LOCATING TELEPHONY LOOP IMPAIRMENTS WITH FREQUENCY DOMAIN REFLECTOMETRY

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by

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Usage of the Internet is growing at a rapid pace. This growing popularity has had an impact on the content available over the Internet. Today, many bandwidth hungry applications are available to end users: music files, streaming audio/video, and video conferencing. To be of practical use, larger bandwidth applications require higher speed Internet connections to the consumer. One method of providing high speed Internet access to a home or small business is through twisted pair telephone cables.

Use of twisted pair to provide high speed Internet service is not without its problems. Many loop impairments exist that hinder high speed service, or prevent it altogether. Purposely placed impairments include loading coils and bridged taps. Unintentional impairments include wire splices and water in the cable casings. The locations of these impairments help the telephone companies (telco's) determine which lines have potential problems in delivering Internet access.

A new technique called frequency domain reflectometry (FDR) has been proposed to locate these loop impairments. This thesis focuses on proving the viability of using FDR to accurately detect loop impairments. A physical device was designed and constructed to use the FDR technique. The device was tested on various telephone
loop configurations. The device was also compared with a commercially available product. The results of the testing indicate FDR is a viable alternative for locating loop impairments. It performs at least as well as commercially available products.
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<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>analog to digital converter</td>
</tr>
<tr>
<td>ADSL</td>
<td>asymmetric digital subscriber loop</td>
</tr>
<tr>
<td>AWG</td>
<td>American wire gauge</td>
</tr>
<tr>
<td>CDSL</td>
<td>Rockwell's consumer digital subscriber loop</td>
</tr>
<tr>
<td>cm</td>
<td>centimeter</td>
</tr>
<tr>
<td>C. O.</td>
<td>central office</td>
</tr>
<tr>
<td>DFT</td>
<td>discrete Fourier transform</td>
</tr>
<tr>
<td>DSL</td>
<td>digital subscriber loop</td>
</tr>
<tr>
<td>DSP</td>
<td>digital signal processing</td>
</tr>
<tr>
<td>EZ-DSL</td>
<td>Cisco's consumer digital subscriber loop</td>
</tr>
<tr>
<td>FDR</td>
<td>frequency domain reflectometry</td>
</tr>
<tr>
<td>FFT</td>
<td>fast Fourier transform</td>
</tr>
<tr>
<td>FM</td>
<td>frequency modulation</td>
</tr>
<tr>
<td>FMCW</td>
<td>frequency modulated continuous wave</td>
</tr>
<tr>
<td>GHz</td>
<td>gigaHertz</td>
</tr>
<tr>
<td>HDSL</td>
<td>high data rate digital subscriber loop</td>
</tr>
<tr>
<td>Hz</td>
<td>Hertz</td>
</tr>
<tr>
<td>IDFT</td>
<td>inverse discrete Fourier transform</td>
</tr>
<tr>
<td>IDSL</td>
<td>integrated services digital network digital subscriber loop</td>
</tr>
<tr>
<td>ISDN</td>
<td>integrated services digital network</td>
</tr>
<tr>
<td>JWI</td>
<td>jumper wire interface</td>
</tr>
<tr>
<td>kHz</td>
<td>kiloHertz</td>
</tr>
<tr>
<td>LMDS</td>
<td>local multi-point distribution system</td>
</tr>
<tr>
<td>MDSL</td>
<td>moderate bit rate digital subscriber loop</td>
</tr>
<tr>
<td>MHz</td>
<td>megaHertz</td>
</tr>
<tr>
<td>MMDS</td>
<td>multi-channel multi-point distribution system</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>------------------------------------------------</td>
</tr>
<tr>
<td>MVL</td>
<td>multiple virtual loop</td>
</tr>
<tr>
<td>op-amp</td>
<td>operational amplifier</td>
</tr>
<tr>
<td>PC</td>
<td>personal computer</td>
</tr>
<tr>
<td>pn</td>
<td>pseudo noise</td>
</tr>
<tr>
<td>RADSL</td>
<td>rate-adaptive digital subscriber loop</td>
</tr>
<tr>
<td>SDSL</td>
<td>single line digital subscriber loop</td>
</tr>
<tr>
<td>SS</td>
<td>spread spectrum</td>
</tr>
<tr>
<td>TDR</td>
<td>time domain reflectometry</td>
</tr>
<tr>
<td>telco</td>
<td>telephone company</td>
</tr>
<tr>
<td>VDSL</td>
<td>very high rate digital subscriber loop</td>
</tr>
<tr>
<td>WWW</td>
<td>world wide web</td>
</tr>
</tbody>
</table>
1. Introduction

1.1 Background

The growth of the Internet, and in particular the World Wide Web (WWW), has been nothing short of exponential over the last seven years. The bandwidth needed by customers has grown just as much. Whereas several years ago the Internet was little more than a medium to exchange text based e-mail, today it provides much more bandwidth hungry applications: multimedia on demand (audio/voice), file downloads (on the order of a few Megabytes is typical), multimedia content streaming (live music, archived video, video conferencing), to name but a few [1].

The need for this ever increasing bandwidth has led to innovations to bring higher speed data transfer to the residential or small business consumer. Cable television providers have been upgrading their plant to include two way data flow, thus providing an option for high speed Internet access [2]. Telephone companies have been developing means to offer high speed data transfer on their existing twisted pair lines that run to virtually every residential home and small business [3]. A third, less common, option is wireless transfer, such as multi-channel multi-point distribution system (MMDS) or local multi-point distribution system (LMDS). This technology is still in its infancy, but is quickly proving itself a viable means of providing high speed data delivery to the customer [4].

Despite content on the Internet growing in bandwidth, these high speed providers, especially the telephone companies (telco's), have been slow to deploy their respective technologies. At the end of 1999, less than 5% of American Internet consumers in the residential and small office markets had high speed Internet access. Approximately 4.5% were connected via cable television companies, and less than 0.5% were connected via the digital subscriber loop (DSL) solution offered by telephone companies
The expected growth in high speed Internet users will increase from less than 2 million at the end of 1999 to over 25.5 million by the end of 2004. By the year 2004 it is expected that DSL will account for 40% of all high speed Internet connections [6]. Figure 1.1 illustrates this growth in number of households connected via high speed Internet.

Along with this growth of DSL comes the associated growth in revenue, from US$580 million in 1999 to over US$7.6 billion in 2004. Figure 1.2 illustrates the growth in revenue from high speed Internet service.

Figure 1.1 High Speed Internet User Projections, 1998-2004, taken from The Strategis Group [6].
1.2 xDSL Basics

Traditional telephone systems were designed with the sole purpose of transmitting voice information over a copper twisted pair wire. For typical voice transmission, the phone lines only had to support a bandwidth of roughly 3.2 kHz - the spectrum ranging from 200 Hz to 3400 Hz. Frequencies higher than this were unused in most cases.

Through the years, technical advancements have been made but almost all were limited to this 3.2 kHz bandwidth. Telephones, dial modems, fax modems, and private line modems all use only the 200 Hz to 3400 Hz spectrum [7]. Such a small bandwidth limits the amount of data that can be sent over the twisted pair.

Recently introduced xDSL systems strive to provide higher data rates by extending beyond the 3.2 kHz of the voice signal spectrum. Higher frequencies are used for data transmission, in both downstream (from headend to customer) and upstream
(customer to headend) directions. Figure 1.3 illustrates the typical spectral usage on a twisted pair employing an xDSL technology.

![Figure 1.3](image)

**Figure 1.3** xDSL Spectral Content on Twisted Pair

Digital subscriber line techniques can be broken down into 3 major categories: the high data rate digital subscriber loop (HDSL) family, the asymmetric digital subscriber loop (ADSL) family, and the very-high rate digital subscriber loop (VDSL) family [8]. These families are further broken down into specific technologies. Some of these technologies have been standardized, and some are variations on the standards, as depicted in Table 1.1.

The HDSL family is an xDSL technology that uses symmetry to accomplish data transfer. The bandwidth dedicated to upstream and downstream traffic is equal, and thus the method is classified as symmetrical. The ADSL family is asymmetric in nature. It provides more bandwidth to the downstream traffic than upstream traffic, making this the most suitable family of xDSL for residential clients. The third and final category is the VDSL family. This family provides very high bit rates and requires very short copper loops, atypical of telephony lines in existence today.

Technology variations within each family is beyond the scope of this thesis. If a twisted pair loop is capable of supporting transmission using the standard DSL
Table 1.1 The xDSL Family

<table>
<thead>
<tr>
<th>HDSL Family</th>
<th>ADSL Family</th>
<th>VDSL Family</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standards</td>
<td>Variations</td>
<td>Standards</td>
</tr>
<tr>
<td>HDSL</td>
<td>MVL</td>
<td>ADSL</td>
</tr>
<tr>
<td>HDSL2</td>
<td>SDSL</td>
<td>RADSL</td>
</tr>
<tr>
<td>MDSL</td>
<td>G.lite</td>
<td>EZ-DSL</td>
</tr>
<tr>
<td>IDSL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

method it is said to “qualify” for this service. It will also qualify for any variations within that family. For example, if a loop qualifies for the ADSL standard, it will also qualify for the CDSL variation.

1.3 Problems Facing Telephone Companies

The fact that growth projections indicate xDSL will serve 40% of high speed Internet users is no guarantee this will happen. Telco’s will only be able to capitalize on increasing demand for high speed Internet service if they can overcome some key obstacles to deployment.

These obstacles to deployment are the copper loop impairments present on the twisted pair line to a subscribers' premises. Some of these impairments to high speed data transfer have been introduced by the telco’s to improve voice service. Loading coils were added to twisted pair lines to extend the range for voice signals, however, these inductors have the effect of severely attenuating high frequencies. Increased wire size (lower gauge) is used at the end of longer loops to reduce voice attenuation, but the impedance discontinuities at the gauge changes reflect high speed data signals. Bridged taps have been added to the twisted pair line making it easier to add and delete customers, but they also introduce an impedance discontinuity at the junction and cause signal reflections [8]. These signal reflections interfere with high speed data transmission. Figure 1.4 illustrates typical impairments in a local loop.
The presence of such impairments to high speed transmission is not the only difficulty facing telco’s. Poor (or non-existent) records of the locations of these impairments make it impossible to tell with certainty where to dig to remove them. In addition to the “known” structural problems, other problems can arise: construction equipment may inadvertently damage wires, rodents may chew into wires, or water may find its way into cable casings. All will cause problems to high speed data transfer, and telephone companies have no way of predicting these occurrences.

The overall result is that telco’s need a cheap, fast, and reliable method for determining if a particular wire loop has impairments to high speed data, and if so, where those impairments are located. By having such a method available, the telephone companies offering high speed Internet service can more quickly get their potential customers connected to the Internet with high speed access.

1.4 System Proposal

The primary objective of this thesis is to provide a proof of concept study of a new technique for determining loop impairments of twisted pair telephone wire. This goal was achieved by constructing a working prototype. The prototype was tested on actual telephone wire, with various degrees of loop impairment.
The technique used is frequency domain reflectometry (FDR). This technique uses a varying frequency sinusoidal waveform injected into the telephone loop under test. The amplitude of the reflected wave is collected and processed with software algorithms to determine the location of the fault.

Once the prototype was built up, it was tested using different line configurations to determine its accuracy. To further offer proof of concept, results obtained were compared with commercially available products to determine if the consumer market would accept such a device.

1.5 Thesis Organization

This thesis is organized into 6 chapters.

Chapter 1 provides background material and motivation for the thesis research. Thesis objectives are also established.

Chapter 2 provides a review of line measurement techniques. Advantages and disadvantages of each will be discussed, in addition to a brief explanation of how each technique works.

Chapter 3 provides a review of transmission line theory. The mathematical concepts behind transmission line theory are discussed: primary constants, travelling waves, and standing waves. The equations pertaining to transmission lines are also developed.

Chapter 4 describes the system design of the thesis project. The hardware design is explored from theory and practical perspectives. Software design and algorithms are also explored and examined from a theoretical and practical side.

Chapter 5 presents the results from the system. Results of simple and complex loop configurations are given. Comparisons are made with existing vendor products.

Chapter 6 draws conclusions based on the results presented. Suggestions for further work are also presented in this chapter.
2. Review of Line Measurement Techniques

Four distinct types of techniques for remote measurement can be found in the literature. Much of the available literature is focused on radar and seismic applications. The four techniques are: time domain reflectometry, frequency domain reflectometry, frequency modulated continuous wave radar, and spread spectrum correlation. An explanation of each technique is provided in this chapter, including advantages and disadvantages, as well as a general block diagram of the specified method. A summary of the advantages and disadvantages of each technique is provided at the end of the chapter.

2.1 Time Domain Reflectometry

Time domain reflectometry (TDR) is the most widely used form of measurement technique for transmission lines. All current equipment vendors for telephone line testing use this technique to perform measurements.

The TDR method involves transmitting an electrical pulse down the transmission line under observation. As long as the impedance of the line remains constant, the pulse suffers only from dispersion and attenuation due to the physical properties of the line itself. Figure 2.1 illustrates these effects on a travelling pulse.

Provided the impedance of the transmission line remains constant, and is properly terminated at the receiving end, the pulse will be completely absorbed by the terminating load. If, on the other hand, there is an impedance discontinuity caused, for example, by a bridged tap, then the pulse will be reflected back toward the transmitter end, as shown in Figure 2.2.

The amplitude and polarity of the reflected pulse are given by the well known reflec-
Figure 2.1 Dispersion/Attenuation of an Electrical Pulse in a Transmission Line

tion coefficient formula, Equation 2.1:

$$\rho = \frac{Z_r - Z_o}{Z_r + Z_o}$$  \hspace{1cm} (2.1)

where $Z_r$ is the impedance seen by the travelling pulse and $Z_o$ is the characteristic impedance of the transmission line. The amplitude just before the pulse reaches the impedance discontinuity is multiplied by this coefficient to give the amplitude (and polarity) of the reflected pulse.

Once the reflected pulse is observed at the transmitter end, information can be gathered from it. The location of the discontinuity is indicated by the time delay in the reflection

$$x = \frac{\nu \tau}{2}$$  \hspace{1cm} (2.2)

where $x$ is the distance from equipment to loop impairment, $\nu$ is the velocity of the pulse in the transmission line and $\tau$ is the time taken for the pulse to be reflected back to the receiver.

The type of discontinuity is indicated by the shape of the returned pulse. Assuming that the electrical properties of the transmission line are known (for example,
wire gauge), then the velocity of propagation of the pulse is known as well as the attenuation per unit distance of the line. By calculating the time needed to receive the reflected pulse, and knowing the velocity of the pulse itself, the distance to the discontinuity can be calculated. Knowing this distance, and the attenuation experienced as a result of travelling this distance, the reflection factor at the discontinuity can be determined from measurements of the returned pulse amplitude. This will provide the effective impedance of the discontinuity using Equation 2.1. Knowing the effective impedance provides knowledge of the type of discontinuity.

Time domain reflectometry has many advantages to its credit. Perhaps TDR's primary advantage over competing techniques is the simplicity of the equipment needed. A step generator and oscilloscope will suffice for relatively short transmission lines, as seen in the block diagram of Figure 2.3. Time domain reflectometry also has the advantage of having a very easy means of determining the location and type of
discontinuity. No signal processing is required to obtain this information.

![TDR System Block Diagram](image)

**Figure 2.3** TDR System Block Diagram

Time domain reflectometry also has many disadvantages. Due to the differing velocity of propagation with frequency, the pulse disperses as it travels down the line. Dispersion increases as the distance the pulse travels increases. The dispersion of the reflected pulse makes determining the peak at the receiver end more difficult; this results in error in determining distance to the discontinuity. The effects of dispersion also prevent the use of narrow pulses. This forces the use of broader pulses, which in turn reduces measurement resolution.

Another disadvantage to TDR is the number of discontinuities that can be detected. Because each discontinuity reduces the power that passes on down the line, it can be easily seen that more than one discontinuity will result in small returned signal amplitude reflections for those occurring after the initial impedance mismatch. These are difficult to detect, and are a leading drawback of existing commercial products.
using TDR.

2.2 Frequency Domain Reflectometry

Frequency domain reflectometry (FDR) uses an approach that is more complex than TDR and it yields somewhat better results. The premise behind FDR is to inject into the transmission line a voltage signal of varying frequency. Similar to the TDR case, if a discontinuity exists on the transmission line the voltage signal will be reflected back to the signal source. This reflected voltage signal will interfere (both constructively and destructively) with the incident waveform, causing a standing wave interference pattern on the transmission line.

The voltage of the standing wave interference pattern is measured at the receiver. As the frequency of the incident voltage signal changes, the standing wave interference pattern will also change, resulting in a different voltage measured at the receiver. If a plot of receiver voltage versus frequency is made, it is observed the voltage will have peaks and troughs of a periodic nature, as depicted in Figure 2.4.

![Figure 2.4](image.png)

**Figure 2.4** Periodicity of Reflected Voltage with Frequency, for an FDR System
The periodic nature of these peaks and troughs contains the information needed to determine where on the transmission line the discontinuity exists. A peak will correspond to some number of wavelengths in the return path from the signal source, to the discontinuity, and back to the source. This could be a non-integral number of wavelengths. At the next peak, the difference in the number of wavelengths in the return path will be exactly one. Figure 2.4 illustrates a received voltage with two peaks at frequency \( f_1 \) and \( 4_2 \). The difference in frequency between the peaks is \( \Delta f \) Hertz. From the argument presented above, there will be exactly one wavelength difference for a wave travelling in the loop between the frequencies of the two loops.

Knowing the difference in frequencies of two consecutive peaks (or troughs), \( \Delta f \), as well as the velocity of propagation of the voltage signal, \( v \), the distance to the discontinuity can be found as follows:

\[
2x = n\lambda_1
\]  
\[
2x = (n + 1)\lambda_2
\]

where \( x \) is the distance from the source to the discontinuity, \( n \) is the number of wavelengths in the return path for \( f_1 \), \( \lambda_1 \) is the wavelength associated with \( f_1 \), and \( \lambda_2 \) is the wavelength associated with \( f_2 \). Rearranging:

\[
\lambda_1 = \frac{2x}{n}
\]  
\[
\lambda_2 = \frac{2x}{n + 1}
\]  
\[
\frac{1}{\lambda_1} = \frac{n}{2x}
\]  
\[
\frac{1}{\lambda_2} = \frac{n + 1}{2x}
\]

From this:

\[
\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = \frac{n + 1}{2x} - \frac{n}{2x}
\]

If \( v \) is constant for both \( f_1 \) and \( f_2 \), then

\[
\frac{f_2}{v} - \frac{f_1}{v} = \frac{1}{2x}
\]
Rearranging and simplifying:

\[ x = \frac{\nu}{2\Delta f} \]  

(2.11)

Thus by knowing the velocity of propagation and difference in frequency between two peaks, the distance to the discontinuity can be found.

A block diagram of a typical FDR system is shown in Figure 2.5. There are 4 major blocks to this system. The first block is the signal generator. This generator provides a sinusoidal signal that steadily increases in frequency. The second block is the hybrid coupler. The hybrid coupler provides directional coupling of the signals of interest. The hybrid coupler couples the transmitted (incident) voltage signal into the transmission line, and prevents it from entering the receiver portion of the circuit. The hybrid coupler also couples the received interference signal into the receiver circuitry. The third block is the analog to digital converter (ADC). The ADC digitizes the returned signal into discrete samples. The fourth block consists of the Fast Fourier Transform, or FFT. The FFT is used to transform these results into a graphical representation of the location of any loop impairments.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fdr_system_block_diagram.png}
\caption{FDR System Block Diagram}
\end{figure}

The FDR method has some advantages that can make the measurements more accurate. One advantage is the ease with which return signal averaging can be accom-
plished. This provides more accurate results by averaging out noise that can corrupt the return signal. Another advantage of FDR is the ability to compensate for line characteristics. Variation in the propagation delay is easily compensated for. Attenuation differences versus frequency are also easily compensated for. Due to the higher frequencies involved in testing the line, testing can be performed while the line is in operation. A final big advantage of FDR is its ability to detect multiple discontinuities more easily than TDR, because of the software and signal processing techniques available.

The FDR method also has some distinct disadvantages. The method of collecting data requires specialized hardware such as a sweep frequency generator and a coupler to remove the reflected signal. Use of a swept frequency generator makes a portable unit more cumbersome. Using a hybrid coupler means developing a very precise balance load to mimic the impedance of the telephone wire over a large range of frequencies (to prevent the transmitted waveform from leaking through to the receiver).

2.3 Frequency Modulated Continuous Wave Radar

Frequency Modulated Continuous Wave (FMCW) radar has traditionally been used in many diverse ways such as navigation radar, altimeters, and survey instruments [9]. This technique might also be used for detecting impairments on a copper loop, with some minor changes to the basic method of operation.

The FMCW radar determines distance to an impairment by using a linearly swept frequency signal and receiving a time delayed reflected signal from the target [10]. In the proposed application, the target would be a loop impairment. The difference in transmitted and received frequency at any instant, referred to as the beat frequency, is related to the time delay in the reflected signal, and hence the distance. Range information can be obtained by determining the frequency of the time domain beat signal [10]. Figure 2.6 illustrates a typical frequency versus time plot. The beat frequency $f_b$ is related to the time $t_d$ it takes for the reflected signal to travel back to the signal source, where the beat measurement takes place.
The range, or distance, to the impairment is then proportional to $f_b$. When there are several reflections, the distance to each can be resolved by performing an FFT on the beat signal [11]. A simplified block diagram of a FMCW radar taken from [12] is shown in Figure 2.7. The linear sweep generator provides a continuous frequency swept waveform to the line under test. The second block consists of a power divider, which splits the transmitted signal into two separate paths: one to the line under test, the other to the mixer. The portion of the transmitted signal that continues down the line goes through a hybrid coupler and through a bandpass filter (to remove any noise and spurious frequency components). Once the reflected waveform travels back to the equipment, it also goes through the bandpass filter, to remove unwanted noise and frequency components. The returned signal is then coupled into the mixer by the hybrid coupler. At the mixer, the beat frequency is determined by mixing the transmitted and reflected signals. The beat frequency signal passes through a low pass filter to remove unwanted spectral components. This filtering is needed to remove the double frequency component from the multiply process, and prevent aliasing at the ADC. The filtered beat frequency is then digitized and processed through an FFT to obtain distance to impairment information.
The advantages to using FMCW radar systems are twofold. The main advantage is the ability to apply digital signal processing (DSP) techniques to the collected data. The FFT will most certainly be executed from the DSP chip, since the FFT is a complex mathematical process. The second advantage is the ability to detect more than one discontinuity on the line. Multiple discontinuities will create multiple beat frequencies, which are easily resolved by the FFT.

