A MASTER-SLAVE TRACTOR GUIDANCE SYSTEM

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Department of Agricultural Engineering
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by
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ABSTRACT

The search for increasing productivity in agriculture has prompted many proposals for automatic and semi-automatic tractor guidance systems. This project involved the investigation and development of a master-slave tractor guidance system. The guidance system developed, which operated the slave tractor, was composed of a sterring control unit and a speed control unit.

Large and small-signal mathematical models indicated that a relatively simple control system would be capable of adequately slaving a tractor-implement unit to an operator controlled unit. An experimental system was constructed and field tested which confirmed this indication.
ACKNOWLEDGEMENTS

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$e_i$  Input voltage
$e_o$  Output voltage
$G_2$  Velocity feedback gain
$f$  Viscous friction
$K_1 K_2$  Angle multiplier constant
$K_2 K_4$  Speed loop potentiometer setting
$K_G$  Gear ratio constant relating linear displacement of the tractor to the angular displacement of the engine crankshaft in feet/radians.
$K_T$  Engine gain, relating output torque to throttle position
$M$  Effective mass
$R_A$  Radius of turn of point A
$R_S$  Radius of turn of center of slave implement
$R$  Resistance
$R_e$  Radius of turn of center of the tractor
$T_G$  Governor time constant
$T$  Torque
$V$  Input voltage
$V_c$  Velocity of point c
$V_D$  Velocity of point D
$V_S$  Slave vehicle speed
$Y_H$  Displacement of point H in Y direction
$Y_A$  Displacement of point A in Y direction
$Y_B$  Displacement of point B in Y direction
$X_H$  Displacement of point H in X direction
\[ Y_c \quad \text{Displacement of point c in Y direction} \]

\[ X_c \quad \text{Displacement of point c in X direction} \]

\[ b \quad \text{Wheelbase of tractor} \]

\[ e \quad \text{Error} \]

\[ l \quad \text{Link length} \]

\[ \alpha \quad \text{Angle between link and longitudinal axis of sensor at A} \]

\[ \beta \quad \text{Angle between link and longitudinal axis of slave tractor} \]

\[ \theta_c \quad \text{Speed controller counter shaft position} \]

\[ \theta_i \quad \text{Angular displacement of implement} \]

\[ \theta_s \quad \text{Apparent guide wheel angle of slave tractor with respect to longitudinal axis of tractor} \]

\[ \theta_T \quad \text{Angle between the tractor center line and X-axis} \]

\[ \dot{\theta}_T \quad \text{Absolute angular velocity of the tractor} \]

\[ \theta_{TM} \quad \text{Angular displacement of master tractor} \]

\[ \theta_{TS} \quad \text{Angular displacement of slave tractor} \]

\[ \phi \quad \text{Direction of motion of the tractor with respect to its center line} \]

\[ \gamma \quad \text{A small angle} \]

\[ \omega \quad \text{Engine speed} \]

\[ \theta_D \quad \text{Dead zone} \]
I. INTRODUCTION

1.1 General

Farmers, as businessmen, are interested in the maximization of profits. Since an individual farmer in many cases has virtually no control over the selling price of his product, profits can only be increased by reducing the unit cost. A primary method of reducing unit cost is to reduce the labour input. A reduction in labour input may be accomplished by placing more productive capacity under the control of each operator. Results obtained thus far through mechanization, as reported by Barber\(^1\) indicate this is a valid approach in the farming industry. In the case of tillage operations reduced labour input has been achieved mainly by reducing the effort required to control the tractor and machine thus allowing the operator to handle larger machines at a higher speed without undue mental or physical stress. A logical extension of this is to allow one operator to control more than one tractor-machine unit.

1.2 Multiple Unit Control

There are two situations in which multiple unit control is advantageous over a single larger unit. Firstly, there is the case of a farming system where various operations are performed having different power requirements. Multiple unit control meets this situation by retaining the flexibility of smaller units, such as maneuverability, ability to perform separate operations simultaneously, and provide an economical source of power other than tractive, while providing the
major advantage of a larger unit. In other words it is conceivable that a farm requiring a medium sized tractor for farmstead operations and harvesting plus a large tractor for tillage could be served just as well by two medium sized tractors if a single operator could employ both of the medium sized tractors simultaneously.

A second situation which arises even when the sole use of a power unit is tractive is the problem of unwieldy sized implements. A maximum practical limit exists to the size of a single unit leaving multiple unit control in some form as the only way to increase the productive power under the control of a single operator.

The effect on tractor costs of widespread adoption of multiple unit control cannot be definitely evaluated. If a reduction in the number of individual models required to cover a broad power spectrum would lead to tangible development and production economics then this would be a further advantage.

A review of the literature related to automatic tractor guidance reveals a number of proposed systems which would allow multiple unit control. Some of these are fully automatic systems while the remainder require the faculties of a human operator, either on a master control vehicle or at a remote location. To differentiate the latter systems they may be termed semi-automatic.

Fully-automatic systems may be further divided into two subclasses: command guidance and preset guidance. Command guidance involves tracking the vehicle from a remote location and sending corrective information to the vehicle. Systems classified as preset guidance are characterized
by having the controlling functions initiated by devices integral with the guided vehicle.

1.2.1 **Fully Automatic systems**

1.2.1(a) **Command Guidance**

Systems of this type are illustrated in Figure 1.1. The following advantages are exhibited by most command guidance systems:

- Good long range stability as a result of absolute position measurement,
- Adaptability to large plane areas and versatility since various programmed courses can be accepted.

The location of the signal transmission link break in computation is immaterial, and would be dependent on the number of signals required at various stages and the practicality of on board computation. Thus it is seen that the sole distinctive feature is the remote tracking and various systems in this subclass are distinguished by the tracking devices employed.

1. (1) Radar Sensing: Radar tracking is highly developed and its suitability as the position sensing portion of a command guidance system has been amply illustrated in the missile field\(^{(19)}\). The precise control of a land vehicle would require a complex sophisticated system especially if operation on rolling terrain was required. Such a system would be characterized by high initial cost and requirement of technical maintenance. Thus it is not presently practical to employ radar sensing in the guidance of an agricultural tractor.
(2) Infra-Red Sensing: A system for automatic guidance of farm tractors where positive information was to be provided by triangulation was suggested by MacHardy\(^{(13)}\). It was further suggested that infra-red detectors, seeking the tractor exhaust pipe, be employed in the triangulation tracking. Preliminary investigations showed\(^{(14)}\) that an infra-red detector was indeed capable of detecting a tractor exhaust pipe. The suitability of using the exhaust pipe in its usual location
remains to be demonstrated. It has been shown by Shukla et al.\(^{(22)}\) that to achieve stable operation at least one sensor (the exhaust pipe in this case is the equivalent of a sensor on a self-contained system) has to be near or ahead of the front axle. Relocation of the exhaust pipe would probably be required for optimum performance. There is no doubt, however, that the successful development of this system is possible using sufficiently accurate components.

The following example, with reference to Figure 1.2, illustrates the degree of accuracy required in the angle measurement. Similar accuracy is required in a radar system.

Suppose that the vehicle is at the extreme corner of a quarter section field the centerline of which is used for the baseline, then \(E_1 = 26^\circ30'\) and \(E_2 = 90^\circ\)

\[
d = \frac{b}{\cot E_1 + \cot E_2}
\]

\[
\frac{\Delta d}{\Delta E_1} = \frac{-b \csc^2 E_1}{(\cot E_1 + \cot E_2)^2}
\]

For \(\Delta d = \pm 1\) ft

\(E_1 = \pm 33.4\) Sec

Since \(\Delta d = \pm 1\) ft, is unacceptably large for row crop work, and an allowance for dynamic error must be made greater static accuracy would be required. Also any initially apparent economic advantage over a radar system would be partially offset by the requirement of having two tracking sensors. Thus it appears that this system is also not
Figure 1.2: Command Guidance System employing two infra-red sensors located at A and B.

Presently economically practical.

(3) Radio Interferometry: It has been suggested that the incremental movement of a vehicle can be detected by mounting a high frequency transmitter on the vehicle and three or four receivers at dispersed locations. By the method of fringe counting incremental radial movements with respect to the receivers can be obtained. The positional accuracy of this system is limited only by attainable frequencies since any capacity of fringe count is readily obtainable. It would appear that the major limitation is again an economic one as complex highly stable
components along with technical maintenance would undoubtedly be 
required. The practicability of this system increases greatly if 
computational facilities are already available.

1.2.1(b) Preset Guidance

The distinguishing feature of preset guidance 
is that the position sensing devices and all control elements are 
contained on the vehicle. As will be seen the various systems falling 
under this classification vary widely in complexity. But in general, 
it can be said that systems of this type are less complex than 
command guidance systems.

(1) Dead Reckoning: In this type of system the vehicle is started 
from a known point and performs all subsequent operations by reference 
to internal standards of heading and distance. A detailed proposal 
for an accessory type automatic pilot mechanism has been made by 
Onshinsky(16). Steering signals would be obtained from a gyroscope 
mounted on a rotatable platform while it was suggested that a fifth 
wheel could be used for distance measurement. The mounting of the 
gyroscope on a rotatable platform was primarily to facilitate programmed 
end turns which would be accomplished by rotating the platform in a 
predetermined pattern. It was also suggested that a marker follower 
type of sensor could be employed in combination with the above.

The construction of a dead reckoning type system for farm tractors 
was reported by Gilmour(7), a British researcher in 1958. No provision 
was made for moving the tractor over one implement width at headlands.
Satisfactory operation was not obtained, the failure being blamed on an insufficiently damped compass and a faulty shuttle value in the steering assembly.

It is improbable that a satisfactory dead reckoning guidance system for farm tractors can be developed. Even if a perfect compass were to be employed lateral displacement disturbances would not be accounted for and considerable errors could accumulate. This is no doubt why Onshinsky has suggested the use of a marker follower in conjunction with the dead reckoning.

(2) Marker Follower: In this type of system a marker of some type is traced out along the edge of each field pass and serves as a position reference for the tractor on the subsequent pass. Several proposals for a marker are suggested by Grovum\(^8\), who reports the successful development of a system of this type using a furrow for the marker. Grovum's work indicated that a completely self contained system with a good long range stability could be developed without elaborate or expensive equipment.

Warner and Harries\(^{24}\) also report on the development of a system using a furrow as the marker. In this case a novel sensing device employing ultrasonic pulse-echo ranging was tried. The equipment functioned satisfactorily at 7 mph on a test track, however, the ultrasonic marker detection proved unsatisfactory in agricultural conditions. It was suggested that optical methods would be better.

A system employing a cord buried a few inches below the surface of the ground was successfully developed by Widden and Blair in
Australia (35). As the tractor travels along the path, the cord is drawn up out of the ground through the guiding mechanism and is reinstalled in the proper position for the subsequent path.

In the case of harvesting machines the edge of the standing crop may be used as the marker, such a system was developed by Parish (17). A hydrostatic drive windrower was fitted with sensors capable of detecting the edge of poor quality alfalfa and was found to perform well under actual adverse field conditions.

(3) Leader Cable: In this type of system, which can be considered as a special case of the marker follower, the magnetic field associated with a buried current carrying conductor is used as the steering reference. Systems have been reported to be operating very satisfactorily in warehouses, assembly lines and feedlot operations (10, 18, 3, 5) where the distances traversed are short and the same path is repeatedly traversed. Working experimental farm tractors employing this form of guidance have been developed in Britain (6) and the U.S.A. (21) and several hundred acres wired for field testing.

