EARLY ACTION INVESTMENT IN THE KYOTO PROTOCOL

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To my parents

PERMISSION TO USE

ABSTRACT

Since uncertainty and irreversibility are inherent, environmental policy involves the problems of timing of implementation. Environmental policy based on cost-benefit analysis using certainty equivalent presents values can be misleading under the combined effect of irreversibility and uncertainty.

Using real options method, the thesis analyzes the timing of early action investment in Canada's Kyoto commitment. Early action investment in emission reductions is irreversible. The thesis uses a simple two-period model, and then lays out a corresponding continuous-time model to show that under technological uncertainty, early action investment should be delayed until more information - the results of R&D - is revealed. In particular, the more uncertain the outcome of research, the more the firm should delay early action investment.

The thesis argues that Canada's Kyoto commitment is well intentioned but not wisely implemented: early action investment on emission reductions may not be efficient. The results suggest that a more gradual Kyoto program would be favourable.

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CONTENTS

CHAPTER 1 INTRODUCTION

Many scientists believe that part of the earth's atmosphere is acting as an insulation traps sufficient infrared radiation to keep the global average temperature in a comfortable range. The insulation is the collection of the six atmospheric gases called "greenhouse gases" (GHGs), which consist of carbon dioxide, methane, nitrous oxide, and fluorinated gases (hydro fluorocarbons, per fluorocarbons and sulphur hexafluoride).

Rising concentration of GHG emissions has been detected, and serious negative consequences have been predicted. Over the past several decades, increasing GHG emissions are direct results of human activities, and are called the enhanced greenhouse effect. The enhanced greenhouse effect will increase the average temperature of the Earth's surface gradually (also known as the global warming effect), and will possibly change the global heating and cooling system. For instance (e.g., see Titenberg, 1996), the increasing concentration of the GHG will possibly increase the temperature of the earth surface to its highest in thousands of years. The consequence is to raise the sea level and affect the agricultural sites, increasing areas of deserts, and trigger a diminished capacity to raise food.

1.1 Kyoto Protocol and Canada

Increasing human interference in the ecological system will bring economic and institutional difficulties. The GHG emissions not only will impose environmental damages on the current public, but also will jeopardize sustainable development in the sense that the next generation will be enjoying fewer natural resources than they would have been. GHG emissions lead to externality. Air is a public good. It is jointly consumed by everyone on the planet and it is unlikely to prevent one from consuming it. There will be market failure in the sense that the scarcity of the clean air can not be reflected by the market price and the marginal condition for optimal resource allocation cannot be met. There is no incentive for individual economic agents, acting alone, to reduce their emissions.

Generally, economists would suggest two categories of policy instruments to correct the failure and to solve the difficulty: an emission trading system and a tax system. For the emission-trading program, the simplest form is that the government issues emission permits for the target level of aggregate emissions. These permits can be "auctioned", or "grandfathered" on the historical emission basis. The chosen method of initial allocation reflects an implicit assignment of the property rights: grandfathering assigns property rights to incumbent polluters; auctioning assigns property rights to the state, and reflects the same "polluter pays principle" embodied in the emission charges. The key to the emission-trading program is that the allocated permits can be traded in an emissions permit market. If the market is "competitive" such that all agents are price-takers, then the marginal abatement cost will be equal to the permit price for all firms.

With emission trading, new international agreements have very large latitude in establishing emission targets and feasible policies. A significant step in this direction is the Kyoto protocol. The Kyoto Protocol to the United Nations Framework Convention on

Climate Change (UNFCCC) was formally adopted by the third session of the Conference of the Parties (COP 3) on 11 December 1997 in Kyoto, Japan. According to the Climate Change Secretariat (2002), the Protocol consists of the five main elements. The first two focus on implementation of costs for developed countries, such as Canada.

The first element of the Protocol, “commitment”, establishes a general legally binding obligation on the industrialized countries (list in Annex I) to reduce emissions for GHG emissions by approximately 5 percent below their 1990 levels in total by the period of 2008-2012. The Annex I countries shall achieve their emissions targets in the (first) commitment period of 2008-2012 and be allowed to emit the assigned amount that was formally established on the baseline of 1990 levels.

The second element of the Protocol is implementation of a significant domestic policy and its measures to achieve emissions target, and supplemented with carbon sinks and market-based mechanisms by Annex I countries. Carbon sinks are the carbon dioxide removed concurrently by the forest and by other ecosystems that will generate removal units to offset emissions. The market-based mechanisms, also known as Kyoto mechanisms, consist of the joint implementation (JI), the clean developed mechanisms (CDM) and the emission trading (ET). The JI will allow Annex I countries to generate emission reduction units against its own commitments by sponsoring emission reduction of carbon sinks projects in other Annex I countries. The CDM allows Annex I countries to earn certified emission reductions by investing in the emission reductions, or sinks projects in the non-Annex I countries. The certified emission reductions generated between 2000 and 2008 can be credited against their commitments under the commitment period, which is called the "early crediting". Banking will be allowed in both JI and CDM. The ET allows Annex I countries to acquire the assigned amount of units, removable units, emission reduction units and certified emission reductions from other Annex I countries.

Canada, as a signatory to the Kyoto Protocol, has committed to reduce GHG emission to 6 percent below its 1990 levels in the first commitment period between 2008 and 2012. This is equivalent to a reduction of 240 Megatons (MT) of the projected business as usual (BAU) emission levels. The Government of Canada (2002) has established some plans to achieve its commitment in three steps. These steps are stated as follows:

1. The action underway is that the emission reductions will be 80 MT, among which 50 MT emission reductions will be bequeathed from Action Plan 2000 and Budget 2001. The Plan and the Budget create market and financial incentives to support the investment in the creation of renewable energies, such as wind power production and hydroelectric production. Investment in transportation and industry sectors is substantial for the emission reductions. Large investments have been made not only to the process innovations, but also to product innovation. For instance, Motor Vehicle Fuel Efficiency Initiative proposes to improve 25% new vehicle fleet fuel efficiency by 2010. The Future Fuel Initiative will develop and demonstrate biofuel technologies and infrastructure for commercializing the fuel cell vehicles.

2. In the first commitment period, new actions are expected to count for 100 MT.

3. The remaining 60 MT gap will be covered by the current or potential reductions, made possible by Partnership Fund and existing research & development (R&D) investments directed at climate change issue.

In sum, Canada plans to reduce emissions as much as 240 MT to meet its commitment. Early action investment in improving energy sufficiency and conducting new technology will generate emission reductions by about 130 MT, which is more than half of the commitment. Emission reduction actions and international emission trading are expected to reduce 76-86 MT. Carbon sinks are expected to count for about 30 MT of GHG emissions.

1.2 The Role of Uncertainty

Many questions arise about the feasibility of the Kyoto. Some opposition leaders in the Canadian government and some Canadian firms are still reluctant to implement the Kyoto Protocol. For example, the firms that heavily depending on fossil fuel in Alberta have opposed the Kyoto. This opposition to the Kyoto clearly indicates that there must be some fundamental problems in the Kyoto that have not yet been solved.

One of the reasons is that the Kyoto protocol is an international convention. Since emission reduction is international public goods, it is available to everyone on the planet. The countries that free ride on emission reductions from other countries will jeopardise the international treaty. It will also present enforcement problems from the practical point of view. Self-monitoring by an individual country is the only practical solution because its government is unlikely to permit international monitoring methods to penalize the countries that exceed their emission quota. In other words, the only enforcement tool that is available is moral obligation or trade sanction.

The relation between a government and a firm is the "principle and agent" relation. As a regulator, the government will seek job security, size of the bureaucracy and emission reductions as its political assets, in order to win elections or extract tangible rents for themselves. The firms will seek a minimum abatement cost in the case of emission reductions. The government can create incentives for the firms to reduce emissions, by three typical instruments: fees, market permits, or liability. A cost-effective policy relied on cost-benefit analysis will set the fee level, or permit price, equal to the marginal abatement cost that is equalized across the firms.

However, the traditional cost-benefit analysis in the context of the Kyoto protocol does not include the impact of uncertainty. On the one hand, as the literature suggests (e.g., see Baranzini, Chesney and Morisset, 2003), the cost-benefit analysis omits the uncertainty surrounding costs and benefits of controlling global warming, and does not consider the possibility of waiting for more information. Early action investment will have irreversible consequences, which may have an option value under uncertainty, in the sense that waiting for information may reduce the total abatement cost over time. Considering this option value, rational firms will have strong incentive to delay early action, which explains why the firms oppose the Kyoto. On the other hand, the uncertainty leads to the principle-agent problem: due to lack of future information, the government may induce more early action investment (or abatements) than what the firms will be willing to undertake (or abate). Thus, the Kyoto target would be much higher than its optimal level, and the Kyoto policy will not be efficiently implemented.

The uncertainty problem requires governments to explicitly deal with global warming in a sustainable way. One important characteristics of a permit system is that, if enforced, the permit system will guarantee the Kyoto commitment regardless the costs. Facing uncertainty, the governments will set the Kyoto target according to the current information and the cost-benefit analysis. Although the permit system will guarantee to meet the Kyoto target, the true levels of emission reduction will be much less than the ones the governments initially thought. As a result, the Kyoto policy will induce a social loss. If the relative slope of marginal damage curve is higher than the slope of marginal cost curve, the social loss will be higher (see Weitzman, 1974). The governments will notice the option value of waiting for more information, thus, will have incentive to wait. This explains why some parties in the Government of Canada oppose the Kyoto.

Using real options, the goal of this thesis is to analyze the timing of implementation of early action investment in the Kyoto Protocol under uncertainty. The irreversibility considered here is associated with the investments made to control emissions. The thesis will implicitly model the technological uncertainty, and demonstrate a flexible early action investment rule for the firms that are facing different levels or magnitude of uncertainty.

1.3 Thesis Organization

The thesis is organized as follows. Chapter 2 will analyze a two-period model, which was developed by Kennedy (2002), and introduce the impact of irreversibility under uncertainty, which is intrinsic to early action investment. Kennedy (2002) concluded that research and planning, which will reveal the viability of prospective technological improvements, is less costly than early action investment in emission reductions. Although his model is implicitly predicated the role of research and development and the new information it would reveal, Kennedy (2002) does not explicitly model the underlying uncertainty which will affect early action investment. Chapter 3 is designed to parameterize the implementation of Chapter 2. Two experiments, the increasing risk and the "bad news principle", will be conducted against a numerical benchmark. These experiments will answer the question of how different level and magnitude of uncertainty will affect investment decisions in the Kyoto protocol. A more rigorous analysis will be developed in Chapter 4, where the real options approach will be used to solve the problem with a continuous-time model. Conclusions will be drawn in Chapter 5.

CHAPTER 2 TWO-PERIOD MODEL

The investment role in the Kyoto is substantial. More than half of the commitment will be reduced using investment on energy efficiency and new technology. In evaluating this investment, firms must pay special attention to decide which information is relevant to the decision at hand and which information is not. Thus, it is important to address:

- What the characteristics of investment are,
- With these characteristics, how uncertainty will affect investment decisions.

This chapter is devoted to these questions. In the first section, some general assumption will be made. In the second section a two-period model which draws on Kennedy (2002) will be introduced. An investment model of cost minimization will be developed. Finally, the role of uncertainty will be demonstrated qualitatively, and the questions above will be answered.

