

A DIGITAL TRANSFORMER DIFFERENTIAL
AND RESTRICTED EARTH FAULT
RELAY

A Thesis

Submitted to the Faculty of Graduate Studies and Research
in Partial Fulfilment of the Requirements
for the Degree of
Master of Science
in the Department of Electrical Engineering
University of Saskatchewan

by

Dennis Vidyavilas Shah
Saskatoon, Saskatchewan

August, 1981

The author claims copyright. Use shall not be made of the material contained herein without proper acknowledgment, as indicated on the following page.

(ii)

The author has agreed that the Library, University of Saskatchewan, shall make this thesis freely available for inspection. Moreover the author has agreed that permission for extensive copying of this thesis for scholarly purposes may be granted by the professor or professors who supervised the thesis work recorded herein or, in their absence, by the Head of the Department or the Dean of the College in which the thesis work was done. It is understood that due recognition will be given to the author of this thesis and to the University of Saskatchewan in any use of material in this thesis. Copying or publication or any other use of the thesis for financial gain without approval by the University of Saskatchewan and the author's written permission is prohibited.

Requests for permission to copy or to make other use of the material in this thesis in whole or in part should be addressed to:

Head of the Department of Electrical Engineering

University of Saskatchewan

Saskatoon, Sask S7N 0W0

CANADA

ACKNOWLEDGEMENTS

The author wishes to thank Dr. M. S. Sachdev for his guidance throughout the course of this work. His advice and assistance in the preparation of this thesis is thankfully acknowledged. This work was partly supported by a scholarship provided by the Saskatchewan Power Corporation.

UNIVERSITY OF SASKATCHEWAN

Electrical Engineering Abstract 81A212

"A DIGITAL TRANSFORMER DIFFERENTIAL AND
RESTRICTED EARTH FAULT RELAY"

Student: D. V. Shah

Supervisor: Dr. M. S. Sachdev

M.Sc. Thesis presented to the College of Graduate
Studies and Research

August 1981.

ABSTRACT

Many kinds of relays are used to detect faults in power transformers. Two of these relays are the restricted earth fault and transformer differential relays. The concepts of harmonic restraint and variable percentage differential characteristic have been widely used in the conventional transformer differential relays. In the past ten years several methods have been proposed for designing digital transformer differential relays.

This project investigates the feasibility of designing a digital differential and restricted earth fault relay. The differential protection segment includes the harmonic restraint and variable percentage differential characteristic features. Non-recursive digital filters are used to determine the fundamental and harmonic frequency components of the transformer currents. The filters have been designed using the least error squares approach, the Fourier series method and the frequency sampling method. The frequency responses of the designed filters have also been investigated.

A digital logic implementing the restricted earth fault and transformer differential relay has been developed. The relay has been tested using simulated internal faults and magnetizing inrush conditions. Some test results are also included.

TABLE OF CONTENTS

Copyright	Page ii
Acknowledgments	iii
Abstract	iv
Table of Contents	v
List of Figures	vii
List of Tables	xiii
1. <u>INTRODUCTION</u>	
1.1 Historical Background	1
1.2 Digital Processor Relays for Transformer Differential Protection	3
1.2.1 Organization of a typical digital relay	4
1.3 Outline of the Thesis	4
2. <u>TRANSFORMER PROTECTION</u>	
2.1 Protection Philosophy	8
2.2 Faults Affecting the Power Transformer	10
2.3 Transformer Protection	13
2.3.1 Gas and oil surge relay	13
2.3.2 Sudden pressure protection	14
2.3.3 Overcurrent relay	14
2.3.4 Restricted earth fault protection	15
2.3.5 Differential protection	17
2.4 Operating Conditions Adversely Affecting Differential Relays	20
2.4.1 Inrush current	20
2.4.2 Mismatch of current transformers	25
2.4.3 Transformer overexcitation	26
2.4.4 Ratio mismatch	26
2.5 Limitations of Transformer Differential Protection Relays	27
2.6 Differential Relay with Percentage Bias and Harmonic Restraint	30
2.7 Simulation of Inrush and Fault Data	33
2.7.1 Magnetizing inrush	33
2.7.2 Internal faults	37
2.7.3 Simultaneous internal fault and inrush condition	39
2.8 Summary	43
3. <u>REVIEW OF PREVIOUSLY PUBLISHED ALGORITHMS</u>	
3.1 Waveshape Identification Technique	45
3.2 Cross-Correlation Technique	45
3.3 Digital Filtering Technique	53
3.3.1 Recursive filtering technique	54
3.3.2 Nonrecursive filtering technique	56
3.4 Summary	58