One disadvantage to FMCW radar is the relative newness to using this technique in a loop impairment situation. In past applications this technology used gigahertz frequencies through the air, not kilohertz frequencies travelling through metallic conductors. As with FDR, a disadvantage of FMCW radar is the specialized equipment needed, such as a swept frequency generator and hybrid coupler.

2.4 Spread Spectrum Correlation

A fourth option for measuring impairments on a line is spread spectrum (SS) correlation. In this technique, a spread spectrum signal is transmitted down the transmission
line and the reflected signals are received and correlated with the transmitted signal.
In the correlator, the correlation is performed as a function of delay in the trans­
mitted signal, and a high correlation value indicates the presence and strength of the
reflected signal. At maximum correlation the time it took for the pulse to travel the
length of the loop and back is known, and hence so is distance.

By definition, a spread spectrum system is one such that the transmitted signal is
spread over a bandwidth much greater than the minimum bandwidth required to
transmit the actual information [13]. Though there are many means to implement
spread spectrum, such as frequency hopping (carrier frequency shifting) and chirp
modulation (pulsed frequency modulation, or pulsed FM), the means focused on here
are the result of direct sequence modulation.

In direct sequence spread spectrum, the data to be transmitted is modulated by a
digital code whose rate is much higher than the rate of the original signal to be
transmitted. The digital code is a pseudo noise (pn) sequence with a noise like
waveform. By spreading the information over a much larger bandwidth, the signal is
much more immune to noise and interference.

In an application such as line measurement, the fact a spread spectrum signal is
highly immune to interference is a very desirable feature. In line measurement of a
twisted pair, attenuation can be severe as the distance measured increases. Using
spread spectrum correlation helps reduce the attenuation problem.

A system block diagram of a direct sequence spread spectrum system is shown in
Figure 2.8 [14]. Since this application uses spread spectrum correlation for ranging
only (and not data transmission), the block diagram can be simplified to that of
Figure 2.9. In this subsequent figure the data modulation and demodulation has
been removed from the block diagram.

The simplified block diagram indicates only one major block of interest, the hybrid
coupler. The hybrid coupler is used to couple the digital code sequence, c(t), into the
line under test. The hybrid coupler also couples the reflected, time-delayed version
of the initial signal, \( z(t) \), into the receiver portion of the test set up.

The two signals \( c(t) \) and \( z(t) \) are correlated using the correlation function given in Equation 2.12:

\[
r(\tau) = \int_{-\infty}^{+\infty} c(t) z(t - \tau) dt
\]  

(2.12)

In the case of using a finite pn sequence length, the integration would take place over the period of the pn sequence \( T_p \):

\[
r(\tau) = \int_{0}^{T_p} c(t) z(t - \tau) dt
\]  

(2.13)

At some time \( \tau \) the correlation function will have a maximum. This corresponds to the time when the reflected signal \( z(t) \) most closely resembles the original signal \( c(t) \).
This time $\tau$ is the time taken for the pn sequence to travel twice the distance to the discontinuity (there and back). If the velocity of the signal is known, the distance to the discontinuity can be found from

$$x = \frac{\nu \tau}{2}$$

(2.14)

where $\nu$ is the velocity of the signal, $\tau$ is the time of the peak correlation value, and $x$ is the distance to the discontinuity.

The biggest advantage to spread spectrum correlation is its immunity to noise. This makes it ideal for telephone wire, as the attenuation can be severe at long distances. Another advantage is being able to use DSP techniques to analyze the collected data and perform the correlation. A final advantage is its ability to detect multiple loop impairments on the line under test. These would show up as multiple correlation peaks at the receiver.

A disadvantage to using a spread spectrum correlation technique is the complexity needed to get a functional spread spectrum system. Another disadvantage is the use of a hybrid coupler. As with FDR, using a hybrid coupler means developing a very precise balance load to mimic the impedance of the telephone wire over a large range of frequencies.
2.5 Summary

All four techniques summarized in this chapter provide an option for measuring the location and type of loop impairments in a twisted pair conductor. Literature searches show that only TDR is used in commercial products designed for detecting loop impairments on a twisted pair. It is assumed that the simplicity of the TDR design makes this the most attractive technique to implement. However, as processing speeds and prices of DSP chips become more attractive, the other three options, especially FDR, will most likely find their way into commercial products designed for detecting loop impairments.

The relative advantages and disadvantages of each technique are provided in Table 2.1 for easy comparison.

Table 2.1 Summary of Various Loop Impairment Detection Techniques

<table>
<thead>
<tr>
<th></th>
<th>TDR</th>
<th>FDR</th>
<th>FMCW Radar</th>
<th>SS Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplicity</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Cost Effectiveness</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Multiple Impairment Detection</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Discontinuity Type Detectable</td>
<td>Yes</td>
<td>Partial</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Easily Compensable</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>
3. Review of Transmission Lines

3.1 Definition

The guided transmission of energy between a source and some distant point requires the use of a transmission medium generally called a transmission line. Usually this medium is a physical structure of some sort (conductors) [15]. Physical implementations of a transmission line can be strip line, coaxial cable or two wire, to name a few. This thesis deals with twisted pair conductors, so the transmission line characteristics will refer to them.

Not all conductors need to be treated as transmission lines. Generally conductors can be considered a transmission line if their lengths are a significant fraction of the wavelength being transmitted [16]. Usually a conductor length of \( \frac{1}{10} \) the wavelength is sufficient for transmission line theory. Calculation of the wavelength can be found from

\[
\lambda = \frac{\nu}{f}
\]  

(3.1)

where \( \lambda \) is the wavelength in m, \( \nu \) is the propagation velocity of the wave on the transmission line in m/s, and \( f \) is the frequency of the wave in Hz.

For a transmitted wave in the GHz range, transmission line theory can be applied to conductors on the order of a centimeter in length. Likewise, for a transmitted wave in the kHz region, transmission line theory needs to be applied for lengths on the order of a few tens of meters in length.

The median length of a twisted pair telephone wire from a field wiring cabinet, otherwise known as a jumper wire interface (JWI), to a home is approximately 600 m, and rarely exceeds 2000 m [17]. For transmission line theory to be applicable when examining twisted pair to the home, frequencies in the kHz range need to be used.
More detail on the physical measurement of the twisted pair line is found in Chapter 4.

3.2 Review of Primary Constants

All transmission lines can be described by their fundamental electrical characteristics, or primary constants. There are four primary constants of a transmission line: series resistance \( r \) (\( \Omega/km \)), series inductance \( l \) (\( mH/km \)), shunt capacitance \( c \) (\( nF/km \)), and shunt conductance \( g \) (in \( \mu S/km \)). The primary constants are used to describe a transmission line equivalent circuit. The T equivalent circuit, illustrated in Figure 3.1, is typically used to represent telecommunication transmission lines [18].

![Figure 3.1 Equivalent T Network of Transmission Line](image)

Series resistance of a transmission line is the result of the non-zero conductivity of the metallic conductors. At small distances conductors are generally assumed to have no resistance. At the larger distances associated with transmission lines these small ohmic losses cannot be ignored.

Series inductance is caused by the magnetic field induced around the transmission line. As the current in the transmission line varies, the magnetic field produced opposes the change in voltage in the transmission line.

Shunt conductance is the result of leaky insulation. In a transmission line, some current will leak through the insulation separating the two conductors. This lost current is characterized by the shunt conductance.
Shunt capacitance occurs between the two metallic conductors. As with any two conductors separated by a finite distance, there is a capacitance between them.

Though primary constants are used to describe the electrical characteristics of the transmission line, in reality they are not constant at all. The primary constants can vary with both frequency and temperature. The text by Reeve [18] provides an excellent breakdown of how these primary constants are affected by both frequency and temperature, as explained in the remainder of this section.

Resistance is affected by both temperature and frequency. At lower frequencies (<1 kHz), the resistance is independent of frequency. At higher frequencies the skin effect begins to increase the resistance of the conductors. Resistance also varies over temperature. Changes of up to 4% occur per 10° C change in temperature.

Inductance changes slightly over frequency and temperature. At frequencies up to 10 kHz the inductance remains constant. Above 10 kHz the inductance decreases slightly due to the skin effect. Over a temperature range of −20° C to +50° C the inductance will change up to 2%.

Capacitance is not affected greatly by either frequency or temperature. Over frequency the capacitance is effectively constant. Over temperature capacitance is also essentially constant, changing only 0.02% per degree Celsius. During manufacture the capacitance is closely controlled, resulting in such low variation.

Conductance is highly dependent on frequency. It increases by a factor of ten for every factor of ten increase in frequency, due to hysteresis in the dielectric. Conductance is negligibly affected by temperature.

The twisted pair wire can have one of two types of filling between the conductors and outer sheath: air core and fill core. Air core cable is used in trunk cables between the central office (C. O.) and the JWI. It is pressurized with air to keep water out. Filled core cable is used between the JWI and the customer location. The jelly fill prevents migration of water into the cable.
Tables 3.1 and 3.2 show the effects of frequency on the primary constants for various gauges of twisted pair wire, at a constant temperature of 20° C, taken from Reeve [18].
**Table 3.1**  Air Core Primary Constants versus Frequency, taken from Reeve [18]

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>$r$ ($\Omega/\text{km}$)</th>
<th>$l$ ($\Omega H/\text{km}$)</th>
<th>$c$ ($\Omega F/\text{km}$)</th>
<th>$g$ ($\Omega S/\text{km}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>19 AWG</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>52.8</td>
<td>0.597</td>
<td>51.5</td>
<td>0.13</td>
</tr>
<tr>
<td>40</td>
<td>61.4</td>
<td>0.568</td>
<td>51.5</td>
<td>5.18</td>
</tr>
<tr>
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<td>0.512</td>
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<td>24.87</td>
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<tr>
<td>772</td>
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<td>0.472</td>
<td>51.5</td>
<td>99.74</td>
</tr>
<tr>
<td>1576</td>
<td>324.5</td>
<td>0.459</td>
<td>51.5</td>
<td>203.41</td>
</tr>
<tr>
<td><strong>22 AWG</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>106.3</td>
<td>0.604</td>
<td>51.5</td>
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</tr>
<tr>
<td>40</td>
<td>112.2</td>
<td>0.581</td>
<td>51.5</td>
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</tr>
<tr>
<td>192</td>
<td>165.4</td>
<td>0.548</td>
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<td>24.87</td>
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<tr>
<td>772</td>
<td>330.7</td>
<td>0.495</td>
<td>51.5</td>
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<td><strong>24 AWG</strong></td>
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<td><strong>26 AWG</strong></td>
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<td>Frequency (kHz)</td>
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<td>22 AWG</td>
<td>24 AWG</td>
<td>26 AWG</td>
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<td>1576</td>
<td>697.8</td>
<td>51.5</td>
<td>203.41</td>
</tr>
</tbody>
</table>
3.2.1 Secondary Constants

Though the primary constants provide a means of describing the transmission line electrically, they are not all that useful in describing the behavior of an electrical signal on that line. The secondary constants, derived from the primary constants, provide such meaningful information as attenuation, phase, and characteristic impedance.

The propagation constant of a transmission line is defined as

\[ \gamma = \sqrt{(r + j\omega l)(g + j\omega c)} \]  
\[ = \alpha + j\beta \]

where \( \gamma \) is the propagation constant, \( r \) is the per unit resistance, \( l \) is the per unit inductance, \( g \) is the per unit conductance, \( c \) is the per unit capacitance, and \( \omega \) is the radian frequency of the waveform on the transmission line. The quantity \( \alpha \) is referred to as the attenuation constant, and provides the amount of attenuation (in Nepers) per unit length of the transmission line. To convert to the more useful dB per unit length, this quantity can be multiplied by 8.686. The quantity \( \beta \) is referred to as the phase constant. This provides the phase delay of a signal on the transmission line, in radians per unit length.

The characteristic impedance is another secondary constant determined from the primary constants of a transmission line. The equation of the characteristic impedance \( Z_0 \) is

\[ Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}} \]

The characteristic impedance of a transmission line is a proportionality constant that relates the instantaneous voltage and the instantaneous current at any point along the transmission line. It is effectively the input impedance of an infinitely long piece of transmission line.

Table 3.3 provides attenuation, phase delay, and characteristic impedance for various gauges of line over frequency.
Table 3.3  Calculated Secondary Constants versus Frequency

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>$\alpha \ (dB_{km})$</th>
<th>$\beta \ (\text{radians}/km)$</th>
<th>$Z_o \ (\Omega/km)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19 AWG</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.1</td>
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<td>4.3</td>
<td>6.8</td>
<td>110</td>
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<td>8.6</td>
<td>26.6</td>
<td>107</td>
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3.3 Travelling Waves

A wave is defined as an oscillation that travels through a medium by transferring energy from one particle or point to another without causing any permanent displacement of the medium [19]. On a transmission line, the energy is transferred down the line in the form of voltage and current. The concept of a wave is important in understanding the standing wave interference that is used to detect loop impairments.

A general equation used to describe the voltage on a conductor of a transmission line can be given by:

\[ \zeta = \eta(x, t) \]  

(3.5)

where \( \zeta \) is the voltage, \( \eta \) is the generalized function, \( x \) is the point along the axis of the transmission line, and \( t \) is the time. Thus, not only is the voltage on the transmission line a function of time, but it is also a function of distance as well.

Consider an arbitrary wave generated at the source \((x = 0)\) as a function of time. The first observation is made at \( x = 0 \), as in Figure 3.2(a). The wave is going to take a finite amount of time to propagate down the transmission line. If another observation of the wave is made at \( x = x_1 \) \((x_1 > 0)\), then the voltage on the transmission line as a function of time will appear as in Figure 3.2(b).

In Figure 3.2(a), the generalized equation to describe the voltage wave is:

\[ \zeta = \eta(0, t) \]  

(3.6)

In Figure 3.2(b), the generalized equation to describe the voltage wave is:

\[ \zeta = \eta(x_1, t) \]  

(3.7)

However, there is a way to simplify this. In the absence of attenuation and distortion the observed waveform is identical to that observed at \( x = 0 \), except for a time delay proportional to \( x_1 \) [20]. The time lag \( t_1 \) is related to how far \((x_1)\) the wave is from the source by the velocity of propagation \((\nu)\) of the wave. Thus

\[ t_1 = \frac{x_1}{\nu}. \]  

(3.8)
Figure 3.2 Plot of Voltage Amplitude versus Time for a Travelling Wave

From this, Equation 3.7 can also be written as

\[
\zeta = \eta(x_1, t) = \eta(0, t - \frac{x_1}{\nu}).
\]  

(3.9)  

(3.10)

This leads to the equation for determining the voltage at any arbitrary location \(x\) as:

\[
\zeta = \eta(0, t - \frac{x}{\nu}).
\]  

(3.11)

Now consider when this generalized function \(\eta\) is sinusoidal. The observed wave at
the generator \((x = 0)\) will be:

\[
\zeta = \eta(0, t) = A \sin(\omega t). \tag{3.12}
\]

The observed wave at any arbitrary point \(x\) along the transmission line, using Equation 3.11 will be:

\[
\zeta = A \sin[\omega(t - \frac{x}{\nu})] \tag{3.14}
\]

\[
= A \sin(\omega t - \frac{\omega x}{\nu}) \tag{3.15}
\]

\[
= A \sin(\omega t - kx) \tag{3.16}
\]

where \(k = \omega/\nu\) is defined as the wave number. Equation 3.16 defines the voltage of a sinusoid wave travelling along a transmission line at a distance \(x\) and time \(t\).

### 3.4 Standing Waves

Standing waves occur when a wave travelling in one direction interferes (either constructively or destructively) with a wave travelling in the opposite direction. In the case of a transmission line, this wave travelling in the opposite direction is the result of a reflection at an impedance discontinuity.

Many types of discontinuities will cause a reflection of an incident wave back to the source. A bridge tap, improper termination impedance, an open circuit at the termination end, or a short circuit at a termination end will all cause a reflection. Consider a voltage pulse travelling on a transmission line terminated in a short circuit, as in Figure 3.3 [21].

The voltage at the point of discontinuity will be the sum of the incident voltage \(V_1\) and the reflected voltage \(V_2\):

\[
V_r = V_1 + V_2 \tag{3.17}
\]

Likewise, the current at the point of discontinuity will be the sum of the incident
current $I_1$ and the reflected current $I_2$:

$$I_r = I_1 + I_2 \quad (3.18)$$

The impedance of the discontinuity will be related to $V_r$ and $I_r$ by:

$$Z_r = \frac{V_r}{I_r} \quad (3.19)$$

Reworking this equation yields:

$$Z_r = \frac{V_1 + V_2}{I_1 + I_2} \quad (3.20)$$

$$= \frac{\frac{V_1}{Z_0} - \frac{V_2}{Z_0}}{\frac{1}{Z_0}} \quad (3.21)$$

$$= \frac{V_1 + V_2}{Z_o} \frac{1}{V_1 - V_2} \quad (3.22)$$

Solving this for $V_2$ gives

$$V_2 = V_1 \frac{Z_r - Z_o}{Z_r + Z_o}. \quad (3.23)$$
Finally, taking the ratio of reflected to incident voltages leads to the reflection factor

\[ \rho = \frac{V_2}{V_1} = \frac{Z_r - Z_0}{Z_r + Z_0} \]  

(3.24) 

(3.25)

The reflection factor from the example of Figure 3.3 will be -1 since the load impedance \( Z_r \) is 0.

From Equation 2.1, the reflection factor \( \rho \) of Figure 3.3 will be -1 for a short circuit termination. The reflected pulse has a negative amplitude with respect to the incident pulse.

Consider a sinusoidal wave travelling toward the shorted end of the transmission line. Equation 3.16 from Section 3.3 describes the voltage amplitude of a sinusoidal voltage travelling in the positive \( x \) direction (away from the source). Denoting \( \zeta_+ \) to be the wave travelling in the positive \( x \) direction

\[ \zeta_+ = A \sin(\omega t - kx) \].  

(3.26)

The reflected wave travelling in the negative \( x \) direction is given by

\[ \zeta_- = -A \sin(\omega t + kx) \].  

(3.27)

The reflection factor of -1 has caused the amplitude shift from \( A \) to \( -A \). The change in direction has caused the argument change from \( -kx \) to \( kx \).

The sum of these two waves (incident and reflected) will be the observable voltage pattern on the transmission line at a given time \( t \) and given position \( x \). This result is found from their sums:

\[ \zeta = \zeta_+ + \zeta_- \]  

(3.28)

\[ = A[\sin(\omega t - kx) - \sin(\omega t + kx)]. \]  

(3.29)

Using the trigonometric relation

\[ \sin \alpha - \sin \beta = 2 \sin\left(\frac{\alpha - \beta}{2}\right) \cos\left(\frac{\alpha + \beta}{2}\right) \]  

(3.30)
then

$$\zeta = -2A \sin(kx) \cos(\omega t).$$  \hspace{1cm} (3.31)

This equation does not correspond to a travelling wave; instead it represents a wave amplitude that varies as $2A \sin(kx)$ along the length $x$ of the transmission line. This is illustrated in Figure 3.4. At different times the voltage at any particular point will vary as shown. Thus standing waves are created by interfering travelling waves going in opposite directions.

![Figure 3.4 Standing Wave Due to Short Circuit on Transmission Line](image)

The discontinuity does not have to be such that there is a polarity change in the reflected waveform. Any discontinuity that prevents the incident waveform from being completely absorbed will cause a reflected wave, and any reflected wave will interfere with the incident waveform to create a partial standing wave.
3.5 Summary

This chapter provided a review of transmission line characteristics: primary constants, secondary constants, travelling waves, and standing waves.

The primary constants of typical twisted pair lines were presented, along with their variations over temperature and frequency. Tables 3.1 and 3.2 show how the primary constants vary with frequency.

The secondary constants were then derived from the primary constants: attenuation, phase delay, and characteristic impedance. The results of Table 3.3 indicate attenuation increases with frequency. This is an effect that ultimately needs addressing (more on this in Chapter 4). Characteristic impedance is important in understanding why loop discontinuities cause wave reflections. Changes in characteristic impedance will result in a travelling wave being partially (or even fully) reflected.

Travelling waves were then introduced as a means to explain standing waves. The concept of a standing wave is important in understanding what exactly is being measured with the FDR device. This will be explained in greater detail in Chapter 4.
4. System Design

Chapter 2 provided different approaches to the problem of determining the electrical and structural characteristics of a telephone line. Chapter 3 introduced the theory behind transmission lines. Chapter 4 will take this background information and develop the system design.

The first section of this chapter will address the system theory behind the thesis design. The second section of this chapter will go into the hardware necessary to capture the raw data needed for the line analysis. The third, and final, section of this chapter will explain the techniques used to analyze the data and provide graphical results that characterize the telephone line.

4.1 System Theory

The system theory section provides an overview of the theoretical aspects of the proposed design. The discrete Fourier transform, windowing, and warping theory are all covered.

