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UMI
NUMERICAL ANALYSIS OF THE MECHANICAL
BEHAVIOR OF COLLAPSING EARTH DAMS
DURING FIRST RESERVOIR FILLING

A Thesis
Submitted to the Faculty of Graduate Studies and Research
in partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy
in the
Department of Civil Engineering
University of Saskatchewan
Saskatoon

by
Jose Henrique Feitosa Pereira

Spring 1996

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UNIVERSITY OF SASKATCHEWAN
College of Graduate Studies and Research

SUMMARY OF DISSERTATION
Submitted in partial fulfillment
of the requirements for the

DEGREE OF DOCTOR OF PHILOSOPHY
by
Jose Henrique Feitosa Pereira

Department of Civil Engineering
University of Saskatchewan
Spring 1996

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Numerical Analysis of the Mechanical Behavior of Collapsing Earth Dams during First Reservoir Filling

The primary objective of this dissertation is the development of practical procedures to better understand the mechanical behavior of small collapsing dams during the first reservoir filling. To achieve this objective, the present research study was conducted from both experimental and theoretical bases.

The influence of the wetting-induced collapse on the mechanical and hydraulic properties for the residual soil compacted as a metastable-structured material was experimentally investigated. Volume changes and the water coefficient of permeability were investigated using the triaxial permeameter system where the stress state variables were independently controlled. As-compacte residual soil specimens were consolidated isotropically after which measurements of total volume change, water content change, and coefficient of permeability were made at specified matric suctions following a wetting stress path. Shear strength was investigated using the modified shear box equipment. As-compacte residual soil specimens were consolidated and sheared at specified stress states. The experimental data were analyzed to define constitutive relationships for the metastable-structured soil.

The procedure developed to simulate the mechanical behavior of a metastable soil following a wetting stress path couples stress equilibrium and water flow using a generalized form of the theory of consolidation for unsaturated soils. The modified Mohr-Coulomb failure criterion is utilized to define the failure conditions in soil elements within the dam. The model takes account of the varying permeability of the collapsing soil when following a wetting stress path. Finite element analyses were found to reproduce analytical/numerical solutions for the consolidation of both saturated and unsaturated soils. Finite element analyses were also performed to study the post-filling performance of small dams. These analyses indicate realistic predictions in comparison to field experience and allow a better understanding of the failure mechanisms of small collapsing dams in Northeast Brazil.
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ABSTRACT

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CHAPTER 1

INTRODUCTION

1.1 General

The prediction of the stability and performance of earth dams in the phases of construction, first reservoir filling and long-term operation, requires a knowledge of the mechanical and hydraulic properties of the component materials in the embankment. The performance of an earth embankment depends upon its response to applied loadings and imposed hydraulic gradients. The associated numerical analyses are more complex in cases where the soils which comprise the embankment undergo significant changes in mechanical and/or hydraulic properties as a result of a change in pore-water pressure. Increases in the water content of the soil are associated with water flow through the earth dam and these increases may occur either during the first reservoir filling of the embankment or during a rainy season. Earth dams may go towards failure as a result of changes in the mechanical behavior of the soil during saturation.

There are two soil types which undergo significant volume changes during saturation. These soil types are:

a.) Collapsible soil: a soil which undergoes a sudden reduction of its volume during the saturation of its voids.

b.) Swelling soil: a soil which undergoes an increase of its volume during saturation.

A collapsible soil is commonly referred to as a metastable-structured soil. An increase in pore-water pressure results in swelling for a stable-structured soil, whereas an increase in pore-water pressure may cause a volume decrease for a metastable-structured soil (Barden et al. 1969). The research literature is in agreement that collapse is a behavior that
any unsaturated soil may undergo under particular conditions of stress and saturation. (Jaky, 1948; Barden, 1965; Dakshamurthy, 1979; Maswoswe, 1985; Steadman, 1987; Tadepalli, 1990; Lawton et al. 1991a). In turn, swelling is a behavior associated with mineralogy and chemical composition of the clay fraction of the soil (Mitchell, 1976). This research program deals with the collapse behavior of compacted soils.

According to previous authors (Barden et al. 1973; Mitchell, 1976), four factors are needed in order to produce the collapse in a soil:

i.) An open, partially unstable, unsaturated fabric,

ii.) A high enough net total stress that will cause the structure to be metastable,

iii.) A bonding or cementing agent that stabilizes the soil in the unsaturated condition,

iv.) The addition of water to the soil which causes the bonding or cementing agent to be reduced and the interaggregate or intergranular contacts to fail in shear, resulting in a reduction in total volume of the soil mass.

The research literature is in agreement that in compacted cohesionless soils the metastable bonding is typically provided by capillary suction, although some post-compaction cementing can also occur. In compacted, cohesive soils a more complex model is proposed. According to this complex model, the collapse behavior of compacted and cohesive soils depends on several factors: the percentage of fines (especially clay fraction), the initial water content, the initial dry density, and the energy and process used in compaction (Jennings and Burland, 1962; Barden et al. 1973; Booth, 1977; Hodek and Lovell, 1979). The influence of each factor on soil collapse is later discussed in this report.

In volumetric terms, a soil is considered as a three-phase material composed of a soil skeleton with voids filled by gas (usually air) and a liquid (usually water). Terzaghi’s (1936) theory provides the idealization of a two-phase material for the saturated condition, wherein water flows, and the soil skeleton comes to equilibrium under imposed loads. The unsaturated condition is established when air is present in the soil voids. Current practice in
geotechnique recognizes an unsaturated soil as a four-phase material composed of air, water, soil skeleton and contractile skin (Fredlund and Morgenstern, 1977). Under this idealization, two phases can flow (i.e., air and water) and two phases come to equilibrium under imposed loads (i.e., soil skeleton and contractile skin).

The principles governing the flow of water through soils are well established. Liquid water flows in proportion to the water head gradients (Croney and Coleman, 1961). Temperature effects and vapor phase transfer can be neglected for problems analyzed under isothermal conditions. Darcy’s law has been used to describe the water flow through soils in both the saturated and unsaturated conditions (Fredlund et al. 1993). The soil permeability with respect to the water is significantly affected by changes in the void ratio and changes in the water content of a soil (Brooks and Corey, 1964; Lambe et al. 1979; Mualem, 1986). The variation of the soil permeability with void ratio for saturated conditions is well established (Lambe et al. 1979). For unsaturated soils, water flows through the pore spaces filled with water; therefore the degree of saturation is an important factor. The available data suggests a rapid decrease of the soil coefficient of permeability as the degree of saturation decreases (Gardner, 1961, Brooks and Corey, 1964). Huang (1994) summarized these findings as follows:

"The soil permeability of a deformable, unsaturated soil should be a function of void ratio, which is a measure of void space in a soil, and the degree of saturation, which is the decimal fraction ratio of the water-filled space to the entire void space".

The establishment of the soil permeability as a function of stress state variables governing the soil behavior is another reasonable alternative for engineering practice.

The principles governing mechanical behavior of a saturated soil are well established with respect to both theoretical and practical aspects (Terzaghi, 1943; Lambe et al. 1979). In Terzaghi’s (1936) theory, the mechanical behavior of a saturated soil is governed by the principle of effective stress. Initial attempts to extend such a theory to unsaturated soils had limited success (Bishop et al. 1963). In recent years, a more sound
theory has been established by Fredlund and Morgenstern (1976, 1977). The theories for unsaturated soils have been experimentally investigated to accommodate the key aspects involved with their mechanical behaviors. These theories are consistent with a multiphase, continuum mechanics approach and describe the mechanical behavior of an unsaturated soil as a function of two independent sets of stress variables. These are: the net normal stress ($\sigma - u_a$) and the matric suction ($u_a - u_w$) where $\sigma$ is the total normal stress, $u_a$ is the pore-air pressure and $u_w$ is the pore-water pressure. In these theories, the saturated condition is a special case where the “effective stress” becomes the governing stress state variable. The collapsing behavior of soils during saturation is one of the complex aspects to be developed through application of the unsaturated soil theory. The prediction of the post-filling performance of small collapsing earth dams is one of the engineering problems depending upon these developments.

The mechanical and hydraulic behaviors of a compacted soil are strongly dependent upon the volume-mass soil properties, namely void ratio ($e$), gravimetric water content ($w$), and degree of saturation ($S$). The unsaturated soil mechanics theory provides sound relationships between changes in volume-mass properties and changes in the stress state of a compacted soil (Fredlund and Morgenstern, 1977). Such relationships can be developed by means of laboratory tests simulating specific field conditions of loading and/or deformation. The analysis and interpretation of laboratory test results are the best means of checking, adjusting and/or extending the theory.

The primary objective of this research is to develop a better understanding of the mechanical behavior of small dams (height smaller than 10 meters) constructed using collapsing soils (i.e., collapsing dams). The condition of primary importance is the first reservoir filling. The behavior to be modeled must take into account the changes in both the net normal stress ($\sigma - u_a$) and matric suction ($u_a - u_w$) within the earth dam caused by the transient unsaturated-saturated water flow which follows the first reservoir impounding.
1.2 Collapsing earth dams during first reservoir filling

Before the first reservoir filling, the stress state in an earthfill dam is a function of the energy of compaction, the water content, the geometry of the dam, and the type of material used to construct the embankment. The energy of compaction, the water content during compaction, and the material used for construction define the initial dry density of the soil. The initial dry density, the water content, and the geometry of the dam allow for the computation of the total stresses within the embankment in accordance with elasticity theory. The initial pore-air pressure of a small dam can be assumed as equal to atmospheric pressure (Sandroni, 1985; Miranda, 1988; Alonso et al. 1985a). The pore-water pressure can be estimated either from the soil water content versus pore-water pressure relationship, or from direct measurements using as-compacted soil specimens.

During the first reservoir filling, earth dams suffer deformations caused by changes in the stress state within the embankment (Wilson and Marsal, 1979). Large deformations (i.e., volume reductions due to soil saturation) occur rapidly in earth dams constructed at a water content far below the optimum moisture and with low energy of compaction (Peterson and Iverson, 1953; Sherard, 1963). Such deformations were primarily attributed to the combined effects of increases in the soil self weight and soil compressibility as the water content increases. These effects can cause the total collapse of an earth dam. The failure is usually associated with mechanisms of hydraulic fracturing and/or progressive rupture of the embankment slopes. The mechanisms involved are illustrated in Figures 1.1 and 1.2. It is reasonable to conceive that failure of dams can occur as a combination of both factors illustrated.

Immediately after the first reservoir impounding, water flows through the dam in a transient manner in accordance with the driving hydraulic gradient, the fluid characteristics, the existing boundary conditions, and the hydraulic properties of the soil. As the water flows,
the soil in the dam undergoes volume changes in response to changes in total stress and matric suction. Volume changes imply changes in both the mechanical and hydraulic properties of the materials within the dam. In addition, volume changes can generate pore-water pressures and alter the transient flow regime within the dam embankment. A further advance of the water flow into the dam results in a new configuration of mechanical and hydraulic properties. This dynamic process puts in action a complex process in terms of both mechanical behavior and water flow. This is even true for an earth structure which is homogeneous. The dynamic process is transient and occurs until the establishment of steady-state conditions. In cases where significant volume changes occur within the embankment, the dam can reach failure conditions (see Figures 1.1 and 1.2) even before the establishment of steady-state water flow. Therefore, the solution to the problem requires a mathematical model coupling mechanical equilibrium and water flow in collapsing dams in order that geotechnical engineers can analyze this kind of problem. Exact solutions do not exist and numerical methods must be developed.

The Finite Element Method (FEM) is a powerful numerical method, which has successfully been used to solve a variety of engineering problems. The method can easily handle irregular geometry, varying properties, and different types of boundary conditions. The Finite Element Method was initially developed for solving problems involving the elasticity theory for homogeneous, isotropic and elastic media. Currently, it is being used to analyze the mechanical behavior of heterogeneous structures even when the component materials are anisotropic and inelastic. Such generalizations are possible by using convenient combinations of incremental techniques and solutions for elasticity theory. The understanding of the physical phenomena involved in a problem is of paramount importance to the development of a numerical model using the Finite Element Method.
Studies have been attempted to explain the collapse of dams during first reservoir filling as a result of changes in the "effective stress" and in the stress-strain relationships of the material within the dam (Nobari and Duncan, 1972; Young et al. 1982; Lourens and Czapla, 1987). Recently, studies have attempted to explain these phenomena by using the two independent stress state variables and a continuum mechanics approach for unsaturated soil (Miranda, 1988; Menescal, 1991; Lloret and Ledesna, 1993). However, these studies have experienced difficulties in reproducing available experimental results in terms of the stress-strain behavior of collapsing soils during saturation. This is particularly true for the condition of collapse of the soil structure under Ko-conditions. Under these conditions, a collapsing soil undergoes an increase in mean net confining stress (Maswoswe, 1985; Lawton
et al. 1991a). Therefore, there exists a need for a better understanding of the mechanical behavior of collapsing soils in view of current theories for unsaturated soils. An auxiliary objective of the present research program is to develop a theoretical understanding of the behavior of a collapsing soil during saturation by using the theory of consolidation for unsaturated soils presented by Fredlund and Rahardjo (1993).

Collapse potential is a particularly significant consideration in earth dam design because of the certainty that post-construction wetting will occur. The documentations in the literature confirms that wetting-induced collapse has caused several earth dams to fail (Peterson et al. 1953; Leonards et al. 1963; Leonards et al. 1984; Miranda, 1988).

Miranda (1988) reported that failures resulting from cracks initiated by wetting-induced collapse in earth dams located in the semi-arid region of Northeast Brazil are so common that these dams are publicly referred to as “Alka-Seltzer” dams. These dams are constructed by using standard design procedures based on the experience of otherwise successful contractors. “Alka-Seltzer” dams are normally constructed with residual soil derived from gneiss. Due to local conditions, these dams often are constructed without the necessary amount of water being added and with inadequate compaction (i.e., poorly compacted). It is worth noting that dams built with the same soil at an acceptable water content and with a sufficiently high compaction energy (i.e., well compacted) perform well (i.e., stable dams). Based on field experience and using a simplified, uncoupled analysis, Miranda (1988) modeled the failure of “Alka-Seltzer” dams during first reservoir filling as a consequence of hydraulic fracturing and piping through wetting-induced cracks (see Fig. 1.1). However, such an idealization does not correspond to available field observations which associate the failure of “Alka-Seltzer” dams to both mechanisms illustrated in Figures 1.1 and 1.2. Therefore, there is a need for a better understanding of the failure mechanism of “Alka-Seltzer” dams to allow the adequate design of such collapsing earth dams. The different behavior of these small dams when well and poorly constructed is used to further evaluate the coupled numerical model proposed in this research.
1.3 Objectives and scope of this study

The primary purpose of this research is to investigate the post-filling behavior of small collapsing dams as constructed in Northeast Brazil. This study has the following objectives.

1.) To derive a mathematical model to handle the prediction of the mechanical behavior of a metastable soil during saturation. Such a model couples mechanical equilibrium and water flow in soils and is based on the theory for consolidation of unsaturated soils.

2.) To develop a numerical procedure, by using the finite element method, for solving the coupled stress and flow equations subject to boundary conditions common to a small earth dam during the transient unsaturated-saturated seepage that follows the first reservoir filling.

3.) To experimentally define the mechanical behavior and hydraulic properties of a residual soil of gneiss compacted at two conditions which reflect the actual field situation in Northeast Brazil.

4.) To achieve a better understanding of the behavior of an “Alka-Seltzer” dam during the transient unsaturated-saturated seepage that follows its first reservoir impounding. Such an understanding is achieved by applying the numerical model proposed in this research study to the analysis of the post-filling performance of a typical “Alka-Seltzer” dam.

1.4 Thesis organization

There are eight chapters in this thesis. This first chapter briefly introduces the problem of collapsing dams during the first reservoir filling. It describes the coupling of mechanical equilibrium and water flow required for the solution of the problem in analysis. Chapter 1 also emphasizes the need for a theoretical understanding of the mechanical
behavior of collapsing soils during saturation. The scope and objectives of the present research study are also outlined.

The second chapter deals with the pertinent literature related to the problem in analysis. Emphasis is given to the study of collapsivity in compacted soils. The mechanical behavior and hydraulic properties of collapsing soils are discussed based on available experimental and theoretical findings. Analysis of experimental data in light of the current theory allows a better theoretical understanding of the behavior of collapsing soils during saturation. The hydraulic properties of collapsing soils are also discussed under the point of view that such a soil is a deformable porous medium during saturation. This chapter also presents a review of earthfill collapse, approaching the problem from a phenomenological point of view. A brief review of seepage in unsaturated soils and post-filling behavior of collapsing earth dams is also included. Finally, a short review of numerical procedures involved in coupled solutions is performed.

The theoretical developments of the coupled solution for collapsing dams are presented in the third chapter. The coupled finite element model is developed using Galerkin's weighted residual method. The numerical model is developed for the plane-strain condition which is assumed to exist in a small earth dam. The formulations are presented and discussed under theoretical and practical points of view.

The fourth chapter outlines the details of the experimental program planned to define the mechanical and hydraulic properties of the residual soil compacted at two conditions (i.e., as a stable-structured soil and as a metastable-structured soil). This experimental program is based upon available data and preliminary laboratory testing results related to small dams constructed in Northeast Brazil. The equipments to be utilized are described and the relevant procedures for conducting the experiments are also outlined.

The fifth chapter presents the results obtained from the experimental program. Results related to both mechanical and hydraulic properties measurements are presented. This chapter also presents discussions and analyses performed to define the constitutive models
for the residual soil of gneiss compacted at two different conditions. Detailed results are documented in the appendices.

In the sixth chapter, the numerical model is applied to solutions of several geotechnical problems involving both saturated and unsaturated soil. Such solutions are utilized to demonstrate the model’s ability to handle coupled solutions under saturated and unsaturated conditions. The ability of the model to handle collapsing behavior of unsaturated soils is also demonstrated. Finally, the numerical model is applied to the analysis of the post-filling performance of a small earth dam constructed in Northeast Brazil.

In the seventh chapter, the numerical model is applied to the analysis of collapsing earth dams during first reservoir filling. This chapter demonstrates how the experience obtained with failures of small dams constructed in Northeast Brazil may be used to further evaluate the model. It is demonstrated that the use of the model provides a better understanding of the failure mechanisms of collapsing dams in Northeast Brazil.

The eighth chapter summarizes the conclusions of the present research study. The recommendations for further studies are also provided.
CHAPTER 2

LITERATURE REVIEW

The stability of an earth dam depends on the mechanical behavior of the component soils. The mechanical behavior of an unsaturated soil can change significantly by changing its pore-pressure conditions. The influence of the pore-water on the soil behavior is often explained in terms of the degree of saturation in engineering practice. The degree of saturation may be altered by a imposed water flow, and the rate of change of the degree of saturation is a function of the hydraulic properties (i.e., coefficient of permeability) of the soil. The relative movement between the pore-water and the soil grains is also strongly influenced by the coefficient of permeability of the soil matrix. This movement of water, if not properly controlled, can contribute significantly to the destabilization of a soil structure.

The prediction of the stability of an earth dam requires a knowledge of the mechanical behavior and the hydraulic properties of the component soils. Based on concepts of continuum mechanics, the mechanical behavior and flow properties of soils can be conveniently expressed in terms of the stress state variables. The soil behavior can be described by means of constitutive relationships linking deformation state variables to stress state variables. These concepts allow for a unified analysis of the soil in terms of its unsaturated or saturated conditions. Terzaghi's principle of effective stress can be seen as a special case of the stress state variables controlling the behavior of soils in general. The flow properties can be either directly related to the stress state variables or, alternatively, to the volume-mass soil properties.

This bibliographic summary has as its main objective, a review of the performance of a collapsing earth dam during its first reservoir filling. This phase is characterized by the
development of the transient saturated-unsaturated water flow regime up to the steady-state condition.

The performance of a dam during first reservoir filling requires that the behavior of the component soils during the transient condition going towards saturation be predicted. This behavior can be described by using a theory capable of describing the behavior of the soil in both the unsaturated and saturated conditions. The theory should also provide a smooth transition between these two conditions. This chapter includes a review of the following subjects:

a.) the characterization of the unsaturated soil in terms of its mechanical behavior and hydraulic properties as functions of its degree of saturation;

b.) a unified theory encompassing both saturated and unsaturated soils. This theory is first presented for a saturated soil and later generalized for an unsaturated soil. Emphasis is given to the formulation of equations describing the mechanical and hydraulic properties of the soil;

c.) the collapsing soil is described as a special behavior that can be experienced by any unsaturated soil depending upon the loading condition. The collapse mechanism is illustrated as a soil collapse behavior, described in a phenomenological sense by using the continuum mechanics approach proposed by Fredlund and Morgenstern (1976, 1977). Emphasis is given to compacted collapsing soils as required for the present research;

d.) previous studies are used to show the effects of soil collapse on both the mechanical and the hydraulic behaviors of collapsing earth dams.

The stability of earth dams is the main focus of this literature review. Numerical modelling currently being used to analyze transient saturated-unsaturated processes in porous medium are also discussed. The emphasis is given to the use of the finite element method.
2.1 Importance of a theory embracing unsaturated soil mechanics

Soil mechanics for saturated soils has been applied successfully to the prediction of the mechanical behavior of saturated soils for many decades. It has also been applied to explain the mechanical behavior of coarse materials in a dry condition. For these extremes conditions, the soil can be considered as a two phase material where a solid porous structure has its voids completely filled with either water (i.e., saturated condition) or air (i.e., dry condition). The pore-fluid can flow in response to an applied gradient. Changes in volume or shear strength of the porous medium are a result of changes in the stress state of the soil. The pore-fluid flow is controlled by the coefficient of permeability of the porous medium as well as by the existent boundary conditions.

The mechanical behavior of soils in their unsaturated condition, where only a fraction of the pore-voids is filled by water, can not be satisfactorily described by using classical soil mechanics principles for saturated soils. The need for a theory embracing unsaturated soils is well justified since a large portion of the earth's surface (i.e., approximately 33 %) is considered arid and semi-arid (Fredlund and Rahardjo, 1993). A specific example emphasizing this need has been cited by Jones and Holtz (1973) where it is mentioned that in the United States about 2.3 billions of dollars are spent on construction related to expansive soils. In terms of collapsible soils, Lawton et al. (1991a) report that there are a substantial number of lawsuits, especially in California, within the past decade related to damage resulting from the collapse settlements of compacted fills. In Brazil, even without accurate statistics about the collapsing soils problem, it has been reported that there is considerable costs associated with this problem each year (Miranda, 1988; Mendonca, 1991).

The large expenditure is mainly due to the large occurrence of both residual and sedimentary tropical soils and the large number of small compacted embankments.
2.1.1 Influence of the degree of saturation on the behavior of soil

The degree of saturation forms a primary reference to the definition of unsaturated soil conditions.

Bear (1979) classifies the soil in terms of the degree of saturation as follows:

a.) "pendular saturation"; the state characterized by a very low degree of saturation. The water is retained in menisci formed around the grain contact points. These menisci do not form a continuous water phase. The air phase is continuous. Figure 2.1a illustrates this water state;

b.) "funicular saturation"; the state characterized by the coexistence of continuous air and water phases. Figure 2.1b illustrates this saturation condition;

c.) "insular air (or occluded) saturation"; the state where the pore air loses its continuity and some parts become trapped in the water as air bubbles. Figure 2.1c illustrates this state.

Figure 2.1 Possible water saturation states (after Bear, 1979).
Alonso and Lloret (1985), based on previous studies and theoretical considerations, suggested the existence of three different types of behavior for analyzing the compressibility of an unsaturated soil under undrained conditions. The following classification system was used.

a.) High degree of saturation ($S > 90\%$): the air-water mixture is more rigid than the soil skeleton. In this condition, any external loading provokes the same increment of pressure in both the air and water phases. This increment of pressure in both phases is equal to the external loading applied. It can be used for the formulation of equations for the compressibility of the air-water mixture. Increasing and instantaneous deformations can be expected with decreasing degrees of saturation.

b.) Degree of saturation between 90\% and 70\%: the compressibility of the mixture is similar to that of the soil skeleton. The compressibility of the unsaturated soil now involves the structural behavior of the soil skeleton.

c.) Degree of saturation less than 70\%: the air-water mixture is highly compressible and the compressibility of the soil depend on the structural behavior of its skeleton.

Khogo et al. (1993), based on Bear’s (1979) research, report that soil has various pore sizes. If the suction exceeds the air entry value, $S_e$, and air enters the pores, the pore-fluid will empty from the larger pores. The smaller pores remain saturated until a higher suction is reached. Therefore, in soils the "pendular", "funicular", and "insular" conditions coexist except for suction lower than the air entry value and at extremely high suctions (i.e., the extreme conditions of saturation).

In general terms, the above mechanistic models emphasize the importance of the air phase in the soil voids. The following summary can be made.

a.) When the air phase is continuous in the soil structure, the fluid flow depends on the air permeability through the voids. Therefore, the soil compression is rapid. The matric suction, $(\mu_a - \mu_w)$, reaches high values and the water phase firmly adheres to the soil skeleton. This fact has considerable influence on the mechanical behavior of the soil. An
increase on the degree of saturation produces a decrease in matric suction which is dependent upon the soil structure. This process may yield in an abrupt collapse of the soil structure.

b.) When the air phase is in the form of occluded bubbles, the soil permeability with respect to the water controls the fluid flow through the voids. The matric suction has an insignificant influence and the water phase can flow, as in the consolidation of a saturated soil. In fine soils (e.g., clays and silts) the presence of occluded air bubbles can produce local differences in compressibility. Even small amounts of occluded air bubbles can make the pore air-water mixture more compressible.

2.1.2 The principle of effective stress

Terzaghi, (1943) presented the most important concept of soil mechanics for saturated soils, the effective stress principle:

"The stresses in any point of a section through a mass of soil can be computed from the total principal stresses $\sigma_1, \sigma_2, \sigma_3$ which act at this point. If the voids of the soil are filled with water under a stress, $u_w$, the total principal stresses consist of two parts. One part, $u_w$, acts in the water and in the solid in every direction with equal intensity. The balance $\sigma'_1 = \sigma_1 - u_w, \sigma'_2 = \sigma_2 - u_w, \text{ and } \sigma'_3 = \sigma_3 - u_w$ represents an excess over the neutral stress, $u_w$, and it has its seat exclusively in the solid phase of the soil. All the measurable effects of a change in stress, such as compression, distortion and a change in shearing resistance are exclusively due to changes in the effective stress $\sigma'_1, \sigma'_2, \text{and} \sigma'_3$. (Fredlund and Morgenstern, 1977).

Terzaghi related the volume change and the shear strength of a saturated soil to the stress state by using only one equation independent of the soil parameters. Terzaghi's proposal was later confirmed by experiments conducted by Rendulic (1936), Taylor (1944), Bishop and Eldin (1950).
Lambe and Whitman (1959) conducted an analysis of the effective stresses principle and concluded that from a theoretical viewpoint it is valid for coarse soils (e.g., sand or gravel). However, they mention the necessity for more research for fine soils, mainly clayey soils due to:

a.) unknown contact areas;
b.) adhesion between particles;
c.) doubts in the meaning of the term "pore-pressure".

Skempton (1961) suggested the use of two independent effective equations for calculating the volume change and shear stresses in saturated porous soils:

i.) for shear strength:

\[ \sigma' = \sigma - (1 - a \frac{tan \psi}{tan \phi'}) \mu_w \]  \hspace{1cm} [2.1]

and

ii.) for volume changes:

\[ \sigma' = \sigma - (1 - a \frac{C_s}{C}) \mu_w \]  \hspace{1cm} [2.2]

where:

\[ a \] = contact area between particles per unit of transverse area;
\[ \psi \] = intrinsic friction angle of the soil particles;
\[ \phi' \] = internal friction angle of the soil;
\[ C_s \] = compressibility of the soil particles;
\[ C \] = compressibility of the soil structure.

Skempton's (1961) equations resulted in a major conceptual modification to Terzaghi's equations because of the inclusion of soil parameters. This fact transforms Terzaghi's original equations into constitutive relationships. In practical terms, the results
will be the same if both the contact area, \( a \), and the relationship, \( C_s/C \), have insignificant values.

Bishop and Blight (1963), based on these additional contributions, presented another version to the effective stress principle: "The effective stress is by definition the total stress and pore pressure function that controls the mechanical effects of stresses changing, as volume change and shear strength, of a soil. The effective stress principle may be used to attest that such function exist, with defined parameters, under a set of specific conditions".

In summary it was concluded that even under the evidence that the effective stress principle does not appear to explain phenomena such as secondary consolidation and thixotropy, it may be considered to be the proper concept for studying the behavior of saturated soils.

### 2.2 Saturated soil

The prediction of the mechanical behavior and hydraulic properties of a saturated soil is well established in both theory and practice. The soil is considered a two phase continuum, where soil particles form a structure whose voids are filled with water. The soil particles and water phase are essentially incompressible in engineering practice.

Under the above idealization the continuity requirement for a saturated soil, deforming under an applied stress gradient, can be written as,

\[
\frac{\Delta V_v}{V_o} = \frac{\Delta V_w}{V_o}
\]  

[2.3]

where:

\( V_o \) = initial overall volume of the saturated soil element

\( V_v \) = volume of soil voids

\( V_w \) = volume of water
\[ \Delta V_v = \text{total volume change of the soil structure} \]
\[ \Delta V_w = \text{volume change of the water phase in the referential soil element}. \]

The above relationships shows that only the volume changes associated with one phase must be measured, while the other can be computed. In practice the total volume change of the soil structure is always monitored. This volumetric change can be expressed by using a x-, y-, z- Cartesian referential system.

\[ \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\Delta V_v}{V_0} \]  \hspace{1cm} [2.4]

where:
\[ \varepsilon_v = \text{is the volumetric strain, or deformation variable,} \]
\[ \varepsilon_x, \varepsilon_y, \text{ and } \varepsilon_z = \text{the normal strains associated with the x-, y-, and z-axis, respectively.} \]

2.2.1 Mechanical behavior

The mechanical behavior of a saturated soil, in terms of both volume changes and shear strength, can be predicted using Terzaghi's effective stress variable.

2.2.1.1 Volume change behavior

Based on the previous continuity requirement, it is well established in soil mechanics that volume changes in a saturated soil are primarily the result of water flowing in or out of the soil voids. Constitutive relationships for volume change behavior have been established between effective stress and soil deformation as required in engineering practice.
Constitutive relationships may be formulated in terms of moduli or compressibility. These forms are familiar in soil mechanics and are often used in the solution of soil mechanics problems. These forms can be used in either a discrete or incremental mode.

**Moduli form**

A semi-empirical approach can be used to present the moduli form of constitutive equations for a saturated soil (Fredlund and Rahardjo, 1993). This approach involves several assumptions which are based on experimental evidences from observing the behavior of many materials (Chou and Pagano, 1967). The constitutive relationships for the saturated soil structure can be formulated in accordance with a generalized Hooke's law by using the effective stress variable, \((\sigma - u_w)\). For an isotropic and linearly elastic soil structure, the constitutive relations for normal strains in the \(x\), \(y\), and \(z\)-directions have the following incremental form:

\[
de_x = \frac{d(\sigma_x - u_w)}{E} - \frac{\mu}{E} d(\sigma_y + \sigma_z - 2u_w) \tag{2.5}
\]

\[
de_y = \frac{d(\sigma_y - u_w)}{E} - \frac{\mu}{E} d(\sigma_x + \sigma_z - 2u_w) \tag{2.6}
\]

\[
de_z = \frac{d(\sigma_z - u_w)}{E} - \frac{\mu}{E} d(\sigma_x + \sigma_y - 2u_w) \tag{2.7}
\]

where:

- \(\sigma_x\) = total normal stress in the \(x\)-direction
- \(\sigma_y\) = total normal stress in the \(y\)-direction
- \(\sigma_z\) = total normal stress in the \(z\)-direction
- \(E\) = modulus of elasticity or Young's modulus for the soil structure
- \(\mu\) = Poisson's ratio.
The constitutive equations for incremental shear deformations are,

\[ d\gamma_{xy} = \frac{d\tau_{xy}}{G} \]  \hspace{1cm} [2.8]

\[ d\gamma_{xz} = \frac{d\tau_{xz}}{G} \]  \hspace{1cm} [2.9]

\[ d\gamma_{zx} = \frac{d\tau_{zx}}{G} \]  \hspace{1cm} [2.10]

where:

\( \tau_{xy} \) = shear stress on the x-plane in the y-direction (i.e., \( \tau_{xy} = \tau_{yx} \))

\( \tau_{yz} \) = shear stress on the y-plane in the z-direction (i.e., \( \tau_{yz} = \tau_{zy} \))

\( \tau_{zx} \) = shear stress on the z-plane in the x-direction (i.e., \( \tau_{zx} = \tau_{xz} \))

\( G = E/(2(1+\mu)) \) is the shear modulus.

The elasticity modulus (\( E \)), and the Poisson ratio (\( \mu \)) in the above equations are defined with respect to a change in the effective stress, \( (\sigma - u) \). The above constitutive equations can also be applied to situations where the stress versus strain curves are nonlinear. Incremental procedures using small stress/strain are commonly used in engineering to apply the above linear and elastic relationships to more general nonlinear cases.

**Compressibility form**

In this form, the volumetric changes, for a three-dimensional loading, are related to effective stress changes in incremental form as follows,

\[ d\varepsilon_v = m_v d(\sigma_{mean} - u) \]  \hspace{1cm} [2.11]

where:

\( d\varepsilon_v = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z \)
$$m_v = 3 \frac{(1 - 2\mu)}{E},$$ is the coefficient of volume change for saturated soil.

$$\sigma_{mean} = \text{average total stress } ((\sigma_x + \sigma_y + \sigma_z)/3).$$

### 2.2.1.2 Shear strength

The shear strength of a saturated soil can be described by combining the Mohr-Coulomb failure criterion and the Terzaghi's effective stress principle as follows,

$$\tau_f = c' + (\sigma_f - u_w) \tan \phi'$$  \hspace{1cm} (2.12)

where:

- $$\tau_f$$ = shear stress on the failure plane at failure
- $$c'$$ = effective cohesion
- $$(\sigma_f - u_w)$$ = normal effective stress on the failure plane at failure
- $$\sigma_f$$ = total normal stress on the failure plane at failure
- $$u_w$$ = pore-water pressure at failure
- $$\phi'$$ = effective angle of internal friction.

Equation 2.12 defines a straight line on a 2-dimensional plot. This line is commonly referred to as the Mohr-Coulomb failure envelope. This envelope represents possible combinations of shear stress and effective normal stress on the failure plane at failure. The use of effective stresses with the Mohr-Coulomb failure criterion has proven to be satisfactory in engineering practice associated with saturated soils.

The failure envelope can be modified to accommodate non-linearity associated to changes in soil structure during shearing, (e.g., the hyperbolic model proposed by Duncan and Chang, 1970). Non-linear strength behavior can be introduced through the soil-
parameters, c' and \( \phi' \). These parameters can be expressed as functions of the effective stress on the failure plane.

### 2.2.2 Water flow and hydraulic properties

The flow of water in saturated soil is commonly described using Darcy's law.

\[
v_w = -k_w \frac{\partial h}{\partial y}
\]  \[2.13\]

where:

- \( v_w \) = macroscopic water discharge velocity
- \( k_w \) = coefficient of permeability with respect to the water phase
- \( \partial h/\partial y \) = hydraulic head gradient in the y-direction, which can be designated as \( i_{wy} \).

The saturated coefficient of permeability is relatively constant for a stable soil. Equation 2.13 can also be written for the x- and z-directions. The negative sign on the equation indicates that water flows in the direction of a decreasing hydraulic head.

The coefficient of permeability with respect to the water phase, \( k_w \), is a measure of the space available for water to flow through the soil. The coefficient of permeability depends on the fluid and the properties of the porous medium.

There are relationships in the literature between the soil coefficient of permeability and the void ratio for a saturated soil, (e.g., Terzaghi and Peck, 1948; Lambe and Whitman, 1979). These two referenced works are widely used in engineering practice. Terzaghi and Peck (1948) presented an equation which was suggested by Casagrande. The saturated coefficient of permeability is expressed in terms of void ratio as:
\[ k_s = 1.4e^2 k_{0.85} \]  \[2.14\]

where:

\[ k_s = \text{saturated coefficient of permeability.} \]
\[ k_{0.85} = \text{saturated coefficient of permeability at a void ratio of 0.85.} \]

The above equation, formulated based on the experimental data of clean medium and fine sands, is often used in dimensioning drainage systems for earth structures.

In a more general approach, Lambe and Whitman (1979) published numerous experimental results from a variety of soils. Based upon these data, it was concluded that the saturated coefficient of permeability, \( k_s \), expressed in a logarithmic scale versus the void ratio, \( e \), generally appears close to a straight line for most soils. The saturated coefficient of permeability, \( k_s \), versus \([e^3/(1+e)]\) relationship also indicates essentially a straight line form.

### 2.3 Unsaturated Soil

The study of unsaturated soils was not adequately considered before 1960 by researchers. The 1960’s register publications of research results by Bishop and co-workers in the Imperial College (Bishop, 1960; Bishop et al, 1960; and Bishop and Donald, 1961). Excellent research was also conducted by Matyas and Radhakrishna (1968). These works provided significant advances in an understanding of the fundamentals of a general soil mechanics which would embrace both saturated and unsaturated soils.

During 1970’s, except for the works of Fredlund in Canada (Fredlund, 1976; Fredlund and Morgenstern, 1976, 1977; and Fredlund, 1979), it appears that there was little substantial advances in the area of unsaturated soils theory.

During the 1980’s and 1990’s, studies in the area of unsaturated soils theory have come from several parts of the world and it must be emphasized that, at the present time, well established research groups in Canada, England, France, India, Spain, Singapore and United States, are widely recognized. In South America, until few years ago, foundations on
expansive soils was the main subject of research in Brazil and Argentina (Santos Neto, 1990). Currently, design and construction of irrigation projects in arid and semi-arid regions have required that special attention be given to solution of geotechnical problems involving both swelling and collapsing soils.

### 2.3.1 Mechanical behavior

Unsaturated soils may either collapse or swell as they go towards saturation. The collapsing behavior of soils has been used as an indication of the limitation of a generalized single effective stress principle embracing both saturated and unsaturated soils. From a continuum mechanics point of view, the unsaturated soil requires the definition of an appropriate set of stress state variables governing deformation. Following is a brief review of the development of a general theory for the analysis of the mechanical behavior of unsaturated soils. Analogous to soil mechanics for saturated soils, there has been an attempt to develop the “principle of effective stress” which would be valid for unsaturated soil.

#### 2.3.1.1 Principle of effective stress for unsaturated soil

Bishop (1959) attempted to extend the effective stress principle for unsaturated soils. Bishop modified Terzaghi's equation by including the pore-air pressure and a soil property as follows:

\[
\sigma' = \sigma - u_a + \chi (u_a - u_w)
\]  \[2.15\]

where:

\[
\chi = \text{Bishop's parameter}
\]

\[
u_a = \text{air pore-pressure}.
\]
Attempts to justify anomalous experimental soil behavior led Bishop (1960) to affirm that:

"The $\chi$ value is equal to 1 for saturated soils and zero for dry soils. Intermediary values of $\chi$ depend upon the degree of saturation, but they will be influenced by factors as: soil structure, drying or wetting cycles, and stress history".

Experimental tests (Bishop et al 1960; Donald, 1960; Bishop and Donald, 1960) seemed to confirm this trend.

Jennings and Burland (1962) were the first to question Bishop's equation. The results from tests under both unsaturated and saturated conditions were compared for the same external loading. It was concluded that there was not a unique relationship between void ratio and effective stress, as defined by Bishop's equation, for almost all the tests conducted on unsaturated soils below a critical degree of saturation. Their conclusion was mainly based on experimental results where specimens suffered a reduction in void ratio (i.e., collapse) when saturated in oedometric tests. Such results were not consistent with the before mentioned effective stress principle.

Coleman (1962) suggested the use of "reduced" stress variables, $(\sigma_1 - u_a)$, $(\sigma_3 - u_a)$, and $(u_a - u_w)$, to represent the axial, confining, and pore-water pressures, respectively, in triaxial tests.

Bishop and Blight (1963) admitted that Bishop's equation could be used more accurately for shear strength behavior than for volume changes. This was justified by the fact that the shear strength is primarily dependent on interparticle forces, while the volume change is mainly dependent upon the stress path. It was concluded that volume changes would be better related with the stress variables $(\sigma - u_a)$ and $(u_a - u_w)$.

Burland (1965) added additional criticism to Bishop's equation by stating that it was not completely correct to compose, $(\sigma - u_a)$ and $(u_a - u_w)$, into a single effective stress from the microscopic view point. This insistence was later ruled of little value since effective
stress must be essentially defined from a continuum mechanic viewpoint (Fredlund and Morgenstern, 1976; Atkinson and Bransby, 1978), and not from the microscopic viewpoint.

Matyas and Radhakrishna (1968) stated that an equation for effective stresses should:

a.) satisfy the extreme dry and saturated conditions of a soil;

b.) describe the mechanical behavior, volume changes and shear strength, of a soil due to changes on the imposed stress state should be predictable in terms of effective stresses, irrespective of the manner in which the total stress and pore-pressure were changed;

c.) and that corrections of such an equation should be verified experimentally.

In their work they define state parameters as the sufficient physical variables that can be used to completely describe the state or condition of a soil element, without the necessity of referring to its history of stresses. The state of a soil element may be graphically represented as a point in a system of coordinate axis representing the state parameters. This point is called a state point and its displacement, when the element state changes, is called the state path. All the possible state paths would form the state surface of the soil. It was proposed that changes in the void ratio and degree of saturation of an unsaturated soil must be expressed as functions of the stress variables, \((\sigma - u_d)\) and \((u_a - u_w)\), forming constitutive and three-dimensional surfaces similar to those presented by Bishop and Blight (1963). Figure 2.2 shows the state surfaces related with the void ratio and degree of saturation from experimental tests performed by Matyas and Radhakrishna (1968).

Different stress paths are illustrated from a state point A in Figure 2.2. These paths are obtained by changing the degree of saturation and/or the stress variable \((\sigma - u_a)\). It can be seen that the stress path ABB' represents the observation from Jennings and Burland (1962) that there is a reduction in void ratio during the saturation of the soil.

Matyas and Radhakrishna (1968) faced difficulties in explaining some results where soil specimens showed expansive behavior associated with a reduction in the degree of
saturation. In order to guarantee uniqueness of the surface states, the following restrictions were paced on the stress paths:

a.) the degree of saturation is a non-decreasing parameter,

b.) the soil is not permitted to swell.

![Diagram](image)

Figure 2.2 Void Ratio and Degree of Saturation constitutive surfaces for a mixture of flint and kaolin under Ko loading. (a). Void ratio state surface; (b). Degree of saturation state surface. (from Matyas et al. 1968).

These restrictions arise from the fact that there is hysteresis in the soil structure due to loading and unloading, as well as, wetting and drying, introduces non-unique characteristics to the soil behavior. The work of Matyas and Radhakrisna (1968) is essentially on a collapsing soil for the range of loadings and suctions utilized. Their excellent experimental results and qualitative analysis serve to demonstrate the limited value of
Bishop's single valued effective stress equation. In quantitative terms, their analysis was done using a microscopic viewpoint, as was done before by Burland (1965). Their work confirmed the adequacy of using two independent stress variables in formulating the mechanical behavior of unsaturated soils.

A general theory for unsaturated soils was presented by Fredlund and Morgenstern (1976, 1977). Their approach was based on the continuum mechanical viewpoint. The unsaturated soil is considered a four-phase system constituted of: air, water, solid skeleton, and contractile skin (i.e., interface air-water, where acts the surface tension). Fredlund (1979) justifies this proposal by showing that the contractile skin satisfied the requisites of a well-defined phase, since it presents characteristics as follows:

a.) different properties from those ones of the adjacent phases (air and water);

b.) well defined boundary surfaces.

From both a theoretical and experimental basis, it was recommended that use be made of the stress variables proposed by Matyas and Radhakrishna (1968), (i.e., \( \sigma - u_a \), and \( u_a - u_w \)).

2.3.1.2 Continuity requirements for unsaturated soil

The continuity requirement for an unsaturated soil, deforming under an applied stress gradient, can be expressed as proposed by Fredlund and Rahardjo (1993).

\[
\frac{\Delta V_v}{V_0} = \frac{\Delta V_w}{V_0} + \frac{\Delta V_a}{V_0} + \frac{\Delta V_c}{V_0}
\]  \hspace{1cm} \text{[2.16]}

where:

\( V_0 = \) initial overall volume of the saturated soil element

\( V_v = \) volume of soil skeleton voids

\( V_w = \) volume of water phase
\[ V_a = \text{volume of air phase} \]
\[ V_c = \text{volume of contractile skin.} \]

By neglecting the volume changes in the contractile skin, the above relationship showed that only the volume changes associated with two phases must be measured, while the third can be computed. In practice changes in void volume and water phase volume are usually measured.

By using a \( x \)-, \( y \)-, and \( z \)-Cartesian coordinate system and referencing deformation to an elemental volume, the total volumetric deformation of an unsaturated soil can be expressed as,

\[ \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\Delta V_v}{V_o} \tag{2.17} \]

Assuming infinitesimal deformations, the same elemental volume can be used for the measurement of water phase volume changes. The deformation variable associated with the water phase is defined as \( \Delta V_w/V_o \).

### 2.3.1.3 Volume change behavior

Unsaturated soil is a four-phase continuum. Under an applied gradient stress two phases can flow; namely, air and water phases, while two phases come to equilibrium: namely, soil structure and contractile skin. As before mentioned, volume changes associated with the contractile skin can be neglected. The three phases constitutive relationships can be formulated by relating volume changes to changes in stress state variables.

The constitutive relationships are again presented using a moduli and a compressibility form. In the compressibility form both the soil structure and water phase
volume changes are expressed as functions of the stress state variables by means of volume change parameters.

When changes in total volume are represented by changes in void ratio, \( e \), and changes in water content are represented by the changes in degree of saturation, \( S \), then the compressibility form represents the state surfaces as defined by Matyas and Radhakrisna (1968).

Both moduli and compressibility forms of the constitutive relationships can be used in discrete or incremental models. Focus is here given to formulations which use the set of independent stress variables, \((u_a - u_w)\) and \((\sigma - u_a)\).

**Moduli form**

Coleman (1962) suggested the following expressions for volumetric and deviatoric strains for states of stress involving changes in isotropic as well deviatoric components.

\[
-d\frac{dV}{V_0} = C_{21}d(u_a - u_w) + C_{22}d(\sigma - u_a) + C_{23}d(\sigma_1 - \sigma_3) \tag{2.18}
\]

\[
-d(\varepsilon_1 - \varepsilon_3) = C_{31}d(u_a - u_w) + C_{32}d(\sigma - u_a) + C_{33}d(\sigma_1 - \sigma_3) \tag{2.19}
\]

where:

- \( C_{ij} \) = compressibility parameters
- \( \sigma - u_a \) = isotropic stress component
- \( \sigma_1 - \sigma_3 \) = deviatoric stress component
- \( V \) = current overall volume of the element
- \( \sigma_1, \sigma_3 \) = are the major and minor principal stresses
- \( \varepsilon_1, \varepsilon_3 \) = are the major and minor normal strain components.
Coleman (1962) mentioned that the "C" parameters in these equations (i.e., compressibility) depend on the state of stress history.

For the water volume change Coleman (1962) suggested a similar relationship.

\[ \frac{dV_w}{V_o} = C_{11}d(u_a - u_w) + C_{12}d(\sigma - u_a) + C_{13}d(\sigma_1 - \sigma_3) \]  \[ (2.20) \]

Fredlund (1979), later updated by Fredlund and Rahardjo (1993), presented constitutive relationships in the moduli form for an unsaturated soil, as an extension of the semi-empirical equations used for a saturated soil. In these relationships the deformation variables for total volume change and water volume change are associated to the stress state variables by means of elastic modulus. Assuming the soil as an isotropic, linear and elastic material, the constitutive relationships are, in accordance with the generalized Hooke's law, as follows:

\[ \varepsilon_x = \frac{(\sigma_x - u_a)}{E} - \frac{\mu}{E} (\sigma_y + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H} \]  \[ (2.21) \]

\[ \varepsilon_y = \frac{(\sigma_y - u_a)}{E} - \frac{\mu}{E} (\sigma_x + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H} \]  \[ (2.22) \]

\[ \varepsilon_z = \frac{(\sigma_z - u_a)}{E} - \frac{\mu}{E} (\sigma_x + \sigma_y - 2u_a) + \frac{(u_a - u_w)}{H} \]  \[ (2.23) \]

where:

\[ H = \text{elasticity modulus for the soil structure relative to a change in } (u_a - u_w) \]

\[ E = \text{elasticity modulus for the soil structure associated to a change in } (\sigma - u_a). \]

The constitutive equations associated with the shear deformations are the same as presented, in section 2.2.1.1, for a saturated soil.
For the water phase constitutive relationship the same semi-empirical approach was used.

\[ \frac{dV_w}{V_o} = \frac{(\sigma_x - u_a)}{E_w} + \frac{(\sigma_y - u_a)}{E_w} + \frac{(\sigma_z - u_a)}{E_w} + \frac{(u_a - u_w)}{H_w} \]  \[ 2.24 \]

where:

- \( E_w \) = water volumetric modulus associated with a change in \((\sigma - u_a)\)
- \( H_w \) = water volumetric modulus associated with a change in \((u_a - u_w)\).

The above forms of the constitutive equations can be applied to non-linear stress versus strain analysis by means of incremental procedures currently in use in engineering practice. Equation 2.24 indicates the significant effect of hysteresis on the volumetric water content of the soil medium since \( H_w \) is dependent on the matric suction stress path (i.e., wetting or drying path).

Alonso et al. (1988), in a similar manner to Fredlund (1979), presented their incremental soil structure stress-strain relationship in the form:

\[ de = D^{-1} d\sigma^* + de_o \]  \[ 2.25 \]

where:

- \( \sigma^* = \sigma - mu_a \), with \( \sigma = \{\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz}\} \) and \( m^t = \{1,1,1,0,0,0\} \)
- \( de^T = [\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}] \) is the vector of infinitesimal strains
- \( de_o = \) is the vector of initial strains
- \( D = \) is the elasticity matrix.

In Eq. 2.25, the strain increments are obtained as a sum of the effect of the net stress changes, \( D^{-1} d\sigma^* \), and the volumetric strains induced by suction changes (i.e., \( \varepsilon_o \)). It was suggested that \( \varepsilon_o \) may be determined, for isotropic or oedometric stress conditions, from the
void ratio state surface. The elasticity matrix $D$ is defined by means of an isotropic and non-linear elastic $[K, G]$ model. The tangent compressibility modulus, $K$, (or bulk modulus), is derived from the state surface for total volume change. It is suggested that the tangent shear modulus $G$, can be derived from a hyperbolic stress-strain model (Duncan and Chang, 1970), in accordance with some experimental evidence provided by Brull (1980). Therefore, expressions for $K$ and $G$ are as follows:

$$
\frac{1}{K} = \frac{de_v}{d(\sigma - u_a)} = \frac{1}{1 + e_o} \frac{de}{d(\sigma - u_a)} \quad [2.26]
$$

where:

$\sigma - u_a$ = vertical stress on an oedometer test or mean total stress in a triaxial test

$e_o$ = initial void ratio.

and for $G$:

$$
G = [G_o + M(u_a - u_w)][1 - \frac{(\sigma_1 - \sigma_3)R}{(\sigma_1 - \sigma_3)_f}]^2 \quad [2.27]
$$

In Eq. 2.27 the initial shear modulus $G_o$ is assumed to increase linearly with the applied suction parameter, $M$, as a constant. $(\sigma_1 - \sigma_3)_f$ is an appropriate failure criterion for an unsaturated soil. $R$, which is the Duncan’s and Chang’s (1970) index failure, is a constant close to 1.

**Compressibility form**

Figure 2.3 presents one of the first proposed constitutive surfaces to describe volume change as a function of net isotropic stress, $(\sigma - u_a)$, and suction $(u_a - u_w)$ (Bishop and
Blight, 1963). It shows that upon wetting the soil swells at a low mean stress and collapses at a higher mean stress.

![Diagram](image)

Figure 2.3 Volume Changes under equal all-round pressure plotted in a void space ratio (after Bishop and Blight, 1963).

Fredlund and Rahardjo (1993) presented the compressibility equations, for soil structure and water phase, as follows:

\[
d\varepsilon_v = \frac{dV}{V_o} = m_1^i d(\sigma_{\text{mean}} - u_a) + m_2^i d(u_a - u_w)
\]  

[2.28]

where:

\[
d\varepsilon_v = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z
\]

\[
m_1^i = 3 \frac{(1 - 2\mu)}{E}, \text{is the coefficient of volume change with respect to net normal stress,}
\]

\[
m_2^i = \frac{3}{H}, \text{is the coefficient of volume change with respect to matric suction.}
\]

and for the water phase,
\[
\frac{dV}{V_0} = m_1^w d(\sigma_{\text{mean}} - u_a) + m_2^w d(u_a - u_w)
\]  \[2.29\]

where:

\[m_1^w = \frac{3}{E_w},\]

is the coefficient of water volume change with respect to net normal stress

\[m_2^w = \frac{1}{H_w},\]

is the coefficient of water volume change with respect to matric suction.

Figure 2.4 illustrates the constitutive surfaces as described by Fredlund's and Rahardjo's (1993) constitutive equations. In these state surfaces both compressive net normal stresses and matric suction are assumed as positive. This implies negative signs for the four parameters \(m_1^s, m_2^s, m_1^w\) and \(m_2^w\). According to Fredlund and Rahardjo (1993) the four negative parameters characterize a stable soil (i.e., a soil that decreases in total volume and water content due to increases in any stress variable). In turn, the stable soil swells upon decrease in any stress variables. These sign conventions are also valid for the correspondent elasticity parameters.

Lloret and Alonso (1985) analyzed confined and isotropic compression test data using a best-fit analysis and presented some analytical expressions for void ratio and degree of saturation constitutive surfaces. Their optimum expressions were as follows:

\[
e = a + b \log(\sigma - u_a) + c \log(u_a - u_w) + d \log(\sigma - u_a) \log(u_a - u_w)
\]  \[2.30\]

for void ratio, and

\[
S = a' - [c' + d'(\sigma - u_a)]Th[b'(u_a - u_w)]
\]  \[2.31a\]

or
\[ S = (1 - e^{a + (u_s - u_w)})[b_s + c_s (\sigma - u_s)] \]  

for degree of saturation, 

where: 

\[ a, b, c, d, a', b', c', d', a_s, b_s, c_s = \text{constants resulting from the best-fit equations} \]

\[ e = \text{the natural base.} \]

(a) \hspace{2cm} (b)

Figure 2.4 Three-dimensional constitutive surfaces for an unsaturated soil. (a) soil structure constitutive surface; (b) water phase constitutive surface (from Fredlund and Rahardjo, 1993).

Alonso and Lloret (1985) reported that equation 2.30 results in a warped shape and may therefore reproduce a joint swelling-collapse behavior as in figure 2.3. In their best-fit analysis, difficulties were experienced in adjusting Eq. 2.31 to the measured values of degree
of saturation. These difficulties were attributed in part to the scatter from experimental
difficulty in measuring very small water volume changes.

Equations 2.30 and 2.31 are in general terms similar to Fredlund and Rahardjo's
equations (Eqs. 2.28 and 2.29) where the soil parameters \( m_f^s, m_2^s, m_1^w \) and \( m_2^w \) are
considered to be functions of the two independent stress state variables.

### 2.3.1.4 Shear strength

The shear strength of an unsaturated soil can be formulated in terms of independent
stress state variables (Fredlund et al. 1978). The stress state variables, \((\sigma - u_a)\) and \((u_a - u_\omega)\),
have been shown to be the most advantageous combination for practice. Using these stress
variables, the shear strength equation is written as follows:

\[
\tau_f = c' + (\sigma_f - u_a)_f \tan \phi' + (u_a - u_\omega)_f \tan \phi^b \tag{2.32}
\]

where:

- \( c' \) = intercept of the "extended" Mohr-Coulomb failure envelope on the
  shear stress axis where the net normal stress and the matric suction
  at failure are equal to zero; it is also referred to as "effective
  cohesion"
- \( (\sigma_f - u_a)_f \) = net normal stress state on the failure plane at failure
- \( u_{af} \) = pore-air pressure on the failure plane at failure
- \( \phi' \) = angle of internal friction associated to the net normal stress
  variable, \( (\sigma_f - u_a)_f \)
- \( (u_a - u_\omega)_f \) = matric suction on the failure plane at failure
- \( \phi^b \) = angle indicating the rate of increase of shear strength relative to the
  matric suction, \( (u_a - u_\omega)_f \).
Equation 2.32 defines a plane. This plane is defined by a three-dimensional plot which has the shear stress, \( \tau \), as the ordinate and the two stress state variables, \((\sigma - u)\) and \((u_a - u_w)\), as abscissas. The failure plane envelope can be modified to accommodate non-linearity associated with the soil parameters involved, \(c'\), \(\phi'\), and \(\phi_b\). Experimental evidence (Gan and Fredlund, 1978) has demonstrated that the shear parameters \(c'\) and \(\phi'\) are relatively constant for stable-structured soils. In turn, \(\phi_b\) changes due to changes in matric suction (Gan and Fredlund, 1978; Fredlund and Rahardjo, 1993; Vanapalli, 1994). For a metastable soil it seems reasonable to realize non-linear behavior for the shear parameters \(c'\), \(\phi'\) and \(\phi_b\) friction angles. This topic will be discussed later in this thesis.

### 2.3.2 Flow law and hydraulic properties

In an unsaturated soil both the air and the water phases can flow. Therefore, an analysis of fluid flow requires that appropriate coefficients be used to relate the flow rate to a driving potential for both phases. The hydraulic head is the driving potential for the water phase.

"The driving potential causing flow in the water phase has the same form for both saturated and unsaturated soils". (Freeze and Cherry, 1979).

Water flows from a point of high total head to a point of low total head, regardless of whether the pressures head are positive or negative.

The flow of air with a continuous phase is governed by either a concentration or pressure gradient. The pressure gradient is most commonly considered as the only driving potential for the air phase. Both Fick's and Darcy's law have been used to describe the flow of air through a porous media. The relative movement of air and water through an unsaturated porous medium is a function of the porosity, degree of saturation, pore-size distribution, and specific properties of the fluids as density and viscosity. It is quite complex to quantify and express all these factors using stress state variables. The flow of water and air through an
unsaturated soil is highly dependent on the degree of saturation. Figure 2.5a, presented by Bear (1972) is used to qualitatively illustrate this dependency. A similar quantitative illustration, Figure 2.5b, is shown by Barden and Pavlakis, (1971) for a compacted soil.

Figure 2.5 Permeability of unsaturated soils. a.) Typical relative permeability curves (from Bear, 1972); b.) Coefficients of permeability with respect to air phase, $k_a$, and water phase, $k_w$, as a function of the gravimetric water content for the Westwater soil (from Barden and Pavlakis, 1971).

The permeability to the air phase of compacted soils decreases as the soil water content or degree of saturation increases (Langfelder, Chen and Justice, 1968; and Barden and Pavlakis, 1971). But, as illustrated on Figure 2.5.b, such studies shows that the soil permeability to air remain significantly greater than the soil permeability to water for all water contents. Therefore, most problems involving unsaturated soils consider the air at constant atmospheric since air pressure gradients are rapidly dissipated.
2.3.2.1 Coefficient of permeability with respect to water phase

According to past experiments (Childs and Collis-George, 1950), Darcy's law also applies for the flow of water through unsaturated soils. Their results indicated that the soil coefficient of permeability with respect to water is a function of the water content (or matric suction) in the unsaturated soil. Lloret and Alonso (1980) and Fredlund (1981) reported that the soil permeability with respect to water must be expressed as a function of the volumemass soil properties.

Fredlund and Rahardjo, (1993) emphasized that in an unsaturated soil, the hydraulic conductivity is affected by combined changes in the void ratio and the degree of saturation (or water content) of the soil. As a soil becomes unsaturated, air first replaces some of the water in the large pores and this causes the water to flow through the smaller pores with an increasing tortuosity to the flow path. A further increase in the matric suction of the soil leads to a further decrease in the pore volume occupied by water. As a result, the soil permeability with respect to water decreases rapidly as the space available for water flow reduces.

Some unsaturated soils do not undergo significant changes in void ratio in response to changes in the stress state variables. For these soils, the soil permeability to water can be expressed as a function of their degree of saturation. Such soils are usually referred to as non-deformable soils. A change in matric suction generally produces a more significant change in water content than does a change in net normal stress. Therefore, the soil-water characteristic curve has been used to derive semi-empirical relationships for the soil permeability to water as a function of the soil matric suction (Gardner and Fireman, 1958; Gardner, 1958; Brooks and Corey, 1964; van Genuchten, 1980; Mualem, 1986; Fredlund and Xing, 1994). Huang (1994) presents an extensive literature review of these relationships. Relationships between the soil permeability with respect to the water and matric suction are strongly dependent on the stress path, (i.e., wetting and drying paths produces different relationships). However,
hysteresis has relatively little influence in the relationships between the soil coefficient of permeability with respect to the water and the degree of saturation for soils with a non-deforming soil structure.

