QUANTIFYING THE UNCERTAINTY ASSOCIATED WITH LONG TERM MAINTENANCE CONTRACTS

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ABSTRACT

Long term maintenance contracts are emerging as an alternative for state agencies to manage their infrastructure assets. For the owner, long term maintenance contracts establish a deterministic schedule for maintenance costs over a fixed time horizon. It has been documented that contractors are willing to accept the risks associated with long term maintenance contracts when provided with the information necessary to assess the potential risk and the freedom to provide innovative solutions to address these risks.

The objective of this research was to develop a generic framework to quantify the financial risk faced by a contractor in bidding a long term maintenance contract for public sector infrastructure. To accomplish this goal, a methodology was developed to take generic infrastructure asset performance curves, maintenance treatment costs, and minimum performance criteria as inputs to calculate the present value of the expected maintenance costs for a long term maintenance contract. The probability distribution associated with these predicted costs can then be applied by a contractor (in conjunction with their risk tolerance) to establish the appropriate tender price. By adjusting the input parameters, the contractor can determine the sensitivity of the optimal maintenance strategy to model inputs. The sensitivity analysis allows the contractor to determine the inputs that must be controlled to ensure success as well as to identify the areas which could potentially provide the greatest opportunity for savings. The framework developed in this research is a generic mathematical methodology, applicable to all forms of public sector infrastructure. To illustrate its application, a roadway pavement management problem was selected.

The methodology to accomplish the research objective was quite straight forward. The first step in the process was to generate transition probabilities from infrastructure asset performance curves. These transition probabilities provided a mathematical representation of asset deterioration and the effects of maintenance and rehabilitation activities throughout the term of the contract. From the transition probabilities, an optimal maintenance strategy was determined. The optimal maintenance strategy was modelled over a

ten year time horizon (the typical length of a long term maintenance contract). From the ten year model, expected costs, variance, and in turn the risk associated with a project (individual maintenance segments in a contract) were determined. The sum of the expected project costs is equal to the expected total cost of the long term maintenance contract. The variance of expected costs of a long term maintenance contract can be determined in a similar manner. Thus, if the risk associated with individual maintenance segments can be determined, the risk associated with a long term maintenance contract can be determined. A series of sensitivity studies were also included to determine the sensitivity of the optimal maintenance strategy to changes in asset performance, maintenance costs, or performance constraints.

In general, the methodology performed well. It was observed, as would be expected, that reducing the rate of asset deterioration reduced maintenance costs. Similarly, increasing treatment effectiveness resulted in a decrease in overall maintenance costs. The maintenance strategies for each scenario were quite similar. The only real change between scenarios was the frequency with which the treatments were applied.

A limitation to this study was the use of a Markov process to create numeric representations of asset deterioration. The Markov process overestimated early deterioration and underestimated deterioration late in the lifecycle of the asset. It is suggested that a semi-markov model would be better suited to model performance curves with the geometric characteristics of the performance curves included in this research; curves with little or no slope for the first few years of the asset's design life.

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A dissertation requires a significant amount of sacrifice and effort by both the researcher and those around him or her. This dissertation is no different. My wife and daughter have spent a lot of evenings keeping each other company while I toiled away on my research. I look forward to spending with them the evenings and weekends that never seemed available these past few years.

I would be remiss to not acknowledge my parents' contribution to this accomplishment. I am not sure if they should be blamed or thanked. But I do know that they gently pointed me in the direction of engineering with the hope that it was a profession that I would enjoy. Since I can not imagine myself in any other career, they must have provided some good guidance. I would also like to point out that after only one semester of engineering, it was my mother that suggested a Ph.D. in engineering was an achievable goal for myself. It is unfortunate that she was unable to see me accomplish this goal, but her memory provided a significant amount of motivation when completing this dissertation appeared to be an insurmountable task.

I would like to thank Dr. Gordon Sparks for supporting my choice to enter the work force full time and work on my research on weekends and evenings. The support that he provided to make sure that I was able to complete my dissertation will never be forgotten. I am sure he has had better students, but I doubt he has had to supervise any for as long as he supervised me.

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Chapter 1

INTRODUCTION

North American society has a significant investment in its infrastructure. The mature state of public sector infrastructure requires a proactive approach to asset management. An emerging asset management practise is the long term maintenance contract wherein a private sector contractor maintains a public sector asset over a fixed term for a predetermined fixed price. The key benefit of long term maintenance contracts is that it introduces private sector efficiencies and innovations to the maintenance of public sector assets. A secondary benefit is that these contracts effectively transfer performance risk from the public to private sector. Unless the contractor can accurately assess the risk, a significant contingency fee would have to be included as part of the bid price; potentially eliminating any benefits expected to accrue due to private sector involvement.

This chapter further develops these concepts and outlines the research methodology that was followed in developing a methodology to quantify the risk faced by contractors in bidding long term contracts.

1.1 Background

Much of North America's public sector assets like pavement, bridges, and sewer and water systems have reached a mature state. In the past decade, the focus has shifted from new construction to maintaining and rehabilitating existing infrastructure. This new focus has lead to what has been termed "asset management."

The Federal Highway Administration has an excellent working definition for asset management (Bloom, 1999):

> "Asset management is a systematic process of maintaining, upgrading, and operating physical assets cost effectively. It combines engineering principles with sound business practices and economic theory, and it provides tools to facilitate a more organized, logical approach to decision-making. Thus, asset management provides a framework for handling both short- and long-range planning."

The Federal Highway Administration has estimated that the United States' federal highway system is a cumulative one trillion dollar investment (Bloom, 1999). On a per capita basis, this is approximately \$3400US. From a Canadian perspective, the city of Winnipeg recently determined their public works infrastructure has a replacement value of five billion dollars and represents a cumulative investment of fourteen billion dollars (Winnipeg, 2000). On a per capita basis, these amounts would be approximately \$7400CDN and \$17,700CDN respectively.

One component of asset management gaining popularity worldwide is the implementation of long term maintenance contracts for a variety of public sector assets. Under typical long term maintenance contracts public sector asset owners (such as highway departments and city engineering departments) contract out the maintenance of an asset to private sector contractors for a fixed term for some fixed price. A typical long term maintenance tender might include 200 kilometers of highway, made up of ten or more homogeneous sections for a ten year contract period. The winning contractor must meet specified performance goals and is responsible for all maintenance short of catastrophic failure. The total bid for this contract would be in the order of 100 million dollars.

Achieving better value for their infrastructure investment is the primary motivation of public sector owners when they contract out asset maintenance. Long term maintenance contracts provide an environment where efficiency and innovation are rewarded. The contractor's key motivator is profit. The owner will set minimum performance standards that must be met. The contractor is then given the freedom to implement whatever means necessary (in his view) to achieve these standards. Research has shown that under this regimen contractors will use innovative methods or new technology and techniques to provide greater efficiencies and to increase the probability that they achieve the specified standards.(Owen, 2000)

A secondary benefit of long term maintenance contracts and the focus of this research is risk transfer. Clemen (1990) provides a concise definition of risk; it is the chance of monetary loss. In a long term maintenance contract a significant component of the normal risk associated with ownership is transferred by the owner (public sector) to the contractor (private sector). From the owner's perspective maintenance is now a deterministic cost paid at some fixed interval. The only remaining uncertainties are catastrophic failure or contractor bankruptcy; both of which are insurable events.

The contractor's financial risks are largely dictated by the uncertainty associated with the asset's performance.[1] The asset's performance is in turn influenced by environmental conditions, asset utilization, and construction material performance. The contractor is able to control, or at least influence, asset performance. For other uncertain quantities, such as environmental conditions or utilization, he is strictly a bystander. To properly manage all risk, the contractor must be able to quantify the risks associated with the contract. He must also be able to measure his ability to modify or control these risks. Research indicates that unless the contractor is able to adequately quantify contract uncertainty and risk he will not be able to manage these risks.(Gallagher and Mangan, 1998)

Quantifying contract risk allows the contractor to see the range of possible outcomes (and their associated costs). This provides the contractor with a rational, repeatable, and defensible basis for selecting the appropriately sized contingency fee when bidding for the contract. Measuring the contractor's ability to influence risk through innovation, new technology, or efficiencies allows the contractor to strategically allocate resources to either

[1]The contractor's financial risks are also influenced by input costs, but the focus of this research is on the impact that asset performance has on financial risk. Discussion on the influence of input costs will be limited to the sensitivity analysis included in Chapter 7.

increase expected return, reduce risk, or increase asset performance. In effect, this allows the contractor to manage the process.

1.2 Objectives

The objective of this research was to develop a methodology to quantify the uncertainty associated with long term maintenance contracts for public sector infrastructure from the contractor's perspective.

The methodology developed included two new contributions to the field of asset management:

- quantifying the risk associated with long term maintenance contracts, and
- measuring the change to the optimal maintenance strategy due to changes in expected asset performance, uncertainty in asset performance, and unit costs of maintenance and rehabilitation costs over time.

The research also extends the state of the art in generating transition probability matrices from performance curves. The performance curves are bounded to define the range of probable asset performance.

1.3 Scope

The scope of this study was limited to the development of a generic asset management model. This model was applied to a basic pavement management problem for a homogeneous section of highway. This model generated an optimal management strategy based on level of service constraints and asset performance curves. The impact on the optimal strategy due to improved or better technology was illustrated by adjusting the performance curves and developing a new optimal management strategy. Saskatchewan Highways and Transportation provided both historic (maintenance) and subjective (pavement performance) data for this research.

1.4 Methodology

The methodology followed to achieve the research objectives can be broken down into four components.

1.4.1 Generating transition probability matrices

Transition probability matrices were generated from pavement performance curves. The key assumption is that these curves were available from one of three sources: historical data collection, mechanistic models, or expert judgment. The transition probability matrices generated from the performance curves were for routine maintenance. Transition probability matrices for the other treatments were based on routine maintenance values. Once transition probability matrices were developed for all combinations of treatments and distresses, the matrices were aggregated into project level matrices - one matrix for each treatment. The project level matrices were the basis of the linear programming formulation of the pavement management model.

1.4.2 Linear programming formulation

Two linear programming models were generated in this research. The first model (the pilot study) was included for illustrative purposes. The pilot study included three distresses each with three condition states (excellent, good, and poor) and three maintenance treatments. The second model was a full scale model and included six distresses, three condition states (excellent, good, poor) and ten maintenance treatments (one of which was routine maintenance).

1.4.3 Illustrating the impact on an optimal strategy due to changes in model inputs

The structure of a linear programming model is such that there are only three possible model input changes: changes to the coefficients of the objective function (treatment costs), changes to the constraints (performance limits), and changes to the technology coefficients (adjusting the asset performance curves). This research was limited to investigating only the impact the changes in treatment costs and performance curves had on the optimal maintenance policy (and its associated cost). Specifically, the five changes studied were:

- flattening of a performance curve (reduced deterioration rate),
- tightening the bounds on a performance curve (improved quality control),
- simultaneous flattening and tightening of a performance curve,
- adjusting the cost of a maintenance treatment but maintaining its original performance curve, and
- adjusting both the maintenance costs and performance curves.

1.4.4 Quantifying risk associated with a long term maintenance contract

The risk associated with a long term maintenance contract can be quantified by calculating the risk associated with maintaining an individual homogeneous highway section. A long term maintenance contract consists of a portfolio of such highway sections. It will be shown that the risk associated with the long term maintenance contract is the sum of the risks associated with the individual sections.

1.5 Organization of thesis

This thesis has been divided into seven sections.

1.5.1 Introduction

The first chapter serves as an overview of the topic of long term maintenance contracts and how it fits into the general area of infrastructure asset management.

1.5.2 Literature review

The second chapter is a literature review. The objective of this chapter is to provide the reader with a background on the domain of this research and to identify how the research contributes to the state of the art of asset management.

1.5.3 Developing TPMs from performance curves

The third chapter demonstrates the conversion of asset performance curves with confidence intervals to Markovian transition probability matrices (TPM). This chapter begins with a short primer on Markov processes to illustrate the importance of the transition probability matrix to the model. This is followed by the development of the methodology necessary to derive transition probabilities directly from assets receiving only routine maintenance. Transition probabilities for other treatments are derived from the routine maintenance probabilities.

1.5.4 Developing the linear programming formulation of the asset management problem

The fourth chapter discusses the development of the linear programming model used in this research to solve the asset management problem. The chapter starts with a brief introduction to linear programming. The linear programming formulation of the asset management problem is developed next. To better illustrate how this model was developed a basic asset management problem is included. This chapter concludes with a brief discussion on alternative algorithmic approaches to solving a Markovian decision process.

1.5.5 Solving the linear programming model given some typical asset performance curves

Chapter five includes a full scale model of the asset management problem solved as a linear program. Typical performance curve data are used to solve a full scale (project level) asset management problem.

1.5.6 Sensitivity analysis

The sixth chapter introduces the concept of sensitivity analysis within the context of asset management. Asset performance curves are adjusted and their confidence intervals reduced to simulate an improvement in performance or better quality control during the construction process. The impact that these changes have on the model inputs and the optimal tactical decision are illustrated.

1.5.7 Discussion of results, conclusions, recommendations and future research

The last chapter acts as a summary to this research. Results are summarized, discussed, and conclusions are made. The chapter also includes recommendations for areas the author feels would provide the most potential for future research.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

The objective of this research was to develop a methodology to quantify the uncertainty associated with long term maintenance contracts from the contractor's perspective. This ultimate goal was accomplished by building on two key elements. The first element was the development of an asset management model that generated an optimal maintenance and rehabilitation strategy for homogeneous segments of pavement. The second element was the incorporation of predictive pavement performance data from one (or all) of the following potential data sources: mechanistic models, empirical data, or expert opinion. The probabilistic nature of the inputs resulted in a probabilistically based optimal maintenance strategy for each pavement segment. The uncertainty associated with these strategies was used to quantify the overall uncertainty associated with the long term contract. Figure 2.1 illustrates this process and will act as a visual reference for the structure of the following literature review.

2.2 Model development

A long term maintenance contract typically encompasses a series of homogeneous segments of pavement. The uncertainty associated with the costs of a long term maintenance

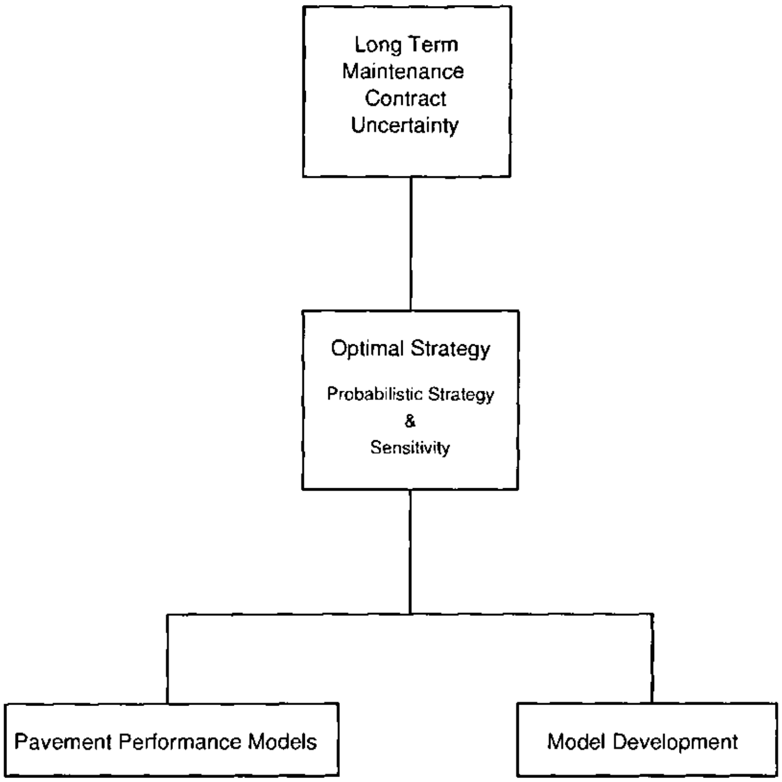

Figure 2.1: Schematic layout of literature review.

contract is a function of the uncertainty associated with the costs of the optimal maintenance strategy of each segment. These optimal strategies are generated from pavement management models. This suggests that pavement management models (and in general asset management models) are cornerstone components to the quantification of long term maintenance contract uncertainty.

There has been a significant amount of work published describing the variety of pavement management models. Many of these models can trace some component of their lineage to Howard's (1960) example of how Markov decision processes can be utilized to optimize the asset replacement problem.

The first published work concerned specifically with optimizing the pavement management decision was by Mahoney et al. (1978). Mahoney et al. developed an integer programming based model to determine an optimal maintenance strategy by maximizing the overall maintenance effectiveness for a set of highway segments under a fixed budget. Several state agencies across the world have adopted either the software (RAMS) or the general approach: state of Texas, Denmark, Australia, New Zealand, and several Canadian provinces (Stein et al., 1987). In practise, Mahoney's integer programming model is applied once the state agency's budget is determined. The budget levels are often determined through the use of a network optimization planning tool such as the one developed by Golabi et al. (1982).

Golabi was commissioned by the Arizona Department of Transportation (ADOT) to develop a pavement management system to better focus their maintenance, rehabilitation, and repair dollars. The end result of this work was the first pavement management system to include a linear optimization model. This was the first application of the Markovian decision process concepts in the area of pavement management. The impact of this work was very substantial (over 14 million dollars saved in its first year of application on a budget of 52 million dollars). It was so innovative that it was awarded the Franz Edelman award in 1982 (the annual award for outstanding application in the field of operations research/management science - as awarded by TIMS/ORSA now known as INFORMS, the Institute for Operations Research and the Management Sciences). The true impact of Golabi's contribution to this domain can be measured by the number of pavement management models that have built upon this model (Golabi et al., 1982).

Since many of the models to be discussed in this chapter are structured similarly to Golabi's work, it only makes sense to take a closer look at this model before reviewing the work of others.

As stated previously, Golabi modelled the pavement management problem as a Markov decision process (MDP) and then solved the model by formulating it as a linear programming problem. To accommodate the variance in highway utilization, pavement design, and environmental factors within the Arizona network, the network was aggregated so segments with similar characteristic were grouped into cohorts. Each cohort was then analyzed individually. Consequently a global optimization across the network did not occur. The solution generated was optimal with respect to each specific cohort. It should be noted that the performance model applied by Golabi is truly a project level formulation; each cohort is modelled as if it is an individual pavement segment. The optimal maintenance strategy is then applied across the network segments that are part of the applicable cohort. As is the case with most pavement management models in use today, identity preservation of individual pavement segments was not possible.

Golabi applied a two phased approach to optimizing ADOT's network. Each phase included similar optimization models. The first model determined the optimal maintenance strategy over the long term. The long term or steady state strategy for the network was then fed into a short term, finite time horizon model. The steady state condition of the network was determined by goals or constraints that were to be achieved given annual budgeting and planning constraints. The ultimate goal of the models was to provide a maintenance strategy where ADOT would know the optimal maintenance action to take when a pavement was in a specific condition state.

Key to each model was the concept of condition state. To be able to determine the necessary maintenance activity or treatment to maintain or repair the pavement, it was necessary to define the existing condition of the asset. In its simplest form an asset's condition could be described as being in excellent, good or poor condition; specific condition state ranges were defined based on the attribute in question. For instance, a pavement would be considered to be in an unacceptable condition with respect to roughness if the roughness was greater than 256 inches per mile or 4.04 meters per kilometers) . Golabi selected to measure pavement condition over four distresses; each with either three or five condition states: roughness (3), cracking (3), change in cracking over a one year period (3), and index to first crack (5). In combination these four distresses and their individual condition states result in an aggregate condition index of $3 \cdot 3 \cdot 3 \cdot 5 = 135$ condition states. Notice the extensive use of cracking as an indicator of pavement condition in this model. If there is a significant failing in Golabi's initial efforts it would be in the representation of pavement condition. Under any maintenance environment there is a relationship between asset condition and maintenance action; the most appropriate cure can not be selected unless the distress can be identified. In the context of pavement management, the distresses selected to represent the asset's condition should relate to the types of maintenance actions that are available. Although each maintenance treatment was not explicitly described, it is doubtful that all seventeen used in the model were designed to specifically address pavement cracking or roughness.

It should also be pointed out that Golabi, and all others who have published their MDP based approaches to pavement management have all elected to optimize their formulation via linear programming. It has been demonstrated that the MDP problem can be solved via algorithms that take less computational effort than the linear programming formulation.(Hastings, 1973) But, given the convenience of implementing off the shelf linear programming software technology, coupled with the ever increasing analytical capability of leading edge linear programming software and desktop computing power, makes custom development of an esoteric algorithm both inconvenient and uneconomical.

Thompson generated a model based closely on Golabi's work for Finland's Road and Waterways Administration.(Thompson et al., 1987) The significance of their work was that they were the first to include user costs as part of the objective function. Before this, pavement management models only minimized agency costs. Unfortunately at the time of publication (1987) Thompson's management system was still under development and the optimal maintenance policy's sensitivity to user costs were not included in Thompson's study.

The Connecticut Department of Transportation was another early developer of a pavement management system based on Golabi's work (Davis and Dine, 1989). Davis's model was based on Golabi's finite time horizon model, and is similar to the Finnish model. This study's model also included user costs as part of its objective function. What made this work significant was the novel approach in representing pavement performance. In previous work, the transition probability matrix describing the pavement's transition from one state to the next was fixed over time. In contrast model Davis and Dine modelled the effects of treatments utilizing two transition probability matrices. One matrix represented the immediate effects on the pavement, and one matrix the pavement's performance after the treatment is completed. From the author's perspective this deconstructionist approach where the treatment and then the subsequent effect of the treatment were modelled makes it much easier for those with little experience with Markov decision processes to better understand or relate to the mathematical representation of the pavement's behaviour. Mathematically the effectiveness is trivial. Once the two transition probability matrices are combined the net effect is a single transition probability matrix describing the pavement's behaviour over the course of a year; treatment and one year's worth of deterioration. As was the case in Thompson's work, Davis did not discuss the actual application of this model nor the source of user cost data.

There has been some discussion on the benefits of approaching the pavement management decision in the typical two phase approach; budget optimization and then network allocation (Gendreau, 1987). Unfortunately there are few models where both the budget optimization and network allocation have been combined into one model. Consequently, the benefits of either approach has not been compared analytically or empirically. The first model to include identity preservation was by Mbwana and Turnquist (1995). Once again, Mbwana and Turnquist's basic model is similar to Golabi's formulation but instead of aggregating similar road segment data, individual segment performance is modelled. Consequently transition probability data must be included for each network segment.

From a data collection perspective this additional level of detail does not increase the amount of necessary data collection. Regardless whether the model is aggregate or disaggregate, the performance data must be collected, stored and analyzed for all components of the network. Aggregation affects model size. In Mbwana's model each segment's maintenance policy is being optimized simultaneously. Thus each segment must have its own performance model expressed in terms of a transition probability matrix. This means that there is a transition probability matrix for each segment and for each treatment that could be applied to each segment. Under this type of model formulation, it would make sense to implement the semi-Markovian formulation by Nesbitt and Sparks (Nesbitt and Sparks, 1987). The semi-Markovian formulation of the pavement management problem is significantly more efficient than the Markovian system. This efficiency is due to the fact that the semi-Markovian formulation requires less data. The Markov model follows the assumption that events (or to be more exact, observations of events) occur at discrete intervals. The dimensionality of the problem is dictated by the number of condition states and the number of time periods in the analysis. In contrast the semi-Markovian formulation models the transition between condition states, as in the Markovian formulation, but the transitions do not occur over fixed time intervals. The assumption in the semi-Markovian formulation is that there is an underlying probabilistic distribution associated with the time required for a transition to occur from one state to the next. The state that the pavement will be in at time t is dictated by this underlying distribution (known as the holding time distribution). The net effect is that the dimensionality of the semi-Markovian formulation is proportional to the number of condition states only. This efficiency is further compounded by the fact that the discrete time nature of the Markov model often requires the use of a dummy states to accommodate the operational realities that are being modelled. The semi-Markov model has no such requirement, further reducing the model's size.

Had Mbwana implemented his system using a semi-Markov formulation the data storage requirements would have been reduced, and model performance would have been improved. In spite of the fact that a more efficient representation was possible, Mbwana did in fact extend the state of the art with this increase in the level of detail included in the network formulation. It should be noted that Mbwana's formulation also included user costs as part of the objective function. Unfortunately the user (and agency) costs included in this model were hypothetical values generated for illustrative purposes rather than a real world application.

The Oklahoma Department of Transportation (ODOT) took a different approach to improve the effectiveness of its network planning model (Chen et al., 1995). Once again the model was similar to Golabi's; roads of similar characteristics and usage were aggregated into cohorts. In constrast this model aggregated each road type being optimized individually, all cohorts were analyzed simultaneously in the same model. As was the case in the Mbwana model this allowed the ODOT to include the interactive effects (and costs) of maintenance across the whole network. It should be noted that this model followed a performance model similar to that of Davis and Dine (1989). Two transition probability matrices were used to represent the effects of treatment; one for the first year after treatment, and one for the pavement's remaining life. This approach is an attempt to compensate for the basic Markovian assumption that there is no memory in the system; future performance is not affected by past experience. There is some validity to this approach, but there are two key problems with this approach. The first problem is that this approach increases the required level of data collection (data for the initial year's performance, as well as data for pavement's performance over the remainder of its life). Data collection for the classical form is onerous enough; Chen's model only makes a difficult task more difficult. The second problem is that there is no guarantee that this additional data is anymore accurate than the data necessary for the classic model. Unfortunately, Chen does not take the opportunity to test the validity or benefits of this new modelling approach.

Validation for Golabi's work was provided by Wang et al. (1993) who reevaluated Golabi's model ten years after its initial development. The group found that the Markov model's prediction of pavement performance compared satisfactorily to actual data. They also noted that the crack change index provided little insight and was consequently dropped from the analysis. This reduced the size of the model formulation and allowed ADOT to move the pavement management system from a mainframe environment to a desktop PC based optimization system.

Not all of the research in this area has focused on a Markov decision process based formulation. The following are two papers that have taken approaches that are borrow from the domain of control theory. The first paper of this nature is by Markow et al. (1987). Markow's work includes an objective function and constraints that are both time varying. Although this work was quite thorough in the issues it covered, little additional insight was provided and no practical application of the work was mentioned.

Ravvirala and Grivas have also developed a non-Markovian approach to the asset management problem (Ravirala and Grivas, 1995). Their approach is called the state increment method. In the Markovian model the pavement is modelled to make changes from state i to state j based on some probability p_{ij} over a fixed time interval. In the state increment method it is assumed that we can select a treatment that will allow the pavement to make the transition from one condition state to another; the only uncertainty is the time necessary for this transition to occur. There are some similarities between the semi-Markov model and the state increment model. The key difference is that in the semi-Markovian formulation the pavement will make a transition from its existing state to one of several possible future states under some probabilistic distribution (i.e. the transition from an excellent state could be to either a good state or a poor state). This uncertainty results in an enumeration of the probability of making the transition between all states and for all treatments. This enumeration is not necessary for the state increment method and thus provides the state increment method with a computational advantage.

The state increment model has been included as part of the New York State Thruway Authority's pavement management system. This indicates that the model can be applied in a real world environment. Given the potential benefits of this approach it is interesting that its application is not more prevalent in the literature. This may be the result of two possible issues. First off, the state increment method is definitely less intuitive than the Markovian model. The second factor may be that most agencies have already developed

Markovian based models and are comfortable with their performance. Regardless of the cause, the net effect is that the state increment method has not been widely adopted.

In general, research into the Markovian decision process model formulation has built upon Golabi's work with ADOT. As confidence in the model has improved, and the computational power available to researchers and state agencies has increased, ever increasingly complex models have been developed and considered. The pavement management models developed in this research were based on Golabi's long term (steady state) formulation (developed in Chapter 4). The focus of this research was to extend the underlying Markovian model in a fashion that has not been previously demonstrated (i.e. quantifying and analyzing risk).

2.3 Pavement performance models

Pavement management systems have three components: a database which includes pavement maintenance, repair, and rehabilitation treatments as well as pavement performance data; an optimization component that allows the agency to optimize or at least compare maintenance strategies across its network; and a front end that facilitates data entry and post optimization analysis and reporting. As illustrated in Figure 2.1 (page 10), this research is concerned with the pavement performance model and the data required by such models. This next sections reviews how data for performance models have historically been generated with regressions techniques, how data is generated for Markovian based systems, and finally research trends within this domain are discussed.

Historically acquisition of new data and application of existing data are the two most contentious issues associated with the application of new pavement management systems. Quite often the necessary data are not available. Often, the cost of collecting the necessary data is significantly than higher expected (OECD, 1987). Under these conditions expert judgement is often used as the data source (Wang et al., 1993). Experts familiar with local pavement conditions will generate a set of pavement performance predictions. This data is used as part of the pavement management system until field data is collected.

The field data is incorporated with the expert data utilizing Bayesian updating (Lu and Madanat, 1994).

Typically pavement performance is measured based on a serviceability index such as the Pavement Condition Index (PCI) (Jackson et al., 1987). In other sitations, data is generated using a mechanistic-empirical based model where the general form of the pavement's performance curve is known based on basic mechanistic properties. Actual model coefficients are then generated through regression analysis of in situ performance data (Rauhut and Gendell, 1987).

Nesbitt and Sparks (1987) probably sum up the problem with regression based models the best. Regression models attempt to assign a deterministic relationship to a probabilistic deterioration mechanism. Pavement deterioration must be represented probabilistically and not deterministically because of the following key factors:

- the mechanistic (physics based) causes of pavement distress are not well known; we can model stress, strain, and deflection but not fatigue, cracking, rutting, etc. (Rauhut and Gendell, 1987),

- pavement is under a continuous barrage of random factors such as traffic loads, utilization, and weather. Even if direct response models existed, the performance over a fixed period would reflect these effects of the uncertain factors, and

- pavement is a heterogeneous material. Over any stretch of significant distance (i.e. 10 metres) one will find that pavement performs differently throughout, even in so called homogeneous control sections.

Regardless of the data's source, it is safe to say that deterministic performance models are limited in their applicability.

By their very nature, optimization models based on the Markovian decision process rely on probabilistic pavement performance models. Some systems such as that developed by Feighan (1989) are based on an aggregate performance index such as the PCI. The key problem with including indices such as PCI is that they are really attempts at objectively measuring serviceability ratings such as user comfort. What is important in a pavement management system is a method to measure pavement performance in such a way that each combination of distress conditions identifies a unique optimal treatment. Aggregate indices can not do this because multiple combinations of distresses can generate the same PCI rating. This is another reason why Golabi's (1982) research was so innovative. He introduced the concept of uniquely identifying pavement condition by distress ratings. From the Arizona Department of Transportation's perspective the crack-centric distresses that Golabi chose have stood the test of time (Wang et al., 1993). Others have selected their own set of distresses (Thompson et al., 1987; SHT, 2000) in an attempt to map more precisely the relationship between pavement condition and maintenance treatments.

Individual distresses should be selected to describe a pavement's condition so that unique maintenance techniques can be identified to best maintain, repair, or rehabilitate the pavement's condition. These distresses are modelled via performance curves. One should note that pavement performance curves are not deterministic entities; they are really just representations of the performance expected to occur over the pavement's lifetime (Cook and Kazakov, 1987). There are performance curves for each distress. These curves act as the base data for a non-linear optimization model. This model generates a transition probability matrix that represent the pavement's performance in the optimization model. This process is covered quite well by Butt et al. (1987). A key difference between the work done by Butt et al. and that which is covered in this research is that Butt generates multiple transition probability matrices from each performance curve. The use of multiple transition probability matrices improves the fit between the performance curves and the transition probability matrix. Unfortunately the size of the model grows proportionally with the number of transition probability matrices for each performance curve. In contrast the Markov process model adopted in this research requires only one transition probability matrix for each performance curve. Consequently, the transition probability models generated in this research underestimate asset conditions in the near term and overestimates its condition in the long term. In other words, if one were to plot the expected condition of the asset over time, the model predicts a pavement in worse condition than one would normally expect to observe in the field, early in the asset's life; late in the asset's life, the model would predict that the pavement would be in better condition than one would expect to observe in the field; results consistent with those of Butt et al.

The direction for future research in this area is highly dependent on which performance modelling technique becomes the standard approach. It is interesting to see that regression techniques continued to appear in the literature (Ben-Akiva and Gopinath, 1995) throughout the last decade in spite of the observations of Nesbitt and Sparks (1987). The semi-Markov model appears to be the logical extension of the typical Markovian based model so prevalent today. But, in spite of its appearance in the literature in the late 1980s, little work has been published supporting the application of the semi-Markov model in the pavement management domain. The state increment method also appears to show promise in the pavement management field. It promises minimal data requirements relative to both the Markovian and semi-Markovian formulations. Unfortunately, its application appears limited to the original research team from Rensselaer Polytechnic Institute. The focus of this research is on the quantifying the risk associated with the optimal maintenance program generated from a pavement management system. The Markovian model was selected as the basis for the asset performance model largely because of its prevalence in the literature, its adoption in industry, as well as the relative ease with which it could be used to develop a basic pavement management system. Had the focus of this research been to extend the state of the art with respect to generating optimal maintenance strategies within a pavement management system then either the semi-Markovian or state increment methods would have been explored. It should be noted that the methodology outlined in this research is not dependent on the modelling technique used to develop an optimal maintenance policy. If either the semi-Markov or state increment models are found to be of use in an applied environment they can be incorporated into the general process described herein.

2.4 Optimal maintenance strategies

Formulating the pavement management problem as a Markov decision process and then solving the problem with linear programming is common throughout the literature. Two issues that are rarely discussed are the probabilistic nature of the optimal maintenance strategy and the sensitivity, or susceptibility to change, given a change in inputs.

Few researchers explicitly address the uncertainty associated with the optimal maintenance strategy in their pavement management model. At best, the stochastic nature is implied given that the policy's cost is an expected value (Golabi et al., 1982). As shown in Chapter 4, the linear programming formulation used to solve most (if not all) Markovian decision process based pavement management systems generate a steady-state probabilistic distribution for the optimal maintenance policy. From this distribution one can easily determine both the expected value as well as the variance associated with this solution. The only researchers to consider the variability associated with the optimal maintenance strategy applied dynamic programming to generate their optimal maintenance policy (O. Omar and Kikukawa, 1993; Feighan et al., 1987). Dynamic programming does not directly generate the probabilistic distribution associated with the optimal solution. To quantify the variance, both research teams (O. Omar and Kikukawa, 1993; Feighan et al., 1987) applied simulation. The inputs were varied over the appropriate probabilistic distributions and the dynamic programming model was solved repeatedly.

Sensitivity is discussed in most pavement models (Golabi et al., 1982; Butt et al., 1987; Chen et al., 1995; Davis and Dine, 1989). Often the investigation is limited to determining the impact changes in budgeting constraints have on the network's condition. The alternative approach is to adjust the network's condition to determine the impact on the necessary budget.

Due to the nature of the linear programming sensitivity analysis can be classified in one of three categories:

- changes in either the objective function (budget),
- constraints (network condition),

- or technology coefficients (pavement performance) (Winston, 1991).

Typically only the first two of the three possible investigations are ever considered; methodically making changes to the technology coefficients is a time consuming process due to the significant number of technology coefficients. For the research included in this thesis, pavement performance models are based on performance curves. These curves provide an efficient form for representing performance. Modification of these curves by changing their slope or tolerances allows us to efficiently adjust the performance models, and provide a simple way to compare the change in pavement performance (change in slope or change in variance) with change in budget.

2.5 Long term maintenance contracts

Research in the domain of long term maintenance contracts is recent. The emergence of this research was due to the trend to privatize long term maintenance for pavement and other types of public sector works throughout the world (Transit, 2001). Unfortunately much of this research was of a subjective nature, and focused largely on the the opportunities and importance of managing risk.

A typical example illustrating the subjective nature of early long term maintenance contract research can be found in (Liddle, 1997). In general, Liddle makes some good observations and suggestions, but there was no empirical data or mathematical models provided to support his conclusions.

Gallagher and Mangan (1998) were the first to directly address risk in the context of long term maintenance contracts. Unfortunately their paper is based on anecdotal evidence, and does not include any methodology to measure or quantify risk. Based on the Gallagher and Mangan's experiences in the Australian pavement industry (Gallagher the public sector and Mangan the private sector) they identified that the key to success in managing the risk associated with long term maintenance contracts is to be able to measure the condition state of the asset. Measuring the state of the asset allows one to understand the pavement's behaviour and thus evaluate its condition effectively so that it can be repaired. An additional benefit is a contractor's performance can be appropriately monitored and evaluated. Observations based on first hand knowledge (such as the quote below) sets this paper apart from the conjecture included in Liddle's work.

> "...contractors have shown a willingness to accept greater risks on larger projects where the margins and opportunities for innovation may be greater and where they have control of materials management and hence quality of the subgrade support, the pavement and sub-soil drainage systems, and the type and design of the pavement structure."

Owen also provides insight based on practical first hand knowledge as an engineer for Transit New Zealand (Owen, 2000). In New Zealand the management of the road network involves the client, the consultant, and the contractor. The client is the owner (the state) of the road network. The consultant is responsible for the achieving performance goals (through both design, and communicating the needs of the client to the pool of potential contractors). The contractor is responsible for the maintenance of the network segments. Owen notes that by giving a contractor the necessary leeway, the contractor will use innovative methods or new technology and techniques to provide greater efficiencies and consequently will achieve the specified end results with a significantly higher probability. He also notes that it is important to highlight areas where difficulties may be encountered and to acquire and present as much historic data as possible. This will allow all contractors to adequately include these risks as part of their tendered contract price. The benefit of this approach is twofold; it provides all parties with a more accurate estimate of the true future costs of the contract, and it eliminates possible misunderstandings at a later date.

The anecdotal evidence put forward by Gallagher and Mangan (1998) as well as by Owen (2000) provide excellent support for the relevance of the research presented in this thesis. The end product of this research is a tool which will provide the public sector owner and the private sector contractor with the tools necessary to understand and measure the uncertainty in expected costs associated with a long term maintenance contract.

Emery was the first researcher to publish multiple papers on long term maintenance contract research (Emery, 2000a,b). In contrast to the previous papers, both of Emery's papers are analytical in nature.

Emery's first paper looks at how the introduction of long term maintenance contracts would impact a country's pavement industry (Emery, 2000a). Emery's examination is based on a simulation model. The model includes both the marketplace (bitumen sales) and the industry (contractors and bitumen sellers). The presence of long term maintenance contracts was simulated by taking a percentage of the total roads out of the annual general market. This resulted in changes in both the bitumen supplier market share as well as contractor market share. The net effect was that long term maintenance contracts will create an environment where larger contractors will have a significant advantage. In addition, utilization of plant equipment by suppliers will be significantly affected.

In general, this paper was not directly pertinent to the research presented in this thesis. It was included in part because it was one of the first analytical papers in the area of long term maintenance contracts. What can be drawn from this research is that long term maintenance contracts will have a significant effect on the economic environment in which they are introduced. This suggests that economic factors such as changes in bitumen costs should be considered in post optimization sensitivity analysis.

Emery's (2000b) second paper evaluated the appropriate warranty period for a long term maintenance contract. The investigation was based on a sequential decision analysis model. His conclusion was that the optimal warranty period should be somewhere between 50 and 60 percent of the pavement's design life. The basic problem with this approach is that the pavement owner (the public) would not obtain the key benefits of a long term maintenance contract. A pavement's performance during the first half of its life is quite linear; in effect it is deterministic. During the second half of its life the deterioration is quite non-linear. Setting the initial warranty period to the first half of the pavement's life would allow contractors bidding on the tender to assume little of the risk associated with the pavement's expected performance. As noted by Gallagher and Mangan (1998), exposure to risk is often what drives innovation.

Under Emery's (2000b) recommendation, a second contract to maintain the pavement's remaining life is necessary. Because of the possible extreme non-linear performance, contractors will include contingency fees to ensure they are fiscally prepared for the significant uncertainty associated with this phase of the asset's life. Given the importance of local knowledge, the contractor that provided the maintenance for the initial maintenance contract will have a significant advantage over the other contractors. The net effect is that a monopoly-like environment exists. One of the benefits of long term maintenance contracts is to instill a competitive environment for pavement management. This will not occur with warranty periods set to half the pavement's design life.

In general it would be safe to say that in the early stages of long term maintenance contract research was more likely to be subjective than objective. As long term maintenance contracts became more commonplace, more researchers became experienced with the practical issues and conjecture was replaced with observed and anecdotal evidence. Once a need for hard analysis was recognized, analytical frameworks were developed and will continue to be developed to quantify the benefits of long term maintenance contracts. The research contained in this thesis is intended to extend the state of the research in this domain.

Chapter 3

GENERATING TRANSITION PROBABILITIES

3.1 Introduction

The primary objective of this chapter is to illustrate how transition probability matrices (TPM) can be generated from pavement performance curves. This chapter begins with an introduction to Markov decision processes which will establish the important role transition probabilities play in the asset performance model developed in this research. The next topic covered in this chapter will be a description of how transition probability matrices can be generated from asset performance curves; curves that describe an asset's condition over time with respect to a specific distress. The chapter concludes with a description of how transition probabilities for each distress-treatment combination are converted into asset level transition probability matrices.

3.2 Markov decision processes

Markov processes are a useful model for studying the state of a system, or the transitions between states over time. From this description it is clear that the key elements of any Markov process are states and transitions.

State is a description of condition, such as hot or cold, or above 20°C or below 20°C. Transition is simply the movement between states, say from hot to cold. Time is used as a point of reference; what is going to happen next, or after t time intervals? Markov processes revolve around fixed time intervals. Generically these time intervals are called epochs. In practise they can be of any length, the only requirement is that the interval length must be fixed throughout the study.

The classic example for a Markov process is a frog sitting in a pond filled with lily pads (Howard, 1960). In this example, each pad in the pond represents a state of the system. If there is a finite number of pads in the pond, the system we are describing is a finite state system. If we were to check the pond every five minutes to observe the frog's location each epoch in the model would be equivalent to five minutes in real time. The likelihood of the the frog making a transition from pad i to pad j is p_{ij}.[1] Figure 3.1 is a simple schematic describing the transition from one state to the next.

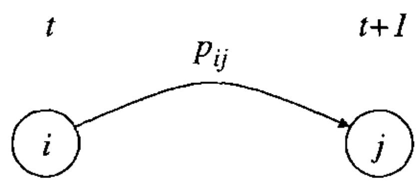

Figure 3.1: The transition between two states.

The transition probabilities p_{ij} are stored in a matrix where the rows represent the present state, and the columns the future state. In the matrix below, the probability of making a transition from state 1 to state 2 is 0.3 (row 1, column 2).

		To		
		1	2	3
	1	0.7	0.3	0
From	2	0	0.6	0.4
	3	0	0	1

[1]For the most part the standard notation in this thesis will be to use a subscript/superscript notation for all probabilities. When this is not possible, an inline notation such as $p(i, j)$ will be used.

Typically the transition probability matrix is denoted as P and the individual elements of the matrix are referenced via the notation p_{ij} where i indicates the row and j indicates the column of the matrix element. The schematic equivalent to the transition matrix shown above can be found in Figure 3.2.

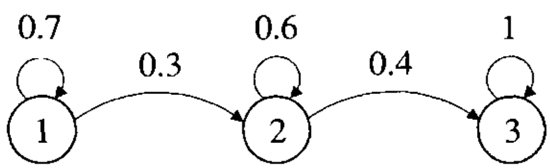

Figure 3.2: Schematic describing the state transitions for a three state model.

A slightly more complex state transition schematic is illustrated in Figure 3.3. In this figure it can be seen that the possible transitions the frog could make over two epochs (time steps). The schematic starts with the frog on some pad i. This schematic illustrates the frog's potential location after one and two time epochs.

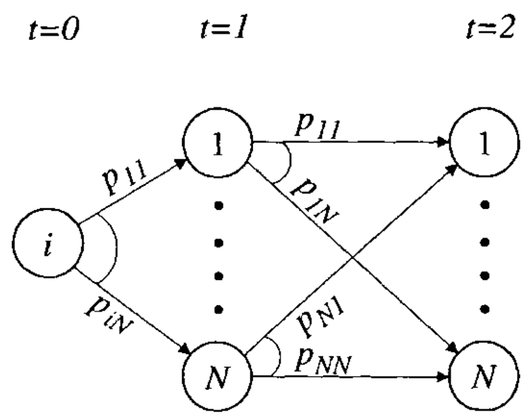

Figure 3.3: The possible transitions over two epochs.

Assuming that the frog never leaves the pond, all that is known for certain is that the frog could be on any pad from pad 1 to pad N. The transition probabilities can be found adjacent to the state transition arcs (p_{i1} and p_{iN}). Since we assume that the frog never leaves the pond the sum of the transition probabilities from pad i to any other pad for any epoch must equal 1:

$$\sum_{j=1}^{N} p_{ij} = 1 \ \forall\, i.$$

The probability of the frog leaping to a specific pad in the immediate future can be calculated from its present location. If $s_i(0)$ describes the likelihood of the frog sitting on pad i initially, then the probability of the frog sitting on pad 1 in the immediate future can be found by the relationship $s_1(1) = p_{i1}s_i(0)$; where p_{i1} is the probability of making the transition from state i to state 1. The general form of this relationship describes the probability of being in state j at time $t+1$ given the probability of being in state i at time t:

$$s_j(t+1) = \sum_{i=1}^{N} p_{ij}s_i(t). \tag{3.1}$$

Equation 3.1 gives the probability of being in a specific state one epoch in the future. This is useful if the frog's position at some time t is known with certainty. In the situation where only the probability distribution associated with the frog's position is known, a state vector is necessary to describe the distribution ($S(t)$). Note that $s_i(t)$ is an element of state vector $S(t)$. Similarly, p_{ij} is an element of the (transition) probability matrix P. The state vector equation predicting the state of the system in the next epoch is

$$S(t+1) = S(t)P. \tag{3.2}$$

The following example further illustrates these concepts. Assume that a pavement can be in one of three condition states: excellent (E), good (G), or poor (P). Each year routine maintenance is applied to the pavement. Historic data shows that the transition probability matrix below describes the pavement's yearly transition between each state

$$\begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.6 & 0.4 \\ 0 & 0 & 1 \end{bmatrix}.$$

These transitions can be represented schematically as shown in Figure 3.4. If the pavement is initially in excellent condition, the likelihood of it being in either excellent, good,

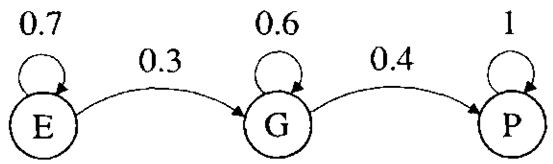

Figure 3.4: Pavement transitions under routine maintenance.

or poor condition after one year can be calculated.

$$S(1) = S(0)P = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.6 & 0.4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.3 & 0 \end{bmatrix}$$

After one year, the pavement will be in excellent condition with a 70 percent probability or it will be in good condition with a 30 percent probability. After two years the pavement's condition will further deteriorate. Once again the probability distribution associated with its condition can be calculated. For small problems such as this, a schematic representation is often useful. Figure 3.5 illustrates the deterioration of the pavement.

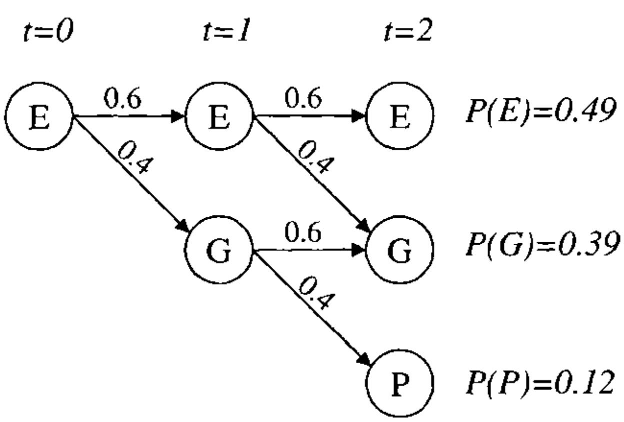

Figure 3.5: State space schematic of pavement deterioration.

$$S(2) = S(1)P = \begin{bmatrix} 0.7 & 0.3 & 0 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.6 & 0.4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.49 & 0.39 & 0.12 \end{bmatrix}$$

Given that the pavement's condition will continue to deteriorate over time, eventually the pavement will reach a poor condition state. The only uncertainty is in how long this deterioration will take. In any asset management environment the asset is maintained, repaired or rehabilitated following some maintenance schedule or policy. In the example problem rehabilitation is not an option. In fact, given the model as shown, there is no opportunity to introduce a change in the pavement's (and in general a system's) performance. However, as will be illustrated below, the Markov decision process provides the necessary tools to model repair and rehabilitation.

As the name implies, the Markov decision process is effectively a Markov process with a decision component. The Markov decision process introduces multiple transition probability matrices to the model. Each transition probability matrix is associated with a decision alternative. In the pavement example routine maintenance, rehabilitation and repair would all be decision alternatives. The objective of a Markov decision process is to determine which alternative to select given the system's present state. For instance, if the pavement is in excellent or good condition routine maintenance may be an appropriate selection. If the pavement is in poor condition a more intense form of maintenance would be necessary. This set of decision alternatives (routine maintenance if the pavement is in excellent or good condition, and a more intense form of maintenance when it is in poor condition) is known as a policy. An optimal policy is the set of decision alternatives that will provide the best outcome over the long term.

In the previous example the pavement was only allowed to deteriorate; no upstream transitions (from a poor state to a good or excellent state) were allowed. To illustrate a basic Markov decision process a second maintenance option (generically labelled repair) will be introduced. The success rate of a repair is dependent on the pavement's condition. If the pavement is in poor condition and it is repaired, the likelihood of it being in excellent, good, or poor condition one year in the future is 0.2, 0.6, and 0.2 respectively. The transitions for the pavement in each initial condition state are represented in the matrix below and schematically in Figure 3.6. The transitions in Figure 3.6 can be represented by

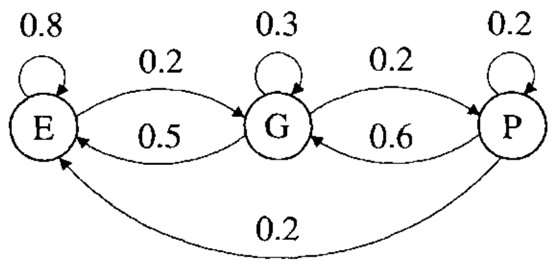

Figure 3.6: Transition probabilities for a repaired pavement

the following transition probability matrix:

$$\begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.5 & 0.3 & 0.2 \\ 0.2 & 0.6 & 0.2 \end{bmatrix}.$$

If the two maintenance options (routine maintenance and repair) cost the same to implement it would only make sense to repair the pavement regardless of its condition. It is unlikely that a more extensive maintenance technique would cost the same as routine maintenance. It becomes apparent then that a Markov decision process must also include a cost component so that maintenance options can be compared based on both the impact on the system and the net costs or benefits associated with their implementation. In this case, there is a cost associated with each maintenance option or treatment. For illustrative purposes assume that the cost of routine maintenance is $\$1/\mathrm{m}^2$ and repair is $\$8/\mathrm{m}^2$. Once costs are included, it is less obvious when either alternative is most appropriate. Needless to say, the complexity of the problem increases with the number of alternatives available.

For now, solving this problem shall be left for Chapter 4. Chapter 4 will introduce the linear programming formulation of the Markov decision process.

3.3 Generating transition probabilities from performance curves

It was established in the previous section that transition probability matrices are an important part of any Markov decision process model. What is yet to be discussed is how these matrices are generated. This section will begin with a brief discussion on performance curves, and will then describe how the beta distribution can be used to estimate probability distributions on pavement performance over the course of the pavement's lifespan. These probability estimates are then used to estimate transition probability matrices. The model used to generate these probabilities is covered in the last part of this section.

3.3.1 Performance curves

Performance curves are a compact way of describing asset performance over time. As illustrated in Figure 3.7 time is measured along the x-axis and the distress is measured along the y-axis. The data for performance curves can come from a variety of sources: empirical data, mechanistic-empirical data, and expert opinion. Typically, performance curves only trace out the asset's expected performance. Note that the use of "expected" is in the colloquial sense and not the statistical sense. Colloquially, an "expected" event is the one that will occur the most frequently; statistically this is the mode. In a statistical sense, an "expected" value is equivalent to the mean.

