ELECTRON TEMPERATURE AND DENSITY MEASUREMENTS
IN A LOW PRESSURE PLASMA
WITH TOROIDAL MAGNETIC FIELD

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of
Master of Science
in the College of Arts and Science
University of Saskatchewan

by

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Saskatoon, Saskatchewan
August 1962

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ACKNOWLEDGMENTS

The author wishes to gratefully acknowledge the assistance and supervision given by Dr. H. M. Skarsgard in the execution of this project, and also to thank Mr. L. T. Shepherd for his kind assistance during all phases of this work.

Financial aid in the form of two N.R.C. Studentships is gratefully acknowledged.
ABSTRACT

Double probe measurement of electron temperature and density have been made in hydrogen and helium afterglows in the pressure range from 1.0 to 60 microns Hg. in the presence of a toroidal magnetic field. Criteria for proper probe operation in the magnetic field have been proposed and followed closely in the case of the measurements in helium.

The electron density and temperature decay rates have been determined in both hydrogen and helium. The most outstanding feature of these was the anomalously fast cooling observed in the early afterglow.

Electron densities have also been measured in an argon plasma under conditions employed in the plasma betatron experiment.
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A. INTRODUCTION

A plasma betatron has recently been constructed in this laboratory in which a plasma is produced in a torus and then a betatron field applied to accelerate the electrons in the gas to energies of a few Mev.

Since the operation of the betatron depends quite critically on the density of ionization in the torus at the beginning of the betatron cycle, it is important to be able to measure the density (electrons/cm$^3$) and so to be able to know the conditions under which the betatron is operating at any time. The electron temperature is also of some interest in this experiment.

Besides the measurement of electron density and temperature for the plasma betatron experiment it seemed interesting to try to measure these quantities during the afterglow of discharges in the same apparatus. In particular, a measurement of the decay of temperature in the afterglow was expected to yield useful data on electron interaction with neutral gas atoms. The low pressure at which the system could be operated was expected to give a temperature decay of conveniently long duration. The relatively high degree of ionization afforded by the large pulsed power input to the discharge was also expected to alleviate the measurement difficulties.

No previous measurements of electron temperature and density have been made under conditions similar to those in the plasma betatron apparatus. Consequently, there was little knowledge of the difficulties or results to be expected.
The method which was employed to measure the electron density and temperature involves the use of electric probes or Langmuir probes. While there are other techniques for measuring electron density there are no other known techniques for measuring electron temperature for conditions in the plasma betatron apparatus. Even the electric probe technique is far from satisfactory. The theory has only been worked out for use of these probes in plasma in the absence of magnetic fields whereas in the plasma betatron apparatus there are magnetic fields present. It was therefore necessary to determine conditions under which the electric probes could be expected to give intelligible results in a magnetic field.

Langmuir\(^2\) developed the basic theory of electric probes and made measurements on plasmas in discharge tubes as early as 1924. The probes used by Langmuir are now called single probes and suffer from the disadvantage that rather large currents are drawn from the plasma. This can, in some cases, cause a significant perturbation in the plasma itself. Thus, the measuring method sometimes affected the quantity being measured. This effect was largely overcome by Johnson and Halter in 1950. They developed a modified version of the Langmuir probe technique which is referred to as the "Langmuir Double Probe."\(^3\)

No revolutionary improvements in probe systems have been made since that time although many refinements in probe construction are possible. Probe theory has been improved somewhat by Bohn\(^4\) and other investigators.
The probe method for measuring ionization density and electron temperature consists basically of introducing a small probe into the plasma and measuring the current that flows to the probe as a function of the probe-to-plasma potential. As will be shown the electron current to the probe is a function of the electron temperature as well as the electron density. The positive ion current to the probe is a function of the density also. These two facts are the basis of temperature and density determinations by the Langmuir probe methods.
THE LANGMUIR PROBES

Because of the unsatisfactory state of the theory of electrostatic probes - especially when employed in the presence of a magnetic field, for which case no satisfactory analysis has yet been carried out, it is worthwhile to review reasonably thoroughly the present state of the theory. To begin with it is useful to point out some characteristics of a plasma which are intimately connected with the behaviour of the electric probes.

A. CHARACTERISTICS OF A PLASMA

A plasma has as one of its basic properties a strong tendency toward electrical neutrality. Electric fields tend to be excluded from the plasma except for a skin effect. If the number of electrons deviates appreciably from the number of ions in the plasma, the electric potentials set up will be many times larger than the thermal energy of the particles, and the particles will quickly move in such a way as to restore electrical neutrality. When electrical neutrality exists \( n_e \), the electron density, must be equal to \( n_i \), the ion density, for the case where the ions are singly ionized. The plasma will behave in such a way that this relationship is maintained.

Consider a plasma enclosed by a container of arbitrary shape whose walls are everywhere at the same potential. Also, assume that no power is being put into the plasma so that no new ions are being created. Since the electrons are much more mobile than the ions, the electrons would strike the wall at a much greater rate than the ions.
However, this will cause the plasma to move to a positive potential with respect to the walls. The resultant potential difference between the plasma and the walls will tend to retard the electrons from reaching the wall, and will grow to a value where the electron current to the wall will just equal the ion current to the wall. The electron and ion densities will then decay together at a rate mostly dependent on the ion mobility. The potential will take the form shown in Fig. 1, where \( V \) is the potential between the plasma body and the wall; \( V \) will usually be about three to six times larger than the electron energy in electron volts.

If the situation occurs where one part of the wall is at a different potential than another, the plasma will float at a potential somewhat positive with respect to the most positive wall area of significant size.

The thickness of the layer between wall and plasma across which the potential drop occurs is of major interest.

Let us consider a completely separate case where we have a plasma with a region where no positive ions are present. We limit ourselves to a situation where the electric field is everywhere parallel to the \( \mathbf{x} \)-axis. The electric potential \( U \) is then determined by Poisson's law which becomes

\[
\frac{d^2 U}{dx^2} = 4\pi n_e e
\]

where \( U \) is in e.s.u. If \( W \) denotes the potential energy of an electron equal to \(-eU\), then the change of \( W \) in crossing a slab of width \( x \) is given by

\[
\Delta W = -2\pi n_e e^2 x^2
\]
for the case where the electric field vanishes on one side of the slab. If we now denote by the symbol $h$ the value of $\alpha$ for which the absolute value of $\Delta W$ equals $\frac{1}{2} kT$, the mean kinetic energy per particle in one direction, we have

$$h = \left(\frac{kT}{4\pi n_e e^2}\right)^{\frac{1}{2}} = 6.90 \left(\frac{T}{n_e}\right)^{\frac{1}{2}}$$

where $h =$ Debye shielding distance (cm.)

$k =$ Boltzmann constant (ergs/°K)

$T =$ temperature (°K)

$e =$ electronic charge (e.s.u.)

The "Debye shielding distance" is the name given to the boundary layer or sheath which the plasma forms when in contact with a solid surface.

For ordinary laboratory plasma with say, $n = 10^{12}$ cm$^{-3}$ and $T = 2 \times 10^4$ the Debye shielding distance is equal to $10^{-3}$ cm. which is usually small compared to the plasma dimensions. The potential drop occurs across this thin layer while the body of the plasma will be at the same potential throughout.

**B. THE SINGLE PROBE**

The single probe method is based on the Boltzmann relation and the plasma sheath which the ionized gas forms around any conductor in the plasma. If a probe is immersed in the plasma and the plasma to probe voltage swept about the plasma potential, the current that flows to the probe will be found to have three fairly distinct regions $3$. (See Fig. 2)
FIG. 2.  Probe Current vs. Voltage.
(i) For large negative probe voltages the current to the probe will be practically independent of voltage and will consist of only positive ions. The electrons will not have sufficient velocity to reach the negative probe. The positive ion current may increase slightly at more negative voltages due to increase in the sheath thickness at the larger voltages.

(ii) The central part of the current vs. voltage characteristic is the region where the current changes rapidly with voltage and is the region from which the electron temperature is determined. In this region the electron current is determined by the velocity distribution of the electrons and for a Boltzmann distribution is given to a fairly good approximation by

$$ i_e = A J_0 e^{V_p/kT_e} $$

where

$$ A = \text{probe area (cm)} $$
$$ J_0 = \text{random space current density} $$
$$ V_p = \text{potential of probe with respect to plasma (a negative number)} $$

For a derivation of a more exact expression for the probe current see Appendix I.

Taking logarithms of (1)

$$ \ln i_e = \frac{eV_p}{kT_e} + \ln (AJ_0) $$

so that a plot of $\ln i_e$ vs. $V_p$ will yield a straight line of slope $\frac{e}{kT_e}$ giving the electron temperature.

(iii) As the voltage becomes more positive, a point will be reached where the current practically ceases to increase as the voltage is increased. This occurs when the potential is such that all the electrons which strike the sheath are collected. The current is then limited by the random electron space current in the plasma.
Ideally the electron density can be found from the saturation electron current. However the single probe method suffers from the fact that the saturation current tends to be rather large and may perturb the plasma. Unless particular care is taken to eliminate this perturbation both the temperature and electron density measurement may involve serious errors.

C. THE DOUBLE PROBE

The double probe consists of two probes connected by a variable voltage source. (See Fig. 3(a).) The probe circuit is isolated from ground so that no net current can flow to the probes. That is, the total number of electrons being collected must at all times equal the total number of ions being collected. The electron current can never exceed the ion current which is usually smaller by a factor of one hundred or more compared to the saturation electron current to a single probe. This limits the double probe to quite small currents and greatly reduces the likelihood of disturbing conditions in the plasma.

Consider the potential diagram in Fig. 3(b). Here both probes are at the same potential. If we consider a uniform plasma, then there will be no current in the external circuit and the ion current must equal the electron current to each probe. The probes will be floating at a potential called the "floating potential", such that most of the electrons are reflected back into the plasma by the electric fields in the sheath.

