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UMI
ELASTIC PHOTON SCATTERING FROM DEUTERIUM

A Thesis
Submitted to the College of Graduate Studies and Research
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
in the
Department of Physics and
Engineering Physics
University of Saskatchewan

by
David Lee Hornidge
Spring, 1999

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0-612-43514-8
UNIVERSITY OF SASKATCHEWAN
College of Graduate Studies and Research

SUMMARY OF DISSERTATION
Submitted in partial fulfillment
of the requirements for the

DEGREE OF DOCTOR OF PHILOSOPHY

by

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Spring, 1999

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ABSTRACT

Tagged photons in the energy range $E_\gamma = 84.3 - 104.5$ MeV were scattered elastically from liquid deuterium and detected in the large volume Boston University NaI (BUNI) spectrometer. The NaI detector has an energy resolution of $\sim 2\%$ at 100 MeV which was sufficient to separate the elastic and inelastic scattering contributions. A 5-point angular distribution covering the range $\theta_\gamma = 35^\circ - 150^\circ$ was measured. Using a modified impulse approximation calculation in addition to a rigorous calculation done by Levchuk, estimates of the nucleon polarizabilities in the deuteron were obtained.
ACKNOWLEDGMENTS

I would like to thank my supervisor, Dennis Skopik, for everything he has done for me during my tenure at the lab. His support and friendship are greatly appreciated; without him, none of this would have been possible and I would not be where I am today.

Henry Caplan deserves thanks for getting me started in nuclear physics.

Many thanks go to Ru Igarashi for helping with all of my stupid questions at all hours of the day and night. This work is as much his as it is mine. I would also like to thank Jack Bergstrom, Norm Kolb, Rob Pywell, and Jerry Feldman for their help with all sorts of physics and experimental questions. Bira van Kolck deserves thanks for his discussions on theory.

I must thank Ed Tomusiak and Tom Steele for their patience in putting up with an experimentalist doing a Master's degree in theoretical physics.

I would like to thank my fellow experimentalist Brad Warkentin for his many discussions on the finer points of data analysis.

Thanks must go to Lavina Carter for keeping the lab running smoothly.

Of course, thanks go to all of the scientists and technical staff at SAL whose help was invaluable in the completion of both the experiment and thesis.

There is also a significant number of graduate students and post-docs at the lab whose fellowship I am grateful for. They are Ken Garrow, Trevor Fulton, Darren Spelay, Joseph Taylor, Che Knisely, Darren White, Don Tiller, Kara Keeter, and Mohamed Benmerrouche. I must also thank Grant O'Rielly and Martin Karlsson for their many discussions at Earl’s, in addition to Jeff Wishart for putting up with me in our office.

Derek Harnett deserves thanks for his friendship in addition to showing me
that life as a “man in a suitcase” is something to look forward to.

Special thanks must go to Terry Pilling, who endured my office antics for the better part of four years. His unique sense of humour and late night discussions played an integral part in maintaining my sanity.

Finally, I must thank my mother, Paige Finney, and the rest of my family for their support and friendship throughout.
DEDICATION

This thesis is dedicated to the memory of my grandfather, Charles Grandison Finney.
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<td>Boston University NaI</td>
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<td>CAMAC</td>
<td>Computer Aided Measurement And Control</td>
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Chapter 1

INTRODUCTION

Elastic photon scattering from deuterium can, in principle, yield basic information on the substructure of the deuteron and hence the nucleons themselves. Due to practical limitations, however, the ability to perform such a measurement has only recently become available. Up to now, there was only one measurement on this reaction below 70 MeV [Luc94]. The main experimental difficulty arises from the necessity of resolving the elastic peak from the inelastic contribution.

One of the main reasons for conducting elastic photon scattering from deuterium is to determine the electric ($\alpha$) and magnetic ($\beta$) polarizabilities of the neutron. These fundamental structure constants have been the subject of considerable interest in recent years. There is a significant amount of data on the proton polarizabilities obtained exclusively from Compton scattering. While these measurements are in general agreement, this is not the case for the neutron. Most of the neutron data show inconsistencies, which need to be resolved.

The goals of this thesis were threefold. First of all, we wanted to obtain angular distributions for elastic cross sections in the 100 MeV region with good statistical accuracy, as it had never been done before. Second, using various complex theoretical calculations, it was hoped that the electric and magnetic polarizabilities of the neutron could be extracted. Finally, the effects of meson exchange currents (MEC) in such a simple nucleus as the deuteron were to be investigated.
1.1 Polarizabilities

The electric and magnetic polarizabilities measure the response of a system to external electric and magnetic fields, \( \mathbf{E} \) and \( \mathbf{B} \), respectively. The induced electric or magnetic dipole moment, \( \mathbf{p} \) or \( \mathbf{m} \), of a system, under such an electric or magnetic field is proportional to the field in question with the constant of proportionality being the polarizability:

\[
\mathbf{p} = \alpha \mathbf{E}. \tag{1.1}
\]
\[
\mathbf{m} = \beta \mathbf{B}. \tag{1.2}
\]

It is important to note that these differ from the intrinsic dipole moments of a system, \( \mathbf{d} \) and \( \mathbf{\mu} \). Further, the potential energy of the system is altered by an amount, \( \Delta E \), due to the intrinsic and induced dipole moments:

\[
\Delta E = -\mathbf{d} \cdot \mathbf{E} - \frac{1}{2} \alpha |\mathbf{E}|^2 - \mathbf{\mu} \cdot \mathbf{B} - \frac{1}{2} \beta |\mathbf{B}|^2. \tag{1.3}
\]

From (1.3), it is apparent that the intrinsic moments and polarizabilities characterize the first and second order responses to an external field, respectively. It is important to note, however, that the polarizabilities characterize the first order response of the internal structure of the system.

The electric polarizability can be thought of as the "stretchability" of the system in an electric field and the magnetic polarizability as the "alignability" of the internal magnetic moments in a magnetic field. This is illustrated in Figure 1.1. Note that the magnetic polarizability has two contributions: a paramagnetic piece from the magnetic moments aligning mentioned above, and a diamagnetic piece resulting from an induced current creating a magnetic field in opposition to the external magnetic field (Lenz’ Law).
Figure 1.1: Conceptual illustration of (a) the electric polarizability, \( \alpha \), and (b) the magnetic polarizability, \( \beta \). Note that there are both paramagnetic and diamagnetic contributions to the magnetic polarizability.

We must note that the polarizabilities measured by photon scattering (the generalized polarizabilities) are different from the static (or proper) polarizabilities mentioned above:

\[
\tilde{\alpha} = \alpha + \Delta \alpha, \quad (1.4)
\]
\[
\tilde{\beta} = \beta + \Delta \beta, \quad (1.5)
\]

where the differences arise from the recoil of the nucleon and retardation effects [Pet64, Sch80, Mak90].

1.1.1 Measuring Polarizabilities

Over the last forty years, there have been numerous experiments undertaken to determine the polarizabilities of the proton, while those experiments to
determine the neutron values are more recent and fewer in number. This section gives a brief synopsis of the current state of both the proton and neutron polarizabilities.

**Proton**

Measurements of the proton polarizabilities have been done exclusively through Compton scattering. For sufficiently low energy, the elastic proton cross section can be written as a model-independent low-energy expansion (LEX) in the laboratory frame of reference (or lab frame) [Pet81]:

\[
\frac{d\sigma}{d\Omega}(\omega, \theta) = \frac{d\sigma}{d\Omega}^{\text{Born}}(\omega, \theta) - \frac{r_0}{2 \omega \omega'} \left( \frac{\omega'}{\omega} \right) \left[ (\bar{\alpha}_p + \bar{\beta}_p)(1 + \cos \theta)^2 + (\bar{\alpha}_p - \bar{\beta}_p)(1 - \cos \theta)^2 \right], \quad (1.6)
\]

where \(\omega\) and \(\omega'\) are the incident and scattered photon energies, \(r_0\) is the classical proton radius, \(\theta\) is the lab angle of the scattered photon, and \(\frac{d\sigma}{d\Omega}^{\text{Born}}(\omega, \theta)\) is the cross section for a point proton with internal structure added in the form of an anomalous magnetic moment [Pow49]. The structure dependent terms are the lowest energy terms due to internal structure in the form of polarizabilities, \(\bar{\alpha}_p\) and \(\bar{\beta}_p\). Looking at the angular dependence of (1.6), we can see that at forward angles, the cross section is sensitive only to the sum of the polarizabilities, while at the back angles it is sensitive to the difference. In addition, at 90 degrees, the cross section is sensitive only to the electric polarizability.

In order to extract information on the polarizabilities using the LEX approach, it is necessary to conduct an experiment at an energy high enough to detect the deviation between the LEX and the Born cross section, but at the same time, the energy must be low enough that the LEX is still valid. In order to determine where the LEX is no longer appropriate, we compare it to a
dispersion-relation (DR) calculation of L'vov [L'v81] that is valid, in principle, at all energies. Referring to Figure 1.2, we can see that the LEX begins to break down at about 100 MeV, suggesting that the ideal region for measuring nucleon polarizabilities through Compton scattering is 50–100 MeV. For measurements made at higher energies, theoretical uncertainties arise due to the model dependence introduced in extracting the polarizabilities.

Further, the proton polarizabilities are constrained by the Baldin sum rule [Bal60] such that their sum is given by a model-independent dispersion-relation in the following way [Pet81]:

\[
\bar{\alpha}_p + \bar{\beta}_p = \frac{1}{2\pi^2} \int_{m_*}^{\infty} \frac{\sigma^p_\omega d\omega}{\omega^2},
\]

\[
= 14.2 \pm 0.5,
\]

in units of \(10^{-4} \text{ fm}^3\) (these units are implicitly understood hereafter), where
\( \sigma_{\gamma}^{p}(\omega) \) is the total photoabsorption cross section for the proton. The integral is done with the existing photoabsorption data up to about 1.5 GeV and then continued to infinity with a reasonable theoretical ansatz.

Referring to Part (a) of Figure 1.3, we can see that the measurements of the proton polarizability are in general agreement.

Combining the existing Compton scattering data\(^{1}\) [Bar75, Fed91, Zie92, Hal93, Mac95], MacGibbon et al. [Mac95] performed an analysis with the DR method of L'vov [L'v97] resulting in a global average for the proton polarizabilities of

\[
\bar{\alpha}_{p} = 12.1 \pm 0.8 \pm 0.5, \quad (1.8)
\]

\[
\bar{\beta}_{p} = 2.1 \pm 0.8 \pm 0.5, \quad (1.9)
\]

where the first error is the combined statistical and systematic, and the second is due to the model dependence of the DR extraction method. Note that the errors for \( \bar{\alpha}_{p} \) and \( \bar{\beta}_{p} \) are anti-correlated due to the sum rule constraint.\(^{1}\)

**Neutron**

The majority of measurements of the electric polarizability of the neutron have been done by low-energy neutron scattering from the Coulomb field of a heavy nucleus. From (1.3), we can derive the interaction potential for an induced electric dipole in an electric field:

\[
V = -\frac{1}{2} \alpha_{n} |E|^{2} = -\frac{1}{2} \alpha_{n} \frac{(Ze)^{2}}{r^{4}}.
\]

\(^{1}\)Two early measurements [Oxl58, Gol60] were omitted due to their large uncertainties.

\(^{1}\)Because the sum is constrained by the sum rule and the difference is extracted experimentally, once the individual polarizabilities are obtained, their errors are not independent.
Figure 1.3: Experimental status of the nucleon polarizabilities. (a) A plot of \( \tilde{\alpha}_p - \tilde{\beta}_p \) where the sum is constrained by the Baldin sum rule, \( \tilde{\alpha}_p + \tilde{\beta}_p = 14.2 \). (b) The electric polarizability of the neutron, \( \alpha_n \), where the circles are from low-energy neutron scattering measurements, and the square is from quasi-free Compton scattering.
However, in order to determine $\alpha_n$, one must separate the effects of this interaction from those of the much stronger effects of nuclear potential scattering. This is done by expanding the low-energy total scattering cross section, $\sigma_s$, in terms of the neutron momentum, $k$:

$$\sigma_s(k) = \sigma(0) + ak + bk^2 + ck^4 + \ldots$$  \hspace{1cm} (1.10)

It can be shown that the coefficient $a$ of the linear term depends exclusively on the neutron polarizability while the other terms depend on neutron-nucleus potential scattering. Once a measurement is made of the total cross section and the effects of absorption, neutron-electron scattering, spin-orbit scattering, and resonances are removed, it is possible to fit the remaining cross section to the expansion in (1.10). A number of these measurements have been made [Sch88, Koe88, Sch91, Koe95] and their resulting values of $\alpha_n$ are given in Part (b) of Figure 1.3. We can see that there is considerable disagreement between the extracted values of $\alpha_n$.

In a similar fashion to the proton, the corresponding model-independent dispersion-relation sum rule for the neutron can be written [Pet81]

$$\alpha_n + \beta_n = \frac{1}{2\pi^2} \int_{m_e}^{\infty} \frac{\sigma_n^\gamma(\omega)d\omega}{\omega^2},$$ \hspace{1cm} (1.11)

$$= 15.8 \pm 0.5,$$

where $\sigma_n^\gamma(\omega)$ is the total photoabsorption cross section for the neutron. Using this constraint and the value for $\alpha_n$, one can obtain the value for $\beta_n$.\footnote{This is possible because the difference between the generalized polarizability and the proper polarizability for the neutron is very small (on the 5% level) [Kar98].}

An alternate method of measuring the neutron polarizabilities is through the use of the quasi-free (QF) Compton scattering reaction $d(\gamma, \gamma' n)p$ in which the scattered photon is detected in coincidence with the recoil neutron. In
certain kinematic regions, the proton behaves as a spectator and the scattering is primarily from the neutron. There has been one measurement done on this reaction using bremsstrahlung photons with an endpoint of 130 MeV [Ros90]. However, due to poor statistics, the resulting determination of the electric polarizability of the neutron effectively gives only an upper limit, $\bar{\alpha}_n = 10.7^{+3.3}_{-10.7}$. The formalism for this reaction was later worked out by Levchuk et al. [Lev94, Lev97] and it was suggested that higher energies would minimize background processes and increase sensitivity to the polarizabilities. A measurement was recently performed at the Saskatchewan Accelerator Laboratory (SAL) using tagged photons in the energy range of 240–260 MeV, but the results have yet to be reported.

A third method to determine the polarizability of the neutron is through the elastic Compton scattering reaction $d(\gamma, \gamma')d$. Apart from the measurement detailed in this thesis, there are only two other measurements of this reaction, both at lower energy: one conducted at Illinois [Luc94] and one recently performed at Lund [Lun98]. Because of the poor statistics of the Illinois experiment, the resulting extracted values of the polarizabilities had substantial uncertainties. The Lund measurement has not yet yielded results. The main drawback in attempting to extract the neutron polarizability from the $d(\gamma, \gamma')d$ reaction is that the deuteron amplitude is sensitive to the sum of both the proton and neutron polarizabilities, so it is necessary to subtract the proton polarizabilities in addition to separating out contributions from meson exchange currents and other nuclear effects.

1.2 Classical Photon Scattering

To see the benefits of Compton scattering from deuterium, it is instructive to first look at classical photon scattering from the proton, neutron, and finally
the deuteron.

The basic idea is that the electromagnetic field of an incident photon with energy \( \omega \) induces a proton with charge \( e \) and mass \( M_p \) to oscillate and in turn re-radiate the photon in a different direction, given by \( \theta \). If we assume that the proton has no internal structure, then the cross section is given by the Thomson scattering formula:

\[
\frac{d\sigma^p}{d\Omega} (\theta) = \left( \frac{e^2}{M_p c^2} \right)^2 \left( \frac{1 + \cos^2 \theta}{2} \right),
\]

\[
= r_0^2 \left( \frac{1 + \cos^2 \theta}{2} \right),
\]

(1.12)

where \( r_0 \) is the classical radius of the proton. Including structure in the form of the internal proton-resonance energy, \( \omega_p \), and width, \( \Gamma_p \), the cross section takes the form

\[
\frac{d\sigma^p}{d\Omega} (\omega, \theta) = r_0^2 \left| 1 - \frac{2\omega}{\omega_p^2 - \omega^2 - i\omega\Gamma_p} \right|^2 \left( \frac{1 + \cos^2 \theta}{2} \right).
\]

(1.13)

For an incident photon energy much smaller than the resonance energy, i.e. \( \omega << \omega_p \), the cross section reduces to

\[
\frac{d\sigma^p}{d\Omega} (\omega, \theta) = r_0^2 \left( 1 - \frac{2\omega^2}{\omega_p^2} \right)^2 \left( \frac{1 + \cos^2 \theta}{2} \right).
\]

(1.14)

Further, \( \omega_p \) can be written in terms of the proton electric polarizability in the following way:

\[
\tilde{\alpha}_p = \frac{2r_0}{\omega_p^2},
\]

(1.15)
so that the cross section becomes

\[
\frac{d\sigma^p}{d\Omega} (\omega, \theta) \simeq \left( r_0^2 - 2\tilde{\alpha}_p \omega^2 \right) \left( \frac{1 + \cos^2 \theta}{2} \right),
\]

where the Thomson term interferes with the polarizability term giving an energy dependence of \( \omega^2 \).

For the neutron, there is no net charge and the Thomson term vanishes, leaving

\[
\frac{d\sigma^n}{d\Omega} (\omega, \theta) = r_0^2 \left| \frac{2\omega^2}{\omega_n^2} \right|^2 \left( \frac{1 + \cos^2 \theta}{2} \right),
\]

\[
= \tilde{\alpha}_n^2 \omega^4 \left( \frac{1 + \cos^2 \theta}{2} \right).
\]

(1.17)

We can see that the polarizability appears in the order of \( \omega^4 \), yielding a much smaller cross section at low energies. Thus, the neutron cross section is much smaller than the corresponding proton cross section, in addition to being less sensitive to the polarizability than the proton.

Considering the deuteron, whose cross section is derived from the coherent sum of the proton and neutron amplitudes, we have

\[
\frac{d\sigma^d}{d\Omega} (\omega, \theta) = \left[ r_0 - \omega^2 (\tilde{\alpha}_p + \tilde{\alpha}_n) \right] \left( \frac{1 + \cos^2 \theta}{2} \right),
\]

\[
\simeq \left[ r_0^2 - 2r_0 \omega^2 (\tilde{\alpha}_p + \tilde{\alpha}_n) \right] \left( \frac{1 + \cos^2 \theta}{2} \right).
\]

(1.18)

Now, the interference between the polarizability term and the Thomson amplitude causes \( \tilde{\alpha}_n \) and \( \tilde{\alpha}_p \) to appear to the same order in energy, \( \omega^2 \). Therefore, the cross section for the deuteron is actually more sensitive to the neutron polarizability than a free neutron.

This simple classical picture does not include magnetic polarizabilities.
because it only involves electric excitations.

1.3 Simple Field-Theoretic Calculation

In order to get an idea of how polarizabilities affect the cross section in a more realistic approach, we can perform a simple field-theoretic calculation that includes only the Thomson term for the proton, the lowest order polarizability terms for both the proton and neutron, and MEC in the form of the zero energy limit of the two-photon amplitude (TPA) [Luc94].

The amplitudes for elastic scattering of a photon of energy $\omega$ through angle $\theta$ from the proton and neutron at low energies are given by

$$f_p(\omega, \theta) = [-r_0 + \tilde{\alpha}_p \omega^2] g_{E1}(\theta) + [\tilde{\beta}_p \omega^2] g_{M1}(\theta), \quad (1.19)$$

$$f_n(\omega, \theta) = [\tilde{\alpha}_n g_{E1}(\theta) + \tilde{\beta}_n g_{M1}(\theta)] \omega^2. \quad (1.20)$$

where

$$g_{E1}(\theta) = \epsilon \cdot \epsilon'.$$  \quad (1.21)

$$g_{M1}(\theta) = (\epsilon \times k) \cdot (\epsilon' \times k').$$  \quad (1.22)

are the angular factors for $E1$ and $M1$ transitions, with $\epsilon$ and $k$ being the polarization and wave number of the incident photon (primes are for the scattered photon). Note that (1.19) and (1.20) differ only by the Thomson term in the proton amplitude; the remaining, common terms are the lowest order contributions from nucleon substructure in the form of the generalized electric and magnetic polarizabilities, $\tilde{\alpha}$ and $\tilde{\beta}$.

In the impulse approximation (IA), the assumption is made that there is no interaction between nucleons during scattering. Thus, the amplitude for
Compton scattering from a deuteron is just

\[ f_{IA}(\omega, \theta) = f_p(\omega, \theta) + f_n(\omega, \theta), \]

\[ = \left\{ \left[ -r_0 + \bar{\alpha}_p \omega^2 \right] g_{E1}(\theta) + \left[ \bar{\beta}_p \omega^2 \right] g_{M1}(\theta) \right\} F_1(q), \]

\[ + \left[ \bar{\alpha}_n g_{E1}(\theta) + \bar{\beta}_n g_{M1}(\theta) \right] \omega^2 F_1(q), \]

\[ = \left\{ \left[ -r_0 + \bar{\alpha} \omega^2 \right] g_{E1}(\theta) + \left[ \bar{\beta} \omega^2 \right] g_{M1}(\theta) \right\} F_1(q), \] (1.23)

where \( \bar{\alpha} \) and \( \bar{\beta} \) are the total polarizabilities of the nucleus, which can be written in terms of the isospin-averaged polarizabilities \( \bar{\alpha}_N \) and \( \bar{\beta}_N \), in the following way:

\[ \bar{\alpha} = \bar{\alpha}_p + \bar{\alpha}_n = 2\bar{\alpha}_N, \]

\[ \bar{\beta} = \bar{\beta}_p + \bar{\beta}_n = 2\bar{\beta}_N, \]

and \( F_1(q) \) is the one-body charge form factor that takes into account finite size effects in the deuteron with \( q \) being the momentum transferred to the deuteron. The nominal free values of the isospin-averaged polarizabilities can be arrived at by taking the free proton values from (1.8) and (1.9), and the "best" free neutron values from Ref. [Sch91]:

\[ \bar{\alpha}_N^{free} = \frac{1}{2}(12.1 + 12.0) \simeq 12.0. \] (1.24)

\[ \bar{\beta}_N^{free} = \frac{1}{2}(2.1 + 3.8) \simeq 3.0. \] (1.25)

In order to make the model more realistic, we include MEC contributions to the amplitude in the form of the zero energy limit of the TPA,

\[ B(0, \theta) = -\frac{1}{2} \kappa r_0 g_{E1}(\theta) F_2(q). \] (1.26)
where $\kappa$ is the MEC enhancement factor derived from the Thomas-Reiche-Kuhn (TRK) classical dipole sum rule and $F_2(q)$ is the two-body charge form factor, of which a reasonable first approximation is given by [Wri85]

$$F_2(q) = [F_1(q/2)]^2.$$  (1.27)

Note that this naive calculation does not preserve gauge invariance or the low-energy theorem (LET) that must give the Thomson cross section at zero energy. The MEC contribution is necessary to ensure that gauge invariance is satisfied by cancelling the low-energy limit of the resonance terms in more complex calculations (see Ref. [Luc94]).

Including MEC in this crude fashion, the modified scattering amplitude becomes

$$f(\omega, \theta) = f_{IA}(\omega, \theta) - \frac{1}{2} \kappa r_0 g_{E1}(\theta) F_2(q).$$

$$= \left\{ \left[-r_0 + \bar{\lambda} \omega^2 \right] g_{E1}(\theta) + \left[\bar{\beta} \omega^2 \right] g_{M1}(\theta) \right\} F_1(q)$$

$$- \frac{1}{2} \kappa r_0 g_{E1}(\theta) F_2(q).$$

$$= \left\{ \left[-r_0 + \bar{\lambda} \omega^2 \right] F_1(q) - \frac{1}{2} \kappa r_0 F_2(q) \right\} g_{E1}(\theta)$$

$$+ \left[\bar{\beta} \omega^2 F_1(q) \right] g_{M1}(\theta).$$

$$= A_{E1} g_{E1}(\theta) + A_{M1} g_{M1}(\theta).$$

Averaging over the initial photon polarization, $\lambda$, and summing over the final photon polarization, $\lambda'$, the differential cross section can be obtained

$$\frac{d\sigma}{d\Omega}(\omega, \theta) = \frac{1}{2} \left( \frac{\omega'}{\omega} \right) \sum_{\lambda, \lambda'} |f(\omega, \theta)|^2,$$

$$= \left( \frac{\omega'}{\omega} \right) \left\{ A_{E1}^2 + A_{M1}^2 \left( \frac{1 + \cos \theta}{2} \right) + 2 A_{E1} A_{M1} \cos \theta \right\}. \quad (1.28)$$
where the following identities were used:

\[ \frac{1}{2} \sum_{\lambda, \lambda'} |g_{E1}(\theta)|^2 = \frac{1}{2} \sum_{\lambda, \lambda'} |g_{M1}(\theta)|^2 = \left( \frac{1 + \cos \theta}{2} \right), \]  \hspace{1cm} (1.29)

\[ \frac{1}{2} \sum_{\lambda, \lambda'} g_{E1}(\theta) g_{M1}(\theta) = \cos \theta. \]  \hspace{1cm} (1.30)

The results of this simple calculation are given in Figure 1.4. It is important to note that this calculation did not address nuclear resonance effects, assumed the deuteron was spinless when it is, in fact, spin-1, and included MEC in a very obtuse manner. However, the effects of both MEC and polarizabilities are readily visible: increasing the MEC factor, \( \kappa \), tends increase the overall level of the cross section, while decreasing \( \bar{\alpha}_N - \bar{\beta}_N \) enhances the cross section at back angles.