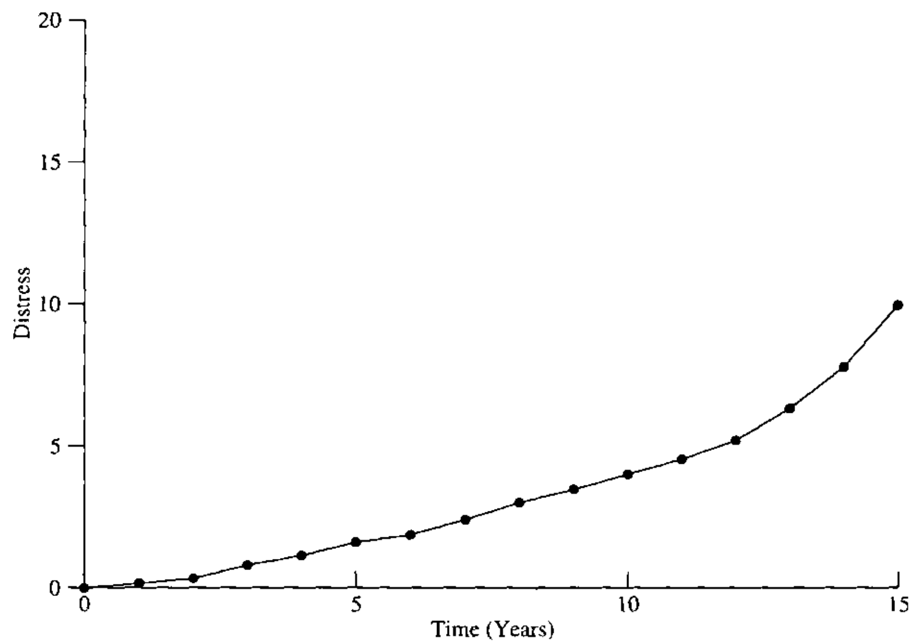

Figure 3.7: A typical asset performance curve.

Performance curves do not describe a certain event; they are estimates of future behaviour. Given the inherent uncertainty associated with a typical performance curve, it is only reasonable that performance bounds should be included. These bounds provide an envelope for reasonably expected performance. For the purposes of this thesis, the curve that is typically drawn to represent pavement performance is labelled the nominal curve. This curve traces the mode of the implied performance distributions over time. An upper bound curve (UB) provides the envelope for the worst possible performance one would expect from the asset. Conversely a lower bound (LB) performance curve defines the asset's best possible performance over its lifespan. Figure 3.8 illustrates these three curves for a pavement performance graph describing rutting.

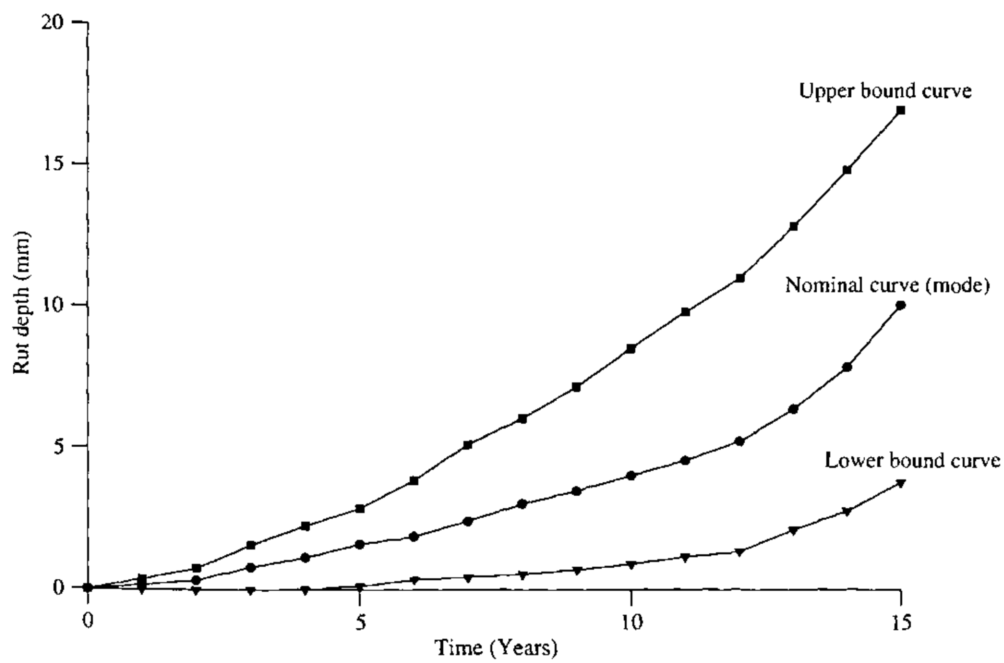

Figure 3.8: A pavement performance curve with performance bounds.

3.3.2 The beta distribution

There is uncertainty associated with any pavement performance curve. The upper and lower bound curves provide estimates of the pavement performance limits but they do not explicitly define the uncertainty associated with the asset's performance over its lifespan. Taking sections (or slices of the distribution) at yearly intervals throughout the lifespan of the asset generates a series of probability distributions as illustrated in Figure 3.9. This section will address how the beta distribution can be used to represent these probability distributions using only the lower bound, nominal, and upper bound performance curves.

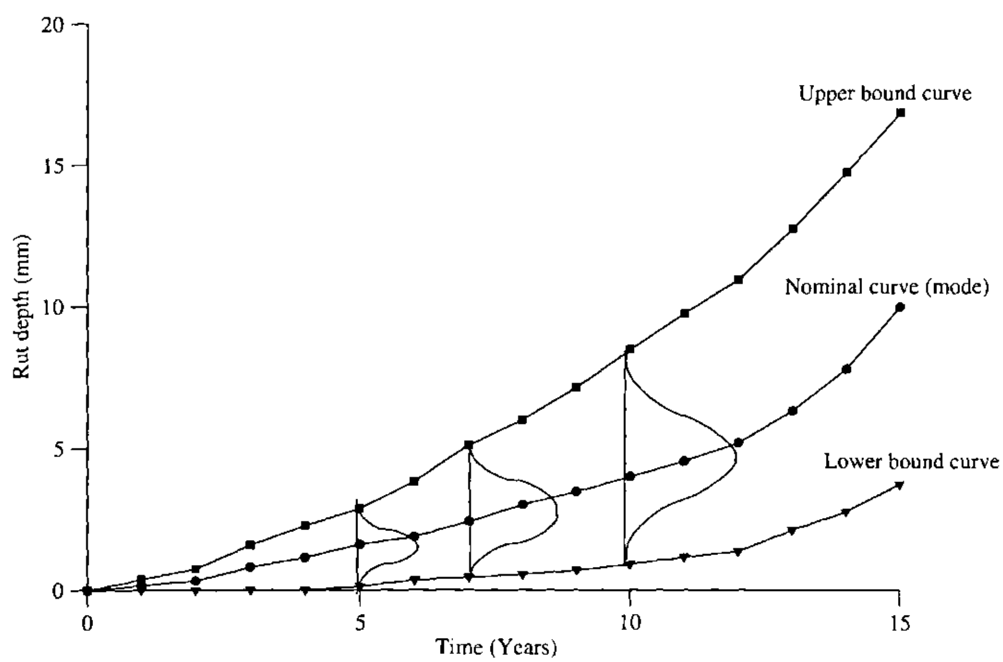

Figure 3.9: A pavement performance curve with implied probability distributions.

As is illustrated in Figure 3.9, there are implied probability distributions at each point in the pavement's lifespan. The first step in determining these probability distributions is to select a set of discrete time steps to provide a basis for analyzing the pavement performance curve. For this problem the natural interval is a one year period. At each interval there will be three pieces of information about the asset: lower bound performance, nominal performance, and upper bound performance. The objective is to generate a probability distribution based solely on these three pieces of information.

The beta distribution is a distribution that is often used to estimate probability distributions (Holloway, 1979). The beta distribution is selected for two reasons: it can take on a variety of shapes, and it requires only two parameters n and r. The n parameter determines the shape of the distribution. As illustrated in 3.10 the larger n, the taller and narrower the distribution.

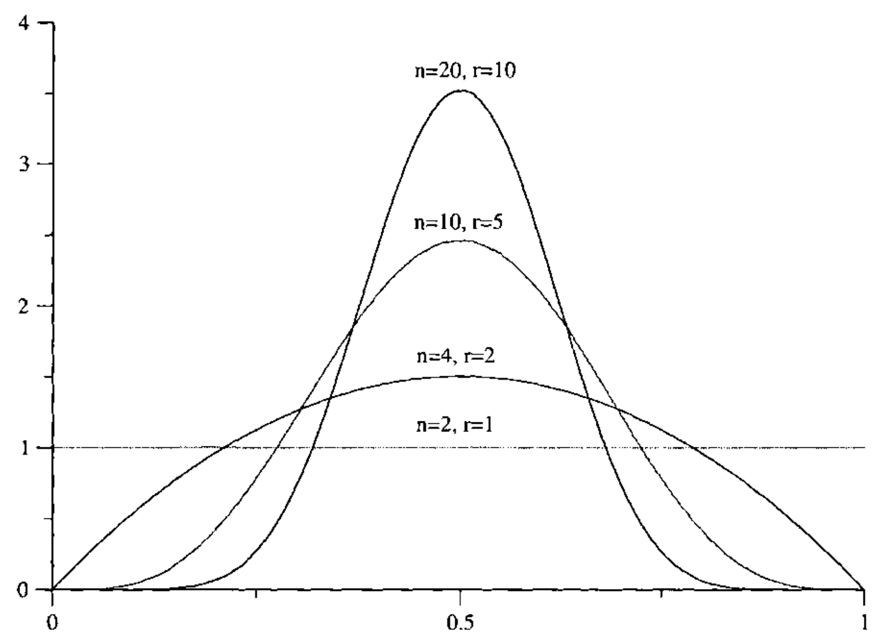

Figure 3.10: Plotting a symmetrical beta distribution for various values of n and r.

Figure 3.11 illustrates how the ratio of n/r determines the skew of the distribution

(Clemen, 1990).

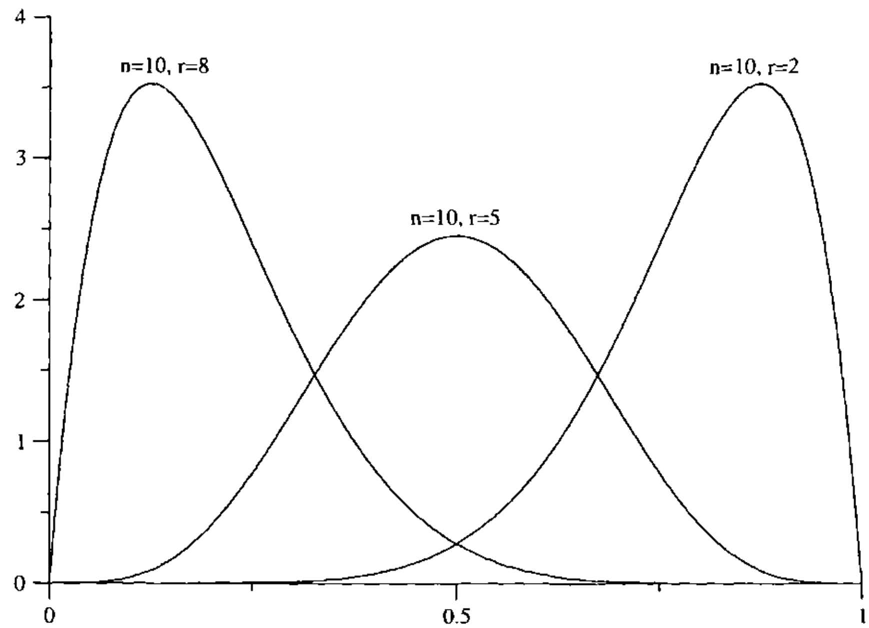

Figure 3.11: Beta distributions with various degrees of skew.

To be able to generate a beta distribution to approximate the probability distribution at each discrete interval in the pavement's lifespan n and r must be determined for the corresponding lower bound, nominal, and upper bound values.

The beta distribution is defined over the range of 0 to 1. This suggests that the range from the lower bound to the upper bound must be normalized. This can be easily done by defining the lower bound to be 0 and the upper bound to be 1. The location of the point on the nominal curve will be defined as q_n and consequently will have a value within the range of 0 to 1.

By definition the lower bound and upper bound define the range of all likely pavement performance at each time interval. Chebyshev's theorem states that regardless of the distribution type at least $1 - 1/k^2$ of the distribution lies within k standard deviations of the mean (Lial et al., 1991). This means that $1 - 1/3^2 = 8/9$ of a distribution would lie within 3 standard deviations from the mean. This suggests that since the normalized space from the lower bound to the upper bound has a width of 1 and the probabilistic distribution has a width of $6\sigma_n$ (where σ_n represents a standard deviation in the normalized space) a value for σ_n can be calculated as follows[2]:

$$\begin{aligned} UB - LB &= 6\sigma n \\ 1 &= 6\sigma_n \\ \frac{1}{6} &= \sigma_n. \end{aligned}$$

Consequently the variance in the normalized space can be defined as $\sigma_n^2 = \frac{1}{36}$.

The variance of the beta distribution is

$$\sigma_\beta^2 = \frac{r(n-r)}{n^2(n+1)},$$

and the normalized space has been defined as a beta distribution, $\sigma_n^2 = \sigma_\beta^2$. This in turn yields

$$\frac{1}{36} = \frac{r(n-r)}{n^2(n+1)} \tag{3.3}$$

$$n^2(n+1) = 36r(n-r). \tag{3.4}$$

Unfortunately there are still two unknowns (n and r) and only one equation (Equation 3.4).

As previously stated a point on the nominal curve corresponds to the mode for the asset's performance distribution at that point in time (which has been assumed to be a beta distribution). The mode for a beta distribution as given in terms of r and n is

$$Mode_\beta = \frac{(r-1)}{(n-2)} \tag{3.5}$$

[2]This line of reasoning is similar to the task duration estimates used in the PERT project scheduling method (Taha, 1987).

(Holloway, 1979). By equating the mode of the beta distribution (equation 3.5) to q_n (the point on the nominal curve in the normalized space) the function relating n and r to q_n is

$$q_n = \frac{(r-1)}{(n-2)}. \tag{3.6}$$

To be able to define either n or r in terms of q_n equation 3.6 must be restated in terms of r and q_n and then this new relationship must be substituted into equation 3.4

$$n = \frac{r-1}{q_n} + 2 \tag{3.7}$$

$$(\frac{r-1}{q_n} + 2)^2 \left[(\frac{r-1}{q_n} + 2) + 1\right] = 36r \left[(\frac{r-1}{q_n} + 2) - r\right]. \tag{3.8}$$

The objective was to determine a completely general relationship between r and q_n. This would provide a method to directly calculate r (and in turn n) for any given value of q_n. Unfortunately solving the general solution to equation 3.8 was found to be quite difficult and computational tools such as Maple or Mathcad were required. The general solutions from these applications were impractical; they were extremely long and included both real and imaginary roots. Since a completely general solution could not be found, the alternative approach was to determine the solution numerically.

The difficulty with a numeric solution is that it is not completely general; it must be evaluated on a case by case basis to determine the limitation of its validity. There is also a risk that the solution found is not unique. In this case, multiple solutions were expected since the order of equation 3.8 is greater than one. Thus before accepting a numerically derived solution it must be verified for both feasibility and uniqueness.

The numeric approach selected to determine the roots of equation 3.8 was Newton's method. Newton's method was selected because of its simplicity (and thus ease of implementation) and its performance (it converged quickly for all test cases). Newton's method is essentially a three step process that applies a gradient descent technique. As illustrated in Figure 3.12 there are three basic steps to Newton's method.

1. Select a point on the function $[x_1\ ,\ f(x_1)\ ]$.

2. Determine the slope of the function at this point and project back along this slope to the x-axis to determine the next x value - (x_2).

3. If $f(x_2) = 0$ then the you have found the solution, otherwise go back to the first step.

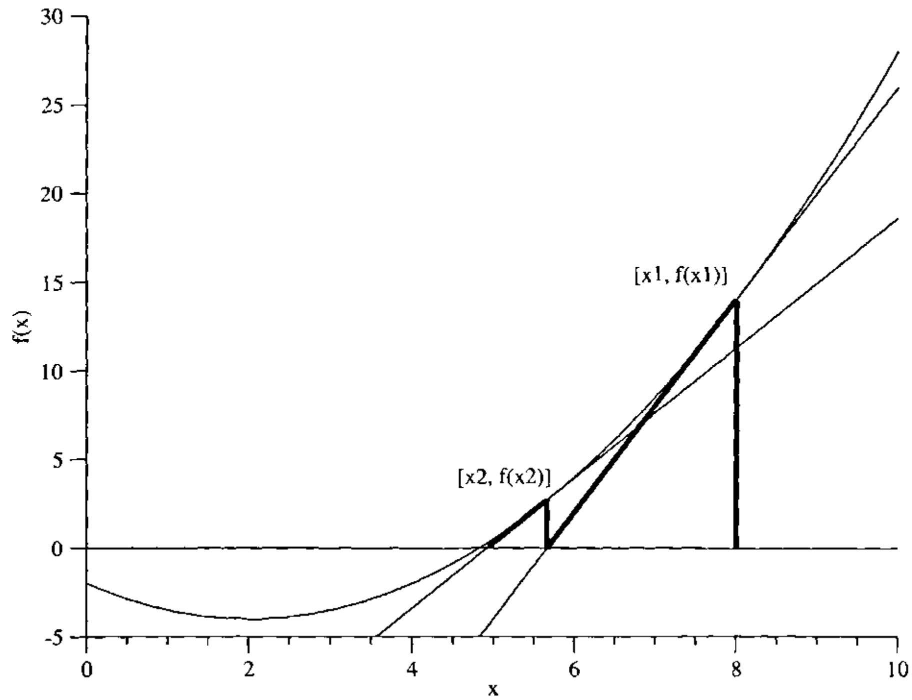

Figure 3.12: Newton's method

To determine the slope at any point along the function Newton's method requires the first derivative of the function. Equation 3.9 illustrates the function and Equation 3.10 the first derivative.

$$f(r) = A^2(A+1) - 36r(A-r) \tag{3.9}$$

$$f'(r) = 2\frac{BC}{q_n} + \frac{B^2}{q_n} - 36A - 72 + 36r - 36r\frac{1}{q_n - 1} \tag{3.10}$$

where

$$\begin{aligned} A &= \frac{r-1}{q_n} + 2 \\ B &= A + 2 \\ C &= A + 3. \end{aligned}$$

Newton's method will determine the roots of $f(r)$. But at this point it was still uncertain whether there were multiple roots within the valid domain of q_n (0 to 1). To determine the number of roots which fall within the range of feasibility, plots of $f(r)$ vs r were generated for a various values of q_n. Over the range of feasible values of r ($r > 1$) it was found that there was only one real root for all values of q_n. Figure 3.13 illustrates how plotting the values of these roots against q_n provided insight into the relationship between q_n and r. Typically $r = f(q_n)$ increases with the value of q_n; once $q_n > 0.687$ r decreases with increases in q_n. Figure 3.13 indicates that by (arbitrarily) selecting a value of 5 for the initial x value the convergence will be to a root that is feasible within the defined constraints. For Newton's method to find an infeasible root, it would have to pass through the feasible root first. Since Newton's method stops at the first root it finds, an infeasible solution will not be found.

3.3.3 Calculating discrete probabilities

The beta distribution is a continuous distribution. Calculating transition probabilities from a performance curve requires discrete probabilities; one probability for each condition state. This implies the necessity of determining discrete probabilities from the continuous beta distribution.

The condition states for a pavement were earlier described qualitatively as excellent, good, and poor. To properly quantify the probability of the pavement being in a specific condition state, discrete distress ranges associated with each qualitatively defined state must be established. For example, if rutting is the distress in question, the ranges shown in Table 3.1 could be selected as the distress levels corresponding to the qualitative condition

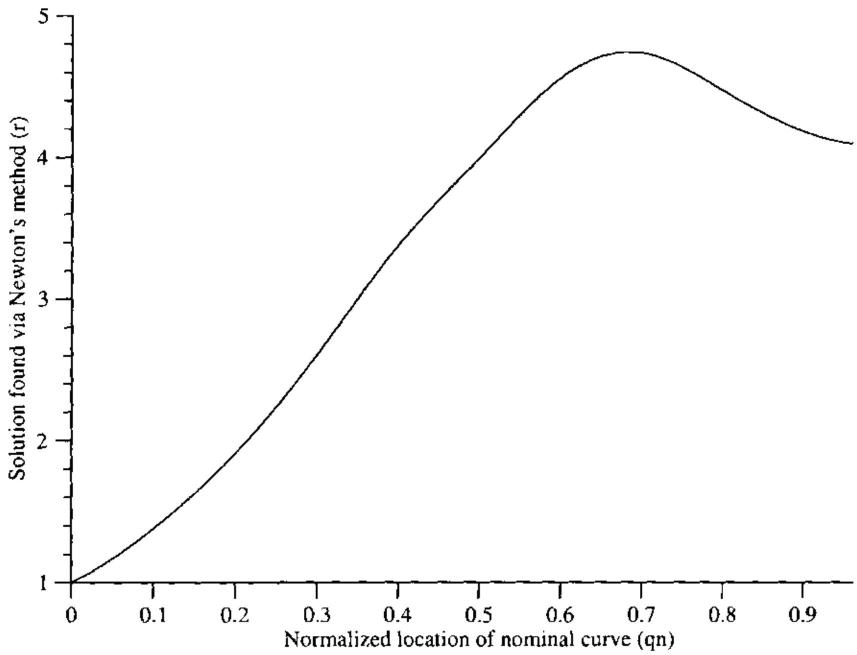

Figure 3.13: Plotting r over q_n.

states. The boundaries between each condition state effectively act as integration limits. Figure 3.14 illustrates how the area under the beta distribution between these bounds determines the discrete probability for each condition state.

Table 3.1: Condition state ranges for pavement rutting.

Condition State	Minimum (mm)	Maximum (mm)
Excellent	0	5
Good	5	11
Poor	11	22.5

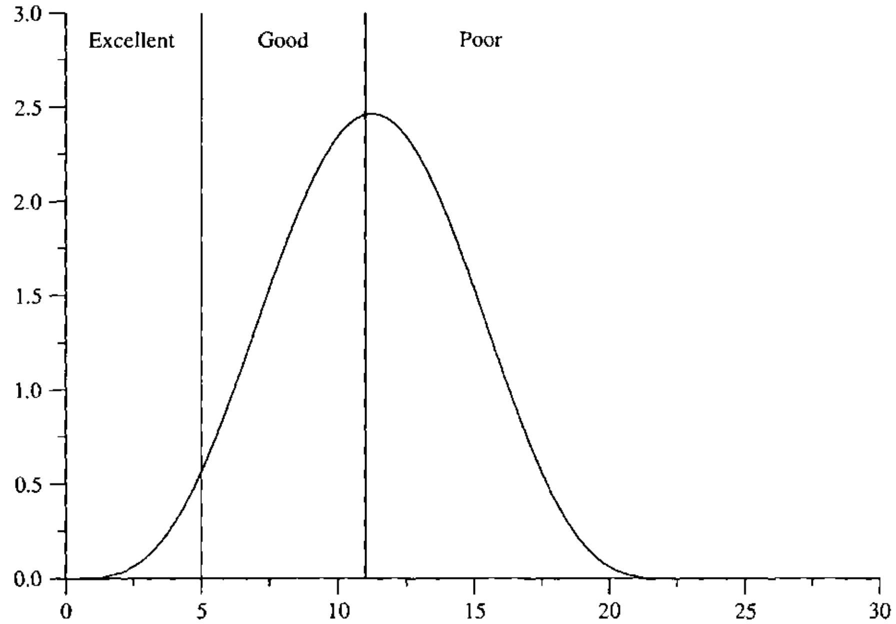

Figure 3.14: Integrating under a pavement performance curve.

The best way to illustrate how to generate the discrete probabilities is to work through an example. Figure 3.15 illustrates a distress curve showing how rutting changes over time. One should note that this set of distress curves includes the condition state boundaries (as described in Table 3.1).

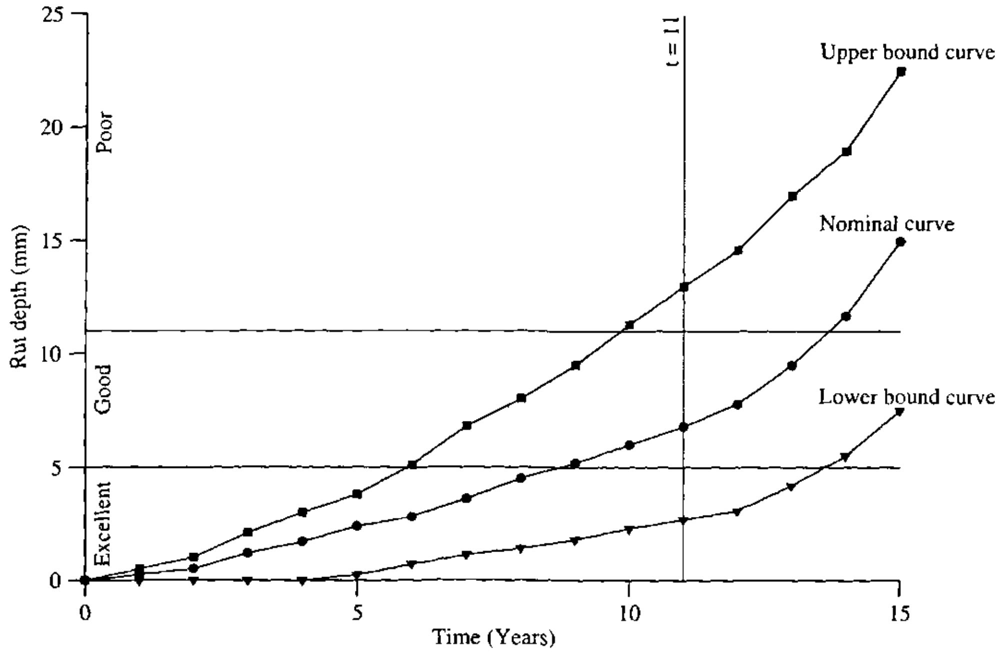

Figure 3.15: Distress curve for example problem - increase in rutting over time.

As an example, time step $t = 11$ will be analyzed. From Figure 3.15 the lower bound, nominal, and upper bound values can be determined. These values can be found in Table 3.2. The first step is to locate the mode (q_n) in the normalized space:

Table 3.2: Distress levels associated with data points.

Data Point	Abbreviation	Rut Depth (mm)
Lower Bound	LB	2.7
Nominal	Nominal	6.8
Upper Bound	UB	13

$$q_n = \frac{Nominal - LB}{UB - LB} = \frac{6.8 - 2.7}{13 - 2.7} = 0.39806.$$

The next calculation is to determine r with Newton's method. When $q_n = 0.39806$ the function and its derivative are:

$$\begin{aligned} f(r) &= 15.855r^3 - 57.825r^2 + 17.842r - 0.1279 \\ f'(r) &= 47.564r^2 - 115.65r + 17.842. \end{aligned}$$

If the initial guess for x_1 is 5 then $f(x_1) = 625.58$. Given that x_1 and $f(x_1)$ are known, it is necessary to project along the tangent to the function back towards the x-axis to determine at where this projection intercepts the x-axis. The slope of the tangent is $f'(x_1) = 628.69$. The equation for the tangent line is:

$$\begin{aligned} y &= mx + b \\ f(x_1) &= f'(x_1)x_1 + b \\ b &= f(x_1) - f'(x_1)x_1 \\ b &= -2517.86 \\ y &= 625.58x - 2517.86. \end{aligned}$$

The tangent line intercepts the x-axis when:

$$\begin{aligned} 0 &= f'(x_1)x_2 + b \\ x_2 &= \frac{-b}{f'(x_1)} \\ x_2 &= 4.0049. \end{aligned}$$

The final step is to check whether $f(x_2) = 0$. If $f(x_2) = 0$ then this is the root when $q_n = 0.39086$. Checking the function shows that $f(x_2) = -23.7592$. Since $f(x_2) \neq 0$ the process must continue until convergence is reached. Table 3.3 summarizes the calculations for finding the root. The solution is $r = 3.306$.

Table 3.3: Summary of Newton's root calculations

Step	x_1	$f(x_1)$	$f'(x_1)$	b	x_2	$f(x_2)$
1	5	625.5767	628.6864	-2517.8551	4.0049	-23.7592
2	4.0049	162.5788	317.5740	-1109.2879	3.4930	-63.4089
3	3.4930	32.6355	194.2064	-645.7285	3.3250	-67.7503
4	3.3250	2.9818	159.1457	-526.1713	3.3062	-68.0075
5	3.3062	0.0349	155.4031	-513.7625	3.3060	-68.0103
6	3.3060	+2.812E-06	155.3584	-513.6147	3.3060	-68.0103
7	3.3060	-1.781E-10	155.3584	-513.6147	3.3060	-68.0103
8	3.3060	+1.170E-13	155.3584	-513.6147	3.3060	-68.0103
9	3.3060	-1.246E-13	155.3584	-513.6147	3.3060	-68.0103
10	3.3060	+1.170E-13	155.3584	-513.6147	3.3060	-68.0103
11	3.3060	-1.246E-13	155.3584	-513.6147	3.3060	-68.0103

Since r and q_n are known n can be calculated:

$$n = \frac{r-1}{q_n} + 2 = \frac{3.3060 - 1}{0.39806} + 2 = 7.7931.$$

Given that we know n and r we can calculate a beta distribution (f_β) for all values over the range $0 < q \leq 1$, where

$$f_\beta(q \mid r, n) = \frac{\Gamma(n)}{\Gamma(r)\Gamma(n-r)} q^{r-1}(1-q)^{n-r-1}.$$

Plotting this function over all values of q results in a continuous probability distribution as illustrated in Figure 3.16.

The final step is to determine the discrete probabilities associated with each condition state (excellent, good, and poor). The values for the boundaries selected for the condition states for the are tabulated in Table 3.4. Thus, if rutting depth is 5 mm or less

Table 3.4: Condition state boundaries for example problem

Condition State	Minimum (mm)	Maximum (mm)
Excellent	0	5
Good	5	11
Poor	11	22.5

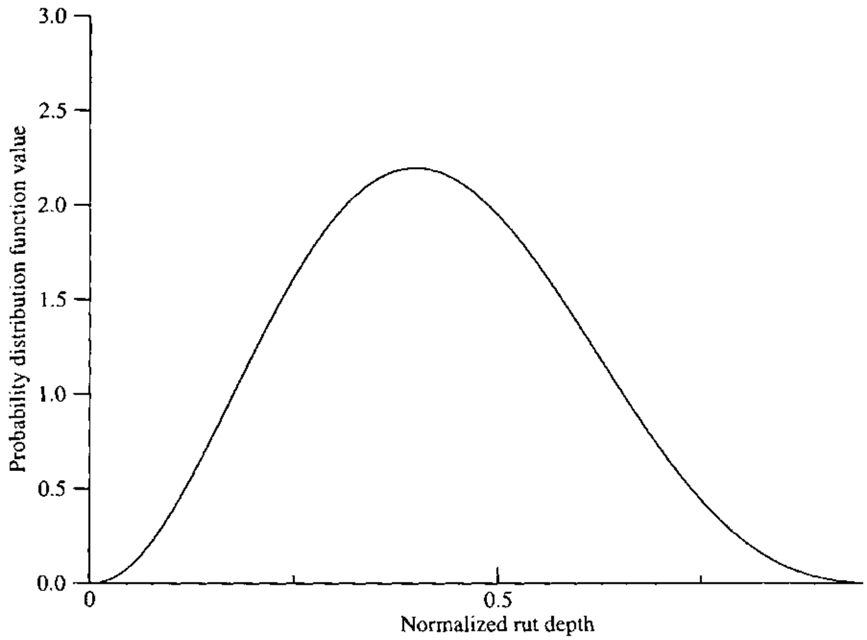

Figure 3.16: Beta distribution for example problem.

then the pavement is in excellent condition with respect to rutting. Figure 3.16 describes the distribution over the normalized space. To calculate the condition state probabilities we must divide the distribution into discrete intervals (see Figure 3.17). Integrating under the curve over the condition state ranges generates discrete probability values of $P(Excellent) = 0.1206$, $P(Good) = 0.8691$, and $P(Poor) = 0.0104$.

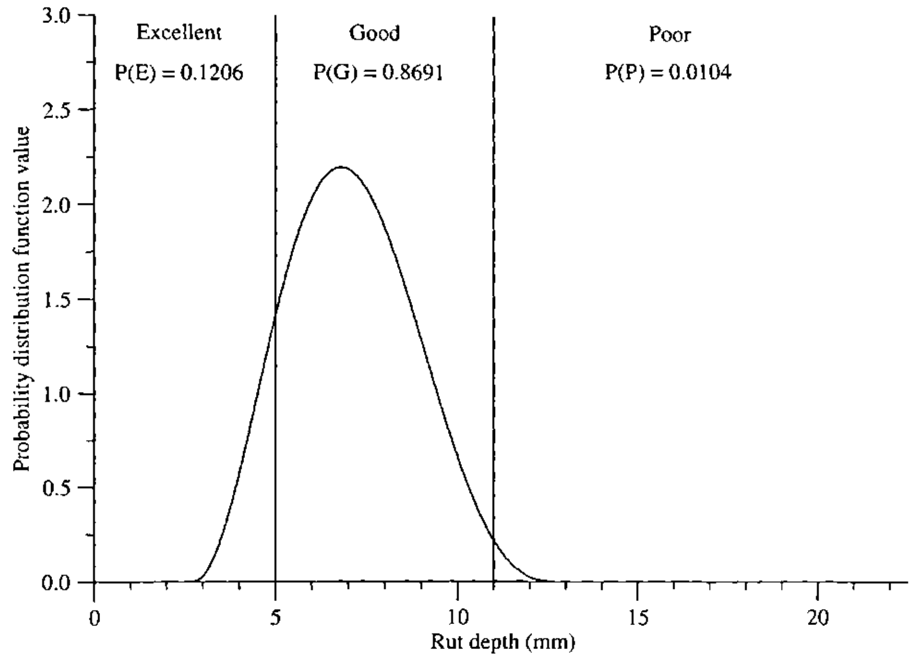

Figure 3.17: Remapping beta distribution to real space.

3.3.4 Determining a representative value for a condition state

The ultimate objective is to turn discrete probabilities from each time step into a transition probability matrix. Accomplishing this requires one additional piece of information, a representative value for each condition state.

Generating transition probability matrices requires an expected condition state value for each time period. In other words, given three condition states and the probability of being in each condition state, what is the average condition of the pavement? The standard form for an expected value calculation is $\Sigma p_i s_i$ where p_i is the discrete probability of being in state i and s_i is the value associated with being in state i. The appropriate value to represent each condition state range has not been discussed. For condition state ranges where the range is clearly defined as in the rutting example illustrated in Figure 3.18 an appropriate approach would be to take a point in the middle of the range. For this example the representative values would be $s_E = (0+5)/2 = 2.5$, $s_G = (5+11)/2 = 8$, and $s_P = (11+22.5)/2 = 16.75$. For situations where the boundaries are not as obvious, as illustrated in Figure 3.19 a different process is necessary.

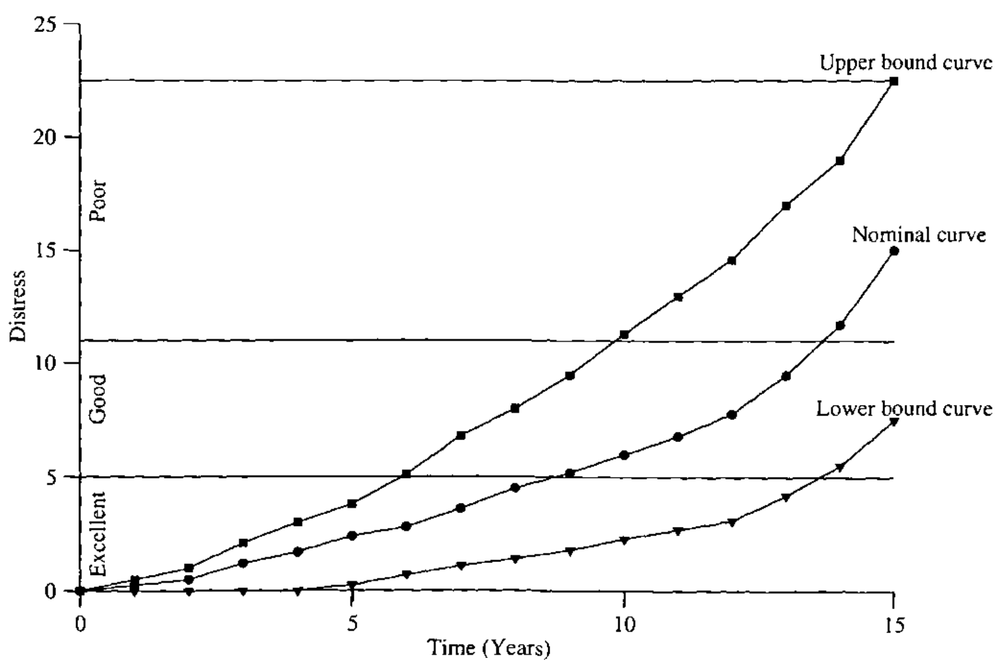

Figure 3.18: Example of bounded condition states

The method developed to determine representative values for unbounded condition states such as those illustrated in Figure 3.19 relies on the concept of duality. Duality

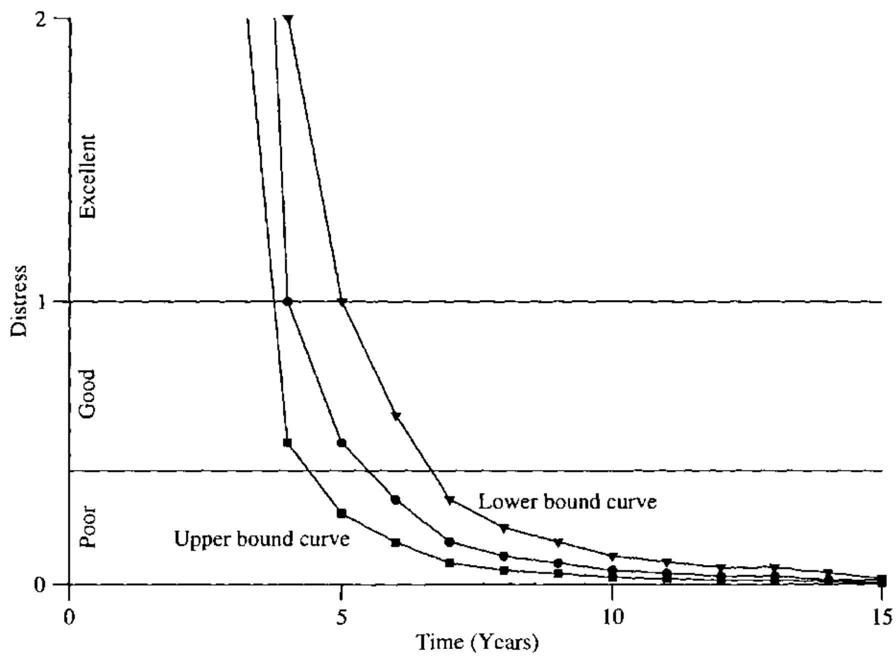

Figure 3.19: Example of an unbounded condition state

exists when there are two ways of representing the same environment. The best pavement example is cracking. When measuring pavement condition with respect to cracking, one approach is to evaluate the average distance between cracks. In a pavement that is crack free the space between cracks is infinite. Over time as the number of cracks increase, the average space between cracks decreases. This phenomenon can be illustrated graphically as in Figure 3.20.An alternative way of measuring condition with respect to cracking is to count the number of cracks over a fixed length of road. This approach is illustrated in Figure 3.21. Although both approaches measure the same distress, the resulting figures are quite different. In Figure 3.20, only the good and poor condition states are finitely bounded; the excellent condition state has no upper limit. A representative value for the good and poor condition states can be readily determined. In Figure 3.21 the excellent and good condition states are cleanly bounded. In the dual space the representative value is $Dual_E = (100 + 0)/2 = 50$, or 50 cracks per kilometer. The equivalent value in the primal space would be an average crack spacing of $1000m/500\ cracks = 200m/crack$. Thus for the situation where a condition state is unbounded, one of two choices must be made; either permanently switch to a dual representation, or use the dual to determine a representative value, and then remap this value to the primal space.

3.3.5 Calculating transition probabilities for routine maintenance

Calculating the transition probability matrix relies on a non-linear programming model. The objective of the model is to determine the transition probability matrix which best approximates a pavement's performance under routine maintenance for a specific distress. The general methodology described herein can be applied to determine a transition probability matrix of any size. The specific implementation applied in this research is based on a three condition state transition probability matrix.

A non-linear programming model is made up of two types of equations; constraints, and the objective function. The constraints define the range of solutions which are applicable to the scenario being modelled. The objective function is an equation that represents an objective criteria from which to compare the net gain or loss associated with each fea-

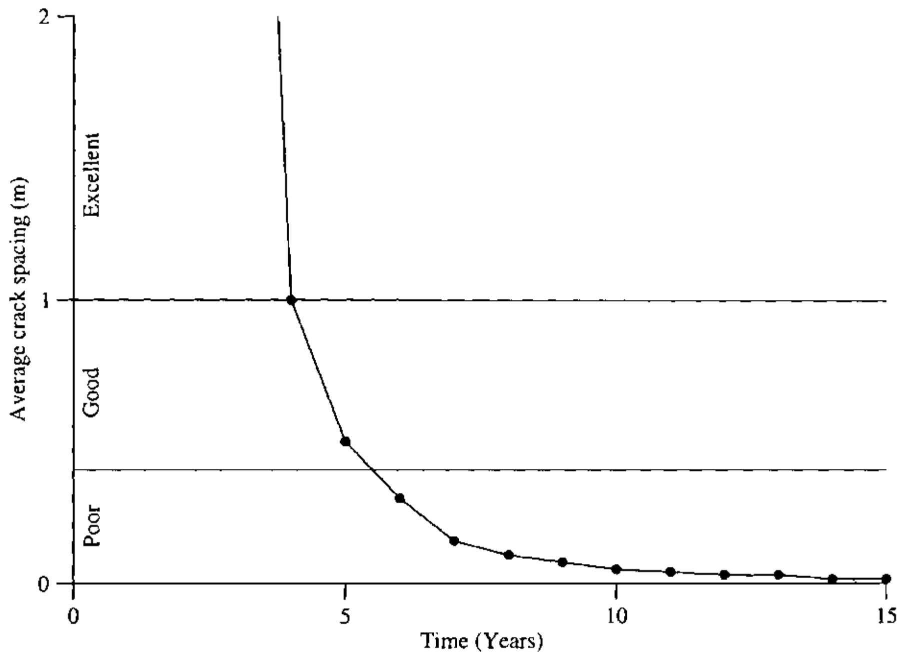

Figure 3.20: Graphing pavement cracking as the average distance between cracks.

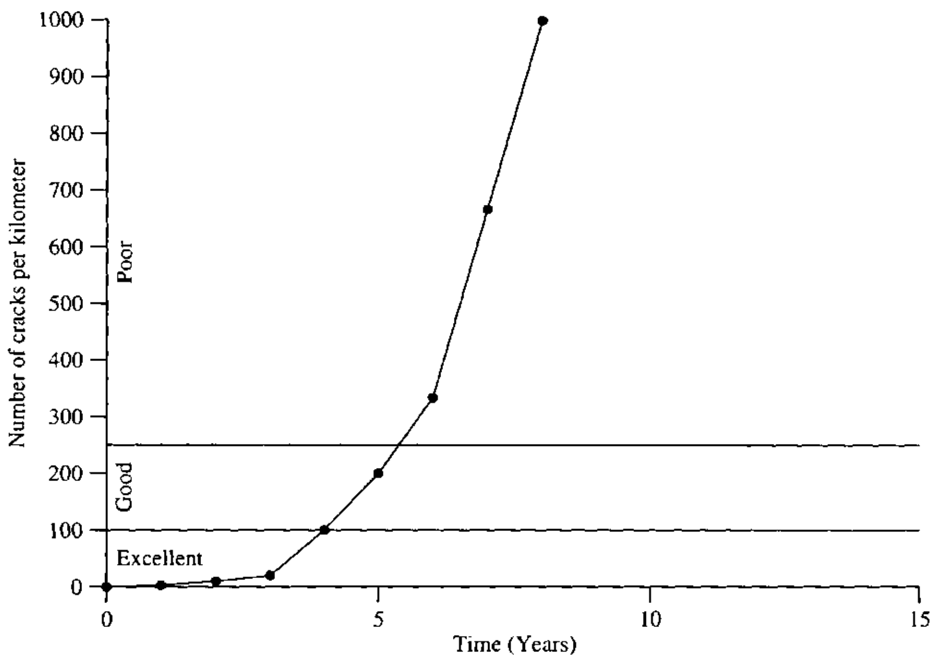

Figure 3.21: Graphing pavement cracking as the number of cracks per kilometer.

sible solution; this allows the identification of the best solution. Both the constraints and objective function contain what are known as decision variables. These variables are adjusted with each solution iteration in search of the combination which allows the model to meet all feasibility conditions and provides an optimal solution.

In this research, the decision variables are the transition probabilities p_{ij}. Since the pavement performance is modelled under routine maintenance the transitions between states is limited to downstream transitions. In other words, since routine maintenance will never create a situation where the pavement condition improves, transitions can only be made to a state equal or worse than its present condition. The following equation constrains the transitions so only downstream transitions can occur:

$$p_{ij} = 0, \ \ \forall\, j < i.$$

Typically transition states should be monotonically decreasing; the probability of making the transition from condition state i to condition state $i+1$ should be greater than the transition from condition state i to condition state $i+2$. This constraint can be modelled as follows:

$$p_{ij} \geq p_{i(j+1)}, \ \ \forall\, i, j.$$

The last two constraints are basic bookkeeping constraints. Because we are modelling transition probabilities, the sum of any row must equal one, and all probabilities must be between 0 and 1:

$$\begin{aligned} \sum_j p_{ij} &= 1, \ \ \forall\, i \\ 0 \leq \ p_{ij} &\leq 1. \end{aligned}$$

The objective is to minimize the difference between observed pavement performance (the performance curves) and predicted performance (pavement performance predicted by the transition probability matrix generated by the non-linear programming model). This is found by calculating the expected value (at each time step) for pavement performance from both the observed and predicted models.

Before the process can be illustrated some terms must first be defined. The representative state for each condition state shall be defined as S_i. For the rutting example $S_i \epsilon \{2.5, 8, 16.75\}$.

As stated previously the model has two sets of condition state probabilities, the observed and the predicted probabilities. The observed probabilities are the discrete probabilities derived from the beta distribution, and will be defined as $\beta_i(t)$. Note that the time element is required as the probability distribution will change over time. The predicted probabilities shall be defined as $\widehat{\beta}_i(t)$. The predicted condition state probability calculation is based on the predicted condition state probabilities and the decision variables. The exception is that the initial predicted probabilities are set equal to the initial observed distribution

$$\begin{aligned} \widehat{\beta}_i(0) &= \beta_i(0), \ \forall \ i \\ \widehat{\beta}_i(t) &= \sum_j \widehat{\beta}_j(t-1) p_{ij}, \ \forall \ i, t > 0. \end{aligned}$$

The objective function is based on a comparison of the average condition of the pavement over time. The average observed value will be defined as $\overline{D}_i(t)$ where

$$\overline{D}(t) = \sum_i \beta_i(t) S_i, \ \forall \ t. \tag{3.11}$$

Similarly, the average predicted value shall be defined as $\widehat{D}(t)$ where

$$\widehat{D}(t) = \sum_i \widehat{\beta}_i(t) S_i(t), \ \forall \ t. \tag{3.12}$$

Now that the necessary terms have been defined, the objective function can be defined:

$$\min \sum_i \left[\overline{D}(t) - \widehat{D}(t)\right]^2 \tag{3.13}$$

$$\min \sum_t \left[\sum_i \beta_i(t) S_i(t) - \sum_i \widehat{\beta}_i(t) S_i(t) \right]^2 . \tag{3.14}$$

Consolidating the equations in canonical form provides the following formulation:

$$\min \sum_t \left[\sum_i \beta_i(t) S_i(t) - \sum_i \widehat{\beta}_i(t) S_i(t) \right]^2$$

Subject to

$$\begin{aligned} p_{ij} &= 0, \ \ j < i \\ p_{ij} &\geq p_{i(j+1)}, \ \ \forall \ j \\ \sum_j p_{ij} &= 1 \\ 0 \leq \ p_{ij} &\leq 1. \end{aligned}$$

Based on the graph for the rutting example, as illustrated in Figure 3.15, the observed probabilities and the predicted probabilities for the optimal transition probability matrix can be found in Table 3.5.

3.4 Transition probability matrices for other treatments

To this point the procedures necessary to generate a transition probability matrix for routine maintenance have been established. Most if not all pavement performance curves describe a pavement's change in distress over time under routine maintenance. To be able to accommodate all maintenance treatments an additional procedure must be included.

A treatment will have one of three effects on a pavement: little or no more effectiveness than routine maintenance, one hundred percent effectiveness (i.e. the pavement is like brand new), and somewhere in between. In the first case, where the treatment's effectiveness is essentially the same as routine maintenance, the transition probability matrix for this treatment can be considered the same as that of routine maintenance. In situ-

Table 3.5: Observed and predicted condition state probabilities

Time	β_E	β_G	β_P	$\overline{D}$	$\widehat{\beta_E}$	$\widehat{\beta_G}$	$\widehat{\beta_P}$	$\widehat{D}$	$\left[\overline{D}-\widehat{D}\right]^2$
0	1	0	0	2.5	1	0	0	2.5	0
1	1	0	0	2.5	0.964	0.036	0.000	2.698	0.0393
2	1	0	0	2.5	0.929	0.065	0.006	2.925	0.1809
3	1	0	0	2.5	0.896	0.087	0.017	3.174	0.4544
4	1	0	0	2.5	0.863	0.105	0.032	3.439	0.8815
5	1	0	0	2.5	0.832	0.118	0.049	3.715	1.4757
6	1	0	0	2.5	0.802	0.128	0.069	3.998	2.2439
7	1	0	0	2.5	0.773	0.136	0.091	4.285	3.1873
8	0.9858	0.0142	0	2.5783	0.746	0.141	0.114	4.574	3.9835
9	0.8962	0.1038	0	3.0710	0.719	0.144	0.137	4.863	3.2098
10	0.7337	0.2663	0	3.9648	0.693	0.146	0.162	5.149	1.4010
11	0.5734	0.4266	0	4.8460	0.668	0.146	0.186	5.432	0.3430
12	0.4131	0.5869	0	5.7279	0.644	0.146	0.211	5.710	0.0003
13	0.1997	0.7951	0.0052	6.9323	0.621	0.144	0.235	5.983	0.9012
14	0.0575	0.8787	0.0638	8.0629	0.598	0.142	0.259	6.250	3.2859
15	0.0033	0.6500	0.3467	10.0406	0.577	0.140	0.283	6.511	12.4584
								$\Sigma =$	34.0470

ations where the benefit is one hundred percent effectiveness, the pavement will make a transition from its existing state to one of excellent condition (as shown in the matrix below)

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

For the situation where partial effectiveness occurs the transition probabilities must be generated manually (usually with the aid of expert judgement) or through the use of historic data. When expert judgement is applied, the transition probability matrix for routine maintenance acts as a frame of reference. The expert essentially adjusts the routine maintenance matrix based on their personal opinion/experience.

3.5 Asset level transition probability matrices

The transition probability matrices that have been discussed so far have all been for a specific distress-treatment combination (e.g. rutting-routine maintenance). The final pavement management model requires a mathematical description of the the asset's performance under all distresses simultaneously. To accomplish this each distress must be combined in such a way that the end result is a condition state scale that allows unique identification of asset condition.

To establish this universal condition state scale transition probability matrices must be combined for each distress into a single transition probability matrix. This single matrix represents the pavement's performance given a common maintenance treatment (e.g. routine maintenance, crack filling, etc.). In other words, instead of a separate transition probability matrix for each distress-treatment combination (distress level transition probability matrices) there will be unique transition probability matrices for each treatment (asset level transition probability matrices).

To combine the transition probability matrices it must be assumed that the behaviour for each distress under a treatment (such as routine maintenance) is independent. The assumption is not necessarily true, but it is a basic assumption that underlies all published work in this area (RTA, 1995). Without this basic assumption, one would have to know the effectiveness of the treatment given the combination of distresses present at any point in time.

The first step in this process is to establish a mapping between the condition states in the distress level transition probability matrices (excellent, good, and poor) with the condition states in the asset level transition probability matrices. In the situation where there are six distresses and three condition states for each distress, the asset level transition probability matrices will have $3^6 = 729$ condition states. In general, the number of condition states for the asset level transition probability matrices is equal to $conditionstates^{distress\ count}$ (assuming that each distress has the same number of condition states). The next step is to enumerate all combinations of condition states for each distress. A partial enumeration can be found in Table 3.6.

Table 3.6: A partial enumeration of distress level condition states.

Condition State	Rutting	Deterioration	Depth	Cracking	Surface	Roughness
1	E	E	E	E	E	E
2	E	E	E	E	E	G
3	E	E	E	E	E	P
4	E	E	E	E	G	E
5	E	E	E	E	G	G
6	E	E	E	E	G	P
7	E	E	E	E	P	E
8	E	E	E	E	P	G
9	E	E	E	E	P	P
10	E	E	E	G	P	E

A good example would be to calculate the probability of making the transition from state 2 to state 5 under routine maintenance. Table 3.6 illustrates that state 2 corresponds to the combination {1,1,1,1,1,2} and state 5 corresponds to the combination {1,1,1,1,2,2}. This means that we need the distress level transition probabilities for rutting for $\{i = 1, j = 1\}$ and deterioration for $\{i = 1, j = 1\}$ and so on. The transition probability matrices can be found in Table 3.7 and the sample calculation in Table 3.8. The asset level transition probabilities are simply the product of these distress level probabilities.