Fig. 3(c) shows qualitatively the situation which is obtained when probe 2 is made positive with respect to probe 1. The ion
currents will remain effectively the same as in Fig. 3(b) since both probes are in a potential well for the ions and all the ions that strike the probe sheath should be collected. The total electron current must equal the total ion current, probe 2 will simply collect a larger share of the electrons than probe 1. If the applied voltage is large the point will be reached where all the electron current flows to one probe. The other probe will be at a potential such that electrons cannot reach it, but even in this case the ion currents remain the same as before, since both probes will still be potential wells for the ions. This case of a large applied voltage causes the saturation current for the double probe to flow in the external circuit.

D. DOUBLE PROBE ANALYSIS

In the following analysis some additional symbols are used. These are:

\[
\begin{align*}
J_{01} & = \text{electron space current in the plasma adjacent to probe 1} \\
J_{02} & = \text{electron space current in the plasma adjacent to probe 2} \\
T_e & = \text{electron temperature} \\
V_1 & = \text{probe to plasma potential for probe 1} \\
V_2 & = \text{probe to plasma potential for probe 2} \\
V_c & = \text{possible small variation in plasma potential due to inhomogeneities.}
\end{align*}
\]

The potential diagram for the general case on which the analysis is based is given in Fig. 3(d).

Since the net current to the system must be zero

\[
i_{+1} + i_{+2} = 2i_p = i_e + i_{e2}
\]
If we substitute for \( i_{e_1} \) and \( i_{e_2} \) from equation (1)

\[
\xi i_p = A_1 j_{o_1} e^{\phi V_1} + A_2 j_{o_2} e^{\phi V_2}
\]

(3)

where \( \phi = \frac{e}{kT} \), \( V_1 \) and \( V_2 \) are negative numbers in all cases.

From Fig. 3(d) we obtain the following relation

\[
V_1 + V_c = V_2 + V_d
\]

or \( V_1 = V_2 + V_d - V_c \)

(4)

Substituting (4) into (3) we have

\[
\xi i_p = A_1 j_{o_1} e^{\phi (V_2 + V_d - V_c)} + A_2 j_{o_2} e^{\phi V_2}
\]

We then divide through by \( i_{e_2} \)

\[
\frac{\xi i_p - 1}{i_{e_2}} = \frac{A_1 j_{o_1}}{A_2 j_{o_2}} e^{\phi (V_d - V_c)} + 1
\]

or

\[
\frac{\xi i_p - 1}{i_{e_2}} = \frac{A_1 j_{o_1}}{A_2 j_{o_2}} e^{\phi (V_d - V_c)}
\]

(5)

If we take logarithms we have

\[
\ln \left( \frac{\xi i_p - 1}{i_{e_2}} \right) = \ln \left( \frac{A_1 j_{o_1}}{A_2 j_{o_2}} e^{-\phi V_c} \right) + \phi V_d
\]

(6)

We assume that the first term on the right will be a constant during the time of any one measurement. A plot of \( \ln \left( \frac{\xi i_p - 1}{i_{e_2}} \right) \) vs. \( V_d \) will then yield a straight line of slope \( \phi = \frac{e}{kT} \) from which the temperatures can be found.

Consideration of equation (6) shows that the slope of the line should be unaffected by differences in probe areas, electron random currents, or plasma potential differences as long as these quantities do not change during the time interval when the probe characteristic is being obtained.
Plotting $\ln\left(\frac{\xi}{i_{ez}} - 1\right)$ vs. $V_d$ is a rather laborious method of determining the electron temperature. Johnson and Malter have worked out simpler methods. The method which they call the "equivalent resistance method" is derived as follows:

Equation (5) can be written

$$\frac{\xi}{i_{ez}} - 1 = \beta e^{\phi V_d}$$

where $\beta = \frac{A_1 J_0}{A_2 J_0} e^{-\phi V_c} = \text{constant}$

Then $i_{ez} = \frac{\xi}{\beta e^{\phi V_d} + 1}$

If the derivative with respect to $V_d$ is taken and evaluated at $V_d = 0$ we have

$$\left(\frac{dI_{ez}}{dV_d}\right)_{V_d = 0} = \frac{\xi}{\beta (\beta + 1)^2} \phi$$

Since $\phi = \frac{e}{kT}$

$$\frac{kT e}{\xi} = \frac{e}{\beta (\beta + 1)^2} \left(\frac{dV_d}{dI_{ez}}\right)_{V_d = 0}$$

where $\frac{3kT}{2 e}$ is the thermal energy of the electrons in electron volts.

Before we can use the above equation we must have a value for $\beta$.

From equation (7) when $V_d = 0$, we have

$$\beta = \left(\frac{\xi}{i_{ez}} - 1\right)_{V_d = 0}$$

If the probes are of equal area and the plasma is homogeneous $\xi$, the total ion current to the probes, should be just double the electron current to one probe, which is $i_{ez}$, when $V_d = 0$. We assume that this is the case so that $\beta = 1$ at all times.
Equation (8) then becomes

\[ \frac{kT_e}{e} = \frac{\sum i_p}{4 \left( \frac{dV_i}{di} \right)} \quad V_i = 0 \] (9)

which is in a useful form.

Johnson and Malter have also derived a relation from which the ion density may be calculated. They assume that \( \sum i_p \), the ion current collected, depends only on

(i) The sheath area of the probes
(ii) The random space current density of the ions

and draw up a solution on this basis.

Bohm, in a later analysis, has shown that the density is given more accurately by

\[ n = \frac{i_{+\perp}}{0.4 A e \sqrt{2 kT_e}} \] (10)

where \( i_{+\perp} = \frac{1}{2} \sum i_p \).

The form of this equation indicates that the ion current to the probe depends on the electron temperature. This is because there is some penetration of the electric field into the plasma beyond the ordinary sheath and the magnitude of the potential drop beyond the sheath depends on the electron temperature.
A. CRITERIA FOR PROPER PROBE MEASUREMENTS

The usual criteria for accurate measurements by means of probes are as follows:

(1) There should be no ionization occurring in the sheath.

The double probes float at a potential which is always negative with respect to the plasma. As electrons move into the sheath they experience a retarding force which stops all but the most energetic ones from reaching the probe. The average energy of electrons within the sheath is therefore less than that of electrons in the main plasma, so that electrons will not be very effective in producing ionization within the sheath.

The ions are accelerated toward the probe by the electric fields in the sheath and might be effective in producing ionization were it not for the fact that the mean free path of the ions is usually very much larger than the sheath thickness. The probability of a collision thus becomes rather low whenever the pressure and density of the plasma are such as to make the ion mean free path large compared to the sheath thickness.

(ii) There should be no secondary emission at the probe surface.

Secondary emission from surfaces can occur either by bombardment by electrons or ions. Since the electrons reach the probe with low velocities they will be unlikely to produce secondary emission. The ions, on the other hand, reach the probe with energies which may be as high as 30 or 40 ev. Fortunately the yield of secondary electrons per ion is
low, of the order of $10^{-2}$ so that the error introduced by this should be small.

The photoelectric effect could also be a potential source of error. The resonance lines of the gases used are in the range from 10 to 20 e.v. Photons of these energies are quite effective in removing electrons from the probes, having a yield as high as $10^{-1}$ per photon incident. However, consideration of the total energy input to the plasma shows that even if all the energy went into photons the effect would just be noticeable. Since by no means all the energy can go into production of photons it would seem that the photoelectric effect is not a serious source of error in these experiments.

(iii) The sheath thickness should be small compared to the probe radius.

The previous analysis of the probe characteristics in chapter II assumes the ion current to each probe to remain constant. One factor which tends to make the ion current vary is change in sheath thickness. The sheath thickness does vary with the applied voltage and this will in turn cause a change in the effective collecting area of the probe. The effect can be made insignificant by ensuring that the sheath thickness is small compared to the probe radius so that a fractional change in the sheath thickness will constitute only insignificant change in the total sheath area.

(iv) The sheath thickness plus the probe radius should be small compared to the particle mean free path.

The probe method assumes that the number of ions which strike the
probe is governed by the random motion of the ions. If this is to be true, the probe must cause a negligible disturbance in the ion space distribution. A rather naive treatment of this problem follows.

Consider a spherical collector immersed in a gas of particles with mean free path $\bar{\lambda}$. Ions striking the probe will on the average originate from a point whose distance from the probe is also $\bar{\lambda}^{(4)}$. Now consider the density at a point of distance $\bar{\lambda}$ from the probe. The density here, if the probe were not present would be made up on ions passing the point from all directions. When the probe is present it cuts off all particles moving toward the point in a cone of solid angle equal to $\frac{\text{probe cross-sectional area}}{4\pi \bar{\lambda}^2} = \frac{\pi a^2}{4\pi \bar{\lambda}^2} = \frac{1}{4} \left(\frac{a}{\bar{\lambda}}\right)^2$

The density is reduced by the above factor which indicates that $a$ must be smaller than $\bar{\lambda}$ if the density is not to be seriously reduced in the neighborhood of the probe. A more sophisticated treatment is given by Bohm.

(v) Plasma oscillations should not be present.

Plasma oscillations are present in many plasmas. These tend to cause the plasma to be lost to the walls more quickly than would be expected and also cause the density and energy of the particles to vary in space and time in an unpredictable manner.

Plasma oscillations are of three main types: electrostatic, electromagnetic and magnetohydrodynamic.

These types of waves have all been analyzed in certain cases and detected in plasmas.

The presence of oscillations would probably make the probe
measurements of density and temperature unreliable, especially since
the oscillations might make the velocity distribution non-Maxwellian.
The probe theory assumes and is dependent on a Maxwellian distribution
of velocities. The presence of oscillations is not easy to detect, and
in any case, not much can be done to eliminate them.

(vi) There should be no magnetic field in the plasma.

The probe theory, as discussed so far, applies strictly only to
the case of a plasma in the absence of a magnetic field. Introduction
of a magnetic field has two main effects on the plasma that are of
importance in probe measurements:

(i) Diffusion of particles across the magnetic
field is hindered. This means that the current the probe can draw
without disturbing the plasma is reduced.

