with a constant input and if that input is suddenly reduced to zero, then the subsequent variation in the output $\delta$ is given by a polynomial of the form

$$a_2 \delta^{11} + a_1 \delta^1 + \delta = 0 \quad C.1$$

where $a_1$ and $a_2$ are constants determined by the system characteristics and $\delta$ is the output variable. The roots of the system characteristic equation are plotted in Figure C.1 and $a_1$ and $a_2$ are defined by Equations C.2 and C.3.

$$a_1 = 2\alpha/(a^2 + b^2) \quad C.2$$

$$a_2 = 1/(a^2 + b^2) \quad C.3$$

$$\omega^2 = a^2 + b^2 \quad C.4$$

In Figure C.1 $\omega$ is the system natural frequency and $\zeta$ is the damping ratio. The weighting factors of the Krasovskii general error criterion which is a minimum for a sinusoidal output with a damping ratio of .707 are given by Equations C.5, C.6 and C.7.

$$\tau_1^2 = 0 \quad C.5$$

$$\tau_2^4 = 1/(a^2 + b^2)^2 = 1/\omega^4 \quad C.6$$
FIG.-C.1 CHARACTERISTIC EQUATION ROOTS OF AN UNDER-DAMPED STABLE SECOND ORDER SYSTEM.
\[ \tau_3^6, \ldots \tau_n^{2n} = 0 \]

Therefore the error criterion which is a minimum when the damping ratio of the output variable of a regulation process is .707 is given by Equation C.8.

\[ J = \int (\delta^2 + \tau_2^4 (\delta^{11})^2) dt \]

The time response of the hydro-electric generation frequency control cannot be faithfully described by a second order polynomial, however, the time response in the region of optimum controller settings does approach a damped sinusoid.

The description of the output frequency variation as given in Figure 5.1 is given by Equation C.9. Observation and mathematical analysis suggest that \( A \), the amplitude approximates the form stated below.

\[ \omega(t) = Ae^{-\xi \omega t} \sin \omega \sqrt{1-\xi^2} t \]

where \( \omega(t) \) is the frequency deviation caused by a load disturbance,

\[ A = K_1/\omega(1-\xi^2) \]

and \( K_1 \) is a constant.

The following operations are performed on Equation C.9. It is differentiated twice and each derivative is squared and integrated with respect to time. Equations C.10 to C.14 are the results.
\[ w^1(t) = A \omega e^{-\zeta \omega t} \cos \left( \omega \sqrt{1-\zeta^2} \ t + \psi \right) \]  
\[ w^{11}(t) = - A \omega e^{-\zeta \omega t} \sin \left( \omega \sqrt{1-\zeta^2} \ t + 2\psi \right) \]
\[ \int_0^\infty w(t)^2 \, dt = \frac{A^2}{4\omega} \left( \frac{1}{\zeta} + \sin(-\psi) \right) = Q_4 \]
\[ \int_0^\infty (w^1(t))^2 \, dt = \frac{A^2 \omega}{4} \left( \frac{1}{\zeta} - \sin(\psi) \right) = Q_5 \]
\[ \int_0^\infty (w^{11}(t))^2 \, dt = \frac{A^2 \omega^3}{4} \left( \frac{1}{\zeta} + \sin(3\psi) \right) = Q_6 \]

In Equations C.10 to C.14 \( \psi = \tan^{-1}(\zeta/\sqrt{1-\zeta^2}) \).

Therefore \( \tau_2^4 = (Q_4/Q_5)^2 \). When the error criterion as given by Equation C.8 is minimized using the above value of \( \tau_2^4 \) the frequency deviation of the hydro-electric generator resembles an overly damped sinusoid. The reasons for this is that in the actual system a frequency offset exists and the time response contains components in addition to the damped sinusoid. If the square of \( Q_5/Q_6 \) is used as the weighting factor, \( \tau_2^4 \), better results are realized.

Figure C.2 is a plot of \( J \), Equation C.8 with \( (Q_5/Q_6)^2 \) substituted for \( \tau_2^4 \), as a function of damping ratio \( \zeta \) and natural frequency \( \omega \) for a process defined by Equation C.9.

From examination of Figure C.2 it is seen that the optimization search converges to the higher natural frequencies. For example, if two different sets of governor settings exist which give an optimum damped frequency deviation, this optimization search technique will converge to the one with the higher natural frequency and thus to the response with the shorter time-to-steady-state.
FIG.-C.2 ERROR CRITERION OF AN IDEALIZED MODEL OF A HYDRO-ELECTRIC POWER GENERATOR.

\[ J = \int_0^\infty (\Delta \omega^2 + \zeta^2 (\omega^2) ) \, dt \]