4. <u>DESIGN OF NON-RECURSIVE FILTERS</u>	59
4.1 <u>Least Error Squares Approach</u>	
4.1.1 Least error squares solution and the pseudoinverse of a rectangular matrix	61
4.1.2 Filter designs	68
4.1.3 Frequency response of a digital filter	71
4.2 <u>Fourier Series Approach</u>	81
4.2.1 Filters designed by the Fourier series method	89
4.3 <u>Frequency Sampling Approach</u>	93
4.3.1 Filters designed by the frequency sampling method	96
4.4 <u>Window Functions</u>	101
4.5 <u>Filter Evaluation</u>	101
4.6 <u>Summary</u>	105
 5. <u>DIFFERENTIAL AND RESTRICTED EARTH FAULT RELAY DESIGN</u>	
5.1 <u>Transformer Differential Relay</u>	106
5.2 <u>Restricted Earth Fault Protection</u>	110
5.3 <u>Digital Relay Design</u>	112
5.3.1 Starting relay design	113
5.3.2 Restricted earth fault logic	115
5.3.3 Differential protection logic	117
5.3.4 Relay characteristic	123
5.4 <u>Digital Relay Algorithm</u>	128
5.4.1 Starting element	128
5.4.2 Restricted earth fault relay	129
5.4.3 Differential relay	130
5.5 <u>Off-line Testing</u>	137
5.6 <u>Summary</u>	154
 6. <u>CONCLUSIONS</u>	155
 7. <u>REFERENCES</u>	158
 8. <u>APPENDICES</u>	
Appendix A	161
A.1 Simulated Magnetizing and Fault Current Waveforms	161
A.2 Elements of the Left Pseudoinverse of Rectangular Matrix	161
Appendix B; Application of the Window Function to Digital Filters	169
Appendix C; Approximating the Variable Percentage Differential Characteristics	173

LIST OF FIGURES

	Page
Figure 1.1 Functional block diagram of a digital processor relay.	5
Figure 2.1 A typical power system and its zones of protection.	9
Figure 2.2 Restricted earth fault scheme for a delta/bye transformer.	16
Figure 2.3 Differential current protection scheme for a delta/bye transformer.	19
Figure 2.4 Hysteresis loop for transformer iron core.	22
Figure 2.5 Residual flux and magnetizing inrush phenomena.	24
Figure 2.6 Earth fault on the bye winding of a delta/bye transformer.	29
Figure 2.7 Amount of winding protected when transformer is resistance earthed.	29
Figure 2.8 Variable percentage differential characteristic.	32
Figure 2.9 Equivalent circuit for modelling magnetizing inrush current.	39
Figure 2.10 Idealized saturation curve for transformer iron core.	34
Figure 2.11 Magnetizing inrush current waveform.	38
Figure 2.12 Interturn fault in a power transformer.	40

Figure 2.13	Equivalent circuit for modelling simultaneous internal fault and inrush currents.	40
Figure 2.14	Simultaneous inrush and internal fault current waveform.	42
Figure 3.1	Sine and cosine time domain orthogonal functions.	47
Figure 3.2	Time domain orthogonal square waves.	49
Figure 4.1	Frequency response of the 60 Hz digital filter designed by the least error squares approach whose coefficients are listed in column one of Table 4.2.	73
Figure 4.2	Frequency response of the 60 Hz digital filter designed by the least error squares approach whose coefficients are listed in column two of Table 4.2.	73
Figure 4.3	Frequency response of the 120 Hz digital filter designed by the least error squares approach whose coefficients are listed in column three of Table 4.2.	74
Figure 4.4	Frequency response of the 120 Hz digital filter designed by the least error squares approach whose coefficients are listed in column four of Table 4.2.	74
Figure 4.5	Frequency response of the 60 Hz digital filter designed by the least error squares approach whose coefficients are listed in column one of Table 4.3.	76
Figure 4.6	Frequency response of the 60 Hz digital filter designed by the least error squares approach whose coefficients are listed in column two of Table 4.3.	76

Figure 4.7	Frequency response of the 120 Hz digital filter designed by the least error squares approach whose coefficients are listed in column three of Table 4.3.	77
Figure 4.8	Frequency response of the 120 Hz digital filter designed by the least error squares approach whose coefficients are listed in column four of Table 4.3.	77
Figure 4.9	Average value of the composite filter response.	80
Figure 4.10	Standard deviation of the composite filter response.	80
Figure 4.11	Ideal frequency response of a band-pass filter.	82
Figure 4.12	The 'even' frequency response characteristics of an ideal band-pass filter.	87
Figure 4.13	The 'odd' frequency response characteristics of an ideal band-pass filter.	87
Figure 4.14	Frequency responses of the fundamental frequency 'even' and 'odd' band-pass filters designed by the Fourier series method.	91
Figure 4.15	Frequency response of the second harmonic frequency 'even' and 'odd' band-pass filters designed by the Fouruer series method.	92
Figure 4.16	Frequency samples of the 'even' frequency response characteristics of an ideal band-pass filter.	97
Figure 4.17	Frequency samples of the 'odd' frequency response characteristics of an ideal band-pass filter.	97

(x)

Figure 4.18	Frequency response of the 60 Hz digital filter designed by the frequency sampling method whose coefficients are listed in column one of Table 4.6.	99
Figure 4.19	Frequency response of the 60 Hz digital filter designed by the frequency sampling method whose coefficients are listed in column two of Table 4.6.	99
Figure 4.20	Frequency response of the 120 Hz digital filter designed by the frequency sampling method whose coefficients are listed in column three of Table 4.6.	100
Figure 4.21	Frequency response of the 120 Hz digital filter designed by the frequency sampling method whose coefficients are listed in column four of Table 4.6.	100
Figure 5.1	Connection diagram for differential protection of a delta/bye transformer.	107
Figure 5.2	Current balancing for restricted earth fault protection of the wye winding of a delta-wye transformer.	111
Figure 5.3	Variable percentage differential characteristics.	126
Figure 5.4	Parameters of the variable percentage differential characteristic.	127
Figure 5.5	Overview of the transformer differential and restricted earth fault protection.	132
Figure 5.6	Flowchart used for implementing the restricted earth fault protection.	133
Figure 5.7	Flowchart used for detecting inrush currents.	134