(ii) The particles are constrained to move in
helical paths whose radii are the Larmor radii of the particles.
This has the effect of reducing the mean free path perpendicular to
the magnetic field to approximately one Larmor radius.

The motion of the electrons and ions along the magnetic field is
unaffected. Also, the random space currents and velocity distributions
should remain the same as for the case where no magnetic field is
present.

It seems reasonable to assume that if the probe is made small
enough, the motion of the ions and electrons in its neighborhood would
approximate straight line motion even though their motion as a whole were
actually helical. In this case the operation of the probe should
certainly be unhindered by the magnetic field.
We have introduced an additional criterion which is

\[ a + h < r_c \]

where \( r_c \) is the Larmor radius of the particles in the magnetic field.

The above criterion is often impossible to meet in practice and is probably stricter than necessary. In these experiments the criterion was satisfied if possible, however, the probes appear to work equally well even if the probe plus sheath radius is a few times larger than the Larmor radius of the particles.

Criteria such as \( h < r_c \) have been suggested by other workers but no real tests or their validity have been made so far as we can ascertain and the system being used for this project is unsuitable for such tests. It is anticipated that some work along these lines may be done in the near future.

A suitable theory for the electric probes in a magnetic field would require a complete and self-consistent analysis of the potentials, charged particle densities and current densities in the neighborhood of the probe. No such analysis has yet been carried out.
A. EXPERIMENTAL

(i) The Plasma Container

The plasma betatron apparatus employed is shown schematically in Fig. 4. The plasma is contained in a pyrex glass torus of major dia. = 38 cm and minor dia. = 6 cm. Provision is made for introducing probes and pressure gauges, etc. through ground glass joints. The inner surface of the torus was coated with a thin, high resistance graphite coating for all the measurements reported here except for those in helium, which were made in an uncoated torus.

The torus is evacuated by means of a mercury vapour diffusion pump and a mechanical pump in series. A liquid nitrogen cold trap is used at all times to prevent pump vapours from getting into the torus. The lowest attainable pressure was about $5 \times 10^{-6}$ mm. Hg, which is approximately a factor of 100 lower than the lowest pressure used in any experiment.

Pressure measurement was by means of a calibrated ionization gauge in the range 0 - 5 microns. Above 5 microns a McLeod gauge was used. The McLeod gauge had an associated cold trap to prevent Hg. vapour from getting into the system.

(ii) Gas Purity

The characteristics of a plasma are rather sensitive to impurities. These impurities may get into the discharge either through leaks in the walls of the vacuum chamber or they may be removed from the inside walls of the chamber by bombardment during the discharge. For best results the vacuum chamber should be completely free from leaks and should be constructed so that it may be baked out over a period of days to remove all foreign atoms from the walls.
FIG. 4. The Betatron Apparatus
The present system was designed to function as a plasma betatron. In the betatron experiment, gas purity is not critically important so no facilities for bake-out were incorporated.

The method used in this experiment, to provide as high a gas purity as possible without baking, was to pump the system continuously over a period of days at the minimum pressure obtainable, which was about \(5 \times 10^{-6}\) mm. Hg. Discharge cleaning was also employed during this period, that is, a few hundred discharges were initiated to remove as much of the foreign gases from the walls as possible. The system was then flushed out with the gas to be used in subsequent experiments, pumped down to the desired pressure and held at that pressure by balancing the pumping speed and the inflow of the gas. All experiments were conducted with a continuous flow of gas through the torus.

The hydrogen and helium gases used were obtained from The Matheson Co. Inc., East Rutherford, New Jersey. The specified impurity content was less the 0.2 mole per cent for hydrogen and less than 0.01 mole per cent for helium.

(iii) The Confining Field

The confinement and discharge fields are pulsed in the sequence shown in Fig. 5. The timing is controlled by conventional delay and trigger circuits. Each timing sequence is initiated manually.

The plasma is confined by the pulsed azimuthal magnetic field provided by firing a bank of condensers through coils on the torus. The circuit is slightly overdamped. The pulse is long enough to provide an essentially steady magnetic field during the time of experiment.
FIG. 5. Timing Sequence
An inherent property of toroidal systems, such as that used in these experiments, is the fact that the confining magnetic field will have a gradient whose direction and magnitude are given by

\[
\frac{\nabla B}{B} \sim - \frac{R}{R^2}
\]

where \( R \) is the major radius of the torus.

A single particle will drift in this inhomogeneous field at a velocity given by

\[
v_p = \frac{v_z}{2\omega_c R}
\]

where \( v_p \) = drift velocity

\( v_z \) = velocity perpendicular to the magnetic field

\( \omega_c \) = cyclotron frequency

Since the magnetic field lines in a torus also have curvature there will be a second drift due to this given by

\[
v_p = \frac{v_{||}}{\omega_c R}
\]

where \( v_{||} \) is the velocity along the lines of force.

For a particle of given sign the two drifts are in the same direction. Adding the two drifts gives

\[
v_p = \frac{1}{\omega_c R} \left( \frac{v_z^2}{2} + v_{||} \right)
\]

for the drift velocity of a particle unhindered by interactions with other particles or fields.

The direction of drift is opposite for positive and negatively charged particles so that this drift tends to separate the ions and electrons. Unless the plasma is surrounded by a good conductor, an
electric field is set up in the plasma which prevents the charges from separating further. However, this electric field results in the particles drifting to the walls due to the $E \times B$ drift which occurs when a charged particle is in a region of crossed/magnetic fields. In this case both ions and electrons drift at the same velocity, given by $10^6 E/B$, and in the same direction. This does not result in charge separation so the drift can go on indefinitely. This is thought to be the most likely mechanism for loss of the plasma to the torus walls.

(iv) The r.f. Fields

Breakdown is accomplished by means of an r.f. oscillator (3 Mc.) inductively coupled to the plasma. The inductance of the oscillator tank circuit is made up of two turns around the torus. This forms a transformer-like arrangement with the plasma itself acting as the secondary winding.

Breakdown with this arrangement will occur at pressures of a few microns and higher.

To get breakdown at lower pressures, an additional oscillator, or in some cases the same oscillator was connected to two electrodes on the inside surface of the torus. With the arrangement breakdown could be obtained down to about 0.25 microns.

The r.f. oscillator pulse length was variable from 0 - 1000 microseconds. The amplitude of the r.f. is large enough so that the total field around the torus is of the order of a few thousand volts. The r.f. oscillators produce a plasma of density about $5 \times 10^6$/cm. The density produced is not very dependent on pressure or the strength of the magnetic field so long as these are sufficient to secure breakdown.
Probe measurements were made both during the r.f. and in the afterglow. In some cases the r.f. oscillator was not capable of producing the density needed to make measurements in the afterglow possible at as late a time as desired. For these cases the betatron field was applied to produce additional ionization. The betatron field will raise the density an order of magnitude in a few microseconds.

For the electron cooling measurements it was essential that the oscillator and betatron circuit did not continue to oscillate after the time when the shutting-off pulses were applied. To ensure that this was the case, the betatron was critically damped and the r.f. near critically damped so that oscillations would be effectively ended after a few cycles.

(v) The Probes

A schematic diagram of one of the double probes is shown in Fig. 6. Spherical copper probes varying in size from 0.25 to 3.0 mm. in diameter have been used in this experiment with what seem to be fairly satisfactory results. Some measurements were made with machined aluminum probes also, but these could not be constructed as small as was necessary.

In general, as small probes as possible are most desirable but difficulty in construction has imposed the lower limit of 0.25 mm. in size and in some cases larger probes had to be used to collect a measurable current. In some cases the criteria for probe size cited earlier were well satisfied. In others the criteria were not so well
FIG. 6. The Probe
satisfied and although the probes yielded similar voltage - current characteristics less confidence must be placed in the results.

The probes have been made up from No. 40 copper wire, vacuum sealed through glass by a method first demonstrated by Housekeeper. A sphere can be produced on the end of the wire by heating it till it flows. The surface tension will then produce a fairly good sphere. If the sphere is passed directly from a gas flame into a beaker of water, oxidation can practically be avoided. If some oxidation does occur it can be removed by a solution of ammonium chloride.

No serious difficulty with vacuum leaks occurred in the probes described above. They have the advantage that waxes and other material of high vapour pressure is eliminated. Since the system is operated for periods of only a few hundred microseconds in length, and the ions are not heated appreciably, there is no problem with the probes burning up.

(vi) The Probe Circuit

The probe circuit developed is shown in Fig. 7. The electron energy and density vary rapidly in the pulsed system. The total elapsed time from breakdown to the disappearance of the ionization is usually about 1500 microseconds. To resolve the temperature and density variations in time, the probe voltage must be swept at a frequency high enough so that its period is short compared to the time variations in the plasma. For this reason probe voltage frequencies chosen were in the range from 2 kc. to 40 kc. For the sake of convenience a sinusoidal voltage waveform was used in all experiments.
Isolating Transformer

Audio frequency signal in here.

+ 300 v.

- 300 v.

All tubes are 12AX7
A is a double beam oscilloscope with two differential amplifiers

FIG. 7. The Probe Circuit
The voltage waveform was generated by an audio oscillator and then amplified. The output stage is a cathode follower of fairly low output impedance which drives the probes through an isolation transformer. This transformer must have low secondary to core capacitance if a.c. probe currents of a few microamperes with a non-sinusoidal waveform are to be measured accurately.

To achieve this, the transformer secondary was wound with a single layer of No. 40 wire, separated from the core by about 1 cm. at all points. Although this tends to destroy the voltage regulation of the transformer, it is nevertheless well worthwhile since the current waveform is much improved due to the lower capacitance to the core.

The total capacitance of the probes, connections and secondary transformer winding to its surroundings is about 30 mmf.

The inputs of four cathode followers are connected to the probe circuit. Since the potential of the probe circuit floats, two cathode followers are necessary to measure the voltage between the probes, and two are required to measure the current flowing between the probes. The outputs of the two pairs of cathode followers are fed into two Tektronix difference amplifiers. Probe current and voltage are then displayed simultaneously on a double beam Tektronix oscilloscope.