For a deformable soil, the void ratio may change significantly in response to changes in the stress state. The literature presents some experimental results that can be used to illustrate the combined effect of void ratio and degree of saturation on the soil coefficient of permeability with respect to the water. Previous works are dated from the 1960s (Mitchell et al. 1965; Laliberte et al. 1966).

Lloret and Alonso (1980), derived equations for the consolidation process of unsaturated soils. An empirical equation was presented to estimate the soil permeability with respect to the water as a function of the void ratio and degree of saturation as follows:

\[ k_w = k_{o}(S,e_o)10^{a(e-e_o)} \]  \hspace{1cm} \text{[2.33]}

where:

\( k_{o}(S,e_o) \) = permeability function at a void ratio of \( e_o \)

\( e_o \) = the initial (or reference) void ratio

\( \alpha \) = the slope of the linear relationship \( \log_{10} k_w \) versus \( e \) for constant \( S \).

It is suggested that \( \alpha \) can be obtained for \( S = 1 \) using the oedometer test.

The authors report that Eq. 2.33 was based on a model presented by Bear (1972) for unsaturated soils.

Chang and Duncan (1983) presented the water coefficient of permeability as a function of both void ratio and degree of saturation.

\[ k_w = k_o G_t H_s \]  \hspace{1cm} \text{[2.34]}

where:
\[ G_e = \frac{e^3_{e}}{1+e^3_{e}} / \frac{e^3_{e}}{1+e_{o}} \text{, or } G_e = \exp \frac{e}{b} / \exp \frac{e_{o}}{b}, = \text{factor depending on void ratio.} \]

\[ H_s = \left( \frac{S - S_f}{1 - S_f} \right)^b, \text{ factor relating } k_w \text{ to degree of saturation, based on Singh (1965).} \]

\[ b = \text{experimental index.} \]

"A constant index, \( \delta \) (e.g., 3) was suggested in the study. However, it may change with the void ratio, at least to some degree, according to the experimental data presented by Laliberte et al. (1966). As a result, the influence of the void ratio on the coefficient of permeability is included in the factor \( H_s \). The parameters, \( \delta \) and \( S_f \) were suggested as 3 and 0, respectively in the case that experimental data are not available to define the most appropriate value of \( H_s \)." (Huang, 1994).

Huang (1994) developed theoretical functions for the soil coefficient of permeability for a deformable unsaturated soil. Two versions were presented, one for a compressible soil and another for a swelling soil. A compressible soil is defined as a soil where changes in the net normal stress produces significant volume changes. Swelling is defined as the soil which produces significant volume changes as a result of changes in matric suction. The developed functions are as follows:

i.) for a compressible soil (in terms of void ratio and degree of saturation) the equation is expressed as:

\[ k_w = k_{so} S \left[ \delta + c(e - e_{o}) \right] b(e - e_{o}) \]  \[ 10^{b(e - e_{o})} \]  \[ [2.35] \]

and,

ii.) for a swelling soil (in terms of void ratio and matric suction), the equation is as follows.

\[ k_w = k_{so} 10^{b(e - e_{o})} \]  \[ [2.36a] \]
for $(u_a - u_w) \leq (u_a - u_w)_{bo} 10^{a(e-e_o)}$

and,

$$k_w = k_{so} 10^{b(e-e_o)} \left[ \frac{(u_a - u_w)_{bo} 10^{a(e-e_o)} (u_o - u_o)}{(u_o - u_w)} \right]^{[\delta_o + c(e-e_o)] [\lambda_o + d(e-e_o)]}$$

[2.36b]

for $(u_a - u_w) > (u_a - u_w)_{bo} 10^{a(e-e_o)}$

where:

$(u_a - u_w)_{bo} =$ the air entry pressure at the void ratio, $e_o$

$k_{so} =$ saturated coefficient of permeability at a void ratio $e_o$

$S_e =$ effective degree of saturation

$b =$ the slope of the line $\Delta log k_{so} / \Delta e$

$a =$ the slope of $\Delta log (u_a - u_w) / \Delta e$

$(u_a - u_w)_{b} =$ air entry pressure estimated from the effective degree of saturation, $S_e$, versus matric suction curve,

$\lambda_o =$ $\lambda$ at a void ratio, $e_o$

$\lambda =$ pore size distribution index. The slope of the log $S_e$ versus $\log(u_a - u_w)$ relationship at a constant void ratio

$d =$ the slope of the relationship $\lambda$ versus $e$

$\delta_o =$ $\delta$ at a void ratio $e_o$

$\delta =$ slope of the relative coefficient of permeability versus effective degree of saturation.

$c =$ the slope of the relationship $\delta$ versus $e$.

Equations 2.35 and 2.36 represent a considerable contribution towards the prediction of a soil permeability function for a deformable unsaturated soil. However, Huang’s (1994) findings are based on laboratory tests conducted along drying stress paths. Therefore, some
efforts are required to extend Huang's (1994) equations to a deformable soil following a wetting stress path.

2.3.2.2 Coefficient of permeability with respect to the air phase

A modified form of Fick's law is usually applied to describe the air flow through an unsaturated porous medium (soil).

\[ J_a = D_a^* \frac{\partial u_a}{\partial y} \]  \hspace{1cm} [2.37]

where:

\( du_a/\partial y \) = pore-air pressure gradient in the y-direction (or similarly in the x- and z-directions).

\( J_a \) = mass rate of air flowing across a unit area of the soil

\( D_a^* = D_a \frac{d(\rho_a(1-S)n)}{du_a} \), coefficient of transmission, a function of the volume mass properties of the soil (S, n), the air density (\( \rho_a \)) and the transmission constant for air flow through a soil (\( D_a \))

\( \rho_a \) = air density related to the absolute air pressure (gas law)

\( n \) = porosity of the soil.

Equation 2.37 has been used in geotechnical engineering to describe air flow through a soil, Blight (1971). As demonstrated by Fredlund and Rahardjo (1993), Eq. 2.37 can be conveniently modified to the form,

\[ v_a = -D_a^* g \frac{\partial h_a}{\partial y} \]  \hspace{1cm} [2.38]

where:

\( h_a \) = pore-air pressure head
\[ \frac{dh_a}{dy} = \text{pore-air pressure head gradient in the y-direction.} \]

Equation 2.38 has the same form as Darcy’s equation for the air phase:

\[ v_a = -k_a \frac{\partial h_a}{\partial y} \quad [2.39] \]

where the relationship between \( D_a^* \) and the soil coefficient of permeability with respect to the air phase, \( k_a \), is defined as follows:

\[ k_a = D_a^* g \quad [2.40] \]

Similarly to the coefficient of permeability with respect to water, the coefficient of permeability with respect to air is a function of the fluid (in this case air) and soil volume-mass properties. However, unlike the water, the air properties can no longer be considered constants. Density and viscosity of the air are functions of the absolute air pressure.

The present research study involves unsaturated soils where the air is at constant atmospheric pressure. Therefore, no further developments on air permeability are required since air flow is not a relevant process.

### 2.3.3 Collapsing soils

The addition of water to some soils under load produces a considerable reduction in volume and consequently, significant settlement (Barden et al., 1973). These materials are referred to as collapsible soils. These soils, at their natural water content, are capable of supporting heavy loads without significant deformation. But, once water starts infiltrating their voids, there may be a sudden loss in their capacity to support the applied load. There is a
rearrangement of the soil structure with an almost instantaneous reduction in volume, and consequently, immediate settlement.

The term, "collapse" is also referred to as the irreversible change in volume of the initially unsaturated and loaded soil during the saturation of its voids. This involves the simultaneous expulsion of air by the infiltrating water. The expressions, "subsidence" and "hydrocompaction" are also used in the literature as synonyms of "collapse". The term "wetting-induced collapse" is often applied to endorse the collapsing behavior as a result of a saturation of the soil voids.

Based on a review of the literature, Lawton et. al. (1991a, 1991b), summarized the following observations regarding the "wetting-induced collapse" in soils:

a.) Four conditions are necessary for collapse to occur in a soil (Barden et. al., 1973; Mitchel, 1976);

i.) an open, partially unstable, partially saturated fabric;

ii.) a high enough total stress so that the structure is metastable;

iii.) a sufficiently large soil suction or the presence of a bonding or cementing agent that stabilizes the soil in its unsaturated condition;

iv.) the addition of water to the soil which reduces the soil suction or softens or destroys the bonding agents, thereby causing shear failures at the interaggregate or intergranular contacts.

b.) For any set of conditions, the amount of collapse generally decreases by increasing pre-collapse water content, increasing pre-collapse dry density, or decreasing overburden pressure (Holtz, 1948; Booth, 1977; Cox, 1978).

c.) For any soil there are combinations of initial dry density, molding water content, and overburden pressure at which no volume change will occur when the soil is inundated (Booth, 1977; Cox, 1978).
d.) For a given soil there appears to be a critical molding water content above which no collapse will occur. For some soils the critical water content is above standard Proctor optimum water content (Hilf, 1956; Barden et al. 1969).

e.) There is a critical degree of saturation for a given soil above which negligible collapse will occur regardless of the magnitude of the pre-wetting overburden pressure (Mishu, 1963; Booth, 1977).

Collapsing soils are sometimes classified either as “truly” or as “conditionally” collapsible soil (Clemence and Finbarr, 1981; Popescu, 1986). The “truly” collapsible soil needs only its self weight and saturation to collapse. The “conditionally” collapsible soil needs, in addition to saturation, the existence of a minimum “critical” load in order to collapse. From this point, a collapsible soil can be saturated and then collapsed. Actually, as soon as the soil becomes saturated, the expression collapsing behavior becomes meaningless. To clarify this point it should be emphasized that collapsing behavior happens only in unsaturated soil (Barden et al. 1973; Mitchel, 1976), and involves the simultaneous expulsion of air from the soil voids, induced by the infiltrating water.

The literature is vast on tentative explanations of the collapsing behavior of soils. Any explanation requires a knowledge of the structure of the soil at both the macro and micro levels. In terms of the microstructure, existent explanations often associate the collapsing behavior to the genesis of the soil (Dudley, 1970; Rogers, 1995; Feda, 1995). The geneses of collapsible soils are broadly classified into two groups: natural collapsing soil and man-made, (i.e., compacted) collapsible soil. The prediction of the performance of a collapsing earth structure in engineering practice requires the modelling of the collapsible soil behavior in a phenomenological manner. Such an approach allows the use of the theory of unsaturated soils mechanics for the prediction of the collapsing behavior. This means that the mechanical behavior of a collapsible soil must be predicted in terms of the stress states variables. According to the literature (Dudley, 1970; Barden et al. 1973; Mitchel, 1976; Miranda, 1988; Alfi, 1984; Alonso et al. 1987) this modelling requires an understanding of the behavior of

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the collapsible soil in terms of its macro- and micro-structures. Alonso (1993), summarized previous studies and stated that the microstructure is widely recognized as important information to explain the behavior of collapsing soils despite the fact that it lacks a simple quantitative descriptor to be used in practice.

In the present study, the emphasis is on the review of previous studies related to the combined effects of changes in both matric suction and net normal stress, in compacted soils. Some studies on natural collapsible soils, considered to be relevant (i.e., mainly in terms of the microstructure), are briefly discussed. The microstructure of natural soils may remain intact in embankments compacted at low compaction energy. Thereby, it is important in engineering practice to have knowledge of the structure of natural collapsible soils.

2.3.3.1 Occurrences

Collapsible soils are metastable and must have an open structure, that is the soil particles must be in an open packing which is capable of a closer packing. A granular material with angular particles compacted on the dry side of optimum can form a structure which is capable of significant further densification, but the classic collapsible soils are natural materials where the combination of particle type and sedimentation mechanism combine to give collapsibility (Rogers, 1995).

Aeolian deposits

Aeolian deposits are referred to as soils that are transported and deposited by wind. These deposits are usually composed by either silt or fine sand size particles, or even a combination of these particles. If the silt size particles are predominant, the deposit is called loess. In loess, silt and varying amounts of sand, clay, cementing agents, and other materials
are arranged in an open and cohesive fabric. Such deposits are characteristic of arid regions. The loess usually collapses when inundated by water, while under its self weight.

**Water laid deposits**

Water laid deposits are originally deposited by flash floods or mud flows. Such deposits present varying amounts of clay size particles. Depending on the amount of clay and the overburden pressure, these soils can collapse or swell during saturation. Bull (1964) analyzed the influence of the clay fraction on the volume change behavior during saturation for these natural soils. Based on extensive analysis of the experimental evidence, in general, the maximum collapse occurs when the clay-size particles are approximately 12% by mass of the soil while swelling may occur when the clay-size particles exceed 30%. Bull’s (1964) data were primarily for soils with low dry densities.

**Residual soil deposits**

Residual collapsible soils originate from the disintegration of parent rocks and the mechanical alteration of their components. The leaching of soil components during cycles of saturation-drainage yield a metastable structure to these soils. It has been observed that these soils have higher void ratios in places where the rainfall is high (Maswoswe, 1980). The resulting final structure is highly dependent on the mineralogy of the parent rock.

**Man-made fills**

Man-made structures such as embankments, road fills and earth dams often exhibit collapse when the soils are compacted dry of optimum. The collapsible behavior is usually associated with sandy and silty soils. Barden et al. (1973) and Cox (1979) demonstrate that
compacted clays can also exhibit collapse. It is now accepted that any type of soil compacted at "dry of optimum" conditions and at a low density may develop a collapsible fabric or metastable structure. The collapsibility of compacted soils depend upon the void ratio (or dry density), water content, grain size distribution, and stress state of the soil.

2.3.3.2 Soil structure and collapse mechanism

Collapsible soils typically possess an open type of structure with many void spaces, giving rise to a metastable structure. Collapsible soils have been traditionally described as loose, primarily granular soils with varying amounts of dry clay, silt, or other cementing materials acting as bond and holding the granular particles together.

For cohesionless soils, capillary action plays the role as the main bond. An increase in saturation gradually eliminates the suction holding the particles in position and provokes a local reduction of the stress and, consequently, the shear strength. As a result, there is a process of "local rupture" where particles slide over their neighbors and go into the voids of the original structure. This process causes a sudden reduction of the soil volume.

The soil collapse concept has been expanded to clayey soils after embankments of compacted clayey soils experienced settlements upon wetting. These embankments were reported to possess soils compacted at "poor conditions", (i.e., compacted at dry of optimum condition and at low densities).

It is now known that a collapsible clayey soil, when loaded at natural water content, can maintain its original structure with no significant volume change. This fact is attributed to action of strong bonds between the coarse particles which prevent relative movements. At this natural water content the small changes in the soil volume, due to external loading, could be attributed to the compression of the fine soil, (i.e., clay bonds, between the coarser material). Jennings and Knight (1957) reported that at a low water content, the soil structure is supported by microforces of shear strength (i.e., bonds) which are highly dependent upon
capillary action. With the increase of the water content the bonds start losing strength and at a "critical water content" the structure collapses. Figure 2.6 illustrates the collapse mechanism according to Jennings and Knight (1957).

![Diagram of collapsible soil structure](image)

Figure 2.6 Structure of a collapsible soil before and after inundation. a) loaded structure at natural water content, b) loaded structure after complete inundation (from Jennings and Knight, 1957).

Lambe (1958a, 1958b), presented a model to explain the behavior of compacted clayey soils. According to this model, an open, flocculated structure is obtained when the soil is compacted at dry of optimum conditions, and a closed-packed dispersed structure results when the soil is compacted at wet of optimum conditions. Earthfill collapse was explained as a result of the compression behavior of the open, flocculated structure when a clay compacted at dry of optimum condition is saturated for the first time (Barden et al. 1969).

Jennings and Burland (1962) proposed a conceptually different model for compacted clayey soils. This model was later studied by means of scanning electronic microscopy by Barden et al. (1973) and Booth (1977). In this model, the remolded soil is assumed to consist of a mixed structure composed of brittle coarse particles and aggregations of fine particles that can be either brittle or plastic, depending on the water content. The
engineering behavior of the compacted mass depends primarily on the distribution and size of the interaggregate and intra-aggregate pores rather than the concentration and distribution of solids. The interaggregate pores are considerably larger than the intra-aggregate pores and can be controlled during the compaction process, whereas the intra-aggregate voids are quite small and are little affected by compaction. The aggregates compacted on the dry side of optimum are brittle and shrunken. Compaction results in the rearrangement of the granular particles and aggregates in much the same way as for a granular material. With the addition of water, the aggregates increase in volume and soften with the absorption of water to the clay particles. The resulting volume change is positive, zero, or negative depending on the prewetting dry density and stress state of the soil. For low densities or high stresses, collapse can occur as the softened, plastic aggregates are remolded at the aggregate-granular or aggregate-aggregate contacts and the soil is rearranged into a denser state of packing. For high densities or low stresses, an increase in volume (i.e., swelling) will occur. The aggregates compacted on the optimum wet-side are plastic and swollen. The aggregates are remolded during compaction to produce a soil with small interaggregate spaces and large intra-aggregate spaces. The addition of water primarily fills the small interaggregate pores with little swelling of the aggregates and little densification or collapse occurring.

The soil structure is highly dependent on the compaction conditions for a compacted soil. Matyas and Radhakrisna (1968) reported that the initial structure of a compacted specimen is adequately specified by the molding water content and the compactive effort, provided that other factors such as the curing period and temperature effects are kept constants. It is also stated that changes in the structure of a soil element during the process of deformation under external loads, have the following implications:

a.) changes in suction pressure due to structural rearrangement on a microscopic scale,

b.) the influence on the stress-deformation relationship on a macroscopic scale.
Matyas and Radhakrisna (1968) also reported that these two factors contribute to the non-uniqueness of the volume change behavior of a soil element following two different stress paths. However, it was recognized that changes in suction and changes in applied net stress have two independent effects on the rigidity of the soil structure as suggested by M.I.T. (1963). In this way the macroscopic (phenomenological) behavior of the soil can be related to the stress state.

Dudley (1970) presented a summarized description of metastable structured soils which are granular and fine grained. It was explained that a collapsing soil possesses an open structure consisting of bulky grains held together in a honeycomb type of fabric by means of a cementing agent or force (capillarity) at the contact points of the bulky grains. Various bonds between particles are possible. The most recognizable forces are capillary suction, silt and/or clay bridges or buttresses, and cementing agents. Any combination of these bonding sources and the various types of collapsing soils structures will form a potential collapsing mechanism.

In terms of granular soils, Dudley’s (1970) research emphasized the role of capillary suction in sandy and silty soils. The menisci were described to develop when a sandy soil dries, providing an increase in the apparent strength of the soil. The menisci were formed when the remaining water (at negative pressure) withdraws into the narrows spaces close to the junction of the soil grains. The apparent high soil strength was then explained to result from an increased effective stress at the contact grains which was far superior to the total stress applied to the soil. The wetting-induced collapse is explained as a total rearrangement of the open soil structure due to the reduction of the negative water pressure. Consequently, there is a decrease in the effective stress with a corresponding reduction in the apparent strength of the soils. For a sandy soil with high dry density, the reduction in negative pore-water pressure does not produce a significant effect. In cases of silt binders, the mechanism of collapse was explained in the same way. It was stated that the capillary forces act in both
silt/silt and silt/sand contacts increasing the apparent strength of the soil. Figures 2.7 to 2.11 illustrates the more common types of metastable structures described in literature.

Figure 2.7 illustrate the action of capillary forces holding sand or silt grains. This type of arrangement usually occurs during drying process. The remaining water is kept close to the contact points of coarse particles originating bonds due to the capillary forces.

![Figure 2.7 Schematic arrangement of sand or silt grains bonded by capillary action](from Dudley, 1970).

Figure 2.8 illustrates a soil structure consisting of sand grains being held together by fine-silt-sized particles. Barden et al. (1973) called this type of arrangement as fine silt bond.

![Figure 2.8 Schematic arrangement of sand and silt grains](from Dudley, 1970).

The presence of clay-sized particles implies a variety of structural arrangements. These arrangements are a function of the origin of the soil and the relative amount of clay particles and coarser grains. Dudley (1970) and Barden et al. (1973) reported that when the
bonding material is formed in-place by autogenesis it could form a parallel plate onion-skin type of effect around the sand grains as illustrated in Figure 2.9.

![Diagram of sand grains aggregated by clay bonds](image)

Figure 2.9 Schematic Arrangement of sand grains aggregated by clay bonds originated by autogenesis (from Dudley, 1970).

Knight (1960) proposed a structural arrangement in which clay-sized particles would form a random flocculated structure giving a buttress type of support to bulky grains, as shown on Figure 2.10. This arrangement was confirmed by Knight (1960) with the aid of an optical microscopic and by Barden et al. (1973) with the aid of a scanning electron microscope.

Another type of arrangement involving clay-sized particles is presented in Figure 2.11. In this arrangement, clay aggregates are connected to each other by clay bridges. These bridges can also connect to coarser particles such as sand and silt particles.

Barden et al. (1973) reported that clay bridges and buttresses bonding silt and sand particles or even clay peds are the most common arrangement in collapsing soils. The structural arrangement of the clay plates is a function of their geologic origin and the history.
of the soil. In all cases, the clayey fraction shows a high dry strength that is reduced when saturation takes place. This reduction in strength can also be related in a general way to changes in matric suction, but on a microscopic level the van der Waals attraction, the double layer repulsion, and the adsorbed water should be taken into account.

![Diagram of sand grain and clay sized particles](image)

Figure. 2.10 Schematic arrangement of ring buttresses (from Dudley, 1970).

![Diagram of clay bridges and clay or silt aggregates](image)

Figure. 2.11 Schematic arrangement of soil structures bonded by clay bridges (from Dudley, 1970; Clemence and Finbarr, 1981; Popescu, 1986).

Cementing agents can also act as bonding factors (Barden et al., 1973; Alfi, 1984). In soils presenting cementing agents the rate of settlement depends on the rate of dissolution of the cementing agent. In this case, collapse may not be able to be directly related to changes in matric suction.
Maswoswe (1985) performed shear tests on the Lower Cromer Till, compacted dry of standard AASHTO optimum condition and under different stress paths, and concluded that the collapse was associated with localized shear failures rather than an overall shear failure of the soil mass. In his research, reference was made to the Jennings and Burland’s clay “aggregate” model to describe the collapsing behavior of the compacted till.

Vaughan (1985) reported that the best experiment to determine the collapsibility of a soil is that of isotropic loading. Depending on the type of soil and the level of load applied, it is possible to provoke the global shearing of the soil, instead of inducing localized shearing.

Miranda (1988), based on Jennings and Burland’s model, explained the collapse of fills as the result of the compression and distortion of weak "grains (aggregates)" of clay when water is applied. It was concluded that the reduction in strength of the clay aggregates can be related to matric suction in a phenomenological sense. It is added, referring to Dudley (1970), that on a microscopic level, the van der Waals attractions, the double layer repulsion, and the adsorbed water should be taken into account.

Miranda (1988) reported that the “aggregate” clay macrostructure is confirmed by the reduced apparent cohesion found in compacted clay materials of small homogeneous dams constructed in Northeast Brazil. Sometimes it is impossible to make an observation pit with vertical walls in the embankment, because the material behaves like a granular material.

In summary, the collapse process occurs during saturation of the soil, due to the reduction in the shear strength of bonds that hold the metastable structure of the soil. In some cases an increase in external loading during saturation is required, to overcome the shear strength of the bonds. The shear strength of the bonds is closely related to the capillary forces (i.e., matric suction). The collapse process causes a sudden destruction of the original structure, and the new structure presents different mechanical and hydraulic properties. This change in structure is closely related to the initial structural characteristics and history of stresses to the point of collapse (Jennings and Knight, 1957; Burland, 1965; Dudley, 1970; Mitchel, 1976; Miranda, 1988; Lawton et. al. 1991b).
The reduction in shear strength at the microstructure level of the clay bonds still needs further study. Lambe’s (1958a) and Jennings’ and Burland’s (1960) models have been used in a combined manner (Dudley, 1970; Barden et al. 1973; Alfi, 1984; Alonso et al. 1985) and also in a separated manner (Maswoswe, 1985; Miranda, 1988) to explain the collapse mechanism in clayey soils as a result of “local shear failure” at microstructure level. Lambe’s (1958a) model appears to be more adequate to explain soil collapse at the microstructure level as being triggered by the destruction of clay bonds (Rogers, 1995). In turn, Jennings and Burland’s model appears to be more appropriate at the macroscopic level, since the clay aggregated behavior seems to be closely related to matric suction. The two models can be seen as complementary when Lambe’s model is used to explain the bond collapse of the clay aggregates in Jennings’ and Burland’s model. Despite these differences, some conclusions can be outlined for clayey structured collapsible soils.

a.) In the case of clay films around soil particles (see Figures 2.9 and 2.11), where the clay particles are arranged in a dispersed way, the addition of water provokes a separation of the clay particles. Consequently, the reduction in the strength of the soil structure leads to the process of finding a new equilibrium structure.

b.) In the case of clay bonds in a flocculated arrangement, the addition of water causes a reduction of the capillary forces and provokes a reduction of the ionic concentration of the pore fluid (Osipov et al. 1995). The reduction of the ionic concentration causes an increase of the repulsive forces between the clay particles, and consequently a decrease in the shear strength of the cohesive bonds. Dudley (1970), suggested that the strength of this type of bond is a function of the salt concentration, void ratio of its flocculated structure and probably temperature.

c.) The above two cases are related to the collapse at the bond level. The collapse of the clay aggregates, may be associated at the microlevel with both mechanisms explained in items a.) and b.). In this case, soil collapse is the resultant of a reduction in the void ratio after the destruction of the clay aggregates.
From previous studies, excluding the case of cemented soils, it can be concluded that the collapse mechanism is triggered by a wetting process. In this way, an increase in the degree of saturation means additional collapse. From both the microscopic and macroscopic points of view, collapse is associated with changes in the independent stress state variables (i.e., matric suction and net normal stress). It would seem that the macroscopic approach is sufficient to engineering practice. However, the microscopic point of view may be useful in explaining the progress of soil collapse as matric suction decreases (or as partial wetting occurs).

2.3.3.3 Influence of the saturation in the wetting-induced soil collapse

The conventional process of inundation of the soil specimen does not allow for the analysis of the influence of the degree of saturation on the wetting-induced collapse. An accurate method to evaluate this influence must be done with experiments where matric suction is decreased in controlled stages to reach a zero value. Previous studies have emphasized the evolution of soil collapse as the degree of saturation increases. There is, however, a critical degree of saturation for a given soil above which negligible collapse will occur regardless of the magnitude of the pre-wetting overburden pressure (Mishu, 1963; Booth, 1973; Booth, 1977).

Lawton et al. (1991b), reported that Lawton (1986) found an average critical degree of saturation of 92% after soaking 36 single oedometer specimens of a clayey sand under a variety of loading and as-compacted conditions. It was also reported: “complete or near-complete saturation may be achieved by back-pressures saturation techniques; however. Witsman and Lovell (1979) determined that collapse is essentially completed during soaking and little additional collapse occurs during “backpressure saturation”.

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Houston et al. (1993) performed tests by partially wetting specimens in oedometer rings. As a result of these tests, one alternative partial wetting process was proposed. The procedure involved the removal of the applied load during the addition of each increment of water and allowing the degree of saturation to equilibrate before the load was reapplied. This technique was applied to 3 compacted specimens of a sandy silt (ML) with a plasticity index ranging from 12 to 15. The results are illustrated in Figure 2.12 as dashed curves. The solid curve represents the matric suction as function of the degree of saturation which was determined independently. It was noted that for a given soil, the position of the partial wetting curve is dependent upon the initial water content, density, confining stress, and maximum past total confining stress applied.

Figure 2.12 Partial collapse due to partial wetting curves for three ML soils (from Houston et al. 1993).

The above analysis is justified by emphasizing that wetting-induced collapse occurs while the clay bonds and clay aggregates are softened and collapsed. As the soil is given free access to water in an inundation procedure, the bonds and aggregates reach near-complete saturation first while the macropores between large particles are still in an unsaturated
condition. Another explanation is related to the existence of trapped air bubbles in the soil voids.

2.3.3.4 Rheological aspects of soil collapse

When a collapsible soil reaches the specified conditions of saturation and loading, the collapse occurs in a very short time. The literature contains numerous tentative explanations regarding the progress of soil collapse during saturation. Some of the conclusions are biased by the particular problem under analysis. As a summary, and according to literature review, the rate of deformation during the wetting-induced soil collapse is controlled by:

a.) the permeability of the soil with respect to water, (Jennings and Knight, 1962).

b.) the combined effect of soil permeability with respect to air and water phases.
   (Lloret and Alonso, 1980),

c.) the magnitude of the applied load, (Villar et al. 1981),

d.) the composition of the pore-fluid, (Clemence and Finbarr, 1981),

e.) the amount of clay and carbonated agents, (Popescu, 1986),

f.) the rate of infiltration of the water in the voids of the soil, (Houston et al. 1988).

g.) the rate of reduction of matric suction in the soil structure (Tadepalli et al. 1992).

In section 2.3.3.2, it was concluded that the rate of collapse was dependent on the rate of breaking the bonds that hold the metastable soil structure. Actually, excluding Popescu’s (1986) analysis and Clemence’s and Finbarr’s (1981) analysis which involved natural or man-made cemented soils, the remainder of the results are from compacted specimens and can be summarized by Tadepalli et al.’s (1992) findings: “Matric suction appears to be the agent which provides the stability to a metastable-structured soil”. As soon as the matric suction decreases, the soil structure starts to collapse. The rate of collapse is
primarily controlled by the rate of decrease in matric suction. The increase of the rate of collapse with the magnitude of the applied load, as proposed by Villar et. al. (1981), cannot be generalized for the different soil structures. It might happen that the increase in applied load changes the soil structure significantly eliminating partially or totally the metastable condition. Depending on the initial conditions of the compacted soil, the rate of collapse can either increase or decrease with the increase of the applied load.

Lloret and Alonso (1980) performed tests on a collapsible, compacted clayey sand in an ordinary oedometer cell without controlling pore-air or pore-water pressures. The specimen had an initial matric suction of 50 kPa and was consolidated under a vertical load of 30 kPa. After consolidation, the specimen was soaked from both ends. The resulting settlement curve is shown in Figure 2.13. It was suggested that on wetting, the collapse occurs in three different phases. The first deformation is characterized by a slow rate of settlement, explained as due to the sudden reduction in air coefficient of permeability at both saturated ends. In the second phase, settlement progresses at an increased rate on a logarithmic scale. This fact was explained as a result of the progressive increase in the soil coefficient of permeability with respect to water as the suction dissipates with time. In the final phase, the settlement again progresses at a slow rate while the soil permeability becomes almost constant along the saturated specimen.

Popescu (1986) performed tests on natural loess specimens which had a small percentage of clay. The tests were performed without controlling or measuring suction, in oedometer rings where the specimens were first loaded and then inundated from the bottom. The collapse versus time measurements as well as the collapse versus applied load are shown in Figure 2.14.

To explain the progress of collapse with time, Popescu (1986) divided the phenomenon of collapse in two different phases. The first phase was termed "unstable", and was characterized by a gradual decrease in the rate of deformation over a short period of time. The second phase was characterized by an almost constant and low rate of deformation
and lasted longer than the first phase. The first phase was attributed to the destruction of bonds that were completely dependent of suction, while the second phase was more related to dissolution of carbonated bonds and secondary compression.

Figure 2.13 Time settlement curve for a collapsible soil (from Lloret and Alonso, 1980).

Figure 2.14 Collapse versus time on natural loess specimens (from Popescu, 1986).
Tadepalli et al. (1992) conducted tests in a specially designed oedometer with matric suction measurements. An Indian Head silty sand was statically compacted in an oedometer ring. Small-tip tensiometers were used to measure the matric suction at different points along the sample. The specimen was then consolidated, at a constant water content, under a constant vertical load. After consolidation, the soil specimen was inundated (generally from the bottom). During inundation, the matric suction and volume decrease were measured simultaneously at various elapsed times. Figure 2.15 shows the results obtained during the inundation of compacted Indian Head silty sand.

Figure 2.15 shows that the collapse occurred over a short period of time. Most important in Fig. 2.15 is the fact that the collapse occurred in accordance with a decrease in matric suction. The rate of collapse was high and totally dependent on the dissipation of the matric suction in the specimen. There was a unique relationship between changes in the matric suction and the total volume change during collapse. It was evident that any bonds acting at the contact points were highly dependent on suction.

2.3.3.5 Mechanical behavior of a collapsible soil

Engineering practice needs phenomenological approaches or analyses, in order to develop practical solutions for general problems. A collapsible soil is an unsaturated soil and consequently must be studied using the theory for unsaturated soils. A soil collapse model must be able to properly reproduce the progress and magnitude of soil collapse as a function of the two independent stress variables, matric suction and net normal stress, governing the mechanical behavior of unsaturated soils.

Despite some references to the applied loads, Section 2.3.3.2 mainly focused on soil collapse behavior as a function of decreasing matric suction during the wetting process. However, as required by the theory for unsaturated soils, soil collapse must also be understood as a function of the external loading (or net normal stress). While the process of
collapse is primarily a function of the decrease in matric suction. The magnitude of the collapse is also dependent on the magnitude of the applied load.

![Graph](image)

Figure 2.15 Matric Suction and total volume changes versus time after inundation of a compacted specimen of Indian Head silty sand (from Tadepalli et al. 1992).

There is a gradual increase in compressibility as well as a gradual decrease in shear strength of a collapsible soil during the saturation process (Jennings and Knight, 1957; Dudley, 1970; Clemence and Finbarr, 1981; Maswoswe, 1985; Lloret and Alonso, 1985; Popescu, 1986; Miranda, 1988; Fredlund and Rahardjo, 1993). There is also a consensus that after the collapse the soil possesses a different structure from the original clayey bonds controlling the soil behavior at the pre-collapse condition.
An earth dam structure constructed of a collapsing soil presents a complex mechanical behavior during the transient unsaturated-saturated process that takes place during the first reservoir filling. The wetting-front advance causes changes in both compressibility and shear strength of the collapsible soil. The post-construction state of equilibrium is altered and load transfer occurs in a progressive manner in the earth dam structure between zones of different compressibility. Differential settlement takes places as a combined result of load transfer and differential stiffness between adjacent zones in the dam. In terms of shear strength, a collapsing soil element in the dam may experience two different and simultaneous effects. The first effect is due to a matric suction decrease which leads to a decrease in the soil shear strength. The second effect is associated with the net normal stress that can either decrease or increase on the soil element. Therefore, depending on its relative position in the dam a soil element can experience either an increase or a decrease in shear strength. The collapsible soil fabric can also play an important role in the shear strength.

The above paragraphs emphasize the importance of collapsible soil modelling in the prediction of the mechanical behavior of a collapsing earth structure. The soil modelling must be able to adequately represent the compressibility as well as the shear strength of the collapsible soil as function of the stress state variables controlling the behavior of an unsaturated soil.

**Volume change behavior**

Some compacted collapsible soils do not undergo collapse during saturation. These situations involve cases where the applied load is insufficient to provoke the collapse. These soils can collapse (i.e., suffer a complete rearrangement in fabric) when the applied load is increased. After saturation, this rearrangement is the well known consolidation of a saturated soil and can be dealt using conventional theories.
Some compacted collapsible soils, even with a low clay content, may exhibit swelling characteristics when saturated under low loads. Such a behavior has been explained (Villar et. al. 1981; Maswoswe, 1985; Miranda, 1988) by using Jennings and Burland’s (1960) model. The swelling behavior is explained as due to the absorption of water by the clay aggregates, with no significant change in terms of the coarser particles.

The research literature has defined the important features of the collapsing behavior of a compacted soil. Some of these characteristics related to compacted clayey sands, are briefly described. Jennings and Knight (1957) proposed the double-oedometer test to define the potential for soil collapse. From this test, the soil collapse is defined, for a given vertical load, as the difference in volumetric deformation between two identical specimens loaded under different saturation conditions. One of the specimens is loaded at “natural” or “as-compacted” conditions while the other is saturated and then loaded. The following conclusions from the authors are nowadays widely recognized and used in engineering practice of problems involving collapsing soils:

a.) the magnitude of collapse is a function of the vertical load. This collapse magnitude increases up to a specific vertical load and then reduces to zero value at high vertical load when the wetted curve coincides with the “as-compacted” curve,

b.) at a given vertical load, the “as compacted” specimen will collapse to the wetted curve. For additional loadings, this specimen will follow the saturated curve.

From the “double-oedometer” test results, Jennings and Knight (1957) proposed the following constitutive relationship for a collapsing soil during saturation.

\[ \Delta \varepsilon_v = \beta \frac{3\mu^2 - 2\mu + 1}{(1 - \mu)^2} \sigma_v (u_a - u_w) \_o \]  \[ 2.41 \]

where:

\( \sigma_v = \) the vertical stress applied to the specimen during saturation (kPa).

\( (u_a - u_w)_o = \) initial matric suction in the specimen (kPa),
\( \Delta \varepsilon_v = \) the vertical strain (i.e., the volumetric strain) corresponding to the difference between the void ratios in unsaturated and saturated curves of the double oedometer test at the vertical stress \( \sigma_v \).

\( \beta = \) a constant coefficient relating volumetric deformation and matric suction at a given vertical stress in a double oedometer test (kPa\(^{-2}\)).

Defining \( \beta \) as a constant assumes that the volumetric strain is linearly related to the suction variation.

Escario et al. (1973) performed modified oedometer tests on three series of specimens of a clayey sand from Madrid having an optimum water content of 11\% and maximum AASHTO density of 1.98 Mg per m\(^3\). The pore-air and pore-water pressures were controlled independently. All three series of specimens were compacted to the same density (i.e., 80\% of maximum AASHTO density). The three series were compacted at different water contents. The first series was compacted at a water content of 3\%. The other series were compacted at water contents of 6\% and 8\% respectively. Similar trends of collapse versus matric suction were obtained for each series tested. The test results for the specimens compacted at water content of 3\% are illustrated in Fig. 2.16.

Figure 2.16 shows different curves corresponding to different vertical loads which were applied to the specimens. The specimens were consolidated under an initial suction equal to the "compaction matric suction" value. The "compaction matric suction" was gradually decreased in stages until the specimen was completely flooded. According to Fig. 2.16, it was reported that settlement caused by the suction decrease process are not large until relatively low suction values were attained. A further reduction in suction accelerated the process considerably. It was observed that there was a slight increase in soil compressibility due to the increase in water content. It was also observed that upon complete flooding, the total final settlement for a given vertical load reached essentially the same magnitude, independent of the initial water content (or initial matric suction). Another important observation from these tests is the non-linear relationship between the progress of collapse
and the decrease in matric suction. Similar non-linear relationships between deformation and matric suction were also reported by other authors (Barden et al. 1969; Booth, 1973).

![Figure 2.16 Settlements curves versus matric suction under different vertical stresses for a collapsing soil specimen (from Escario et al. 1973).](image)

Masagoszwe (1985) reports several suction controlled triaxial tests in low plasticity sand clay (Lower Cromer Till). The soil properties are as follows: $w_L = 25\%$, $w_p = 12\%$; clay content = 17\%; percentage of sand > 50\%; clay activity = 0.71. Specimens were statically compacted dry of optimum but relatively close to optimum water content. A typical stress path involved vertical loading at a constant water content followed by suction reduction (soaking) at a constant vertical stress. These are Ko (i.e., oedometer) type tests performed in a triaxial cell. Radial strains were monitored and the stress state was modified to ensure Ko conditions. As a result, the state of stress in the sample was always known. Masagoszwe (1985) conducted tests on specimens compacted at three different initial void ratios (i.e., $e_o =$
0.66, e_o = 0.46 and e_o = 0.33). Specimens reflecting low compactive efforts (i.e., e_o = 0.66) presented significant wetting-induced collapse under applied vertical loads of about 190 kPa. A non-linear relationship between wetting-induced collapse and matric suction was reported. In addition, Maswoswe (1985) observed that during collapse the horizontal stresses on the soil-specimens had to be increased to ensure Ko-conditions. In structural terms, this implies an increase in Poisson ratio of the collapsing soil during saturation. Similar increases in Poisson ratio for collapsing loess is also reported by Handy (1995).

Lawton et. al. (1991a) presented an analysis involving the three-dimensional aspects of soil collapse. A clayey sand containing by weight about 15% of particles finer than 0.002 mm was tested. The testing program consisted of triaxial collapse tests on nominally identical specimens compacted to 85% of modified AASHTO maximum dry density at an optimum water content of 10%. Under these conditions, the compacted specimens were slightly expansive when wetted at low applied loads and collapsible at high loads. A procedure similar to the double-oedometer test was designed to test the soil collapse under anisotropic stresses applied using triaxial equipment. Different principal stress ratios, (σ_1/σ_3), were imposed to pairs of specimens where the “as-compacted” and “wetted” curves were defined by the “double-triaxial tests”. The authors concluded that the magnitude of volumetric strain resulting from a change in stress state or from wetting, depended on the mean normal total stress and was independent of the principal stress ratio. However, the individual components of volumetric strain (i.e., axial and radial strain) depended on principal stress-ratio. For a given mean normal total stress, the magnitude of axial collapse increased and the magnitude of radial collapse decreased with an increasing stress ratio. Figures 2.17, 2.18 and 2.19 illustrate Lawton’s et al. (1991a) findings.

Lawton’s et al. (1991a) research provides valuable information for the modelling of the collapsing soil behavior since it relates the total volumetric wetting-induced collapse to the mean net normal total stress. In addition, it illustrates that depending on the applied stress ratio a collapsing soil can even undergo expansion to the direction of the minor total stress.
Figure 2.17 Volumetric strain as function of mean normal total stress for soaked and as-compacted triaxial tests (from Lawton et al. 1991a).

Figure 2.18 Axial strain difference as function of mean normal total stress for triaxial tests (from Lawton et al. 1991a).
Figure 2.19 Radial strain difference as function of mean normal total stress for triaxial tests (from Lawton et al. 1991a).

during saturation. Such an information is in agreement with the data previously presented by Maswoswe's (1985) and Handy's (1995). Besides, Lawton et al. (1991a) demonstrate that a collapsible soil is an unsaturated soil whose behavior is strongly dependent on the stress path since that either changes in matric suction or net normal stresses can originate collapse deformations. Therefore, the definition of a laboratory program for modelling the collapsing soil behavior must be guided by the choice of stress paths which best reproduce the field conditions.

In a phenomenological way (and according to Fredlund and Rahardjo, 1993) the theory for unsaturated soil for the volume change of a collapsible soil can be characterized by using the compressibility form presented in section 2.3.1.3. According to that information, and based on Lawton et al.'s (1991a) findings, the moduli form of the constitutive surface
equations for a collapsible soil can be written as a function of the average total stress and matric suction. The volumetric deformation of the soil structure can be expressed as,

\[ d\varepsilon_v = m_1^i d(\sigma_{\text{mean}} - u_a) + m_2^i d(u_a - u_w) \]  \[2.28\]

where:

\[ m_1^i = \frac{3(1-2\mu)}{E}, \text{ and } m_2^i = \frac{3}{H} \]

The water phase volume change can be expressed as,

\[ \frac{dV_w}{V_o} = m_1^w d(\sigma_{\text{mean}} - u_a) + m_2^w d(u_a - u_w) \]  \[2.29\]

where:

\[ m_1^w = \frac{3}{E_w}, \text{ and } m_2^w = \frac{1}{H_w} \]

A collapsible soil exhibits a volume decrease as a result of a reduction in matric suction. According to Eq. 2.28, if the net normal stress is kept constant, the soil structure must undergo a volume decrease due to a decrease in matric suction. By using the conventions suggested by Fredlund and Rahardjo (1993) (section 2.3.1.3), this implies that a collapsible soil has a positive compressibility modulus associated with a change in matric suction, (i.e., \(m_2^w\)). Depending on the initial degree of saturation and the reduction in porosity during collapse, it is possible that the volumetric water content may decrease as the matric suction decreases. In this case, the compressibility water phase modulus associated with a change in matric suction is also positive, (i.e., \(m_2^w\)). The change in the initial volume, \(V_o\), emphasizes the necessity of using an incremental procedure to analyze problems associated to collapsing soils.
The moduli form of the constitutive relationships presented by Fredlund and Rahardjo, (1993) (see Equations 2.21, 2.22, and 2.23 in section 2.3.1.3) is required to discuss the stress path followed by a collapsible soil during saturation. Fredlund’s and Rahardjo’s (1993) moduli form of constitutive equations assume isotropic mechanical properties for an unsaturated soil. This implies a positive value for the isotropic elasticity modulus for the soil structure relative to a change in matric suction (i.e., H) for a collapsible soil, since $m_z$ is positive. An isotropic modulus H results in isotropic wetting-induced collapse of a soil element in response to a decrease in matric suction, independent on the total stress state applied on the soil element. In consequence, the isotropic formulation predicts reductions in lateral (i.e., radial) stresses for a soil specimen which undergoes wetting-induced collapse under a constant vertical load and under Ko-conditions. Such a prediction contradicts the previously discussed available experimental data reported by Maswoswe, (1985) and Handy, (1995). Previous studies suggest that isotropic wetting-induced soil collapse can occur on a soil specimen under an isotropic total stress state (Lawton et al. 1991a). In addition, Lawton et al. (1991a) findings predict that during triaxial wetting-induced collapse a soil specimen undergoes anisotropic deformations which are functions of the applied anisotropic stress state. Therefore, an stress induced anisotropic modulus H appears to be a reasonable preliminary alternative to the theory of unsaturated soils to properly model the collapsing behavior of a soil during saturation. An stress induced anisotropic formulation for the elasticity form of the constitutive equations for a collapsing soil is proposed in the present research study. By using a xyz-Cartesian system the proposed equations are as follows:

\[
\varepsilon_x = \frac{(\sigma_x - u_a)}{E} - \frac{\mu}{E} (\sigma_y + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H_z} \tag{2.41}
\]

\[
\varepsilon_y = \frac{(\sigma_y - u_a)}{E} - \frac{\mu}{E} (\sigma_x + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H_y} \tag{2.42}
\]
\[ \varepsilon_z = \frac{(\sigma_z - u_s)}{E} - \frac{\nu}{E} (\sigma_z + \sigma_y - 2u_a) + \frac{(u_a - u_w)}{H_z} \]

where:

- \( H_i = H/(1+H\chi_i) \), is the elasticity modulus for the soil structure in the \( i \)-direction relative to a change in matric suction (i.e., \( u_a - u_w \))
- \( H \) = isotropic elasticity modulus (function of the mean net total stress) for the soil structure relative to a change in \( u_a - u_w \)
- \( \chi_i \) = stress induced anisotropic collapse factor in the \( i \)-direction (function of the stress ratios \( \sigma_i/\sigma_j \) and \( \sigma_i/\sigma_k \)) for the soil structure relative to a change in \( u_a - u_w \)
- \( i, j, k \) = directions of a three-orthogonal coordinate system (e.g., \( x, y, z \)).

In addition and in consequence of the assumption that the volumetric wetting-induced collapse is a unique function of the applied mean net stress, comparison between Eq. 2.28 and Eqs. 2.41 to 2.43 allows to define the following relationships for \( H_i \)'s and \( \chi_i \)'s. \( \chi_i \)'s are non-linear soil parameters with values equal to zero for an isotropic stress state. Additional research on these parameters is required in order to better characterize the behavior of an unsaturated soil during collapse.

\[ m_i^r = \frac{1}{H_x} + \frac{1}{H_y} + \frac{1}{H_z} \]

\[ x_z + x_y + x_z = 0 \]

Further theoretical developments of the stress induced anisotropic constitutive equations for the modelling of collapsing behavior of soils are later discussed in Chapter 3. Applications of the modelling of collapsible soil behavior are presented in Chapters 6 and 7.
Shear strength

The linear form of the shear strength equation for an unsaturated soil was presented as:

\[ \tau_{ff} = c' + (\sigma_f - u_a)_f \tan \phi' + (u_a - u_w)_f \tan \phi_b \]  \[2.32\]

The definition of the soil strength parameters \(c', \phi'\) and \(\phi_b\) becomes more complex for a collapsible soil. The main complexity is associated with the metastable structure of the soil. Depending upon the total stress applied to the soil, a metastable structure can change to a stable structure when collapse occurs. A typical collapsible soil usually has a fraction of clay between 10% and 20%, while the remainder is composed of coarser material; mainly silt and fine sand. As discussed before, the clay particles may be present in different forms in the soil structure. Collapse occurs as a result of shear failures at water-softened interaggregate or intergranular contacts, or the softening and distortion of clay aggregates.

In a drained direct shear test, a soil specimen is assumed to remain under a constant net normal stress and matric suction. Depending on the matric suction and the vertical stress applied, the shearing process can induce a progressive and gradual transference of the shearing forces from the clay bonds and/or clay aggregations to the coarser particles at the failure plane. For high values of matric suction clay aggregations may remain intact after shearing and behave similar to sand. For low values of matric suction, the clay bonds and/or aggregations may collapse, depending on the net normal stress. The shearing of clay bonds and/or aggregations means a decrease in the soil cohesion and might mean an increase in the frictional strength on the failure plane. The magnitude and rate of this transference depends on factors such as: applied stress state, initial condition of the specimen, and the rate of shearing.
The influence of soil collapse on shear strength still needs additional research (Schmertmann, 1976; Alonso et. al., 1985; El-Sohby et. al., 1987; Handy, 1995). From previous studies it seems reasonable to evaluate the shear strength of a collapsible soil in an experimental manner. This means that the shear parameters $c'$, $\phi'$ and $\phi^b$, required for Eq. 2.32 must be obtained using a range of stress state variables and stress paths compatible with the problem being analyzed.

2.3.3.6 Fluid flow and hydraulic properties

Darcy's and Fick's laws are usually used as the flow laws for the water and air phases even when there are changes in the structure of a collapsing soil during saturation (Lloret et. al., 1980; Lourens and Czapla, 1987; Miranda, 1988). The non-uniqueness of the state surface for degree of saturation poses the main problem for the definition of the constitutive parameters for both the air and water phases. For conditions where the saturation of the soil voids occurs in a monotonic wetting manner, such as in the case of the first reservoir filling of collapsing earth dams, the problem is minimized and unique constitutive parameters can be defined. The constitutive parameters can be defined for any point of the specific wetting surface state. In experimental terms, a hydraulic gradient can be imposed to the soil at a specific state point and the hydraulic properties defined. The applied gradient should not alter the saturation condition of the soil element.

The above reasoning can be applied for both air and water phases. The main difficulties arise from the practical problems associated with such experiments. The problems involved in such experiments can be stated as follows:

a.) the small amount of water that flows through the soil at low degree of saturation (Alonso et. al. 1985; Fredlund et. al. 1993; Juca, 1993)

b.) the diffused air flowing through the high air entry values porous stones (Fredlund et. al. 1993)
c.) long times are required for performing these kind of test (Fredlund et. al. 1993)

d.) the time for moisture equilibrium is affected by the path followed, the suction gradient and the impedance of the drainage system (Fredlund et. al. 1993; Juca, 1993)

e.) at a low water content, the water in soil does not make good contact with the drainage systems in order to facilitate continuous flow. In this case, a major part of the water movement occurs by vapor transfer under a low velocity. (Juca, 1993).

Juca (1993) performed permeability tests on three different compacted soils; a gray clay with 72% of clay fraction, a red clay with 35 % of clay, and a clayey sand with 6 % of clay fraction. The specimens were compacted at maximum dry standard Proctor density and dry of optimum conditions, for the clayey sand at minus 2.3% dry of optimum conditions. At this condition, the clayey sand showed an initial matric suction of 70 kPa. The matric suction ranged from 0 up to 450 kPa during the measurement of the hydraulic conductivity of the soils. The hysteresis behavior of the suction-water content curves under wetting-drying cycles is shown in Fig. 2.20.

![Graph showing hysteresis behavior of tested soils](image)

Figure 2.20 Hysteretic behavior of the tested soils (from Juca, 1993)
Some features illustrated in Figure 2.20 are: a.) The sharp desaturation of the clayey sand from an increase in suction from 0 to 70 kPa. For both drying and wetting paths, the degree of saturation was less than 40% at 100 kPa of matric suction; b.) the wetting path did not reach the 100% degree of saturation at zero matric suction; c.) the hysteretic behavior of the soil decreased with a decrease in clay content. In fact, it was attributed to the shrinkage behavior associated with the clay fraction; d.) the residual water content for the drying curve of the clayey sand was reached before 100 kPa of suction.

The coefficient of permeability of the specimens were measured and Fig. 2.21 illustrates the hydraulic behavior of the compacted clayey sand. It can be seen that the permeability decreased by 5 orders of magnitude for a variation of suction from 0 to less than 100 kPa of matric suction.

![Hydraulic conductivity of a clayey sand](image)

**Figure 2.21** Hydraulic conductivity of a clayey sand (from Juca, 1993).

Juca (1993) explained the higher values of permeability for the 0 to 70 kPa range by suggesting that there may be a sharp transition in this range. Oscillations on the measured
values from 100 to 450 kPa were justified by the experimental problems previously mentioned. Juca's (1993) experiments were conducted on unloaded soil specimens which did not collapse during saturation despite their metastable structure. Therefore, these experiments did not take into account the influence of the soil collapse on the hydraulic properties of the compacted clayey sand.

**Permeability to water of a metastable-structured soil**

The literature makes use of relationships where the soil permeability with respect to air and water phases are functions of void ratio and/or degree of saturation (Lloret and Alonso, 1980; Miranda, 1988; Fredlund et. al. 1993; Alonso et. al. 1995).

Lloret et al. (1980) used relationships between soil coefficient of permeability with respect to water and the volume-mass properties of an collapsing soil. This relationship is used in a numerical model to simulate the collapsing behavior of a compacted clayey silt during wetting-induced collapse in an oedometer test.

Miranda (1988) made use of the drying soil-water characteristic curve of the collapsible soil to develop the relationship between the soil permeability with respect to water and matric suction by using the Brooks and Corey's method (1964). In this approach, both the change in overall volume and the hysteresis effects were neglected. Miranda (1988) used this relationship in a numerical model used to predict the mechanical behavior of small earth dams during the first filling of the reservoir.

Depending on the magnitude of the soil collapse, an abrupt change in the soil structure may affect the soil coefficient of permeability of the soil in its saturated conditions. Available methods express the unsaturated water permeability as function of the saturated water coefficient of permeability and the soil-water characteristic curve (Brooks and Corey, 1964; van Genuchten, 1980). For a collapsible soil the saturated coefficient of permeability may be highly affected by the applied net normal stress. Existing equations relating the
coefficient of permeability and void ratio may be not valid for a collapsible soil (Lambe and Whitman, 1979).

A collapsible soil usually maintains an open structure that allows an almost complete desaturation of the soil at relatively low matric suctions. This means that the soil has a low permeability to the water phase and a high permeability to the air phase at low levels of matric suction. When saturating the collapsing soil may undergo collapse depending on the applied net normal stress. Theoretically, this process can be controlled using experiments which apply discrete changes in the stress states variables.

Huang (1994) performed tests on a clayey silt by using a triaxial permeameter. From his experiments, it was possible to obtain a relationship between the soil coefficient of permeability with respect to water and the stress state variables for a deformable soil. The drying stress path was used and the specimens were loaded and consolidated at increasing net normal stresses. Volume change measurements were made and the void ratio was controlled and related to the saturated coefficient of permeability. Based on Huang’s (1994) findings it appears that the soil coefficient of permeability with respect to water of a collapsible soil requires further research in order to well define and formulate a theoretical basis.

**Permeability to air of a metastable-structured soil**

Lloret and Alonso (1980) used Darcy’s law (i.e., Eq. 2.39) for the relationship between the soil permeability with respect to air and soil volume-mass properties when simulating the behavior of a collapsible soil during “wetting-induced” collapse. Experimental values were not available to compare with calculated values, and their analysis was limited to a qualitative assessment showing the gradual decrease in the air coefficient of permeability as saturation increases.

Alonso et. al. (1985) emphasized that for most practical problems involving unsaturated soils, the air phase can be considered to remain at atmospheric conditions and air
flow can be neglected, thus eliminating the need for the air coefficient of permeability determination.

Miranda (1988) performed a numerical analysis of a small collapsing earth dam. The compacted soil was initially at a low degree of saturation. It was assumed that the air phase remained at constant atmospheric conditions during the post-filling period. This assumption was justified by the open structure of the compacted collapsing soil as well as by his assumption regarding one-directional advance of water flow after filling of the reservoir.

2.4 Collapsing earth dams

Before the first filling the state of stress within the embankment is a function of the type of material used in the construction, the water content and dry density of the compacted material, the compaction process and energy, and the geometry of the dam.

During the first filling of the reservoir, large deformations can occur rapidly in collapsing dams (Peterson and Iverson, 1953; Miranda, 1988). Large differential settlement can results in internal cracking and the water flowing through these cracks may lead to hydraulic failure of the embankment (Sherard, 1986; Miranda, 1988).

Miranda (1988) reported that during the drought of 1979 through 1983 about 20,000 dams were built, enlarged or rehabilitated in the State of Ceara', in northeast Brazil, by the Emergency Program. A commission of engineers examined 720 of these dams and concluded that about 80 percent were going to fail in the next rainy season. It was reported that the compaction of the material was, in general, very deficient, almost always without the use of water. The reason for this type of construction was explained as follows: "the difficulty to provide water to satisfy the most elementary necessities of the people did not permit the use of such a precious liquid in the construction of dams".
2.4.1 Stability analysis and hydraulic behavior of earth dams

The conventional design of earth dams requires that stability analysis for the earth dam be performed for three distinct phases of the dam life, namely, the after-construction, under steady-state condition after reservoir filling, and the eventual rapid drawdown. Conventional stability analysis involves the use of limit equilibrium method to evaluate the stability of both upstream and downstream slopes of the dam. Safety factors must satisfy minimum requirements. The last two phases of design require that a seepage analysis be performed in order to properly assess the safety of the dam. Seepage analyses used to be performed using a conventional graphical net flow net or more recently, by means of more realistic numerical procedures (Newman, 1973; Papagiannakis, 1982; Lam et al. 1987).

The limit equilibrium and seepage analysis are performed by using separated procedures, and then are combined to define the factors of safety and hydraulic behavior for the earth dam. Limit equilibrium methods provide a powerful means of evaluating overall stability. The analyses are based on an assumed sliding wedge or curve slip surface. The most serious shortcoming of the limit equilibrium method is that it provides no information about the failure mechanism. These methods do not take account of the stress-strain behavior of the soil. There is no indication when yield starts or how it develops. Improvements on slope stability analyses have been achieved by using the limit equilibrium method in combination with stress-strain analysis from a finite element method (Naylor, 1982; Fredlund, 1991; Faria. 1993).

Stability analysis of earth structures requires the formulation of a failure criterion. The failure criterion establishes the conditions at which the shear strength of the soil is reached. Constitutive relationships for the soil phases are then required for the post-failure soil behavior. Soil mechanics practice have traditionally simplified stability analysis of earth structures to the plane strain condition. In this two-dimensional simplification the influence
of stresses from the direction normal, (e.g., z-axis), to the plane (e.g., xy-plane) are neglected. Under this assumption the factor of safety of an earth dam can be evaluated by using the limit equilibrium method.

In the analysis of the stability of an earth structure the limit equilibrium method requires that two assumptions be satisfied. The first assumption is the definition of the geometry of a potential slip surface. The second assumption is about the stress distribution along a given potential slip surface. The strength of finite element based methods lies in its potential to answer these two questions. For the first assumption, a stress distribution from a stress-strain analysis can be used to calculate local factors of safety by comparing the shear stress to the shear strength at any point in the dam. The distribution of local factors of safety can be used as a guideline for the definition of the failure mechanism and potential slip surfaces in the dam. The second assumption is answered in a direct way by the stress distribution in the dam that results from a finite element analysis.

The overall stability of an earth dam can be evaluated by using the stress field produced by a finite element analysis in conjunction with a conventional limit equilibrium method. This combined procedure is sometimes called the “enhanced limit method”, (Naylor, 1982; Faria, 1993). Figure 2.22a illustrates the geometry and stress components along a trial slip surface. Figure 2.22b shows the importance of properly constituted non-linear analysis. where: i.) the dashed line shows the variation of shear strength, \( \tau_f \), along AB; ii.) the actual shear stress distribution is indicated by the full line.

A properly constituted non-linear analysis should produce a shear stress distribution lying entirely under or on the strength line. However, for linear analysis, the shear stresses may exceed the strength line in regions of local yield.

Based on its original form (i.e., Eq. 2.32) and on Fig. 2.22a, the Mohr-Coulomb failure criterion for the two-dimensional condition can be expressed as follows:
\[ \tau_f = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_n) \tan \phi^b \]  \[ \text{[2.32]} \]

where:

\[ (\sigma_n - u_a) = (\sigma_x - u_a) \sin^2 \theta + (\sigma_y - u_a) \cos^2 \theta - 2\tau_{xy} \sin \theta \cos \theta \]

---

Figure 2.22 Shear stress and shear strength distribution along a trial slip surface
(from Faria, 1993).

