Portfolio Diversification for Long Holding Periods:

How Many Stocks Do Canadian Investors Need?

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ABSTRACT

The number of stocks required to achieve diversification has been under discussion for over four decades. Traditionally, it is viewed that between 8 to 20 stocks are adequate for a 'well' diversified portfolio based on American studies, and 30 to 50 stocks based on a Canadian study. The majority of the past literature has used American data with a focus on the short-term investment horizon. Cleary and Copp's (1999) paper is the only study that utilized Canadian data with an emphasis on the short-term investment horizon.

To fill this void, this thesis examines the cumulative rates of return over a 20-year investment horizon by randomly investing $100,000 initially across 100 Canadian firms. The results of the simulation illustrate the probability distributions of the shortfall risks for individuals who own fewer than 100 stocks. To see if diversifying across industry groups reduces the shortfall risk faced by investors, a similar simulation is completed for investing randomly across Canada's four prime industry groups.

The empirical results of this thesis suggest that the standard recommendation of 8 to 20 is inadequate for a long-term Canadian investor. More than 80 Canadian companies are required to obtain a shortfall risk amount of less than 5% ($57,929) of the 100-stock portfolio when investing randomly in Canadian companies.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Many people prefer selecting individual stocks for their own portfolios rather than investing in pre-packaged portfolios through mutual funds. This type of investing is especially preferable for investors who wish to combine a long-term investment horizon with a buy-and-hold strategy. The advantages of this type of strategy include the avoidance of ongoing fund management fees, and the realization of capital gains and losses for particular securities is at the discretion of the owner.

Maintaining and monitoring a well-diversified portfolio of investments usually means being diversified globally, but this thesis assumes that Canadian investors will maintain a one hundred percent Canadian content portfolio. The reasoning behind this assumption is twofold. First, the dividend tax credit for non-registered investments is an advantage that Canadian investors obtain from investing in Canadian companies (see Appendix A on the dividend tax credit). Second, registered savings plans in Canada require that foreign content be limited to less than 30% of the total investment portfolio[1]. The restriction on foreign content has to be maintained and monitored at all times by Canadian investors. For the two reasons listed above, the portfolios constructed for analysis in this thesis are comprised of one hundred percent Canadian content.

An important decision of the optimal number of stocks to own for a 'well'-diversified portfolio must be addressed for investors who choose to maintain their own portfolios. Newbould and Poon (1993, 1996) survey a number of finance textbooks which all concluded

[1] Source: Revenue Canada Guide "RRSP and Other Registered Plans for Retirement – 2000" T4040.

that between eight and twenty stocks, in a portfolio, are sufficient to be adequately diversified. However, Newbould and Poon, as well as other academic papers such as Elton and Gruber (1977), Statman (1987), Cleary and Copp (1999), de Vassal (2001), and Domian, Louton, and Racine (2002), suggest the number of stocks should be greater than twenty to be adequately diversified. In existing literature, studies have been completed using American data, with the exception of Cleary and Copp (1999) who use Canadian data. The majority of the studies, including Cleary and Copp (1999), focus only on the short-term investment horizon. After reading these studies, the following question remains unanswered: 'What is the optimal number of Canadian stocks necessary to form a well-diversified portfolio for a *long-term* investment horizon?'. This thesis will examine this question by calculating the 20-year cumulative returns of portfolios constructed by randomly selecting from 100 large Canadian firms. The number of companies in each portfolio ranges from 4 to 100. The results will measure the shortfall risk faced by investors who own fewer than 100 companies. Finally, to see if diversifying across industry groups reduces shortfall risk faced by investors, portfolios of 4, 8, 12… and, 96 equally-weighted stocks are formed by randomly selecting one, two, three, etc. stocks simultaneously across four individual industry groups. To support the findings in the thesis, an in depth literature review relevant to the study is also performed.

The remainder of the thesis is organized as follows. Chapter 2 introduces how risk is measured and the definition of shortfall risk and also reviews the empirical evidence of past literature. Chapter 3 introduces the methodology and data used in the thesis. Chapter 4 presents the results of the tests and analysis of the findings. The final chapter, Chapter 5, offers concluding remarks and summarizes the thesis.

CHAPTER 2

PRIOR RESEARCH

This chapter reviews the relevant prior research in portfolio diversification and the studies that address the decision of the optimal number of stocks needed to be 'well' diversified. It will also examine the methods for measuring risk in portfolio management. The chapter is organized as follows. Section 1 reviews measuring risk and shortfall risk in portfolio management, Section 2 reviews the prior empirical research, and Section 3 summarizes the literature.

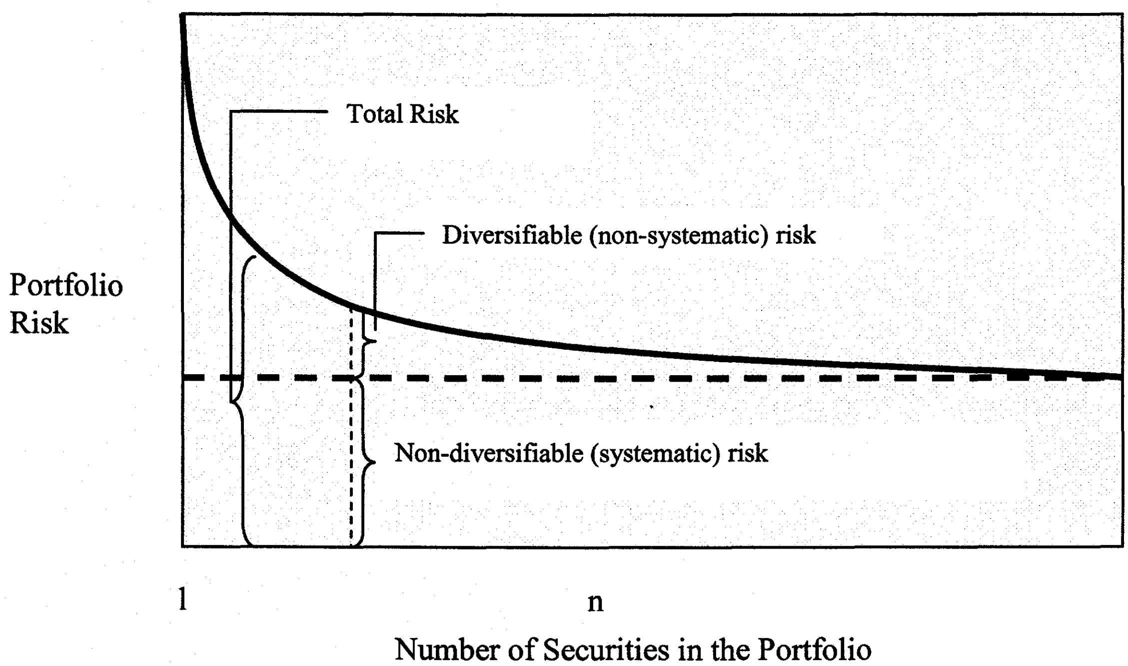
2.1 MEASURING RISK AND SHORTFALL RISK IN PORTFOLIO MANAGEMENT

The most commonly used measures of risk are standard deviation and beta. The standard deviation is the square root of the variance. The variance and standard deviation measure the dispersion of possible rates of return around the expected rate of return. The larger the variance and standard deviation for an expected rate of return, the greater the dispersion of expected returns and the greater the uncertainty, or risk, of the investment. When the return is of perfect certainty, there is no variance of return because there is no deviation from the expectations and hence, no risk or uncertainty.

The overall risk of an individual stock is the total of the *systematic* or non-diversifiable risk and *unsystematic* or diversifiable risk. The portion of an asset's total variance attributable to the variability of the total market portfolio is also referred to as an asset's *systematic risk* and sometimes referred to as *market risk*. This is commonly measured by beta. The systematic risk or beta measures the co-movement of the asset with the market portfolio. In short, the *systematic* or market risk results from general market and economic conditions that affect all companies in the economy. *Unsystematic* risk is the firm-specific risk that affects only one company. The objective of diversification in portfolio management is to reduce the overall risk of the portfolio by reducing the unsystematic risk through diversification. Systematic risk cannot be reduced. Thus, a 'well' diversified portfolio should only be susceptible to systematic risk, as virtually all of the unsystematic risk is reduced through diversification.

Figure 1 illustrates the concept of declining non-systematic risk in a portfolio of securities. Notice, as the number of securities is added to a portfolio, the non-systematic risk decreases until the total risk of the portfolio approaches its systematic risk. At this point, portfolio's total risk equals the total market risk of the portfolio and it is said to be 'well' diversified.

Figure 1 Systematic and Non-systematic Risk in Portfolio Diversification



Source: Cleary and Jones, 2000, Investment: Analysis and Management, Canadian Edition (John Wiley & Sons Canada, Toronto), p 240.

Sortino and van der Meer (1991) comment that the standard deviation and beta are not relevant measures of risk for many investment situations because they do not capture what the investor has at stake. For many investors, there is a minimum return that must be earned in order to accomplish some investment objective. The requirement of a minimum return for long-term investors is very important. The minimum return must be met as often these investments are the investor's life savings and they may be dependent on these savings to live through their retirement years. Simply put, risk averse investors require that the minimum return must be earned in order to prevent unfavourable outcomes from occurring. The standard deviation, as a measure of risk, only captures the risk associated in achieving the mean. However, the standard deviation is totally unrelated to the potential of unfavourable outcomes when an investment falls short of the minimum return investment objective. This 'falling short' of the investment objective is known as the *shortfall risk.* The shortfall risk is the most significant risk for risk-averse investors as it captures the unfavourable outcome of falling short of the investment objective. The shortfall risk measures the risk of falling short of a target investment objective and emphasizes the 'safety-first' rule, which limits the risk of unfavourable outcomes. For example, a 'safety-first' criterion, first developed by Roy (1952), states that the best portfolio is the one that has the smallest probability of producing a return below some specified level. If R_P is the return on the portfolio, and R_L is the level below which the investor does not wish the returns to fall, according to Roy's criteria:

minimize $\text{Prob}(R_P<R_L)$.

For the purpose of this thesis, the emphasis is on the shortfall risk as a measurement of risk because it captures the unfavourable outcome of falling short of the investment objective and it emphasizes the 'safety first' rule.

2.2 PRIOR EMPIRICAL RESEARCH

Markowitz (1959) is the first researcher to derive the relationship between the expected portfolio variance and the number of securities. The following formula states that, when equally weighted portfolios are constructed from randomly selected securities, the expected portfolio variance is:

$$E(\sigma_p^2) = \frac{1}{N}\overline{\sigma^2} + \frac{N-1}{N}\overline{cov(i,j)}$$

Where $E(\sigma_p^2)$ = expected variance of a portfolio;

$\overline{\sigma^2}$ = average variance for all stocks in the population;

$\overline{cov(i,j)}$ = average covariance between all stocks in the population;

N = number of securities in the portfolio.

Since the development of Markowitz's formula, many researchers in the 1960s and early 1970s used empirical simulations to examine the relationship between the expected portfolio variance and the number of securities. One of the earlier studies by Evans and Archer (1968) examines semi-annual observations of 470 stocks listed on the Standard & Poor's index from January 1958 to June 1967. This means a holding period of 9.5 years. Evans and Archer measure the unsystematic variation (risk) of a portfolio by the standard deviation of return from the average return for that portfolio. Then they run a large-scale simulation and plot the average standard deviation of return against the number of securities in the portfolio. Evans and Archer find that much of the unsystematic variation is eliminated by the time the 8th security is added to the portfolio. They also conclude that due to the higher transaction cost that is incurred when the number of stocks increases, a portfolio composed of greater than 10 stocks is economically unjustified since the impact of the marginal stock on the portfolio variance is negligible.

Shortly after the Evans and Archer study, Fisher and Lorie (1970) examine the frequency distribution of "wealth ratios" of investments for different-sized (i.e., 1, 2, 8, 16, 32, 128) portfolios using New York Stock Exchange stocks from 1926 to 1965, with equal initial investments made in each stock in a portfolio. They analyze the holding period of 1 to 20 years. The "wealth ratio" is the ratio of the value of an investment at the end of the period to the amount initially invested. Fisher and Lorie believe that it is extremely difficult to understand the significance of differences among annual rate of returns for long holding periods; and therefore, they used the "wealth ratio" in their study. To clarify this, consider an investment that earns 5 percent per annum compounded annually, versus 10 percent per annum over a forty-year period. The resulting wealth ratios would be approximately 7 and 45, respectively. This is noticeably different as the wealth ratio produced by the 10 percent annual return is 543 percent greater than the ratio produced by the 5 percent annual return. The wealth ratios capture the considerable wealth differences between the two returns over the forty-year period whereas the rate of return does not. Fisher and Lorie conclude in their study that, as the number of stocks in the portfolio increases, the ability to reduce dispersion decreases. The dispersion is measured by the variance of wealth ratios. According to Fisher and Lorie's study, approximately 40 percent of achievable reduction is obtained by holding two stocks in a portfolio; 80 percent, by holding eight stocks; 90 percent, by holding sixteen stocks; 95 percent, by holding thirty-two stocks; and 99 percent, by holding 128 stocks.