The cathode followers are operated without grid resistors. This was done because the plasma floating potential may move one or two hundred volts from ground potential and even a large resistor will conduct a current to ground comparable to the probe current itself under these conditions. The grids and probe circuit may charge up,
due to grid current, in the time between discharges. However, the
initiation of a discharge brings the circuit back to the proper
potential very quickly.

(vii) Physical Quantities Measured

Calculation of the electron temperature and density from the
probe characteristics requires the measurement of two quantities.
(See equations (9) and (10).) These are \( \xi_{i_r} \), which is the saturation
current, and \( \frac{dV}{di} \) at \( V_d = 0 \) which may be thought of as the resistivity
of the plasma around the point \( V_d = 0 \). It has been found convenient
to measure these quantities separately.

The method used has been as follows:

(a) Measurement of \( \xi_{i_r} \)

A fairly large voltage, sufficient to produce current saturation,
is applied to the probes. The saturation current and applied voltage
for a representative discharge are displayed on the oscilloscope and
photographed. The saturated current will have a decaying waveform,
due to the fall-off in density as time goes on, and will approximate
a clipped sine wave in form. (See Fig. 8(a).)

(b) Measurement of \( \frac{dV}{di} \) at \( V_d = 0 \)

The technique developed for measurement of this quantity is as
follows. The voltage is set at a value much smaller than that necessary
to produce current saturations and a second photograph obtained which
shows the voltage and current simultaneously for a discharge similar
to that used in (a). From this photograph \( \frac{dV}{di} \) or the resistivity, in
the neighborhood of \( V_d = 0 \) can be obtained. (See Fig. 8(b).)
Fig. 8(a) The upper trace is the saturated probe current $\Sigma i_p$. The lower trace is the voltage $V_d$ applied across the probes.

Fig. 8(b) Measurement of $\frac{dV}{di}$. The upper trace is the probe current $\Delta i$ and the lower trace is the probe voltage $\Delta V$. 
(viii) Probable Errors

A small systematic error is introduced when $\frac{dV}{di}$ is measured over a finite interval in the neighborhood of $V_d = 0$ rather than at the point $V_d = 0$. In practice the voltage, for measurement of $\frac{dV}{di}$, was set at a value such that the current produced was 25% or less of the saturated probe current. The error introduced can be calculated and shown to be 4% or less. Also, the error will always be in a direction which will make the calculated electron energy too small.

The estimated random error in the measured quantities is given in Table I.

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Estimated Random Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dV$</td>
<td>10%</td>
</tr>
<tr>
<td>$di$</td>
<td>10%</td>
</tr>
<tr>
<td>$i$</td>
<td>15%</td>
</tr>
<tr>
<td>probe area</td>
<td>6%</td>
</tr>
</tbody>
</table>

Errors in the first three quantities are mostly due to the difficulty in making exact measurements on the oscillograms. The error in the probe area is partly due to the fact that the probes are not perfectly spherical, so that the surface area is in some doubt.

Combining the pertinent quoted random and systematic errors gives a probable error of 23% in the temperature measurements and a probable error of 19% in the density measurements. The experiments, in every case,
involve plotting a curve from a number of points, each of which is subject to the probable error of 23% (temperature) or 19% (density). The curve itself should, of course, have somewhat better accuracy.
CHAPTER V

ELECTRON DENSITY MEASUREMENTS

Electron density measurements were made, by means of double probes, both during the time the r.f. breakdown field was applied and during the afterglow of the discharge.

Density determination requires that two quantities be known; the probe saturation current and the electron temperature or energy.

From equation (10),

\[ n = \frac{i_+}{0.4Ae} \sqrt{\frac{m+}{2kTe}} \]

where \( i_+ = \frac{1}{2} \Sigma i_p \)

The saturated probe current could be measured in all cases but the electron energy could not be measured during the r.f. pulse. Density measurements during this region are based on an assumed value for the electron temperature.

The previously cited criteria for proper probe operation in a magnetic field were:

\[ h < a \]

\[ h + a < r_c \]

These may be written

\[ h < a < r_c \]

keeping in mind the fact that there is evidence that the criterion \( h + a < r_c \) is stricter than necessary.

When possible these criteria were satisfied. However, larger probes had to be used in many cases to secure a measurable probe current. A table giving the value for \( a \) and typical values for \( h \) are forthcoming.
and $r_c$ for the various gases as well as the magnetic field most often used with each is given below:

<table>
<thead>
<tr>
<th>Gas</th>
<th>Magnetic Field</th>
<th>$h$</th>
<th>$a$</th>
<th>$r_c$ (ion)</th>
<th>$r_c$ (electron) (le.v.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>40 gauss</td>
<td>0.0096</td>
<td>0.35</td>
<td>14.3 mm</td>
<td>0.85 mm</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>700</td>
<td>0.09</td>
<td>2.8</td>
<td>0.575</td>
<td>0.048</td>
</tr>
<tr>
<td>Argon</td>
<td>1000</td>
<td>0.021</td>
<td>1.0</td>
<td>1.8</td>
<td>0.034</td>
</tr>
</tbody>
</table>

**A. Density Measurements in Helium**

Measurements were made in the afterglow of helium. Since plasmas with fairly high density were easily obtained with rather low magnetic fields, the probe criteria were well satisfied as indicated in Table II. In this case the probe behaviour is expected to be well described by the theory outlined in Chap. III and unaffected by the magnetic field.

Furthermore, the electron-collision frequency in the afterglow was sufficiently high to ensure that the electron velocity distribution was closely Maxwellian at all times.

Representative curves showing the density fall-off with time are shown in Fig. 9. These indicate a more rapid fall-off in density at first, when the electron energy is higher, followed by a region where the logarithmic plot becomes a straight line, indicating an exponential decay.

The times required for various mechanisms to reduce the density by a factor $1/e$ have been computed and are compared with the observed loss times in Table III. The figures given for a field of 40 gauss.
FIG. 9. Density vs. Time in Helium Afterglow
TABLE III
HELIUM PLASMA

<table>
<thead>
<tr>
<th>Electron Energy (ev)</th>
<th>Loss time (single particle drift)</th>
<th>Loss time (diffusion)</th>
<th>Loss time (recombination)</th>
<th>Loss time (experimental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>14.3 μsec</td>
<td></td>
<td></td>
<td>17.5 μsec</td>
</tr>
<tr>
<td>0.6</td>
<td>19.0</td>
<td></td>
<td></td>
<td>19.2</td>
</tr>
<tr>
<td>0.4</td>
<td>28.8</td>
<td>450 μsec</td>
<td>147 μsec</td>
<td>21.0</td>
</tr>
<tr>
<td>0.3</td>
<td>38.1</td>
<td></td>
<td></td>
<td>28</td>
</tr>
<tr>
<td>0.2</td>
<td>57.2</td>
<td></td>
<td></td>
<td>56</td>
</tr>
<tr>
<td>0.15</td>
<td>75.7</td>
<td></td>
<td></td>
<td>79</td>
</tr>
</tbody>
</table>

The loss time for recombination given above is that for dissociative recombination by the mechanism

\[ \text{He}_2^+ + e = \text{He} + \text{He} \]

and is calculated on the basis that all the ions present are of the type \( \text{He}_2^+ \) which has a larger recombination coefficient than \( \text{He}^+ \).

Since helium does not ordinarily form molecules in a low pressure discharge it is expected that only a small percentage of the ions will be of this type. The loss time given above is therefore the minimum loss time that could occur by recombination.

The loss time for diffusion is several times longer than the observed loss time. The loss time for diffusion as shown in Table III was calculated using the measured ambipolar diffusion coefficient for a thermal electron distribution, corrected for the magnetic field. Actually the loss due to diffusion will probably be somewhat higher.
since the diffusion coefficient may be expected to become larger at higher electron energies. Unfortunately, information is not available to compute the diffusion loss rate at other than thermal energies. If ambipolar diffusion were an important loss mechanism, the density decay rate should be a function of pressure. Study of the curves shown in Fig. 9 shows that this is not so. We conclude that diffusion may be a small contributing factor but not the controlling factor in the loss rate of the plasma.

The loss times for drift given in Table III are those that would occur if the electron drifted, unhindered, in the inhomogeneous magnetic field. This is usually referred to as the "single particle drift".

As discussed previously in Chap. IV, Sec. A (iii) the single particle drift results in separation of the positive and negative charges since the ions and electrons drift in opposite directions. For the case of the torus ions will drift upward and electrons downward or vice versa depending on the direction of the confining magnetic field. The separation of charge results in an electric field being set up in a direction which tends to hinder the single particle drift. A schematic diagram showing the directions of the fields and drifts is given in Fig. 10.

The electric polarization field in combination with the confining magnetic field will then produce a crossed field drift in the radially outward direction given by

\[ \vec{v}_d = \frac{\vec{E} \times \vec{B}}{B^2} \]
Cross-section of Chamber

FIG. 10. Field and Drift Directions
To experience the drift effects discussed above a particle must have a collision frequency low enough so that it does actually rotate around the magnetic field lines, i.e. if a particle has a collision frequency much higher than its cyclotron rotation frequency (many more than one collision per revolution) one would not expect the drifts to apply. For the pressures and magnetic fields used in the helium measurements ions of typical energy make from 2 to 6 collisions per revolution of cyclotron rotation. The motion of the ions is consequently practically unaffected by the presence of the magnetic field. On the other hand electrons of typical energy make a few revolutions per collision. The electrons will therefore be strongly affected by the magnetic field and may be expected to drift as discussed above. In summary the situation that occurs in the torus is thought to be as follows:

After an initial transient the electrons and ions achieve an equilibrium drift in an outward and downward direction (referring to Fig. 10). The ions are not hindered by the magnetic field in this case but by their finite mobility in the gas. The drift loss rate is determined by many factors such as electron energy, magnetic field, the size of the currents tending to neutralize the polarization field both within the plasma and in the high resistance torus coating, and the mobility of the ions.

