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Date
DIFFUSION AND DRIFT LOSSES AND ELECTRON COOLING
IN HELIUM AFTERGLOWS CONFINED BY
A TOROIDAL MAGNETIC FIELD

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by
Orlando Duane Olson

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This thesis describes theoretical and experimental investigations of the decay of electron density and temperature in an afterglow confined by a toroidal magnetic field. Measurements were carried out in helium afterglows by means of double floating probes. The plasma density was high enough so that coulomb interactions were dominant at all times. An analysis of the rate of diffusion of a plasma across a magnetic field is presented and compared with experimental results. For $B \leq 0.0160 \text{ Wb/m}^2$ diffusion is the dominant density decay mechanism and the observed rate of diffusion agrees with the calculated rate within a factor of 2 for a wide range of electron densities and energies. At higher magnetic fields drift in the inhomogeneous magnetic field becomes the dominant loss mechanism. An analysis of the drift effect is presented and comparisons with the experimental results are made. Theory and experiment agree within a factor of about 2 on the average but the results are less conclusive than in the case of diffusion. Observed electron energy decay rates are compared with calculated cooling rates due to elastic collisions with ions and neutral atoms. For sufficiently low discharge power and neutral gas pressure the agreement is reasonably good. However for higher pressures and larger discharge powers the cooling rate is slower than that expected from recoil cooling alone. There is some evidence that this slow cooling is due to heating of the electrons through interactions with metastable atoms during the afterglow.
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CHAPTER 1
INTRODUCTION

1.1 The problem

In recent years an understanding of the mechanisms by which a plasma can move across a magnetic confining field to the walls of a chamber has been the object of considerable experimental and theoretical work. This activity has been stimulated by the investigation of plasmas in thermonuclear research. Difficulties in confining plasmas at high temperatures have prompted scientists to begin a study of the problem by examining the loss mechanisms for low energy plasmas in considerable detail.

Theoretical and experimental studies have shown that there are numerous instabilities which can drastically enhance the loss of plasma to the chamber walls. However, in many cases the plasma is stable and is lost by more ordinary mechanisms such as diffusion, volume recombination, or drift in an inhomogeneous magnetic field.

Most of the work reported in this thesis is connected with investigations of the rate of loss of a helium plasma in a toroidal system. Diffusion, and drift due to the inhomogeneous magnetic field inherent in this geometry, were the controlling plasma loss mechanisms for the conditions under which the experiments were carried out. By a proper choice of the experimental conditions it was possible to study these processes individually. The measurements, which were made in the afterglow of a pulsed discharge, also yielded information concerning the rate of decay of electron temperature in helium afterglows.

Qualitative discussions of the diffusion, drift, and electron cooling mechanisms, and outlines of the previous work on these topics follow.
Most attention is devoted to diffusion since much more previous work has been reported on diffusion than on drift and electron cooling. Since this thesis deals with diffusion in afterglows the previous work in this area is considered in most detail.

1.2 Electron cooling

In low temperature plasma experiments such as those considered here a plasma is produced by partially ionizing a gas within a container of some sort. During the active discharge period (when power is being fed into the plasma) the degree of ionization is usually a few percent or less, and the average electron energy is of the order of a few electron volts. When the power input to the plasma ceases the afterglow period begins. The electron energy decays during the afterglow and tends to approach the temperature of the ions and neutrals which usually remain near room temperature at all times. Various processes such as collisions (which may be elastic, inelastic or superelastic), recombination, diffusion or drift can in general affect the electron cooling rate.

Ingraham (1963) has measured the rate of decay of electron temperature in a helium afterglow by means of a transient microwave pyrometer. Comparison of the observed cooling rates with those predicted due to elastic collisions showed that the observed cooling rates were slower in most cases than could be explained on the basis of elastic collisions only. This was attributed to the action of metastable atoms which give up their excitation energy to the electrons during the afterglow period. This explanation is supported by the observation that for conditions chosen so that metastable atoms were de-excited early in the afterglow, the cooling rate approached that predicted by elastic collisions.
1.3 Drift in the homogeneous magnetic field

The term "drift" here refers to the motion of the centre of gyration of a charged particle in a direction perpendicular to the magnetic field. Particles which have a Larmor frequency greater than their collision frequency experience this effect in the presence of an inhomogeneous magnetic field. In toroidal geometry this drift results in a loss of the plasma to the container walls and under certain conditions this loss mechanism can dominate over other loss mechanisms.

Bostick and Levine (1955) made afterglow measurements on the rate of decay of plasma density in a metallic torus with a magnetic confining field. It was observed that the time constant for density decay increased with magnetic field strength up to a certain point and then decreased. This behavior was considered by Bostick and Levine to be evidence of anomalous diffusion. Golant, Danilov and Zhilinskii (1963) have suggested that this behavior may have been the result of drift effects which Bostick and Levine did not take into account. Golant, Danilov and Zhilinskii have analyzed the drift problem for the case of a dielectric plasma container. They applied the results of this analysis to Bostick and Levine's experiment and found some evidence that drift may have caused the unexpectedly high loss rates at large magnetic fields. However, their analysis is not strictly applicable since Bostick and Levine employed a metallic chamber.

Golant, Danilov and Zhilinskii (1963) have also measured the plasma decay rate in a curved discharge tube which approximated a section of a torus. At large magnetic fields the decay rate was controlled by drift effects and good agreement between theory and experiment was demonstrated.
1.4 Diffusion

1.4.1 Introduction Under ordinary conditions in experiments such as those considered here the ions and electrons recombine much more quickly at the walls of the container than in the plasma volume. Hence the plasma tends to assume a configuration in which the particle concentration has a maximum at the centre of the chamber and falls off toward the walls. This density gradient results in particle flow toward the container walls. If other phenomena, such as drift in an inhomogeneous magnetic field or volume recombination, are not effective in removing the plasma, a measurement of the rate of decay of the plasma density provides information which can be used directly to evaluate the diffusion coefficient, i.e. the coefficient relating particle flow per unit area to the density gradient.

In a stable plasma the diffusion coefficient is determined by collision processes. In some experiments diffusion at a rate too high to be explained by collision processes has been observed. This abnormally high rate of diffusion is believed to be caused by plasma oscillations and is generally referred to as "anomalous" diffusion. Anomalous diffusion is not treated in this thesis.

The collisions which control collisional diffusion are of two types: charged particle-neutral particle collisions which have a relatively small radius of interaction, and coulomb collisions (collisions between two charged particles) which have a large radius of interaction. In plasmas in which the percentage ionization of the gas is less than about 0.01 percent, called weakly ionized gases, charged particle-neutral particle interactions dominate while plasmas with a much greater degree of ionization, called highly ionized gases, are governed by coulomb interactions and behave somewhat differently.
Most of the experiments on diffusion in low temperature plasmas have been carried out on helium. Helium is preferred because of its reasonably simple energy level system and the fact that it remains in atomic form under most conditions. Also, neutral helium atoms have a low cross-section for electron attachment so that experiments are not complicated by the presence of negative ions. Furthermore, the required information on collision processes is more accurately known for helium than for most other gases.

1.4.2 Diffusion in a weakly ionized gas with $\vec{B} = 0$ The mechanisms involved in this type of diffusion are well understood and the dependence of the diffusion coefficient on ion and electron temperature and neutral gas pressure has been clearly demonstrated. The numerical value of the diffusion coefficient for a helium plasma is in some doubt however.

The best direct measurements of the diffusion coefficient are probably those of Oskam and Mittlestadt (1963) whose measurements yield the value $D_a = 0.0421 \text{ m}^2 \text{ sec}^{-1}$ at a pressure of 1 torr and a temperature of 273°K. This value is in excellent agreement with an earlier measurement by Kerr and Leffel (1962) who used the same method. Elaborate precautions were taken by Oskam and Mittlestadt to assure purity of the helium gas used. Since the diffusion coefficient is directly related to the ion mobility, the diffusion coefficient can also be obtained from measurements of the mobility of $\text{He}^+$ ions in helium. Mobility measurements carried out by Hornbeck (1951) Biondi and Chanin (1954) and Chanin and Biondi (1957) yield values of the diffusion coefficient in good agreement with Oskam and
Mittlestadt’s value. There are, however, a number of other measurements which give higher values for the diffusion coefficient. Oskam and Mittlestadt suggest that these discrepancies are caused by impurities in the helium gas used or by mistaking He$_2^+$ for He$^+$. It is known that He$_2^+$ has a higher mobility than He$^+$ and hence a plasma with He$_2^+$ present will exhibit a higher diffusion coefficient than one in which only He$^+$ is present.

1.4.3. Diffusion in arc plasmas Arc plasmas are produced by passing a narrow beam of high energy electrons through a vessel containing neutral gas. This produces a cloud of secondary ionization around the electron beam. Bohm, Burhop, Massey and Williams (1949) carried out some early experiments with arc plasmas in magnetic fields. The electron beam was set up parallel to the magnetic field and the resulting charged particle density distribution was determined by means of probes. The diffusion coefficient could then be calculated from the density distribution.

Bohm, Burhop, Massey and Williams concluded that diffusion took place at a rate orders of magnitude larger than collision theory would indicate. However, Simon (1959) Zharinov (1960) and Tonks (1960) later showed that the diffusion mechanism in this geometry was more complicated than the model used by Bohm et al and that the observed diffusion rate could be explained on the basis of collisional diffusion by a treatment that took end-effects in the plasma container into account.

1.4.4 Diffusion in a steady discharge Measurements on the rate of diffusion in the positive column of a steady discharge indicate diffusion in agreement with collision theory for low magnetic field strengths. Lehnert
(1958) first observed that there was a critical field strength above which diffusion took place at an anomalously high rate. Kadomtsev and Nedospasov (1960) have developed a theory according to which a plasma, with a directed current flowing through it, develops a helical instability for magnetic fields above a certain value. This theory seems to explain the observed effects reasonably well.

1.4.5. Diffusion in afterglows

In most cases anomalous diffusion has been associated with applied electric fields or a directed current in the plasma. In afterglows these conditions are not present. Also, little or no ionization takes place so that the plasma is expected to be in a relatively quiescent state. Consequently afterglow diffusion is of special interest. Experiments have been carried out by several workers on both weakly and strongly ionized helium plasmas in various geometries and at various magnetic field strengths.

Syrgii and Granovskii (1959) have investigated plasma decay in the afterglow of a partially ionized gas contained in a linear discharge tube. The measurements were made by means of Langmuir probes. Qualitative agreement with the theory was found for magnetic fields \(< 0.1 \, \text{Wb/m}^2\). Very little change in the decay time was observed for magnetic fields larger than 0.1 \, \text{Wb/m}^2.\) The authors suggested that this behaviour was due to loss of the plasma by recombination. However, Golant and Zhilinskii (1960) later pointed out that the discrepancies might be due to the increased diffusion caused by coulomb collisions which were not taken into account by Syrgii and Granovskii.
Golant and Zhilinskii (1960, 1962) used a microwave technique to investigate plasma decay in magnetic fields up to 0.25 Wb/m². A straight cylindrical discharge tube was placed in a cylindrical waveguide and the density was inferred from the shift in phase of the microwaves propagating along the axis. The plasma decay with time was observed to be exponential for densities below $10^{15} - 10^{16} \text{m}^{-3}$ which indicates that in this region diffusion was controlled by charged particle–neutral particle collisions. The diffusion coefficient also exhibited the expected $1/B^2$ dependence on magnetic field strength. However, absolute values of the diffusion coefficient were higher than calculated values by a factor ranging from 2 to 10. In the higher density range ($10^{16} - 10^{17} \text{m}^{-3}$) where coulomb collisions are dominant, the diffusion coefficient is expected to be proportional to $1/B^2$ and approximately proportional to the density. This dependence was observed for magnetic fields up to about 0.15 Wb/m². Excellent quantitative agreement between experimental and calculated values of the diffusion coefficient was obtained but only by adjusting the calculated collision frequencies upward by a factor of 2 or 3. Recombination was assumed to be the dominant density decay mechanism at larger fields.

Ichimaru, Iida, Sekeguchi and Yamada (1962) used a microwave method similar to that discussed above to investigate diffusion in an afterglow. At the pressures employed (0.3 - 10 torr) the density decay was exponential for densities $< 10^{17} \text{m}^{-3}$ indicating that diffusion controlled by collisions with neutral particles was dominant. At densities $>10^{17} \text{m}^{-3}$ the density decay was not exponential and indicated that diffusion controlled by coulomb collisions was dominant in this region. Comparisons between theory and experiment showed that the observed diffusion coefficient was 3 or 4 times lower than the theoretical value.
Anisimov, Vinogradov, Golant and Konstantinov (1962) used a "free space" method in investigating the decay of a high density plasma produced by a pulsed induction or electrode discharge in a glass cylinder of 0.03 m diameter. The plasma was probed simultaneously by three microwave beams at frequencies below cutoff to determine the density distribution, while the average density was deduced from the phase shift of a transmitted beam. These data were verified by measurements of the distribution of optical emission from the plasma. In the absence of a magnetic field the electron density distribution was found to be approximately described by a zero order Bessel function as expected from diffusion theory for a weakly ionized gas. The presence of a magnetic field caused a flattening of the density profile which is also expected from theory. At magnetic fields below 0.1 Wb/m², good agreement between experiment and theory was obtained, but again only by adjusting the collision frequencies to higher values by a factor of 2 or 3. At magnetic fields greater than 0.10 to 0.15 Wb/m² the decay rate was in good agreement with that observed by Motley and Kuckes (1961) in the B-1 stellarator at fields of 3 Wb/m². Under these conditions the decay was found to be controlled by three-body recombination involving an interaction between an ion and two electrons. (Motley and Kuckes 1961; Hinnov and Hirschberg 1961).

Bostick and Levine (1955) carried out afterglow experiments on helium in a toroidal chamber. Their metal torus was rectangular in cross-section and served simultaneously as a resonator for two different microwave frequencies. One of the frequencies was pulsed at high power to produce the discharges. The other frequency was used to determine
the shift in the resonant frequency of the cavity and hence the mean electron density. The observed time constant for density decay exhibited the pressure dependence expected for diffusion over the range from 50 to 350 torr and increased with the magnetic field strength at small fields. At larger magnetic fields the time constant for density decay decreased. This was at first considered to be evidence of anomalous diffusion but may actually have been caused by drift which was not considered by Bostick and Levine (see section 1.3).

