OPEN CHANNEL TURBULENT BOUNDARY LAYERS
AND WALL JETS ON ROUGH SURFACES

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ABSTRACT

For many industrial and environmental flows, the momentum and convective heat transfer rates at the surface are determined by the turbulence structure in the near-wall region. Although many flows of practical interest occur on rough surfaces, our ability to predict rough wall turbulent flows lags far behind the corresponding technology for smooth surfaces. This provides reasonable grounds for additional refined rough wall measurements with the expectation of improving our physical understanding of practically relevant turbulent flows.

This thesis reports a comprehensive experimental investigation of wall roughness effects on the characteristics of a turbulent boundary layer and wall jet. The measurements are obtained for a hydraulically smooth as well as three geometrically different rough surfaces using a LDA system. For the smooth wall measurements, data are obtained in the viscous sublayer which then allow the wall shear stress to be accurately determined. Data presented include the streamwise and wall-normal components of the mean velocity and their fluctuations, Reynolds shear stress as well as distributions of turbulence kinetic energy budgets and mixing length. An insightful presentation of the results requires that the correct scaling laws must be used. In the case of the turbulent boundary layer, the appropriateness of the classical log law proposed by Millikan (1938) to model the overlap region of the mean flow as well as the recent power laws proposed by Barenblatt (1993) and George and Castillo (1997) is also examined. The present results are interpreted within the context of the ‘wall similarity
hypothesis', which states that, outside the roughness sublayer, turbulent motions are independent of wall roughness at sufficiently high Reynolds numbers.

The boundary layer results show that, irrespective of the specific surface conditions, the power law proposed by George and Castillo (1997) has important advantages over the log law both in modeling the mean velocity profiles as well as predicting the wall shear stress. The results also show that the characteristics of the turbulence structure and the transport terms depend on the specific geometry of the roughness elements, which suggests that rough wall turbulence models must explicitly account for the specific geometry of the roughness elements in order to predict the mixing characteristics accurately. This promises to provide significant challenges to rough wall turbulence models. In the case of turbulent wall jet, it is observed that surface roughness increases the inner layer thickness but the jet half-width does not show any important sensitivity to surface roughness. The fact that the spread rate is not altered by surface roughness suggests that a wall jet is a complex flow in which the mechanisms of near-wall damping are not the same as in a simple boundary layer. The spread rates for the jet half-width are higher than the values obtained in earlier measurements. This may be attributed to the high background turbulence levels in the present flow. It is also observed that the streamwise evolution of the mean flow is nearly independent of initial conditions when scaled using the exit kinematic momentum.
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DEDICATION

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NOMENCLATURE

ACRONYMS
AIP: Asymptotic Invariance Principle
DNS: Direct Numerical Simulation
LDA: Laser Doppler Anemometer
LES: Large Eddy Simulation
NOTAR: No Tail Rotor
PIV: Particle Image Velocimetry
SNR: Signal-to-noise-ratio
STOL: Short Takeoff and Landing
STOVL: Short Takeoff and Vertical Landing
VISA: Variable-interval-space-averaging
VITA: Variable-interval-time-averaging

ENGLISH SYMBOLS
A: power law constant
b: slot height (mm)
b_{ij}: stress anisotropy tensor
B: additive constant in log law (also a power law constant)
C: power law constant
C_f: skin friction coefficient
C_i: power law constant
C_o: power law constant
d: pipe diameter (mm)
f_i: dimensionless functional relationship for the inner layer
f_o: dimensionless functional relationship for the outer layer
F: flatness factor
h: depth of flow (mm)
H: boundary layer shape factor
J: jet exit momentum flux (kg/s^2)
k: average roughness height (mm)
k: turbulent kinetic energy (m²/s²)
K1: maximum velocity decay constant
K2: maximum velocity decay constant
L: mixing length (mm)
M: jet exit momentum flux (kg/s²)
N: number of samples
P: pressure (N/m²)
R: pipe radius (mm), also as correlation coefficient in uncertainty analysis
Re: Reynolds number
Re₆: Reynolds number based on jet exit (bulk) velocity and slot height
Re₇: Reynolds number based on jet exit (maximum) velocity and slot height
Re₈: Roughness Reynolds number based on average roughness height and friction velocity
Re₉: Reynolds number based on inner layer thickness and maximum velocity
Re₁₀: Reynolds number based on momentum thickness
Re₁₁: Reynolds number based on depth of flow and friction velocity
S: skewness factor
u: streamwise turbulence intensity (m/s)
U: streamwise component of mean velocity (m/s)
Uₕ: skin friction velocity (m/s)
v: vertical turbulence intensity (m/s)
V: vertical component of mean velocity (m/s)
w: spanwise turbulence intensity (m/s)
W: spanwise component of mean velocity (m/s)
x: streamwise distance (m)
y: vertical or wall-normal distance (m)
y₁₂: jet half-width (mm)
y₆: virtual origin (mm)
yₘ: inner layer thickness of a wall jet (mm)
z: spanwise distance (m)
GREEK SYMBOLS

α: power law exponent
β: power law exponent
δ: boundary layer thickness (mm)
δ*: boundary layer displacement thickness (mm)
δ*: Reynolds number based on boundary layer thickness and friction velocity
δ_{ij}: stress anisotropy tensor
ΔB*: roughness function
γ: power law exponent
κ: Von Karman constant
μ: absolute viscosity (N.s/m²)
ν: kinematic viscosity (m²/s)
ν*: eddy viscosity (m²/s)
Π: Coles wake parameter
θ: boundary layer momentum thickness (mm)
ρ: density (kg/m³)
σ_0: error in beam-crossing angle (percent)
τ_w: wall shear stress (N/m²)

SUBSCRIPTS

j: jet exit
max: maximum
u: streamwise component
v: wall-normal/vertical component

SUPERSCRIPTS

+ : normalization by viscous units
CHAPTER 1

INTRODUCTION

The study presented in this thesis pertains to experimental investigations of complex near-wall turbulent flows. Specifically, the structure of both a turbulent boundary layer and wall jet created in an open channel flow on smooth and rough surfaces is examined. From the perspective of near-wall turbulence research, a turbulent wall jet is a near-wall flow that is a degree more complex than a canonical turbulent boundary layer. An insightful interpretation of the wall jet data requires that the structure of the turbulent boundary layer must be examined first. In open channel flows, when experiments are required to be carried out at low Froude numbers, i.e. in the sub-critical range, there is a limitation of working at low velocities and, therefore, low values of Reynolds number based on momentum thickness. In this case, low Reynolds number effects on the turbulence structure must be examined. Furthermore, systematic investigation of surface roughness and low Reynolds number effects on the turbulence structure requires that the correct scaling laws be used to analyze the results.

In this chapter, some features of turbulence with specific reference to its complexity and practical importance are introduced. The basic equations of motion and motivations for near-wall turbulence research are presented. An overview of low Reynolds number effects and scaling issues in near-wall turbulent flows is also
presented. With regard to surface roughness, some characteristic features, definitions and terminology are introduced. Some nomenclature and applications of a turbulent wall jet are also presented. In order to facilitate discussion and comparison to a zero-pressure gradient turbulent boundary layer and fully developed duct flow in subsequent chapters, some characteristics of open channel flows are discussed. The overall objectives and scope of the present study are also outlined.

1.1 GENERAL REMARKS

1.1.1 Turbulence

Fluid flow turbulence is a phenomenon encountered in many scientific and technological disciplines: in industrial and environmental flows, combustion, aerodynamics, meteorology, hydrodynamics and oceanography. It presents some of the most difficult problems both in the fundamental understanding of its physics and in applications, some of which are still unresolved in spite of extensive research for well over a century. Indeed, turbulence has been characterized as the last, great, unsolved problem of classical physics by several physicists including the late Nobel Prize winner Richard Feynman (Zhou and Speziale, 1998). Horace Lamb (1916), one of the leading hydrodynamicists of the last century, after discussing all the branches of hydrodynamics known to him, finally had to deal with turbulence and remarked: "It remains to call attention to the chief outstanding difficulty of our subject" (see Bradshaw, 1990).
There are extremely different points of view on turbulence when it is viewed as a fluid flow phenomenon, all of which have in common its complexity. In his book 'Turbulence in Fluids', Marcel Lesieur (1987) gave the following notion

_The flows one calls ‘turbulent’ may possess fairly different dynamics, may be three-dimensional or sometimes quasi-two-dimensional, may exhibit well organized structures or otherwise. A common property which is required of them is that they should be able to mix transported quantities much more rapidly than if only molecular diffusion processes were involved._

Turbulence is characterized by its richness in scales, randomness and enhanced mixing property. From a practical point of view, its ability to enhance mixing is certainly the most important. An engineer, for instance, is mainly concerned with the knowledge of turbulent drag and/or heat transfer coefficients. In spite of its complex features as well as frustration and challenges that are usually encountered, turbulence continues to receive considerable research attention as evidenced in the amount of human and material resources dedicated to studying it in recent years.

### 1.1.2 Equations of Motion and Notation

In the present study, Cartesian coordinates are adopted: \((x, y, z)\) are used to denote streamwise, vertical or wall-normal, and spanwise directions, respectively. The components of mean velocity and turbulence fluctuations in these directions are denoted by \((U, V, W)\) and \((u, v, w)\), respectively. In Cartesian tensor notation, the mean and fluctuating values in the positive \(x_i\) direction are denoted by \(U_i\) and \(u_i\), respectively. Here,
and in subsequent chapters, \( i = 1, 2, 3 \) denote the streamwise, vertical and spanwise direction, respectively. Furthermore, the suffix summation conventions usually adopted when discussing Cartesian tensor are implied. The superscript \( '+' \) is used to represent quantities in wall units. For example, \( U^+ = U/U_\tau \), \( u^+ = u/U_\tau \) and \( y^+ = y U_\tau/\nu \), where \( U_\tau \) is the friction velocity (to be defined later) and \( \nu \) is the kinematic viscosity.

On the basis of the continuum fluid assumption (e.g. Townsend, 1976), the dynamics of turbulence is adequately described by the continuity and Navier-Stokes equations. Using tensor notation, the continuity and the Reynolds-averaged Navier-Stokes equations for an incompressible fluid are, respectively, given by

\[
\frac{\partial U_i}{\partial x_i} = 0 \tag{1.1}
\]

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial U_i}{\partial x_j} - \langle u_i u_j \rangle \right) \tag{1.2}
\]

where \( P \) is the thermodynamic pressure, \( \rho \) is the fluid density, \( \nu \) is the kinematic viscosity, and \( \langle u_i u_j \rangle \) denotes turbulent or Reynolds stresses.

1.1.3 Motivation for Near-Wall Turbulence Research

Many fluid flow processes encountered in engineering, environmental, and industrial applications are turbulent and are directly influenced by a solid boundary. Most of the flow dynamics take place in the near-wall region. In simple wall-bounded flows, for example, the peak values of the Reynolds shear stress and production of turbulence kinetic energy occur in the near-wall region. Furthermore, heat, mass and momentum
transfer rates at the surface are controlled by the turbulent flow structure in the near-wall region. In near-wall turbulence, the wall shear stress is one of the most important parameters of interest. An accurate determination of the wall shear stress is important because, as will be seen in Chapter 2, most of the scaling laws involve the friction velocity or wall shear stress. The wall shear stress is also of primary importance from a practical perspective since it is directly related to the drag force. Progress in near-wall turbulence research could lead to a better understanding of the turbulence structure and skin friction characteristics so that momentum, heat, and mass transfer rates at the wall can be accurately predicted. Consequently, near-wall turbulence has attracted considerable research attention and continues to be of current research interest.

1.1.4 Canonical and Complex Near-wall Flows

In the present study, a turbulent boundary layer is defined as a near-wall flow in which the following two regions can be identified: 1) a very thin layer in the immediate neighborhood of the wall in which the velocity gradient normal to the wall ($\partial U/\partial y$) is very large and viscosity exerts an essential influence, and 2) an outer region where viscous effects are unimportant. The term canonical flow will refer to a zero pressure gradient turbulent boundary layer (at low freestream turbulence) or fully developed flow in ducts with smooth surfaces. Because of their simplicity, both in physics and geometry, canonical flows are the most widely researched flows.

Although results obtained from these studies have improved our physical understanding of the near-wall turbulence structure, many practical flows are much more
complex. For example, in flow over an airfoil or turbine blade, rapid changes in pressure gradients and surface curvature may occur. Blowing or suction may also be applied at some point along the airfoil as a form of boundary layer control. In industrial applications, the mass transfer of interest may relate to corrosion of the wall material, in which case corrosion is responsible for generating surface roughness. For environmental flows involving dispersion of waste products in rivers and streams, the streambed is always rough. These extra strains, e.g. pressure gradient, surface roughness and curvature, may introduce additional physics which complicates the turbulence structure. For the purpose of accurate prediction of practically relevant near-wall turbulent flows, additional research on relatively more complex flows are necessary.

1.1.5 Research Methodologies

Near-wall turbulence has been extensively investigated using experimental techniques, and theoretical and numerical analyses, i.e. turbulence modeling and direct numerical simulation (DNS). Experimental investigation, using both single and multi-point measurements as well as flow visualization, is the approach most widely used in turbulence research.

Turbulence modeling, of varying complexity, has also played a significant role in turbulence research. The low-Reynolds number two-equation models have been widely used in predicting a variety of turbulent flows encountered in practice. Eddy-viscosity models have regained popularity as components of two-layer turbulence models (Rodi et al., 1993). For engineering flows dominated by complex flow physics, the Reynolds
stress transport equation becomes a viable alternative because the various terms in the governing equations are treated exactly.

The arrival of high performance computing resources in recent years has opened the possibility of direct numerical simulation (DNS) of turbulent flows by solution of the three-dimensional Navier Stokes equations. The DNS has been very successful in calculating relatively low Reynolds number turbulent flows with simple geometry. The DNS has also the capability to adequately resolve the diffusive sublayer and accurate statistics obtained from DNS have provided insight into the physics of turbulence. In some sense, DNS can be regarded as a companion (or numerical) experiment to the actual physical experiment because it can generate much comprehensive information on the turbulence structure of the flow field. Even though DNS is not yet a technique for complex engineering flow computations, the data obtained from DNS calculation, together with physical experimental data, have enhanced the development of physically correct turbulence models.

1.1.6 Measuring Devices

In the area of experimental investigation of turbulent flows, conventional instruments such as thermal anemometers and Pitot-tubes are most extensively employed. However, the suitability of Pitot-tubes and conventional thermal anemometers for velocity measurements in regions of high turbulence level has been repeatedly questioned. Furthermore, the spatial resolution of Pitot-tube and thermal probes is usually poor and it
is difficult to make measurement in the viscous sublayer especially if cross-wires are employed.

With the arrival of optical devices such as particle image velocimetry (PIV) and laser Doppler anemometer (LDA), and recent advances in signal processing technology, it is now possible to obtain sophisticated measurements, which provide remarkable insight into the near-wall turbulence structure. Since optical anemometers are non-intrusive, they exert minimal interference on the flow field. The LDA is also suitable for near-wall measurements and in regions of high local turbulence intensity. In contrast to Pitot-tube and thermal probes, the LDA exhibits a linear velocity-frequency relationship and constant instrument sensitivity. In spite of these attractive features, care is required in the proper use of the LDA.

1.2 REYNOLDS NUMBER EFFECTS AND SCALING ISSUES
Reynolds numbers encountered in practice are usually very high. Due to hardware and equipment limitations, Reynolds numbers of the flows investigated experimentally or numerically are orders of magnitude lower than those encountered in practice. It is therefore important to know whether results obtained from relatively low Reynolds number experiments or numerical calculations can be extrapolated to the higher Reynolds numbers encountered in engineering and environmental applications.

The concept of Reynolds number similarity has been widely used in fluid dynamics research. When similarity assumptions are applied to near-wall turbulent
flows, they imply that individual turbulence statistics obtained from different facilities and at different Reynolds numbers will collapse onto a single curve when they are made dimensionless using the proper velocity and length scales. Implicit in the above assumption is that the proper scaling laws need be identified in a systematic investigation of Reynolds number effects. In near-wall turbulence research, two possible velocity scales are the friction velocity \( U_c = \left[ \frac{\tau_w}{\rho} \right]^{1/2} \), where \( \tau_w \) and \( \rho \) are the wall shear stress and fluid density, respectively) and freestream velocity \( U_e \). The ratio \( U_c/U_e \), which is directly related to the skin friction coefficient, is known to be Reynolds number dependent. It should be pointed out that an inaccurate estimate of the friction velocity may mask any systematic examination of Reynolds number effects. Although high Reynolds number experiments will be useful in examining what the (upper) limits are, low Reynolds number experiments are required for sorting differences in scaling laws because of the more rapid variation of \( U_c \) and \( U_e \) in this regime (George and Castillo, 1997). Another motivation for low Reynolds number experiments is that the viscous sublayer is relatively thick so that a more accurate estimate of the friction velocity can be made using data obtained in the viscous sublayer. In this case, the influence of measurement uncertainties is kept minimal.

1.3 SURFACE ROUGHNESS AND THE WALL SIMILARITY

HYPOTHESIS

1.3.1 Definitions and Terminology

Before discussing roughness and its effects on the turbulence structure, some terms and notations that are frequently encountered in this and subsequent chapters are defined.
Figure 1.1 is a schematic of a rough surface and also defines some geometrical features of the roughness elements. In this figure, $k$ denotes a representative average roughness height; $y_p$ is the wall-normal distance measured from the top plane of the roughness elements; and $y_o$ is the virtual origin and represents the distance between $y_p$ and the location at which the mean velocity goes to zero (i.e. $U = 0$). Therefore, for a rough surface, the effective wall normal distance $y = y_p + y_o$. Other related terminology for $y_o$ includes fluid-dynamic height origin (e.g. Raupach et al., 1991) or error-in-origin (Perry et al., 1969; Bandyopadhyay, 1987). Perry et al. (1969) suggested that $y_o$ is a measure of the interaction between the mean flow and roughness. For a given roughness height, the virtual origin must satisfy the following constraint: $0 < y_o < k$. The exact value of $y_o$, however, depends on the roughness elements and other geometric factors. For sand grain roughness, for example, the data compiled by Nezu and Nakagawa (1993) suggest $y_o = 0.15 - 0.3k$. Krogstad et al. (1992) reported a value of $y_o = 0.25k$ for their wire mesh roughness.

![Figure 1.1: Definition sketch of roughness elements](image)

The term roughness sublayer is the counterpart of the viscous sublayer in a smooth wall turbulent boundary layer. It refers to the entire layer which is dynamically influenced by length scales associated with roughness elements. Typically, it extends
from the wall to 2 - 5 roughness heights, i.e. \( y \leq 5k \) (Raupach et al., 1991). The roughness Reynolds number is defined as \( \text{Re}_k = kU_\text{c} \sqrt{\nu} \). Physically, it represents the ratio of a typical roughness length scale to the viscous length scale.

1.3.2 Some Characteristics of Surface Roughness

Following the classical sand roughness experiments by Nikuradse (1933), it is generally accepted that for \( \text{Re}_k > 5 \), the surface is considered as being hydraulically rough. It is important to recognize that a flow that is hydraulically smooth in one sense may be fully rough from other perspectives. For example, in the case of high Prandtl and Schmidt number fluids, the diffusive sublayer is extremely thin so that it may lie entirely within the momentum viscous sublayer. While such a surface may be considered hydraulically smooth from the point of view of momentum transport, the roughness elements may likely protrude beyond the thermal diffusive sublayer.

Turbulent flow over rough surfaces is a complex phenomenon. In the vicinity of the roughness elements, the flow is spatially heterogeneous, may be three-dimensional and no longer parallel to the ground plane (see Figure 1.1). In this case, spatial (rather than time) averaging is desirable although this is difficult to carry out in physical experiments. The passage of eddy motion over the roughness elements causes local eruptions which may increase the vertical mass/momentum interchange (Gatski, 1985). Details of the turbulent structure in this region are also controlled by the specific geometry of roughness elements. The local turbulence intensity close to the roughness elements is usually high in which case standard techniques such as cross-wire
anemometry suffer from substantial measurement errors that are often difficult to diagnose and correct. These complicating features may explain the relatively slow progress in our physical understanding of rough wall turbulent boundary layers in comparison to the smooth wall counterpart. Furthermore, rough wall turbulence research has not benefited much from DNS because of the additional complexity introduced by the geometry of roughness elements as well as intricate physics of the flow.

Although the average height is very often used as the characteristic length scale for roughness elements, other length scales and geometric factors such as aspect ratio, roughness element dimensions and element separation may considerably influence the dynamics of flow over rough surfaces. For example, the rough wall data compiled by Bandyopadhyay (1987) showed that the upper critical value of $Re_k$ above which the flow regime is fully rough decreases as the span-to-height aspect ratio ($l/k$) increases. In the case of evenly distributed sand grains, the upper critical value is about 55 to 70 while it is approximately 12 – 15 for an aspect ratio ($l/k$) greater than 12. Since most of the roughness parameters depend on the specific geometry of roughness elements, it is relatively difficult to develop a unifying theory for rough wall turbulent flows.

1.3.3 Types of Surface Roughness

In considering the effects of surface roughness, often the specific characteristics of the roughness elements have been given minimal attention. Two main types of roughness have been identified in the literature. Following the terminology of Perry et al. (1969), these are referred to as k-type or d-type roughness. If the roughness function (i.e. the
parallel shift between smooth wall and rough wall velocity profiles on a semi-
logarithmic plot) depends on Re_k, it is termed k-type roughness. Experiments have
shown that the k-type scaling is not obeyed by transverse grooved surfaces when the
cavities are narrow or on a smooth surface with a series of depressions. This type of
roughness, as opposed to the sand-grain or k-type roughness, scales with outer variables,
i.e. the boundary layer thickness, δ, or the pipe diameter, d, and is known as d-type.
Depending on the ratio of the groove spacing to height, such a surface roughness may
reduce the non-dimensional Reynolds stress near the surface. Of particular importance to
wall turbulence control are small-scale longitudinal grooves (or riblets) which can
produce local shear reduction of up to 50 percent and net drag reduction of the order of
10 percent (Walsh, 1982; Walsh and Lindemann, 1984).

1.3.4 The Wall Similarity Hypothesis

In his book 'The Structure of Turbulent Shear Flows', Townsend (1976) states

While geometrically similar flows are expected to be dynamically similar if their
Reynolds numbers are the same, their structures are also very nearly similar for
all Reynolds numbers which are large enough to allow (fully) turbulent flow.

This is one of the several statements of the Reynolds number similarity implied in
Section 1.2. Perry and Abell (1977) extended the above notion to rough wall boundary
layers. A more general statement of the similarity concept, which is referred to as the
"wall similarity hypothesis" was given by Raupach et al. (1991) as follows
Outside the roughness (or viscous) sublayer, the turbulent motions in a boundary layer at high Reynolds number are independent of the wall roughness and the viscosity, except for the role of the wall in setting the velocity scale $U_\infty$, the height $y$ and the boundary layer thickness $\delta$.

The above notion suggests that the effects of surface roughness are confined to the immediate vicinity of the roughness elements so that the turbulence structure over a significant portion of the boundary layer should be unchanged in spite of substantial alterations to the surface characteristics of the wall. This has important implications for rough wall turbulence models as will be discussed in subsequent chapters.

1.4 TURBULENT WALL JETS

1.4.1 Definition and Nomenclature

A turbulent wall jet is a shear flow directed along a wall, where by virtue of the initially supplied momentum, at any downstream station, the streamwise velocity over some region within the flow exceeds that in the external stream (Lauder and Rodi, 1981). A wall jet also constitutes part of more complex flows as in the case of the flow regime downstream of an impinging jet.

A definition sketch that also serves to define some of the flow nomenclature is shown in Figure 1.2. In this figure, $x$ and $y$ denote distances in the streamwise and vertical directions, respectively; $U$ and $V$ are the streamwise and vertical components of the mean velocity; $U_j$ is the jet exit velocity; $b$ is the slot height; $U_m$ is the local maximum velocity; $y_m$ and $y_{1/2}$, respectively, denote the vertical locations where $U_m$ and
0.5U_m occur. In the present study, y_m and y_{1/2} will be referred to as the inner thickness and the jet half-width, respectively.

![Figure 1.2: Schematic of a turbulent wall jet](image)

The flow field is traditionally divided into two regions: an inner layer which extends from the wall to the point of maximum velocity (i.e. y ≤ y_m), and an outer region which stretches from the point of maximum velocity to the outer edge of the flow (i.e. y > y_m). In this context, a turbulent wall jet may be thought of as a composite flow made up of two interacting shear layers: an inner region, which possesses many of the characteristics of a turbulent boundary layer, and an outer region, which, though influenced by the solid wall, is structurally similar to a free plane jet. The interaction between the small-scale dominated inner layer and the large-scale dominated outer layer creates a complex structure that is characterized by intense mixing. This region is still poorly understood and poses the greatest challenge to numerical models.

1.4.2 Applications of Wall Jets

The turbulent wall jet has received considerable research attention, prompted mainly by its important and diverse technological applications, e.g. in boundary layer control and
film cooling technology. Among the various types of control techniques, boundary layer control is probably the oldest and most economically important (Gad-el-Hak, 1996). The turbulent wall jet has been suggested as the most preferred and straightforward flow separation control technique applied to military fighters and STOL transports (Gad-el-Hak and Bushnell, 1991). By using a wall jet to alter the locations and strength of vortices formed at aircraft wingtips, wing/body junctions as well as around slender bodies such as missiles and aircraft fuselages, the amount of lift and drag are effectively modified. A tangential jet blowing over the upper surface of a rounded trailing-edge airfoil is also employed to set an effective Kutta condition by fixing the location of separation. This technique is also used to stabilize a trapped vortex and has been employed to achieve increased super-manoeuvrability of helicopters and controllability of aircraft flying at low angles of attack. Thus wall jet flows may be found on airplane wings (e.g., F-104 Starfighter, A-6 Crusader) and more recently on helicopter tail booms (NOTAR).

Investigation of a wall jet in a cross flow has been stimulated primarily by problems of interaction between lifting jets and crosswinds used by VSTOL aircraft. Aerodynamic interaction between a hovering aircraft and the ground environment has also been recognized to be a dominant factor in the successful development of VSTOL technology. In order to determine the impact an aircraft has on the surrounding environment while hovering in ground effect, adequate knowledge of important design parameters such as surface temperature and pressure at impingement, acoustic noise and velocity decay of the ground plane wall jet is necessary. Sufficient understanding of the
velocity characteristics is also necessary to avoid unnecessary aerodynamic loading on
ground personnel, buildings, and other aircraft.

In order to improve the thermal performance of modern gas turbines used in either
aircraft engines or power production systems, specifications for turbine inlet temperature
continue to increase while cooling airflow is kept minimal. The specific power of a gas
turbine also depends on turbine inlet temperature. In spite of noticeable progress made in
turbine blade metallurgy, a reasonable lifetime of turbine blades can be ensured only if
an efficient surface-cooling mechanism is employed. Film cooling has been suggested
(MacMullin et al., 1989; Lakehal et al., 1998) as one of the most efficient cooling
methods for such devices. It is claimed that this method is more efficient than internal
convection cooling because of the relatively low heat-transfer characteristics of air. The
wall jet has been identified as one of the most efficient film cooling devices in gas
turbine applications.

Other widespread applications of wall jets for heat and mass transfer modification
can be found in the automobile defroster and deflectors used in conditioned air-
circulation systems (Launder and Rodi, 1983). The design and position of such
deflectors become especially crucial in large-scale one-of-a-kind applications such as
found in a concert auditorium. Turbulent wall jets are also of particular interest in
agriculture to improve air circulation in poultry houses (Blackwell et al., 1990), and in
many other industrial applications to effect enhanced drying, leaching of solids and
toughening of glass.
Investigation of wall jets, aside from practical applications, has also drawn considerable fundamental interest in the past because it has the characteristics of both a boundary layer and a free jet. The wall jet has, therefore, been identified as a prototypical flow for investigating the physics of complex near-wall turbulence as well as improving our physical understanding of the interaction between a boundary layer and free shear flow. Furthermore, the influence of elevated freestream turbulence on fluid dynamics and convective heat transfer has recently been recognized as a major factor in turbine blade design (MacMullin et al., 1989). In this respect, a wall jet has also been used in such research efforts to simulate the high free-stream turbulence encountered in turbomachinery.

1.5 SOME CHARACTERISTICS OF OPEN CHANNEL FLOWS

The structure of most wall-bounded flows is considered to be similar. However, there are some specific and important differences among these flows. Since the present study pertains to measurements in an open channel flow and the results will be compared to other near-wall turbulent flows, some of the important and unique characteristics of open channel flows are summarized below.

1. In an open channel boundary layer, the maximum streamwise mean velocity may occur below the free surface (Tominago et al., 1989; Nezu and Nakagawa, 1993; Shi et al., 1999). This unique feature is referred to as ‘velocity dip’, and is attributed to secondary flows (Nezu and Nakagawa, 1993).
2. Near the free surface of an open channel flow, the background turbulence level is substantially higher than freestream turbulence intensities reported in typical wind tunnel experiments.

3. Similar to zero pressure gradient turbulent boundary layers (e.g. Gad-el-Hak and Bandyopadhyay, 1994), the existing open channel flow literature indicates that the outer wake parameter ($\Pi$) shows a Reynolds number dependence. The LDA data of Nezu and Rodi (1986) suggested an asymptotic value of $\Pi = 0.2$. This value is considerably lower than the asymptotic value of $\Pi = 0.62$ reported by Coles (1987).

4. In open channel flows, the vertical motions are restrained in the interfacial or free surface region by the damping effect of the free surface (e.g. Komori et al., 1993; Borue et al., 1995). The DNS results of Komori et al. (1993) indicate that the turbulent kinetic energy of the vertical motion is re-distributed to the spanwise and streamwise motions through the pressure fluctuation. This causes an increased stress anisotropy in the vicinity of the free surface of an open channel flow in comparison to outer edge of canonical turbulent boundary layers.

1.6 SUMMARY

Near-wall turbulence is a complex fluid flow phenomenon. The skin friction behavior is of both practical and theoretical interest. For flow over rough surfaces, the presence of vortical structures, which are present in the roughness-element wakes, further complicates the turbulence structure especially close to the roughness elements. The high local turbulence intensity close to the roughness elements suggests that conventional
thermal anemometers may be making important measurement errors close to the wall where most of the flow dynamics occur.

Although the physics of canonical near-wall flows is relatively well understood, mainly due to refined measurements and direct numerical simulations (DNS), our physical understanding of practically relevant (i.e. complex turbulent flows) is deficient. Low Reynolds number effects and scaling issues remain important research questions. Although the wall similarity hypothesis would be very attractive for turbulence modeling, since it suggests that the turbulence structure over smooth and rough surfaces is essentially the same, its validity needs critical verification. This provides reasonable grounds for a systematic experimental investigation of surface roughness and its effect on the near-wall turbulence structure. Furthermore, by treating roughness as a modification of the inner layer, or the outer region of a turbulent wall jet as a modification of the outer layer of a turbulent boundary layer, an improved understanding of the interaction of the inner and outer layers may be realized. Therefore, investigation of a turbulent boundary layer and a wall jet over smooth and rough surfaces, apart from practical motivations, would also promote a better understanding of complex turbulent flow.

1.7 OBJECTIVES AND SCOPE

1.7.1 Objectives

The purpose of this research is to examine the structure of turbulent boundary layers and wall jets on smooth and different types of rough surfaces with an overall objective of
improving our physical understanding of the near-wall turbulence structure. The objectives are:

1. To examine low Reynolds number effects and scaling laws for turbulent boundary layers. The understanding obtained from these results is then used to accomplish the principal objective which is stated next.

2. To examine the interaction between the inner and outer layers of a turbulent boundary layer and wall jet on smooth and rough surfaces.

1.7.2 Scope

The scaling laws for the mean velocity and its higher order moments, as well as the relevant experimental and numerical literature on smooth and rough wall turbulent boundary layers and wall jets are reviewed in Chapter 2. In the light of our current understanding of these flows, further refinements are made to the objectives stated in Section 1.7.1. An overview of the LDA system, description of experimental facilities and surface roughness as well as instrumentation and experimental details are given in Chapter 3. In Chapter 4, one-component smooth and rough wall velocity measurements in turbulent boundary layers are reported. The sets of data presented in this chapter are used to examine low Reynolds number effects, scaling issues and effects of the specific roughness geometry on the turbulence structure. The effects of surface roughness on higher order turbulence statistics such as Reynolds stresses, triple correlation as well as the energy budget, mixing length and eddy viscosity distributions are discussed in Chapter 5. The understanding obtained regarding the turbulence structure on smooth and
rough wall turbulent boundary layers is used as the basis for interpreting the relatively more complex wall jet data in Chapter 6. A summary, the major conclusions and contributions from the present research are given in Chapter 7.
CHAPTER 2

LITERATURE REVIEW

Scaling laws for the mean velocity and turbulence statistics are reviewed in this chapter. In the case of the overlap region of the mean turbulent boundary layer, the scaling laws proposed by classical theories as well as recent power laws formulated by Barenblatt (1993) and George and Castillo (1997) are considered. The techniques used to determine the wall shear stress are discussed. Both conventional and recent scaling laws proposed to describe the streamwise evolution of turbulent wall jets are also discussed. Finally, the recent and relevant experimental and numerical studies on turbulent boundary layers and wall jets are reviewed.

2.1 THEORETICAL ANALYSIS

2.1.1 Turbulent Boundary Layers

Scaling laws derived from theoretical analysis have played a significant role in interpreting near-wall experimental data. It is generally accepted that the dynamics of a turbulent flow is described by the Navier-Stokes equation, i.e. Eqn. (1.2). For near-wall turbulent flows, the two-layer concept forms the basis of interpreting events and also constructing mathematical models. According to the two-layer concept, the flow structure consists of two distinct regions: 1) an inner layer, i.e. viscous sublayer and buffer region, where viscous effects dominate; and 2) an outer region where inertial
effects dominate. At sufficiently high Reynolds numbers, classical theories (e.g. asymptotic expansion (Millikan, 1938) and mixing length (Prandtl, 1932)), suggest an overlap region between the inner and outer layers. On the other hand, more recent analyses propose power laws to describe the overlap region. In the following subsections, scaling laws for the inner and outer layers as well as the overlap region are reviewed. Some of the available techniques used to determine the wall shear stress are also discussed.

