

EVALUATION OF BORDER DYKE SYSTEMS

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by

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## INTRODUCTION

Irrigation is the application of water to soil for the purpose of supplying moisture necessary for the growth of a crop. Irrigation has been practiced since the beginning of civilization and its importance in the present world is well recognized as a means of developing a profitable agriculture. The rapid growth in the world population and the consequent need for additional food supplies are making irrigation absolutely necessary in the world today.

In surface irrigation by flooding, two general methods are used; uncontrolled flooding or controlled flooding. In irrigating by uncontrolled flooding, or wild flooding, water is applied to an area which has undergone no prior land preparation. In most cases, this unsystematic method produces inefficient irrigation. Low spots in the field are over-irrigated whereas high areas are under-irrigated. In the controlled flooding method, predetermined rates and quantities of water are applied to areas which have been prepared for irrigation. This method of irrigation includes border dyke, border ditch, and border check systems. For proper design of these systems, the size of stream is balanced against the intake rate of the soil, the total depth of water to be stored in the root zone, and the area to be covered by the stream.

With the completion of the South Saskatchewan Dam, it is expected that the irrigated acreage of the province will increase substantially. Further, it is expected that most of the new area coming under irrigation will be irrigated by some surface irrigation method.

One of the most common and diversified of the surface irrigation methods, which is adaptable to both close growing and row crops, is the



border dyke or border strip method. In this method, water is introduced to land which is bounded by low, flat levees or dykes which extend in the direction of the steepest slope.

The present procedures used in design of border dyke systems are empirical. The experimental data reported in the literature were obtained from systems whose soils, topography and other conditions differ widely from those which will be encountered in the South Saskatchewan River Development Project. Thus, it is questionable whether these data can be applied directly to the design of systems in Saskatchewan. There is a great need for experimental data from systems installed on areas which are similar to those that will be encountered under the proposed project.

The study reported in this thesis was undertaken in an attempt to satisfy some of these needs. Experimental data collected from existing border dyke systems located in Saskatchewan are presented. In addition, a rational approach for evaluating several factors affecting the design of these systems for example, soil intake rate, is given.

## LITERATURE REVIEW

## Border Dyke Irrigation

Israelsen and Hansen (18)<sup>1</sup>, suggest that surface irrigation systems should be designed to accomplish the following objectives:

1. Store the required water in the root zone of the soil and reduce deep percolation.
2. Minimize soil erosion, runoff of irrigation water, labour requirements and land used.
3. Provide for beneficial use of runoff water.
4. Fit and adapt the system to field boundaries and soil topography.
5. Obtain reasonably uniform application of water and a favourable salt balance.
6. Facilitate the use of machinery for land preparation, cultivating, furrowing, harvesting, etc.

The extent to which the design requirements are fulfilled is in turn governed by the economics of the farming operation. The design of the system is made exceedingly complex when consideration is given to both factors. On this matter, Davis (8) pointed out that, one should choose the most desirable objective first and then attempt to meet the other objectives through manipulation of the variables affecting the performance of the system.

In border dyke irrigation, a water stream of suitable size is applied on a strip of levelled land, bounded by low flat levees, which extend in the direction of steepest slope. The size of an individual

<sup>1</sup>/Numbers in brackets refer to appended references

strip may vary from 30 ft to 60 ft in width and from 300 ft to 1300 ft in length. The width is generally governed by the slope of the land and the amount of water that can be carried throughout the strip. It is recommended that the strip should have a uniform slope and should be level transversely so that the advancing sheet of water covers the entire width of land bounded by the border dyke (18).

Generally, it is recommended that the slope of the border strip should not exceed three percent. However, in special cases, on soils where erosion is not a problem, slopes as high as  $7\frac{1}{2}$  percent may be irrigated by this method. The size of stream to be applied to a border strip may range from  $1/2$  to 10 cfs depending on the soil type, the size of the border and the type of crop.

#### Present Design Criteria

The design of border dyke systems is at the present time dependent on empirical relationships and procedures. Foremost among these procedures is the unit-stream approach suggested by Criddle et al (7). In the unit-stream approach, it is assumed that the size of the irrigation stream is proportional to the border strip area. Under this assumption, once the proper unit-stream has been determined for a given slope, soil and depth of application, the actual size of the irrigating stream for any set of border strip dimensions is merely the product of the unit stream and the number of unit areas in the strip. A unit area in this case is considered to be one hundred square feet, and a unit-stream is a stream required to irrigate a strip area one foot wide by one hundred feet long.

The authors have provided several tables and figures derived from experimental data collected in the United States from which unit streams

can be selected for various depths of water application, basic soil intake rates and slopes. Once the unit stream size has been selected, the maximum length of run can be calculated by dividing the maximum allowable stream size per foot of border strip by the unit-stream size.

#### Factors Affecting the Design of Border Dyke Systems

Attempts to incorporate a more rational approach in design, that is, from reason to result is complicated by the complex interrelationships between the many factors which influence surface flow and the intake phenomenon. The hydraulics of overland flow represent a case of unsteady state, non uniform flow. Meyers (22) stressed that the difficulty in analyzing surface irrigation is not so much due to the difficulty in determining the intake of soil, but more so due to a lack of knowledge concerning the fluid mechanics of surface irrigation. He suggests that serious errors can result from the use of flow equations which do not apply to the situation in hand.

Hansen (15) suggested the following basic variables are involved in the hydraulics of surface irrigation (see Fig. 1):

1. Size of stream,
2. Soil intake rate,
3. Depth of water to be applied,
4. Rate of advance,
5. Length of run,
6. Slope of land surface,
7. Surface roughness,
8. Shape of flow channel,

9. Depth of flow, and
10. Erosion hazards.

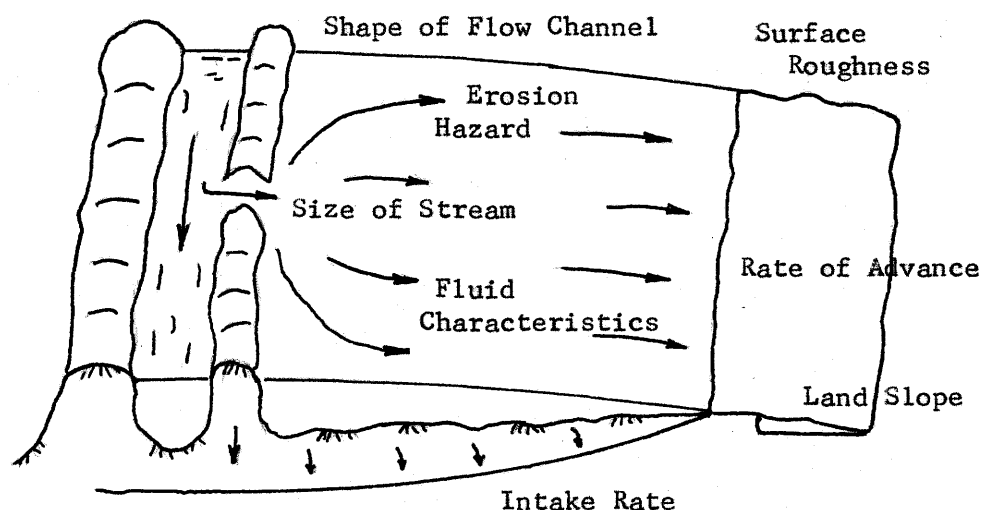


Fig. 1. Schematic view of border irrigation illustrating the basic variables involved in the hydraulics of surface irrigation.

#### Size of Stream

The selection of a stream size is influenced to some extent by all other factors which affect the design of surface irrigation systems. The interrelationship of these factors and stream size are discussed in subsequent paragraphs of this thesis.

In selecting the maximum permissible stream size, consideration must be given to two factors; the erosiveness of the soil and the depth of flow. In no case should a stream size be used which would produce scouring in the strip or overtop the dyke. Conversely, the stream should not be so small to permit meanders to develop in the strip so as to produce non uniform coverage of the strip with water.

### Soil Intake Rate, Depth and Time of Application

The rate at which water enters the soil is one of the most important factors influencing the design of an irrigation system. The soil intake rate varies with many factors such as: soil type, the surface depth of water, the temperature of the water and soil, vegetative cover, soil moisture content and others.

Many investigators, for example, Bondurant (4), Edlefsen and Bodman (9), Free, Browning and Musgrave (11), Horton (16), Kohnke (20), Philip (24), Shull (26) and others have conducted investigations to measure and describe the intake phenomenon. The intake rate of any soil can be described mathematically as a function of time. Christiansen, Bishop and Fok (6) suggest that the functional relationship is generally assumed to follow one of three forms:

$$\begin{array}{lll} f = Kt^n & . . . . . & 1. \\ f = c + Kt^n & . . . . . & 2. \\ f = c + Ke^{-rt} & . . . . . & 3. \end{array}$$

in which  $f$  is the intake rate at anytime,  $t$ . The constants, coefficients, and exponents,  $K$ ,  $c$ ,  $n$  and  $r$  characterize the soil and are evaluated from field data.

For surface irrigation purposes, the form of the relationship given by the simple power equation (Eqn. 1) is generally accepted. This results because of the simplicity of the relationship and because it has been found experimentally that within the time required to irrigate most soils, the relationship is valid (7). The total time,  $T$ , required to replace a given depth of water,  $d$ , to the soil profile can

be calculated by integration of Eqn. 1, to obtain:

$$T = \left[ \frac{d(n+1)}{K} \right]^{\frac{1}{n+1}} \dots \dots \dots 4.$$

When intake rate is plotted with time on logarithmic paper, K, equals the intake rate intercept at unit time, and n, equals the slope of the line.

#### Rate of Advance and Length of Run

##### General

In border dyke irrigation, it is necessary to consider the rate at which a sheet of water advances down the strip and the rate at which it recedes from the strip. A plot of time versus the distance of the advancing and receding wet fronts are called advance curves and recession curves respectively. An analysis of the advance and recession curves for a border strip provides some insight of the performance of a system.

The shape of the advance curve is affected by many factors such as stream size, soil intake rate, border slope and the physical properties of the border. When a constant input is applied to a border dyke, the rate of advance slows from its initial rapid movement. As the movement of the front approaches a near stationary position, the total intake of water by the soil and the border input are approximately equal. The effect of increasing the input to a border is to increase the rate of advance and, hence, increase the distance the front moves before the effect of soil intake becomes predominant. As the slope of the border is increased the velocity of flow is increased and the time required for a wet front to travel a given distance is reduced.

The shape of the recession curve is governed by the same factors that affect the advance curve and, in addition, the depth of water in surface storage. In general, the shape of the recession curve follows the shape of a flattened S-curve. Immediately after water to the strip is turned off, the water ponded on the surface begins to move from areas adjacent to the supply canal by sheet flow and intake to the soil. On these areas, the rate of recession is very rapid. Gradually, with time, the advance of the wet front virtually stops. When this occurs, the component of surface flow in the recession process becomes negligible and the removal of surface storage occurs as soil intake. As the effect of surface flow diminishes, the rate of recession decreases. In the length of the strip in which surface water is removed primarily by soil intake the rate of recession tends to increase. The increase in rate of recession at the lower ends of the strip is explained by the fact that the depth of ponded water is very small and disappears very rapidly. In addition, the fact that various parts of the strip have been wetted for different periods of time provides that the soil intake rate along the strip increases with distance.

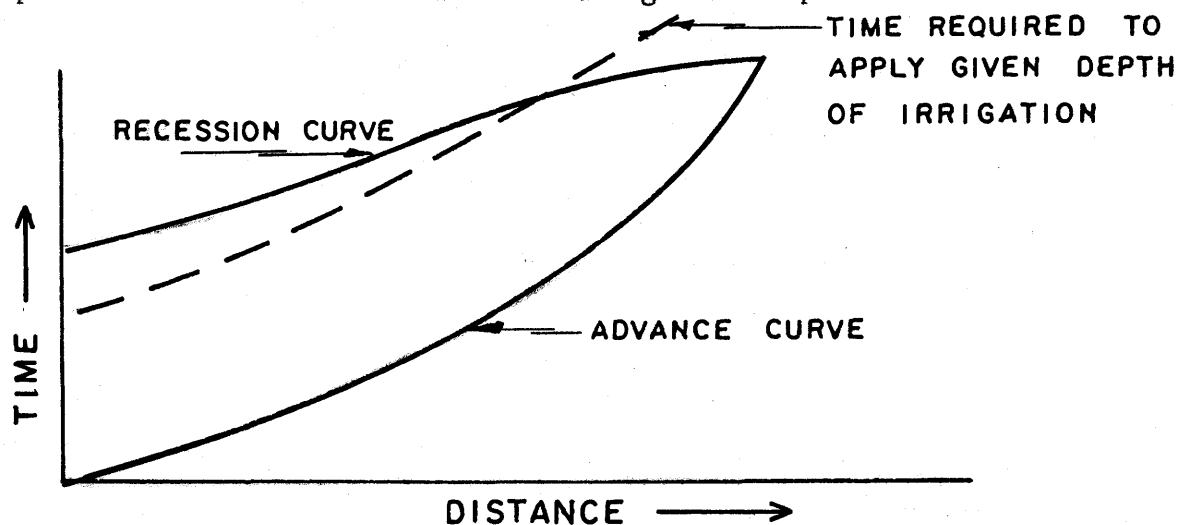


Fig. 2. Schematic advance and recession curves



Fig. 2 is a schematic of a normal set of advance and recession curves obtained from a border strip. Superimposed on the figure is a line drawn parallel to the advance curve to represent the time required to apply the depth of irrigation. To obtain an uniform application of water throughout the strip, water should cover all parts of the field for a time equal to the irrigation period. That is, the recession and advance curves should be parallel at a distance apart equal to the irrigation time. If this condition is fulfilled the infiltration opportunity time at all points along the strip is the same. If the line defining the irrigation time falls above the recession curve, this represents a case in which insufficient water has been added to fulfil the irrigation requirements. If the "irrigation time" curve falls below the recession curve, water will be lost to deep percolation and the efficiency of the system will be reduced.

Practically, however, it is almost impossible to obtain a completely uniform depth of water application through the total length of strip. As the influence of the soil intake rate increases it tends to steepen the advance curve and flatten the recession curve. However, even though the times between the recession and advance curves become shorter at the lower end of the strip, the effect on irrigation efficiency may not be severe. This results because the soil intake rate varies as some power of time. Hence, a given reduction in infiltration opportunity time does not cause a proportional decrease in the depth of water applied. To prevent excessive runoff the inflow must be stopped before the advancing water front has reached the lower end of the field and for reasonably uniform application this point of cutoff must occur

before the advance curve becomes too steep. The time of water cutoff sets the original spacing between the curves and thereby establishes the depth of water application.

#### Mathematical relationships for length of run

The design of border dyke systems would be greatly enhanced if the advance curve could be expressed mathematically. Such an expression would permit the engineer to estimate the performance of a system prior to its construction. Of necessity, the development of this relationship would also require a basic understanding of the principal factors which affect the phenomenon. Efforts in research then could be devoted to obtaining experimental values for the coefficients needed to solve the basic equations. In this aspect, certain areas, for example, soil types, may be grouped or characterized for irrigation on the basis of the values of the coefficients.

The rate of advance of wetted front in a border strip has been found to follow the relationship,

$$t = CD^m \quad \dots \dots \dots 5.$$

in which

t = time,

C = coefficient,

D = distance, and

m = exponent.

A given coefficient, C, and exponent, m, are valid to describe the curve throughout most of the advance. However, as the curve approaches a static position, that is, where input equals intake to soil and the advance is very slow, the value of these constants change.

Hansen (15) reported two approaches for determination of rate of advance:

1. Hydrodynamic approach, and
2. Intake related to rate of advance.

In the hydrodynamic approach, the expression for rate of advance can be developed from the shape of water surface profile. The design and prediction of the behaviour of the system can also be made from the soil intake rate. A summary of some of the more recent works to describe the rate of advance curves mathematically is given in subsequent paragraphs.

Lewis and Milne

In 1938, Lewis and Milne (21), presented an expression to define the rate of advance of a wet front in a border strip as a function of time, depth of water in the strip, input and soil intake rate. In their analysis, they assumed that the soil intake rate varied exponentially with time. Thus the depth of water which had penetrated into the soil was calculated as,

$$d = c(1 - e^{-rt}) \quad \dots \dots \dots 6.$$

where  $d$  = depth of water penetrated  
to the soil at a given point,

$c$  = final intake rate,

$r$  = constant, and

$t$  = time.

The equation for the rate of advance of the wet front is given as,

$$D = \frac{q}{Lr(c+y)} \left\{ t - \frac{1}{K} (1 - e^{-Kt}) \right\} \dots \dots \dots 7.$$

where  $D$  = length of advance,

$q$  = inflow to the strip

$L$  = width of the strip, and

$y$  = depth of flow (assumed constant)

$K = \frac{r(c+y)}{y}$  and,

$e$  = base of natural logarithms.