- Figure 5.8 Flowchart used to detect a possible fault condition within the zone of the transformer. 135
- Figure 5.9 Flowchart used to implement the variable percentage characteristics. 136
- Figure 5.10 Second harmonic (plot A) and third harmonic (plot B) components of the magnetizing inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the Fourier series method. 142
- Figure 5.11 Second harmonic (plot A) and third harmonic (plot B) components of the simultaneous internal fault and inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the Fourier series method. 143
- Figure 5.12 Second harmonic (plot A) and third harmonic (plot B) components of the magnetizing inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the frequency sampling method. 144
- Figure 5.13 Second harmonic (plot A) and third harmonic (plot B) components of the simultaneous internal fault and inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the frequency sampling method. 145
- Figure 5.14 Second harmonic (plot A) and third harmonic (plot B) components of the magnetizing inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the least error squares approach. 146

- Figure 5.15 Second harmonic (plot A) and third harmonic (plot B) components of the simultaneous internal fault and inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the least error squares approach. 147
- Figure B.1 Frequency response of the 60 Hz digital filter with the Hamming's weighting applied to the filter coefficients. 172
- Figure C.1 Two-piece piecewise linear differential relay characteristics. 176
- Figure C.2 The true and approximate differential protection characteristics. 176

LIST OF TABLES

	Page
Table 4.1 Combination of variable parameters used in the design of filters by the least error squares approach.	62
Table 4.2 The elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental, second and third harmonic frequencies.	70
Table 4.3 The elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of fundamental, second, third and fourth harmonic frequencies.	70
Table 4.4 Filter coefficients of the fundamental and second harmonic digital filters designed by the Fourier series method.	90
Table 4.5 Design specifications for the fundamental and second harmonic digital filters.	98
Table 4.6 Filter coefficients of the fundamental and second harmonic digital filter designed by the frequency sampling method.	98
Table 4.7 Digital filter performance indices for the designed filters.	104
Table 5.1 Analysis of the differential current and the corresponding threshold values of the variable percentage differential characteristic for a digital relay that uses non-recursive filters designed by the frequency sampling method.	150

Table 5.2	Analysis of the differential current and the corresponding threshold values of the variable percentage differential characteristic for a digital relay that uses non-recursive filters designed by the least error squares method.	152
Table 5.3	The restricted earth fault setting and the restricted earth fault relay operating current ($I_n - 3I_0$) obtained by non-recursive digital filters designed by the frequency sampling method and the least error squares method.	153
Table A.1	Instantaneous values of the simulated magnetizing inrush, simultaneous magnetizing inrush and winding fault and winding fault current waveforms sampled at 720 Hz.	162
Table A.2	Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental and second harmonic frequencies.	166
Table A.3	Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental and second harmonic frequencies.	166
Table A.4	Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental, second, third and fourth harmonic frequencies.	167
Table A.5	Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated three terms representing the decaying d.c. component and components of the fundamental, second, third and fourth harmonic frequencies.	167

Table A.6 Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental, second, third and fourth harmonic frequencies.

168

Table A.7 Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental, second, third and fourth harmonic frequencies.

168

Table B.1 Coefficients of the fundamental frequency band-pass filters before and after the application of the Hamming window function.

172

1. INTRODUCTION

1.1 Historical Background

One of the major functions of protective devices is to detect the occurrence of a power system fault, such as a short circuit, and to isolate the faulted section preventing deterioration of the power system operating state. Early developments in this regard were the development of fuses which were the first automatic devices to be used for isolating the faulted equipment. They were, and still are being used effectively for the protection of distribution circuits and subtransmission lines. The major disadvantage of using a fuse is that it has to be replaced every time it interrupts current in a circuit. Another major disadvantage of fuses is that they cannot discriminate easily the faulted zones from the unfaulted zones in a complex power system.

A significant improvement in power system protection was achieved with the introduction of automatic circuit-breakers with trip coils energized by system currents. This was soon followed by the development of protective relays whose operation initiated tripping of circuit breaker(s). The early relays which responded to excessive short circuit currents, were of the attracted

armature type. Protective devices with greater sensitivity, selectivity and speed were developed to satisfy the requirements of increasing complexity of power systems. The development of induction disc relays was followed by the balanced beam and induction cup types. Use of individual relays culminated in the development of complex protective schemes incorporating several relays for performing sophisticated protection and control tasks.

Solid state relays were introduced in 1950's. In their early stages of development, these relays used thermionic components. They did not, however, receive widespread acceptance because of the high failure rates of the thermionic components. Later developments used semiconductor technology. Because of lower failure rates of the semiconductors and smaller power consumption, many solid state relays were developed and implemented. Solid state devices have been used in the design of many protective schemes that include phase and amplitude comparators. These devices require minimum maintenance and are free from several inherent disadvantages of the electromechanical relays.

Rapid advances in digital processor technology have prompted the application of these devices to industrial process control. The use of real time digital computers for system protection was first proposed in 1966 by Last and

Stalewsky⁸. In a remarkable paper, Rockefeller¹⁵ proposed an overall philosophy for using a digital computer to protect all the equipments in a substation. Mann and Morrison¹⁶ later demonstrated the feasibility of protecting a transmission line by a digital computer. Many researchers are of the view that the long range economic trend favors the use of digital processors for protection and control. New and ingenious algorithms and processing techniques have already been proposed¹⁰.

