NONEQUILIBRIUM TRANSPORT PROCESSES IN
WEAKLY IONIZED PLASMAS

A Thesis Submitted to the College of
Graduate Studies and Research
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
in the Department of Physics and Engineering Physics
University of Saskatchewan
Saskatoon

By
Eugene S. Fourkal
Spring 1999

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SUMMARY OF DISSERTATION

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of the requirements for the

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Abstract

The nonlocal electron and ion transport processes in weakly ionized plasmas are studied. The goal is to consider the most general ordering when the mean free path for the charged species is arbitrary with respect to the characteristic length scale in plasma and characteristic frequency is arbitrary with respect to collision frequency of electron component, we present a rather general method of solving the Boltzmann equation, which is based on the expansion of the total distribution function (DF) in the series of the eigenfunctions of the collision operator. The coefficients in this expansion are related to the different velocity moments of the distribution function. The expansion of the DF in terms of the eigenfunctions of the collision operator is equivalent to the expansion in the series of a parameter \( \varepsilon \) which is a measure of spatial and temporal uniformity (\( \varepsilon = k \lambda; \varepsilon = \omega/\nu \)). As this parameter increases (the mean free path becomes larger and/or the characteristic length of plasma inhomogeneity decreases and/or characteristic frequency of plasma inhomogeneity increases), the larger number of terms should be included in the expansion procedure and in the limiting collisionless case all harmonics (all moments) must be included. The obtained infinite system of equations for the expansion coefficients is solved in terms of the continued fraction representation. We have calculated transport coefficients of a weakly ionized
plasma that describe the relaxation processes for the arbitrary uniformity parameter $\varepsilon$. In this case the transport coefficients become integro-differential operators acting on the lower moments (density, temperature, mean velocity, and external fields). As an example we consider the anomalous absorption of the electromagnetic wave by a weakly ionized plasma or anomalous skin effect. Unlike the classical skin effect in which the electric field is monotonously (exponentially) decaying inside of the conductive medium (plasma, etc.), in the anomalous case, the nonmonotonous decay occurs and there are regions where the absolute value of the electric field can increase that correspond to the negative absorption of the wave energy. This effect is a direct consequence of the influence of the thermal electron motion on the electric conductivity coefficient and cannot be obtained from the fluid regime (small $\varepsilon$).

For ion component, nonlocal ion transport in a weakly ionized plasma with a strong electric field is analyzed. It is assumed that charge-exchange interactions are the main mechanism of ion scattering. Ion density and drift velocity are determined for nonuniform time varying electric field by using both the direct solution of the kinetic equation and the Chapman-Enskog type approach. The ion mean velocity is calculated in terms of a nonlocal ion mobility that is an integro-differential operator applied to the electric field. Ion density and drift velocity exhibit resonant behavior when $\omega \simeq kW_0$, which corresponds to the resonance between ions moving with average velocity $W_0$ and wave traveling with the phase velocity $\omega/k$. 
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Chapter 1

Introduction

1.1 Introduction

The charged particle transport processes in weakly ionized plasmas play a fundamental role in gas discharge physics and its applications, in particular, for a variety of plasma sources used in science and technology. Knowledge of particle transport is required for the design of low temperature plasma reactors for material processings, radio-frequency (RF) lighting sources, as well as for understanding fundamental properties of nonequilibrium discharges. The ensemble-averaged dynamic behavior of an assembly of charged particles in a background gas under the influence of a space-time varying field may be described at a microscopic level by the time-dependent distribution function $f(v, r, t)$ (where $v$ is the velocity, $r$ is the position, and $t$ is the time). Knowledge of this function is required for determining the macroscopic properties of the system. Given the initial state of the assembly, the distribution function at any other time may be obtained from either the analytical solution of a kinetic equation
such as a Boltzmann equation (BE)

\[ \partial_t f + \mathbf{v} \cdot \nabla_r f + \frac{q}{m} \mathbf{E} \cdot \nabla_v f = C(f), \tag{1.1} \]

where \( \partial_t = \frac{\partial}{\partial t}, \nabla_r = \frac{\partial}{\partial \mathbf{r}}, \nabla_v = \frac{\partial}{\partial \mathbf{v}}, \ C(f) \) is the collision integral, \( q \) is the electric charge, \( m \) is the mass and \( \mathbf{E} \) is the electric field, or from numerical simulations. Once the distribution function is found, desired macroscopic properties (density mean velocity, temperature, etc.) can be calculated by averaging the corresponding microscopic properties over the distribution. An alternative method for obtaining the macroscopic properties of the plasma is a moment approach [1]. In one form or another, this method has been used to treat nonlocal transport in gas discharges for the past 75 years [2, 9, 63]. In general, an exact description of the assembly requires an infinite set of moments (this is equivalent to the fact that we need an infinite set of moments to specify the distribution function \( f \)). At the macroscopic level, that can be described by the "state vector" (of infinite dimension) \( \mathbf{S} \) whose components are moments of the distribution function. These moments form a hierarchy (infinite set) of equations obtained by taking different, properly weighted velocity moments of the Boltzmann equation [2]. The equations for the first three moments [namely, density \( n(\mathbf{r}, t) \), mean energy \( \bar{e}(\mathbf{r}, t) \), and average velocity \( \mathbf{u}(\mathbf{r}, t) \)] are

\[ \partial_t n + \nabla \cdot (n\mathbf{u}) = -\nu n \tag{1.2} \]

\[ \partial_t (n\bar{e}) + \nabla \cdot (e\mathbf{v}) - q\mathbf{E} \cdot n\mathbf{u} = -\nu_e n\bar{e} \tag{1.3} \]

\[ \partial_t (nu) + \nabla \cdot (n\mathbf{v}) - \frac{q}{m} E n = -\nu_m nu \tag{1.4} \]

where the bracket implies an average over the distribution, \( e = \frac{m u^2}{2} \), and \( \nu, \nu_e, \) and \( \nu_m \) are the effective-ionization, energy exchange, and momentum exchange frequencies. These frequencies are defined by

\[ -\nu n = \int C(f) \, dv \]
\[ -\nu_t n \bar{c} = \int \frac{1}{2} m v^2 C(f) \, dv \]  
(1.6)

\[ -\nu_m n u = \int v C(f) \, dv \]  
(1.7)

where \( C(f) \) is the total collision integral. Because of the difficulty in obtaining solutions to the equations for the infinite state vector \( S \), a description in terms of the finite number of moments is desirable. Unfortunately, any finite set of moment equations is not determinate. For example, the set of Eqs. (1.2-1.4) contains unknown averages over the distribution function (quantities in brackets) and unknown rates [Eqs.(1.5-1.7)]. To calculate these unknowns and thus arrive at a determinate set of equations for \( S_N \), one has to find the proper closure method in which the higher unknown moments are expressed through the lower moments. A similar problem arises in classical gas kinetics, which has been a subject of much investigation [3, 4, 5]. For some cases in gas kinetics [namely those, for which the external forces are zero and the spectrum of the corresponding (linearized) collision operator \( C(f) \) is either known or significant properties are known], it is possible to obtain formal (exact) expressions for \( f \), i.e., expansions in terms of the eigenvectors of \( C(f) \) [3, 4, 5]. The coefficients in this expansion method are related to different moments of the distribution function. When the distribution function is substituted into the Boltzmann equation one obtains an infinite system of linearized equations for the expansion coefficients or moments. For example the first three moments of the distribution function for a gas of Maxwell molecules (molecules, that interact with each other, with the inverse fifth power of distance), namely the density, mean velocity and temperature satisfy the following system of linearized equations:

\[ \frac{\partial n}{\partial t} + \frac{\partial V}{\partial x} = 0 \]  
(1.8)

\[ \frac{\partial V}{\partial t} + \frac{\partial \Pi_{xx}}{\partial x} + \frac{\partial T}{\partial x} + \frac{\partial n}{\partial x} = 0 \]  
(1.9)
\[
\frac{\partial T}{\partial t} + \frac{2}{3} \frac{\partial q_x}{\partial x} + \frac{2}{3} \frac{\partial V}{\partial x} = 0
\]  \hspace{1cm} (1.10)

The equations (1.8-1.10) are written in normalized variables \(n/n_0, T/T_0, p/p_0, \Pi_{xx}/p_0,\)
\(q/p_0 v_T, \omega t, \) and \(kx,\) where \(\omega^2 = v_T^2 k^2, v_T^2 = T_0/m.\) The right-hand sides of the
system are equal to zero which is a direct consequence of the three-fold degeneracy
of the lowest eigenvalue of the collision operator, or the conservation of the number
of particles, their total momentum and energy. Assuming the plane wave solution
\[
b_{\alpha\beta} = \left( \begin{array}{c}
    n \\
    V \\
    T \\
    \Pi_{xx} \\
    q_x
\end{array} \right) = \hat{b}_{\alpha\beta} e^{\sigma t - i k x} \hspace{1cm} (1.11)
\]

for all variables, where the \(\hat{b}_{\alpha\beta}\) are constants, we obtain
\[
\sigma n - ikV = 0 \hspace{1cm} (1.12)
\]
\[
\sigma V - ikn - ikT - ik \Pi_{xx} = 0 \hspace{1cm} (1.13)
\]
\[
\sigma T - \frac{2ik}{3} V - \frac{2ik}{3} q_x = 0 \hspace{1cm} (1.14)
\]

This system of algebraic equations contains undefined quantities, namely the stress
tensor \(\Pi_{xx}\) and the heat flux vector \(q_x.\) In the limit of slow spacial variations of the
higher moments (small \(k\)) we can neglect the spatial derivatives of the stress tensor
and heat flux vector to obtain
\[
\sigma n - ikV = 0 \hspace{1cm} (1.15)
\]
\[
\sigma V - i kn - ikT = 0 \hspace{1cm} (1.16)
\]
The above system of equations, the Euler equations, is completely determinate and a solution can be given in terms of eigenmodes of the dispersion relation:

\[
D^{(1)}(\sigma, k) = \det \begin{pmatrix}
\sigma & -ik & 0 \\
-ik & \sigma & -ik \\
0 & -\frac{2}{3}ik & \sigma
\end{pmatrix} = 0
\] (1.18)

The solution of the dispersion equation (1.18) has three roots which are

\[
\sigma = 0, \quad \sigma = \pm \sqrt{\frac{5}{3}}ik
\] (1.19)

Since the real part of \(\sigma\) is equal to zero in the approximation of a small wave number \(k\), there is no decay in such a system and the Euler equations give the nondecaying sound propagation solution. In the next-order expansion in a wave vector \(k\), the heat flux vector and the stress tensor cannot be neglected and in order to resolve the system we need to supplement the system with additional equations for the heat flux and stress tensor to give

\[
\sigma n - ikV = 0
\] (1.20)

\[
\sigma V - ikn - ikT - ik\Pi_{xx} = 0
\] (1.21)

\[
\sigma T - \frac{2ik}{3}V - \frac{2ik}{3}q_x = 0
\] (1.22)

\[
(\sigma - \lambda_{02}) \Pi_{xx} - \frac{4}{3}ikV - \frac{8}{15}ikq_x + 2\sqrt{\frac{21}{35}}ikb_{03} = 0
\] (1.23)

\[
(\sigma - \lambda_{11}) q_x - \frac{5}{2}ikT - ik\Pi_{xx} - \sqrt{\frac{10}{3}}ikb_{20} + \sqrt{\frac{14}{3}}b_{12} = 0
\] (1.24)

where \(b_{20}, b_{12}, b_{03}\) are higher moments of the distribution function, \(\lambda_{02}\), and \(\lambda_{11}\) are the eigenvalues of the collision operator, which are always negative. Again, discarding
the moments \( b_{20}, b_{12}, b_{03} \) in equations for the stress tensor and heat flux we arrive at the system of five equations that has five unknowns. The dispersion relation for the eigenmodes is

\[
D^{(2)}(\sigma, k) = 0 = \det \begin{pmatrix}
\sigma & -ik & 0 & 0 & 0 \\
-ik & \sigma & -ik & -ik & 0 \\
0 & \frac{-2ik}{3} & \sigma & 0 & \frac{-2ik}{3} \\
0 & \frac{-4ik}{3} & 0 & (\sigma - \lambda_{02}) & \frac{-8ik}{15} \\
0 & 0 & \frac{-5ik}{2} & -ik & (\sigma - \lambda_{11})
\end{pmatrix}
\]  

(1.25)

The solution of the dispersion equation (1.25) has five roots which are (for small \( k \) and large \( \lambda \)) [4]

\[
\sigma = \frac{k^2}{\lambda_{11}} + O(k^4).
\]  

(1.26)

\[
\sigma = i\sqrt{\frac{5}{3}} \left[ k + k^3 \left( -\frac{2}{5\lambda_{11}\lambda_{02}} + \frac{1}{10\lambda_{11}^2} + \frac{8}{15\lambda_{02}^2} \right) \right] + k^2 \left( \frac{1}{3\lambda_{11}} + \frac{2}{3\lambda_{02}} \right) + O(k^4)
\]

(1.27)

\[
\sigma = \frac{\lambda_{11}}{k_{21}} \frac{k^2}{\lambda_{11}} \left[ \frac{5}{3} + \frac{8\lambda_{11}}{15(\lambda_{11} - \lambda_{02})} \right] + O(k^4), \quad \sigma = \lambda_{02} - \frac{k^2}{\lambda_{02}} \left[ \frac{4}{3} + \frac{8\lambda_{02}}{15(\lambda_{02} - \lambda_{11})} \right] + O(k^4)
\]  

(1.28)

In this approximation, there appears the real part of \( \sigma \) which signifies the modes decay processes. The branch given by \( \sigma = \frac{k^2}{\lambda_{11}} \) is purely decaying and is related to the heat conduction. The sound propagation branch in this approximation is dispersive and decaying as opposed to the Euler equations where it is nondispersive and nondecaying. One can proceed further and calculate the dispersion relation to higher powers of the wave number \( k \). As we see, the dispersion relation can be presented as a power series in a wave number \( k \). The inclusion of the higher moments leads to the higher powers of \( k \), and to a higher decay rate, which is physical, since
the higher moments have shorter decay times in the strongly collisional limit, so that the higher the moment is the shorter the decay time is. In general, the solution to the infinite system of equations will give the dispersion relations for all propagation modes possible in a considered system. The number of modes is determined by the number of eigenroots $\lambda_{a,\beta}$ of the collision integral $C(f)$. For the gas of Maxwell molecules there are one real and two complex branches that emanate from the origin ($\sigma = 0$). The purely real branch leads to a strictly decaying mode which is connected to the heat conduction mode, whereas the two complex branches have propagation in addition to decay. As it was said earlier these two conjugate branches correspond to ordinary sound propagation. There are other complex branches which indicates the existance of additional propagating modes. Also, there are additional purely decaying modes. These latter modes are distinguished from the hydrodynamic modes (modes that start from the origin $\sigma = 0$) in that they have a negative real part even in the limit of $k \to 0$. This results in an immediate exponential decay in time for all such modes. The solution of the Boltzmann equation includes all modes and may be thought of as a succession of epochs, or temporal layers on the $\sigma$ plane. Each epoch is determined by a "folding time" $\lambda_{a,\beta}^{-1}$. After each of these times a cluster of modes becomes, roughly speaking, $e^{-1}$ of its initial value. Asymptotically, any particular mode is negligible compared to its neighbor with the larger characteristic times, so that only modes with a life time larger than a given time scale, $\lambda_{a,\beta}^{-1} > \tau_c$, have to be retained if one is interested in approximate dynamics accurate on a time scale $\tau_c$. The hydrodynamic mode has an infinite life time, and all other modes are asymptotically small compared to it [5]. When the gas becomes more rarefied, the eigenvalues of the collision operator become smaller (since they are proportional to the density of particles), the mode damping decreases, and the density of modes in the particular
time scale increases. Physically it means that one needs to include more modes to properly describe the system in the desired time scale.

In contrast to classical gas dynamics, very little is known about the properties of the operator \((q/m) \mathbf{E} \cdot \nabla_v - C(f)\) for the case of charged particles in a weakly ionized plasma. In this case, the nature of the interactions between charged species and background makes it very difficult to gain information as to the properties of these operators. Because of this, a more physical approach was proposed [6]. The key to this approach is the use of information from the macroscopic equations to affect the truncation. First, the moments are ordered according to their characteristic relaxation rates. This step requires \textit{a priori} assumptions about the relative magnitude of these scales. Equations (1.2-1.4) have been ordered according to their characteristic times. These times are (in decreasing magnitude): \(\tau\) (effective charged species production/loss time \(= \nu^{-1}\)), \(\tau_e\) (energy-exchange time \(= \nu_e^{-1}\)), and \(\tau_m\) (momentum-exchange time \(= \nu_m^{-1}\)). Next, the number of moments in the state vector \(S_N\) is determined from physical considerations, and from the scale of the desired description. Alternatively, the number of moments that are used determines the coarseness of the macroscopic description. This is because the model is only valid for time scales of the order of the smallest characteristic time contained in the finite set of equations. Finally, the distribution function \(f\) which satisfies Eq.(1.1), contains information to all orders of time greater than a microscopic collision time. This time is, in general, much shorter than the characteristic times in any finite set of moment equations. As far as the moment equations, \(f\) contains "too much information". Thus, to obtain a determinate set of moment equations, it is sufficient to use an \(f\) which only contains information in the time scale of the moment equations. In other words it means that this distribution function, the macroscopic distribution function \(f_M\), is a function of
particle velocity and the finite number of moments

\[ f = f_M (v, n(r, t), V(r, t), T(r, t), ...) \]  \hspace{1cm} (1.29)

This distribution function obeys a "macroscopic-kinetic equation", which can be derived by substituting Eq.(1.29) into the microscopic Boltzmann kinetic equation [7, 8]. The equation of evolution for \( f_M \), together with the finite set of moment equations, form a closed set. This set can be used to describe the nonequilibrium dynamics of the charged species in a time scale corresponding to the characteristic times of the moment equations.

A few words should be said about the time evolution of the system. The most coarse-grained description is valid for times in the order of \( \tau \). From Eqs.(1.2)-(1.4), since \( \nu < \nu_e < \nu_m \), there is a time for which the mean energy and average momentum (and all higher moments) of the charged particles have relaxed to a state of quasiequilibrium where their subsequent variation is in the scale of \( \tau \) i.e., the scale of the density variations. For such times, the macroscopic evolution of the system can be described in a single time scale. Thus, \( S_1 = [n(r, t)] \); that is the macroscopic state vector contains a single moment, the density. By analogy with classical gas kinetics, the time regime for which this description is valid (the longest time scale) is named the hydrodynamic regime. However, in contrast to gas kinetics, the properties of this state can be derived from a single macroscopic variable (instead of three), the density. Progressively less coarse-grained levels of description can be defined by systematically using an additional moment in the state vector. This assumes that the characteristic times in the moment equations are not degenerate. In this case, there are times for which the higher-order moments have relaxed to a state in which their scale of variation is the same as the moments being used in the characterization of the macroscopic state. If there was a degeneracy (for example \( \nu_e^{-1} = \nu_m^{-1} \)), then
the corresponding moments must be collectively taken as components of the state vector. For electrons/ions in weakly ionized plasmas, the characteristic times are not in general, degenerate. Thus, the next less coarse-grained levels of description are in terms of $S_2 = [\bar{n}(r, t), \bar{\varepsilon}(r, t)]$ and $S_3 = [\bar{n}(r, t), \bar{\varepsilon}(r, t), \mathbf{V}(r, t)]$. These are valid for times in the order of $\nu_{ \varepsilon }^{-1}$ and $\nu_{m}^{-1}$ correspondingly.

### 1.2 Two-term Approximation

One of the most widely used moment techniques for solving the Boltzmann kinetic equation for electron component is the expansion of the total distribution function in the series of the spherical harmonics

$$f(r, v, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_{l,m}(r, v, t) Y_{l,m}(\theta, \phi).$$

The advantage of this expansion comes from the fact that the spherical functions are the eigenfunctions of the elastic electron-neutral atom collision operator (in the approximation when the neutrals can be considered to be motionless). Low order truncation of the spherical harmonic representation (1.30) of the velocity distribution function $f(r, v, t)$ of a swarm of electrons in a neutral gas generally reflects the belief (or the hope) that $f$ is nearly isotropic in $v$ space or at least that it has this property over the range of velocities $v$ which controls the transport coefficients of the swarm. The physical basis for this can be seen from elementary mechanics: An electron of mass $m$ and a neutral atom (molecule) of mass $M$ exchange a fraction $\sim 2m/M << 1$ of their energy in an elastic collision and thus, even if the swarm is driven through the gas by a strong electric field, energy and momentum gained from the field are efficiently distributed in all directions through the action of such collisions, which have the effect of randomizing directions of electron velocity vectors $v$, without significantly
altering their magnitude. For practical purposes, it was necessary to make either the truncated representation

\[ f(r, v, t) = \sum_{l=0}^{l_{\text{max}}} \sum_{m=-l}^{l} f_{l,m}(r, v, t) Y_{l,m}(\theta, \phi), \quad (1.31) \]

that is, to assume \( f_{l,m} = 0 \) if \( l > l_{\text{max}} \), or to assume some other property of these higher-order expansion coefficients. For electrons it has been traditional [1] to set \( l_{\text{max}} = 1 \), the so-called two-term approximation. With this approximation, the Boltzmann equation (1.1) can be solved in a relatively straightforward manner by introducing the expansion (1.31) into the Boltzmann equation, multiplying by \( Y_{0,0} \) and \( Y_{1,(-1,0,1)} \) respectively, and integrating over the angle space yielding the following equations for \( f_0 \) and \( f_1 \)

\[ \frac{\partial f_0}{\partial t} + \frac{v}{3} \nabla_r \cdot \mathbf{f}_1 + \frac{1}{3v^2} \frac{\partial}{\partial v} \left( v^2 \frac{q}{m} \mathbf{E} \cdot \mathbf{f}_1 \right) = C_0(f) \quad (1.32) \]

\[ \frac{\partial f_1}{\partial t} + v \nabla_r f_0 + \frac{q}{m} \mathbf{E} \frac{\partial f_0}{\partial v} = C_1(f) \quad (1.33) \]

\( C_0(f) \) and \( C_1(f) \) are the corresponding collision integrals of zero and first order which are found by performing the above integration on the collision term of the Boltzmann equation. The system of two equations (1.32) and (1.33) can be solved to give expressions for the velocity distribution function in a two-term approximation.

This procedure is clearly invalid if \( f \) departs substantially from spherical symmetry in \( v \) space, and then one has to systematically investigate solutions of the Boltzmann equation by successively incrementing \( l_{\text{max}} \) until some convergence criterion is met.

In contemplating a description in terms of the two-term approximation, we arrive at the following validity criteria for the approach:

a) The electron velocity directed along the field \( u \) should be much less than the random velocity \( v \). The terms with higher-order harmonics are proportional to the
appropriate powers of the ratios $u/v$. These ratios serve as the small parameters for series expansion of the distribution function $f(r,v,t)$. The indicated condition is satisfied in most practically interesting cases.

b) The characteristic length scale of the field $(1/k)$ should be sufficiently larger than the electron mean free path. The situation with this condition is more delicate since in low pressure plasma discharges, the electron mean free path may exceed the discharge dimension. In this case the two-term approximation becomes invalid and one should include higher order terms in the expansion.

In terms of the moment approach, discussed in the previous section, the two-term approximation is a truncation procedure in which only scalar (density, temperature, etc.) and vector (mean velocity, heat flux, etc.) moments are taken into consideration. This means, that the two-term approach cannot describe the relaxation of all higher moments such as the pressure tensor, energy weighted pressure tensor, etc. The relaxation of the lower moments (mean velocity, heat flux vector) is described in the local sense, by the two-term approximation, meaning that heat flux vector is proportional to the temperature gradient, the particle flux is proportional to the density gradient and the electric field.

In other words the two-term approximation does not take into account the thermal electron motion, thus it neglects an important physical phenomenon such as the wave-particle interaction, which is the main relaxation mechanism in collisionless or near collisionless plasma discharges. Therefore new analytical methods are needed in order to correctly describe the nonequilibrium transport processes in weakly ionized plasmas for a wide range of discharge parameters.
1.3 The Chapman-Enskog Procedure

As it was mentioned earlier the main problem in the method of moments is to introduce the proper closure procedure, based on the physical considerations of the problem, which would solve prior unsolved system. One of the methods that allows to properly close the system of equations is the Chapman-Enskog procedure. This method consists in the following steps:

Step 1 consists in writing the Boltzmann equation as

$$\left( \frac{\partial}{\partial t} + \widehat{D} \right) f = \frac{1}{\varepsilon} \widehat{C}(f)$$ (1.34)

where the dimensionless parameter $\varepsilon << 1$, $\widehat{D} = \nu \cdot \frac{\partial}{\partial \nu} + K \cdot \frac{\partial}{\partial \nu}$ ($K$ is a force acting on a particle). Setting $\varepsilon << 1$ is equivalent to stipulating that the system is dominated by large collision frequency.