1.5 The nature of the experiments reported in this thesis

The afterglow experiments performed by the author and discussed in this thesis were all carried out on helium afterglows at pressures from 0.026 to 0.038 torr. Helium was chosen because of the advantages discussed in section 1.4.1 and because most of the previous work has been done with helium. The interest in diffusion or drift or electron cooling is in the mechanism itself rather than in the characteristics of any particular gas. Afterglow measurements were preferred because the complication of the instability due to a directed current is thereby avoided.

Toroidal geometry was used. In experiments on diffusion across a magnetic field, the two geometries used previously have been toroidal systems and long cylindrical tubes. Both of these systems approximate an endless system. This is necessary in experiments of this type if losses due to diffusion across the magnetic field are to be dominant over losses due to diffusion along the field. Diffusion along the magnetic field lines is unhindered by the magnetic field.
A toroidal system was available in the plasma betatron already constructed at the University of Saskatchewan (Skarsgard 1959). As pointed out earlier the decay of the plasma in this system is due to a combination of diffusion and drift in the inhomogeneous magnetic field. For sufficiently weak fields the diffusion process was dominant whereas for sufficiently strong fields the drift process was most important. Hence it was possible to study each process individually by proper choice of the magnetic field strength. The use of a toroidal system made it possible to determine whether or not the "anomalous diffusion" observed by Bostick and Levine (1955) in a similar geometry could be explained by drift processes.

A highly ionized gas was used at all times so that coulomb collisions were dominant.

Langmuir double floating electric probes were used in order to measure the electron energy and density simultaneously. In all the previous experiments, cited above in section 1.4, the electron temperature was assumed to be in equilibrium (at approximately 0.025 ev) with that of the ions and neutrals. This is in accordance with calculations on energy transfer through elastic collisions which indicate that in most experimental systems electron cooling should take place much more quickly than electron loss to the walls. However, recent experiments (Ingraham 1963; Olson and Skarsgard 1963) indicate that for some reason—probably metastable activity—the electron temperature in many cases does not fall off as quickly as expected. Since the diffusion rate is approximately proportional to electron temperature it is very important to have access to this quantity if valid comparisons are to be made between theory and experiment. Hence the electron
temperature was measured in conjunction with the electron density in the hope that some of the discrepancies reported previously might be resolved.

1.6 The experimental system

A complete description of the experimental system has been given previously (Olson 1962). For convenience a schematic diagram of the system (Figs. 1(a) and 1(b)) is given here together with a brief discussion of its operation.

The plasma was contained in a glass torus (Fig.1(a)) which is part of the plasma betatron apparatus. The plasma betatron can be used to feed large amounts of power into the plasma in a short time and thus is useful in producing a plasma of density such that coulomb interactions are dominant in the afterglow.

The timing sequence for the various fields employed, shown in Fig.1(b), is controlled by conventional pulse and delay circuits. The magnetic confining field $\vec{B}$ is provided by 18 equally spaced solenoidal coils wound on formers enclosing the torus. Initial breakdown of the gas inside the torus is achieved by inductively coupling a radio frequency field $\vec{E}_{rf}$ into the chamber. Frequencies from 2 to 15 Mc/sec were employed. The plasma densities provided by the rf field were not high enough, at low gas pressures, to permit suitable measurements in the afterglow. However, the much larger betatron field $\vec{E}_b$ could be used to increase the intensity of the discharge to the desired level. At 0.02 to 0.04 torr, about 1% of the gas was observed to be ionized at a time 50 microseconds after the power input to the plasma was terminated. Extrapolating the density back to the
Fig. 1(a) The toroidal plasma chamber and the fields used in producing a discharge.
Fig. 1(b) The timing sequence for a single discharge.
beginning of the afterglow indicates that the degree of ionization would approach 10% at that time. These densities are such that coulomb interactions are strongly dominant throughout the period of measurement.

Both the rf and betatron circuits were well damped to ensure that power was not fed into the plasma during the afterglow period. The timing sequence was adjusted so that the afterglow period coincided with the peak of the $\hat{B}$ waveform. This produced an essentially constant confining field throughout the first millisecond of the afterglow period, during which the measurements were carried out.

It was not feasible to bake out the vacuum system. Instead, the torus was pumped down to $10^{-6}$ torr, then filled with helium and several hundred intense discharges initiated with helium constantly flowing through the system in order to remove as much as possible of the foreign gas from the chamber walls. All the experiments were conducted with a continuous flow of helium through the system. The gas intake was balanced against throttled pumping speed to give the desired pressure, which was measured by means of a McLeod gauge. Liquid nitrogen traps were used to prevent contaminants from the pumps and pressure gauge from entering the system. The helium employed had a specified impurity content of less than 0.01 mole per cent.

1.7 Coordinates and units

Position in the torus is defined by the coordinates $r$, $\theta$, and $z$, as shown in Fig. 2(a) with $z$ taken to be parallel to the local direction of $\hat{B}$. For analytical purposes the torus is approximated by a cylinder of infinite length as shown in Fig. 2(b).
Fig. 2(a) Coordinates in the torus.

Fig. 2(b) Coordinates in the infinite cylinder.
The MKS system of units is used in all derivations and calculations in this thesis unless it is otherwise noted.

1.8 Symbols

\begin{itemize}
  \item \(a\) - radius of chamber cross section
  \item \(A\) - probe area
  \item \(\vec{B}\) - magnetic confining field
  \item \(B_0\) - collision integral
  \item \(C\) - arbitrary constant
  \item \(d\) - parameter denoting average distance particles must drift to reach the chamber wall
  \item \(D\) - diffusion coefficient for a single species of particles in the absence of a magnetic field
  \item \(D_e\) - diffusion coefficient for electrons in the absence of a magnetic field
  \item \(D_i\) - diffusion coefficient for ions in the absence of a magnetic field
  \item \(D_{a}\) - ambipolar diffusion coefficient in the absence of a magnetic field
  \item \(D_{a}^{P}\) - ambipolar diffusion coefficient perpendicular to the magnetic field
  \item \(e\) - electronic charge
  \item \(E\) - electric field
  \item \(E_{rf}\) - radio frequency electric field
\end{itemize}
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$\mathcal{E}_b$ - betatron accelerating field

$f$ - Maxwellian velocity distribution function

$G$ - fraction of excess energy transferred in a collision between two particles

$h$ - Debye length

$k$ - Boltzmann constant

$\lambda$ - mean free path

$\ln \Lambda$ - factor describing cutoff of collision integral for coulomb interactions

$L$ - characteristic diffusion length

$m$ - mass of a particle

$n$ - particle density

$p$ - pressure in torr

$P_c$ - probability of collision per unit path length

$R$ - mean radius of the torus (see Fig. 2(a))

$s^{-k}$ - most probable speed for a Maxwell distribution

$T$ - temperature (° K) of a Maxwellian velocity distribution

$\bar{u}$ - average energy of a species of particles

$\bar{v}$ - macroscopic velocity

$V_d$ - voltage applied across the probes

$V_{\alpha d}$ - drift velocity of particles of type $\alpha$ due to gradient and curvature in the magnetic field

$w$ - microscopic velocity of a particle

$\varepsilon_o$ - permittivity of free space

$\Gamma$ - particle flux
\( \lambda \) - fraction of electron energy transferred per collision  
\( \mu \) - mobility coefficient  
\( \nu_{\alpha \beta} \) - collision frequency for momentum transfer for a particle of type \( \alpha \) interacting with particles of type \( \beta \)  
\( \rho \) - Larmor radius  
\( \Sigma i_p \) - peak to peak saturated probe current  
\( \tau_{\text{diff}} \) - time constant for density decay due to diffusion  
\( \tau_{\text{drift}} \) - time constant for density decay due to drift  
\( \tau_{\text{calc}} \) - calculated time constant for density decay due to drift and diffusion combined  
\( \tau_{\text{obs}} \) - observed time constant for density decay  
\( \omega \) - Larmor frequency
2.1 Introduction

In this chapter we begin by reviewing free diffusion of a single species of particle as well as ambipolar diffusion of two different species of charged particles. An exact solution of the diffusion equation which involves the time constant for density decay is obtained for these two cases. We then proceed to derive the diffusion coefficient for ambipolar diffusion perpendicular to a magnetic field. Interactions between all species of particles are included. The various terms which appear in the expression for the diffusion coefficient are discussed and finally a description is given of an approximate method by which a time constant for density decay may be obtained under these conditions.

2.2 Diffusion of a weakly ionized gas with $\mathbf{B} = 0$

2.2.1 Free diffusion of a single species of particle  Consider a gas made up of particles of type 1 diffusing through a gas of particles of type 2 which have no macroscopic velocity. Each of the two types of particles may be either charged or uncharged. Assume that the only external force which the particles of type 1 experience is that due to collisions with particles of type 2, i.e. in the case of charged particles of type 1 there are no applied electric fields and the particle density is low enough so that the Debye shielding distance is large compared to the dimensions of the container. Consider the case in which a steady state has been established, the density of particles of type 1 is much less than that of the type 2 particles, and only the particles of type 1 have a density gradient.
Suppose that the density gradient is parallel to the negative x-direction. Then the particles of type 1 will on the average be moving in the x-direction. The rate of transfer of momentum to particles of type 1 from particles of type 2 can be expressed as

\[ \frac{d(n_1 m_1 v_{i1})}{dt} = -n_1 m_1 v_{i1} \psi \]

where \( n_1 \) refers to particle concentration, \( m_1 \) is the particle mass, \( v_1 \) is the macroscopic velocity of the particles of type 1 and \( \psi \) is a constant (for small \( v_1 \)) which satisfies the equation. \( \psi \) is called the "collision frequency for momentum transfer" for particles of type 1 interacting with particles of type 2.

If no electric fields are present the momentum transfer must be balanced by the pressure gradient of the particles of type 1, i.e.

\[ \frac{d(n_1 kT_1)}{dx} = -n_1 m_1 v_{i1} \psi \]

If the temperature of the gas is not dependent on the coordinates we can write

\[ v_{i1} = -\frac{kT_1}{m_1 \psi} \frac{dn_1}{dx} \]

Generalizing to three dimensions gives for the particle flux

\[ (2.1) \quad \vec{\Gamma}_1 = n_1 \vec{v}_1 = -\frac{kT_1}{m_1 \psi} \nabla n_1 \]

Since the diffusion coefficient is defined as the coefficient relating the particle flux to the density gradient we have for the diffusion coefficient

\[ D = \frac{kT_1}{m_1 \psi} \]

Substituting in (2.1) we obtain

\[ (2.2) \quad \vec{\Gamma}_1 = -D \nabla n_1 \]
If particles of type 1 are not being produced or destroyed in the volume under consideration the continuity equation is

\[ \nabla \cdot \Gamma_1 = - \frac{\partial n_1}{\partial t} \]

Taking the divergence of both sides of (2.2) we obtain an equation for the time variation of density of particles of type 1,

(2.3) \[ \frac{\partial n_1}{\partial t} = D \nabla^2 n_1 \]

This is generally called a "diffusion equation". For a cylindrical geometry of infinite length this equation can be solved by the method of separating the variables. The solution is (see for example Brown 1959)

(2.4) \[ n_i = \sum_{k=1}^{\infty} C_k J_0 \left( \frac{n}{L_k} \right) e^{-\tau \lambda_k} \]

where \( J_0 \) denotes the Bessel function. The index \( k \) denotes the different possible diffusion modes and \( L_k \) is called the "characteristic diffusion length" of the chamber. Using the boundary condition \( n = 0 \) when \( r = a \) we have for the first, and slowest, diffusion mode

(2.5) \[ L = \frac{a}{2.405} \]

where \( a \) is the radius of the cylinder and

(2.6) \[ \tau = \frac{L^2}{D} \]

The subscript 1 has been dropped for convenience. Thus the density at any point in the chamber decays with the time constant given in (2.6) if the slowest diffusion mode only is present. Since the higher modes decay more
quickly they tend to become insignificant after the density decay has been in progress for a time comparable to $\tau$.

2.2.2 Ambipolar diffusion with no magnetic field  In section 2.2.1 we considered the free diffusion of one species of particle. In a plasma for which the sheath thickness is much smaller than the container dimensions, diffusion of the two species of charged particles, electrons and ions, takes place simultaneously. The two species interact by means of an electric field which is set up in order to equalize the diffusion rates of the two kinds of particles. Diffusion under these conditions is called ambipolar. The expressions for the electron and ion fluxes are (Allis 1956)

$$\mathbf{\nabla}_e = \mathbf{-D}_e \mathbf{\nabla} n_e - n_e \mu_e \mathbf{E}$$

(2.7)

$$\mathbf{\nabla}_i = \mathbf{-D}_i \mathbf{\nabla} n_i + n_i \mu_i \mathbf{E}$$

(2.8)

where $\mu_e$ and $\mu_i$ are the mobilities of the particles and $\mathbf{E}$ is the electric field.

One of the basic properties of a plasma such as that considered here is its strong tendency to establish electrical neutrality over the plasma volume. The plasma, since it contains large numbers of free charges, tends to assume a charge distribution which balances out any potential difference which is larger than the particle energy ($< 1 \text{ eV}$ in the experiments considered in this thesis). Calculation shows that at densities of $10^{18} \text{ m}^{-3}$, for example, a difference of only 1% in the density of the ions and electrons would produce potentials of the order of hundreds of volts in a plasma body of the size considered in this thesis. Hence $n_e \approx n_i$ and $\mathbf{\nabla} n_e \approx \mathbf{\nabla} n_i$.
to a good approximation under the experimental conditions considered in this thesis. This implies that $\vec{r}_e = \vec{r}_i$ which is then equivalent to the assumption that $\vec{v}_e = \vec{v}_i$. Using these assumptions and eliminating $\vec{E}$ in equations (2.7) and (2.8) we obtain

$$\nabla \nabla \eta = - \left( \frac{D_e \mu_i + D_i \mu_e}{\mu_e + \mu_i} \right) \nabla \nabla \eta$$

which relates the particle flux to the density gradient for either of the two oppositely charged particle species. By analogy with equations (2.1) and (2.2) the ambipolar diffusion coefficient $D_a$ is defined by

$$D_a = \frac{D_e \mu_i + D_i \mu_e}{\mu_e + \mu_i}$$

This result was first obtained by Schottky (1924). It can be shown by methods exactly similar to those used to derive equation (2.6) that when there is no creation or destruction of ions, the time constant for density decay due to ambipolar diffusion with no magnetic field is given by

$$\tau = \frac{L^2}{D_a}$$

2.3 Ambipolar diffusion in a magnetic field

The problem of the effect of an impressed magnetic field on the diffusion rate of a plasma can be attacked from several points of view. Townsend (1912, 1915, 1938, 1947) and Huxley (1937) used an approximate method based on mean free paths to determine the diffusion rate. More detailed analyses have been carried out by Chapman and Cowling (1939), Cowling (1945), Allis (1956) and Gershmann (1956). The latter analyses were based on a solution of the Boltzmann equation and dealt with the case of a weakly ionized gas.
Golant (1960, 1963) has considered the case of a highly ionized gas, using both the averaged equations of motion and a method based on the calculation of individual particle displacements.