4.1.1 Discrete Fourier Transform

Continuous, or analog, waveforms are defined as having a measurable value at all points in time. Discrete signals, on the other hand, exist only at discrete points in time, and are undefined at all other times. They are best described as a set of weighted impulses [22]. There are advantages and disadvantages to working in either domain. With the advent of powerful personal computers (PC's), and mathematical software to run on them, it is possible to do more manipulation of discrete signals than ever before.
A non-periodic signal in time can be represented as a continuum of frequency components [23]. The result is the well-known Fourier transform equation

\[
X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt
\]  

(4.1)

where \( X(f) \) is the frequency spectrum of a time signal \( x(t) \). This equation applies to continuous signals in the analog domain. The time domain signal \( x(t) \) must have finite energy in order for its Fourier transform to exist [24]. This condition is satisfied if:

\[
E = \int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty
\]  

(4.2)

where \( E \) is the normalized energy. All physically realizable waveforms satisfy this condition, thus all physically realizable waveforms are Fourier transformable.

The discrete version of the Fourier transform is known as the discrete Fourier transform (DFT). The equation for the DFT is

\[
X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}
\]  

(4.3)

where \( k \) defines the frequency component, \( X(k) \) defines the \( k \)th frequency component value, \( n \) is the sample number, \( x(n) \) is the \( n \)th discrete sample, and \( N \) is the total number of samples in the collected data.

There are many means of implementing the DFT given by Equation 4.3. Much time and effort has been spent trying to make the DFT computationally efficient [25]. These efficient algorithms are known collectively as fast Fourier transforms, or FFT's.

The Fourier transform, as given in Equation 4.1 or Equation 4.3, transforms a time domain signal into its frequency domain components. The inverse is also a valid transformation, that is transforming a frequency domain signal into its time domain components. This is accomplished through the inverse Fourier transform.

The inverse Fourier transform for an analog signal is given by

\[
x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df
\]  

(4.4)
and the equation of the inverse discrete Fourier transform (IDFT) is given by

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{\frac{j2\pi kn}{N}}. \]  \hspace{1cm} (4.5)

One of the useful properties of the Fourier transform is the duality principle [23]. This property states that if

\[ x(t) \Leftrightarrow X(f) \]

then

\[ X(t) \Leftrightarrow x(-f) \]

In other words, if a given time domain waveform \( x(t) \) has a Fourier transform in the frequency domain \( X(f) \), then a time domain waveform \( X(t) \) will have a Fourier transform in the frequency domain of \( x(-f) \). Thus, if a sinusoid is present in the time domain, its Fourier transform will be two impulses in the frequency domain. If a frequency domain spectrum has a sinusoidal shape, then the inverse Fourier transform will be two impulses in the time domain.

4.1.2 Windowing

The results of a Fourier transform, whether analog or discrete, will only be ideal if the function representing the data is known over a time span of \(-\infty\) to \(+\infty\). Likewise, the results of the inverse Fourier transform will only be ideal if the data is represented by a function spanning the frequency range from \(-\infty\) to \(+\infty\) [26]. In the real world this is obviously impossible; only a small portion of a signal can be collected and analyzed.

The DFT is based on periodicity of the signal. That is, to achieve proper results the sampled data must be periodic within the observation window of the DFT. If this is not the case, then the periodic extension of the signal will exhibit discontinuities [27].
Data that is periodic within the observation window will contain only two non-zero components in its DFT, if the periodic waveform is sinusoidal (recall the Fourier transform for a sinusoid is a two line spectra). However, the discontinuity exhibited by a non-periodic signal will have the effect of distorting the spectral components of the DFT. This distortion is known as spectral leakage.

Consider a sinusoidal signal. If the sampled data of that signal is periodic within the observation window, then no discontinuities exist, as in Figure 4.1. An extension of the sampled data is smooth and continuous. Since the data is periodic within the observation window, the DFT exhibits a line spectra, as illustrated in Figure 4.2.

Consider the same sinusoidal signal, only this time the sampled data is not periodic within the observation window, as in Figure 4.3. In this case it has been sampled at a different rate (with number of samples remaining the same), so the observation window no longer covers an exact period of the waveform. An extension of the sampled data is neither smooth nor continuous - there is a discontinuity at the boundary of the observation window. The resulting DFT exhibits spectral leakage, as in Figure 4.4.

Windowing alleviates this problem. Windowing refers to the use of a weighted function applied to the sampled data to help reduce the effects of spectral leakage. The common result of windowing is that sampled data tapers off at the extremities, so the periodic extension of the data is continuous. Extensive research has gone into devising windows, along with their relative advantages and disadvantages [27], [28].

Figure 4.5 illustrates the effect of using the well-known Hanning window on the signal of Figure 4.3. The resulting DFT is shown in Figure 4.6. The use of the Hanning window has reduced the spectral leakage considerably.

It is common practice in digital signal processing to apply windows to a set of sampled data. The obvious advantage is to reduce the spectral leakage of the DFT and isolate the spectral components to near the true fundamental. The main disadvantage is a broadening of the components near the fundamental, reducing the frequency resolu-
Figure 4.1 Periodic Signal within Observation Window

Figure 4.2 DFT of Periodic Signal within Observation Window
Figure 4.3 Non-Periodic Signal within Observation Window

Figure 4.4 DFT of Non-Periodic Signal within Observation Window
Figure 4.5  Non-Periodic Signal within Observation Window, Hanning Window Applied

Figure 4.6  DFT of Non-Periodic Signal within Observation Window, Hanning Window Applied
tion. This is an effect which cannot be avoided, and in this application it is better to reduce the spectral leakage than to keep the resolution as high as possible.

The Hanning window is used because it provides sharper roll off in the frequency domain than other available windows in MATLAB. This allows peaks that exist further from the primary peak to be detected. The tradeoff is the widening of the peak near the primary, resulting in a loss of resolution of peaks in the immediate vicinity.

4.1.3 Warping

In Section 3.4 it was shown that a standing wave can be created in a transmission line if there is a single frequency waveform applied to the line and the line has a discontinuity that causes a wave reflection. If the voltage applied at the discontinuity is held constant, the voltage at the transmitting end of the standing wave will vary as distance from the discontinuity is changed. The same is also true if the input frequency changes, and the distance to the discontinuity is fixed. Thus, by varying the input frequency, and measuring the amplitude ratio of the input and reflected wave, a periodic (with frequency) pattern will emerge.

If the wave propagation velocity is constant over frequency, the equation expressing the velocity is given by

\[ \nu = \frac{2\pi f}{\beta} \] (4.6)

where \( \nu \) is the velocity of propagation, \( f \) is the frequency of the waveform, and \( \beta \) is the phase constant of the transmission line. Recall from Section 3.3 that \( \beta \) does not vary linearly with frequency. This results in a velocity of propagation that is not constant over frequency.

Equation 2.11 relates the distance to a discontinuity in terms of velocity of propagation, and the difference in frequency between peaks:

\[ x = \frac{\nu}{2\Delta f} \] (4.7)

A non-constant velocity of propagation changes the period of variation in the fre-
frequency domain. After taking the DFT of the waveform this will increase the spectral spreading, making it harder to calculate the distance to the fault or faults. Higher resolution on the fault location can be obtained from the DFT if a correction factor is applied to the data in order to take into account the non-constant velocity of propagation. This process is called warping.

The derivation of the warping equation is left to Appendix B, but the result is given as

\[
\chi = \frac{\sqrt{\frac{1}{2}(\sqrt{(r^2 + w^2l^2)(g^2 + w^2c^2)} - rg + w^2lc)}}{2\pi} \tag{4.8}
\]

where \( r \) is the resistance per unit length, \( l \) is the inductance per unit length, \( g \) is the conductance per unit length, \( c \) is the capacitance per unit length, and \( \omega \) is the radian frequency.

This factor is applied to the independent axis data collected by the hardware of the device. For each frequency point collected, a unique \( \chi \) value is calculated from Equation 4.8 for that frequency. The frequency point is then multiplied by the \( \chi \) factor to warp the axis. This converts the independent axis of the data from frequency to inverse wavelength \((1/\lambda)\).
4.2 Hardware Design

This section details the various blocks used in the hardware portion of the data gathering equipment. There are four major portions to the hardware, as shown in Figure 4.7: the hybrid coupler, the post amplifier, signal reintroduction, and data collection. The first block is the hybrid coupler. This is used to isolate the transmitted and received (reflected) voltages from the transmission line. After the reflected voltage is taken from the hybrid coupler, it is put through the second block, the post amplifier. The purpose of the amplifier is to compensate for increased line loss at higher frequency. After the post amplifier a portion of the original signal is reintroduced via a resistor, the third block. This helps ensure the “true” peak of the reflected data is captured. The fourth block consists of the network analyzer, which acts as both a signal source and a means of collecting the data.

![Network Analyzer Data Collection](image)

**Figure 4.7** Detailed Hardware Block Diagram of FDR System Set-up
4.2.1 Frequency Selection

Before any discussion on the hardware is given, a brief explanation on the input frequency range is needed. Recall from Section 3.1 that in order for transmission line theory to be applicable, the length of conductor needs to be on the order ($\sim \frac{1}{10}$) of a wavelength. The length of practical telephone lines could be anywhere from 50 to 5000 m [17]. This means a frequency range on the order 100's of kHz to 10's of MHz needs to be used in order for transmission line theory to be applicable.

Due to the large range of frequencies required, the frequency range (and hence distance) were divided into two distinct groups. For line lengths of 50 to 800 m, a frequency sweep range of 1 MHz to 5 MHz was used. For line lengths of 800 m or more, a frequency sweep range of 100 kHz to 1 MHz was used. These results were empirically determined from tests performed in the lab, and will be discussed further in Chapter 5.

4.2.2 Hybrid Coupler

The purpose of the hybrid coupler is to isolate the transmitted and received (or reflected) signals. One end of the line under test is the only interface point to the equipment and both signals are present on the line. In order to gain valuable information from the reflected signal, it must be isolated from the transmitted signal to avoid unnecessary interference.

Hybrid couplers are already used extensively in telephony systems. Telephones employ hybrid couplers to separate the voice signals of the two people talking. The integrated services digital network (ISDN) also uses hybrid couplers to separate data signals. For this setup, however, a more accurate coupler was needed to capture the interference signal.

Hybrid couplers can be constructed as passive or active devices. Passive couplers use transformers and discrete resistors, capacitors, and inductors to provide isolation. Active couplers use operational amplifiers (op-amps) to provide isolation. A block
diagram of a generalized hybrid coupler is shown in Figure 4.8.

![Generalized Diagram of a Hybrid Coupler](image)

**Figure 4.8** Generalized Diagram of a Hybrid Coupler

The hybrid coupler can be thought of as a four port device. If each port is properly terminated \((Z_{TX} = Z_{RX}, Z_B = Z_O)\) then any energy applied to a port is equally divided between adjacent ports; no energy will flow to the opposite port.

It is relatively simple to match \(Z_{TX}\) and \(Z_{RX}\), but it is much more difficult to match \(Z_B\) to \(Z_O\). As explained in Section 3.2.1, the complex impedance of twisted pair telephone wires varies over frequency, so a simple passive circuit configuration will not suffice.

There is difficulty in accurately modelling the line impedance for a large ratio of upper and lower frequencies [29]. As a result, two balance impedances were calculated, based on the frequency ranges described in Section 4.2.1. A MATLAB computer program was written to calculate the circuit values needed to match the characteristic impedance, \(Z_O\), of the twisted pair. The code was designed to calculate values that produced a frequency response between that of 24 and 26 AWG wires. The balance impedances could then be used in most practical telephone loop situations. Appendix A contains a complete code listing.
The calculated networks are shown in Figures 4.9(a) and 4.9(b) for the 100 kHz to 1 MHz (800 m or longer) and 1 MHz to 5 MHz (50 m to 800 m) ranges, respectively. The component values are rounded to the nearest standard 5% value for both capacitors and resistors, without significant error in impedance matching. The components were chosen so that the network will have both magnitude and phase responses between that of 26 and 24 AWG wire. This allows the device to be used on typical urban phone lines.

The actual circuit make up of the hybrid coupler needs to take into account a single to differential conversion. The signal source is single ended, while the twisted pair interface is differential. The hybrid coupler layout presented in Figure 4.10 accomplishes this task.

The amplifiers need to be capable of driving high capacitance loads, such as twisted pair telephone lines at the frequencies of interest.
Figure 4.9 Calculated Balance Impedances for Two Frequency Ranges

(a) $Z_B$ for 100 kHz to 1 MHz
(b) $Z_B$ for 1 MHz to 5 MHz

Figure 4.10 Finalized Hybrid Coupler Circuit Used in FDR Device
4.2.3 Signal Reintroduction

Under ideal circumstances the hybrid coupler will be perfectly matched to the line under test. Figure 4.11 shows the phasor of the reflected signal. The spiralling nature of the diagram is due to the increased attenuation with frequency.

![Figure 4.11 Phasor Diagram of Reflected Signal, Under Ideal Conditions](image)

When the angle $\beta$ of the phasor is $0^\circ$ the reflected signal will have a peak. When the angle $\beta$ of the phasor is $180^\circ$ the reflected signal will have a minimum. These are shown in Figure 4.12. These results come from how the reflected signal interacts with the reference signal (the original transmit signal). Under ideal conditions this phasor will have an angle $\alpha$ of $0^\circ$. The maximum will occur when the two phasors line up, when $\beta$ is $0^\circ$.

In reality the balance impedance of the hybrid coupler will not perfectly match the line under test. This results in some leakage of the original signal through the hybrid coupler to the receiver side. This leakage signal will vary in both magnitude and phase, relative to the signal source, since the impedance match will also be a function of frequency. A possible phasor diagram of the leakage signal is illustrated in Figure 4.13.
Figure 4.12 Plot of Reflected Signal, Under Ideal Conditions

Figure 4.13 Phasor Diagram of Leakage Signal
This leakage has the result of affecting the reference signal. The phasor diagram of Figure 4.14 shows how the leakage signal will produce a resultant reference signal with a varying angle $\alpha$ (dependent on frequency $f$). When the reflected signal interferes with this resultant reference the measured peak will occur when $\beta = \alpha$. This will shift the peak away from the spot it would normally have been expected. Figure 4.15 shows how the peak has shifted away from the expected location because of the angle $\alpha$ in the resultant reference.

![Figure 4.14 Phasor Diagram of Reference Signal, with Hybrid Coupler Leakage](image1)

![Figure 4.15 Plot of Reflected Signal, with Hybrid Coupler Leakage](image2)
This problem can be reduced, however. By adding more of the original signal back into the loop, the effects of the hybrid coupler leakage can be reduced. Figure 4.16 shows a phasor diagram when more of the original transmit signal is reintroduced. The phasor corresponding to the reference signal has a much larger magnitude compared to that from Figure 4.14. The resultant reference is much closer to the true reference. The resultant angle $\alpha$ varies much less with the larger reference signal than with the smaller reference. The measured peak will still occur when $\beta = \alpha$, but $\alpha$ is going to vary much closer to 0°. Thus, the observed peak is going to be much closer to where the peak is expected.

Figure 4.17 shows a phasor diagram of the returned signal and the reference signal. Figure 4.18 shows the resulting measured signal at the receiver. Due to the variance of the reference angle $\alpha$, the peak occurs at a frequency of $f_3$. As $\alpha$ varies less and less, the peak will occur closer to its proper location.

**Figure 4.16** Phasor Diagram of Reference Signal, with Hybrid Coupler Leakage and Signal Reintroduction
Figure 4.17 Phasor Diagram of Reference Signal and Reflected Signal, Producing the Resultant Measured Signal at Receiver

Figure 4.18 Plot of Reflected Signal, with Hybrid Coupler Leakage and Signal Reintroduction
4.2.4 Post Amplifier

The attenuation characteristic of twisted pair telephone wire is such that attenuation increases with frequency, as depicted in Table 3.3. The received signal measured at the hybrid coupler will appear to be a sinusoidal wave modulated with an exponentially decaying signal. Amplifying the signal with gain that increases with frequency will remove this effect, and give a near constant amplitude.

The amplifier was designed to “window” the data. Section 4.1.2 showed how the use of windows helps reduce spectral leakage. The amplifier was designed to give an effect similar to this: tapered at either end of the data sequence. The frequency response of the amplifier attenuates at lower frequencies and provides a constant gain at higher frequencies.

Two amplifiers were designed. One to accommodate the 100 kHz to 1 MHz range and the other to accommodate the 1 MHz to 5 MHz range. Both amplifiers consisted of a low pass filter with gain cascaded with a high pass filter with gain, designed and adjusted to give the desired waveform shaping. Standard filter design was performed through steps obtained in a filter design handbook [30].

Figures 4.19 and 4.20 illustrate the effects of the post amplifier. An unterminated loop 2000 m long was used to gather this data. Figure 4.19 illustrates the received signal without any post amplification. The signal starts out strong, but has an obvious exponential decay. Figure 4.20 illustrates the received signal, this time with post amplification. There is an attenuation of lower frequencies and an amplification of higher frequencies. The overall effect is to taper the signal more of a taper at either end.
Figure 4.19 Reflected Signal Without Post Amplification

Figure 4.20 Reflected Signal With Post Amplification
4.2.5 Network Analyzer and Data Collection

The network analyzer used was a Hewlett-Packard 4195A model. This was set up to have a driving output of swept frequencies and a voltage input. The maximum number of points the analyzer can handle is 401, so this was used as the default.

The driver of the network analyzer was connected directly to the input of the prototype board. From there it was split to two input paths: the hybrid coupler and the signal re-introduction resistor. Both of these blocks have been previously explained.

The input to the network analyzer is single ended, yet the test points on the hybrid coupler are differential. To make the transition, a differential to single-ended voltage probe was used. This is manufactured by Tektronix, part number P5205. This probe was connected to an oscilloscope, Tektronix model TDS544A. From the oscilloscope, an auxiliary output was connected to the network analyzer. In this fashion the differential signal from the hybrid coupler was converted to a single-ended source for the input of the network analyzer.

Each input sample was converted to discrete form within the network analyzer. This was saved to a floppy diskette and processed using software algorithms to be presented next.
4.3 Software Design

This section details the various blocks of the software algorithms. The software is needed to perform the digital signal processing on the collected data. All software modules were written with MATLAB. The five major blocks of the software are shown in Figure 4.21. The first block is known as baseline compensation. This process takes into account a “baseline” reading of the device when an “infinite” (ideal) line is connected to the unit. The second block is frequency warping. This digital signal processing (DSP) warping algorithm accounts for variations in the velocity of propagation, as explained in Section 4.1.3. The third block re-samples the warped data to keep the discrete samples on an even, periodic basis. The fourth block is the Fast Fourier Transform (FFT). This is a common algorithm that normally converts data points, sampled in time, to their respective frequency spectrum. In this application the FFT provides the inverse function and converts data points, sampled in frequency, to their respective time function. The fifth and final block is amplitude/distance estimation. This block estimates the true location of the peak resulting from the FFT. Recall from Section 4.1.2 that most signals will exhibit spectral leakage. When this occurs, the true “peak” will be located between two of the DFT values.

4.3.1 Baseline Compensation

In any real hardware system, many factors can influence the electrical response of a circuit. Component variations, stray capacitance, and stray inductance are some of the factors that can deviate the response away from what is theoretically expected. One means of eliminating these errors in the electrical response is to determine and then remove them from the final response.

In addition to these component variations, this design is unique in its use of a circuit to mimic the complex impedance of twisted pair conductors. Recall from Section 4.2.2 that a complex circuit was designed for each of the two input frequency ranges used. Due to differences in the designed balance impedance and the characteristic impedance, some “reflection” will occur at the receiver due to the difference between
the balance impedance and the line impedance.

To overcome this problem, baseline compensation was used. This involves measuring the receiver response when an "infinite" length of line is attached as the line under test. In reality, this infinite line consists of a very long line, relative to the range of the device, terminated in the same balance impedance as the hybrid coupler. For the experimental set up a line length of 6000 m was used, terminated with the proper $Z_B$.

The result obtained is then the "baseline" of the hybrid coupler. This is a unique set of data, different for each hybrid coupler. When an actual measurement is taking place, the resultant data will be modulated with the baseline data. To obtain the true reflection data of the line under test, the baseline data must be subtracted. Figure 4.22 shows data collected without baseline compensation, Figure 4.23 shows the device baseline, and Figure 4.24 shows the data after baseline compensation has been applied. Note the measurements shown also show the amplification of the higher
frequencies from the post amplifier, as explained in Section 4.2.4.

The baseline can be attributed to imprecise balance impedance match with the line under test. This mismatch will result in some of the transmit signal entering the receiver side via the hybrid coupler. This leakage signal will vary slightly with frequency in both magnitude and phase (see Section 4.2.3). The leakage signal will result in the baseline wander accounted for here.
Figure 4.22 Reflected Signal With No Baseline Compensation Applied

Figure 4.23 Baseline Signal of FDR System
Figure 4.24 Reflected Signal With Baseline Compensation Applied
4.3.2 Warping

The warping function was described in Section 4.1.3, and given as Equation 4.8. For convenience the equation is repeated below:

\[
\chi = \frac{\frac{1}{2}(\sqrt{r^2 + w^2l^2})(g^2 + w^2c^2) - rg + w^2lc}{2\omega}
\]  

(4.9)

The warping factor is used to compensate for differences in the velocity of propagation due to changes in frequency. Each discrete frequency point is multiplied by its warping factor value (as provided by Equation 4.9). This will convert the independent axis data from frequency to inverse wavelength. Subsequent FFT results of the warped data will convert the independent axis to meters (in much the same way as the FFT traditionally converts the independent axis from time to frequency). A plot of the warping factor versus frequency is shown in Figure 4.25.

![Figure 4.25 Warping Factor of FDR System Set-up Versus Frequency](image)
At higher frequencies, such as those used by the input frequency range of this device, the effects of this factor are subtle. To better visualize the effects of the warping factor, a low frequency waveform is simulated with the effects of the warping factor.