Table 3.7: Distress level transition probability matrices for routine maintenance.

$$\underset{\text{Rutting}}{\begin{bmatrix} 0.932 & 0.068 & 0 \\ 0 & 0.817 & 0.182 \\ 0 & 0 & 1 \end{bmatrix}} \quad \underset{\text{Deterioration}}{\begin{bmatrix} 0.932 & 0.068 & 0 \\ 0 & 0.817 & 0.182 \\ 0 & 0 & 1 \end{bmatrix}} \quad \underset{\text{Depth}}{\begin{bmatrix} 0.949 & 0.051 & 0 \\ 0 & 0.783 & 0.217 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\underset{\text{Cracking}}{\begin{bmatrix} 0.769 & 0.231 & 0 \\ 0 & 0.516 & 0.484 \\ 0 & 0 & 1 \end{bmatrix}} \quad \underset{\text{Surface}}{\begin{bmatrix} 0.929 & 0.071 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}} \quad \underset{\text{Roughness}}{\begin{bmatrix} 0.935 & 0.065 & 0 \\ 0 & 0.791 & 0.209 \\ 0 & 0 & 1 \end{bmatrix}}$$

This procedure must be repeated for all elements of each asset level transition probability matrix. In the example there are 6 distresses and 3 condition states. This means that for each treatment there are $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6 = 729$ condition states. Since the matrix is

Table 3.8: Sample calculation for asset level transition probabilities

Distress	Row	Column	Probability
Rutting	1	1	0.932
Transitive Cracking - Deterioration	1	1	0.932
Transitive Cracking - Depth	1	1	0.949
Cracking	1	1	0.769
Surface	1	2	0.071
Roughness	2	2	0.791
		$\Pi =$	0.0356

square there are $(3^6)^2 = 729^2 = 531,441$ elements in each asset level transition probability matrix.

3.6 Conclusion

This chapter began with an introduction to Markov processes where the concepts of state and transition were defined, and the importance of the transition probability matrix was established. Once the importance of transition probability matrices was established, the bulk of the chapter was spent developing the methodology necessary to generate a transition probability matrix from asset performance curves. The performance curves used to generate a transition probability matrix typically model performance under routine maintenance. The pavement management model requires transition probability matrices for all treatments. The process necessary to generate transition probability matrices for the other treatments was covered next. The chapter concluded with a description of how to generate asset level transition probability matrices.

Chapter 4

MARKOV DECISION PROCESSES AS A LINEAR PROGRAM

4.1 Introduction

There are a variety of techniques that could be applied when solving the Markovian decision process model introduced in the previous chapter. This research relied on a linear programming based solution. This chapter introduces linear programming concepts (both graphically and mathematically), discusses how sensitivity analysis can be applied to linear programming models, and develops the linear programming form of the Markovian decision process. The chapter will conclude with a brief discussion on other analytical approaches for solving Markov decision processes.

4.2 Introduction to linear programming

The standard form for a linear programming model is as follows: find the x_i such that

$$\begin{aligned} \text{max} \quad & \textstyle\sum c_i x_i \\ \text{subject to} \quad & \textstyle\sum a_{ij} x_i \quad \leq b_j \ \forall \ j \end{aligned}$$

where $\sum c_i x_i$ is the objective function and $\sum a_{ij} x_i \leq b_j$ are the constraints. For problems where the objective is to minimize costs the optimal solution will consist of a minimized objective function. The basic mechanics and concepts of linear programming will be introduced by solving a simple example problem graphically.

Traditional Toys by Todd makes wooden puzzles for educational toy stores. Todd is looking to expand his product line by adding bird houses. Net revenue from each puzzle is currently \$6 and bird houses are expected to generate \$12. Todd can spend only 30 hours each week building toys. Of these 30 hours he must be either doing carpentry work or finishing work. Since Todd prefers finishing twice as much as carpentry he usually only spends 10 hours each week on carpentry. Each puzzle takes 2 hours of carpentry and 1 hour of finishing. Bird houses take 1 hour of carpentry and 3 hours of finishing. For Todd to maximize his revenues, how many bird houses and puzzles should he be making each week?

Todd's objective is to maximize his revenues, and his decision is to determine how many bird houses and puzzles to make. If the decision variables are defined as x_1 for the number of puzzles and x_2 for the number of bird houses, then the objective function is $6x_1 + 12x_2$. It is assumed that only positive quantities of puzzles and birdhouses can be made. Todd's only constraint is time. He only has 10 hours of week to do carpentry and 20 hours per week to complete his finishing. The functions that describe these constraints are $2x_1 + x_2 \leq 10$ and $x_1 + 3x_2 \leq 20$. Formulating this problem as a standard linear programming model gives:

$$\begin{aligned} \max \quad & 6x_1 + 12x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \quad \leq 10 \\ & x_1 + 3x_2 \quad \leq 20. \end{aligned}$$

Since this model consists of only two decision variables it can be represented graphically as illustrated in Figure 4.1. This figure shows the region defined by the carpentry constraint; any combination of puzzles and bird houses that require a total of 10 hours or less of carpentry time. Figure 4.2 introduces the finishing constraint. The first step is to determine feasible values for x_1 and x_2. Typically the starting point is to set the decision variables equal to 0 as illustrated in Figure 4.3. The next step is to move the objective function line through one of the adjacent vertices. Moving to an adjacent vertex must result in an increase in the objective function's value. If this does not occur then the present vertex corresponds to the optimal solution.

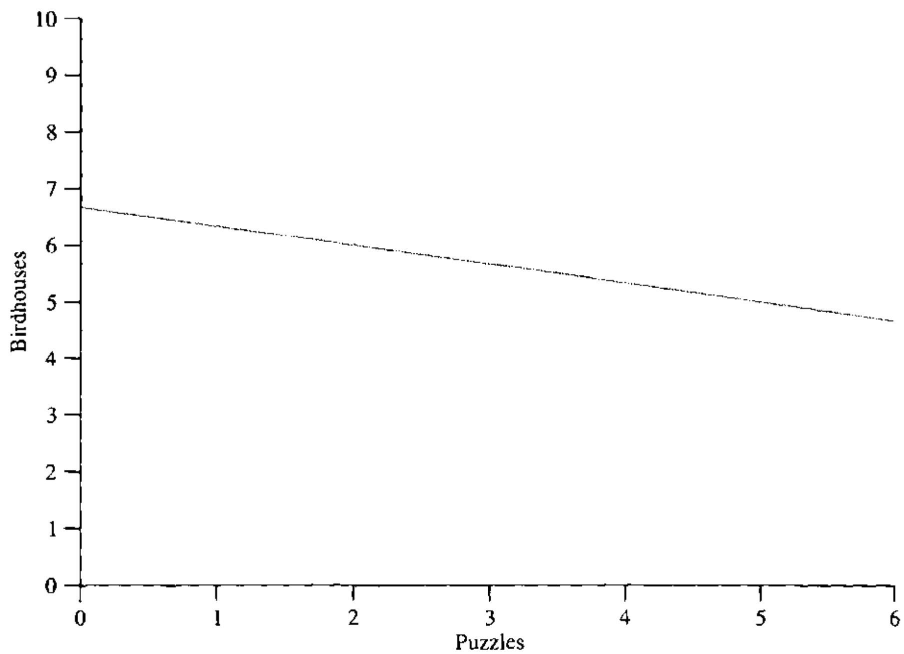

Figure 4.1: The carpentry constraint

Figure 4.4 shows progressively higher objective functions plotted on the constrained solution space. The optimal solution has a value of Z=84, and intersects the vertex (2,6); the optimal production plan is to produce 2 puzzles and 6 bird houses each week for a total net revenue of $84.

This problem will now be solved using the standard simplex algorithm. Once again,

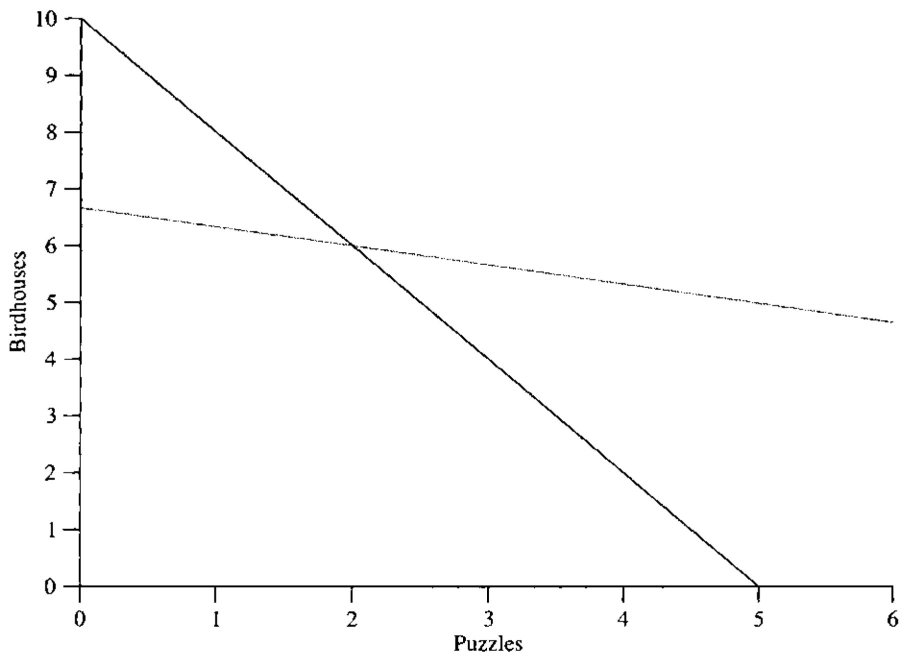

Figure 4.2: The carpentry and finishing constraints

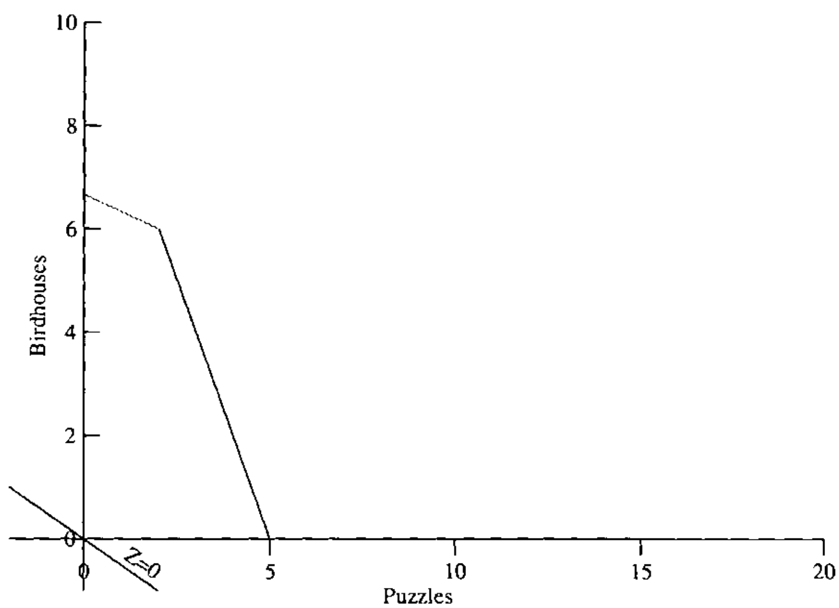

Figure 4.3: Placing the objective function within our constrained problem space.

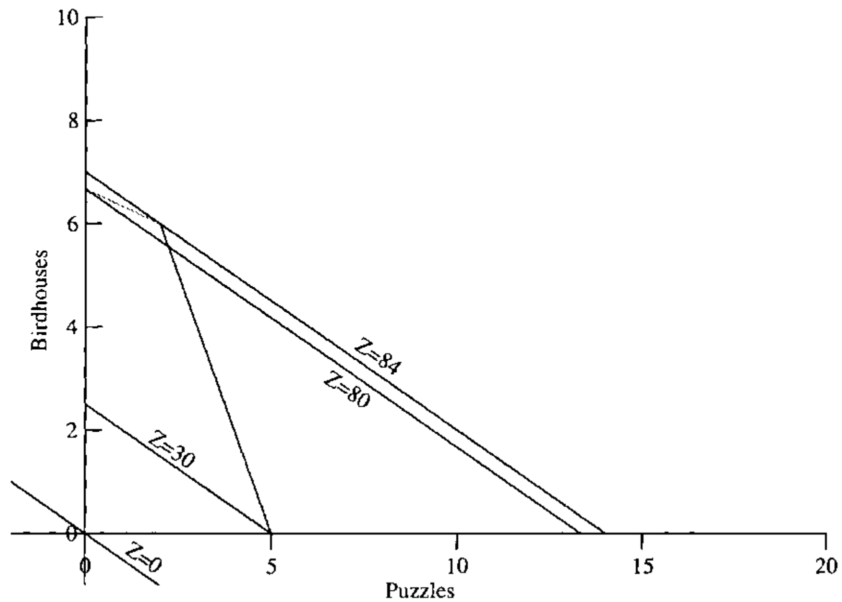

Figure 4.4: The optimized solution.

the problem in standard form is:

$$\begin{aligned} \max \quad & 6x_1 + 12x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \quad \leq 10 \\ & x_1 + 3x_2 \quad \leq 20. \end{aligned}$$

The numeric solution to a linear programming problem requires the problem to be formulated as a series of algebraic equations. Inequalities are converted to equalities by the introduction of slack variables. These equations are often placed in a tableau as shown in Table 4.1.

Table 4.1: Initial tableau

	z	x_1	x_2	s_1	s_2	Constraint
Objective function:	1	-6	-12			0
Carpentry:		2	1	1		10
Finishing:		1	3		1	20

A solution will have two sets of variables: basic and non-basic. The number of non-basic variables is equal to the number of variables, in this case 4, less the number of linear equations, in this case 2. There can be only one basic variable in each equation.

The usual starting point is to set the decision variables to zero; effectively drawing the objective function through the origin (making x_1 and x_2 non-basic variables and the slack variables basic).

The optimal solution is found by incrementally improving each solution one step at a time. Improvement occurs by determining which non-basic variable must be swapped for a basic variable to increase the objective function.

Initially x_1 and x_2 are non-basic. Since x_2 has the most negative coefficient it will increase the objective function more than x_1. By introducing x_2 into the basis (i.e. make it one of the basic variables) there is the risk of finding a non-feasible solution. To prevent a non-feasible solution from being selected the ratio test is performed. In the ratio test each constraint constant is divided by the new basic variable's coefficient (for that constraint equation). For this example $10/1 = 10$ and $20/3 = 6.67$. Any value of x_2 greater than 6.67 generates an infeasible solution. Thus x_2 will be made basic for the finishing constraint. By applying elementary row operations to each row we get the new tableau found in Table 4.2.

Table 4.2: Second tableau

	z	x_1	x_2	s_1	s_1	Constraint
Objective function:	1	-2			4	80
Carpentry:		5/3		1	-1/3	10/3
Finishing:		1/3	1		1/3	20/3

As an aside it should be noted that this solution corresponds to the solution where the objective function intersects the upper left corner of the constrained space in Figure 4.4. The new basic variables are now $\{s_1, x_2\}$. Since there is still a negative coefficient in the objective function, the objective function can be further improved. The ratio test gives $10/3 \cdot 3/5 = 2$ and $20/3 \cdot 3 = 20$. Thus, the binding constraint is the carpentry constraint. Once again elementary row operations are performed given that x_1 is basic in the carpentry constraint. The final solution can be found in Table 4.3. Table 4.3 is the optimal tableau because there are no negative coefficients in the objective function. As was the case in the graphical solution, the maximum revenue is $84/week generated by a production plan of 2 puzzles and 6 birdhouses per week.

Table 4.3: Final tableau

	z	x_1	x_2	s_1	s_2	Constraint
Objective function:	1			6/5	18/5	84
Carpentry:		1		3/5	-1/5	2
Finishing:			1	-1/5	2/5	6

4.3 Sensitivity analysis

Sensitivity analysis is essentially a measurement in the change to the optimal decision (i.e. outputs) due to changes in the values of input variables. These changes could be due to changes in the available resources, objective function, or production functions. Since it is difficult to predict future operating environments sensitivity analysis is an important part of any modelling exercise.

Sensitivity analysis usually focuses on changes to the objective function (in this case revenues from puzzles or birdhouses) or available resources (in this case carpentry time or finishing time). This is largely due to the nature of the linear programming model; the results due to changes in the objective functions or constraints can often be derived directly from an existing optimal solution. Changes in a production function always require a new optimal solution to be generated. Typical questions that could be addressed through sensitivity analysis include:

1. How many puzzles and bird houses should Todd make if the revenue from a puzzle was was $4? *(Puzzles = 0, Bird houses = 6.6667)* What if revenue was $22? *(Puzzles = 2, Birdhouses = 6)*

2. How much revenue from each birdhouse would be needed so only birdhouses would be made? *($18)*

3. If Todd worked fifty percent of the time on carpentry work would he generate more revenue? *(No. Total revenue would be $72.)* What would be his weekly production? *(Puzzles = 6, Birdhouses = 3)*

4. How much more revenue would Todd generate if he would be able to spend 10 more hours each week on both carpentry and finishing? *(Revenue would be $132, an increase of $48. Puzzles = 6, Birdhouses = 8)*

5. If a new sander were to cut finishing time in half, what would this be worth to Todd (i.e. he would still spend 20 hours per week, he would just finish twice as

many puzzles and birdhouses)? *(Total revenues would be \$120. The new sander would generate an additional \$36/week. Puzzles = 1, Birdhouses = 10)*

4.4 Markov decision processes as a linear program

The previous chapter introduced the Markov decision process model, but stopped just short of solving a problem. This section formulates and solves a Markov decision process as a linear programming problem.

4.4.1 Markov decision processes as a recursive function

As stated earlier, Markov decision processes are made up of three key elements: states, transitions, and actions. The standard formulation for a Markov decision process is as a recursive function

$$v_i(t+1) = \min_{k} \left[q_i^j + \sum_{j=1}^{N} p_{ij}^k v_j(t) \right] \tag{4.1}$$

where

$$q_i^k = \sum_{j=1}^{N} p_{ij}^k r_{ij}^k$$

and p_{ij}^k is the probability of making transitions from state i to state j under action k. From an asset management perspective, action k is maintenance treatment k. Since the objective to minimize cost, maintenance treatment k will be selected based on the treatment which provides the lowest cost maintenance strategy. The term r_{ij}^k is the reward (and in this example the cost) associated with making the transition from condition state i to transition state j. In the pavement models considered in this dissertation, maintenance costs are dictated strictly by the initial condition state (condition state i) and not the destination state. In other words if a pavement begins in an poor condition state and finishes in a good condition state, the maintenance costs are dictated by the initial condition (poor) not the final condition state (good). The last term to be defined is $v_j(t)$, which is the total expected reward in state i after t periods (given that an optimal maintenance plan has been followed).

4.4.2 Markov decision processes as a linear programming model

Markov decision processes can be easily represented with a linear programming model (Osaki and Mine, 1970). Applying a linear programming model reduces the need for customized programming and allows the use of off the shelf (albeit, very specialized) technology to obtain the optimal maintenance solution. This simplifies both model development and ongoing support efforts. This section derives the linear programming formulation equivalent to the standard model developed in the previous section.

The first element to be defined is the decision variable x_i^k; the probability of being in state i at steady state given that treatment k was applied to the pavement when it was in state i. The cost of applying maintenance treatment k when the pavement is in state i is c_i^k. The objective function is thus

$$\sum_i \sum_k c_i^k x_i^k.$$

There are two sets of constraints for this model. The first constraint ensures that the sum of all steady state probabilities equals one:

$$\sum_i \sum_k x_i^k = 1.$$

The second constraint describes the steady state condition of the system. At steady state the probability of being in state x_i^k is the same from one epoch to the next. If $X(t)$ describes the system's condition state for some time state t then $X(t) = X(t-1)P$ where P is the transition probability matrix that corresponds to this optimal maintenance policy. At steady state the probability distribution for the condition states from epoch to epoch is equal ($X(t) = X(t+1)$). Thus, the time step index can be removed from the notation ($X = XP$) with no loss of clarity. Since, x_i^k is an element of the vector X and p_{ij}^k is an element of P, the steady state constraint is

$$\sum_k x_j^k = \sum_i \sum_k x_i^k p_{ij}^k \;\; \forall \;\; j.$$

The following linear programming model is equivalent to the recursive function in Equation 4.1:

$$\begin{aligned} \min \quad & \sum_i \sum_k c_i^k x_i^k \\ \text{subject to} \quad & \sum_i \sum_k x_i^k = 1 \\ & \sum_k x_j^k = \sum_i \sum_k x_i^k p_{ij}^k \;\; \forall \;\; j. \end{aligned}$$

The only difficulty with this model (and it is also a problem with the recursive formulation as well) is that the least cost policy is inherently the least effective; the pavement will deteriorate to the worst possible condition state. To prevent the wholesale deterioration of the pavement, performance constraints can be added to the model. In this case two constraints are included. The first constraint ensures that a minimum percentage of the asset is maintained in excellent condition

$$\sum_{i \in E} \sum_k x_i^k \geq \varepsilon$$

where ε is the minimum percentage of the asset to be maintained in excellent condition. We will also include a constraint to limit the percentage of the pavement that reaches a poor condition state

$$\sum_{i \in E} \sum_k x_i^k \leq \Pi$$

where Π is the percentage of the pavement that can not be allowed to reach a poor condition state.

4.4.3 A linear programming model without performance constraints

To better illustrate the implementation of the linear programming formulation, this section reviews a basic example. The data for this example can be found in Neudorf (1989). The first instance of this example does not include any performance limits. The next section in this chapter reviews sensitivity analysis. Performance limits are introduced in these illustrative examples.

For this example problem there are only three condition states (state 1, state 2, and state 3) and three maintenance treatments (treatment 1, treatment 2, and treatment 3). The transition probability matrices for each treatment are found in Table 4.4. The costs associated with each treatment-condition state combination can be found in Table 4.5.

Table 4.4: Transition probabilities for example problem (p_{ij}^k)

$$\begin{array}{ccc} \text{Treatment 1} & \text{Treatment 2} & \text{Treatment 3} \\ \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0 & 0.6 & 0.4 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0 & 0.8 & 0.2 \\ 0 & 0.1 & 0.9 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0.9 & 0.1 & 0 \\ 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$$

Table 4.5: Treatment costs in $/m^2 ($c_i^k$)

	Treatment 1	Treatment 2	Treatment 3
State 1	1	7	30
State 2	3	12	35
State 3	8	18	40

The intial tableau based on this data can be found in Table 4.6.

Table 4.6: Linear programming tableau for a Markov decision process

	i=1, k=1	i=1, k=2	i=1, k=3	i=2, k=1	i=2, k=2	i=2, k=3	i=3, k=1	i=3, k=2	i=3, k=3	Const
Objective function (c_i^k)	1	7	30	3	12	35	8	18	40	
State 1 (p_{i1}^k)	0.4	0.2	0.2	0	0	-0.9	0	0	-0.8	0
State 2 (p_{i2}^k)	-0.4	-0.2	0	0.4	0.2	0.9	0	-0.1	-0.2	0
State 3 (p_{i3}^k)	0	0	0	-0.4	-0.2	0	0	0.1	1	0
	1	1	1	1	1	1	1	1	1	1

This model cannot be solved using the basic simplex algorithm because the basic simplex algorithm requires a ratio based on the constraint values. In this formulation the constraints are equal to 0; dividing these constraints by a technology coefficient is not going to identify the pivot value. For a concise description of the modified solution process the reader should see Neudorf (1989). For the remainder of this chapter the linear programming models are solved with an optimizer such as those found in most spreadsheet software packages. The optimal policy for our example can be found in Table 4.7. The table shows the probability that the pavement will be in each condition state and the treatment that should be applied to the pavement when it is in that condition state. The "B" in Table 4.7 indicate that these states are in the basis, but the probability of being in either state 1 or state 2 at steady state is effectively 0. Table 4.8 shows the calculation for the average annual maintenance costs at steady state. For this maintenance strategy, the average maintenance cost will be $8/m^2. Under this strategy the pavement will be in condition state 3 at steady state.

Table 4.7: Optimal policy for linear programming example

	Treatment 1	Treatment 2	Treatment 3
State 1	B		
State 2	B		
State 3	1		

Table 4.8: Objective function calculations

	i=1, k=1	i=1, k=2	i=1, k=3	i=2, k=1	i=2, k=2	i=2, k=3	i=3, k=1	i=3, k=2	i=3, k=3	Total
Objective function (c_i^k)	1	7	30	3	12	35	8	18	40	
Steady state (x_i^k)							1			8

4.4.4 Linear programming with performance constraints

A performance constraint will be now be added to the basic linear programming formulation. The performance constraint limits the likelihood of the pavement reaching condition state 3 to ten percent is

$$\sum_k x_3^k \leq 0.1.$$

The tableau for the reformulated problem can be found in Table 4.9 and the resulting optimal policy in Table 4.10. Table 4.10 shows that the policy is stochastic; more than one treatment is optimal when the pavement is in a specific condition state. When the pavement is in condition state 2 treatment 1 should be applied 0.25/(0.25+0.1385)=64.35% of the time and treatment 2, 0.1385/(0.25+0.1385)=35.65% of the time. The average maintenance cost at steady state is \$10.108/m^2/year, as shown in Table 4.11.

Table 4.9: Initial tableau - limiting condition state 3 probabilities

	i=1, k=1	i=1, k=2	i=1, k=3	i=2, k=1	i=2, k=2	i=2, k=3	i=3, k=1	i=3, k=2	i=3, k=3	Const
Objective function (c_i^k)	1	7	30	3	12	35	8	18	40	
State 1 (p_{i1}^k)	0.4	0.2	0	0	0	-0.9	0	0	-0.8	0
State 2 (p_{i2}^k)	-0.4	-0.2	0	0.4	0.2	0.9	0	-0.1	-0.2	0
State 3 (p_{i3}^k)	0	0	0	-0.4	-0.2	0	0	0.1	1	0
	1	1	1	1	1	1	1	1	1	1
Limit poor condition							1	1	1	0.1

Table 4.10: Optimal policy - limiting condition state 3 probabilities

	Treatment 1	Treatment 2	Treatment 3
State 1	0.5115		
State 2	0.2500		0.1385
State 3			0.1000

Table 4.11: Objective function calculations - limiting condition state 3 probabilities

	i=1, k=1	i=1, k=2	i=1, k=3	i=2, k=1	i=2, k=2	i=2, k=3	i=3, k=1	i=3, k=2	i=3, k=3	Total
Objective funcion (c_i^k)	1	7	30	3	12	35	8	18	40	
Steady state (x_i^k)	0.5115			0.25		0.1385			0.1	10.1077

4.4.5 Sensitivity relative to maintenance cost adjustments

This example illustrates the influence that can be introduced to a model by adjusting the maintenance costs of the original linear programming formulation (i.e. without performance constraints). In the original solution the optimal maintenance policy was to apply treatment 1 regardless of the pavement's condition. In this new model the cost of applying treatment 1 to a pavement in condition state 3 is set prohibitively high so that it is unlikely this treatment will be selected. Table 4.12 contains the initial tableau for this model and Table 4.13 the optimal policy. The optimal policy shows that Treatment 1 should be applied when the pavement is in condition state 1 and 2 and treatment 3 when in condition state 3. Table 4.14 shows the steady state probability for each condition state as well as the corresponding maintenance cost. The average annual maintenance cost at steady state is $\$9/\text{m}^2/\text{year}$. This is an increase of 1 $\$/\text{m}^2/\text{year}$ over the original model, and resulted in a policy change where treatment 3 is now applied when the pavement is in condition state 3.

Table 4.12: Initial tableau - setting a high treatment cost

	i=1, k=1	i=1, k=2	i=1, k=3	i=2, k=1	i=2, k=2	i=2, k=3	i=3, k=1	i=3, k=2	i=3, k=3	Const
Objective function (c_i^k)	1	7	30	3	12	35	9999	18	40	
State 1 (p_{i1}^k)	0.4	0.2	0.2	0	0	-0.9	0	0	-0.8	0
State 2 (p_{i2}^k)	-0.4	-0.2	0	0.4	0.2	0.9	0	-0.1	-0.2	0
State 3 (p_{i3}^k)	0	0	0	-0.4	-0.2	0	0	0.1	1	0
	1	1	1	1	1	1	1	1	1	1

Table 4.13: Optimal policy - setting a high treatment cost

	Treatment 1	Treatment 2	Treatment 3
State 1	0.3636		
State 2	0.4545		
State 3			0.1818

Table 4.14: Optimal objective function calculations - setting a high treatment cost

	i=1, k=1	i=1, k=2	i=1, k=3	i=2, k=1	i=2, k=2	i=2, k=3	i=3, k=1	i=3, k=2	i=3, k=3	Total
Objective funcion (c_i^k)	1	7	30	3	12	35	9999	18	40	
Steady state (x_i^k)	0.3636			0.4545					0.1818	9

4.4.6 Sensitivity relative to perfomance improvement

This sensitivity example illustrates the impact that a change in pavement performance will introduce. Specifically the benefit of improved pavement performance will be examined. Imagine the scenario where a pavement's deterioration is slowed. The question is what impact would a more durable pavement have on a maintenance budget? A pavement which is more durable will have transition probabilities that increase for the "better" condition states relative to the base case transition probability matrix. Essentially the likelihood of staying in the existing condition state has increased. Table 4.15 shows both the base case and improved transition probability matrices. The intial tableau can be found in Table 4.16. Once again a performance constraint has been included and once again the policy is stochastic as illustrated in Table 4.17. The original maintenance cost was \$10.108/m^2/year and the new cost is \$7.877/m^2/year, this is a decrease in cost of \$2.231/m^2/year. Thus, if the party responsible for maintaining the pavement could acquire pavement materials that performed as in our model, it would be worth up to \$2.231/m^2/year in maintenance costs. Chapter 5 introduces the procedure necessary to calculate the present value of these annual maintenance savings over the life of a maintenance contract.

4.5 Alternative solutions to the Markov decision process

The traditional approach to solving a Markov decision process is via Howard's policy iteration algorithm (Howard, 1960). This research did not apply this technique. To ensure that a complete treatise on the topic of Markov decision processes has been assembled, a

Table 4.15: Transition probability matrix comparison

Treatment 1	Treatment 2	Treatment 3
$\begin{bmatrix} 0.6 & 0.4 & 0 \\ 0 & 0.6 & 0.4 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0.2 & 0 \\ 0 & 0.8 & 0.2 \\ 0 & 0.1 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0.9 & 0.1 & 0 \\ 0.8 & 0.2 & 0 \end{bmatrix}$
Treatment 1a	Treatment 2a	Treatment 3a
$\begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.7 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.85 & 0.15 & 0 \\ 0 & 0.85 & 0.15 \\ 0 & 0.15 & 0.85 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0.95 & 0.05 & 0 \\ 0.85 & 0.15 & 0 \end{bmatrix}$

Table 4.16: Initial tableau - improving pavement performance

	i=1, k=1	i=1, k=2	i=1, k=3	i=2, k=1	i=2, k=2	i=2, k=3	i=3, k=1	i=3, k=2	i=3, k=3	Const
Objective function (c_i^k)	1	7	30	3	12	35	8	18	40	
State 1 (p_{i1}^k)	0.3	0.15	0	0	0	-0.95	0	0	-0.15	0
State 2 (p_{i2}^k)	-0.3	-0.15	0	0.3	0.15	0.95	0	-0.15	-0.15	0
State 3 (p_{i3}^k)	0	0	0	-0.3	-0.15	0	0	0.15	1	0
	1	1	1	1	1	1	1	1	1	1
Limit poor condition							1	1	1	0.1

Table 4.17: Optimal policy - improving pavement performance

	Treatment 1	Treatment 2	Treatment 3
State 1	0.4987		
State 2	0.3333		0.0680
State 3			0.1000

Table 4.18: Objective function calculations - improving pavement performance

	i=1, k=1	i=1, k=2	i=1, k=3	i=2, k=1	i=2, k=2	i=2, k=3	i=3, k=1	i=3, k=2	i=3, k=3	Total
Objective funcion (c_i^k)	1	7	30	3	12	35	8	18	40	
Steady state (x_i^k)	0.4987			0.3333		0.068			0.1	7.8787

description of the procedure has been included. The policy iteration algorithm consists of two key steps; the value determination operation and the policy improvement routine. The first step, (the value determination operation) is based on determining the solution to a set of simultaneous equations found in

$$g + v_j = c_i + \sum_{i=1}^{N} v_i, \quad \text{where} \quad v_n = 0$$

for a given policy. The initial policy is usually selected via a greedy algorithm where the maintenance policy is set equal to a group of treatments that provides the lowest cost maintenance solution. Once there is a solution for g and v_1 to v_{N-1} it must be determined whether any other maintenance strategy would provide a lower cost strategy. This part of the algorithm is called the policy improvement routine. If after solving equation 4.2 for all i, the lowest cost policy is the same as the existing policy then it is known the existing policy is optimal.

$$\underset{k}{Min} \quad \left\{ c_i^k + \sum_j v_j p_{ij}^k \right\} \quad \forall \ i \tag{4.2}$$

The policy iteration algorithm is a more efficient algorithm than the simplex method for solving Markov decision process problems. In the simplex method each vertex of the solution space corresponds to a maintenance strategy. When searching for an improved strategy, the simplex method only evaluates adjacent vertices (strategies). Adjacent strategies will share $n - 1$ maintenenance treatments. Hence improvement is effectively one treatment at a time. In contrast the policy iteration algorithm is not as limited in its search for potentially new policies; as many as three treatments could change from strategy to strategy.

Both Hastings (1973) and Osaki and Mine (1970) propose hybrid solutions to Markov decision processes. But, given the efficiencies of today's linear programming engines and the significant desktop computing power available, it is difficult to justify the time and effort necessary to implement customized algorithms when off the shelf technology can be implemented with minimal customization and minimal impact with respect to optimization performance.

4.6 Conclusion

The objective of this chapter was to illustrate how a Markov decision process could be formulated and solved as a linear programming problem. To accomplish this task this chapter began with basic introduction to linear programming. Although quite cursory, the introduction included both graphical and numeric examples to illustrate the simplex algorithm. In any business environment, models are built on estimates and best guesses. It is important that all models be tested over a range of possible scenarios. This testing and the measurement in the changes to the results is called sensitivity analysis. The section on linear programming was followed by a section on how sensitivity analysis can be applied to a linear programming model and the kinds of questions and scenarios that can be evaluated with linear programming models. Once the background concepts were introduced the focus shifted to modelling the Markov decision process as a linear programming problem. The section began with a brief review on the traditional formulation of a Markov decision process as a recursive function. This was then followed by the linear programming formulation. To reinforce the various concepts introduced in this section (including sensitivity analysis) a variety of examples were covered. The chapter then concluded with a brief discussion on alternative techniques for solving Markov decision processes, and why the linear programming formulation was selected for this research.

Chapter 5

CALCULATING THE RISK ASSOCIATED WITH PAVEMENT PERFORMANCE

5.1 Introduction

The ultimate objective of this research was to quantify the uncertainty in costs associated with long term maintenance contracts from the contractor's perspective. The previous two chapters discussed the mechanics associated with capturing the uncertainty associated with pavement performance and determining the optimal (least cost) maintenance strategy given that performance. This chapter will illustrate the procedure necessary to capture the financial risk associated with an individual project as well as that associated with a long term maintenance contract (a portfolio of projects).

5.2 Defining risk

Risk can be defined in a variety of ways. For purpsoses of this research, the most appropriate definition comes from Clemen (1990) "risk is the chance of monetary loss." It has been shown that an optimal (least cost) maintenance strategy can be determined for a pavement with a given performance (i.e. set of distress curves) and minimum level of service criteria. The model that generates this strategy also provides the average annual cost at steady state for this strategy. What is not known is:

- How long will it take for the system to reach steady state?
- What is the expected maintenance cost over a specific time horizon?
- What is the variance associated with this expected cost?

The first question allows the contractor to identify if the contract is long enough to allow the pavement to reach steady state under the proposed maintenance strategy. The last two questions (expected cost and variance) are critical pieces of information from a contractor's perspective; expected cost and variance effectively quantify the break even, risk neutral tender price, and its associated risk.

5.3 Performance under the optimal maintenance strategy

In a Markovian decision process the system's development over time can be described by the condition state vector $\pi(t)$. Given a transition probability matrix P and the initial condition state vector (usually $\pi(0)$) the state of the system can be determined recursively for any point in time through the following relationship $\pi(t+1) = \pi(t)P$.

Based on this relationship, the expected cost of maintaining the pavement at each epoch ($Z(t)$) can be determined through the dot product of the condition state vector and the maintenance cost vector associated with the optimal maintenance policy (δ) where

$$Z(t) = \pi(t)\delta.$$

From this calculation the expected annual cost and its associated variance throughout the life of the contract can be found. *This leads to the question, how is the optimal transition probability matrix formed?*

5.3.1 Determining the optimal transition probability matrix

The optimal maintenance strategy determines which treatment should be applied when the pavement is in a specific condition state. The transition probability matrix is constructed by selecting the row from the transition probability matrix associated with each treatment-condition state tuple. For instance with an optimal strategy of $\{1, 3, 2\}$ treatment 1 would be applied when the pavement is in condition state 1, treatment 3 when the pavement is in condition state 2, and treatment 2 when the pavement is in condition state 3. Consequently the optimal transition probability matrix would be created by selecting row 1 from the transition probability matrix for treatment 1, row 2 from the transition probability matrix for treatment 3 and row 3 from the transition probability matrix for treatment 2.

There is a possibility that the optimal strategy will be stochastic; there is more than one possible treatment for a specific condition state. When there is more than one possible treatment for a condition state, a weighted average of each applicable set of transition probabilities is used to generated the corresponding row in the optimal transition probability matrix.

An example will help clarify the above description. Consider the optimal maintenance strategy in Table 5.1 and the transition probability data in Table 5.2.

Table 5.1: Optimal maintenance strategy

	Treatment 1	Treatment 2	Treatment 3
State 1	0.5115		
State 2	0.2500		0.1385
State 3			0.1000

Determining the first and third row of the optimal transition probability matrix is simply a matter of extracting the probabilities from Table 5.2 corresponding to state 1-treatment 1 ($i = 1, k = 1$) and state 3-treatment 3 ($i = 3, k = 3$). Thus the first row of the optimal transition probability has transition probabilities of $\begin{bmatrix} 0.6 & 0.4 & 0 \end{bmatrix}$ and the third row of the optimal transition probability matrix is equal to $\begin{bmatrix} 0.8 & 0.2 & 0 \end{bmatrix}$. If the

Table 5.2: Transition probabilities

i	k	$j=1$	$j=2$	$j=3$
1	1	0.6	0.4	0
	2	0.8	0.2	0
	3	1	0	0
2	1	0	0.6	0.4
	2	0	0.8	0.2
	3	0.9	0.1	0
3	1	0	0	1
	2	0	0.1	0.9
	3	0.8	0.2	0

pavement is in condition state 2, treatment 1 is applied part of the time and treatment 3 part of the time. The ratio for applying these treatments is simply the relative proportion of each probability. Thus, treatment 1 would be selected $\frac{0.25}{0.1385+0.25} = 64.35\%$ of the time and treatment 2 35.65% of the time. These weightings are also used to calculate the appropriate row probabilities for the transition probability matrix. Row 2 in the optimal transition probability matrix is equal to:

$$0.6435 \times Row(i=2,k=1) + 0.3565 \times Row(i=2,k=3)$$

$$0.6435 \begin{bmatrix} 0 & 0.6 & 0.4 \end{bmatrix} + 0.3565 \begin{bmatrix} 0.9 & 0.1 & 0 \end{bmatrix} = \begin{bmatrix} 0.3208 & 0.4218 & 0.2574 \end{bmatrix}.$$

The optimal transition probability matrix is then

$$\begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.3208 & 0.4218 & 0.2574 \\ 0.8 & 0.2 & 0 \end{bmatrix}.$$

A similar calculation is used to generate the maintenance cost vector associated with the optimal strategy. The maintenance costs when the pavement is in condition state 1 and 3 are 1 and 40. The maintenance cost associated with state 2 is then the weighted average of these two costs is then $0.6435 \cdot 1 + 0.3565 \cdot 40 = 14.904$. The vector describing the maintenance costs is then

$$\begin{bmatrix} 1 & 14.904 & 40 \end{bmatrix}.$$

5.3.2 Calculating the average annual maintenance cost

The optimal transition probability matrix is necessary to calculate the average annual maintenance cost (and the associated variance) which in turn determines the time necessary to reach steady state. The following example will illustrate how these values are calculated.

Condition state probabilities can be determined by the relationship $X(t+1) = X(t)P$. If the initial condition state vector is $X(0) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ (where the pavement is initially in excellent condition) and the optimal transition probability matrix is

$$\begin{bmatrix} 0.6 & 0.4 & 0 \\ 0 & 0.6 & 0.4 \\ 0.8 & 0.2 & 0 \end{bmatrix}$$

then,

$$X(1) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0 & 0.6 & 0.4 \\ 0.8 & 0.2 & 0 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 & 0 \end{bmatrix}.$$

By defining the cost vector (C) for the optimal maintenance strategy as $\begin{bmatrix} 1 & 3 & 40 \end{bmatrix}$ the expected cost at each epoch can be calculated with the following vector equation

$$\begin{aligned} E(X^t) &= X(t)C^T \\ E(X^1) &= \begin{bmatrix} 0.6 & 0.4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 40 \end{bmatrix} = 1.8. \end{aligned}$$

The variance calculations are equally as straightforward. Variance is defined as $Var(X) = E\left(X - E(x)\right)^2$. This relationship can be reduced to $Var(X) = E(X^2) - [E(X)]^2$. Thus to calculate $Var(X^t)$ we need $E\left((X^t)^2\right)$ which is simply $X(t)C^{2^T}$ where C^2 is a vector made up of elements of vector C, each of which is squared. The variance is then

$$\begin{aligned}
Var(X^t) &= X(t)C^{2^T} - \left[X(t)C^T\right]^2 \\
Var(X^1) &= \begin{bmatrix} 0.6 & 0.4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 9 \\ 1600 \end{bmatrix} - \left[\begin{bmatrix} 0.6 & 0.4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 40 \end{bmatrix} \right]^2 \\
&= 4.2 - 3.24 \\
Var(X^1) &= 0.96.
\end{aligned}$$

Table 5.3 shows the condition state probabilities as well as the corresponding expected value and variance calculations over time. Figure 5.1 is a plot of the expected maintenance costs over time given that the pavement began in condition state 1. The expected annual maintenance cost for other initial starting conditions can also be calculated. Figure 5.2 shows how maintenance costs change depending on the pavement's initial condition. Note that regardless of the starting condition the systems all eventually reach the same steady state value. It should be pointed out that the steady state average annual maintenance cost is equal to the value of the objective function in the optimized linear programming model.

5.3.3 Total expected costs and variance

When a contractor is bidding on a long term maintenance contract the key concern is the timing and costs of maintenance over the life of the contract. This section develops the formulas necessary to calculate the expected value of the maintenance costs as well as the associated variance. To simplify the development of this formulation the traditional non-discounted form of these equations are be developed first and then extended to include discounting. The first formula developed is the expected value of the total costs associated with the contract. The base data are the condition state probabilities for each condition state.

Table 5.3: Expected value and variance calculations

Time	π_1^t	π_2^t	π_3^t	$E(X)$ $\$/m^2$	$E(X^2)$ $(\$/m^2)^2$	$Var(X)$ $(\$/m^2)^2$
0	1	0	0	1	1	0
1	0.6000	0.4000	0.0000	1.8000	4.2000	0.9600
2	0.3600	0.4800	0.1600	8.2000	260.6800	193.4400
3	0.3440	0.4640	0.1920	9.4160	311.7200	223.0589
4	0.3600	0.4544	0.1856	9.1472	301.4096	217.7383
5	0.3645	0.4538	0.1818	8.9962	295.2643	214.3334
6	0.3641	0.4544	0.1815	8.9875	294.8601	214.0857
7	0.3637	0.4546	0.1818	8.9978	295.2709	214.3105
8	0.3636	0.4546	0.1818	9.0006	295.3854	214.3752
9	0.3636	0.4545	0.1818	9.0003	295.3754	214.3702
10	0.3636	0.4545	0.1818	9.0000	295.3643	214.3640
11	0.3636	0.4545	0.1818	9.0000	295.3628	214.3632
12	0.3636	0.4545	0.1818	9.0000	295.3634	214.3635
13	0.3636	0.4545	0.1818	9.0000	295.3637	214.3636

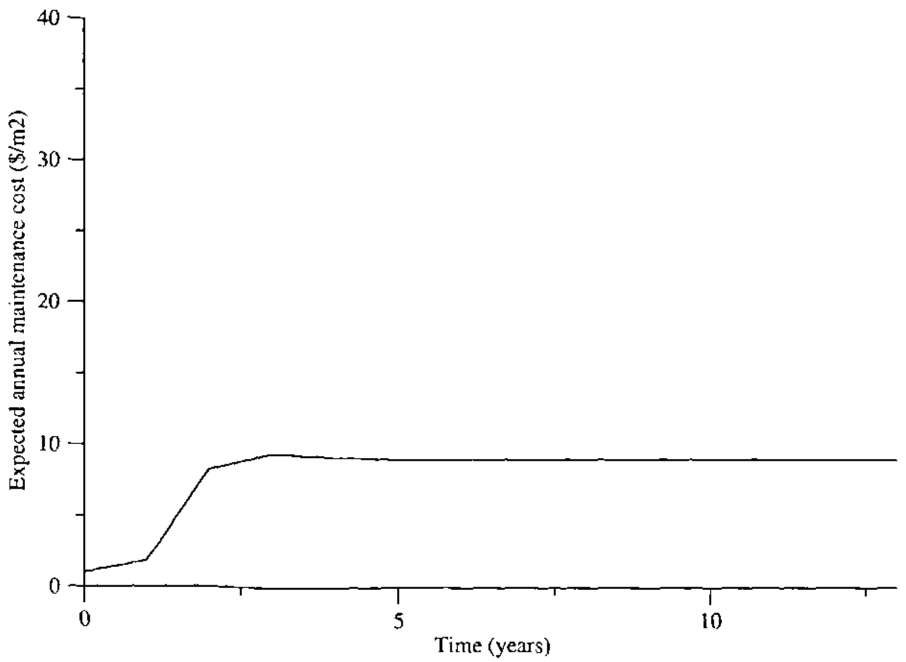

Figure 5.1: Expected annual maintenance cost over time when pavement begins in condition state 1

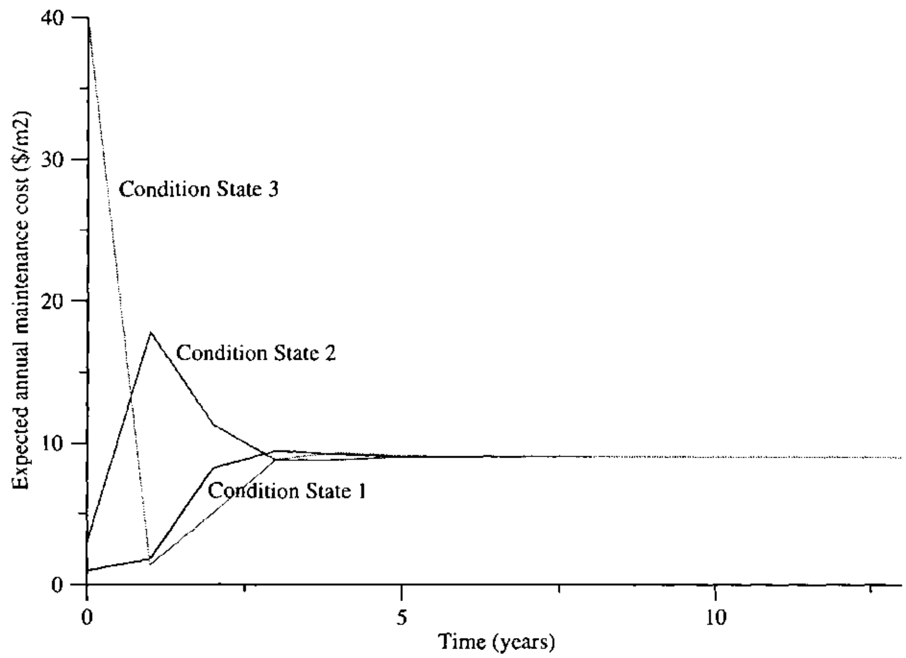

Figure 5.2: Expected annual maintenance costs as a function of time and initial pavement condition

The first term to be defined is X^t; the random variable describing the condition state distribution as a vector at time t. The maintenance cost when the system is in state i is defined as g_i. For this formulation, maintenance costs are independent of time; the costs are strictly a function of the condition state. The probability of being in condition state i at time t is $P(X^t = i)$. The expected maintenance cost at any time t is then

$$E(X^t) = \sum_i P(X^t = i)g_i. \tag{5.1}$$

It can be shown that $E\left[\sum X^t\right] = \sum\left[E(X^t)\right]$ regardless whether X^t represents dependent or independent condition state vectors (Grassmann, 1981). Consequently the expected total cost can be calculated as follows:

$$E\left[\sum_t X^t\right] = \sum_t \sum_i P(X^t = i)g_i. \tag{5.2}$$

Variance of the total cost can not be generated by summing the variance for each year unless the condition of the pavement each year is independent of the condition in the previous year. This is not the case for a Markov process. Therefore, the variance of the total cost must account for the covariance as shown in equation 5.3

$$Var\left[\sum_{t=1}^{m} X^t\right] = \sum_{t=1}^{m} Var(X^t) + 2\sum_{t=1}^{m}\sum_{r=1}^{t} Cov(X_r, X_t) \tag{5.3}$$

where the covariance can be calculated by

$$Cov(X_r, X_t) = \sum_i \sum_j (g_i - \mu_r)(g_j - \mu_n)\pi_i^r P_{ij}^{n-r}, \;\; t > r. \tag{5.4}$$

Note that μ_r is the expected maintenance cost for time step r, μ_n is the expected maintenance cost for time step n, π_i^r is the condition state probability for state i at time step r and P^{n-r} is the $(n-r)^{th}$ power of the transition probability matrix; P_{ij}^{n-r} is the transition probability from state i to state j associated with this matrix. The variance for each time step can be calculated via

$$\begin{aligned} Var(X^t) &= E\left((X^t)^2\right) - \left[E(X^t)\right]^2 \\ &= \sum_i P(X^t = i)g_i^2 - \left[P(X^t = i)g_i\right]. \end{aligned}$$

Calculating the covariance can be found more efficiently using the iterative relationship developed by Grassmann Grassmann (1987). Grassman has shown that

$$\sum_{t=1}^{m}\sum_{r=1}^{t} Cov(X_r, X_t) = \sum_{t=1}^{m}\sum_{j\epsilon S}(g_i - u_t)W_j^t \tag{5.5}$$

where,

$$W_j^0 = 0$$

$$W_j^t = \sum_{i\epsilon S}\left(W_i^{t-1} + (g_i - u_{t-1})\pi_i^{t-1}\right)P_{ij} \tag{5.6}$$

The only term in this equation not yet defined is p_{ij} which is simply the transition probability from state i to state j.

The variance for each time step can be calculated via

$$\begin{aligned} Var(X^t) &= E\left((X^t)^2\right) - \left[E(X^t)\right]^2 \\ &= \sum_i P(X_i^t)g_i^2 - \left[P(X_i^t)g_i\right]. \end{aligned}$$

The following sample calculations will illustrate the usefulness of Grassmann's recursive approach to calculating covariance. A sample calculation using the traditional approach will act as the base case. This example will only look at three time steps. The optimal transition probability matrix is

$$\begin{bmatrix} 0.6 & 0.4 & 0 \\ 0 & 0.6 & 0.4 \\ 0.8 & 0.2 & 0 \end{bmatrix}$$

and the associated maintenance costs are $\begin{bmatrix} 1 & 3 & 40 \end{bmatrix}$. Table 5.4 shows the probability distribution at each time step as well as the expected maintenance costs and variance.