2.1 The Model

2.1.1 General Assumption

For simplicity, this thesis will concentrate on one representative firm. This firm will be risk neutral and will compete in competitive input and output markets. The general assumption under a deterministic setting is that the firm has a perfect foresight of the future. This will ensure the firm to behave according to intrinsic logic rather than ad hoc assumptions, or the rules of thumb. In a stochastic setting, it is assumed that the firm has rational expectations. Thus, facing future contingency, the firm will continuously adjust its investment plans grounded in the new information as the information is scarce and no information should be wasted. The investment plans may be erroneous and suffer from continuously favourable or unfavourable shocks, however, the rational forecast will be unbiased. The probability distribution of the estimated future total abatement cost will concentrate around the true value of the abatement cost, which is unknown. As a result, the firm's optimization problem can be simplified as optimizing the expected total abatement cost.

2.1.2 Investment and Emission Reductions

The technological abatements will yield some different time profiles of emission reductions due to investment lags and learning effect. This diversity will be modeled into two stylized types: research and planning investment and capital investment. Technological changes will have had different effects on the technological abatement, and those effects will have occurred with different lags. The research and planning investment are the investment in knowledge creation. The accumulation of knowledge usually takes time, and thus the research and planning investment will yield some emission reductions by its cumulative level. The capital investment refers to the money paid to purchase the capital assets or fixed assets such as equipments or machineries, updating maintenance, building wind or hydroelectric station. The capital investment will yield emission reductions concurrently. The past capital investment also yield emission reductions due to the learning effect.

Following Kennedy (2002), let x denote the research and planning investment and y denote the capital investment. Denote r as the technological abatement. The technological abatement for early action period (2003-2008) and the first commitment period (2008-2012) are

$$r_1 = \theta_{11} y_1^{1/2}, \tag{2.1}$$

$$r_2 = \theta_{21} y_1^{1/2} + \theta_{22} y_2^{1/2} + \alpha_{21} x_1^{1/2}, \tag{2.2}$$

where θ_{ij} is the positive parameter for emission reductions in ith period by capital investment y_j and α_{21} is the emission reduction parameter of research and planning investment x_1 in the first commitment period. The positive parameters θ_{ij} and α_{21} are the shift parameters that parameterize the effectiveness of the investments; they determine the marginal technological abatement of capital on emission reductions partially. For any given amount of investment, the greater the parameter (θ_{ij} or α_{21}), the more emission reductions will be generated than otherwise. For example if $\theta_{21} > \theta_{11}$, where θ_{11} is the shift parameter of investment and θ_{21} is the shift parameter of cumulative investment, a given amount of cumulative capital investment will yield more emission reductions than new capital investment will. This can be explained as learning effect associated with new technology because the cumulative capital investment yields more emission reductions without new economic resources engaged in knowledge creation. The investments on the right of Equation (2.1) and (2.2) are all in square root form, which models technological abatement function with diminishing returns. Replicating the input in the Kyoto might not double the emission reductions. In the case of knowledge creation, replicating the research and planning investment without research interaction will cause the same set of discoveries happens twice. The capital investment is assumed to follow the conventional economy; it is diminishing returns. Finally, an equality of the equation simply indicates that the technological abatement is the total emission reductions with capital investment, cumulative research and planning investment, and cumulative capital investment.

2.1.3 Control Cost

The cost functions are assumed quadratic in abatement actions. A large deviation from the origin will induce a high penalty cost, while a smaller deviation will induce a lower penalty cost. Recall that there are two stages: stage one (the early action period) is from 2002 to 2008 and stage two (the first commitment period) is from 2008 to 2012. The cost function in the early action period can be described as

$$c_1 = c_1(x_1, y_1, e_1) = x_1 + y_1 + \gamma\, z_1^2 + p_1 e_1, \qquad (2.3)$$

where γ is a positive parameter, z_1 is behavioural abatement, p_1 is permit price, and e_1 is the actual emissions in stage one. The model assumes that the purchase price of capital is embedded in the cost function, and is fixed at one. The behavioural abatement cost γz_1^2 is in quadratic form in order to emphasize a higher impact of this cost to the total cost. There will be no incentive to undertake behavioural abatement in stage one if there is any charge for emissions ($if\ p_1 = 0$). In the first commitment period (stage two), the cost function is arranged as

$$c_2 = c_2(y_2, e_2; p_2) = y_2 + \gamma\, z_2^2 + p_2 e_2. \qquad (2.4)$$

Denote b as emissions under BAU levels. By definition,

$$e = b - (r + z)\,, \tag{2.5}$$

such that the quantity of total abatement will cover the gap between emissions under BAU levels and actual emissions. Substitute Equation (2.1) and (2.2) into (2.3) and (2.4) respectively to obtain

$$c_1 = c_1(x_1, y_1, e_1) = x_1 + y_1 + \gamma(b_1 - r_1 - e_1)^2 + p_1 e_1\,, \tag{2.6}$$

$$c_2 = c_2(y_2, e_2) = y_2 + \gamma(b_2 - r_2 - e_2)^2 + p_2 e_2\ . \tag{2.7}$$

The firm will make decisions sequentially. In this model, the fist commitment period will be the terminal stage, at which the stage one action will have become history to the firm. At this stage, the firm's problem will be to minimize its cost by choosing the optimal amount of investment and emissions, given the early action in stage one. The technological abatement in stage two is a combination of early action and capital investment in stage two, because the research and planning investment in stage one takes effect in stage two, and the capital investment in stage one will still be effective in stage two. In the early action period, the firm minimizes the overall cost over the two periods by choosing early action to smooth emission reductions.

2.2 Optimal Investment Rule

Denote J as the minimized cost. The representative firms will minimize their costs over the two stages with a time discount rate that is constant and equals to ρ. The minimization problems in stage two and stage one are

$$J_2(x_1, y_1) = \underset{y_2, e_2}{Min}\ c_2(y_2, e_2), \tag{2.8a}$$

and

$$J_1 = \underset{x_1, y_1, e_1}{Min}\left\{c_1(x_1, y_1, e_1) + \frac{1}{1+\rho}\,\mathrm{E}\,J_2(x_1, y_1)\right\}. \qquad (2.8b)$$

Note that in stage two, x_1 , y_1, and e_1 will be given in the sense that x_1 , y_1, and e_1 have been chosen in stage one and what have been chosen in stage one can not be changed in stage two. Whatever the initial choice will be, the remaining choices such as y_2, e_2 should be able to minimize the sub-problem starting in the following stage.

In stage two, the firm will choose investment and emissions to minimize the concurrent cost. Specifically, the firm will solve Equation (2.8a) in stage two. At an interior solution, unless can sell (or issue) permits, the first order condition will be

$$2\gamma\ z_2 = p_2. \qquad (2.9)$$

Equation (2.9) states that the optimal emissions occur regardless the value of firm specific parameter γ , as long as the marginal behavioural abatement cost equals to the permit price p_2. The permit price is an exogenous variable determined by domestic marginal abatement cost or world permit market. Specifically, if Canada will do the Kyoto alone, the permit price is the domestic marginal abatement cost. Otherwise, the permit market will be determined in the international emission trading. As a result, the marginal behavioural abatement cost $2\gamma\ z_2$ will be equalized across the firms. Equation (2.9) determines the optimal behavioural abatement such that $z_2 = \frac{p_2}{2\gamma}$.

The first order condition of Equation (2.8a) with respect to y_2 yields

$$1 - p_2 r_2'(y_2) = 0. \qquad (2.10)$$

Recall that the purchase price of capital is fixed as one. This equation states that the market value of the extra technological abatement per unit of capital investment is the purchase price of capital investment. The optimal investment can be derived according to Equation (2.10) such that $y_2 = \left(\frac{1}{2}\theta_{22} p_2\right)^2$. Insert the optimal behavioural abatement and investment back into Equation (2.8a), yields the minimized cost for stage two:

$$J_2 = -p_2^2\left(\frac{\theta_{22}^2}{4} + \frac{1}{4\gamma}\right) + p_2\left(b_2 - \alpha_{21}x_1^{1/2} - \theta_{21}y_1^{1/2}\right). \tag{2.11}$$

Equation (2.11) carries the optimal investment and the optimal behavioural abatement. The minimized cost is shown in Figure 2.1.

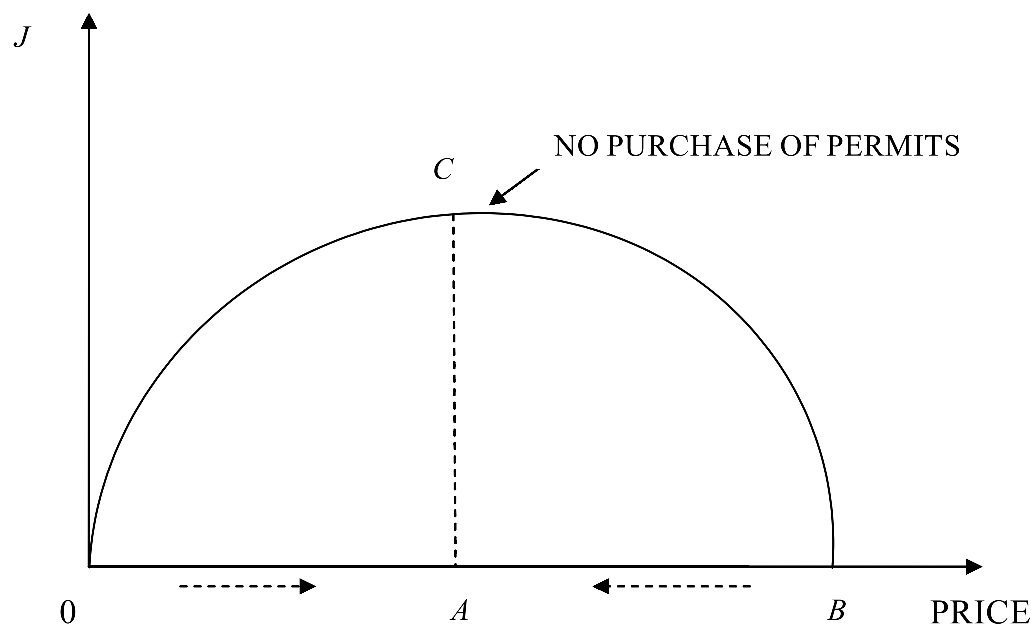

Figure 2. 1. Minimized Cost

The parabolic minimized cost curve cuts the price axis at two points; price converges from the left and the right to the middle (Point A) where permit price coincides with the equilibrium price $p_2 = \dfrac{\left(b_2 - \alpha_{21} x_1^{1/2} - \theta_{21} y_1^{1/2}\right)}{\dfrac{\theta_{22}^2}{2} + \dfrac{1}{2\gamma}}$. The equilibrium price also carries the optimal investment and the optimal behavioural abatement; the numerator is BAU emission levels minus the technological abatement resulting from early action, and the investment and behavioural abatement in stage two are embedded in the denominator. Rearrange the denominator and insert the optimal investment and behavioural abatement back into the equilibrium price yields $z_2 + r_2 = b_2$; at the equilibrium (Point C), the marginal abatement cost is the market clearing permit price, and the permit trading does not occur at this point. In other words, under perfect competition assumption, this equilibrium price is the marginal abatement cost such that $p_2 = \dfrac{2(b_2 - r_2)}{\dfrac{1}{\gamma}}$. The permit price might converge to the equilibrium price from the right: from Point B to Point A. A permit price higher than the equilibrium price indicates that $z_2 + r_2 > b_2$, so that the firm will buy permits and the price offered will be higher than the equilibrium price due to a higher private marginal abatement cost. If the permit price is lower than the equilibrium, it converges from the left: from the origin to Point A. As a result, the firm will sell permits and be willing to trade at a lower price assuming there is a market at which the firm can issue permits.