The advantages of this type of system are the possibility of extremely accurate steering, especially important in row crop work, excellent long range stability and a minimum of control hardware. The major disadvantage is again an economic one, the cost of imbedding the wire grid in the soil. This cost can to some extent be reduced by employing a sensor which is capable of making more than one pass on each leader cable. Multiple passes may be accomplished by mounting a
moveable sensor on a boom or by employing off-the-wire guidance as described by Brooke \(^{(2)}\). However it is probable that the cost of embedding even the reduced number of cables required by the present state of the art would be at the moment uneconomic for western Canadian farmers.

A second disadvantage of this system is that the same route must always be traversed which in many farming operations would be undesirable. This currently may be overcome by embedding a second grid, but only, of course, at greatly increased cost. Further developments in off-the-wire guidance may, however, allow a programmed course to be followed.

(4) Beam Rider: A fourth type of system in this general classification is the so-called "beam rider" technique which has been applied in the guided-missile field. Various types of beams could conceivably be used; electromagnetic, infra-red, visible light or laser. The laser has the advantage of a high intensity non divergent beam being readily obtainable. In 1962 a gas laser was used to accurately guide a large tunnelling machine during a New Mexico irrigation project \(^{(12)}\). It is reported that after an advance of a mile and a half the cutting machine had not strayed more than 5/8 in. from its planned course.

The use of this technique for farm tractor guidance would be primarily suited to a switchback mode of operation. The beam, or more probably a verticle plane, generator would be mounted on a robot placed at one end of the field and advance one implement width after each pass. Attaining a sufficiently accurate angular position of the
beam after each move would probably prove quite expensive. A suitable beam generator would also be an expensive item. For example, the cost of the laser unit used to guide the tunneling machine mentioned above was $2,000.

The anticipated high cost of this system combined with its limited flexibility (straight line passes, relatively level land) makes it unsuitable for general tractor guidance applications.

1.2.2 Semi Automatic

(1) Radio Control: Radio control of land vehicles from a remote location has been extensively developed. Radio control of a farm tractor was accomplished in 1945(9). Since then more sophisticated multi-channel radio control systems have been developed. In 1959 a group of Russian researchers built a multi-channel radio-controlled tractor employing hydraulic actuators as power devices(15). A similar unit was designed and constructed at the University of Nebraska, employing dual audio modulators, which allowed two control signals to be transmitted simultaneously rather than one as was the case for the Russian design.

While radio remote control by a human operator has little to offer in itself, except for operation under hazardous conditions, it forms an integral part of many proposed control systems. Thus the overall performance of the system is dependent upon the performance of the radio link.
(2) Marker Follower: Fully automatic marker follower systems have been described above, however it may be advantageous to provide for a remote control override actuated by an operator on a second unit. In this way the provision for making end turns automatically could be eliminated thus partly offsetting the cost of the radio link. It would also be beneficial to be able to restart the tractor from a remote location if it should stop due to becoming lost.

(3) Dead Reckoning: The marker follower system may be difficult to implement for large implement widths. The use of a dead reckoning guidance system with occasional remote control corrections being made by the operator of a second outfit would be more suitable. Headland turns could also be made by remote control.

(4) Master-Slave: This system was suggested by Larson(11) where two vehicles are connected by a physical link of variable length, as shown in Figure 1.3. The angular displacement of the guide wheels of the slave tractor would be a function of the offset angle, link length and angular displacement of the guiding wheels on the master tractor. Larson indicated that two separate directional control systems would be needed. A position maintenance system would be used for constant bearing operation while a motion duplication procedure would be employed for making turns. Although a two mode system has a certain advantage, a system in which the position maintenance portion could be suitably modified so that it would guide the slave vehicle in turns, would be
Figure 1.3: System using an offset link, operating in an offset position.

less complex and thus less expensive and less prone to failure.

Ideally a master-slave system would allow a single operator to control both units by simply driving one of them in the normal manner. It is also conceivable that such a system could be extended to more
than one slave vehicle.
2. OBJECTIVE

Consideration of the pros and cons of the various types of systems and work which has already been done resulted in the conclusion that the master-slave type system warranted further investigation since

1) It satisfies the primary objective of reducing labor input per unit of output.

2) It has a great deal of flexibility in that the tractors may be used for purposes other than tillage; as separate units, if this is desirable in the overall farm system. The slave tractor could readily be fitted with a magnetic guidance head for performing farmstead operations automatically.

3) It increases the feasibility of using a magnetic guidance fully automatic system for field operations by increasing the maximum wire spacing thus reducing the major cost of that type of system, and

4) No attempt had been made to construct an experimental system.

The objective of this project was to further investigate the feasibility of the master-slave tractor guidance system followed by the design, construction and field testing of an experimental unit.
3. PERFORMANCE CRITERIA

The ideal system performance wise would track perfectly under all conditions and for any manoeuver of the master tractor. This is obviously unobtainable using a conventional tractor and trailed implement combination since in some cases it is physically impossible.

The degree of attainment of the ideal depends upon the complexity of the system. The degree of complexity is limited by costs, both initial and for maintenance, which must be comparatively low in agricultural applications. A further limitation on complexity is that the system must be easily serviced. Thus the following relaxed minimum performance criteria are specified for the two common modes of field operation prevalent in western Canada, which are switchback with headlands and perimeters. These are shown in Figure 3.1.

For switchback tillage operation a satisfactory system would be one in which skipping and overlapping are comparable to that obtained by a human operator with a steady state overlap of less than one foot under all reasonable operating conditions. A steady state skip is obviously not tolerable. Large radius turns in either direction should not produce skipping although the overlap may then be allowed to increase. Headland turns, i.e. small radius turns with raised implements should be readily navigable but accurate tracking not required. The steady state link length should remain constant regardless of changes in the steady state speed of the master tractor.
Figure 3.1: Paths followed for switchback (a) and perimeter (b) field operations.
For perimeter operation the above criteria are applicable with the added restriction that relatively small radius turns, in one direction, with the implement engaged should be made with a minimum of overlap required to prevent any skipping.
4. PROPOSED SYSTEM

4.1 Principle

The proposed system, a variation of Larson's proposal, is illustrated in Figure 4.1. Inspection of the case where a constant bearing tracking error exists reveals that

\[ \alpha = f(e, t) \]  
\[ \beta = f(e, t, \theta_s) \]

Now if the speed of one or both of the tractors were controlled so as to keep \( t \) constant then

\[ \alpha = f(e, t) \]  
\[ \beta = f(e, \theta_s) \]

Since both \( \alpha \) and \( \beta \) are a function of the error either one or both of them could be used for steering control. \( \beta \) however is also a function of the rate of correction since

\[ \frac{de}{dt} = f(\theta_s, V_s) \]

It would thus appear advantageous to employ both \( \alpha \) and \( \beta \) since it may then be possible to vary the effect of \( e \) and \( \dot{e} \) semi-independently. The use of either angle alone is further rejected since for the anticipated required link lengths \( \alpha \) alone would tend to cause excessive overshoot on a turn while \( \beta \) alone would result in severe corner cutting.

In order to obtain corrective steering action from the error signals provided by \( \alpha \) and \( \beta \) it is necessary that

\[ \phi_s = f(\alpha, \beta) \]
Figure 4.1: System using a calinear link operating in an offset position (a) with no error, and (b) with an error e.
the simplest case being

\[ \phi_s = K_1 \alpha + K_2 \beta \]  

(4.7)

where \( K_1 \) and \( K_2 \) are positive constants. Qualitative inspection of Figure 4.1 reveals that this will result in the desired type of response. Quantitative estimates of performance are necessary, however, to determine if this proposal will result in a satisfactory performance.

4.2 Mathematical Models

Two mathematical models of the proposed system were developed. A simplified model, to determine the range of response available to small inputs or disturbances, and a more elaborate model to investigate small-radius curve following ability. In both models, which were purely kinematic, two assumptions were made:

1) It was assumed that any necessary control elements were ideal, that is had an instantaneous response. In particular that the guiding wheels of the slave vehicle could be made any desired value instantaneously.

2) That a wheel rolls in the direction in which it is pointed.

4.2.1 Small Signal Model

The physical system represented by the small signal model is illustrated by Figure 4.2, where inputs are limited to small \( y \) displacements of point A. This is representative of two situations; an actual input of this form or a lateral displacement disturbance of the slave vehicle. The output is the \( y \) displacement of point B. It
Figure 4.2: Physical system represented by small signal model.

should be noted that the response of a trailed implement, not included in this model, would have a smaller amplitude than point B.

By inspection of Figure 4.3 and recalling that at any instant the motion of a point may be considered as combined translation and rotation or as pure rotation about the point O, and that the angular velocity of any point with respect to any other point is equal to the absolute angular velocity of the tractor (\( \dot{\theta}_T \)), the following motion
Figure 4.3: Tractor kinematics

describing equations may be written:

\[ \dot{\theta}_T = \frac{V_c}{r_c} \]  \hspace{1cm} (4.8)

\[ r_c = \frac{b}{\tan \phi} \]  \hspace{1cm} (4.9)

\[ \theta_T = \frac{V_c}{b} \tan \phi \]  \hspace{1cm} (4.10)

\[ x_c = V_c \cos \theta_T \]  \hspace{1cm} (4.11)
\begin{align}
\dot{y}_C &= V_C \sin \theta_T \quad (4.12) \\
\dot{x}_B &= x_C - b \theta_T \sin \theta_T \quad (4.13) \\
\dot{y}_B &= y_C + b \theta_T \cos \theta_T \quad (4.14)
\end{align}

Also, from Figure 4.1(b)
\begin{align}
\alpha &= \sin^{-1} \left( \frac{y_A - y_B}{\ell} \right) - \theta_{TM} \quad (4.15) \\
\beta &= \sin^{-1} \left( \frac{y_A - y_B}{\ell} \right) - \theta_{TS} \quad (4.16)
\end{align}

Letting \( \phi_s = k_1 \alpha + k_2 \beta \)
\begin{equation}
\theta_{TS} = \frac{V_C}{b} \tan \left( k_1 \sin^{-1} \left( \frac{y_A - y_B}{\ell} \right) - \theta_{TM} \right) + k_2 \left( \frac{y_A - y_B}{\ell} - \theta_{TS} \right) \quad (4.17)
\end{equation}

considering the case where \( \theta_{TM}(t) = 0, y_A(t) \) is small, \( y_B(0) = 0, \theta_{TS}(0) = 0 \), then all angles will remain small and for a small angle \( \gamma \)
\begin{align}
\sin \gamma &= \tan \gamma = \gamma \\
\cos \gamma &= 1
\end{align}

making these substitutions equations (4.14) and (4.17) then simplify to
\begin{align}
\dot{y}_B &= V_C \theta_{TS} + b \theta_{TS} \quad (4.18) \\
\dot{\theta}_{TS} &= \frac{V_C}{b} \left( k_1 \left( \frac{y_A - y_B}{\ell} \right) + k_2 \left( \frac{y_A - y_B}{\ell} - \theta_{TS} \right) \right) \quad (4.19)
\end{align}

Laplace transforming and dropping the subscript \( s \)
\begin{align}
s \dot{y}_B &= V_C \theta_T + b \theta_T \quad (4.20) \\
se_T &= \frac{V_C}{b} \left( \frac{k_1}{\ell} y_A - \frac{k_1}{\ell} y_B + \frac{k_2}{\ell} y_A - \frac{k_2}{\ell} y_B \right) - k_2 \frac{V_C}{b} \theta_T \quad (4.21)
\end{align}
from (4.20)

\[ \theta_T = \left( \frac{S Y_B}{V_c + b S} \right) \quad (4.22) \]

Substituting (4.22) into (4.21) and putting in transfer function form gives

\[ \frac{Y_B}{Y_A} = \frac{(V_c^2/b\ell)(K_1 + K_2)(1 + b/V_c S)}{S^2 + \left( \frac{V_c K_2}{b} + \frac{V_c}{\ell}(K_1 + K_2) \right)S + \frac{V_c^2}{b\ell}(K_1 + K_2)} \quad (4.23) \]

which is seen to be a second order system with an additional zero.