Table 5.4: Expected maintenance costs and variance

	t=0	t=1	t=2	t=3
π_1^t	1	0.6	0.36	0.344
π_2^t	0	0.4	0.48	0.464
π_3^t	0	0	0.16	0.192
E(X)	1	1.8	8.2	9.416
E(X^2)	1	4.2	260.68	311.72
Var(X)	0	0.96	193.44	223.06

The variance of the totals can be found by calculating

$$Var(X^1 + X^2 + X^3) = \sum_{t=1}^{3} Var(X^t) + 2\left[Cov(X^1, X^2) + Cov(X^1, X^3) + Cov(X^2, X^3)\right] \tag{5.7}$$

where the covariance is

$$Cov(X^r, X^n) = \sum_i \sum_j (g_i - \mu_r) g_j \pi_i^r P_{ij}^{n-r}, n > r. \tag{5.8}$$

For $Cov(X^1, X^2)$

$$g = \begin{bmatrix} 1 \\ 3 \\ 40 \end{bmatrix}$$

$$\mu_1 = 1.8$$

$$\pi^1 = \begin{bmatrix} 0.6 \\ 0.4 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0 & 0.6 & 0.4 \\ 0.8 & 0.2 & 0 \end{bmatrix}.$$

M^1 will be defined as the results from the product $(g_i - \mu_r)(g_j)$ for all i, j and can be found in Table 5.5. M^2 will then be defined as $\pi_i^1 \cdot P_{ij}$ for all i, j which incidentally is the joint probability matrix for X^1 and $X^2(P(X^1 = i, X^2 = j))$:

$$M^2 = \begin{bmatrix} 0.36 & 0.24 & 0 \\ 0 & 0.24 & 0.16 \\ 0 & 0 & 0 \end{bmatrix}$$

Table 5.5: Covariance calculation (X^1, X^2) - M^1

	$g_1 = 1$	$g_2 = 3$	$g_3 = 40$
$g_1 - \mu_1 = -0.8$	-0.8	-2.4	-32
$g_2 - \mu_1 = 1.2$	1.2	3.6	48
$g_2 - \mu_1 = 38.2$	38.2	114.6	1528

The covariance between time step 1 and 2 is then

$$Cov(X^1, X^2) = \sum_i \sum_j M_{ij}^1 M_{ij}^2 = 7.68. \tag{5.9}$$

For $Cov(X^1, X^3)$

$$g = \begin{bmatrix} 1 \\ 3 \\ 40 \end{bmatrix}$$

$$\mu_1 = 1.8$$

$$\pi^1 = \begin{bmatrix} 0.6 \\ 0.4 \\ 0 \end{bmatrix}$$

$$P^2 = PP = \begin{bmatrix} 0.36 & 0.48 & 0.16 \\ 0.32 & 0.44 & 0.24 \\ 0.48 & 0.44 & 0.08 \end{bmatrix}$$

$$M^1 = \begin{bmatrix} -0.8 & -2.4 & -32 \\ 1.2 & 3.6 & 48 \\ 38.2 & 114.6 & 1528 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 0.216 & 0.288 & 0.096 \\ 0.128 & 0.176 & 0.096 \\ 0 & 0 & 0 \end{bmatrix}$$

where M^1 is the same as in the previous covariance calculation and M^2 is the joint probability matrix for X^1 and X^3

$$Cov(X^1, X^3) = \sum_i \sum_j M^1_{ij} M^2_{ij} = 1.4592. \tag{5.10}$$

For $Cov(X^2, X^3)$

$$g = \begin{bmatrix} 1 \\ 3 \\ 40 \end{bmatrix}$$

$$\mu_2 = 8.2$$

$$\pi^2 = \begin{bmatrix} 0.36 \\ 0.48 \\ 0.16 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0 & 0.6 & 0.4 \\ 0.8 & 0.2 & 0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} -7.2 & -21.6 & -288 \\ -5.2 & -15.6 & -208 \\ 31.8 & 95.4 & 1272 \end{bmatrix}$$

$$Cov(X^2, X^3) = \sum_i \sum_j Mij^1 Mij^2 = -41.9712. \tag{5.11}$$

Table 5.6: Covariance calculation (X^2, X^3) - M^1

	$g_1 = 1$	$g_2 = 3$	$g_3 = 40$
$g_1 - u_2 = -7.2$	0.216	0.144	0
$g_2 - u_2 = -5.2$	0	0.288	0.192
$g_3 - u_2 = 31.8$	0.128	0.032	0

Now that all of the variables have been determined (the variances can be found in Table 5.4 and the covariances in equations 5.9, 5.10, and 5.11), the variance of the total cost can be calculated from equation 5.7

$$\begin{aligned} Var(X^1 + X^2 + X^3) &= (0.96 + 193.44 + 223.0594) + \\ &\quad 2(7.68 + 1.4592 - 41.9712) \\ &= 351.795. \end{aligned}$$

$$\begin{aligned} E(X^0 + X^1 + X^2 + X^3) &= 1 + 1.8 + 8.2 + 9.416 \\ &= 20.416. \end{aligned}$$

Calculating the covariance under Grassmann's recursive fomulation is quite straight forward once one establishes a systematic procedure. The first step is to calculate W_j^t (where j is the condition state and t the time step) as formulated in equation 5.6.

The best approach is to calculate W_j^t in two steps. First calculate $(g_i - u_{t-1})\pi_i^{t-1}$ (for simplification purposes this term will be labelled Ω_i^{t-1}) for each state and time step, and then calculate W_j^t for each state and time step. Once W_j^t is determined, the variance must be calculated as defined in equation 5.5.

Thus, if $(g_i - u_t)W_i^t$ is calculated for all condition states, and then summed for all condition states and time steps the covariance for the total maintenance costs will be known. Table 5.7 shows the calculations for the same three time step period in the previous example.

Table 5.7: Covariance calculations

	$t=0$	$t=1$	$t=2$	$t=3$	Totals
π_1^t	1	0.600	0.360	0.344	
π_2^t	0	0.400	0.480	0.464	
π_3^t	0	0.000	0.160	0.192	
$E(X^t) = \mu_t$	1	1.800	8.200	9.416	20.416
$E([X^t]^2)$	1	4.200	260.680	311.720	
$Var(X)$	0	0.960	193.440	223.059	417.459
$Std(X)$	0	0.980	13.908	14.935	
$[g_1 - \mu_t]\pi_1^{t-1}$		0	-0.480	-2.592	
$[g_2 - \mu_t]\pi_2^{t-1}$		0	0.480	-2.496	
$[g_3 - \mu_t]\pi_3^{t-1}$		0	0.000	5.088	
W_1^t	0	0	-0.288	2.496	
W_2^t	0	0	0.096	-1.536	
W_3^t	0	0	0.192	-0.960	
$[g_1 - \mu_t]\pi_1^{t-1}W_1^t$	0	0	2.074	-21.006	
$[g_2 - \mu_t]\pi_2^{t-1}W_2^t$	0	0	-0.499	9.855	
$[g_3 - \mu_t]\pi_3^{t-1}W_3^t$	0	0	6.106	-29.361	
Cov	0	0	7.680	-40.512	-32.832

From Table 5.7 the total expected cost is

$$E_t(\sum_t X^t) = 20.416$$

and the variance of the total cost is

$$\begin{aligned} Var(\sum X^t) &= 417.459 + 2(-32.832) \\ &= 351.795. \end{aligned}$$

As would be expected these two methods generate the same results.

5.3.4 Total expected costs and variance with discounting

By definition, long term maintenance contracts are for extended periods of time. In addition these contracts are significant in their monetary value. The combination of significant amounts of money being spent over extended periods of time implies that the time value of money (discounting) should be accounted for in the calculation of the total costs of a long term maintenance contract. If discounting is ignored, then future costs of maintenance will be seriously undervalued, and short term decision making would wrongly lead to defering maintenance required today to the future. Given the extended time horizons associated with long term maintenance constracts, the total expected cost and variance must include a discount rate that is applicable throughout the life of the contract. The following is a derivation of the formulas necessary to calculate the present value of the total expected cost of maintenance contract as well as the associated variance.

It has been shown in section 5.3.3 (equation 5.1) that the expected cost associated with a maintenance strategy is

$$E(X^t) = \sum_i P(X^t = i) g_i,$$

where g_i is equal to the maintenance cost of being in state i at any time t. If the discount factor β is defined as $\beta = \frac{1}{1+R}$ where R is the discount rate, then the present value of the expected cost at time t is

$$\begin{aligned} E_{PV}(X^t) &= \sum_t P(X^t = i)\beta^t g_i^t \\ &= \beta^t \sum P(X^t = i) g_i^t \\ &= \beta^t E(X^t). \end{aligned}$$

It was also shown in section 5.3.3 (equation 5.2) that the expected value of the total cost of a project can be found by

$$E\left[\sum_t X^t\right] = \sum_t E(X^t)$$

$$E\left[\sum_t X^t\right] = \sum_t \sum_i P(X^t = i) g_i.$$

If the discount factor is applied to each expected value then

$$\begin{aligned}
E_{PV}\left[\sum_t X^t\right] &= \sum_t \beta^t \sum_i P(X^t = i) g_i \\
E_{PV}\left[\sum_t X^t\right] &= \sum_t \beta^t E(X^t = i) \\
E_{PV}\left[\sum_t X^t\right] &= \sum_t E_{PV}(X^t).
\end{aligned}$$

As stated in section 5.3.3 the non-discounted variance equation (5.3) was defined as

$$Var\left[\sum_{n=0}^{m-1} X_n\right] = \sum_{n=0}^{m-1} Var\left[X_n\right] + 2\sum_{n=0}^{m-1}\sum_{r=0}^{n-1} Cov\left[X_r, X_n\right]$$

The equivalent discounted version of this equation must be developed in parts. The first step is to focus on the variance at each time step.

$$\sum_{n=0}^{m-1} Var\left[X_n\right]$$

Variance can be determined by

$$\begin{aligned}
Var(X^t) &= E\left(\left[X^t - E(X^t)\right]^2\right) \\
Var(X^t) &= E\left[(X^t)^2\right] - \left[E(X^t)\right]^2.
\end{aligned}$$

The discounted value of $E\left[(X^t)^2\right]$

$$\begin{aligned}
E_{PV}\left[(X^t)^2\right] &= \sum_i \left[g_i \beta^t\right]^2 P(X^t = i) \\
&= (\beta^t)^2 \sum_i \left[g_i\right]^2 P(X^t = i) \\
E_{PV}\left[(X^t)^2\right] &= \beta^{2t} E\left[(X^t)^2\right] \\
E_{PV}(X^t) &= \beta^t E(X^t)
\end{aligned}$$

$$\left[E_{PV}(X^t)\right]^2 = \left[\beta^t E(X^t)\right]^2$$
$$Var_{PV}(X^t) = \beta^{2t} E\left[(X^t)^2\right] - \left[\beta^t E(X^t)\right]^2$$

$$E_{PV}\left[(X^t)^2\right] = \sum_i \left[g_i \beta^t\right]^2 P(X^t = i)$$
$$E_{PV}\left[(X^t)^2\right] = (\beta^t)^2 \sum_i \left[g_i\right]^2 P(X^t = i)$$
$$E_{PV}\left[(X^t)^2\right] = \beta^{2t} E\left[(X^t)^2\right].$$

Similarly, the present value of $[E(X^t)]^2$is

$$E_{PV}(X^t) = \beta^t E(X^t)$$
$$\left[E_{PV}(X^t)\right]^2 = \left[\beta^t E(X^t)\right]^2.$$

Consolidating these values determines the sum of the variances for each time step

$$Var_{PV}(X^t) = \beta^{2t}\left[E\left[(X^t)^2\right] - \left[E(X^t)\right]^2\right]$$
$$Var_{PV}(X^t) = \beta^{2t}\left[Var(X^t)\right].$$

The discounted version of Grassmann's recursive covariance function is the last element to be defined. Grassmann's original function for W_j^t was as follows

$$\sum_{t=1}^{m}\sum_{r=1}^{t} Cov(X_r, X_t) = \sum_{t=1}^{m}\sum_{j \epsilon S}(g_i - \mu_t)W_j^t$$

where

$$W_j^0 = 0$$

$$W_j^t = \sum_{i \epsilon S}\left(W_i^{t-1} + (g_i - \mu_{t-1})\pi_i^{t-1}\right) P_{ij}. \tag{5.12}$$

All values in equation 5.12 deal with present values for time step t except for μ_{t-1} which is the average maintenance cost for the previous time step. Since this amount is moved forward in time its value must be increased by β^{-1}

$$W_{PV_j}^{t} = \sum_{k \epsilon S} \left[W_k^{m-1} + \left(g_k - \frac{\mu_{m-1}}{\beta} \right) \pi_k^{m-1} \right] P_{kj}.$$

The covariance calculations deal with the covariance at each time step. These must be discounted to a present value

$$\sum_{n=0}^{m-1} \sum_{r=0}^{n-1} Cov_{PV}\left[X_r, X_n\right] = \sum_{n=0}^{m-1} \beta^n \sum_j \left(g_j - \mu_n\right) W_{PV_j}^n.$$

Since discounted formulas have been developed for both the variance and covariances at each time step, they can be consolidated to determine the present value of the variance for the total maintenance cost:

$$Var\left[\sum_{n=0}^{m-1} \beta^n X_n\right] = \sum_{n=0}^{m-1} \beta^{2n} Var\left[X_n\right] + 2 \sum_{n=0}^{m-1} \beta^n \sum_j \left(g_j - \mu_n\right) W_{PV_j}^n.$$

Once again these calculations are illustrated with an example. Table 5.4 provides the source data, but this time the costs are calculated over a ten year period. Table 5.8 consists

Table 5.8: Expected cost and variance calculations over a ten year time horizon

t	π_1^t	π_2^t	π_3^t	$E(X^t)$ $\$/m^2$	$Var(X^t)$ $(\$/m^2)^2$	$Var_{PV}(X^t)$ $(\$/m^2)^2$	$E_{PV}(X)$ $\$/m^2$
0	1	0	0	1	0	0	0.926
1	0.600	0.400	0.000	1.800	0.960	0.823	1.667
2	0.360	0.480	0.160	8.200	193.440	142.184	7.593
3	0.344	0.464	0.192	9.416	223.059	140.565	8.719
4	0.360	0.454	0.186	9.147	217.738	117.637	8.470
5	0.364	0.454	0.182	8.996	214.333	99.278	8.330
6	0.364	0.454	0.182	8.987	214.086	85.016	8.322
7	0.364	0.455	0.182	8.998	214.310	72.964	8.331
8	0.364	0.455	0.182	9.001	214.375	62.574	8.334
9	0.364	0.455	0.182	9.000	214.370	53.646	8.334
10	0.364	0.455	0.182	9.000	214.364	45.991	8.333
Totals				83.545	1921.036	820.678	77.359

of the base calculations (expected value and variance over a ten year period) that would be generated from a optimal maintenance strategy and Table 5.9 illustrates the covariance

Table 5.9: Discounted covariance calculations

t	Ω_1^t	Ω_2^t	Ω_3^t	W_1^t	W_2^t	W_3^t	$[g_1 - \mu_t]W_1^t$	$[g_2 - \mu_t]W_2^t$	$[g_3 - \mu_n]W_3^t$	Cov_{PV}
0				0	0	0	0	0	0	0
1	-0.080	0.000	0.000	-0.048	-0.032	0.000	0.038	-0.038	0.000	0.000
2	-0.566	0.422	0.000	-0.369	-0.012	0.156	2.654	0.060	4.966	6.584
3	-2.828	-2.811	4.983	2.193	-1.944	-1.129	-18.459	12.475	-34.528	-32.160
4	-3.154	-3.327	5.727	3.102	-2.627	-2.108	-25.275	16.150	-65.048	-54.520
6	-3.177	-3.047	5.504	2.294	-3.043	-2.336	-18.324	18.218	-72.451	-45.723
8	-3.170	-3.054	5.504	1.755	-3.708	-2.570	-14.040	22.251	-79.677	-38.610
9	-3.171	-3.055	5.506	1.499	-4.037	-2.705	-11.991	24.224	-83.862	-35.832
10	-3.171	-3.055	5.506	1.237	-4.364	-2.837	-9.896	26.183	-87.939	-33.189
									Total	-327.08

calculations for each time step.

5.4 Quantifying risk

In the previous section the expected value for the total maintenance costs in present dollars over a ten year time horizon was calculated to be $77.359/m^2 and the standard deviation was $12.90/m^2 (equivalent to a variance of 166.52 ($/m^2)^2). The question that must be addressed now is how is this information useful to the contractor?

When submitting a price for a long term maintenance contract the contractor will submit a price that will account for expected project cost, profit and the uncertainty associated with the project costs. If the contractor is large enough that reasonable cost overruns on the project will not bankrupt the company and the contractor is risk neutral then the contractor's bid will be equal to the expected value of the total cost of the contract plus some amount to allow a reasonable rate of return (which in turn is influenced by such factors as the number of competing bids).

Few decision makers are risk neutral in the face of significant uncertainty. Thus, it is useful to have a tool which will quantify this uncertainty. By generating the probability distribution associated with total maintenance costs through our knowledge of the mean and variance of these costs we can provide a tool which will measure this uncertainty.

Take the distribution shown in Figure 5.3. This distribution is the cumulative probability distribution for an expected present value cost of $77.36/m^2 with a standard deviation of $12.90/m^2. By plotting the cumulative probability of the project costs we can directly determine the probability the costs will exceed some value x. Imagine a scenario where a contractor wishes to submit a price where there is only a twenty percent chance that the project's costs will exceed the tendered price. The contractor can determine the bid amount to meet this criteria by drawing a horizontal line at the eighty percent point on the cumulative probability axis to the cumulative probability curve, and then projecting a vertical line downward to the project cost axis. Figure 5.4 illustrates this scenario. Based on a cumulative probabilty of eighty percent, the corresponding maintenance cost is $88.23/m^2. This information could then be used to determine the probability distribution across expected project profits as a function of bidding price.

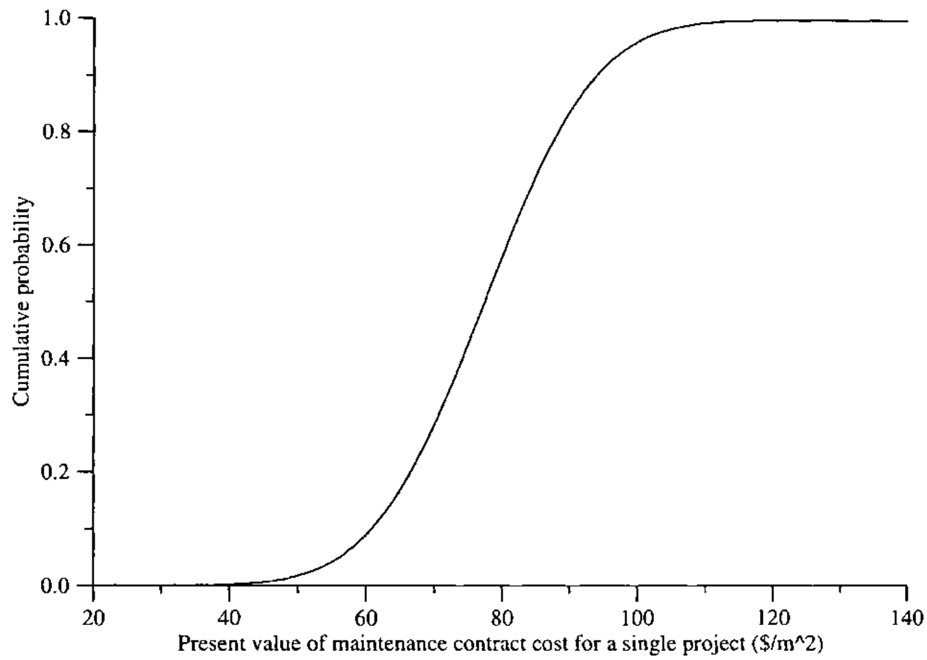

Figure 5.3: Cumulative probability distribution for expected cost of $77.369/m^2$ and variance of 166.52 $(\$/m^2)^2$.

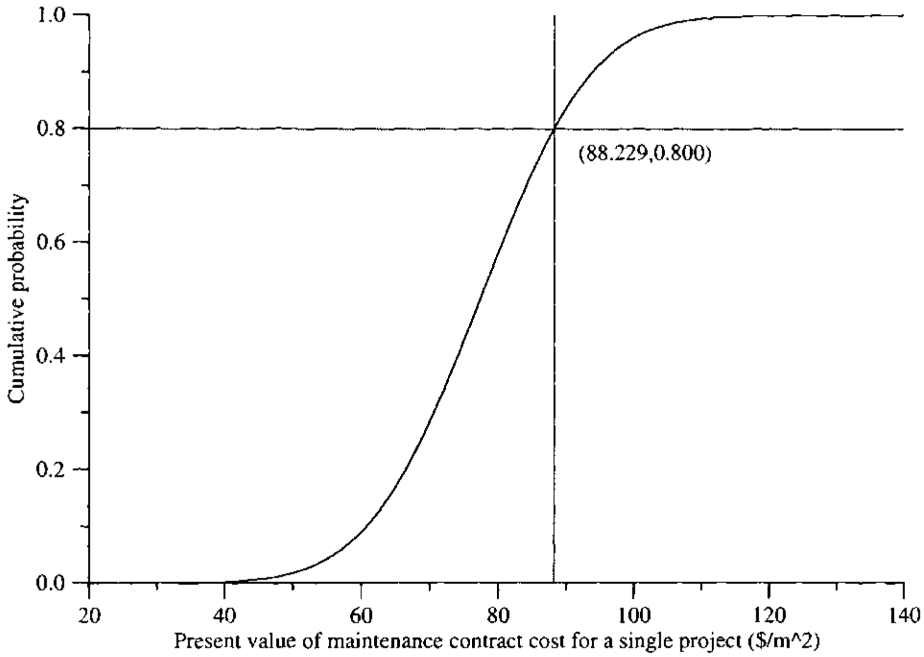

Figure 5.4: Selecting a cumulative percentage to estimate maintenance costs for a project.

5.5 Quantifying risk for a long term maintenance contract

Long term maintenance contracts are simply a portfolio of maintenance projects. From a contractor's perspective the total cost and its associated variance are key pieces of information. From the total cost and variance, the contractor can assess expected cost of a long term maintenance contract and the risk associated with that contract.

This dissertation has established a procedure for the calculation of the total expected cost and the associated variance for a project. If it can be assumed that each project is independent and indentically distributed, then the sum of the costs for all projects is approximately normally distributed; this is a result of the central limit theorem (Grassmann, 1981). Consequently the total cost of all the projects can be found by simply summing the total expected cost of each project. Similarly, the total variance can be determined by summing the variance associated with each project.

Individual projects are determined based on uniformity; each project must be a homogeneous section with uniform usage and environment. This suggests that the independence assumption is reasonable. Thus, total expected costs and variance can be determine via summation of expected project costs and variance. If the projects are found to be influenced by a common influence such as the weather or traffic conditions, then the uncertainties associated with the performance are conditional. If this is the case, the total expected costs and total variance can still be determined through the use of conditional expectations (the expected value conditioned on a specific event) (Grassmann, 1981).

5.6 Conclusions

The key objective of this chapter was to illustrate how the risk associated with a long term maintenance contract can be quantified. The first step in this process was to illustrate how an optimal transition probability matrix can be derived from the results generated by the optimal maintenance schedule. From this transition matrix it was illustrated how pavement condition can be predicted under the optimal maintenance policy given an initial condition state profile. The data generated by this predictive model illustrates both the time necessary to reach steady state as well as the expected maintenance costs (and the associated variance). Before applying the data to calculate risk, the formula for calculating the present value of the expected total project cost and its variance had to be developed.

Chapter 6

APPLYING THE METHODOLOGY TO A FULL SCALE MODEL

6.1 Introduction

This chapter is an overview of the methodology used to quantify the financial risk asssociated with long term maintenance contracts from a contractor's perspective. Essentially, this chapter will be an application of the techniques introduced in the three previous chapters. The results will act as a base case for sensitivity analysis comparisons in the following chapter.

The structure of this chapter will closely follow the schematic in Figure 6.1. The chapter will begin with an introduction to the distresses, treatments and performance curves. Performance curves typically represent the pavement's performance under routine maintenance. From the performance curves transition probability matrices can be directly developed, but for the other treatments, expert judgement (or some other methdology) is required. The mathematical model that generates the optimal maintenance strategy requires transition probability matrices that are treatment specific. These aggregate matrices are generated and then the optimal maintenance strategy is determined. The optimal maintenance strategy provides the lowest cost steady state maintenance strategy; essentially the strategy defining which treatment to apply to a project when the pavement is in

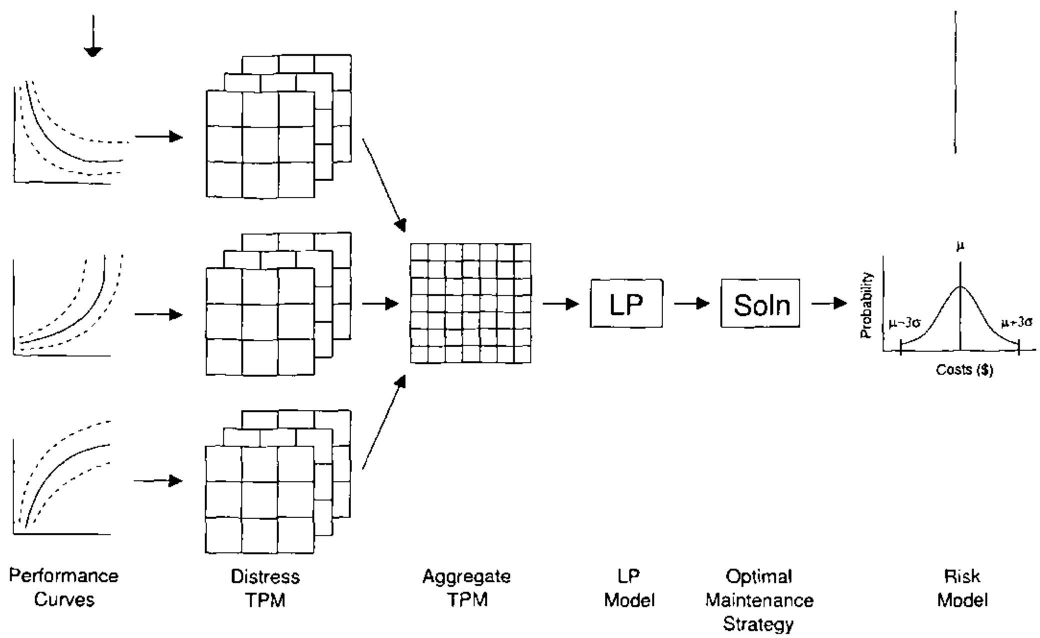

Figure 6.1: A schematic of the methodology to quantify the uncertainty associated with long term maintenance contracts

a specific condition. From this strategy the following can be determined:

- the time the pavement takes to reach steady state,
- the present value of the maintenance costs over a finite time horizon,
- as well as the financial risk associated with the various bid prices for this project.

6.2 Model inputs

The model that generates the optimal maintenance strategy has three components: treatment costs, pavement performance, and performance constraints. This section will review the treatments (types and costs) and the distresses (which describe pavement performance over time) used in the base model.

6.2.1 Distresses

Pavement management models (and in general, asset management models) require distress indices or metrics to describe how pavement (assets) deteriorate over time. The key concerns in describing asset deterioration are severity and extent of various distresses. Severity describes the level or intensity of the distress whereas extent describes the percentage of the asset that is distressed. The set of distresses applied in the base model have been selected from the 2000 Saskatchewan Highways and Transportation Surface Condition Rating Manual (SHT, 2000) and can be found in Table 6.1 and the corresponding condition state limits are found in Table 6.2.

Table 6.1: Distresses included in full scale model

Number	Distress	Abbreviation
1	Rutting	RUT
2	Transverse Cracking - Deterioration	DET
3	Transverse Cracking - Depth	DEP
4	Surface	SRF
5	Roughness	RUF
6	Fatigue block cracking	CRK

Table 6.2: Distress limit

Number	Distress Abbreviation	Measure	Excellent	Good	Poor
1	RUT	rut depth (mm)	< 5	5 - 11	> 11
2	DET	crack width (mm)	< 3	3 -10	> 10
3	DEP	crack depth (mm)	< 3	3 - 10	> 13
4	SRF	pickouts/m^2	< 10	10 - 50	> 50
5	RUF	IRI (mm/m)	< 10	10 - 100	> 100
6	CRK	block size (m)	> 1	0.4 - 1	< 0.4

The pavement management model requires each distress to be described by condition states. Increasing the number of condition states increases the accuracy of any deterioration model. Unfortunately, the exponential nature of the model makes relatively detailed models unsolvable without significant computing power. Given the number of treatments (six), a three condition state model was selected to ensure tractability. These states were labelled as excellent, good, and poor. Quantitatively the boundaries between states were determined on a distress by distress basis. It should be noted that Saskatchewan Highways and Transportation (SHT) uses a four condition state system (None, Slight, Moderate and Extreme). Given this research applied a three condition state model, the SHT condition state boundaries were modified by the author on an arbitrary basis. Consequently the condition state boundaries found in this chapter bear only a moderate similarity to the SHT guidelines.

For completeness, each distress will be defined and then the boundary between condition states quantified.

Rutting is defined by SHT as longitudinal surface depressions. These depressions develop in the wheel paths due to repeated load applications. Rutting severity is measured in terms of depth. A rut depth less than 5 mm is considered to be excellent and a depth greater than 11 mm is poor. Good is defined as any level of rutting between these two extremes.

Transverse cracking describes the situation where cracks run at right angles to a pavement's centreline. Transverse cracking is a unique distress in that two specific characteristics are measured; crack width and crack depth. Crack severity is measured by crack width. A crack width less than 3 mm is considered excellent and a width greater than 10 mm is poor. The second characteristic of transverse cracks is the depth of each crack. A pavement with a crack depth of less than 3 mm is considered to be in excellent condition. A depth of more than 13 mm is considered to be poor.

The technical definition for how surface condition is measured is "an assessment of the pavement surface with respect to ravelling, segregation and loss of aggregate." Basically this is an attempt to measure how much of the original surface has been lost. The severity of surface distress is measured in terms of pickouts per square metre; excellent is less than 10 pickouts per square metre and poor is greater than 50 pickouts per square metre.

Roughness is normally used as a measure of riding comfort. Severity is measured using the International Roughness Index (IRI) format of millimetres of accumulated displacement per metre (mm/m). The IRI is a profile-based roughness index which is linearly proportional to roughness. The severity limits chosen for this research were any amount less than 10 mm/m was considered to be excellent and any amount greater than 100 mm/m would be poor.

Fatigue block cracking is a catch-all classification for any cracking mechanism that affects a roadway's performance and structural capacity. Fatigue cracking creates blocks of pavement, subsequently the severity is measured in terms of the length of the short side (the less or the length or width) of the block. If the short side of the blocking is greater than 1 metre the pavement is considered in excellent condition relative to fatigue cracking. Pavements with blocks with a short side less than 0.4 metres are considered to be in poor condition.

6.2.2 Treatments

Treatments are used to either slow or reverse the deterioration associated with distresses that occur over time. The list of treatments applied in this research were supplied by SHT. It should be noted that the maintenance costs provided by SHT did not vary with asset condition. In other words, it did not cost any more to apply the same treatment to an asset in very poor condition as one in excellent condition. To illustrate that the model applied in this research could accommodate the more general case where pavement maintenance costs varied with asset condition, a linear scaling of costs was assumed. The base assumption was that a pavement in the worst condition state would cost thirty percent more to maintain than one in the best condition state. Maintenance costs were calculated based on this function

$$SHT\ cost \times \left[1 + \frac{condition\ state}{3^6} \times 0.3\right].$$

The treatments and the base costs can be found in Table 6.3.

Table 6.3: Treatments included in full scale model

Number	Treatment	Abbreviation	Unit Cost ($\$/m^2$)
1	Routine maintenance	ROU	0.02
2	Thermopatch	THR	0.44
3	Flush seal	FLU	0.22
4	Spot seals	SPS	0.30
5	Strip seals	STS	0.72
6	Micro surface	MSU	2.97
7	Full seal	FUS	1.31
8	Spot overlay and seal	SOS	3.27
9	Thin overlay	THO	9.02
10	Structural overlay	STO	20.08

6.2.3 Performance under routine maintenance

Severity and extent of distresses are classifications for measuring pavement deterioration. Our list of treatments define the methods available to retard or reverse the deterioration. The performance curves which will be introduced in this section are graphical summaries of pavement performance with respect to a specific distress-treatment combination. Typically performance curves are generated for pavement performance under routine maintenance; this will be the case for this section as well.

The data source for performance curves can include empirical data, mechanistic (physics) predictive models, or subjective/expected data (expert judgement). The curves introduced in this section are based on expert judgement, but have also been modified to illustrate that a variety of performance curve shapes can be accommodated. Tables 6.4 through 6.9 provide the numeric performance data illustrated in Figures 6.2 through 6.7. There is a one to one matching between figures, tables and distresses.

6.3 Calculating transition probabilities from performance curves

As can be seen in Figure 6.1, once the performance curves are established, the next step is to determine the transition probabilities. The mechanics for calculating the transition probabilities for any distress under routine maintenance were explicitly covered in Sec-

Table 6.4: Performance data for pavement rutting

Time	Nominal (mm)	LB (mm)	UB (mm)
0	0	0	0
1	0.25	0	0.5
2	0.5	0	1.0
3	1.2	0	2.1
4	1.7	0	3.0
5	2.4	0.25	3.8
6	2.8	0.7	5.1
7	3.6	1.1	6.8
8	4.5	1.4	8.0
9	5.2	1.8	9.5
10	6.0	2.3	11.3
11	6.8	2.7	13.0
12	7.8	3.1	14.6
13	9.5	4.2	17.0
14	11.7	5.5	19.0
15	15	7.5	22.5

Table 6.5: Performance data for transverse cracking - deterioration (crack width)

Time	Nominal (mm)	LB (mm)	UB (mm)
0	0	0	0
1	0.1	0.05	0.4
2	0.3	0.1	0.75
3	0.5	0.18	1.0
4	0.7	0.25	1.5
5	0.8	0.35	2.0
6	1.0	0.5	2.5
7	1.4	0.65	3.0
8	1.6	0.75	3.64
9	2.0	0.8	4.27
10	2.4	0.95	5.4
11	2.75	1.0	6.71
12	3.5	1.4	8.60
13	5.0	1.8	10.85
14	7.0	2.5	13.0
15	10	4	15

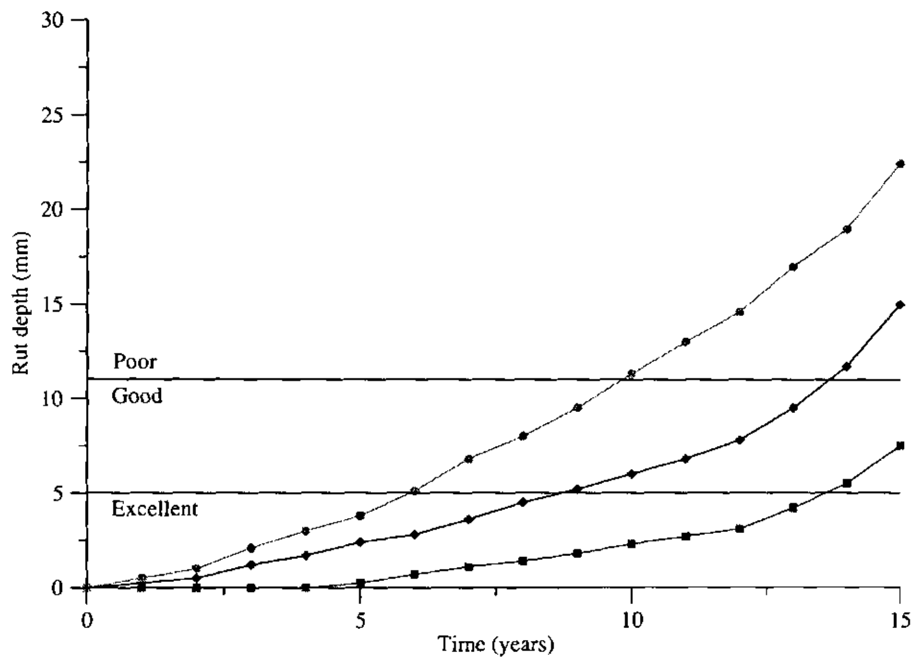

Figure 6.2: Increase in rut depth over time

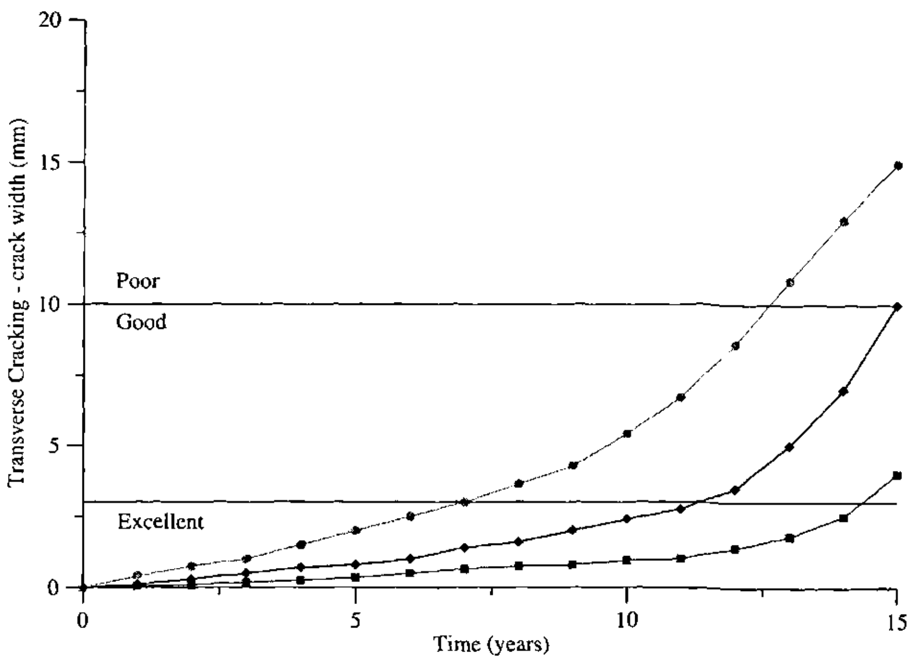

Figure 6.3: Transverse cracking - increase in crack width over time

Table 6.6: Performance data for transverse cracking - depth

Time	Nominal (mm)	LB (mm)	UB (mm)
0	0.00	0.00	0.00
1	0.65	0.28	1.00
2	1.30	0.56	2.00
3	1.95	0.84	3.00
4	2.60	1.12	4.00
5	3.25	1.40	5.00
6	3.90	1.68	6.00
7	4.55	1.96	7.00
8	5.20	2.24	8.00
9	5.85	2.52	9.00
10	6.50	2.80	10.00
11	7.15	3.08	11.00
12	7.80	3.36	12.00
13	8.45	3.64	13.00
14	9.10	3.92	14.00
15	9.75	4.20	15.00

Table 6.7: Performance data for pavement cracking

Time	Nominal (mm)	LB (mm)	UB (mm)
0	999	999	999
1	30.000	60.000	15.000
2	10.000	20.000	5.000
3	5.000	10.000	2.500
4	1.000	2.000	0.500
5	0.500	1.000	0.250
6	0.300	0.600	0.150
7	0.150	0.300	0.075
8	0.100	0.200	0.050
9	0.075	0.150	0.038
10	0.050	0.100	0.025
11	0.040	0.080	0.020
12	0.030	0.060	0.015
13	0.030	0.060	0.015
14	0.015	0.040	0.010
15	0.015	0.020	0.005

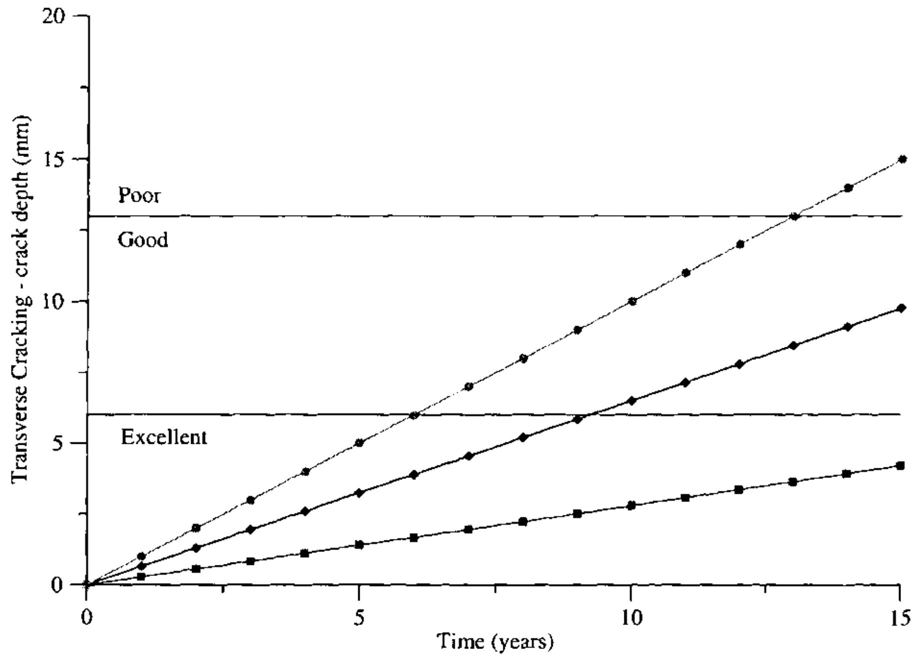

Figure 6.4: Transverse cracking - increase in crack depth over time

Table 6.8: Performance data for surface condition

Time	Nominal (pickouts/m^2)	LB (pickouts/m^2)	UB (pickouts/m^2)
0	0	0	0
1	1.8	0.6	2.3
2	2.5	1.5	5.0
3	4.0	2.0	7.0
4	5.0	3.0	9.5
5	7.5	5.0	12.0
6	9.0	7.0	15.0
7	12.0	9.0	18.0
8	15.0	11.0	21.0
9	18.0	13.0	25.0
10	21.0	15.0	28.0
11	25.0	18.0	32.0
12	28.0	21.0	38.0
13	33.0	25.0	44.0
14	40.0	28.0	50.0
15	50.0	34.0	60.0

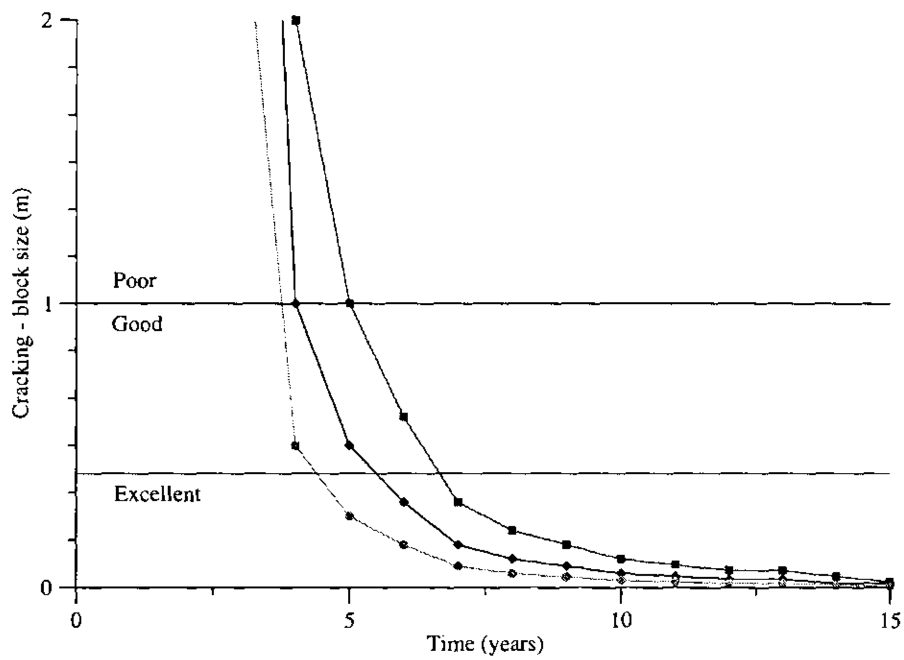

Figure 6.5: Increase in cracking over time

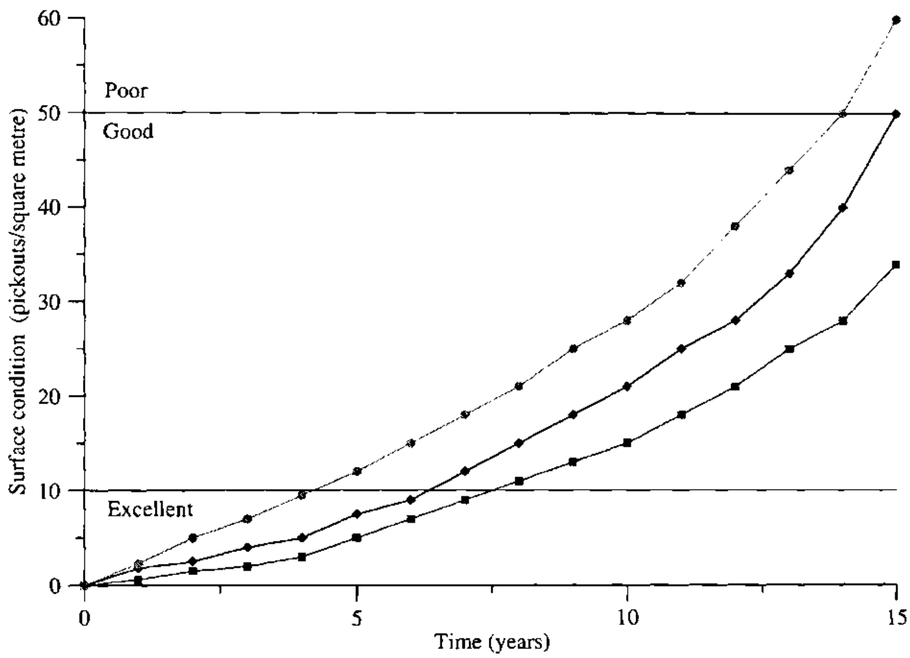

Figure 6.6: Change in surface condition over time

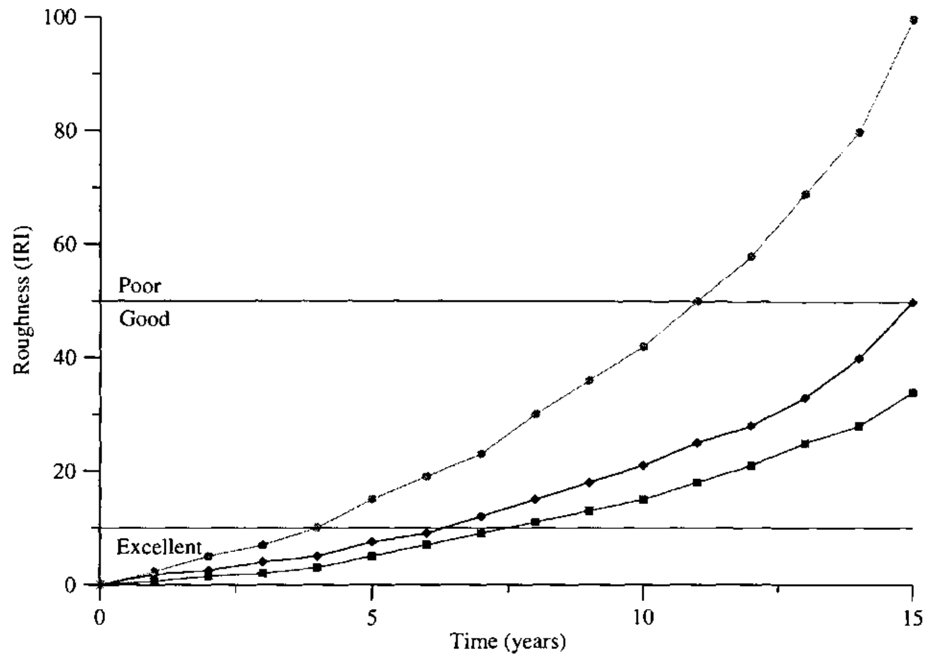

Figure 6.7: Increase in roughness over time

Table 6.9: Performance data for roughness

Time	Nominal (mm/m)	LB (mm/m)	UB (mm/m)
0	0.0	0.0	0.0
1	1.8	0.6	2.3
2	2.5	1.5	5.0
3	4.0	2.0	7.0
4	5.0	3.0	10.0
5	7.5	5.0	15.0
6	9.0	7.0	19.0
7	12.0	9.0	23.0
8	15.0	11.0	30.0
9	18.0	13.0	36.0
10	21.0	15.0	42.0
11	25.0	18.0	50.0
12	28.0	21.0	58.0
13	33.0	25.0	69.0
14	40.0	28.0	80.0
15	50.0	34.0	100.0

tion 3.3. The transition probabilities determined from the the performance data introduced in Section 6.2.3 can be found in Table 6.10. The matrices in Table 6.10 model pavement deterioration under routine maintenance. Transition probabilities for other forms of treatment must also be determined; this is often done by relying on the judgement of an experienced pavement engineer.

The following methodology for generating transition probabilities for other treatments was selected to provide a consistent process. The importance of maintaining a consistent approach will become very clear in Chapter 7 where the sensitivity analysis will be reviewed. The procedure consisted of three phases. The first step was to generate a set of ten generic transition probability matrices. The thought process behind this step was that a treatment can have one of three effects: return an asset to excellent condition; regardless of its initial condition, it can have no more effect than routine maintenance; and it can be somewhere in between these two extremes. The objective was to methodically define a

Table 6.10: Transition probabilities under routine maintenance

Distress	Transition Probability Matrix
Rutting	$\begin{bmatrix} 0.932 & 0.068 & 0.000 \\ 0.000 & 0.817 & 0.183 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Transverse cracking - deterioration	$\begin{bmatrix} 0.960 & 0.040 & 0.000 \\ 0.000 & 0.832 & 0.168 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Transverse cracking - depth	$\begin{bmatrix} 0.949 & 0.051 & 0.000 \\ 0.000 & 0.783 & 0.217 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Cracking	$\begin{bmatrix} 0.769 & 0.231 & 0.000 \\ 0.000 & 0.516 & 0.484 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Surface condition	$\begin{bmatrix} 0.930 & 0.070 & 0.000 \\ 0.000 & 0.691 & 0.309 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Roughness	$\begin{bmatrix} 0.935 & 0.065 & 0.000 \\ 0.000 & 0.791 & 0.209 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$

discrete continuum in a generic fashion. The routine treatment matrix was defined as

$$TPM_1 = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & 1 \end{bmatrix}$$

where a, b, c, d, and e are the transition probabilities for routine maintenance. In other words, if the asset is initially in condition state 1, the probability of making the transition to condition state 2 is b. Similarly the probability of making the transition from condition state 2 to condition state 3 is e. The matrix describing full replacement (i.e. regardless of its initial state the asset is always returned to an excellent condition state) is then

$$TPM_{10} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

The eight other transition probability matrices which map the transition between these two matrices were defined somewhat arbitrarily by the author, but attempts were made to maintain monotonicity. Table 6.11 illustrates the ten generic transition probability matrices.

Before these matrices could be applied, a full enumeration of treatments and distresses was required as summarized in Table 6.12. Once the treatment-distress pairings were generated, the effectiveness of each treatment with respect to each distress was considered and arbitrarily ranked on a scale of one to ten. A score of one corresponds to a treatment which is only as effective as routine maintenance. A score of ten corresponds to a treatment that is equivalent to full replacement. Table 6.12 maps the effectiveness of each treatment to each distress. By having such a mapping, the appropriate generic TPM from Table 6.11 can be selected for each treatment-distress pairing.

The final step in this process is the application of the rankings. For example, take the combination of distress 4 (surface) and treatment 5 (strip seals). For these pairing it was

Table 6.11: Generic transition probability matrices

Label	Matrix
TPM_1	$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & 1 \end{bmatrix}$
TPM_2	$\begin{bmatrix} a & b & c \\ 0.6 & 0.4 & 0 \\ 0.6 & 0.35 & 0.05 \end{bmatrix}$
TPM_3	$\begin{bmatrix} a & b & c \\ 0.65 & 0.35 & 0 \\ 0.65 & 0.30 & 0.05 \end{bmatrix}$
TPM_4	$\begin{bmatrix} a & b & c \\ 0.7 & 0.3 & 0 \\ 0.7 & 0.25 & 0.05 \end{bmatrix}$
TPM_5	$\begin{bmatrix} a & b & c \\ 0.75 & 0.25 & 0 \\ 0.75 & 0.20 & 0.05 \end{bmatrix}$
TPM_6	$\begin{bmatrix} a & b & c \\ 0.80 & 0.20 & 0 \\ 0.80 & 0.15 & 0.05 \end{bmatrix}$
TPM_7	$\begin{bmatrix} a & b & c \\ 0.85 & 0.15 & 0 \\ 0.85 & 0.15 & 0.05 \end{bmatrix}$
TPM_8	$\begin{bmatrix} 0.95 & 0.05 & 0 \\ 0.90 & 0.10 & 0 \\ 0.90 & 0.05 & 0.05 \end{bmatrix}$
TPM_9	$\begin{bmatrix} 1 & 0 & 0 \\ 0.95 & 0.05 & 0 \\ 0.90 & 0.10 & 0 \end{bmatrix}$
TPM_{10}	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Table 6.12: Establishing the effectiveness of each treatment as applied to each distress

Treatment	Rutting	Deterioration	Depth	Surface	Roughness	Cracking
Routine	1	1	1	1	1	1
Thermopatch	2	2	5	2	1	6
Flush seal	1	1	1	2	2	1
Spot seals	2	2	2	2	1	1
Strip seals	3	2	1	4	3	1
Micro surface	5	4	1	5	7	6
Full seal	1	2	1	6	8	2
Sport overlay and seal	4	8	4	7	4	4
Thin Overlay	7	6	8	8	9	8
Structural Overlay	10	10	10	10	10	10

determined by the author that Matrix 4 would be applicable. From Table 6.11 Matrix 4 has the following structure:

$$\begin{bmatrix} a & b & c \\ 0.7 & 0.3 & 0 \\ 0.7 & 0.25 & 0.05 \end{bmatrix}$$

The routine maintenance transition probability matrix for crackings can be found in Table 6.10

$$\begin{bmatrix} 0.930 & 0.070 & 0 \\ 0 & 0.691 & 0.309 \\ 0 & 0 & 1 \end{bmatrix}.$$

The transition probability matrix for cracking when a flush seal is applied consists of the top row of the routine maintenance transition probability matrix and the bottom two rows of Matrix 4 from the list of generic transition probability matrices

$$\begin{bmatrix} 0.930 & 0.070 & 0 \\ 0.700 & 0.300 & 0 \\ 0.700 & 0.250 & 0.05 \end{bmatrix}.$$

This process is repeated for all combinations of treatments and distresses in Table 6.12 and requires some form of expert or subjective judgement.

