Error Correction Model Estimation of the Canada-US Real Exchange Rate

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ABSTRACT

Using the error correction model, we link the long-run behavior of the Canada-US real exchange rate to its short-run dynamics. The equilibrium real exchange rate is determined by the energy and non-energy commodity prices over the period 1973Q1-1992Q1. However such a single long-run relationship does not hold when the sample period is extended to 2004Q4. This breakdown can be explained by the break point which we find at 1993Q3. At the break point, the effect of the energy price shocks on Canada's real exchange rate turns from negative to positive while the effect of the non-energy commodity price shocks is constantly positive. We find that after one year 40.03% of the gap between the actual and equilibrium real exchange rate is closed. The Canada-US interest rate differential affects the real exchange rate temporarily. The Canada's real exchange rate depreciates immediately after a decrease in Canada's interest rate and appreciates next quarter but not by as much as it has depreciated.

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List of Abbreviations

N: Nontradables produced in Canada.

C: Commodities produced in Canada.

N^*: Nontradables produced in the US

M: Manufactured goods produced in the US

Subscript N, C: Canada's nontradable and commodity sector respectively.

Subscript N^*, M: the US nontradable and manufacturing sector respectively.

Superscript *: the US

P_N: Nominal price of N.

P_C: Nominal price of C.

P_{N*}: Nominal price of N^*.

P_M: Nominal price of M.

p_N: Relative price of N in terms of M.

p_C: Relative price of C in terms of M.

p_{N*}: Relative price of N^* in terms of M.

K: Capital.

L: Labor.

r: Real interest rate.

w: Real wage rate.

Y: Real output.

k: Capital-labor ratio.

Π: Present value of real profit.

MPK: Marginal productivity of capital.

MPL: Marginal productivity of labor.

$\hat{w}$: Percentage change in w.

$\hat{p}_N$: Percentage change in p_N.

μ_{LN}: Labor's share in the nontradable sector.

μ_{LC}: Labor's share in the commodity sector.

P: Price index or price level.

E: Nominal exchange rate expressed as home currency price of foreign currency.

q: Real exchange rate expressed as home currency price of foreign currency.

M^s: Money supply.

M^d: Money demand.

R: Nominal interest rate.

E^e: Expected nominal exchange rate.

RFX: Real Canadian per US dollar exchange rate.

COM: Real non-energy commodity price.

ENE: Real energy price.

RDIFF: Canada-US nominal interest rate differential.

DRFX: First difference of RFX.

DCOM: First difference of COM.

DENE: First difference of ENE.

DINF: Canada-US inflation differential.

AN: Amano and van Norden (1995).

I (1): Integrated with order one.

DGP: Data-generating process.

AR(1): Autoregressive process of order 1.

Chapter 1 Model Specification

1.1 Introduction

Over the past thirty years of Canada's experience with free exchange rates, we have witnessed a significant variation in the Canada-US exchange rate from a low of 62 cents in November of 2001 to a high of 1.09 in November of 2007. Since the ratio of the US price level to the Canadian price level is very stable over the same interval, the high variability of nominal exchange rates has been directly correlated with high variability of real exchange rates.

The nature of this relationship between nominal and real exchange rates has played an important role in the Canadian debate regarding the optimal exchange rate regime. Courchene and Harris (1999) have argued that there is no compelling evidence to suggest that macroeconomic fundamentals are driving the real exchange rate, but rather that speculative behavior unrelated to fundamentals is the cause of the volatility of nominal exchange rates. That is, the correlation between nominal and real exchange rates is evidence of causality running from nominal to real exchange rates. As a consequence, Canada has suffered significant periods of misalignment of real exchange rates with their attendant real adjustment costs. In their view, fixing the Canada-US exchange rate would eliminate volatility of nominal and real exchange rates and by extension eliminate real adjustment costs.

In contrast, proponents of free rates such as Laidler (1999) employ the argument of Friedman (1953) that the underlying cause of nominal exchange rate volatility is shifts in real macroeconomic fundamentals that require real exchange rate adjustments. That is, causality runs from real exchange rate volatility to nominal exchange rate volatility. In this case, fixing the nominal exchange rate would force the adjustment required by real shocks to domestic prices and wages that are much less flexible than nominal exchange rates in the short run. Therefore, fixing the exchange rate would impose greater real adjustment costs on the economy in the face of real fundamental disturbances.

In order to shed some light on this controversy, in this thesis I extend and estimate a fundamental model of real exchange rate determination first proposed by Amano and van Norden (1995). Amano and van Norden, hereafter, AN find that commodity terms of trade, the price of Canada's exported commodities divided by the price of its imported manufactured goods, play a key role in explaining the Canada-US real exchange rate movements. In their study, they split the terms of trade into two components, energy and non-energy terms of trade. Their results of cointegration tests, the single-equation method proposed by Hanson (1990) and the Johansen-Juselius system approach proposed by Johansen and Juselius (1990), show that the real exchange rate is cointegrated with energy and non-energy terms of trade. This cointegrating relationship indicates that the non-energy commodities and energy terms of trade have a long-run effect on the variations in the Canada-US real exchange rate, or the long-run equilibrium values of the real exchange rate are determined by these terms of trade.[1] Then they estimate the error correction model using the nonlinear least-squares approach proposed by Phillips and Loretan (1991) which estimates both the cointegrating relationships and error correction model simultaneously.

After successively omitting variables with insignificant t-statistics, AN construct a simple equation, called the Bank of Canada's exchange rate equation. This equation expresses the changes in the real exchange rate as a function of energy and non-energy terms-of-trade and an interest rate differential which is defined as the difference between Canada's and the US gaps between short- and long-term interest rates. One of the findings from the estimated equation is that an increase in the non-energy terms of trade causes Canada's real exchange rate to appreciate while an increase in the energy terms of trade causes it to depreciate. The

[1] Chen and Rogoff (2003) also find that for Canada there is a long-run cointegrating relationship between the real exchange rate and commodity prices, but relatively weak co-movement in the short run. However they do not include energy prices in their analysis.

other finding is that the interest rate differential has a transitory effect and an increase in it causes Canada's real exchange rate to appreciate immediately.

AN's equation fits the data so well that it can forecast the most important turning points over the sample period 1973M1 to 1992M2. Its out-of-sample forecast performance is better than a random walk. The results of their research also uncover several facts. First, energy price shocks can account for the greatest portion of real exchange rate variance among all the explanatory variables. Secondly, energy price and commodity price shocks could reinforce or cancel one another. The net movement of the real exchange rate depends on the net effect of different shocks. Thirdly, large persistent price shocks have more significant effects than large short-lived shocks. Finally, the interest rate differential appears to play a smaller role than do the terms of trade.

The Bank of Canada's exchange rate equation obtained by AN, however, raises two questions. First, contrary to their expectation, AN find that higher energy prices lead to a real depreciation of the Canadian dollar even though the United States is more dependent on energy imports than Canada. This result is counter to the view of the Canadian dollar as a petro-currency. In their study, they failed to explain the mystery. Secondly, is the model durable? With the passage of time, can it still track the movements in real exchange rates reasonably well? Laidler and Aba (2001) estimated the Bank of Canada equation for the period 1973-2000 with three separate coefficients on each of the energy and non-energy commodity prices for the 1970s, 1980s and 1990s. Their main finding is that the positive effect of non-energy commodity prices seems to decline from decade to decade while the effect of energy prices changes from negative for the first two decades to insignificant for the 1990s. Using cointegration tests with a structural break, Issa, Lafrance and Murray (2006) find that 1993Q3 is a break point at which the sign of the effect of energy prices on the Canadian dollar turned from negative to positive. The break, they suggest, can be attributed to the growing importance of energy exports caused by the deregulation of Canadian energy sector and the North American Free Trade Agreement.

In my study, using quarterly data from 1973 to 2004, I investigate whether it is still the energy and non-energy commodity prices that determine the Canada-US real exchange rate in the long run, whether the interest differential is still powerful in explaining the short-run dynamics of the Canada-US real exchange rate, and whether a structural change occurs during the sample period.

The thesis is organized as follows: In Sections 1.2 and 1.3 we review the economic theories on how commodity price shocks affect the real exchange rate in the long run and how the real exchange rate deviates from its equilibrium value due to sticky price level in the short run. Section 2.1 introduces the set of variables investigated and describes the data used. Section 2.2 demonstrates the time series econometric methods applied to examine the models described in Sections 1.2 and 1.3. In Section 2.3 we check the effectiveness of the methods by comparing the results of AN with ours using the sample period 1973Q1 to 1992Q1. Chapter 3 presents the results of the sample period 1973Q1 to 2004Q4 with and without a structural break. Chapter 4 summarizes.

1.2 Relationship of the Real Exchange Rate, Terms of Trade and Productivity Changes

The model reviewed in this section is based on Obstfeld and Rogoff (1996) and Chen and Rogoff (2003). It can explain how terms of trade affect the long run behavior of the Canada real exchange rate. Assume Canada is a small open economy relative to the world market. Thus Canada has no power to determine the prices of its exported and imported products. All price shocks in our study are exogenous.

Assume Canada produces nontradables (N) and commodities (C) and it exports C to the US. The US produces nontradables (N^*) and manufactured goods (M). Canada imports M from the US. Then C, M and N are consumed in Canada while C, M and N^* are consumed in the US. Let P_N, P_{N*}, P_C and P_M denote the nominal prices of N, N^*, C and M. Their relative prices in terms of M are p_N, p_{N*} and p_C, respectively. Assume there are two inputs for each sector of N, C, M and N^*: capital (K) and labor (L). Capital can migrate between countries and industries while labor is free to migrate only between industries. As a result, the real interest rate (r), the cost of capital, is equalized all over the world while the real wage rate (w), the cost of labor, is equalized across industries within a country. Both r and w, like the real prices, are in terms of M.

Canada's constant return production functions for N and C can be written as

$$Y_N = A_N F(K_N, L_N) \tag{1.1a}$$

$$Y_C = A_C G(K_C, L_C) \tag{1.1b}$$

where Y denotes real output, subscripts N and C denote the nontradable sector and commodity sector respectively, and A denotes an exogenously varying productivity coefficient, which measures changes in technology. The exogeneity of A implies that A does not change with K or L. F and G describe how N and C are produced from K and L. We define the capital-labor ratio (k) as

$$k_i = \frac{K_i}{L_i}$$

where subscript i refers to sector. Then the per labor outputs can be expressed as

$$f(k_N) = F(k_N, 1), \tag{1.2a}$$

$$g(k_C) = G(k_C, 1) \tag{1.2b}$$

Y_N and Y_C can be rewritten as

$$Y_N = A_N L_N f(k_N) \tag{1.1c}$$

$$Y_C = A_C L_C g(k_C) \tag{1.1d}$$

Discounted by the constant world interest rate r, the present value of real profit (Π) in the nontradable sector in period t is

$$\Pi_N = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} [p_{N,s} A_{N,s} L_{N,s} f(k_{N,s}) - w_s L_{N,s} - \Delta K_{N,s+1}] \tag{1.3a}$$

and in the commodity sector is

$$\Pi_C = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} [p_{C,s} A_{C,s} L_{C,s} g(k_{C,s}) - w_s L_{C,s} - \Delta K_{C,s+1}] \tag{1.3b}$$

where $\Delta K_{i,s+1}=K_{i,s+1}-K_{i,s}$. We assume that there is no depreciation of the capital stock. We also assume that the capital which is used this period must have been accumulated by the end of last period while labor can be adjusted in each period.

By first-order condition for the profit-maximization problem, in the nontradable sector we have

$$\frac{\partial \Pi_N}{\partial K_{N,\,s+1}}=\frac{\sum_{s=t}^{\infty}(\frac{1}{1+r})^{s-t}[p_N A_N L_{N,s} f(k_{N,s})-w_s L_{N,s}-(K_{N,s+1}-K_{N,s})]}{\partial K_{N,\,s+1}}=0$$

$$\Rightarrow -1+\frac{1}{1+r}[p_N A_N L_{N,s+1}\frac{\partial f_{N,s+1}}{\partial k_{N,s+1}}\frac{\partial k_{N,\,s+1}}{\partial K_{N,s+1}}+1]=0$$

$$\Rightarrow \ -1+\frac{1}{1+r}[p_N A_N L_{N,s+1} f'(k_{N,s+1)})\frac{1}{L_{N,s+1}}+1]=0$$

$$\Rightarrow \ p_N A_N f'(k_{N,s+1})-r=0$$

$$\Rightarrow \ p_N A_N f'(k_{N,s+1})=r \qquad (1.4a)$$

and

$$\frac{\partial \Pi_N}{\partial L_{N,s}}=\frac{\sum_{s=t}^{\infty}(\frac{1}{1+r})^{s-t}[p_N A_N L_{N,s} f(k_{N,s})-w_s L_{N,s}-(K_{N,s+1}-K_{N,s})]}{\partial L_{N,\,s}}=0$$

$$\Rightarrow \frac{\partial[p_N A_N L_{N,s} f(k_{N,s})-w_s L_{N,s}]}{\partial L_{N,s}}=0$$

$$\Rightarrow p_N A_N (\frac{\partial L_{N,s}}{\partial L_{N,s}} f(k_{N,s})+L_{N,s}\frac{\partial f(k_{N,s})}{\partial L_{N,s}})-w_s=0$$

$$\Rightarrow \ p_N A_N (f(k_{N,s})+L_{N,s}\frac{\partial f_{N,s}}{\partial k_{N,s}}\frac{\partial k_{N,s}}{\partial L_{N,s}})-w_s=0$$

$$\Rightarrow \ p_N A_N (f(k_{N,s})+L_{N,s} f'(k_{N,s})(-\frac{K_{N,s}}{(L_{N,s})^2}))-w_s=0$$

$$\Rightarrow \ p_N A_N (f(k_{N,s})-f'(k_{N,s})\frac{K_{N,s}}{L_{N,s}})-w_s=0$$

$$\Rightarrow \ p_N A_N (f(k_{N,s})-f'(k_{N,s})k_{N,s})=w_s \qquad (1.4b)$$

where $p_N A_N f'(k_N)$ equals $\frac{\partial p_N Y_N}{\partial K_N}$ which is the marginal productivity of capital (MPK_N)

and $p_N A_N(f(k_N)\text{-}f'(k_N)k_N)$ equals $\frac{\partial p_N Y_N}{\partial L_N}$ is the marginal productivity of labor (MPL_N). Equations (1.4a) and (1.4b) show that the profit maximization conditions are that MPK equals the world interest rate and MPL equals the prevailing real wage rate.

Similarly, in the commodity sector, we have

$$p_C A_C g'(k_{C,s+1})=r \qquad (1.4c)$$

$$p_C A_C (g(k_{C,s})\text{-}g'(k_{C,s})k_{C,s})=w_s \qquad (1.4d)$$

We assume all the productivity coefficients A_i are constant. According to (1.4c), an increase in the relative price of commodities in Canada leads to an increase in the MPK in the commodity sector. In order to keep MPK equal to the exogenous and constant world interest rate, the effect of the rise in p_C must be neutralized by a higher capital-labor ratio if MPK is decreasing in the level of capital utilized. These increases in the commodity price and capital-labor ratio drive up MPL in the commodity sector. The real wage rate must rise as well to satisfy Equation (1.4d). In Canada's nontradable sector, at least one of the nontradable price and capital-labor ratio must rise to satisfy Equation (1.4b) in response to the increased real wage rate. Similar to Equation (1.4c), Equation (1.4a) shows that the nontradable price and capital-labor ratio must move in the same direction to keep MPK equal to the constant world interest rate. As a result, both the nontradable price and capital-labor ratio increase. In other words, to survive in the market the nontradable producers can reduce the burden of the higher wage cost by increasing the nontradable price and the proportion of capital used. Therefore, the effect of the commodity price shocks is transmitted to Canada's nontradable prices through changes in the real wage level in Canada.

We assume that the US has the same production functions as Canada. Repeating the procedure for Equations (1.4a), (1.4b), (1.4c) and (1.4d), we have

$$p_N^* A_N^* f'(k_{N,s+1}^*)=r \qquad (1.5a)$$

$$p_N^* A_N^* (f(k_{N,s}) - f'(k_{N,s}^*) k_{N,s}^*) = w^* \quad (1.5b)$$

$$A_M^* g'(k_{M,s+1}^*) = r \quad (1.5c)$$

$$A_M^* (g(k_{M,s}^*) - g'(k_{M,s}^*) k_{M,s}^*) = w_s^* \quad (1.5d)$$

where the superscript * refers to the US. As we can see from Equations (1.5a) to (1.5d), any change in p_C cannot affect either w* or p_N^* by virtue of the assumption that commodities are not produced in the US.