Root contours for the characteristic equation, with \( b = 7 \) ft, as \( K_1 \) and \( K_2 \) are varied are shown for link lengths of 10 ft and 20 ft in Figures 4.4 and 4.5 respectively. Since the model is entirely kinematic the information obtained from these plots is independent of the velocity employed in obtaining them. A velocity of 5 fps was chosen as being representative of an actual field velocity.

The effect of the finite zero is to decrease the time to peak and increase the overshoot of the step response. The magnitude of this effect is dependent upon the location of the zero with respect to the roots and the origin and thus cannot be determined until an initial root location has been chosen.

In choosing suitable root locations two factors must be considered; the desired transient response and the ability to reject a disturbance. Most systems are designed with an underdamped transient response as it is usually advantageous to pay the price of reasonable overshoots and
$b = 7 \text{ FT.}$
$L = 10 \text{ FT.}$
$V = 5 \text{ FPS.}$

Figure 4.4: Root contours of simplified model characteristic Eqn. as $K_1$ and $K_2$ are varied. Note: Relative positions are independent of velocity.
Figure 4.5: Root contours of simplified model characteristic Eqn. as $K_1$ and $K_2$ are varied. Note: Relative positions are independent of velocity.

$b = 7$ FT.
$L = 20$ FT.
$V = 5$ FPS.
oscillations to gain a faster time response. In the above transfer function model however this would not always be desirable as a step input $y_A$ is representative of two physical conditions; firstly an actual input at point A, and secondly a lateral step displacement disturbance at point B. These two conditions dictate opposing transient characteristics.

If the input represents a physical input in the direction of the land larger than the overlap then a long wave length, which corresponds to a very low damped natural frequency, is required to avoid a miss. The overshoot if present must also be minimal to avoid a miss. A sluggish response is also desirable in rejecting any impulse displacement disturbances at point A.

In contrast an input representing a disturbance at point B would be best corrected by an underdamped response of higher frequency as the trailed implement will not then follow the disturbance, nor will it be appreciably affected by the oscillations. An underdamped response is also suitable for an actual input at point A which is away from the land, although it would result in unnecessary overlap.

Since practically a true step input at A is not attainable, and an approximate step will probably not occur often an underdamped response would be most suitable. Root locations cannot however be chosen on this basis alone, even for constant bearing operation, since the ability of the system to accommodate a constant disturbance is dependent upon the values of $K_1$ and $K_2$. 
A physical example of a constant disturbance is operation on a sidehill in which case a value of \( \phi \) other than zero is necessary to prevent the tractor from sliding down the hill. In the proposed system such a constant non zero angle \( \phi \), the slip angle, can only be obtained when an error exists. Required slip angles for satisfactory field operation are not known, however as an example for \( \varepsilon = 20 \text{ ft} \) if the minimum slip angle is allowed to be 10°, which corresponds to a side slip of approximately 17.5 ft per 100 ft, for an error of 1 foot then \((K_1 + K_2)\) must be greater than 3.5. If \( \varepsilon \) is reduced to 10 ft then \((K_1 + K_2)\) must be greater than only 1.8.

From the above discussion it would appear that suitable values for \( K_1 \) and \( K_2 \) could be selected for constant bearing operation. It would be desirable if parameters giving satisfactory constant bearing operation would also satisfy the requirements for small radius turns, thus eliminating the need for a motion duplication mode of operation. A less simplified model than the above is required to investigate this possibility.

4.2.2 Large Displacement Model

In order to investigate the effect of varying \( K_1 \) and \( K_2 \) on the curve following ability of the proposed system a more complex model was set up. Since it is the position of the implement trailed by the slave tractor which is of paramount interest, the implement was included in this model.
The following equations describe the motion of an implement such as that shown in Figure 4.6.

\[
\dot{x}_H = \frac{1}{h} (\sin \theta_I \dot{x}_H - \cos \theta_I \dot{y}_H) \quad (4.24)
\]

\[
V_D = (\cos \theta_I \dot{x}_H + \sin \theta_I \dot{y}_H) \quad (4.25)
\]

\[
\dot{x}_D = \cos \theta_I V_D \quad (4.26)
\]

\[
= \cos^2 \theta_I \dot{x}_H + \sin \theta_I \cos \theta_I \dot{y}_H \quad (4.27)
\]

\[
\dot{y}_D = \sin \theta_I V_D \quad (4.28)
\]

\[
= \sin \theta_I \cos \theta_I \dot{x}_H + \sin^2 \theta_I \dot{y}_H \quad (4.29)
\]

The point H is common to the implement and tractor and thus its motion is determined by the motion of the tractor. From inspection of Figure 4.3,

\[
\dot{x}_H = V_c \cos \theta_T + \theta_T \sin \theta_T \quad (4.30)
\]

\[
\dot{y}_H = V_c \sin \theta_T - \theta_T \cos \theta_T \quad (4.31)
\]

Note: \( \theta_T \) as shown is negative.

The above equations along with those previously presented for the simplified model are sufficient to describe the motion of the implement when the motion of point A is specified. However, in the case of large displacements, the simplifying approximations made in the previous analysis do not hold. Since the unsimplified equations would be difficult to solve, a numerical method of solution was employed.
Figure 4.6: Kinematics of a trailed implement.

Numerous numerical methods for solving ordinary differential equations exist. However for small sets of equations where a limited number of solutions are required the Runge-Kutta method has an advantage, especially when a digital computer is being used to perform the calculations, since it is self-starting and does not require explicit definitions of derivatives beyond the first. The method is explained in Reference 20, and its application to this problem is illustrated in Appendix A.
The program was set up so that point A could be caused to follow an inside or outside curve of varying degree and radius. Since the master and slave trailed implements follow paths of different lengths their speeds must be adjusted if the link length is to be kept constant or within specified bounds. In the small model this was accomplished by making the slave tractor speed a step function of the link length. The path for a 90° turn and its physical significance can best be understood by referring to Figure 4.7. Figure 4.8 is a plot of a typical result of this analysis. Plots for all values of \( K_1 \) and \( K_2 \) as determined by inspection are presented in Table 4.1.

Inspection of Table 4.1 reveals several important points. A constant value of 0.5 for \( K_1 \) would be satisfactory. The values of \( K_2 \) for inside and outside turns are different but constant for a given type of turn. None of the combinations appear compatible with those which would be chosen for a desirable small signal response. In the case of small radius turns however the angles \( \alpha \) and \( \beta \) are relatively large compared with those encountered in an essentially constant bearing operation. This suggests the employment of nonlinear angle measurement sensors, having an attenuated output at large angles thus effectively decreasing \( K_1 \) and/or \( K_2 \) for small radius turns. No computational trials were made to investigate the effect of non-linear sensors, due to the lack of knowledge of the types and strengths of disturbances which would be encountered in field conditions. Also as the radius of turn is decreased the assumption that a wheel (in particular the guiding wheels) rolls in the direction in which it is
Figure 4.7: System represented by large displacement model.
**Figure 4.8:** Predicted slave implement paths as $K_1$ and $K_2$ are varied.
### TABLE 4.1

Optimum Values for $K_1$ and $K_2$ Predicted From Model

<table>
<thead>
<tr>
<th>Type of Turn</th>
<th>Turn Angle deg.</th>
<th>$R_A$, ft</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside</td>
<td>90</td>
<td>15</td>
<td>0.5</td>
<td>0.8</td>
<td>$L(0) = 20$ ft for all turns</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>15</td>
<td>0.5</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>24</td>
<td>0.5</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Inside</td>
<td>90</td>
<td>24</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>8</td>
<td>0.4</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>24</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>40</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

pointed becomes less and less valid. The slip angle occurring during a turn is an unknown function of the radius of turn, the load on the tractor, soil characteristics, and characteristics of the tractor itself such as wheelbase, static front wheel weight and weight transfer.

Despite the apparent discrepancies in suitable coefficient values, it appeared that a variation of the proposed system would be feasible. Thus it was decided to proceed with the construction of a flexible experimental physical system in which parameters could be readily varied over a wide range and thus determine the effects of disturbances and slip angles under actual field conditions.
4.3 **Speed Control**

In order to keep the link length within bounds it is necessary, as explained above, to vary the speed of one or both of the tractors. Controlling the speed of both tractors has definite advantages in a practical system, however for experimental purposes it is desirable to limit the automatic speed control to the slave tractor due to the reduction in hardware required. Also, having all of the apparatus on a single tractor allows any other available tractor to be used as the master.

The proposed speed control is shown in block diagram form in Figure 4.9. Determination of the feasibility of this proposal requires a knowledge of the response of the tractor, i.e. the form of $G_T(S)$. Furthermore the design of the compensation requires that $G_T(S)$ be known quite accurately. This is, of course, peculiar to a specific tractor and is most readily and reliably obtained experimentally using frequency response methods.

It has been implied above that a transfer function approach is appropriate. Valid representation of an input-output relationship by a transfer function requires that the relationship be linear within the range of interest. Many real-world devices cannot be said to be linear even within a restricted range, however if their nonlinearity is small within a range about an operating point the transfer function approach may be advantageously employed.
Figure 4.9: Block diagram of proposed speed control loop.
Although the design of a system requires quantitative knowledge of the transfer functions which are to represent the system their form if known provides insight and allows qualitative observations to be made. If it is assumed that both an engine and a governor may be represented by first order systems as was done by Shepovalov(23), then the desirability and drawbacks of various control methods may be investigated. An engine may be considered as a torque source driving a load composed of inertia and viscous drag. The inertia includes the rotating mass of the engine and power train as well as the inertia due to longitudinal motion, the latter being a function of the gear ratio. The viscous drag represents, along with the internal friction of the engine and power train, the drawbar load of the implement, considering only variations around an operating point, and thus would also be dependent upon the gear ratio. This is illustrated in Figure 4.10, and may be expressed mathematically as follows:

For the Governor \( G(S) = \frac{1}{T_G S + 1} \) \( \quad (4.32) \)

For the Engine \( G(S) = K_T \) \( \quad (4.33) \)

Also \( T = M\omega + f\omega \) \( \quad (4.34) \)

so that \( T(S) = (MS + f)\omega(S) \) \( \quad (4.35) \)

or \( \omega(S) = \frac{1}{T} \frac{1}{MS + f} \) \( \quad (4.36) \)

and \( \frac{R(S)}{C} = \frac{KT/M_T}{S^2 + \left( \frac{1}{T_G} + \frac{f}{M} \right)S + \frac{f + KT_K}{MT_G}} \) \( \quad (4.37) \)
Figure 4.10: Block diagram of tractor dynamics.
Equation (4.37) indicates the expected form of \( G_T(S) \). If this is placed in the proposed speed control loop, replacing the engine-governor-gear ratio and dynamics blocks, and the speed control drive motor is assumed to be a pure integration then the following loop transfer function results:

\[
G_H(S) = \frac{K_T/MT_G}{S^2(S^2 + \frac{1}{T_G} + \frac{f}{M}S + \frac{f + K_T K_G}{MT_G})} \cdot H(S) \quad (4.38)
\]

The effect of the zero due to the term \((T_G S + 1)\) would not be significant in the range of interest if \( T_G \) is sufficiently small which would be expected. If such is the case then it would no doubt be further superfluous as other factors are neglected such as time delay for increased fuel to reach the cylinders after throttle opening. It is proposed that \((T_G S + 1)\) be neglected and the loop transfer function be written as:

\[
G_H(S) = \frac{K_T/MT_G}{S^2(S^2 + \frac{1}{T_G} + \frac{f}{M}S + \frac{f + K_T K_G}{MT_G})} \cdot H(S) \quad (4.39)
\]

Assuming (4.39) to be a realistic indication of the transfer function it is noted that although \( H(S) \) cannot be a simple gain factor for reasons of stability there is the possibility of attaining stability with a feedback compensator of the form.
\[ G_c(S) = \frac{T_2S + 1}{T_2S + 1} \]  

(4.40)

where \( T_1 \gg T_2 \). Such a compensator may have to be readily variable to compensate for changes in gear ratio.