Diffusion of a fully ionized gas across a magnetic field has been investigated by means of the hydromagnetic equations by Spitzer (1952, 1962), Simon (1955) and others. Braginski (1957), Rosenbluth and Kaufman (1958) and others have approached the problem by attempting a more exact solution of the Boltzmann equation than that represented by the hydromagnetic equations, while Longmire and Rosenbluth (1956), Golant (1963a) and others used a method based on the calculation of individual particle displacements.

The analysis of diffusion which is presented here is based on the averaged equations of motion of the electrons and ions. The analysis is similar to the one carried out by Golant (1963) but avoids one of the approximations made by him. The effect of a gradient in the electron temperature is also included. The procedure used has the advantage of simplicity, but provides no information concerning the velocity distribution of the particles. We assume a Maxwellian velocity distribution—a condition which should be closely satisfied by the afterglows of interest here. Interactions between all species of particles are included so that the results are expected to be valid for a wide range in the degree of ionization.

We consider a partially ionized gas in the presence of a magnetic confining field. We restrict ourselves to magnetic fields which are not so large that the mechanics of collisions are affected. (i.e. the Larmor
radius is much larger than the distance at which one particle can influence another). We also require that all particles experience many collisions during the time required for the plasma parameters such as density and energy to change significantly and that \( \mathcal{h} \ll l \) and \( \rho \ll l \), where \( \mathcal{h} \) is the Debye length, \( l \) is the smallest dimension of the chamber and \( \rho \) is the Larmor radius of the particles.

For convenience we assume the plasma to be contained in a cylinder of infinite length with coordinates as shown in Fig. 2(b), and assume that all quantities are constant in the z-direction which is taken parallel to the axis of the cylinder and the confining magnetic field. We assume proportionality, i.e.

\[
(2.12) \quad \frac{\nabla n_e}{n_e} = \frac{\nabla n_i}{n_i}
\]

and congruence in the radial direction,

\[
(2.13) \quad \nu_{en} = \nu_{ih}
\]

but place no restriction on \( \nu_{ee} \) and \( \nu_{ie} \). Particle density is denoted by \( n \), velocity by \( \nu \), and the subscripts \( e \) and \( i \) denote quantities pertaining to the electrons and ions respectively.

For the sake of convenience we begin by assuming the temperature to be independent of position in the chamber; the effect of a temperature gradient is taken into account later. The averaged equations of motion for a species of particles denoted by \( \alpha \) can then be written

\[
(2.14) \quad n_\alpha \mathcal{m}_\alpha \left[ \frac{\partial \mathbf{v}_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla \mathbf{v}_\alpha \right] = n_\alpha \mathcal{E}_\alpha \left\{ \mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B} \right\} - \mathcal{K}_\alpha \nabla n_\alpha - n_\alpha \mathcal{m}_\alpha \sum_{\alpha'} \mathcal{\nu}_{\alpha \alpha'} (\mathbf{v}_\alpha - \mathbf{v}_{\alpha'})
\]
where $m_\alpha$ is the particle mass, $Z_\alpha e$ is the charge, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $k$ is the Boltzmann constant, and $T_\alpha$ is the temperature of the Maxwellian distribution. $\nu_{\alpha\beta}$ is the collision frequency for momentum transfer for particles of type $\alpha$ interacting with particles of type $\beta$.

The last term on the right of equation (2.14) represents the friction force experienced by the particles of type $\alpha$ due to collisions with other particles. This quantity in general is complex and can depend on the magnetic field strength as well as the density and temperature of the various particles involved. In effect we are using the approximation which assumes that the frictional force is proportional to the relative velocity of the two species. This requires that the macroscopic velocity of the particles be small compared to their thermal velocities. Terms of the form $\nu_{ee}$ or $\nu_{ii}$ (for which $\alpha = \beta$) are not taken into account here since collisions between particles of the same species do not change the average momentum of the species and hence result in no net momentum transfer.

The determination of a diffusion coefficient can be simplified by considering a stationary state. The terms on the left side of (2.14) then vanish. The diffusion coefficient obtained in this way does not, however, depend on the macroscopic velocity and therefore applies to non-stationary states as well, provided only that the other conditions are satisfied.
where it is assumed that the neutrals (denoted by subscript n) have zero macroscopic velocity. If we assume symmetry about the axis of the cylinder we can solve for the velocity components of the ions and electrons to obtain

$$\nu_{i\theta} = \frac{-\omega_i}{\nu_{ie} + \nu_{in}} \nu_{i\lambda} + \frac{\nu_{ie}}{\nu_{ie} + \nu_{in}} \nu_{e\theta}$$

$$\nu_{e\theta} = \frac{\omega_e}{\nu_{ei} + \nu_{en}} \nu_{e\rho} + \frac{\nu_{ei}}{\nu_{ei} + \nu_{en}} \nu_{i\rho}$$

$$\nu_{i\rho} = \frac{Ze}{m_i \nu_{in}} E_{\rho} + \frac{\omega_i}{\nu_{in}} \nu_{i\theta} - \frac{k T_i}{n_i m_i \nu_{in}} \frac{\partial n_i}{\partial \rho}$$

$$\nu_{e\rho} = \frac{-e}{m_e \nu_{en}} E_{\rho} + \frac{\omega_e}{\nu_{en}} \nu_{e\theta} - \frac{k T_e}{n_e m_e \nu_{en}} \frac{\partial n_e}{\partial \rho}$$

where $\omega_i = eB/m_i$ and $\omega_e = eB/m_e$ are the ion and electron Larmor frequencies (from here on we assume $Z = 1$). Employing (2.12) and (2.13) we can reduce (2.17), (2.18), (2.19), and (2.20) to five equations in the five unknowns: $\nu_{e\theta}$, $\nu_{i\theta}$, $\nu_{e\rho}$, $E_{\rho}$, and $\frac{1}{n_e} \frac{\partial n_e}{\partial \rho}$. The equations are more easily written if we make the substitutions
Equations (2.17) to (2.20) then become, making use of (2.12) and (2.13),

\begin{align*}
\mathbf{u}_{e\theta} &= a_1 \mathbf{u}_e + a_2 \mathbf{u}_{e\phi} \\
\mathbf{u}_{e\phi} &= a_3 \mathbf{u}_e + a_4 \mathbf{u}_{e\phi} \\
\mathbf{u}_e &= a_5 \mathbf{E}_e + a_6 \mathbf{u}_{e\theta} + a_7 \frac{1}{n_e} \frac{\partial n_e}{\partial \eta} \\
\text{and} \\
\mathbf{u}_{e\lambda} &= a_8 \mathbf{E}_e + a_9 \mathbf{u}_{e\theta} + a_{10} \frac{1}{n_e} \frac{\partial n_e}{\partial \lambda}
\end{align*}
Eliminating $E_r$ from (2.23) and (2.24) we have

$$ \nu_{e\phi} (a_8 - a_5) = a_6 a_8 \nu_{e\phi} - a_5 a_7 \nu_{e\phi} + (a_7 a_8 - a_5 a_{10}) \frac{1}{n_e} \frac{\partial n_e}{\partial n} $$

while from (2.21) and (2.22)

$$ \nu_{e\phi} = \left( \frac{a_1 + a_5 a_7}{1 - a_5 a_4} \right) \nu_{e\phi} $$

and

$$ \nu_{e\phi} = \left( \frac{a_1 + a_5 a_7}{1 - a_5 a_4} \right) \nu_{e\phi} $$

(2.26) and (2.27) may be substituted into (2.25) to obtain

$$ \nu_{e\phi} = \frac{a_7 a_8 - a_5 a_{10}}{(a_8 - a_5) - \left[ \frac{a_6 a_8 (a_7 + a_5 a_7)}{1 - a_5} - \frac{a_5 a_7 (a_2 + a_4)}{1 - a_5} \right]} \frac{\partial n_e}{\partial n} $$

We now consider separately the expression in square brackets (denoted by $\phi$)

and substitute for the $a$'s from the definitions given previously to obtain

$$ \phi = \frac{-e}{m_e \nu_{en}} \frac{\omega_e}{\nu_{en}} \left( \frac{\nu_{ei} + \nu_{ei} + \nu_{ei}}{\nu_{en} + \nu_{ei}} \right) + \frac{e}{m_i \nu_{in}} \frac{\omega_i}{\nu_{en}} \left( \frac{\nu_{ei} - \nu_{ei} + \nu_{ei}}{\nu_{en} + \nu_{ei}} \right) $$

which is equivalent to

$$ \phi = \frac{e}{m_e \nu_{en}} \frac{\omega_e}{\nu_{en}} \left( \frac{\nu_{ei} + \nu_{ei} - \nu_{ei}}{\nu_{en} + \nu_{ei}} \right) $$

Now $\frac{\omega_e}{m_e} = \frac{\omega_i}{m_i}$ so (2.29) can be written

$$ \phi = \frac{e}{m_e \nu_{en}} \frac{\omega_e}{\nu_{en}} \left( \frac{\nu_{ei} + \nu_{ei}}{\nu_{en} + \nu_{ei}} \right) $$

which is equivalent to

$$ \phi = \frac{e}{m_i \nu_{en}} \frac{\omega_i}{\nu_{en}} \left( \frac{\nu_{ei} + \nu_{ei}}{\nu_{en} + \nu_{ei}} \right) $$

Now $\mu_i = \frac{e}{m_i \nu_{en}}$ and $\mu_e = \frac{e}{m_e \nu_{en}}$ are the mobilities of the ions and
electrons for the case in which there is no magnetic field present (Allis 1956). Hence (2.31) can be written in the form

\[
\phi = (\mu_i + \mu_e) \left( \frac{\omega_i \omega_e}{\nu_i \nu_e + \nu_e \nu_i + \nu_e \nu_e} \right)
\]

If we substitute (2.32) for the expressions within the square brackets in equation (2.28) and at the same time substitute for the remaining a's in equation (2.28), noting that \( \frac{\hat{A} \nu_i}{m_i \nu_i} = D_i \) and \( \frac{\hat{A} \nu_e}{m_e \nu_e} = D_e \), we obtain

\[
\eta_e \nu_e = \frac{D_i \mu_e + D_e \mu_i}{(\mu_i + \mu_e) - (\mu_i + \mu_e) \left( \frac{\omega_i \omega_e}{\nu_i \nu_e + \nu_e \nu_i + \nu_e \nu_e} \right)} \frac{\partial n_e}{\partial n}
\]

Using the relation given in equation (2.10) and putting \( \Gamma_e = \eta_e \nu_e \), (2.33) becomes

\[
\Gamma_e = - \left( \frac{D_e}{1 + \frac{\omega_i \omega_e}{\nu_i \nu_e + \nu_e \nu_i + \nu_e \nu_e}} \right) \frac{\partial n_e}{\partial n}
\]

which relates the particle flux to the density gradient. Hence the diffusion coefficient perpendicular to the magnetic field, \( D^P_a \), is given by

\[
D^P_a = \frac{D_e}{1 + \frac{\omega_i \omega_e}{\nu_i \nu_e + \nu_e \nu_i + \nu_e \nu_e}}
\]

where \( D_a \) is the diffusion coefficient that would occur under the same conditions without a magnetic field. We use the term "B-factor" to denote the denominator of the above expression since this is the factor by which the magnetic field modifies the diffusion rate perpendicular to the magnetic field. Diffusion in the direction along the field lines is unaffected by the presence of the magnetic field.
The expression for $D_P^a$ in (2.35) is similar to one derived by Golant (1963) for a highly ionized gas except for the term $v_{ie} v_{en}$ which is missing in Golant's result, as a consequence of an approximation made in his analysis. Neglecting this term is a good approximation in most cases since the conservation of momentum demands that $n_i m_i v_{ie} = n_e m_e v_{ei}$ or, if $n_i = n_e$,

$$v_{ie} = \frac{m_e}{m_i} v_{ei} \tag{2.36}$$

From (2.36), $v_{ie} \ll v_{ei}$, which makes $v_{ie} v_{en} \ll v_{ei} v_{in}$ in most cases.

Equation (2.35) gives familiar results for certain special cases:

Case (i) $B = 0$

In this case $\omega_e = \omega_i = 0$ and we have $D_P^a = D_a$. Thus in the absence of a magnetic field the diffusion coefficient is independent of coulomb collisions even if the gas is highly ionized. This can be understood in the following way. Consideration of equations (2.17) and (2.18) shows that $v_{ee} = v_{ie} = 0$ for $B = 0$; hence the electrons and ions have no macroscopic velocity with respect to each other and no net momentum transfer takes place on the average as a result of collisions between them.

Case (ii) $v_{ei} \ll v_{en}$ and $v_{ie} \ll v_{in}$

This case corresponds to a weakly ionized gas in which interactions with neutral particles are dominant. Equation (2.35) under these conditions leads to the familiar result (see, for example, Golant 1963)

$$D_P^a = \frac{D_a}{1 + \frac{v_{ei} v_{en}}{v_{in} v_{in}}} = \frac{D_a}{1 + \mu_e \mu_i B^2}$$

Case (iii) $v_{en} = 0$ and $v_{in} = 0$ (fully ionized gas)

Under these conditions equation (2.35) becomes an indeterminate ex-
pression of the form $D^P_a = \frac{\alpha}{\omega}$. However, by modifying the basic equations (2.15) and (2.16) to suit this case it can be shown that they lead to the diffusion coefficient derived from the hydromagnetic (two-fluid) equations for a fully ionized gas (Spitzer 1962).