From Equations (1.1a) and (1.1b), the nontradable and commodity outputs in terms of manufactured goods are

$$p_N Y_N = p_N A_N F(K_N, L_N) \quad (1.1e)$$

$$p_C Y_C = p_C A_C G(K_C, L_C) \quad (1.1f)$$

Total differentiating Equations (1.1c) and (1.1d) and substituting $\frac{\partial p_i Y_i}{\partial K_i} = MPK_i$ and $\frac{\partial p_i Y_i}{\partial L_i} = MPL_i$ give

$$p_N Y_N = MPK_N K_N + MPL_N L_N \quad (1.1g)$$

$$p_C Y_C = MPK_C K_C + MPL_C L_C \quad (1.1h)$$

To satisfy the profit-maximization conditions which are Equations (1.4a), (1.4b), (1.4c) and (1.4d), Equations (1.1g) and (1.1h) can be rewritten as

$$p_N Y_N = rK_N + wL_N$$

$$p_C Y_C = rK_C + wL_C$$

Substituting for Y_N and Y_C from Equations (1.1c) and (1.1d) and dividing both sides by L_N and L_C respectively gives the zero-profit conditions as follows

$$p_N A_N f(k_N) = rk_N + w \quad (1.6a)$$

$$p_C A_C f(k_C) = rk_C + w \quad (1.6b)$$

Log-differentiating (1.6a) yields

$$d\log p_N A_N f(k_N) = d\log(w + rk_N) \tag{1.6c}$$

$$\text{the left-hand side} = d(\log p_N + \log A_N + \log f(k_N))$$
$$= d\log p_N + d\log A_N + d\log f(k_N)$$
$$= \frac{dp_N}{p_N} + \frac{dA_N}{A_N} + \frac{df(k_N)}{f(k_N)}$$

Substituting $f'(k_N) = \frac{df(k_N)}{dk_N}$ from Equation (1.4a) and solving for $df(k_N)$ gives

$$df(k_N) = \frac{rdk_N}{p_N A_N}$$

Substituting for $df(k_N)$ on the left-hand side yields

$$\text{left-hand side} = \frac{dp_N}{p_N} + \frac{dA_N}{A_N} + \frac{rdk_N}{p_N A_N f(k_N)}$$
$$= \frac{dp_N}{p_N} + \frac{dA_N}{A_N} + \frac{rk_N}{p_N A_N f(k_N)} \frac{dk_N}{k_N}$$

$$\text{right-hand side} = \frac{d(w + rk_N)}{w + rk_N}$$
$$= \frac{dw + rdk_N}{w + rk_N}$$

By eq.(1.6a) $p_N A_N f(k_N) = rk_N + w$, substituting for $w + rk_N$ on the right-hand side yields

$$\text{right-hand side} = \frac{dw + rdk_N}{p_N A_N f(k_N)}$$
$$= \frac{w}{p_N A_N f(k_N)} \frac{dw}{w} + \frac{rk_N}{p_N A_N f(k_N)} \frac{rdk_N}{rk_N}$$

Therefore equation (1.6c) can be rewritten as

$$\frac{dp_N}{p_N} + \frac{dA_N}{A_N} + \frac{rk_N}{p_N A_N f(k_N)} \frac{dk_N}{k_N} = \frac{w}{p_N A_N f(k_N)} \frac{dw}{w} + \frac{rk_N}{p_N A_N f(k_N)} \frac{rdk_N}{rk_N}$$

$$\Rightarrow \frac{dp_N}{p_N} + \frac{dA_N}{A_N} = \frac{w}{p_N A_N f(k_N)} \frac{dw}{w} \tag{1.6d}$$

Let us define percentage changes in A_N, w, p_N and labor’s share as follows

$$\hat{A}_N = \frac{dA_N}{A_N} \tag{1.7a}$$

$$\hat{w} = \frac{dw}{w} \tag{1.7b}$$

$$\hat{p}_N = \frac{dp_N}{p_N} \tag{1.7c}$$

$$\mu_{LN} = \frac{w}{p_N A_N f(k_N)} \tag{1.7d}$$

Equation (1.6d) can be rewritten as

$$\hat{p}_N + \hat{A}_N = \mu_{LN}\,\hat{w} \tag{1.8a}$$

Similarly, in the commodity sector, we have

$$\hat{p}_C + \hat{A}_C = \mu_{LC}\,\hat{w} \tag{1.8b}$$

Solving for $\hat{w}$ from Equation (1.8b) gives

$$\hat{w} = \frac{\hat{p}_C + \hat{A}_C}{\mu_{LC}}$$

Substituting for $\hat{w}$ into Equation (1.8a) gives

$$\hat{p}_N + \hat{A}_N = \mu_{LN}\frac{\hat{p}_C + \hat{A}_C}{\mu_{LC}}$$

$$\Rightarrow \hat{p}_N = \frac{\mu_{LN}}{\mu_{LC}}(\hat{p}_C + \hat{A}_C) - \hat{A}_N \tag{1.9a}$$

Since we have assumed that A_C and A_N are constant, equation (1.9a) can be reduced to

$$\hat{p}_N = \frac{\mu_{LN}}{\mu_{LC}}\hat{p}_C \tag{1.9b}$$

From Equation (1.9b), we can see that the relative nontradable price moves by the same percentage as the relative commodity price if labor's share in the nontradable sector equals that in the commodity sector. Generally the commodity sector is more capital intensive than the nontradable sector, that is, $\frac{\mu_{LN}}{\mu_{LC}} > 1$. As a result, the effect of commodity price shocks on p_N should be reinforced.

Repeating the procedure for the US, we have

$$\hat{p}_N^* + \hat{A}_N^* = \mu_{LN}^* \hat{w}^* \qquad (1.10a)$$

$$\hat{A}_M^* = \mu_{LM}^* \hat{w}^* \qquad (1.10b)$$

Then we have

$$\hat{p}_N^* = \frac{\mu_{LN}^*}{\mu_{LM}^*} \hat{A}_M^* + \hat{A}_N^* = 0 \text{ if } \hat{A}_M^* = \hat{A}_N^* = 0 \qquad (1.11)$$

Equation (1.11) shows that changes in the price of the US nontradables are not related to the commodity prices.

Basic economic theory tells us that any price is determined by both the supply-side and demand-side factors. We have discussed the supply side above. Now we move to the demand side. We assume that the aggregate utility function for Canada is in Cobb-Douglas form

$$U=C_N^{\alpha}C_M^{\beta}C_C^{1-\alpha-\beta} \qquad (1.12)$$

where U is aggregate utility, C_i is aggregate consumption of N, M or C, and α, β as well as 1-α-β are non-negative constants. Then the price index P consistent with the Cobb-Douglas utility function is also a geometric average of P_N, P_M and P_C, with weights α, β and 1-α-β respectively.

$$P=P_N^{\alpha}P_M^{\beta}P_C^{1-\alpha-\beta}$$

Dividing P by P_M, we have

$$p=p_N^{\alpha}p_C^{1-\alpha-\beta} \qquad (1.13)$$

where p is the Canadian price index in terms of M.

Log-differentiating Equation (1.13), we have

$$\mathrm{dlog}p=\mathrm{dlog}(p_N^{\alpha}p_C^{1-\alpha-\beta})$$

$$\Rightarrow \frac{dp}{p}=\alpha\frac{dp_N}{p_N}+(1-\alpha-\beta)\frac{dp_C}{p_C}$$

$$\Rightarrow \hat{p} = \alpha\,\hat{p}_N + (1-\alpha-\beta)\,\hat{p}_C$$

Substituting for $\hat{p}_N$ from Equation (1.9b) into the above equation yields

$$\hat{p} = \alpha \frac{\mu_{LN}}{\mu_{LC}} \hat{p}_C + (1-\alpha-\beta)\hat{p}_C$$

$$= \alpha(\frac{\mu_{LN}}{\mu_{LC}} - 1)\hat{p}_C + (1-\beta)\hat{p}_C \qquad (1.14)$$

We assume that Canada and the US have the same consumption pattern. This implies that the weights of prices of nontrables, manufactured goods and commodities in the price index are identical for these two countries. Repeating the procedure above for the US, we obtain

$\hat{p}^* = \alpha \hat{p}_N^* + (1-\alpha-\beta)\hat{p}_C$, which, using Equation (1.11), yields

$$\hat{p}^* = (1-\alpha-\beta)\hat{p}_C \qquad (1.15)$$

Equation (1.14) shows that if the relative commodity price rises by one percent, Canada's price index will rise by 1-β percent if $\frac{\mu_{LN}}{\mu_{LC}} = 1$ or greater than 1-β percent if $\frac{\mu_{LN}}{\mu_{LC}} > 1$. On the other hand, the US price index will rise by 1-α-β which is smaller than Canada's because changes in the relative commodity price do not affect the relative US nontradable prices. The decrease in the Canada-US price level ratio (p^*/p) implies that one Canadian consumption bundle can exchange for more US consumption bundles and thus the Canadian dollar experiences a real appreciation against the US dollar. Therefore the relative commodity prices have a negative effect on the real exchange rate defined as p^*/p.

1.3 Deviation of Relative Exchange Rate from Its Long-run Value

The theories reviewed in this section are based on the model described by Krugman and Obstfeld (2000). The nominal and real exchange rates are related by the condition (1.16)

$$E = q\frac{P}{P^*} \qquad (1.16)$$

where E is nominal exchange rate, q is real exchange rate, P is the home price level, or the Canadian price level in our study, and P^* is the foreign price level, or the US price level. E is expressed as the home currency price of foreign currency, or CAD/USD.

The condition for equilibrium in the money market is:

$$M^s = M^d \qquad (1.17a)$$

where M^s is money supply and M^d is money demand. Assume that M^s is controlled by the central bank. The money demand is determined by

$$M^d = \underset{+}{P} \times L(\underset{-}{R}, \underset{+}{Y}) \qquad (1.17b)$$

where L is real money demand and R is nominal interest rate. Other things equal, a decrease in R or an increase in Y causes an increase in M^d.

The uncovered interest parity condition shows that assets denominated in all currencies must offer an identical expected rate of return measured in comparable terms when the foreign exchange market is in equilibrium. For given domestic interest rate R, foreign interest rate R^* and expected nominal exchange rate E^e, the interest parity condition can be used to determine the current equilibrium nominal exchange rate.

$$R - R^* = (E^e - E)/E \qquad (1.18)$$

We assume that both Canadian and US price levels cannot change in the short run. Suppose there is an increase in the money supply in Canada while the US money supply remains unchanged. Then the interest rate in Canada declines to clear its money market. Equation (1.18), the uncovered interest parity condition, shows that the Canadian dollar depreciates immediately (E rises) and goes up higher than its long-run expected value. As a result, Canada's real exchange rate depreciates as well (q rises) due to the sticky domestic price level so that the real exchange rate moves away from its constant long-run value.

However as the domestic price level starts to rise in response to the money supply increase, the domestic interest rate must rise with a given output level to keep the domestic money market in equilibrium. Canada's nominal exchange rate thus appreciates to clear the foreign exchange market. Canada's real exchange rate starts to appreciate as well to approach its constant long-run value due to the price adjustment and the appreciation of the Canadian dollar. Once the domestic price level rises proportionally to the increase in the stock of money, and the nominal exchange rate depreciates proportionally to the increase in stock of money, the interest rate differential and real exchange rate return to their long-run values. Therefore, a domestic money supply increase makes Canada's real exchange rate deviate from its long-run value shortly after the change of monetary policy, but in the long run, this change has no effect on the Canada-US real exchange rate.

1.4 Error Correction Model

The theories discussed in Sections 1.2 and 1.3 tell us that real commodity price shocks have a long-run effect on the movement in the real exchange rate while monetary shocks have a transitory effect. In order to investigate the behavior of the Canada-US real exchange rate, we need a model to combine these two effects. The error correction model (ECM) put forward by Granger and Weiss (1983) is a model that relates the variation in a time series integrated of order one (I(1)) to its long-run determinants which are cointegrated with the time series, and its short-run factors which are stationary. In Chapter 2 we investigate the time series properties of the relevant variables and estimate the ECM.

Chapter 2 Diagnostic Testing and Re-estimation of the AN Model

2.1 Data Definition and Sources

The variables under consideration include the real exchange rate (RFX), the real price of non-energy commodities (COM), the real price of energy (ENE) and interest rate differential (RDIFF). All the price variables are measured in the real terms and in logarithms. The definitions of all the variables are as follows:

$$\text{RFX}=\log\left[\text{Canada-US exchange rate(CAD/USD)}\frac{\text{US GDP deflator}}{\text{Canadian GDP deflator}}\right] \tag{2.1a}$$

$$\text{COM}=\log\left[\frac{\text{non-energy commodity price index}}{\text{US GDP deflator}}\right] \tag{2.1b}$$

$$\text{ENE}=\log\left[\frac{\text{energy commodity price index}}{\text{US GDP deflator}}\right] \tag{2.1c}$$

$$\text{RDIFF}=\text{3-month yield on prime corporate paper in Canada-3-month yield on commercial paper in US} \tag{2.1d}$$

As shown in Equation (2.1a), RFX is the measure of the Canadian goods price of the US goods. We take logarithms for RFX, COM and ENE so that any difference between two consecutive periods measures the percentage change in those variables. The reason we choose 3-month interest rates is that they best match quarterly RFX which we investigate.

As we can see from above equations, the data collected include the Canada-US nominal exchange rate, the US GDP deflator, the Canadian GDP deflator, the non-energy commodity price index, the energy commodity price index, 3-month yield on prime corporate paper in Canada, and 3-month yield on commercial paper in the US. The nominal exchange rate is the monthly average of the noon daily spot rate in CAD/USD, the price of the US dollar in terms of the Canadian dollar. The US GDP deflator is the seasonally adjusted implicit price deflator with the base year 2000. The Canadian GDP deflator is the implicit price deflator at market prices with the base year 1997. The non-energy and energy commodity price indexes developed by the Bank of Canada are fixed-weight indexes of the spot or transaction prices of commodities produced in Canada and sold in world markets in US dollar terms with the base years 1982-1990. The weight is based on the average value of Canadian production of the commodity over the 1988-1999 period.[2] Non-energy commodities consist of food (grains and oilseeds, livestock, and fish) and industrial materials (metals, minerals, and forest products). Energy commodities consist of crude oil, natural gas and coal. All these data except the US GDP deflator come from CANSIM II while the US GDP deflator comes from the US Bureau of Economic Analysis.[3] In this Chapter, the sample period is from 1973Q1 to 1992Q1 and in Chapter 3 from 1973Q1 to 2004Q4.

There are three major differences in the definition of some variables compared to AN. First of all, the US and Canadian GDP deflators are used to convert nominal values to real values. Since in Section 1.2 we assumed that as a small open economy, Canada has no power to determine the prices of its exported and imported products and we defined the real commodity price in terms of the manufactured goods produced in the US, the real commodity price shocks are exogenous to Canada. To measure real exogenous price shocks, we use the US GDP deflator, the price index of goods and services produced in the US instead of its CPI, the price index of those consumed in the US, to calculate ENE and COM due to the fact that some goods and services consumed in the US are produced in Canada. The advantage of the US GDP deflator is that it can exclude the effect of any production change in Canada on real price shocks. That is also the reason that Laidler and Aba (2001) suggest that GDP deflator is more appropriate than the CPI in this situation. Accordingly, we use the US and Canadian GDP deflators to obtain real exchange rates. All the data collected are monthly except the US and Canadian GDP deflators which are quarterly. We thus take the average of those monthly observations to get quarterly ones and then calculate those quarterly variables. Secondly, we

[2] [2]The description of the variables is from the website of the Bank of Canada: http://www.bank-banque-canada.ca/en/rates/commod.htm.

[3] "CANSIM is Statistics Canada's database providing access to current and historical time series data collected on a wide variety of subjects. In 2001, Statistics Canada released CANSIM II, an updated version of CANSIM." The quotation about CANSIM is from the website of the library of the University of Saskatchewan: https://library.usask.ca/data/business/cansim.

use the price index of non-energy and energy commodities deflated by the US GDP deflator, instead of terms of trade, to capture the economic shocks to the Canada-US real exchange rate. Chen and Rogoff (2003) point out that sluggish price adjustment and incomplete pass-through make standard terms-of-trade variables inappropriate to measure exogenous shocks to which the real exchange rate would respond. Thirdly, our RDIFF is defined as the gap between 3-month yields on commercial paper in Canada and in the US. AN measure the interest rate differential with the difference between Canada's and the US's gaps between short- and long-term nominal interest rates. But they find that the effect of RDIFF changes slightly when short, long or both of them are chosen to construct RDIFF.

It is instructive to present data on Canada's energy and non-energy commodity exports to motivate their roles in real exchange rate determination. As Figure 2.1a shows, the share of energy to Canada's total exports varies from 8% to 17% over the period 1973-2004. The share of non-energy commodities has been decreasing from more than 50% in the 70s to around 35% after 2000. The importance of energy and non-energy commodity exports can be better understood by decomposing Canada's net exports. Figure 2.1b shows that net exports of commodities decide Canada's net trade position. In most years, net exports of commodities exceed net imports of other merchandise, so Canada always has positive net exports. What's more, Canada's net exports of energy and non-energy commodities appear to increase over time.

