INVESTIGATION OF SPRING VALVES

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ABSTRACT

This thesis presents a study of spring valves with the ultimate aim of using them as high pressure hydraulic flow-control valves for agricultural applications. A closed helical coil spring, when pressurized, will provide a controlled flow between the coils when extended or laterally deflected a given distance. Tests revealed severe longitudinal oscillations when operating in the extension mode, resulting in a very low fatigue life. The lateral deflection mode did not exhibit these oscillations but did exhibit a surge phenomenon at higher pressures.

A four-way closed center spring valve was built using four externally pressurized springs operating in the lateral deflection mode. A unique cable actuation system provided good proportional control with minimum deadzone and tolerancing. Tests showed that performance was acceptable up to approximately 1,000 psi but surges at higher pressures deteriorated performance.

As a result of this study it is concluded that spring valves have applications at moderate pressures but operation at higher pressures is not practical.

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LIST OF SYMBOLS

A	Increase in circumferential area (or open area).
A_D	Increase in circumferential area due to spring deflection.
A_e	Increase in circumferential area due to spring extension.
A_i	Initial circumferential area.
A_o	Orifice area at end of spring.
A_T	Total circumferential area of deflected spring.
A_{TV}	Total circumferential area due to deflection in shear.
A_V	Increase in circumferential area due to deflection in shear.
a	Cross-sectional area of wire.
B	A constant used for calculating A_D.
C_1	A couple which is dependent on θ and is applied to a coil of a spring.
C_2	A couple which is independent of θ and is applied to a coil of a spring.
C_d	An arbitrary coefficient of discharge.
C_F	A couple replacing F_a and L_2.
D_h	Hydraulic diameter.
E	Modulus of elasticity.
$(EI)_s$	Equivalent (EI) of a spring acting as a beam.
F_a	Actuation force required to deflect a spring.
F_A	Axial force acting at any point on a coil.
F_F	Equivalent actuation force at free length (L_1).
F_H	Horizontal force acting at any point of a coil.

F_v	Vertical force acting at any point of a coil.
G	Modulus of rigidity.
g_c	Gravitational Constant.
I	Moment of inertia.
J	Polar moment of inertia.
K	Spring rate (or spring constant).
L_o	An arbitrary fixed length along a spring.
L_1	Free length of the spring.
L_2	Length beyond L_1 at which force is applied to spring to cause deflection.
l_c	Arc length of a portion of a coil.
M_1	A constant moment used in calculating $(EI)_S$.
M_b	Moments acting on equivalent beam.
M_v	Vertical moment acting on a coil.
m	An integer for setting θ'.
N_r	Reynolds number.
n	Number of active coils in free length of spring.
P_d	Drain pressure.
P_s	Supply pressure.
Q	Hydraulic flow through a spring.
R	Mean radius of a coil.
R'	An arbitrary radius less than or equal to R.
R_s	Radius of curvature of equivalent beam.
r	Radius of wire.
T	Torques acting on a coil.
t	Time in seconds.
U	Internal strain energy.

V	Velocity of the oil ($V = Q/A$).
V_H	Horizontal shear forces acting on a coil.
V_V	Vertical shear forces acting on a coil.
x	Variable length along a spring (used to integrate over).
Δ	Deflection of spring at point of force application.
Δ_F	Deflection of spring at free length.
Δ_V	Vertical deflection of coil.
ΔP	Pressure drop across a spring.
θ	Angular position about a coil from point of application of F_V.
θ_1	Angular deflection of coil (a function of θ).
θ_2	Angular deflection of coil (independent of θ).
θ_T	Total angular deflection of equivalent beam.
θ_u	Angular deflection (or slope) at any point along the equivalent beam.
θ_V	Angular deflection of equivalent beam in shear.
θ'	Limit of integration for θ_1 or θ_2.
μ	Poisson's ratio.
μ_o	Absolute viscosity of oil.
ρ	Density of oil.
ν	Kinematic viscosity ($\nu = \mu_o/\rho$).

CHAPTER I

INTRODUCTION

1.1 General

As the demands on hydraulic control systems have become increasingly severe the emphasis has generally been on high dynamic performance rather than low cost. The primary reason for the high cost of conventional control valves is the close machining required to minimize the leakage and accurately meter the flow. This in turn requires elaborate filtering in order to remove foreign particles from the oil that are likely to wear the spool surfaces. Many applications, such as in the agricultural industry, do not need high dynamic performance as much as for the system to be reliable, low cost and able to operate in dirty environments. Agricultural equipment most commonly uses four-way manual on-off spool valves. Although relatively cheap to manufacturers in large quantities, they still require good filtering and also, being on-off, it is often difficult to attain satisfactory position control. It would, therefore, be desirable to develop a low cost proportional flow-control valve which is not dependent on close tolerances.

Several advantages of hydraulic control systems over electrical or mechanical systems are:

1. They exhibit a large torque to inertia ratio.
2. They exhibit a large mechanical stiffness.
3. The fluid carries away heat that is generated.
4. The fluid acts as a lubricant giving longer life to mating mechanical surfaces.
5. Fluid power is easily transmitted over medium distances.

For these reasons, hydraulic control systems have been used in ever expanding roles in various industries. Unfortunately, there are also disadvantages such as:

1. Components can be more expensive because close machining tolerances are required.
2. Components are susceptible to dirt and other contamination.
3. Elaborate power supplies are required.
4. Hydraulic fluids are a fire hazard.
5. Such systems have nonlinear operating characteristics.

1.2 Literature Survey

Some development work has been done on low-cost valves for use in low pressure pneumatic systems. Lichtarowicz and Kiessling [(1,2)*] have developed pneumatic valves using the principle of deforming '0' rings to control the flow around a rod. It is felt, however, that this principle could not be successfully applied to a high pressure hydraulic valve. The '0' rings would deteriorate rapidly under high pressure oil and would require considerable deadzone for positive shutoff.

A more practical proportional device which has had some success in hydraulic control valve applications is the fluidic vortex valve [(3)]. It is simple, rugged and cheap but also has some disadvantages. Positive shutoff is not possible since the outlet flow is equal to the control flow when the supply flow is shut off, resulting in a continuous power consumption. The control pressure also needs to be larger than the supply pressure. A vortex valve is, of necessity, a two-stage valve since some device

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* Superscribed numbers in parentheses refer to references in the List of References

must be included which meters the control flow. Spring valves have been used for this purpose [3] and were found to give satisfactory results. It therefore appeared logical to develop a four-way single-stage flow control valve using only spring valves.

Spring valves were first proposed as pneumatic valves by De Bruyne [4]. Their operation is based on the principle that a closely coiled helical spring will allow a controlled flow between the coils whenever the spring is extended or laterally deflected a given distance (Figure 1). The small input displacement characteristics are similar to flapper nozzle valves but no close tolerancing is required and physical damage is therefore less likely. Deadzone is not a problem and there is very little leakage when the spring is closed. Supply pressure can be applied either externally or internally provided pretension is sufficient to hold the spring closed. The valves are easily manufactured by soldering one end of a spring rigidly in a hollow plug while the other end is closed by means of a solid plug. Specific springs can readily be chosen to meet particular requirements.

1.3 Objectives

The ultimate aim of this study was to develop a simple, low-cost, single-stage, four-way hydraulic flow-control valve using helical coil springs. Many modern hydraulic control systems operate with a maximum supply pressure of 2,500 psi and a corresponding flow of about 8 gpm. Therefore, a control valve, using springs, should also be able to operate in this range. It is also desirable to have automatic positive shutoff which, in the case of spring valves, implies external pressure acting on the springs. There should also be minimum deadzone during actuation. All of these objectives should be achieved with a minimum of close tolerancing.

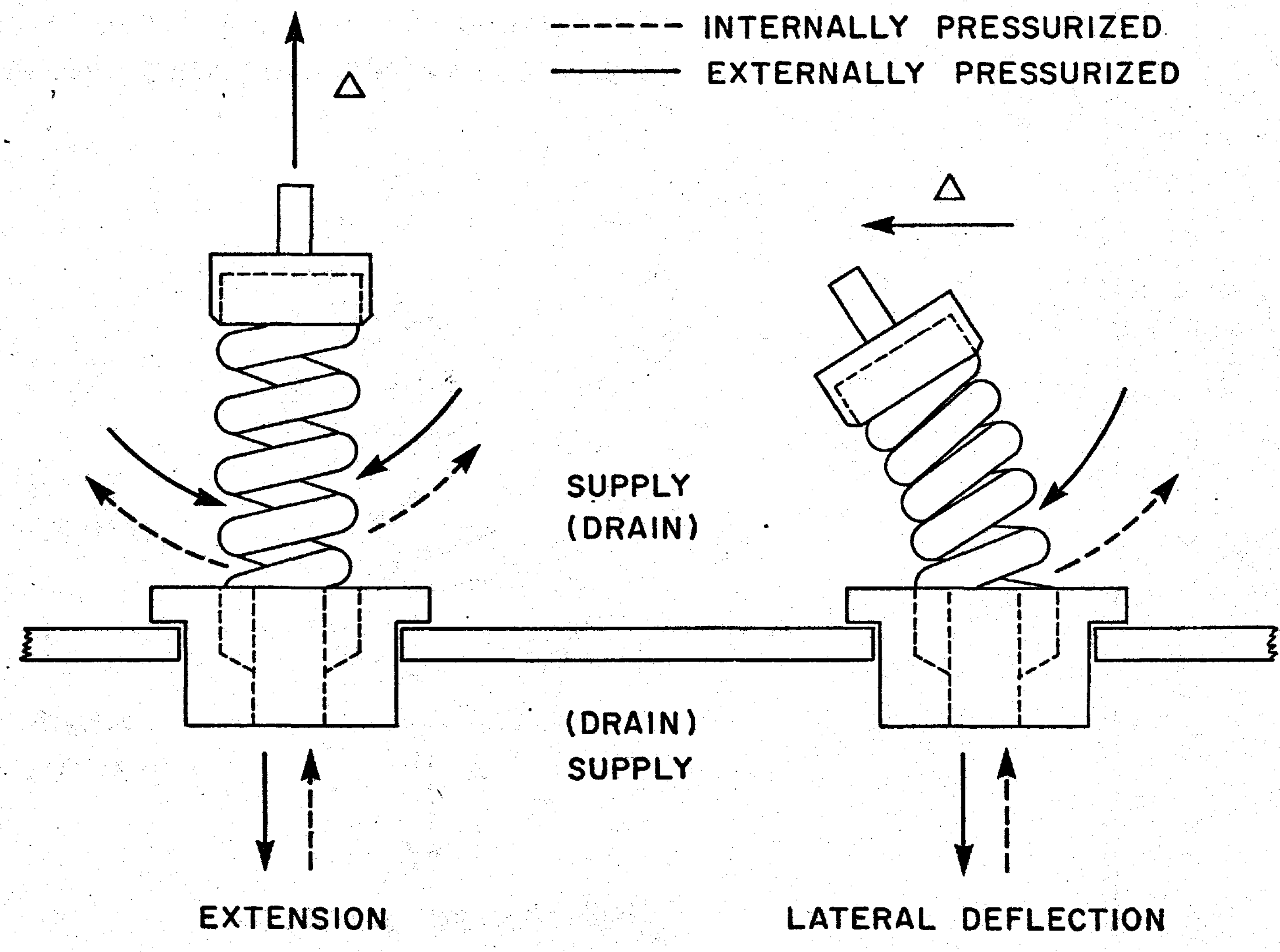


Figure 1. Schematic of a Single Spring Valve

The aims of the investigation were thus to:

1. determine the best type of spring configuration.
2. develop and evaluate a simple four-way high pressure flow control valve using helical coil springs.

CHAPTER II

THEORETICAL DEVELOPMENTS

In order to understand the significance of the experimental results, a theoretical analysis of the springs first had to be done. Of greatest importance was the need to know what the effect of any given force acting on a spring would be. By assuming a linear relationship between all forces and the corresponding deflections, several equations were developed by using energy methods (see Appendix A and B).

All forces were assumed to act on a portion of a single coil. Longitudinal forces gave rise to an extension of the spring or an angle of twist of a coil. By considering the spring as an equivalent beam and equating the angles of twist, an equivalent $(EI)_s$ for a spring was calculated[5,6,7].

By using $(EI)_s$ it was possible to calculate the deflection of any portion of a helical coil spring assuming it acted as an equivalent beam (Figure 2). It was assumed that lateral forces acting on the coils were negligible; therefore, the equivalent shear was also negligible. Similarly, all deflections were assumed sufficiently small so that linearity in the calculations was valid. For any given lateral deflection the angle of twist, and hence the radius of curvature, could be calculated along the length of the spring. Integrating the mean circumference of the spring over its arc length gave the circumferential area of the spring for any given deflection. Subtracting the initial area of $2\pi RL_1$ from this calculated area yielded the area of interest; that is, the increase in the circumferential area between the coils as a function of

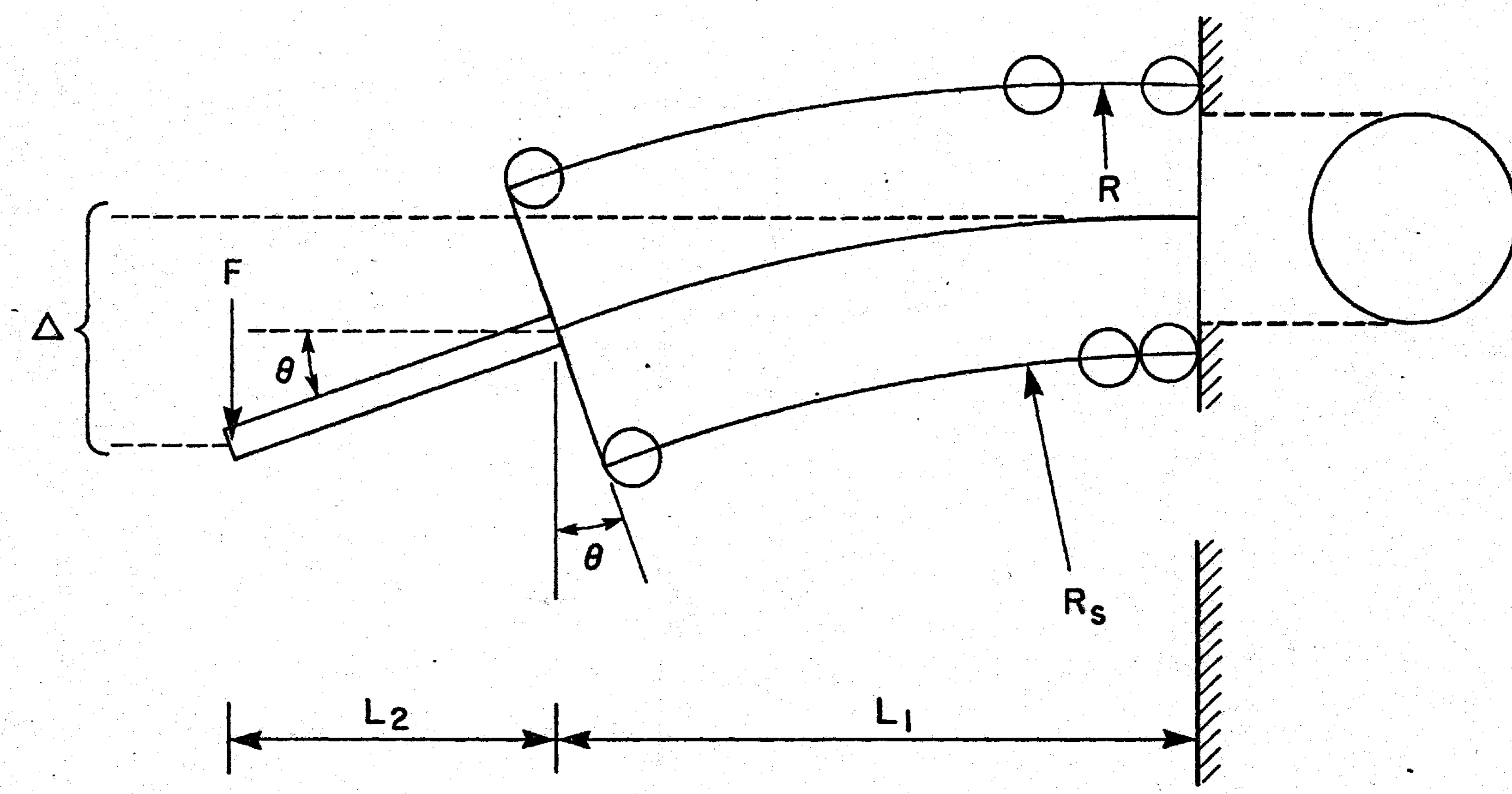


Figure 2. A Spring as an Equivalent Beam

the lateral deflection (A). This was found to be:

$$A = \Delta \left\{ \frac{\pi R^2 \left[L_1 + 2L_2\right]}{\left[\left(\frac{L_1^2}{3}\right) + L_1 L_2 + L_2^2\right]} \right\} \qquad (1)$$

If Δ is considered to be the longitudinal extension of the spring, the equation for the increase in the circumferential area would then be:

$$A = \Delta\, 2\pi R \qquad (2)$$

Having calculated the increase in the circumferential area of a spring it was then possible to determine the equations of flow through that spring [8]. From Bernoulli's equation:

$$Q = C_d\, A \sqrt{(2g_c/\rho)(\Delta p)} \qquad (3)$$

All terms in the equation could be readily obtained with the exception of the coefficients of discharge for the springs. These had to be experimentally determined. This was best done by plotting curves of the coefficients of discharge versus Reynolds numbers for various springs. The coefficients of discharge were measured by rewriting Equation 3 as follows:

$$C_d = \frac{Q}{A \sqrt{(2g_c/\rho)(\Delta p)}} \qquad (4)$$

In order to calculate the Reynolds number a suitable characteristic length needed to be defined. A quantity called the hydraulic diameter (D_h) was defined as follows [8]:

$$D_h = \frac{4\ \text{(the area through which the fluid flows)}}{\text{(the perimeter surrounding that area)}} \qquad (5)$$

For a helical coil spring this reduced to:

$$D_h = \frac{2\ A\ r}{\pi\ R\ L_1} \qquad (6)$$

Therefore the Reynolds number was:

$$N_r = \frac{V\,D_h}{\nu}$$

The derivation of these equations was necessary in order to analyse the results of the experimentally obtained flow-pressure curves. Once the characteristics of individual spring valves were defined it was then possible to define the performance of any possible combination of spring valves, such as in a four-way valve. Manifold and line losses could be easily taken into account by again using Bernoulli's equation or by modifying the coefficients of discharge used to calculate the performance of the individual springs. By linearizing these results about an operating point it would be possible to obtain a transfer function for a four-way spring valve.