1.2 Digital Processor Relays for Transformer Differential Protection

In 1969, Rockefeller proposed a method for identifying magnetizing inrush from transformer faults and proposed an algorithm for transformer differential protection. Since then several schemes have been reported in the literature^{7,11,24}. The object of this project is to study the previously proposed algorithms and design a digital transformer differential relay that might be suitable for replacing the presently used electromechanical and solidstate devices. The relay design can be divided into two main tasks; the design of digital filters and the development of relay algorithm and program.

1.2.1 Organization of a typical digital relay

Figure 1.1 is a block diagram which depicts the major processes associated with a digital relay. An analog input subsystem accepts ac signals from power system transducers, such as conventional current transformers and capacitive current and voltage transformers. The analog signals are processed by lowpass filters to bandlimit the signals. The processed signals are sampled at a prespecified rate. The desired cut-off frequency of the analog filters and the sampling rate are inter-dependent. All analog signals are sampled simultaneously by sample and hold devices, are converted to equivalent digital numbers and are then transferred to the processor memory/cpu. The processor stores, organizes, computes and makes decision based on the sampled values of the signals.

1.3 Outline of the Thesis

This thesis is organized into six chapters and three appendices. Chapter 2 reviews briefly the philosophy generally used for protecting power system elements. Faults affecting power transformers and methods used to detect these faults are outlined briefly. Transformers usually experience magnetizing inrush when switched on to an energy source. This condition is a transient which lasts for a short duration only and has to be distinguished from

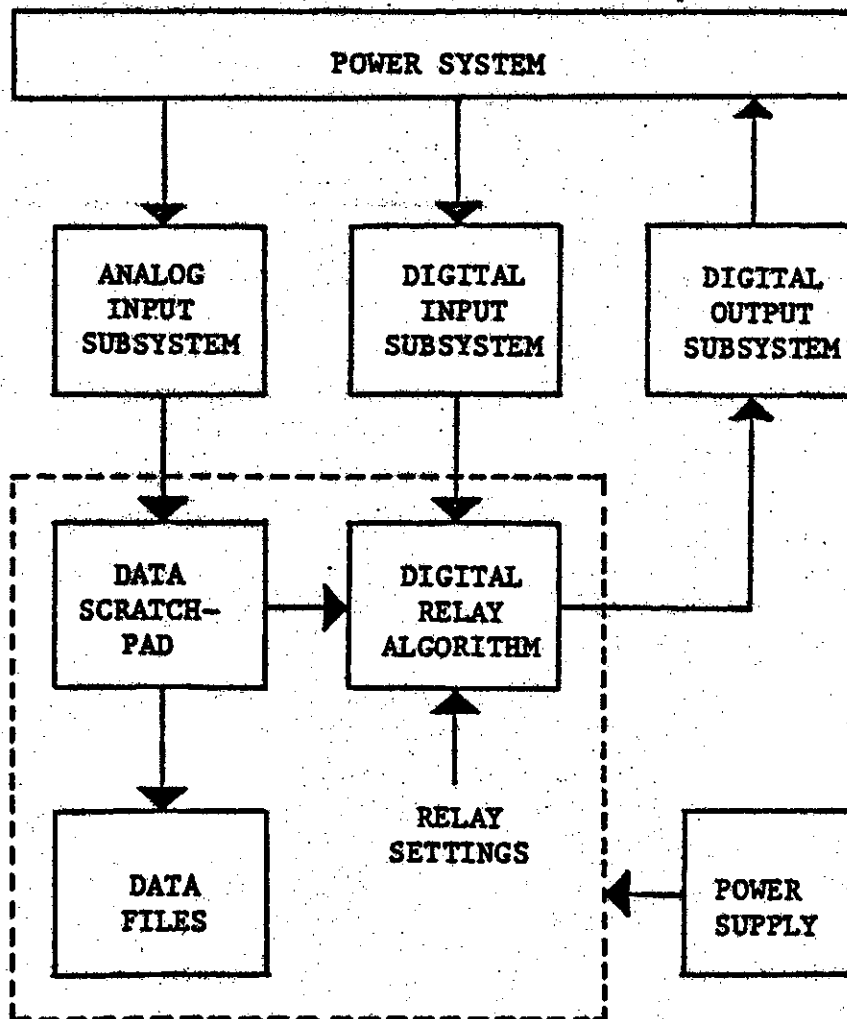


Figure 1.1 Functional block diagram of a digital processor relay.

internal faults. Methods of mathematically simulating internal faults, magnetizing inrush and simultaneous fault and magnetizing inrush are described. Some system data was generated using these methods and was later used in testing the relay.

Chapter 3 presents the review of the previously published techniques used for digital differential protection of transformers. Algorithms reviewed in this chapter use a simple waveshape identification method and digital filtering techniques. Both recursive and non-recursive filters have been used in the work reported in the literature.

Chapter 4 briefly presents three techniques of designing non-recursive digital filters. These techniques are based on the least error squares approach, Fourier series and frequency sampling methods. Frequency response of the digital filters designed by these methods are examined. Methods of evaluating the filter responses are also described in this chapter.

The design philosophies used in the proposed digital relay are discussed in Chapter 5. The development of the variable percentage and harmonic restraint characteristic is also presented. Also outlined is the relay algorithm. Three versions of the relay were programmed; each using

digital filters designed by one of the three techniques described in Chapter 4. The relay was tested using the simulated data generated in Chapter 2. Some test results are included in this chapter.

Chapter 6 presents a brief summary and conclusions of the studies reported in this thesis. A list of references is given in Chapter 7. The references are listed in alphabetical order of the last name of the first author.