2.2 Econometric Analysis of Time Series

The error correction model explains how a nonstationary series, which is supposed to be the RFX variable in our study, tends to approach its long-run equilibrium determined by other nonstationary series which are cointegrated with RFX, and how its short-run dynamics are affected by stationary series. In Section 2.2.1, we test the stationarity of the series involved in our study using the Augmented Dickey-Fuller (ADF) test and Phillips-Perron (PP) tests. At the same time, we try to discover the data-generating process (DGP) of each series as knowledge of the DGP helps us determine the specification of the cointegration test and the

Figure 2.1a Canada's shares of energy and non-energy commodities to total exports

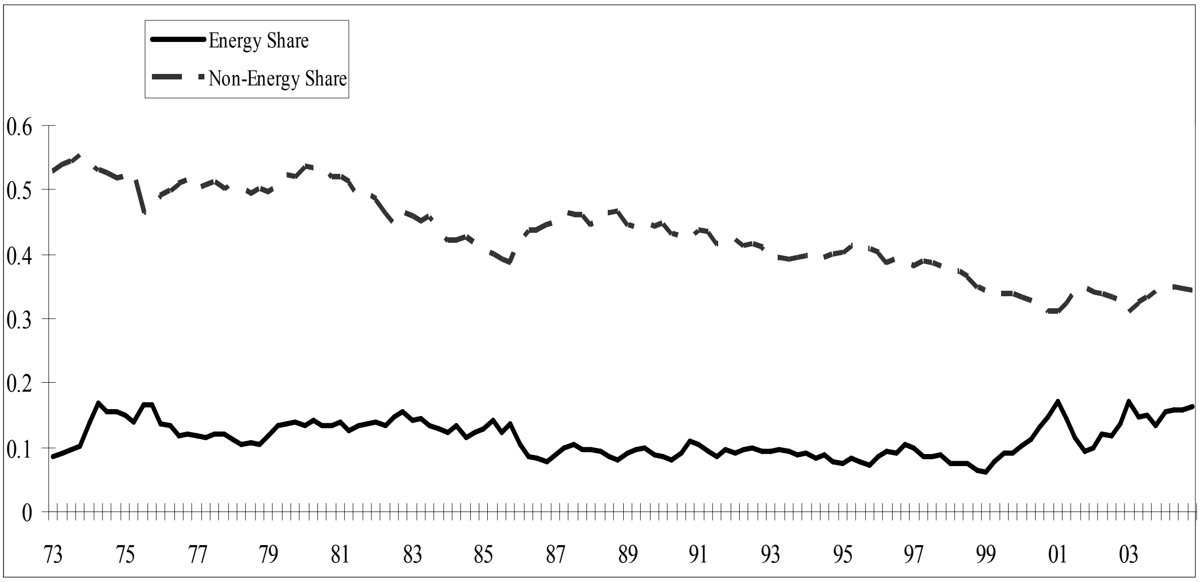
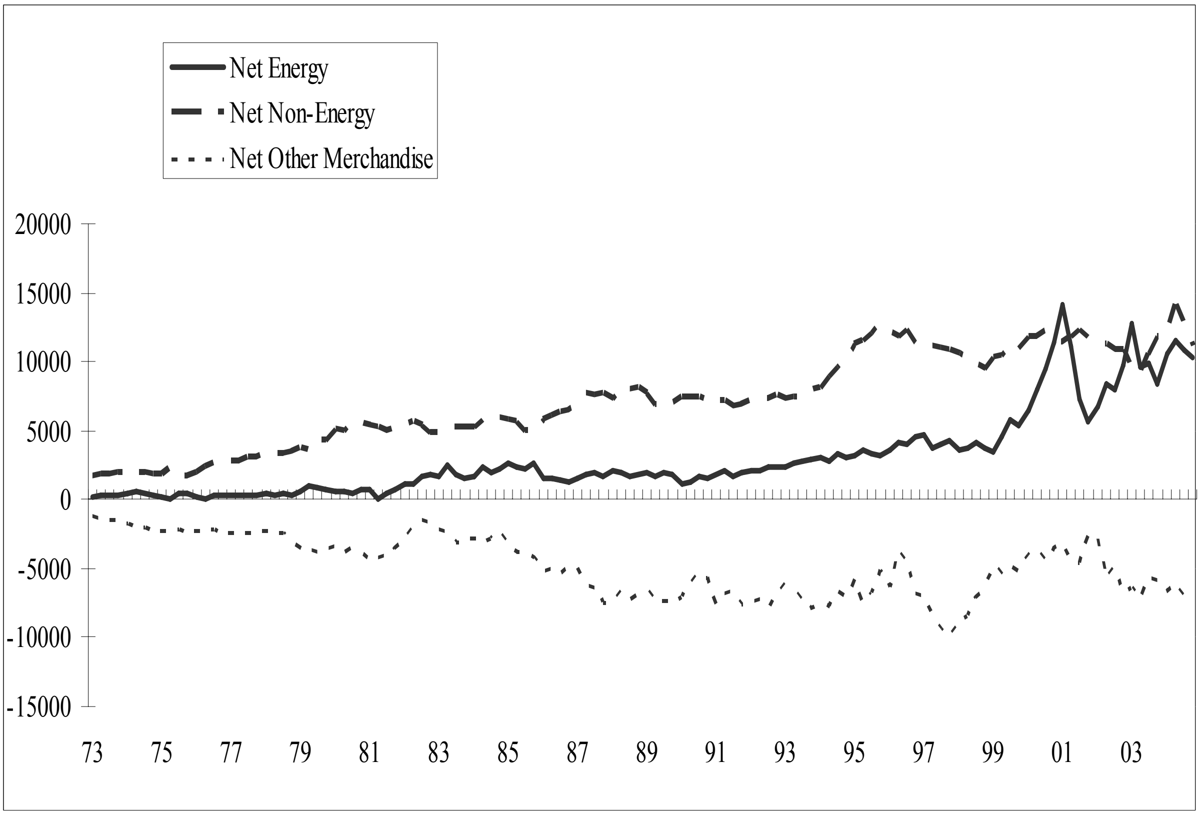


Figure 2.1b Canada's decompositions of net exports



ECM. If RFX is nonstationary and it is not the only nonstationary variable, we conduct the augmented Engle-Granger (AEG) test and Johansen procedure to find out whether RFX is cointegrated with other nonstationary series in Section 2.2.2. If there exist one or more cointegrating relationships, we use a two-step procedure to estimate the cointegrating regression in Section 2.2.2 and then estimate the ECM in Section 2.2.3.

2.2.1 Unit Root Tests

Our first step is to test for a unit root in RFX, the dependent variable. If it does have a unit root, it is nonstationary and we can decompose it into two components: one is its trend component that includes a stochastic trend and perhaps a deterministic trend; the other is its stationary component. Then we need to find two groups of factors to explain the behavior of RFX. One group shares the common stochastic trend with RFX. The other captures its short-run dynamics. The next step is to test COM, ENE, and RDIFF for evidence of a unit root. Those nonstationary variables are candidates for the first group which determines the long-run equilibrium of RFX. The stationary variables might be in the second group which explains the short-run deviations of RFX from its equilibrium value. RFX is stationary if it does not contain a unit root. In this case, any shock to RFX is temporary because RFX tends to revert to its long-run mean level and the effect of any shock will die out over time.

Actually, to detect the true DGP is still a challenging topic in time series econometrics. We start with plotting the time series under study. A plot usually gives us some clues about the pattern of the series such as the presence of a trend, the existence of a constant mean, and how much it moves and down, but this graphical analysis can not distinguish a unit root process from a trend stationary or from a near unit root process. We need a more precise econometric method which is known as unit root test to determine how a time series y_t is generated and whether this series has a unit root. A framework of three widely used models developed by Bhargava (1986) will be introduced to test for a unit root.

In case I, y_t can be represented as

$$y_t=\gamma_0+\gamma_1 t+\mu_t \quad (2.2a)$$

$$\mu_t=\rho\mu_{t-1}+e_t \quad (2.2b)$$

where t is time, μ_t is an error term which is an autoregressive process of order 1 (AR(1)), e_t is a stationary error term, γ_0 is a constant, γ_1 is the coefficient of time, and ρ is the coefficient of autocovariance. Substituting for μ_t from Equation (2.2b) into Equation (2.2a) gives

$$y_t=\gamma_0+\gamma_1 t+\rho\mu_{t-1}+e_t$$

Substituting for μ_{t-1} from Equation (2.2a) into the above equation gives

$$\begin{aligned} y_t&=\gamma_0+\gamma_1 t+\rho(y_{t-1}-\gamma_0-\gamma_1(t-1))+e_t \\ &=\gamma_0(1-\rho)+\gamma_1\rho+\gamma_1(1-\rho)t+\rho y_{t-1}+e_t \\ &=\delta_0+\delta_1 t+\rho y_{t-1}+e_t \end{aligned}$$

or

$$\Delta y_t=\delta_0+\delta_1 t+\delta_2 y_{t-1}+e_t \quad (2.2c)$$

where $\delta_0 = \gamma_0(1-\rho) +\gamma_1\rho$, $\delta_1 = \gamma_1(1-\rho)$, and $\delta_2 = 1-\rho$.

In case II, y_t can be represented as

$$y_t=\gamma_0+\mu_t \quad (2.3a)$$

$$\mu_t=\rho\mu_{t-1}+e_t \quad (2.3b)$$

Repeating the procedure for Equation (2.2c), we have

$$\begin{aligned} y_t&=\gamma_0+\rho(y_{t-1}-\gamma_0)+e_t \\ &=\gamma_0(1-\rho)+\rho y_{t-1}+e_t \\ &=\delta_0+\rho y_{t-1}+e_t \end{aligned}$$

or

$$\Delta y_t=\delta_0+\delta_2 y_{t-1}+e_t \quad (2.3c)$$

Where $\delta_0 = \gamma_0(1-\rho)$ and $\delta_2 = 1-\rho$.

In case III, y_t can be represented as

$$y_t=\mu_t \quad (2.4a)$$

$$\mu_t=\rho\mu_{t-1}+e_t \tag{2.4b}$$

Repeating the procedure for Equation (2.2c), we have

$$\begin{aligned} y_t &= \rho y_{t-1}+e_t \\ &= \rho y_{t-1}+e_t \end{aligned}$$

or

$$\Delta y_t=\delta_2 y_{t-1}+e_t \tag{2.4c}$$

Where δ_2 = 1-ρ.

The null hypothesis of unit root test is $\delta_2 = 0$ or $\rho = 1$. If the null cannot be rejected, y_t has a unit root or it is nonstationary. If the null can be rejected, y_t does not have a unit root or it is stationary. In case I, the failure of rejection of the null implies that δ_1 is also equal to zero because $\delta_1 = \gamma_1\delta_2$. Then the DGP under the null is a random walk with a drift:

$$\begin{aligned} y_t &= \delta_0+y_{t-1}+e_t \\ &= \delta_0+(\delta_0+y_{t-2}+e_{t-1})+e_t \\ &= y_0+\delta_0 t+\sum_{i=1}^{t} e_i \end{aligned} \tag{2.2d}$$

As we can see in Equation (2.2d), y_t is I(1), that is, its first difference is stationary but the mean of its first difference cannot be zero as δ_0 is not supposed to be zero in this case. The economic meaning of e_t is a random shock on y_t. $\sum_{i=1}^{t} e_i$ is called the stochastic trend which indicates that any random shock has a persisting effect on the current and future values of y. The rejection of the null in case I means that y_t is a trend-stationary process, or that y_t moves around its time trend. In case II, the failure of rejection of the null implies that δ_0 is also equal to zero because $\delta_0 = \gamma_0\delta_2$. Then the DGP under the null is a random walk:

$$\begin{aligned} y_t &= y_{t-1}+e_t \\ &= y_{t-2}+e_{t-1}+e_t \\ &= y_0+\sum_{i=1}^{t} e_i \end{aligned} \tag{2.3d}$$

As we can see in Equation (2.3d), y_t is I(1) and its first difference is a stationary process with zero mean. The rejection of the null in case II means that y_t is a stationary process around a constant. In case III, the DGP under the null is the same as Equation (2.3d). The rejection of the null means that y_t is a stationary process with zero mean.

Since the t-ratio for the unit root null does not follow conventional normal distribution, Dickey and Fuller (1979) first calculated the critical values depending on the form of regression and sample size. In most empirical studies however, the error terms in Equations (2.2c), (2.3c) and (2.4c) are found to be serially correlated. Based on the assumption that Δy_t is an AR process, the ADF test suggests adding lagged values of the dependent variable Δy_t to equations (2.2c), (2.3c) and (2.4c):

$$\Delta y_t=\delta_0+\delta_1 t+\delta_2 y_{t-1}+\sum_{i=1}^{n}\lambda_i \Delta y_{t-i}+e_t \tag{2.2}$$

$$\Delta y_t=\delta_0+\delta_2 y_{t-1}+\sum_{i=1}^{n}\lambda_i \Delta y_{t-i}+e_t \tag{2.3}$$

$$\Delta y_t=\delta_2 y_{t-1}+\sum_{i=1}^{n}\lambda_i \Delta y_{t-i}+e_{tt} \tag{2.4}$$

where λ is the coefficient of the lagged Δy_t and n is the lag length. The tests based on equations (2.2), (2.3) and (2.4) are called the ADF test for case I, case II, and case III, respectively. The critical values of the ADF test are the same as those provided by Dickey and Fuller (1979). We use the Akaike information criterion 2 (AIC2) to select the optimal n. Pantula, Gonzalez-Farias and Fuller (1994) pointed out that AIC2 can avoid the problem of size distortion caused by AIC.

To solve the problem of autocorrelation of the error terms in Equations (2.2c), (2.3c) and (2.4c), Phillips and Perron (1988) proposed a nonparametric approach of modifying the statistics. This adjusted test statistic has the same asymptotic distribution as the ADF test statistic. MacKinnon (1994) calculates the critical values for these two well-known unit root tests. The results of the Monte Carlo study by DeJong, Nankervis, Savin and Whiteman (1992) show: as error terms in Equations (2.2c), (2.3c) and (2.4c) appear to be negatively autocorrelated, the PP test tends to reject the null hypothesis of a unit root while the power of the ADF test slightly drops; on the other side, as the errors appear to be positively autocorrelated, the power of the PP test drops a little while the rejection frequency of the ADF test slightly increases. Therefore the ADF test is more reliable in the presence of negative autocorrelation of the errors while the PP test is more reliable in the presence of positive autocorrelation. Therefore, their findings help to decide which test is preferred in such situations that the ADF test and PP test give conflicting results.

In case I or II, testing for the joint hypothesis can also be used to find the presence of a unit root in y_t and its DGP. The null hypothesis for Equation (2.2) is $\delta_1 = \delta_2 = 0$ and the null for Equation (2.3) is $\delta_0 = \delta_2 = 0$. Dickey and Fuller (1981) provide the critical values of Φ_1 statistic for Equation (2.3)and Φ_3 for Equation (2.2). These Φ statistics, formed in exactly the same way as ordinary F-test, are:

$$\Phi i=\frac{(RSS_{R,i}-RSS_{UR,i})/r}{RSS_{UR,i}/(T-k)}$$

where $RSS_{R,i}$ = the residual sum of squares from the restricted model

$RSS_{UR,i}$ = the residual sum of squares from the unrestricted model

i =type of Φ

r = number of restrictions

T = number of usable observations

k= number of parameters estimated in the unrestricted model

To use the Bank of Canada equation, here are the necessary conditions: RFX, COM and ENE are nonstationary and RDIFF is stationary.

2.2.2 Cointegration Tests and Cointegrating Regression

Since we are interested in the Canada-US real exchange rate movements, in this section we will explain how the long-run equilibrium value of RFX can be determined by other nonstationary variables if RFX is nonstationary itself. Engle and Granger (1987) state that the linear combination of two or more nonstationary variables that is stationary can be interpreted as a long-run equilibrium relationship among these variables. The sense of their statement is that the stochastic trend in RFX, which cannot be predicted directly, can be eliminated by the stochastic trends in those nonstationary variables that are cointegrated with RFX so that RFX is affected by those variables permanently.

Before conducting cointegration tests and estimating cointegrating vectors, we need to determine the appropriate form of the cointegrating regression which should be consistent with the DGPs of variables involved. A general cointegrating regression for I (1) variables can be written as

$$Y_t=\beta_0+\beta_1'X_t+\beta_2 t+e_t \tag{2.5}$$

where Y_t is RFX in our study, X_t is an n-dimensional vector of nonstationary variables which are cointegrated with Y_t, β_1 is an n-dimensional coefficient vector, $\begin{pmatrix}1\\-\beta_1\end{pmatrix}$ is the n+1-dimensional cointegrating vector and e_t is a stationary error term. As we can see in Equation (2.2d), the value of a nonstationary variable is a function of time t. So if any of those nonstationary variables tested for cointegration is generated by Equation (2.2d), or a random walk with a drift, a time t should be included in the cointegrating regression. Otherwise e_t cannot be stationary in this case and we have a problem of model specification error because the omitted variable t would be in e_t. If all those nonstationary variables are generated by Equation (2.3d), or a random walk, t should be excluded from the cointegrating regression because none of the variables is related to t.