2.1.1.1 Scaling Law for the Inner Layer

In the classical approach, dimensional analysis of the dynamical equations and boundary conditions leads to a scaling law for the mean velocity profile. In the immediate vicinity of a solid boundary, the flow dynamics is assumed to depend on the distance from the wall ($y$), the wall shear stress ($\tau_w$) and the fluid properties, i.e. kinematic viscosity ($\nu$) and density ($\rho$). From dimensional considerations, the following dimensionless functional relationship is obtained for the mean velocity

$$U^* = f_i[y^*, \delta^*]$$  \hspace{1cm} (2.1a)

where $U^* = U/U_\tau$, $y^* = yU_\tau/\nu$ and $U_\tau = [\tau_w/\rho]^{1/2}$ is the friction velocity. The parameter $\delta^*$ ($= \delta U_\tau/\nu$) is a Reynolds number based on the boundary layer thickness ($\delta$) and the friction velocity, and indicates the ratio of the outer to the inner length scales. If the dimensionless functional relationship $f_i$ is independent of Reynolds number, i.e.

$$U^* = f_i[y^*]$$  \hspace{1cm} (2.1b)

it implies complete similarity exists in the inner region. Eqn. (2.1b) is commonly referred to as the universal law of the wall.
The velocity distribution in the near-wall region (i.e. the viscous sublayer and lower part of the buffer region) will be of considerable interest in determining the skin friction and also for constructing a composite velocity profile. Using a Taylor series expansion together with the continuity equation and no-slip condition at the wall, the mean velocity can be expressed by the following relation

\[ U^+ = y^+ + c_4 y^{+4} + c_5 y^{+5} + \text{HOT} \quad (2.2) \]

where the coefficients \( c_4 \) and \( c_5 \) may vary slightly with Reynolds number and \( \text{HOT} \equiv \) higher order terms. Recent LDA measurements (Eriksson et al., 1998) suggested \( c_4 = -0.0003 \pm 0.0001 \) while George and Castillo (1997) proposed a value of \( c_5 = 13.5 \times 10^{-6} \).

For a rough surface, the characteristic length scales may also include the average roughness height, \( k \), and any additional length scales needed to completely characterize the roughness. If the viscous length \( (\nu/U_t) \) and the average roughness height \( (k) \) are chosen as the only relevant length scales in order to preserve the generality of flow over both smooth and rough surfaces (Raupach et al., 1991), one can define a roughness Reynolds number, \( \text{Re}_k (= kU_t/\nu) \).

### 2.1.1.2 Scaling Law for the Outer Layer

In the outer region, the wall acts to retard the local velocity in a way that is independent of viscosity \( (\nu) \), but dependent on the distance from the wall \( (y) \), the boundary layer thickness \( (\delta) \) and an outer velocity scale \( U_o \). In the case of a fully developed duct flow, the length scale is given by the radius \( (R) \). It is important to note that, in contrast to the inner layer, no equivalent theory has been proposed by the classical theories for the outer
layer. On the basis of experimental evidence and the need to attain similarity in the outer region, the mean velocity profile for this region is often presented in a defect form. According to classical theories, the velocity scale for both turbulent boundary layers and duct flows is the friction velocity, i.e. $U_o = U_f$. The recent theory derived for a canonical zero-pressure gradient turbulent boundary layer by George and Castillo (1997) showed that the proper outer velocity scale is the freestream velocity, i.e. $U_o = U_e$. The velocity distribution in the outer region is given by

$$\frac{U_o - U}{U_o} = f_o \left[ \frac{y}{\delta}, \delta' \right]$$  \hspace{1cm} (2.3a)

where $f_o$ expresses the dimensionless functional relationship. If $f_o$ is independent of Reynolds number, complete similarity exists in the outer region, i.e.

$$\frac{U_o - U}{U_o} = f_o \left[ \frac{y}{\delta} \right]$$  \hspace{1cm} (2.3b)

### 2.1.1.3 Scaling Laws for the Overlap Region

At a sufficiently high Reynolds number ($\delta'$), classical theories suggest an overlap region between the inner and outer layers where both layers interact. In this region, the inner length scale ($v/U$ or $k$) is presumably too small to control the dynamics of the flow, and the outer length scale ($\delta$) is presumably too large to be effective (Tennekes and Lumley, 1972). If this occurs, the dynamics of the flow is independent of all length scales except the distance from the wall ($y$).
The scaling law for the mean velocity in the overlap region has been of considerable interest to the fluid dynamics community because it leads directly to a skin friction relation. In the overlap region, the scaling law for the mean velocity is obtained by matching the inner and outer scaling laws. The specific form of the scaling law in this region depends on the additional assumptions made in the course of the matching process. The classical theories (Millikan, 1938; Clauser, 1954; Panton, 1990) propose a log law for both duct flows and turbulent boundary layers. The recent pipe flow analysis (Barenblatt, 1993) and zero-pressure gradient theory proposed by George and Castillo (1997) indicate that the overlap region is described by a power law. Long and Chen (1981) and George and Castillo (1997) showed that although the mean velocity in the overlap region is logarithmic in the case of pipe flows, the scaling law for turbulent boundary layers is entirely different.

For a turbulent boundary layer, Long and Chen (1981) remarked that it is strange that the overlap region between the viscous inner and outer layers which is characterized by inertia does not depend on both inertia and viscosity, but only on inertia. They suggested that this might be a consequence of improperly matching two layers which do not overlap. They also showed that irrespective of Reynolds number, there exists a ‘mesolayer’ which intrudes between the inner and outer layers and prevents the overlap of the classical theory. In spite of these recent developments, the log law continues to be the more preferable scaling law used in the analysis of both turbulent boundary layers and duct flows. Sreenivasan (1989) argued that although the power law used by engineers to describe the mean velocity profile has been discredited by scientists ever
since Millikan (1938) derived the log law from asymptotic arguments, the basis for the power law is *a priori* as sound as that for the logarithmic law. George and Castillo (1997) pointed out that it is very difficult to distinguish a logarithmic law from a weak power law using experimental data alone since one can be expanded in terms of the other.

In spite of specific differences among researchers, it appears that a power law is more suitable for low Reynolds number flows (e.g. Djenidi et al., 1997; George and Castillo, 1997; Zagarola et al., 1997). The boundary layer analysis of George and Castillo (1997) showed that the overlap region consists of a mesolayer ($30 < y^+ < 300$) and an inertial sublayer ($y^+ > 300$). It was argued that the logarithmic portion of a boundary layer (i.e., $50 < y^+ < 150$) is just a portion of the mesolayer. Based on empirical evidence, Zagarola et al. (1997) proposed that for pipe flows, the mean velocity consists of two distinct regions, a power law region for $50 \leq y^+ \leq 500$ or $0.1R^+$ (the upper limit being dependent on Reynolds number), and a log law region for $500 \leq y^+ \leq 0.1R^+$. Recent refined measurements and DNS results at low Reynolds numbers showed that the overlap region gradually disappears as the Reynolds number decreases. It follows that at low Reynolds numbers, a log law region may not appear. This has important implications for low Reynolds number flows (especially on a rough surface as will be shown in this study) because without a well-defined log law region the usefulness of the Clauser plot technique to determine the skin friction is severely diminished.
2.1.1.3.1 The Logarithmic Law

According to classical theories (Millikan, 1938; Clauser, 1954), the inner and outer layers can be matched in the limit of infinite Reynolds number, i.e. assuming complete similarity, to obtain the following log law for smooth-wall turbulent flows

\[ U^* = \frac{1}{\kappa} \ln y^* + B \]  

(2.4)

In Eqn. (2.4) the log law constants (i.e. the von Karman constant, \( \kappa \) and the additive constant, \( B \)) are assumed to be universal and independent of Reynolds number. The exact values differ slightly from one researcher to the other; in the present study, the following values are adopted: \( \kappa = 0.41 \) and \( B = 5.0 \).

For a rough wall boundary layer, the mean velocity profile may be written in the following form

\[ U^* = \frac{1}{\kappa} \ln y^* + B - \Delta B^* \]  

(2.5)

where \( \Delta B^* \) is the roughness function which represents the (parallel) shift between smooth-wall and rough-wall velocity profiles on a semi-logarithmic plot. The specific value of \( \Delta B^* \) depends on the roughness Reynolds number as well as the roughness geometry. As mentioned earlier, the log law is the most widely used scaling law for both turbulent duct flow and boundary layers, and as such is the formulation presented in most undergraduate fluid mechanics texts. It also forms the basis of the Clauser chart technique used to determine the wall shear stress.
2.1.1.3.2 Power Laws

Over the past decade, power laws have received increasing attention as an alternative formulation for the mean velocity profile in boundary layer flows. Various types of power law formulations have emerged in recent years depending on the specific assumptions made. In the present study, the formulations proposed by Barenblatt (1993) and George and Castillo (1997) are considered.

Barenblatt (1993) [BP]

The power law by Barenblatt (1993) was specifically formulated for pipe flows. He explained the theoretical basis of both the log law and power law, and offered an argument in favor of a power law to describe the mean velocity. His formulation is based on an incomplete similarity assumption for the overlap region, which implies that the flow in this region is Reynolds number dependent. The power law proposed by Barenblatt (1993) is of the form

\[ U^+ = C(y^+)^\alpha \]

(2.6)

where \( C \) and \( \alpha \) are constants that vary slowly with Reynolds number. The power law constants are given by the following asymptotic expansions

\[ \alpha = \frac{a_1}{\ln Re} + \frac{a_2}{(\ln Re)^2} + \ldots \]

(2.7)

\[ C = c_1 \ln Re + c_2 + \frac{c_3}{\ln Re} + \ldots \]

(2.8)

On the basis of the pipe flow experiments of Nikuradse (1933), Barenblatt and Prostokishin (1993), hereafter denoted as [BP], proposed the following values: \( a_1 = 1.5 \),
\[ c_1 = 1 / \sqrt{3} \text{ and } c_2 = 2.5, \text{ where only the first term and the first two terms are retained for } \alpha \text{ and } C, \text{ respectively. More recently, Zagarola et al. (1997) used their super-pipe data, which covers the range of } 31 \times 10^3 \leq \text{Re} (= 2RU/\nu) \leq 35 \times 10^6 \text{, to recalibrate the power law constants. At lower Reynolds numbers, their values of C are significantly lower than the values of [BP] while their values of } \alpha \text{ are higher than the values of [BP].} \]

*George and Castillo (1997) [GC]*

George and Castillo (1997), hereafter denoted by [GC], used what they termed the Asymptotic Invariance Principle (AIP) to formally derive a different power law for the overlap region of the canonical zero-pressure gradient boundary layer. They assumed complete similarity in the inner and outer layers in the limit of infinite Reynolds numbers. According to their theory, the appropriate velocity scales for the inner and outer layers are \( U_i \) and \( U_o \), respectively. It should be mentioned that unlike the classical theory, which assumed the outer velocity scale \( U_o \) to be identical to the friction velocity (i.e. \( U_o = U_f \)), George and Castillo (1997) derived \( U_o = U_c \) from similarity considerations. Since the ratio of inner and outer velocity scales (i.e. \( U_i/U_o \)) is Reynolds number dependent, it follows that the overlap region must admit Reynolds number dependence except in the limit of infinite Reynolds number. Using a near-asymptotic analysis in the overlap region, they showed that the mean velocity is described by a power law at large Reynolds numbers. In inner and outer coordinates, respectively, their form of the power law becomes

\[ U^+ = C_i (y^+ + a^-)^{\gamma} \quad (2.9) \]
\[
\frac{U}{U_e} = C_o \left( \frac{y + a}{\delta} \right)^\gamma
\]  \hspace{1cm} (2.10)

The coefficients \(C_i\) and \(C_o\) as well as the exponent \(\gamma\) are dependent on the Reynolds number \(\delta^+\). In the above relations, the parameter \(a\) (or \(a^+\)) represents a shift in the origin for measuring \(y\), associated with the growth of the mesolayer region \((30 \leq y^+ \leq 300)\). They pointed out that the asymptotic approach of \(\gamma\) to a small value makes it possible to approximately recover the log law relation of the classical theories. In this case, the additive constant in the classical log law is identical to \(C_i\), which may vary from 7 to 10. Note, however, that these values are substantially higher than the typical value of \(B = 5.0\), but fall within the range of \(4 \leq B \leq 12\) reported in some low Reynolds number experiments. As noted by [GC], neither the near-wall profile (Eqn. (2.2)) nor the overlap profile (Eqn. (2.9)) is valid at \(y^+ = 15\). They proposed the following composite velocity profile to describe the mean velocity in the viscous sublayer, the buffer region and the overlap region
\[
U^+ = \left[ y^+ + c_4 y^{-d} + c_5 y^{+5} \right] \exp(-dy^+) + \nonumber \\
C_i y^{+7} \left[ 1 + \gamma a^{-1} y^{-1} + \frac{1}{2} \gamma(y - 1)a^{-2}y^{+2} \right] \left[ 1 - \exp(-dy^+) \right] \hspace{1cm} (2.11)
\]

where \(a^+ = 16\) and \(d\) is a damping parameter chosen as \(d = 8 \times 10^{-3}\) to fix the transition from the viscous wall region to the overlap region at \(y^+ = 15\).

2.1.1.4 Determination of Shear Stress

In any near-wall turbulence research, one of the most important parameter to determine is the wall shear stress, and hence the friction velocity. An accurate determination of the
wall shear stress is important from practical point of view and also in view of its relevance in scaling the mean velocity as well as turbulence quantities. Following George and Castillo (1997), one may physically view the wall shear stress as measuring the forcing of the inner flow by the outer, or alternatively, as measuring the retarding effect of the inner flow on the outer. This would suggest a strong interaction between the wall shear stress and the outer flow structure so that consideration must be given to the specific flow structure in the outer layer in an accurate determination of the friction velocity.

The methods used to determine the wall shear stress include direct measurement (e.g. with a floating element gauge), performing a momentum balance, extrapolating the Reynolds shear stress to the wall, or by fitting the mean velocity to a standard profile. If the Reynolds number is high enough for a well-defined overlap region to exist, the wall shear stress is commonly determined by fitting the logarithmic profile (Eqn. 2.4) to the mean velocity data. This approach is known as the Clauser plot technique. The use of the Clauser plot technique is well established for turbulent flow over a smooth surface at low-freestream turbulence intensity, and has also been assumed to be valid in high freestream turbulence flows (Hancock and Bradshaw, 1983; Thole and Bogard, 1996). Although the Clauser plot technique (Eqn. 2.5) has been used in some earlier rough wall boundary layers, some studies demonstrated that a Clauser technique for rough wall boundary layers may not be reliable. Perry et al. (1969) remarked that due to two additional roughness variables (i.e. the roughness shift, $\Delta B^+$ and the virtual origin, $y_0$), the Clauser plot technique for finding the wall shear stress would be inaccurate.
As mentioned earlier, the overlap region is negligibly small at low Reynolds numbers, especially on a rough surface. In this case a more reliable estimate of the wall shear stress can be made by fitting to the mean velocity data in both the overlap and outer regions. For a turbulent boundary layer developing over a rough surface, the complete velocity profile is given by

\[ U^- = \frac{1}{\kappa} \ln y^+ + B - \Delta B^+ + \frac{2\Pi}{\kappa} w\left(\frac{y}{\delta}\right) \]  

(2.12)

where \( \Pi \) is Coles’ wake parameter and \( w \) is a universal function of \( y/\delta \). Eqn. (2.12) indicates that description of a measured velocity profile on a rough wall requires the determination of four parameters, namely: \( U_0, \Delta B^+, \Pi \) and \( y_0 \). A reduction in the number of parameters to be fitted is obtained by choosing to work with the defect form of the velocity profile given by Eqn. (2.3). By subtracting \( U^- \) from its value \( U_e^- \) at the edge of the boundary layer, the roughness parameter \( \Delta B^+ \) is eliminated and Eqn. (2.12) becomes

\[ U_e^- - U^- = \frac{2\Pi}{\kappa} \left[ w(1) - w\left(\frac{y}{\delta}\right)\right] - \frac{1}{\kappa} \ln\left(\frac{y}{\delta}\right) \]  

(2.13)

which indicates that the velocity deficit in the outer region is strongly dependent on the magnitude of the wake parameter \( \Pi \). The wake parameter is generally regarded as dependent on streamwise location. Coles (1956) initially proposed that for a smooth-wall zero pressure gradient turbulent boundary layer, \( \Pi \) would be 0.55 at high Reynolds numbers, but later (1987) gave an asymptotic value of 0.62. The recent smooth wall experiments by Osaka et al. (1998) exhibited a Reynolds number dependence for \( \Pi \).
However, an asymptotic value of 0.62 was observed at sufficiently high Reₜ, where Reₜ is the Reynolds number based on boundary layer momentum thickness θ. For sub-critical smooth-wall open channel flows, Nezu and Rodi (1986) also reported a Reynolds number dependence, but indicated that the wake parameter remains nearly constant at Π = 0.2 at sufficiently high Reynolds numbers. Xinyu et al. (1995) made LDA measurements in super-critical open channel flows at varying bed slopes and obtained a value of Π = 0.3.

A commonly used form of Eqn. (2.13) for the velocity distribution in zero pressure gradient boundary layers on a rough wall is Hama's (1954) formulation. For small values of y/δ, Eqn. (2.13) is dominated by the logarithmic term and is therefore written as

$$U_e^- - U^* = -\frac{1}{\kappa} \ln \left( \frac{y}{\delta^* U_e^-} \right) - 0.6 \quad (y/\delta^* U_e^- \leq 0.045)$$  \hspace{1cm} (2.14)

For larger values of y/δ, the wake contribution dominates and Hama proposed the following function

$$U_e^- - U^* = 9.6 \left[ 1 - \left( \frac{10y}{3\delta^* U_e^-} \right) \right]^2 \quad (y/\delta^* U_e^- > 0.045)$$  \hspace{1cm} (2.15)

In both cases, the displacement thickness δ*, is used as the reference length scale. Eqns. (2.14) and (2.15) connect smoothly at y/δ* U_e^- = 0.045 or y/δ = 0.15. Bandyopadhyay (1987) suggests that the Hama profile could be fitted to obtain a reliable estimate of U_e irrespective of the surface. He also argued that since the Clauser technique matches the profile in the logarithmic region, which is thin, there are only a few data points to work
with. In contrast, the profile matching using Hama's formulation covers virtually the entire region.

It has, however, been observed (e.g. Bandyopadhyay, 1987; Perry et al., 1987; Krogstad et al., 1992) that the value of friction velocity \( U_f \), obtained from the Hama formulation (Eqns. (2.14) and (2.15)) is consistently higher than that obtained from either a momentum balance or by extrapolating the Reynolds stress to the wall. Bradshaw (1987) suggested that this may be due to the strength of the wake, as implied by Eqn. (2.15), being too small. With recent evidence of the dependence of \( \Pi \) on \( Re_\theta \), roughness and (high) turbulence levels, the usefulness of a defect law such as that of Hama which fixes the value of \( \Pi \) may be limited for the present experimental conditions. As will be shown subsequently, incorrect wake strength may contaminate an estimate of the skin friction coefficient, and hence the roughness shift, in rough wall flows.

As an alternative to Hama's formulation, Krogstad et al. (1992) employed a correlation that does not implicitly fix \( \Pi \) but rather allows its value to be optimized. They used the formulation proposed by Finley et al. (1966), and later used by both Granville (1976) and Hancock and Bradshaw (1983), namely

\[
\frac{w}{\delta} \left( \frac{y}{\delta} \right) = \frac{1}{2\Pi} \left[ \left( 1 + 6\Pi \right) - \left( 1 + 4\Pi \right) \left( \frac{y}{\delta} \right) \right] \left( \frac{y}{\delta} \right)^2
\]  

(2.16)

Eqn. (2.16) is the simplest polynomial satisfying the two boundary conditions (correct slope and function values) both near the wall and the boundary layer edge. Krogstad et al. (1992) combined Eqns. (2.13) and (2.16) to obtain
\[ f = \frac{U}{U_e} = 1 \frac{U_t}{\kappa U_e} \left\{ \ln \left( \frac{y}{\delta} \right) - \left( 1 + 6\Pi \frac{y^3}{\delta} \right) + \left( 1 + 4\Pi \frac{y^3}{\delta} \right) \right\} \] (2.17)

which is a more sophisticated expression for the mean velocity profile which can be fitted to the experimental data to obtain the optimized values of \( U_t, \Pi \) and \( y_0 \). Of special importance is the explicit determination of the wake strength \( \Pi \), and the expectation of a more accurate estimate of the friction velocity, \( U_t \).

For a smooth wall turbulent boundary layer, if a sufficient number of data points is obtained in the linear viscous sublayer, i.e. the near-wall region where \( U^+ = y^+ \), a more accurate estimate of the wall shear stress can be obtained using the relation

\[ \tau_w = \mu \frac{dU}{dy} \] (2.18)

Recent LDA measurements indicate that the linear profile (i.e. \( U^+ = y^+ \)) is strictly valid only for \( y^+ \leq 4 \). This requirement is too stringent to be met in many physical experiments, especially if Pitot-tube and hot-wire probes are used. On the other hand, by fitting a polynomial (Eqn. (2.2)) to the velocity data in the near-wall region, the useful extent of the viscous region in determining the wall shear stress can be increased. Durst et al. (1998) used a fifth order polynomial to describe their near-wall mean velocity profiles. In the present study, Eqn. (2.2) truncated at the fifth order, i.e.

\[ U^+ = y^+ + c_4 y^4 + c_5 y^5 \] (2.19)

is adopted in determining the wall shear stress for the smooth wall data.
The power laws proposed by Barenblatt (1993) and George and Castillo (1997) were also used to derive skin friction relations. The skin friction relation obtained from the formulation proposed by Barenblatt (1993) was shown (see Djenidi et al., 1997) to be of the following form

\[
\frac{U_\tau}{U_e} = \frac{1}{\exp(3/2\alpha)} \left[ \frac{\exp(3/2\alpha)}{C} \right]^\alpha
\]

(2.20)

For the power law derived by George and Castillo (1997), the skin friction relation was also shown to be a power law and is of the form

\[
\frac{U_\tau}{U_e} = \left[ \frac{C_0}{C_1} \right]^\beta \left[ \frac{U_e \delta}{v} \right]^\gamma
\]

(2.21)

Djenidi et al. (1997) applied the theory of Barenblatt (1993) to low Reynolds number smooth-wall boundary layers. They simply defined \( \text{Re} = U_e \delta / v \) and were able to obtain skin friction velocities that agreed with the corresponding values measured by a Preston tube to within ±1.5 percent.

2.1.1.5 Scaling the Turbulence Quantities

Although the classical theories proposed \( U_\tau^2 \) as the appropriate scale for the Reynolds shear stress, the scaling laws for the other turbulence quantities are, in general, less obvious. Most of the earlier boundary layer analyses adopted \( U_\tau^2 \) for normalizing the normal Reynolds stress components and \( U_\tau^3 \) for the various terms in the energy budget (see for example, Krogstad and Antonia, 1999). According to the analysis of George and Castillo (1997), the proper velocity scale for the normal Reynolds stresses is \( U_e^2 \) while the shear stresses were shown to scale on \( U_\tau^2 \). Their analysis also showed that the triple
correlations, stress production as well as dissipation scale on the mixed velocity scale, \( U_c^2 U_e \).

### 2.1.2 Turbulent Wall Jets

#### 2.1.2.1 Scaling the Transverse Profiles

The scaling law for the mean velocity in the overlap region of a turbulent wall jet has drawn considerable controversy and continues to be of current research interest. Many wall jet investigators assumed a similarity between the inner regions of a wall jet and a turbulent boundary layer so that the overlap region is also described by the classical log law. However, there is a considerable inconsistency in the log law constants (i.e. the von Karman constant \( \kappa \) and additive constant \( B \)) reported by various investigators. A summary of the log law constants reported in some earlier studies is given in Table 2.1. The technique used to determine the wall shear stress is also given.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Technique</th>
<th>( \kappa )</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Myers et al. (1963)</td>
<td>Clauser plot</td>
<td>0.41</td>
<td>4.9</td>
</tr>
<tr>
<td>Kruka and Eskinazi (1964)</td>
<td>Preston tube</td>
<td>0.48</td>
<td>11.4</td>
</tr>
<tr>
<td>Pai and Whitelaw (1969)</td>
<td>Razor blade</td>
<td>0.52</td>
<td>9.0</td>
</tr>
<tr>
<td>Alcaraz et al. (1977)</td>
<td></td>
<td>0.56</td>
<td>8.0</td>
</tr>
<tr>
<td>Wygnanski et al. (1992)</td>
<td>Velocity gradient, momentum balance, Preston tube</td>
<td>0.41</td>
<td>5.5 (9.5)</td>
</tr>
<tr>
<td>Karlsson et al. (1993)</td>
<td>Velocity gradient at the wall</td>
<td>0.41</td>
<td>5.0</td>
</tr>
<tr>
<td>Abrahamsson et al. (1994)</td>
<td>Velocity gradient at the wall</td>
<td>0.41</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of log law constants for turbulent wall jets
It is clear from Table 2.1 that while some investigators (e.g. Karlsson et al., 1993; Abrahamsson et al., 1994) observed log law constants identical to the values used in boundary layer analysis, others (e.g. Kruka and Eskinazi, 1964; Pai and Whitelaw, 1969) did not. It should be noted that the data reported by Wygnanski et al. (1992) demonstrated a universality of the slope ($\kappa$) but the additive constant ($B$) showed Reynolds number dependence. MacMullin et al. (1989) (not shown) reported a tendency of the slope of the log region to decrease with increasing turbulence intensity and downstream distance.

A number of reasons have been proposed to explain the discrepancies reported in the literature regarding the log law constants. Launder and Rodi (1983) attributed the disparity in the log law constants to possible errors in measuring the wall shear stress and hence the friction velocity $U_f$. Another possible reason is attempting to fit the log law over too wide a portion of the inner region. It is important to note that non-existence of a well-defined log law region with universal log law constants has important implications for both experimentalists and numerical analysts. For example, use of the Clauser plot technique to determine the wall shear stress as well as the conventional "wall function" used to resolve the near-wall region in near-wall flow computations cannot be employed in wall jet research.

Some wall jet researchers argued that the inner region of a wall jet has a character quite different from that of a turbulent boundary layer. Hammond (1982) indicated that there is no well-defined log law region for the wall jet. He presented an analysis of the
complete velocity and proposed a composite velocity profile to describe the mean velocity profile from the wall up to the edge of the flow. George et al. (2000) extended their AIP analysis (originally applied to a turbulent boundary layer) to derive similarity theory for turbulent wall jets. Their analysis showed that the inner region of a turbulent wall jet is identical to that of a turbulent boundary layer. More specifically it was shown that Eqns. (2.2), (2.9) and (2.10) can be used to describe the inner region. The scaling laws for the turbulence quantities for turbulent wall jets were also shown to be identical to those they obtained for a turbulent boundary layer.

2.1.2.2 Skin Friction Correlation

A problem plaguing wall jet analysis stems from the difficulties of measuring the wall shear stress. A summary of the skin friction relations is given in the review article of Launder and Rodi (1981). The skin friction correlations reported show considerable scatter. Some of the inconsistencies reported in the literature are attributed to the lack of two-dimensionality, to the thinness of the inner layer and to poor experimental techniques (Wygnanski et al., 1992). Many devices commonly used to determine the wall shear stress, and hence the skin friction, in turbulent boundary layers rely on the validity of the log law. However, as demonstrated in Table 2.1, the universality of the log law constants has been repeatedly questioned. Because the skin friction coefficient is such a minor contributor to the wall jet growth rate, attempts (e.g. Schwarz and Cosart, 1961) to estimate the skin friction from a momentum balance often give highly implausible results. Furthermore, the shear stress falls off so rapidly with distance that it
is usually not possible to determine the wall shear stress by extrapolating the Reynolds shear stress to the wall (Lauder and Rodi, 1981).

Bradshaw and Gee (1962) reported skin friction measurements using a Preston tube and proposed the following skin friction correlation:

\[ C_f = 0.0315 \text{Re}_{m}^{-0.182} \]  
(2.22)

where \( \text{Re}_{m} \) is the local Reynolds number based on \( U_m \) and \( y_m \). Among the skin friction correlations available prior to the review of Lauder and Rodi (1981), Eqn. (2.22) appears to be the most satisfactory correlation for a wall jet in stagnant surroundings in the range \( 3 \times 10^3 < \text{Re}_{m} < 4 \times 10^4 \). Hammond (1982) derived the following 'optimum' skin friction formula for the plane wall jet

\[ C_f = 0.06675 \text{Re}_{m}^{-0.258} \]  
(2.23)

Eriksson et al. (1998) also used their LDA data to develop the following skin friction relation

\[ C_f = 0.0179 \text{Re}_{m}^{-0.113} \]  
(2.24)

George et al. (2000) showed that the skin friction law for a turbulent wall jet is also a power law and is identical to that derived for turbulent boundary layers (i.e. Eqn. (2.21)).

2.1.2.3 Streamwise Development

The streamwise evolution of the flow has traditionally been scaled using the slot height \( (b) \) and the exit velocity \( (U_j) \). According to Lauder and Rodi (1981), the growth rate of
the jet half-width and the decay of maximum velocity are, respectively, given by the following relations

\[
\frac{dy_{1/2}}{dx} = 0.073 \pm 0.002
\]  \hspace{1cm} (2.25)

\[
\left[ \frac{U_j}{U_m} \right]^2 = K_1 \left[ \frac{x}{b} \right] + K_2
\]  \hspace{1cm} (2.26)

where \(K_1\) and \(K_2\) are presumed to be constants. Whenever the above scaling laws are used both the velocity decay and spread rates showed Reynolds number and facility dependence. Recent measurements reported growth and decay rates that showed important sensitivity to Reynolds number as well as to the types of measuring devices used. For example, the velocity data reported by Wygnanski et al. (1992) showed distinct Reynolds number dependence. Abrahamsson et al. (1994) reported a spread rate that varied from 0.075 to 0.081, depending on the exit Reynolds number, while Schneider and Goldstein (1994) reported values in the range 0.074 – 0.082, depending on the measuring devices used.

Narasimha et al. (1973) suggested that scaling of the relevant distances by the characteristic dimension of the nozzle and exit velocity might be erroneous. Instead, they proposed scaling the streamwise evolution of the flow by the momentum flux \(J = U_j^2 b\) and kinematic viscosity \(\nu\) of the fluid. The more recent parametric analysis by Wygnanski et al. (1992) and the wall jet similarity theory proposed by George et al. (2000) support Narasimha's suggestion. Following Narasimha's suggestion, the
maximum velocity decay and spread of the wake half-width \( y_{1/2} \) are shown (e.g. Wygnanski et al., 1992; George et al., 2000) to be power laws of the form

\[
\frac{U_m v}{J} = A \left[ \frac{x J}{v^2} \right]^{-\alpha} \tag{2.27}
\]

\[
\frac{y_{1/2} J}{v^2} = B \left[ \frac{x J}{v^2} \right]^{-\beta} \tag{2.28}
\]

where \( A, B, \alpha \) and \( \beta \) are constants that may depend slightly on initial conditions. From similarity considerations, George et al. (2000) showed that the local maximum velocity \( (U_m) \) and the jet half-width \( (y_{1/2}) \) are also related by a power law as follows

\[
\frac{U_m v}{J} = C \left[ \frac{y_{1/2} J}{v^2} \right]^\gamma \tag{2.29}
\]

where \( C \) and \( \gamma \) are constants that may depend on initial conditions. Using the LDA measurements of Eriksson et al. (1998), they recommended the following values: \( C = 1.85 \) and \( \gamma = -0.528 \).

2.2 PREVIOUS STUDIES

2.2.1 Turbulent Boundary Layers

2.2.1.1 Reynolds Number Effects

Reynolds number effects have been the focus of a number of previous near-wall turbulent flow studies (Spalart, 1988; Antonia et al., 1990, Durst et al., 1998). Purtell et al. (1981) investigated Reynolds number effects in a zero pressure gradient turbulent boundary layer. The Reynolds numbers examined were in the range \( 450 < Re_\theta < 5100 \). Their results showed that the overlap region did not disappear even at the lowest \( Re_\theta \).
examined. They observed that the outer wake parameter showed a distinct Reynolds number dependence for $\text{Re}_g < 2000$. In inner coordinates, distributions of the streamwise turbulence intensity were similar for $y^+ < 15$ while a much greater degree of similarity was noted when the boundary layer thickness was used as the normalizing length scale. Wei and Willmarth (1989) made measurements in a fully developed channel over a wide range of Reynolds numbers and concluded that the region of Reynolds number similarity is limited to $y^+ \leq 10$. Harder and Tideman (1991) reported measurements in a fully developed channel flow and observed that for $y^+ \leq 50$, their profiles are independent of Reynolds number. So et al. (1996) investigated Reynolds number effects in zero pressure gradient turbulent boundary layers ($1410 \leq \text{Re}_g \leq 15400$) as well as fully developed channel and pipe flows ($180 \leq \text{Re}_c \leq 8760$) using Reynolds stress models. The results show that Reynolds number effects are very distinct. In the inner region, these effects are less distinct for turbulent boundary layers in comparison to pipe and channel flows. The LDA measurements by Ching et al. (1995) at $400 \leq \text{Re}_g \leq 1320$ showed that the effect of Reynolds number is felt down into the viscous sublayer.

One of the most refined sets of near-wall measurements was made by Durst et al. (1998). Their measurements were made in a fully developed channel flow using a high resolution LDA. The Reynolds numbers (based on bulk velocity and channel width) varied from $2500$ to $9800$. After applying all known corrections to their data, they observed that the streamwise turbulence intensity scaled on inner variables for $y^+ \leq 50$. Furthermore, the peak value of the profiles was found to be $2.55$, independent of Reynolds number. Osaka et al. (1998) examined Reynolds number effects in turbulent
boundary layers. They observed a reasonable collapse of the mean velocity in the near-wall region. The $u^+$ profiles showed Reynolds number independence for $y^+ \leq 20$ and the peak values were found to be insensitive to Reynolds number. More recently, Balachandar and Ramachandran (1999) reported LDA measurements in open channel at $180 < Re_\theta < 480$, thus extending the database to lower values of $Re_\theta$. They identified an overlap region, albeit narrow, with $\kappa$ that is independent of $Re_\theta$. Within the range of Reynolds number considered, they observed the outer wake parameter to decrease with increasing $Re_\theta$.

In spite of some specific differences among findings of previous investigators, Reynolds number effects in turbulent boundary layers are weak for $Re_\theta > 3000$ (Antonia et al., 1990). An excellent review of Reynolds number effects in wall-bounded flows was made by Gad-el-Hak and Bandyopadhyay (1994). They showed that even at the highest Reynolds number flows available in the literature turbulence quantities scaled using inner variables show Reynolds number effects.