1.4 Rigorous Field-Theoretic Calculations

In order to extract the polarizabilities from data, it is necessary to have a rigorous calculation that includes nucleon substructure in the form of polarizabilities and nucleon resonances, in addition to nuclear resonances and meson exchange currents. Over the last few years, there has been considerable theoretical interest in the \( d(\gamma, \gamma')d \) reaction, leading to many calculations. This section will summarize four such calculations.

1.4.1 Karakowski et al. [Kar98]

This calculation was done using non-relativistic diagrammatic perturbation theory. The differential cross section was calculated non-relativistically with
Figure 1.4: Simple field-theoretic calculation of $d(\gamma, \gamma')/d \theta$. (a) Angular distributions for an incident photon energy of 94.2 MeV. (b) Energy distributions for a lab angle of 150 degrees. The upper curves are for $\kappa = 0.95$ while the lower curves are for $\kappa = 0.35$. The dashed curves are for nominal free nucleon polarizabilities, i.e. $\tilde{\alpha}_N = 12.0$ and $\tilde{\beta}_N = 3.0$, and the solid curves are for $\tilde{\alpha}_N = 9.0$ and $\tilde{\beta}_N = 6.0$. 
Fermi’s golden rule

\[ d\sigma = \frac{2\pi}{\hbar} \delta(\Delta E) \times \frac{1}{\text{initial flux}} \times (\text{number of final states}) \times |\mathcal{T}_{fi}|^2, \]

(1.31)

and

\[ \mathcal{T}_{fi} = \langle d_f, \gamma_f | \mathcal{H}_{\text{int}} | d_i, \gamma_i \rangle = \sum_C \frac{\langle d_f, \gamma_f | \mathcal{H}_{\text{int}} | C \rangle \langle C | \mathcal{H}_{\text{int}} | d_i, \gamma_i \rangle}{E_{d_i} + \hbar \omega_i - E_C + i\epsilon}, \]

(1.32)

where \( d_{i(f)} \) represents all of the quantum numbers needed to describe the deuteron in the initial (final) state, \( \gamma_{i(f)} \) represents the corresponding quantities for the initial (final) photon, and \( C \) represents an intermediate np or d state. Further, the intermediate state may also contain photons and/or mesons.

The interaction Hamiltonian used to generate the various diagrams\(^7\) was given by

\[
\mathcal{H}_{\text{int}} = \sum_{j=m,p} \left[ \frac{e^2}{2M_j} A^2(x_j) - \frac{e_j}{M_j} \mathbf{A}(x_j) \cdot \mathbf{p}_j - \frac{e(1 + \kappa_j)}{2M_j} \mathbf{\sigma}_j \cdot (\nabla_j \times \mathbf{A}(x_j)) \right. \\
- \frac{1}{2} \tilde{a}_j \left( \frac{\partial \mathbf{A}(x_j)}{\partial t} \right)^2 - \frac{1}{2} \tilde{b}_j (\nabla_j \times \mathbf{A}(x_j))^2 \\
+ \frac{i f_{\pi} e_{\pi}}{m_{\pi}} (\mathbf{\sigma}_j \cdot \mathbf{A}(x_j)) (\tilde{\tau}_j \cdot \tilde{\phi}_j(x_j)) + \frac{f_{\pi} \hbar}{m_{\pi}} (\mathbf{\sigma}_j \cdot \nabla_j) (\tilde{\tau}_j \cdot \tilde{\phi}_j(x_j)) \\
\left. + \frac{1}{\hbar^2} \int d^3x e^2 A^2(x_j) [\phi_+(x_j)\phi_-(x_j) + \phi_-(x_j)\phi_+(x_j)] \right]. \]

(1.33)

The corresponding diagrams are listed in Figure 1.5. The Thomson, or seagull, term is represented by Figure 1.5 (a), while Figure 1.5 (b) contains all other one-body interactions, which are at least of order \( \omega \). It is important to note that the polarizabilities are included in this diagram (terms 4 and 5 in the Hamiltonian).

---

\(^7\)The formulation of Karakowski was slightly different from this in that the substitution \( \mathcal{H}_{\text{int}} = - \int J(\xi) \cdot \mathbf{A}(\xi) d^3\xi \) was made, and charge density and current operators were defined.
Figure 1.5: Feynman diagrams included in the calculation of Karakowski et al.
The resonance terms, which result from a photon being absorbed at one point, exciting a low-lying intermediate state, and then being re-emitted at another point, are represented by Figures 1.5 (c) and 1.5 (d). They were the most tedious terms involved in the calculation. Meson exchange contributions were included with Figures 1.5 (e)–(g) and turned out to have little effect at the energies considered. A relativistic correction was made for the spin-orbit effect by adding the following term to the Hamiltonian:

\[
\mathcal{H}_{\text{int}}^{\text{RC}} = \sum_{j=n,p} \frac{e_j(1 + 2\kappa_j)}{4M_j^2} e_j \mathbf{\sigma}_j \cdot \left[ A(x_j) \times \frac{d}{dt} A(x_j) \right] .
\]  

(1.34)

In addition, a correction was made for the recoil of the deuteron.

All of the diagrams were calculated by expanding the photon wavefunctions into partial waves in conjunction with realistic Bonn-potential deuteron wavefunctions. The Green’s functions for intermediate states were calculated numerically. Finally, checks were made for the calculation with two separate methods. First, certain parts of the amplitude were compared with total photoabsorption data via the optical theorem, and found to be in good agreement. Second, gauge invariance was shown to be satisfied by arriving at the Thomson result for zero energy. Results of the calculation for an initial photon energy of 95 MeV with various polarizability values are given in Figure 1.6.

1.4.2 Levchuk et al. [Lev97]

The differential cross section for \( d(\gamma, \gamma') d \) in this calculation was again obtained with the diagrammatic approach. However, it was done in a slightly different fashion: more MEC degrees of freedom were added and rescattering in the intermediate state was considered. For photon scattering from the deuteron
Figure 1.6: Karakowski et al. calculation of $d(\gamma, \gamma')d$ as a function of lab angle for an initial photon energy of 95 MeV. The solid curve is for $\bar{\alpha}_N = 12$ and $\bar{\beta}_N = 2$, the dotted curve is for $\bar{\alpha}_N = 10$ and $\bar{\beta}_N = 4$, and the dashed curve is for $\bar{\alpha}_N = 14$ and $\bar{\beta}_N = 0$.

we can write the differential cross section as [Are86]

$$
\frac{d\sigma}{d\Omega}(\omega, \theta) = \frac{1}{6} \left( \frac{\omega'}{\omega} \right) \sum_{\lambda' m' \lambda m} \left| T^{fi}_{\lambda' m' \lambda m}(k', k) \right|^2.
$$

(1.35)

where $\lambda$ and $m$ are the spin states for the photon and deuteron, respectively (primes indicate the final state), and $T^{fi}_{\lambda' m' \lambda m}(k', k)$ is the scattering amplitude. The factor of $\frac{1}{6}$ comes from averaging over the initial and summing over the final spin states of the photon and deuteron. Now, the amplitude can be separated into two parts:

$$
T^{fi}_{\lambda' m' \lambda m}(k', k) = R_{\lambda' m' \lambda m}(k', k) + S_{\lambda' m' \lambda m}(k', k),
$$

(1.36)
where \( R_{\lambda'\lambda m}(k', k) \) is the resonance term and \( S_{\lambda'\lambda m}(k', k) \) is the seagull term. It is important to note that this separation is purely formal, as the two terms are not separately gauge invariant [Are86].

The resonance amplitude (see Figure 1.7) is generally composed of both real and imaginary pieces, and involves the off-shell vertex, \( \Gamma_d \), of deuteron photodisintegration. Figure 1.7 (a) corresponds to the free propagation of the intermediate nucleons, while Figure 1.7 (b) illustrates \( NN \) rescattering in the intermediate state and contains the deuteron pole contribution. The nuclear Hamiltonian \( H_0 \) used to find the deuteron wavefunctions and off-shell \( NN \)-scattering amplitudes is given by

\[
H_0 = T + V = \sum_{i=1}^{2} \frac{p_i^2}{2M_N} + \sum_{\alpha=\pi,\eta,\delta,\sigma,\omega,\rho} V_{OBE}^\alpha,
\]

where the potentials \( V_{OBE}^\alpha \) are non-relativistic Bonn One Boson Exchange potentials (OBEP) which depend on the nucleon momentum, \( p_i \), and include meson exchanges with \( \alpha = \pi, \eta, \delta, \sigma, \omega, \) and \( \rho \). The photodisintegration vertex, \( \Gamma_d \), includes the same exchanges as the OBEP, and is given by the one- and two-body contributions, respectively (see Figure 1.8). Figure 1.8 (b) involves the photon-meson-nucleon-nucleon vertices, which are given explicitly in Figure 1.9.
Figure 1.8: Photodisintegration vertex for deuterium.

Figure 1.9: Photon-meson-nucleon-nucleon vertices. The symbol $\bar{N}$ stands for the antiparticle part of the nucleon propagator.

The two-photon amplitude is real and corresponds to a photon being absorbed and emitted at the same moment within the energy scale involved. Two-photon amplitude diagrams are shown in Figure 1.10. Figure 1.10 (a) represents the one-body piece, $S^{[1]}$, of the TPA and was derived from the kinetic energy part of the Hamiltonian:

$$
\epsilon_1^{*}\epsilon_2^{[1]}(k', k)\epsilon_j = \frac{e^2}{M_N} g_{E1}(\theta).
$$

(1.38)
The polarizabilities can be written as a contribution to the one-body seagull [L'v92] and were included in the following way:

$$\varepsilon'^* S^{[1]}_{ij} (k', k) \varepsilon_j = -2 \times 4 \pi \omega' \left[ \bar{\alpha}_N g_{E1}(\theta) + \bar{\beta}_N g_{M1}(\theta) \right].$$

(1.39)

The two-body TPA contributions are given in Figures 1.10 (b)-(d) where the

![Diagram](image)

(a) (b) (c) (d)

Figure 1.10: The two-photon or seagull amplitude.

same exchanges as the $VNN$ potentials were included.

In summary, the nucleon polarizabilities appear in the one-body TPA term and were used in conjunction with the Baldin sum-rule to give one free parameter, $\alpha_N - \beta_N$. It is hoped that adjusting this parameter will allow estimation of the data, and in turn give values for the isospin-averaged nucleon polarizabilities. Results of the calculation with various values for the polarizabilities are given in Figure 1.11. We can see that including nucleon substructure in the form of polarizabilities suppresses the cross section as a whole.
Figure 1.11: Levchuk et al. calculation of $d(\gamma, \gamma')d$ as a function of centre-of-mass (CM) angle, $\theta^*$, for an incident photon energy of 95 MeV. The dotted curve is for $\alpha_N = \beta_N = 0$, the dash-dotted curve is for $\alpha_N = 4$ and $\beta_N = 11$, the dashed curve is for $\alpha_N = 7.5$ and $\beta_N = 7.5$, and the solid curve is for nominal free nucleon polarizabilities of $\alpha_N = 12.0$ and $\beta_N = 3.0$.

Further, decreasing $\tilde{\alpha}_N - \tilde{\beta}_N$ has the effect of strengthening the back angle cross section.

1.4.3 Wilbois et al. [Wil95]

The main emphasis of this work was the role of the $NN$ interaction and meson exchange currents. The theoretical framework was based on the work of Weyrauch [Wey90].

The hadronic wavefunctions were calculated using the Bonn OBEPQ-R potential to include rescattering contributions consistent with exchange contributions. Through the use of Siegert's theorem and the exploitation of gauge
Figure 1.12: Wilbois et al. calculations of $d(\gamma, \gamma)d$ for an incident photon energy of 100 MeV and $\alpha_N = \beta_N = 0$.

conditions for the generalized polarizabilities, the majority of MEC and two-body contributions were included in the amplitude. Explicit MEC terms were included for $\pi$ and $\rho$ exchanges and polarizabilities were added identically to Levchuk by including the modification to the one-body seagull amplitude given in (1.39). Results of this calculation are given in Figure 1.12. However, it is important to note that for energies in the range that is relevant to this thesis, i.e. 100 MeV, the calculation was done with the polarizabilities switched off ($\alpha_N = \beta_N = 0$).

1.4.4 Beane et al. [Bea99]

This calculation was done in the context of a special case of chiral perturbation theory ($\chi$PT) called heavy baryon $\chi$PT, or HB$\chi$PT. For an in-depth
treatment of $\chi$PT, refer to Donoghue [Don92]. Briefly, $\chi$PT is an effective theory modeling quantum chromodynamics (QCD) at low energies. It incorporates the symmetries of QCD, while at the same time it is a perturbative theory in which higher order terms are suppressed by an expansion in the small pion momentum, $Q$. At energies significantly lower than the chiral-symmetry breaking scale $\Lambda_{\chi SB} \approx M_N$, the electromagnetic interactions of pions and nucleons can be described systematically by $\chi$PT. The dynamical effects of all other mesons are accounted for through local pionic operators. Ideally, the coefficients of these operators are fit to one experiment and then used to predict another experiment.

Calculations have been done involving nucleon Compton scattering [Ber92], and to order $Q^3$ they predict reasonable values for the nucleon polarizabilities:

$$\bar{\alpha}_p = \bar{\alpha}_n = 12.2. \quad (1.40)$$

$$\bar{\beta}_p = \bar{\beta}_n = 1.2. \quad (1.41)$$

To include the $O(Q^4)$ terms that effect the polarizabilities, it is necessary to fix independently some undetermined parameters [Ber93].

When a nucleus is involved, the nuclear binding energy introduces a new energy scale that is small compared to a typical hadronic scale. This causes infrared singularities in the $A$-nucleon reducible Feynman diagrams evaluated in the static approximation [Wei90]. To remedy this, a modified power counting scheme is adopted in which $\chi$PT is used to generate an effective potential consisting of $A$-nucleon irreducible graphs, which is solved through iteration using the Lippmann-Swinger equation [Wei90].

This calculation used HB$\chi$PT to calculate an irreducible scattering kernel to order $Q^3$, and realistic deuteron wavefunctions were then sewn to it as seen
in Figure 1.13. Results for both $\mathcal{O}(Q^2)$ and $\mathcal{O}(Q^3)$ are given for an incident photon energy of 95 MeV in Figure 1.14.

Figure 1.13: Schematic diagram of the Beane et al. calculation of $d(\gamma, \gamma')d$. Baryon $\chi$PT was used to derive the irreducible kernel, $I$, and then external deuteron wavefunctions, $\Psi_d$, were sewn on to it to give the matrix element.
Figure 1.14: Beane et al. calculations of $d(\gamma, \gamma')d$ as a function of lab angle for an incident photon energy of 95 MeV. The dashed curve is to order $Q^2$ while the solid curve is to order $Q^3$. It is important to note that because $\chi$PT predicts values for the polarizabilities, there are no free parameters in this theory.
Figure 1.14: Beane et al. calculations of $d(\gamma, \gamma')d$ as a function of lab angle for an incident photon energy of 95 MeV. The dashed curve is to order $Q^2$ while the solid curve is to order $Q^3$. It is important to note that because $\chi$PT predicts values for the polarizabilities, there are no free parameters in this theory.
Figure 2.1: The Saskatchewan Accelerator Laboratory.
energy compression system (ECS) was installed at the end of the LINAC to reduce the energy spread of the electron beam produced by the LINAC by a factor of ten. Details are given in Laxdal [Lax80].

Once the electrons have been injected into the PSR, they are tuned so that a small number are extracted at a constant rate, thus producing an electron beam with a much higher duty cycle than the LINAC and an energy spread further improved by an order of magnitude (\(\sim 0.01\%\)). A comprehensive description of the PSR is given by Dallin [Dal90].

Upon extraction from the ring, the electrons are sent either into Experimental Area Two (EA2) for photon experiments (as was the case with this measurement) or into Experimental Area Three (EA3) for use in electron scattering experiments.

2.2 Photon Tagging

2.2.1 The General Principle of Photon Tagging

The continuous nature of the bremsstrahlung distribution (as seen in Figure 2.2) has been the source of many problems in previous photon experiments. It is impossible to discern the energy of photons on an event-by-event basis, resulting in incomplete kinematics. Also, accurate determination of the photon flux is difficult, and a timing reference signal corresponding to the creation of the photon is not possible.

Photon tagging was first suggested independently by Camac at Cornell and Koch at Illinois, but because of low duty factor pulsed-beam machines and slow electronics, it was not feasible until recently. High duty cycle machines are essential for photon tagging experiments in order to minimize the rate of accidental coincidences while also keeping data rates at levels the electronics...
Figure 2.2: Bremsstrahlung spectrum with tagged energy range.

can handle.

A schematic diagram of the basic principle of photon tagging in the production of quasi-monochromatic photons is given in Figure 2.3. It involves a coincidence between the post-bremsstrahlung electron and the detected photo-reaction product(s) caused by the bremsstrahlung photon. After the electron has emitted bremsstrahlung, it is momentum analyzed in a magnetic spectrometer and then detected in the spectrometer focal plane. Since the energy of the initial electron, $E_e$, is known, as is the energy of the detected electron, $E_{e'}$, the energy of the photon that caused the event in the detector, $E_\gamma$, is given by simple energy conservation: $E_\gamma = E_e - E_{e'}$. In this way, the photon is
Figure 2.3: Schematic diagram of the photon tagging technique.

tagged; its energy is well defined, the photon flux is accurately determined, and a timing reference signal corresponding to the creation of the photon can be obtained. One of the drawbacks to photon tagging experiments is that, even with high duty cycle machines, the beam current is required to be lower than that for pure bremsstrahlung experiments, which can result in longer running time for experiments with small cross sections.

2.2.2 The SAL Tagger

Once the electrons entered EA2 (see Figure 2.4), they impinged upon a 115 μm (0.13% radiation length) aluminum foil radiator where a small fraction of them produce a thin cone of bremsstrahlung photons with energies up to that of the electron beam. Those electrons that did radiate endure a greater bend from the tagger magnet and are deflected into the tagger focal plane array; electrons that do not radiate travelled on to a beam dump. The tagged bremsstrahlung
photons were then collimated, and continued on to the target. As the photon tagger at SAL is documented by Vogt et al. [Vog93], only a brief description is given here.

The focal plane array consists of 63 scintillation counters arranged in two rows with a 50% overlap resulting in 62 channels as seen in Parts (a) and (b) of Figure 2.5. A tagger event is generated by a coincidence between the signals of two overlapping counters. Reading coincidences instead of counter signals effectively suppresses the photomultiplier dark current and room background (which would otherwise show up when the tagger is run at low count rates) and increases the energy resolution (to about 1%).
Figure 2.5: Arrangement of the focal plane scintillators. (a) Detector array and support structure. (b) Staggered scintillator arrangement.
Table 2.1: Tagged photon energies.

<table>
<thead>
<tr>
<th>Central No.</th>
<th>Central Momentum (P₀)</th>
<th>Central Photon Energy (% (MeV/c)</th>
<th>Photon Energy (MeV)</th>
</tr>
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<td>104.61 ± 0.14</td>
<td></td>
</tr>
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</tr>
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<tr>
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<tr>
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<td>62</td>
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The tagger setting (which determines the strength of the magnetic field), focal plane shift, and beam energy define the energy of the electrons sampled, and thus determine the bite of the bremsstrahlung spectrum tagged. Note that for a given beam energy, it is possible to vary the tagger setting and focal plane shift in order to tag a different photon range. The set of tagger parameters and the corresponding photon energies for the maximum electron beam energy of this measurement are listed in Table 2.1. For this configuration, the bremsstrahlung-weighted average photon energy was 94.2 MeV. A conceptual view of the bremsstrahlung spectrum generated for this experiment and the tagger bite are shown in Figure 2.2.

In order to determine a coincidence between an electron detected in the focal plane and the corresponding reaction product, X, the difference in their arrival times was recorded with a time-to-digital converter (TDC). The focal plane signals would have reached the tagger coincidence electronics before those from the reaction products, but were delayed. Thus, the experiment arm, or X-arm, signal started the tagger TDC and the electron in the focal plane stopped it. Each channel of the subsequent TDC spectrum contained a true, or prompt, coincidence peak superimposed on a background of random events. These channels were shifted so that the prompt peaks coincided, and then summed together. The TDC spectrum for the 150 degree deuterium data is given as an example in Figure 2.6. The width of the TDC spectrum, known as the hardware coincidence resolving time, was set to roughly 40 ns for the SAL tagger. During the preliminary set-up for the experiment, hardware delays were adjusted so that the prompt peak appeared in the centre of the timing spectrum.
Figure 2.6: Sample tagger TDC spectrum.

2.2.3 Tagging Efficiency Measurements

For accurate measurement of absolute cross sections, it is essential to determine the photon flux. With the SAL photon tagger, some of the photons that are produced at the radiator in the tagged energy range are eliminated from the beam through collimation and thus are not incident on the target. The fraction of bremsstrahlung photons that reach the target is called the tagging efficiency.

This efficiency depends on the bremsstrahlung opening angle (which is related to the electron beam energy), the beam position, and the collimator size. Since the tuning and positioning of the beam have a significant effect on the number of photons passing through the collimators, it was necessary to take
tagging efficiency measurements at regular intervals during the experiment. To facilitate this, a lead glass Čerenkov detector (assumed to be 100% efficient in detecting photons in the tagged energy range) was moved into the beam and the electron rate was decreased several orders of magnitude to reduce the rate of accidental coincidence with background while also keeping data rates at a manageable level. A run was done with the radiator in place and then another was performed with the radiator out to correct for non-radiator related background in the focal plane and lead glass. The ratio of true counts in the lead glass detector to the number of electrons detected in each focal plane channel gave the channel efficiency:

\[
\varepsilon_r = \frac{\text{interrupts}^{\text{rad in}} - R \times \text{interrupts}^{\text{rad out}}}{\text{scaler counts}^{\text{rad in}} - R \times \text{scaler counts}^{\text{rad out}}}, \tag{2.1}
\]

where "interrupts" is the number of coincidences between the Čerenkov detector and the focal plane channel, "scaler counts" is the total counts registered in the focal plane channel, and R is given by

\[
R = \frac{\text{livetime}^{\text{rad in}}}{\text{livetime}^{\text{rad out}}},
\]

with the livetime being that period of time in which the electronics were capable of accepting an interrupt.

The efficiency did vary by a small amount over the entire experiment, due to small differences in the beam set-up, yielding an average value of 0.533.
2.2.4 Duty Factor Determination using the Tagger

The duty factor, as mentioned in Section 2.1, is a quantitative measure of the beam continuity. It is defined as

\[
df = \frac{\langle I(t)^2 \rangle}{\langle I^2(t) \rangle},
\]

(2.2)

where \( I(t) \) is the beam current and \( \langle f(t) \rangle \) is

\[
\langle f(t) \rangle = \frac{1}{T} \int_0^T dt \ f(t),
\]

the time averaged value of the function \( f(t) \) over a given period of time \( T \). It is instructive to look at the variation of the beam current (in terms of the covariance, \( \sigma_I \)) to get a "feel" for what (2.2) means. We can write

\[
\sigma_I^2 = \langle I^2(t) \rangle - \langle I(t) \rangle^2,
\]

so that

\[
df = \frac{\langle I(t)^2 \rangle}{\langle I^2(t) \rangle} = 1 - \frac{\sigma_I^2}{\langle I^2(t) \rangle}.
\]

Thus, for large fluctuations in the current, \( \sigma_I^2 \) approaches \( \langle I^2(t) \rangle \) resulting in a duty factor close to zero. Conversely, for a constant current with small \( \sigma_I^2 \), the duty factor approaches one.

The duty factor monitor used in conjunction with the SAL tagger is explained in Vogt et al. [Vog94]. It is shown that for two well-separated channels in the tagger focal plane, \( A \) and \( B \), with coincidence resolving time \( \tau \), the duty
factor is given by

\[ df = \frac{N_A N_B T}{N_{AB}} \]  

(2.3)

where \( N_A \) and \( N_B \) are the number of events in channels \( A \) and \( B \) respectively, and \( N_{AB} \) is the number of coincidences between the two channels.

In this way, the SAL tagger was used to measure the duty factor of the beam quite accurately. For this experiment, it ranged from 50–90% with a time-averaged value of approximately 65%.

2.3 The Experiment Arm

The experimental arm consisted of the target (carbon or LD\(_2\)), the photon spectrometer, the pair monitor, the X-arm electronics and the data acquisition system. In this section we will examine the X-arm in detail.

2.3.1 Targets

Two targets were used in this experiment: a solid graphite block for the \(^{12}\text{C}(\gamma, \gamma')^{12}\text{C} \) measurements and LD\(_2\) for the \( d(\gamma, \gamma')d \) measurements.