Chapter 8 includes three appendices. Appendix A contains the data representing magnetizing inrush and fault current waveforms that are used in the off-line testing of the digital relay. This appendix also lists the elements of the filters designed by the least error squares approach. Appendix B briefly describes the use of weighting functions with non-recursive digital filters. Appendix C shows how the variable percentage differential characteristic can be implemented effeciently reducing computations in the relay algorithm.

2. TRANSFORMER PROTECTION

The philosophy of power system protection is briefly described in this chapter. Faults affecting the performance of power transformers are identified. Methods used for detecting transformer faults and isolating the affected transformer from the remaining system are reviewed with particular emphasis on differential current relaying. The operating conditions which adversely affect the performance of differential relays are outlined. Limitations of transformer differential protection are also examined. Mathematical models for simulating current waveform representing magnetizing inrush, winding faults and simultaneous magnetizing inrush and winding faults are then established.

2.1 Protection Philosophy

Fault sensing and switching devices are provided in power systems to limit equipment damage during faults and overloads, and to minimize the disturbing effects of faults on the system²⁵. This is accomplished by dividing the system into protective zones. A zone usually includes one major element and is separated from the remaining system by circuit breakers as shown in Figure 2.1. Protective relays

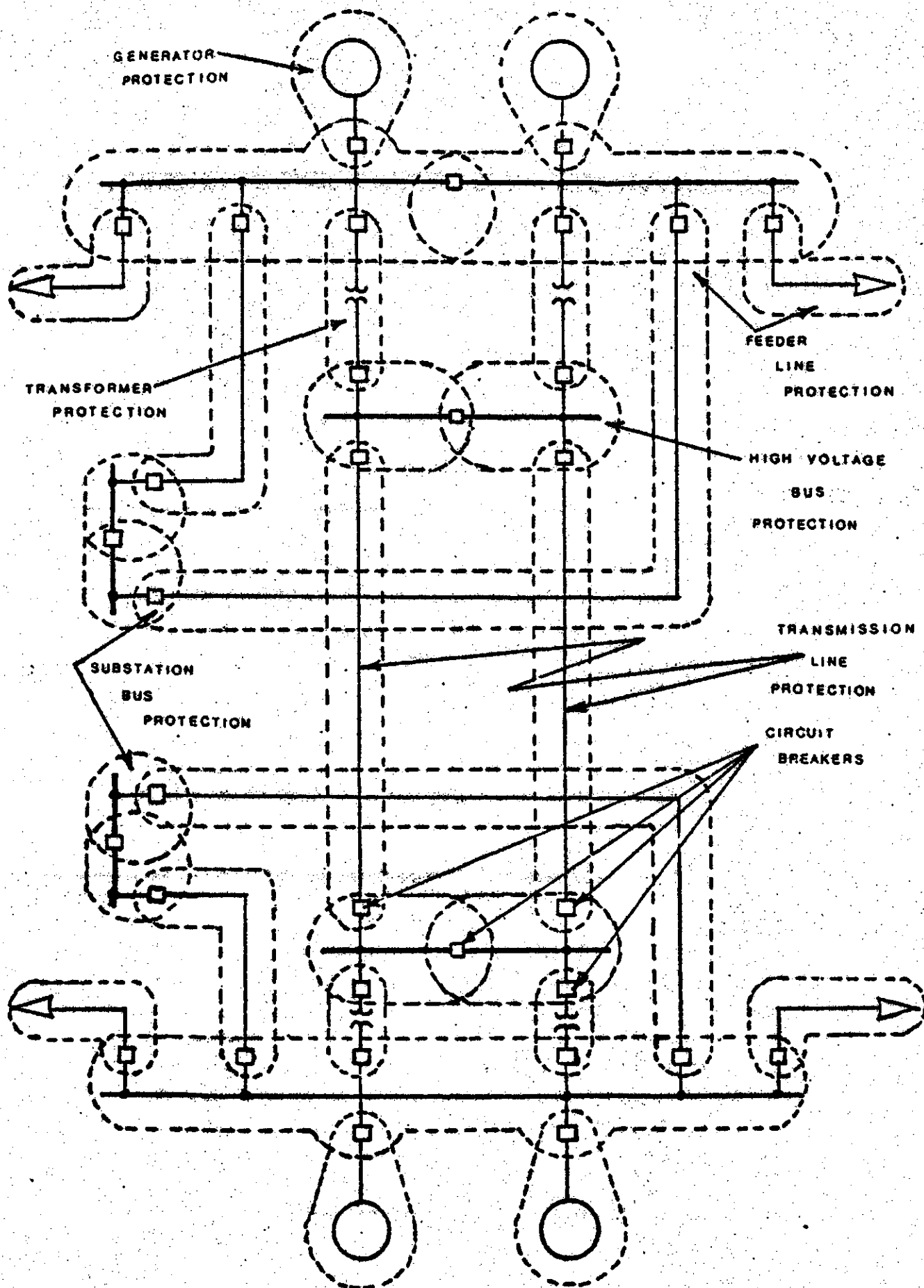


Figure 2.1 A typical power system and its zones of protection.

are deployed to achieve two objectives. The first objective is to detect the existence of a fault and identify the zone in which it has occurred. The second objective is to open circuit breakers to isolate the faulted zone. In addition to these actions, the protective relay may alert an operator and start equipment for recording the system currents and voltages during the fault.