Mao (1970) examines the means, variances, and co-variances of the securities with a theoretical analysis to form the optimal portfolio. This optimal portfolio maximizes the ratio (θ) between the expected value and standard deviation of the excess return over the risk-free rate of interest given by the following equation:

$$\theta = \left(\frac{\mu}{\sigma}\right)_{portfolio} = \frac{\sum_{i=1}^{n} h_i \mu_i}{\left(\sum_{i=1}^{n} h_i^2 \sigma_i^2 + 2\sum_{i} \sum_{<\ j} h_i h_j \sigma_{ij}\right)^{1/2}}$$

Where θ = the ratio between the expected value and standard deviation of the excess return over the risk-free rate of interest;

μ_i = mean of the i^{th} security;

σ_i^2 = variance of the i^{th} security;

σ_{ij} = covariance between the i^{th} and j^{th} securities;

h_i = fraction of portfolio invested in security i.

Mao finds that when the average correlation coefficient between securities is set at a realistic value of 0.5, only 3 securities are needed to capture 50 percent of the maximum benefits of diversification, and no more than 17 securities are required to obtain 90 percent of the benefits of diversification[2]. When the average correlation between securities is as low as 0.2, 90 percent of the benefits of diversification are achieved with only 34 stocks. Mao comments that the cost of diversification increases with the number of securities. This cost is usually increasing at an increasing rate. For example, as the number of securities are added to a portfolio, the costs of transactions,

[2] The typical correlation among U.S. securities is about 0.5 to 0.6, Reilly and Brown (2000). The average correlation among the 100 Canadian stocks in this study is approximately 0.437771.

information, and management increases, thus, the benefits resulting from diversification increase with the number of securities, but at a decreasing rate.

Contrary to the claims of the previous studies, Elton and Gruber (1977) ascertain that more stocks are required. Elton and Gruber claim that the earlier studies define risk improperly. Prior studies define risk solely by the dispersion of a portfolio return around the mean return of that portfolio. They disregard the risk associated with the probability that the mean return on the portfolio held will be different from the return in the market. Simply put, the risk of holding a single security rather than the market is not just due to the variability of that security's return, but is also due to the uncertainty of what the average return on that security will be. Elton and Gruber illustrate their results with the total risk, which is composed of both the variance of the return on the portfolio of N securities from the portfolio's expected return, and the variation caused by the difference between the expected return on the portfolio and the expected return for the population. Elton and Gruber use the weekly returns from 150 to 3,290 securities selected from the New York and American Stock exchange over the period June 1971 to June 1974 to calculate the variances. Elton and Gruber's study concludes that the total risk declines with the addition of securities in portfolios. The major declines in total risk occur at the smaller portfolios levels, which is similar to the earlier studies that used variance as a measure of total risk. For example, the 10 security portfolios' variance of an equally weighted population portfolio (EWPP) is 11.033, which is one-fourth of the single security portfolios' variance of EWPP, 46.811. Even though the 10 security portfolios represent a major decrease, the total risk for the 10 security portfolios is still 156 percent of the minimum total risk of 7.07 for 3,290 securities. Elton and Gruber's results also indicate that for actual risk in a portfolio to be only 20 percent higher than the minimum total risk requires 28 securities; only 10 percent higher, 60 securities, and only 5 percent higher, 110 securities. Elton and Gruber observe that even though total risk decreases at a decreasing rate as more securities are added, it is clear that the decrease may still be of importance to portfolio management. For example, a 15 stock portfolio has 32 percent more risk than a 100 stock portfolio. This means that diversification beyond 15 stocks is significant.

Using the data from Fisher and Lorie (1970), Levy (1979) applies the second-degree stochastic dominance (SSD) approach of analysis to 1, 5, 10, and 20-year periods (see Appendix B on SSD). Levy finds that for the 1- and 5-year periods, the 128 stock portfolios dominate all other portfolios. Nevertheless, for periods beyond the 5-year time horizon, it is difficult to determine dominance among the portfolios. For the 20-year period, Levy comments that portfolio diversification beyond eight securities may not be worthwhile. Levy's reasoning is based on the hypothesis that diversification over time reduces the importance of diversification between securities[3].

Eight years after Levy's study, Statman (1987) uses a holding period of one year, demonstrates that for a portfolio to be 'well' diversified, 30 stocks are ideal for a borrowing investor and 40 stocks for a lending investor. In other words, an investor who wants to borrow at a higher than the risk-free rate to invest in more than 100% of the portfolio benchmark must include at least 30 stocks for a 'well' diversified portfolio. An investor who invests in the risk-free asset must also include at least 40 stocks for a well-

[3] Diversification over time means a portfolio is diversified not only between the stocks but also diversified over time. This means the longer the time period, the more diversified a portfolio is.

diversified portfolio. Statman reaches these conclusions by utilizing a 500-stock portfolio as a benchmark and the security market line to allow for borrowing and lending.

These earlier studies have influenced textbook recommendations about the number of stocks required to achieve adequate diversification in a portfolio. Newbould and Poon (1993, 1996) survey 14 published textbooks written in the 1980s and 1990s for the U.S. market. The result of their survey concludes that between 8 and 20 stocks are sufficient to achieve diversification. In one of the textbooks, Francis (1991) states that 10 to 15 stocks are adequate as further diversification is "superfluous diversification" and should be avoided[4]. More recently, Fabozzi (1999) reports that about 20 randomly selected stocks in a portfolio are sufficient to diversify away almost completely the unsystematic risk, leaving the portfolio with only the systematic or market risk.

While the U.S. textbooks tend to agree that diversification can be achieved with a relatively limited number of stocks, this is not true for Canadian texts. Sharpe, Alexander, Bailey, Fowler, and Domian (2000) comment that a portfolio of 30 or more randomly selected securities constitutes a 'well' diversified portfolio. This portfolio will have a relatively small amount of unsystematic risk and thus, the total risk will be slightly greater than the amount of market risk that is present. Bodie, Kane, Marcus, Perrakis, and Ryan (1999) point out that the correlation between the security returns limits the power of diversification. When the correlation coefficient is zero (uncorrelated), a 100-security portfolio has a standard deviation of 5%, which is still significant when considering the potential of a near zero standard deviation. For a more realistic correlation of 0.40, that same 100-security portfolio has a standard deviation of 31.86%. They conclude that this

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[4]According to Francis, "superfluous diversification" usually results in portfolio management problems such as impossibility of good portfolio management, purchase of lackluster performers, high search cost, and high transaction costs.

standard deviation is high, yet it is very close to the non-diversifiable systematic risk in the infinite-sized universe portfolio, which has a standard deviation of 31.62%. At this point, when the correlation coefficient is equal to 0.40, further diversification is of modest value. Finally, Cleary and Jones (2000) point out that there is a difference between American and Canadian evidence for the number of securities required to eliminate most or all of the unsystematic risk. They note that approximately 30 to 40 securities are adequate for a well-diversified portfolio based on U.S. evidence, yet recent Canadian research suggests that more than 70 or more stocks are required to obtain a well-diversified portfolio. The reason for the difference is due to the high proportion of resource-based companies listed on the TSE and the high concentration of Canadian stocks within a few industries[5].

Newbould and Poon (1993, 1996) argue that the standard recommendation of between 8 to 20 stocks to achieve diversification is flawed because a typical investor has only one portfolio. A typical investor is unwilling to jeopardize his or her funds on the basis of an average outcome of a large number of equal-size portfolios. Having one portfolio from a particular universe of portfolios exposes the investor to an additional source of risk. This additional source of risk is that there is no guarantee the investor will be at the mean. Newbould and Poon (1993, 1996) present the 95% confidence intervals for both the risk and the return for a holding period of one month. They conclude that for a risk averse investor to be within 10% of the population average return and 10% of the population average risk, he or she would need a minimum of 60 stocks. Another risk

[5] For example, the TSE financial services companies comprised close to 24.07% of the market capitalization of the TSE 300 composite index as of December 31, 2000, while the TSE sub-indices for metals and minerals, gold and precious minerals and oil and gas companies accounted for 23.29%.

averse investor who wants to be within 5% of the average return and 20% of the average risk would necessitate more than 100 stocks.

Levy and Livingston (1995) investigate the gains from diversification taking into account transaction costs and superior information. This superior information would bring about a superior asset and, thus, result in a higher mean return and or lower standard deviation and or lower correlation between the individual assets. The results in their paper suggest that if all securities have the same covariances and a correlation coefficient of .50 or larger, a portfolio size of 10 securities could attain about 90% of the potential benefits from diversification. If both a superior asset and transaction costs were to exist in the portfolio, the optimal number of stocks in a well-diversified portfolio reduces. The superior asset could significantly reduce the gains from diversification for comparatively small amounts of special information. This happens because the cost disadvantage of adding securities is larger.

Cleary and Copp (1999) is the only study that uses Canadian data. Cleary and Copp use stocks listed on the Toronto Stock Exchange between the periods of January 1985 to December 1997. They analyze an investment horizon of one month. Stocks that have complete total return information available for the time period are randomly selected to simulate equally-weighted portfolios ranging from one to more than 200 stocks per portfolio. Cleary and Copp's method of analysis includes survivorship bias, because stocks of companies that merged with other companies or were delisted from the TSE are not included in the analysis. Cleary and Copp's analysis shows that risk is reduced by 46% for a 10-stock portfolio, 53% for a 20-stock portfolio, and 56% for a 30-stock portfolio. The reduction in risk obtained from the 10-stock portfolio represents

68% of the total risk reduction attainable with all 222 stocks, 78% for the 20-stock portfolio, and 84% for the 30-stock portfolio. Diversifying even further can eliminate more of the unsystematic risk. For example, 90% of the total risk reduction benefits can be attained from a 50-stock portfolio, 95% from a 90-stock portfolio, and 99.6% from a 200-stock portfolio. Cleary and Copp conclude that 30 to 50 Canadian stocks are required to capture most of the benefits of diversification, but do not state an exact number of stocks required in a well-diversified portfolio. Their reasoning is that the benefits of diversification must be weighed against the cost of excessive diversification, which is in the form of transaction costs and monitoring costs. These costs arise from tracking a large number of stocks.

A current study by de Vassal (2001) uses the Russell 1000 index as a benchmark to randomly select portfolios ranging from 1 to 100 securities between the 1992 to 1999 period which results in an investment horizon of seven years. De Vassal analyzes the "downfall" risk of the different portfolios and concludes that a conservative investor, who does not want to have greater than a 40% chance of under-performing the market by 25%, should own a minimum of 50 stocks. A less risk-averse investor who would be satisfied with a lower downside constraint of underperforming the market, possibly by 50%, should include about 15 stocks. De Vassal also points out that increasing the number of stocks held can significantly reduce risk of underperforming the stock market. The other advantage of increasing the number of stocks held would relate to the fact that stock returns are positively skewed. In other words, the maximum loss that a stock can incur is negative 100%, but the potential gain is unlimited. "Super performers" (earning greater than 300% returns) are rare and the likelihood of acquiring "super performers" increases as additional stocks are added.

Finally, a more recent study by Domian, Louton, and Racine (2002) suggests that the typical 8 to 20 stocks may be insufficient for long-term investors. Domian, Louton, and Racine used returns on the 100 stocks comprising the S&P 100 Index from the beginning of 1979 through December 1998. Randomly selecting from the S&P 100 Index, portfolios of less than 100 stocks were formed and held for 5 and 20 years. The results of their study show the percentile of the distribution of the ending wealth for the different-sized portfolios from an initial $100,000 investment. The ending wealth of the different-sized portfolios held for 5 and 20 years were compared to the S&P 100 Index ending wealth to illustrate the different levels of shortfall risk. Domian, Louton, and Racine conclude that over 60 stocks are required to reduce the shortfall risk below 10% for a 20-year investment period. Even when considering a shorter investment horizon of only 5 years, portfolios exceeding 40 stocks are required to achieve the benefits of diversification.