Calculations of the undamped natural frequencies of springs was also obtained but was of little practical use for the following reasons. The forcing function, which is due to the flow passing by the coils, does not act at one point on the spring but is distributed along its length in some manner. Damping would also be significant due to the high viscosity of oil as compared with air. It would be difficult to adequately define a realistic damping term in order to calculate the damped natural frequency of a spring.

A number of other calculations were also done but not included since they are of only minor importance. The total force required to deflect a spring was calculated assuming that the physical characteristics of the spring and the flow and pressure forces acting on it were known. Mechanical operating limits were also calculated since the stresses which occur whenever a spring is deflected determines the life expectancy of that spring. For a laterally deflected spring the maximum stresses occur in the first coil since that is where the maximum bending moment occurs. From this the maximum allowable forces, deflections or areas can be readily calculated. Fatigue life calculations can also be done and would tend to reduce the maximum allowable stresses for any given life expectancy. Techniques are readily available for performing the above calculations;[5,6,7] however, the results should be viewed with caution. Because of the many simplifying assumptions that have to be made, the theoretical solutions should only be used to give an indication of the order of magnitude of the actual results. The actual performance of spring valves can only be determined by experimentally testing individual springs and noting where the results differ from those predicted by theory.

CHAPTER III

PRELIMINARY TESTS

3.1 Extension Mode Tests

Having completed the necessary theoretical developments it was then possible to run preliminary experimental tests in order to determine which type of spring configuration would be most suitable for use in a four-way spring valve. A high pressure filter casing was modified (Figures 3, 4, 5 and 6) so that a variety of springs could be extended or laterally deflected any desired distance. It was also possible to pressurize the springs either externally or internally. Instrumentation allowed the flowrate and absolute or differential pressure across the spring to be measured. A variety of springs (Table I) were tested in various modes of operation.

It was initially felt that extending the springs would be more practical than laterally deflecting them. The flow between the coils would be symmetrical and therefore probably give better performance. Also, the extensions needed for any desired coil opening would be small and therefore a four-way valve could be built as a compact unit. The experimental results quickly proved that this arrangement was impractical.

All of the springs tested in the extension mode were observed to enter a high frequency resonance at extensions which were necessary in order to produce the desired flow rates. All possible external sources which may have contributed to the resonance were minimized, such as removing all air from the system and disconnecting the accumulators.

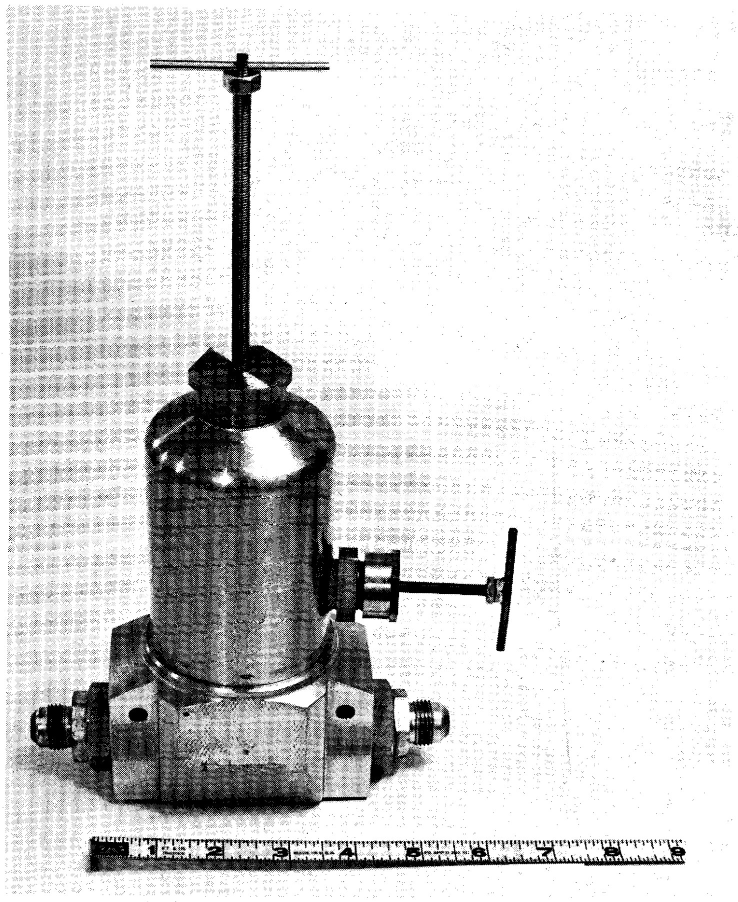


Figure 3. High Pressure Test Apparatus

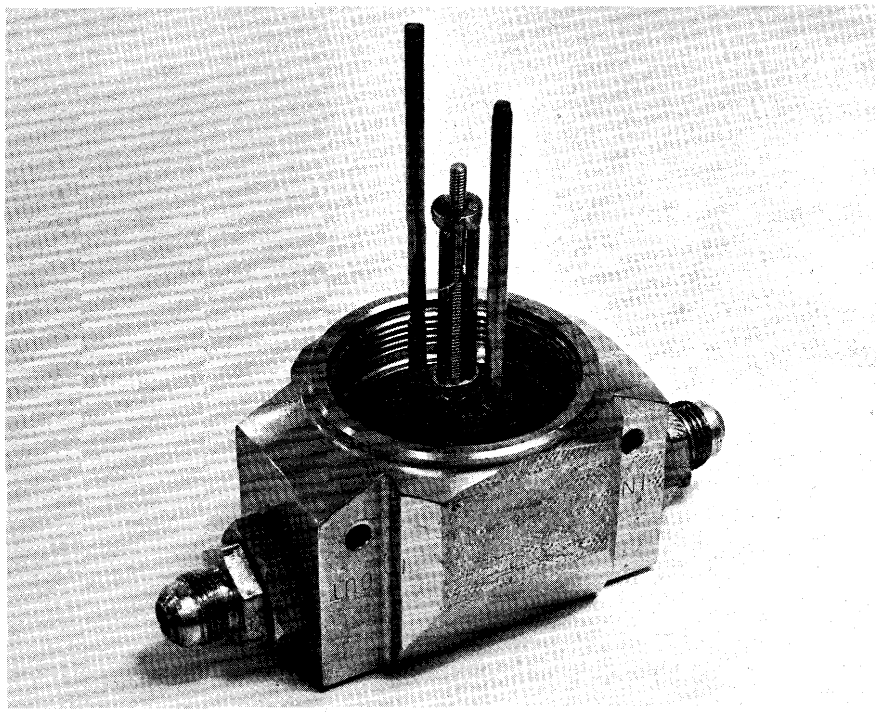


Figure 4. Base of Test Apparatus

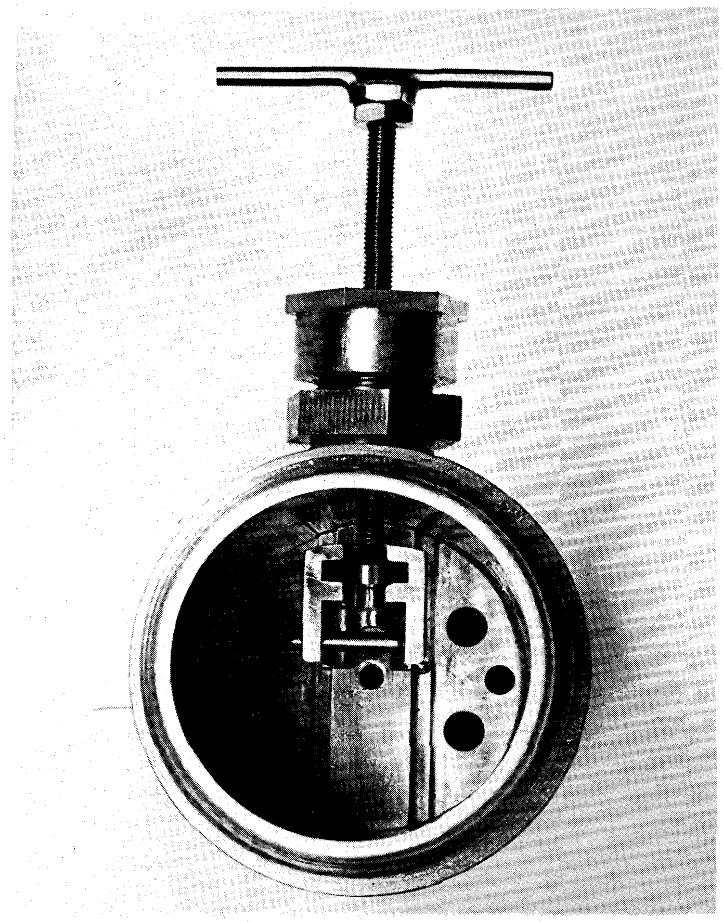


Figure 5. Casing of Test Apparatus

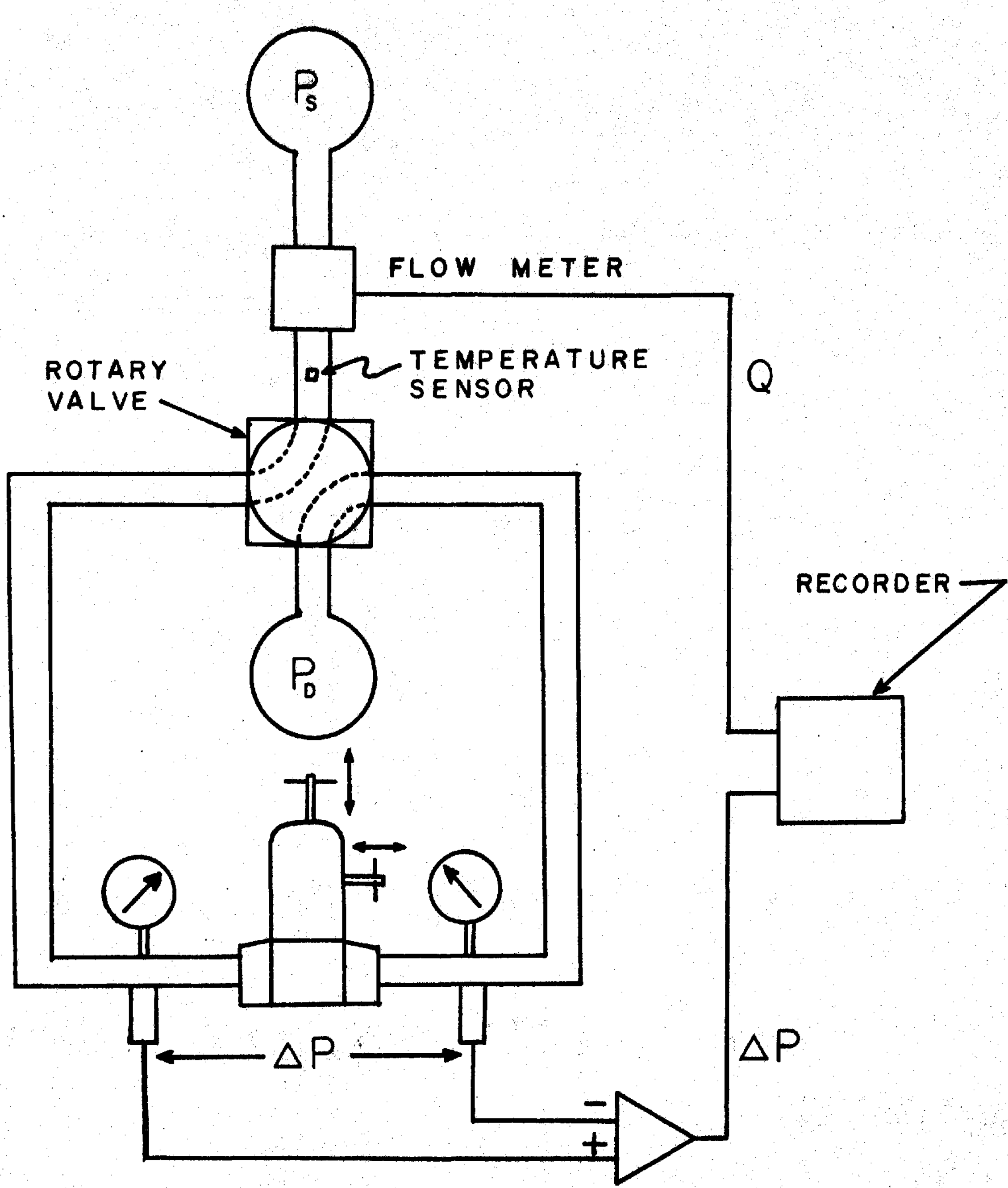


Figure 6. Preliminary Testing Arrangement

Table I. Springs Used in the Preliminary Tests

Spring Number	Outside Diameter (in)	Wire Diameter (in)	Free Length, L_1 (in)	Spring Rate Per Coil $\left(\frac{lb_f}{in/coil}\right)$	Mode of Operation	Comments
1	0.125	0.0263	0.7	670	D	rapidly saturated
2 a	0.185	0.03275	0.5	427	E,D	rapidly fatiqued due to oscillations in extension mode
b			0.1			
3 a	0.1875	0.031	1.0	387	E	-ΔP=1/3 Ps, higher frequency oscillations at higher Ps. 3 guides installed.
b			0.2		E	-sudden surge noted at higher Ps as well as the high frequency oscillations. 3 guides installed.
c			0.2		D	-gave good control.
4	0.240	0.051	0.5	685	D	gave good control.
5	0.250	0.026	1.2	63.5	E	3 guides installed oscillations and collapsed at high Ps due to coils overlapping.
6 a	0.250	0.031	0.4	138	D	surge increased at larger Ps and had negligible hysteresis on return.
b			0.2		E	slight leakage flow on return but eliminated by reducing Ps to '0' for instant.
7 a	0. 250	0.037	0.8	300	E	Small flow, then oscillations, then approached saturation without oscillations. Magnitude of oscillations decreased as L_1 decreased. Worse for external Ps than internal Ps.
b			0.5		E	
c			0.2		E	
8 a	0.255	0.046	0.43	466	E	-surge present at higher Ps. Oscillations worse for internal Ps than external Ps.
b			0.43		D	-worked satisfactorily with no surge.
9	0.30	0.050	0.4	400	D	pronounced surge; rapid step response with some overshoot.
10	0.340	0.03516	2.8 2	81.7	E,D	collapsed due to pressure ;lateral and longitudinal oscillations.

D = Lateral deflection mode of operation (only pressurized springs externally).
E = Extension mode of operation (pressurized springs externally and internally).

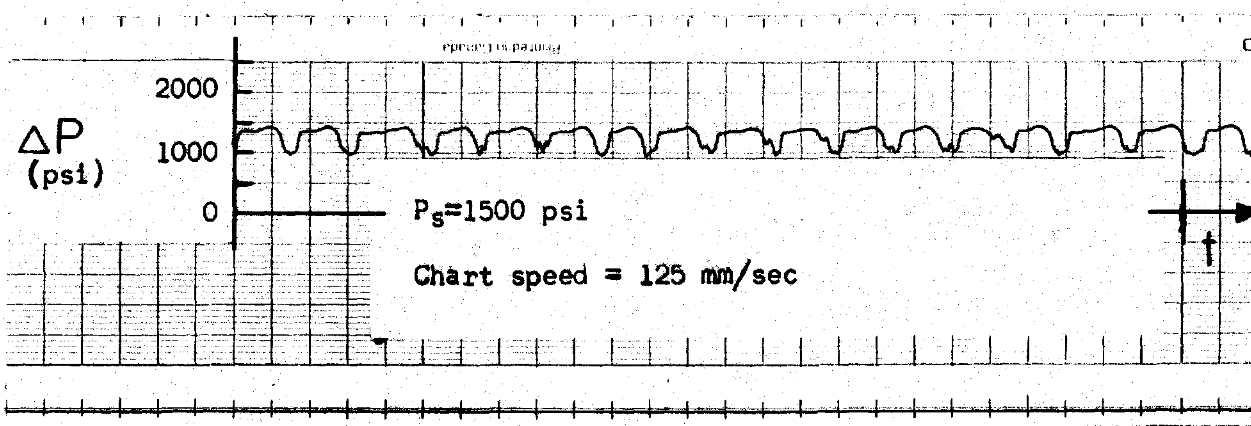
All springs tested up to 1500 to 2000 psi

Nevertheless, the high frequency oscillations persisted. A test on one particular spring (Table I, #3a) showed differential pressure oscillations in the order of 15.7 H_z with an amplitude of about 450 psi when the supply pressure was set at 1,500 psi (Figure 7). At a supply pressure of 2,000 psi the differential pressure oscillations occurred at about 115 H_z and 500 psi pressure amplitude.