In stage one, the firm minimizes the cost over the two stages by choosing the concurrent investments as in Equation (2.8b). Given the minimized cost in stage two as in Equation (2.11), by the first order condition, the optimal investments of both types are

$$x_1 = \left(\frac{\alpha_{21} p_2}{2(1+\rho)}\right)^2 \tag{2.12}$$

$$y_1 = \left(\frac{1}{2}\theta_{11} p_1 + \frac{\theta_{21} p_2}{2(1+\rho)}\right)^2. \tag{2.13}$$

Substitute (2.12) and (2.13) into (2.8b) yield the overall minimized abatement cost. It is similar to the minimized cost in stage two so that only the optimal investment rule is given.

2.3 Uncertainty and Investment

2.3.1 The Characteristics of Early Action Investment

The first characteristic of investment is an ability to wait to invest. The firm can choose the timing of investment freely such that investment is a right rather than an obligation. A typical traditional approach in the global warming context (as well as in the Kyoto context) is the cost-benefit analysis, which based on the net present value approach. An underlying assumption of the net present value approach is that the investment possess a once or never proposition. If the firm do not choose to undertake an investment now, then the firm will never invest later. In the case of abatement cost, the firm's problem is to minimize the net present value of abatement cost once and for all. However, in a dynamic setting, the firm will be able to make sequential decisions due to the waiting ability. In particular, in stage two, the firm's problem will be to choose the rate of investment and the level of emissions in order to minimize the control cost regardless what have been chosen, and in stage one, the firm's problem is to minimize the overall cost until the end of Kyoto period. Hence, the abatement cost over the two periods will be optimized sequentially and recursively.

The second characteristic is that the investment is, or at least partially, irreversible. In the context of environmental economics, there are two typical topics of irreversibility. First, pollution damage is irreversible in the sense that the GHG emissions can only be removed slowly and naturally. Second, investments on emission reduction projects are irreversible: once undertaken, the investment will be too costly to be recouped or transferred. This thesis will deal with the second form. Emission reduction projects will not yield an additional output to the society, but will produce clean air to the Kyoto. The firm will invest in the projects aimed at improving energy efficiencies or the ones aimed at developing new cleanup technologies. Once the firm invests at a higher rate, the investments to the projects can be accumulated very quickly up to an optimal level such that a bad decision might lead to overinvestment, and the cumulated investments will only fall back to the optimal level slowly by a time discount and depreciation.

The third characteristic of investment is that future payoffs, negative or positive, is uncertain. In particular, the thesis will narrow down the type of uncertainty to be the technological uncertainty, the one that is essential for Canada to achieve its commitment, and the one associated with the arrival of new information on research over time. Since uncertain outcomes of the research process will affect future relative productivity of capital investment, the uncertainty will be surrounding capital investment in order to keep the model in the later chapters as simple as possible.[1]

2.3.2 The Combined Effect under Uncertainty

Under uncertainty, the firm's investment decisions are subject to the combined effect of the three characteristics. The combined effects of uncertainty and irreversibility will induce an option value, which is the value of retaining the choice to a later period in time. As literature suggests (e.g., see Saphores, 2000), Weisbrod (1964) first introduced option value concept and argued that, in the case of irreversible decision of choice, the flexibility to choose the timing of the decision, which creates an option value, should be included in the cost-benefit analysis. Cicchetti and Freeman (1971) and Schmalensee (1972) explained option value as a risk premium of risk averter, but the model failed if option value is negative. Arrow and Fisher (1974) and Hendry (1974) suggested that, regardless risk preferences, the uncertainty associated with an arrival of new information together with irreversible consequences of a decision would introduce an option value (or quasioption value). The option value is identified as the value of information that is conditional on whether retaining the option or not. The option value could be negative if the choice is not binary.

[1] Of course, the future rewards of research and planning investment is uncertain. However, uncertainty will not induce delays on the research and planning investment. The firm will keep investing in knowledge creation (research and planning investment) because research and planning reveals more information. Under uncertainty, the new information will be scarce but useful for decisions making. In other words, there is no opportunity cost of undertaking research and planning investment today instead of later. This is not the case is capital investment. The uncertainty surrounding the capital investment is a reflection of technology uncertainty. The future relative productivity of capital investment is uncertain because the outcome of research and planning investment is uncertain. By waiting, the firm can gain more information through time. The uncertainty will thus affect the decision on capital investment. It makes sense that uncertainty defined in this thesis is uncertain future reward on capital investment for the sake of simplification.

The real options approach (such as Dixit and Pindyck (1994)) provides a new view to explain Weisbrod's option value concept. The investment made on real asset has option-like characteristics: ability to wait, irreversibility and uncertainty. With the joint effect of these characteristics, the uncertainty could be implicitly modeled into the expected payoff of the real asset. In particular, a rational firm will make decisions sequentially due to its ability to wait and irreversibility. On the one hand, the opportunity cost of emission is the market permit price, which creates an incentive to reduce emissions and to invest in the emission reduction projects. On the other hand, the firm is aware that it will be costly to reverse investment plans, and that information will come in time. With irreversibility, the technological uncertainty will create a risk that the firm may overabate if future relative productivity is improving, or may underabate if future relative productivity is slacking. The risk will lead to a suboptimal abatement level, and will create an opportunity cost of committing the Kyoto target. Instead, the firm might delay its investment in order to avoid the risk. The choice of investing now forgoes the opportunity of investing to a later date when the forthcoming information on the future economic condition will be valuable to the firm. In other words, waiting to invest may create a positive value (or option value) facing irreversible abatement process. Thus, the firm will reduce the target cumulative investment, and wait for more information that is valuable.

In the context of real options approach, the optimal investment rule can be obtained by the contingent claim analysis, or equivalently, the dynamic programming (DP) analysis.

In this thesis, the DP approach will be used.[2] A discount rate that is equivalent to the risk free rate of return is assumed.

2.4 Summary

Canada will achieve its commitment in the Kyoto protocol mainly through investments in the emission reductions. In a deterministic setting, the optimal early action investment rule can be derived by optimizing the abatement cost function. In reality, early action investment decisions are subject to the combined effect of investment attributes: ability to wait, irreversibility, and technological uncertainty. Given irreversible consequences of early action investment, the flexibility of decisions making will be counted as a positive value (i.e. option value), and the firm will postpone its investment to a more propitious date.

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[2] In the thesis, it is assumed that the spanning assumption holds. Generally, if can be traded in the financial market, the emission permits would be the spanning asset that is, in principle, perfectly correlated with the evolution of early action investment. The DP approach is equivalent to assuming that the spanning asset (e.g. emission permits) has a risk free rate of return. However, the spanning assumption might be too strong in reality.

CHAPTER 3 NUMERICAL EXAMPLES

In the previous chapter, it is argued that early action investment has three characteristics: irreversibility, ability to wait and uncertainty. The firms' investment decisions should be flexible because of the joint effects of these attributes.

To show the impact of uncertainty on investment decisions, some numerical examples will be developed, and the value of waiting to invest (i.e. option value) will be examined in this chapter. In particular, two experiments of early action investments will be conducted under two different circumstances: under increasing risk, and under the world of bad news and good news. These experiments will show that waiting can avoid a downside risk; thus, there will be an option value of delaying early action investment. Section 3.1 lays out a numerical benchmark from Kennedy (2002). Section 3.2 presents a simulation of investments when the firm is facing an increasing risk. Section 3.3 presents a simulation of investments when the firm is facing two possible outcomes: the bad news and the good news.

3.1 Numerical Benchmark

The purpose of this calibration is to re-examine the numerical benchmark of Kennedy (2002). This calibration is required for the purpose of comparison between certainty case, and uncertainty case, which will be developed in section 3.2 and 3.3.

Following Kennedy (2000), the model is disaggregated into five sectors: electricity generation, industry, residential and agriculture, transportation, and “other”. Data on emissions is taken from the Analysis and Modeling Group (1999: Annex C, C-25) and the Energy Research Group (2000: Table 4-8). The Analysis and Modeling Group (1999) has projected BAU emissions in six sectors from 1990 to 2020, and has reported approximately every five years. First, the annual BAU emissions are derived from the growth rate of the Analysis and Modeling Group (1999). Then the data of BAU emissions from year 2002 to 2008 is added to become the calibrated data of BAU emissions in stage one.

Kennedy (2000) has stressed that the BAU and the reduced emissions in stage two can be calibrated from the Energy Research Group (2000) who has estimated the least abatement cost with a national wide marginal abatement cost of $120 and a discount rate of 10%. Under this least-cost scheme, the estimated BAU emissions and the emission reductions in six sectors were forecasted for year 2010. Kennedy (2000) had also expanded the annual emission data to 5 years. A scale factor of 1.004 was chosen to ensure the extrapolated total equals the budget.

The parameters in the model, namely θ_{ij} and α_{21}, are derived in the following ways. First, the capital investment in stage two is arbitrarily assumed to be the numeraire abatement method. This is because the parameters vary among sectors and periods. By assuming a numeraire, the other parameters will become some relative values. According to Kennedy (2000), the parameters can be calibrated according to these relationships:

$\alpha_{21} = \theta_{11} = \theta_{22}$, $\theta_{21} = 1.1\theta_{22}$, and $\gamma = 2\theta_{22}^{-2}$. These values imply that abatement due to research and planning and capital investment (in both periods) have the same marginal cost, that the learning effect associated with early capital investment is 10%, and that behavioural abatement is twice as costly as technological abatement. Second, the numeraire parameter θ_{22} is derived by using marginal abatement cost of $120. Third, the result of all other parameters is based on the relationships with the numeraire parameter θ_{22}.

Table 3. 1--Optimal Solutions under Different Settings

Variable	Deterministic (i)	Stochastic			
		Increasing risk		"Bad News Principle"	
		(ii)	(iii)	(iv)	(v)
Random variable (d)	0	0.5	0.75	0.50	0.50
Probability (q)	—	0.50	0.50	0.50	0.75
Research & planning investment	7302	7302	7302	7302	7302
Capital investment in stage one	8835	8243	7339	2209	4970
Capital investment over two stages	20594	20002	19099	13968	16729
Behavioural abatement costs	5880	5880	5880	5880	5880
Emissions in period 1	4135	4142	4154	4242	4188
Emission reductions in period 2 due to EA	433	425	412	315	374
Emissions in period 2	2825	2833	2846	2943	2884
Permit price	120	120	120	120	120
Expected Compliance Cost	33776	33184	32280	27149	29910

Notes: Investments and costs are given in million dollars. Emissions and abatements are reported in megatons. Permit price is given in dollars.
*Source*s: Kenney (2002) for deterministic and author's calculation for stochastic results.

The aggregate optimal investments (including research and planning investment, and capital investment), behavioural abatement cost, emissions, and total abatement cost are presented in column (i) of Table 3.1. Kennedy (2002) considered the co-benefit of emission reductions, while the thesis does not. The compliance cost is approximately 33 billion dollars. The compliance cost minus the co-benefit will yield the same result as Kennedy (2002).