Step 2 consists in the introduction of the expansion

$$f = f^{(0)} + \varepsilon f^{(1)} + \varepsilon^2 f^{(2)} + \cdots$$ (1.35)

where $f$ satisfies the moment relations

$$n = \int f d^3v, \ nV = \int \nu f d^3v, \ \frac{3}{2} nT = \int \frac{m w^2}{2} f d^3v$$ (1.36)

and where $w = \nu - V$, is the random velocity.

Step 3 stipulates that the variables $n, V, T$ are determined by $f^{(0)}$, whereas terms in the series (1.35) corresponding to $f^{(i)}, i > 0$, contribute to the higher moments $q$ (heat flux vector), $\Pi$ (stress tensor). Thus we write

$$\int f^{(0)} \begin{pmatrix} 1 \\ \nu \\ w^2 \end{pmatrix} d^3v = \begin{pmatrix} n \\ nV \\ 3nT/m \end{pmatrix}$$ (1.37)
\[
\int f^{(i)} \begin{pmatrix} 1 \\ \nu \\ w^2 \end{pmatrix} d^3\nu = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad i > 0
\] (1.38)

\[
q = \sum_i \varepsilon^i q^{(i)} = \frac{1}{2} \sum_i \varepsilon^i \int f^{(i)} \nu mw^2 d^3w
\] (1.39)

\[
\Pi = \sum_i \varepsilon^i \int f^{(i)} \nu w d^3w.
\] (1.40)

Substituting the series (1.35) into the Boltzmann equation (1.34) and equating the coefficients of powers of \( \varepsilon \) we find

\[
\hat{C} \left( f^{(0)} \right) = 0
\] (1.41)

\[
\hat{C} \left( f^{(l)} \right) = \left( \frac{\partial f^{(l-1)}}{\partial t} + \hat{D}_j f^{(l-1)} \right), \quad l > 0.
\] (1.42)

The zeroth order solution is the so-called local Maxwellian given by

\[
f^{(0)} = n(r, t) \left( \frac{m}{2\pi T(r, t)} \right)^{3/2} \exp \left( -\frac{m(v - V(r, t))^2}{2T(r, t)} \right).
\] (1.43)

In this manner we find that the zeroth-order solution in the small mean free path approximation is the local Maxwellian \( f^{(0)} = f_M \). This solution may be used to obtain a zeroth-order set of hydrodynamic equations for the purpose of evaluating \( n(r, t), V(r, t), T(r, t) \). To construct these, we first calculate the heat conductivity \( q \) and the stress tensor \( \Pi \). Substituting \( f^{(0)} \) into (1.39) and (1.40) one obtains in the zeroth approximation

\[
q^{(0)} = 0.
\] (1.44)

\[
\Pi^{(0)} = Ip.
\] (1.45)
In the lowest order, there is no heat flow and the pressure tensor is diagonal. Substituting these values into the moment equations for density

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{V} = 0$$  \hspace{1cm} (1.46)$$

mean velocity

$$mn \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = qn\mathbf{E} - \nabla p$$  \hspace{1cm} (1.47)$$

and temperature

$$\left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \left( \frac{p}{n^{5/2}} \right) = 0$$  \hspace{1cm} (1.48)$$

Solution to these equations gives $n = n(r, t), \mathbf{V} = \mathbf{V}(r, t), T = T(r, t)$, which when substituted in (1.43) completely determines $f^{(0)}$.

The first-order solution is obtained from Eq.(1.42)

$$\hat{C} \left( f^{(1)} \right) = \left( \frac{\partial f^{(0)}}{\partial t} + \hat{D} f^{(0)} \right)$$  \hspace{1cm} (1.49)$$

Since the dependence of $f^{(0)}$ is through the zero-order hydrodynamic moments $n, \mathbf{V},$ and $T$ the spatial and temporal derivatives of it are

$$\frac{1}{f^{(0)}} \frac{\partial f^{(0)}}{\partial t} = \left[ \frac{1}{n} \frac{\partial n}{\partial t} + \frac{2w}{v_t^2} \frac{\partial \mathbf{V}}{\partial t} + \left( \frac{w^2}{v_t^2} - \frac{3}{2} \right) \frac{1}{T} \frac{\partial T}{\partial t} \right].$$  \hspace{1cm} (1.50)$$

$$\frac{1}{f^{(0)}} \mathbf{v} \cdot \nabla f^{(0)} = \mathbf{v} \cdot \left[ \frac{1}{n} \nabla n + \frac{2w}{v_t^2} \nabla \mathbf{V} + \left( \frac{w^2}{v_t^2} - \frac{3}{2} \right) \frac{1}{T} \nabla T \right].$$  \hspace{1cm} (1.51)$$

Replacing time derivatives by equations for $n, \mathbf{V},$ and $T$ gives

$$\hat{C} \left( f^{(1)} \right) = f^{(0)} \left[ \left( \frac{w^2}{v_t^2} - \frac{5}{2} \right) w \cdot \nabla \ln T + \frac{2}{v_t^2} \left( \mathbf{ww} - \frac{1}{3} w^2 \mathbf{I} \right) : \nabla \mathbf{V} \right] + K \cdot \frac{\partial f^{(0)}}{\partial \mathbf{v}}$$  \hspace{1cm} (1.52)$$

which is a linear inhomogeneous integral equation for $f^{(1)}$. Solution to this equation will give the first-order correction to the local Maxwellian distribution. The first-order corrections to the heat flux vector and the stress tensor are

$$\mathbf{\Pi}^{(1)} = m \int \mathbf{ww} f^{(1)} d^3\mathbf{v} , \quad q^{(1)} = \int \frac{m w^2}{2} \mathbf{w} f^{(1)} d^3\mathbf{v}$$  \hspace{1cm} (1.53)$$
Extending this procedure further it is possible to find higher order correction terms as well. A few words should be said about the convergence of the Chapman-Enskog expansion method. It is a well known fact that the Chapman-Enskog expansion yields a series which is asymptotic to an actual solution of the Boltzmann equation [3]. It is not convergent in general, therefore the sum should be truncated at a finite number of terms. The number of terms in the expansion is determined by the fact that the remainder in the expansion must be small in parameter $\varepsilon$. For the strongly collisional case it is sufficient to calculate the distribution function to the first order in $\varepsilon$. When the characteristic length/time scales of the system's inhomogeneities become smaller the larger number of terms must be taken into account, and in the limiting, collisionless case one would have to solve an infinite number of partial differential equations.

Therefore, in this thesis we propose a different approach to the solution of the Boltzmann kinetic equation for the electron component in a weakly ionized plasma, which would describe the relaxation processes in a system for any collisionality regimes, including collisionless. Expanding the total distribution function in terms of the eigenfunctions of the collision operator and regrouping some terms in the expansion, so that the lowest order moments such as density, temperature and mean velocity constitute the local Maxwellian distribution and all the rest is the deviation from the local Maxwellian, we will obtain the inhomogeneous system of equations for the expansion coefficients, which is solved in terms of an infinite continued fraction representation. The proposed method successfully surmounts the closure problem, since we solve the complete infinite set of equations for the expansion coefficients.
1.4 Nonlocal Transport Phenomena and Heating in Weakly Ionized Plasmas

The methods presented in the previous sections are mostly concerned with the finding of a solution to the Boltzmann kinetic equation. Once a solution (distribution function) is found, one can determine different moments of the distribution function and subsequently the transport coefficients of the system. This will completely determine the transport processes taking place in the system. Transport of particles and energy is often a critical issue defining the performance of plasma based reactors and lighting devices. In this thesis we demonstrate that in the less dense plasmas, where the collisionless effects of thermal particle motion are important, transport properties are no longer determined solely by particles collisions. In fact, noncollisional mechanisms, such as Landau wave-particle interaction become important. We show, that such collisionless mechanisms with necessity require inclusion of higher order moments such as viscosity, etc. Moreover, these collisionless mechanisms can be described in terms of higher order moments.

As an example we will consider power deposition mechanisms in plasma discharges. The heating of electrons by time-varying fields is fundamental to the operation of radio frequency (RF) and microwave plasma discharges. In a uniform oscillating electric field, \( E = \text{Re} \, E_0 e^{-i\omega t} \), a single electron has a coherent velocity of motion that lags the phase of the electric field force \(-eE\) by 90°. Hence, the time average power transferred from the field to the electron is zero. Electron collisions with other particles destroy the phase coherence of the motion, leading to a net transfer of power. For an ensemble of \( n \) electrons per unit volume, it is usual to introduce the macroscopic current density \( J = e n u \), with \( u \) the macroscopic electron velocity, and to relate the amplitudes of \( J \)
and $E$ through a local conductivity: $J_0 = \sigma_p E_0$ where

$$\sigma_p = e^2 n/m (\nu_m + i\omega)$$  \hspace{1cm} (1.54)

is the plasma conductivity and $\nu_m$ is the energy-independent electron collision frequency for momentum transfer. The electric conductivity coefficient (1.54) can be obtained from the two-term approximation procedure discussed in the previous section.

In the "fluid" (collisional) approach, the average electron velocity $u$ still oscillates coherently but lags the electric field by less than $90^\circ$, leading to an ohmic power transfer per unit volume

$$P_{ohm} = \frac{1}{2} \text{Re} \mathbf{J}_0 \cdot \mathbf{E}_0^* = \frac{1}{2} |E_0|^2 \text{Re} (\sigma_p) = \frac{1}{2} |J_0|^2 \text{Re} (\sigma_p^{-1}) .$$

Although the average velocity is coherent with the field, the fundamental mechanism that converts electric field energy to thermal energy is the breaking of the phase-coherent motion of individual electrons by collisions: the total force (electric field force plus that due to collisions) acting on an individual electron becomes spatially nonuniform and nonperiodic in time.

These observations suggest that a spatially nonuniform electric field by itself might lead to electron heating, even in the absence of interparticle collisions, provided that the electrons have thermal velocities sufficient to sample the field inhomogeneity. This phenomenon has been well-known in plasma physics since Landau [10, 11] demonstrated the collisionless damping of an electrostatic wave in a warm plasma, and is variously referred to in the literature as collisionless, stochastic or anomalous heating or dissipation. Since that time, collisionless dissipation has been studied extensively in fusion and space plasma physics. However, within the last decade and with the increased emphasis on industrial applications of low pressure gas discharges, it has
become evident that collisionless dissipation phenomena are fundamental to RF and microwave discharges. As will be seen in the following chapters, collisionless interactions leading to electron heating can be a basic feature of warm plasmas having space dispersion. The electron response (J) at some point in the plasma is defined not only by the field (E) at that point, but by an integrated effect over the neighboring space. Due to the spatial variation, the time-varying field seen by an individual "thermal" electron is nonperiodic. The electron can lose phase coherence with the field (which is strictly periodic), resulting in stochastic interaction with the field and collisionless heating.

In almost all discharges, the spatial variation of the time-varying field is strongly nonuniform, with a low field in the bulk of the plasma and one or more highly localized field regions (RF sheaths), near the plasma boundaries. An electron, being confined by the dc ambipolar and boundary sheath potential in the discharge, interacts repeatedly with the high field regions, but interacts only weakly during its drift through the plasma bulk. This suggested a dynamical model to investigate the energy transfer and loss of phase coherence: a ball bounces elastically back and forth between a fixed and an oscillating wall. The model was first introduced by Fermi [12] to explain the origin of cosmic rays. The process in which the ball repeatedly interacts with the oscillating wall, resulting in phase randomization and stochastic heating, is known as Fermi acceleration and this acceleration model was adapted for understanding collisionless heating in weakly ionized gas discharges [13].

Another way of looking at collisionless electron heating has been discussed by Surendra and Daivie [14] and by Turner [15]. They showed that an approximation to the heating can be obtained within the macroscopic (fluid) theory by incorporating pressure effects that arise during the expansion and contraction of the sheath in a
nonhomogeneous plasma model. The pressure effect is caused by the difference in plasma density and electron energy between the bulk plasma and the near-sheath plasma. When the sheath expands, electrons flow into the adjacent bulk plasma and are compressed. At the same time, electrons are rarefied as they flow into the opposite, collapsing sheath. Turner showed that these simultaneous rarefactions and compressions of the electron gas produce nonequilibrium thermal disturbances, and the net work done is not zero. In this way, kinetic effects are incorporated approximately into a fluid model. Solving the fluid equations with the electron energy balance equation, Turner verified this approach using PIC simulations (particles in cell), and showed [16] that a weak transverse dc magnetic field can induce a transition between the pressure heating and ohmic heating.

A moments method, developed in the present thesis will be applied to analyzing the plasma heating mechanisms and they will be expressed through the moments (stress tensor, mean velocity, electric current) of the distribution function. It will be shown that the stochastic (collisionless) plasma heating can be solely explained on the basis of the hybrid fluid/kinetic method, in which the kinetic effects of the wave-particle interaction are incorporated in higher moments.

1.5 Summary of Findings

In this thesis, an analytical technique is developed for solving a nonequilibrium Boltzmann kinetic equation. The technique is based on the hybrid fluid/kinetic description which allows to obtain the linear closure procedure between the higher moments (viscosity, heat flux) and the lower moments (temperature, mean velocity, density) for
the most general case, when the charged particle mean free path is arbitrary with respect to the system's spatial and temporal inhomogeneities. The developed approach includes the kinetic effects of wave-particle interactions, which can not be recovered from the conventional methods (gradient expansion, velocity moments, two-term approximation methods). The complete set of transport coefficients of a weakly ionized plasma is obtained for the most general collisionality regime. It is shown that the transport coefficients in this case are the nonlocal integro-differential operators of space and time variables acting on the lower hydrodynamic moments. The anomalous penetration of the electromagnetic waves into the semi-infinite weakly ionized plasma is considered. The mechanisms of the plasma heating in a system are analyzed and expressed through the known hydrodynamic values (density, mean velocity, temperature, pressure, stress tensor).
Chapter 2

Nonlocal Electric Conductivity and Anomalous Skin Effect in a Weakly Ionized Plasma

2.1 Introduction

In this chapter the electric conductivity of a weakly ionized plasma in a time dependent nonuniform electric field when the main mechanism of electron scattering is elastic collisions with neutrals is considered and nonclassical (anomalous) penetration of the electric field into the system is analyzed.

We are interested in a situation when the electron mean free path is not small compared to the characteristic length of the external electric field inhomogeneity, and the electron collisional frequency is not necessary large compared to the characteristic frequency of the electric field oscillations. For such conditions effects of the thermal electron motion become important so that electron conductivity becomes a nonlocal
operator both in space and in time [17, 18]. This modifies the mechanism of the electric field penetration into a plasma that is no longer local but rather becomes nonlocal. It is usually referred to as anomalous skin effect [19, 20]. These effects have been observed experimentally in gas discharges [21, 22]. They are becoming increasingly important as gas discharges for plasma processing and lighting move toward the lower pressure regimes. In this chapter we will develop an approach which allows to uniformly describe both strongly collisional and collisionless regimes. Such approach is especially important for description of the electron transport in inert gases where the differential cross section of the electron-neutral atom interaction exhibits nonmonotonous behavior with the electron energy (the Ramsauer effect) [23] so that electrons of different energies could be in different collisionality regimes.

The traditional procedure of solving the Boltzmann kinetic equation for electrons is to expand the electron distribution function in a series of spherical functions and then truncate this representation retaining only a certain number of terms [1]. This series expansion is equivalent to the expansion of the distribution function in the parameter $k\nu/(i\omega - \nu)$ where $k$ is the wave number, $\omega$ is the frequency of the external electric field, $\nu$ is the collision frequency and $\nu$ is the electron velocity. Often, the so called two-term approximation [19, 23, 24] is implemented to solve the Boltzmann equation. The applicability of the two-term expansion to the problem of the electron conductivity is limited to the case when the parameter $k\nu_T/(i\omega - \nu)$ is sufficiently small, i.e. the electron motion is strongly collisional, $\nu > \omega$, $\nu > k\nu_T$, or the oscillation frequency is large, $\omega > \nu, \omega > k\nu_T$. In this approximation, the perturbed distribution function and, respectively, the electric conductivity do not depend on the wave vector $k$. Thus, the electric conductivity is a local quantity and does not take into account the thermal electron motion. It is worth noting also that the two-term
approximation does not describe higher order moments such as electron heat flux which is important for the problem of anomalous heating [25, 26]. The two-term approximation can be further improved by including a few more terms in the spherical harmonic expansion [27]. An approach that includes the complete infinite hierarchy of spherical harmonics and allows to calculate the perturbed distribution function for arbitrary values of the parameter \( k\nu / (i\omega - \nu) \) is developed. This chapter will concentrate on the nonlocal effects in the perturbed distribution function due to the external electric field and neglect effects of inhomogeneity of the equilibrium electron density due to the equilibrium ambipolar potential. Thus we neglect effects of energy relaxation and associated nonlocal effects due to particle trapping in the ambipolar potential of a bounded plasma column [19, 24]. The latter effects can be described within the two-term approximation and are not considered in this thesis.

Nonlocal electron conductivity and associated anomalous skin effect have also been investigated [17, 19, 28, 29] by solving the Boltzmann kinetic equation with simplified BGK type collision operator in the form

\[
C(f) = -\nu(v)(f - f_M),
\]

(2.1)

where \( \nu(v) \) is the effective collision frequency, \( f_M \) is the Maxwellian distribution. Using (2.1) one can solve the linearized kinetic equation for perturbations without making the expansion in spherical harmonics. Such an approximation may be used for electrons of very low energies where the differential cross section of electron-atom interaction does not depend on the electron energy and is isotropic in space. At higher energies of electrons the cross section of electron-neutral interaction becomes velocity dependent and anisotropic in space. This is usually the most typical situation for electron scattering in inert gases with Ramsauer effect. In this paper we consider the realistic case when the differential cross section is anisotropic and approximation (2.1)
is not valid. As an example we use argon gas where the Ramsauer effect manifests itself in a sharp decrease in the magnitude of the transport cross section as the energy of electrons increases. In the region of the minimum energy, the spatial motion of electrons may become important even if it can be neglected for other energy ranges.

2.2 Solution of the Electron Kinetic Equation in the Inhomogeneous and Time Dependent Electric Field

The Boltzmann kinetic equation for electron distribution function in a weakly ionized plasma with the external time-varying and inhomogeneous electric field is

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{eE(r,t)}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = C(f). \tag{2.2}
\]

Here \( C \) is the collisional integral of interaction between electrons and atoms given by [30]

\[
C(f) = \int v_{rel} (f' f_1' - f f_1) \frac{d\sigma}{d\Omega} d^3 p_1 d\Omega, \tag{2.3}
\]

where \( v_{rel} \) is the relative velocity between electrons and atoms, \( f, f_1 \) and \( f', f_1' \) are the electron and neutral atom distribution functions before and after the collision, \( d\sigma/d\Omega \) is the differential cross section of electron-neutral collisions. In the present paper we neglect the electron-electron and electron-ion interactions, since the neutral atom density is considerably higher then that of the electrons and ions.

We neglect the effects of energy transfer between electrons and atoms because of the large mass difference between the two species. Thus atoms are motionless, and their distribution function is given by

\[
f_1' = f_1 = N \delta (v_{1x}) \delta (v_{1y}) \delta (v_{1z}), \tag{2.4}
\]
where $N$ is the neutral atom density. Substituting the expression (2.4) into (2.3) we get

$$C = N \int (f(t, r, v') - f(t, r, v)) v \frac{d\sigma}{d\Omega} d\Omega. \quad (2.5)$$

As mentioned above we neglect the effects of spatial distribution of the ambipolar potential, so that the electron equilibrium function $F_0(v)$ can be taken in the Maxwellian form with a uniform density. Under the influence of the external electric field, the electron distribution function departs from the equilibrium so that the total electron distribution function can be represented in a form [1]

$$f(v, r, t) = F_0(v) + \sum_{l,m} f_{l,m}(v) Y_{l,m}(\theta, \phi) e^{i(kx - \omega t)}, \quad (2.6)$$

where $v = |v|$, $f_{l,m}(v)$ is a function of the absolute value of electron velocity, $Y_{l,m}(\theta, \phi)$ is the spherical harmonic function and $F_0$ is the Maxwellian distribution function. We assume that the electric field is in the $z$ direction, $E = E\hat{z}$, and its variation is in the $x$ direction, $k = k\hat{x}$.

Substituting (2.6) into (2.5) and (2.2), and expanding the electric field in a Fourier series, one gets in the linear approximation with respect to the electric field

$$-i\omega \sum_{l,m} f_{l,m}(v) Y_{l,m}(\theta, \phi) + ikv \sin \theta \cos \phi \sum_{l,m} f_{l,m}(v) Y_{l,m}(\theta, \phi)$$

$$= \frac{eE(\omega, k)}{m} \cos \theta \frac{\partial F_0}{\partial v} - \sum_{l,m} \nu_l f_{l,m}(v) Y_{l,m}(\theta, \phi), \quad (2.7)$$

where $\nu_l$ is the $l$–th order collision frequency defined by [1]

$$\nu_l(v) = N v \int (1 - P_l(\cos \theta)) \frac{d\sigma(v, \theta)}{d\Omega} d\Omega, \quad (2.8)$$

where $P_l(\cos \theta)$ are the Legendre polynomials. If the differential cross section does not depend upon the poloidal scattering angle $\theta$, then all $\nu_l$ are equal, except for $\nu_0$. The characteristic frequency $\nu_0$ describes the energy relaxation that is neglected here.
due to the large mass difference between the electron and neutral atoms. If \( \nu_l = \text{const} \) for all \( l \) then there is no need of expanding the electron distribution function in the series of spherical functions and one can solve the Boltzmann kinetic equation with the collision term given by (2.1).

If the differential cross section is a function of \( \theta \), equation (2.7) leads to coupled equations for different spherical harmonics. Multiplying (2.7) by \( Y_{l,m}^*(\theta, \phi) \) and integrating over the solid angle we obtain

\[
-i \omega f_{l,m} + \frac{1}{2} ikv \Gamma_{l,m} - \sqrt{\frac{4 \pi}{3} \frac{eE(\omega, k)}{m}} \delta_{l,1} \delta_{m,0} \frac{\partial F_0}{\partial v} = -\nu_l f_{l,m},
\]

where

\[
\Gamma_{l,m} = \sqrt{\frac{(l - m + 1)(l - m + 2)}{(2l + 1)(2l + 3)}} f_{l+1,m-1} + \sqrt{\frac{(l - m)(l - m - 1)}{(2l + 1)(2l + 3)}} f_{l-1,m+1} - \sqrt{\frac{(l + m - 1)(l + m)}{(2l - 1)(2l + 1)}} f_{l-1,m-1} - \sqrt{\frac{(l + m + 2)(l + m + 1)}{(2l - 1)(2l + 3)}} f_{l+1,m+1}.
\]

The standard two-term approach consists in retaining in (2.9) only the first term, \( f_{1,0} \), and neglecting all the higher angular harmonics. This is justifiable in a strongly collisional limit, but in weakly collisional regimes effects of the higher angular harmonics of the distribution function become important. These effects can be accounted for by a direct solution of the infinite system (2.9) for \( f_{1,0}(\omega, k, v) \) in the form

\[
f_{1,0}(\omega, k, v) = -\sqrt{\frac{4 \pi}{3} \frac{eE(\omega, k)}{m}} \frac{1}{\partial v (i\omega - \nu_1(v))} H_1(v, k, \omega).
\]

The effects of the higher-order spherical harmonics are included in the continued fraction

\[
H_l(v, k, \omega) = 1 + C_{l+1}/(1 + C_{l+2}/1 + C_{l+3}/ \cdots),
\]

(2.12)
with coefficients

\[ C_l = \frac{(l^2 - 1) k^2 v^2}{(4l^2 - 1)(i\omega - \nu_l)(i\omega - \nu_{l-1})}. \]  

(2.13)

The similar method of incorporating of the higher-order spherical harmonics was used in [31, 32] for the problem of electron-ion collisions.