We use the AEG test, a single equation method, and the Johansen procedure, a system method to do the cointegration test. The AEG test is a residual-based test which conducts a unit root test, usually the ADF test, for the estimated error term e_t in Equation (2.5). The failure to reject the unit root null hypothesis indicates that the linear combination of the variables is not stationary, and the null hypothesis of no cointegration cannot be rejected either. We then conclude that there does not exist a cointegrating relationship. If e_t appears to be stationary, we conclude that Y_t and the variables in X_t are cointegrated. There are some disadvantages of the AEG test we need to mention here. First, the AEG test is a two-step procedure. Any bias from the first step of estimation can be carried to the second step of the ADF test. Secondly, for a system of n nonstationary variables in which n is greater than 2, there can be as many as n-1 linearly independent cointegrating relationships. But the AEG test, which is based on one equation, cannot help us to determine the number of cointegrating relationships.

If we look at n nonstationary series as a system, the Johansen procedure starts with the vector autoregression (VAR) model:

$$Y_t = A_0 + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + e_t$$

where Y_{t-k} is an n-dimensional vector of I(1) variables for k =0,1,…,p, A_0 is an n-dimensional constant vector, A_j is a $n \times n$ matrix of parameters for j = 1,2,…,p, p is the lag length, and e_t is an n-dimensional vector of stationary error terms. Subtracting Y_{t-1} from each side gives

$$\Delta Y_t = A_0 + (A_1 - I) Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + e_t$$

Adding and subtracting $A_p Y_{t-p-1}$ on the right-hand side gives

$$\Delta Y_t = A_0 + (A_1 - I) Y_{t-1} + A_2 Y_{t-2} + \cdots + (A_{p-1} + A_p) Y_{t-p-1} - A_p \Delta Y_{t-p-1} + e_t$$

Then adding and subtracting $(A_{p-1}+A_p)Y_{t-p-2}$ on the right-hand side gives

$$\Delta Y_t = A_0 + (A_1 - I) Y_{t-1} + A_2 Y_{t-2} + \cdots + (A_{p-2} + A_{p-1} + A_p) Y_{t-p-2} - (A_{p-1} + A_p) \Delta Y_{t-p-2} - A_p \Delta Y_{t-p-1} + e_t$$

Continuing adding and subtracting $(\sum_{i=j}^{p} A_i) Y_{t-i}$ gives

$$\begin{aligned} \Delta Y_t &= A_0 + (-I + \sum_{i=1}^{p} A_i) Y_{t-1} - (\sum_{i=2}^{p} A_i) \Delta Y_{t-1} - \cdots - A_p \Delta Y_{t-p-1} + e_t \\ &= A_0 + \Pi Y_{t-1} - B_2 \Delta Y_{t-1} - \cdots - B_p \Delta Y_{t-p-1} + e_t \end{aligned} \qquad (2.6)$$

where $\Pi = -I + \sum_{i=1}^{p} A_i$, $B_j = \sum_{i=j}^{p} A_i$ for j = 2,…,p, and I is an $n \times n$ identity matrix. The number of different cointegrating relationships is determined by the rank of Π (rank(Π)) which equals the number of Π's characteristic roots that differ from zero. In practice, two statistics λ_{trace} and λ_{max} are widely used to check the significance of the number of independent cointegrating relationships:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{n} \ln(1-\hat{\lambda}_i) \quad (2.7a)$$

$$\lambda_{max}(r, r+1) = -T\ln(1-\hat{\lambda}_{r+1}) \quad (2.7b)$$

where $\hat{\lambda}_i$ is the descending estimated values of the characteristic roots (also called eigenvalue) obtained from the estimated Π, T is the number of usable observations, and r is the number of cointegrating relationships. The statistic λ_{trace} is to test the null hypothesis that the number of distinct cointegrating vectors is less than or equal to r against the alternative that the number is more than r. If the estimated λ_{trace} is greater than the critical value at a specific significance level, we conclude that there are more than r cointegrating relationships in the system. Otherwise we conclude that there are at most r cointegrating relationships. The null hypothesis of the maximum eigenvalue test is that the number of distinct cointegrating vectors is r and its alternative is that there are r+1 cointegrating relationships. If we cannot reject the null, we conclude that there are r cointegrating relationships. The rejection of the null indicates that there are r+1 cointegrating relationship. For both these statistics, we start with r=0 and then continue with r=1, r=2… until r=n-1. Usually the value of the first r which cannot be rejected is the number of cointegrating relationships. MacKinnon, Haug and Michelis (1999) calculate the critical values of λ_{trace} and λ_{max} statistics for several specifications of regression models based on Equation (2.6) taking account of different forms of deterministic trend.

If there are more than one cointegrating relationships in the system, the Johansen procedure can be used to estimate the cointegrating regressions. If only one cointegrating relationship exists, we use Saikkonen’s (1991) dynamic ordinary least squares (DOLS) with Newey-West heteroscedasticity and autocorrelation consistent (HAC) standard error and covariance to estimate those long-run parameters. Using DOLS we will estimate Equation (2.5) with $e_t = \sum_{j=-k_1}^{k_2} b_j \Delta X_{t-j} + v_t$ ($j \neq 0$) where ΔX_{t-j} is an n-dimensional vector of leads or lags of first differences of X_t, v_t is an independent and identically distributed (i.i.d.) process with zero mean and constant variance, or white noise error term, k_1 is the number of leads, k2 is the number of lags and b_j is an n-dimensional coefficient vector. Since all those elements of X_t are I(1), their first differences must be stationary. This implies that they cannot have a long-run effect on Y_t. As a result, we cannot include them in our cointegrating relationship. Maddala and Kim (1998) summarize many Monte Carlo studies on the estimates of cointegrating parameters and conclude that for a system with only one cointegrating relationship, a linear model with leads and lags like DOLS, is better than any other method. DOLS performs better than the AEG because DOLS can correct the bias in the estimates from static regressions caused by superconsistency. Empirical studies suggest that the Johansen estimates exhibit high variances. If the error variance is not constant or the error terms are serial correlated, the OLS estimates are no longer efficient. The Newey-West procedure, which is designed to handle both heteroscedasticity and autocorrelation, is able to obtain efficient standard errors of OLS estimates. Therefore, the Newey-West HAC estimates do not change the value of OLS estimates, but the t ratios which affect the significance of estimated parameters.

The economic theory discussed in Section 1.2 suggests that in the long run, the real commodity price shocks have a positive effect on the Canadian real exchange rate due to the fact Canada is a commodity exporter to the US. So we expect that RFX is cointegrated with COM and ENE and that the estimated coefficients of COM and ENE are both negative, as RFX is the reciprocal of the Canadian real exchange rate.

2.2.3 Error Correction Model

In this section we introduce the model which explains the fluctuations of an I(1) variable by combining its long-run behavior with its short-run dynamics. If RFX is cointegrated with the prices of energy and non-energy commodities, as we have expected, there is a long-term equilibrium relationship among them. In the short run however, RFX may deviate from its equilibrium value. This deviation, which is called the equilibrium error, can be used to tie the long-run value of RFX to its short-run variation. An important theorem known as the Granger representation theorem (Granger and Weiss 1983) states that if a set of I (1) variables are cointegrated, they can be expressed as ECM which can be written as

$$\Delta Y_t=\alpha_1(Y_{t-1}-\beta_1'X_{t-1}-\beta_0-\beta_2 t)+\sum_{i=1}^{m}\gamma_i'Z_{t-i}+\alpha_0+\varepsilon_t \qquad (2.8)$$

where ΔY is ΔRFX in our study, the change in the real exchange rate from one quarter to the next, X_{t-1} is an n-dimensional vector of other I(1) variables which are cointegrated with RFX, Z_t is an l-dimensional vector of the first differences of all the nonstationary variables in the system and other stationary variables which can explain the short-run movement in RFX, α_0 is a constant, α_1 is the coefficient of the speed of adjustment, m is the number of lags of Z and ε_t is a white noise error term. The component in the parentheses, $Y_{t-1}-\beta_1'X_{t-1}-\beta_0-\beta_2 t$, is the one-period lagged value of deviation of Y from its long-run equilibrium. The specification of ECM, or the presence of a constant α_0 depends on the DGP and the means of the series in the model. If all nonstationary variables are generated by Equation (2.3d) and the mean of any stationary variable is equal to zero, there is no reason to put a constant in the model. Otherwise, a constant should be included when we estimate the model by OLS. A plot of its residuals can help us check its performance.

In Equation (2.8) the coefficient of the speed of adjustment measures how quickly equilibrium is restored. If Y_{t-1} is above its equilibrium value of $\beta_1'X_{t-1}+\beta_0+\beta_2 t$, Y starts falling in the next period to correct this equilibrium error. Similarly, if Y_{t-1} is below the equilibrium, Y starts rising in the next period. As a result, α_1 is expected to be negative. The following formula makes α_1 more straightforward to understand

$$(1+\alpha_1)^t=1-P \qquad (2.9)$$

where t is the number of quarters and P is the percentage of the gap between the actual and equilibrium real exchange rate to be closed. If t=4, the P obtained is the proportion of adjustment completed within one year. On the other hand, if P=0.50, the calculated t is the half-life of the adjustment.

Besides the expected negative coefficient of the speed of adjustment, we also expect that the estimated coefficients of the one-period lagged RDIFF, which measures the transitory effect of the monetary factor, is negative. Other positive coefficients of lagged RDIFF, which reflect the subsequent reversion of the real exchange rate, might be found.

2.3 Results for the Sample Period 1973Q1 to 1992Q1

In AN's study, they employ the non-linear least-squares approach of Phillips and Loretan (1991) to simultaneously estimate the cointegrating vector and ECM. Instead, a two-step procedure is used in our study. We estimate the cointegrating vector by DOLS and then estimate ECM by OLS. In order to examine whether our method can work as well as AN's, in this section we try to replicate the results they obtained using the same time period as they did—1973Q1 to 1992Q1.

2.3.1 Unit Root Tests

Before conducting formal tests for a unit root, we plot the time series under study to get an intuitive feel about the likely nature of these time series. As we can see in Figures 2.2a and 2.2c, neither RFX nor ENE exhibits a clear trend or moves up and down around a constant mean. The most likely possibility is that they are both a random walk process. The initial impression from Figure 2.2b is that COM seems to be decreasing over the sample period. Its downward trend perhaps suggests that the mean of COM is changing over time and thus COM might not be stationary. Another guess from the downward trend in COM is that COM could be a stationary process around a time trend. Figure 2.2d shows that RDIFF seems to have a constant mean and its variance does not change much, so it is the most likely candidate to be stationary.

Figure 2.2a RFX

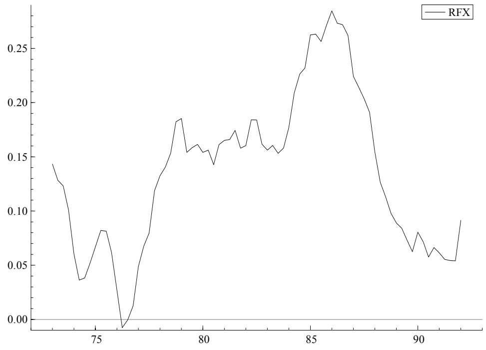


Figure 2.2b COM

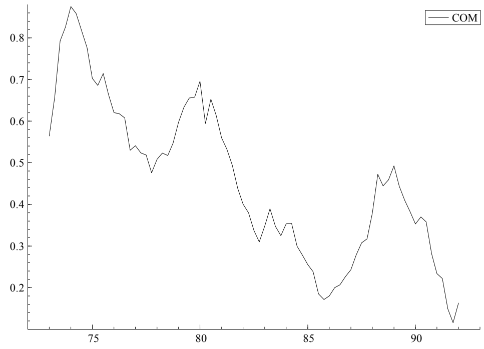


Figure 2.2c ENE

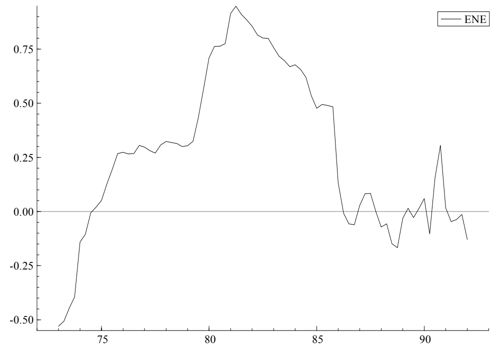


Figure 2.2d RDIFF

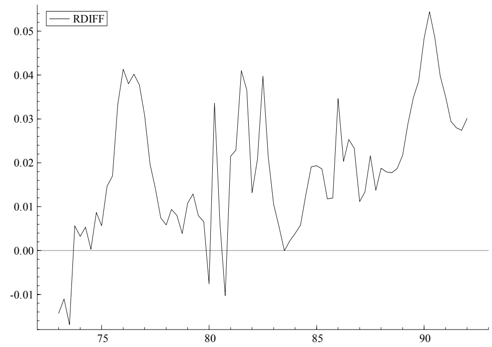


Table 2.1 presents the results of the ADF and PP tests and the joint hypothesis test. Table 2.2 reports the results of the t test for zero means and it helps find the appropriate DGP of each series. First let us look at the results for RFX since the movement of RFX is the key concern of our research and its stationarity property is crucial in deciding the specification of the regression model used to explain its behavior. As seen in Table 2.1, all the results of the three tests for a unit root point to the conclusion that RFX contains a unit root. In each of the three cases, the p values of the ADF and PP tests are both greater than 0.10 and thus the null of a unit root cannot be rejected at the 10 percent significance level. Both the estimated Φ_3 and Φ_1 statistics are below the 10 percent critical values and they support the results of the ADF and PP test that RFX is nonstationary.

But the results shown in Table 2.1 are not enough for us to choose the appropriate DGP of RFX because the failures of the rejections of the null of case I suggest that RFX is a random walk with a drift while the failures of the rejections of the null of cases II and III suggest that RFX is a random walk without a drift. As mentioned in Section 2.2.1, the mean of the first difference of a random walk process without a drift must be zero, but this is not the case for a random walk process with a drift. Therefore we can distinguish these two processes by examining the mean of the first difference of the series. Table 2.2 shows that the mean of the first difference of RFX (DRFX) is a very small negative number. The absolute value of the computed t statistic is much lower than the critical values at conventional significance levels. The null of zero mean then cannot be rejected. As a result, we conclude that RFX is a random walk process.

Table 2.1 Tests for a unit root and DGP

Specification	RFX	COM	ENE	RDIFF	RFX	COM	ENE	RDIFF
	ADF test				PP test			
Case I	-2.32	-2.68	-1.90	-3.03	-5.43	-14.16	-4.33	**-24.01
Case II	-2.29	-1.35	-1.25	*-2.64	-5.45	-2.88	-5.97	**-19.42
Case III	-0.78	-1.38	-1.11	-0.96	-1.27	-1.17	-3.44	-5.45
	p value[2]							
Case I	0.42	0.25	0.65	0.13	0.79	0.21	0.87	**0.03
Case II	0.17	0.61	0.65	*0.084	0.39	0.67	0.35	**0.01
Case III	0.39	0.16	0.24	0.31	0.42	0.44	0.20	0.11
	Number of lags[3]							
Case I	8	6	5	8	8	6	5	8
Case II	8	7	8	8	8	7	8	8
Case III	3	7	8	4	3	7	8	4
	Φ_3 (δ_1=δ_2=0)				5%		10%	
Case I	2.79	3.61	3.23	4.59		6.49		5.47
	Φ_1 (δ_0=δ_2=0)				5%		10%	
Case II	2.64	1.38	0.85	*3.86		4.71		3.86

1. * and ** represent significance at the 10 and 5 percent levels respectively.
2. p values of estimated δ_2 in Equations (2.2), (2.3) and (2.4) are computed using the critical values provided by Mackinnon (1994).
3. The maximum number of lags is 8 which is twice the frequency of 4. The optimal lag length from the ADF test is also used for the PP test.

Table 2.2 Test for zero mean

Variables	Mean	SE	t value[1]
RDIFF	0.018	0.015	1.20
DRFX	-0.00068	0.018	-0.038
DCOM	-0.0053	0.043	-0.12
DENE	0.0053	0.091	0.058

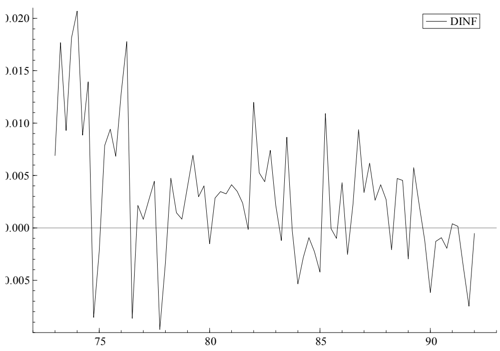
1. The 5 percent critical value of t statistic for a sample size of 60 is 2.00, and the 10 percent value is 1.67.

The presence of a unit root in RFX implies that RFX consists of a stochastic trend and a stationary component. Its stochastic trend can be purged by the combination of other stochastic trends. Those variables with such a stochastic trend in this combination can affect RFX permanently. The short-run dynamics of RFX can be explained by some stationary variables. So the next step is to find those potential long-run and short-run factors of RFX by examining the stationarity property of COM, ENE and RDIFF.

As we can see in Table 2.1, for both COM and ENE, in each case the unit root null of the ADF and PP tests and the joint hypothesis test cannot be rejected at the 10 percent significance level. These failures of rejection of the null indicate that both COM and ENE have a unit root. Now we turn to Table 2.2 to decide their appropriate DGP. The first differences of both COM and ENE, denoted as DCOM and DENE respectively, are very close to zero. For both DCOM and DENE, we cannot reject the null of zero mean because the absolute values of the computed t statistics are both lower than the critical values. Therefore we conclude that both COM and ENE are a random walk processes.