3.2 Increasing Risk

Increasing risk is an economic application of nonuniqueness of density function. According to Rothschild and Stiglitz (1970), increasing risk is an increasing variability of random variable, or random variable becomes riskier. In other words, increasing risk refers to a wider movement of the random variable, while the probability of doing so will not change. Since in a continuous distribution, the probability of any individual point on the real line is zero, the values of the density function of the random variable can be changed arbitrarily at a sequence of points without changing the probability distribution of that random variable. Technically, a sequence of steps can be taken to shift the probability weight from the center of the density function to the tail while keeping the mean, and is called a "mean preserving spread."

Uncertain payoffs of capital investment can be inferred to the choice of random variable. To model a technological uncertainty, let a given unit of capital investment yield technological abatement such that $\theta_{21}(\varepsilon\, y_1)^{1/2}$, where ε is a random variable with mean one. The technological abatement $\theta_{21}(\varepsilon\, y_1)^{1/2}$ together with the random variable ε describes an evolution of uncertain future payoffs. Intuitively, $\varepsilon\, \mathrm{y}_1$ is the past investment, y_1, rescaled to the efficiency units of the current technology y_2. The random variable ε reflects the outcome of the research process for the productivity of new investments relative to the productivity of the past investments. The minimization problem is

$$J_1 = \underset{x_1, y_1}{Min}\left\{ c_1(x_1, y_1) + \frac{1}{1+\rho} \mathrm{E}\, J_2(x_1, y_1) \right\}, \quad (3.1)$$

where $J_2(x_1, y_1) = \underset{y_2, e_2}{Min}\, c_2(y_2, e_2)$.

The first order condition will ensure that the optimal capital investment and behavioural abatement in stage two will be determined by permit price and prospective parameters rather than uncertainty. The expected minimized cost in stage two will be subjected to uncertainty such that

$$EJ_2(x_1, y_1) = -p_2^2\left(\frac{\theta_{22}^2}{4} + \frac{1}{4\gamma} \right) + p_2\left(b_2 - \alpha_{21} x_1^{1/2} - \theta_{21} E(\varepsilon\, y_1)^{1/2} \right). \quad (3.2)$$

In stage one, the firm will minimize the total cost over both stages by choosing research and planning investment x_1 and capital investment y_1. The optimal x_1 and y_1 are found by substituting Equation (3.2) into Equation (3.1) and setting the derivative of the right hand side of Equation (3.1) equal to zero. The optimal capital investment will satisfy

$$1 = \frac{1}{2}\theta_{11} p_1 y_1^{-1/2} + E\frac{1}{2}\theta_{21} p_2 y_1^{-1/2}\left(\sqrt{\varepsilon}\right). \quad (3.3)$$

It is obviously that the second term on the right, $\frac{1}{2}\theta_{21} p_2 y_1^{-1/2}\sqrt{\varepsilon}$, is concave in random variable ε. As a result, the optimal capital investment y_1 will decrease under increasing risk. Under a mean preserving spread in risk, the reason of such decreasing is that, with

ε uncertain, the expected marginal technological abatement of capital investment (equivalent to MPK) will change less than $\frac{1}{2}\theta_{21} p_2 y_1^{-1/2}\sqrt{E(\varepsilon)}$.[3] Thus, the firm will undertake less capital investment y_1 and increase its expected marginal technological abatement of capital investment, in order to drive back to the fixed relationship in Equation (3.3).

The numerical example is conducted in two steps: first step is changing the probability of random variable while keeping the mean constant; second step is changing the movement of the random variable in order to shift the probability weight. The parameters θ_{ij}, α_{21} and γ remain fixed at the benchmark levels because the parameters are derived from the least-cost scheme. They indicate the fact that learning effect exists in stage two, and that behavioural abatement is more costly than technological abatement.

Suppose the random variable ε increases to 1+d with probability 0.5, the future payoff will increase to $\frac{1}{2}\theta_{21}\overline{Y_1}^{1/2}$, where $\overline{Y_1} = (1+d)y_1$. Suppose ε fall to 1-d with probability 0.5, and the future payoff will decrease to $\frac{1}{2}\theta_{21}\underline{Y_1}^{1/2}$, where $\underline{Y_1} = (1-d)y_1$. The mean of ε will remain fixed at the initial level. By solving the minimization problem of Equation (3.1), the optimal capital investment will be:

$$y_1 = \left(\frac{1}{2}\theta_{11} p_1 + \frac{\theta_{21} p_2}{2(1+\rho)}\left(0.5\sqrt{1+d} + 0.5\sqrt{1-d}\right)\right)^2 \qquad (3.4)$$

[3] This argument utilizes Jensen's Inequality.

The numerical results of this experiment are presented in column (ii) and (iii) of Table 3.1. Column (ii) shows the results when d=0.50. When the second step is pursued, the value of d changes to 0.75 and the numerical results are given in column (iii).[4] As expected, the capital investment in stage one decreases with mean preserving spread: the firm will delay capital investment y_1 under increasing risk. Another interesting result is that the minimized compliance cost decreases under increasing risk. Column (ii) and (iii) show that, if all combinations are equally likely, the higher the level of uncertainty, the lower the level of early action investment and the minimized cost. The lower minimized cost comes entirely from option value that occurred by delaying investment (waiting to invest) and adopting flexible investment schedules. Similar to Weibrod's definition, this flexibility can be counted as an option value in the sense that it will force the firm to avoid the downside risk caused by uncertainty and, thus, induce the minimized cost lower.

3.3 The "Bad News Principle"

The size of uncertainty may vary, and random variable may move asymmetrically. The outcome of research investment on emission reductions may be a "good news" (an upward movement) or a "bad news" (a downward movement). If bad news happens, the firm will regret of what it has invested in research, and the investment will not yield as much abatement as expected; the good news is otherwise.

[4] Wider movement of d, or, "riskier".

Technically, the bad news can be conceived as not delaying the capital investment such that $\overline{Y_1} = y_1$ with probability q, and the good news is $\underline{Y_1} = 0$ with probability 1-q, where the fraction d ranges between 0 and 1. The firm's problem is to minimize the expected cost in Equation (3.1). The expected minimized cost in stage two is

$$EJ_2(x_1, y_1) = -p_2^2\left(\frac{\theta_{22}^2}{4} + \frac{1}{4\gamma}\right) + p_2\left(b_2 - \alpha_{21}x_1^{1/2} - q\theta_{21}\overline{Y_1}^{1/2}\right). \tag{3.5}$$

Substituting Equation (3.5) into Equation (3.1), and taking the first order condition, it yields the optimal investment

$$y_1 = \left(\frac{1}{2}\theta_{11}p_1 + q\frac{\theta_{21}p_2}{2(1+\rho)}\right)^2. \tag{3.6}$$

The optimal capital investment y_1 will not depend on 1-q nor d, but on q and other exogenous parameters. The result restates Bernanke's "Bad News Principle": the decision of whether to invest in stage one does not depend on how good the good news might be (optimal investment y_1 is independent of the probability of good news, 1-q), but on how bad the bad news might be (optimal investment y_1 depends on q).

A numerical example is given by letting d=0.50, q=0.50 and q=0.75. The value of the parameters, θ_{ij}, α_{21} and γ, is the same as what they are in the numerical benchmark. The calibrated results are reported in column (iv) and (v) of Table 3.1. As expected, if the bad news is not too bad, the firm tends to hold up the capital investment. As the probability of bad news increases, the firm tends to increase the capital investment in stage one. In other words, the more likely to receive a bad news, the less incentive the firm has to postpone its investment. This result indicates that the option value (or the value of waiting) is the result of avoiding the downside risk in the sense that the firm will delay the timing of capital investment in order to avoid the occurrence of the bad news.

3.4 Summary

In a stochastic setting, some numerical examples are developed under increasing risk, and in a world of good news and bad news. Both experiments show that the firm tends to delay the capital investment in stage one in order to gain more information that is propitious. Since delaying investment will reduce a downside risk, holding up the capital investment the firm will gain an option value, which is the value of information that can be revealed with research and planning, and this will reduce the minimized abatement cost.

CHAPTER 4 A CONTINUOUS-TIME MODEL

The previous chapters suggest that facing technological uncertainty the firm shall delay early action investment that has option-like characteristics. There are some theoretical and numerical limitations, however, of the model in the previous two chapters. First, the two-period model is too simple to gain insights on investment decisions constantly revised over time. Second, the movement and the probability of the random variable (d) can only be chosen arbitrarily such that the random payoffs do not evolve continuously over time and, thus, the model fails to address investment decisions under the ongoing uncertainty.

A more general model will be presented in this chapter under the framework of Dixit and Pindyck (1994).[5] They have developed a general investment decision rule in a stochastic dynamic setting and concluded that when future economic conditions are uncertain and investment is irreversible, facing a downside risk, firms should not invest until more information is revealed.

The goal of this chapter is to determine the optima path of investment and its timing, and to address:

- How a firm should manage the technological uncertainty when delaying investment on emission reduction is possible.

[5] In most of the real options literature, uncertainty is captured implicitly by a Brownian motion because it is then relatively simple to derive an optimal investment and a value of waiting. In this thesis, the stochastic process in Equation (4.1) is custom-built according with the technological uncertainty.

- What the target investment level (optimal investment) should be, and when the firm should invest.

4.1 Continuous Model

4.1.1 Technological uncertainty

Capital investment will follow a continuous-time stochastic process. Uncertainty will occur in every step of emission reduction research and project: initiation, processing, and completion. The rational firm will observe the realization of existing productivity and choose an investment level, which will transform into capital in the future. Let Y be the cumulative capital stock depreciating at a rate of δ , and y be an instantaneous capital investment. The evolution of the capital stock is modelled to take the form

$$dY = (y - \delta Y)dt + \upsilon \, dW \, , \tag{4.1}$$

where $dW = \varepsilon \sqrt{dt}$.

The term dW is the increment of a standard Wiener process with ε , a normally distributed random variable with a mean of zero and a variance of σ^2. The depreciation rate δ and the positive parameter υ are exogenous.

The first term on the right hand side of Equation (4.1) is the expected rate of investment. In the deterministic case, the increment of cumulative capital investment is the net capital investment. The second term is a random component. The parameter υ parameterizes the risks with respect to uncertain outcomes of research and planning: the greater the value of the parameter, the higher the level of the technological uncertainty that will result in higher penalties or gains. There will not be any uncertainty if $\upsilon = 0$. The Wiener process

$W(t)$ has standard characteristics as follows: it is a Markov process, it has independent increments, and the change in the process is normally distributed over finite time interval. The variance σ^2 provides a measure of spread, or dispersion, caused by favourable or unfavourable exogenous shocks; a positive ε implies a favourable shock while a negative ε indicates an unfavourable one. For simplicity, I will assume that the variance is as small as one because a rational firm will be continuously responding to shocks by revising its investment schedules, thus, will be unlikely to suffer from a big shock resultant a large σ^2.

The randomness of dW is entirely from the ongoing uncertainty. The increment of cumulative capital stock changes even if the new investment does not, which suggests that an ongoing technological uncertainty will change the effective level of cumulative capital stock regardless of what the firm does. The strong Markov property of the Wiener process $W(t)$ will demonstrates this condition. A continuous-time Markov process possesses the strong Markov property in the sense that the probability distribution of the stochastic process depends on its current value only. In the case of research and planning, a past research discovery on emission reductions incorporates into current knowledge accumulation quickly, and future knowledge accumulation will be determined by current research discovery and knowledge accumulation only. Since research on emission reductions will resolve uncertainty, the capital investment schedules will depends on the pace and the amount of knowledge accumulation on emission reductions, and the evolution of knowledge accumulation will lead to stochastic process of capital investment.