To find an expression for the perturbed electron distribution function, one has to calculate the continued fraction \( H_1(v, k, \omega) \) which in turn requires the knowledge of the \( l \)-th order collision frequencies \( \nu_l \). If the electron-neutral atom interaction is of the polarization type, which is assumed in the present thesis, then the collision frequency \( \nu_l \) rapidly converges to a value which does not depend on \( l \).

\[ \nu_\infty(v) = \lim_{l \to \infty} \nu_l(v). \]  

(2.14)

In this case the continued fraction \( H_1(v, k, \omega) \) can be approximated by (see Subsection 2.2.1)

\[
H_1(\omega, kv, \nu) = 1 + \frac{1}{5} \frac{k^2 v^2}{(i\omega - \nu_2(v)) (i\omega - \nu_1(v))} \times \frac{\left( \sqrt{1 + k^2 v^2 / (i\omega - \nu_\infty(v))^2} + 1 \right)}{\sqrt{1 + k^2 v^2 / (i\omega - \nu_\infty(v))^2} + 1 + 16/35 k^2 v^2 / ((i\omega - \nu_2(v)) (i\omega - \nu_\infty(v)))},
\]

(2.15)

where

\[
\nu_1(v) = N v \int (1 - \cos \theta) \frac{d\sigma(v, \theta)}{d\Omega} d\Omega,
\]

(2.16)

\[
\nu_2(v) = \frac{3}{4} N v \int (1 - \cos 2\theta) \frac{d\sigma(v, \theta)}{d\Omega} d\Omega,
\]

(2.17)

\[
\nu_\infty(v) = N v \int \frac{d\sigma(v, \theta)}{d\Omega} d\Omega.
\]

(2.18)
Substituting Eq.(2.15) into Eq.(1.33) we obtain the expression for the electron perturbed distribution function in the form

$$f_{1,0}(k, \omega, v, \theta) = -\frac{eE(\omega, k)}{m} \frac{\partial F_0}{\partial v} \frac{\cos \theta}{(i\omega - \nu_1(v)) H_1(\omega, kv, \nu)}$$  \hspace{1cm} (2.19)

where $H_1(\omega, kv, \nu)$ is given in Eq. (2.15). In the limit, $k = 0$ we obtain the local electron distribution function from Eq. (2.19), investigated in [23]. We have parametrized the perturbed distribution function $f_{1,0}$ by three collisional frequencies $\nu_1, \nu_2$, and $\nu_\infty$. As shown in the next section, this approximation satisfactorily describes the case of electron scattering from Argon gas. The parametrization (2.15) can be easily generalized for the arbitrary dependence of the collisional cross section $d\sigma(v, \theta)/d\Omega$, provided that the condition (2.14) is satisfied. It is worth noting, that both analytical representation and results of computer modeling and/or experimental data can be used for $d\sigma(v, \theta)/d\Omega$ to determine the parameters $\nu_1, \nu_2$, and $\nu_\infty$ required for (2.19).

### 2.2.1 Approximate Expressions for the Continued Fraction

The continued fraction $H_1$ has the form

$$H_1 = 1 + \frac{C_2}{1} + \frac{C_3}{1} + \frac{C_4}{1} + \ldots$$  \hspace{1cm} (2.20)

The $C_l$ coefficients are

$$C_l = \frac{(l^2 - 1) k^2 \nu^2}{(4l^2 - 1) (i\omega - \nu_l) (i\omega - \nu_{l-1})},$$  \hspace{1cm} (2.21)

where $\nu_l$ is given by (2.45). For large $l$ coefficients $C_l$ converge to a constant

$$C_l \approx \frac{1}{4} \frac{k^2 \nu^2}{(i\omega - \nu_\infty)^2} = \frac{1}{4} \nu^2,$$  \hspace{1cm} (2.22)
where \( x = k\nu / (i\omega - \nu_\infty) \). For constant value of \( C_i \) the following identity holds true
\[
\sqrt{1 + x^2} = 1 + \frac{1}{2} + \frac{x^2}{4} + \frac{x^2}{4 + \frac{x^2}{4 + \frac{x^2}{4 + \cdots}}}.
\] (2.23)

Keeping a few first terms exact and replacing the rest with the approximate expression \( C_i \approx x^2/4 \) the function \( H_1 \) can be represented asymptotically
\[
H_1^N = 1 + C_2/1 + C_3/1 + \ldots + C_N/1 + \frac{x^2}{4}/1 + \frac{x^2}{4}/1 + \ldots
\] (2.24)

Then
\[
H_1^N = 1 + C_2/1 + C_3/1 + \ldots + C_N/1 + \left(\frac{\sqrt{1 + x^2} / 2 - 1/2}{1 + x^2/2 - 1/2}\right).\] (2.25)

Approximating \( C_4 \) and higher coefficients by an expression (2.22) and using expansion (2.23), one finds for \( H_1 \) (2.20)
\[
H_1 \simeq 1 + \frac{C_2}{1 + \frac{x^2}{4 + \frac{x^2}{4 + \frac{x^2}{4 + \cdots}}}} = 1 + \frac{1}{5} \frac{k^2\nu^2 / (i\omega - \nu_2(\nu))(i\omega - \nu_1(\nu))}{\sqrt{1 + k^2\nu^2 / (i\omega - \nu_\infty(\nu))^2} + 1 + 16/35k^2\nu^2 / (i\omega - \nu_2(\nu))(i\omega - \nu_\infty(\nu))}.
\] (2.26)

The accuracy of the expression (2.26) can be improved by approximating \( C_5 \) and higher coefficients by an expression (2.22), whereas \( C_1, C_2, C_3 \) and \( C_4 \) are calculated according to (2.21). As it will be shown in section 2.3 the approximation (2.26) provides a very reasonable fit to the exact continued fraction (2.20).
2.3 Effective Collisional Frequencies and Ramsauer Effect in Argon

To calculate the perturbed distribution function (2.19) one has to specify the form of the collisional integral and then calculate \( \nu_t \). The \( l \)th-order collision frequencies are given by Eq.(2.8) and are fully determined, provided that the differential cross-section is known. As noted before, in this thesis we consider the elastic electron-atom polarization interaction as the main channel of electron scattering from an atom. In general, the differential cross section of electron neutral atom interaction is given by

\[
\frac{d\sigma}{d\Omega} = |f(\theta)|^2,
\]

(2.27)

where \( f(\theta) \) is the scattering amplitude. The scattering amplitude can be expressed in terms of the phase shifts \( \delta_l \) [43]

\[
f(\theta) = \sum_{l=0}^{\infty} \frac{1}{2i k} (2l + 1)(e^{2i\delta_l} - 1) P_l(\cos \theta),
\]

(2.28)

where \( k \) is the electron wave number, \( P_l(\cos \theta) \) are Legendre polynomials. If the ground state of the atom does not have a permanent electric quadrupole moment (which is the case for rare-gas atoms), the leading term in the electron-atom interaction potential is of the polarization type [44]. The solution of the Schrodinger equation with the polarization potential \( V \sim 1/r^4 \) gives the values of the phase shifts (and consequently, the differential cross section) which describe the experimental data only for low energies (less than 0.1-0.2 eV for argon gas). For higher electron energy, the interaction potential between an electron and atom deviates from the polarization form, so that the higher order terms in the multipole expansion \( (1/r^6, ... , etc) \) of the interaction potential become important. According to the modified effective range theory [44] the phase shifts can be represented as a series in the \( k a_0 \) parameter...
(which is the measure of the energy of the incident electron), where $a_0$ is the Bohr radius. Coefficients of this expansion are chosen to fit the experimental data. Following this procedure O’Malley [33] has obtained the following expressions for the phase shifts

\[ \tan \delta_0 = -Lk - \frac{\pi \beta k^2}{3a_0} - \frac{4}{3a_0} \beta Lk^3 \ln(ka_0) + Dk^3 + O(k^5a_0^5), \quad (2.29) \]

\[ \tan \delta_1 = \frac{\pi}{15a_0} \beta k^2 + A_1 k^3 + O(k^4a_0^4), \quad (2.30) \]

\[ \tan \delta_l = \pi \left[ (2l + 3)(2l + 1)(2l - 1)a_0 \right]^{-1} \beta k^2 + O(k^4a_0^4), \text{ for } l > 1. \quad (2.31) \]

where for Argon gas

\[ L = -1.7a_0, \quad \beta = 11a_0^3, \quad D = 49.368a_0^3, \quad A_1 = -8a_0^3. \quad (2.32) \]

The expressions given by (2.29), (2.30), (2.31) provide a good fit for the electron-argon atom cross sections in the energy range between 0 and 0.7 eV.

To extend these expressions into the higher energy region we introduce the higher order terms in the modified effective range theory expansion

\[ \tan \delta_0 = -Lk - \frac{\pi \beta k^2}{3a_0} - \frac{4}{3a_0} \beta Lk^3 \ln(ka_0) + Dk^3 + \sum_{l=4}^{\alpha} \beta_l k^l, \quad (2.33) \]

\[ \tan \delta_1 = \frac{\pi}{15a_0} \beta k^2 + A_1 k^3 + \sum_{l=4}^{\mu} \gamma_l k^l, \quad (2.34) \]

\[ \tan \delta_2 = \frac{\pi}{105} \beta k^2 + \sum_{l=2}^{\delta} j_l k^{2l}, \quad (2.35) \]

\[ \tan \delta_l = \pi \left[ (2l + 3)(2l + 1)(2l - 1)a_0 \right]^{-1} \beta k^2. \quad l > 2 \quad (2.36) \]

New coefficients $\beta_l$, $\gamma_l$, and $j_l$ are chosen to fit the experimentally measured phase
shifts [36] to give for Argon gas

\[
\begin{align*}
\beta_4 &= -900.1a_0^4 & \gamma_4 &= -23.287a_0^4 & j_2 &= 0.77a_0^4 \\
\beta_5 &= 18121.4a_0^5 & \gamma_5 &= 354.605a_0^5 & j_3 &= 0.85a_0^6 \\
\beta_6 &= -150462.988a_0^6 & \gamma_6 &= -2386.8a_0^6 \\
\beta_7 &= 687021.5a_0^7 & \gamma_7 &= 9514.9a_0^7 \\
\beta_8 &= -1938761.7a_0^8 & \gamma_8 &= -23018.38a_0^8 \\
\beta_9 &= 3533198.63a_0^9 & \gamma_9 &= 34197.286a_0^9 \\
\beta_{10} &= -4174116.25a_0^{10} & \gamma_{10} &= -30531.53a_0^{10} \\
\beta_{11} &= 3090292.45a_0^{11} & \gamma_{11} &= 15039.7a_0^{11} \\
\beta_{12} &= -1303452.58a_0^{12} & \gamma_{12} &= -3141.58a_0^{12} \\
\beta_{13} &= 239014.83a_0^{13}.
\end{align*}
\]

Then, the scattering amplitude \( f(\theta) \) and differential cross section \( d\sigma/d\Omega \) are

\[
f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) \delta_l P_l(\cos \theta) = \widehat{A} - \frac{\pi \beta k}{2a_0} \sin \frac{\theta}{2} + \widehat{B} P_1(\cos \theta) + \widehat{C} P_2(\cos \theta), \quad (2.37)
\]

\[
\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \widehat{A}^* \widehat{A} - \frac{\pi \beta k}{2a_0} \left\{ \widehat{A} + \widehat{A}^* \right\} \sin \frac{\theta}{2} + \left\{ \widehat{A} \widehat{B}^* + \widehat{A}^* \widehat{B} \right\} P_1(\cos \theta) \right.
\]

\[
- \frac{\pi \beta k}{2a_0} \left\{ \widehat{B} + \widehat{B}^* \right\} \sin \frac{\theta}{2} P_1(\cos \theta) + \frac{\pi^2 \beta^2 k^2}{4a_0^2} \sin^2 \frac{\theta}{2} + \widehat{B} \widehat{B}^* P_2(\cos \theta)
\]

\[
+ \widehat{C} \widehat{C}^* P_2^2(\cos \theta) + \left\{ \widehat{A} \widehat{C}^* + \widehat{A}^* \widehat{C} \right\} P_2(\cos \theta) - \frac{\pi \beta k}{2a_0} \left\{ \widehat{C} + \widehat{C}^* \right\} \sin \frac{\theta}{2} P_2(\cos \theta)
\]

\[
+ \left\{ \widehat{B} \widehat{C}^* + \widehat{B}^* \widehat{C} \right\} P_1(\cos \theta) P_2(\cos \theta). \quad (2.38)
\]

Here \( P_1(\cos \theta), P_2(\cos \theta) \) are the first and second order Legendre polynomials,

\[
\widehat{A} = \frac{e^{i\delta_0}}{k} \sin \delta_0 + \frac{\pi \beta k}{3a_0}. \quad (2.39)
\]
\[ \widehat{B} = \frac{3e^{i\delta_1}}{k} \sin \delta_1 - \frac{\pi \beta k}{5a_0}, \]  
(2.40)

\[ \widehat{C} = \frac{5e^{i\delta_2}}{k} \sin \delta_2 - \frac{\pi \beta k}{21a_0}. \]  
(2.41)

The expansion

\[ \sin \frac{\theta}{2} = -2 \sum_{l=0}^{\infty} \frac{P_l(\cos \theta)}{(2l+3)(2l-1)} \]  
(2.42)

was used in derivation of formula (2.37) and (2.38). The differential cross section (2.38) gives a very good description of the electron scattering by the Argon atom in the energy range from 0 to 10 eV. This can be extended to even higher energies by including the next order terms in the expansion for the phase shifts. The differential cross section (2.38) is a function of the electron velocity \(v\) and the scattering angle \(\theta\). For some gases (e.g. Argon), the scattering length \(L\) is negative and the transport cross section \(\sigma_{tr}\) defined as

\[
\frac{\sigma_{tr}}{4\pi} = \frac{1}{4\pi} \int (1 - \cos \theta) d\sigma = \frac{\widehat{A}\widehat{A}^*}{3} + \frac{\widehat{B}\widehat{B}^*}{5} - \frac{2}{5} \frac{\pi \beta k}{a_0} \left\{ \widehat{A} + \widehat{A}^* \right\}
+ \frac{18 \pi \beta k}{105a_0} \left\{ \widehat{B} + \widehat{B}^* \right\} - \frac{2\pi \beta k}{105a_0} \left\{ \widehat{C} + \widehat{C}^* \right\} - \frac{1}{3} \left\{ \widehat{A}\widehat{B}^* + \widehat{A}^*\widehat{B} \right\}
- \frac{2}{15} \left\{ \widehat{B}\widehat{C}^* + \widehat{B}^*\widehat{C} \right\} + \frac{1}{6} \frac{\pi^2 \beta^2 k^2}{a_0^2}
\]  
(2.43)

has a minimum at \(v \approx v_c\) [34] where

\[
v_c = -\frac{12\hbar La_0}{5\pi m\beta}.
\]  
(2.44)

The sharp decrease in the transport cross section of electron-atom interaction is known as the Ramsauer effect. As an example we shall use Argon gas for the present calculations. The angular dependence of the differential cross section for different energies given by (2.38) is shown in Fig.(2.1). The transport cross section (2.43) of
Figure 2.1: The differential cross section of the electron-argon atom scattering versus the scattering angle for different electron energies. The solid, dotted, dashed, dashed-dotted lines represent the electron energy of 0.1, 1, 5, and 10 eV, respectively.
Figure 2.2: The momentum transfer scattering cross section of an electron on argon versus the electron energy. •: Nakanishi et al. [38]; —: present results
Figure 2.3: The effective collision frequencies $\nu_1$, $\nu_2$, and $\nu_\infty$ versus the electron energy. The neutral atom density is $10^{15}$ cm$^{-3}$. The solid line represent $\nu_1$, the dotted $\nu_2$, and the dashed $\nu_\infty$. 
electron scattering in argon as a function of the incident electron energy is shown in Fig.(2.2). Using the definition of the \(l\)-th order collision frequency (2.8) and Eq.(2.38) for the differential cross-section, one obtains

\[
\frac{\nu_1(u)}{N_v} = 4\pi \frac{\pi^3 \beta^2 k^2}{a_0^3} \left( 1 - \delta_{l,0} + \frac{\delta_{l,1}}{3} \right) - \frac{4\pi}{3} \left\{ \hat{A} \hat{B}^* + \hat{A}^* \hat{B} \right\} \delta_{l,1} \\
- \frac{2\pi^2 \beta k}{a_0} \left\{ \hat{B} + \hat{B}^* \right\} \left( \frac{2}{15} - \frac{2(l + 1)}{(2l + 1)^2(2l + 5)(2l + 3)} \right) \\
- \frac{2\pi^2 \beta k}{a_0} \left\{ \hat{B} + \hat{B}^* \right\} \left( \frac{2l}{(2l + 1)^2(2l - 3)(2l - 1)} \right) \\
+ 4\pi \hat{B} \hat{B}^* \left( \frac{1}{3} - \frac{1}{3} \delta_{l,0} - \frac{2}{15} \delta_{l,2} \right) + 2\pi \hat{C} \hat{C}^* \left( \frac{2}{5} - \frac{2}{5} \delta_{l,0} - \frac{4}{35} \delta_{l,2} - \frac{4}{35} \delta_{l,4} \right) \\
- \frac{4\pi}{2} \left\{ \hat{A} \hat{C}^* + \hat{A}^* \hat{C} \right\} \delta_{l,2} - 2\pi \left\{ \hat{B} \hat{C}^* + \hat{B}^* \hat{C} \right\} \left( \frac{4}{15} \delta_{l,1} + \frac{6}{35} \delta_{l,3} \right) \\
- \frac{\pi^2 \beta k}{a_0} \left\{ \hat{C} + \hat{C}^* \right\} \left( -\frac{4}{105} - \frac{2}{(2l + 3)(4l^2 - 1)} \right) \\
- \frac{\pi^2 \beta k}{a_0} \left\{ \hat{C} + \hat{C}^* \right\} \left( \frac{6(l + 1)^2}{(2l + 1)^2(2l + 3)^2(2l - 1)} \right) \\
- \frac{\pi^2 \beta k}{a_0} \left\{ \hat{C} + \hat{C}^* \right\} \left( \frac{6l(l - 1)}{(2l + 1)(2l - 3)(2l - 1)^2(2l - 5)} \right) \\
- \frac{\pi^2 \beta k}{a_0} \left\{ \hat{C} + \hat{C}^* \right\} \left( \frac{6(l + 1)(l + 2)}{(2l + 1)(2l + 3)^2(2l + 5)(2l + 7)} \right) \\
- \frac{\pi^2 \beta k}{a_0} \left\{ \hat{C} + \hat{C}^* \right\} \left( \frac{6l^2}{(2l + 1)^2(2l - 1)^2(2l + 3)} \right).
\]

(2.45)

Fig.(2.3) shows the effective collision frequencies \(\nu_1, \nu_2,\) and \(\nu_\infty\) versus the electron energy. One can see the nonmonotonous behavior of the effective frequencies with the electron energy which is typical for the Ramsauer effect. With an explicit form for the effective collision frequencies (2.45) we can readily calculate the continued fraction \(H_1\) and subsequently the perturbed electron distribution function. The approximation
Figure 2.4: The complete continued fraction $H_1$ computed numerically and the approximation (2.26) as a function of $kv/\omega$ parameter for $\nu_1/\omega = 0.1$. The solid line: the real part of the complete continued fraction $H_1$; the dotted line: the real part of the approximation (2.26); the dashed line: the imaginary part of the complete continued fraction $H_1$; the dashed-dotted line: the imaginary part of the approximation (2.26).
Figure 2.5: The complete continued fraction $H_1$ computed numerically and the approximation (2.26) as a function of $k v / \omega$ parameter for $\nu_1 / \omega = 5$. The solid line: the real part of the complete continued fraction $H_1$; the dotted line: the real part of the approximation (2.26); the dashed line: the imaginary part of the complete continued fraction $H_1$; the dashed-dotted line: the imaginary part of the approximation (2.26).
(2.15) reasonably well reproduces the exact continued fraction (2.12) in the wide range of collisionality regimes. The comparison of the full continued fraction (2.12) computed numerically with the approximation (2.15) is given in Figs. (2.4) and (2.5) for different values of the parameter $\nu_1/ \omega$.

2.4 Nonlocal Plasma Conductivity

The Fourier component of the plasma conductivity is defined from the relation

$$j(\omega, k) = -e \int v_z f_{1,0} d^3 v = \sigma(\omega, k) E(\omega, k).$$

(2.46)

Using (2.19) one obtains

$$\sigma(\omega, k) = -\frac{4\pi ne^2}{3T} \left( \frac{m}{2\pi T} \right)^{\frac{3}{2}} \int_0^\infty \frac{v^4 \exp\left( -\frac{mv^2}{2T} \right)}{(i\omega - \nu_1(v)) \mathcal{H}_1(\omega, kv, \nu)} dv$$

(2.47)

The effective frequencies defined in this thesis reasonably reproduce the theoretical and experimental data in the energy region from 0 to 10 eV. Integration in the expression (2.47) is done over the entire velocity region from 0 to $\infty$. Since the main contribution to the integral in Eq. (2.47) comes from velocities that are close to the thermal, the error introduced by electrons with energies higher than 10 eV is exponentially small for plasmas with electron temperature $T_e < 10$ eV. The electron plasma conductivity $\sigma$ given by Eq. (2.47) is a complicated function of $\omega$ and $k$. Converted back to a configuration space $z$ and $t$, $\sigma(z, t)$ becomes a nonlocal integro-differential operator in space and time. It means that the conductivity is a function of the electric field throughout the entire skin layer and as it will be shown in the next section it causes the nonmonotonous behavior of the electric field inside of a plasma.

For the case when $\nu = 0$, the plasma conductivity can be easily calculated from
Figure 2.6: The collisionless plasma conductivity (2.48) and plasma conductivity (2.47) calculated in the limit $\nu \to 0$. The solid line: the real part of the collisionless plasma conductivity (2.48); the dotted line: the real part of plasma conductivity (2.47) calculated in the limit $\nu \to 0$; the dashed line: the imaginary part of the collisionless plasma conductivity (2.48); the dashed-dotted line: the imaginary part of plasma conductivity (2.47)
Figure 2.7: The nonlocality parameters $\zeta$ and $\lambda$ versus electron energy for the neutral atom density $N = 10^{15}$ cm$^{-3}$, electron density $n_e = 10^{12}$ cm$^{-3}$, and the frequency of the external electric field $\omega = 6.78$ MHz. The solid line represents $\zeta$ and the dotted line represents $\lambda$. 
the collisionless kinetic equation giving [55]

\[
\sigma_0 (\omega, k) = -\frac{i\omega^2_{pe}}{4\pi k v_T} Z \left( \frac{\omega}{k v_T} \right),
\]

(2.48)

where \( Z(x) \) is the plasma dispersion function

\[
Z(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-t^2)}{t-x} dt.
\]

In Fig.(2.6) we compare the exact analytical result (2.48) obtained in the collisionless regime with our approximation (2.47) in the limit \( \nu \to 0 \). This comparison demonstrates that Eq.(2.47) accurately describes the plasma response for a wide range of the collisionality regimes and wide range of values of the parameter \( \omega/kv_T \).

The continued fraction \( H_1 \) in Eq.(2.47) describes nonlocal effects due to a spatial dependence of the conductivity operator \( \sigma \). One can introduce the following criterion when the nonlocal effects are important

\[
\zeta \equiv \left| \frac{k^2 v_T^2/((i\omega - \nu_2(v))(i\omega - \nu_1(v)))(\sqrt{1 + k^2 v_T^2/(i\omega - \nu_\infty(v))^2 + 1})}{\left((\sqrt{1 + k^2 v_T^2/(i\omega - \nu_\infty(v))^2 + 1}) + 16/35 k^2 v_T^2/((i\omega - \nu_2(v))(i\omega - \nu_\infty(v)))\right)^{1/2}} \right| > 1.
\]

(2.49)

Qualitatively we can assume that \( \nu_1 = \nu_2 = \nu_\infty \). Then expression (2.49) can be rewritten as

\[
\frac{k^2 v_T^2}{\omega^2 + \nu_1^2} \left((1 + k^2 v_T^2/ (\omega^2 + \nu_1^2))^{1/2} + 1\right) > \left((1 + k^2 v_T^2/ (\omega^2 + \nu_1^2))^{1/2} + 1\right) + 16/35 k^2 v_T^2/ (\omega^2 + \nu_1^2).
\]

Introducing a parameter \( \eta \)

\[
\eta = \frac{k^2 v_T^2}{\omega^2 + \nu_1^2}
\]

(2.50)

one obtains from (2.49)

\[
\eta \left(1 + \left(1 + \eta \right)^{1/2}\right) > 1 + \left(1 + \eta \right)^{1/2} + \frac{16}{35} \eta.
\]

(2.51)
Thus the nonlocal effects are important for

\[ \eta > 1.2 \]  

(2.52)

or

\[ \eta = \frac{k^2 v_T^2}{\omega^2 + \nu^2} > 1. \]  

(2.53)

The characteristic wave vector \( k \) can be approximated by the inverse skin depth in the local case

\[ k = \delta^{-1} = \text{Re} \left( \frac{4\pi i\omega\sigma/c^2}{} \right)^{1/2}, \]  

(2.54)

where \( \sigma \) is the local conductivity. By using the classical local expression for the electron conductivity

\[ \sigma = \frac{\omega_p^2}{4\pi (\nu - i\omega)}, \]  

(2.55)

one obtains from (2.53) and (2.54) the well known nonlocality parameter \( \Lambda \) introduced in [17, 28]

\[ \Lambda = \left( \frac{\omega_p v_T}{c} \right)^2 \frac{\omega}{(\omega^2 + \nu^2)^{3/2}}. \]  

(2.56)

The expression (2.55) is valid however only in the limit when the effective collisional frequency \( \nu \) does not depend on the electron velocity. It has been pointed out in [23] that for the velocity dependent collisional frequency \( \nu(v) \) the expression (2.55) does not provide an adequate description of the electron conductivity \( \sigma \). In the latter case, more general expression

\[ \sigma = -\frac{4\pi ne^2}{3T} \left( \frac{m}{2\pi T} \right)^{3/2} \int_0^\infty v^4 \exp \left( -\frac{mv^2}{2T} \right) \frac{dv}{(i\omega - \nu_1(v))} \]  

(2.57)

has to be used for evaluation of the local conductivity. Respectively, the more general expression (2.57) has to be used in (2.54) to evaluate the characteristic wavelength \( k \). The parameters \( \zeta \) (2.49) with \( k \) calculated by using the local conductivity \( \sigma \) from
(2.57) and \( \Lambda \) (2.56) are plotted in Fig. (2.7) as functions of the electron energy for the same values of the electron and neutral atoms density and frequency of the external field. As one can see there is a noticable difference between these two parameters. The parameter \( \Lambda \) considerably overestimates the nonlocality region for this particular case of argon gas.