Figure 4.26 illustrates a simulated signal that has been affected by changes in velocity of propagation due to frequency. As frequency increases, the peaks of the interference signal move closer together relative to the peaks at lower frequencies. To compensate for this effect, the warping factor is applied to create a signal with much more uniform period, as shown in figure 4.27. This aids in the accuracy of the FFT results.
Figure 4.26 Example Signal Prior to Warping Application

Figure 4.27 Example Signal After Warping Application
4.3.3 Resampling

The third software block is the resampling algorithm. Recall that the warping function will compress or expand points in the discrete sample stream, based on the frequency of the point. This causes an uneven spacing in the warped, discrete sample sequence. The uneven spacing prevents standard DFT algorithms from providing correct results, as the DFT is based on evenly spaced samples.

Many techniques exist that use an iterative process to achieve the true DFT of an irregularly spaced set of data [31]. These techniques are useful for complex signals that contain components right up to one-half the Nyquist frequency. The signals processed by the FDR device are smooth and continuous, so the added complexity was not needed. Instead, a simple linear interpolation routine was designed instead.

The algorithm consists of two primary steps. The first step is calculating the new x-axis points (defined in this section as \( X \)). The second step is calculation of the new y-axis points (defined in this section as \( Y \)) to correspond with the newly created x-axis points.

To determine the new x-axis points, a sample size needs to be chosen. The distance between the two points closest on the x-axis is defined as the sample size for this application. This ensures the newly sampled data will be a subset of the existing collected data. The two points that fit this criteria turn out to be the last two points of the existing x-axis, \( x_N \) and \( x_{N-1} \) (where \( N \) is the array size of the existing x-axis). The new x-axis is then created using this sample size. The initial point of the new x-axis corresponds to the initial point of the existing x-axis (\( X_1 = x_1 \)). Thus, the newly created x-axis points are defined to be:

\[
X_n = x_1 + n(x_N - x_{N-1})
\]  

(4.10)

where \( n \) is the sample number and \( 2 \leq n \leq N \).

Once the new x-axis has been created, the corresponding y-axis values need to be calculated using linear interpolation. The new y-axis value \( Y_n \) that corresponds to
the new x-axis point $X_n$ is found from:

$$Y_n = \frac{(y_{n-1} - y_n)(X_n - x_{n-1})}{x_{n-1} - x_n} + y_{n-1}$$  \hspace{1cm} (4.11)

A full code listing of this algorithm can be found in Appendix A.

Figure 4.28 is a “stem” plot (a discrete sequence plot) of an original data sequence and its resampled version. The data was obtained from a 400 m open circuit loop. The far left stem corresponds to the initial data point of both the original and resampled data sequence. By definition these two are the same point. Towards the right the effects of the resampling process are more pronounced. The newly sampled point $X_{15}$ lies between $x_{14}$ and $x_{15}$, according to the value calculated from Equation 4.10. The amplitude of $X_{15}$, which is the value $Y_{15}$, is found from Equation 4.11.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{resampling_plot.png}
\caption{Illustration of Resampling}
\end{figure}

4.3.4 Fast Fourier Transform

One of the major software components calculates the time domain response from the frequency domain response. Recall the equation used to calculate the DFT, given as
Equation 4.3. This equation provides frequency component information \( X(k) \) from a given time domain sample sequence \( x(n) \). The inverse of the DFT, the inverse discrete Fourier transform, or IDFT, is defined by Equation 4.5. This equation provides the time domain components \( x(n) \) from a given frequency domain sample sequence \( X(k) \). Also recall from Section 4.1.1 that much effort has been made to make the DFT computationally efficient through the use of algorithms known as fast Fourier transforms, or FFT's. The software written to analyze the collected data was written for MATLAB, so it makes sense to make use of the FFT algorithm already provided by MATLAB. A brief explanation of the benefits of using an FFT to calculate the DFT are given below.

To calculate the DFT from first principles (using Equation 4.3 or Equation 4.5) will require \( N^2 \) complex multiplications [25]. For large numbers of samples the number of multiplications needed can grow very large. Indeed, even for the small 401 samples processed in this experimental setup, the results need well over 150000 complex multiplications.

Previously it was mentioned that much time and effort has been spent trying to make the DFT more efficient. One of the FFT algorithms used to accomplish this is the Radix-2 algorithm. This is the same algorithm used by MATLAB to compute an FFT [32]. The idea is to break the input data into ever smaller components, until only a series of 2 point DFT calculations need be performed. The results are subsequently used in successive fashion until all the spectral components have been calculated.
When implemented on a DSP chip, the benefits of using an FFT algorithm to compute the DFT are substantial, especially with increasing sample numbers. Table 4.1 from [25] shows the reduction in complexity that comes with using an FFT algorithm. Implementation of an FFT algorithm in this application would yield a speed improvement in the DFT calculation by a factor of approximately 100.

Table 4.1  Comparison of Computational Complexity for the Direct Computation of the DFT Versus FFT

<table>
<thead>
<tr>
<th>Number of Points (N)</th>
<th>Complex Multiplications in Direct Computation of DFT, ((N^2))</th>
<th>Complex Multiplications in FFT Algorithm, ((\frac{N}{2}\log_2 N))</th>
<th>Speed Improvement Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>16</td>
<td>4</td>
<td>4.0</td>
</tr>
<tr>
<td>8</td>
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<tr>
<td>1024</td>
<td>1048576</td>
<td>5120</td>
<td>204.8</td>
</tr>
</tbody>
</table>
### 4.3.5 Peak Estimation

Recall from Section 4.1.1 the equation for the discrete Fourier transform (DFT) is

\[ X(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{i2\pi kn}{N}} \quad (4.12) \]

As was discussed, if the frequency of the sampled data is not an integral multiple of the bin frequency size \( \frac{f_s}{N} \), then the resultant DFT will exhibit spectral leakage. For a single sinusoid, this leakage follows a \( \frac{\sin \frac{\pi}{N}}{\sin \frac{\pi}{N}} \) profile.

For a single frequency waveform the true spectral line will fall somewhere between the two largest spectral components \((l, l+1)\). It is desirable to determine this true spectral line location, along with its amplitude. Some work has already been done in algorithms to provide high accuracy results of these parameters [33].

#### Fundamental

The algorithm from [33] explains how to obtain the fundamental frequency. This fundamental can be found from

\[ f_1 = (l + \frac{\alpha}{1+\alpha})(\frac{1}{NT_s}) \quad (4.13) \]

where \( l \) is the left component sample value, \( \alpha \) is defined as the ratio of the magnitudes of the two spectral components in question (\( \alpha = \frac{|X(l+1)|}{|X(l)|} \)), \( N \) is the number of samples present in the data, and \( T_s \) is the sample period.

A major assumption of this formula (assuming a pure sine wave)

\[ |X(l)| = 0.5A_1 \frac{\sin \pi \delta}{\sin \frac{\pi \delta}{N}} \quad (4.14) \]

\[ \approx 0.5A_1 \frac{\sin \pi \delta}{N} \quad (4.15) \]

where \( A_1 \) is the DFT amplitude of the unknown fundamental, and \( \delta \) is defined as \( \frac{\alpha}{\alpha+1} \).

The paper states a value of \( N \geq 1024 \) will provide an error of no more than 0.015% if Equation 4.15 is used instead of Equation 4.14. This is, in fact, an over estimation.
An error of no more than 0.00016% results. Thus, the authors of this paper have greatly overstated the resolution needed for accurate results. Using $N = 401$, as is the hardware limitation in this experimental setup, the error will be no more than 0.0011%. Thus, the use of Equation 4.15 is justified while using the resolution of this experimental setup. Appendix C contains a proof that the error is much less than stated by [33].

**Amplitude**

The amplitude calculation described in [33] proved accurate, but is difficult to visualize. Using this work, a slightly easier method was developed. Consider the FFT plot of a non integral frequency as in Figure 4.29.

As noted previously, the spectral shape is a $\frac{\sin \pi x}{\sin \frac{\pi x}{N}}$ curve, with the peak occurring at the now known fundamental frequency, $f_1$. The value of the FFT magnitude will be $\frac{\sin \pi \delta}{N} A$, where $A$ is the unknown amplitude of the FFT of the fundamental. Rearranging,

$$A = \left| X(l) \right| \left| \frac{\pi \delta}{N \sin \pi \delta} \right|$$

(4.16)
where $|X(l)|$ is the magnitude of the $l$th DFT spectral component, $\delta$ is defined as $\frac{\alpha}{\alpha+1}$, and $N$ is the number of samples in the DFT.

Another solution to determine both location and amplitude is to zero pad the sequence before taking an FFT. The added zeros will provide more samples in the frequency domain. Location and amplitude of the peak is more easily accomplished by inspection with more samples.

### 4.4 Summary

This chapter explained in detail the various components of the design. These components were broken down into three sections: theory, hardware, and software.

The theory section provided background information on converting a discrete sequence from the frequency domain to the time domain via the discrete Fourier transform (DFT). Also presented was a section on windowing. Windowing improves the DFT results for non-periodic signals within the DFT observation window. A closer look at warping, and its effect on the data was also examined. All of these processes help shape the data for better results in subsequent processes.

The hardware section focused on the physical equipment used to collect data from a line under test. The problems encountered and addressed included the hybrid coupler (used to separate the reflected signal from the line under test), the post amplifier (used to compensate for higher attenuation at higher frequencies), and signal reintroduction (to alleviate the effects of leakage through the hybrid coupler).

The software section focused on various algorithms used to help gather useful information from the collected data. The problems encountered and addressed included baseline compensation (removing unwanted noise from the system), warping (removing dependencies on waveform propagation velocity), and resampling (compensating for the effects of warping causing uneven sample spacing).
5. System Results

The motivation behind the thesis was to develop better loop qualifying test equipment. This chapter provides the results of the research for some typical telephone wire configurations. These results are also compared to competitive products for various line configurations. Conclusions are drawn from the tests and comparisons with competitive products are made.

The first section describes the experimental test set up. The second section contains test results of simple loop configurations. These involve no bridge taps. The third section consists of complex loop configurations including one or two bridge taps. The fourth section makes direct comparisons to competing commercial products. Finally, the fifth section draws conclusions from the results presented in the chapter.

5.1 Experimental Setup

The experimental setup consists of the hardware and software portions of the design. The hardware portions of the setup consist of the common general blocks presented in section 4.2. Figure 5.1 illustrates the connections between these blocks. The frequency generator used is a Hewlett-Packard 4195A network analyzer. The radio frequency (RF) output of the network analyzer is used as the signal source. Depending on the line distance, the output is set to sweep in either 100 kHz to 1 MHz, or in 1 MHz to 5 MHz intervals. Each frequency range consists of 401 frequency points evenly divided between the two-end point frequencies. The power level of the RF waveform was set to 15 dBm, the maximum power available from the analyzer. When used with a long twisted pair this allows the most sensitive measurement of reflections.

The RF signal from the network analyzer is injected into the frequency domain reflectometry (FDR) device, as described in greater detail in section 4.2.2. The line under
Figure 5.1 Full Laboratory Hardware Set Up

test also interfaces to the FDR device, and consists of various loop configurations described later in this chapter. These configurations are constructed using segments of 24 AWG twisted pair telephone wire. The segments are in fixed lengths of 100 m and 400 m.

The differential signal observed across the observation points of the hybrid coupler circuit was converted to a single-ended signal through the use of a differential probe, in this case a Tektronix P5205. The differential leads of the probe were attached to the observation points of the hybrid coupler, located within the FDR device. The output of the differential probe was connected to a RDS544A Tektronix oscilloscope. The output port of the oscilloscope was fed into the post amplifier of the FDR device. The signal from the output of the post amplifier was used as the RF input signal of the network analyzer. After a complete sweep was made of the line under test, the data collected on the network analyzer was copied to diskette.

The copied data was run through a MATLAB program located on a personal computer.
The MATLAB program incorporates all the software functionality discussed in the previous chapter (baseline compensation, warping, re-sampling, fast Fourier transform, and peak estimation). A full code listing is found in Appendix A. Plots were generated by the code to aid in the graphical analysis by the user. In addition, a displayed message was given of the estimated distance and amplitude of the FFT peak(s).

5.2 Simple Loops

5.2.1 Set-up

Initially a simple “loop” was set up. This type of loop consisted of a twisted pair that contained no bridge taps, and had uniform characteristic impedance throughout the line. The loop was terminated in a termination impedance $Z_T$. Figure 5.2 shows the simple loop connected to the FDR device and terminated in $Z_T$.

![Figure 5.2 Simple Loop Configuration Set Up](image)

The variables in this simple loop test setup are loop length and termination impedance. Table 5.1 shows the results of using a 100 kHz to 1 MHz input frequency range. The loop lengths used in the testing were 400 m, 800 m, 1200 m, 1600 m, 2000 m, 2400 m, and 2800 m. For each loop length the termination impedance was varied between $\infty \Omega$, 680 $\Omega$, and 0 $\Omega$. Table 5.2 shows the results of using a 1 MHz to 5 MHz input frequency range. The loops lengths used were 400 m, 800 m, 1200 m, and 1600 m. For each loop length the termination impedance was varied between $\infty \Omega$, 680 $\Omega$, and 0 $\Omega$. Both tables also include the reflection factor for the termination impedances,
calculated at 1 MHz.

Examination of the data presented in Tables 5.1 and 5.2 indicate that each input frequency range provides useful results for different distance ranges. An input frequency range of 100 kHz to 1 MHz yielded useful data for loop lengths of approximately 800 m or more, as indicated in Table 5.1. An input frequency range of 1 MHz to 5 MHz yielded useful data for loop lengths of approximately 800 m or less, as indicated by Table 5.2. Based on these results, the distance cutoff was empirically determined to be around 800 m.

The estimated FFT magnitudes are based on a normalized FFT result. The results from the analysis software normalize the highest peak of the FFT result to unity. This explains why it is possible for the estimated FFT magnitude from a termination impedance of 680 Ω is larger than with an open or shorted circuit. Normally one would expect a smaller FFT magnitude with a 680 Ω termination impedance.
Table 5.1  Simple Loop Configuration Summary of Results,  
100 kHz to 1 MHz Test Frequency Range

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>$Z_T$ (Ω)</th>
<th>$\rho$ (at 1 MHz)</th>
<th>Estimated Distance (m)</th>
<th>Estimated FFT Magnitude</th>
<th>Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>$\infty$</td>
<td>+1</td>
<td>200</td>
<td>1.4124</td>
<td>-200</td>
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<tr>
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<td>200</td>
<td>1.4002</td>
<td>-201</td>
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<td>1200</td>
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<td>+1</td>
<td>200</td>
<td>1.4105</td>
<td>-200</td>
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<tr>
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<td>-200</td>
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<td>2800</td>
<td>$\infty$</td>
<td>+1</td>
<td>200</td>
<td>1.4124</td>
<td>-200</td>
</tr>
<tr>
<td>400</td>
<td>680</td>
<td>+0.74</td>
<td>200</td>
<td>1.4105</td>
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</tr>
<tr>
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<td>0</td>
<td>-1</td>
<td>200</td>
<td>1.4124</td>
<td>-200</td>
</tr>
</tbody>
</table>

* The calculation algorithm gave spurious results at long distances. Low distance spurs were used in the calculation of the impairment distance. Visual inspection of the display gives an indication of the expected result.
Table 5.2  Simple Loop Configuration Summary of Results, 1 MHz to 5 MHz Test Frequency Range

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>$Z_T$ (Ω)</th>
<th>$\rho$ (at 1 MHz)</th>
<th>Estimated Distance (m)</th>
<th>Estimated FFT Magnitude</th>
<th>Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
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<td>-1</td>
<td>32</td>
<td>1.4765</td>
<td>-1568*</td>
</tr>
</tbody>
</table>

* The calculation algorithm gave spurious results at long distances. Low distance spurs were used in the calculation of the impairment distance. Visual inspection of the display gives an indication of the expected result.
5.2.2 Explanation of Frequency Ranges Used

The difference in results between the two frequency ranges can be explained by two factors. The first factor to consider is attenuation through the loop as a function of frequency. Recall from Table 3.3 that as frequency of a signal in a telephone loop increases, so does the attenuation (per unit distance) the signal experiences. This helps explain why use of a larger frequency range (1 MHz to 5 MHz) is limited to distances of no more than 800 m. Beyond this distance the return signal is too severely attenuated to measure accurately.

Figure 5.3 illustrates the measured signal of a 400 m open circuit line using an input frequency range of 1 MHz to 5 MHz. Figure 5.4 illustrates the estimated distance of the same loop. The signal as it appears in Figure 5.3 becomes more attenuated with frequency, yet is still easily visible - no noise at all. The resulting FFT (Figure 5.4) shows a sharp peak at the loop length.

Figure 5.5 illustrates the measured signal of a 1600 m open circuit line using a 1 MHz to 5 MHz input frequency range. Figure 5.6 illustrates the estimated distance of the same loop. The signal as it appears in Figure 5.5 is nothing more than noise, most likely due to hybrid coupler impedance mismatch. No discernable information can be gathered from this signal. As expected, the estimated distance graph provides no useful information from this noisy signal. The peak at roughly 33 m is purely spurious.

Note the sharper edges to the bottom peaks relative to the upper peaks in Figure 5.3. This gives rise to the slight peak in the FFT results at around the 800 m position in Figure 5.4. As the reflected signal becomes much smaller than the reference transmit signal, the amplitude variations become more sinusoidal.

The sharper edges on the bottom peaks are the result of the geometry of the phasors that result in the measured signal. Figure 5.7 shows the phasor diagram of the signals involved. The measured signal is the result of the reference and returned signal interfering. When $\alpha$ is small (at around $0^\circ$), a change in $\alpha$ will result in a small
Figure 5.3 400 m Open Circuit, 1 MHz to 5 MHz Test Frequency, Raw Data

Figure 5.4 400 m Open Circuit, 1 MHz to 5 MHz Test Frequency, Estimated Distance
Figure 5.5 1600 m Open Circuit, 1 MHz to 5 MHz Test Frequency, Raw Data

Figure 5.6 1600 m Open Circuit, 1 MHz to 5 MHz Test Frequency, Estimated Distance
change in $\beta$. The magnitude of the measured signal will not vary by much. This gives rise to the more rounded top side peaks. When $\alpha$ is close to $180^\circ$, a change in $\alpha$ will result in a more pronounced change to $\beta$. The magnitude of the measured will vary considerably more. This gives rise to the sharper peaks along the bottom.

![Figure 5.7 Phasor Diagram of Resultant Measurement Signal, Showing Relation to Reference and Returned Signals](image)

The second factor to consider in the difference in results between the two frequency ranges is the number of "crests" returned for a given distance. For a specific line length, the larger the frequency range used, the greater the number of returned crests. Consider Equation 2.11, reproduced here for convenience:

$$x = \frac{v}{2\Delta f}$$

(5.1)

$\Delta f$ is the difference in frequency between two returned crests. Theoretically this is enough to provide accurate FFT results, but the higher the number of crests, a more rectangular window function can be used resulting in higher resolution. Inclusion of as many crests as possible can be achieved by increasing the overall input frequency range. Thus, in this setup, using the $100$ kHz to $1$ MHz input frequency range ($900$ kHz) will result in relatively poor resolution, while using the $1$ MHz to $5$ MHz frequency range ($4$ MHz) will result in relatively good resolution (better by a factor of $\sim 4.5$).
Figure 5.8 illustrates the measured signal of a 400 m open circuit line using a 100 kHz to 1 MHz input frequency range. The resultant FFT (estimated distance) is provided in Figure 5.9. Figure 5.10 illustrates the measured signal of the same 400 m open circuit line using a 1 MHz to 5 MHz input frequency range. The resultant FFT (estimated distance) is provided in Figure 5.11. Since there are more cycles of amplitude variation in Figure 5.11, the resulting FFT has more points over the distance range 0 to 3000 m.

Notice the difference in the number of returned crests for the two input frequency ranges. The 100 kHz to 1 MHz input frequency range yields four crests, while the 1 MHz to 5 MHz range yields sixteen crests. The resulting difference in resolution can be seen because of this. Figure 5.9 appears very piecewise, while Figure 5.11 is much smoother.
Figure 5.8 400 m Open Circuit, 100 kHz to 1 MHz Test
Frequency, Raw Data

Figure 5.9 400 m Open Circuit, 100 kHz to 1 MHz Test
Frequency, Estimated Distance
Figure 5.10 400 m Open Circuit, 1 MHz to 5 MHz Test Frequency, Raw Data

Figure 5.11 400 m Open Circuit, 1 MHz to 5 MHz Test Frequency, Estimated Distance
5.2.3 Error

Table 5.1 shows the error in estimated loop length compared with the known length for selected loop lengths using the 100 kHz to 1 MHz input frequency range. The results seem to be estimating approximately 170 m too high.

Table 5.2 shows the error in the estimated loop length compared with the known length for selected loop lengths using the 1 MHz to 5 MHz input frequency range. The results seem to be estimating (on average) 55 m too high.

A thorough examination of the equipment setup was performed to determine if there was an error in the data collection process. The data from Tables 5.1 and 5.2 seem to suggest the fixed length segments of 400 m are closer to 405 m. This conclusion is drawn from the increase in error by an additional 5 m as loop length increases by 400 m. However, this would not account for the large overall estimation error.

Some additional twisted pair cable was used to connect the twisted pair spool to the FDR device. This added no more than 5 m to the overall loop length under test. This is not enough to account for the overall estimation error.

The analysis software was also examined to determine if there was an error in the process. The warping equation was re-calculated and found to be correct. The conversion of FFT results to distance measurement was also re-examined and found to be correct. The lone suspect to be found was in the primary constants used to calculate the warping factor.

The primary constants as a function of frequency were generated from the tabulated values in Tables 3.1 and 3.2. These values were graphed and curve fit to an equation for use in the analysis software. The equations and the subsequent curve fittings can be found in Appendix D. The equation was used to generate the warping factor. This equation may be a source of error, as it may not characterize the primary constants as a function of frequency adequately enough.

