Serviceability–based Design Approach for Reinforced Embankments on Soft Clay

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College of Graduate Studies and Research
In partial fulfillment of the requirements for the
Degree of Master of Science
In the Department of Civil and Geological Engineering
University of Saskatchewan
Saskatoon

by

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ABSTRACT

The mechanism of soil-reinforcement interaction for a reinforced embankment on soft clay has been explored by conducting a parametric study using a coupled non-linear elastoplastic finite element program. One of the major issues in the design of a reinforced embankment on soft clay is the magnitude of tension that can be mobilized in the geosynthetic reinforcement. Previous research using geotechnical centrifuge modelling and present research using finite element modelling has confirmed that the tension mobilized in the reinforcement is only of the order of active lateral thrust in the embankment. The parametric study has revealed that the soil-reinforcement interaction mechanism depends on the ratio of embankment height to the depth of the clay layer. The embankment behaves similar to a rigid footing in case of deep clay deposit. In this case, the failure mechanism is similar to a slip circle and there is very little contribution from the clay-reinforcement interface towards the mobilization of reinforcement tension. However, if the depth of clay deposit is small, the soil-reinforcement interaction mode is similar to direct shear failure and slip surface is located close to the clay-reinforcement interface. In this case, the contribution of clay-reinforcement interface towards the tension mobilized in the reinforcement is higher and therefore, the contribution of the reinforcement towards overall stability of the embankment is greater. Based on the results of the parametric study a novel serviceability criterion is proposed that aims to limit the lateral deformation of the clay foundation at the toe of the embankment by limiting the allowable mobilized tension in the reinforcement. A simple procedure for the evaluation of the efficiency of soil-reinforcement interface for reinforced embankments on soft clays is also proposed. The validity of the proposed serviceability criterion and the design charts was successfully tested using two field case studies. Sackville test embankment constructed to failure in 1989 and a levee test section that remained serviceable after construction in 1987 at Plaquemine, Louisiana were able to confirm the validity of the serviceability criterion proposed in the present study.
ACKNOWLEDGEMENTS

I would like to express my gratitude and appreciation to my supervisor Dr. Jitendra Sharma for the guidance, encouragement and support that he provided throughout the duration of this study. I have always found him open for discussion and ideas, as he seemed to be approachable even at odd times. His patience to discuss things made it easier for me to express freely my problems relating to research or otherwise. His advice and thorough critique during the preparation of this thesis are greatly appreciated.

I would also like to thank the Department of Civil and Geological Engineering at University of Saskatchewan for giving me the opportunity to join the department and cherish the beautiful Saskatoon, which otherwise I would have never been able to discover. “The City of Bridges” in the “Land of Living Skies” has made me wonder if life could be so serene and peaceful anywhere else. I am grateful to Natural Sciences and Engineering Research Council of Canada (NSERC) for funding my project.

I greatly appreciate my course instructors for broadening my geotechnical prospective from Rock to Soil, Unsaturated to completely Saturated and Geosynthetics in Field to its Numerical Modelling. I would also like to thank Dr. Amir Rahim of CRISP Consortium Limited, for his timely and patient help. The clarifications and explanations he provided regarding on the use of SAGE-CRISP version 4.0 during early stages of finite element modelling were extremely helpful.

In the end I would like to acknowledge ardent support of my family, who have always encouraged me to achieve the set goals. I am lucky that just before I embarked on my research program, I met my wife and best friend, Roop, who has become the panorama of my life. Without my family being on my side, completing this dissertation would not have been possible.
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<th>Roman</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_r$</td>
<td>Area of cross-section per meter width of the reinforcement</td>
<td>m²/m</td>
</tr>
<tr>
<td>$b$</td>
<td>Half width of the footing</td>
<td>m</td>
</tr>
<tr>
<td>$B_c$</td>
<td>Crest width of the embankment</td>
<td>m</td>
</tr>
<tr>
<td>$c$</td>
<td>Intercept of Mohr-Coulomb failure envelope with shear stress axis</td>
<td>kPa</td>
</tr>
<tr>
<td>$C_v$</td>
<td>Coefficient of consolidation</td>
<td>m²/s</td>
</tr>
<tr>
<td>$d$</td>
<td>Maximum drainage path</td>
<td>m</td>
</tr>
<tr>
<td>$D$</td>
<td>Depth of the clay layer</td>
<td>m</td>
</tr>
<tr>
<td>$e$</td>
<td>Void ratio</td>
<td>-</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
<td>kPa</td>
</tr>
<tr>
<td>$E_o$</td>
<td>Young’s modulus of the sand embankment</td>
<td>kPa</td>
</tr>
<tr>
<td>$E_r$</td>
<td>Young’s modulus of the reinforcement</td>
<td>kPa</td>
</tr>
<tr>
<td>$E_{slip}$</td>
<td>Young’s modulus of slip element</td>
<td>kPa</td>
</tr>
<tr>
<td>$E_U$</td>
<td>Undrained Young’s modulus of the clay foundation</td>
<td>kPa</td>
</tr>
<tr>
<td>$E_{UO}$</td>
<td>Undrained Young’s modulus at the clay-reinforcement interface</td>
<td>kPa</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus</td>
<td>kPa</td>
</tr>
<tr>
<td>$G_{slip}$</td>
<td>Shear modulus of slip element</td>
<td>kPa</td>
</tr>
<tr>
<td>$G_U$</td>
<td>Undrained shear modulus</td>
<td>kPa</td>
</tr>
<tr>
<td>$H$</td>
<td>Height of the embankment</td>
<td>m</td>
</tr>
<tr>
<td>$H_c$</td>
<td>Critical height of unreinforced embankment</td>
<td>m</td>
</tr>
<tr>
<td>$H_f$</td>
<td>Failure height of the embankment</td>
<td>m</td>
</tr>
<tr>
<td>$H_{CRITICAL}$</td>
<td>Critical height of the reinforced embankment</td>
<td>m</td>
</tr>
<tr>
<td>$J$</td>
<td>Reinforcement tensile modulus (stiffness)</td>
<td>kN/m</td>
</tr>
<tr>
<td>$k_h$</td>
<td>Horizontal permeability</td>
<td>m/s</td>
</tr>
<tr>
<td>$k_{ht}$</td>
<td>Horizontal permeability in tension crack region</td>
<td>m/s</td>
</tr>
<tr>
<td>$k_v$</td>
<td>Vertical permeability</td>
<td>m/s</td>
</tr>
<tr>
<td>$k_{vt}$</td>
<td>Vertical permeability in tension crack region</td>
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<td>--------</td>
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<tr>
<td>$K$</td>
<td>Elastic bulk modulus</td>
<td>kPa</td>
</tr>
<tr>
<td>$K_o$</td>
<td>Coefficient of earth pressure at rest</td>
<td>-</td>
</tr>
<tr>
<td>$K_{oNC}$</td>
<td>Coefficient of earth pressure at rest for 1-D normal consolidation</td>
<td>-</td>
</tr>
<tr>
<td>$K_{oU}$</td>
<td>Coefficient of earth pressure at rest for 1-D unloading</td>
<td>-</td>
</tr>
<tr>
<td>$K_n$</td>
<td>Normal stiffness of the slip element</td>
<td>kPa</td>
</tr>
<tr>
<td>$K_s$</td>
<td>Shear stiffness of the slip element</td>
<td>kPa</td>
</tr>
<tr>
<td>$K_{sres}$</td>
<td>Residual stiffness of the slip element after slip</td>
<td>kPa</td>
</tr>
<tr>
<td>$m_C$</td>
<td>Rate or increase of undrained shear strength with depth</td>
<td>kPa/m</td>
</tr>
<tr>
<td>$m_E$</td>
<td>Rate or increase of Young’s modulus with depth</td>
<td>kPa/m</td>
</tr>
<tr>
<td>$m_v$</td>
<td>Coefficient of volume compressibility</td>
<td>m²/kN</td>
</tr>
<tr>
<td>$N$</td>
<td>Soil constant</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>Factor of gravitational acceleration, centrifuge model scale factor</td>
<td>-</td>
</tr>
<tr>
<td>$OCR$</td>
<td>Over consolidation ratio</td>
<td>-</td>
</tr>
<tr>
<td>$OCR_{max}$</td>
<td>Maximum value of the overconsolidation ratio</td>
<td>-</td>
</tr>
<tr>
<td>$p'$</td>
<td>Mean effective stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$p_c$</td>
<td>Maximum preconsolidation pressure</td>
<td>kPa</td>
</tr>
<tr>
<td>$p_f$</td>
<td>Effective mean stress at failure</td>
<td>kPa</td>
</tr>
<tr>
<td>$p_{max}$</td>
<td>Maximum previous effective mean stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$p'_{x}$</td>
<td>Peak mean effective stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$q$</td>
<td>Deviatoric stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$q_f$</td>
<td>Deviatoric stress at failure</td>
<td>kPa</td>
</tr>
<tr>
<td>$q_{max}$</td>
<td>Maximum previous deviatoric stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$q_u$</td>
<td>Ultimate bearing capacity</td>
<td>kPa</td>
</tr>
<tr>
<td>$R$</td>
<td>Serviceability criterion - dimensionless parameter</td>
<td>-</td>
</tr>
<tr>
<td>$s$</td>
<td>Shear stress in slip element</td>
<td>kPa</td>
</tr>
<tr>
<td>$S$</td>
<td>Slope of tension cut-off line in $q - p'$ space</td>
<td>-</td>
</tr>
<tr>
<td>$S_{Emb}$</td>
<td>Embankment side slope</td>
<td>-</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Settlement at any time ($t$)</td>
<td>m</td>
</tr>
<tr>
<td>$S_U$</td>
<td>Undrained shear strength</td>
<td>kPa</td>
</tr>
<tr>
<td>$S_{UMOB}$</td>
<td>Mobilized undrained shear strength</td>
<td>kPa</td>
</tr>
<tr>
<td>$S_{ult}$</td>
<td>Ultimate settlement</td>
<td>m</td>
</tr>
<tr>
<td>Greek</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>---------</td>
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<td>-------------</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>Horizontal displacement of the clay foundation</td>
<td>m</td>
</tr>
<tr>
<td>$\delta_v$</td>
<td>Vertical displacement of the clay foundation</td>
<td>m</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Normal strain</td>
<td>%</td>
</tr>
<tr>
<td>$\varepsilon_a$</td>
<td>Allowable compatible strain in the reinforcement</td>
<td>%</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Angle of friction</td>
<td>°</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Stress ratio ($= q/p'$)</td>
<td>-</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Specific volume of the soil at critical state at $p'=1kPa$</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Shear strain</td>
<td>%</td>
</tr>
<tr>
<td>$\gamma_{bulk}$</td>
<td>Bulk unit weight of soil</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>Bulk unit weight of water</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>$H$</td>
<td>Slope of the Hvorslev line in $q - p'$ space</td>
<td>-</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Slope of the swelling line in $V - ln p'$ space</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Slope of the consolidation line in $V - ln p'$ space</td>
<td>-</td>
</tr>
<tr>
<td>$A$</td>
<td>Plastic volumetric strain ratio ($=(\lambda - \kappa)/\lambda$)</td>
<td>-</td>
</tr>
<tr>
<td>$M$</td>
<td>Slope of the critical state line in $q - p'$ space</td>
<td>-</td>
</tr>
<tr>
<td>$M_{PS}$</td>
<td>Plane strain value of $M$</td>
<td>-</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Total normal stress</td>
<td>kPa</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>$\sigma'$</td>
<td>Effective normal stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$\sigma_h'$</td>
<td>Effective horizontal stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$\sigma_v'$</td>
<td>Effective vertical stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$\sigma_{v_{\text{max}}}'$</td>
<td>Previous maximum effective vertical stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear stress</td>
<td>kPa</td>
</tr>
<tr>
<td>$\tau_{\text{slip}}$</td>
<td>Shear strength of the slip element</td>
<td>kPa</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Dimensionless parameter used for estimating the $\varepsilon_a$</td>
<td>-</td>
</tr>
</tbody>
</table>
1 INTRODUCTION

1.1 Background

During the past two decades, the use of geosynthetic reinforcement installed at the base of an embankment on soft clay has gained popularity. It is now widely accepted as an efficient method of overcoming the problem of potential short-term instability of the clay foundation arising from the generation of high excess pore pressures. The factor of safety for an embankment constructed over a soft clay deposit is lowest at the end of embankment construction. Its long-term stability is usually satisfactory due to the gain in the shear strength of soft clay due to consolidation. Therefore, the short-term instability controls the design of such embankments.

Many researchers have studied the behaviour of geotextile-reinforced embankments over soft foundations. The behaviour has been explored using field trials (Olivera 1982; Barsvary et al. 1982; Rowe et al. 1984; Lockett and Mattox 1987; Bassett and Yeo 1988; Fowler et al. 1990; Bergado et al. 1994; Rowe et al. 1996; Chai et al. 2002). Centrifuge model tests have contributed significantly towards improving the understanding of the behaviour of geotextile-reinforced embankments (Ovesen and Krarup 1983; Taniguchi et al. 1988; Terashi and Kitazume 1988; Zhang and Chen 1988; Bolton and Sharma 1994; Mandal and Joshi 1996b; Sharma and Bolton 1996a). Finite element technique has also been used in the past to analyze reinforced embankments (Rowe 1982; Rowe and Soderman 1985a; Rowe and Soderman 1986; Mylleville and Rowe 1991; Sharma and Bolton 1996b; Hinchberger and Rowe 1998; Varadarajan et al. 1999; Sharma and Bolton 2001).

One of the major issues in the design of a reinforced embankment on soft clay is the magnitude of tension that can be mobilized in the geosynthetic reinforcement. Most design methods predict large magnitudes of mobilized tension whereas measurements
from field trials and laboratory tests have indicated that the magnitude of mobilized
tension is small. Previous research using geotechnical centrifuge modelling has
confirmed that the tension mobilized in the reinforcement is only of the order of lateral
thrust in the embankment. This can be attributed to the limited available shear strength
of the clay-reinforcement interface.

Clearly, the failure of a structure marks the end of its life but another limit worthy of
consideration is the serviceable state of the structure, which marks the end of its useful
life, even though it is nowhere near failure. Consideration should be given to the
serviceability limit state of a structure beyond which it ceases to be capable of carrying
out the function for which it was designed. Numerous limit equilibrium-based design
methods have been proposed by various authors to assess the stability of geotextile-
reinforced embankments founded on soft clayey deposits (Jewel 1982; Fowler 1982;
Hird 1986; Leshchinsky and Smith 1989; Low et al. 1990; Kaniraj and Abdullah 1992;
Bergado et al. 1994; Mandal and Joshi 1996a; Low and Tang 1997, Hird et al. 1997;
Palmeira et al. 1998).

The limit equilibrium analysis is generally preferred due to its mathematical simplicity
and in spite of it having several drawbacks when compared to, for example, the finite
element analysis. For example, limit equilibrium-based design methods do not take into
account the effect of system deformation on the soil-reinforcement interaction and
neglect the redistribution of stresses in the embankment due to the presence of
reinforcement. In general, the embankment satisfies overall equilibrium only at the
expense of exceeding the available shear strength at the clay-reinforcement interface.
This results in the embankment undergoing large deformations and therefore, becoming
unserviceable. Therefore, there is a need for the incorporation of a serviceability
criterion into the design process that can limit the shear stress applied at the clay-
reinforcement interface. The development of such a criterion is the main goal of this
project.
1.2 State-of-the-Art Practice

Several attempts have been made by various researchers to incorporate the essential components of soil-reinforcement interaction and ensure serviceability of the embankment as well by limiting the allowable reinforcement strain at failure for use with a limit equilibrium analysis (Rowe and Soderman 1985b; Mylleville and Rowe 1991; Hinchberger and Rowe 2003). The maximum allowable tensile strain approach proposed by Rowe and Soderman (1985b) has found widespread acceptance in the geotechnical community. An effort is made in the present research to develop a simple and versatile design procedure for reinforced embankments on soft clay that can be incorporated into a limit equilibrium-based design method. A new serviceability criterion is proposed that aims to improve upon the approach proposed by Rowe and Soderman (1985b) by taking into account the rate of lateral deformation of the clay layer vis-à-vis the rate of embankment construction.

1.3 Research Objectives

Main objectives for the research undertaken are as follows:

1. To identify the various mechanisms of soil-reinforcement interaction for reinforced embankments on soft clay;
2. To identify the parameters influencing the deformation mechanism of reinforced embankments and mobilized tension in the reinforcement;
3. To develop a serviceability criterion that can be incorporated into a limit equilibrium based design process which would limit the shear stresses applied at the clay-reinforcement interface;
4. To prepare two design charts based on the above-mentioned serviceability criterion:
   a. A design chart to select the maximum height of the embankment that can be constructed safely over a clay layer of given strength.
A design chart to estimate the maximum tension that can be mobilized without sacrificing the serviceability of the embankment.

These objectives are achieved by conducting a numerical parametric study using a coupled non-linear elastoplastic finite element program in combination with back analysis of several published case histories (field trials, large-scale tests and centrifuge tests).

1.4 Research Outline

Initially, an extensive review of the literature published to date was conducted. The main emphasis of this review was on field trials, centrifuge modelling, finite element modelling, design of reinforced embankments and serviceability-based design. Salient points that emerged from the literature review are presented in Chapter 2.

Chapter 3 describes the finite element mesh and the material parameters and models used in the present study. Validation of the software and calibration of the model, by back-analyzing the results of the centrifuge model tests, are also reported in this chapter.