The scatter among measurements has been attributed, in part, to resolution problems associated with measuring techniques and inaccuracies in diagnostic instruments. For example, in the case of hot-wire measurements, Johansson and Alfredsson (1983) showed that the maximum value of the normalized streamwise turbulence intensity decreases from $u^+ = 2.9$ for $l^+ = 2.5$ to $u^+ = 2.1$ for $l^+ = 100$, where $l^+$ is length of the hot-wire probe in wall units. In a related study, Johansson and Alfredsson (1986) examined the effects of Reynolds number and probe length on $u^+$ distributions.
They found that in the near-wall region \((y^+ < 30)\), distributions of \(u^+\) showed a dependence on probe length but were independent of Reynolds number. Gad-el-Hak and Bandyopadhyay (1994) recommended that for reliable measurement of turbulence quantities, especially in the vicinity of the wall, probe lengths less than the viscous sublayer thickness are required. Some of the scatter observed previously can also be attributed to inaccurate values of \(U_c\). Note that a \(U_c\) value that is 5 percent too high will pull the \(u^+\) profile down and to the right with an overall error in the distribution that is higher than 5 percent.

### 2.2.1.2 Surface Roughness Effects

Subsequent to the classical sand grain pipe flow experiments of Nikuradze (1933), a number of rough wall turbulent boundary layer measurements have been reported (Perry et al., 1969; Antonia and Luxton, 1971; Bandyopadhyay, 1987; Perry et al., 1987; Hirota et al., 1993). A comprehensive review of both theoretical and experimental knowledge of rough wall turbulent boundary layers was given by Raupach et al. (1991). Furuya and Fujita (1967) reported measurements on sand grain roughness and wire-screen with different pitch-to-diameter ratio, \(t/d\). In the case of the wire-screen data, they observed that the effect of roughness increases to a maximum for \(5 < t/d < 9\) and decreases when \(t/d > 10\).

In most of the earlier rough wall investigations, minimal attention was given to the specific form of the outer layer, i.e. the outer wake component. Mills and Hang (1983) remarked that extensive rough wall turbulent boundary layer experiments carried out at
Stanford University gave skin friction coefficients that deviated from the Prandtl-Schlichting (1934) formulation by as much as 25 percent. They attributed the disparity to the neglect of the role of the wake component of the velocity profile in the Prandtl-Schlichting formulation. A number of previous rough wall experiments were re-evaluated by Tani (1987) and the values of \( \Pi \) obtained fell in the range of 0.4 – 0.7. The d-type roughness experiment of Osaka and Mochizuchi (1988) at \( \text{Re}_\theta = 5300 \) gave \( \Pi = 0.68 \). Recent boundary layer experiments by Krogstad et al. (1992) on a rough surface indicated \( \Pi = 0.7 \), which is distinctly different from the asymptotic value proposed by Coles.

A number of rough wall boundary layer calculations have been reported. Cebeci and Chang (1978) and Krogstad (1991) used eddy viscosity and mixing length models, respectively, to compute the mean velocity. Tarada (1990) and Zhang et al. (1996) employed different low Reynolds number k-\( \varepsilon \) models and observed fair agreement between calculations and experiments. Patel (1998) used k-\( \varepsilon \) and k-\( \omega \) models to calculate both the mean velocity and Reynolds shear stress. Predictions of the mean flow, especially the roughness shift, were comparable to experimental data but the Reynolds shear stress was in error over most of the boundary layer.

Although the global effect of surface roughness on the mean flow is relatively well understood, considerable inconsistencies are reported regarding roughness effects on higher order moments. Grass (1971) reported rough wall measurements using the hydrogen-bubble technique at different values of the roughness Reynolds number. He
observed that outside the roughness sublayer, $v^-$ is invariant of wall conditions. Wood and Antonia (1975) concluded from their investigation that the influence of surface roughness is confined to the wall region. Sabot et al. (1977) reported large differences in the spanwise $w^+$ and vertical $v^-$ components of the Reynolds stress although the streamwise component $u^+$ was observed to be independent of wall conditions. Raupach (1981) made cross-wire measurements over cylindrical roughness elements arranged in different patterns. It was found that outside the roughness sublayer, second-order moments when normalized by $U_r$ are universal and independent of surface roughness. However, the third-order moments as well as production and turbulence diffusion terms in the energy budget showed important sensitivity to the specific roughness concentration.

Measurements over uniform spheres were reported by Ligrani and Moffat (1985). The roughness Reynolds number considered varied from transitionally rough to fully rough regimes. Their results showed a lack of collapse in $u^-$ and $v^-$ but the Reynolds shear stress $\langle u^+ v^- \rangle$ and the correlation coefficients were invariant with $Re_k$ and freestream conditions. It was also observed that, for both transitionally and fully rough regimes, the diffusion terms are altered by surface roughness. Furthermore, turbulence production caused by Reynolds shear stress increases with increasing roughness Reynolds number. The d-type rough wall experiment of Osaka and Mochizuki (1988) also showed that significant differences exist between smooth and rough wall measurements even at $y = 0.6\delta$. Krogstad et al. (1992) compared measurements over a smooth surface and wire screen roughness. Their results showed that $u^+$ is not sensitive
to the surface condition but the $v^+$ profile over the rough surface is significantly higher than observed for the smooth surface. The LDA measurements over smooth and d-type roughness reported by Djenidi et al. (1996) showed important differences between the smooth and rough wall data at significant distances from the surfaces.

Mazouz et al. (1994) investigated the turbulence structure over different types of surface roughness with varying roughness geometry. It was observed that the skewness of the streamwise velocity fluctuation showed distinct dependence on the span-to-height ratio of the roughness elements. However, skewness of the vertical velocity fluctuation as well as the streamwise and vertical components of the flatness factor did not show any important sensitivity to roughness geometry. Mazouz et al. (1998) compared measurements over a smooth wall and a k-type roughness generated using square cross-sectioned two-dimensional bars. Their results revealed that $u^+$ profiles are independent of surface roughness but $v^+$ and $w^+$ profiles over the entire channel are lower on the rough wall than the corresponding smooth wall data. Krogstad and Antonia (1999) made measurements on wire mesh and lateral rods of equivalent roughness shift ($\Delta B^+$). They found that the distributions of Reynolds stresses depend on the specific form of surface roughness. Using u-v quadrant analysis they also showed that the near-wall diffusion is highly dependent on the surface geometry. The triple products also showed distinct roughness dependence.

The stress anisotropy tensor $b_{ij}$ is an important turbulence parameter. Here, $b_{ij} = \langle u_i u_j \rangle / 2k - \delta_{ij} / 3$ where $2k = u^2 + v^2 + w^2$ is the turbulence kinetic energy and $\delta_{ij}$ is the
Kronecker delta so that $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$, otherwise. Although the shear stress anisotropy ($b_{12}$) in turbulent duct flows appears to be independent of surface roughness, the streamwise ($b_{11}$) and vertical ($b_{22}$) stress tensors were found to be significantly higher for a rough surface than observed for a smooth surface (Sabot et al., 1977; Mazouz et al., 1998). Compared to the DNS results of Spalart (1988), the rough wall boundary layer measurements reported by Shafi and Antonia (1995) showed a reduction in the normal stress anisotropy tensors. This is in contrast to the observations made in fully developed channel flows.

2.2.1.3 Effects of Elevated Freestream Turbulence

As mentioned in Chapter 1, the background turbulence levels near the free surface of an open channel boundary layer are relatively higher than reported for canonical zero pressure gradient turbulent boundary layers. In order to facilitate discussion and comparison with the existing literature, some related boundary layer experiments conducted at elevated freestream turbulence intensity are briefly reviewed. For smooth-wall boundary layers at elevated turbulence intensity ($Tu$), Bradshaw (1978) argued that the log law holds when there is local equilibrium in the near wall region. Hancock and Bradshaw (1989) measured various terms in the turbulence energy transport equation at $Tu \leq 6$ percent and found the boundary layer to be in local equilibrium. Thole and Bogard (1996) extended the existing smooth-wall data to turbulence intensity values as high as $Tu = 20$ percent. Among other findings, they confirmed the validity of the log law at high freestream turbulence and noted significant alterations of the outer region of the boundary layer. Based on the measured velocity spectrum, they found that at $Tu = 20$
percent, the freestream turbulence penetrates deep into the wall region. Experimental
evidence also suggests that the strength of the wake is strongly altered at high freestream
turbulence levels. Blair (1983) and Hancock and Bradshaw (1983, 1989) showed that as
the freestream turbulence increases, the outer region of the boundary layer exhibits a
depressed wake region. At a turbulence level of \( Tu = 5 \) percent, for example, the wake
was essentially nonexistent. In the recent smooth-wall study of Thole and Bogard (1996),
an asymptotic value of \( \Pi = -0.5 \) was observed.

2.2.1.4 Turbulent Boundary Layer in Open Channel Flows

There is a considerable amount of literature on turbulent boundary layers in open
channel flow, see for example, Steffler et al. (1983), Nezu and Rodi (1986) and
Tominago et al. (1989). An excellent review of the literature existing prior to 1993 is
given by Nezu and Nakagawa (1993). More recent studies include the LDA
measurements reported by Xinyu et al. (1995), Balachandar and Ramachandran (1999)
as well as LES and DNS results of Komori et al. (1993) and Borue et al. (1995) and Shi
et al. (1999).

In open channel boundary layers, the Moody chart (with pipe diameter replaced by
four times the hydraulic diameter) has been recommended for the prediction of the skin
friction (see for example ASCE Task Force, 1963). Other techniques widely used for
skin friction measurements in open channel flows include \( U_\tau = ghS^{1/2} \), where \( g \) is the
acceleration due to gravity, \( h \) is the depth of flow and \( S \) denotes the channel slope.
Although the friction velocity determined using this relation is found in many previous
experiments to be in fair agreement with the values obtained using other techniques, it is important to note that the former gives an average value rather than a local one. As rightly pointed out by Nezu and Nakagawa (1993), the value of $U_c$ determined using the channel slope may not be adequate for the evaluation of turbulence characteristics.

Similar to other near-wall flows, distributions of the mean velocity are often interpreted in the context of inner and outer scaling laws discussed in Section 2.1. Nezu and Rodi (1986) and many other researchers indicated that the overlap region is well described by a logarithmic law with universal constants identical to those used in boundary layer analysis. The proper outer velocity and length scales are the maximum velocity ($U_c$) and depth of flow ($h$). In order to facilitate comparison to earlier canonical turbulent boundary layers, the boundary layer thickness ($\delta$), which is defined as the vertical distance at which $U = 0.99U_c$, is adopted as the outer length scale.

### 2.2.2 Turbulent Wall Jets

Some of the earliest measurements in a turbulent wall jet include those of Forthmann (1934) and Sigalla (1958) over smooth surfaces, and the rough wall measurements reported by Rajaratnam (1965) and Sakipov et al. (1975). The extensive wall jet literature existing prior to 1981 was critically reviewed by Launder and Rodi (1981, 1983). Some of these studies are summarized in Table 2.2. In Table 2.2, $Re_j$ is the Reynolds number based on exit velocity ($U_j$) and slot height ($b$), $x$ is streamwise distance relative to the exit, $dy_{1/2}/dx$ is the growth rate of the jet-half-width, $M(x)$ denotes the local momentum flux and $J (= U_j^2b)$ is the jet momentum based on exit conditions.
### Table 2.2: Summary of some earlier wall jet studies

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Re/1000</th>
<th>x/b</th>
<th>dy/1/2/dx</th>
<th>M(x)/J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forthmann (1934)</td>
<td>53</td>
<td>3 - 33</td>
<td>0.082</td>
<td>1.0</td>
</tr>
<tr>
<td>Sigalla (1958)</td>
<td>20 - 40</td>
<td>4 - 70</td>
<td>0.064</td>
<td>0.66</td>
</tr>
<tr>
<td>Bradshaw &amp; Gee (1962)</td>
<td>6.1</td>
<td>339 -</td>
<td>0.071</td>
<td>0.43-0.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1459</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Schwartz &amp; Cosart (1961)</td>
<td>13.5 – 42</td>
<td>29 - 85</td>
<td>0.056 – 0.085</td>
<td>0.60 – 0.80</td>
</tr>
<tr>
<td>Myers et al. (1963)</td>
<td>7.1 - 56</td>
<td>12 - 190</td>
<td>0.077</td>
<td>0.65</td>
</tr>
<tr>
<td>Gartshore &amp; Hawaleshka (1964)</td>
<td>30.8</td>
<td>18 - 124</td>
<td>0.065</td>
<td>0.77</td>
</tr>
<tr>
<td>Tailland &amp; Mathieu (1967)</td>
<td>11 - 25</td>
<td>33- 200</td>
<td>0.075</td>
<td>0.89</td>
</tr>
<tr>
<td>Verhoff (1970)</td>
<td>10 - 12</td>
<td>57 - 410</td>
<td>0.077 - 0.082</td>
<td>0.61 - 0.73</td>
</tr>
<tr>
<td>Wygnanski et al. (1992)</td>
<td>3.7-19</td>
<td>0 - 140</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>Abrahamsson et al. (1994)</td>
<td>10-20</td>
<td>0 - 150</td>
<td>0.075 - 0.081</td>
<td>0.8 - 1.0</td>
</tr>
<tr>
<td>Schneider &amp; Goldstein (1994)</td>
<td>14</td>
<td>43 - 110</td>
<td>0.074 - 0.082</td>
<td>0.80</td>
</tr>
<tr>
<td>Eriksson et al. (1998)</td>
<td>10</td>
<td>0 - 200</td>
<td>0.078</td>
<td>0.85 - 1.0</td>
</tr>
<tr>
<td>Venas et al. (1999)</td>
<td>15.2</td>
<td>125</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Despite the large body of literature existing at that time, the review articles revealed that accurate, consistent and comprehensive data sets were lacking and the physics of the flow was still not well understood. Some of the important observations and conclusions drawn from these review articles are as follows:

1. Almost all the measurements considered in these reviews were obtained using Pitot-tubes and hot-wires with an obvious spatial-resolution limitation, especially when cross-wires are used. Accurate and reliable near-wall data were scarce.
2. Many of the studies reported in the literature lack two-dimensionality. This was attributed, in part, to inaccurate measurement of the mean exit velocity.

3. The log law constants and skin friction correlation showed considerable scatter.

4. Scaling the streamwise evolution of the flow with the slot height and exit velocity showed considerable scatter. On the basis of the spread rates available at that time, a value of 0.073 ±0.002 was recommended.

5. Turbulence measurements were scarce. Only a few experimenters measured all the Reynolds stress components. Furthermore, higher order statistics such as triple products and energy budgets were not sufficiently known.

A number of measurements have been reported subsequent to the reviews of Launder and Rodi. These studies attempt to address some of the important research questions that were unanswered. Dakos et al. (1984) investigated a heated wall jet on both plane and curved surfaces. Measurements reported include Reynolds stresses, heat fluxes and triple velocity correlations. Wygnanski et al. (1992) reported measurements over a wide range of Reynolds numbers using hot-wire probes. The streamwise turbulence intensity data showed significant Reynolds number dependence. At a given inlet Reynolds number, it was also observed that distribution of turbulence intensity varied appreciably with streamwise distance for 60 < x/b < 120, although most studies reported similarity in both mean and turbulence quantities for x/b > 20. Karlsson and co-workers (e.g. Karlsson et al., 1993; Eriksson et al., 1998) reported one of the most comprehensive measurements using high spatial resolution LDA. They were able to resolve the mean velocity down to y^+ = 1. The mean and Reynolds stress data reported
by Eriksson et al. (1998) showed similarity for $x/b \geq 40$. It was also shown that the total production of turbulence caused by the normal stresses is small compared to shear stress production term. Furthermore, the production of turbulence caused by shear stress showed two peaks, one very close to the wall and a relatively higher one in the vicinity of $y = y_{1/2}$.

Schneider and Goldstein (1994) made measurements using LDA, Pitot-tube and hot-wires. In contrast to measurements obtained using the LDA, the mean velocity data obtained using hot-wires did not go to zero at large distances from the wall. The mean data obtained using the Pitot-tube were unacceptably low in the outer region. This appears to be characteristic of all Pitot-tube measurements reported in the literature. The Reynolds stresses obtained using the LDA were significantly higher than the data obtained using cross-wires. In a related study, Venas et al. (1999) compared their pulsed hot-wire data to the LDA measurements of Karlsson et al. (1993) and Schneider and Goldstein (1994) as well as measurements obtained by Abrahamsson et al (1994) in the same facility but with conventional hot-wires. In the case of the mean velocity distribution in the outer region, they observed good agreement between LDA and the pulsed-wire data but these profiles were quite different from the profiles obtained using conventional thermal anemometry. The Reynolds stresses obtained using LDA and pulsed hot-wire showed good agreement but were found to deviate significantly from the measurements obtained from the conventional hot-wire over most part of the flow.
Accurate prediction of the turbulent wall jet has been a major challenge to turbulence modelers. The unique characteristic of zero shear not coincident with the zero mean velocity gradient suggests that gradient transport models may not be appropriate for computation of turbulent wall jets. Launder and Rodi (1983) summarized some of the earlier wall jet computations. Gerodimos and So (1997) assessed some of the existing near wall two-equation (k-ε and k-ω) models for their ability to replicate the mixing behavior between the outer jet-like layer and inner wall layers. Using the experimental data of Karlsson et al. (1992) and Wygnanski et al. (1992), they concluded that all the models are capable of replicating the Reynolds number effects. However, prediction of the near wall asymptotic behavior, spread rate and decay of maximum velocity was poor. Yamamoto (1997) used a multiple-time-scale Reynolds stress to compute the plane wall jet measurements of Irwin (1973). The model gave a reasonable prediction for the mean velocity distribution and the spread rate. The spanwise and vertical stresses were in good agreement with measurements but streamwise stress and shear stresses were over predicted. More recently, Vasic (1999) compared the performance of two equation and Reynolds stress models to the measurements reported by Karlsson et al. (1992). It was concluded that the Reynolds stress model successfully predicted the velocity decay, but results from the two-equation models were in error. None of the models was able to predict the skin friction reasonably well. Furthermore, the Reynolds stress models gave a superior prediction of the Reynolds stress but predictions from the two-equation models were unacceptable over most region of the flow.
2.3 STATE OF KNOWLEDGE AND REFINEMENT IN OBJECTIVES

In the previous sections of this chapter, the scaling laws for the mean and turbulence quantities were reviewed. Some of the widely used skin friction relations were summarized. In view of the strong interaction between the inner and outer layer, it was emphasized that skin friction correlations which explicitly takes the specific structure of the outer flow into consideration should be preferred. The existing literature on the turbulent wall jet as well as Reynolds number and surface roughness effects in turbulent boundary layers was briefly reviewed.

Regarding Reynolds number effects, no definitive statement could be made as to the extent to which it persists. Some of the inconsistencies can be explained by poor spatial resolution and inaccurate skin friction measurements. Although the friction velocity is used exclusively to scale both the mean velocity and turbulence statistics, most of the scaling laws proposed by George and Castillo (1997) suggest otherwise. It will be of interest to see how the flow structure varies with Reynolds number when the recent theory is used to analyze the data.

A summary of the distributions of turbulence intensities and Reynolds shear stress outside the roughness sublayer in some studies is given in Table 2.3. It is clear from this table that, in spite of extensive research efforts, the present state of knowledge regarding roughness effects on the turbulence structure is contradictory.
Table 2.3: Summary of state of knowledge regarding roughness effects on turbulence structure (ZPG = zero-pressure gradient)

Most of the earlier rough wall measurements were made using hot-wires which may have been affected by the high local turbulence levels in the vicinity of roughness elements. In some of the previous experiments, only rough wall measurements are conducted and the results compared with smooth wall data conducted at different Re₉ or in different facilities. As is well known, turbulent flows are very sensitive to initial or boundary conditions so that measurements obtained in different facilities or at different conditions may not be similar in all details. This may suggest that with exception of wall conditions, attempts should be made to match all other initial conditions as much as possible so that definite conclusions could be drawn with regards to surface roughness effects on the turbulence structure. Furthermore, the existing rough wall literature indicates that accurate measurement of skin friction, especially at low Reynolds numbers, still poses a challenge to experimentalists.

In the light of recent LDA and pulsed hot-wire measurements, it appears the scatter in earlier turbulent wall jet data can be attributed, at least in part, to the well-known problems of Pitot-tube and conventional hot-wires close to the wall and in regions of
high local turbulence intensity. Although studies reported in recent years attempt to address some of the open questions and issues raised by the review articles by Launder and Rodi (1981, 1983), some important and practically relevant research questions remain unanswered. For example, higher order statistics such as triple correlations remain unknown while information on energy budgets remains limited. Although most practical flow systems in which wall jets are found are hydraulically rough, measurements of turbulent wall jets on rough surfaces are rather scarce. Perhaps with the exception of the Pitot-tube measurements reported by Rajaratnam (1965) and Sakipov et al. (1975), the effects of surface roughness on the hydrodynamic characteristics of turbulent wall jets are not known.

On the basis of our current understanding on turbulent boundary layers and wall jets, the objectives of this study are re-stated as follows:

1. To examine the appropriateness of the different scaling laws proposed for the mean velocity and turbulence statistics in a turbulent boundary layer.
2. To determine reliable methods for the evaluation of skin friction in near-wall turbulent flows over smooth and rough surfaces.
3. To investigate the effects of wall roughness on the mean and turbulence statistics in open channel turbulent boundary layers and wall jets using different types of roughness elements.
4. To provide benchmark data for rough wall turbulent boundary layers and wall jets in open channel for the purpose of developing practical turbulence models.
CHAPTER 3

INSTRUMENTATION AND EXPERIMENTAL DETAILS

In this chapter, an overview of the LDA system is given. Some of the problems, which may introduce significant measurement errors, are also reviewed in this chapter. Descriptions of materials used to create the surface roughness as well as the test facilities, instrumentation, and measurement procedure are given. Experimental details and summaries of test conditions for both the boundary layer and wall jet experiments are also presented.

3.1 THE LASER DOPPLER ANEMOMETER

Laser Doppler anemometry is the measurement of fluid velocity by detecting the Doppler frequency shift of laser light that has been scattered by small particles moving with the fluid. A laser Doppler anemometer (LDA) system consists of a laser source, an optical arrangement, a photo-detector that converts light into electrical signals and a signal processor. The various components are discussed in Appendix A.

3.2 ERRORS IN LDA MEASUREMENTS

In spite of the non-intrusive characteristic of the LDA and its suitability for turbulent flow measurements, its potential to provide highly accurate measurements is sometimes not realized because of some inherent problems. Some of the well-known sources of
measurement errors include velocity bias, presence of multiple particles in the measuring volume, gradient broadening, and errors due to noise and non-orthogonality of beam crossing. These errors are discussed in Appendix B. The results of preliminary experiments conducted to examine some of these effects are also reported in Appendix B. Steps taken to minimize or correct possible measurement errors are also discussed.

With regard to velocity bias, experiments were conducted using three different sampling schemes (Appendix B.1). The results showed that the maximum deviation observed for each statistic is comparable to the corresponding measurement uncertainties. Based on experimental investigations of Johnson and Barlow (1989) (see Appendix B.2), it is inferred that the streamwise component of mean velocity as well as streamwise and vertical components of turbulence fluctuations are nearly independent of the spanwise dimension of the probe volume. However, the Reynolds shear stress may be underestimated by as much as 12 percent.

Analytical treatments and experiments carried out by Durst et al. (1995, 1998) and Eriksson et al. (1999) are summarized in Appendix B.3. Their results suggest that the effects of gradient broadening on mean and turbulence quantities are negligible for the present system and experimental conditions. They also showed that errors due to noise are negligible except in the immediate vicinity of the wall (Appendix B.4). In the present study, steps are taken to minimize such errors.

In order to obtain data very close to the wall, the fiber-optic probe is pitched towards the wall at an angle β. Preliminary experiments were conducted to examine the
angle of tilt ($\beta$) on the mean velocities and higher order turbulence statistics (Appendix B.5). It is concluded that for $\beta \leq 5^\circ$, except for the vertical turbulence fluctuations, the turbulence statistics do not show any significant dependence on the angle of tilt.

3.3 EXPERIMENTAL SET-UP

3.3.1 The Open Channel Flume

The boundary layer experiments were conducted in a rectangular cross-section open channel flume. A schematic of the flume is shown in Figure 3.1. The flume is 0.8 m wide, 0.6 m deep, and 10 m long. The sidewalls of the flume were made of transparent tempered glass to facilitate velocity measurements using a laser Doppler anemometer. A contraction and several stilling arrangements used to reduce any large-scale turbulence in the flow preceded the straight section of the channel. The channel bottom was made of brass and the slope was adjustable. For the present experiments, the channel bottom was horizontal. The experiments were conducted on a hydraulically smooth and three different types of surface roughness. The various rough surfaces are described in Section 3.3.3.

3.3.2 The Wall Jet Facility

Figure 3.2 shows an overview of the set-up for the wall jet experiments. The important dimensions are also indicated. The wall jet test facility was screwed on to the bottom of the open channel flume described in Section 3.3.1. The inlet of the nozzle was placed 3 m downstream of the channel contraction. The nozzle has a contraction ratio of 9 to 1 and was designed following Morel (1975) in order to avoid flow separation. Depending
Fig. 3.1: A schematic of the open channel flume

Fig. 3.2: A schematic of the wall jet facility
on the exit jet velocity $U_j$, the water level downstream the exit varied from 350 to 400 mm above the floor of the test facility. The ratio of slot thickness ($t$) to the slot height ($b$) was $t/b = 0.6$ while the width ($w$) of the slot to the slot height was $w/b = 79$. A weir downstream kept the water level constant. The slot exit was preceded with straw packing to reduce any large-scale disturbance in the approaching flow. The wall jet experiments were conducted on a smooth wall and a rough surface created from sand grains as described next.

### 3.3.3 Description of Surface Roughness

In order to examine the effect of surface roughness on near-wall flows, three geometrically different types of surface roughness were employed in addition to a hydraulically smooth surface:

![Rough Surfaces](image)

Figure 3.3: Pictures showing sections of (a) perforated plate (PF) and (b) wire mesh (WM) rough surfaces

A 1.4-mm thick and 1.5 m long sheet with circular perforations arrayed in a hexagonal pattern. The perforation diameter was 2.2 mm with a 4.0-mm spacing between centers. This configuration gives an openness ratio of approximately 43
percent. A picture of a section of the perforated plate (PF) is shown in Figure 3.3a.

A uniformly and closely distributed 1.2-mm nominal diameter sand grains (SG). The sand grains were coated on to a 1.75-m long plywood sheet using double-sided tape.

A 1.3-m long stainless steel wire mesh (WM). The mesh was made of 0.6 mm diameter wires with 7.0 mm centerline spacing, giving a ratio of centerline spacing to wire diameter of about 12. A section of the wire mesh roughness is shown in Figure 3.3b.

3.4 INSTRUMENTATION AND MEASUREMENT PROCEDURE

The velocity measurements were obtained using a single- and two-component fiber-optic probe LDA system. The LDA system is powered by a 300 mW Argon-Ion laser (Dantec Inc.). The optical elements include a 40 MHz Bragg cell to remove directional ambiguity, a 1.96 beam expansion unit, a beam splitter, a color separator and a 500 mm focusing lens. The laser beam is separated into green ($\lambda = 514.5$ nm) and blue light ($\lambda = 488$ nm). The two-component system uses a four-beam two-color configuration arranged at right angles to each other. The measuring volume dimensions (based on the $e^{-2}$ light intensity cut-off point) for the present configuration were 0.12 mm x 0.12 mm x 1.4 mm.

A photo-multiplier (PM) configured in backscatter mode is used to collect scattered light received from the measuring volume. The optical and operating parameters of the LDA system are summarized in Table 3.1.
<table>
<thead>
<tr>
<th>Wavelength of laser (nm)</th>
<th>514.5 (green), 488 (blue)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of laser beam (mm)</td>
<td>1.35</td>
</tr>
<tr>
<td>Focal length of the transmitting lens (mm)</td>
<td>400 (Series B), 500 (all other tests)</td>
</tr>
<tr>
<td>Beam separation (mm)</td>
<td>38 (Series B), 74.5 (all others)</td>
</tr>
<tr>
<td>Number of fringes</td>
<td>36</td>
</tr>
<tr>
<td>Fringe spacing (μm)</td>
<td>5.422 (Series B), 3.2842 (all others)</td>
</tr>
</tbody>
</table>
| Measuring volume dimensions | 0.19 x 0.19 x 4.09 mm$^3$ (Series B)  
0.12 x 0.12 x 1.4 mm$^3$ (all other tests) |
| Beam expansion unit | 1.96 (for all tests except Series B) |
| Bragg cell | 40 MHz |

**Table 3.1: Optical and operating parameters of the LDA**

In the present sets of experiments, no artificial seeding was used since there were enough scattering particles (i.e. naturally occurring hydrosols) in the flow. The use of naturally and uniformly occurring seeding is expected to minimize velocity bias towards higher velocities (McLaughlin and Tideman, 1973). Scattered light from the measuring volume is digitally processed with a 58N40 Flow Velocity Analyser (FVA) that is interfaced to a microcomputer using a 58G110 PC Interface board. The measurement process, data acquisition and data processing are controlled by type 46S51 FLOware, which is a user-friendly professional software package developed by DANTEC. The trigger of the LDA system is set in such a way that no signal is obtained when one of the laser beams is blocked. The bandwidth parameters affecting the arrival and transit time clock rates, the optical shift and hardware filter values were set to 'best choice values' according to the recommendations of DANTEC. With these settings, the influence of noise on the measured data is expected to be minimal. Particular attention was paid to the validation parameters that affect whether data are converted by FLOware and
whether invalid data can exist in the converted data file. In this regard, both velocity channels were enabled and the rejection levels were set in accordance with the present optical parameters and the flow conditions following the suggestions made by DANTEC.

The transmitting optical elements were cleaned before the commencement of the tests. Prior to the measurement of each set of data, the bottom wall as well as the sidewall of the test facility were cleaned. These were found necessary in order to minimize extraneous light scattered from particles distributed throughout the illuminating beams. Prior to each measurement series, data were acquired in a repetitive mode, which means that acquisition is performed on line with data displayed on the acquisition window. This mode is used to examine the quality of data, fringe count errors, and signal-to-noise ratio (SNR) validation level error. In the event of unsatisfactory error levels or poor quality of data, the necessary optical parameters and validation levels are reset. Karlsson et al. (1993) and Durst et al. (1995) have pointed out that even a small misalignment of the fiber-optics probe on the order of 1° in the x-y plane could cause large errors in vertical component of the mean velocity and its fluctuation. With this in mind, care was taken to minimize any possible misalignment. In most of the experiments described below, the probe was pitched towards the bottom wall but in a way that no significant measurement errors are introduced. On the basis of preliminary results, which are summarized in Appendix B.1, no correction for velocity bias was applied.
3.5 EXPERIMENTAL DETAILS

3.5.1 Boundary Layer

Three sets of boundary layer experiments were conducted. The first set (Series A) was conducted on a hydraulically smooth surface at five different Reynolds numbers. In order to stay in the sub-critical range (i.e. Froude number less than unity), it was not possible to attain Reynolds numbers based on momentum thickness higher than 3300. In spite of this limitation, these measurements allowed some scaling issues and Reynolds number effects to be examined. The major objective of the second (Series B) and third (Series C) sets of boundary layer experiments was to examine the effects of surface roughness using a single- and two-component LDA, respectively. As will be discussed in Chapter 5, the configuration of the present two-component LDA system could not permit measurements very close to the wall. In the boundary layer experiments described below, the change in water surface elevation was less than 1 mm over a streamwise distance of 600 mm implying a negligible pressure gradient. No surface waves were observed at the free surface. Extensive preliminary experiments showed that the variations of the mean and turbulence quantities across the channel (i.e. spanwise direction) are comparable to the measurement uncertainties at the middle third of the channel. In all the experiments reported in the next sections, the measurements were acquired at the centerline of the channel.

3.5.1.1 Series A: Reynolds Number Effects

As mentioned above, the object of these measurements is to investigate Reynolds number effects on the streamwise component of the mean velocity and its higher order turbulence statistics. The measurements were made on a hydraulically smooth surface at
five different freestream velocities. The depth of flow was kept constant at $h = 100$ mm and the channel aspect ratio ($AR = B/h$) was 8. To ensure a fully turbulent boundary layer, a 1-mm diameter rod located 4.5 m downstream of the contraction and spanned the width of the flume was used to trip the flow. The measurements were obtained at 750 mm (i.e. 750d) downstream of the trip. For most of the experiments in this series, the LDA probe was tilted at $\beta = 2^\circ$ towards the bottom wall. The validated data rate varied from 6 to 10 Hz close to the wall and approximately 50 Hz at distances remote from the wall. The maximum duration of data acquisition at each measuring location was set to 750 seconds while the maximum sample size was set to 10000. Typical sample size at a measuring point varied from 5000 to 10000.

The test conditions are summarized in Table 3.2. In this table, $U_c$ denotes the local maximum velocity, $Tu$ is the turbulence intensity ($u/U_c$) at the outer edge of the boundary layer ($y = \delta$), $Re_\theta$ and $Re_h$ are the Reynolds number based on the momentum thickness $\theta$ and depth of flow, respectively. $Re_c (= hU_c/\nu)$ is the Reynolds number based on the friction velocity and depth of flow, $\delta^-$ is the Reynolds number based on the boundary layer thickness and the friction velocity, $H$ is the boundary layer shape factor, $C_f$ is the skin friction coefficient, and $l_y^- (= l_y U_c/\nu)$ is the vertical dimension of the probe measuring volume in viscous units. The present values of $l_y^-$ are adequate to resolve both the mean and turbulence statistics down to the wall (Gad-el Hak and Bandyopadhyay, 1994). According to the investigation of Johnson and Barlow (1989), the spanwise dimensions of the measuring volume in wall units ($11 < l_x^- < 39$) are not expected to cause any significant effect on the mean and turbulence intensity. On the
basis of $Re_h$, the present flows may be considered as low Reynolds number turbulent boundary layers. The present values of $Re_h$ are comparable to most open channel flow data available in the literature. The range of $Re_c$ considered herein is, however, considerably higher than those reported for fully developed (closed) duct flows.

<table>
<thead>
<tr>
<th>Test</th>
<th>$U_c$ (cm/s)</th>
<th>$Tu$ (%)</th>
<th>$Re_o$</th>
<th>$Re_h$</th>
<th>$H$</th>
<th>$C_r$ ($x10^3$)</th>
<th>$\delta^-$</th>
<th>$Re_c$</th>
<th>$I_y^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-SM1</td>
<td>15.3</td>
<td>2.0</td>
<td>750</td>
<td>15,300</td>
<td>1.30</td>
<td>5.33</td>
<td>440</td>
<td>790</td>
<td>1.0</td>
</tr>
<tr>
<td>A-SM2</td>
<td>20.9</td>
<td>2.4</td>
<td>1080</td>
<td>20,900</td>
<td>1.31</td>
<td>4.76</td>
<td>560</td>
<td>1020</td>
<td>1.3</td>
</tr>
<tr>
<td>A-SM3</td>
<td>34.5</td>
<td>2.1</td>
<td>1450</td>
<td>34,500</td>
<td>1.29</td>
<td>4.36</td>
<td>930</td>
<td>1610</td>
<td>2.0</td>
</tr>
<tr>
<td>A-SM4</td>
<td>54.2</td>
<td>2.0</td>
<td>2400</td>
<td>54,200</td>
<td>1.25</td>
<td>4.23</td>
<td>1450</td>
<td>2410</td>
<td>3.0</td>
</tr>
<tr>
<td>A-SM5</td>
<td>62.1</td>
<td>3.2</td>
<td>3250</td>
<td>62,100</td>
<td>1.25</td>
<td>4.01</td>
<td>1610</td>
<td>2780</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of test conditions for Series A.