In their works, the authors also presented a second equation for advance based on the assumption that,

$$d = at + c(1 - e^{-rt}) \quad \dots \dots \dots 8.$$

With the depth of penetration defined in this manner, the equation of advance becomes:

$$D = \frac{q}{La} \left[ 1 - e^{-\beta t} \cosh \xi t + \left( \frac{a}{y\xi} - \frac{\beta}{\xi} \right) e^{-\beta t} \sinh \xi t \right] \dots \dots 9.$$

where  $a$  = coefficient,

$$\beta = \frac{a + cr + yr}{2y}, \text{ and}$$

$$\xi = \frac{\sqrt{(a + cr + yr)^2 - 4ary}}{2y}$$

The works of the authors, although highly creditable, have not been widely accepted. Several factors have contributed to the limited use of these equations. Needless to say, the mathematical complexities of the equations are probably among the most important of these factors. Further, the assumed forms of the functional relationship between soil intake rate and time and the assumption of a constant depth of flow throughout the front could be seriously questioned. In addition, the

solution of the equations demand prior information of the soil intake rate and surface storage. Further, the influence of surface roughness and slope on the rate of advance are not considered.

### Israelsen

Israelsen (17) presented a simplified approach to border analysis. He derived a mathematical expression for the time required to cover a given area as a function of the average surface depth of water, the soil intake rate, input and wetted area. The expression given by Israelsen is:

$$t = 2.303 \frac{y}{f} \log \frac{q}{q-fA} \dots\dots\dots 10.$$

in which:

A = area covered with water at anytime, t,

f = intake rate,

q = input rate,

t = time after water was turned onto the  
land, and

y = average depth of water flowing over  
the land.

The principal assumptions underlying the development of the equation were that the soil intake rate and surface storage depth remained constant with time. Inasmuch as these assumptions were invalid, calculations made using expressions as Eqn. 10 can be in gross error. For this reason, the work has not gained wide acceptance.

Hall

In 1956, Hall presented a simple, numerical method of predicting the rate of advance of water in a border check. Essentially the method used to obtain the advance of the water is a numerical integration to satisfy the law of conservation of matter. That is, the quantity of water flowing into the check during any time increment is equaled to increments of storage produced in the form of intake and surface storage. The equation of advance given by Hall is:

$$\Delta x_i = \frac{Q\Delta t - (a_1\Delta x_i + \dots + a_n\Delta x_{i-2})}{b(kd_1 + cy_0 + e)} \dots \dots \dots 11.$$

in which,

$\Delta x_i$  = increment of advance,

$Q$  = input rate,

$b$  = width of border,

$\Delta t$  = time increment to travel  $\Delta x_i$ ,

$a_1, a_2$  = factors determined from intake  
properties of the soil,

$k, c$  = geometric constants,

$d_1$  = depth of intake at end of first  
time increment,

$y_0$  = normal flow depth, and

$e$  = depth correction factor.

For  $\Delta x_1$  the solution is simply,

$$\Delta x_1 = \frac{Q\Delta t}{b(kd_1 + cy_0 + e)} \dots \dots \dots 12.$$

The value of  $\Delta x_1$  is obtained from Eqn. 12 and substituted into Eqn. 11 for  $i = 2$  to obtain  $\Delta x_2$ . This value is substituted in turn in Eqn. 11 for  $i = 3$  to obtain  $\Delta x_3$ .

To apply the relationship given by Hall requires information concerning the soil intake rate, the unevenness of the border strip and some estimate of the surface roughness. As pointed out by the author, the main advantage of the method is its simplicity and the advantage that the necessary calculation can be completed very rapidly.

#### Christensen, Bishop and Fok

Like Israelsen, Christensen, Bishop and Fok (6) presented a direct approach to relate the soil intake rate to the rate of advance of a wet front without considering the shape of the free water surface. In their analysis, the average depth of water on the surface is assumed constant throughout the strip. The length of run,  $D$ , in time,  $T$ , is given by the expression,

$$D = \frac{qT}{d_a + y_s} = \frac{\frac{qT}{KT^{n+1}}}{(n+1)(n+2)} + y_s \quad \dots \dots \dots 13.$$

in which,

$q$  = input to the strip,

$d_a$  = average depth of water intake,

$y_s$  = equivalent average depth of

water on the surface, and

$K, n$  = coefficient and exponent of intake equation respectively.

Equation 13 is, in reality, the continuity equation of flow in a strip of unit width. An interesting feature of the equation is the term  $d_a$ , the average depth of water infiltrated throughout the strip. This depth is determined by integrating the accumulative depth equation (see Eqn. 4) and dividing by the time. In actuality the depth thus obtained is only an approximation of the average depth of water absorbed over the length of the strip. Theoretically, the equation will give the average depth only when the rate of advance is a linear function of time.

Although the validity of Eqn. 13 may be questioned because of the simplified assumptions used in its development, the authors report several cases in which it has been successfully applied to field data.

#### Tinney and Basset

Tinney and Basset (27) studied the movement of a shallow liquid front over an impervious bed. They reported that the shape of a gradually tapering two dimensional front for laminar flow can be defined by the equation,

$$\frac{z}{y_0} \sin \theta = \tanh^{-1} \left( \frac{y}{y_0} \right) - \frac{y}{y_0} \dots \dots \dots 14.$$

where,

$y_0$  = normal depth of flow,

$y$  = depth of flow in the front,

$z$  = coordinate of measurement of  $y_0$

with the origin at the tip of

the front, and

$\sin \theta$  = slope of the bed.



For turbulent flow, the shape of the front is defined by an equation of the form,

$$\frac{\Sigma}{y_0} \sin \theta \left( \frac{n_m}{n_0} \right)^b = \tanh^{-1} \left( \frac{y}{y_0} \right) - \frac{y}{y_0} \dots \dots \dots 15.$$

in which,

$n_m$  = Manning's "n", and

$n_0$  = Manning's n for a particular  
turbulent profile ( $n_0 = 0.02$ ).

In reviewing Eqns. 14 and 15 it is obvious that the terminal shape of the wet front, under laminar flow conditions, is a function only of normal depth. For turbulent flow, the shape is also influenced by the relative roughness of the channel bed.

The authors also noted that when a liquid is introduced slowly into a channel, it first decelerates and acquires a long, gradually tapering profile. Thereafter, it advances at a constant velocity equal to the average velocity in the upstream uniform section. That is, the advancing stream of water over an impervious bed can be considered as consisting of two phases,

- (1) An initial section of unsteady, non-uniform flow  
which is gradually transformed to,
- (2) An essentially steady state flow for slopes greater  
than zero.

The authors submit that the work is preliminary and the findings reported may be of considerable importance for hydraulic analysis in future investigations but are not expected to have field application at this time.



### Slope, Depth of Flow and Surface Roughness

The flow in border dyke systems represents a case of unsteady state, non uniform overland flow. Overland flow may range from purely laminar for small detention depths to purely turbulent over smooth slopes.

Theoretical and empirical considerations of the overland flow regime in surface runoff studies indicate flow equations of the form,

$$q = By^aS^b \dots\dots\dots 16.$$

where,

$q$  = discharge rate per unit width,

$y$  = depth of flow,

$S$  = slope of the land, and

$B, a$  and  $b$  = coefficient and exponents respectively.

The magnitude of the coefficient,  $B$ , a retardance coefficient, and the exponents,  $a$  and  $b$ , vary widely according to roughness of the land surface, soil intake rate and the Reynolds Number. As pointed out by Izzard (19), the retardance coefficient varies inversely with surface roughness. When laminar flow conditions exist throughout the flow distance the values for  $a$  and  $b$  will be 3.0 and 1.0 respectively. In cases where the flow is turbulent for part of the distance these values will be reduced. For completely turbulent flow the values for  $a$  and  $b$  are 1.67 and 0.5 respectively.

Bowman (5) found that shallow flow of irrigation water through vegetation is either turbulent or transitional, the stage between laminar and turbulent. He suggested that the form of flow equation given by Eqn. 16 is applicable for flow in surface irrigation systems.

Bartels (1) and Monson (23) have successfully applied Manning's equation to characterize surface irrigation flow. Manning's equation is Eqn. 16 with  $B = 1.49/n_m$ , in which  $n_m$  is a roughness factor. These investigators found that the roughness factor,  $n_m$  varied inversely with the velocity of flow, in addition to surface roughness.

The above discussions suggest that the form of equation given by Eqn. 16 is generally accepted to describe the flow in surface irrigation systems. According to the equation, the velocity of flow varies inversely with surface roughness and directly as some power of the depth of flow and slope. In that the shape of the advance and recession curves are directly related to the velocity of flow it follows that the slopes of these curves would decrease with a decrease in surface roughness or an increase in input or slope. Further, the equation suggests the higher the input the greater the depth of flow.

#### Shape of the Flow Channel and Erosion Hazards

For channel flow, because the flow rate varies as the hydraulic radius; the ratio of the flow area to the wetted perimeter, the shape of the channel is very important. Hence in furrow and corrugations this factor is an important consideration. However, in border dyke systems, the ratio of depth of flow to the width of the dyke is generally very small and changes in depth of flow produce but small changes in the hydraulic radius. For these systems, only slight error is introduced to the calculated velocities if it is assumed the depth of flow is equal to the hydraulic radius. In general, because border dykes are placed on relatively wide spacings and the cross slopes are small, the shape of the flow channel is not considered to be an important factor affecting the flow in these systems.

It is difficult to evaluate quantitatively the erosional potential of a soil. The eroding and transporting power of sheet flow are functions of the depth and velocity of flow for a given size, shape and density of soil particle or aggregate in given condition. Generally, the erosional hazard of a soil is classified in accordance to its ability to resist a given velocity of flow, the maximum permissible velocity. Fortier and Scobey (10) suggest the following maximum velocities for use in open ditch design (see Table 1).

Table 1. Allowable velocities for open ditches

Soil type	Maximum velocity clear water (fps)
Fine Sands and Sandy Loam	1.5
Silt Loam, Loams, non Colloidal Sediment	2.0
Colloidal Clay and Fine Gravel	2.5
Coarse Gravel, Shales, Hardpans	4.0

Ree (25) in his work in Oklahoma found the velocity of flow in vegetated watercourses varied with the type of vegetation and bed slope. Table 2 summarizes the permissible velocities for the more common grass species as suggested by Ree.

Although the permissible velocities given in Tables 1 and 2 are average velocities to be applied to the design of open channels, they do provide an index of the relative erosiveness of soils. In design of surface irrigation systems, if experimental data concerning the erosion of soil are not available, calculations of the expected velocities for

Table 2. Permissible velocities for channels lined with vegetation

	Permissible Velocity, fps					
	Erosion Resistant Soils			Easily Eroded Soils		
	% Slope			% Slope		
	0-5	5-10	>10	0-5	5-10	>10
Bermuda Grass	8	7	6	6	5	4
Kentucky Bluegrass	7	6	5	5	4	3
Smooth Brome						
Alfalfa	3.5			2.5		
Crab grass						
Annuals for Temporary Protection	3.5			2.5		

the given input should be calculated by Manning's Equation (see Eqn. 16). The calculated velocity should be compared with tabulated results, and, if there is doubt about the erosion hazard the input should be reduced.

#### Advantages and Disadvantages of Border Dyke Irrigation

Border dyke irrigation is an excellent method of irrigating lands which have been properly levelled and prepared. Under these conditions, the method will provide reasonably high water application efficiencies, the labor requirements for operation of the system are low and, large water streams can be used safely. Other advantages to be realized are that the method provides for undisturbed farming operations and it is adaptable for irrigating most close growing crops.

As pointed out in the preceding paragraph, this method of irrigation is only efficient when the land is well prepared. This factor limits its use to fairly level topography. On rough topography, the cost

of land preparation may be so high to render the system economically unfeasible. In addition, the fact that the land must be prepared restricts the method to deep soils. It may be impractical to use this method on light textured soils because the lengths of run must be short.

## OBJECTIVES

The primary objectives of the study were:

1. To investigate the performance of some existing border dyke systems in Saskatchewan,
2. To obtain information which could be used as design criteria for future installations, and
3. To study the hydraulics of flow in border dyke systems and, where possible, to establish relationships between pertinent variables and the flow phenomenon.

## INVESTIGATION, RESULTS AND DISCUSSION

### Field Measurements

During the summer of 1962 and 1963, several field tests were conducted on border dyke systems located at Saskatoon, Outlook and Battleford. At these sites, a total of seven border strips were investigated. Each border strip differed from any other in either its length, slope or width. A summary of the pertinent dimensions of each border on which tests were conducted is given in Table 3.

Table 3. Physical properties of border strips

Border Property	Outlook		Saskatoon			Battleford	
	1	2	1	2	3	1	2
Length (ft)	650	650	600	475	475	800	800
Width (ft)	30	30	33	33	33	27	27
Slope (%)	0.27	0.52	1.97	2.20	2.30	0.80	0.80

All strips were cropped to an established forage crop of either alfalfa or alfalfa-brome grass mixture.

### Field Tests

In total, seventeen irrigation trials were conducted. The differences between individual field trials on a given strip were that different inputs were used in each test. A summary of these inputs is given in Table 4.



Table 4. Summary of input rates applied to different border strips

Location	Border No.	Input Imp. gpm
Outlook	1	105
		157
		160
		315
	2	110
		165
		168
		370
Saskatoon	1	183
		262
	2	154
		237
	3	212
Battleford	1	187
		208
	2	71
		96

In conducting the field trials, a constant input was applied to the border, and the following observations were made:

1. Input,
2. Rate of advance,
3. Change of surface storage, and
4. Rate of recession

### Measurement of Input

In tests conducted at Battleford and Saskatoon, input to the strips was measured by passing water through 3-inch Parshall flume. The flume was equipped with a Stevens Type F water level recorder to provide a continuous record of input for the test period.

At Outlook, water was introduced to the border strips through a 9-inch square box turnout, which was fitted with a metal gate. The discharge characteristics of these turnouts for various heads and gate openings, were determined in the hydraulics laboratory prior to their placement in the field. Measurements of the head and gate openings were taken at random times during the test run. These readings were converted to discharge readings by using the calibration chart to obtain an average discharge for the test.

### Rate of Advance

The rate of advance of the wet front was observed by recording the time required for the wetting front to travel a given distance. In most tests, the strip was staked to 50 ft or 100 ft stations to facilitate this measurement.

### Surface Storage Measurements

The rate of ponding or accumulation of surface storage with time was measured at different stations during each test run. In 1962, these observations were obtained by measuring the depth to the surface of the water sheet from a fixed height with a scale at random time during the period of test. This method of measuring storage although it provides reasonably accurate results, was very time consuming to perform. To

facilitate this measurement in subsequent tests conducted in 1963, a simple gaging well was constructed.

The gaging well was constructed from a  $1\frac{1}{2}$ -inch diameter steel tube which was perforated at the bottom (see Fig. 3). A float assembly consisting of a wax-treated cork fitted with a thin copper wire rod marked in inches, was used to indicate the water level in the well.

Prior to conducting the trials, several gaging wells were installed at different stations in the border dyke. Then, the wells were filled with water to the ground level, and the zero gauge readings for each station was noted.

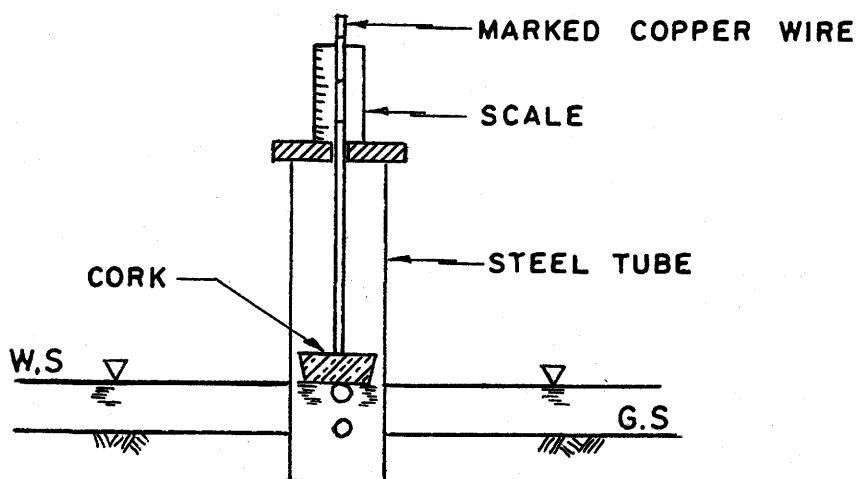


Fig. 3 Gaging well for storage measurements

#### Recession Curve

On completion of the rate of advance test, the water supply to the strip was turned off, and the rate at which the sheet of water receded from the strip was observed and recorded. In both the advance and recession tests, if the shape of the wet front was very irregular

at a given station, an average time for each of the advance and the recession was recorded.

#### Soil Moisture Measurements

In several tests, an attempt was made to study the application efficiency of the border system. For this purpose, soil moisture measurements were taken at 50-ft or 100-ft stations along the strip before and after the completion of each test. The soil moisture measurements were made with the neutron moisture meter. Wherever possible these observations were taken at one-foot increments of depth to a depth of six feet.

#### Rate of Advance and Recession

The results of the field investigations pertaining to the rates of advance and recession tests are presented in Figs. 4 to 7. A discussion of the curves obtained at the individual stations is presented below.

#### Outlook Border Strip Tests

Essentially three inflow rates were used in the field tests on each of the Outlook border strips. On border 1, which has a slope of 0.27 percent, the low input rate (105 gpm) produced a steep advance curve which effectively limits the length of run to approximately 400 ft (see Fig. 4). Over this distance, the depth of intake is fairly constant at about 3 in but beyond 400 ft it decreases rapidly<sup>1/</sup>. If the water inflow had been stopped in time to prevent the movement of water beyond 400 ft the depth of water applied would have been unreasonably small. The two

<sup>1/</sup> Soil intake estimates are calculated from soil intake rate equations which are developed later in this thesis.