The outline of the finite element parametric study is documented in Chapter 4. This chapter includes the development of a serviceability criterion to limit the lateral deformation of the clay foundation at the toe of the embankment. The results and comparisons of the present study are documented in this chapter. In the end a design procedure based on the proposed serviceability criterion is developed.

The results obtained by the parametric study are compared with two well-documented case studies in Chapter 5. Chapter 6 discusses major findings of the research and presents the conclusions that can be derived from the present research. Some suggestions for future research on this topic are also presented in Chapter 6.
2 LITERATURE REVIEW

2.1 Field Trials

Measurements obtained from the instrumented full-scale reinforced test embankments provide valuable data for understanding the behaviour of reinforced embankment, validating theories and the development of improved design methods. Several publications exist that document such instrumented field trials and large scale testing of reinforced embankments on soft clay. One of the earliest instrumented embankment reported by Olivera (1982) and the Sackville test embankments constructed to failure in 1989, along with other case studies have been documented in this section.

Olivera (1982) reported construction of a deep highway embankment over swamp soil using nonwoven geotextiles. Settlement values, horizontal and vertical displacements and the pore pressures were recorded for reinforced and unreinforced sections. Geotextile prevented excessive fill consumption for the construction of the embankment and the savings were reported to be in the range of 50% as compared to unreinforced section.

The case history of two instrumented sections of road embankments built over soft organic deposits using various geotextiles for separation and reinforcement was documented by Barsvary et al. (1982). In order to observe field performance, some field instruments were installed to monitor embankment settlement and geotextile elongation. Maximum observed elongation was less than 10% in all the cases and strain gauges registered a gradual decrease in elongation during the months following construction. It was postulated that the membrane effect of the geotextile was fully utilized in preventing rotational failure. In addition, the mobilized tensile strength of the geotextile and the soil-fabric friction assisted in restraining the fill from lateral spreading.
Rowe et al. (1984) described the design, instrumentation, and field performance of two instrumented sections of a geotextile-reinforced embankments forming part of an extension to Bloomington Road, located between Leslie Street and Highway 404 near Aurora, Ontario, Canada. A polypropylene, monofilament woven fabric and a strong, twisted, slit film, polypropylene woven fabric were used to reinforce the less compressible and the more compressible section of the deposit, respectively. It was concluded that the use of a single layer of geotextile was insufficient to prevent large shear deformations in these deep, compressible peat deposits primarily because the geotextile was not sufficiently stiff.

Construction of a 6.62 m high bridge approach embankment over weak marsh deposits in Mobile, Alabama using geosynthetics was documented by Lockett and Mattox (1987). Lightweight polypropylene geogrid and nonwoven polyester geotextile were used in tandem to permit the construction of a sand blanket across the marsh, which served as a working platform for the installation of wick drains. Wick drains accelerated the consolidation process, resulting in improved embankment stability due to gain in shear strength. No evidence of embankment cracking, lateral spreading or shear displacements in the marsh during or after embankment construction was reported.

Bassett and Yeo (1988) described the construction of a geogrid reinforced embankment over a deep soft clay deposits at Stanstead Abbots, U.K. Several inclinometers, hydraulic and pneumatic piezometers and horizontal profile gauge were used at this trial embankment. Geogrid reinforcement was instrumented with load cells and strain measuring devices. The magnitude of maximum tension was measured to be 16 kN/m, which was significantly less than the recommended operational values of 25 to 30 kN/m.

The results of the geotechnical instrumentation, testing and monitoring of a fabric reinforced dyke at New Bedford harbour, U.S.A. were presented by Fowler et al. (1990). The dyke was instrumented with slope inclinometers, settlement plates, piezometers, monitoring wells, and stability poles. The maximum horizontal movement observed in the inclinometers was 7.6 cm and total downward displacement of the settlement plates
was in the 0.9 m to 1.2 m range. Strain gauges were cemented to the fabric before the placement of fill and indicated a maximum strain of 5.5 - 7 % at the end of embankment construction. Many gauges were reported to be damaged during fabric installation and dike construction operations. The authors reported that the dike was built economically and performed well.

Bergado et al. (1994) reported the performance of two full-scale test embankments, with and without geotextile-reinforcement, on soft Bangkok clay. One embankment was reinforced by multiple layers of low-strength, nonwoven, needle-punched geotextile and the other embankment was reinforced by a single layer of high-strength, composite nonwoven/woven geotextile placed directly on the natural ground surface. An unreinforced embankment was also built nearby as a control embankment. Reinforcement strain of about 2-3.5% were observed corresponding to an embankment height of 4.2 m. Maximum strain of 12% was measured before embankment failure at 6 m height. They concluded that high-strength geotextile as basal reinforcement can reduce the plastic deformation in the underlying foundation soil and increase the collapse height of the embankment on soft ground.

The behaviour of a geotextile-reinforced test embankment constructed to failure over soft compressible soil at Sackville, New Brunswick was reported by Rowe et al. (1996). A multifilament polyester woven geotextile with ultimate strength of 216 kN/m was sewn in the factory into a 23 m x 30 m rectangular section and instrumented in the field with electrical, electromechanical, and mechanical strain gauges. The test embankment was constructed with a series of berms that ensured that failure was directed to north, due to site limitations. Locally available fill material (gravely silty sand with some clay) was used for embankment construction. A good quality fill material 0.3 – 0.5 m was placed below and above the geotextile to ensure good bond between the geotextile and the surrounding soil. Instrumentation consisted of piezometers, settlement plates, augers, heave plates, inclinometers and a total pressure cell. In general, the Sackville test embankment failed while the embankment height was maintained at 8.2 m and the reinforcement strain was estimated to be between 8.6 % and 13 %. The authors
concluded that the geotextile either yielded or otherwise underwent a local failure near the centre line of the embankment at about 8.2 m height of embankment. No excavation was done to check the validity of this conclusion.

Chai et al. (2002) described a case history of both reinforced and unreinforced embankments built-to-failure on soft subsoil at Lian-Yun-Gang, China. The foundation soil consisted of a 2.0 m thick clay crust underlain by 8.5 m thick soft clay layer. Each embankment had a base width of 42 m with a side slope of 1V: 1.75H. A berm with a width of about 8.0 m was built on both sides. Sandy clay was used as a fill material with a unit weight of 19 kN/m$^3$, with an average filling rate of 0.1 m/day. The reinforcement stiffness used was in the range of 800 - 1600 kN/m with strength equal to 40 kN/m. The embankments were instrumented with surface settlement gauges, piezometer points, and casings for lateral displacement measurement. The unreinforced embankment failed at a fill thickness of 4.04 m, while the reinforced embankment failed at a fill thickness of 4.35 m. This relatively small increase in the collapse height of the embankment compares well with the Mylleville and Rowe (1991) observation that the inclusion of geotextile reinforcement with low stiffness, over a foundation with surface crust gives rise to minimal increase in the collapse height of the reinforced embankment as compared to the unreinforced embankment.

Literature review of reinforced and unreinforced embankments over soft clay foundations confirmed the fact that geotextile can prevent excessive fill consumption and reduce deformations. The embankment height can potentially be raised by a considerable amount compared with the case of unreinforced embankments, if geotextile reinforcement of appropriate stiffness is employed. Most of the field trials studied used settlement plates, slope inclinometers and piezometers to monitor system deformation and excess pore water pressures due to the embankment construction. However, few measured the tension mobilized in the reinforcement. These case studies proved that the magnitude of mobilized tension in the reinforcement is quite small of the order of 50 kN/m to 100 kN/m. The important fact that the geotextile reinforcement restrains the fill from spreading was emphasized in all case studies.
2.2 Centrifuge Modelling

Centrifuge modelling combines the ease of management and the economy of conducting a small-scale test with correct stress levels that could otherwise be achieved only in a large-scale test or a field trial. Design methods and analysis can be validated using data obtained from these idealized models under controlled conditions. Several researchers have studied the behaviour of reinforced embankments constructed on soft clay foundations by using the technique of centrifuge modelling.

Ovesen and Krarup (1983) presented the results of centrifuge tests performed to demonstrate the influence of geotextile-reinforcement in the stability and settlements of embankments on soft, normally consolidated clay. The test procedure adopted for the model tests and scaling problems were documented. The stability of the embankment was found to improve considerably with the reinforcement. In none of the tests, failure occurred in the geotextile reinforcement even though, in some of the tests, tensile strength of geotextile was very low. This indicates that fairly low values of tension were mobilized in the reinforcement.

Centrifugal model tests were performed on embankments reinforced with nonwoven fabric by Taniguchi et al. (1988). The embankments studied were divided into two series, one with inclined face and the other with vertical face, as shown in Figure 2-1. In latter case, four sandbags, 2.5 cm in diameter, protected the vertical slope. The sandbag and reinforcing fabric were sewn together. A nearly circular slip surface was observed when a fabric reinforced embankment collapsed or had large deformation due to tilting or application of vertical loads. It was concluded that, for both cases (inclined and vertical side walls), the reinforcing effect of the fabric could be enhanced by increasing the reinforcement length or the number of layers of geotextile reinforcement.
Terashi and Kitazume (1988) investigated the behaviour of embankments over a fabric-reinforced normally consolidated clay and unreinforced clay using the geotechnical centrifuge. Three different models were set up as follows: 1) Nonwoven geotextile was spread in excess of the entire width of the embankment, 2) Small containing dike, reinforced, preceded the construction of the main body of embankment, unreinforced and 3) Foundation load acted on the top surface of a reinforced embankment over an overconsolidated clay. It was observed that in the case of reinforced embankments constructed on normally consolidated clay, foundation failure preceded overall rotational failure, which included rupture of the reinforcement fabric or reinforcement-soil interface failure. The authors concluded that the mechanical behaviour of the reinforced embankment and the function of the reinforcement are influenced by the geometric configuration, the strength of the clay and the loading condition.

Zhang and Chen (1988) reported centrifuge model tests on reinforced embankments. A weak clay layer was simulated using remoulded clay slurry that was consolidated in-flight to undrained shear strength of about 2-3 kPa. The model reinforcement and the model embankment were placed on top of the model foundation prior to the starting of the centrifuge. In-flight photographic measurements were taken to determine the deformation behaviour of the embankment. Zhang and Chen concluded that the reinforcement significantly increases the embankment stability.

Bolton and Sharma (1994) carried out a parametric study using the technique of centrifuge modelling in order to replicate field behaviour. Several tests were carried out
at 1:40 scale. The type of reinforcement and the depth of clay foundation were varied. The effect of wick drains in the clay foundation was also studied. Direct measurement of tensions induced in the reinforcement was carried out using load cells constructed on the reinforcement. The measurement of clay displacements was done by taking in-flight photographs of the black plastic markers installed on the front surface of the clay block. They concluded that the stiffness and the surface characteristics of the reinforcement are more important than its ultimate strength. They also noted that the increase in undrained shear strength due to the presence of wick drains was more significant from the point of view of embankment stability as compared to tension mobilized in the reinforcement. The maximum tension mobilized in the reinforcement was measured in the range of 30 to 70 kN/m (prototype scale), which was approximately of the same magnitude as the outward lateral thrust within the embankment.

High quality data obtained by the centrifuge models in a controlled environment can certainly be used for the verification of analysis and design methods. However, scaling laws are required to relate the centrifuge model parameters to the full scale prototype. The centrifuge case studies provided concrete evidence of the fact that the tension mobilized in the reinforcement is quite small and is comparable to the magnitude of outward lateral thrust within the embankment. Failure in most cases was reported at the reinforcement-soil interface rather than the actual rupture of the reinforcement fabric. The shear strength of the clay, the embankment slope and the loading rate were found to affect the deformation mechanism of the reinforced embankment.

2.3 Finite Element Modelling

The finite element technique is well recognized as a powerful analysis and design tool and numerous examples of its application to the analysis and the design of reinforced embankments can be found in the literature. It can be used to improve our understanding of observed behaviour in field trials; to model the complete response of a reinforced embankment up to collapse; to investigate changes in construction procedures and the nature of the system; and to examine the effects of the above changes on the system.
The factors affecting the performance of a test embankment constructed at Pinto Pass, Alabama were examined by Rowe (1982). The efficiency of geotextile reinforcement was studied considering settlements, horizontal movements, membrane forces and embankment stability. The results of this theoretical study indicated that fabric does increase the stability of the embankment. It was observed that force in the fabric and the degree of mobilization increased with increasing fabric stiffness. However, for fabric with low to moderate stiffness, extremely large deformations may occur prior to the fabric reaching its tensile capacity, and in these cases, failure may be deemed to have occurred prior to the rupture of the fabric. This observation can be related to the mobilization of high shear stresses at the soil-reinforcement interface. When the entire available shear strength at the interface is mobilized, slip occurs along the foundation-reinforcement interface and the system deforms without mobilizing further tension in the reinforcement.

Rowe and Soderman (1985a) examined the stability and deformations of geotextile reinforced embankments constructed on peat, underlain by a firm base. The stabilizing effect of the geotextile was shown to increase with the increase in geotextile modulus and the effect was more significant for shallower deposits. It was concluded that the effective means of improving the performance of embankments over peat is to use high stiffness geotextile reinforcement in conjunction with lightweight fill.

Rowe and Soderman (1986) extended the behaviour of geotextile-reinforced embankments on peat underlain by a firm base, to the case of peat underlain by a layer of very soft clay. The results indicated that situations might occur where the use of even a very stiff geotextile may not be sufficient to ensure stability of low embankments constructed from granular fill. It was concluded that the most satisfactory means of improving the performance of embankments on these very poor foundations is to use geotextile reinforcement in conjunction with lightweight fill.
Mylleville and Rowe (1991) used finite element analyses to examine the effect of geosynthetic modulus (axial stiffness) on the behaviour of reinforced embankments constructed on very soft clay deposits with and without a higher strength surface crust. They showed that the failure mechanism for the heavily reinforced embankment resembled the bearing capacity failure of a semi-rigid footing. They also observed that if the higher strength surface crust exists then the effect of the crust dominates, even when a high modulus geosynthetic is used.

Sharma and Bolton (1996b) studied the behaviour of reinforced embankments on soft clay by back-analyzing the results of centrifuge model tests using a fully coupled non-linear finite element program. They pointed out importance of incorporating the stress-induced anisotropy of 1-d consolidated clay foundation. When the embankment is constructed, the clay in the passive zone (away from the embankment) swells, the clay in the shear zone (underneath the slope of the embankment) experiences approximately 90° rotation of the strain path and the clay in the active zone (underneath the shoulder of the embankment) experiences a complete reversal of the strain path. Therefore, they divided the subsoil into active, simple shear and passive zone by specifying different $S_U$ values in these three zones, with the ratios 1: 0.644: 0.61, respectively. The results of the centrifuge model tests and their predictions made using finite element analyses were shown to be in good agreement. They found that the magnitude of tension induced in the reinforcement is very sensitive to the magnitude and distribution of undrained shear strength of the clay foundation. Small variation in the undrained shear strength of the clay was shown to cause significant variation in the magnitude of tension induced in the reinforcement. They attributed this observation to the relatively large proportion of stabilizing moment provided by the $S_U$ as compared to that provided by the tension mobilized in the reinforcement.

Hinchberger and Rowe (1998) studied stages 1 and 2 of the Gloucester test embankment using a fully coupled finite element model. The elliptical cap model was used for the time-dependant plastic (viscoplastic) foundation soil and parameters were obtained from
laboratory tests. The measured and calculated settlements were generally in good agreement for the two stages of Gloucester test embankment construction.

Varadarajan et al. (1999) conducted a parametric study of a reinforced embankment-foundation system using coupled elastoplastic finite element analysis. The parameters studied were reinforcement stiffness, shear strength of the clay foundation, depth of foundation and drainage condition. For smaller foundation depth, they noted that the effect of reinforcement stiffness was enhanced, as was the force in the reinforcement and the height of the embankment. They reported that increasing reinforcement stiffness follows the pattern of diminishing returns in terms of increase in the embankment height. The effectiveness of reinforcement with high stiffness was shown to depend on the magnitude of shear strength of clay-reinforcement interface.

Sharma and Bolton (2001) compared the results of centrifuge tests with those from non-linear coupled elastoplastic finite element analyses. They reported that for the reinforced embankments on soft clay installed with wick drains, magnitude of maximum tension in the reinforcement was slightly higher than that for the case of no wick drains. Also, the distribution of tension in the reinforcement was reported to be much more localized under the shoulder of the embankment for the wick drain case as compared to the case with no wick drains. They attributed this effect to the reduction of the lateral spread of the clay foundation and the localization of the excess pore water pressures underneath the embankment for the case with wick drains.

Finite element literature provides substantial evidence of the stabilizing effect of the geotextile reinforcement underneath an embankment constructed over soft clay deposit. Most of the time, system failure was reported prior to the rupture of the fabric due to the limited available shear strength at the soil-reinforcement interface. Therefore, interface elements form an integral part of the finite element model in order to model the slip along the soil-reinforcement interface. The important parameters such as reinforcement stiffness, undrained shear strength, depth of foundation and drainage condition need to
be studied further to completely understand the soil-reinforcement interaction mechanism for reinforced embankments on soft clay.

### 2.4 Design of Reinforced Embankments

#### 2.4.1 Limit Equilibrium-based Design Methods

Analyzing the stability of earth structures of earth structures, by discretizing the potential sliding mass into slices was introduced in the early 20th century. Numerous limit equilibrium-based design methods have been proposed by various authors to assess the stability of geotextile-reinforced embankments founded on soft clayey deposit. The various limit equilibrium-based design methods propose different failure surfaces i.e., circular failure, composite failure and/or log spiral.