2.5 Anomalous Skin Effect in a Weakly Ionized Plasma

In this section we analyze the electric field penetration into a semi infinite plasma \( x > 0 \) and calculate the surface impedance when the spatial motion of electrons becomes important. For the sake of simplicity we assume that the electric field is parallel to the \( z \) axis and it's spatial variation is in the \( x \). Adopting the specular reflection of the electrons at the boundary \( x = 0 \), the electric current density can be written as [41]

\[
\mathbf{j}_z (x) = \int_0^\infty dx' \left[ \Sigma (x - x') + \Sigma (x + x') \right] \mathbf{E}_z (x')
\]  

or, extending the definition to \( \mathbf{E}_z (x) \) into the domain \( x < 0 \) by

\[
\mathbf{E}_z (-x) = \mathbf{E}_z (x)
\]

we have

\[
\mathbf{j}_z (x) = \int_{-\infty}^\infty dx' \Sigma (x - x') \mathbf{E}_z (x')
\]  

where

\[
\Sigma (x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \sigma (|k|, \omega) e^{ikx}.
\]
Combining Eq.(2.60) with the Maxwell equations

\[ \nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \]  
\[ \nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \mathbf{j} \]  

we get

\[ -\frac{\partial E_z}{\partial x} = \frac{i\omega}{c} B_y \]  
(2.64)

\[ \frac{\partial B_y}{\partial x} = \frac{4\pi}{c} \int_{-\infty}^{\infty} dx' \Sigma (x - x') E_z (x') . \]  
(2.65)

In Eq.(2.65), we have neglected the displacement current, since it is small in comparison with the current term, for inductively coupled plasma discharges. Since \( E_z (x) \) is an even function with respect to \( x \), Eq.(2.64) indicates that \( B_y (x) \) is an odd function: \( B_y (x) \) is discontinuous at \( x = 0 \) as \( B_y (+0) = -B_y (-0) \). Differentiating Eq.(2.64) with respect to \( x \) and substituting Eq.(2.65) in it and taking account of the foregoing discontinuity, we obtain

\[ \frac{\partial^2 E_z}{\partial x^2} + \frac{4\pi \omega}{c^2} \int_{-\infty}^{\infty} dx' \Sigma (x - x') E_z (x') = \frac{2i\omega}{c} B_y(+0) \delta (x) \]  
(2.66)

where \( \delta (x) \) is the Dirac’s delta function. This differential equation may be solved through Fourier transformation to give

\[ E_z (x) = -i \frac{\omega}{\pi c} B_y (+0) \int_{-\infty}^{\infty} \frac{\exp [ikx] dk}{k^2 - 4\pi i\omega \sigma (\omega, |k|) / c^2}. \]  
(2.67)

The surface impedance \( \Upsilon \) is defined as

\[ \Upsilon = \frac{E_z (0)}{B_y (0)} = -i \frac{\omega}{\pi c} \int_{-\infty}^{\infty} \frac{dk}{k^2 - i4\pi \omega \sigma (|k|\omega / c^2)}, \]  
(2.68)

where \( \sigma (|k|, \omega, T) \) is the Fourier component of plasma conductivity. The nonlocality
Figure 2.8: The amplitude of the electric field $E$ as a function of the distance $x$ for different values of the parameter $\varsigma$. The solid line: $\varsigma = 0.9 \,(N_{at} = 10^{15} \text{ cm}^{-3},\ n_e = 10^{12} \text{ cm}^{-3},\ \omega = 6.78 \text{ MHz},\ T_e = 0.5 \text{ eV});\$ the dotted line: $\varsigma = 0.9 \,(N_{at} = 10^{15} \text{ cm}^{-3},\ n_e = 10^{13} \text{ cm}^{-3},\ \omega = 6.78 \text{ MHz},\ T_e = 1 \text{ eV});\$ and the dashed line: $\varsigma = 30.3 \,(N_{at} = 10^{15} \text{ cm}^{-3},\ n_e = 10^{14} \text{ cm}^{-3},\ \omega = 3.89 \text{ MHz},\ T_e = 0.6 \text{ eV}).$
Figure 2.9: The surface impedance versus the electron energy. The solid line: the real part of the system's impedance for the neutral atom and electron densities of $N_{at} = 10^{15}$ cm$^{-3}$, $n_e = 10^{12}$ cm$^{-3}$; the dashed line: the imaginary part of the system's surface impedance for the neutral atom and electron densities of $N_{at} = 10^{15}$ cm$^{-3}$, $n_e = 10^{12}$ cm$^{-3}$. The dotted line: the real part of the system's impedance for the neutral atom and electron densities of $N_{at} = 10^{16}$ cm$^{-3}$, $n_e = 10^{12}$ cm$^{-3}$; the dashed line: the imaginary part of the system's surface impedance for the neutral atom and electron densities of $N_{at} = 10^{16}$ cm$^{-3}$, $n_e = 10^{12}$ cm$^{-3}$. Electric field frequency is $\omega = 6.18$ MHz.
parameter $\zeta$ shows that in both limits of low and high frequency as well as low and high temperature cases the penetration of the electromagnetic waves into a plasma can be described as a classical skin effect (exponential decay of an electric field inside of a plasma) and the surface impedance can be calculated by integrating the expression (2.68) with the conductivity given by (2.57) which finally yields [42]

$$Z = (1 - i) \frac{\omega}{8\pi \sigma}.$$  \hspace{1cm} (2.69)

For the non-local case ($\zeta > 1$) however, the formula (2.69) is not applicable and one has to solve the problem of propagation of the electromagnetic wave, when the current at a given point is determined by the field distribution within the electron free path distance. The electric field and impedance numerically calculated from (2.68) and (2.67) are shown in Figs.(2.8) and (2.9). In the local case $\zeta \leq 1$ the electric field amplitude monotonically decays with the distance (approximately as a damped exponent). For the nonlocal case, $\zeta > 1$ on the contrary, the non-monotonic field decay occurs. Such nonmonotonous behavior has been found previously [17] and recently detected experimentally [21]. For the gas pressures of the order of $10^{-2}$ Torr the system's surface impedance becomes a nonmonotonous function of temperature, which is the result of the Ramsauer effect.

### 2.6 Summary

In the present chapter, we have considered nonlocal electron kinetics in a weakly ionized plasma subject to the time and space dependent electric field when the main channel of electron scattering is the electron-neutral atom interaction. Since the differential cross section of electron scattering depends on the poloidal angle, the collision integral $C(f)$ cannot be presented in the local form $C(f) = -\nu(\nu) f(r, v, t)$. 
In this case the perturbed electron distribution function is expanded in the series of spherical harmonics. In low collisionality regimes effects of electron thermal motion become essential so that the higher harmonics are important. This leads to the infinite system of coupled equations for the separate harmonics. We have developed the procedure which allows to solve this infinite hierarchy in terms of the continued fraction and found a finite parametric representation for this continued fraction. Unlike the two-term approximation which only accounts for the first term $f_1$ in the spherical modes expansion, the approach developed in the present thesis leads to a space and time dependent electron distribution function that describes electron kinetics when the electron's mean free path is comparable or exceeds the characteristic length scale of an external electric field inhomogeneity. The developed approach uniformly describes the low and high collisionality regimes and can be applied to a wide class of electron-neutral interaction processes with an angular dependence of the differential cross sections, in particular, for inert gases exhibiting the Ramsauer effect. The electron distribution function was used to find the nonlocal electric conductivity, the surface impedance of a semi-infinite plasma and to analyze the anomalous penetration of the electric field into Argon plasma. Unlike the local conductivity which leads to the exponential (local) decay of the electric field, the nonlocal conductivity leads to non-monotonous dependence of the amplitude of the electric field on the penetration depth (anomalous skin effect).

It is shown that the Ramsauer effect manifests itself in the nonmonotonous behavior of the impedance for small temperatures and becomes noticeable for the gas pressures of the order of $10^{-2}$ Torr. We have introduced the nonlocality parameter $\zeta$ which determines the boundary between the local, $\zeta \ll 1$, and nonlocal, $\zeta > 1$, regimes.
Chapter 3

Ion Transport in an Inhomogeneous and Time Dependent Electric Field

3.1 Introduction

Ion transport processes are important for modeling of electric discharges in gases and determine many properties of laboratory and ionospheric plasmas. Ion transport in an electric field has been the subject of numerous experimental and theoretical studies [46, 47]. In a number of cases, ion transport properties can be determined in terms of the ion mobility [48] (which defines ion drift velocity in the electric field) and diffusion coefficient (which describes ion diffusion due to inhomogeneous ion density). Such approach is justified for equilibrium conditions when the total ion flux consists of only mobility and diffusion fluxes, and the diffusion and mobility are related by the Einstein equation. This assumes that the ion flux is determined by local values of
the electric field $E$, ion density $n$, and gradient of ion density $\nabla n$. In nonequilibrium plasmas, the ion distribution function is far from the Maxwellian distribution, and ion flux is no longer local so that it cannot be described in terms of local parameters such as $E$, $n$, and $\nabla n$.

Deviations of the distribution function from the equilibrium Maxwellian are often due to effects of strong electric fields. In this case, additional fluxes could occur that are not reduced to standard mobility and diffusion, but rather depend on higher order spatial and time derivatives of the electric field and ion density. In general case, the nonlocal fluxes are given by the integro-differential operators applied to the electric field and ion density.

In the following sections we consider ion transport in a space-time dependent electric field $\tilde{E}(z,t)$ with charge exchange interactions as the main mechanism of ion scattering. The perturbed electric field is superimposed on the equilibrium electric field $E_0$ which is constant in time and space. The amplitude of the perturbed electric field is assumed to be small compared to the equilibrium field, $E_0 > \tilde{E}(z,t)$. Such electric field can occur in the sheath region of the RF driven capacitive discharge [52]. It is assumed that the equilibrium electric field is strong, $qE_0\lambda > T$, where $q$ is the ion charge, $\lambda$ is the ion mean free path, $T$ is the temperature of the neutral gas. Thus, an average energy acquired by ions from the electric field between consecutive collisions is larger than the mean thermal energy of neutral particles. We also assume that plasma is weakly ionized so that Coulomb collisions between ions are unimportant, $\nu \gg \nu_{\text{Coul}}$, where $\nu$ is the effective frequency of charge-exchange collisions, and $\nu_{\text{Coul}}$ is the frequency of Coulomb collisions. Under these conditions, the ion distribution function becomes non-Maxwellian, and the ion effective temperature (mean average energy) greatly exceeds the temperature of the neutral gas. Such conditions are
typical for a number of gas discharges [53].

The main objective of the present chapter is to obtain expressions for ion mobility and diffusion coefficients when the electric field varies in space and time and the ion distribution function is strongly nonequilibrium. Similar problem was considered in [54], where nonlocal corrections to the ion transport were found in the limit of slow time dependence $\omega/\nu < 1$ ($\omega$ is the characteristic frequency of the electric field variations) and weak inhomogeneity $\lambda/L < 1$, where $L \simeq k^{-1}$ is the characteristic length scale of the electric field. By means of the perturbation technique it has been shown that in the case of the slow varying (in time and space) electric field, there are additional ion fluxes that are proportional to the first derivatives of the electric field in time and space. In the present thesis, we derive the expression for ion transport fluxes for arbitrary values of the parameters $\omega/\nu$ and $\lambda/L$. In the general nonlocal case these fluxes are described by integro-differential operators applied to the electric field and to the ion density. In this chapter we employ two different approaches to determine ion transport fluxes. The first method is based on a direct solution of the linearized kinetic equation, when the perturbation of the ion density and ion flow are found by integration of the perturbed distribution function. In this case the total ion flux is given by the generalized mobility operator applied to the electric field. In the second approach we determine ion flux in terms of the lowest order moment (density) and the electric field. This approach is similar to the Chapman-Enskog procedure when the distribution function is represented as a sum of the “dynamical equilibrium” part and the deviation. The dynamical part is determined by the time dependent electric field and the total ion density. The deviation part thus does not contribute to the density moment. The recast kinetic equation is solved for the deviation part, and the ion flux is found in terms of the particle density and electric field.
If one finds the ion density using the continuity equation and then substitutes it into an expression for the ion drift velocity found by using the Chapman-Enskog type approach, one will obtain an expression for the ion drift velocity found by using the method of a direct solution of the Boltzmann equation, when the perturbation of the ion density and ion flow are found by integration of the perturbed distribution function.

3.2 Ion Kinetic Equation

We consider the ion velocity distribution function \( f(v,z,t) \) that is described by the Boltzmann equation

\[
\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial z} + \frac{qE}{m} \frac{\partial}{\partial v} f = J(f),
\]

where \( J \) is the collision integral for interaction of ions with the neutral particles, \( q \) and \( m \) are the ion charge and mass respectively. We assume that the main channel for ion scattering by neutral particles is a resonant charge exchange leading to an exchange of the velocities of the colliding particles, and that the external electric field is unidirectional. The ion drift velocity along a strong electric field appreciably exceeds the thermal velocity of the neutral particles, whereas the transverse component of ion velocity is of the order of the thermal velocity of neutrals. Therefore we can neglect the transverse component of the ion drift velocity as well as the diffusion processes in the direction perpendicular to the external electric field. A one dimensional approximation is typically employed in analysis of the sheath region of capacitively driven radio frequency discharges [20].

In the range of \( E/N \) values of interest, the charge exchange cross section \( \sigma \) can be assumed to be independent of the particle velocity. Then the collision integral has
the form [55]

\[ J(F, f) = \sigma \int \{ F(v)f(v') - F(v')f(v) \} \, |v - v'| \, dv'. \]  

(3.2)

For strong electric field, \(|q|E\lambda > T\), one can neglect the temperature of the neutral gas so that the distribution function of neutral atoms is approximated by

\[ F = N\delta(v), \]  

(3.3)

where \(\delta(v)\) is the Dirac’s delta function and \(N\) is the neutral gas density. Then, the collisional integral takes the form

\[ J(f) = -N\sigma |v| f + N\sigma \Pi \delta(v), \]  

(3.4)

where

\[ \Pi = \int |v| f \, dv, \]  

(3.5)

with \(|v|\) indicating the magnitude of the ion velocity.

The Boltzmann equation (3.1) with the collisional integral (3.4) can be integrated by the method of characteristics resulting in the general solution in the form

\[ \Phi(z_0, v_0, c) = 0. \]  

(3.6)

Here \(\Phi\) is an arbitrary differentiable function of integrals of motion \(z_0, v_0,\) and \(c\). Equations of characteristics in the case of a unidirectional external electric field are

\[ v_0 = v - \int \frac{qE(z, t)}{m} \, dt, \]  

(3.7)

\[ z_0 = z - \int v(t) \, dt, \]  

(3.8)

\[ c = \exp \left( \frac{mN\sigma}{q} \int \frac{|v|}{E(z, t)} \, dv \right). \]  

(3.9)
Then, the general solution for (3.1) can be written

\[ f = \exp \left( -\frac{mN\sigma}{q} \int \frac{|v| dv}{E(z,t)} \right) \Psi \left( z - \int v(t) dt, v - \int \frac{qE(z,t)}{m} dt \right), \quad (3.10) \]

where \( \Psi \) is an arbitrary function. The explicit form of this function is determined by initial conditions. For the steady-state uniform external electric field the ion distribution function is [56]

\[ f_0 = \frac{2n_0}{\pi W_0} \Theta(v) \exp \left[ -\frac{mN\sigma v^2}{2qE_0} \right], \quad (3.11) \]

where \( n_0 \) is the ion density, \( \Theta(v) \) is the Heaviside step function, and \( W_0 \) is the ion drift velocity in the uniform time-independent external electric field

\[ W_0 = \left( \frac{2\eta E_0}{m\pi N\sigma} \right)^{1/2}. \quad (3.12) \]

In the Sections to follow, we consider perturbations to the distribution function (3.11) due to the time and spatial dependence in the electric field and find the perturbed ion density and velocity.

### 3.3 Nonlocal Ion Mobility

In this section we solve the kinetic equation for ions in a time and space varying external electric field. The electric field is represented as

\[ E(z,t) = E_0 + \tilde{E}(z,t), \]

where \( E_0 = \text{const} \) is the equilibrium electric field, and \( \tilde{E}(z,t) = \tilde{E} \exp(ikz - i\omega t) \) is the perturbation. The ion velocity distribution function is sought in the form

\[ f(v) = f_0(v) + \tilde{f}(z,t,v), \quad (3.13) \]
where \( f_0(v) \) is a stationary solution of the Boltzmann equation given by Eq. (3.11), \( \tilde{f}(z, t, v) \) is the perturbation due to a non-uniform and time dependent field. Linearizing (3.1) and changing the units so that \( E_q/m \to E \) leads to

\[
\frac{\partial}{\partial t} \tilde{f} + v \frac{\partial}{\partial z} \tilde{f} + N\sigma |v| \tilde{f} = -E_0 \frac{\partial}{\partial v} \tilde{f} - \tilde{E} \frac{\partial}{\partial v} f_0 + N\sigma \tilde{\Pi} \delta(v),
\]

where

\[
\tilde{\Pi} = \int_{-\infty}^{\infty} |v| \tilde{f} dv,
\]

represents a phase space particle source located at \( v = 0 \). In Fourier components Eq. (3.14) reads

\[
-i\omega \tilde{f} + ikv \tilde{f} + N\sigma |v| \tilde{f} = -E_0 \frac{\partial}{\partial v} \tilde{f} - \tilde{E} \frac{\partial}{\partial v} f_0 + N\sigma \tilde{\Pi} \delta(v).
\]

In the region \( v < 0 \), solution to (3.16) has the form

\[
\tilde{f}^-(v, \omega, k) = C^-(v, \omega, k) \Delta(v) \Omega(v, \omega, k),
\]

where \( \Delta(v) \) and \( \Omega(v, \omega, k) \) are given by

\[
\Delta(v) = \exp \left( \frac{N\sigma v^2}{2E_0} \right),
\]

\[
\Omega(v, \omega, k) = \exp \left[ \frac{i}{E_0} \left( \omega v - \frac{k v^2}{2} \right) \right]
\]

From the condition that \( \tilde{f}^-(v, \omega, k) \) be bounded at \( |v| \to \infty \), it follows that \( C^-(v, \omega, k) = 0 \), or \( \tilde{f}^-(v, \omega, k) = 0 \).

When \( v > 0 \), the solution to (3.16) has the form

\[
\tilde{f}^+(v, \omega, k) = C^+(v, \omega, k) \Delta^{-1}(v) \Omega(v, \omega, k).
\]

The function \( C^+(v, \omega, k) \) is given by

\[
C^+(v, \omega, k) = -\frac{\tilde{E}}{E_0} \int v \frac{\partial f_0}{\partial v} \exp \left( \frac{N\sigma v^2}{2E_0} \right) \exp \left[ -\frac{i}{E_0} \left( \omega v - \frac{k v^2}{2} \right) \right] dv + C,
\]
where $C$ is the integration constant and

$$
\frac{\partial f_0}{\partial v} = -\frac{4n_0 v}{\pi^2 W_0^3} \exp \left( -\frac{v^2}{\pi W_0^2} \right).
$$

(3.22)

The phase space particle source $\tilde{\Pi}$ coincides with the ion perturbed flux $\tilde{n\bar{W}}$, since the ion distribution function is defined only for the positive values of $v$. Substituting (3.22) into (3.21) and making integration we get

$$
C^+(v, \omega, k) = \frac{4n_0 \tilde{E}}{\pi^2 W_0^3 i k} \Omega^*(v, \omega, k) + \left( \frac{1 + i}{2} \right) \frac{4n_0 \tilde{E}}{\pi^2 W_0^3 E_0} \frac{\omega}{k} \left( \frac{\pi E_0}{k} \right)^{1/2} \exp \left[ -\frac{ik}{2E_0} \left( \frac{\omega}{k} \right)^2 \right]
\times \text{erf} \left( \frac{(1 - i)}{\sqrt{2}} \left( \frac{k}{2E_0} \right)^{1/2} \left( v - \frac{\omega}{k} \right) \right) + C,
$$

(3.23)

where $\text{erf}(z)$ is the error function

$$
\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) dt.
$$

Expression (3.20) with (3.23) determines the perturbed velocity distribution function of ions $\tilde{f}(v, \omega, k)$. It is defined for velocities $v > 0$. The integration constant $C$ is found from the matching condition

$$
E_0 \tilde{f} \big|_{v=+\varepsilon} + \tilde{E} f_0 \big|_{v=+\varepsilon} = N\sigma \tilde{n\bar{W}},
$$

(3.24)

obtained by integration of Eq. (3.16) over the velocity space from $v = -\varepsilon$ to $v = +\varepsilon$ when $\varepsilon \to 0$. Substituting (3.20) into (3.24) we obtain the integration constant $C$

$$
C = \frac{N\sigma \tilde{n\bar{W}}}{E_0} - \frac{\tilde{E}^2}{E_0} \frac{2n_0}{\pi W_0} \frac{4n_0 \tilde{E}}{\pi^2 W_0^3 i k} \left( \frac{\pi E_0}{k} \right)^{1/2}
+ \frac{(1 + i)}{2} \frac{4n_0 \tilde{E}}{\pi^2 W_0^3 E_0} \frac{\omega}{k} \left( \frac{\pi E_0}{k} \right)^{1/2}
\times \exp \left[ -\frac{ik}{2E_0} \left( \frac{\omega}{k} \right)^2 \right]
\text{erf} \left( \frac{(1 - i)}{\sqrt{2}} \left( \frac{k}{2E_0} \right)^{1/2} \frac{\omega}{k} \right).
$$

(3.25)