Protective relays are excited by the outputs of current and voltage transformers which are in turn connected to the system element that is to be protected. An auxiliary source provides energy required to open circuit breakers. In power systems, the continuity of supply is an important consideration and precautions are, therefore, taken to avoid unwarranted isolation of the system elements.

2.2 Faults Affecting the Power Transformer

Faults affecting a power transformer can be divided into two categories: through faults and internal faults. Faults outside the protective zone of the transformer are classified as through faults and the faults in the protected zone of the transformer are classified as internal faults. Most faults are associated with excessive currents flowing in the windings of transformers. These currents usually produce mechanical and thermal stresses in the transformer windings and also in the connecting bus bars.

It is not essential to disconnect a transformer immediately from the system when a through fault occurs or when the transformer is overloaded. However, if the fault is not isolated within a predetermined time, the transformer must be disconnected to protect from serious damage.

Usually a transformer can sustain overloads for long periods of time which depend on the permitted temperature rise in the windings and the cooling medium. Excessive overloads cause insulation deterioration and subsequent failure.

External short-circuits are sometimes limited by the transformer reactance only. Since this reactance is small, fault currents in these cases are excessive. If these currents are allowed to persist for more than a few cycles (at 60 Hz), they can cause extensive damage to the transformer.

Internal faults can be subdivided into incipient faults and phase to phase, phase to ground and interturn faults. The main purpose of transformer relays is to identify conditions which arise from faults inside the transformer zone and open circuit breakers to isolate the zone thus minimizing the damage.

Phase to phase faults on the high voltage or low voltage terminals of transformers are rare, but single phase to ground faults in tap changers sometimes develop into phase to phase faults. These faults cause substantial reductions in the system voltage causing further complications.

Phase to ground faults on the high voltage and low voltage windings of transformers are mainly due to bushing failures and flashovers within the transformer tank. The severity of these faults depend on the the design of the transformer and the method of grounding the system neutral.

Interturn faults in the high voltage and low voltage windings cause localized burning of the conductors of the affected coil, and by charring and breakdown of the interturn insulation of the coil. Initial interturn insulation failure does not draw sufficient current from the mains and is hard to detect until the fault extends to embrace a considerable portion of the affected winding. The majority of the internal faults which occur within the winding are interturn faults.

Incipient faults are also internal faults which constitute no immediate hazard. However if they are left undetected they may develop into major faults. Majority of faults in this category are core faults; due to failure of

the insulation between the core laminations and deterioration of transformer oil due to overheating.

*Shall be
commented
14 moisture
metal
particles*

2.3 Transformer Protection

The choice of protection systems selected for application to a transformer is influenced by the size and importance of the transformer. Small transformers are usually protected by overcurrent and instantaneous earth fault relays only. For larger transformers restricted earth fault relays and gas and oil surge detection relays are provided. Differential protection is used for transformers of over 20 MVA rating²⁵.

2.3.1 Gas and oil surge relays

Most power transformers are oil immersed. An internal fault in such a transformer is often accompanied by violent release of gas. The heavy short-circuits rapidly increase the oil temperature to the vaporizing temperature creating pressure surge in the transformer tank forcing oil to the conservator. Most incipient faults can be detected only by monitoring the release of gas. Other types of protection are either nonresponsive or are not sufficiently sensitive.

The device used to detect an incipient fault is known as the Bucholz relay. It is fitted between the transformer

and its conservator tank. It also detects excessive leakage of oil from the transformer tank by detecting a drop in the oil level in the tank. This relay is usually applied to supplement the differential protection of power transformers.

2.3.2 Sudden pressure protection

Some transformers have a gas cushion instead of a conservator tank. The Bucholz relay can not be applied in these cases and is replaced by a sudden pressure relay. This device is mounted on the tank, above the oil level and monitors the rate of rise of the pressure in the transformer. It does not operate on static pressure or small pressure changes resulting from normal operation of the transformer. The sudden pressure relay is more sensitive to light internal faults than the differential relay. Its operating time varies from 1/2 cycle to 37 cycles depending on the severity of the fault²⁵.

2.3.3 Overcurrent relay

A fault external to a transformer results in an overload which can cause transformer failure if the fault is not cleared promptly. Overcurrent relays are often used to isolate the transformer before it is damaged. In the case of small transformers, overcurrent relays are used for both

overload and fault protection. An extremely inverse time-overcurrent characteristic is preferable for overload and light faults, with an instantaneous overcurrent unit for heavy faults. On large transformers, overcurrent relays provide backup for the differential relays. Overcurrent relays are simple and inexpensive, but their application is limited by their low resolution of the setting and the necessity of providing time delay for coordination of relays.

2.3.4 Restricted earth fault relay

The most common type of faults in power transformers are single phase to ground faults. These can be best detected by using restricted earth fault relays. The principle used is current balance. (A typical restricted earth fault relay applied to the wye side of a three phase two winding transformer is shown in Figure 2.2. In this relay, the three line currents are balanced against the current in the transformer neutral. For a line to ground fault outside the zone of the restricted earth fault relay, the neutral current is the same as $3I_0$ at the transformer terminals. Current in the operating coil of the restricted earth fault relay is, therefore, zero. However, in case of a phase to ground fault in the protected zone, the neutral current will not be balanced by $3I_0$ at the transformer terminals. Current would, therefore, flow in the restricted