An accepted design method for systems containing a relay where the remainder of the system loop may be considered linear is a modification of the Nyquist technique. The relay is represented by a describing function \( G_p(jw) \) independent of frequency and dependent upon the amplitude of the input. The describing function of a relay with dead zone is derived in Appendix C.

In employing the Nyquist technique if the phase-gain locus encircles the critical point \((-1)\) instability is indicated, and if the locus passes through the critical point a marginally stable system is indicated. In the modification the critical point is expanded into the locus of \( \frac{1}{G_D} \). Encirclement of this locus by the phase-gain locus of the remainder of the loop indicates instability, while intersection of the two loci indicates the existence of a limit cycle. The magnitude of the limit cycle is determined by the magnitude of \( G_D \) at the point of intersection while the frequency is determined as that required to obtain the phase lag in the remainder of the system which is indicated by the intersection point.

The plots showing the form of an uncompensated and suitable compensated system are given in Figure 4.11.
Figure 4.11: Nyquist diagram for (a) uncompensated system,
(b) suitably compensated system.
Replacing the speed control drive motor with a fast acting proportional controller would eliminate the instability problem but a limit cycle would still possibly be present. The extra difficulty due to the integral type controller is justified since this results in a type 2 system and a steady state link length error of zero.
5. PHYSICAL SYSTEM

5.1 Description

For the experimental system a physical link was employed to connect the master and slave tractors, namely a cable wound upon a counter-weighted drum. At points A and B (Figure 4.1) single turn potentiometers fitted with a light arm having a loop at its extremity through which the cable passed, gave a measure of the angles \( \alpha \) and \( \beta \).

The steering wheels were positioned by driving the manual steering wheel with a D.C. motor through a chain and sprocket reduction. A measurement of \( \phi \) was obtained from a ten-turn potentiometer directly coupled to the steering drive motor. This drive is shown in Figure 5.1.

Speed control was accomplished by actuating the governor input with a PMDC gearmotor and chain and sprocket drive. The input to the controller was obtained from a ten-turn potentiometer directly coupled to the drum on which the cable was wound.

5.2 Component Selection and Design

5.2.1 Cable Tension Apparatus

The requirements of a physical link are that it be sufficiently "rigid" to operate the potentiometers and have the ability to vary in length over a wide range. A tensioned cable combined with a drum from which it may be wound in or out fulfills these requirements. Furthermore a shielded multiple conductor cable may be employed, facilitating signal transmission between the units. The steady state
Figure 5.1(a): Top view of Steering Drive
Figure 5.1(b): Side view of steering drive and speed control drive.
link length is also readily changed without any compromise in allowable deviations which is advantageous on an experimental machine.

The tension in the cable must be controlled so that it does not become so slack, except momentarily, as to be incapable of operating the sensors or so taut that it fails. Early failure, due to bending fatigue at guide rollers or the drum, is possible at tensions much less than that required to fail the cable in pure tension. Maximum cable life will be obtained by using a tension just sufficient to provide positive sensor operation. Three possible methods of maintaining the tension are discussed below.

The most obvious method for precise tension control is to actually measure the tension and drive the drum accordingly. The tension could be measured by looping the cable over an idler anchored by a spring, so that the idler displacement would be proportional to the tension. A signal from a displacement transducer would then cause the drum to be driven in the appropriate direction by means of an electrical or hydraulic motor.

This method would allow the tension to be controlled within close limits and thus the low static tension desired could be obtained.

An alternate method dispenses with the tension measurement at the expense of an increase in static tension. A constant torque would be applied to the drum by means of an electrical torque motor, "Constant torque" spring motor, or a hydraulic motor supplied with constant pressure. The hydraulic motor would be most readily employed on tractors.
equipped with constant pressure hydraulic systems. The spring motor has the advantage of being independent of the tractor and thus allows the tensioning device to be self contained.

Accelerating and decelerating of the motor and drum would be a result of the differences between the torque applied by the motor and that resulting from the tension in the cable. A drum of low inertia and friction is required to reduce the cable tension deviations required and thus in turn the required static tension. Shock loading could be minimized by passing the cable over a spring mounted idler.

A third method is illustrated schematically in Figure 5.2. This device is similar to method two above except that the opposing torque is provided by a pair of cables, tensioned by a weight box, wound upon small diameter drums integral with the link cable drum. The link cable tension deviation must now accelerate the weight box as well as the drum, however the motor is dispensed with. This method appeared to have distinct advantages for an experimental system. Primarily it is completely independent of the tractor on which it is mounted and thus it could be built and tested without reference to any specific tractor. Secondly the static tension of the link cable is readily varied by changing the weight of the weight box, a feature not readily provided by a spring motor. A major disadvantage of this system is the limited range of link length obtainable while keeping the weight box displacement reasonable. The link cable to weight box displacement ratio can be increased only with the expense of increased friction and a larger
Figure 5.2: Method of operation of link cable tensioning device

weight. Some problems could also be encountered when driving over rough ground since the force required for vertical acceleration of the weight box must be provided by variations in the tension of the supporting cables.

The versatility of this method was felt to outweigh its disadvantages and a tensioning device of this type was constructed based on the following design criteria:
- Minimum static tension capability of 25 lbs.
- Maximum weight of weight box 250 lbs.
- Minimum cable length deviation of 25 ft.
- Maximum weight displacement of 3 ft.
- Drum displacement of less than 10 revolutions
- Provision for addition of a shock absorber.

The drum was constructed of aluminum allowing a rigid, rugged construction with low interia. The drum was mounted in sealed self-aligning ball bearings. The self-aligning feature of the bearings allowed the main frame to be of one piece welded construction with loose dimensional tolerances. All guide rollers which the link cable passed over were standard idler pulleys having integral sealed ball bearings.

The completed tensioning device is shown mounted on a tractor in Figure 5.3.

5.3 Steering Control

5.3.1 Angle Measurement Transducers

As noted in the general description above these transducers were single turn potentiometers provided with an arm actuated by the cable. In Section 4.2.2. (page 29) the possible desirability of non-linear sensors having a reduced gain at the larger angles was pointed out. Figure 5.4 illustrates how this was accomplished. For small angles the centre of turning of the link is effectively behind the
Figure 5.3(a): Front view of cable tensioning apparatus.
Figure 5.3(b): Side view of cable tensioning apparatus.
potentiometer axis so that the arm turns through a larger angle than the cable. For larger angles the cable contacts the guide rollers and the contact point becomes the centre of turning. The arm now turns through a smaller angle than the link cable providing the desired attenuation.

This difference in turning centres however resulted in a problem as the cable tended to bind in the loop at large angles. It was not desirable to overcome this by increasing the loop width as this increases the dead zone. A satisfactory solution was to use a spring for one side of the loop as shown in Figure 5.4. The spring was rigid enough to operate the potentiometer but flexible enough to allow the cable to extend and retract without damaging the arm.

The figure also illustrates that the degree of change in gain is adjustable by moving the potentiometer longitudinally and that the guide spacing determines the range of the high gain portion. The form of the input output relationship is shown in Figure 5.5. Figure 5.6 shows the \( \alpha \)-transducer mounted on a cultivator for offset operation.

The output signal from the transducer at the master end of the cable (\( \alpha \)-sensor) is transmitted through the cable as is the supply. The signal and supply connections to the rotating end of the cable are made by means of a brush and slip ring assembly. Two diametrically opposed brushes are used for each conductor to reduce the possibility of signal loss. The brush and slip ring assembly can be seen in Figure 5.7.
Figure 5.4: Angle Measurement Transducer
Figure 5.5: Input-output relationship for angle measurement transducer.
Figure 5.6: a transducer mounted for offset operation.
Figure 5.7: Close-up of cable tensioning apparatus showing cable length transducer, brush and slip ring assembly and β transducer.
Commercial automotive alternator brush and holder assemblies mounted on a plexiglass plate form the brush assembly. The slip ring assembly was constructed by pressing brass tubing over thick walled plexiglass tubing and then cutting away the unwanted brass.

5.3.2 Steering Drive

Most farm tractors are fitted with power steering which employs a hydraulic ram to position the guiding wheels. Two possibilities exist for positioning the guiding wheels automatically. Oil flow to the positioning ram may be controlled directly by an electro-hydraulic valve. Or, alternatively, the manual steering wheel may be mechanically driven.

Direct control of the oil flow easily provides a fast response as little inertia is present, however its adaptation to existing systems of the assist type having a value integral with the ram is not practical. Mechanically driving the steering wheel is adaptable to any type of system, including non-power. For power systems a small torque is sufficient to drive the steering wheel however a motor with considerable reserve torque for acceleration is required to provide a fast response.

The tractor available for test purposes was fitted with a valve integral with the ram so mechanical drive of the manual steering wheel employing an electric motor was selected. A 1/8 h.p. 200 RPM gear motor in conjunction with a chain reduction provided ample torque for acceleration while driving the system at near capacity rate. The speed of
response for this type of drive, while maintaining full power assist, is limited by the retraction speed of the ram with the valve fully open. The drive is shown installed on the test tractor in Figure 5.1.

5.3.3 Wheel Position Feedback

A wheel position feedback signal may be obtained by connecting a potentiometer to any link in the steering mechanism which has a fixed relationship to the guiding wheels. Since the steering system on the tractor available for test was of the assist type, as opposed to hydrostatic, a fixed relationship exists between the manual steering wheel position and thus also the drive motor, and the position of the guiding wheels. Therefore the most convenient method of obtaining a feedback signal was to directly couple a multi-turn potentiometer to the driving motor shaft.

The relationship between potentiometer output and guide wheel position (φ) was determined experimentally. The tractor was placed on a concrete floor and a chalk line drawn on the floor directly below and parallel to the rear axle. A specially fabricated "square" was mounted on one front wheel which facilitated the extending of a string normal to the plane of the wheel. The intersection of this string with the chalk line was marked and the output of the potentiometer noted for a number of positions. Since the intersection point is the nominal centre of rotation the angle φ of an imaginary guide wheel located in the centre of the front axle may be obtained from the relationship
\[ \phi = \tan^{-1} \frac{b}{R} \]  

The results of this calibration are shown in Figure 5.8. The slope of the best fit line shown in the figure was considered to be a satisfactory representation of the relationship throughout its entire range. The non-zero intercept is due to potentiometer wheel zero misalignment and is of no consequence.

Since this feedback potentiometer has a different input-output relationship than the \( \alpha \) and \( \beta \) sensors a gain is required to make it equivalent.