2.3 CHAPTER SUMMARY

Table 1 summarizes the prior literature relevant to portfolio diversification. From the table, as discussed in Domian, Louton, and Racine (2002), the previous studies fall into one of two categories in assessing the benefits of diversification. One category assesses the benefits of diversification by measuring the risk using the variance of one stock portfolio relative to the variance of successively larger portfolios. The studies that fall into this category include Evans and Archer (1968), Mao (1970), Francis (1991), Fabozzi (1999) and Cleary and Copp (1999). The general consensus of this category, with the exception of Cleary and Copp, is that a relatively small number of stocks are required to achieve the full benefits of diversification. The studies that fall into the second category are Fisher and Lorie (1970), Elton and Gruber (1970), Levy (1979), Statman (1987), Newbould and Poon (1996, 1999), de Vassal (2001), and Domian, Louton and Racine (2002). This category uses more sophisticated methodologies in measuring risk in assessing the benefits of diversification. This category favours the recommendation that more than the traditional 8 to 20 stocks are needed to obtain a 'well' diversified portfolio. The majority of the studies focuses on the short-term investment horizon. The exceptions to this are the studies by Fisher and Lorie (1970), Levy (1979), and Domian, Louton, and Racine (2002), who looked at an investment horizon of 20 years.

Despite the various views of past papers and researchers, the question of how many stocks to hold in a diversified portfolio for a Canadian long-term investor with a buy-and-hold strategy is still largely unanswered. This thesis will try and address this question by using shortfall risk, introduced in the Domian, Louton, and Racine (2002) study, as the measurement of risk to assess the benefits of diversification.

Table 1
Review of Diversification Literature: How many stocks does it take to be 'well' diversified?

Authors	Time Period Analyzed	Duration of Investment Period	Recommendation (number of stocks)
Cleary and Copp (1999)	January 1985 - December 1997	one month	30-50
De Vassal (2001)	January 1992 - December 1999	7 years	50, for less than 40% chance of under-performing the market by 25%; 15, for less than 50%
Domian, Louton, and Racine (2002)	January 1979 - December 1998	5 and 20 years	greater than 40 for 5 years; greater than 60 for 20 years
Elton and Gruber (1977)	June 1971 - June 1974	one week	much greater than 15
Evans (1970)	January 1958 - June 1967	9.5 years	n/a
Evans and Archer (1968)	January 1958 - June 1967	9.5 years	8; 10 has marginal benefits = marginal costs
Fabozzi (1999)	June 1960 - May 1970	5 and 10 years	20
Fisher and Lorie (1970)	1926 - 1965	1 - 20 years	8, for 80% of achievable reduction in dispersion; 16, for 90% reduction
Francis (1991)	January 1958 - June 1967	9.5 years	10-15
Levy and Livingston (1995)	theoretical	annual	10, for identical covariances and correlations ≥ .5; much greater than 10, if covariances are unequal
Levy (1979)	Fisher and Lorie's data	1 - 20 years	8, for 20 year horizon; 128, for 1- 5 year horizons
Mao (1970)	theoretical	annual	17, for 90% of benefits; 34, if correlations are .2
Markowitz (1959)	theoretical	n/a	n/a

Table 1 (continued)
Review of Diversification Literature: How many stocks does it take to be 'well' diversified?

Newbould and Poon (1993)	January 1988 – December 1990	One month	much greater than 20, depending on investor risk preference, desired confidence level
Newbould and Poon (1996)	January 1987 - December 1993	one month	25 - 80, for a portfolio of large stocks; 40 - 100, for a portfolio of small stocks
Statman (1987)	1979 - 1984	annual	30, for an investor who borrows to invest in the benchmark portfolio; 40, for a lending investor

SOURCE: Domian, Dale L., David A. Louton, and Marie D. Racine, 2002, Portfolio Diversification for Long Holding Periods: How Many Stocks Do Investors Need?, Forthcoming in *Studies in Economics and Finance.*

CHAPTER 3

THEORETICAL FRAMEWORK, DATA AND METHODOLGY

The data and methodology used in the thesis are examined in this chapter. Section 1 discusses the data used and some of the problems encountered. Section 2 reviews the methodology.

3.1 DATA AND RELATED ISSUES

The data are comprised of monthly returns on 100 companies chosen from the TSE 300 composite index from the beginning of 1981 to the end of 2000[6]. The source of the data is from the Canadian Financial Markets Research Centre (CFMRC)[7]. To obtain the largest 100 company returns from the TSE 300 composite index poses several problems. The first problem is that the TSE 300 composite index contains a high proportion of resourced-based companies and, if selected, would result in a concentration of returns based on that particular industry. To address this problem, the TSE 300 composite index's 14 indices are divided into four categories according to the traditional TSE 100 composite index: resource, industrial, consumer and interest rate sensitive[8]. As a result, the top 100 companies are comprised of the top 25 firms from each industry group.

The other problem in collecting the returns for the 100 largest companies is that obtaining the returns for the entire 20-year period is not feasible. The reason being is that the returns are often not available as many of these companies, from the initial starting period of 1981, were either merged, acquired, or simply delisted for various reasons. To address this problem, the merged, acquired or delisted companies are assumed to be reinvested into the next largest company in the appropriate category at the relevant point

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[6] The reason for selecting 100 stocks as the upper bound for the portfolio is twofold. First, to replicate the U. S. study done by Domian, Louton, and Racine (2002), an index of 100 stocks is required. Second, the total number of stocks that are broadly traded with complete data on returns available on the TSE for the entire twenty years is limited. This means that thinly traded stocks available in the TSE encounter data continuity problems and as a result the total number of stocks available becomes limited.

[7] As of April 15, 2002, the Toronto Stock Exchange changed its abbreviation to TSX and as of May 1, 2002, the TSE 300 composite index name was changed to S&P/TSX composite index.

[8] The TSE 100 is not used to select companies for this thesis as it was first introduced in 1993 and the TSE 300 was first introduced in 1977.

in time[9]. For example, one thousand dollars is invested into Company X in industry group Y at the beginning of 1981. Subsequently, Company X, for various reasons, is delisted in March of 1983. At the time, Company X has a cumulative net loss of 90%. This means, according to this assumption, that one hundred dollars from the initial one thousand dollar investment is available to be invested into the next largest company in industry group Y as of March 1983.

The largest 25 companies are selected from the TSE 300 composite index for each of the four industry groups. These companies are selected based on the individual company's relative weight on the TSE 300 composite index at the beginning of January 1981. This is determined by *The TSE Review*[10]. Since many data points are missing throughout the holding period, one requirement has to be met for a company to be chosen. For example, from the CFMRC database, numerous companies have strings of missing observations for at least two consecutive months at the start of the sample. To be included in the initial 100-stock portfolio, the largest 25 companies from each category have to have at least 11 months of monthly return data in the initial year of 1981.

As mentioned previously, many data points are missing throughout the holding period. To prevent survivorship bias in the future years, any company, which contains 3 or more months of missing returns within a 12-month time frame, is reinvested into the next largest company in that industry group at the relevant point in time[11]. For example, if Company X contains only 9 months of returns within the 12-month time frame, Company X is automatically reinvested into the next largest company in that particular

[9] Throughout the entire 20-year period, this occurred 59 times.
[10] *The TSE Review* is a monthly journal that contains all the rêlative weight of the companies, that are part of the TSE 300 composite, and the market capitalization of the companies.
[11] The Canadian study by Cleary and Copp (1999) demonstrates survivorship bias.

industry group at the appropriate point in time. This procedure prevents survivorship bias by retaining companies which have full data points, as well as companies that have only a few missing data points. For all other missing monthly returns, the TSE 300 composite index's monthly returns are used[12].

Cumulative returns, which include reinvestment of dividends, are calculated for the 100 companies over a twenty-year holding period. The cumulative returns are calculated using the following equation:

$$CR_i = \prod_{t=1}^{240} (1 + R_{i,t}) - 1$$

where CR_i = cumulative return for company i

and $R_{i,t}$ = return for company i in month t

The cumulative returns range from negative 0.999854 (i.e., a 99.9854% loss) to positive 92.34349 (i.e., a 9234.349% gain) as shown in Table 2 below. The sample statistics include a mean return of 10.58572, a median of 3.585692 (the average of the 50th and 51st returns), and a skewness of 2.638144. The positive skewness is not surprising since returns are bounded by 100% losses yet potential gains are unlimited.

[12] Out of the complete 42,000 data points, 105 of the TSE 300 composite index's monthly returns are used.

Table 2 Sample Statistics for the 100-Stock Portfolio

PERCENTILE OF DISTRIBUTION	CUMULATIVE RETURNS	DESCRIPTIVE STATISTICS	
1%	-0.998585	Mean	10.58572
5%	-0.994408	Median	3.585692
10%	-0.966153	Std. Dev.	17.64199
25%	0.385227	Skewness	2.638144
50%	3.585692	Maximum	92.34349
75%	15.75008	Minimum	-0.999854
90%	28.15303		
95%	48.22704		
99%	88.31857		
100%	92.34349		

3.2 METHODOGY

There are numerous portfolio combinations of k stocks from a set of n stocks. The exact number of combinations is given by the binomial factor.

$$C_{n,k} = \frac{n!}{k!(n-k)!}$$

For example, randomly choosing a portfolio of 10 stocks from the initial 100 companies would result in 17.3 trillion different combinations. A more convenient way to analyze the risk of portfolios with different levels of holdings from a set of n stocks is to use a simulation approach that will show the range of possible outcomes.

With the aid of computer simulation, portfolios of 4, 8, 12, ..., and 96 equally-weighted stocks are formed by randomly selecting without replacement from the 100-stock portfolio. For each portfolio size, 500,000 different combinations are formed, and ending wealth is computed from an initial investment of \$100,000 held for 20 years. A sample of the computer commands used to construct the various portfolios over the period of 1981-2000 is presented in Appendix C. Transactions costs are not considered here, as they are insignificant when buying and holding the stocks for the period of 20 years. For example, if an investor were to invest in 100 stocks at the beginning of 1981 at an average transaction cost of \$50 per trade, this would result in \$5,000. The \$5,000 costs of transacting is 5% of the \$100,000 would result an initial investment amount of \$95,000. The costs associated with transacting for a 100-stocks portfolio would have a future value of \$57,929 over a 20-years investment horizon[13]. This clearly indicates that the transactions costs are 5% of the ending wealth of the portfolio. When this is compared

[13] The \$100,000 invested in the 100-stocks portfolio grew to \$1,158,572 and the \$95,000 invested in the same portfolio grew to \$1,100,643. The future value of the \$5,000 is the difference between the two amounts.

to a passive mutual fund like an Index Fund, where there are also transactions costs for purchasing and trading plus fund management fees which can range from a .86% to a 1.62% per year, the transactions costs are marginal for the 20-year individual investor[14].

To see if there are any differences in investing across industries, portfolios of 4, 8, 12, … and 96 equally-weighted stocks are formed by randomly selecting one, two, three, etc. stocks simultaneously across the four individual industry groups of 25 stocks. Similar to the previous case, 500,000 different combinations are formed and the ending wealth is computed from an initial investment of $100,000 held for 20 years.

[14]The source for the management expense ratios is obtained from the *Scotia Mutual Fund, Semi-annual Report, June 30, 2002, Scotia Securities.*

CHAPTER 4

EMPIRICAL RESULTS

This chapter presents the findings of the thesis. Section 1 analyzes the shortfall risk when investing randomly for different-sized portfolios over the investment horizon from 1981 to 2000. Section 2 analyzes the shortfall risk when investing across the four industry groups over the same period. Finally, a thorough comparison of the two methods of investing is done in Section 2.

4.1 INVESTING RANDOMLY

An equally-weighted portfolio invested in all 100 stocks would earn a 20-year cumulative rate of return equal to a mean of 10.58572 (1,058.572% see Table 2)[15]. The initial investment of $100,000 would grow to $1,158,572 during the 20-year holding period. Table 3 depicts the various portfolios and the ending wealth for different percentiles of the distribution. The percentiles of the distribution simulated by this thesis represent the risk an individual investor faces. For example, at the 1st percentile, a 20-stock portfolio investor has a 1 in 100 chance of obtaining an ending wealth less than $474,321 in the 20-year holding period. That same investor has a 1 in 4 chance of obtaining an ending wealth less than $901,359. This is risky considering the missed potential of obtaining an ending wealth of $1,158,572 with a 100% probability for the 100-stock portfolio. As illustrated in Table 3, the ranges and standard deviations decline as the size of the portfolio increases. The 20-stock and 96-stock portfolios' ranges are $2,856,951 and $286,526 and the standard deviations are $352,819 and $36,031, respectively. These numbers are not surprising since the smaller the portfolio, the more unsystematic risk is incurred.