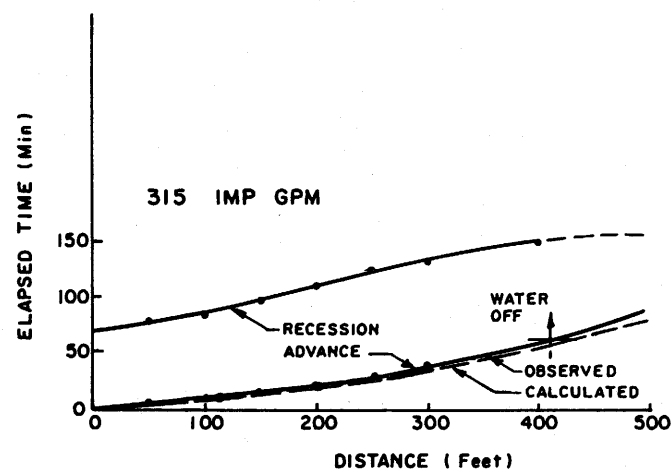
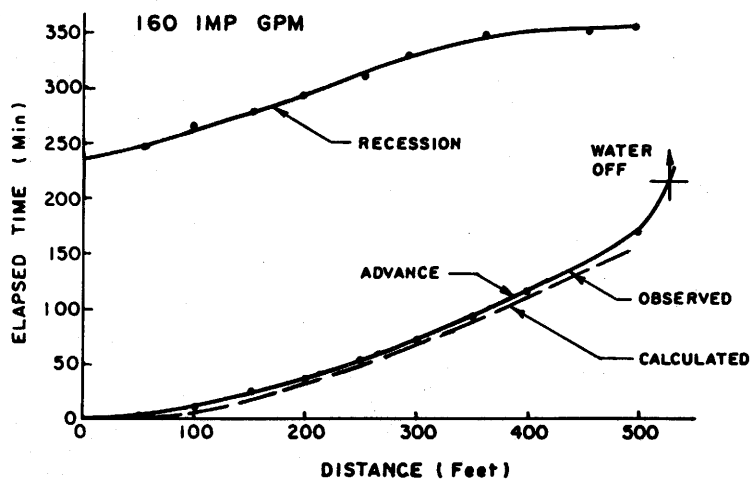
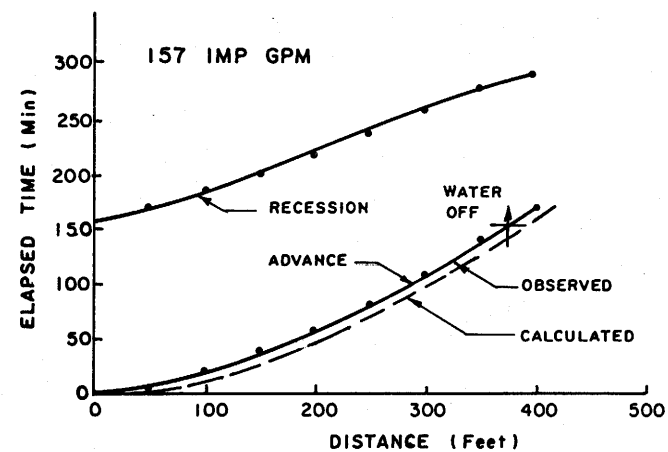
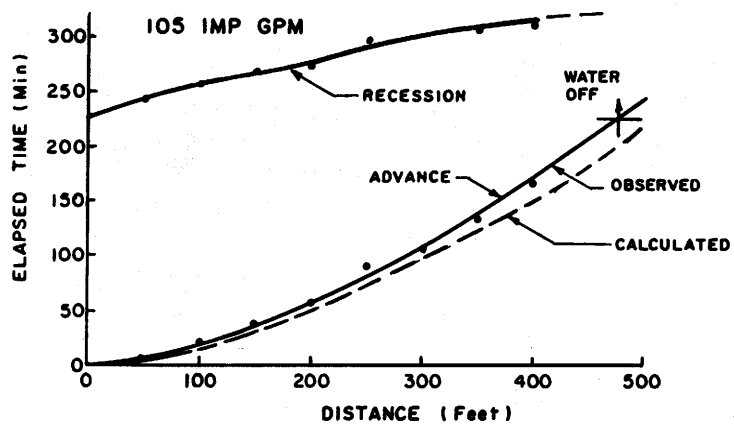


FIG. 4 RATE OF ADVANCE AND RECESSION CURVES FOR BORDER STRIP No. 1, OUTLOOK.

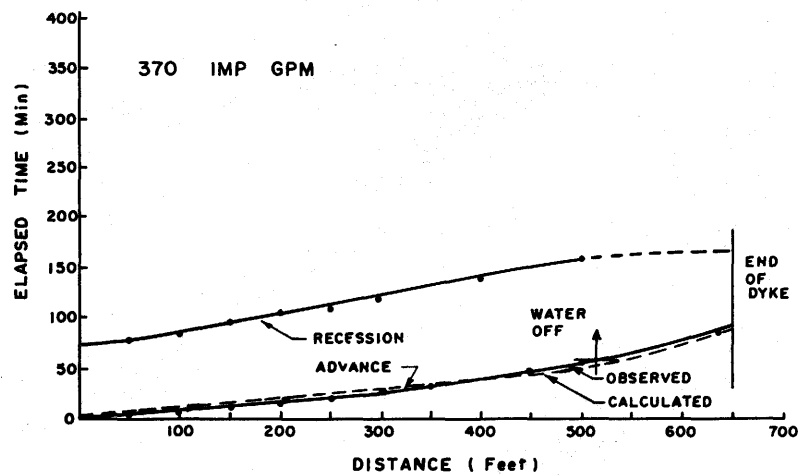
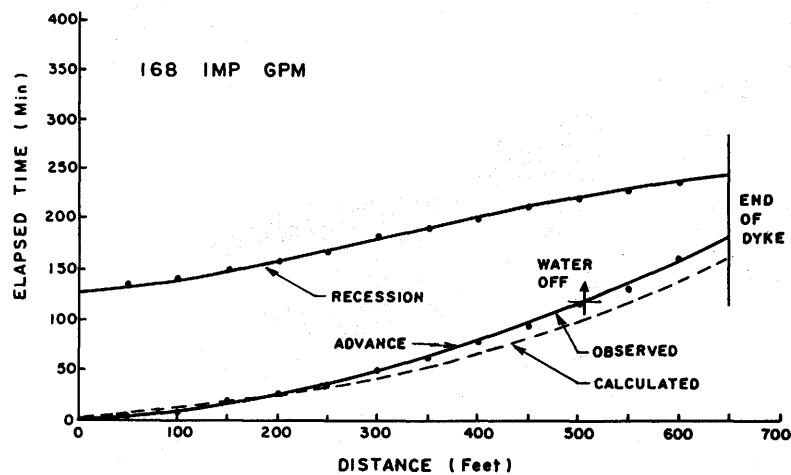
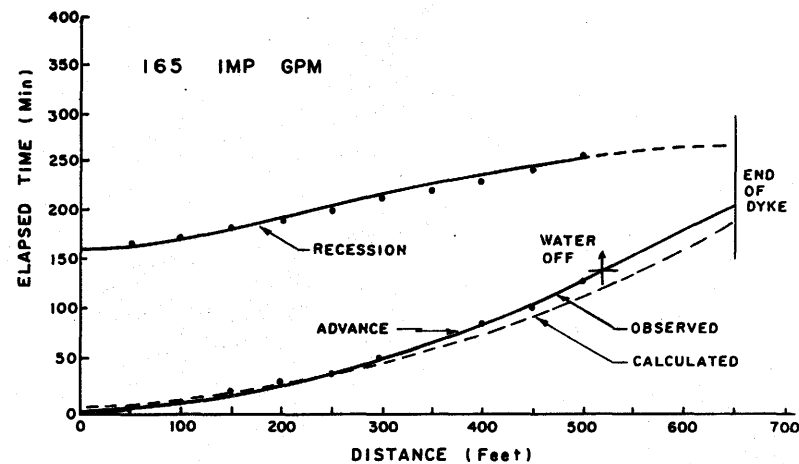
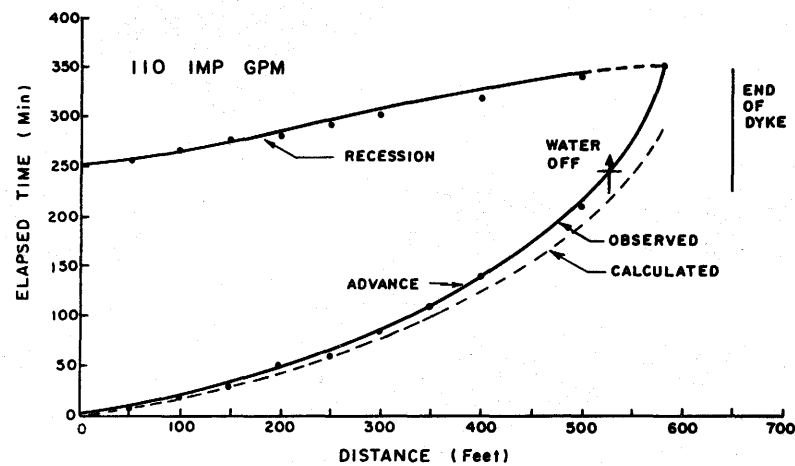


FIG. 5 RATE OF ADVANCE AND RECESSION CURVES FOR BORDER STRIP No. 2, OUTLOOK.

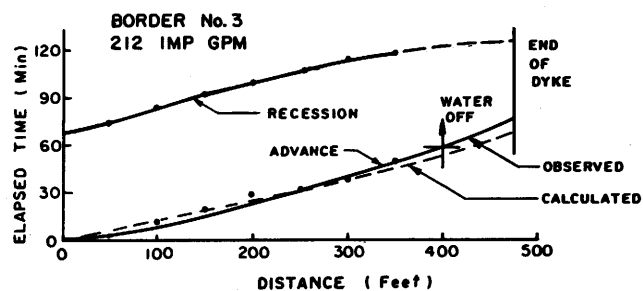
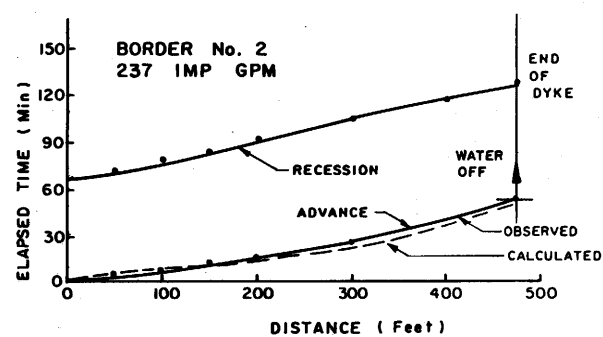
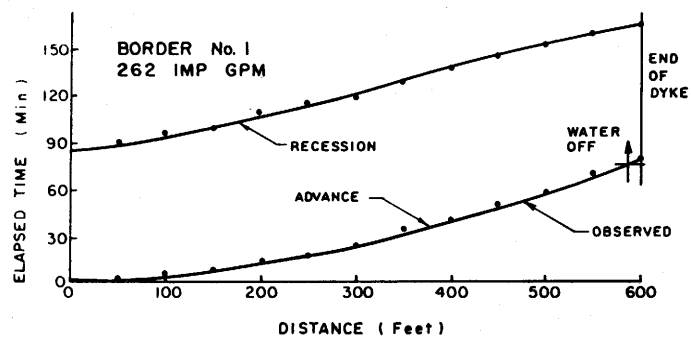
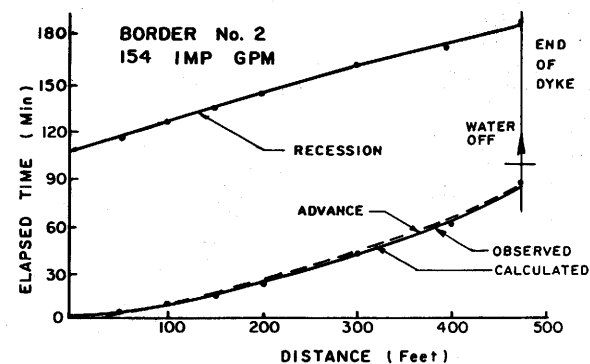
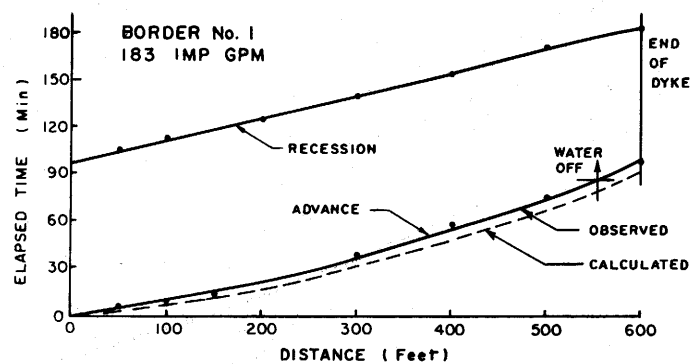


FIG. 6 RATE OF ADVANCE AND RECESSION CURVES FOR BORDER STRIPS, SASKATOON.

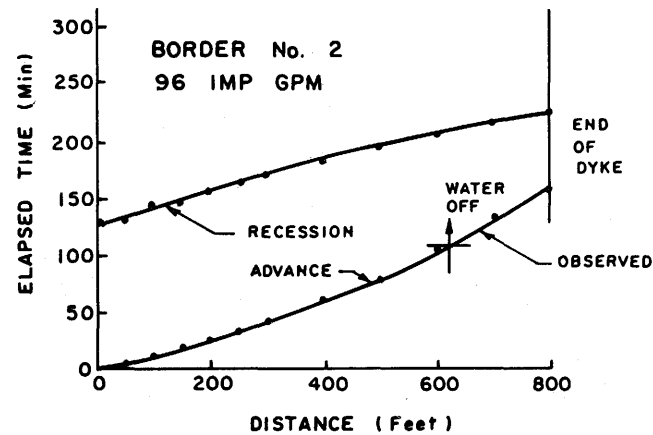
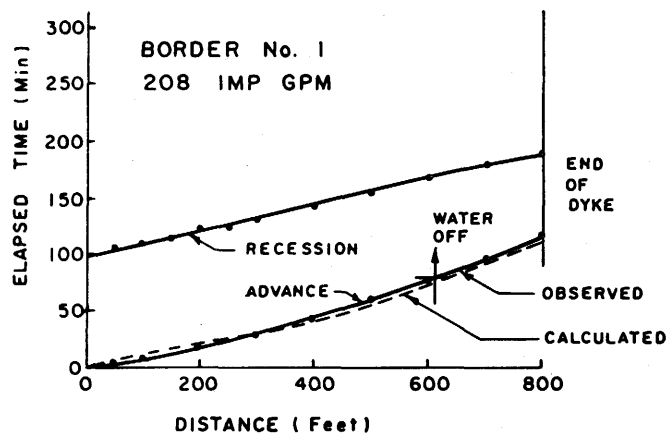
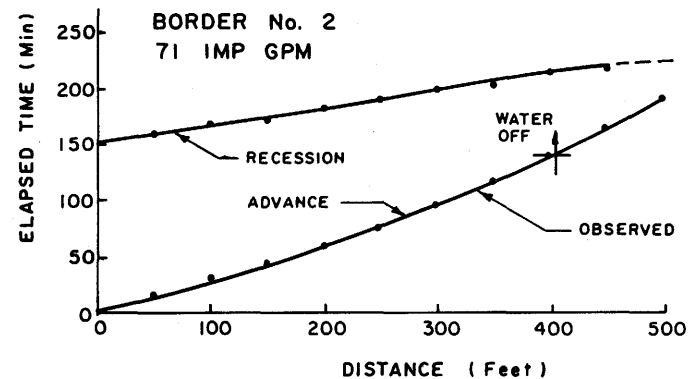
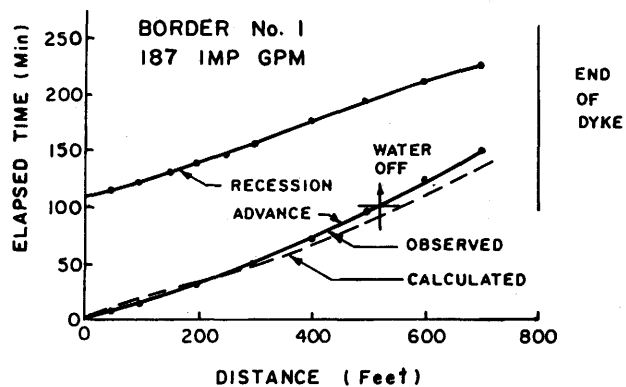


FIG. 7 RATE OF ADVANCE AND RECESSION CURVES FOR BORDER STRIPS, BATTLEFORD.



tests with intermediate flow rates around 160 gpm produce a flatter advance curve and a uniform application near 4 in to distances of 500 ft. Beyond this distance the application would decrease rapidly. The test using an input rate of 315 gpm gave a very uniform but low application over a distance of 500 ft and the length of field could have been extended, probably to 600 ft, without seriously affecting the uniformity of application to give a total intake near 3 in.

On border No. 2, where the slope is 0.52 percent and the soil type<sup>1/</sup> similar to that encountered on border 1, the low input rate (110 gpm) again gave a steep advance curve which limits the length of field to about 400 ft (see Fig. 5). Had the input been stopped in time to prevent runoff beyond this distance, a low application would again have resulted. The intermediate flow rates (165 gpm) while producing a relatively uniform application over a field length of 500 ft, but not beyond, gave a water application of only about 2 in. The high flow rate (370 gpm) gave a uniform, low application of 1.6 in over the 650-ft length and it is probable that a somewhat longer field might be used to give a larger application.