6.4 Creating aggregate transition probabilities

The model that generates the optimal maintenance strategy requires a single transition probability matrix for each treatment. To accomplish this we must aggregate the distress transition probability matrices from section 6.3. In a nutshell the procedure is as follows:

- map the condition states of the distress level transition probability matrices to the aggregate transition probability matrix through enumeration, and
- calculate the product of the distress level transition probabilities based on the distress to aggregate mapping.

The mapping from the distress level to aggregate level is the same for all treatments. Thus the distress level tuple $\{1,1,1,1,1,1\}$ maps to $\{1\}$ at the aggregate level. Similarly $\{3,3,3,3,3,3\}$ maps to $\{3^6\}$. Table 6.13 is an example that shows a sample of the necessary mapping; and Table 6.14 illustrates the probability calculations.

Table 6.13: A partial mapping of the distress level condition states to aggregate level condition states

Aggregate State	Rutting	Deterioration	Depth	Surface	Roughness	Cracking
289	2	1	2	3	1	1
290	2	1	2	3	1	2
291	2	1	2	3	1	3
292	2	1	2	3	2	1
293	2	1	2	3	2	2
294	2	1	2	3	2	3
295	2	1	2	3	3	1
296	2	1	2	3	3	2
297	2	1	2	3	3	3
298	2	1	3	1	1	1
299	2	1	3	1	1	2
300	2	1	3	1	1	3

Table 6.14: Aggregate transition probability sample calculations

	Rutting	Deterioration	Depth	Surface	Roughness	Cracking	Aggregate
From	2	2	2	3	3	1	376
To	3	3	2	3	3	1	700
p(i,j)	0.183	0.168	0.783	1	1	0.769	0.01851

6.5 Calculating the optimal maintenance strategy

Chapter 4 introduced the linear programming formulation of the Markov decision process. The example problem in Chapter 4 consisted of 1 distress with 3 condition states, 3 treatments and 1 performance constraint. The size of this model was trivial; 9 variables and 54 coefficients (36 of those being non-zero coefficients). Fortunately the linear programming formulation is scalable and essentially the same formulation was applied to the full scale version of this problem. The full scale model consists of 6 distresses with 3 condition states, 10 treatments, and 2 performance constraints. The size of this model is large (but not extraordinary given the capabilities of today's solvers); 3^6*10=7290 variables and over 5.3 million coefficients (although only 600,000 were non-zero). As an aside, given this model's formulation, the number of variables in the model is determined by the relationship $c^d t$ where c is the number of condition states, d is the number of distresses and t is the number of treatments. The total number of coefficients is dictated by the function $(c^d t)(c^d + k + 1)$ where k is the number of performance constraints added to the model. As one can see, increasing the number of distresses causes the model to grow exponentially.

Because the formulation was changed slightly with the addition of an additional performance constraint, it would be useful to include the exact optimization model:

$$\begin{aligned}
\min \; & \sum_{i=1}^{729} \sum_{k=1}^{10} c_i^k x_i^k \\
\text{subject to} \; & \sum_{i=1}^{729} \sum_{k=1}^{10} x_i^k = 1 \\
& \sum_{k=1}^{10} x_j^k = \sum_{i=1}^{729} \sum_{k=1}^{10} x_i^k p_{ij}^k \quad \forall \; j \\
& \sum_{i \epsilon E} \sum_{k} x_i^k \geq 0.2 \\
& \sum_{i \epsilon P} \sum x_i^k \leq 0.05.
\end{aligned}$$

In this model two performance constraints have been included. In the original model the only performance constraint limited the steady state solution to ten percent of the pavement reaching a poor condition state. The new model has two performance limits; at least twenty percent of the pavement must be in excellent condition and no more than five percent of the pavement can be in poor condition. Additional performance limits could be added to the model with minimal impact on solution times.

In the pilot study model (the example problem in section 4.4.3) the condition states were explicitly labelled excellent, good, and poor. For the full scale model the concepts of excellent, good, and poor were really only applicable to the distress level transition probability matrix. At the aggregate transition probability matrix level, the concepts of excellent, good and poor were less clearly defined. For the purposes of this research a scoring system was put into place where excellent, good and poor at the distress level were worth one, two and three points respectively. When combined at the aggregate level, a condition state of $\{E, E, E, E, E, E\}$ would be worth $1 + 1 + 1 + 1 + 1 + 1 = 6$ points. Similarly $\{P, P, P, P, P, P\}$ would be worth $3 + 3 + 3 + 3 + 3 + 3 = 18$ points. The boundaries for excellent, good, and poor were arbitrarily set; a score of nine or less was Excellent and poor was a score of fourteen or more.

6.6 Average annual maintenance costs

The linear programming model generates the optimal maintenance strategy. The objective function will equal the average annual cost at steady state. The cost at steady state is useful information, but to the owner or contractor responsible for maintaining the pavement a key concern is the maintenance cost each year leading up to steady state.

One of the outputs of the linear programming model is the optimal maintenance strategy. This strategy outlines which treatment to apply when the pavement is in a specific condition state. For the pilot model, creating an optimal maintenance strategy table was trivial (3 states and 3 treatments). For the full scale model the equivalent report easily exceeded ten pages. From an operational perspective these reports are important; they direct the field staff on how to react to pavement conditions as they develop. Table 6.15 is a high level summary of this data. What Table 6.15 illustrates is the likelihood that each treatment would be applied. For instance, at steady state routine maintenance would be applied 87.22% of the time. From a planning perspective the information that is important are the projected maintenance costs under the optimal maintenance strategy.

Table 6.15: Optimal maintenance strategy summary

Treatment	Abbreviation	Probability
Routine maintenance	ROU	0.8722
Thermopatch	THR	0.0018
Flush seal	FLU	0.0532
Spot seal	SPS	0.0728

To calculate the annual maintenance costs we need to generate the transition probability matrix that corresponds to the optimal maintenance strategy. Once again, for the pilot study this was a trivial task accomplished by inspection. For the full scale problem the procedure was automated (as described in Section 5.3.1. Even using relatively powerful desktop computing technology (Pentium III-900 MHz with 640MB of memory) this process could take in excess of 20 hours to complete.

Once completed, the optimal transition probability matrix provided the necessary data to calculate the average annual costs (as well as other statistics) over the life of the pavement. Figure 6.8 shows the increase in maintenance costs over time for a pavement that begins in condition state 1 (where condition state 1 is equivalent to a pavement where each individual distress is in excellent condition) and is treated following the optimal maintenance policy. The model that has been developed can also accommodate a stochastic starting point. Most calculations in this research calculate the steady state maintenance costs when the pavement begins in a condition state equivalent to a brand new pavement. In addition, the initial condition state is known with certainty. Because the transition probability matrix associated with the optimal maintenance policy is also associated with a steady state probability vector, the initial condition state is actually irrelevant to the steady state conditions. Figure 6.9 illustrates the average annual maintenance costs for a pavement where the initial condition is not known with certainty; all that is known for certain is that the initial state could be any state from 11 to 729 with equal likelihood. In both Figure 6.8 the steady state maintenance costs are $0.0579/m^2. This is an extreme example, but it does illustrate that the asset's condition at steady state is independent of the asset's initial condition when the optimal maintenance strategy is applied. In practise it is unlikely that these results could be replicated. The time periods necessary for the system to reach steady state are extremely long when compared to the maintenance/operating environment required under most situations. But, in general, by following the optimal maintenance policy the pavement will reach steady state, and steady state is independent of the initial starting condition of the pavement.

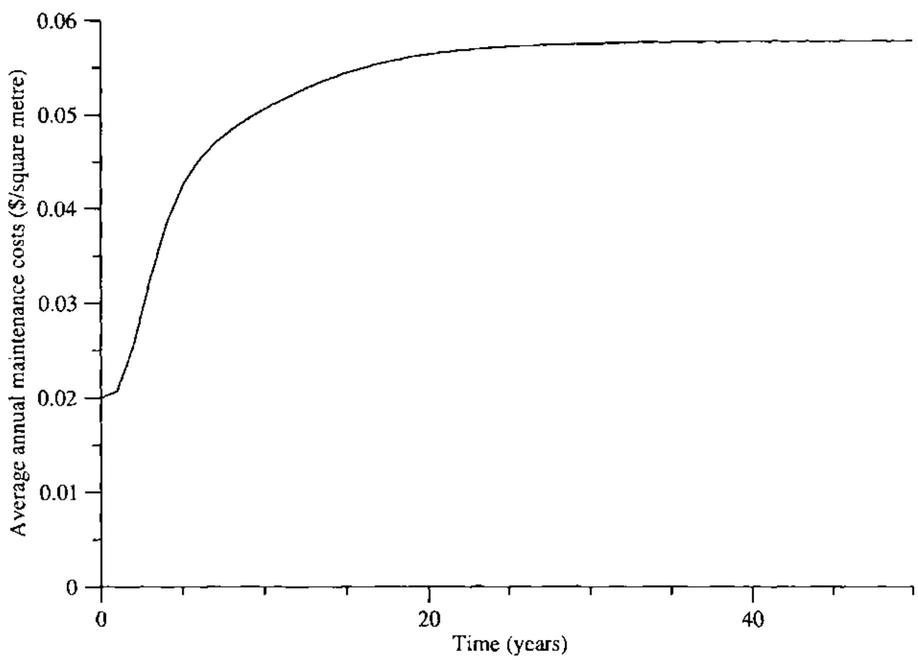

Figure 6.8: Maintenance costs over time for a pavement starting in condition state 1

and Figure 6.9

Once again, a key concern here are the costs over the life of the maintenance contract. Figures 6.8 and 6.9 show the annual costs for a single project over an extended period. Fo-

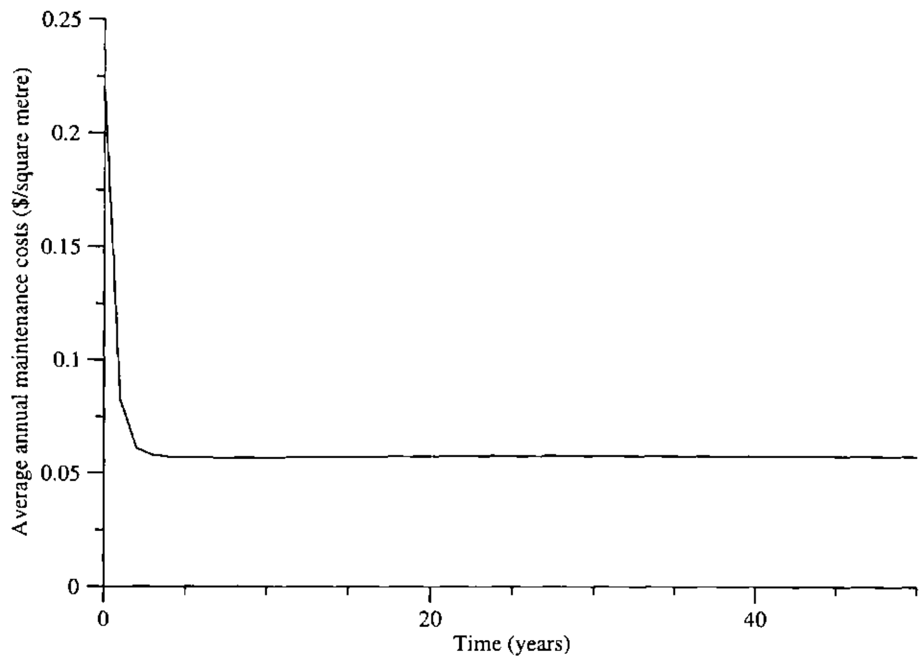

Figure 6.9: Maintenance costs over time for a pavement not starting in excellent condition

cusing strictly on this project (and not on an overall maintenance contract consisting of a number of such projects), the total expected costs of maintaining this stretch of pavement is the sum of the average annual maintenance costs over the maintenance period. To calculate the present value of these costs we need to apply a discount rate. For this research a discount rate of 8% was selected. Table 6.16 includes the original and discounted average annual maintenance costs corresponding to the first ten years of Figure 6.8. The total expected present worth cost of maintaining this pavement over ten years at a discount rate of 8% is \$0.2740/m^2.

Table 6.16: Average annual maintenance costs for first ten years of project's life.

Time (years)	Exp(X) (\$/m^2)	Exp_{PV}(X) (\$/m^2)
0	0.02	0.02
1	0.020679	0.019147
2	0.025439	0.021810
3	0.032337	0.025670
4	0.038256	0.028119
5	0.042427	0.028875
6	0.045183	0.028473
7	0.047057	0.027457
8	0.048451	0.026177
9	0.049602	0.024813
10	0.050622	0.023448
Totals	0.420053	0.27399

6.7 Quantifying project risk

Risk was previously defined as the likelihood of losing money. The costs calculated in the previous section were average costs. Thus it is known that there is some chance that these costs may be lower or higher. The objective now is to calculate the chance of these higher costs. As was illustrated in Section 5.4 both the expected value and variance are necessary to quantify the risk. Table 6.17 shows the non-discounted and discounted results for the expected value, variance and covariance over a ten year period. The total discounted variance can be found by summing the discounted annual variance plus twice the discounted covariance.

$$Var_{PV}\left(\sum_{n} X_n\right) = \sum_{n} Var_{PV}\left(X_n\right) + 2\sum_{n}\sum_{r} Cov_{PV}\left(X_r, X_n\right) \quad where \;\; r < n$$

Table 6.17: Discounted cost statistics

Time (years)	E(X) (\$/m^2)	Var(X) (\$/m^2)^2	Cov (\$/m^2)^2	E_{PV}(X) (\$/m^2)	Var_{PV}(X) (\$/m^2)^2	Cov_{PV} (\$/m^2)^2
0	0.02000	0.00000	0.00000	0.02000	0.00000	0.00000
1	0.02068	0.00013	0.00000	0.01915	0.00011	0.00000
2	0.02544	0.00124	0.00001	0.02181	0.00091	0.00001
3	0.03234	0.00285	0.00010	0.02567	0.00180	0.00008
4	0.03826	0.00421	0.00020	0.02812	0.00227	0.00015
5	0.04243	0.00517	0.00026	0.02888	0.00240	0.00018
6	0.04518	0.00583	0.00028	0.02847	0.00232	0.00018
7	0.04706	0.00632	0.00028	0.02746	0.00215	0.00017
8	0.04845	0.00671	0.00026	0.02618	0.00196	0.00014
9	0.04960	0.00704	0.00022	0.02481	0.00176	0.00011
10	0.05062	0.00734	0.00017	0.02345	0.00158	0.00008
Totals	0.42005	0.04684	0.00178	0.27399	0.01725	0.00110

The net result is that we know that the discounted expected value of the total costs is \$0.2740/m^2 with a standard deviation of \$0.1395/m^2.

Figure 6.10 is the cumulative probability distribution for our mean and standard deviation. From this distribution a contractor can determine the probability that told maintenance costs will be less than or equal to specific unit costs. The procedure is quite straight forward. The first step is to select a unit cost and then project vertically from the x-axis to the distribution. From the distribution one projects horizontally to the vertical axis. The vertical axis determines the probability that the average total maintenance costs will not be exceeded. The complementary probability $(1 - x\%)$ is the probability of the costs exceeding this amount. This is by definition the project risk.

As a simple example take Figure 6.10. If a contractor was to select a cost of \$0.40/m^2 as a tender price then the probability that maintenance costs will be less than or equal to this amount is roughly 81.7%. The probability of maintenance costs exceeding this amount would be 18.3%.

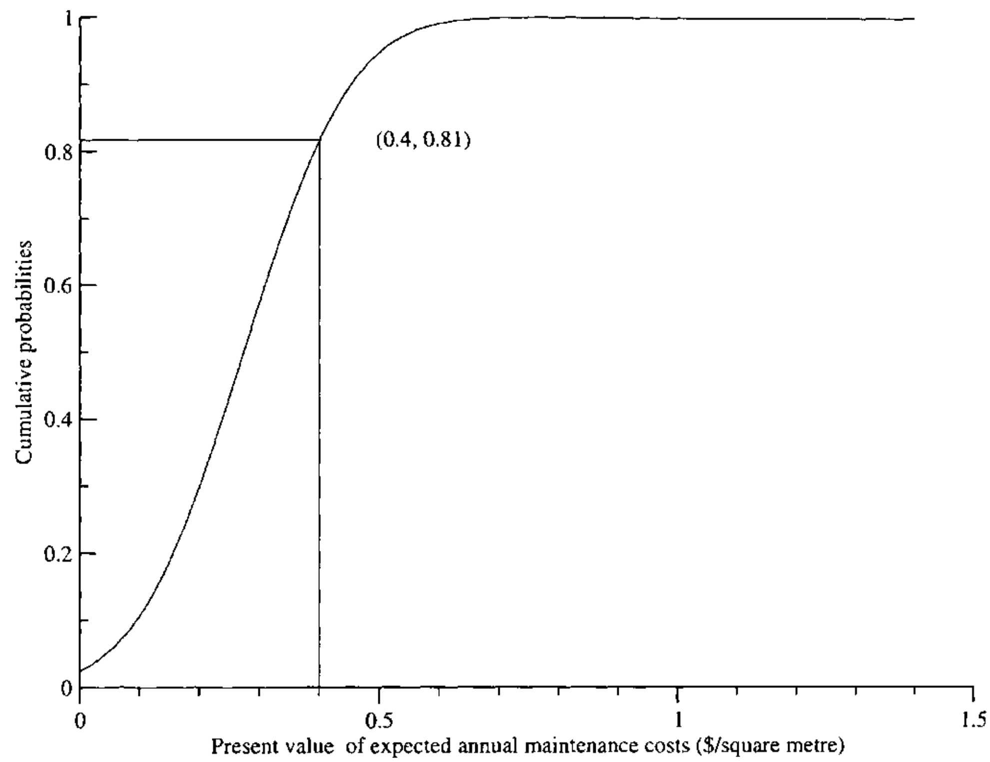

Figure 6.10: Cumulative probability distribution for base model

To further extend this example assume that the contractor is bidding on a project with a length of 100km. If there are roughly 7000m^2/km of roadway then there is a total of 700x10^{3}m^2 of pavement in this project. If the present worth of the expected maintenance costs over a ten year period at a discount rate of 8% is \$0.2740/m^2, this translates to a present value of $700x10^3m^2 * \$0.2740/m^2 = \$191,800$. By a similar calculation the standard deviation in unit costs (\$0.1395/m^2) is \$97,650. If the contractor is contemplating a bid of \$0.40/m^2 this would translate to a present worth of $700x10^3m^2 * \$0.40/m^2 = \$280,000$. The expected profit for this bid would be $\$280,000 - \$191,800 = \$88,200$. Given that the expected profit is known (\$88,200) and so is the standard deviation (\$97,650), a distribution for the present value of the expected profit can be generated. Figure 6.11 illustrates the cumulative probability distribution for profit given a bid price of \$0.40/m^2 (\$280,000).

6.8 Conclusions

The objective of this chapter was to apply the techniques introduced in previous chapters to a full scale model. The analysis began with the performance curves for six distresses. These performance curves were converted to transition probability matrices for routine maintenance and in turn nine other maintenance treatments. The transition probabilities for the various distress-treatment combinations were the basis for the aggregate transition probability matrices necessary for the linear programming model that generated the optimal maintenance strategy. From the optimal maintenance strategy we were able to determine the steady state average annual maintenance costs (\$0.0579/m^2) as well as the average annual maintenance costs before steady state was reached. By calculating the present value of these maintenance costs (and the associated variance) the expected value (and standard deviation) for the total maintenance costs over a fixed time horizon were also determined (\$0.2740/m^2 and \$0.1395/m^2 respectively).

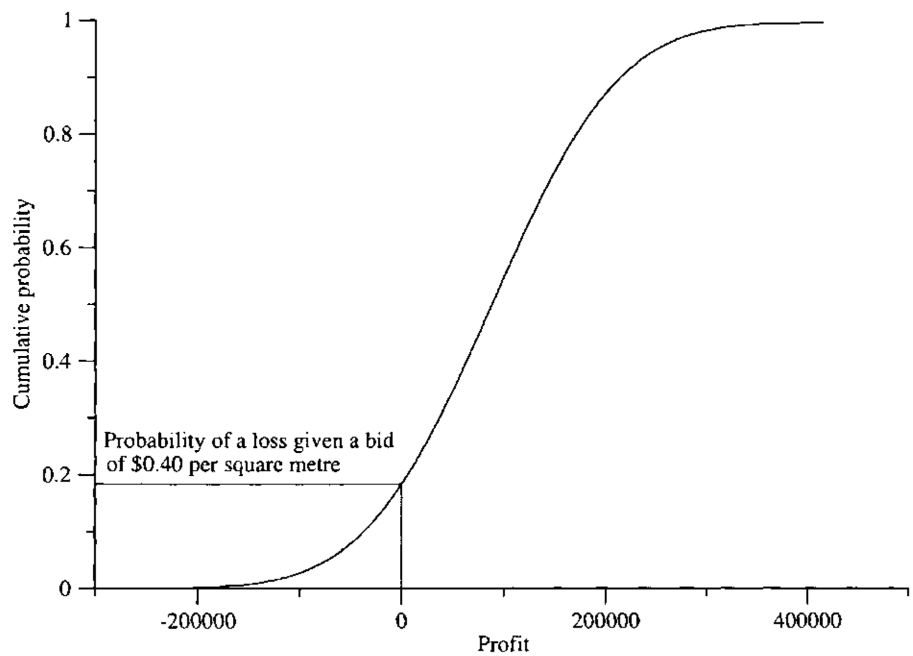

Figure 6.11: The probability distribution for profit based on a bid price of $\$0.40/\mathrm{m}^2$ for a typical project in a long term maintenance contract.

Chapter 7

SENSITIVITY ANALYSIS

7.1 Introduction

In its simplest form sensitivity analysis is an investigation of how changes in the values of a model's input variables impact the results. Sensitivity analysis allows us to identify the variables that have the most influence on the results. This insight provides direction for both operations and research. From an operations perspective the parameters that have the least or most tolerance to change can be identified. From a research perspective we can identify the parameters or performance variables that should be investigated so future performance can be influenced the most.

The objective of this chapter is to investigate the impact that changes in pavement performance, input costs, performance constraints, and maintenance treatment effectiveness have on the results relative to the base case. The sensitivity analysis described herein were a series of *what if* investigations where nine specific changes were made. The impact of these changes were noted relative to the base case and, where applicable, to each other. The analysis will illustrate the potential impact due to changes in these variables and will consequently act as the basis for suggestions for further research. The eight models (as well as the base case) that were investigated are identified in Table 7.1. It should be noted that the original intent was to only investigate the impact of changes in pavement performance (models 2, 3, and 4). But after some deliberation it was apparent that model sensitivity relative to maintenance treatment costs would provide some insight. As will be noted in each analysis, the optimal maintenance strategy appeared to be relatively insensitive to the changes introduced with models 2, 3, and 4. Consequently the approach taken in subsequent studies was to introduce model changes (increases in input costs, changes in performance constraints, and changes in treatment effectiveness) in search of what model component would impact the maintenance strategy the most. The description of each analysis follows the same basic structure. The first step is to identify how the values for the parameters in this model differed from the base model. This is followed with a brief discussion of the scenario each change is supposed to simulate or represent. The results due to these changes are then described and discussed relative to the base case and where applicable to other studies. Each section concludes with a review on how these changes affected the risk associated with a specific base tender price (where applicable).

Table 7.1: Sensitivity analysis models

Model	Description
Base	Base case
2	Improved performance
3	Reduced performance variance
4	Improved performance and reduced performance variance
5	Reduce difference treatment costs
6	Ten percent increase in all input costs
7	Change in performance constraints
8	Twenty percent increase in all input costs
9	Increased effectiveness of treatments

7.2 Models

7.2.1 Base case

The model introduced in Chapter 6 was used as the base case. For reference a review of the base case parameters is included. There were six distresses in each model. The distresses are identified in Table 7.2. Each model uses the same six distresses, and unless noted, the same performance data (transition probabilities). Similarly, each model includes the same set of ten treatments as identified in Table 7.3.

Table 7.2: Distresses

Number	Distress	Abbreviation
1	Rutting	RUT
2	Tranvserve cracking - deterioration	DET
3	Transverse cracking - depth	DEP
4	Cracking	CRK
5	Surface condition	SRF
6	Roughness	RUF

Table 7.3: Treatments

Number	Treatment	Abbreviation	Unit Costs ($\$/m^2$)
1	Routine maintenance	ROU	0.02
2	Thermopatch	THR	0.44
3	Flush seal	FLU	0.22
4	Spot seals	SPS	0.30
5	Strip seals	STS	0.72
6	Micro surface	MSU	2.97
7	Full seal	FUS	1.31
8	Spot overlay and seal	SOS	3.27
9	Thin overlay	THO	9.02
10	Structural overlay	STO	20.08

Table 7.3 describes the treatments, the abbreviation for each treatment and the base cost for each treatment. As a reminder, treatment costs were increased linearly to illustrate the fact that the model applied in this research could accommodate the more general case where treatment costs vary with the condition of the pavement. A more detailed description of this function can be found in Chapter 6.

The general structure of the model was fixed throughout the study. The model is a mathematical representation of pavement performance (change in distress over time), maintenance costs, and performance constraints:

$$\min \sum_{i=1}^{729} \sum_{k=1}^{10} c_i^k x_i^k$$

$$
\begin{aligned}
\text{subject to } \sum_{i=1}^{729}\sum_{k=1}^{10} x_i^k &= 1 \\
\sum_{k=1}^{10} x_j^k &= \sum_{i=1}^{729}\sum_{k=1}^{10} x_i^k p_{ij}^k \quad \forall\ j \\
\sum_{i\epsilon E}\sum_{k} x_i^k &\geq 0.2 \\
\sum_{i\epsilon P}\sum x_i^k &\leq 0.05.
\end{aligned}
$$

The optimal maintenance policy and the associated steady state costs can be found in Table 7.4. The expected maintenance cost at steady state was calculated to be \$0.0579/m^2. Based on the optimal maintenance policy and a pavement starting in new condition the present value of the annual average maintenance costs over a ten year period at an 8% discount rate was found to be \$0.274/m^2 and the standard deviation was \$0.1395/m^2. Based on a rough estimate of 7400 square metres per kilometer of pavement on a two way, two lane roadway the discounted expected costs are \$2027.60/km with a standard deviation of \$1032.30/km.

Table 7.4: Summary of the optimal maintenance strategy for the base case model

Treatment	Abbreviation	Probability
Routine maintenance	ROU	0.8722
Thermopatch	THR	0.0018
Flush seal	FLU	0.0532
Spot seal	SPS	0.0728
Cost		\$0.0579/m^2

7.2.2 Improved asset performance

The first analysis in the sensitivity analysis was designed to determine the impact of improving asset performance. From an application perspective, improving asset performance curves was defined as slowing the deterioration rate of the pavement. Effectively this analysis investigated the benefit of a longer lasting or more durable pavement.

From a graphical perspective better performing assets have flatter performance curves. Flatter performance curves were generated by adjusting the Lower Bound, Nominal and

Upper Bound values of the base case by one half, two thirds and three quarters respectively. For instance, in the base case, rutting value at time step 6 were 1.4mm, 4.5mm, and 8mm. For this analysis the rutting values were 0.7mm, 3mm, and 6mm. This was done for each distress with the exception of cracking. The nature of the cracking performance curve was such that adjustments to the bounds had minimal impact on the transition probability matrix. Given that changes to the inputs had little to no effect on the outputs, the inputs remained unchanged. The transition probability matrix that resulted from the conversion of these new curves to transition probabilities can be found in Table 7.5. Note that the base case probabilities have also been included for a consistency check.

If an asset is deteriorating slowly there will be a bias in the transition probability matrix towards the higher level condition states. In other words, the probability of the pavement leaving its present state will be quite low and the diagonal elements of the transition probability matrices will generally be large. The larger the diagonal elements the lower the deterioration rate. The net result is that the routine maintenance transition probability matrices in this analysis should have diagonal elements with larger probabilities relative to the base case probabilities. By inspection we can see that this is the case in Table 7.5.

As was shown in Chapter 6, the routine maintenance transition probabilities are the basis for calculating the transition probabilities for the other treatments. Consequently the changes to the routine maintenance transition probabilities due to flattening the performance curves also impacted the transition probabilities of the other treatments. This analysis focused strictly on adjustments to the performance data. The maintenance costs and performance constraints were not changed. As would be expected, the results in Table 7.6 show that a better performing pavement costs less to maintain.The present value of the average annual maintenance costs over a ten year period as well as the standard deviation were also calculated. These values are summarized in Table 7.7.

Figure 7.1 illustrates the cumulative probability curves associated with the mean and standard deviation for the present value of the total maintenance costs over a ten year time horizon for both the base case and the improved performance model. Figure 7.1 illustrates that the improved performance model provides a maintenance strategy which

Table 7.5: Transition probabilities generated from improved performance curves (Model 2)

Distress	Base case	Improved performance
Rutting	$\begin{bmatrix} 0.932 & 0.068 & 0.000 \\ 0.000 & 0.817 & 0.183 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.964 & 0.036 & 0.000 \\ 0.000 & 0.832 & 0.168 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Trans. crack. - deterioration	$\begin{bmatrix} 0.960 & 0.040 & 0.000 \\ 0.000 & 0.832 & 0.168 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.973 & 0.027 & 0.000 \\ 0.000 & 0.836 & 0.164 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Transverse crack. - depth	$\begin{bmatrix} 0.949 & 0.051 & 0.000 \\ 0.000 & 0.783 & 0.217 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.980 & 0.020 & 0.000 \\ 0.000 & 0.821 & 0.179 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Cracking	$\begin{bmatrix} 0.769 & 0.231 & 0.000 \\ 0.000 & 0.516 & 0.484 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.769 & 0.231 & 0.000 \\ 0.000 & 0.516 & 0.484 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Surface condition	$\begin{bmatrix} 0.930 & 0.070 & 0.000 \\ 0.000 & 0.691 & 0.309 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.936 & 0.064 & 0.000 \\ 0.000 & 0.749 & 0.251 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Roughness	$\begin{bmatrix} 0.935 & 0.065 & 0.000 \\ 0.000 & 0.791 & 0.209 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.946 & 0.054 & 0.000 \\ 0.000 & 0.820 & 0.180 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$

Table 7.6: Optimal maintenance strategy costs for improved performance as compared to the base case analysis (Model 2)

Treatment	Base case	Improved performance
Routine maintenance	0.8722	0.9081
Thermopatch	0.0018	0.0240
Flush seal	0.0532	0.0304
Spot seals	0.0728	0.0374
Strip seals		0.0001
Cost	\$0.0579/m^2	\$0.0512/m^2

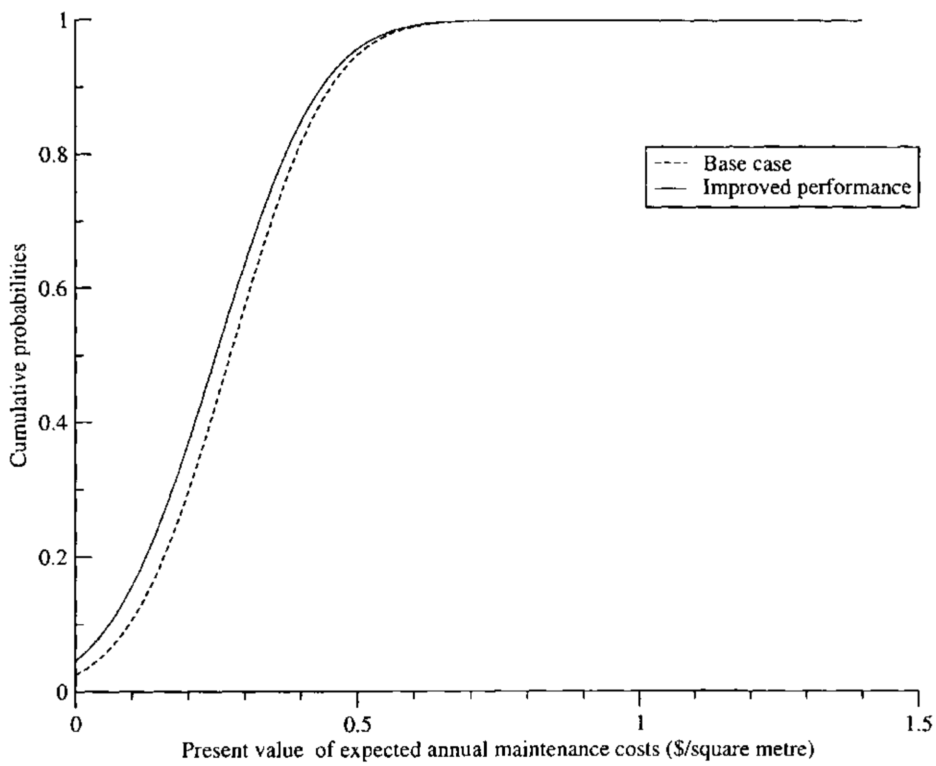

Figure 7.1: Cumulative probability curve comparison of the risk associated with the base case and improved performance (Model 2)

Table 7.7: Present value of annual expected maintenance costs over a ten year period for the improved performance (Model 2)

Time (years)	Discounted expected costs ($\$/m^2$)	Discounted variance $(\$/m^2)^2$	Discounted covariance $(\$/m^2)^2$
0	0.0200	0.0000	0.0000
1	0.0194	0.0002	0.0000
2	0.0220	0.0012	0.0000
3	0.0242	0.0018	0.0001
4	0.0253	0.0022	0.0002
5	0.0254	0.0023	0.0003
6	0.0247	0.0022	0.0003
7	0.0237	0.0020	0.0003
8	0.0225	0.0019	0.0003
9	0.0212	0.0017	0.0004
10	0.0200	0.0015	0.0004
Total	0.2484	0.0170	0.0023

stochastically dominates the base case; regardless of the unit cost selected, the flattened performance curves will provide a less risky proposition. As a comparison, if the contractor were to submit a tender price of $\$0.4/m^2$, the probability of the maintenance costs not exceeding this amount would be 0.85 (Figure 7.2). In other words, the probability of a contractor losing money with a $\$0.4/m^2$ tender price is 15%. More succinctly, the risk associated with a $\$0.4/m^2$ bid is 15%. In comparison, the risk associated with the base case was 18%.

7.2.3 Reducing performance variance

The next analysis was an investigation of how reducing the variance in pavement performance (tightening the performance curves) would impact on maintenance costs and risk. Reducing the variance in pavement performance could be accomplished by improving quality control on pavement inputs (such as aggregates) or construction practise.

To replicate this reduction in uncertainty the Lower Bound data for each distress was increased by twenty five percent and the Upper Bound data was decreased by twenty five percent. As an example, if the rutting distress for time step 8 had a Lower Bound value of 1.4mm and an Upper Bound value of 8mm, the values for this analysis would

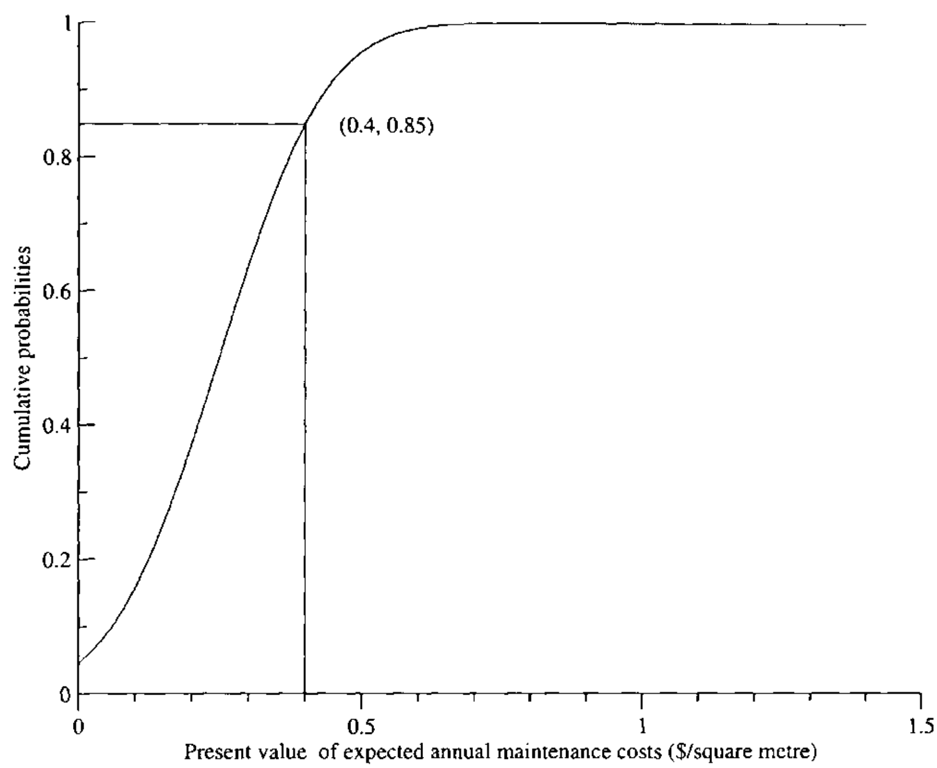

Figure 7.2: The risk associated with the improved performance for a specific unit cost (Model 2)

be 1.75mm and 6mm respectively. The transition probability matrices generated from the new performance curves are summarized in Table 7.8.

Table 7.8: Transition probabilities generated from reducing performance variance (Model 3)

Distress	Base case	Reducing performance variance
Rutting	$\begin{bmatrix} 0.932 & 0.068 & 0.000 \\ 0.000 & 0.817 & 0.183 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.934 & 0.066 & 0.000 \\ 0.000 & 0.801 & 0.199 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Trans. crack. - deterioration	$\begin{bmatrix} 0.960 & 0.040 & 0.000 \\ 0.000 & 0.832 & 0.168 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.961 & 0.039 & 0.000 \\ 0.000 & 0.820 & 0.180 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Transverse crack. - depth	$\begin{bmatrix} 0.949 & 0.051 & 0.000 \\ 0.000 & 0.783 & 0.217 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.943 & 0.057 & 0.000 \\ 0.000 & 0.768 & 0.232 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Cracking	$\begin{bmatrix} 0.769 & 0.231 & 0.000 \\ 0.000 & 0.516 & 0.484 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.769 & 0.231 & 0.000 \\ 0.000 & 0.516 & 0.484 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Surface condition	$\begin{bmatrix} 0.930 & 0.070 & 0.000 \\ 0.000 & 0.691 & 0.309 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.927 & 0.073 & 0.000 \\ 0.000 & 0.734 & 0.266 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Roughness	$\begin{bmatrix} 0.935 & 0.065 & 0.000 \\ 0.000 & 0.791 & 0.209 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.935 & 0.065 & 0.000 \\ 0.000 & 0.734 & 0.266 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$

Once again it is possible to perform a consistency check on the data. It was previously stated that flattening the performance curves would increase the magnitude of the diagonal elements. Slowing down the deterioration decreases the likelihood of the pavement leaving its existing condition state. Reducing the variance is essentially improving the pavement's performance consistency. To replicate this consistency the expectation was that the probability weighting of the of the non-diagonal elements must increase. As can be seen in Table 7.8 the transition probabilities for this analysis is somewhat consistent with the expected behaviour. For the most part, element (1,1) for the transition probability matrices remain unchanged (no more than a 0.1% change). Element (2,2) and consequently element (2,3) of the transition probability matrices appeared to change as expected. The exception was surface condition; element (2,2) increased in probability.

The transition probabilities for the other nine treatments were based on the routine maintenance transition probabilities. Treatment costs and performance constraints were not adjusted for this analysis.

The optimal maintenance strategy and the associated steady state average maintenance costs are summarized in Table 7.9.

Table 7.9: Optimal maintenance strategy costs for reducing performance variance as compared to the base case analysis (Model 3)

Treatment	Base case	Reducing performance variance
Routine maintenance	0.8722	0.8663
Thermopatch	0.0018	0.0010
Flush seal	0.0532	0.0577
Spot seals	0.0728	0.0750
Cost	$\$0.0579/m^2$	$\$0.0592/m^2$

The results were initially surprising; one would expect that with reduced variance the average costs would not increase. But, as can be seen in Table 7.9 this was not the case. The expected maintenance costs at steady state increased from $\$0.0579/m^2$ to $\$0.0592/m^2$. By tightening the performance curves the chance of an unlikely (but favourable) outcome from occurring is reduced. The end result is that the failure rate increased and in turn so did the maintenance cost.

In retrospect, reducing variance (i.e. increasing quality control) would not be expected to reduce the service life of the asset. Improved quality control should only tighten the lower bound, not both the upper and lower bounds on asset performance. So, although the intent of the model adjustment was to replicate or simulate the effects of increased quality control, the true effect was strictly a reduction in the variance.

The present value of the average annual maintenance costs as well as the variance and covariance can be found in Table 7.10. It should be noted that the present value of the total expected maintenance costs are less for this analysis ($0.2702/m^2$) than for the base case. This apparent inconsistency is due to the fact that the rate which the asset reaches steady state is slower than for the base case. This (apparent) paradox suggests that the expected steady state maintenance cost is not necessarily a metric which can be used in isolation. Figure 7.3 compares the cumulative probability distributions for this analysis as well as the base case. For this analysis the cumulative probability curve intersects the base case curve at approximately $0.2/m^2$. This crossover can be attributed to the higher cost but lower variance of the results for this model. Consequently the relative risk associated with each (i.e. whether one scenario is preferred over the other) is a function of the selected unit cost.

Figure 7.4 provides a method to calculate the risk associated with a bid cost of $0.40/m^2$. The probability of the costs exceeding $0.40/m^2$ is 16%. For the base case, the probability of the costs exceeding $0.40/m^2$ is 18%.

Table 7.10: Present value of annual expected maintenance costs over a ten year period for reducing performance variance (Model 3)

Time (years)	Discounted expected costs ($/m^2)	Discounted variance ($/m^2)^2	Discounted covariance ($/m^2)^2
0	0.0200	0.0000	0.0000
1	0.0190	0.0001	0.0000
2	0.0204	0.0006	0.0000
3	0.0239	0.0014	0.0001
4	0.0266	0.0020	0.0001
5	0.0280	0.0022	0.0001
6	0.0282	0.0022	0.0001
7	0.0276	0.0022	0.0001
8	0.0267	0.0020	0.0001
9	0.0255	0.0018	0.0000
10	0.0243	0.0016	-0.0000
	0.2702	0.0162	0.0006

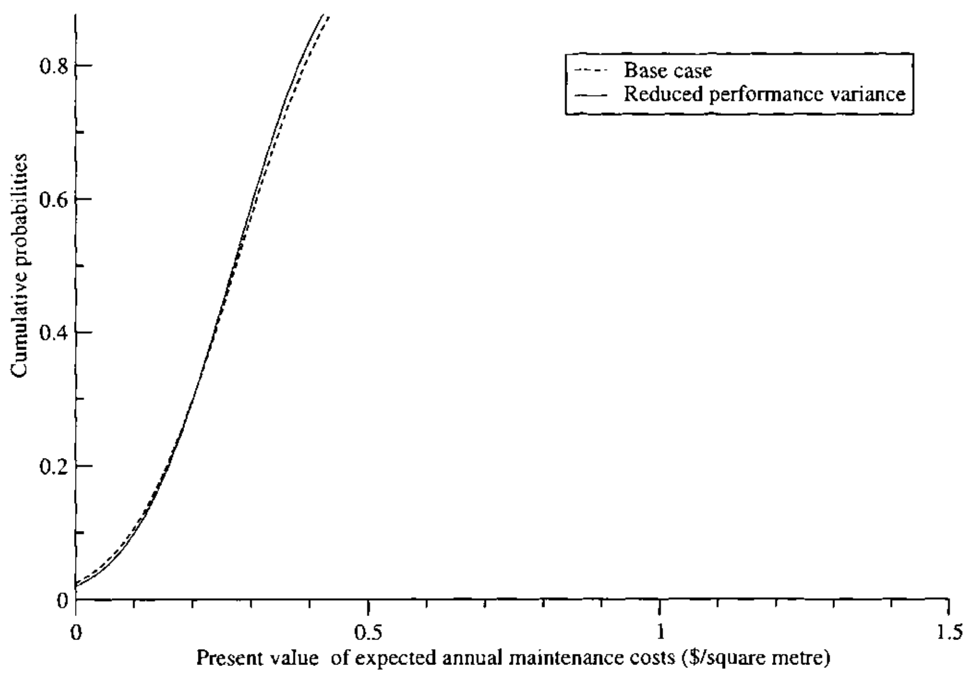

Figure 7.3: Cumulative probability curve comparison of the risk associated with the base case and reduced performance variance (Model 3)

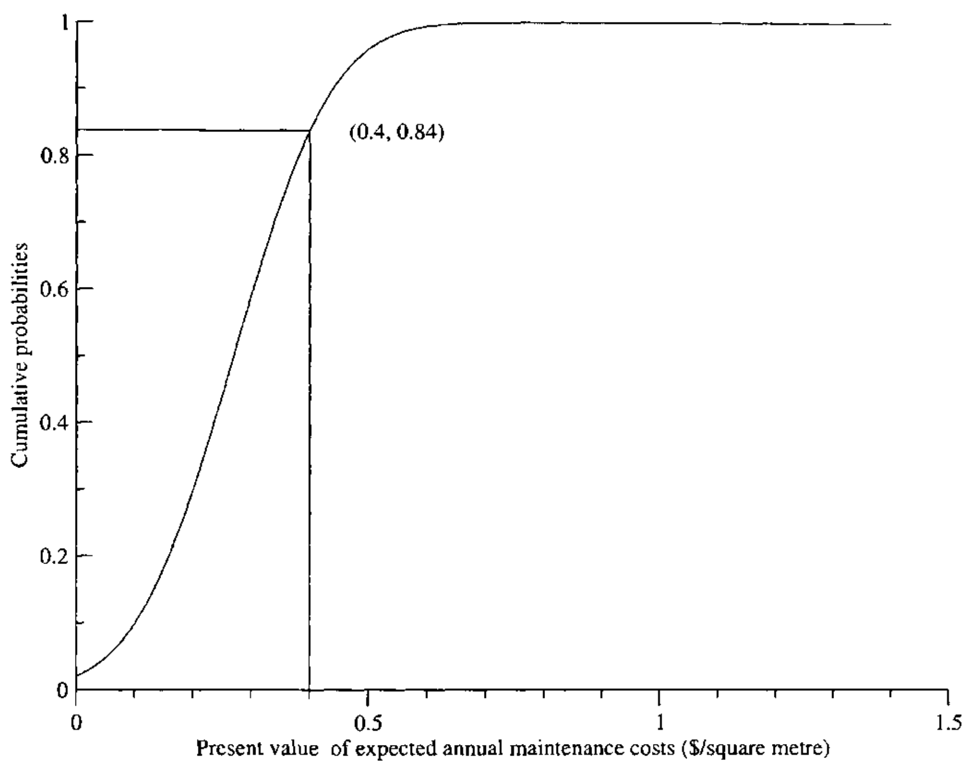

Figure 7.4: The risk associated with reducing performance variance for a specific unit cost (Model 3)

7.2.4 Improving performance and reducing performance variance

The third analysis in this analysis was designed to investigate the benefits of improving asset performance and reducing performance variance. From a practical perspective any material, process or design that introduces performance improvement will probably also provide a more consistent product. The previous two studies separated these components to help identify the individual contribution that each made to the simultaneous application of both forms of improvement.

The transition probabilities for this analysis were adjusted in two stages. The first adjustment was to flatten the Lower Bound, Nominal and Upper Bound values by applying the same factors as in Section 7.2.2 (one third, one half, and three quarters) to each distress, except for cracking. The second step was to tighten the curves as described in Section 7.2.3 (decrease the Upper Bound values by twenty five percent and increase the Lower Bound value by twenty five percent). The transition probability matrices that were generated from the adjusted values (as well as the base case) are shown in Table 7.11.

The treatment costs and the performance constraints for this model were consistent with the base case model. The average maintenance cost at steady state was significantly lower than both the base case and the first sensitivity analysis. The optimal strategy has been summarized in Table 7.12. Note that in spite of the fact that only flattening the performance curves brought about a 11.6% decrease in costs and tightening the curves brought about a 2.2% increase in costs, the combination of flattening and tightening curves resulted in a 16.2% decrease. The present worth of the expected value of the total maintenance costs over a ten year period and 8% discount rate was calculated to be $0.2407/m^2. Table 7.13 shows the discounted expected cost, variance and covariance. The standard deviation associated with the expected value is $\sqrt{0.130 + 2(0.0036)} = \$0.1421/m^2$. Figure 7.5 compares the cumulative probability distribution for the base case and this analysis. As was the case when the performance curves were flattened the cumulative probabilities for this analysis stochastically dominates the base case's cumulative probability curve.

Figure 7.6 shows that for this model the risk associated with the standard $0.40/m^2

Table 7.11: Transition probabilities for improved performance and reduction in performance variance (Model 4)

Distress	Base case	Improved performance and reduction in performance variance
Rutting	$\begin{bmatrix} 0.932 & 0.068 & 0.000 \\ 0.000 & 0.817 & 0.183 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.965 & 0.035 & 0.000 \\ 0.000 & 0.820 & 0.180 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Trans. crack. - deterioration	$\begin{bmatrix} 0.960 & 0.040 & 0.000 \\ 0.000 & 0.832 & 0.168 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.972 & 0.028 & 0.000 \\ 0.000 & 0.826 & 0.174 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Transverse crack. - depth	$\begin{bmatrix} 0.949 & 0.051 & 0.000 \\ 0.000 & 0.783 & 0.217 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.984 & 0.016 & 0.000 \\ 0.000 & 0.821 & 0.179 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Cracking	$\begin{bmatrix} 0.769 & 0.231 & 0.000 \\ 0.000 & 0.516 & 0.484 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.769 & 0.231 & 0.000 \\ 0.000 & 0.516 & 0.484 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Surface condition	$\begin{bmatrix} 0.930 & 0.070 & 0.000 \\ 0.000 & 0.691 & 0.309 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.929 & 0.071 & 0.000 \\ 0.000 & 0.734 & 0.266 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$
Roughness	$\begin{bmatrix} 0.935 & 0.065 & 0.000 \\ 0.000 & 0.791 & 0.209 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 0.941 & 0.059 & 0.000 \\ 0.000 & 0.796 & 0.204 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$

Table 7.12: Optimal maintenance strategy costs for improved performance and reduction in performance variance as compared to the base case analysis (Model 5)

Treatment	Base case	Tightened performance curves
Routine maintenance	0.8722	0.9042
Thermopatch	0.0018	0.0053
Flush seal	0.0532	0.0476
Spot seals	0.0728	0.0428
Cost	\$0.0579/m^2	\$0.0485/m^2

Table 7.13: Present value of annual expected maintenance costs over a ten year period for improved performance and reduction in performance variance (Model 4)

Time (years)	Discounted expected costs (\$/m^2)	Discounted variance (\$/m^2)^2	Discounted covariance (\$/m^2)^2
0	0.0200	0.0000	0.0000
1	0.0194	0.0002	0.0000
2	0.0213	0.0009	0.0000
3	0.0235	0.0015	0.0002
4	0.0246	0.0018	0.0004
5	0.0245	0.0018	0.0005
6	0.0238	0.0017	0.0006
7	0.0227	0.0015	0.0006
8	0.0215	0.0014	0.0005
9	0.0203	0.0012	0.0005
10	0.0190	0.0011	0.0004
Total	0.2407	0.0130	0.0036

tender price is 13%.