\(^{12}\text{C}\)

The carbon target used in this experiment was a block of graphite with length 10.85 ± 0.05 cm, width 7.65 ± 0.05 cm, thickness 5.15 ± 0.05 cm having mass of 680.6 ± 0.5 g, and yielding a density of 1.592 ± 0.020 g/cm\(^3\). This resulted in \((4.11 \pm 0.07) \times 10^{23}\) nuclei/cm\(^2\) target centres per unit area, also known as the target density. Refer to Section 3.4.2 for the actual calculation of target densities. The block was moved in and out of the beam using a target ladder system (see Figure 2.7). A remote system was used so that the target ladder
Figure 2.7: A schematic diagram of the $^{12}$C target ladder. Note that the beam is incident normal to the plane of the page.

could be controlled from the data acquisition or counting room to minimize down-time while switching to target empty or tagging efficiency runs. Although the ladder frame and lattice were rotated at different detector angles to ensure that they did not intercept scattered photons, the graphite slab was always kept normal to the beam line.

**LD$_2$**

The LD$_2$ target utilized an active cryogenic cooling system that maintained the deuterium at a temperature slightly less than its 20 K liquefaction point.

The target cell itself consisted of a mylar cylinder with rounded ends $12.7 \pm 0.1$ cm long, $10.2 \pm 0.1$ cm in diameter, and $0.20$ mm thick. With a beam spot
3.5 cm in diameter centred on the target axis, the effective thickness of the target cell was calculated to be 12.16 ± 0.22 cm. The LD$_2$ had a density of 0.162 g/cm$^3$, yielding a target density of (5.90 ± 0.15) × 10$^{23}$ nuclei/cm$^2$.

The containment surrounding the target cell and refrigeration system was made of aluminum thick enough to withstand evacuation necessary to thermally isolate the cell. Further isolation of the target cell was accomplished by wrapping it in ten layers of 20 nm thick aluminized mylar. To minimize background events caused by the target apparatus, the ends of the aluminum containment tube were situated well past the target cell and had mylar windows for the photon beam to pass through.

For the initial target fill, D$_2$ gas was leaked into the system where it was cooled by the helium refrigerator unit on a cold finger, until condensation occurred in the reservoir (see Figure 2.8). Once the reservoir was adequately cooled, the condensate dripped down the fill line to the target cell, slowly cooling it through evaporation. The vapour then circulated up through the vent line to the cold finger, where re-condensation occurred. Eventually, the target cell was cold enough to hold liquid. The refrigerator was required during regular operation of the target, as the cell was not perfectly isolated, causing LD$_2$ to continually boil off, travelling up the vent line where it re-condensed on the cold finger.

The reservoir was also used for storing the LD$_2$ during target empty runs. In order to move the LD$_2$ to the reservoir for target empty runs, the fill/empty valve in the vent line was closed. This precluded the gaseous D$_2$ in the vent line from reaching the refrigerator and thus pressure built up, forcing the LD$_2$ out of the target cell and back into the reservoir.

The entire target setup hung from rails attached to the ceiling of EA2 to facilitate movement along the beam line. This was necessary to change the
position of the target when the photon spectrometer was changed to different angles because of the limited area in EA2 (see Figure 2.4).

The status of the D$_2$ in the cell was monitored through the use of two 1000 $\Omega$ carbon resistors: one situated at each of the top and bottom of the cell. The target valve and vacuum equipment were manipulated using a master control panel situated in the counting room via a pump cart in EA2. Further details of this target system can be found in Amendt [Ame91].
2.3.2 The Photon Spectrometer

Photons scattered from the target were detected in the large-volume, total-absorption Boston University NaI (BUNI) gamma ray spectrometer [Mil88]. Since the inelastic contribution to $d(\gamma, \gamma')d$ begins only 2.22 MeV below the elastic peak, it was essential that the photon detector have at least 2% resolution at 100 MeV, which BUNI did. The excellent resolution of BUNI is mainly due to the fact that its large size enables it to effectively contain 100% of the electromagnetic showers created by the incident photons.

Because NaI crystals are difficult to grow, the main section of BUNI is composed of five optically isolated pieces of NaI, each in two segments of length 35.6 cm and 20.3 cm respectively: the core, whose two cylindrical segments are 26.7 cm in diameter, and four quadrants that form an 11.4 cm thick annulus around the core (see Figure 2.9). Surrounding the quadrants is a six segment BC 400 plastic scintillator cosmic ray veto annulus. The core has seven photomultiplier tubes (PMTs) attached to it, the quadrants each have three, and the plastic annulus segments each have two, for a total of 31. The high voltage (HV) unit used to power the PMTs was a LeCroy 1440 High Voltage System controlled through the data acquisition computer. In addition to PMTs, the NaI and plastic scintillators all have ports for fibre optic cables used in the gain tracking system. For structural support, the annulus and NaI segments that made up BUNI proper were surrounded by a thin aluminum casing.

The plastic annulus did an excellent job of vetoing cosmic rays incident on the top and sides of the detector, but it was unable to stop cosmic rays coming from the front or back. To mitigate these effects, six more 2.54 cm thick plastic scintillator paddles (each connected to a PMT) were put on the

---

1The front and back segments are not optically isolated as the photomultiplier tubes collect light from both.

45
Figure 2.9: The BUNI spectrometer. (a) Radial view with photomultiplier tube arrangement. (b) Longitudinal view with physical dimensions of the NaI crystal and plastic cosmic ray veto annulus.
outside of BUNI so that three of these detectors hung over the front and back, respectively (see Figure 2.10). The front middle paddle doubled as a charge particle veto, since it was situated directly behind the aperture in the shielding.

![Diagram](image)

**Figure 2.10**: BUNI plastic cosmic ray veto arrangement. Although only one of each is shown, there are three front and three back paddle vetoes.

In addition, another plastic scintillator veto (attached to a PMT) was situated outside the shielding and used strictly for vetoing charged particles entering through the shielding aperture.

Due to the many sources of backgrounds in EA2, it was necessary to put a large amount of shielding around BUNI. There were two main levels of shielding: a large box built into the table that BUNI rested on, and removable walls added to the outside of the box. The box itself consisted of lead and steel slabs bolted together with a combined thickness of 17.78 cm. The additional shielding walls were made of 10.16 cm thick lead bricks inside a 0.34 cm thick steel housing. Several sections were cut out of the movable shielding and lead box
to allow BUNI to be sufficiently close to the target without interfering with the beam (refer to Figure 2.4). In addition to the lead and steel used to reduce the electromagnetic background, there were sheets of borated foam glued to the interior of the box and spread loosely around BUNI to stop thermal neutrons in EA2.

In order to ensure that the calibration of BUNI was correct, it was necessary to monitor the gains\(^5\) of each of the photomultiplier tubes on the NaI crystal, as the gains were inclined to drift over a relatively short period of time while in use. To this end, a light emitting diode (LED) flasher system was used. A Xenon bulb flasher device was originally constructed for this purpose, but it tended to be unstable, so it was replaced with an LED driven by a pulse generator. The LED was optically coupled to the core and quadrants by a bundle of fibre optic cables (three to the core and two each to the quadrants as shown in Part (a) of Figure 2.9). Had the LED system been stable, this would have been sufficient. However, as the light output from the LED also drifted over time, it was necessary to send flashes into a small NaI reference detector which, in turn, was monitored using a weak Th-C source. In this way, the reference detector normalized the flasher signal so that it could be used to calibrate the BUNI photomultiplier tubes (see Figure 2.11).

2.3.3 Pair Monitor

In order to measure the luminosity of the beam, a pair of plastic scintillation detectors were set up in EA2 at 170 cm from the target and 42.3° from the beam line. The two detectors, together known as a “pair monitor,” measured the atomic pair-production yields from the target. In addition to being sensitive to the luminosity of the beam, the pair monitor generated signals that were

\(^5\)The gain of a PMT relates the voltage output from it to the amount of light collected by it.
Figure 2.11: Schematic diagram of the BUNI gain tracking system [Iga93].
directly correlated to the beam-dependent background, allowing it to be used for monitoring pile-up, an effect that arises from the coincidence of random events with actual events.

A thorough treatment of the pair monitor is given in Warkentin [War99a].

### 2.4 Signal Processing and Data Acquisition

The X-Arm electronics, triggers and data acquisition system are discussed in this section. As mentioned previously, the tagger electronics are documented in Vogt et al. [Vog93], so they are not examined here.

The PMTs from BUNI generate electrical pulses that had to be converted into a form which could be interpreted by the experimenter. It was the function of the electronics and data acquisition system to do this conversion. The analog signals from the PMTs were split into two parts. The first set was converted into logic signals for the generation of triggers. These triggers were then used to inform the acquisition electronics to read out the second, delayed set of signals. This accomplished, the data acquisition system analyzed, displayed and also recorded the data to hard disk for further analysis.

#### 2.4.1 X-Arm Electronics

The X-arm electronics are divided into five parts: the core and aperture veto, the quadrants, cosmic ray vetoes, gain monitoring, and pair monitor. For the most part, the electronics consisted of standard nuclear instrumentation modules (NIM) and computer assisted measurement and control (CAMAC) modules.
Core and Aperture Veto

The output pulses from the core NaI phototubes were immediately directed to a 10× PMT amplifier with two sets of outputs. One wave was sent to analog-to-digital converters (ADCs), and the other was sent to constant-fraction discriminators (CFDs) for use in timing reference signals and generation of triggers and ADC gates. ADCs change the electrical signal from the PMT to a positive integer that can be recorded by the data acquisition system using a gate that tells when to start analyzing the pulse and when to stop. A discriminator takes an analog signal and outputs a logic signal if the PMT pulse is above a certain threshold, otherwise it does nothing. A schematic diagram of the core electronics is given in Figure 2.12.

The first group of signals was delayed by 300 ns of cable (to allow sufficient time for the generation of gates), passively split and then sent to both peak sensing and charge integrating ADCs. The former type of ADC was used for measuring the energy deposited in the core, while the latter was used for monitoring pile-up. Peak sensing ADCs measure the maximum pulse height of the PMT output signal in the gate, whereas charge integrating ADCs, as the name implies, integrate the entire charge in a pulse while the gate is open. Because of this, the former is much less sensitive to pile-up than the latter so that, when they were used in conjunction with one another, a measure of the pile-up in the core was obtained. The second group of core signals from the PMT amplifier went to a CFD (with a 60 ns internal delay), a scaler and then a TDC.

CFDs are used because, unlike leading-edge discriminators, there is no timing jitter from different size pulses, since the CFD fires at a constant fraction of the pulse height. The only drawback to using CFDs is that the internal delay must be adjusted to match the pulse shapes of the detector material.
Figure 2.12: Schematic diagram of the core NaI and aperture veto electronics.
(e.g. plastic or NaI). The scaler was used to count the number of times the discriminator fired, and the TDC was used to make a self-timing peak for the core events.

The aperture veto PMT signal was also sent to an amplifier. One of the outputs was delayed and sent to a charge integrating ADC, while the other was sent to a CFD (with a 2 ns internal delay). One output from the CFD went to a scaler while the other was delayed and then sent to a logical fan-in, fan-out (FIFO) module with one signal being sent to a TDC.

The core trigger signal was the result of a logical AND of the CFD output of core tube #1 and the inverted output of the aperture veto CFD. This was done so that only valid core events were sent to the trigger electronics, i.e. if the aperture veto registered an event above the discriminator threshold, it was assumed to be a charge particle and thus rejected to lower the rate of spurious events. From the AND gate module, one signal was sent to the trigger electronics and a second was sent to the SAL tagger unit (STU), a custom built module that handled all of the X-arm and tagger interrupts (again refer to Vogt et al. [Vog93]).

Quadrants

The electronics for the quadrants were similar to those of the core (see Figure 2.13). As the electronics set-up for each quadrant was identical, only one is examined here. The signals from each of the three PMTs were first sent to a PMT amplifier and split in two. One set of outputs was delayed and passively split into three signals: two high gain and one low gain. The low gain and one of the high gain signals were sent to separate charge integrating ADCs, while the remaining high gain signal was sent to a peak sensing ADC. The peak sensing ADC was used for measuring the energy deposited by the
Figure 2.13: Schematic diagram of the quadrant NaI electronics.

The majority of quadrant events (< 10 MeV), the low gain charge integrating ADC was used for measuring the high energy events (up to 30 MeV) and the high gain charge integrating ADC was compared to the peak sensing ADC for pile-up monitoring. The second set of signals from the PMT amplifier was sent to a CFD, scaler and TDC. The CFD signal from the centre tube of each quadrant was sent to the triggering electronics.

The quadrant trigger signal was the result of a logical OR of the four centre tubes. These signals were then also delayed, prescaled and fanned-out into two; one signal was sent to the trigger electronics and the second was sent to the STU. The prescaler was needed because of the low quadrant CFD...
threshold and the fact that the radioactive source flux could not be regulated (see Section 2.4.2) which caused an extremely high data rate during quadrant calibrations. Because of the low thresholds and resulting sensitivity to background, the CFD output of quadrant centre tube #2 was sent to a rate meter in the counting room to monitor the low energy background in EA2.

**Cosmic Ray Vetoes**

The plastic cosmic ray vetoes can be divided into two parts: the annulus veto and six external paddles. The electronics for a single segment of the annulus veto are given in Figure 2.14. The signals from both tubes were summed and then split using a linear FIFO module; one signal was delayed and sent to

![Diagram of cosmic ray veto annulus electronics]

Figure 2.14: Schematic diagram of the cosmic ray veto annulus electronics. The cosmic ray veto paddle electronics were identical, except that the paddles contained only one PMT each, obviating the need for a FIFO.

...a charge integrating ADC while the other was sent to a CFD (with a 10 ns delay), scaler, and TDC. The signals from the tubes were added in hardware to simplify the circuitry, as the resolution of the annulus was not crucial in detecting cosmic rays. The electronics for the external paddles were identical to the annulus except for the hardware sum. It is important to note that there
were no triggers from the cosmic ray veto system.

**Gain Monitoring**

The electronics for the gain monitoring system had two major components: the NaI reference detector electronics and the triggered output from the pulse generator controlling the LED flasher (see Figure 2.15).

The signal from the reference NaI was sent through a PMT amplifier and split in two. One output was delayed and went to a charge integrating ADC while the other was sent to a CFD (with 2 ns delay—timing was not crucial), scaler, and TDC. A second output from the CFD was prescaled, delayed, and then fanned-out into four. One of the these pulses was sent to a scaler, another was sent to the trigger electronics, a third was sent to the STU, and a fourth was sent to a logical OR with the external trigger from the flasher pulse generator to generate a forced tagger signal. This signal was used to ensure that the tagger would register an event whenever the reference NaI or the flasher fired, even if there were no electrons in the tagger focal plane.

The external trigger from the flasher pulse generator was sent to a logical fan-out where it was split into four. One output was sent to a scaler, the second was sent to a logical OR with the reference detector for the forced signal, the third was sent to the trigger electronics, and the fourth was sent to a gate generator on the way to the STU.

**Pair Monitor**

The signals from the two plastic scintillator tubes were actively split using a linear FIFO, with one set being delayed and sent to a charge integrating ADC as seen in Figure 2.16. The other pair went to a CFD with the output of each, as well as the logical AND of both, going to a scaler and TDC. The output of
Figure 2.15: Schematic diagram of the gain monitoring system electronics.
Figure 2.16: Schematic diagram of the pair monitor electronics.

the AND was prescaled and then sent to the trigger electronics and STU.

Because the pair monitor events were not correlated to Compton events, they were used to generate pedestals. A pedestal event occurs when the ADC is read out and there is no signal present, thus representing the actual zero value of the ADC. The reason the pedestal peak does not coincide with channel zero of the ADC is because of its inherent positive bias.

2.4.2 Triggers and Ancillary Electronics

There were five types of trigger events used in this experiment: the core (which only fired in the absence of an aperture veto hit), the NaI reference detector, the LED pulse generator, the pair monitor, and the quadrants. A logical OR of the first four triggers was used for the X-arm trigger (or X-trigger) under normal running conditions while the quadrant triggers were included for quadrant calibration runs (see Part (a) of Figure 2.17). The core tube #1
Figure 2.17: Schematic diagram of the trigger and ancillary electronics. (a) X-trigger electronics. (b) X-ref electronics. (c) Scaler inhibit electronics.

CFD threshold was set at roughly 20 MeV, as it was the main trigger used during normal running, and it was necessary to eliminate the multitude of low energy background events. The thresholds on the quadrant events were set much lower (about 400 keV) because they were used only during quadrant calibrations (with the beam off) where the important source of events was a series of peaks from a radioactive Th-C source (with the 2.614 MeV peak being the most important).

As well as trigger and detector electronics, there was also a set used to generate ADC gates (see Part (b) of Figure 2.17). Once the X-trigger signal had been sent to the tagger and the STU decided that it was a valid event, i.e. there were electrons with the appropriate timing in the focal plane, the tagger sent a reference signal (denoted X-ref) back to the X-arm electronics. The
tagger X-ref signal was sent through a variable delay, then to a gate generator and finally through a series of logical FIFO modules before heading to the ADC modules. Note that the X-ref signal was also used to stop the self-timing TDCs.

While the ADCs were processing events, they were unable to accept new information, so it was necessary to inhibit the scalers during this time. The tagger electronics produced such an inhibit signal that was fanned-out and then sent to the scalers (see Part (c) of Figure 2.17). Since EA2 is situated directly in line with the LINAC and ECS, there was a considerable amount of background produced when the LINAC was accelerating electrons. To minimize these effects, a signal from the electron gun trigger and a radio frequency (RF) veto signal were sent to the tagger which, in turn, distributed an inhibit to the X-arm electronics.

2.4.3 Data Acquisition

It was the function of the data acquisition system to take digital signals from the electronics modules and put them into a form that the experimenter could use. To this end, an arrangement consisting of a hardware system and the LUCID software package [SAL95] was developed by SAL staff.

A schematic diagram of the data acquisition hardware is given in Figure 2.18. It consisted of CAMAC crates in which CAMAC modules (ADCs, TDCs, and scalers) resided, a CAMAC crate controller for each crate, a Motorola 68040 VME-based single board front-end computer\[\text{VME stands for Versa Module Europe.}\] a SUN workstation with a 4 Gbyte hard disk, and a CD writer. The information passed from the modules through the CAMAC crate to the crate controller which, in turn,
passed it to the VME via the serial highway. From there the VME, along with the workstation, handled the on-line analysis, display and writing to hard disk. When the disk contained a sufficient amount of data, it was compressed and written to compact disc (CD).

LUCID controlled the hardware system by setting up a data stream with three distinct parts: the READER, the LOOKER and the WRITER. It was necessary to write three separate, high-level description files in the LUCID program language to interface with the system, along with a file containing complicated C routines. The LUCID files were translated into C, linked together with the C routine file, and then compiled on the SUN workstation.
The READER was then downloaded onto the VME where it was able to retrieve the data from the crate controllers and pass it on to the workstation where the LOOKER enabled viewing and analysis of the data on-line and the WRITER copied it to hard disk. LUCID was controlled with a user interface program (called the Xlucid manager) that also ran on the workstation. Typical commands were to record data, pause, resume, and stop acquisition. As there were certain events common to all tagged photon experiments, standard format description file templates were used. These files included the necessary tagger components, to which the specific X-arm code was appended.

Since the VME uses the READER to collect data from the CAMAC crate controllers, it is in this description file that the instructions for data collection reside. These instructions are in the form of trigger commands. In this experiment, there were six types of trigger events: startup which occurred when the VME received a beginrun signal from the Xlucid manager, wakeup which occurred when a resumerun signal was received, sleep which occurred when a suspendrun signal was received. tagdata which occurred when the STU sent a look-at-me (LAM) signal to the VME through the STU interface module, scalerread which occurred every 15 seconds or when a suspendrun signal was received, and goodbye which occurred when an endrun signal was received by the VME. Note that the tagdata event type encompassed all of the relevant tagger/X-arm events.

The LOOKER was used to sample and analyze data on-line. The main purpose of this was two-fold: first, it was used to give beam diagnostics in the form of the duty factor and the tagger channel rates, and second, it was used to ensure that the tagger and X-arm detectors and electronics were functioning properly. A secondary purpose of the on-line analysis was to give a rough idea of yields to determine run times. In order to compute these yields
on-line, it was necessary to calculate kinematics and make various cuts on the
data and energy calibrations. In general, the LOOKER is CPU-intensive and
cannot analyze all of the data without impeding the rate of data acquisition;
the fraction of data that did pass through the LOOKER was called the looker
fraction. Because of the relatively simple LOOKER code for this experiment
coupled with the low data rate, the looker fraction was close to 100%, excepting
tagging efficiency runs.

The WRITER was a simple program that told LUCID to write the data
to one of two partitions on a 4 Gbyte hard disk. The data were then copied
to another computer (containing the CD writer) and compressed. Once there
was 650 Mbyte of compressed data, a CD was "burned." Over the course of
the entire experiment 120 CDs were burned, amounting to approximately 240
Gbytes of uncompressed data.

2.5 Calibrations

In this section we will discuss the two different types of BUNI calibrations
done during the experiment: the zero degree calibration and the quadrant
calibrations.

2.5.1 Zero Degree Calibration

The zero degree (or in-beam) calibration was done once, in the middle of the
experiment, in order to obtain both the lineshape of BUNI and an energy cali-
bration for the core PMTs for use in gain tracking. Since the energy deposited
in the core was, in general, over 50 MeV and the core discriminator thresholds
were roughly 20 MeV, it was necessary to use the photon beam for energy
calibration of the core tubes, as the range of available radioactive sources is
5–10 MeV.
For the actual calibration, BUNI was rotated and translated such that it was facing into the beam line. Lead was then stacked in front of the aperture in order to shield the detector while the beam was tuned to acceptable current levels. This was done to ensure that the data rate was low enough for the electronics could handle, and also to ensure that the PMTs were not overloaded, which would cause their gains to drift during the calibration. Once a very low current was obtained, the lead was removed and data acquisition began.

2.5.2 Quadrant Calibrations

Similar to the zero degree calibration, quadrant calibrations were needed to energy calibrate the quadrant PMTs. Since the thresholds for the quadrants were set at ~400 keV, it was possible to use a small Th-C source with a 2.614 MeV peak for calibration. An example of the Th-C spectrum in one of the quadrants is given in Figure 2.19. The source was placed in the aperture where it was directed at the front face of the core NaI so that it caused showering to occur in both the core and quadrants. A switch on one of the trigger electronics modules was also thrown, allowing the quadrants to become part of the main trigger. It was determined early on in the experiment that the gain tracking system was behaving erratically, so it was deemed necessary to increase the frequency of quadrant calibrations from one every few days to at least one a day in order to give a baseline for comparison of flasher pulse height. In this way, it was possible to bypass the reference NaI detector and use the Th-C peaks in the quadrants to give a direct measure of quadrant gain drift, while at the same time calibrating the LED light output drift for use in core gain computation.
Figure 2.19: Sample quadrant Th-C calibration spectrum.

2.6 Consistency Tests

Because of the length of the run, it was decided that the 90 degree point for carbon should be measured twice as a check of reproducibility. This was done between the 60 and 35 degree measurements. The set-up was identical to that of the initial 90 degree point and data were taken until adequate statistics were obtained.

In addition to the above test, $^{12}$C data were taken at different electron flux rates (while BUNI was at 90 degrees) to ensure that the normalization was not rate dependent.
2.7 Run Logistics

The necessary procedures for experimental set-up and moving the photon spectrometer angle will be discussed, as will general run considerations.

2.7.1 Set-Up

The testing and set-up of the X-arm electronics were done over a four month period preceding the run. When EA2 was ready, BUNI, the spectrometer table and lead box, the extra shielding, and the electronics were craned into it. BUNI was then surveyed into the box, with attention being paid to centering it laterally and moving it as far forward as possible while leaving room for the borated foam shielding and paddle vetoes. The box and table were then surveyed into position so that the distance from the beam line to the front face of BUNI itself was 100 cm and it sat at 90°. Lateral surveying was done using an existing trace of the beam line on the floor of EA2 and a mark was used to centre the target. Measurements were made and a line was drawn perpendicular to the beam line, with which the BUNI table was aligned using plumb lines. Once BUNI was at the proper angle and distance, it was raised to beam height and levelled. The extra lead shielding walls were then added to the box and table. The mylar LD$_2$ target cell was centred in the beam line before the aluminized mylar wrappings and vacuum containment were added. Once properly aligned and assembled, the LD$_2$ target system could be rolled out of the way to make room for the carbon target ladder, which was surveyed into place each time it was needed.

With the detector in place, a final set of calibrations was done with the quadrants and the core. The Th-C source was placed against the front face of BUNI (as in the quadrant calibrations) and the core signals were amplified a further tenfold. Using the resulting ADC peaks, the core gains were matched.
A similar procedure was done using the quadrants, except that the signals were not amplified because of their low discriminator thresholds.

2.7.2 Angle Changes

The main concern in changing the detector angle was moving BUNI, the table, box, and shielding, because their combined weight exceeded fifty tonnes. To facilitate moving, the table was raised using four hydraulic jacks, so that a tracked "crawler" was able to fit under each corner. Similar to the initial set-up, for each angle, measurements were made and a line was drawn for alignment. BUNI was then translated and rotated manually using various block-and-tackles attached to both the detector and the EA2 walls. During movement, special attention was paid to the HV and signal cables to ensure that they were not pulled or severed. For the case when the angle was changed from obtuse to acute, the extra shielding on the front face of the table was rotated 180° so that the cut-away portion coincided with the beam.