For the \( \alpha \) and \( \beta \) Sensors \( K = \frac{270^\circ}{30V} = 9^\circ/V \)

For the \( \phi \) potentiometer \( K = \frac{3.4^\circ}{V} \)

therefore the gain required is \( \frac{3.4}{9} = 0.38 \)

5.3.4 **Summer Relay**

In order to effect the relationship \( \phi = K_1\alpha + K_2\beta \), where \( K_1 \) and \( K_2 \) are now understood to be functions of \( \alpha \) and \( \beta \) respectively due to the non-linear sensors, it is necessary to force the sum \( (K_1\alpha + K_2\beta - \phi) \) to zero. This was accomplished by means of a summing amplifier driving a transistorized relay which controlled the steering drive motor. Figure 5.9 is a schematic diagram. A high input impedance operational amplifier was used as the summing device. A high input impedance was desirable in this case since it facilitated high impedances throughout the system; lowering the demands on the regulated power supply, which was a pair of low cost integrated circuits. Although
Figure 5.8: Calibration of guidewheel position feedback potentiometer.

\[ \text{SLOPE} = \frac{46}{13.5} = \% \]
Figure 5.9a: Schematic circuit diagram of steering control system.

Figure 5.9b: Block diagram of steering control loop.
an all-transistorized relay results in higher losses in the power circuit than an electromechanical one. It is more trouble free and reliable. The final stage power transistors were mounted in heat sinks which enabled them to carry the stalled motor current indefinitely. The possibility of stalling the drive motor was unlikely but always present as a result of turning the system on without having the engine running or a minor malfunction causing the steering-gear to saturate. A schematic circuit diagram of the transistorized relay is shown in Figure 5.10.

The operation of the relay is described in Appendix D where it is shown that it has a fixed dead zone of ± 0.6V. The dead zone of the combined summer and relay, however, may be made any desired value by proper selection of the amplifier input and feedback resistors. The maximum allowable dead zone is determined by the allowable steady state error. In the case of a steady state error the input to the summer-relay is $K_1\alpha + K_2\beta$ where $\alpha$ and $\beta$ are equal ($\phi$ must be zero for constant bearing steady state). Assuming $K_1 = K_2 = 0.7$ then

$$e_i = 1.4 \ (\text{in})$$

$$\alpha = \frac{1/4}{20} = 0.0125 \quad (5.3)$$

and

$$e_i = 1.4 \times 0.0125V = .1115V \quad (5.4)$$

Now $\pm e_i G$ must equal ± 0.6V where $G = R_f/R_i$ the amplifier gain.

therefore

$$G = \frac{0.6}{.115} = 5.21$$
Figure 5.10: Steering control system transistorized relay.

Q3 = Q7 = 2N3055  Q1 = Q5 = 2N1307  D1 = HD184D  R4 = 1K  R8 = 10K  
R7 = 8.2K  Q4 = 2N1306  R1 = R2 = 3.3K  R5 = 1.8K  R9 = 22K  
Q2 = Q6 = 2N2219  R3 = 0.68K  R6 = 33  F1 = F2 = Load
5.3.5 **Velocity Feedback**

Close control of the static error required a very small dead zone. This can be illustrated by calculating the angular displacement of the steering drive motor corresponding to the dead zone ($\theta_D$). For a dead zone of 0.1115V, a ten turn potentiometer excited by ±15V and the required attenuation of 0.379, this is

$$\theta_D = 0.1115V \times \frac{1}{0.379} \times \frac{10\text{rev}}{30V} = 0.098 \text{rev}. \quad (5.5)$$

It was anticipated that this could result in a limit cycle without some form of rate feedback. A velocity signal could be obtained by employing a tach-generator coupled to the motor or differentiating the position feedback signal. Neither of these were used however. A third alternative less expensive than a tachometer and less noise sensitive than "true" differentiating was employed in the form of a velocity simulation circuit, similar to that described by Grovum\(^8\). This circuit shown in Figure 5.11 provides a signal which very closely approximates the velocity of the loaded motor. The actual step response of the motor and load, obtained by recording the output of the position potentiometer, is shown in Figure 5.12, along with the theoretical output of the velocity simulation circuit. Inspection of the response indicates that it could be described by a first order transfer function and thus rate feedback would not necessarily be required. The relay driving amplifier however was fitted with a large feedback capacitor to avoid noise thus introducing a further lag.
Figure 5.11: Velocity Simulation Circuit
STEERING MOTOR AND LOAD 2

STEP RESPONSE CHART SPEED = 1"/SEC.
±15V ON TEN TURN POT, COUPLED DIRECT TO MOTOR.
SPAN = 20V.

Figure 5.12: Recorded step response of steering motor and load, and theoretical output of velocity simulation circuit.
The necessity of rate feedback was confirmed when its removal resulted in a limit cycle.

The magnitude of the feedback signal was adjustable by means of a dual potentiometer which allowed various responses to be readily obtained. The amount of velocity feedback required can be estimated for a two step response and final position error of zero for a step input of sufficient magnitude to result in the motor attaining a steady state velocity. The desired response is shown in Figure 5.13.

The change in output voltage due to coasting is $\Delta e$ and if this is greater than $2e_z$ then $G_2$, the velocity feedback gain, must be greater than zero to prevent a multi-step response. Furthermore if $\Delta e$ is greater than $e_z$ then $G_2$ must be greater than zero to result in zero error.

At the time of turnoff $e_s = e_z$

or $G_1(E_1 - e_0) - G_2 e_V = e_z \quad (5.6)$

Also $E_1 - e_0 = \Delta e \quad (5.7)$

so $G_2 = \frac{(\Delta e)G_1 - e_z}{e_V} \quad (5.8)$

Now $\Delta e$ may be determined from the step response curve and thus $G_2$ evaluated using equation (5.7). From Figure 5.12, $\Delta e$ was estimated to be $1.3V$ at the potentiometer which is attenuated to $0.494V$; $e_V$ can be seen from Figure 5.11 to be $12V$

Substituting the numerical values gives

$$G_2 = \frac{0.494(5.2) - 0.6}{12} = 0.208$$
Figure 5.13: Desired response of servomechanism to step input.

\[ e_s = -G_1 (E_i - \Theta_o) - G_2 e_v \]
5.4 Speed Control

The objective of the speed control is to keep the steady state link length constant and dynamic excursions within bounds. Obviously this is limited by the response of the tractor and load which are fixed. The primary limitation is the speed range available by changes in only the governor setting. This not only defines the steady state operating point limits, but also seriously affects the dynamic response when the operating point is close to the limit.

The design of the speed control involved determining suitable compensation such that the link length dead zone would be less than a prescribed minimum while avoiding a detrimental limit cycle. Also to be designed was a governor actuator for use in the experimental determination of the frequency response of the engine-governor combination and which could be easily adapted as the actuator in the completed system. A PMDC gearmotor and chain and sprocket drive proved to be a satisfactory actuator.

5.4.1 Open Loop Uncompensated Frequency Response

5.4.1(a) Experimental Determination of Engine and Governor Frequency Response

The manual speed control mechanism on the Massey Ferguson 165 tractor available for test purposes is shown schematically in Figure 5.14. A test was made to determine the degree of linearity in the relationship of unloaded engine speed (output) to the position of shaft "A" (input). See Figure 5.14. A potentiometer was coupled to
Figure 5.14: Manual Speed Control Mechanism on Massey Ferguson 165 Tractor.
Figure 5.15: PTO Speed as a function of input speed command.

Key:
- • - INCREASING
- X - DECREASING

LINEARITY TEST
SPEED V.S. SHAFT "A" POSITION
(See Fig. 5.14)
shaft A to indicate its position, while the speed was measured by means of a D.C. tachometer generator coupled to the P.T.O. shaft. The results of this test are shown in Figure 5.15. Part of the upper end hysteresis is due to the flexible coupling between the potentiometer and shaft A. Although the relationship is non linear in the operating range of interest it was not felt that rejection of a frequency response transfer function method of approach was warranted on these grounds alone.

In order to experimentally determine the frequency response of the engine governor combination it was necessary to provide a sinusoidal input to shaft A. In order to accomplish this a governor actuator employing a permanent magnet field D.C. gearmotor with further chain and sprocket reduction was constructed to replace the manual control lever. This unit is shown schematically in Figure 5.16. It was anticipated that this drive could later be used in the experimental automatic speed control system especially since it provided a means of readily changing the gain by changing sprockets.

Figure 5.17 shows, in a block diagram, the arrangement employed in the experimental determination of the frequency response the input (I) countershaft position ($\theta_c$) and the output speed were recorded simultaneously on a multichannel recorder. A single turn potentiometer rigidly coupled to the chain reduction countershaft was used to obtain $\theta_c$ as well as provide the indicated feedback. The feedback was required to hold the operating point constant at an engine speed of 1600 RPM.
Figure 5.16: Governor Actuator employed during the experimental determination of $G_T(s)$. 

SAME AXIS AS MANUAL CONTROL LEVER (SEE FIG. 5.14)

TO SHAFT "A" - (SEE FIG. 5.14)

PMDC MOTOR

COUNTER SHAFT

CHAIN
Figure 5.17: Block diagram for experimental determination of $G_t(S)$. $I$, $\theta_c$, and speed were recorded on a multichannel recorder.
Initial trials without the feedback resulted in a slow drift of the operating point due to the imbalance of the load.

The results of this test are shown graphically in Figures 5.18, 5.19 and 5.20. From the experimental results the engine-tachometer combination was considered from inspection to be adequately represented by the transfer function

\[ W = \frac{K_E (4.7)^2}{\theta_c \left( S^2 + 0.7(4.7)S + (4.7)^2 \right)} \]  

(5.8)

In the same manner, but considering the magnitude ratio only, the transfer function for the drive motor would be

\[ \theta_c = \frac{K_M}{V} \frac{S(0.15S + 1)}{(4.7)^2} \]

(5.9)

5.4.1(b) Effect of Gear Ratio

The overall loop contains in addition to the experimentally determined functions the gear ratio relating no load ground speed to engine speed which are shown in Table 5.1. The expected effects of changes in gear ratio other than a simple change of gain are discussed in Section 4.3.1.

5.4.1(c) Cable Length Transducer

Also included in the loop is the cable length transducer which was a ten turn potentiometer with ± 15 Volt supply directly coupled to the cable drum. The transfer function is thus given by

\[ \frac{\Delta E}{\Delta L} (S) = \frac{30V}{10 \text{ turns} \pi D} \]
Figure 5.18: Experimental determination of frequency response of the governed tractor engine.
Figure 5.19: Experimentally Determined Frequency response of the Governor Controller Motor.
Figure 5.20: Phase Lag of the Governed Tractor Engine vs. Frequency.
<table>
<thead>
<tr>
<th>Range</th>
<th>Gear</th>
<th>M.P.*</th>
<th>fps</th>
<th>mph</th>
<th>(K_n) (ft/e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOW</td>
<td>1</td>
<td>L</td>
<td>1.51</td>
<td>1.032</td>
<td>9.02 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>1.97</td>
<td>1.345</td>
<td>11.78 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>L</td>
<td>2.56</td>
<td>1.542</td>
<td>15.3 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>2.97</td>
<td>2.025</td>
<td>17.7 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>L</td>
<td>4.15</td>
<td>2.835</td>
<td>24.8 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>5.45</td>
<td>3.71</td>
<td>32.5 x 10^{-3}</td>
</tr>
<tr>
<td>HIGH</td>
<td>1</td>
<td>L</td>
<td>6.05</td>
<td>4.12</td>
<td>36.1 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>7.91</td>
<td>5.39</td>
<td>47.3 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>L</td>
<td>9.35</td>
<td>6.37</td>
<td>55.9 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>11.90</td>
<td>8.11</td>
<td>71.0 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>L</td>
<td>16.65</td>
<td>11.35</td>
<td>97.4 x 10^{-3}</td>
</tr>
</tbody>
</table>

* MP = Multipower
L = Low range
H = High range
Since D the diameter of the drum was 1 ft.

\[ \frac{E_0}{\Delta L}(s) = 0.955 \text{ Volts/ft.} \]

Figure 5.21 is a block diagram of the loop while the overall loop response for two gear ratios bounding the range of interest are shown in Figure 5.22. The observed phase angles are shown and the gear ratio has been assumed to be a simple gain. The mass and viscous drag of the tractor and implement have been neglected.