In the majority of the tests the oscillations were considerably more severe when pressurized externally than when pressurized internally. It was observed that the magnitude of the oscillations was proportional to the length of the spring, the coil diameter and the supply pressure. All of the springs tested were found to have a fatigue life varying from about thirty seconds to ten minutes, which was totally unacceptable. Although an improvement in performance was obtained by using shorter springs, this was not practical since the extension per coil became too large and the springs were then permanently distorted or broken.

A general pattern was noted in all of the tests as the springs were slowly extended, then returned to the closed position. As the spring was first cracked open, no oscillations were present. A slight increase in the extension produced sudden oscillations. These oscillations continued, but diminished, with increasingly larger extensions. The reverse process took place as the spring was closed.

It was felt that the oscillations may have been lateral so three close-fitting guides were placed around a spring. Free extensional motion was still allowed. Upon pressurizing it was found that the pressure oscillations were as severe as before. As mentioned previously, the oscillations were less severe when the spring was internally pressurized, but this mode of operation was not felt to be practical for use in a flow



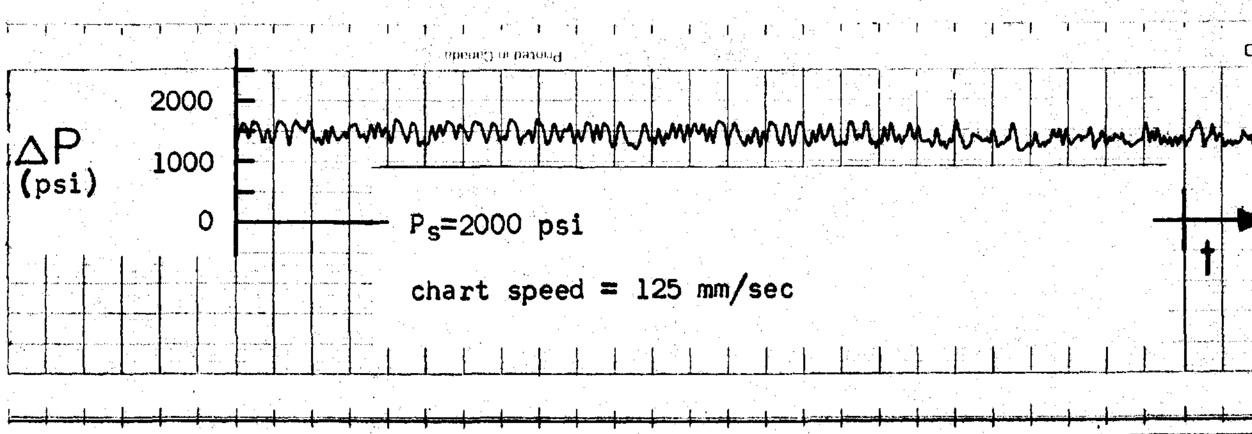


Figure 7. Oscillations of an Externally Pressurized Spring in the Extension Mode (Table I, #3a)

control valve. Considerable force (under internal pressurization) had to be applied to prevent leakage flow when completely shut off, particularly for large diameter springs. This would not allow automatic positive shutoff in a four-way spring valve.

It was noticed, however, that if the spring was allowed to extend in an unrestrained manner when internally pressurized, there were no oscillations present. This characteristic implied that an arrangement of this sort could be effectively used as a simple check valve with no need to machine a valve seat as in present check valves. In this case, as in all the above cases, it was found that the stiffer the spring and the larger the wire diameter to coil diameter ratio the better the spring was able to withstand large pressures without distorting.

3.2 Low Pressure Visual Tests

In view of the above extension mode results, it was felt that the oscillations were caused by longitudinal waves travelling from end to end of the spring. In order to check the validity of this assumption and take corrective action, if possible, a low pressure plastic casing was built (Figure 8). The casing could be pressurized to 140 psi using either air or water and the resulting oscillations could be studied with a stroboscope. Visual observations readily indicated that, although random lateral oscillations did occur, the most serious problem was, in fact, due to high frequency longitudinal travelling waves.

The flow forces which caused the above results were not satisfactorily explained by the theoretical equations. Two possible explanations

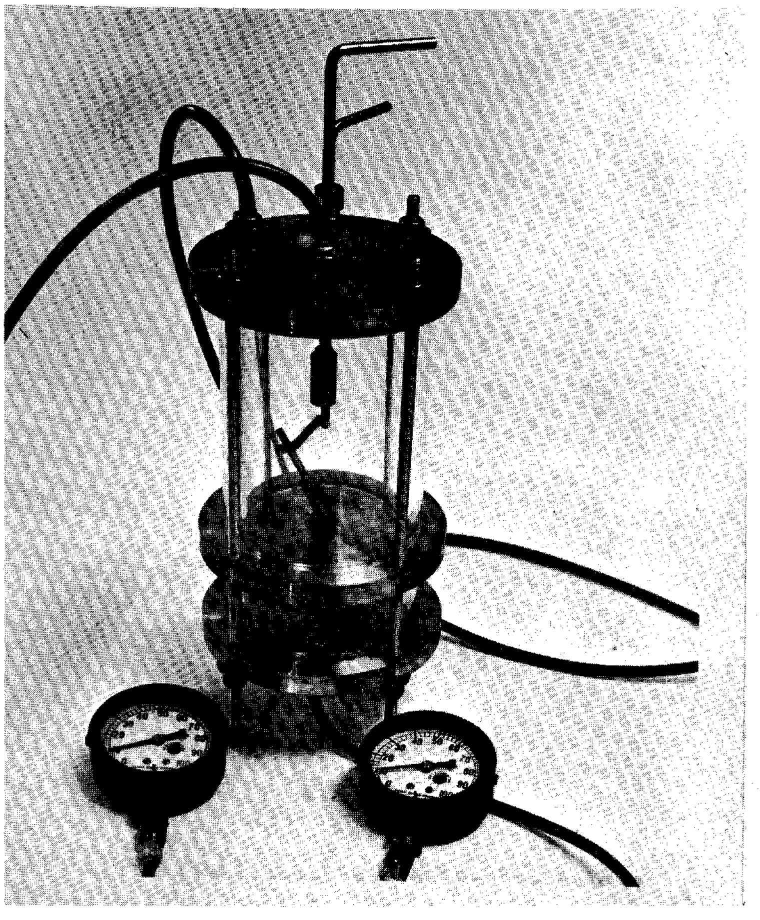


Figure 8. Low Pressure Plastic Casing (For Visual Observations)

can be proposed in an attempt to understand how the flow forces could set up oscillations in the coils. The first, and most likely, is that vortex shedding downstream of the coils generated rapidly alternating flow forces acting on the rear portion of the coils. The second possibility is that the flow between any two coils could rapidly alternate between the laminar and turbulent regimes. At very small openings the flow remained completely laminar and the forces perpendicular to the flow acting on either side of a particular coil were equal. At very large openings the flow was completely turbulent and again these forces were approximately equal. It was felt that the transition from one flow regime to the other caused the problems. For the flow passing any given coil, the perpendicular forces acting on the coil were larger for laminar than for turbulent flow. This forced the coil over, reducing the effective area in the turbulent region and increasing it in the laminar region. At some point the flow regimes reversed because the area changes resulted in changing Reynold's numbers. When this occurred, the forces were reversed and the coils began to move back again. This proposed flow-induced phenomenon was self-sustaining and resulted in high frequency oscillations of the coils. Regardless of the cause of these oscillations they could be readily propogated from coil to coil. This rapidly degenerated into longitudinal waves observed travelling at what was assumed to be the dampened natural frequency of the spring. Calculations of the undampened natural frequency indicated a much higher frequency than was observed (i.e. 300 H_z)(Table I, #3a) (Figure 7).

Tests were then carried out in the lateral deflection mode of operation. Only external pressurization was allowed in this mode.

Random lateral oscillations were observed but there were no longitudinal waves travelling from end to end of the spring. Each coil seemed to be vibrating independently of all the other coils. This was due to the fact that all coils were in point contact with one another and this restraint prevented longitudinal waves from developing. As a result, good proportional control was obtained in the lateral deflection mode without noticeable pressure oscillations in the output. An important aspect of this was that the spring should no longer fatigue after only a few minutes of operation in high pressure oil.

3.3 Lateral Deflection Mode Tests

High pressure tests were then conducted in the lateral deflection mode. As expected, no sign of pressure oscillations could be detected in the output; however, one problem did appear. The sensitivity, which was assumed to be the change of pressure and flow for any given change of deflection, increased as the supply pressure was increased (Figure 9). This was most noticeable at the mid-range of deflections. At larger pressures this degenerated into step changes in the output, noticed as surges in the flow and differential pressure curves. This phenomenon was thought to be due to a pressure-induced change of shape of the spring at medium deflections. It was noticed, however, that the performance of the valve did not seem to depend on the length of the spring as in the deflection mode tests.

Step response tests were also carried out in order to determine how rapidly a deflected spring would automatically shut off at high pressures (Figure 10). This time could be determined by deflecting a spring and

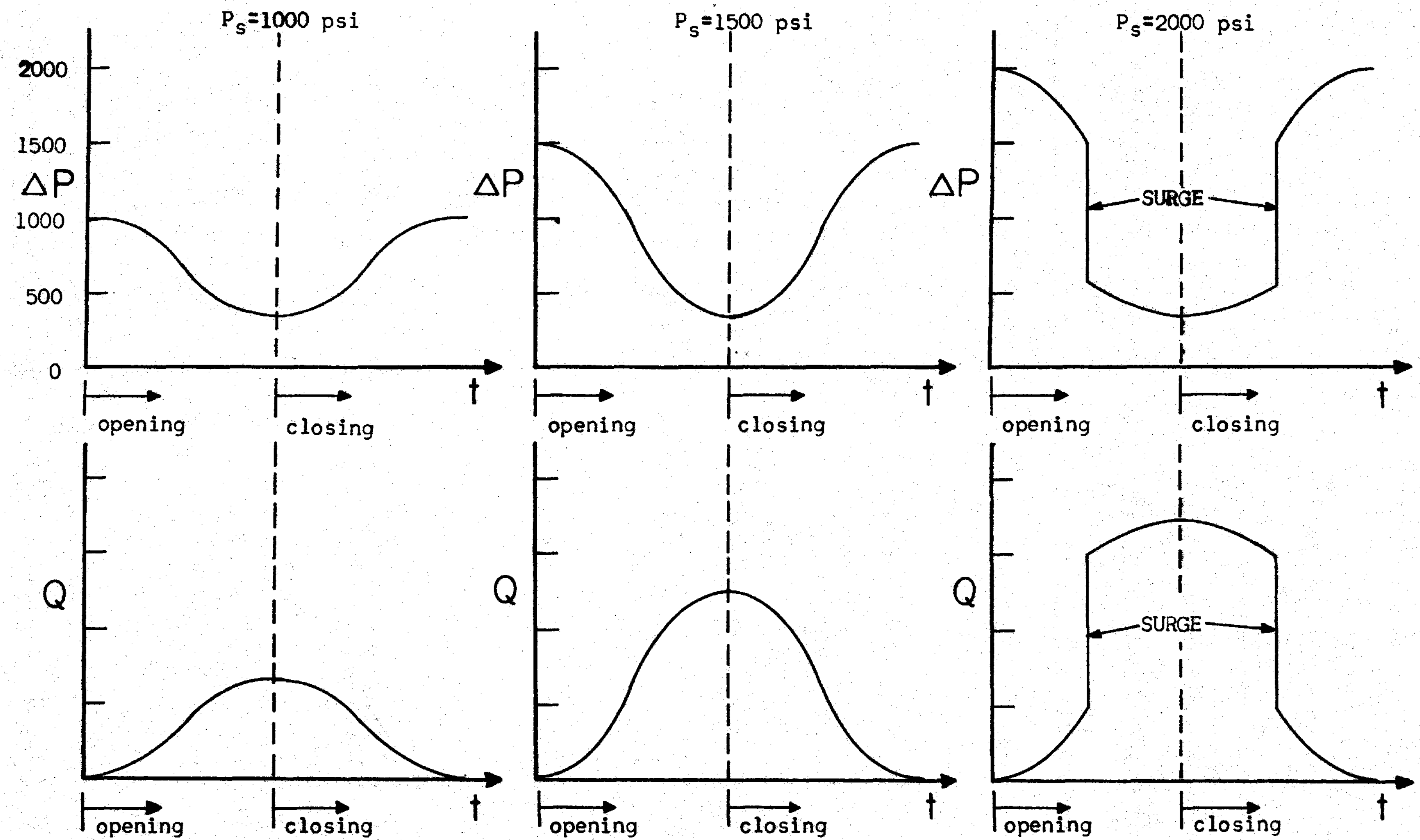
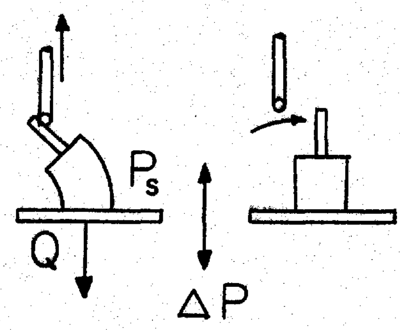


Figure 9. Typical Sensitivity Curves of Flow and Pressure for a Deflected Spring



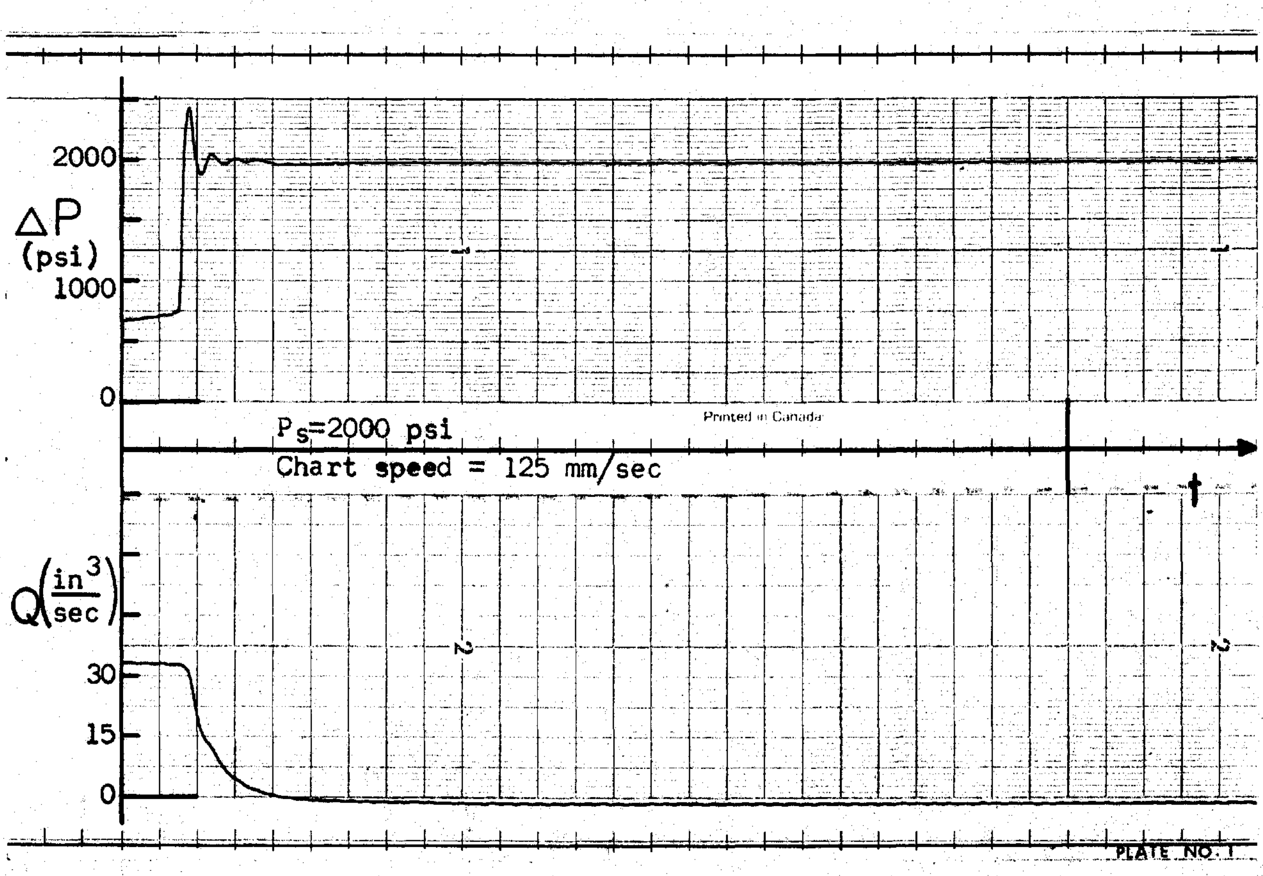


Figure 10. Step Response of a Deflected Spring (Table I, #9)

instantly releasing it. Tests indicated that a spring would completely shut off in less than 0.10 second after passing through about four rapidly decaying oscillations. Detailed frequency response tests were not necessary, however, since manual valves only operate at low frequencies.