2.7.3 Run Considerations

At the beginning of data acquisition at a new angle, a set of tests were conducted to determine an acceptable beam current, as it was desirable to run with as high an electron rate as possible while minimizing background. Note that a measure of electron rate in the tagger focal plane was given by the scaler rate of channel 29, noted as the e29-rate. There were four main background-related problems that were considered: first, a large amount of pile-up in BUNI tended to decrease the resolution, secondly, a signal-to-noise (S/N) ratio in the tagger TDC spectrum of less than one made it difficult to locate TDC peak position and determine peak resolution, thirdly, systematic errors that were caused by backgrounds scale with rate, and lastly, an excessive background rate tended
to swamp the electronics, causing an inordinate amount of dead-time. A good indication of the low energy background (comprised of neutrons, electrons and untagged photons) in EA2 was given by a rate meter connected to the CFD from one of the quadrants. During the preliminary running at each angle, these factors were taken into account and the rate was adjusted accordingly.

As well, it was necessary to determine the amount of time spent running with the target empty in order to minimize error in the yield. The target full versus empty ratio was calculated with

$$\frac{t_e}{t_f} = \sqrt{\frac{S_f(Y_e^p + r_{pa}^2 Y_e^a)}{S_e(Y_f^p + r_{pa}^2 Y_f^a)}} \quad (2.4)$$

where $S$ is the total tagger channel sum and $Y^p$ and $Y^a$ are the prompt and accidental yields, $r_{pa}$ is the prompt-accidental scaling ratio, with the $f$ and $e$ subscripts denoting the full and empty test runs respectively. A derivation (2.4) is given in Appendix B.

After some preliminary running, it was deemed necessary to measure the luminosity of the beam, so a pair monitor (as described in Section 2.3.3) was set up in the forward direction.

Data acquisition was terminated for an angle when there were roughly 5% statistics for the yield in the appropriate region of interest.

### 2.8 Run Summary

A summary of run specifications including run number, target, lab angle, channel-averaged instantaneous electron rate, $\bar{r}_e$,** and target to detector distance is given in Table 2.2. The data were taken over a period of seven months from April–October, 1996. All data were taken with an electron beam energy

---

**A measure of the instantaneous rate is given by dividing the time-averaged rate by the duty factor.
of 135.19 MeV. Note that gaps in the run ranges occur because of test runs and calibrations.

Table 2.2: Data point specifications as a function of run number.

<table>
<thead>
<tr>
<th>Run Range</th>
<th>Target</th>
<th>$\theta$ (deg)</th>
<th>$\bar{\tau}_e$ (MHz)</th>
<th>Target to BUNI Distance (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>819-919</td>
<td>$^{12}\text{C}$</td>
<td>90.0</td>
<td>1.698</td>
<td>100.0</td>
</tr>
<tr>
<td>1004-1272</td>
<td>d</td>
<td>90.0</td>
<td>1.465</td>
<td>100.0</td>
</tr>
<tr>
<td>1292-1490</td>
<td>$^{12}\text{C}$</td>
<td>120.0</td>
<td>1.367</td>
<td>133.0</td>
</tr>
<tr>
<td>1627-1830</td>
<td>d</td>
<td>120.0</td>
<td>1.319</td>
<td>133.0</td>
</tr>
<tr>
<td>1870-2165</td>
<td>d</td>
<td>150.0</td>
<td>1.074</td>
<td>137.0</td>
</tr>
<tr>
<td>2166-2261</td>
<td>$^{12}\text{C}$</td>
<td>150.0</td>
<td>1.061</td>
<td>137.0</td>
</tr>
<tr>
<td>2340-2453</td>
<td>$^{12}\text{C}$</td>
<td>60.0</td>
<td>0.869</td>
<td>133.0</td>
</tr>
<tr>
<td>2454-2744</td>
<td>d</td>
<td>60.0</td>
<td>1.370</td>
<td>133.0</td>
</tr>
<tr>
<td>2745-2764</td>
<td>$^{12}\text{C}$</td>
<td>90.0</td>
<td>1.048</td>
<td>100.0</td>
</tr>
<tr>
<td>2873-2884</td>
<td>$^{12}\text{C}$</td>
<td>90.0</td>
<td>0.800</td>
<td>100.0</td>
</tr>
<tr>
<td>2890-2897</td>
<td>$^{12}\text{C}$</td>
<td>90.0</td>
<td>4.764</td>
<td>100.0</td>
</tr>
<tr>
<td>2918-2996</td>
<td>$^{12}\text{C}$</td>
<td>35.0</td>
<td>0.352</td>
<td>134.0</td>
</tr>
<tr>
<td>2997-3279</td>
<td>d</td>
<td>35.0</td>
<td>0.552</td>
<td>134.0</td>
</tr>
</tbody>
</table>
Chapter 3

DATA ANALYSIS

This chapter gives an overview of the data analysis, involving five main areas: data reduction, calibrations, extraction of yields, normalization, and differential cross sections. In addition, inelastic contributions and systematic errors will be discussed. It is important to note that the carbon data was analyzed in detail by Warkentin [War99a].

3.1 Data Reduction

The first step in the analysis was to reduce the data to a manageable amount; only a small fraction of the 240 Gbytes on CD consisted of relevant Compton events. A large percentage were cosmic rays and charged particles, in addition to scattered photons with energies outside the region of interest (ROI) for this experiment. It was necessary to eliminate as much of these events as possible without discarding good data. Of the remaining events, the majority were from the reference NaI and LED flasher. It was desirable to extract the useful information and then discard these events, leaving only valid scattering events. Further, it was expedient to extract applicable information from the quadrant calibration and tagging efficiency runs. To this end, special off-line LOOKER and C routine filter files were written.

All of the data were run through the filter software. For the production runs (target full, target empty, and full-current radiator out), the filtering process rejected data through three "cuts." The first rejected events with hits above a specific threshold in any of the six plastic paddles or the six plastic annulus
segments. This removed a large portion of the cosmic ray events in addition to charged particles passing through the shielding aperture and front middle plastic paddle. This cut is illustrated in Figure 3.1. The second cut, used to eliminate cosmic rays, was in the form of an upper limit imposed on the ratio of energy deposited in the quadrants versus the total amount of energy deposited in NaI.\textsuperscript{1} Because cosmic rays are minimum ionizing particles, the energy they deposit in a medium is proportional to the distance travelled through that

\textsuperscript{1}Because only the NaI portion of BUNI (the quadrants and core) were used to determine the energy of photon events, the total energy deposited in BUNI refers to the energy deposited in NaI only.
medium (see Figure 3.2). For this reason, cosmic rays tended to deposit a large amount of energy in the quadrants compared to Compton events. A two-dimensional histogram showing the aforementioned ratio plotted against the total energy deposited in BUNI is given in Figure 3.3. It clearly shows two bands of cosmic ray events, corresponding to one and two quadrant hits. The ratio cut was set at 50% for the filtered data. The final cut imposed by the filtering software was on the total energy deposited in BUNI. Since Compton events had energy in the range 90 – 105 MeV, a “loose” energy cut of 50 – 130 MeV was placed on the data. The effect of these three cuts on the production runs was to reduce the number of events by about 99%.

In addition to removing a large portion of superfluous events for each run, the filtering LOOKER extracted the peak positions of the flasher in the
reference NaI and each BUNI NaI PMT, the pedestal positions for each of the ADCs, and wrote them to a set of *dump files* for further analysis. Since

![Graph](image)

**Figure 3.3:** Two dimensional histogram illustrating cosmic ray events. The filtering software rejected events with 50% or more energy deposited in the quadrants in addition to events with energy outside the region of interest.

there were two types of ADCs for the core (peak sensing and charge integrating), and both high and low energy ADCs for the quadrants, cross-calibration factors were calculated in the filtering process and saved to the dump files. In addition, a histogram of the energy spectrum with the Th-C peaks in the reference NaI detector was written to a dump file for each run for subsequent fitting. It was unnecessary to fit the flasher peaks and pedestals to locate their positions because they were both well-defined, clean, and narrow; and thus
could be calculated within the filtering software.

Once extracted, this information was discarded and the remainder of the data were written to hard disk. In this way, 78 Gbytes of compressed data (240 Gbytes uncompressed) were reduced to 650 Mbytes, which fit on a hard disk for later analysis.

For tagging efficiency data, the filter software extracted information needed in the calculation of channel-by-channel efficiencies, and saved it to dump files. For the quadrant calibration data, the peak positions of the flasher in each quadrant tube were extracted, and a histogram of the Th-C spectrum was saved to dump files for each run. Note that filtered data were not written to hard disk for non-production runs. A flow chart of the filtering process is given in Figure 3.4.

Figure 3.4: Function of the filtering software on (a) production data and, (b) tagging efficiency and quadrant calibration data.
3.2 Calibrations

Calibrations were needed to determine the initial gains, or base gains, for the BUNI photomultiplier tubes, and also to ensure that this calibration was valid for the entire experiment. Because the run continued over such a long period of time (6.5 months), and the output voltages of the PMTs tended to drift while in use, it was necessary to "track" these drifts and then correct for them. Two types of calibrations were needed: a zero degree calibration to set the initial gains of the core tubes, and quadrant calibrations that were used to set the quadrant tube gains. Initially, quadrant calibrations were done every few days, but it was determined that the gain tracking system was behaving erratically, and as a result the frequency of calibrations was increased to help compensate for this.

3.2.1 Zero Degree Calibration

As mentioned in Section 2.5.1, BUNI was put directly into the photon beam for calibration. The detector energy spectrum or response function of the calibration run is shown in Figure 3.5. Note that all 62 tagger channels were shifted to the highest tagger channel energy (104.61 ± 0.14 MeV) and summed together in one histogram to increase the amount of counts, decreasing the statistical error. For the in-beam calibration, it was not essential to sum the channels because of the large number of counts, but for the deuterium data, it was requisite. For a mono-energetic photon beam incident on the detector, ideally we would get a delta function at that energy in the BUNI response function. Since the NaI crystal was not perfect (and also due in small part to the tagger channels' bin size of ~300 keV), the output energy distribution had a finite width. We can see from Figure 3.5 that the energy resolution was 1.95 MeV, sufficiently small enough to resolve the inelastic contributions to
both the deuterium and carbon lineshapes, which begin 2.2 MeV and 4.4 MeV below the elastic peak, respectively.

For initial gain matching of each of the seven PMTs attached to the core, the peak sensing ADC histogram yielded a spectrum similar (but not nearly as clean, since it was only one core tube) to Figure 3.5. The gains for each PMT were adjusted until the BUNI lineshape agreed with that of an Electron Gamma Shower (EGS) simulation [Nel85]. These simulations were done by Warkentin [War99a]. Agreement between the lineshape and simulation was checked by minimizing the reduced $\chi^2$ calculated between the two (explained in Section 3.2.3). It was sufficient to approximate these basegains because of
the shifting done later on to correct for deficiencies in the gain tracking system. The basegains calculated this way were for the peak sensing ADC; gains for the charge integrating ADC were calculated using a cross-calibration factor obtained through filtering.

3.2.2 Quadrant Calibrations

Once the filtering software retrieved Th-C energy spectra from the quadrant and reference NaI data, it was necessary to fit them in order to extract the 2.614 MeV peak positions. Because of the enormous number of spectra, the Physics Analysis Workstation (PAW) software package [CER93] was used. A sample of the fitting done to find the position of the peak in a quadrant calibration spectrum is given in Figure 3.6. For most of the spectra, it was sufficient to use an exponential function along with a constant to describe the background, and three separate Gaussians to estimate the three highest energy peaks; the fourth peak was not used as it was partially cut off by the threshold. In some cases, when the low energy neutron background in EA2 was especially high during production runs, the NaI tended to become activated, causing low energy background that persisted even when the beam was off for calibration runs. This background washed out some of the lower energy peaks, making the above fitting technique inadequate. Under these circumstances, the fit was modified to use only one or two Gaussians and the background fit was adjusted accordingly.

Once the Th-C positions were known, they were used in conjunction with the flasher peak positions to calculate a set of basegains for each quadrant calibration run:

\[
QHBGAIN[i] = \frac{2.614}{QHBPK[i] - QHBPED[i]},
\]

(3.1)
Figure 3.6: Fitting of Th-C peaks in a quadrant calibration histogram. The spectrum was fitted with three Gaussians, an exponential, and a constant. The position of 2.614 MeV peak was sufficient for use in the gain tracking analysis.

where QHBGAIN is the high gain (low energy) peak sensing basegain, QHBPK is the ADC channel of the calibration peak, and QHBPED is the corresponding ADC pedestal. Again, the basegains for the charge integrating ADCs (low and high gain) were obtained via cross-calibration factors, in the same way as the core.

3.2.3 Gain Calculation

The gain calculation was, in theory, a fairly simple task but because the gain tracking system malfunctioned, it turned into a long, arduous exercise. One of
the indicators that the gain tracking system was deficient was that the light output from the LED varied between the reference NaI and BUNI in ways that were improbable; the flasher peak in the reference NaI made drastic excursions from its base position while those peaks in the BUNI PMTs did not, resulting in gains that were obviously incorrect. This effectively rendered the reference NaI data useless.

Because of the length of the experiment, the electron beam energy shifted a small amount over the course of the run. Thus, in order to sum histograms for a specific angle, it was necessary to shift the data to one beam energy. For simplicity, the data for all angles were shifted to the maximum beam energy of the experiment, 135.19 MeV. This correction was done prior to the gain analysis.

**Standard Scheme**

Even though it was known that the reference NaI data were corrupted, it was useful to calculate the gains with the original procedure to get a “feel” for what exactly was happening.

The quadrant gains for a production run were calculated using the base information (gains, mean flasher peak positions, and pedestal positions) from the quadrant calibration applicable to that run:

\[
QHGAIN[i] = QHBGAIN[i] \times QGDRIFT[i],
\]

(3.2)

where QGDRIFT is the gain drift in the respective quadrant PMT. This factor can be split into two parts:

\[
QGDRIFT[i] = QFDRIFT[i] \times NFDRIFT,
\]
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of the resulting BUNI energy spectra for each angle. The carbon data were used in this capacity because of the high cross section and resulting profusion of counts. The resulting response function resolutions and peak positions for carbon are listed in Table 3.1 by run range and lab angle. With the knowledge

Table 3.1: Carbon lineshape parameters obtained using the standard gain calculation scheme.

<table>
<thead>
<tr>
<th>Run Range</th>
<th>θ (deg)</th>
<th>Resolution (MeV)</th>
<th>Peak Position (MeV)</th>
<th>Compton Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>819–919</td>
<td>90.0</td>
<td>2.51</td>
<td>106.8</td>
<td>103.6</td>
</tr>
<tr>
<td>1292–1490</td>
<td>120.0</td>
<td>2.90</td>
<td>108.6</td>
<td>103.2</td>
</tr>
<tr>
<td>2166–2261</td>
<td>150.0</td>
<td>2.34</td>
<td>103.7</td>
<td>102.8</td>
</tr>
<tr>
<td>2340–2453</td>
<td>60.0</td>
<td>2.16</td>
<td>103.6</td>
<td>104.1</td>
</tr>
<tr>
<td>2745–2764</td>
<td>90.0</td>
<td>1.93</td>
<td>100.4</td>
<td>103.6</td>
</tr>
<tr>
<td>2918–2996</td>
<td>35.0</td>
<td>2.09</td>
<td>101.5</td>
<td>104.4</td>
</tr>
</tbody>
</table>

that the resolution should be roughly 1.9 MeV, and with the theoretically predicted values of the peak positions derived in Appendix A, we can see that the original gain scheme is indeed inadequate.

**Modifications**

Due to the obvious problems with the standard gain calculation method, some alternate procedures were implemented. The first thing that was done was to use the quadrant calibration information to obtain gains for the quadrants.

For the quadrant gains, the flasher information was initially discarded altogether. The quadrant gains were obtained for each production run (with run number “r”) using a source peak position (QHPK) calculated with a linear interpolation between the adjacent quadrant calibration peak positions (labelled

---

\footnote{In order to calculate these quantities, accidental and target empty subtractions were made similar to, but not as complicated as, those of Sections 3.3.2 and 3.3.3.}
\[ QHGAIN[i] = \frac{2.614}{QHPK[i] - QHPED[i]}, \]

where

\[ QHPK[i] = \left( \frac{QHBPK2[i] - QHBPK1[i]}{r2 - r1} \right) \times (r - r1) + QHBPK1[i]. \]

The source peak positions, and hence gains, were assumed to drift from one quadrant calibration value to another as a function of time. Because it was difficult to calculate an actual run-time, the run number was used as an approximation.

A second method of calculating the quadrant gains (labelled with a subscript F) was used where the flasher output was assumed to be constant between calibration runs, attributing the flasher drift entirely to changes in the quadrant PMT gains. Therefore, the gains were written

\[ QHGAINF[i] = QHBGAIN[i] \times QFDRIFT[i]. \]

To check the validity of the flasher information, these two methods were checked against each other. Because they should ideally be the same, or at the very least the difference between them on a tube-by-tube basis should be constant, the comparison is a good test of flasher behaviour. There were significant differences in the methods, so the former was used for the quadrant gains.

For the core, there was only one calibration, so it was necessary to use the flasher information to some degree. The first thing done was to calculate the core gains using the base gains and the uncorrected flasher drift in the core.
tubes:

$$CGAIN[i] = CBGAIN[i] \times CFDRIFT[i].$$  \hfill (3.4)

The resolutions and peak positions for the carbon data were calculated using

Table 3.2: Carbon lineshape parameters obtained with gain Set A.

<table>
<thead>
<tr>
<th>Run Range</th>
<th>$\theta$ (deg)</th>
<th>Resolution (MeV)</th>
<th>Peak Position (MeV)</th>
<th>Compton Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>819–919</td>
<td>90.0</td>
<td>2.72</td>
<td>106.3</td>
<td>103.6</td>
</tr>
<tr>
<td>1292–1490</td>
<td>120.0</td>
<td>2.39</td>
<td>104.6</td>
<td>103.2</td>
</tr>
<tr>
<td>2166–2261</td>
<td>150.0</td>
<td>2.18</td>
<td>103.4</td>
<td>102.8</td>
</tr>
<tr>
<td>2340–2453</td>
<td>60.0</td>
<td>2.04</td>
<td>104.4</td>
<td>104.1</td>
</tr>
<tr>
<td>2745–2764</td>
<td>90.0</td>
<td>2.17</td>
<td>102.3</td>
<td>103.6</td>
</tr>
<tr>
<td>2918–2996</td>
<td>35.0</td>
<td>1.89</td>
<td>103.8</td>
<td>104.4</td>
</tr>
</tbody>
</table>

the interpolated quad gains and the uncorrected core gains (Set A), and are listed in Table 3.2. From these peak positions and resolutions, it is obvious that these gains did poorly. For this reason, a correction was made using the quadrant gain and flasher information.

Table 3.3: Carbon lineshape parameters obtained with core gains corrected for flasher drift (B).

<table>
<thead>
<tr>
<th>Run Range</th>
<th>$\theta$ (deg)</th>
<th>Resolution (MeV)</th>
<th>Peak Position (MeV)</th>
<th>Compton Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>819–919</td>
<td>90.0</td>
<td>2.79</td>
<td>106.4</td>
<td>103.6</td>
</tr>
<tr>
<td>1292–1490</td>
<td>120.0</td>
<td>2.49</td>
<td>103.9</td>
<td>103.2</td>
</tr>
<tr>
<td>2166–2261</td>
<td>150.0</td>
<td>2.26</td>
<td>102.6</td>
<td>102.8</td>
</tr>
<tr>
<td>2340–2453</td>
<td>60.0</td>
<td>2.18</td>
<td>103.7</td>
<td>104.1</td>
</tr>
<tr>
<td>2745–2764</td>
<td>90.0</td>
<td>1.97</td>
<td>102.4</td>
<td>103.6</td>
</tr>
<tr>
<td>2918–2996</td>
<td>35.0</td>
<td>1.98</td>
<td>102.9</td>
<td>104.4</td>
</tr>
</tbody>
</table>
To make a flasher correction for the core gains, both of the two quadrant gain methods were put to use. It was apparent that the flasher peaks in a few of the quadrant tubes were behaving in a similar manner; the remainder were random. This was discovered in the comparison of the two quadrant gain methods mentioned previously. A difference was taken for each tube in the following way:

$$QHCORR[i] = |QHGAIN[i] - QHGAIN_F[i]|,$$

and if this value was within some small tolerance of that for another tube, i.e.

$$|QHCORR[i] - QHCORR[j]| < \text{tolerance},$$

then the flasher output in these tubes was thought to be behaving in the same way. It was determined that tubes 1, 2, 3, and 8 satisfied the above criterion. The flasher correction for the core was then taken as an average of the ratios of the two gain methods for the four tubes.

$$FLASHCOR[i] = \frac{1}{4} \sum_{i=1,2,3,8} QHGAIN_F[i] \cdot QHGAIN[i].$$

Using this correction and the resulting gains, Set B, a new set of peak positions and resolutions were calculated. They are listed in Table 3.3. Again, these gains did a poor job of reproducing proper resolution and peak position for any of the angles.

Because of the failure of both systems, an alternate method was attempted; the gains calculated for run 556 were used for all of the carbon data. Surprisingly, the resolution was better, in general, than the previous methods. The peak positions were, however, unacceptable. To remedy this, factors were found
Table 3.4: Carbon lineshape parameters obtained with a modified version of the core gains from run 556 (C).

<table>
<thead>
<tr>
<th>Run Range</th>
<th>$\theta$ (deg)</th>
<th>Resolution (MeV)</th>
<th>Peak Position (MeV)</th>
<th>Compton Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>819–919</td>
<td>90.0</td>
<td>1.97</td>
<td>103.8</td>
<td>103.6</td>
</tr>
<tr>
<td>1292–1490</td>
<td>120.0</td>
<td>1.96</td>
<td>103.1</td>
<td>103.2</td>
</tr>
<tr>
<td>2166–2261</td>
<td>150.0</td>
<td>2.06</td>
<td>102.5</td>
<td>102.8</td>
</tr>
<tr>
<td>2340–2453</td>
<td>60.0</td>
<td>1.85</td>
<td>103.5</td>
<td>104.1</td>
</tr>
<tr>
<td>2745–2764</td>
<td>90.0</td>
<td>2.25</td>
<td>103.8</td>
<td>103.6</td>
</tr>
<tr>
<td>2918–2996</td>
<td>35.0</td>
<td>1.93</td>
<td>103.8</td>
<td>104.4</td>
</tr>
</tbody>
</table>

for the middle run of each range that put the peaks in their proper positions.

Table 3.5: Comparison of lineshape parameters for LD$_2$ data with the three gain computation methods.

<table>
<thead>
<tr>
<th>Gain Set</th>
<th>Run Range</th>
<th>$\theta$ (deg)</th>
<th>Resolution (MeV)</th>
<th>Peak Position (MeV)</th>
<th>Compton Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1004–1272</td>
<td>90.0</td>
<td>2.86</td>
<td>104.3</td>
<td>99.1</td>
</tr>
<tr>
<td></td>
<td>1627–1830</td>
<td>120.0</td>
<td>2.95</td>
<td>97.2</td>
<td>96.5</td>
</tr>
<tr>
<td></td>
<td>1870–2165</td>
<td>150.0</td>
<td>3.07</td>
<td>99.1</td>
<td>94.8</td>
</tr>
<tr>
<td></td>
<td>2454–2744</td>
<td>60.0</td>
<td>12.38</td>
<td>101.7</td>
<td>101.8</td>
</tr>
<tr>
<td></td>
<td>2997–3279</td>
<td>35.0</td>
<td>3.09</td>
<td>101.9</td>
<td>103.6</td>
</tr>
<tr>
<td>B</td>
<td>1004–1272</td>
<td>90.0</td>
<td>1.93</td>
<td>101.7</td>
<td>99.1</td>
</tr>
<tr>
<td></td>
<td>1627–1830</td>
<td>120.0</td>
<td>2.34</td>
<td>96.9</td>
<td>96.5</td>
</tr>
<tr>
<td></td>
<td>1870–2165</td>
<td>150.0</td>
<td>1.62</td>
<td>94.8</td>
<td>94.8</td>
</tr>
<tr>
<td></td>
<td>2454–2744</td>
<td>60.0</td>
<td>2.50</td>
<td>102.2</td>
<td>101.8</td>
</tr>
<tr>
<td></td>
<td>2997–3279</td>
<td>35.0</td>
<td>1.37</td>
<td>102.5</td>
<td>103.6</td>
</tr>
<tr>
<td>C</td>
<td>1004–1272</td>
<td>90.0</td>
<td>1.97</td>
<td>99.0</td>
<td>99.1</td>
</tr>
<tr>
<td></td>
<td>1627–1830</td>
<td>120.0</td>
<td>2.28</td>
<td>97.3</td>
<td>96.5</td>
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<tr>
<td></td>
<td>1870–2165</td>
<td>150.0</td>
<td>1.57</td>
<td>94.5</td>
<td>94.8</td>
</tr>
<tr>
<td></td>
<td>2454–2744</td>
<td>60.0</td>
<td>1.95</td>
<td>102.2</td>
<td>101.8</td>
</tr>
<tr>
<td></td>
<td>2997–3279</td>
<td>35.0</td>
<td>16.22</td>
<td>105.1</td>
<td>103.6</td>
</tr>
</tbody>
</table>

86
Then, using a third order polynomial fit, gain adjustment factors were found for each run. Combining the resultant core gains and the interpolated quadrant gains into a third set of gains, Set C, a new set of energy spectra were produced. The corresponding response function parameters are given in Table 3.4. We can see that this method does fairly well in reproducing both resolution and peak position.