The safety factor is defined as the ratio of the area under the strength curve to the area under the computed shear stress curve.

\[ F_s = \frac{\int \tau_f dl}{\int \tau_n dl} \]  \[ \text{[2.46]} \]

where:

\[ \tau_n = (\sigma_y - \sigma_x) \sin \theta \cos \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta) \]  \[ \text{[2.47]} \]

"This definition is equivalent to that used in conventional limit analysis where, at least for circular slips, it is usually expressed as the ratio of righting to disturbing moment", (Faria, 1993).
The conventional procedures for stability analyses have been successfully applied through the years in the design and construction of dams with stable-structured soils. However, both slope stability and seepage problems have been experienced when collapsing materials are present in the dam structure. Following are some examples from literature.

2.4.2 Failures of collapsing earth dams during reservoir filling

The literature contains considerable information related to wetting-induced soil collapse to failure of both large and small homogeneous earth dams. Failure mechanisms are associated with the collapse of the wetted and then softened soil in the upstream zone in the dam. Collapse in that zone alters the initial (i.e., after-reservoir filling) static equilibrium in the dam leading the structure to search for a new equilibrium configuration. This results in load being transferred between adjacent zones of the dam with different stiffness. Differential settlements and cracking are the main consequence of the load transfer in the earth dam. Seepage through cracks gradually spreads the collapse to other dam zones and a complex problem is posed to engineering practice.

Peterson and Iverson (1953) reported the behavior of two low earth dams built in Western Canada in the late 1930’s and early 1940’s that failed. Another dam suffered distress as a result of collapse. It was reported that soon after reservoir filling, the dams start showing upstream movements and evidence of differential settlements along the crest. These movements and settlements were associated with wetting-induced collapse of the wetted upstream zone of the embankment.

Sherard (1953) presented the analysis of the first filling of the Rector Creek dam. This dam presented longitudinal cracks in the downstream slope and significant movement in the upstream direction at the beginning of the reservoir filling. With a rise of the water level, the dam returned to its end-of-construction’s position. Sherard described the non-uniform.
wetting-induced collapse of the fill as the key component for the movements and cracks observed.

Marsal (1960) described the behavior of the Cuauhtemoc dam which presented significant differential settlements and associated fissures, at the beginning of the reservoir filling. It was concluded that the wetting-induced collapse of the upstream zone of the dam was the responsible for the problems which occurred.

Leonards and Narain (1963) reported on the analysis for cracking of four earth dams built in California. It was concluded that in three dams, the primary causal factor was differential settlements resulting from wetting-induced collapse within the embankment material.

Holestol et al. (1965) describe the performance of the Venemo dam in Norway. The occurrence of differential settlements in the dam was reported during the first filling. It was concluded that the wetting-induced collapse in one zone of the dam was proportional to the acting stress state, and this fact was the primary reason for the differential settlements observed.

Leonard and Davidson (1984) also mentioned the wetting-induced collapse as a key component of the collapse of Teton Dam in Idaho in 1976.

Miranda (1988) reported that piping failures resulting from hydraulic cracking initiated by wetting-induced collapse in earth dams located in the semi-arid region of northeast Brazil are so common that these dams are publicly referred to as “Alka-Seltzer” dams. Evidences of differential settlements associated with superficial fissures are also reported in some of these dams, Miranda (1983).

All of these cases lead to the conclusion that in each dam there is a contiguous zone, often quite extensive, of relatively dry and loose compacted material existing prior to distress or failure. In the specific case of “Alka-Seltzer” dams, Miranda (1988) reported that failed Brazilian earth dams were typically constructed with soil compacted at its natural water content (usually 4% to 6%) because of the high cost of transporting water to the site.
Compaction was generally light (i.e., dry density of about 80% to 90% of the maximum standard Proctor dry density), with resulting degrees of saturation of about 10%. In contrast, dams built with similar soils but compacted at optimum water content condition (i.e., 14% to 16%) and at maximum standard Proctor dry density generally performed well.

The distress in these dams typically occurred during or shortly after the first reservoir filling. Failure or reservoir leakage in the Brazilian dams was primarily attributed to piping through holes that were created by arching above the collapsed soil (Miranda, 1988). In addition to the hydraulic failures, there was often the occurrence of fissures and cracks that represent risks for the stability of the whole dam. It would appear that collapsing dams failures are usually associated with either piping due to hydraulic fracturing or general instability of the shell slopes, or a combination effect of both.

2.4.3 Analysis of collapsing earth dams

"Deformation of unsaturated soil depends on the intensity of the applied load and wetting history of soil. Temperature changes are also relevant in soil behavior. The flow of water, air and heat will induce changes in the fluid pressures and temperature which will result in soil deformations and changes in degree of saturation. The three effects will alter the stress state. On the other hand, any stress and fluid pressure change will modify the permeabilities and thermal conductivity of the soil. In conclusion, the flow of air, water and heat and the deformation phenomena are highly coupled", Lloret and Ledesna, (1993).

The general solution for the analysis of mechanical behavior of a collapsing earth dam during its first reservoir filling involves the dynamic interdependence of transient pore-fluid flow, (i.e., air and water, and the soil structure equilibrium).
2.4.3.1 Physics involved in the problem of collapsing dams

The basic physics equations for the component soil phases of an unsaturated soil can be used in the prediction of complex flow-deformation problems. A rigorous treatment of the isothermal fluid flow-deformation problem in soil mechanics requires the simultaneous solution of the following basic equations:

a.) mechanical equilibrium

b.) water continuity

c.) air continuity

A rigorous formulation of the three-dimensional flow-deformation problem in soil mechanics was first proposed by Biot (1941) to analyze the consolidation of soils saturated by a compressible pore-fluid. In this particular case, the compressible pore-fluid represented a mixture of water and trapped air bubbles. Biot expressed his equations as follows:

a.) equilibrium equations:

\[ \sigma_{ij,j} + b_i = 0 \]  \hspace{1cm} [2.48]

where:

\[ \sigma_{ij} \] = the total stress tensor

\[ b_i \] = the body forces.

b) Water continuity equation.

For a compressible pore fluid, the volume decrease of a soil element is the sum of the net flux of water into the soil element plus the decrease in elemental volume of the pore-fluid. The continuity equation in differential form can be written in a simplified form (Duncan and Chang, 1983) as follows,
\[-v_{ij} + \dot{w}_{ij} - \frac{\dot{u}}{Q} = 0\]  \hspace{1cm} [2.49]

where:

- \(v_i\) is the superficial velocity vector.
- \(\dot{w}_{ij}\) = rate of volumetric strain change of the soil element (i.e., time derivative).
- \(\dot{u}\) = rate of change of excess pore-water in the pore-fluid.
- \(1/Q\) = compressibility of the fluid.

Closed-formed solutions of the three-dimensional consolidation theory developed by Biot (1941, 1955, 1956) were obtained only for simple cases due to the complexity of the governing equations.

Lloret and Alonso (1980) proposed a simplified procedure to analyze the consolidation of unsaturated soils. In their procedure, the continuity equations for the air and water phases are formulated and solved simultaneously. The deformation analysis is performed separately by using the constitutive relationship of the soil structure. Their equations were expressed as follows.

a) Air continuity equation:

\[\frac{\partial}{\partial t} \left[ Q_a n (1 - S + H_c S) \right] + \nabla \cdot \left[ Q_a v_a + Q_a H_c v_w \right] = 0\]  \hspace{1cm} [2.50]

where:

- \(Q_a\) = mass density of air
- \(n\) = porosity
- \(S\) = degree of saturation
- \(H_c\) = Henry's constant \((nH_cS\) is the volume of dissolved air in the pore-water phase)
- \(v_a = v_a^i \mathbf{i} + v_a^j \mathbf{j} + v_a^k \mathbf{k}\), macroscopic velocity vector of air
- \(v_w = v_w^i \mathbf{i} + v_w^j \mathbf{j} + v_w^k \mathbf{k}\), macroscopic velocity vector of water
\[ \nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \], the divergence operator.

b.) Water continuity equation:

\[ \frac{\partial (Q_w nS)}{\partial t} + \nabla \cdot (Q_w v_w) = 0 \tag{2.51} \]

where:

\[ Q_w = \text{mass density of water.} \]

In engineering practice, water can be considered as an incompressible material. (i.e., \( Q_w \) is constant) and Eq. 2.51 can be written as:

\[ \frac{\partial (nS)}{\partial t} + \nabla \cdot (v_w) = 0 \tag{2.52} \]

or in the form,

\[ \frac{\partial (\theta_w)}{\partial t} + \nabla \cdot (v_w) = 0 \tag{2.53} \]

where:

\[ \theta_w (= nS), \text{ is the volumetric water content.} \]

Lloret and Alonso (1980) used their equations to produce a numerical solution for the consolidation of an unsaturated soil. The numerical procedure was also used to simulate the wetting-induced swelling and collapsing soil behavior.

Dakshanamurthy et al. (1984) extended Biot's theory for unsaturated soils by explicitly including the air as a continuous and compressible phase. The basic physics equations used were as follows.
a.) Equilibrium equations:

\[ \sigma_{y,j} + b_i = 0 \quad [2.54] \]

b.) Air continuity equation:

\[ \frac{\partial}{\partial t} \left[ Q_n (1 - S + HcS) \right] + \nabla \cdot \left[ J_a \right] = 0 \quad [2.55] \]

where:

\[ J_a = \text{mass rate of air flowing across a unit area of the soil.} \]

c.) Water continuity equation:

\[ \frac{\partial (\theta_w)}{\partial t} + \nabla \cdot (\mathbf{v}_w) = 0 \quad [2.56] \]

The application of the above equations was illustrated to a one-dimensional case. Similar derivations were later presented by Alonso et al. (1988), Lloret and Ledesna (1993).

2.4.3.2 Numerical analysis applied to earth dam structures

The flow-deformation equations can be solved in two distinct ways. The first way is called uncoupled solution, where the fluid flow problem is solved separate from the mechanical equilibrium problem. The interdependence of the equations is made in a iterative manner where the flow problem is solved for a given time step and the resultant pore-fluid pressures are used as input in a stress-strain analysis. In turn, stresses from the stress-strain analysis may be used in the evaluation of the hydraulic properties for the next time-step of the fluid-flow analysis. This procedure is sometimes called the “staggered” approach.
The second way is known as a coupled solution which involves the simultaneous solution of fluid-flow and mechanical equilibrium. In this approach, the dynamic interdependence between the flow problem and the stress-strain problem is fully considered.

Zienkiewski (1984) presents a classification of coupled problems in two distinct categories:

i.) Class I contains problems in which the various domains overlap other totally or partially. Here, the coupling occurs in the differential equations normally referring to different physical phenomena.

ii.) Class II contains problems in which coupling occurs only on domain interfaces and clearly this coupling will occur only through the imposed boundary conditions. In this class of solution, two further sub-classifications can be identified: (a) problems in which different physics (and/or dependent variables) occur in different domains; and (b) problems in which identical physics and dependent variables are specified in different domains.

The problem of a collapsing dam clearly belongs to the Class I category of problem. Here, the displacements of the soil structure are strongly dependent on the pore-pressure distribution (i.e., air and water) and in turn the phenomena of fluid flow, giving the equations from which the pore pressures can be found, is coupled to the rate at which displacements squeeze the fluid out of the pores. The domains overlap completely, and the coupling disappears at steady state conditions. When the steady state condition is reached, the pore-pressure fields can be determined simply through the solution of the uncoupled, seepage equations.

The solution of problems related to flow and deformation in unsaturated porous medium have involved many researchers over the last two decades. The following paragraphs show some typical examples of numerical modelling that have been extensively used in coupled or uncoupled analysis in the assessment of the stability or hydraulic behavior of earth dams. It can be seen that progress has been made towards the better reproduction of the
behavior of the soil and on the complexity of the problem undertaken. This advance is associated with the progresses in numerical techniques and in computer capabilities.

Nobari and Duncan (1972) performed numerical analysis, by using the finite element method, to analyze the mechanical behavior of collapsing rockfill dams with a central clay core, during first reservoir filling. The clay core is considered impermeable during the first reservoir filling and the upstream dam collapse is explained as due to the breaking of highly stressed contact points of the rockfill material when wetted. The problem was solved in an uncoupled manner with the water level being raised at several steps. The collapse in a soil element was simulated as a combination of two steps: a.) in the first step there was an internal stress relaxation due to wetting in the element. This resulted in non-equilibrium conditions when compared with existent external stresses; b.) in the second step, the difference between the external and internal stresses was applied to the element with increased compressibility (i.e., softened material), in order to restore the equilibrium conditions. The authors’ results showed a good correlation with the behavior of the Oroville dam when using their method.

Newman (1973) developed the two-dimensional finite element program called UNSAT2 to analyze the water flow in saturated-unsaturated porous media. This program can be used for the solution of water flow problems in soil mechanics. Applications to the simulation of transient saturated-unsaturated processes in earth dams were illustrated to demonstrate the efficiency of the program. This type of solution is convenient for flow-deformation analysis which makes use of the “staggered” approach.

Chang and Duncan (1983) performed analyses of the consolidation of soft clay cores during the construction of rockfill zoned dams. The same basic equations proposed by Biot (1941) were used in their formulation. It is assumed that the pore-fluid was a compressible air-water mixture, where the compressibility decreased until full saturation was achieved. The soil was considered to be non-linear, obeying a critical state model. The efficiency of the
procedure was demonstrated by simulating the mechanical behavior of New Melones dam. There was good agreement between the calculated and measured results.

Miranda (1988) presented an uncoupled analysis of the mechanical behavior of "Alka-Seltzer" dams during first reservoir filling. The transient seepage analysis was performed by using the program UNSAT2, and the stress-strain analysis was performed by using the program UNSTRUCT developed by Miranda (1988). The soil was considered to be linear elastic with elasticity modulus changing as a function of the matric suction. It was assumed that for a given vertical load, the wetting induced collapse was linearly related to the matric suction. The seepage analysis was simulated in a transient manner providing during each time step for the decrease in matric suction due to the advance of the "wetting-front". The time step decrease in matric suction was used to evaluate the corresponding collapse deformation based on laboratory data. This deformation was transformed into equivalent forces and applied in the softened (i.e., more compressible) soil structure. With this procedure Miranda (1988) concluded that hydraulic fracturing was the major factor causing "Alka-Seltzer" dam to fail.

Alonso et al. (1988) performed analyses on the consolidation of zoned earth dams with a clay core, during construction. The analyses were carried out using a "staggered" approach. Parametric analyses were performed to evaluate the influence of the initial water content of the compacted soil on the pore-pressure developed. A 90 meter high dam was used as the main example. No comparisons with field measurements were presented.

Some comments about uncoupled solutions

Some advantages associated with uncoupled solutions are as follows:

i.) A separate solution of fluid flow, usually only for the water phase, and the mechanical equilibrium problem allows the use of existing programs developed to solve
independently the flow analysis and the stress-strain analysis. It will only requires a linking of the software files.

ii.) In general, the computational efficiency increases due to the smaller bandwidth of the resulting systems of equations.

iii.) The non-linearity involved in the soil-parameters are separated in two different solutions. This requires less numerical efforts in terms of procedures for non-linear solutions.

Some disadvantages associated with uncoupled solutions are as follows:

i.) The dynamic interaction of the soil phases is not considered. Therefore, load transfer between these phases cannot be simulated by the uncoupled solution. The Mandel-Cryer effect cannot be simulated by using an uncoupled solution. Therefore, this kind of solution is limited to "simpler" problems.

ii.) The multi-stage character of this solution requires additional effort to estimate the time steps to be used in the separated solutions in order to minimize their influence on the final result. This influence increases with the non-linearity involved in the separated solutions.

**Some comments about coupled solutions**

The problem of collapsing dams during first reservoir filling is transient. The numerical solution requires a time-discretization of the system of the three basic equations. When solved simultaneously the solution is said to be coupled. Some benefits of a coupled solution are as follows:

i.) The solution involves all the relevant aspects of the phenomena in analysis, (i.e., the dynamic interaction between fluid flow and mechanical behavior of the soil structure).
ii.) This solution results in a compact program that can be based in previous stress-strain analysis programs. General characteristics of finite element programs are maintained.

ii.) The search for an adequate time-step procedure involves both problems of flow and stress-strain analysis involved, resulting in a better way to automatize the procedure for any kind of problem. This means less dependence and interference from the user.

iv.) The capability of computers and an understanding of the physical models are increasing rapidly, and soon the more complex aspects of any physical phenomena can be solved in a coupled mode.

Some problems associated with coupled solutions are as follows:

i.) The combined non-linearities from both stress and flow analysis demand additional efforts, in terms of numerical and computational procedures, as compared to uncoupled solutions.

ii.) There is an increase in time-processing of the numerical solution. This is a result of the larger bandwidth for the non-symmetric matrix of the system of equations.

In a general way, the problems associated with coupled solutions are usually associated with present numerical and computational limitations. But, these kind of limitations are being reduced dramatically through the advance of numerical techniques and the development of high-speed processing computers.

2.5 Summary

In engineering practice, the collapsing behavior occurs from either silty or clayey sands compacted at low density and at dry of optimum conditions. The dry density and water content during compaction are important factors in the establishment of a metastable
structure for the compacted soil. Soils with small percentage of clay fraction (i.e., SC, ML) produce collapse behavior under low compactive efforts.

Partial wetting results in partial collapse. From a macroscopic point of view, the rate of soil collapse is mainly controlled by the rate of decrease of matric suction. In turn, the magnitude of collapse is mainly controlled by the magnitude of the net normal stress. The microstructure of the collapsing soil might be used to provide a better understanding of the factors controlling the rate of collapse.

The magnitude of the volumetric soil collapse during wetting is a function of the octaedral stress (i.e., mean normal stress). From this point of view, pure shearing does not induce collapse. Experimental results also assume that the collapse does not occur due to overall shearing of the collapsible soil but rather due to local shear in the interparticles bonds.

During wetting, soil collapse occurs over a very short time provided the infiltrating water is sufficient to saturate the clay bonds holding the metastable soil structure. The soil structure does not need to reach complete saturation to suffer maximum collapse. It is only necessary that the degree of saturation reach a critical value which corresponds to near-saturation of the bonds holding the metastable structure.

Conventional methods of assessment of stability and the hydraulic behavior of earth dams are not satisfactory for the evaluation of the safety factors for collapsing dams. In collapsing dams, the critical condition is established during the first reservoir filling rather than the reservoir-operation phase. Collapsing dams undergo complex mechanical behavior during their first reservoir filling. The stability and hydraulic behavior of such structures can best be evaluated by means of a coupled flow-deformation analysis.

Fredlund’s and Rahardjo’s (1993) theory for consolidation of unsaturated soils in its more generalized form (i.e., introducing an anisotropic behavior of an unsaturated soils element in response to a change in its matric suction), appears to be able to reproduce the stress-strain behavior of a collapsing soil during saturation. The need of such an anisotropic behavior is based on previous experimental works on collapsing soils (Maswoswe, 1985;
Lawton et al. 1991a; Handy, 1995). The mechanical behavior of a stable-structured soil is a particular case of this more generalized theory.

Constitutive relationships for the collapsing soil can be developed by using the two independent stress state variables that controls the behavior of any unsaturated soil. The mechanical properties and hydraulic properties can be formulated as functions of volume-mass properties. In a more direct approach, where hysteresis is negligible, the functions can be written in terms of the stress state variables. Constitutive relationships for both the mechanical behavior and hydraulic properties of collapsing soils are required in order to predict the behavior of collapsing earth structures.

The coupled flow and stress equations involved in the post-filling behavior of a collapsing earth dam requires a numerical solution. In the present research study, the Finite Element Method will be utilized to the development of a numerical model dealing with such a solution. Unlike numerical solutions previously developed (Miranda, 1988; Lloret and Ledesna, 1993), the numerical solution presented in this research study includes the anisotropic behavior of a collapsing soil in response to changes on matric suction. The detailed development of the numerical model is presented in Chapter 3 of this Thesis.
CHAPTER 3

THEORY

3.1 Introduction

Rigorous analysis of the mechanical behavior of collapsing earth dams during first reservoir filling requires the coupled solution of the transient saturated/unsaturated seepage and the soil equilibrium problems. These problems must be formulated and solved by using the theory of soil mechanics for unsaturated soils.

3.1.1 General

A rigorous formulation for two- and three-dimensional consolidation of unsaturated soils requires that the continuity equations for the air and the water phases be coupled with the equilibrium equations for the soil. Transient processes of air and water flow alter the equilibrium conditions in an unsaturated soil since they change the stress state in the porous media. In consequence, volume change occurs as the soil structure searches for a new equilibrium configuration. In turn, these volume changes alter the hydraulic properties (i.e., storage and permeability) of the soil structure and thus affect the transient processes of air and water flow through the porous medium. The interdependence of the air and water flow equations and the soil equilibrium equations is more pronounced in cases where both mechanical and hydraulic properties are highly influenced by the existent stress state.

The theory for the analysis of the mechanical behavior of “Alka-Seltzer” dams during first reservoir filling is based on a general coupled solution for consolidation of unsaturated soils. This theory for unsaturated soils was originally presented by Biot (1941) and later modified (i.e., by explicitly including the air phase) by Dakshanamurthy et al.
(1984) and later updated by Fredlund and Rahardjo (1993). In this chapter, the theory is first presented in its general three-dimensional form where the continuity equations are developed for both air and water as continuous phases. The two-dimensional plane strain condition is then characterized and the theory is reformulated for the analysis of the first reservoir filling of "Alka-Seltzer" dams. The effect of a continuous air phase at constant atmospheric pressure condition is considered. The solution for the plane strain case is solved using the finite element method. Galerkin's residual weighted method is used for the spatial discretization of the continuum and a finite difference scheme is used for the temporal discretization. An incremental form is used for the constitutive relationships for the soil structure, water and air phases. The deformations are assumed to be infinitesimal. The soil is considered to be an incrementally isotropic, linear and elastic material in terms of mechanical properties related to changes in net normal stresses. The soil is assumed to be an incrementally stress-induced anisotropic, linear and elastic material in terms of mechanical properties related to changes in matric suction (see section 2.3.3.5). The soil is assumed to be anisotropic in terms of its flow properties.

3.1.2 Small earth dams during first reservoir filling

In order to formulate the theory for the behavior of a small earth dam during its first reservoir filling, it is necessary to have:

a.) a definition of the basic equations governing the phenomena involved;

b.) the constitutive relationships for the component phases of the soil in terms of both mechanical behavior and pore-fluid (e.g., water in this case) flow properties;

c.) the initial stress state conditions in the dam, (i.e., $\sigma - u_a$ and $u_a - u_w$), at the end-of-construction phase;

d.) the essential and natural boundary conditions required for the basic equations (i.e., equilibrium and water continuity) governing the phenomena involved;
e.) the change in the stress state immediately after the first filling of the reservoir,
   (i.e., the effect of the water pressure at the upstream slope of the dam);

f.) constitutive relationships for the soil in terms of both mechanical and hydraulic
   properties;

3.2 Basic equations of physics

A rigorous analysis of the mechanical behavior of an unsaturated soil requires the
 coupling of the following system of equations: a.) water phase continuity equation; b.) air
 phase continuity equation; c.) static equilibrium of the overall soil medium. These equations
 are presented for a referential elemental volume. This referential volume is used as a spatial
 element (i.e., Eulerian description) for the water and air phases. The elemental volume can
 also be used as a fixed element of mass with respect to the referential initial configuration
 (i.e., Lagrangian description) for the soil. Due to the assumption of infinitesimal
deformations, the Lagrangian and Eulerian descriptions give essentially the same results.
Using the infinitesimal deformation approach, it is possible to avoid the extra complexity of
using the strain and stress tensors appropriate to large deformations and strains.

3.2.1 Water continuity equation

Let us consider a unit volume of porous media, as shown in the x-, y-, and z-
coordinate system in Fig. 3.1. The water flow continuity equation can be expressed,
according to Freeze and Cherry (1979), as follows:

$$\frac{\partial (q^w nS)}{\partial t} + \nabla \cdot (q^w v^w) = 0$$  \[3.1\]

where:
\( n = \text{porosity} \)

\( S = \text{degree of saturation} \)

\( q_w = \text{mass density of water} \)

\( v_w = v_w^x i + v_w^y j + v_w^z k, \) macroscopic (i.e., rate of flow through a unit area)

velocity vector of water

\( \nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k, \) the divergence operator

![Diagram of elemental referential volume for water flow through a porous media](image)

**Figure 3.1** Elemental referential volume for water flow through a porous media

(from Freeze and Cherry, 1979).

In engineering practice, water is commonly considered as an incompressible material, (i.e., \( q_w \) is constant), and Eq. 3.1 can be written as,

\[
\frac{\partial (nS)}{\partial t} + \nabla \cdot (v_w) = 0
\]

[3.2a]

Another possible form for Eq. 3.2a is as follows:
\[
\frac{\partial (\Theta_w)}{\partial t} + \nabla \cdot (v_w) = 0 \tag{3.2b}
\]

where:
\[
\Theta_w = nS, \text{ which is the volumetric water content.}
\]

Involving the assumption of infinitesimal deformations, Eq. 3.2b can be written as,
\[
\frac{\partial (V_w / V_o)}{\partial t} + \nabla \cdot (v_w) = 0 \tag{3.3}
\]

where:
\[
V_w = \text{the current water volume in the referential element}
\]
\[
V_o = \text{the referential volume of the element at the current time-step}
\]
\[
\frac{\partial (V_w / V_o)}{\partial t} = \text{net volume change in water volume per unit volume of the soil.}
\]

### 3.2.2 Air continuity equation

Similarly to the water continuity equation, and based on Figure 3.2, the air continuity equation can be expressed, according to Fredlund and Rahardjo (1993), as follows:

\[
\frac{\partial}{\partial t} \left[ Q_a n (1 - S + H_c S) \right] + \nabla \cdot [J_a] = 0 \tag{3.4}
\]

where:
\[
Q_a = \text{mass density of air}
\]
\[
H_c = \text{Henry's volumetric coefficient of solubility (i.e., } nHS \text{ is the volume of dissolved air/per volume of pore-water)}
\]
\[
J_a = \text{mass rate of air flowing across a unit area of the soil.}
\]
By using the assumption of infinitesimal deformations, Eq. 3.4 can be expressed in the form.

\[ \frac{\partial (M_a/V_o)}{\partial t} + \nabla \cdot [J_a] = 0 \] \hspace{1cm} [3.5]

where:

- \( M_a \) is the air mass in the soil element, \( q_aV_a \)
- \( V_a \) is the air volume in the soil element, \( n(1-S+HS)V_o \)
- \( \frac{\partial (M_a/V_o)}{\partial t} \) is net change in air mass per unit volume of the soil element.

![Figure 3.2 Elemental referential volume for air flow through a porous media.](image)

The air phase is a highly compressible medium and its density is a function of the air pressure. By combining the expressions, \( M_a = q_aV_a \) and \( V_a = n(1-S+HS)V_o \) into Eq. 3.5, the air continuity equation can be expanded to the form,

\[ q_a \frac{\partial (V_a/V_o)}{\partial t} + n(1-S+HS) \frac{\partial q_a}{\partial t} + \nabla \cdot [J_a] = 0 \] \hspace{1cm} [3.6]
3.2.3 Equilibrium of the soil element

The static equilibrium of a soil element can be expressed in a condensed form, as presented by Lloret and Alonso (1980), as follows:

\[
\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0 \quad [3.7]
\]

where:

\[\sigma_{ij} = \text{the total stresses}\]
\[x_i = \text{the directional system coordinates}\]
\[b_i = \text{the body forces}.\]

In a \(x\)-, \(y\)-, and \(z\)-coordinate system, Eq. 3.7 can be expanded as follows,

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + b_x = 0 \quad [3.8a]
\]

\[
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + b_y = 0 \quad [3.8b]
\]

\[
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0 \quad [3.8c]
\]

3.3 Constitutive relationships and laws of motion

The displacement vector, \(\mathbf{u}\), the air and water pore-pressures (i.e., \(u_a\) and \(u_w\)) are usually chosen as the basic variables of the flow/deformation problem. To solve the basic
equations presented in item 3.2, a set of constitutive relationships and laws of motion are necessary.

For the soil element equilibrium equation it is necessary to define constitutive relationships linking volume/displacement changes to the stress state variables. Additionally, the shear strength constitutive relationship is necessary in problems where there is a potential risk of failure of the soil. In these cases both a failure criterion and a post-failure behavior have to be defined.

For the air and water phases basic equations, it is necessary to define: a.) constitutive equations relating volume changes in these phases to changes in stress state variables; b.) laws of motion relating the driving potential to the rate of flow; c.) constitutive equations for the laws of motion.

3.3.1 Volume change behavior

A volume change constitutive relationship links the deformation state variables to the stress state variables. According to the theory presented in section 2.3 (Chapter 2), the deformation variables for an unsaturated soil element are the changes in total volume (i.e., soil structure), changes in water volume and changes in air volume. The stress state variables are \((\sigma - u_d)\) and \((u_a - u_w)\).

3.3.1.1 Soil structure constitutive relationship

The constitutive relations are herein presented using the elasticity form of the semi-empirical approach discussed in chapter 2. These equations describe the stress versus strain relationships for an unsaturated soil. Under triaxial loading conditions, the equations were presented in section 2.3.1.3 in Eqs. 2.21 to 2.26.
The complete set of equations can be written in an incremental form using a \( x-, y-, z- \), coordinate system as follows:

\[
de \varepsilon = D^{-1}d(\sigma^*) + h_s d(u_a - u_w) \tag{3.9}
\]

where:

\[
\sigma^* = \sigma - m u_s
\]

\[
h_s^T = \left[ \frac{1}{H_x}, \frac{1}{H_y}, \frac{1}{H_z}, 0, 0, 0 \right]
\]

\[
\varepsilon = \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix}, \quad D^{-1} = \begin{bmatrix}
1 & -\mu & -\mu & 0 & 0 & 0 \\
-\mu & 1 & -\mu & 0 & 0 & 0 \\
-\mu & -\mu & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2(1 + \mu) & 0 & 0 \\
0 & 0 & 0 & 0 & 2(1 + \mu) & 0 \\
0 & 0 & 0 & 0 & 0 & 2(1 + \mu)
\end{bmatrix}, \quad \sigma = \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix}
\]

\[
m^T = [1, 1, 1, 0, 0, 0].
\]

Equation 3.9 can be used to express the stress versus strain relationship since it resembles the general equation, (i.e., \( de = D^{-1}d(\sigma^*) + de_0 \)), often used for structural analysis. Additionally, Eq. 3.9 also provides a suitable form for computational purposes (Zienkiewicz, 1975). In this form, strain increments, \( de \), are obtained as a sum of the strains due to the increments of total stress (i.e., \( \sigma^* = \sigma - m u_s \)) and the volumetric strains, \( de_0 \), induced by suction changes. As an alternative, the solution of deformation/flow problems can be accomplished using a staggered (i.e., uncoupled) approach to Equation 3.3 and 3.9 (Miranda, 1988; Lloret et al. 1993).

By using the normal strains from Eq. 3.9, the increment in volumetric strain, \( de_v \), is equal to \( (de_x + de_y + de_z) \). The volumetric strain, \( de_v \), can be written as follows:

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\[ d\varepsilon_v = m_1^s d(\sigma_{\text{mean}} - u_a) + m_2^s d(u_a - u_w) \]  
\[ \text{where:} \]
\[ \sigma_{\text{mean}} = \text{the mean total stress, (i.e., } (\sigma_x + \sigma_y + \sigma_z)/3) \]
\[ m_1^s = 3 \frac{(1-2\mu)}{E} \]
\[ m_2^s = \frac{3}{H_s} = \frac{1}{H_x} + \frac{1}{H_y} + \frac{1}{H_z}. \]

Equation 3.10 shows the compressibility form for the volumetric deformation of an unsaturated soil (Fredlund and Morgenstern, 1976). This compressibility form was previously presented in section 2.3.1.3 in chapter 2. The soil moduli parameters, \( m_1^s \) and \( m_2^s \), have well defined meanings in geotechnical engineering. For unsaturated soils, an analogy can be made to the compressibility parameter \( m_v \) used in soil mechanics for saturated soils.

By using Eq. 3.9, the increments of net normal stress can be expressed as function of strain increments and matric suction in the form:

\[ d\sigma = Dd\varepsilon - D_s d(u_a - u_w) \]  
\[ \text{where:} \]
\[ D_s = Dh_s. \]

Alternatively, from Eq. 3.10, an increment of mean net normal stress can be expressed as a function of volumetric strain and matric suction as follows:

\[ d(\sigma_{\text{mean}} - u_a) = \frac{1}{m_1^s} d\varepsilon_v - \beta d(u_a - u_w) \]  
\[ \text{where:} \]
\[ \beta = m_2^s / m_1^s. \]
Equation 3.12 is a convenient and useful form for net normal stresses to be applied in a compressibility approach where volumetric changes are independent of shear stresses. This equation is later used to express the relationship between total volumetric strain, \(de_v\), and volumetric deformations for the other phases; namely, the water phase \((dV_w/V_o)\) and the air phase, \((dV_a/V_o)\).

### 3.3.1.2 Water phase constitutive relationship

The constitutive relationship for the water phase defines the water volume change in a soil element for a change in the stress state variables. This constitutive equation can be written as a linear combination of the stress state variable changes, and has the following incremental form as previously defined in Eq. 2.24.

\[
\frac{dV_w}{V_o} = \frac{d(\sigma - u_a)}{E_w} + \frac{d(\sigma - u_a)}{E_w} + \frac{d(\sigma - u_a)}{E_w} + \frac{d(u_a - u_w)}{H_w} \quad [3.13a]
\]

Equation 3.13a can also be written, using compressibility type variables as follows.

\[
\frac{dV_w}{V_o} = m_1^w d(\sigma_{mean} - u_a) + m_2^w d(u_a - u_w) \quad [3.13b]
\]

where:

\[
m_1^w = \frac{3}{E_w}, \text{ water phase compressibility parameter related to a change in } (\sigma - u_a)
\]

\[
m_2^w = \frac{1}{H_w}, \text{ water phase compressibility parameter related to a change in } (u_a - u_w).
\]
By using the equations for mean net normal stress, (i.e., Eq. 3.12), Eq. 3.13b can be expressed as a function of the total volumetric strain and the matric suction.

\[
\frac{dV^w}{V_o} = \beta_{w1} \frac{de}{v} + \beta_{w2} d(u_a - u_w) \tag{3.14}
\]

where:

\[
\beta_{w1} = \frac{E}{[E_w(1-2\mu)]} \text{ or, in the compressibility form, } \beta_{w1} = \frac{m^w}{m^i}.
\]

\[
\beta_{w2} = \frac{1}{H_w} - \frac{3\beta}{E_w} \text{ or, in the compressibility form, } \beta_{w2} = \frac{m^w}{m^i} - \frac{m^w m^i}{m^i}.
\]

Equation 3.14 is a suitable form for the coupled solution since it expresses the water volumetric deformation as a function of the total volumetric strain/displacements, and the pore fluid pressures, (i.e., \(u_a\) and \(u_w\)).

### 3.3.1.3 Air phase constitutive relationship

The pore-air is a highly compressible phase and it requires a constitutive relationship for the air density as a function of the air pressure. Assuming that the air behaves as an ideal gas, its density, \(\rho_a\), can be expressed as,

\[
\rho_a = \frac{\omega_a}{RT} u_a \tag{3.15}
\]

where:

\[\omega_a\] = molecular mass of air (kg/kmol)

\[R = \text{universal (molar) gas constant [i.e., 8.31432 J/(mol.K)]}\]

\[T = \text{absolute temperature (i.e., } T = t^o + 273.16) \text{ in Kelvin degrees (}^\circ\text{K})\]

\[t^o = \text{temperature in Celsius degrees (}^\circ\text{C})\]
\[ \bar{u}_a = \text{absolute pore-air pressure (i.e., } \bar{u}_a = u_a + \bar{u}_{atm} \text{) (kPa)} \]
\[ u_a = \text{gauge-pore air pressure (kPa)} \]
\[ \bar{u}_{atm} = \text{atmospheric pressure (i.e., 101 kPa or 1 atm).} \]

A second constitutive relationship for the air phase defines the air volume change in the soil element, to changes in the stress state variables. Similar to the soil structure and water phase, this relationship can be expressed, in an incremental elasticity-type form as follows,

\[
\frac{dV_a}{V_o} = \frac{d(\sigma_x - u_a)}{E_a} + \frac{d(\sigma_y - u_a)}{E_a} + \frac{d(\sigma_z - u_a)}{E_a} + \frac{d(u_a - u_w)}{H_a} \quad [3.16a]
\]

or, in a compressibility form as,

\[
\frac{dV_a}{V_o} = m^a_1 d(\sigma_{\text{mean}} - u_a) + m^a_2 d(u_a - u_w) \quad [3.16b]
\]

where:
\[ m^a_1 = \frac{3}{E_a} \text{, air phase compressibility parameter related to a change in } (\sigma - u_a) \]
\[ m^a_2 = \frac{1}{H_a} \text{, air phase compressibility parameter related to a change in } (u_a - u_w). \]

By combining the equations for net normal stresses, (i.e., Eq. 3.10), with Eq. 3.16b, the incremental volume change in the air phase can be written as:

\[
\frac{dV_a}{V_o} = \beta_{a1} d\epsilon_v + \beta_{a2} d(u_a - u_w) \quad [3.17]
\]

where:
\[ \beta_{a1} = \frac{E}{[E_a(1-2\mu)]} \]
\[ \beta_{a2} = \frac{1}{H_a} - 3\beta/E_a \]
The continuity equation, (i.e., Eq. 2.16, expressed in section 2.3.1.2 as \( \frac{dV_v}{V_o} = \frac{dV_w}{V_o} + \frac{dV_a}{V_o} \)), must always be satisfied, and therefore the parameters, \( \beta_{a1} \) and \( \beta_{a2} \), can be expressed as functions of \( \beta_{w1} \) and \( \beta_{w2} \), as follows:

\[
\frac{dV_w}{V_o} = \beta_{w1} d\varepsilon_v + \beta_{w2} d(u_a - u_w) \tag{3.14}
\]

\[
+ \frac{dV_a}{V_o} = \beta_{a1} d\varepsilon_v + \beta_{a2} d(u_a - u_w) \tag{3.17}
\]

\[
d\varepsilon_v = \frac{dV_v}{V_o} = (\beta_{w1} + \beta_{a1}) d\varepsilon_v + (\beta_{w2} + \beta_{a2}) d(u_a - u_w) \tag{3.18}
\]

Therefore,

\[
\beta_{w1} + \beta_{a1} = 1 \tag{3.19}
\]

and,

\[
\beta_{w2} + \beta_{a2} = 0 \tag{3.20}
\]

Equations 3.19 and 3.20 are useful in solving the problems associated with the measurement of air phase constitutive parameters. These relationships are in response to the fact that the volume of air is a function of the porosity and the degree of saturation of the soil (i.e., \( V_a = n(1-S+HS)V_o \), as used in Eq. 3.5).

In summary, the volume change constitutive relationships require the definition of the following soil parameters, \( E, \mu, \) and \( H_x, H_y, H_z \), for the soil structure, and \( E_w \) and \( H_w \), for the water phase. These five parameters can be computed from laboratory experiments in which controlled stress paths are imposed on soil specimens while total volume and water volume changes are monitored.
3.3.2 Flow laws and hydraulic properties

The description of fluid flow requires a law of motion and a constitutive relationship linking the driving potential to either the specific discharge or the mass rate flow of the fluid. The definition of the permeability of the unsaturated soil with respect to both air and water phases are required for the description of the fluid flow through an unsaturated soil.

3.3.2.1 Water phase flow

Water flow through a saturated/unsaturated soil can be described by a generalized Darcy's law (see section 2.3.2) as follows (with z-coordinate as the vertical direction):

\[ v_w = -k_w \nabla \left( \frac{u_w}{\gamma_w + z} \right) \tag{3.21} \]

As discussed in section 2.3.3.6, for an unsaturated soil the water coefficient of permeability can be expressed as a function either of the matric suction (Gardner, 1958; Brooks and Corey, 1964) or the volume-mass soil properties (Lloret and Alonso, 1980; Fredlund and Rahardjo, 1993; Huang, 1994). Relationships between the water coefficient of permeability and void ratio (Lambe and Whitman, 1969) are sometimes used for the saturated condition of the soil, but generally it is left constant.

3.3.2.2 Air phase flow

Air flow through an unsaturated soil can be described using a generalization of Fick's law (see section 2.3.2) as follows:
\[ J_a = -D_a^* \nabla u_a \]  

[3.22]

In the above equation the coefficient of air diffusion, \( D_a^* \), can be related to the air coefficient of permeability, (i.e., \( k_a \) in Darcy's approach), by using Eq. 2.40 (i.e., \( D_a^* = k_a / g \)). The air coefficient of permeability is usually expressed as a function of the volume-mass properties (Brooks and Corey, 1964; Barden and Pavlakis, 1971; Yoshimi and Osterberg, 1963). In these equations the air coefficient of permeability is mainly affected by changes in the degree of saturation (section 2.3.2.2). Under saturated conditions the air coefficient of permeability becomes zero.

In problems where the air can be considered to remain constant at atmospheric pressure, the air flow equation does not need to be considered as a flow phase. In these cases the air coefficient of permeability does not need to be evaluated.

3.4 Coupled equations for consolidation of an unsaturated soil

For a three-dimensional deformation-fluid flow problem in unsaturated soils, there are three unknown deformation variables to be determined, (i.e., the volumetric changes for the three phases; soil-structure, \( \delta e_v \), water phase \( d(V_w/V_r) \) and air phase \( d(V_a/V_r) \)). These volumetric changes are linked to the stress state variables by means of the constitutive equations and must obey the previously presented governing equations (i.e., Eqs. 3.7, 3.1 and 3.4). The governing equations can be combined with the constitutive relationships (i.e., Eqs. 3.9, 3.13 and 3.16) and flow laws (i.e., Eqs. 3.21 and 3.22) to explicitly define the coupled equations governing the phenomena of consolidation of unsaturated soils in terms of the basic variables involved. The coupled equations can be solved simultaneously. The primary unknowns are better defined as the displacements in the x-, y- and z-direction, (i.e., \( u, v \) and \( w \)), and the pore fluid pressures, (i.e., \( u_a \) and \( u_w \)).
3.4.1 Equilibrium equations for an unsaturated soil

The equilibrium equations for an unsaturated soil can be expressed in terms of displacements and pore fluid pressures by replacing the stress tensor increment, (i.e., \( d\sigma^* \)), by its expression from the system of constitutive equations, (i.e., Eq. 3.11). This procedure results in the following equations:

i) stress equilibrium in the x-coordinate direction,

\[
G \nabla^2 u + \left[ \frac{G}{(1-2\mu)} \right] \frac{\partial \varepsilon_v}{\partial x} - \beta \frac{\partial (u_a - u_w)}{\partial x} + \frac{\partial u_a}{\partial x} + b = 0
\]  
[3.23a]

ii) stress equilibrium in the y-coordinate direction,

\[
G \nabla^2 v + \left[ \frac{G}{(1-2\mu)} \right] \frac{\partial \varepsilon_v}{\partial y} - \beta \frac{\partial (u_a - u_w)}{\partial y} + \frac{\partial u_a}{\partial y} + b = 0
\]  
[3.23b]

iii) stress equilibrium in the z-coordinate direction,

\[
G \nabla^2 w + \left[ \frac{G}{(1-2\mu)} \right] \frac{\partial \varepsilon_v}{\partial z} - \beta \frac{\partial (u_a - u_w)}{\partial z} + \frac{\partial u_a}{\partial z} + b = 0
\]  
[3.23c]

where:

\( G = E/2(1+\mu) \) is the shear modulus

\[
\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}
\]
3.4.2 Water phase continuity equation

Combining the water continuity equation, (i.e., Eq. 3.3), and Darcy's flow law, (i.e., Eq. 3.21), results in the following equation.

\[
\frac{\partial (V_w / \theta_w)}{\partial t} = \nabla \cdot [k \nabla (u_w / \gamma_w + z)] \tag{3.24}
\]

Equation 3.24 is often referred to as Richard's equation when the water coefficient of permeability and the volumetric water content, \( \theta_w \), (i.e., \( V_w / V_o \)), are expressed in terms of the negative pore-water pressure (or pressure head) in the unsaturated soil.

The water phase continuity equation can be expressed in terms of displacements and pore fluid pressures by equating the time-derivative of the water constitutive equation (i.e., Eq. 3.14) to the water flow equation (i.e., Eq. 3.24), expressed in terms of pore-water pressure head.

\[
\beta_w \frac{\partial e_v}{\partial t} + \beta_w \frac{\partial (u_a - u_w)}{\partial t} = \nabla \cdot [k \nabla (u_w / \gamma_w + z)] \tag{3.25}
\]

3.4.3 Air phase continuity equation

Combining the air continuity equation (i.e., Eq. 3.6), the constitutive relationship for the air density (i.e., Eq. 3.15), and the Fick's law of motion (i.e., Eq. 3.22), results in the following equation for the incremental volumetric change in the air phase:

\[
\frac{\partial (V_a / V_o)}{\partial t} = \left( \frac{1}{(\omega_a / RT) u_a} \right) \left[ \nabla \cdot (D_a^* \nabla u_a) \right] - \frac{(1 - S + HS)n}{u_a} \frac{\partial u_a}{\partial t} \tag{3.26}
\]
The air phase continuity equation can be expressed in terms of displacements and pore fluid pressures by equating the time-derivative of the air constitutive equation (Eq. 3.17) to the unit volume flow rate expressed by Eq. 3.26.

\[
\beta_{a1} \frac{\partial \varepsilon}{\partial t} + \beta_{a2} \frac{\partial (u_a - u_w)}{\partial t} = \left( \frac{1}{(\omega / RT)u_a} \right) \nabla \cdot \left( D_a^* \nabla u_a \right) - \frac{(1 - S + HS)n \partial u_a}{u_a} \frac{\partial u_a}{\partial t}
\]

[3.27]

3.4.4 Summary

The coupled fluid flow-deformation analysis in and unsaturated soil involves the simultaneously solution of the following system of equations.

i.) stress equilibrium in the x-coordinate direction,

\[
G \nabla^2 u + \left[ \frac{G}{(1 - 2\mu)} \right] \frac{\partial \varepsilon}{\partial x} - \beta \frac{\partial (u_a - u_w)}{\partial x} + \frac{\partial u_a}{\partial x} + b_x = 0
\]

[3.23a]

ii.) stress equilibrium in the y-coordinate direction,

\[
G \nabla^2 v \left[ \frac{G}{(1 - 2\mu)} \right] \frac{\partial \varepsilon}{\partial y} - \beta \frac{\partial (u_a - u_w)}{\partial y} + \frac{\partial u_a}{\partial y} + b_y = 0
\]

[3.23b]

iii.) stress equilibrium in the z-coordinate direction,

\[
G \nabla^2 w + \left[ \frac{G}{(1 - 2\mu)} \right] \frac{\partial \varepsilon}{\partial z} - \beta \frac{\partial (u_a - u_w)}{\partial y} + \frac{\partial u_a}{\partial z} + b_z = 0
\]

[3.23c]
iv.) the water phase continuity equation,

$$\beta_{w1} \frac{\partial \varepsilon_v}{\partial t} + \beta_{w2} \frac{\partial (u_a - u_{w})}{\partial t} = \nabla \cdot \left[ k \nabla (u_w / \gamma_w + z) \right]$$  \[3.25\]

v.) the air phase continuity equation,

$$\beta_{a1} \frac{\partial \varepsilon_v}{\partial t} + \beta_{a2} \frac{\partial (u_a - u_{w})}{\partial t} = \left( \frac{1}{(\omega_a / RT)u_a} \right) \nabla \cdot \left( D_a^* \nabla u_a \right)$$  \[3.27\]

$$- \frac{(1 - S + HS)n}{u_a} \frac{\partial u_a}{\partial t}$$

3.5 Coupled solution for a plane strain analysis of an earth dam

The two-dimensional, plane strain formulation for consolidation of unsaturated soils is based on the general three-dimensional consolidation theory previously presented in this chapter. The equations are developed for the solution of the coupled deformation/flow problem occurring during the first reservoir filling of small collapsible earth dams. Additional assumptions need to be introduced and the stress state and basic equations are specialized to the plane-strain condition. The objective of the solution is to analyze the mechanical behavior of small collapsing earth dams during the transient saturated-unsaturated water flow process which takes places immediately after the first reservoir filling and ends when a steady-state condition is reached.

The equations are developed by neglecting the air phase continuity equation. It is assumed that the air pressure is always atmospheric during the transient water flow process. As previously explained in chapter 1, this assumption is reasonable based on the nature of the
open structure of the compacted soil in the collapsing dam. It is also justifiable in that the flow of water into the dam is primarily from the upstream towards the downstream slope of the embankment. As later discussed in Chapter 5, the condition of entrapped air only occurs when the collapsing soil approaches saturation (i.e., at high degree of saturation) where the pore-fluid can be assumed to possess a relatively low compressibility (see section 5.4). Therefore, as the water advances into the dam, there is a gradual expulsion of the pore-air through the soil voids which remain with air at atmospheric condition. In the following, the pore-air pressure term is kept in the other two basic equations, only for consistency with the formulated theory. In the general 3-dimensional case, the 2-dimensional formulation is presented in an incremental form in the constitutive relationships for both the soil structure and water phase. The mechanical behavior of the soil is assumed to be isotropic, linear and elastic in the incremental formulation. In terms of water flow, the water coefficient of permeability is assumed to be anisotropic as a result of the compaction process. The preferential permeability would be in the horizontal direction.

3.5.1 Basic equations for two-dimensional problems

The basic equations formulated for the water phase (i.e., Eq. 3.1) and for the equilibrium of the soil element (i.e., Eq.3.7), are independent of the number of dimensions of the problem. For the plane strain or plane stress condition, the basic equations can be simplified for the two-dimensional form. This simplification implies that, for both water flow and equilibrium equations, one-direction (e.g., z-direction) can be neglected. In terms of water flow, this means that the z-velocity component is negligible. In structural terms, this means that there is no deformation (plane strain) or stress (plane stress) in the z-direction. In the following formulation the y-coordinate axis coincides with the vertical direction.
3.5.1.1 Water continuity equation

The general water continuity equation maintains its original form, (i.e., Eq. 3.3). The change for the two-dimensional condition is illustrated by the simplified form of the water flow velocity vector.

\[ \frac{\partial (V_w / V_o)}{\partial t} + \nabla \cdot [v_w] = 0 \]  
\[ \text{[3.3]} \]

where:

\[ v_w = v_x^w \hat{i} + v_y^w \hat{j}, \text{ macroscopic velocity vector of water (i.e., rate of flow through a unit area).} \]

3.5.1.2 Equilibrium equations

For the two-dimensional case the equilibrium equations (i.e., Eq.3.8) can be written as follows:

\[ \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + b_x = 0 \]  
\[ \text{[3.28a]} \]

\[ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + b_y = 0 \]  
\[ \text{[3.28b]} \]

3.5.1.3 The plane strain condition

The structural restriction of allowing deformation at only in one plane, (e.g., strain in the xy-plane) defines the plane strain condition. Assuming y as the vertical direction and x as the horizontal direction, the following boundary conditions apply.
\[ \varepsilon_i = 0 \quad [3.29a] \]

Consequently, the volumetric strain can be written as follows.

\[ \varepsilon_v = \varepsilon_i + \varepsilon_j \quad [3.29b] \]

Introducing Eq. 3.29a into the system of constitutive equations, (i.e., Eqs. 3.9), and restricting the net normal stress to the z-direction results in the following equation.

\[ d(\sigma_z - u) = \mu (\varepsilon_i + \varepsilon_j - 2u) - \frac{E}{H} d(u_z - u_w) \quad [3.30] \]

### 3.5.2 Constitutive relationships and flow laws for plane strain case

Following is a description of the constitutive relationships for the water phase and the soil structure under plane strain conditions. The same sequence for the derivation is used as was presented for the three-dimensional case.

#### 3.5.2.1 Volume change behavior

Suitable constitutive relationships, for the case of plane strain must be presented for the soil structure and the water phase.

**Soil structure constitutive relationship**

Substituting Eq. 3.30 into the three-dimensional stress strain relationships (i.e., Eq. 3.11) results in the following plane strain stress-strain relationships in the xy-plane.
\[
\begin{align*}
\begin{align*}
\sigma^* &= \sigma - \mathbf{m} u_a = 
\begin{bmatrix}
\sigma_x - u_a \\
\sigma_y - u_a \\
\tau_{xy}
\end{bmatrix}, \\
\mathbf{m} &= [1, 1, 0], \\
\mathbf{E} &= \left\{ \begin{array}{c}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{array} \right\} = 
\begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial x} \\
\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}
\end{bmatrix}, \\
\varepsilon &= \text{the vector of total strains}, \\
u, v &= \text{displacements in x- and y-directions respectively,}
\end{align*}
\end{align*}
\]
where:

\[
\begin{align*}
\mathbf{D} &= \frac{E(1-\mu)}{(1+\mu)(1-2\mu)} \begin{bmatrix}
1 & \frac{\mu}{1-\mu} & 0 \\
\frac{\mu}{1-\mu} & 1 & 0 \\
0 & 0 & \frac{1-2\mu}{2(1-\mu)}
\end{bmatrix}, \text{is the constitutive tangent matrix for plane strain condition,}
\end{align*}
\]

\[
\begin{align*}
\mathbf{D}_z &= \frac{E(1-\mu)}{(1+\mu)(1-2\mu)} \begin{bmatrix}
1 & \frac{\mu}{1-\mu} \left( \frac{1}{H_z} + \frac{1}{H_z} \right) \\
\frac{1}{H_x} & \frac{\mu}{1-\mu} \left( \frac{1}{H_y} + \frac{1}{H_y} \right) & 0
\end{bmatrix}
\end{align*}
\]
The incremental volumetric strain, (i.e., \( d\varepsilon_v = d\varepsilon_x + d\varepsilon_y \)), can be modified by combining Eq. 3.10 (i.e., \( d\varepsilon_v \), for three-dimensional deformation) and Eq. 3.30 (i.e., \( d\sigma_z \) for plane strain conditions) to give the form:

\[
d\varepsilon_v = m_1^v d(\sigma_{ave} - u_a) + m_2^v d(u_a - u_w)
\]  \[3.32\]

where:

\[
\sigma_{ave} = \text{average total normal stress for two-dimensional loading},
\]

(i.e., \( (\sigma_x + \sigma_y)/2 \)),

\[
m_1^v = \frac{2(1 + \mu)(1 - 2\mu)}{E},
\]

\[
m_2^v = \frac{1}{H_x} + \frac{1}{H_y} + \frac{2\mu}{H_z}.
\]

The above \( m_1^v \) and \( m_2^v \) are the compressibility parameters for the plane strain condition.

**Water phase constitutive relationship**

The water phase constitutive equation for plane strain conditions can be formulated by combining Eq. 3.13a of the three-dimensional water phase constitutive equation with Eq. 3.30 (i.e., \( d\sigma_z \) for plane strain conditions).

\[
\frac{dV_w}{V_o} = m_1^w d(\sigma_{ave} - u_a) + m_2^w d(u_a - u_w)
\]  \[3.33\]

where:

\[
m_1^w = \frac{2(1 + \mu)}{E_w}
\]

\[
m_2^w = \left[ \frac{1}{H_w} - \frac{(E/H)}{E_w} \right]
\]
Substituting the increments in normal stresses from Eq. 3.31 into Eq. 3.33, the water phase constitutive equation can also be expressed in terms of increment in volumetric strain and changes in matric suction.

\[
\frac{dV_w}{V_0} = \beta_{w1} de_v + \beta_{w2} d(u_a - u_w) \quad [3.34]
\]

where:

\[
\beta_{w1} = \left[ \frac{E}{E_w (1-2\mu)} \right]
\]

\[
\beta_{w2} = \left[ \frac{I}{H_w} - \frac{E}{E_w} \frac{1}{H_z} - \frac{1}{E_w (1-2\mu)} \left( \frac{1}{H_z} + \frac{1}{H_y} + \frac{2\mu}{H_z} \right) \right]
\]

or, in compressibility form,

\[
\beta_{w1} = \frac{m_i^w}{m_i^s}
\]

\[
\beta_{w2} = m_i^w - \frac{m_i^w m_i^s}{m_i^s}.
\]

### 3.5.2.2 Shear strength behavior of the unsaturated soil

The definition of a failure criterion establishes the conditions at which the shear strength of the soil is reached. Post-failure behavior must define the new volume change behavior of the failed material as a function of the stress state variables and/or deformation variables. The failure of the earth structure might render the structure no longer be an intact continuum.

Fredlund and Gan (1988) extended the Mohr-Coulomb failure criteria to unsaturated soils. The shear strength equation for an unsaturated soil was expressed as follows (see section 2.3.1.3):

\[
\tau_{ff} = c' + (\sigma_n - u_a) f_{\tan \phi'} + (u_a - u_w) f_{\tan \phi^b}
\]  
[2.32]
The above equation includes in an elementary manner the effect of shear strength increasing with matric suction. The required parameters, (i.e., $c'$, $\phi'$ and $\phi^b$) can be determined from laboratory tests. Alternatively, for a non-deformable soil the $\phi^b$ parameter can be estimated from the soil-water characteristic curve (Vannapali, 1994).

As discussed before in section 2.3.3.5, a metastable structural soil requires the definition of its shear strength parameters in an experimental way. Previous studies (Maswoswe, 1985; El-Sohby et. al. 1987) alerted to the high influence of the microstructure on the shear strength. The amount of clay fraction and arrangement of clay bonds at interparticle contacts can render a variety of shear strength behavior for a compacted metastable soil structure. An experimental program can properly define the shear strength behavior of a specific collapsible soil. The shear strength must be defined for the range of stress state variables expected in the problem under analysis. The laboratory program elaborated for the present research study is presented in details in chapter 4.

3.5.2.3 Flow law and hydraulic properties for the water phase

The generalized Darcy's law, presented previously in section 3.3.2, is independent of the number of dimensions of the problem. For the xy-coordinate system (i.e., with y in the vertical direction) in plane strain conditions, the water flow equation (Eq. 3.22) can be adjusted as follows:

$$v_w = -k_w \nabla (u_w / \gamma_w + y) \quad [3.35]$$

The determination of the water coefficient of permeability for an unsaturated soil can face the problem of a combined effect of degree of saturation and void ratio as a result of changes in stress state (see section 2.3.2). The inclusion of void ratio changes seems
important in cases where the soil must be treated as a deformable porous medium. Experimental evidence, (Huang, 1994), has demonstrated that even for a deformable and unsaturated porous medium, the degree of saturation is the main factor controlling the water coefficient of permeability. The influence of void ratio is of some importance only at high degrees of saturation, (Lambe and Whitman, 1969).

The prediction of water coefficient of permeability for a metastable structural soil can be done in an approximately way by combining the saturated coefficient of permeability, (i.e., \( k_s \)), and the wetting soil-water characteristic curve (Miranda, 1988). This prediction does not include the effect of loading, (i.e., the collapse of the soil structure), in the determination of the water coefficient of permeability. An improvement in defining the water coefficient of permeability can be accomplished by means of experiments which simulate the stress paths which occur during the saturation of the collapsible soil. These stress paths must simulate the combined effect of net normal stress and matric suction during saturation. For the present research program an experimental study was used to define soil water permeability as a function of the stress state variables, (i.e., \( k_w(\sigma - u_a, u_a - u_w) \)). The water coefficient of permeability is defined for the range of stress state variables expected in the problem under study. The laboratory program elaborates on the present program research and is presented in details in chapter 4.