3.4 Initial Conclusions

Several conclusions could be drawn from the preliminary tests discussed above. Springs operating in the extension mode would be very unreliable as flow control valves although they could be used as simple check valves. A four-way flow control valve would probably give best results if the springs were operated in the lateral deflection mode. Although surge was a problem it was hoped that this could be largely overcome by using larger diameter springs and deflecting them less to provide any given flow. This would also stress the springs less and therefore tend to increase their expected life. The lengths of the springs should also be kept as short as possible so as to minimize the size of the four-way valve. The optimum size appeared to have a free length about equal to the outside diameter of the springs. A four-way valve could then be designed and built on the lateral deflection mode of operation. Far fewer problems would be present and their possible solutions more easily obtainable.

CHAPTER IV

FINAL DESIGN AND TESTING

4.1 Design of a Four-Way Spring Valve

A simple, inexpensive, four-way, closed center hydraulic flow control valve using four springs operating in the lateral deflection mode was constructed (Figures 11 and 12). The principle of operation was that two cables were wrapped around an actuator shaft and thereby deflected a corresponding pair of springs. These springs metered the flow from the supply, to the load, to the drain. Deflecting the actuator shaft in the other direction deflected an opposite pair of springs and metered the flow in the reverse direction (Figure 13).

The valve was constructed from a solid steel cylinder. A hole to accept the actuator shaft was drilled from end to end through the center of the manifold. At about half the outside radius of the manifold four symmetric chambers, each having a smaller diameter at about half their total depth, were drilled and tapped to accept the plugs holding the springs. Two adjacent chambers of the four were drilled from one end whereas the other two were drilled from the other end (Figures 11 and 12). Load Port 1 was drilled diametrically through the casing so that it intersected the top portion of Chamber 2 and the lower portion of Chamber 4. Load Port 2, at right angles to, but not intersecting load Port 1, was similarly drilled so that the remaining two chambers were interconnected. An offset drain part was drilled so that the bottoms of Chambers 1 and 2 were interconnected. Finally, a large diameter supply port was drilled from the side in such a manner that the actuator shaft hole and the tops

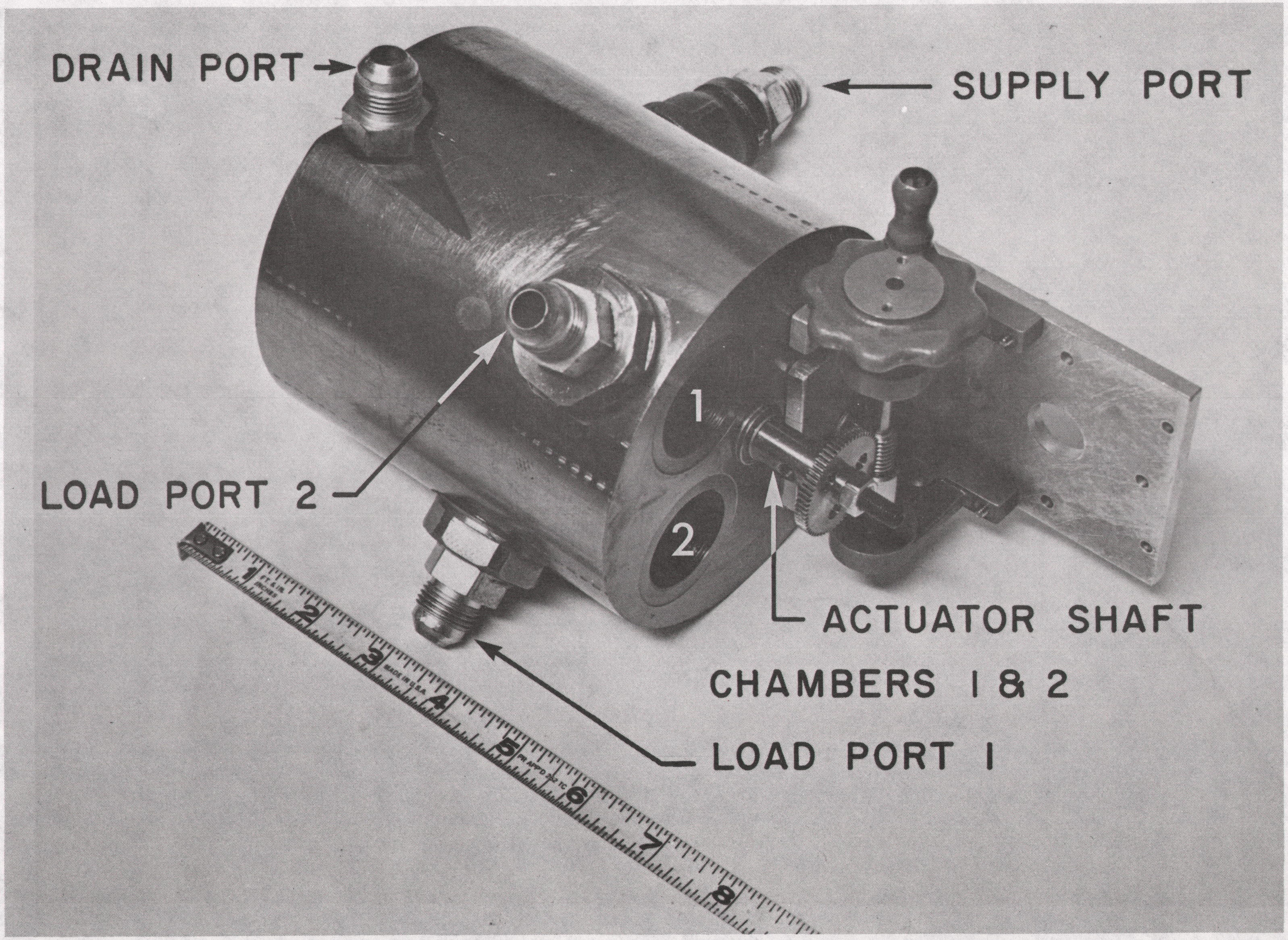


Figure 11. Manifold Front of a Four-Way Spring Valve

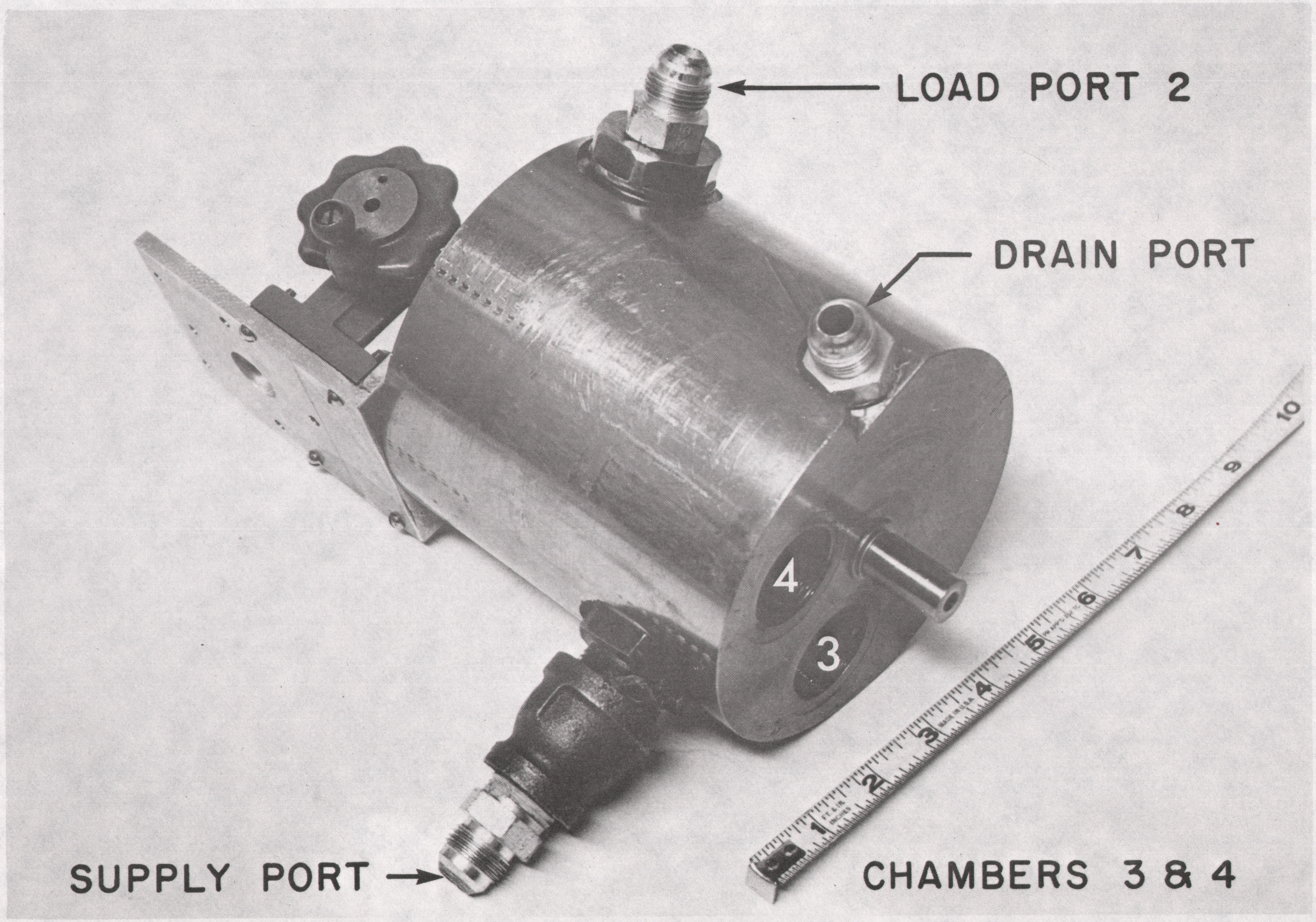


Figure 12. Manifold Rear of Four-Way Spring Valve

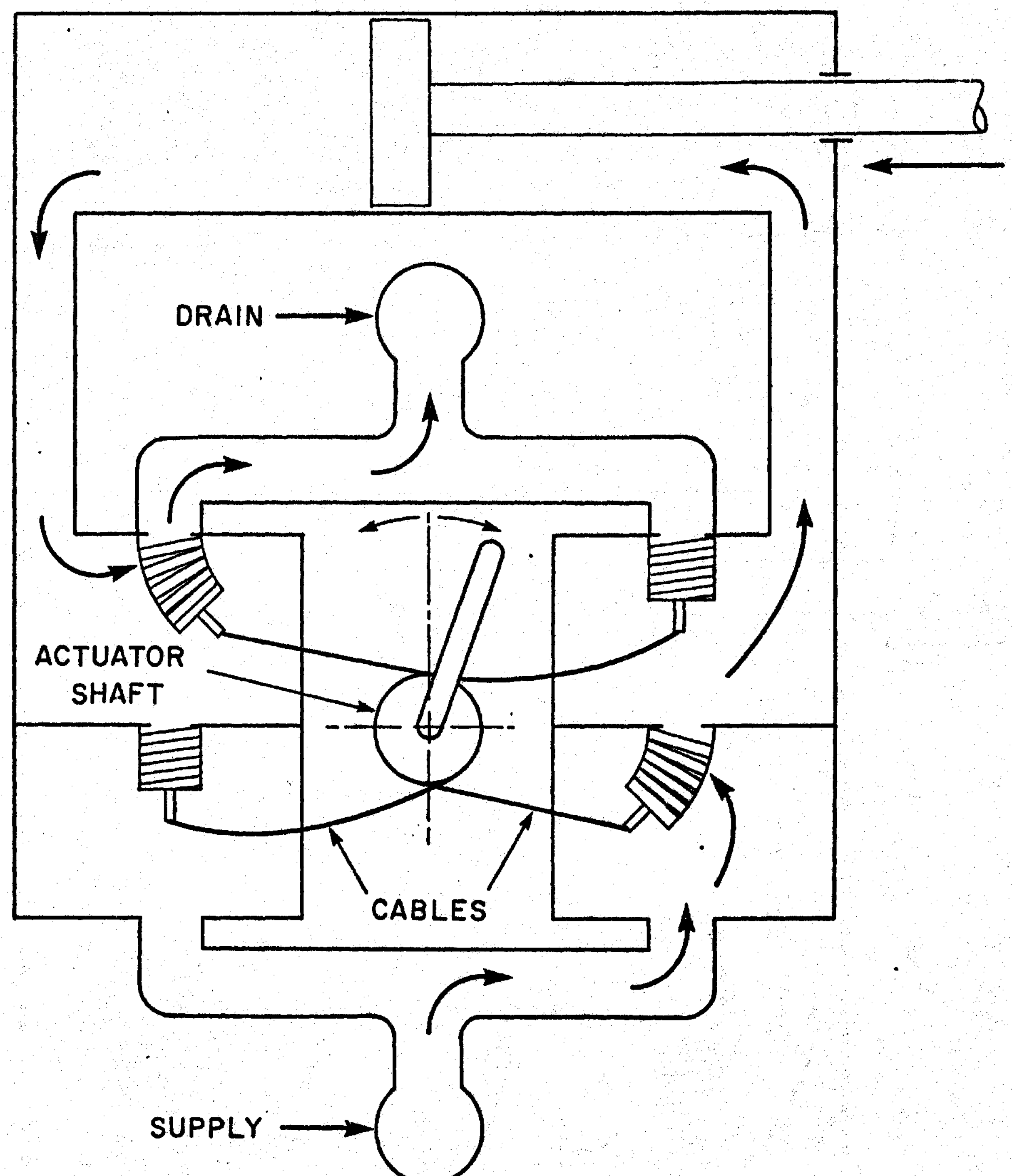


Figure 13. Four-Way Spring Valve Schematic

of Chambers 3 and 4 were all intersected.

Threaded plugs with teflon seals were used to cap the four chambers which contained the springs. By using this type of arrangement there was no need to machine mating faces or use special gaskets or stud bolts in order to assemble the manifold, thereby reducing manufacturing costs. The most unique feature of the design was the method of actuating the springs. The half inch diameter actuator shaft only needed four 'O' rings in order to isolate the ends of the manifold, the load ports and the supply port. This was the only portion of the manifold that required reasonably close tolerancing in order to minimize leakage around the 'O' rings. Since no end forces were present snap rings on each end of the actuator shaft were sufficient to hold it in place. Four small diameter holes were drilled and recessed along the length of the shaft in order to correspond to the tips of the four springs when they were in place. Small diameter steel cables were run through these holes and wrapped around the shaft in the appropriate directions. The actuator shaft was then held in the neutral position while the cables were pulled taut and soldered to the tips of the springs which had previously been inserted into the chambers. In this manner there was minimum deadzone present with no need for close tolerancing.