The shortfall risk is the most significant for risk-averse investors. The shortfall risk measures the risk of falling short of a targeted investment and emphasizes the "safety-first" rule. Considering that the 100-stock portfolio has a 100% probability of obtaining an ending wealth of $1,158,572, it is safe to assume that the target investment does not fall short of 5% (or $57,929) of the guaranteed amount of $1,158,572. This clearly indicates that a portfolio investment cannot exceed a shortfall risk in the amount

[15] The TSE 300 total return index's cumulative return which includes dividends is 613.7103% over the 20-year investment horizon

of \$57,929. The 5% is chosen because it is customary choice for tests of statistical significance. Table 4 and Figure 2 depict the different amounts of shortfall risk for different-sized portfolios in percentages. Table 5 and Figure 3 depict the different amounts of shortfall risk for different-sized portfolios in dollar values. From these tables and figures, it is evident that there is a significant reduction in shortfall risk as the number of stocks increase in the portfolios. For example, at the 1st percentile of the distribution, the 20-stock portfolios have a shortfall risk of 59.06% or \$684,251 below the wealth obtained from owning the 100-stock portfolio. Whereas, for the 96-stock portfolios, at the 1st percentile of the distribution, the shortfall risk is much lower at 9.61% or \$111,353 below the wealth obtained from owning the 100-stock portfolio. Even though increasing the number of stocks reduces the shortfall risk, it is apparent from the tables and figures, that at the 1st percentile of the distribution none of the portfolios reach the target investment of a shortfall risk of less than 5% or \$57,929.

Consider Tables 4 and 5. A 20-stock portfolio investor who happens to be in the 25th percentile of the distribution would acquire a shortfall risk of 22.20% or \$257,213. This again illustrates that 20-stock portfolios are not adequately diversified. Progressively increasing the portfolio size reduces risk but still reveals ample shortfall risk. For instance, as Table 5 illustrates, more than one-quarter of the portfolios for the 40- to 50-stock portfolios are more than \$100,000 below the wealth of the 100-stock portfolio. This indicates that diversification beyond the 40- to 50-stock portfolios is justifiable. Finally, based on the 25th percentile of the distribution, more than 80 stocks are required to reduce the shortfall risk amount to the target investment of less than 5% (or \$57,929) of the ending wealth obtained from the 100-stock portfolio.

4.2 INVESTING ACROSS FOUR INDUSTRY GROUPS

Diversifying across the four industry groups reduces the unsystematic risk to the portfolios, as the correlations between the securities across the four groups are usually smaller than the correlations of securities that are randomly selected. Table 6 depicts the various portfolios' ending wealth for different percentiles of distribution for the 20-year holding period when investing $100,000 initially across the four industry groups. Comparing Table 3 and Table 6, there is a greater risk for an individual investor who invests randomly than for an investor who invests across the four industry groups. For example, at the 1st percentile, a 16-stock portfolio investor who invests across the four industry groups has a 1 in 100 chance of obtaining an ending wealth of less than $424,418 in the 20-year holding period. This compares to an ending wealth of less than $408,564 for an investor who invests randomly. The difference in total dollar value is $15,854 over the 20-year holding period. This represents approximately 3.88% of the total ending wealth for the investor who invests randomly. The larger standard deviations and ranges also indicate that investing randomly is riskier. As shown in Table 3 and Table 6, the standard deviation for the 16-stock portfolios is $393,401 for an investor who invests across the four industry groups, versus $404,383 for an investor who invests randomly. The ranges are $3,064,172 and $3,376,089, respectively. Progressively increasing the portfolio size reduces the difference between the standard deviations in the two methods of investing. For example, for the 72-stock portfolios, the standard deviation is $106,837 when investing across the four industry groups and $110,133 when investing randomly. The difference is $3,296. This difference is considerably less since the difference between the standard deviations when investing randomly and investing across industry groups for the 16-stock portfolios is $10,982. This example also shows that even increasing the size of the portfolios, more risk, measured with standard deviation, is incurred when an investor invests randomly.

The shortfall risk for the two methods of investing is still very significant for risk-averse investors. Table 7 (Figure 4) and Table 8 (Figure 5) depict the different amounts of shortfall risk for different-sized portfolios, in percentage and dollar values, when investing randomly across the four industry groups. Using Table 4 (Figure 2) and Table 5 (Figure 3), which illustrates investing randomly, one is able to compare the shortfall risk for the two method of investing. It is evident by these four tables and figures, that the shortfall risk faced by individual investors who invest randomly is greater than individuals who invest across industry groups. For example, at the 1st percentile of distribution, the 16-stock portfolios invested across the four industry groups have a shortfall risk of 63.37% or $734,154 below the wealth obtained from owning the 100-stock portfolio. Whereas, the same-sized portfolios invested randomly have a shortfall risk of 64.74% or $750,008 below the wealth acquired from the 100-stock portfolio. Slowly increasing the portfolio size reveals that investing across the four industry groups is still preferable to investing randomly as less shortfall risk is incurred. For example, again at the 1st percentile of distribution, the 32-stock portfolios invested across the four industry groups have a shortfall risk of 45.17% or $523,383 versus 46.82% or $542,400 for the 32-stock portfolios invested randomly.

Figure 6 and Figure 7 compare the 1st, 5th, 10th, and 25th percentiles of distribution, in percentages (Figure 6) and in dollars (Figure 7), of the shortfall risk for investing randomly and across the four industry groups. Notice that the shortfall risk differences between the two methods of investing gradually decreases as the portfolio size increases. This gradual decrease is representative of all the percentiles of the distributions. For example, using Figures 6 and 7 along with Tables 4, 5, 7, and 8, at the 1st percentile of the distribution, the 52-stock portfolios have a shortfall risk of 33.37% or \$386,606 for investors who invest randomly. Again, at the 1st percentile of the distribution, of the 52-stock portfolios obtained from investing across industry groups, the shortfall risk incurred is 32.35% or \$374,805. The difference between the two methods of investing is 1.02% or \$11,801. The shortfall risk differences between the two methods of investing become considerably less at all levels of percentiles as the portfolio size increases. Consider the 64-stock portfolios. At the 1st percentile of the distribution, the shortfall risk differences between investing randomly and across the four industry groups is 0.81% or \$9,312. At the 76-stock portfolio, the difference is only 0.68% or \$7,909. Figures 6 and 7 and Tables 4, 5, 7, and 8 also illustrate that the shortfall risk differences between the two methods of investing are slightly greater at the lower percentile of the distribution for all portfolio sizes. An example of this can be seen with the 40-stock portfolios. At the 1st percentile of distribution, the difference between the portfolios is 1.41% or \$16,343; at the 5th percentile of distribution, the difference is 0.91% or \$10,591; at the 10th percentile of distribution, the difference is 0.59% or \$6,893; and at the 25th percentile of distribution, the difference is 0.39% or \$4,609.

Even though investing across the four industry groups shows less shortfall risk than investing randomly, this shortfall is still considerable. For example, a 56-stock investor has a 1 in 100 chance of obtaining an ending wealth of less than \$806,485. This ending wealth is 30.39% or \$352,087 less than the wealth obtained from owning the 100-

stock portfolio (see Table 7 and 8). Progressively increasing the portfolio size reduces the shortfall risk as indicated in Table 7 and Table 8. For example, a 60-stock investor has a 1 in 100 chance of obtaining a shortfall risk of 28.32% or $328,115 below the wealth obtained from owning the 100-stock portfolio. The shortfall risk incurred by the larger portfolio is reduced but is still considerable. For example, at the 1st percentile of distribution, a 92-stock portfolio investor has a 1 in 100 chance of obtaining an ending wealth of less than $1,013,831 in the 20-year holding period. The shortfall risk for the 92-stock portfolio investor is 12.49% or $144,741 below the wealth obtained from owning the 100-stock portfolio. A 96-stock investor has a 1 in 100 chance of obtaining an ending wealth of less than $1,047,656. The shortfall risk is 9.57% or $110,916 below the wealth obtained in owning the 100-stock portfolio. Both cases are risky considering the missed potential of obtaining an ending wealth of $1,158,572 with a 100% probability for the 100-stock portfolio. Consider a more optimistic scenario where the investor happens to fall in the 25th percentile of the distribution. More than 80 stocks is needed to reduce the shortfall risk amount to less than 5% or $57,929 over the 20-year investment horizon which is similar to the randomly invested portfolios.

Table 3
Ending Wealth of Different-Sized Portfolios, 1981-2000

The figures below show ending wealth from $100,000 invested at the beginning of 1981. Portfolios of 4, 8, 12,…, 96 stocks are randomly chosen from 100-stock portfolio. For each portfolio size, 500,000 combinations are formed. Selected percentiles are given from the distribution of ending wealth.

Percentile of Distribution	Number of Stocks											
	4	8	12	16	20	24	28	32	36	40	44	48
1%	$83,218	$216,237	$321,934	$408,564	$474,321	$530,529	$575,615	$616,172	$655,054	$686,004	$715,692	$745,00[illegible]
5%	185,755	360,720	481,253	561,714	625,796	675,946	715,943	751,517	781,900	809,215	834,839	857,05[illegible]
10%	263,551	471,857	584,851	662,911	720,633	765,444	800,063	831,136	857,788	881,064	901,720	919,96[illegible]
25%	507,146	703,807	799,000	860,992	901,359	932,225	954,729	974,973	992,733	1,006,408	1,019,584	1,031,01[illegible]
50%	929,560	1,056,954	1,102,042	1,123,051	1,133,736	1,141,218	1,144,469	1,148,785	1,152,350	1,154,062	1,156,005	1,157,57[illegible]
75%	1,582,389	1,523,708	1,458,839	1,419,646	1,388,835	1,366,556	1,346,643	1,330,909	1,318,511	1,305,802	1,294,908	1,285,03[illegible]
90%	2,437,852	1,990,979	1,810,689	1,706,047	1,632,213	1,577,700	1,533,336	1,496,791	1,469,298	1,442,621	1,418,553	1,397,22[illegible]
95%	2,896,551	2,283,149	2,030,007	1,883,243	1,781,146	1,706,104	1,646,295	1,598,080	1,558,343	1,522,895	1,490,803	1,462,46[illegible]
99%	3,835,599	2,857,720	2,451,297	2,222,012	2,065,210	1,947,057	1,855,238	1,779,899	1,721,785	1,668,285	1,619,062	1,576,76[illegible]
Mean	1,157,148	1,159,106	1,158,277	1,159,592	1,159,148	1,159,298	1,157,929	1,157,933	1,159,116	1,158,592	1,158,337	1,158,18[illegible]
Std. Dev.	863,128	598,612	477,806	404,383	352,819	313,723	282,976	256,999	235,630	216,568	199,014	183,55[illegible]
Range	6,845,248	4,623,768	4,066,996	3,376,089	2,856,951	2,515,088	2,307,627	2,009,398	1,795,820	1,750,153	1,586,023	1,435,02[illegible]

Table 3 (continued)
Ending Wealth of Different-Sized Portfolios, 1981-2000

The figures below show ending wealth from $100,000 invested at the beginning of 1981. Portfolios of 4, 8, 12,…, 96 stocks are randomly chosen from 100-stock portfolio. For each portfolio size, 500,000 combinations are formed. Selected percentiles are given from the distribution of ending wealth.

Percentile of Distribution	Number of Stocks											
	52	56	60	64	68	72	76	80	84	88	92	96
1%	$771,966	$797,424	$819,886	$842,657	$865,296	$887,685	$909,270	$932,656	$956,923	$986,115	$1,010,901	$1,047,219
5%	878,261	898,336	916,644	934,372	952,030	968,613	986,009	1,003,194	1,020,842	1,039,761	1,060,669	1,085,849
10%	937,958	954,379	969,855	984,220	995,936	1,012,388	1,026,663	1,040,535	1,054,816	1,069,687	1,086,366	1,105,001
25%	1,041,616	1,051,113	1,060,307	1,069,206	1,077,305	1,084,984	1,093,120	1,101,219	1,109,434	1,117,388	1,126,916	1,140,808
50%	1,158,657	1,160,225	1,160,967	1,162,220	1,163,119	1,163,699	1,164,701	1,165,058	1,165,762	1,166,107	1,167,430	1,168,013
75%	1,275,577	1,267,120	1,259,482	1,252,124	1,244,817	1,237,411	1,230,496	1,223,072	1,215,481	1,207,526	1,198,151	1,185,674
90%	1,378,202	1,359,841	1,343,706	1,327,547	1,312,607	1,298,031	1,283,135	1,268,238	1,253,020	1,236,710	1,218,275	1,195,892
95%	1,437,023	1,412,517	1,391,116	1,369,724	1,350,135	1,330,905	1,311,311	1,291,880	1,271,984	1,250,829	1,227,913	1,199,099
99%	1,539,988	1,504,063	1,413,526	1,441,999	1,413,526	1,384,701	1,356,937	1,329,585	1,301,797	1,272,588	1,240,399	1,206,285
Mean	1,158,222	1,158,314	1,158,457	1,158,541	1,158,667	1,158,496	1,158,708	1,158,675	1,158,720	1,158,490	1,158,564	1,158,520
Std. Dev.	169,423	156,014	144,018	132,252	121,001	110,133	99,159	88,070	76,861	65,141	52,044	36,031
Range	1,367,743	1,198,815	1,149,302	1,058,488	954,342	868,205	745,602	698,016	629,428	517,883	410,635	286,526

Table 4
Shortfall Risk of Different-Sized Portfolios (Percentages), 1981-2000

The figures below show the deviation of wealth from the 100-stock portfolio for different-sized portfolios indicated in percentages at selected percentile of distributions.