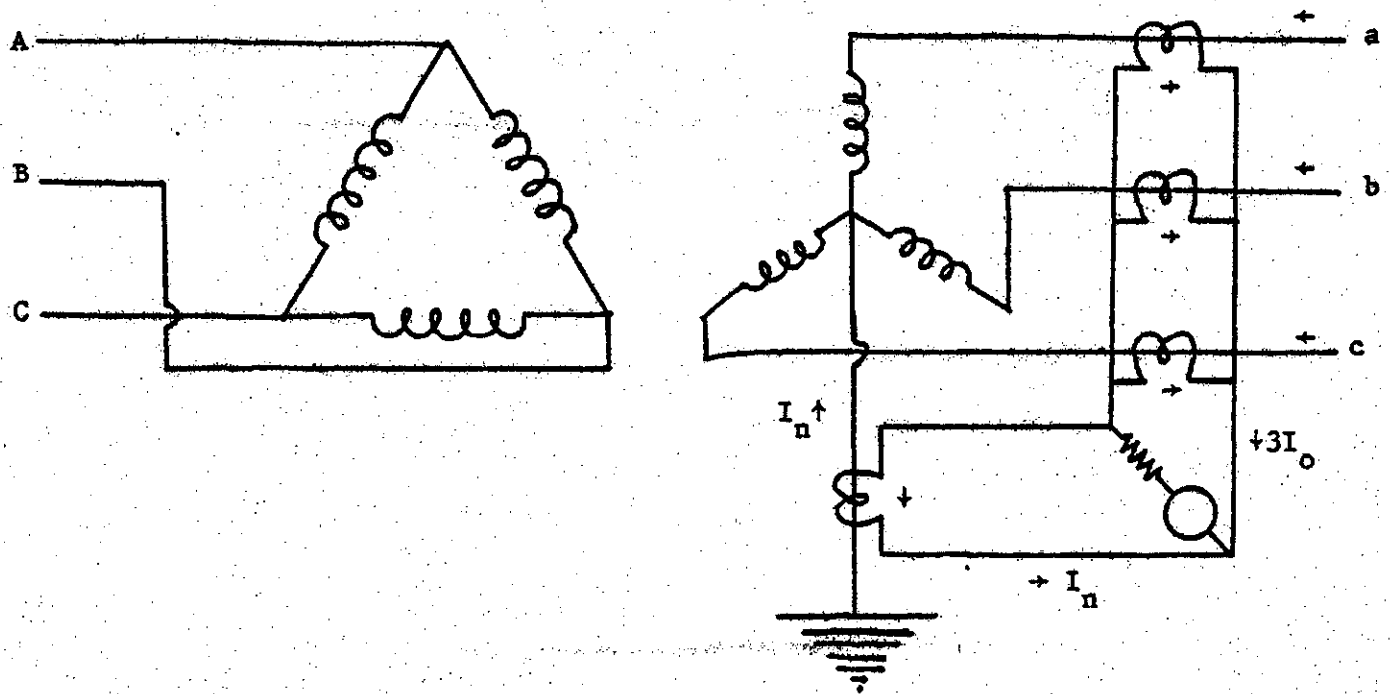


Figure 2.2 Restricted earth fault scheme for a delta/wye transformer.

earth fault relay which would operate. This restricted earth fault protection will, however, not respond to two phase and three phase faults on the wye side which may be inside or outside the protected zone of the relay.

The extent of the winding protected by a restricted earth fault relay applied to the wye side of a transformer depends on the selected rating of the neutral earthing resistor and the relay setting.

The restricted earth fault protection applied to the delta side of a wye/delta transformer, the three line c.t.'s are connected in parallel with the operating element of the relay. This scheme responds to earth faults on the delta side but does not respond to earth faults on the wye side of the transformer.) ✓

2.3.5 Differential protection

Manufacturers of transformers usually recommend that a percentage differential relay be used for all power transformers of 1000 kVA and higher ratings ^{5,25}. The principle of differential protection compares the outputs of current transformers installed on the primary and secondary sides of a transformer. The connections are selected in such a manner that no current flows in the operating element of the relay during normal power flow conditions and

external faults. Figure 2.3 depicts a typical differential protection arrangement for a three phase two winding transformer. In case of the wye/delta transformer, the line currents on the primary and the secondary side of the transformer are not in phase with each other, but are displaced by 30 degrees. To eliminate this discrepancy the c.t. secondaries are connected in delta on the wye connected side and are wye connected on the delta connected side. Also, the currents at the terminals of a one to one ratio wye/delta transformer are equal. The outputs of the delta connected c.t.'s are $\sqrt{3}$ times larger than the outputs of the wye connected c.t.'s. This difference is compensated by using suitable c.t. ratios.

During normal operating conditions and external faults, the transformer primary and secondary currents are balanced in phase and magnitude. The c.t. secondary currents, therefore, circulate only through the restraining element of the differential relay. A change in the power flow conditions which upsets the balance causes current to flow in the operating winding of the relay.