The four-way spring valve was then mounted and instrumented (Figures 13, 14, 15 and 16). Four quarter-inch diameter springs were used for the first test. The load was a reversible hydraulic motor which gave a good visual indication of the proportionality of the valve. Results of the test were as expected. Proportionality was very good; however, surge became pronounced at pressures above about 1,000 psi. Deadzone, leakage, and the required actuation force all increased somewhat with an increasing

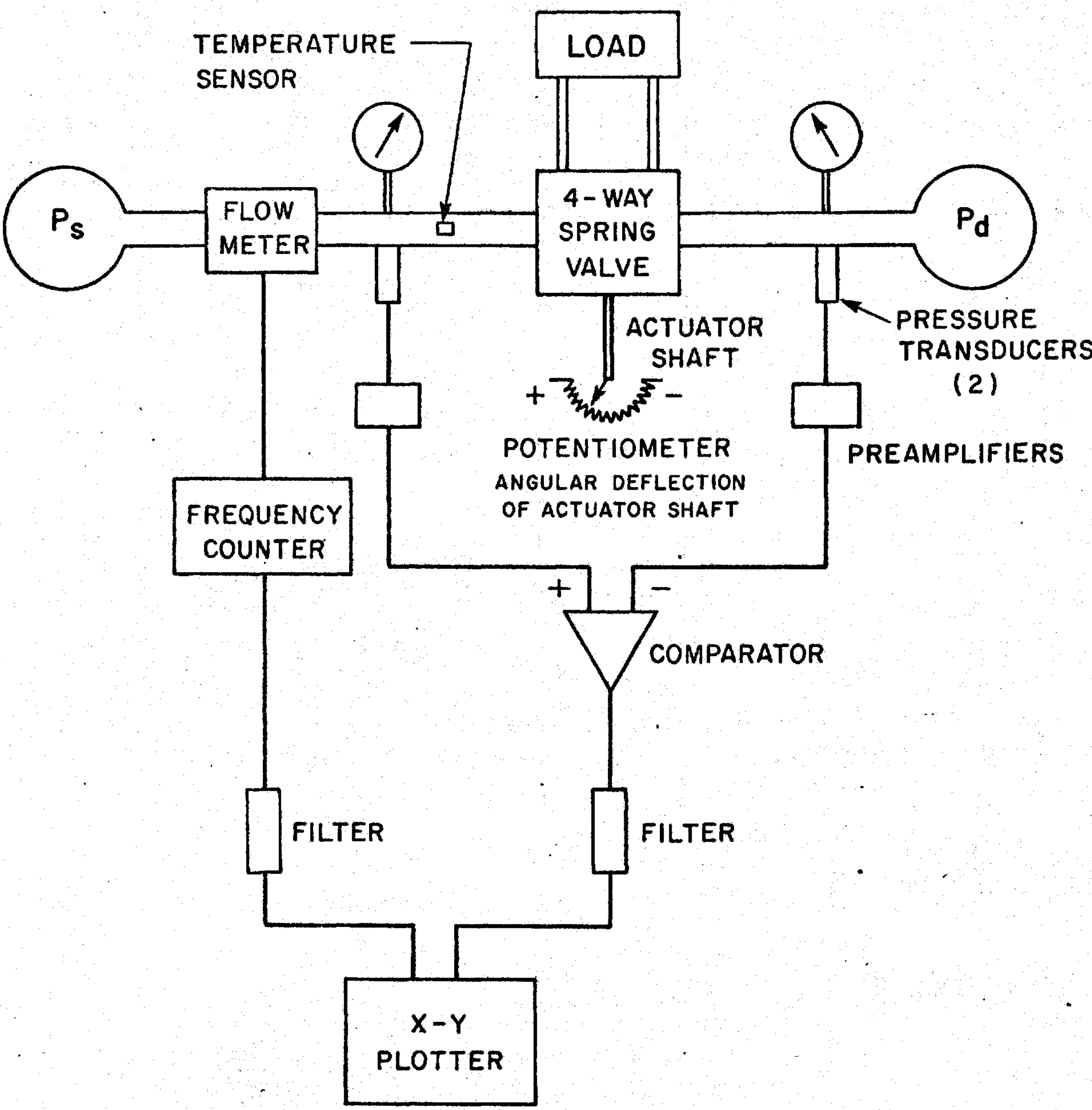


Figure 14. Final Testing Arrangement

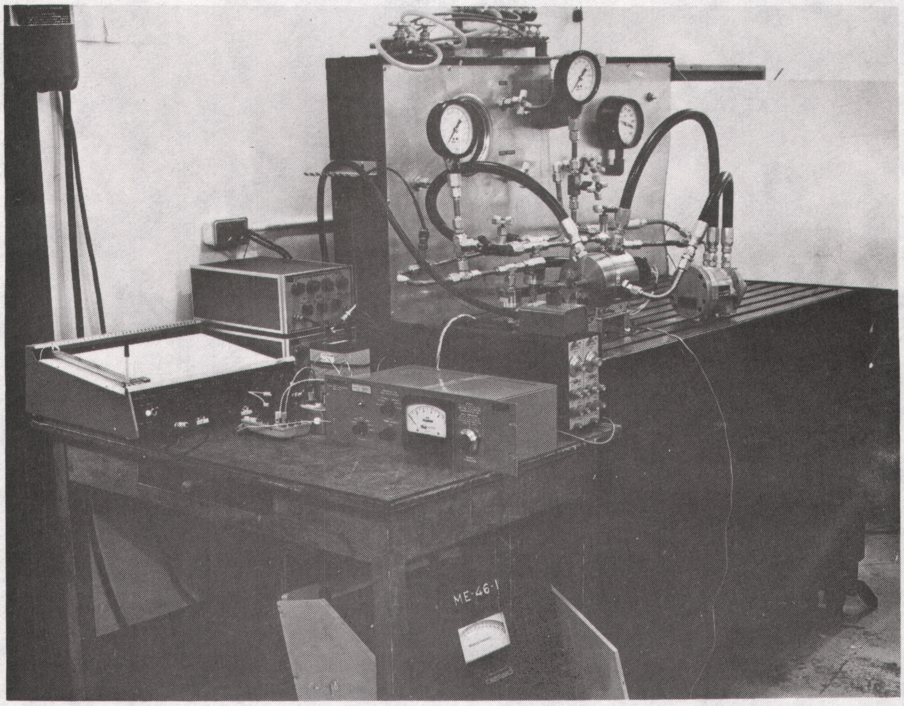


Figure 15. Four-Way Spring Valve Test Stand

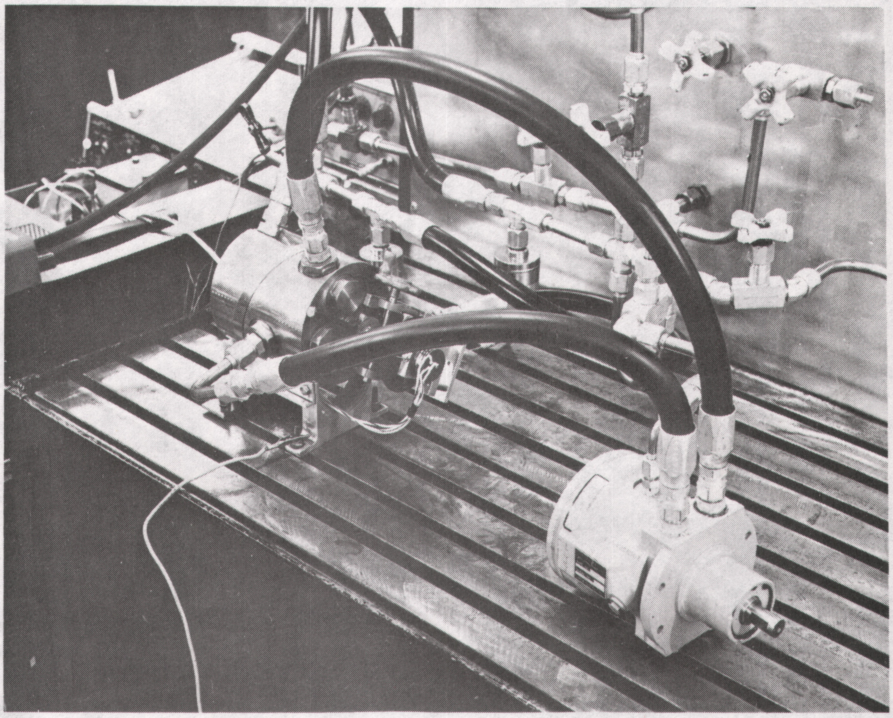


Figure 16. Four-Way Spring Valve and Hydraulic Motor

supply pressure, but they did not appear to be too serious. It was found that the pump could only deliver a maximum of about 8 gpm to the system. This design assumed that the supply pressure exceeded the load pressure which always exceeded the drain pressure. If for any reason this was not so, it would be desirable to place check valves in the supply and drain lines to prevent reverse flow. Having proved that the basic principle of operation worked satisfactorily in a four-way valve, it was then necessary to study the performance of individual springs in detail in order to choose springs which would provide optimum performance.

4.2 Testing Procedure and Results

The manifold was prepared for detailed testing of individual springs by first blocking off the load ports and one of the chambers on the supply pressure side. This forced the flow to follow one path through the manifold. A flow-pressure curve was then obtained in order to determine the losses through the manifold. Individual springs were then placed in the open chamber and flow-pressure curves were again obtained. By subtracting the losses through the manifold it was possible to obtain flow-pressure curves for the spring valve by itself. In this manner it was possible to run tests on a number of springs with varying parameters (Table II) (Figure 17).

Each spring was tested in two phases. First, the spring deflection was set by fixing the angular deflection of the actuator shaft. The supply pressure, and hence the pressure differential, was slowly increased to some maximum value and then decreased to zero (Figure 18). This was performed for several equal increments of actuator angle settings.

Table II. Springs Used in the Final Tests

Spring Number	Outside Diameter (in)	Wire (in) Diameter	Free Length L_1 (in)	Cap Length L_2 (in)	Spring Rate Per Coil $\left(\frac{lb_f}{in/coil}\right)$	A		B			C	
						N_r	C_d	N_r	C_d	$\pm D$	N_r	C_d
1	0.625	0.069	0.12	0.66	203	90	0.21	575	0.36	0.02	440	0.41
2	0.50	0.063	0.34	0.66	288	45	0.34	230	0.52	0.14	155	0.60
3	0.375	0.055	0.40	0.62	430	33	0.30	200	0.42	0.08	150	0.46
4	0.30	0.050	0.40	0.25	400	40	0.26	215	0.39	0.08	160	0.42
5	0.25	0.037	0.36	0.23	300	42	0.34	200	0.44	0.06	140	0.47
6	0.25	0.037	0.11	0.25	300	170	0.65	1000	0.69	0.25	490	0.78
7	0.1875	0.031	0.09	0.23	387	140	0.44	860	0.48	0.05	570	0.50
8	0.70	0.1333	0.43	0.52	3400	25	0.19	165	0.17	0.10	80	0.20
9	0.625	0.069	0.405	0.66	203	52	0.22	300	0.42	0.02	240	0.44
10	0.50	0.063	0.34	0.60	288	53	0.38	210	0.31	0.05	100	0.34
11	0.30	0.050	0.40	0.25	400	80	0.26	350	0.20	0.10	180	0.28

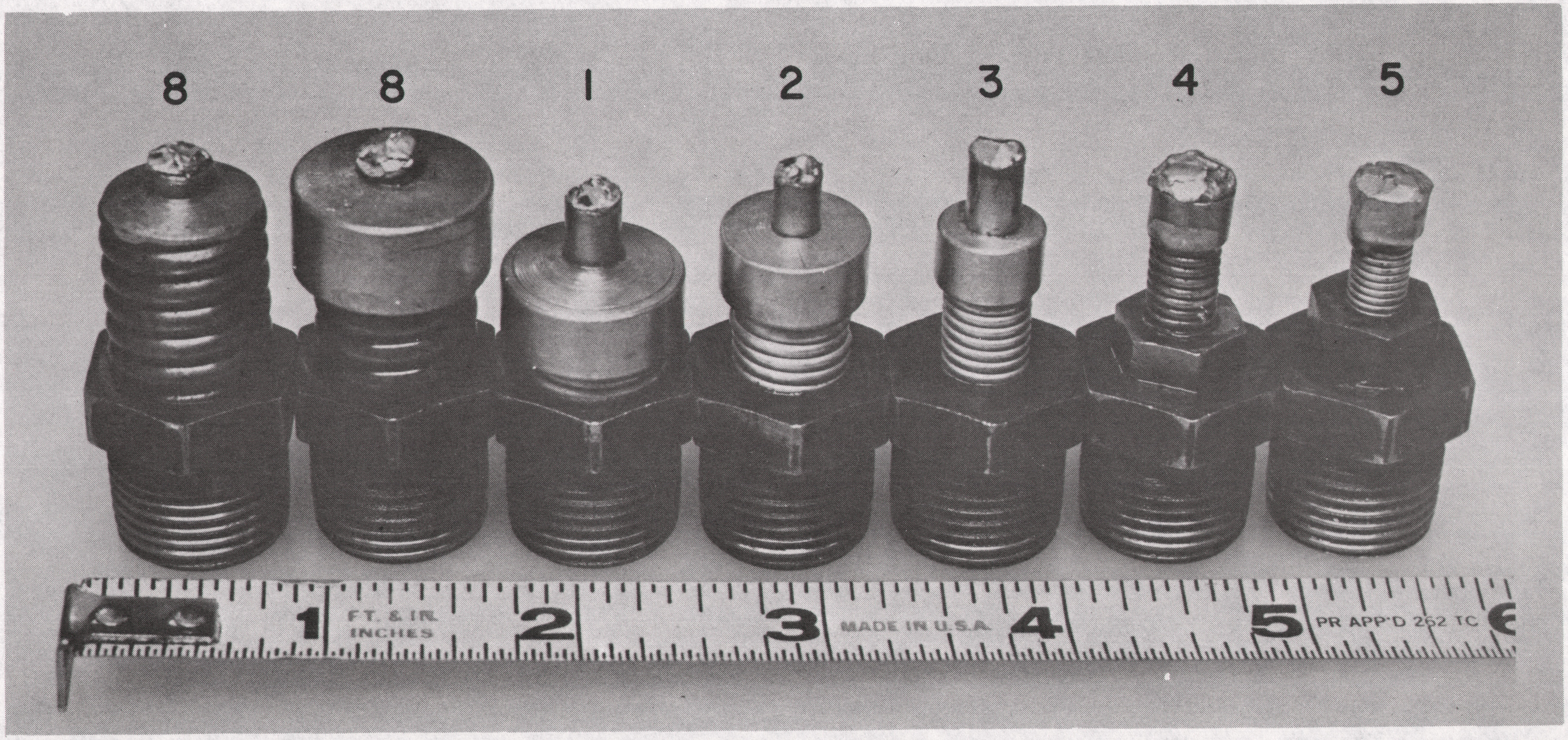


Figure 17. Sample of Tested Springs (Table II)

In the second phase, the supply pressure was set and the actuator angle was slowly increased to some maximum angle and back again (Figure 19). At points which exhibited unusual features, the corresponding actuation angle was noted. This process was repeated for several supply pressures of equal increments. Thus a family of flow-pressure curves for any particular spring was obtained on an x-y plotter.

The flow-pressure curves of all the tests followed a particular pattern (Figures 18 and 19), the differences between springs being primarily in magnitude. No spring was found which would give satisfactory results much above about 1,000 psi. Surge was still present, even in stiff springs with a comparatively large wire diameter to coil diameter ratio (i.e.: >0.2) (Table II, #8). The form of these flow-pressure curves was very non-linear, particularly at large flows and pressures. Due to the high pressures involved it was not possible to directly observe these phenomena and also, because the results were so non-linear it was not possible to correlate the results with the theoretical developments (see Appendix A and B). It was felt that these non-linearities were all due to pressure-induced distortions of the springs in the form of the coils being progressively forced over one another.

A flow-pressure curve of the form shown and labelled in Figure 18 was obtained by fixing the angular deflection and varying the supply pressure. The curves closely followed predicted theory to Point 'a' where a slight distortion occurred due to the second coil being forced slightly over the first. The flow continued to increase steadily to Point 'c' where saturation flow occurred through the spring (or the pump saturated). Continuing to increase the pressure resulted in a series of steps as indicated by Points 'd' and 'f'. These steps were due to progressive coils being

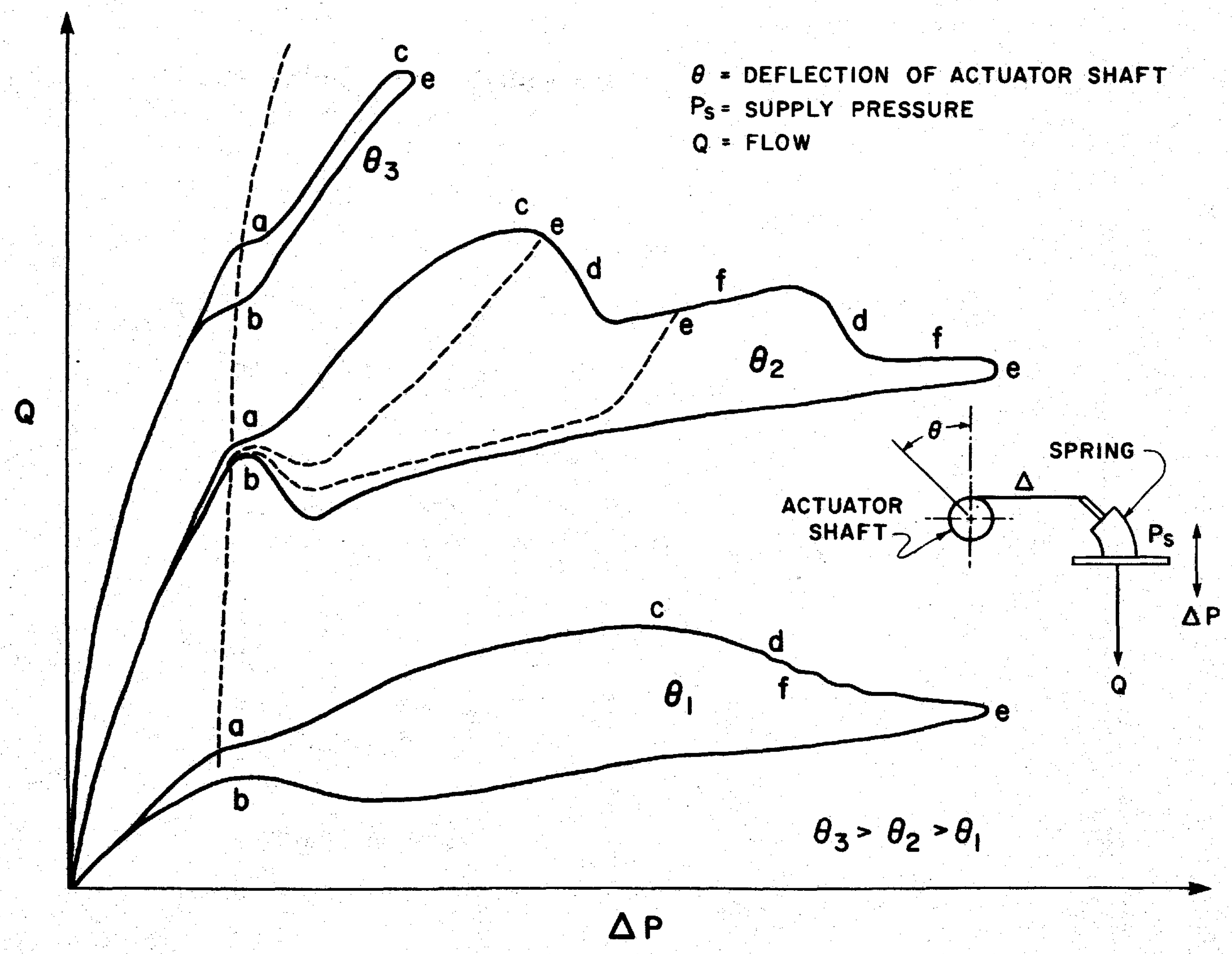


Figure 18. Flow-Pressure Curves (Constant θ) (Variable P_s)