Percentile of Distribution	Number of Stocks											
	4	8	12	16	20	24	28	32	36	40	44	48
1%	92.82%	81.34%	72.21%	64.74%	59.06%	54.21%	50.32%	46.82%	43.46%	40.79%	38.23%	35.70%
5%	83.97%	68.87%	58.46%	51.52%	45.99%	41.66%	38.20%	35.13%	32.51%	30.15%	27.94%	26.02%
10%	77.25%	59.27%	49.52%	42.78%	37.80%	33.93%	30.94%	28.26%	25.96%	23.95%	22.17%	20.60%
25%	56.23%	39.25%	31.04%	25.69%	22.20%	19.54%	17.59%	15.85%	14.31%	13.13%	12.00%	11.01%
	52	56	60	64	68	72	76	80	84	88	92	96
1%	33.37%	31.17%	29.23%	27.27%	25.31%	23.38%	21.52%	19.50%	17.40%	14.89%	12.75%	9.61%
5%	24.19%	22.46%	20.88%	19.35%	17.83%	16.40%	14.89%	13.41%	11.89%	10.25%	8.45%	6.28%
10%	19.04%	17.62%	16.29%	15.05%	14.04%	12.62%	11.39%	10.19%	8.96%	7.67%	6.23%	4.62%
25%	10.09%	9.28%	8.48%	7.71%	7.01%	6.35%	5.65%	4.95%	4.24%	3.55%	2.73%	1.53%

Table 5
Shortfall Risk of Different-Sized Portfolios (Dollars), 1981-2000

The figures below show the deviation of wealth from the 100-stock portfolio for different-sized portfolios indicated in dollar values at selected percentile of distributions.

Percentile of Distribution	Number of Stocks											
	4	8	12	16	20	24	28	32	36	40	44	48
1%	$1,075,354	$942,335	$836,638	$750,008	$684,251	$628,043	$582,957	$542,400	$503,518	$472,568	$442,880	$413,572
5%	972,817	797,852	677,319	596,858	532,776	482,626	442,629	407,055	376,672	349,357	323,733	301,514
10%	895,021	686,715	573,721	495,661	437,939	393,128	358,509	327,436	300,784	277,508	256,852	238,609
25%	651,426	454,765	359,572	297,580	257,213	226,347	203,843	183,599	165,839	152,164	138,988	127,553
	52	56	60	64	68	72	76	80	84	88	92	96
1%	$386,606	$361,148	$338,686	$315,915	$293,276	$270,887	$249,302	$225,916	$201,649	$172,457	$147,671	$111,353
5%	280,311	260,236	241,928	224,200	206,542	189,959	172,563	155,378	137,730	118,811	97,903	72,723
10%	220,614	204,193	188,717	174,352	162,636	146,184	131,909	118,037	103,756	88,885	72,206	53,571
25%	116,956	107,459	98,265	89,366	81,267	73,588	65,452	57,353	49,138	41,184	31,656	17,764

Table 6
Ending Wealth for Different-Sized Portfolios When Investing Randomly Across Four Industry Groups, 1981-2000

The figures below show the ending wealth from $100,000 invested at the beginning of 1981. Portfolios of 4, 8, 12,…, 96 stocks are randomly chosen from randomly selecting one, two three, etc. stocks simultaneously across the four individual industry groups. For each portfolio size, 500,000 combinations are formed. Selected percentiles are given from the distribution of ending wealth.

Percentile of Distribution	Number of Stocks											
	4	8	12	16	20	24	28	32	36	40	44	48
1%	$85,767	$228,299	$339,006	$424,418	$491,987	$547,072	$594,452	$635,189	$668,268	$702,347	$731,814	$758,42
5%	193,976	381,992	498,949	579,874	641,582	689,764	730,489	764,153	794,318	819,806	844,232	865,10
10%	278,448	490,580	604,526	677,950	733,553	775,784	811,380	841,014	865,918	887,957	908,883	926,73
25%	532,427	724,145	814,570	868,837	908,086	937,692	961,117	979,109	996,245	1,011,017	1,022,878	1,034,56
50%	952,311	1,064,913	1,102,836	1,119,254	1,131,309	1,139,535	1,144,901	1,148,302	1,151,691	1,154,367	1,156,209	1,157,65
75%	1,588,859	1,500,702	1,446,211	1,409,203	1,381,700	1,360,322	1,341,873	1,327,126	1,314,023	1,302,606	1,291,134	1,282,07
90%	2,372,886	1,963,417	1,794,489	1,690,682	1,618,455	1,567,413	1,524,657	1,490,534	1,460,552	1,435,120	1,411,468	1,390,42
95%	2,858,186	2,260,167	2,009,080	1,865,669	1,763,492	1,692,406	1,634,503	1,587,846	1,546,891	1,512,130	1,481,873	1,453,52
99%	3,840,174	2,822,993	2,430,272	2,195,652	2,041,033	1,925,766	1,834,579	1,764,862	1,704,766	1,650,058	1,606,134	1,565,47
Mean	1,160,685	1,159,356	1,159,018	1,157,917	1,158,132	1,158,634	1,158,678	1,158,507	1,158,688	1,158,796	1,158,533	1,158,45
Std. Dev.	842,365	581,006	464,386	393,401	343,013	305,405	274,933	250,427	228,812	210,151	193,425	178,52
Range	6,354,140	4,506,759	3,747,646	3,064,172	2,761,413	2,547,689	2,173,564	2,037,522	1,763,808	1,641,340	1,653,839	1,383,05

Table 6 (continued)
Ending Wealth for Different-Sized Portfolios When Investing Randomly Across Four Industry Groups, 1981-2000

The figures below show the ending wealth from $100,000 invested at the beginning of 1981. Portfolios of 4, 8, 12,…,96 stocks are randomly chosen from randomly selecting one, two three, etc. stocks simultaneously across the four individual industry groups. For each portfolio size, 500,000 combinations are formed. Selected percentiles are given from the distribution of ending wealth.

Percentile of Distributions	Number of Stocks											
	52	56	60	64	68	72	76	80	84	88	92	96
1%	$783,767	$806,485	$830,457	$851,969	$874,101	$895,848	$917,179	$939,681	$960,869	$992,197	$1,013,831	$1,047,65
5%	886,356	904,526	923,179	939,631	956,857	973,825	990,699	1,007,165	1,024,078	1,042,525	1,062,767	1,088,30
10%	943,991	959,592	974,775	988,826	1,002,159	1,016,344	1,030,263	1,043,509	1,057,001	1,071,856	1,088,421	1,108,40
25%	1,044,540	1,053,760	1,063,347	1,071,612	1,078,815	1,087,067	1,094,904	1,102,846	1,110,724	1,119,290	1,128,678	1,140,70
50%	1,159,457	1,160,285	1,161,437	1,162,393	1,162,693	1,163,827	1,164,359	1,165,185	1,165,784	1,166,368	1,166,706	1,167,49
75%	1,273,266	1,264,949	1,257,014	1,249,461	1,242,212	1,235,420	1,228,026	1,221,229	1,213,639	1,205,535	1,196,400	1,184,70
90%	1,371,834	1,355,190	1,338,495	1,323,110	1,307,469	1,293,390	1,278,842	1,264,917	1,249,999	1,234,277	1,216,693	1,195,20
95%	1,428,910	1,406,152	1,384,016	1,363,279	1,342,945	1,324,854	1,306,095	1,287,825	1,268,872	1,248,503	1,226,277	1,198,75
99%	1,527,658	1,494,563	1,462,057	1,432,372	1,404,552	1,377,429	1,351,749	1,324,892	1,298,669	1,270,252	1,239,438	1,203,36
Mean	1,158,549	1,158,470	1,158,590	1,158,520	1,158,036	1,158,536	1,158,536	1,158,664	1,158,597	1,158,594	1,158,495	1,158,57
Std. Dev.	164,570	152,188	139,889	128,577	117,535	106,837	96,098	85,668	74,894	63,307	50,587	34,94
Range	1,291,923	1,224,819	1,095,864	1,013,336	902,251	833,847	764,039	699,765	604,822	511,673	406,429	264,75

Table 7
Shortfall Risk for Different-Sized Portfolios When Investing Randomly Across Four Industry Groups (Percentages), 1981-2000

The figures below show the deviation of wealth from the 100-stock portfolio for different-sized portfolios indicated in percentages at selected percentile of distributions.

Percentile of Distribution	Number of Stocks											
	4	8	12	16	20	24	28	32	36	40	44	48
1%	92.60%	80.29%	70.74%	63.37%	57.54%	52.78%	48.69%	45.17%	42.32%	39.38%	36.83%	34.54%
5%	83.26%	67.03%	56.93%	49.95%	44.62%	40.46%	36.95%	34.04%	31.44%	29.24%	27.13%	25.33%
10%	75.97%	57.66%	47.82%	41.48%	36.68%	33.04%	29.97%	27.41%	25.26%	23.36%	21.55%	20.01%
25%	54.04%	37.50%	29.69%	25.01%	21.62%	19.06%	17.04%	15.49%	14.01%	12.74%	11.71%	10.70%
	52	56	60	64	68	72	76	80	84	88	92	96
1%	32.35%	30.39%	28.32%	26.46%	24.55%	22.68%	20.84%	18.89%	17.06%	14.36%	12.49%	9.57%
5%	23.50%	21.93%	20.32%	18.90%	17.41%	15.95%	14.49%	13.07%	11.61%	10.02%	8.27%	6.07%
10%	18.52%	17.17%	15.86%	14.65%	13.50%	12.28%	11.07%	9.93%	8.77%	7.48%	6.05%	4.33%
25%	9.84%	9.05%	8.22%	7.51%	6.88%	6.17%	5.50%	4.81%	4.13%	3.39%	2.58%	1.54%

Table 8
Shortfall Risk for Different-Sized Portfolios When Investing Across Four Industry Groups (Dollars), 1981-2000

The figures below show the deviation of wealth from the 100-stock portfolio for different-sized portfolios indicated in dollars at selected percentile of distributions.

Percentile of Distribution	Number of Stocks											
	4	8	12	16	20	24	28	32	36	40	44	48
1%	$1,072,805	$930,273	$819,566	$734,154	$666,585	$611,500	$564,120	$523,383	$490,304	$456,225	$426,758	$400,151
5%	$964,596	$776,580	$659,623	$578,698	$516,990	$468,808	$428,083	$394,419	$364,254	$338,766	$314,340	$293,466
10%	$880,124	$667,992	$554,046	$480,622	$425,019	$382,788	$347,192	$317,558	$292,654	$270,615	$249,689	$231,839
25%	$626,145	$434,427	$344,002	$289,735	$250,486	$220,880	$197,455	$179,463	$162,327	$147,555	$135,694	$124,008
	52	56	60	64	68	72	76	80	84	88	92	96
1%	$374,805	$352,087	$328,115	$306,603	$284,471	$262,724	$241,393	$218,891	$197,703	$166,375	$144,741	$110,916
5%	$272,216	$254,046	$235,393	$218,941	$201,715	$184,747	$167,873	$151,407	$134,494	$116,047	$95,805	$70,269
10%	$214,581	$198,980	$183,797	$169,746	$156,413	$142,228	$128,309	$115,063	$101,571	$86,716	$70,151	$50,171
25%	$114,032	$104,812	$95,225	$86,960	$79,757	$71,505	$63,668	$55,726	$47,848	$39,282	$29,894	$17,871

Figure 2
Shortfall Risk for Different-Sized Portfolios When Investing Randomly, 1981-2000
(Shortfall in Percentages)