For silt loam soils, as those encountered at Outlook, it is apparent that flow rates of approximately 100 gpm to a 33-ft border will give uniform applications of about 2 in to 3 in on field lengths up to 400 ft and that the slope of the border if it varies only between reasonable limits, will have little influence on the uniformity of application. An input rate of 160 gpm will give uniform application of water on border lengths of 500 ft but the depth applied will decrease from about 4 in to 2 in as the slope increases from 1/4 to 1/2 percent. Higher flow rates of 300 gpm

<sup>1/</sup> The textural composition of the soils encountered on each border strip are given in Appendix C.

or more may be used on field lengths of 600 ft or more where application rates of less than 2 in are acceptable.

### Efficiency Calculations

One method of evaluating the performance of an irrigation system is to evaluate its water application efficiency. The water application efficiency is the ratio of the volume of water stored in the crop root zone to the volume of water applied, expressed as a percent. The water application efficiencies for several tests conducted at Outlook are tabulated in Table 5.

Table 5. Water application efficiencies for different inflows on Borders 1 and 2, Outlook.

Border No.	Input (gpm)	Border Length Considered (ft)	Applied <sup>a</sup> Depth (in)	Soil <sup>b</sup> Moisture Stored (in)	Consumptive <sup>c</sup> Use (in)	Total Stored (in)	Water Application Efficiency (%)
1	110	580	4.94	3.44	0.80	4.24	86
	165	650	2.33	1.46	0.64	2.10	90
	370	640	2.62	0.95	0.80	1.75	67
2	105	500	5.02	3.57	0.80	4.37	87
	315	500	2.60	0.38	0.80	1.18	46

<sup>a</sup>/ Average depth of flow over border length considered computed from inflow measurements plus rainfall

<sup>b</sup>/ Depth of moisture stored in a 4-ft zone calculated from soil moisture readings

<sup>c</sup>/ Estimated from bellani plate readings and by the Thornthwaite method

For the tests using both intermediate and low flows the efficiency is about 90 percent. Such high efficiencies are to be expected as long as the flow rates do not produce excessive runoff and if the tests are not extended in time so as to permit deep percolation losses. The fact that inputs in excess of 300 gpm resulted in low efficiencies is indicative of

the fact that runoff did occur in these tests.

#### Saskatoon Border Strip Tests

All of the borders in the Saskatoon tests were on a slope of approximately two percent and had a maximum length of run of 600 ft or less. The input rates to the borders varied from 154 gpm to 237 gpm and because of the steep slopes and relatively low infiltration rate they may all be considered as representing a high input rate. In all cases, the depths of water application over the available border length are very uniform (see Fig. 6). In all tests the actual water intake was extremely low, varying from 1/2 in to 1.3 in and even lower intakes would have occurred had water been shut off early enough to prevent runoff. It is evident that input rates in the range of the tests would have produced uniform depths of water applications on longer lengths of run than were available and this would lead to somewhat larger intakes. The limit to which length of run could be extended without affecting uniformity of application is uncertain. Theoretically, a lower input rate would cause a slower advance, and hence permit a greater water intake. However, on slopes as steep as those under test, it is anticipated that it would be difficult to maintain coverage across the border width with inputs much lower than those used.

Generally speaking, it will be difficult or impossible to apply normal irrigation amounts, 3 in to 4 in at one irrigation, with a reasonable application efficiency to steep borders containing soils having relatively low intake rates, similar to those tested at Saskatoon. When border dyke irrigation is used under such conditions minimum lengths of run in excess of 700 ft appear to be essential. In addition the lowest

practical flow rates will be necessary and this factor emphasizes the need for precise land preparation on such steep slopes. The above discussion applies to situations where the entire irrigation is to be applied by a constant input. Perhaps, under the given conditions the use of a cut back stream would enable efficient irrigations of shorter lengths of run.

#### Battleford Border Strip Tests

The border strips tested at Battleford were 800 ft in length and sloped at 0.80 percent. The soils at this location were significantly heavier textured than those soils encountered at Saskatoon or Outlook, containing up to 30 percent clay and have therefore much lower infiltration rates. Border inputs used ranged from 70 gpm to 208 gpm. The lower input rate was insufficient to give complete coverage and this resulted in the rapid recession for that test (see Fig. 7). The input rate of 96 gpm is probably close to the minimum practical flow rate and the border length of 800 ft is apparently too long for this inflow. With an input of 96 gpm, the water intake was quite uniform at 1.5 in for the first 600 ft and decreased to less than 1 in at 800 ft. The use of larger streams, near 200 gpm gave uniform applications of 1 inch or less for the full border length.

To irrigate soils having such low intake rates on slopes of  $3/4$  percent or larger it is obviously impossible to obtain uniform application with a normal application depth of 3 in or 4 in. Lengths of run under such conditions should be limited to about 600 ft and minimum practical inflows should be used to obtain maximum application depth. Larger inputs may be used on border strips of 800 ft or more where light applications of

water are acceptable. It is apparent that these heavy soils can only be irrigated effectively on relatively flat slopes, probably less than 1/2 percent, where the advance rate will be slowed and where low flow rates will still provide complete coverage behind the advance front. As discussed previously, it may be advisable to investigate the feasibility of using a cutback stream to irrigate these soils.

#### Evaluation of Constants of the Advance Curve

As shown by other investigators, the advance curve can be described mathematically by a simple power equation as given by Eqn. 5. The least squares estimators of the coefficient,  $C$ , and the exponent,  $m$ , were obtained from the experimental data by a regression analysis. These values are summarized in Table 6.

It is difficult to attach much practical significance to the values of the coefficients. However, the magnitudes of the exponents do provide some insight of the advance characteristics. As the value of the exponent decreases, the slope of the advance curve becomes flatter and the time required for the wet front to traverse a given distance decreases.

In comparing the values of the exponent,  $m$ , given in Table 6 the results point out that,

1. For a given border strip, the rate of advance of a wet front increases with an increase in input and,
2. At a given location, the rate of advance of a wet front increases with an increase in the slope of the strip.

Table 6. Physical and mathematical properties of the advance curves

Location	Border No.	Input Imp. GPM	Slope %	MC	Advance Curve	
					Coefficient <sup>a</sup> (C)	Exponent (m)
Outlook	1	105	0.27	12.9	0.026	1.46
		157		--	0.015	1.57
		160		18.1	0.013	1.52
		315		16.9	0.031	1.25
	2	110	0.52	13.1	0.047	1.32
		165		19.0	0.022	1.37
		168		14.9	0.035	1.29
		370		15.2	0.073	1.06
Saskatoon	1	183	2.0	28.5	0.030	1.25
		262		32.9	0.040	1.15
	2	154	2.20	9.9	0.028	1.27
		237		11.7	0.081	1.04
	3	212	2.30	--	0.037	1.24
Battleford	1	187	0.80	--	0.116	1.08
		208		20.7	0.049	1.16
	2	71	0.80	--	0.254	1.04
		96		--	0.08	1.12

<sup>a</sup>/The units on coefficient, C, give advance in min. with D in ft

Because the amount of data available is limited no attempt was made to relate the values of the coefficients or exponents to some physical characteristics of the area. Gray and Beer (13) have shown that several factors other than input, slope and soil intake, for example, soil moisture content, degree of soil cracking, surface sealing and compaction influence the advance of a wet front in furrows. Similarly, it might be assumed that these same factors would influence the advance in

border dyke systems.

Fig. 8 shows the values of the coefficient,  $C$ , and exponent,  $m$ , plotted with discharge for the data collected at Saskatoon and Outlook. This plot is given to serve as an aid in selecting a combination of these constants for use in system design at sites of comparable conditions under which the tests were conducted. No explanation can be given to account for the parabolic shape of the curves. A similar observation was noted by Beer (2) in plotting the same characteristics obtained from furrow irrigation systems.

#### Mass Storage Curves

Inasmuch as the rate of advance of a wet front and soil intake rate can be expressed as a function of time, it is logical to assume that the accumulation of sheet flow or surface storage could also be expressed as some function of time. The mass curves for different test runs are plotted in Figs. 9 to 11. The following procedure was used to develop the curves,

1. From the observations of surface storage taken at stations along the border strip, plottings were made of the longitudinal profiles of the wet front at various times after water had been introduced to the strip (see Figs. B-1 to B-4). The area enclosed by each curve represents the volume of water per unit width which has accumulated on the strip up to the given time.
2. The areas under each profile were determined by planimetry and the mass curves were plotted as shown in Figs. 9 to 11.

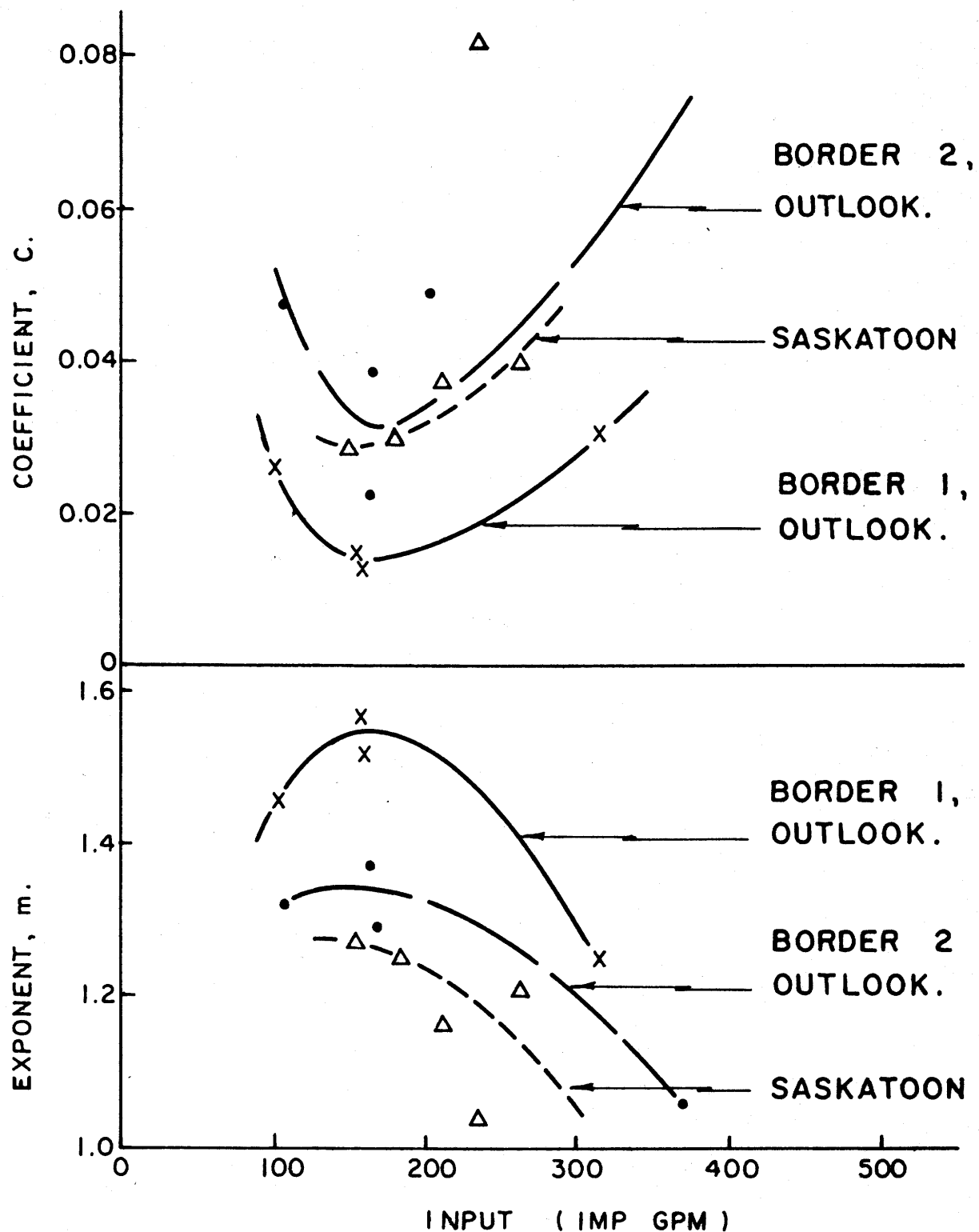


FIG. 8 VARIATION IN COEFFICIENT, C, AND EXPONENT, m, FROM RATE OF ADVANCE  $t = C D^m$  FOR DIFFERENT INPUTS.



## Discussion

No attempt is made in this thesis to give a complete discussion concerning the properties of the mass storage curves and those of the longitudinal profiles of the wet front. These data are supplemental to the current study. Essentially the data are listed for two reasons, (a) the mass curves are used later in the study to develop soil intake rate equations and (b) to report the data in hopes they will precipitate additional study and analyses.

Before passing, there are several pertinent comments which can be made from direct observations of the curves,

1. The rate of accumulation of surface storage diminishes with time to a very low rate. This characteristic is very pronounced on the mass curves developed for the low inputs. Probably, it represents the condition which evolves when the advance of the wet front essentially ceases. Any additional accumulation of storage and advance of the wet front after this time is dependent on the rate of decrease in the soil intake rate.
2. On a given strip, as the rate of input is increased, the rate of accumulation of storage increases. This characteristic can be explained by considering the hydraulics of flow. In prismatic channels, the depth of flow and velocity of flow must increase with an increase in discharge (see Eqn. 16).
3. The assumption that the depth of surface storage is constant on a strip, as used in many of the mathematical expressions for rate of advance, does not apply for short lengths of run,

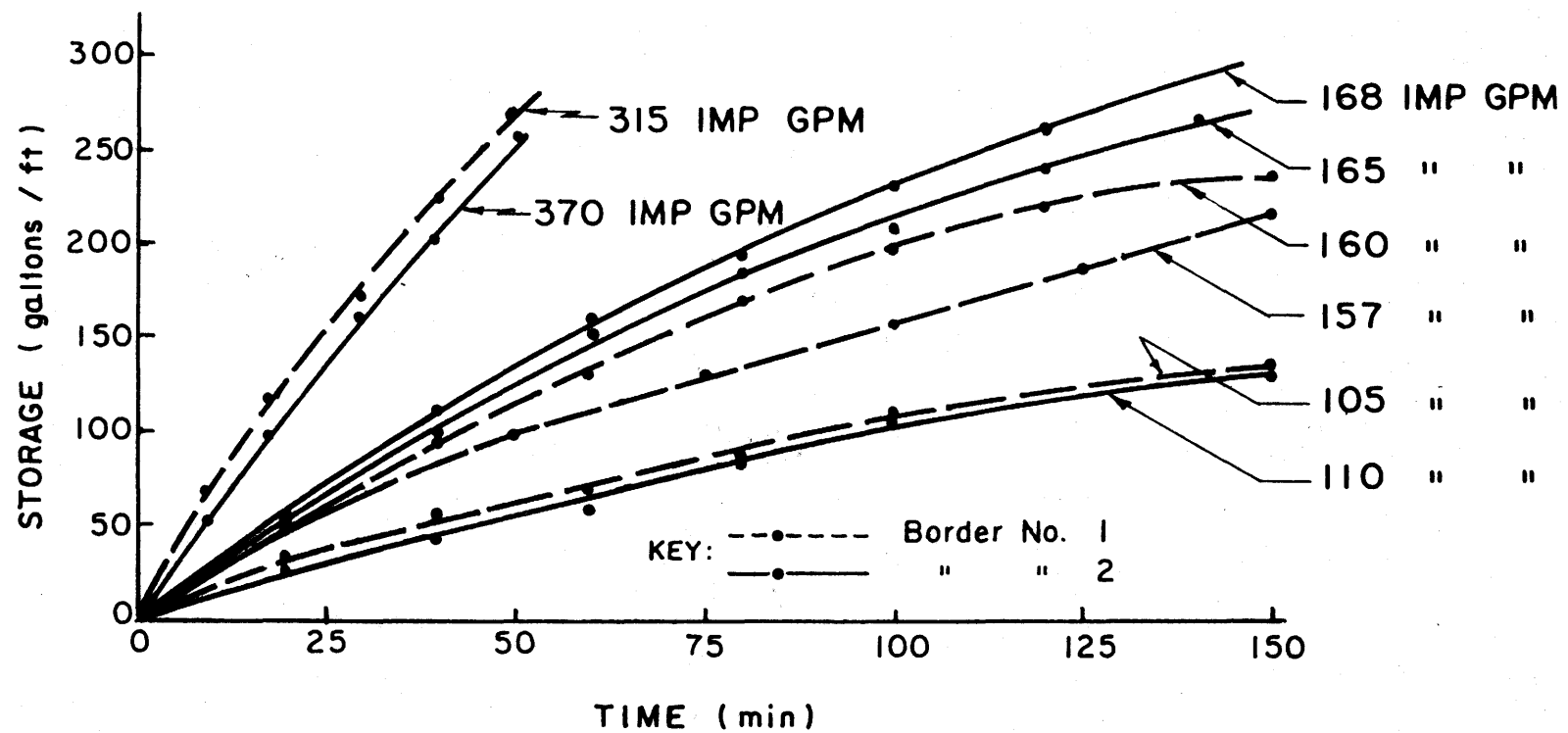


FIG. 9 MASS STORAGE CURVES, OUTLOOK.