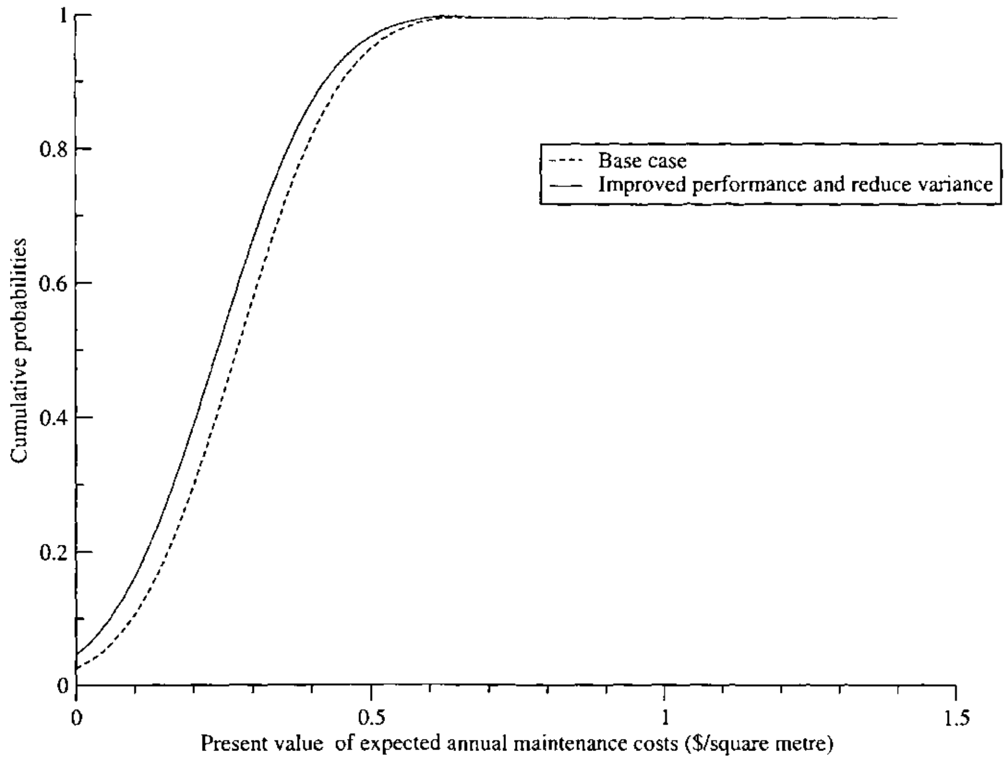

Figure 7.5: Cumulative probability curve comparison of the risk associated with the base case and the improved performance and reduction in performance variance (Model 4)

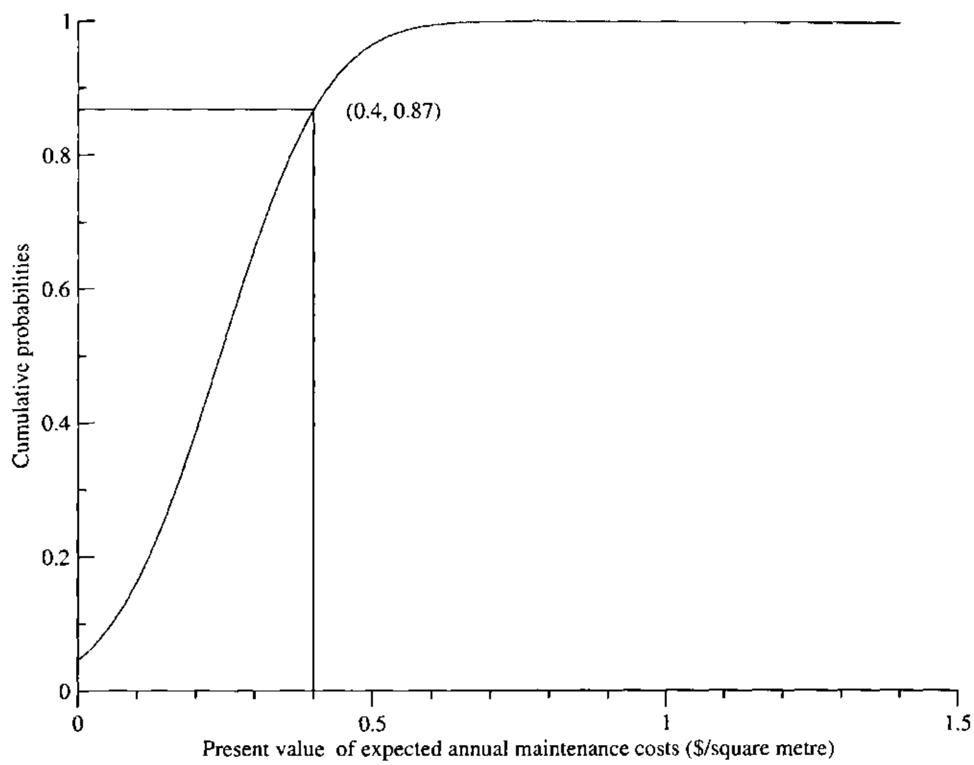

Figure 7.6: The risk associated with the improved performance and a reduction in performance variance for a specific unit cost (Model 4)

7.2.5 Reduction in the difference between treatment costs

The fourth analysis focused on changes in maintenance costs. Typically an increase in the cost of oil will result in an increase in pavement treatment costs. The objective of this analysis was to determine the impact changes in input costs have on the optimal strategy and in turn the maintenance costs.

Treatment costs were the only model adjustment made in this analysis. The base case treatment cost data as well as the data applied in this analysis can be found in Table 7.14. As a reminder to the reader, treatment costs were increased linearly to simulate the relationship between increasing treatment costs and pavement deterioration.

The cost changes shown in Table 7.14 were adjusted in an effort to reduce the difference between the treatments costs. The objective of this approach was twofold. The first objective was to illustrate that the impact of increases in treatment costs could be measured with this methodology. The second objective was an attempt to maximize the impact on the optimal maintenance strategy. As noted in the previous models, the optimal maintenance strategies have been fairly consistent from analysis to analysis. It was thought that if the lower cost treatments were given cost increases and the higher cost treatments were reduced in cost, a change in the treatments included in the optimal maintenance strategy would occur.

Table 7.14: Treatment costs for reduction in variance of treatment costs (Model 5)

Number	Treatment	Base	Model 5	% change
1	Routine maintenance	0.02	0.03	50.0%
2	Thermopatch	0.44	0.50	13.6%
3	Flush seal	0.22	0.30	36.4%
4	Spot seals	0.30	0.40	33.3%
5	Strip seals	0.72	0.80	11.1%
6	Micro surface	2.97	2.50	-15.8%
7	Full seal	1.31	1.10	-16.0%
8	Spot overlay and seal	3.27	3.00	-8.25%
9	Thin overlay	9.02	6.00	-33.5%
10	Structural overlay	20.08	10.00	-50.2%

Given the base summary strategy and the unit treatment costs in Table 7.14 an estimate for the base model would be \$0.5178/m^2 and a cost \$0.07215/m^2 for Model 5 if the strategy did not change.

Table 7.15 illustrates the calculations and shows that extrapolating the costs would result in a 35.6% increase. After solving the model with the new input costs the price increase was actually a 36.3% increase in costs. This hardly appears to be an improved maintenance strategy.

Table 7.15: Extrapolating expected treatment costs based on an optimal maintenance strategy

Treatment	Probability	Base case costs (\$/m^2)	New costs (\$/m^2)
Routine maintenance	0.8722	0.02	0.03
Thermopatch	0.0018	0.44	0.50
Flush seal	0.0532	0.22	0.30
Spot seals	0.0728	0.30	0.40
Strategy costs		0.05178	0.07215

Table 7.16 demonstrates that the same basic four treatments did not change from the base model. What did change was the relative frequency of their application. The unexpected result was that in spite of the fifty percent increase in the cost to apply routine maintenance, there was a two percent increase in the frequency of its application.

Table 7.16: Optimal maintenance strategy costs for reducing the difference in treatment costs as compared to the base case analysis (Model 5)

Treatment	Base case	Reducing the difference in costs
Routine maintenance	0.8722	0.8910
Thermopatch	0.0018	0.0355
Flush seal	0.0532	0.0421
Spot seals	0.0728	0.0314
Cost	\$0.0579/m^2	\$0.0789/m^2

The discounted total average cost and standard deviation for a ten year time horizon were \$0.3276/m^2 and \$0.1400/m^2 respectively. Figure 7.7 shows the comparison of the cumulative probability distribution of the base case and this model.

It is interesting to note that in spite of the relative change in treatment costs there was not a significant change in the applied treatments, only in the relative likelihood of

Table 7.17: Present value of annual expected maintenance costs over a ten year period for reduced difference in treatment cost (Model 5)

Time (years)	Discounted expected cost ($\$/m^2$)	Discounted variance $(\$/m^2)^2$	Discounted covariance $(\$/m^2)^2$
0	0.0300	0.0000	0.0000
1	0.0280	0.0000	0.0000
2	0.0275	0.0004	0.0000
3	0.0282	0.0010	0.0000
4	0.0293	0.0017	0.0000
5	0.0302	0.0022	0.0001
6	0.0309	0.0027	0.0001
7	0.0313	0.0029	0.0001
8	0.0313	0.0030	-0.0000
9	0.0309	0.0030	-0.0001
10	0.0301	0.0029	-0.0002
Total	0.3276	0.0197	-0.0001

the application of existing treatments. This suggests that the efficacy of the treatments is driving the maintenance strategy at least as much as the costs.

The risk associated with a tender price of $\$0.40/m^2$ was 30% as illustrated in Figure 7.8.

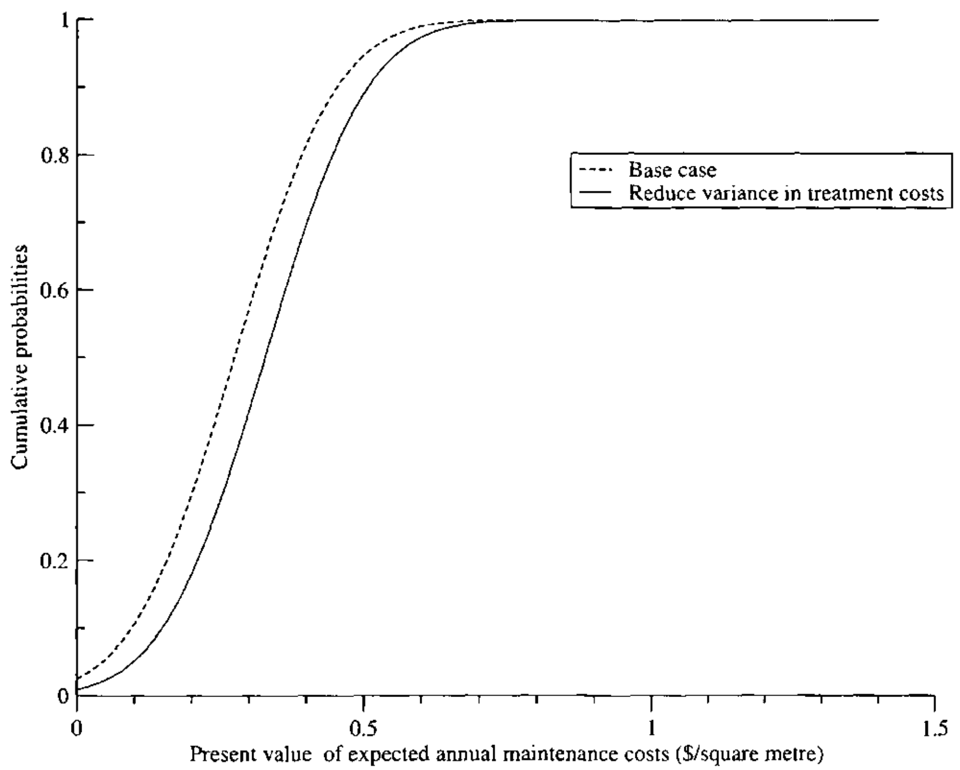

Figure 7.7: Cumulative probability curve comparison of the risk associated with the base case and the reduction in variance of treatment costs (Model 5)

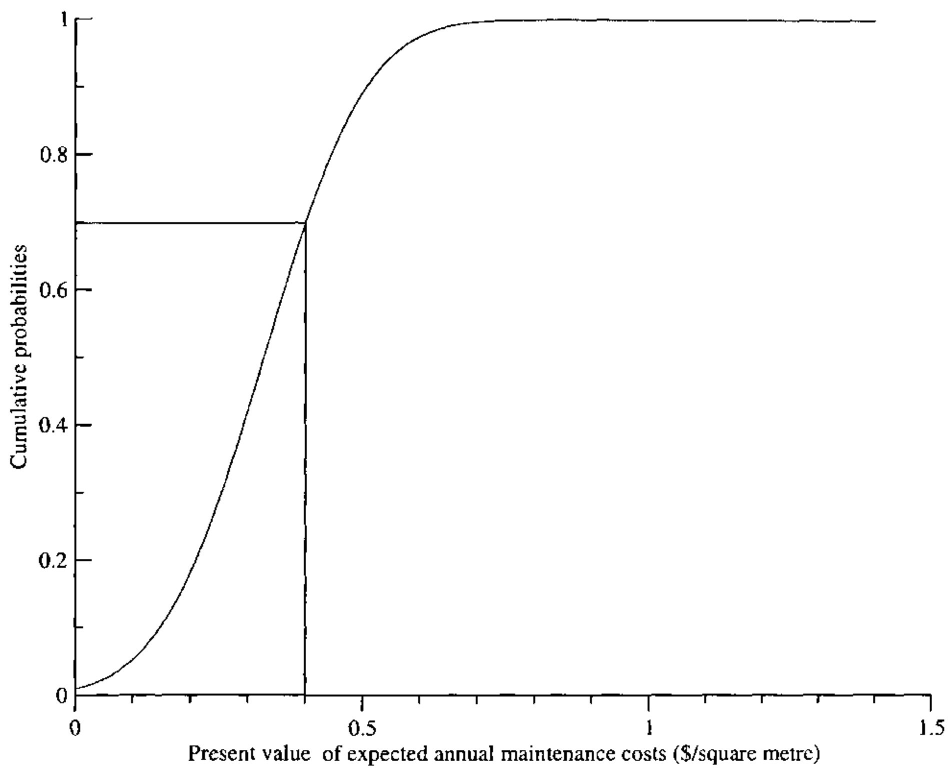

Figure 7.8: The risk associated with the reduction in variance in treatment costs for a specific unit costs (Model 5)

7.2.6 Increase in treatment costs by ten percent

The objective of this analysis was to investigate the impact of significantly increasing the routine maintenance treatment cost (by a factor of eleven) as well as a ten percent increase in the other treatment costs (except for structural overlay). The treatment costs for the base case and this analysis can be found in Table 7.18. Once again, the rational behind this study was to investigate the level to which input costs needed to be increased so that a significant change in the optimal maintenance strategy would occur. In spite of an order of magnitude increase, routine maintenance was still the treatment with the lowest unit cost and consequently was still the predominant maintenance treatment (Table 7.19). Once again this suggests that both cost and relative effectiveness is driving the selection of the optimal maintenance policy.

Table 7.18: Treatment costs for increasing treatment costs by ten percent (Model 6)

Number	Treatment	Base	Model 6
1	Routine maintenance	0.02	0.22
2	Thermopatch	0.44	0.48
3	Flush seal	0.22	0.24
4	Spot seals	0.30	0.33
5	Strip seals	0.72	0.79
6	Micro surface	2.97	3.27
7	Full seal	1.31	1.44
8	Spot overlay and seal	3.27	3.60
9	Thin overlay	9.02	9.92
10	Structural overlay	20.08	22.09

Table 7.19: Optimal maintenance strategy costs for a ten percent increase in treatment costs as compared to the base case analysis (Model 6)

Treatment	Base case	Increase in treatment costs
Routine maintenance	0.8722	0.8358
Thermopatch	0.0018	0.0000
Flush seal	0.0532	0.0358
Spot seals	0.0728	0.1284
Cost	\$0.0579/m^2	\$0.2440/m^2

The order of magnitude increase in the routine maintenance treatment costs had a

Table 7.20: Present value of annual expected maintenance costs over a ten year period for a ten percent increase in treatment costs (Model 6)

Time (years)	Discounted expected costs ($\$/m^2$)	Discounted variance $(\$/m^2)^2$	Discounted covariance $(\$/m^2)^2$
0	0.2203	0.0000	0.0000
1	0.2135	0.0011	0.0000
2	0.2015	0.0013	0.0004
3	0.1883	0.0012	0.0006
4	0.1752	0.0011	0.0006
5	0.1629	0.0010	0.0006
6	0.1512	0.0008	0.0005
7	0.1404	0.0007	0.0005
8	0.1302	0.0006	0.0004
9	0.1208	0.0005	0.0004
10	0.1120	0.0005	0.0004
	1.8163	0.0089	0.0043

significant impact on the steady state maintenance costs. The costs for this analysis increased to $\$0.2440/m^2$ as compared to the base case cost of $\$0.0579/m^2$. The time to reach steady state was similar (28 years for the base case and 24 years for this model). The increase in routine maintenance costs reduced the frequency of its application. Thermopatching was no longer applied in the optimal strategy and spot overlay and sealing was used roughly 75% more often than in the base case. Given the significant increase in the steady state maintenance costs it was reasonable to expect that the present value of the total maintenance costs over a ten year horizon would increase. The expected cost for a ten year contract was $\$1.816/m^2$ with a standard deviation of $\$0.1325/m^2$. A graph comparing the cumulative probability of this analysis to that of the base case was not included given the significant difference in costs between these two models ($\$0.274/m^2$ versus $\$1.816/m^2$). Similarly, it did not make sense to calculate the risk associated with a tender bid of $\$0.4/m^2$ as was done in the previous studies. For this analysis a tender bid of $\$2/m^2$ was selected. In Figure 7.9 we can see that the risk that maintenance costs would exceed $\$2/m^2$ is roughly 15%.

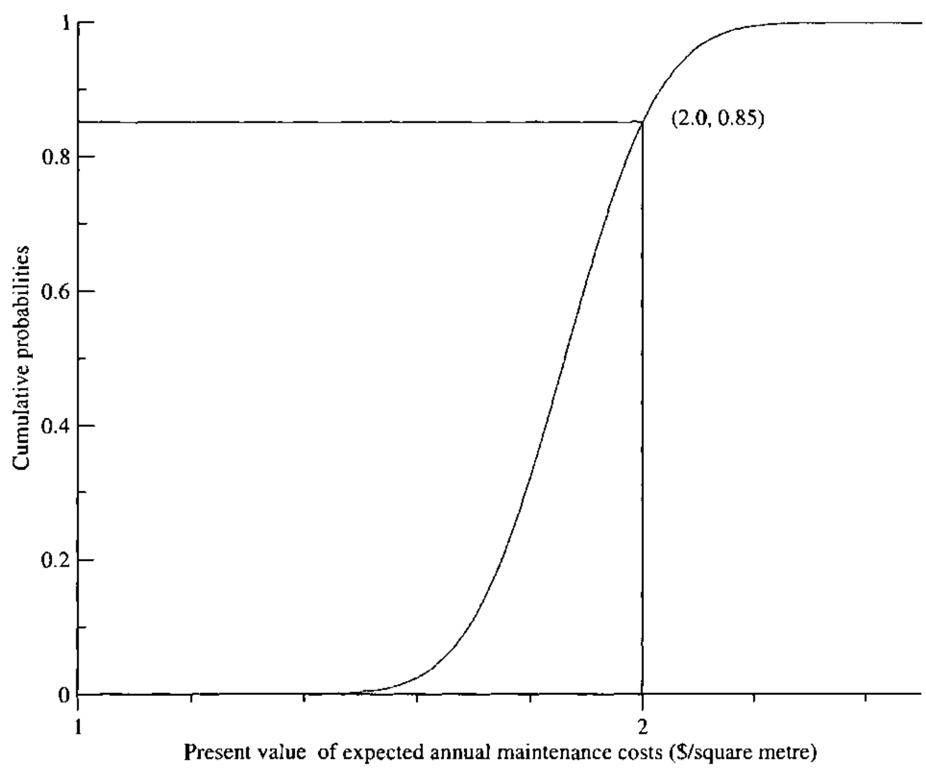

Figure 7.9: The risk associated with a ten percent increase in costs (Model 6)

7.2.7 Changing a performance constraint

The objective of this analysis was to investigate the impact of changing a performance constraint. The previous studies investigated the impact that changes in pavement performance (technology coefficients) and cost (objective function) had on the results. An investigation into the sensitivity of a linear programming model would not be complete unless the impact due to changes in the right hand side of a constraint were also examined. In this analysis the constraint that limited the likelihood that a pavement would reach a poor condition at steady state was adjusted. Specifically, the base model limited the probability that that pavement would reach a poor condition state to five percent. In this analysis the pavement was not allowed to reach a poor condition (i.e. a zero tolerance policy).

From a practical perspective model constraints are not always a result of physical or resource requirements. They can also can be generated by policies based on political, legal, or societal needs. From a budgetary perspective the financial impact of these policies must be quantified.

The inputs to this model were the same as those in the base case except for the constraint adjustment which is the focus of this analysis.

Table 7.21 summarizes the optimal maintenance policy. From Table 7.21 it appears that to ensure the pavement does not reach a poor condition routine maintenance was applied less often, and flush seals and spot seals more often. The shift from a five percent to zero tolerance policy resulted in a 39% increase in steady state maintenance costs. This increase in costs is illustrated in Table 7.21.

Table 7.21: Optimal maintenance strategy costs for a change in performance constraints (Model 7)

Treatment	Base case	Change in performance constraints
Routine maintenance	0.8722	0.7840
Thermopatch	0.0018	0.0052
Flush seal	0.0532	0.0873
Spot seals	0.0728	0.1235
Cost	$0.0579/m^2$	$0.0805/m^2$

Table 7.22 illustrates the present value of the expected maintenance costs for the zero tolerance policy ($0.3919/m^2$) The present value costs represent a 43% increase over the present worth of the average annual costs of the base model ($0.2740/m^2$).

Table 7.22: Present value of annual expected maintenance costs over a ten year period for a change in performance constraints (Model 7)

Time (years)	Discounted expected costs ($\$/m^2$)	Discounted variance $(\$/m^2)^2$	Discounted covariance $(\$/m^2)^2$
0	0.0200	0.0000	0.0000
1	0.0207	0.0005	0.0000
2	0.0274	0.0023	0.0001
3	0.0350	0.0038	0.0007
4	0.0406	0.0046	0.0012
5	0.0434	0.0048	0.0016
6	0.0441	0.0046	0.0017
7	0.0432	0.0042	0.0016
8	0.0414	0.0037	0.0015
9	0.0392	0.0033	0.0013
10	0.0368	0.0028	0.0011
	0.3919	0.0345	0.0108

Figure 7.10 shows that the base case stochastically dominates the results from this analysis for the present value of the expected annual maintenance costs above $0.10/m^2$. Had the variance of this model been the same or lower as the base case, stochastic dominance would have occurred throughout.

Figure 7.11 illustrates the risk associated with our standard $0.40/m^2$ tender price. For this analysis, the risk that the standard price will be exceeded is 49%. That is a significant risk. As a reminder, the risk associated with the base case was 18%. Effectively the zero tolerance policy has increased the risk from a chance of one in five (for the base case) to just slightly less than a coin toss.

7.2.8 Increasing treatment costs by twenty percent

The title of this analysis is a bit of a misnomer. Once again, the objective for this analysis was to determine if adjusting treatment costs by twenty percent would introduce a different set of maintenance treatments. In an effort to reduce the omnipresence of routine

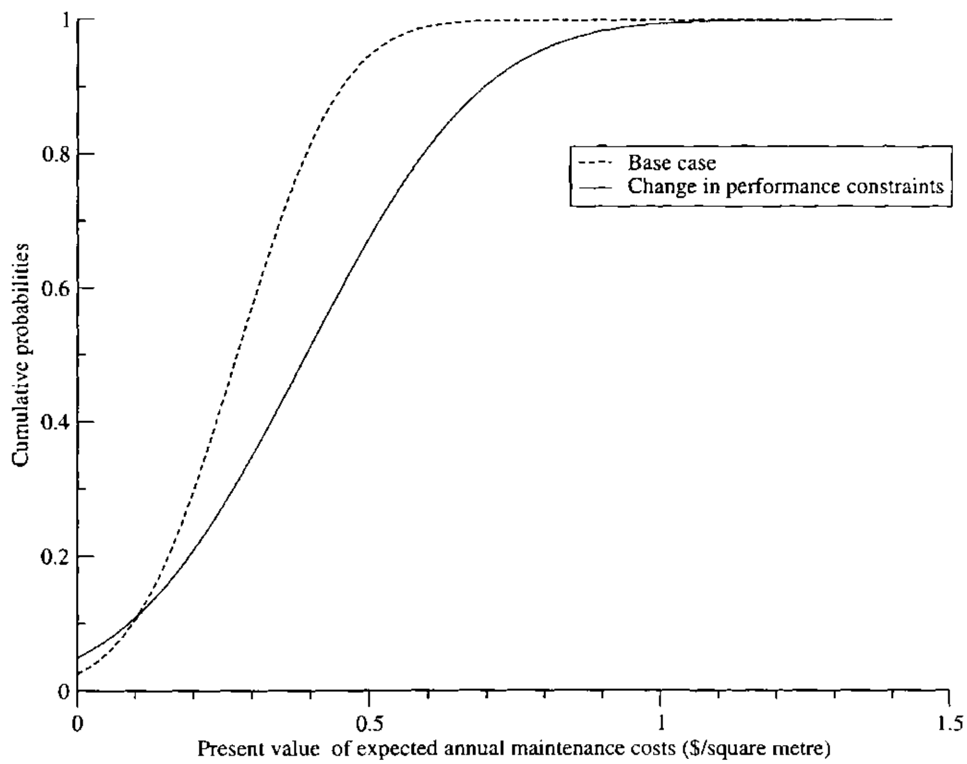

Figure 7.10: Cumulative probability curve comparison of the risk associated with a change in performance constraints and the base case (Model 7)

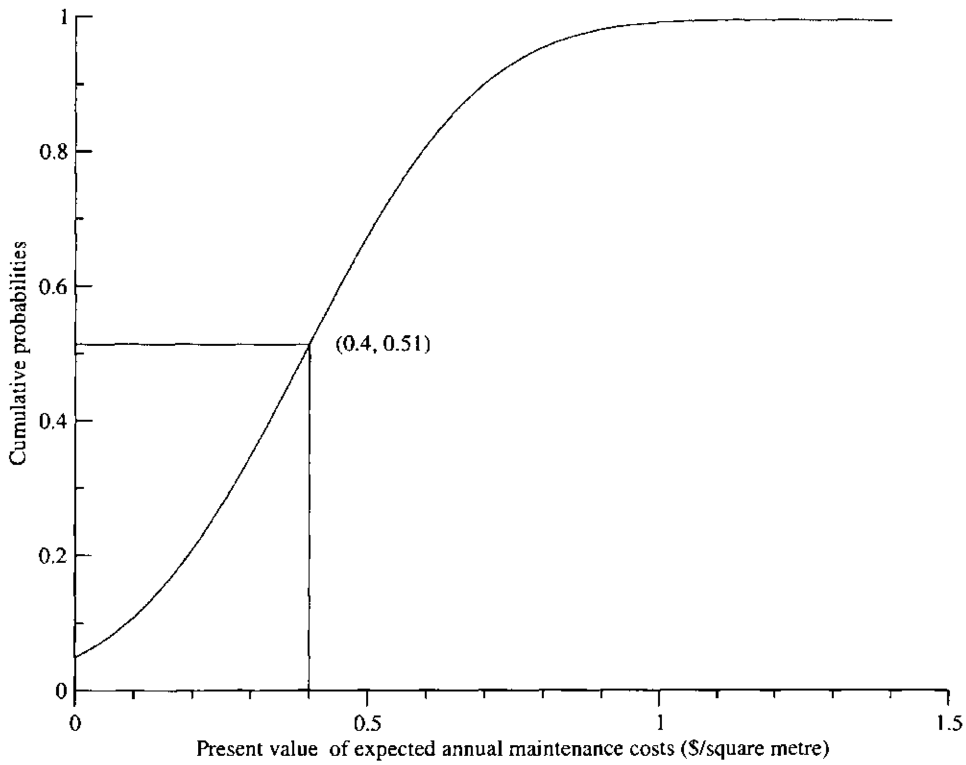

Figure 7.11: The risk associated with a change in performance constraints and the base case (Model 7)

maintenance in the optimal maintenance strategy, its costs were increased by fifty percent. Table 7.23 is a summary of the optimal maintenance strategy. In spite of an increase in the treatment costs, the optimal maintenance strategy is effectively the same as for the base case. The key exception is that in this analysis there is a slight (but effectively zero percent likelihood) that strip seals would be applied. Table 7.24 illustrates the present value of the annual expected maintenance costs over a ten year period at a discount rate of 8%.

Table 7.23: Optimal maintenance strategy costs for a treatment cost increase of twenty percent (Model 8)

Treatment	Base case	Increase in treatment costs
Routine maintenance	0.8722	0.8711
Thermopatch	0.0018	0.0004
Flush seal	0.0532	0.0533
Spot seals	0.0728	0.0752
Strip seals		0.0000
Cost	$\$0.0579/m^2$	$\$0.0750/m^2$

Table 7.24: Present value of annual expected maintenance costs over a ten year period for a change in performance constraints (Model 8)

Time (years)	Discounted expected costs ($\$/m^2$)	Discounted variance $(\$/m^2)^2$	Discounted covariance $(\$/m^2)^2$
0	0.0300	0.0000	0.0000
1	0.0286	0.0001	0.0000
2	0.0316	0.0014	0.0000
3	0.0362	0.0027	0.0001
4	0.0390	0.0035	0.0002
5	0.0396	0.0036	0.0003
6	0.0387	0.0035	0.0003
7	0.0371	0.0032	0.0002
8	0.0351	0.0029	0.0002
9	0.0331	0.0025	0.0001
10	0.0311	0.0022	0.0001
	0.3800	0.0256	0.0016

Figure 7.12 shows that the base case stochastically dominates the results from this analysis. That is not surprising given the much higher total expected costs and only slightly higher variance in these costs. As far as the risk calculation, Figure 7.13 illustrates the likelihood that a tender price of $\$0.40/m^2$ would be exceeded is 45%.

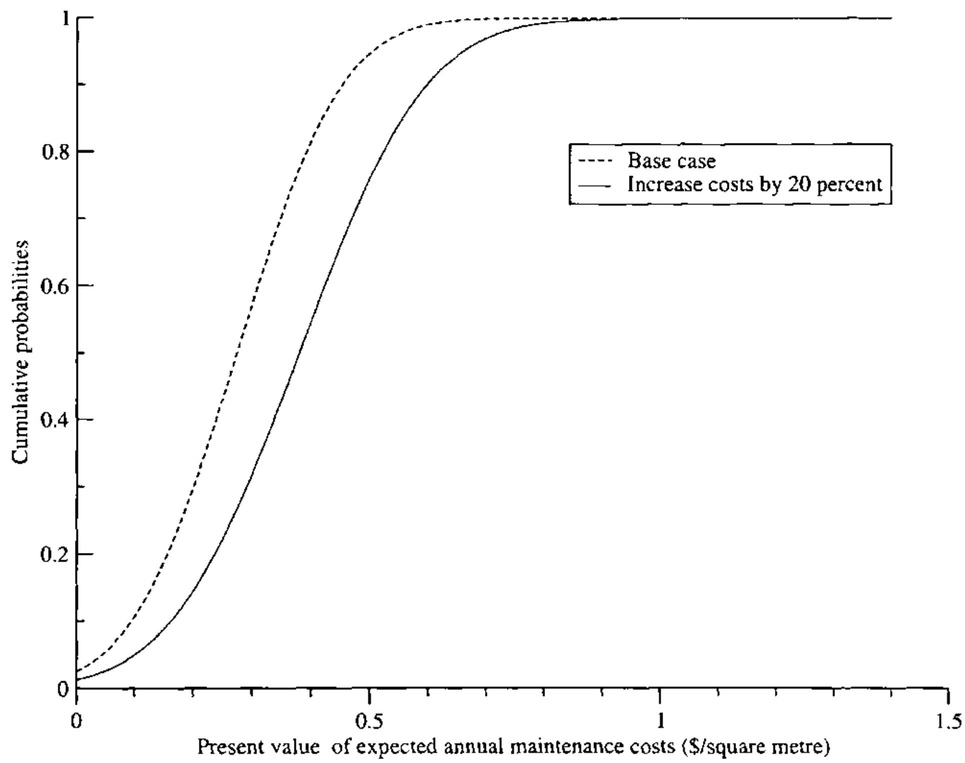

Figure 7.12: Cumulative probability curve comparison of the risk associated with the base case and for an increase of twenty percent for treatment costs (Model 8)

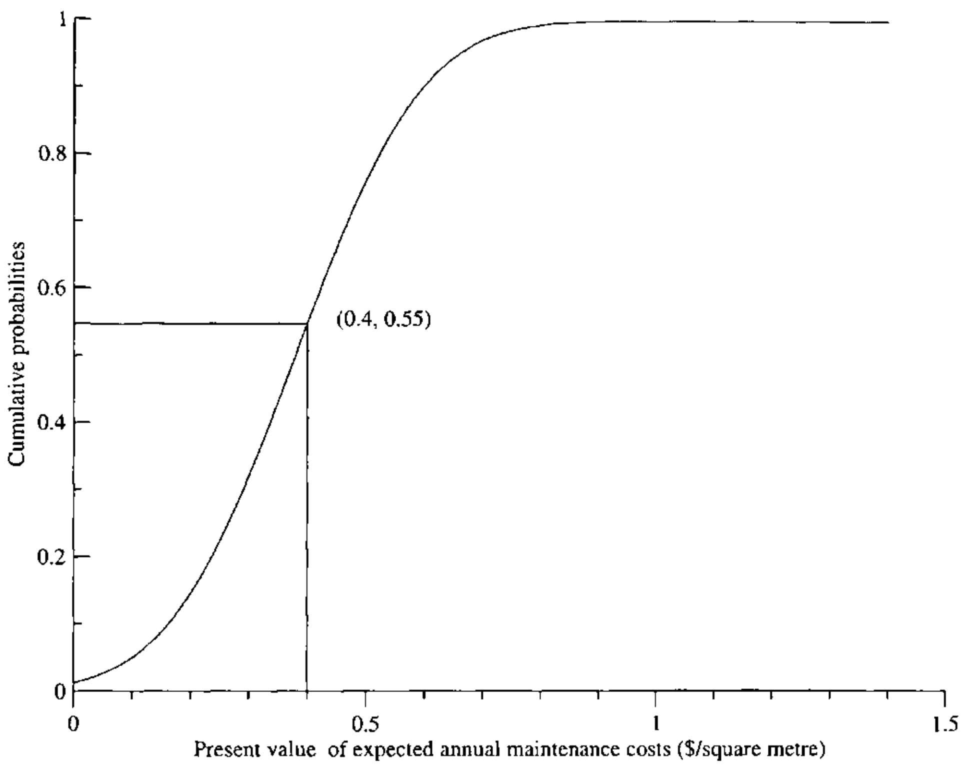

Figure 7.13: The risk associated with the base case and for an increase of twenty percent for treatment costs (Model 8)

7.2.9 Increasing the effectiveness of maintenance treatments

The objective of this analysis was to investigate the impact that increasing treatment effectiveness would have on the optimal maintenance strategy. The base case model was modified to reflect increased treatment effectiveness by adjusting the table of generic maintenance treatments found in Section 6.3. The original and modified data can be found in Table 7.25. These modified transition probability matrices were used to generate transition probability matrices from the routine maintenance transition probabilities.

Table 7.26 is a summary of the optimal maintenance strategy. The key observation about the optimal maintenance strategy is that there is very little change from the base case. The difference is more discernible when the present value of the total expected costs of the two studies are compared. For the base case, the total expected cost (and the accompanying standard deviation) was \$0.2740/m^2 (\$0.1395/m^2). For this analysis the expected cost (and standard deviation) was \$0.2375/m^2 (\$0.1186/m^2), a decrease of 13% in expected costs and 15% decrease in the standard deviation. In spite of similar maintenance costs at steady state there is a difference in the present value calculation. This can be attributed to the fact that this model reaches steady state more slowly than the base case. Figure 7.14 shows that the cumulative probability distribution for this model stochastically dominates the base case. This domination can be attributed to the lower expected costs and standard deviation.

Figure 7.15 shows that the risk that project costs will exceed \$0.40/m^2 is roughly 9%, half the risk for the base case (18%).

Table 7.25: Generic transition probability matrices as modified for sensitivity analysis

Label	Orginal Matrix	Modified matrices
TPM_1	$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & 1 \end{bmatrix}$	
TPM_2	$\begin{bmatrix} a & b & c \\ 0.6 & 0.4 & 0 \\ 0.6 & 0.35 & 0.05 \end{bmatrix}$	
TPM_3	$\begin{bmatrix} a & b & c \\ 0.65 & 0.35 & 0 \\ 0.65 & 0.30 & 0.05 \end{bmatrix}$	
TPM_4	$\begin{bmatrix} a & b & c \\ 0.7 & 0.3 & 0 \\ 0.7 & 0.25 & 0.05 \end{bmatrix}$	
TPM_5	$\begin{bmatrix} a & b & c \\ 0.75 & 0.25 & 0 \\ 0.75 & 0.20 & 0.05 \end{bmatrix}$	$\begin{bmatrix} a & b & c \\ 0.90 & 0.10 & 0 \\ 0.9 & 0.05 & 0.05 \end{bmatrix}$
TPM_6	$\begin{bmatrix} a & b & c \\ 0.80 & 0.20 & 0 \\ 0.80 & 0.15 & 0.05 \end{bmatrix}$	$\begin{bmatrix} a & b & c \\ 0.90 & 0.10 & 0 \\ 0.90 & 0.05 & 0.05 \end{bmatrix}$
TPM_7	$\begin{bmatrix} a & b & c \\ 0.85 & 0.15 & 0 \\ 0.85 & 0.15 & 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.98 & 0.02 & 0 \\ 0.95 & 0.05 & 0 \\ 0.90 & 0.05 & 0.05 \end{bmatrix}$
TPM_8	$\begin{bmatrix} 0.95 & 0.05 & 0 \\ 0.90 & 0.10 & 0 \\ 0.90 & 0.05 & 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.01 & 0 \\ 0.95 & 0.05 & 0 \\ 0.98 & 0.01 & 0.01 \end{bmatrix}$
TPM_9	$\begin{bmatrix} 1 & 0 & 0 \\ 0.95 & 0.05 & 0 \\ 0.90 & 0.10 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0.99 & 0.01 & 0 \\ 0.97 & 0.02 & 0.01 \end{bmatrix}$
TPM_{10}	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	

Table 7.26: Optimal maintenance strategy costs for increased treatment effectiveness (Model 9)

Treatment	Base case	Increase in treatment costs
Routine maintenance	0.8722	0.8875
Thermopatch	0.0018	0.0201
Flush seal	0.0532	0.0454
Spot seals	0.0728	0.0470
Cost	$0.0579/m^2	$0.0574/m^2

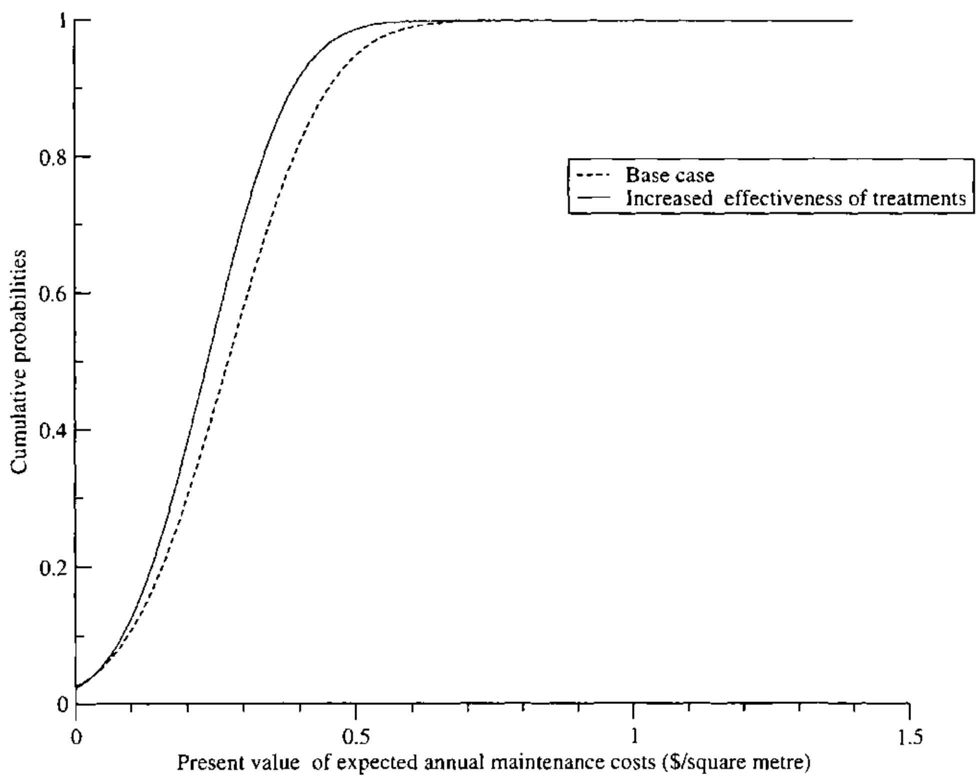

Figure 7.14: Cumulative probability curve comparison of the risk associated with the base case the case where effective of maintenance treatments are increased (Model 9)

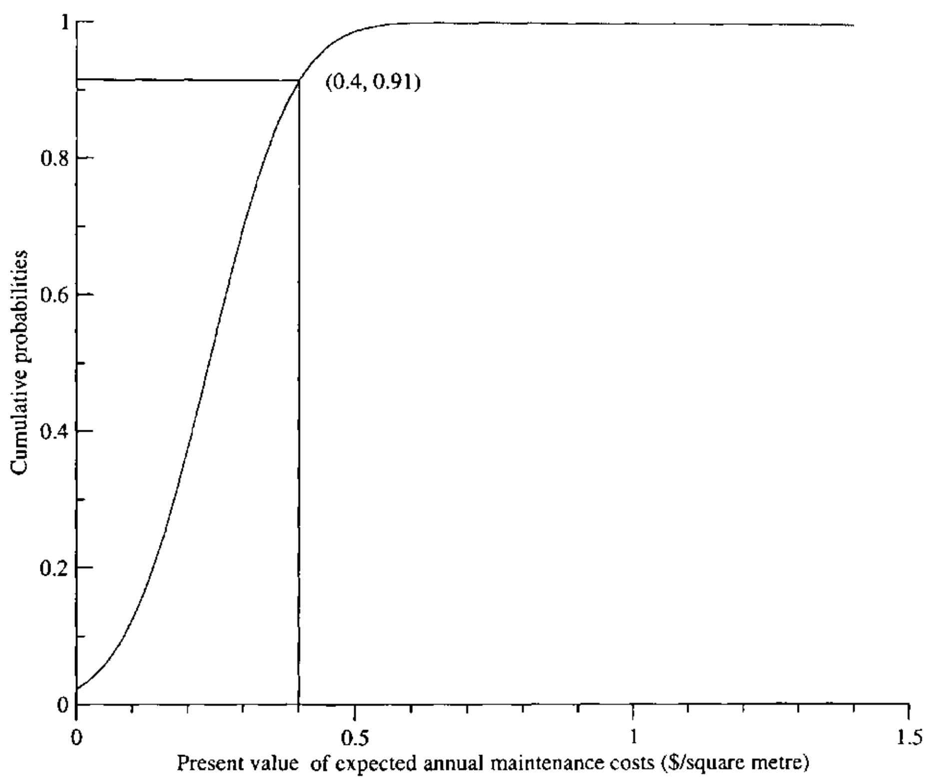

Figure 7.15: The risk associated with the base case and the case where effective of maintenance treatments are increased (Model 9)

7.3 Conclusions

There are three components of a linear programming model that can be adjusted as part of a sensitivity analysis; technology coefficients (treatment costs), the objective function (treatment costs), and the right hand side of the modelling constraints (performance constraints). The following are the observations generated from these studies.

The most consistent element between these studies was the optimal maintenance strategy. There were three maintenance treatments present in each analysis; routine maintenance, flush seal, and spot seals. Thermopatching was selected in all but one strategy (although the likelihood of it being applied in three of the nine cases was less than one percent).

It would be fair to say that the effectiveness of the routine maintenance treatments is what drove the optimal maintenance policies. Had the routine maintenance treatments been less effective it is expected that some of the higher cost treatments would have been introduced to the optimal maintenance strategies.

It would also be accurate to say that because routine maintenance was relied upon so heavily in all strategies (all strategies applied routine maintenance at least 78% of the time), the maintenance costs were driven largely by the cost of routine maintenance. There was an expectation that significant increases in routine maintenance costs would reduce its presence in the optimal strategy. As was illustrated in Model 6, this was not necessarily the case. This suggests that a cost-to-effectiveness ratio is what drove the selection of the maintenance treatments.

The last general observation that can be made was that the expected maintenance cost at steady state is a poor metric for comparing maintenance strategies. The time to steady state, and the rate at which steady state is reached, can be quite variable. This can result in a strategy with a lower present value cost (based on a fixed time horizon, such as the ten year horizon used throughout this chapter) having a higher steady state cost. A perfect example is a comparison of the results from Models 2 and 9. In Table 7.27 we can observe that although Model 2 has a lower maintenance cost at steady state, the discounted expected costs and the associated standard deviation are less. The net result is that the risk associated with Model 9 is much less than Model 2.

Table 7.27: Comparing optimal maintenance policy costs ($\$/m^2$)

Model	Description	Steady state costs	Discounted costs	Discounted standard deviation	Risk of costs exceeding $\$0.4/m^2$
Base		0.0579	0.2740	0.1395	18%
2	Improved performance	0.0512	0.2484	0.1470	13%
9	Increase treatment effectiveness	0.0574	0.2375	0.1186	9%

Chapter 8

RESEARCH SUMMARY, RESEARCH LIMITATIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

8.1 Research summary and conclusions

The objective of this research was to develop a methodology to quantify the uncertainty associated with long term maintenance contracts for public sector infrastructure from the contractor's perspective. This objective was clearly accomplished. The risk from the contractor's perspective was quantified based on asset performance curves, maintenance treatment costs and a set of minimum performance constraints. In addition to the stated objectives, this research was also able to identify and measure the change to an optimal maintenance strategy due to changes in any of the model inputs.

Long term maintenance contracts are emerging as a viable alternative for state agencies to manage their infrastructure assets. From the owner's perspective, long term maintenance contracts establish a deterministic schedule for the payments (costs) necessary to meet a specific performance standard over a known time span. From the contractor's perspective, the contract establishes the performance standard that must be met. If the contract is performance based, the contractor is given the freedom to develop and implement the necessary procedures to meet the performance guidelines. The transfer of risk from the owner to contractor injects an element of potential reward for successful innovation into the asset management environment.

It is in the best interests of both the owner and the contractor that the risk associated with long term maintenance contracts can be quantified. The objective of this research was to develop a methodology to quantify the financial risks (faced by the contractor) associated with the uncertainty in the performance of the asset. To accomplish this goal, a framework was developed to take pavement performance curves, treatment costs, and minimum performance criteria, and calculate the present value of the expected maintenance costs for a long term maintenance contract. The probability distribution associated with these predicted costs would then be applied (in conjunction with the contractor's risk tolerance) to establish the appropriate tender price. By adjusting the input parameters, the owner/contractor can determine the sensitivity of the optimal maintenance strategy to the value of model inputs. The sensitivity analysis allows the owner/contractor to determine the input variables that must be controlled to ensure a success as well as identify the areas which could potentially provide the greatest opportunity for savings.

The methodology for this research parallels the data flow necessary to quantify the risk. The process was divided into seven steps:

1. Develop a procedure for generating transition probability matrices from routine maintenance performance curves (Chapter 3).
2. Create transition probability matrices for non-routine maintenance treatments (Chapter 3).
3. Create aggregate transition probability matrices (Chapter 3).
4. Formulate and solve the pavement management model as a linear program (Chapter 4).

5. Calculate the risk associated with a single project. This was completed by calculating the present value of the total expected costs and the standard deviation of these costs for a specific project. The costs and in turn the risks for a portfolio of projects (which is equivalent to a long term maintenance contract) is found by summing the expected costs and variance of the individual projects (Chapter 5).

6. Apply framework to a full scale problem which then acted as a base case for future studies (Chapter 6).

7. Investigate the impact changes in the model inputs have on the optimal maintenance strategy (sensitivity analysis). The outcomes from this analysis provided many of the suggestions for future research. Sensitivity analysis is the tool that the owner/contractor would use to identify areas for efficiency improvements or productive innovation.

Many observations were recorded during this research. The first set of observations relate to how transition probabilities were generated from pavement performance curves. The first observation was that the long flat performance curves that were used to describe pavement performance were not well modelled by a Markov process. The horizontal nature of the early phases of these curves resulted in the Markov model overestimating initial deterioration and underestimating later stage deterioration. Mathematically this resulted in a strong diagonal bias in the transition probability matrices.

The second observation that came out of the transition probability calculations was that increasing the number of condition states at the distress level appeared to be beneficial.

The size of a linear programming model is defined by the number of variables and the number of restrictions on the model. For this research, the linear programming model grows proportionally to KS^D where K is the number of maintenance treatments, S is the number of condition states, and D is the number of distresses. Given the present model requires approximately 60 MB of memory ($S = 3$, $D = 6$) increasing the number of states from three to five increases the memory requirements to 1286 MB of memory; this is within the addressable range of leading edge desktop computers.

The next observations were generated from the work done on modelling the Markov decision process as a linear program. The first observation is that solving a Markov decision process as a linear program worked quite well. A state of the art solver (ILOG, 2001) and a mathematical modelling package (Software, 2001) were applied "off the shelf" with minimal effort to generate a robust modelling solution. In fact with today's technology, solving the actual problem takes a trivial amount of time relative to the time spent waiting for the data to be loaded into the model and storing the solution.

The linear programming model generated an optimal maintenance strategy at steady state. Unfortunately for most of the models the time necessary to reach steady state was better measured in decades than years. Given that a typical long term maintenance contract is ten years, the strategies generated by the model were not practical. A better approach would have been to control the pavement's performance over the length of the contract.

Calculating project risk was one of the significant contributions of this research. The risk associated with a long term maintenance contract was based on the risk associated with each individual project. To calculate this risk Grassmann's (Grassmann, 1987) recursive function for calculating the covariance associated with a Markov process was modified to include discounting. Once the total expected costs (and the associated variance) were found the risk associated with the long term maintenance contract could be calculated.

Sensitivity analysis consisted of making several adjustments to the base case performance model and identifying the impact these changes had on the model. Table 8.1 shows the summary of the optimal maintenance strategies for each model. Table 8.2 illustrates the expected maintenance costs at steady state as well as the present value (and the associated variance) of the expected total maintenance costs based on a ten year period. From Table 8.1 we can see that only five of the ten possible treatments were applied in the various sensitivity studies. Of those five, one was essentially never applied, and the other was applied less than one percent of the time in five of the studies.

Table 8.1: Optimal maintenance policy summary for each sensitivity analysis

Model	Routine maintenance	Thermopatching	Flush seal	Spot seals	Strip seals
Base	0.87217	0.00183	0.05325	0.07275	
2	0.90812	0.02394	0.03042	0.03741	0.00009
3	0.86633	0.00101	0.05768	0.07498	
4	0.90418	0.00535	0.04764	0.04284	
5	0.89099	0.03555	0.04210	0.03137	
6	0.83575		0.03581	0.12843	
7	0.78403	0.00520	0.08728	0.12349	
8	0.87109	0.00038	0.05327	0.07525	0.00000
9	0.88748	0.02014	0.04540	0.04698	

Table 8.2: Costs associated with each optimal maintenance strategy

Model	Steady state costs ($/m^2)	Discounted expected value ($/m^2)	Discounted variance (($/m^2)^2)
Base	0.0579	0.2740	0.0195
2	0.0512	0.2484	0.0216
3	0.0592	0.2702	0.0173
4	0.0485	0.2407	0.0203
5	0.0789	0.3276	0.0195
6	0.2440	1.8163	0.0176
7	0.0805	0.3919	0.0560
8	0.0750	0.3800	0.0288
9	0.0574	0.2375	0.0141

There was a heavy reliance on routine maintenance in all strategies. The general conclusion was that the model over-represented the effectiveness of routine maintenance. This was true even when the routine maintenance costs were increased by an order of magnitude. The heavy reliance on routine maintenance as a treatment resulted in routine maintenance treatment costs driving the cost of the optimal maintenance strategies.

Most models took more than ten years (the typical length of a long term maintenance contract) to reach steady state. This resulted in some situations where the steady state treatment costs would increase (decrease) relative to the base model and the present value of a typical maintenance contract would decrease (increase). The first conclusion derived from this observation was that the two measures (steady state costs and present value costs) were not necessarily good metrics on their own; the two should be used simultaneously. The second conclusion was that the steady state conditions predicted by each model would be difficult to achieve in an applied environment. The time horizons were too long to be practical. For a more realistic strategy it would be necessary to create a model that represents the same planning horizon as the long term maintenance contract (not the steady state horizon). If the maintenance strategy suggested by this new model were unachievable, then this would suggest that the fault is not in the modelling process; the ten year horizon may not be the appropriate length for a long term maintenance contract.

The next set of observations were generated from the sensitivity analysis.

The objective of the first sensitivity analysis was to determine the impact of reducing pavement deterioration rates (for all distresses). From a performance curve perspective reducing deterioration was equivalent to geometric flattening of the performance curves. All studies that included reduced rates of deterioration (Models 2, and 4) resulted in decreases in maintenance costs (both steady state and present value). This suggests that reducing pavement deterioration is an important objective from both the owner's and contractor's perspective.

The second analysis performed reviewed the benefits of creating a pavement that was more consistent in its performance; the performance was more predictable and thus the bounds on the performance curves were tightened. When the performance curves were tightened (Model 3) the steady costs increased but the present value decreased. When applied in conjunction with the flattened performance curves (Model 4) there was a substantial decrease in both the steady state and present value costs relative to the base case. This suggests that there is some benefit to a more consistent pavement, but the true benefit is when the pavement is both more durable and more consistent.