Calculations based on the mobility of the ions show that an electric field $E \approx 0.45$ volts/cm would be sufficient to move the ions to the wall at a rate which would lead to a characteristic loss
time of 30 microseconds as for the central values in Table III. Furthermore, calculation shows that a field of this order would also lead to an $\mathbf{E} \times \mathbf{B}$ drift of the electron consistent with the characteristic loss time of 30 $\mu$sec. Though too complicated for a complete quantitative analysis, this is thought to be a very probable explanation of the density decay in Helium.

B. DENSITY MEASUREMENTS IN HYDROGEN

Densities were also measured in the afterglow of hydrogen plasmas. In hydrogen it was found necessary to use a much stronger magnetic field to obtain suitable densities. Fields of 600-1000 gauss were used. Also, the size of the probes had to be increased a great deal to obtain a measurable probe current. The result was that the probe criterion

$$h + a < r_c$$

was not nearly satisfied and even the criterion

$$h < r_c$$

was not met. However, the probes still yielded characteristics that seemed normal and temperature measurements that appeared just as reasonable as those for helium where the probe criteria were well satisfied. The hydrogen measurements are therefore included, with the qualification that they are not so reliable as those in helium.

Curves, showing the measured density decay with time for hydrogen plasmas, are given in Fig. 11. These show little, if any, variation in the decay rate with electron temperature. The decay appears to be approximately exponential.
FIG. II. Density vs. Time for H₂ Afterglow
The time required for various mechanisms to reduce the density by a factor of $1/e$ has been calculated and tabulated in Table IV.

**TABLE IV**

**HYDROGEN PLASMA**

<table>
<thead>
<tr>
<th>Electron Energy (eV)</th>
<th>Loss time (single particle drift)</th>
<th>Loss time (diffusion)</th>
<th>Loss time (Recombination)</th>
<th>Loss time (experimental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>221 $\mu$s</td>
<td>1000 $\mu$s</td>
<td>1000 $\mu$s</td>
<td>80 $\mu$s</td>
</tr>
<tr>
<td>0.8</td>
<td>250</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>298</td>
<td>1000 $\mu$s</td>
<td>1000 $\mu$s</td>
<td>80 $\mu$s</td>
</tr>
<tr>
<td>0.4</td>
<td>378</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>752</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>999</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For hydrogen the loss times for recombination and diffusion are long, so these are not considered to be important loss mechanisms. Diffusion in this case is much slower than the case for helium due to the magnetic field being much larger and the pressure being much lower so that the collisions which are necessary to move the particles across the magnetic field are not so frequent.

It is interesting to note that the characteristic loss time in hydrogen ($\sim$80 microseconds at 0.4 eV) is much less than the loss time for single particle drift in the inhomogeneous magnetic field (378 microseconds). If the electron loss is explained on the basis of an $\mathbf{E} \times \mathbf{B}$ drift a polarization field of $\mathbf{E} \approx 2$ volts/cm is needed.
The situation for hydrogen differs somewhat from that for helium in that the ion cyclotron frequency in hydrogen is large compared to the collision frequency. The consequent reduction in mobility means that the ions should now experience an \( \frac{E \times B}{B^2} \) drift just as the electrons do. The particles should come out more nearly in the direction of \(-\nabla B\) than in the case of helium. This could possibly be checked by observation of recombination light but so far this has not been attempted. Conceivably the polarization field \( E \) could also be measured as a check on the loss mechanism. This is difficult since it is necessary to measure small voltage gradients in the presence of much larger plasma potentials.

In the foregoing discussion of the plasma decay rates in the plasma used in these experiments no attention has been given to the possibility that plasma oscillations may be responsible for moving the charges across the field lines. These could possibly be present but the difficulty in treating them theoretically or in detecting them experimentally would make any treatment that could be given here unsatisfactory.

C. DENSITY MEASUREMENTS IN ARGON

In the plasma betatron experiments an argon plasma is usually used. Since a knowledge of the plasma density is of importance an effort was made to measure the electron density under the conditions pertaining to this experiment.
The r.f. pulse which is used to ionize the gas, has a decaying envelope so that the density of the plasma falls steadily during the pulse. Trigger circuits are provided so that the r.f. may be cut off sharply at any desired time during this decay and the betatron accelerating cycle begun within a microsecond. Since the betatron may be started at any desired time in the r.f. pulse it is necessary to know the density that is present throughout most of the r.f. pulse.

Initially we hoped to make density measurements in the afterglow and extrapolate these back to the time when the r.f. was shut off. This proved too difficult due to the fast initial density decay which seems to occur at these pressures (0.5 micron), the small probe current collected and the presence of large transient fluctuations in the plasma potential. This method was abandoned in favour of measurements during the r.f. pulse where larger probe currents are available and the density is not decaying as fast as in the afterglow.

Measurements of density with probes required knowledge of the electron temperature. However, measurements of the electron temperature during the r.f. pulse were impossible due to the noise present and also the large fluctuations in plasma potential. In any case, there is some doubt about the value of the probe temperature measurements during the r.f. since the distribution would probably not be Maxwellian. It has been necessary to make an estimate of the electron temperature which is rather uncertain since no information is available on electron energies at the large values of $E/p$ present in this case.
The electron energy was estimated to be 15 e.v. which is equal to the ionization potential of argon. This estimate is based on the assumption that if the electron energy were higher than this the percentage ionization of the gas (1% in this case) should be higher than that observed. Fortunately the density varies as the reciprocal of the square root of the electron energy, so that a given error in the assumed electron temperature does not introduce so large an error in the density.

The saturated probe current could be measured with fairly good accuracy during the r.f. pulse. Densities were computed from the saturated probe current and the assumed electron temperature. The results obtained by this method are shown in Fig. 12.

The results obtained by this method can be compared with the densities obtained indirectly in the plasma betatron experiment \(^1\). The density values from the betatron experiment are based on the assumption that the betatron accelerates all the electrons initially inside the region where there is betatron focussing. The density is then computed from the rate of rise of the current.

Densities obtained from the betatron experiment in this way are lower by a factor of six or seven than those obtained from probe measurements.

In practice it is possible that not all the electrons are accelerated, so that the densities obtained in this way are actually a lower limit on the plasma density. Also there is some loss of electrons in the interval between the end of the r.f. pulse and the
Argon: 0.4 microns

Curve A: 200 gauss
B: 100 gauss
C: 200 gauss with tank circuit damped.

FIG. 12. Density During r.f. Pulse
beginning of the betatron accelerating cycle. In fact, there is some
evidence that there may be an anomalously fast fall-off in density in
the initial interval after the r.f. is shut off.

These things, together with the possibility of some error in the
probe measurements are probably able to explain the discrepancy in the
measurements.
CHAPTER VI

ELECTRON TEMPERATURE IN THE AFTERTGLOW

Electron temperature measurements were made in the afterglow of hydrogen and helium plasmas. Fig. 13 shows typical density and temperature decay curves. The temperature of the electrons, which is of the order of 10 e.v. during the r.f. pulse, decays during the first few hundred microseconds of the afterglow through various processes discussed below. The ions are assumed to remain fairly close to room temperature at all times since their large mass and resultant low velocities prevent them from picking up much energy in the r.f. field. Any energy that the ions do pick up is quickly shared with the much more numerous neutral gas atoms. In an elastic collision with a neutral gas atom an ion will lose \( \frac{1}{2} \) of its excess energy on the average. An electron, on the other hand, loses only the fraction \( \frac{2m}{M} \) of its excess energy in an average elastic collision.

The recoil cooling of the electrons due to collision with neutral atoms can be simply expressed in terms of the electron-neutral collision frequency and the fractional energy given up per collision provided the electron velocity distribution is known. If recoil cooling dominates over other possible energy transfer processes measurements of the electron temperature as a function of time in the afterglow can lead to determination of basic recoil data.

Since both the collision frequency and the average energy loss per collision are known for hydrogen the cooling rate for the electrons in a hydrogen plasma under given conditions can be calculated. Measurement
Helium: 61 microns

FIG. 13. Electron Energy and Density Decay
of the cooling rate should agree with the calculations if the method of measurement is satisfactory and if collision cooling is in fact the dominant cooling mechanism.

For helium the collision frequency has been measured and the electron energy loss per electron-neutral collision is expected to be simply the energy transfer one would calculate for a simple two-body elastic collision since the helium atom does not have excitation levels close to the ground state. The electron should then lose a fraction $\frac{2m}{M}$ of its excess energy at each collision.

Initially the intention was to test the temperature measurement technique on hydrogen and helium afterglows. If the experimental temperature decay agreed with that calculated from known recoil data the technique could then be employed with some confidence in an investigation of other gases for which recoil data were unknown.

Temperature decay measurements actually made on hydrogen and helium afterglows showed (as is discussed later) that recoil cooling was not ordinarily the dominant energy transfer process. Several alternative contributing processes can be thought of, although it is not possible to estimate accurately the importance of many of them. Following a rigorous theoretical analysis of recoil cooling a somewhat qualitative discussion will be given of other possible energy transfer processes.

A. POSSIBLE PROCESSES CONTRIBUTING TO TEMPERATURE DECAY

(i) Recoil Cooling

Brown and co-workers give reliable measured values, for hydrogen,
for the collision probability $P_m$ and the fraction, $G$, of excess energy lost by an electron in collision with a gas molecule. For a perfectly elastic collision $G = \frac{2m}{M}$ but the hydrogen molecule has easily excited vibrational and rotational states so that the average energy transfer per collision is a few times higher than this. The cooling rate $\frac{d\bar{u}}{dt}$ where $\bar{u}$ is the average electron energy, can be calculated for a Maxwellian distribution using Brown's results.

The electron velocity distribution is assumed to be Maxwellian at all times. This can be shown to be a plausible assumption since for typical conditions in the afterglow the rate of energy transfer between the electrons themselves is by far the fastest process taking place.