3.5.1.2 Series B: 1-D Smooth and Rough Wall Experiments

In this set of experiments, measurements were obtained on a hydraulically smooth and three geometrically different rough surfaces. The rough surfaces consist of the perforated plate (PF), sand grain (SG), and wire mesh (WM) described in Section 3.3.3. For each surface condition, measurements were made at three different velocities and depths of flow. To ensure a turbulent boundary layer, a trip was located 3.5 m downstream of the contraction and spanned the width of the flume. The trip was composed of 3-mm (median diameter) pebbles glued to the bottom of the channel as a 40-mm long strip. The perforated plate (PF) and sand grain roughness (SG) were located at about 1.1 m downstream of the trip, while the wire mesh screen was located 1.2 m downstream of the trip.
For convenience, a reference axial position \( (x = 0) \) was located 1.3 m downstream of the trip. For the smooth surface (SM), measurements were made at an axial station of \( x = 0.50 \) m. For the wire mesh roughness (WM), measurements were made at axial locations of \( x = 0.30 \) and 0.50 m for each test condition. The measurements on the perforated plate and sand grain roughness were conducted at \( x = 0.10, 0.25 \) and 0.52 m, for each test condition. The data reported in this study are measurements obtained at \( x = 0.50 \) or 0.52 m. A summary of the important test conditions for this set of measurements is given in Table 3.3.

<table>
<thead>
<tr>
<th>Test</th>
<th>Type of surface</th>
<th>Depth, ( h ) (mm)</th>
<th>( U_e ) (m/s)</th>
<th>( \text{Tu} (%) ) (at ( y = \delta ))</th>
<th>( \delta ) (mm)</th>
<th>( \theta ) (mm)</th>
<th>( H )</th>
<th>( Re_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-SM 1</td>
<td>smooth</td>
<td>100</td>
<td>0.737</td>
<td>3.1</td>
<td>46</td>
<td>3.56</td>
<td>1.29</td>
<td>2625</td>
</tr>
<tr>
<td>B-SM 2</td>
<td>smooth</td>
<td>80</td>
<td>0.331</td>
<td>3.3</td>
<td>48</td>
<td>4.17</td>
<td>1.33</td>
<td>1380</td>
</tr>
<tr>
<td>B-SM 3</td>
<td>smooth</td>
<td>50</td>
<td>0.463</td>
<td>2.4</td>
<td>40</td>
<td>3.37</td>
<td>1.31</td>
<td>1750</td>
</tr>
<tr>
<td>B-PF 1</td>
<td>perforated</td>
<td>100</td>
<td>0.702</td>
<td>3.3</td>
<td>42</td>
<td>3.49</td>
<td>1.35</td>
<td>2450</td>
</tr>
<tr>
<td>B-PF 2</td>
<td>perforated</td>
<td>75</td>
<td>0.604</td>
<td>3.2</td>
<td>40</td>
<td>3.73</td>
<td>1.35</td>
<td>2250</td>
</tr>
<tr>
<td>B-PF 3</td>
<td>perforated</td>
<td>50</td>
<td>0.482</td>
<td>3.1</td>
<td>35</td>
<td>2.90</td>
<td>1.41</td>
<td>1400</td>
</tr>
<tr>
<td>B-SG 1</td>
<td>sand grain</td>
<td>100</td>
<td>0.622</td>
<td>3.2</td>
<td>50</td>
<td>4.20</td>
<td>1.37</td>
<td>2620</td>
</tr>
<tr>
<td>B-SG 2</td>
<td>sand grain</td>
<td>75</td>
<td>0.495</td>
<td>3.5</td>
<td>41</td>
<td>3.97</td>
<td>1.37</td>
<td>1970</td>
</tr>
<tr>
<td>B-SG 3</td>
<td>sand grain</td>
<td>50</td>
<td>0.448</td>
<td>2.5</td>
<td>39</td>
<td>3.48</td>
<td>1.38</td>
<td>1560</td>
</tr>
<tr>
<td>B-WM 1</td>
<td>wire mesh</td>
<td>100</td>
<td>0.773</td>
<td>2.9</td>
<td>45</td>
<td>5.07</td>
<td>1.50</td>
<td>3920</td>
</tr>
<tr>
<td>B-WM 2</td>
<td>wire mesh</td>
<td>75</td>
<td>0.675</td>
<td>2.8</td>
<td>42</td>
<td>4.80</td>
<td>1.50</td>
<td>3240</td>
</tr>
<tr>
<td>B-WM 3</td>
<td>wire mesh</td>
<td>50</td>
<td>0.519</td>
<td>2.5</td>
<td>40</td>
<td>4.97</td>
<td>1.52</td>
<td>2580</td>
</tr>
</tbody>
</table>

Table 3.3: Summary of test conditions for Series B
3.5.1.3 Series C: 2-D Smooth and Rough Wall Experiments

The measurements in Series C were obtained to examine the effects of surface roughness on the turbulence structure using a two-component LDA. The use of a two-component LDA allowed the Reynolds shear stress, the mixing length and eddy viscosity as well as the turbulent energy budget to be examined. Measurements were obtained on a smooth surface (SM), sand grain (SG) and wire mesh (WM) roughness. A trip composed of 3-mm (median diameter) pebbles glued to the bottom of the channel as a 40-mm long strip was located 5.0 m downstream of the contraction. The measurements were made at an axial location of 1.0 m downstream of the trip. For the smooth wall measurements, the probe was slightly pitched towards the bottom wall. Coincident data rates of 7 Hz very near the wall and 30 to 50 Hz away from the wall were typical depending on the local velocity. The maximum duration of data acquisition was set to 1500 seconds. Depending on the local velocity and distance away from the wall, typical sample size at a measuring point varied from 10000 to 20000. Due to a hardware limitations, two-component measurements could not be made close to the wall. In order to resolve the streamwise velocity statistics down to the wall, one-component LDA measurements were also made for each test condition. The streamwise turbulence statistics obtained from the two-component measurements were compared to the corresponding measurements made using the one-component LDA. For each test condition, the two sets of data were found to agree within measurement uncertainties.

A summary of some important flow parameters is given in Table 3.4. The probe dimension in the wall-normal direction was in the range $1.8 \leq l_y^* \leq 3.5$ which is high
enough to permit reliable measurements of both mean velocity and turbulence intensity. The spanwise dimensions in wall units are in the range $22 < l_z < 41$. Based on the results of Johnson and Barlow (1989), the present $u^+$ and $v^+$ profiles will be unaffected but $<u^+v^+>$ may be underestimated by as much as 12 percent. Inasmuch as the spanwise dimensions in wall units are approximately constant for smooth and rough wall experiments at similar freestream conditions (i.e. for SML and SGL; and also for SMH, SGH and WMH), underestimation of $<u^+v^+>$ is expected to be similar. Because the purpose is to examine the effects of surface roughness on the turbulence structure, no attempts are made to correct the Reynolds shear stress for probe volume effects.

<table>
<thead>
<tr>
<th>Test</th>
<th>$U_c$ (cm/s)</th>
<th>$h$ (mm)</th>
<th>$\delta$ (mm)</th>
<th>$\theta$ (mm)</th>
<th>$Re_h \times 10^3$</th>
<th>$Re_\theta$</th>
<th>$H$</th>
<th>$l_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-SMH</td>
<td>49.2</td>
<td>120</td>
<td>38</td>
<td>3.87</td>
<td>59.0</td>
<td>1900</td>
<td>1.31</td>
<td>31</td>
</tr>
<tr>
<td>C-SML</td>
<td>34.8</td>
<td>180</td>
<td>60</td>
<td>5.90</td>
<td>62.6</td>
<td>2050</td>
<td>1.28</td>
<td>22</td>
</tr>
<tr>
<td>C-SGH</td>
<td>53.1</td>
<td>120</td>
<td>37</td>
<td>4.11</td>
<td>63.7</td>
<td>2180</td>
<td>1.37</td>
<td>38</td>
</tr>
<tr>
<td>C-SGL</td>
<td>35.1</td>
<td>180</td>
<td>63</td>
<td>6.20</td>
<td>63.2</td>
<td>2180</td>
<td>1.34</td>
<td>25</td>
</tr>
<tr>
<td>C-WMH</td>
<td>53.4</td>
<td>120</td>
<td>38</td>
<td>4.91</td>
<td>64.0</td>
<td>2600</td>
<td>1.49</td>
<td>41</td>
</tr>
</tbody>
</table>

Table 3.4: Test conditions and boundary layer parameters for Series C

### 3.5.2 Wall Jet Experiments

The wall jet experiments were conducted on a smooth wall (SM) and a sand grain (SG) rough surface. It is impossible to create a turbulent wall jet in a still surrounding in an open channel flow. As will be shown in Chapter 6, the outer edge of the present wall jet is characterized by high background turbulence levels and recirculating flow. Notwithstanding these effects, the results obtained from the flows considered here are
similar in many respects to existing data in the literature. In wall jet experiments, it is well known that inaccurate measurements of the exit velocity profile make a significant contribution to lack of conformity of the local momentum flux to two-dimensionality (Launder and Rodi, 1981). In view of this and also due to the importance of the exit momentum flux in scaling the streamwise evolution of the mean flow, measurements were obtained at the jet exit ($x/b = 0$) in the case of the smooth wall tests.

Two sets of wall jet measurements were made. The first set (Series D) pertains to single-component measurements on a smooth wall and a sand grain rough surface. Measurements at the exit as well as several locations downstream of the jet exit were made. This allowed the effects of surface roughness on the streamwise component of the mean velocity and its higher order moments to be made. This set of data is also used to examine the streamwise evolution of the wall jet. In the second set of experiments (Series E), two-component measurements on smooth and sand grain rough surfaces are reported. Measurements obtained in Series E enabled turbulence statistics such as Reynolds shear stress, triple correlations, and energy budgets to be analyzed.

3.5.2.1 Series D: 1-D Smooth and Rough Wall Experiments

In this set, measurements were obtained at the slot and several distances up to 100 slot heights downstream of the exit. For the smooth wall measurements, the probe was tilted about 2° towards the bottom wall. The turbulence intensity in the central region of the jet exit varied from 3 to 6 percent. The sampling rate varied from 7 Hz in regions of low velocity to 80 Hz in regions of high local velocities. The maximum sampling time and maximum sample size at a measuring location were set to 1000 seconds and 10000,
respectively. Typical sample sizes varied from 5000 to 10000. The boundary layer thickness at the exit is approximately 2 to 3 mm.

Depending on the number of downstream locations traversed, the duration of one set of experiments was in excess of 100 hours. For each test condition, a minimum of 10 measurements of the mass flow rate was obtained using an electronic weighing tank. The standard deviation of the mean bulk velocity \( U_b \) calculated from the mass flow rate varied from 2 to 5 percent. The higher values were typical for experiments that ran for a longer period of time.

The important test parameters are summarized in Table 3.5. Here, \( U_j \) is the maximum velocity at the jet exit, \( U_b \) is the bulk mean velocity determined from mass flow rate measurements. \( Re_j \) and \( Re_b \) are the Reynolds number based on exit conditions \( (U_j \text{ and } b) \) and \( (U_b \text{ and } b) \), respectively. The momentum thickness determined from the exit velocity profiles is denoted as \( \theta \) and \( H \) is the boundary layer shape factor.

<table>
<thead>
<tr>
<th>Test</th>
<th>( U_j ) (m/s)</th>
<th>( U_b ) (m/s)</th>
<th>( Re_j )</th>
<th>( Re_b )</th>
<th>( \theta ) (mm)</th>
<th>( H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-SM1</td>
<td>1.389</td>
<td>1.202</td>
<td>14000</td>
<td>12100</td>
<td>0.43</td>
<td>1.70</td>
</tr>
<tr>
<td>D-SM2</td>
<td>1.054</td>
<td>0.868</td>
<td>10000</td>
<td>8700</td>
<td>0.41</td>
<td>1.68</td>
</tr>
<tr>
<td>D-SM3</td>
<td>0.759</td>
<td>0.595</td>
<td>7500</td>
<td>6000</td>
<td>0.44</td>
<td>1.89</td>
</tr>
<tr>
<td>D-SG1</td>
<td>1.394</td>
<td>1.185</td>
<td>14000</td>
<td>12000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D-SG2</td>
<td>1.204</td>
<td>0.997</td>
<td>12000</td>
<td>10000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D-SG3</td>
<td>0.721</td>
<td>0.584</td>
<td>7200</td>
<td>5900</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5: Summary of test parameters for Series D
3.5.2.2 Series E: 2-D Smooth and Rough Wall Experiments

In this set of experiments, single- and two-component velocity measurements were obtained on smooth and rough surfaces. For the two-component measurements, the probe was pitched about 3° towards the bottom wall so that data could be obtained closer to the wall. Following the preliminary results summarized in Appendix B.2, it is concluded that no significant errors were caused in the measurements of U, V, u and <uv>. The vertical turbulence fluctuation, i.e. v, may be slightly contaminated, but as noted in Appendix B.2, any possible error may be comparable to the corresponding measurement uncertainties in v. The sampling rate varied from 7 Hz in regions of low velocity to 60 Hz in regions of high local velocities. The maximum sampling time and sample size at each measuring location were set to 1500 seconds and 15000, respectively. Depending on the local velocities typical sample size varied from 10000 to 15000.

<table>
<thead>
<tr>
<th>Test</th>
<th>(U_j) (m/s)</th>
<th>(U_b) (m/s)</th>
<th>(Re_j)</th>
<th>(Re_b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-SM1</td>
<td>1.341</td>
<td>1.146</td>
<td>13400</td>
<td>11500</td>
</tr>
<tr>
<td>E-SM2</td>
<td>1.023</td>
<td>0.857</td>
<td>10300</td>
<td>9000</td>
</tr>
<tr>
<td>E-SG1</td>
<td>1.304</td>
<td>1.117</td>
<td>13100</td>
<td>12000</td>
</tr>
<tr>
<td>E-SG2</td>
<td>0.985</td>
<td>0.828</td>
<td>9900</td>
<td>8500</td>
</tr>
</tbody>
</table>

Table 3.6: Summary of test parameters for Series E

A summary of some important test parameters is given in Table 3.6. For Test E-SM1, measurements were obtained at \(x/b = 0, 10, 30, 40, 50, 60, 70, 80\) and 100. For each of Test E-SM2, E-SG1, E-SG2, measurements were obtained at \(x/b = 30\) and 50.
The analyses and results of the above experiments are reported in the next three chapters. In Chapter 4, the boundary layer measurements obtained in Series A and B are discussed while the results for Series C are discussed in Chapter 5. The wall jet data obtained in Series D and Series E are discussed in Chapter 6.

3.6 Uncertainty Estimates

In this section, statistical uncertainties, at the 95 percent confidence level, are presented for the mean velocities and turbulence fluctuations. A more complete uncertainty analysis is presented in Appendix C. For the boundary layer, measurement uncertainty in the mean velocities (U and V) and the turbulence intensities (u, v) is less than 1 percent. Close to the wall, the error in u is estimated to be about 4 percent. The maximum uncertainty in the Reynolds shear stress is about 12 percent. These uncertainty bounds are similar to those obtained by Schwarz et al. (1999). In the inner layer of the wall jet, the estimates are similar to those outlined for the turbulent boundary layers. The uncertainty in the outer layer is substantially higher due to the local turbulence intensity as well as reduction in sample size. Typical estimates in the outer region are as follows: ± 2.5 percent for the mean velocities, ± 5 – 10 percent for the turbulence intensity and Reynolds shear stress.
CHAPTER 4

SURFACE ROUGHNESS AND LOW REYNOLDS NUMBER EFFECTS ON THE STREAMWISE VELOCITY COMPONENT

In this chapter, smooth and rough wall measurements of the streamwise component of the mean velocity and its higher order turbulence statistics are reported. The techniques used to determine the wall shear stress for both the smooth and rough wall data are discussed. Measurements obtained in Series A are used to compare the conventional scaling laws with the more recent theory proposed by George and Castillo (1997). This also allows the effect of Reynolds number on the mean and turbulence quantities to be examined. The effects of surface roughness on the mean velocity and turbulence intensity are examined using the measurements obtained in Series B. Finally, the appropriateness of the power laws proposed by Barenblatt (1993) and George and Castillo (1997) to describe the mean velocity profile on both smooth and rough surfaces is assessed. The wall shear stress values obtained from these power laws are also compared to those obtained from other reliable and widely used techniques.

4.1 Determination of Wall Shear Stress

An accurate determination of the wall shear stress (or the friction velocity) is important because of its relevance in scaling the mean velocity and turbulence quantities. For flow over a smooth surface, a reliable estimate of the wall shear stress can be obtained from
the velocity gradient at the wall. Because of the thinness of the linear viscous sublayer, sufficient data could not be obtained in this region for some of the smooth wall experiments. Consequently, the velocity gradient at the wall, whenever possible, as well as fourth \((U^+ = y^+ + c_4 y^{+4})\) and fifth \((U^+ = y^+ + c_4 y^{+4} + c_5 y^{+5})\) order polynomial fits to the near-wall data are used to estimate the friction velocity for the smooth wall data.

<table>
<thead>
<tr>
<th>Test</th>
<th>(U_e) (cm/s)</th>
<th>(Re_θ)</th>
<th>(δ^−)</th>
<th>(U_τ) (cm/s)</th>
<th>(ΔU_{max}^−)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-SM1</td>
<td>15.3</td>
<td>750</td>
<td>440</td>
<td>0.79</td>
<td>0.14</td>
</tr>
<tr>
<td>A-SM2</td>
<td>20.9</td>
<td>1080</td>
<td>560</td>
<td>1.02</td>
<td>0.28</td>
</tr>
<tr>
<td>A-SM3</td>
<td>34.5</td>
<td>1450</td>
<td>930</td>
<td>1.61</td>
<td>0.36</td>
</tr>
<tr>
<td>A-SM4</td>
<td>54.2</td>
<td>2400</td>
<td>1450</td>
<td>2.41</td>
<td>0.44</td>
</tr>
<tr>
<td>A-SM5</td>
<td>62.1</td>
<td>3250</td>
<td>1610</td>
<td>2.78</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of friction velocity and deviation of measured \(U^+\) from log law profile at \(y^+ = δ^−\)

The near wall data and the corresponding linear as well as the fourth and fifth order polynomial fits to some of the smooth wall data sets (A-SM1, A-SM2, A-SM3) are shown in Figure 4.1a. The coefficient \(c_4 = -0.00027\) is in good agreement with the value of \(-0.0003\) (±0.0001) suggested by Eriksson et al. (1998); the value of \(c_5 = 13.4 \times 10^{-6}\) is 0.7 percent lower than that proposed by George and Castillo (1997). The slight difference between the present value for \(c_5\) and that recommended by George and Castillo (1997) may be due to Reynolds number effects. Figure 4.1a shows that the experimental data agree fairly well with the linear fit for \(y^+ ≤ 5\), while agreement is good
Fig. 4.1a: Determination of $U_\tau$ for smooth wall data: Linear and polynomial fits to near-wall data

Fig. 4.1b: Determination of $\Pi$ and $U_\tau$ for smooth and rough wall data (Lines indicate fits to Eqns. 2.13 and 2.16)
for $y^+ \leq 14$ in the case of the fifth order polynomial. The values of $U_\tau$ and the skin friction coefficients for measurements obtained in Series A are given in Table 4.1. The values of $U_\tau$ determined by fitting a fifth order polynomial to the near-wall data agree with the corresponding value obtained from the velocity gradient (whenever possible) to within $\pm 1.5$ percent. In Table 4.1, $\Delta U_{\text{max}}$ denotes the deviation of the mean velocity at $y^+ = \delta^+$. The relevance of this parameter will be discussed in a later section.

For the rough wall data, the values of $U_\tau$ and $\Pi$ were determined following the optimization procedure outlined in Section 2.1.4. The optimization was carried out by fitting Eqn. (2.13) to the experimental data, using Eqn. (2.16) for the wake function. Specifically, the iterated values of $U_\tau$ and $\Pi$ that gave the best fit to Eqn. (2.13) while ensuring a log-linear relation with $\kappa = 0.41$ were sought. As mentioned earlier, the present technique does not implicitly fix the value of the wake parameter $\Pi$. Instead it allowed the value of $\Pi$ to be optimized with the expectation of ensuring a reliable estimate of $U_\tau$ to be made. Since the wall shear stress is influenced by the outer layer, the correlation adopted here is expected to yield some important advantages over Hama’s formulation, as will be shown subsequently. Compared to the Clauser plot method, which uses only data in the overlap region, the technique adopted here uses more data points in the course of the profile matching since data in both the overlap and outer regions are employed. For the smooth wall data in Series B, $U_\tau$ and $\Pi$ were obtained using the above optimization technique. The friction velocity $U_\tau$ was also determined independently from the velocity gradient at the wall. This allowed a comparison between
$U_t$ obtained from the two different methods to be made. The difference between the $U_t$ values as obtained from the velocity gradient and optimization technique was less than 4 percent for all cases.

<table>
<thead>
<tr>
<th>Test</th>
<th>$Re_0$</th>
<th>$U_t$ (cm/s)</th>
<th>$C_f$ ($\times 10^3$)</th>
<th>$\Pi$</th>
<th>$\Delta B^+$</th>
<th>$Re_k$</th>
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<tr>
<td>B-SM1</td>
<td>2625</td>
<td>3.24</td>
<td>3.80</td>
<td>0.11</td>
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</tr>
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<td>B-SM2</td>
<td>1380</td>
<td>1.55</td>
<td>4.39</td>
<td>0.10</td>
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<td></td>
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<td>0.35</td>
<td></td>
<td></td>
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<td>0.36</td>
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<td>4.20</td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-SG1</td>
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<td>4.50</td>
<td>0.25</td>
<td>2.8</td>
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<td>4.22</td>
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<td>27</td>
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<tr>
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<td>25</td>
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<td>25</td>
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<td>5.56</td>
<td>0.45</td>
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</tr>
</tbody>
</table>

Table 4.2: Summary of skin friction velocity and wake parameter for Series B

In Figure 4.1b, fits of Eqn. (2.13) and (2.16) to some of the experimental data (Tests B-SM2, B-PF1, B-SG3 and B-WM2) are shown. Following Press et al. (1987), the chi-square distribution of the fitted curve and the experimental data was computed in the region $0.1 \leq y/\delta \leq 1$ as a quantitative measure of the goodness-of-fit for the plots shown in Figure 41b. It was noted that the fits gave a good representation of the experimental data at 99.5 percent confidence level.
The $U_r$ and $\Pi$ values obtained for the smooth and rough wall data are also summarized in Table 4.2. The present values of $\Pi$ obtained for the smooth wall data are similar to the value of 0.1 and 0.16 reported by Kirkgoz and Ardichoglu (1997) and Nezu and Rodi (1986), respectively, in open channel flow experiments at similar test conditions. It is, however, important to note that the present smooth wall values of $\Pi$ are significantly lower than the suggested value of 0.55 for a typical smooth wall zero pressure gradient boundary layer. This difference is most likely due to both the free surface effects present in open channel flow and the elevated turbulence levels. Similar to the observations made by Krogstad et al. (1992) for a zero pressure gradient boundary layer, Table 4.2 shows a clear variation in the relative strength of the wake with the type of roughness. Specifically, the smooth plate has the lowest wake strength ($\Pi = 0.1$) while the wire mesh has the highest strength ($\Pi = 0.48$). This observation suggests that the effects of surface roughness are not confined to the wall region but affects the outer flow more than implied by the ‘wall similarity hypothesis’.

4.2 MEAN VELOCITY DISTRIBUTIONS

4.2.1 Mean Profiles in Outer Coordinates

The mean velocity profiles for the smooth wall data (Series A) in outer coordinates are shown in Figure 4.2a. The characteristic ‘blunt’ profile typical of a turbulent boundary layer is clearly evident. Figure 4.2a shows that the mean profile becomes more ‘full’ as $Re_a$ increases. The velocity dip for this set of data is minimal
Fig. 4.2. Distributions of mean velocity in outer variables
(a) Smooth data at various \( Re_\theta \)
(b) Smooth and rough wall data
Figure 4.2b compares the smooth and rough wall data (Series B) in outer coordinates. Here only the data up to $y = 1.25\delta$ are shown while the inset shows the profiles up to the free surface for some of the data sets. The rough wall data are less ‘full’ when compared to the smooth wall profile. It is also clear from this figure that the wire mesh exhibits the highest deviation from the smooth profile while the perforated plate shows the least deviation. This suggests that even though the wire itself has the smallest diameter (0.6 mm compared with 1.2 mm for the sand grain, and 1.4 mm for the perforation depth), the mesh roughness exhibits the greatest resistance for the present set of test conditions. Further evidence of this trend will be discussed in subsequent sections. The inset shows that the mean data in the vicinity of the free surface are about 94 to 96 percent of the corresponding local maximum value. Compared to measurements obtained in Series A (see Figure 4.2a), the inset shows that the characteristic velocity dip is more extreme for Series B.

4.2.2 Mean Velocity Defect Profiles

The velocity defect profiles for Series A are shown in Figure 4.3. In Figure 4.3a, the conventional velocity scale $U_z$ is used while the freestream velocity $U_e$ is used as the scaling velocity in Figure 4.3b. In both figures, the boundary layer thickness, $\delta$ is used to normalize the vertical distance. In Figure 4.3a, no systematic Reynolds number effects can be observed for $y \geq 0.02\delta$. On the other hand, Figure 4.3b which uses the scaling proposed by George and Castillo (1997), shows a small but systematic decrease in the mean velocity defect at similar $y/\delta$ as $Re_\delta$ increases.
Fig. 4.3: Distribution of mean velocity defect
(a) inner coordinate; (b) outer coordinate
The distributions of the velocity defect for the smooth wall data sets (Series B) are compared with the results of Thole and Bogard (1996) at both lower and higher freestream turbulence values in Figure 4.4a. The solid line is a fit to Hama’s function (Eqns. (2.14) and (2.15)) which is representative of a correlation which fixes the value of \( \Pi \) implicitly. It should be noted that the study of Thole and Bogard (1996) considered a boundary layer with significant and sustained freestream turbulence, while our data pertains to an open channel flow where the notion of “freestream turbulence” becomes ambiguous. Even though our study considered a boundary layer in an open channel, the present smooth data fall within the envelope of Thole and Bogard’s boundary layer data for a zero pressure gradient.

In Figure 4.4a, the defect velocity profiles for all data sets are consistently lower than would be predicted by Hama’s functions. In fact, if in the course of the optimization technique, one insists on the smooth wall data (Test B-SM3) following the Hama fit by fixing \( \Pi = 0.55 \), a \( U_c \) value of about 0.0165 m/s (compared with 0.0210 m/s obtained from Eqn. (2.13) and (2.16)) would be predicted. This would give a skin friction coefficient that is 40 percent lower than otherwise obtained. The friction velocity obtained from Hama’s formulation for the Test B-SM3 data would also require a slope of \( \kappa^\prime = 3.2 \) and an additive constant of \( C = 7.2 \) for the experimental data to follow the log law. Furthermore, the universality of \( U^+ = y^+ \) would be invalidated. It should be recalled that for the smooth case, the value of \( U_c \) obtained from the present optimization procedure closely matched that determined from the slope of the velocity profile at the wall. Following Bradshaw (1987), one may attribute the lower value of \( U_c \) obtained from
Fig. 4.4a. Mean defect profiles for smooth wall

Fig. 4.4b. Velocity defect profiles for smooth and rough surfaces, solid line is a fit to Hama profile ($\Pi = 0.55$)
the Hama function (and hence $C_D$) to the strength of the wake, as implied by Eqn. (2.15), being too large for the present experiments. This result leads one to conclude that use of a correlation such as Hama's which implicitly fixes the wake strength at $\Pi = 0.55$ is an erroneous approach for an open channel boundary layer as well as other turbulent boundary layers with a weak wake component (i.e. a low $\Pi$ value).

In Figure 4.4b the smooth and rough wall data in Series B are shown. The strengths of the wake produced by the perforated plate and sand grain roughness are nearly equal ($\Pi = 0.24 - 0.36$), while the wire mesh has the strongest wake strength. Consistent with the $\Pi$ values summarized in Table 4.2, the wire mesh profile which has the highest $\Pi$ value follows the Hama formulation most closely. One may conclude from Figure 4.4b that the velocity profiles for the rough wall boundary layers being studied are significantly different from the smooth case in the outer part of the flow.

### 4.2.3 Mean Profiles in Inner Coordinates

Figure 4.5a shows the distributions of the mean profiles in inner coordinates for the smooth wall data (Series A). The log law constants adopted are $\kappa = 0.41$ and $B = 5.0$. The collapse of the profiles at various $Re_\theta$ is to be expected in the viscous sublayer as well as in the overlap region. As $Re_\theta$ increases, the extent over which the experimental data collapse onto the logarithmic law increases. This is consistent with the trend with $\delta^-$ shown in Table 4.1.
Fig. 4.5. Velocity distribution in inner variables
(a) Smooth wall data at various Reₙ
(b) Comparison between smooth and rough wall data
The Reynolds number effects in the outer region can be examined from the deviation \( \Delta U_{\text{max}}^- \) between the mean data and the log law profile at \( y^- = \delta^- \). The maximum deviation, \( \Delta U_{\text{max}}^- \), is related to the strength of the wake, \( \Pi \) as follows: 
\[
\Delta U_{\text{max}}^- = 2\Pi/\kappa.
\]
The values of \( \Delta U_{\text{max}}^- \) are summarized in Table 4.1. Using this relation, the corresponding values of the wake parameter are \( \Pi = 0 \) and \( 0.1 \) at \( \text{Re}_b = 750 \) and \( 3250 \), respectively. The evident reduction of the wake strength for all the profiles may be attributed to the relatively higher turbulence levels in the outer channel flow as well as the free surface effect.

The rough wall data are compared with the smooth wall profiles in Figure 4.5b. The effect of surface roughness is to shift the velocity profile down and to the right relative to the profile on a smooth wall. The shift in the velocity profile, and to a lesser degree the shape of the profile, is strongly dependent on the type of roughness. The roughness function \( (\Delta B^-) \) of each profile and the corresponding roughness Reynolds number \( (\text{Re}_k = kU_f/\nu) \) are also summarized in Table 4.2. For the tests conducted on the perforated plate (Tests B-PF1, B-PF2 and B-PF3), no noticeable shift \( (\Delta B^-) \) was observed suggesting that the mean velocity profile is essentially similar to the smooth wall data except for the strength of the wake. A possible explanation for this observation is that the flow within the cavities (i.e. the perforations) does not interact substantially with the bulk flow.

Figure 4.5b also shows that even though the mesh diameter is only half as thick as the average diameter of the sand grains, it gave the highest roughness function. It should
be noted that for flow over rough surfaces, the total drag consists of both viscous and form drag. The contribution of the form drag would depend on the onset of vortex shedding process, which in turn depends on the specific geometry of the roughness elements. Furthermore, Bandyopadhyay (1987) indicated that the critical roughness Reynolds number \( (Re_k) \) beyond which the flow regime becomes fully rough decreases with increasing span-to-height \( (l/k) \) ratio of the roughness elements. The critical values of \( Re_k \) for sand grain \( (l/k = 1) \) and the present wire mesh \( (l/k = 12) \) are approximately 55 and 15, respectively (see Bandyopadhyay, 1987; Figure 26). This may explain the higher roughness effect observed for the wire mesh compared to the sand grain data. The velocity profiles for the wire mesh roughness plotted in Figure 4.5b show a kink and a dramatic change in slope at \( y^+ = 15 \). These features are possibly an artifact of the periodic vortices shed over the wire. Obviously, the mean velocities in this region represent the time-averaged values of these periodic vortical structures.

### 4.2.4 Comparison Between Log Law and Power Laws

Consideration is now turned to the appropriateness of using a power law to describe the mean velocity in the overlap region. The results for some of the tests (Tests A-SM2, C-SMH, C-SGL, C-SGH, C-WMH, B-WM1) are shown and discussed. For the power law formulation of Barenblatt (1993), the initial values of the constants \( \alpha \) and \( C \) were computed using Eqn. (2.7) and (2.8). However, an improved fit to the experimental data was obtained by slightly modifying both \( \alpha \) and \( C \). The modified values, which were also used to compute \( U_{-2} \), are summarized in Table 4.3. For C-SMH, no modification was necessary for \( \alpha \) but the modified value of \( C \) was found to be 0.8 percent lower than the
initial estimate. For A-SM2, on the other hand, the difference between the modified values of $\alpha$ and C, and those determined from Eqn. (2.7) and (2.8) was 3.1 and 4.9 percent, respectively. In view of the sensitivity of the skin friction velocity to the values of the power law constants, these modest differences between the modified and initial values for $\alpha$ and C are significant.

In the case of the formulation proposed by George and Castillo (1997), the value of $C_o$ needs to be prescribed. It was observed that a more accurate estimate of $U_{\tau2}$ is obtained by setting $C_o = 1.00$. Although $C_o$ depends on Reynolds number, the range of Reynolds numbers considered here is too narrow for any variation in $C_o$ to be important. Therefore, a value of $C_o = 1.00$ is assumed for both the smooth and rough-wall analysis reported herein. The constants used in fitting the power law formulation of George and Castillo (1997) are also summarized in Table 4.3.

<table>
<thead>
<tr>
<th>Test</th>
<th>$\alpha$</th>
<th>C</th>
<th>$U_{\tau2}$</th>
<th>$\Delta U_{\tau}$ (%)</th>
<th>$\gamma$</th>
<th>$C_i$</th>
<th>$U_{\tau2}$</th>
<th>$\Delta U_{\tau}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-SM2</td>
<td>0.166</td>
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<td>1.0</td>
<td>0.133</td>
<td>8.8</td>
<td>1.03</td>
<td>1.0</td>
</tr>
<tr>
<td>C-SMH</td>
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<td>8.14</td>
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<td>0.139</td>
<td>8.7</td>
<td>2.20</td>
<td>1.3</td>
</tr>
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<td>C-SGL</td>
<td>0.184</td>
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<td>0.180</td>
<td>6.0</td>
<td>1.69</td>
<td>5.0</td>
</tr>
<tr>
<td>C-SGH</td>
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<td>5.2</td>
<td>2.65</td>
<td>2.9</td>
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<td></td>
<td>0.260</td>
<td>2.76</td>
<td>3.94</td>
<td>4.6</td>
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</table>

Table 4.3: Summary of power law constants for [BP] and [GC] and friction velocity
Figures 4.6a and 4.6b show the velocity distributions for A-SM2 and C-SMH, respectively, as well as the corresponding fits to the power law derived by Barenblatt (1993) and the composite profile proposed by George and Castillo (1997). The logarithmic law (with $\kappa = 0.41$ and $B = 5.0$) is also shown for comparison. The power law of Barenblatt (1993) and the log law adequately represent the mean profile in the overlap region. With the exception of a ‘kink’ observed at $y^* = 16$, the composite profile of George and Castillo (1997) closely matches the velocity from the wall up to $y^* = \delta^*$. As mentioned earlier, the ‘kink’ at $y^* = 16$ may be attributed to the fact that neither Eqn. (2.2) nor Eqn. (2.9) is valid in the neighborhood of $y^* = 15$.