3.5.3 Coupled equations for plane strain conditions

The solution of the coupled flow/deformation problem under plane strain conditions for small earth dams involves, in essence, the simultaneous solution of the equilibrium equations, (i.e., Eq. 3.28), and the continuity equation for the water phase, (i.e., Eq. 3.3). The primary unknowns are better defined as the displacements in the x-, y-direction, (i.e., \( u \), and \( v \)), and the pore water pressure, (i.e., \( u_w \)).
3.5.3.1 Equilibrium equations

Similar to the general 3-D case, the plane strain condition can have the equilibrium equations, (i.e., Eq. 3.28), in terms of displacements and pore-water pressure by replacing the stress vector, \( d\sigma \), by the system of constitutive equations, (i.e., Eq. 3.32) and expanding to the following form.

i) stress equilibrium equation in the x-direction

\[
\frac{\partial}{\partial x} \left( c_{11} \frac{\partial u}{\partial x} + c_{12} \frac{\partial u}{\partial y} \right) + c_{33} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - d_x^s \frac{\partial (u - u_w)}{\partial x} + \frac{\partial a}{\partial x} + b_x = 0 \quad [3.36a]
\]

ii) stress equilibrium equation in the y-direction

\[
c_{33} \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( c_{12} \frac{\partial u}{\partial x} + c_{22} \frac{\partial v}{\partial y} \right) - d_y^s \frac{\partial (u - u_w)}{\partial y} + \frac{\partial a}{\partial x} + b_y = 0 \quad [3.36b]
\]

where:

\[
c_{11} = c_{22} = \frac{E(1-\mu)}{(1+\mu)(1-2\mu)}
\]

\[
c_{12} = \frac{E\mu}{(1+\mu)(1-2\mu)}
\]

\[
c_{33} = \frac{E}{2(1+\mu)}
\]

\[
d_x^s = c_{11} \left( \frac{1}{H_x} + \frac{\mu}{(1-\mu)H_y} + \frac{\mu}{(1-\mu)H_z} \right)
\]

\[
d_y^s = c_{11} \left( \frac{\mu}{(1-\mu)H_x} + \frac{1}{H_y} + \frac{\mu}{(1-\mu)H_z} \right)
\]
3.5.3.2 Water phase continuity equation

Similar to the procedure described in section 3.4.2 the continuity equation for the water phase can be obtained by equating the time-derivative of the water phase constitutive equation (i.e., Eq. 3.34) to the net mass of water flow rate as described by Darcy's law. The resulting equation is as follows:

$$\beta_w \frac{\partial e_v}{\partial t} + \beta_w \frac{\partial (u_a - u_w)}{\partial t} = \nabla \cdot [k \nabla (\frac{u_w}{\gamma_w} + y)]$$

[3.37]

3.5.3.3 Summary of the coupled equations

The coupled water flow-deformation equations for a plane strain analysis involving unsaturated soils require the simultaneously solution of the following system of equations, herein expressed using the primary unknown variables (i.e., $u$, $v$, $u_a$ and $u_w$).

i) stress equilibrium equation in the $x$-direction

$$\frac{\partial}{\partial x} \left( c_{11} \frac{\partial u}{\partial x} + c_{12} \frac{\partial v}{\partial y} \right) + c_{33} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - s \frac{\partial (u_a - u_w)}{\partial x} + \frac{\partial u_a}{\partial x} + b_x = 0 \quad [3.36a]$$

ii) stress equilibrium equation in the $y$-direction

$$c_{33} \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( c_{12} \frac{\partial u}{\partial x} + c_{22} \frac{\partial v}{\partial y} \right) - s \frac{\partial (u_a - u_w)}{\partial y} + \frac{\partial u_a}{\partial y} + b_y = 0 \quad [3.36b]$$

iii) water phase continuity equation
\[
\beta_{w1} \frac{\partial e_v}{\partial t} + \beta_{w2} \frac{\partial (u_a - u_w)}{\partial t} = \nabla \cdot \left[ k \nabla \left( \frac{u_w}{\gamma_w} + y \right) \right]
\]  

where:

\[
c_{11} = c_{22} = \frac{(1 - \mu)E}{(1 + \mu)(1 - 2\mu)}
\]

\[
c_{12} = \frac{\mu E}{(1 + \mu)(1 - 2\mu)}
\]

\[
c_{33} = \frac{E}{2(1 + \mu)}
\]

\[
a_x^s = c_{11} \left( \frac{1}{H_x} + \frac{\mu}{(1 - \mu)H_y} + \frac{\mu}{(1 - \mu)H_z} \right)
\]

\[
a_y^s = c_{11} \left( \frac{\mu}{(1 - \mu)H_x} + \frac{1}{H_y} + \frac{\mu}{(1 - \mu)H_z} \right)
\]

\[
\beta_{w1} = \left[ \frac{E}{E_w(1 - 2\mu)} \right]
\]

\[
\beta_{w2} = \left[ \frac{1}{H_w} - \frac{E / H_x}{E_w} - \frac{E}{E_w(1 - 2\mu)} \left( \frac{1}{H_x} + \frac{1}{H_y} + \frac{2\mu}{H_z} \right) \right]
\]

3.5.4 Finite element solution for the system of coupled equations

The problem must be discretized in terms of both time and space. The procedure herein presented performs the spatial discretization by using the Galerkin's weighted residual method and a two-level finite difference technique to perform the time discretization.

3.5.4.1 Spatial discretization

The primary unknowns are: \(u, v, u_a\) and \(u_w\). These can be approximated in the domain of the problem using the following numerical expressions,

In the vector form the displacements can be written as,
\[ u(x,y,t) \equiv \sum_{j=1}^{m} \Phi_j(x,y)u_j(t) = \Phi \bar{u} \]  

[3.38]

The pore-fluid pressures can also be written in the vector form,

\[ u_w(x,y,t) \equiv \sum_{j=1}^{n} \phi_j(x,y)u_{w,j}(t) = \Phi \bar{u}_w \]  

[3.39a]

\[ u_a(x,y,t) \equiv \sum_{j=1}^{n} \phi_j(x,y)u_{a,j}(t) = \Phi \bar{u}_a \]  

[3.39b]

where:

\[ \Phi = [\Phi_1, \Phi_2, \ldots, \Phi_m] \]

\[ \Phi_j = \phi_j \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ u_j = \begin{bmatrix} u_j \\ v_j \end{bmatrix} \]

\[ \Phi = [\phi_1, \phi_2, \ldots, \phi_n] \]

\[ n = \text{number of nodal points for displacement} \]

\[ m = \text{number of nodal points for pore-fluid pressures} \]

\[ \phi_j(x,y) = \text{shape function for displacements} \]

\[ j = \text{index for node number} \]

\[ \phi_j(x,y) = \text{shape function for pore-fluid pressures (i.e., } u_w, \text{ and } u_a) \]

\[ u(t)_j = \text{nodal value of the } x\text{-displacement at specific time } t \]

\[ v(t)_j = \text{nodal value of the } y\text{-displacement at specific time } t \]

\[ u_{w,j}(t) = \text{nodal value of the water pressure at specific time } t \]

\[ u_{a,j}(t) = \text{nodal value of the air pressure at specific time } t. \]

Strains must be defined in terms of nodal displacements. Assuming small strains in the domain of the problem, \( \Omega \), the strains can be discretized as:
\[ \varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \sum_{j=1}^{m} \mathbf{B}_j (x, y) \mathbf{u}_j (t) = \mathbf{B} \tilde{\mathbf{u}} \]

where:

\[ \mathbf{B}_j = \begin{bmatrix} \frac{\partial \Phi_j}{\partial x} & 0 \\ 0 & \frac{\partial \Phi_j}{\partial y} \\ -\frac{\partial \Phi_j}{\partial y} & \frac{\partial \Phi_j}{\partial x} \end{bmatrix} \]

From the above approximations, it can be seen that different shape functions can be assumed for the primary unknowns. The primary unknowns have different continuity requirements. The displacement variables require higher order shape functions than do the pore-fluid pressure variables. Some authors, (Reddy, 1985; Lloret and Ledesna, 1993), prefer the use of quadratic functions for displacements and linear functions for pore-fluid pressures. Some authors prefer to use higher order shape functions for both displacement and pore-fluid pressures since high pore-fluid pressure gradients might be involved in the problem being analyzed, (Desai, 1975; Zienkiewicz, 1989). Zienkiewicz (1989) recommends the use of pore-fluid pressure shape functions of the same order or lower than the displacements. In the Galerkin's weighted residual method the weighting functions can be used as the same shape functions used to approximate the unknown variables.

**Spatial discretization for the equilibrium equations**

An alternative way to solve the system of equilibrium equations in terms of finite element approximation, is to use the principle of virtual work. "The virtual work statement is simply a 'weak form' of equilibrium equations.", Zienkiewicz (1977). By using the virtual
work principle, it is not necessary to apply the Galerkin’s weighted residual method to the system of equilibrium equations (i.e., Eq. 3.36).

Let us consider a problem where a domain, \( \Omega \), has boundary surface, \( S \), and \( S \) is composed of two complementary parts \( S_1 \) and \( S_2 \). Assuming that stresses, \( \tau_p \), are acting on \( S_1 \) and displacements, \( u \), are imposed on \( S_2 \), then for any arbitrary virtual compatible displacement, \( \delta u \) (i.e., \( \delta u = 0 \) on \( S_2 \)), the equilibrium equations can be expressed as follows:

\[
\int_{\Omega} \delta \varepsilon^T \sigma d\Omega - \int_{\Omega} \delta u^T b d\Omega - \int_{S_2} \delta u^T \tau_p dS = 0
\]  

where:

\[
b^T = [b_x, b_y] \text{ is the vector of body forces.}
\]

The problem being analyzed is a transient process and Eq. 3.41 must be valid for any incremental deformation, defined by a time-step basis. That equation can be expressed in the incremental, time-derivative, form as follows:

\[
\int_{\Omega} \delta \varepsilon^T \frac{\partial \sigma}{\partial t} d\Omega - \int_{\Omega} \delta u^T \frac{\partial b}{\partial t} d\Omega - \int_{S_2} \delta u^T \frac{\partial \tau_p}{\partial t} dS = 0
\]  

From equations 3.40 and 3.38 respectively, the compatible virtual vectors for strain and displacements can be expressed as follows:

\[
\delta \varepsilon^T = \delta \bar{u}^T \Phi^T
\]  

\[
\delta u^T = \delta \bar{u}^T T
\]  

Substituting Eqs. 3.43 and Eq. 3.44, into Eq. 3.42 results:
\[
\int_{\Omega} B^T \frac{\partial \sigma}{\partial t} d\Omega - \int_{\Omega} \Phi^T \frac{\partial b}{\partial t} d\Omega - \int_{S_2} \Phi^T \frac{\partial \tau}{\partial t} dS = 0
\]  \hspace{1cm} \text{[3.45]}

The total stress, \( \sigma \), can be expressed in terms of net normal stresses, \( \sigma^* \) (i.e., \( \sigma - u_a \)), by expanding the first integral of Eq. 3.45 to the form,

\[
\int_{\Omega} B^T \frac{\partial \sigma}{\partial t} d\Omega = \int_{\Omega} B^T \left[ \frac{\partial \sigma}{\partial t} - \frac{m}{a} \frac{\partial u}{\partial t} + m \frac{\partial u}{\partial t} \right] d\Omega
\] \hspace{1cm} \text{[3.46]}

Substituting Eq. 3.46, the constitutive stress-strain relationships, (i.e., Eq. 3.31), the strain-displacement relationship, (i.e., Eq. 3.40), and the approximation equations for pore-fluid pressures, (i.e., Eqs. 3.39a and 3.39b) into Eq. 3.45.

\[
\int_{\Omega} B^T \left[ \frac{\partial u}{\partial t} + D_s \Phi \frac{\partial u}{\partial t} + D_s \Phi \frac{\partial u}{\partial t} + m \Phi \frac{\partial u}{\partial t} \right] d\Omega - \int_{\Omega} \Phi^T \frac{\partial b}{\partial t} d\Omega - \int_{S_2} \Phi^T \frac{\partial \tau}{\partial t} dS = 0
\] \hspace{1cm} \text{[3.47]}

The primary unknown variables, \( \bar{u} \), \( \bar{u}_a \), and \( \bar{u}_w \), are the vectors of nodal values (dependent only on time). Equation 3.47 can be rewritten in the form,

\[
\frac{\partial \bar{u}}{\partial t} \int_{\Omega} B^T dB d\Omega + \frac{\partial \bar{w}}{\partial t} \int_{\Omega} B^T D_s \Phi d\Omega + \frac{\partial \bar{a}}{\partial t} \int_{\Omega} B^T [m - D_s] \Phi d\Omega = \int_{\Omega} \Phi^T \frac{\partial b}{\partial t} d\Omega + \int_{S_2} \Phi^T \frac{\partial \tau}{\partial t} dS
\] \hspace{1cm} \text{[3.48]}

In the matrix form, Eq. 3.48 can be written as,
\[
[DK] \ddot{u} + [CW] \ddot{u}_w + [CA] \ddot{u}_a = \{F\} \\
\text{[3.49]}
\]

where:

\[\dot{x} = \text{represents the time derivative of } x\]

\[[DK] = \iint_B^T B d\Omega, \text{ is the soil structure stiffness matrix}\]

\[[CW] = \iint_{\Omega} B^T D_s \Phi d\Omega, \text{ is the stiffness matrix related to water phase coupling}\]

\[[CA] = \iint_{\Omega} B^T (m - D_s) \Phi d\Omega, \text{ is the stiffness matrix related to air phase coupling}\]

\[\{F\} = \iint_{\Omega} \Phi^T \frac{\partial b}{\partial t} d\Omega + \iint_{S_2} \Phi \frac{\partial \tau}{\partial t} dS, \text{ is the load vector related to volumetric forces and external stresses.}\]

For the case of earth structures where the pore-air pressure can be considered as being constant and/or at atmospheric conditions, the term \(\ddot{u}_a\) is equal to zero and the system of equations can be reduced to:

\[\[DK\] \ddot{u} + [CW] \ddot{u}_w = \{F\} \quad \text{[3.50]}\]

**Spatial discretization for the water phase continuity equation**

The partial discretization of the water equation will be performed using the basic water flow equation as defined by Eq. 3.3. Later, the water phase constitutive relationship (i.e., Eq. 3.34) is used to express incremental changes in volumetric water content as function of changes in volumetric strain and changes in matric suction. Both equations are reproduced
as follows, and their combination results in the water continuity equation. The volumetric water content variable, $\theta_w$, will be used to replace the term $d(V_w/V_o)$ for convenience.

$$\frac{\partial (\theta_w)}{\partial t} + \nabla \cdot (V_w) = 0 \quad [3.2b]$$

$$d\theta_w = \frac{V_w}{V_o} (\beta w1 d\xi) + \beta w2 d(u_a - u_w) \quad [3.34]$$

Assuming that the domain boundary surface, $S$, for the water phase, is composed of areas $S_1$ and $S_2$ with the following properties: i.) $S_1 \cap S_2 = \emptyset$ and, ii.) $S_1 \cup S_2 = S$. Pore-water pressure and water flux are imposed exclusively on the surfaces $S_1$ and $S_2$ as follows:

$$v_w n = \lambda \text{ at surface } S_2 \quad [3.51a]$$

$$u_w = \phi_w \text{ at surface } S_1 \quad [3.51b]$$

where:

$n$ = the unit vector normal to the boundary.

$\phi_w$ = prescribed pore-water pressure on $S_1$.

Integrating Eq. 3.2b over the domain $\Omega$ by using the Galerkin's weighted residual method, the following weighted integrals should vanish in the domain of integration.

$$\int_\Omega \Phi^T \frac{\partial \theta_w}{\partial t} d\Omega + \int_\Omega \Phi^T \nabla \cdot (V_w) d\Omega = 0 \quad [3.52]$$

Using the identity,

$$\Phi^T \nabla (V_w) = \nabla (\Phi^T (V_w)) - (\nabla \Phi)^T (V_w) \quad [3.53]$$
Equation 3.52 becomes,
\[
\oint_{\Omega} \Phi^T \frac{\partial \theta}{\partial t} w \, d\Omega + \oint_{\Omega} \nabla^T [\Phi^T v \w] \, d\Omega - \oint_{\Omega} [\nabla \Phi]^T v \w \, d\Omega = 0 \tag{3.54}
\]

Applying the divergence theorem to the second term of Eq. 3.54 produces the following results.
\[
\oint_{\Omega} \Phi^T \frac{\partial \theta}{\partial t} w \, d\Omega - \oint_{\Omega} (\nabla \Phi)^T v \w \, d\Omega + \oint_{S} \Phi^T v \w \cdot \mathbf{n} \, dS = 0 \tag{3.55a}
\]
or, in the expanded form,
\[
\oint_{\Omega} \Phi^T \frac{\partial \theta}{\partial t} w \, d\Omega - \oint_{\Omega} (\nabla \Phi)^T v \w \, d\Omega + \oint_{S_1} \Phi^T v \w \cdot \mathbf{n} \, dS + \oint_{S_2} \Phi^T \lambda \, dS = 0 \tag{3.55b}
\]

Equation 3.55b is obtained from Eq. 3.55a by splitting the boundary term into two terms, one on \( S_1' \) and the other on \( S_2' \), and substituting the natural boundary condition (i.e., Eq. 3.51a) into the term of \( S_2' \). One note that if the choice of \( u_w \) is so restricted as to satisfy the forced boundary conditions \( u_w \) equals to \( \varphi_w \) on \( S_1' \), one can omit the last term of Eq. 3.55b by restricting the choice of \( \Phi \) to functions which give \( \phi_j (x,y) \) equals to zero on \( S_1' \). Therefore, Eq. 3.55b can be written as,
\[
\oint_{\Omega} \Phi^T \frac{\partial \theta}{\partial t} w \, d\Omega - \oint_{\Omega} (\nabla \Phi)^T v \w \, d\Omega + \oint_{S_2} \Phi^T \lambda \, dS = 0 \tag{3.56}
\]
Applying Darcy's law to the second term of Eq. 3.56 gives,

\[
\int_{\Omega} \Phi^T \frac{\partial \theta}{\partial t} w \, d\Omega + \int_{\Omega} \left( \nabla \Phi \right)^T \left( \frac{1}{\gamma_w} k_w \nabla u_w + k_w \nabla y \right) d\Omega + \int_{S_i} \Phi^T \lambda dS = 0 \quad [3.57]
\]

The time-derivative of the water constitutive equation (i.e., Eq. 3.34) can be written in the form,

\[
\frac{\partial \theta}{\partial t} w = \beta_w \frac{\partial \varepsilon}{\partial t} + \beta_w \frac{\partial (u_a - u_w)}{\partial t} \quad [3.58]
\]

and expanded to the following form by using the strain-displacement relationship, Eq. 3.40, and the approximations using the element shape functions, defined in Eqs. 3.38, and 3.39.

\[
\frac{\partial \theta}{\partial t} w = \beta_w m^T B \frac{\partial \bar{u}}{\partial t} + \beta_w \frac{\partial u_a}{\partial t} - \beta_w \frac{\partial u_w}{\partial t} \quad [3.59]
\]

Substituting the time derivative of the water constitutive equation (i.e., Eq. 3.59), into the first term of Eq. 3.57 and neglecting the air phase terms, gives,

\[
\frac{\partial \bar{u}}{\partial t} \int_{\Omega} \beta_w \Phi^T m^T B d\Omega - \frac{\partial \bar{u}}{\partial t} \int_{\Omega} \beta_w \Phi^T \lambda dS + \int_{\Omega} \left( \nabla \Phi \right)^T \left( \frac{1}{\gamma_w} k_w \nabla \bar{u}_w + k_w \nabla y \right) d\Omega + \int_{S_i} \Phi^T \lambda dS = 0 \quad [3.60]
\]

Equation 3.60 can be rearranged to the following form,
In matrix form, the water continuity equation can be written as:

\[
[HW][\ddot{\mathbf{u}}_w] + [WK] \begin{bmatrix} \ddot{\mathbf{u}} \\ \dot{\mathbf{u}}_w \end{bmatrix} - [TW] \begin{bmatrix} \mathbf{u} \\ \dot{\mathbf{u}}_w \end{bmatrix} = \{FW\}
\]

where:

\[
HW = \int_{\Omega} (\nabla \Phi)^T \left( \frac{1}{\gamma} k_w \nabla \Phi \right) d\Omega, \text{ is the hydraulic conductivity matrix}
\]

\[
WK = \int_{\Omega} \beta w_1 \Phi^T m^T B d\Omega, \text{ is the mass matrix related to soil structure coupling}
\]

\[
TW = \int_{\Omega} \beta w_2 \Phi^T \Phi d\Omega, \text{ is the mass matrix}
\]

\[
FW = -\int_{\Omega} (\nabla \Phi)^T (k_w \nabla y^2) d\Omega - \int_{S_z} \Phi^T \lambda dS, \text{ vector related to the gravity forces and unit flux across } S_z.
\]

**Summary of the system of coupled equations**

By neglecting the air phase, the system of coupled equations can be written in the following form:

\[
[DK] \ddot{\mathbf{u}} + [CW] \dot{\mathbf{u}}_w = \{F\}
\]
\[
\text{[HW]} \begin{bmatrix} \ddot{w} \end{bmatrix} + \text{[WK]} \begin{bmatrix} \ddot{u} \end{bmatrix} - \text{[TW]} \begin{bmatrix} \dddot{u}_w \end{bmatrix} = \{FW\} \quad [3.62]
\]

In a condensed form, this system can be additionally written as:

\[
\begin{bmatrix} A \end{bmatrix} \{w\} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} \ddot{w} \end{bmatrix} = \{T\} \quad [3.63]
\]

where:

\[
\begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \text{HW} \end{bmatrix}, \quad \begin{bmatrix} B \end{bmatrix} = \begin{bmatrix} \text{DK} & \text{CW} \\ \text{WK} & -\text{TW} \end{bmatrix}
\]

\[
\{w\} = \begin{bmatrix} \ddot{u} \\ \dddot{u}_w \end{bmatrix}, \quad \{T\} = \begin{bmatrix} F \\ \text{FW} \end{bmatrix}.
\]

### 3.5.4.2 Time discretization

The transient analysis can be performed by using a finite difference discretization. For a linear problem, the matrixes \([A]\) and \([B]\) and the force vector \(\{T\}\) are solved as being constants with respect to time. In a general solution, these matrixes are non-linear, being dependent on the stress state variables and/or the material properties of the soil. A two-time level scheme is used in the general formulation herein developed. For this type of integration, the system of differential equations is evaluated at time \((t+\theta\Delta t)\), as follows,

\[
\begin{bmatrix} A \end{bmatrix}_t + \theta\Delta t \begin{bmatrix} w \end{bmatrix}_t + \theta\Delta t + \begin{bmatrix} B \end{bmatrix}_t + \theta\Delta t \begin{bmatrix} \frac{\partial w}{\partial t} \end{bmatrix}_t + \theta\Delta t = \begin{bmatrix} T \end{bmatrix}_t + \theta\Delta t \quad [3.64]
\]
A linear variation of the vector of unknown variables, (i.e., \( w \)), in the time increment between \( t \) and \( (t+\Delta t) \) can be assumed:

\[
\{w\}_t + \theta \Delta t = (1 - \theta) \{w\}_t + \theta \{w\}_{t + \Delta t} \tag{3.65}
\]

The time derivative of the vector of unknown variables, (i.e., \( \dot{w} \)), is expressed in the form:

\[
\left[ \frac{\partial \{w\}}{\partial t} \right]_{t + \theta \Delta t} = \frac{\{w\}_{t + \Delta t} - \{w\}_t}{\Delta t} \tag{3.66}
\]

Equation 3.64 can be solved by substituting in equations 3.65 and 3.66.

\[
\left[ \theta [A]_t + \theta \Delta t + \frac{1}{\Delta t} [B]_t + \theta \Delta t \right] \{w\}_t + \Delta t = \{T\}_t + \theta \Delta t - \left[ (1 - \theta) [A]_t + \theta \Delta t - \frac{1}{\Delta t} [B]_t + \theta \Delta t \right] \{w\}_t \tag{3.67a}
\]

Equation 3.67a can be rearranged to the following form.

\[
[\Delta t \theta [A]_t + \theta \Delta t + [B]_t + \theta \Delta t] \{w\}_t + \Delta t = \Delta t \{T\}_t + \theta \Delta t - \Delta t (1 - \theta) [A]_t + \theta \Delta t - [B]_t + \theta \Delta t \{w\}_t \tag{3.67b}
\]

The matrixes \([A]_t + \theta \Delta t\), \([B]_t + \theta \Delta t\) and the vector \(\{T\}_t + \theta \Delta t\) depend on the unknowns and must be approximated as:

\[
[A]_t + \theta \Delta t = [A(\{w\}_t + \theta \Delta t)] = [A((1 - \theta) \{w\}_t + \theta \{w\}_{t + \Delta t})] \tag{3.68a}
\]
\[
[B]_t + \theta \Delta t = [B(\{w\}_t + \theta \Delta t)] = [B((1 - \theta) \{w\}_t + \theta \{w\}_{t + \Delta t})] \tag{3.68b}
\]
\[ [T]_{t+\theta\Delta t} = [T(\{w\}_{t+\theta\Delta t})] = [T((1-\theta)\{w\}_{t} + \theta\{w\}_{t+\Delta t})] \quad [3.68c] \]

An iterative procedure must be used when solving the nonlinear Equation 3.67b. The simplest procedure is a direct iteration method:

\[ \{w\}_{t+\Delta t} = [C]^{-1}\{f\} \quad [3.69] \]

where:

"i" is the last iteration number

\[ [C] = \left[ \Delta \theta [A]_{t}^{\prime} + \theta \Delta t [B]_{t}^{\prime} \right] \]

\[ \{f\} = \left\{ \Delta \{T\}_{t}^{\prime} - \left[ \Delta \theta (1-\theta) [A]_{t}^{\prime} + \theta \Delta t - [B]_{t}^{\prime} + \theta \Delta t \right] \{w\}_{t} \right\} \]

The iterative process stops when a previously defined accuracy is reached. The chosen time interval plays a fundamental role in the rate of convergence.

Additionally, Eq. 3.67b can be written in a global form as follows,

\[ [AG] \{w\}_{t+\Delta t} = \{FG\}_{t} \quad [3.70] \]

where:

\[ [AG] = \left[ \Delta \theta [A]_{t} + \theta \Delta t [B]_{t} \right] \]

\[ \{FG\}_{t} = \left\{ \Delta \{T\}_{t} + \theta \Delta t - \left[ \Delta \theta (1-\theta) [A]_{t}^{\prime} + \theta \Delta t - [B]_{t}^{\prime} + \theta \Delta t \right] \{w\}_{t} \right\} \]

3.5.4.3 Some comments on the use of the coupled solution

The theory presented in section 3.5.4 uses the finite element method in conjunction with the finite difference in the numerical modelling of a two-dimensional coupled water flow-deformation problem applied to an unsaturated soil. The use of such a modelling in the solution of a specific problem requires time and experience to be performed. Some time can
be saved by using previous experience accumulated from past and similar studies. In the following some aspects of the particular solution for collapsing dams are discussed based on previous studies.

a.) The assumption of infinitesimal deformations has been used in the formulation. Hence, there is some error involved in the solution since it neglects a change in the elemental volume, \( dV_0 \). For a rigid soil, the assumption of infinitesimal deformation is fulfilled without any restriction. For a deformable porous medium, the study presented by Carter (1977) examined the importance of non-linear geometric effects in geotechnical analysis. "The conclusion of the study was that the 'linear' assumption of small strains and small displacements is usually satisfactory for the solution of geotechnical problems. In most cases, the normal infinitesimal strain assumption leads to an over-estimation of deformations when compared to the use of a finite deformation theory.". Britto and Gunn (1985).

Alternatively, geometric non-linearities can be included in accordance to Britto and Gunn (1985) who, quoting to Cook (1981), mentioned the advantages of the updating the continuum geometry based in a deformation criterion. The update of the geometry corresponds to a first approximation of an up-dated Lagrangian formulation.

In the present model, the up-dating of the continuum geometry is used based on a deformation criterion.

b.) The solution requires appropriate boundary conditions along all the boundaries of the domain for the three primary unknowns (i.e., displacements, \( u, v \), and pore-water pressure, \( u_w \)) in an independent manner (i.e., each primary unknown has its specific boundary conditions). The boundary conditions related with the primary unknown (i.e., geometric or essential boundary condition) act in one part of the boundary surface of the domain. The boundary conditions related with the derivative of the primary unknown (i.e., natural boundary conditions) act on the complementary part of the surface.

The problem of a collapsing dam is characterized by a transient evolution of a 'wetting-front'. This fact might require up-dating of pore-water essential boundary conditions.
before the steady-state conditions be reached (i.e., a "free surface" at the downstream slope). The up-dating can be performed by using the following steps suggested by Papagiannakis (1982).

i.) definition of the boundary surfaces which might suffer changes during the transient seepage process;

ii.) definition of a set of criteria under which the boundary conditions change;

iii.) definition of a numerical procedure, to check the set of criteria and perform the changes in boundary conditions during the transient ‘wetting-front’ advance process;

c.) The solution of the system \([AG]w_{t+\Delta t} = FG_t\) results in the computation of the vector \(w_{t+\Delta t}\) by using the previous vector \(w_t\). It is necessary to know the initial conditions for the three unknowns involved in the problem. In the particular problem involving the coupled flow-deformation analysis of an unsaturated earth structure, it is necessary to know the initial conditions related to displacements, and water and air pressures at time \(t\) equal to zero (i.e., at the beginning of the transient process).

d.) The transient solution requires the choice of an appropriate time step, \(\Delta t\). In theory, for two-step scheme level, the convergence of the solutions will be stable for any choice of time step since \(\theta \geq 0.5\), although it is still possible to have instability in the solution. Usually it is appropriate to choose \(\theta\) equal to 1 in order to have unconditionally stable solution for any size of time steps (Booker and Small, 1975).

Britto and Gunn (1985), using linear elements to solve the consolidation problem of saturated soils emphasized two specific points. The first point was associated with a preliminary estimate of the coefficient of consolidation, \(c_v\), of the soil. This value is required in order to provide an estimate of the total time required for pore-pressure dissipation.

The second point is explained by the paragraph extracted from the cited reference:

"Figure 3.3 illustrates the isochrone of water pore-pressure moving in from the boundary up to the point denoted by A. Points below A have not yet experienced any change
in pore-water pressure due to the draining boundary. For saturated soils, it can be shown that the time taken, \( t \), is given by

\[ l = \sqrt{12c_v t} \quad [3.71] \]

where \( l \) is the distance to point A from the boundary. If \( l \) is the normal distance of the first pore-pressure node from the boundary, then \( t \) specifies the minimum time step that can be specified. This can be explained as follows. The selected element allows for a linear variation in pore-water pressure. If a time step \( t_1 \) is less than this time interval \( t \) then the drainage would have taken place up to point B. An attempt by the analysis to accurately model this situation would generate a pore pressure at A equal to \( \Delta \sigma' \) which is greater than the applied vertical pressure. In order to compensate for this error, a smaller pore pressure is generated in the next node. This results in the zigzag distribution, shown in Figure 3.3".

![Diagram](image)

Figure 3.3 Pore pressures distribution after first step of analysis with short time step. (after Britto and Gunn, 1985).

Britto's and Gunn's (1985) analysis was performed for a saturated soil with a constant water permeability, \( k_w \), and a constant coefficient of volume change, \( m_v \), then
resulting in a constant value for $c_w$. For unsaturated soils, the selection of the initial time step to satisfy the coupled flow-deformation solution is more complex. This is a result of the low and non-linear water coefficient of permeability, $k_w$, and the existence of four different compressibility coefficients, $m_1^s$, $m_2^s$, $m_1^w$, and $m_2^w$ controlling the consolidation process. In the present research study a trial and error procedure was performed in the search of an initial time that satisfy the physical reality of the problem. The use of smaller elements near the draining boundary assists in minimize the problem.

e.) In the present research study the mechanical behavior of the soil is simulated by means of state surfaces as proposed by Matyas and Radhakrisna (1968). On failure the soil is considered to be a perfectly plastic material (i.e., a relatively low elasticity modulus, $E$, is assumed to the elasticity matrix $D$ for a failed soil element) with non-linear shear strength parameters defined from modified direct shear tests. The failure criteria utilized is the extended Mohr-Coulomb model proposed by Fredlund and Gan (1978). The mobilization of shear strength (i.e., $\frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f}$) is utilized to define failure conditions in a soil element.

f.) At saturated conditions the soil behaves as an isotropic and non-linear elastic material. The mechanical behavior of a saturated soil element is then controlled by the effective stress acting on the soil skeleton. The soil transition from unsaturated to saturated condition requires that the compressibility parameters $m_1^s$, $m_2^s$, $m_1^w$, and $m_2^w$, change to the coefficient of volume change $m_v$ for the saturated soil. This transition must be performed in a smooth manner to avoid disturbance of the stress state in the soil. In numerical terms this transition can be better controlled by defining smaller time steps for the transient process.

3.6 Applications of the numerical modelling

Applications of the numerical model to coupled stress and flow analyses in unsaturated soil mechanics are presented and discussed in chapters 6 and 7 in this thesis.
CHAPTER 4

LABORATORY PROGRAM

4.1 Introduction

A laboratory program was designed with the main objective of defining the mechanical behavior and hydraulic properties of a compacted collapsing soil gradually saturated by water. The saturating of the collapsing soil would simulate the transient unsaturated-saturated water flow during the first reservoir filling of a small dam. The range of stress state variables and stress paths used were defined considering the following factors:

a.) the initial condition of the compacted soil in terms of both net normal stress and matric suction. The selected stress state corresponded to the after construction/after reservoir filling stress state in a small typical earth dam;

b.) the gradual decrease in matric suction during the development of the transient water flow process before steady-state conditions are reached;

c.) the results from previous uncoupled analyses (Miranda, 1988) utilized to simulate the mechanical behavior of "Alka-Seltzer" dams during the first filling of the reservoir.

The analysis of the mechanical behavior of a collapsing earth dam during its first reservoir filling requires an understanding of the constitutive relationships for the collapsing soil. The state surfaces (i.e., void ratio and degree of saturation versus stress state variables relationships) are the constitutive relationships required for the volume change behavior of a collapsing soil (Matyas and Radhakrishna, 1968). The shear strength constitutive relationship for the collapsible soil is defined by the shear parameters required for the extended Mohr-Coulomb failure criterion (Fredlund et al. 1978).
The analysis of the hydraulic behavior of a collapsing earth dam involves a prediction of the flow of water through a deforming unsaturated soil. The flow law requires that the coefficient of permeability, $k_w$, be defined as a function of either the stress state variables or the volume-mass properties of the soil. The coefficient of permeability corresponds to the constitutive relationship for water flow.

In the present research study, the definition of the mechanical behavior and hydraulic properties for the collapsing soil required the development of a two phase laboratory program:

i.) The first phase consisted of a program of conventional laboratory testing with the objective of defining the compaction conditions of the collapsing soil. During this phase, previous studies on “Alka-Seltzer” dams (Miranda, 1988) were used in the definition of the compaction conditions of the collapsing soil. Additional studies were performed on the mechanical behavior and hydraulic properties of the same soil compacted under non-collapsing conditions. The stable compaction conditions were defined with the intent of looking forward to alternative solutions applicable to the design of “Alka-Seltzer” dams.

ii.) The second phase consisted of performing a series of laboratory tests on the collapsing soil to define its volume change state surfaces, extended Mohr-Coulomb failure criterion, and coefficient of permeability for the range over which the stress state variables change. In order for this second phase to be performed, the first phase needed to be completed. The second phase was the main objective of the laboratory program. Chapter 4 first presents the results and preliminary analysis of the first phase and then defines a series of laboratory tests for the second phase.

4.2 Testing material

The soil used in the present research study is a residual soil derived from a granitic gneiss of the Ceara’ group and is representative of a typical soil used in the construction of
“Alka-Seltzer” dams in Northeast Brazil. This material was used by Miranda (1988) in the analysis of failure mechanisms of “Alka-Seltzer” dams during first reservoir filling.

4.2.1 Characterization of the Testing material

The soil is a residual silty sand. The index properties are presented in Table 4.1. This table was compiled based on data obtained from Miranda (1988) and by the author.

Table 4.1 Index properties of the soil (Miranda*, 1988; Pereira, 1994).

<table>
<thead>
<tr>
<th>Soil</th>
<th>Residual silty sand derived from gneiss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>County of Pacatuba in the State of Ceará/Brazil</td>
</tr>
<tr>
<td>Natural water content</td>
<td>2 - 4 %(*)</td>
</tr>
<tr>
<td>Typical grain size distribution</td>
<td>Sand - 52 %; 54%(*)</td>
</tr>
<tr>
<td></td>
<td>Silt - 35 %; 28%(*)</td>
</tr>
<tr>
<td></td>
<td>Clay - 13%; 13%(*)</td>
</tr>
<tr>
<td></td>
<td>D10 - 0.0006 mm</td>
</tr>
<tr>
<td></td>
<td>D30 - 0.016</td>
</tr>
<tr>
<td></td>
<td>D60 - 0.22 mm</td>
</tr>
<tr>
<td>Atterberg limits</td>
<td>Liquid limit, (w_l) = 29; 33(*)</td>
</tr>
<tr>
<td></td>
<td>Plastic limit, (w_p) = 17; 25(*)</td>
</tr>
<tr>
<td></td>
<td>Plasticity index, PI = 12; 8(*)</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>(G_s = 2.64; 2.64(*)</td>
</tr>
<tr>
<td>Unified soil classification system</td>
<td>SW-SM; well-graded sand with silt</td>
</tr>
</tbody>
</table>
connecting the larger particles of sand and silt. The formation of clay aggregations in a compacted soil is a function of the amount of clay and the water content during compaction.

![Graph showing grain size distribution](image)

Figure 4.1 Grain size distribution for the residual soil.

Laboratory tests to define the constitutive relationships for the collapsing soil were performed on statically compacted specimens using material passing the 2.0 mm sieve. The amount of material larger than 2 mm is less than 2%. This procedure was used in order to obtain compacted specimens of a conventional size for consolidation tests and shear tests. Miranda (1988) also used the same procedure.

**4.2.2 Properties of the compacted testing soil**

Engineering practice has established that compaction of the soil at optimum conditions of the standard AASHTO energy generally results in a satisfactory small dam and
in a stable-structured compacted soil. Previous studies (see section 2.3.3) have illustrated the following points about clayey or silty sand compacted at dry of optimum conditions:

a.) when a silty or clayey sand is compacted dry of optimum water content and at a low dry density, it often has a metastable structure (Dudley, 1970; Barden et. al. 1973).

b.) when a soil is compacted dry of optimum water content, the denser the compacted soil the less its collapsing behavior (Popescu, 1986; Tadepalli, 1990).

c.) when a soil is compacted at a given dry density, its compressibility increases as the compaction water content increases (Booth, 1977; Tadepalli, 1990; Lawton et al. 1991a).

d.) the initial matric suction in a compacted collapsing soil is primarily a function of its compaction water content rather than its dry density, (Booth, 1977; Lin, 1995).

The present research program is also related to the study of alternative solutions for “Alka-Seltzer” dams. Adverse site conditions will be considered as determinantal factors in laying out the testing program (see section 1.1). Some design alternatives for collapsing dams have already been discussed elsewhere (Sherard, 1953; Miranda, 1983,1988; Pereira, 1985).

A first alternative solution for an “Alka-Seltzer” dams is to design it as a homogeneous embankment where the soil is compacted dry of optimum water content, at a low dry density. A second alternative is to design the dam as a zoned dam where the soil is compacted at optimum AASHTO standard energy condition in one vertical zone, while the remainder of the embankment is compacted dry of optimum water content at a low dry density. The laboratory program focuses on defining the mechanical behavior and hydraulic properties of the soil at two different compaction conditions in order that the suggested alternatives can be studied. The two conditions are as follows:

a.) optimum water content condition of the soil with the AASHTO standard compaction energy;

b.) dry of optimum water content condition and at a low dry density. This condition would reflect the situation corresponding to the construction of an “Alka-Seltzer” dam. This condition results in a collapsible soil.
The dry compaction condition resulting in a collapsing soil for the range of net normal stresses occurring in an “Alka-Seltzer“ dam can be defined. Specimens extracted from “Alka-Seltzer” dams in northeast Brazil demonstrate that the average dry density is about 14.75 kN/m$^3$ (Miranda, 1988). An initial water content was selected such that the initial matric suction could be measured and controlled using equipment available in the laboratory at the University of Saskatchewan.

Miranda (1988) suggested the use of a gravimetric water content of 6.5 % and a dry density of 14.75 kN/m$^3$. The soil would have a collapsing behavior and the matric suction was estimated from a soil-water characteristic curve to be of about 5050 kPa. The initial water content of the soil specimens to be used in the present research study was selected on a trial and error basis. Using the dry density of 14.75 kN/m$^3$, specimens were compacted at gravimetric water contents of 8.5 %, 9.5 %, and 10.5 %. The measured matric suction for specimens compacted at water contents of 8.5% and 9.5 % were higher than 600 kPa (i.e., limit of the available null-pressure plate). The matric suction of 370 kPa was measured for the specimen compacted at a water content of 10.5%. The collapsing behavior of the soil compacted at 9.5 % and 10.5 % was checked by using double oedometer tests (Jennings and Knight, 1957), according to the procedure ASTM-D5333 - 92. In terms of the magnitude of volumetric collapse, the results were similar. However, there was a slight reduction in the magnitude of the soil collapse for the soil specimen compacted at 10.5%. The verification of a significant amount of collapse, the initial measured matric suction, and the practical aspect of workability of the specimen, led to the choice of a gravimetric water content of 10.5 %. The results of the double oedometer test on specimens with a water content of 10.5 % are presented later in this chapter.

Figure 4.2 shows the compaction curve obtained by using the AASHTO's standard energy. It also illustrates the proposed point (i.e., at a gravimetric water content of 10.5 % and a dry density of 14.75 kN/m$^3$) used in the present research study.
Figure 4.2 also illustrates that the initial condition of the collapsing soil corresponds to a water content of minus 4% dry of optimum conditions and a density of 90% of the corresponding point on the AASHTO standard compaction energy curve. The relative density of 90% reflects the actual conditions, in terms of equipment availability, at the site of the construction of Alka-Seltzer dams. Miranda's (1988) suggested point, at a water content of 6.5%, is also illustrated in Figure 4.2. A preliminary extrapolation of the compaction curve allows a verification of Miranda's (1988) suggested point. Such a extrapolation also shows Miranda's (1988) suggested point reaches the AASHTO standard compaction energy curve.

![Compaction curve of the residual soil of gneiss.](image)

Figure 4.2 Compaction curve of the residual soil of gneiss.

The justification for the study of the soil in two different compaction conditions is illustrated with the aid of Figure 4.3. This figure presents possible alternative solutions wherein the collapsing earth dam was reinforced by a narrow zone of soil compacted at
optimum conditions. These solutions were based on the failure mechanisms of collapsing
dams as presented on Figure 1.1 in chapter 1.

![Diagram of upstream reinforcement and central reinforcement](Image)

a.) Upstream reinforcement.

b.) Central reinforcement

Figure 4.3 Alternative solutions for collapsing earth dams

Figure 4.3a illustrates an upstream reinforcement of the collapsing dam that would
improve the stability of the upstream slope during its first reservoir filling. This alternative
also would reduce risks of "piping" in the collapsing dam. Figure 4.3b shows a central
reinforcement of the collapsing dam with the main objective to improve the safety of the
embankment against "piping". The efficiency and cost of these alternatives are functions of
the thickness of the reinforced zone compacted at optimum conditions. Miranda (1988) proposed a solution involving a central reinforcement based on an simplified uncoupled numerical analysis that indicated that piping was the phenomenon associated with the failure of “Alka-Seltzer” dams.

4.2.2.1 Properties of the soil compacted at dry of optimum conditions

Figure 4.4 shows the variation of volumetric strain with respect to the applied vertical stress (i.e., collapse potential of the soil tested) obtained from the double oedometer tests. The soil specimens were statically compacted, in the oedometer ring, to a dry density of 14.75 kN/m$^3$ and at a gravimetric water content of 10.5%. The range of vertical stresses used for the study was 0 to 800 kPa. However, the range of interest for practical conditions is lower than 200 kPa (i.e., based on the geometry of the dam and previous studies from Miranda, 1988).

Figure 4.4 shows that the soil specimen did not present any collapsing behavior for applied vertical stresses lower than 50 kPa. A vertical stress of 100 kPa produced soil collapse amounting to 3.0%. The volumetric collapse reached about 7.2% at a vertical stress of 200 kPa.

The soil presented low compressibility when loaded under unsaturated conditions. Upon saturation by inundation at a vertical stress of 400 kPa, the measured soil collapse was greater than 11% and closely coincided with the saturated stress path curve. An increase to 800 kPa in the vertical stress resulted in a similar response for both saturated stress path curves. When unloaded to 25 kPa of vertical stress there was essentially no strain recovery of the specimen volume. The soil exhibited a rigid behavior when unloading.

Figure 4.5 illustrates the double oedometer test results for the collapsing compacted soil specimen in terms of void ratio and volumetric water content versus vertical stress.
Fig. 4.4 Volumetric deformation versus vertical stress relationship for the collapsible soil.
Figure 4.5 Double oedometer test results for the collapsible soil.
Figure 4.5a reflects the results already presented for volumetric deformation in Fig. 4.4. In Figure 4.5b, the volumetric water content of the soil specimen following the unsaturated path was plotted by assuming that the soil specimen maintained a constant gravimetric water content before it was saturated at the vertical stress of 400 kPa. This assumption can be justified by the short time required for the load increments and by the isolated condition of the specimen during the experiment. The volumetric water content of the unsaturated curve showed negligible changes for vertical stresses from 0 to 200 kPa.

Figure 4.6 presents the double oedometer test results, in terms of volumetric deformation, for the soil compacted at a water content of 6.5% (i.e., -8% dry of optimum) and at a dry density of 14.75 kN/m³. These results were obtained by Miranda (1988).

![Graph showing volumetric strain vs. vertical stress](image)

Figure 4.6 Volumetric deformation and collapse potential of the residual soil compacted at the water content of 6.5%. (Miranda's, 1988).

Comparing the soil collapse behavior at water contents of 6.5% and 10.5% (in Fig. 4.4) the following observations were made:
a.) the loading of the unsaturated specimens showed no differences for vertical loads from 0 up to 400 kPa. A higher compressibility of the wetter soil specimens was expected. However, the problem may be partly attributable to the lateral friction between the soil specimen and the oedometric ring. Other potential factors are the random variations in the soil specimens and, to a lesser extent, changes in the microstructure produced by the different compaction water content.

b.) the influence of the initial water content was not significant in terms of the magnitude of collapse for vertical stresses from 100 to 200 kPa. As expected, the wetter specimens presented less collapse than the drier specimens. At vertical stresses less than 100 kPa, the wetter specimens showed a significant decrease in collapse. Such differences can also be partly justified by: i.) the side friction between the oedometer ring and soil specimens, mainly for the low vertical loads; and ii.) random variations in specimens and changes in microstructure due to the different compaction water content.

Falling-head permeability tests were also performed on the soil specimens prepared at a gravimetric water content of 10.5%. These tests were performed on the saturated soil specimens during the double oedometer test and after equilibrium was achieved for each step of the vertical applied load. Figure 4.7 illustrates the results in terms of the logarithm of the coefficient of permeability versus the void ratio. The vertical stress applied at each corresponding void ratio is also presented. If the fabric of a saturated soil remains constant during consolidation, a linear relationship between the logarithm of coefficient of permeability versus void ratio can be expected (Lambe and Whitman, 1979).

Figure 4.7 illustrates a non-linear variation between the logarithm of coefficient of permeability and the void ratio. Such a non-linearity may be attributed to the changes in soil fabric due to the collapse behavior (see section 2.3.2.1). Figure 4.7 also illustrates that the linearity of the curve presents a significant influence of the soil collapse on the coefficient of permeability only beyond a vertical load of 200 kPa.
Figure 4.7 Falling-head permeability tests for the collapsing soil at a gravimetric water content of 10.5%.
Figure 4.8 shows an unloaded drying soil-water characteristic curve for the collapsing soil. This test was performed to determine the storage capacity of the collapsing soil under increasing values of suction. This result was utilized to estimate, in a preliminary basis, the range of matric suction to be used in the isotropic consolidation tests performed in a subsequent phase of the laboratory program for this research. The tests were performed using Tempe-cell and pressure-plate apparatus and were based on the (ASTM D 3152-72). When using the Tempe-cell test, it was possible to apply a starting matric suction of 1.0 kPa for the first step of the soil-water characteristic curve. The pressure-plate tests had to start with a matric suction of 10 kPa since the soil specimens could not be removed from the pressure chamber at lower matric suction.

\[
\gamma_d = 14.75 \text{ kN/m}^3
\]
\[
\omega(%) = 10.5 \%
\]

Figure 4.8 Drying soil-water characteristic curve of the collapsible soil.
The drying soil-water characteristic curve showed that the soil specimen started desaturating at a matric suction of about 3.0 kPa. The degree of saturation of the specimens dropped to values of less than 50% for suction values of about 60 kPa. The results indicates a residual degree of saturation could be about 40%.

Figure 4.8 also shows the initial matric suction (i.e., 370 kPa) of the collapsing compacted soil measured by using null-tests. The as-compacted initial degree of saturation of 36.7% was never reached by the drying process used for defining the soil-water characteristic curve, even for an applied matric suction of 500 kPa.

From the drying soil-water characteristic curve it can be inferred that the coefficient of permeability would reach extremely low values at a matric suction around 100 kPa. A long time would be required to attain a steady-state water flow condition in the as-compacted soil specimen at low applied hydraulic gradients. The time for equilibrium would also depend on the dimensions of the compacted specimen. Using the drying soil-water characteristic curve and the saturated coefficient of permeability value, an estimate of the coefficient of permeability for the unsaturated soil as a function of the applied matric suction was performed using Brooks’ and Corey’s (1964) equation. The analysis indicates a residual degree of saturation of about 34%. Figure 4.9 illustrates the coefficient of permeability versus matric suction relationship.

Figure 4.9 indicates that under an applied matric suction of around 370 kPa, the unsaturated coefficient of permeability might reach a magnitude of about $10^{-13}$ m/s. Under an applied matric suction of 80 kPa, the coefficient of permeability would reach a magnitude of about $10^{-11}$ m/s. Under a wetting stress path, a very small increase in the volumetric water content from the initial condition of the soil (i.e., matric suction equals to 370 kPa) up to an applied matric suction of 80 kPa was expected since hysteresis should prevent high increases in volumetric water content of the soil specimen.
Figure 4.9 Estimate of the coefficient of permeability from the soil-water characteristic curve by using Brooks’ and Corey’s (1964) method.

4.2.2.2 Properties of the soil compacted at optimum conditions

Figure 4.10 shows the results of a double-oedometer test performed on the soil compacted at optimum conditions. It illustrates the changes in volumetric deformations with respect to the applied vertical stresses. At the optimum condition, the residual soil of gneiss did not present any collapsing behavior. The loading of the unsaturated soil practically coincided with the loading of the saturated soil for vertical loads from 25 to 800 kPa. Up to an applied vertical stress of 200 kPa, the soil reached a vertical (i.e., volumetric) deformation of less than 3%. The soil along the unsaturated path was inundated at a vertical stress of 400 kPa but did not show any additional deformation.
Figure 4.10 Volumetric deformation and collapse potential for the soil compacted at optimum condition of the AASHTO standard energy.
Figure 4.10 shows results which are similar to those presented by Miranda (1988). Therefore, the residual soil presents a stable-structured behavior when compacted at optimum conditions of the AASHTO standard energy.

Figure 4.11 shows the double oedometer test results for the stable-structured soil in terms of void ratio and volumetric water content changes. Figure 4.11a shows the variation of the void ratio with the applied vertical stress. These results are similar to those presented in Figure 4.10. Figure 4.11b illustrates the double-oedometer test results for the soil compacted at optimum conditions in terms of volumetric water content and changes in vertical stress. The volumetric water content for the curve of unsaturated soil loading was again plotted with the assumption that the soil gravimetric water content remained constant as the applied vertical stress was increased from 0 to 400 kPa. At this vertical stress the soil specimen was inundated and allowed to reach saturation for a period of 24 hours. Figure 4.11b shows that the volumetric water content versus applied vertical stress curves for both unsaturated and saturated paths coincided (i.e., before inundation) at an applied vertical stress of 400 kPa.

Fig. 4.11a.) Void ratio.
Fig. 4.11b.) Volumetric water content.

Figure 4.11 Double oedometer test results for the soil compacted at optimum conditions of AASHTO standard compaction energy.

Falling-head permeability tests were also performed on the soil specimens compacted at optimum conditions. The tests were performed for each load step along the saturated stress path of the double-oedometer test. Figure 4.12 shows the test results obtained. The logarithmic variation of the coefficient of permeability versus the void ratio is linear. In contrast to the dry of optimum conditions (Fig. 4.7), the saturated coefficient of permeability was relatively constant for the void ratio variation from 0.417 (i.e., at 25 kPa of vertical stress) to 0.36 (i.e., at 800 kPa of vertical stress). The soil fabric appears to be independent of the applied range vertical loads.
Figure 4.12 Falling-head permeability test results for the soil compacted at optimum condition.
Figure 4.13 shows the drying curves performed on soil specimens compacted at optimum conditions. The soil-water characteristic curve is presented in terms of degree of saturation versus matric suction and illustrates the drainage/storage capacity of the soil at the optimum conditions.

![Graph showing soil-water characteristic curve](image)

Figure 4.13 Drying soil-water characteristic curve of the stable soil.

Figure 4.13 also illustrates that the soil starts desaturating under an applied matric suction of about 10 kPa for specimens compacted at optimum conditions. The residual state conditions were not reached even for an applied matric suction value of 300 kPa. The initial matric suction of 30 kPa, measured by using null-tests, is also shown in Figure 4.13. The as-compactsed condition presents a degree of saturation of about 90%. By using the drying soil-water characteristic curve, this value would correspond to an applied matric suction of about 40 kPa. At 300 kPa, the soil had a degree of saturation higher than 82%. Due to the low
soil compressibility, this drying soil-water characteristic could be used to predict the coefficient of permeability for the range of stress state variables to occur in an small collapsing dam. Under a wetting process, even the saturated coefficient of permeability could be used as a reasonable estimate, as did Miranda (1988), for representing the unsaturated coefficient of permeability of the soil compacted at optimum conditions.

4.2.2.3 Summary of the first phase of laboratory testing

The preliminary laboratory testing program consisted of the following tests performed on the residual soil of gneiss: i.) soil characterization and compaction test by using the AASHTO Standard compaction energy, ii.) double-oedometer tests for the soil compacted at two different compaction conditions, iii.) falling-head water permeability tests, performed with the soils at a saturated condition and under different vertical loads, for the two differently compacted soils, and iv.) drying soil-water characteristic curves for the residual soil of gneiss compacted at different conditions.

Double oedometer tests on specimens compacted with a dry of optimum water content showed collapsing behavior for a vertical stress of about 50 kPa. The deformation due to the soil collapse increased when the vertical stress was increased in steps from 50 to 400 kPa. The soil had been compacted at minus 4% dry of optimum water content and at a relative density of 90% of the AASHTO standard energy compaction curve. When the soil was compacted at optimum conditions of the AASHTO standard compaction energy, it behaves as a stable-structured soil with low compressibility for the range of vertical stresses from 25 to 800 kPa.

Falling-head permeability tests were conducted on the compacted soil specimens along the saturated path of the double-oedometer test. The saturated coefficient of permeability of the soil compacted with a water content dry of optimum conditions varied from \( k_w = 1.5 \times 10^{-6} \) to \( k_w = 4.0 \times 10^{-9} \) m/s, for applied vertical stresses from 25 kPa to 800 kPa.
A relationship between the logarithm of the coefficient of permeability versus the void ratio was non-linear. This behavior was attributed to the change in soil fabric along the saturated path as the soil gradually collapses under increasing vertical stresses. For the soil compacted with a water content reflecting optimum conditions, the coefficient of permeability varied from $k_w = 2.3 \times 10^{-9}$ to $k_w = 5.5 \times 10^{-10}$ m/s, for the applied vertical stress range of 25 to 800 kPa. The relationship between the logarithm of the coefficient of permeability and the void ratio was linear and illustrated the stable structure of the soil when compacted at optimum conditions.

The drying soil-water characteristic curve of the soil compacted at a water content dry of optimum suggested that the soil desaturated up to a degree of saturation of about 40% when the matric suction was increased to 80 kPa (see Fig. 4.8). Low values for the coefficient of permeability would be expected for suction values higher than 80 kPa. Brooks and Corey's (1964) method was used to estimate the unsaturated coefficient of permeability. A coefficient of permeability equal to $10^{-11}$ m/s was predicted at a matric suction of 80 kPa. At the as-compactcd initial matric suction of 370 kPa, the unsaturated coefficient of permeability was predicted to be of about $10^{-13}$ m/s.

4.3 Laboratory program for definition of the constitutive relationships of the soil

The first phase of the laboratory program defined the compaction conditions for the collapsing soil. The order of magnitude of the soil collapse as a function of the applied vertical stress was determined by means of conventional double oedometer tests. These tests do not allow any prediction of the evolution of the soil collapse as a function of the gradually decreasing matric suction during the saturation of the soil specimen.

The second phase of the laboratory program was designed to incorporate the mechanical behavior and hydraulic properties of the two compaction conditions defined in
the first phase. Therefore, this phase required the use of equipment which allowed the independent control of matric suction and total stress. The preliminary results from the first phase were used as guidelines for identifying the range of stress state variables of interest with respect to practical field loads and equipment limitations. The preliminary results could also be used to estimate orders of magnitude of the soil collapse and the coefficient of permeability to be expected for the second phase. However, the main concern in this phase of testing was the prediction of the mechanical behavior of the collapsing soil during saturation.

4.3.1 Evaluation of the collapsing soil behavior during saturation

Analysis of the available data from the first phase along with the knowledge of the available equipment in the laboratory suggested the following guidelines for the second phase of the laboratory program. The program was required:

1.) To define the volume change constitutive relationships for the compacted collapsing soil by using isotropic consolidation tests performed in the triaxial permeameter built by Huang (1994). The stress paths utilized should be based on the loading conditions and the wetting process occurring with the collapsing soil during the first reservoir filling of “Alka-Seltzer” dams.

2.) To define the extended Mohr-Coulomb failure envelope for the collapsing soil by using the modified shear box built by Gan and Fredlund (1988).

3.) To define the coefficient of permeability for the collapsing soil along the wetting stress paths used for measurements of volume changes. The coefficient of permeability could be measured by the triaxial permeameter using the constant head, steady-state, controlled head method, as done by Huang (1994).

The mechanical behavior of a collapsing soil is highly dependent on the stress path (see section 2.3.3). The present research study required that all the tests be performed in steps of increasing degrees of saturation (i.e., decreasing matric suction). Such procedures implied
that the compacted specimens would be initially at an matric suction of 370 kPa. The first step for the increasing matric suction to be applied on the unsaturated soil specimen was defined based on the following information:

a.) Escario et al. (1973) presented studies on the collapsing behavior of a compacted clayey sand during saturation. The soil specimens were loaded at a given net confining stress (in the range of 100 to 600 kPa and gradually saturated until the matric suction reached a 0 kPa value (see Figure 2.16 in Chapter 2). Some results pointed out that for specimens at an initial matric suction of 350 kPa, the soil specimens only started collapsing when the matric suction was reduced to less than 60 kPa.

b.) The minimum coefficient of permeability measurable using the triaxial permeameter is $10^{-11}$ m/s (Huang, 1994). Juca (1993) illustrated that for a compacted clayey sand, the measured values of coefficient of permeability were practically constant and about $10^{-12}$ m/s after reaching the residual degree of saturation (see Figure 2.20 in Chapter 2). In the first phase of this laboratory program, the Brooks and Corey’s (1964) method was utilized to predict the coefficient of permeability versus matric suction relationship for the collapsing soil (see Figure 4.9) using a drying soil-water characteristic curve. A coefficient of permeability of $10^{-11}$ m/s was predicted at a matric suction of about 80 kPa. A small increase in volumetric water content from the initial condition of the soil up to an applied matric suction of 80 kPa is expected under a wetting stress path. Therefore, it was decided to conduct the first test to measure the water coefficient test at an applied matric suction of 90 kPa. More details of this testing procedure are available in section 4.7.1.

The practical range of isotropic net normal stress on “Alka-Seltzer” dams will generally be lower than 200 kPa. Hence, a range of 20 to 200 kPa was chosen as the testing range. The steps of isotropic net normal stresses used are detailed in section 4.7.1.

For the modified direct shear tests, a range from 0 to 100 kPa of matric suction values was chosen. This range was mainly based on the fact that after the residual degree of saturation has been reached, there is no significant increase in shear strength (Escario et al.,
1986; Vanapalli, 1994). The range of vertical net normal stress was defined from 20 to 200 kPa based on the geometry of the dam. Wetting stress paths were also used in these tests. The steps for the stress state variables are later detailed in section 4.7.2.

4.3.2 Evaluation of the stable soil behavior during saturation

The mechanical behavior of the soil compacted at optimum conditions was evaluated by using the following procedures.

a.) The volume change behavior was estimated from double oedometer tests, using a procedure similar to that used by Miranda (1988). This procedure was explained in section 2.3.1.3 in chapter 2. The soil is considered as an elastic and linear material. The soil structure compressibility parameter related to the net normal stress, \( m_1^s \), is defined directly from a linear best-fit from the stress-strain curve of the double oedometer test. The soil structure compressibility parameter related to matric suction, \( m_2^s \), is assumed to be zero based on the double oedometer test results (see Figure 4.10). The Poisson ratio, \( \mu \), is estimated as a constant value equal to 0.3 according to Miranda, (1988). The water phase compressibility parameter related to the net normal stress, \( m_1^w \), is assumed to be zero due to the low compressibility of the soil structure. The water phase compressibility parameter related to the matric suction, \( m_2^w \), is calculated as a non-linear parameter from the drying soil-water characteristic curve.

b.) The extended Mohr-Coulomb failure envelope is defined using the saturated shear parameters, \( c' \) and \( \phi' \), from conventional direct shear tests. The angle \( \phi^b \) is defined using the values of \( c' \) and \( \phi' \) and the soil-water characteristic curve according to Vanapalli (1994).

c.) The unsaturated coefficient of permeability of the stable soil could be considered a constant value equal to the unloaded saturated coefficient of permeability. In this research
study, van Genuchten's (1980) method is used to estimate the unsaturated coefficient of permeability for the stable soil based on the drying soil-water characteristic curve.