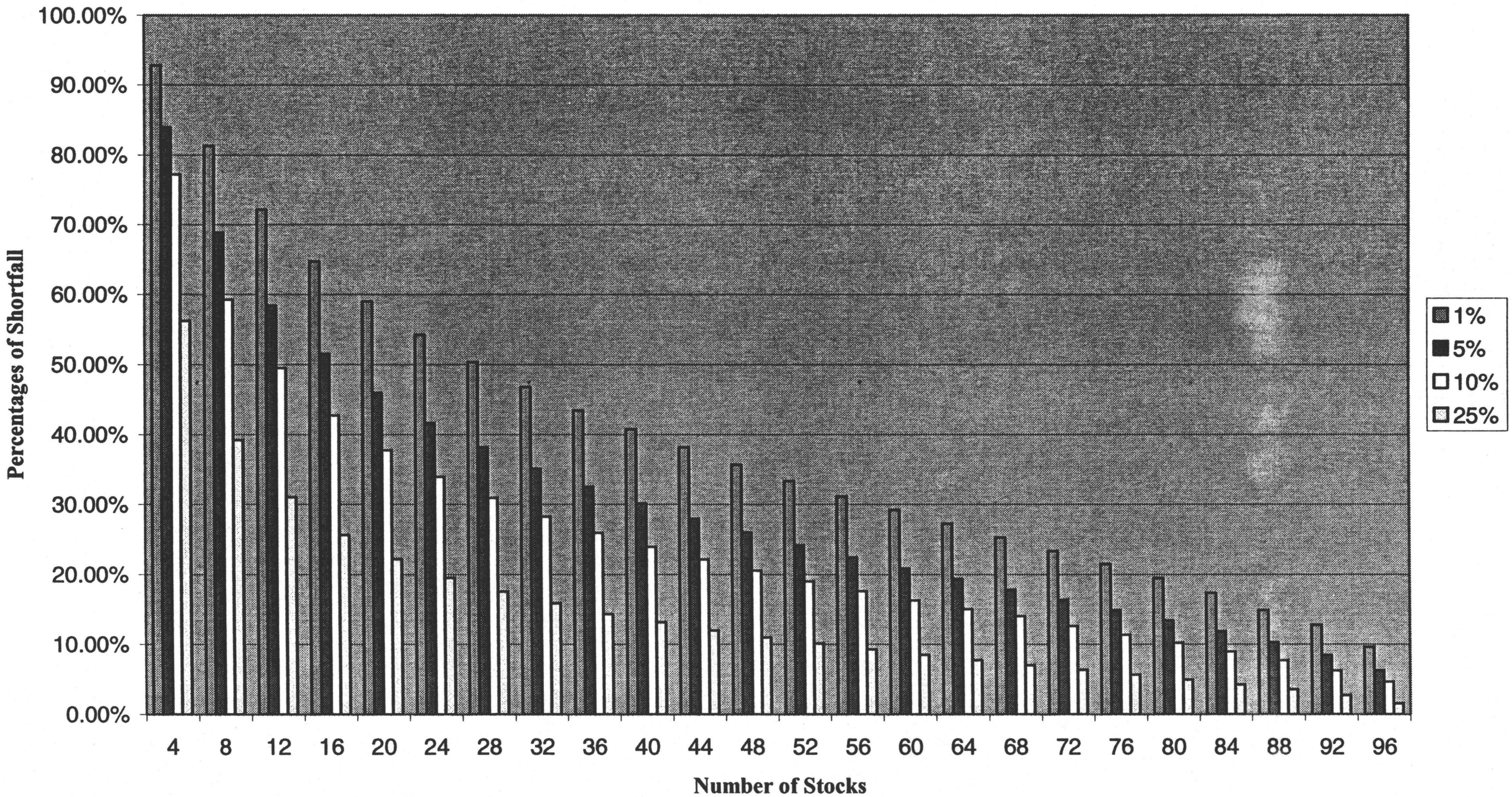


Figure 3
Shortfall Risk for Different Sized Portfolios When Investing Randomly, 1981-2000
(Shortfall in Dollar Values)

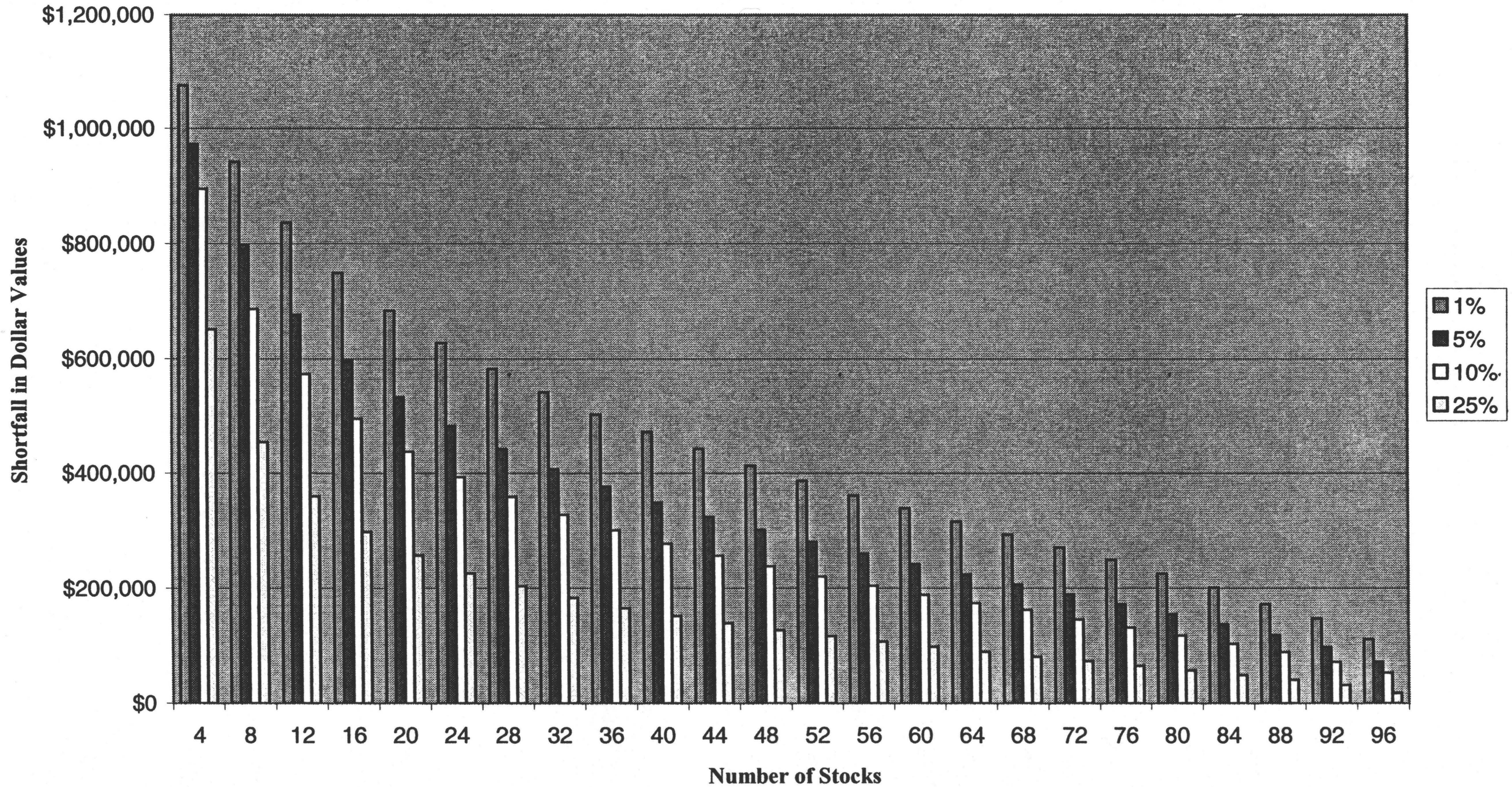


Figure 4
Shortfall Risk for Different Sized Portfolios When Investing Randomly Across Four Industry Groups, 1981-2000
(Shortfall in Percentages)

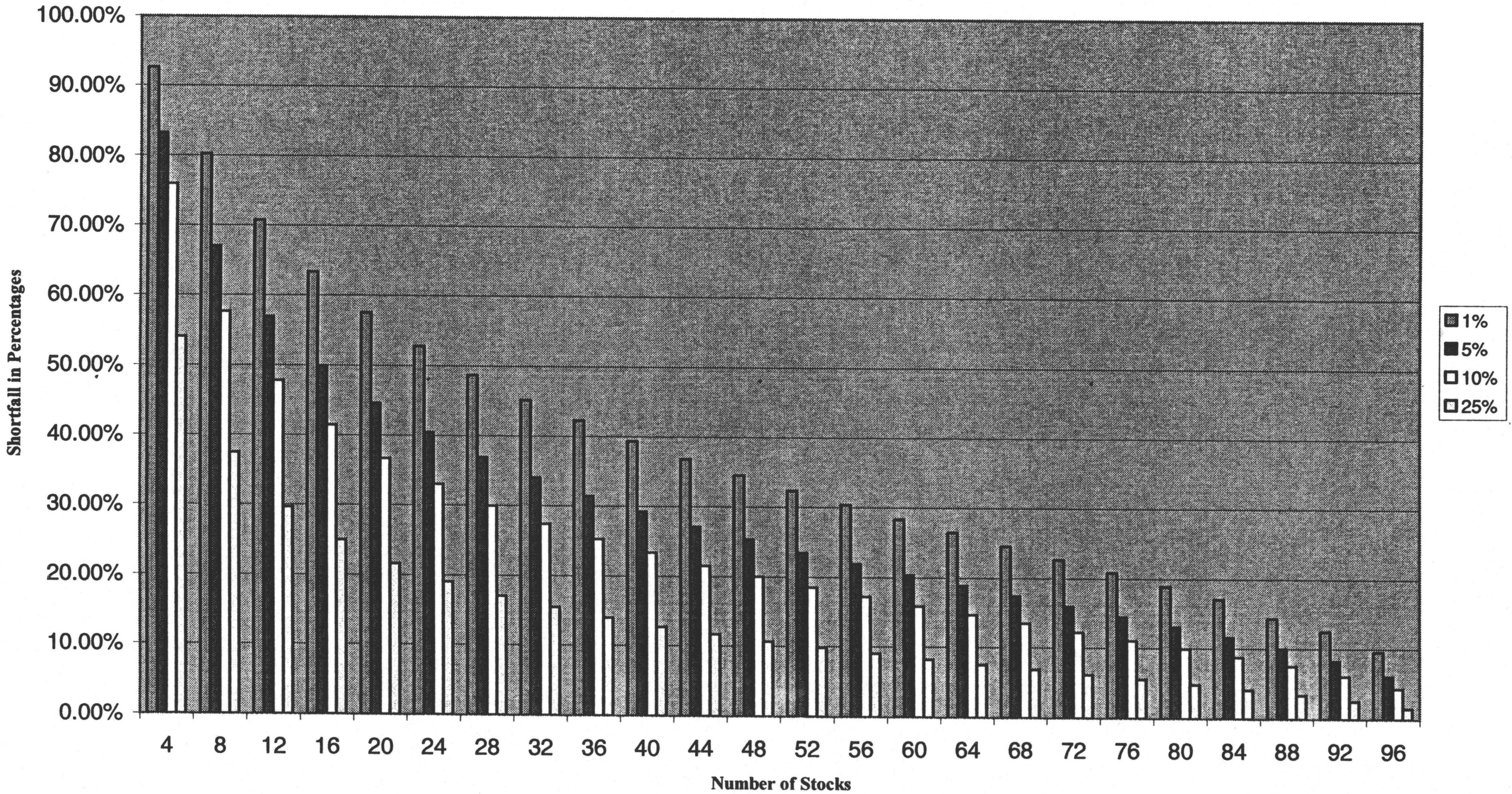


Figure 5
Shortfall Risk for Different-Sized Portfolios When Investing Randomly Across Four Industry Groups
1981-2000
(Shortfall in Dollar Values)