If we neglect the cooling effects of drift and diffusion, i.e. the plasma is contained by the magnetic field, the Boltzmann equation can be written

$$\frac{df(v)}{dt} = B_o$$ \hspace{1cm} (11)

where $f(v)$ is the velocity distribution and $B_o$ is the collision integral.

For $f(v)$ we use the expression

$$f(v) = \frac{1}{\pi^{3/2} K^{3/2}} e^{-Kv^2}$$

where $K = \frac{3m}{4\bar{u}}$

also, $B = \frac{1}{v} \frac{d}{dv} \left[ v^5 G Vm \left( f + \frac{2kT}{m} \frac{df}{d(v^2)} \right) \right]$ \hspace{1cm} (12)

Here $G$ has been substituted for the factor $\frac{2m}{M}$ in Allis' equation (which assumes purely elastic collisions) and $V_m = P_m v$ is the collision frequency for momentum transfer.
To find the rate of change of the average energy of the distribution we multiply both sides of (11) by \(4\pi v^2 \frac{mv^2}{2}\) and integrate over velocity space.

\[
2\pi m \int_0^{\infty} v^4 \frac{\partial f}{\partial t} \, dv = 2\pi m \int_0^{\infty} v^4 B_0 \, dv \tag{13}
\]

Consider the first integral in (13)

\[
\frac{\partial f}{\partial t} = \frac{2(\pi)^{-\frac{3}{2}} k_{\frac{3}{2}} e^{-kv^2}}{\pi}
\]

\[
= \frac{2}{\pi^{-\frac{3}{2}} k_{\frac{3}{2}}} \frac{du}{dt} \left[2e^{-kv^2} + K v^2 e^{-kv^2}\right]
\]

The first integral then becomes

\[
2\pi^{-\frac{1}{2}} k_{\frac{3}{2}} \frac{du}{dt} \left[\int_0^{\infty} 2v^4 e^{-kv^2} \, dv + \int_0^{\infty} K v^6 e^{-kv^2} \, dv\right] \tag{14}
\]

The integrals within the square brackets in equation (14) can be solved by making the substitution \(K v^2 = y\) and putting them into the general form

\[
A \int_0^{\infty} y^{(x-1)} e^{-y} \, dy = A \Gamma(x) \tag{15}
\]

where \(A\) is a constant and \(\Gamma(x)\) is the Gamma Function.

If this operation is carried out we have

\[
2\pi m \int_0^{\infty} v^4 \frac{\partial f}{\partial t} \, dv = 2\pi^{-\frac{1}{2}} m k_{\frac{3}{2}} \frac{du}{dt} \left(-\frac{3}{4} + \frac{5}{4}\right) K^n \Gamma\left(\frac{5}{2}\right)
\]

Now using \(K = \frac{3m}{4u}\) and \(\Gamma\left(\frac{5}{2}\right) = \frac{3\sqrt{\pi}}{4}\)

we have

\[
2\pi m \int_0^{\infty} v^4 \frac{\partial f}{\partial t} \, dv = \frac{3\sqrt{\pi}}{2}
\]

Substituting this result into (13) we have

\[
\frac{\partial u}{\partial t} = 2\pi m \int_0^{\infty} v^4 B_0 \, dv \tag{16}
\]

Before we solve the second integral in (13) we make the following substitutions in (12)

\[
v^3 G v_m = \propto v^\beta
\]
where \( \alpha \) and \( \beta \) are constants carefully chosen to give a close fit to the previously measured values of \( G \) and \( \mathcal{V}_m \).

Also \( \frac{d}{d(v)} = \frac{d}{dv} \cdot \frac{dv}{d(v)} = \frac{1}{2v} \frac{d}{dv} \)

Hence \( \frac{df}{dv} = -2Kv \)

Then \( f + \frac{2kT}{2mv} \frac{df}{dv} = f \left( 1 - \frac{u_g}{u} \right) \)

where \( u_g \) is the average energy of the gas molecules.

Hence \( B_0 = \left( \frac{1 - u_g}{u} \right) \frac{d}{dv} \left[ \alpha v^{\beta} f \right] \)

If we substitute for \( f \), carry out the indicated differentiation and put the resulting expression into (16) we have

\[
\frac{d\bar{u}}{dt} = \frac{2\pi m}{2\pi k} \left( 1 - \frac{u_g}{u} \right) \left[ \int_0^\infty K^{\frac{\beta}{2}} e^{-\frac{k\beta}{2} f} dv - 2 \int_0^\infty v^{\beta+1} K^{\frac{\beta}{2}} e^{-k\beta f} dv \right]
\]

These integrals can also be put into the form of (15) by the substitution \( Kv^{\beta} = y \). The result will be

\[
\frac{d\bar{u}}{dt} = \frac{m\alpha}{4\pi} \left( 1 - \frac{u_g}{u} \right) \left[ \beta \frac{e^{\beta + 1}}{2} \Gamma \left( \frac{\beta + 2}{2} \right) - K \Gamma \left( \frac{\beta + 2}{2} \right) \right]
\]

For the case of hydrogen

\( \alpha = 7.21 \times 10^{-9} \) \( \left( \frac{2m}{M} \right) \) \( \rho \)

where \( \rho \) is the pressure in microns Hg. normalized to \( 0^\circ \text{C} \) and

\( \beta = 5 \)

Substituting these numerical values into (17) we have

\[
\frac{(d\bar{u})}{(dt)} \text{hydrogen} = 2.31 \times 10^{-4} \left( 1 - \frac{u_g}{u} \right) \bar{u}^2 P
\]

A similar expression for Helium was obtained

\[
\frac{(d\bar{u})}{(dt)} \text{helium} = 675 \left( 1 - \frac{u_g}{u} \right) \bar{u}^2 P
\]

where the value for \( P_m \) was obtained from Gould and Brown \(^{16}\) and it was assumed that \( G = \frac{2m}{M} \).
Equations (18) and (19) were used to calculate the cooling rate curves labelled theoretical in Figs. 14.1, 14.2, 15.1 and 15.2.

(ii) Drift Cooling

As discussed previously, a single charged particle will drift in the inhomogeneous toroidal magnetic field with a velocity given by

$$v_D = \frac{1}{\omega_c R} \left( \frac{1}{2} v_\perp^2 + v_\parallel^2 \right)$$

or if the energy is equally distributed in the three degrees of freedom

$$v_D = \frac{2}{3} \frac{u}{\omega_c R}$$

where $u$ is the energy of the particular particle under consideration.

The above relation shows that the velocity of drift of a single charged particle is proportional to its energy. If particles are lost to the chamber wall through this drift mechanism it is apparent that the more energetic particles will be lost at a greater rate than the slower ones. This preferential loss of the higher energy electrons will lower the average energy of the group. The rate at which the average energy of the electrons is lowered by this mechanism will now be investigated.

For a Maxwellian velocity distribution the normalized density of particles with speeds between $v$ and $v + dv$ is

$$dN(v) = 4\pi^{-\frac{3}{2}}K \frac{v^2}{\gamma^2} e^{-\gamma v^2} dv$$

where

$$\int dN(v) = 1$$

Let the loss rate per electron as a function of velocity be given by $\gamma_D = Cv^\beta$ where $C$ is an arbitrary constant and $\beta$ is also a constant.
For the case of pure single-particle drift (eq. 20) we have $\beta = 2$. However as has been discussed previously (ch. 5) a polarization electric field is expected to develop which leads to an altered drift loss of electrons. The velocity dependence of this altered drift is unknown; in fact if a pure $E \times B/B^2$ electron drift developed it should be independent of velocity so that $\beta = 0$, and no drift cooling would result. We will consider here the general case of $\beta$ unspecified.

The rate of cooling, for a maxwell distribution, is then given by the following integral.

$$\frac{d\bar{n}}{dt} = \int \nu_b (u - \bar{u}) \, dN(v)$$

$$= 4\pi \nu^2 \frac{m v^2}{2} \left[ \int_0^\infty \frac{4 + \beta}{v^2} e^{-Kv^2} \, dv - \bar{u} \int_0^\infty \frac{2 + \beta}{v^2} e^{-Kv^2} \, dv \right]$$

By making the substitution $Kv^2 = x$ these integrals can be put in the form $A \int_0^\infty y^{(x-1)} e^{-y} \, dy$ whose solution is $A \Gamma(x)$.

The above equation then becomes

$$\frac{d\bar{n}}{dt} = 4\pi \nu^2 \frac{m v^2}{2} \left[ \frac{K}{2} \int_0^\infty \frac{5 + \beta}{v^2} \, dv - \bar{u} \frac{3 + \beta}{v^2} \, \int_0^\infty \frac{3 + \beta}{v^2} \, \Gamma \left( \frac{3 + \beta}{2} \right) \right]$$

We wish to relate the cooling rate to the density decay rate. Thus

$$\frac{1}{n} \frac{dn}{dt} = \int_0^\infty C v^\beta \frac{m v^2}{2} \frac{3 + \beta}{v^2} \, e^{-Kv^2} \, dv$$

This can be solved in a manner exactly analogous to the previous integrals.

The result is

$$\frac{1}{n} \frac{dn}{dt} = \frac{4C}{3\pi} \frac{K}{\frac{3 + \beta}{2}} \Gamma \left( \frac{3 + \beta}{2} \right)$$

We then divide (21) by (22) and simplify by using the relations $K = \frac{3m}{4\pi}$ and

$$\Gamma(x) = (x-1) \Gamma(x-1)$$
The result is
\[
\frac{\frac{du}{dt}}{\frac{1}{n} \frac{dn}{dt}} = \frac{\beta}{3} \cdot \bar{u}
\]
or
\[
\frac{1}{u} \frac{du}{dt} = \frac{\beta}{3} \left( \frac{1}{n} \frac{dn}{dt} \right)
\]
which relates the rate of cooling to the density decay where the probability of the loss of an electron is proportional to \( \bar{v} \).