### 4.4 Equipment utilized to performing the soil testing

The present laboratory program was planned such that available testing equipment would be used to study both the mechanical behavior and the hydraulic properties of the compacted soil. The volume change behavior and the coefficient of permeability for the unsaturated and collapsing compacted soil were defined using the triaxial permeameter cell developed by Huang (1994). Some minor modifications were done to minimize leakages. Changes were also performed to eliminate noise in the data acquisition system.

The extended Mohr-Coulomb failure envelope for the collapsing soil was defined using the modified direct shear box developed by Gan et al. (1988). The saturated shear strength envelope for the stable soil was defined using the conventional direct shear box.

#### 4.4.1 Triaxial permeameter cell

Detailed information about the triaxial permeameter cell is available in Huang (1994). Some information about the testing procedure is detailed here. The equipment can independently control the total stress, $\sigma$, the pore-air pressure, $u_a$, and the pore-water pressure, $u_w$. This ability ensures that the stress state variables within the specimen are properly defined. For the present research study, the experimental range of matric suction was limited from 0 to 90 kPa. The experimental range of net normal stress, (i.e., $\sigma - u_a$), was defined from 20 to 200 kPa, based in the range of stresses in the “Alka-Seltzer” dams.

The triaxial testing system has the facility of a chosen hydraulic gradient on the soil specimen. The differential head is precisely measured and the inflow and outflow of water are controlled. Previous performance showed that the system could ensure that leakages were
insignificant in comparison to the inflow and outflow of water. The system also allows the flushing out of diffused air bubbles and the measurement of the volume of the diffused air. The accuracy of the flow water volume measurements is highly influenced by the diffused air volume.

The triaxial testing system can measure the total volume changes so that the current volume of the specimen at any stage is known. This is particularly important when the soil collapses due to an increase in degree of saturation along a wetting stress path.

4.4.1.1 Description of the triaxial permeameter cell

The general assembly of the triaxial permeameter cell is shown in Figure 4.14. The cell consists of a steel cylindrical cover, an aluminum base plate and an aluminum loading cap. The base plate contains a 10.13 mm thick, 90.0 mm diameter, 100 kPa high flow high air entry disc. A commercial epoxy was used to seal the high air entry disc onto the base plate.

The aluminum loading cap also contains a 10.13 mm thick, 88.0 mm diameter, 100 kPa high flow high air entry disc. A 2.0 mm wide and 2.0 mm deep groove, surrounding the high air entry disc, was designed for applying the pore-air pressure to the specimen.

The pore-air pressure was applied through the pore-air pressure line (i.e., P.A.P line in Fig. 4.14) to the pore-air pressure groove on the loading cap. A uniform pore-air pressure can thus be established within the specimen.

The hydraulic gradient across the specimen was imposed by applying different pore-water pressures to the upper and lower ends of the specimen. Both the pedestal and the loading cap had a pore-water pressure line (i.e., P.W.P. in Fig. 4.14) so that the pore-water pressures applied to both ends could be controlled independently. It is possible that the distribution of the pore-water pressure across the unsaturated specimen might not be linear. To minimize this problem, the differential heads used were in a range of 4 to 5 kPa. With this gradient, a linear pore-water pressure distribution within the specimen was assumed.
Figure 4.14 General assembly of the triaxial permeameter cell (after Huang, 1994)
Both the pedestal and the loading cap were provided with spiral flushing grooves to flush out air bubbles that might have been trapped or accumulated as a result of diffusion. In addition, these spiral grooves can also expedite the application of the water pressure to the specimen.

The deformation of the specimen was measured using a "non-contacting displacement measuring system" manufactured by the Kaman Sciences Corporation at Colorado Springs, Colorado/USA (Huang, 1994). As shown on Fig. 4.14, two of the non-contacting strain transducers were laterally arranged to measure changes in diameter while the third one was installed vertically to monitor the change in the height of the specimen.

For the lateral transducers, aluminum foil targets were attached to the rubber membrane opposite to the transducers using a thin O-ring. For the third transducer, the aluminum top cap was used as the target. The aluminum targets were made from four folds of heavy duty commercial aluminum foil, $8 \times 10^{-3}$ mm thick and $20 \times 20$ mm in area. The lateral deformations were combined with the vertical deformation so that the change in total volume of the specimen could be measured continuously.

### 4.4.1.2 Plumbing layout for the triaxial permeameter system

Figure 4.15 shows the plumbing board used to support the operation of the triaxial permeameter cell. A total of six lines are connected to the triaxial permeameter cell. The upper and lower pore-water pressure lines were controlled by the pressure regulators C and D respectively, and then transferred to water pressure within an air/water tank. The water pressure was then transmitted through the corresponding volume change indicators to, after crossing the high air entry disc, the ends of the specimen.

The water pressure at the lower pore-water pressure line was monitored using a pressure transducer (Fig. 4.15). The differential pressure applied to the top and bottom of the specimen was measured using a highly accurate differential pressure transducer manufactured
by Instruments Division, Bell & Howell Corporation (Huang, 1994). It has a range of ±14 kPa and a resolution of 0.015 kPa (i.e., 1.5 mm of water column). The water pressure at the upper pore-water pressure line was obtained by combining the pressure measurements at the lower pore-water pressure line and the measurements of the differential pressure.

Two conventional twin-burette volume change indicators were installed in upper and lower pore-water pressures lines, respectively, in order to permit the measurement of the inflow and outflow water volumes. Small-bore burettes with a volume of 10 cm³ and 0.02 cm³ resolution were used as the central tube for the volume change indicators.

The changes in differential pressure induced by changes in the elevations of the water/kerosene interface within the volume change indicators were accurately monitored by the data acquisition system. Therefore, the differential pressure transducer was not only used to measure the initial differential pressure, but also to monitor the change in the differential pressure induced by changes in water volume in the burettes. In addition, the differential pressure transducer could be used to monitor the change in the differential pressure induced by fluctuations in the applied pore-water pressures.

The pore-air pressure line was controlled by using regulator A. Before it was applied to the triaxial permeameter cell, the air pressure was stored in a tank and monitored by a pressure transducer. The air eventually arrived at the pore-air pressure groove on the loading cap.

The cell pressure line was controlled by using regulator B. The air pressure was routed through a storage tank and then transmitted to the triaxial cell. The cell pressure acted as a confining pressure on the soil specimen. A pressure transducer monitored the cell pressure line. Through the spiral flushing grooves on the loading cap or the base plate, the diffused air bubbles were flushed out from the pore-water pressure compartments into the diffused air volume indicators (Fredlund, 1973). The measured volume of water, combined with the measured diffused air volume, allowed the inflow and outflow water volumes to be calculated.
Figure 4.15 Plumbing layout for the triaxial permeability tests (after Huang, 1994)
The pressure transducers, along with the strain transducers, were connected to a data acquisition system assembled at the Engineering Shop in the University of Saskatchewan (Huang, 1994). The "Notebook" software (an interface program produced by Laboratory Technologies Corporation, Wilmington, M.A.) was used to record and store the data in the hard disk of a microcomputer. In addition, pressure and deformation data were displayed on the monitor for observation during the tests. The burette readings were taken manually.

4.4.2 Modified direct shear box

The modified direct shear apparatus for testing unsaturated soils was designed and built by Gan and Fredlund (1988). This equipment has been used to define the shear strength envelope of a variety of different soils (Gan and Fredlund, 1988).

4.4.2.1 Description of the modified direct shear box

The modified direct shear equipment designed to perform drained testing in unsaturated soil is shown in Figure 4.16. In Figure 4.16a, the high air entry ceramic disk has been removed. The grooved water chamber serves as support for the high air entry disk. The grooves also facilitate the flow of water from the water compartment to the exit port below the ceramic disk and are useful for flushing purposes. A 6.0 mm thick ceramic disk with an air entry value of 500 kPa and a coefficient of permeability of $5 \times 10^{-10}$ m/s was used.

The cylindrical stainless steel pressure chamber can sustain internal air pressures of up to 1000 kPa. In the present program, only air pressures of up to 100 kPa were necessary. The shear force is applied by pistons directly to the shear box.
Figure 4.16 Modified direct shear apparatus for testing unsaturated soils. a.) Plan view of the pressure chamber; b.) cross-sectional view B-B of a direct shear box and pressure chamber (from Gan et al., 1988).
The matric suction was applied by maintaining a constant air pressure in the chamber and by keeping the high air entry disk in a saturated condition. The base of the high air entry disk was connected to a calibrated burette which provides a maximum 6 kPa water pressure. This burette was also used to monitor water volume changes in the soil specimens.

During the tests, the diffused air at the base of the high air entry disk was removed by flushing below base plate according to procedures recommended by Gan (1986) and Vannapali (1984). The periodic removal of the diffused air was necessary to ensure the water movement into the sample. Both the water equilibration with time and the vertical dial gauge reading with time were used as indicators to confirm the end of the consolidation/collapsing phase of the specimen.

4.5 Performance of the equipment utilized in the research study

Drained testing on unsaturated soils is time-consuming, requiring that the equipment satisfy minimum requirements to furnish satisfactory results. A parallel analysis of the partial results is essential to provide an indication of the performance of the equipment.

4.5.1 Performance of the triaxial permeameter system

The triaxial permeameter system was calibrated prior to testing. The coefficients of permeability for the saturated high air entry disks were measured. Checking of leakages from the plumbing lines required a two-week period of observation. The measured one week leakage from the plumbing lines located outside the triaxial permeameter was insignificant (less than 0.08 cm³). For the plumbing lines within the cell, the leakage from the connection joints (Huang, 1994) could be significantly minimized by replacing the plastic tube of the top flushing line with a flexible copper tube. The leakage from the gaps between the rubber
membrane and both the cap and the pedestal (Huang, 1994) was minimized by using tight O-rings pressed by "Hose-clamps".

In the present research, unlike in Huang’s (1994) study, a wetting process was used for all the testing. Therefore, the minimized air leakages from the gaps could be neglected while the soil specimen was unsaturated. For the saturated condition, the duration of the permeability tests was less than two hours. Therefore, the leakage was less than 0.01 cm³ during the testing. In reference to the air leakage, Huang (1994) stated:

"The air leakage due to the difference in the cell air pressure and the pore-air (or pore-water) pressure directly affected the accuracy of the inflow and outflow water volume measurements when the specimen was in a saturated state. However, the effect of this air leakage was reduced considerably when the specimen became unsaturated. The reason for this is that new air channels had developed when the matric suction exceeded the air entry pressure of the specimen. The air leakage from the cell would then mix with the air within the specimen and would tend to increase the pore-air pressure. On the other hand, the pore-air pressure was controlled and maintained at a constant value. As a result, the leakage of the air neither affected the flow water volumes nor the matric suction when the specimen was unsaturated."

In the present research study the specimens approached saturation only at very low values of matric suction.

4.5.2 Performance of the modified direct shear box

The main problem associated with the modified direct shear testing was related to the removal of diffused air from below the bottom air entry disk. However, the use of the conventional procedure was sufficient to overcome the problem and guarantee the execution of the drained shear strength testing.
4.6 Testing procedures

The axis-translation technique (Hilf, 1956) was used in all tests in this second phase laboratory program. As mentioned before, two main pieces of equipment were used in this program: the triaxial permeameter system and the modified direct shear test. Some tests, performed by using the conventional direct shear box (i.e., soil at optimum conditions), followed usual soil testing procedures.

4.6.1 Testing procedure for the triaxial permeameter system

The triaxial permeameter tests were conducted in order to evaluate both the volume change behavior and the coefficient of permeability at different net normal stresses and matric suction levels. The total volume change was measured using three strain transducers, and the water volume change was measured by using the twin-burettes (Figure 4.16). The coefficient of permeability, as a function either of the volume-mass properties or the stress state variables, could be obtained from these tests.

The high air entry disks were saturated and their coefficients of permeability, with magnitudes of about 2 x 10^{-8} m/s, were measured using a constant head permeability test. Unsaturated and statically compacted specimens, 44.8 mm high and 101.1 mm in diameter, were then assembled into the triaxial permeameter cell. After the specimen was properly assembled, with all the valves shut-off, the following steps were performed.

a.) A confining pressure (i.e., token load) of 5 kPa was applied to the specimen. After equilibration of the three strain transducer readings (usually less than 1 minute), both the desired isotropic net confining stress and the matric suction were applied to the specimen in three subsequent phases. In the first phase, the desired confining pressure was applied to the specimen in a drained loading. At this stage, despite the valves of both air pressure and water pressure lines remaining closed, air could escape through the air pressure line on the
loading top cap. The water in the specimen remained at its low negative pressure and could not flow. In the second phase, the air pressure line was opened and adjusted with the cell pressure in an incremental way, keeping constant the applied isotropic confining pressure (i.e., cell pressure minus air pressure). At this point in the test, the valve of the water pressure was still closed. In the third phase, the water pressure lines, already adjusted to the hydraulic pressure gradient, were opened. The air pressure line and cell pressure line were adjusted to their final values by keeping constant the net confining stress. The opening of the water pressure lines connected the ends of the specimen to the twin burettes, and the inflow and outflow of water could be controlled. Under these applied stress state conditions, the specimen was allowed to saturate until equilibrium was reached. Then both the total volume change and volumetric water change of the specimen were controlled by using the twin burettes and the strain transducers up to the complete equilibration.

b.) The coefficient of permeability was measured by using the differential head applied to the ends of the specimen and monitoring the inflow and outflow water volumes. The differential head was applied simultaneously to the last phase of the loading process described in the above item a.) in order to maintain an approximately constant stress state in the specimen. This procedure also helped to save time when performing the tests. The readings for the inflow and outflow volumes were taken regularly from the volume change indicators. Once the difference between the inflow and outflow water volumes was small (10% of the total average flow volume), this step of the test was considered complete. The coefficient of permeability for the unsaturated condition (i.e., corresponding to that initial stress state) could then be calculated by using the data obtained during the corresponding time interval.

c.) After equilibration at the previous applied stress state, the matric suction was then decreased while the net confining stress was kept constant. To ensure monotonic loading, both the pore-air pressure and the confining pressure were decreased gradually and simultaneously by the same amount.
d.) The specimen was allowed to saturate under the newly applied matric suction. The differential head was maintained on the ends of the specimen at the same time. The total volume change and the inflow and outflow water volumes were monitored and regularly recorded. Once the difference between the inflow and outflow water volumes, for a defined interval of time, was smaller than a established tolerance, the specimen was considered to be at equilibrium under the applied matric suction. The data from this particular time was used to calculate the coefficient of permeability for the applied stress state condition. The volume of diffused air was regularly flushed and measured. Appropriate corrections were applied to the inflow and outflow water volumes to ensure accuracy in the permeability measurements.

e.) Steps c.) and d.) were repeated at lower matric suction values in the same specimen until the matric suction became zero. In all the steps, simultaneous plots of both degree of saturation and void ratio of the specimens were performed in order to define their current values. These plots made it possible, for example, to determine if the specimen had reached complete saturation at zero matric suction. All the samples tested (i.e., at confining net pressures of 20 kPa, 50 kPa, 100 kPa, and 200 kPa), presented a final degree of saturation of less than 95% and required a back-pressure (i.e., 2-4 kPa) to reach their complete saturation.

f.) After saturation, the coefficient of permeability was defined for that particular net confining stress. This stress is referred to as the confining effective stress. At the saturated condition, the criterion of equilibration was decreased to 5% of the average flow volume in the particular period of measurement. The effective stress was then increased and the sample was allowed to consolidate along a saturated stress path. As a general rule, attempts were made to consolidate the specimens with initial net confining stress lower than 100 kPa to a confining pressure of 100 kPa. The specimen with the initial net normal stress of 100 kPa was consolidated up to 200 kPa. An attempt was made to consolidate the specimen with initial net normal stress of 200 kPa to a confining pressure of about 300 kPa.
Approximately 8 weeks were required for the first test (i.e., with a net confining stress of 20 kPa) due to the problems with diffused air and adjustments of the procedures to be used in the following tests. Only this first test and the last test which followed a net confining pressure of 200 kPa had an initial stage with an applied matric suction of 90 kPa. On average, 5 weeks were required for performing each of the other tests by using the triaxial permeameter equipment.

4.6.2 Testing procedures for the modified direct shear box

The modified direct shear tests were performed to define the extended Mohr-Coulomb failure envelope for the collapsing soil. The tests were performed in both saturated and unsaturated conditions using the modified direct shear box. This procedure was used to avoid equipment influences on the results of shear strength of such a sensitive soil.

The high air entry disk at the base of the shear box was properly saturated according to the procedures and recommendations suggested by Gan (1986) and Vanapalli (1994). The 51 x 51 x 21 mm specimens were extruded from a 100 mm diameter by 25 mm thick statically compacted specimen. The specimen was placed in the direct shear equipment by pushing it from the mold into the direct shear box.

After the specimen was properly assembled in the shear box, the following procedure was performed:

a.) The vertical deflection of the specimen was monitored using a LVDT adjusted on the loading system. A predetermined vertical load was applied to the specimen and the desired matric suction was applied by maintaining a constant air-pressure in the chamber, while keeping the system completely saturated and connected to the burette open to the atmosphere. In the burette, a water level resulting in an average back-pressure of 6 kPa at the base of the air entry disk was maintained. This back-pressure was considered in the definition of the air pressure in the chamber.
b.) The specimen was then allowed to equilibrate under the predetermined vertical net normal stress and applied matric suction. Changes in total volume (i.e., LVDT readings), were periodically recorded by using the previously mentioned "Notebook" software. Changes in water content were calculated by reading periodically the acting water levels in the burette (with an accuracy of 0.05 ml). Evaporation on the top of the burette was minimized by using a plastic seal. A needle, crossing the seal, was used to ensure the atmospheric conditions in the top of the burette. Consolidation was allowed until the complete equilibration of both total volume changes and volumetric water content of the specimen was achieved.

c.) After equilibration (an average of 5 days), the specimen was sheared at a constant rate under drained conditions. Based on results presented in literature (Escario and Saez, 1986; Fredlund and Rahardjo, 1993), a shear rate of 2.4 mm/day was found to be satisfactory. All the tests were performed in a single stage. The total volume change and the volumetric water change of the specimen were monitored by using the same procedures as those of the consolidation stage. The matric suction range selected was from zero to 100 kPa.

The above procedure was used for sixteen specimens which were subject to four different matric suction and four different net normal stress. On average, one week was required to test each sample.

4.7 Testing program

The testing program was designed to evaluate the mechanical behavior and hydraulic properties of the collapsing compacted soil. The material at optimum condition was tested only to define its saturated Mohr-Coulomb failure envelope.

The experiments were planned as three different series. The first series of tests were performed using the triaxial permeameter to define the volume change behavior (i.e., state surfaces) and the coefficient of permeability of the collapsing soil for the range of stress states of interest. The second series was performed to define the shear strength behavior of
the collapsing soil. The third series was to study the saturated shear strength envelope for the material compacted at optimum conditions using the conventional direct shear test.

The range of net total stresses used was defined based on the real problem to be analyzed. Figure 4.17 shows, in a schematic way, the guidelines used for the definition of the range of net normal stresses used in the laboratory program for testing the collapsing soil.

![Diagram](image)

Figure 4.17 Guidelines for the definition of the range of net normal stresses for the laboratory program.

### 4.7.1 Triaxial permeameter testing program

The triaxial permeameter allowed the measurement of both the volume change behavior and the coefficient of permeability of the specimen under a wetting stress path. The first test, at a confining pressure of 20 kPa, was the pilot test. The purpose of this test was to observe the overall performance of the equipment and to evaluate the applicability of the planned testing procedure. Subsequent tests were based on previous results.

The soil specimen was first loaded to the desired net normal stress for each test. The pore-air and pore-water pressures were then adjusted to the defined matric suction and the
specimen was allowed to equilibrate under the applied stress state. A hydraulic pressure
gradient was then applied to the ends of specimen. The equilibrium involved the total volume
change of the specimen, the water volume change in the specimen and the water flow through
the specimen. The stress state variables applied to the specimens are illustrated in Table 4.2.

Table 4.2 Stress state variables for the triaxial permeameter tests on the collapsing soil.

<table>
<thead>
<tr>
<th>Test Step</th>
<th>Stress State Variable (kPa)</th>
<th>PT1* (kPa)</th>
<th>PT2* (kPa)</th>
<th>PT3* (kPa)</th>
<th>PT4* (kPa)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-6</td>
<td>(σ - uₐ)</td>
<td>20</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>(uₐ-uₜₜ)</td>
<td>370</td>
<td>370</td>
<td>370</td>
<td>370</td>
<td>Drained loading at initial conditions</td>
</tr>
<tr>
<td>1</td>
<td>(uₐ-uₜₜ)</td>
<td>90</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(uₐ-uₜₜ)</td>
<td>60</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(uₐ-uₜₜ)</td>
<td>30</td>
<td>0</td>
<td>15</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(uₐ-uₜₜ)</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Saturated</td>
<td>Saturated</td>
<td>Saturated</td>
<td>Saturated</td>
<td>(uₐ-uₜₜ) = 0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(σ - uₜₜ)</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>Consolidation</td>
</tr>
<tr>
<td>6</td>
<td>(σ - uₜₜ)</td>
<td>100</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Consolidation</td>
</tr>
</tbody>
</table>

*PTi - Permeability Test number i.

4.7.2 Direct shear testing program

The direct shear tests were performed in two different series as detailed earlier. In
the first series, modified direct shear tests were carried out to define the shear strength failure
plane for the collapsing soils. In the second series, conventional direct shear tests were
performed to define the saturated shear strength envelope for the soil compacted at optimum condition.

4.7.2.1 Modified direct shear testing program

A wetting path was followed for each test. The unsaturated specimen was first loaded to the desired net normal stress. In the following step the pore-air pressure and pore-water pressure (i.e., the back pressure) were adjusted to the defined matric suction. The water was allowed to infiltrate and the specimen was then consolidated until equilibrium was reached. After consolidation, the specimens were sheared under drained conditions. The shear tests were performed according to the program presented in the following Table 4.3.

Table 4.3 Stress state variables for the modified direct shear tests on the collapsing soil.

<table>
<thead>
<tr>
<th>Stress State Variable (kPa)</th>
<th>Plane 1 (kPa)</th>
<th>Plane 2 (kPa)</th>
<th>Plane 3 (kPa)</th>
<th>Plane 4 (kPa)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma - u_{w}$</td>
<td>25</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>Saturated Specimen</td>
</tr>
<tr>
<td>$u_{a} - u_{w}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$u_{a} - u_{w} = 25$ kPa</td>
</tr>
<tr>
<td>$u_{a} - u_{w}$</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>$u_{a} - u_{w} = 50$ kPa</td>
</tr>
<tr>
<td>$u_{a} - u_{w}$</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>$u_{a} - u_{w} = 100$ kPa</td>
</tr>
<tr>
<td>$u_{a} - u_{w}$</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

4.7.2.2 Conventional direct shear tests

For each test, the statically compacted and unsaturated specimen was first saturated and consolidated at the desired net normal stress. The shearing stage, after consolidation, was followed in accordance with the procedure described previously. The stress state variables for
different specimens (Shear Test for Specimen at Optimum Condition) are illustrated in the following Table 4.4.

Table 4.4 Stress state variables for the conventional direct shear testing used to shearing the soil compacted at optimum condition of AASHTO Standard energy.

<table>
<thead>
<tr>
<th>Stress State Variable (kPa)</th>
<th>STSOC1 (kPa)</th>
<th>STSOC2 (kPa)</th>
<th>STSOC3 (kPa)</th>
<th>STSOC4 (kPa)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(σ - u_w)</td>
<td>25</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>Saturated specimen</td>
</tr>
</tbody>
</table>
CHAPTER 5

LABORATORY TESTING RESULTS AND SOIL MODELLING

5.1 Introduction

This Chapter is divided into two parts. Part one presented the laboratory test results corresponding to the laboratory program outlined in chapter 4. Modelling of the collapsing soil behavior in terms of mechanical behavior and hydraulic properties is addressed in the second part.

The volume change behavior and coefficient of permeability of the collapsing soil are established as functions of the stress state variables using the results obtained from triaxial permeameter tests. The modified direct shear tests were used to define the extended Mohr-Coulomb failure criterion for the collapsing soil. Conventional direct shear test equipment was utilized to define the saturated shear strength parameters of the soil compacted at optimum water content conditions. All tests were performed using wetting stress paths as explained in Chapter 4. The data considered relevant for the soil models are organized for presentation in part one of this chapter. Detailed results of the testing program are documented in the appendices. Additional details on the performance of both the triaxial permeameter system and the modified direct shear equipment, as well as the experimental difficulties encountered during the tests, are also summarized in part one.

The presentation of the testing program in part one of this Chapter follows the same format as the program described in Chapter 4. The test results related to the definition of the models to simulate the mechanical behavior and hydraulic properties for the collapsing soil, are described in part two of this Chapter.
PART ONE - LABORATORY TEST RESULTS

5.2 Triaxial permeameter testing results

The constitutive relationships for both volume change and the hydraulic behaviors of the compacted collapsing soil were defined using the triaxial permeameter test. To establish these relationships, four triaxial permeability tests (i.e., TPT1, TPT2, TPT3, and TPT4) were conducted on four statically compacted collapsing soil specimens. Table 4.2 showed the stress state variables used for testing. Each soil specimen, at its as-compacted initial condition and at an initial matric suction of 370 kPa, was isotropically loaded under a previously specified net normal confining pressure. The pore-air pressure was controlled in a drained mode. The specimens were allowed to consolidate at various steps of decreasing matric suction (i.e., following a wetting stress path) until saturation was reached. The applied net confining pressure was maintained constant. A pilot test was conducted with a net confining pressure of 20 kPa (see section 4.2.2.3). This test started by applying a matric suction of about 90 kPa in the as-compact soil specimen. This was the first step towards saturation. The changes in the total volume of the specimens were monitored during each step of the tests. The outflow and inflow of water to the specimen was also monitored in order to determine the changes in water content. This allowed the computation of the coefficient of permeability at each applied matric suction. The first test was used to define all the testing procedures involving both assembling of the soil specimen in the triaxial permeameter and monitoring of the soil specimen volume changes.

The specimen for the pilot test had a height of 54.8 mm and a diameter of 101.1 mm. For the remaining specimens (i.e., TPT2, TPT3, and TPT4) their dimensions were 44.8 mm in height and 101.1 mm in diameter. The change in height was necessary in order to reduce the equilibration time. The specimen did not collapse upon saturation in the first test, and the triaxial system seemed to perform satisfactorily in terms of water flow and the
measurement of the increase in water content in the specimen. The volume of water volume inflow to be measured at matric suctions higher than 30 kPa was small. The specimen was allowed to equilibrate at matric suctions equal to 90, 60, 30, and 0 kPa for this first test. However, the sample did not reach complete saturation at a zero value of matric suction. Under an applied back-pressure of about 4 kPa, the specimen reached a calculated degree of saturation of 100%. The coefficient of permeability was measured on the saturated specimen. The specimen was then consolidated to a confining pressure of 50 kPa. The consolidation process appeared to function satisfactorily for about 1 hour of the test. Then air started to infiltrate into the soil specimen. This problem was caused by the slippage of the “Hose-clamp” attached to the bottom porous stone. The experiment was halted, the specimen was removed and its volume-mass properties were measured.

The second test, TPT2, corresponding to a net confining pressure of 50 kPa, was conducted using the previously used assembling procedure and initial drained loading defined by the pilot test. The second test was started by applying a matric suction of 60 kPa in the soil specimen. This was the first step towards saturation. The specimen showed a negligible reduction in total volume from the initial condition (i.e., 50 kPa of net confining stress and 370 kPa of matric suction) to the first imposed state of stress (i.e., 50 kPa of net confining pressure and 60 kPa of matric suction). The test was then continued by allowing the soil specimens to equilibrate at matric suction of 30 and 0 kPa.

In the second test, the specimen collapsed at a matric suction between 30 and 0 kPa. Unfortunately, no intermediate matric suction steps were performed between 30 and 0 kPa. The actual value of matric suction where the collapse occurred was later estimated with the assistance of the other test results (see section 5.4.1). The coefficients of permeability for each step of matric suction were calculated based on the established water flow. As in the previous test, the second specimen required a back-pressure of about 4 kPa to reach saturation. The coefficient of permeability was determined on the soil specimen after saturation. Following saturation, the specimen was consolidated to a confining pressure of
100 kPa. The saturated coefficient of permeability was determined for the specimen at a confining effective stress of 100 kPa. A comparison between the total volume change of the specimen and the amount of water outflow from the specimen showed that the system of non-contact transducers used in the measurement of volume changes of the specimen, was accurate. The permeability test was completed one hour after the confining pressure of 100 kPa was applied. The specimen was removed and its volume-mass properties were determined.

The third test, TPT3, corresponding to a confining net pressure of 100 kPa, was conducted using the procedures established during the first two tests. Based on the previous results, it was decided to commence the third test under an applied matric suction of 60 kPa. As in the second test, this specimen did not undergo a significant reduction in volume when the matric suction was decreased from the initial matric suction of 370 to 60 kPa. The reduction in total volume was larger than had been observed for the two first tests. The test was continued with matric suctions decreasing from 60 to 30 to 10 to 5 to 0 kPa. Measurements of both total volume and water volume changes were performed without experimental difficulties.

In the third test, the specimen showed a significant collapse between 60 and 30 kPa of matric suction. Therefore, intermediate matric suction steps of 10 and 5 kPa were performed. The coefficients of permeability were calculated for each step of matric suction based on the established water flow. Similar to the earlier tests, the third specimen required a back-pressure of about 4 kPa to reach complete saturation. The coefficient of permeability was measured on the saturated specimen. The specimen was then consolidated to a confining pressure of 200 kPa. The saturated coefficient of permeability at a confining effective stress of 200 kPa was determined. The test performed well for the first 30 minutes, but then air started entering the specimen through the ends of the rubber membrane. The test was then stopped. The specimen was removed and volume-mass properties were determined.
The fourth test, TPT4, corresponding to net a confining pressure of 200 kPa, was conducted similar to the previous specimens. It was decided to commence at an applied matric suction of 90 kPa. The relative reduction in total volume was larger for this test when the matric suction was decreased from 370 kPa to 90 kPa. In comparison to the first step of the third test, the soil had started collapsing before the matric suction had reached 90 kPa. The test was continued in steps of matric suction from 90 to 60 to 30 to 10 to 5 to 0 kPa in order to better define the collapsing behavior of the soil. Measurements of both total volume and water volume changes performed well for this test. The coefficient of permeability was measured after all values of matric suction.

In the fourth test, there was a gradual collapse of the soil when the matric suction was changed from 90 to 10 kPa. No additional collapse was observed when the matric suction was decreased from 10 to 0 kPa. As with the previous tests, the fourth specimen did not reach complete saturation at a matric suction of 0 kPa. In contrast to the other tests, the soil specimen did not reach complete saturation even with a back-pressure of about 4 kPa. The “saturated” coefficient of permeability was determined for a degree of saturation of about 96%. An attempt to consolidate the soil specimen at a confining pressure of 300 kPa failed since the air started entering the soil specimen. The test was halted, the specimen was removed and its volume-mass properties were determined.

Table 5.1 summarizes the volume-mass properties of the initial and final stress state conditions of the specimens tested. The final degree of saturation was calculated based on the measured gravimetric water content and the final void ratio corresponding to the final step of each experiment (i.e., saturated specimen).

5.2.1 Volume change behavior of the collapsing soil

Table 5.2 presents a summary of the changes in both total volume and water phase volume versus matric suction for the four soil specimens tested. The deformation variables
were previously defined in section 3.2.1 as the ratio between the corresponding volume change (i.e., either total volume or water volume) and the referential volume, \( V_0 \). In Table 5.2, the deformation variables were calculated using an updated referential volume. The updating procedure herein utilized the wetting stress path and the total volume of the specimen at the beginning of each applied matric suction, as the referential volume. This procedure implied that there was a decreasing referential volume of the soil specimen as the matric suction was gradually reduced. The decrease in referential volume upon wetting was proportional to the applied net confining pressure.

Table 5.1 Summary of the volume-mass properties corresponding to the initial and final conditions of the specimens tested in the triaxial permeability tests

<table>
<thead>
<tr>
<th>Test Number</th>
<th>*TPT1</th>
<th>**TPT2</th>
<th>***TPT3</th>
<th>****TPT4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial void ratio, ( e_i )</td>
<td>0.754</td>
<td>0.754</td>
<td>0.754</td>
<td>0.754</td>
</tr>
<tr>
<td>Final void ratio, ( e_f )</td>
<td>0.748</td>
<td>0.740</td>
<td>0.648</td>
<td>0.558</td>
</tr>
<tr>
<td>Initial water content, ( w_i ) (%)</td>
<td>10.50</td>
<td>10.50</td>
<td>10.50</td>
<td>10.50</td>
</tr>
<tr>
<td>Final water content, ( w_f ) (%)</td>
<td>27.72</td>
<td>26.96</td>
<td>24.00</td>
<td>21.71</td>
</tr>
<tr>
<td>Initial degree of saturation, ( S_i )</td>
<td>36.5</td>
<td>36.5</td>
<td>36.5</td>
<td>36.5</td>
</tr>
<tr>
<td>Final degree of saturation, ( S_f )</td>
<td>98.5</td>
<td>96.2</td>
<td>97.8</td>
<td>95.6</td>
</tr>
<tr>
<td>Confining net pressure, ( \sigma_u ) (kPa)</td>
<td>20</td>
<td>50</td>
<td>100</td>
<td>200</td>
</tr>
</tbody>
</table>

*consolidated at saturated condition with 50 kPa of confining pressure  
**consolidated at saturated condition with 100 kPa of confining pressure  
***consolidated at saturated condition with 200 kPa of confining pressure  
****consolidated at saturated condition with 300 kPa of confining pressure

Table 5.2 presents the deformation variables with respect to a starting matric suction of 370 kPa. These values correspond to the initial drained loading of the specimen. More
details are available in section 4.6.1. The deformation variables were calculated using the assumption that the initial loading was drained in terms of the air phase and that both gravimetric water content and matric suction in the soil specimen remained constant. The small decrease in total volume of the unsaturated specimen upon the initial loading caused a slight increase in the volumetric water content. This gives rise to the initial water phase deformation variable. The initial total volume change was defined for a time of 2 minutes after the net confining pressure had been applied. The choice of this elapsed time interval was based on the unsaturated loading results from the double oedometer tests (4.2.2.1).

Table 5.2 Summary of the volume changes for the total volume and water content of the collapsing soil under a wetting stress path in the triaxial permeability tests

<table>
<thead>
<tr>
<th>$u_a-u_w$ (kPa)</th>
<th>TPT1</th>
<th>TPT2</th>
<th>TPT3</th>
<th>TPT4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$dV_v/V_o$</td>
<td>$dV_w/V_o$</td>
<td>$dV_v/V_o$</td>
<td>$dV_w/V_o$</td>
</tr>
<tr>
<td>370</td>
<td>-.0034</td>
<td>.0006</td>
<td>-.0075</td>
<td>.0012</td>
</tr>
<tr>
<td>90</td>
<td>-.0034</td>
<td>.0286</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>60</td>
<td>-.0034</td>
<td>.0383</td>
<td>-.0076</td>
<td>.0316</td>
</tr>
<tr>
<td>30</td>
<td>-.0034</td>
<td>.0567</td>
<td>-.0079</td>
<td>.0482</td>
</tr>
<tr>
<td>10</td>
<td>-.0034</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-.0034</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0</td>
<td>-.0034</td>
<td>.269</td>
<td>-.0165</td>
<td>.2620</td>
</tr>
</tbody>
</table>

Figures 5.1, 5.2, 5.3 and 5.4 show the deformation variables versus matric suction relationships using two different coordinate systems. Figures 5.5 and 5.6 illustrate the influence of the updating the referential volume as compared to the use of the initial volume of the specimen as the referential volume on deformation variables.

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Figure 5.1 Volumetric deformation versus matric suction relationship for the collapsing soil under wetting paths on an arithmetic scale.
Figure 5.2 Volumetric deformation versus matric suction relationship for the collapsing soil under wetting paths on a semi-log scale.
Figure 5.3 Volumetric water content deformation versus matric suction relationships for the collapsing soil under wetting paths on an arithmetic scale. The unloaded curve is the drying soil-water characteristic curve.
Figure 5.4 Volumetric water content deformation versus matric suction relationships for the collapsing soil under wetting paths on an semi-log scale. The unloaded curve is the drying soil-water characteristic curve.
Figures 5.1 and 5.2 show the changes in total volume and illustrate that the collapsing behavior is a function of both net normal stress and matric suction. A typical collapse behavior is illustrated by following a stress path of decreasing matric suction at a given net confining pressure. Such a stress path shows that there are three distinct phases in the collapse mechanism. In the first phase, at relatively high matric suctions, the soil does not collapse and only small deformations occur in response to a decrease in matric suction. In the second phase, at intermediate matric suctions, large deformations are observed in response to a decrease in matric suction. In the third phase, at low matric suctions, there is an absence of deformations as the matric suction is reduced to zero.

Figures 5.3 and 5.4 show the relationships between changes in water volume and the applied stress state during the saturation of the soil. These figures illustrate a typical behavior for a wetting soil-water characteristic curve. The curve is independent of the collapsing behavior of the soil. The influence of soil collapse is only noticed as the soil approaches complete saturation: The lower the porosity of the collapsed soil, the less is the increase in water content in response to further decreases in matric suction.

Figures 5.5 and 5.6 illustrate that for the level of soil collapse reached (i.e., about 10% of volumetric deformation), either the initial volume or the updated volume can be used to calculate the deformation variables. The use of an updated referential volume is conservative in terms of soil collapse (see section 3.5.4.3). An alternative way to present changes in both the total volume and the water volume of the soil specimens is the use of the volume-mass soil properties. The following tables and graphics illustrate the changes in volume-mass properties of the collapsing soil specimens according for the tests performed. The volume-soil properties are calculated using an updated referential volume.

Table 5.3 summarizes changes in both void ratio and degree of saturation versus matric suction for the four soil specimens tested.
Figure 5.5 Volumetric deformation versus matric suction relationships for the collapsing soil under wetting paths.
Figure 5.6 Volumetric water deformation versus matric suction relationships for the collapsing soil under wetting paths.
Table 5.3 Summary of the volume-mass soil properties, void ratio (\(e\)) and degree of saturation (\(S\)) of the collapsing soil under a wetting stress path in the triaxial tests.

<table>
<thead>
<tr>
<th>(u_a-u_w) (kPa)</th>
<th>TPT1</th>
<th>TPT2</th>
<th>TPT3</th>
<th>TPT4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(e)</td>
<td>(S)</td>
<td>(e)</td>
<td>(S)</td>
</tr>
<tr>
<td>370</td>
<td>.7483</td>
<td>0.370</td>
<td>0.7408</td>
<td>0.374</td>
</tr>
<tr>
<td>90</td>
<td>.7483</td>
<td>0.437</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>60</td>
<td>.7483</td>
<td>0.460</td>
<td>0.7406</td>
<td>0.445</td>
</tr>
<tr>
<td>30</td>
<td>.7483</td>
<td>0.503</td>
<td>0.7400</td>
<td>0.485</td>
</tr>
<tr>
<td>10</td>
<td>.7483</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>.7483</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0</td>
<td>.7483</td>
<td>1.00</td>
<td>0.7251</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Three-dimensional plots of void ratio and degree of saturation versus the stress state variables constitute the state surfaces as defined by Matyas and Radhakrishna (1968). Figures 5.7 and 5.8 show both void ratio and degree of saturation versus matric suction for the four collapsing specimens tested under different net confining pressures. Figure 5.8 shows that the soil specimens represent similar increases in degree of saturation, as the matric suction was reduced from the initial condition (i.e., matric suction equal to 370 kPa) to 0 kPa, irrespective of differences in the soil collapse induced by the applied net confining pressures.

Table 5.4 summarizes the changes in both porosity and volumetric water content versus matric suction for the four collapsing specimens tested. Figures 5.9 and 5.10 illustrate the porosity and volumetric water content versus matric suction relationships, respectively, for the four specimens.

The results of total volume changes and water content volume changes are combined and shown in Figure 5.11. Soil collapse progresses with the gradual saturation of
the soil specimen. Irrespective of the net confining pressure, the soil collapse was completed before the soil specimen reached complete saturation. Booth. (1973); Lawton, et. al., (1991b); Houston et. al., (1993), have observed similar results for compacted silty and clayey sands (see section 2.3.3.3).

Table 5.4 Summary of the volume-mass soil properties, porosity (n) and volumetric water content (θ_w) of the collapsing soil under a wetting stress path in the triaxial permeability tests

<table>
<thead>
<tr>
<th>u_d-u_w (kPa)</th>
<th>n</th>
<th>θ_w</th>
<th>n</th>
<th>θ_w</th>
<th>n</th>
<th>θ_w</th>
<th>n</th>
<th>θ_w</th>
</tr>
</thead>
<tbody>
<tr>
<td>370</td>
<td>0.428</td>
<td>0.158</td>
<td>0.4255</td>
<td>0.1592</td>
<td>0.4237</td>
<td>0.1597</td>
<td>0.4224</td>
<td>0.1601</td>
</tr>
<tr>
<td>90</td>
<td>0.428</td>
<td>0.187</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>60</td>
<td>0.428</td>
<td>0.197</td>
<td>0.4254</td>
<td>0.1895</td>
<td>0.4224</td>
<td>0.1888</td>
<td>0.4051</td>
<td>0.1870</td>
</tr>
<tr>
<td>30</td>
<td>0.428</td>
<td>0.215</td>
<td>0.4253</td>
<td>0.2062</td>
<td>0.4190</td>
<td>0.2134</td>
<td>0.3927</td>
<td>0.2061</td>
</tr>
<tr>
<td>10</td>
<td>0.428</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.4040</td>
<td>0.2513</td>
<td>0.3755</td>
<td>0.2521</td>
</tr>
<tr>
<td>5</td>
<td>0.428</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.3932</td>
<td>0.2993</td>
<td>0.3754</td>
<td>0.2839</td>
</tr>
<tr>
<td>0</td>
<td>0.428</td>
<td>0.428</td>
<td>0.4203</td>
<td>0.4203</td>
<td>0.3932</td>
<td>0.3932</td>
<td>0.3752</td>
<td>0.3752</td>
</tr>
</tbody>
</table>

According to the literature review presented in sections 2.3.3 and 4.2.2, the open and metastable structure of a compacted collapsing soil with a low clay content containing larger particles (i.e., sand, silt and/or clay aggregations), is kept together by connecting bonds (e.g., clay and/or silt bridges or buttresses). The stability of such an open structure is a function of capillary action and internal microforces acting in the connecting bonds and clay aggregations. At a given net confining stress, the soil collapses in response to a reduction in the matric suction and an increase in water content in the connecting bonds and clay aggregations. Both mechanisms occur during soil saturation.
Figure 5.7 Void ratio versus matric suction suction relationships for the collapsing soil under wetting stress paths.
Figure 5.8 Degree of saturation versus matric suction relationships for the collapsing soil under wetting stress paths.
Figure 5.9 Porosity versus matric suction relationships for the collapsing soil under wetting stress paths.
Figure 5.10 Volumetric water content deformation versus matric suction relationships for the collapsing soil under wetting paths. The unloaded curve is the drying soil-water characteristic curve.
Figure 5.11 Collapse versus degree of saturation for the collapsing soil under wetting stress paths.
Results such as those presented in Figure 5.11 lead to the conclusion that any explanation of the collapsing behavior of soil structure must be related to the net normal stress. A metastable soil can saturate without collapse of its structure under a low net confining stress (e.g., $\sigma - u_n = 20$ kPa in Fig. 5.11). Also, under a given net normal stress, a metastable-structured soil can show a considerable increase in its degree of saturation without there being any collapse of the soil structure (e.g., $\sigma - u_n = 50$ and 100 kPa in Fig. 5.11). Further discussions on the volume change behavior of the collapsing soil are provided in section 5.4.1.1.

5.2.2 Coefficient of permeability of the collapsing soil

Table 5.5 presents a summary of the coefficient of permeability measurements as a function of the stress state variables imposed during the four collapsing tests. The values for 0 kPa of matric suction correspond to full saturation of the soil specimen (i.e., after applying a back-pressure of 4 kPa). The coefficient of permeability for the initial condition (i.e., at matric suction of 370 kPa) was estimated from the drying soil-water characteristic curve using the Brooks and Corey’s (1964) method as presented in section 4.2.2.1 (see Fig. 4.9). The coefficient of permeability versus matric suction relationships, corresponding to the four different net confining stresses, are presented on an arithmetic scale (Figure 5.12) and a semi-logarithmic scale (Figure 5.13). Figure 5.13 shows that, under a wetting stress path, the coefficient of permeability for each soil specimen varied in a manner similar to its degree of saturation change (see Figure 5.8).

The small amounts of water flow involved made it difficult to determine the influence of gradual soil collapse on the unsaturated coefficient of permeability of the soil specimens. A qualitative analysis suggests that the soil collapse produces local increases (i.e., at microstructure level) in the degree of saturation. These changes affect the coefficient of permeability of the collapsing soil since they change the pore-size distribution of the soil
structure. The soil collapse might also generate internal hydraulic gradients which alter the water flow paths through the soil structure. During the wetting process with simultaneous soil collapse there is a gradual transfer of water from the microstructure to the macrostructure in the soil specimen (Alonso et al. 1985).

Table 5.5 Summary of the measurements of the coefficient of permeability \((k_w)\) of the collapsing soil under a wetting stress path in the triaxial permeability tests

<table>
<thead>
<tr>
<th>((u_r-u_w)) (kPa)</th>
<th>Coefficient of permeability (values shown are to be multiplied by (10^9) m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TPT1</td>
</tr>
<tr>
<td>370*</td>
<td>0.0001</td>
</tr>
<tr>
<td>90</td>
<td>0.005</td>
</tr>
<tr>
<td>60</td>
<td>0.014</td>
</tr>
<tr>
<td>30</td>
<td>0.021</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>0**</td>
<td>600</td>
</tr>
</tbody>
</table>

*Coefficients of permeability based on the drying soil-water characteristic curve using the Brooks and Corey’s (1964) method.

**The soil was saturated by using a back-pressure of 4 kPa.

Figures 5.12 and 5.13 also show that the gradual soil collapse affects only the unsaturated coefficient of permeability of the soil specimen at low values of matric suction. Despite the small amounts of water flow measured and considering the range of stress state used, the available results suggest that:

i.) the unsaturated coefficient of permeability of the collapsing soil is primarily a function of its degree of saturation. For the range of confining stresses used in the present
study, the degree of saturation of the collapsing soil was primarily a function of the applied matric suction;

ii.) the saturated coefficient of permeability is a function of the void ratio of the soil and consequently of the amount of collapse occurring along the wetting path. At low values of matric suction there is a transitional zone where the influence of soil collapse starts to play a significant role in the degree of saturation of the soil.

5.3 Direct shear testing results

Direct shear tests are presented as two series of tests, as described in section 4.7.2. The first series of shear tests results is for the soil compacted dry of optimum water content (i.e., with a metastable structure). These tests were performed using a modified direct shear apparatus to determine the extended Mohr-Coulomb failure envelope of the collapsing soil. The second series of shear tests results is for the soil compacted at optimum water content conditions (i.e., with a stable structure). Conventional direct shear tests were performed to determine the saturated shear strength parameters of the soil in a stable condition.

5.3.1 Modified direct shear test results

Modified direct shear tests were conducted on the collapsing soil in accordance with the testing program presented in Table 4.3. Each soil specimen was compacted at the defined dry of optimum water content condition and consolidated under the previously specified stress state conditions. The specimens were allowed to achieve equilibrium with respect to total volume change and water content volume change. Wetting stress paths were used for all tests. The soil specimens were then sheared under drained conditions after consolidation. All the tests were performed as single stage direct shear tests.
Figure 5.12 Coefficient of permeability versus matric suction relationships for the collapsing soil under wetting stress paths on an arithmetic scale. The unloaded curve corresponds to Brooks' and Corey's (1964) method.
Figure 5.13 Coefficient of permeability versus matric suction relationships for the collapsing soil under wetting stress paths on a semi-log scale. The unloaded curve corresponds to Brooks' and Corey's (1964) method.
The period of time for the consolidation phase varied for each soil specimen and was mainly dependent on the applied matric suction. The following criteria were used for deciding the equilibrium conditions of the soil specimen with respect to consolidation:

a.) stabilization of the volumetric (i.e., vertical) deformation of the soil specimen was achieved when the increase in settlement for a 24 hours time interval was less than 5% of the total (i.e., accumulated) settlement measured for that specimen;

b.) stabilization of the water volume change in the specimen was achieved when the increase in calculated volumetric water content, for a period of 24 hours, was less than 2% of the total volumetric water content change. For the tests under zero matric suction consolidation was considered to be complete when the soil specimen was found to be saturated. The saturation condition was defined by means of calculations taking into account the water volume entering the specimen and the volumetric change of the soil specimen. Soil specimens under an applied matric suction of 0 kPa required about two days to achieve complete saturation. Soil specimens under an applied matric suction of 100 kPa required about eight days to achieve equilibrium conditions. Soil specimens under an applied matric suction of 25 and 50 kPa required periods of 5 to 6 days to achieve equilibrium conditions.

Figures 5.14 to 5.21 show the behavior of the collapsing soil specimens during the consolidation phase in terms of both total volume change (i.e., in the form of vertical deformation) and water phase volume change (i.e., using the degree of saturation). These results correspond to the four applied vertical stresses (i.e., 25, 50, 100 and 200 kPa). A comparison among the results obtained for the saturated specimens in the direct shear box and those obtained from the double oedometer tests (see Fig. 4.4) illustrated that the former results (i.e., direct shear results) over-estimated the soil collapse for the vertical stresses of 50, 100 and 200 kPa. These differences can be attributed to such factors as:

a.) an initial vertical pressure of about 12 kPa was applied to the soil specimens by the self weight of the frame used for assembling the load-deflection system in the direct shear apparatus;
b.) there may have been additional vertical deflection due to the penetration of the
grooved metallic top-cap placed on the soil specimens compacted at a low density in the
direct shear box; and

c.) the random effect of potential gaps between the soil specimen and the shear box
(i.e., the soil specimens for shearing were carved from a larger specimen and put into the
shear box rather than compacted in the ring as was the case for the specimens in the double-
oodrometer tests);

The above factors did not allow a quantitative analysis of the volumetric changes of
the soil specimens during the consolidation phase of the direct shear tests. However, these
factors did not pose any difficulties in the determination of the equilibrium conditions of the
soil specimens, either at the saturated or the unsaturated condition during the consolidation
phase. For the volumetric water content changes, the main problems were associated with the
small volumes of water to be measured for applied matric suctions higher than 25 kPa. Proper
contact between the bottom surface of the soil specimen and the bottom high air entry value
porous disc was also a concern during the execution of these tests.

On a qualitative basis, Figures 5.14 to 5.21 show that the measured volumetric
changes corresponding to the consolidation of the soil specimens in the direct shear box were
similar to the measured results obtained from the triaxial permeameter tests (Figures 5.1 to
5.8). Despite the inherent differences in confining conditions, as compared to the triaxial
permeameter system, the $K_0$-consolidation in the shear box can be utilized to reinforce the
following characteristics of the collapsing soil behavior for the soil tested:

i.) when a collapsing soil is saturated under a vertical stress of 25 kPa, the dry of
optimum compacted soil does not present a collapsing behavior;

ii.) there is a nonlinear soil collapse behavior with respect to a given vertical load for
decreasing matric suctions. Similar to the isotropic confining condition, the $K_0$-consolidation
indicated a critical value of matric suction where the soil starts collapsing;
Figure 5.14 Consolidation of the collapsing soil under net vertical stress of 25 kPa (wetting from the initial matric suction of 370 kPa).
Figure 5.15 Degree of saturation versus time for the collapsing soil specimen consolidated at a net vertical stress of 25 kPa (wetting from the initial matric suction of 370 kPa).
Figure 5.16 Consolidation of the collapsing soil specimen under a net vertical stress of 50 kPa (wetting from the initial matric suction of 370 kPa).
Figure 5.17 Degree of saturation versus time for the collapsing soil specimen consolidated under a net vertical stress of 50 kPa (wetting from the initial matric suction of 370 kPa).
Figure 5.18 Consolidation of the collapsing soil specimen under a net vertical stress of 100 kPa (wetting from the initial matric suction of 370 kPa).
Figure 5.19 Degree of saturation versus time for the collapsing soil specimen consolidated under a net vertical stress of 100 kPa (wetting from the initial matric suction of 370 kPa).
Figure 5.20 Consolidation of the collapsing soil specimen under net vertical stress of 200 kPa (wetting from the initial matric suction of 370 kPa).
Figure 5.21 Degree of saturation versus time for the collapsing soil specimen consolidated under a net vertical stress of 200 kPa (wetting from the initial matric suction of 370 kPa).
iii.) the soil collapse was completed before the soil specimen reached complete saturation.

Figures 5.22 to 5.25 show the shear stress versus shear displacement relationships for the collapsing soil under vertical stresses of 25, 50, 100 and 200 kPa, respectively. The shear tests were performed under drained conditions, at applied matric suctions of 100, 50, 25 and 0 kPa for each vertical stress. Figures 5.26 to 5.29 show the vertical deflection versus shear displacement relationships corresponding to the shear tests under vertical stresses of 25, 50, 100 and 200 kPa, respectively. Figures 5.30 and 5.31 summarize the shear strength behavior of the collapsing soil at saturated conditions.

Figures 5.26 to 5.29 illustrate the collapsing behavior of the soil specimens. These specimens behave either as a loose sand (i.e., compresses during shear) or as compact sand (i.e., dilates during shear) depending upon the applied stress state (i.e., net normal stress and matric suction). At saturated conditions (i.e., at a matric suction of 0 kPa) for all the applied vertical stresses, the soil compresses during shear. Under vertical stresses of 25 and 50 kPa and at matric suctions ranging from 25 to 100 kPa, the soil specimens dilate during shear. The same behavior was observed for soil specimen under a vertical stress of 100 kPa and at a matric suction of 100 kPa. Under a vertical stress of 200 kPa, the soil compresses during shear for matric suctions ranging from 25 up to 100 kPa.

Figures 5.22 to 5.25 illustrate that, irrespective of the volume changes induced in either the consolidation and shear phases, the shear strength of the collapsing soil increases as a combined function of the vertical stress and the applied matric suction. Despite the fact that this conclusion reflects the anticipated behavior of any unsaturated soil, some comments are warranted to explain the combined results of volumetric changes and shear strength for the collapsing soil. The following comments attempt to explain the shear strength behavior of a compacted collapsing soil with low clay content, which is compacted at dry of optimum water content:
i.) The structure of an uncemented compacted collapsing soil with a low clay content has been described (Dudley, 1970; Alfi, 1984; Miranda, 1988) as constituted of clay aggregations (i.e., macropeds) and coarser grains (i.e., sand and silt) connected by bonds of finer particles (i.e., clay and silt particles). The strength of such bonds and clay aggregations is highly dependent upon capillary action. Any external loading generates shear stresses at the connecting bonds. The unsaturated soil does not collapse as long as the local shear strengths of the connecting bonds are higher than the acting shear stresses. During saturation, both the clay aggregations and the connecting bonds soften and weaken due to the reduction in matric suction. The soil collapses as the local shear strength at the connecting bonds is overcome by the shear stresses induced by the applied loading. The soil microstructure is complex in terms of pore-size distribution (Matyas and Radhakrishna, 1968; Lawton et. al, 1991b; Alonso et. al, 1993; Khogo et al. 1993), and each connecting bond is sheared according to its local shear strength. The breaking of connecting bonds and/or clay aggregations implies the local redistribution of stresses among the remaining clay aggregations and coarser particles. The soil skeleton can reach a new equilibrium depending on the external load and the remaining strength of the connecting bonds and clay aggregations. The breaking of connecting bonds and clay aggregations occurs in a progressive manner as the matric suction decreases and/or the acting external load increases.

ii.) The double oedometer test performed on the collapsing soil (see Figure 4.4) illustrates that there is a gradual increase in soil collapse when the vertical stress is increased from 25 to 800 kPa. This suggests that the metastable structure (i.e., soil skeleton supported by clay aggregations and connecting bonds) of the collapsing soil remains partially intact even under vertical stresses as high as 400 kPa. At a vertical stress of 800 kPa, the void ratio of the collapsing soil is about 0.45, which is close to the void ratio of the soil compacted at optimum conditions.

iii.) In the consolidation phase, each soil specimen was brought to equilibrium conditions under a defined stress state following a wetting stress path. The behavior of the
collapsing soil during consolidation is illustrated in Figures 5.14 to 5.21. There is a progressive increase in the total soil collapse (i.e., at matric suction of 0 kPa) when the vertical stress applied to the soil specimens is increased from 25 to 200 kPa. As discussed above, the saturation of the specimen, even under an applied vertical stress of 200 kPa, is not enough to destroy the structure of the collapsing soil. Under a given net vertical stress, the higher the applied matric suction, the higher the amount of remaining bonds and clay aggregations binding together the structure of the collapsing soil. This fact leads to the conclusion that each soil specimen (i.e., possessing a specific metastable structure after consolidation) will present a specific behavior during the shearing phase.

iv.) The shearing phase induces shear stresses and consequently alters the existing equilibrium stress state in the connecting bonds and clay aggregations along the failure surface. Despite the well known limitations of direct shear tests to provide quantitative information on either stress distribution or stress-strain characteristics of a soil, a qualitative analysis can be done on the test results. The imposed stress state around the failure surface can induce local shearing of connecting bonds and/or clay aggregations. Additional soil collapse occurs when the induced local shear stress overcomes the available local shear strength. As the connecting bonds and/or clay aggregations are sheared, the remaining clay aggregations and coarser particles behave as a granular soil along the failure plane. Depending on the net confining stress (i.e., a direct function of the applied stress state) and the interlocking of the granular structure (i.e., remaining from the consolidation phase), a soil specimen can undergo either dilation (see Figs. 5.26 and 5.27) or compression (see Figs. 5.28 and 5.29) during shearing.

v.) Additional collapse of the soil structure implies a gradual transfer of shear stresses from connecting bonds and clay aggregations on the failure plane, to coarser particles and remaining clay aggregations around this failure plane. Continuous breaking of connecting bonds and clay aggregations will result in a continuous decrease in void ratio which generally results in an increase in shear strength. The shearing of the connecting bonds
should predominate at low shear deformations, since the shearing of the soil skeleton is necessary in order for shear displacement to take place along the failure plane.

vi.) Under unsaturated conditions, the collapsing soil specimens present a stiff soil skeleton that behaves like an elastic rigid body at low shear deformations. The shear displacements start as the soil skeleton strength (i.e., the connecting bonds and/or clay aggregations along the failure plane) is overcome by the induced shear stress. The mobilized failure surface might present an irregular shape and the soil specimen will tend to dilate as shear displacement continues. The dilating behavior is a combined function of the vertical stress and the strength of the soil skeleton along the irregular failure surface.

Figures 5.22, 5.23 and 5.24, corresponding to vertical stresses of 25, 50 and 100 kPa respectively, illustrate a well-defined primary shear strength peak at low shear deformation for all the specimens tested, irrespective of the applied matric suctions. Such a behavior, although not so well-defined, was also observed for the soil specimens under a vertical stress of 200 kPa (see Fig. 5.25). Similar behavior has been observed in artificially cemented sand specimens (Lefebvre, 1995) and in naturally cemented loess specimens (Lin, 1995). In the present study, the cementing effect represents the breaking of connecting bonds holding the metastable soil skeleton. Figures 5.22 to 5.25 also illustrate that the shear behavior of the soil specimens after the primary peak, is a function of the combined effects of the applied vertical stress and the matric suction. The continuous contracting behavior of the saturated soil specimens (Figs. 5.30 and 5.31), as opposed to the equivalent strain-hardening stress-strain curves, is related to a continuous decrease in void ratios during shear, leading to a continuous increase of shear stress in these high initial void ratio collapsing soils. The strain-hardening behavior of the non-collapsing soil specimens (i.e., tested under vertical stresses of 25 and 50 kPa and at matric suctions higher than 20 kPa) is related to the interlocking effects in the soil. In general, the direct shear tests show the role of matric suction on the shear strength of soil specimens compacted dry of optimum water content, at a low dry densities. The form of the extended Mohr-Coulomb failure envelope for collapsing soil is discussed in section 5.4.1.2.
Figure 5.22: Shear strength of the collapsing soil at a net vertical stress of 25 kPa and at different matric suctions.
Figure 5.23 Shear strength of the collapsing soil at a net vertical stress of 50 kPa and at different matric suctions.
Figure 5.24 Shear strength of the collapsing soil at a net vertical stress of 100 kPa and at different matric suctions.
Figure 5.25 Shear strength of the collapsing soil at net vertical stress of 200 kPa and at different matric suctions.
Figure 5.26  Vertical displacement versus shear displacement of the collapsing soil at net vertical stress of 25 kPa.
Figure 5.27  Vertical displacement versus shear displacement of the collapsing soil at net vertical stress of 50 kPa.
Figure 5.28 Vertical displacement versus shear displacement of the collapsing soil at a net vertical stress of 100 kPa.