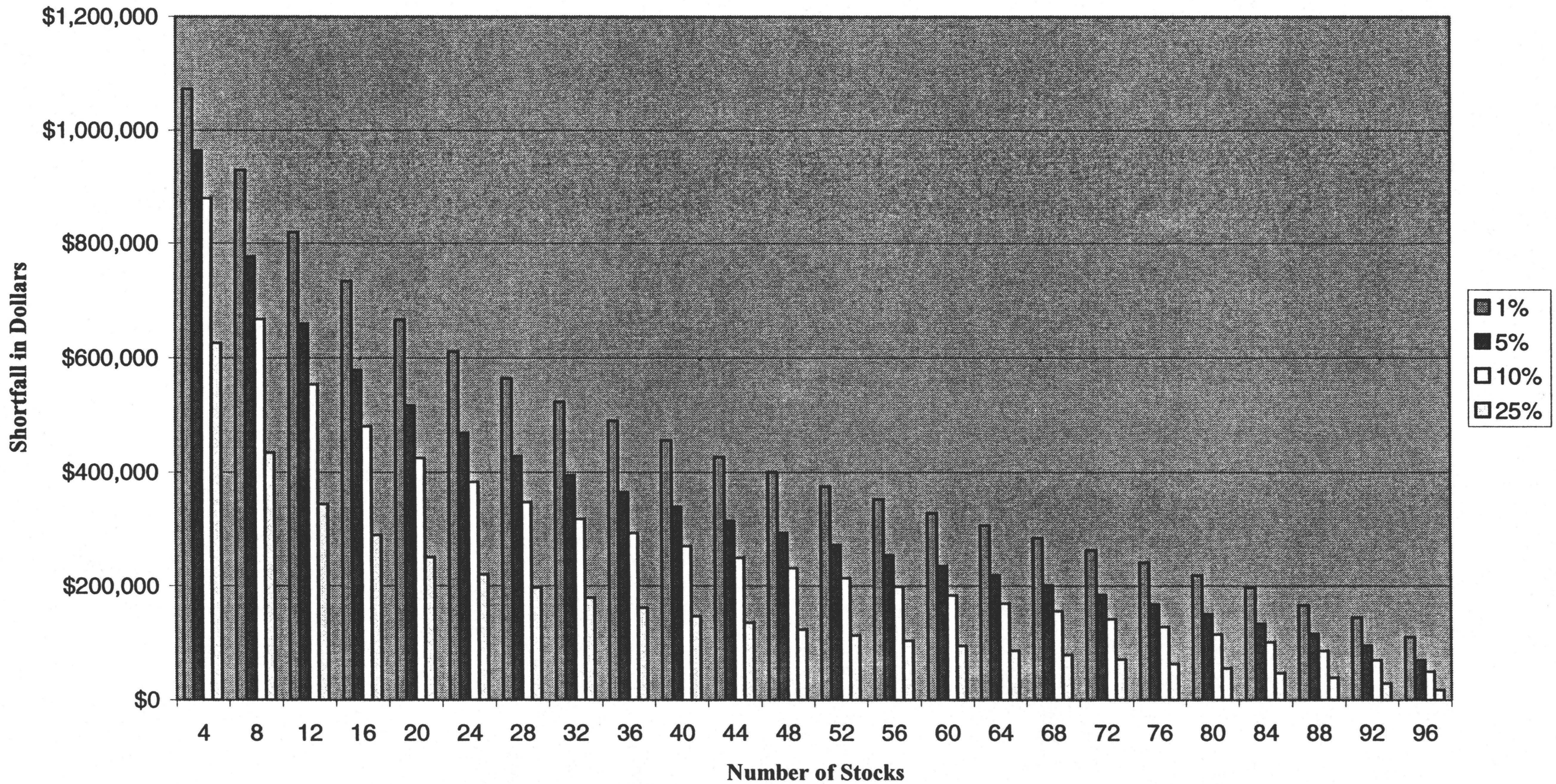


Figure 6
Comparison of the Shortfall Risk in Percentages at Selected Percentile of the Distribution
1981-2000
(Investing Randomly Versus Across Industries)

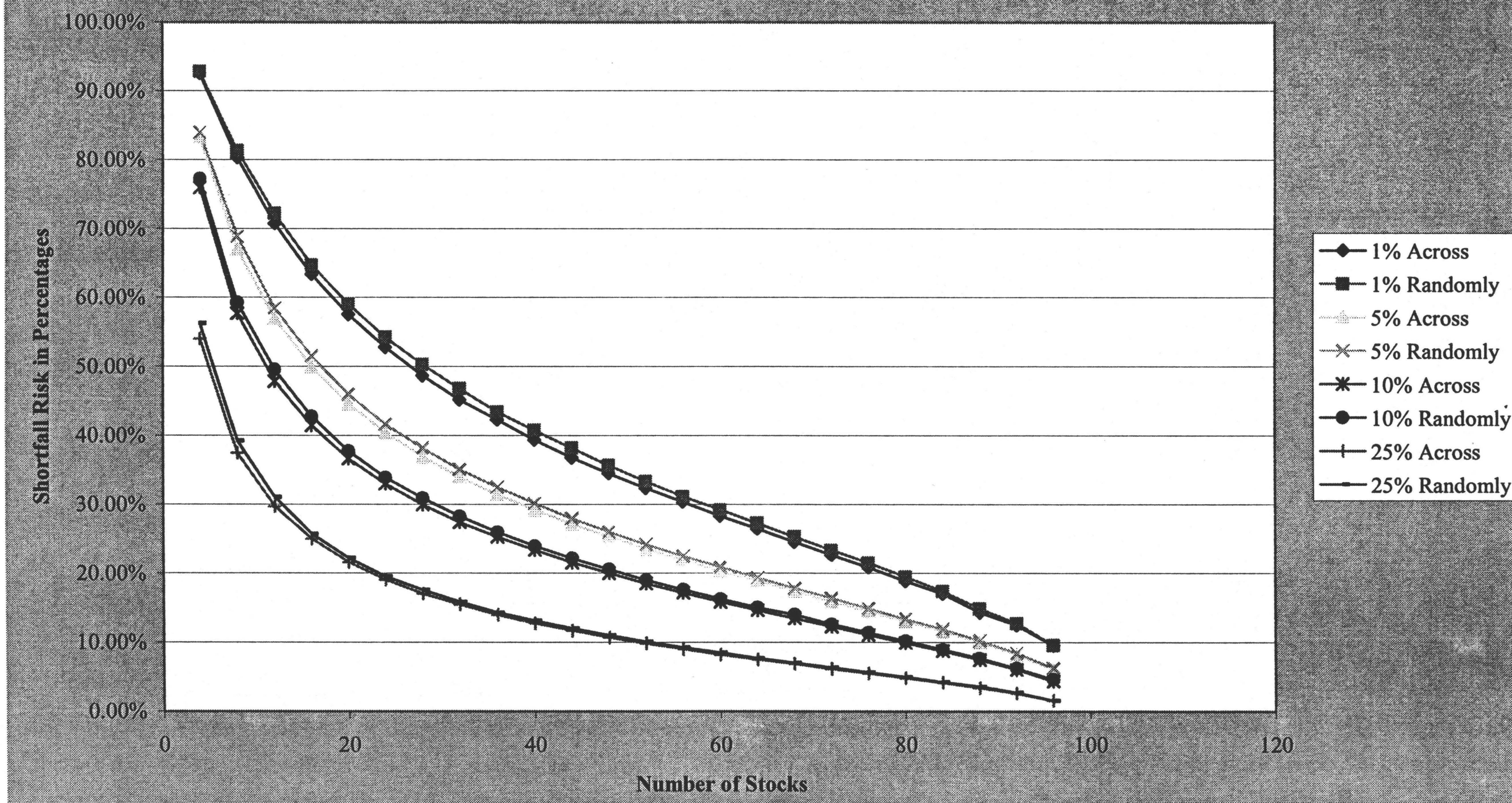
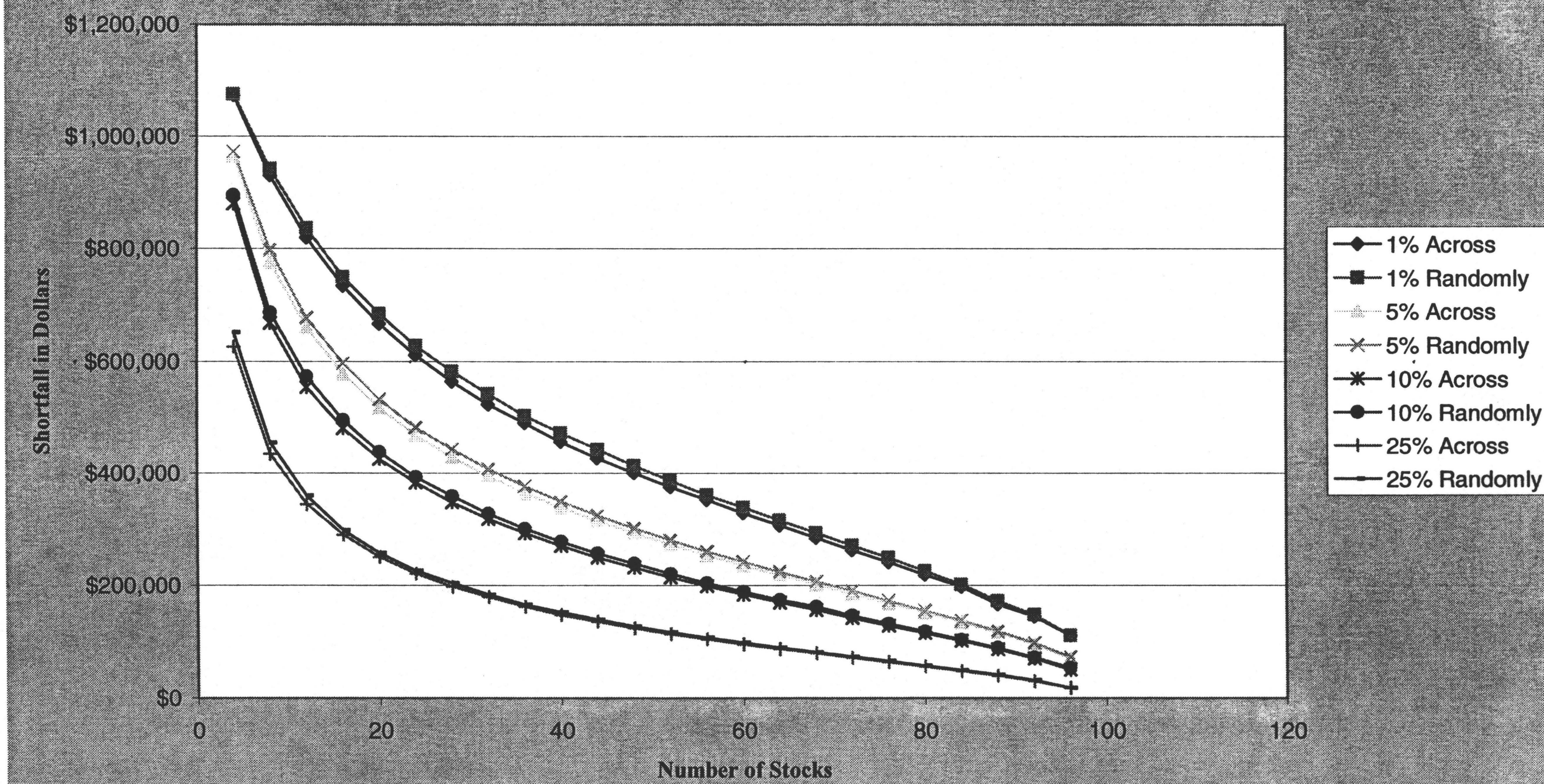


Figure 7
Comparison of the Shortfall Risk in Dollars at Selected Percentile of the Distribution
1981-2000
(Investing Randomly Versus Across Industries)



CHAPTER 5

SUMMARY AND CONCLUSIONS

This chapter gives the summary of findings and the implications of the finding in section 1. Finally, in section 2, the limitations and suggestions for future research are presented.

5.1 SUMMARY OF FINDINGS AND IMPLICATIONS

Practitioners and academics agree that risk-averse investors should own diversified portfolios to reduce the overall risk of their holdings. However, the number of stocks required to achieve diversification has been under discussion for over four decades. Traditionally, it has been viewed that 8 to 20 stocks is sufficient for a 'well' diversified portfolio based on U.S. studies, and 30 to 50 stocks based on a Canadian study. The results in this paper suggest many more companies are required for a 'well' diversified portfolio.

The greatest risk that a long-run investor faces is the potential shortfall risk from not owning an adequate number of stocks in one's portfolio. Increasing the number of stocks in his/her portfolio greatly reduces the shortfall risk faced by individual investors. Based on the probability distribution, and investing randomly across Canadian companies over 80 stocks is required to reduce the shortfall risk below 5% of the 100-stock portfolio for the 20-year holding period. This is marginally different from the U. S. study by Domian, Louton, and Racine (2002) which illustrates that over 60 stocks are needed to reduce the shortfall risk below 10% for a 20-year investment horizon. The differences could be attributed to the much larger companies in the United States relative to those that make up the TSE 300.

Diversifying across the four industry groups instead of diversifying randomly in Canadian companies reduces the shortfall risk. The differences in shortfall risk faced by investing across the four industry groups versus randomly investing are considerable when comparing the smaller-sized portfolios. As the size of the portfolios increases, the differences in shortfall risk faced by the two methods of investing declines and gradually become less and less significant at the larger portfolio sizes. Similarly to the randomly invested portfolios, more than 80 Canadian companies are needed to obtain a shortfall risk amount of less than 5% ($57,929) of the 100-stock portfolio when investing across the four industry groups.

In conclusion, this thesis suggests that investors who hold the recommended 8-20 stocks are more likely to obtain a substantially higher amount of shortfall risk for long-term investors. Increasing and diversifying across an array of stocks that are held can significantly reduce the shortfall risk. Another advantage to holding a larger diversified portfolio relates to the positive skewness of stock returns. The maximum loss that a stock can incur is negative 100%, but the maximum return on a stock in unlimited. Increasing one's portfolio size increases the likelihood of acquiring a remarkable performer and therefore reducing the overall shortfall risk of the portfolio.

5.2 LIMITATIONS AND SUGGESTIONS FOR FUTURE RESEARCH

This thesis provides evidence that long-term Canadian investors should increase the number of stocks in their portfolio beyond the traditional 8-20 stocks to reduce the shortfall risk incurred. However, there are a few questions that remain unanswered. For example, how much money does an individual investor require to apply such a strategy? The issue of transactions costs versus the benefits of diversification must be weighed. An investor would only want to form his/her own portfolio if the transactions costs are only marginal compared to the total dollar amount of the trade. Otherwise, the investor would not benefit as the transactions costs would absorb most of the benefits of diversification.