For RDIFF, unlike other variables, the results shown in Table 2.1 do not reach a consensus whether RDIFF contains a unit root. In case I, conflicting results are reported. The ADF test and the joint hypothesis test show that the null of a unit root cannot be rejected at the 10 percent significance level while the PP test shows that the null can be rejected. In case II, all three tests agree that RDIFF is a stationary process. The p values of the ADF and PP tests are both below 0.10 and the computed Φ_3 is equal to the 10 percent critical value so we can reject the null of a unit root at the 10 percent significance level. In case III, the ADF and PP tests suggest that RDIFF appears to be nonstationary for the p values of both the tests are higher than 0.10. Thus there is some evidence to support that RDIFF is a random walk process either with a drift or without a drift. At the same time there is other evidence to suggest that RDIFF could be a stationary process around either a time trend or a constant. Then we try to test for a unit root on a series which can shed some light on this confusing situation. The Canada-US inflation differential (DINF) is such a series in that the real interest rate differential equals the difference between the nominal interest rate differential and the inflation differential. Under the assumption that the real interest rate differential is stationary, RDIFF must be stationary if DINF does not contain a unit root. Otherwise RDIFF cannot be stationary. As we can see in Figure 2.2e, apparently DINF does not change with time and it appears to move around a constant mean.

Figure 2.2e DINF



The results shown in Table 2.3 confirm our first impression from Figure 2.1e that DINF does not contain a unit root. For the ADF and PP test, DINF always appears to be stationary no matter which specification is chosen for it. The highest p value of the ADF test is lower than 10 percent and the p values of the PP test are very close to zero. Both the estimated Φ_3 and Φ_1 statistics are higher than the 5 percent critical values. Thus there is strong evidence to support that DINF is a stationary process. Accordingly, we conclude that RDIFF is stationary. As Figure 2.2d shows, RDIFF does not seem to have a trend. What's more, it does not make economic sense that the interest rate differential changes with time. For these two reasons we argue that RDIFF is a stationary process around a drift rather than a trend-stationary process. Table 2.2 reports that the mean of RDIFF is slightly above zero and the computed t value is even lower than the 10 percent critical value. Thus the null of zero mean cannot be rejected. As a result, the mean of RDIFF is not significantly different from zero. That may explain why the null of a unit root can be rejected in case II, but cannot in case III. Therefore we conclude that RDIFF is a stationary process around zero.

Table 2.3 Tests for a unit root on DINF

	Case I	Case II	Case III
	Statistic		
ADF test	**-3.83[1]	**-4.08	*-1.77
PP test	**-59.45	**-51.50	**-47.81
	p value[2]		
ADF test	**0.015	**0.0011	*0.073
PP test	**0.0000	**0.0000	**0.0000
	Number of lags[3]		
ADF test	2	4	8
PP test	2	4	8
	Joint hypothesis test		
Φ_3 ($\delta_1=\delta_2=0$)	**7.35		
Φ_1 ($\delta_0=\delta_2=0$)		**8.87	

1. * and ** represent significance at the 10 and 5 percent levels respectively.
2. p values of estimated δ_2 in Equations (2.2), (2.3) and (2.4) are computed using the critical values provided by Mackinnon (1994).
3. The maximum number of lags is 8 which is twice the frequency of 4. The optimal lag length from the ADF test is also used for the PP test.

2.3.2 Cointegration Tests

Since COM and ENE each has a unit root, they are able to determine the long-run equilibrium of RFX if the combination of their stochastic trends can remove the stochastic trend in RFX. Cointegration tests can help us find out whether COM and ENE are those and only those factors to have a long-run effect on RFX. As we mentioned in Section 2.2.2, the specification of the cointegrating equation is determined by the DGP of the variables involved. RFX, COM and ENE are all a random walk without a drift. Their values are not related to time. Therefore, a time trend should not be included in the cointegrating regression.

Table 2.4 presents the results of cointegration tests. The p value of the AEG test is 0.14 so the null hypothesis of no cointegration cannot be rejected at the 10 percent significance level.

However, the Johansen test provides strong evidence that COM and ENE are cointegrated with RFX. The p values of both the trace test and the maximum eigenvalue test on the null hypothesis that there is no cointegrating relation in the system are less than 0.10. So the absolute value of both the estimated statistics are greater than the absolute value of the 10 percent critical values. We can thus reject the null of no cointegration at the 10 percent significance level. Neither of the p values of the trace and the maximum eigenvalue tests on the null that there exists one cointegrating relationship is lower than 0.10. Therefore we cannot reject the null hypothesis of one cointegrating relation at the 10 percent significance level. The Johansen test provides some supporting evidence that there is one and only one cointegrating relationship among the series RFX, COM and ENE. It also implies that there cannot exist any other factors that have a long-run effect on RFX because if we did omit one of them, we could not find the cointegrating relationship as above.

Table 2.4 Tests for cointegration

	AEG test			
	Test statistic	p value[1]		
	-3.28	0.14456		
	Johansen test			
	Trace statistics	p value[2]	λ_{max} statistics	p value
Less than 1	**36.62	**0.035	**24.56	**0.024
Less than 2	12.062	0.44	10.084	0.33
Less than 3	1.98	0.78	7.56	0.78

1. The AEG test uses the ADF test to test on residuals from the cointegrating regression. p value for the AEG test is computed using the coefficients in Mackinnon (1994).
2. p values for the Johansen test is computed using the coefficients in Mackinnon, Haug and Michelis (1999).

For the purpose of avoiding the AEG test's problem of superconsistency and the Johansen procedure's problem of high variance, we use DOLS with Newey-West HAC standard error and covariance to obtain efficient OLS estimates of cointegrating equation. According to our assumption of a small open economy, COM and ENE cannot be determined by Canada and they are exogenous variables. As mentioned above, a time trend should not be included in the cointegrating relationship as all the series RFX, COM and ENE are a pure random walk process. Then the cointegrating equation based on Equation (2.5) can be written as

$$RFX_t=\beta_0+\beta_C COM_t+\beta_E ENE_t+e_t \qquad (2.5a)$$

where $e_t=\sum_{j=-k_1}^{k_2} b_j \Delta X_{t-j}+v_t$ ($j\neq 0$, $k_1=k_2=2$), $X=(COM, ENE)'$. Stock and Watson (1993) suggest that the length of the lags and leads of first differences of X depends on sample size. In their Monte Carlo experiment, they set k_1 and k_2 to 2 for a sample size of 100 and to 3 for a sample size of 300. Therefore we set k_1 and k_2 to 2 as our sample size is 77.

Table 2.5 reports the estimates of the cointegrating equation. The p values of all the estimated parameters are much lower than 0.05 and this means that the estimates of β_0, β_C and β_E are all statistically significant. The adjusted R^2 of 0.50 indicates that approximately 50% of the variation in the quarterly equilibrium Canada-US real exchange rate is explained by the prices of energy and non-energy commodities. These estimated long-run effects suggest that a 1% increase in the non-energy commodity price results in a 0.21% appreciation of Canada's real exchange rate against the US while a 1% increase in the energy price results in a 0.11% depreciation of the real exchange rate. Figure 2.3a shows that the predicted real exchange rate by equation (2.5a) fits the data quite well in that the actual exchange rates are close to the regression line during most of the sample time period. Figure 2.3b plots the residuals of the regression which seem to be an i.i.d. process with zero mean.

Table 2.5 Cointegrating equation estimates

	Coefficient	Std. Error	t-Statistic	p value
β_0	0.21	0.039	**5.42	**0.00
β_C	-0.21	0.077	**-2.77	**0.0075
β_E	0.11	0.030	**3.53	**0.0008
Adjusted R^2	0.50			

Figure 2.3a Actual and predicted RFX

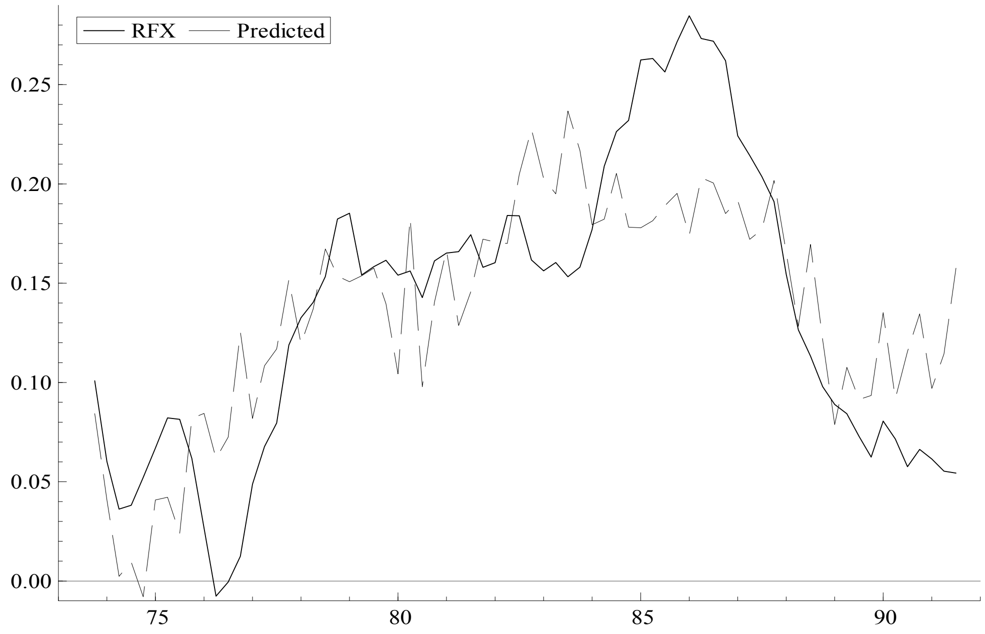
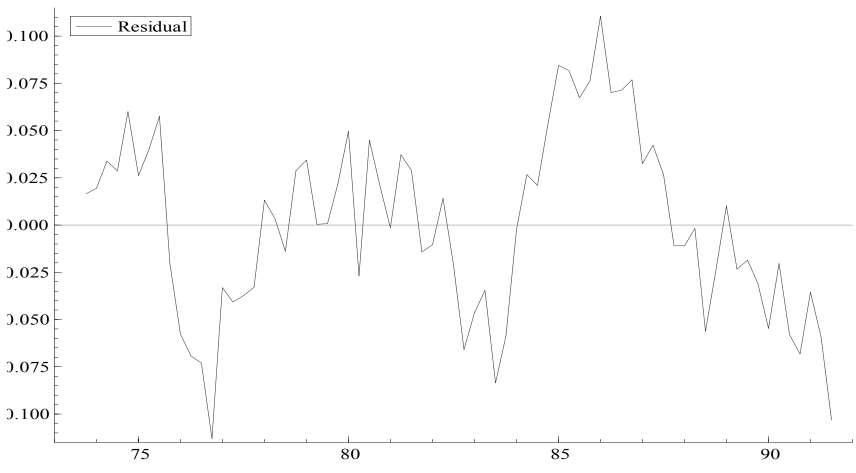


Figure 2.3b Residuals of the cointegrating regression



2.3.3 Error Correction Model

We find that over the sample period 1973Q1-1992Q1, COM and ENE have a long-run effect on RFX and we have estimated this equilibrium relationship. But in the short run, RFX might depart from its equilibrium. We can use ECM to link this equilibrium error to the long-run value of RFX. As mentioned in Section 2.2, the DGPs of the RFX, COM and ENE help determine the specification of ECM. The results in Section 2.3.1 show that RFX, COM, and

ENE are pure random walk processes and that the stationary variable, RDIFF, has a zero mean as well. Therefore a constant should not be included in the regression model represented by Equation (2.8). The ECM to be estimated is

$$\Delta RFX_t=\alpha_1(RFX_{t-1}-\beta_0-\beta_C COM_{t-1}-\beta_E ENE_{t-1})+\sum_{i=1}^{m}\gamma_{RDi}RDIFF_{t-i}+ \sum_{i=1}^{m}\gamma_{RFi}DRFX_{t-i}+\sum_{i=1}^{m}\gamma_{Ci}DCOM_{t-i}+\sum_{i=1}^{m}\gamma_{Ei}DENE_{t-i}+\varepsilon_t \tag{2.8a}$$

The numbers of the lags of stationary variables are selected by using a testing-down procedure. First we estimate equation (2.8a) with 4 lags. As shown in Table 2.6, none of the estimated γ_{RD4}, γ_{RF4}, γ_{C4} and γ_{E4} is statistically significant at the conventional levels. Then we estimate Equation (2.8a) with 3 lags. Still none of the estimated γ_{RD3}, γ_{RF3}, γ_{C3} and γ_{E3} is statistically significant. We continue to estimate Equation (2.8a) with two lags. The estimated γ_{RD2} and γ_{C2} are significant but the other two are not. Meanwhile, the estimated γ_{C1} and γ_{E1} are not significant. The estimates of Equation (2.8a) with one lag show that γ_{C1} and γ_{E1} are still not significant. At the same time, the adjusted R^2 drops from 0.46 to 0.37 relative to the regression with 2 lags. Then we conclude that $RDIFF_{t-2}$ and $DCOM_{t-2}$ must play an important role in explaining the dynamics of the Canada-US real exchange rate. The estimated α_1, γ_{RD1} and γ_{RF1} are statistically significant in all cases at the 5 and 10 percent levels. Therefore Equation (2.8a) can be reduced to

$$\Delta RFX_t=\alpha_1(RFX_{t-1}-\beta_0-\beta_C COM_{t-1}-\beta_E ENE_{t-1})+\gamma_{RD1}RDIFF_{t-1}+\gamma_{RF1}DRFX_{t-1} +\gamma_{RD2}RDIFF_{t-2}+\gamma_{C2}DCOM_{t-2}+\varepsilon_t \tag{2.8b}$$

Table 2.7 reports the estimates of Equation (2.8b). Except for the estimated γ_{C2} which is significant at the 10 percent level, the other coefficient estimates are statistically significant at the 5 percent level. The results indicate that the estimated Equation (2.8b) can approximately account for 47% of the quarter-to-quarter changes in the Canada-US real exchange rate. Based on Equation (2.9), the speed of adjustment -0.088 implies that after one year 30.82% of the gap between the actual and equilibrium real exchange rate is closed or alternatively the half-life is 7.52 quarters. The different signs of estimated γ_{RD1} and γ_{RD2} reflect the deviation

Table 2.6 Estimates of ECM with 4, 3, 2 and 1 lag(s)

Variable	4 lags		3 lags		2 lags		1 lag	
	estimate	p value	estimate	p value	estimate	p value	estimate	p value
α_1	**-0.95	0.034	**-0.091	0.022	**-0.073	0.046	**-0.13	0.000
γ_{RD1}	**-0.4801	0.013	**-0.49	0.006	**-0.54	0.001	**-0.18	0.035
γ_{RF1}	**0.50	0.001	**0.50	0.000	**0.42	0.000	**0.43	0.000
γ_{C1}	-0.043	0.46	-0.028	0.60	-0.040	0.42	0.010	0.84
γ_{E1}	0.0075	0.72	0.0094	0.64	0.018	0.34	0.0063	0.74
γ_{RD2}	0.37	0.12	*0.40	0.077	**0.39	0.025		
γ_{RF2}	-0.14	0.39	-0.15	0.26	-0.61	0.58		
γ_{C2}	-0.082	0.15	-0.85	0.10	**-0.096	0.045		
γ_{E2}	-0.25	0.27	-0.22	0.29	-0.21	0.26		
γ_{RD3}	-0.11	0.66	-0.0393	0.83				
γ_{RF3}	0.20	0.16	0.17	0.16				
γ_{C3}	0.048	0.39	0.057	0.26				
γ_{E3}	-0.0002	0.99	0.0046	0.81				
γ_{RD4}	0.11	0.58						
γ_{RF4}	-0.023	0.85						
γ_{C4}	0.053	0.34						
γ_{E4}	0.0007	0.97						
Adjusted R^2	0.36		0.44		0.46		0.37	

of the real exchange rate from its equilibrium caused by changes in monetary factors and the reversion afterwards: the Canadian dollar depreciates by 0.54% just after a 100bp decrease in Canada's interest rate if the US interest rate stays constant and then will appreciate by 0.40% in one quarter. So the immediate depreciation caused by the decrease in Canada's interest rate overshoots the equilibrium rate and the Canadian dollar will appreciate to approach its long run value in the next quarter. The estimated γ_{RF1} of 0.41 implies that approximately 40% of the previous change in the real exchange rate persists in the current time period. The estimated γ_{C2} of -0.083 implies that a change in the non-energy price causes Canada's real exchange rate to appreciate in two quarters. Figure 2.4a shows that the predicted value of ΔRFX fits the data quite well. The regression line captures most of the turning points occurring over the sample period. The residuals plotted in Figure 2.4b appear to move up and down quite evenly around the mean.