Vertical displacement (mm)

Shear displacement (mm)

- Matric suction = 100 kPa
- Matric suction = 50 kPa
- Matric suction = 25 kPa
- Matric suction = 0.0 kPa
Figure 5.29 Vertical displacement versus displacement of the collapsing soil at net vertical stress of 200 kPa.
Figure 5.30 Shear strength of the collapsing soil at saturated condition and under different vertical stresses.
Figure 5.31 Vertical displacement versus shear displacement of the collapsing soil at saturated condition.
5.3.2 Conventional direct shear test results

Conventional direct shear tests were conducted on soils compacted at optimum Standard AASHTO energy conditions as outlined in the testing program presented in Table 4.4. Each soil specimen was compacted at optimum water content conditions, placed in the direct shear test box and saturated for 24 hours. After saturation, the soil specimen was consolidated under a defined vertical stress and sheared under drained conditions. These stable-structured soil specimens were sheared using single stage direct shear tests.

The stable-structured soil specimens showed small vertical deflections for the range of vertical stresses from 25 to 200 kPa during the consolidation phase. This is similar to the results from the double oedometer tests (see Fig. 4.10) Figure 5.32 shows the shear stress versus shear displacement behavior of the stable-structured soil under saturated conditions. These results show a relatively high shear strength for the stable-structured soil as compared to the collapsing soil (Fig. 5.30).

PART TWO - SOIL MODELLING

Part two of Chapter 5 is related to the definition of soil models and functions required for the simulation of the mechanical and hydraulic behaviors of both the collapsing and stable-structured soils used in the present research study. The soil functions define the mathematical relationships between the soil properties and the stress state variables. The soil functions are defined using the laboratory test results presented in Part One of this Chapter.

The modelling of the mechanical behavior of both the metastable and the stable-structured soils involves the definition of relationships for simulation of both volume change and shear strength behavior. The volume change state surfaces (Matyas and Radhakrishna, 1968) are the required models to define void ratio changes of the unsaturated soil. A
Figure 5.32 Shear strength versus shear displacement of the stable soil at saturated condition.
mathematical functional is required for the extended Mohr-Coulomb failure envelope (Fredlund and Gan, 1988).

The modelling of the hydraulic properties of an unsaturated soil where the air phase is at atmospheric condition (i.e., at constant pressure) requires the definition of a mathematical function between the soil coefficient of permeability and the stress state variables.

5.4 Collapsing soil modelling

The laboratory results for the collapsing compacted soil of this study were presented in sections 5.2 (i.e., volume changes measurements and soil permeability) and 5.3.1 (i.e., shear strength tests). In this section, the laboratory test results are processed for the definition of the required collapsing soil models. Previous soil models (Brooks and Corey, 1964; Fredlund, 1979; van Genuchten, 1980; Lloret et. al., 1985; Miranda, 1988; Lloret et. al, 1993) were used as guidelines for modelling collapsing soil behavior. The soil models were defined by using best-fit analyses in the search for functional relationships that could capture the essential characteristics of the behavior of the soil as observed from the available experiments.

The best-fit analyses were performed by using version 1.0 of the software SigmaPlot developed by Jardel Scientific (SigmaPlot, 1995). This software performs a best-fit analysis using an equation containing up to 25 parameters and 10 independent variables of available data. Additionally, a maximum of 25 parameter constraints can be used. The Marquardr-Leveberg algorithm was used to determine the parameters which minimize the sum of the squares of the differences between the dependent variable values in the equation and the observed values.

The definition of the soil functions or models had as its main guideline the search for continuous mathematical functions which describe the available data. To provide suitable
analytical expressions for the soil behavior, as required by the theory presented in Chapter 3, the following criteria were established:

a.) the state surfaces (i.e., functional relationships for void ratio and degree of saturation) had to result in continuous $C^1$ functions (i.e., functions with continuous first derivatives) in order to allow the continuity of the compressibility parameters (i.e., $m'_s$, $m''_s$, $m'_w$, $m''_w$) over the range of stress state variables utilized;

b.) the models for shear strength and coefficient of permeability could result in continuous $C^0$ functions since the numerical modelling (Chapter 3) does not require the use of derivatives of such models. A proposed model, which is a $C^0$ function, is later described in section 5.4.1.1.

5.4.1 Mechanical behavior of the collapsing soil

The modelling of the mechanical behavior of the collapsing soil was developed by using two different models. The first model was associated with the volume change behavior of the collapsing soil based on the available data from the triaxial permeameter tests. The second model was related to the prediction of the shear strength behavior of the collapsing soil based on the available data from the modified direct shear tests.

5.4.1.1 Volume change behavior of the collapsing soil

The theory in Chapter 3 referred to the state surfaces for the unsaturated soil which was defined by compressibility parameters $m'_s$, $m''_s$, $m'_w$, $m''_w$. The parameters are to be defined from the mathematical models expressing the volume change behavior of a collapsing soil. The compressibility parameter $m'_s$ can be defined (Fredlund and Rahardjo, 1993) as follows:
\[ m_1' = \left( \frac{dV_v}{V_0} \right) \frac{d \varepsilon_v}{d \sigma^*} = \frac{d \varepsilon_v}{d \sigma^*} \]  

where:

\( dV_v \) = change in total volume of the soil element due to a change in \( d \sigma^* \)

\( V_0 \) = referential volume of the soil element

\( d \varepsilon_v \) = is the volumetric deformation (i.e., \( dV_v/V_0 \))

\( d \sigma^* \) = increment of mean net normal stress (i.e., \( [\sigma_x+\sigma_y+\sigma_z - 3u_a]/3 \)).

The assumption of a constant referential volume (see section 3.2), for a defined increment of volumetric deformation, allows the expression of volumetric deformation to be written as follows:

\[ d \varepsilon_v = \frac{de}{1+e_0} \]  

where:

\( e_0 \) = initial (i.e., referential) void ratio of the soil element

\( de \) = change in void ratio due to \( d \sigma^* \).

Equations 5.1 and 5.2 can be combined to express the compressibility parameter \( m_1' \) as a function of the void ratio.

\[ m_1' = \frac{1}{1+e_0} \frac{de}{d \sigma^*} \]  

In a similar manner, the compressibility parameter \( m_2' \) can be expressed in the form,

\[ m_2' = \frac{1}{1+e_0} \frac{de}{d(u_a - u_w)} \]
where:

\[ d(u_a - u_a) = \text{increment in matric suction} \]

\[ de = \text{change in void ratio due to } d(u_a - u_a). \]

Equations 5.3 and 5.4 illustrate that a functional relationship for the void ratio state surface provides a meaning to define the compressibility parameters \( m^1 \) and \( m^2 \).

The water phase compressibility parameter \( m^w \) can be defined as a function of void ratio and degree of saturation of the soil element by using the following procedure (Fredlund and Rahardjo, 1993):

\[
m^w = \frac{d\theta_w}{d\sigma^*} = \frac{d(Sn)}{d\sigma^*} \tag{5.5}
\]

where:

\[ d\theta_w = \text{increment of volumetric water content in the soil element (i.e., } \frac{dV_w}{V_0} \text{) due to a change in net confining stress (i.e., } d\sigma^*. \]

\[ S = \text{degree of saturation of the soil element} \]

\[ n = \text{porosity of the soil element}. \]

The assumption of a constant referential volume for the soil element for a specific increment of net confining stress, combined with the relationship between porosity and void ratio (i.e., \( n = e/(1+e) \)), allows the expression of the compressibility parameter \( m^w \) in the form,

\[
m^w = \frac{S}{1+e_0} \frac{de}{d\sigma^*} + \frac{e}{1+e_0} \frac{dS}{d\sigma^*} \tag{5.6}
\]

where:

\[ e = \text{current void ratio of the soil element} \]
\[ S = \text{current degree of saturation of the soil element} \]
\[ de = \text{change in void ratio due to } d\sigma^* \]
\[ dS = \text{change in degree of saturation due to a change in mean net confining stress.} \]

Using a similar procedure to that above, the compressibility parameter \( m_2^w \) can be expressed as follows:

\[
m_2^w = \frac{S}{1 + e_0} \frac{de}{d(u_a - u_w)} + \frac{e}{1 + e_0} \frac{dS}{d(u_a - u_w)} \quad [5.7]
\]

where:

\[ d(u_a - u_a) = \text{is an incremental change in matric suction} \]
\[ de = \text{change in void ratio due to } d(u_a - u_a) \]
\[ dS = \text{change in degree of saturation due to } d(u_a - u_a). \]

Equations 5.6 and 5.7 illustrate the need for a functional relationship for the state surfaces in order to define the compressibility parameters \( m_1^w \) and \( m_2^w \). The derivatives \( \frac{de}{d\sigma^*} \), \( \frac{dS}{d\sigma^*} \), \( \frac{de}{d(u_a - u_w)} \) and \( \frac{dS}{d(u_a - u_w)} \) can be defined from the state surfaces equations.

**Total volume change behavior of the collapsing soil**

Figure 5.7 has shown that at a given mean net normal stress and under a wetting process, the collapsing soil shows three distinct phases (e.g., curve for net normal stress equals to 100 kPa) in terms of the total deformation versus matric suction relationship.

The first phase of deformation occurs at high matric suctions and is characterized by small volumetric deformations of the soil in response to relatively large decreases in matric
suction. From a phenomenological standpoint, this behavior implies small values for the compressibility parameter, $m^2_s$. From a structural point of view, the soil response can be described as undergoing elastic compression without grain slippage. This elastic response can be visualized as the cementing effect provided to the soil microstructure and the matric suction. Figures 5.22 to 5.25, previously discussed in section 5.3.1, illustrated this cementing effect as the collapsing soil specimens were sheared under unsaturated conditions with a controlled matric suction. This first phase of the collapsing soil deformation is herein termed the “pre-collapse” phase.

The second phase of deformation occurs at intermediate values of matric suction and is characterized by significant volumetric deformations in the collapsing soil as it responds to reductions in matric suction. From a phenomenological standpoint, this behavior implies significant values for the compressibility parameter, $m^2_s$. This behavior can be explained in terms of a combination of further structural rearrangements and by the occurrence of local shearing of both the connecting bonds and clay aggregations. Both are due to a reduction in the matric suction at the microstructure level. At this second phase, the breaking of the connecting bonds and clay aggregations implies that there is a gradual increase in the number of contact points between larger particles (i.e., sand and/or silt particles and even remaining clay aggregations). The rearrangement of the collapsing soil structure in this second phase involved both the macro and the microstructure of the collapsing soil mass. The collapse continues until a new equilibrium configuration is reached. Under a given net normal stress, the new equilibrium configuration does not require the total destruction of all connecting bond clay aggregations in response to an incremental decrease in matric suction. This information and the fact that the soil structure is composed of a full range of combined microstructures (Khogo et. al. 1993; Feda, 1995; Rogers, 1995, Osipov et al. 1995) assist us in understanding the unique relationship between soil collapse and matric suction for an uncemented, compacted, collapsing soil. This second phase of the collapsing soil deformation is herein termed the “collapse” phase.
The third phase of deformation occurs at low matric suctions and requires no additional volumetric deformations for the collapsing soil as it responds to further reductions in matric suction. From a phenomenological standpoint, this phase implies negligible values for the compressibility parameter $m^2$. From a structural standpoint, this behavior can be explained at the microstructure level. In the third phase, the collapsing soil has already reached its full saturation, except for the presence of some amount of trapped air bubbles. The combined influence of an increase in the amount of entrapped air due to the further reduction in the microstructure pore-size distribution and the difficulty of the free water to penetrate in the reduced air-filled pore-spaces under low hydraulic gradients help explain why the higher the net confining stress, the smaller the degree of saturation where the maximum collapse is reached. The small deformations observed for the third phase, for the confining stress of 200 kPa, can be attributed to secondary compression of the soil skeleton. This third phase of the collapsing soil deformation is herein termed the "post-collapse" phase.

Figures 5.2 (in terms of volumetric deformation) and 5.7 (in terms of void ratio) suggest that the collapsing soil behavior can be modelled for the three phases discussed. Figure 5.33 illustrates a proposed model to predict the volume change behavior of the collapsing soil behavior at a given net confining stress, under a saturating stress path.

![Figure 5.33](image)

**Figure 5.33** The volume change behavior of a collapsing soil during the wetting process.
The proposed soil model is composed of three linear equations expressing the void ratio of the soil (on an arithmetic scale) as a function of matric suction (on a logarithmic scale) along a saturating path. The three equations are expressed using curve fitting parameters (i.e., \(d_1\), \(d_2\), \(e_u\), \(e_f\), \((u_a-u_w)_c\), and \((u_a-u_w)_f\)). These parameters are described as follows:

\[
d_1 = \text{slope of volumetric deformation in the pre-collapse phase (i.e., } \frac{de}{d(u_a - u_w)} \text{)}
\]

\[
d_2 = \text{slope of volumetric deformation in the collapse phase (i.e., } \frac{de}{d(u_a - u_w)} \text{)}
\]

\[
e_u = \text{unsaturated void ratio (i.e., for the soil loaded at condition)}
\]

\[
e_f = \text{final void ratio (i.e., after the complete saturation of the soil)}
\]

\[
(u_a-u_w)_c = \text{critical matric suction below what the soil structure starts collapsing}
\]

\[
(u_a-u_w)_f = \text{final matric suction below what the soil structure stops collapsing}
\]

Figure 5.34 illustrates the use of the proposed model in simulating the collapse behavior by using the available data from the triaxial permeameter tests. Unfortunately, the available experimental data allowed only the complete definition of the fitting parameters for the case of a net confining stress of 100 kPa. For the net confining stress of 50 kPa, the available data could be used to define only some of the fitting elements, (i.e., \(e_u\), \(d_1\) and \(e_f\)), due to the lack of additional experimental data for matric suctions between 30 and 0 kPa. The soil test at a net confining stress of 200 kPa allowed only the definition of the elements, \(e_u\), \(d_2\), \(e_f\), and \((u_a-u_w)_f\), since the available data suggested that at a matric suction of 90 kPa, (i.e., the first step utilized in the test), the soil had already collapsed.

The definition of the collapsing soil model, as proposed in Figure 5.33, requires the adoption of several assumptions. Based on the available experimental data and some experimental evidence from the literature (Escario et. al. 1973; see Fig. 2.18 in section...
2.3.3.5), it is assumed that there is a linear relationship between the slope parameters, (i.e., \( d_1 \) and \( d_2 \)) and the net confining stress. This assumption allows the complete definition of the collapsing soil behavior under the net confining stress of 200 kPa, since the slope of the “pre-collapse” phase (i.e., \( d_1 \)) can then be defined. As a first approximation, a linear relationship can be assumed between the final matric suction, \((u_d-u_w)_f\), and the net confining stress. This assumption allows for a complete description of the collapsing soil behavior at a net confining stress of 50 kPa.

The proposed model suggests that at a net confining stress of 50 kPa, the soil will collapse only at a very low matric suction. This means that the connecting bonds and the clay aggregations have to reach a high degree of saturation (i.e., practically eliminating the capillary action) in order to start collapsing. This observation, along with the available soil collapse versus degree of saturation relationship (Fig. 5.11), suggested that at a net confining pressure of 50 kPa, the soil collapses at a degree of saturation higher than 80%.

The combination of relationships between the curve fitting parameters (i.e., \( d_1, d_2, e_u, e_f \), and \((u_d-u_w)_f\)), and the net confining stress allows the use of the proposed model in the description of the collapsing soil behavior along decreasing matric suction stress paths (i.e., a saturation process). Figure 5.35 illustrates the best-fit relationships used for the curve fitting for the void ratio parameters (i.e., \( e_u \) and \( e_f \)). Figure 5.36 illustrates the use of the proposed model in simulating the behavior of the collapsing soil under net confining stresses of 50, 75, 100, 150 and 200 kPa. The available experimental data are also illustrated in Fig. 5.36.

The proposed model is limited to the available data that has been used in the formulation of the curve fitting parameters. For a larger range of net confining stresses, a more complex relationship between the curve fitting parameters and the net confining stress would be required.

The use of the proposed model as input for numerical simulations of collapsing soil behavior is limited by the existence of “corners” as illustrated in Fig. 5.36. Such “corners” imply discontinuities for the compressibility parameters (i.e., \( m_1^e, m_2^e, m_1^w, \) and \( m_2^w \)) along
Figure 5.34 The behavior of the collapsing soil when saturating at a given net confining stress.
Figure 5.35 Void ratio versus net confining stress relationships for the collapsing soil at initial (unsaturated) and final (saturated) conditions.
Figure 5.36 Prediction of the behavior of a collapsing soil soil when saturated under a given net confining stress.
the saturation paths. Attempts to use the proposed model in numerical simulation of soil collapsing behavior suggested the need for a smooth void ratio state surface with continuous first derivatives (i.e., a \( C^1 \) function). For this reason, the proposed model was used in conjunction with the software SigmaPlot to define a smooth void ratio state surface.

The software SigmaPlot requires the definition of a trial equation wherein a dependent variable (e.g., the void ratio) is expressed as a function of several independent variables (e.g., net confining stress and matric suction). The trial equation can involve up to 25 coefficients. The independent variables can be expressed either in a simple linear form or in more complex forms (e.g., as arguments of exponential or logarithm functions).

The search for a smooth equation for the void ratio state surface involved several attempts. One of the first attempts involved the use of Eq. 2.33, which was proposed by Lloret et. al., (1985), for best-fitting the data. Equation 2.33 is a nonlinear equation relating the void ratio as a function of the stress state variables.

\[
e = a + b \log(\sigma - u_a) + c \log(u_a - u_w) + d \log(\sigma - u_a) \log(u_a - u_w) \tag{2.33}
\]

where:

\( a, b, c, d = \) constant coefficients resulting from best-fitting analysis

Equation 2.33, when used as the input trial equation for the software SigmaPlot, did not provide a suitable best-fit to the available data. It was evident that Eq. 2.33 required more flexibility in order to reproduce the changes observed in the behavior of the collapsing soil during saturation. This is particularly true for the transition zone between the pre-collapse and collapse phases. Equation 2.33 can provide a suitable best-fit analytical model for cases where a metastable soil presents a smooth collapsing behavior along the saturation path.

A superior curve-fitting was obtained by using a five parameter logistic function. Equation 5.8 shows this function in terms of void ratio as a function of matric suction at a given matric suction. Figure 5.33 help illustrate the terms in Equation 5.8.
\[ e = e_u + \frac{e_f - e_u}{1 + \left( \frac{u_s - u_w}{c} \right)^{b^a}} \]  

where:

- \( e_u \) = initial void ratio of a soil specimen under a given net confining stress
- \( e_f \) = final void ratio of a soil specimen under a given net confining stress
- \( c \) = matric suction value at the inflection point (i.e., middle point of “collapse” phase)
- \( b \) = slope parameter (i.e., slope of the “collapse” phase)
- \( a \) = the symmetry parameter which makes the logistic function asymmetric

The best-fitting procedure was developed on a trial and error basis using the above logistic function as the input equation for the software SigmaPlot. The following factors were used as guidelines in the establishment of the mathematical model (i.e., void ratio state surface):

i.) the soil began to show collapse behavior beyond a net confining pressure of approximately 50 kPa. Hence, only the experimental data corresponding to net confining pressures from 50 to 200 kPa were used in the development of the soil model. There is a critical net confining pressure of about 40 kPa (Fig. 5.35) below which the soil does not collapse during the saturation process;

ii.) the best-fit relationship for the void ratio curve fitting parameters (i.e., \( e_u \) and \( e_f \)) as illustrated in Fig. 5.35, were used as constraints for the soil model. The use of constraints improves the accuracy and provides more rapid convergence of the algorithm used in SigmaPlot;

iii) preliminary analysis suggested the use of a power function relationship between the slope parameter of Equation 5.8 (i.e., \( b \)) and the net confining pressure. The relationship \( b \)
\( b_i^2 = (\sigma - u_o)^{b_2} \) was used where \( b_1 \) and \( b_2 \) are constants to be defined from the best-fitting analysis. In a similar manner, a second-degree polynomial relationship between the inflection parameter (i.e., \( c \)) and the net confining stress was also adopted. The coefficients of the polynomial equation \( c = c_1\sigma^2 + c_2\sigma + c_3 \) are constants to be defined from the best-fit analysis. Another conclusion from the preliminary analysis was that the symmetry parameter (i.e., \( a \)) can be assumed to be equal to 1 without significant changes in the best-fitting results.

Equation 5.9 is the mathematical model for the void ratio state surface of the collapsing soil resulting from the best-fit analysis of the available data using Eq. 5.8.

\[
e = e_u + \frac{e_f - e_u}{\left[ 1 + \left( \frac{u_o - u_w}{c} \right)^b \right]^c} \tag{5.9}
\]

where:

\[
e_u = 0.7697 - 0.0073 \ln(\sigma^*)
\]

\[
e_f = 1.2264(\sigma^*)^{-0.1399}
\]

\[
c = c_1(\sigma^*)^2 + c_2(\sigma^*) + c_3
\]

\[
b = b_i(\sigma^*)^{b_2}
\]

\[
c_1 = 9.398 \times 10^4
\]

\[
c_2 = 7.465 \times 10^2
\]

\[
c_3 = -4.066
\]

\[
b_1 = 49.01
\]

\[
b_2 = -6.103 \times 10^3
\]

Figure 5.37 shows the computer simulated three-dimensional void ratio state surface. For the transition from the "collapse" phase to the "post-collapse" phase, Eq. 5.9 provides a suitable best-fit for net confining stresses higher than 50 kPa. Figure 5.38 illustrates the best-fit model in terms of void ratio versus matric suction relationships under
different net confining stresses. The available experimental data are also illustrated in Figure 5.38 and show a comparison of the best-fit surface provided by Eq. 5.9.

The compressibility parameter $m_t'$, was previously expressed in equation 3.11, in terms of elastic parameters and for triaxial loading conditions.

$$m_t' = \frac{3(1 - 2\mu)}{E} \quad [3.11]$$

Equation 5.3 allows the evaluation of $m_t'$ by using the void ratio state surface. However, according to Equation 3.11, the definition of $m_t'$ does not allow the evaluation of the elastic parameters (i.e., $E$ and $\mu$), as required by the numerical modelling presented in Chapter 3.

Previous researchers (Miranda, 1988; Alonso et al., 1988, 1995; Lloret et al., 1993) assumed a constant Poisson ratio equal to 0.3 in numerical simulations of the behavior of collapsing earth structures upon saturation. These researchers justified such an assumption by referring to the studies of Clough and Woodward (1967) and Lambe and Whitman (1979), as previously presented in section 2.3.3.5.1. A Poisson ratio equal to 0.3 may reflect the as-compact ed (i.e., at end-of-construction) condition of a loosely compacted embankment (Miranda, 1988; Alonso et al., 1988, Lawton et al., 1991a). However, the changes in the structure of a metastable structured soil upon saturation, results in corresponding changes in the Poisson ratio (Maswoswe, 1985; Lawton, 1991a).

In the present research study, the available laboratory test results were used to define the Poisson ratio as function of the stress state variables. The first step was to establish a relationship between Poisson ratio and the mean normal stress of the collapsing soil at saturated conditions. Such a relationship was defined by combining the triaxial and double-oedometer test results with the experimental observations that: “the volumetric collapse of a soil mass is truly a function of the acting mean total stress (Lawton, 1991a)”.

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Figure 5.39 shows the available consolidation results as superimposed curves where the isotropic test results (i.e., unsaturated and saturated paths) are presented in terms of void ratio versus mean total stress (i.e., $\sigma_m$) and the oedometer test results (i.e., saturated path) are presented in terms of void ratio versus vertical stress (i.e., $\sigma_v$). Based on the isotropic test results, an exponential relationship between the void ratio and the vertical stress was assumed for the saturation path in the double oedometer tests. The exponential relationship was developed by using only the vertical stresses of 100 and 200 kPa, since a vertical stress of 50 kPa did not produce collapse in the oedometer test. Approximate values for mean total stress in the oedometer test at various values of vertical stresses, $\sigma_v$, were determined by selecting values of vertical stress for various void ratios on the oedometer curve. Then the corresponding value of mean net stress on the saturated isotropic curve was determined as illustrated in Figure 5.39. A given combination of mean total stress and vertical stress (i.e., $\sigma_m$ and $\sigma_v$ values) allowed an evaluation of the Poisson's ratio for the saturated collapsing soil (i.e., $\mu_s$) by means of the following equation:

$$\mu_s = \frac{3\sigma_m - \sigma_v}{\sigma_v + 3\sigma_m}$$  \hspace{1cm} [5.10a]

where:

$$\sigma_m = \frac{\sigma_v + 2\sigma_h}{3}$$

$$\sigma_h = \frac{\mu}{1-\mu} \sigma_v$$

Figure 5.40 shows the Poisson ratio versus mean net normal stress obtained from Eq. 5.10a.

The Poisson ratio for the collapsing soil at unsaturated conditions was determined as follows:
Figure 5.37 Void ratio state surface for the collapsing soil.
Figure 5.38 Comparison of available data and soil modelling for the void ratio versus matric suction relationships for the collapsing soil.
Figure 5.39 Void ratio versus confining or vertical stress relationships for the collapsing soil at initial (unsaturated) and final (saturated) conditions.
Figure 5.40 Poisson ratio versus mean total stress relationships for the collapsing soil at saturated conditions.
a.) At a given mean net stress (i.e., net confining stress) a constant Poisson ratio equal to 0.3 is assumed for the “pre-collapse” phase (Figure 5.33). This assumption can be justified by the negligible changes in soil structure for the collapsible soil during this phase;

b.) At a given mean total stress, it is assumed that the Poisson ratio increases with the soil collapse. This implies an increase of the Poisson ratio when the matric suction decreases which reflects experimental evidence (Maswoswe, 1985; Handy, 1995). In the present research study, the same relationship used to simulate the soil collapse versus matric suction (i.e., Eq. 5.9) is used to simulate the change in Poisson ratio in response to a change in matric suction. This assumption implies a variation of the Poisson’s ratio of the collapsing soil from 0.3 (i.e., at the as-compacted conditions) to a value calculated by using Eq. 5.10a when the soil reaches saturated conditions. Therefore, the Poisson ratio for the collapsing soil under unsaturated conditions is calculated as follows,

\[
\mu = 0.3 + \frac{\mu - 0.3}{1 + \left(\frac{\mu_s - \mu_u}{c}\right)^b}
\]

[5.10b]

where:

\[
c = c_1 \sigma^2 + c_2 \sigma + c_3
\]

\[
b = b_1 \sigma^2
\]

\[
c_1 = 9.398*10^4
\]

\[
c_2 = 7.465*10^2
\]

\[
c_3 = -4.066
\]

\[
b_1 = 49.01
\]

\[
b_2 = -6.103*10^2
\]

\[
\mu_s = saturated\ Poisson\ ratio\ calculated\ by\ using\ Eq.\ 5.10a.
\]

The meaning of the parameters are the same as previously discussed in Eq. 5.9.
Water volume change behavior of the collapsing soil

The degree of saturation state surface was also defined using a best-fit analysis similar to that of the void ratio state surface. After several trials, the logistic function provided the best-fit results of the available data. Equation 5.11 presents the mathematical model obtained for the degree of saturation state surface model of the collapsible soil:

\[ S = S_0 + \frac{1 - S_0}{1 + \left( \frac{u_o - u_w}{c} \right)^a} \]  \hspace{1cm} [5.11]

where:

\[ S_0 = a + b \times \ln(\sigma^*) \]
\[ a = 3.541 \times 10^4 \]
\[ b = 3.654 \times 10^3 \]
\[ c = 7.906 \]
\[ d = 9.769 \times 10^1 \]

Figure 5.41 shows the corresponding three-dimensional degree of saturation state surface. Equation 5.11 is a phenomenological model adequate for use in numerical analyses. However, this model does not allow further considerations of the soil microstructure during saturation. The available data suggests that soil collapse produces two different and opposite effects in the saturation of the soil mass. The first effect is the reduction of the void volume of the soil mass which increases its degree of saturation. The second effect is the reduction in the water inflow into the soil mass micropores. The second effect can be enhanced by the fact that the higher the net normal stress, the more difficult it is to reach saturation in the soil specimen. Soil specimens under 20 and 50 kPa of net confining pressure appear to become saturated by using a back pressure of 4 kPa. However, the same back pressure appeared to
Figure 5.41 Degree of saturation state surface for the collapsing soil.
produce a degree of saturation of about 97% for the soil specimen under net confining stress of 200 kPa. This suggests that air leakage into the soil specimen could be partly responsible for this behavior (section 4.5.1). However, the short time associated with the saturation process (i.e., when the back pressure was applied) did not appear to be sufficient to allow enough air inflow to prevent the saturation of the open-structured soil specimen. The following additional factors can be cited to justify the unsaturation of the collapsing soil specimen under 200 kPa of net confining pressure:

a.) internal hydraulic gradients from the soil microstructure to the soil macrostructure caused by the soil collapse which induces local increases in pore-water pressure in the soil microstructure (Alonso et al. 1985; Handy, 1995)

b.) trapped air bubbles in both the micro- and macrostructure of the soil specimen when its degree of saturation was higher than 80 % (Fredlund et al. 1993, Osipov et al. 1995).

From a macroscopic point of view, the available data would suggest that the magnitude of the measured soil collapse (i.e., volumetric deformation less than 10 %, for the range of applied net confining stresses), did not significantly increase the degree of saturation of the soil specimen. A phenomenological model for the degree of saturation already includes both the effect of changes in the soil structure (i.e., pore size changes) and the water inflow when the collapsible soil is saturating. Figure 5.42 shows the resulting best-fit model in terms of the degree of saturation versus matric suction relationship under different net confining stresses. Available experimental data are also shown in Figure 5.42 illustrating the best fit of Eq. 5.11.

5.4.1.2 Shear strength behavior of the collapsing soil

The available shear strength data for the collapsing soil are presented and discussed in section 5.3.1. Figures 5.22 to 5.25 illustrate the predominant strain-hardening behavior of the collapsing soil when it is sheared at unsaturated conditions under a net vertical stress
higher than 25 kPa. The shear strength of the soil specimen allows the definition of an extended Mohr-Coulomb shear strength envelope for a collapsing soil. There appears to be no well-established criteria in the literature, for the definition of failure for strain-hardening soil behavior. The conventional approach is to define failure based on a strain criteria. Such an approach may lead to the definition of shear strength parameters that increase with increasing shear displacement (Terzaghi and Peck, 1967). The choice of a deformation criteria compatible with the field problem at hand minimizes inaccuracies from the use of the strain criteria approach. Based on the fact that the shear tests with a well-defined peak strength maintained their strength for large shear displacements (Figs. 5.22 to 5.24) and on the deformations expected to occur in “Alka-Seltzer” dams, it was assumed that: In the absence of a well-defined peak strength, failure would occur when the shear stress versus shear displacement curve reached a shear displacement of 5 mm. The soil specimen length in the direction of shearing is 51 mm. Similar procedures were used by others in previous studies (Olool, 1994; Campos et al. 1995).

Figure 5.43 shows the shear strength versus matric suction relationships under various net vertical stresses for the collapsing soil. Figure 5.44 shows the shear strength of the collapsing soil plotted against the net vertical stress. From a phenomenological standpoint, the above figures can be used to define the extended Mohr-Coulomb envelope (i.e., a shear strength surface) for the collapsing soil. SigmaPlot was used to define the best-fit coefficients for the extended Mohr-Coulomb failure envelope. Equation 5.12 expresses the resulting best-fit mathematical model for the extended Mohr-Coulomb failure envelope for the collapsing soil.

\[
\tau_{ef} = a_1 + b_1 \sigma + c_1 (u_a - u_w) + d_1 \sigma (u_a - u_w)^p
\]  \hspace{1cm} [5.12]

where:

\[a_1 = -7.893\]

\[b_1 = 0.1944\]
Figure 5.42 Comparison of available data and soil modelling for the degree of saturation versus matric suction relationships of the collapsing soil.
Figure 5.43  Shear strength versus matric suction relationships for different net normal stresses.
Figure 5.44: Shear strength versus net normal stress relationships for different matric suctions.
\[ c_1 = 0.3238 \]
\[ d_1 = 0.09319 \]
\[ p = 0.04307 \]

\((\sigma - u_a)\) = net normal stress at the failure plane.

Equation 5.12 defines the shear strength envelope of the collapsing soil for the range of matric suction from 0 to 100 kPa. The study assumes that at a given net normal stress, the shear strength of the collapsing soil remains constant for the range of matric suctions from 100 to 370 kPa.

Figure 5.45 illustrates the comparison between the best-fit results and the experimental data in terms of shear strength versus matric suction for various net vertical stresses.

5.4.2 Coefficient of permeability of the collapsing soil

Figures 5.12 and 5.13 illustrate that the coefficient of permeability of the collapsing soil specimens with respect to the water, while at unsaturated conditions, did not show significant effect from the net confining stress. However, when the collapsing soil specimens approach saturation, the coefficient of permeability decreases as the net confining is increased from 20 to 200 kPa. As previously discussed in sections 5.2.2 and 5.4.1.1, the degree of saturation of the unsaturated soil specimens depends mainly on the applied matric suction rather than the acting net confining stress. At saturated conditions, the coefficient of permeability of each soil specimen depends mainly on its void ratio which is a direct function of the net confining pressure (see Figure 5.7). Figure 5.13 also shows a close comparison between the available data for the coefficient of permeability and the calculated values (i.e., using Brooks and Corey's (1964) equation along with a drying soil-water characteristic curve) for matric suctions higher than 5 kPa. However, the transition from the
Figure 5.45 Shear strength versus matric suction relationships for different net normal stresses.
unsaturated to the saturated condition (i.e., matric suction from 5 to 0 kPa) requires some changes in the parameters in order that the equation will better simulate the data.

Brooks and Corey’s (1964) equation and the software SigmaPlot were used in best-fit analyses of the available experimental data as function of the stress state variables. A relationship between the saturated coefficient of permeability, \( k_s \), and the net confining stress was previously defined and used as a constraint in the best-fit analysis. Equation 5.13 expresses the best-fit mathematical equation for the water coefficient of permeability, as a function of the stress state variables, of the collapsing soil.

\[
k_w = k_p \left( \frac{\psi_{cr}}{(u - u_w)} \right)^\lambda
\]  

[5.13]

where:

\[
k_w \leq k_s
\]

\[
k_p = -1.39 \times 10^{-7} + 6.259 \times 10^{-8} \text{Ln}(\sigma^*),
\]

\[
k_s = 1.17 \times 10^{-4} - 1.8 \times 10^{-7} \text{Ln}(\sigma^*), \text{ is the saturated coefficient of permeability.}
\]

\[
\psi_{cr} = 3.0
\]

\[
\lambda = 2.90.
\]

Figures 5.46 and 5.47 show the best-fit results in terms of the water coefficient of permeability versus matric suction relationships for different net confining stresses. The figures also show the available data for the net confining stress of 200 kPa in order to show the accuracy of the predicting model.

### 5.5 Modelling of the stable soil

The modelling of both the mechanical behavior and the hydraulic properties of the stable soil (i.e., soil compacted at optimum condition) was the same as previously used for
the metastable-structured soil (section 5.4). The double oedometer test (Figure 4.11) illustrates the low compressibility of the stable-structured soil for the range of vertical stresses from 0 to 200 kPa. This allows the use of the conventional unsaturated soil mechanics theories.

5.5.1 Mechanical behavior of the stable-structured soil

The mechanical behavior of the stable saturated soil requires the definition of two models. The first model describes the volume change behavior while the second model defines the shear strength behavior of the stable-structured soil. Both models were based on the available data from laboratory tests presented in sections 4.2.2.2 and 5.3.2.

5.5.1.1 Volume change behavior of the stable-structured soil

Volume change behavior of the initially unsaturated stable soil involves the definition of models for both the total volume change and the water phase volume change, in response to changes in the acting stress state variables. The models also predict the transition of the stable-structured soil from the unsaturated condition to the saturated condition. The models are in accordance with the theory presented in sections 3.5.4 and 5.4.1.

Total volume change behavior of the stable-structured soil

The modelling of the total volume change of a stable-structured soil element is based on the results of the double oedometer tests (Fig. 4.11) presented in section 4.2.2.2. The results illustrate that the compressibility of a stable-structured soil, under oedometric loading, does not change from the unsaturated to the saturated condition. This implies that the void ratio state surface is independent of matric suction changes (i.e., $m^2_s = 0$) and can be
expressed as a single function of the net confining pressure. For the double oedometer test, the confining stress can be expressed as a function of the vertical stress and the Poisson ratio. The analysis of “Alka-Seltzer” dams (section 4.1) requires the modelling of the void ratio state surface of the stable-structured soil for the range of net vertical stresses from 0 to 200 kPa.

Figure 5.48 shows the resulting best-fit linear model adjusted to the available data from the double oedometer tests performed on a stable-structured soil specimen. The compressibility parameter $m'_x$, from oedometric loading, can be expressed in terms of elastic parameters as follows:

$$
\frac{d\varepsilon_v}{d\sigma_v} = \frac{(1 + \mu)(1 - 2\mu)}{E(1 - \mu)} \quad [5.14]
$$

where:

$m'_x =$ compressibility of the soil structure due to a change in net/effective vertical stress (at saturated condition $m'_x = m_v$)

$\sigma_v =$ net/effective vertical stress

$\mu =$ Poisson ratio of the soil structure

$E =$ Elasticity modulus of the soil structure.

In the present research study, a constant Poisson ratio of 0.3 is assumed for the stable-structured soil. The stable soil allows a low compressibility and does not change its mechanical behavior upon saturation. This information, in conjunction with the low level of confining stress anticipated in “Alka-Seltzer” dams, justifies the above assumption. The same assumption was used by Miranda (1988) and is in agreement with Lambe and Whitman (1969) who concluded that $\mu$ "usually has a relatively small effect upon engineering predictions".
Figure 5.46 Modelling of the water coefficient of permeability versus matric suction relationships for the collapsing soil.
Figure 5.47 Modelling of the water coefficient of permeability versus matric suction relationships for the collapsing soil.
Figure 5.48 Modelling of the total volume change behavior of the residual soil compacted at stable condition.
Water volume change behavior of the stable-structured soil

The double oedometer test showed that under the range of vertical stresses from 25 to 200 kPa, the stable-structured soil specimens underwent negligible (i.e., less than 3%) total volume changes (Fig. 4.10). The as-compact ed stable-structured soil had an initial matric suction of about 30 kPa (Figure 4.14). This information along with anticipated wetting paths for the problem of "Alka-Seltzer" dams allow the use of the soil-water characteristic curve to model the water volume change. The same procedure was used by Miranda (1988). The literature provides a number of mathematical models for describing the soil-water characteristic curve (Brooks and Corey, 1964; van Genuchten, 1980; Fredlund and Xing, 1993). The present research study uses van Genuchten's (1980) equation to define the water volume versus suction for a stable-structured soil. The model was developed using the soil-water characteristic data previously presented in section 4.2.2.2 (Figure 4.14). Van Genuchten's (1980) equation can be expressed as follows:

$$\theta_w = \theta_s + \frac{(\theta_s - \theta_r)}{\left\{ 1 + [\alpha (u_a - u_w)]^n \right\}^m} \tag{5.15}$$

where:

- $\theta_s$ = volumetric water content of the soil element at saturated condition,
- $\theta_r$ = residual volumetric water content of the soil element,
- $\alpha$, $n$ = soil parameters estimated from the soil-water characteristic curve
- $m = 1 anthropology{n}.$

Figure 5.49 shows the results of the best-fit model, including the parameters, based on the soil-water characteristic data for the stable structured soil. From Eq. 5.15, the water phase compressibility parameter $m^2_w$ is defined as follows:
Figure 5.49  Modelling of the volumetric water content for the stable soil using van Genuchten's (1980) equation.

van Genuchten's parameters:
\[ \theta_s = 0.302, \theta_r = 0.245, \]
\[ \alpha = 0.045 \text{ and } n = 2.3 \]
\[ m'_{w} = \frac{d\theta}{d(u_a - u_w)} = -\frac{mna^n(\theta - \theta_r)}{[1 + \alpha(u_a - u_w)]^{n-1}}(u_a - u_w)^{n-1} \]  \[ 5.16 \]

The water phase compressibility parameter $m'_{w}$ is calculated according to Eq. 5.6. The low compressibility of the stable-structured soil implies in low values for $m'_{w}$. These values are consistent with the use of the soil-water characteristic curve to define the water phase volume changes for a stable-structured soil.

5.5.1.2 Shear strength behavior of the stable-structured soil

Figure 5.50 shows the shear strength Mohr-Coulomb envelope of the saturated stable-saturated soil. The envelope was defined from the shear tests results presented in Figure 5.32. Figure 5.50 also illustrate the shear strength parameters, $c'$ and $\phi'$, for the stable-saturated soil at saturated conditions.

In order to define the shear strength of the stable-saturated soil at unsaturated conditions, the study used the closed form equation presented by Fredlund et al., (1995). The solution is based on the general predictive model (Eq. 2.33) and uses the saturated shear strength parameters (i.e., $c'$ and $\phi'$) and the soil-water characteristic curve to predict the unsaturated shear strength.

\[ \tau'_{f} = c' + (\sigma - u_a) \tan \phi' + \tan \phi' \left[ \frac{\theta - \theta_r}{\theta - \theta_r} \right] d(u_a - u_w) \]  \[ 2.33 \]

A closed form solution can be derived from Eq. 2.33 provided the integral in the last term of the equation is solvable. The solution requires an equation to describe the soil-water characteristic curve [i.e., the term $(\theta - \theta_{r,\sigma})/(\theta - \theta_{r,\omega})$] in Eq. 2.33. Another alternative is to use the Brooks and Corey’s (1964) equation.
Figure 5.50 Shear strength envelope of the stable-structured soil at saturated conditions.
\[
\frac{(\theta - \theta_r)}{(\theta_s - \theta_r)} = \left( \frac{u_a - u_w}{u_a - u_w} \right) ^ \lambda
\]

where:

\((u_a - u_w)_b\) = air-entry value.

\(\lambda\) = pore size distribution index.

Combining Eqs. 2.33 and 5.17, Fredlund et al. (1995) suggested the following closed form solution for the unsaturated strength of a soil:

\[
\tau_f = c' + (\sigma - u_a) \tan \varphi' + (u_a - u_w) \tan \varphi' + \frac{(u_a - u_w)_b}{\lambda} \left( \frac{1}{(u_a - u_w)_b^{\lambda-1}} - \frac{1}{(u_a - u_w)^{\lambda-1}} \right) \tan \varphi
\]

Figure 5.51 shows a best-fit analysis of Equation 5.17 to the available soil-water characteristic data. Van Genuchten’s and Brooks and Corey’s best-fit curve are also shown in Fig. 5.51. Figure 5.52 shows the unsaturated shear strength of the stable-structured soil predicted using Equation 5.18. The shear strength curves are predicted for net normal stresses of 0, 100 and 200 kPa.

5.5.2 Coefficient of permeability of the stable-structured soil

In the present study, the coefficient of permeability of the stable soil was modelled using the following equation proposed by van Genuchten (1980):

\[
k_w = k_s \left( \frac{\left\{ -\left[\alpha (u_a - u_w)^n\right]^{1-m} \left[1 + \alpha (u_a - u_w)^n\right]^{-m} \right\}^{\frac{1}{2m}}}{\left[1 + \alpha (u_a - u_w)^n\right]^{\frac{1}{2}}} \right)
\]

where:
Figure 5.51 Modelling of the volumetric water content for the stable soil using Brooks' and Corey's (1964) equation.
Figure 5.52 Shear strength behavior of the stable soil using Fredlund et al.'s (1995) equation.
\[ k_s = \text{saturated coefficient of permeability}, \]

\[ n, \alpha, m = \text{van Genuchten's parameters previously defined in Equation 5.15}. \]

Data from section 4.2.2.2 (see Figure 4.13) allows the estimation of \( k_s \) as equal to \( 1.5 \times 10^{-10} \text{ m/s} \) (average value for the range of vertical stresses from 25 to 200 kPa). Figure 5.53 illustrates the modelling of the coefficient of permeability for the stable-structured soil. The best-fit parameters are also presented in Fig. 5.53.

![Graph showing coefficient of permeability vs. matric suction](image)

**Figure 5.53** Modelling of the coefficient of permeability for the stable-structured soil using van Genuchten's (1980) equation.
CHAPTER 6

APPLICATION OF THE NUMERICAL MODEL TO SIMPLE CASES, LABORATORY TESTS, AND A SMALL EARTH DAM

6.1 Introduction

A finite element model was presented in Chapter 3. To demonstrate the accuracy and applicability of the model, a number of problems were analyzed and the results were compared with available solutions. A computer program, hereafter called COUPSO, was developed based on the theory presented in Chapter 3. COUPSO uses the nine-noded quadrilateral Lagrangian element. A detailed description of the program is presented in Appendix B. The numerical model is first applied to the solution of consolidation problems on saturated soils. The results are compared with analytical and numerical solutions from the literature. Second, the ability of the model to handle soils that undergo collapsing behavior is demonstrated using results of laboratory tests presented by Maswoswe (1985). Third, the proposed numerical model is applied to the analysis of the mechanical behavior of a small earth dam.

6.2 Consolidation of saturated soil

The numerical model is first applied to the one-dimensional consolidation of a homogeneous soil layer as illustrated in Figure 6.1. A load, q, equal to 1000 kN/m² is applied to a single layer of soil which is 1.0 m thick and rests on top of a rigid, impervious base. The soil surface is a free draining boundary and the load is applied on an area of large dimension as compared to the thickness, H. Figure 6.1 also shows the soil parameters (i.e., coefficient of consolidation, c, coefficient of compressibility, m, and coefficient of permeability, k).
Figure 6.1 Consolidation problem of a homogeneous and saturated soil-layer.

Table 6.1 presents the excess heads at mid-height and at the bottom of the layer as determined from an analytical solution (i.e., Terzaghi’s solution according to Lambe and Whitman, 1979), by the finite element program SEEP/W (Geo-Slope, 1991), and by the finite element computer program, COUPSO.

Table 6.1 Comparison of consolidation results of a homogeneous and saturated soil-layer.

<table>
<thead>
<tr>
<th>time (hours)</th>
<th>Head at mid-height (meters)</th>
<th>Head at bottom (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Terzaghi</td>
<td>SEEP/W</td>
</tr>
<tr>
<td>100</td>
<td>88</td>
<td>89.17</td>
</tr>
<tr>
<td>200</td>
<td>74</td>
<td>76.81</td>
</tr>
<tr>
<td>300</td>
<td>64</td>
<td>66.37</td>
</tr>
<tr>
<td>400</td>
<td>55</td>
<td>57.94</td>
</tr>
<tr>
<td>600</td>
<td>42</td>
<td>45.76</td>
</tr>
<tr>
<td>800</td>
<td>33</td>
<td>36.47</td>
</tr>
<tr>
<td>1000</td>
<td>26</td>
<td>29.18</td>
</tr>
<tr>
<td>1200</td>
<td>20</td>
<td>23.37</td>
</tr>
<tr>
<td>1400</td>
<td>15</td>
<td>18.73</td>
</tr>
</tbody>
</table>
Figure 6.2 shows the results of excess pore-water pressure dissipation versus time as determined by SEEP/W and COUPSO. Figures 6.1 and 6.2 show that the finite element model developed in this study closely approximates the available solutions for this problem.

A second application of the numerical model is related to the one-dimensional consolidation of contiguous layers. The problem analyzed was that of two contiguous layers with the same compressibility, the upper one being four times as permeable as the lower one. The results obtained from six analyses and presented for the factor time, \( T \), equal to 0.16 (\( T = c_r \nu / H^2 \)) of the upper layer are shown in Figure 6.3. The solutions used for comparison are:

a.) Schmidt’s (1924) graphical solution;

b.) Luscher’s (1970) analog computer solution;

c.) Harr’s (1967) finite difference solution;

d.) Christian’s and Boehmer’s (1970) finite element solution;

e.) Chang’s (1976) finite element solution.

Figure 6.3 also shows the soil parameters used for the analysis involved in this study. This second application was based on the results presented by Christian and Boehmer (1970) and Chang (1976). The variation of the results can be justified as follows:

i.) Luscher’s (1965) curves were based on an analogue solution. The approximations used in the analysis (i.e., a finite number of analogue elements) led to results which were expected to be too slow;

ii.) Harr’s (1967) explicit finite difference procedure did not consider the compressibility of the material at the interface between the two layers, and it, therefore, led to a faster consolidation;

iii.) Schmidt’s (1924), Christian’s (1970) and Chang’s (1976) solutions included the compressibility of the material at the interface between the two materials and are believed to be accurate. The finite element numerical solution developed during this study is in close agreement with these results. However, it should be emphasized that all the solutions to this problem are approximate.
Figure 6.2 Excess pore-water pressure dissipation on one-dimensional consolidation of a homogeneous single layer.
Figure 6.3 One-dimensional consolidation of two layers with drainage at the most pervious side.
A third application of the numerical model is the prediction of the time versus settlement relationship for the one-dimensional consolidation of a soil profile due to a time-varying load applied to the soil surface. A similar problem was solved by Lambe and Whitman (1979) using an approximate procedure developed by Taylor (1948). Lambe’s and Whitman’s (1979) solution predicted the time-settlement relationship of a clay layer wherein a load, q, increased linearly from 0 to 99 kN/m² during 1 year. Figure 6.4 shows the field conditions and properties of the clay layer of the consolidation problem in analysis.

![Diagram of a stratum of clay](image)

**Figure 6.4** Stratum of clay, loading conditions and properties of the clay layer (after Lambe and Whitman, 1979).

Figure 6.5 shows both the time-varying loading condition and the solutions obtained for the time-settlement relationships of the clay layer for a period of 4 years. The figure also illustrates a good comparison between Lambe’s and Whitman’s (1979) solution and the finite element numerical solution (COUPSO) of the present study. The agreement between these
Figure 6.5 Time versus settlement relationship of a clay layer under a time-varying load.
solutions is anticipated because of the similar procedure of applying the load in an equal number of increments which are defined by a chosen time interval, $\Delta t$. The finite element solution of this study utilized a time interval of 73 days to calculate the results shown in Figure 6.5.

A fourth application of the numerical model was for the analysis of a time versus settlement relationship for a strip footing on a layer of finite thickness. The finite element mesh used for the analysis is shown in Figure 6.6. Both the half width of the footing and the thickness of the clay layer are $a$. The soil was assumed to be linear and elastic, with a shear modulus, $G$, and a Poisson's ratio, $\mu$. The finite element solution is compared to the analytic solution presented by Gibson et al. (1970). Figure 6.7 shows the settlement of the center point of the strip footing plotted against the time factor, $T = c' \sqrt{a^2}$ (as defined by Gibson et al, 1970). The figure shows the close agreement between the finite element numerical solution of this study and Gibson's et al. (1970) closed form solution.

![Figure 6.6 Finite element mesh for a strip footing on a finite layer.](image-url)
Figure 6.7 Surface displacement of a strip footing on a layer of finite thickness.

\[ G \text{ = shear modulus} \]
\[ w(0,t) = \text{displacement at } (x,z)=(0,0) \]
\[ t = \text{time} \]
\[ c' = 2\eta Gk_w/\gamma_w \]
\[ \eta = 1 \]
\[ \mu = 0 \]

\[ \text{Factor time } T = c'/h^2 \]
6.3 Mechanical behavior of unsaturated soils during saturation

The mechanical behavior of a collapsible soil requires stress induced anisotropic constitutive relationships with normal deformations associated with a reduction in matric suction, as previously discussed in section 2.3.3.5. In that section, the constitutive relationships for unsaturated soils, as presented by Fredlund and Rahardjo (1993), were modified to produce an stress induced anisotropic form as follows:

\[
\varepsilon_x = \left(\frac{\sigma_x - u_a}{E}\right) - \frac{\mu}{E} \left(\sigma_y + \sigma_z - 2u_a\right) + \frac{(u_a - u_w)}{H_x} \tag{2.22}
\]

\[
\varepsilon_y = \left(\frac{\sigma_y - u_a}{E}\right) - \frac{\mu}{E} \left(\sigma_x + \sigma_z - 2u_a\right) + \frac{(u_a - u_w)}{H_y} \tag{2.23}
\]

\[
\varepsilon_z = \left(\frac{\sigma_z - u_a}{E}\right) - \frac{\mu}{E} \left(\sigma_x + \sigma_y - 2u_a\right) + \frac{(u_a - u_w)}{H_z} \tag{2.24}
\]

where:

\( H_i = H/(1 + H\chi_i) \) is the elasticity modulus for the soil structure in the i-direction relative to a change in matric suction (i.e., \( u_a - u_w \)),

\( H = \) isotropic elasticity modulus (function of the mean net stress) for the soil structure relative to a change in matric suction,

\( \chi_i = \) anisotropic factor in the i-direction (function of the stress ratios \( \sigma_i/\sigma_j \) and \( \sigma_i/\sigma_k \)) for the soil structure relative to a change in matric suction,

\( i, j, k = \) directions of a three-orthogonal coordinate system (e.g., \( x, y, z \)).

Previous research studies (Maswoswe, 1985; Lawton, et. al, 1991a/1991b; Alonso, 1993) defined the incremental volumetric wetting-induced soil collapse as a sole function of the mean net stress acting on the soil, in response to an incremental change in matric suction. As illustrated in Eq. 2.25, the relationship between the compressibility parameter due to a change in matric suction (i.e., \( m_s^s \)) and the \( H_i \)'s parameters (i.e., \( m_s^s = 1/H_x + 1/H_y + 1/H_z = \)
3/H), as previously defined by Fredlund and Rahardjo (1993) for unsaturated soil, is valid for a collapsible soil. This information suggests that the three-orthogonal anisotropic factors, (i.e., \( \chi_r \), \( \chi_t \), and \( \chi_k \)) are related in a closed form relationship (i.e., \( \chi_i + \chi_t + \chi_k = 0 \)) as presented in Eq. 2.26 in the section 2.3.3.5.

Examples are presented using the computer program COUPSO to simulate the mechanical behavior of an unsaturated soil. In a first example, the computer program COUPSO is used to simulate the behavior of a collapsing soil specimen under Ko-condition when the matric suction is gradually reduced to zero. A second example is used to illustrate the capabilities of the computer program COUPSO to simulate the mechanical behavior of a small earth dam constructed with a stable compacted soil when there is transient unsaturated-saturated water flow through the dam. The analysis of the mechanical behavior of a stable soil compacted dam allows for a better comparison with available information from the literature. Chapter 7 presents the analysis and design of a small collapsing earth dam (i.e., “Alka-Seltzer” dam) during its first reservoir filling.

6.3.1 Collapsing behavior of an unsaturated soil specimen

Maswoswe (1985) reports several suction controlled Ko triaxial tests in low plasticity sand clay (Lower Cromer Till). The soil properties are as follows: \( w_L \) equal to 25%; \( w_p \) equal to 12%; clay content equal to 17%, percentage of sand less than 50%; clay activity equal to 0.71. Specimens were statically compacted dry of optimum but relatively close to optimum water content. A typical stress path involved vertical loading at a constant water content followed by suction reduction (soaking) at a constant vertical stress. These are Ko (i.e., oedometer) type tests performed in a triaxial cell. Radial strains were monitored and the stress state was modified to ensure Ko conditions. As a result, the state of stress in the sample was always known.
Maswoswe (1985) conducted tests on specimens compacted at three different initial void ratios ($e_0 = 0.66$, $e_0 = 0.46$ and $e_0 = 0.33$). Test SK2 is one of the tests that corresponds to low compactive effort ($e_0 = 0.66$). In this test, the specimen was loaded to a vertical stress of 190.4 kPa and then wetted. Table 6.2 illustrates the experimental results from Maswoswe (1985).

Table 6.2 Data results of test SK2 from Maswoswe (1985).

<table>
<thead>
<tr>
<th>Pt.</th>
<th>$(\sigma_s - u_a)$ (kPa)</th>
<th>$(u_s - u_w)$ (kPa)</th>
<th>$(\sigma_h - u_a)$ (kPa)</th>
<th>$e_v$ (%)</th>
<th>Void ratio ($e$)</th>
<th>*$(\sigma_m - u_a)$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0.66</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>190.4</td>
<td>250</td>
<td>50.9</td>
<td>0.0083</td>
<td>0.646</td>
<td>97.4</td>
</tr>
<tr>
<td>C</td>
<td>190.4</td>
<td>220</td>
<td>80.4</td>
<td>0.0278</td>
<td>0.614</td>
<td>117.0</td>
</tr>
<tr>
<td>D</td>
<td>190.4</td>
<td>150</td>
<td>89.6</td>
<td>0.0574</td>
<td>0.565</td>
<td>123.2</td>
</tr>
<tr>
<td>E</td>
<td>190.4</td>
<td>0</td>
<td>110</td>
<td>0.1259</td>
<td>0.451</td>
<td>136.8</td>
</tr>
<tr>
<td>F</td>
<td>220.4</td>
<td>0</td>
<td>120.9</td>
<td>0.1287</td>
<td>0.446</td>
<td>154.0</td>
</tr>
</tbody>
</table>

* $(\sigma_m - u_a) = [ (\sigma_s - u_a) + 2(\sigma_h - u_a)]/3$

The soil parameters are required for the COUPSO computer program. The parameters were defined using the following procedures:

a.) State surfaces for void ratio ($e$) and degree of saturation ($S$).

The available data in Table 6.2 were used to define a plane for the void ratio ($e$) versus the mean net stress, $\sigma_m$, and matric suction, $u_s - u_w$, relationship for the collapsing soil. This plane was adjusted for the stress states corresponding to the initial point (i.e., B) and the final point (i.e., E) of the wetting-induced collapse. The void ratio state surface was established using the following equation:

$$e = a_0 + a_1 (\sigma_m - u_a) + a_2 (\sigma_m - u_a)(u_s - u_w)$$  \[6.1\]

where:
\[ a_0 = 0.66 \]
\[ a_1 = -1.528 \times 10^{-3} \]
\[ a_2 = 5.70 \times 10^{-6}. \]

For the degree of saturation state surface, the relationship from Alonso (1993), based on Maswoswe’s (1985) measurements, was used. In this relationship, Alonso (1993) comments that the influence of applied stresses was small when compared with the effect of suction. The following equation was established as state surface for the degree of saturation:

\[ S = 1 - m \tanh[n(\omega_a - \omega_w)] \]  \[\text{[6.2]}\]

where:

\[ m = 0.64 \text{ and } n = 5.38 \times 10^{-3} \text{ kPa}^{-1}. \]

b.) Poisson ratio (\( \mu \)), coefficient of permeability (\( k_w \)) and factors \( \chi_i \).

Since the horizontal stress increases when the soil collapse under \( K_0 \)-conditions and under a constant vertical stress, the following linear relationship between the Poisson ratio (\( \mu \)) and the matric suction was established based on the initial and final stress states (i.e., points B and E) of the wetting-induced collapse:

\[ \mu = \mu_f + d_o \ast (\omega_a - \omega_w) \]  \[\text{[6.3]}\]

where:

\[ \mu_f = 0.366 \]
\[ d_o = (\mu_o - \mu_f)/(\omega_a - \omega_w)_{\text{max}} \]
\[ \mu_o = 0.21 \]
\[ (\omega_a - \omega_w)_{\text{max}} = 250 \text{ kPa}. \]
The coefficient of permeability (i.e., \( k_w \)) was considered constant and equal to 1.36x10^{-8} \text{ m/s}. The stress-induced anisotropic factors \( \chi_i \) were defined in a trial and error process, using the computer program COUPSO, in such a way that the wetting-induced collapse (i.e., from point B to point E) was exactly reproduced. In this process, equal values were used for the horizontal-direction factors (i.e., \( \chi_x \) and \( \chi_z \)), simulating the confining oedometric conditions. The closed-form relationship \( \chi_v = - (\chi_x + \chi_z) \) (i.e., Eq. 2.26) defined the anisotropic factor in the vertical direction. The trial and error process defined horizontal-direction factors (i.e., \( \chi_x \) and \( \chi_z \)) equal to -1.28.

Figures 6.8 and 6.9 present a comparison between the results obtained using COUPSO and the measured values obtained by Maswoswe (1985). Figure 6.8 shows the capability of COUPSO to reproduce a wetting-induced soil collapse where the vertical stress is kept constant. The intermediate values (i.e., points C and D) were calculated by keeping the previously defined soil parameters and changing the boundary conditions to reproduce the partial wetting-collapse. Figure 6.8 shows that despite the assumptions involved, especially a constant value for the factors \( \chi_i \) and that Maswoswe’s (1985) test is better simulated by an axi-symmetric analysis, the numerical and experimental results are in good agreement. Figure 6.9 illustrates an increase in the mean net stress, as a result of an increase in the net horizontal stress during soil collapse as measured by Maswoswe (1985). These results show the importance of the factors \( \chi_i \) to the constitutive relationships for a collapsing soil. These factors are essential to a numerical model’s ability to reproduce the stress path of a wetting-induced soil collapse.