Using the three corrected sets of gains (A, B, and C), the deuterium data were analyzed and the lineshape parameters tabulated (see Table 3.5); none of the gains did a good job for the whole data set. Because of this, a “quasi-final” gain set was made up of a combination of all three.

At this stage, the BUNI calibration was good enough to continue with random and target empty subtractions. Once these were done, the final adjustments could be made to the gains. For this reason, the final lineshape parameters are included in Section 3.3.3.

3.3 Yield Extraction

In order to calculate the differential cross section, it was essential to first obtain the yield. For this measurement, the yield extraction involved the various background subtractions in addition to the integration of the subtracted energy response function.

3.3.1 Final Cosmic Ray and Charged Particle Reduction

Although the majority of the cosmic ray and charged particle background events were eliminated during filtering, a small amount remained. These remaining events were removed by “tightening” two of the three filter cuts (see Section 3.1). The thresholds on the plastic paddle and annulus ADC histograms were lowered, and the energy ratio threshold was reduced. It is important to
note, however, that the ADC histogram and ratio thresholds could not be reduced indiscriminately for fear of eliminating good events; the effects of these cuts on the energy spectra were monitored to ensure that good data were not thrown away. These cuts effectively eliminated 99.9% of cosmic ray events and a large majority of the charged particles entering BUNI through the shielding aperture.

It is also important to note that off-line analysis of pile-up revealed that it was negligible for all angles and rates.

### 3.3.2 Random Background Subtraction

As mentioned in Section 2.2.2, coincidences between electrons in the tagger focal plane and X-arm events were recorded with the tagger TDC. Again, since the path lengths for both the electron and the scattered photon were constant for each BUNI angle, and because the electron effectively travelled at the speed of light, this coincidence was well defined, and is reflected in the prompt TDC peak (see Figure 3.7 for an example of the tagger TDC spectrum). The energy spectrum generated by events from all 62 channels of the prompt TDC region is given in Part (a) of Figure 3.8. Because of the nature of photon tagging, the prompt, Compton events are superimposed on a "sea" of random events. Using the energy spectrum generated by the accidental TDC region (shown in Part (b) of the same figure), it was possible to subtract the random background events from the prompt spectrum. However, due to the complicated nature of the accidental background, an in-depth treatment was necessary.

An X-arm event starts each of the 62 TDC channels, and because the coincidence resolving time has finite width, more than one channel may be stopped by multiple electron hits in the focal plane. If these electrons occur in channels other than that of the prompt electron, then they contribute to the
Figure 3.7: Tagger TDC spectrum for deuterium at 150 degrees. Both the prompt and accidental regions are illustrated, along with a polynomial fit to the accidental background. Note that this spectrum was made by summing all 62 TDC channels together and increasing the timing bin-width by a factor of four. Further, the prompt and accidental background areas are also illustrated.

uncorrelated, random background. If an electron registers a hit in the same channel as the prompt electron, and it occurs before the prompt electron arrives, then the true coincidence is stolen by the random electron. This happens because a TDC channel can stop only once for a given X-trigger, and thus the higher TDC channels have a lower efficiency because of the greater probability of a random event stopping the TDC within a shorter time. It is important to note that the amount of stolen coincidences is dependent on the instantaneous electron rate in the tagger focal plane. The stolen effect, in addition to
Figure 3.8: Target full energy spectra for deuterium at 150 degrees generated by summing all 62 tagger channels together into one bin. (a) Events from the prompt TDC window. (b) Events from the accidental TDC window.

other rate-dependent effects, can change the shape of the random background in the TDC spectrum if the instantaneous rate is high enough. In addition to reducing the yield. For rates significantly less than 1 MHz, these effects are negligible. The electron rate was low enough in this experiment that the effect of stolen coincidences on the yield was minimal (as shown in Section C.1), but rate-dependent effects did alter the shape of the accidental background in the TDC spectra. Ideally, with a low enough instantaneous rate, the random background would be a flat shelf with the prompt peak sitting on top of it. Rather, as seen in Figure 3.7, the background for the 150 degree deuterium
data (which had a rate of ~ 1 MHz) was convex.

In order to make a random background subtraction using the tagger TDC, it was necessary to ensure that the proper amount of background was subtracted. A fit was made to the accidental background (while ignoring the peak) and a factor was obtained to correct the amount of the accidental subtraction. This correction was done by taking the ratio of the area subtended by the prompt region ($A_p$) versus that of the accidental region ($A_a$); for the spectrum given in Figure 3.7, a correction factor of 0.697 was obtained. This done, the accidental spectrum could be subtracted from the prompt one to give the spectrum for the target full data (an example is shown in Part (a) of Figure 3.9).

For each tagger channel and energy bin, this subtraction can be expressed algebraically as follows:

$$Y_f = Y_f^p - r_{pa} Y_f^a,$$

where $Y$ is the yield, $Y^p$ is the number of prompt counts, $Y^a$ is the number of accidental counts, and $r_{pa}$ is the accidental area correction factor mentioned above:

$$r_{pa} = \frac{A_p}{A_a},$$

and the subscript $f$ stands for target full data. The error in the full yield is given by

$$\sigma_{Y_f} = \left( \sigma_{Y_f^p}^2 + r_{pa}^2 \sigma_{Y_f^a}^2 \right)^{\frac{1}{2}},$$

$$= \left( Y_f^p + r_{pa}^2 Y_f^a \right)^{\frac{1}{2}},$$

where the error in the number of prompt or accidental counts is the statistical.
Figure 3.9: Subtracted energy spectra for deuterium at 150 degrees. (a) Target full showing the deuterium Compton peak. (b) Target empty with a prompt background peak from the mylar target cell.

error, given by the square root. The error in the ratio is negligible.

A similar prescription was followed for the prompt and accidental empty spectra to give a subtracted energy spectrum for target empty data (an example is shown in Part (b) of Figure 3.9).

3.3.3 Target Empty Subtraction

Once the respective target full and empty spectra were obtained, it was possible to make the target empty subtraction. This was necessary because of prompt backgrounds created by Compton scattering and pair-production from
non-target materials. The target empty subtraction for carbon was very small because of the simple nature of the $^{12}$C set-up. For deuterium, however, the target enclosure and cell gave a significant contribution to the background (see Figure 3.9(b)). In order to remove these effects, it was important to subtract the proper amount from the target full spectrum. This was done by normalizing the target empty spectrum by the photon flux, so that for each tagger channel and energy bin, the yield is

$$Y = Y_f - r_{fe} Y_e,$$

where $r_{fe}$ is the ratio of photon flux of full versus empty, given by

$$r_{fe} = \frac{N_{\gamma f}}{N_{\gamma e}}.$$

Using the fact that the photon flux can be written in terms of the tagging efficiency, $\varepsilon_{tag}$, and the electron flux, $S$, the ratio can be written

$$r_{fe} = \frac{\varepsilon_{tag} S_f}{\varepsilon_{tag} S_e} = \frac{S_f}{S_e}.$$

Note that the tagging efficiency will cancel only if it is the same for both target full and empty runs; for this experiment, that was the case. Further, the error in the subtracted yield is given by

$$\sigma_Y = \left( \sigma_{Y_f}^2 + r_{fe}^2 \sigma_{Y_e}^2 \right)^{\frac{1}{2}},$$

where the error in the ratio is negligible due to the large value of the electron flux.
Once the subtracted energy spectra were obtained, final adjustments were made to the gains in order to ensure that the lineshapes agreed with that of the shifted zero degree calibration. This was done by calculating a $\chi^2$ between the two sets of spectra and then plotting them together on the same graph for visual interpretation. Because of the larger amplitude of the zero degree spectrum, it was first necessary to scale it down to the size of the scattered spectrum, with the scaling factor found by taking a ratio of counts in the respective ROIs for the two spectra.

For the 150 degree deuterium data, a comparison of the final energy spectrum with the scaled and shifted in-beam lineshape is shown in Figure 3.10, again noting that these spectra were made by summing all 62 channels of the focal plane together in one bin to increase the statistical accuracy. The reduced $\chi^2$ calculation was made as follows:

$$\chi^2 = \frac{1}{N_{bin} - 1} \sum_{ROI} \left( \frac{Y_{\text{exp}}[i] - Y_{\text{zero}}[i]}{\sigma_{Y_{\text{exp}}}[i]} \right)^2,$$

(3.5)

where $Y_{\text{exp}}$, $\sigma_{Y_{\text{exp}}}$, and $Y_{\text{zero}}$ are the experimental yield, its error, and the zero degree calibration yields respectively, and the sum is over the ROI with the total number of bins summed over being $N_{bin}$. The final set of peak positions for both carbon and deuterium along with their $\chi^2$ values are given in Table 3.6.

### 3.3.4 Yield Integration

Once the random subtraction was finished and the calibration was complete, we needed to integrate the energy bins of the summed and subtracted spectrum.\footnote{To clarify, each of the tagger channels gave the energy of the incident photon while the energy bins of the subtracted spectrum gave the energy of the photon detected in BUNI. Thus, one summed and shifted tagger bin is the equivalent of a mono-energetic photon beam of 94.2 MeV with the scattered photons events showing up in the subtracted spectrum.}
Figure 3.10: Comparison of the 150 degree deuterium energy spectrum with the scaled zero degree calibration lineshape (shaded area). We can see that the inelastic contribution is readily visible. Further, the 2.2 MeV ROI is also shown.

over a specific region of interest depending on the angle and target to obtain the yields for the experiment. For deuterium, a region of interest 2.2 MeV wide starting 1.7 MeV to the left of the Compton energy for that angle was integrated (refer to Figure 3.10). For carbon, the ROI was 4.0 MeV wide starting 2.5 MeV to the left of the appropriate Compton energy. These were chosen to ensure that there were no inelastic contributions contained in the integrated region.

Because the bin widths of the energy spectra to be integrated were 0.25 MeV, it was necessary to make a correction for the highest and lowest energy
Table 3.6: Final lineshape parameters for all data.

<table>
<thead>
<tr>
<th>Gain Set</th>
<th>Run Range</th>
<th>( \theta ) (deg)</th>
<th>Resolution (MeV)</th>
<th>Peak Position (MeV)</th>
<th>Compton Energy (MeV)</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1004-1272</td>
<td>90.0</td>
<td>1.56</td>
<td>98.9</td>
<td>99.1</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>1627-1830</td>
<td>120.0</td>
<td>1.89</td>
<td>96.5</td>
<td>96.5</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>1870-2165</td>
<td>150.0</td>
<td>1.80</td>
<td>94.8</td>
<td>94.8</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>2454-2744</td>
<td>60.0</td>
<td>1.71</td>
<td>101.8</td>
<td>101.8</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>2997-3279</td>
<td>35.0</td>
<td>1.43</td>
<td>103.4</td>
<td>103.6</td>
<td>0.60</td>
</tr>
<tr>
<td>( ^{12} \text{C} )</td>
<td>819-919</td>
<td>90.0</td>
<td>1.90</td>
<td>103.6</td>
<td>103.6</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>1292-1490</td>
<td>120.0</td>
<td>1.99</td>
<td>103.1</td>
<td>103.2</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>2166-2261</td>
<td>150.0</td>
<td>1.97</td>
<td>102.8</td>
<td>102.8</td>
<td>1.98</td>
</tr>
<tr>
<td></td>
<td>2340-2453</td>
<td>60.0</td>
<td>1.96</td>
<td>104.1</td>
<td>104.1</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td>2745-2764</td>
<td>90.0</td>
<td>2.38</td>
<td>103.5</td>
<td>103.6</td>
<td>1.44</td>
</tr>
<tr>
<td></td>
<td>2918-2996</td>
<td>35.0</td>
<td>1.80</td>
<td>104.4</td>
<td>104.4</td>
<td>2.39</td>
</tr>
</tbody>
</table>

bins in order to obtain an accurate measure of the yield. For example, with the 150 degree deuterium data, the ROI was 93.05–95.25 MeV. That meant that the lower energy for the ROI fell between the energy bins of 92.875 and 93.125.\textsuperscript{\textsc{ii}} Thus, a fraction of counts from the 93.125 energy bin were discarded (see Figure 3.11). The same procedure was used for the higher energy limit of \( \text{PAW.} \)

\textsuperscript{\textsc{ii}}LUCID shifted the energy bins by 0.125 MeV to accommodate PAW.
the ROI. Algebraically, the integrated yield for each tagger channel is given by

\[ Y = \sum_i Y[i] + Y_{corr}^{low} + Y_{corr}^{high}, \]  

(3.6)

where the sum is over the energy bins contained in the ROI and the two correction factors are for the lowest and highest energy bins respectively.

Yields for the entire experiment are given in Table 3.7.

Table 3.7: Integrated yields for both targets and all angles. The width of the ROI was 2.2 MeV for deuterium and 4.0 MeV for carbon.

<table>
<thead>
<tr>
<th>Target</th>
<th>Run Range</th>
<th>( \theta ) (deg)</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1004-1272</td>
<td>90.0</td>
<td>590.2 ± 32.2</td>
</tr>
<tr>
<td></td>
<td>1627-1830</td>
<td>120.0</td>
<td>400.0 ± 27.0</td>
</tr>
<tr>
<td></td>
<td>1870-2165</td>
<td>150.0</td>
<td>614.9 ± 35.0</td>
</tr>
<tr>
<td></td>
<td>2454-2744</td>
<td>60.0</td>
<td>572.6 ± 33.4</td>
</tr>
<tr>
<td></td>
<td>2997-3279</td>
<td>35.0</td>
<td>227.0 ± 22.2</td>
</tr>
<tr>
<td>(^12\text{C})</td>
<td>819-919</td>
<td>90.0</td>
<td>3189.6 ± 70.7</td>
</tr>
<tr>
<td></td>
<td>1292-1490</td>
<td>120.0</td>
<td>6220.7 ± 91.4</td>
</tr>
<tr>
<td></td>
<td>2166-2261</td>
<td>150.0</td>
<td>4206.6 ± 73.0</td>
</tr>
<tr>
<td></td>
<td>2340-2453</td>
<td>60.0</td>
<td>3199.7 ± 67.3</td>
</tr>
<tr>
<td></td>
<td>2745-2764</td>
<td>90.0</td>
<td>1489.7 ± 44.9</td>
</tr>
<tr>
<td></td>
<td>2873-2884</td>
<td>90.0</td>
<td>1299.1 ± 40.7</td>
</tr>
<tr>
<td></td>
<td>2890-2897</td>
<td>90.0</td>
<td>1021.4 ± 51.9</td>
</tr>
<tr>
<td></td>
<td>2918-2996</td>
<td>35.0</td>
<td>1458.4 ± 42.5</td>
</tr>
</tbody>
</table>

3.4 Normalization

In order to obtain absolute cross sections, it was necessary to calculate the normalization for each angle and target. The yields were normalized to solid angle, \( d\Omega \), target density, \( t \), photon flux, \( N_\gamma \), and detector efficiency, \( \varepsilon_{det} \).
3.4.1 Solid Angle

The solid angle for a specific BUNI set-up was given by the area subtended by the aperture, \( A \), divided by the square of the distance from the centre of the target to the inner face of the shielding aperture, \( R \), or

\[
d\Omega = \frac{A}{R^2} = \pi \left( \frac{r}{R} \right)^2,
\]

where \( r = 6.35 \pm 0.05 \), the radius of the shielding aperture. The solid angles for each BUNI set-up are given in Table 3.8. Simulations were done by Warkentin [War99a] to check the effect of finite target size on the solid angle;

<table>
<thead>
<tr>
<th>( \theta ) (deg)</th>
<th>( R ) (\pm 0.1 cm)</th>
<th>( d\Omega ) (msr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.0</td>
<td>123.6</td>
<td>8.29 ± 0.13</td>
</tr>
<tr>
<td>60.0</td>
<td>122.6</td>
<td>8.43 ± 0.13</td>
</tr>
<tr>
<td>90.0</td>
<td>89.6</td>
<td>15.78 ± 0.25</td>
</tr>
<tr>
<td>120.0</td>
<td>122.6</td>
<td>8.43 ± 0.13</td>
</tr>
<tr>
<td>150.0</td>
<td>126.6</td>
<td>7.90 ± 0.13</td>
</tr>
</tbody>
</table>

they were found to be negligible for both the carbon and deuterium targets.

3.4.2 Target Density

The target densities for both \(^{12}\text{C}\) and \(\text{LD}_2\) were calculated from physical dimensions using

\[
t = \frac{\Delta x \rho N_A}{m},
\]

(3.8)
where $\Delta x$ is the thickness of the target, $\rho$ is the density, $N_A$ is Avogadro's number, and $m$ is the molar mass. Note that for the LD$_2$ target, an effective thickness was obtained using a simple Monte Carlo simulation to take into account the rounded ends of the mylar cell. The target density values are given in Table 3.9.

<table>
<thead>
<tr>
<th>Target</th>
<th>$m$ (g/mol)</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>$\Delta x$ (cm)</th>
<th>$t \times 10^{23}$ nuclei/cm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>2.0141</td>
<td>0.162 ± 0.003</td>
<td>12.16 ± 0.22</td>
<td>5.90 ± 0.15</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>12.0000</td>
<td>1.59 ± 0.02</td>
<td>5.15 ± 0.05</td>
<td>4.11 ± 0.07</td>
</tr>
</tbody>
</table>

3.4.3 Photon Flux

The photon flux for a specific tagger channel is given by multiplying the tagging efficiency (explained in Section 2.2.3), $\varepsilon_{tag}$, a correction factor due to tagger rate effects (derived in Appendix C), $f_{rate}$, and the measured electron flux registered in the channel scaler, $S$,

$$N_\gamma = \varepsilon_{tag} f_{rate} S. \quad (3.9)$$

These factors were applied on a run-by-run basis to ensure that the appropriate correction was being made. Note that for simplicity, the rate correction factor also included the yield correction from ghost and stolen coincidence effects. The channel-averaged values$^\dagger$ for tagging efficiency, rate correction factor, measured electron flux, and resulting photon flux are given in Table 3.10.

$^\dagger$The channel scalers for each production run were adjusted such that the non-radiator related background was subtracted in a similar fashion to that for tagging efficiency runs (see Section 2.2.3).

$^\dagger\dagger$The average values were bremsstrahlung-weighted using the channel scalers.
Table 3.10: Photon flux.

<table>
<thead>
<tr>
<th>Target</th>
<th>Run Range</th>
<th>$\theta$ (deg)</th>
<th>$\bar{\epsilon}_{tag}$</th>
<th>$\bar{f}_{rate}$</th>
<th>$\bar{S}$ ($\times 10^{13}$)</th>
<th>$\bar{N}_{\gamma}$ ($\times 10^{13}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1004–1272</td>
<td>90.0</td>
<td>0.535</td>
<td>0.977</td>
<td>1.897</td>
<td>0.987</td>
</tr>
<tr>
<td></td>
<td>1627–1830</td>
<td>120.0</td>
<td>0.528</td>
<td>0.982</td>
<td>1.841</td>
<td>0.950</td>
</tr>
<tr>
<td></td>
<td>1870–2165</td>
<td>150.0</td>
<td>0.530</td>
<td>0.986</td>
<td>2.646</td>
<td>1.375</td>
</tr>
<tr>
<td></td>
<td>2454–2744</td>
<td>60.0</td>
<td>0.537</td>
<td>0.982</td>
<td>2.762</td>
<td>1.450</td>
</tr>
<tr>
<td></td>
<td>2997–3279</td>
<td>35.0</td>
<td>0.531</td>
<td>0.991</td>
<td>1.101</td>
<td>0.576</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>819–919</td>
<td>90.0</td>
<td>0.533</td>
<td>0.976</td>
<td>0.521</td>
<td>0.269</td>
</tr>
<tr>
<td></td>
<td>1292–1490</td>
<td>120.0</td>
<td>0.533</td>
<td>0.981</td>
<td>1.548</td>
<td>0.807</td>
</tr>
<tr>
<td></td>
<td>2166–2261</td>
<td>150.0</td>
<td>0.530</td>
<td>0.986</td>
<td>1.143</td>
<td>0.594</td>
</tr>
<tr>
<td></td>
<td>2340–2453</td>
<td>60.0</td>
<td>0.532</td>
<td>0.988</td>
<td>0.743</td>
<td>0.389</td>
</tr>
<tr>
<td></td>
<td>2745–2764</td>
<td>90.0</td>
<td>0.541</td>
<td>0.984</td>
<td>0.224</td>
<td>0.119</td>
</tr>
<tr>
<td></td>
<td>2873–2884</td>
<td>90.0</td>
<td>0.539</td>
<td>0.988</td>
<td>0.204</td>
<td>0.108</td>
</tr>
<tr>
<td></td>
<td>2890–2897</td>
<td>90.0</td>
<td>0.539</td>
<td>0.942</td>
<td>0.163</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>2918–2996</td>
<td>35.0</td>
<td>0.538</td>
<td>0.994</td>
<td>0.210</td>
<td>0.112</td>
</tr>
</tbody>
</table>

3.4.4 Detector Efficiency

Not all of the tagged photons that were elastically scattered from target nuclei into the solid angle subtended by the shielding aperture were contained in the energy spectrum ROI, and thus were not counted in the integrated yield. Some of these photons were detected in BUNI but appeared outside the ROI. If there were no inelastic contributions, the entire lineshape could have been integrated; this is not the case, resulting in small regions of interest for yield integration. Other photons were absorbed in the target, target containment, air between the target and detector, aperture veto, borated foam shielding, and front middle paddle. Because of this, we needed to correct the integrated yield to include all tagged Compton events scattered from target nuclei, labelled $\mathcal{Y}$. 

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This was represented using the detector efficiency, $\varepsilon_{det}$, in the following way:

\[
Y = \varepsilon_{det} Y, \quad \text{(3.10)}
\]

\[
= \varepsilon_{ROI} \varepsilon_{abs} Y,
\]

where $Y$ is the integrated yield for a tagger channel from (3.6), $\varepsilon_{ROI}$ is the efficiency due to the small size of the ROI, and $\varepsilon_{abs}$ is the efficiency with respect to photon absorption. The ROI efficiency was arrived at using the zero degree spectrum Compton shifted to the appropriate energy, as it represents the response of monochromatic photons in BUNI. For the 150 degree deuterium data, the number of counts in the ROI was $0.856 \times 10^6$ while the total number
of counts in the lineshape was $1.381 \times 10^6$, giving a ROI efficiency of

$$\varepsilon_{ROI} = \frac{\sum_{ROI} Y}{\sum Y},$$

$$= \frac{0.856 \times 10^6}{1.381 \times 10^6},$$

$$= 0.622.$$

The absorption efficiency was broken into two parts: that due to absorption of photons in the target and target apparatus, $\varepsilon_{abs}^1$, and that due to absorption of photons in the aperture veto, borated foam, and front middle paddle, $\varepsilon_{abs}^2$. The first absorption factor was obtained with an EGS simulation (see Warkentin [War99a]), with the 150 degree deuterium number being 0.958.\footnote{This number varied as a function of angle because the amount of material the photons traversed changed, in addition to the energy dependence of absorption.}

The second was found by integrating the entire zero degree lineshape, and then taking a ratio using the adjusted tagger scalers:

$$\varepsilon_{abs}^2 = \frac{\sum Y}{\varepsilon_{tag} S},$$

$$= \frac{1.381 \times 10^6}{(0.531)(2.823 \times 10^6)},$$

$$= 0.921.$$

Using these values, the absorption efficiency for the 150 degree deuterium data was found to be 0.882, giving an overall detector efficiency of 0.548 (also shown in Figure 3.12). Detector efficiencies for each target and angle are given in Table 3.11.
Table 3.11: Detector efficiencies and corrected yields for each target and angle.