Table 2.7 ECM estimates

	Coefficient	Std. Error	t-Statistic	p value
α_1	-0.088	0.031	**-2.86	0.006
γ_{RD1}	-0.54	0.15	**-3.71	0.000
γ_{RF1}	0.41	0.094	**4.31	0.000
γ_{RD2}	0.40	0.16	**2.56	0.012
γ_{C2}	-0.083	0.044	*-1.91	0.061
Adjusted R^2	0.47			

Figure 2.4a Actual and predicted DRFX

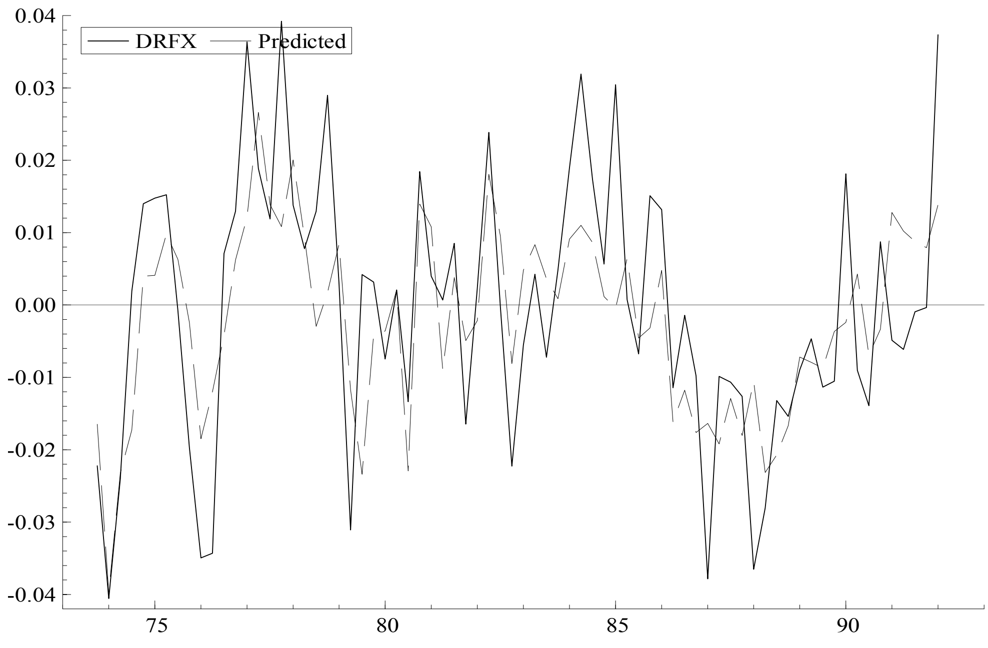
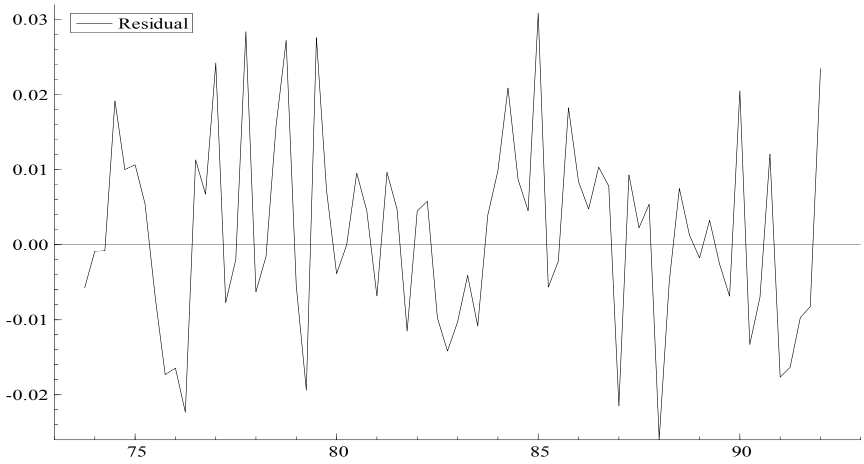


Figure 2.4b Residuals of the estimated ECM



2.3.4 Comparison with Amano and van Norden's (1995) Results

We use different methods to test for a unit root, cointegration and to estimate cointegrating regression and ECM than AN, but those results are still quite close to theirs. They conduct ADF, PP and KPSS to test for a unit root and conclude that the series RFX, COM and ENE are a unit root process without a drift and RDIFF is stationary. We follow the procedure suggested by Bhargara (1986) and the joint hypothesis test to discover DSP and to test for a unit root. We find that RFX, COM and ENE are all pure random walk processes and the RDIFF is a stationary process around zero. They conclude that ENE and COM are cointegrated with RFX after conducting Hansen ADF and PP tests and the JJ test. The Johansen test we conduct gives the same result. Table 2.8 presents the ECM estimates using the non-linear least squares methodology by AN. After successively omitting variables with insignificant t-statistics, they found that the one-period lagged RDIFF was the only factor that has a transitory effect on RFX. We find that one-period lagged and two-period lagged RDIFFs, one-period lagged DRFX and two-period lagged DCOM can significantly affect the short-run deviations of RFX from its equilibrium. Their speed of adjustment α_1 obtained is a little faster than ours. Their estimate of -0.038 implies that 37.1% of adjustment is completed within one year, or a half-life of about one year and a half. Our estimate of -0.088 implies that 30.82% of adjustment is completed within one year, or a half-life of almost two years. The estimated long-run effects on the RFX they obtain have the same signs as ours, but are greater than ours. Their estimated coefficients of constant, COM and ENE are 0.552, -0.811 and

0.223 respectively, higher in absolute value than our estimates of 0.21, -0.21 and 0.11.

Table 2.8 Estimates of ECM by AN

Variable	Coefficient	Std. error	t-statistic	p value
α_1	-0.038	0.011	-3.446	_
β_0	0.552	0.097	5.681	0.000
β_1	-0.811	0.296	-2.736	0.006
β_2	0.223	0.060	3.700	0.000
γ_{RD1}	-0.187	0.043	4.390	0.000

To sum up, our methodology yields similar results to AN's in that all the estimated coefficients we obtain have the same sign as they do and we both find: The prices of energy and non-energy commodities determine the long-run equilibrium of the Canada-US real exchange rate; the Canadian real exchange rate is positively related to non-energy commodity prices and negatively to energy prices; one-period lagged interest rate differentials can account for the short-run dynamics of the real exchange rate. Among these findings only the negative effect of the energy prices is contrary to our expectation. We also find that beside the one-period lagged interest rate differentials, two-period lagged interest rate differentials, one-period lagged changes in real exchange rate and two-period lagged non-energy commodity prices have a transitory effect on the real exchange rate. Our next step is to attempt to investigate whether the model remains valid with an extension of the sample period and figure out the puzzle of the negative effect of the energy prices. So we turn to these questions in Chapter 3.

Chapter 3 Sample Extension and Structural Break Test

In this chapter we try to find out whether COM and ENE still have a long-run effect on RFX and whether ECM model can still link the short-run behavor of RFX to its long-run value when our sample period is extended to the end of 2004. As shown in Chapter 2, higher energy prices weaken the Canadian dollar. This result does not support the theory we discussed in section 1.2 which suggests that higher energy prices strengthen the Canadian dollar when Canada is a main oil exporter to the US. We thus try to solve the puzzle by considering a structural break in the long-run relationship.

3.1 Unit Root Tests and Cointegration Tests Without a Structural Break

3.1.1 Unit Root Tests

As we can see in Figures 3.1a-3.1d which plot the series RFX, COM ENE and RDIFF, RDIFF is still the most likely candidate to be stationary among them. The variance of RFX appears to increase with time as does the variance of ENE. COM seems to continue to trend downward.

Figure 3.1a RFX

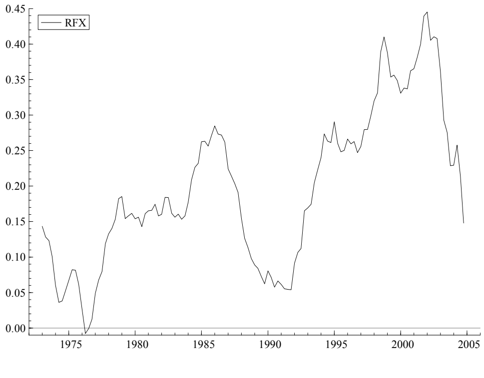


Figure 3.1b COM

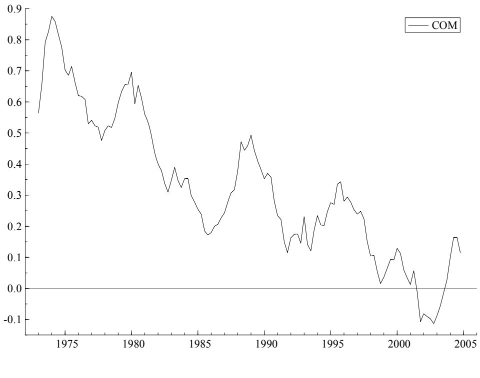


Figure 3.1c ENE

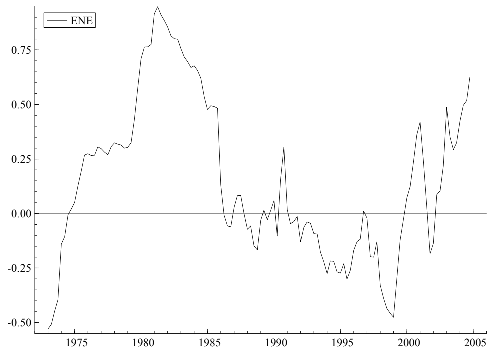


Figure 3.1d RDIFF

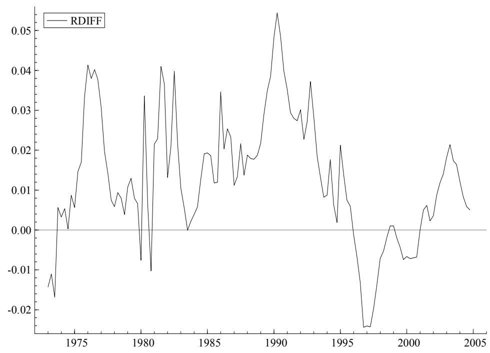


Table 3.1 above presents the results of the tests for a unit root. Table 3.2 below reports the results of the t test for zero means. We first check the stationarity property of RFX. All the results suggest that RFX is nonstationary and it is a pure random walk process. None of the p values of the ADF and PP tests is lower than 0.10 and the computed Φ_1 statistic is below the 10 percent critical value. Thus the null of a unit root cannot be rejected at the 10 percent significance level and RFX is a random walk without a drift. As shown in Table 3.2, the mean of RFX is not significantly different from zero. This confirms the conclusion from Table 3.1.

Our next step is to examine the stationarity property of the other variables in the study. Like RFX, the null of any such test for ENE cannot be rejected at the 10 percent significance level. We conclude that ENE contains a unit root and it is a pure random walk process.

For COM and RDIFF, conflicting results are reported. The results of most tests indicate that COM does contain a unit root while the ADF test for case III shows that the null hypothesis of a unit root can be rejected at the 10 percent significance level. RDIFF is found to be stationary by the PP test while the null of a unit root cannot be rejected at the 10 percent level by the ADF test and the joint hypothesis test.

Table 3.1 Tests for a unit root[1]

Specification	RFX	COM	ENE	RDIFF	RFX	COM	ENE	RDIFF
	ADF test				PP test			
Case II	-2.35	-1.63	-1.58	-2.17	-5.48	-3.85	-7.43	**-21.22
Case III	-0.88	*-1.69[2]	-1.13	-1.35	-1.26	-2.22	-5.36	**-6.55
	p value[3]							
Case II	0.16	0.47	0.49	0.22	0.39	0.56	0.25	**0.0086
Case III	0.34	*0.086	0.23	0.16	0.42	0.30	0.11	**0.019
	Number of lags[4]							
Case II	5	8	5	8	5	8	5	8
Case III	5	8	5	8	5	8	5	8
	Φ_1 ($\delta_0=\delta_2=0$)				5%		10%	
Case II	2.72	2.52	1.48	2.37		4.71		3.86

1. The results for case I are not reported because in Chapter 2 we find that Case I is not an appropriate specification for any of the variables.
2. * and ** represent significance at the 10 and 5 percent levels respectively.
3. p values of estimated δ_2 in Equations (2.3) and (2.4) are computed using the critical values provided by Mackinnon (1994).
4. The maximum number of lags is 8 which is twice the frequency of 4. The optimal lag length from the ADF test is also used for the PP test.

Table 3.2 Test for zero mean

Variables	Mean	SE	t value[1]
RDIFF	0.013	0.016	0.79
DRFX	0.000036	0.022	0.0016
DCOM	-0.0035	0.043	-0.082
DENE	0.0091	0.097	0.094

1. The 5 percent critical value of t statistic for a sample size of 120 is 1.98, and the 10 percent value is 1.66.

As we discussed in Section 2.2, the ADF test is preferable in situations where the residuals of Equations (2.2c), (2.3c) and (2.4c) are negatively autocorrelated while the PP test is preferred in the situations of positively autocorrelated residuals. DeJong, Nankervis, Savin and

Whiteman (1992) suggest that AR(1) process might capture the critical feature of the error terms and their Monte Carlo experiments are thus based on the AR(1) process. We collect the residuals μ_t after estimating Equation (2.4c) for COM and RDIFF and Equation (2.3c) for RDIFF. Then the AR(1) parameters are estimated by regressing μ_t on μ_{t-1}. As seen in Table 3.3 the estimated AR(1) parameter for COM is significantly positive. In such a situation, the PP test is more reliable. Therefore, we can conclude that the COM is nonstationary and it is a random walk without a drift. For RDIFF, the estimated AR(1) parameters are both negative, but they are statistically insignificant at the 10 percent level. These insignificant negative AR(1) parameters suggest that the residuals are not negatively autocorrelated. We still cannot tell which one is better, the ADF test or the PP test for RDIFF.

Table 3.3 AR(1) parameters of the error terms

	AR(1) parameter	P value
COM based on Equation (2.4c)	0.25	**0.005
RDIFF based on Equation (2.3c)	-0.076	0.40
RDIFF based on Equation (2.4c)	-0.11	0.23

As we did in Section 2.3, we need to test for a unit root on DINF. Under the assumption that the real interest differential is stationary, if DINF is stationary, we conclude that RDIFF is stationary. As show in Figure 3.1e, DINF does not display a clear trend. It seems that DINF has a constant mean and it moves around the mean. The results shown in Table 3.4 provide strong evidence that DINF does not have a unit root. The p values of both ADF and PP tests are very close to zero. The computed Φ_1 statistic is much higher than the 10 percent critical value. Therefore RDIFF is a stationary process as well. As we can see in Table 3.2, the mean of RDIFF is slightly above zero, but the computed t statistic is lower than the critical values at conventional levels. The null of zero mean then cannot be rejected. As a result, we conclude that RDIFF is a stationary process with zero mean.

Figure 3.1e

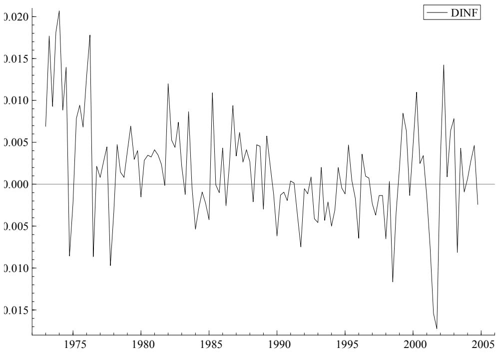


Table 3.4 Tests for a unit root on DINF

	Case II	Case III
	Statistic	
ADF test	**-4.74[1]	**-4.58
PP test	-75.01	**-69.87
	p value[2]	
ADF test	**0.000069	**0.000006
PP test	**0.000000	**0.000000
	Number of lags[3]	
ADF test	2	2
PP test	2	2
	Joint hypothesis test	
Φ_1 ($\delta_0=\delta_2=0$)	**11.31	

1. * and ** represent significance at the 10 and 5 percent levels respectively.
2. p values of estimated δ_2 in Equations (2.3) and (2.4) are computed using the critical values provided by Mackinnon (1994).
3. The maximum number of lags is 8 which is twice the frequency of 4. The optimal lag length from the ADF test is also used for the PP test.

3.1.2 Cointegration Tests

As shown in previous sections, the property and DGP of each time series under study does not change when the sample period is extended to 2004Q4. RFX, ENE and COM are still a pure random walk process and RDIFF is still a stationary process around zero. In this section, we test for the existence of the long-run relationship among the series with a unit root using the AEG and Johansen cointegration tests.

Table 3.5 reports the results of the cointegration tests. The 0.55 p value of the AEG test indicates that the estimated statistic is much lower than the 10 percent critical values. The null hypotheses of no cointegration cannot be rejected at the 10 percent significance level. The Johansen test reconfirms the results of the AEG test. The p values of both the trace test and the maximum eigenvalue test on the null hypothesis that there is no cointegrating relationship in the system are more than 0.70 and thus the estimated statistics are less than the 10 percent critical values. We cannot reject the null hypothesis of no cointegration at the 10 percent significance level. We conclude that RFX is not cointegrated with COM and ENE. Therefore it seems the equilibrium Canada-US real exchange rate cannot be determined by only COM and ENE when the sample period is extended[4].