The effect of an electron temperature gradient can be incorporated in (2.35). If an electron temperature gradient is present equation (2.16) becomes

$$\nabla T_e = -e\eta_e (\vec{E} + \vec{u}_e \times \vec{B}) - \vec{v}_e \left( \eta_e T_e \right) - \eta_e m_e v_e (\vec{u}_e - \vec{v}_e) - \eta_e m_e v_e \nabla T_e$$

The diffusion coefficient, in the presence of a temperature gradient, may be derived from equations (2.15) and (2.37) by a method exactly similar to that used to obtain (2.35). The only difference is that the coefficient $a_{10}$, appearing in equation (2.24) will in this case be given by

$$a_{10} = -\frac{\nabla T_e}{m_e v_e} \left( 1 + \frac{1}{T_e} \frac{\partial T_e}{\partial n} \right)$$

which can be written in the form

$$a_{10} = -D'_e$$

where

$$D'_e = D_e \left( 1 + \frac{1}{T_e} \frac{\partial T_e}{\partial n} \right)$$

The diffusion coefficient, where both a temperature and a density gradient perpendicular to the magnetic field exist, is given by

$$D^f_a = \frac{D'_e \mu_i + D_e \mu_e}{\mu_e + \mu_i} \frac{\omega_e \omega_i}{1 + \frac{\nu_e v_e n_e + \nu_i v_i n_i + \nu_{ei} v_{ei} n_{ei}}$$
In the experiments reported here the temperature was constant over the chamber cross-section (see chapter 6). Hence equation (2.35) was used to compute all diffusion coefficients.

2.4 Collision frequencies

In order to make use of equation (2.35) a knowledge of the collision frequencies and \( D_\alpha \) is required. All the measurements considered here were carried out in helium. For \( \nu_{en} \) in helium we have used the value

\[
(2.39) \quad \nu_{en} = \frac{P_c P}{\bar{v}_e} \text{ sec}^{-1} \quad \left( P_c = 1830 \text{ m}^{-1} \text{ torr}^{-1} \right)
\]

where \( P_c \) is the collision probability, known accurately from measurements by Normand (1930) Ramsauer and Kollath (1932), Gould and Brown (1954). \( p \) is the pressure in torr and \( \bar{v}_e \) is the average electron velocity for the Maxwellian distribution. (2.39) gives a value for \( \nu_{en} \) that is 20% smaller than that used by Golant (1963) or 25% smaller than the value we calculate by putting \( P_c = 1830 \text{ m}^{-1} \text{ torr}^{-1} \) into Golant's expression for \( \nu_{en} \), which was obtained by taking the Maxwellian velocity distribution into account. This discrepancy is not considered important here since in practice terms in (2.35) containing \( \nu_{en} \) have only a small effect on the diffusion coefficient.

\( \nu_{ei} \) was calculated from the dynamical friction coefficient derived by Chandrasekhar (1942, 1943) and Spitzer (1962), from a statistical theory of coulomb interactions. This gives

\[
(2.40) \quad \nu_{ei} = 4\pi n_e \left( \frac{e^2}{4\pi \bar{v}_e m_e} \right)^2 \frac{L_n \Lambda}{\sigma^3} \text{ sec}^{-1}
\]

where \( L_n \Lambda \) is a factor, tabulated by Spitzer, describing the cutoff of the
collision integral. \( \gamma_{ei} \) as defined by (2.40) is the reciprocal of the "slowing down" time calculated by Spitzer. We have employed the average thermal velocity of the electrons in (2.40) to obtain an approximate average collision frequency for a Maxwellian distribution. Values calculated in this manner are 8% lower than those calculated by Golant (1963) by taking the velocity distribution into account.

\( \gamma_{ie} \) was calculated from equation (2.36).

Finally, \( \gamma_{in} \) was calculated from the mobility of He+ in helium, measured by Oskam and Mittlestadt (1963). They conclude that \( \mu_1 = 0.00107 \text{ m}^2 \text{ volt}^{-1} \text{ sec}^{-1} \) at 1 atmosphere pressure and 273 °K which implies a collision frequency

\[
(2.41) \quad \gamma_{in} = 2.85 \times 10^7 p \quad \text{sec}^{-1}
\]

where \( p \) is measured in torr. The mobility value obtained by Oskam and Mittlestadt is lower than that previously reported by Biondi and Brown (1949) and others, but there is some evidence that the higher mobility values obtained by these workers may be due to the presence of He\(_2^+\) and other ions (see section 1.4.2).

2.5 Numerical value for \( D_a \) in helium

The mobility value of Oskam and Mittlestadt can also be used to determine a numerical value for \( D_a \) as a function of gas pressure and electron temperature. If the Einstein relations for a Maxwellian distribution

\[
\frac{D_e}{\mu_e} = \frac{kT_e}{e} \quad \text{and} \quad \frac{D_i}{\mu_i} = \frac{kT_i}{e}
\]
are substituted into equation (2.10) and we note that \( \mu_e \gg \mu_i \) in practice, we have

\[
D_e = \mu_e \frac{k T_e}{e} \left(1 + \frac{T_e}{T_i}\right)
\]  

(2.42)

In the experimental work considered here we assume that the ions remain at the ambient temperature, since they are in good collisional contact with the neutrals. Also, the ion mobility is inversely proportional to the gas pressure. Under these conditions (2.42) becomes

\[
D_e = 0.042 \frac{p}{1 + \frac{T_e}{T_i}} m^2 \text{sec}^{-1}
\]  

(2.43)

where \( p \) is the pressure in torr. This expression was substituted in (2.35) in calculating the diffusion coefficients in this thesis.

2.6 Time constant for density decay due to non-linear diffusion

Under conditions for which coulomb collisions are dominant the diffusion coefficient defined by equation (2.35) will be a function of position in the plasma chamber (since the plasma density varies with position). The differential equation describing the density decay under these conditions is

\[
\frac{\partial n}{\partial t} = \frac{1}{n} \frac{\partial}{\partial n} \left( n D_e \frac{\partial n}{\partial n} \right)
\]  

(2.44)

This equation is not easily solved; hence an approximate method of calculating the theoretical time constant for density decay must be found. For the cases of constant diffusion coefficients dealt with earlier, solutions of the diffusion equation yielded expressions for the time constant for density decay of the form.
(2.45) \[ \tau = \frac{\text{diffusion coefficient}}{l^2} \]

(see equations (2.6) and (2.11)). In order to obtain an approximate time constant for density decay for the case in which the diffusion coefficient varies over the torus cross-section, an average diffusion coefficient to be substituted in (2.45) was calculated by using an "average" density \( \bar{n} = \frac{n_0}{2} \), where \( n_0 \) is the measured density at the centre of the torus. The accuracy of this method for obtaining a theoretical time constant for density decay was checked, in one case, by measuring the particle energy and density profiles in the plasma chamber by means of movable probes. The observed profiles are shown in Fig. 6, Chapter 6. Using these results numerical values for all quantities on the right side of equation (2.44) can be found, which then gives a value for the time constant directly. The time constant obtained by the approximate method (assuming \( \bar{n} = \frac{n_0}{2} \)) was found to be about 20% lower than that obtained directly by the more exact numerical solution. This accuracy was considered sufficient for our purposes.
CHAPTER 3
DRIFT IN THE INHOMOGENEOUS MAGNETIC FIELD

3.1 Introduction

In the experiments with which we are concerned in this thesis we have a plasma, in which collisions are effective, confined by a toroidal magnetic field. Such a field has curved lines of force and an inherent gradient, both of which lead to drift effects that provide an additional mechanism by which the plasma may be lost to the container walls. The contribution of drift losses to the overall density decay rate must be known if the losses due to diffusion are to be correctly evaluated (see section 1.3).

The magnitude of the drift velocity of a single particle due to combined gradient and curvature effects \( V_d' \), is given by (Spitzer 1962)

\[
V_d' = \frac{1}{\omega R} \left( \frac{1}{2} w_{\perp}^2 + w_{\parallel}^2 \right)
\]

where \( \omega \) is the cyclotron frequency of the particle, \( R \) is the radius of curvature of the lines of force, \( w_{\perp} \) is the component of the particle velocity perpendicular to \( B \) and \( w_{\parallel} \) is the component parallel to \( B \). Drifts of the positive and negative particles are oppositely directed with the positive particle drift having the direction \( \mathbf{B} \times \mathbf{v} \) (see Fig. 2(a)). These drift effects tend to separate the charges, producing a polarization electric field in the plasma, and a consequent crossed field drift which causes particles of both signs to move together toward the chamber wall in the radial direction. Chandrasekhar (1960) has treated the case of a fully ionized collisionless plasma in a torus with non-conducting walls and concludes that the plasma undergoes acceleration in the radial direction as the polarization field grows with time. In the analysis
presented here a partially ionized plasma is considered, taking collisions into account, and it is found that the plasma drift rate, under these conditions, quickly reaches a quasi-stationary value.

3.2 Theory

We consider a partially ionized gas contained in a torus with non-conducting walls and confined by a time independent magnetic field. We assume that a particle undergoes many collisions during the time required for the plasma parameters, such as density and energy, to change significantly and that the Debye length and the Larmor radii of the particles are small compared to the chamber dimensions. It is also assumed that the velocity distributions of the various species of particles are Maxwellian at all times. Equation (3.1) then becomes

\[ V_{\alpha d} = \frac{4 \bar{u}_\alpha}{3 m_\alpha \omega_\alpha R} \]

where \( V_{\alpha d} \) is the macroscopic drift velocity of a group of particles of type \( \alpha \) and \( \bar{u}_\alpha \) and \( m_\alpha \) are the average energy and mass of the particles. Now a plasma in which collisions are important has a finite conductivity perpendicular to the magnetic field and hence the polarization electric field due to drift currents grows only to the point where the drift currents are balanced by the mobility currents (see Fig. 3(a) page 43). Once this regime is set up the electric field changes only relatively slowly - at a rate determined by the rate of change of particle energy. This implies that the currents in the plasma will be small and hence to a good approximation

\[ \overrightarrow{U}_e = \overrightarrow{U}_c \]
where \( \vec{V}_d \) and \( \vec{V}_e \) denote the average macroscopic velocity of the ions and electrons, taking both drift and conductivity effects into account.

In order to analyze the drift problem under these conditions we assume that the total macroscopic velocity of a group of particles for which \( \omega/\nu \gg 1 \) (where \( \nu \) is the collision frequency for momentum transfer) can be written as the sum of the velocities due to the various processes, i.e.

\[
\vec{V}_\alpha = \vec{V}_{\alpha d} + \mu_\alpha \vec{E} + \frac{\vec{E} \times \vec{B}}{B^2}
\]

where \( \vec{V}_{\alpha d} \) is the drift velocity given by (3.2), \( \mu_\alpha \) is the mobility of particles of type \( \alpha \) perpendicular to the magnetic field and the sign preceding the mobility term depends on the charge. The last term in (3.4) is the crossed field drift. A particle for which \( \omega/\nu \ll 1 \) does not drift and hence its velocity will consist of a mobility term only.

We employ a right-handed rectangular coordinate system with the \( z \)-axis parallel to the local direction of \( \vec{B} \) and the \( y \)-axis parallel to \( \vec{V}_B \). The electron drift given by (3.2) will then be parallel to the \( x \)-axis. We consider two separate cases which will be referred to as the "weak field" case and the "strong field" case. In the weak field case only the electron drift in the inhomogeneous magnetic field is important while in the strong field case both the electron and ion drifts are taken into account.

**Case (i) Weak magnetic field**

Here we consider the case in which \( \omega_e/(\nu_{en} + \nu_{el}) \gg 1 \) and \( \omega_i/\nu_{in} \ll 1 \), where, for example \( \nu_{en} \) is the collision frequency for momentum transfer of electrons interacting with neutrals and the other collision frequencies are defined similarly. Under these conditions the ion motion is
relatively unaffected by the magnetic field and the macroscopic velocities of the electrons and ions respectively can be written

\[ (3.5) \quad \vec{v}_e = \vec{V}_e - \mu_e \vec{E} + \frac{\vec{E} \times \vec{B}}{B^2} \]

and

\[ (3.6) \quad \vec{v}_i = \mu_i \vec{E} \]

The situation described by these equations is one in which the electrons are drifting to the chamber walls and drawing the ions with them. The electron drift velocity is modified by the small electric field which is set up.

Combining (3.3) and (3.6) yields

\[ (3.7) \quad \vec{E} = \frac{\vec{v}_e}{\mu_e} \]

Substituting (3.7) into (3.5) and writing the components separately we obtain

\[ (3.8) \quad (v_e)_x = V_{e_d} - \frac{\mu_e}{\mu_i} (v_e)_x + \frac{1}{\mu_i B} (v_e)_y \]

and

\[ (3.9) \quad (v_e)_y = - \frac{\mu_e}{\mu_i} (v_e)_y - \frac{1}{\mu_i B} (v_e)_x \]

or

\[ (3.10) \quad (v_e)_y = - \frac{1}{B} \left( \frac{1}{\mu_i + \mu_e} \right) (v_e)_x \]

which gives the ratio of \((v_e)_x\) and \((v_e)_y\). The plasma moves to the container.
wall in a direction making an angle with the x-axis given by

\[ \Theta = \tan^{-1} \left[ -\frac{1}{B} \left( \frac{1}{\mu_i^p + \mu_e^p} \right) \right] \]

Substituting (3.10) into (3.8) we have

\[ (\mathbf{v}_e)_x = \frac{V_{ed} \mu_i^p B}{1 + B^2 (\mu_i^p + \mu_e^p)^2} \]

Now the magnitude of the macroscopic velocity can be written

\[ v_e = \left[ (\mathbf{v}_e)_x^2 + (\mathbf{v}_e)_y^2 \right]^{\frac{1}{2}} = (\mathbf{v}_e)_x \left[ \frac{1 + (\mathbf{v}_e)_y^2/(\mathbf{v}_e)_x^2}{} \right]^{\frac{1}{2}} \]

Substitution from (3.10) and (3.12) gives

\[ \mathbf{v}_e = \frac{V_{ed} \mu_i^p B}{[1 + B^2 (\mu_i^p + \mu_e^p)^2]^{\frac{1}{2}}} \]

which is the velocity at which the plasma moves across the magnetic field.

Since the ions and electrons have no relative macroscopic velocity with respect to each other, collisions between them are not effective in determining their macroscopic velocities, and therefore only collisions with neutrals need to be taken into account. The mobilities (Allis 1956) become

\[ \mu_e^p = \frac{e}{m_e \nu_e} \sim \frac{e \nu_e}{m_e \omega_e} \quad \text{since} \quad \frac{\omega_e}{\nu_e} \gg 1 \]

and

\[ \mu_i^p \sim \frac{e}{m_i \nu_i} \quad \text{since} \quad \frac{\omega_i}{\nu_i} \ll 1 \]

Substitution in (3.13) then gives

\[ \mathbf{v}_e \sim \frac{V_{ed} \omega_i}{\nu_i} \]

or, making use of equation (3.2) we obtain

\[ \mathbf{v}_e \sim \frac{4 \bar{u}_e}{3 R m_i \nu_i} \]
Fig. 3(a) Directions of fields and drift velocity components for case (i).