Jewel (1982) introduced a method for short-term stability analysis of low reinforced embankments. The same factor of safety was applied for both the soil strength and the reinforcement force. Trial slip circle analysis was proposed to get the required reinforcement force needed for equilibrium at the specified target safety factor on soil strength.

Fowler (1982) developed design charts for use in the slope stability analysis of fabric-reinforced embankments. His method eliminated the need for a repetitive trial-and-correction analysis when one design parameter is fixed and others vary. Dimensionless design curves were presented based on several limit equilibrium analysis to determine the fabric strength necessary to maintain equilibrium. For Pinto Pass test section, Mobile Alabama, these design charts were shown to predict tension mobilized in the reinforcement equal to 54 kN/m. However the values measured in the field was 14.6 kN/m.

Hird (1986) presented dimensionless stability charts derived using limit equilibrium method in which the reinforcement is assumed to apply a horizontal force to a potential
sliding mass. Separate factors of safety were defined for the shear strength of the soil and the tensile strength of the reinforcement. No documentation was found for the validation of the charts proposed by Hird (1986).

Leshchinsky and Smith (1989) proposed a method of determining the factor of safety through a simple minimization process and without resorting to statical assumptions. These statical assumptions are primarily based on whether the interslice normal and shear forces are included and the assumed relationship between the interslice forces. However, they selected a kinematically admissible failure mechanism in the strict framework of limit analysis. The failure surface is made of log-spiral in the embankment and a circular arc in the foundation. Physically, the mechanism enabled the “stiff” embankment to break steeply squeezing out the “Soft” foundation material underneath.

Low et al. (1990) proposed a design chart for computing the factor of safety of geotextile-reinforced embankments constructed on soft ground. The analysis considers only rotational failure based on the limit equilibrium method. The concept of trial limiting tangent to the slip circles was used in order to arrive at the critical factor of safety. The authors recognised the fact that the proposed design charts based on extended limit equilibrium solutions would be more meaningful if supplemented by reinforcement strains at working load obtained from parametric studies using a finite element software.

A reliability evaluation procedure was proposed for a limit equilibrium stability model of reinforced embankments on soft ground by Low and Tang (1997). The proposed stability model allows for a tension crack in the embankment, tensile reinforcement at the base of embankment, and a non-linear undrained shear strength profile in the soft ground. The user is free to decide how much tension is mobilized in the reinforcement and whether this force acts in a horizontal direction or tangential to the slip surface. For a known mobilized tension, factor of safety can be calculated using the proposed reliability evaluation procedure.
Borges and Cardoso (2002) analysed the overall stability of geosynthetic-reinforced embankments on soft soils using limit equilibrium method and compared it to the finite element method. They highlighted that the important difference between the two methodologies is related to the overturning and resisting moments and that this difference is larger in embankments with smaller values of the safety factor. This was explained on the basis of the redistribution of stresses inside the soil mass, simulated by the finite element method but not considered in the limit equilibrium method.

The limit equilibrium method of slices is based purely on the principle of statistics, which is the summation of moments, vertical forces, and horizontal forces. According to Krahn (2004) “The missing physics in a limit equilibrium formulation is the lack of a stress-strain constitutive relationship to ensure displacement compatibility. A full understanding of the limit equilibrium method and its limits leads to greater confidence in the use and in the interpretation of the results”. It is to be noted that the limit equilibrium can give reasonably accurate results for the stability of unreinforced embankment. However, great caution and care is required when stress concentrations exist in the potential sliding mass due to the slip surface or due to the soil-structure interaction.

### 2.4.2 Serviceability-based Design Methods

The limit equilibrium method does not provide information concerning the embankment deformations and reinforcement strains that are associated with a given system. The performance of the reinforced embankment will depend on the soil-reinforcement interaction and this interaction will arise from strain compatibility requirements at the interface between the foundation, fill and the reinforcement. Serviceability of the embankment will depend on the magnitude of the deformations and reinforcement strains. A few researchers have proposed methods to limit the maximum allowable tensile strain in the reinforcement to ensure serviceability of the embankment.
Rowe and Soderman (1985b) proposed a method of estimating the short-term stability of reinforced embankments constructed on a uniform, clayey soil deposit. The proposed method tries to ensure the serviceability of the embankment by limiting the maximum allowable tensile strain in the reinforcement. Based on extensive study of unreinforced and reinforced embankments on soft clay using the finite element method, they proposed a design chart to calculate the maximum allowable compatible strain, depending on the foundation stiffness, embankment geometry, depth of the subsoil, and the critical height of the unreinforced embankment. The advantage of this approach is that it maintains the simplicity of simple limit equilibrium techniques while incorporating the effects of soil-geotextile interaction.

Hinchberger and Rowe (2003) presented an approximate method for estimating geosynthetic reinforcement strains at failure and the resultant undrained stability of reinforced embankments constructed on soft clayey foundation soils. Effect of reinforcement stiffness, embankment crest width, undrained shear strength at the foundation surface, rate of increase of undrained shear strength with depth and undrained modulus was studied on reinforcement strains. Design charts were prepared from finite element results, to establish geosynthetic reinforcement strains suitable for design. The design method employs the same procedure of Rowe and Soderman (1985b) for calculating the allowable compatible strain from the unreinforced collapse height.

The maximum allowable tensile strain approach proposed by Rowe and Soderman (1985b) has found widespread acceptance in the geotechnical community. However this method overpredicts the mobilized tension in the geotextile because the strain field for an unreinforced embankment, instead of a reinforced embankment, was used for estimating the maximum allowable strains at the soil-geotextile interfaces. The inclusion of even a relatively weak geotextile at the interface of an unreinforced embankment can alter its strain field considerably without contributing significantly towards increase in the height at collapse. Moreover, the design method does not provide the increase in the embankment height due to the inclusion of the reinforcement at the base of an embankment.
2.5 **Summary**

Numerous design methods exist for unreinforced embankments but only a handful of design methods have been developed for reinforced embankments. At present, a single design method that can predict both critical height of the reinforced embankment and corresponding tension mobilized in the reinforcement in order to keep the embankment serviceable does not exist. The different mechanisms of soil-reinforcement interaction for reinforced embankment on soft clay have not yet been fully understood. Most of the times, system failure is observed prior to the rupture of the fabric due to the limited available shear strength at the soil-reinforcement interface. Therefore, the behaviour of soil-reinforcement interface needs to be characterized by studying different aspects of the problem such as shear strength of the clay, embankment slope, reinforcement stiffness and depth of foundation. The new serviceability criterion presented in this study aims to improve upon the approach proposed by Rowe and Soderman (1985b) by taking into account the rate of lateral deformation of the clay layer vis-à-vis the rate of embankment construction.
3 FINITE ELEMENT ANALYSES

3.1 Introduction

Finite element technique is a powerful tool to study critical aspects of a physical model such as soil-reinforcement interaction, which are difficult to measure using instrumentation. This chapter gives a brief overview of the advantages of finite element method in general and the salient features of the finite element tool (SAGE-CRISP version 4.0) used in the present study. A reliable scientific approach has been adopted for the validation and calibration of the finite element model. The process of comparing finite element result with known theoretical solutions and measured results has been documented in detail.

3.2 Salient Features of SAGE-CRISP Version 4.0

Following are the capabilities and the limitations of SAGE-CRISP ver 4.0 (CRISP Consortium, 2003):

1. It is capable of undrained, drained or fully coupled (Biot, 1941) consolidation analysis of two-dimensional plane strain or axisymmetric (with axisymmetric loading) solid bodies. The finite element program is also capable of analysing three-dimensional problems although in the present version, this capability is only available if a third party pre processor is used.
2. It incorporates critical state soil models along with anisotropic and inhomogeneous elastic and elastic-perfectly plastic models.
3. Goodman type interface element (Goodman et al., 1968) are used in SAGE-CRISP to allow slip to occur between dissimilar materials or materials having a large difference in their stiffness.
4. It allows for automatic generation of a finite element mesh from a super mesh, using either unstructured or structured mesh generation techniques.

5. It is neither suitable for stress cycling, nor is it capable of handling partially saturated conditions. Moreover, it uses the small-strain/small displacement approach and therefore, it is not suitable for large deformation analysis.

6. It uses the incremental (tangent stiffness) approach without any stress corrections when critical state based models are employed. This means that if the number of increments are insufficient, the response will drift away from the true solution. For elastic-perfectly plastic models, however, when yielding occurs the stress state is corrected back to the yield surface at the end of every increment. The unbalanced load arising from this stress correction is re-applied in the subsequent increment. Additionally, at the end of every increment, the strains are subdivided into smaller steps and the stress state is re-evaluated more accurately.

3.3 Program Validation and Calibration

Finite element program should be checked for any uncertainties in its numerical algorithms and the capability of its constitutive models. Finite element results are user dependent and familiarization with the software, its capabilities and limitations is necessary. Hence, the developed model should be validated and calibrated with the known theoretical solutions, field trials, small scale/centrifuge tests and large-scale tests. In the present study, to start with basic element validation tests were conducted. The sensitivity analysis was done to check the performance of the available interface element. Stiffness matrix formulation and solver was verified by simulating a strip footing over a linear elastic soil. Next, two constitutive soil models to be used in the present study – Elasticperfectly plastic Tresca model and an elastoplastic model based on Critical state soil mechanics (Schofield model) were validated by comparing the results of the known analytical solutions with finite element analysis results. Finally, the reinforced embankment finite element model was calibrated by back analyzing the results of centrifuge model tests conducted by Sharma (1994).
3.4 Basic Element Tests

The following building blocks of the finite element model, to be used in the present study of reinforced embankments on soft clay were tested:

a) Quadrilateral (8-noded) with and without excess pore pressure;
b) Triangle (6-noded) with and without excess pore pressure;
c) Bar (3-noded); and,
d) Joint Interface (slip) element (8-noded).

All of the above elements are ‘linear strain’ elements. The variation of displacements is obtained by quadratic (2nd order) shape function, i.e., the variation of strains is obtained by linear (1st order) shape function.

3.4.1 Quadrilateral (8-noded) without Excess Pore Pressure

A single quadrilateral element with unit dimensions was displaced / loaded so as to produce the prescribed stress or strain path. The boundary conditions for the single element are shown in Figure 3-1.

Figure 3-1: Loaded quadrilateral element.
Linear elastic soil parameters Young’s modulus, \( E = 50 \) MPa and Poisson’s ratio, \( \nu = 0.25 \) were assumed for this single element. In Case A - increments of strain were applied to a linear elastic soil (Table 3-1) and the increments of stress were computed (Table 3-3). Similarly in Case B - increments of stress were applied to a linear elastic soil (Table 3-2) and the increments of strain were computed (Table 3-4).

Table 3-1: Case A - Increments of strain applied to quadrilateral element

<table>
<thead>
<tr>
<th>Increment</th>
<th>Applied ( \Delta \varepsilon_x )</th>
<th>Applied ( \Delta \varepsilon_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.001</td>
<td>0.0005</td>
</tr>
<tr>
<td>2</td>
<td>0.001</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Table 3-2: Case B - Increments of stress applied to quadrilateral element

<table>
<thead>
<tr>
<th>Increment</th>
<th>Applied ( \sigma_x ) (kPa)</th>
<th>Applied ( \sigma_y ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>-50</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>-50</td>
</tr>
</tbody>
</table>

Calculations for the stresses were done using the following equations (Timoshenko and Goodier, 1951)—

\[
\Delta \varepsilon_x = (\sigma_x/E) - (\nu \sigma_y/E) - (\nu \sigma_z/E) \quad [3.1]
\]

\[
\Delta \varepsilon_y = -(\nu \sigma_x/E) + (\sigma_y/E) - (\nu \sigma_z/E) \quad [3.2]
\]

\[
\Delta \varepsilon_z = -(\nu \sigma_x/E) - (\nu \sigma_y/E) + (\sigma_z/E) \quad [3.3]
\]

\[
\Delta \gamma_{xy} = 2(1+\nu) \tau_{xy} / E \quad [3.4]
\]

where \( \varepsilon_x, \varepsilon_y \), and \( \varepsilon_z \) are strains in x, y and z direction respectively;

\( \sigma_x, \sigma_y \) and \( \sigma_z \) are normal stresses in x, y and z direction respectively;

\( \gamma_{xy} \) is shear strain in x-y plane; and,

\( \tau_{xy} \) is shear stress in x-y plane.
Table 3-3: Case A - Increments of stress due to applied strain

<table>
<thead>
<tr>
<th>Increment</th>
<th>Solution</th>
<th>$\sigma_x$ (kPa)</th>
<th>$\sigma_y$ (kPa)</th>
<th>$\gamma_{xy}$</th>
<th>$\sigma_z$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SAGE-CRISP</td>
<td>70</td>
<td>50</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Calculated</td>
<td>70</td>
<td>50</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>SAGE-CRISP</td>
<td>140</td>
<td>100</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Calculated</td>
<td>140</td>
<td>100</td>
<td>0</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 3-4: Case B - Increments of strain due to applied stress

<table>
<thead>
<tr>
<th>Increment</th>
<th>Solution</th>
<th>$\varepsilon_x$</th>
<th>$\varepsilon_y$</th>
<th>$\gamma_{xy}$</th>
<th>$\varepsilon_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SAGE-CRISP</td>
<td>1.25E-3</td>
<td>-1.25E-3</td>
<td>2.5E-3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Calculated</td>
<td>1.25E-3</td>
<td>-1.25E-3</td>
<td>2.5E-3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>SAGE-CRISP</td>
<td>2.5E-3</td>
<td>-2.5E-3</td>
<td>5.0E-3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Calculated</td>
<td>2.5E-3</td>
<td>-2.5E-3</td>
<td>5.0E-3</td>
<td>0</td>
</tr>
</tbody>
</table>

3.4.2 Triangle (6-noded) without Excess Pore Pressure

Next two linear strain triangular elements were selected to make a quadrilateral of unit dimensions. The boundary conditions and the prescribed stress or strain path are shown in Figure 3-2. Same linear elastic soil parameters $E = 50$ Mpa and $\nu = 0.25$ were assumed for these two linear triangular elements. In Case A - increments of strain were applied to a linear elastic soil (Table 3-5) and the increments of stress were computed (Table 3-7). Similarly in Case B - increments of stress were applied to a linear elastic soil (Table 3-6) and the increments of strain were computed (Table 3-8).
Table 3-5: Case A - Increments of strain applied to triangular elements

<table>
<thead>
<tr>
<th>Increment</th>
<th>Applied $\Delta \varepsilon_x$</th>
<th>Applied $\Delta \varepsilon_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.001</td>
<td>0.0005</td>
</tr>
<tr>
<td>2</td>
<td>-0.001</td>
<td>-0.0005</td>
</tr>
</tbody>
</table>

Table 3-6: Case B - Increments of stress applied to triangular elements

<table>
<thead>
<tr>
<th>Increment</th>
<th>Applied $\sigma_x$ (kPa)</th>
<th>Applied $\sigma_y$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>-100</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>-100</td>
</tr>
</tbody>
</table>

Table 3-7: Case A - Increments of stress due to applied strain

<table>
<thead>
<tr>
<th>Increment</th>
<th>Solution</th>
<th>$\sigma_x$ (kPa)</th>
<th>$\sigma_y$ (kPa)</th>
<th>$\gamma_{xy}$</th>
<th>$\sigma_z$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SAGE-CRISP</td>
<td>70</td>
<td>50</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Calculated</td>
<td>70</td>
<td>50</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>SAGE-CRISP</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Calculated</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 3-8: Case B - Increments of strain due to applied stress

<table>
<thead>
<tr>
<th>Increment</th>
<th>Solution</th>
<th>$\varepsilon_x$</th>
<th>$\varepsilon_y$</th>
<th>$\gamma_{xy}$</th>
<th>$\varepsilon_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SAGE-CRISP</td>
<td>2.5E-3</td>
<td>-2.5E-3</td>
<td>5.0E-3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Calculated</td>
<td>2.5E-3</td>
<td>-2.5E-3</td>
<td>5.0E-3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>SAGE-CRISP</td>
<td>5.0E-3</td>
<td>-5.0E-3</td>
<td>1.0E-2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Calculated</td>
<td>5.0E-3</td>
<td>-5.0E-3</td>
<td>1.0E-2</td>
<td>0</td>
</tr>
</tbody>
</table>

3.4.3 Quadrilateral (8-noded) with Excess Pore Pressure

The one-dimensional consolidation of an axisymmetric sample as shown in Figure 3-3 was simulated to check the validity of linear strain quadrilateral elements. Linear elastic soil parameters were assumed for this simulation and the parameters are listed in Table 3-9. The sample was assumed to be double drained i.e., draining from both top and bottom boundary.

Figure 3-3: One-dimensional consolidation setup.
Table 3-9: Isotropic elastic parameters

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>1.0E3</td>
<td>kPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>10</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>$\gamma_{bulk}$</td>
<td>0</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>$k_h$</td>
<td>1.0E-6</td>
<td>m/s</td>
</tr>
<tr>
<td>$k_v$</td>
<td>1.0E-6</td>
<td>m/s</td>
</tr>
</tbody>
</table>

where $\gamma_w$ is bulk unit weight of water;
$\gamma_{bulk}$ is bulk unit weight of soil;
$k_h$ is horizontal permeability; and,
$k_v$ is vertical permeability.