In the cooling rate experiments all the variables in (23) are measured except \( \bar{v} \). In principle one could substitute into the above formula and find the value of \( \bar{v} \) necessary to explain the observed cooling rate. If a single value for \( \beta \) were found this would constitute evidence that there was cooling by this mechanism.

(iii) The formation of Particles Differing from the Parent Gas

The formation, in the discharge, of ions and molecules such as He\(_2\), He\(_2^+\) in helium and H\(^+\) or H in hydrogen, which are different from the parent gas, may complicate the cooling phenomena. The experimental conditions were not thought to be conducive to the formation of species of ions and molecules differing from the parent gas, however this possibility cannot be ruled out completely, especially since the betatron produces large electric fields which feed large amounts of pulsed power into the discharge.

Some of these atoms and molecules have large cross-sections which could lead to anomalous cooling effects. For example the cross-section for collision of electrons with atomic hydrogen is believed to be five to ten times larger\(^\text{17}\) than that for collision with He\(_2\) and the recombination coefficient (dissociative) of He\(_2^+\) is also large.
(iv) Metastable Atoms and Molecules

Cooling which is slower than the theoretical value could possibly be caused by the presence of metastable atoms or molecules in the discharge which might give up their excitation energy to the electrons. Cooling measurements were made in the first millisecond of the afterglow which is about equal to typical lifetimes of metastable states. Not enough information is available on the cross-section for production and the lifetimes of metastable states in $H_2$ or $He$ or on the cross-section for de-excitation of metastable by electron collision to make any quantitative treatment possible.

(v) The Effect of Gas Atoms on the Chamber Walls

An effect which seems more likely to be serious is the releasing of impurity atoms from the chamber walls by bombardment during the discharge.

An atomically clean surface consists of atoms of the underlying material only and the binding energy of the atoms in the lattice is usually a few electron volts. Such a surface can be produced in about a second by heating to 2000 - 3000°C. Gas atoms are quickly adsorbed on any clean cold surface. Atoms striking such a surface stick with a probability of about 0.3 to 0.5 until one foreign atom lies on the surface for about each four surface atoms. The binding energy will usually be about 2 to 4 electron volts. Such a layer is called a "monolayer". Successive layers are then formed with lower sticking probability and lower binding energy until the number of atoms escaping due to thermal agitation of the lattice equals the number
arriving per unit time. In a good vacuum heating to 700°C will remove all but the last few monolayers and heating to 2500°C will remove all the monolayers.

A monolayer forms very rapidly and has about $4 \times 10^{14}$ atoms/cm$^2$. This constitutes a considerable reservoir of gas. For the torus used in this experiment one monolayer would have $8.5 \times 10^{17}$ atoms in it. Some of the measurements were made at a pressure of 1 micron Hg, which is equivalent to $10^{17}$ gas atoms in the entire torus. Thus one monolayer contains 8 times more atoms than the torus does.

After pumping a system for some time and flushing with the gas to be used in the experiment the monolayers will consist mostly of the gas to be used in the experiment. However, the possibilities for the discharge to become contaminated by this mechanism are still probably rather high for an unbaked system. It should be noted that prior to the measurements in helium, the chamber (uncoated) was cleaned by many intense discharges. The impurity level for the helium measurements is therefore not expected to be very high.

A second manner in which the above phenomenon could produce anomalous results is by the pressure change which would result from removal of even a part of a monolayer by bombardment during the discharge.

The characteristics of a plasma are in general very sensitive to impurities. The solution to this problem would be to bake the system at very low pressure and then let only very pure gas into the system.
(vi) Plasma Oscillations

Another phenomenon which can cause anomalously fast cooling of the electrons is plasma oscillations. Here the energy of the electron distribution can feed the oscillations and may eventually be radiated or go into additional ionization or heating of the ions.

The phenomena listed above in Sec. (iii) - (vi) are difficult to detect. The experimental techniques used so far are not capable of yielding direct information as to whether these effects are present or not and so it is impossible to decide which effects might be present and which might seriously affect the results obtained in the experiments. It is obvious that collision cooling must take place in any afterglow and it seems reasonable to expect some drift cooling in a geometry such as this. However, when the results differ seriously from those expected on the basis of collision cooling, one is not justified in doing more than pointing out the features of the results obtained and the possible causes of the anomalies that occur.

B. TEMPERATURE MEASUREMENTS IN HELIUM AFTERGLOW

The probe measurements yielded values for electron temperature versus time in the afterglow. Figs. 14.1 and 14.2 show plots of the observed cooling rate versus electron energy as well as the theoretical cooling rate that would result if recoil cooling were the only cooling mechanism. The experimental cooling curves were derived from data tabulated in Appendix II. The calculated cooling rate is based on the assumption that the collisions are elastic i.e. the energy of an electron is reduced by the factor $\frac{2m}{M}$ per collision. (see equation (19).)
Helium: 61 microns

- Theory (recoil cooling)
- Experiment

Helium: 34.5 microns

- Theory (recoil cooling)
- Experiment

**FIG. 14.1 Electron Cooling Rates**
FIG. 14.2 Electron Cooling Rates

Helium: 61 microns
- Theory (recoil cooling)
- Experiment

Helium: 18.2 microns
- Theory (recoil cooling)
- Experiment
Study of the curves in Figs. 14.1 and 14.2 shows that the absolute rate of cooling is actually higher at low pressures than at the high pressures. Since this behaviour is in direct opposition to that expected for collision cooling it is obvious that collision cooling is not the dominant mechanism.

The density decay rates were measured simultaneously with the temperature decay for each of the curves shown in Figs. 14.1 and 14.2. The density decay rates are very similar in all cases. The fact that the density decay rates are similar in all cases while the temperature decay rate fluctuates widely would seem to indicate that the temperature decay is not strongly related to the density decay, i.e. drift cooling is not a likely mechanism to explain these results since the value of $\beta$ (determined using equation (23)) would have to vary widely in the various afterglows.

The remaining possible mechanism are those discussed in Part A, Sections (iii) – (vi).

Interesting features of the measured cooling curves for helium are:

(a) Extremely fast cooling during the first 30 to 80 microseconds of the afterglow.

During the early portion of the afterglow there is very rapid cooling of the electrons – from about 10 e.v. initial temperature to less than 1 e.v. in a time considerably less that 100 $\mu$ sec. While no measurements could be made during the first 30 $\mu$ sec – owing to large fluctuating plasma potentials – the anomalously fast cooling was frequently observed extending to times at which measurements were possible.
It seems impossible to explain this anomalously fast cooling in terms of ordinary interactions between electrons and ions or atoms of helium. There remain two possible explanations - plasma oscillations and impurities.

Although precautions were taken (see section (v) above) to minimize the impurity level one cannot exclude entirely the possibility of anomalously fast cooling due to impure gas in the afterglow. Even if the impurity content were known basic data on electron-ion and electron-molecule interactions is presently so incomplete that reliable estimates of the effects of impurities would be impossible to calculate.

The effect of plasma oscillations is difficult to estimate for a different reason. The particle drifts are so complex that a rigorous analysis of the plasma oscillations which may undergo growth is almost impossible. The conditions, however, resemble somewhat a simpler case considered by Buneman in which there is an initial drift motion of the electrons relative to the ions across a magnetic field. Buneman finds that unstable (growing) plasma oscillations would be expected to destroy the mutual drift at the same time taking energy from the electrons. A process such as this may possibly explain the anomalously fast cooling observed in the early helium afterglow.

Efforts to relate the cooling rate to the loss rate in the manner of equation (23) were not successful indicating that drift cooling was not a significant mechanism in determining the rate of cooling.

(b) A region from about 50 to 500 microseconds. The variable cooling observed in this region may possibly be a consequence of the transition from anomalously fast cooling to slower cooling, especially since the transition does not appear to take place consistently at any given time or electron temperature.
(c) A region from 500 microseconds on, where the cooling appears to be approaching closer to the theoretical value for collision cooling. Unfortunately there is some doubt in the measurements, since the density becomes rather low for accurate probe measurement. There is also uncertainty in the theoretical recoil cooling rate since it depends on the difference between two nearly equal quantities, i.e. $u_3$ and $\bar{u}$ for the electrons, both of which are in some doubt.

C. TEMPERATURE MEASUREMENTS IN HYDROGEN AFTERGLOWS

Probe measurements of the electron temperature decay in hydrogen plasmas were made under conditions where some of the probe criteria were not satisfied. (See Table II, page 37.) Hence not as much confidence can be placed in these measurements as for those in helium.

The experimental cooling rates were determined from the measured data in Appendix II as for helium. These are compared with the calculated recoil cooling rates in Figs. 15.1 and 15.2.

Study of the curves shown in Figs. 15.1 and 15.2 shows that there is one outstanding feature. The experimental rate of cooling is essentially the same in all cases, independent of the pressure, and is considerably slower than that expected due to collision cooling which must of course be present. The results indicate the collision cooling is not the dominant cooling mechanism.

The fact that the rate of temperature decay is essentially the same in all cases, independent of pressure, and that the time constant for density decay is approximately equal in all cases would suggest that drift cooling may be taking place according to equation (23). However, drift cooling added to collision cooling would give a cooling rate faster than the theoretical curves in Figs. 15.1 and 15.2 whereas the observed cooling rate is slower.

Just as for helium it is possible to divide the cooling curve up into three regions:
H$_2$: 7.27 microns

- Theory (recoil cooling)
- Experimental

H$_2$: 1.02 microns

- Theory (recoil cooling)
- Experimental

FIG. 15.1 Electron Cooling Rates
FIG. 15.2 Electron Cooling Rates
(a) During the initial region, of about 250 microseconds, no measurements were made in hydrogen due to large fluctuating plasma potentials. The electrons, during this time, cool from 10 or 15 e.v. down to about 0.5 to 1.0 e.v. which is the temperature observed at the earliest measurements at 250 microseconds. This cooling rate is considerably slower than for Helium where approximately the same cooling takes place in the initial 30 microseconds. However, as noted in part (b) below, the hydrogen cooling rate after 250 microseconds indicates the possibility of metastable atoms tending to keep the electron temperature up. If this is so, it would be possible that the anomalous fast cooling mechanisms (plasma oscillations or impurities) were active in the initial period in hydrogen also, but that their effect was partly masked by "metastable heating".