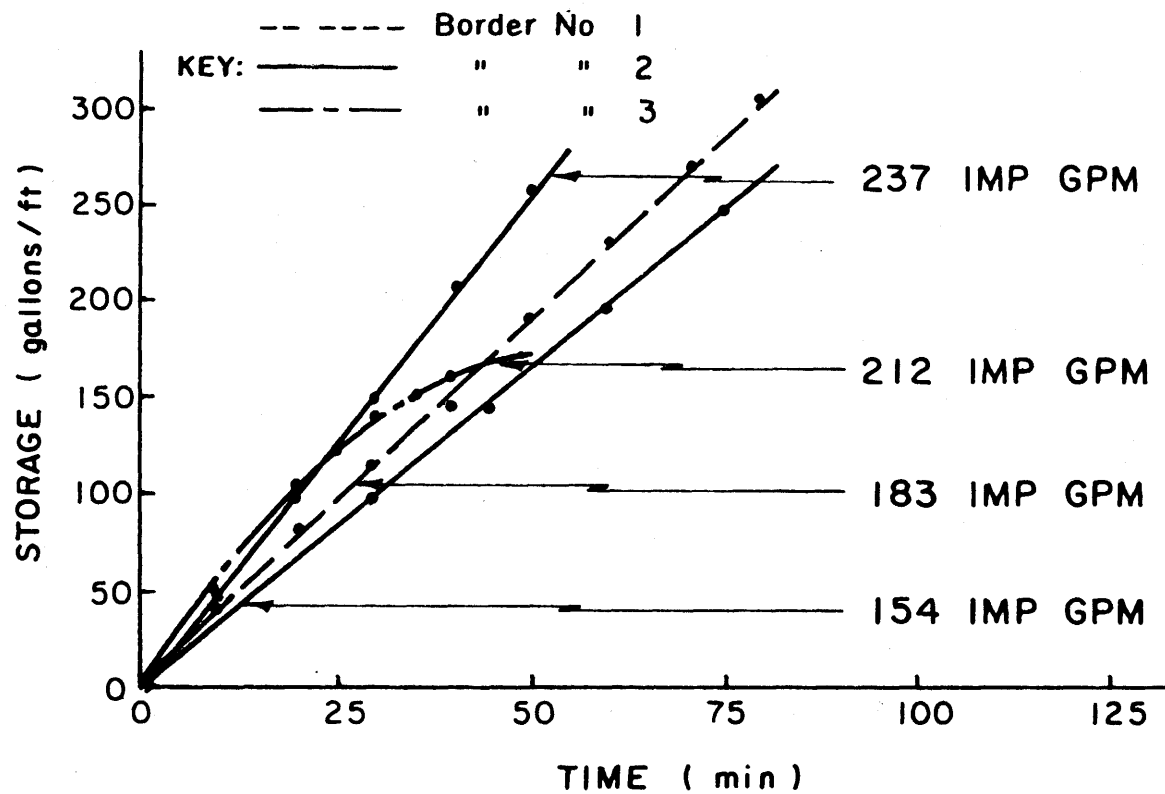


FIG. 10 MASS STORAGE CURVES, SASKATOON.

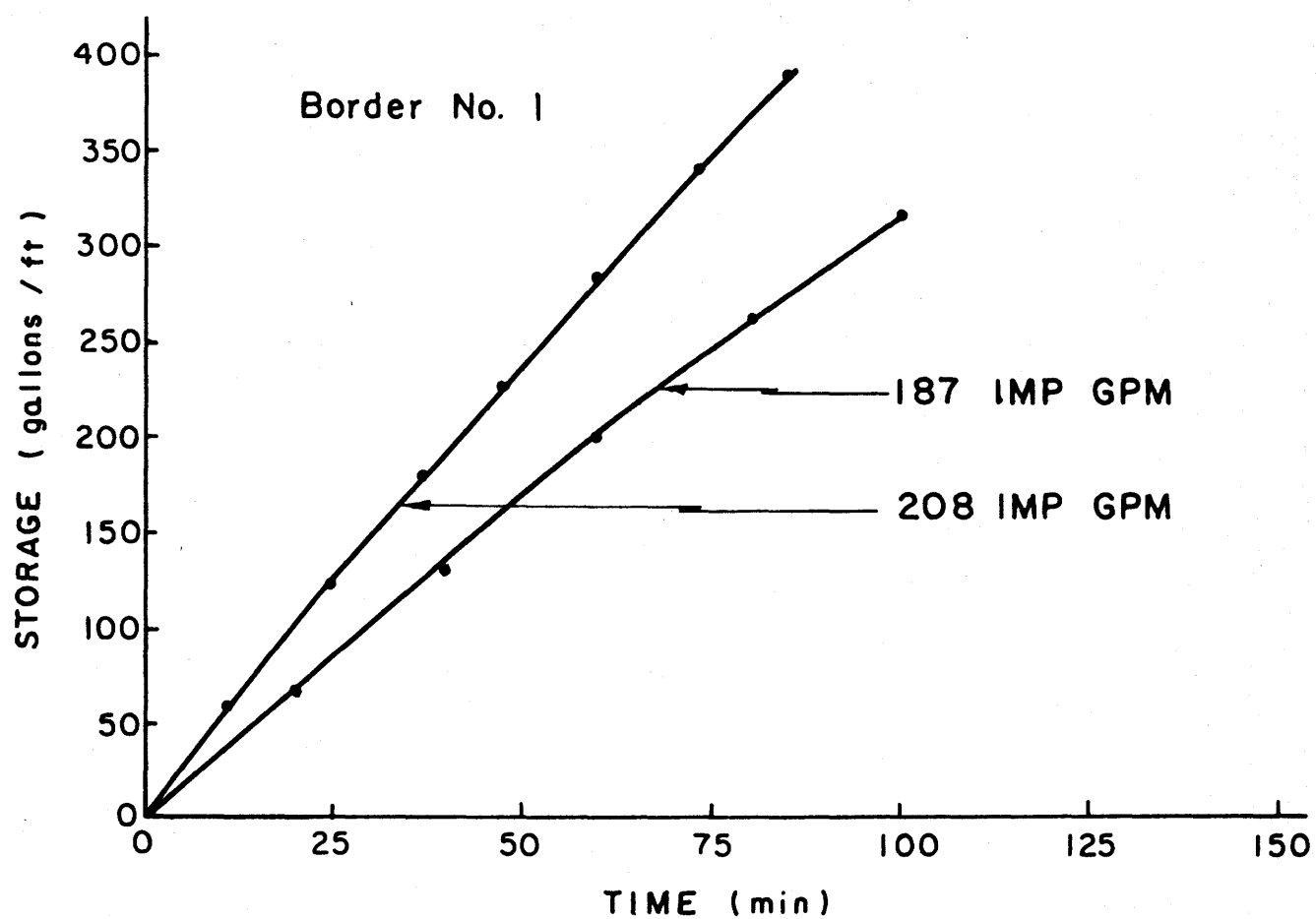


FIG. II MASS STORAGE CURVES, BATTLEFORD.

less than 200 ft to 300 ft. As shown in Figs. B-1 to B-4, for these lengths of run the longitudinal profile of the wet front exhibits considerable curvature (see Appendix B).

These rather casual observations suggest that there is need of a further study of the shape of a wet front over a porous bed. It is believed that a comprehensive analysis of the data contained in Appendix B would prove helpful in establishing a program for these investigations.

### Soil Intake Rate

#### Development of an Equation for Soil Intake Rate

A major disadvantage of the use of infiltrometers for measuring the soil intake rate is that the value is measured over a relatively small sized sample. In effect, for practical purposes, these measurements may be considered point measurements. It is not difficult to understand the inadequacy of applying a point measurement of infiltration for design of a surface irrigation system. Even areas, which contain reasonably uniform, homogeneous, soils would be changed to a heterogeneous mixture of deposits all having different intake characteristics after being land prepared for irrigation. Thus, to obtain an average value of the soil intake rate for the area would necessitate numerous time-consuming point measurements.

Inasmuch as the soil intake rate exerts a pronounced influence on the rate of advance of a wet front in a border strip, it would appear logical to assume that some functional relationship between those factors could be developed. Such a relationship would provide a composite value for intake based on the flow measurements.

Let us consider the water intake to the soil under border dyke irrigation after the wet front has traversed a length  $D_0$  in time,  $T_0$ , (see Fig. 12). Consider also some other point in the strip defined by the coordinates,  $D$ ,  $t$ . At distance  $D$ , the depth of water infiltrated to the soil,  $d$ , is equal to (see Eqn. 4),

$$d = \int_0^{T_0-t} f dt \quad \dots \dots \dots 17.$$

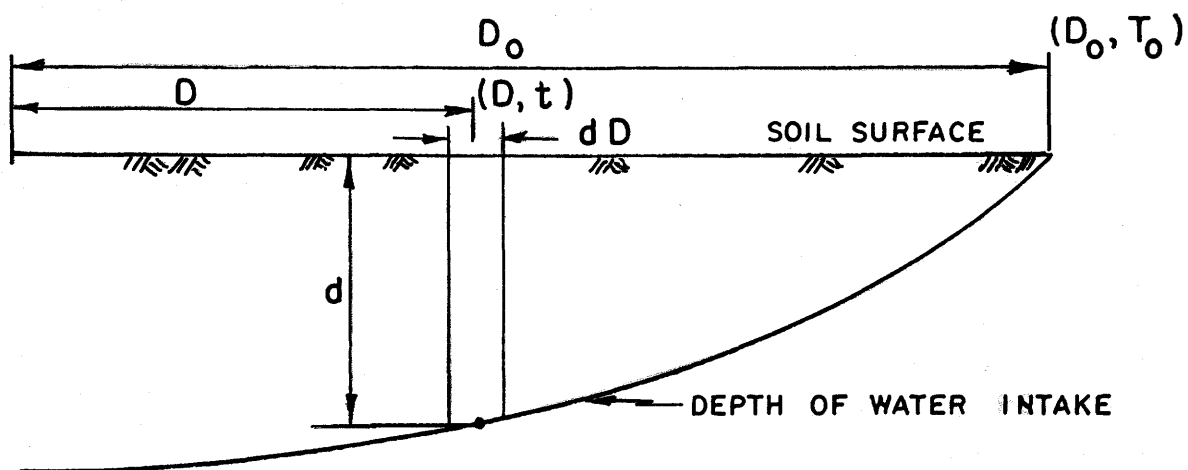


Fig. 12. Schematic diagram of depth of intake during the advance of a wet front

Assuming the intake capacity,  $f$ , varies with time according to a power series (see Eqn. 1), then Eqn. 17 can be rewritten as,

$$d = \int_0^{T_0-t} Kt^n dt = \frac{K}{n+1} (T_0-t)^{n+1} \quad \dots \dots \dots 18.$$

Let us now consider the incremental volume of intake per unit width,  $dV_f$ , contained within a differential length of the strip,  $dD$ .

Mathematically,

$$\begin{aligned} dV_f &= (d) dD \\ &= \frac{K}{n+1} (T_o - t)^{n+1} dD \dots\dots\dots 19. \end{aligned}$$

Thus, the total volume of water intake per unit width,  $V_f$ , in a strip of length  $D_o$  is given as,

$$V_f = \int_0^{D_o} \frac{K}{n+1} (T_o - t)^{n+1} dD \dots\dots\dots 20.$$

In previous discussions, it was shown that the rate of advance of the wet front down a strip follows a simple power law. That is,

$$D = C' t^{m'} \dots\dots\dots 21.$$

Differentiating Eqn. 21 with respect to time one obtains an expression for  $dD$  as,

$$dD = C' m' t^{m'-1} dt \dots\dots\dots 22.$$

Eqn. 22 is now substituted into Eqn. 20 to obtain an integrable equation for  $V_f$  as a function of a single variable, time,

$$V_f = \int_0^{T_o} \frac{K}{n+1} (T_o - t)^{n+1} C' m' t^{m'-1} dt \dots\dots\dots 23.$$

Simplifying Eqn. 23,

$$V_f = G \int_0^{T_o} (T_o - t)^z t^{m'-1} dt \dots\dots\dots 24.$$

in which  $G = \frac{C' m' K}{z}$  = a constant, and

$$z = n + 1$$

The integration of Eqn. 24 is simplified by expanding the term  $(T_0 - t)^z$  according to a binomial expansion assuming  $T_0$  is constant.

$$\begin{aligned}
 (T_0 - t)^z &= T_0^z \left(1 - \frac{t}{T_0}\right)^z \\
 &= T_0^z \left[ 1 - z \left(\frac{t}{T_0}\right) + \frac{z(z-1)}{2!} \left(\frac{t}{T_0}\right)^2 - \frac{z(z-1)(z-2)}{3!} \left(\frac{t}{T_0}\right)^3 \right. \\
 &\quad \left. \dots + \frac{(-1)^r (z!)}{(z-r)!(r!)} \left(\frac{t}{T_0}\right)^r \dots \dots \dots \right] \quad 25.
 \end{aligned}$$

Substituting the equality given by Eqn. 25, into Eqn. 24, one can write:

$$\begin{aligned}
 V_f &= G \int_0^{T_0} \left[ T_0^z t^{m'-1} - z T_0^{z-1} t^{m'} + \frac{z(z-1) T_0^{z-2} t^{m'+1}}{2!} \right. \\
 &\quad \left. - \frac{z(z-1)(z-2) T_0^{z-3} t^{m'+2}}{3!} \dots + \frac{(-1)^r (z!)}{(z-r)!(r!)} T_0^{z-r} t^{m'-1+r} \right] dt \quad \dots 26a. \\
 &= G \left[ \frac{1}{m'} T_0^z t^m - \frac{z}{(m'+1)} T_0^{z-1} t^{m+1} + \frac{z(z-1)}{2!(m'+2)} T_0^{z-2} t^{m+2} \right. \\
 &\quad \left. - \frac{z(z-1)(z-2)}{3! (m'+3)} T_0^{z-3} t^{m+3} \dots \frac{(-1)(z!)}{(m'+r)(z-r)!(r!)} T_0^{z-r} t^{m'+r} \right]_0^{T_0} \quad \dots 26b
 \end{aligned}$$

$$\begin{aligned}
 &= G \left[ \frac{T_0^{z+m'}}{m'} - \frac{z}{(m'+1)} T_0^{z+m'} + \frac{z(z-1)}{2!(m'+2)} T_0^{z+m'} - \frac{z(z-1)(z-2)}{3!(m'+3)} T_0^{z+m'} \right. \\
 &\quad \left. \dots + \frac{(-1)(z!)}{(z-r)!(r!)(m'+r)} T_0^{z+m'} \dots \dots \dots \right] \quad 26c
 \end{aligned}$$



But  $z = n+1$ , therefore,

$$V_f = GT_o^{n+1+m'} \left[ \frac{1}{m'} - \frac{(n+1)}{(m'+1)} + \frac{(n+1)(n)}{2!(m'+2)} - \frac{(n+1)(n)(n-1)}{3!(m'+3)} \right. \\ \left. \dots + \frac{(-1)(n+1)!}{(n+1-r)!(r!)(m'+r)} \right] \dots \dots \dots 27a$$

For simplicity, let the expression contained within the brackets of Eqn. 27a be written as  $\phi(n, m')$  to denote a function of the exponents  $n$  and  $m'$ . That is,

$$V_f = GT_o^{n+1+m'} \phi(n, m') \dots \dots \dots 27b$$

Equation 27a provides an expression from which the volume of intake to the soil per unit width can be calculated for a given length of border strip. The equation is solvable only after prior knowledge of the rate of advance of a wet front and the soil intake rate have been obtained. Since, the properties,  $C'$  and  $m'$  of the advance curve are easily obtained from measurements of the rate of advance, (ie.  $C' = 1/C^{1/m}$  and  $m' = 1/m$ , see Eqn. 5) the problem remains to develop a method for evaluating the soil intake rate properties,  $K$  and  $n$ , during advance tests so as to avoid the necessity of resorting to infiltration measurements.

To accomplish this objective, let us consider the flow per unit width to a strip in terms of the continuity equation. The disposition of input to the strip must include three components, (a) surface or sheet storage, (b) soil intake volume and (c) evaporation. Expressed mathematically,

$$qT_o = V_f + S + E \quad \dots \dots \dots 28.$$

in which,

$q$  = input to a strip of unit width,

$T_o$  = time required for wet front to traverse the strip,

$V_f$  = soil intake volume to a strip of unit width,

$S$  = surface storage volume on a strip of unit width, and

$E$  = evaporation in time  $T_o$  from a strip of unit width.

Because the advance occurs in very short interval of time, it can be assumed that for practical purposes, evaporation is negligible. Thus, Eqn. 28 can be reduced to,

$$qT_o = V_f + S \quad \dots \dots \dots 29.$$

Substituting Eqn. 27b into Eqn. 29,

$$qT_o = GT_o^{n+1+m'} \phi(n, m') + S \quad \dots \dots \dots 30a.$$

or  $GT_o^{n+1+m'} \phi(n, m') = qT_o - S \quad \dots \dots \dots 30b.$

or  $\frac{K}{n+1} T_o^{n+1+m'} \phi(n, m') = \frac{1}{C' m'} (qT_o - S) \quad \dots \dots \dots 30c.$

Equation 30c can be solved for the constants of the intake rate equation,  $K$  and  $n$ , by simultaneous solution when the following characteristics are known:

1. The properties of the rate of advance curve for a constant input rate, and
2. Two values of  $S$  are known at two times during the advance.