As will be discussed later, the success of the power law of Barenblatt (1993) in describing the velocity profile close to $y^* = \delta^*$ is partly due to the negligible wake component observed for the present smooth wall data. It should be noted that in the lower part of the overlap region, i.e. $30 < y^* < 120$ and $40 < y^* < 300$ in Figure 4.6a and 4.6b, respectively, the power and log law profiles are almost indistinguishable from each other. The values of the friction velocity $U_{c2}$ derived from the constants used in fitting Barenblatt (1993) and George and Castillo (1997) are reported in Table 4.3. These values are in excellent agreement with the values determined from the velocity gradient at the wall, the maximum deviation being less than 3 percent.

The measurements obtained on the sand grain roughness are shown in Figure 4.7. Also shown are the corresponding log law and power law profiles. Although $a^*$ (Eqns. 2.9 and 2.11) may depend on the roughness Reynolds number, and even the specific
Fig. 4.6: Log law and power law profiles [BP] and [GC] on smooth surface
Fig. 4.7: Log law and power law profiles [BP] and [GC] for sand grain data
geometry of the roughness elements, a value of $a^* = -16$ was used for all the rough wall data as well. The formulation of George and Castillo (1997) fits the profiles reasonably well up to $y^* = 0.98$. On the other hand, the power law of Barenblatt (1993) and the log law collapse with the experimental data over a more limited range of $y^*$. This can be attributed to the relatively larger wake component observed for the sand grain data in comparison to the smooth wall data. The values of $U_2$ obtained from the skin friction laws derived by Barenblatt (1993) and George and Castillo (1997), and the corresponding deviations from the reference values given in Tables 4.2 and 5.1, are reported in Table 4.3. The values obtained from George and Castillo (1997) are in very good agreement with those reported in Table 4.2 and 5.1, while the values obtained from Barenblatt (1993) are unacceptably high.

In Figure 4.8, some of the velocity distributions obtained on the wire mesh roughness (C-WMH and B-WM1) are presented. It should be noted that the wire mesh data showed wake components ($\Pi = 0.52$ and 0.48 for C-WMH and B-WM1, respectively) that are significantly larger than those observed for the smooth surfaces and also, presumably, in pipe flows. In view of the poor performance of Barenblatt (1993) for the sand grain data, the power law formulation of Barenblatt (1993) was not applied to the wire mesh data. Only the profiles derived from George and Castillo (1997) are shown in the figure. The good agreement between the measurements and fitted profiles is remarkable. For the purpose of comparison, the log law profile to C-WMH is also shown. It should be noted that while the composite profile follows the data for C-WMH up to $y^* = 1500$, the range of applicability of the log law does not extend beyond
Fig. 4.8: Log law and power law profiles [GC] for wire mesh
$y^* = 160$. For Test C-WMH, only 5 data points collapse on the log law. An immediate implication of this observation is that the usefulness of the log law in low Reynolds number flows over rough surfaces reduces as the roughness effect or the wake component increases. In contrast, the power law proposed by George and Castillo (1997) was able to follow the velocity profile over most of the extent of the boundary layer. As shown in Table 4.3, the friction velocities predicted from George and Castillo (1997) formulation agree with the values obtained from the profile matching technique to within ± 5.0 percent.

4.3 TURBULENCE INTENSITY

Figure 4.9a compares the distributions of $u^-$ obtained for Tests A-SM3, A-SM4 and B-SM1 to the boundary layers of Ching et al. (1995) and Osaka et al. (1998), and the fully developed channel data reported by Johansson and Alfredsson (1982) and Harder and Tideman (1991). All profiles (previous/present and channel/boundary layer) collapse reasonably well in the near-wall region ($y^* < 30$). The peak values of the profiles agree to within ± 5 percent. Beyond $y^* = 30$, the data of A-SM4 and B-SM1 which have $Re_\theta$ values similar to the data of Osaka et al. (1998), show consistently higher values than the data of Osaka et al. (1998). This can be attributed, in part, to the characteristic high background turbulence levels in the free surface region of open channel flows.

It is evident from Figure 4.9a that the values of $u^-$ at the free surface of the open channel experiments are comparable to the centerline value of 0.78 ± 10 percent...
Fig. 4.9: Distributions of turbulence intensity
(a) Present and previous canonical boundary layer flows
(b) Present and previous boundary layers at high freestream turbulence
compiled for duct flows by Durst et al. (1998). The present values are, however, significantly higher than those obtained at the outer edge of canonical turbulent boundary layers. It is also observed that the canonical boundary layer profile of Osaka et al. (1998) falls off more rapidly than the profile of A-SM4, perhaps, because of the lower turbulence levels at the outer edge of their flow. The deviations between the open channel data, i.e. Tests A-SM4 and B-SM1, on the one hand, and the fully developed channel measurements of Johansson and Alfredsson (1982) and Harder and Tiderman (1991), may be due to the relatively higher values of Re_h in the open channel data.

Although the data for Tests A-SM4 and B-SM1 were obtained in the same facility, some deviations are apparent between the two sets of data in the vicinity of the free surface. For example, Test B-SM1 shows a slight increase beyond y^+ = 2000. It should be recalled from Figure 4.2 that for measurements in Series B, the velocity dip is more extreme as evidenced in the mean value closest to the free surface being about 96 percent of the local maximum. For Series A, on the other hand, the velocity dip is minimal. It should also be pointed out that the region over which the u^+ profile increases in the outer region corresponds to the region beyond the velocity dip. In this region, both \langle uv\rangle and \partial U/\partial y are negative. As will be shown in Chapter 5, the magnitude of these quantities increases as the velocity dip becomes more extreme. The increase observed in u^+ profiles of Test B-SM1 compared to Test A-SM4 may be due to higher production of turbulence kinetic energy close to the free surface for the former data.
The turbulence intensity for the smooth case is also compared to other boundary layer measurements at elevated freestream turbulence in order to assess the effect of the turbulence intensity of the exterior flow. Although comparisons are made to the boundary layer measurements of Thole and Bogard (1996) and Hancock and Bradshaw (1983) at similar and different freestream turbulence intensities, the present case considers an open channel flow where the flow region outside the boundary layer is somewhat different, both in terms of mean flow structure and turbulence length scale. The distributions of the turbulence intensity for the smooth wall data in Series B as well as the data reported by Thole and Bogard (1996) and Hancock and Bradshaw (1983) are shown in Figure 4.9b. Here, the friction velocity and the boundary layer thickness are used as the normalizing scales. The agreement between the present smooth data and the other studies at comparable intensities appears reasonable. Specifically, the present data and the data of Hancock and Bradshaw (1983) at $Tu = 3.45$ percent are similar in the range $y < 0.75\delta$. Figure 4.9b also suggests that as the freestream turbulence increases, the intensity profile becomes more flat, implying that the outer turbulence is penetrating more deeply into the boundary layer.

Figure 4.10a shows the $u^+$ profiles for the present smooth wall data (Series A) using inner variables. For these profiles, some scatter exists in the near-wall region. In view of the difficulty in determining the position $y = 0$, an uncertainty of about $\pm 0.025$ mm is possible. In wall units, this uncertainty varies from $y^* = 0.3$ for Test A-SM1 to $y^* = 0.8$ for A-SM4. If an uncertainty of $\pm 0.5$ is allowed in $y^+$, the collapse in the near-wall region will improve significantly. Allowing for this uncertainty and also uncertainties in
Fig. 4.10: Turbulence intensity profiles at various $Re_\theta$
(a) inner variables; (b) outer coordinates
$u^*$, Reynolds number similarity may be claimed for $y^* \leq 30$. Compared to the mean flow, for which the profiles collapse up to $y^* = 250$, this implies that Reynolds number effects in the turbulence intensity profiles "penetrate deeper" into the inner region than for the mean profiles. The peak value for the present profiles is $(u^*)_{\text{max}} = 2.73 \pm 0.05$ and these values occur in the range $13 < y^* < 15$, irrespective of $Re_\theta$. These locations are in good agreement with the values observed in open channel LDA experiments and DNS results (e.g. Komori et al., 1993; Borue et al., 1995). From Figure 4.10a, systematic deviations are apparent in the profiles for $y^* \geq 30$.

In high Reynolds number near-wall flows, the turbulence intensity is approximately constant over the constant-stress region. Gad-el-Hak and Bandyopadhyay (1994) remarked that $u^*$ asymptotes to 2 in the constant-stress layer. The present profiles, in good agreement with the trend reported by Purtell et al. (1981) at similar Reynolds numbers, indicate that a constant-$u^*$ region does not exist in low Reynolds number turbulent boundary layers. It is apparent from Figure 4.10a, however, that as $Re_\theta$ increases the values of $u^*$ systematically increase towards the asymptotic value in the region over which the constant-stress is presumed to occur. At $Re_\theta = 750$, for example the value of $u^*$ in this region is about 1.5 while a value of $u^* = 1.82$ is attained at $Re_\theta = 2400$.

Figure 4.10b shows the distributions of streamwise turbulence intensity in outer variables. This is the scaling law derived from the recent boundary layer analysis of George and Castillo (1997). In outer scaling, the profiles collapse fairly well in the near-
wall region (i.e. $y/\delta < 0.01$). The collapse is also remarkable for $y/\delta > 0.2$, in apparent contradiction to the observations made in Figure 4.10a which uses inner scaling. As one would expect (see for example George and Castillo, 1997) the largest Reynolds number effects are evident in the overlap region. Furthermore, there is a tendency for the location at which the peak value of $u$ occurs to move closer to the wall as $Re_\theta$ increases. It is also evident from Figure 4.10b that as $Re_\theta$ increases, the peak value of $u/U_c$ reduces.

Figure 4.11 compares the turbulence intensity profiles on smooth and rough surfaces. In Figure 4.11a, the boundary layer thickness is used as the normalizing length scale, while in Figure 4.11b the viscous length scale is adopted. From Figure 4.11b, the location ($y^*_{\text{max}}$) at which each data set attains its maximum value is confined to the range $10 < y^*_{\text{max}} < 15$. It is also evident from Figure 4.11 that the smooth wall and perforated plate data exhibit slightly higher peak values. The smooth wall data, however, fall more rapidly and beyond $y^* \sim 20$, they become consistently lower than all the other data sets up to $y^* \sim 300$. In general, at similar outer turbulence levels, the turbulence intensity profiles tend to be more flat as the roughness effect increases. It is important to note that, in the constant-stress (or overlap) region, the rough wall data indicate $u^* = 2.3$ which is distinctly higher than the asymptotic value recommended for high Reynolds number smooth wall data. This is clear evidence of the influence of roughness extending beyond the roughness sublayer.
Fig. 4.11. Variation of turbulence intensity on smooth and rough surfaces
(a) outer variables  (b) inner variables
4.4 **SKEWNESS AND FLATNESS**

Investigation of structural information in wall-bounded flows within the context of near-wall turbulence production mechanism has received considerable attention (e.g. Kline et al., 1967; Kim et al., 1971). Low Reynolds number flow visualizations show that low-speed fluid in the near-wall region occasionally erupts violently into the high-speed outer region of the boundary layer. Following Kline et al. (1967) and Kim et al. (1971), this process is referred to as bursting. They also concluded that essentially all the turbulent production occurs during the bursting process. Corino and Brodkey (1969) indicated that the ejection phase of the bursting process is followed by a large-scale motion of upstream fluid that emanates from the outer region and sweeps the wall region of the previously ejected fluid. Subsequent studies (e.g. Raupach, 1981) used conditional sampling methodologies (e.g. VITA, VISA and quadrant analysis) to further our understanding of the turbulent structure. However, the structural information obtained using these techniques could be ambiguous (Gad-el-Hak and Bandyopadhyay, 1994).

The skewness \( S_i = \frac{\overline{u_i'}}{\overline{u}^3} \) and flatness \( F_i = \frac{\overline{u_i^2}}{\overline{u}^2} \) factors, (where \( u_i \) denotes the instantaneous turbulence fluctuation in the positive i-direction), give useful quantitative information regarding the temporal distribution of the velocity fluctuation around its mean value. A non-zero skewness factor indicates the degree of temporal asymmetry of the random fluctuation, e.g. acceleration versus deceleration or sweep versus ejection. Since the skewness retains the sign information, it contains valuable statistical information related to coherent structures and can be used to extract structural information without ambiguity or subjectivity (Gad-el-Hak and Bandyopadhyay, 1994).
A flatness factor larger than 3 is generally associated with a peaky signal as for example that produced by intermittent turbulent events. In this section, the skewness and flatness factors are used to document some structural information regarding open channel turbulent boundary layers.

Distributions of the skewness ($S_u$) and flatness ($F_u$) of the streamwise component of the turbulence fluctuations are shown in Figures 4.12a and 4.12b, respectively. Similar to the refined LDA measurements of Durst et al. (1995) and DNS results of Kim et al. (1986) in fully developed duct flows, the $S_u$ increases in the linear viscous sublayer. The peak value of 0.87 in the present flows is also in good agreement with the value of 0.85 reported by Durst et al. (1995). The large values of $F_u$ in the near-wall region (Figure 4.12b) are a manifestation of intermittent bursting events that take place there. In the near-wall region, the in-rush phase of the bursting cycle brings in high-velocity fluid from the outer layer. This results in large-amplitude positive $u$ fluctuations and high positive values of $S_u$ in the near-wall region (Simpson et al., 1981). Beyond $y^+ = 5$, $S_u$ and $F_u$ values decrease with increasing wall distance. The $S_u$ profiles change sign in the buffer region (at $y^+ = 12$) and exhibit a near-wall dip (a local minimum) at $y^+ = 15 - 20$. The location at which $S_u$ changes sign also corresponds to the position where $F_u$ attains its near-wall minimum value and also to the location where $u^*_{max}$ occurs. Beyond this location, $S_u$ and $F_u$ profiles increase slightly and stay approximately constant in the overlap region. The values of $S_u$ and $F_u$ in this region are close to the Guassian values of 0 and 3, respectively. Consistent with a wider region of overlap at
Fig. 4.12a: Distribution of skewness factor at various Reₙ

Fig. 4.12b: Distribution of flatness factor at various Reₙ
higher $Re_\theta$, there is a tendency for the region of 'near-constant' $S_u$ and $F_u$ to increase with increasing $Re_\theta$.

Although there is some scatter in $S_u$ and $F_u$ profiles in the vicinity of the free surface, these values deviate significantly from the Gaussian values. It is also observed that the skewness profiles show a dip in the neighborhood of $y^+ = \delta^+$ and increase as the free surface is approached. This observation appears to be unique to an open channel flow. In the outer edge of canonical turbulent boundary layers, the flow is intermittent and the flatness factors are high (typically 6 to 7 as reported by Andreopoulos et al., 1984). On the other hand, the $F_u$ data obtained in fully developed duct flows using the LDA and DNS (e.g. Kim et al., 1986; Niederschulte et al., 1990; Durst et al., 1995) show near-constant values in the overlap and the core regions of the channel. Furthermore, typical values of $F_u$ in the overlap and core regions for fully developed flows varied from 3 to 4. The high values of $F_u$ (about 4 – 7) observed close to the free surface in the present study (Figure 4.12b) are presumably a signature of intermittent large-scale negative $u$ fluctuations which occur as a result of the large eddies driving the fluid from the low velocity region.

4.5 TRIPLE CORRELATION

The gradient of the triple correlation $<u^3>$ contributes to the streamwise diffusive flux of the streamwise kinetic energy ($u^3$). The distributions of $<u^3>$ for Series A are shown in Figure 4.13. Inner scaling is used in Figure 4.13a, while the mixed scaling derived by George and Castillo (1997) is adopted in Figure 4.13b. The data for Test B-SM1 are also
Fig. 4.13: Variation of $\langle u^3 \rangle$ with Reynolds number
(a) Inner scaling (b) Mixed scaling derived from AIP
shown for comparison. In Figure 4.13a, the profiles show a reasonable collapse in the near-wall region (i.e. $y^+ < 250$). Here, the peak values as well as the locations of the near-wall maximum ($y^+ \approx 7$) and minimum ($y^+ \approx 20$) are nearly independent of Reynolds number. In the outer region, however, the profiles obtained at lower Reynolds number are closer to zero.

In Figure 4.13b, the distribution obtained from the re-scaled high $Re_\theta$ canonical turbulent boundary layer data obtained by Krogstad and Antonia (1999), i.e. [KA99], is also shown. The trend shown by the present results is similar to that reported by Krogstad and Antonia (1999). Each profile has a maximum value at a location that lies within the viscous sublayer. In the region that corresponds to the buffer region, the profiles show a systematic Reynolds number dependence. More specifically, the absolute values are higher at lower Reynolds numbers. The profiles also change less rapidly with the wall-normal distance at lower Reynolds number in the inner region. The location at which the near-wall minimum occurs is closer to the wall at higher Reynolds numbers. Beyond this location, both the present and earlier measurements show a systematic variation as the Reynolds number becomes higher. All the profiles are nearly flat in the region corresponding to the overlap region. It is also observed that the high Reynolds number profile of Krogstad and Antonia (1999) shows a more extended overlap region in comparison to the open channel flow data.
4.6 SUMMARY

Measurements of the streamwise component of the mean velocity and higher order turbulence statistics were obtained in smooth and rough wall open channel turbulent boundary layers. In analyzing the data, the scaling laws derived from classical theories as well as the recent scaling laws proposed by Barenblatt (1993) and George and Castillo (1997) were used. In order to assess the effect of the moderate turbulence intensity level in the channel flow outside the boundary layer, the data were compared to boundary layer data in the literature at different freestream turbulence intensities. For the smooth wall data, the friction velocity was obtained from the velocity gradient at the wall or by fitting fourth and fifth order polynomials to the near-wall data. In the case of the rough wall data, a velocity defect profile was fitted to each data set to determine the strength of the wake and the skin friction coefficient. In fitting the velocity defect law, a correlation which did not fix the value of \( \Pi \) implicitly, was found to yield a more consistent and accurate estimate for the skin friction coefficient than a formulation such as that of Hama which fixes the value of \( \Pi \).

The power laws proposed by Barenblatt (1993) and George and Castillo (1997) were found to describe the mean velocity for the smooth wall data almost to the outer edge. The values of the friction velocity obtained from the skin friction laws derived from these power laws were in excellent agreement with the corresponding values obtained from other reliable and widely accepted techniques. The power law derived by Barenblatt (1993) was not suitable for modeling the mean velocity profiles over the rough surfaces considered in the present study. Furthermore, the friction velocities
obtained from the corresponding power-law skin friction relation were found to be about 20 to 40 percent higher than the values obtained from other reliable techniques. In contrast, the power law proposed by George and Castillo (1997) was found to do an excellent job of describing the mean velocity over a significant extent of the boundary layer. The values of the friction velocity predicted from their skin friction law were in very good agreement (less than 5 percent variation) with the values obtained from a velocity defect matching technique.

The mean defect profiles do not show any sensitivity to Reynolds number when conventional inner scaling is used. In contrast, the present results show a slight but systematic Reynolds number dependence when the scaling derived from the AIP, i.e. the freestream velocity, is used. In inner coordinates, the turbulence intensity profiles show important dependence on Reynolds number except for $y^+ < 30$. When outer coordinates are used, the turbulence intensity profiles show Reynolds number dependence only in the overlap region. The skewness and flatness factors appear to be independent of Reynolds number but the triple correlation shows distinct Reynolds number dependence.

The effect of roughness on both the mean velocity and, to a lesser extent, the turbulence intensity, varied for the three different roughness elements. The value of the wake parameter, $\Pi$, was also observed to vary with roughness element. These observations are at variance with the “wall similarity hypothesis” which suggests that the effects of surface roughness should be confined to the roughness sublayer. Even though the boundary layer in an open channel flow is influenced by the free surface, many of the
flow characteristics, in particular those that pertain to surface roughness, are similar to those observed in a canonical zero pressure gradient boundary layer.
CHAPTER 5

EFFECTS OF SURFACE ROUGHNESS ON TURBULENCE STRUCTURE

In this chapter, two-dimensional boundary layer measurements on smooth and rough surfaces obtained in Series C are reported. The data presented include the mean velocity, Reynolds stresses, triple correlation, approximate energy budgets as well as mixing length and eddy viscosity. The recent theory proposed by George and Castillo (1997) and conventional scaling laws are used to analyze the data. Comparisons to other smooth and rough wall boundary layer measurements and DNS results are made. The data and discussion presented in this chapter provide insight into the effects of surface roughness on the turbulence structure and its implication for rough wall turbulence models.

5.1 DETERMINATION OF FRICTION VELOCITY

For the smooth wall measurements, the friction velocity was determined using the linear ($U^+ = y^+$) and near-wall polynomial fit ($U^+ = y^+ + c_4 y^{+4} + c_5 y^{+5}$) discussed in Chapter 4 (Section 4.1). The coefficients obtained were $c_4 = -0.28 \times 10^{-3}$ and $c_5 = 13.6 \times 10^{-6}$, which are in good agreement with the values obtained in Chapter 4 and also with the values recommended by Eriksson et al. (1998) and George and Castillo (1997). The friction velocities obtained from the near-wall data are denoted by $U_\tau$ and are summarized in Table 5.1. The power law and skin friction relation proposed by George and Castillo (1997) were also used to determine the friction velocity. These values are denoted by $U_{c2}$.
and are also summarized in Table 5.1. Irrespective of the specific wall conditions, the value of \( U_{\infty} \) is in excellent agreement with the corresponding value \( U_{r} \).

In the case of the rough wall data, the optimization procedure outline in Chapter 4 (Eqns. (2.13) and (2.16)) as well as the skin friction relation proposed by George and Castillo (1997) (Eqn. 2.20) were used to determine the friction velocity. These values are denoted as \( U_{r} \) and \( U_{\infty} \), respectively, in Table 5.1. The values of the wake parameter \( \Pi \) for both the smooth and rough wall data are also summarized in Table 5.1. Similar to the observation made in Chapter 4, the values obtained on the rough wall are significantly higher than the smooth wall data. The roughness Reynolds number \( Re_k \) for each of the rough wall data are also summarized in Table 5.1.

![Table 5.1: Test conditions and boundary layer parameters.](image)

<table>
<thead>
<tr>
<th>Test</th>
<th>( U_{e} ) (cm/s)</th>
<th>( Re_9 )</th>
<th>( U_{r} ) (cm/s)</th>
<th>( U_{\infty} ) (cm/s)</th>
<th>( \Pi )</th>
<th>( Re_k )</th>
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<td>C-C-SMH</td>
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<td>2.23</td>
<td>2.22</td>
<td>0.10</td>
<td></td>
</tr>
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<td>2050</td>
<td>1.57</td>
<td>1.55</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
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<td>2.65</td>
<td>0.30</td>
<td>33</td>
</tr>
<tr>
<td>C-C-SGL</td>
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<td>2180</td>
<td>1.78</td>
<td>1.69</td>
<td>0.25</td>
<td>26</td>
</tr>
<tr>
<td>C-C-WMH</td>
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<td>2600</td>
<td>2.90</td>
<td>2.83</td>
<td>0.52</td>
<td>17</td>
</tr>
</tbody>
</table>

**5.2 MEAN VELOCITY PROFILES**

**5.2.1 Outer Coordinates**

The distributions of the streamwise component of the mean velocity in outer coordinates are shown in Figure 5.1a. At the outer edge, i.e. \( y > \delta \), each velocity profile
Fig. 5.1 Mean velocity in outer coordinates
(a) streamwise; (b) vertical
(Lines represent best fits to experimental data)
shows a slight dip where the local maximum value ($U_\text{\textcircled{L}}$) occurs below the free surface and $\partial U/\partial y$ is negative in the vicinity of the free surface. As expected, the rough-wall profiles are less 'full' compared to the smooth surface. Near the free surface, it is evident that the magnitude of $\partial U/\partial y$ is higher for the lower depths of flow. The distributions of the mean velocity in the vertical direction ($V$) are shown in Figure 5.1b. The gradient $\partial V/\partial y$ is also negative near the free surface. It follows from continuity consideration that $\partial U/\partial x > 0$ (i.e. a slight acceleration) close to the free surface. The velocity dip and hence the magnitude $\partial V/\partial y$ are higher for the wire mesh data than observed for the smooth wall.

For the purpose of subsequent analysis, gradients of both the mean and turbulence quantities (e.g. $\partial U/\partial y$ and $\partial v^3/\partial y$) are required. The procedure of estimating derivatives directly from experimental data point is sensitive to 'noise' and may yield erroneous results. Alternatively, by generating good analytical or functional fits to the experimental data, more reliable derivatives can be obtained. This alternative procedure is adopted in the present analysis. In developing the curve fits, $y$-steps of 0.0005$\delta$ and $y^+$-steps of 0.5 (wall units) were used. To obtain the gradients, the curves were graphically differentiated and smoothed over 5 data points. The 5 data points over which the smoothing was done correspond to 2.5 wall units, which is comparable to the wall-normal dimension of the probe volume in viscous units ($1.8 < l_y^+ < 3.5$). The fits to the mean vertical velocity profiles for C-SMH, C-SGH and WM are shown in Figure 5.1b. An assessment of the goodness of fit for each curve was made by evaluating the coefficient of determination ($R^2$). The value of $R^2$ calculated for each curve was higher.
than 0.99. An alternative assessment using a chi-squared distribution at a 99.5 percent confidence level indicated that the curves are good representatives of the experimental data.

5.2.2 Inner Coordinates

The streamwise component of the mean velocity in inner variables is shown in Figure 5.2a using a semi-log scale. The corresponding logarithmic profiles are shown as lines. The smooth wall data show a good agreement with the logarithmic law in the range $25 < y^+ < 350$. Compared to the smooth wall data, the rough-wall profiles show the expected downward-right shift. However, the region over which experimental data and logarithmic profiles overlap is limited for the rough surfaces. Consistent with the values of $\Pi$ shown in Table 5.1, the wake components for the rough-wall data are much larger than for the smooth wall data.

The data for C-SMH, C-SGH and C-WMH as well as the corresponding composite profiles, i.e. Eqn. (2.11), (dashed lines) are shown in Figure 5.2b. For the composite profiles, the value of $a^+ (= -16)$ recommended by George and Castillo (1997) was adopted. In the viscous sublayer and the buffer region of the smooth wall data, the values of $c_4$ and $c_5$ used in fitting the fifth order polynomial to the near-wall data are adopted. As observed in Chapter 4, the composite profiles match the velocity data from the wall up to $y^+ = 850$ except for the 'kink' observed at $y^+ = 15$. Although the composite profile describes the overlap region reasonably well, it does not extend to the free surface. For the purpose of subsequent analysis, polynomials were also fitted to
Fig. 5.2: Mean profiles in inner coordinates
(a) lines represent log law profiles
(b) lines: solid (polynomial fits); dashed (composite profiles)
Tests C-SMH, C-SGH and C-WMH. The polynomial fits are shown as solid lines in Figure 5.2b. For each test, the polynomial fit is observed to describe the experimental data from the wall to the free surface better than the composite profile.

The mean velocity gradients $\partial U^*/\partial y^*$ obtained from the polynomial fits for C-SMH, C-SGH and C-WMH as well as from the composite profile for C-SMH are shown in Figure 5.3. In the viscous sublayer (i.e. $y^* < 5$) both profiles give $\partial U^*/\partial y^* = 1$ for C-SMH. The data obtained from the composite profile show some discontinuity in the region $10 \leq y^* \leq 30$. This may be attributed to the singularity in Eqn. (8) at $y^* = a^* (= 16)$ and ‘kink’ observed in the mean velocity profile in the neighborhood of $y^* = 15$ (Figure 5.2b). The $\partial U^*/\partial y^*$ profile obtained from the polynomial fit, on the other hand, is reasonably smooth across the entire depth of flow. With the exception of the disparity observed between the two C-SMH profiles in the region $10 \leq y^* \leq 30$, the values of $\partial U^*/\partial y^*$ from both profiles compare favorably. Comparison between $\partial U^*/\partial y^*$ profiles obtained from the composite profiles and the curve fits for the rough data is very good.

The values of $\partial U^*/\partial y^*$ from the DNS data obtained in a channel flow ($h^* = 180$, where $h^* = hU_L/v$) by Kim et al. (1986) were reported by Cenedese et al. (1998). Their profile is also shown in Figure 5.3 for the purpose of comparison. For $y^* < 100$, which represents the viscous sublayer, the buffer region and the overlap region of the DNS data, the deviations between C-SMH and DNS profiles are within $\pm 5$ percent. The present $\partial U^*/\partial y^*$ profiles are nearly independent of wall conditions in the overlap region, i.e. $30 < y^* < 200$. For $y^* > 200$, however, the C-WMH profile is consistently higher.
Fig. 5.3: Distributions of mean velocity gradient
than the smooth wall data. This is consistent with the trend shown by Π values summarized in Table 5.1. In the subsequent analysis, values of ∂U'/∂y' obtained from the polynomial fits are adopted.

5.3 TURBULENCE INTENSITY AND REYNOLDS STRESSES

The distributions of the streamwise turbulence intensity and Reynolds stress are shown in Figures 5.4a and 5.4b, respectively. In Figure 5.4a, the profiles are normalized using the freestream velocity (U∞) which is the correct velocity scale according to the recent theory proposed by George and Castillo (1997), while Figure 5.4b uses the conventional inner scaling (Uτ). Irrespective of the specific wall conditions, the profiles increase slightly with the vertical distance in the free surface region (Figure 5.4b). As remarked in Chapter 1, this is characteristic of open channel flows and has been observed in earlier LDA measurements and DNS results. This phenomenon is attributed to the suppression of vertical turbulence fluctuations at the free surface and a concomitant energy redistribution from the vertical component to the streamwise and spanwise component via pressure-strain (Komori et al., 1993).

The C-WMH profile shows a flat and broad ‘hump’ at y/δ = 0.06 ~ 0.2 (or 65 < y* < 250). According to Ligrani and Moffat (1985) this is a salient feature of a fully rough surface and also represents a region where production of longitudinal turbulence energy is important. They also speculated that the large ‘hump’ may be a result of important ejection-sweep cycle differences due to roughness, which according to the flow visualization results of Grass (1971) are associated with the detailed mechanics of low
Fig. 5.4: (a) Streamwise turbulence intensity in outer coordinates
(b) Streamwise Reynolds stress in inner coordinates
momentum fluid entrainment at the bed surface following the inrush phases. As the roughness effect reduces, the ‘hump’ becomes less flat and less broad. Figure 5.4a shows higher values for the rough surfaces than for the smooth wall. Although the average physical roughness height of C-WMH (i.e. the wire diameter) is only about 50 percent of the value for C-SGH (i.e. the nominal diameter of sand particles), C-WMH shows significantly higher values over most of the flow. In flow past bluff bodies (e.g. Kiya and Matsumura, 1988), the instantaneous velocity is decomposed into a time-mean, a phase-averaged or coherent component and an incoherent or random component. The relatively higher values obtained for the wire mesh may be due to the contribution of the coherent motion associated to the vortices shed from the wire.

The trend observed in Figure 5.4 and subsequent results suggest that the extent to which roughness influences the turbulence structure depends on the specific geometry of the roughness elements. The deviation of the C-WMH profile from the smooth-wall data (C-SMH) persists up to \( y^+ = 0.85 \) which corresponds to about 50 roughness heights away from the wall. This is at variance with the wall similarity hypothesis, which implies that any influence of wall roughness on the turbulent structure should be confined to about 5 roughness heights. In inner coordinates (i.e. Figure 5.4b), the peak values for the present smooth wall data occurred at \( y^+ = 13 \). The smooth wall data are considerably higher than the rough-wall values in the near-wall region (\( y^+ < 30 \)). For \( y^+ \geq 30 \), and allowing for measurement uncertainties, the smooth wall and sand grain profiles at similar freestream conditions are comparable. Compared to the smooth and sand grain data, the wire mesh
profile is consistently higher up to $y^* = 1000$. Curve fits to C-SMH, C-SGH and WMH data are also included for subsequent analysis.

The distributions of the vertical component of turbulence intensity and Reynolds stress are shown in Figure 5.5a and Figure 5.5b, respectively. Due to limitations in spatial resolution, reliable data could not be obtained in the very near-wall region ($y/\delta < 0.02$). DNS results of free surface flows indicate that the vertical fluctuation decreases to zero at the free surface. Furthermore, the decrease is rapid and occurs in a thin region close to the free surface. Limitations in the present system could not allow this region of interest to be captured. Figure 5.5a shows that the rough-wall data are significantly higher than the smooth-wall profiles. The present smooth wall profiles are comparable to earlier measurements. In inner coordinates, i.e. Figure 5.5b, it is also observed that the rough-wall data are distinctly higher than the smooth wall data over a significant part of the boundary layer. Curve fits to the data are also shown in Figure 5.5b.

The distributions of the Reynolds shear stress are shown in Figures 5.6a and 5.6b using outer and inner scaling, respectively. It is of interest to note that the shear stress is negative for all wall conditions in the region where $\partial U/\partial y$ and $\partial V/\partial y$ are negative, i.e. $y/\delta > 1.5$. The peak values of $-\langle u^+ v^+ \rangle$ for the smooth wall data (Figure 5.6b) are about 0.6 to 0.7. Although these values are comparable to the LDA measurements reported by Komori et al. (1993) and Xinyu et al. (1995) at low Reynolds numbers, they are lower than the asymptotic value of 1 (i.e. $-\langle uv \rangle = U_\tau^2$), which is the value to be expected at high Reynolds numbers. Measurements of Reynolds shear stress reported in the
Fig. 5.5: (a) Vertical turbulence intensity in outer variables
(b) Vertical Reynolds stress in inner variables
Fig. 5.6: Reynolds shear stress on smooth and rough surfaces
(a) outer scaling (b) inner scaling
literature showed considerable Reynolds number dependence. For example, the LDA channel data reported by Wei and Willmarth (1989) showed peak values that varied from 0.6 to 0.9. In the turbulent boundary layer measurements reported by Ching et al. (1995), the normalized Reynolds shear stress showed peak values that varied from 0.8 to 1.0. It should be remarked that the LDA systems used by Wei and Willmarth (1989) and Ching et al. (1995) have better spatial resolutions than the system used in the present study. Johnson and Barlow (1989) recommended that a spanwise dimension of the probe volume less than 15 viscous units is required for accurate measurements of the Reynolds shear stress. Unfortunately, this requirement could not be met in this study. It is, therefore, not clear whether the relatively low peak values observed for the present smooth wall data are due to low Reynolds number effects or limitations in spatial resolution.