5.4.1(d) The Relay

The transistorized relay shown in Figure 5.23 is somewhat similar to that employed in the steering loop. The operation is explained in Appendix E. The power transistors were mounted on the chassis which provided a more than adequate heat sink. A dead zone of ± 0.6V is obtained in the same manner as with the steering control relay. The nominal output of ± 12 V results in a maximum describing function value of 12.7 as shown in Figure 5.22.

5.4.2 Closed Loop Operation

5.4.2(a) Required Compensation

Inspection of the uncompensated response in Figure 5.22 or inspection of Figure 5.21 with \( H(S) = K \), makes it obvious that some form of compensation is required as neither a decrease in gain or alternatively an increase in the dead zone will rectify the undesirable situation. However a feedback transfer function of the form

\[ H(S) = K_3 + K_4 S \] (5.10)
Figure 5.21: Block diagram of speed control loop.
Figure 5.22: Open Loop Frequency Response of the Speed Control System
Figure 5.23: Speed control system transistorized relay.

- $R_1 = R_2 = 3.3 \text{K}$
- $R_3 = 0.39 \text{K}$
- $R_4 = 0.22 \text{K}$
- $D_1 = \text{HD1840}$
- $Q_1 = \text{2N1306}$
- $Q_2 = \text{TIP32}$
- $Q_3 = \text{2N1307}$
- $Q_4 = \text{TIP31}$
will, for suitable values of \( K_3 \) and \( K_4 \), rectify the situation as is also shown in Figure 5.22. The suitability of this form of compensation may also be illustrated with Nyquist plots. The form of the uncompensated and compensated Nyquist plots of the linearized portion of the loop are shown in Figure 5.24. An undesirable limit cycle may be avoided by preventing the Nyquist plot of the linear portion of the system from cutting the locus of \(-1/G_o\) in the vicinity of the minimum value of \(1/G_o\) where \(G_o\) is the describing function of the relay.

If the feedback transfer function is expressed as

\[
H(S) = K \left( 1 + \frac{K_4}{K_3} S \right)
\]

then it may readily be seen how \( K_3 \) and \( K_4 \) may be correctly chosen. \( K_3 \) is selected to give a desired cable length dead zone, then \( K_4 \) must have a value which gives the desired phase lead characteristics, i.e. \( K_3/K_4 \) is the corner frequency.

5.4.2(b) Realization of Compensation

A compensator which will provide the desired form is shown schematically in Figure 5.25 and may be represented by the transfer function

\[
\frac{E_o}{E_i}(S) = - \left( \frac{R_1 + R_2}{R_1} + \frac{R_1 R_2 CS}{R_1} \right)
\]

as shown in Appendix F.

In this case \( K_3 = \frac{R_1 + R_2}{R_i} \)  

(5.13)
Figure 5.24: Nyquist diagram for (a) uncompensated system, (b) suitably compensated system.
and \[ K = \frac{R_1 R_2 C}{R_1} \]  \hspace{1cm} (5.14)

For this application however, the required resistances are undesirably large and an addition to the circuit to overcome this was deemed desirable. This is shown in Figure 5.26 which also shows the addition of two attenuating voltage dividers which facilitate rapid adjustments of the compensator characteristics which could be required for different gear ratios.

Considering the attenuators as pure voltage dividers the overall compensator transfer function becomes

\[
\frac{E_o(s)}{E_I} = P_1 \frac{R_F}{R_3} \left( \frac{R_1 + R_2}{R_1} + \frac{R_1 R_2 C}{R_1} \right) - P_2 \frac{R_F}{R_4}
\]

so that \[ K_3 = P_1 \left( \frac{R_F (R_1 + R_2)}{R_3 R_1} \right) - P_2 \left( \frac{R_F}{R_4} \right) \]  \hspace{1cm} (5.16)

and \[ K_4 = P_1 \left( \frac{R_F R_1 R_2 C}{R_3 R_1} \right) \]  \hspace{1cm} (5.17)

where \( P_1 \) and \( P_2 \) are the potentiometer settings taking values between 0 and 1. The effects of varying \( P_1 \) and \( P_2 \) are shown in Figure 5.27 which may be used as a guide in selecting suitable values.

5.4.2.2(2) Cable Length Dead Zone

As stated in the previous section the cable length dead zone is determined by \( K_3 \), which is variable, and thus some leeway is
available. The advantage of using this type of cable length control system, i.e. "zero position error" is only fully realized as the dead zone approaches zero. If \( K_3 \) is chosen equal to 1, then, since the relay dead zone is nominally \( \pm 0.6 \) V the cable length dead zone will be

\[
\pm \frac{0.6 \text{V} \times 10 \text{ turns}}{30 \text{V}} \times \frac{\pi \text{ft}}{\text{turn}} = \pm 0.628 \text{ ft.}
\]

As this was deemed a satisfactorily small dead zone, \( K_3 \) was set at 1.

\[\text{Figure 5.25: Basic Compensator Circuit}\]
Figure 5.26: Circuit diagram of speed control system. The portion of the circuit from $e_i$ to $e_0$ constitutes the feedback compensator.
Figure 5.27: Effect of varying $P_1$ and $P_2$ on the Compensator Characteristics.
6. FIELD TESTING

6.1 Objective

The objective of the field test was two fold. Primarily the testing was to evaluate the constructed system both qualitatively and quantitatively. Secondly it was desired to evaluate the large displacement steering model as a predictor of the prototype response.

6.2 Procedure

6.2.1 Preliminary Qualitative Testing

Preliminary qualitative observation of the system as well as all quantitative tests were made with the slave tractor following directly behind the master tractor as opposed to the offset position. It was readily apparent that the initial steady state link length in excess of 25 ft. was neither necessary or desirable, so it was reduced to approximately 15 ft. which provided satisfactory operation. During this qualitative testing the potentiometers controlling $K_1$ and $K_2$ were varied until settings were found which gave satisfactory operation for large radius turns, of varying degree up to 180° as indicated by coincidence of the markers (described in Section 6.3) in the turn although no measurements of curve entrance and exit characteristics were taken.

Slippage of the guide wheels, the possibility of which had been an area of some concern, was not noticed when making 180° turns with no drawbar load. With a drawbar load side slip was evident at a turn
radius of 25 ft. and was sufficient to prevent the negotiation of a 15 ft. radius turn. These results were sufficiently promising to warrant quantitative tests without further modification of the system.

With respect to stability, both the directional and speed controls appeared quite stable and limit cycles were not observed in either system.

6.2.2 Quantitative Tests

Two conditions may be envisioned in practice which would tax the following ability of the system. Firstly the end turn and secondly a manoeuver required to avoid an obstruction. The first is the already familiar 180° turn and the second can be represented by a sinusoidal path preceded and followed by a constant bearing stretch. The sinusoidal paths used for testing and their arrangement are shown in Figure 6.1.

Since the slave tractor was coupled directly behind the master, the α-sensor was attached to the master tractor drawbar. The path of this point of attachment was thus the desired path of the centre of the slave drawn implement. In conducting the sinusoidal tests the master tractor operator attempted to keep the α-sensor centred over the sinusoidal path which had been marked out on the field with lime. The actual path of the sensor was recorded by means of a spring loaded marker wheel. With an experienced tractor operator the deviation of the actual path from the marked path was insignificant.
Figure 6.1: Layout of sinusoidal paths for field tests.
Figure 6.2: Device for marking path of centre of the slave trailed implement.
The marking out of a path for the master tractor for the 180°
turns was done in a different manner. A base line was initially marked
on the field with lime. The master tractor was driven up to the base
line along a path normal to the base line, corresponding to the
desired radius, and stopped with the sensor over the base line. A
path was then marked out with lime at the radius of the inside guide
wheel of the master tractor. The spring loaded marker wheel on the
drawbar was again used to indicate the actual path.

For both types of tests a marker was also required to determine
the path of the centre of the slave trailed implement, a heavy duty
cultivator. The centre shanks were removed from the cultivator and
replaced with a double disc furrow opener as shown in Figure 6.2. The
marker was positioned longitudinally so that its soil contact point
and the soil contact point of the cultivator wheels would be collinear.

Only a small number of quantitative tests were run due to the
time required to measure and mark out the field. Before conducting
any of the tests the values of $K_1$ and $K_2$ were adjusted to values which
were expected on the basis of the preliminary qualitative tests, to
give reasonably good performance. Optimum use was thus made of the
laboriously marked out paths.

6.3 Results

The first test was for 180° turns both with and without a drawbar
load. The results are summarized in Table 6.1. A second set of 180°
TABLE 6.2

Test 2: Experimental Results for Trailed Implement, 180° Turn.

<table>
<thead>
<tr>
<th>$R_A$, ft</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>$R_s$, ft</th>
<th>Maximum Error, ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.65</td>
<td>0.78</td>
<td>15.5-16.9</td>
<td>+ 1.9</td>
</tr>
<tr>
<td>24(24.2-24.5)</td>
<td>0.65</td>
<td>0.78</td>
<td>24 -25.6</td>
<td>- 0.8</td>
</tr>
<tr>
<td>40(39.9-40.8)</td>
<td>0.65</td>
<td>0.78</td>
<td>39.6-41.1</td>
<td>- 0.9</td>
</tr>
<tr>
<td>60(59-59.6)</td>
<td>0.65</td>
<td>0.78</td>
<td>59.4</td>
<td>+ 0.4</td>
</tr>
<tr>
<td>80(78.6-80)</td>
<td>0.65</td>
<td>0.78</td>
<td>79.1-79.8</td>
<td>± 0.5</td>
</tr>
</tbody>
</table>
The sinusoidal tests proved to be quite severe however the slave was capable of following the path at a reduced amplitude as evidenced by the results in Table 6.3.

TABLE 6.3
Sinusoidal Paths

<table>
<thead>
<tr>
<th>Amplitude (ft)</th>
<th>Cycle Length, ft</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>60</td>
<td>0.7</td>
<td>0.8</td>
<td>Very poor, not measured</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>0.7</td>
<td>0.6</td>
<td>Error less than 6 in.</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>0.7</td>
<td>0.6</td>
<td>Error less than 6 in.</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>0.7</td>
<td>0.6</td>
<td>Trailing implement amplitude = 2 ft*</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>0.7</td>
<td>0.6</td>
<td>Trailing implement amplitude = 4 ft*</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>0.7</td>
<td>0.6</td>
<td>Maximum error less than 6 in.</td>
</tr>
</tbody>
</table>

* No "phase shift" was observed

6.4 Offset Operation

Qualitative tests of offset operation were conducted employing 10 ft, heavy duty cultivators. The cultivator trailed by the master tractor was fitted with a boom on which the $\alpha$-sensor was mounted as shown in Figure 6.3. No further problems were encountered except for an insufficient speed range in the slave tractor when negotiating sharp turns as was expected. This required that the speed of the master tractor
Figure 6.3: General view of master-slave tractors with cultivators in the field.
be manually controlled when making small radius turns of 90° or greater. Limiting turns to inside turns and allowing the slave to stop by disengagement of the clutch would overcome this problem. A better solution performance exercise would be to control the speed by means of an infinitely variable ratio transmission.
7. SUMMARY AND CONCLUSIONS

7.1 Summary

Multiple unit control is one goal of all automatic tractor guidance systems. The master-slave system attains this goal by having one or more units guided, through some form of link, by an operator controlled unit.