<table>
<thead>
<tr>
<th>Target</th>
<th>Run Range</th>
<th>( \theta ) (deg)</th>
<th>( \varepsilon_{ROI} )</th>
<th>( \varepsilon_{abs} )</th>
<th>( \varepsilon_{det} )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1004-1272</td>
<td>90.0</td>
<td>0.622</td>
<td>0.892</td>
<td>0.555</td>
<td>1064.3 ± 58.1</td>
</tr>
<tr>
<td></td>
<td>1627-1830</td>
<td>120.0</td>
<td>0.621</td>
<td>0.892</td>
<td>0.554</td>
<td>722.0 ± 48.7</td>
</tr>
<tr>
<td></td>
<td>1870-2165</td>
<td>150.0</td>
<td>0.622</td>
<td>0.882</td>
<td>0.548</td>
<td>1121.8 ± 63.8</td>
</tr>
<tr>
<td></td>
<td>2454-2744</td>
<td>60.0</td>
<td>0.621</td>
<td>0.892</td>
<td>0.554</td>
<td>1033.6 ± 60.3</td>
</tr>
<tr>
<td></td>
<td>2997-3279</td>
<td>35.0</td>
<td>0.621</td>
<td>0.883</td>
<td>0.549</td>
<td>413.4 ± 40.5</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>819-919</td>
<td>90.0</td>
<td>0.819</td>
<td>0.794</td>
<td>0.650</td>
<td>4907.9 ± 108.9</td>
</tr>
<tr>
<td></td>
<td>1292-1490</td>
<td>120.0</td>
<td></td>
<td>0.806</td>
<td>0.660</td>
<td>9429.3 ± 138.5</td>
</tr>
<tr>
<td></td>
<td>2166-2261</td>
<td>150.0</td>
<td></td>
<td>0.815</td>
<td>0.667</td>
<td>6305.9 ± 109.5</td>
</tr>
<tr>
<td></td>
<td>2340-2453</td>
<td>60.0</td>
<td></td>
<td>0.801</td>
<td>0.656</td>
<td>4879.3 ± 102.6</td>
</tr>
<tr>
<td></td>
<td>2745-2764</td>
<td>90.0</td>
<td></td>
<td>0.794</td>
<td>0.650</td>
<td>2292.2 ± 69.1</td>
</tr>
<tr>
<td></td>
<td>2873-2884</td>
<td>90.0</td>
<td></td>
<td>0.794</td>
<td>0.650</td>
<td>1999.0 ± 62.6</td>
</tr>
<tr>
<td></td>
<td>2890-2897</td>
<td>90.0</td>
<td></td>
<td>0.794</td>
<td>0.650</td>
<td>1571.6 ± 79.9</td>
</tr>
<tr>
<td></td>
<td>2918-2996</td>
<td>35.0</td>
<td></td>
<td>0.808</td>
<td>0.661</td>
<td>2206.0 ± 64.3</td>
</tr>
</tbody>
</table>

3.5 Differential Cross Sections

Once the yield and normalization were obtained, it was possible to calculate the differential cross section. In the lab frame, it is given by the formula

\[
\frac{d\sigma}{d\Omega} = \frac{\gamma}{N, t d\Omega}.
\]

In order to compare with theory calculations, which are usually given in terms of the centre-of-mass, or CM, frame of reference (where CM quantities are signified by an asterisk), it was necessary to multiply the lab cross section by the Jacobian (derived in Appendix A and listed for each angle and target in Table A.1), \( J(\theta) \), so that

\[
\frac{d\sigma}{d\Omega^*} = \frac{\gamma}{N, t d\Omega} J(\theta).
\]
For both targets, the CM differential cross sections are given as a function of CM angle, $\theta^*$, in Table 3.12.

Table 3.12: Elastic differential cross sections in the CM frame as a function of CM angle for both deuterium and carbon.

<table>
<thead>
<tr>
<th>Target</th>
<th>Run Range</th>
<th>$\theta^*$</th>
<th>$\frac{d\sigma}{d\Omega^*}$ (nb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(deg)</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>1004–1272</td>
<td>93.0</td>
<td>11.7 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>1627–1830</td>
<td>122.6</td>
<td>16.1 ± 1.1</td>
</tr>
<tr>
<td></td>
<td>1870–2165</td>
<td>151.5</td>
<td>19.0 ± 1.1</td>
</tr>
<tr>
<td></td>
<td>2454–2744</td>
<td>62.7</td>
<td>13.7 ± 0.8</td>
</tr>
<tr>
<td></td>
<td>2997–3279</td>
<td>36.8</td>
<td>13.5 ± 1.3</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>819–919</td>
<td>90.5</td>
<td>281.3 ± 6.2</td>
</tr>
<tr>
<td></td>
<td>1292–1490</td>
<td>120.5</td>
<td>340.2 ± 5.0</td>
</tr>
<tr>
<td></td>
<td>2166–2261</td>
<td>150.3</td>
<td>331.7 ± 5.8</td>
</tr>
<tr>
<td></td>
<td>2340–2453</td>
<td>60.5</td>
<td>358.2 ± 7.5</td>
</tr>
<tr>
<td></td>
<td>2745–2764</td>
<td>90.5</td>
<td>297.5 ± 9.0</td>
</tr>
<tr>
<td></td>
<td>2873–2884</td>
<td>90.5</td>
<td>285.6 ± 8.9</td>
</tr>
<tr>
<td></td>
<td>2890–2897</td>
<td>90.5</td>
<td>296.8 ± 15.1</td>
</tr>
<tr>
<td></td>
<td>2918–2996</td>
<td>35.3</td>
<td>570.1 ± 16.6</td>
</tr>
</tbody>
</table>

3.6 Inelastic Contributions

As evidenced in Figure 3.10, there was a substantial inelastic contribution to the 150 degree deuterium energy spectrum. It is of interest to see what the cross section for this process is for all angles.

An estimate of the inelastic energy spectrum was made for the data by subtracting the zero degree calibration lineshape from that of the elastic. The resulting inelastic spectrum is shown in Figure 3.13. Then, to integrate the yield, a region of interest was taken starting at the lower end of the elastic ROI, 1.7 MeV below the Compton peak. This was extended a further 10.3 MeV giving a ROI 12.0 MeV wide (see Figure 3.13). Note that the cross
Figure 3.13: Inelastic energy spectrum for the 150 degree deuterium data. Shown is the 12.0 MeV yield integration ROI.

The section calculation was done in a similar manner to that for the elastic except that there was no correction made for ROI efficiency, as there was no inelastic lineshape for comparison. The inelastic yields and resulting cross sections are given in Table 3.13.

Table 3.13: Inelastic differential cross sections in the CM frame as a function of CM angle for deuterium.

<table>
<thead>
<tr>
<th>Run Range</th>
<th>$\theta^*$ (deg)</th>
<th>$\gamma$</th>
<th>$\frac{d\sigma}{d\Omega^*}$ (nb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1004-1272</td>
<td>93.0</td>
<td>329.3 ± 79.4</td>
<td>3.8 ± 0.9</td>
</tr>
<tr>
<td>1627-1830</td>
<td>122.6</td>
<td>234.1 ± 72.5</td>
<td>5.3 ± 1.6</td>
</tr>
<tr>
<td>1870-2165</td>
<td>151.5</td>
<td>514.2 ± 97.2</td>
<td>8.7 ± 1.6</td>
</tr>
<tr>
<td>2454-2744</td>
<td>62.7</td>
<td>386.0 ± 101.5</td>
<td>5.7 ± 1.3</td>
</tr>
<tr>
<td>2997-3279</td>
<td>36.8</td>
<td>87.9 ± 58.4</td>
<td>3.3 ± 1.9</td>
</tr>
</tbody>
</table>
3.7 Beam Consistency Results

The results of the $^{12}\text{C}(\gamma, \gamma')^{12}\text{C}$ rate test are listed in Figure 3.14. We can see that the cross section is not a function of the instantaneous tagger focal plane rate. Further, the measurements done at the beginning and end of the experiment agree, which suggests that the cross section is not a function of time. Both of these factors are useful consistency checks, and lend credibility to the deuterium cross section reported earlier in the thesis.

Figure 3.14: Results of the $^{12}\text{C}(\gamma, \gamma')^{12}\text{C}$ rate test. Carbon cross sections measured at 90 degrees and different instantaneous rates are listed. The square data point was taken at the beginning of the experiment and the circles were taken near the end.
3.8 Systematic Errors

Certain factors contributing to the cross section possessed non-zero systematic uncertainties. Errors in the solid angle and target density were due to measurement errors in the physical dimensions used in their calculation. The photon absorption was assigned an error because of a small inconsistency between the measured and simulated values of $\varepsilon_{\text{abs}}^2$. Uncertainty in the yield was due to the PMT gain calibration used to position the energy spectra. The systematic error for the yield was arrived at by changing the PMT calibration without changing the $\chi^2$ values more than 50%, and then quantifying the effect on the cross section. The effect of gain calibration error would most likely reduce the yield, because valid counts would get shifted out of the integration ROI. Looking at the error in the yield as a function of run number (see Table 3.14) and noting that the zero degree calibration was done during runs 2299–2300, we can see that the greatest systematic errors occur furthest from the in-beam calibration, which is a direct result of the failure of the gain tracking system.

A summary of the systematic errors and their effects are listed in Table 3.14. Note that these errors are summed in quadrature because they are assumed to be independent of each other.

<table>
<thead>
<tr>
<th>Target</th>
<th>Run Range</th>
<th>$\theta$ (deg)</th>
<th>Source of Error (%)</th>
<th>Total (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$t$</td>
<td>$d\Omega$</td>
<td>$\varepsilon_{\text{abs}}$</td>
</tr>
<tr>
<td>d</td>
<td>1004–1272</td>
<td>90.0</td>
<td>2.5</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>1627–1830</td>
<td>120.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1870–2165</td>
<td>150.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2454–2744</td>
<td>60.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2997–3279</td>
<td>35.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>all</td>
<td>all</td>
<td>2.5</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 3.14: Systematic errors in the cross section and their effects.
Chapter 4

RESULTS AND CONCLUSIONS

In this chapter, differential cross sections of the $d(\gamma, \gamma')d$ reaction are presented and compared to various theoretical calculations. The results and discussion of the $^{12}\text{C}(\gamma, \gamma')^{12}\text{C}$ measurement constitutes the thesis of Ref. [War99a], and thus are not given here.

4.1 Comparison of Results with Theory

4.1.1 Simplistic Field-Theoretic Calculation

In order to use the simplistic calculation from Section 1.3 to get an idea of the isospin-averaged polarizabilities in the deuteron, the MEC enhancement factor, $\kappa$, was adjusted until the forward data points were roughly described by the theory curve. This was done because the forward angles are sensitive to the sum of the polarizabilities, which is assumed to follow the Baldin sum rules given by (1.7) and (1.11), so that $\bar{\alpha}_N + \bar{\beta}_N = 15.0 \pm 0.5$. The difference in the polarizabilities was then adjusted until the theory curve gave an estimate of the backward data points, and then the sum rule was used to extract the individual electric and magnetic polarizabilities. For this calculation, the MEC factor that gave good agreement with the forward points was $\kappa = 0.95$, well above the accepted value of $\kappa = 0.35$. With this value, the resulting polarizabilities were found to be $\bar{\alpha}_N = 9.0$ and $\bar{\beta}_N = 6.0$, significantly different from the best free values of $\bar{\alpha}_N = 12.0$ and $\bar{\beta}_N = 3.0$. The experimental results are shown and compared to the simple calculation in Figure 4.1.

In addition, it is instructive to look at the energy dependence of the
Figure 4.1: Comparison of lab differential cross sections for elastic scattering of 94.2 MeV photons from deuterium with a simple field-theoretic calculation. The lower set of curves uses the nominal MEC enhancement factor, $\kappa = 0.35$, while the upper curves use $\kappa = 0.95$. The dashed curves were calculated with the best free values of the nucleon polarizabilities, $\tilde{\alpha}_N = 12.0$ and $\tilde{\beta}_N = 3.0$, while the solid curves use $\tilde{\alpha}_N = 9.0$ and $\tilde{\beta}_N = 6.0$. Errors bars are statistical only.

differential cross section. To do this, we split the 150 degree data into four separate tagger bins\(^1\) of energy 87.2, 93.0, 97.9, and 102.5 MeV, respectively. Further, using the 49 and 69 MeV energy data from Ref. [Luc94], we can look at a range of over 50 MeV. Although neither the 49 MeV or the 69 MeV data contained 150 degree data points, we can make a reasonable fit to the existing points to infer cross sections at this angle. The results are plotted with the current data and compared to the simple calculation in Figure 4.2. We can

\(^1\)It was mentioned earlier that due to statistical reasons, only one energy bin was possible; the profusion of counts and excellent lineshape at 150 degrees allows us to sub-divide the tagger focal plane into four bins.
Figure 4.2: Comparison of lab differential cross sections for elastic photon scattering from deuterium at a lab angle of 150 degrees with a simple field-theoretic calculation. The squares are extrapolated from a fit to data given in Ref. [Luc94], and the circles are data from the present thesis. The lower set of curves use the nominal MEC enhancement factor, $\kappa = 0.35$, while the upper curves use $\kappa = 0.95$. The dashed curves were calculated with the best free values of the nucleon polarizabilities, $\tilde{\alpha}_N = 12.0$ and $\tilde{\beta}_N = 3.0$, while the solid curves use $\tilde{\alpha}_N = 9.0$ and $\tilde{\beta}_N = 6.0$. Error bars are statistical only.

see that the cross section varies much less with energy than the calculation suggests.

Noting this, we can examine the angular distributions from Ref. [Luc94] for 49 and 69 MeV (see Figures 4.3 (a) and Figures 4.3 (b)). Because of the simple nature of the calculation, it is necessary to adjust the MEC enhancement factor so the theory curves agree with the forward points. For the 49 MeV, we use $\kappa = 0.20$, which is lower than the nominal value, while for the 69 MeV data, we find that $\kappa = 0.45$ is requisite. Because of the lower energy, the sensitivity of the cross section at backward angles to $\tilde{\alpha}_N - \tilde{\beta}_N$ is diminished, but it is still
Figure 4.3: Comparison of lab differential cross sections for elastic scattering of (a) 49 MeV and (b) 69 MeV photons from deuterium with a simple field-theoretic calculation. Data are from Ref. [Luc94]. The top curves are for $\kappa = 0.95$. In (a), the lower and middle curves are for $\kappa = 0.20$ and $\kappa = 0.35$, respectively; in (b), they are for $\kappa = 0.35$ and $\kappa = 0.45$, respectively. The dashed curves were calculated with the best free values of the nucleon polarizabilities, $\bar{\alpha}_N = 12.0$ and $\bar{\beta}_N = 3.0$, while the solid curves use $\bar{\alpha}_N = 9.0$ and $\bar{\beta}_N = 6.0$. Error bars are statistical only.
apparent that in the context of the simple calculation, the best free value of $\tilde{\alpha}_N - \tilde{\beta}_N = 9.0$ is too large.

4.1.2 Karakowski et al.

A plot of this calculation along with the data from the present thesis is listed in Figure 4.4. We can see that the calculation underestimates the data at all angles, regardless of the polarizability values. Because MEC are given by $\pi$ exchanges only, and they do not have a large effect in this calculation, it is not surprising that the data are underestimated.

![Figure 4.4: Comparison of lab differential cross sections for elastic scattering of 94.2 MeV photons from deuterium with the calculation of Karakowski et al. The solid curve is for $\tilde{\alpha}_N = 12$ and $\tilde{\beta}_N = 2$, the dotted curve is for $\tilde{\alpha}_N = 10$ and $\tilde{\beta}_N = 4$, and the dashed curve is for $\tilde{\alpha}_N = 14$ and $\tilde{\beta}_N = 0$. Error bars are statistical only.](image)

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4.1.3 Wilbois et al.

Valid comparison of this calculation with the data is not possible because it does not include the nucleon polarizabilities at all. However, at back angles, this curve does surprisingly well (see Figure 4.5), and is higher than the corresponding no-polarizability curve from Ref. [Lev95]. It would be interesting to see the results of this calculation with polarizabilities, as the inclusion of subnucleonic degrees of freedom in this fashion tends to suppress the forward angles relative to the back angles (see Figure 4.6).

Figure 4.5: Comparison of CM differential cross sections for elastic scattering of 94.2 MeV photons from deuterium with the calculation of Wilbois et al. where $\bar{\alpha}_N = \bar{\beta}_N = 0$. Error bars are statistical only.
4.1.4 Levchuk et al.

This is the most comprehensive of the diagrammatic calculations. Meson exchange currents are treated in great detail, as is re-scattering. Thus, we expect it to do the best job in estimating the data.

![Graph](image)

Figure 4.6: Comparison of CM differential cross sections for elastic scattering of 94.2 MeV photons from deuterium with the calculation of Levchuk et al. The dotted curve is for $\bar{\alpha}_N = \bar{\beta}_N = 0$, the dash-dotted curve is for $\bar{\alpha}_N = 4$ and $\bar{\beta}_N = 11$, the dashed curve is for $\bar{\alpha}_N = 7.5$ and $\bar{\beta}_N = 7.5$, and the solid curve is for nominal free nucleon polarizabilities of $\bar{\alpha}_N = 12.0$ and $\bar{\beta}_N = 3.0$. Error bars are statistical only.

Although no adjustment of $\bar{\alpha}_N - \bar{\beta}_N$ gives good agreement with the data, the best agreement is found for a range of 0 to -7 (see Figure 4.6), giving

$$\bar{\alpha}_N = 6 \pm 2, \quad (4.1)$$

$$\bar{\beta}_N = 9 \mp 2, \quad (4.2)$$
for the isospin-averaged nucleon polarizabilities. It is important to note that these are significantly different than the nominal free polarizabilities, $\tilde{\alpha}_N = 12$ and $\tilde{\beta}_N = 3$.

It is interesting to note that Levchuk's calculation does a similar job in estimating the 49 and 69 MeV data of Lucas (see Ref. [Lev97]), where it does a good job at forward angles and is too low at backward angles for nominal values of the nucleon polarizabilities.

4.1.5 Beane et al.

In the last few years, $\chi$PT has proven to be a very successful theory in predicting observables for low-energy pion production and other sub-atomic processes. Because of this, we expect it to do well in describing Compton scattering from deuterium. Looking at a comparison between the theory curves and the data (see Figure 4.7), we can see that they do not agree. However, the change between the $\mathcal{O}(Q^2)$ and $\mathcal{O}(Q^3)$ calculations is rather large, and it seems to be approaching the data. This suggests that the fourth order terms cannot be neglected, and it is hoped that they further bring the curve closer in line with the data.

4.2 Conclusions

We have measured an angular distribution of the elastic photon scattering cross section from deuterium at 94.2 MeV with good statistical accuracy.

Using a naive modified impulse-approximation calculation with two free parameters (an MEC enhancement factor $\kappa$ in addition to $\tilde{\alpha}_N - \tilde{\beta}_N$, the difference in the isospin-averaged nucleon polarizabilities) and the sum rules from (1.7) and (1.11), we have found that $\tilde{\alpha}_N \simeq 9$ and $\tilde{\beta}_N \simeq 6$ with $\kappa = 0.95$, as opposed to the nominal free values of the polarizabilities $\tilde{\alpha}_N = 12$ and $\tilde{\beta}_N = 3$. 

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Figure 4.7: Comparison of lab differential cross sections for elastic scattering of 94.2 MeV photons from deuterium with the calculation of Beane et al. The dashed curve is to order $Q^2$ while the solid curve is to order $Q^3$. It is important to note that because $\chi$PT predicts values for the polarizabilities, there are no free parameters in this theory. Error bars are statistical only.

Further, the accepted value of the MEC enhancement factor for the deuteron is $\kappa = 0.35$, much less than that needed by the simple calculation to estimate the data. Utilizing the data from Ref. [Luc94], the cross section tends to remain relatively constant over the range of 50–100 MeV, whereas the theories suggest that there should be a significant decrease. Although not much stock should be put in these values of $\tilde{a}_N$ and $\tilde{b}_N$, this calculation does show, in a crude fashion, how the polarizabilities and MEC affect the cross section.

Comparing the data to rigorous calculations, we were not able to get good agreement for any of the theories. However, the Levchuk et al. calculation did the best job and we extracted approximate values for the isospin-averaged
polarizabilities of $\tilde{\alpha}_N = 6 \pm 2$ and $\tilde{\beta}_N = 9 \mp 2$. Now, if we assume that because the deuteron is so loosely bound (2.2 MeV) its constituents are not subjected to large nuclear and MEC effects, we can use the best free values of the proton polarizabilities from (1.8) and (1.9), to obtain the neutron polarizabilities:

\begin{align*}
\tilde{\alpha}_n &= 0 \pm 4, \\
\tilde{\beta}_n &= 16 \mp 4.
\end{align*}

(4.3) \hspace{2cm} (4.4)

However, charge symmetry says that the properties of the neutron and proton should be quite similar, which would preclude the above values for the neutron polarizabilities. If we are inclined to take this stance, then we are only able to extract isospin-averaged polarizabilities, suggesting that both the proton and neutron polarizabilities are modified drastically from their free values, in the nuclear medium.

It is hoped that the measurement of quasi-free photon scattering from deuterium [Kol99] that is currently being analyzed, in addition to further theoretical work, will shed some light on the matter of the neutron polarizability.
References


Appendix A
KINEMATICS

The kinematics for Compton scattering will be examined in this appendix as will the Jacobian used to transform the lab angles and differential cross sections to the CM frame.

A.1 Compton Scattering Kinematics

Compton scattering is a two-body reaction that occurs when a photon scatters elastically from a charged particle (as illustrated in Figure A.1). Given a photon scattering from a particle X initially at rest (with four-momenta $k$ and $p$, respectively), we have from four-momentum conservation

$$k + p = k' + p', \quad (A.1)$$

which can also be written in terms of three-momentum and energy conservation as follows

$$k + p = k' + p'. \quad (A.2)$$

$$k_0 + p_0 = k'_0 + p'_0. \quad (A.3)$$

Figure A.1: Compton scattering in the lab frame.
Squaring (A.1) and reducing we have

\[(k + p)^2 = (k' + p')^2,\]
\[k^2 + p^2 + 2k \cdot p = k'^2 + p'^2 + 2k' \cdot p',\]
\[m_X^2 + 2k_0p_0 - 2k \cdot p = m_X^2 + 2k_0p'_0 - 2k' \cdot p'.\]

Noting that the square of a four-vector is the square of the corresponding particle's mass, we have in the lab frame

\[k_0 m_X = k'_0 p'_0 - |p'| |p'| \cos(\theta_\gamma + \theta_X),\]
\[= k'_0 [p'_0 - |p'| (\cos \theta_\gamma \cos \theta_X + \sin \theta_\gamma \sin \theta_X)]. \quad (A.4)\]

Separating (A.2) into x and y components, for the x-direction

\[k_0 = k'_0 \cos \theta_\gamma + |p'| \cos \theta_X\]

so that

\[\cos \theta_X = \frac{k_0 - k'_0 \cos \theta_\gamma}{|p'|}. \quad (A.5)\]

and for the y-direction

\[k'_0 \sin \theta_\gamma = |p'| \sin \theta_X\]

leading to

\[\sin \theta_X = \frac{k'_0 \sin \theta_\gamma}{|p'|}. \quad (A.6)\]

Substituting (A.5) and (A.6) into (A.4), we arrive at

\[k_0 m_X = k'_0 [p'_0 - \cos \theta_\gamma (k_0 - k'_0 \cos \theta_\gamma) + \sin \theta_\gamma (k'_0 \sin \theta_\gamma)],\]
\[= k'_0 [p'_0 - k_0 \cos \theta_\gamma + k'_0 \cos^2 \theta_\gamma + k'_0 \sin^2 \theta_\gamma],\]
\[= k'_0 [p'_0 - k_0 \cos \theta_\gamma + k'_0].\]

Using energy conservation, \(k'_0 + p'_0 = k_0 + p_0 = k_0 + m_X\), we can write

\[k_0 m_X = k'_0 [k_0 + m_X - k_0 \cos \theta_\gamma],\]
\[= k'_0 [m_X + k_0 (1 - \cos \theta_\gamma)],\]

which gives the usual Compton scattering formula for the energy of the scattered photon

\[k'_0 = \frac{k_0}{1 + \gamma (1 - \cos \theta_\gamma)}, \quad (A.7)\]
where \( \gamma = \frac{k_0}{m_x} \). If we define the energy transferred to the target as \( T = k_0 - k_0' \), then

\[
T = k_0 - \frac{k_0}{1 + \gamma(1 - \cos \theta_\gamma)}',
\]

\[
= k_0 \left[ 1 - \frac{1}{1 + \gamma(1 - \cos \theta_\gamma)} \right],
\]

\[
= k_0 \left[ \frac{1 + \gamma(1 - \cos \theta_\gamma)}{1 + \gamma(1 - \cos \theta_\gamma)} \right],
\]

which gives

\[
T = k_0 \left[ \frac{\gamma(1 - \cos \theta_\gamma)}{1 + \gamma(1 - \cos \theta_\gamma)} \right]. \tag{A.8}
\]

For the various targets and angles used in this experiment, the scattered photon energies and energy transfers are given in Table A.1.

### A.2 Jacobians

Jacobians are needed to transform the lab angles and differential cross sections into the CM frame. The Compton scattering reaction in the CM frame is illustrated in Figure A.2. Note that in this frame, \( k^\ast = -p^\ast \) and \( k''^\ast = -p''^\ast \).

![Compton scattering in the CM frame.](image)

Figure A.2: Compton scattering in the CM frame.