(Equivalent Spring Orifice Characteristics)

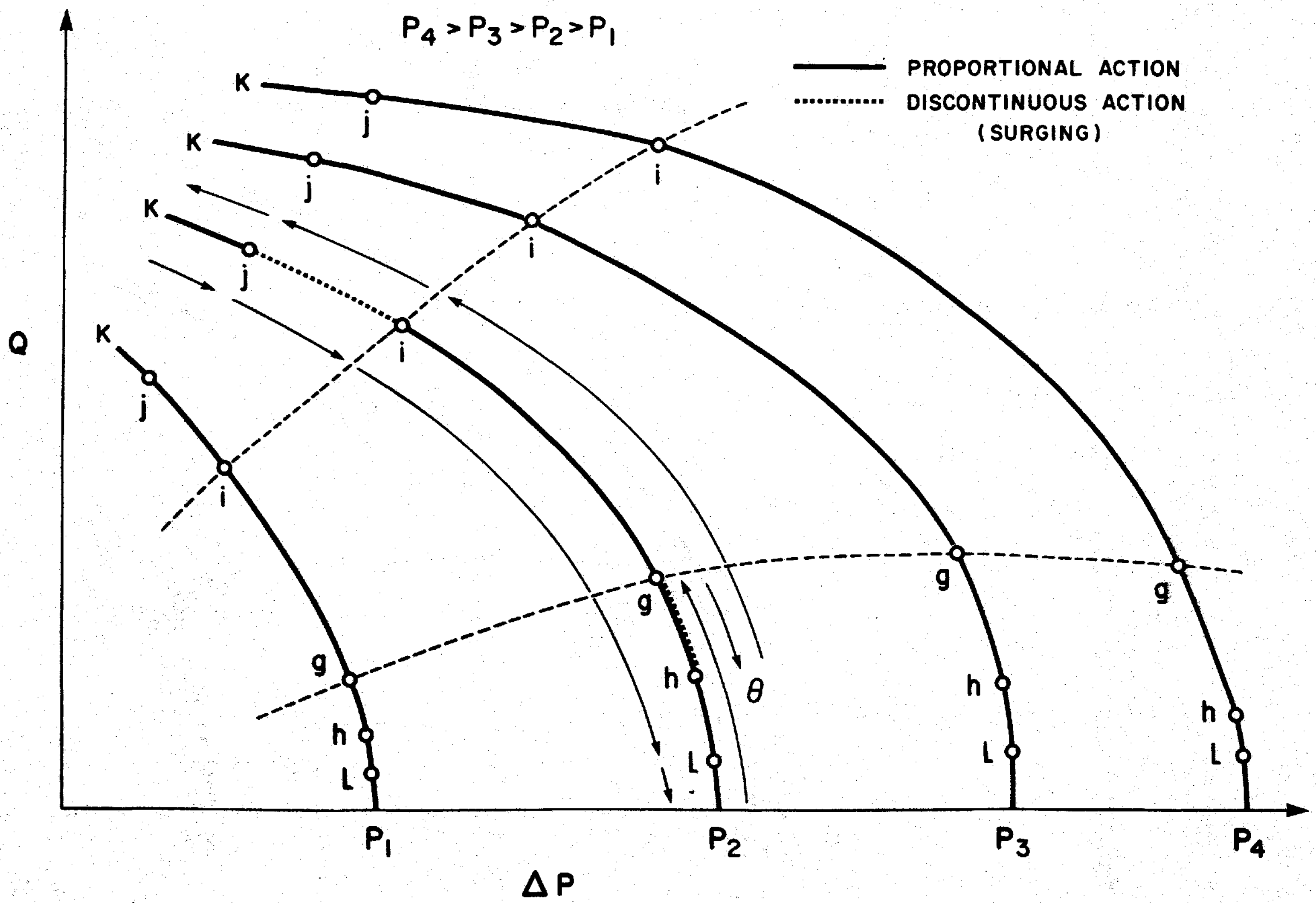


Figure 19. Flow-Pressure Curves (Constant P_s) (Variable θ)
(Equivalent Spring Load Characteristics)

forced over one another by the increasing supply pressure. Decreasing the supply pressure at any Point 'e' resulted in considerable hysteresis until all the coils simultaneously jumped back to Point 'b'. The theoretical curve was then followed back again with little hysteresis. The effects of different actuation angles was also readily apparent. An imaginary dotted line was drawn through all Points 'a' noting that 'a' became more indistinct at smaller flows and pressures. The curves to the left of this dotted line were readily predicted by theory whereas those to the right were very non-linear and were not predictable.

A second set of flow-pressure curves (Figure 19) was obtained by fixing the supply pressure then deflecting the actuator shaft, and hence the spring, to some maximum angle and back again. When the spring was closed the pressure differential was equal to the supply pressure. As the spring was deflected the flow increased while the pressure differential decreased. Having set the supply pressure, several degrees of deflection (about 5^o), this increasing with the supply pressure, were required before any flow was observed. At Point 'g' the flow would often, but not always, jump back to Point 'h'. This occurred several times or not at all along any particular curve for any particular spring. The most likely cause of this phenomenon was that the spring was deflected by an equivalent shear due to the pressure and not a moment as was assumed in the theory. At Point 'g' the spring suddenly jumped to the shape predicted by theory and the flow rate fell to Point 'h'. Although present in tests of individual springs this phenomenon could not be detected in the four-way valve operation and was therefore not considered very serious. Proportionality was maintained to Point 'i' where a sudden jump to Point 'j' was noted, this being the surge mentioned previously.

It, too, was probably due to the coils being forced over one another in some manner. Proportionality was then noted from 'j' to saturation flow at 'K'.

In the process of returning the spring a reverse surge was noted from 'j' to 'i' and then proportionality was maintained down to 'l'. At 'l' the actuator shaft was in the zero deflection position, but there was a small leakage flow due to the high pressure keeping the springs slightly distorted. Momentarily reducing the supply pressure to zero allowed the spring to return to its proper shape without leakage. This problem was not very noticeable in the four-way valve configuration and was therefore not considered serious.

As in the first set of flow-pressure curves, an imaginary dotted line was drawn through all Points 'g' or 'i', whichever it was desirable not to exceed. In this case, all desirable operating points were below this arbitrary dotted line, again noting that the points became indistinct at low flows and pressures.

Because of the nonlinearities present these two sets of flow-pressure curves should not be superimposed. Fixing any two of the three variables involved (ΔP, Q or θ) will not necessarily yield unique solutions for the third variable in either set of curves.

It was felt useful to obtain an indication of the magnitude of the coefficients of discharge for the springs tested (Table II). The only region tested in which such calculations were possible was the region to the left of the dotted line in Figure 18. This was because the area was reasonably well predicted by theory in that region. In actual fact the discharge coefficients were calculated for all points on the curve up to Point 'c'. Under actual operating conditions, the four-way valve followed the curves of Figure 19. For this reason the calculated discharge coefficients were of little practical use.

The coefficients of discharge were calculated along the length of each flow-pressure curve obtained for each spring. In this manner the variations of the coefficients of discharge could be seen for various deflections and flows. These curves were plotted on semi-log paper such that the coefficient of discharge was plotted versus the Reynolds number. The Reynolds number took into account the flowrate, the density and viscosity of the fluid and the physical dimensions of the spring. The resulting curves all tended to take the form shown by the solid lines in Figure 20. A similarity was readily seen between these curves and the corresponding curves of Figure 18.

This new family of curves was approximated by the superimposed continuous curve shown by the central dotted line in Figure 20. Points 'A', 'B' and 'C' corresponded to the average of Points 'a_r', 'b_r' and 'c_r' and represented an average curve for the family of curves obtained for each spring. The distribution of the actual curves about the average curve was indicated by the dotted curves displaced a Distance 'D' from the average curve. The Reynolds numbers and corresponding coefficients of discharge for Points 'A', 'B' and 'C' were included in Table II for the springs tested. The distribution of the coefficients of Discharge, 'D', for each spring was also listed in Table II.

The average values of the coefficients of discharge for all the springs were plotted together as shown by Figure 21. It was seen that the majority of the Reynolds numbers tended to fall into the range of from 50 to 500 which indicated that the flow between the coils remained

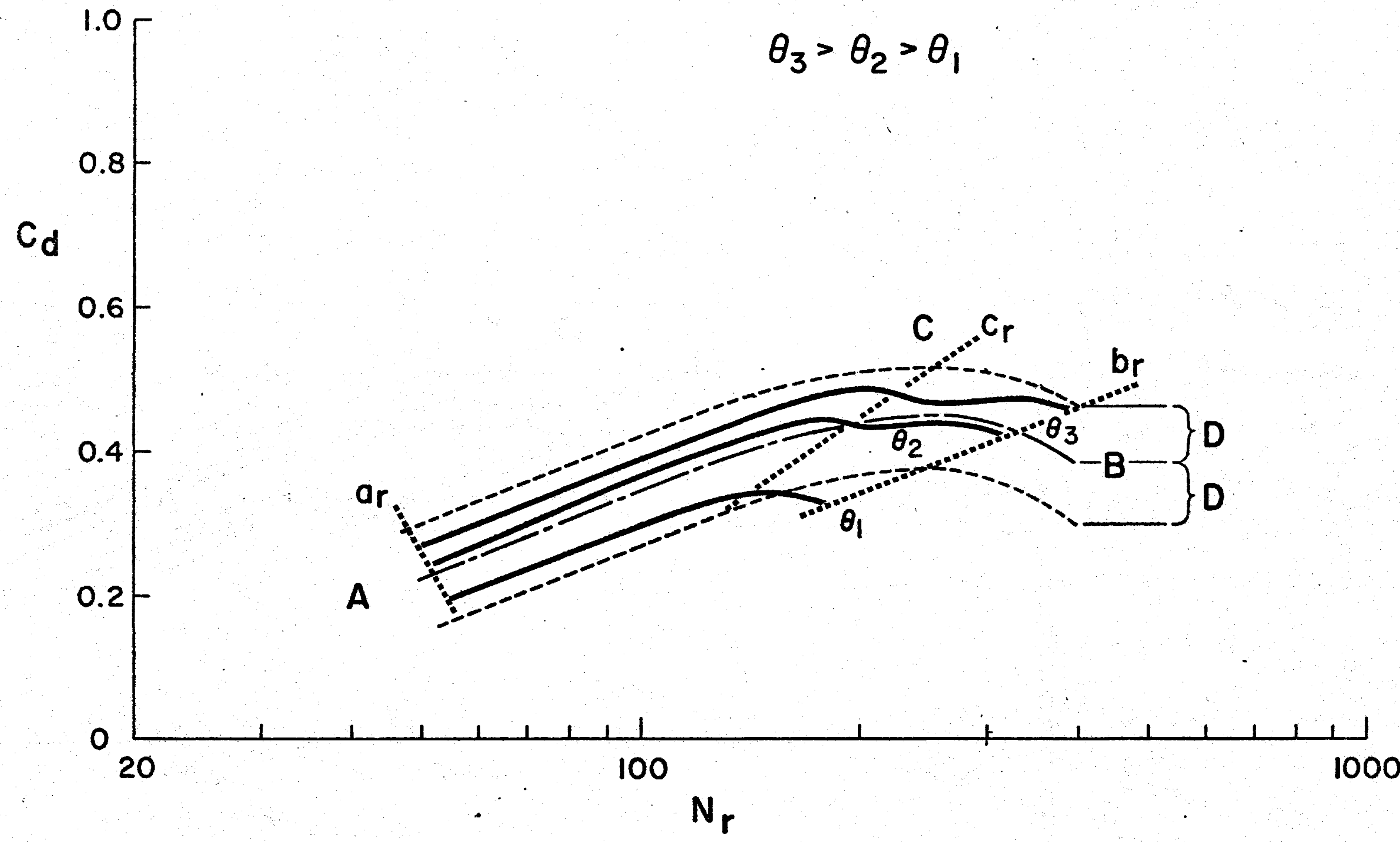


Figure 20. Detailed Discharge Coefficient Curves (and Their Approximation)

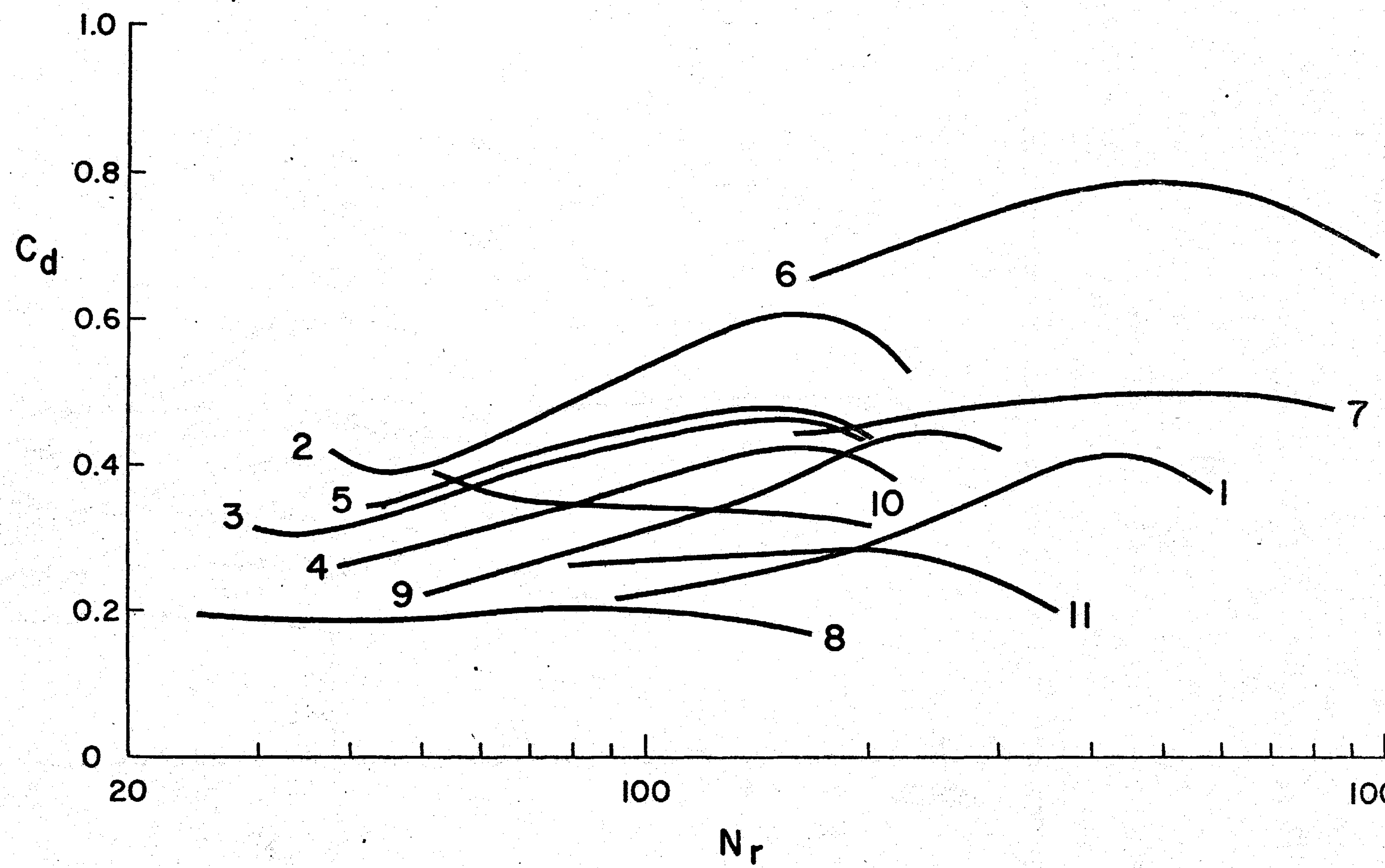


Figure 21. Experimental Discharge Coefficient Curves (Table II)

laminar at all times. The coefficients of discharge generally tended to fall into the range of from 0.2 to 0.6 with an average value of about 0.4. This was also low and indicated that significant energy losses occurred as the flow passed through a spring valve. By comparing these results with the various spring parameters shown in Table II it was felt that no significant trends could be deduced from the curves.

A crude approximation could be made by assuming a coefficient of discharge of 0.4, but as indicated earlier, this would be of little practical use in an operating spring valve. Because of the uncertainty of the shape of the spring for the curves in all other regions no further calculations of the coefficients of discharge were done. It was noted, however, that for the springs tested point 'g' generally occurred at actuator shaft deflections of about 20^{o} and 'i' occurred at angles of about 50^{o}. Since the shaft diameter was 0.50 inch these corresponded to spring deflections of about 0.1 inch and 0.2 inch respectively.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

A helical coil spring valve is a nonlinear, distributed parameter system. As a result, its performance could not be theoretically predicted with any accuracy. For this reason no satisfactory explanation could be proposed to fully describe the experimental results obtained. Considerably more work could be done on this valve, but it was felt that little more would be accomplished.

The lateral deflection mode of operation was chosen because springs operating in the extension mode rapidly oscillated and fatigued. No way could be found to eliminate these oscillations and it appeared that the only potential use for the extension mode of operation was as a simple check valve. Problems may still be present, however, since not enough work was done with this use in mind to be conclusive.