Mathematical models indicated that such a system could be constructed from relatively simple components and still provide satisfactory operation for both switchback and perimeter operations. As a result an experimental system was constructed employing a variable-length link in the form of a cable wound upon a counterweighted drum. The orientation of the cable with respect to the master-slave units provided the inputs to the steering control system of the slave tractor while its length provided the input to a speed control system which attempted to maintain the length constant.

Both the steering and speed control systems were electromechanical relay operated systems. Control was obtained by actuating the manual steering wheel and governor input. Suitable compensation was developed to provide satisfactory field performance, although the speed control system proved to have too small a range of speed variation.

Field tests indicated that the controlling information obtained from the link was adequate and that the concept was sound.
7.2 Conclusions

Despite the limited field testing several definite conclusions may be drawn.

1) The simple system described is capable of controlling a slave tractor under load for constant bearing operation and around turns of 25 foot radius or larger. End turns of less radii are negotiable with no drawbar load.

2) The mathematical model was sufficiently accurate as the basis for an experimental system design.

3) The field results, like those of the model, are not limited to a system employing a physical link.

4) Controlling the speed solely through the engine governor does not provide a large enough speed range. An infinitely variable transmission such as a hydrostatic unit would greatly improve speed control.

5) The system as constructed is most adaptable to implements of narrow width and high power requirements.

7.3 Suggestions For Further Work

The project described here has demonstrated the ability of a simple, low cost control system to adequately control a tractor in a slave relationship. It is felt that refinement work in terms of component development or ornamentation should not be conducted prior to a thorough economic analysis of the cost benefits such a system could provide if adopted on a large scale.
Experimental work with radar techniques currently being conducted by major automobile manufacturers\(^4\) with a view to study collision prevention, will probably result in low cost, mass produced short range distance measurement systems. Such a system could probably be adapted to replace the physical link, and should be investigated.

The combination of the speed control system with a load control to provide optimum engine loading could result in components doing double duty. Further investigations in this area could best be conducted if a thorough understanding of tractor dynamics and degree of loading under field conditions was obtained. An accurate dynamic model of the tractor would also be beneficial in any further guidance work as tractor dynamics appear more significant than deviations due to disturbances from a purely kinematic model.

This project has demonstrated that a minimal amount of information is necessary to adequately control a tractor in a slave arrangement. The use of a collinear link, however, becomes less desirable as implement width increases due to the long rigid boom required on the master implement. Adaptability to large implements would be enhanced by the use of an oblique link provided the same or equivalent information could be readily obtained without expensive computational devices. The practicality of an oblique link is enhanced by keeping the link length constant which depends upon the degree of speed control. This last factor again points out the importance of a full understanding of the tractors dynamic characteristics.
BIBLIOGRAPHY


APPENDIX A

Application of the Runge-Kutta Method

The purpose of the Runge-Kutta method is to obtain an approximate numerical solution for a set of ordinary differential equations. The Runge-Kutta method is an algorithm designed to approximate the Taylor series solutions. The advantage of the Runge-Kutta method over the formal Taylor series solution is that explicit definitions of, nor evaluations of derivatives beyond the first, are not required. Further advantages and disadvantages are presented in Ralston and Wilf (20) along with a comprehensive mathematical discussion.

Outlined below is the calculation procedure for a system of first-order equations, which is a variation of the Runge-Kutta, fourth-order process due to Gill, as presented by Ralston and Wilf (20).

a) Input

(1) The description of the system of \((n + 1)\) first-order equations,

\[ y'_i(x) = f_i(y_0(x), y_1(x), \ldots, y_n(x)), \]

\[ i = 0, 1, 2, \ldots, n \]

where \(y'_0(x) = f_0 = 1\); i.e., \(y_0(x) - x\) is used for convenience of notation and to simplify the form of the process.

(2) The initial conditions, \(y_1(x_0) = y_{i0}, i = 0, 1, 2, \ldots, n\).

b) Order of the Calculation

(1) Let \(j = 1\)

(2) Let \(i = 0\)
(3) Compute:

\[ y_{ij} = k_{ij} = f(y_{0,u-1}, y_{1,j-1}, \ldots, y_{n,j-1}) \]

\[ = f_{1,j-1} \]

(4) Repeat step (3) for \( i = 1, 2, \ldots, n \)

(5) Let \( i = 0 \)

(6) Compute:

\[ y_{ij} = y_{1,j-1} + h \left[ a_j (k_{ij} - b_j q_{1,j-1}) \right] \]

\[ q_{ij} = q_{1,j-1} + e \left[ a_j (k_{ij} - b_j q_{1,j-1}) \right] - c_j k_{ij} \]

where

\[ a_1 = \frac{1}{2} \quad b_1 = 2 \quad c_1 = \frac{1}{2} \]

\[ a_2 = 1 - \frac{1}{2} \quad b_2 = 1 \quad c_2 = 1 - \frac{1}{2} \]

\[ a_3 = 1 + \frac{1}{2} \quad b_3 = 1 \quad c_3 = 1 + \frac{1}{2} \]

\[ a_4 = \frac{1}{6} \quad b_4 = 2 \quad c_4 = \frac{1}{2} \]

Initially, let \( q_{10}(x_0) = 0 \) for all \( i \); thereafter in advancing the solution, let

\[ q_{10}(x_t) = q_{14}(x_{t-1}), t = 1, 2, \ldots \]

(7) Repeat step (6) for \( i = 1, 2, \ldots, n \)

(8) Repeat steps (2) - (7) for \( j = 2, 3 \) and 4

c) Output

\[ y_{14} = \overline{y}_1 (x_0 + h) \]

To advance the solution, repeat steps (11 - 8), letting the current \( y_{14} \) be the initial values \( \dot{y}_{10} \) for the next step.
Following is the program employed to solve the equations of Section 4.2 based on the above outline.

```plaintext
REAL X1, K2, I
DIMENSION Y(50,5), YP(50,5), C(50,5), A(5), B(5), C(5)
I = 15
A(1) = 0.0
A(2) = 1.5
A(3) = 1.0 - SQR(0.5)
A(4) = 1.0 + SQR(0.5)
A(5) = 1.0/6.0
B(1) = 0.0
B(2) = 2.0
B(3) = 1.0
B(4) = 1.0
B(5) = 2.0
C(1) = 0.0
C(2) = 0.5
C(3) = 1.0 - SQR(0.5)
C(4) = 1.0 + SQR(0.5)
C(5) = 0.5
DO 2 KK1 = 2, 5, 3
   C1 = KK1
   K1 = C1/10.0
   DO 2 KK2 = 5, 11, 3
   C2 = KK2
   K2 = C2/10.0
   H = 0.1
   TC 2 K = 1, 2
   M = 0
C SET VALAST NOT EQ. 0
   VALAST = 10.0
   WRITE(5, 100)
   DO 4 J = 1, 5
   DO 4 I = 1, 5
   Y(I, J) = 0.0
   YP(I, J) = 0.0
4 2(I, J) = 0.0
AT = 3.1415
BT = 8.0
HI = 7.0
EL0 = 20.0
AIM = 16.0
WIS = 16.0
Y(9, 1) = -EL0
Y(6, 1) = -EL0 - BT
Y(11, 1) = -EL0 - BT - AT
Y(14, 1) = -EL0 - BT - HI - AT
TIMMAX = 3.1415
R = 24.0
IMAX = 500
```

11 DO 1 J=2,5
   VC= 8.0
   VA=VC/(R+(WIM+WIS)/2.0)
   IF(Y(4,5).GT.TIMMAX) VA = VC
   IF(L.LT.25.0) VA =VA/2.0
   IF(L.LT.25.0.AND.L.GT.20.0.AND.VLAST.LE.5.0) VA = VA/2.0
   VLAST =VA
   IF(L.LT.15.0) VC = VC/2.0
   IF(L.LT.15.0.AND.L.LT.20.0.AND.VCLAST.LE.5.3) VC=VC/2.0
   VCLAST = VC
   YP(I,J) = 1.
   IF(Y(4,5).GT.TIMMAX)GO TO 13
   YP(2,J) = VA*SIN(Y(4,1-J))
   YP(3,J) = VA*COS(Y(4,1-J))
   YP(4,J) = VA/E
   GO TO 14
13 YP(2,J) = VA*SIN(TIMMAX)
   YP(3,J) = VA*COS(TIMMAX)
   YP(4,J) = 0.0
14 YP(5, J) = VC*SIN(Y(7, J-1))
   YP(6,J) = VC*COS(Y(7, J-1))
   ANGLE = Y(15, J-1)
   YP(7,J) = (VC/8I) * SIN(K1 * (ANGLE - Y(4,J-1)) + K2 * (ANGLE
   1 -Y(7,J-1))/ COS(K1 * (ANGLE - Y(4,J-1)) + K2 * (ANGLE
   2 -Y(7,J-1)))
   YP(8,J) = YP(5,J) + BT * COS(Y(7,J-1)) * YP(7,J)
   YP(9,J) = YP(6,J) - BT * SIN(Y(7,J-1)) * YP(7,J)
   YP(11,J) = YP(5,J) - AT * COS(Y(7,J-1)) * YP(7,J)
   YP(12,J) = (CGST(Y(12,J-1)) * YP(10,J) - SIN(Y(12,J-1))
   * YP(11,J) ) / HI
   YP(13,J) = YP(10,J) - HI * YP(12,J) * COS(Y(12,J-1))
   YP(14,J) = YP(11,J) + HI * YP(12,J) * SIN(Y(12,J-1))
   YP(15,J) = ((YP(2,J) - YP(8,J)) * COS(Y(15,J-1)) - (YP(3,J) - YP(9,J))
   * SIN(Y(15,J-1)) ) / 
   SQRT(Y(12,J-1)**2 + Y(3,J-1)**2)**2
   DO 1 I=1,N
   Y(I,J) = Y(1,J-1) + H *(A(I,J) * (YP(I,J) - B(I,J) * Q(I,J)))
   Q(I,J) = Q(1,J-1) + 3. * (A(I,J) * (YP(I,J) - B(I,J) * Q(I,J)))
   1 -C(I,J) * YP(I,J)
   L = SQRT((Y(12,5) - Y(8,5))**2 + (Y(3,5) - Y(9,5))**2)
   ANGLEF = Y(15,5)
   ALPHA = ANGLEF - Y(4,5)
   ETA = ANGLEF - Y(7,5)
   PHI = K1 * ALPHA + K2 * ETA
   WRTE (6,101) Y(1,5), Y(13,5), Y(14,5), L, ALPHA, ETA, PHI,
   1 Y(2,5), Y(3,5), H, Y(4,5), M
   DO 3 I=1,N
   3 Y(I,1) = Q(I,5)
12 IF(Y(4,GT_MMAX.OR. (Y(14,5).LT. -20.0).AND. Y(13,5).GT. 5.0))
   1 GO TO 10
   M = M + 1
   GO TO 11
   10 H = H/2
   2 CONTINUE
   STOP
   100 FORMAT(1H1, 3X, 4HTIME, 6X, 4HDX(D), 6X, 4HY(D), 7X, 1HL, 6X, 5HALPHA, 7X,
   1 4HBETA, 9X, 3HPHI, 8X, 4HX(A), 6X, 4HY(A), 6X, 1LH/1H, 3X, 4HSEC.,
   2 6X, 4HFEET, 6X, 4HFEET, 6X, 4HFEET, 5X, 4HRAD., 8X, 4HRAD., 9X,
   3 4HRAD., 7X, 4HRAD., 6X, 4HFEET//)
   101 FORMAT(1H1, 3F10.3, F8.2, 1P3E12.3, 0P2F10.3, 1P3E12.3, 1E12.3, 13)
<table>
<thead>
<tr>
<th>TIME</th>
<th>X(D)</th>
<th>Y(D)</th>
<th>L</th>
<th>ALPHA</th>
<th>BETA</th>
<th>PHI</th>
<th>X(A)</th>
<th>Y(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEC.</td>
<td>FEET</td>
<td>FEET</td>
<td>FEET</td>
<td>RAD.</td>
<td>RAD.</td>
<td>RAD.</td>
<td>FEET</td>
<td>FEET</td>
</tr>
<tr>
<td>0.100</td>
<td>0.200</td>
<td>0.300</td>
<td>0.400</td>
<td>0.500</td>
<td>0.600</td>
<td>0.700</td>
<td>0.800</td>
<td>0.900</td>
</tr>
</tbody>
</table>
APPENDIX B