### 6.3.2 Mechanical behavior of a small and stable dam

To further evaluate the proposed model (i.e., COUPSO), conditions typical of a small earth dam in Northeast Brazil were analyzed. In this region of Brazil, small dams constructed with residual soil of gneiss compacted at optimum conditions of the standard AASHTO compaction energy withstand the first reservoir filling with little or no cracking
Figure 6.8  Void ratio versus net vertical stress relationship during wetting-induced soil collapse under Ko-conditions.
Figure 6.9  Void ratio versus mean net stress relationship during wetting-induced soil collapse under Ko-conditions.
(Miranda, 1983; Pereira, 1986; Miranda, 1988). The properties of the residual soil of gneiss, when compacted at optimum conditions, have been described in section 4.2.2. Section 5.5 has described the modelling of the soil behavior compacted at those conditions. The proposed numerical model is herein utilized to analyze the post filling mechanical behavior of a small dam constructed with a stable soil when transient unsaturated-saturated water flow takes place. The stability of the dam, evaluated by using local safety factors, is utilized to evaluate the proposed model.

6.3.2.1 Description of the small dam

Figure 6.10 shows a section of a small dam (i.e., h < 10.00 meters) typically constructed in northeast Brazil. Such dams are constructed as homogeneous embankments, often without internal drainage.

![Figure 6.10 Typical cross section of a small dam in Northeast Brazil.](image)

6.3.2.2 Analysis procedure

The theory described in Chapter 3 was used to analyze the post-filling mechanical behavior of the small dam. The initial conditions for the post filling phase of the homogeneous embankment were defined as follows:
a.) The net normal stress distribution corresponding to the end-of-construction phase and first impounding of the reservoir, was calculated using the computer program CONSAT (Pereira, 1986) which was developed for plane-strain analysis and by using an eight-noded, isoparametric, quadrilateral finite element. More details of the construction and first impounding phases are given later in sections 6.3.3.4 and 6.3.3.5.

b.) The initial matric suction of the compacted soil in the cross section was assumed to be equal to 30 kPa. This initial matric suction corresponds to the value measured on as- compacted soil specimens and previously presented in Fig. 4.13. The air phase was assumed to be at a constant atmospheric pressure, as previously discussed in Chapter 3. This last assumption implies that the initial negative pore-water pressure has a magnitude equal to the assumed matric suction.

The concept of state surfaces from Matyas and Radhakrisna (1968) is used to define the compressibility parameters for the compacted soil as previously described in sections 5.4 and 5.5. The shear strength behavior of the compacted soil was based on the general predictive model proposed by Fredlund et al. (1995) which utilizes the saturated shear strength parameters and the soil-water characteristic curve to predict the shear strength of the unsaturated soil. The coefficient of permeability of the soil was defined using the equation proposed by van Genuchten (1980). More details of the soil properties are given later in Section 6.3.2.3.

The post-filling performance of the small dam was simulated as a transient process with water flowing through the homogeneous embankment with respect to time. For each time step the displacements, water pore-pressures, and stresses in the dam were evaluated. This allowed the analysis of both the mechanical and hydraulic behavior of the embankment. Comparison of the water pore-pressures with respect to the minor principal total stress at any given point in the dam allows an evaluation of the risk of hydraulic failure in that point. In order to evaluate the stability of the dam, on a preliminary basis, the shear strength mobilization at internal points of the dam were calculated as follows:
$$SMOB = \left( \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f} \right)$$  \hspace{1cm} \text{[6.4]}$$

where:

$SMOB$ = shear strength mobilization

$(\sigma_1 - \sigma_3) = \text{acting deviatoric stress}$

$(\sigma_1 - \sigma_3)_f = \text{deviatoric stress at failure}$.

### 6.3.2.3 Modelling of the soil properties

The modelling of the soil properties was defined based on the laboratory results previously discussed in sections 4.2.2.2 and 5.5. A brief description of these models is as follows:

a.) State surfaces for void ratio ($e$) and degree of saturation ($S$).

Available data from double-oedometric tests (Fig. 5.49) and the assumption of a constant Poisson ratio equal to 0.3 (see section 5.5.1.1.1) allowed the definition of the following void ratio ($e$) versus mean net stress ($\sigma_m$) relationship for the compacted soil:

$$e = a_0 + a_1 (\sigma_m - \mu_o)$$ \hspace{1cm} \text{[6.5]}$$

where:

$$a_0 = 0.432$$

$$a_1 = -3.23 \times 10^{-4}.$$ 

The above equation is valid for both unsaturated and saturated conditions of the compacted soil which is stable when compacted at optimum conditions of the standard AASHTO energy. This implies that the soil compressibility modulus, $m^2_s$, is equal to zero.
Available data of volumetric water content, previously illustrated in Fig. 5.51 (see section 5.5.1.1.2), in conjunction with the use of the initial void ratio of the soil specimens (i.e., $e_o = 0.432$) allows a definition of the degree of saturation state surface:

$$S = S_0 + \frac{(S_s - S_0)}{\left[1 + [\alpha (u_a - u_w)]^{1/n}\right]^m}$$  \[6.6\]

where:

- $S_0 = 0.811$
- $S_s = 1.00$
- $\alpha = 0.045$
- $n = 2.30$
- $m = 1 - 1/n$.

b.) Shear strength behavior

The following, previously defined Eq. 5.18 (see section 5.5.1.2 and Fig. 5.53), accounting for a variable $\varphi^b$, defines the shear strength behavior of the stable compacted soil:

$$\tau_g = c' + (\sigma - u_a) \tan \varphi' + (u_a - u_w) \tan \varphi' +$$

$$\left(\frac{u_a - u_w}{\lambda - 1}\right) \left(\frac{1}{(u_a - u_w)^{\lambda - 1} - (u_a - u_w)^{\lambda - 1}}\right) \tan \varphi'$$  \[5.18\]

where:

- $c' = 33.8$ kPa
- $\varphi' = 32.6$
- $\lambda = .60$
- $(u_a - u_w)_b = 10$ kPa.

c.) Coefficient of permeability ($k_w$)

The coefficient of permeability is defined by the following and previously presented Eq. 5.19 (see section 5.5.2).
\[
 k_w = k_z \left( \frac{1 - [\alpha (u_a - u_w)]^{n-1} [1 + \alpha (u_a - u_w)^n]^{-m}}{[1 + \alpha (u_a - u_w)^n]^{m/2}} \right)
\]  

[5.19]

where:

\[ k_z = 2.5 \times 10^{-9} \text{ m/s} \]
\[ \alpha = 0.045 \]
\[ n = 2.30 \]
\[ m = 1 - 1/n. \]

6.3.2.4 Analysis

A plane strain condition is assumed to reflect the structural conditions of small dams in Northeast Brazil. Figure 6.11 shows the mesh used for the analysis carried out in the present application. The post reservoir filling behavior of the dam was simulated using the computer program COUPSO.

![Finite element mesh](image)

Scales: Geometry 4.40E+00 meters

Figure 6.11 Finite element mesh for the small and stable earth dam.

The construction phase and first reservoir filling phase were simulated using the computer program CONSAT (Pereira, 1986). This computer program models incremental
loading. Therefore, the construction of a dam can be simulated. Both the CONSAT and COUPSO computer programs have the capability of simulating the filling of a reservoir behind an embankment by specifying appropriate pore pressure boundary conditions on the upstream face of the dam.

The analysis traces the stresses, the displacements, and the pore pressures within the dam throughout construction and until steady state flow conditions were established after the first reservoir filling. The following three phases are considered in the analysis: the construction phase, the reservoir filling phase, and the post-filling phase. The last phase consists of the transient unsaturated-saturated seepage until steady state conditions are established. The analyses are performed in such a way that the final stress state conditions of one phase formed the initial stress state conditions for the subsequent phase. However, because the displacements are not considered in a cumulative way, each phase has an initial configuration based on the initial geometry of the small dam. The main objective of this last procedure is to better visualize the mechanical behavior of the small dam in response to the three phases under analysis. This last procedure can also be justified by the small deformations involved in the analysis. This is particularly true during the construction and first reservoir filling phases.

6.3.2.4.1 The construction phase

The construction of the small dam is simulated in four horizontal layers. Changes in water pore-pressure during construction are neglected in the analysis. The embankment is simulated as an elastic medium with a constant Poisson ratio equal to 0.3 and a constant Young modulus of about 5300 kN/m² (from Eq. 6.5). Only the self weight of the compacted soil, (i.e., unit weight equal to 18.44 kN/m³ as previously presented in Fig. 4.2), generates stresses within the dam during this phase.
Figure 6.12 shows the displacements in the embankment dam at the end-of-construction phase. Figure 6.13 presents the directions of both the major \((\sigma_i - u_a)\) and the minor \((\sigma_j - u_a)\) principal net normal stress within the dam. Figures 6.14 and 6.15 present the major \((\sigma_i - u_a)\) and minor \((\sigma_j - u_a)\) principal net normal stress distributions within the dam.

Figure 6.16 illustrates the percentage of shear strength mobilized within the dam at the end-of-construction phase. It shows small values of shear strength mobilized, reflecting values of local safety factors (i.e., the inverse of the shear strength mobilization) higher than 3.0. This illustrates the satisfactory stability conditions of the small dam at the end-of-construction phase. The local safety factors are the result of the high shear strength of the soil compacted at optimum conditions of the standard AASHTO energy. Under these conditions, the soil presents a high angle of internal friction and a high value of cohesion. Cohesion is composed of the saturated cohesion and the contribution from the initial matric suction of the compacted soil. In general terms, the shear strength mobilization distribution reflects the maximum shear stress distribution in the dam.

Figures 6.17 and 6.18 show the intermediate \((\sigma_2 - u_a)\) and the mean \((\sigma_m - u_a)\) net normal stress distribution at the end-of-construction phase, respectively.

![Diagram of displacement field within the dam at the end-of-construction phase.](image-url)
Figure 6.13 \((\sigma_1 - u_0)\) and \((\sigma_3 - u_0)\) directions within the dam at the end-of-construction phase.

Figure 6.14 \((\sigma_1 - u_0)\) (kPa) distribution within the dam at the end-of-construction phase.

Figure 6.15 \((\sigma_3 - u_0)\) (kPa) distribution within the dam at the end-of-construction phase.
Figure 6.16 Percent of shear strength mobilized distribution within the dam at the end-of-construction phase (i.e., SMOB = 100*([σ₁-σ₃]/[σ₁-σ₃]t)).

Figure 6.17 (σ₂ - uₐ) (kPa) distribution within the dam at the end-of-construction phase.

Figure 6.18 (σₘ - uₐ) (kPa) distribution within the dam at the end-of-construction phase.
6.3.2.4.2 The reservoir filling phase

After the dam has been constructed, the reservoir of the small dam is filled to an elevation of 8.0 meters. In the analysis, it was assumed that the water is raised to that elevation in a short period of time, and that neither water flow into the dam nor changes in water pore-pressure occurred within the embankment. The same assumption was made by Miranda (1988). The program, CONSAT, calculated the stress-strain state resulting from the water load acting on the upstream slope of the embankment immediately after the reservoir is filled.

Figure 6.19 shows the displacements in the dam after the reservoir filling. As expected, the water pressure on the upstream slope causes a horizontal displacement in the downstream direction throughout the dam with higher values within the upstream zone and low values within the downstream zone. The vertical displacement pattern is predominantly downward within the upstream zone due to the vertical component of the water loading. The central part of the dam undergoes upward vertical displacements as a combined effect of the horizontal component of the water loading and the resistance offered by the downstream zone to movement in the downstream direction.

Figure 6.19 Displacement field within the dam after the reservoir filling phase.
Figures 6.20 presents the directions of both the major \((\sigma_1 - u_o)\) and the minor \((\sigma_3 - u_o)\) principal net normal stress within the dam, respectively. Figures 6.21 and 6.22 present the major \((\sigma_1 - u_o)\) and minor \((\sigma_3 - u_o)\) magnitudes of principal net normal stress distributions respectively within the dam immediately after the reservoir filling. Figure 6.20 shows the rotation of stresses within the upstream zone towards the dam reservoir in response to the water loading on the upstream slope. Figures 6.21 and 6.22 illustrate the increase in principal stresses within the upstream zone of the dam as compared to the results at the end-of-construction phase (see Figs. 6.14 and 6.15).

Figure 6.23 shows the shear strength mobilized within the dam after the reservoir filling phase. This figure shows that the shear strength mobilization reflects satisfactory stability of the dam. There is a slight change in the shear strength mobilized within the upstream zone of the dam as a result of the combined and opposite effects caused by the vertical and horizontal components of the applied water loading. The vertical component acts to increase the major principal stress, while the horizontal component acts in the opposite way to increase the minor principal stress. The low values of shear strength mobilized can still be justified by the high values of shear strength of the compacted soil in the dam.

Figures 6.24 and 6.25 show, respectively, the intermediate net principal stress \((\sigma_2 - u_o)\) and the mean net normal stress \((\sigma_m - u_o)\) distribution within the dam immediately after the reservoir filling phase.

Figure 6.20 \((\sigma_1 - u_o)\) and \((\sigma_3 - u_o)\) directions within the dam after the reservoir filling phase.
Figure 6.21 \((\sigma_1 - u_a)\) (kPa) distribution within the dam after the reservoir filling phase.

Figure 6.22 \((\sigma_2 - u_a)\) (kPa) distribution within the dam after the reservoir filling phase.

Figure 6.23 Shear strength mobilized (%) distribution within the dam after the reservoir filling phase.
Figure 6.24 \((\sigma_2 - u_a)\) (kPa) distribution within the dam after the reservoir filling phase.

Figure 6.25 \((\sigma_m - u_a)\) (kPa) distribution within the dam after the reservoir filling phase.

6.3.2.4.3 The post-filling phase

The reservoir was assumed to remain at an elevation of 8.0 meters until steady state conditions were established. The coupled process of transient unsaturated-saturated water flow and stress-strain analysis of the small dam was simulated using the computer program COUPSO. Equation 5.19 defined the vertical water coefficient of permeability of the compacted soil in the embankment. According to design procedures in Northeast Brazil (Miranda, 1983; Sherard, 1963), the dam was considered to be anisotropic with a horizontal
water coefficient of permeability equal to 10 times the vertical coefficient of permeability. The transient water flow analysis was performed in accordance with the following time discretization: 11 time steps of 5 days, 11 time steps of 55 days, and 4 time steps of 605 days. Therefore, the analysis required a period of 3025 days until the steady state water flow conditions were reached.

Figures 6.26 to 6.57 show the results obtained from the analysis. Such results illustrate pore pressures, displacements, stresses, and shear strength mobilized within the dam corresponding to periods of 20 days, 40 days, 330 days, and 3025 days.

The results of the coupled transient water flow and stress strain analyses are herein examined to:

a.) analyze the development of deformations within the dam during the transient water flow through the dam;

b.) analyze the development of stresses within the dam during the transient water flow;

c.) analyze the shear strength mobilized and the possibility of hydraulic fracturing within the dam during the transient water flow.

**Pore-water pressures**

Figures 6.26 and 6.27 illustrate pore-water pressure distributions within the dam during the early stages of the transient water flow. Figure 6.26 and Figure 6.27 correspond, respectively, to 20 and 40 days after the first filling of the reservoir. At these stages the water flow saturates only the zone of the dam near the upstream slope. The water flow is impeded by the low coefficient of permeability in the unsaturated portion of the dam which remains essentially at its initial matric suction. These behavior results in high pore-water pressures at the toe of the upstream slope.
Figure 6.28 shows the pore-water pressure distributions within the dam after 330 days of the filling of the dam reservoir. At this stage, the water flow has advanced such that the upstream slope is saturated. The water flow is well established in a smooth pattern throughout the upstream zone of the dam. At this stage, only areas near the downstream slope still retain their initial values of matric suction.

![Diagram of pore pressure distribution after 20 days of reservoir filling.]

Figure 6.26 Pore pressure (kPa) distribution after 20 days of reservoir filling.

![Diagram of pore pressure distribution after 40 days of reservoir filling.]

Figure 6.27 Pore pressure (kPa) distribution after 40 days of reservoir filling.
Figure 6.28  Pore pressure (kPa) distribution after 330 days of reservoir filling.

Figure 6.29 presents the pore-water pressure distribution within the dam after 3025 days of the filling of the dam reservoir. At this stage, the water flow has reached steady state conditions. The water emerges on the downstream slope as illustrated by the seepage face. The extension of the seepage face is predominantly a function of the permeability anisotropy assumed for the dam.

Figure 6.29  Pore pressure (kPa) distribution after 3025 days of reservoir filling.

The pore-water pressure distribution under steady state conditions is consistent with the technical literature (Sherard, 1963; Singh and Sarma, 1976; Freeze and Cherry, 1979).
Deformations

The relatively rigid compacted soil did not allow high deformations to occur within the dam. This is illustrated in Figures 6.30, 6.31, 6.32, and 6.33. Despite this fact, the results can be used to illustrate the general performance of the embankment during transient water flow.

Immediately after the reservoir filling, the water load on the upstream slope created a downstream and downward movement within the upstream zone of the dam (Fig 6.19). With the development of the water flow through the embankment, the displacement pattern gradually changed showing an upward movement within this zone (Figs. 6.30, 6.31 and 6.32).

Figure 6.30 Displacement distribution after 20 days of reservoir filling.

Figure 6.31 Displacement distribution after 40 days of reservoir filling.
Figure 6.32 Displacement distribution after 330 days of reservoir filling.

Figure 6.32 illustrates the general displacement pattern within the dam when the upstream slope is saturated. This figure shows the predominance of the upward component of displacement within the dam. It also shows the upstream tendency of displacement within the embankment zone near the upstream slope. The remainder of the dam moves to the downstream direction. This behavior can be explained by the combined effects of seepage, an increase in the self-weight of the soil, and buoyant uplift forces:

a.) with the development of the water flow through the embankment, the water pressure is gradually distributed as seepage forces spread throughout the saturated zone of the dam. These forces, inducing downstream direction movements on the embankment, become progressively smaller with the spreading of the saturated zone and the corresponding reduction of hydraulic gradients (Figures 6.26 to 6.28).

b.) with the decrease of the seepage forces, the buoyant uplift forces in the saturated zone dominate the movements within the dam (Nobari, 1972; Chang, 1976; Pereira, 1986; Miranda, 1988) and cause upward movement, particularly within the upstream zone;

c.) besides the seepage and buoyant uplift forces, there is an increase in the self-weight of the soil due to saturation. Although small, since the initial degree of saturation of the compacted soil was higher than 90%, this effect is transmitted to the upstream direction.
of movement in the dam causing a "bulging" of the upstream slope. As expected, this effect gradually decreases from the bottom to the top of the dam;

d.) it is also worth noting that the compacted soil was assumed to be a stable soil. Therefore, no collapse occurred when the soil was saturated.

Figure 6.33 illustrates the movement pattern within the dam when steady state conditions were reached 3025 days after the reservoir filling. This general pattern is similar to results presented in the literature (Alberro and Leon, 1971; Chang, 1976; Miranda, 1988).

![Phreatic line]

**Figure 6.33** Displacement distribution after 3025 days of reservoir filling.

**Stresses**

Immediately after the reservoir filling, the water load on the upstream slope increased the net principal stresses within the upstream zone of the dam (Fig 6.20). Figures 6.34 to 6.53 present the principal stress distributions within the dam, in both magnitude and direction, with the development of the water flow through the embankment. These figures present the net principal stresses within the unsaturated zone and the effective stresses within the saturated zone. The fact that the soil is stable implied that the net normal stresses at a soil element within the dam did not change as a result of a decrease of the matric suction in that element. In turn, the effective stress state at any point within the saturated zone obeys the Terzaghi's principle of effective stresses.

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Figure 6.34 Principal stress directions distribution after 20 days of reservoir filling.

Figure 6.35 $\sigma_1$ (kPa) distribution after 20 days of reservoir filling.

Figure 6.36 $\sigma_2$ (kPa) distribution after 20 days of reservoir filling.
Figure 6.37 $\sigma_2$ (kPa) distribution after 20 days of reservoir filling.

Figure 6.38 $\sigma_m$ (kPa) distribution after 20 days of reservoir filling.

Figure 6.39 Principal stress direction distribution after 40 days of reservoir filling.
Figure 6.40 $\sigma_1$ (kPa) distribution after 40 days of reservoir filling.

Figure 6.41 $\sigma_3$ (kPa) distribution after 40 days of reservoir filling.

Figure 6.42 $\sigma_2$ (kPa) distribution after 40 days of reservoir filling.
Figure 6.43 $\sigma_m$ (kPa) distribution after 40 days of reservoir filling.

Figure 6.44 Principal stress direction distribution after 330 days of reservoir filling.

Figure 6.45 $\sigma_f$ (kPa) distribution after 330 days of reservoir filling.
Figure 6.46 $\sigma_3$ (kPa) distribution after 330 days of reservoir filling.

Figure 6.47 $\sigma_2$ (kPa) distribution after 330 days of reservoir filling.

Figure 6.48 $\sigma_m$ (kPa) distribution after 330 days of reservoir filling.
Figure 6.49 Principal stress distribution after 3025 days of reservoir filling.

Figure 6.50 $\sigma_1$ (kPa) distribution after 3025 days of reservoir filling.

Figure 6.51 $\sigma_3$ (kPa) distribution after 3025 days of reservoir filling.
Figures 6.52 \( \sigma_2 \) (kPa) distribution after 3025 days of reservoir filling.

Figures 6.53 \( \sigma_m \) (kPa) distribution after 3025 days of reservoir filling.

Figures 6.44 to 6.48, corresponding to the time step of 330 days after reservoir filling, illustrate the general stress distribution pattern within the dam. These figures show the reduction of the principal effective stresses in the saturated part of the dam in response to the phreatic line advance. As expected, the stress state behavior is closely related to the displacement field within the dam. Therefore, the stress distributions can be explained by the same effects of seepage, increase of soil self-weight and buoyant uplift forces previously used to justify the displacement pattern within the dam.
The decrease in net stresses in the downstream zone are due to the upward strain caused by the buoyant uplift forces. This decrease does not occur in an isotropic manner for all the principal stresses, since the Poisson ratio is not equal to 0.5. The main reduction in stress occurs in the direction of the upward strain. Therefore, depending on the zone of the dam, the main reduction can occur either to the major or to the minor principal stress. Figures 6.34 to 6.53 suggest that the main reduction happens to the major principal stresses since this is the direction which is predominately vertical. This analysis implies a reduction of deviatoric stresses within most parts of the embankment dam (e.g., the central part).

Figures 6.44 and 6.49 illustrate that negative minor principal stresses (i.e., tension) occurred near the upstream slope of the embankment. However, such negative stresses presented values of low magnitudes (i.e., magnitudes less than 10.0 kPa) and appear as isolated points along that slope. At these points, the major principal stress has positive values of magnitudes less than 10 kPa.

**Shear strength mobilized and hydraulic fracturing**

Figures 6.54 to 6.57 present the gradual changes of shear strength mobilization distribution within the dam with the development of the water flow through the embankment. These gradual changes can be explained as the combined action of two opposite effects:

a.) the decrease in shear strength (i.e., $(\sigma_1 - \sigma_3)_{fr}$) of the soil in the dam due to the gradual reduction of its cohesion (i.e., the contribution from matric suction) in conjunction with the reduction of the minor principal stress;

b.) a gradual decrease in the deviatoric stress as the water flow advances through the dam as previously discussed in this section.

According to the results illustrated in Figs. 6.54 to 6.57, the embankment dam has satisfactory stability against failure.
The effective stresses are analyzed to verify the possibility of hydraulic cracking. For the analysis adopted in this study, the criterion was that hydraulic fracturing may occur in a soil element when tensile effective stress in one direction (i.e., the minor effective stress is negative) is higher than the soil cohesion. As previously discussed in Section 6.3.2.4.3, isolated points appeared near the upstream toe of the dam wherein the minor principal effective stresses are negative. The compacted soil of the dam presents a cohesion of about 33.8 kPa which is several times higher than the minimum negative stress that appeared. The conclusion is that there is no risk of hydraulic fracturing in the dam.

Figure 6.54 Shear strength mobilized (%) distribution after 20 days of reservoir filling.

Figure 6.55 Shear strength mobilized (%) distribution after 40 days of reservoir filling.
Figure 6.56 Shear strength mobilized (%) distribution after 330 days of reservoir filling.

Figure 6.57 Shear strength mobilized (%) distribution after 3025 days of reservoir filling.

6.4 Summary

In Chapter 6, the numerical model proposed in this research was applied to solve problems involving both saturated and unsaturated soils. First, the developed computer program COUPSO was applied to the solution of consolidation of saturated soils. Both one-dimensional and two-dimensional problems were solved and the numerical solutions were compared to previous solutions from the literature.
Second, the ability of the program COUPSO to handle the behavior of collapsing soils during saturation was demonstrated. COUPSO is used to model a \( K_0 \) triaxial test presented by Maswaswe (1985). The results (Figs. 6.8 and 6.9) showed the consistency between the measured and calculated values. This analysis also pointed to the importance of the consideration of anisotropic parameters \( H_x, H_y, \) and \( H_z \) (i.e., the elastic modulus of the soil associate to a change in matric suction) to calculate correctly the horizontal stresses.

Finally, the program COUPSO was applied to analyze the post filling behavior of a small dam constructed in Northeast Brazil. In that region, small dams are constructed with residual soil derived from gneiss. It has been observed that small dams built with this soil at the optimum standard AASHTO energy conditions survive the first reservoir filling with no or little cracking.

Specimens of the residual soil, compacted at optimum conditions, were tested in the laboratory to define both the mechanical and the hydraulic properties (see Sections 4.3.2, 5.5 and 6.3.2.3) of the soil. Both the construction and the first impounding phases of the dam were simulated by using the program CONSAT (Pereira, 1986). The stress state at the end of the after reservoir filling phase, as resulted from CONSAT, was used as the initial stress state condition to simulate the post-filling behavior of the dam.

The computer program COUPSO was applied to simulate in a coupled process the transient water flow and stress-strain behavior that follows the first impounding of the dam reservoir. The post-filling behavior was simulated by using the time discretization: 11 time steps of 5 days, 11 time steps of 55 days, and 4 time steps of 605 days. These time steps totaled 3025 days which were necessary for the establishment of water flow steady state conditions.

The water pore pressures and stress-strain states within the dam were examined to:

a.) follow the progress of the transient water flow until steady state conditions were reached;

b.) follow the development of displacements within the embankment;
c.) evaluate the shear strength mobilized and possibility of hydraulic fracturing.

The results for the post-filling behavior (Figures 6.26 to 6.57) illustrated the capability of the program COUPSO to analyze the mechanical behavior of a small earth dam with the development of the transient water flow. The analysis concluded that the small earth dam has a satisfactory stability against both shear failure and hydraulic fracturing.

In the analysis, the soil compacted at optimum standard AASHTO conditions were assumed to be stable according to the laboratory test results. Therefore, the unsaturated soil has a compressibility modulus related to matric suction (i.e., $m_2^s$) equal to zero during the analysis. This implies an infinite value for the elastic modulus $H$. In order to avoid numerical difficulties a relatively low value of about $10^{-14}$ kPa is assumed to $H$ in the analyses here performed. For the saturated condition, the compressibility modulus $m_2^s$ was equal to the compressibility parameter $m_v$ of the saturated soil. In Chapter 7 the program COUPSO is utilized to analyze the mechanical behavior of the post-filling phase of a small earth dam constructed with a collapsing soil.
CHAPTER 7

POST-FILLING PERFORMANCE OF AN ALKA-SELTZER DAM

7.1 Introduction

In this Chapter the computer program COUPSO is utilized to analyze the post-filling performance of a small collapsing dam similar to those constructed in Northeast Brazil. In this region of Brazil, failure of small dams constructed with residual soil of gneiss, compacted at a low dry density and drier than the optimum water content conditions, of the standard AASHTO compaction energy, are common (DAER, 1983; Pereira, 1985; Miranda 1988). Failure of such dams occurs after the first reservoir filling is completed and before the establishment of a steady-state flow condition (DAER, 1983). In opposition, as previously discussed in section 6.3.2, small dams constructed with the residual soil of gneiss compacted at optimum conditions of the standard AASHTO compaction energy generally survive their first reservoir filling without stability risks. The prediction of the post-filling performance of a small collapsing dam (i.e., an “Alka-Seltzer” dam), is used to further evaluate the numerical model, COUPSO.

7.2 Description of the problem

Figure 6.10 (see Section 6.3.2) illustrates a typical section of a small earth dam (i.e., with \( h \leq 10 \) meters) constructed in Northeast Brazil. Such dams are constructed as homogeneous embankments, often without internal drainage. In 1983, the State of Ceara’, the Department of Roads created a task force of five civil engineers to study the consequence of frequent small dam failures on the safety of the state roads. Such dams were reported to fail
in a short time after their first reservoir filling. The commission’s report (DAER, 1983) stated:

a.) “during the drought of 1979 through 1983, about 20,000 small dams were built, enlarged, or rehabilitated in the State of Ceara’ by the Emergency program”;

b.) “the commission examined 720 dams and concluded that about 80 percent were going to fail in the next rainy season”;

c.) about the quality of the construction: “The compaction of the material is, in general, very deficient, almost always without use of water”;  

d.) explaining why the dams were constructed with a deficiency of water: “The difficulty to provide water to satisfy the most elementary necessities of the people did not permit the use of such a precious liquid in the construction of dams”.

These dams are constructed with residual soil derived from gneiss, which is a silty sand with low plasticity as previously described in Section 4.2. Chapter 4 presented the complete characterization and compaction properties of a sample of soil which was taken from the Borrow Area of the Cauipe Dam in the Municipality of Pacatuba, State of Ceara’ in Northeast Brazil. Soil from the same location was used in the study performed by Miranda (1988).

7.3 Analysis procedure

The computer program COUPSO was used to analyze the post-filling performance of an “Alka-Seltzer” dam. The initial conditions for the post-filling phase of the homogeneous embankment were defined as follows:

a.) The net normal stress distributions corresponding to the end-of-construction phase and first impounding of the reservoir were calculated by using the computer program CONSAT (Pereira, 1986). More details of the construction and first impounding phases are given later in section 7.5.
b.) The initial matric suction of the compacted soil in the entire cross-section was assumed to be equal to 370 kPa. This matric suction corresponds to the value measured on as-compact soil specimens and previously presented in Fig. 4.8 (Section 4.2.2.1). It is further assumed that the air phase is at constant atmospheric pressure conditions, as previously discussed in Chapter 3. This last assumption implies that the initial negative pore-water pressure has a magnitude equal to the assumed matric suction.

The concept of state surfaces from Matyas and Radhakrisna (1968) is used to define the compressibility parameters for the compacted soil as previously described in sections 5.4 and 5.5. The shear strength behavior of the collapsing compacted soil is derived from available data (see Section 5.3.1) by using a best-fitting procedure as previously described in section 5.4.1.2. The coefficient of permeability of the soil is also defined from available data (see Section 5.2.2) by using a best-fitting procedure as previously described in Section 5.4.2. More details of the soil properties are given later in Section 7.4.

The post-filling performance of the small dam is simulated in a transient process where water flows through the homogeneous embankment according to a defined time discretization. In each time step the displacements, the water pore-pressures, and the stresses in the dam are evaluated. The analysis allows both the mechanical and hydraulic behavior of the embankment to be visualized. Comparison of the water pore-pressure with the minor principal total stress in any given point in the dam allows for the evaluation of the risk of a hydraulic failure in that point.

The shear strength mobilized at internal points of the dam is calculated as follows:

\[ SMOB = \left( \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_f)} \right) \]  \hspace{1cm} [6.4]

where:

- \( SMOB \) = shear strength mobilization
- \( (\sigma_1 - \sigma_f) \) = acting deviatoric stress
\[ (\sigma_1 - \sigma_3) = 2*(c* \cos \phi + \sigma_3 * \sin \phi) \] is the deviatoric stress at failure

\[ \phi = \frac{\partial \tau_{ff}}{\partial \sigma} = \tan^{-1}[b_1 + d_1 \cdot (u_a - u_w)] \] is the friction angle (from Eq. 5.12)

\[ c = a_1 + c_1 \cdot (u_a - u_w) \] is the cohesion of the soil (from Eq. 5.12).

The shear strength mobilized (i.e., SMOB) is used as a criterion to assume post-failure behavior at internal points of the dam. Different post-failure behaviors are used to define the collapsing soil conditions in the saturated and unsaturated portions within the dam. A high compressibility is assumed for a soil element which reaches failure within the dam (Dunlop and Duncan, 1970). Such an assumption is consistent with the "perfectly" plastic behavior of the saturated collapsing soil when sheared under low octahedral stresses (see Figures 5.22, 5.23 and 5.24). In turn, it is assumed that an unsaturated soil element at failure conditions keeps its compressibility behavior as a function of the applied stress state (Matyas and Radhakrisna, 1968; Lawton et al. 1991a). Such an assumption allows an unsaturated soil element to increase its shear strength by increasing its mean net normal stress when collapsing (Maswoswe, 1985; Handy, 1995). This assumption reflects a more realistic approach to the "hardening" shear strength behavior of an unsaturated collapsing soil specimen (see Figs. 5.22, 5.23 and 5.24). Besides, such an assumption presents two advantages. The first one is that the concept of void ratio state surfaces remains valid for the unsaturated collapsing soil at failure. The second advantage is that numerical difficulties are avoided by maintaining the continuity of the void ratio state surface. Such an assumption is valid for cases where the soil does not suffer significant changes in its void ratio during wetting-induced collapse (Miranda, 1988; Alonso, 1993).

### 7.4 Modelling of the soil properties

The collapsing soil properties are based on the laboratory results previously discussed in sections 4.2.2.1 and 5.1. A brief description of these models is as follows:
a.) State surfaces for void ratio, \( e \), Poisson ratio, \( \mu \), and degree of saturation, \( S \).

Available data from triaxial permeameter tests allowed the definition of Eq. 5.9 (see Section 5.4.1) as expressing the void ratio, \( e \), versus mean net stress, \( \sigma_m \), relationship for the collapsing compacted soil as follows:

\[
e = e_u + \frac{e_f - e_u}{1 + \left( \frac{(u_o - u_w)}{c} \right)^b}
\]

[5.9]

where:

\[
e_u = 0.7697 - 0.0073 \ln(\sigma_m)
\]

\[
e_f = 1.2264 (\sigma_m)^{-0.1359}
\]

\[
c = c_1(\sigma_m)^2 + c_2(\sigma_m) + c_3
\]

\[
b = b_1(\sigma_m)^{b_2}
\]

\[
c_1 = 9.398 \times 10^{-4}
\]

\[
c_2 = 7.465 \times 10^{-2}
\]

\[
c_3 = -4.066
\]

\[
b_1 = 49.01
\]

\[
b_2 = -6.103 \times 10^{-1}
\]

The above equation is valid for both unsaturated and saturated conditions of the collapsing compacted soil for \( \sigma_m \) higher than 10 kPa. Under saturated conditions, \( \sigma_m \) represents the mean effective stress and the soil behavior is defined using equation of the final void ratio (i.e., \( e_f \)) for \( \sigma_m \) higher than 43 kPa. The equation of the initial void ratio (i.e., \( e_u \)) is utilized for values of \( \sigma_m \) less than 43 kPa (see Fig. 5.35). This implies that the soil is stable for \( \sigma_m \) lesser than 43 kPa (i.e., its compressibility modulus \( m^2 \) is equal to zero).

As previously discussed in section 5.4.1 the Poisson ratio of the collapsing soil is defined by Eq. 5.10 b as follows:
\[ \mu = \mu_u + \frac{\mu_f - \mu_u}{1 + \left( \frac{u_a - u_w}{c} \right)^b} \]  

[5.10b]

where:

\( \mu_u = 0.3 \)
\( \mu_f = 0.0915 \ln (\sigma_m) - 0.0207 \)
\( c = c_1 (\sigma_m)^2 + c_2 (\sigma_m) + c_3 \)
\( b = b_1 (\sigma_m)^2 \)
\( c_1 = 9.398 \times 10^{-4} \)
\( c_2 = 7.465 \times 10^2 \)
\( c_3 = -4.066 \)
\( b_1 = 49.01 \)
\( b_2 = -6.103 \times 10^1 \)

A constant Poisson ratio of 0.3 is assumed for the collapsible soil under mean net/effective stresses (i.e., \( \sigma_m \)) less than 43 kPa.

Available data from triaxial permeameter tests allowed the definition of Eq. 5.11 (see Section 5.4.1) to express the degree of saturation (i.e., \( S \)) versus matric suction (i.e., \( u_a - u_w \)) relationship for the unsaturated collapsing compacted soil.

\[ S = S_0 + \frac{1 - S_0}{1 + \left( \frac{u_a - u_w}{c} \right)^d} \]

[5.11]

where:

\( S_0 = a + b \ln (\sigma_m) \)
\( a = 3.541 \times 10^{-1} \)
\( b = 3.654 \times 10^{-3} \)
\( c = 7.906 \)
\( d = 9.769 \times 10^{-1} \)
The above equation reflects a very steep soil-water characteristic curve for the collapsible soil (See Fig. 5.42).

b.) Shear strength behavior

Available data allowed the definition of the previously presented Equation 5.12 (see section 5.4.1 and Fig. 5.45), illustrating the shear strength behavior versus stress state variables relationship for the collapsing compacted soil.

\[
\tau_{\gamma} = a_i + b_i (\sigma - u_a) + c_i (u_a - u_w) + d_i (\sigma - u_a)(u_a - u_w)^p \\
\]

where:

\begin{align*}
    a_i &= -7.893 \\
    b_i &= 1.944 \times 10^{-1} \\
    c_i &= 3.238 \times 10^{-1} \\
    d_i &= 9.319 \times 10^{-2} \\
    p &= 4.307 \times 10^{-2}
\end{align*}

As previously illustrated in Fig. 5.45, the present research study assumes that at a given net normal stress the shear strength of the collapsing soil remains constant for the range of matric suctions from 100 to 370 kPa. Equation 5.12 predicts negative shear strength under low net stresses and under low matric suction. Therefore, a minimum shear strength of 5 kPa for the collapsing soil is also assumed. This assumption is based on the available data (see Fig. 5.45).

c.) Coefficient of permeability \( (k_w) \)

Equation 5.13 which was based on available data from the triaxial permeameter tests defined the following the coefficient of permeability function (see section 5.4.2 and Fig. 5.46).

\[
k_w = k_p \left( \frac{\psi}{(u_a - u_w)} \right) \lambda \]

where:
\[ k_w \leq k_z \]
\[ k_z = c + d \ln (\sigma_m) \]
\[ k_p = a + b \ln (\sigma_m) \]
\[ a = -1.40 \times 10^{-7} \]
\[ b = 6.26 \times 10^{-8} \]
\[ c = -1.17 \times 10^{-6} \]
\[ d = -1.80 \times 10^{-7} \]
\[ \lambda = 2.90. \]

Equation 5.13 is a very steep water coefficient permeability function (see Fig. 5.47) and its use causes convergence difficulties for numerical models.

d.) Stress induced anisotropic collapse factors \( \chi_i \).

The anisotropic factors \( \chi_i \)'s (i.e., \( \chi_x \), \( \chi_y \), and \( \chi_z \)) were defined in a trial and error process using the computer program COUPSO and the available double-oedometer tests (see Fig. 4.5 in Section 4.2.2.1). This trial and error procedure consisted of the reproduction of the wetting-induced soil specimen collapse under Ko-conditions and under an applied vertical stress of 200 kPa. In this process, equal values were assumed for the horizontal-direction factors (i.e., \( \chi_x \) and \( \chi_y \)), simulating the confining oedometric conditions. The closed-form relationship \( \chi_y \) equal to minus \( \chi_x + \chi_z \) (i.e., Eq. 2.26) defined the anisotropic factor in the vertical direction. A value of approximately minus 2.60 was calculated for the anisotropic factors \( \chi_x \) and \( \chi_z \) from the above trial and error procedure.

Figure 7.1 illustrates the simulation of the wetting-induced soil collapse by using the above defined anisotropic factors. It shows the wetting-induced soil collapse of specimens under vertical loads of 100 and 200 kPa and under Ko-loading conditions. These results demonstrate the generality of the anisotropic collapse factors in simulating the collapsing soil behavior under different vertical loads. The present research study assumes the above defined anisotropic factors for the analysis of the small collapsing dam. The assumption of plane strain conditions for the small dam is justified in a first approximation for the use of the anisotropic collapse factor \( \chi_x \) for the direction normal to the cross section being analyzed.
Figure 7.1 Void ratio versus mean total stress relationships for the collapsing soil during saturation.
(i.e., z-direction). The assumption of the same anisotropic parameter for the other horizontal direction (i.e., x-direction) is conservative since a reduced value would decrease the wetting-induced soil collapse within the small dam.

Table 7.1 shows results from the trial and error procedure utilized to define anisotropic parameters simulating the wetting-induced collapse of the soil specimen (i.e., reduction in matric suction from 370 to 0 kPa) illustrated in Figure 7.1. The intermediate values (i.e., points B, C, D and E) were calculated by keeping the defined soil parameters and changing the boundary conditions to reproduce the partial wetting-collapse.

Table 7.1 Data results of double oedometer tests on collapsing soil specimens.

<table>
<thead>
<tr>
<th>Point</th>
<th>( (u_{z}-u_{w}) ) (kPa)</th>
<th>Void ratio ( (e) )</th>
<th>( \sigma_{h} ) (kPa)</th>
<th>( \sigma_{v} ) (kPa)</th>
<th>( \sigma_{m} ) (kPa)</th>
<th>( \sigma_{m} ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>( \sigma_{v} = 100 )</td>
<td>( \sigma_{v} = 200 )</td>
<td>( \sigma_{v} = 100 )</td>
<td>( \sigma_{v} = 200 )</td>
<td>( \sigma_{v} = 100 )</td>
<td>( \sigma_{v} = 200 )</td>
</tr>
<tr>
<td>A</td>
<td>370</td>
<td>0.735</td>
<td>0.730</td>
<td>42.8</td>
<td>85.7</td>
<td>62.0</td>
</tr>
<tr>
<td>B</td>
<td>200</td>
<td>0.735</td>
<td>0.730</td>
<td>42.8</td>
<td>85.7</td>
<td>62.0</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>0.735</td>
<td>0.727</td>
<td>42.8</td>
<td>99.0</td>
<td>62.0</td>
</tr>
<tr>
<td>D</td>
<td>50</td>
<td>0.735</td>
<td>0.705</td>
<td>42.8</td>
<td>123.5</td>
<td>62.0</td>
</tr>
<tr>
<td>E</td>
<td>10</td>
<td>0.733</td>
<td>0.610</td>
<td>47.5</td>
<td>170.0</td>
<td>65.0</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0.678</td>
<td>0.604</td>
<td>64.2</td>
<td>174.0</td>
<td>76.0</td>
</tr>
</tbody>
</table>

* stresses in kPa
** Initial stress \( \sigma_{h} \) is calculated using a Poisson ratio equal to 0.3.

The numerical analyses were performed by discretizing the soil specimen, which has a height equal to 0.02 meters, using a mesh of 5 equal quadrilateral finite elements with heights of 0.004 meters and bases of 0.03 meters. The analyses required time steps varying from 10 to 0.5 seconds. The larger time steps were used for cases where the applied boundary conditions kept the soil specimen with stress paths essentially in the pre-collapse phase (see Section 5.4.1.1). The smaller time steps were required when the soil specimen approached
complete saturation. In this wetting-induced collapse test it is worth noting the monotonic stress path followed by the collapsing soil specimen. For this loading condition the concept of stress state surfaces is fulfilled and reasonable predictions of changes in void ratio and degree of saturation can be expected.

7.5 Analysis of an “Alka-Seltzer” dam

A plane strain condition is assumed to reflect the structural conditions of small dams in Northeast Brazil. Figure 7.2 shows the finite element discretization used for the analysis carried out in the present application. The post-reservoir filling behavior of the dam was simulated using the computer program COUPSO. The construction phase and first reservoir-filling phase were simulated using the computer program CONSAT (Pereira, 1986). The analysis traced the stresses, the displacements, and the pore pressures within the dam throughout construction, reservoir-filling, and transient seepage until a characterization of the dam failure had been established.

![Finite element mesh for a small collapsing earth dam.](image)

Figure 7.2 Finite element mesh for a small collapsing earth dam.
The following three phases were considered in the analysis: the construction phase, the reservoir filling-phase, and the post-filling phase. The last phase consisted of the transient unsaturated-saturated seepage through the dam after the first reservoir filling. The analyses were performed in such a way that the final stress state conditions of one phase formed the initial stress state conditions for the subsequent phase. However, the displacements are not considered in a cumulative manner and each phase had an initial configuration based on the initial geometry of the small dam. Such a procedure helps one to better visualize the mechanical behavior of the small collapsing dam for each of the three phases under analysis (see Section 6.3). This last procedure can also be justified by the small deformations involved in the analysis. This is particularly true during the construction and first reservoir-filling phases. Emphasis is given to the transient seepage phase since the construction and first reservoir-filling phases are similar to the small and stable dam previously presented in Section 6.3.2. Sections 7.5.1 and 7.5.2 are useful to illustrate that an “Alka-Seltzer” dam presents a satisfactory structural stability for the construction and first reservoir filling phases.

7.5.1 The construction phase

The construction of the small collapsing dam was simulated in five horizontal layers. Changes in water pore-pressure during construction were neglected in the analysis. The embankment was simulated as an elastic medium with a constant Poisson ratio equal to 0.3 and a constant Young modulus of about 5800 kN/m$^2$ (from Eq. 5.9). Only the self weight of the compacted soil, (i.e., unit weight equal to 14.75 kN/m$^3$), generates stresses within the dam during this phase. Figure 7.3 shows the displacements within the dam at the end-of-construction phase. Figure 7.4 presents the directions of both the major $(\sigma_1 - u_o)$ and the minor $(\sigma_3 - u_o)$ principal net normal stresses within the dam. Figures 7.5 and 7.6 present the major $(\sigma_1 - u_o)$ and minor $(\sigma_3 - u_o)$ principal net normal stress distributions within the dam.
Figure 7.3 Displacement field within the dam at the end-of-construction phase.

Figure 7.4 $(\sigma_1 - u_a)$ and $(\sigma_3 - u_a)$ directions within the dam at the end-of-construction phase.

Figure 7.5 $(\sigma_1 - u_a)$ (kPa) distribution within the dam at the end-of-construction phase.
Figure 7.6 $(\sigma_3 - u_e)$ (kPa) distribution within the dam at the end-of-construction phase.

Figure 7.7 illustrates the shear strength mobilized within the dam at the end-of-construction phase. It shows values of maximum shear strength mobilized which reflect the local safety factors (i.e., the inverse of the shear strength mobilized) higher than 1.5. As expected, high values of shear strength mobilized are concentrated in the low central part of the dam reflecting the maximum shear stress distribution within the dam.

Figure 7.7 Percent of shear strength mobilized distribution within the dam at the end-of-construction phase.
Figure 7.7 illustrates a satisfactory structural stability of the small collapsing dam at the end-of-construction phase. Despite the deficiency in compaction conditions, the collapsing soil presents a considerable shear strength at unsaturated conditions. The local safety factors reflect the effect of the matric suction on the structural stability of the collapsing dam. Under unsaturated conditions the collapsing soil presents a low friction angle and a high value of cohesion (see Fig. 5.45). Cohesion is composed of a small value of effective cohesion and a high contribution from the initial matric suction of the soil.

Figures 7.8 and 7.9 show the intermediate ($\sigma_2 - u_a$) and the mean ($\sigma_m - u_a$) net normal stress distribution at the end-of-construction phase respectively. Figure 7.9 illustrates values of mean net normal stress lower than those obtained for the stable dam in section 6.3.2.4 (see Fig. 6.18). Such results reflect the low density of the soil compacted at dry of optimum conditions.

![Figure 7.8](image_url)  
*Figure 7.8 ($\sigma_2 - u_a$) (kPa) distribution within the dam at the end-of-construction phase.*

![Figure 7.9](image_url)  
*Figure 7.9 ($\sigma_m - u_a$) (kPa) distribution within the dam at the end-of-construction phase.*
7.5.2 The reservoir filling phase

After the dam has been constructed, its reservoir is filled to an elevation of 8.0 meters. In the analysis, it is assumed that the water is raised to the full supply level in a short period of time. Neither water flow into the dam nor changes in water pore-pressure are allowed to occur within the embankment. The same assumption was made by Miranda (1988). The program, CONSAT, calculated the stress-strain state resulting from the water load acting on the upstream slope of the embankment immediately after the reservoir is filled.

Figure 7.10 shows the displacements in the dam after the reservoir filling. Similar to the stable dam (see Figure 6.19), the water pressure on the upstream slope produces a downstream direction displacement pattern within the dam. Higher displacements occur within the upstream zone and lower values within the downstream zone as a result of the spreading of the water pressure to the foundation and the downstream slope of the dam.

![Displacement field within the dam after the reservoir filling phase.](image)

Figure 7.10 Displacement field within the dam after the reservoir filling phase.

Figure 7.10 illustrates that the vertical displacement pattern is predominantly downward within the upstream zone due to the vertical component of the water loading. The central part of the dam undergoes upward vertical displacements as a combined effect of the
horizontal component of the water loading and the resistance offered by the downstream zone to movement in the downstream direction.

Figures 7.11 presents the directions of both the major ($\sigma_1 - u_0$) and the minor ($\sigma_3 - u_0$) principal net normal stress within the dam respectively. Figure 7.11 shows the rotation of stresses within the upstream zone towards the dam reservoir in response to the water loading on the upstream slope. Figures 7.12 and 7.13 present the major ($\sigma_1 - u_0$) and minor ($\sigma_3 - u_0$) magnitudes of principal net normal stress distributions respectively within the dam immediately after the reservoir filling. It is also illustrated that the major and minor principal stresses are practically coincident with the vertical and horizontal stresses, respectively, at any point within the lower half of the central part and the upstream slope of the dam.

![Figure 7.11](image.png)

**SCALES:**
- Stresses: $1.294 \times 10^2$ kPa
- Geometry: $4.40 \times 10^6$ meters

**CONVENTION:**
- Compression
- Traction

Figure 7.11 ($\sigma_1 - u_0$) and ($\sigma_3 - u_0$) directions within the dam after the reservoir filling phase.

Figures 7.12 and 7.13 illustrate the increase in principal stresses within the upstream zone of the dam as compared to the results at the end-of-construction phase (see Figures 7.5 and 7.6).

Figure 7.14 shows the shear strength mobilized within the dam after the reservoir filling phase. This figure shows that the shear strength mobilized reflects satisfactory stability of the dam. There is a slight change in the shear strength mobilized within the upstream zone.
of the dam as a result of the combined and opposite effects caused by the vertical and horizontal components of the applied water loading. The vertical component acts to increase

\[ \sigma_1 - u_\alpha \] (kPa) distribution within the dam after the reservoir filling phase.

\[ \sigma_2 - u_\alpha \] (kPa) distribution within the dam after the reservoir filling phase.

Percent of shear strength mobilized distribution within the dam after the reservoir filling.
the major principal stress, while the horizontal component acts to increase the minor principal stress. The vertical component of the applied water loading is almost completely transmitted to the foundation of the upstream slope. In turn, the horizontal component is transmitted into the downstream direction within the dam and resisted by the downstream slope. This justifies the increase in minor principal stress within the lower central part of the dam and consequently the reduction in shear strength mobilized values. The satisfactory values of shear strength mobilized are still results of the high cohesion of the unsaturated as-compact soil within the dam.

Figures 7.15 and 7.16 show, respectively, the intermediate net principal stress (i.e., $\sigma_2 - u_a$) and the mean net normal stress (i.e., $\sigma_m - u_a$) distribution within the dam immediately after the reservoir-filling phase. The small values of mean net stress (i.e., values less than 70 kPa) within the dam reflect the combined effect of the low dry density and low Poisson ratio of the collapsing soil. These values also suggest that upon saturation, little settlement (i.e., soil collapse) must occur at the outer shell of the small earth dam (see Fig. 5.38). Higher collapse deformations should be expected at the central part of the dam due to the higher values of mean net stress there existent. Increase in self weight of the collapsing soil due to saturation will contribute to further collapse deformations within the dam.

![Diagram](image.png)

Figure 7.15 $(\sigma_2 - u_a)$ (kPa) distribution within the dam after the reservoir filling phase.
Figure 7.16 \((\sigma_m - u_p)\) (kPa) distribution within the dam after the reservoir filling phase.

### 7.5.3 The post-filling phase

The reservoir was assumed to remain at an elevation of 8.0 meters during the transient-water-flow analysis. The coupled process of transient unsaturated-saturated water flow and stress-strain analysis of the small dam was simulated using the computer program COUPSO. The soil models from Section 7.4 were used to simulate the behavior of the collapsing soil in the dam. According to design procedures in Northeast Brazil (Sherard, 1963; DAER, 1983), the small dam was considered to be anisotropic with a horizontal coefficient of permeability equal to 10 times the vertical coefficient of permeability.

#### 7.5.3.1 The numerical performance of the constitutive soil models

Numerical difficulties (i.e., non-convergence) arose during attempts to simulate the post-filling behavior of an “Alka-Seltzer” dam while using the soil properties models as defined in Section 7.4. Despite successive refinements in both spatial and temporal discretizations, a redefinition of those models was required. Analysis of the initial mean net normal stress distribution in conjunction with preliminary numerical results suggested the following guidelines for modifications of the soil properties models.
a.) The initial mean net stress distribution (i.e., the mean net stress after reservoir filling) presents values in the range of 20 to 60 kPa in the upstream zone of the small dam. Such values experience a reduction during early stages of seepage, as discussed later in this section. The void ratio state surface, as described by Eq. 5.9, produces a discontinuity in its first derivative at the mean net stress of about 43 kPa (see Fig. 7.1). Such discontinuities are produced at the intersection of the different curves utilized to simulate the “unsaturated” (i.e., \( e_u \)) and “saturated” (i.e., \( e_s \)) stress paths for the collapsing soil. In addition, Eq. 5.13 produces very steep void ratio versus matric suction relationships for mean net stresses between 40 and 70 kPa as the soil approaches saturation (see Figure 5.38). This situation results in convergence difficulties and oscillations (especially in terms of wetting-induced soil collapse) for a numerical model.

b.) Equation 5.11 generates a steep degree of saturation versus matric suction relationships. The steepness is independent of the existing mean net normal stress distribution and occur as the soil approaches saturation (see Fig. 5.42). Preliminary numerical results indicated that smoothing the steepness could minimize the convergence problems occurring in the numerical solution.

c.) The coefficient of permeability versus matric suction relationship also produces a relationship as the soil approaches saturation. Such a function does not experience significant influence of the mean net stress distribution (see Fig. 5.48 and Eq. 5.13). Preliminary numerical analysis indicated that a combined smoothing of both the degree of saturation and the coefficient of permeability functions provided superior performance of the numerical model.

In the present research study, the primary concern is the prediction of the structural stability of the collapsing dam rather than time predictions. Such stability is dependent upon the net normal stress distributions, as well as the pore-pressure distribution, within the dam. The collapsing soil behavior is primarily dependent upon the matric suction which plays a fundamental role on the net normal stress distributions within the dam. In turn, the hydraulic
characteristics of the collapsing soil play a fundamental role on the pore-pressure distribution and are not significantly affected by the net normal stress distributions. A compromise would be to use moderately steep soil models, which would maintain the basic characteristics of the original models, and yet produce reasonable results.

Based on the above guidelines, the modelling of the soil properties (i.e., models for void ratio, Poisson ratio, anisotropic collapse factors, degree of saturation and coefficient of permeability) were modified as follows.

i.) Void ratio model.

A modified void ratio versus stress state variables function (i.e, a slightly modified Equation 5.9) is presented in Equation 7.1 as follows:

\[
e = e_u + \frac{e_f - e_u}{1 + \left(\frac{(u_a - u_w)}{c}\right)^b}
\]

[7.1]

where:

\[
e_u = 0.7697 - 0.0073 \ln(\sigma_m)
\]

\[
e_f = 0.752 - 0.142 / [1 + (\sigma_m/75)^{-3.5}]
\]

\[
c = c_1(\sigma_m)^2 + c_2(\sigma_m) + c_3
\]

\[
b = b_1(\sigma_m)^{b_2}
\]

\[
c_1 = 9.398 \times 10^4
\]

\[
c_2 = 7.465 \times 10^2
\]

\[
c_3 = 11.0
\]

\[
b_1 = 39.0
\]

\[
b_2 = -6.103 \times 10^1
\]

Equation 7.1 is a moderately steep void ratio model as the soil approaches saturation. When compared to Eq. 5.9, it presents advantages such as: a.) elimination of the discontinuity in derivative of the original function by expressing the "saturated" void ratio (i.e., \(e_f\)) by using a logistic type function; b.) smoothing of the void ratio versus matric
suction relationship due to a reduction in the exponential parameter "b", in conjunction with an increase of the parameter "c". Additionally, Equation 7.1 keeps the essential soil collapsing characteristic that the smaller the mean net normal stress, the smaller will be the matric suction wherein the soil starts collapsing. Equation 7.1 was derived for the range of mean net normal stresses from 20 to 150 kPa. Figure 7.17 illustrates a comparison between the modified void ratio function and the available experimental data. Figure 7.17 shows both the "unsaturated" and the "saturated" loading stress path for the void ratio versus mean net normal stress relationships. Figure 7.18 shows the void ratio versus matric suction relationships for the modified void ratio model. Figure 7.19 shows the same results in terms of volumetric deformation versus mean net normal stress relationships.

ii.) Poisson ratio model.

In order to maintain the characteristics previously discussed (see section 5.4.1.1) for the Poisson ratio, Equation 5.10b is updated by using the same parameters "b" and "c" from Eq. 7.1. Such a procedure maintains a proportionality between the Poisson ratio and the wetting-induced soil collapse. A constant Poisson ratio of 0.3 is assumed for the collapsible soil under mean net/effective stresses (i.e., $\sigma_m$) less than 30 kPa.

iii) Degree of saturation and coefficient of permeability models.

Equations 5.11 and 5.13 are steep functions for degree of saturation and coefficient of permeability, respectively. The functions represent the hydraulic properties for the collapsing soil and are intrinsically related in the numerical solution by the water continuity equation (see Section 3.5.3.2). Therefore, such equations have a balanced effect on the performance of a numerical model predicting water flow through a porous medium. Both equations predict smooth increases of the dependent function when the matric suction decreases from 370 to 30 kPa (see Figures 5.42 and 5.47). These equations also ensure a significant increase of the hydraulic properties when the matric suction decreases from 30 to 0 kPa. The sharp change in derivative for a matric suction of 30 kPa represents a point of
Figure 7.17 Modelling of the void ratio versus mean net normal stress relationships for the collapsing soil.

\[ e_u = -0.0073 \ln(\sigma) + 0.7697 \]

\[ e_f = 1.2264\sigma^{-0.1359} \]

\[ e_f = 0.752 - 0.142 / [1 + (\sigma/75)^{-3.5}] \]
Figure 7.18 Modelling of the void ratio versus matric suction relationship for the collapsing soil.
Figure 7.19 Volumetric deformation versus Mean net normal stress relationships.
concern for convergence of a numerical model. Additionally, their steep behavior as the soil approaches saturation requires refinements in both temporal and spatial discretizations.

In the present research study, a reasonable function was defined for the degree of saturation state surface by changing the "c" parameter in equation 5.13 from 7.906 to a value of about 20. In addition, the numerical model performed in a superior manner by increasing the initial degree of saturation (i.e., $S_o$) to a constant value (i.e., independent of $\sigma_m$) of about 0.375. Figure 7.20 illustrates the modification in the degree of saturation state surface.

A reasonable function for the coefficient of permeability was derived by reducing the exponential parameter $\lambda$, in Eq. 5.13, from 2.9 to 2.1. Figure 7.21 illustrates the comparison between the original (i.e., Eq. 5.19) and the modified coefficient of permeability functions.

iv.) Anisotropic factors (i.e., $\chi_i$) model.

The anisotropic factors $\chi_i$'s (i.e., $\chi_x$, $\chi_y$, and $\chi_z$) were redefined by using the new soil models in combination with a trial and error process as previously performed in Section 7.4. This trial and error procedure consisted of the reproduction of the wetting-induced soil specimen collapse under Ko-conditions with an applied vertical stress of 100 kPa. A value of about minus 1.95 was calculated for the anisotropic factors $\chi_x$ and $\chi_z$. The wetting-induced soil collapse behavior at Ko-conditions with a constant vertical stress of 100 kPa is similar to the that previously presented in Figure 7.1.

v.) Summary of the compressibility parameters for the collapsing soil.