Fig. 3(b) Directions of fields and drift velocity components for case (ii).
Case (ii) Strong magnetic field

Here we consider the case for which $\omega_e / (\nu_{en} + \nu_{ei}) \gg 1$ and $\omega_i / (\nu_{in} + \nu_{ie}) \gg 1$. In this case both ions and electrons tend to drift in the inhomogeneous magnetic field (see Fig. 3(b)). A polarization electric field will be set up but again it will grow only to a value such that the mobility current balances the drift current; at this point the electric field becomes quasi-stationary. Under these conditions the macroscopic velocities of the electrons and ions are

\begin{equation}
\vec{v}_e = \vec{v}_{ed} - \mu_e^p \vec{E} + \frac{\vec{E} \times \vec{B}}{B^2} \tag{3.16}
\end{equation}

and

\begin{equation}
\vec{v}_i = \vec{v}_{id} + \mu_i^p \vec{E} + \frac{\vec{E} \times \vec{B}}{B^2} \tag{3.17}
\end{equation}

Combining (3.3), (3.16), and (3.17) gives

\begin{equation}
\vec{v}_{ed} - \vec{v}_{id} - \mu_e^p \vec{E} - \mu_i^p \vec{E} = 0 \tag{3.18}
\end{equation}

Now $\vec{V}_{ed}$ and $\vec{V}_{id}$ are oppositely directed so that

\begin{equation}
|\vec{V}_{ed} - \vec{V}_{id}| = V_{ed} + V_{id}
\end{equation}

and hence

\begin{equation}
\vec{E} = \frac{V_{ed} + V_{id}}{\mu_e^p + \mu_i^p} \tag{3.19}
\end{equation}

The components of (3.16) are

\begin{equation}
(v_x^e) = V_{ed} - \mu_e^p \left( \frac{V_{ed} + V_{id}}{\mu_e^p + \mu_i^p} \right) \tag{3.20}
\end{equation}
and

\begin{equation}
(\mathbf{v}_e)_{\parallel} = -\frac{E}{B} = -\frac{1}{B} \left( \frac{V_{ed} + V_{id}}{\mu_e + \mu_i} \right)
\end{equation}

The resultant velocity is

\[ \mathbf{v}_e = \left[ (\mathbf{v}_e)_x^2 + (\mathbf{v}_e)_y^2 \right]^{\frac{1}{2}} \]

or

\begin{equation}
\mathbf{v}_e = \frac{1}{\mu_e + \mu_i} \left[ \left( \mu_i^p V_{ed} - \mu_e^p V_{id} \right)^2 + \left( \frac{V_{ed} + V_{id}}{B} \right)^2 \right]^{\frac{1}{2}}
\end{equation}

As in case (i) the ions and electrons are moving together so that the mobilities in (3.22) are given approximately by (Allis 1956)

\[ \mu_e^p = \frac{e}{m_e V_{en}^2} \left( 1 + \left( \frac{\omega_e}{\nu_{en}} \right)^2 \right) \]

and since for the case under consideration \( \omega_e / \nu_{en} > 1 \) and \( \omega_i / \nu_{in} > 1 \), we can write \( B \mu_i^p \approx \nu_{in} / \omega_i \) and \( B \mu_e^p \approx \nu_{en} / \omega_e \). Also, in practice \( \nu_{en} / \omega_e \ll \nu_{in} / \omega_i \) and \( \mu_e^p \ll \mu_i^p \). Under these conditions (3.22) may be written

\begin{equation}
\mathbf{v}_e \approx \frac{\omega_i}{\nu_{in}} \left( V_{ed} + V_{id} \right)
\end{equation}

Substituting from (3.2) and noting that \( m_0 \omega_e = m_1 \omega_i \) we obtain for the velocity at which the plasma crosses the magnetic field

\begin{equation}
\mathbf{v}_e \approx \frac{4(\mathbf{v}_e + \mathbf{v}_i)}{3 \tau m_0 \nu_{in}}
\end{equation}

Golant (1963) has considered drift in a toroidal geometry under the same conditions as those pertaining to this strong field case. The pro-
procedure used was the solution of the averaged equations of motion for the electrons and ions. The resulting drift velocity found by Golant is identical to the value given by our equation (3.24).

Comparing (3.15) and (3.24) we note that the expressions for the macroscopic drift velocity are similar in both cases except for the factor $\vec{u}_1$ which does not appear in (3.15). This is to be expected since in case (i) only the electron drift, which is proportional to $\vec{u}_e$, is effective in setting up the electric field whereas in case (ii) both ion and electron drifts contribute to the electric field. In cases for which the average electron energy is much higher than the average ion energy equations (3.15) and (3.24) yield essentially the same result.

It is interesting to note that, in both cases discussed above, the rate of drift is independent of the magnetic field strength. Since the diffusion rate varies inversely as the square of the magnetic field strength, this would indicate that in general diffusion could be expected to be the dominant loss mechanism at low magnetic fields and drift could be expected to become dominant at higher fields. This is borne out by the experimental data.

Measurement of the electric field set up by the drift process (see equations (3.7) and (3.19)) could be used as a further check on the validity of the drift analysis presented above. If (3.15) is substituted in (3.7) we obtain

$$E = \frac{4 \vec{u}_e}{3 \pi e}$$

for the magnitude of the electric field in the "weak field" case. The
direction of \( E \) will be given by (3.11) in this case. Similarly, if (3.2) is substituted in (3.19) and we assume that \( m_e \nu_{en} \ll m_i \nu_{in} \) we obtain

\begin{equation}
(3.26) \quad E = \frac{4}{3 \rho_e} \frac{\omega_i}{\nu_{in}} \left( \bar{u}_e + \bar{u}_i \right)
\end{equation}

for the magnitude of the electric field in the "strong field" case. Here the electric field will have the direction \( \vec{\nu} B \times \vec{B} \), from consideration of (3.13) and the fact that the drift of equation (3.1) is in the direction \( \vec{B} \times \vec{\nu} B \) for a positive particle.

For the experimental conditions under consideration in this thesis the electric field intensity predicted by (3.25) and (3.26) is 20 volts/m or less which could perhaps produce a detectable difference in the floating potential of two probes separated by a few cm at points along the electric field. No measurements of this type were attempted in these experiments since the drift theory was developed after the experiments were performed, and the experimental system was considerably modified by that time.
CHAPTER 4
ELECTRON COOLING THEORY

4.1 Introduction

A typical electron temperature decay curve is shown in Fig. 5, Chapter 6. The temperature of the electrons, which is of the order of a few eV during the period when power is being fed into the plasma, decays to approximately the ion temperature within about 200 microseconds of the beginning of the afterglow period. The ions are expected to remain close to the ambient temperature at all times since their large mass and low velocity prevents them from gaining much energy in the rf or betatron fields. Also, since an ion on the average loses one half of its excess energy in an elastic collision with a neutral atom, the ion and neutral temperatures can be assumed to be equal by the time afterglow measurements begin.

There are several processes which can affect the rate of cooling of the electron gas. An analysis of one of these, cooling due to elastic collisions, will be carried out. The remaining processes are discussed qualitatively.

Electron cooling through elastic collisions results in general from a combination of electron - ion collisions and electron - neutral collisions. In a weakly ionized gas the cooling rate is determined mainly by electron - neutral collisions while in a strongly ionized gas electron - ion collisions are dominant.

4.2 Electron cooling due to collisions with neutrals

In this section the electron cooling rate due to electron - neutral collisions is calculated by a simple integration over the electron velocity distribution. An alternative treatment based on the Boltzmann equation
Collisions of low energy electrons with helium atoms in the ground state are elastic since the helium atom does not have low-lying energy levels. Under these conditions the electrons lose, on the average, a fraction \( \lambda = 2m_e m_h^{-1} \left( 1 - \bar{u}_n / \bar{u}_e \right) \) of their energy in a single collision (Beketi and Brown 1958). \( \bar{u}_n \) and \( \bar{u}_e \) are the average energies of the neutrals and electrons respectively. We also have for the collision frequency for momentum transfer in helium (see section 2.4)

\[
(4.1) \quad \nu_{en} = P_e P v
\]

With this information, the rate of cooling can be calculated provided the electron velocity distribution is known. The electron velocity distribution is expected to be very nearly Maxwellian at all times in the afterglow since the rate of transfer of energy between the electrons themselves is by far the fastest process taking place. The rate of cooling of the electrons is then given by

\[
(4.2) \quad \left( \frac{d\bar{u}_e}{dt} \right)_{e-n} = -\int_0^\infty \left[ \lambda \nu_{en}(v) \bar{u}_n \right] f(v) dv
\]

\( f(v) \) is the Maxwellian velocity distribution, which we write in the normalized form

\[
\frac{\nu}{\sqrt{\pi}} 5^{\frac{3}{2}} v^2 e^{-\nu^2}
\]

where \( \left( \frac{1}{3} \right)^{\frac{1}{2}} = \left( \frac{4 \bar{u}_n}{3 m_e} \right)^{\frac{1}{2}} \) is the most probable electron velocity. Substituting in equation (4.2) we obtain

\[
(4.3) \quad \left( \frac{d\bar{u}_e}{dt} \right)_{e-n} = - \frac{m_e^+}{m_i} \left( 1 - \bar{u}_e / \bar{u}_m \right) \frac{4 P_e P}{\sqrt{\pi}} \int_0^\infty 5^{\frac{3}{2}} v^2 e^{-\nu^2} dv
\]
By using the substitution $y = sv^2$ the integral in (4.3) can be written in the form

$$A \int_{y_0}^{\infty} y^{x-1} e^{-y} dy = A \Gamma(x)$$

where $A$ is a constant and $\Gamma(x)$ is the Gamma function. If this operation is carried out we have for the cooling rate due to electron-neutral interactions

$$\left( \frac{d \bar{v}_e}{dt} \right)_{c-n} = -\frac{m_i}{m_e} \left( |1 - \frac{\bar{v}_n}{\bar{v}_e}| \right) \gamma(\bar{v}_e)_{c-n} \bar{v}_e$$

or

$$(4.4) \quad \left( \frac{d \bar{v}_e}{dt} \right)_{c-n} = -\frac{16}{3\pi} \frac{m_i}{m_e} \left( |1 - \frac{\bar{v}_n}{\bar{v}_e}| \right) \gamma(\bar{v}_e)_{c-n} \bar{v}_e$$

where $(\gamma(\bar{v}_e)_{c-n})$ is the collision frequency of electrons with velocity equal to the most probable speed, $\bar{v}_e = \left( \frac{4\pi m_e}{3kT_e} \right)^{\frac{1}{2}}$, for the Maxwellian distribution.

4.3 Cooling due to coulomb interactions

The effect of cooling due to coulomb interactions between electrons and ions can be calculated from results obtained by Spitzer (1940, 1962) using a statistical theory of coulomb interactions. For Maxwell velocity distributions

$$(4.5) \quad \left( \frac{d \bar{v}_e}{dt} \right)_{c-I} = \frac{\bar{v}_i - \bar{v}_e}{\tau_{eq}}$$

where

$$(4.6) \quad \tau_{eq} = \frac{3 m_i (kT_e)^{\frac{3}{2}}}{8 (2\pi)^{\frac{3}{2}} n_i m_e^{\frac{3}{2}}} \left( \frac{4\pi e^2}{e^z} \right)^2 \frac{1}{ln \Lambda}$$
is the time of equipartition. In applying this result $n_i$ is replaced by $n_o$, the measured ion density at the centre of the chamber where $T_e$ is measured simultaneously.

4.4 Other processes affecting the rate of decay of electron temperature

4.4.1 Drift cooling Loss of particles by drift processes can reduce the average electron energy if the more energetic electrons in the velocity distribution are carried to the container wall more quickly than those of lower energy. In the toroidal geometry used in the experiments reported in this thesis particle losses due to drift involve a combination of drift due to the gradient and curvature in the magnetic field and the crossed field drift (see Chapter 3).

The drift rate due to the gradient and curvature in the magnetic field is proportional to particle energy and hence preferential loss of the high energy electrons and electron cooling will result if this process is dominant in moving the electrons to the chamber wall. The crossed field drift rate is independent of energy so that cooling will not result in cases where this is the dominant transport process. The relative importance of these two processes determines the amount of cooling that will take place due to drift, and can be found from an examination of the direction in which the resultant drift takes place.

If conditions are such that the analysis in case (i) Chapter 3 is applicable the direction of drift is given by equation (3.11) Small values of $\Theta$ indicate that drift mechanisms which can cause cooling are effective while values of $\Theta$ in the neighborhood of 90° indicate that the plasma motion toward the wall is caused primarily by the crossed field drift which does
not result in electron cooling. Equation (3.11) can be written

\[
\Theta = \tan^{-1} \left[ -\frac{1}{B(\mu_i^R + \mu_e^R)} \right] = \tan^{-1} \left[ -\frac{\omega_i}{\nu_\infty + \nu_e} \right]
\]

Since the conditions for validity of the analysis in this case are \( \omega_i/\nu_\infty \ll 1 \) and \( \nu_e/(\nu_\infty + \nu_e) \gg 1 \), the denominator of the expression in brackets is much less than 1, hence \( \Theta \) has a value approaching 90°. This indicates that drift losses are not effective in causing cooling of the electrons in this case.

If conditions are such that the analysis in case (ii) Chapter 3 is applicable the mobilities of both electrons and ions is low. A comparison of the x and y components of the resultant drift velocity (equations (3.20) and (3.21)) then shows that the crossed field drift is the dominant mechanism for removal of the electrons and electron cooling effects due to drift are again insignificant.

The conclusions regarding the unimportance of drift cooling in the experiments reported in this thesis can be put on a somewhat more quantitative basis. An analysis has been carried out (Olson 1962) in which the rate of loss of the electrons was assumed to be proportional to the electron energy. This corresponds to a situation in which drift losses are due to the gradient and curvature drifts only. Consequently this analysis leads to the fastest possible cooling rate due to drift under any conditions.

The result obtained is

\[
(4.7) \quad \frac{1}{\nu_e} \frac{d\nu_e}{dt} = \frac{2}{3} \frac{1}{n_e} \frac{dn_e}{dt}
\]
which indicates that even in this case energy decay should go at a slower rate than the density decay. In the experiments reported in this thesis the energy decay actually took place at a rate several times faster than the density decay. Hence drift cooling could not have contributed significantly to the observed cooling rate.