RESULTS OF LARGE DISPLACEMENT MODEL STUDIES

The following predicted paths were plotted from the results of the computer program described in Appendix A.
$K_1 = 0.2$, $K_2 = 1.1$

$R = 24'$

INSIDE TURN
IMPLEMENT WIDTHS 16'

Fig B-1
$K_1 = 0.4, \quad K_2 = 0.3$

$K_1 = 0.4, \quad K_2 = 0.4$

$K_1 = 0.4, \quad K_2 = 0.5$

$R_A = 8'$

INSIDE TURN

IMPLEMENT WIDTHS 16'$

Fig. B-2
INSIDE TURN

\[
\begin{align*}
K_1 &= 0.5 \\
K_2 &= 0.4, 0.5, 0.6. \\
R_A &= 40' \\
\text{IMPLEMENT WIDTHS 16'}
\end{align*}
\]

Fig. B-7
$R_A = 8'$

INSIDE TURN

IMPLEMENT WIDTHS 16"
$K_1 = 0.5$, $K_2 = 0.5$

$K_1 = 0.5$, $K_2 = 0.8$

$K_1 = 0.5$, $K_2 = 1.1$

$R_A = 24'$

INSIDE TURN

IMPLEMENT WIDTHS 16'

Fig. B-5
$K_1 = 0.3, \quad K_2 = 0.3$

$K_1 = 0.3, \quad K_2 = 0.4$

$K_1 = 0.3, \quad K_2 = 0.5$

Fig. B-6
$R_A = 24'$

$K1 = 0.4$

$K2 = 0.4, 0.5, 0.6$

INSIDE TURN

IMPLEMENT WIDTHS 16'

Fig. B-7
\( R_A = 24' \)

\( K_1 = 0.4 \)

\( K_2 = 0.4, 0.5, 0.6 \)

**INSIDE TURN**

**IMPLEMENT WIDTHS 16'**
\[ R = 24' \]
\[ K_1 = 0.5 \]
\[ K_2 = 0.4, 0.5, 0.6 \]

**INSIDE TURN**

**IMPLEMENT WIDTHS 16'**
\( R_a = 24' \)
OUTSIDE TURN
IMPLEMENT WIDTHS 16'

\[ K_1 = 0.2, \quad K_2 = 0.5 \]
\[ K_1 = 0.2, \quad K_2 = 0.8 \]
\[ K_1 = 0.2, \quad K_2 = 1.1 \]

Fig. B-10
$K_I = 0.5, K_2 = 0.5$

$K_I = 0.5, K_2 = 0.8$

$K_I = 0.5, K_2 = 1.1$

$R_A = 15'$

OUTSIDE TURN

IMPLEMENT WIDTHS 10'

Fig. 8-11
\[ K_1 = 0.2, \quad K_2 = 0.5 \]
\[ K_1 = 0.2, \quad K_2 = 1.1 \]
\[ K_1 = 0.2, \quad K_2 = 0.8 \]

\[ R_A = 15' \]

OUTSIDE TURN

IMPLEMENT WIDTHS 10'

Fig. B-12
\( R_A = 15' \), IMPLEMENT WIDTH = 10'

- \( K1 = 1.1, K2 = 0.6 \)
- \( K1 = 1.1, K2 = 0.8 \)
- \( K1 = 1.1, K2 = 1.1 \)
\[ R_A = 15' \quad \text{IMPLEMENT WIDTH} = 10' \]
APPENDIX C

The Describing Function for a Relay With Dead Zone

The input-output relationship for a relay with dead zone is shown in Figure C-1, while Figure C-2 shows the steady state sinusoidal response. Since the steady state output is a continuous periodic wave it may be represented by a Fourier series. For the describing function approach only the fundamental components of the output function are considered. The describing function for a non-linear element is then the ratio of the fundamental component of the output
function to the input function. In the case of the relay with dead zone no phase shift is present so that the describing function becomes the amplitude ratio.

From the figures

\[ \text{Input: } x = x_m \sin \omega t \quad \text{(C.1)} \]
\[ \text{Output: } y = y = 0 \quad 0 < \omega t < \alpha \quad \text{(C.2)} \]
\[ y = y_m \quad \forall \omega t < \frac{\pi}{2} \quad \text{(C.3)} \]

where
\[ \alpha = \sin^{-1} \frac{D}{x_m} \quad \text{(C.4)} \]

It is now necessary to compute the fundamental components of the output function, \( f(t) \).

Now, \( f(t) = \frac{1}{2} A_0 + A_1 \cos t + B_1 \sin \omega t \quad \text{(C.5)} \)

Since there is no static bias \( A_0 = 0 \)

Also since the function is odd \( A_1 = 0 \)

So
\[ f(t) = B_1 \sin \omega t \quad \text{(C.6)} \]

\( B_1 \) may be evaluated from
\[
B_1 = \frac{1}{2 \pi} \int_{0}^{2\pi} f(t) \sin \omega t \, d(\omega t) \quad \text{(C.7)}
\]
\[
= \frac{4}{\pi} \int_{0}^{\pi/2} y \sin \omega t \, d(\omega t) = \frac{4Y}{\pi} \left( -\cos \omega Y \right) \bigg|_{0}^{\pi/2}
\]
\[ = \frac{4Y \cos \alpha}{\pi} \quad \text{(C.8)} \]
Since $X \sin \alpha = D$

$$\cos \alpha = \frac{X^2 - D^2}{X^2} \quad (C.9)$$

then

$$G_D = \frac{B_1}{X} = \frac{4Y}{\pi X} \left(1 - \left(\frac{D}{X}\right)^2\right) \quad (C.10)$$

Equation (C.10) is the describing function of a relay with a dead zone.

In order to determine the maximum value of $G_D$ it is convenient to first rewrite (C.10) as

$$G_D = \frac{4Y}{\pi X} \frac{X^2 - D^2}{X^2} \quad (C.11)$$

Differentiating

$$\frac{dG_D}{dX} = \frac{4Y}{\pi} \left[\frac{1}{X^2} - \frac{1}{(X^2 - D^2)^{1/2}} - \frac{3X}{2(X^2 - D^2)^{3/2}}\right] \quad (C.12)$$

Setting $\frac{dG_D}{dX} = 0$

$$X^{-1}(X^2 - D^2)^{-1/2} = 2X^{-3}(X^2 - D^2)^{1/2} \quad (C.13)$$

Solving for $X$ find

$$X = \sqrt{2} D \quad (C.14)$$

Substituting equation (C.11) into equation (C.10) gives the maximum value as

$$G_D = \frac{4Y}{\pi 2D^2} \sqrt{2D^2 - D^2} = \frac{2Y}{\pi D} \quad (C.15)$$
APPENDIX D

Operation of the Steering Control Relay

The purpose and location of the relay whose operation is described in this Appendix is given in Section 5.3.4. The relay, in actuality a high gain amplifier, is shown schematically in Figure 5.10. When $e_1 = 0$, all transistors are reversed biased and are thus in the off state. The diodes $D_1$ and resistors $R_2$ result in the emitter junctions of $Q_1$ and $Q_4$ being biased by 0.3V (The forward voltage drop of the diodes), independent of emitter current. The transistors $Q_1$ or $Q_4$ will thus not begin to conduct until $e_s$ reaches -0.3V or +0.3V respectively. Since the base emitter forward drop is also approximately 0.3V, the effective dead zone is ± 0.6V.

As $e_i$ becomes more negative than -0.6V, $Q_1$ will begin to conduct, resulting in $Q_2$ also becoming forward biased and conducting. The emitter current of $Q_2$ supplies the base current of $Q_3$, the power transistor, which has a collector current capability of 15 Amps. The diodes $D_2$ protect the power transistor from back emf. and transients. The capacitors filter commutator noise.

When $e_i$ becomes more positive than +0.6V a similar action to that described above, takes place in the lower portion of the circuit with $Q_4$ acting as an inverter.
APPENDIX E

Operation of the Speed Control Relay

The purpose and location of the relay described in this Appendix is given in Section 5.4.1(d). The relay is shown schematically in Figure 5.23. When $e_i = 0$, all transistors are reverse biased and thus in the off state. The resistors $R_2$ and $R_3$ combined with the diodes $D_1$ reverse bias $Q_1$ and $Q_3$ by 0.13V, resulting in an absolute dead zone of $\pm 0.3V$. The base emitter forward drop of $Q_1$ and $Q_3$ adds a further 0.3V so that the effective dead zone is approximately $\pm 0.6V$.

Since the two halves of the circuit are mirror images this action is identical except for polarity. When $e_i$ becomes sufficiently positive to cause $Q_1$ to conduct, the reverse bias on $Q_2$, the power transistor, is reduced. A further increase in $Q_1$ collector current due to a further slight increase in $e_i$ causes $Q_2$ to saturate.
APPENDIX F

Development of Compensator Transfer Function

In this Appendix the transfer function of the basic building block of the speed control system compensator is derived. A schematic circuit diagram is given in Figure 5.25.

By inspection

\[ e_1 = i_1(R_1 + R_1) - i_2R_2 = e_0 \]  \hspace{1cm} (F.1)

If the amplifier is assumed to have an infinite gain then the emf. at A may be assumed to be zero, so that

\[ e_1 = i_1R_1 \]  \hspace{1cm} (F.2)

also

\[ V_c = -i_1R_1 \]  \hspace{1cm} (F.3)

\[ V_c = \frac{1}{c} \int (i_1 - i_2) \, dt \]  \hspace{1cm} (F.4)

where \( V_c \) = the capacitor voltage.

Taking the Laplace transform (F.1) becomes

\[ E_i(s) - I_1(s)(R_1+R_1) - I_2(s)R_2 = E_o(s) \]  \hspace{1cm} (F.5)

(F.3) and (F.4) give

\[ -I_1(s)R_1 = \frac{1}{Cs}(I_1(s) - I_2(s)) \]  \hspace{1cm} (F.6)

which can be solved for \( I_2(s) \) yielding

\[ I_2(s) = I_1(s)(1+R_1CS) \]  \hspace{1cm} (F.7)

Substituting from (F.7) for \( I_2(s) \) in (F.5) yields

\[ E_i(s) - I_1(s)(R_1+R_1) - I_1(s)(1+R_1CS)R_2 = E_o(s) \]  \hspace{1cm} (F.8)
But from (F.2)
\[ I_1(s) = \frac{E_i}{R_1} \]  

(\text{F.9})

Substituting from (F.9) for \( I_1(s) \) in (F.8) yields
\[
E_4(s) \left( 1 - \frac{R_1 + R_1}{R_1} - \frac{(1+R_1CS)R_2}{R_1} \right) = E_0(s)
\]  

(\text{F.10})

from which
\[
\frac{E_0}{E_4}(s) = - \left( \frac{R_1 + R_2}{R_1} + \frac{R_1R_2CS}{R_1} \right)
\]  

(\text{F.11})