Expressions (3.25), (3.23) and (3.20) completely determine the perturbed ion distribution function $\tilde{f}(v, \omega, k)$. Multiplying (3.20) by $v$ and integrating it over the velocity space one obtains an expression for the perturbed ion flux $\tilde{n\bar{W}}$

$$
\tilde{n\bar{W}} = \frac{\Pi^1}{1 - N\sigma \lambda_2(\omega, \chi)/E_0},
$$

(3.26)
Figure 3.1: The perturbed ion density (3.30) versus a parameter $\omega/W_0 N\sigma$. The solid and dashed lines represent the real and imaginary parts of it, respectively.
Figure 3.2: The perturbed ion velocity (3.31) versus a parameter $\omega/W_0 N\sigma$. The solid and dashed lines represent the real and imaginary parts of it, respectively.
\[ \Pi^1 = \frac{2n_0 \bar{E}}{\pi W_0^i k} - \frac{4n_0 \bar{E}}{\pi^2 W_0^i k} \lambda_2 + \frac{(1 + i) 4n_0 \bar{E}}{2 \pi^2 W_0^i k} \frac{\omega}{k} \left( \frac{\pi E_0}{k} \right)^{1/2} \]
\[ \times \exp \left[ -\frac{i k}{2 E_0} \left( \frac{\omega}{k} \right)^2 \right] \Phi + \frac{(1 + i) 4n_0 \bar{E}}{2 \pi^2 W_0^i k} \frac{\omega}{k} \left( \frac{\pi E_0}{k} \right)^{1/2} \]
\[ \times \exp \left[ -\frac{i k}{2 E_0} \left( \frac{\omega}{k} \right)^2 \right] \text{erf} \left( \frac{(1 - i)}{\sqrt{2}} \left( \frac{k}{2 E_0} \right)^{1/2} \frac{\omega}{k} \right) \lambda_2 - \frac{2n_0 \bar{E}}{\pi W_0 E_0} \lambda_2. \] (3.27)

\[ \Phi = \int_0^\infty v \text{erf} \left( \frac{(1 - i)}{\sqrt{2}} \left( \frac{k}{2 E_0} \right)^{1/2} \left( v - \frac{\omega}{k} \right) \right) \Delta^{-1}(v) \Omega(v, k, \omega) dv, \] (3.28)

\[ \lambda_2 = \int_0^\infty v \Delta^{-1}(v) \Omega(v, \omega, k) dv = \frac{E_0}{i \chi} + \frac{E_0 \omega}{i \chi} \left( \frac{\pi i \chi}{2 E_0} \right)^{1/2} \exp \left[ \frac{i \chi}{2 E_0} \left( \frac{\omega}{\chi} \right)^2 \right] \]
\[ \times \left\{ 1 + \text{erf} \left( \frac{\omega}{\chi} \left( \frac{i \chi}{2 E_0} \right)^{1/2} \right) \right\}, \] (3.29)

and \( \chi = k - iN\sigma \). Ion perturbed distribution function is found by substituting Eq. (3.27) into Eqs. (3.26), (3.25), (3.23) and finally Eq. (3.20). Equation (3.26) defines the perturbed ion flux. Ion density and velocity are found by using equations

\[ \tilde{n}(\omega, k) = \int_0^\infty \tilde{f}(\omega, k, v) dv, \] (3.30)

\[ \frac{\tilde{W}(\omega, k)}{W_0} = -\frac{\tilde{n}(\omega, k)}{n_0} + \frac{\tilde{W}(\omega, k)}{W_0 n_0}. \] (3.31)

Integration in (3.30) and (3.26) can be done numerically. The normalized ion density \( \tilde{n}/n_0 \) and velocity \( \tilde{W}/W_0 \) calculated from (3.30) and (3.31) are shown in Fig. (3.1) and Fig. (3.2) as functions of the parameter \( \omega/W_0 N\sigma \) for a given value of the parameter \( k/N\sigma \). The ion perturbed density and velocity exhibit resonant behavior when \( \omega = kW_0 \) due to the contribution of ions moving with the wave phase velocity \( \omega/k \). The
Figure 3.3: The resonant part of the normalized ion perturbed density \((1.2)\) versus a parameter \(\omega/kW_0\), when \(k/N\sigma = 0.5\). The solid and dotted lines represent the real and imaginary parts of it.
Figure 3.4: The resonant part of the normalized ion perturbed density (1.2) versus a parameter $\omega/kW_0$, when $k/N\sigma = 0.1$. The solid and dotted lines represent the real and imaginary parts of it.
resonant part of the perturbed ion density \( \tilde{n}_{res} \) can be represented in a form

\[
\frac{\tilde{n}_{res}(\omega, k)}{n_0} = \tilde{L}(\omega, k) \frac{\tilde{E}(\omega, k)}{E_0},
\]

where the explicit form of \( \tilde{L}(\omega, k) \) is given by

\[
\tilde{L}(\omega, k) = \frac{2}{\pi W_0^2} \frac{W^1}{1 - 2Q_1(\omega, k)/(\pi W_0^2)} Q(\omega, k),
\]

where

\[
W^1 = \frac{W_0 N\sigma}{ik} - \frac{2}{\pi W_0} \left\{ 1 + \frac{N\sigma}{ik} \right\} Q_1(\omega, k) + \frac{4}{\pi W_0^2} \frac{\omega}{k} \left( \frac{N\sigma}{2k} \right)^{1/2} \exp \left[ -\frac{ik}{\pi N\sigma} \left( \frac{\omega}{k W_0} \right)^2 \right] \text{erf} \left( \frac{(1 - i) \sqrt{2}}{\pi N\sigma} \left( \frac{k}{2} \right)^{1/2} \frac{\omega}{k W_0} \right) Q_1(\omega, k) + \frac{4}{\pi W_0^2} \frac{\omega}{k} \left( \frac{N\sigma}{2k} \right)^{1/2} \exp \left[ -\frac{ik}{\pi N\sigma} \left( \frac{\omega}{k W_0} \right)^2 \right] \Phi,
\]

\[
Q_1(\omega, k) = \frac{\pi W_0^2 N\sigma}{2i\chi} (1 + \Sigma(\omega, k))
\]

\[
Q(\omega, k) = \frac{\pi W_0}{2} \left( \frac{N\sigma}{i\chi} \right)^{1/2} \exp \left[ \frac{i\chi}{\pi N\sigma} \left( \frac{\omega}{\chi W_0} \right)^2 \right] \left\{ 1 + \text{erf} \left( \frac{\omega}{\chi W_0} \left( \frac{i\chi}{\pi N\sigma} \right)^{1/2} \right) \right\}.
\]

where \( \Sigma \) is

\[
\Sigma(\omega, k) = \frac{\omega}{\chi W_0} \left( \frac{i\chi}{N\sigma} \right)^{1/2} \exp \left( \frac{i\chi}{\pi N\sigma} \left( \frac{\omega}{\chi W_0} \right)^2 \right) \left\{ 1 + \text{erf} \left( \frac{\omega}{\chi W_0} \left( \frac{i\chi}{\pi N\sigma} \right)^{1/2} \right) \right\}
\]

and \( \Phi \) is defined in Eq. (3.28). Function \( \tilde{L}(\omega, k) \) is plotted in Figs. (3.3) and (3.4). As one can see, parameter \( N\sigma \) determines the resonance width of the curve. The resonance curve becomes wider as the parameter \( N\sigma \) decreases.

The nonresonant part of ion density is given by

\[
\frac{\bar{n}_{nonres}}{n_0} = \tilde{T}(\omega, k) \frac{\tilde{E}(\omega, k)}{E_0}
\]
where \( \tilde{T}(\omega, k) \) is given by

\[
\tilde{T}(\omega, k) = \frac{N\sigma}{ik} + \frac{(1+i)}{2} \frac{4n_0 \tilde{E} \omega}{\pi^2 W_0^3 E_0 k} \left( \frac{\pi E_0}{k} \right)^{1/2} \exp \left[ -\frac{ik}{2E_0} \left( \frac{\omega}{k} \right)^2 \right] \Gamma(\omega, k) \\
+ Z(\omega, k) Q(\omega, k)
\]  

(3.39)

where

\[
\Gamma(\omega, k) = \int_0^\infty \operatorname{erf}\left( \frac{(1-i)}{\sqrt{2}} \left( \frac{k}{2E_0} \right)^{1/2} \left( v - \frac{\omega}{k} \right) \right) \Delta^{-1}(v) \Omega(v, k, \omega) \, dv
\]  

(3.40)

\[
Z(\omega, k) = \frac{(1+i)}{2} \frac{4\omega}{\pi^2 W_0^3 k} \left( \frac{\pi E_0}{k} \right)^{1/2} \exp \left[ -\frac{ik}{2E_0} \left( \frac{\omega}{k} \right)^2 \right]
\times \operatorname{erf}\left( \frac{(1-i)}{\sqrt{2}} \left( \frac{k}{2E_0} \right)^{1/2} \frac{\omega}{k} \right) - \frac{2}{\pi W_0} \frac{2N\sigma}{\pi W_0 ik}.
\]  

(3.41)

In the limit of a slowly varying electric field, \( k/N\sigma < 1 \), and \( \omega/W_0 N\sigma < 1 \), the expression (3.20) for the perturbed distribution function can be simplified and integration in expressions (3.30) and (3.31) can be done analytically. In this case, to the first order in the parameters \( ik/N\sigma \) and \( i\omega/W_0 N\sigma \) one obtains

\[
\tilde{n}(\omega, k) = -\frac{1}{2} \frac{n_0 \tilde{E}}{E_0} + \frac{1}{8 N\sigma E_0} \tilde{E}(\omega, k) - \frac{4}{3\pi} \frac{n_0 i\omega}{W_0 E_0 N\sigma} \tilde{E}(\omega, k), \quad \text{(3.42)}
\]

\[
\tilde{W}(\omega, k) = W_0 \frac{\tilde{E}(\omega, k)}{2E_0} - \frac{1}{8 N\sigma E_0} \tilde{E}(\omega, k) - \frac{(3\pi - 8)}{6\pi} \frac{W_0}{W_0 N\sigma E_0} \tilde{E}(\omega, k). \quad \text{ (3.43)}
\]

In real space \((z, t)\), the ion density and ion flow velocity are given by

\[
W^{(1)}(z, t) = W_0 + \frac{W_0}{2} \frac{\tilde{E}(z, t)}{E_0} - D_\parallel \frac{1}{E_0} \frac{\partial E(z, t)}{\partial z} - D_t \frac{1}{W_0} \frac{\partial E(z, t)}{E_0 \partial t}, \quad \text{ (3.44)}
\]

\[
n^{(1)} = n_0 - \frac{1}{2} \frac{n_0 \tilde{E}}{E_0} + \frac{1}{8 N\sigma E_0} \frac{n_0 \partial E(z, t)}{\partial z} + \frac{4}{3\pi} \frac{n_0}{W_0 E_0 N\sigma} \frac{\partial E(z, t)}{\partial t}. \quad \text{ (3.45)}
\]

The coefficients

\[
D_\parallel = \frac{1}{8 N\sigma}, \quad \text{ (3.46)}
\]
Figure 3.5: (a) The real part of the Pade approximant (3.53) of the ion perturbed velocity versus a parameter $\omega/W_0 N \sigma$. (b) The real part of the nonresonant portion of the ion perturbed velocity versus a parameter $\omega/W_0 N \sigma$. 
Figure 3.6: (a) The imaginary part of the Pade approximant (3.53) of the ion perturbed velocity versus a parameter $\omega/W_0 N\sigma$. (b) The imaginary part of the nonresonant portion of the ion perturbed velocity versus a parameter $\omega/W_0 N\sigma$. 
and

\[ D_t = \frac{(3\pi - 8)}{6\pi} \frac{W_0}{N\sigma}, \]  

(3.47)

have the dimensions of diffusion coefficients and describe nonlocal and inertial fluxes (to first order in \(ik/N\sigma\) and \(i\omega/W_0N\sigma\)).

The second order (in \(ik/N\sigma\) and \(i\omega/W_0N\sigma\)) expressions for \(W^{(2)}(\omega, k)\) and \(n^{(2)}(\omega, k)\) are

\[
W^{(2)}(\omega, k) = W_0 + \frac{W_0}{2} \frac{\tilde{E}(\omega, k)}{E_0} - \frac{1}{8} \frac{W_0i k}{N\sigma E_0} \tilde{E}(\omega, k)
\]

\[- \frac{(3\pi - 8)}{6\pi} \frac{W_0}{W_0 N\sigma E_0} \tilde{E}(\omega, k) + \frac{63}{48} \frac{W_0 k^2}{(N\sigma)^2 E_0} \tilde{E}(\omega, k)
\]

\[- \frac{5}{8} \frac{\omega k}{(N\sigma)^2 E_0} \tilde{E}(\omega, k) + \frac{79}{12\pi} \frac{\omega^2}{W_0 (N\sigma)^2 E_0} \tilde{E}(\omega, k), \]  

(3.48)

\[
n^{(2)}(\omega, k) = - \frac{1}{2} \frac{n_0 E}{E_0} + \frac{1}{8} \frac{n_0 i k}{E_0 N\sigma} \tilde{E}(\omega, k) - \frac{4}{3\pi} \frac{n_0 i \omega}{W_0 N\sigma E_0} \tilde{E}(\omega, k)
\]

\[+ \frac{1}{32} \frac{n_0 k^2}{(N\sigma)^2 E_0} \tilde{E}(\omega, k) - \frac{2}{3\pi} \frac{n_0 \omega k}{(N\sigma)^2 W_0 E_0} \tilde{E}(\omega, k)
\]

\[+ \frac{32}{9\pi^2} \frac{W_0}{W_0 (N\sigma)^2 E_0} \tilde{E}(\omega, k). \]  

(3.49)

In \((z, t)\) space, \(W^{(2)}(\omega, k)\) and \(n^{(2)}(\omega, k)\) have the form

\[
W^{(2)}(z, t) = W_0 + \frac{W_0}{2} \frac{\tilde{E}(z, t)}{E_0} - D_{\parallel} \frac{1}{E_0} \frac{\partial E(z, t)}{\partial z} - D_t \frac{1}{W_0} \frac{\partial E(z, t)}{\partial t} - \frac{63}{48} \frac{W_0}{(N\sigma)^2 E_0}
\]

\[\times \frac{\partial^2 E(z, t)}{\partial z^2} - \frac{5}{8} \frac{1}{(N\sigma)^2 E_0} \frac{\partial^2 E(z, t)}{\partial z \partial t} - \frac{79}{12\pi} \frac{1}{W_0 (N\sigma)^2 E_0} \frac{\partial^2 E(z, t)}{\partial t^2}, \]  

(3.50)

\[
n^{(2)}(z, t) = n_0 - \frac{n_0 E}{E_0} + \frac{n_0}{8 E_0 N\sigma} \frac{\partial E(z, t)}{\partial z} + \frac{4}{3\pi W_0 N\sigma} \frac{\partial E(z, t)}{\partial t} - \frac{1}{32} \frac{n_0}{(N\sigma)^2 E_0} \frac{\partial^2 E(z, t)}{\partial z^2}
\]

\[\times \frac{2}{3\pi W_0 (N\sigma)^2 E_0} \frac{\partial^2 E(z, t)}{\partial z \partial t} - \frac{32}{9\pi^2} \frac{n_0}{W_0^2 (N\sigma)^2 E_0} \frac{\partial^2 E(z, t)}{\partial t^2}. \]  

(3.51)
Figure 3.7: (a) The real part of the Pade approximant (3.52) of the ion perturbed density versus a parameter $\omega/W_0 N \sigma$. (b) The real part of the nonresonant portion of the ion perturbed density versus a parameter $\omega/W_0 N \sigma$. 
Figure 3.8: (a) The imaginary part of the Pade approximant (3.52) of the ion perturbed velocity versus a parameter \( \omega/W_0 N \sigma \). (b) The imaginary part of the nonresonant portion of the ion perturbed density versus a parameter \( \omega/W_0 N \sigma \).
By using the expansions (3.44) (3.45), one can construct the two-point ($\omega = 0$ and $\omega = \infty$, $k = 0$ and $k = \infty$) Padé type approximants for the exact expressions (3.30) and (3.31). Matching the expressions for small $\omega$ and $k$ (in the first order) and the unperturbed values for $\omega = \infty$, $k = \infty$ we obtain

\[
\begin{align*}
n(z,t) &= n_0 - n_0 \left( 1 + \frac{1}{4N\sigma} \frac{\partial}{\partial z} + \frac{8}{3\pi W_0 N\sigma} \frac{\partial}{\partial t} \right)^{-1} \frac{E(z,t)}{2E_0}, \\
W(z,t) &= W_0 + W_0 \left( 1 + \frac{1}{4N\sigma} \frac{\partial}{\partial z} + \frac{3\pi - 8}{3\pi W_0 N\sigma} \frac{\partial}{\partial t} \right)^{-1} \frac{E(z,t)}{2E_0}.
\end{align*}
\tag{3.52, 3.53}
\]

These expressions reasonably approximate the non-resonant parts of the ion perturbed velocity and density, but fail to recover the resonant contribution. Figs. (3.5), (3.6) (3.7), (3.8) show the non-resonant parts of ion perturbed velocity and density along with their Padé approximations (3.52) and (3.53).

### 3.4 The Chapman-Enskog Type Approach

In this section we solve the Boltzmann equation (1.2) by using the Chapman-Enskog type approach. In this approach the ion distribution function is presented by using the Ansatz

\[
f(v, z, t) = F_{eq}(v, z, t) + \tilde{F}(v, z, t),
\tag{3.54}
\]

where

\[
F_{eq}(v, z, t) = \frac{2n(z,t)}{\pi \xi(z,t)} \exp \left[- \frac{v^2}{\pi \xi^2(z,t)} \right] \Theta(v),
\tag{3.55}
\]

\[
\xi(z,t) = \sqrt{\frac{2E(z,t)}{\pi N\sigma}},
\]

is the dynamical equilibrium function which corresponds to the equilibrium function with a time and space dependent external electric field, and $\tilde{F}(v, z, t)$ is the deviation.
The dynamical part of the distribution function is determined by the equation

\[ E(z,t) \frac{\partial F_{eq}}{\partial v} = -N \sigma |v| F_{eq} + N \sigma Q_{eq} \delta(v), \tag{3.56} \]

where

\[ Q_{eq} = \int_0^\infty v F_{eq}(v,z,t) dv. \tag{3.57} \]

We require that the total ion density be determined only by the dynamical distribution function \( F_{eq}(v,z,t) \), so that

\[ n(z,t) = \int \left( F_{eq}(v,z,t) + \bar{F}(v,z,t) \right) dv = \int F_{eq}(v,z,t) dv. \tag{3.58} \]

This imposes the following constraint on the deviation

\[ \int \bar{F}(v,z,t) dv = 0. \tag{3.59} \]

The purpose of this approach is to determine the ion distribution function in terms of the gradients of the lower moment (ion density) plus the gradients of the external electric field. Solution to equation (1.2) with the assumption of a unidirectional external electric field is sought in the form (3.54) with normalization conditions (3.58) and (3.59). As in the previous case we suppose that

\[ E(z,t) = E_0 + \bar{E}(z,t). \]

Then the linearized version of Boltzmann Eq. (1.2) with the Ansatz (3.54) reads

\[ \frac{\partial \tilde{F}}{\partial t} + v \frac{\partial \tilde{F}}{\partial z} + E_0 \frac{\partial \tilde{F}}{\partial v} = -N \sigma |v| \tilde{F} - \frac{\partial F_{eq}}{\partial t} - v \frac{\partial F_{eq}}{\partial z} + N \sigma \bar{Q} \delta(v), \tag{3.60} \]

where \( \bar{Q} \) is

\[ \bar{Q} = \int_0^\infty v \tilde{F}(v,z,t) dv, \tag{3.61} \]
and

\[
\frac{\partial F_{eq}}{\partial t} = \frac{\partial F_{eq}}{\partial n} \frac{\partial n}{\partial t} + \frac{\partial F_{eq}}{\partial \xi} \frac{\partial \xi}{\partial t}, \tag{3.62}
\]

\[
\frac{\partial F_{eq}}{\partial z} = \frac{\partial F_{eq}}{\partial n} \frac{\partial n}{\partial z} + \frac{\partial F_{eq}}{\partial \xi} \frac{\partial \xi}{\partial z}. \tag{3.63}
\]

Substituting (3.62) and (3.63) into (3.60), and solving it for \( v > 0 \) and \( v < 0 \) as in the previous section, one gets in the linear approximation

\[
\bar{F}^+(v, \omega, k)/\Delta^{-1}(v) = -\frac{2}{\pi W_0} n(\omega, k) + \frac{2n_0}{\pi W_0^2} \xi(\omega, k) - \frac{4n_0}{\pi^2 W_0^4} \left( v^2 - \frac{2E_0}{ik} \right) \]

\[
\times \xi(\omega, k) + \frac{1}{2} \frac{i\omega}{\pi^2 W_0^4} k \sqrt{\frac{\pi E_0}{k}} \xi(\omega, k) \exp \left[ -\frac{ik}{2E_0} \left( \frac{\omega}{k} \right)^2 \right] \]

\[
\times \Omega(v, \omega, k) \text{erf} \left( \frac{1 - i}{\sqrt{2}} \sqrt{\frac{k}{2E_0}} \left( v - \frac{\omega}{k} \right) \right) + C_1(\omega, k) \Omega(v, \omega, k) \]

\[
+ \frac{N\sigma}{E_0} \bar{Q}\Omega(v, \omega, k), \tag{3.64}
\]

\[
\bar{F}^-(v, \omega, k) = 0, \tag{3.65}
\]

where \( \Delta^{-1}(v) \) and \( \Omega(v, \omega, k) \) are defined in Eqs.(3.18) and (3.19). The part of the ion flux \( \bar{Q} \) pertaining to the deviation part of the ion distribution function as well as \( C_1(\omega, k) \) are the two unknown values in expression (3.64). To find them, one needs to construct a system of two equations. One of these equations is the constraint (3.59), which after the integration takes the form

\[
\lambda_1 C_1(\omega, k) = n(\omega, k) - \frac{4n_0E_0 \xi(\omega, k)}{(\pi W_0^3)ik} - \frac{8n_0\omega \xi(\omega, k) \Psi(\omega, k)}{\pi^2 W_0^4 k} - \lambda_1 \frac{N\sigma}{E_0} \bar{Q}, \tag{3.66}
\]

where

\[
\Psi(\omega, k) = \frac{1}{2} \frac{\sqrt{\pi E_0}}{k} \exp \left[ -\frac{ik}{2E_0} \left( \frac{\omega}{k} \right)^2 \right] \]

\[
\times \int_0^\infty \text{erf} \left( \frac{1 - i}{\sqrt{2}} \sqrt{\frac{k}{2E_0}} \left( v - \frac{\omega}{k} \right) \right) \Delta^{-1}(v)\Omega(v, \omega, k)dv, \tag{3.67}
\]
\[ \lambda_1 = \left(\frac{\pi E_0}{2i\chi}\right)^{1/2} \exp\left[\frac{i\chi}{2E_0} \left(\frac{\omega}{\chi}\right)^2\right] \left(1 + \text{erf}\left(\frac{\omega}{\chi} \left(\frac{i\chi}{2E_0}\right)^{1/2}\right)\right). \]

(3.68)

The second equation is obtained by multiplying Eq. (3.64) by \(v\) and integrating it over the velocity space to give

\[ \tilde{Q} = \int_0^\infty v \tilde{F}(k, \omega, v) dv = \frac{-n_0 \xi(\omega, k)}{1 - \frac{N\sigma}{E_0} \lambda_2} - \frac{W_0 n(\omega, k)}{1 - \frac{N\sigma}{E_0} \lambda_2} + \frac{4n_0 E_0 \xi(\omega, k)}{(\pi W_0^2) \left(1 - \frac{N\sigma}{E_0} \lambda_2\right) i k} \]

\[ + \frac{8n_0 \omega \xi(\omega, k) \Xi(\omega, k)}{(\pi^2 W_0^4) \left(1 - \frac{N\sigma}{E_0} \lambda_2\right) k} + \frac{C_1 \lambda_2}{1 - \frac{N\sigma}{E_0} \lambda_2}. \]

(3.69)

where

\[ \Xi(\omega, k) = \frac{(1 + i)}{2} \sqrt{\frac{\pi E_0}{k}} \exp\left[\frac{-i k}{2E_0} \left(\frac{\omega}{k}\right)^2\right] \]

\[ \int_0^\infty v \text{erf}\left(\frac{(1 - i)}{\sqrt{2}} \sqrt{\frac{k}{2E_0}} (v - \frac{\omega}{k})\right) \Delta^{-1}(v) \Omega(v, \omega, k) dv, \]

(3.70)

\(\lambda_2\) is defined by Eq.(3.29). Solving this system of two equations we obtain an expression for \(\tilde{Q}\)

\[ \tilde{Q} = -n_0 \xi(\omega, k) - W_0 n(\omega, k) + \frac{4n_0 E_0}{\pi W_0^2} i k \xi(\omega, k) + \frac{8n_0}{\pi^2 W_0^4} \frac{\omega}{k} \xi(\omega, k) \Xi(\omega, k) \]

\[ - \left(-n(\omega, k) + \frac{4n_0}{\pi W_0^3} i k \xi(\omega, k) + \frac{8n_0}{\pi^2 W_0^4} \frac{\omega}{k} \xi(\omega, k) \Psi(\omega, k)\right) \frac{\lambda_2}{\lambda_1}. \]

(3.71)

The total ion drift velocity is determined from

\[ W(\omega, k) = \frac{\int v f dv}{\int f dv} = \frac{\int v \left(F_{eq} + \tilde{F}\right) dv}{\int f dv} = \xi(\omega, k) + \frac{\tilde{Q}}{n(\omega, k)}. \]

(3.72)

To the first order of parameters \(ik/N\sigma, i\omega/W_0 N\sigma\), the expression (3.72) in the \((z, t)\) space reads

\[ W(z, t) = \xi(z, t) - \frac{1}{2} \frac{W_0}{N\sigma} \frac{\partial \ln n(z, t)}{\partial z} - \left(1 - \frac{2}{\pi}\right) \frac{\partial \ln n(z, t)}{N\sigma \partial t} \]

\[ - \frac{3}{4N\sigma} \frac{\partial \xi(z, t)}{\partial z} - \frac{2}{3\pi W_0 N\sigma \partial t}. \]

(3.73)
The hydrodynamic ion velocity consists of two parts. One is induced by an external electric field and the other is of diffusive nature. Substituting the temporal derivative of ion density from the continuity equation (3.77) into Eq. (3.73) we obtain the ion drift velocity expressed in terms of both, an external electric field along with its derivatives and the gradient of ion number density

\[ W(z, t) = \xi - D_t \frac{\partial \ln \gamma}{W_0 \partial t} - D_{||} \frac{\partial \ln \gamma}{\partial z} - \frac{4 - \pi}{2\pi} \frac{W_0}{N\sigma} \frac{\partial \ln n}{\partial z}, \]  

(3.74)

where \( \gamma = \frac{E(z, t)}{N} \), and

\[ D_t = \frac{W_0}{3\pi N\sigma}, \]

(3.75)

\[ D_{||} = \frac{8 - \pi}{8\pi} \frac{W_0}{N\sigma}. \]

(3.76)

In the linear approximation, coefficients \( D_t \) and \( D_{||} \) coincide with ones found in [54].