### Intake Rate Calculations

The coefficient,  $K$ , and exponent,  $n$ , of the intake rate equation (see Eqn. 1) were determined for each test run by simultaneous solution of Eqn. 30c using the rate of advance and surface storage data. An example of this calculation is given in Appendix A. Table 7 summarizes the results of these calculations.

Table 7. Calculated values of the constant,  $K$ , and exponent,  $n$ , of the soil intake equation,  $f = Kt^n$

Location	Border No.	Input (Imp gpm)	Slope (%)	Moisture Content (% Volume)	Intake Equation <sup>a/</sup> Coefficient $K$	Exponent $n$
Outlook	1	105	0.27	12.9	7.42	-0.56
		157		--	11.07	-0.52
		160		18.1	7.45	-0.47
		315		16.9	8.50	-0.63
	2	110	0.52	13.1	8.33	-0.67
		165		19.0	6.96	-0.58
		168		14.9	6.80	-0.61
		370		15.2	6.06	-0.91
Saskatoon	1	183	2.00	28.5	2.82	-0.72
	2	154	2.20	9.9	2.62	-0.68
		237		11.7	2.60	-0.83
	3	212	2.30	--	4.06	-0.45
Battleford	1	187	0.78	--	5.37	-0.90
		208	0.78	20.2	4.50	-0.78

<sup>a/</sup>The values of the coefficient and exponent give soil intake rate in in/hr when time is expressed in min.

### Outlook

Figures 13 and 14 show the intake rate curves for the tests conducted at Outlook. These curves were plotted according to the values of the constants given in Table 7.

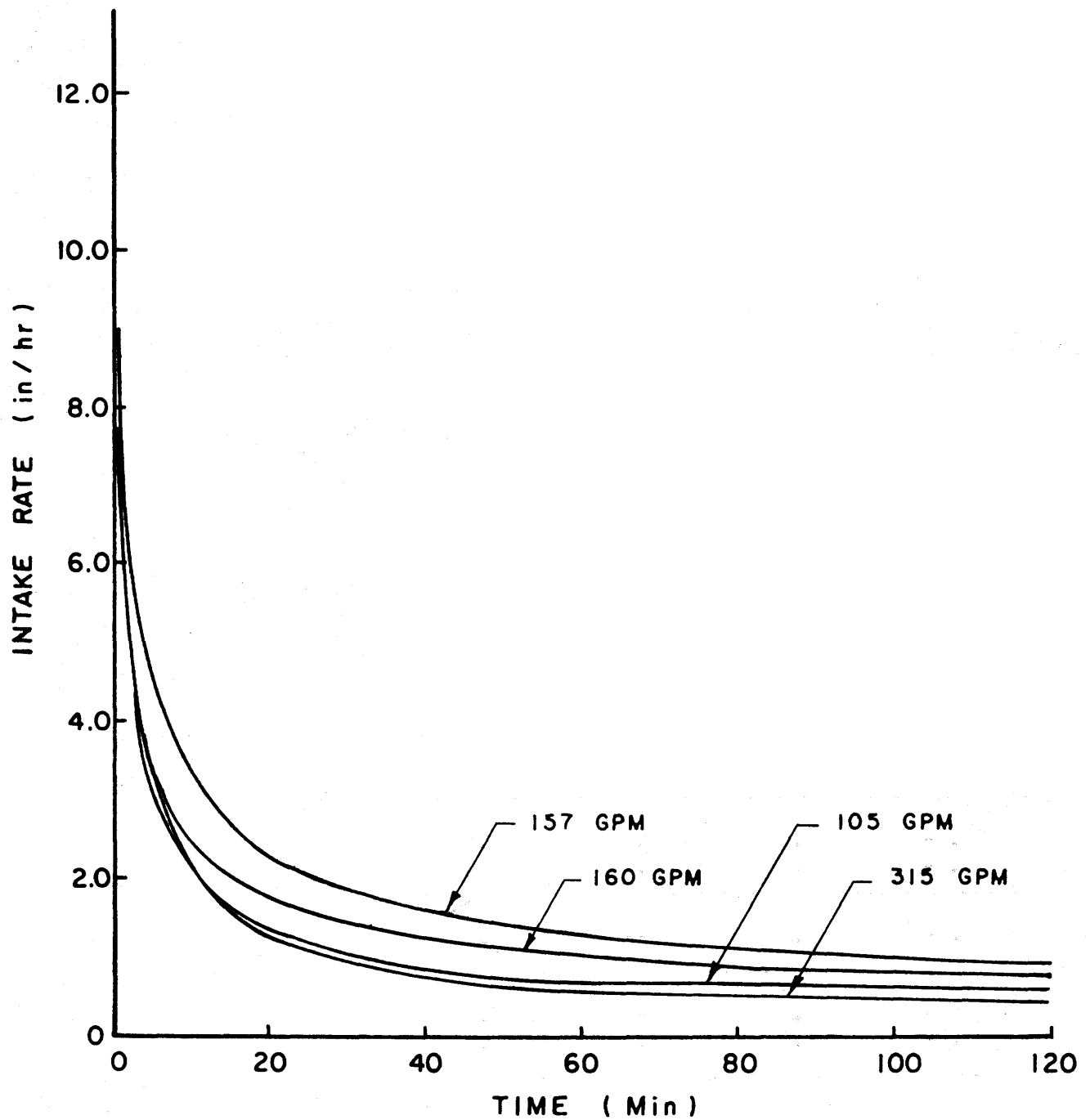


FIG. 13 INTAKE RATE CURVES FOR DIFFERENT INPUTS, BORDER 1, OUTLOOK.

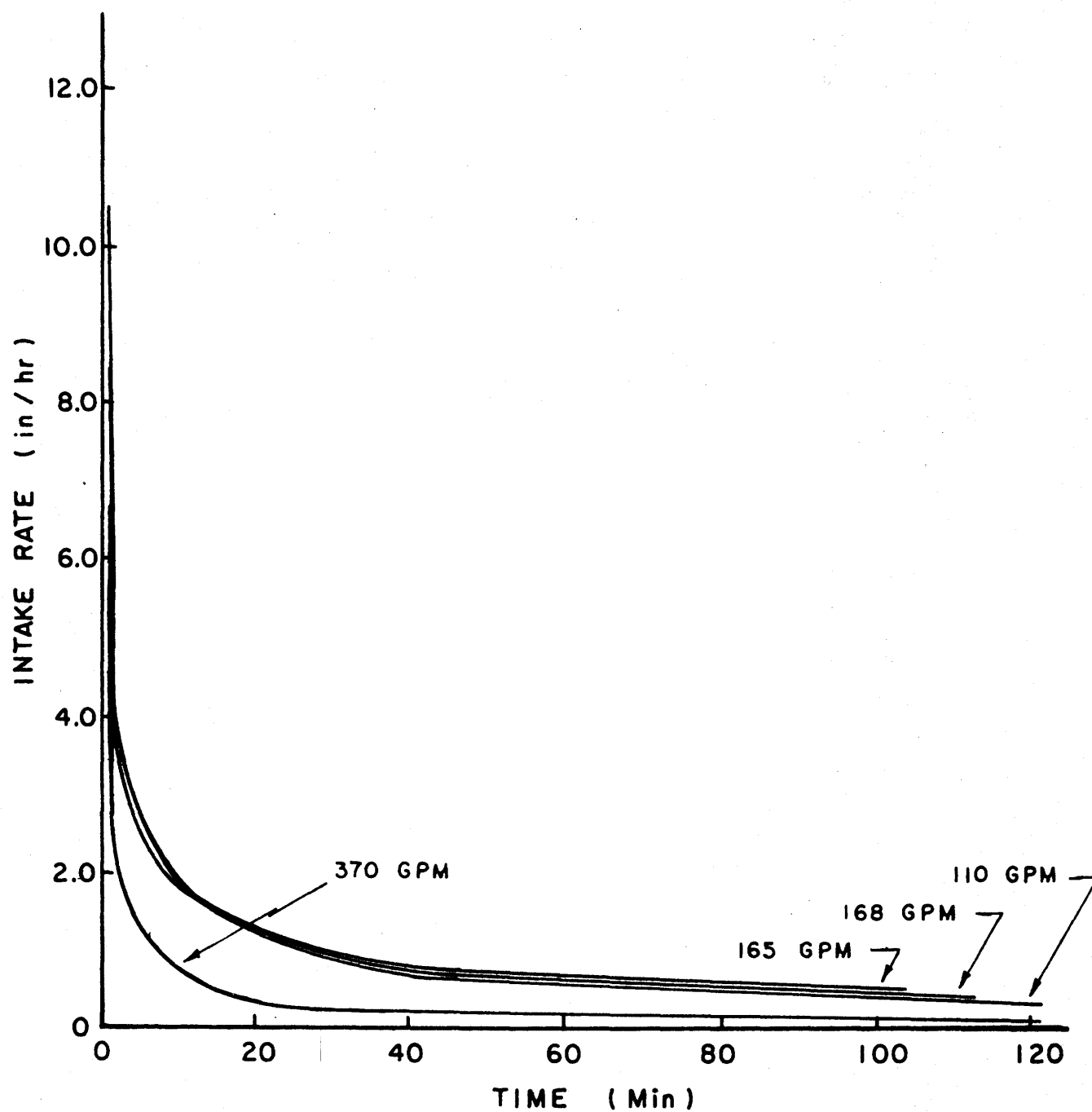


FIG. 14 INTAKE RATE CURVES FOR DIFFERENT INPUTS, BORDER 2, OUTLOOK.

In comparing the slopes of the intake curves,  $n$ , for a given border strip, it is evident that for narrow ranges of input 100 gpm to 170 gpm there is remarkable consistency in the values of the slope. There is, however, a tendency for the slopes to increase at the higher inputs ( 370 gpm). This trend is not completely unsuspected, and may possibly be explained by giving consideration to the depth of water in storage and the effective head producing intake under different discharges.

It is easily realized that for a given border strip the depth of water ponded increases with increased inputs (see Figs. B-1 to B-4). During the initial stages of the intake process (near zero time), the depth of water penetration is small. Under these conditions, the acting hydraulic head governs, to a large extent, the rate of water entry to the soil. The greater the hydraulic head the higher the intake rate. Inasmuch as hydraulic head is proportional to the depth of water ponded on the surface, it follows that the higher initial intake rates should be associated with the higher inputs. As time progresses, the hydraulic gradient rapidly approaches unity. As pointed out by Bodman and Colman (3), in time, the hydraulic gradient becomes relatively ineffectual as it influences the rate of water entry at the surface. Moreover the authors suggest that the rate of entry is governed by the total soil moisture potential gradient which exists within the wetted profile.

Assuming the above reasoning to be correct, it follows that the effect of the high initial intake rates at the higher inputs would be to increase the slope of the intake curves. The fact that the instantaneous "initial" intake rates are high is substantiated by the fact that the

intercept values,  $K$ , (at a time of 1 minute) are very high. Further, the steepness of the slopes of the curve may, in fact, substantiate the idea of a rapid decrease with time of the effectiveness of the hydraulic gradient in governing intake.

In general, the intake rates obtained from border 2 are somewhat lower than those obtained from border 1.

#### Saskatoon

The intake rate curves calculated from trials conducted at Saskatoon are shown in Fig. 15. As shown in the Fig. 15 and Table 7, there is close agreement between curves obtained on borders 1 and 2 at inputs of 183 gpm and 154 gpm respectively. It can be observed from the results obtained from border 2 that the slope of the intake curve increases with an increase in input. A similar observation was noted in the results obtained at Outlook.

No explanation can be given to explain the reason the intake curve obtained from border 3 differed so widely from the curves obtained on borders 1 and 2. Presumably the soil type was reasonably homogeneous over the area (see Tables C-3 and C-4). Perhaps the result simply points out normal variations in the intake properties of soils which may be produced by land preparation procedures prior to installation of the system.

#### Battleford

The slopes of the intake curve tabulated for Battleford in Table 7 indicate some discrepancy in the values for the two tests; -0.90 compared to -0.78 for inputs of 187 gpm and 208 gpm, respectively. Superficially this difference in the values may appear to be highly significant. An appreciation of the practical importance of this difference can be

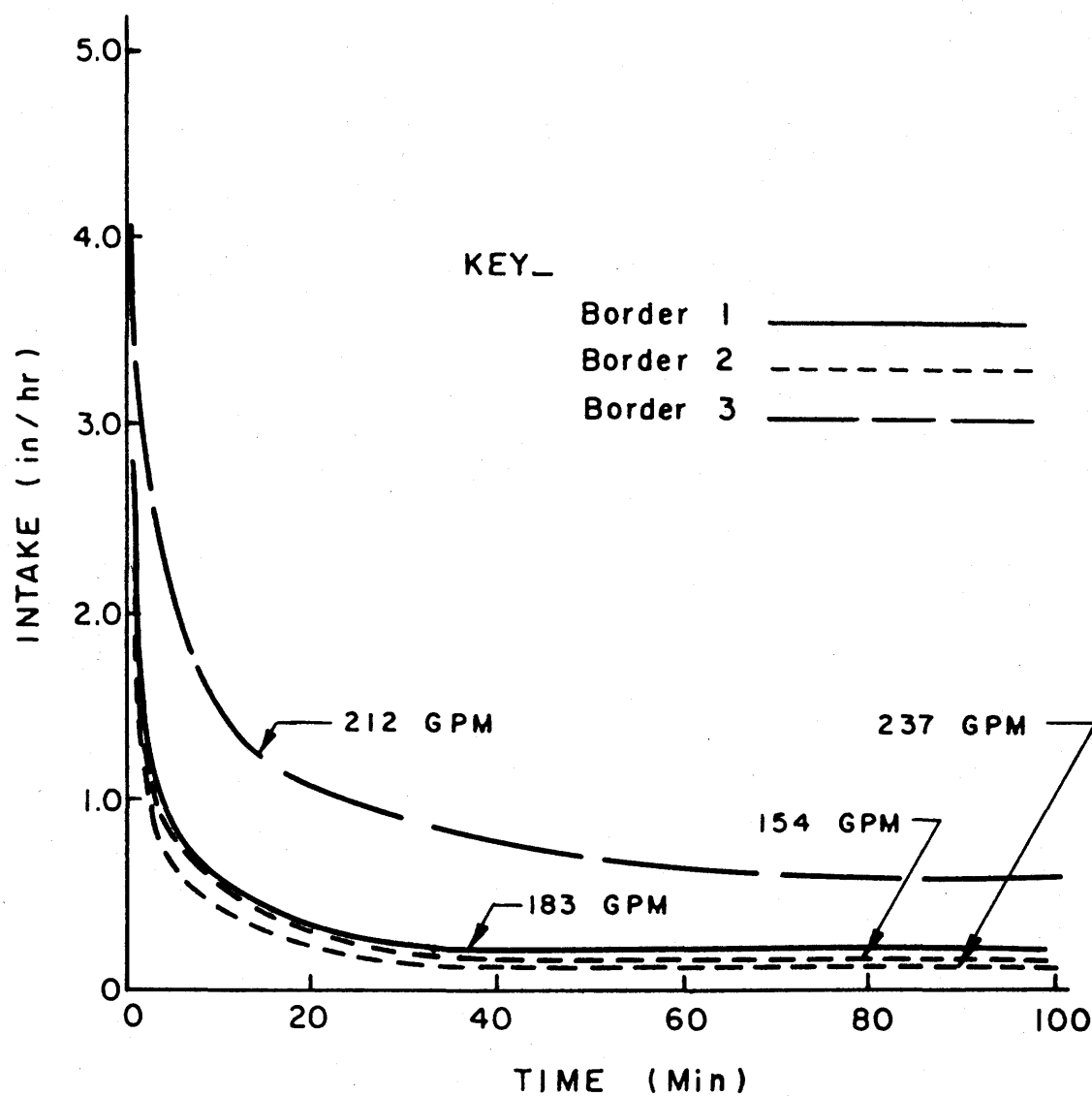


FIG.15 INTAKE RATE CURVES FOR DIFFERENT INPUTS, BORDERS 1, 2, & 3, SASKATOON.



obtained from the plot of the two curves shown in Fig. 16. The figure shows the two curves are in close agreement such that differences in the area enclosed by each - which denotes the depth of water infiltrated - is very small. It follows that such minor differences in depths of application would have little practical importance when applying the data to system design.

The intake rates exhibited by the soils at Battleford are lower than those encountered at either Saskatoon or Outlook. This result is consistent with the difference in the soil types at the sites. The soils at Battleford are considerably more heavier textured than the soils encountered at Outlook or Saskatoon.

#### Evaluation of Soil Intake Rate Determinations

The applicability of the soil intake rate calculations were tested by two methods:

1. The measured rate of advance curves were compared with calculated curves determined by substitution of the intake data into the equation for advance given by Christensen, Hansen and Fok (see Eqn. 13).
2. The depth of water intake, calculated by integration of the intake rate equation over the time between the recession and advance curve was compared with the measured depth of soil moisture.