The manifold design for the four-way valve worked very successfully and appeared to meet all the desired requirements. It was found that the method of mounting the individual springs was easily accomplished and their subsequent actuation by cables up to the point of surge gave excellent results. This actuation technique provided very good proportional control and allowed positive shutoff with minimum deadzone and leakage, as desired.

Surging of the springs was by far the most serious problem with the design. Some form of guiding mechanism could be placed beside each spring in order to control the deflections more accurately. These guides may be able to restrain the motion of the coils sufficiently to eliminate the surge. To be completely successful, the guides would probably have to be attached directly to the individual coils. Both these possibilities would be difficult to implement because construction and alignment problems would be severe. These two possible solutions were therefore not attempted since increased complexity would lead to an increased cost.

A possible method of eliminating leakage flow due to minor distortions of the springs in the closed position would be to apply a thin film of rubber or plastic to the coils. When the coils compress, this film would also compress, providing a better seal. Deadzone and leakage of the springs, however, did not appear to be as significant when operating as a four-way valve than when tested individually since the total pressure drop was distributed across two springs instead of one. Dropping the pressure across two springs instead of one, however, did not significantly improve the surging. It was still significant in all tests at supply pressures of 1,000 psi and completely unmanageable at 2,000 psi. This indicated that spring valves could only be used as single stage valves with supply pressures not exceeding about 750 psi to 1,250 psi; depending on the magnitude of surge allowed. This was far below the operating pressures of most agricultural equipment in use today. As such, spring valves were not considered practical for the applications originally envisioned.

Although not practical as a single-stage high pressure hydraulic flow control valve, spring valves would be very applicable as proportional control valves where pressures below about 1,000 psi are used. They are relatively simple and cheap to construct and dirt in the fluid would have little effect on the performance. Although not investigated, they would appear to be very good valves to use for controlling pneumatic systems. By carefully choosing the type of 'O' rings used on the actuator shaft, no lubrication would be required. As has been indicated by previous research [3], spring valves could possibly be used successfully as the first stage of a two stage valve since the flow and pressure requirements are generally not as severe as in a single stage valve.

In conclusion, spring valves would be practical in many applications, but care should be exercised in order to avoid the problems that were found to exist.

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APPENDIX A

SPRING DEFLECTIONS

A1 Introduction

In order to study spring valves the effect of any given force acting on a spring should be known. All forces can be assumed to act on a portion of a single coil giving a corresponding deflection. The deflection can then be calculated for a single coil or, using equivalent beam theory, for any number of coils in a spring. From this deflection, it is possible to obtain an expression for the increase in the circumferential area (i.e. open area) of the spring for any given increase in its deflection. This is desirable so that all terms in the equations of flow of a spring valve will be defined. Calculating the open area of a spring for any given deflection also allows curves of Reynolds numbers versus discharge coefficients to be obtained. In this manner the overall performance of any given spring valve can be determined.

A2 Deflection of a Coil

Energy methods are used to determine the effect of any force or combination of forces acting on a helical coil spring (Figure 22). The force at any point in a spring can be expressed in terms of the strain energy stored in the spring and the deflection at that point. The most general expression for this relationship is [5]:

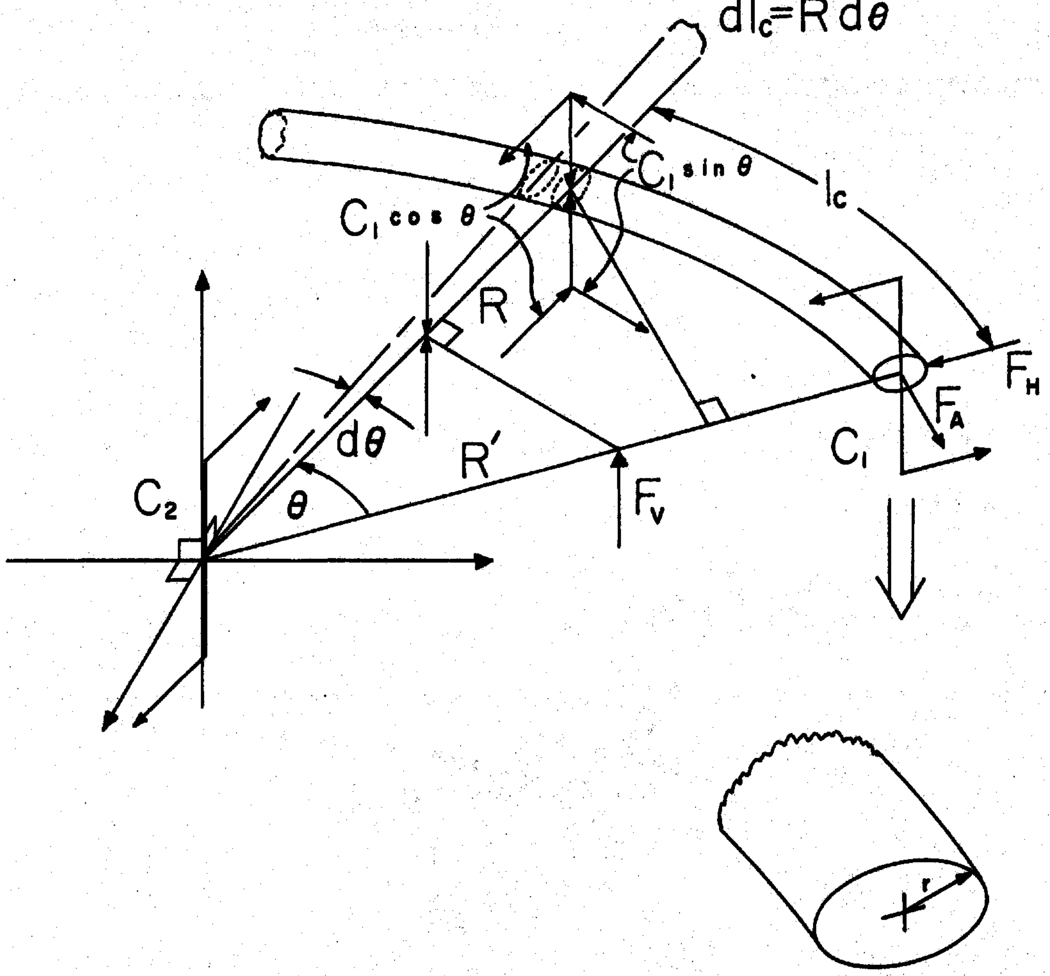

Figure 22. A Section of a Helical Coil

$$U = \frac{1}{2}\,\frac{F_A^{\,2}\,lc}{a\;E} + \int_0^{lc} \frac{1}{2}\,\frac{V_v^{\,2}}{a\;G}\,dlc + \int_0^{lc} \frac{1}{2}\,\frac{V_H^{\,2}}{a\;G}\,dlc +$$

$$\int_0^{lc} \frac{1}{2}\,\frac{M_v^{\,2}}{E\;I}\,dlc + \int_0^{lc} \frac{1}{2}\,\frac{M_H^{\,2}}{E\;I}\,dlc + \int_0^{lc} \frac{1}{2}\,\frac{T^2}{JG}\,dlc$$

where:

- $F_{A,V,H}$ = mutually perpendicular axial, vertical and horizontal forces acting at any point of a coil.
- $V_{V,H}$ = shear forces acting on a coil.
- $M_{V,H}$ = moments acting on a coil.
- T = the torque acting on a coil.
- U = the strain energy stored in the coil.
- l_c = the arc length of the coil.
- a = πr^2.
- I = $\pi r^4/4$.
- J = $2\ I = \pi r^4/2$.
- E = modulus of elasticity.
- G = modulus of rigidity.

The deflection of any point can be obtained by differentiating the strain energy with respect to a force acting in the desired direction at that point. Similarly, the angle of twist can be obtained by differentiating the strain energy with respect to a couple at that point. If these forces or couples are imaginary they can be made equal to zero once the partial derivatives have been obtained. Therefore:

$$\Delta_{V,H} = \frac{\partial\ U}{\partial F_{V,H}}$$

and

$$\theta_{1,2} = \frac{\partial U}{\partial C_{1,2}}$$

Assume an arbitrary portion of a coil has forces acting on it as is shown in Figure 22. Also assume that the deflections are relatively small so that the change of shape of the coil is negligible, resulting in a linear relationship between the forces and their corresponding deflections. Axial forces are not included and by the previous assumption the deflections are sufficiently small so that any resulting axial components can be ignored. As indicated by Wahl[7] this is a reasonable assumption until fairly large helix angles are obtained.

From Figure 22 the following equations can be obtained:

$$M_H = F_H R \sin \theta$$

$$V_H = F_H \cos \theta$$

$$M_V = F_V R' \sin \theta + C_1 \sin \theta$$

$$V_V = F_V$$

$$T = F_V (R - R' \cos \theta) - C_1 \cos \theta + C_2$$

Since most of the springs of interest do not have a relatively large wire diameter with respect to coil diameter, it is reasonable to neglect the effects of the shear forces. The simplified equation for the strain energy is now:

$$U = \int_0^{lc} \frac{1}{2} \frac{M_H^2}{E\,I} dlc + \int_0^{lc} \frac{1}{2} \frac{M_V^2}{E\,I} dlc + \int_0^{lc} \frac{1}{2} \frac{T^2}{JG} dlc$$

The horizontal deflections will be considered separately; therefore, the following equations can be written:

$$\Delta_V = \int_0^{lc} \frac{M_V}{EI} \frac{\partial M_V}{\partial F_V} dlc + \int_0^{lc} \frac{T}{JG} \frac{\partial T}{\partial F_V} dlc$$

and

$$\theta_{1,2} = \int_0^{lc} \frac{M_V}{EI} \frac{\partial M_V}{\partial C_{1,2}} dlc + \int_0^{lc} \frac{T}{JG} \frac{\partial T}{\partial C_{1,2}} dlc$$

but $dlc = Rd\Theta$

Therefore

$$\Delta_v = \int_0^{\Theta'} \frac{M_v}{EI} \frac{\partial M_v}{\partial F_v} R \, d\, \Theta + \int_0^{\Theta'} \frac{T}{JG} \frac{\partial T}{\partial F_v} R \, d\, \Theta$$

and

$$\theta_{1,2} = \int_0^{\Theta'} \frac{M_v}{EI} \frac{\partial M_v}{\partial C_{1,2}} R \, d\, \theta + \int_0^{\theta'} \frac{T}{JG} \frac{\partial T}{\partial C_{1,2}} R \, d\, \theta$$

where θ' is an arbitrary upper limit of integration. It should be pointed out that C_1 is a function of θ; therefore, the resulting θ_1 gives the apparent angle of twist of the coil with respect to some external reference. C_2 is not a function of θ and therefore θ_2 gives the absolute or total angle of twist from one end of the coil to the other end. The following partial derivatives can now be obtained:

$$\frac{\partial M_v}{\partial F_v} = R' \sin \theta$$

$$\frac{\partial T}{\partial F_v} = R - R' \cos \theta$$

$$\frac{\partial M_v}{\partial C_1} = \sin \theta$$

$$\frac{\partial T}{\partial C_1} = - \cos \theta$$

$$\frac{\partial M_v}{\partial C_2} = 0$$

$$\frac{\partial T}{\partial C_2} = 1$$

Now let: $C_1 = C_2 = 0$

This leaves:

$$M_v = F_v R' \sin\theta$$

$$T = F_v (R - R' \cos\theta)$$

The general expressions for Δ_v, θ_1 and θ_2 can now be evaluated. First considering Δ_v:

$$\Delta_v = \frac{1}{EI}\int_0^{\theta'} \left[F_v R' \sin\theta\right] \left[R' \sin\theta\right] R\, d\theta +$$

$$\frac{1}{JG}\int_0^{\theta'} \left[F_v (R - R' \cos\theta)\right] \left[R - R' \cos\theta\right] R\, d\theta$$

$$\Delta_v = \frac{F_v R R'^2}{E I} \int_0^{\theta'} \left[\sin^2\theta\right] d\theta +$$

$$\frac{F_v R}{JG} \int_0^{\theta'} \left[R^2 - 2R R' \cos\theta + R'^2 \cos^2\theta\right] d\theta$$

$$\Delta_v = \frac{F_v R R'^2}{E I} \left[\frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta\right]_0^{\theta'} +$$

$$\frac{F_v R}{JG} \left[R^2\theta - 2 R R' \sin\theta + R'^2 \left(\frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta\right)\right]_0^{\theta'}$$

Therefore, the general expression for Δ_v is:

$$\Delta_v = \frac{F_v R R'^2}{EI} \left[\tfrac{1}{2}\,\theta' - \tfrac{1}{2}\,\text{Sin}\,\theta'\,\cos\theta'\right] +$$

$$\frac{F_v\ R}{JG}\left[R^2\,\theta' - 2\,R\,R'\sin\theta' + R'^2\left(\tfrac{1}{2}\,\theta' + \tfrac{1}{2}\sin\theta'\cos\theta'\right)\right]$$

Similarly for θ_1:

$$\theta_1 = \frac{1}{EI}\int_0^{\theta'} \left[F_v\,R'\sin\theta\right]\left[\sin\theta\right] Rd\theta +$$

$$\frac{1}{JG}\int_0^{\theta'} \left[F_v\,(R - R'\cos\theta)\right]\left[-\cos\theta\right] Rd\theta$$

$$\theta_1 = \frac{F_v\ RR'}{E\ I}\int_0^{\theta'} \left[\sin^2\theta\right] d\theta +$$

$$\frac{F_v\ R}{JG}\int_0^{\theta'} \left[R'\ \cos^2\theta - R\cos\theta\right] d\theta$$

$$\theta_1 = \frac{F_v\ RR'}{E\ I}\left[\tfrac{1}{2}\,\theta - \tfrac{1}{2}\sin\theta\cos\theta\right]_0^{\theta'} +$$

$$\frac{F_v\ R}{J\ G}\left[R'\left(\tfrac{1}{2}\,\theta + \tfrac{1}{2}\sin\theta\cos\theta\right) - R\sin\theta\right]_0^{\theta'}$$

Therefore the general expression for θ_1 is:

$$\theta_1 = \frac{F_v\ RR'}{E\ I}\left[\tfrac{1}{2}\,\theta' - \tfrac{1}{2}\sin\theta'\cos\theta'\right] +$$

$$+ \frac{F_v\ R}{JG}\left[R'\left(\tfrac{1}{2}\,\theta' + \tfrac{1}{2}\sin\theta'\cos\theta'\right) - R\sin\theta'\right]$$

Similarly for θ_2:

$$\theta_2 = \frac{1}{EI}\int_0^{\theta'} \left[F_v\ R'\ \sin\theta\right]\left[o\right]\ R\ d\ \theta\ +$$

$$\frac{1}{JG}\int_0^{\theta'} \left[F_v\ (R - R'\ \cos\theta)\right]\ \left[1\right]\ \ R\ d\ \theta$$

$$\theta_2 = 0 + \frac{F_v\ R}{JG}\int_0^{\theta'} \left[R - R'\ \cos\theta\right] d\ \theta$$

$$\theta_2 = \frac{F_v\ R}{JG}\left[R\ \theta - R'\ \sin\theta\right]_0^{\theta'}$$

Therefore the general expression for θ_2 is:

$$\theta_2 = \frac{F_v\ R}{JG}\left[R\ \theta' - R'\ \sin\theta'\right]$$

Note that:

$\sin\theta' = 0$

for $\theta' = m(\pi/2)$

where $m = 0, 2, 4, 6, \ldots$

and $\cos\theta' = 0$

for $\theta' = m(\pi/2)$

where $m = 1, 3, 5, 7, \ldots$

Therefore $\sin\theta'\ \cos\theta' = 0$

for $\theta' = m(\pi/2)$

where $m = 0, 1, 2, 3, 4, \ldots$

The following reductions can be made:

Let $R' = 0$

Therefore:

$$\Delta_v = \frac{F_v R^3 \left[\theta'\right]}{JG}$$

$$\theta_1 = \frac{F_v R^2}{JG} \left[- \sin \theta'\right]$$

$$\theta_2 = \frac{F_v R^2 \left[\theta'\right]}{JG}$$

Notice that $\Delta_v = R\theta_2$ which is expected.

Let $R' = R$

Therefore:

$$\Delta_v = \frac{F_v R^3}{E I} \left[\tfrac{1}{2} \theta' - \tfrac{1}{2} \sin \theta' \cos \theta'\right] +$$

$$\frac{F_v R^3}{JG} \left[\frac{3}{2} \theta' - 2 \sin \theta' + \tfrac{1}{2} \sin \theta' \cos \theta'\right]$$

$$\theta_1 = \frac{F_v R^2}{EI} \left[\tfrac{1}{2} \theta' - \tfrac{1}{2} \sin \theta' \cos \theta'\right] +$$

$$\frac{F_v R^2}{JG} \left[\tfrac{1}{2} \theta' - \sin \theta' + \tfrac{1}{2} \sin \theta' \cos \theta'\right]$$

and

$$\theta_2 = \frac{F_v R^2}{JG} \left[\theta' - \sin \theta'\right]$$

Let $\theta' = 2\pi$

as well as $R' = R$

Therefore for one full turn of a coil:

$$\theta_1 = \frac{F_v R^2}{EI} \left\{ \left[\pi - 0\right] + \frac{EI}{JG} \left[\pi + 0 - 0\right] \right\}$$

$$\theta_1 = \frac{F_v R^2 \pi}{EI} \left[1 + \frac{EI}{JG} \right]$$

but:

$$J = 2I = 2\left(\frac{\pi r4}{4}\right) = \frac{\pi r4}{2}$$

and by definition: (5,6)

$$E = 2G\ (1 + \mu)$$

where μ = Poisson's ratio.