4.1.2 The Abatement Function and the Cost Function

The technological abatement and the cost function correspond to the two- period model in Chapter 2. Instantaneous technological abatement corresponding to Equation (2.1) and Equation (2.2) becomes

$$r = \theta_1 y^{1/2} + \theta_2 Y^{1/2}, \tag{4.2}$$

where $\theta_i \geq 0$ and $i = 1, 2$.

The technological abatement in Equation (4.2) is slightly different from the one in Chapter 2. In this chapter, it is assumed that the time path of research and planning investment is fixed, perhaps at its optimal level. The research and planning investments are made for the purpose of knowledge creation. For a single representative firm, the knowledge accumulation will not likely appreciate or depreciate without the flow of new investment. As a result, there will be no value of waiting for investing in research and planning, and only by investing the firm can reveal propitious information. Kennedy (2002) has stressed that research and planning investment is less costly than capital investment as it reveals the viability of the prospective technological improvement. Thus, the firm will invest in the knowledge creation at a maximum rate until the cumulative research and planning investment reaches an optimal level. There is also the likelihood that research and planning reflects activity beyond the control of the firm, which is exogenous to the investment decision. Therefore, it makes sense to assume the cumulative research and planning investment fixed.

The behavioural abatement is not included in the continuous-time model because the model will focus more on investment issues. Another reason is that the expected behavioural abatement is determined by equating the expected marginal abatement cost to the permit price, thus, the expected behavioural abatement will not change under technological uncertainty. The numerical examples in the previous chapters show that the firm tends to change the level of investment and the actual emissions in response to the uncertainty.

Recall that b is the BAU emission levels. The difference between BAU emissions and abatement will be an emission permit, for which the firm has to pay. The total abatement cost corresponding to Equation (2.3) and (2.4) is

$$C = y + p(b - r). \tag{4.3}$$

This equation implies that the total abatement cost is the sum of investment and actual emission payments beyond the Kyoto target. Substituting Equation (4.2) into (4.3) yields per-period total abatement cost function, corresponding to Equation (2.6) and (2.7),

$$C = y + p\left(b - \theta_1 y^{1/2} - \theta_2 Y^{1/2}\right). \tag{4.4}$$

4.2 The Mechanism of Dynamic Programming

In the context of DP, the subsequent decisions should proceed optimally for the sub-problem starting at the next instant regardless the current decisions. A rational firm will choose an optimal path of investment that will be independent of time (time invariant) and will minimize the abatement cost.

The dynamic adjustment around the equilibrium can be explained with DP. The assumption that the firm has a perfect foresight or rational expectations is crucial since it will ensure that an optimal investment decision will take into account a change of the state of the world. In a deterministic setting, current investments will affect future abatement cost through capital stock accumulation, which will entail an extra shadow value. In a stochastic case, both current investment and uncertainty will induce an extra shadow value on the future cost. The optimal investment rule will satisfy the condition that the value of marginal unit of cumulative capital stock equal to the cost saving. In particular, the influence of uncertainty and irreversibility require the cost saving to include the option value of waiting for more information, rather than irreversibly invest in the unit.

The spirit of DP approach is the Bellman equation. Denote J as a collection of minimized costs. Recall that C is the cost that accrues all at once. Let a prime denote investment at the next instant. The Bellman Principle of Optimality at time interval Δt is given as

$$J(Y) = \underset{y}{Min}\left\{C(Y, y)\Delta t + (1 + \rho\Delta t)^{-1} \mathrm{E}\left[J(Y') \middle| Y, y\right]\right\},$$

where $Y' = Y + \Delta Y$,

and ρ is a discount rate. The equation says that the remaining investment choice y' is subsumed in the minimized cost $J(Y')$ grounded in the current information, thus only y remains to be chosen optimally. Why does the firm optimize its expected cost under uncertainty? In the deterministic case, the expectation symbol can be simply eliminated. In the stochastic setting, under the rational expectations assumption, the firm is aware of the underlying stochastic process of Y, and the expectation of the future minimized cost $J(Y')$ will be equal to the unknown true value of the minimized cost.

As shown in Appendix A.1, the continuous-time Bellman equation can be rewritten as

$$\rho J(Y) = \underset{y}{Min}\{C(Y, y) + 1/dt\,\mathrm{E}(dJ)\}. \tag{4.5}$$

The discounted abatement cost is the sum of a minimum per period cost and the direct effect of the change in time of expected minimum abatement cost. The structure of the last term on the right of Equation (4.5), $1/dt\,\mathrm{E}(dJ)$, can be presented clearly by applying Ito's lemma on such that

$$dJ/dt = (y - \delta\ Y)J_Y dt \tag{4.6}$$

$$1/dt\ EdJ = (y - \delta\ Y)J_Y dt + \frac{1}{2}\upsilon^2 J_{YY}. \tag{4.7}$$

Equation (4.6) and (4.7) describe as a direct effect of the change in time on the expected minimum cost in the deterministic setting and the stochastic setting, respectively. Substituting Equation (4.6) or Equation (4.7) into Equation (4.5) yields the explicit form of the Bellman equation in the deterministic setting,

$$\rho J(Y) = \underset{y}{Min}\{C(Y, y) + (y - \delta\ Y)J_Y\}, \tag{4.8}$$

and Hamilton-Jacobi-Bellman (HJB) in the stochastic setting,

$$\rho J(Y) = \underset{y}{Min}\left\{C(Y, y) + (y - \delta\ Y)J_Y + \frac{1}{2}\upsilon_Y^2 J_{YY}\right\}. \tag{4.9}$$

4.3 Deterministic Case

This section restricts the problem in the deterministic setting in order to narrow down the discussion of the effect of uncertainty. In the deterministic case, the firm is risk neutral and has perfect foresight. The firm will make decisions based on intrinsic logic rather than relying on ad hoc assumptions, or rules of thumb, and will exhibit optimization behaviour. The firm's problem is to discover an optimal investment rule to minimize the total abatement cost over Kyoto periods

The minimized abatement cost under deterministic setting, $\widetilde{J}$, is denoted differently than the one under stochastic case. The firm's problem is to solve the Bellman equation (4.8), which is a combination of the dynamic framework and the static sub-problem. A dynamic movement of the system is introduced by the shadow price of cumulative capital stock $\widetilde{J}_Y$ over time. A direct effect of the change by time in cumulative capital stock on the minimized cost $\widetilde{J}$ will be $1/dt\left(d\widetilde{J}_Y\right)$. The static sub-problem is solved with a first order condition in the same way in a common static problem. An optimal investment rule can be found by equating the value of marginal unit of cumulative capital stock to the cost saving C_y. Thus, the DP combines a dynamic framework and a static optimization problem in order to yield an optimal path of investment.

The algebra in Appendix A.2 shows the details of the optimization of the cumulative capital investment. The optimal investment rule must satisfy the partial differential equations below:

$$\dot{y} = \frac{-\frac{1}{2}\theta_2 pY^{-1/2} + (\rho + \delta)\left(1 - \frac{1}{2}\theta_1 py^{-1/2}\right)}{\frac{1}{4}\theta_1 py^{-3/2}} \tag{4.10}$$

$$\dot{Y} = y - \delta\, Y. \tag{4.11}$$

Given the initial condition $Y(0) = Y_0 \geq 0$ and $y(0) = y_0 \geq 0$, solving the differential Equation (4.10) and (4.11) directly will induce tedious results. Instead, a phase diagram, as presented in Figure 4.1, is utilized to illustrate the stability of the model and the characteristics of the solutions.

The Equations (4.10) and (4.11) imply that the capital investment will converge to equilibrium and will remain there. This can be easily shown in Figure 4.1. The loci $\dot{Y} = 0$ and $\dot{y} = 0$ divide the nonnegative orthant into four isosectors. The locus $\dot{Y} = 0$ is a straight line with a slope of δ. The locus $\dot{y} = 0$ consists of two lines: the horizontal axis and curve $y = \left[\frac{\frac{1}{2}(\rho + \delta)\theta_1 p}{(\rho + \delta) - \frac{1}{2}\theta_2 pY^{-1/2}}\right]^2$. The two loci intersect at Point $\tilde{E}(\tilde{Y}^*, \tilde{y}^*)$, or the origin (0, 0).