The equations used are extrapolated to a frequency of 5 MHz, even though the final
tabulated point is at approximately 1.5 MHz. This could cause problems if the primary constants do not change with frequency as predicted by the curve fit equations. As well, the equations used at times do not even pass through the known data points. This also indicates there may be problems with the curve fit equation used.

To overcome this overestimation error, it would be advisable to take measurements of the primary constants versus frequency to fill out the tabulated data (i.e. more than five points should be used for curve fitting). This will improve the accuracy of the equations used in calculating the warping factor. Additionally, a "calibration factor" may be needed for units in the field to overcome the overestimation problem.

5.2.4 Graphical Results

The following figures show the estimated distance displays for an open circuit loop of varying lengths. The first three figures (5.12, 5.13, and 5.14) correspond to results obtained using a 1 MHz to 5 MHz input frequency range. Notice the lengths of 400 m and 800 m are easily observed, but the 1200 m length is much smaller in amplitude. Reasons for this were discussed earlier in this section.

Figure 5.12 shows the peak at around 400 m, as expected. Also indicated on the figure are the second and third harmonics of the 400 m peak. As discussed previously in this section, the sharper crests of the measured signal (refer to Figure 5.3) give rise to harmonic content on the FFT. Figure 5.13 also contains some harmonic content at approximately 1600 m.

The remaining 7 figures (5.15, 5.16, 5.17, 5.18, 5.19, 5.20, 5.21) correspond to results obtained using a 100 kHz to 1 MHz input frequency range. Again, note the 400 m length does not provide good results. Starting at 800 m to about 2400 m the results clearly indicate the location of discontinuity. At 2800 m the FDR device seems to start having trouble with noise, as evidenced by the large peak at around the 200 m mark (clearly erroneous). Though the device still picks up on the peak at 2800 m.

Figure 5.15 indicates a peak at approximately 400 m, along with an 800 m harmonic.
Figure 5.16 indicates the expected 800 m peak, along with both its second and third harmonics at approximately 1600 m and 2400 m, respectively. Figure 5.17 shows the 1200 m peak. It also has a second harmonic peak visible at approximately 2400 m. The remaining figures show no harmonics, as they are not visible on the 0 to 3000 m scale.

The harmonic content visible in subsequent figures is a troublesome feature. There is a distinct possibility a user of the FDR device will confuse a harmonic with an actual impairment peak. To help reduce the problem of harmonics, more of the reference (transmitted) signal could be reintroduced into the system. This will have the effect of “softening” the sharp edges of the bottom peaks. See Section 5.2.2 for more information on the cause of this problem.
Figure 5.12 400 m Open Circuit, 1 MHz to 5 MHz Input Frequency Range

Figure 5.13 800 m Open Circuit, 1 MHz to 5 MHz Input Frequency Range
Figure 5.14 1200 m Open Circuit, 1 MHz to 5 MHz Input
Frequency Range

Figure 5.15 400 m Open Circuit, 100 kHz to 1 MHz Input
Frequency Range
Figure 5.16 800 m Open Circuit, 100 kHz to 1 MHz Input Frequency Range

Figure 5.17 1200 m Open Circuit, 100 kHz to 1 MHz Input Frequency Range
Figure 5.18 1600 m Open Circuit, 100 kHz to 1 MHz Input Frequency Range

Figure 5.19 2000 m Open Circuit, 100 kHz to 1 MHz Input Frequency Range
**Figure 5.20** 2400 m Open Circuit, 100 kHz to 1 MHz Input Frequency Range

**Figure 5.21** 2800 m Open Circuit, 100 kHz to 1 MHz Input Frequency Range
5.3 Complex Loops

5.3.1 Set-up

The next set of tests involved complex loops. These loops involved varying lengths of line, with one or two bridge taps. The terminating impedance was matched to the input frequency range of 100 kHz to 1 MHz, and consisted of the circuit of Figure 4.9(a). Figure 5.22 shows how a complex loop is constructed.

A maximum of two bridge taps were used because in practise there are rarely more than 2 bridge taps to a telephone loop. Additionally, loading coils are rarely used in an urban setting, so these were not tested against either.

![Complex Loop Configuration Set Up](image)

**Figure 5.22** Complex Loop Configuration Set Up

The variables in the complex loop test set up are length of the primary line (a+c+e), length of the bridge taps (b,d), and location of the bridge taps (a, a+c). Table 5.3 indicates the test parameters for the complex loop configuration. Differing loop lengths and bridge tap lengths are indicated. In all cases, the test frequencies used were 100 kHz to 1 MHz due to the length of the lines involved. The table also contains the estimated distance values of the initial discontinuity, along with the estimated FFT magnitude values of the initial discontinuity. Graphical results of some selected tests are shown at the end of this section.
Table 5.3  Complex Loop Configuration Summary of Results

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>Interpolated Distance (m)</th>
<th>Interpolated FFT Magnitude</th>
<th>Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>800</td>
<td>1312</td>
<td>1.5254</td>
<td>+112</td>
</tr>
<tr>
<td>1200</td>
<td>0</td>
<td>0</td>
<td>300</td>
<td>800</td>
<td>1285</td>
<td>1.5640</td>
<td>+85</td>
</tr>
<tr>
<td>1200</td>
<td>0</td>
<td>0</td>
<td>800</td>
<td>400</td>
<td>1362</td>
<td>1.5458</td>
<td>+162</td>
</tr>
<tr>
<td>400</td>
<td>1200</td>
<td>800</td>
<td>200</td>
<td>800</td>
<td>506</td>
<td>1.5495</td>
<td>+106</td>
</tr>
<tr>
<td>400</td>
<td>800</td>
<td>400</td>
<td>200</td>
<td>800</td>
<td>506</td>
<td>1.5420</td>
<td>+106</td>
</tr>
<tr>
<td>400</td>
<td>800</td>
<td>400</td>
<td>300</td>
<td>800</td>
<td>506</td>
<td>1.5420</td>
<td>+106</td>
</tr>
</tbody>
</table>

5.3.2 Results

The first three entries of Table 5.3 were set up with a single bridge tap of varying length. A scale drawing of the three configurations follows, along with the graphical estimation and a brief description.

The scale drawing of Figure 5.23 shows the complex loop configuration from the first entry of Table 5.3. The resulting estimation is displayed in Figure 5.24. The point labelled 1 in Figure 5.24 corresponds to the initial discontinuity located at 1200 m from the FDR device. This peak should be the strongest in level for two reasons:

- the discontinuity is the closest
- the most signal energy will be reflected from this point

The point labelled 2 in Figure 5.24 corresponds to the reflection off the open end of the bridge tap, located 1300 m from the device. This signal is somewhat smaller in amplitude due to increased distance and reduced signal energy.

Other peaks are expected, corresponding to lengths of 1400 m, 1500 m, 1600 m, and so on. These result from multiple reflections off the primary discontinuity and bridge
tap. These peaks would be of a fairly small amplitude, and are likely lost within the skirt of the two visible peaks.

\[ \text{Figure 5.23 Complex Loop, Single Bridge Tap Length 100 m} \]

\[ \text{Figure 5.24 Complex Loop, Single Bridge Tap Length 100 m, Estimated Distance} \]

The scale drawing of Figure 5.25 shows the complex loop configuration from the second entry of Table 5.3. The resulting estimation is displayed in Figure 5.26. The point labelled 1 in Figure 5.26 corresponds to the initial discontinuity located at 1200 m from the FDR device. The point labelled 2 corresponds to the reflection off the open end of the bridge tap, located 1500 m from the device. The point labelled 3 corresponds to the second reflection off the open end of the bridge tap, located 1800 m from the FDR device. Missing from the figure is the third reflection off the bridge.
tap, located at 2100 m. The peak at 2400 m is the second harmonic component of the 1200 m peak, labelled as point 4.

![Complex Loop, Single Bridge Tap Length 300 m](image)

**Figure 5.25** Complex Loop, Single Bridge Tap Length 300 m

![Normalized FFT Magnitude](image)

**Figure 5.26** Complex Loop, Single Bridge Tap Length 300 m, Estimated Distance

The scale drawing of Figure 5.27 shows the complex loop configuration from the third entry of Table 5.3. The resulting estimation is displayed in Figure 5.28. The point labelled 1 in Figure 5.28 corresponds to the initial discontinuity located at 1200 m from the FDR device. The point labelled 2 corresponds to the reflection off the open end of the bridge tap, located 2000 m from the device. Another peak is expected at
Figure 5.27 Complex Loop, Single Bridge Tap Length 800 m

Figure 5.28 Complex Loop, Single Bridge Tap Length 800 m, Estimated Distance
2800 m, corresponding to the second reflection off the bridge tap. However, a simple loop of length 2800 m pushes the limit of the FDR device, so it is likely this peak has been lost due to extremely low signal strength.

The last three entries of Table 5.3 show some double bridge tap results. The results are not as good as the results measured from a simple loop or single bridge tap, but the results indicate successful resolution of two bridge taps in the loop.

The scale drawing of Figure 5.29 shows the complex loop configuration from the fourth entry of Table 5.3. The resulting estimation is shown in Figure 5.30. The point labelled 1 in Figure 5.30 corresponds to the initial bridge tap location 400 m from the FDR device. The point labelled 2 corresponds to the harmonic of the 400 m peak. Point 3 corresponds to the second bridge tap at a location 1200 m from the device. Point 4 is the reflection off the 200 m open tap, for a total distance of 1400 m. Point 5 is the reflection off the 1200 m open tap, for a total distance of 1600 m.

The scale drawing of Figure 5.31 shows the complex loop configuration for the fifth entry of Table 5.3. The resulting estimation is shown in Figure 5.32. The point labelled 1 in Figure 5.32 corresponds to the initial bridge tap 400 m from the device. Point 2 corresponds to the second bridge tap located 800 m from the device. This peak will also include the second harmonic of the 400 m peak. The third point corresponds to the reflection off the open end of the second bridge tap, located 1000 m from the device. The fourth point corresponds to the reflection off the open end of the first bridge tap, located 1200 m from the device. The area indicated by point 5 includes multiple bounces off the bridge taps, at distances of 1400 m, 1600 m, and 1800 m. The sixth point corresponds to the second reflection off the 800 m bridge tap, a distance of 2000 m.

The scale drawing of Figure 5.33 shows the complex loop configuration for the last entry of Table 5.3. The resulting estimation is shown in Figure 5.34. The point labelled 1 in Figure 5.34 corresponds to the initial bridge tap 400 m from the device. Point 2 corresponds to the second bridge tap located 800 m from the device. The
Figure 5.29 Complex Loop, Double Bridge Tap Lengths 1200 m, 200 m

Figure 5.30 Complex Loop, Double Bridge Tap Lengths 1200 m, 200 m, Estimated Distance
Figure 5.31 Complex Loop, Double Bridge Tap Lengths 800 m, 200 m

Figure 5.32 Complex Loop, Double Bridge Tap Lengths 800 m, 200 m, Estimated Distance
Figure 5.33 Complex Loop, Double Bridge Tap Lengths 800 m, 300 m

Figure 5.34 Complex Loop, Double Bridge Tap Lengths 800 m, 300 m, Estimated Distance
third point corresponds to the reflection off the open end of the second bridge tap, located 1100 m from the device. The fourth point corresponds to the reflection off the open end of the first bridge tap, located 1200 m from the device.

As with the results presented in Section 5.2.1, the estimated distances all seem to be consistently high. Refer to Section 5.2.3 for possible explanations for this overestimation.

5.4 Competitor Comparisons

Many companies have developed and are currently selling DSL loop qualifiers to telephone companies to help them determine the location of loop impairments. These loop qualifiers all employ time domain reflectometry (TDR) to help determine location and type of loop impairment. Some manufacturers of these devices include Tempo [34] (Tektronix sold their copper cable test division to Tempo in 2001), Consultronics [35], and Fluke [36]. In the course of investigating these various companies, only Tempo (Tektronix) provided results of their equipment to potential customers. Though this limited any direct comparison to other equipment manufacturers, the results of other vendors should not be too far off those of Tempo (Tektronix). These results provided an excellent means of directly comparing the lab prototype with a commercially successful product.

The Tektronix web site did contain an excellent application note [37] on TDR and their corresponding product, the TS100. With Tektronix’s subsequent sale of its copper test division to Tempo, the application note can be found in a somewhat different form [38]. In this document they describe and show graphical results of their product under two line conditions.

The first line configuration consists of a 640 m long loop, unterminated, as depicted in Figure 5.35.

The second line configuration consists of a 640 m long loop, unterminated. There is a bridge tap of length 320 m located at the halfway point of the loop. This tap is
also unterminated. This is depicted in Figure 5.36.

Both of these loop configurations were set up in the lab, and the experimental device was used to measure the discontinuities. The results indicate the FDR device outperforms the TDR device.

![Image](image-url)

**Figure 5.35** Tektronix Example 640 Meter Open Circuit

![Image](image-url)

**Figure 5.36** Tektronix Example 640 Meter Open Circuit

Line, Bridge Tap at 320 Meters

Figure 5.37 shows a composite display of data obtained from the Tektronix website overlayed with results from the FDR device. The figure shows direct comparison of the simple open circuit shown in Figure 5.35. At the impairment location the FDR device generates a sharp peak to indicate the location. The TDR device generates an inflection to indicate the location. Both are adequate to show the location of the loop impairment.

Figure 5.38 also shows a composite display of data obtained from the Tektronix website overlayed with results from the FDR device. The figure shows direct comparison of the bridge tap in Figure 5.36. The bridge tap location is indicated by a sharp
Figure 5.37 630 Meter Open Circuit, TDR versus FDR Comparison (from Tektronix website)

Figure 5.38 630 Meter Open Circuit Line, Bridge Tap at 320 Meters, TDR versus FDR Comparison (from Tektronix website)
peak generated by the FDR device. By comparison, the TDR provides an inflection point at the impairment. Both are easily identifiable. A sharp peak is also visible as a result of the reflection off the open end of the bridge tap. By comparison another inflection point, though much more difficult to notice relative to the first inflection point, is displayed by the TDR device.

These results indicate both FDR and TDR are acceptable in detecting a single loop impairment, such as an improperly terminated loop. With a single bridge tap, both methods are acceptable in determining the location of the tap. However, FDR appears to have better success in indicating the length of that bridge tap.

5.5 Summary

In section 5.2.1 the FDR device was run through a series of simple loops to determine the range and accuracy of the unit. These tests also provided empirical evidence of the frequency ranges switch point. The results provide the following summary:

- range of ~ 2400 meters
- input frequency range breakpoint of ~ 800 m

In Section 5.3 the FDR device was run through a series of complex loop configurations, consisting of one or more bridged taps. The results were encouraging. Single bridge taps were easily seen, and two bridge taps could be made out with a little effort. The results provided the following summary:

- resolution of better than 100 meters
- single bridge taps easily discernable
- double bridge taps detectable

In Section 5.4 the FDR device was directly compared with a commercially available TDR device, the Tektronix TS100. Both a single loop and complex single bridge tap
loop were compared. The results also show some advantages of the FDR device over the TDR device:

- better visual indication of a single discontinuity
- better visual indication of a single bridge tap
6. Conclusions

6.1 Summary

The growing popularity of the Internet has brought about tremendous changes to the content it provides. Today, bandwidth hungry applications such as streaming multimedia, music files, and video conferencing are common to end users. The need for this ever increasing bandwidth has led to innovations to bring higher speed data transfer to the residential and small business consumer.

One means of providing high speed Internet access to the consumer is through the use of existing twisted pair telephone lines owned by telcos. There are, however, problems that need to be addressed before a successful high speed Internet access connection can be provided: locations of loading coils, bridge taps, and gauge changes all need to be known.

This thesis focused on the development and validation of a new technique for detecting the location of such impairments. The proposed measurement technique, called frequency domain reflectometry (FDR), was investigated as an alternative to other, more common detection methods. The FDR technique uses a varying frequency sinusoidal waveform injected into the telephone loop under test. The reflected waveform interferes with the transmitted waveform, the resultant waveform is captured, and it is processed to determine impairment location(s).

The project was focused into two major components: hardware and software. The hardware was designed and built to separate and store the magnitude of the interference signal. This was accomplished with a hybrid coupler designed for minimal reflections over a broad range of frequencies. A post amplifier helped amplify the received interference signal, and provided some filtering to the signal as well. A net-
work analyzer was used for both generating the swept input frequencies to the unit, and collecting and storing the magnitude of the interference signal.

The software was designed to compensate for known problems in the collection of data. It also performed some analysis on the collected data to determine loop impairment location(s). The software initially performed a baseline compensation on the collected data. This compensation technique took into account the "measured" interference signal when the line was of "infinite" length. This was subtracted from subsequent data measurements to provide the data. A warping factor was also developed and applied to the data to convert the independent axis from \( f_3 \) to \( t \). This eliminated the frequency dependence of the velocity of propagation - providing a more accurate Fast Fourier Transform (FFT). Fundamental and amplitude calculation algorithms were developed and applied to the collected data to detect the true FFT peak(s), and hence true distance(s) to the loop impairment.

Tests were performed on the FDR device using various loop configuration of 26 and 24 AWG twisted pair telephone wire. Planned and practical configurations were used to test the ability of the FDR device to detect loop impairments, and to test the accuracy of the device itself.

### 6.2 Results and Conclusions

The FDR device was initially tested to determine accuracy and attainable distance ranges. The overall range of the device was determined to be approximately 2400 m. A majority of customers are within this range from a jumper wire interface (JWI), making the range of the device acceptable for use in commercial applications.

The accuracy of the device was not as good as hoped. Using an input frequency range of 100 kHz to 1 MHz, the FDR device overestimated by approximately 170 m. As discussed in Chapter 5, the reasons for this overestimation are not precisely known. However, the overestimation seemed to be fairly constant, so it would be possible to introduce a compensation for this error to get more accurate results.
The FDR device was then tested on detecting multiple loop impairments. The results indicated multiple discontinuities are detectable, with discerning peaks at the locations of impairments. These results also indicated a resolution of better than 100 m.

The detection of loop impairments was compared with results of a device employing time domain reflectometry (TDR). The device was the TS100 manufactured by Tempo. The loop impairments used were provided by Tempo in an application note. The results indicated the FDR device performs at least as well as the commercially available product.

6.3 Future Work

The scope of the thesis was to prove the FDR technique for finding loop impairments was a viable alternative to the TDR technique. The results presented in this thesis indicate loop impairment detection is possible using FDR. However, with these results, no indication is made for whether or not the loop impairment(s) will affect the quality of Internet service. Future work in this area should include developing a database to cross reference results to quality of service. These results could be gathered from existing service records along with field analysis of the given lines.

The device in its current form is not a marketable product. The input frequency was generated from a network analyzer. The data collection and sampling also used the network analyzer. A network analyzer is too expensive and cumbersome to use in the field. Work needs to be done to integrate this functionality into a much smaller unit. Additionally, a DSP chip should be investigated to run the software algorithms within the unit. This will permit real time results, instead of collecting data in the field and analyzing it elsewhere on a PC.

Future work should also focus on improving the accuracy and resolution of the FDR device. To be a truly competitive product, both accuracy and resolution need to be improved upon. Additional hardware and/or software would be needed to achieve
this (through more sensitive measurements or better processing algorithms).

Applications of FDR need not be limited to qualifying twisted pair telephone loops. In fact, FDR is already used extensively in the characterization of optical loops. Other, more imaginative, applications could involve FDR as an ultrasound alternative or radar based alternative, applications that both use variations on TDR.
References


[34] “Cable Tester for Telephone Applications.”, Summary sheet on TelScout TS100 Time Domain Reflectometer, URL: http://www.tempocomm.com/tek_ts100.htm, June 27, 2002.


A. MATLAB Code

A.1 MATLAB Code For Calculating Hybrid Coupler Balance Impedance

```matlab
%******************************************************************************

% Z_CALCULATOR

%******************************************************************************

Written by: Terence Monteith.

Written for: TRLabs, for work on thesis project.

Purpose: This program will calculate R,C values for a three branch, series RC network.

Notes:

1. The program expects three files that contain frequency (freq.txt), real impedance (rl.txt), and imaginary impedance (im.txt). The data should be in a column vector (i.e. only one value per line).

2. The user specifies a high and low range for the R and C values, and the program will use them in its calculation algorithm.

3. The program is explicitly designed to run 3 branches only. Anything different will require modifications to the loops.

Revisions: Original code TWM 12/21/99

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The information contained herein is confidential property of TRLabs. The use, copying, transfer or disclosure of such information is prohibited except by express written agreement with TRLabs.

%******************************************************************************

% initialize parameters

% miscellaneous initializations

clear; % clear all variables
```
flops (0);       % clear flop counter
MSE = Inf;       % highest possible mean square error number

%****************************************************************************** load files ****************************
%**** inform user ****
disp('');
disp('');
disp('***************************************************************

disp('*

disp('* Z Calculator Program

disp('*

disp('*

disp('* Version 1.00

disp('*

disp('*

disp('* (C) 1999 by Terence Monteith. All Rights Reserved. *

disp('*

disp('***************************************************************

disp(' '); disp('Loading data files.....');

%**** load frequency file ****
load freq.txt;
F = freq';       % convert to row vector

%**** load real impedance file ****
load rl.txt;
R = rl';         % convert to row vector

%**** load imaginary impedance file ****
load im.txt;
I = im';         % convert to row vector

%**** convert to a complex impedance ****
Z = R + i*I;

%****************************************************************************** user data ****************************
%**** prompt user for component ranges ****
disp('');
R.Max.Abs = input('Enter highest expected resistance (in ohms): ');
R.Min.Abs = input('Enter lowest expected resistance (in ohms): ');
C.Max.Abs = input('Enter highest expected capacitance (in pF): ');
C.Min.Abs = input('Enter lowest expected capacitance (in pF): ');
disp(' ');}
%**** calculate rough step size ****
R.Step.Rough = (R.Max.Abs - R.Min.Abs)/4; % (1/4 of the total component range)
C.Step.Rough = (C.Max.Abs - C.Min.Abs)/4; % ...