The ion density and flux satisfy the continuity equation

\[ \frac{\partial n}{\partial t} + \frac{\partial (nW)}{\partial z} = 0. \]

(3.77)

Substituting (3.72) into (3.77) and after Fourier transformation, one derives an expression for the ion perturbed density

\[ \frac{\tilde{n}(\omega, k)}{n_0} = \frac{\tilde{F}(\omega, k)}{\omega/k - \lambda_2/\lambda_1}, \]

(3.78)

where

\[ \tilde{F}(\omega, k) = \frac{2\tilde{E}}{\pi W_0 i k} \left( 1 - \frac{1}{W_0} \frac{\lambda_2}{\lambda_1} \right) + \frac{4n_0 \tilde{E}}{\pi^2 W_0^3 E_0 k} \frac{\omega}{k} \Xi(\omega, k) - \frac{4n_0 \tilde{E}}{\pi^2 W_0^3 E_0 k} \frac{\omega}{k} \frac{\lambda_2}{\lambda_1} \Psi(\omega, k). \]

(3.79)

Expression (3.78) coincides with the expression for the perturbed ion density for ions determined by the Eq. (3.30).
3.5 Summary

In the present chapter, we have considered nonlocal ion transport in a weakly ionized plasma placed in a strong electric field when the ion velocity distribution function is nonequilibrium. The ion distribution function is presented as a sum of two parts. One is the stationary ion distribution and the other is the deviation due to the perturbed external electric field. The nonequilibrium steady-state ion distribution sustained by the strong homogeneous electric field has a beam-like form with uniform ion density and uniform ion flow. We consider the ion flux due to the imposed perturbation of the electric field which varies in time and space. An expression for the ion distribution function for arbitrary values of \( \omega \) and \( k \) is obtained. In the case when the ion mean free path is not small compared to the characteristic length scale of an external electric field inhomogeneity, the perturbed ion flux and density can no longer be expressed in terms of local parameters such as \( E \) (the electric field) and \( n \) (ion density), but rather are represented through an integro-differential operator acting on the electric field. It is shown that the perturbed ion flux and density exhibit resonant behavior when \( \omega \simeq kW_0 \) which is the resonance between the average ion velocity and wave phase velocity. The form of the resonant curve is determined by the parameter \( N\sigma \). The curve becomes steeper and narrower as the parameter \( N\sigma \) increases. The resonant part of ion density and velocity when \( \omega \simeq kW_0 \) is much larger than that of the nonresonant part.

The results of the present chapter can be used in the modeling of ion transport in a plasma sheath region of the capacitive RF discharges, where there exists a situation when the constant electric field in a sheath region is superimposed by time and space varying electric field created by the external source. The ion density, that has been obtained in the present thesis can be used to solve the Poisson’s equation to find the
potential in a sheath region.
Chapter 4

Hybrid Fluid/Kinetic Description of the Transport Processes in Weakly Ionized Plasmas

4.1 Introduction

Electron transport processes in weakly ionized plasmas play a fundamental role in gas discharge physics and its applications, in particular, for a variety of plasma sources used in science and technology. In the present paper we consider the electron transport processes in a low temperature plasma (0 – 10 eV.) placed in a nonuniform (in time and space) electric field when the main mechanism of electron scattering is elastic collisions with neutrals.

We are interested in a situation when the electron mean free path is not small compared to the characteristic length scale of the system's inhomogeneity, and the electron collisional frequency is not necessarily large compared to the frequency of the
external field. For such a condition, the thermal electron motion becomes important, so that the transport coefficients become the nonlocal operators acting both in space and time. This modifies the mechanisms of the thermal and electric conductivities, viscosity, diffusion in a system and these mechanisms become increasingly important as gas discharges for plasma processing and lighting move toward the lower pressure regimes. In these regimes the usual hydrodynamic approach is not adequate and one has to use the hybrid fluid/kinetic approach in order to realistically describe the relaxation processes that occur in the considered system.

The unification of both approaches is of the greatest importance in the considered problem, since it will allow us to uniformly describe the system's behavior in any collisionality regimes and to include the kinetic effects such as Landau wave-particle resonance (in collisionless regime) as well. We note that the untruncated fluid moment equation system (equations for density $n$, flow velocity $\mathbf{V}$, temperature $T$, stress tensor $\Pi$, etc.) is equivalent to the Boltzmann kinetic equation (taking different velocity moments of it will give us the hierarchy of moment equations). Therefore the kinetic information has not been lost in such a hierarchy. The applicability of such a set of closed fluid equations (e.g., $n, \mathbf{V}, T$) is determined by the closure relations (expressing the stress tensor $\Pi$ and the heat flux $\mathbf{q}$ in terms of lower moments $n, \mathbf{V}$, and $T$). In the usual fluid approach it is the high collision frequency assumption ($\nu \gg \partial/\partial t$) used in obtaining closure relations that restricts its validity to the high collisionality (hydrodynamic) regime. Therefore all the kinetic effects can be included in fluid moment equations if we do not exclude the kinetic effects in calculating the closure relations. In this case both approaches are unified. Closed fluid/kinetic equations can then be used in deriving expressions for the transport coefficients of the considered system.
We use a Chapman-Enskog-like procedure to introduce the kinetic effects into the fluid equations. This method involves an approach in which the deviation of the distribution function from the thermodynamic equilibrium is expanded in the gradients of the hydrodynamic moments, or in other words, the total electron distribution function is decomposed into two parts - a time and space dependent Maxwellian \( f_M \), plus a deviation \( F \). The Chapman-Enskog Ansatz assumes that the spatial and temporal dependence in \( f_M \) is through the hydrodynamic quantities \( n(r, t), T(r, t) \) and \( \mathbf{V}(r, t) \). Substituting the first three fluid moment equations (for \( n(r, t), T(r, t) \) and \( \mathbf{V}(r, t) \)) into the Boltzmann kinetic equation, one obtains a kinetic equation for the departure part \( F \). The departure part in turn has to satisfy certain constraints which come from the fact that the first three moments \( n(r, t), T(r, t) \) and \( \mathbf{V}(r, t) \) have to be determined by the dynamic Maxwellian distribution function \( f_M \). These constraints will give us the higher moments \( \Pi \) and \( \mathbf{q} \) of the distribution function expressed through the lower hydrodynamic moments \( n, T \) and \( \mathbf{V} \) from which we will find the appropriate transport coefficients. This procedure is equivalent to the velocity moment approach in which all of the moments are taken into the description of the state of the system in such a way, that the higher moments (stress tensor \( \Pi \), heat flux vector \( \mathbf{q} \), ...) are expressed in terms of 3 lower moments \( (n(r, t), T(r, t), \mathbf{V}(r, t)) \). It should be mentioned here, that the standard Chapman-Enskog procedure is the asymptotic series in which the total distribution function is expanded in a parameter \( \varepsilon \) which represents the scale of the mean free path. For example, in the zeroth order it gives the local Maxwellian distribution, which determines the first three moments: density, mean velocity, and temperature (all higher moments are zero). In the first order of \( \varepsilon \) one obtains the first order correction to the local Maxwellian function which will give nonzero viscosity and heat flux. Since the series will not be convergent in general, the sum should be
truncated at a finite number of terms. The number of terms in a series is determined by the fact that the remainder of the expansion should be small in $\epsilon$. As a parameter $\epsilon$ increases, the number of terms needed to properly describe the system should increase as well. In the limiting, collisionless case one would have to solve an infinite number of partial differential equations to obtain the total distribution function, which would be a formidable task. The approach that is developed in this chapter circumvents this problem by introducing a technique in which the total distribution function is developed in a series of the collision operator eigenfunctions with the coefficients being the different moments of the distribution function. This method will allow us to obtain an infinite system of equations for the expansion coefficients and an exact solution in terms of the infinite continued fractions will be found.

We neglect the influence of the induced magnetic field on the system, since it is of the order of $v/c$ and $v << c$. We also ignore processes which change the number density of the various species, e.g., ionizing collisions as well as inelastic collisions and the energy transfer between electrons and neutrals, because of the large mass difference between both species (the rate of energy transfer between electrons and neutrals is of the order of $m_e/M_{at}$, where $m_e$ is an electron mass, $M_{at}$ is the mass of an atom).

4.2 The Hierarchy of the Distribution Function Expansion

The Boltzmann kinetic equation for the electron distribution function in a weakly ionized plasma is

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \frac{e\mathbf{E}(r,t)}{m} \cdot \nabla f = \tilde{C}(f),$$

(4.1)
where \( \hat{C} \) is linearized collision integral. We neglect the electron-electron and electron-ion interactions, since the neutral atom density is considerably higher than that of the electrons and ions. We also neglect the effects of energy transfer from electrons to atoms because of the large mass difference between the two species. Thus atoms can be considered to be motionless, and their distribution function is given by

\[
f'_1 = f_1 = N \delta (v_{1x}) \delta (v_{1y}) \delta (v_{1z}),
\]

where \( N \) is the neutral atom density. Substituting the expression (2.4) into (2.3) we obtain the linearized collision operator,

\[
\hat{C} (f) = N \int (f (t, r, v') - f (t, r, v)) v \frac{d\sigma}{d\Omega} d\Omega.
\]

(4.3)

In the approximation when the neutral atoms distribution function is not changed by collisions with electrons and the differential cross-section of the electron scattering by the neutral atom does not depend on the azimuthal angle \( \phi \), the orthonormal eigenfunctions of the collision operator (4.3) are [1]

\[
\Psi_{r, \ell, m} = L^{\ell+1/2} \left( \frac{v^2}{v'_t} \right) v^l P_l^{|m|} (\cos \theta) e^{im\phi}/N_{r, \ell, m}^{1/2}
\]

(4.4)

where \( L^{\ell+1/2} \left( \frac{v^2}{v'_t} \right) \) are Laguerre polynomials, \( P_l^{|m|} (\cos \theta) \) are associated Legendre polynomials, \( \theta \) and \( \phi \) are the angular variables in spherical velocity space, \( N_{r, \ell, m}^{1/2} \) is the normalization constant equal to

\[
N_{r, \ell, m} = \frac{2^{l+1} \Gamma (r + l + 3/2) (l + |m|)!}{\sqrt{\pi} r! (2l + 1) (l - |m|)!}.
\]

(4.5)

Since

\[
Y_{l, m} (\theta, \phi) = (-1)^m \sqrt{\frac{(2l + 1) (l - m)!}{4\pi (l + m)!}} P_l^{|m|} (\cos \theta) e^{im\phi}
\]

(4.6)

where \( Y_{l, m} (\theta, \phi) \) are the spherical harmonics, the Eq. (4.4) can be written in a form

\[
\Psi_{r, \ell, m} = (-1)^m \sqrt{\frac{4\pi^{3/2} r!}{2^{l+1} \Gamma (r + l + 3/2)}} L^{|m|+1/2} \left( \frac{v^2}{v'_t} \right) v^l Y_{l, m} (\theta, \phi).
\]

(4.7)
The functions (4.7) satisfy the eigenrelation,
\[
\widehat{C}(\Psi_{r,l,m}) = \nu_{r,l,m}\Psi_{r,l,m}
\] (4.8)

For elastic electron-neutral atom collisions the eigenvalues are \(r(2l+1)\) fold degenerate—not depending on the subscripts \(r\) and \(m\) at all and are given by Eq.(2.8) (degeneracy comes from the fact that the differential cross section of the elastic electron-neutral atom interaction is independent of azimuthal angle \(\phi\) and we neglected the energy transfer between electrons and neutrals). Since functions (4.4) form a complete set, we can expand the unknown electron distribution function \(f\) in the \(\Psi_{r,l,m}\)
\[
f = \sum_{r,l,m} b_{r,l,m}(r, t) \Psi_{r,l,m}; \quad b_{r,l,m}(r, t) = \int e^{-\nu^2/\nu_i^2} f \Psi_{r,l,m}^* d^3\nu.
\] (4.9)

Since the expansion coefficients \(b_{r,l}(r, t)\) are related to the moments of the distribution function, the expansion (4.9) can be written in a form
\[
f = f_{M_0} (a + v \cdot b + (v\nu - v^2/3\Gamma) : B + \cdots)
\] (4.10)

where \(f_{M_0}\) is the Maxwellian distribution, \(v\) is the electron total velocity, \(a, b, B\) are
\[
a = \sum_{r=0}^{\infty} b_{r,0,0}(r, t) \Psi_{r,0,0} = \sum_{r=0}^{\infty} \alpha_r L_r^{1/2}(x)
\] (4.11)
\[
v \cdot b = \sum_{r=0, m=-1}^{\infty} b_{r,1,m}(r, t) \Psi_{r,1,m} \Rightarrow b = \sum_{r=0}^{\infty} \xi_r L_r^{3/2}(x)
\] (4.12)
\[
(v\nu - v^2/3\Gamma) : B = \sum_{r=0, m=-2}^{\infty} b_{r,2,m}(r, t) \Psi_{r,2,m} \Rightarrow B = \sum_{r=0}^{\infty} \delta_r L_r^{5/2}(x)
\] (4.13)

where \(x = v^2/v_T^2\), Using the definition for different moments (density, velocity, pressure etc.) given by
\[
n = \int f d^3\nu, \quad V = \frac{\int v f d^3\nu}{n}, \quad p = \frac{m}{3} \int v^2 f d^3\nu, \quad q = T \int v \left(\frac{v^2}{v_T^2} - \frac{5}{2}\right) f d^3\nu,
\]
\[
\Pi = m \int (v\nu - v^2/3\Gamma) f d^3\nu, \quad \Theta = \int \frac{mv^2}{2} (v\nu - v^2/3\Gamma) f d^3\nu \ldots
\] (4.14)
we can find the unknown values for $\alpha_n, \xi_n, \delta_n$ and so on. For example, using the definitions for the density $n$, pressure $p$, mean velocity $V$, heat flux $q$, stress tensor $\Pi$, and energy weighted stress tensor $\Theta$, we find the first two scalar $\alpha_0$, $\alpha_1$, vector $\xi_0$, $\xi_1$ and tensor $\delta_0$, $\delta_1$ quantities to be

$$
\alpha_0 = 1 + \frac{\bar{n}}{n_0}, \quad \alpha_1 = -\frac{\bar{T}}{T_0}, \quad \xi_0 = \frac{2\bar{V}}{v_T^2}, \quad \xi_1 = -\frac{2\bar{q}}{5p_0}, \quad \delta_0 = \frac{2\bar{\Pi}}{mn_0 v_T^4}, \quad \delta_1 = -\frac{2\bar{\Theta}}{7T_0} \tag{4.15}
$$

where $\bar{n}, \bar{T}, \bar{V}, \bar{q}, \bar{\Pi}, \bar{\Theta}$ are the perturbed density, temperature, hydrodynamic velocity, heat flux vector, stress tensor and energy weighted stress tensor. Doing this procedure for all higher tensorial moments, we obtain the following form of the distribution function (4.10)

$$
f/f_{M0} = \left(1 + \frac{\bar{n}}{n_0} L_0^{(1/2)} - \frac{\bar{T}}{T_0} L_1^{(1/2)} + \cdots\right) + \frac{2v}{v_T} \cdot \left(\bar{V} L_0^{(3/2)} - \frac{2}{5p_0} \bar{q} L_1^{(3/2)} \right)
+ 2\frac{v^2 - v^2 / 3I}{mn_0 v_T^4} : \left(\bar{\Pi} L_0^{(5/2)} - \frac{2}{7T_0} \bar{\Theta} L_1^{(5/2)} + \cdots\right) + \cdots \tag{4.16}
$$

Regrouping some terms in (4.16) we obtain

$$
f/f_{M0} = \left(1 + \frac{\bar{n}}{n_0} L_0^{(1/2)} - \frac{\bar{T}}{T_0} L_1^{(1/2)} + \frac{2v}{v_T^2} \cdot \bar{V} L_0^{(3/2)} \right) + \frac{2v}{v_T} \cdot \left(-\frac{2}{5p_0} \bar{q} L_1^{(3/2)} \right)
+ 2\frac{v^2 - v^2 / 3I}{mn_0 v_T^4} : \left(\bar{\Pi} L_0^{(5/2)} - \frac{2}{7T_0} \bar{\Theta} L_1^{(5/2)} + \cdots\right) + (\chi + \cdots) + \cdots \tag{4.17}
$$

where $\chi$ is one of the higher scalar moment. In the linear approximation, the first bracket on the right of the Eq. (4.17) constitutes the so-called local Maxwellian distribution function $f_M$ which has the form

$$
f_M = n(r,t) \left(\frac{m}{2\pi T(r,t)}\right)^{3/2} \exp \left(-\frac{m(v - V(r,t))^2}{2T(r,t)}\right). \tag{4.18}
$$

The remaining terms in Eq. (4.17) constitute the deviation part $F$ of the distribution function from the local Maxwellian. The perturbed distribution function consists of
all higher moments (scalar, vector, tensor) and can be written in a form

$$F = \sum f_{l,m}(r,v,t)Y_{l,m} (\theta, \phi)$$  \hspace{1cm} (4.19)

where all the scalar moments are incorporated in

$$f_{0,0} = \sum_{r=2}^{\infty} b_{r,0,0} L_r^{1/2} (x),$$  \hspace{1cm} (4.20)

vector moments in

$$f_{1,-1} = -\sum_{r=1}^{\infty} \sqrt{\frac{\pi^{3/2} r!}{\Gamma (r+5/2)}} b_{r,1,-1} L_r^{3/2} (x),$$  \hspace{1cm} (4.21)

$$f_{1,0} = \sum_{r=1}^{\infty} \sqrt{\frac{\pi^{3/2} r!}{\Gamma (r+5/2)}} b_{r,1,0} L_r^{3/2} (x),$$  \hspace{1cm} (4.22)

$$f_{1,1} = -\sum_{r=1}^{\infty} \sqrt{\frac{\pi^{3/2} r!}{\Gamma (r+5/2)}} b_{r,1,1} L_r^{3/2} (x).$$  \hspace{1cm} (4.23)

The tensor moments (all ranks starting from 2) are incorporated in $f_{2,m}, f_{3,m} \ldots$ where $m = -l, -l + 1, \ldots l$.

### 4.3 The Chapman-Enskog - Like Equation

The hydrodynamic equations for the electron density $n$, flow velocity $V$, and temperature $T$ are

$$\frac{\partial n}{\partial t} + \nabla \cdot n V = 0$$  \hspace{1cm} (4.24)

$$mn \left( \frac{\partial V}{\partial t} + V \cdot \nabla V \right) = -en E - \nabla p - \nabla \cdot \Pi + R$$  \hspace{1cm} (4.25)

$$\frac{3}{2} n \left( \frac{\partial T}{\partial t} + V \cdot \nabla T \right) = -p \nabla \cdot V - \Pi : \nabla V - \nabla \cdot q + Q_e$$  \hspace{1cm} (4.26a)
where $E$ is an external electric field, $p$ - electron pressure,

$$p = \frac{m}{3} \int w^2 f(r, t, v) \, d^3v$$

(4.27)

$\Pi$ - stress tensor,

$$\Pi_{\alpha\beta} = m \int \left( w_{\alpha} w_{\beta} - \frac{1}{3} w^2 \delta_{\alpha\beta} \right) f(r, t, v) \, d^3v$$

(4.28)

$R$ - the change of the electron momentum as a consequence of collisions with neutrals,

$$R = m \int v C(f(r, t, v)) \, d^3v$$

(4.29)

$q$ - the heat flux,

$$q = \frac{m}{2} \int w^2 w f(r, t, v) \, d^3v$$

(4.30)

and $Q_e$ - is the collisional heat generation in electrons as a result of collisions with neutrals

$$Q_e = \int \frac{mw^2}{2} C(f(r, t, v)) \, d^3v$$

(4.31)

where $v' = v - V$ is the random electron velocity. These equations represent respectively the conservation of particles, momentum and energy.

Because the total distribution function of electrons in weakly ionized plasmas is decomposed into two parts - a local Maxwellian $f_M$ plus a deviation part $F$,

$$f(r, v, t) = f_M(r, v, t) + F(r, v, t)$$

(4.32)

the latter has to satisfy the following constraints:

$$\int \{1, v, v^2\} F \, d^3v = 0$$

(4.33)

since the first three moments are determined by the local Maxwellian distribution function $f_M$. Substituting Eq. (4.32) into the Boltzmann kinetic equation (1.1) and
using the fluid moment equations (4.24)-(4.26a), one obtains a kinetic equation for the deviation part $F$ - the Chapman-Enskog-like equation

$$\frac{\partial F}{\partial t} + \mathbf{v} \cdot \nabla F - \frac{e}{m} \mathbf{E} \cdot \nabla_\mathbf{v} F = C(f_M + F) - \frac{m}{T} \left( \mathbf{w} \frac{w_2}{3} \right) : \nabla \nabla f_M$$

$$+ \mathbf{w} \cdot \left( \nabla \cdot \Pi - \mathbf{R} \right) \frac{f_M}{p} + \left( \frac{m w^2}{3T} - 1 \right) \left( \Pi : \nabla \mathbf{v} \right) \frac{f_M}{p}$$

$$+ \frac{f_M}{p} \left( \frac{m w^2}{3T} - 1 \right) \left( \nabla \cdot \mathbf{q} - Q_e \right)$$

$$- \left( \frac{m w^2}{2T} - \frac{5}{2} \right) \mathbf{w} \cdot \nabla_T \frac{f_M}{T}$$

(4.34)

Eq. (4.34) is an exact equation. No approximations have been made. In the classical limit, the high collisionality approximation allows one to eliminate all but the first, the second and the last terms on the right of Eq. (4.34). Linearizing Eq. (4.34), we obtain the following version of it for the perturbed distribution function $F$

$$\frac{\partial F}{\partial t} + \mathbf{v} \cdot \nabla F - C(\tilde{f}_M + F) = - \frac{m}{T_0} \left( \mathbf{v} \mathbf{v} - \frac{v^2}{3} \right) : \nabla \nabla f_{M_0} + \mathbf{v} \cdot \left( \nabla \cdot \Pi - \mathbf{R} \right) \frac{f_{M_0}}{p_0}$$

$$+ \left( \frac{2v^2}{3v_T^2} - 1 \right) f_{M_0} \frac{f_{M_0}}{p_0} \nabla \cdot \mathbf{q} - \left( \frac{v^2}{v_T^2} - \frac{5}{2} \right) \mathbf{v} \cdot \nabla_T \frac{f_{M_0}}{T_0}$$

(4.35)

where $v_T^2 \equiv 2T_0/m$, $p_0 = n_0 T_0$, $f_{M_0}$ is the nonshifted lowest-order Maxwellian distribution function and the perturbed Maxwellian is given by

$$\tilde{f}_M = f_{M_0} \left[ \frac{\tilde{n}}{n_0} + \left( \frac{v^2}{v_T^2} - \frac{3}{2} \right) \frac{\tilde{T}}{T_0} + \frac{m}{T_0} \mathbf{v} \cdot \tilde{V} \right].$$

(4.36)

The collisional heat generation term $Q_e$ has been dropped in Eq. (4.35), since [57]

$$Q_e = - \mathbf{R} \cdot \mathbf{V}$$

(4.37)

and it is a nonlinear function of the flow velocity $\mathbf{V}$. A few words should be said about the applicability of the linearization procedure in Eq.(4.35). The linearization means
that we restrict ourselves to the situation when the perturbed density $\tilde{n}$, temperature $\tilde{T}$ and mean velocity $\tilde{V}$ are small in comparison with their equilibrium values. Since the hydrodynamic moments can be expressed in terms of the electric field potential, the linearized equation (4.35) for the distribution function describes the system which is influenced by relatively weak electric field for which the condition $eE\lambda_e << T_e$ should be satisfied.