4.4.2 Effect of recombination on electron cooling Recombination processes favour removal of electrons from the low energy part of the distribution and thus would tend to slow the cooling rate. However, as discussed in Chapter 6, recombination does not contribute significantly to the density decay rates in the experiments considered here, hence recombination cannot be effective in determining the electron cooling rate.

4.4.3 Effect of metastable atoms Electron cooling which is slower than the predicted value due to elastic collisions alone can be caused by the presence of metastable atoms which may give up their excitation energy to the electrons during the afterglow. The helium atom has metastable states at 19.8 and 20.6 eV with long lifetimes. There are two alternative processes, involving these metastable atoms, which can add energy to the electrons. Ionizing collisions may occur between two metastable atoms, resulting in the production of energetic electrons. Alternatively, electrons may interact superelastically with metastable atoms. An analysis of the relative importance of these two processes in the experiments reported in this thesis is given in Appendix B.
Double floating probes were used to measure the electron temperature and density as a function of time in the afterglow. Since these quantities decay with time (in about 1 millisecond in this experiment) it was necessary to sweep the probe voltage at a rate which would produce several probe characteristics during this time. An ac method was used. A detailed discussion of the probe operation has been given previously (Olson 1962, p. 4-20) and the ac measurement technique and the probe circuit have also been described (Olson 1962, p. 29-35). For the sake of completeness a brief discussion is included here.

The electron energy is related to the probe characteristics by (Johnson and Malter 1950)

\[ \frac{\hbar T_e}{e} = \frac{\Sigma i_p}{4} \left( \frac{dV_d}{di} \right)_{V_d=0} \]

\( \Sigma i_p \) is the peak-to-peak saturation probe current, i.e. the sum of the saturation currents to both probes, and \( \left( \frac{dV_d}{di} \right)_{V_d=0} \) is the effective resistivity of the plasma at the point where the potentials of the two probes are equal. \( V_d \) denotes the potential applied across the probes. The electron density is determined from (Bohm, Burhop, Massey and Williams 1949; Johnson and Malter 1950)

\[ n_e = \frac{\Sigma i_p}{0.8 Ae} \left( \frac{m_i}{2 \hbar T_e} \right)^{1/2} \]

where \( A \) is the probe area which is assumed to be the same for both probes and the plasma is assumed to be singly ionized and charge neutralized (elec-
tron and ion densities equal).

Equations (5.1) and (5.2) are derived for the case that \( \lambda << \lambda + a \ll \bar{\lambda} \) (where \( \lambda \) is the Debye length, \( a \) is the probe radius and \( \bar{\lambda} \) is the smallest of the mean free paths of the ions and electrons), \( T_e >> T_i \) and \( E = 0 \). The required relationship between the temperatures is satisfied as long as the electron temperature is more than approximately twice the ion temperature according to Bohm, Burhop, Massey and Williams (1949). Bohm et al have also shown that when \( a + h \ll \bar{\lambda} \) the current drawn by the probe can be supplied by diffusion from the surrounding regions without significantly disturbing the particle concentration.

In the experimental work considered here most of the measurements were carried out in the presence of a magnetic field such that \( \omega_c >> \nu_{ei} + \nu_{en} \). Under these conditions the electron mean free path perpendicular to the magnetic field is effectively reduced to the Larmor radius. Consequently we have replaced the requirement given above with a similar one, namely that

\[
(5.3) \quad \lambda << \lambda + a \ll \bar{\rho}_e
\]

where \( \bar{\rho}_e \) is the average Larmor radius of the electrons. The presence of a magnetic field does not change the random particle flux within the plasma. When (5.3) is satisfied, the motion of the particles in the neighborhood of the probe is essentially straight line motion and the particle collection should not be hindered by the magnetic field. The criterion (5.3) may be overstrict since motion of the particles along the lines of force is unaffected by the magnetic field. Also, the double probes collect only electrons from the high energy "tail" of the Maxwellian velocity distribution which will have Larmor radii larger than \( \bar{\rho}_e \). In the absence of complete
information on the behavior of probes in a magnetic field it was considered wise to adhere to criterion (5.3) even though it may be overstrict.

The physical dimensions of the probes must be impossibly small if criterion (5.3) is to be satisfied at magnetic fields larger than about 0.02 Wb/m² for electrons with the energies of interest here. Thus in order to satisfy (5.3) the probe method is limited to rather special conditions. Since reliable measurements required a magnetic field of less than approximately 0.02 Wb/m² it was necessary to perform the measurements at low pressure in order that diffusion would be dependent on the magnetic field. To satisfy criterion (5.3) probes made of 0.025 mm diameter tungsten wire (Fig. 4) were used for most of the measurements. A few measurements at 0.0040 Wb/m² were carried out by means of 0.3 mm diameter spherical copper probes which also satisfied (5.3) at these low fields. Since the pulsed discharge produced in the experiments considered here lasts only a few hundred microseconds there is no problem with the probes heating up.

Assuming that the particle collection is correctly described by (5.1) and (5.2), the estimated probable error in a single measurement is 25% for temperature measurements and 20% for density measurements. Curves determined from a set of points should have somewhat better accuracy.
Fig. 4  Probe construction.
Chapter 6

Results and Discussion

6.1 Density decay due to diffusion and drift

Measurement of the electron temperature and density as a function of time in the afterglow yielded decay curves similar to those shown in Fig. 5. Curves of this type were obtained for afterglows at magnetic fields of 0.0040, 0.0080, 0.012, 0.016, 0.042 and 0.086 Wb/m² at pressures in the vicinity of 0.03 torr.

The main object of the experimental work carried out was to compare the observed decay rate of the plasma with that predicted by the diffusion and drift theories developed in Chapters 2 and 3, for various experimental conditions. The results are shown in Table I. $\tau_{\text{obs}}$, the observed time constant for density decay, defined by

$$\frac{1}{\tau_{\text{obs}}} = \frac{1}{n_e} \frac{dn_e}{d\tau}$$

was obtained directly from the slope of the density decay curves and is compared with $\tau_{\text{calc}}$, defined by

$$\frac{1}{\tau_{\text{calc}}} = \frac{1}{\tau_{\text{diff}}} + \frac{1}{\tau_{\text{drift}}}$$

where $\tau_{\text{diff}}$, the time constant for density decay by diffusion is calculated from equation (2.45) and $\tau_{\text{drift}}$, the time constant for density decay due to drift effects is calculated from the relationship

$$\tau_{\text{drift}} = \frac{d}{U_d}.$$ 

$U_d$ is the drift rate (from equation (3.15) or (3.24)) and $d$ is the distance to the chamber wall. In most cases the detailed shape of the density profile is not known. Therefore the correct effective value of $d$
Fig. 5 Electron energy and density decay in a helium afterglow.
TABLE I

COMPARISON OF CALCULATED AND MEASURED TIME CONSTANTS FOR DENSITY DECAY

<table>
<thead>
<tr>
<th>Field and pressure</th>
<th>Electron energy (eV)</th>
<th>Density ( \times 10^{-17} \text{m}^{-3} )</th>
<th>B-Factor</th>
<th>( \tau_{\text{diff}} ) (( \mu \text{sec} ))</th>
<th>( \tau_{\text{drift}} ) (( \mu \text{sec} ))</th>
<th>( \tau_{\text{calc}} ) (( \mu \text{sec} ))</th>
<th>( \tau_{\text{obs}} ) (( \mu \text{sec} ))</th>
</tr>
</thead>
<tbody>
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<td>0.0040 Wb/m²</td>
<td>0.13</td>
<td>169</td>
<td>1.03</td>
<td>40</td>
<td>665</td>
<td>38</td>
<td>74</td>
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<tr>
<td>0.038 torr</td>
<td>0.11</td>
<td>72</td>
<td>1.04</td>
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<td>1260</td>
<td>66</td>
<td>89</td>
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<td>1.82</td>
<td>106</td>
<td>1130</td>
<td>97</td>
<td>162</td>
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<tr>
<td>0.11</td>
<td>4.7</td>
<td>2.42</td>
<td>175</td>
<td>1510</td>
<td>157</td>
<td>233</td>
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<tr>
<td>0.09</td>
<td>2.0</td>
<td>3.54</td>
<td>286</td>
<td>1750</td>
<td>280</td>
<td>217</td>
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<tr>
<td>0.012 Wb/m²</td>
<td>0.16</td>
<td>145</td>
<td>1.27</td>
<td>66</td>
<td>1040</td>
<td>62</td>
<td>62</td>
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<tr>
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<td>106</td>
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<td>99</td>
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<tr>
<td>0.012 Wb/m²</td>
<td>0.10</td>
<td>21</td>
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<td>1660</td>
<td>125</td>
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<tr>
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**TABLE I (CONTINUED)**

<table>
<thead>
<tr>
<th>Field and pressure</th>
<th>Electron energy (eV)</th>
<th>Density ( x 10^{-17} \text{ (m}^{-3}\text{)} )</th>
<th>B-Factor</th>
<th>( \tau_{\text{diff}} ) (( \mu\text{sec} ))</th>
<th>( \tau_{\text{drift}} ) (( \mu\text{sec} ))</th>
<th>( \tau_{\text{calc}} ) (( \mu\text{sec} ))</th>
<th>( \tau_{\text{obs}} ) (( \mu\text{sec} ))</th>
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<tr>
<td>0.50</td>
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<td>15.0</td>
<td>43.4</td>
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<td>0.16 ( \text{Wb/m}^2 )</td>
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<td>900*</td>
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<td>150</td>
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<tr>
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<td>89.2</td>
<td>2810</td>
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<td>77.7</td>
<td>3100</td>
<td>561</td>
<td>475</td>
<td>301</td>
</tr>
<tr>
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<td>70.6</td>
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<td>677</td>
<td>566</td>
<td>337</td>
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<tr>
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<td>92.5</td>
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<td>645</td>
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<td>164</td>
<td>9200</td>
<td>763</td>
<td>705</td>
<td>209</td>
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</tbody>
</table>

* These values were calculated using equation (3.15) although the conditions for its validity were not strictly satisfied.
cannot be accurately determined. In these calculations \( d \) has been taken equal to the radius of the chamber cross-section; this is consistent with a density profile that peaks at the centre of the chamber. \( \tau_{\text{calc}} \) is thus the time constant for density decay due to a combination of drift and diffusion. Equation (6.2) contains the assumption that the loss rates due to drift and diffusion are additive. \( \tau_{\text{diff}} \) and \( \tau_{\text{drift}} \) are also shown separately in Table I in order to indicate the relative importance of diffusion and drift in each case. The \( B \) - factor is the numerical value of the denominator of equation (2.35), i.e. the factor by which the magnetic field is expected to reduce the diffusion rate.

The measurements at magnetic fields from 0.0040 to 0.016 Wb/m\(^2\) were made under conditions for which the strict probe criterion (equation (5.3)) was satisfied. In the measurements at 0.042 and 0.086 Wb/m\(^2\) this probe criterion was not satisfied so that these results must be considered much less reliable. They are retained however in order to provide some check on the theory of particle losses due to drift effects since it is only at these highest magnetic fields that drift losses dominate over diffusion losses.

The data in Table I can be divided into three general regimes according to the mechanism by which the plasma is lost:

(a) At 0.0040 Wb/m\(^2\) the density decay is controlled by diffusion due to collisions with neutral particles. The \( B \) - factor under these conditions is approximately equal to unity. Consequently neither the magnetic field nor coulomb collisions are effective in determining the diffusion rate even though the gas is highly ionized. Terms involving coulomb collision fre-
quencies occur only in the denominator of equation (2.35).

(b) As the magnetic field is increased the diffusion mechanism is altered until in the 0.016 Wb/m² measurements for example, the B - factor is in the range from 10 to 20 and the diffusion rate is controlled by coulomb collisions and the magnetic field.

(c) Finally, as the magnetic field is increased to 0.086 Wb/m² a third regime is reached in which drift effects largely determine the loss rate of the plasma.

The experiments cover a wide range of conditions. Observed densities varied from 3.3 × 10¹⁶ to 1.69 × 10¹⁹ m⁻³, the electron temperature varied from 0.61 to 0.069 eV and the B - factor varied from 1 to 22.8 in the measurements at fields from 0.004 to 0.016 Wb/m².

The calculated and measured time constants, \( \tau_{\text{calc}} \) and \( \tau_{\text{obs}} \) (Table I), agree within a factor of 2 for all except three of the measurements taken with magnetic fields from 0.004 to 0.016 Wb/m². This agreement is probably as good as can be expected. \( \tau_{\text{calc}} \) is based on calculations involving collision frequencies which are not known to high accuracy. Also under conditions for which diffusion is controlled by coulomb collisions our method (equation (2.45)) of obtaining \( \tau_{\text{diff}} \) is inexact. Finally, the analysis in terms of the averaged equations of motion is inherently approximate.

At the higher magnetic fields for which the density decay is controlled by drift in the inhomogeneous magnetic field, the comparison between theory and experiment must be considered less conclusive since the probe measurements are not expected to be so reliable here. Nevertheless the observed dependence of the drift loss rate on the particle energies appears to agree reasonably well with the theory. The theoretical loss rates are consistently lower -
by a factor of approximately 2 on the average than the experimentally determined loss rates. This discrepancy may be due, at least in part, to an improper choice of $d$ in equation (6.3). When the particle loss is controlled by drift the peak in the density profile is displaced from the centre toward the outside of the torus. Consequently the effective value of $d$ in (6.3) is something less than the radius of the cross-section actually employed.

In general, recombination may influence the decay of plasma density. Dissociative recombination by the process

$$\text{He}_2^+ + e \longrightarrow \text{He} + \text{He}$$

would be capable of producing density decay rates of the order of those observed here if most of the ions present were those of the \text{He}_2^+ type. However, at the low pressures employed, the number of \text{He}_2^+ ions is expected to be very low. Furthermore the density decay rates do not show the variation with density expected for a recombination process. Hence this recombination process has been neglected.

A second recombination process which can produce high density decay rates in a dense plasma is three-body recombination by the mechanism

$$\text{He}^+ + 2e \longrightarrow \text{He'} + e \longrightarrow \text{He} + e + h\nu$$

Aleshkovskii and Granovskii (1962) have reported the following loss rate due to this process:

$$\frac{d\eta_e}{dt} = -1.17 \times 10^{-29} \eta_e^k \text{ m}^{-3} \text{ sec}^{-1}$$

(6.4)
where \( T_e \) is expressed in eV, \( k = 2.6 \pm 0.1 \) and \( \beta = 2.8 \pm 0.2 \). Substitution of the measured values of \( n_e \) and \( T_e \) from Table I in (6.4) yields decay rates which are negligible compared to the observed density decay rates for most of the measurements. Furthermore the density decay shows no similarity to the form predicted by equation (6.4). Since density decay by recombination processes other than the two cited above take place at much slower rates, recombination has been neglected.