It is of interest to note that in the wall region, i.e. \( y^+ < 200 \), the data for C-SML \( (l_\gamma^+ = 21) \) are slightly higher than those for C-SMH \( (l_\gamma^+ = 32) \). It should also be pointed out that the data rate for C-SML was lower than that obtained for C-SMH. The relatively higher peak observed for C-SML may be attributed to the smaller value of \( l_\gamma^+ \) and lower data rate for C-SML in comparison to C-SMH. If the peak for C-SMH is increased by 12 percent to 'correct' for a possible underestimation of the measured shear stress (Johnson and Barlow, 1989), the peak values would then be comparable to previous boundary layer flows at comparable Reynolds numbers. No correction was applied to the data plotted in Figure 5.6 since the values of \( l_\gamma^+ \) at similar freestream conditions do not vary much among the smooth and rough-wall data so that any possible effects of \( l_\gamma^+ \) on \( \langle uv \rangle \) will be nearly independent of wall conditions.
Irrespective of the scaling used, the shear stress shows important sensitivity to the specific wall conditions. For example, in inner scaling, Figure 5.6b shows peak values about 30 and 60 percent higher for C-SGH and C-WMH compared to C-SMH. The distinction among the various profiles is observed over most of the boundary layer. For the purpose of subsequent analysis, curve fits to C-SMH, C-SGH and C-WMH data are also shown. The curves describe the experimental data over the depth of flow reasonably well except for C-WMH where the curve is slightly higher than the experimental data for $y^+ > 1000$.

5.4 SHEAR STRESS CORRELATION COEFFICIENT

The distributions of the correlation coefficient are shown in Figure 5.7. The data obtained on the smooth surface are lower than those obtained for the rough surfaces in the inner region ($y/\delta < 0.2$) but independent of wall conditions for $y/\delta > 0.2$. The peak values are in the range $0.35 \pm 0.02$, which are lower than observed in high Reynolds number boundary layer flows. As observed earlier, $u^+$ and $v^+$ obtained for C-SMH are similar to earlier measurements but the present peak values of $\langle u^+ v^+ \rangle$ are lower than most of the existing boundary layer data. A 12 percent increase to account for any possible effect of the long spanwise dimension of the measurement volume would improve agreement between the profiles and typical high Reynolds number canonical boundary layer profiles. However, the profiles shown in Figure 5.7 compare favorably to the LDA data of Xinyu et al. (1995) whose Reynolds shear stress profiles are similar in magnitude to the present data and also to values inferred from measurements and the DNS data of Komori et al. (1993).
Fig. 5.7: Distribution of shear correlation
5.5 STRESS ANISOTROPY TENSOR

In order to quantify the differences between the stress distribution on smooth and rough surfaces, the stress anisotropy tensor $b_{ij}$ was evaluated, where $b_{ij} = \langle u_i u_j \rangle / q - 1/3 \delta_{ij}$, $q = 2k = u^2 + v^2 + w^2$, and $\delta_{ij} = 1$ if $i = j$ and 0 otherwise. The streamwise and vertical components of the tensor are denoted, respectively, by $b_{11}$ and $b_{22}$, while $b_{33}$ and $b_{12}$ denote the spanwise and the shear components, respectively. Since the spanwise stress was not measured, these values were approximated. Spanwise data for open channel flows are scarce. The few measurements and DNS results show considerable scatter. Following earlier boundary layer results, the following approximation was used: $\nu^* = K(u^* + v^*)^2$, where commonly used value of $K$ for high Reynolds number boundary layers is 0.5 (see e.g., Antonia and Luxton, 1971; Cutler and Johnston, 1989). The DNS results of Spalart (1988) at $Re_\theta = 1410$ (see Rodi et al., 1993) and the open channel flow analysis of Nezu and Nakagawa (1993) suggest $K = 0.4$. The more recent boundary layer measurements of Skare and Krogstad (1994) also showed a preference for $K = 0.4$. In the present analysis, $K = 0.4$ was adopted.

Figure 5.8a shows the distributions of $b_{11}$ for both smooth and rough wall data, while $b_{22}$ and $b_{12}$ are shown in Figure 5.8b and 5.8c, respectively. The values of $b_{11}$ obtained on the smooth surface show higher values for $y/\delta < 0.2$. For each surface, $b_{22}$ shows a trend that is opposite to $b_{11}$. Figure 5.8a and 5.8b suggest that surface roughness reduces the anisotropy close to the wall. Compared to the smooth wall data, Figure 5.8c shows that the magnitude of $b_{12}$ is higher for the rough wall data close to the wall. The negative peak values of $b_{12}$ are respectively 0.10 and 0.11 for the smooth and rough wall
Fig. 5.8: Distribution of stress anisotropy tensor
measurements. These values are about 5 to 10 percent lower than the smooth and rough wall measurements obtained by Antonia and Luxton (1971) but approximately 30 to 40 percent lower than the typical value of 0.14 for relatively high Reynolds numbers.

Shafi and Antonia (1995) compared their rough wall boundary layer to the smooth wall DNS results of Spalart (1988). In the inner layer, the trends observed for \(b_{11}\) and \(b_{22}\) are qualitatively similar to those noted in the present study. In the case of \(b_{12}\), they found no important differences between the smooth and rough wall data. Mazouz et al. (1998) made a similar comparison between smooth and rough wall measurements in a channel. They also reviewed a number of previous channel and pipe flow measurements over smooth and rough surfaces. In contrast to the present observation and that of Shafi and Antonia (1995), they concluded that the stress tensors \(b_{11}\) and \(b_{22}\) in duct flows over rough surfaces are higher than the corresponding smooth wall values although \(b_{12}\) profiles were not significantly affected by surface roughness.

Figure 5.8d shows the distributions of \(\Delta b_{ij} = (b_{ij}^s - b_{ij}^r)/b_{ij}^s\) where superscript \(s\) and \(r\) stand for smooth and rough, respectively. For C-WMH, \(\Delta b_{11}\) decreases from 17 percent at \(y/\delta = 0.04\) to 5 percent at \(y/\delta = 0.15\). The corresponding values for \(\Delta b_{22}\) at these locations are 20 and 7 percent, respectively. Compared to the smooth wall data, \(b_{12}\) for C-WMH indicates deviations as high as 60 percent at \(y/\delta = 0.04\) and about 10 percent at \(y/\delta = 0.15\). In each case the data for C-SGH are slightly lower than the data for C-WMH.
5.6 **SKEWNESS AND FLATNESS FACTORS**

The distributions of the streamwise ($S_u$) and vertical ($S_v$) skewness factors for the smooth and rough wall data are shown in Figure 5.9a and 5.9b using inner and outer coordinates, respectively. The streamwise ($F_u$) and vertical ($F_v$) components of the flatness factors are plotted in Figure 5.9c and 5.9d using inner and outer coordinates, respectively. For the smooth wall data, $S_u$ data are positive close to the wall ($y^+ < 12$), and decrease with increasing wall distance. The high positive values of $S_u$ in this region are possibly due to the arrival of high-speed fluid (i.e., acceleration-dominated events) from regions away from the wall. The $S_u$ profile for C-SMH changes sign at $y^+ = 12$ which also corresponds to the point of maximum $u^+$ (Figure 5.4b) and location of minimum $F_u$ (Figure 5.9c). In the overlap region, the smooth wall data are negative suggesting that large negative signals occur much more frequently. Beyond the overlap region, $S_u$ initially decrease to a local minimum ($y^+ = 800$) before increasing towards the free surface. The large negative values of $S_u$ in the outer region are evidence of deceleration-dominated events.

Earlier measurements and DNS results in smooth wall turbulent boundary layers and fully developed duct flows (e.g. Kim et al., 1986; Durst et al., 1995; Gunther et al., 1998) show that $S_v$ is positive in the viscous sublayer, shows a dip in the buffer region and remains positive for $y^+ > 30$. However, the data reported by Kreplin and Eckelmann (1979) did not show any region of negative $S_v$ in the buffer region. Reliable data for $S_v$ could not be obtained for $y^+ < 30$ in the present measurements. With the exception of a few data points, $S_v$ data are consistently positive over a significant portion of the
Fig. 5.9: Distribution of skewness and flatness factors
(a), (c) inner coordinates; (b), (d) outer coordinates
boundary layer ($y^+ < 900$). For $120 < y^+ < 600$, $S_v$ is almost the reverse of $S_u$. The present smooth wall data for $S_u$ and $S_v$ are qualitatively similar to canonical turbulent boundary layers and duct flows (e.g. Kim et al., 1986; Andreopoulos et al., 1984, Durst et al., 1995; Gunther et al., 1998) in the inner layer. The present data are, however, distinctly different from canonical boundary layer flows in the outer region which may be due to free surface effects.

Consideration is now turned to the effects of surface roughness on the skewness and flatness factors. The trends shown by the rough wall data are qualitatively similar to that observed for the smooth wall data. Although $v^+^2$ (Figure 5.5) shows sensitivity to wall conditions, $S_v$ appears to be independent of the wall condition. This observation is similar to that made by Mazouz et al. (1994) over smooth and grooved surfaces. However, $S_u$ profiles show important sensitivity to surface roughness for $y^+ < 700$. In contrast to the negative values obtained on the smooth surface (C-SMH), the data obtained for C-SGH is approximately zero over most part of the overlap region. In the case of C-WMH, $S_u$ is consistently positive for $y^+ < 120$. This may suggest that, in contrast to the smooth wall data, the overlap region of the wire mesh data are dominated by high-speed fluid. Raupach (1981) and Mazouz et al. (1994) also reported higher values of $S_u$ for rough surfaces compared to smooth wall data in the inner region.

With the exception of the large positive values observed in the viscous sublayer for C-SMH, $F_u$ for both smooth and rough walls are close to the Guassian value of 3. Irrespective of the wall condition, the $F_v$ profiles are higher than the Guassian value over the entire depth of flow but the data in the overlap region do not deviate much from 3.
The large values of $F_u$ and $S_u$ for C-SMH in the viscous sublayer are manifestation of intermittent bursting events that take place there. The large values of $F_u$ and $F_v$ and the corresponding large negative values of $S_u$ and $S_v$ in the outer region are signatures of intermittent large-scale negative fluctuations which occur as a result of the large eddies driving the fluid from the low velocity region. The relatively larger values of $F_v$ compared to $F_u$ close to the free surface may suggest that $v$ signals are more intermittent than $u$ signals near the free surface. The $u$ and $v$ signals in the vicinity of the free surface do not show any sensitivity to wall conditions.

5.7 TRIPLE CORRELATION

The gradients of the third-order turbulence statistics are important because they are associated with the transfer and redistribution of turbulent kinetic energy. In the present study, the following triple products are measured: $<u^3>$, $<v^3>$, $<u^2v>$ and $<v^2u>$. Following the analysis of George and Castillo (1997), the triple products are normalized by $U_c^2 U_e$.

The distribution of $<u^3>$, which is associated with the transport of $<u^2>$ by the turbulent motion in the streamwise direction, is shown in Figure 5.10a. The distributions of $<u^2v>$ and $<v^3>$, which represent turbulent transport of $<u^2>$ and $<v^2>$, respectively, in the vertical direction are shown in Figures 5.10b and 5.10c. The turbulent work done by the Reynolds shear stress in the vertical direction is represented by $<v^2u>$ and is shown in Figure 5.10d. The magnitude of $<u^3>$ is significantly larger than the other triple products. The trends shown by $<u^3>$ and $<v^2u>$, i.e. Figures 5.10a and 5.10d, are
Fig. 5.10: Distributions of triple correlation (normalized by $U_x U_y$)
qualitatively similar. Both sets of data are positive only in the vicinity of the wall \((y/\delta < 0.04)\). They show two dips at \(y/\delta = 0.03\) and 0.4. The trends shown by \(<u^2v>\) and \(<v^3>\), i.e. Figure 5.10b and 5.10c, which make most of the contribution to turbulent diffusion in the energy budget are opposite to those observed for \(<u^3>\) and \(<v^2u>\). The profiles of \(<v^3>\) and \(<u^2v>\) are positive over most of the boundary layer. These profiles exhibit two peaks, which are located at \(y/\delta = 0.03\) and 0.4, i.e. positions at which \(<u^3>\), and \(<v^2u>\) showed their dips. The peak values for \(<u^2v>\) are higher (about twice) than for \(<v^3>\). Except for possible Reynolds number effects, the present data are qualitatively similar in the inner region to earlier canonical boundary layer measurements (e.g. Bandyopadhyay and Watson, 1988; Krogstad and Antonia, 1999). The present profiles are, however, distinctly different from the canonical boundary layer data both in shape and sign in the outer layer, possibly due to free surface effects.

From Figures 5.10a and 5.10c, it is apparent that the rough wall data (C-SGH and C-WMH) are considerably higher than the smooth wall data in the inner layer. The effect of roughness on \(<u^3>\) is confined to \(y/\delta = 0.1\) but it persists up to \(y/\delta = 0.5\) in the case of \(<v^3>\). In the outer layer, the profiles are almost independent of specific wall conditions. In the case of \(<u^2v>\) and \(<uv^2>\), roughness effects appear to be more important in the intermediate region where the magnitude of the data obtained on the rough surfaces is higher than the smooth wall data.

For the purpose of modeling the turbulence diffusion term in the kinetic energy equation, it is the derivatives of the triple products with respect to \(y\) rather than the
actual values that are required. Figure 5.11a shows a plot of the sum of $<v^3>$ and $<u^2v>$. These terms are the major contributors to the turbulence diffusion term in the energy budget. The location of the outer (and larger) peak is closer to the wall as the roughness effect increases. The corresponding curves used to obtain their derivatives are also shown. The profiles for the rough surfaces are much higher than the smooth wall profile in the inner region. As will be shown subsequently (Figure 5.11b), the non-zero values of $\partial(<u^2v+v^3>/\partial y$ in the vicinity of the free surface suggest a non-negligible turbulence diffusion near the free surface.

5.8 ENERGY BUDGETS

Consideration is now turned to the turbulent kinetic energy budget. For an incompressible fluid, the exact transport equation for turbulent kinetic energy ($k$) is given by (e.g. Hinze, 1975)

$$\frac{Dk}{Dt} = -\frac{\partial}{\partial x}(u,(p+<u^2>,>) - <u,u,> \frac{\partial u_1}{\partial x} + v \frac{\partial}{\partial y} \frac{\partial <u,u,>}{\partial x} - \sqrt{\left(\frac{\partial u_1}{\partial x}\right)^2 + \left(\frac{\partial u_2}{\partial y}\right)^2} \right)$$ (5.1)

where, I denotes total advection of turbulence kinetic energy; II denotes pressure and turbulence diffusion; III represents turbulence production by the mean flow; IV denotes viscous diffusion; and V represents viscous dissipation.

In the present analysis, the following approximations are adopted:

i. The mean flow is two-dimensional so that $W = 0$ and $\partial W/\partial z = 0$. From continuity consideration, it follows that $\partial U/\partial x = -\partial V/\partial y$. 

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Fig. 5.11: (a) Triple correlation \(<v^3> + <u^2v>\)
(b), (c), (d) elements of energy budget
ii. Measurement of $\partial U/\partial z$ showed that this term is negligibly small.

iii. According to the DNS results for canonical turbulent boundary layers by Spalart (1988) and open channel boundary layer by Komori et al. (1993), the viscous diffusion may be neglected since its importance is restricted to the viscous region near the wall.

iv. Measurements of the pressure diffusion term are scarce. Based on the DNS results of Komori et al. (1993) in open channel flow, the pressure diffusion term is estimated to be small compared to the other terms.

v. The shear stress $\langle \nu w \rangle$ is estimated to be negligibly small compared to $\langle u v \rangle$.

On the basis of the above assumptions, the approximate two-dimensional steady state transport equation for turbulent energy is given by

$$
\left[ U \frac{\partial k}{\partial x} + V \frac{\partial k}{\partial y} \right] = - \langle uv \rangle \frac{\partial U}{\partial y} + \langle v' \rangle - \langle u' \rangle \frac{\partial V}{\partial y} \frac{\partial (\langle v k \rangle)}{\partial y} + \left[ \frac{\partial (\langle v k \rangle)}{\partial y} \right] - \left[ v \left( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial x} \right) \right]
$$

where I = advection of energy by the mean flow; II = production by shear and normal stresses; III = turbulent diffusion; and IV = viscous dissipation. Some of the above terms could not be directly measured and are approximated as discussed subsequently.

a. Advection

The advection of energy by the vertical component of the mean velocity, i.e. $V \partial k/\partial y$, was directly measured but $U \partial k/\partial x$ could not be measured. According to the measurements of Skare and Krogstad (1994), $U \partial k/\partial x$ and $V \partial k/\partial y$ are of the same order.
of magnitude but opposite in sign so that the total advection by the mean flow should be small. The channel data on smooth and rough surfaces reported by Hirota et al. (1992) showed that the total advection term is indeed negligible across the channel. The boundary layer measurements on smooth and rough surfaces reported by Antonia and Luxton (1971) and Kroghstad and Antonia (1999) also showed similar results.

b. Production

Turbulence production caused by streamwise and vertical components of the normal stresses, i.e. \( u^2 \partial U/\partial x \) and \( v^2 \partial V/\partial y \), were computed. Since the mean flow is assumed to be two-dimensional and \( \partial W/\partial z = 0 \), \( w^2 \partial W/\partial z \), which was not measured is assumed to be negligibly small compared to \( u^2 \partial U/\partial x \) and \( v^2 \partial V/\partial y \). Subsequent to assumption (v) above, the main contributor to turbulence production caused by shear stress is \( \langle uv \rangle \partial U/\partial y \). This term was also measured directly.

c. Turbulence Diffusion

Turbulence diffusion by the streamwise and spanwise components of turbulence fluctuation, i.e. \( \partial \langle uk \rangle /\partial x \) and \( \partial \langle wk \rangle /\partial z \) were not measured. These terms are assumed to be negligible compared to \( \partial \langle vk \rangle /\partial y \). Following previous approaches (e.g. Bradshaw, 1967; Antonia and Luxton, 1971; Kroghstad and Antonia, 1999), \( \partial \langle vk \rangle /\partial y \) is approximated by \( 0.75 \partial (\langle u^2 \rangle + \langle v^3 \rangle) /\partial y \).
d. Dissipation Rate

The dissipation term was not measured but is obtained from the net energy imbalance. Strictly speaking, the imbalance comprises the dissipation and all neglected terms (e.g. pressure and viscous diffusion). However, the neglected terms are assumed to be small compared to the dissipation rate.

All the terms considered in the following discussions are obtained by fitting curves to the experimental data. These curve fits have been discussed in earlier sections. Following the analysis by George and Castillo (1997), all the terms in the energy budget are normalized by $U_x U_z^2 / \delta$.

Figure 5.11b shows the distributions of advection and turbulence diffusion terms for both the smooth and rough surfaces. In the region $0.1 \leq y/\delta \leq 0.35$, the diffusion terms on all surfaces are positive (i.e. gain) while the profiles are negative (loss) for $0.4 \leq y/\delta \leq 1$. In the region $1 \leq y/\delta \leq 2$, the profiles show minimal gains and beyond this region, slight losses are observed for each surface. Close to the free surface, the diffusion terms are small but not negligible, e.g. compared to the production term (Figure 5.11c). As the roughness effect increases, the magnitude of turbulence diffusion also increases. Furthermore, the locations of the innermost peaks and dips on the rough surfaces are closer to the wall compared to the smooth wall profiles.

The advection of kinetic energy by the vertical component of mean velocity, i.e. $V \partial k / \partial y$, is negative (loss) for $y/\delta < 2$. These terms are slightly positive beyond $y/\delta = 2$.
but are smaller than the magnitude of the corresponding diffusion terms in this region. Over most of the flow, the magnitude of the approximate advection term obtained on the rough surfaces is higher than the corresponding smooth wall data. Distributions of the production terms, \(-<uv> \partial U/\partial y\) and \(<u^2> \partial U/\partial x\) are shown in Figure 5.11c. For the purpose of clarity, \(<v^2> \partial V/\partial y\) is not shown. Irrespective of wall condition, \(<u^2> \partial U/\partial x\) and \(<v^2> \partial V/\partial y\) are nearly the same but opposite in sign so that their sum is negligibly small compared to the individual normal production terms as well as production caused by the Reynolds shear stress. Figure 5.11c shows that at \(y/\delta = 0.1\), \(<u^2> \partial U/\partial x\) is about 30 to 50 percent of the corresponding \(<uv> \partial U/\partial y\). Compared to the smooth wall data, production caused by normal and shear stresses obtained on the rough walls show significantly higher values over most part of the flow. In the inner layer, the higher values observed for the rough surfaces may be attributed to the higher stress observed for the rough surfaces (Figures 5.4-5.6). In the outer layer where the stresses are nearly independent of wall conditions, the higher production observed on the rough surfaces may be attributed to higher values of \(\partial U/\partial y\) for the rough surfaces (see Figure 5.3).

The energy imbalance is estimated as the difference between production and turbulence diffusion. On basis of the assumptions made earlier, i.e. viscous diffusion and pressure diffusion terms are negligible, and also in view of earlier experimental evidence that advection of turbulence by the mean flow is negligibly small over most part of the flow, one would expect the imbalance to approximate the dissipation. Figure 5.11d shows the distributions of turbulence production, diffusion and dissipation (i.e. imbalance) for smooth (C-SMH) and wire mesh (C-WMH) data. The data obtained for
C-SGH are not shown for the purpose of clarity. However, the profiles for C-SGH fall between the corresponding profiles for the smooth (C-SMH) and wire mesh (C-WMH) data. The dissipation rate for the smooth surface is considerably lower than the corresponding data obtained for the wire mesh over most region of the flow. For $0.1 \leq y/\delta \leq 0.3$, dissipation is nearly equal to production, irrespective of the specific wall condition. However, a balance between production and dissipation rate deteriorates in the region $y/\delta > 0.4$.

5.9 MIXING LENGTH AND EDDY VISCOSITY

Although the methodology of modelling turbulent flows via mixing length and eddy viscosity does not incorporate the exact physical processes, it has been successful in predicting simple shear flows on smooth and rough surfaces (e.g. Cebeci and Chang, 1978; Antonia et al., 1991). Furthermore, one-equation eddy viscosity models have recently regained popularity as components of two-layer models where an eddy viscosity model is used to resolve the near-wall region and more general models such as $k$-$\varepsilon$ or Reynolds-stress-equation models are employed outside the wall region (Rodi et al., 1993). Within this context, the effects of surface roughness on the distributions of the mixing length and eddy viscosity are examined in this section.

The mixing length and eddy viscosity depend on the relative magnitude of the turbulent shear stress and the mean velocity gradient. In order to facilitate discussion on the mixing length and eddy viscosity, the parameter $R = <uv>/\partial U/\partial y$ for C-SMH, C-SGH and C-WMH is plotted in Figure 5.12a. Since two-component measurements
Fig. 5.12: (a), (b) Distributions of eddy viscosity on smooth and rough surfaces (c), (d) Distributions of mixing length on smooth and rough surfaces
could not be obtained very close to the wall, the values of R for \( y^+ < 30 \) may not be reliable. In the inner region where \( \partial U'/\partial y^+ \) is almost independent of wall conditions, the higher values of R observed in Figure 5.12a for the rough surfaces are due to the relatively higher shear stress obtained on the rough surfaces (Figure 5.6). In the outer region, \( \partial U'/\partial y^+ \) is considerably lower for C-SMH (Figure 5.3) so that R is higher for C-SMH than observed for C-SGH and C-WMH.

The parameter R also represents the ratio of eddy viscosity \( (\nu_t) \) to molecular viscosity \( (\nu) \), i.e. \( R = \nu_t/\nu \). According to Rodi et al. (1993), this quantity is an indicator of the influence of viscous effects on the flow and therefore if Reynolds number effects are important. The data deduced from Spalart's (1988) DNS results at \( Re_\theta = 670 \) and 1410 (Rodi et al., 1993) are also shown in Figure 5.12a for comparison. Close to the wall the agreement among all the data sets is good. In the overlap region, the present profiles do not deviate significantly from the DNS data at \( Re_\theta = 1410 \). The present data and DNS results show that the eddy viscosity in the outer region is much dependent on \( \partial U'/\partial y^+ \). The DNS results at \( Re_\theta = 1410 \) as well as C-SGH and C-WMH, which have relative higher \( \Pi \) values and presumably higher values of \( \partial U'/\partial y^+ \) in the outer region, show lower values compared to their respective peaks which occur in the overlap region. In contrast, the DNS results at \( Re_\theta = 670 \) and C-SMH, which have low \( \Pi \) values and presumably lower values of \( \partial U'/\partial y^+ \) show much higher values in the outer region. Figure 5.12b shows distributions of the eddy viscosity in outer coordinates. It should be recalled that \( \partial U'/\partial y^+ \) and \( <u^+v^+> \) are both negative in the free surface region. The rise in
the profiles for \( y/\delta > 1.2 \) in Figure 5.12b are caused by the negative values of the shear stresses in this region.

The distributions of the mixing length \( L = -\langle u^+v^+ \rangle^{1/2}/\partial U/\partial y \) are shown in Figures 5.12c and 5.12d using inner and outer scaling, respectively. Rodi et al. (1993) computed the mixing length distributions from the DNS data of Spalart (1988). These data are also shown in Figure 5.12c. For \( y^+ < 80 \), the agreement between the present data and the DNS results is reasonable. In general, the data for C-SMH and the DNS results show similar trends. This similarity may be due to the fact that for these flow conditions, \( \partial U^+/\partial y^+ \) tends to zero much faster than \( \langle u^+v^+ \rangle \).

The smooth and rough wall boundary layer measurements obtained by Antonia and Luxton (1971) and Krogstad and Antonia (1999) are compared to the present data in Figure 5.12d. Close to the wall, all measurements (both earlier and present) are approximately described by the relation \( L = 0.41y \). The present and earlier measurements are in good agreement for \( y/\delta < 0.3 \). Although \( \langle u^+v^+ \rangle \) for C-SMH is lower than the earlier smooth wall profiles, the C-SMH data are higher in the outer region due to their characteristic low values of \( \partial U/\partial y \) in this region. The \( \langle u^+v^+ \rangle \) profile as well as \( \Pi \) for C-WMH are comparable to the data reported by Antonia and Luxton (1971) and Krogstad and Antonia (1999). These features may explain the good agreement between C-WMH and the earlier data set up to \( y/\delta = 0.6 \). Due to the dip in velocity close to the free surface, the data for C-WMH are distinctly different from the boundary layer profiles.
5.10 SUMMARY

Two-component LDA measurements of turbulent boundary layers created in an open channel on smooth and rough surfaces are reported in this chapter. The data presented in this chapter indicated some important distinctions between a boundary layer in open channel flow and a canonical zero-pressure gradient boundary layer in the outer region, i.e. $y/\delta > 1$. These differences are likely due to free surface effects as well as the characteristic high background turbulence levels in open channel flows. However, there are strong similarities between a turbulent boundary layer in open channel flow and a zero pressure counterpart in the wall region which allow one to draw general conclusions regarding the effects of surface roughness on near-wall turbulence structure.

The present measurements show that surface roughness effects are not confined to the roughness sublayer as implied by the wall similarity hypothesis. With regard to the mean flow, the effect of surface roughness penetrates deep into the outer edge of the flow and substantially increases the value of the wake parameter $\Pi$ over the corresponding smooth wall value. For the low Reynolds numbers considered herein, the relatively higher values of $\Pi$ observed for the rough surfaces result in a limited overlap region which may render the use of Clauser plot technique of determining the skin friction unreliable.

The results presented in this chapter indicate that the turbulence intensities and the Reynolds stresses are sensitive to the specific wall roughness. For the streamwise component, the data obtained on the smooth and sand grain roughness do not show
important differences. Although the physical diameter of the wire mesh is smaller than the sand grain diameter, the data obtained on the wire mesh are significantly larger than the values observed for the smooth surfaces. The vertical component of the normal stresses and the shear stress show even greater sensitivity to wall condition. Distributions of stress anisotropy tensors also show dependence on surface roughness in the inner layer. It was found that roughness promotes a tendency toward isotropy close to the wall. The triple products involving the streamwise and vertical components of turbulent fluctuations were computed. When these statistics are normalized by the scaling suggested by George and Castillo (1997) they show sensitivity to surface roughness. The streamwise component of skewness factor shows higher values for the rough surfaces than for the smooth surface but the vertical component of skewness factor as well as the flatness factors do not show any sensitivity to surface roughness.

The distributions of turbulent diffusion, production and dissipation showed a strong dependence on the specific rough elements. These observations suggest that rough wall turbulence models must explicitly account for the specific geometry of roughness elements in order to accurately predict the transport characteristics of the flow. This promises to provide significant challenges to turbulence models. The distributions of the eddy viscosity and mixing length show that the Reynolds shear stress and the velocity gradient dominate these quantities in the inner and outer regions, respectively. Consequently, the mixing length and eddy viscosity are higher for the rough surfaces in the inner region while the relatively lower values of velocity gradient for the smooth surface cause significant increase in these parameters in the outer region.
CHAPTER 6

CHARACTERISTICS OF A TURBULENT WALL JET ON SMOOTH AND ROUGH SURFACES

Wall jet measurements over smooth and rough surfaces are reported in this chapter. The data reported include the mean velocities, Reynolds stresses, triple products and distributions of the energy budget as well as mixing length and eddy viscosity. Compared to earlier studies, the present flows are significantly modified by reverse flow as well as high background turbulence levels close to the free surface. In order to facilitate comparison with previous works, issues regarding quality of flow and two-dimensionality are discussed. With regards to the streamwise development of the mean flow, both conventional scaling and scaling laws proposed by Narasimha et al. (1973) are applied. Some of the scaling laws used for the boundary layer analysis in Chapters 4 and 5 are applied to the inner region of the wall jet. The inner and outer regions of the wall jet are compared to the structure of a turbulent boundary layer and turbulent free plane jet, respectively.

6.1 FLOW QUALIFICATION

6.1.1 Exit Profiles

Since the streamwise evolution of the flow may depend on initial conditions such as the slot momentum, it is important to document the exit profiles. Figure 6.1 shows distributions of the mean and fluctuating streamwise components of the velocity for
Fig. 6.1a: Mean velocity profiles at jet exit

Fig. 6.1b: Streamwise turbulence fluctuation at jet exit
some of the smooth wall tests. The velocity data and vertical distance are normalized by the centerline mean velocity $U_j$ and slot height $b$, respectively. The profiles become more ‘full’ as the exit Reynolds number increases. In contrast to many experiments (e.g. Karlsson et al., 1993 [KEP]; Abrahamsson et al., 1994 [AJL]; Schneider and Goldstein, 1994 [SG]) where top-hat velocity profiles have been reported, the present mean profiles are flat only over the central 30 to 40 percent of the slot. The streamwise turbulence intensity is flat, to within ±5 percent, over the middle 20 percent of the slot. The centerline turbulence intensity varies from 3 to 5 percent, which is an order of magnitude higher than values reported in the literature. A closer examination of the lower half of the turbulence intensity indicates a Reynolds number effect, where the peak value decreases with increasing exit Reynolds number. Furthermore, the peak value occurred closer to the wall at a higher Reynolds number.

<table>
<thead>
<tr>
<th>Test</th>
<th>$U_j$ (m/s)</th>
<th>$U_b$ (m/s)</th>
<th>$U_o$ (m/s)</th>
<th>$Re_j$</th>
<th>$J$ (Kg/s²)</th>
<th>$M_o$ (Kg/s²)</th>
</tr>
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<tbody>
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<td>1.190</td>
<td>14000</td>
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<td>1.11</td>
<td>0.84</td>
</tr>
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<td></td>
<td>14000</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td>D-SG2</td>
<td>1.204</td>
<td>0.997</td>
<td></td>
<td>12000</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>D-SG3</td>
<td>0.721</td>
<td>0.584</td>
<td></td>
<td>7200</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>E-SM1</td>
<td>1.371</td>
<td>1.146</td>
<td>1.115</td>
<td>13700</td>
<td>1.88</td>
<td>1.37</td>
</tr>
<tr>
<td>E-SM2</td>
<td>1.023</td>
<td>0.857</td>
<td></td>
<td>10300</td>
<td>1.05</td>
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</tr>
<tr>
<td>E-SG1</td>
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<td>1.117</td>
<td></td>
<td>13100</td>
<td>1.70</td>
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</tr>
<tr>
<td>E-SG2</td>
<td>0.985</td>
<td>0.828</td>
<td></td>
<td>9700</td>
<td>0.97</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Summary of exit conditions
The characteristics of the mean profiles at the slot are summarized in Table 6.1. In this table, $U_j$ denotes the maximum velocity; $U_b$ is the bulk velocity obtained from mass flow rate measurement using an electronic weighing tank; $U_o$ is the mean velocity obtained by integrating the exit profile; the slot momentum $J = \rho U_j^3 b$ and $M_o = \rho \Sigma U^2 \Delta y$ is a finite difference approximation of the exit momentum flux. The ratio $\lambda = U_j / U_o$ in the present experiments varied from 1.17 to 1.25. These values are higher than the value of 1.10 reported by Schneider and Goldstein (1994) and other wall jet studies but are comparable to the values of 1.15 to 1.20 obtained by Durst et al. (1998) in their low Reynolds number channel flow experiments. The differences between the bulk velocity determined from mass flow rate measurement ($U_b$) and the corresponding value obtained by integrating the exit velocity profile ($U_o$) were less than 3 percent. It is important to note from Table 6.1 that the value of the exit momentum flux $J$ is about 30 to 40 percent higher than the corresponding value $M_o$ determined by integrating the exit velocity profile. In view of possible dependence of streamwise development on initial conditions such as the exit velocity profile or source momentum (Lauder and Rodi, 1981; George et al., 2000 [GAEKLW]), the values of $M_o$ are preferred for scaling purposes in subsequent analysis.

6.1.2 Effects of Reverse Flow and Flow Development

Wall jet experiments conducted in small enclosures are often influenced by secondary flows. The streamwise evolution of the flow may depend on secondary flows so that a reliable calculation of decay and growth rates will be obtained only if data in the region of minimal secondary flow effects are considered. The present wall jets are created in an
open channel and are characterized by a significant reverse flow and high background turbulence intensity, especially at distances remote from the slot.