Therefore substituting:

$$\theta_1 = \frac{F_v R^2 \pi}{EI} \left[1 + \frac{2G\ (1 + \mu)\ I}{2\ I\ G} \right]$$

$$\theta_1 = \frac{F_v R^2 \pi}{E\ I} \left[2 + \mu \right]$$

Let:

n = the number of active coils.

$L_1 = n(2r)$ = the free length of the spring.

Define $M_1 = F_v R$ = a constant moment.

Therefore, for any number of turns:

$$\theta_1 = \frac{M_1\ Rn\ \pi}{E\ (\pi r4/4)} \left[2 + \mu \right]$$

$$\theta_1 = \frac{4\ M_1\ Rn}{E\ r^4} \left[2 + \mu \right]$$

Since θ_1 is the apparent angular deflection of the last coil it is valid to assume that the end of the spring as a whole also deflects through an angle very close to θ_1. Based on this assumption an equivalent (EI) for the spring can be obtained, assuming that it acts as a simple beam.

Let:

$(EI)_s$ = the equivalent (EI) of the spring assuming that it is a beam.

Therefore, let:

$$\theta_1 = \frac{M_1 L_1}{(EI)s}$$

where (EI)s is the only unknown.

Therefore:

$$\frac{M_1 L_1}{(EI)s} = \frac{4 M_1 R n}{E r^4} (2 + \mu)$$

and remembering that $L_1 = 2nr$

$$(EI)s = \frac{E r^4 2 n r}{4 R n (2 + \mu)}$$

The general expression for (EI)s is:[7]

$$(EI)s = \frac{E r^5}{2 R (2 + \mu)}$$

A3 Equivalent Beam Deflections

Assume that a spring is mounted and deflected as is shown in Figure 23. The spring can be treated as an equivalent beam and the deflection at any point can be easily found. Energy methods are again used to calculate the deflections. It is assumed that the deflections are small enough that all the lengths remain constant and therefore linear. The moments for the equivalent beam approximation are:

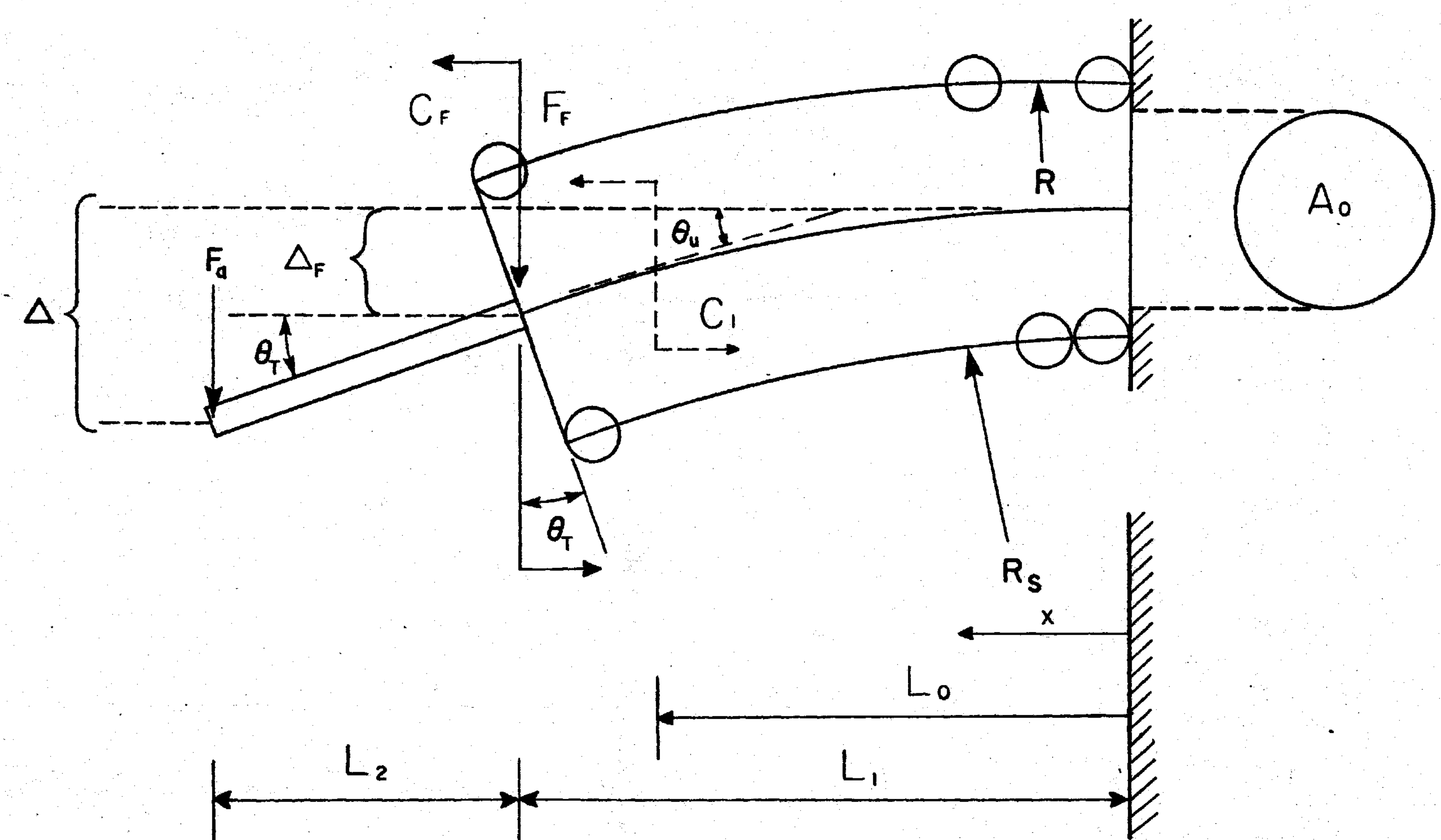

Figure 23. A Spring as an Equivalent Beam

$$M_b = F_a\ (L_1 + L_2 - x) + F_F\ (L_1 - x) + C_F + C_1$$

for $x = 0$ to $x = L_o$

Therefore:

$$\frac{\partial M_b}{\partial F_a} = (L_1 + L_2 - x)$$

$$\frac{\partial M_b}{\partial C_1} = 1$$

$$M_b = F_a\ (L_1 + L_2 - x) + F_F\ (L_1 - x) + C_F$$

for $x = L_o$ to $x = L_1$

Therefore:

$$\frac{\partial M_b}{\partial F_a} = (L_1 + L_2 - x)$$

$$\frac{\partial M_b}{\partial C_1} = 0$$

and

$$M_b = F_a\ (L_1 + L_2 - x)$$

for $x = L_1$ to $x = L_1 + L_2$

Therefore:

$$\frac{\partial M_b}{\partial F_a} = (L_1 + L_2 - x)$$

$$\frac{\partial M_b}{\partial C_1} = 0$$

For the purpose of calculations replace F_a by an equal force F_F and a couple C_F where:

$F_F = F_a$ in magnitude and

$$C_F = F_a L_2 = F_F L_2$$

let $C_1 = 0$ and $F_a = 0$

Δ = the deflection at the free end of the spring.

$$\Delta = \int_o^{L_1} \frac{M_b}{(EI)s} \frac{\partial M_b}{\partial F_a} dx + \int_{L_1}^{L_1 + L_2} \frac{M_b}{(EI)s} \frac{\partial M_b}{\partial F_a} dx$$

$$\Delta = \frac{1}{(EI)s} \int_o^{L_1} \left[F_a (L_1 + L_2 - x) + F_F (L_1 - x) + C_F \right]$$

$$\left[L_1 + L_2 - x \right] dx +$$

$$\frac{1}{(EI)s} \int_{L_1}^{L_1 + L_2} \left[F_a (L_1 + L_2 - x) \right] \left[L_1 + L_2 - x \right] dx$$

but $F_a = 0$ and $C_F = F_F L_2$

Therefore:

$$\Delta = \frac{1}{(EI)s} \int_o^{L_1} \left[F_F (L_1 - x) + F_F L_2 \right] \left[L_1 + L_2 - x \right] dx + 0$$

$$\Delta = \frac{F_F}{(EI)s} \int_o^{L_1} \left[L_1 + L_2 - x \right]^2 dx$$

$$\Delta = \frac{F_F}{(EI)s} \int_o^{L_1} \left[L_1^2 + L_2^2 + x^2 + 2L_1L_2 - 2L_1x - 2L_2x \right] dx$$

$$\Delta = \frac{F_F}{(EI)s} \left[\frac{x^3}{3} + L_1^2x + L_2^2x + 2L_1L_2x - \frac{2}{2} L_1x^2 - \frac{2}{2} L_2x^2 \right]_o^{L_1}$$

$$\Delta = \frac{F_F}{(EI)s} \left[\frac{L_1^3}{3} + L_1^3 + L_1L_2^2 + 2L_1^2L_2 - L_1^3 - L_1^2L_2 \right]$$

$$\Delta = \frac{F_F}{(EI)s}\left[\frac{L_1^{\,3}}{3} + L_1^{\,2}L_2 + L_1L_2^{\,2}\right]$$

Since the magnitudes of F_F and F_a are equal, let $F_a = F_F$.

Therefore:

$$\Delta = \frac{F_a}{(EI)s}\left[\frac{L_1^{\,3}}{3} + L_1^{\,2}L_2 + L_1L_2^{\,2}\right]$$

Note that if $L_2 = 0$

$$\Delta = \frac{F_a}{(EI)s}\left[\frac{L_1^{\,3}}{3}\right]$$

which is the deflection at the end of a simple beam.

θ_u = the angular deflection or slope at any point along the spring.

θ_T = the final slope at the end of the spring.

$$\theta_u = \int_0^{L_o} \frac{M_b}{(EI)s}\frac{\partial M_b}{\partial C_1}\,dx + \int_{L_o}^{L_1} \frac{M_b}{(EI)s}\frac{\partial M_b}{\partial C_1}\,dx$$

$$\theta_u = \int_0^{L_o} \frac{1}{(EI)s}\left[F_a\,(L_1 + L_2 - x) + F_F\,(L_1 - x) + C_F + C_1\right]$$

$$\left[1\right]\,dx\ +$$

$$\int_{L_o}^{L_1} \frac{1}{(EI)s}\left[F_a\,(L_1 + L_2 - x) + F_F\,(L_1 - x) + C_F\right]$$

$$\left[0\right]\,dx$$

but again $C_1 = 0$

$F_a = 0$

and $C_F = F_F\,L_2$

Therefore:

$$\theta_u = \frac{F_F}{(EI)s} \int_0^{L_o} \left[L_1 + L_2 - x\right] dx + 0$$

$$\theta_u = \frac{F_F}{(EI)s} \left[L_1 x + L_2 x - \frac{x^2}{2}\right]_0^{L_o}$$

$$\theta_u = \frac{F_F}{(EI)s} \left[L_1 L_o + L_2 L_o + \frac{L_o^2}{2}\right]$$

but $F_a = F_F$ in magnitude; therefore,

$$\theta_u = \frac{F_a}{(EI)s} \left[L_1 L_o + L_2 L_o + \frac{L_o^2}{2}\right]$$

but let $L_o = L_1$; therefore,

$\theta_u = \theta_T$ and therefore

$$\theta_T = \frac{F_a}{(EI)s} \left[L_1 L_2 + \frac{L_1^2}{2}\right]$$

Note that if $L_2 = 0$

$$\theta_T = \frac{F_a}{(EI)s} \left[\frac{L_1^2}{2}\right]$$

which is the slope at the end of a simple beam.

A4 Area Calculations

It is now possible to calculate the increase in the circumferential area of a deflected spring. Let the orifice area at the end of the spring be defined as:

$$A_o = \pi(R-r)^2$$

$$A_o = \pi(R^2 - 2Rr + r^2)$$

Let the initial circumferential area be:

$A_i = 2 \pi R L_1$

Let A_T be the total circumferential area when the spring is deflected. Therefore the increase in the circumferential area (i.e. open area) due to the deflection is:

$A_D = A_T - A_i$

Let R_s be the radius of curvature of the equivalent beam. Therefore:

$R_s \, d\,\theta_u = d\,L_o$ and therefore:

$$\frac{1}{R_s} = \frac{d\,\theta_u}{d\,L_o} = \frac{F_a}{(EI)s}\left[L_1 + L_2 - L_o\right]$$

Therefore:

$$A_T = 2 \pi R \int_o^{\theta_T} \left[R_s + R\right] d\,\theta_u$$

$$A_T = 2 \pi R \int_o^{L_1} \left[R_s + R\right]\left[\frac{1}{R_s}\right] d\,L_o$$

$$A_T = 2 \pi R \int_o^{L_1} \left[1 + \frac{R}{R_s}\right] d\,L_o$$

$$A_T = 2 \pi R \int_o^{L_1} \left[1 + \frac{R\,F_a}{(EI)s}\,(L_1 + L_2 - L_o)\right] d\,L_o$$

$$A_T = 2 \pi R \left[L_o + \frac{R\,F_a}{(EI)s}\,(L_1 L_o + L_2 L_o - \frac{L_o^2}{2})\right]_o^{L_1}$$

$$A_T = 2 \pi R \left[L_o + R\,\theta_u\right]_o^{L_1}$$

Therefore:

$$A_T = 2\pi R\left[L_1 + \frac{R F_a}{(EI)s}\left(L_1L_2 + \frac{L_1^2}{2}\right)\right]$$

Therefore:

$$A_T = 2\pi R\left[L_1 + R\,\theta_T\right]$$

$$A_T = 2\pi R L_1 + 2\pi R^2\,\theta_T$$

$$A_T = A_1 + 2\pi R^2\,\theta_T$$

Therefore:

$$A_D = A_1 + 2\pi R^2\,\theta_T - A_1$$

$$A_D = 2\pi R^2\,\theta_T$$

The increase in circumferential area is therefore:

$$A_D = 2\pi R^2\left[\frac{F_a}{(EI)s}\left(L_1L_2 + \frac{L_1^2}{2}\right)\right]$$

Since both the deflection Δ and the area A_D are known, the term $F_a/(EI)s$ can be eliminated when the two equations are combined. This leaves A_D as a function of Δ and several constants.

$$A_D = \Delta\left\{\frac{2\pi R^2\left[\frac{L_1^2}{2} + L_1L_2\right]}{\left[\frac{L_1^3}{3} + L_1^2\,L_2 + L_1L_2^2\right]}\right\}$$

$$A_D = \Delta\left\{\frac{2\pi R^2\left[\frac{L_1}{2} + L_2\right]}{\left[\frac{L_1^2}{3} + L_1\,L_2 + L_2^2\right]}\right\}$$

For convenience this equation can be rewritten as:

$A_D = \Delta\ B$ where:

$$B = \left\{ \frac{\pi\ R^2 \left[L_1 + 2\ L_2\right]}{\left[\frac{L_1^2}{3} + L_1\ L_2 + L_2^2\right]} \right\}$$

A5 Equivalent Shear and Axial Calculations

The above area equation is based on the original assumption that a vertical force F_v acting on an individual coil produces torques and vertical moments. From this an equivalent (EI) for a spring was obtained. This in turn allowed a calculation of the deflection of the spring assuming that the deflection was entirely due to a moment caused by force F_a.

In a similar manner a horizontal force F_H produces horizontal moments in an individual coil. From this an equivalent (Ga) for a spring can be obtained. This would allow a calculation of the deflection of the spring assuming that it is entirely caused by a shear force which in turn is caused by force F_a. It is valid to assume a deflection due to shear alone because the external pressure acting on the capped free end of the spring compensates for any tendency to produce a moment.

Since no angular deflection takes place at the free end it can be assumed that $\theta_T = 0$ for any Δ. The equivalent shear is constant along L_1; therefore, a simplified development can be used to calculate the areas. It is valid to define a new angle θ_v such that:

$$\theta_V = \tan^{-1} \left(\frac{\Delta}{L_1}\right)$$

Define A_V as the increase in circumferential area due to deflection in shear. Define A_{TV} as the total circumferential area due to deflection in shear. Therefore:

$$A_V = A_{TV} - A_i$$

where:

$$A_{TV} = \frac{2\pi R L_1}{\cos\theta_V}$$

Therefore:

$$A_V = \frac{2\pi\ R\ L_1}{\cos\ \theta_V} - 2\pi R\ L_1$$

$$A_V = 2\pi R\ L_1 \left(\frac{1}{\cos\ \theta_V} - 1\right)$$

For the small deflections which are actually required it is noted that A_D is about two orders of magnitude larger than A_V. Although in actual fact both these developments should be combined for a single solution, it is not possible because the pressure and flow forces acting on an actual spring cannot be theoretically determined in enough detail. This means that the shapes assumed above are not valid for many of the operating conditions of any actual spring. The pressure and flow forces predominate resulting in unpredictable results.

As a final note, if the force F_a were to act along the axis of the spring causing the coils to extend a distance Δ, the increase in circumferential area, A_e, in the extension mode would be:

$$A_e = 2\ \pi\ R\ \Delta$$