#### Calculated and Observed Rate of Advance Curves

The soil intake rate was related to the rate of advance of water over the land by means of the equation suggested by Christiansen, Bishop

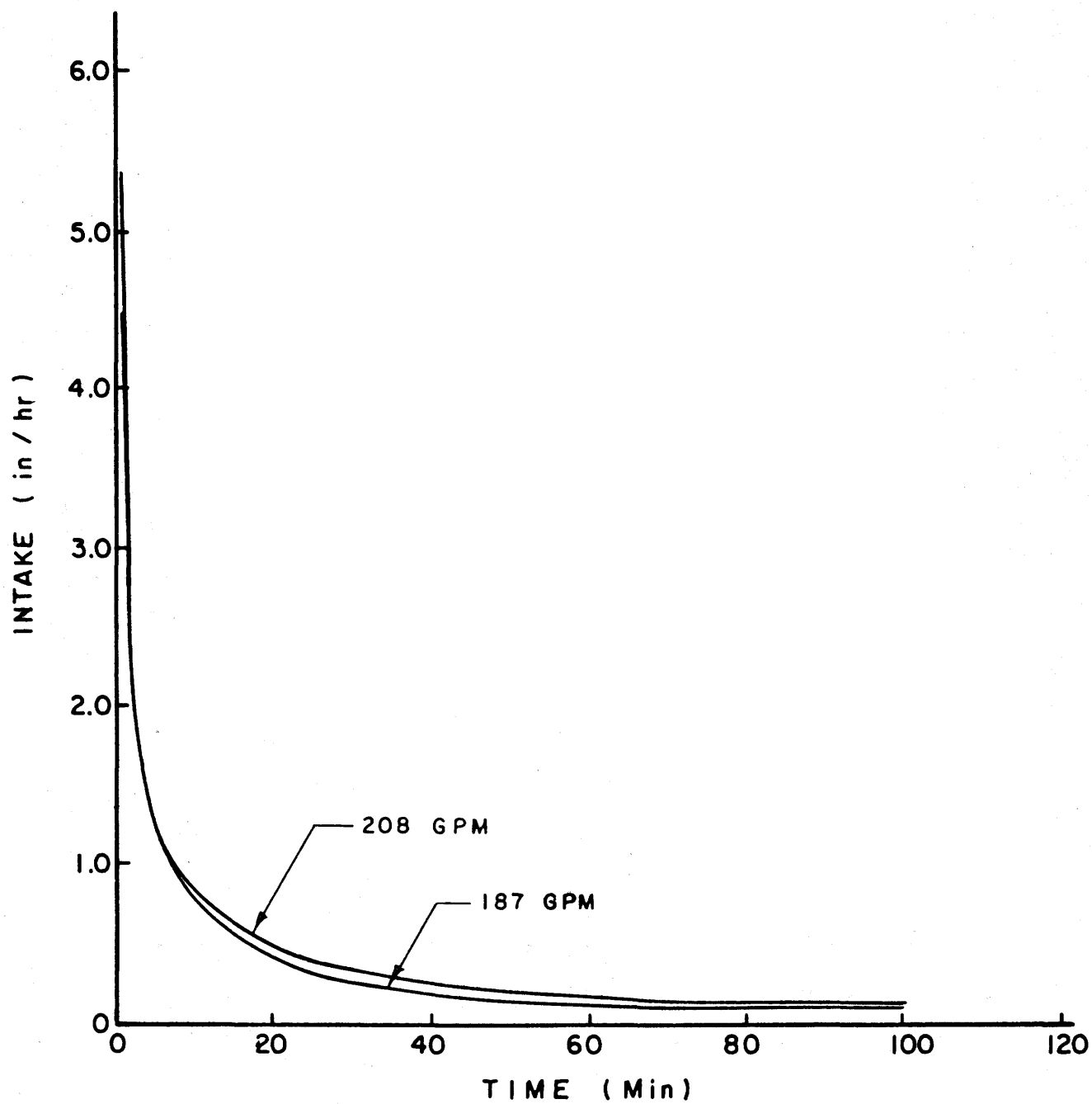


FIG. 16 INTAKE RATE CURVES FOR DIFFERENT INPUTS,  
BORDER 1, BATTLEFORD.

and Fok (see Eqn. 13). In solving this equation, the equivalent average depths of surface flow at any time were calculated from the mass storage curves (see Figs. 9, 10 and 11). A sample of this calculation is given in Appendix A.

The advance curves, calculated in the above matter are plotted with the measured curves as shown in Figs. 4, 5, 6 and 7. As shown in the figures; generally, the calculated and observed advance curves show close agreement. On all curves, the times required for the wet front to move given distances, as calculated from the equation, agree within 15 percent of the observed values. Similarly, the calculated distances of advance agree within 10 percent of the observed values. The magnitudes of these differences are within the expected accuracy of the inflow measurements. In fact, the agreement between the two curves for the Saskatoon and Battleford tests is better than that obtained from tests conducted at Outlook. At Saskatoon and Battleford, Parshall flumes were used for measurement of inflow whereas at Outlook the measurements were made with a calibrated rectangular turnout box. It would be expected that the Parshall flumes would provide a more accurate measure of input than the turnout.

As shown in the figures, at each location the magnitude of the discrepancy between the observed and calculated advance curves increase with a decrease in input. This tendency probably reflects the differences in the uniformity of the depth of surface flow on the strip. At low input rates, the depth of surface flow through the longitudinal profile of the advancing wet front decreases quite rapidly with distance whereas, at larger inputs this change in depth is much more uniform. Inasmuch as the

development of the advance equation (Eqn. 13) is predicated on the assumption of an uniform depth of surface storage it would be expected that the agreement between the observed and calculated curves would be better for the higher inputs. Similarly, one would expect a more uniform depth of surface flow in a direction transverse to the direction of flow under high discharge.

In summary, it can be stated that the calculated and observed rate of advance curves for all tests agree very closely. It is questionable whether, in any instance, the differences between the curves are of such magnitude to affect the depth of water applied or the distance of advance to be of practical importance. The agreement between the curves substantiate the validity of the soil intake rate determination and the use of Eqn. 13 for predicting rate of advance.

#### Depth of water intake

The average depths of water calculated from soil intake rate and rate of advance and recession data and determinations made from soil moisture data for the border strips at Outlook are given in Table 8. In the table, the rainfall, (col. 4) during the interval between soil moisture measurements is assumed to be totally effective in increasing soil moisture and, hence, has been added to col. 3 to give the total intake depth, col. 5. The evapotranspiration, col. 7, was estimated from irrigation gage measurements and application of the Thornthwaite Method.

In the table the sum of the average depth of water stored in the border strip determined from soil moisture measurements (col. 6) plus evapotranspiration is compared to the sum of the depth of water calculated

Table 8. Comparison of calculated and measured average depths of water stored in border strips

1	2	3	4	5	6	7	8	9
Border No.	Input (gpm)	Average Calculated Depth of Soil Intake (in)	Rain-fall (in)	Total Intake (in)	Soil Moisture Stored (in)	Evapo-transpiration (in)	Total Stored (in)	Ratio <sup>a</sup>
1	105	2.73	2.04	4.77	3.57	0.80	4.37	0.92
	315	2.00	0.21	2.21	0.38	0.80	1.18	0.53
2	110	2.34	2.04	4.38	3.44	0.80	4.24	0.97
	165	2.14	--	2.14	1.46	0.64	2.10	0.98
	370	1.66	0.21	1.87	0.95	0.80	1.75	0.94

<sup>a</sup>/ Ratio of Total Stored (col. 9) to Total Intake (col. 5)

from the soil intake data plus rainfall as the ratio of the two amounts (col. 9). In four of the five tests, the agreement between the two depths was extremely good; as denoted by the fact the values of the ratios are close to unity. It is believed that the poor result obtained from the one test, border 1 (315 gpm) can be attributed to an inaccurate soil moisture determination rather than an error in the soil intake rate. This reasoning arises because of the small amount of soil moisture measured.

It is realized that only limited weight can be given to the results because of the assumptions involved in the calculations. Nevertheless, the results do validate to limited extent the soil intake rate determinations.

#### Summary

The principal findings presented in prior paragraphs concerning the soil intake rate determinations and calculated and observed rate of advance curves are condensed for review,

1. For irrigation purposes, the variation of the soil intake rate with time can be adequately defined by a simple power equation (see Eqn. 1).
2. Evaluation of the constant and exponent of the soil intake rate equation can be accomplished from rate of advance and surface storage measurements from Eqns. 27a and 29. These equations were developed by applying the equation of continuity to a border strip of unit width.
3. The equation of advance proposed by Christiansen, Bishop and Fok (see Eqn. 13) provided a reasonable prediction of the advance when the soil intake characteristics, as determined under item 2, were used in the calculation. The close agreement between the observed and calculated advance curves substantiates the validity of the intake calculations.
4. Close agreement was obtained between the average depths of water applied calculated from soil intake rate and rate of advance and recession curves. Compared with determinations made from soil moisture measurements for several tests conducted at Outlook.

The foregoing statements are predicated on the basis of the results obtained in the study. Further work is required to gain more conclusive evidence in support or rebuttal of these results. Such studies should be designed to include a much wider range of pertinent variables. That is, the investigations should be conducted on borders of different lengths, slopes, widths and soil types under a wide range of input rates. In these studies, every effort should be made to substantiate the intake rate calculations by soil moisture measurements.

## SUMMARY AND CONCLUSIONS

Several tests were conducted on existing border dyke systems located at Outlook, Saskatoon and Battleford. In these investigations an attempt was made to obtain data which would be useful as design criteria for systems installed under comparable conditions at different sites and, to study the factors influencing flow in a border dyke.

From an analysis of the data, the following conclusions were derived.

1. Rate of advance and recession curves are extremely useful for evaluating the performance and selecting an operating procedure for border dyke systems.
2. For the soils tested, several combinations of input and length of run could be used to produce an uniform application of irrigation water. It is difficult, however, to select a suitable combination of these factors which would provide uniform applications of normally required depths (3 in to 4 in). This difficulty suggests that the maximum permissible slopes of surface irrigation systems should be considered in terms of the soil intake rate.
3. For low input rates and/or short lengths of run the longitudinal profile of the wet front exhibits considerable curvature and, the assumption of a constant depth of surface storage is not valid. Conversely, for high input rates and/or long lengths of run the assumption of constant depth will not introduce serious error to length of run computations.

4. The form of the soil intake rate equation given by the power equation,  $f = Kt^n$ , was found to be applicable to border dyke systems.
5. The coefficient,  $K$ , and exponent,  $n$ , of the soil intake rate equation can be evaluated from rate of advance and surface storage measurements by a rational approach to the hydraulics of flow based on the continuity equation. In this analysis, the soil intake rate equation was expanded by the binomial theorem to obtain an expression for volume of intake.
6. Under surface irrigation, the slopes of the intake capacity curve - denoted by the values of the exponent,  $n$ , - are steep and tend to increase with an increase in input.
7. The rational equation for length of advance presented by Christiansen, Bishop and Fok provides a reliable estimate of the advance curve.



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## APPENDIX A

## Example Calculations of Rate of Advance, Soil Intake

## Rate and Length of Advance

The example calculations presented in this Appendix are based on data procured from border 2, Outlook for an input of 370 gpm.

## Rate of Advance

According to Eqn. 5, the advance of a wet front in a border strip can be described by the power equation

$$t = CD^m$$

Accordingly, the equation can be linearized by expressing the variables as logarithms. That is,

$$\ln t = \ln C + m \ln D \quad \dots \dots \dots \quad A-1$$

By performing a simple linear regression analysis on the data, the least squares estimators of the coefficient, C, and exponent, m, to fit the model given by Eqn. A-1 can be obtained. These estimators can be calculated from the expressions,

$$m = \frac{N \sum \ln D \ln t - \sum \ln D \sum \ln t}{N \sum (\ln D)^2 - (\sum \ln D)^2} \quad \dots \quad A-2$$

and

$$\ln C = \frac{\sum (\ln D)^2 \sum \ln t - \sum \ln D \sum \ln D \ln t}{N \sum (\ln D)^2 - (\sum \ln D)^2} \quad A-3$$

in which N = number of variables.

## Example Calculation

Table A-1. Rate of advance data

Time t (min)	ln t	Distance D (ft)	ln D
0	--	0	--
5	1.609	50	3.912
9	2.197	100	4.605
14	2.639	150	5.011
19	2.944	200	5.298
24	3.178	250	5.522
31	3.434	300	5.704
38	3.638	350	5.858
44	3.784	400	5.992
<hr/>			
N = 8			
$\sum \ln t = 23.424$		$\sum \ln D = 41.901$	
$\sum \ln t \ln D = 126.355$		$\sum (\ln D)^2 = 222.923$	

From Eqn. A-2,

$$m = \frac{(8)(126.355) - (41.901)(23.424)}{8(222.923) - (41.901)^2}$$

$$m = \underline{1.059}$$

From Eqn. A-3,

$$\ln C = \frac{(222.923)(23.424) - (41.901)(126.355)}{8(222.928) - (41.901)^2}$$

$$\ln C = -2.621$$

$$C = \underline{0.073}$$

Therefore, the equation of advance is,

$$t = 0.073D^{1.059}$$

## Soil Intake Rate Coefficients

According to Eqn. 30c, the continuity equation for flow to a border strip per unit width can be written as,

$$\frac{K}{n+1} T_o^{n+1+m'} \phi(n, m') = \frac{1}{C' m'} (q T_o - S)$$

Suppose the equation is written for two different times of advance,  $T_A$  and  $T_B$ , in which the surface storage amounts are  $S_A$  and  $S_B$ , respectively. Then,

$$\frac{K}{n+1} T_A^{n+1+m'} \phi(n, m') = \frac{1}{C' m'} (q T_A - S_A) \dots \quad A-4$$

and

$$\frac{K}{n+1} T_B^{n+1+m'} \phi(n, m') = \frac{1}{C' m'} (q T_B - S_B) \dots \quad A-5$$

Hence, the ratio of Eqn. A-4 to Eqn. A-5 is,

$$\frac{T_A^{n+1+m'}}{T_B^{n+1+m'}} = \frac{(q T_A - S_A)}{(q T_B - S_B)} \dots \dots \dots \quad A-6$$

Solving for Exponent "n"

Given:  $T_A = 10$  min

$T_B = 50$  min

$m = 1.059$

$q = 12.33$  gals/ft

$m' = 1/m = 0.944$

$S_A = 55$  gal/ft

$S_B = 255$  gal/ft

Substituting the given values into Eqn. A-6 one obtains

$$\left(\frac{10}{50}\right)^{n+1.944} = \frac{(12.33)(10) - 55}{(12.33)(50) - 255}$$

$$(0.20)^{n+1.944} = 0.189$$

or

$$(n+1.944)(\ln(0.20)) = \ln(0.189)$$

$$n = \frac{\ln(0.189) - 1.944}{\ln(0.20)}$$

$$n = -0.909$$

Solving for "K"

$$\text{Given: } T_B = 50 \text{ min}$$

$$S_B = 255 \text{ gal/ft}$$

$$q = 12.33 \text{ gal/ft}$$

$$n = -0.909$$

$$m' = 0.944$$

$$C = 0.073$$

$$C' = 1/(C)^{1/m} = 11.82$$

From Eqn. A-5

$$V_{fA} = (12.33)(50) - 255 = 361.7 \text{ gal/ft}$$

Therefore, from Eqn. 27a,

$$361.7 = \frac{(11.82)(0.944)}{0.091} (K)(50)^{1.035} \left[ \frac{1}{0.944} - \frac{(0.091)}{(1.943)} - \frac{(0.909)(0.091)}{(2.943)(2)} \right. \\ \left. - \frac{(0.909)(0.091)(1.906)}{(3.943)(6)} \dots \dots \dots \right]$$

$$361.7 = \frac{(11.82)(0.944)(K)(50)^{1.035}(0.992)}{0.091}$$

$$K = \frac{(0.091)(361.7)}{(11.82)(0.944)(50)^{1.035}(0.992)}$$

$$K = 0.053 \text{ gal/ft}$$

The intake equation is thus,

$$f = 0.053 t^{-0.91} \text{ gpm/ft}$$

or

$$f = 6.06 t^{-0.91} \text{ in/hr}$$

#### Calculated Length of Advance

The calculated advance curve was determined from Eqns. 13 and 14. To perform these calculations, the equivalent average depth of water on the surface  $y_s$  was evaluated from Fig. 1. Table A-2 summarizes these calculations.

Table A-2: Length of advance calculations

Time t (min)	Input Rate per ft q (gpm)	Intake Depth d <sub>a</sub> (ft)	Surface Storage y <sub>s</sub> L (ft <sup>3</sup> )	Length of Advance	
				Calculated (ft)	Observed (ft)
0	12.33	--	--	--	--
10		0.104	8.4	110	105
20		0.110	16.9	205	200
30		0.115	24.8	302	300
40		0.115	32.9	393	385
50		0.12	41.0	482	450

## APPENDIX B

## Surface Storage Curves

The longitudinal profile of the wet front at different times during advance are plotted as shown in Figs. B-1 to B-4.