The modified void ratio and degree of saturation state surfaces allow for the definition of the compressibility parameters for the collapsing soil (i.e., $m_1^i,m_2^i,m_1^w$ and $m_2^w$). Figures 7.22 to 7.25 illustrate the compressibility parameters versus stress state variables relationships for the collapsing soil in the present research study. These figures reflect the continuity of the state surfaces (i.e., in terms of first derivative), particularly for the range of mean net normal stresses from 20 to 150 kPa. Such a continuity is an essential requirement for minimizing convergence difficulties for a numerical model.
Figure 7.20 Modelling of the degree of saturation versus matric suction relationship for the collapsing soil.
Figure 7.21 Modelling of the water coefficient of permeability versus matric suction relationships for the collapsing soil.
Figure 7.22 Compressibility parameter $m_1^3$ versus mean net normal stress relationship.
Figure 7.23 Compressibility parameter $m^2$ versus matric suction relationship for different net normal stresses.
Figure 7.24 Compressibility parameter $m_1$ versus mean net normal stress relationship.
Figure 7.25 Compressibility parameter $m_2^w$ versus matric suction relationship for different mean net normal stresses.
Figure 7.22 show that a compressibility parameter $m_i^s$ equal to minus $10^{-3}$ kPa$^{-1}$ is reasonable for a saturated soil element that reaches failure conditions. A higher compressibility would also be suitable. However, higher compressibilities added numerical difficulties to the solution since the failure criterion represented a discontinuity in the state surfaces. Nevertheless, the assumed compressibility parameter of $10^{-3}$ kPa$^{-1}$ proved to be satisfactory to reproduce the mechanical behavior of an “Alka-Seltzer” during the transient seepage phase.

7.5.3.2 Analysis of the post-filling behavior of an “Alka-Seltzer” dam

Figure 7.26 illustrates the spatial discretization used in the seepage analysis. The transient seepage analysis was performed in accordance with a time discretization consisting of time steps varying from 0.70 to 0.20 days depending upon the convergence requirements. An initial time step of about 17 days established the initial pore-water pressure conditions within the embankment after imposition of the water pore-pressure boundary conditions at the upstream slope face of the dam (see section 3.5.5). The initial time step is dependent upon the spatial discretization and is necessary for overcoming initial oscillations in the numerical model (Britto and Gunn, 1985). In the present research study, such time steps reduced the matric suction of soil elements near to the upstream face from 370 to values of about 200 kPa. Such reductions did not induce significant changes in the deformation variables within the dam due to the low compressibility parameters of the collapsing soil for that range of matric suctions (see Figures 7.22 to 7.25). The numerical analysis was carried out to simulate a period of about 150 days. This time proved to be sufficient to characterize the mechanical behavior of an “Alka-Seltzer” dam after the first reservoir filling. Despite remaining oscillations, the results are satisfactory to the analysis of the post-filling behavior of an “Alka-Seltzer” dam. Further improvements are beyond the scope of the present study.
7.26 Finite element mesh for the post-filling phase.
Immediately after the reservoir filling, the water load on the upstream slope created a downstream and downward movement within the upstream zone of the dam (Fig 7.10). At the transient seepage phase, the displacement pattern gradually changes as a combination of the following effects.

a.) The seepage forces spreading within the saturated zone of the dam, being transmitted to the unsaturated part of the dam. Such forces induce downstream movements within the dam (see Figs 6.26 to 6.29).

b.) The increase in the soil self-weight due to saturation. The as-compacted collapsing soil has an initially low volumetric water content. Therefore, the soil saturation results in an increase of the soil self weight which induces downward movements within the dam when the soil approaches saturation.

c.) The unsaturated soil undergoes a collapsing behavior for mean net normal stresses higher than 30 kPa (see Figs. 7.17 and 7.19). A given soil element starts collapsing as its matric suction drops to a critical value. Such a critical value is proportional to the mean net normal stress applied on the soil element (see Fig. 7.18). An “Alka-Seltzer” dam presents an initial mean net normal stress distribution within the range of 10 to 70 kPa (i.e., after the reservoir filling phase as illustrated in Fig. 7.16). For this range of mean net normal stresses, it is expected that the soil will collapse when the matric suction decreases to values of about 50 kPa (see Figs. 7.18).

d.) The buoyant uplift pressures acting within the saturated zone of the dam and inducing upward strains within the whole dam embankment (see Fig. 6.26 to 6.29). Such buoyant uplift pressures induces reductions of the mean net normal stresses within the unsaturated zone of the dam. Such reductions in normal stresses minimize the soil collapse behavior as the soil approaches saturation.

e.) A soil element within the dam suffers a substantial reduction of its shear strength in response to a decrease in its matric suction, especially as the soil approaches saturation (see Fig. 5.46). This implies that a soil element within the dam can reach failure conditions
due to saturation. Figures 5.22 to 5.24 has illustrated the high compressibility of a soil specimen which reaches failure conditions. A highly compressible soil element within the dam result in additional settlement in response to increases in the soil element self weight. It also result in additional horizontal deformations in response to horizontal forces (e.g., from relative sliding between elements of different stiffness.).

f.) Load transfer occurs between adjacent elements of different stiffnesses during the transient seepage through the collapsing porous medium. In opposition to a one-dimensional Ko-conditions problem (see Fig. 7.1), such a phenomena is particularly true for the seepage phase of an “Alka-Seltzer” dam wherein the compacted soil has a mechanical behavior that is primarily a function of its matric suction.

The above effects occur simultaneously in a coupled stress and flow analysis. Therefore, a collapsing dam presents a complex behavior during the transient water seepage. The complex behavior requires an analysis by stages (i.e., time intervals) wherein the current stress-strain state conditions of the dam are explained by combining the effects of the water flow advance into the dam and the previously existing stress state.

Figures 7.27 to 7.58 show the results obtained from the analysis. Such results illustrate pore pressures, displacements, stresses, and shear strength mobilization within the dam; corresponding to periods of about 30 days, 55 days, 100, days and 145 days after the reservoir filling of an “Alka-Seltzer” dam. Periods of time less than 30 days did not show relevant changes in deformations within the dam. The following sections present results and discussions for each of the above time intervals hereafter named as stages 1, 2, 3, and 4, respectively.

The results of the coupled transient water flow and stress strain analyses are herein examined to:

a.) analyze the development of stress state variables within the dam during the transient water flow;
b.) analyze the development of deformations within the dam during the transient water flow through the dam;

  c.) analyze the shear strength mobilized and the possibility of hydraulic fracturing within the dam during the transient water flow;

  d.) analyze the overall structural stability of the small collapsing dam;

  e.) elaborate preliminary suggestions for a proposed solution for small collapsing earth dam constructed in Northeast Brazil.

**Stage 1: 30 days after first reservoir filling**

Figure 7.27 shows the pore-water pressure distribution within the dam 30 days after its first reservoir filling. At this stage, the water flow saturates only the outer shell of the upstream slope of the dam. The water flow is impeded by the low coefficient of permeability in the unsaturated portion of the dam which remains essentially at its initial matric suction. A high hydraulic gradient is developed at the downstream side of the phreatic line as a result of the steep slope of the permeability function. As later illustrated, the high hydraulic gradient driving the advancing water front remains for all the stages hereafter analyzed. This hydraulic gradient combined with the assumed soil permeability anisotropy results in a seepage wherein the phreatic line advances parallel to the upstream face of the small dam.

Figure 7.28 shows the displacement pattern within the dam 30 days after its first reservoir filling. Immediately after the first reservoir filling (see Fig. 7.16) the unsaturated soil within the upstream toe of the dam represents a mean net normal stress of about 50 kPa. As the soil in that zone approached saturation the upstream soil zone suffers collapse. The unsaturated soil also undergoes an increase in self weight of about 2.6 kN/m$^3$ (i.e., of about 20% of the initial dry density). The saturation also increases the unsaturated soil compressibility and decreased its shear strength. When combined, these effects induce additional settlement of the outer shell upstream zone of the dam. Such settlement increases
in an accumulative manner from the bottom to the top part of the outer upstream shell. Since the bottom part of the dam is supported by a rigid foundation, the soil settlement results in the bulging of the upstream toe of the dam. In contrast to the settlement pattern, the buoyant uplift pressures induce upwards movements to the upstream and saturated part of the dam. Figure 7.28 indicates that the uplift buoyant pressures predominate within the upstream toe zone. However, the uplifting effects are gradually minimized towards the upper saturated part of the dam due to the cumulative settlement of the outer shell of the dam. The unsaturated part of the dam presents a slight tendency for downstream movements in response to seepage forces applied into the dam.

Figures 7.29 to 7.31 present the principal stress distributions within the dam, in both magnitude and direction, 30 days after first reservoir filling. These figures present the net normal principal stresses within the unsaturated zone and the effective stresses within the saturated zone. In opposition to a stable soil (see Section 6.3.2.4), the net normal stresses on a collapsing soil element can increase in response to a decrease in its matric suction (see Figure 7.1). A soil element which is unsaturated has its mechanical behavior governed by two stress state variables (i.e., net normal stresses and matric suction). In turn, the effective stress state at any point within the saturated zone obeys the Terzaghi's principle of effective stresses.

Figures 7.29 shows that, despite the increase in the soil self weight during saturation, the initially unsaturated upstream zone suffers a substantial reduction in major and minor principal stresses within the saturated zone. Tensile stresses (see Fig. 7.31) of small magnitude spread throughout the saturated zone. Such a behavior demonstrates the predominance of the uplifting water pressures at this stage within the saturated zone. Figure 7.29 also demonstrates that points at the unsaturated zone did suffer little or no changes in their net normal principal directions after 30 days of reservoir filling. This is particularly true for the bottom part of the upstream zone of the dam. Such behavior is a result of the low compressibility of the unsaturated soil at suctions higher than 50 kPa and at mean net normal stresses lower than 70 kPa (see Fig. 7.22). Figure 7.27 illustrated that only a narrow zone at
Figure 7.27 Pore-water pressure distribution 30 days after first reservoir filling.

Figure 7.28 Displacement distribution 30 days after first reservoir filling.
Figure 7.29 Principal stress directions distribution 30 days after first reservoir filling.

Figure 7.30 \( \sigma_1 \) (kPa) distribution 30 days after first reservoir filling.
the downstream side of the phreatic line is under matric suctions less than 60 kPa. Therefore, small changes were expected to occur only at unsaturated soil elements which are close to the phreatic line.

Figure 7.30 shows the major net normal-effective principal stress distribution within the dam 30 days after its first reservoir filling. Figure 7.30 shows a reduction of the major net normal principal stress at the current stage as compared to the initial stress state at the after reservoir-filling phase (see Fig. 7.12). Changes in the major net principal stresses (i.e., \( \sigma_f - u_d \)), which is nearly in the vertical direction, are mainly reductions induced by the buoyant uplift pressure acting in the saturated zone. Such a reduction is transmitted to the minor net principal stress in a minimized manner due to the low Poisson ratio (i.e., of about 0.30) of the unsaturated soil. Figure 7.30 shows the previously discussed decrease in effective stresses within the saturated zone of the dam.

Figure 7.31 shows the minor net normal-effective principal stress distribution within the dam. Reductions in the minor net normal stress distribution are observed in a narrow zone close to the phreatic line. This is particularly true within the bottom upstream zone of the dam. Such reductions are the consequence of the combined effect of the uplifting water pressures and the bulging of the upstream face of the dam as previously discussed in this section. There is no evidence of increasing minor net normal stresses during collapse (i.e., due to the anisotropic factor collapse). Such a result might be attributed to the low level of mean net normal stresses within the collapsing dam. In addition, both the structural condition and the highly compressible saturated upstream toe of the dam would minimize any increase in horizontal stresses due to the soil collapse in that zone. In opposition to the stable dam (see Figures 6.36 and 6.41), the saturated zone has low effective stresses, mostly tensile stresses, as previously discussed in this section.

Figure 7.32 shows the distribution of shear strength mobilized within the collapsing dam at the current stage. Shown is the increase in shear stress mobilized within the upstream zone of the dam, particularly the saturated zone. At this stage, only the upstream toe presents
Figure 7.32 Percent of shear strength mobilized distribution 30 days after first reservoir filling.
shear strength mobilized values of 100 percent. Figure 7.32 also shows the significant effect of the water flow advance in the shear strength mobilized distribution. In contrast to the stable dam (see Figures 6.54 and 6.55), the above results reflect the low shear strength of the collapsing soil under saturated conditions. Minor changes of mobilized shear strength within the unsaturated downstream zone are the result of the redistribution of the major and minor net normal stresses. The unsaturated and upstream zone of the dam presents a sharp decrease in shear strength mobilized from the phreatic line towards the downstream zone of the dam. Such reduction reflects mainly the increase of the shear strength of the collapsing soil due to the contribution of the matric suction, and to a lesser extent, the redistribution of major and minor net principal stresses. Values of shear strength mobilized less than 100 percent are observed within the upper part of the saturated zone. Such values are a combined effect of the assumption of a minimum cohesion of 5 kPa for the collapsing material (see Section 7.4) and the low values of deviatoric stresses within that zone.

Figure 7.33 shows the intermediate net normal-effective principal stress distribution within the dam. Such a distribution presents a general pattern similar to the minor net normal/effective stress distribution within the dam with low effective stresses in the saturated zone. Similar to the minor stress state, increases in intermediate net normal stresses during collapse (i.e., due to the anisotropic factor collapse) are not observed within the saturated upstream toe of the dam.

Figure 7.34 shows the mean net normal-effective principal stress distribution within the dam. This distribution presents a pattern similar to the minor net normal-effective stress distribution within the dam with low effective stresses in the saturated zone. Such a result is mainly a consequence of the minor changes in net normal-effective principal stresses that occurred within the dam as previously discussed. Figure 7.34 suggests that additional soil collapse is expected to occur within the lower half of the upstream zone for subsequent advances of the transient water flow.
Figure 7.33 $\sigma_z$ (kPa) distribution 30 days after first reservoir filling.

Figure 7.34 $\sigma_m$ (kPa) distribution 30 days after first reservoir filling.
Stage 2: 55 days after first reservoir filling

Figure 7.35 shows the pore-water pressure distribution within the dam 55 days after its first reservoir filling. At this stage the water flow has saturated a considerable portion of the upstream part of the dam. The saturated zone presents essentially a hydrostatic pore-water pressure distribution, while both the unsaturated central and the downstream parts of the dam retain their original values of matric suction. The seepage pattern illustrates a high hydraulic gradient at the downstream side of the phreatic line which remains parallel to the upstream face of the small dam as previously discussed for stage 1.

Figure 7.36 shows the displacement pattern within the dam at the current stage. It shows the increase of bulging of the upstream toe of the dam as well as along the upstream face of the dam. Such a pattern is a combined effect of the collapse of the lower upstream zone of the dam, the increase of the soil self weight along the entire saturated zone, and the increase in compressibility of the saturated soil. At this stage, the uplifting water pressures are still predominant within the saturated zone, especially in areas near to the upstream face of the dam, despite the collapse and increase in self weight experienced by the soil in that zone. However, as expected, such a predominance is gradually minimized from the upstream face towards the phreatic line. The upper part of the saturated zone presents a downstream and downward movement pattern reflecting the combined action of the seepage forces and an increase in compressibility of the soil in that area. Soil elements within that zone have reached failure conditions at the current stage. Figure 7.36 shows that the unsaturated part of the dam presents a tendency of downstream and downward movements in response to seepage forces into the dam.

Figures 7.37 to 7.39 present the principal stress distributions within the dam, in both magnitude and directions at the current stage. Figure 7.37 shows the spreading of the low effective stresses zone within the saturated zone as a consequence of the predominance of the uplifting water forces at this stage. The upper part of the saturated zone experiences higher
values of effective stresses than does the lower saturated part as a consequence of the
previously discussed displacement pattern within the saturated zone. Figure 7.37
demonstrates that points at the unsaturated zone suffered little or no changes in their net
normal principal directions 55 days after their first reservoir filling. These results reflect the
low compressibility of the unsaturated soil.

Figure 7.38 shows the major net normal-effective principal stress distribution within
the dam 55 days after its first reservoir filling. It shows additional reductions of the major net
normal principal stress as compared to the previous stage, especially at the upstream zone
nearby and downstream of the phreatic line. Similar to the previous stage, changes in the
major net normal principal stress (i.e., $\sigma_1 - u_0$), which is nearly in the vertical direction, are
mainly a reduction caused by the buoyant uplift pressures acting in the saturated zone. These
reductions are transmitted to the minor net principal stress in a minor way due to the low
Poisson ratio (i.e., of about 0.30) of the unsaturated soil. Figure 7.38 shows the previously
discussed decrease in effective stresses within the saturated zone of the dam.

Figure 7.39 shows the minor net normal-effective principal stress distribution within
the dam at this stage. Reductions in the minor net normal stress distribution are observed in a
narrow zone nearby and downstream of the phreatic line, particularly within the bottom
upstream zone of the dam. These reductions are the result of the combined effect of the
uplifting water pressures and the bulging of the upstream face of the dam, as previously
discussed in this section. The analysis does not indicate increasing minor net normal stresses
during collapse (i.e., due to the anisotropic factor collapse). The analysis suggests that such
increase could be prevented by the combined effect of the structural conditions of the dam
and the spreading (i.e., towards the downstream part of the dam) of the highly compressible
zone within the upstream toe of the dam.

Figure 7.40 presents the distribution of shear strength mobilized within the
collapsing dam 55 days after its first reservoir filling. It shows the increase in shear strength
mobilized within the upstream zone of the dam, particularly the saturated zone, as compared
Figure 7.35 Pore-water pressure (kPa) distribution 55 days after first reservoir filling.

Figure 7.36 Displacement distribution 55 days after first reservoir filling.
Figure 7.37 Principal stress directions distribution 55 days after first reservoir filling.

Figure 7.38 $\sigma_1$ (kPa) distribution 55 days after first reservoir filling.
Figure 7.39 $\sigma_3$ (kPa) distribution 55 days after first reservoir filling.

Figure 7.40 Percent of shear strength mobilized distribution 55 days after first reservoir filling.
to the previous stage. At this stage the 100 percent shear strength mobilized zone comprises the upstream and saturated lower part of the dam, as well as a localized area within the upper part of the saturated zone. This increase in the failed zone reflects the reduction of the shear strength of the compacted soil due to saturation. Minor changes in mobilized shear strength within the unsaturated downstream zone are results of the redistribution of major and minor net normal stresses, as previously discussed in this section. Similar to the previous stage, the unsaturated and upstream zone of the dam presents a sharp decrease in mobilized shear strength mobilized from the phreatic line towards the downstream zone of the dam.

Figure 7.41 shows the intermediate net normal-effective principal stress distribution within the dam. It illustrates reductions of intermediate net normal-effective stresses within the saturated zone of the dam and the unsaturated zone nearby the phreatic line. In general, Fig. 7.41 shows results similar to the previously discussed minor net normal-effective stress distribution, especially within the unsaturated zone (see Fig. 7.39). However, the assumed plane strain condition prevents movements in the cross section direction. In addition, minor increase in intermediate principal effective stresses happened at this stage due to the anisotropic soil collapse. Therefore, the intermediate net normal-effective principal stress distribution presents higher effective stresses within the saturated zone than those corresponding to the minor net normal-effective stresses.

Figure 7.42 shows the mean net normal-effective principal stress distribution within the dam. This distribution is similar to the previously discussed net normal-effective principal stress distributions and reflects the effects of the buoyant uplifting forces within the saturated zone. Figure 7.42 suggests that additional soil collapse is expected to occur within the lower half of the upstream zone with subsequent advances of the transient water flow.
Figure 7.41 $\sigma_z$ (kPa) distribution 55 days after first reservoir filling.

Figure 7.42 $\sigma_m$ (kPa) distribution 55 days after first reservoir filling.
Stage 3: 100 days after first reservoir filling

Figures 7.43 shows the pore-water pressure distribution within the dam 100 days after its first reservoir filling. At this stage the phreatic line still remains parallel to the upstream face since there is no sign of dissipation of the hydraulic gradient which is driving the water flow advance. The saturated part of the dam presents essentially hydrostatic pressures due to its high coefficient of permeability.

Figure 7.44 shows the displacement pattern within the dam 100 days after its first reservoir filling. At this stage the transient water flow has advanced to a point wherein the factors inducing settlement start predominating over the buoyant uplifting forces within the saturated part of the dam. The upward displacement trend within the saturated zone from the two previous stages is now replaced by a complex pattern. This pattern presents a transition from upward and upstream movements at the upstream face of the dam to downward and upstream movements near the phreatic line. As a general pattern, the saturated upstream slope of the dam starts sliding towards the dam reservoir. As previously described for stage 1, the factors inducing settlement include: i.) the increase of the soil self weight within the upstream zone of the dam due to saturation; ii.) the increase of the soil compressibility due to saturation; iii.) the collapse of the lower upstream part of the dam due to saturation and, iv.) the increase of compressibility of saturated soil elements which reach failure conditions.

Figure 7.44 shows a pronounced bulging of the upstream face of the dam at this stage. It also shows that the unsaturated zone experiences small downward and downstream movements due to the advance of the saturated zone. Such small displacements reflect the stiffness of the unsaturated zone within the dam which remains at high matric suctions except in a narrow zone near the phreatic line (see Fig. 7.43).

Figures 7.45 to 7.47 present the principal stress distributions within the dam, in both magnitude and directions, 100 days after its first reservoir filling. Figure 7.45 shows the spreading of the low effective stresses zone within the saturated zone, especially at the lower
part of the dam, at this stage. The low effective stresses within the lower part of the dam are a combined effect of the uplifting water forces, the bulging of the upstream zone of the dam and the load transfer from the highly compressible and collapsing saturated zone to the rigid unsaturated zone. Such a load transfer is better visualized by comparing the stress distributions from the previous stage (i.e., Figs. 7.38 and 7.39) with the ones at the present stage (i.e., Figs. 7.46 and 7.47). Figures 7.45 to 7.47 indicate that the upper saturated part of the dam kept relatively high values of major effective stresses and suffered decreases in its values of minor effective stresses at the current stage.

Figures 7.45 to 7.47 demonstrate that points at the unsaturated zone and nearby the phreatic line suffered increases in their net normal principal stresses. Such increases are consequence of the load transfer from the highly compressible saturated zone as previously discussed. The rest of the unsaturated zone did suffer minor changes in both magnitude and directions of its net normal principal directions as compared with the previously discussed stage 2.

Figure 7.48 presents the distribution of shear strength mobilized within the collapsing dam 100 days after its first reservoir filling. It shows the spread throughout the saturated zone of the 100 percent of shear strength mobilized zone. Such a spreading is a combined effect of the reduction of matric suction and the decrease of minor principal stress within the saturated zone as previously discussed. At this stage, the 100 percent shear strength mobilized comprises the entire upstream and saturated lower part of the dam as well as a localized area within the upper part of the saturated zone. An area at the middle and saturated part of the dam did not reach failure conditions. However, this area is isolated and surrounded by soil elements at failure conditions. Therefore, such an area has its movement pattern governed by the surrounding failed zone. The unsaturated zone suffered an increase in shear strength mobilized within the central part of the dam. This increase reflects the increase of major net normal stresses in that zone due to the load transfer previously discussed. The downstream slope of the dam suffered minor changes of shear strength mobilized due to the
Figure 7.43 Pore-water pressure (kPa) distribution 100 days after first reservoir filling.

Figure 7.44 Displacement distribution 100 days after first reservoir filling.
Figure 7.45 Principal stress directions distribution after 100 days of reservoir filling

Figure 7.46 \( \sigma_1 \) (kPa) distribution after 100 days of reservoir filling.
Figure 7.47 $\sigma_3$ (kPa) distribution 100 days after first reservoir filling.

Figure 7.48 Percent of shear strength mobilized distribution 100 days after first reservoir filling.
high values of matric suction existing there. Similar to the previous stage, there is a sharp
decrease in mobilized shear strength mobilized from the phreatic line towards the
downstream zone of the dam.

Figure 7.49 shows the intermediate principal stress distribution within the dam. At
this stage such a distribution has a pattern similar to the minor net normal/effective stress
distribution within the dam, especially at the central part of the dam, due to the predominance
of the load transfer within that zone. Similar to the previous stage, the upstream toe zone of
the dam presents values of intermediate effective stresses higher than the minor effective
stresses values. Such results are a combined effect of the assumed structural condition of the
dam and, to a lesser extend, the influence of the anisotropic collapse factor in the cross
section direction.

Figure 7.50 shows the mean stress distribution within the dam. This distribution
presents an increase of mean net normal stress within the central part of the dam. This
increase reflects the load transfer from the highly compressible and saturated zone to the rigid
unsaturated zone. The general pattern predicts additional soil collapse within the central part
of the dam, especially the lower half, for subsequent advances of the transient water flow into
the dam. The soil collapse will happen as a consequence of the relatively high mean net
normal stress values existing within the lower unsaturated central part of the dam.

Stage 4: 145 days after first reservoir filling

Figures 7.51 shows the pore-water pressure distribution within the dam 145 days
after its first reservoir filling. At this stage the upstream slope of the dam has been saturated
and the upper part of the phreatic line is near the downstream slope face. As previously
discussed, this pattern is a consequence of the hydraulic anisotropy of the soil combined with
the high hydraulic gradient which is driving the transient water flow through the dam
embankment. However, comparison between the positions of the phreatic line at the previous stage (i.e., Fig. 7.43) and at the current stage (i.e., Fig. 7.51) indicates a rotation of the phreatic line wherein the lower part of that line has advanced more than its upper part. Such a rotation is a consequence of the slight dissipation of the hydraulic gradient dissipation at the upper and unsaturated part of the dam as illustrated in Fig. 7.51. The steep coefficient of permeability function keeps a high hydraulic gradient within the unsaturated zone near the phreatic line.

Figure 7.51 indicates a critical condition for the dam stability since the water can emerge from the top and downstream face of the dam for further advances of the transient water flow. Papagiannakis (1982) analyzed the influence of anisotropy on steady-state flow in dam embankments. In his studies, a smooth coefficient of permeability function was utilized. Despite this, it is concluded that a ratio of permeability anisotropy (i.e., $k_x/k_y$) higher than 9 can result in a situation where the phreatic line emerges at the downstream face of the dam. Referring to Casagrande (1937), he added that such a situation “is an undesirable condition that may in the course of time lead to partial or complete failure of the dam”. In the present research study the situation is worsened due to the steep coefficient of permeability function of the collapsing compacted soil. Miranda (1983) reports that failure of Alka-Seltzer dams is related to piping and hydraulic fracturing within the dam embankment. Such a statement was based on information from local people living in areas where an “Alka-Seltzer” dam had reached failure conditions. In summary, it had been reported that the water had emerged from the downstream slope and after a short time the entire dam was being carried out, as a mudflow, by the running water. The present research study has demonstrated that the above condition is fairly depicted in Figure 7.51.

Figure 7.52 shows the displacement pattern within the dam 145 days after its first reservoir filling. This stage reflects the complete failure of the upstream slope of the dam embankment. The displacement pattern can be explained by the same mechanisms previously
Figure 7.51 Pore-water pressure (kPa) distribution 145 days after first reservoir filling.

Figure 7.52 Displacement distribution 145 days after first reservoir filling.
discussed for stage 3. At this stage, the lower central part of the dam has collapsed due to saturation and the upstream slope is sliding towards the dam reservoir.

Figure 7.52 illustrates that the unsaturated zone experiences small downward and downstream movements due to the advance of the saturated zone. Such small displacements reflect the stiffness of the remaining unsaturated zone within the dam. Similar to the previous stage, the unsaturated zone remains at high matric suctions except in a narrow zone nearby the phreatic line (see Fig. 7.51).

Figures 7.53 to 7.55 present the principal stress distributions within the dam, in both magnitude and directions, 145 days after its first reservoir filling. Figure 7.53 illustrates rotations of effective principal stresses within the saturated zone as a consequence of the displacement pattern at this stage. Major effective stress within the saturated zone inclines towards the downstream slope, especially at the lower part of the dam, in response to the predominance of a downward and downstream displacement pattern within that zone. Figure 7.53 also shows rotations of net principal stresses within the unsaturated zone in response to the sliding of the highly compressible saturated zone. This is particularly true for the zone near the phreatic line.

Figures 7.53 to 7.55 also illustrate the increase in net principal stresses within the unsaturated and lower half of the dam due to the load transfer from the highly compressible saturated zone. Figure 7.55 illustrates that in response to the upstream slope sliding of the dam, tensile stresses at the upper part of the dam within both saturated and unsaturated zones occur. This might result in vertical cracking of the top downstream part of the dam. Despite their nearly vertical orientation, such a cracking pattern might constitute preferential seepage paths which contribute to further advances of the water flow towards the downstream slope face of the dam. In addition, such cracking zones contribute to the structural instability of the dam. At this stage, the saturated zone present increases in principal effective stresses distributions due to the sliding of the upstream slope. Tensile stresses are mainly restricted to zones near the upstream face of the dam. This reflects a good condition of the dam against
hydraulic fracturing since positive effective stresses predominate throughout the saturated zone.

Figure 7.56 presents the distribution of shear strength mobilized within the collapsing dam 145 days after its first reservoir filling. At this stage, the failed zone within the dam has been spread throughout the saturated zone. Similar to the previous stage, such a spreading is a combined effect of the reduction of matric suction and the decrease of minor principal stress within the saturated zone. An area of the middle and saturated part of the dam retained its previous values of shear strength mobilized less than 100 percent (see Fig. 7.48). Such an area suffered essentially translation since the previous stage 3, therefore no additional strains happened within that area. As previously discussed in this section, such an area has its movement pattern governed by the surrounding failed zone. The unsaturated zone suffered an increase in shear strength mobilized within the central part of the dam. This increase reflects the increase of major net normal stresses in that zone due to the load transfer previously discussed. The downstream slope of the dam suffered minor changes of shear strength mobilized due to the high values of matric suction existing there. Similar to the previous stage, there is a sharp decrease in mobilized shear strength mobilized from the phreatic line towards the downstream zone of the dam.

In terms of stability, the current stage has demonstrated the failure of an "Alka-Seltzer" dam during its first reservoir filling. The failure mechanism was triggered by the low shear strength and the high compressibility of the collapsing soil at saturated conditions. The upstream slope has reached failure conditions and presented a pronounced sliding towards the dam reservoir before steady state conditions had been reached. Sherard (1963) stated that "for the full reservoir state, only the downstream portion of the dam needs to be analyzed since an upstream slope slide during full reservoir is conceivable only if the strength of the foundation were to be reduced very greatly by wetting. Because of this, and since there are no records of upstream slope slides in earth dams when the reservoir was full, we can assume that upstream slides occur only during construction or following reservoir drawdown". Such a concept has
Figure 7.53 Principal stress directions distribution 145 days after first reservoir filling.

Figure 7.54 $\sigma_1$ (kPa) distribution 145 days after first reservoir filling.
Figure 7.55 $\sigma_3$ (kPa) distribution after 145 days of reservoir filling.

Figure 7.56 Percent of shear strength mobilized distribution 145 days after first reservoir filling.
been utilized through the years in the engineering practice of the design of homogeneous earth dams (Singh and Sharma, 1976; DAER, 1983; Miranda, 1988). The present research study has demonstrated that the failure of an “Alka-Seltzer” is associated with the failure of its upstream slope at full reservoir condition.

Figure 7.57 shows the intermediate principal stress distribution within the dam. At this stage, such a distribution has a pattern similar to the minor net normal/effective stress distribution within the dam, especially at the central part of the dam, due to the predominance of load transfer within that zone. In a general pattern, the intermediate net-normal principal stresses are higher than the minor net normal-effective stresses. This reflects the assumed plane strain condition of the dam combined with increase of intermediate stresses due to the anisotropic collapse factor. The analysis indicates positive effective stresses throughout the saturated part of the dam, except for a narrow zone near the upstream face. Therefore, there is no risks of hydraulic failure in the cross section direction of an “Alka-Seltzer” dam.

Figure 7.58 shows the mean stress distribution within the dam at the current stage. This distribution presents an increase of mean net normal stress within the central part of the dam. Such an increase reflects the load transference from the highly compressible and saturated zone to the rigid unsaturated zone. The general pattern predicts additional soil collapse within the central part of the dam, especially the lower half, for subsequent advances of the transient water flow into the dam. The soil collapse will happen as a consequence of the values of mean net normal stresses existing within the lower central half of the dam.

7.6 Summary

In Chapter 7, the numerical model proposed in this research was applied to analyze the post-filling mechanical behavior of an “Alka-Seltzer” dam as constructed in Northeast Brazil. In that region, small dams are constructed with residual soil derived from gneiss. It has been observed that small dams built with this soil compacted at lower density and drier
Figure 7.57 $\sigma_2$ (kPa) distribution 145 days after first reservoir filling.

Figure 7.58 $\sigma_m$ (kPa) distribution 145 days after first reservoir filling.
than optimum standard AASHTO energy conditions fail in a short time after their first reservoir filling.

Specimens of the residual soil, compacted at optimum conditions, were tested in the laboratory to define both the mechanical and the hydraulic properties (see Sections 4.3.2, 5.4) of the soil. Both the construction and the first impounding phases of the dam were simulated by using the program CONSAT (Pereira, 1986). The stress state at the end of the after reservoir filling phase, as resulted from CONSAT, was used as the initial stress state condition to simulate the post-filling behavior of the dam.

The computer program COUPSO was applied to simulate in a coupled process the transient water flow and stress-strain behavior that follows the first impounding of the dam reservoir. The post-filling behavior was simulated by using the time discretization: 1 initial time step of about 17 days, subsequent time steps varying from 0.70 to 0.20 days. The analyses were performed over a period of 150 days. This time was necessary to demonstrate the failure mechanism of an “Alka-Seltzer” dam as the transient water flow takes place through the dam.

The water pore pressures and stress-strain states within the dam were examined:

a.) to follow the progress of the transient water flow to analyze the post-filling performance of an “Alka-Seltzer” dam in order to define the mechanisms involved in the failure of such a dam embankment;

b.) to follow the development of displacements within the embankment with the water flow advance, especially in terms of settlements due to the collapsing soil behavior during saturation;

c.) to evaluate the shear strength mobilized and the possibility of hydraulic fracturing within the dam during the transient water flow. Such an analysis indicates the performance of the dam embankment in terms of its structural stability.

The results for the post-filling behavior (Figures 7.27 to 7.58) illustrated the capability of the program COUPSO to analyze the mechanical behavior of an “Alka-Seltzer”
dam with the development of the transient water flow. The analysis indicated that an "Alka-Seltzer" dam suffers an upstream slope failure at full reservoir condition when the transient water flow takes place. The analysis indicated a satisfactory performance of the "Alka-Seltzer" against hydraulic fracturing. The numerical analysis herein performed reproduces realistic post-filling behavior of an "Alka-Seltzer" dam as constructed in Northeast Brazil.

The stability analysis of alternative solutions for an "Alka-Seltzer" dam is a subject for further research. Alternative solutions must be evaluated in both structural stability and hydraulic performance aspects. Chapter 8 summarizes the conclusions of the present study and presents some suggestions for future research regarding the stability analysis of "Alka-Seltzer" dams.
CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

This study was undertaken to develop practical procedures to better understand of the mechanical behavior of small collapsing dams during the first reservoir filling. Such procedures take into account the changes in the stress state variables (i.e., \( \sigma - u_a \) and \( u_a - u_w \)) caused by the transient unsaturated-saturated flow through the embankment. To achieve this objective, the present research study was conducted from both experimental and theoretical bases.

a.) An experimental program was planned and conducted to define the mechanical and hydraulic behaviors of a residual soil of gneiss. This material is a typical soil utilized in the construction of small dams in Northeast Brazil. When this residual soil is compacted at a low density, dry of optimum standard AASHTO conditions, it behaves as a metastable-structured soil and undergoes wetting-induced collapse under applied mean net normal stresses higher than 40 kPa. In turn, when compacted at standard optimum AASHTO conditions, this residual soil behaves as a stable-structured material for the range of mean net normal stresses expected in a small dam. The laboratory program focused primarily on the definition of the mechanical and hydraulic behaviors of the residual soil under metastable-structured conditions.

b.) The theory for consolidation of unsaturated soils according to Fredlund and Rahardjo (1993) was applied to simulate the mechanical behavior of a metastable-structured soil following a wetting stress path. This theory was utilized to develop an incremental and practical procedure for performing finite element analysis on the behavior of a small collapsing dam during transient unsaturated-saturated seepage which follows the first filling of the reservoir.
The developed procedure couples the effects of both stress equilibrium and water flow using Fredlund's and Rahardjo’s (1993) theory of consolidation for unsaturated soils. The analysis takes into account the non-linear stress-strain behavior of a metastable soil using the concept of state surfaces proposed by Matyas and Radhakrisna (1968). The modified Mohr-Coulomb failure criterion proposed by Fredlund et al. (1978) is utilized to define the failure conditions in soil elements within the dam. The model takes account of the varying permeability of the collapsing soil when following a wetting stress path.

The developed finite element procedure requires the definition of constitutive relationships for the soil expressing the void ratio, the degree of saturation, the Poisson ratio, the water coefficient of permeability and the shear strength, as functions of the stress state variables governing the soil behavior (i.e., \( \sigma - u_d \) and \( u_a - u_w \)). As results, the numerical model calculates pore-water pressures, displacements, stresses and shear strength mobilized distributions within an earth structure at defined time steps of a transient analysis.

8.1 Conclusions

The following conclusions can be drawn from the present research study:

1.) Fredlund’s and Rahardjo’s (1993) theory for consolidation of unsaturated soils in its more generalized form (i.e., introducing an anisotropic behavior of an unsaturated soil element in response to a change in its matric suction), can be used to simulate the stress-strain behavior of a collapsing soil during saturation. The need to use anisotropic behavior is based on previous experimental works on collapsing soils (Maswoswe, 1985; Lawton et al. 1991a; Handy, 1995). The mechanical behavior of a stable soil is a special case of this more generalized theory presented in this thesis.

2.) A conventional triaxial permeameter system is suitable for measuring the total volume change, water content changes, and the water coefficient of permeability of a collapsing soil specimen under a constant confining pressure, following a wetting stress path.
However, problems arise in the measurement of the saturated water coefficient of permeability (i.e., \( k_s \)) since the soil specimen being tested required back-pressures in the order of 2 to 4 kPa in order to achieve a degree of saturation of 100 percent.

3.) At a given net confining pressure and following a wetting stress path, the metastable-structured soil follows three distinct phases in terms of total deformation versus matric suction.

i.) The first phase occurs when the unsaturated soil is subjected to high values of matric suction. In this phase, a metastable-structured soil suffers small volumetric deformations in response to decreases in its matric suction. At this phase, no slippage occurs between the grains of the unsaturated soil, and the soil structure remains intact. In the present research study, this phase is termed the “pre-collapse” phase.

ii.) The second phase which follows the “pre-collapse phase” occurs as the unsaturated soil experiences intermediate values of matric suction. In this phase, a metastable soil can suffer a significant decrease in its total volume (i.e., soil collapse) in response to decreases in its matric suction. In this phase, the metastable structure of the unsaturated soil is altered due to the breakage of bonds connecting larger particles within the unsaturated soil. In the present research study, this phase is termed the “collapse” phase.

iii.) The third phase follows the “collapse” phase and occurs as the unsaturated soil approaches saturation. In this phase, a metastable soil does not suffer significant, further decreases in its total volume in response to decreases in its matric suction. From a practical point of view, the soil structure remains unaltered within this phase. In the present research study, this phase is termed the “post-collapse” phase.

4.) The higher the net confining pressure applied, the higher will be the wetting-induced volumetric collapse suffered while testing the metastable material. In addition, the
present study suggests that the higher the net confining stress applied on a soil element, the higher will be the matric suction at which the "collapse" phase starts, and the higher will be the matric suction at which the "collapse" phase ends.

5.) Available data suggest that the wetting-induced soil collapse produced two different and opposite effects in the saturation process while testing the unsaturated metastable-structured soil. The first effect is the reduction of the void ratio which increases the degree of saturation of the collapsing material. The second effect is the reduction in the water flow into the soil structure due to the increasing amount of trapped air within the microstructure of the metastable-structured soil.

6.) The shear strength versus matric suction relationships for a metastable-structured soil is complex and highly affected by the wetting-induced soil collapse. Available data suggests that at high matric suctions (i.e., at the "pre-collapse" phase) a metastable-structured soil maintains its opened structure intact. In turn, the available data suggests that in the "collapse" phase the metastable soil structure changes due to the breakage of bonds and aggregations along the shearing plane.

7.) The finite element model developed as part of this study is sufficiently versatile to allow the incorporation of the most relevant theoretical aspects involved in the consolidation of both saturated and unsaturated soils. As a unique characteristic, it presents the incorporation of anisotropic soil behavior to allow for the simulation of the stress-strain behavior of a metastable-structured soil during saturation. The anisotropic behavior has certain inherent limitations which are common to this type of model.

8.) The computer model COUPSO, which has been developed to solve the finite element equations, is relatively easy to use. COUPSO was developed in such a way that either further models or changes of soil models can be easily accommodated. Applications in Chapter 6 demonstrate the applicability and accuracy of the computer program COUPSO to the solution of geotechnical problems involving coupled stress and flow in both saturated and unsaturated soils. The ability of the COUPSO to model Ko-conditions triaxial tests on
metastable soils is demonstrated. The results show reasonable accuracy to the measured data. Besides, the results show the importance of taking into account the wetting-induced collapse when considering the soil as an anisotropic material relative to changes in matric suction.

9.) In Chapter 6, the computer program COUPSO was also applied to analyze the behavior of a small and stable dam constructed in Northeast Brazil. In the Northeast Brazil, it has been observed that small dams built with a residual soil of gneiss compacted at standard AASHTO conditions survive their first reservoir filling without cracks or at least without significant cracks. The results show stress-strain states within the stable dam reflecting satisfactory stability of the dam during the transient seepage phase which follows the first impounding of the reservoir. Stability is also demonstrated for the steady-state condition. The results also reflect the expected behavior of the dam in accordance with the technical literature (Nobari and Duncan, 1972; DAER, 1983; Miranda, 1988).

10.) In Chapter 7, the computer program COUPSO was applied to analyze the behavior of an “Alka-Seltzer” constructed in Northeast Brazil. In that region of Brazil, it has been observed that small dams built with a residual soil of gneiss compacted at lower density and drier than standard AASHTO conditions fail a short time after their first reservoir filling. It has been reported that such dams suffer many cracks and reach failure conditions before steady-state conditions are reached (DAER, 1983; Miranda, 1988). Results obtained from COUPSO demonstrate the structural instability of an “Alka-Seltzer” dam during transient seepage flow following the first reservoir filling of the reservoir. The analysis indicates a progressive failure associated with a sliding of the upstream slope of the dam in response to the water advance into the embankment. The results reflect the combined effect of the decrease in shear strength and the collapsing behavior of the metastable-structured soil in response to a decrease in matric suction. At saturated conditions, the metastable-structured soil presents a low shear strength and a high compressibility.

11.) The results obtained from the model application described in Chapters 5 and 6 are consistent with the observed behavior of small earth dams built in Northeast Brazil with
residual soil derived from gneiss. On the whole, it can be concluded that the numerical analysis techniques proposed in this research study give realistic results for the cases analyzed.

12.) Based on the results of the analysis performed, it seems likely that these analysis procedures may be used to predict pore-water pressures, stresses and movements in small collapsing dams during any stage after the beginning of construction of the dam and may be useful for selecting desirable instrument locations in small dams, and to help in interpreting the results of instrumentation studies. Perhaps the greatest value of these procedures is in connection with instrumentation studies.

8.2 Recommendations

The following suggestions for future research arise from this study:

1.) Additional studies on the anisotropic collapsing factors (i.e., $\chi_{t*}'$s factors) are required. More research is needed to completely understand these factors and their influence on the stress-strain behavior of a metastable-structured soil. A laboratory program using an oedometer cell which allows the measurement of lateral stresses is required for further advances on this subject.

2.) Available triaxial permeameters are not totally suitable for measuring the compressibility and hydraulic properties of a metastable-structured soil under net confining pressures higher than 200 kPa. The laboratory program conducted in this study suggested that minor adjustments, specially in terms of air leakage from the loading cap, are required in order to carry out such measurements. The measurement of water coefficient of permeability for matric suctions higher than 90 kPa is also a subject for further research.

3.) A detailed investigation into the sensitivity and accuracy of the finite element model utilized in the present study was not carried out. There is ample scope to study these aspects and improve the model.
4.) Due to the flexibility of the finite element model, complex boundary conditions, material heterogeneity and material anisotropy regarding permeability can be easily handled. Hence, there are many problems that can be better analyzed using the proposed model. Among them are the following ones:

i.) settlement of shallow and deep foundation on collapsing soils;

ii.) post-filling performance of irrigation canals constructed on either natural or compacted collapsing soil deposits.

5.) Other research is recommended to develop the model to apply to swelling soils. The computer program COUPSO is already able to handle earthfill or natural deposits of soil that expand when the matric suction is decreased. However, it would be necessary to define adequate soil models for the swelling behavior of soils.

6.) The following preliminary suggestions for the design of an “Alka-Seltzer” dam have arisen from the present research study.

i.) The stability of the upstream slope of the dam needs to be improved. An improvement can be achieved by constructing a stability berm at the upstream side of the dam (see Fig. 8.1). Such a berm must be constructed with the residual soil compacted at optimum standard AASHTO compaction energy conditions since it provides a better stability against failure (see Figures 6.54 to 6.57).

ii.) The stability of the downstream slope also needs to be improved. In this case an improvement can be achieved by constructing an internal drainage system to collect the water flow within the central part of the dam. Such a drainage system should keep the downstream slope in an unsaturated condition. An additional alternative, if necessary, is the construction of another stability berm at the downstream toe of the dam (see Fig. 8.1).

iii.) The structural stability of the dam can be improved by flattening both the upstream and downstream slopes of an “Alka-Seltzer” dam.
Figure 8.1 Preliminary proposal for design of an “Alka-Seltzer” dam.

The stability analysis of alternative solutions for an “Alka-Seltzer” is a subject for further research. Alternative solutions can be evaluated with respect to both structural stability and hydraulic performance.
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APPENDIX A

LABORATORY PROGRAM RESULTS FOR THE COLLAPSING SOIL

The data for triaxial permeameter tests, which determine the mechanical and hydraulic properties of the residual gneiss compacted at a metastable-structured condition, are presented in this appendix. The data obtained from the isotropic permeameter tests are listed. These data are first presented to illustrate the state surfaces of the collapsing soil (i.e., void ratio and degree of saturation state surfaces). The data for the controlled head water permeability tests are then outlined.

The volume change behavior of the metastable-structured collapsing soil is described by the state surfaces for void ratio and degree of saturation. For incremental changes in soil element volume phases (i.e., water volume and total volume changes), the state surfaces for void ratio, $e$, and degree of saturation, $S$, are calculated as follows:

$$e = e_0 + \Delta e, (1 + e_0)$$

$$S = \frac{G_s \cdot w}{e}$$

where:

- $e_0$ = initial void ratio of the soil element
- $\Delta e$ = total volume change of the soil element
- $G_s$ = specific gravity
- $w$ = water content in the soil element.

In a steady-state, controlled head permeability test, the calculation of the coefficient of permeability is formulated as following:

$$k_w = \frac{H_s/A_s}{\frac{\Delta h\Delta t}{Q_{ave}} - \frac{L_{up}}{k_{up}A_{up}} - \frac{L_{low}}{k_{low}A_{low}}}$$
where:
\[ k_w = \text{water coefficient of permeability (cm/s)} \]
\[ H_s = \text{height of the soil specimen (cm)} \]
\[ A_s = \text{cross section area of the soil specimen (\pi D_s^2/4 in cm}^2) \]
\[ D_s = \text{diameter of the soil specimen (cm)} \]
\[ \Delta h = \text{differential head (cm)} \]
\[ \Delta t = \text{time interval (s)} \]
\[ Q_{ave} = \text{mean value of inflow volume and outflow volume (}(Q_{in} + Q_{out})/2 \text{ in cm}^3 ) \]
\[ L_{up} = \text{length of the upper high air entry disc (1.013 cm)} \]
\[ A_{up} = \text{cross section of the upper high air entry disc (60.13 cm}^2) \]
\[ k_{up} = \text{coefficient of permeability of the upper high air entry disc (60.13 cm}^2 ) \]
\[ L_{low} = \text{length of the lower high air entry disc (0.94 cm)} \]
\[ A_{low} = \text{cross section of the lower high air entry disc (60.13 cm}^2) \]
\[ k_{low} = \text{coefficient of permeability of the lower high air entry disc (60.13 cm}^2) \).

Both the inflow volume, \(Q_{in}\), and the outflow volume, \(Q_{out}\), should take into account the volume of the diffused air, \(V_{da}\), and the deformation of the differential pressure transducer, \(V_{dpt}\). That is,
\[
Q_{in} = Q_{reading\, (in)} + V_{da\, (in)} + V_{dpt}
\]
\[
Q_{out} = Q_{reading\, (out)} - V_{da\, (out)} - V_{dpt}
\]

Item A.1 presents the volume change behavior of the soil specimen in terms of both water volume and total volume changes. Item A.2 presents the water coefficient of permeability of the soil specimens by following wetting stress paths. The laboratory test results are presented for the four different net confining stresses of 20, 50, 100 and 200 kPa.

Figure A.1 illustrates the performance of the triaxial permeameter system in the consolidation of the specimen TPT2 (i.e., wetted at an applied net confining stress of 50 kPa) at saturated conditions and under a confining stress of 100 kPa.
Figure A.2 illustrates the development of the soil collapse for the specimens consolidated under applied net confining stresses of 100 and 200 kPa. It shows the evolution of the soil collapse in terms of axial and radial components as well as the total volumetric collapse when the soil specimen is gradually wetted. The measured values for axial and radial components of soil collapse at a given net confining stress appears to confirm Lawton et al.'s (1991) findings that the soil collapses isotropically under an isotropic net stress state.
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* back-pressure of 2-4 kPa
## TRIAXIAL PERMEAMETER TESTS

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<th>H&lt;sub&gt;x&lt;/sub&gt;(cm)</th>
<th>D&lt;sub&gt;x&lt;/sub&gt;(cm)</th>
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*back-pressure of 2-4 kPa
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* back-pressure of 2-4 kPa
Figure A.1 Consolidation of the specimen TPT2 at saturated conditions and under a confining stress of 100 kPa.

\[ k_w = 2.50 \times 10^{-7} \text{ m/s} \]
Figure A.2 Total volume, axial and radial deformations versus matric suction during soil collapse.
APPENDIX B

COMPUTER PROGRAM "COUPSO" FOR THE FINITE ELEMENT ANALYSIS OF THE POST-FILLING BEHAVIOR OF SMALL COLLAPSING DAMS.

Identification

This appendix contains the calling tree, the flow chart and a brief description of the computer program, COUPSO. This computer program, which consists of a MAIN program and 22 subroutines, was developed during the course of the present research study. COUPSO uses the concept of incremental loading in order to simulate changes which occur in the earth structure (Clough and Woodward, 1966; Britto and Gun, 1985). The computer program COUPSO is a Finite Element Program for solving 2-D coupled consolidation problems in unsaturated soil mechanics. This program is formulated by using a general theory which couples stress equilibrium and water flow in unsaturated soils (Fredlund and Rahardjo, 1993). The saturated condition of the soil is a particular case of the general theory. The computer program COUPSO utilizes 2-dimensional nine-noded finite elements. Interpolation is performed by using Lagrange's polynomials. Integration is performed by using Gauss-Legendre quadrature. The matrix of linear equations is stored in core by skylines and solved by using a LU decomposition according to a procedure developed by Dhat and Touzot (1982).

Purpose

The purpose of this computer program COUPSO is to calculate the stresses, strains, pore-pressures and displacements in small dams by simulating the stage that follows the
reservoir filling, when the transient long term seepage develops. It can also be utilized to analyze consolidation of soils in both saturated and unsaturated conditions.

Sequence of operations

A short description of the function of each subroutine is given below. The main program monitors all operations by calling the subroutines to perform the analysis. Figure B1 presents the calling tree for the developed procedure. Figure B2 shows the flowchart for the sequence of operations. A description of the variables used in the flowchart is also provided.

Subroutine DATAIN: This subroutine deals with the input and pre-processing of all data for the transient seepage analysis. This subroutine handles the input of the analysis type. The available options are flow analysis or consolidation analysis in either saturated or unsaturated soils. This subroutine also reads all the control data (e.g., number of Gauss Points to be used in the analysis), the nodal data, the element data, the connectivity data, the soil properties data, the boundary conditions data, the concentrated loading data, the geometry data of the structure in analysis, and the time-discretization data.

Subroutine TENSI: This subroutine calculates the initial stress conditions for the earth structure for cases wherein the initial stresses are not available. The stresses are calculated at the Gauss points of each finite element in the soil structure. For cases wherein there exists initial stresses from a previous analysis (e.g., from the simulation of the reservoir filling phase) this subroutine keeps these existent values. In addition, the subroutine TENSI calculates the principal stresses and shear strength mobilized at the Gauss points by using the subroutines TEPRIN and MOBLZ respectively.

Subroutine GAUSS2: This subroutine call GAUSS in order to assign weights and locations to the Gauss points for the planes elements used. The available options are nine, four
and one Gauss points. The reduced integration (i.e., using four Gauss points) as suggested by Zienkiewski (1977) provided better results in the present research study.

Subroutine GAUSS: This subroutine assigns weights and locations to the Gauss points for a Lagrangian one-dimensional element.

Subroutine SHAPE2Q: This subroutine establish shape functions and their derivatives for a two-dimensional nine-noded Lagrangian element. This subroutine calls SHAPE1D and is used to interpolate nodal values of displacements and pore-pressures to the Gauss points of each element. This subroutine calculates both shape function and corresponding derivative values at the Gauss points of each element. These interpolated values are used to calculate the element matrixes and force vectors for the finite element model.

Subroutine SHAPE1D: This subroutine establish shape functions and their derivatives for Gauss points of an one-dimensional 3-noded Lagrangian element.

Subroutine TEPRIN: This subroutine calculates principal stresses (i.e., $\sigma_1$ and $\sigma_2$) by using the plane stress state of a point (i.e., $\sigma_x$, $\sigma_y$, and $\tau_{xy}$). This subroutine is used to calculate the principal stresses and their orientation relative to the horizontal axis coordinate (i.e., $x-x$) at each Gauss point in the finite elements.

Subroutine MOBLZ: This subroutine calculates the shear strength mobilized values to the Gauss points. The shear strength mobilized is evaluated based on the modified Mohr-Coulomb shear failure criterion as proposed by Fredlund et al. (1988). This subroutine assigns complete failure conditions to a Gauss point under tensile stresses (i.e., when $\sigma_1$ and $\sigma_2$ are both tensile stresses). A tensile strength equivalent to the existing cohesion is assigned to a Gauss point wherein only the minor principal stress is a tensile stress.
Subroutine GLOBAL: This subroutine formulates the global system of linear equations (i.e., \( A \times x = B \)) for the numerical model for each iteration of the transient analysis. In order to formulate the system of linear equations this subroutine calls ASSEMAATRIX to obtain all the individuals matrixes and force vectors for the coupled system (i.e., the stiffness matrix, the coupling matrixes, the conductance matrix, the water mass matrix and the forces vectors related to both soil structure and water phase). This subroutine GLOBAL calls NBC2 and ESSBC to apply natural and essential boundary conditions respectively to the system of linear equations.

Subroutine TIMESCH: This subroutine carries out the updating of the time step. For the first time step of the analysis TIMESCH is called by GLOBAL to estimate a initial time step. For the subsequent steps TIMESCH is called by the MAIN program.

Subroutine ASSEMAATRIX: This subroutine formulates the individuals matrixes and force vectors of the coupled system of equations (i.e., the stiffness matrix, the coupling matrixes, the conductance matrix, the water mass matrix and the forces vectors related to both soil structure and water phase).

Subroutine ELEM2Q: This subroutine formulates the matrixes and force vectors for each finite element of the earth structure in analysis. This subroutine calls the subroutine GAUSS2, SHAPE2Q, and JACBN2 in order to in order to properly calculate the shape functions and their derivatives to the Gauss points. ELEM2Q calls SOILPARAM to obtain the soil properties and the increase in the soil self weight due to saturation (i.e., if necessary) at each Gauss point in the finite elements in the soil structure.

Subroutine SOILPARAM: This subroutine calls VCPARAM and PERMEAB to assign to the Gauss points both the mechanical and hydraulic properties for the soil either at saturated or at unsaturated conditions. It also calculates the local increase in self
weight of an unsaturated soil element due to an increase in its degree of saturation. The material properties and self weight increases are calculated by using the average stress state of each Gauss point. During the transient analysis the average stress state at a Gauss point is calculated by using the existent (i.e., initial) stress state and the newly calculated stress state.

Subroutine VCPARAM: This subroutine calculates the elastic parameters (i.e., \( E, E_w, H, \) and \( H_w \)) corresponding to a specified point stress state for the soil at either saturated or unsaturated conditions. VCPARAM calls STATESUR to obtain the volume-mass soil properties (i.e., degree of saturation and void ratio) as well as the derivatives of both the state surfaces relative to a specified stress state (i.e., \( \frac{de}{d\sigma}, \frac{de}{d(u_a - u_w)}, \frac{dS}{d\sigma}, \) and \( \frac{dS}{d(u_a - u_w)} \)) for an unsaturated soil element at unsaturated conditions.

Subroutine STATESUR: This subroutine computes the volume-mass soil properties (i.e., degree of saturation, \( S \), and void ratio, \( e \), for a specified stress state of an unsaturated soil element. STATESUR also calculates the derivatives of both degree of saturation and void ratio state surfaces relative to a specified stress state (i.e., \( \frac{de}{d\sigma}, \frac{de}{d(u_a - u_w)}, \frac{dS}{d\sigma} \) and \( \frac{dS}{d(u_a - u_w)} \)).

Subroutine PERMEAB: This subroutine computes the water coefficient of permeability of the soil in either unsaturated or saturated conditions for a specified stress state. This subroutine uses either the Brooks’ and Corey’s (1964) method or the van Genuchten’s (1980) method in order to calculate the water coefficient of permeability of a soil element.

Subroutine JACBN2: This subroutine calls SHAPE2Q in order to calculate the Jacobian of the transformation between global and local coordinates system for a specified point in a finite element.
Subroutine NBC2: This subroutine applies the natural boundary conditions (i.e., stresses at the domain boundary and/or prescribed water flow) for each time step of the transient analysis. This subroutine calls GAUSS and SHAPE1D in order to integrate the distributed natural boundary conditions along the side of the finite element.

Subroutine ESSBC: This subroutine applies the essential boundary conditions data (i.e., the prescribed displacements and prescribed pore-water pressures) by using the algebraic method.

Subroutine MASOLSKY: This subroutine solves a linear system of equations of the form $A(x) = B$. $A$ is a $(N \times N)$ matrix, $X$ is a vector of unknowns and $B$ is a vector of $N$ elements. It is required to find the solution $X$. This subroutine stores the matrix $A$ in skylines and calls the subroutine SOL to solve the system of equations.

Subroutine SOL: This subroutine solves a system of linear equations stored in core by skylines. The solution can be obtained for either symmetric or nonsymmetric system of equations. This subroutine call the subprogram function SCAL to perform the scalar product of vectors.

Function SCAL: This function computes the scalar product of two vectors.

Subroutine STRESSES: This subroutine calculates the strains, stresses and pore-water pressures at the Gauss points by using the calculated nodal values of displacements and pore-water pressures. For each iteration in a transient analysis the stresses over an element are calculated by using the same soil properties used to calculate the element matrixes in the subroutine ELEM2Q.

The output of the results is processed in the MAIN program. For defined time intervals (i.e., defined in the input data subroutine DATAIN) the output results are:
a.) Nodal values of x- and y-displacements; nodal values of water pore-presures and water total heads.

b.) For each Gauss point in the finite elements the analysis presents: coordinate stresses and strains (i.e., $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$, and $\gamma_{xy}$); principal stresses and maximum shear stress (i.e., $\sigma_1$, $\sigma_2$, and $\tau_{\text{max}}$); orientation of the major principal stress relative to the x-x axis coordinate; and shear strength mobilized.

Variables in the flowchart

NSTEP: Number of the current time step during the transient analysis.
NS: Maximum number of time-steps to be performed.
STEP: Updated total time performed in the transient analysis.
TDAIYS: Maximum number of days to be simulated in the transient analysis.
DUW: Maximum difference in terms of nodal pore-water pressure between two successive iterations in the current time step of the transient analysis. This variable, DUW, is used to checking the convergence at each time step.
DDP: Maximum difference in terms of total nodal displacement between two successive iterations in the current time step of the transient analysis. This variable, DDP, is also used to checking the convergence at each time step.
TOLUW: Tolerance assumed for the nodal pore-water pressure in the iterative process required for the non-linear transient analysis.
TOLP: Tolerance assumed for the nodal total displacement in the iterative process required for the non-linear transient analysis.
NITER: Number of the iteration being performed.
NIMAX: Number of iterations to be performed in one time step.
DT: Time interval in the current time step.
UW(i): Value of a nodal pore-water pressure in the current iteration “i”.
TD(i): Value of a nodal total displacement in the current iteration “i”.

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Figure B1 Calling tree information for COUPSO.
Figure B2 Flowchart for COUPSO.