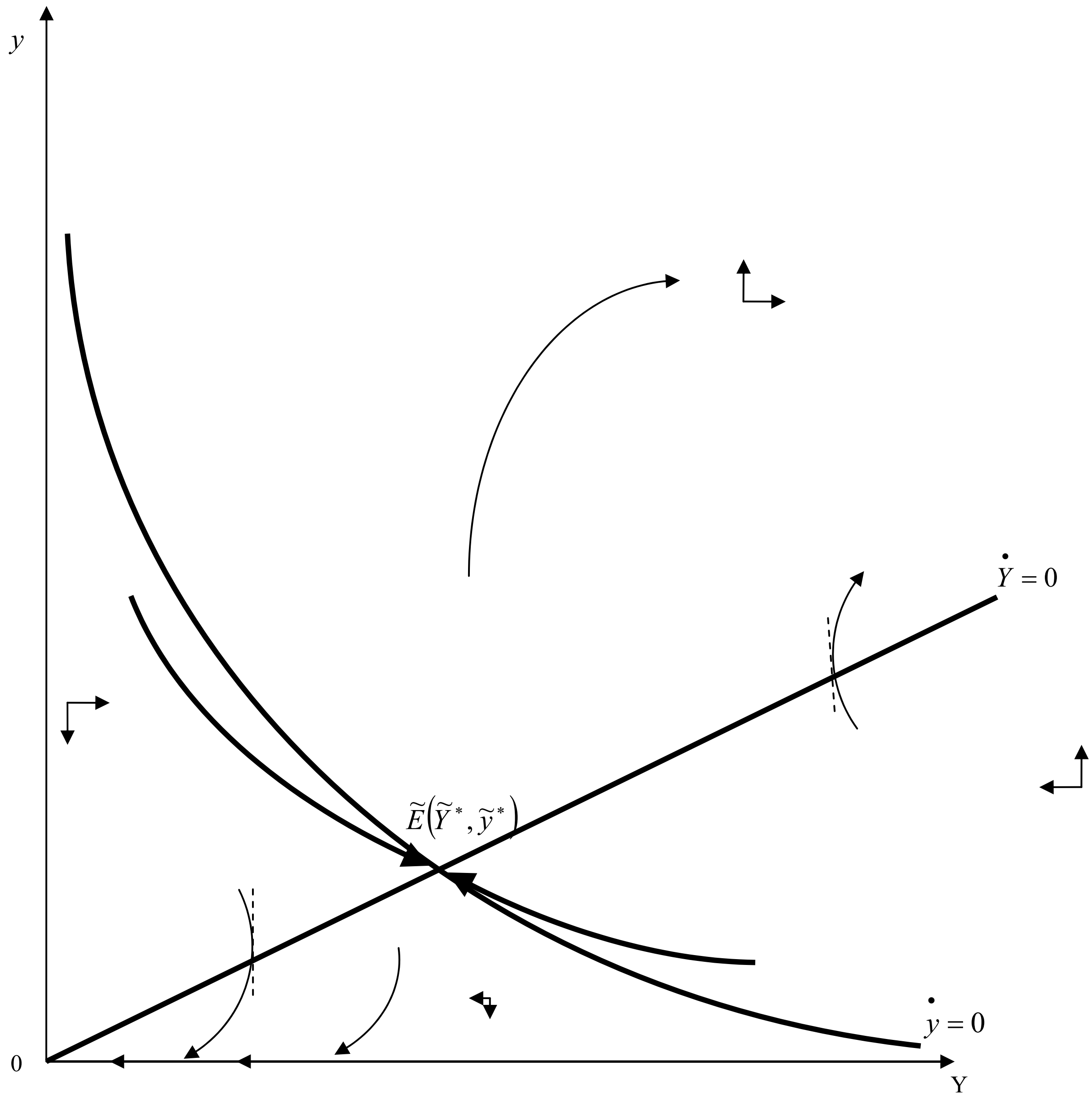

Figure 4. 1. Phase Diagram for Capital Investment

Point $\tilde{E}$ is the long-run equilibrium. When the initial capital stock is low, the firm will invest at a high initial rate of investment in order to approach $\tilde{Y}^*$. Or, when the initial capital stock is high, the firm will invest at a low rate until the capital stock depreciates to $\tilde{Y}^*$. With perfect foresight, the firm will invest at such a level that is consistent with imtertemporal optimization, or exists along the stable path, in order to approach the equilibrium $\widetilde{E}$. Intuitively, the equilibrium, as the intersection of the two loci, represents a point where neither the rate of investment nor the capital stock is changing over time. Once the capital stock cumulated to its optimal level, the optimal investment rule solves the minimization problem, and the dynamics of the system has no incentive to move to elsewhere but to stay at the equilibrium. However, because the two loci become arbitrarily small near the equilibrium, it will take an infinite amount of time to reach the equilibrium.

The conditional equilibrium (0, 0) exists under two possible conditions: first, both the rate of investment and the cumulative capital stock are zero, and second, the permit price is zero. In other words, there is no penalty to emit and the firm has no incentive to invest in emission reductions. The rate of investment will become zero, and the cumulative capital stock will eventually depreciate to zero because of the discount rate and the appreciation rate.

4.4 Stochastic Solutions

In the stochastic setting, uncertainty and irreversibility induce more features compared to those in the deterministic case. First, technological uncertainty is embedded in the stochastic process of Y. The optimal path of the cumulative capital stock Y is determined in the model, and what can be taken exogenous is the position of the unique optimal trajectory $y^*(Y)$. Second, the firm's problem is to minimize the expected abatement cost over time, balancing the cost C, which accrues at once and the expected cost,

$\mathrm{E}[J(Y')|Y, y]$, accrues thereafter over time. Third, the variable J and the stochastic process of Y are not differentiable with respect to time because their time derivatives do not exist. Instead, Ito's lemma, which is intuitively the stochastic extension of the chain rule, and the differential operator $(1/dt)Ed(\bullet)$ will be used to solve the minimization problem.

A direct effect of a change in capital investment on the minimized cost is the differential of HJB equation with respect to Y such as

$$\rho J_Y = C_Y + (y - \delta\, Y)J_{YY} - \delta\, J_Y + \frac{1}{2}\upsilon^2 J_{YYY}. \tag{4.12a}$$

Using Ito's lemma, Equation (4.12a) can be rewritten as

$$\rho J_Y = C_Y - \delta\, J_Y + 1/dt\mathrm{E}d(J_Y) \tag{4.12b}$$

The optimal rate of investment that minimize the cost can be obtained by taking the first order condition of HJB equation such as

$$J_Y = -C_y. \tag{4.13}$$

Equation (4.13) states that the value of a marginal unit of Y is the cost saving C_y. Rather than in the deterministic case, with uncertainty, Equation (4.13) is not differentiable with respect to time. Instead, the differential operator $(1/dt)\mathrm{E}d(\bullet)$ is used on Equation (4.13) such that:

$$\begin{aligned} 1/dt\mathrm{E}d(J_Y) &= -1/dt\mathrm{E}d(C_y) \\ &= -1/dt\mathrm{E}\left[C_{yy}dy + \frac{1}{2}C_{yyy}(dy)^2\right] \end{aligned} \tag{4.14}$$

Along the stable path, the optimal rate of investment $y^* = y^*(Y)$ can be expanded as

$$1/dt\,\mathrm{E}(dy)^2 = 1/dt\,\mathrm{E}(y_Y dY)^2 = y_Y^2 \upsilon^2 . \tag{4.15}$$

Substitute Equation (4.13), (4.14), and (4.15) into Equation (4.12b) and rearrange, will yield the condition of optimal rate of investment. Thus, the optimal investment rule satisfies

$$\frac{1}{dt}Edy = \left(\frac{y}{\frac{1}{4}\theta_1 p} \right) \left[\left(-\frac{1}{2}\theta_2 p Y^{-1/2} + (\rho + \delta) \right) y^{1/2} - \frac{1}{2}(\rho + \delta)\theta_1 p + \left(\frac{3}{8}\theta_1 p y_Y^2 \upsilon^2 \right) y^{-2} \right] \tag{4.16}$$

$$1/dt\,\mathrm{E}dY = y - \delta\, Y . \tag{4.17}$$

4.5 The Effect of Uncertainty

The optimal path of capital investment is required to satisfy the differential Equations (4.16) and (4.17). As in the deterministic case, a phase diagram will be presented in order to illustrate the effect of technological uncertainty. The associated calculation is provided in Appendix A.2. The phase diagram is presented in Figure 4.2, along with the $\dot{y} = 0$ locus for the purpose of comparison.

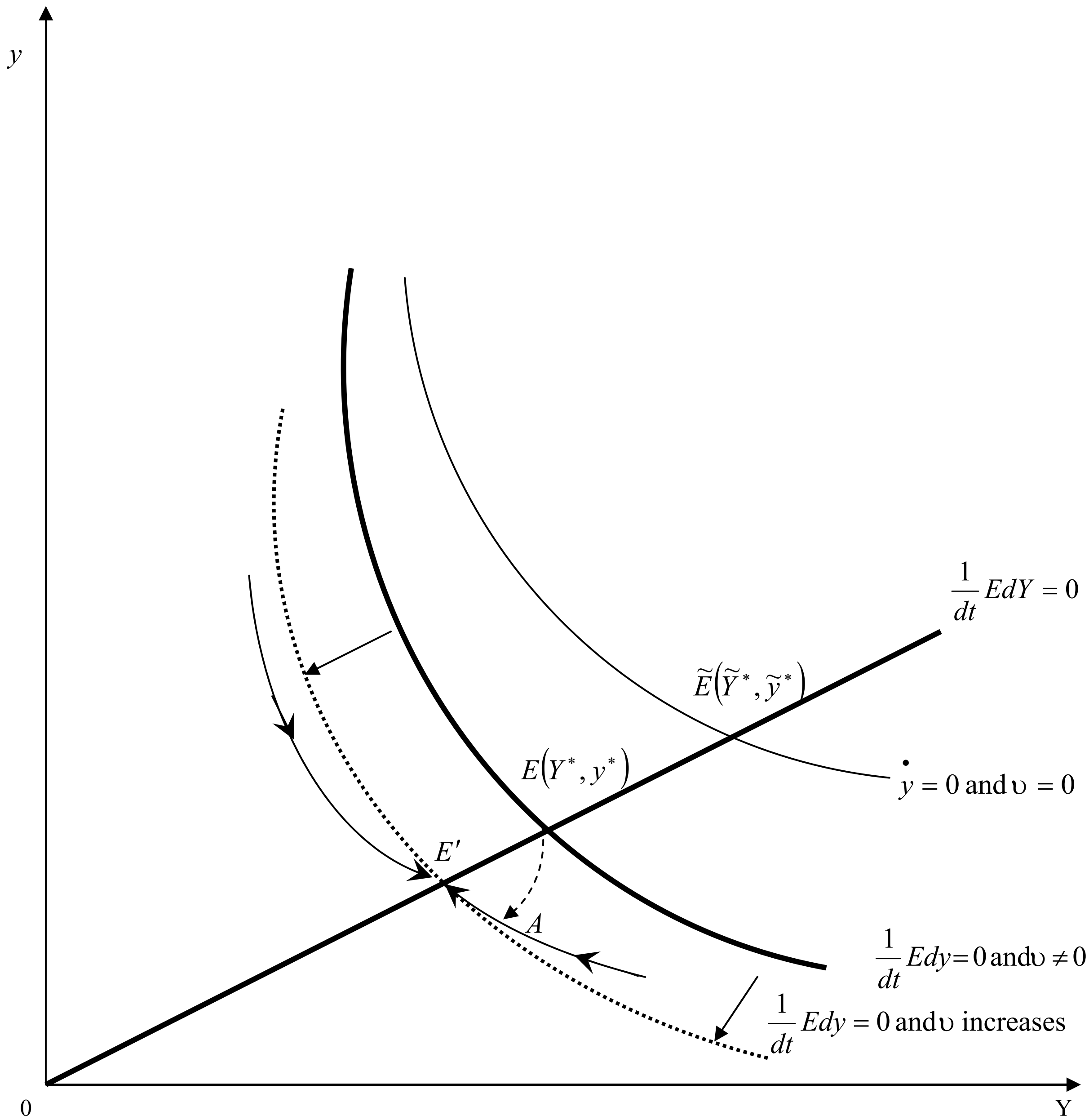

Figure 4. 2. The Effect of Technological Uncertainty

The thickly drawn loci in Figure 4.2 divide the nonnegative plan (Y, y) into four isosectors with unique sign. The locus $1/dtEdY = 0$ duplicates $\dot{Y} = 0$ in deterministic case, and $1/dt\,Edy = 0$ defines two lines: the horizontal axis at $y = 0$, and the curve that satisfies $Y^{1/2} = \frac{1}{2}\theta_3 py^{1/2} \Big/ \left[-\frac{1}{2}(\rho + \delta)\theta_1 p + (\rho + \delta)y^{1/2} + \left(\frac{3}{8}\theta_1 py_Y^2\upsilon^2\right)y^{-2} \right]$. At Point $E(Y^*, y^*)$, the long-run equilibrium, the rate of investment and the capital stock will remain at a steady state.

The parameter υ parameterizes the risks with respect to uncertain outcome of research and planning. It is somewhat similar to the random variable d in Chapter 3. A higher value of υ implies more risk in research and planning, otherwise it implies less risk. One extreme case is when υ approaches zero, thus, Equation (4.16) becomes exactly the same as Equation (4.10), and the boundary of the stochastic case and the deterministic case will meet.

The increment of υ can be inferred as an unanticipated permanent increase in research funds. Large research projects, in most of the cases, involve more uncertainty, and require more research funds. Suppose the firm is initially at the long run equilibrium E. When the government announces that more federal research fund on emission reductions is available for firms, υ will increase, and this will affect the $1/dtEdy = 0$ locus according to Equation (4.16). An increment in υ means that, for a given capital stock Y, $1/dt\,Edy$ is higher than the initial level. The rate of investment will decrease immediately to force the dynamics of y to drop to Point A when the shock occurs. Intuitively, the force that pushes the dynamics of Y and y move to A is the opportunity cost of irreversible investing, rather than waiting. In other words, under uncertainty and irreversibility, the firm will retain an option value by gaining more information. The value of waiting, or option value, will create an incentive for the firm to hold up the investment schedule. Finally, the dynamic adjustment of Y and y will move to a new long-run equilibrium at Point E', and the optimal investment rule will stay at a lower level. Therefore, an exogenous increase in research fund will have a reverse effect: it will not create more incentive to invest, but will induce a sudden drop of the rate of investment, and will result in a lower optimal investment rate and lower optimal capital stock. In other words, increasing risk (as υ increases) will reduce the optimal stock of capital, and the firm tends to wait for more information.