### 4.4 Solution of the Chapman-Enskog-Like Equation with the Model Collision Operator

Eq. (4.19) in Fourier components reads

$$ F(r, v, t) = \sum_{l,m} Y_{l,m}(\theta, \phi) \int f_{l,m}(k, \omega, v)e^{i(k\cdot r-\omega t)}d^3k d\omega, \quad (4.38) $$

where $v = |v|$, $f_{l,m}(k, \omega, v)$ is a function of the absolute value of electron velocity. For the sake of simplicity we assumed that the electric field $E = E_x(x, t)\hat{x} + E_z(x, t)\hat{z}$, and all the inhomogeneities are in $x$ direction, that is $k = k\hat{x}$, $T = T(x), V = \hat{x}V_x(x) + \hat{z}V_z(x), n = n(x)$. Under the assumption (4.33) the collision term (2.1) is [1]

$$ \tilde{C} \left( f_M + F \right) = -\frac{2\nu_1 \tilde{V}_x v_{fM0}}{u_T^2} \cos \theta - \frac{2\nu_1 \tilde{V}_z v_{fM0}}{u_T^2} \sin \theta \cos \phi - \sum_{l=0,m}^{\infty} \nu_l f_{l,m} Y_{l,m}(\theta, \phi) \quad \text{ (4.39)} $$

where $\nu_l$ is the $l$-th order collision frequency given by Eq.(2.8). The electron-electron collisions are neglected, since the electron density of the considered plasma discharges is much less than that of neutrals (the electron density is approximately 1% of the density of neutrals). The characteristic frequency $\nu_0$ describes the energy relaxation between electrons and neutrals (energy transfer from electrons to neutrals). It is
neglected here due to large mass difference between the electron and neutral atoms. If \( \nu_l = \text{const} \) (the differential cross section is independent of the scattering angles) for all \( l \), than there is no need of expanding the electron distribution function in the series of spherical functions and one can solve the Boltzmann kinetic equation with the collision term given by

\[
C(F) = -\nu(v)F
\]

(4.40)

This collisional term can adequately describe the considered system only in the limit of very small electron energies (in this limit there is a very slight dependence of the differential cross section on both the scattering angle and electron energy) [58].

Substituting Eq. (4.38) into Eq. (4.35) and Fourier transforming the latter, we obtain

\[
-\omega \sum_{l,m} f_{l,m}(k, \omega, v) Y_{l,m}(\theta, \phi) + ikv \sin \theta \cos \phi \sum_{l,m} f_{l,m}(k, \omega, v) Y_{l,m}(\theta, \phi)
\]

\[
= -\sum_{l,m} \nu_l f_{l,m}(k, \omega, v) Y_{l,m}(\theta, \phi) - \frac{2\nu_1 \bar{V}_z v f_{M0}}{v_l^2} \cos \theta - \frac{2\nu_1 \bar{V}_z v f_{M0}}{v_l^2} \sin \theta \cos \phi
\]

\[- \frac{2f_{M0}}{v_l^2} v^2 \sin \theta \cos \theta \cos \phi \left(ik\bar{V}_z\right) - \frac{2f_{M0}}{v_l^2} v^2 \sin^2 \theta \cos^2 \phi \left(ik\bar{V}_z\right)
\]

\[+ \frac{2f_{M0}}{3v_l^2} v^2 \left(ik\bar{V}_z\right) + \frac{v f_{M0}}{p_0} \cos \theta \left(ik\bar{I}_{xx}\right) + \frac{v f_{M0}}{p_0} \sin \theta \cos \phi \left(ik\bar{I}_{xx}\right)
\]

\[- \frac{v R_z f_{M0}}{p_0} \sin \theta \cos \phi - \frac{v R_z f_{M0}}{p_0} \cos \theta + \left(\frac{2v^2}{3v_f^2} - 1\right) \frac{f_{M0}}{p_0} (ik\bar{q}_z)
\]

\[- \left(\frac{v^2}{v_f^2} - \frac{5}{2}\right) \frac{v f_{M0}}{T_0} \sin \theta \cos \phi \left(ik\bar{T}\right).
\]

(4.41)

If the differential cross section is a function of \( \theta \), equation (4.41) leads to coupled equations for different spherical harmonics. Multiplying (4.41) by \( Y_{l,m}^*(\theta, \phi) \) and integrating it over the solid angle we obtain

\[-i\omega f_{l,m} + \frac{1}{2} ikv \Gamma_{l,m} + \sqrt{\frac{4\pi}{3}} \frac{2\nu_1 \bar{V}_z v f_{M0}}{v_l^2} \delta_{l,1,0} + \sqrt{\frac{8\pi}{3}} \frac{\nu_1 \bar{V}_z v f_{M0}}{v_l^2} (\delta_{l,1,\delta_{m,-1}} - \delta_{l,1,\delta_{m,1}})
\]
\begin{align}
\Gamma_{l,m} &= \sqrt{\frac{(l-m+1)(l-m+2)}{(2l+1)(2l+3)}} f_{l+1,m-1} + \sqrt{\frac{(l-m)(l-m-1)}{(2l+1)(2l-1)}} f_{l-1,m+1} \\
&\quad - \sqrt{\frac{(l+m-1)(l+m)}{(2l-1)(2l+1)}} f_{l-1,m-1} - \sqrt{\frac{(l+m+2)(l+m+1)}{(2l+1)(2l+3)}} f_{l+1,m+1}.
\end{align}

The infinite system of equations (4.42) is solved to give the following expressions for the perturbed electron distribution functions $f_{0,0}$ and $f_{1,0}$, and $f_{1,-1}$

\begin{align}
\begin{split}
f_{0,0} &= \sqrt{4\pi} \frac{f_{M0}}{p_0 i k} \left( \frac{1}{H_1(k,\omega,\nu)} - 1 \right) \left( \frac{1}{H_1(k,\omega,\nu)} - 1 \right) \tilde{\Pi}_{xx} \\
&\quad + 4\sqrt{\pi} \frac{f_{M0}(i\omega - \nu_1)}{v_T^2 i k} \left( \frac{1}{H_1(k,\omega,\nu)} - 1 \right) \left( \tilde{V}_x \right) - \frac{4\sqrt{\pi} k v^2 f_{M0}}{3 v_T^2 \omega} \frac{H_1(k,\omega,\nu)}{H_1(k,\omega,\nu)} \\
&\quad - \frac{4\sqrt{\pi} v_1 f_{M0}}{v_T^2 k^2} \left( \frac{1}{H_1(k,\omega,\nu)} - 1 \right) \left( ik \tilde{V}_x \right) + \sqrt{4\pi} \left( \frac{v^2}{v_T^2} - \frac{5}{2} \right) \frac{f_{M0}}{T_0} \\
&\quad \times \left[ \frac{1}{H_1(k,\omega,\nu)} - 1 \right] \tilde{T} - \frac{2\sqrt{4\pi}}{3} \left( \frac{v^2}{v_T^2} - \frac{3}{2} \right) \frac{f_{M0}}{p_0} \left( \frac{k}{\omega H_1(k,\omega,\nu)} \right) \tilde{q}_x
\end{split}
\end{align}

\begin{align}
\begin{split}
f_{1,0} &= \sqrt{\frac{16\pi}{3}} \frac{f_{M0} v}{v_T} \left( \frac{1}{H_2(k,\omega,\nu)} - 1 \right) \tilde{V}_x + \sqrt{\frac{4\pi}{3}} \frac{2\nu_1 v f_{M0}}{v_T^2} \left( \frac{1}{(i\omega - \nu_1) H_2(k,\omega,\nu)} \tilde{V}_x \right)
\end{split}
\end{align}
\[ + \sqrt{\frac{4\pi}{3}} \frac{f_{M0} v}{p_0} \left( \frac{1}{(i\omega - \nu_1)} H_2(k, \omega, v) R_z \right) - \sqrt{\frac{4\pi}{3}} \frac{f_{M0} v}{p_0} \left( \frac{ik}{(i\omega - \nu_1)} H_2(k, \omega, v) \tilde{\Pi}_{zz} \right) \]

(4.45)

\[
f_{1,-1} = \frac{3}{2} \sqrt{\frac{8\pi}{3}} \frac{f_{M0} i\omega}{p_0 k^2 v} \left( \left[ 1 - \frac{1}{H_1(k, \omega, v)} \right] R_z \right) + \frac{3}{2} \sqrt{\frac{8\pi}{3}} \frac{f_{M0} \omega}{k v} \left( \left[ 1 - \frac{1}{H_1(k, \omega, v)} \right] \tilde{\Pi}_{xx} \right) + \frac{3}{2} \sqrt{\frac{8\pi}{3}} \frac{f_{M0} i\omega}{p_0 k^2 v} \left( \left[ 1 - \frac{1}{H_1(k, \omega, v)} \right] \tilde{V}_x \right) - \sqrt{\frac{8\pi}{3}} \frac{f_{M0} v}{p_0} \frac{i\omega}{H_1(k, \omega, v)} \tilde{V}_x + \frac{3}{2} \sqrt{\frac{8\pi}{3}} \frac{3\nu_1 i\omega f_{M0}}{v_k^2 k^2 v} \left( \left[ 1 - \frac{1}{H_1(k, \omega, v)} \right] \tilde{V}_z \right) - \frac{3}{2} \sqrt{\frac{8\pi}{3}} \frac{3 v^2}{v_T^2 - \frac{5}{2}} \frac{f_{M0} \omega}{T_0} \frac{1}{H_1(k, \omega, v)} \tilde{V}_z \times \left( \left[ 1 - \frac{1}{H_1(k, \omega, v)} \right] \tilde{T} \right) + \frac{8\pi}{3} \left( \frac{v^2}{v_T^2} - \frac{3}{2} \right) \frac{f_{M0}}{p_0 v} \left( 1 - \frac{1}{H_1(k, \omega, v)} \right) \tilde{q}_x. \]

(4.46)

The effects of the higher-order spherical harmonics are included in the continued fractions \( H_1 \) and \( H_2 \)

\[ H_1(v, k, \omega) = 1 + C_1/(1 + C_2/1 + C_3/\cdots), \]  

(4.47)

\[ H_2(v, k, \omega) = 1 + D_2/(1 + D_3/1 + D_4/\cdots), \]  

(4.48)

with coefficients

\[ C_l = \frac{l^2 k^2 v^2}{(4l^2 - 1) (i\omega - \nu_l) (i\omega - \nu_{l-1})} \]  

(4.49)

\[ D_l = \frac{(l^2 - 1) k^2 v^2}{(4l^2 - 1) (i\omega - \nu_l) (i\omega - \nu_{l-1})}. \]  

(4.50)

To find an expression for the perturbed electron distribution functions \( f_{0,0}, f_{1,0}, f_{1,-1} \), one has to calculate the continued fractions \( H_1(v, k, \omega) \) and \( H_2(v, k, \omega) \) which in turn
requires the knowledge of the $l$–th order collision frequencies $\nu_l$. If the electron-neutral atom interaction is of the polarization type, then the collision frequency $\nu_l$ rapidly converges to a constant value $\nu_{\infty} (v)$. In this case the continued fractions (4.47) and (4.48) can be approximated by

$$H_1 (\omega, kv, \nu) = 1 + \frac{1}{3} \frac{k^2 v^2}{i \omega (i \omega - \nu_1 (v))} \times \frac{\left( \sqrt{1 + k^2 v^2 / (i \omega - \nu_{\infty} (v))^2} + 1 \right)}{\sqrt{1 + k^2 v^2 / (i \omega - \nu_{\infty} (v))^2} + 1 + 8/15 k^2 v^2 / ((i \omega - \nu_1 (v)) (i \omega - \nu_{\infty} (v)))}$$

(4.51)

$$H_2 (\omega, kv, \nu) = 1 + \frac{1}{5} \frac{k^2 v^2}{(i \omega - \nu_2 (v)) (i \omega - \nu_1 (v))} \times \frac{\left( \sqrt{1 + k^2 v^2 / (i \omega - \nu_{\infty} (v))^2} + 1 \right)}{\sqrt{1 + k^2 v^2 / (i \omega - \nu_{\infty} (v))^2} + 1 + 16/35 k^2 v^2 / ((i \omega - \nu_2 (v)) (i \omega - \nu_{\infty} (v)))}$$

(4.52)

with effective collisional frequencies, given by (2.45).

4.5 The Transport Coefficients of Weakly Ionized Plasmas

Eq. (4.42) has five unknown values ($\bar{\Pi}_{xx}, \bar{\Pi}_{zz}, R_x, R_z, \bar{q}_x$). To find them one has to use the equations of constraints (4.33). Since there are only three constraints imposed on the perturbed part of the distribution function, one has to supplement two additional equations in order to resolve the system. These additional equations are,
the definitions for the electron momentum change \( R_x \) and \( R_z \), so that the perturbed distribution function has to satisfy the following equations

\[
R_z = m \int v_z C(f) \, dv^3 = -\frac{8\pi m V_z}{3v_T^2} \int_0^\infty \nu_1 v^4 f_{M0} dv - \frac{4\pi}{3} m \int_0^\infty \nu_1 v^3 f_{1,0} dv \tag{4.53}
\]

\[
R_x = -\frac{8\pi m V_z}{3v_T^2} \int_0^\infty \nu_1 v^4 f_{M0} dv - \frac{8\pi}{3} m \int_0^\infty \nu_1 v^3 f_{1,-1} dv \tag{4.54}
\]

\[
\int_0^\infty v^2 f_{0,0} dv = \int_0^\infty v^3 f_{1,-1} dv = 0 \tag{4.55}
\]

\[
\int_0^\infty v^3 f_{1,0} dv = 0 \tag{4.56}
\]

\[
\int_0^\infty v^4 f_{0,0} dv = 0 \tag{4.57}
\]

Having obtained expressions for the perturbed electron distribution functions we can readily derive equations to determine the transport coefficients of the system. Substituting expressions (4.44), (4.45) and (4.46) into equations of constraints (4.55), (4.56) and (4.57) and using the definition for the \( z \)th (4.53) and \( x \)th (4.54) components of the electron momentum change \( \mathbf{R} \), one derives the system of equations for \( R_x, R_z, \Pi_{xz}, \Pi_{zx} \) and \( q_x \) solving which gives

\[
R_x = -\zeta \tilde{V}_z - \tau n_0 \left( ik \tilde{T} \right) \tag{4.58}
\]

\[
\tilde{\Pi}_{xz} = -\eta_{xz} \left( ik \tilde{V}_z \right) - \zeta n_0 \left( \tilde{T} \right) \tag{4.59}
\]

\[
\tilde{q}_z = \tau p_0 \tilde{V}_z - \zeta \left( ik \tilde{T} \right) \tag{4.60}
\]

\[
R_z = -\xi \tilde{V}_z \tag{4.61}
\]

\[
\Pi_{xz} = -\eta_{xz} \left( ik \tilde{V}_z \right) \tag{4.62}
\]
As one can see from Eqs. (4.58) and (4.54) the transfer of momentum from neutrals to electrons by collisions \( \mathbf{R} = \mathbf{R}_V + \mathbf{R}_T \) is made up of two parts: the force of friction \( \mathbf{R}_V \) and a thermal force \( \mathbf{R}_T \), which arises by virtues of a gradient in the electron temperature and dependence of the collision frequency on electron velocity.

It is interesting to note that the above equations could be obtained using the definitions for heat flux and viscosity [59] which have the form

\[
\tilde{\Pi}_{xx} = m \int \left( \frac{v_x^2 - v_z^2}{3} \right) F d^3v
\]

\[
\tilde{\Pi}_{xz} = m \int v_x v_z F d^3v
\]

\[
\tilde{q}_x = T \int v_x \left( \frac{v_z^2}{v_T^2} - \frac{5}{2} \right) F d^3v.
\]

Both methods are equivalent in description and give the same results. Eqs. (4.58), (4.59), (4.60), (4.61), (3.27) allows one to define the nonlocal transport coefficients such as the thermal force coefficient \( \tau \), the stress force coefficient \( \varrho \) induced by the temperature gradient, the heat pinch coefficient \( r \), the thermal \( \chi \), electrical \( \sigma_x, \sigma_z \) conductivity coefficients, transverse diffusion \( D \), and viscosity \( \eta_{xx}, \eta_{xz} \) coefficients, given by the following expressions

\[
\tau = \frac{4\pi m i \omega}{p_0 k^2} \left[ \frac{E^{(1)} \left( A^{(3)} B^{(2)} - A^{(2)} B^{(3)} \right) + E^{(2)} \left( A^{(1)} B^{(3)} - A^{(3)} B^{(1)} \right)}{A^{(3)} B^{(1)} - A^{(1)} B^{(3)}} \right]
\]

\[
+ \frac{4\pi m i \omega}{p_0 k^2} \frac{E^{(3)} \left( A^{(2)} B^{(1)} - A^{(1)} B^{(2)} \right)}{A^{(3)} B^{(1)} - A^{(1)} B^{(3)}}
\]

\[
\eta_{xx} = \frac{4\pi m i \omega}{p_0 k^4} A^{(1)} B^{(1)} \left[ \frac{E^{(1)} \left( B^{(3)} / B^{(1)} \Delta - A^{(3)} / A^{(1)} \Upsilon \right) + E^{(3)} \left( \Upsilon - \Delta \right)}{A^{(3)} B^{(1)} - A^{(1)} B^{(3)}} \right] - \frac{\Psi}{k^2}
\]

\[
+ \frac{1}{k^2} \frac{B^{(3)} A^{(1)} \Delta - A^{(3)} B^{(1)} \Upsilon}{A^{(3)} B^{(1)} - A^{(1)} B^{(3)}}
\]

\[
\varrho = \left( -\frac{A^{(3)} B^{(2)} - A^{(2)} B^{(3)}}{A^{(3)} B^{(1)} - A^{(1)} B^{(3)}} + \tau \right)
\]
\[ r = \frac{3i\omega}{2p_0k^2} A^{(1)} B^{(1)} \frac{(\Delta - \Upsilon)}{[A^{(3)} B^{(1)} - A^{(1)} B^{(3)}]} \]  
(4.69)

\[ \chi = \frac{3n i\omega [A^{(2)} B^{(1)} - A^{(1)} B^{(2)}]}{2k^2 [A^{(3)} B^{(1)} - A^{(1)} B^{(3)}]} \]  
(4.70)

\[ \eta_{xx} = nm \frac{D^{(1)}}{k^2 D^{(2)}} \]  
(4.71)

where

\[ \Delta = -i\omega mn + nm \frac{k^2 F^{(2)}}{3\omega A^{(1)}} \]  
(4.72)

\[ \Upsilon = -i\omega mn + nm \frac{k^2 G^{(2)}}{3\omega B^{(1)}} \]  
(4.73)

\[ \Psi = \frac{8\pi m}{3v_T^2} C^{(3)} - \frac{8\pi mw^2}{v_T^2 k^2} E^{(1)} \]  
(4.74)

and the closure coefficients \( A^{(1)} ... G^{(3)} \) are given in the Subsection 4.5.1.

The electric conductivity and diffusion coefficients can be found from Eq. (4.25). The \( z \)th and \( x \)th components of this equation are

\[ mn (-i\omega \tilde{V}_z) = -enE_z - (ik\tilde{\Pi}_{xz}) + R_z \]  
(4.75)

\[ mn (-i\omega \tilde{V}_x) = -enE_x - (ik\tilde{\Pi}_{zx}) - (ik\tilde{\Pi}_{xz}) + R_x. \]  
(4.76)

Substituting expressions (4.61) and (4.62) into Eq. (4.75) we obtain

\[ i_x = -\frac{3n_0\omega}{mik^2} \frac{[A^{(3)} B^{(2)} - A^{(2)} B^{(3)}]}{[B^{(3)} F^{(2)} - A^{(3)} G^{(2)}]} \left( ik\tilde{T} \right) - \frac{3\omega}{mik^2} \frac{[A^{(3)} B^{(1)} - A^{(1)} B^{(3)}]}{[B^{(3)} F^{(2)} - A^{(3)} G^{(2)}]} \left( ik\tilde{p} \right) \]

\[ -\frac{3\omega n_0 e}{mik^2} \frac{[A^{(3)} B^{(1)} - A^{(1)} B^{(3)}]}{[B^{(3)} F^{(2)} - A^{(3)} G^{(2)}]} E_x \]  
(4.77)

\[ i_x = \frac{4\pi e}{3T_0} (D^{(2)} E_x) \]  
(4.78)

where \( i_x \) and \( i_z \) are the electron fluxes due to both, the external electric field, density and temperature inhomogeneities in \( x \) direction. Eqs. (4.77) and (4.78) define the
nonlocal electric conductivity $\sigma_z$ and $\sigma_x$, and transverse diffusion $D$ coefficients of the considered system, given by

$$\sigma_x = \frac{3\omega e^2}{m k^2} \frac{[A^{(3)} B^{(1)} - A^{(1)} B^{(3)}]}{[B^{(3)} F^{(2)} - A^{(3)} G^{(2)}]}$$  \hspace{1cm} (4.79)

$$\sigma_z = -\frac{4\pi e^2}{3 T_0} D^{(2)}$$  \hspace{1cm} (4.80)

$$D = \frac{3 T_0 \omega}{m k^2} \frac{[A^{(3)} B^{(1)} - A^{(1)} B^{(3)}]}{[B^{(3)} F^{(2)} - A^{(3)} G^{(2)}]}.$$  \hspace{1cm} (4.81)

Converted back to space-time variables the transport coefficients become the integro-differential operators acting on temperature, density, flow velocity and external electric field. Expression (4.80) for the nonlocal transverse electrical conductivity coincides with the one found in [58].

It is necessary to stress that there exists a simple relation between the heat pinch coefficient $r$, thermal force coefficient $t$ and stress force coefficient $\varrho$ given by

$$r = t - \varrho$$  \hspace{1cm} (4.82)

The existence of Eq. (4.82) is a direct consequence of the Onsager symmetry principle. What is interesting to note is that Eq. (4.82) holds true not only for strongly collisional case, but for any values of the nonlocality parameter $k v_T/(i \omega - \nu)$[32].

When nonlocality is taken into account, the transverse transport coefficients do not coincide with the longitudinal coefficients ($\sigma_z \neq \sigma_x$) and ($\eta_{zz} \neq \eta_{xx}$). The electron fluxes become anisotropic. As it will be shown in the next section the anisotropy vanishes in the hydrodynamic limit. The nonlocal, collisional heat generation in electrons is

$$Q = -\mathbf{R} \cdot \mathbf{V} = -R_z V_z - R_x V_x$$  \hspace{1cm} (4.83)

The heat generated as a result of viscosity is

$$Q_{vis} = -\Pi_{\alpha \beta} \frac{\partial V_\alpha}{\partial x_\beta} = -\Pi_{zz} \frac{\partial V_z}{\partial x} - \Pi_{xx} \frac{\partial V_x}{\partial x}$$  \hspace{1cm} (4.84)
where

\[ R_x (x, t) = \frac{-1}{(2\pi)^2} \int_0^\infty \zeta e^{-i(k_x - \omega t)} \chi (k, \omega) \, dk \, d\omega - \frac{n}{(2\pi)^2} \int_0^\infty \tau e^{-i(k_x - \omega t)} (i k T (k, \omega)) \, dk \, d\omega \]  

\[ R_z (x, t) = \frac{-1}{(2\pi)^2} \int_0^\infty \xi e^{-i(k_z - \omega t)} \chi (k, \omega) \, dk \, d\omega \]  

\[ \Pi_{xx} (x, t) = \frac{-1}{(2\pi)^2} \int_0^\infty \eta e^{-i(k_x - \omega t)} (i k \chi (k, \omega)) \, dk \, d\omega \]  

\[ \Pi_{xx} (x, t) = \frac{-1}{(2\pi)^2} \int_0^\infty \eta e^{-i(k_x - \omega t)} (i k \chi (k, \omega)) \, dk \, d\omega - \frac{n}{(2\pi)^2} \int_0^\infty \xi e^{-i(k_z - \omega t)} (i k \tau (k, \omega)) \, dk \, d\omega \]  

where

\[ \zeta = -\Psi - \frac{4\pi m i \omega}{p_0 k^2} \left[ E^{(1)} \left( A^{(3)} B^{(1)} \chi - B^{(3)} A^{(1)} \Delta \right) + E^{(3)} \left( \Delta - \chi \right) \right] \]  

\[ \xi = n m \frac{C^{(1)}}{D^{(2)}} \]  

As one can see there are several mechanisms of electron heating in a system. First, is the collisional mechanism, which is dominant in a collisional operating regimes (local), second is the compressional/viscous mechanism which is dominant in collisionless regimes (nonlocal) when the kinetic effects are important (wave-particle interaction).