The average degree of consolidation ($U_{avg}$) at time $t$ for constant initial pore water pressure $u_i$ over the depth of the layer is given by (Das, 1999)—

$$U_{avg} = \left( 1 - \int_0^H \frac{u}{u_i} \, dz \right) = 1 - \sum_{m=0}^\infty \frac{2}{M^2} e^{-M^2 T_v}$$  \[3.5\]

where $T_v$ is the time factor which is a dimensionless number;
$i$ is time;
$C_v$ is the coefficient of consolidation;
$m_v$ is coefficient of volume compressibility; and,
d is maximum drainage path (H/2 for two-way drainage).

Figure 3-4 compares the average degree of consolidation ($U_{avg}$) calculated from the above equations and the one computed by SAGE-CRISP. The $U_{avg}$ values from SAGE-CRISP were calculated using the following equation:
\[ U_{\text{avg}} = \frac{S_t}{S_{\text{ult}}} \]  

where \( S_t \) is settlement at any time \( t \); and,

\( S_{\text{ult}} \) is the ultimate settlement.

![Graph showing comparison between theoretical solution and SAGE-CRISP for average degree of consolidation.

Figure 3-4: Comparison of average degree of consolidation.

### 3.4.4 Triangle (6-noded) with Excess Pore Pressure

Figure 3-5 shows the loading and boundary conditions for the simulation of the one-dimensional consolidation using linear strain triangular elements. Linear elastic parameters listed in Table 3-9 were used for the soil. The simulation was conducted by allowing the sample to drain from both top and bottom boundaries. Figure 3-6 compares the average degree of consolidation (\( U_{\text{avg}} \)) calculated from the theoretical equations and the one computed by SAGE-CRISP.
Figure 3-5: One-dimensional consolidation setup – triangular elements.

Figure 3-6: Comparison of average degree of consolidation – triangular elements.
3.4.5 Bar (3-noded)

The 3-noded bar elements provided in SAGE-CRISP have displacement degrees of freedom (only) and are limited to carrying axial force. Both ends make pinned connections with other elements and thus cannot transmit moments. This 3-noded bar element can be used to simulate a geotextile where the bar must lie between triangular or quadrilateral elements. In order to check the accuracy of the 3-node bar element, it was loaded as shown in the Figure 3-7. Material parameters specified for bar element are also shown in Figure 3-7, where $A_r$ is area of cross-section per meter width of bar element. Dummy quadrilateral element was placed on both side of this 3-noded bar element, with $E$ value close to zero. The load $F$ was applied in two steps of 10 kPa each. The resultant axial stress vs. axial strain plot obtained by SAGE-CRISP is shown in Figure 3-8.

![Figure 3-7: Boundary conditions and material parameters for the bar element.](image1)

![Figure 3-8: Resultant axial stress vs. axial strain plot.](image2)
3.4.6 Interface (Slip) Element

The factors that influence the shear strength parameters and stress-displacement characteristics of interfaces are surface roughness of the reinforcement, composition and moisture content of soil, relative density of soil, magnitude of normal stress, and boundary conditions in the direction normal to the interface plane. The parameters required for the slip elements are: intercept of Mohr-Coulomb failure envelope with the shear stress axis, \(c\); Friction Angle, \(\phi\); Normal Stiffness, \(K_n\); Shear Stiffness, \(K_s\); Residual Shear Stiffness, \(K_{sres}\) and Thickness, \(t_{slip}\). The constitutive relationships for an interface element are shown in Figure 3-9.

![Figure 3-9: Constitutive behaviour of interface elements.](image)

Slip element should be assigned normal and shear stiffness which are consistent with the continuum material(s) either side of it. The \(K_n\) and \(K_s\) value should be calculated from:

\[
K_n = \frac{E_{slip}(1-\nu)}{(1+\nu)(1-2\nu)} \quad [3.11]
\]

\[
K_s = \frac{G_{slip}}{2(1+\nu)} \quad [3.12]
\]

Limiting shear stress \((\tau_{slip})\) is calculated as:

\[
\tau_{slip} = c + \sigma \tan \phi \quad [3.13]
\]

where \(E_{slip}\) is young’s modulus of slip element;
\(G_{slip}\) is shear modulus of slip element; and,
\(\sigma\) is normal stress.
Until the limiting shear stress is reached, the slip element behaves as an elastic material i.e., the slip elements bind the soil to the structure. However if the limiting shear stress is reached, the shear modulus is set equal to the limiting value of $K_{res}$, which permits relative slip between soil and the structure. The thickness of the slip element ($t_{slip}$), normally specified to be 1/10 to 1/100 of the length of the slip element, is used for calculating the relative displacements ($\delta$) of the nodes from the shear strains ($\gamma_{slip}$) as follows:

\[
\gamma_{slip} = s / K_s
\]

where $s$ is shear stress calculated for the slip element

\[
\delta = \gamma_{slip} \cdot t_{slip}
\]

An interface element was introduced in between these two 8–noded plane strain quadrilateral element with unit lengths and high stiffness. The boundary conditions for the elements are shown in Figure 3-10.

**Figure 3-10: Boundary conditions for the interface and quadrilateral elements.**

Linear elastic parameters, $E = 5E+8$ kPa and $\nu = 0.2$ were assumed for the quadrilateral elements and the interface parameters are listed in Table 3-10.
The top element was loaded/displaced so as to produce the prescribed stress or strain path. The applied displacement was equal to 0.01 m. Using above equations and the interface parameters in Table 3-10, the expected value of $\tau_{\text{slip}}$, $\gamma_{\text{slip}}$ and $\delta$ can be calculated as follows:

$$\tau_{\text{slip}} = c + \sigma \tan \phi$$

$$\Rightarrow \quad \tau_{\text{slip}} = 1 + 100 \times \tan(30)$$

$$\Rightarrow \quad \tau_{\text{slip}} = 58.73 \text{ kPa}$$

$$\gamma_{\text{slip}} = \frac{s}{K_s}$$

$$\Rightarrow \quad \gamma_{\text{slip}} = \frac{58.73}{3.75E+03}$$

$$\Rightarrow \quad \gamma_{\text{slip}} = 1.57E-02$$

$$\delta = \gamma_{\text{slip}} \times t_{\text{slip}}$$

$$\Rightarrow \quad \delta = 1.57E-02 \times 0.1$$

$$\Rightarrow \quad \delta = 1.57E-03 \text{ m}$$

Comparison of the result using SAGE-CRISP and calculated values can be seen in Table 3-11. Theoretical values and the values calculated by SAGE-CRISP are very close.

### Table 3-11: Comparison of the interface element test

<table>
<thead>
<tr>
<th>Solution</th>
<th>$\tau_{\text{slip}}$ (kPa)</th>
<th>$\gamma_{\text{slip}}$ (%)</th>
<th>$\delta$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increment 1</td>
<td>SAGE-CRISP</td>
<td>58.74</td>
<td>0.0160</td>
</tr>
<tr>
<td></td>
<td>Calculated</td>
<td>58.73</td>
<td>0.0157</td>
</tr>
</tbody>
</table>

### 3.5 Testing of Stiffness Matrix Formulation and Solver

The stiffness matrix formulation and solver of the finite element program was checked by simulating strip footing over a linear elastic soil. In 1885, Boussinesq determined the
stresses due to a point load on the surface within a semi-infinite, homogeneous, isotropic mass with linear stress-strain relationship. The stresses due to surface loads distributed over a particular area can be obtained by integration from the point load solutions. Contours of the vertical stress beneath a strip footing form what is commonly known as a pressure bulb. The shape of the pressure bulb in a homogeneous linear elastic soil and vertical and horizontal stresses at one section has been used to compare the results of the plane strain analyses of a strip footing.

Figure 3-11 shows the finite element mesh used to model a flexible strip footing over a linear elastic soil with $E = 100$ kPa and $\nu = 0.45$. The footing load is applied as a unit normal stress so that the resulting contours can be presented as a ratio of the applied stress. Progressive mesh refinement was undertaken to check the convergence of this mesh to “true” solution.

Figure 3-11: Strip footing over a linear elastic soil.
The analytical solution for loading on an infinite strip as shown in Figure 3-12 was obtained from Poulos & Davis (1974) by using the following equations:

\[ \sigma_v = \left( \frac{p}{\pi} \right) \left( \alpha + \sin \alpha \cos (\alpha + 2\delta) \right) \]  
\[ \alpha = \tan^{-1} \left( \frac{(x+b)}{y} \right) - \delta \quad \text{ (in radians)} \]  
\[ \delta = \tan^{-1} \left( \frac{(x-b)}{y} \right) \quad \text{ (in radians)} \]

Figure 3-12: Uniform loading on an infinite strip (Poulos & Davis, 1974).

Figure 3-13 shows the vertical stress contours beneath the strip footing. The contours have the correct general shape and the values are close to the values predicted by the closed form solution. Close to the base of the mesh, the agreement is not so good. This is due to the fact that for Boussinesq’s solution, the soil is considered semi-infinite but for the SAGE-CRISP analysis the soil layer is of finite thickness.
Figure 3-13: Vertical stress contour beneath a strip footing.

Figure 3-14 shows a comparison between the closed form solution and vertical stress and horizontal stress values obtained using SAGE-CRISP at section A-A’ (Figure 3-11).

Figure 3-14: Comparison of vertical and horizontal stress at section A-A'.
3.6 Testing Soil Models

The soil models available in SAGE-CRISP and to be used in the present study were verified by comparing the results of the known analytical solutions with the results of finite element analysis.

3.6.1 Elastic-Perfectly Plastic Model

The ultimate bearing capacity \( q_u \) is known as the pressure that would cause shear failure of the supporting soil immediately below and adjacent to a foundation. The derivation of the exact solution, using plasticity theory, for the ultimate bearing capacity of a strip footing on the surface of a weightless soil has been given by Atkinson (1981). For the undrained condition in which the shear strength is given by \( S_U \):

\[
q_u = (2 + \pi) S_U = 5.14 S_U \tag{3.19}
\]

A rigid strip footing of 1 m width was loaded on a foundation soil as shown in Figure 3-15. The footing load was applied as a fixed displacement \( \delta_V \) in the vertical direction. Elastic-perfectly plastic model with Tresca yield criterion was used for the undrained plane strain analyses. The parameters used for the foundation soil were: \( E = 1\text{E}5 \) kPa and \( S_U = 100 \) kPa but the unit weight was assumed to be zero.
Figure 3-15: Rigid strip footing over an elastic-perfectly plastic soil.

Higher order cubic strain triangular elements were used to get a close approximation with the analytical solution. The resulting vertical stress ($\sigma_V$) due to the prescribed loading ($\delta_V$), underneath the footing at the integration points is shown in Figure 3-16. An average of resulting vertical stresses ($\sigma_V$) was taken for any given loading ($\delta_V$), in order to plot a graph between average $\sigma_V$ and $\delta_V$ shown in Figure 3-17.

![Graph showing $\sigma_V$ vs $\delta_V$](image)

Figure 3-16: Resulting $\sigma_V$ underneath the footing due to the prescribed $\delta_V$. 

38
For a rigid footing in this problem, the ultimate bearing capacity \( q_u \) equal to 514 kPa can be obtained from the Equation 3.19. This is identical to the SAGE-CRISP value of 514 kPa obtained from Figure 3-17.

### 3.6.2 Schofield Model

The Schofield model was presented by Schofield in 1980. It is essentially a Cam-clay model with a Hvorslev surface and a tension cut-off on the dry side of the critical state line (Figure 3-18). The clay foundation that is heavily overconsolidated at the top surface can be effectively modelled using Schofield model. The presence of a Hvorslev surface limits the strength of the top of the clay foundation, since it is likely to yield on the dry side of the critical state line.
The yield function for Cam-clay surface, as illustrated in Figure 3-18, can be expressed in terms of mean effective stress, $p'$ and deviator stress, $q$ as follows (Britto and Gunn, 1987):

$$F = \frac{q}{Mp'} + \ln \left( \frac{p'}{p'_{\times}} \right) - 1$$  \[3.20\]

where $M$ is slope of the critical state line in $q - p'$ space; and,

$$p'_{\times} = p'_{c}$$  \[3.21\]

The peak mean stress, $p'_{\times}$, is related to the pre-consolidation pressure, $p'_{c}$, by:

$$\ln p'_{\times} = \ln p'_{c} - 1$$, or,

$$p'_{\times} = \frac{p'_{c}}{2.71828}$$  \[3.22\]

The yield function for the Hvorslev surface was obtained from the following expression (Britto and Gunn, 1990):

$$q = (M - H) p'_{\times} \left( \frac{p'}{p'_{\times}} \right)^{\kappa / \lambda} + H p'$$  \[3.23\]

where $H$ is slope of the Hvorslev line in $q - p'$ space;

$k$ is slope of the swelling line in $V - \ln p'$ space; and,

$\lambda$ is slope of the consolidation line in $V - \ln p'$ space.
Schofield model can be verified by comparing theoretical predictions with results from SAGE-CRISP for triaxial compression simulations. A consolidated undrained triaxial tests and drained triaxial tests were simulated in SAGE-CRISP on two different specimens – one was lightly over-consolidated and the other was heavily over-consolidated soil specimen. Results from SAGE-CRISP were compared with theoretical results obtained from governing equations of Schofield model. Figure 3-19 shows the triaxial test specimen and the critical state soil material properties used in the finite element analyses. Because of symmetry of triaxial setup, only a quarter section was considered for this axisymmetric problem. Load was applied as a specified normal stress, which is equivalent to a stress-controlled test.

![Figure 3-19: Triaxial test specimen setup and critical state parameters.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>3.953</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\gamma_{\text{bulk}}$</td>
<td>20</td>
</tr>
<tr>
<td>$H$</td>
<td>0.59</td>
</tr>
<tr>
<td>$S$</td>
<td>2</td>
</tr>
<tr>
<td>$k_h$</td>
<td>2.50E-07</td>
</tr>
<tr>
<td>$k_v$</td>
<td>1.50E-07</td>
</tr>
</tbody>
</table>

**Undrained Triaxial Simulation**

Two simulations were conducted in which the soil sample was isotropically consolidated to 200 kPa and then isotropically unloaded to a mean normal stress of 150 kPa for the
Test 1 and isotropically unloaded to a mean normal stress of 50 kPa for the Test 2. Hence the sample is lightly over-consolidated for Test 1 and heavily over-consolidated for Test 2. These two tests were done to make the sample yield on both wet and dry side of the critical state line respectively. The loading was simulated up to the failure point.

**Results of lightly over-consolidated sample - Test 1:**

Over consolidation ratio (OCR) is given by the ratio of preconsolidation pressure $p'_c$ and mean effective stress at the start of the test $p'$. To summarize the result of lightly over-consolidated sample ($OCR = 1.33$) Test 1, Figure 3-20 shows the effective stress path (ESP) and total stress path (TSP) on $q-p'$ and $q-p$ space. The initial and final yield loci were calculated using a spreadsheet. The ESP and TSP were obtained from SAGE-CRISP. The ESP was vertical in the elastic phase AB at constant $p' (=150$ kPa) until the stress state reached the initial yield locus at point B. When yielding began at point B, the ESP turned towards left, until it reached the critical state line (CSL) at point F. Point $F(p'_f, q_f)$ in Figure 3-20 as computed by SAGE-CRISP was equal to F(83.1kPa, 83.1kPa).

![Figure 3-20: Undrained compression test on lightly overconsolidated soil.](image-url)
The failure point \( F(p', q_f) \) for lightly over-consolidated sample can be calculated using the expression given by Britto and Gunn (1987):

\[
p' = \exp \left( \left( \Gamma - V_o \right) / \lambda \right) \\
q_f = M p_f' \\
V_o = N - \lambda \ln (p'_c / p'_o) + \kappa \ln \left( p'_c / p'_o \right)
\]

where

- \( \Gamma \) is specific volume of the soil at critical state at \( p' = 1 \text{kPa} \);
- \( V_o \) is specific volume at the start of the test;
- \( p'_o \) is mean effective stress at the start of the test; and,
- \( N \) is a soil constant.

From the above equations, the failure point \( F(p', q_f) \) value was calculated equal to \( F(82.85 \text{kPa}, 82.85 \text{kPa}) \) which is very close to the value computed by SAGE-CRISP.

As anticipated, the total stress path (TSP) in Figure 3-20 is a straight line at slope 3V:1H. Deviator stress \( q \) and deviatoric strain \( e_q \) are plotted in Figure 3-21. When yielding begins at B (Figure 3-20), extra plastic shear strains occur and there is a sharp drop in stiffness (Figure 3-21). When stress ratio \( \eta = q/p' \) becomes equal to the value of \( M \), it acts as a limit to the undrained effective stress path. For Test 1 when stress ratio \( \eta \) was equal to \( M \), the implied values of deviator stress \( q \) and mean effective stress \( p' \) imposed asymptote towards which the stress:strain curve (Figure 3-21) was tended.

Plot between pore water pressure \( u \) and deviator strain \( e_q \) is shown in Figure 3-22. The values obtained from SAGE-CRISP were superimposed on to the expected curve. The change in pore pressure \( \Delta u \) is calculated from change in mean effective stress \( \Delta p' \) and the corresponding increment of the total mean stress \( \Delta p \) as follows:

\[
\Delta u = \Delta p - \Delta p'
\]

Similar to the \( q:e_q \) curve the \( u:e_q \) curve became an asymptote to the failure point \( F \), when the \( \eta \) reached the value of \( M \). All of these responses are in accordance with anticipated behaviour of a lightly over-consolidated clay sample.
Figure 3-21: Plot between deviator stress ($q$) and deviator strain ($\varepsilon_q$).

Figure 3-22: Plot between pore water pressure ($u$) and deviator strain ($\varepsilon_q$).