%**** calculate coarse step size ****
R.Step.Coarse = R.Step.Rough/2; % (1/2 of rough step size)
C.Step.Coarse = C.Step.Rough/2; % ...

%**** calculate medium step size ****
R.Step.Medium = R.Step.Coarse/2; % (1/2 of coarse step size)
C.Step.Medium = C.Step.Coarse/2; % ...

%**** calculate fine step size ****
R.Step.Fine = R.Step.Medium/2; % (1/2 of fine step size)
C.Step.Fine = C.Step.Medium/2; % ...

%**** calculate smooth step size ****
R.Step.Smooth = R.Step.Fine/2; % (1/2 of smooth step size)
C.Step.Smooth = C.Step.Fine/2; % ...

%%%%%%%%%%%%%%%%%%%%% initialize branch impedance values %%%%%%%%%%%%%%%%%%%%
%**** initialize branch 1 parameters ****
R1 = 0;
R.Min1 = R.Min.Abs;
R.Max1 = R.Max.Abs;
C1 = 0;
C.Min1 = C.Min.Abs;
C.Max1 = C.Max.Abs;

%**** initialize branch 2 parameters ****
R2 = 0;
R.Min2 = R.Min.Abs;
R.Max2 = R.Max.Abs;
C2 = 0;
C.Min2 = C.Min.Abs;
C.Max2 = C.Max.Abs;

%**** initialize branch 3 parameters ****
R3 = 0;
R.Min3 = R.Min.Abs;
R.Max3 = R.Max.Abs;
C3 = 0;
C.Min3 = C.Min.Abs;
C.Max3 = C.Max.Abs; % initial maximum resistance of loop counter for C3

%****************************************************************************** rough component selection ****************************************
%**** user information ****
disp('Calculating ROUGH Values.... please be patient.....');

%**** loop the rough values to obtain estimate of R and C ****
    for c3 = C.Min3:C.Step.Rough:C.Max3
            for c2 = C.Min2:C.Step.Rough:C.Max2
                for r1 = R.Min1:R.Step.Rough:R.Max1
                    for c1 = C.Min1:C.Step.Rough:C.Max1
                        Zb1 = r1-i./(2*Pi*F*c1*le-12); % calculate branch impedance 1
                        Zb2 = r2-i./(2*Pi*F*c2*le-12); % calculate branch impedance 2
                        Zb3 = r3-i./(2*Pi*F*c3*le-12); % calculate branch impedance 3

                        Zt = (Zb1.*Zb2)./(Zb1+Zb2); % calculate parallel impedance
                        Zt = (Zt.*Zb3)./(Zt+Zb3); % ...
                        Error = log10(Z)-log10(Zt); % use logs to find error

                        if (mean(abs(Error)) <= MSE) % compare with current LMSE...
                            MSE = mean(abs(Error)); % ...new LMSE
                            R1 = r1; % ...new R1 value
                            C1 = c1; % ...new C1 value
                            R2 = r2; % ...new R2 value
                            C2 = c2; % ...new C2 value
                            R3 = r3; % ...new R3 value
                            C3 = c3; % ...new C3 value
                        end
                    end
                end
            end
        end
    end
end

%****************************************************************************** coarse component selection ****************************************
%**** user information ****
disp('Calculating COARSE Values.... please be patient.....');

%**** calculate the maximum and minimum R,C values ****
R\_Max1 = R1 + R\_Step\_Rough; % maximum R1 value
if (R\_Max1 > R\_Max\_Abs)
    R\_Max1 = R\_Max\_Abs;
end

R\_Min1 = R1 - R\_Step\_Rough; % minimum R1 value
if (R\_Min1 < R\_Min\_Abs)
    R\_Min1 = R\_Min\_Abs;
end

C\_Max1 = C1 + C\_Step\_Rough; % maximum C1 value
if (C\_Max1 > C\_Max\_Abs)
    C\_Max1 = C\_Max\_Abs;
end

C\_Min1 = C1 - C\_Step\_Rough; % minimum C1 value
if (C\_Min1 < C\_Min\_Abs)
    C\_Min1 = C\_Min\_Abs;
end

R\_Max2 = R2 + R\_Step\_Rough; % maximum R2 value
if (R\_Max2 > R\_Max\_Abs)
    R\_Max2 = R\_Max\_Abs;
end

R\_Min2 = R2 - R\_Step\_Rough; % minimum R2 value
if (R\_Min2 < R\_Min\_Abs)
    R\_Min2 = R\_Min\_Abs;
end

C\_Max2 = C2 + C\_Step\_Rough; % maximum C2 value
if (C\_Max2 > C\_Max\_Abs)
    C\_Max2 = C\_Max\_Abs;
end

C\_Min2 = C2 - C\_Step\_Rough; % minimum C2 value
if (C\_Min2 < C\_Min\_Abs)
    C\_Min2 = C\_Min\_Abs;
end

R\_Max3 = R3 + R\_Step\_Rough; % maximum R3 value
if (R\_Max3 > R\_Max\_Abs)
    R\_Max3 = R\_Max\_Abs;
end
R._Min3 = R3 - R.Step.Rough; % minimum R3 value
if (R._Min3 < R.Min.Abs)
    R._Min3 = R.Min.Abs;
end

C._Max3 = C3 + C.Step.Rough; % maximum C3 value
if (C._Max3 > C.Max.Abs)
    C._Max3 = C.Max.Abs;
end

C._Min3 = C3 - C.Step.Rough; % minimum C3 value
if (C._Min3 < C.Min.Abs)
    C._Min3 = C.Min.Abs;
end

%***** loop the coarse values to obtain estimate of R and C *****
    for c3 = C._Min3:C.Step.Coarse:C._Max3
                for r1 = R._Min1:R.Step.Coarse:R._Max1
                    for c1 = C._Min1:C.Step.Coarse:C._Max1
                        Zb1 = r1-i./(2*pi*F*c1*1e-12); % calculate branch impedance 1
                        Zb2 = r2-i./(2*pi*F*c2*1e-12); % calculate branch impedance 2
                        Zb3 = r3-i./(2*pi*F*c3*1e-12); % calculate branch impedance 3
                        Zt = (Zb1.*Zb2)./(Zb1+Zb2); % calculate parallel impedance
                        Zt = (Zt.*Zb3)./(Zt+Zb3); % ...
                        Error = log10(Z)-log10(Zt); % use logs to find error
                        if (mean(abs(Error)) <= MSE) % compare with current LMSE...
                            MSE = mean(abs(Error)); % ...new LMSE
                            R1 = r1; % ...new R1 value
                            C1 = c1; % ...new C1 value
                            R2 = r2; % ...new R2 value
                            C2 = c2; % ...new C2 value
                            R3 = r3; % ...new R3 value
                            C3 = c3; % ...new C3 value
                        end
                    end
                end
            end
        end
    end
end
end
%****************************** medium component selection ******************************

%**** user information ****

disp('Calculating MEDIUM Values..... please be patient.....');

%**** calculate the maximum and minimum R,C values ****
R.Max1 = R1 + R.Step_Coarse; % maximum R1 value
if (R.Max1 > R.Max_Abs)
    R.Max1 = R.Max_Abs;
end

R.Min1 = R1 - R.Step_Coarse; % minimum R1 value
if (R.Min1 < R.Min_Abs)
    R.Min1 = R.Min_Abs;
end

C.Max1 = C1 + C.Step_Coarse; % maximum C1 value
if (C.Max1 > C.Max_Abs)
    C.Max1 = C.Max_Abs;
end

C.Min1 = C1 - C.Step_Coarse; % minimum C1 value
if (C.Min1 < C.Min_Abs)
    C.Min1 = C.Min_Abs;
end

R.Max2 = R2 + R.Step_Coarse; % maximum R2 value
if (R.Max2 > R.Max_Abs)
    R.Max2 = R.Max_Abs;
end

R.Min2 = R2 - R.Step_Coarse; % minimum R2 value
if (R.Min2 < R.Min_Abs)
    R.Min2 = R.Min_Abs;
end

C.Max2 = C2 + C.Step_Coarse; % maximum C2 value
if (C.Max2 > C.Max_Abs)
    C.Max2 = C.Max_Abs;
end

C.Min2 = C2 - C.Step_Coarse; % minimum C2 value
if (C.Min2 < C.Min.Abs)
    C.Min2 = C.Min.Abs;
end

R.Max3 = R3 + R.Step.Coarse; % maximum R3 value
if (R.Max3 > R.Max.Abs)
    R.Max3 = R.Max.Abs;
end

R.Min3 = R3 - R.Step.Coarse; % minimum R3 value
if (R.Min3 < R.Min.Abs)
    R.Min3 = R.Min.Abs;
end

C.Max3 = C3 + C.Step.Coarse; % maximum C3 value
if (C.Max3 > C.Max.Abs)
    C.Max3 = C.Max.Abs;
end

C.Min3 = C3 - C.Step.Coarse; % minimum C3 value
if (C.Min3 < C.Min.Abs)
    C.Min3 = C.Min.Abs;
end

%*** loop the medium values to obtain estimate of R and C ***
    for c3 = C.Min3:C.Step.Medium:C.Max3
            for c2 = C.Min2:C.Step.Medium:C.Max2
                for r1 = R.Min1:R.Step.Medium:R.Max1
                    for c1 = C.Min1:C.Step.Medium:C.Max1
                        Zb1 = r1−i./(2*π*F*c1*1e−12); % calculate branch impedance 1
                        Zb2 = r2−i./(2*π*F*c2*1e−12); % calculate branch impedance 2
                        Zb3 = r3−i./(2*π*F*c3*1e−12); % calculate branch impedance 3

                        Zt = (Zb1.*Zb2)./(Zb1+Zb2); % calculate parallel impedance
                        Zt = (Zt.*Zb3)./(Zt+Zb3); % ...

                        Error = log10(Zt)−log10(Zt); % use logs to find error

                        if (mean(abs(Error)) <= MSE) % compare with current LMSE...
                            MSE = mean(abs(Error)); % ...new LMSE
                            R1 = r1; % ...new R1 value
                            C1 = c1; % ...new C1 value
                        end

                    end
                end
            end
        end
    end
end

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R2 = r2;  % ...new R2 value
C2 = c2;  % ...new C2 value
R3 = r3;  % ...new R3 value
C3 = c3;  % ...new C3 value
end
end
end
end

calculating FINE values.... please be patient.......

disp('Calculating FINE Values..... please be patient.....');

%**** calculate the maximum and minimum R,C values ****
R_Mx1 = R1 + R_Step.Medium;  % maximum R1 value
if (R_Mx1 > R_Mx.Abs)
   R_Mx1 = R_Mx.Abs;
end
R_Min1 = R1 - R_Step.Medium;  % minimum R1 value
if (R_Min1 < R_Min.Abs)
   R_Min1 = R_Min.Abs;
end
C_Mx1 = C1 + C_Step.Medium;  % maximum C1 value
if (C_Mx1 > C_Mx.Abs)
   C_Mx1 = C_Mx.Abs;
end
C_Min1 = C1 - C_Step.Medium;  % minimum C1 value
if (C_Min1 < C_Min.Abs)
   C_Min1 = C_Min.Abs;
end
R_Mx2 = R2 + R_Step.Medium;  % maximum R2 value
if (R_Mx2 > R_Mx.Abs)
   R_Mx2 = R_Mx.Abs;
end
R_Min2 = R2 - R_Step.Medium;  % minimum R2 value
if (R_Min2 < R_Min.Abs)
\[ R_{\text{Min}2} = R_{\text{Min.Abs}}; \]

\begin{verbatim}
end

C_{\text{Max}2} = C2 + C._{\text{Step.Medium}}; \text{ maximum } C2 \text{ value}
if (C_{\text{Max}2} > C_{\text{Max.Abs}})
    C_{\text{Max}2} = C_{\text{Max.Abs}};
end

C_{\text{Min}2} = C2 - C._{\text{Step.Medium}}; \text{ minimum } C2 \text{ value}
if (C_{\text{Min}2} < C_{\text{Min.Abs}})
    C_{\text{Min}2} = C_{\text{Min.Abs}};
end

R_{\text{Max}3} = R3 + R._{\text{Step.Medium}}; \text{ maximum } R3 \text{ value}
if (R_{\text{Max}3} > R_{\text{Max.Abs}})
    R_{\text{Max}3} = R_{\text{Max.Abs}};
end

R_{\text{Min}3} = R3 - R._{\text{Step.Medium}}; \text{ minimum } R3 \text{ value}
if (R_{\text{Min}3} < R_{\text{Min.Abs}})
    R_{\text{Min}3} = R_{\text{Min.Abs}};
end

C_{\text{Max}3} = C3 + C._{\text{Step.Medium}}; \text{ maximum } C3 \text{ value}
if (C_{\text{Max}3} > C_{\text{Max.Abs}})
    C_{\text{Max}3} = C_{\text{Max.Abs}};
end

C_{\text{Min}3} = C3 - C._{\text{Step.Medium}}; \text{ minimum } C3 \text{ value}
if (C_{\text{Min}3} < C_{\text{Min.Abs}})
    C_{\text{Min}3} = C_{\text{Min.Abs}};
end

%**** loop the fine values to obtain estimate of R and C ****
for r3 = R_{\text{Min3}}:R._{\text{Step.Fine}}:R_{\text{Max3}}
    for c3 = C_{\text{Min3}}:C._{\text{Step.Fine}}:C_{\text{Max3}}
        for r2 = R_{\text{Min2}}:R._{\text{Step.Fine}}:R_{\text{Max2}}
            for c2 = C_{\text{Min2}}:C._{\text{Step.Fine}}:C_{\text{Max2}}
                for r1 = R_{\text{Min1}}:R._{\text{Step.Fine}}:R_{\text{Max1}}
                    for c1 = C_{\text{Min1}}:C._{\text{Step.Fine}}:C_{\text{Max1}}
                        Zb1 = r1\cdot i/(2\pi F* c1*1e-12); \text{ calculate branch impedance 1}
                        Zb2 = r2\cdot i/(2\pi F* c2*1e-12); \text{ calculate branch impedance 2}
                        Zb3 = r3\cdot i/(2\pi F* c3*1e-12); \text{ calculate branch impedance 3}
                    end
                end
            end
        end
    end
end
\end{verbatim}
\[ Z_t = \frac{Z_{bb}Z_{b2}}{Z_{bl} + Z_{b2}}; \]  % calculate parallel impedance
\[ Z_t = \frac{Z_t \cdot Z_{b3}}{Z_t + Z_{b3}}; \]  % ...

Error = \log_{10}(Z) - \log_{10}(Z_t);  % use logs to find error

if (mean(abs(Error)) \leq \text{MSE})  % compare with current LMSE...
    \text{MSE} = \text{mean}(\text{abs}(\text{Error}));  % new LMSE
    R1 = r1;  % ...new R1 value
    C1 = c1;  % ...new C1 value
    R2 = r2;  % ...new R2 value
    C2 = c2;  % ...new C2 value
    R3 = r3;  % ...new R3 value
    C3 = c3;  % ...new C3 value
end
end
end
end
end

%****************************************************************************** smooth component selection ******************************************************************************
%*** user information ***
disp('Calculating SMOOTH Values..... please be patient.....');

%**** calculate the maximum and minimum R,C values ****
R\_Maxl = Rl + \text{R\_Step\_Fine};  % maximum R1 value
if (R\_Maxl > R\_Max\_Abs)
    R\_Maxl = R\_Max\_Abs;
end

R\_Minl = Rl - \text{R\_Step\_Fine};  % minimum R1 value
if (R\_Minl < R\_Min\_Abs)
    R\_Minl = R\_Min\_Abs;
end

C\_Maxl = C1 + \text{C\_Step\_Fine};  % maximum C1 value
if (C\_Maxl > C\_Max\_Abs)
    C\_Maxl = C\_Max\_Abs;
end

C\_Minl = C1 - \text{C\_Step\_Fine};  % minimum C1 value
if (C\_Minl < C\_Min\_Abs)
    C\_Minl = C\_Min\_Abs;

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R_{\text{Max}2} = R_2 + \text{R.\text{Step.Fine}}; \ % \ maximum \ R_2 \ value
if \ (R_{\text{Max}2} > R_{\text{Max.Abs}})
\quad R_{\text{Max}2} = R_{\text{Max.Abs}};
end

R_{\text{Min}2} = R_2 - \text{R.\text{Step.Fine}}; \ % \ minimum \ R_2 \ value
if \ (R_{\text{Min}2} < R_{\text{Min.Abs}})
\quad R_{\text{Min}2} = R_{\text{Min.Abs}};
end

C_{\text{Max}2} = C_2 + \text{C.\text{Step.Fine}}; \ % \ maximum \ C_2 \ value
if \ (C_{\text{Max}2} > C_{\text{Max.Abs}})
\quad C_{\text{Max}2} = C_{\text{Max.Abs}};
end

C_{\text{Min}2} = C_2 - \text{C.\text{Step.Fine}}; \ % \ minimum \ C_2 \ value
if \ (C_{\text{Min}2} < C_{\text{Min.Abs}})
\quad C_{\text{Min}2} = C_{\text{Min.Abs}};
end

R_{\text{Max}3} = R_3 + \text{R.\text{Step.Fine}}; \ % \ maximum \ R_3 \ value
if \ (R_{\text{Max}3} > R_{\text{Max.Abs}})
\quad R_{\text{Max}3} = R_{\text{Max.Abs}};
end

R_{\text{Min}3} = R_3 - \text{R.\text{Step.Fine}}; \ % \ minimum \ R_3 \ value
if \ (R_{\text{Min}3} < R_{\text{Min.Abs}})
\quad R_{\text{Min}3} = R_{\text{Min.Abs}};
end

C_{\text{Max}3} = C_3 + \text{C.\text{Step.Fine}}; \ % \ maximum \ C_3 \ value
if \ (C_{\text{Max}3} > C_{\text{Max.Abs}})
\quad C_{\text{Max}3} = C_{\text{Max.Abs}};
end

C_{\text{Min}3} = C_3 - \text{C.\text{Step.Fine}}; \ % \ minimum \ C_3 \ value
if \ (C_{\text{Min}3} < C_{\text{Min.Abs}})
\quad C_{\text{Min}3} = C_{\text{Min.Abs}};
end

%**** \ loop \ the \ smooth \ values \ to \ obtain \ estimate \ of \ R \ and \ C ****
for \ r_3 = R_{\text{Min}3}; \text{R.\text{Step.Smooth}}; R_{\text{Max}3}
for c3 = C.Min3:C.Step_Smooth:C.Max3  
  for r2 = R.Min2:R.Step_Smooth:R.Max2  
    for c2 = C.Min2:C.Step_Smooth:C.Max2  
      for r1 = R.Min1:R.Step_Smooth:R.Max1  
        for c1 = C.Min1:C.Step_Smooth:C.Max1  
          
          Zb1 = r1-i./(2*pi*F*ch1*1e-12); % calculate branch impedance 1
          Zb2 = r2-i./(2*pi*F*C2*1e-12); % calculate branch impedance 2
          Zb3 = r3-i./(2*pi*F*C3*1e-12); % calculate branch impedance 3

          Zt = (Zb1.*Zb2)./(Zb1+Zb2); % calculate parallel impedance
          Zt = (Zt.*Zb3)./(Zt+Zb3); % ...

          Error = log10(Z)-log10(Zt); % use logs to find error

          if (mean(abs(Error)) <= MSE) % compare with current LMSE...
            MSE = mean(abs(Error)); % ... new LMSE
            R1 = r1; % ... new R1 value
            C1 = c1; % ... new C1 value
            R2 = r2; % ... new R2 value
            C2 = c2; % ... new C2 value
            R3 = r3; % ... new R3 value
            C3 = c3; % ... new C3 value

          end  
        end  
      end  
    end  
  end  
end

%****************************** display calculated R and C value to user *********************
%**** display blank line ****
disp(' '); 
%**** display branch 1 ****
Temp.Text = 'The calculated effective resistance R1 is '; 
Temp.Text = [Temp.Text, num2str(R1)]; 
Temp.Text = [Temp.Text, ' ohms. '];
disp(Temp.Text);

Temp.Text = 'The calculated effective capacitance C1 is '; 
Temp.Text = [Temp.Text, num2str(C1)]; 
disp(Temp.Text);
%**** display branch 2 ****
Temp.Text = 'The calculated effective resistance R2 is ';
Temp.Text = [Temp.Text, num2str(R2)];
Temp.Text = [Temp.Text, ' ohms. '];
disp(Temp.Text);

Temp.Text = 'The calculated effective capacitance C2 is ';
Temp.Text = [Temp.Text, num2str(C2)];
disp(Temp.Text);

%**** display branch 3 ****
Temp.Text = 'The calculated effective resistance R3 is ';
Temp.Text = [Temp.Text, num2str(R3)];
Temp.Text = [Temp.Text, ' ohms. '];
disp(Temp.Text);

Temp.Text = 'The calculated effective capacitance C3 is ';
Temp.Text = [Temp.Text, num2str(C3)];
disp(Temp.Text);

%**** display error ****
disp(' ');
Temp.Text = 'The MSE: ';
Temp.Text = [Temp.Text, num2str(MSE)];
if (MSE < 0.005)
    Temp.Text = [Temp.Text, ' (!!!HIGH!!! Degree of Confidence in Calculated Values) '];
disp(Temp.Text);
elseif (MSE < 0.02)
    Temp.Text = [Temp.Text, ' (!!!MEDIUM!! Degree of Confidence in Calculated Values) '];
disp(Temp.Text);
    disp('Run again with the values above as basis for new threshold values.');
else
    Temp.Text = [Temp.Text, ' (!!!LOW!!! Degree of Confidence in Calculated Values) '];
disp(Temp.Text);
    disp('Run again with the values above as basis for new threshold values. ');
end

%**** display flops used ****
Temp.Text = 'The number of flops used: ';
Temp.Text = [Temp.Text, num2str(flops)];
disp(Temp.Text);
A.2 MATLAB Code For Main Software Algorithm

```
clear;

load X1.TXT -ascii;  % load the independent axis of collected data
load Data1.TXT -ascii;  % load collected data
load Base1.TXT -ascii;  % load collected baseline

x_axis = X1';  % set up x-axis vector
y_axis = Data1';  % set up y-axis vector

```

Written by: Terence Monteith.