In conclusion, the optimal capital investment and its cumulative level will be reduced due to the technological uncertainty. Technological uncertainty is a consequence of research and planning, which is embedded in the parameter υ. The results also suggest that, any exogenous increase in research funds will induce an immediate decrease in the rate of investment. This conclusion supports the validity of the numerical examples in Chapter 3, thus, the firm will have a strong incentive to delay capital investment until sufficient information is revealed.

4.6 Permit Price Uncertainty

The United States represents 32% of Annex I baseline (1990) emissions and is expected to be a net buyer, mostly from Russia or Ukraine. However, the U.S., the largest prospective buyer, had decided to withdraw from the Kyoto. Consequently, the permit price under Annex I country will drop dramatically due to the repudiation, and this jeopardizes the stability and the existence of a tradable permit system. However, there is still a chance that the U.S. will re-negotiate the Kyoto because provision has been made in order to keep the Kyoto alive, or the U.S. will not ratify the Kyoto at all. This section considers the early action investment decision of Canadian firms, given the U.S. will persistently change its position in the Kyoto protocol for the regulatory purpose.

Beyond technological uncertainty, whether the U.S. will be absent from the Kyoto is still uncertain, which will directly lead to permit price uncertainty. Suppose the U.S. has a fifty-fifty chance to ratify the Kyoto protocol, and the permit price uncertainty will not have a combined effect with other uncertainty. A phase diagram in Figure 4.3 illustrates the impact of permit price uncertainty on early action investment. Suppose before the first commitment period, the U.S. make a final announcement that it will not ratify the Kyoto at all. The permit price will consequently reduce to $\hat{p}$ $(\text{where } \hat{p} > 0)$. The permit price will not drop to zero since no one would have any reason to "sell" them for nothing. There is also the prospect that potential sellers, if few in number, will act more politically. The $1/dt\, Edy = 0$ locus will consequently shift to the left according to

$$Y^{1/2} = \frac{1}{2}\theta_2 p y^{1/2} \Big/ \left[-\frac{1}{2}(\rho + \delta)\theta_1 \hat{p} + (\rho + \delta) y^{1/2} + \left(\frac{3}{8}\theta_1 p y_Y^2 \upsilon^2 \right) y^{-2} \right]. \tag{4.18}$$

The new long-run equilibrium will be $\hat{E}\left(\hat{Y}^*, \hat{y}^*\right)$. Otherwise, if the U.S. will ratify the Kyoto, the equilibrium will stay where it is at $\mathrm{E}\left(Y^*, y^*\right)$.

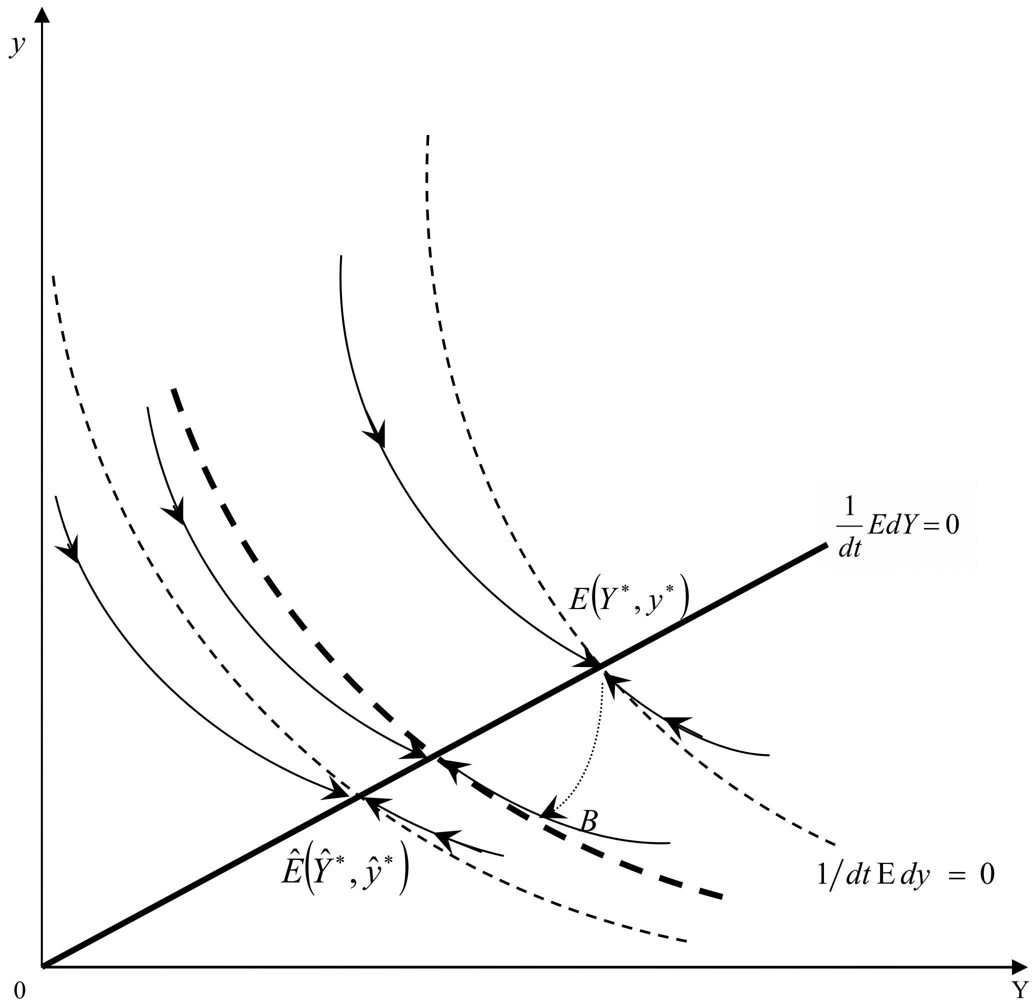

Figure 4. 3. The Role of the U.S. in the Kyoto Protocol

Under the influence of uncertainty and irreversibility, though there is fifty-fifty chance to move either to equilibrium $\hat{E}\left(\hat{Y}^*,\hat{y}^*\right)$ or to $E\left(Y^*,y^*\right)$, the locus $1/dt\,Edy=0$ will shift more than half way from E, and the new locus will be closer to the locus with equilibrium $\hat{E}$. In fact, the firm will have rational expectations about the U.S.'s choice but rather hazy about when the U.S. will make a final decision. Before the U.S. makes the decision, y will drop to Point B, and the dynamics of Y and y will carry them to the stable path around the thickly drawn dashed curve. If the U.S. eventually agrees, the dynamics of Y and y will be pushed back to the long-run equilibrium E, and the uncertainty will only effect the investment decision as a temporary shock. On the other hand, if the U.S. does not agree, the price will drop to $\hat{p}$, and the dynamics of Y and y will move to a new equilibrium at Point $\hat{E}$ and remain there. The lower the permit price $\hat{p}$, the lower the equilibrium $\hat{E}$ will be, and the more investment will the firm want to defer. An extreme case is y drops to the horizontal axis in the sense that the repudiation has such large influence that the international tradable permit system will finally collapse and the firm has no incentive to invest in emission reductions at all. The fact that it is costly to reduce capital holdings will induce the firm to invest much lesser because the option value (the value of waiting) will cause the firm to invest less. Therefore, the U.S. decision will have a substantial impact on Canadian firms' capital holdings in the Kyoto as the U.S. may re-negotiate the Kyoto protocol. The firms will hold up early action investment until the U.S. states its position in the Kyoto protocol.

4.7 Summary

This chapter lays out the Kyoto investment rule in general. The main finding of the continuous-time model in this chapter supports the conclusion in Chapter 3: under uncertainty, firms will delay the early action investment. An evolution of capital investment is a direct result of uncertain outcomes of research and planning. By waiting, firms will gain an option value through the accumulation of propitious information. The results also suggest that either a large sum of Canadian federal fund, or uncertainty surrounding the position of the United States in the Kyoto protocol, will reduce the optimal early action investment.

CHAPTER 5 CONCLUSION

This thesis is concerned with applying the real option theory to early action investment of the Kyoto Protocol under uncertainty. The early action investment is stylized as capital investment and research and planning investment. The main characteristics of these investments are irreversibility, ability to wait, and uncertainty of future payoffs. With these characteristics, the investment decisions will be highly sensitive to uncertainties such as technological uncertainty and price uncertainty. The main finding of the thesis is that firms should delay the capital investment of early action until propitious information is revealed.

The thesis demonstrates that the Kyoto protocol is well intentioned but not wisely implemented. Uncertainty and irreversibility will affect the effectiveness of Kyoto as well as the optimal capital holding on emission reduction projects. A consequence of low-level optimal early action investment will be low-level optimal emission reductions, while Canada has to meet its commitment, which is set at a much higher level than the level of optimal emission reductions. This will induce a large social loss, and will create a heavy financial burden in the future. The role of the United States in the Kyoto protocol increases complication of the firms' investment decisions. When the US's decision is negative, this thesis suggests Canadian firms not to invest in emission reductions projects. Canada shall reduce its emissions gradually rather than setting its rigid commitment target. The Kyoto protocol requires many countries to meet some fixed targets within a short period. This is not feasible for most firms such that more modification is required to the Kyoto protocol. For example, the Kyoto commitment period can be extended to a later date.

There are many possible extensions of this thesis. An interesting future research topic would be a derivation of the optimal investment when a joint decision on research and capital investment occurs. The probability distribution of cumulative capital investment might depend jointly on current cumulative capital investment and knowledge accumulation. The problem can also be extended to deal with technological uncertainty with research interaction (industry specific), or, to deal with other uncertainties such as a permit price uncertainty. It will also be interesting to study the research and development process in more detail.

APPENDIX

A.1 A Derivation of the Bellman Equation

Denote J as the value function of the state variable X and Y. The early action and the Kyoto periods will last at time interval Δt with discounted rate of ρ. The discrete-time Bellman equation gives

$$J(Y) = \underset{y}{Min}\left\{C(Y, y)\Delta t + (1 + \rho\Delta t)^{-1} \mathrm{E}\left[J(Y')\middle| Y, y\right]\right\}, \qquad \text{(A.1)}$$
$$\text{where: } Y' = Y + \Delta Y.$$

The firm chooses the rate of investment over time to obtain an expected minimum cost in the whole time interval. The term $(1 + \rho\Delta t)^{-1}$ is a discount factor for the time interval. Multiply the discount factor through and rearrange, it yields

$$\rho\Delta t J(Y) = \underset{y}{Min}\{C(Y, y)(1 + \rho\Delta t)\Delta t + \mathrm{E}(\Delta J)\}. \qquad \text{(A.2)}$$

Divide both sides of Equation (A.2) by Δt, and let Δt approach zero, yields

$$\rho J(Y) = \underset{y}{Min}\{C(Y, y) + 1/dt\, \mathrm{E}(dJ)\}. \qquad \text{(A.3)}$$

As Δt goes to zero, the Bellman equation becomes continuous. Equation (A.3) is the continuous time Bellman equation, and is equivalent to Equation (A.1).