Since the total cross section should be invariant, i.e. \( d\sigma = d\sigma^\ast \), then

\[
\left( \frac{d\sigma}{d\Omega} \right) d\Omega = \left( \frac{d\sigma}{d\Omega^\ast} \right) d\Omega^\ast.
\]
or

\[ \left( \frac{d\sigma}{d\Omega} \right) = \left( \frac{d\sigma}{d\Omega^*} \right) \frac{d\Omega}{d\Omega^*}. \]

so that the Jacobian to transform the differential cross section from lab to CM frames is given by

\[ J(\theta^*) = \frac{d\Omega}{d\Omega^*}. \]

To arrive at an expression for \( J(\theta^*) \), we must look at

\[ J(\theta^*) = \frac{d\Omega}{d\Omega^*}, \]

\[ = \frac{\sin \theta d\theta d\phi}{\sin \theta^* d\theta^* d\phi^*}, \]

\[ = \frac{d(-\cos \theta)}{d(-\cos \theta^*)}, \]

\[ = \frac{d}{d(\cos \theta^*)} \cos \theta, \]

where \( d\phi = d\phi^* \) because there is no \( \phi \) dependence in the lab to CM conversion. In order to calculate the Jacobian, it is instructive to look at the square of the total four-momentum while again noting the invariance of the square of a four-vector,

\[(k + p)^2 = (k^* + p^*)^2.\]

\[k^2 + p^2 + 2k \cdot p = k'^2 + p'^2 + 2k^* \cdot p^*,\]

\[k \cdot p = k^* \cdot p^*,\]

\[k_0 m_X = k_0^* p_0^* - k^* \cdot p^*,\]

\[k_0 m_X = k_0^* \sqrt{k_0^* m_X^2 + m_X^2 + k_0^* 2},\]

\[k_0 m_X = k_0^* W^*,\]

where

\[W^* = \sqrt{k_0^* m_X^2 + m_X^2 + k_0^* 2}\]

is the total CM energy. Solving (A.9) for \( k_0^* \) yields

\[k_0^* = \left( \frac{m_X}{2k_0 + m_X} \right)^{\frac{1}{2}} k_0.\]

(A.10)
Now, to find relations between $\cos \theta$ and $\cos \theta^*$ it is useful to inspect

\begin{align*}
(k - k')^2 &= (k^* - k^*)^2, \\
k^2 + k'^2 - 2k \cdot k' &= k^{*2} + k'^{*2} - 2k^* \cdot k'^*, \\
k \cdot k' &= k^* \cdot k'^*, \\
k_0k'_0 - k \cdot k' &= k_0^*k'_0 - k^* \cdot k'^*, \\
k_0k'_0 - k_0k'_0 \cos \theta &= k_0^*k'_0 - k_0^*k'^_0 \cos \theta^*, \\
k_0k'_0(1 - \cos \theta) &= k_0^*k'_0(1 - \cos \theta^*),
\end{align*}

but $k_0^* = k'_0$ leading to

\[k_0k'_0(1 - \cos \theta) = k_0^2(1 - \cos \theta^*).\]

Using (A.7) and (A.10), we have

\[\frac{k_0^2m_X}{m_X + k_0(1 - \cos \theta)}(1 - \cos \theta) = \frac{k_0^2m_X}{2k_0 + m_X}(1 - \cos \theta^*),\]

which can be reduced to

\[\cos \theta = \frac{m_X \cos \theta^* + k_0(1 + \cos \theta^*)}{m_X + k_0(1 + \cos \theta^*)}. \tag{A.11}\]

Thus the Jacobian for converting the differential cross section from the lab to CM frame in terms of $\theta^*$ is

\[J(\theta^*) = \frac{d}{d(\cos \theta^*)} \left[ \frac{m_X \cos \theta^* + k_0(1 + \cos \theta^*)}{m_X + k_0(1 + \cos \theta^*)} \right],\]

\[= \frac{m_X(2k_0 + m_X)}{[m_X + k_0(1 + \cos \theta^*)]^2}. \tag{A.12}\]

Solving (A.11) for $\cos \theta^*$ gives the angle transformation for lab to CM frames

\[\cos \theta^* = \frac{m_X \cos \theta - k_0(1 - \cos \theta)}{m_X + k_0(1 - \cos \theta)},\]

which can be substituted into (A.12) to give the Jacobian in terms of the lab angle $\theta$,

\[J(\theta) = \frac{[m_X + k_0(1 - \cos \theta)]^2}{m_X(2k_0 + m_X)}. \tag{A.13}\]

For the various targets and angles used in this experiment, the Jacobians and CM scattering angles are given in Table A.1.
Table A.1: Compton energies, Jacobians and CM angles for $k_0 = 104.61$ MeV.

<table>
<thead>
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<th>Target</th>
<th>$m_X$ (u)</th>
<th>$\theta$ (deg)</th>
<th>$\theta^*$ (deg)</th>
<th>$k'_0$ (MeV)</th>
<th>$T$ (MeV)</th>
<th>$J(\theta)$</th>
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<td>99.09</td>
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Appendix B

DERIVATION OF TARGET FULL VERSUS EMPTY RUN-TIME RATIO

A derivation of the suggested ratio of time spent running target full versus empty is given. Note that this is a modified version of the treatment given in Appendix D of Igarashi [Iga93].

The yield for given region of interest is given by

\[ Y = Y_f - \frac{N_{\gamma f}}{N_{\gamma e}} Y_e, \quad (B.1) \]

where \( Y_f \) and \( Y_e \) are the respective target full and empty yields and \( N_\gamma \) is the corresponding photon flux needed to produce the yield. The full and empty yields can each be written in terms of the prompt and accidental yields and the prompt-accidental scaling ratio, \( r_{pa} \), so that (B.1) reads

\[ Y = (Y_f^p - r_{pa} Y_f^a) - \frac{N_{\gamma f}}{N_{\gamma e}} (Y_e^p - r_{pa} Y_e^a). \]

Defining the following:

\[ F^p = \frac{Y_f^p}{N_{\gamma f}}, \]
\[ F^a = \frac{Y_f^a}{N_{\gamma f}}, \]
\[ E^p = \frac{Y_e^p}{N_{\gamma e}}, \]
\[ E^a = \frac{Y_e^a}{N_{\gamma e}}, \]

using the photon flux rates, \( R_f \) and \( R_e \), and the full and empty times \( t_f \) and \( t_e \), we can write

\[ Y = (F^p - r_{pa} F^a) R_f t_f - \frac{R_f t_f}{R_e t_e} (E^p - r_{pa} E^a) R_e t_e, \]
\[ = R_f t_f (F^p - r_{pa} F^a - E^p + r_{pa} E^a). \]

As well, the variance of the yield can be written

\[ \sigma_Y^2 = \sigma_{Y_f}^2 + \left[ \frac{N_{\gamma f}}{N_{\gamma e}} \right]^2 \sigma_{Y_e}^2, \]
\[ = (F^p + r_{pa} F^a) R_f t_f + \left[ \frac{R_f t_f}{R_e t_e} \right]^2 (E^p + r_{pa} E^a) R_e t_e, \]
\[ = R_f t_f \left[ (F^p + r_{pa} F^a) + \frac{R_f t_f}{R_e t_e} (E^p + r_{pa} E^a) \right]. \]
It is desirable to minimize the relative error in the yield, $\sigma_Y/Y$: however, for convenience we work with the square:

$$
\frac{\sigma_Y^2}{Y^2} = \frac{R_f t_f \left[ (F^p + r_{pa}^2 F^a) + \frac{R_L t_f}{R_{L}} (E^p + r_{pa}^2 E^a) \right]}{\left[ R_f t_f (F^p - r_{pa} F^a - E^p + r_{pa} E^a) \right]^2},
$$

$$
= \frac{(F^p + r_{pa}^2 F^a) + \frac{R_L t_f}{R_{L}} (E^p + r_{pa}^2 E^a)}{R_f t_f (F^p - r_{pa} F^a - E^p + r_{pa} E^a)^2}.
$$

(B.2)

Since we are trying to ascertain the ratio of full-empty time that will minimize the error in the yield, we express the target full time as a function of the empty time and the total time available, $t = t_f + t_e$, so that

$$
\frac{\sigma_Y^2}{Y^2} = \frac{(F^p + r_{pa}^2 F^a) + \frac{R_f (t - t_e)}{R_{L}} (E^p + r_{pa}^2 E^a)}{R_f (t - t_e) (F^p - r_{pa} F^a - E^p + r_{pa} E^a)^2},
$$

$$
= \frac{1}{R_f (F^p - r_{pa} F^a - E^p + r_{pa} E^a)^2} \left[ \frac{(F^p + r_{pa}^2 F^a)}{t - t_e} + \frac{R_f (E^p + r_{pa}^2 E^a)}{R_{L} t_e} \right].
$$

Now, taking the derivative with respect to $t_e$ and setting it to zero, we have

$$
\frac{\partial}{\partial t_e} \left( \frac{\sigma_Y^2}{Y^2} \right) = \frac{1}{R_f (F^p - r_{pa} F^a - E^p + r_{pa} E^a)^2} \times \left[ \frac{(F^p + r_{pa}^2 F^a)}{(t - t_e)^2} - \frac{R_f (E^p + r_{pa}^2 E^a)}{R_{L} t_e^2} \right],
$$

$$
= 0,
$$

which gives an equation quadratic in $t_e$,

$$
\left( 1 - \frac{\alpha}{R} \right) t_e^2 + \frac{2 \alpha t}{R} t_e - \frac{\alpha^2}{R} = 0,
$$

(B.3)

where we have defined the following:

$$
\alpha = \frac{R_f}{R_{L}},
$$

$$
R = \frac{F^p + r_{pa}^2 F^a}{E^p + r_{pa}^2 E^a}.
$$

The relevant solution to (B.3) is

$$
t_e = t \frac{\sqrt{\frac{\alpha}{R}}}{1 + \sqrt{\frac{\alpha}{R}}},
$$

$$
= (t_f + t_e) \frac{\sqrt{\frac{\alpha}{R}}}{\sqrt{1 + \frac{\alpha}{R}}},
$$

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which gives
\[ \frac{t_e}{t_f} = \sqrt{\frac{\alpha}{R}} \] (B.4)

for the suggested run-time ratio.

For a set of full and empty test runs of length \( \tau_f \) and \( \tau_e \) respectively, we can write

\[ \alpha = \frac{R_f}{R_e} = \frac{N_{\gamma f} \tau_e}{N_{\gamma e} \tau_f}, \]
\[ R = \frac{F^p + r_{pa}^2 F^a}{E^p + r_{pa}^2 E^a} = \frac{(Y_f^p + r_{pa}^2 Y_f^a) \tau_e}{(Y_e^p + r_{pa}^2 Y_e^a) \tau_f}, \]

so that (B.4) becomes

\[ \frac{t_e}{t_f} = \sqrt{\frac{N_{\gamma f} \tau_e}{N_{\gamma e} \tau_f} \frac{(Y_e^p + r_{pa}^2 Y_e^a) \tau_e}{(Y_f^p + r_{pa}^2 Y_f^a) \tau_f}}, \]
\[ \frac{N_{\gamma f} (Y_e^p + r_{pa}^2 Y_e^a)}{N_{\gamma e} (Y_f^p + r_{pa}^2 Y_f^a)}, \] (B.5)

If we rewrite the photon flux in terms of the tagging efficiency, \( \varepsilon_{tag} \), and the electron flux, \( S \),

\[ N_{\gamma} = \varepsilon_{tag} S, \]

then (B.5) takes the form

\[ \frac{t_e}{t_f} = \sqrt{\frac{\varepsilon_{tag} S_f (Y_e^p + r_{pa}^2 Y_e^a)}{\varepsilon_{tag} S_e (Y_f^p + r_{pa}^2 Y_f^a)}}, \]
\[ \frac{S_f (Y_e^p + r_{pa}^2 Y_e^a)}{S_e (Y_f^p + r_{pa}^2 Y_f^a)}, \] (B.6)
Appendix C

TAGGER CORRECTIONS

As mentioned in Section 2.2.2, the tagger focal plane counters have an overlap of 50%, with a hit in a particular tagger channel generated by a coincidence between the signals of two overlapping counters. When the tagger is run at high rates (≥ 1 MHz), the overlapping of counters and dead-time of the updating discriminators used in the coincidence electronics (see Vogt [Vog93]) can cause some problems. These effects are a function of the instantaneous electron rate, a measure of which is given by taking the time-averaged rate and dividing by the duty factor. A simple explanation for using the instantaneous rate instead of the average rate is that these effects depend on event rates at any given moment; when there is no current there is no effect. This appendix examines three rate-dependent problems in detail: stolen coincidences, ghosts, and adjacent doubles.

C.1 Stolen Coincidences

This treatment is a modified version of that written by Pywell [Pyw94].

If a random electron registers a hit in the same channel as the prompt electron, and it occurs before the prompt electron arrives, then the true coincidence is stolen by the random electron. This happens because a TDC channel can stop only once for a given X-trigger, and thus the higher TDC channels have a lower efficiency because of the greater probability of a random event stopping the TDC within a shorter time.

In an experiment of duration $T$ where $N_e$ is the number of bremsstrahlung electrons (and photons) that can produce an X-trigger and $\sigma$ is the true reaction probability (including all detector and tagging efficiencies), the number of X-triggers is $N_X = \sigma N_e$. If we define $N_{Xi}$ and $N_{ei}$ as the number of X-triggers and electrons falling on channel $i$ respectively, then we can write the measured reaction probability for a specific channel as

$$\sigma_i^M = \frac{P_{Xi}N_{Xi}}{P_{ei}N_{ei}},$$

where $P_{Xi}$ is the probability of an electron in channel $i$ arriving at the coincidence peak actually stopping the TDC and $P_{ei}$ is the probability of an electron in channel $i$ registering a count in the scaler. Thus the correction factor needed is

$$f_{\text{stolen}}^i = \frac{P_{Xi}}{P_{ei}}.$$
so that $\sigma = \sigma_i^M / f_{stolen}^i$. The correction factor is written this way because it is
easier to correct the scalers, and not the yield, on a run-by-run basis.

It is also important to define $T_p$ as the time between the opening of the
tagger edge-triggered coincidence gate and the coincidence peak in the TDC
spectrum, and $\tau_d$, the dead-time which is the output width of the updating
discriminator.

There are three distinct situations that need to be examined:

1. $\tau_d > T_p$,
2. $\tau_d < \frac{T_p}{2}$,
3. $\frac{T_p}{2} < \tau_d < T_p$.

In all three cases, the probability that an electron count will be seen in the
scaler is the probability that there was no count in the previous time interval
$\tau_d$. This is because the updating discriminator prevents a new leading edge
from appearing in the time interval $\tau_d$.

According to Poisson statistics, the probability of getting $n$ counts when
the mean number of counts is $\lambda_i = tr_i$, with $t$ being the time interval and $r_i$
the instantaneous count rate, is

$$P_i(n) = \frac{\lambda_i^n e^{-\lambda_i}}{n!}, \quad (C.1)$$

so that the probability of getting no counts registering in the scaler for channel
$i$ is

$$P_{ei} = P_i(0) = e^{-\tau_d r_i}. \quad (C.2)$$

C.1.1 Case 1: $\tau_d > T_p$

The TDC spectrum is given in Figure C.1. The probability of the TDC regist-
ERING a count at $X$ is the probability of no count in the previous time interval
$\tau_d$ since it will be prevented by the updating discriminator output. Thus,

$$P_{Xi} = P_i(0) = e^{-\tau_d r_i},$$

giving

$$f_{stolen}^i = \frac{P_{Xi}}{P_{ei}} = \frac{e^{-\tau_d r_i}}{e^{-\tau_d r_i}} = 1,$$
Figure C.1: Stolen coincidence TDC spectrum for Case 1.

and there is no correction for this case.

C.1.2 Case 2: $\tau_d < \frac{T_p}{2}$

Figure C.2: Stolen coincidence TDC spectrum for Case 2.

The TDC spectrum is given in Figure C.2. For an electron to be observed in the TDC spectrum at X, there must be no count in the time interval $T_p$, as such a count would prematurely stop the TDC. However, it is possible for a count arriving in the time interval C (say at time $t'$) to not prevent a count at X, if it was prevented from stopping the TDC by dead-time generated by a count arriving in the time interval D.

If $P_A$ is defined as the probability of getting one or more counts in the time
interval $A$, etc., then we can write

$$ P_{X_i} = P_A \cdot P_B \cdot \left( P_C + P_C \cdot P_D \right), $$

with

$$ P_A = e^{-\tau_d r_i}, $$
$$ P_B = e^{-\left(T_p - 2\tau_d\right) r_i}, $$
$$ P_C = e^{-\tau_d r_i}, $$
$$ P_C = 1 - e^{-\tau_d r_i}. $$

The time interval $D$ varies with the position, $t'$, of a count in $C$ (i.e. $t_D = \tau_d - t'$). The probability that a count arrives in time interval $D$ is $1 - e^{-\tau_d r_i}$ so that the probability, $P_D$, that a count in $C$ is prevented is the average over $\tau_d$ of the probability of a count in the time interval $D$, expressed by the integral

$$ P_D = \frac{1}{\tau_d} \int_0^{\tau_d} \left( 1 - e^{-\left(\tau_d - t'\right) r_i} \right) dt', $$

$$ = 1 - \frac{1}{\tau_d r_i} \left( 1 - e^{-\tau_d r_i} \right), $$

$$ \approx \frac{1}{2} \tau_d r_i, $$

to first order in $\tau_d r_i$. Solving for $P_{X_i}$ we have

$$ P_{X_i} = e^{-\tau_d r_i} e^{-\left(T_p - 2\tau_d\right) r_i} \left[ e^{-\tau_d r_i} + \left(1 - e^{-\tau_d r_i}\right) \frac{1}{2} \tau_d r_i \right], $$

$$ = e^{-\tau_d r_i} \left[ 1 - (T_p - \tau_d) r_i \right] + \mathcal{O}(r_i^2), $$

which gives

$$ f'_{stolen} = \frac{P_{X_i}}{P_{ei}}, $$

$$ \approx \frac{e^{-\tau_d r_i} \left[ 1 - (T_p - \tau_d) r_i \right]}{e^{-\tau_d r_i}}, $$

$$ \approx 1 - (T_p - \tau_d) r_i, $$

to first order in $r_i$.

**C.1.3 Case 3: $\frac{T_p}{2} < \tau_d < T_p$**

The TDC spectrum is given in Figure C.3. This case is very similar to Case 2 except that

$$ P_{X_i} = P_A \cdot \left( P_C + P_C \cdot P_D \right), $$

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Figure C.3: Stolen coincidence TDC spectrum for Case 3.

with

\[ \overline{P}_A = e^{-\tau_d r_i}, \]
\[ \overline{P}_C = e^{-(T_p - \tau_d) r_i}, \]
\[ P_C = 1 - e^{-(T_p - \tau_d) r_i}, \]

and

\[ P_C = \frac{1}{T_p - \tau_d} \int_0^{T_p - \tau_d} (1 - e^{-(\tau_d - t) r_i}) \, dt'. \]
\[ = 1 + \frac{e^{-\tau_d r_i}}{(T_p - \tau_d) r_i} \left( 1 - e^{(T_p - \tau_d) r_i} \right). \]
\[ = \left[ \frac{\tau_d^2 - (T_p - 2\tau_d)^2}{2(T_p - \tau_d)} \right] r_i + O(r_i^2). \]

Thus,

\[ P_{Xi} = e^{-\tau_d r_i} \left[ e^{-(T_p - \tau_d) r_i} + \left( 1 - e^{-(T_p - \tau_d) r_i} \right) \right] \]
\[ \times \left[ 1 + \frac{e^{-\tau_d r_i}}{(T_p - \tau_d) r_i} \left( 1 - e^{(T_p - \tau_d) r_i} \right) \right]. \]
\[ = e^{-\tau_d r_i} \left[ 1 - (T_p - \tau_d) r_i \right] + O(r_i^2), \]

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which gives

\[ f_{\text{stolen}}^i = \frac{P_{X_i}}{P_{e_i}}, \]

\[ = e^{-\tau d r_i} [1 - (T_p - \tau_d) r_i], \]

\[ = 1 - (T_p - \tau_d) r_i, \]

the same correction factor as Case 2.

C.1.4 Conclusions

Ideally, we would want the prompt peak situated within \( \tau_d \) of the opening of the coincidence gate (Case 1), so that a stolen correction is unnecessary. For those cases where this is not possible, the correction factor to first order in the rate is given by

\[ f_{\text{stolen}}^i = 1 - (T_p - \tau_d) r_i. \]  \hspace{1cm} (C.3)

Although the stolen coincidence correction factor was derived for a specific channel, it is generally sufficient to calculate it based on a channel-averaged instantaneous rate, \( \bar{r}_e \), since the channel rates are approximately equal.

C.1.5 Stolen Coincidences and This Measurement

This measurement used channel-by-channel stolen correction factors. The prompt peak positions, channel-averaged instantaneous rates\(^\dagger\) (which give a good measure of the individual channel rates), and corresponding correction factors are listed in Table C.1. The dead-time associated with the updating discriminators was 15 ns. We can see that these factors are small except for the correction to the high-rate 90 degree point.

C.2 Ghosts

This section is based on work done by Pywell [Pyw98].

If an electron registers a hit in channel \( i - 1 \) and \( i + 1 \), we can see from Figure C.4 that a false hit will register in channel \( i \) as well. This is a ghost event. Because of the complicated nature of these events, it is instructive to examine them two different ways, with and without dead-time.

C.2.1 Simplistic Case with No Dead-Time

In an experiment of duration \( T \) where \( N_e \) is the number of bremsstrahlung electrons (and photons) that can produce an X-trigger \( \sigma \) is the reaction

\(^\dagger\)These rates are also averaged over all of the runs for each angle.
Table C.1: Stolen coincidence correction factors.

<table>
<thead>
<tr>
<th>Target</th>
<th>Run Range</th>
<th>$\theta$ (deg)</th>
<th>$\bar{f}$ (MHz)</th>
<th>$T_p$ (ns)</th>
<th>$f_{\text{stolen}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1004-1272</td>
<td>90.0</td>
<td>1.465</td>
<td>22.1</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>1627-1830</td>
<td>120.0</td>
<td>1.365</td>
<td>20.1</td>
<td>0.993</td>
</tr>
<tr>
<td></td>
<td>1870-2165</td>
<td>150.0</td>
<td>1.071</td>
<td>19.3</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td>2454-2744</td>
<td>60.0</td>
<td>1.400</td>
<td>19.6</td>
<td>0.994</td>
</tr>
<tr>
<td></td>
<td>2997-3279</td>
<td>35.0</td>
<td>0.582</td>
<td>23.2</td>
<td>0.996</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>819-919</td>
<td>90.0</td>
<td>1.698</td>
<td>22.1</td>
<td>0.989</td>
</tr>
<tr>
<td></td>
<td>1292-1490</td>
<td>120.0</td>
<td>1.359</td>
<td>20.1</td>
<td>0.993</td>
</tr>
<tr>
<td></td>
<td>2166-2261</td>
<td>150.0</td>
<td>1.056</td>
<td>19.3</td>
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<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
<td>2873-2884</td>
<td>90.0</td>
<td>0.804</td>
<td>21.3</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td>2890-2897</td>
<td>90.0</td>
<td>4.755</td>
<td>21.3</td>
<td>0.970</td>
</tr>
<tr>
<td></td>
<td>2918-2996</td>
<td>35.0</td>
<td>0.353</td>
<td>23.2</td>
<td>0.997</td>
</tr>
</tbody>
</table>

probability (including all detector and tagging efficiencies), the number of X-triggers is $N_X = \sigma N_e$. If $f_i$ is defined as the fraction of electrons falling on channel $i$, then the number of electrons falling on channel $i$ is $N_{ei} = f_i N_e$ and the number of X-triggers in channel $i$ is

$$N_{Xi} = \sigma f_i N_e,$$

$$= \sigma f_i \frac{N_X}{\sigma},$$

$$= f_i N_X,$$

![Electron direction](image)

Figure C.4: Tagger counter layout.
as expected.

When the TDC for channel $i$ gets a start signal, it is because there was an X-trigger, i.e. the TDC will start $N_{X_i}$ times during the experiment. Often the X-trigger has nothing to do with the electron that stops the TDC. These are the random coincidences that we see in the TDC spectrum. If the X-trigger is from an electron in channel $i$, then the event will occur in the prompt peak, unless prevented by a random or ghost event.

As mentioned previously, if the X-trigger is from an electron adjacent to channel $i$ then there is a finite probability (if it is in coincidence with an electron in the other adjacent channel) that the event will appear as a ghost in channel $i$.

If we integrate the region in the TDC spectrum which contains totally the prompt peak and ghosts, and then subtract the random coincidences, a number is obtained, say $Y_i$. If the number of electrons recorded in the scaler is $S_i$, then the measured reaction probability in channel $i$ for the experiment would be

$$\sigma_i^M = \frac{Y_i}{S_i}.$$  

In a perfect world without ghosts, $Y_i = N_{X_i}$ and $S_i = N_{e_i}$, so that

$$\sigma_i^M = \frac{N_{X_i}}{N_{e_i}} = \sigma,$$

as we would hope.