Distributions of the mean velocity and Reynolds stresses at various downstream locations were examined in order to identify the region over which modification of the turbulence structure by the reverse flow is minimal. Figures 6.2a and 6.2b, respectively, show streamwise and vertical components of the mean profiles for Test E-SM1 in the region $10 \leq x/b \leq 100$. The mean velocities are normalized by $U_m$ while the vertical distance is normalized by $y_{1/2}$. The profiles are shown up to $y/y_{1/2} = 5$ so that data close to the free surface, where effects of return flow are expected to be most extreme, could be examined. Figure 6.2a shows that for $x/b \geq 30$, reverse flow is present. At $x/b \leq 60$, the magnitude of the reverse or return flow in the vicinity of $y/y_{1/2} = 4$ is less than 5 percent of the local maximum velocity $U_m$. With increasing downstream distance, the influence of the return flow becomes more extreme. At $x/b = 80$ and 100, the negative velocities in the vicinity of $y/y_{1/2} = 3$ are 13 and 30 percent, respectively, of the local maximum value. It should be pointed out that data reported in most of the earlier studies terminate at $y/y_{1/2} = 2$. There is considerable scatter among the mean velocity distributions obtained by various researchers in the outer region of the flow ($y/y_{1/2} \geq 1.3$). Note that if consideration is limited to data in the region $y/y_{1/2} \leq 1.5$, it would be concluded from Figure 6.2a that no effects of flow reversal are present. Figure 6.2a also shows that the mean streamwise profiles at $x/b \geq 30$ collapse reasonably well. With regard to the vertical mean velocity profiles, Figure 6.2b shows that there are no systematic deviations among the profiles obtained at $x/b \leq 70$ in the region $y/y_{1/2} \leq 1.5$. 

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Fig. 6.2: Mean velocity at various downstream locations for Test E-SM1 ($Re_b = 11500$)
(a) streamwise (b) vertical component
At $x/b > 70$, the influence of the reverse flow becomes important. In contrast to the streamwise component for which secondary flow effects are limited to $y/y_{1/2} > 1.5$, Figure 2b shows that $V$ could be modified up to the wall.

The distributions of the Reynolds stresses are shown in Figures 6.3 using outer scaling. Figures 6.3a and 6.3b show that the streamwise and wall-normal components of turbulence fluctuations collapse reasonably well in the region $30 \leq x/b \leq 60$. The effects of secondary flow become severe beyond $x/b = 70$. At these downstream distances, the turbulence levels in the outer region increase appreciably and penetrate almost down to the wall. On the other hand, distributions of the shear stress (Figure 6.3c) do not show any systematic trend in the region $y/y_{1/2} < 0.8$ for $30 \leq x/b \leq 80$. It is concluded from Figure 6.3c that for $x/b \leq 80$, the effects of flow reversal on the Reynolds shear stress are limited to $y/y_{1/2} > 0.8$.

There are a number of studies available in the literature for which secondary flows appear to be less severe than observed in the present study, yet the Reynolds stresses fail to collapse in the region where the mean profiles become self-preserving. For example, in the wind tunnel measurements reported by Wygnanski et al. (1992) [WKH], the normalized streamwise turbulence intensity increases with downstream distance. All three components of the normal stresses as well as shear stresses reported by Dakos et al. (1984) in a co-flowing plane turbulent wall jet also indicate a similar trend. A semi-bounded flow like a turbulent wall jet may not be strictly self-preserving. Very often, the lack of self-preservation for a wall jet is attributed to the fact that the maximum velocity
Fig. 6.3: Distributions of (a) Streamwise turbulence intensity
(b) Vertical turbulence intensity (c) Reynolds shear stress
$U_m$ and the Reynolds stresses decay at different rates with downstream distance. However, the data reported by Irwin (1973), [KEP] and [AJL] and many others identified a region over which the turbulence stresses are nearly self-preserving. It seems reasonable to speculate that the lack of collapse observed in the present flow at $x/b \geq 70$ may be due to the influence of reverse flow in the outer region.

6.1.3 Test for Two-dimensionality

For free plane jets Kotsovinos (1976) suggested that the following constraints: $w/b > 10$ to 20 and $x/w < 2$, where $w$ is the slot width, would ensure a satisfactory two-dimensionality in the plane of symmetry. This criterion suggests that for a given slot height and exit Reynolds number, a jet created in a large enclosure is more likely to conform to two-dimensionality for larger downstream distances than a corresponding flow created in a smaller enclosure. For a turbulent wall jet, a number of criteria such as uniformity in mean velocity data in the crosswise direction (e.g. Gartshore and Hawaleshka, 1964; [WKH]) have been proposed to examine two-dimensionality. Launder and Rodi (1981) recommended that conservation of two-dimensional momentum flux should be used as a more critical criterion. If the recommendation of Kotsovinos (1976) is adopted, satisfactory two-dimensionality can be claimed for all the measurement locations considered herein. A similar conclusion is arrived at if spanwise variation in streamwise component of the mean velocity is adopted. In the present study the recommendation made by Launder and Rodi (1981) is adopted as the principal test in examining the two-dimensionality of the flow.
The variation of local momentum flux with streamwise distance is shown in Figure 6.4. Figure 6.4a shows the momentum flux ($M_u$) due to the mean velocity alone while Figure 6.4b shows the fractional contribution of streamwise turbulence fluctuations ($M_a$). For a given test condition, the maximum momentum flux occurred at $x/b = 30 – 50$ rather than at the slot. The maximum values are approximately 20 to 30 percent higher than the corresponding value ($M_o$) obtained at the slot. A review of measurements obtained in plane free jet by Ramaprian and Chandrasekhara (1985) showed a similar trend. For the experiments reviewed in that paper, it was observed that the maximum local momentum flux could be as high as 55 percent higher than the exit value. Following Ramaprian and Chandrasekhara (1985), the increase in momentum flux can be attributed to the presence of negative pressure supported by the turbulence fluctuations in the cross-stream velocity component. It can be shown from the momentum equation that even small pressure changes can account for significant changes in momentum flux. For each set of data shown in Figure 6.4a, the local momentum flux is normalized by the maximum value.

In contrast to a plane free jet where the momentum integral is considered to be a conserved quantity, there is a continuous loss of streamwise momentum flux with streamwise distance in the case of a turbulent wall jet. Although part of the losses is often attributed to the presence of the solid surface, it is generally considered that a solid wall contributes very little to the total momentum loss. Launder and Rodi (1981) estimated that about 8 percent of the exit momentum flux could be attributed to frictional losses at $x/b = 100$. In the present analysis, losses due to wall friction are estimated to be less than 10 percent. This would suggest that satisfactory two-
Fig. 6.4: Streamwise momentum flux at various downstream locations
(a) data from mean velocity profiles only
(b) contribution from turbulence fluctuation
dimensionality may be realized if the following condition holds: $M_u \geq 0.9M_\infty$ or 75 percent of the local maximum momentum flux. If the above constraint is used, satisfactory two-dimensionality can be claimed for $x/b \leq 100$ (Figure 6.4a). For each test, the axial locations at which the above condition is fulfilled correspond to the position at which the mean velocity profiles show satisfactory self-similarity. The farthest downstream distance at which the local value of $M_u$ is greater than 90 percent of the exit momentum flux reduces as the Reynolds number decreases.

The fractional contribution of $u$ increases from a value of 0.5 percent at the slot to as high as 25 percent at large downstream distances (Figure 6.4b). The large values far downstream are due to the high turbulence levels at these locations. In the study of [AJL], which was conducted in a large enclosure to minimize secondary flow effects, the streamwise turbulence intensity contributes about 10 percent to the total local momentum flux in the self-preserving region. It is clear from Figure 6.4b that at $x/b \leq 70$, $M_w(M_u+M_U)$ does not exceed 12.5 percent. For each test, the axial locations at which $u$ contributes less than 12.5 percent of the total momentum flux coincide with the locations at which the $u$ profiles retain the characteristic wall jet shape.

6.2 STREAMWISE EVOLUTION OF THE MEAN FLOW

6.2.1 Velocity Decay

The variation of normalized maximum velocity $U_m$ with streamwise distance is shown in Figure 6.5. In Figure 6.5a, $U_j$ and $b_3$ are used as the appropriate velocity and length scales. Here, $b_3 = (b - 2\theta)$, where $\theta$ is the exit boundary layer momentum thickness.
Fig. 6.5: Variation of maximum velocity with streamwise distance
(a) conventional (b) kinematic momentum scaling (Lines denote fits to Eqn. 2.27)
The choice of $b_s$, rather than $b$, is expected to account for the shape of the exit mean profiles (Ramaprian and Chandrasekhara, 1985). Irrespective of the wall conditions, the data obtained at the lowest Reynolds number decay more rapidly. The data reported by [WKH] and [AJL] also showed a similar Reynolds number dependence. The data of [KEP] is in fair agreement with the present smooth wall data. At similar streamwise location, the rough wall data are slightly lower than the corresponding smooth wall data. The deviation of the rough wall data from the smooth wall data is greatest in the region of flow development $(x/b \leq 20)$.

The variation of $U_m$ with streamwise distance for the smooth wall data using the exit kinematic momentum $M_o$ (i.e. Eqn. (2.27)), which is the proper scaling according to Narasimha et al. (1973), is shown in Figure 6.5b. Only data in the region where the mean velocity profiles are nearly self-preserving and the effects of reverse flow minimal are considered here. In contrast to the systematic Reynolds number effect observed in Figure 6.5a, the choice of kinematic momentum as the scaling parameter makes the decay rate independent of Reynolds number. Also shown in Figure 6.5b are best fits of Eqn. (2.27) to the present data and those of [WKH]. The values of the power law exponent ($\alpha$) for both sets of experiments are within 4 percent of each other but the multiplicative constants ($A$) are significantly different. These differences may possibly be due to different initial conditions such as Reynolds number and source momentum. The near constancy of $\alpha$ suggests universality of Eqn. (2.27). Figure 6.5b also supports a power law variation of $U_m^2$ with $x$ in the self-preserving region.
6.2.2 Growth of Inner Layer and Jet Half-width

The variation of the inner layer thickness, $y_m$, with streamwise distance for smooth and rough wall data is shown in Figure 6.6. An accurate determination of $y_m$ is generally difficult. In the neighborhood of $y_m$, y-steps for smooth and rough surfaces were 1.0 mm and 1.5 mm, respectively. The uncertainty in $y_m$ is estimated to be less than 10 percent. It is clear from Figure 6.6 that wall jet flow over a rough surface has a considerably thicker inner layer. More specifically, the local value of $y_m$ is about 40 and 25 percent higher for the rough wall data at $x/b = 30$ and $x/b = 70$, respectively.

The variation of the jet half-width $y_{1/2}$ with $x$ for smooth and rough surfaces is shown in Figure 6.7a using the conventional scaling law. In sharp contrast to the sensitivity of inner thickness to surface condition, the spread rate for the jet half-width is nearly independent of the surface condition. Similar to some earlier studies, (e.g. [AJL, WKH]), the spread rate shows a Reynolds number dependence. More specifically, the spread rate increases with decreasing Reynolds number. The present growth rates vary from 0.085 to 0.090 for the highest and lowest Reynolds numbers, respectively. The present growth rates are higher than the value of 0.0073 recommended by Launder and Rodi (1981) for plane wall jets but lower than those obtained in free plane jet studies. As will be shown in a later section, the relatively higher spread rates obtained for the present study may be due to the high turbulence levels and turbulence production in the outer region of the flow.

Figure 6.7b shows the plots of $y_{1/2}$ with $x$ for the smooth wall data using the scaling recommended by Narasimha et al. (1973), i.e. Eqn. (2.28). In contrast to the
Fig. 6.6: Inner layer thickness for smooth and rough surfaces
Fig. 6.7: Variation of half-width with streamwise distance (a) conventional (b) kinematic momentum scaling (Lines denote fits to Eqn. 2.28)
distinct Reynolds dependence observed in Figure 6.7a, the use of kinematic momentum to scale the data renders the spread rate for the jet half-width nearly independent of Reynolds number. The best fits of Eqn. (2.28) to the present data and those of [WKH] are in good agreement with each other. The power law exponents are within 2 percent of each other. Furthermore, the value of $\beta$ is less than 1 indicating a non-linear spread rate of $y_{1/2}$ in accordance of the recent similarity theory proposed by [GAEKLW].

A plot of $U_m$ versus $y_{1/2}$ for the smooth wall data is shown in Figure 6.8. The best fit of Eqn. (2.29) to the present data set and a fit with the constants recommended by [GAEKLW] are also shown. The values of $C$ in both fits are identical and the difference between the power law exponents ($\gamma$) is less than 1.5 percent. This slight difference may be due to a dependence on initial conditions. The present value of $\gamma$ is less than -0.5 as required by similarity consideration [GAEKLW].

### 6.3 TRANSVERSE PROFILES

#### 6.3.1 Mean Velocity Distributions

##### 6.3.1.1 Determination of Wall Shear Stress

In view of the existing debate regarding the scaling law for the overlap region of the mean velocity, an accurate and independent determination of wall shear stress is critical. The technique used to determine the friction velocity for the smooth wall jet is similar to that used in the boundary layer analysis (Chapter 4), i.e. using the velocity gradient at the wall or fitting a fifth order polynomial to the near-wall data. The near-wall data and corresponding linear and fifth order polynomial fits are shown in Figure 6.9a.
Fig. 6.8: Variation of maximum velocity with jet half-width
(Lines represent fits to Eqn. 2.29)
Fig. 6.9a: Linear and polynomial fits to near-wall data

\[ U' = y' \]

\[ U' = y' - 0.00027y^{-4} + 13.6 \times 10^{-5}y^3 \]

Fig. 6.9b: Variation of skin friction coefficient with Reynolds number
The similarity between these data and those reported in Figure 4.1a implies that the mean velocity profile for a turbulent boundary layer and wall jet is identical in the near-wall region.

Figure 6.9b shows the distribution of skin friction coefficient $C_f$ with Reynolds number $Re_m (= U_m y_m/v)$. The skin friction correlations of Bradshaw and Gee (1962), Hammond (1982) and Eriksson et al. (1998) are shown for comparison. The present $C_f$ values are in good agreement with the correlation proposed by Bradshaw and Gee (1962) and Eriksson et al. (1998). However, the present sets of data at $Re_m < 3000$ are in better agreement with Bradshaw and Gee's correlation, while the data at $Re_m > 5000$ are better described by the correlation of Eriksson et al (1998). Hammond's correlation is in fair agreement with the present data for $Re_m > 7000$ but would substantially over predict the present values of $C_f$ for $Re_m \leq 7000$. For the range of $Re_m$ considered herein, the skin friction coefficients obtained for the rough surface are in the range $0.008 \leq C_f \leq 0.012$. These values are substantially higher than the corresponding values obtained on a smooth surface at similar $Re_m$.

6.3.1.2 Mean Profiles in Outer Coordinates

The mean profiles in the self-preserving region for some of the smooth wall experiments are shown in Figure 6.10a in outer coordinates, i.e. $U_m$ and $y_{1/2}$. The profiles collapse reasonably well. The inset shows a comparison between the present data and the hot-wire data of [WKH] and [AJL] as well as LDA data of [KEP]. The agreement among the present and previous data sets is excellent up to $y/y_{1/2} = 1.5$. The agreement
Fig. 6.10: Mean velocity profiles in outer coordinates
(a) smooth wall (b) comparison between smooth and rough wall data
between the present data and the LDA data of [KEP] is reasonable up to \( y/y_{1/2} = 2.5 \). However, the LDA data sets show lower values at the outer edge of the flow compared to the hot-wire data sets. This trend is consistent with previous comparisons made by [SG] and Venas et al. (1999) and has been attributed to instrument limitations although secondary flow effects could not be completely ruled out. As remarked earlier, the effects of reverse flow on the mean velocity profiles is not any more severe than in previous studies where attempts were made to minimize secondary flow effects.

The mean velocity profiles in outer coordinates for the rough wall experiments are shown in Figure 6.10b. A smooth wall profile is also shown for comparison. The smooth and rough wall data collapse reasonably well in the outer part \( (y/y_{1/2} > 0.5) \) of the flow. However, systematic and significant deviations are observed close to the wall. As already shown in Figure 6.6, the locations at which the maximum mean velocity occurred are farther removed from the wall in the case of the rough-wall profiles than for the smooth wall data. The data in the region \( y/y_{1/2} \leq 0.4 \) is shown as an inset so that the near-wall region can be more closely examined. Similar to the observations made in Chapters 4 and 5 for turbulent boundary layer, the smooth wall profiles are more full than the corresponding rough wall profiles in the inner region. Furthermore, the rough wall profiles become less full as the roughness effect increases. For a turbulent boundary layer, it is claimed that surface roughness enhances entrainment of irrotational flow into the inner layer owing to higher surface drag over a rough surface (Krogstad et al., 1992). It may be speculated that the relatively thicker inner layer \( (y_{m}) \) observed for a turbulent wall jet over a rough surface (Figures 6.6 and 6.10b) is caused by the same mechanism.
6.3.1.3 Mean Profiles in Inner Coordinates

Distributions of the mean velocity in inner coordinates are shown in Figure 6.11a for the smooth wall data. A data set of [KEP] is also shown for comparison. The present data, in excellent agreement with those of [KEP], show that a well-defined log region does exist although the extent of the overlap region is relatively shorter than observed for a turbulent boundary layer (Chapters 4 and 5). Launder and Rodi (1981) suggested that the narrow overlap region identified above may be due to incursions of large-scale eddies from the outer shear layer bearing a shear stress of opposite sign. The present data also show that as the slot Reynolds number increases, the extent of the log law region increases slightly.

Although Reₘ increases with increasing streamwise distance for a given Reₑ, there is no noticeable increase in the log law region with increasing streamwise distance. This observation should be contrasted to a turbulent boundary layer for which the overlap region increases indefinitely with increasing x (or δ⁺). As can be inferred from the values of <uv>/Uₘₐₓ² (Figure 6.3c) and Uₑ²/Uₘₐₓ² (Figure 6.9b), the wall shear stress appears to decrease faster than the shear stress in the outer region. As a consequence, the relative strength of the outer flow and its encroachment on the inner layer increase progressively with downstream distance (Launder and Rodi, 1981).

In accordance with the similarity theory proposed by [GAELKW], the results presented in Figures 6.9a and 6.11a suggest a striking similarity between the inner region of a wall jet and that of the turbulent boundary. Since the overlap region of a wall
Fig. 6.11: Mean velocity in inner coordinates
(a) smooth surface (b) rough surface

$U^* = 2.44 n y^- + 5.0$

$U^* = y^-$
jet is well described by a logarithmic law with values of \( \kappa \) and B that are universal and identical to those used in boundary layer analysis, the use of the Clauser plot technique to estimate the skin friction is justified for a turbulent wall jet. From a practical point of view, the limited extent of the log law region could lead to significant error unless caution is exercised so that the region over which data is considered is not too wide. For the same reason, turbulent wall jet modelers can make use of the 'wall function' that is widely used in boundary layer computations.

Determination of friction velocity for a turbulent wall jet over a rough surface is a challenging task. For Test D-SG1, an attempt was made to use a momentum balance from one-component measurements at \( x/b = 30, 33, 35, 40 \) and 64 to evaluate the wall shear stress. The value of skin friction obtained from this method was considerably lower than the corresponding values obtained from the near-wall data for the smooth wall data at similar \( \text{Re}_m \). This is not surprising since previous attempts to determine the wall shear stress from momentum balance were unsuccessful even for wall jets over smooth surfaces (Schwarz and Cosart, 1961). The present smooth wall data suggest a well-defined overlap region with universal log law constants (Figure 6.11a). Furthermore, Figure 6.10 shows that the inner region of the rough wall data is larger than for the smooth wall data. Therefore, the existence of a log law region with universal constants for the smooth wall and the evidence of a relatively thicker inner layer for the rough wall data are used to justify a Clauser chart technique for the rough wall data. The uncertainty in \( U_c \) determined from this approach could be high, perhaps of the order of 10 percent.
The velocity profiles in inner coordinates for the rough wall data are shown in Figure 6.11b. As expected and has been observed in boundary layer studies, the rough wall data show a downward-right shift with respect to the log law profile for a smooth surface. At similar x/d, the roughness shift increases with increasing Reynolds number. For a given slot Reynolds number, the roughness effect increases with increasing downstream distance.

6.3.2 Reynolds Shear Stress and Turbulence Intensities

The Reynolds stresses are shown in Figures 6.12 and 6.13. Distributions of the streamwise turbulence intensity obtained using one-component LDA are shown in Figure 6.12. The use of the one-component system allowed measurements closer to the wall than when the two-component LDA system is used. In tests for which both one- and two-component measurements were made, the profiles obtained using one-component compared favorably to the streamwise component of the corresponding two-component measurements. In all cases the deviation was less than 5 percent, which is comparable to the variation in U_b obtained from mass flow rate measurements.

In Figure 6.12a, the present measurements are compared to the hot-wire data of [AJL] and the LDA measurements reported by [KEP] and [SG]. The present data set is in good agreement with the profile of [KEP] for y/y_{1/2} < 1.5. The outer peak values and their corresponding locations are similar in both studies. The data compiled by Ramaprian and Chandrasekhara (1985) for a turbulent free plane jet indicated peak values that vary from 0.22 to 0.31, which are substantially higher than observed in Figure 6.12a. In the outer part of the flow, however, the present profile shows
Fig. 6.12: Distributions of streamwise turbulence intensity
(a) present and previous smooth data in outer coordinates
(b) present and previous smooth data in inner coordinates
(c), (d) present smooth and rough data in outer coordinates
Fig. 6.13: Vertical turbulence fluctuations

(a) comparison to previous data

(b) comparison between smooth and rough data
significantly higher turbulence levels. Compared to the present profiles, the data obtained by [AJL] are in good agreement in the inner region but are lower in the outer region. The LDA data of [SG] show persistently higher levels in the inner region but their peak value is located closer to the wall.

In inner coordinates, Figure 6.12b shows a good agreement between the present data and the profile obtained by [KEP]. The value of the inner peak and the corresponding location were found to be $u_{max}^+ = 3$ and $y^+ = 14 - 15$, in good agreement with the boundary layer data reported in Chapters 4 and 5. It is concluded from Figure 6.12b that distribution of streamwise turbulence intensity in the inner region is similar for both a turbulent wall jet and boundary layer.

Figure 6.12c compares the present smooth and rough wall data in outer coordinates. The smooth and rough wall profiles are similar in the outer part of the flow. The values of the outer peak and their wall-normal locations at similar axial locations appear to be independent of surface conditions. In order to examine any possible effect of surface roughness on the inner region, the smooth and rough wall profiles are shown in Figure 6.12d for $y/y_{1/2} < 0.25$. It is observed that surface roughness increases the turbulence intensity in comparison to the smooth wall data.

The wall-normal component of turbulence fluctuations is compared to the data reported by [KEP] and [AJL] in Figure 6.13a using outer scaling. In contrast to the streamwise fluctuation and Reynolds shear stress, the vertical turbulence fluctuation decreases monotonically from its maximum value, i.e. at $y/y_{1/2} = 0.7 - 0.9$, towards the
wall. The present data are consistently higher than the earlier measurements over the entire depth of flow. The peak values for the present profiles are about 0.17 to 0.19, compared to typical values of 0.18 to 0.24 reported for a turbulent free jet (Ramaprian and Chandrasekhara, 1985). It should be noted from [KEP]'s profile that the turbulence fluctuation goes to zero rapidly in a thin region close to the wall. The LDA system could not allow data acquisition very close to the wall. The data obtained closest to the wall in the present tests are substantially higher than those obtained by [KEP]. This may suggest that the high background turbulence levels observed in the outer layer penetrate deeper into the wall region than indicated by the streamwise turbulence intensity.

A comparison between the present smooth and rough data at x/b = 50 is shown in Figure 6.13b. The relatively higher values observed for Test E-SG2 in the near-wall region may be due to a severe encroachment of the high turbulence levels close to the free surface. With regard to possible roughness effects, it appears reasonable to compare Test E-SM1, E-SM2, and E-SG1, all of which have similar turbulence levels in the outer region. In contrast to the observation made for the streamwise turbulence intensity, these profiles do not show any systematic roughness effects. Since data could not be obtained very close to the wall as was obtained for the streamwise component, it is likely that the region over which roughness effect dominates was not captured. It should be recalled, however, that for the boundary layer data reported in Chapter 5, wall roughness effects are noticed at farther distances from the wall in the case of the vertical component than for the streamwise turbulence intensity. It is important to note that there are two competing effects modifying the structure of the flow: 1) wall roughness which modifies the flow away from the wall, and 2) high turbulence levels whose influence is from the
more energetic outer region and towards the near-wall region. The interface between the inner and outer layers may dampen these effects to different extents. In connection with the data shown in Figure 6.13a, it seems reasonable to speculate that the effects of high turbulence levels dominate down to the near-wall region. This may not always be the case since the roughness effect of the sand grain used in this study is minimal. In this regard, it will be of interest to investigate interaction between elevated freestream turbulence and surfaces with significantly greater roughness effects or lower freestream turbulence than considered here.

Figure 6.14a shows a comparison between the present Reynolds shear stress profiles and data obtained by [KEP], [AJL] and [SG]. For \( y/y_{1/2} < 0.5 \), all the profiles, both present and previous, collapse to within measurement uncertainties. Important differences are, however, observed among the different sets of data in the outer region. The outer peak values for the present profiles are higher than those reported by [KEP] and [AJL] but are comparable to the values obtained by [SG]. For the data reported by [AJL], the outer peak is located at \( y/y_{1/2} = 0.66 \), while the present data and the other LDA data (i.e. [KEP] and [SG]) indicate a peak at \( y/y_{1/2} = 0.8 - 0.9 \). For a turbulent free plane jet, the data compiled by Ramaprian and Chandrasekhar (1985) indicated peak values that vary from 0.02 to 0.026, which are higher than observed in the present measurements. Compared to the smooth data, Figure 6.14b shows no important sensitivity to wall conditions.
Fig. 6.14: Distributions of Reynolds shear stress
(a) present and previous smooth wall data
(b) present smooth and rough wall data
6.3.3 Triple Correlation

The distributions of the following triple correlation: \(<u^3>\), \(<u^2v>\), \(<v^3>\) and \(<v^2u>\) normalized by \(U_m^3\) for Tests E-SM2 (\(x/b = 30, 50\)) and E-SG2 (\(x/b = 50\)) are shown in Figure 6.15a. The trends observed here are qualitatively similar to the hot-wire data reported by Irwin (1973) and Dakos et al. (1984). In contrast to the observation made for the boundary layer data in Chapter 5, the wall jet profiles do not show any systematic dependence on surface conditions. In the outer region, the profiles pass through zero at \(0.80 \leq y/y_{1/2} \leq 0.95\) which is close to the location at which the maximum Reynolds stresses occurred (Figure 6.14). The locations of zero crossing in the measurement reported by Irwin (1973) and Dakos et al. (1984) are \(0.7 \leq y/y_{1/2} \leq 0.8\) and \(0.9 \leq y/y_{1/2} \leq 0.97\). Each profile shows an inner and outer peak.

6.3.4 Energy Budgets

In computing the energy budgets for the wall jets, the approximations and assumptions made in Chapter 5 for the turbulent boundary layers are applied. Measurements of turbulence energy budgets for wall jets are scarce. Furthermore, there are no DNS data for a turbulent wall jet so that not all the assumptions and approximations implied in the following discussion can be justified. These comments notwithstanding, the subsequent discussion provides some insight into the turbulence energy budgets. As indicated in Chapter 5, the use of experimental data and their derivatives may give rise to significant errors so that curve fits to data points and their derivatives are used here. In order to show the quality of curve fits used in the subsequent analysis, experimental data and their corresponding fits for two sets of data are shown in Figure 6.16. Figures 6.16a and
Fig. 6.15: Triple correlation (normalized by $U_m^3$)
Fig. 4.16: Mean and turbulence data and corresponding curve fits
(a) mean (b) turbulence data for Test E-SM2 x = 30b
(c) mean (d) turbulence data for Test E-SG2 x = 50b
6.16b, respectively, show the mean and Reynolds stresses for Test E-SM2 (x = 30b) while Figures 6.16c and 6.16d show the corresponding distributions for Test E-SG2 (x = 50b). An assessment of goodness-of-fit using $R^2$ and chi-square analysis showed that the curves describe the experimental data reasonably well over most of the flow.

Figure 6.17 plots the sum of $<u^2v>$ and $<v^3>$, which are the major contributing correlations to turbulence diffusion in the turbulence kinetic energy transport equation. The corresponding best fit to each set of data is also shown. It is clear that energy is transported away from the two peaks. The dip between the inner and outer peaks occurred at $y/y_{1/2} = 0.3 - 0.4$. The differences between the smooth and rough wall data are probably due to the higher turbulence levels observed for the rough data (Fig. 6.14b, for example) rather than wall roughness effects.

Distributions of energy production caused by the shear stress $<uv>\partial U/\partial y$ and vertical normal stress $v^2\partial V/\partial y$ are shown in Figure 6.18a. The profiles of $<uv>\partial U/\partial y$ reported by Eriksson et al. (1999) [EKP] is also shown for comparison. In each case, two peaks can be inferred for the production caused by shear stress. The inner peak is relatively higher than the outer peak. In each case the outer peak occurred in the vicinity of maximum shear stress, i.e. $y/y_{1/2} = 0.8 - 0.9$. Over most of the flow, the vertical stress production is small compared to that caused by shear stress. The production caused by the streamwise normal stress $u^2\partial U/\partial x$ nearly balances the vertical contribution except close to the wall so that the total production by normal stresses does not make any important contribution to turbulence energy production except close to the wall.
Fig. 6.17: Distributions of $(\langle v^3 \rangle - \langle u^2 v \rangle)U_m^{-3}$ for smooth and rough wall data
Fig. 6.18: Turbulence kinetic energy budget
(a) production by normal and shear stress
(b) advection and turbulence diffusion
Production caused by shear stress changes sign (i.e. becomes slightly negative) in the near-wall region but the total production does not change sign owing to the contribution from the normal stresses. Since the mean velocity profiles for the present data and the data obtained by [KEP] collapse (Figure 6.10a) and the Reynolds shear stresses are similar for $y/y_{1/2} < 0.5$, the good agreement between the present values of $<uv>\partial U/\partial y$ and the corresponding profile for [EKP] is not surprising. The higher values in the outer region for the present sets of data are consistent with the trends observed for the shear stress in Figure 6.14a. The peak values for the present profiles are about 0.13 - 0.15 which are about 10 percent higher than reported by [EKP], but are comparable to the free plane jet data reported by (Ramaprian and Chandrasekhara, 1985).

Distributions of turbulence diffusion and advection by the vertical component of the mean velocity (Figure 6.18b) are qualitatively similar to those reported by Irwin (1973). A significant amount of turbulence is diffused outward from the inner peak and inward from the outer peak. The present diffusion terms at $y/y_{1/2} = 0.1 - 0.2$ are several times higher than the data reported by Irwin (1973). This and the high values for $y/y_{1/2} > 1.8$ are most likely due to the scatter observed in Figure 6.17. Data in this range may not be reliable.

The dissipation term was approximated by the net imbalance. The values close to the wall are implausible mainly due to the high values of the diffusion terms discussed in the previous paragraph. A comparison among the various energy budgets for Test E-SM2 ($x = 50b$) and the wall jet data of Irwin (1973) as well as the plane free jet
measurements reported by Bradbury (1965) is shown in Figure 6.19. It is striking to note that with the exception of the advection term, the present profiles fall within the envelop of the corresponding wall jet and free jet data obtained by Irwin (1973) and Bradbury (1965), respectively. This is consistent with the spread rate observed in the present study the values of which are intermediate to typical values of 0.073 and 0.1 reported for wall jet and free jets, respectively.

6.3.5 Mixing Length and Eddy Viscosity

Distributions of mixing length and eddy viscosity are shown for Test E-SM2 at x/b = 30 and 50 in Figure 6.20. It should be recalled that the mean velocity is maximum in the neighborhood of y/y_{1/2} = 0.15 – 0.17 which may explain the singularity observed in this region. The near-wall data (inset) indicate a region of rapid increase that is followed by a region of near-constant distribution and then a rapid decrease towards zero. The mixing length distribution for a free jet calculated by Bradbury (1965) is also shown in Figure 6.20a. Beyond the location of maximum mean velocity, the present data and calculation show a similar trend. For 0.5 ≤ y/y_{1/2} ≤ 2, the distributions are nearly constant and similar.

Figure 6.20b shows the eddy viscosity from the present measurements as well as the hot-wire data of Bradbury (1965) and LDA measurements reported by Ramaprian and Chandrasekhara (1985). The wall jet and free jet data are comparable in the outer region.
Fig. 6.19: Comparison between present data and previous plane wall and free jets
Fig. 6.20: (a) Distributions of mixing length for wall jet and free jet
(b) Distributions of eddy viscosity for wall jet and free jet
6.4 SUMMARY

Measurements of turbulent wall jets on smooth and rough surfaces in an open channel were obtained using a LDA. Although the present flows are modified by reverse flow and high background turbulence intensity, the turbulence structure is similar to previous studies in which secondary flow effects are minimal. In analyzing the streamwise evolution of the flow, both conventional and scaling laws proposed by Narasimha et al. (1973), Wygnanski et al. (1992) and George et al. (2000) are used.

The results show that the inner layer is relatively thicker for the rough surface but the jet half-width is independent of wall conditions. Similar to earlier findings, application of the conventional scaling law makes the velocity decay and spread rate of the jet half-width Reynolds number dependent. The spread rates observed in this study are considerably higher than reported in earlier investigations. On the other hand, the decay and spread rates do not show any important dependence on Reynolds number when kinematic momentum scaling is adopted. The fact that the spread rate is not altered by surface roughness supports the premise of a previous numerical study (Gu and Bergstrom, 1994) that a wall jet is a complex flow in which the mechanisms of damping are not the same as in a simple turbulent boundary layer. The present study also provides support for a power law decay and growth rates for the mean velocity and jet half-width. Furthermore, the power law constants found in the present study are in good agreement with the values obtained in some earlier measurements and those recommended from the similarity theory proposed by George et al. (2000).
The inner region of a turbulent wall jet is similar to that of a turbulent boundary layer. In contrast to some earlier arguments, a well-defined log law region with universal log law constants was identified. Therefore, the popular Clauser chart technique for skin friction measurements and the use of wall functions to resolve the near wall region are valid, at least in principle. The skin friction coefficients over a rough surface are significantly higher than obtained over a smooth wall. One other important effect of surface roughness on the mean flow is that it displaces the location of maximum velocity farther away from the wall. The streamwise turbulence intensity in the near-wall region is found to be considerably higher for the rough wall than for the smooth surface. These results, in contradiction to the wall similarity hypothesis, show that the effects of surface roughness on the structure of a turbulent wall jet penetrate beyond the roughness sublayer. However, the normal turbulence intensity and shear stress do not show any important sensitivity to surface roughness. Since this is inconsistent with the findings made for turbulent boundary layers (Chapters 4 and 5), it is speculated that the high turbulence levels near the free surface may be making severe encroachment down to the near-wall region.