Written for: TRLabs, for work on thesis project.

Purpose: This program serves as the main routine that controls the various functions that provide compensation and analysis of the collected data.

Notes:
1. The independent axis of collected data should be called X.TXT (case sensitive)
2. The collected data should be called DATA.TXT (case sensitive)
3. The collected baseline data should be called BASELINE.TXT (case sensitive)
4. This program uses Figure 1 for original data, Figure 2 for warped data, Figure 3 for re-sampled data

Revisions:
00 - Original code TWM 03/21/00.
01 - accomodate new ANALYSIS.m file by adding varaible for returned amplitude TWM 03/23/00.
02 - accomodate new ANALYSIS.m file by adding parameter to indicate which analysis range (0.1 to 1 MHz or 1 to 5 MHz) is being used.

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```
base_comp = Basel';  % set up baseline compensation vector

%****************************** calculate and plot original data ***************************
%**** perform baseline compensation ****
y_axis = y_axis - base_comp;

y_axis_2 = (y_axis/max(abs(y_axis)))';

save results3 y_axis_2 / ascii;

for i = (401+1): (2*401)
x_axis(i) = x_axis(401) + (i-401)*(x_axis(2)-x_axis(1));
y_axis(i) = 0;
end

figure (10);
subplot(212);
plot(x_axis,y_axis);
axis([0 15e5 -0.025 0.025]);
xlabel('Frequency');
ylabel('Voltage');

load Testx.TXT -ascii;
load Testdata.TXT -ascii;
load Testbase.TXT -ascii;

x_axis = Testx';  % load the independent axis of collected data
y_axis = Testdata';  % load collected data
base_comp = Testbase';  % load collected baseline

x_axis = Testx';  % set up x-axis vector
y_axis = Testdata';  % set up y-axis vector
base_comp = Testbase';  % set up baseline compensation vector

y_axis = y_axis - base_comp;

y_axis_2 = (y_axis/max(abs(y_axis)))';

save results3 y_axis_2 / ascii;

for i = (401+1): (2*401)
x_axis(i) = x_axis(401) + (i-401)*(x_axis(2)-x_axis(1));
y_axis(i) = 0;
end