In general case, one has to include both cases in order to properly describe the electron heating in weakly ionized plasmas. The obtained transport coefficients can be used to solve the hydrodynamic equations for density, velocity and energy, to obtain the electron density and temperature profiles for the nonlocal case.
4.5.1 The Closure Relations for Calculations of the Transport Coefficients

The closure coefficients for calculations of the transport coefficients are:

\[ A^{(1)} = \int_0^\infty v^2 \left( \frac{1}{H_1(\omega, kv, \nu)} - 1 \right) f_{M0} dv, \]  
(4.91)

\[ A^{(2)} = \int_0^\infty v^2 \left( \frac{v^2}{v_T^2} - \frac{5}{2} \right) \left( \frac{1}{H_1(\omega, kv, \nu)} - 1 \right) f_{M0} dv, \]  
(4.92)

\[ A^{(3)} = \int_0^\infty v^2 \left( \frac{v^2}{v_T^2} - \frac{3}{2} \right) \frac{f_{M0}}{H_1(\omega, kv, \nu)} dv, \]  
(4.93)

\[ B^{(1)} = \int_0^\infty v^4 \left( \frac{1}{H_1(\omega, kv, \nu)} - 1 \right) f_{M0} dv, \]  
(4.94)

\[ B^{(2)} = \int_0^\infty v^4 \left( \frac{v^2}{v_T^2} - \frac{5}{2} \right) \left( \frac{1}{H_1(\omega, kv, \nu)} - 1 \right) f_{M0} dv, \]  
(4.95)

\[ B^{(3)} = \int_0^\infty v^4 \left( \frac{v^2}{v_T^2} - \frac{3}{2} \right) \frac{f_{M0}}{H_1(\omega, kv, \nu)} dv, \]  
(4.96)

\[ E^{(1)} = \int_0^\infty \nu_1 v^2 \left( \frac{1}{H_1(\omega, kv, \nu)} - 1 \right) f_{M0} dv, \]  
(4.97)

\[ E^{(2)} = \int_0^\infty \nu_1 v^2 \left( \frac{v^2}{v_T^2} - \frac{5}{2} \right) \left( \frac{1}{H_1(\omega, kv, \nu)} - 1 \right) f_{M0} dv, \]  
(4.98)

\[ E^{(3)} = \int_0^\infty \nu_1 v^2 \left( \frac{v^2}{v_T^2} - \frac{3}{2} \right) \left( \frac{1}{H_1(\omega, kv, \nu)} - 1 \right) f_{M0} dv, \]  
(4.99)

\[ C^{(1)} = \int_0^\infty \frac{\nu_1 v^4 f_{M0}}{(i\omega - \nu_1) H_2(\omega, kv, \nu)} dv, \]  
(4.100)
\[ D^{(1)} = \int_0^\infty v^4 \left( \frac{1}{H_2(\omega, k\nu, \nu)} - 1 \right) f_{M_0} dv, \quad (4.101) \]

\[ D^{(2)} = \int_0^\infty \frac{v^4 f_{M_0}}{(i\omega - \nu_1) H_2(\omega, k\nu, \nu)} dv, \quad (4.102) \]

\[ F^{(1)} = \int_0^\infty v^2 (i\omega - \nu_1) \left( \frac{1}{H_1(\omega, k\nu, \nu)} - 1 \right) f_{M_0} dv, \quad (4.103) \]

\[ F^{(2)} = \int_0^\infty \frac{v^4}{H_1(\omega, k\nu, \nu)} f_{M_0} dv, \quad (4.104) \]

\[ G^{(1)} = \int_0^\infty v^4 (i\omega - \nu_1) \left( \frac{1}{H_1(\omega, k\nu, \nu)} - 1 \right) f_{M_0} dv, \quad (4.105) \]

\[ G^{(2)} = \int_0^\infty \frac{v^6}{H_1(\omega, k\nu, \nu)} f_{M_0} dv, \quad (4.106) \]

\[ G^{(3)} = \int_0^\infty v^4 \nu_1 \left( \frac{1}{H_1(\omega, k\nu, \nu)} - 1 \right) f_{M_0} dv \quad (4.107) \]

### 4.6 The Transport Coefficients in the Fluid Limit

**Case \((\nu \gg \omega \gg k\nu_T)\)**

The transport coefficients obtained in the previous section are complicated functions of the electron temperature \(T\), frequency \(\omega\) and wave number \(k\). But one can easily obtain the hydrodynamic limiting case by assuming that \(\nu \gg \omega\), \(\nu \gg k\nu_T\). In this case the closure coefficients \(A^{(1)} - G^{(3)}\) are

\[ A^{(1)} = \frac{k^2}{3i\omega} \int_0^\infty \frac{v^4 f_{M_0}}{\nu_1} dv, \quad A^{(2)} = \frac{k^2}{3i\omega} \int_0^\infty \frac{v^4 (\frac{\nu^2}{\nu_1} - \frac{\nu}{\nu_1}) f_{M_0}}{\nu_1} dv, \quad A^{(3)} = 0. \quad (4.108) \]
\[ B^{(1)} = \frac{k^2}{3i\omega} \int_0^\infty \frac{v^6 f_{M0}}{\nu_1} dv, \quad B^{(2)} = \frac{k^2}{3i\omega} \int_0^\infty \frac{v^6 \left( \frac{v^2}{\nu_1^2} - \frac{5}{2} \right) f_{M0}}{\nu_1} dv, \quad B^{(3)} = \frac{3p_0}{4\pi m}. \]  
\[ E^{(1)} = \frac{p_0 k^2}{4\pi m i\omega}, \quad E^{(2)} = 0, \quad E^{(3)} = \frac{p_0 k^2}{4\pi m i\omega}, \]  
\[ C^{(1)} = -\frac{3}{4\pi m}, \quad D^{(1)} = -\frac{k^2}{5} \int_0^\infty \frac{v^6 f_{M0}}{\nu_1 \nu_2} dv, \quad D^{(2)} = -\int_0^\infty \frac{v^4 f_{M0}}{\nu_1} dv, \]  
\[ F^{(1)} = -\frac{p_0 k^2}{4\pi m i\omega}, \quad F^{(2)} = \frac{3p_0}{4\pi m}, \quad G^{(1)} = -\frac{15nk^2 v^4}{48\pi i\omega}, \]  
\[ G^{(2)} = \frac{15nv^4}{16\pi}, \quad G^{(3)} = \frac{15nk^2 v^4}{48\pi i\omega}. \]  

Using above expressions, the local thermal force coefficient is
\[ \tau_{lc} = \frac{A^{(2)}}{A^{(1)}} = \left[ \int_0^\infty \frac{v^4 \left( \frac{v^2}{\nu_1^2} - \frac{5}{2} \right) f_{M0}}{\nu_1} dv \right] \div \left[ \int_0^\infty \frac{v^4 f_{M0}}{\nu_1} dv \right]. \]  

The heat pinch coefficient \( r \) is
\[ r_{lc} = \frac{3i\omega}{2p_0 k^2} \frac{B^{(1)} (\Delta - \Upsilon)}{B^{(3)}} = \left[ \int_0^\infty \frac{v^4 \left( \frac{v^2}{\nu_1^2} - \frac{5}{2} \right) f_{M0}}{\nu_1} dv \right] \div \left[ \int_0^\infty \frac{v^4 f_{M0}}{\nu_1} dv \right]. \]  

The stress force coefficient is
\[ \theta_{lc} = 0. \]  

The coefficient of thermal conductivity is
\[ \chi_{lc} = \frac{2\pi m}{3T_0} \left\{ \int_0^\infty \frac{v^6 \left( \frac{v^2}{\nu_1^2} - \frac{5}{2} \right) f_{M0}}{\nu_1} dv - \tau \left( \int_0^\infty \frac{v^6 f_{M0}}{\nu_1} dv \right) \right\}. \]  

The stress tensor in the collisional limit has a form
\[ \Pi_{i,j} = \eta \left( V_{i,j} - \frac{2}{3} \delta_{i,j} \frac{\partial V_k}{\partial x_k} \right). \]
Figure 4.1: The local thermal force coefficient as a function of electron energy
Figure 4.2: The local thermal conductivity coefficient as a function of electron energy. The neutral atom density is $N = 10^{15} \text{ cm}^{-3}$, the electron density is $n_e = 10^{12} \text{ cm}^{-3}$. 
Figure 4.3: The local viscosity coefficient as a function of electron energy. The neutral atom density is $N = 10^{15} \text{ cm}^{-3}$, the electron density is $n_e = 10^{12} \text{ cm}^{-3}$
Figure 4.4: The normalized local electric conductivity coefficient as a function of electron energy.
Figure 4.5: The local transverse diffusion coefficient as a function of electron energy. The neutral atom density is $N = 10^{15} \text{ cm}^{-3}$. 
where

\[ V_{i,j} = \frac{\partial V_i}{\partial x_i} + \frac{\partial V_i}{\partial x_j} \]  (4.119)

with viscosity coefficient given by

\[ \eta_{lc} = \frac{nm D^{(1)}}{k^2 D^{(2)}} = \frac{nm}{5} \left[ \int_0^\infty \frac{v^6 f_{M0}}{\nu_1 \nu_2} dv \right] / \left[ \int_0^\infty \frac{v^4 f_{M0}}{\nu_1} dv \right]. \]  (4.120)

The local electric conductivity coefficients are

\[ \sigma_x = \sigma_z = \frac{4\pi e^2}{3T_0} \int_0^\infty \frac{v^4 f_{M0}}{\nu_1} dv = \sigma_{lc}. \]  (4.121)

The transverse diffusion coefficient is

\[ D_{lc} = \frac{4\pi}{3n_0} \int_0^\infty \frac{v^4 f_{M0}}{\nu_1} dv. \]  (4.122)

The local collisional and viscous heat generation in electrons are

\[ Q = -R_z V_z - R_x V_x = \frac{J_z^2 + J_x^2}{\sigma_0} + \tau n \frac{\partial T}{\partial x} V_x \]  (4.123)

\[ Q_{vis} = -\Pi_{xz} \frac{\partial V_z}{\partial x} - \Pi_{xx} \frac{\partial V_x}{\partial x} = \eta \left[ \left( \frac{\partial V_z}{\partial x} \right)^2 + \frac{4}{3} \left( \frac{\partial V_x}{\partial x} \right)^2 \right] \]  (4.124)

where \( J_z \) and \( J_x \) are the electron current densities in the \( x \) and \( z \) directions. The behavior of the local transport coefficients greatly depends on the collision frequency. Figs. (4.1), (4.2), (4.3), (4.4), (4.5) show the local thermal force, thermal conductivity, viscosity, electric conductivity, diffusion coefficients of Argon as the functions of the electron temperature. As one can see the nonmonotonous behavior of the transport cross section with electron energy (the Ramsauer effect) in Argon gas modifies the thermal force coefficient, that is if the collision frequency were the simple power function of the electron velocity then the thermal force coefficient would not depend on the electron energy as it is for an electron-ion plasma. However, in the present case
not only does it depend on the energy, but it can also be positive (the thermal force is in the opposite direction to the temperature gradient) or negative (the thermal force is codirectional with the temperature gradient) depending on the value of electron temperature.

In the case when \( \nu \) is independent of electron velocity, the local transport coefficients can be calculated analytically to give the known results [61]

\[
\tau = 0, \quad \tau = 0, \quad \chi = \frac{5nT}{2m\nu}, \quad \eta = \frac{nT}{\nu}, \quad \sigma = \frac{ne^2}{m\nu}, \quad D = \frac{v_T^2}{2\nu}.
\]  

(4.125)

As was mentioned earlier, the above coefficients can be used to describe the transport processes in a weakly ionized plasma only in a case of very low energies of the electrons (low electron temperature) and high density of neutrals, so that the condition \( \nu \gg \omega \gg k\nu_T \) still holds.

### 4.7 Nonlocal Limit Case \((\nu, \omega \ll k\nu_T \text{ and } \omega \gg k\nu_T, \nu)\)

To calculate the transport coefficients \((4.66)-(2.53), (4.80)-(2.38)\) in both cases we have to obtain the asymptotic form for the continued fractions \(H_1\) and \(H_2\). In these cases it is possible to obtain the exact asymptotics for the continued fractions \(H_1\) and \(H_2\). First, taking the limit \(\nu, \omega \ll k\nu_T\) in the expressions for the exact continued fractions \((4.47)\) and \((4.48)\) we have

\[
H_1 = -\frac{2k\nu}{\pi i\omega},
\]  

(4.126)

\[
H_2 = -\frac{4}{3\pi i\omega - \nu_1}. \tag{4.127}
\]

The closure and transport coefficients in this limit are

\[
A^{(1)} = -\frac{n}{4\pi}, \quad A^{(2)} = \frac{n}{4\pi}, \quad A^{(3)} = \frac{i\omega m}{8\sqrt{\pi k\nu_T}}, \tag{4.128}
\]
\[ B^{(1)} = -\frac{3}{4\pi m}, \quad B^{(2)} = \frac{i \omega n v_T^2}{8\sqrt{\pi} kv_T}, \quad B^{(3)} = -\frac{i \omega n v_T^2}{8\sqrt{\pi} kv_T}, \]  
(4.129)

\[ E^{(1)} = -\int \nu_1 v^2 f_{M0} dv, \quad E^{(2)} = -\int \nu_1 v^2 \left( \frac{v^2}{v_T^2} - \frac{5}{2} \right) f_{M0} dv \]  
(4.130)

\[ E^{(3)} = -\int_0^\infty \nu_1 v^2 \left( \frac{v^2}{v_T^2} - \frac{3}{2} \right) f_{M0} dv, \]  
(4.131)

\[ C^{(1)} = -\frac{3\pi}{4k} \int_0^\infty \nu_1 v^3 f_{M0} dv, \quad D^{(1)} = -\frac{3}{4\pi \frac{n T_0}{m}}, \quad D^{(2)} = -\frac{3}{8\sqrt{\pi}} \frac{n v_T}{k}, \]  
(4.132)

\[ F^{(1)} = -\int_0^\infty v^2 (i \omega - \nu_1) f_{M0} dv, \quad F^{(2)} = -\frac{i \omega n v_T}{4\sqrt{\pi} k}, \quad G^{(1)} = -\int_0^\infty v^4 (i \omega - \nu_1) f_{M0} dv, \]  
(4.133)

\[ G^{(2)} = -\frac{i \omega n v_T^3}{2\sqrt{\pi} k}, \quad G^{(3)} = -\int_0^\infty v^4 \nu_1 f_{M0} dv, \]  
(4.134)

\[ \tau = -\frac{48\sqrt{\pi}}{5n_0 kv_T} \int_0^\infty \nu_1 v^2 \left( \frac{v^2}{v_T^2} - \frac{3}{2} \right) f_{M0} dv \approx 0, \quad q = \frac{2}{5}, \quad r = -\frac{2}{5}, \quad \chi = \frac{9}{5\sqrt{\pi} kv_T}, \]  
(4.135)

\[ \eta_{xx} = \frac{2\sqrt{\pi} m n v_T^2}{5kv_T}, \quad \eta_{zz} = \frac{mn v_T^2}{\sqrt{\pi} kv_T}, \quad \sigma_x = \frac{7ne^2}{2\sqrt{\pi} m^2 kv_T}, \quad \sigma_z = \frac{ne^2}{m kv_T}, \quad D = \frac{7}{4\sqrt{\pi} kv_T}. \]  
(4.135)

The calculated transport coefficients coincide with the ones found by [59, 32] in the adiabatic limit case for the electron-ion plasma. The coincidence of the transport coefficients in the adiabatic limit for electron-neutral and electron-ion plasma is not surprising, since this limit can be considered to be collisionless and the main mechanism of the transport processes in both systems is the wave-particle interaction (Landau damping). It is worth noting that in the present case, the higher moments (viscosity, heat flux) as well as the diffusive electron flux are proportional to the perturbed lower moments (temperature, density, mean velocity) (in the local case
the higher moments are proportional to the gradients of the corresponding lower moments).

In the case when \( \omega \gg k v_T, \nu_e \), the continued fractions \( H_1 \) and \( H_2 \) are given by

\[
H_1 = 1 - \frac{1}{3} \frac{k^2 v^2}{\omega^2},
\]

\[
H_2 = 1 - \frac{1}{5} \frac{k^2 v^2}{\omega^2},
\]

and the transport coefficients are

\[
\tau = \frac{4\pi m}{3 p_0 i \omega} \int v_1 v^4 \left( \frac{v^2}{v_T^2} - \frac{5}{2} \right) f_{M0} dv = 0, \quad \rho = \frac{4\pi m}{3 p_0 i \omega} \int v_1 v^4 \left( \frac{v^2}{v_T^2} - \frac{5}{2} \right) f_{M0} dv = 0,
\]

\[
\begin{align*}
\tau &= \frac{-5 k^2 v_T^2}{6 \omega^2} = 0, \quad \eta_{xx} = \eta_{zz} = \frac{-5 m n v_T^2}{6 i \omega}, \quad \chi = \frac{-5 m n v_T^2}{4 i \omega}, \quad \sigma_x = \sigma_z = \frac{-n e^2}{m i \omega},
\end{align*}
\]

\[
D = -\frac{v_T^2}{2 i \omega}
\]

(4.138)

As we can see, in this limiting case the transport coefficients are purely imaginary, so that there is no dissipative (irreversible) processes in a system, which is not an unexpected result since there are no collisions \( \omega >> \nu \) as well as there is no wave-particle interaction \( \omega/k >> v_T \).

4.8 Summary

In this chapter, we have considered nonlocal electron transport processes in weakly ionized plasmas when the main mechanism of electron scattering is the electron-neutral atom interaction. The approach that was used to find the transport coefficients of the considered system is a moment procedure, similar to the Chapman-Enskog expansion method, which allowed us to determine the higher moments (stress
tensor $\Pi$, heat flux vector $\mathbf{q}$) in terms of the lower moments (density $n$, temperature $T$ and mean velocity $\mathbf{V}$) and consequently to obtain the transport coefficients. Because the standard Chapman-Enskog method is essentially the expansion of the total distribution function in a small parameter $\varepsilon$ which represents the scale of the mean free path, it can not be used to describe the system in the collisionless case, since an increase in the value of $\varepsilon$ leads to the inclusion of the larger number of terms in the expansion (the number of terms should be such that the remainder of the expansion must be small in $\varepsilon$, so that the series is asymptotically convergent) and in the collisionless limit one would have to solve the system of an infinite number of equations, which would be a formidable task for the Chapman-Enskog expansion method. The modification to the Chapman-Enskog procedure developed in this chapter is rather mathematical which lies in the expansion of the total distribution function in the complete set of the eigenfunctions of the collision operator, which allowed us to solve the obtained infinite system of equations for the expansion coefficients. The obtained total distribution function is represented by the infinite continued fraction (which is a function of $\omega$ and $k$) and functional representation for the continued fraction was found. Transport coefficients are the complicated functions of the electron temperature $T$, wave number $k$ and frequency $\omega$ in a Fourier space. Converted back to the space-time variables they become the integro-differential operators acting on the lower moments.

In the limit $k\nu_T \ll \nu_e$, $\omega \ll \nu_e$ we obtain the local transport coefficients which are the nonmonotonous functions of the electron energy and as in the case of the thermal force coefficient it can be codirectional or antiparallel to the temperature gradient which is a direct consequence of rather complicated dependence of the differential cross section of electron-neutral atom interaction on electron energy. Such behavior of the
thermal force coefficient drastically modifies the total heat flux in the system which under the certain circumstances, investigated in the present thesis can flow toward the plasma center. In the opposite case, when $k\nu_T \gg \nu_e$ the transport processes are due to the wave-particle interaction (Landau damping mechanism) and unlike the local case where the higher moments (heat flux, viscosity) are expressed in terms of the gradients of the corresponding lower hydrodynamic moments (temperature, density, mean velocity) it is shown here that in the adiabatic case ($\nu, \omega \ll k\nu_T$) the higher moments are proportional to the perturbed lower moments. For practical purposes one can approximate expressions for the transport coefficients by Pade approximants and subsequently solve the hydrodynamic equations for density, mean velocity and energy to find the profiles for electron density and temperature.
Chapter 5

Conclusions

The modeling of gas discharge plasmas plays an important role in understanding the basic physical properties of the plasma and aids plasma source design. One of the main difficulties in self-consistent modeling consists in the enormous difference between the characteristic space, time, and energy scales governing the plasma particle kinetics and profiles of the fields. In spite of considerable progress in the past years, the gap between desirable aims and the present abilities of models is still wide, and the development of effective approaches to this problem is an important task. The treatment of electron and ion kinetics accounting for spatial and temporal nonlocality is frequently the most laborious part of self-consistent models.

In the present thesis we considered the nonlocal electron and ion kinetics in weakly ionized plasmas. For electron component, we presented a rather general method of solving the Boltzmann equation, which is based on the expansion of the total distribution function (DF) in the series of the collision operator eigenfunctions. The coefficients in this expansion are related to the velocity moments of the distribution function. The obtained infinite system of equations for the expansion coefficients was
solved in terms of the continued fraction representation and transport coefficients of a weakly ionized plasma were found. Inclusion of all terms in the expansion is necessary for the proper closure relation between the higher and lower moments of the distribution function. Any truncation in the system will introduce errors in the resulting total distribution function, since the expansion of the DF in terms of the collision operator eigenfunctions is equivalent to its expansion in the series of a parameter $\varepsilon$ which is a measure of spatial and temporal uniformity ($\varepsilon = k\lambda; \varepsilon = \omega/\nu$). As this parameter increases (system becomes more rarefied or the characteristic length/time scales of the system's spatial/temporal inhomogeneities become smaller), the larger number of terms should be included in the expansion procedure and in the limiting collisionless case all harmonics (all moments) must be included. Our method effectively takes into account an infinite series of moments and the obtained transport coefficients describe the relaxation processes for the arbitrary uniformity parameter $\varepsilon$. In this case the transport coefficients become integro-differential operators acting on the lower moments (density, temperature, mean velocity, and external fields). The inclusion of the thermal electron motion drastically modifies the transport properties of the system. As an example we considered the anomalous absorption of the electromagnetic wave by a weakly ionized plasma or anomalous skin effect. Unlike the classical skin effect in which the electric field is monotonously (exponentially) decaying inside of the conductive medium (plasma, etc.), in the anomalous case, the nonmonotonous decay occurs and there are regions where the absolute value of the electric field can increase. This effect is a direct consequence of the influence of the thermal electron motion on the electric conductivity coefficient and can not be recovered from the fluid regime (small $\varepsilon$). Other transport coefficients are modified as well, and in order to obtain the density and temperature profiles in the system for the most general uniformity case
one will have to solve equations for density, mean velocity and temperature with the proper closure for heat flux vector, and stress tensor, which are expressed through the found transport coefficients.

Ion kinetics in a weakly ionized plasma placed in strong time and space dependent electric field was considered in Chapter 3. Because of the different interaction potential as well as comparable masses between ions and neutrals, the ion distribution function significantly differs from that of the electrons. In a strong time and space independent electric field for which the condition $qE_0 \lambda > T$ applies, where $q$ is the ion charge, $\lambda$ is the ion mean free path, $T$ is the temperature of the neutral gas, the ion distribution function is anisotropic (in velocity space) and strongly nonequilibrium with uniform ion density and ion flow. In this case the ion transport is determined in terms of ion mobility and ion drift velocity $W_0$. We considered the ion flux due to the imposed perturbation of the electric field which varies in space and time, and derived the expression for ion transport fluxes for arbitrary values of the parameters $\omega/\nu$ and $\lambda/L$, where $\omega$ is the frequency of the external electric field, $\nu$ is the collision frequency, $\lambda$ is the ion mean free path and $L$ is the characteristic length scale of the system's inhomogeneity. In this case the fluxes are described by integro-differential operators acting on the electric field. We showed that the perturbed ion flux and density exhibit resonant behavior when $\omega = kW_0$ which is the resonance between the average ion velocity and wave phase velocity. These results can be used in the modeling of ion transport in a plasma sheath region of the capacitive RF discharges, where there exists a situation when the constant electric field in a sheath region is superimposed by time and space varying electric field created by the external source. The ion density, that has been obtained in Chapter 3 can be used to solve the Poisson's equation to find the potential in a sheath region.
Bibliography