Denote $\tilde{J}$ as the minimized cost in the deterministic setting. Equation (A.3) can be rewritten as:

$$\rho\tilde{J}(Y) = \underset{y}{Min}\left\{C(Y, y) + 1/dt\left(d\tilde{J}\right)\right\} \qquad \text{(A.4)}$$

A.2 An Optimal Path in the Deterministic Setting

Differentiate both sides of Equation (4.8) with respect to Y yields

$$\rho \tilde{J}_Y = C_Y + (y - \delta\, Y)\tilde{J}_{YY} - \delta\, \tilde{J}_Y . \tag{A.5}$$

Notice that by Ito's lemma, the expansion of $d\tilde{J}_Y$ will become

$$1/dt\left(d\tilde{J}_Y\right) = (y - \delta\, Y)\tilde{J}_{YY} .$$

Insert the $d\tilde{J}_Y$ expansion back to Equation (A.5) yields

$$\rho \tilde{J}_Y = C_Y + 1/dt\left(d\tilde{J}_Y\right) - \delta\, \tilde{J}_Y . \tag{A.6}$$

Minimize Equation (4.8) with respect to y yields

$$\tilde{J}_Y = -C_y . \tag{A.7}$$

As $\tilde{J}_Y$ and C_y is differentiable with respect to time, Equation (A.7) can be rewritten as

$$1/dt\left(d\tilde{J}_Y\right) = -dC_y/dt = -\left(\frac{dC_y}{dy}dy\right)\Big/ dt = -C_{yy}\frac{dy}{dt} . \tag{A.8}$$

Combine Equation (A.6), (A.7) and (A.8) to eliminate $\tilde{J}_Y$, yields $\dot{y}$ such that

$$\begin{aligned}\dot{y} &= \frac{C_Y + (\rho + \delta)C_y}{C_{yy}} \\ &= \frac{-\frac{1}{2}\theta_2 p Y^{-1/2} + (\rho + \delta)\left(1 - \frac{1}{2}\theta_1 p y^{-1/2}\right)}{\frac{1}{4}\theta_1 p y^{-3/2}} \\ &= \left(\frac{y}{\frac{1}{4}\theta_1 p}\right)\left[\left(-\frac{1}{2}\theta_2 p Y^{-1/2} + (\rho + \delta)\right)y^{1/2} - \frac{1}{2}(\rho + \delta)\theta_1 p\right].\end{aligned} \tag{A.9}$$

The initial condition are given as $Y(0)=Y_0 \geq 0$ and $y(0)=y_0 \geq 0$. The differential equations $\dot{Y}=y-\delta Y$ and Equation (A.9) are difficult to solve analytically. However, the system of capital investment can be described with phase diagram.

Equation $\dot{Y}=y-\delta Y$, Equation (A.9) and the initial condition can be used for characterizing the solution of the capital investment system in the nonnegative orthant of the (Y,y) plane as shown in Figure 4.1. The $\dot{Y}=0$ locus is the straight line $y=\delta\,Y$ with slope δ. The $\dot{y}=0$ locus requires that y=0, or

$$y=\left[\frac{\frac{1}{2}(\rho+\delta)\theta_1 p}{(\rho+\delta)-\frac{1}{2}\theta_2 p Y^{-1/2}}\right]^2, \tag{A.10}$$

which is decreasing and convex. There are two Points at which both $\dot{Y}=0$ and $\dot{y}=0$. First is at the origin (0, 0), and second is at the intersection of curve $y=\delta\,Y$ and the curve represented by Equation (A.10); hence, Point $\tilde{E}(\tilde{Y}^*,\tilde{y}^*)$ is where $\tilde{Y}^*=\left[\frac{(\rho+\delta)\delta^{-1/2}\theta_1 p+\theta_2 p}{2(\rho+\delta)}\right]^2$ and $\tilde{y}^*=\delta\tilde{Y}^*$. The loci $\dot{y}=0$ and $\dot{Y}=0$ define four regions (isosectors) where the sign of $\dot{y}$ and $\dot{Y}$ are uniquely determined inside each region. To see this, note that $\partial\dot{y}/\partial Y>0$ and $\partial\dot{Y}/\partial y>0$ which imply that as Y increases, $\dot{y}$ increases, and as y increases, $\dot{Y}$ increases. The trajectories across the loci will change the signs. Hence, we can define the sign of each region as shown in Figure 4.1. The arrows on the curves indicate the directions of admissible trajectories in each region. The equilibrium point (0, 0) is conditional stable: only the trajectories that start in the south region, and some trajectories in the west region cross over $\dot{Y} = 0$ will converge to this point. The trajectory that start in north region and some trajectories in the east region cross over $\dot{Y} = 0$ will not converge to any equilibrium. As the trajectories cover over the entire space of the west and the east region, there must be one stable path will approach the equilibrium $\tilde{E}\left(\tilde{Y}^*, \tilde{y}^*\right)$ because the trajectories in the west region move to southeast and the ones in the east region moves to the southwest.

A.3 An Optimal Path in the Stochastic Setting

With Ito's lemma on $\mathrm{E}dJ(Y)$ and drop the terms of a higher order more than one will yield

$$\mathrm{E}dJ(Y) = \left((y - \delta\, Y) J_Y + \frac{1}{2} \upsilon^2 J_{YY} \right) dt \, . \tag{A.11}$$

Substitute (A.11) into (A.3) yields:

$$\rho J(Y) = \underset{y}{Min} \left\{ C(Y, y) + (y - \delta\, Y) J_Y + \frac{1}{2} \upsilon^2 J_{YY} \right\} \tag{A.12}$$

Differentiate both side of Equation (A.12) with respect to Y such that:

$$\rho J_Y = C_Y + (y - \delta Y) J_{YY} - \delta J_Y + \frac{1}{2} \upsilon^2 J_{YYY} \tag{A.13}$$

By using the Ito's lemma, Equation (A.13) can be rewritten as:

$$\rho J_Y = C_Y - \delta\, J_Y + 1/dt\, Ed\left(J_Y\right) \tag{A.14}$$

The investment y will minimize the right hand side of the Equation (A.12). The first order condition will be

$$J_Y = -C_y . \tag{A.15}$$

Applying differential operator on both sides of Equation (A.15) and using Ito's lemma to expand C_y yields

$$1/dt\mathrm{E}d(J_Y) = -1/dt\mathrm{E}d(C_y) = -1/dt\mathrm{E}\left[C_{yy}dy + \frac{1}{2}C_{yyy}(dy)^2\right]. \tag{A.16}$$

Notice that $y = y^*(Y)$ goes along the trajectory, thus, dy can be expanded. Drop the terms with order higher than one becomes

$$1/dt\,\mathrm{E}(dy)^2 = 1/dt\,\mathrm{E}(y_Y dY)^2 = y_Y^2\upsilon^2 . \tag{A.17}$$

Insert Equation (A.15) to (A.16) into Equation (A.14) to eliminate J_Y such that:

$$-(\rho+\delta)C_y = C_Y - C_{yy}(1/dt\,Edy) - \frac{1}{2}C_{yyy}y_Y^2\upsilon^2$$

Rearrange yields:

$$\begin{aligned}\frac{1}{dt}Edy &= \frac{(\rho+\delta)C_y + C_Y - \frac{1}{2}C_{yyy}y_Y^2\upsilon^2}{C_{yy}}\\ &= \frac{-\frac{1}{2}\theta_2 pY^{-1/2} + (\rho+\delta)\left(1-\frac{1}{2}\theta_1 py^{-1/2}\right) + \frac{3}{8}\theta_1 py^{-5/2}y_Y^2\upsilon^2}{\frac{1}{4}\theta_1 py^{-3/2}}\\ &= \left(\frac{y}{\frac{1}{4}\theta_1 p}\right)\left[\left(-\frac{1}{2}\theta_2 pY^{-1/2} + (\rho+\delta)\right)y^{1/2} - \frac{1}{2}(\rho+\delta)\theta_1 p + \left(\frac{3}{8}\theta_1 py_Y^2\upsilon^2\right)y^{-2}\right]\end{aligned} \tag{A.18}$$

There are two lines will satisfy $\frac{1}{dt}Edy = 0$: one is the y=0 line; and the other one is

$$\left(-\frac{1}{2}\theta_2 pY^{-1/2} + (\rho+\delta)\right)y^{1/2} - \frac{1}{2}(\rho+\delta)\theta_1 p + \left(\frac{3}{8}\theta_1 p y_Y^2 \upsilon^2\right)y^{-2} = 0.$$

The shape of $\frac{1}{dt}Edy = 0 \quad (y \neq 0)$ is discussed in the following way. According to Equation (A.18), the points on locus $\frac{1}{dt}Edy = 0 \quad (y \neq 0)$ will satisfy

$$Y^{1/2} = \frac{1}{2}\theta_2 p y^{1/2} \Big/ \left[-\frac{1}{2}(\rho+\delta)\theta_1 p + (\rho+\delta)y^{1/2} + \left(\frac{3}{8}\theta_1 p y_Y^2 \upsilon^2\right)y^{-2}\right]. \qquad \text{(A.19)}$$

Note that Equation (A.19) is the stochastic version of the Equation (A.10). For convenience, rewrite (A.10) such that:

$$\tilde{Y}^{1/2} = \frac{1}{2}\theta_2 p \tilde{y}^{1/2} \Big/ \left[-\frac{1}{2}(\rho+\delta)\theta_1 P + (\rho+\delta)\tilde{Y}^{1/2}\right] \qquad \text{(A.20)}$$

For given amount of y (or $\tilde{y}$), Y is smaller than $\tilde{Y}$. As y goes to infinity, the limit $\left(\frac{3}{8}\theta_1 p y_Y^2 \upsilon^2\right)y^{-2}$ will approach zero. This indicates that as y increases, Y will be closer and closer to $\tilde{Y}$. The $\frac{1}{dt}\mathrm{E}dy = 0$ and $(y \neq 0)$ locus will be infinitely close to the $\dot{y} = 0$ locus as y increases, and the $\frac{1}{dt}\mathrm{E}dy = 0$ and $(y \neq 0)$ locus will go further away from the locus $\dot{y} = 0$. Thus, the locus $\frac{1}{dt}\mathrm{E}dy = 0$ and $(y \neq 0)$ can be drawn. Another locus $\frac{1}{dt}\mathrm{E}dY = 0$ is the same straight line as the one in the deterministic case $y = \delta Y$. The equilibrium $E(Y^*, y^*)$ is smaller than $\widetilde{E}(\widetilde{Y}^*, \widetilde{y}^*)$. The phase diagram can therefore be presented in Figure 4.2.

A.4 Main Data and Parameters

Table A.1--Data and Parameters

	b_1	b_2	Least Cost	γ	θ_{11}	θ_{21}	θ_{22}	α_{21}
Electricity	688	646	257	0.89	1.50	1.50	1.65	1.50
Industry	1471	1279	1130	2.32	0.93	0.93	1.02	0.93
Residential and Agriculture	295	235	191	7.89	0.50	0.50	0.55	0.50
Transportation	1112	995	775	1.57	1.13	1.13	1.24	1.13
Others*	784	577	472	3.27	0.78	0.78	0.86	0.78
Total	4350	3731	2825					

*Others include commercial & institutional, carbon sinks of forest and agricultural.
Source: Kennedy (2002).

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