Models 5, 6 and 8 investigated the impact of increasing costs. As was noted earlier, there was little adjustment made to the basic maintenance strategy largely due to the effectiveness of routine maintenance.

Model 7 illustrated the budgetary impact that changes in policy can have. In the base model up to five percent of the pavement was allowed to reach a poor condition. In this study, the tolerance was reduced to zero and the impact was a 39 % increase in the steady steady costs and 43% increase in the present value costs. This suggests that when performance limits are being established the cost associated with these limits must be examined.

The last model in the study (Model 9) examined the impact of improving treatment effectiveness. The results showed that improving treatment effectiveness had a minor effect on the steady state costs (a less than one percent decrease) but provided the largest decrease (13%) in the present value costs. This was a reasonable result; the more effective the treatment, the less often anything but routine maintenance is going to be applied.

8.2 Research limitations

In spite of the general success of this research, there were some limitations that should be identified. The limitations are presented in an order which matches the general layout of this research.

The first limitation was identified when formulating the non-linear programming model to generate transition probability matrices. In this model the expected value of the pavement's distress must be calculated. An expected value calculation requires a probability and a state value. For this research, the value selected for each condition state (excellent, good, and poor). For bounded condition states, the state value was simply the average value for the condition state bounds. For an unbouned condition state, this was not possible (the average of a finite value and an infinite value is an infinite value). For these condition states, a dual representation was found and an appropriate condition state value was calculated. This dual value was then mapped to the primal space.

Probably the most significant limitation to this study was the application of Markov processes to model the distresses included in this study. In general, the performance curves in this study were relatively long and flat. Consequently the pavement stayed in the initial condition state for an extended period. This could be resolved one of two ways. One approach would be to increase the number of condition states. The practical limit to the number of condition states (due to computational issues) would be five; any more and significant structural changes to the model would be necessary. Alternatively a different modelling technique could have been applied. The suggested approach would have be to use a semi-Markov model to model pavement deterioration. The additional benefit of a semi-Markov formulation is its compactness. This would allow an increase in the number of condition states, or keep the same number of condition states and solve the problem more quickly.

A fundamental weakness of this model and all other Markov process based pavement management models in the literature, is the assumption that the distresses behave independently. With the independence assumption, it is easy to generate the aggregate transition probability matrices; they are simply the product of the distress level transition probability matrices. If there is an area where the Markov process representation of pavement performance breaks down, it is with respect to this assumption.

As far as solving the Markov decision process as a linear program, this went quite well. The only real technical difficulty arose when the performance constraints were added to the model. Once these constraints were added a top end solver (CPLEX) was required to generate a solution. From a modelling perspective, solving for the steady state solution resulted in an impractical solution. The optimal maintenance strategies required between one and three decades to reach steady state. This was an impractical amount of time especially considering that a typical long term maintenance contract is ten years. Once again, the overly effective routine maintenance treatments had some contribution to these results. If a pavement is only slowly deteriorating, it will take a long time for the pavement to reach steady state. Regardless of the cause, it would make sense to reformulate the problem so that a ten year planning horizon is modelled. Annual performance constraints throughout the life of the contract could be included as control points.

In this research, the risk associated with a long term maintenance contract was not explicitly calculated. The assumption was that if each project is independent we could calculate the risk associated with the long term maintenance contract by summing the expected costs and variance of each project. The independence assumption is relatively reasonable, but is it valid? From an engineering perspective the independence assumption was reasonable for a first attempt at solving this problem, but in the long term it is important to be able to identify the factors that would condition the probabilistic performance between projects. The list of potential factors would include traffic, weather, soil conditions.

The sensitivity analysis in this research was simply a series of *what if* studies. There was no research into establishing a trend or relationship between the change in input parameters and model outputs. The *what if* scenarios provided a jumping off point for future research. A suggested approach would be to begin with a typical high, medium, low deterministic sensitivity analysis to determine which models deserve further study. A more thorough examination of the remaining models would result in a mapping between the change in inputs with model outputs.

Another area where the sensitivity analysis could have been extended was an investigation into the impact associated with changes in only one distress. This kind of analysis would help identify if any of the distresses had any more or less impact on the optimal maintenance strategy. A similar study could also have been performed for the effectiveness of each treatment.

Another shortcoming of this research was that the results from each study could not accurately be compared. The likelihood of each scenario occurring was not considered. Hence, the amount of improvement in pavement performance in Model 2 may be only half as likely as the reduction in pavement performance in Model 3. The parametric values selected were chosen either to illustrate a point with respect to the changes to the input, or with the objective of making substantial changes to the model outputs (again for illustrative purposes). All that can really be stated is that the change in input 'A' resulted in a change of output 'B'. It would be useful to select a set of parametric changes that would all occur with the same level of feasibility. This would allow cross study comparison.

8.3 Future research

Future research suggestions were generated from the research summary and research limitations. The order of these suggestions is in no particular order.

1. Exploit leading edge desktop computing technology and increase the number of condition states from three to five. This would improve the Markov processes ability to represent the pavement performance.

2. Apply a semi-Markov model to generate transition probabilities from the routine maintenance performance curves. The expectation is that a semi-markov model would better represent the pavement performance curves included in this study.

3. Better integration of the techniques used to generate transition probability curves with the pavement management optimization model would be useful. In its existing state the two are separate entities. At the very least there needs to be a method in place to automate the transfer of data. In its present form the process has a strong manual component. By integrating the two procedures the impact of changes in the performance curves to the optimal maintenance strategy can be directly measured.

4. There needs to be more research into what value most accurately represents each condition state. The importance of this is reduced if we are able to increase the

number of condition states; the narrower the condition state range, the closer the mean is to the true representative value.

5. Investigate whether it is more accurate to create a dual representation of performance curves that will have an unbounded condition state, and then use this representation throughout the study. The present approach is to create a dual representation and then map the results/findings from the dual space to the primal space.

6. Create a linear programming formulation of the pavement performance model which has a finite planning horizon equal to the length of the long term maintenance contract. This generates an optimal maintenance strategy which could be followed in an applied environment.

7. A finite horizon planning model will require the model to explicitly include a discount rate in the model. Consequently we could investigate the model's sensitivity to the discount rate.

8. Sensitivity analysis should be investigated over a range of values for the input parameters. This would allow us to establish a relationship between changes in inputs and outputs.

9. Investigate the validity of the independence assumption when combining distresses to generate the aggregate transition probability matrices.

10. Similarly, the validity of the independence assumption between projects in a long term maintenance project must be determined.

11. Given a contractor's risk tolerance and the probability distribution associated with the expected cost of a long term maintenance contract, we should be able to quantify the optimal tender price the contractor should submit.

12. Similarly, if a contractor is bidding on a series of long term maintenance contracts, what is the best bidding strategy? Long term maintenance contracts are substantial

endeavors. The contractor must protect himself from the good fortune of winning multiple tenders.

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Appendix A

AN OPTIMAL MAINTENANCE STRATEGY FOR THE BASE CASE MODEL

Optimal Strategy for Base Model

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
1	ROU	RUT1DET1DEP1CRK1SRF1RUF1	0.02	0.0001817
2	ROU	RUT1DET1DEP1CRK1SRF1RUF2	0.02	0.0002348
3	ROU	RUT1DET1DEP1CRK1SRF1RUF3	0.02	0.0765855
4	ROU	RUT1DET1DEP1CRK1SRF2RUF1	0.02	0.0007023
5	ROU	RUT1DET1DEP1CRK1SRF2RUF2	0.02	0.0003358
6	ROU	RUT1DET1DEP1CRK1SRF2RUF3	0.02	0.0297729
7	ROU	RUT1DET1DEP1CRK1SRF3RUF1	0.02	0.0004907
8	FLU	RUT1DET1DEP1CRK1SRF3RUF2	0.22	0.0001757
9	FLU	RUT1DET1DEP1CRK1SRF3RUF3	0.22	0.0147408
10	ROU	RUT1DET1DEP1CRK2SRF1RUF1	0.02	0.0000917
11	ROU	RUT1DET1DEP1CRK2SRF1RUF2	0.02	0.0000839
12	ROU	RUT1DET1DEP1CRK2SRF1RUF3	0.02	0.0238250
13	ROU	RUT1DET1DEP1CRK2SRF2RUF1	0.02	0.0003683
14	FLU	RUT1DET1DEP1CRK2SRF2RUF2	0.22	0.0001158
15	FLU	RUT1DET1DEP1CRK2SRF2RUF3	0.22	0.0049278
16	FLU	RUT1DET1DEP1CRK2SRF3RUF1	0.22	0.0001177
17	FLU	RUT1DET1DEP1CRK2SRF3RUF2	0.22	0.0000343
18	FLU	RUT1DET1DEP1CRK2SRF3RUF3	0.22	0.0028352
19	ROU	RUT1DET1DEP1CRK3SRF1RUF1	0.02	0.0000516
20	ROU	RUT1DET1DEP1CRK3SRF1RUF2	0.02	0.0000451
21	FLU	RUT1DET1DEP1CRK3SRF1RUF3	0.22	0.0063324
22	FLU	RUT1DET1DEP1CRK3SRF2RUF1	0.22	0.0000785
23	FLU	RUT1DET1DEP1CRK3SRF2RUF2	0.22	0.0000242
24	FLU	RUT1DET1DEP1CRK3SRF2RUF3	0.22	0.0004984
25	FLU	RUT1DET1DEP1CRK3SRF3RUF1	0.22	0.0000196
26	FLU	RUT1DET1DEP1CRK3SRF3RUF2	0.22	0.0000055
27	FLU	RUT1DET1DEP1CRK3SRF3RUF3	0.22	0.0000413
28	ROU	RUT1DET1DEP2CRK1SRF1RUF1	0.02	0.0001310
29	ROU	RUT1DET1DEP2CRK1SRF1RUF2	0.02	0.0000826
30	ROU	RUT1DET1DEP2CRK1SRF1RUF3	0.02	0.0170203
31	ROU	RUT1DET1DEP2CRK1SRF2RUF1	0.02	0.0002616
32	ROU	RUT1DET1DEP2CRK1SRF2RUF2	0.02	0.0001107
33	ROU	RUT1DET1DEP2CRK1SRF2RUF3	0.02	0.0069395
34	FLU	RUT1DET1DEP2CRK1SRF3RUF1	0.22	0.0000827
35	ROU	RUT1DET1DEP2CRK1SRF3RUF2	0.02	0.0000510
36	ROU	RUT1DET1DEP2CRK1SRF3RUF3	0.02	0.0158935
37	ROU	RUT1DET1DEP2CRK2SRF1RUF1	0.02	0.0000478
38	ROU	RUT1DET1DEP2CRK2SRF1RUF2	0.02	0.0000285
39	ROU	RUT1DET1DEP2CRK2SRF1RUF3	0.02	0.0062653
40	FLU	RUT1DET1DEP2CRK2SRF2RUF1	0.22	0.0000890
41	ROU	RUT1DET1DEP2CRK2SRF2RUF2	0.02	0.0000323
42	ROU	RUT1DET1DEP2CRK2SRF2RUF3	0.02	0.0021777
43	FLU	RUT1DET1DEP2CRK2SRF3RUF1	0.22	0.0000222
44	FLU	RUT1DET1DEP2CRK2SRF3RUF2	0.22	0.0000095
45	FLU	RUT1DET1DEP2CRK2SRF3RUF3	0.22	0.0021797
46	ROU	RUT1DET1DEP2CRK3SRF1RUF1	0.02	0.0000214
47	ROU	RUT1DET1DEP2CRK3SRF1RUF2	0.02	0.0000167
48	ROU	RUT1DET1DEP2CRK3SRF1RUF3	0.02	0.0049461
49	FLU	RUT1DET1DEP2CRK3SRF2RUF1	0.22	0.0000094
50	ROU	RUT1DET1DEP2CRK3SRF2RUF2	0.02	0.0000083
51	ROU	RUT1DET1DEP2CRK3SRF2RUF3	0.02	0.0016722
52	FLU	RUT1DET1DEP2CRK3SRF3RUF1	0.22	0.0000021
53	FLU	RUT1DET1DEP2CRK3SRF3RUF2	0.22	0.0000019

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
54	FLU	RUT1DET1DEP2CRK3SRF3RUF3	0.22	0.0003629
55	ROU	RUT1DET1DEP3CRK1SRF1RUF1	0.02	0.0001264
56	ROU	RUT1DET1DEP3CRK1SRF1RUF2	0.02	0.0000789
57	ROU	RUT1DET1DEP3CRK1SRF1RUF3	0.02	0.0214317
58	ROU	RUT1DET1DEP3CRK1SRF2RUF1	0.02	0.0001275
59	ROU	RUT1DET1DEP3CRK1SRF2RUF2	0.02	0.0000741
60	ROU	RUT1DET1DEP3CRK1SRF2RUF3	0.02	0.0094202
61	FLU	RUT1DET1DEP3CRK1SRF3RUF1	0.23	0.0000293
62	ROU	RUT1DET1DEP3CRK1SRF3RUF2	0.02	0.0000392
63	ROU	RUT1DET1DEP3CRK1SRF3RUF3	0.02	0.0318076
64	ROU	RUT1DET1DEP3CRK2SRF1RUF1	0.02	0.0000399
65	ROU	RUT1DET1DEP3CRK2SRF1RUF2	0.02	0.0000254
66	ROU	RUT1DET1DEP3CRK2SRF1RUF3	0.02	0.0079730
67	FLU	RUT1DET1DEP3CRK2SRF2RUF1	0.23	0.0000213
68	ROU	RUT1DET1DEP3CRK2SRF2RUF2	0.02	0.0000152
69	ROU	RUT1DET1DEP3CRK2SRF2RUF3	0.02	0.0034312
70	FLU	RUT1DET1DEP3CRK2SRF3RUF1	0.23	0.0000037
71	FLU	RUT1DET1DEP3CRK2SRF3RUF2	0.23	0.0000048
72	ROU	RUT1DET1DEP3CRK2SRF3RUF3	0.02	0.0067103
72	FLU	RUT1DET1DEP3CRK2SRF3RUF3	0.23	0.0004520
73	SPS	RUT1DET1DEP3CRK3SRF1RUF1	0.31	0.0000131
74	ROU	RUT1DET1DEP3CRK3SRF1RUF2	0.02	0.0000171
75	ROU	RUT1DET1DEP3CRK3SRF1RUF3	0.02	0.0210581
76	FLU	RUT1DET1DEP3CRK3SRF2RUF1	0.23	0.0000014
77	ROU	RUT1DET1DEP3CRK3SRF2RUF2	0.02	0.0000070
78	ROU	RUT1DET1DEP3CRK3SRF2RUF3	0.02	0.0089826
79	FLU	RUT1DET1DEP3CRK3SRF3RUF1	0.23	0.0000001
80	FLU	RUT1DET1DEP3CRK3SRF3RUF2	0.23	0.0000015
81	FLU	RUT1DET1DEP3CRK3SRF3RUF3	0.23	0.0038394
82	ROU	RUT1DET2DEP1CRK1SRF1RUF1	0.02	0.0002158
83	ROU	RUT1DET2DEP1CRK1SRF1RUF2	0.02	0.0001226
84	ROU	RUT1DET2DEP1CRK1SRF1RUF3	0.02	0.0190057
85	ROU	RUT1DET2DEP1CRK1SRF2RUF1	0.02	0.0004777
86	ROU	RUT1DET2DEP1CRK1SRF2RUF2	0.02	0.0001875
87	ROU	RUT1DET2DEP1CRK1SRF2RUF3	0.02	0.0067523
88	FLU	RUT1DET2DEP1CRK1SRF3RUF1	0.23	0.0001357
89	ROU	RUT1DET2DEP1CRK1SRF3RUF2	0.02	0.0000814
90	ROU	RUT1DET2DEP1CRK1SRF3RUF3	0.02	0.0148659
91	ROU	RUT1DET2DEP1CRK2SRF1RUF1	0.02	0.0000788
92	ROU	RUT1DET2DEP1CRK2SRF1RUF2	0.02	0.0000430
93	ROU	RUT1DET2DEP1CRK2SRF1RUF3	0.02	0.0060473
94	FLU	RUT1DET2DEP1CRK2SRF2RUF1	0.23	0.0001660
95	ROU	RUT1DET2DEP1CRK2SRF2RUF2	0.02	0.0000563
96	ROU	RUT1DET2DEP1CRK2SRF2RUF3	0.02	0.0020312
97	FLU	RUT1DET2DEP1CRK2SRF3RUF1	0.23	0.0000403
98	FLU	RUT1DET2DEP1CRK2SRF3RUF2	0.23	0.0000160
99	FLU	RUT1DET2DEP1CRK2SRF3RUF3	0.23	0.0019651
100	ROU	RUT1DET2DEP1CRK3SRF1RUF1	0.02	0.0000331
101	ROU	RUT1DET2DEP1CRK3SRF1RUF2	0.02	0.0000247
102	ROU	RUT1DET2DEP1CRK3SRF1RUF3	0.02	0.0052399
103	FLU	RUT1DET2DEP1CRK3SRF2RUF1	0.23	0.0000147
104	ROU	RUT1DET2DEP1CRK3SRF2RUF2	0.02	0.0000138
105	ROU	RUT1DET2DEP1CRK3SRF2RUF3	0.02	0.0018128
106	FLU	RUT1DET2DEP1CRK3SRF3RUF1	0.23	0.0000032
107	FLU	RUT1DET2DEP1CRK3SRF3RUF2	0.23	0.0000032

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
108	FLU	RUT1DET2DEP1CRK3SRF3RUF3	0.23	0.0003924
109	ROU	RUT1DET2DEP2CRK1SRF1RUF1	0.02	0.0000547
110	ROU	RUT1DET2DEP2CRK1SRF1RUF2	0.02	0.0000330
111	ROU	RUT1DET2DEP2CRK1SRF1RUF3	0.02	0.0047986
112	ROU	RUT1DET2DEP2CRK1SRF2RUF1	0.02	0.0001414
113	ROU	RUT1DET2DEP2CRK1SRF2RUF2	0.02	0.0000554
114	ROU	RUT1DET2DEP2CRK1SRF2RUF3	0.02	0.0018260
115	FLU	RUT1DET2DEP2CRK1SRF3RUF1	0.23	0.0000382
116	ROU	RUT1DET2DEP2CRK1SRF3RUF2	0.02	0.0000247
117	ROU	RUT1DET2DEP2CRK1SRF3RUF3	0.02	0.0059812
118	SPS	RUT1DET2DEP2CRK2SRF1RUF1	0.31	0.0000110
119	SPS	RUT1DET2DEP2CRK2SRF1RUF2	0.31	0.0000064
120	ROU	RUT1DET2DEP2CRK2SRF1RUF3	0.02	0.0017040
121	SPS	RUT1DET2DEP2CRK2SRF2RUF1	0.31	0.0000452
122	ROU	RUT1DET2DEP2CRK2SRF2RUF2	0.02	0.0000153
123	ROU	RUT1DET2DEP2CRK2SRF2RUF3	0.02	0.0006135
124	FLU	RUT1DET2DEP2CRK2SRF3RUF1	0.23	0.0000117
125	FLU	RUT1DET2DEP2CRK2SRF3RUF2	0.23	0.0000048
126	FLU	RUT1DET2DEP2CRK2SRF3RUF3	0.23	0.0007009
127	SPS	RUT1DET2DEP2CRK3SRF1RUF1	0.32	0.0000026
128	SPS	RUT1DET2DEP2CRK3SRF1RUF2	0.32	0.0000020
129	ROU	RUT1DET2DEP2CRK3SRF1RUF3	0.02	0.0018293
130	SPS	RUT1DET2DEP2CRK3SRF2RUF1	0.32	0.0000033
131	SPS	RUT1DET2DEP2CRK3SRF2RUF2	0.32	0.0000027
132	THR	RUT1DET2DEP2CRK3SRF2RUF3	0.46	0.0003536
133	FLU	RUT1DET2DEP2CRK3SRF3RUF1	0.23	0.0000008
134	FLU	RUT1DET2DEP2CRK3SRF3RUF2	0.23	0.0000007
135	FLU	RUT1DET2DEP2CRK3SRF3RUF3	0.23	0.0000626
136	ROU	RUT1DET2DEP3CRK1SRF1RUF1	0.02	0.0000316
137	SPS	RUT1DET2DEP3CRK1SRF1RUF2	0.32	0.0000140
138	ROU	RUT1DET2DEP3CRK1SRF1RUF3	0.02	0.0051996
139	SPS	RUT1DET2DEP3CRK1SRF2RUF1	0.32	0.0000283
140	ROU	RUT1DET2DEP3CRK1SRF2RUF2	0.02	0.0000187
141	ROU	RUT1DET2DEP3CRK1SRF2RUF3	0.02	0.0022272
142	SPS	RUT1DET2DEP3CRK1SRF3RUF1	0.32	0.0000068
143	ROU	RUT1DET2DEP3CRK1SRF3RUF2	0.02	0.0000121
144	ROU	RUT1DET2DEP3CRK1SRF3RUF3	0.02	0.0098870
145	SPS	RUT1DET2DEP3CRK2SRF1RUF1	0.32	0.0000040
146	SPS	RUT1DET2DEP3CRK2SRF1RUF2	0.32	0.0000021
147	ROU	RUT1DET2DEP3CRK2SRF1RUF3	0.02	0.0017756
148	SPS	RUT1DET2DEP3CRK2SRF2RUF1	0.32	0.0000060
149	SPS	RUT1DET2DEP3CRK2SRF2RUF2	0.32	0.0000033
150	THR	RUT1DET2DEP3CRK2SRF2RUF3	0.47	0.0004293
151	SPS	RUT1DET2DEP3CRK2SRF3RUF1	0.32	0.0000015
152	SPS	RUT1DET2DEP3CRK2SRF3RUF2	0.32	0.0000014
153	SPS	RUT1DET2DEP3CRK2SRF3RUF3	0.32	0.0009698
154	SPS	RUT1DET2DEP3CRK3SRF1RUF1	0.32	0.0000006
155	SPS	RUT1DET2DEP3CRK3SRF1RUF2	0.32	0.0000007
156	SPS	RUT1DET2DEP3CRK3SRF1RUF3	0.32	0.0016478
157	SPS	RUT1DET2DEP3CRK3SRF2RUF1	0.32	0.0000003
158	SPS	RUT1DET2DEP3CRK3SRF2RUF2	0.32	0.0000007
159	THR	RUT1DET2DEP3CRK3SRF2RUF3	0.47	0.0004536
160	SPS	RUT1DET2DEP3CRK3SRF3RUF1	0.32	0.0000001
161	SPS	RUT1DET2DEP3CRK3SRF3RUF2	0.32	0.0000002
162	SPS	RUT1DET2DEP3CRK3SRF3RUF3	0.32	0.0001669

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
163	ROU	RUT1DET3DEP1CRK1SRF1RUF1	0.02	0.0001547
164	ROU	RUT1DET3DEP1CRK1SRF1RUF2	0.02	0.0000947
165	ROU	RUT1DET3DEP1CRK1SRF1RUF3	0.02	0.0027967
165	SPS	RUT1DET3DEP1CRK1SRF1RUF3	0.32	0.0030181
166	ROU	RUT1DET3DEP1CRK1SRF2RUF1	0.02	0.0001571
167	ROU	RUT1DET3DEP1CRK1SRF2RUF2	0.02	0.0000919
168	ROU	RUT1DET3DEP1CRK1SRF2RUF3	0.02	0.0045305
169	FLU	RUT1DET3DEP1CRK1SRF3RUF1	0.24	0.0000360
170	ROU	RUT1DET3DEP1CRK1SRF3RUF2	0.02	0.0000480
171	ROU	RUT1DET3DEP1CRK1SRF3RUF3	0.02	0.0192853
172	ROU	RUT1DET3DEP1CRK2SRF1RUF1	0.02	0.0000491
173	ROU	RUT1DET3DEP1CRK2SRF1RUF2	0.02	0.0000307
174	ROU	RUT1DET3DEP1CRK2SRF1RUF3	0.02	0.0036418
175	FLU	RUT1DET3DEP1CRK2SRF2RUF1	0.24	0.0000243
175	SPS	RUT1DET3DEP1CRK2SRF2RUF1	0.32	0.0000000
176	ROU	RUT1DET3DEP1CRK2SRF2RUF2	0.02	0.0000185
177	ROU	RUT1DET3DEP1CRK2SRF2RUF3	0.02	0.0018198
178	FLU	RUT1DET3DEP1CRK2SRF3RUF1	0.24	0.0000040
179	FLU	RUT1DET3DEP1CRK2SRF3RUF2	0.24	0.0000058
180	ROU	RUT1DET3DEP1CRK2SRF3RUF3	0.02	0.0046205
181	SPS	RUT1DET3DEP1CRK3SRF1RUF1	0.32	0.0000159
182	ROU	RUT1DET3DEP1CRK3SRF1RUF2	0.02	0.0000202
183	ROU	RUT1DET3DEP1CRK3SRF1RUF3	0.02	0.0115951
184	FLU	RUT1DET3DEP1CRK3SRF2RUF1	0.24	0.0000015
185	ROU	RUT1DET3DEP1CRK3SRF2RUF2	0.02	0.0000085
186	ROU	RUT1DET3DEP1CRK3SRF2RUF3	0.02	0.0050637
187	FLU	RUT1DET3DEP1CRK3SRF3RUF1	0.24	0.0000001
188	FLU	RUT1DET3DEP1CRK3SRF3RUF2	0.24	0.0000019
189	FLU	RUT1DET3DEP1CRK3SRF3RUF3	0.24	0.0023863
190	ROU	RUT1DET3DEP2CRK1SRF1RUF1	0.02	0.0000305
191	SPS	RUT1DET3DEP2CRK1SRF1RUF2	0.32	0.0000139
192	ROU	RUT1DET3DEP2CRK1SRF1RUF3	0.02	0.0024389
193	SPS	RUT1DET3DEP2CRK1SRF2RUF1	0.32	0.0000233
194	ROU	RUT1DET3DEP2CRK1SRF2RUF2	0.02	0.0000166
195	ROU	RUT1DET3DEP2CRK1SRF2RUF3	0.02	0.0012613
196	SPS	RUT1DET3DEP2CRK1SRF3RUF1	0.32	0.0000055
197	ROU	RUT1DET3DEP2CRK1SRF3RUF2	0.02	0.0000108
198	ROU	RUT1DET3DEP2CRK1SRF3RUF3	0.02	0.0063070
199	SPS	RUT1DET3DEP2CRK2SRF1RUF1	0.32	0.0000046
200	SPS	RUT1DET3DEP2CRK2SRF1RUF2	0.32	0.0000025
201	ROU	RUT1DET3DEP2CRK2SRF1RUF3	0.02	0.0009906
202	SPS	RUT1DET3DEP2CRK2SRF2RUF1	0.32	0.0000038
203	SPS	RUT1DET3DEP2CRK2SRF2RUF2	0.33	0.0000027
204	THR	RUT1DET3DEP2CRK2SRF2RUF3	0.48	0.0002701
205	SPS	RUT1DET3DEP2CRK2SRF3RUF1	0.33	0.0000008
206	SPS	RUT1DET3DEP2CRK2SRF3RUF2	0.33	0.0000012
207	SPS	RUT1DET3DEP2CRK2SRF3RUF3	0.33	0.0006647
208	SPS	RUT1DET3DEP2CRK3SRF1RUF1	0.33	0.0000009
209	SPS	RUT1DET3DEP2CRK3SRF1RUF2	0.33	0.0000011
210	SPS	RUT1DET3DEP2CRK3SRF1RUF3	0.33	0.0011034
211	SPS	RUT1DET3DEP2CRK3SRF2RUF1	0.33	0.0000002
212	SPS	RUT1DET3DEP2CRK3SRF2RUF2	0.33	0.0000007
213	THR	RUT1DET3DEP2CRK3SRF2RUF3	0.48	0.0003243
214	SPS	RUT1DET3DEP2CRK3SRF3RUF1	0.33	0.0000000
215	SPS	RUT1DET3DEP2CRK3SRF3RUF2	0.33	0.0000002

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
216	SPS	RUT1DET3DEP2CRK3SRF3RUF3	0.33	0.0001333
217	SPS	RUT1DET3DEP3CRK1SRF1RUF1	0.33	0.0000094
218	SPS	RUT1DET3DEP3CRK1SRF1RUF2	0.33	0.0000034
219	ROU	RUT1DET3DEP3CRK1SRF1RUF3	0.02	0.0068117
220	SPS	RUT1DET3DEP3CRK1SRF2RUF1	0.33	0.0000037
221	ROU	RUT1DET3DEP3CRK1SRF2RUF2	0.02	0.0000066
222	ROU	RUT1DET3DEP3CRK1SRF2RUF3	0.02	0.0030950
223	SPS	RUT1DET3DEP3CRK1SRF3RUF1	0.33	0.0000008
224	SPS	RUT1DET3DEP3CRK1SRF3RUF2	0.33	0.0000040
225	SPS	RUT1DET3DEP3CRK1SRF3RUF3	0.33	0.0035179
226	SPS	RUT1DET3DEP3CRK2SRF1RUF1	0.33	0.0000008
227	SPS	RUT1DET3DEP3CRK2SRF1RUF2	0.33	0.0000003
228	SPS	RUT1DET3DEP3CRK2SRF1RUF3	0.33	0.0008660
229	SPS	RUT1DET3DEP3CRK2SRF2RUF1	0.33	0.0000003
230	SPS	RUT1DET3DEP3CRK2SRF2RUF2	0.33	0.0000007
231	SPS	RUT1DET3DEP3CRK2SRF2RUF3	0.33	0.0002716
232	SPS	RUT1DET3DEP3CRK2SRF3RUF1	0.33	0.0000001
233	SPS	RUT1DET3DEP3CRK2SRF3RUF2	0.33	0.0000004
234	SPS	RUT1DET3DEP3CRK2SRF3RUF3	0.33	0.0002686
237	SPS	RUT1DET3DEP3CRK3SRF1RUF3	0.33	0.0002131
239	SPS	RUT1DET3DEP3CRK3SRF2RUF2	0.33	0.0000001
240	SPS	RUT1DET3DEP3CRK3SRF2RUF3	0.33	0.0000200
242	SPS	RUT1DET3DEP3CRK3SRF3RUF2	0.33	0.0000000
243	SPS	RUT1DET3DEP3CRK3SRF3RUF3	0.33	0.0000014
244	ROU	RUT2DET1DEP1CRK1SRF1RUF1	0.02	0.0000680
245	ROU	RUT2DET1DEP1CRK1SRF1RUF2	0.02	0.0000607
246	ROU	RUT2DET1DEP1CRK1SRF1RUF3	0.02	0.0261990
247	ROU	RUT2DET1DEP1CRK1SRF2RUF1	0.02	0.0001259
248	ROU	RUT2DET1DEP1CRK1SRF2RUF2	0.02	0.0000706
249	ROU	RUT2DET1DEP1CRK1SRF2RUF3	0.02	0.0105264
250	ROU	RUT2DET1DEP1CRK1SRF3RUF1	0.02	0.0001266
251	ROU	RUT2DET1DEP1CRK1SRF3RUF2	0.02	0.0000734
252	ROU	RUT2DET1DEP1CRK1SRF3RUF3	0.02	0.0219676
253	ROU	RUT2DET1DEP1CRK2SRF1RUF1	0.02	0.0000337
254	ROU	RUT2DET1DEP1CRK2SRF1RUF2	0.02	0.0000241
255	ROU	RUT2DET1DEP1CRK2SRF1RUF3	0.02	0.0091287
256	FLU	RUT2DET1DEP1CRK2SRF2RUF1	0.24	0.0000403
257	ROU	RUT2DET1DEP1CRK2SRF2RUF2	0.02	0.0000198
258	ROU	RUT2DET1DEP1CRK2SRF2RUF3	0.02	0.0032994
259	FLU	RUT2DET1DEP1CRK2SRF3RUF1	0.24	0.0000165
260	FLU	RUT2DET1DEP1CRK2SRF3RUF2	0.24	0.0000091
261	FLU	RUT2DET1DEP1CRK2SRF3RUF3	0.24	0.0029685
262	ROU	RUT2DET1DEP1CRK3SRF1RUF1	0.02	0.0000207
263	ROU	RUT2DET1DEP1CRK3SRF1RUF2	0.02	0.0000173
264	ROU	RUT2DET1DEP1CRK3SRF1RUF3	0.02	0.0083245
265	FLU	RUT2DET1DEP1CRK3SRF2RUF1	0.24	0.0000070
266	ROU	RUT2DET1DEP1CRK3SRF2RUF2	0.02	0.0000067
267	ROU	RUT2DET1DEP1CRK3SRF2RUF3	0.02	0.0029669
268	FLU	RUT2DET1DEP1CRK3SRF3RUF1	0.24	0.0000014
269	FLU	RUT2DET1DEP1CRK3SRF3RUF2	0.24	0.0000014
270	FLU	RUT2DET1DEP1CRK3SRF3RUF3	0.24	0.0006452
271	ROU	RUT2DET1DEP2CRK1SRF1RUF1	0.02	0.0000341
272	ROU	RUT2DET1DEP2CRK1SRF1RUF2	0.02	0.0000225
273	ROU	RUT2DET1DEP2CRK1SRF1RUF3	0.02	0.0067379
274	ROU	RUT2DET1DEP2CRK1SRF2RUF1	0.02	0.0000457

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
275	ROU	RUT2DET1DEP2CRK1SRF2RUF2	0.02	0.0000238
276	ROU	RUT2DET1DEP2CRK1SRF2RUF3	0.02	0.0028455
277	FLU	RUT2DET1DEP2CRK1SRF3RUF1	0.25	0.0000171
278	ROU	RUT2DET1DEP2CRK1SRF3RUF2	0.02	0.0000154
279	ROU	RUT2DET1DEP2CRK1SRF3RUF3	0.02	0.0097484
280	SPS	RUT2DET1DEP2CRK2SRF1RUF1	0.33	0.0000075
281	SPS	RUT2DET1DEP2CRK2SRF1RUF2	0.33	0.0000046
282	ROU	RUT2DET1DEP2CRK2SRF1RUF3	0.02	0.0024748
283	SPS	RUT2DET1DEP2CRK2SRF2RUF1	0.33	0.0000095
284	ROU	RUT2DET1DEP2CRK2SRF2RUF2	0.02	0.0000052
285	ROU	RUT2DET1DEP2CRK2SRF2RUF3	0.02	0.0010063
286	FLU	RUT2DET1DEP2CRK2SRF3RUF1	0.25	0.0000027
287	SPS	RUT2DET1DEP2CRK2SRF3RUF2	0.34	0.0000021
288	SPS	RUT2DET1DEP2CRK2SRF3RUF3	0.34	0.0012310
289	SPS	RUT2DET1DEP2CRK3SRF1RUF1	0.34	0.0000025
290	SPS	RUT2DET1DEP2CRK3SRF1RUF2	0.34	0.0000020
291	SPS	RUT2DET1DEP2CRK3SRF1RUF3	0.34	0.0012532
292	SPS	RUT2DET1DEP2CRK3SRF2RUF1	0.34	0.0000008
293	SPS	RUT2DET1DEP2CRK3SRF2RUF2	0.34	0.0000013
294	SPS	RUT2DET1DEP2CRK3SRF2RUF3	0.34	0.0004731
295	FLU	RUT2DET1DEP2CRK3SRF3RUF1	0.25	0.0000002
296	SPS	RUT2DET1DEP2CRK3SRF3RUF2	0.34	0.0000003
297	SPS	RUT2DET1DEP2CRK3SRF3RUF3	0.34	0.0001079
298	ROU	RUT2DET1DEP3CRK1SRF1RUF1	0.02	0.0000310
299	SPS	RUT2DET1DEP3CRK1SRF1RUF2	0.34	0.0000142
300	ROU	RUT2DET1DEP3CRK1SRF1RUF3	0.02	0.0082589
301	SPS	RUT2DET1DEP3CRK1SRF2RUF1	0.34	0.0000160
302	ROU	RUT2DET1DEP3CRK1SRF2RUF2	0.02	0.0000130
303	ROU	RUT2DET1DEP3CRK1SRF2RUF3	0.02	0.0036328
304	SPS	RUT2DET1DEP3CRK1SRF3RUF1	0.34	0.0000035
305	ROU	RUT2DET1DEP3CRK1SRF3RUF2	0.02	0.0000096
306	ROU	RUT2DET1DEP3CRK1SRF3RUF3	0.02	0.0172759
307	SPS	RUT2DET1DEP3CRK2SRF1RUF1	0.34	0.0000047
308	SPS	RUT2DET1DEP3CRK2SRF1RUF2	0.34	0.0000025
309	SPS	RUT2DET1DEP3CRK2SRF1RUF3	0.34	0.0013842
310	SPS	RUT2DET1DEP3CRK2SRF2RUF1	0.34	0.0000021
311	SPS	RUT2DET1DEP3CRK2SRF2RUF2	0.34	0.0000020
312	SPS	RUT2DET1DEP3CRK2SRF2RUF3	0.34	0.0005954
313	SPS	RUT2DET1DEP3CRK2SRF3RUF1	0.34	0.0000004
314	SPS	RUT2DET1DEP3CRK2SRF3RUF2	0.34	0.0000009
315	SPS	RUT2DET1DEP3CRK2SRF3RUF3	0.34	0.0016909
316	SPS	RUT2DET1DEP3CRK3SRF1RUF1	0.34	0.0000010
317	SPS	RUT2DET1DEP3CRK3SRF1RUF2	0.34	0.0000013
318	SPS	RUT2DET1DEP3CRK3SRF1RUF3	0.34	0.0016596
319	SPS	RUT2DET1DEP3CRK3SRF2RUF1	0.34	0.0000001
320	SPS	RUT2DET1DEP3CRK3SRF2RUF2	0.34	0.0000006
321	SPS	RUT2DET1DEP3CRK3SRF2RUF3	0.34	0.0007064
322	SPS	RUT2DET1DEP3CRK3SRF3RUF1	0.34	0.0000000
323	SPS	RUT2DET1DEP3CRK3SRF3RUF2	0.34	0.0000001
324	SPS	RUT2DET1DEP3CRK3SRF3RUF3	0.34	0.0002916
325	ROU	RUT2DET2DEP1CRK1SRF1RUF1	0.02	0.0000357
326	ROU	RUT2DET2DEP1CRK1SRF1RUF2	0.02	0.0000253
327	SPS	RUT2DET2DEP1CRK1SRF1RUF3	0.34	0.0032615
328	ROU	RUT2DET2DEP1CRK1SRF2RUF1	0.02	0.0000674
329	ROU	RUT2DET2DEP1CRK1SRF2RUF2	0.02	0.0000339

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330	ROU	RUT2DET2DEP1CRK1SRF2RUF3	0.02	0.0021730
331	SPS	RUT2DET2DEP1CRK1SRF3RUF1	0.34	0.0000246
332	ROU	RUT2DET2DEP1CRK1SRF3RUF2	0.02	0.0000211
333	ROU	RUT2DET2DEP1CRK1SRF3RUF3	0.02	0.0085153
334	SPS	RUT2DET2DEP1CRK2SRF1RUF1	0.34	0.0000089
335	SPS	RUT2DET2DEP1CRK2SRF1RUF2	0.34	0.0000055
336	SPS	RUT2DET2DEP1CRK2SRF1RUF3	0.34	0.0010314
337	SPS	RUT2DET2DEP1CRK2SRF2RUF1	0.34	0.0000155
338	SPS	RUT2DET2DEP1CRK2SRF2RUF2	0.34	0.0000064
339	ROU	RUT2DET2DEP1CRK2SRF2RUF3	0.02	0.0007134
340	SPS	RUT2DET2DEP1CRK2SRF3RUF1	0.34	0.0000045
341	SPS	RUT2DET2DEP1CRK2SRF3RUF2	0.34	0.0000027
342	SPS	RUT2DET2DEP1CRK2SRF3RUF3	0.34	0.0010235
343	SPS	RUT2DET2DEP1CRK3SRF1RUF1	0.34	0.0000031
344	SPS	RUT2DET2DEP1CRK3SRF1RUF2	0.34	0.0000024
345	SPS	RUT2DET2DEP1CRK3SRF1RUF3	0.34	0.0007295
346	SPS	RUT2DET2DEP1CRK3SRF2RUF1	0.34	0.0000011
347	SPS	RUT2DET2DEP1CRK3SRF2RUF2	0.34	0.0000012
348	SPS	RUT2DET2DEP1CRK3SRF2RUF3	0.34	0.0003751
349	SPS	RUT2DET2DEP1CRK3SRF3RUF1	0.34	0.0000002
350	SPS	RUT2DET2DEP1CRK3SRF3RUF2	0.34	0.0000003
351	SPS	RUT2DET2DEP1CRK3SRF3RUF3	0.34	0.0000902
352	SPS	RUT2DET2DEP2CRK1SRF1RUF1	0.34	0.0000065
353	SPS	RUT2DET2DEP2CRK1SRF1RUF2	0.34	0.0000042
354	SPS	RUT2DET2DEP2CRK1SRF1RUF3	0.34	0.0007937
355	SPS	RUT2DET2DEP2CRK1SRF2RUF1	0.34	0.0000136
356	ROU	RUT2DET2DEP2CRK1SRF2RUF2	0.02	0.0000075
357	ROU	RUT2DET2DEP2CRK1SRF2RUF3	0.02	0.0006090
358	SPS	RUT2DET2DEP2CRK1SRF3RUF1	0.34	0.0000042
359	ROU	RUT2DET2DEP2CRK1SRF3RUF2	0.02	0.0000049
360	SPS	RUT2DET2DEP2CRK1SRF3RUF3	0.34	0.0014788
361	SPS	RUT2DET2DEP2CRK2SRF1RUF1	0.34	0.0000011
362	SPS	RUT2DET2DEP2CRK2SRF1RUF2	0.34	0.0000007
363	SPS	RUT2DET2DEP2CRK2SRF1RUF3	0.34	0.0002538
364	SPS	RUT2DET2DEP2CRK2SRF2RUF1	0.34	0.0000037
365	SPS	RUT2DET2DEP2CRK2SRF2RUF2	0.35	0.0000015
366	SPS	RUT2DET2DEP2CRK2SRF2RUF3	0.35	0.0001338
367	SPS	RUT2DET2DEP2CRK2SRF3RUF1	0.35	0.0000010
368	SPS	RUT2DET2DEP2CRK2SRF3RUF2	0.35	0.0000006
369	SPS	RUT2DET2DEP2CRK2SRF3RUF3	0.35	0.0001728
370	SPS	RUT2DET2DEP2CRK3SRF1RUF1	0.35	0.0000002
371	SPS	RUT2DET2DEP2CRK3SRF1RUF2	0.35	0.0000002
372	SPS	RUT2DET2DEP2CRK3SRF1RUF3	0.35	0.0001728
373	SPS	RUT2DET2DEP2CRK3SRF2RUF1	0.35	0.0000002
374	SPS	RUT2DET2DEP2CRK3SRF2RUF2	0.35	0.0000002
375	SPS	RUT2DET2DEP2CRK3SRF2RUF3	0.35	0.0000473
376	SPS	RUT2DET2DEP2CRK3SRF3RUF1	0.35	0.0000001
377	SPS	RUT2DET2DEP2CRK3SRF3RUF2	0.35	0.0000001
378	SPS	RUT2DET2DEP2CRK3SRF3RUF3	0.35	0.0000100
379	SPS	RUT2DET2DEP3CRK1SRF1RUF1	0.35	0.0000032
380	SPS	RUT2DET2DEP3CRK1SRF1RUF2	0.35	0.0000014
381	SPS	RUT2DET2DEP3CRK1SRF1RUF3	0.35	0.0006699
382	SPS	RUT2DET2DEP3CRK1SRF2RUF1	0.35	0.0000024
383	SPS	RUT2DET2DEP3CRK1SRF2RUF2	0.35	0.0000021
384	SPS	RUT2DET2DEP3CRK1SRF2RUF3	0.35	0.0003571

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
385	SPS	RUT2DET2DEP3CRK1SRF3RUF1	0.35	0.0000006
386	SPS	RUT2DET2DEP3CRK1SRF3RUF2	0.35	0.0000016
387	SPS	RUT2DET2DEP3CRK1SRF3RUF3	0.35	0.0013760
388	SPS	RUT2DET2DEP3CRK2SRF1RUF1	0.35	0.0000004
389	SPS	RUT2DET2DEP3CRK2SRF1RUF2	0.35	0.0000002
390	SPS	RUT2DET2DEP3CRK2SRF1RUF3	0.35	0.0001648
391	SPS	RUT2DET2DEP3CRK2SRF2RUF1	0.35	0.0000005
392	SPS	RUT2DET2DEP3CRK2SRF2RUF2	0.35	0.0000003
393	SPS	RUT2DET2DEP3CRK2SRF2RUF3	0.35	0.0000514
394	SPS	RUT2DET2DEP3CRK2SRF3RUF1	0.35	0.0000001
395	SPS	RUT2DET2DEP3CRK2SRF3RUF2	0.35	0.0000002
396	SPS	RUT2DET2DEP3CRK2SRF3RUF3	0.35	0.0001223
397	SPS	RUT2DET2DEP3CRK3SRF1RUF1	0.35	0.0000000
398	SPS	RUT2DET2DEP3CRK3SRF1RUF2	0.35	0.0000001
399	SPS	RUT2DET2DEP3CRK3SRF1RUF3	0.35	0.0001254
400	SPS	RUT2DET2DEP3CRK3SRF2RUF1	0.35	0.0000000
401	SPS	RUT2DET2DEP3CRK3SRF2RUF2	0.35	0.0000001
402	SPS	RUT2DET2DEP3CRK3SRF2RUF3	0.35	0.0000352
403	SPS	RUT2DET2DEP3CRK3SRF3RUF1	0.35	0.0000000
404	SPS	RUT2DET2DEP3CRK3SRF3RUF2	0.35	0.0000000
405	SPS	RUT2DET2DEP3CRK3SRF3RUF3	0.35	0.0000127
406	ROU	RUT2DET3DEP1CRK1SRF1RUF1	0.02	0.0000302
407	SPS	RUT2DET3DEP1CRK1SRF1RUF2	0.35	0.0000142
408	SPS	RUT2DET3DEP1CRK1SRF1RUF3	0.35	0.0004563
409	SPS	RUT2DET3DEP1CRK1SRF2RUF1	0.35	0.0000180
410	ROU	RUT2DET3DEP1CRK1SRF2RUF2	0.02	0.0000148
411	ROU	RUT2DET3DEP1CRK1SRF2RUF3	0.02	0.0012994
412	SPS	RUT2DET3DEP1CRK1SRF3RUF1	0.35	0.0000041
413	ROU	RUT2DET3DEP1CRK1SRF3RUF2	0.02	0.0000110
414	ROU	RUT2DET3DEP1CRK1SRF3RUF3	0.02	0.0099981
415	SPS	RUT2DET3DEP1CRK2SRF1RUF1	0.35	0.0000050
416	SPS	RUT2DET3DEP1CRK2SRF1RUF2	0.35	0.0000028
417	SPS	RUT2DET3DEP1CRK2SRF1RUF3	0.35	0.0002769
418	SPS	RUT2DET3DEP1CRK2SRF2RUF1	0.35	0.0000023
419	SPS	RUT2DET3DEP1CRK2SRF2RUF2	0.35	0.0000020
420	SPS	RUT2DET3DEP1CRK2SRF2RUF3	0.35	0.0002611
421	SPS	RUT2DET3DEP1CRK2SRF3RUF1	0.35	0.0000004
422	SPS	RUT2DET3DEP1CRK2SRF3RUF2	0.35	0.0000010
423	SPS	RUT2DET3DEP1CRK2SRF3RUF3	0.35	0.0010074
424	SPS	RUT2DET3DEP1CRK3SRF1RUF1	0.35	0.0000012
425	SPS	RUT2DET3DEP1CRK3SRF1RUF2	0.35	0.0000015
426	SPS	RUT2DET3DEP1CRK3SRF1RUF3	0.35	0.0008468
427	SPS	RUT2DET3DEP1CRK3SRF2RUF1	0.35	0.0000001
428	SPS	RUT2DET3DEP1CRK3SRF2RUF2	0.35	0.0000006
429	SPS	RUT2DET3DEP1CRK3SRF2RUF3	0.35	0.0003925
430	SPS	RUT2DET3DEP1CRK3SRF3RUF1	0.35	0.0000000
431	SPS	RUT2DET3DEP1CRK3SRF3RUF2	0.35	0.0000001
432	SPS	RUT2DET3DEP1CRK3SRF3RUF3	0.35	0.0001803
433	SPS	RUT2DET3DEP2CRK1SRF1RUF1	0.35	0.0000033
434	SPS	RUT2DET3DEP2CRK1SRF1RUF2	0.35	0.0000014
435	SPS	RUT2DET3DEP2CRK1SRF1RUF3	0.35	0.0001850
436	SPS	RUT2DET3DEP2CRK1SRF2RUF1	0.35	0.0000021
437	SPS	RUT2DET3DEP2CRK1SRF2RUF2	0.35	0.0000020
438	SPS	RUT2DET3DEP2CRK1SRF2RUF3	0.35	0.0001957
439	SPS	RUT2DET3DEP2CRK1SRF3RUF1	0.35	0.0000005

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440	SPS	RUT2DET3DEP2CRK1SRF3RUF2	0.35	0.0000015
441	SPS	RUT2DET3DEP2CRK1SRF3RUF3	0.35	0.0009445
442	SPS	RUT2DET3DEP2CRK2SRF1RUF1	0.35	0.0000004
443	SPS	RUT2DET3DEP2CRK2SRF1RUF2	0.35	0.0000002
444	SPS	RUT2DET3DEP2CRK2SRF1RUF3	0.35	0.0000738
445	SPS	RUT2DET3DEP2CRK2SRF2RUF1	0.35	0.0000003
446	SPS	RUT2DET3DEP2CRK2SRF2RUF2	0.36	0.0000003
447	SPS	RUT2DET3DEP2CRK2SRF2RUF3	0.36	0.0000307
448	SPS	RUT2DET3DEP2CRK2SRF3RUF1	0.36	0.0000001
449	SPS	RUT2DET3DEP2CRK2SRF3RUF2	0.36	0.0000001
450	SPS	RUT2DET3DEP2CRK2SRF3RUF3	0.36	0.0000866
451	SPS	RUT2DET3DEP2CRK3SRF1RUF1	0.36	0.0000001
452	SPS	RUT2DET3DEP2CRK3SRF1RUF2	0.36	0.0000001
453	SPS	RUT2DET3DEP2CRK3SRF1RUF3	0.36	0.0000806
454	SPS	RUT2DET3DEP2CRK3SRF2RUF1	0.36	0.0000000
455	SPS	RUT2DET3DEP2CRK3SRF2RUF2	0.36	0.0000001
456	SPS	RUT2DET3DEP2CRK3SRF2RUF3	0.36	0.0000249
457	SPS	RUT2DET3DEP2CRK3SRF3RUF1	0.36	0.0000000
458	SPS	RUT2DET3DEP2CRK3SRF3RUF2	0.36	0.0000000
459	SPS	RUT2DET3DEP2CRK3SRF3RUF3	0.36	0.0000101
460	SPS	RUT2DET3DEP3CRK1SRF1RUF1	0.36	0.0000007
461	SPS	RUT2DET3DEP3CRK1SRF1RUF2	0.36	0.0000003
462	SPS	RUT2DET3DEP3CRK1SRF1RUF3	0.36	0.0004974
463	SPS	RUT2DET3DEP3CRK1SRF2RUF1	0.36	0.0000003
464	SPS	RUT2DET3DEP3CRK1SRF2RUF2	0.36	0.0000006
465	SPS	RUT2DET3DEP3CRK1SRF2RUF3	0.36	0.0002394
466	SPS	RUT2DET3DEP3CRK1SRF3RUF1	0.36	0.0000001
467	SPS	RUT2DET3DEP3CRK1SRF3RUF2	0.36	0.0000004
468	SPS	RUT2DET3DEP3CRK1SRF3RUF3	0.36	0.0002606
469	SPS	RUT2DET3DEP3CRK2SRF1RUF1	0.36	0.0000001
470	SPS	RUT2DET3DEP3CRK2SRF1RUF2	0.36	0.0000000
471	SPS	RUT2DET3DEP3CRK2SRF1RUF3	0.36	0.0000632
472	SPS	RUT2DET3DEP3CRK2SRF2RUF1	0.36	0.0000000
473	SPS	RUT2DET3DEP3CRK2SRF2RUF2	0.36	0.0000001
474	SPS	RUT2DET3DEP3CRK2SRF2RUF3	0.36	0.0000209
475	SPS	RUT2DET3DEP3CRK2SRF3RUF1	0.36	0.0000000
476	SPS	RUT2DET3DEP3CRK2SRF3RUF2	0.36	0.0000000
477	SPS	RUT2DET3DEP3CRK2SRF3RUF3	0.36	0.0000199
480	SPS	RUT2DET3DEP3CRK3SRF1RUF3	0.36	0.0000156
482	SPS	RUT2DET3DEP3CRK3SRF2RUF2	0.36	0.0000000
483	SPS	RUT2DET3DEP3CRK3SRF2RUF3	0.36	0.0000015
486	SPS	RUT2DET3DEP3CRK3SRF3RUF3	0.36	0.0000001
487	ROU	RUT3DET1DEP1CRK1SRF1RUF1	0.02	0.0000577
488	ROU	RUT3DET1DEP1CRK1SRF1RUF2	0.02	0.0000383
489	ROU	RUT3DET1DEP1CRK1SRF1RUF3	0.02	0.0311273
490	ROU	RUT3DET1DEP1CRK1SRF2RUF1	0.02	0.0000464
491	ROU	RUT3DET1DEP1CRK1SRF2RUF2	0.02	0.0000308
492	ROU	RUT3DET1DEP1CRK1SRF2RUF3	0.02	0.0136931
493	FLU	RUT3DET1DEP1CRK1SRF3RUF1	0.26	0.0000257
494	ROU	RUT3DET1DEP1CRK1SRF3RUF2	0.02	0.0000315
495	ROU	RUT3DET1DEP1CRK1SRF3RUF3	0.02	0.0447629
496	ROU	RUT3DET1DEP1CRK2SRF1RUF1	0.02	0.0000171
497	ROU	RUT3DET1DEP1CRK2SRF1RUF2	0.02	0.0000121
498	ROU	RUT3DET1DEP1CRK2SRF1RUF3	0.02	0.0113911
499	SPS	RUT3DET1DEP1CRK2SRF2RUF1	0.36	0.0000055