6.2 Electron temperature and density profiles

The density and temperature profiles shown in Fig. 6 were obtained from measurements made at various points across the torus cross-section. Due to the construction of the probes only slightly more than half of a complete profile could be obtained. The density and temperature profiles were measured in order (a) to check whether or not the density profile at 0.0040 Wb/m\(^2\) was of the shape expected due to linear diffusion; (b) to obtain a numerical solution of equation (2.44) and hence a check on the approximate method of obtaining the time constant in equation (2.45); (c) to provide some verification of the assumption that the electron temperature was constant (at any given time) over the torus cross-section.

At 0.0040 Wb/m\(^2\) a density profile of Bessel function shape is expected since drift is insignificant and diffusion is linear for such a low magnetic field. The measured profile differs noticeably from the expected shape. One difference is that the measured particle concentration does not decrease so rapidly toward the chamber wall as the expected Bessel function shape. However the discrepancy between the observed and theoretical density grad-
Fig. 6 Electron energy and density profiles in the torus with a magnetic field of 0.0040 Wb/m² (Fig. 6(a)) and 0.016 Wb/m² (Fig. 6(b)). 

r is the distance in cm from the centre of the torus in the median plane.

- - - - - denotes the expected Bessel function profile.
ients is not so large that the theoretical time constant for density decay would be seriously affected. An additional discrepancy occurs early in the afterglow at which time the measured density profile peaks at a point displaced from the centre toward the outside of the torus. This can possibly be explained in the following manner: Since the drift rate is proportional to the particle energy it may be that drift, although negligible during the afterglow at 0.0040 Wb/m$^2$, is quite effective during the breakdown period when the particle energy is high, so that an afterglow is produced which is initially more dense near the outside wall of the torus. As far as the density profile at 0.016 Wb/m$^2$ is concerned it is expected to be shifted outward at all times since drift is an important factor here, even in the afterglow.

The measured electron temperature profiles (Fig. 6) are consistent with the assumption, made earlier, that the electron temperature is constant throughout the torus at any given time, when it is taken into account that the probable error in a single measurement has been estimated at $\pm 25\%$.

6.3 Electron energy decay in the afterglow

A typical electron energy decay curve is shown in Fig. 5. It is of interest to determine whether or not the observed cooling rates are in agreement with those predicted due to elastic collisions alone (equations (4.4) and (4.5)). It turns out that agreement is found only at low gas pressures and when a minimum of power is used in breaking down the gas and forming the plasma. These are conditions which tend to minimize the number of metastable atoms present in the afterglow. The expected cooling
rate due to elastic collisions calculated from equations (4.4) and (4.5) is compared in Table II with the observed electron cooling rate in after-glow of plasmas which were formed with a minimum amount of power input.

TABLE II

**EXPERIMENTAL AND THEORETICAL RATES OF DECAY OF AVERAGE ELECTRON ENERGY**

<table>
<thead>
<tr>
<th>$P$ (torr)</th>
<th>$\bar{u}_e$ (ev)</th>
<th>$(d\bar{u}_e/dt)_e$-ion (ev/sec)</th>
<th>$(d\bar{u}_e/dt)_e$-n (ev/sec)</th>
<th>$(d\bar{u}<em>e/dt)</em>\text{theor}$ (ev/sec)</th>
<th>$(d\bar{u}<em>e/dt)</em>\text{obs}$ (ev/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.018</td>
<td>0.40</td>
<td>$11.8 \times 10^3$</td>
<td>$1.5 \times 10^3$</td>
<td>$12.3 \times 10^3$</td>
<td>$10.0 \times 10^3$</td>
</tr>
<tr>
<td>0.018</td>
<td>0.25</td>
<td>$9.2 \times 10^3$</td>
<td>$0.7 \times 10^3$</td>
<td>$9.9 \times 10^3$</td>
<td>$9.6 \times 10^3$</td>
</tr>
<tr>
<td>0.018</td>
<td>0.15</td>
<td>$7.4 \times 10^3$</td>
<td>$0.3 \times 10^3$</td>
<td>$7.7 \times 10^3$</td>
<td>$3.2 \times 10^3$</td>
</tr>
<tr>
<td>0.035</td>
<td>0.50</td>
<td>$10.2 \times 10^3$</td>
<td>$4.1 \times 10^3$</td>
<td>$14.3 \times 10^3$</td>
<td>$7.1 \times 10^3$</td>
</tr>
<tr>
<td>0.035</td>
<td>0.30</td>
<td>$7.3 \times 10^3$</td>
<td>$1.8 \times 10^3$</td>
<td>$9.1 \times 10^3$</td>
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</tr>
<tr>
<td>0.035</td>
<td>0.15</td>
<td>$1.7 \times 10^3$</td>
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<td>0.061</td>
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<td>$5.7 \times 10^3$</td>
</tr>
<tr>
<td>0.061</td>
<td>0.20</td>
<td>$6.4 \times 10^3$</td>
<td>$1.6 \times 10^3$</td>
<td>$8.0 \times 10^3$</td>
<td>$2.1 \times 10^3$</td>
</tr>
</tbody>
</table>

Columns 3 and 4 of Table II show the separate cooling rates due to electron - ion collisions and electron - neutral atom collisions while column 5 gives the combined cooling rate due to both interactions.
The cooling due to electron-ion interactions is the most important one, especially at the lowest pressure. With the exception of the lowest energy points at 0.035 and 0.061 torr there is rough agreement (within a factor of about 2) between the theoretical and experimental cooling rates. The theoretical cooling rate is in all cases somewhat higher than the experimental cooling rate. However it appears that over most of the energy range the transfer of electron energy is predominantly due to the equipartitioning of energy between the energetic electrons and the less energetic helium atoms and ions. The agreement between theory and experiment is best at the lowest pressure (0.013 torr) where metastable atoms should be able to reach the wall and be de-excited earlier in the afterglow. The large discrepancies between the theoretical and observed cooling rates at the lowest electron energies probably result from the following effect. The calculated cooling rate varies as the difference between the electron energy and that of the ions and neutrals. Both the measured electron energy and the assumed value for the ions and neutrals (0.04 eV) are open to some error. When the electron energy approaches that of the ions and neutrals the percentage error in the difference between these two energies can become large.

The rate of decay of the average electron energy, in afterglows of discharges formed at high power, was in all cases observed to be much slower than that expected due to elastic collisions alone. Since electron cooling due to elastic collisions must certainly be taking place at all times this indicates that a mechanism which increases the average energy of the electrons is in operation. The fact that high power input to the discharge
(which would tend to produce a larger metastable population) and high
pressure (which prevents metastables from easily reaching the wall and
being de-excited) enhance the slow cooling effect is considered good ev-
dence that metastable action is the mechanism responsible. This effect
has also been observed by Ingraham (1963) and Ingraham and Brown (1963).

If recoil cooling and metastable action are the only two processes
effective in determining the electron cooling rate we can write

\[
(6.5) \quad \frac{d\bar{u}_e}{dt}_{\text{obs}} = \left( \frac{d\bar{u}_e}{dt} \right)_{e-n} + \left( \frac{d\bar{u}_e}{dt} \right)_{e-i} + \left( \frac{d\bar{u}_e}{dt} \right)_m
\]

where \( \frac{d\bar{u}_e}{dt}_{\text{obs}} \) is the observed cooling rate and \( \frac{d\bar{u}_e}{dt}_m \) is the rate of
heating of the electrons due to metastable activity. The number of meta-
stable atoms present at any time in the afterglow is not known and hence the
cooling rate cannot be predicted. However, the magnitude of the heating ef-
fect of the metastables at any time in the afterglow can be obtained from
equation (6.5) since it is just the difference between the observed cooling
rate and that due to recoil cooling. The dependence of the metastable heat-
ing effect on electron energy and density and gas pressure and its variation
with time can, in some cases, be used to identify the process by which meta-
stable atoms transfer their energy to the electrons and to obtain informat-
ion about the cross-sections for interaction of the metastables with each
other and the plasma particles. An analysis of this type has been carried
out on the experimental cooling results which indicate significant meta-
stable heating (see Appendix B). The results of the analysis are not
consistent. The rate of heating, in combination with the electron density,
indicates that superelastic collisions between an electron and a metastable atom are the dominant heating process. However, the rate of decay of electron heating is not consistent with the rate at which metastables would be used up by this process.

6.4 Conclusions

The results shown in Table I are considered to be good evidence that the mechanisms by which the electron density decays are collision controlled diffusion and drift in the toroidal magnetic field. The accuracy of the measurements and theoretical analysis was not good enough to provide very exact information, but should have revealed any significant departure from theory in the behavior of the plasma. For the particular conditions under which these experiments were carried out it seems that the diffusion was not anomalous.

The agreement found between theory and experiment under conditions for which the diffusion coefficient is independent of position (B ≤ 0.0080 Wb/m²) is somewhat better than that obtained by Golant and Zhilinskii (1960, 1962) for the weakly ionized case. The improvement has probably resulted from taking the measured electron temperature into consideration. Since Golant and Zhilinskii's microwave method yielded no information concerning the electron temperature, they assumed that the ion and electron temperature were in equilibrium. In our experiments such an assumption would have resulted in large errors.

Diffusion controlled by coulomb collisions (highly ionized gas in a
magnetic field) dominates the $0.012$ and $0.016 \text{ Wb/m}^2$ measurements, although drift is equally important early in the $0.016 \text{ Wb/m}^2$ afterglow. Theory and experiment agree here within a factor of about 2 for $B$-factors as large as 20. Golant and Zhilinskii (1962) obtained good agreement under similar conditions, but only by assuming a value of 8 for the coulomb logarithms, whereas calculation indicates a value ranging from 2.5 to 7.

The agreement between the measured and calculated time constants for density decay at the higher magnetic field strengths ($0.042$ and $0.086 \text{ Wb/m}^2$) although not very good, indicates that drift losses largely determine the density decay in these cases. Some method of determining a better value for the factor $d$ in equation (6.3) would probably improve the agreement between theory and experiment.

The measurements of electron energy decay indicate that when the power used in producing the plasma is low the observed cooling rate is in reasonably good agreement with that expected due to recoil alone. The dependence of the cooling rate on input power to the plasma and neutral gas pressure points strongly to metastable atoms as the cause of the slow cooling observed at higher discharge power input.
APPENDIX A

ELECTRON COOLING DUE TO ELECTRON - NEUTRAL COLLISIONS

In this appendix the rate of cooling of electrons due to elastic collisions with helium atoms is calculated making use of an expression for the collision integral derived by Allis (1956). If collisions are the only mechanism affecting the electron velocity distribution $f(v)$ the Boltzmann equation can be written in the simple form

\[ \frac{df(v)}{dt} = B_o \]  

where $B_o$ is the collision integral which is given by

\[ B_o = \frac{1}{v} \frac{d}{d(v^2)} \left[ v^3 G \sqrt{\frac{m_e}{m}} \left( f + \frac{2 k T_e}{m_e} \frac{df}{d(v^2)} \right) \right] \]

where $G$ is the fraction of its excess energy transferred by an electron per collision. $G = 2m_e/m$ for helium. It is assumed in equation (A.2) that the particles have Maxwellian velocity distributions. For $f(v)$ we use the normalized form of the Maxwellian distribution in velocity space

\[ f(v) = \frac{1}{\pi S^3} e^{-Sv^2} \]

where

\[ S = \frac{3m_e}{4 \bar{v}_e} \]

$S^{-3}$ is the most probable speed for the distribution. To find the rate of change of the average energy of the distribution we multiply both sides of (A.1) by

\[ 4 \pi v^2 \left( \frac{m_e v^2}{2} \right) \]  

and integrate over velocity space.

\[ 2 \pi m_e \int_0^\infty v^4 \frac{df}{dt} dv = 2 \pi m_e \int_0^\infty v^4 B_o dv. \]
Consider the first integral in (A.5)
\[ \frac{df}{dt} = \frac{d}{dt}(\pi^{-\frac{3}{2}} \gamma_e^{-\frac{3}{2}} e^{-sv^2}) \]
\[ = \pi^{-\frac{3}{2}} \gamma_e^{-\frac{3}{2}} \frac{d\bar{u}_e}{dt} \left[ -\frac{3}{2} e^{-sv^2} + sv^2 e^{-sv^2} \right]. \]

The first integral then becomes
\[ I_1 = \frac{2m_e s^{\frac{3}{2}}}{\sqrt{\pi} \bar{u}_e} \frac{d\bar{u}_e}{dt} \left[ \int_{-\infty}^{\infty} v^4 e^{-sv^2} dv + \int_{0}^{\infty} sv^2 e^{-sv^2} dv \right]. \]

The integrals within the square brackets can be solved by making the substitution \( sv^2 = y \) and putting them in the general form

(A.6) \[ A \int_{0}^{\infty} y^{(x-1)} e^{-y} dy = A \Gamma(x) \]

where \( A \) is a constant and \( \Gamma(x) \) is the Gamma function. If this operation is carried out we have
\[ 2\pi m_e \int_{0}^{\infty} v^4 \frac{df}{dv} dv = \frac{2\pi^{-\frac{3}{2}} m_e s^{\frac{3}{2}}}{\bar{u}_e} \frac{d\bar{u}_e}{dt} \left( -\frac{3}{4} + \frac{5}{4} \right) s^{-\frac{5}{2}} \Gamma\left(\frac{5}{2}\right). \]

Now using \( s = \frac{3m_e}{4\bar{u}_e} \) and \( \Gamma(5/2) = \frac{3\sqrt{\pi}}{4} \) we have
\[ 2\pi m_e \int_{0}^{\infty} v^4 \frac{df}{dv} dv = \frac{d\bar{u}_e}{dt}. \]

Substituting this result in (A.5) we have

(A.7) \[ \frac{d\bar{u}_e}{d\gamma} = 2\pi m_e \int_{0}^{\infty} v^4 B_\gamma dv. \]

Before we solve the integral in (A.7) we make the substitution

(A.8) \[ v^3 G \gamma_{en} = \alpha v^\beta \]

in the expression for \( B_\gamma \) (equation (A.2)). Also

(A.9) \[ \frac{d}{d(v^*)} = \frac{d}{dv} \cdot \frac{dv}{d(v^*)} = \frac{1}{2v} \frac{d}{dv} \]
and

\[(A.10) \quad \frac{df}{d\nu} = -2s\nu f.\]

Then

\[(A.11) \quad f + \frac{2kT_e}{2m_e\nu} \frac{df}{d\nu} = f \left( 1 - \frac{\bar{u}_m}{\bar{u}_e} \right)\]

where \(\bar{u}_m\) is the average energy of the gas molecules. Hence

\[B_e = \frac{(1 - \frac{\bar{u}_m}{\bar{u}_e})}{2\nu^2} \frac{d}{d\nu} \left( \alpha\nu^\beta f \right).\]

If we substitute for \(f\), carry out the indicated differentiation and put the resulting expression into \((A.7)\), we have

\[(A.12) \quad \frac{d\bar{u}_e}{d\tau} = \frac{2\pi m_e \alpha}{2m^2} \left( 1 - \frac{\bar{u}_m}{\bar{u}_e} \right) \left[ \int_0^{\infty} \nu \nu e^{-\nu^2} \nu^\beta\nu^\beta+1 d\nu - 2 \int_0^{\infty} \nu^\beta+3 \nu \bar{u}_e^{-\nu^2} d\nu \right].\]

These integrals can also be put in the form of \((A.6)\) by the substitution \(\nu^2 = y\). The result is

\[(A.13) \quad \frac{d\bar{u}_e}{d\tau} = \frac{m_e \alpha}{2\pi} \left( 1 - \frac{\bar{u}_m}{\bar{u}_e} \right) \left\{ \frac{\beta}{2} \Gamma \left( \frac{\beta+1}{2} \right) - \frac{\beta}{2} \Gamma \left( \frac{\beta+3}{2} \right) \right\} \]

For helium we have \(\nu_{em} = P_c B_P\) (see section 2.4). Substituting for \(G\) and \(\nu_{em}\) in \((A.8)\) we obtain \(\alpha = 2m_e P_c P/m_i\) and \(\beta = 4\).