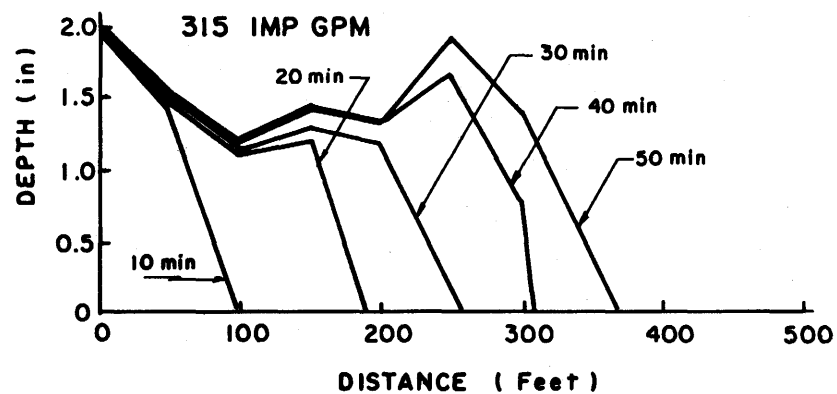
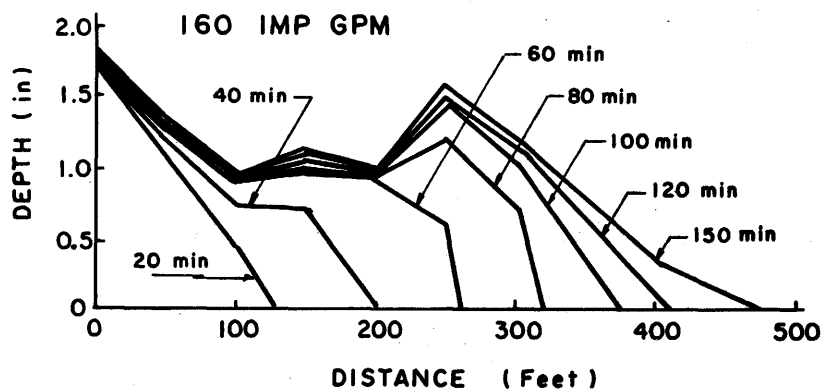
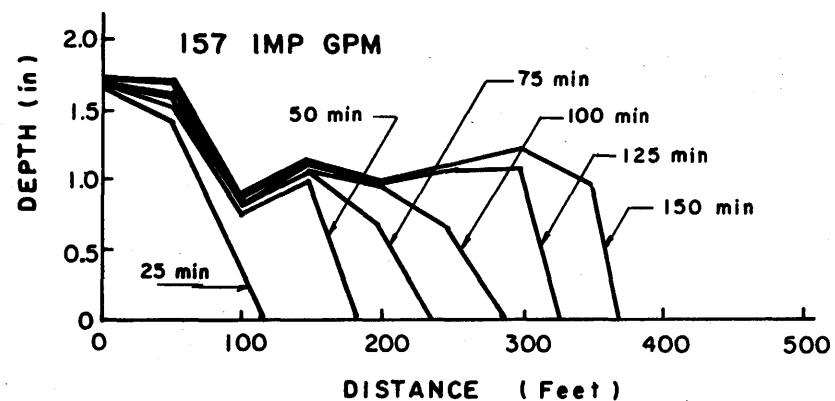
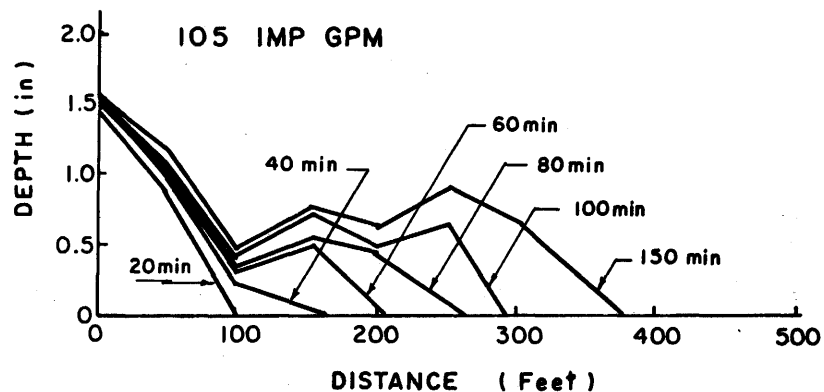


FIG. B-1 LONGITUDINAL PROFILE OF WET FRONT, BORDER No. 1 OUTLOOK.

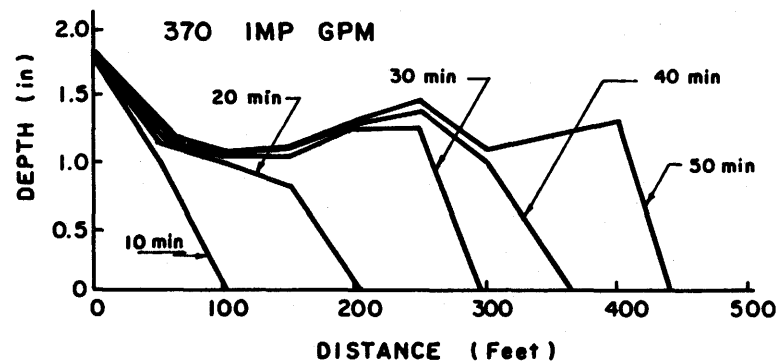
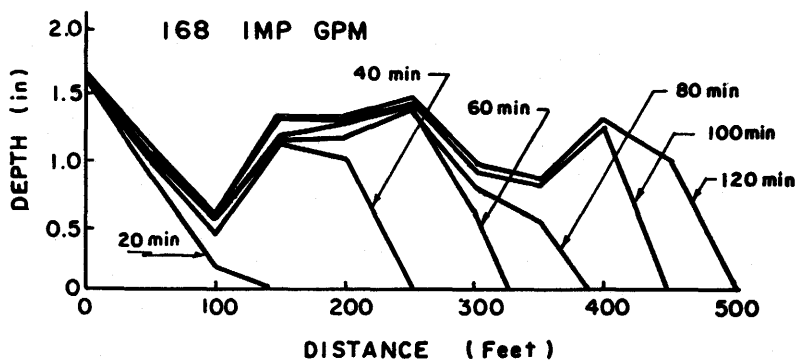
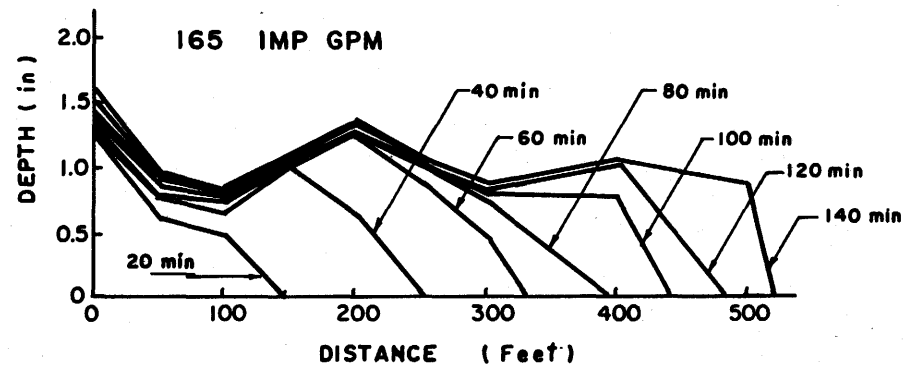
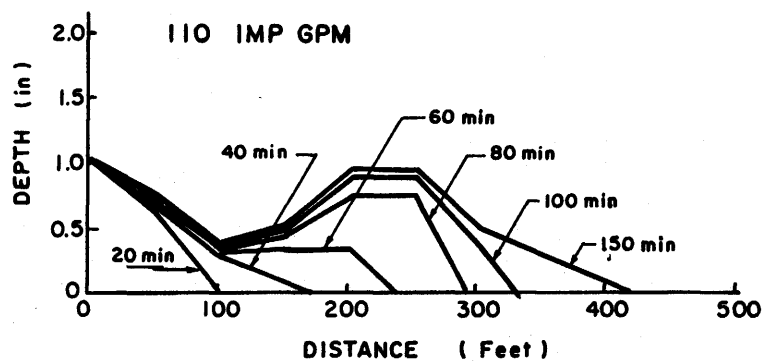


FIG. B-2 LONGITUDINAL PROFILE OF WET FRONT, BORDER No. 2 OUTLOOK.

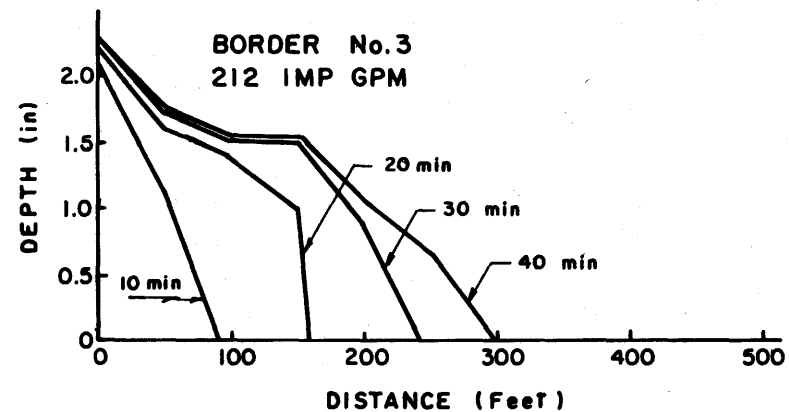
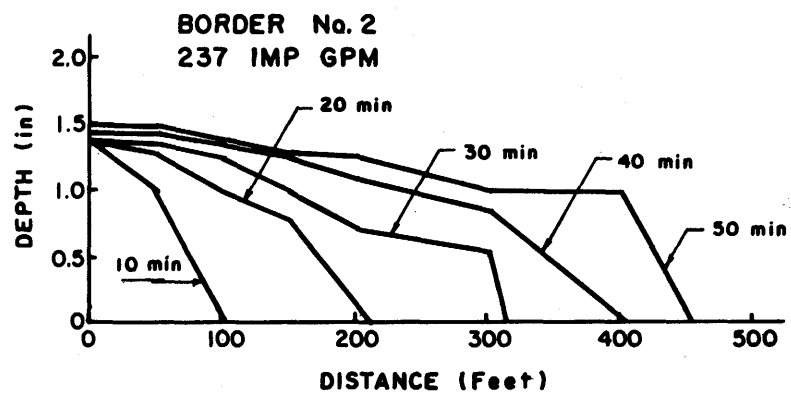
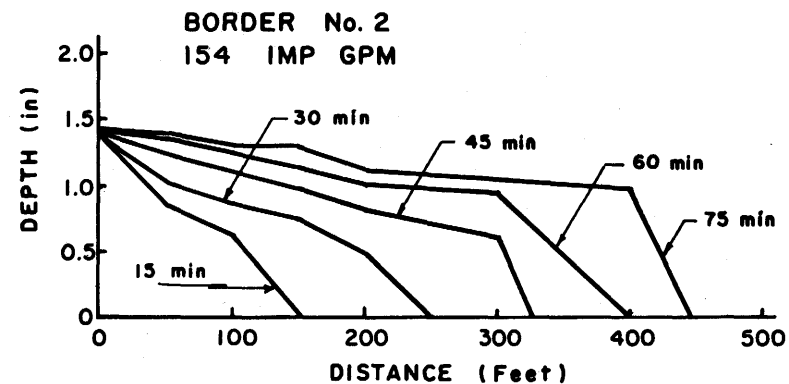
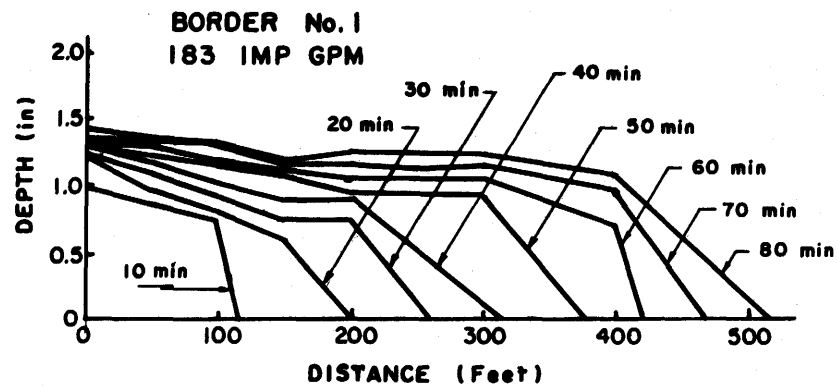


FIG. B-3 LONGITUDINAL PROFILE OF WET FRONT, SASKATOON.

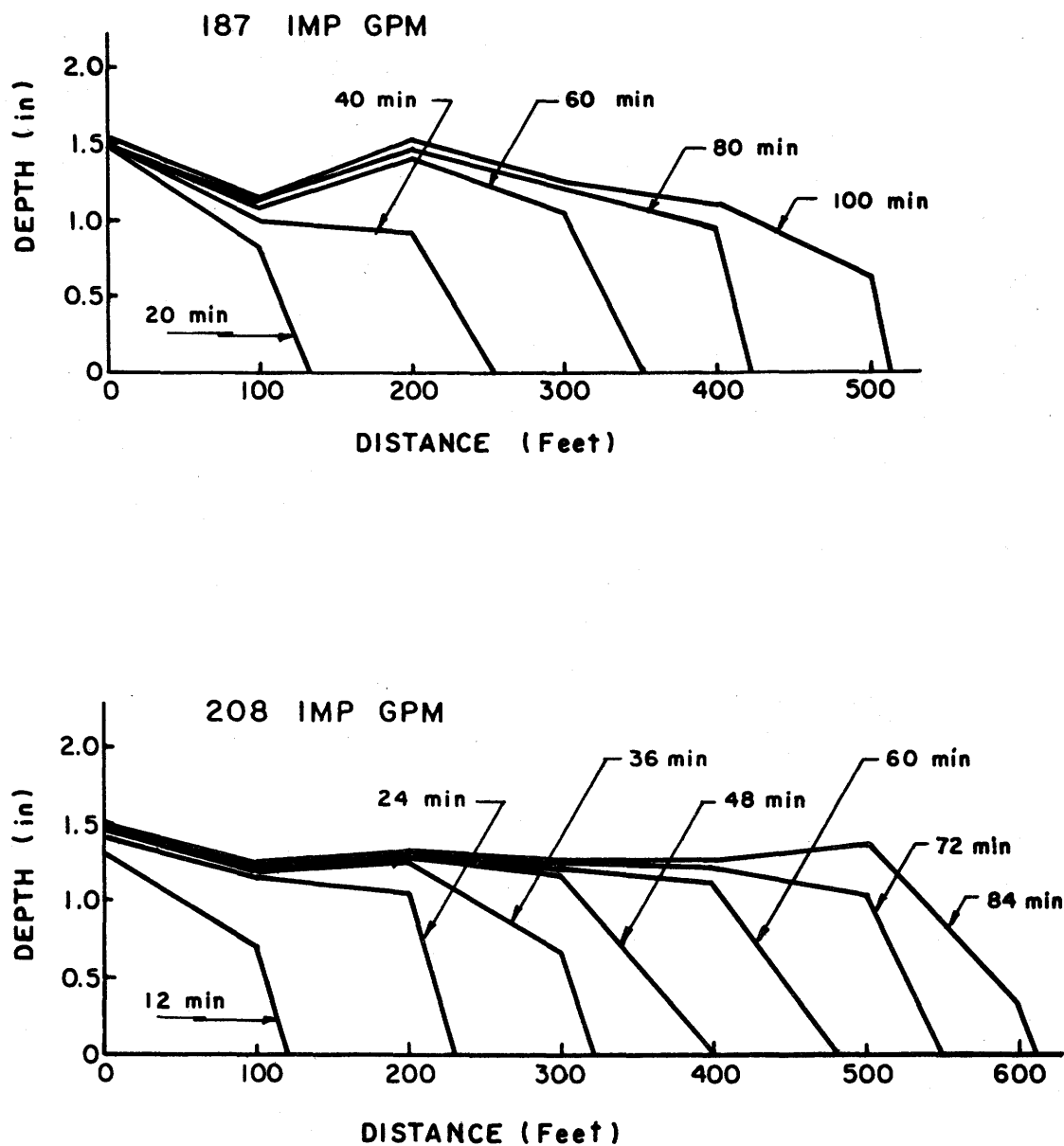


FIG. B-4 LONGITUDINAL PROFILE OF WET FRONT, BORDER No. 1 BATTLEFORD.

## APPENDIX C

Textural Classifications of Soils for  
the Different Border Strips

Table C-1. Mechanical analysis of soils of border 1,  
Outlook

Depth Interval (ft)	Sand (%)	Silt (%)	Clay (%)	Texture <sup>a/</sup>
0 - 1	35	60	5	SiL
1 - 2	34	53	13	SiL
2 - 3	42	36	22	L
3 - 4	40	33	27	CL
4 - 6	57	28	15	SL

<sup>a/</sup>Textural symbols: SiL - silt loam, L - Loam  
CL - clay loam, SiCL - silty clay loam and  
SL - sandy loam

Table C-2. Mechanical analysis of soils of border 2,  
Outlook

Depth Interval (ft)	Sand (%)	Silt (%)	Clay (%)	Texture
0 - 1	40	56.5	3.5	SiL
1 - 2	33	57.5	9.5	SiL
2 - 3	36	48	16	L
3 - 4	46	41	13	L



Table C-3. Mechanical analysis of soils of border 1, Saskatoon

Depth Interval (ft)	Sand (%)	Silt (%)	Clay (%)	Texture
0 - 1	68	22.5	9.5	SL
1 - 2	53	31	16	SL
2 - 3	55	31.5	13.5	SL
3 - 4	64	22.5	13.5	SL

Table C-4. Mechanical analysis of soils of border 2, Saskatoon

Depth Interval (ft)	Sand (%)	Silt (%)	Clay (%)	Texture
0 - 1	68	22.5	9.5	SL
1 - 2	64	26	10	SL
2 - 3	68	24	8	SL
3 - 4	75	17	8	SL

Table C-5. Mechanical analysis of soils of border 1, Battleford

Depth Interval (ft)	Sand (%)	Silt (%)	Clay (%)	Texture
0 - 1	16	58	26	SiL
1 - 6	13	55	32	SicL