Results of heavily over-consolidated sample - Test 2:

Figure 3-23 shows the effective stress path (ESP) and total stress path (TSP) on $q-p'$ and $q-p$ space, for over-consolidated sample ($OCR = 4$) – Test 2. The first phase of loading
was purely elastic AB with no change in mean effective stress $p' = 50$ kPa until the Hvorslev surface was reached at point B. After point B, the ESP moved up and to the right, until it reached the CSL at point F. Point $F(p'_f, q_f)$ in Figure 3-23 as computed by SAGE-CRISP was equal to $F(69.5$ kPa, 69.5 kPa). High yield ratios were observed at this point of simulation, suggesting the failure of the sample.

![Figure 3-23: Undrained compression test on heavily overconsolidated soil.](image)

The failure point $F(p'_f, q_f)$ for heavily over-consolidated sample can be hand calculated using the expression given by Britto and Gunn (1987):

\[
p'_f = p'_o (OCR)^\Lambda \exp (-\Lambda) \quad [3.29]
\]

\[
q_f = M p'_f \quad [3.30]
\]

\[
OCR = p'_c / p'_o \quad [3.31]
\]

\[
\Lambda = 1 - (\kappa / \lambda) \quad [3.32]
\]

where $\Lambda$ is the plastic volumetric strain ratio.

From these equations, the failure point $F(p'_f, q_f)$ value was calculated equal to $F(68.98$ kPa, 68.98 kPa) which is close to the computed value by SAGE-CRISP.
The $q$ vs. $\varepsilon_q$ and $u$ vs. $\varepsilon_q$ diagrams are presented in Figure 3-24 and Figure 3-25, respectively. The shear stress, $q$, reaches a maximum value of 69.5 kPa and then remain essentially constant. The pore-water pressure increases until the effective stress reaches the Hvorslev surface. Then it decreases to zero and becomes negative as the effective stress path moves towards the critical state line. All of these responses are in accordance with anticipated behaviour of over-consolidated clay.
**Drained Triaxial Simulation**

Next two simulations were carried while allowing the sample to drain from the top boundary. The soil sample was isotropically consolidated to 200 kPa and then isotropically unloaded to a mean normal stress of 150 kPa for the Test 3 and isotropically unloaded to a mean normal stress of 40 kPa for the Test 4.

**Results of lightly over-consolidated sample - Test 3:**

To summarize the result of lightly over-consolidated sample (OCR = 1.33) Test 3, Figure 3-26 shows the effective stress path (ESP) and total stress path (TSP) on $q-p'$ and $q-p$ space. The ESP increased at a slope of 3V:1H until the line intersected the CSL at failure point F. Point $F(p'_f, q_f)$ in Figure 3-26 as computed by SAGE-CRISP was equal to F(224 kPa, 224 kPa).

![Drained compression test on lightly overconsolidated soil.](image)

**Figure 3-26: Drained compression test on lightly overconsolidated soil.**
It is straightforward to calculate the failure point \( F(p'_f, q_f) \) for lightly over-consolidated sample by calculating the intersection of the ESP and the CSL:

\[
q = 3(p' - p'_o) \quad [3.33]
\]
\[
q = M p' \quad [3.34]
\]
giving \( p'_f = \frac{3 p'_o}{3 - M} \) and \( q_f = \frac{3 M p'_o}{3 - M} \)

From these equations, the intersection point \( F(p'_f, q_f) \) value was calculated equal to \( F(225 \text{ kPa}, 225 \text{ kPa}) \) which is close to the computed value by SAGE-CRISP.

Figure 3-27 shows a plot between \( q \) vs. \( \varepsilon_q \). When the stress state reaches point \( F \) (Figure 3-26) \( \eta (= q/p') \) becomes equal to the value of \( M \). At this point, unlimited plastic shear strains develop with no plastic volumetric strain. With no plastic volumetric strain, the yield locus remains of constant size. Since there is just one point \( F \) where the ESP intersects the yield locus, plastic shearing continues at constant effective stresses, and the loading can precede no further (Wood, 1990).

![Figure 3-27: Plot between deviator stress (q) and deviator strain (\( \varepsilon_q \)).](image)
Results of heavily over-consolidated sample - Test 4:

Figure 3-28 shows the effective stress path (ESP) and total stress path (TSP) on $q-p'$ and $q-p$ space, for heavily over-consolidated sample ($OCR = 5$) – Test 4. The ESP increased at a slope of $3V: 1H$ until the line intersected the Hvorslev surface at point B. With $\eta$ greater than the value of $M$ at point B, plastic strains are associated with contraction rather than expansion of the yield locus. Therefore the ESP traversed back to the CSL towards failure point F. Point F($p'_f$, $q_f$) in Figure 3-28 as computed by SAGE-CRISP was equal to F(59.8 kPa, 59.8 kPa).

![Figure 3-28: Drained compression test on heavily overconsolidated soil.](image)

The hand calculation for the intersection point F($p'_f$, $q_f$) using equations 3.33 and 3.34 was done to be F(60 kPa, 60 kPa). The value obtained by SAGE-CRISP is very close to this calculation. The plot between $q$ vs. $\varepsilon_q$ is shown in Figure 3-29. After the initial elastic rise in $q$ and $\varepsilon_q$, further plastic shearing is associated with a drop in $q$ and an increase in $\varepsilon_q$ towards the limiting values corresponding to point F (Figure 3-28).
Figure 3-29: Plot between deviator stress \( q \) and deviator strain \( \varepsilon_q \).

3.7 Calibration by Back-analysis of Centrifuge Tests

Sharma (1994) conducted 1:40 scale centrifuge model tests of reinforced embankments on soft clay, using the Cambridge University 10 m balanced beam centrifuge. Figure 3-30 shows the schematic diagram of the centrifuge test. Due to the inherent symmetry of the embankment about its centreline, only half side of it was modelled. The basic concept of the centrifuge test is that a 1:\( n \) scale model can be tested in a geotechnical centrifuge in a radial acceleration field of \( n \) times normal gravity so that self-weight stresses are the same at corresponding points in the model and in the prototype. Centrifuge modeling combines the ease of management and the economy of conducting a small-scale test with correct stress levels that could otherwise be achieved only in a large-scale test or a field trial. Design methods and analysis tools can be validated using data obtained from idealized models under controlled conditions. Scaling laws are required to relate the model parameters to those of the full-scale prototype. Table 3-12 lists some important scaling laws.
A Prototype of an Embankment  

A Centrifuge model of an Embankment

Figure 3-30: Schematic diagram of the centrifuge test.

Table 3-12: Centrifuge scaling relationships (after Sharma, 1994)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value in Prototype</th>
<th>Value in Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear dimension</td>
<td>1</td>
<td>$1/n$</td>
</tr>
<tr>
<td>Area</td>
<td>1</td>
<td>$1/n^2$</td>
</tr>
<tr>
<td>Volume</td>
<td>1</td>
<td>$1/n^3$</td>
</tr>
<tr>
<td>Stress</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Force</td>
<td>1</td>
<td>$1/n^2$</td>
</tr>
<tr>
<td>Moment</td>
<td>1</td>
<td>$1/n^3$</td>
</tr>
<tr>
<td>Displacement</td>
<td>1</td>
<td>$1/n$</td>
</tr>
<tr>
<td>Strain</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Consolidation time</td>
<td>1</td>
<td>$1/n^2$</td>
</tr>
</tbody>
</table>
3.7.1 Outline of the Finite Element Model Calibration

Six analyses were carried out in order to calibrate the finite element model by back analysis of the centrifuge model tests. The details of these six analyses are given in Table 3-13. It is to be noted that full-scale prototype finite element models were employed to analyse the results of the corresponding centrifuge model tests.

Table 3-13: Details of the finite element models

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Centrifuge Model (Sharma, 1994)</th>
<th>Depth of Clay (m)</th>
<th>Reinforcement</th>
<th>Model used for Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNREINF_8D_S</td>
<td>JSS8</td>
<td>8</td>
<td>Unreinforced</td>
<td>Schofield</td>
</tr>
<tr>
<td>GTX_8D_S</td>
<td>JSS7</td>
<td>8</td>
<td>Geotextile</td>
<td>Schofield</td>
</tr>
<tr>
<td>GTX_4D_S</td>
<td>JSS12</td>
<td>4</td>
<td>Geotextile</td>
<td>Schofield</td>
</tr>
<tr>
<td>UNREINF_8D_T</td>
<td>JSS8</td>
<td>8</td>
<td>Unreinforced</td>
<td>Tresca</td>
</tr>
<tr>
<td>GTX_8D_T</td>
<td>JSS7</td>
<td>8</td>
<td>Geotextile</td>
<td>Tresca</td>
</tr>
<tr>
<td>GTX_4D_T</td>
<td>JSS12</td>
<td>4</td>
<td>Geotextile</td>
<td>Tresca</td>
</tr>
</tbody>
</table>

The centrifuge model tests (Sharma, 1994) selected for the calibration involved fast construction of the geotextile-reinforced embankment over a soft clay layer. From previous field trials (Barsvary et al., 1982; Bassett and Yeo, 1988) and finite element analyses (Rowe, 1982; Hird and Kwok, 1990) it has been established that after overcoming the danger of short-term failure, the reinforcement does not help much to prevent the consolidation settlements of the embankment. The settlement of the embankment due to the consolidation of the foundation usually does not cause stability problem, because the soil upon consolidation gains shear strength. The short-term stability problem can be successfully modelled using undrained plane strain analysis.
Initially, Schofield model was employed for the clay foundation to analyse the problem. Next, a much simpler Elastic-perfectly plastic model with Tresca yield criterion was employed. The results obtained using both the models were compared with the results obtained from centrifuge tests (Sharma, 1994). The purpose of simulating the centrifuge model results using two different soil models was, if the results with both the soil models are comparable then the parametric study can be conducted using a simple elastic-perfectly plastic - Tresca Model. The following benefits will be achieved using a simple elastic-perfectly plastic model –

1. Fewer parameters are needed for Tresca model as compared to the Schofield model.
2. Computational time is reduced significantly, allowing more analyses to be conducted.
3. Design charts would depend on one soil parameter - undrained shear strength.
4. Elastic-perfectly plastic model is more popular in the industry due to its simplicity and the in-situ model parameters are easy to obtain.
5. Design charts would be simple and user friendly.

### 3.7.2 Finite Element Mesh Simulating a Full-scale Prototype

Figure 3-31 shows the details of the finite element meshes used for the back-analyses. It is a 2-D plane strain mesh with boundary conditions similar to the centrifuge models with all dimensions 40 times those of the centrifuge models, simulating a full-scale prototype. The unstructured mesh generator was used to generate automatically triangular elements, from the created super meshes with defined super nodes grading. The elements representing the 6 m high embankment were added in-place in seven layers, to simulate its construction over a clay foundation. The reinforcement was discretized using three-noded bar elements and interface elements were used to simulate clay-reinforcement and embankment-reinforcement interfaces as shown in Figure 3-31.
Figure 3-31: Finite element meshes used in the back-analyses.
3.7.3 Material Models and Parameters

Present research objective was to conduct a parametric study for reinforced embankments on soft clay and further advance the results obtained by Sharma (1994). Detailed description of the tests conducted to obtain the material parameters can be found in Sharma (1994).

3.7.3.1 Clay Foundation

One-dimensionally consolidated clay exhibits stress-induced anisotropy and such behaviour can be modelled accurately only when an anisotropic constitutive model is used to represent the clay foundation. However, in the present study an isotropic soil model with an average value of undrained shear strength $S_U$ (usually obtained from simple shear test or in-situ vane shear test) was used to model the clay foundation. Table 3-14 lists the critical state parameters for the clay foundation reported by Sharma (1994). The clay foundation used in the centrifuge model was heavily overconsolidated at the top with OCR ranging from 33 at the surface to around 4 at a depth of 3 m.

Table 3-14: Critical state parameters for the clay foundation (Sharma, 1994)

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\lambda$</th>
<th>$M$</th>
<th>$\Gamma$</th>
<th>$\nu$</th>
<th>$\gamma_{bulk}$</th>
<th>$k_v$</th>
<th>$k_h$</th>
<th>$H$</th>
<th>$S$</th>
<th>$k_{ht}$</th>
<th>$k_{vt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.028</td>
<td>0.187</td>
<td>0.755</td>
<td>3.0</td>
<td>0.3</td>
<td>16.3</td>
<td>1.28E-9</td>
<td>0.50E-9</td>
<td>0.59</td>
<td>2.0</td>
<td>2.5E-7</td>
<td>1.5E-7</td>
</tr>
</tbody>
</table>

For the Schofield model, assuming that all other parameters are constant for a given soil, the $S_U$ value depends only on the value of the maximum isotropic preconsolidation pressure $p_c'$. In order to specify the average $S_U$ profile, the value of $p_c'$ was adjusted based on the values of maximum preconsolidation pressures inferred from the vane shear strength profiles.
In order to model the clay foundation as an elastic-prefectly plastic material with a Tresca yield criterion, the required parameters were obtained by making a linear approximation of the average undrained shear strength ($S_U$) and average undrained young’s modulus ($E_U$) profile obtained from Schofield model. Figure 3-32 shows an example of the linear approximation made for an average $S_U$ and $E_U$ profile obtained from Schofield model for $p_c'$ equal to 100 kPa. The corresponding value of $S_{U0}$, $m_c$, $E_{U0}$ and $m_E$ was then used for modelling the clay foundation using Tresca yield criterion. It was found that the model gave reasonably accurate estimates of the deformation of the clay foundation as well as predicted the magnitude and the distribution of excess pore water pressures in the clay foundation accurately.

![Figure 3-32: An example of the linear approximation of average $S_U$ and $E_U$ profile.](image)

3.7.3.2 Sand Embankment

The sand embankment was modelled as elastic-perfectly plastic material with a Mohr-Coulomb yield criterion and associated flow rule. Table 3-15 gives all the parameters specified for this soil model. A small value of cohesion $c = 1$ kPa was specified in order to avoid numerical instabilities at the toe of the embankment. The Young’s modulus of the embankment was modelled as linearly increasing with the depth of the embankment with maximum Young’s Modulus $E_o = 10000$ kPa at the base of the embankment. No
testing was done to establish the stiffness parameters in view of the fact that most of the embankment reaches yield condition when approaching full height (Hird et al. 1990).

<table>
<thead>
<tr>
<th>$E_o$ (kPa)</th>
<th>$\nu$</th>
<th>$c$ (kPa)</th>
<th>$\gamma_{bulk}$ (kN/m$^3$)</th>
<th>$\phi$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>0.3</td>
<td>1</td>
<td>16.0</td>
<td>35</td>
</tr>
</tbody>
</table>

### 3.7.3.3 Reinforcement

A high stiffness geotextile reinforcement was considered for all the analyses. Linear elastic 3-noded bar elements that are capable of taking only axial force were used to model the geotextile reinforcement. The parameters specified for the reinforcement are tabulated in Table 3-16.

<table>
<thead>
<tr>
<th>$E_r$ (kPa)</th>
<th>$\nu$</th>
<th>$A_r$ (m$^2$/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7E+6</td>
<td>0.35</td>
<td>0.00212</td>
</tr>
</tbody>
</table>

Axial stiffness of a geotextile is usually expressed as force per unit width per unit strain (kN/m) that is commonly referred as the Reinforcement Modulus, $J$. In the formulation of bar elements in SAGE-CRISP version 4.0, $J$ is equal to the product of $E_r$ and $A_r$, where $E_r$ is the Young’s modulus of the reinforcement and $A_r$ is the area of cross-section per meter width of the reinforcement.

### 3.7.3.4 Soil-Reinforcement Interfaces

The soil-reinforcement interfaces were modelled using the six-noded quadrilateral interface elements based on an elastic-perfectly plastic model. The clay-reinforcement
interface was considered undrained with shear strength equal to 0.8 times the undrained shear strength of clay \((0.8 \times S_{uo})\). The sand-reinforcement interface was considered drained with friction angle equal to that of sand embankment. The values of \(K_n\) and \(K_s\) were selected from the shear and normal stiffness of adjacent soils. Table 3-17 gives the soil-reinforcement interface parameters.

### Table 3-17: Soil-reinforcement interface parameters specified

<table>
<thead>
<tr>
<th>Interface</th>
<th>(c)</th>
<th>(\phi)</th>
<th>(K_n)</th>
<th>(K_s)</th>
<th>(K_{sres})</th>
<th>(t_{slip})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay-geotextile</td>
<td>(0.8 \times S_{uo})</td>
<td>0</td>
<td>3500</td>
<td>1000</td>
<td>10</td>
<td>0.04</td>
</tr>
<tr>
<td>Sand-geotextile</td>
<td>0</td>
<td>35</td>
<td>13000</td>
<td>3800</td>
<td>40</td>
<td>0.04</td>
</tr>
</tbody>
</table>

### 3.8 Comparison of Back-analysis Results

#### 3.8.1 Unreinforced Embankment and 8 m Deep Clay Layer

Figure 3-33 shows the horizontal displacements of the nodes near the toe of the embankment with respect to embankment construction using the Tresca and the Schofield model. The analysis UNREINF_8D_T (using Tresca model) predicted onset of failure after 89% of embankment was constructed, which is comparable to the result obtained by the analysis UNREINF_8D_S (using Schofield model) in which the failure is predicted to have occurred at 89% of embankment construction and centrifuge test results (Sharma, 1994) that reported failure, approximately after 85% of the embankment was constructed. Clearly, the elastic perfectly plastic - Tresca model with an average S_U profile for the clay foundation gives fairly accurate results using fewer increments for the short term undrained problem as compared to the Schofield model.
Figure 3-33: Horizontal displacement under the toe of the embankment.