If we include ghosts, $Y_i = N_{X_i} + N_{Gi}^{TDC}$ where $N_{Gi}^{TDC}$ is the number of ghost stops in TDC channel $i$. We can write

$$N_{Gi}^{TDC} = N_{X_i-1} P_{i+1}^{hit} + N_{X_i+1} P_{i-1}^{hit},$$

where $P_{i}^{hit}$ is the probability of at least one electron hit in channel $i$. Note that

$$P_{i}^{hit} = 1 - e^{-tr_i} \approx 1 - (1 - tr_i) \approx tr_i$$

for nominal tagger conditions of $t = 15$ ns and $r_i \leq 2$ MHz. Thus,

$$N_{Gi}^{TDC} = N_{X_i-1} t_{0i} r_{i+1} + N_{X_i+1} t_{0i} r_{i-1},$$

where $t_{0i}$ is the minimum time difference between channel $i - 1$ and $i + 1$ that can cause a ghost in channel $i$. Rewriting, we have

$$r_i = \frac{N_{e_i}}{T} = \frac{f_i N_e}{T} = f_i R_{sum},$$

where $R_{sum}$ is the instantaneous rate of hits in the entire focal plane:

$$R_{sum} = \sum_{i=1}^{62} r_i.$$
Note that the ghost TDC will have timing close to the prompt peak because it was caused by a true X-trigger in one of the adjacent channels. We can now write

\[ Y_i = N_{X,i} + N_{X,i+1}^{TDC}, \]
\[ = N_{X,i} + N_{X,i-1}t_{0i}r_{i+1} + N_{X,i+1}t_{0i}r_{i-1}, \]
\[ = f_i N_X + f_{i-1} N_X t_{0i} f_{i+1} R_{sum} + f_{i+1} N_X t_{0i} f_{i-1} R_{sum}, \]
\[ = N_X \left( f_i + 2 f_{i-1} f_{i+1} R_{sum} t_{0i} \right), \]
\[ = f_i N_X \left[ 1 + 2 \frac{f_{i-1} f_{i+1}}{f_i} R_{sum} t_{0i} \right]. \] (C.4)

There will also be ghosts in the scalers. As there is no requirement for a coincidence with an X-trigger for a ghost to be counted, the number of ghost counts is simply the number of times an electron in channel \( i - 1 \) is within the time \( t_{0i} \) of an electron in channel \( i + 1 \). Therefore \( S_i = N_{et} + N_{Gi}^S \) with

\[ N_{Gi}^S = N_{i-1} t_{i+1} t_{0i} = N_{i+1} t_{i-1} t_{0i}, \]
\[ = f_{i-1} N_e f_{i+1} R_{sum} t_{0i} = f_{i+1} N_e f_{i-1} R_{sum} t_{0i}, \]
\[ = f_{i-1} f_{i+1} \frac{N_X}{\sigma} R_{sum} t_{0i}, \]

so that

\[ S_i = f_i \frac{N_X}{\sigma} + f_{i-1} f_{i+1} \frac{N_X}{\sigma} R_{sum} t_{0i}, \]
\[ = \frac{N_X}{\sigma} \left[ f_i + f_{i-1} f_{i+1} R_{sum} t_{0i} \right]. \]

Now, we can write the measured reaction probability as

\[ \sigma_i^M = \frac{Y_i}{S_i}, \]
\[ = \frac{N_X \left( f_i + 2 f_{i-1} f_{i+1} R_{sum} t_{0i} \right)}{\frac{N_X}{\sigma} \left[ f_i + f_{i-1} f_{i+1} R_{sum} t_{0i} \right]}, \]
\[ = \frac{\sigma \left[ 1 + 2 \frac{f_{i-1} f_{i+1}}{f_i} R_{sum} t_{0i} \right]}{1 + \frac{f_{i-1} f_{i+1}}{f_i} R_{sum} t_{0i}}. \] (C.5)

Rewriting (C.5), we have

\[ \sigma_i^M = \sigma \left[ 1 + 2 \frac{f_{i-1} f_{i+1}}{f_i} t_{0i} \right], \]
\[ \frac{1}{1 + \frac{f_{i-1} f_{i+1}}{f_i} t_{0i}}. \]

Assuming that the electron rate is approximately the same for adjacent...
channels, i.e. \( r_{i+1} \approx r_{i-1} \approx r_i \), we have

\[
\sigma_i^M \approx \sigma \left[ \frac{1 + 2r_i t_{0i}}{1 + r_i t_{0i}} \right] \approx \sigma [1 + r_i t_{0i}].
\]

It is important to remember that this is a first order calculation that does not include dead-time. It does show, however, that without any correction we would expect a reaction probability that is too high with the yield increasing linearly with rate.

### C.2.2 Full Calculation Including Dead-Time

To include dead-time, we must treat both the TDC and the scalers more carefully. It is instructive to again refer to Figure C.4.

If we let \( \tau_j \) be the output width of the discriminator for counter \( i \), then

\[
\tau_j = \begin{cases} 
\tau_0 \approx 20 \text{ ns} & j = \text{odd} \\
\tau_f \approx 15 \text{ ns} & j = \text{even}
\end{cases}
\]

where the back counters have a slightly larger value. Because of the nature of the updating discriminators, the dead-time of a channel is actually variable. If two electrons hit the same channel in short succession (< 15 ns), then the actual dead-time for the next electron in that channel would be in the range of 15–30 ns as seen in Figure C.5. This is not a problem, as the chance of

![Figure C.5: Effective channel dead-time.](image)

three electrons arriving in one channel in less than 30 ns is extremely small \( (P \approx 10^{-6}) \). Therefore it is appropriate to use an effective dead-time given by the output width of the front counter (\( \tau_f \approx 15 \text{ ns} \)).

#### Yield Correction

The timing for a ghost would occur as in Figure C.6 where \( t_{mi} \) is the minimum width signal in channel \( i \) needed to get through the electronics and be counted.
Figure C.6: Timing for a ghost event in channel i.

in the scaler or registered in the TDC (about 5 ns). An output will appear in
cchannel i whenever the coincidence between i - 1 and i + 1 is within the range
of time t_{0i}. We can see that

\[ t_{0i} = \tau_{j+1} - t_{ms} + \tau_j - t_{ms}, \]
\[ = \tau_{j+1} + \tau_j - 2t_{ms}, \]
\[ \approx 25 \text{ ns}. \]

Further, the timing for ghost events is variable. Because the back and front
counters have different output widths, the timing for ghost events in even and
odd channels differ slightly. Using the timing of the prompt channel, \( t_p \), and
the timing of the random event causing the ghost, \( t_a \), the various scenarios for
the ghost timing, \( t_g \), are given in Table C.2. An example of a TDC spectrum

Table C.2: Timing for ghost events.

<table>
<thead>
<tr>
<th>Ghost Channel</th>
<th>( t_a - t_p ) (ns)</th>
<th>( t_g - t_p ) (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>odd</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt; 15</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>5 - 15</td>
<td></td>
<td>0 - 10</td>
</tr>
<tr>
<td>-10 - 5</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>(&lt; -10</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>even</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt; 10</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>-5 - 10</td>
<td></td>
<td>-5 - 10</td>
</tr>
<tr>
<td>-15 - 5</td>
<td></td>
<td>-5</td>
</tr>
<tr>
<td>(&lt; -15</td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

with ghost events emphasized is given in Figure C.7.
Figure C.7: An example of ghosts in the tagger TDC spectrum.

In making a correction to the yield, it is important to ensure that only those ghost events appearing in the prompt window are included in the correction. In addition, because the ghost events occur as much as 10 ns to the right of the prompt peak, we must ensure that any ghost events in the accidental window are taken into account in the accidental subtraction. Thus, if we define $r_{pg}$ as the probability of the ghost occurring inside the prompt region, then the channel yield is given by

$$Y_i = f_i N_X \left[ 1 + 2r_{pg} \frac{f_{i-1} f_{i+1}}{f_i} R_{sum} \tau_0 \right].$$  \hspace{1cm} (C.6)

**Scaler Correction**

Ignoring ghosts, the number of electrons registered by the scaler in channel $i$ is

$$S_i = N_{c_i} \times \text{(probability of no electron in previous } \tau_d) ,$$

$$= N_{c_i} e^{-r_i \tau_d},$$

$$\simeq N_{c_i} (1 - r_i \tau_d).$$

Note that only the channel rate, $r_i$, appears in this equation, not the counter rate which is larger due to the contribution from adjacent channels. Electrons in the back counter from the adjacent channel only extend the updating discriminator output and do not steal the front counter leading edge. Thus the
count in channel \( i \) is not stolen. Electrons in the front counter from the adjacent channel simply move the timing of the overlap output but do not steal it from the scaler (see Figure C.8).

![Figure C.8](image)

Figure C.8: Effect of the updating discriminator. (a) The back counter fires from an electron in the adjacent channel. (b) The front counter fires from an electron in the adjacent channel.

The rate of ghost counts in channel \( i \) is simply the rate of overlaps between channels \( i - 1 \) and \( i + 1 \) within time interval \( t_{0i} \). It is convenient to split this rate into two parts: where the timing of the count entering the scaler is determined by counter \( j \), and where the timing is determined by counter \( j + 1 \). Again, it is useful to refer to Figures C.4 and C.6 for this discussion.

In the first case, the number of leading edges in counter \( j \) due to electrons in channel \( i - 1 \) is given by

\[
N_{i-1} \times \text{probability of no hit in counter } j = N_{i-1} e^{-\left( r_i + r_{i-1} \right) \tau_j}.
\]

Notice that the rate used in the dead-time correction is \( r_i + r_{i+1} \), since an electron in channel \( i \) will steal the leading edge from counter \( j \), i.e. we must use the rate for counter \( j \). The probability of finding an electron in channel \( i + 1 \) that overlaps this leading edge by at least the time \( t_{mi} \) is

\[
P(n \geq 1) = \overline{P}(0),
= 1 - P(0),
= 1 - e^{-r_{i+1}(\tau_{j+1} - t_{mi})}.
\]

so that the number of ghosts with timing determined by counter \( j \) is

\[
N_{i-1} e^{-\left( r_i + r_{i-1} \right) \tau_j} \left( 1 - e^{-r_{i+1}(\tau_{j+1} - t_{mi})} \right).
\]

By a similar argument, the number of ghosts with timing determined by
counter $j + 1$ is
\[
N_{i+1}e^{-(r_i+r_{i+1})\tau_j + 1} \left( 1 - e^{-r_i (\tau_j - t_{mi})} \right).
\]
Thus, the total number of ghosts is
\[
N_{Gi}^S = N_{i-1}e^{-(r_i+r_{i-1})\tau_j} \left( 1 - e^{-r_{i+1} (\tau_{j+1} - t_{mi})} \right) 
+ N_{i+1}e^{-(r_i+r_{i+1})\tau_{j+1}} \left( 1 - e^{-r_{i-1} (\tau_j - t_{mi})} \right),
\]
\[
= N_e \left( f_{i-1} g_1 + f_{i+1} g_2 \right),
\]
\[
= f_1N_e \left( \frac{f_{i-1}}{f_i} g_1 + \frac{f_{i+1}}{f_i} g_2 \right),
\]
where
\[
g_1 = e^{-(r_i+r_{i-1})\tau_j} \left( 1 - e^{-r_{i+1} (\tau_{j+1} - t_{mi})} \right),
\]
\[
g_2 = e^{-(r_i+r_{i+1})\tau_{j+1}} \left( 1 - e^{-r_{i-1} (\tau_j - t_{mi})} \right).
\]

Note that if a real electron occurs in channel $i$ in coincidence with a ghost, the real electron will be counted and the ghost will be stolen as already accounted for in the dead-time correction. Therefore, the real events and ghosts are mutually exclusive and the total rate in channel $i$ scaler is just the sum, i.e.
\[
S_i = N_{ei}e^{-r_i \tau_4} + N_{Gi}^S. \tag{C.7}
\]

Now, if we include the yield correction from (C.6), the total correction factor is
\[
\sigma_i^M = \frac{f_1N_e \left( 1 + 2r_{pg} \frac{h_i f_{i+1}}{h_i} R_{sum t_{0i}} \right)}{N_e e^{-r_i \tau_4} + N_{Gi}^S},
\]
\[
= \frac{f_1N_e \left( 1 + 2r_{pg} \frac{h_i f_{i+1}}{h_i} R_{sum t_{0i}} \right)}{f_1N_e \left( e^{-r_i \tau_4} + \frac{h_i}{h_{i-1}} g_1 + \frac{h_i}{h_{i+1}} g_2 \right)},
\]
\[
= \sigma \left( 1 + 2r_{pg} \frac{h_i f_{i+1}}{h_i} R_{sum t_{0i}} \right) \left( e^{-r_i \tau_4} + \frac{h_i}{h_{i-1}} g_1 + \frac{h_i}{h_{i+1}} g_2 \right),
\]
so that the true reaction probability is given by
\[
\sigma = \frac{\sigma_i^M}{f_{ghost}},
\]

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with

\[ f_{\text{ghost}}^i = \frac{1 + 2r_{pg} \frac{t_{i-1} + t_{i+1}}{t_i} R_{\text{run}}t_0}{e^{-r_{rd} + \frac{t_{i-1}}{t_i} g_1 + \frac{t_{i+1}}{t_i} g_2}}. \] (C.8)

To ensure that this correction is applicable to the data in question, it should be applied to the channel scalers, \(S_i\), on a run-by-run basis.

C.2.3 Ghosts and This Measurement

In order to make the ghost correction, it was first necessary to arrive at a value for the probability of a ghost appearing in the prompt peak. Reorganizing the odd and even channels from Table C.2, and assuming that the distribution of ghosts in time was even, the probability of a ghost appearing in different regions was calculated (see Table C.3). Using these values, and the prompt window width of 5.4 ns centred around the peak, the probability of a ghost appearing in the prompt region was obtained in the following way:

\[ r_{pg} = \left( \frac{2.7 \text{ ns}}{10 \text{ ns}} \right) (0.40) + 0.30 + \left( \frac{2.7 \text{ ns}}{5 \text{ ns}} \right) (0.10), \]
\[ = 0.462. \]

Further, no correction was made for ghosts in the accidental region because it was assumed that the background fit was over a large enough, "ghost-free" range that they were not included in the subtraction.

The channel-averaged ghost correction factors averaged over all runs for each angle, \( \tilde{f}_{\text{ghost}} \), are given in Table C.4. Although they do not represent the actual corrections to the cross sections, they give a very good idea of the effect of ghosts on the data.

C.3 Adjacent Doubles

This work was done with the assistance of Warkentin [War99b].
Table C.4: Average ghost correction factors.

<table>
<thead>
<tr>
<th>Target</th>
<th>Run Range</th>
<th>$\theta$ (deg)</th>
<th>$\tilde{r}_e$ (MHz)</th>
<th>$\bar{f}_{\text{ghost}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1004–1272</td>
<td>90.0</td>
<td>1.465</td>
<td>1.020</td>
</tr>
<tr>
<td></td>
<td>1627–1830</td>
<td>120.0</td>
<td>1.365</td>
<td>1.018</td>
</tr>
<tr>
<td></td>
<td>1870–2165</td>
<td>150.0</td>
<td>1.071</td>
<td>1.015</td>
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<td></td>
<td>2454–2744</td>
<td>60.0</td>
<td>1.400</td>
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<td>1.007</td>
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<td>$^{12}$C</td>
<td>819–919</td>
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<td>1.698</td>
<td>1.022</td>
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<td>90.0</td>
<td>4.755</td>
<td>1.076</td>
</tr>
<tr>
<td></td>
<td>2918–2996</td>
<td>35.0</td>
<td>0.353</td>
<td>1.005</td>
</tr>
</tbody>
</table>

An adjacent double event occurs when two adjacent tagger channels register hits within the same coincidence gate. They are caused by either one electron, called geometric doubles, or two electrons.

C.3.1 One Electron

Because of the angle of electrons incident on the focal plane counters (refer to Figure C.4), a small number of electrons actually “clip” a second channel and thus register events in two channels at once. Because these geometric doubles occur in equal amounts in both the yield and the scalers, and are not rate-dependent, they do not have an effect on the reaction probability:

$$\sigma^M = \frac{Y^M}{S^M},$$

$$= \frac{Y + rY}{S + rS},$$

$$= \frac{Y(1 + r)}{S(1 + r)},$$

$$= \sigma,$$

where $r$ is the ratio of events that are one-electron doubles. Further, because the events are caused by the same electron, they have identical timing.
C.3.2 Two Electrons

The two-electron events we are concerned with are the result of a random electron registering an event in a channel adjacent to the prompt channel within a short time (15 ns). Because of the configuration of the updating discriminators for front and back counters, it is possible for a random electron to shift the timing of the prompt electron outside of the prompt peak, stealing it from the yield.

It is useful to split adjacent doubles into those where the back counter fires due to an electron in an adjacent counter, and those where the front counter fires due to an electron in an adjacent counter (refer to Figure C.8). In the first case, there is no change in the timing of either the prompt or accidental channel. In the second, however, if the accidental event comes first, the prompt event is shifted from 0 to 5 ns earlier. In addition, if the prompt electron comes before the accidental, the timing of the accidental event will be shifted closer to the prompt peak. The overall effect is a partial cancellation.

The measured yield in channel $i$ will be given by $Y_i = N_{Xi} - N_{Xi}^{double}$, where $N_{Xi}^{double}$ is the number of prompt events shifted out of the prompt region by a random adjacent double minus the number of random events shifted into the prompt region by a prompt event in the adjacent channel. We can write

$$N_{Xi}^{double} = r_{pd} N_{Xi} P_{i+1}^{hit} - r_{ad} N_{Xi+1} P_{i}^{hit}$$

for odd channels, and

$$N_{Xi}^{double} = r_{pd} N_{Xi} P_{i-1}^{hit} - r_{ad} N_{Xi-1} P_{i}^{hit}$$

for even channels, where the probability of an adjacent double occurring is given by

$$P_{i}^{hit} = 1 - e^{-r_{pd}r_{i}}$$

$r_{pd}$ is the probability of a prompt event being shifted out of the prompt region, and $r_{ad}$ is the probability of an accidental event being shifted into the prompt region. Since the scalers are not affected, the measured reaction probability can be written

$$\sigma^{M} = \frac{Y_i}{S_i} = \frac{N_{Xi} - r_{pd} N_{Xi} P_{i+1}^{hit} + r_{ad} N_{Xi+1} P_{i}^{hit}}{S_i} = \frac{f_i N_X (1 - r_{pd} P_{i+1}^{hit} + r_{ad} \frac{f_{i+1}}{f_i} P_{i}^{hit})}{f_i N_x} = \sigma \left(1 - r_{pd} P_{i+1}^{hit} + r_{ad} \frac{f_{i+1}}{f_i} P_{i}^{hit}\right)$$

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Thus, the true reaction probability is given by

\[ \sigma = \frac{\sigma^M}{f_{\text{double}}} \]

with

\[ f_{\text{double}} = 1 - r_{pd}P_{i\pm 1}^{\text{hit}} + r_{ad}\frac{f_{i\pm 1}}{f_i}P_{i}^{\text{hit}}, \]  \hspace{1cm} (C.9)

where the "plus" is for odd channels and the "minus" is for even ones.

Using a similar analysis to that of the ghosts, a summary of the timing for shifted prompt events, \( t_{p'} \), and shifted accidental events, \( t_{a'} \), is given in Table C.5.

<table>
<thead>
<tr>
<th>( t_a - t_p ) (ns)</th>
<th>( t_{p'} - t_p ) (ns)</th>
<th>( t_{a'} - t_p ) (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 - 15</td>
<td>-</td>
<td>0 - 10</td>
</tr>
<tr>
<td>0 - 5</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>-5 - 0</td>
<td>-5 - 0</td>
<td>-</td>
</tr>
<tr>
<td>-15 - -5</td>
<td>-5</td>
<td>-</td>
</tr>
</tbody>
</table>

### C.3.3 Adjacent Doubles and This Measurement

In order to make a correction for this measurement, we needed to obtain the net yield that was shifted out of the prompt region. First, the timing distribution for shifted prompt counts was calculated (see Table C.6). With these

<table>
<thead>
<tr>
<th>Case</th>
<th>( t' - t_p ) (ns)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>accidental</td>
<td>0 - 10</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>prompt</td>
<td>-5 - 0</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td>-5</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

values, and the prompt window width of 5.4 ns centred around the peak, the probability of a prompt event being shifted outside the prompt region was
calculated in the following way:

\[
    r_{pd} = \frac{2}{3} + \left(\frac{2.3 \text{ ns}}{5 \text{ ns}}\right)^{\frac{1}{3}},
\]

\[
    = 0.82.
\]

For the probability of an random event being shifted into the prompt region, we first had to take into account that for the events shifted from 0 – 5 ns to 0, there was a finite probability that they were already inside the prompt region, given by

\[
    \left(\frac{2.7 \text{ ns}}{10 \text{ ns}}\right)^{\frac{1}{3}} = 0.18.
\]

Thus, the probability needed was

\[
    r_{ad} = \frac{1}{3} - 0.18 + \left(\frac{2.7 \text{ ns}}{10 \text{ ns}}\right)^{\frac{2}{3}},
\]

\[
    = 0.33.
\]

The channel adjacent double correction factors averaged over all of the runs for each angle, \(\bar{f}_{\text{double}}\), are given in Table C.7. Again, although they do not represent the actual correction factors, which were applied on a run-by-run basis to the channel scalers, they do give a good idea of the overall effect of adjacent doubles on this measurement.

Table C.7: Average adjacent double correction factors.

<table>
<thead>
<tr>
<th>Target</th>
<th>Run Range</th>
<th>(\theta) (deg)</th>
<th>(\bar{r}_e) (MHz)</th>
<th>(\bar{f}_{\text{double}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1004–1272</td>
<td>90.0</td>
<td>1.465</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>1627–1830</td>
<td>120.0</td>
<td>1.365</td>
<td>0.991</td>
</tr>
<tr>
<td></td>
<td>1870–2165</td>
<td>150.0</td>
<td>1.071</td>
<td>0.992</td>
</tr>
<tr>
<td></td>
<td>2454–2744</td>
<td>60.0</td>
<td>1.400</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>2997–3279</td>
<td>35.0</td>
<td>0.582</td>
<td>0.996</td>
</tr>
<tr>
<td>(^{12}\text{C})</td>
<td>819–919</td>
<td>90.0</td>
<td>1.698</td>
<td>0.989</td>
</tr>
<tr>
<td></td>
<td>1292–1490</td>
<td>120.0</td>
<td>1.359</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>2166–2261</td>
<td>150.0</td>
<td>1.056</td>
<td>0.992</td>
</tr>
<tr>
<td></td>
<td>2340–2453</td>
<td>60.0</td>
<td>0.868</td>
<td>0.994</td>
</tr>
<tr>
<td></td>
<td>2745–2764</td>
<td>90.0</td>
<td>1.048</td>
<td>0.992</td>
</tr>
<tr>
<td></td>
<td>2873–2884</td>
<td>90.0</td>
<td>0.804</td>
<td>0.994</td>
</tr>
<tr>
<td></td>
<td>2890–2897</td>
<td>90.0</td>
<td>4.755</td>
<td>0.967</td>
</tr>
<tr>
<td></td>
<td>2918–2996</td>
<td>35.0</td>
<td>0.353</td>
<td>0.997</td>
</tr>
</tbody>
</table>
C.4 Summary of Rate Corrections

In order to apply both the stolen and ghost correction factors to the same data, it is necessary to ensure that we do not over correct the scalers. Looking at (C.2) and (C.7), we can see that the scaler correction for stolen leading edges is made in both cases; it is simplest to undo the stolen scaler correction in the following way:

\[ f_{\text{stolen}}^i \rightarrow f_{\text{stolen}}^i \cdot e^{-r_i \tau_d}. \]

Thus, the overall correction factor is given by

\[
f_{\text{rate}} = f_{\text{stolen}}^i f_{\text{ghost}}^i f_{\text{double}}^i = \left\{ \left[ 1 - (T_p - \tau_d) r_i \right] e^{-r_i \tau_d} \right\} \left( 1 + 2r_{pg} \frac{f_{i-1}/f_i + 1}{f_i} R_{\text{sum}} t_{0i} \right) \left( e^{-r_i \tau_d} + \frac{f_{i-1}/f_i + 1}{f_i} g_1 + \frac{f_i + 1}{f_i} g_2 \right) \right. \\
\left. \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \left( 1 - r_{pd} P_{i+1}^{\text{hit}} + r_{ad} \frac{f_{i+1}/f_i + 1}{f_i} P_{i}^{\text{hit}} \right) \right),
\]

and should be applied to each tagger channel for each individual run.

The combined channel-averaged rate-correction factors for this measurement, \( \bar{f}_{\text{rate}} \), are given in Table C.8. Again, we must note that they are not the actual correction factors used on the data, as the actual corrections were made to the scalers on a channel-by-channel and run-by-run basis.

<table>
<thead>
<tr>
<th>Target</th>
<th>Run Range</th>
<th>( \bar{\theta} ) (deg)</th>
<th>( \bar{r_e} ) (MHz)</th>
<th>( \bar{f}_{\text{stolen}} )</th>
<th>( \bar{f}_{\text{ghost}} )</th>
<th>( \bar{f}_{\text{double}} )</th>
<th>( \bar{f}_{\text{rate}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1004–1272</td>
<td>90.0</td>
<td>1.465</td>
<td>0.968</td>
<td>1.020</td>
<td>0.989</td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td>1627–1830</td>
<td>120.0</td>
<td>1.365</td>
<td>0.975</td>
<td>1.018</td>
<td>0.991</td>
<td>0.982</td>
</tr>
<tr>
<td></td>
<td>1870–2165</td>
<td>150.0</td>
<td>1.071</td>
<td>0.979</td>
<td>1.015</td>
<td>0.992</td>
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<tr>
<td></td>
<td>2454–2744</td>
<td>60.0</td>
<td>1.400</td>
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<tr>
<td></td>
<td>2997–3279</td>
<td>35.0</td>
<td>0.582</td>
<td>0.988</td>
<td>1.007</td>
<td>0.996</td>
<td>0.991</td>
</tr>
<tr>
<td>( ^{12} \text{C} )</td>
<td>819–919</td>
<td>90.0</td>
<td>1.698</td>
<td>0.966</td>
<td>1.022</td>
<td>0.989</td>
<td>0.976</td>
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<td>1292–1490</td>
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<td>0.990</td>
<td>0.981</td>
</tr>
<tr>
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<td>1.015</td>
<td>0.992</td>
<td>0.986</td>
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<td>0.978</td>
<td>1.015</td>
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<td>0.983</td>
<td>1.011</td>
<td>0.994</td>
<td>0.988</td>
</tr>
<tr>
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<td>1.076</td>
<td>0.966</td>
<td>0.937</td>
</tr>
<tr>
<td></td>
<td>2918–2996</td>
<td>35.0</td>
<td>0.353</td>
<td>0.992</td>
<td>1.005</td>
<td>0.997</td>
<td>0.994</td>
</tr>
</tbody>
</table>