(b) A region where the cooling rate is independent of pressure and much slower than the theoretical recoil cooling rate. The presence of metastable atoms or molecules here seems quite likely since some mechanism which can supply energy to the electron distribution is needed to explain the observed cooling. Though

(c) A region where the cooling appears as if it might approach the recoil cooling rate but where both the measurements and theory become inaccurate as explained previous in the case of helium. (see Sec. B, part (c)).
CHAPTER VII

CONCLUSIONS

Plasma densities and temperatures have been measured under a variety of conditions in hydrogen, helium and argon discharges by means of the Langmuir double probe technique. The rates of temperature and density decay in hydrogen and helium afterglows have also been measured by this technique.

An attempt has been made to explain the observed decay rates of density and temperature on the basis of simple mechanisms. Some success has been achieved in explaining the density decays on the basis of particle drifts in the inhomogeneous magnetic confining field used in these experiments. Little success has been achieved in explaining the temperature decay rates. It is thought that these are probably controlled by phenomena too complex to analyze with the methods available for this experiment.

An anomalously fast cooling has been observed in the early afterglow. While impurities can not be ruled out as the cause of this anomalous cooling it is possible that plasma oscillations are responsible for it. There is reason to expect plasma oscillations in plasma in which the electrons drift relative to the ions. This mutual drift is present in systems employing a toroidal magnetic field and results directly from the inhomogeneity in the field. Since a toroidal system is one commonly employed in thermonuclear work the identification of an anomalous fast cooling inherent in such systems would be of some interest. Further investigation of the phenomenon should be carried out.

- It has become clear from this work that the inherent lack of effective plasma confinement in toroidal magnetic fields can complicate
experiments designed to measure specific data on electron interactions with ions and gas molecules. Generally speaking the confinement time must be long compared with the characteristic time for the phenomenon being investigated if useful measurements are to be made.

It is also apparent that the purity of gas studied is of major importance. Since it is in general impossible to calculate completely the effect of a small impurity one should aim at as high a level of purity as possible. Discharge cleaning and system bake-out facilities are useful in this respect.
APPENDIX I

THE CORRECT EXPRESSION FOR THE CURRENT TO A PROBE

The usual Langmuir probe theory is based on the assumption that the electron current to a probe immersed in a plasma is given by

\[ i = C e^{\frac{eV}{kT}} \quad (1) \]

The exact expression for the current differs somewhat from this expression as will be shown below. As a preliminary to deriving the exact expression for the current to the probe for a maxwellian distribution, consider a probe immersed in a gas of particles \( N/cm^3 \) all moving at one identical velocity \( \bar{v} \). We wish to calculate \( \Gamma \), the unit area of number of particles which strike a probe of radius \( a \), per second.

All particles within a distance \( r = \bar{v} \) could possibly strike the probe during the next second. Their probability of striking the probe depends on the solid angle subtended by the probe at the particular radius \( r \), of the particle. This is given by the cross-sectional area of the probe divided by the area of a sphere of radius \( r \) which is

\[ \frac{\pi a^2}{4 \pi r^2} \]

The number of particles within the sphere \( r = \bar{v} \), which may possibly strike the probe in the next second is given by

\[ \int_0^{\bar{v}} N \frac{\pi \bar{v}}{4 \pi r^2} dr \]

The number of particles that will strike the probe is given by the same integral with the probability of a particle striking the
probe included in the integrand

\[ \int \frac{\pi a^2}{(\pi r^2)^{3/2}} \, dr = \pi a^2 N v \]

Now the probe area is given by \( A = 4\pi r^2 \). We then have

\[ \Gamma = \frac{N v A}{4} \]

which is the accepted result for the case of particles all moving at the same velocity.

It can be shown that the above result is also true for a Maxwellian distribution of velocities where \( \bar{v} \) is the average velocity of the distribution for the case where there is no retarding field around the probe or the particles are not charged.

We now consider the case of a Maxwellian velocity distribution of charged particles in the neighborhood of a probe surrounded by a potential field such that the particles tend to be prevented from reaching the probe, as in the case of a Langmuir probe.

Let the Maxwellian distribution be given by

\[ \frac{dN}{dv} = \frac{4N}{\sqrt{\pi}} \beta^3 v^2 e^{-\beta^2 v^2} \]

where \( \beta = \sqrt{\frac{m}{2kT}} = \text{constant} \)

Now compute the current to the probe due to particles in a particular small velocity interval \( dv \). Let the mean velocity in the interval be \( v \). We then have a situation analogous to the one treated previously, i.e. effectively a single particle velocity with the density of particles of this velocity being given by \( \frac{dN}{dv} \) dv rather than
N as in the previous case. We then have

\[ \int_0^\infty \frac{dN}{dv} \frac{4 \pi a^2}{4 \pi r^2} \, 4 \pi r^2 \, dr = \frac{dN}{dv} \pi a^2 v. \]

To find \( \int \) to a probe due to particles with velocities between \( v \) and \( v' \) expression (2) is integrated from \( v \) to \( v' \).

\[ \int v \, dv = \int \frac{Av}{4} \, dN \, dv \]

For a probe immersed in a plasma with potential such that electrons of velocity < \( v \) cannot reach it the particle current is given by

\[ \Gamma = \int_0^\infty \frac{Av}{4} \, dN \, dv \]

Where the sheath is infinitely thin the lower limit on the integral is defined by \( -eV = \frac{1}{2} mv^2 \)

where \( e \) = electronic charge

\( V \) = probe to plasma potential (always a negative number)

Using the expression for the Maxwell distribution (4) becomes

\[ \Gamma = \int_0^\infty \frac{AV}{4} \frac{dN}{4} \beta^3 v^2 e^{-\beta^2 v^2} \, dv \]

This integral can be solved by making the substitution \( \beta^2 v^2 = x \)

and integrating by parts.

The result is

\[ \Gamma = \frac{AN}{2 \beta} e^{\frac{-\beta^2 v^2}{2}} \left[ \beta^2 v^2 + 1 \right] \]
If we then substitute for \( v \) in terms of relation (5) and use the relation \( \beta = \left( \frac{m}{2kT} \right)^{\frac{1}{2}} \) we have

\[
\Gamma = \frac{AN}{2\sqrt{\pi}} \sqrt{\frac{2kT}{m}} \ e^{\frac{eV}{kT}} \left[ 1 - \frac{eV}{kT} \right] \tag{6}
\]

Equation (6) gives \( \Gamma \) in terms of the probe to plasma potential \( V \).

That the above expression is correct can be checked by calculating \( \Gamma \) when \( V = 0 \). The result should be

\[
\frac{NA\bar{v}}{4}
\]

as in the simple case treated at the beginning of this appendix.

Substitution of \( \bar{v} = \sqrt{\frac{2kT}{m}} \) which is the expression for the average velocity of a Maxwellian distribution into equation (6) gives

\[
\Gamma = \frac{NA\bar{v}}{4} e^{\frac{e\bar{v}}{kT}} \left[ 1 - \frac{e\bar{v}}{kT} \right] \tag{7}
\]

which reduces to

\[
\Gamma = \frac{NA\bar{v}}{4}
\]

for the case where \( V = 0 \).

Equation (7) in terms of amperes is given by

\[
i = \frac{NA\bar{v}}{4} e^{\frac{e\bar{v}}{kT}} \left[ 1 - \frac{e\bar{v}}{kT} \right] \tag{8}
\]

This differs from (1) by the factor \( \left[ 1 - \frac{e\bar{v}}{kT} \right] \).

We now derive an expression for the electron temperature in terms of (8)

\[
\frac{di}{dv} = \frac{NA\bar{v}}{4} e^{\frac{e\bar{v}}{kT}} \left[ \frac{-e}{kT} + \left( 1 - \frac{e\bar{v}}{kT} \right) \frac{e}{kT} \right]
\]

\[
= \frac{NA\bar{v}}{4} e^{\frac{e\bar{v}}{kT}} \frac{e}{kT} \left( - \frac{e\bar{v}}{kT} \right)
\]

\[
= \frac{i \left( \frac{e}{kT} \right) \left( - \frac{e\bar{v}}{kT} \right)}{\left( 1 - \frac{e\bar{v}}{kT} \right)}
\]
We can then write

\[
\frac{kT}{e} = -i \frac{dV}{di} \left[ \frac{eV}{kT - eV} \right]
\]

(9)

where \( V \) is as assumed previously a negative number.

This is in contrast to the expression for the temperature derived from (1) which is

\[
\frac{kT}{e} = i \frac{dV}{di}
\]

(10)

Unfortunately (9) cannot be used to calculate the electron temperature since \( V \) is ordinarily not known.

Equation (10) will give approximately the correct result only if \( eV > kT \).

For the case of the double probes, which collect only a small fraction of the saturation current \( \frac{eV}{kT} \) will be of the order of 4 or 5 and hence the error should be 20% or less when equation (10) is used to calculate the temperature.

For the case of a single probe, where the electron current to the probe is much higher, an error of about 50% may be expected when the electron temperature is calculated from the slope at the centre of the probe characteristic, which is a rather serious error.
APPENDIX II

The measured electron temperature versus time for hydrogen and helium afterglows are tabulated below for various pressures. Two sets of data are given for the same pressure in some cases to illustrate the lack of reproducibility in some of the results.

A. ELECTRON TEMPERATURE IN HELIUM AFTERGLOWS

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<th>Time (microseconds)</th>
<th>Electron Temperature (e.v.)</th>
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Electron Temperature Data (cont'd.)

Pressure = 34.5 microns Hg.

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Electron Temperature Data (cont'd.)

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B. ELECTRON TEMPERATURE IN HYDROGEN AFTERGLOWS

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Electron Temperature Data (cont'd.)

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Pressure = 1.1 microns Hg.
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