Miniature pore pressure transducers were installed in the original centrifuge model JSS8 (Sharma, 1994) at seven different locations. Table 3-18 gives the measured excess pore pressures at the end of embankment construction and simulated excess pore pressures after 88.95% of embankment construction. Comparing the pore pressure response at different locations of pore pressure transducers it can be easily seen that analysis UNREINF_8D_T predicted the pore pressure response reasonably well.

Table 3-18: Measured (Sharma, 1994) and simulated excess pore pressures.

<table>
<thead>
<tr>
<th>Location</th>
<th>PPT1</th>
<th>PPT2</th>
<th>PPT3</th>
<th>PPT4</th>
<th>PPT5</th>
<th>PPT6</th>
<th>PPT7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured (kPa)</td>
<td>33</td>
<td>24</td>
<td>29</td>
<td>42</td>
<td>&gt;40</td>
<td>70</td>
<td>&gt;65</td>
</tr>
<tr>
<td>UNREINF_8D_T (kPa)</td>
<td>19</td>
<td>25</td>
<td>34</td>
<td>45</td>
<td>44</td>
<td>50</td>
<td>65</td>
</tr>
</tbody>
</table>
3.8.2 Reinforced Embankment and 8 m Deep Clay Layer

Table 3-19 compares the measured (Sharma, 1994) parameters at the end of embankment construction and results obtained by the analyses GTX_8D_S and GTX_8D_T. Sign convention adopted for the displacement assumes positive values for displacements coinciding with the direction of the coordinate axes (see Figure 3-31). It can be seen in Table 3-19, the direction and magnitude of the maximum horizontal and vertical displacement values in the clay foundation are quite comparable for the centrifuge model and corresponding finite element models GTX_8D_S and GTX_8D_T. It is also apparent from Table 3-19 that mobilized tension in the reinforcement predicted by the analyses GTX_8D_S and GTX_8D_T is comparable. Therefore it is reconfirmed that finite element model being studied gives comparable results with both critical state soil model and Elastic perfectly plastic model, for fast undrained analysis.

Table 3-19: Comparison of measured (Sharma, 1994) and simulated results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Location</th>
<th>Measured</th>
<th>GTX_8D_S</th>
<th>GTX_8D_T</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{v_{\text{max}}}$</td>
<td>Near shoulder of the embankment</td>
<td>-0.11 m</td>
<td>-0.12 m</td>
<td>-0.13 m</td>
</tr>
<tr>
<td>$\delta_{v_{\text{max}}}$</td>
<td>Near toe of the embankment</td>
<td>+0.11 m</td>
<td>+0.12 m</td>
<td>+0.11 m</td>
</tr>
<tr>
<td>$\delta_{h_{\text{max}}}$</td>
<td>Near shoulder of the embankment</td>
<td>-0.15 m</td>
<td>-0.12 m</td>
<td>-0.12 m</td>
</tr>
<tr>
<td>$\delta_{h_{\text{max}}}$</td>
<td>Near toe of the embankment</td>
<td>-0.20 m</td>
<td>-0.18 m</td>
<td>-0.18 m</td>
</tr>
<tr>
<td>$(T_{\text{MOB}})_{\text{max}}$</td>
<td>Near centre line of the embankment</td>
<td>60 kN/m</td>
<td>90 kN/m</td>
<td>100 kN/m</td>
</tr>
</tbody>
</table>

Excess pore pressures in clay just after embankment construction as documented by Sharma (1994) can be seen in Table 3-20. The excess pore pressure prediction obtained by the corresponding finite element model GTX_8D_T can also be seen in Table 3-20. Comparing the results it was found that the magnitudes of pore pressure in clay foundation are similar at the end of embankment construction for the analysis and the centrifuge model.
Table 3-20: Measured (Sharma, 1994) and simulated excess pore pressures.

<table>
<thead>
<tr>
<th>Location</th>
<th>PPT1</th>
<th>PPT2</th>
<th>PPT3</th>
<th>PPT4</th>
<th>PPT5</th>
<th>PPT6</th>
<th>PPT7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured (kPa)</td>
<td>-</td>
<td>24</td>
<td>33</td>
<td>45</td>
<td>49</td>
<td>64</td>
<td>65</td>
</tr>
<tr>
<td>GTX_8D_T (kPa)</td>
<td>10</td>
<td>18</td>
<td>26</td>
<td>45</td>
<td>40</td>
<td>55</td>
<td>66</td>
</tr>
</tbody>
</table>

Shear stresses profile for soil-reinforcement interfaces obtained from GTX_8D_T are superimposed on the results obtained from GTX_8D_S in Figure 3-34. From Figure 3-34 effective length of the reinforcement was calculated, for which the shear stresses in the sand-reinforcement and clay-reinforcement interface complement each other and help the reinforcement to mobilize more tension. The shear stress profile is same for both the analysis and the effective length of the reinforcement is similar as well. Integrating the shear stresses in slip elements obtained from Figure 3-34 for per unit width, we can obtain the tension in the reinforcement. The difference in the clay-reinforcement and sand-reinforcement interfaces was seen to be maximum underneath the centre of the embankment slope. It can be inferred from Figure 3-34 that slip did not occur at any of the interfaces because shear stresses at both the interfaces were only about 50% of the strength of the interfaces.

![Shear stresses at soil-reinforcement interface.](image)

Figure 3-34: Shear stresses at soil-reinforcement interface.
### 3.8.3 Reinforced Embankment and 4 m Deep Clay Layer

Table 3-21 compares the measured (Sharma, 1994) parameters at the end of embankment construction and results obtained by the analyses GTX_4D_S and GTX_4D_T. Close agreement has been found in the direction and magnitude of the maximum horizontal and vertical displacement values in the clay foundation between the centrifuge model and corresponding finite element models GTX_4D_S and GTX_4D_T. The maximum mobilized tension in the reinforcement predicted by the analyses GTX_4D_S and GTX_4D_T is in excellent agreement to the measured value in centrifuge test.

**Table 3-21: Comparison of measured (Sharma, 1994) and simulated results.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Location</th>
<th>Measured</th>
<th>GTX_4D_S</th>
<th>GTX_4D_T</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{v \text{max}}$</td>
<td>Near centre line of the embankment</td>
<td>- 0.02 m</td>
<td>- 0.03 m</td>
<td>- 0.01 m</td>
</tr>
<tr>
<td>$\delta_{v \text{max}}$</td>
<td>Near toe of the embankment</td>
<td>+ 0.03 m</td>
<td>+ 0.05 m</td>
<td>+ 0.03 m</td>
</tr>
<tr>
<td>$\delta_{h \text{max}}$</td>
<td>Near shoulder of the embankment</td>
<td>- 0.04 m</td>
<td>- 0.04 m</td>
<td>- 0.04 m</td>
</tr>
<tr>
<td>$\delta_{h \text{max}}$</td>
<td>Near toe of the embankment</td>
<td>- 0.05 m</td>
<td>- 0.06 m</td>
<td>- 0.06 m</td>
</tr>
<tr>
<td>$(T_{MOB})_{\text{max}}$</td>
<td>Near centre line of the embankment</td>
<td>30 kN/m</td>
<td>40 kN/m</td>
<td>30 kN/m</td>
</tr>
</tbody>
</table>

It can be seen from Table 3-22 that the excess pore pressures at the end of embankment construction obtained from centrifuge model (Sharma, 1994) and finite element model GTX_4D_T are quite comparable. It is interesting to note that the magnitude and extent of pore pressures was much less for the analysis GTX_4D_T as compared to the pore pressure distribution for analysis GTX_8D_T (Table 3-20).

**Table 3-22: Measured (Sharma, 1994) and simulated excess pore pressures.**

<table>
<thead>
<tr>
<th>Location</th>
<th>PPT1</th>
<th>PPT2</th>
<th>PPT3</th>
<th>PPT4</th>
<th>PPT5</th>
<th>PPT6</th>
<th>PPT7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured (kPa)</td>
<td>2</td>
<td>7</td>
<td>18</td>
<td>44</td>
<td>44</td>
<td>34</td>
<td>77</td>
</tr>
<tr>
<td>GTX_4D_T (kPa)</td>
<td>5</td>
<td>9</td>
<td>20</td>
<td>45</td>
<td>44</td>
<td>40</td>
<td>81</td>
</tr>
</tbody>
</table>
Figure 3-35 compares the shear stresses obtained from analysis GTX_4D_S and analysis GTX_4D_T. It can be seen from Figure 3-35 that shear stresses in the soil-reinforcement interfaces at the end of embankment construction are quite comparable. Effective length of the reinforcement was also calculated of the same magnitude from both the analysis. Comparing Figure 3-34 with Figure 3-35 it can be seen that the area under the curves is more for 8 m deep clay layer case and hence the mobilized tension in the reinforcement was higher.

![Figure 3-35: Shear stresses at soil-reinforcement interfaces.](image)

### 3.9 Summary

The ability of finite element model to accurately reflect field conditions should essentially depend on the ability of the constitutive model to represent real soil behaviour and the appropriate boundary conditions applied at the various stages of construction. In this chapter SAGE-CRISP has been validated with known theoretical solutions and it has been found that reasonable results can be achieved by using the Elastic-perfectly plastic model with Tresca yield criterion for the soft clay foundation, in
short term instability problems. It was found that the Tresca model gave reasonably accurate estimates of the deformation of the clay foundation, pore-water pressure distribution, the magnitude of tension mobilized in the reinforcement and the distribution of shear stress in the soil-reinforcement interfaces. In order to accurately quantify the effect of reinforcement on the deformation mechanism of reinforced embankment on soft clay, a parametric study was conducted. The details of the parametric study are presented in the next chapter.
4 PARAMETRIC STUDY

4.1 Introduction

The concept and the development of the serviceability criterion for the reinforced embankments on soft clay have been described in detail in this chapter. The different deformation mechanisms for reinforced embankment on soft clay are highlighted. Design charts are presented based on the serviceability criterion and parameters, such as, the depth of clay layer, the undrained shear strength at the clay-reinforcement interface, the rate of change of undrained shear strength with depth, the embankment slope and the reinforcement stiffness. Illustrative examples have been provided highlighting the application of the proposed design procedure.

4.2 Finite Element Model

After the successful calibration of the initial finite element model (Figure 3-31), following changes were incorporated in the model before conducting the parametric study:

a) The left hand side boundary in the initial model was closer to the toe of embankment to reflect the boundary conditions prevailing in the centrifuge models. However, this boundary was taken away from the toe of the embankment to reflect the boundary conditions prevailing in the field.

b) Interface elements were introduced at the bottom boundary of the finite element model as well. The purpose of using interface elements at the bottom horizontal boundary was to limit the available shear strength in case of localization of shear strains at this boundary. True deformation mechanism is difficult to capture if the elements at the bottom of clay foundation are fixed. This is especially important in case of a shallow clay layer. In reality there exists an interface between the
hard strata (or bed rock) and the clay foundation. This interface cannot possibly have strength greater than the undrained shear strength of clay at the corresponding depth. Including slip elements at this interface ensures limited shear strength should the failure surface goes through this interface. Hird et al. (1990) also used the similar interface elements at the boundary to model the collapse of reinforced embankment on soft clay. This interface is considered undrained with shear strength equal to 0.8 times the undrained shear strength of clay at the bottom. Table 4-1 lists the parameters specified for the boundary interface.

Table 4-1: Parameters for the interface at the bottom horizontal boundary

<table>
<thead>
<tr>
<th>Interface</th>
<th>$c$</th>
<th>$\phi$</th>
<th>$K_n$</th>
<th>$K_s$</th>
<th>$K_{sres}$</th>
<th>$t_{slip}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary</td>
<td>0.8 x $S_U$</td>
<td>0</td>
<td>3500</td>
<td>1000</td>
<td>10</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Figure 4-1 shows the details of the generic 2-D plane strain mesh used for the parametric study. The elements representing the embankment were added in-place in six layers, each of 1 m thickness, to simulate its construction over a clay foundation. Beyond a height of 6 m, the increase in height of the embankment was simulated by placing a normal surcharge at the top of the embankment until failure of the clay foundation was achieved. The reinforcement was discretized using three-noded bar elements and interface elements were used to simulate clay-reinforcement and embankment-reinforcement interfaces. Interface elements were also used at the bottom horizontal boundary of the clay foundation as shown in Figure 4-1.
Figure 4-1: Finite element mesh with boundary conditions and details at the interfaces.

4.3 Outline of the Parametric Study

In the case of a reinforced embankment, the soil-reinforcement interface is one of the important factors influencing the behaviour of the structure. Accurate quantification of the effect of the reinforcement requires complete understanding of the soil-
reinforcement interaction modes. These modes have been found to influence the foundation-deformation pattern. Therefore, in the present parametric study it was examined if the ratio of embankment height to the depth of the clay layer ($H/D$ ratio) has any influence on the soil-reinforcement interaction mechanism. Several analyses were carried out in the present study by varying the depth of clay layer $D$ and increasing the embankment height $H$ up to the point of failure. These analyses were done for different available undrained shear strength at the interface ($S_{UO}$) and the rate of increase of undrained shear strength ($m_c$) with depth. Next the effect of different embankment slope ($S_{Emb}$) and the reinforcement stiffness ($J$) was studied. The undrained young’s modulus of clay ($E_U$) was not varied during the analyses, since at $R \geq 0.1$ most of the clay foundation is yielding and plasticity governs the magnitude of tension mobilized in the reinforcement. Table 4-2 gives the detail of the above parameters.

Table 4-2: Parametric range

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ (m)</td>
<td>24, 22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 3</td>
</tr>
<tr>
<td>$S_{UO}$ (kPa)</td>
<td>8, 10, 12, 14, 16, 18, 20</td>
</tr>
<tr>
<td>$m_c$ (kPa/m)</td>
<td>0, 1, 2</td>
</tr>
<tr>
<td>$S_{Emb}$</td>
<td>1.5, 1.8, 2.5, 3.0</td>
</tr>
<tr>
<td>$J$ (kN/m)</td>
<td>1431, 2862, 5724, 11448</td>
</tr>
</tbody>
</table>

Each of the above analyses were given a unique identifier, of the form “$D$-$S_{UO}$-$m_c$-$S_{Emb}$-$J$”. For example, analysis 8-14-1.8-5724 corresponds to a model with $D = 8$ m, $S_{UO} = 14$ kPa, $m_c = 1$ kPa per meter depth, $S_{Emb} = 1.8$ and $J = 5724$ kN/m.
4.4 Establishing Serviceability Criterion

Several careful finite element analyses were performed to establish the serviceability criterion that will have a general applicability for all the cases included in the parametric study. The embankment height $H$ was increased up to the point of failure for each of the analyses. For all the analyses, it was found that after a certain height of embankment (denoted by $H_{CRITICAL}$) is reached, onset of plastic yielding in clay foundation takes place, resulting in increased lateral deformation of the clay foundation near the toe of the embankment. If the embankment construction is continued beyond $H_{CRITICAL}$, the embankment approaches failure rapidly. To capture the above-mentioned behaviour of the embankment, a criterion that characterizes the serviceability-limit state of the embankment is proposed. The basic idea behind the proposed serviceability criterion is that if the embankment is to remain serviceable, the ratio of the rate of lateral displacement at the toe of the embankment to the rate of embankment construction must be kept below a certain value. The serviceability criterion is defined in terms of a dimensionless parameter $R$:

$$ R = \frac{\Delta (\delta_h)}{\Delta H} \quad [4.1] $$

In Equation 4.1, $\Delta (\delta_h)$ is the change in the lateral displacement at the toe in response to a change in the height of the embankment ($\Delta H$). Typical plot between $H$ and $R$ can be seen from Figure 4-2 showing the result of the analysis 14-14-1-1.8-5724. It can be seen from Figure 4-2 that when $R > 0.1$, there is no appreciable change in the height of the embankment but there is a significant increase in the horizontal displacement at the toe. For the analysis 14-14-1-1.8-5724, for $R = 0.1$, $H_{CRITICAL}$ is equal to 7 m and mobilized tension in the reinforcement ($T_{MOB}$) is equal to 125 kN/m. It is proposed that the embankment remains serviceable at $R \leq 0.1$. 

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Figure 4-2: Typical plot between embankment height \((H)\) vs. \(R\) near the toe of the embankment.

The critical height of the embankment \((H_{\text{CRITICAL}})\) was obtained for the various parametric analyses, by employing the previously discussed serviceability criterion i.e., limiting the parameter \(R\) to a value of 0.1. This \(H_{\text{CRITICAL}}\) corresponds to the height of reinforced embankment which can be constructed over a clay deposit of a given undrained shear strength, without causing serviceability problems. Further, for each of the parametric analysis \(T_{\text{MOB}}\) corresponding to \(H_{\text{CRITICAL}}\) was obtained.