Hence

\[(A.14) \quad \frac{d\bar{u}_e}{d\tau} = \frac{2m_e^2 P}{2\pi m_i} \left( 1 - \frac{\bar{u}_m}{\bar{u}_e} \right) \leq \frac{2m_e^2 P}{2\pi m_i} \left[ 2 \Gamma(3) - \Gamma(4) \right].\]

Substituting from \((A.4)\) we obtain the cooling rate of the electron distribution due to elastic collisions with the neutral particles,

\[(A.15) \left( \frac{d\bar{u}_e}{d\tau} \right)_{e-n} = -\frac{16m_e}{3\sqrt{\pi} m_i} \nu_{em} \bar{u}_e \left( 1 - \frac{\bar{u}_m}{\bar{u}_e} \right)\]
where \( \nu_{\text{en}} \nu_e \) is the collision frequency of electrons with velocity equal to the most probable speed, \( \nu_e = (2kT_e/m_e)^{\frac{3}{2}} \), for the electron distribution. The result (A.15) is identical to the cooling rate obtained in equation (4.4) by a simple integration over the Maxwellian electron velocity distribution. This is not surprising since the assumption of a Maxwellian electron velocity in the solution based on the Boltzmann equation means that no new information is contained in this solution.
APPENDIX B
CALCULATIONS ON THE ELECTRON HEATING DUE TO
METASTABLE ACTIVITY IN THE AFTERGLOW

The observed electron cooling rate in afterglows of discharges formed at high power input and high gas pressure is much slower than that expected due to elastic collisions alone. The difference between the expected electron cooling rate due to elastic collisions and the observed cooling rate corresponds to a rate of increase of the average electron energy. Metastable activity in the afterglow is thought to be the most likely source of this effect. The rate of increase of average electron energy \( \left( \frac{d\bar{u}_e}{dt} \right) \) during the afterglow is shown in Fig. 7 for various plasma conditions. For convenience in this analysis we denote \( \left( \frac{d\bar{u}_e}{dt} \right) \) by the symbol \( H \). The rate of decay of \( H \) can be calculated from a knowledge of the mechanism by which the metastable atoms transfer energy to the electrons and a knowledge of the dominant mechanism by which the destruction of the metastable atoms occurs.

Helium has two metastable states with long lifetimes. These are the \( ^2S(20.61 \text{ eV}) \) and the \( ^3S(19.8 \text{ eV}) \) states. Phelps (1955) has determined the cross section for conversion of the \( ^1S \) state to the \( ^3S \) state, by collision with thermal electrons, to be \( 3 \times 10^{-18} \text{ m}^2 \). This indicates that the \( ^1S \) state would have a lifetime of less than 1 microsecond under the conditions of the experiments reported in this thesis. Hence we consider only the \( ^3S \) state.

Energy can be transferred to the electrons by an ionizing collision
Fig. 7 The electron heating as a function of time in the afterglow.
between two metastable atoms or by superelastic collisions between electrons and metastable atoms. The cross section for ionizing collisions between two metastable atoms by the process

\[
\text{He}^m + \text{He}^m \rightarrow \text{He} + \text{He}^+ + e + \text{K.E.}
\]

has been measured by Phelps and Molnar (1953) and found to be \(10^{-18} \text{ m}^2\) (The symbol \(\text{He}^m\) denotes a helium atom in a metastable state). The superelastic collision process is

\[
\text{He}^m + e \rightarrow \text{He} + e + \text{K.E.}
\]

The cross section for superelastic collisions of electrons with the \(2^3S\) metastable helium atoms has been calculated according to the principle of "detailed balancing" from the inelastic cross section for excitation of the \(2^3S\) state. The latter cross section was measured by Schulz and Fox (1957). The two cross sections are related by the expression

\[
\sigma_{se}(u_e-1.8) = \frac{u_e}{u_e-1.8} \sigma_{in}(u_e)
\]

\(\sigma_{se}\) is the superelastic cross section and \(\sigma_{in}\) is the inelastic cross section. This gives the superelastic cross section as a function of \(u_e\), the electron energy after the superelastic collision. The value of \(\sqrt{v_e} \sigma_{se}\) for a Maxwellian electron velocity distribution was then found by numerical integration. For average electron energies in the range from 0.1 to 0.3 eV the value of \(\sqrt{v_e} \sigma_{se}\) obtained was in the range from \(3 \times 10^{-15}\) to \(4.4 \times 10^{-15}\) m\(^3\) sec\(^{-1}\).

The validity of the assumption of a Maxwellian energy distribution requires that the rate of transfer of energy between the electrons themselves be large compared to the rate of transfer of energy from the electrons to other particles. The characteristic time required for energy transfer between
the electrons is given by (Spitzer 1962)

$$\tau_{ee} = \frac{m_e^2}{8 \cdot 0.714 \cdot n_e} \left( \frac{4 \pi \epsilon^2}{e^2} \right)^{3/2},$$

$$\tau_{ee} \approx 10^{-9} - 10^{-10} \text{ sec}$$
for the experiments reported here. The characteristic time for energy transfer between electrons and ions is of the order of $10^{-6} \text{ sec}$ while for elastic collisions with neutrals the time is even larger.

De-excitation of the $2^3S$ metastable state produces an electron with energy above 19.8 eV. This electron is then capable of giving up its energy by inelastic collision with an atom in the ground state, thus creating a new metastable atom. However this process should not be important since, for the experimental conditions considered here, the mean time required for a collision of this type is

$$\tau_{in} = \frac{1}{n_e v_n v_e} \approx 10^{-4} \text{ sec}$$

which is very long compared to $\tau_{ee}$, a measure of the time required for an electron to move out of the energy range where inelastic collisions can take place. Hence the assumption of a Maxwellian electron velocity distribution is justified.

The relative importance of superelastic collisions and the metastable-metastable ionizing collisions discussed above can be evaluated from the respective cross sections. Calculations indicate that superelastic collisions are the dominant mechanism for transfer of energy from the metastable atoms to the electrons under the conditions of the experiments reported in this thesis. Sample calculations are given below.

For a typical case we have $H = 1.17 \times 10^5 \text{ eV sec}^{-1}$, $n_e = 7.43 \times 10^{18} \text{ m}^{-3}$,
\( \mu_e = 0.267 \text{ eV} \) and \( p = 0.0288 \text{ torr} \). Under these conditions energy is being fed into the plasma at the rate
\[
1.17 \times 10^5 \times 7.43 \times 10^{18} = 8.68 \times 10^{23} \text{ eV m}^{-3} \text{ sec}^{-1}.
\]
In each superelastic collision the electrons receive 19.8 eV. Hence if superelastic collisions are the dominant heating mechanism metastable atoms are destroyed at the rate

\[
\frac{dn_m}{dt} = -\frac{8.68 \times 10^{23}}{19.8} = -4.34 \times 10^{22} \text{ m}^{-3} \text{ sec}^{-1}.
\]

Now, \( \frac{dn_m}{dt} = -n_e n_m \sigma_{sc} \nu_e \).

For this case \( \sigma_{sc} \nu_e = 4.18 \times 10^{-15} \text{ m}^3 \text{ sec}^{-1} \). Solving for \( n_m \) we obtain

\[
n_m = 1.4 \times 10^{18} \text{ m}^{-3}
\]

for the density of metastable atoms required to produce the observed heating effect. With this metastable density ionizing metastable-metastable collisions of the type discussed above would occur at a rate given approximately by

\[
\text{ionizations/sec} = n_m^2 \sigma_{m-m} \bar{v}_m
\]

where \( \sigma_{m-m} \) is the cross section for the interaction and \( \bar{v}_m \) is the average metastable velocity. Substituting numerical values we obtain

\[
(B.2) \quad \frac{1}{2} \frac{dn_m}{dt} = -(1.4 \times 10^{18})^2 \times 10^{-15} \times 1.28 \times 10^3
\]

\[
= -2.51 \times 10^{21} \text{ m}^{-3} \text{ sec}^{-1}.
\]

The ionization potential of helium is 24.5 eV so that each ionization adds \((2 \times 19.8) - 24.5 = 15.1 \text{ eV}\) to the electron energy. Hence meta-
stable - metastable interactions would feed energy into the electron distribution at the rate

\[ 15.1 \times 2.51 \times 10^{21} = 3.79 \times 10^{22} \text{ eV m}^{-3} \text{ sec}^{-1} \]

so that the superelastic mechanism is approximately 20 times more effective under these conditions.

Metastable atoms are destroyed by interactions with each other and with electrons and also by diffusion to the chamber wall and subsequent de-excitation there. For the typical case under discussion the loss rate due to superelastic collisions is given by (B.1). The loss rate due to diffusion is

\[ \left( \frac{dn_m}{d\tau} \right)_{\text{diff}} = -\frac{D_m n_m}{p L^2} \]

where \( D_m = 0.047 \text{ m}^2 \text{ sec}^{-1} \) is the diffusion coefficient at a pressure of 1 torr (Phelps 1955.), \( p \) is the pressure in torr and \( L^2 = 1.93 \times 10^{-4} \text{ m}^2 \) is the square of the characteristic diffusion length for the plasma chamber used in these experiments. Then

\[ \left( \frac{dn_m}{d\tau} \right)_{\text{diff}} = -\frac{0.0470 \times 1.4 \times 10^{12}}{0.0288 \times 1.93 \times 10^{-4}} = -1.18 \times 10^{22} \text{ m}^{-3} \text{ sec}^{-1} \]

which indicates that the loss rate due to diffusion in this case is about 25% of the loss rate due to superelastic collisions.

A test of the analysis given above can be made by calculating the expected rate of change of \( H \) with time and comparing this with the slope of the curves in Fig. 7 deduced from the experimental data. The heating due to superelastic collisions is

\[ H = n_m \bar{v} e V \times 19.8 \text{ eV sec}^{-1} \]
Taking logarithms of both sides

\[ \ln H = \ln n_m + \ln \frac{\sigma_{se} v_e}{v_e} + \ln 19.8. \]

Differentiating (B.6) we obtain

\[ \frac{d(\ln H)}{dt} = \frac{1}{n_m} \frac{dn_m}{dt} \]

since \( \frac{\sigma_{se} v_e}{v_e} \) is approximately constant in the energy range (\( \bar{u}_e = 0.1 - 0.3 \) eV) covered by the curves shown in Fig. 7. If superelastic collisions are the dominant mechanism by which metastable atoms are destroyed the rate of destruction is

\[ \frac{dn_m}{dt} = - n_e n_m \frac{\sigma_{se} v_e}{v_e}. \]

Substituting this expression in (B.7) we obtain

\[ \frac{d(\ln H)}{dt} = - n_e \frac{\sigma_{se} v_e}{v_e}. \]

The rate of decay of \( \ln H \) will actually be larger than that given in (B.8) because of the destruction of metastable atoms by other processes. For the typical case under discussion \( n_e = 7.43 \times 10^{18} \) m\(^{-3}\) and \( \frac{\sigma_{se} v_e}{v_e} = 4.18 \times 10^{-15} \) m\(^3\) sec\(^{-1}\) from which we obtain

\[ \frac{d(\ln H)}{dt} = - 3.11 \times 10^4 \text{ sec}^{-1}. \]

The observed value, given by the slope of the curve at the point P on the curve in Fig. 7 is

\[ \left( \frac{d(\ln H)}{dt} \right)_{obs} = - 6.27 \times 10^3 \text{ sec}^{-1}. \]
which differs by a factor of about 5 from the calculated value. Physically this means that the observed heating effect falls off only about one-fifth as fast as expected from a consideration of the rate at which metastable atoms must be destroyed in order to produce the observed heating.

The observed heating can be explained on the basis of the mechanisms discussed above only if the cross sections for both superelastic collisions and metastable - metastable interactions are much smaller than the values used. This does not seem probable. In any case the analysis given above is inadequate to explain in a self-consistent manner the slow electron cooling observed in afterglows of high power discharges. Nevertheless the observed dependence of the slow electron cooling on discharge power and pressure suggests that some form of metastable heating is responsible.

Tynes and Brady (1964) have recently reported rate coefficients for an additional process

$$\text{He}^m + \text{He}^m \rightarrow (\text{He}_2^+)' + e + \text{K.E.}$$

($$(\text{He}_2^+)'$$ denotes an excited ionized molecule) which under certain conditions can be the dominant mechanism by which metastable atoms influence the afterglow. Applying their results to the experiments reported in this thesis indicates that this heating mechanism is of comparable importance to the superelastic process. However the same difficulty arises as before since the calculated rate of destruction of the metastable atoms indicates that the heating effect should fall off much faster than is observed.
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