Table 3.5 Tests for cointegration

	AEG test			
	Test statistic	p value[1]		
	-2.38	0.55		
	Johansen test			
	Trace statistics	p value[2]	λ_{max} statistics	p value[2]
Less than 1	19.45	0.76	11.19	0.73
Less than 2	8.26	0.80	5.34	0.86
Less than 3	2.92	0.60	2.92	0.60

1. p value for the AEG test is computed using the coefficients in Mackinnon (1994).
2. p values for the Johansen test are computed using the coefficients in Mackinnon, Haug and Michelis (1999).

3.2 Error Correction Model with a Structural Change in the Long-Run Relationship

3.2.1 Cointegration Tests and Cointegrating Regression with a Structural Change

The conventional cointegration tests assume that the long-run relationship among the I(1)

[4] We cannot reject the null hypothesis that there is no cointegrating relationship between RFX and COM or RFX and ENE either using the AEG and Johansen tests.

variables is constant over time. However the failure of the rejection of the null hypotheses of no cointegration could result from the presence of a structural shift over the sample period. The Monte Carlo experiments by Gregory, Nason and Watt (1996) show that the in-sample power of the cointegration tests based on the conventional ADF test drops considerably when there is a structural break in the long-run relationship. Then it is necessary to test for a structural break once we find that COM, ENE and RFX are not cointegrated with the extension of the sample period to 2004. Issa, Lafrance and Murray (2006) find a break point in 1993Q3 at which the sign of the relationship between energy prices and the Canadian dollar changes from negative to positive. They suggest that this sign change is associated with the growing importance of energy exports by Canada, due to the deregulation of the Canadian energy sector and the implementation of North American Free Trade Agreement. We follow their procedure proposed by Quintos (1995) to find a break point except we use a standard Wald test on the coefficients of the dummy variables instead of a modified Wald test. Our results confirm that there is a break point in 1993Q3. Also at this point, the effect of the energy prices on the Canadian dollar shifted from negative to positive.

In her study, Quintos (1995) estimates the regression with dummy variables to split the sample period into two sub-periods and tests the significance of the coefficients of the dummy variable for each point from 15% to 85% of the sample period. She suggests that the break point is among those with peak and significant modified Wald statistics which are χ^2 distributed if the nonstationary variables are all I(1). Following this procedure, we estimate the equation as follows for each point from 1978Q1 to 1999Q4

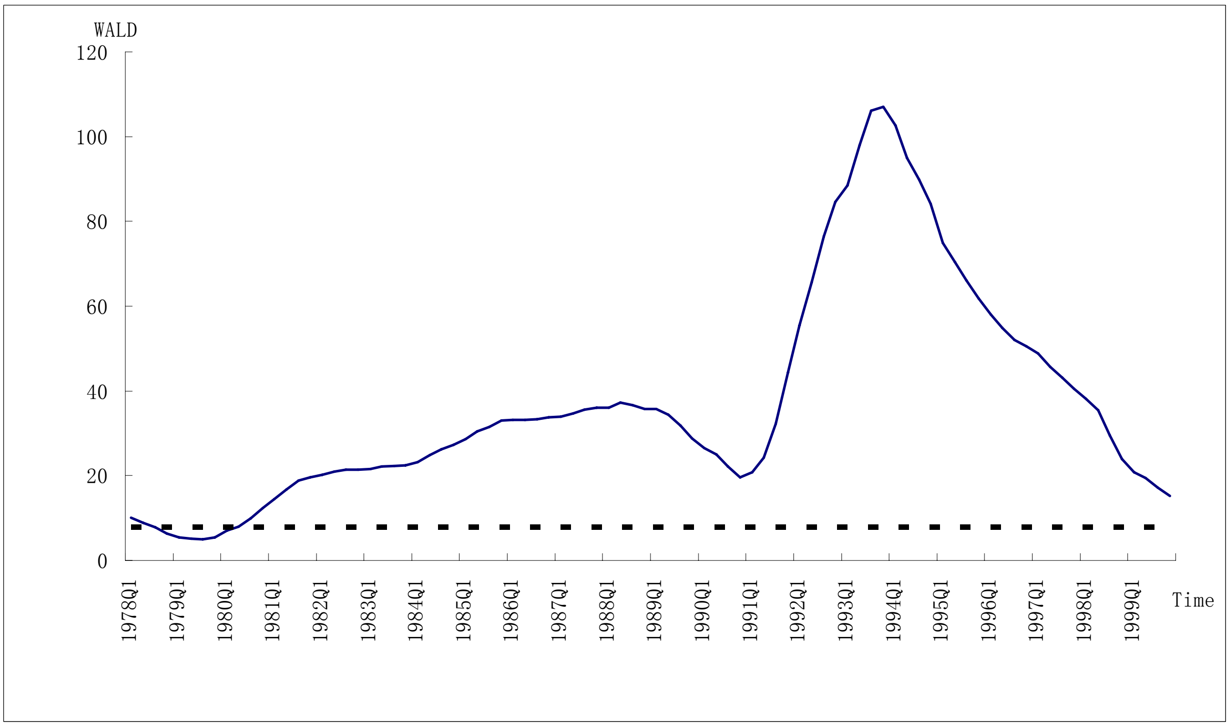
$$RFX_t=\beta_0+\beta_C COM_t+\beta_E ENE_t+\theta_0 D_t+\theta_C(D_t COM_t)+\theta_E(D_t ENE_t)+\mu_t \qquad (2.5b)$$

where μ_t is an error term and $D_t = 1$ for t > the tested point

= 0 otherwise

As seen in Figure 3.2, the two peak values are at 1993Q3 and 1993Q4. The dotted line indicates the critical value at 5 percent significant level. The p values of these two points are very close to zero. Since two adjacent points cannot both be break points, we choose 1993Q3 as this break point is also found by Issa, Lafrance and Murray (2006).

Figure 3.2 WALD statistics for a structural break



After finding a break point, Quintos carries out the residual based test and the Johansen test for cointegration for each sub-period. We then conduct the AEG and Johansen tests separately for two sub-periods 1973Q1-1993Q3 and 1993Q4-2004Q4. Tables 3.6a and 3.6b present the results of the cointegration tests. As we can see in table 3.4a, the p value of the AEG test is slightly higher than 0.05 and we can reject the null hypothesis of no cointegration at the 10 percent significance level. The Johansen test supports the results of the AEG test that RFX is cointegrated with COM and ENE. The p values of both the trace and the maximum eigenvalue tests on the null hypothesis that there is no cointegrating relation in the system are less than 0.05. This indicates that both the estimated statistics are greater than the 10 percent critical values. We can reject the null hypotheses of no cointegration at the 10 percent significance level. Neither the p values of the trace and the maximum eigenvalue tests on the null that there exists one cointegrating relation is lower than 0.10 and we cannot reject the null hypothesis of one cointegrating relationship at the 10 percent significance level. Therefore we conclude that there is one and only one cointegrating relationship among the series RFX, COM and ENE for the period 1973Q1-1993Q3. This also explains our finding in Chapter 2 that the equilibrium RFX is determined by COM and ENE for the period 1973Q1-1992Q1 which is part of our first sub-period.

Table 3.6a Tests for cointegration: 1973Q1-1993Q3

	AEG test			
	Test statistic	p value[1]		
	*-3.74	*0.050		
	Johansen test			
	Trace statistics	p value[2]	λ_{max} statistics	p value
Less than 1	**36.84	**0.033	**24.31	**0.026
Less than 2	12.53	0.40	10.79	0.27
Less than 3	1.74	0.83	1.74	0.83

Table 3.6b Tests for cointegration: 1993Q4-2004Q4

	AEG test			
	Test statistic	p value[1]		
	-2.34	0.57		
	Johansen test			
	Trace statistics	p value[2]	λ_{max} statistics	p value
Less than 1	24.99	0.40	16.06	0.29
Less than 2	8.94	0.74	5.14	0.88
Less than 3	3.79	0.44	3.79	0.44

1. p value for the AEG test is computed using the coefficients in Mackinnon (1994).
2. p values for the Johansen test are computed using the coefficients in Mackinnon, Haug and Michelis (1999).

However, for the later sample period all the p values in Table 3.6b are more than 0.10. It seems that RFX, COM and ENE are not cointegrated for the period 1993Q4-2004Q4. But the short period of the second sub sample could cause lower power of the AEG test. Andrade, Bruneau and Gregoir (2005) argue that for a sample period with a structural break, if we conduct Johansen tests separately for the two sub-periods, the test power for the second period is so low that we sometimes fail to reject the null of no cointegration when a cointegrating relationship does exist. They explain that the effect of the first period data generating process on the initial value of the second period causes the low test power for the second period. In addition, Issa, Lafrance and Murray (2006) find some evidence of cointegration after 1993Q3 using Johansen maximum eigenvalue test. So our next step is to estimate the cointegrating vector with a structural break at 1993Q3.

Using DOLS, we estimate the equation as follows for the period 1973Q1-2004Q4

$$RFX_t=\beta_0+\beta_C COM_t+\beta_E ENE_t+\theta_0 D_t+\theta_C(D_t COM_t)+\theta_E(D_t ENE_t)+e_t \quad (2.5c)$$

where $D_t = 1$ for $t > 1993Q3$

= 0 otherwise

Table 3.7 reports the estimates of the cointegrating equation. The p values of all the estimated parameters are much lower than 0.05. These low p values indicate that all the estimated parameters are statistically significant. The adjusted R^2 of 0.79 indicates that approximately 79% of variation in the quarterly equilibrium Canada-US real exchange rate is explained by the energy and non-energy commodity prices. The estimated long-run effects suggest: (1) For the period 1973Q1-1993Q3, a 1% increase in the non-energy commodity price results in a 0.20% appreciation of Canada's real exchange rate while a 1% increase in the energy price results in a 0.094% depreciation. (2) For the period 1993Q4-2004Q4, the intercept increases by 0.18 relative to the earlier sub-period. A 1% increase in the non-energy commodity price results in 0.33% additional appreciation of Canada's real exchange rate relative to the first sub-period. It is very interesting to find that a 1% increase in the energy price results in a 0.21% appreciation relative to the first sub-period. This positive effect of the energy prices on the Canadian real exchange rate makes the effect of the estimated ENE change from negative over the first sub-period to positive over the second sub-period. In total, a 1% increase in the energy price results in a 0.116% appreciation of the Canadian real exchange rate after 1993Q3. At the same time, a 1% increase in the non-energy commodity price results in a total 0.52% appreciation of the real exchange rate. Figure 3.3a shows that the predicted real exchange rate in equation (2.5c) fits the data very well in that the regression line is very close to the actual exchange rate during most of the sample time period. Figure 3.3b plots the residuals of the regression which seems to be an i.i.d. process with zero mean.

Table 3.7 Cointegrating equation estimates

	Coefficient	Std. Error	t-Statistic	P value
β_0	0.20	0.036	5.46	0.0000
β_C	-0.20	0.064	-3.06	0.0028
β_E	0.094	0.025	3.69	0.0004
θ_0	0.18	0.040	4.37	0.0000
θ_C	-0.33	0.086	-3.76	0.0003
θ_E	-0.21	0.061	-3.43	0.0008
Adjusted R^2	0.79			

Figure 3.3a

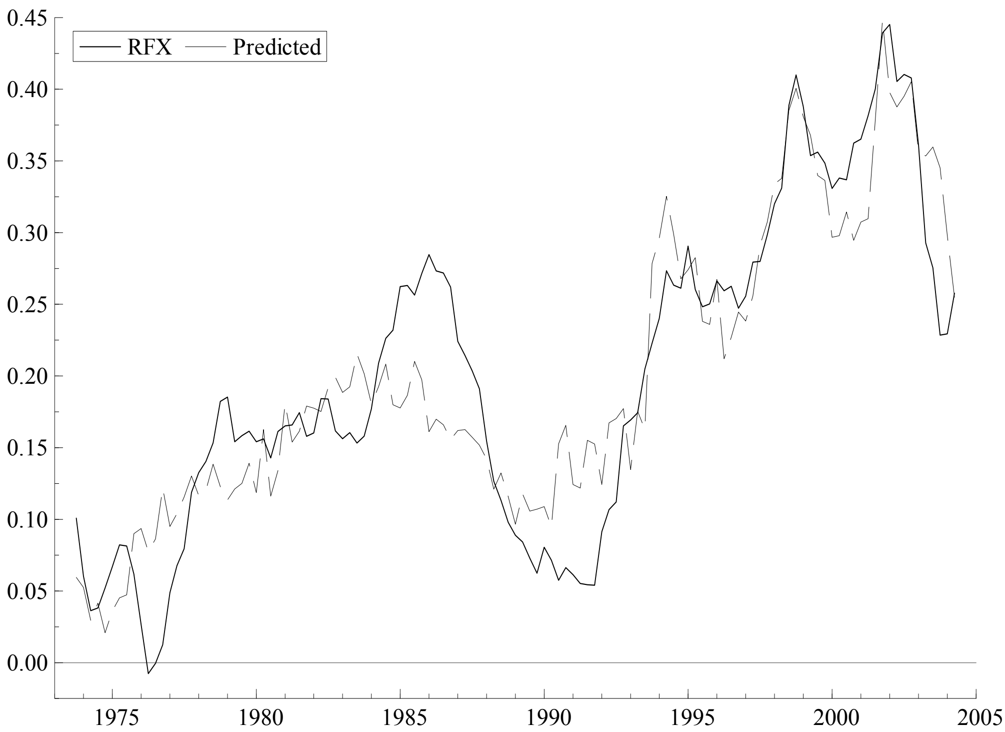
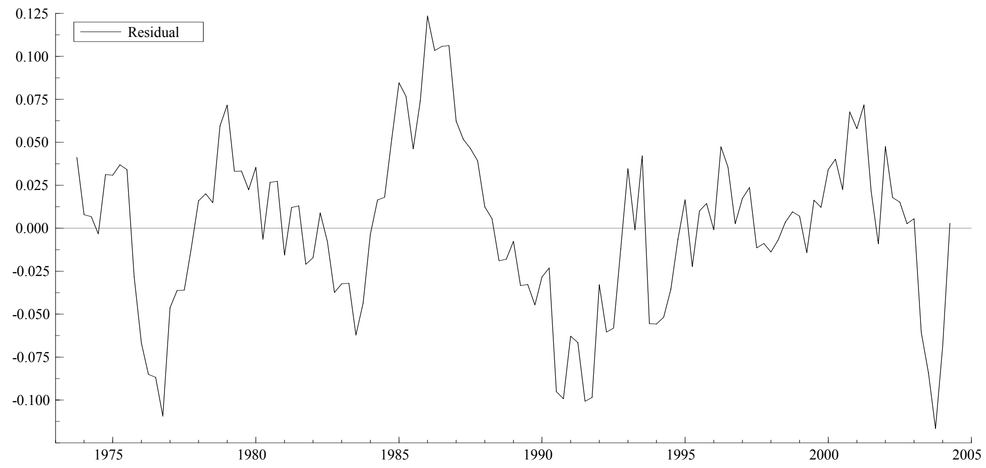


Figure 3.3b



3.2.2 Error Correction Model

In previous section we find that with a break point at 1993Q3, the equilibrium Canada-US real exchange rate is determined by COM and ENE for the period 1973Q1-2004Q4. In this section we try to find out whether ECM is still valid for the extended period. We assume that there is no structural change in ECM, that is, the speed of adjustment stays constant over the whole period. In order to select the appropriate numbers of lags and specification of ECM, we start with the most general ECM which can be written as

$$\Delta RFX_t=\alpha_1(RFX_{t-1}-\beta_0-\beta_C COM_{t-1}-\beta_E ENE_{t-1}-\theta_0 D_{t-1}-\theta_C(D_{t-1}COM_{t-1}) -\theta_E(D_{t-1}ENE_{t-1}))+\sum_{i=1}^{m}\gamma_{RDi}RDIFF_{t-i}+\sum_{i=1}^{m}\gamma_{RFi}DRFX_{t-i} +\sum_{i=1}^{m}\gamma_{Ci}DCOM_{t-i}+\sum_{i=1}^{m}\gamma_{Ei}DENE_{t-i}+\varepsilon_t \quad (2.8c)$$

Table 3.8 reports the estimates of Equation (2.8c) with 1, 2, 3 and 4 lags. As shown in the table, none of the estimated parameters of the four-period lagged variables γ_{RD4}, γ_{RF4}, γ_{C4} and γ_{E4} is statistically significant at the conventional levels. Then we remove all these variables and estimate Equation (2.8c) with 3 lags. The 0.005 p value of the estimated γ_{RF3} indicates that this coefficient is significant at the conventional levels. We continue to estimate Equation (2.8c) with 2 lags. The decreased adjusted R^2 from 0.29 to 0.26 suggests that $DRFX_{t-3}$ cannot be omitted from the model. The results also show that the estimated γ_{RD2} is significant but the other three are not. The adjusted R^2 of the estimated equation with one lag drops from 0.26 to

Table 3.8 Estimates of ECM with 4, 3, 2 and 1 lag(s)