Another question to be considered is would this thesis' findings be different if the study was completed over a different time period, such as 50 years ago for example? The varying economic conditions of the different time periods compared to the period studied in this thesis may yield different results.

Finally, to conclude this chapter, a few extensions to this thesis are presented. One possible extension could be to use the second degree stochastic dominance approach as in Levy's (1979) study for the methodology and compare the results to the current existing findings of the thesis. This would allow us to observe if there are any differences in the overall results. Another extension to consider would be to use the TSE 300 as the benchmark instead of using the 100 stocks as the benchmark as the return data becomes available in the future to see if the results changes.

APPENDIX A

THE DIVIDEND TAX CREDIT

The dividend tax credit is applied only to dividends paid by Canadian corporations. There are two goals in the treatment of dividends in Canada. First, corporations pay dividends from after-tax income so to avoid double taxation, the dividend tax credit shelters dividends from the full tax in the hands of the shareholders. Second, the dividend tax credit applies only to dividends paid by Canadian corporations. The end result is to encourage Canadian investors to invest in Canadian firms as opposed to foreign firms.

To see how dividends are taxed, one will assume an individual investor holds common shares in a Canadian corporation and receives $1,000 in dividends. Table A.1 illustrates the dividend tax credit calculation. The steps follow the instruction on the federal tax returns. The actual dividends are grossed up by 25 percent and the federal tax is calculated on the grossed up figure. Then a dividend tax credit of 13⅓ percent of the grossed up dividend is subtracted from the federal tax to obtain the federal tax payable. The provincial tax (for Saskatchewan in this example) is calculated and is added to the federal tax payable to obtain the total tax payable.

The result is a total tax payable in the amount of $289.55. The dividend tax credit reduced the amount of tax payable and making dividends a more attractive income for Canadian investors. For example, if the individual investor had received $1,000 in interest income, he/she would have a tax bill in the amount of $445.00. The effective tax rate on the $1,000 dividend is 28.96% versus 44.5% on interest income. In conclusion, dividends paid by Canadian corporations are taxed far more lightly than interest income.

Table A.1 Dividend Tax Credit for Saskatchewan Residents in the Top Bracket (over $103,001) for 2002

Dividends	$1,000.00
Gross up at 25%	250.00
Grossed up dividend	1,250.00
Federal tax at 29%	362.50
Less dividend tax credit (13⅓%×$1,250)	(166.70)
Federal tax payable	195.80
Provincial tax payable[16]	93.75
Total tax payable	289.55

Source: Adapted from KPMG, *Individual Federal and Provincial Tax Rates*, 2002

[16] The Saskatchewan Provincial tax is calculated as follow:

Dividends	$1,000.00
Gross up at 25%	250.00
Grossed up dividend	1,250.00
Provincial tax at 15.5%	193.75
Less dividend tax credit (8%×$1,250)	(100.00)
Provincial tax payable	93.75

APPENDIX B

B.1 FIRST-ORDER STOCHASTIC DOMINANCE

An alternative to mean variance analysis is stochastic dominance. There are three levels of stochastic dominance that have varying degrees of assumptions. For the purpose of this thesis, only first- and second-order stochastic dominance will be examined.

Before examining first- and second-order stochastic dominance, a brief explanation of dominance will be discussed. Investment A is said to dominate over investment B when an individual is certain to receive greater wealth from investment A in every (ordered) state of nature. For example, consider Table B.1. According to the definition of dominance, investment A dominates investment B because, regardless of which outcome occurs, A will always yield a higher return than B. If the market conditions are average, then A will return 9% and B will return 8%. If the market conditions are very good, investment A will return 11% whereas B will return 10%. Under any market conditions, investment A is guaranteed to yield a higher return than investment B and therefore investment A dominates investment B.

Table B.1 Outcomes Associated with Alternative Market Conditions

Market Conditions	Outcomes A	Outcomes B
Very good	11	10
Good	10	9
Average	9	8
Poor	8	7
Very poor	7	6

First-order stochastic dominance assumes investors prefers more to less. Now consider the choices given in Table B.2. Investment A has the better outcome but it is no longer certain that an investor will do worse by investing in B. For example, there is a one in three chance for an investor to obtain 11% return if an investor invests in B and there is an equal probability of obtaining 10% or 8% return if an investor invests in A. However, the likelihood that the investor will obtain a lower return is higher with investment B. In this example, investment A is preferable to investment B, not because the investor will always yield a higher return like the previous example, but rather because for all returns, the probability of obtaining that return or less (doing worse) are as high with B as with A, or higher.

Table B.2 Two Investment Alternatives: Outcomes and Associated Probabilities

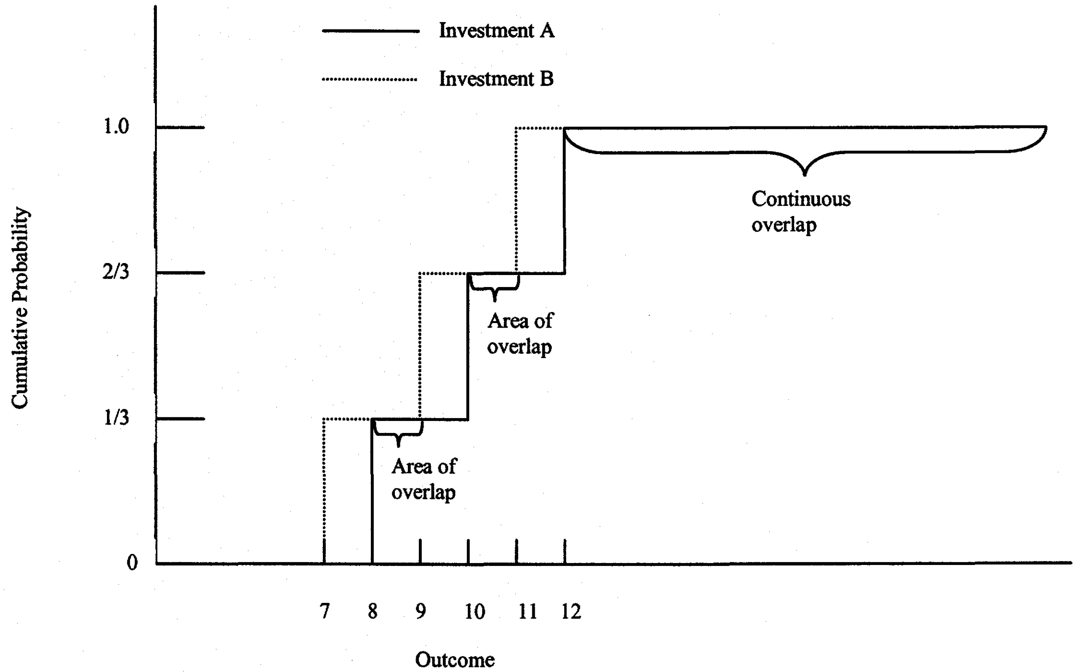
Investment A		Investment B	
Outcome	Probability	Outcome	Probability
12	1/3	11	1/3
10	1/3	9	1/3
8	1/3	7	1/3

First-order stochastic dominance can also be illustrated with the cumulative probability table. The cumulative probability is the likelihood of obtaining a given return or less. Table B.3 shows the cumulative probability of any particular return. Figure B.1 illustrates the example shown in Table B.3. Notice investment B lies above and to the left of investment A at all levels of return. This implies that the odds of obtaining any return or less are as high with B as A, or higher.

Table B.3 A Cumulative Probability Distribution

	Odds of Obtaining a Return Equal to or Less Than That Shown in Column 1	
Return	A	B
7	0	1/3
8	1/3	1/3
9	1/3	2/3
10	2/3	2/3
11	2/3	1
12	1	1

Figure A.1 Cumulative Frequency Function for Investment A and B



The above example illustrates the concept of first-order stochastic dominance. The formal theorem states: If investors prefer more to less, and if the cumulative probability of investment A is never greater than the cumulative probability of investment B and sometimes less, then investment A is preferred to investment B.

B.2 SECOND-ORDER STOCHASTIC DOMINANCE

Second-order stochastic dominance assumes that, in addition to investors preferring more to less, they are also risk averse. Being a risk averse investor means that the investor must be compensated for bearing risk. It arises when each increment in return is less valuable to the investor than the last. An example of second-order stochastic dominance is illustrated in Table B.4, which shows two possible investments, each with four equally likely outcomes. Table B.5 shows the cumulative probability for these two investments. Looking at the lower returns, one cannot select between investment A and investment B using first-order stochastic dominance. For example, at a return of 5%, investment B has a higher probability of a poor return compared to investment A. However, at 8%, investment A has a higher probability of a poorer return. In order to be able to choose between these two investments, one has to be able to decide whether the higher probability of low returns between the range of 5% and 6% for B is more important than the higher probability of a low return from investment A in the range of 8% to 9%. Using second-order stochastic dominance, where one assumes the investor is risk averse, along with preferring more to less, one can make such a decision. A risk averse investor selects investment A over investment B because the 1% increment in return from 5% to 6% is more valuable than the 1% increment in return from 8% to 9%. When an investor is risk averse, then he/she should be willing to lose 1% in return at the higher level of return in order to obtain an extra 1% at a lower return level. The above example illustrates the concept of second-order stochastic dominance and implies investment A dominates investment B.

Table B.4 Two Investment Alternatives: Outcomes and Associated Probabilities

A		B	
Outcome	Probability	Outcome	Probability
6	¼	5	¼
8	¼	9	¼
10	¼	10	¼
12	¼	12	¼

Table B.5 The Sum of the Cumulative Probability Distribution

	Cumulative Probability		Sum of Cumulative Probability		Sum of the Sums of Cumulative Probabilities	
Return	A	B	A	B	A	B
4	0	0	0	0	0	0
5	0	¼	0	¼	0	¼
6	¼	¼	¼	½	¼	¾
7	¼	¼	½	¾	¾	1½
8	½	¼	1	1	1¾	2½
9	½	½	1½	1½	3¼	4
10	¾	¾	2¼	2¼	5½	6¼
11	¾	¾	3	3	8½	9¼
12	1	1	4	4	12½	13¼

The following is the formal theorem for second-order stochastic dominance which states: If investors prefer more to less, are risk averse, and the sum of the probabilities for all returns is never more with investment A than investment B and sometimes less, then A dominates B.

APPENDIX C

The following is a description of the Regression Analysis of Time Series (RATS version 4.2) computer simulation commands used to construct 500,000 portfolios of 4, 8, 12, …, and 96 equally-weighted stocks over the period of 1981 – 2000. Below this description is a portion of the actual RATS commands used for the test. The description starts out at the *'do i=1,500000'* command where the simulation starts. This is done 500,000 times for each portfolio indicated by the 500,000. The next three lines of commands generate a method for randomly selecting integers from 1 to 100 and ordering them according to the normal distribution. First, the *'set draws 1 100 = % ran(1.0)'* command creates a series named *draws* with a length of 100 individual elements. The 100 individual elements of the series are randomly drawn from a normal distribution. Second, the *'set entry 1 100 = t'* creates a series named *entry* containing numbers between 1 to 100. Third, the *'order draws 1 100 entry'* command orders the series named *draws.* The *draws* series consist of randomly drawn numbers from a normal distribution and is associated with an entry number. The result of the previous command generates a method for randomly selecting integers from 1 to 100 from a series named *entry* which is associated with the series *draws.* When the series *draws* is being ordered the *entry* series are re-arranged randomly. The next few lines of commands let us create the portfolios. The *'return(fix(entry(t)))'* randomly re-orders the 100 cumulative return according to the series *entry*. The series *entry* contain random real numbers between 1 and 100. The *fix* command puts the real numbers back to integers. The command *'set crandom 1 100 = return(fix(entry(t))'* creates a series named *crandom* with 100 length of elements. Each element is a firm's cumulative return. The *'acc crandom 1 100 crettot'* sums up the series *crandom* and produces a new series named *'crettot'*. The new series *crettot* is the sum of the cumulative returns in the *crandom* series. For example, the *crettot(n)* is the sum of the first n cumulative returns. The four-stock portfolios as in the example below are compiled by taking the first 4 stocks cumulative returns and are divided by 4 to find the average in the *'com port4(i) = crettot(4)/4'* command. The averaging of the return is equivalent to an equally-weighted portfolio of four stocks.

```
do i=1,500000
 set draws 1 100 = %ran(1.0)
 set entry 1 100 = t
 order draws 1 100 entry
 set crandom 1 100 = return(fix(entry(t)))
 acc crandom 1 100 crettot
 com port4(i) = crettot(4)/4
end do i
```

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