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
500	ROU	RUT3DET1DEP1CRK2SRF2RUF2	0.02	0.0000059
501	ROU	RUT3DET1DEP1CRK2SRF2RUF3	0.02	0.0049756
502	FLU	RUT3DET1DEP1CRK2SRF3RUF1	0.27	0.0000020
503	ROU	RUT3DET1DEP1CRK2SRF3RUF2	0.02	0.0000046
504	ROU	RUT3DET1DEP1CRK2SRF3RUF3	0.02	0.0114939
505	SPS	RUT3DET1DEP1CRK3SRF1RUF1	0.36	0.0000072
506	ROU	RUT3DET1DEP1CRK3SRF1RUF2	0.02	0.0000104
507	ROU	RUT3DET1DEP1CRK3SRF1RUF3	0.02	0.0333611
508	SPS	RUT3DET1DEP1CRK3SRF2RUF1	0.36	0.0000005
509	ROU	RUT3DET1DEP1CRK3SRF2RUF2	0.02	0.0000036
510	ROU	RUT3DET1DEP1CRK3SRF2RUF3	0.02	0.0144377
512	FLU	RUT3DET1DEP1CRK3SRF3RUF2	0.27	0.0000014
513	FLU	RUT3DET1DEP1CRK3SRF3RUF3	0.27	0.0064340
514	ROU	RUT3DET1DEP2CRK1SRF1RUF1	0.02	0.0000152
515	ROU	RUT3DET1DEP2CRK1SRF1RUF2	0.02	0.0000107
516	ROU	RUT3DET1DEP2CRK1SRF1RUF3	0.02	0.0073271
517	SPS	RUT3DET1DEP2CRK1SRF2RUF1	0.36	0.0000075
518	ROU	RUT3DET1DEP2CRK1SRF2RUF2	0.02	0.0000066
519	ROU	RUT3DET1DEP2CRK1SRF2RUF3	0.02	0.0032292
520	ROU	RUT3DET1DEP2CRK1SRF3RUF1	0.02	0.0000053
521	ROU	RUT3DET1DEP2CRK1SRF3RUF2	0.02	0.0000075
522	ROU	RUT3DET1DEP2CRK1SRF3RUF3	0.02	0.0145058
523	SPS	RUT3DET1DEP2CRK2SRF1RUF1	0.36	0.0000019
524	SPS	RUT3DET1DEP2CRK2SRF1RUF2	0.36	0.0000013
525	ROU	RUT3DET1DEP2CRK2SRF1RUF3	0.02	0.0024896
526	SPS	RUT3DET1DEP2CRK2SRF2RUF1	0.36	0.0000007
527	SPS	RUT3DET1DEP2CRK2SRF2RUF2	0.37	0.0000009
528	SPS	RUT3DET1DEP2CRK2SRF2RUF3	0.37	0.0006418
529	SPS	RUT3DET1DEP2CRK2SRF3RUF1	0.37	0.0000004
530	SPS	RUT3DET1DEP2CRK2SRF3RUF2	0.37	0.0000007
531	SPS	RUT3DET1DEP2CRK2SRF3RUF3	0.37	0.0015667
532	SPS	RUT3DET1DEP2CRK3SRF1RUF1	0.37	0.0000004
533	SPS	RUT3DET1DEP2CRK3SRF1RUF2	0.37	0.0000006
534	SPS	RUT3DET1DEP2CRK3SRF1RUF3	0.37	0.0024341
535	SPS	RUT3DET1DEP2CRK3SRF2RUF1	0.37	0.0000000
536	SPS	RUT3DET1DEP2CRK3SRF2RUF2	0.37	0.0000003
537	SPS	RUT3DET1DEP2CRK3SRF2RUF3	0.37	0.0008550
539	SPS	RUT3DET1DEP2CRK3SRF3RUF2	0.37	0.0000001
540	SPS	RUT3DET1DEP2CRK3SRF3RUF3	0.37	0.0003552
541	SPS	RUT3DET1DEP3CRK1SRF1RUF1	0.37	0.0000070
542	ROU	RUT3DET1DEP3CRK1SRF1RUF2	0.02	0.0000061
543	ROU	RUT3DET1DEP3CRK1SRF1RUF3	0.02	0.0170642
544	SPS	RUT3DET1DEP3CRK1SRF2RUF1	0.37	0.0000016
545	ROU	RUT3DET1DEP3CRK1SRF2RUF2	0.02	0.0000039
546	ROU	RUT3DET1DEP3CRK1SRF2RUF3	0.02	0.0076057
547	SPS	RUT3DET1DEP3CRK1SRF3RUF1	0.37	0.0000011
548	SPS	RUT3DET1DEP3CRK1SRF3RUF2	0.37	0.0000030
549	SPS	RUT3DET1DEP3CRK1SRF3RUF3	0.37	0.0076949
550	SPS	RUT3DET1DEP3CRK2SRF1RUF1	0.37	0.0000005
551	SPS	RUT3DET1DEP3CRK2SRF1RUF2	0.37	0.0000005
552	SPS	RUT3DET1DEP3CRK2SRF1RUF3	0.37	0.0016819
553	SPS	RUT3DET1DEP3CRK2SRF2RUF1	0.37	0.0000001
554	SPS	RUT3DET1DEP3CRK2SRF2RUF2	0.37	0.0000004
555	SPS	RUT3DET1DEP3CRK2SRF2RUF3	0.37	0.0006216
556	SPS	RUT3DET1DEP3CRK2SRF3RUF1	0.37	0.0000001

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
557	SPS	RUT3DET1DEP3CRK2SRF3RUF2	0.37	0.0000002
558	SPS	RUT3DET1DEP3CRK2SRF3RUF3	0.37	0.0005854
561	SPS	RUT3DET1DEP3CRK3SRF1RUF3	0.37	0.0001771
563	SPS	RUT3DET1DEP3CRK3SRF2RUF2	0.37	0.0000000
564	SPS	RUT3DET1DEP3CRK3SRF2RUF3	0.37	0.0000217
566	SPS	RUT3DET1DEP3CRK3SRF3RUF2	0.37	0.0000000
567	SPS	RUT3DET1DEP3CRK3SRF3RUF3	0.37	0.0000025
568	ROU	RUT3DET2DEP1CRK1SRF1RUF1	0.02	0.0000126
569	SPS	RUT3DET2DEP1CRK1SRF1RUF2	0.37	0.0000064
570	ROU	RUT3DET2DEP1CRK1SRF1RUF3	0.02	0.0042605
571	SPS	RUT3DET2DEP1CRK1SRF2RUF1	0.37	0.0000083
572	ROU	RUT3DET2DEP1CRK1SRF2RUF2	0.02	0.0000074
573	ROU	RUT3DET2DEP1CRK1SRF2RUF3	0.02	0.0024554
574	SPS	RUT3DET2DEP1CRK1SRF3RUF1	0.37	0.0000026
575	ROU	RUT3DET2DEP1CRK1SRF3RUF2	0.02	0.0000070
576	ROU	RUT3DET2DEP1CRK1SRF3RUF3	0.02	0.0134023
577	SPS	RUT3DET2DEP1CRK2SRF1RUF1	0.37	0.0000015
578	SPS	RUT3DET2DEP1CRK2SRF1RUF2	0.37	0.0000009
579	ROU	RUT3DET2DEP1CRK2SRF1RUF3	0.02	0.0014496
580	SPS	RUT3DET2DEP1CRK2SRF2RUF1	0.37	0.0000007
581	SPS	RUT3DET2DEP1CRK2SRF2RUF2	0.37	0.0000007
582	SPS	RUT3DET2DEP1CRK2SRF2RUF3	0.37	0.0004642
583	SPS	RUT3DET2DEP1CRK2SRF3RUF1	0.37	0.0000002
584	SPS	RUT3DET2DEP1CRK2SRF3RUF2	0.37	0.0000006
585	SPS	RUT3DET2DEP1CRK2SRF3RUF3	0.37	0.0013766
586	SPS	RUT3DET2DEP1CRK3SRF1RUF1	0.37	0.0000003
587	SPS	RUT3DET2DEP1CRK3SRF1RUF2	0.37	0.0000004
588	SPS	RUT3DET2DEP1CRK3SRF1RUF3	0.37	0.0017223
589	SPS	RUT3DET2DEP1CRK3SRF2RUF1	0.37	0.0000000
590	SPS	RUT3DET2DEP1CRK3SRF2RUF2	0.37	0.0000002
591	SPS	RUT3DET2DEP1CRK3SRF2RUF3	0.37	0.0006503
593	SPS	RUT3DET2DEP1CRK3SRF3RUF2	0.37	0.0000001
594	SPS	RUT3DET2DEP1CRK3SRF3RUF3	0.37	0.0002751
595	SPS	RUT3DET2DEP2CRK1SRF1RUF1	0.37	0.0000012
596	SPS	RUT3DET2DEP2CRK1SRF1RUF2	0.37	0.0000007
597	ROU	RUT3DET2DEP2CRK1SRF1RUF3	0.02	0.0000000
597	SPS	RUT3DET2DEP2CRK1SRF1RUF3	0.37	0.0004735
598	SPS	RUT3DET2DEP2CRK1SRF2RUF1	0.37	0.0000007
599	SPS	RUT3DET2DEP2CRK1SRF2RUF2	0.37	0.0000010
600	SPS	RUT3DET2DEP2CRK1SRF2RUF3	0.37	0.0002942
601	SPS	RUT3DET2DEP2CRK1SRF3RUF1	0.37	0.0000003
602	SPS	RUT3DET2DEP2CRK1SRF3RUF2	0.37	0.0000010
603	SPS	RUT3DET2DEP2CRK1SRF3RUF3	0.37	0.0012518
604	SPS	RUT3DET2DEP2CRK2SRF1RUF1	0.37	0.0000001
605	SPS	RUT3DET2DEP2CRK2SRF1RUF2	0.37	0.0000001
606	SPS	RUT3DET2DEP2CRK2SRF1RUF3	0.37	0.0001576
607	SPS	RUT3DET2DEP2CRK2SRF2RUF1	0.37	0.0000001
608	SPS	RUT3DET2DEP2CRK2SRF2RUF2	0.38	0.0000001
609	SPS	RUT3DET2DEP2CRK2SRF2RUF3	0.38	0.0000452
610	SPS	RUT3DET2DEP2CRK2SRF3RUF1	0.38	0.0000000
611	SPS	RUT3DET2DEP2CRK2SRF3RUF2	0.38	0.0000001
612	SPS	RUT3DET2DEP2CRK2SRF3RUF3	0.38	0.0001161
613	SPS	RUT3DET2DEP2CRK3SRF1RUF1	0.38	0.0000000
614	SPS	RUT3DET2DEP2CRK3SRF1RUF2	0.38	0.0000000
615	SPS	RUT3DET2DEP2CRK3SRF1RUF3	0.38	0.0001193

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
617	SPS	RUT3DET2DEP2CRK3SRF2RUF2	0.38	0.0000000
618	SPS	RUT3DET2DEP2CRK3SRF2RUF3	0.38	0.0000383
620	SPS	RUT3DET2DEP2CRK3SRF3RUF2	0.38	0.0000000
621	SPS	RUT3DET2DEP2CRK3SRF3RUF3	0.38	0.0000152
622	SPS	RUT3DET2DEP3CRK1SRF1RUF1	0.38	0.0000003
623	SPS	RUT3DET2DEP3CRK1SRF1RUF2	0.38	0.0000003
624	SPS	RUT3DET2DEP3CRK1SRF1RUF3	0.38	0.0007116
625	SPS	RUT3DET2DEP3CRK1SRF2RUF1	0.38	0.0000001
626	SPS	RUT3DET2DEP3CRK1SRF2RUF2	0.38	0.0000003
627	SPS	RUT3DET2DEP3CRK1SRF2RUF3	0.38	0.0003321
628	SPS	RUT3DET2DEP3CRK1SRF3RUF1	0.38	0.0000001
629	SPS	RUT3DET2DEP3CRK1SRF3RUF2	0.38	0.0000002
630	SPS	RUT3DET2DEP3CRK1SRF3RUF3	0.38	0.0003250
631	SPS	RUT3DET2DEP3CRK2SRF1RUF1	0.38	0.0000000
632	SPS	RUT3DET2DEP3CRK2SRF1RUF2	0.38	0.0000000
633	SPS	RUT3DET2DEP3CRK2SRF1RUF3	0.38	0.0000702
634	SPS	RUT3DET2DEP3CRK2SRF2RUF1	0.38	0.0000000
635	SPS	RUT3DET2DEP3CRK2SRF2RUF2	0.38	0.0000000
636	SPS	RUT3DET2DEP3CRK2SRF2RUF3	0.38	0.0000271
637	SPS	RUT3DET2DEP3CRK2SRF3RUF1	0.38	0.0000000
638	SPS	RUT3DET2DEP3CRK2SRF3RUF2	0.38	0.0000000
639	SPS	RUT3DET2DEP3CRK2SRF3RUF3	0.38	0.0000247
642	SPS	RUT3DET2DEP3CRK3SRF1RUF3	0.38	0.0000074
645	SPS	RUT3DET2DEP3CRK3SRF2RUF3	0.38	0.0000009
648	SPS	RUT3DET2DEP3CRK3SRF3RUF3	0.38	0.0000001
649	SPS	RUT3DET3DEP1CRK1SRF1RUF1	0.38	0.0000056
650	SPS	RUT3DET3DEP1CRK1SRF1RUF2	0.38	0.0000020
651	ROU	RUT3DET3DEP1CRK1SRF1RUF3	0.03	0.0033859
652	SPS	RUT3DET3DEP1CRK1SRF2RUF1	0.38	0.0000015
653	ROU	RUT3DET3DEP1CRK1SRF2RUF2	0.03	0.0000036
654	ROU	RUT3DET3DEP1CRK1SRF2RUF3	0.03	0.0024490
655	SPS	RUT3DET3DEP1CRK1SRF3RUF1	0.38	0.0000003
656	SPS	RUT3DET3DEP1CRK1SRF3RUF2	0.38	0.0000027
657	SPS	RUT3DET3DEP1CRK1SRF3RUF3	0.38	0.0044302
658	SPS	RUT3DET3DEP1CRK2SRF1RUF1	0.38	0.0000004
659	SPS	RUT3DET3DEP1CRK2SRF1RUF2	0.38	0.0000002
660	SPS	RUT3DET3DEP1CRK2SRF1RUF3	0.38	0.0004046
661	SPS	RUT3DET3DEP1CRK2SRF2RUF1	0.38	0.0000001
662	SPS	RUT3DET3DEP1CRK2SRF2RUF2	0.38	0.0000003
663	SPS	RUT3DET3DEP1CRK2SRF2RUF3	0.38	0.0002063
664	SPS	RUT3DET3DEP1CRK2SRF3RUF1	0.38	0.0000000
665	SPS	RUT3DET3DEP1CRK2SRF3RUF2	0.38	0.0000002
666	SPS	RUT3DET3DEP1CRK2SRF3RUF3	0.38	0.0003368
669	SPS	RUT3DET3DEP1CRK3SRF1RUF3	0.38	0.0000668
672	SPS	RUT3DET3DEP1CRK3SRF2RUF3	0.38	0.0000097
675	SPS	RUT3DET3DEP1CRK3SRF3RUF3	0.38	0.0000013
676	SPS	RUT3DET3DEP2CRK1SRF1RUF1	0.38	0.0000003
677	SPS	RUT3DET3DEP2CRK1SRF1RUF2	0.38	0.0000001
678	SPS	RUT3DET3DEP2CRK1SRF1RUF3	0.38	0.0001821
679	SPS	RUT3DET3DEP2CRK1SRF2RUF1	0.38	0.0000001
680	SPS	RUT3DET3DEP2CRK1SRF2RUF2	0.38	0.0000003
681	SPS	RUT3DET3DEP2CRK1SRF2RUF3	0.38	0.0001426
682	SPS	RUT3DET3DEP2CRK1SRF3RUF1	0.38	0.0000000
683	SPS	RUT3DET3DEP2CRK1SRF3RUF2	0.38	0.0000002
684	SPS	RUT3DET3DEP2CRK1SRF3RUF3	0.38	0.0002412

Condition State	Treatment Abbreviation	Condition State Abbreviation	Treatment Cost $/square metre/year	Probability of Treatment being applied at Steady State
685	SPS	RUT3DET3DEP2CRK2SRF1RUF1	0.38	0.0000000
686	SPS	RUT3DET3DEP2CRK2SRF1RUF2	0.38	0.0000000
687	SPS	RUT3DET3DEP2CRK2SRF1RUF3	0.38	0.0000218
688	SPS	RUT3DET3DEP2CRK2SRF2RUF1	0.38	0.0000000
689	SPS	RUT3DET3DEP2CRK2SRF2RUF2	0.39	0.0000000
690	SPS	RUT3DET3DEP2CRK2SRF2RUF3	0.39	0.0000119
693	SPS	RUT3DET3DEP2CRK2SRF3RUF3	0.39	0.0000183
696	SPS	RUT3DET3DEP2CRK3SRF1RUF3	0.39	0.0000036
699	SPS	RUT3DET3DEP2CRK3SRF2RUF3	0.39	0.0000005
702	SPS	RUT3DET3DEP2CRK3SRF3RUF3	0.39	0.0000001
707	SPS	RUT3DET3DEP3CRK1SRF2RUF2	0.39	0.0000000
708	SPS	RUT3DET3DEP3CRK1SRF2RUF3	0.39	0.0000030
710	SPS	RUT3DET3DEP3CRK1SRF3RUF2	0.39	0.0000000
711	SPS	RUT3DET3DEP3CRK1SRF3RUF3	0.39	0.0000008
717	SPS	RUT3DET3DEP3CRK2SRF2RUF3	0.39	0.0000002
719	SPS	RUT3DET3DEP3CRK2SRF3RUF2	0.39	0.0000000
720	SPS	RUT3DET3DEP3CRK2SRF3RUF3	0.39	0.0000001

Appendix B

METHODOLOGY OVERVIEW

B.1 Introduction

This appendix is an overview of the methodology used to convert pavement performance curves to the transition probability matrices (TPM) required to generate an optimal pavement management strategy. The document follows the flow of information through the modelling process. The asset modelled is a fictitious section of highway which is a part of a portfolio of projects in a long term maintenance contract. The model consists of six distresses (each with 3 condition states) and ten treatments.

Condition states:	Excellent (E), Good (G), Poor (P)
Distresses:	Rutting, Transverse Cracking-Deterioration,
	Transverse Cracking-Depth, Cracking Surface, Roughness
Treatments:	Routine maintenance, Thermopatch, Flush seal,
	Spot seal, Strip seals, Microsurface, Full seal,
	Spot overlay and seal, Thin overlay, Structural overlay

B.2 Converting performance curves to TPM

Converting asset performance curves to transition probability matrices is a multi-step procedure and has been automated in an Excel spreadsheet. The illustrations included in this document are specifically for the rutting distress.

The first step in creating the transition probability matrix (TPM) is entering the performance curve data and the defining limits for each condition state.

The three curves shown in Figure B.1 represent (from top to bottom) the upper bound, nominal and lower bound performance curves. The nominal curve is the asset performance that is most likely to occur. The upper bound is intended to put a bound on the asset's worst possible performance. The lower bound is a representation of the asset's best possible performance. Throughout the life of this asset there is an implied uncertainty about the asset's performance. This uncertainty is modelled with a beta distribution. A beta distribution can be described by two parameters (n and r). This is similar to the normal distribution which can be described by its mean and variance.

The shaded areas in the two tables to the left are user input areas. All other information is generated by formulas or programming. The user enters the required data and presses the "Solve for TPM" button in the lower right hand corner. The transition probability matrix (lower left hand corner) is automatically generated. The following figures illustrate the "behind the scenes" processes required to complete this step.

Figure B.2 is the worksheet where n and r are calculated. Note that q is the normalized value of a point on the nominal curve over the range between the lower bound and upper bound curves. This calculation is not explicitly shown on this screen.

A user-defined function was programmed to take q as an input and generate r which in turn allows us to find n. Also of note are the gamma function columns ([r], [n], [n-r], [n]/{[r][n-r]}). For non-integer values of n and r the gamma function must be used to calculate the beta distribution. If it could be guaranteed that n and r would always be integers then the model would be simplifed by using factorials when calculating beta.

The spreadsheet in Figure B.3 is significantly more complex than the one shown in Figure B.2. The table at the top of the screen is the calculation of the beta distribution at increments of 0.05 for each time step. The table relies on the previously calculated n and r parameters. The plot at the bottom of the spreadsheet is a beta distribution. The users can select a time step in the shaded cell (cell C20) and the corresponding row is extracted

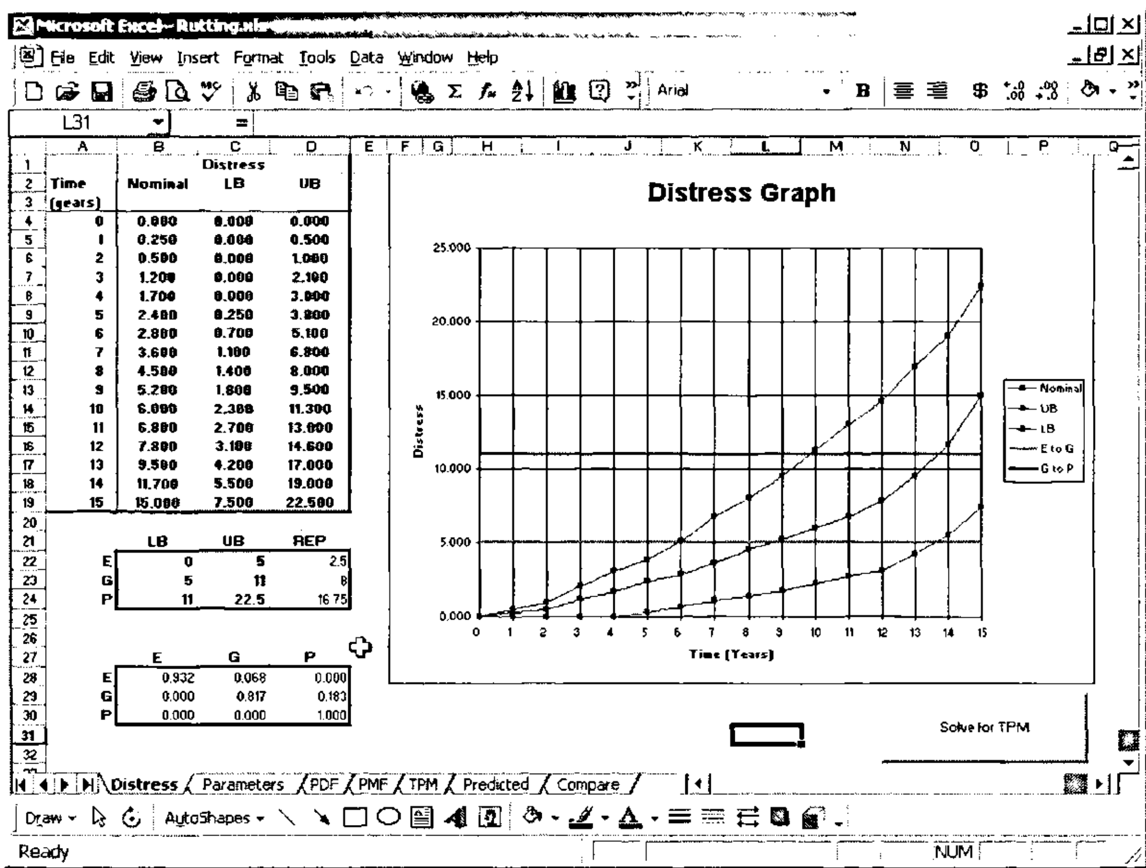
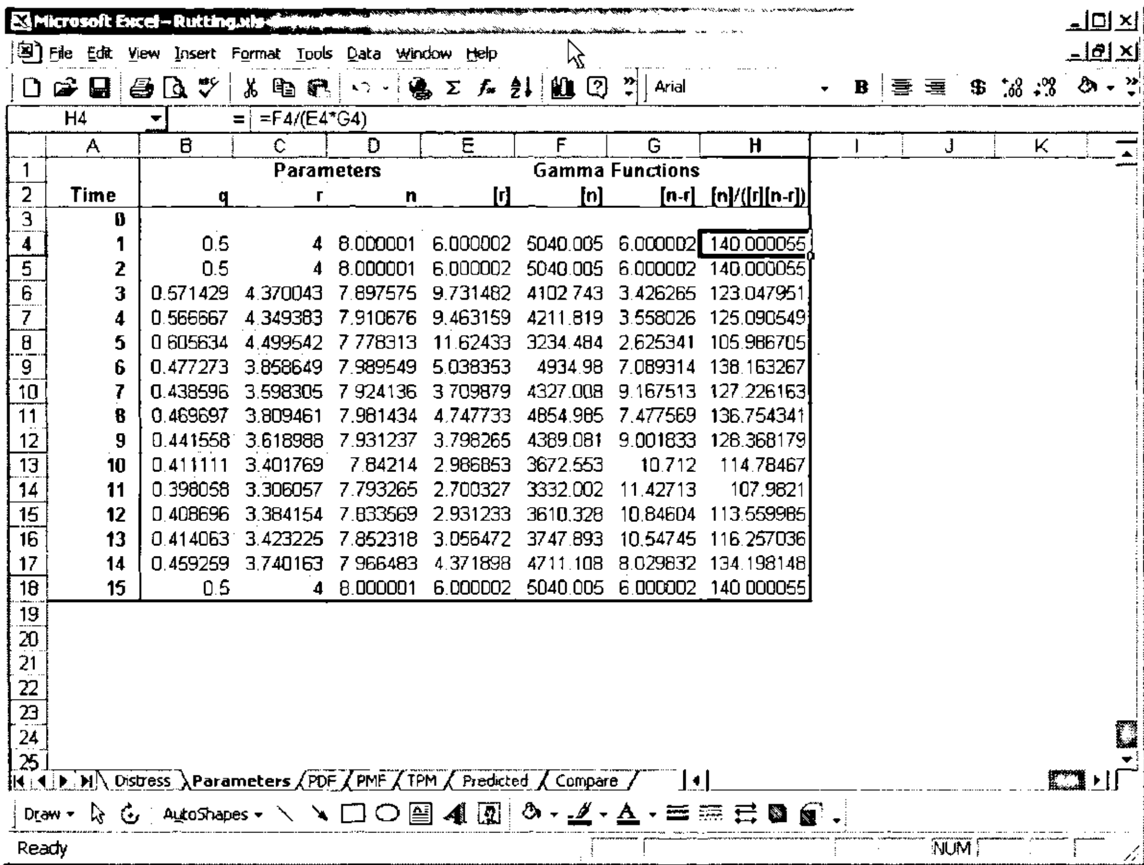


Figure B.1: Performance curve data



Time	q	r	n	[r]	[n]	[n-r]	[n]/([r][n-r])
0							
1	0.5	4	8.000001	6.000002	5040.005	6.000002	140.000055
2	0.5	4	8.000001	6.000002	5040.005	6.000002	140.000055
3	0.571429	4.370043	7.897575	9.731482	4102.743	3.426265	123.047951
4	0.566667	4.349383	7.910676	9.463159	4211.819	3.558026	125.090549
5	0.605634	4.499542	7.778313	11.62433	3234.484	2.625341	105.986705
6	0.477273	3.858649	7.989549	5.038353	4934.98	7.089314	138.163267
7	0.438596	3.598305	7.924136	3.709879	4327.008	9.167513	127.226163
8	0.469697	3.809461	7.981434	4.747733	4854.985	7.477569	136.754341
9	0.441558	3.618988	7.931237	3.798265	4389.081	9.001833	128.368179
10	0.411111	3.401769	7.84214	2.986853	3672.553	10.712	114.78467
11	0.398058	3.306057	7.793265	2.700327	3332.002	11.42713	107.9821
12	0.408696	3.384154	7.833569	2.931233	3610.328	10.84604	113.559985
13	0.414063	3.423225	7.852318	3.056472	3747.893	10.54745	116.257036
14	0.459259	3.740163	7.966483	4.371898	4711.108	8.029832	134.198148
15	0.5	4	8.000001	6.000002	5040.005	6.000002	140.000055

Figure B.2: Calculating the beta distribution parameters

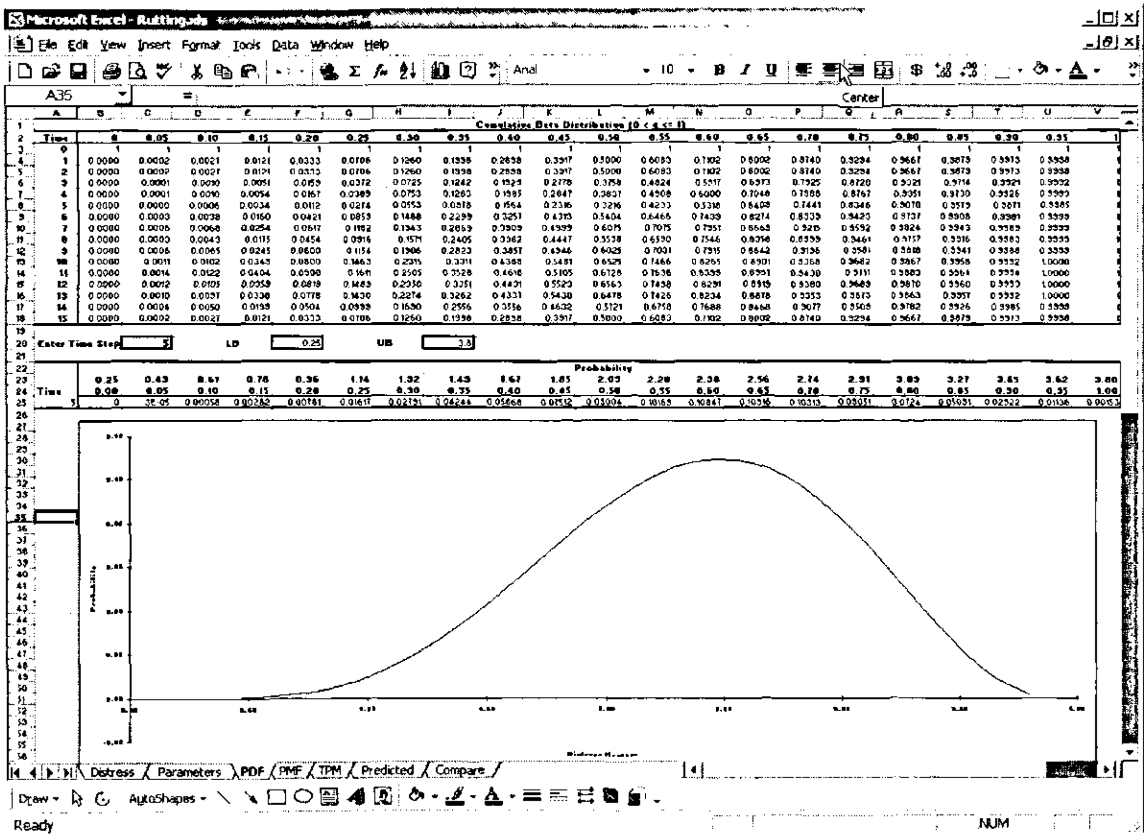


Figure B.3: Beta distribution calculations from the table and plotted. The plot is used as a tool to help visually verify that the beta distribution is shaped as expected.

The fourth step in this process is to find the discrete probabilities associated with each condition state. This research was completed using three condition states; Excellent, Good, and Poor. If rut depth is less than or equal to 5 mm the pavement is considered to be in Excellent condition. If the rut depth is between 5 mm and 11 mm it is in Good condition. Any rutting of 11 mm or more indicates the pavement is in Poor condition. The data in the middle table is extracted from the table in Figure B.3. A user-defined function was designed to integrate under the beta distribution curve over the appropriate range limits for each condition state. For instance, the Excellent range is from 0 to 5 mm, so the software integrates the probabilities over this range.

Note that there are two sets of discrete probabilities for each condition state; one for a beta distribution and one for an exponential distribution. Performance curves can be quite skewed. It was determined that if the nominal and lower bound curves are equal (or nearly equal) for a time step then the exponential distribution better represents the probability distribution for that point in time. It was easier to calculate the values for an exponential distribution at all time steps and then select when it would applicable than to just calculate the beta or exponential distribution probabilities on an as needed basis.

Similarly to the previous spreadsheet, the user can enter a time step and see what the discrete probability distribution looks like.

Figure B.5 shows the spreadsheet where the transition probability matrices are generated. A non-linear programming model is used to generate the transition probability matrix. Without delving too deeply into the model formulation, we are attempting to minimize the difference between the performance model that we generate from our observed discrete probabilities and the performance model that our transition probability matrix generates.

There are three tables and one bar graph in this screen. The top two tables are the transition probability matrix (left) and the condition state ranges (right). The bottom table consists of two halves. The left half (Observed) is a copy of the discrete probabilities

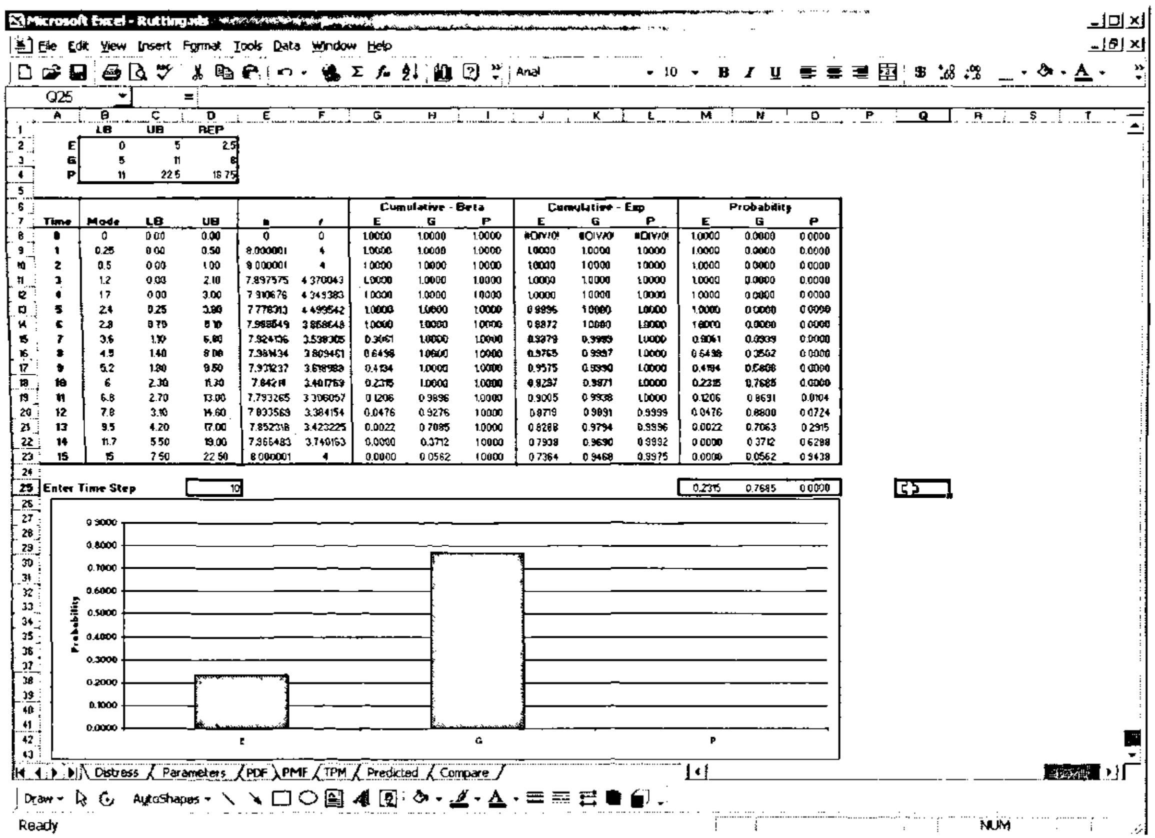


Figure B.4: Calculating the beta distribution probability mass function

	E	G	P	Row Sum		LB	UB	REP
E	0.932045	0.067955495	0	1		0	5	2.5
G	0	0.817028534	0.182971466	1		5	11	8
P	0	0	1	1		11	22.5	16.75

Representative Values

2.5 8 16.75

	Observed				Predicted				
Time	Obs_E	Obs_G	Obs_P	Exp	Pre_E	Pre_G	Pre_P	Exp	Diff Sq
0	1	0	0	2.5	1	0	0	2.500	
1	1	0	0	2.5	0.932	0.068	0.000	2.874	0.139693
2	1	0	0	2.5	0.869	0.119	0.012	3.331	0.690409
3	1	0	0	2.5	0.810	0.156	0.034	3.846	1.811408
4	1	0	0	2.5	0.755	0.183	0.063	4.398	3.604276
5	1	0	0	2.5	0.703	0.200	0.096	4.973	6.115161
6	0.999995	5.15818E-06	0	2.500028	0.656	0.212	0.133	5.557	9.343391
7	0.906136	0.093864318	0	3.016254	0.611	0.217	0.172	6.140	9.760904
8	0.649799	0.350201464	0	4.426108	0.569	0.219	0.211	6.717	5.24802
9	0.419408	0.580592353	0	5.693258	0.531	0.218	0.251	7.281	2.51996
10	0.231528	0.768465641	6.67954E-06	6.726656	0.495	0.214	0.291	7.828	1.212348
11	0.120583	0.869065789	0.010350928	7.427363	0.461	0.208	0.330	8.355	0.860931
12	0.047572	0.879986065	0.072441522	8.372215	0.430	0.202	0.369	8.861	0.239208
13	0.002165	0.706340191	0.29149496	10.53867	0.401	0.194	0.405	9.345	1.425401
14	0	0.371185859	0.628814141	13.50212	0.373	0.186	0.441	9.805	13.66862
15	0	0.056192396	0.943807604	16.25832	0.348	0.177	0.475	10.242	36.19794
								Sum	92.838

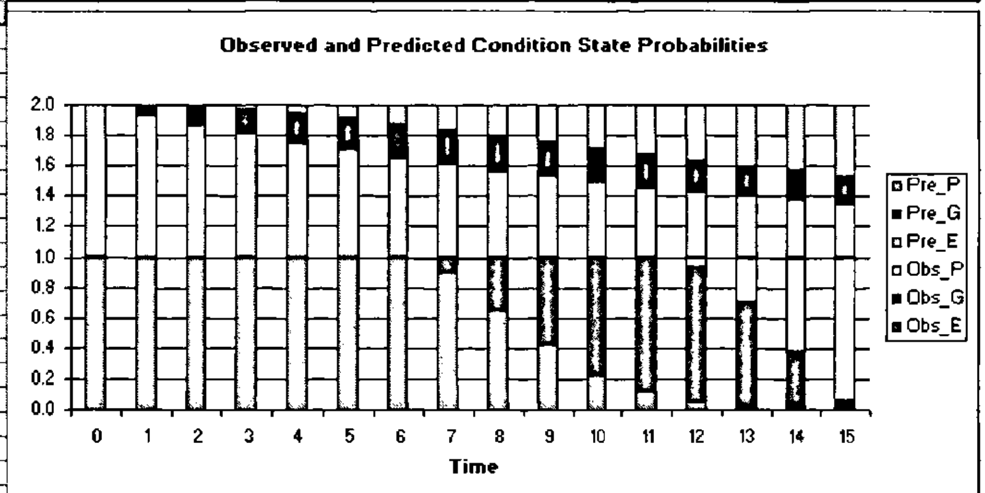


Figure B.5: Determining the transition probability matrix with a non-linear programming model calculated in the previous spreadsheet (Figure B.4). The probabilities in the right half (Predicted) are calculated from the transition probability matrix. The last column (to the right) is the difference between the expected value calculation of both sides of the table (squared). The sum of these differences is to be minimized; in effect this column represents the objective function. The solver adjusts the values of the transition probability matrix until a set number of iterations or convergence is achieved. The optimal transition probability matrix values are shown here and on the first spreadsheet (Figure B.1).

The graph at the bottom of Figure B.5 is a visual comparison for the observed and predicted probabilities.

The last spreadsheet (Figure B.6) is included so the user can better visualize the relationship between the performance models(observed and predicted). Figure B.6 shows that the transition probability matrix underestimates the asset's condition in the early phases of its life and overestimates its condition late in its life.

B.3 Generating TPM for other treatments

The previous section described the process for creating transition probability matrices for various distresses under routine maintenance conditions. Generating transition probability matrices for other forms of maintenance, repair, or rehabilitation (which we shall generically label as treatments) is a manual process. There were 10 treatments selected for this model (see the glossary for a complete list) one of which was routine maintenance. The transition probability matrices for the remaining nine treatments are completed on a case by case basis. The transition probability matrix for each treatment used routine maintenance as its base case. The affect each treatment would have on the distress is considered and the routine maintenance transition probability matrix was adjusted accordingly. In situations where the treatment would have no effect on the distress, then the routine matrix would not be modified. Where the distress would be returned to an Excellent condition the transition probability would become 1. Since this process is manual there are no figures illustrating the process.

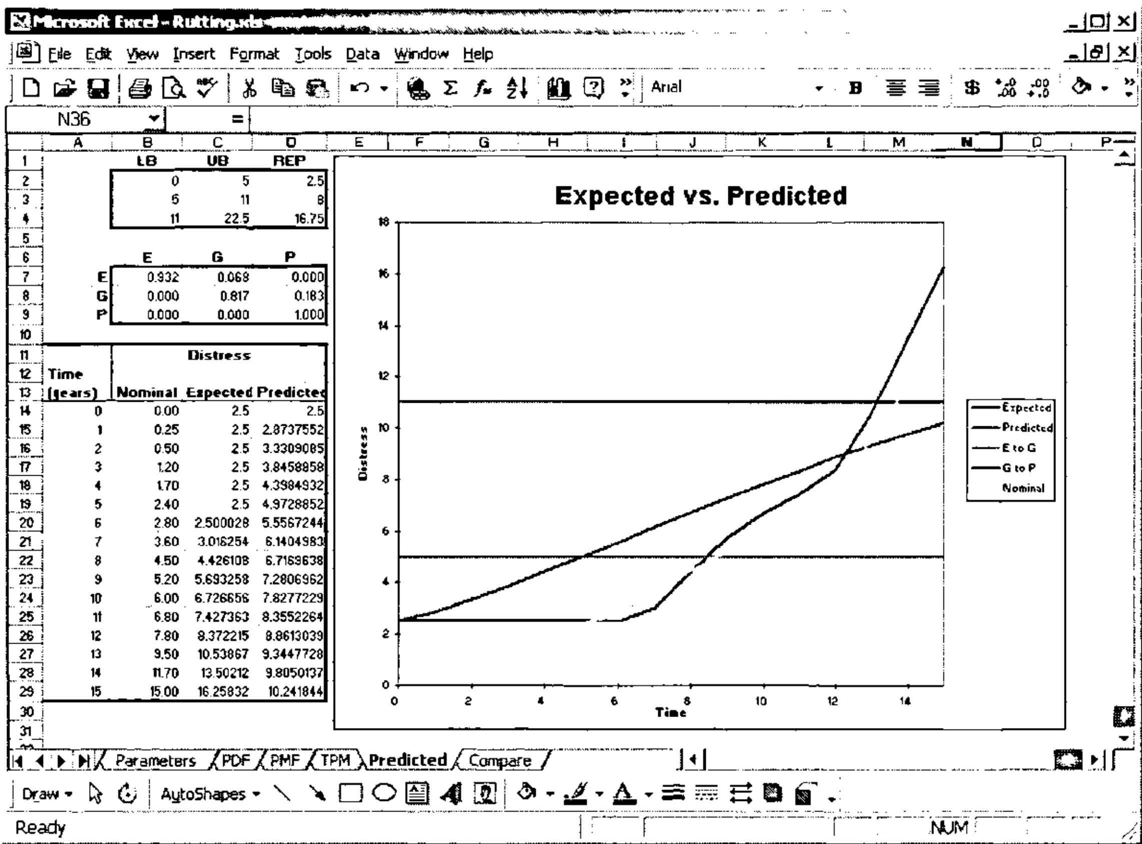


Figure B.6: Visual comparison of nominal, expected, and predicted performance curves

B.4 Creating aggregate TPM for all treatments

The transition probability matrices (TPM) that have been developed are for individual distresses under specific treatment regimes. These matrices are 3x3 in size. The optimization model that will be described in the next section requires an aggregate transition probability matrix for each distress. The aggregate matrix is created by mapping the permutation of condition states for each distress to a single dimension. Since there are 6 distresses each with 3 condition states there are 36=729 different condition states for the aggregate matrix. Mapping the condition states of the project level transition probability matrices to the aggregate level transition probability matrix is simply a matter of enumerating all combinations of states. The size of the aggregate transition probability matrix is 36x36= 729x729 for a total of 531,441 data elements. Initially the project level transition probability matrices are entered into the database system. The aggregate transition probability matrix is created by running a small Visual Foxpro program.

B.5 Optimization

A linear programming model determines the optimal maintenance tactic. This tactic identifies the least cost action (treatment) to take given the asset is in a specific condition. The model consists of two major components; the objective function and the constraints (see the glossary for a brief explanation of the linear programming model). The data for the model is derived in the previous steps. The model is generated by a series of Visual Foxpro programs and stored in a database. In the optimization procedure the linear programming data is extracted and passed to the optimization engine (XA). Figure B.7 illustrates a typical database environment and optimization run.

Model generation takes approximately 20 minutes and optimization approximately 5 minutes on a 350MHz AMD K-2 processor with 320MB of memory (running Windows 95). Reviewing the model results is quite time consuming. A series of Crystal Reports were developed to allow consistent extraction of pertinent model data (Figure B.8).

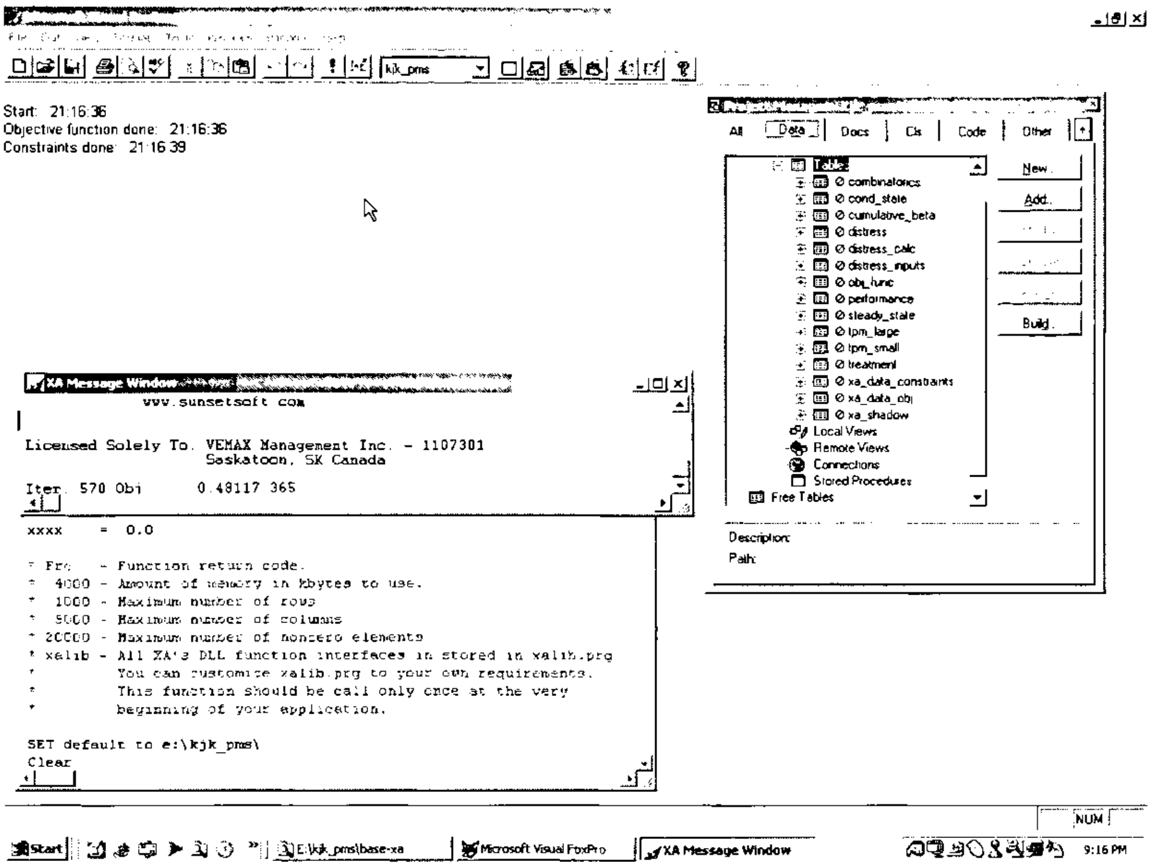


Figure B.7: Linear programming optimization in progress within database environment

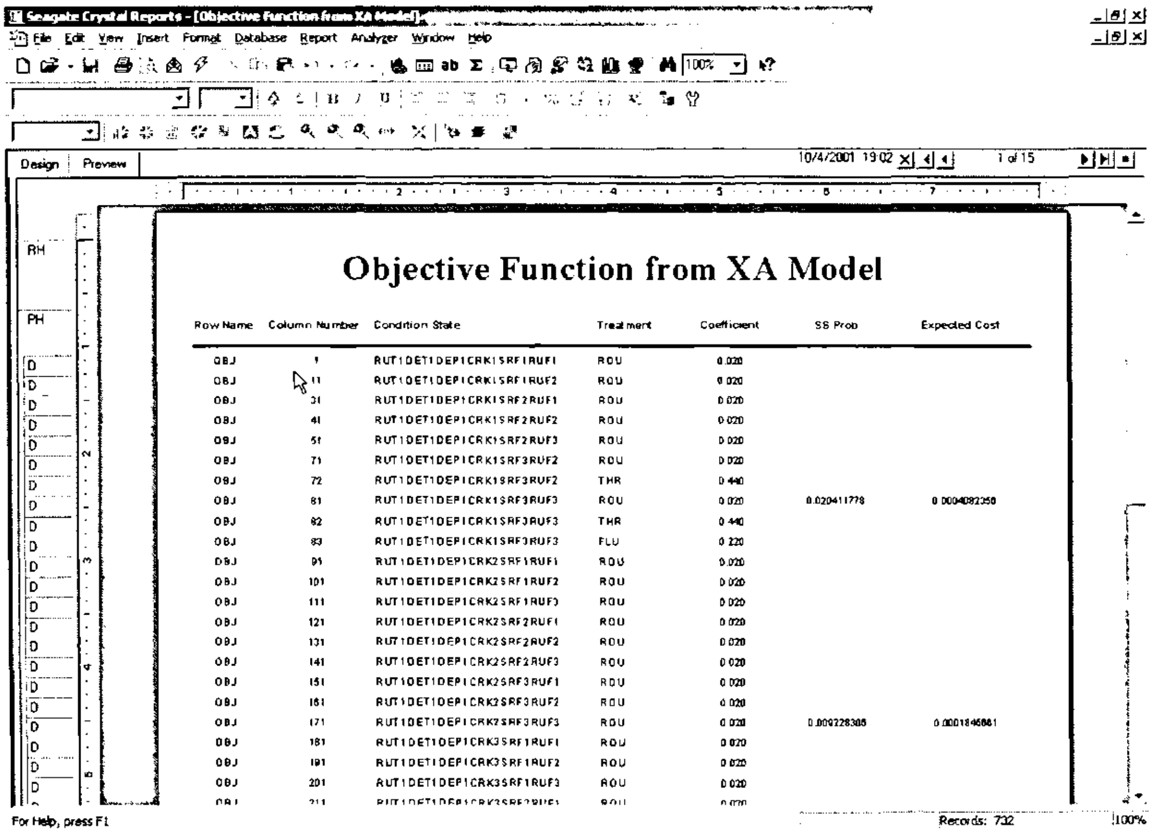


Figure B.8: Analyzing model results with Crystal Reports