subplot(211);
plot(x_axis,y_axis);
axis([0 7e5 -4e-4 4e-4]);
```
xlabel('Frequency');
ylabel('Voltage');

%***** plot data ****
figure(1);
plot(x_axis, y_axis);
title('Plot of Original Data, Baseline Compensated');
xlabel('Frequency (Hz)');
ylabel('Voltage (V)');
axis([0,1.5e6, -.025, .025]);

%********************** warp axis and plot results **********************
%**** call warp function ****
x_axis = warp(x_axis,24);

%***** plot data ****
figure(2);
plot(x_axis, y_axis);
grid on;
zoom on;
hold on;
title('Plot of Original Data, Baseline Compensated, Warping Applied');
xlabel('Lambda--1 (m--1)');
ylabel('Voltage (V)');

%****************************** re-sample data and plot results ******************************
%**** call re-sample function ****
[x_axis, y_axis] = resample1(x_axis, y_axis);

%***** plot data ****
figure(3);
plot(x_axis, y_axis);
grid on;
zoom on;
title('Plot of Original Data, Baseline Compensated, Warping Applied, Resampled');
xlabel('Lambda--1 (m--1)');
ylabel('Voltage (V)');

%****************************** analyse results ******************************************
[distance, magnitude] = analysis(x_axis, y_axis,'NON',1);

%***** display results ****
% fancy header
% distance to fault
[temp.text, 'Estimated distance to fault: '];
[temp.text, num2str(round(distance))];
[temp.text, ' m. '];
disp(temp.text);

% magnitude of FFT
[temp.text, 'Estimated spectral magnitude: '];
[temp.text, num2str(magnitude)];
[temp.text, ' units. '];
disp(temp.text);
function x_axis = warp(frequency, gauge);

%s---------- initialize parameters ----------------------------

% initialize primary constants ****
if (gauge == 26)
    % unit resistance (ohms/kft)
    r = 82.490828 - 0.00083896972.*frequency + 7.7360828e-5.*frequency.*log(frequency) ...
end
- 7.2662322e-10.*(frequency.^2) + 4.3271278e-11.*(frequency.^2).*log(frequency);

% unit inductance (H/kft)
l = 1e-3*(0.13560646 + 0.0045586458./(1 + 0.0045586458.*0.029993745.*frequency) ...
+ 0.042974221./(1 + 0.042974221.*2.4978083e-5.*frequency));

% unit conductance (S/kft)
g = 1e-6*(0.056975378 + 3.9604488e-5.*frequency - 9.7592923e-20.*(frequency.^3) ...
    - 0.0064187937.*log(frequency) - 0.017247963./frequency);

elseif (gauge == 24)

% unit resistance (ohms/kft)
r = 51.454871 - 0.00081480804.*frequency + 8.4036765e-5.*frequency.*log(frequency) ...
    - 2.9944726e-7.*(frequency.^1.5) + 4.7368441e-11.*(frequency.^2);

% unit inductance (H/kft)
l = 1e-3*(0.13270211 + 0.0034384324./(1+0.0034384324.*0.033834621.*frequency) ...
+ 0.044967001./(1+0.044967001.*3.1971495e-5.*frequency));

% unit conductance (S/kft)
g = 1e-6*(0.056975378 + 3.9604488e-5.*frequency - 9.7592923e-20.*(frequency.^3) ...
    - 0.0064187937.*log(frequency) - 0.017247963./frequency);

else

dispstring = 'Invalid gauge type ';
dispstring = [dispstring , num2str(gauge)];
dispstring = [dispstring , '. Use 24 or 26. Returned result is invalid. '];
disp(dispstring);
end

%**** initialize radian frequency ****
w = 2*pi*frequency;

%******************************************************************************* calculate warping factor ********************************************************
%**** calculate beta from primary constants (will have value in 1/kft) ****
term1 = r.^2 + (w.^2).*l.^2;
term2 = g.^2 + (w.^2).*((15.7e-9).^2);
term3 = sqrt(term1.*term2);

beta = sqrt(0.5*(term3 - r.*g + (w.^2).*l.*15.7e-9));

%**** convert to 1/m ****
beta = beta/304.8; % 304.8 m in 1 kft

%**** warp factor ****
warpfactor = beta./w;  % see pg 14 of log book #1 for derivation

%***************************************************************************** return warped data ****************************
x_axis = frequency.*warpfactor;


A.4 MATLAB Code For Resampling Data Function

function [x..new,y..new] = resample(x,y);

%******************************************************************************************
% Written by: Terence Monteith.
% Written for: TRLabs, for work on thesis project.
% Purpose: This function file will "resample" a set of data points that have an
independent axis consisting of irregularly spaced sample points. This will
be accomplished by computing new independent axis points (evenly spaced),
and linearly interpolating the data at those new points.
% Notes: 1. The input arguments x and y should be row vectors.
% 2. The lengths of inputs x and y should be the same.
% 3. The resample size is defined to be the smallest distance between two
points on the input independent axis, in this case the last two points
of the vector.
% 4. The returned vectors will be the same length as original input vectors.
% Revisions: 00 - Original code TWM 02/21/00.
% Copyright: Copyright 2000 TRLabs. All Rights Reserved.
% The information contained herein is confidential property of TRLabs. The
% use, copying, transfer or disclosure of such information is prohibited
% except by express written agreement with TRLabs.
%******************************************************************************************

%************************************************************** initialize parameters **************
N = length(x); % number of points in vectors
sample.size = x(length(x))-x(length(x)-1); % resample step size
points = floor ((max(x)-min(x))/sample.size); % number of points in new data sets
overlap = 0; % for overlap explanation see pg 129 of ...
% ... log book #1

%************************************************************** calculate new points **************


%**** set initial points ****
y_axis(1) = y(1);
x_axis(1) = x(1);

%**** loop for remaining points ****
for i = 2:points
    % calculate new independent axis points
    x_axis(i) = x_axis(1) + sample_size*(i-1);

    % determine if overlap occurred
    if (x_axis(i) < x(i-1-overlap))
        overlap = overlap+1; % overlap occurred, increment counter
    end

    % use linear interpolation to find new dependent axis points
    y_axis(i) = ((y(i-1-overlap)-y(i-1-overlap))*(x_axis(i)-x(i-1-overlap)))/(x(i-overlap)-...;
                 x(i-1-overlap)))+y(i-1-overlap);
end

%**** truncate end points ****
y_axis = y_axis(1:N);
x_axis = x_axis(1:N);

%****************************************************************************** return new vectors **************************************************************************
x_new = x_axis;
y_new = y_axis;
A.5 MATLAB Code For Analyzing Data Function

%*******************************************************************************
% % ANALYSIS
% %
% % Written by: Terence Monteith.
% %
% % Written for: TRLabs, for work on thesis project.
% %
% % Purpose: This function file will analyze a set of data points representing the
% inverse wavelength (independent axis) and the voltage (dependent axis). A
plot will be generated of the FFT response, as will significant distances
calculated by an algorithm to detect the peaks (discontinuities) of the
plot.
%
% % Notes: 1. The input arguments x and y should be row vectors.
% 2. The window input should be one of the following:
%   BAR - Bartlett window
%   BLA - Blackman window
%   HAM - Hamming window
%   HAN - Hanning window
%   KAI - Kaiser window
%   NON - no windowing
%   Anything else will default to no windowing.
% 3. Due to FFT properties, the length of data vectors resulting from FFT
will be reduced by approximately half the length of input vectors.
% 4. The distance calculating algorithm has been written for a straight line
single discontinuity problem.
% 5. The plot will be displayed as Figure 100.
%
% % Revisions: 00 - Original code TWM 02/21/00.
% 01 - Normalized FFT to largest component TWM 03/21/00.
% 02 - Removed FFT normalization TWM 03/23/00.
%   - added simple analysis algorithm for straight cable, single
% discontinuity TWM 03/23/00.
%   - added a returned parameter amplitude TWM 03/23/00.
% 03 - added a deharmonize section to remove harmonic content from the FFT
graph TWM 03/28/00.
%
% % Copyright: Copyright 2000 TRLabs. All Rights Reserved.
% %
% The information contained herein is confidential property of TRLabs. The
use, copying, transfer or disclosure of such information is prohibited
function [distance, magnitude] = analysis(x, y, window, range);

%*****************************************************************************
% initialize parameters*****************************************************************************
N = length(x); % number of points in vectors
sample_size = x(2) - x(1); % step size in pre-FFT domain

%*****************************************************************************
% window data*****************************************************************************
if (window == 'BAR')
    w = bartlett(N);
    y = y.*w';
elseif (window == 'BLA')
    w = blackman(N);
    y = y.*w';
elseif (window == 'HAM')
    w = hamming(N);
    y = y.*w';
elseif (window == 'HAN')
    w = hanning(N);
    y = y.*w';
elseif (window == 'KAI')
    w = kaiser(N);
    y = y.*w';
end

%*****************************************************************************
% set up FFT results*****************************************************************************
%**** calculate FFT ****
Y = fft(y);
Y = Y/N; % normalize FFT results
Y = Y(1:round(N/2)); % use only half of data due to mirroring
Y = Y/max(abs(Y));

B = fir1(5, 0.35);
A = 1;
Y = filter(B, A, Y);
Y = Y/max(abs(Y));
%***** calculate independent axis *****
D = 0:1:(round(N/2)-1); % same number of points as FFT
D = D/(2*N*sample.size); % normalize independent axis to show distance TO fault...
% ... not total distance travelled.

YY = abs(Y)';

%*************************************************************************** plot results ***************************************************************************
figure(100);
plot(D,abs(YY),'-b'); % use a stem plot instead of continuous
hold on;
grid on;
zoom on;
title('Distance to Fault Measurement');
xlabel('Distance (m)');
ylabel('Normalized FFT Magnitude');
axis([0 3000 0 max(abs(Y))]);

%*************************************************************************** return distance/amplitude calculations ***************************************************************************
%***** !!!!!!!! NOTE: SIMPLE STRAIGHT LINE ALGORITHM!!!!!!!! *****

%*************************************************************************** determine position and value for peak spike ***************************************************************************
Y(1) = 0; % not concerned with 0 distance component
Y(2) = 0;
[S,I] = sort(abs(Y)); % need to use index I
p1 = I(length(I)); % position of spike

%*************************************************************************** interpolate distance ***************************************************************************
%***** determine next largest peak *****
if (abs(Y(p1-1)) > abs(Y(p1+1)))
    % left peak is next
    p2 = p1; % leftmost point is p1, according to algorithm
    p1 = p2-1; % ...
else
    % right peak is next
    p2 = p1+1; % leftmost point is still p1
end

%***** generate needed parameters *****
alpha = abs(Y(p2))/abs(Y(p1)); % alpha
delta = alpha/(1+alpha); % delta
%**** interpolate distance ****
distance = (p1+delta-1)*1/(2*N*sample_size); % divide by 2 here because distance to...
% ... fault is half distance travelled.

%**************************** interpolate magnitude ****************************
%**** interpolate magnitude ****
magnitude = abs(Y(p1))*pi*delta)/abs(sin(pi*delta));
B. Warping Factor Equation Derivation

Equation 4.6 provides the velocity of propagation for a wave travelling in a transmission line. It is repeated here for convenience:

\[ \nu = \frac{2\pi f}{\beta} \]  \hspace{1cm} (B.1)

In section 4.1.3 it was stated that this velocity of propagation is not constant over frequency. Thus, to use a frequency variant source to locate discontinuities, the effects of this non constant velocity of propagation must be accounted for. The effect of changing the collected data to account for this variance in wave velocity is referred to as “warping”.

The derivation of the “warping” factor described in Equation 4.8 is now presented:

\[ \nu = \frac{2\pi f}{\beta} \]  \hspace{1cm} (B.2)

\[ = \lambda f \]  \hspace{1cm} (B.3)

Also,

\[ \frac{1}{\lambda} = \frac{\beta}{2\pi} \]  \hspace{1cm} (B.4)

\[ \frac{f}{\nu} \]  \hspace{1cm} (B.5)

Since the impedance equation is periodic with \( \beta x \), and \( x \) will be a constant (since the distance to the discontinuity is fixed), then the impedance equation must be periodic with \( \beta \), and hence periodic with \( \frac{1}{\lambda} \).

Therefore, to warp the frequency axis, the frequency must be divided by the velocity of propagation of the wave at that frequency.
C. Error Calculation For DFT Peak Amplitude Estimation

The authors of the peak estimation algorithm [33] have used some approximations in their derivation of the location of the peak amplitude. To make these approximations some assumptions were made as to the number of data points needed to obtain accurate results. The following proof shows the original authors have greatly overstated their assumption. A much smaller number of data points can be used to still obtain the error the authors feel is adequate for use.

According to [33], the largest two spectral components of a signal are given by:

\[
S(l) = -j0.5A_1e^{j(a\delta+\phi_1)}\frac{\sin(\pi\delta)}{\sin(\frac{\pi\delta}{N})}
\]  \hspace{1cm} (C.1)

\[
S(l) = -j0.5A_1e^{j(a(\delta-1)+\phi_1)}\frac{\sin(\pi(\delta - 1))}{\sin(\frac{\pi(\delta-1)}{N})}
\]  \hspace{1cm} (C.2)

where \(A_1\) is the amplitude of the fundamental frequency, \(\phi_1\) is the phase of the fundamental frequency, \(\delta\) refers to the fractional portion of the bin the fundamental resides in \((0 \leq \delta < 1)\), and \(a = \frac{\pi(N-1)}{N}\).

The assumption made by the authors is the sine term of the denominators of Equations C.1 and C.2 can be replaced by their respective arguments. They claim for \(N = 1024\) this will result in an error of no more than 0.015%.

Table C.1 shows the error from replacing the sine term with its argument, for \(N = 1024\) samples. Table C.2 shows the error from replacing the sine term with its argument, for \(N = 401\) samples.
**Table C.1** Error Resulting in Replacement of Sine Term with Sine Argument, $N = 1024$

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\sin\left(\frac{\pi \delta}{N}\right)$</th>
<th>$\frac{\pi \delta}{N}$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0000000000</td>
<td>0.0000000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.1</td>
<td>0.000306796</td>
<td>0.000306796</td>
<td>0.000002</td>
</tr>
<tr>
<td>0.2</td>
<td>0.000613592</td>
<td>0.000613592</td>
<td>0.000006</td>
</tr>
<tr>
<td>0.3</td>
<td>0.000920388</td>
<td>0.000920389</td>
<td>0.000014</td>
</tr>
<tr>
<td>0.4</td>
<td>0.001227184</td>
<td>0.001227185</td>
<td>0.000025</td>
</tr>
<tr>
<td>0.5</td>
<td>0.001533980</td>
<td>0.001533981</td>
<td>0.000039</td>
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<tr>
<td>0.6</td>
<td>0.001840775</td>
<td>0.001840777</td>
<td>0.000056</td>
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<tr>
<td>0.7</td>
<td>0.002147572</td>
<td>0.002147573</td>
<td>0.000077</td>
</tr>
<tr>
<td>0.8</td>
<td>0.002454367</td>
<td>0.002454369</td>
<td>0.000100</td>
</tr>
<tr>
<td>0.9</td>
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<td>0.002761165</td>
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<tr>
<td>1.0</td>
<td>0.003067957</td>
<td>0.003067962</td>
<td>0.000157</td>
</tr>
</tbody>
</table>

**Table C.2** Error Resulting in Replacement of Sine Term with Sine Argument, $N = 401$

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\sin\left(\frac{\pi \delta}{N}\right)$</th>
<th>$\frac{\pi \delta}{N}$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0000000000</td>
<td>0.0000000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.1</td>
<td>0.000783439</td>
<td>0.000783440</td>
<td>0.000010</td>
</tr>
<tr>
<td>0.2</td>
<td>0.001566879</td>
<td>0.001566879</td>
<td>0.000041</td>
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<tr>
<td>0.3</td>
<td>0.002350317</td>
<td>0.002350319</td>
<td>0.000092</td>
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<tr>
<td>0.4</td>
<td>0.003133753</td>
<td>0.003133758</td>
<td>0.000164</td>
</tr>
<tr>
<td>0.5</td>
<td>0.003917188</td>
<td>0.003917198</td>
<td>0.000256</td>
</tr>
<tr>
<td>0.6</td>
<td>0.004700620</td>
<td>0.004700637</td>
<td>0.000368</td>
</tr>
<tr>
<td>0.7</td>
<td>0.005484049</td>
<td>0.005484077</td>
<td>0.000501</td>
</tr>
<tr>
<td>0.8</td>
<td>0.006267475</td>
<td>0.006267517</td>
<td>0.000655</td>
</tr>
<tr>
<td>0.9</td>
<td>0.007050898</td>
<td>0.007050956</td>
<td>0.000829</td>
</tr>
<tr>
<td>1.0</td>
<td>0.007834315</td>
<td>0.007834396</td>
<td>0.001023</td>
</tr>
</tbody>
</table>
The maximum error using 1024 samples would occur at $\delta = 1$. The corresponding error is $\sim 0.00016\%$. The maximum error using 401 samples would occur at $\delta = 1$. The corresponding error is $\sim 0.0010\%$. Thus, the error introduced by replacing the sine terms with their respective arguments has been greatly overstated by the authors of [33].
D. Primary Constant Curve Fitted Equations

The warping factor requires a calculation of the parameter $\beta$ over a number of different frequencies. The tabulated values of the primary constants in Tables 3.1 and 3.2 do not suffice for these calculations. As a result, the tabulated values were plotted, curve fitted, and the curve generating equation determined. These equations allowed a more accurate means of calculating the parameter $\beta$ than had the values been interpolated manually for each frequency point.

Of the four primary constants, only three were fitted to an equation. The per unit capacitance was not fitted because it stays essentially constant over frequency. Equations for the remaining three primary constants were calculated, for both 24 and 26 AWG. These equations are used in the MATLAB function warp.m to calculate $\beta$.

The results obtained here are suspect in the error of results presented in Chapter 5. In all cases the data points end at 1.4 MHz, yet are extrapolated out to 5 MHz. Additionally, some of the curves do not properly fit the data points used to generate the curve fits. Future work should properly assess whether these curve fit equations are appropriate for use in the software algorithms.
D.1 Resistance

D.1.1 24 AWG

The curve fitted equation for the per unit resistance of 24 AWG twisted pair is:

\[ r = (51.4) - (7.4 \times 10^{-4} f) + (7.7 \times 10^{-5} f \ln f) - (2.7 \times 10^{-7} f^{1.5}) + (4.2 \times 10^{-11} f^2) \] (D.1)

where \( r \) is the per unit resistance and \( f \) is the frequency in Hz.

The plot of this equation is given in Figure D.1. For reference purposes, the tabulated data points are indicated on the graph.

![Figure D.1 Curve Fit for Resistance of 24 AWG](image)
D.1.2 26 AWG

The curve fitted equation for the per unit resistance of 26 AWG twisted pair is:

\[
r = (82.4) + (2.0 \times 10^{-4} f) - (4.6 \times 10^{-12} f^2 \ln f) + (2.2 \times 10^{-14} f^{2.5}) - (4.2 \times 10^{-3} f^{0.5} \ln f)
\]

(D.2)

where \( r \) is the per unit resistance and \( f \) is the frequency in Hz.

The plot of this equation is given in Figure D.2. For reference purposes, the tabulated data points are indicated on the graph.

![Figure D.2 Curve Fit for Resistance of 26 AWG](image)
D.2 Inductance

D.2.1 24 AWG

The curve fitted equation for the per unit inductance of 24 AWG twisted pair is:

\[ l = (0.13) + \frac{3.1 \times 10^{-3}}{1 + (3.1 \times 10^{-3})(4.2 \times 10^{-2})f} + \frac{4.5 \times 10^{-2}}{1 + (4.5 \times 10^{-2})(4.1 \times 10^{-5})f} \]  (D.3)

where \( l \) is the per unit inductance and \( f \) is the frequency in Hz.

The plot of this equation is given in Figure D.3. For reference purposes, the tabulated data points are indicated on the graph.

![Figure D.3 Curve Fit for Inductance of 24 AWG](image)
D.2.2 26 AWG

The curve fitted equation for the per unit inductance of 26 AWG twisted pair is:

\[ l = (0.14) + \frac{4.8 \times 10^{-3}}{1 + (4.8 \times 10^{-3})(2.5 \times 10^{-2})f} + \frac{4.3 \times 10^{-2}}{1 + (4.3 \times 10^{-2})(2.4 \times 10^{-5})f} \quad \text{(D.4)} \]

where \( l \) is the per unit inductance and \( f \) is the frequency in Hz.

The plot of this equation is given in Figure D.4. For reference purposes, the tabulated data points are indicated on the graph.

**Figure D.4** Curve Fit for Inductance of 26 AWG
D.3 Conductance

D.3.1 24/26 AWG

The curve fitted equation for the per unit conductance of both 24 and 26 AWG twisted pair is:

\[ g = (5.7 \times 10^{-2}) + (4.0 \times 10^{-5} f) - (9.8 \times 10^{-20} f^3) - (6.4 \times 10^{-3} \ln f) - \left( \frac{1.7 \times 10^{-2}}{f} \right) \]  

(D.5)

where \( g \) is the per unit conductance and \( f \) is the frequency in Hz.

The plot of this equation is given in Figure D.5. For reference purposes, the tabulated data points are indicated on the graph.

![Figure D.5 Curve Fit for Conductance of 24/26 AWG](image-url)
E. System Schematic Diagrams
From network analyzer

Reintroduced Signal

Signal A

Signal B

U101
HA2539

R101
820R

C102
100n

R103
1K2

C104
100n

U102
HA5503

R102
820R

C103
100n

U103
HA2539

R104
1K5

C105
100n

U104
HA5503

R105
820R

C106
100n

R106
1K2

C107
100n
post emphasis amplifier 100 kHz to 1 MHz range

post emphasis amplifier 1 MHz to 5 MHz range

Reintroduced Signal

to network analyzer input
F. Selected Data Sheets

F.1 Intersil Video Buffer Chip, HA5033
250MHz Video Buffer

The HA-5033 is a unity gain monolithic IC designed for any application requiring a fast, wideband buffer. Featuring a bandwidth of 250MHz and outstanding differential phase/gain characteristics, this high performance voltage follower is an excellent choice for video circuit design. Other features, which include a minimum slew rate of 1000V/μs and high output drive capability, make the HA-5033 applicable for line driver and high speed data conversion circuits.

The high performance of this product is a result of the Intersil Dielectric Isolation process. A major feature of this process is that it produces both PNP and NPN high frequency transistors which makes wide bandwidth designs, such as the HA-5033, practical. Alternative process methods typically produce a lower AC performance.

Features

- Differential Phase Error .................. 0.02 Degrees
- Differential Gain Error .................. 0.03%
- High Slew Rate .................. 1100V/μs
- Wide Bandwidth (Small Signal) .................. 250MHz
- Wide Power Bandwidth .................. DC to 17.5MHz
- Fast Rise Time .................. 3ns
- High Output Drive .................. ±10V With 100Ω Load
- Wide Power Supply Range .................. ±5V to ±16V
- Replace Costly Hybrids

Applications

- Video Buffer
- High Frequency Buffer
- Isolation Buffer
- High Speed Line Driver
- Impedance Matching
- Current Boosters
- High Speed A/D Input Buffers
- Related Literature
  - AN548, Designer's Guide for HA-5033

Ordering Information

<table>
<thead>
<tr>
<th>PART NUMBER (BRAND)</th>
<th>TEMP. RANGE (°C)</th>
<th>PACKAGE</th>
<th>PKG. NO.</th>
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<tbody>
<tr>
<td>HA2-5033-2</td>
<td>-55 to 125</td>
<td>12 Pin Metal Can</td>
<td>T12.C</td>
</tr>
<tr>
<td>HA2-5033-5</td>
<td>0 to 75</td>
<td>12 Pin Metal Can</td>
<td>T12.C</td>
</tr>
<tr>
<td>HA3-5033-5</td>
<td>0 to 75</td>
<td>8 Ld PDIP</td>
<td>E8.3</td>
</tr>
<tr>
<td>HA4PS033-5</td>
<td>0 to 75</td>
<td>20 Ld PLCC</td>
<td>N20.35</td>
</tr>
<tr>
<td>HA9P033-5 (H50335)</td>
<td>0 to 80 (Note 3)</td>
<td>8 Ld PSOP</td>
<td>M8.15A</td>
</tr>
</tbody>
</table>

Pinouts
Absolute Maximum Ratings

- Voltage Between V+ and V-Pins: 40V
- DC Input Voltage: V+ to V-
- Output Current (Peak, 50ms On/1 Second Off): ±200mA

Operating Conditions

- Temperature Ranges:
  - HA-5033-2: -55°C to 125°C
  - HA-5033-5 (Note 3): 0°C to 70°C
  - HA9PS033-5 (Notes 1, 3): -40°C to 80°C

CAUTION: Stresses above those listed in "Absolute Maximum Ratings" may cause permanent damage to the device. This is a stress only rating and operation of the device at these or any other conditions above those indicated in the operational sections of this specification is not implied.

NOTES:
1. Maximum power dissipation, including load conditions, must be designed to maintain the maximum junction temperature below 175°C for the metal can package, and below 150°C for the plastic packages (See Figure 5).
2. $I_{\text{JA}}$ is measured with the component mounted on an evaluation PCB board in free air.
3. Maximum operating temperature in the PSOP package is limited to 60°C, for $V_{\text{SUPPLY}} = \pm 12V$ to prevent the junction temperature from exceeding 150°C. The maximum operating temperature may have to be derated further depending on the output load condition. The operating temperature may be increased if the HA9PS033 is operated at lower $V_{\text{SUPPLY}}$. For example, the quiescent operating temperature may be increased to 75°C by operating at $V_{\text{SUPPLY}} = \pm 9.7V$. See Figure 5 for more information.

Thermal Information

- Thermal Resistance (Typical, Note 2)
  - $R_{\text{JA}}$ (PowerW) $R_{\text{JC}}$ (PowerW)
  - Metal Can Package: 15
  - PDIP Package: 12
  - PSOP Package (Note 4): 30
  - PLCC Package: 60

Electrical Specifications

- $V_{\text{SUPPLY}} = \pm 12V$, $R_S = 500\Omega$, $R_L = 100\Omega$, $C_L = 10\mu F$, Unless Otherwise Specified

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>TEST CONDITIONS</th>
<th>TEMP. (°C)</th>
<th>MIN</th>
<th>TYP</th>
<th>MAX</th>
<th>MIN</th>
<th>TYP</th>
<th>MAX</th>
<th>UNITS</th>
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<td>Full</td>
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<td>5</td>
<td>19</td>
<td>6</td>
<td>15</td>
<td>mV</td>
<td></td>
<td></td>
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<tr>
<td>Average Offset Voltage Drift</td>
<td>Full</td>
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<td>33</td>
<td>-</td>
<td>33</td>
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HA-5033

Electrical Specifications

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POWER SUPPLY CHARACTERISTICS:

| Supply Current            | 25               | 21        | 25        | 21        | 25    | mA    |
|----------------------------|------------------|-----------|-----------|-----------|-------|
| Full                       | 21               | 30        | 21        | 30        |       | mA    |
| Power Supply Rejection Ratio | Full             | 54        | 54        |           | dB    |
| Harmonic Distortion        | \( V_{IN} = 1 V_{RMS} \) at 100kHz | 25        | <0.1      | <0.1      | %     |

NOTES:
5. \( V_{SUPPLY} = \pm 15V, V_{OUT} = \pm 10V, R_L = 1k\Omega \).
6. Differential gain and phase error are nonlinear signal distortions found in video systems and are defined as follows: Differential gain error is defined as the change in amplitude at the color subcarrier frequency as the picture signal is varied from blanking to white level. Differential phase error is defined as the change in the phase of the color subcarrier as the picture signal is varied from blanking to white level. \( R_L = 300\Omega \).

Test Circuits and Waveforms

**FIGURE 1. SLEW RATE AND SETTLING TIME**

**FIGURE 2. TRANSIENT RESPONSE**

**FIGURE 3. SETTLING TIME AND SLEW RATE**

**FIGURE 4. RISE TIME AND OVERSHEAT**

\( V_{IN} = 15V, R_L = 10\, \text{k}\Omega \)

\( V_{OUT} = 10V \)
**Test Circuits and Waveforms** (Continued)

![Waveform Diagram](image)

\[ V_{\text{OUT}} = \frac{V_{\text{IN}}}{2} \]

\[ V_{\text{OUT}} = 20 \text{mV} \]

\[ V_{\text{IN}} = 50 \text{mV} \]

\[ V_{\text{IN}} = 0 \text{V} \]

\[ T_A = 25^\circ \text{C}, R_S = 50 \Omega, R_L = 1000 \Omega \]

**Pulse Response**

---

**Schematic Diagram**

---

**Application Information**

**Layout Considerations**

The wide bandwidth of the HA-5033 necessitates that high frequency circuit layout procedures be followed. Failure to follow these guidelines can result in marginal performance.

Probably the most crucial of the RF/video layout rules is the use of a ground plane. A ground plane provides isolation and minimizes distributed circuit capacitance and inductance which will degrade high frequency performance. This ground plane shield can also incorporate the metal case of the HA-5033 since pin #2 is internally tied to the package. This feature allows the user to make metal to metal contact between the ground plane and the package, which extends shielding, provides additional heat sinking and eliminates the use of a socket. IC sockets contribute inter-lead capacitance which limits device bandwidth and should be avoided.

---

For the PDIP, pin 8 can be tied to either supply, grounded, or simply not used. But to optimize device performance and improve isolation, it is recommended that this pin be grounded.

Other considerations are proper power supply bypassing and keeping the input and output connections as short as possible which minimizes distributed capacitance and reduces board space.

**Power Supply Decoupling**

For optimum device performance, it is recommended that the positive and negative power supplies be bypassed with capacitors to ground. Ceramic capacitors ranging in value from 0.01\(\mu\)F to 0.1\(\mu\)F will minimize high frequency variations in supply voltage. Solid tantalum capacitors 1\(\mu\)F or larger will optimize low frequency performance.
HA-5033

It is also recommended that the bypass capacitors be connected close to the HA-5033 (preferably directly to the supply pins).

Figure 5 is based on:

\[ P_{\text{DMAX}} = \frac{T_{J\text{MAX}} - T_A}{\theta JA} \]

Where: \( T_{J\text{MAX}} \) = Maximum Junction Temperature of the Device
\( T_A \) = Ambient Temperature
\( \theta JA \) = Junction to Ambient Thermal Resistance

**Typical Applications** (Also see Application Note AN548)

**Figure 6.** VIDEO COAXIAL LINE DRIVER 50Ω SYSTEM

**Figure 7.** VIDEO GAIN BLOCK

\[ T_A = 25^\circ C, R_S = 50\Omega, R_M = R_L = 50\Omega \]

\[ V_O = V_{IN} \left( \frac{R_L}{R_L + R_M} \right) = \left( \frac{3}{2} \right) V_{IN} \]

**POSITIVE PULSE RESPONSE**

\[ T_A = 25^\circ C, R_S = 50\Omega, R_M = R_L = 50\Omega \]

\[ V_O = V_{IN} \left( \frac{R_L}{R_L + R_M} \right) = \left( \frac{1}{2} \right) V_{IN} \]

**NEGATIVE PULSE RESPONSE**
Typical Performance Curves

**Figure 8. Input Offset Voltage vs Temperature**

**Figure 9. Input Bias Current vs Temperature**

**Figure 10. Supply Current vs Temperature**

**Figure 11. Slew Rate vs Temperature**

**Figure 12. Slew Rate vs Load Capacitance**

**Figure 13. Slew Rate vs Load Capacitance**
Typical Performance Curves (Continued)

FIGURE 14. GAIN ERROR vs INPUT VOLTAGE

FIGURE 15. GAIN ERROR vs INPUT VOLTAGE

FIGURE 16. GAIN ERROR vs TEMPERATURE

FIGURE 17. $V_{IN} - V_{OUT}$ vs $I_{OUT}$

FIGURE 18. Y - PARAMETERS PHASE vs FREQUENCY

FIGURE 19. Y - PARAMETER MAGNITUDE vs FREQUENCY
Typical Performance Curves (Continued)

**Figure 20. Power Supply Rejection Ratio vs Frequency**

**Figure 21. Total Harmonic Distortion vs Frequency**

**Figure 22. Total Harmonic Distortion vs Input Voltage**

**Figure 23. Output Voltage Swing vs Load Resistance**

**Figure 24. Output Swing vs Frequency (Note)**

**Figure 25. Output Swing vs Frequency (Note)**

**NOTE:**
This curve was obtained by noting the output voltage necessary to produce an observable distortion for a given frequency. If higher distortion is acceptable, then a higher output voltage for a given frequency can be obtained. However, operating the HA-5033 with increased distortion (to the right of curve shown), will also be accompanied by an increase in supply current. The resulting increase in chip temperature must be considered and heat sinking will be necessary to prevent thermal runaway. This characteristic is the result of the output transistor operation. If the signal amplitude or signal frequency or both are increased beyond the curve shown, the NPN, PNP output transistors will approach a condition of being simultaneously on. Under this condition, thermal runaway can occur.
Die Characteristics

DIE DIMENSIONS:
51 mils x 67 mils x 19 mils
1300µm x 1700µm x 483µm

METALLIZATION:
Type: Al, 1% Cu
Thickness: 16kÅ ±2kÅ

PASSIVATION:
Type: Nitride (Si₃N₄) over Silox (SiO₂, 5% Phos.)
Silox Thickness: 12kÅ ±2kÅ
Nitride Thickness: 3.5kÅ ±1.5kÅ

SUBSTRATE POTENTIAL (Powered Up):
Unbiased

TRANSISTOR COUNT:
20

PROCESS:
Bipolar Dielectric Isolation

Metallization Mask Layout

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F.2 Intersil Very High Slew Rate Operational Amplifier, HA2539
600MHz, Very High Slew Rate Operational Amplifier

The Intersil HA-2539 represents the ultimate in high slew rate, wideband, monolithic operational amplifiers. It has been designed and constructed using the Intersil High Frequency Bipolar Dielectric Isolation process and features dynamic parameters never before available from a truly differential device.

With a 600V/μs slew rate and a 600MHz gain bandwidth product, the HA-2539 is ideally suited for use in video and RF amplifier designs, in closed loop gains of 10 or greater. Full ±10V swing coupled with outstanding AC parameters and complemented by high open loop gain makes the devices useful in high speed data acquisition systems.

For further design assistance please refer to Application Note AN541 (Using the HA-2539 Very High Slew Rate Wideband Operational Amplifiers) and Application Note AN566 (Thermal Safe-Operating-Areas For High Current Operational Amplifiers).

For military grade product information, the HA-2539/883 data sheet is available upon request.

### Features
- Very High Slew Rate ......................... 600V/μs
- Open Loop Gain .......................... 15kV/V
- Wide Gain-Bandwidth (AV ≥ 10) ............... 600MHz
- Power Bandwidth ......................... 9.5MHz
- Low Offset Voltage ....................... 8mV
- Input Voltage Noise ................. 8nV/√Hz
- Output Voltage Swing ............. ±10V
- Monolithic Bipolar Dielectric Construction

### Applications
- Pulse and Video Amplifiers
- Wideband Amplifiers
- High Speed Sample-Hold Circuits
- RF Oscillators

### Pinout

**HA-2539 (PDIP, CERDIP) TOP VIEW**

+IN 1 -IN 4
NC 2 NC 13 NC
V- 3 NC 12 NC
NC 4 NC 11 NC
NC 5 +V 10
NC 6 NC 9 NC
NC 7 OUTPUT 8

NOTE: No-Connection (NC) leads may be tied to a ground plane for better isolation and heat dissipation.

### Ordering Information

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<td>F14.3</td>
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<td>HA3-2639-5</td>
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<td>14 Ld PDIP</td>
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Absolute Maximum Ratings

Supply Voltage Between V+ and V-Terminals .................. 35V
Differential Input Voltage ..................................... 6V
Peak Output Current .......................................... 50mA
Continuous Output Current ..................................... 33mA RMS

Operating Conditions

Temperature Range
HA-2539-2 .............................................. -65°C to 125°C
HA-2539-5 .............................................. 0°C to 70°C

CAUTION: Stresses above those listed in "Absolute Maximum Ratings" may cause permanent damage to the device. This is a stress only rating and operation of the device at these or any other conditions above those indicated in the operational sections of this specification is not implied.

NOTES:
1. Maximum power dissipation with load conditions must be designed to maintain the maximum junction temperature below 125°C for the ceramic package and below 150°C for the plastic package. By using Application Note AN556 on Safe Operating Area equations, along with the thermal resistances, proper load conditions can be determined. Heat sinking is recommended above 75°C.
2. $\theta_{JA}$ is measured with the component mounted on an evaluation PCB board in free air.

Electrical Specifications

$V_{SUPPLY} = \pm 15V$, $R_L = 1k\Omega$, $C_L < 10pF$. Unless Otherwise Specified

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<td>8.7</td>
<td>9.5</td>
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**HA-2539**

**Electrical Specifications** \( V_{SUPPLY} = \pm 15V, R_L = 1k\Omega, C_L < 10pF, \) Unless Otherwise Specified (Continued)

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<td>7</td>
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<td>Overshoot</td>
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**POWER REQUIREMENTS**

| Supply Current             | Full | -   | 20  | -   | 20  | -   | mA    |
| Power Supply Rejection Ratio (Note 9) | Full | 60  | 70  | -   | 60  | 70  | dB    |

**NOTES:**

3. \( R_L = 1k\Omega, V_C = \pm 10V. \)
4. \( V_{CM} = \pm 100V. \)
5. \( V_C = 90\text{mV}. \)
6. \( A_V = 10. \)
7. Full Power Bandwidth guaranteed based on slew rate measurement using: \( \text{FPBW} = \frac{\text{Slew Rate}}{2 \times V_{PEAK}}. \)
8. Refer to Test Circuits section of data sheet.
9. \( V_{SUPPLY} = \pm 8V, -15V \) and \( +15V, -5V. \)
10. Guaranteed range for output voltage is \( \pm 10V. \) Functional operation outside of this range is not guaranteed.

**Test Circuits and Waveforms**

![Test Circuit Diagram](image)

**NOTES:**

11. \( V_C = \pm 15V. \)
12. \( A_V = \pm 10. \)
13. \( C_L \leq 10pF. \)

**FIGURE 1. TEST CIRCUIT**

![Waveform A](image)

Vertical Scale: \( A = 0.5V/\text{Div.}, B = 5.0V/\text{Div.} \)
Horizontal Scale: 50ns/Div.

**FIGURE 2. LARGE SIGNAL RESPONSE**

![Waveform B](image)

Vertical Scale: Input = 10mV/Div., Output = 50mV/Div.
Horizontal Scale: 20ns/Div.

**FIGURE 3. SMALL SIGNAL RESPONSE**

---

**InterSili**

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Test Circuits and Waveforms (Continued)

Schematic Diagram

NOTES:

15. Load capacitance should be less than 10pF.
16. It is recommended that resistors be carbon composition and that feedback and summing network ratios be matched to 0.1%.
17. SETTLE POINT (Summing Node) capacitance should be less than 10pF. For optimum settling time results, it is recommended that the test circuit be constructed directly onto the device pins. A Tektronix 568 Sampling Oscilloscope with 5-3A sampling heads is recommended as a settle point monitor.

FIGURE 4. SETTLING TIME CIRCUIT
Typical Applications

![Typical Applications Diagrams]

*Figure 5. Frequency Compensation by Overdamping

*Figure 6. Stabilization Using $Z_N$

*Figure 7. Reducing DC Errors: Composite Amplifier

*Figure 8. Differential Gain Error (3%) HA-2539 20dB Video Gain Block

Typical Performance Curves

![Typical Performance Curves Diagrams]

*Figure 9. Input Offset Voltage and Bias Current vs Temperature

*Figure 10. Input Noise Voltage and Noise Current vs Frequency
Typical Performance Curves (Continued)

FIGURE 17. OUTPUT VOLTAGE SWING vs LOAD RESISTANCE

FIGURE 18. NORMALIZED AC PARAMETERS vs TEMPERATURE

FIGURE 19. SETTLING TIME FOR VARIOUS OUTPUT STEP VOLTAGES

FIGURE 20. POWER SUPPLY CURRENT vs TEMPERATURE
Die Characteristics

DIE DIMENSIONS:
62 mils x 76 mils x 19 mils
1575µm x 1930µm x 483µm

METALLIZATION:
- Type: Al, 1% Cu
- Thickness: 16kÅ ±2kÅ

PASSIVATION:
- Type: Nitride (Si₃N₄) over Silox (SiO₂, 5% Phos.)
- Silox Thickness: 12kÅ ±2kÅ
- Nitride Thickness: 3.5kÅ ±1.5kÅ

SUBSTRATE POTENTIAL (Powered Up):
- V-

TRANSISTOR COUNT:
- 30

PROCESS:
- Bipolar Dielectric Isolation

Metallization Mask Layout

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