4.5 Deformation Mechanisms

Deformation mechanism for a reinforced embankment constructed over a deep clay layer was found to be different from a reinforced embankment constructed over a shallow clay layer. Rotational failure was observed for deep clay layer, whereas the direct shear type failure was observed for shallow clay layer.
4.5.1 Deformation Mechanism for Deep Clay Layer

Figure 4-3 and Figure 4-4 show the displacement vectors and the contours of maximum shear strain for the analysis 24-14-1-1.8-5724 with embankment constructed up to a height of 8 m. Slip surface obtained from maximum shear strain plot was superimposed on the displacement vectors. It can be seen from Figure 4-3 and Figure 4-4 that the clay foundation has nearly failed by undergoing a rotational slip due to the construction of the embankment. Shear strains in excess of 30% are seen to occur along the circular slip surface. It is important to note that the failure surface passes at a shallow depth and the increase in shear strength of clay with depth beyond this shallow depth does not contribute to prevent the failure.

\[
D - S_U - m_c - S_{Emb} - J
\]

Analysis 24-14-1-1.8-5724

![Figure 4-3: Displacement vectors for the deep clay layer.](image-url)
The distribution of shear stresses at the clay-reinforcement interface for different
$H / H_{CRITICAL}$ ratios is shown in Figure 4-5. A $H / H_{CRITICAL}$ ratio greater than 1 signifies
that the embankment has been constructed to height beyond the critical height of the
embankment. Such a failure mechanism is less than ideal for the mobilization of tension
in the reinforcement. It can be clearly seen from Figure 4-5 that the shear stresses at the
clay-reinforcement interface were much less than the available shear strength at the
interface.
4.5.2 Deformation Mechanism for Shallow Clay Layer

Figure 4-6 and Figure 4-7 show the displacement vectors and the contours of maximum shear strain for the analysis 3-14-1-1.8-5724 with embankment constructed up to a height of 11 m. Slip surface obtained from maximum shear strain plot was superimposed on the displacement vectors. It can be seen from Figure 4-6 and Figure 4-7 that the developing rotational slip failure is interrupted by the boundary between the clay foundation and the stiff bedrock. As a result, the mode of failure of the clay foundation is similar to that observed in a direct simple shear test.
Figure 4-6: Displacement vectors for the shallow clay layer.

Figure 4-7: Contours of maximum shear strain for the shallow clay layer.

The distribution of shear stresses at the clay-reinforcement interface for different $H / H_{\text{CRITICAL}}$ ratios is shown in Figure 4-8. The clay layer is getting squeezed from underneath the embankment and therefore, it is imposing considerable frictional drag on the reinforcement at the clay-reinforcement interface. It can be clearly seen from Figure 4-8 that the available shear strength at the interface has been completely mobilized on the completion of embankment. The shear stresses exceeding the value of available shear strength at the toe of the embankment should be ignored. This effect is due to the fact that bar elements used for modelling reinforcement are capable of taking
compressive axial force. In reality, geotextile reinforcement will simply crumple at the toe of the embankment. Direct shear mechanism is ideal for efficient mobilization of tension in the reinforcement as almost the entire length of the reinforcement participates in the tension mobilization process. For such cases, it is generally not a problem to keep the embankment serviceable. Therefore, only the cases showing rotational failure of clay foundation will be considered for subsequent presentation and analysis of results.

![Shear stresses at clay-reinforcement interface for the shallow clay layer.](image)

Figure 4-8: Shear stresses at clay-reinforcement interface for the shallow clay layer.

### 4.6 Effect of Undrained Shear Strength

Figure 4-9 to Figure 4-11 show the variation between the undrained shear strength ($S_{UO}$) of clay available at the interface and the critical height of the embankment ($H_{CRITICAL}$) for the cases $m_c = 0$ (undrained shear strength constant with depth), $m_c = 1$ and $m_c = 2$ (undrained shear strength increasing with depth), respectively. It is to be noted that these
plots are for an embankment with slope equal to 1.8 and reinforcement stiffness \((J)\) equal to 5724 kN/m. It was found that \(H_{\text{critical}}\) increased with increase in \(S_{UO}\) and the non-linearity of the relationship between \(H_{\text{critical}}\) and \(S_{UO}\) increases as \(m_C\) increases. The rate of increase in \(H_{\text{critical}}\) with increasing \(S_{UO}\) was greater for smaller depths of clay, which can be due to the transition of deformation mechanism from a rotational failure for a deep clay layer to a translational failure for a shallow clay layer.

\[m_C = 0\]

\(S_{Emb} = 1.8\)
\(J = 5724 \text{ kN/m}\)

**Figure 4-9:** \(S_{UO}\) vs. \(H_{\text{critical}}\) for clay with \(m_C = 0\).
Figure 4-10: \( S_{UO} \) vs. \( H_{CRITICAL} \) for clay with \( m_c = 1 \).

Figure 4-11: \( S_{UO} \) vs. \( H_{CRITICAL} \) for clay with \( m_c = 2 \).
4.7 Effect of Embankment Slope

The embankment slope ($S_{Emb}$) was varied from 1.5 to 3.0, in order to incorporate its effect on the $H_{CRITICAL}$ and $T_{MOB}$. Figure 4-12 show the plots between $S_{UO}$ vs. $H_{CRITICAL}$ for $S_{Emb}$ equal to 1.5, 1.8, 2.5 and 3.0. The differences of geometry cause variation of both magnitude and pattern of deformation. As $S_{Emb}$ increased from 1.5 to 3 (i.e., embankment slope became gentler) the circular slip surface was seen passing through the deeper clay layers. It can be seen from Figure 4-12 as $S_{Emb}$ increases, the embankment becomes more stable and hence the $H_{CRITICAL}$ increases. However, if the depth is sufficiently small, additional constraints are introduced on the lateral deformation of clay foundation and greater benefits in terms of higher $H_{CRITICAL}$ are observed.
Figure 4-12: $S_{U0}$ vs. $H_{CRITICAL}$ for different $S_{Emb}$ values.
4.8 Effect of Reinforcement Stiffness

The effect of reinforcement stiffness on $H_{CRITICAL}$ and $T_{MOB}$, was studied by changing the original reinforcement stiffness ($J$) to $J/4$, $J/2$ and $2J$ for the previous analyses i.e., $J$ value was changed from 5724 to 1431, 2862 and 11448 kN/m, respectively. Figure 4-13 show the plot between $S_{UO}$ vs. $H_{CRITICAL}$ for different reinforcement stiffness values. Comparing these plots it can be seen that upon lowering the reinforcement stiffness, the value of $H_{CRITICAL}$ reduces. It was also observed that for a given $m_{C}$ value, as the reinforcement stiffness increases transition is seen in the deformation mechanism from rotational slip failure to direct shear failure for shallower clay deposits.
Figure 4-13: \( S_{uo} \) vs. \( H_{critical} \) for different \( J \) values.

In order to increase \( H_{critical} \), \( J \) has to increase and at the same time \( T_{mob} \) must also be high. Figure 4-14 shows the plot between Reinforcement Stiffness (\( J \)) vs. \( H_{critical} \) for
the analyses with $D = 10\ \text{m}$, $S_{UO} = 8\ \text{kPa}$, $m_{C} = 2\ \text{kPa/m}$ and $S_{Emb} = 1.8$. Figure 4-15 shows the plot between Reinforcement Stiffness ($J$) vs. $T_{MOB}$ for the same analyses. It can be seen from Figure 4-14 and Figure 4-15 that increasing $J$, there is a small increase in $H_{CRITICAL}$ but large increase in $T_{MOB}$. Therefore if high $J$ reinforcement is used, it also needs to be high strength reinforcement. Hence moderate stiffness and strength results in optimal design.

![Figure 4-14: Reinforcement Stiffness ($J$) vs. $H_{CRITICAL}$.](image)

![Figure 4-15: Reinforcement Stiffness ($J$) vs. $T_{MOB}$.](image)
4.9 Mobilized Tension in the Reinforcement

Figure 4-16 shows the plot between the critical height of the embankment ($H_{CRITICAL}$) and the tension mobilized in the reinforcement ($T_{MOB}$) for different undrained shear strengths ($S_{UD}$) of clay available at the interface. Results show good correlation between $H_{CRITICAL}$ and $T_{MOB}$ for the high stiffness geosynthetic reinforcement (5724 kN/m), which can be attributed to the limited available shear strength at the clay-reinforcement interface. It can be seen from Figure 4-16 that varying $S_{Emb}$ had little effect on the $H_{CRITICAL}$ vs. $T_{MOB}$ plot. Next the reinforcement stiffness was varied from the original case ($J = 5724$ kN/m); the $T_{MOB}$ for different reinforcement stiffness values is given by the plot shown in Figure 4-17.

![Figure 4-16: $H_{CRITICAL}$ vs. $T_{MOB}$ plot for $J = 5724$ kN/m.](image-url)
It is to be noted that the tension mobilized in the reinforcement reduces with the decrease in the reinforcement stiffness. This is because the reinforcement with lower stiffness value has to deform more in order to mobilize the same amount of tension as compared to stiffer reinforcement. Figure 4-18 and Figure 4-19 shows the shear stresses at soil-reinforcement interfaces for the analysis 14-14-1-1.8-J with reinforcement stiffness $J$ equal to 11448 kN/m and 1431 kN/m respectively. It can be seen from these two figures that stiffer reinforcement induces higher shear stresses, tending to induce more tension, from both the embankment and the foundation interfaces. The shear stress results obtained are in accordance with the results documented by Hird and Kwok (1990).
Figure 4-18: Shear stresses at soil-reinforcement interfaces for $J = 11448$ kN/m.

Figure 4-19: Shear stresses at soil-reinforcement interfaces for $J = 1431$ kN/m.
4.10 Critical Height of an Unreinforced Embankment

The gain in $H_{CRITICAL}$ due to reinforcement at the base of an embankment constructed on top of soft clay was studied by employing the serviceability criterion to unreinforced embankment. Figure 4-20 shows plots between $S_{UO}$ vs. $H_{CRITICAL}$ for $S_{Emb}$ equal to 1.5, 1.8, 2.5 and 3.0 for an unreinforced embankment. Comparing Figure 4-20 with Figure 4-12 it can be easily recognised that installing reinforcement at the base of an embankment increases $H_{CRITICAL}$. 
Figure 4-20: $S_{UO}$ vs. $H_{CRITICAL}$ for different $S_{Emb}$ values.
4.11 Suggested Design Procedure

Based on the results shown in Figure 4-9 to Figure 4-20 a simple design procedure is suggested as follows:

Obtain Critical Height of the Unreinforced Embankment

Estimate $H_{CRITICAL}$ based on the undrained shear strength of the clay foundation from Figure 4-20. If the required embankment height is less than unreinforced $H_{CRITICAL}$ then no reinforcement is required. However if the required embankment height is greater than unreinforced $H_{CRITICAL}$ then go to the next step.

Obtain Critical Height of the Reinforced Embankment

Estimate $H_{CRITICAL}$ based on the undrained shear strength of the clay foundation at the interface using Figure 4-9 to Figure 4-13. This $H_{CRITICAL}$ would correspond to the height of reinforced embankment, which upon construction over a clay deposit of a given undrained shear strength will not cause any serviceability issues.

Corresponding Mobilized Axial Strain in the Reinforcement

Once $H_{CRITICAL}$ is obtained, Figure 4-17 can be used to estimate the required tensile strength of the reinforcement. Based on the limited available shear strength at the clay-reinforcement interface, this is the tension that can be mobilized in the reinforcement of any given stiffness.

4.12 An Example

Suppose an 8 m high embankment with 1.8H: 1V is to be constructed over an 18 m deep clay deposit using high stiffness geotextile reinforcement ($J = 5000$ kN/m). The
undrained shear strength of clay deposit at the surface is 12 kPa and the rate of increase of undrained shear strength with depth is 1 kPa/m. The fill parameters are $\gamma_{bulk} = 16$ kN/m$^3$ and $\phi = 35^\circ$. Following the serviceability-based design approach,

Obtain $H_{CRITICAL}$: From Figure 4-13, for $S_{Emb} = 1.8H: 1V$ and $S_{UO} = 12$ kPa, $H_{CRITICAL}$ is equal to 6.5 m.

Estimate $T_{MOB}$: From Figure 4-16, for $H_{CRITICAL} = 6.5$ m, $T_{MOB} = 93.5$ kN/m.

Hence to ensure serviceability of the embankment, the embankment height should not be increased beyond 6.5 m, which is the critical height of the embankment. Even with a high stiffness geotextile in place, maximum safe mobilized tension in the reinforcement would only be 93.5 kN/m, which can be attributed to limited available shear strength at the clay-reinforcement interface. Hence, in order to achieve the desired embankment height, one would first need to incorporate a ground improvement technique that would increase the available shear strength of clay foundation (Leroueil et al. 1990).

### 4.13 Comparison of Present Method with Rowe and Soderman (1985b) Method

Figure 4-21 shows the geometry and material properties of a typical reinforced embankment problem on soft clay. Initially the Rowe and Soderman method will be applied to the problem stated herein. The results will be compared to the serviceability criterion results proposed in this study.
Figure 4-21: A typical reinforced embankment problem on soft clay.

Rowe and Soderman (1985b) defined the allowable compatible strain ($\varepsilon_a$) as the maximum strain developed in a reinforcement having negligible modulus prior to the collapse of the embankment. Based on finite element study they proposed a dimensionless parameters $\Omega$ defined as:

$$\Omega = \left( \gamma_{\text{bulk}} H_C / S_U \right) \left( S_U / E_U \right) \left( D / B_c \right)_e^2$$  \[4.2\]

where

- $H_C$ is the critical height of the unreinforced embankment;
- $E_U$ is the undrained young’s modulus of the soil;
- $B_c$ is the crest width of the embankment;
- $(D / B_c)_e$ is the effective depth to crest width ratio; and,

$$(D / B_c)_e = \begin{cases} 
0.2 & \text{for } D / B_c < 0.2 \\
D / B_C & \text{for } 0.2 \leq D / B_c \leq 0.42 \\
0.84 - D / B_c & \text{for } 0.42 < D / B_c \leq 0.84 \\
0 & \text{for } 0.84 < D / B_c
\end{cases}$$
$E_U$ is given by the relationship $E_U = 3G_U$; where $G_U = 62\text{p}''$. Therefore $E_U$ is approximately equal to 4830 kPa based on initial value of $G_U = 1610 \text{kPa}$. $(D / B_c)_{e}$ was found equal to 0.284 for $D / B_c = 0.555$.

**Case A:** $S_UO = 14 \text{kPa}; \; m_C = 0 \text{kPa/m}$

Critical height of the unreinforced embankment ($H_C$) was found equal to 5.2 m for Case A from Figure 4-20. Substituting the above values in Equation 4.2, $\Omega$ is equal to 0.0012. Using the graph between allowable compatible strain, $\varepsilon_a$, and dimensionless parameter, $\Omega$, presented by Rowe and Soderman (1985):

$$\varepsilon_a = 1.5 \% \quad \Rightarrow \quad T_{MOB} = 86 \text{kN/m}$$

Using proposed serviceability criterion:

$H_{CRITICAL} = 4.8 \text{ m}$ \hspace{1cm} [see Figure 4-9]

$T_{MOB} = 62.5 \text{kN/m}$ \hspace{1cm} [see Figure 4-16]

**Case B:** $S_UO = 14 \text{kPa}; \; m_C = 1 \text{kPa/m}$

For Case B the Critical height of the unreinforced embankment ($H_C$) was obtained equal to 6.3 m. Substituting the above values in Equation 4.2, $\Omega$ is equal to 0.0020. Using the graph the between allowable compatible strain, $\varepsilon_a$, and dimensionless parameter, $\Omega$, presented by Rowe and Soderman (1985b):

$$\varepsilon_a = 2.5 \% \quad \Rightarrow \quad T_{MOB} = 143 \text{kN/m}$$

Using proposed serviceability criterion:

$H_{CRITICAL} = 7.1 \text{ m}$ \hspace{1cm} [see Figure 4-10]

$T_{MOB} = 125.9 \text{kN/m}$ \hspace{1cm} [see Figure 4-16]

Comparing the results for cases A and B, it can be seen that the proposed serviceability criterion gives a lesser value of $T_{MOB}$ as compared to the design method proposed by Rowe and Soderman in 1985. Additionally the present design method gives the critical height of the reinforced and unreinforced embankment that can be constructed over a soft clay deposit.
4.14 Summary

Soil-reinforcement interaction mechanism is one of the important aspects to be considered for the design of reinforced embankments on soft clay. The parametric study has revealed that the soil-reinforcement interaction mechanism depends on the ratio of embankment height to the depth of the clay layer (H/D ratio). In the case of an embankment constructed over a deep clay deposit, the failure mechanism is similar to a slip circle and there is poor mobilization of tension in the reinforcement. However, if the embankment is constructed over a shallow clay layer, the soil-reinforcement interaction mode is similar to the failure mechanism in the direct shear test and mobilization of tension occurs approximately along the entire length of the reinforcement. In this case, the tension mobilized in the reinforcement is higher and therefore, the contribution of the reinforcement towards overall stability of the embankment is greater. For the cases where rotational failure of the clay foundation is more likely, the results presented in the study have shown that there is a direct relationship between the increment in embankment height and the rate of increase in horizontal displacement near the toe. A serviceability criterion proposed in this chapter can prevent the shear failure at the clay-reinforcement interface and the accompanying large deformations. Development of serviceability criterion was the main aim of the present research. Design charts were generated easily. A relationship can be established between the critical height of the embankment and the tension mobilized in the reinforcement for a given reinforcement stiffness, which can be attributed to the limited available shear strength at the clay-reinforcement interface. Tension mobilized in the reinforcement increases with the increase in the reinforcement stiffness because stiffer reinforcement induces higher shear stresses, tending to induce higher tensions, from both the embankment and the foundation interfaces. However, if there is a need to construct an embankment of height greater than the critical height proposed for the available undrained shear strength, then other options such as wick drains in combination with geosynthetic reinforcement, use of light weight fill or stage construction should be considered to limit the deformation of the clay foundation. The increased degree of consolidation of the clay foundation achieved during the embankment construction in case of wick drains would ensure
higher available shear strength of clay foundation (Sharma and Bolton, 2001). The outcome of the present study is a simple and versatile design procedure for reinforced embankments on soft clay that may eliminate some of the inherent limitations of a limit equilibrium-based design process.
5 RESULTS VALIDATION: FIELD COMPARISON

5.1 Introduction

Measurements obtained from the previous studies involving full-scale instrumented test embankments provide valuable data for understanding embankment behaviour, investigating the validity of theories and assumptions used for analysis and design, and the development of improved design methods (Rowe et al., 2001). In order to validate the suggested serviceability-based design procedure, the results from the present parametric study was compared with results of two previously published field trials. The first field comparison was based on the test embankment constructed to failure at Sackville, New Brunswick, Canada. The second field comparison was based on a fabric reinforced levee test section at Plaquemine Parish, Louisiana, U.S.A.