Comparison to earlier wall jet and free jet investigations show some similarity between the energy budgets. A term by term comparison showed that almost all the energy terms obtained in this study are intermediate to corresponding wall jet and free jet data. This observation and the high values obtained for the Reynolds stress in the outer region may explain the present spread rate being higher than the wall jet data available in the literature and lower than corresponding values obtained in free jet experiments.
CHAPTER 7

SUMMARY, CONCLUSIONS, CONTRIBUTIONS AND FUTURE WORK

In this chapter, a summary and the major conclusions and contributions of the present study are given. Some important implications of the present findings for near-wall turbulence models are also discussed. Finally, recommendations for future work are outlined.

7.1 SUMMARY

A program of study was undertaken to provide further insight into near-wall turbulence structure. Specifically, the effects of wall roughness on an open channel turbulent boundary layer and a turbulent wall jet were investigated. The experiments were conducted using a single and a two-component LDA system. The results were interpreted using the conventional scaling laws and the recent theories proposed for turbulent boundary layers and wall jets. The boundary layer and wall jet experiments and results are summarized in the following sub-sections.

7.1.1 Turbulent Boundary Layers

The boundary layer measurements were obtained on a smooth surface, and surface roughnesses created from three geometrically different roughness elements (i.e. sand grains, perforated plate and wire mesh) so that the specific geometry of wall conditions
on the turbulence structure could be examined. The effects of low Reynolds number on
the turbulence structure were also examined. The data presented in this study include
mean velocity, turbulence intensities, Reynolds stresses and triple correlations, skewness
and flatness factors, as well as distributions of approximate turbulence kinetic energy
budgets, mixing length, and eddy viscosity. The rough wall data were interpreted within
the context of the wall similarity hypothesis, which suggests similarity between the
turbulence structures on both smooth and rough surfaces, except in the roughness
sublayer.

Most of the techniques available for the determination of the wall shear stress were
discussed. In the case of flow over a smooth surface, it was shown that, the useful extent
of the viscous sublayer in determining the wall shear stress could be increased without
sacrificing accuracy, by fitting a fifth order polynomial to the near-wall data. In the case
of the rough wall data, a velocity defect profile was fitted to each data set to determine
the strength of the wake and the skin friction coefficient. In fitting the velocity defect
law, a correlation which did not fix the value of \( \Pi \) implicitly but allowed its value to be
optimized, was found to yield a more consistent and accurate estimate for the skin
friction coefficient than other formulations which fix the value of \( \Pi \). The friction laws
proposed by Barenblatt (1993) and George and Castillo (1997) were also applied to the
smooth and rough wall turbulent boundary layers.

In analyzing the boundary layer data, both conventional scaling laws and recent
theories were considered. Specifically, the classical logarithmic law and the power laws
formulated by Barenblatt (1993) and George and Castillo (1997) were used to model the overlap region of the mean velocity profiles over smooth and rough surfaces. The present study is the first to extend these power laws to rough wall turbulent boundary layers. The values of the friction velocity obtained from the power laws proposed by Barenblatt (1993) and George and Castillo (1997) were compared to those obtained from the near-wall data and velocity profile matching technique. For the turbulence quantities, the friction velocity and scaling laws proposed by George and Castillo (1997) were applied.

Even though the boundary layer in an open channel flow is influenced by the free surface, the results presented in this study showed that many of the flow characteristics, in particular those that pertain to surface roughness, are similar to those observed in a canonical zero pressure gradient boundary layer. With regard to Reynolds number effects on the turbulence structure, the application of different scaling laws gave different conclusions. The use of inner scaling laws showed that similarity for the streamwise turbulence intensity is limited to \( y^+ < 30 \). On the other hand, the scaling law proposed by George and Castillo (1997), i.e. outer scaling laws, suggested similarity in the very near-wall region and the outer layer, but the overlap region showed Reynolds number dependent. The present results also indicated that the effect of surface roughness on the turbulence structure is not confined to the roughness sublayer as implied by the wall similarity hypothesis. Instead, surface roughness increases the turbulence fluctuations, Reynolds stresses, triple correlations and the components of the turbulent kinetic energy budgets. It was also observed that the extent to which the mean and
turbulence quantities are modified by surface roughness depends on the specific geometry of the roughness elements.

7.1.2 Turbulent Wall Jets

The wall jet measurements were obtained on a smooth surface and surface roughness created from sand grains. The data presented in this study include mean velocity, turbulence intensities, Reynolds stresses and triple correlations as well as distributions of approximate turbulence kinetic energy budgets, mixing length and eddy viscosity. The streamwise evolution was analyzed using both conventional scales and scaling laws proposed by Narasimha et al. (1973), Wygnanski et al. (1992) and George et al. (2000).

For the smooth wall data, the friction velocity was independently determined from velocity gradient at the wall or by fitting a fifth order polynomial to the data in the region $y^+ < 15$. This was critical to an independent examination of whether a well-defined log law region with universal constants exists or not. In the case of the rough wall data, the friction velocity was obtained using a Clauser plot technique.

Although the present wall jet measurements were somewhat modified by reverse flow and high background turbulence intensity close to the free surface, the mean velocity and turbulence quantities were found to compare favorably to other reliable data in the literature. The effects of surface roughness appear to modify the mean flow significantly as evidenced in higher skin friction coefficients as well as a thicker inner layer and more rapid velocity decay for flows over a rough surface compared to a smooth surface. With the exception of the streamwise turbulence intensity in which case the near-wall data were found to be considerably higher for the rough wall, none of the
other turbulence statistics showed any important sensitivity to surface roughness. Comparison to earlier wall jet and free jet investigations showed some similarity between the energy budgets. A term by term comparison showed that, except for the advection term, all the energy terms obtained in this study are within the envelop of earlier wall jet and free jet data.

7.2 CONCLUSIONS

The major conclusions of the present study are summarized in the following subsections.

7.2.1 Turbulent Boundary Layers

1. The scaling laws proposed by George and Castillo (1997) are more suitable for examining low Reynolds number effects.

2. The mean profiles showed a systematic Reynolds number effect as evidenced in the systematic variation of the outer wake parameter with Reynolds number. In inner coordinates, the present results indicate Reynolds number similarity for the turbulence intensity in the region $y^+ < 30$. Application of the scaling law proposed by George and Castillo (1997) suggests that Reynolds number effects on the turbulence intensity are confined to the overlap region.

3. The power laws proposed by Barenblatt (1993) and George and Castillo (1997) are excellent alternatives to the logarithmic law profiles for a smooth wall data. In the case of rough wall turbulent boundary layers, the formulation proposed by George and Castillo (1997) is more suitable than the log law and the power law formulated by Barenblatt (1993), both in modeling the velocity data and prediction of skin friction.
4. The effect of surface roughness on the mean velocity profiles extends to the outer edge of the flow and increases the value of the wake parameter \( \Pi \) over the corresponding smooth wall data. The \( \Pi \) values also depend on the specific geometry of the roughness elements.

5. The effects of surface roughness on the turbulence quantities such as the Reynolds stresses, triple correlations, and the distributions of the energy budgets, eddy viscosity and mixing length are not confined to the roughness sublayer as implied by the wall similarity hypothesis. Instead, surface roughness modifies the turbulence structure outside the roughness sublayer in a way that depends on the specific geometry of the roughness elements.

6. The present results show that surface roughness substantially increases the peak values of the wall-normal component of the turbulence fluctuations and the Reynolds shear stress. Furthermore, the stress tensor and the turbulence diffusion term in the turbulence kinetic energy equation are significantly modified by the specific geometry of the roughness elements. These findings suggest that for rough wall turbulence models to be able to predict the transport properties accurately, they must explicitly account for the specific geometry of the roughness elements.

7.2.2 Turbulent Wall Jets

1. The spread of the jet half-width and the decay of the maximum velocity for the turbulent wall jets show distinct Reynolds number dependence when they are scaled using the slot height and exit velocity. However, the spread and decay
rates are independent of Reynolds number when kinematic momentum is used as
the appropriate scaling parameter. This observation supports the use of the
kinematic momentum for scaling the streamwise evolution of a turbulent wall jet.

2. The spread rates for the jet half-width obtained in the present experiments are
significantly higher than the values reported in previous measurements.

3. The skin friction coefficient as well as inner thickness and maximum velocity
decay are larger for rough surfaces compared to a smooth surface. However, the
spread rate for the jet half-width is independent of wall roughness.

4. The present smooth wall data show a striking similarity between a turbulent
boundary layer and a turbulent wall jet in the inner region. The mean velocity
profiles are well modeled by a logarithmic law with universal constants that are
identical to values used in boundary layer analysis. The distributions of mixing
length and eddy viscosity in the outer region are similar to classical turbulent free
jet data in the literature.

5. The similarity between the inner region of a turbulent boundary layer and that of
a wall jet provides evidence for the appropriateness of ‘wall functions’ for
turbulent wall jet calculations.

6. With the exception of the streamwise component of the turbulence fluctuations,
the statistics do not show any important sensitivity of surface roughness. This
observation suggests that the turbulent turbulent wall jet is a complex flow in
which the mechanism of damping is not the same as in a relatively simpler
turbulent boundary layer.

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7.3 CONTRIBUTIONS

The present study provides additional insight into roughness effects on low Reynolds number turbulent boundary layers. The major contributions of this study are summarized as follows:

1. A complete and comprehensive set of rough wall measurements in open channel turbulent boundary layers.
2. The first study to apply the power laws proposed Barenblatt (1993) and George and Castillo (1997) to rough wall turbulent boundary layers.
3. The first comprehensive study of turbulent wall jets on a rough surface.

7.4 RECOMMENDATIONS FOR FUTURE WORK

On the basis of the above conclusions and our current understanding of near-wall turbulent structure, the following recommendations are relevant for future work:

1. The usefulness of the power formulations to model the mean velocity depends on one’s ability to accurately determine the power law constants. In this regard, additional theoretical analysis and refined measurements at higher Reynolds numbers and with larger roughness elements are required to calibrate the power law constants.
2. Application of very high spatial resolution LDA systems and multi-point devices such as PIV would be useful to explore the turbulence structure in the immediate vicinity of the roughness element, and also the free surface region of an open channel flow.
3. Investigation of a turbulent wall jet with varying roughness effects and freestream turbulence intensities will provide further insight into the interaction between the inner and outer layers.
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APPENDIX A

AN OVERVIEW OF A LDA

In this section, an overview of the various components of a LDA system is presented. The parameters and condition that deserve particular considerations for accurate flow measurements are also indicated.

A.1 Laser Source

Laser light has some attractive characteristics that make it very suitable for LDA application. A laser beam is both monochromatic, i.e. its energy is concentrated in extremely narrow bandwidth, and coherent which means all emitted radiation has the same phase, both in space and in time. A laser beam is also highly collimated and has a plane of minimal cross section referred to as the ‘waist’ of the beam. The light intensity across the beam has a Gaussian distribution. The beam diameter is defined as the distance between points where the light intensity has dropped to $e^{-2}$ of the maximum value. In selecting a laser source for application, important consideration is given to power requirements, cost and the light frequency. The light frequency is particularly important because it affects the fringe spacing, the photo-detector quantum efficiency and scattering from particles. The amount of power scattered defines the signal level and the signal-to-noise ratio (SNR). The beam diameter is also an important parameter because it influences the measuring volume dimensions.

A.2 Transmitting Optics and Measuring Volume

The transmitting optics consists of a beam splitter, a Bragg cell and a convergent lens or mirror. It may also contain filters, polarizers, beam path equalizers and a beam expander. For a dual beam system, which is the arrangement used in the present study, an intense, highly collimated light beam from the laser is divided into two coherent beams of equal power by the beam splitter. As is well known, the Doppler frequency is not dependent on the sign of the direction of the velocity. Therefore, a positive and a
negative velocity of the same magnitude will result in the same Doppler shift. To remove directional ambiguity, the laser beams are frequency shifted by Bragg cells. The two coherent beams are transmitted through a focusing lens and directed into the flow. Ideally, the two beams should intersect at the focal point of the lens, i.e. their respective beam waists. At the point of intersection, the two beams give rise to an interference pattern or fringes. The spatial region from which measurements are obtained is essentially the intersection of these beams. This region is referred to as the measuring volume. The measuring volume is defined by the locus of $e^{-2}$ intensity points and is ellipsoidal in shape. The dimensions of the control volume depend on the wavelength $\lambda$ of the laser beam and the optic parameters. The number of fringes and the fringe spacing within the measuring volume depend on the optical parameters and the size of the measuring volume.

A.3 The Receiving Optics and Photo-detectors

When a scattering particle passes through a measuring volume, it scatters or reflects the incident light. The scattered light is collected by a set of receiving lenses and focused onto a photo-detector. The photo-detector utilizes the ‘photoelectric effect’, i.e. the absorption of photons and emission of electrons to convert the optical signal into an electrical signal for processing. The photo-current is subject to several sources of noise: shot noise which may be due to random fluctuations in the rate of collected photons and background illumination, and electronic or thermal noise which may be due to amplification of current within the photo-detector or in external amplifiers. Among the various types of photo-detectors in use currently, photo-multipliers (PM) yield the best signal-to-noise ratio (SNR) because of their nearly noise-free internal amplification (Durst and Sender, 1990). According to the analysis of Durst and Sender (1990), PM is most suitable for the range of velocities considered in the present study. The present system uses a photo-multiplier (PM).

A.4 Signal Processing Systems

The selection of a signal processor for fluid flow measurements depends on the type of signal generated, for example, high or low particle density and also on flow
information desired. The appearance of the photo-detector output signal depends on the collected light intensity, on the number of particles crossing the measuring volume at any one time and also on the scattering characteristics of the particles. At extremely low particle density, the signal consists of a train of pulses corresponding to individual collected particles. In this case, special techniques are required to recover the Doppler frequency. On the other hand, if the particle density is extremely high so that many particles are present in the measuring volume at any time, the Doppler signal is continuous but its phase and amplitude would vary randomly. This randomness introduces an additional error in the Doppler frequency measurement, called ‘ambiguity noise’. If the particle density is sufficiently large to provide quasi-continuous signal but low enough for the measuring volume to contain at most a single scattering particle at any time, the signal received by the photo-detector will consist of a series of ‘bursts’ corresponding to particle crossing. Each burst can be viewed as an amplitude-modulated sinusoidal function of frequency $f_0$. The amplitude modulation depends on the light intensity variation within the measuring volume, while differences between bursts reflect differences between particle sizes and crossing paths. A particle passing through the measuring volume will cross a certain number of fringes per unit time. Using a suitable signal processor, the signals are processed for the determination of the Doppler frequency. If the frequency $f_0$ of fringe crossing is known, the velocity of the particle is given by

$$U = C f_0$$  \hspace{1cm} (A.1)

where $C$ is a calibration factor which depends on the optical parameters. Eqn (A.1) shows a linear velocity-frequency relationship.

A.5 Seeding

Scattering particles are the basic source of the Doppler signal. The particles may typically be 0.1 to 10 $\mu$m in diameter. According to Adrian (1983), the scattering particles have more influence on the quality of the signal than any other component of the system. For example, the signal strength can be increased by $10^2$ to $10^4$ by increasing the particle diameter from several tenths of a micron to several microns. Improvements
of these orders of magnitude are difficult, expensive, or perhaps, impossible to achieve by increasing the laser power or otherwise improving the optical system.

The velocity measured by the LDA system is that of the scattering particle. Therefore only if the scattering particles faithfully follow any changes of the flow velocity can one expect the measurements to yield velocity data that accurately represent the flow velocity. If the scattering particles are too large or if their density is too high then, as a result of inertia, they may not respond to velocity changes sufficiently rapidly. The aerodynamic size of a scattering particle, which is a measure of its ability to faithfully follow the flow, is one of the most important properties of an individual scattering particle. The signal-to-noise-ratio (SNR) that it produces is also important. A high SNR requires that the particle is an effective scatterer. The concentration and uniformity of the particle population also play important roles. Ideally, particles that have the same density as the fluid, large effective area in regard to scattering power, very uniform properties from one particle to the other, easily controlled concentration, and low expense are desirable. For liquid flows, the velocities are usually small and the primary limitation on particle size comes from the settling velocity rather than the ability to follow the flow. In water flows, naturally occurring hydrosols are convenient and often yield satisfactory results.

An ideal system considers a single particle in the measuring volume at any one time. In densely seeded flows, with the receiving optics configured in backscatter mode, validation of Doppler signal from multiple particles within the measuring volume may contaminate the accuracy of flow measurements. This is particularly true if the measuring volume is long. If the particle concentration is low, the streamwise and vertical velocity fluctuations would be independent of the spanwise extent of the LDA measuring volume. Another positive side of relatively low data rate is that it is unlikely to measure multiple particles in one time window.
APPENDIX B
ERRORS IN LDA MEASUREMENTS

In this section, some of the common errors encountered in LDA measurements are discussed. Procedures required to correct or minimize these errors are also outlined. Preliminary experiments conducted to verify some of the errors are also reported and discussed.

B.1 Velocity Bias
A burst mode or individual realization LDA operates on signals generated by single particles passing through the measurement volume and produces measurement of the velocity of the particle while it is in the control volume. During periods of relatively high velocity, more particles are measured per unit time than in periods of relatively low velocity. The arrival rate of the measurable particle is, in general, not statistically independent of the flow velocity which brings them to the measurement volume (Edward, 1987). If the flow statistics are calculated by simply summing the velocities of all the measured particles and dividing by the number of particles, i.e. particle averaging, the statistics may be seriously biased.

In turbulent flows, velocity bias occurs when the particle measurement rate, $f_p$, is correlated to the magnitude of the instantaneous velocity vector $U_i$ at a point in the flow field (McLaughlin and Tideman, 1973). Many corrections and sampling strategies have been proposed to eliminate velocity bias (e.g. McLaughlin and Tideman, 1973; Barnett and Bently, 1974; Buchave, 1975; Stevenson and Thompson, 1982). Attempts have also been made by a number of researchers to experimentally verify some of these analytical studies. The results obtained are inconclusive as to the magnitude and even the existence of velocity bias. These experimental results notwithstanding, many LDA users routinely correct for velocity bias. It should, however, be noted that if bias does not occur, such routine bias corrections could lead to significant errors.
Giel and Barnett (1978) experimentally examined statistical bias in a confined subsonic air jet. They compared velocity parameters obtained by averaging individual realization laser velocimeter data to measurements obtained using pitot-tube and hot-wire probes. It was concluded that no consistent bias exists. Stevenson et al. (1982) reviewed most of the existing experimental studies. They also reported measurements in a rearward facing step at various locations characterized by different levels of turbulence. For low turbulence level (of the order of 1 percent), the results obtained using ensemble average or particle averaging was found to be identical to time average values. At higher turbulence intensity (25 and 35 percent), mean velocity obtained using particle averaging was significantly biased at low particle arrival rates. Adams and Eaton (1988) made measurements in a backward-facing step using LDA, pulsed hot-wire and thermal tuft. They concluded that depending on the particle rate and signal-to-noise ratio, the particle average might not be biased. It was remarked that in such a situation, the use of bias-elimination algorithm such as the 1-D and 2-D McLaughlin-Tideman correction or residence-time weighting scheme would result in 'over-correction' of the bias. In processing their LDA data, Adams and Eaton (1988) used three different sampling schemes including particle averaging. Their results show that the data obtained using particle averaging compared most favorably with those obtained using the thermal tuft.

In view of the above discussions, preliminary experiments were conducted to examine if serious velocity bias exists. The data were analyzed using the following three different sampling algorithms.

1. Unweighted or particle averaging (PA)
2. Residence-time weighting (RT)
3. Inter-arrival time weighting (IT).

The results obtained for one set of experiments are shown in Figure B.1 and B.2. In order to quantify the differences among the three data sets, the standard deviation was computed at each measurement location. Figure B.1a shows the distributions of the
streamwise component of the mean velocity. As this figure and the subsequent plots show, the individual profiles compared with each other reasonably well. It should also be noted that in most cases, the data obtained using particle averaging (PA) lie between those obtained using residence-time weighting (RT) and inter-arrival time weighting (IT). Compared to the freestream velocity \( (U_c) \), the variation among the three sets of data varies from 0.9 percent in the vicinity of the wall to 0.03 percent at the outer edge of the flow. Figure B.1b shows the distributions of the vertical component of the mean velocity, \( V \). The maximum and minimum variations among the data sets are \( 0.026V_{\text{max}} \) close to the wall and \( 0.003V_{\text{max}} \) in the outer region.

Figure B.1c and B.1d show plots of the streamwise \((u)\) and vertical \((v)\) components of the turbulence intensity. Compared to the peak values, the standard deviation among the three sets of data varies from 1.6 percent (close to the wall) to 0.2 percent (near the free surface) for \( u \) and from 1.8 percent (close to the wall) to 0.4 percent (near the free surface) for \( v \).

Figure B.2a and B.2b show distributions of the Reynolds shear stress and stress correlation coefficient, respectively. Close to the wall, the Reynolds shear stress data sets agree to within 4.8 percent of the peak value and 0.1 percent of the peak value in the outer part of the flow. The maximum variation observed in the correlation coefficient was 4.1 percent of the peak value. The streamwise and vertical skewness distributions are shown in Figure B.2c and B.2d, respectively. The standard deviation at each y-location is compared with the Gaussian value of 3. The deviations were generally less than 5 percent and 10 percent, respectively for the streamwise and vertical components. For each turbulence statistics discussed above, the maximum deviation among the three sets of data is comparable to the corresponding statistical uncertainty estimates shown in Appendix C.
Fig. B1: Data processing using different sampling schemes
(a), (b) mean velocity profiles
(c), (d) turbulence fluctuations from various sampling schemes
Fig. B.2: Data processing using different sampling schemes
(a) Reynolds shear stress (b) shear stress correlation
(c), (d) skewness factors
B.2 Multiple Particles in Measuring Volume

In practice more than one particle may be present in the measuring volume so the photo-detector usually receives light scattered from particles distributed throughout the illuminating beams. In densely seeded flows or in a long measuring volume, the probability of the presence of multiple particles within the measuring volume is high. When multiple particles are present in the measuring volume, Doppler signal on the U and V channel may be validated simultaneously but may not come from the same particle. This may cause the Reynolds shear stress to be underestimated.

Johnson and Barlow (1990) investigated the effect of spanwise dimension \( l_z^* = l_z U_f/v \) on two-component LDA measurements in a turbulent boundary layer at \( \text{Re}_\theta = 1440 \). The spanwise dimensions considered were in the range \( 7 \leq l_z^* \leq 44 \) and the sampling rate was set to 25 Hz. The measurements were compared to the DNS results of Spalart at \( \text{Re}_\theta = 1410 \). They concluded that the streamwise component of mean velocity as well as streamwise and vertical components of velocity fluctuations is nearly independent of the spanwise dimension. It was observed that the Reynolds shear stress decreases as \( l_z^* \) increases. The strongest dependence was observed nearest to the wall (\( y^* < 10 \)) where the values obtained using \( l_z^* = 6.7 \) and 43.6 were found to be 30 and 50 percent, respectively, lower than the DNS results of Spalart (1988). At \( y^* = 38 \) and 71, the probe with \( l_z^* = 43.6 \) gave values that were 12 percent lower than obtained from \( l_z^* = 6.7 \). They recommended that accurate measurement of the Reynolds shear stress requires a spanwise extent of the measuring volume to be less than 15 viscous units.

B.3 Gradient Broadening

Due to the finite size of the measuring volume, LDA data are not really point measurements but integrated in space over the measuring volume. Finite volume size may cause large velocity gradients and may also present difficulty in accurately locating the wall (\( y = 0 \)). If there exists a non-negligible mean velocity gradient in the measuring volume, the resulting probability function will be broadened and skewed. As a result, time-averaged turbulence properties, especially in the vicinity of the wall, will show a

The effects of finite measuring volume on measured mean and turbulence quantities were discussed by Durst et al. (1995, 1998) and Eriksson et al. (1999). They developed the following correction formulas

$$U_{i,o} = U_{i,n} + \frac{d_{mv}^2}{32} \left( \frac{d^2 U_{i,n}}{dy^2} \right) + \text{HOT}$$ (B.1)

$$\langle u_i^2 \rangle_o = \langle u_i^2 \rangle_n + \frac{d_{mv}^2}{16} \left( \frac{dU_{i,n}}{dy} \right)^2 + \text{HOT}$$ (B.2)

where $i$ is the $i^{th}$ velocity component; subscript $n$ and $o$ represent the quantity actually measured and the corrected data, respectively; $d_{mv}$ denotes the probe volume in the vertical direction, and HOT = higher order terms. The above expressions show that correction for the mean velocity depends on the second derivative of the mean velocity while the correction for the turbulence intensity is proportional to the gradient of the mean velocity. In view of the linearity of the instantaneous streamwise velocity in the viscous sublayer, it follows from Eqn. (3.3) that the streamwise turbulence intensity when normalized by the local mean velocity has the following limiting behavior

$$\left( \frac{u}{U} \right) = \left( \left( \frac{u}{U} \right)^2 + \frac{d_{mv}^2}{16y^2} \right)^{1/2} \text{ as } y \to 0$$ (B.3)

Eriksson et al. (1999) concluded from their wall jet data ($d_{mv}^- = 1.1$) that outside $y^- = 6$, gradient broadening can be neglected. The near-wall measurements reported by Durst et al. (1995) revealed that the effect of measuring volume on the streamwise turbulence intensity is negligible for $y^+ \geq 2.5$. 

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B.4 Errors due to noise

Erroneous contribution to LDA measurements may be due to electronic noise resulting from signal processing equipment as well as light scattered from small impurities on the solid wall and test windows. Errors due to noise have been treated by Durst et al. (1995) and Eriksson et al. (1999). The near-wall data of Durst et al. (1995) showed that the effect of electronic noise is negligible for $y^+ \geq 2$. Eriksson et al. (1999) made a systematic investigation of system error in their wall jet measurements. They found that the effect of system error on vertical component of turbulence fluctuations is about 10 percent at $y^+ = 4$, becoming larger as the wall is approached. The influence of system noise on the Reynolds shear stress, however, was found to be negligible. Error due to extraneous sources such as impurities on solid walls and test windows are difficult to quantify.

B.5 Non-orthogonality

In LDA measurements, it is a common practice to tilt or pitch the fiber-optic probe towards the wall in order to obtain velocity data closer to the wall (Karlsson et al., 1993, Swain and Schultz, 1999). When the probe is pitched towards the wall at an angle $\beta$, the measured vertical component will be tilted relative to the normal of the wall. This may contaminate the velocity data, especially if $\beta$ is large. Karlsson et al. (1993) discussed these and other effects and developed formulas to correct quantities actually measured. It was shown that for $\beta \leq 3.3^\circ$, the value of the vertical component of turbulence fluctuation is artificially increased by 6 percent at $y^+ = 3$ but the influence of tilt can be neglected for $y^+ \geq 6$. The effects of $\beta$ on the streamwise component of the mean and turbulence intensity were found to be negligible.

In the present study, two sets of preliminary experiments were conducted to study the effects of $\beta$ on the mean velocities and their higher order moments. One of the experiments was conducted in a smooth-wall turbulent boundary layer using a single-component LDA. The tilt angles were in the range $0^\circ \leq \beta \leq 3^\circ$. The results for the mean velocity and turbulence intensity in inner coordinates are shown in Figure B.3a and B.3b, respectively. The data for skewness and flatness are shown in Figure B.3c and
Fig. B.3: Mean and turbulence statistics in a boundary layer at various angles of tilt
(a) mean profiles (b) turbulence intensity (c) skewness (d) flatness factor
B.3d, respectively. It is observed that deviations among profiles obtained at different angles of tilt are within measurement uncertainties. It is therefore concluded that for \( \beta \leq 3^\circ \), the streamwise component of mean velocity, turbulence intensity as well as skewness and flatness factors are independent of \( \beta \).

Two-component measurements in a turbulent wall jet were also made for \( 0^\circ \leq \beta \leq 5^\circ \). The results for the mean velocities (\( U \) and \( V \)) and Reynolds stresses (\( u, v \) and \( <uv> \)) are shown in Figure B.4. With the exception of the vertical component of the velocity fluctuations, deviation among the profiles are within measurement uncertainties. It is therefore concluded that no significant dependence on tilt for \( \beta \leq 5^\circ \).
Fig. B4: (a) Mean velocity profiles at various angles of tilt
(b) Turbulence intensity and shear stress at various angles of tilt
APPENDIX C

UNCERTAINTY ANALYSIS

The uncertainty analysis presented below are based on the methodology outlined by Kline and McClintock (1953) and Moffat (1988). A 95 percent confidence interval is assumed in the following analysis. The main source of error in LDA measurements is the uncertainty in the beam spacing calculation or how accurately the processor can deduce the frequency present in each burst. This, in turn, depends on an accurate determination of the beam-crossing angle (Yanta and Smith, 1973; Schwarz et al., 1999) and the signal-to-noise ratio (Castro, 1986). In addition to the above considerations, the uncertainty in statistical quantities will also depend on both the sample size (N) and rms level.

In the present analysis, consideration is given to the following:

1. Except in the immediate vicinity of a solid wall, a photomultiplier has very high signal-to-noise-ratio (SNR).
2. The Doppler signal is band-pass filtered to remove the pedestal (non-oscillating part) from the signal. The sensitivity to noise is further reduced by the use of a three level detection scheme.
3. Stringent validation levels are used in course of data acquisition to reject spurious data.
4. The fringe bias angle is expected to be minimal by the application of a frequency shift of 40MHz (Durst et al., 1993).

A methodology for estimating uncertainty in LDA measurements was developed by Yanta and Smith (1973) and Schwarz et al. (1999). They derived the following relations for the streamwise and vertical components of the mean velocity, respectively:

\[ \frac{\sigma_u}{U} = \left[ \left( \sigma_o \right)^2 + \frac{1}{N} \left( \frac{u}{U} \right)^2 \right]^{\frac{1}{2}} \]  

(C.1)
\[
\frac{\sigma_v}{U} = \left[ \left( \sigma_o \right)^2 + \frac{1}{N} \left( \frac{v}{U} \right)^2 \right]^{\frac{1}{2}} \tag{C.2}
\]

The corresponding expressions for the streamwise and vertical components of turbulence fluctuations and the Reynolds shear stress are, respectively, given by:

\[
\frac{\sigma_u}{u} = \left[ \left( \sigma_o \right)^2 + \frac{1}{2N} \right]^{\frac{1}{2}} \tag{C.3}
\]

\[
\frac{\sigma_v}{v} = \left[ \left( \sigma_o \right)^2 \left( \frac{\langle uv \rangle}{v^2} \right)^2 + \frac{1}{2N} \right]^{\frac{1}{2}} \tag{C.4}
\]

\[
\frac{\sigma_{\langle uv \rangle}}{\langle uv \rangle} = \left[ \left( \sigma_o \right)^2 \left( 1 + \frac{u^2}{\langle uv \rangle} \right)^2 + \frac{1}{N} \left( \frac{2}{R} \right)^2 \right]^{\frac{1}{2}} \tag{C.5}
\]

where \( \sigma_o \) is the error due to uncertainty in the determination of the beam-crossing angle, \( N \) is the number of samples and \( R \) is the shear stress correlation coefficient.

Following Schwarz et al. (1999), a value of \( \sigma_o = 0.4 \) percent is adopted in the present analysis. Typical estimates of uncertainties for the mean and fluctuating quantities are given in Table C.1 using the test conditions for Test C-SMH. The values summarized in this table are very similar to those reported by Schwarz et al. (1999) in their boundary layer LDA measurements.

<table>
<thead>
<tr>
<th>Region</th>
<th>U (%)</th>
<th>V (%)</th>
<th>u (%)</th>
<th>v (%)</th>
<th>\langle uv \rangle (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near-wall (( y^+ &lt; 15 ))</td>
<td>0.5</td>
<td>0.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overlap (( y^+ \approx 60 ))</td>
<td>0.4</td>
<td>0.4</td>
<td>0.7</td>
<td>0.6</td>
<td>12.0</td>
</tr>
<tr>
<td>Outer (( y^+ \approx 750 ))</td>
<td>0.4</td>
<td>0.4</td>
<td>0.6</td>
<td>0.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table C.1: Typical uncertainty estimates for Test C-SMH

In the inner region of the wall jet, the error estimates are comparable to those reported in Table C.1. However, the uncertainties in U, V and \( \langle uv \rangle \) in the outer edge of the wall jet are considerably higher due to the high local turbulence levels and lower
data rates. Typical uncertainty estimates in the outer region of the wall jet are as follows:
± 2.5 percent for $U$ and $V$; ± 5 to 10 percent for $u$, $v$, and $<uv>$.

<table>
<thead>
<tr>
<th>$y$</th>
<th>± 0.025 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>± 5 mm</td>
</tr>
<tr>
<td>$y_m$</td>
<td>± 10 %</td>
</tr>
<tr>
<td>$y_{1/2}$</td>
<td>± 5 %</td>
</tr>
<tr>
<td>$\theta$</td>
<td>± 5 %</td>
</tr>
<tr>
<td>$U_\tau$</td>
<td>± 2.5 % for a smooth surface</td>
</tr>
<tr>
<td></td>
<td>± 5 - 10 % for a rough surface</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>± 0.05</td>
</tr>
<tr>
<td>Triple products</td>
<td>± 10 %</td>
</tr>
<tr>
<td>$S_u$, $S_v$</td>
<td>± 10 %</td>
</tr>
<tr>
<td>$F_u$, $F_v$</td>
<td>± 15 %</td>
</tr>
<tr>
<td>Energy budgets</td>
<td>± 25 - 30 % for dissipation</td>
</tr>
<tr>
<td></td>
<td>± 15 - 20 % for all other terms</td>
</tr>
<tr>
<td>Mixing length $L$</td>
<td>± 12 - 15 %</td>
</tr>
<tr>
<td>Eddy viscosity $\nu_t$</td>
<td>± 12 - 15 %</td>
</tr>
</tbody>
</table>

Table C.2: Typical uncertainty estimates

It should be pointed out that the uncertainty estimates summarized in Table C.1 do not consider errors due to electronic noise. The signal-to-noise-ratio is expected to decrease as the wall is approached because of the decrease of the velocity and also due to extraneous reflection from the wall. This would in turn increase the uncertainty in the turbulence statistics in the vicinity of the wall. Ching et al. (1995) obtained repeated measurements at a given $y^-$ in the near-wall region. Their results showed that for $y^- < 15$, the uncertainty in $u$ and $v$ are, respectively, ± 4 and 9 percent. Typical estimates for other parameters at 95 percent confidence level are summarized in Table C.2.