Variable	4 lags		3 lags		2 lags		1 lag	
	estimate	P value	estimate	P value	estimate	P value	estimate	P value
α_1	**-0.91	0.030	**-0.10	0.012	**-0.089	0.021	**-0.12	0.002
γ_{RD1}	**-0.59	0.007	**-0.56	0.006	**-0.61	0.003	**-0.20	0.031
γ_{RF1}	**0.41	0.00	**0.40	0.000	**0.39	0.000	**0.38	0.000
γ_{C1}	-0.040	0.43	-0.041	0.38	-0.054	0.24	-0.029	0.51
γ_{E1}	-0.16	0.41	-0.012	0.54	-0.011	0.56	-0.016	0.40
γ_{RD2}	0.42	0.12	0.42	0.12	**0.46	0.026		
γ_{RF2}	*-0.19	0.07	*-0.17	0.093	-0.070	0.46		
γ_{C2}	-0.032	0.52	-0.030	0.53	-0.042	0.36		
γ_{E2}	-0.0034	0.87	-0.0081	0.68	-0.013	0.50		
γ_{RD3}	-0.14	0.60	-0.0021	0.99				
γ_{RF3}	**0.31	0.004	**0.27	0.005				
γ_{C3}	0.0027	0.96	0.0016	0.97				
γ_{E3}	0.0097	0.64	-0.0032	0.87				
γ_{RD4}	0.187	0.41						
γ_{RF4}	-0.047	0.65						
γ_{C4}	-0.0068	0.89						
γ_{E4}	0.020	0.32						
Adjusted R^2	0.25		0.29		0.26		0.24	

0.24 relative to the estimated equation with 2 lags. Similarly, $RDIFF_{t-2}$ must play an important role in explaining the dynamics of the Canada-US real exchange rate. It is worth mentioning here that the estimated α_1, the speed of adjustment, is negative and statistically significant in each case we have estimated. It is a good sign that ECM works. Like α_1, both γ_{RD1} and γ_{RF1} are statistically significant in all cases at the 10 percent level. Therefore equation (2.8c) can be reduced to

$$\Delta RFX_t=\alpha_1(RFX_{t-1}-\beta_0-\beta_C COM_{t-1}-\beta_E ENE_{t-1}-\theta_0 D_{t-1}-\theta_C(D_{t-1}COM_{t-1}) -\theta_E(D_{t-1}ENE_{t-1}))+\gamma_{RD1}RDIFF_{t-1}+\gamma_{RF1}DRFX_{t-1} +\gamma_{RD2}RDIFF_{t-2}+\gamma_{RF3}DRFX_{t-3}+\varepsilon_t \quad (2.8d)$$

Table 3.9 reports the estimates of Equation (2.8d). All the estimated parameters are statistically significant at the 5 percent significance level. The adjusted R^2 implies that the estimated equation can approximately account for 31% of the quarter-to-quarter changes in the real exchange rate. Based on Equation (2.9), the speed of adjustment -0.12 implies that after one year, 40.03% of the gap between the actual and equilibrium real exchange rate is closed or a half-life of 5.42 quarters. The different signs of estimated γ_{RD1} and γ_{RD2} show that the Canadian dollar depreciates by 0.55% just after a 100bp decrease in Canada's interest rate if the US interest rate stays unchanged and then will appreciate by 0.43% in one quarter, so finally the Canadian dollar still depreciate slightly. The estimated γ_{RF1} of 0.55 implies that approximate 55% of the previous change in real exchange rate stays in the current time period. The estimated γ_{RF3} indicates that a change in real exchange rate three quarters before affects the current real exchange rate. Figure 3.4a shows that the fluctuation of the estimated ΔRFX is very close to the actual one and the estimated ΔRFX moves within a smaller range. It is impressive that our model can capture most of the actual turning points. The residuals plotted in Figure 3.4b appear to be a stationary process with a zero mean.

Table 3.9 ECM estimates

	Coefficient	Std. Error	t-Statistic	P value
α_1	-0.12	0.035	-3.314	0.001
γ_{RD1}	-0.55	0.19	-2.98	0.004
γ_{RF1}	0.40	0.079	5.06	0.000
γ_{RD2}	0.43	0.19	2.24	0.027
γ_{RF3}	0.24	0.081	3.00	0.003
Adjusted R^2	0.31			

Figure 3.4a

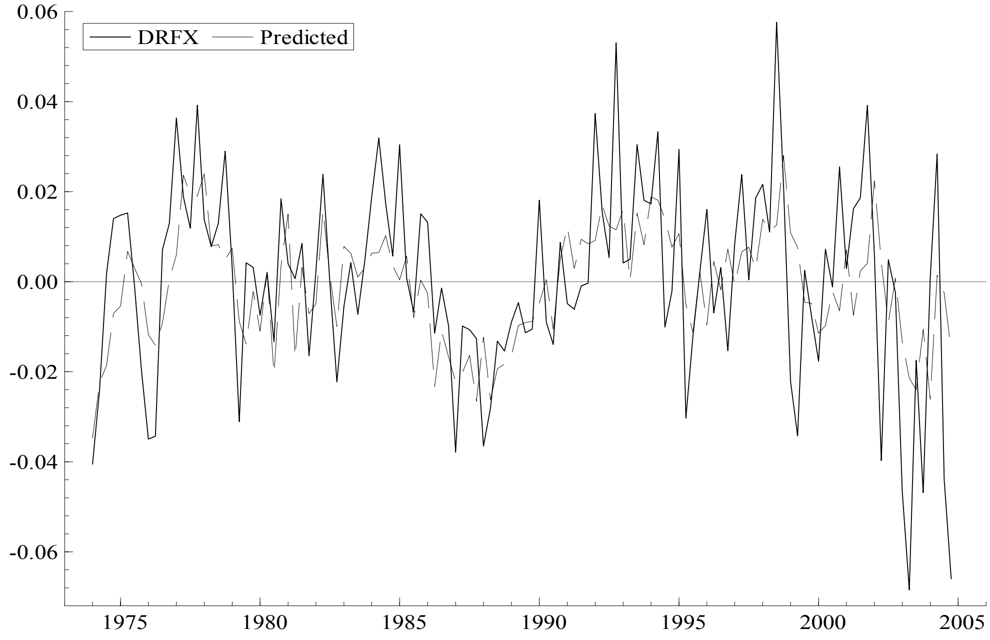
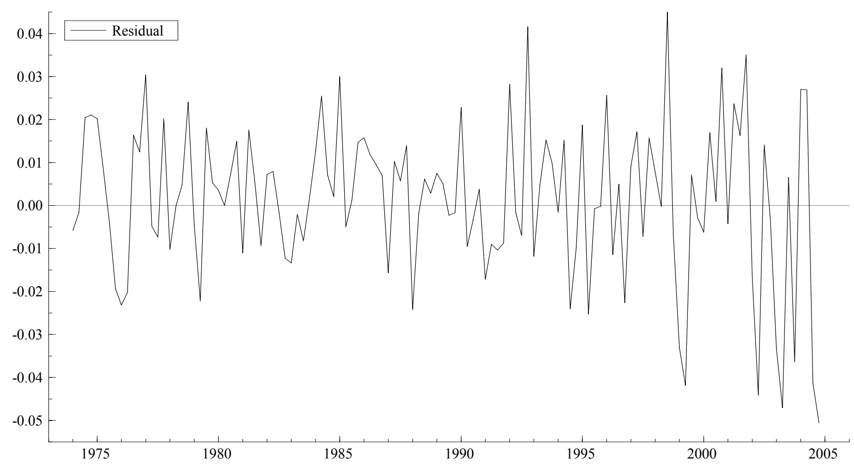


Figure 3.4b



3.2.3 Comparison with the Results for 1973Q1 to 1992Q1

The structural break of 1993Q3 we have obtained for the extended period implies that one stable cointegrating relationship exists for the period 1973Q1 to 1992Q1. A comparison of Tables 2.5 and 3.7 indicates the estimated cointegrating vector for the period 1973Q1 to 1992Q1 is very similar to that for the first sub-period 1973Q1 to 1993Q3. For the second sub-period, we find that the positive effect of real non-energy commodity prices on the equilibrium Canada-US real exchange rate is greater than its previous effect. The effect of energy prices on the Canadian dollar changes from negative to positive after 1993Q3.

The speed of adjustment α_1 for the period 1973Q1 to 2004Q4 is faster than that for the period 1973Q1 to 1992Q1. The adjustment percentage within one year has increased from 30.82% to 40.03%, or the half-life is reduced from 7.52 quarters to 5.42 quarters. The short-run effects of one-period and two-period lagged RDIFF and one-period lagged DRFX on the short-term dynamics of the Canada-US real exchange rate are relatively stable in that for both the periods, their estimated coefficients are statistically significant and very close to each other. We also find that the short-run effects of the two-period lagged DCOM and the three-period lagged DRFX are not stable. The estimated coefficient of the two-period lagged DCOM is statistically significant over the period 1973Q1 to 1992Q1 but it is not significant for the extended period. The estimated coefficient of the three-period lagged DRFX is significant for the period 1973Q1 to 2004Q4 but it is insignificant for the period 1973Q1 to 1992Q1.

For both the periods 1973Q1-1992Q1 and 1973Q1-2004Q4, the positive effect of the non-energy commodity prices on the Canadian real exchange rate and the deviation of the real exchange rate when the interest rate differential changes are consistent with our expectations which come from the theories discussed in Chapter 1. As Issa, Lafrance and Murray (2006) point out, the negative effect of energy prices before 1993Q3 can be accounted for by Canada's energy policies during that time and its positive effect is a result of deregulation of those policies and the implementation of North American Free Trade Agreement. We analyze the impact of deregulation in Chapter 4.

Chapter 4 Interpretation of Results and Summary

4.1 Analysis of Deregulation

According to Canadian Energy Chronology, from 1973-84 the focus of Canadian policy was to ensure energy security through government intervention to manage self-sufficiency. From 1984-94, Canada's energy policies turned to pursue market-led development through deregulation.[5] Replacing government administrative prices of oil and natural gas with market-driven prices helped create a competitive environment that benefits both producers and consumers. Decontrol on energy exports encouraged trade by Canada, and relaxation of rules for foreign investment in the oil and gas industry promoted the growth of the industry.

In April 1984, the Canada/B.C. Agreement covering petroleum pricing was amended. The price of oil from infill wells and production was to be determined by the New Oil Reference Price, which is basically the world price, replacing the Special Old Oil Price. Since November 1984, negotiated prices, instead of government-set prices, were applied to Canadian natural gas exporters. In January 1985, oil price deregulation and flexible natural gas pricing were approved in Quebec. In March 1985, the federal/provincial governments signed the Western Accord which deregulated Canadian crude oil prices. Besides, the federal-provincial agreement removed import subsidies, export taxes on crude and oil products, the petroleum compensation charge, and controls on oil exports. In October, another federal-provincial agreement on Natural Gas Prices Markets and Prices introduced a more flexible system of natural gas pricing which became effective at the beginning of November 1986. Since then, natural gas prices were allowed to be negotiated between sellers and buyers in both domestic and export markets. The agreement also loosened export license

[5] All information in this section are from the Natural Resources Canada website at http://www2.nrcan.gc.ca/es/es/EnergyChronology/index_e.cfm and the Office of the Auditor General of Canada website at http://www.oag-bvg.gc.ca/DOMINO/REPORTS.NSF/html/coo3aa_e.html

condition. In November 1990, the Petro-Canada Privatization Act was passed and in June 1991, Petro-Canada offered its first round of shares. In March 1992, new rules for foreign investment in the oil and gas industry removed the minimum 50% Canadian ownership of the upstream oil and gas industry. In June 1993, the Canadian Ownership Requirement Repeal Act removed the minimum 50% Canadian ownership for the issuance of frontier oil and gas production licenses and eliminated the process of review and approval for transfers of ownership in a frontier oil and gas production license or shares. In October 1993, two orders which had restricted natural gas exports to northern California were revoked in response to an agreement to resolve Canada-California dispute.

These initiatives were designed to create more market oriented response of energy exports to world oil prices. The 1993 structural break in the relationship between world oil prices and the real exchange rate uncovered in this study suggests that deregulation was successful. That is, prior to 1993, a rise in world oil prices lowered Canada's real exchange rate whereas, past 1993, this relationship turned positive as one would expect in a market environment.

4.2 Summary

The economic theories reviewed in Chapter 1 suggest that the long-run equilibrium of the real exchange rate can be determined by the real price of exportables. As a main commodity exporter to the US, Canada experiences an appreciation of its real exchange rate against the US as real commodity prices increase. In the short-run, monetary factors can account for the deviation of the real exchange rate from its equilibrium value. Canada's real exchange rate depreciates shortly after the Bank of Canada increases monetary supply when the price level in Canada has not changed. The real exchange rate starts to appreciate to approach its long-run value when the price level in Canada starts to rise in response to the money supply increase. An error correction model has been applied in examining these long-run and short-run effects on the Canada-US real exchange rate. The findings of our empirical study, which support several previous studies, help to explain the movement of the Canada-US real exchange rate.

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To check whether the econometric method for estimating the ECM model and the variables we use to measure the real shocks and the monetary policies of Canada and the US work well, we replicate the AN model using the data for the period 1973Q1 to 1992Q1 and find that the estimated coefficients they obtain have the same sign as we do. We discover that RFX, COM and ENE follow a pure random walk process and RDIFF is a stationary process with zero mean. Considering that none of the DGPs of RFX, COM and ENE has a time trend or a drift, it is appropriate to exclude a time trend from the cointegrating relationship when it is tested and estimated. The cointegration tests show that RFX are cointegrated with COM and ENE. This implies that the long-run equilibrium of RFX is determined by COM and ENE with the assumption of exogeneity of COM and ENE. Then we find that an increase in COM results in an appreciation of the real value of the Canadian dollar relative to the US dollar while an increase in ENE results in a depreciation of the real Canada-US exchange rate. The findings we obtain from the estimated ECM are as follows: first of all, after one year 30.82% of the gap between the actual and equilibrium Canada-US real exchange rate is closed, or it takes 7.52 quarters to close 50% of the gap; second, the Canadian dollar overdepreciates immediately after a decrease in the interest rate in Canada and then appreciates to move toward its equilibrium in one quarter; finally, the one-period lagged changes in RFX and the two-period lagged changes in COM can explain the short-run dynamics of RFX.

When we extend the sample period to the end of 2004, the cointegration tests fail to reject the null hypothesis of non-cointegration. It seems that there is no cointegrating relationship among RFX, COM and ENE for the period 1973Q1 to 2004Q4. However, we find a structural break in 1993Q3 which can explain the failure of the conventional cointegration test. Our estimated long-run relationship shows that an increase in COM strengthens the Canadian dollar and its positive long-run effect has been greater since 1993Q3. The effect of ENE on the Canadian dollar has changed from negative to positive at the break point 1993Q3. After one year 40.03% of the gap between the actual and equilibrium Canada-US real exchange rate is closed, or it takes 5.42 quarters to close 50% of the gap. The interest rate differential still has an short-run effect on the Canada-US real exchange rate. Canada’s real exchange rate depreciates immediately after a decrease in Canada’s interest rate and appreciates next quarter but not by as much as it has depreciated. The one-period lagged and the three-period lagged changes in RFX can explain the short-run dynamics of RFX.

In summary, our results generally support most of the theoretical predictions such as the positive effect of real non-energy commodity prices on the Canadian real exchange rate and the deviation of the real exchange rate from its long-run value caused by sticky price level in the short-run when the interest rate differential changes. Before 1993Q3 real energy price shocks have a negative effect which is inconsistent with the theory, but after that time they have a positive effect, reflecting the cumulative effect of the deregulation of the energy sector past 1984.

Our results suggest that the Canada-US real exchange rate fluctuations are driven by fundamentals, energy and non-energy commodity prices. So it is relative price volatility, rather than a flexible exchange rate system, that is driving the volatility of Canada's nominal exchange rate. In other words, the volatility of Canada's nominal exchange rate cannot be eliminated by fixing the exchange rate. Thus, the results of this study weaken the core for fixing the Canada-US exchange rate.

This study also suggests the possibility that Canada might be presently experiencing the symptoms of Dutch Disease (Corden and Neary 1982). The phenomenon of Dutch Disease refers to the possibility of a contraction of the manufacturing sector in a country with a booming natural resource sector caused by an increase in the resource price. Evidence of Dutch Disease is a sharp appreciation of the real exchange rate which places the manufacturing sector at a competitive disadvantage. The recent increase in the world price of oil to almost $100 per barrel can be expected to generate an increase in Canada's real exchange rate and a subsequent contraction of the manufacturing sector.

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Appendix A

Data Sources

Canada-US exchange rate
Source: CANSIM II, Table Number-1760064, Series Level-V37426

US GDP deflator
Source: Bureau of Economic Analysis, Table 1.1.9.

Canadian GDP deflator
Source: CANSIM II, Table Number-3800003, Series Level-V1997756

Non-energy commodity price index
Source: CANSIM II, Table Number-1760001, Series Level-V36383

Energy commodity price index
Source: CANSIM II, Table Number-1760001, Series Level-V36384

3-month yield on prime corporate paper in Canada
Source: CANSIM II, Table Number-1760043, Series Level-V122491

3-month yield on commercial paper in the US
Source: CANSIM II, Table Number-1760044, Series Level- V122141