5.2 Sackville Test Embankment

A fully instrumented geosynthetic reinforced test embankment was constructed to failure on a soft clay deposit near Sackville, New Brunswick, Canada. The behaviour of the Sackville test embankment has been described in detail in the literature by Rowe et al. (1991) and Rowe and Hinchberger (1998), hence only the necessary details will be repeated here. Figure 5-1 shows the average soil strength profile obtained from in situ cone penetration and field vane test reported by Hinchberger and Rowe (2003). It can be noted from the soil strength profile that the available shear strength ($S_{UO}$) at the clay-reinforcement interface is approximately 18 kPa and the rate of increase of undrained shear strength ($m_c$) with depth is 2 kPa/m.
The geometry of the original Sackville test embankment was not symmetric along the centre line, with berm constructed on one side. However to get an approximate result an embankment of similar geometry modelled from the centre line was constructed on the Sackville foundation soil. A finite element model of the reinforced embankment constructed over the Sackville foundation soil is shown in Figure 5-2. An embankment of intended 1H:1V side slope was constructed up to 9.5 m high on the foundation soil under undrained conditions. The foundation soil was assumed to have an impermeable rigid boundary with slip interface at a depth of 14 m.
Table 5-1 gives the embankment fill and geotextile properties used in the Sackville test embankment as documented by Rowe and Hinchberger (1998). The properties of the first 0.7 m of the fill were different from those of the remaining fill. The reinforcement in the present analysis was placed directly on the clay foundation and the reinforcement stiffness was taken as 1920 kN/m.

Table 5-1: Embankment fill and geotextile properties
(Rowe and Hinchberger, 1998)

<table>
<thead>
<tr>
<th>Embankment fill</th>
<th>First 0.7 m</th>
<th>Remainder of fill</th>
<th>Geotextile</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>0</td>
<td>17.5</td>
<td>ε_f 13 %</td>
</tr>
<tr>
<td>φ’</td>
<td>43</td>
<td>38</td>
<td>J 1920</td>
</tr>
<tr>
<td>ψ</td>
<td>8</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>γ_bulk</td>
<td>18.0</td>
<td>19.6</td>
<td></td>
</tr>
<tr>
<td>ν</td>
<td>0.35</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5-3 shows the plot between the horizontal displacement near the toe vs. increase in unreinforced embankment height. The failure height of an unreinforced embankment constructed on the Sackville foundation was found to be 7.1 m as seen from the
Rowe et al. (2001) reported that the clay foundation yielded at an unreinforced embankment height of 6.25 m. The failure was documented to be of a progressive nature and construction was stopped when very rapid movements were observed at a height of 7.25 m.

![Unreinforced Embankment Failure height, $H_f = 7.1$ m](image)

**Figure 5-3:** Horizontal displacement near the toe vs. increase in unreinforced embankment height.

Rowe and Hinchberger (1998) reported that the reinforced section of the Sackville test section reached its maximum net embankment height at a fill thickness of approximately 8.2 m. As the fill was advanced from a thickness of 5.7 m to a thickness of 8.2 m, the maximum reinforcement strain increased from 5.4 % to 8.6 % (Rowe and Gnanendran, 1994). The reinforcement strain of 5.4 % correspond to 82 kN/m tensile force and reinforcement strain of 8.6 % correspond to 146 kN/m, which was obtained from axial load versus elongation curve (Hinchberger, 1996). At a fill thickness of 8.2 m, the embankment continued to deform under conditions of approximately constant effective stress. Horizontal displacements near the toe of the reinforced embankment for the present finite element analysis are shown in Figure 5-4. The failure height of the reinforced embankment was found to be 8.1 m from Figure 5-4. The embankment height ($H$) was plotted vs. $R$ near the toe of the reinforced embankment in Figure 5-5. The reinforcement strain for the failure height of 8.1 m was found to be equal to 6 %.
(90 kN/m) from Figure 5-5. A close approximation was obtained by using a simple finite element model constructed on the Sackville foundation.

From Figure 5-5 the value of $H_{CRITICAL}$ was found equal to 7.7 m and $T_{MOB}$ was equal to 60 kN/m. Thus if the embankment would have been constructed up to 7.7 m height, the embankment would have remained stable and serviceable. The tension mobilized in the reinforcement would then be equal to 62 kN/m that corresponds to 4% strain.

Figure 5-4: Horizontal displacement near the toe vs. increase in reinforced embankment height.

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Figure 5-5: Plot between embankment height ($H$) vs. $R$ near the toe of the embankment.

Comparison with the Design Charts:

Figure 5-6 and Figure 5-7 show design plots for $D = 14$ m, $S_{UO} = 8$ to 20 kPa, $m_c = 2$, $S_{Emb} = 1.5$ and $J = 2000$ kN/m. It can be seen from Figure 5-6 that for $S_{UO}$ equal to 18 kPa, the value of $H_{CRITICAL}$ is 9.2 m. The $\gamma_{bulk}$ used for the design charts is 16 kN/m$^3$; therefore for $\gamma_{bulk}$ equal to 19.6 kN/m$^3$ the corresponding $H_{CRITICAL}$ is 7.55 m. The $T_{MOB}$ was found equal to 70 kN/m from Figure 5-7.
Figure 5-6: $S_{UO}$ vs. $H_{CRITICAL}$

Figure 5-7: $H_{CRITICAL}$ vs. $T_{MOB}$
5.3 Fabric Reinforced Test Embankment, Plaquemine, Louisiana, U.S.A.

Fowler and Edris Jr (1987) described the results of an instrumented high strength woven geotextile fabric reinforced levee test section. The purpose of the test section was to determine the technical feasibility, construction practicality, and the economic feasibility of geotextile reinforced levees for future projects involving very soft, highly compressible foundations. The Levee test section was instrumented with settlement plates, slope inclinometers, and piezometers. Resistive potentiometers and strain gauges were attached to the embedded fabric. It was proposed to raise the existing levee cross section from elevation +2.4 m NGVD (National Geodetic Vertical Datum) to a proposed elevation of +4.4 m NGVD as shown in Figure 5-8. The actual and the assumed undrained shear strength profile are also shown in Figure 5-8. The fill material used for the embankment construction was sand with unit weight of 16 kN/m$^3$. The embankment was constructed with slope equal to 1V:3H on the side of the original levee. Geotextile with reinforcement modulus ($J$) equal to 4200 kN/m was used to reinforce the new levee section.
Fowler and Edris Jr (1987) reported all movements in the top 9.1 m of the foundation material. Maximum horizontal displacements indicated by the inclinometers varied from 18 cm to 33 cm. The results from all the strain gauges were consistent and the strain in the transverse direction varied from 0.5% to 2.0%. From the load strain plot, for a range of 0.5 to 2 percent strain, the fabric load of about 44 to 105 kN/m width was reported. From the above results it can be concluded that the lateral displacements were controlled at a levee height of 4.1 m above ground level. The construction of levee was reported to be technically and economically successful.

Using Serviceability Criterion:

Obtain $H_{\text{CRITICAL}}$: For $D = 20$ m; $S_{UO} = 7.2$ kPa; $m_C = 1$; $S_{Emb} = 3$ and $J = 4200$ kN/m the $H_{\text{CRITICAL}}$ was found equal to 4.5 m from Figure 5-9.
Obtain $T_{MOB}$: Corresponding to $H_{CRITICAL} = 4.5$ m, the $T_{MOB}$ can be estimated from Figure 5-10, which is 50 kN/m. From serviceability criterion approach, the calculated critical height of the embankment is 4.5 m. The final height of the constructed levee was 4.1 m and hence it remained serviceable. The calculated tension mobilized in the reinforcement is also close to the measured reinforcement tension in the original levee.

![Figure 5-9: $S_{UO}$ vs. $H_{CRITICAL}$](image)

![Figure 5-10: $H_{CRITICAL}$ vs. $T_{MOB}$](image)
5.4 Summary

The serviceability-based design approach has been validated by comparing the results of the parametric study with published data from two instrumented field trials. The failure heights of the unreinforced and reinforced embankment on Sackville foundation soil and reinforcement strains, obtained by the finite element analyses were in good agreement with the field values documented by Rowe et al. (1991). Based on serviceability-based design approach, $H_{\text{CRITICAL}}$ and $T_{\text{MOB}}$ were proposed for a reinforced embankment of given geometry and reinforcement stiffness, to remain serviceable on Sackville foundation soil. Another fabric reinforced embankment test section was reported to have remained serviceable after the completion of the embankment construction by Fowler and Edris Jr (1987). The given data was used to calculate $H_{\text{CRITICAL}}$ and $T_{\text{MOB}}$ from the design charts. It was found that the constructed height of the embankment was close to the critical height of the embankment and it remained serviceable. The tension mobilized in the reinforcement ($T_{\text{MOB}}$) was also close to the measured range in the field. These results show that these easy to use design charts incorporating serviceability-based design approach can be successfully used to design stable and serviceable reinforced embankments.
6 CONCLUSIONS

6.1 Summary

A simple and versatile design procedure has been developed for reinforced embankments on soft clay using finite element modelling. A numerical parametric study using a coupled non-linear elastoplastic finite element program was carried out to explore the behaviour of reinforced embankments on soft clay. One of the objectives of the parametric study was to relate the lateral deformation of the clay layer to the amount of tension mobilized in the reinforcement. The effects of the depth of clay layer, the undrained shear strength available at the clay-reinforcement interface, the rate of increase of undrained shear strength of the clay layer with depth, the embankment slope and the stiffness of the reinforcement were investigated. A novel serviceability criterion was developed based on the results of the parametric study. This criterion limits the lateral deformation of the clay foundation at the toe of the embankment by limiting the allowable mobilized tension in the reinforcement. Based on this criterion, a simple procedure for the evaluation of the efficiency of soil-reinforcement interface for reinforced embankments on soft clays was proposed. This procedure limits the magnitude of mobilized tension in the reinforcement calculated by a limit equilibrium-based design method on the basis of lateral deformation of the clay foundation near the toe of the embankment and ensures serviceability of the embankment.

Scientific method (Barbour and Krahn, 2004), which outlines the four processes: Observe, Measure, Explain and Verify was rigorously adopted in the present study. Basic element tests were performed on all the building block elements that became the part of the final finite element model. Comparing the finite element results with known theoretical solutions validated the stiffness matrix formulation and solver of the software and constitutive soil models. The centrifuge model tests (Sharma, 1994) were back-analysed at prototype scale to validate the finite element program SAGE-CRISP.
Version 4.0 (Crisp Consortium, 2003). It was found that the model gave reasonably accurate estimates of the deformation of the clay foundation as well as the magnitude and the distribution of excess pore water pressures in the clay foundation.

Several finite element analyses were performed to establish the serviceability criterion that will have a general applicability for all the cases included in the parametric study. It was found that rate of change of horizontal displacement is a function of change (increase) in embankment height. After a certain height of the embankment (denoted by \( H_{\text{CRITICAL}} \)) is reached, the rate of horizontal displacement of the clay foundation with respect to the rate of embankment construction increases rapidly. This indicates the onset of plastic yielding in clay foundation that will eventually lead to failure if embankment construction is continued. If the embankment were to remain serviceable, this rate of horizontal displacement at the toe of the embankment must be kept below a certain value. Therefore, a parameter ‘\( R \)’ defined as \( R = \frac{\Delta(\delta_h)}{\Delta H} \) was used to limit the rate of change of horizontal displacements with respect to embankment construction. It was seen that the embankment remains serviceable at \( R \)-values of \( \leq 0.1 \). The critical height of the embankment (\( H_{\text{CRITICAL}} \)) was obtained for the various parametric analyses for \( R = 0.1 \). This \( H_{\text{CRITICAL}} \) corresponds to the height of a reinforced embankment that can be constructed over a clay deposit of a given undrained shear strength without causing serviceability problems. Furthermore, for each of the parametric analysis mobilized tension in the reinforcement (\( T_{\text{MOB}} \)) corresponding to \( R = 0.1 \) was obtained.

In order to validate the proposed serviceability criterion, the results from the present parametric study were compared with the two field trials reported previously by Rowe et al. (1991) and Fowler and Edris Jr (1987), respectively. All material parameters chosen for the validation of the serviceability criterion were obtained from the published data. In the case of the field trial reported by Rowe et al. (1991), application of the proposed serviceability criterion gave a value for the critical height of the embankment (\( H_{\text{CRITICAL}} \)) that was close to the pre-failure embankment height. For the case of field trial reported by Fowler and Edris Jr (1987), application of the proposed serviceability criterion gave a value for the critical height of the embankment (\( H_{\text{CRITICAL}} \)) that was close to the design
embankment height. In addition, for this field trial, the tension mobilized in the reinforcement ($T_{MOB}$) given by the serviceability criterion was close to measured values of tension in the reinforcement.

### 6.2 Conclusions

Based on the results of the parametric study, the following conclusions can be drawn:

1. Soil-reinforcement interaction mechanism depends on the ratio of embankment height to the depth of the clay layer ($H/D$ ratio). In the case of an embankment constructed over a deep clay deposit, the failure mechanism is more like a slip circle and there is poor mobilization of tension in the reinforcement. However, if the embankment is constructed over a shallow clay layer, the soil-reinforcement interaction mode is similar to direct shear failure and almost the entire length of the reinforcement participates in the mobilization of tension. In this case, the tension mobilized in the reinforcement is higher and therefore, the contribution of the reinforcement towards overall stability of the embankment is greater.

2. A direct correlation exists between the increment in embankment height and the rate of increase in horizontal displacement of the clay layer underneath the toe of the embankment. Based on this correlation, a serviceability criterion was proposed that limits the height of the embankment with respect to the rate of horizontal displacement in the clay layer underneath the toe of the embankment.

3. Using the proposed serviceability criterion, it was possible to establish a correlation between the critical height of the embankment and the mobilized tension in the reinforcement for clay layers that were deeper than 4 m. This correlation was found to be independent of the undrained shear strength profile of the clay layer and the side slope of the embankment. This can be attributed to the fact that for all the clay layers deeper than 4 m, no slip was observed at the clay-reinforcement interface. In other words, the deformation mechanism was
rotational with deformations concentrated along a circular slip surface passing through the clay layer.

4. Tension mobilized in the reinforcement was found to increase with the increase in the reinforcement stiffness. This is because a stiffer reinforcement attracts higher shear stresses at its interfaces with the embankment and the clay foundation, thereby mobilizing higher tension.

5. The benefit of increasing the reinforcement stiffness excessively follows a pattern of diminishing returns. A stiff reinforcement attracts more tension but does not provide an appreciable increase in the height of the embankment. It is possible to achieve an economical design using a moderately stiff and moderately strong reinforcement.

6. The outcome of the present study is a simple and versatile design procedure for reinforced embankments on soft clay with a built-in serviceability criterion that may eliminate some of the inherent limitations of a limit equilibrium-based design process.

6.3 Suggestion for Future Work

The $T_{MOB}$ was plotted against the $H_{CRITICAL}$ for different $J$ values in Figure 4-17. It was found that the above design chart did not normalize very well upon plotting $(T_{MOB}/J)$ vs. $H_{CRITICAL}$ or $(T_{MOB}/J)$ vs. $S_{UO}$. Preliminary investigation has revealed that $(T_{MOB}/J)$ if plotted against undrained shear strength mobilized ($S_{UMOB}$) at $R = 0.1$ gives a unique curve as shown in Figure 6-1. The $S_{UMOB}$ was obtained by first selecting the failure surface from the contours of maximum shear strains in the clay foundation and then superimposing this failure surface on the contours of mobilized shear strength in the clay foundation for embankment height equal to $H_{CRITICAL}$. The average $S_{UMOB}$ can then be obtained by taking weighted mean of the mobilized shear strengths along the failure surface. The results obtained from this preliminary investigation contradict those that...
can be obtained from a traditional limit equilibrium-based analysis. In case of a limit equilibrium analysis, there is no way of establishing relative contributions of the clay layer and the reinforcement towards the stability of the embankment. Most limit equilibrium-based design methods give a higher value of tension in the reinforcement if the mobilized shear strength of the clay foundation is reduced by means of a factor of safety. It is recommended that this hypothesis should be investigated further.

**Data set corresponds to:**

\[ D = 14 \text{ m} \]
\[ S_{UO} = 10, 14, 20 \text{ kPa} \]
\[ m_C = 0, 1, 2 \text{ kPa/m} \]
\[ S_{Emb} = 1.8 \]
\[ J = 1431, 2862, 5724, 11448 \text{ kN/m} \]

**Figure 6-1:** \( T_{MOB} / J \) vs. \( S_{UMOB} \) plot.
REFERENCES