The presence of differential current or a sudden change in its magnitude is not, by itself, an infallible indication of the presence of a fault in the transformer zone. In practice, the operation of a differential protection relay is adversely affected by several factors which are discussed

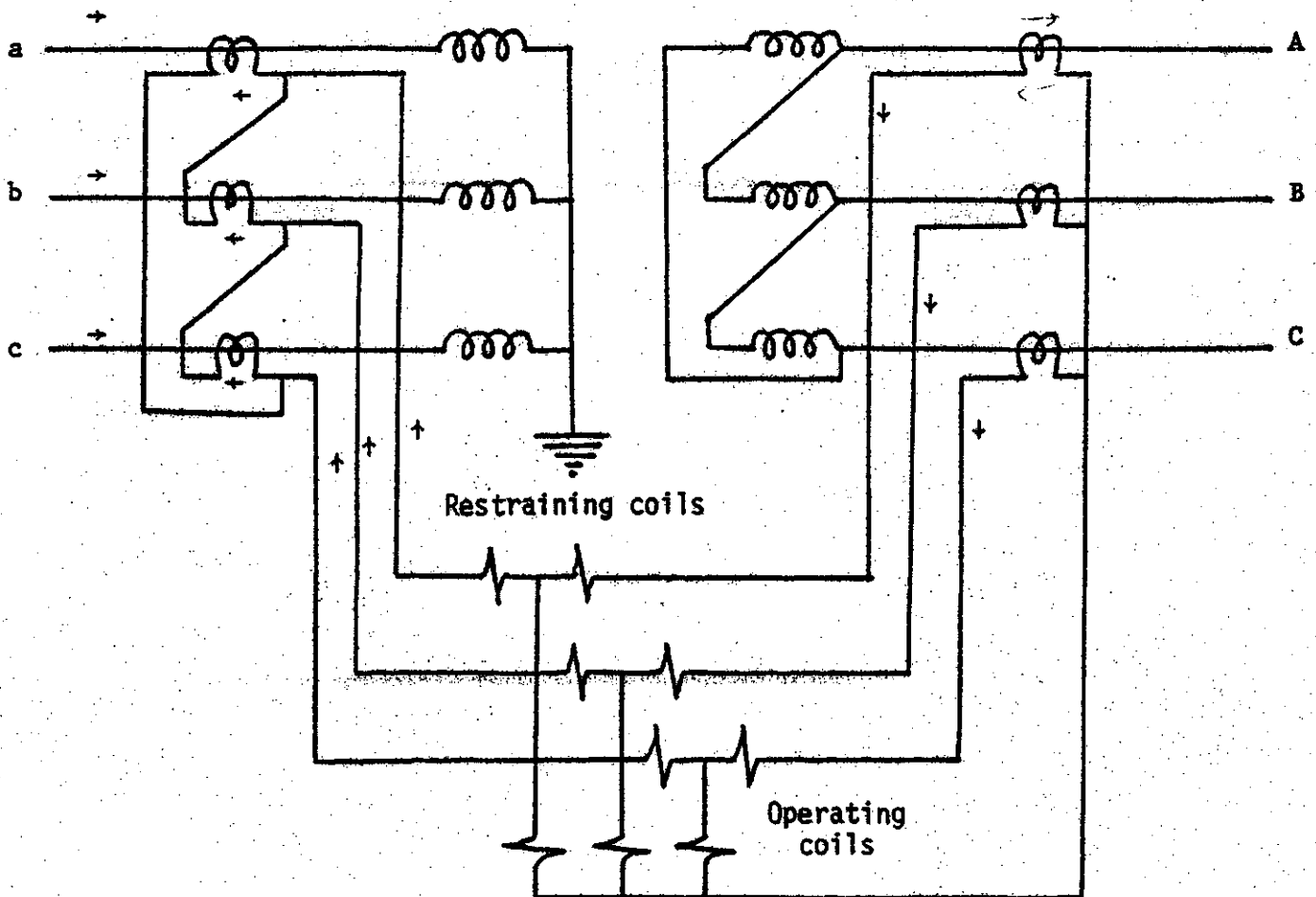


Figure 2.3 Differential current protection scheme for a delta/ye transformer.

in the next section.

2.4 ~~Operating Conditions Adversely Affecting Differential~~ Relays

The operation of transformer differential relays is adversely affected by four major non-linear phenomena. These are: magnetizing inrush, differences in the characteristics of c.t.'s used on the primary and secondary side of the transformer, overexcitation and c.t. ratio mismatch^{5,25}. These phenomena cause flow of currents in the operating coils of differential relays even when the transformers are not faulted. Consequently, transformers may be removed from service. Transformer differential relays with percentage and harmonic restraint features are immune to the adverse effects of these phenomena. The phenomena of magnetizing inrush, c.t. mismatch, overexcitation and ratio mismatch are briefly examined in this section. The objective is to illustrate the significance of including the percentage bias and harmonic restraint features in a relay design.

2.4.1 Inrush current

A transformer may experience magnetizing inrush current of peak values which may be several times the rated full load current. The magnitudes of inrush currents depend on²²

- (i) the excitation impedance which is very low when the transformer core is saturated.
- (ii) the residual flux in the transformer core.
- (iii) the instant of the voltage waveform when the transformer is connected to the power source.

Due to the saturation of the transformer core, the shape of the inrush current wave is highly distorted. Since the inrush currents flow in either the primary or the secondary windings of a transformer, they appear to the differential relays as operating currents.

When a transformer is energized from an a.c. power source, the waveform of the steady state flux in the core is in quadrature with the source voltage E . It is well known that if an alternating supply is switched to a highly inductive circuit at the instant when the voltage is passing through zero, flux doubling will occur²². If a transformer is energized at an instant of the voltage waveform which normally corresponds to the residual flux density within the core, the switching in would be a smooth continuation of a previous operation and will not give rise to magnetic transients. However, in practice, the instant of switching is uncontrolled, and therefore magnetizing transients are unavoidable. To understand the phenomenon, consider Figure 2.4 which shows a typical B/H curve of a transformer core.

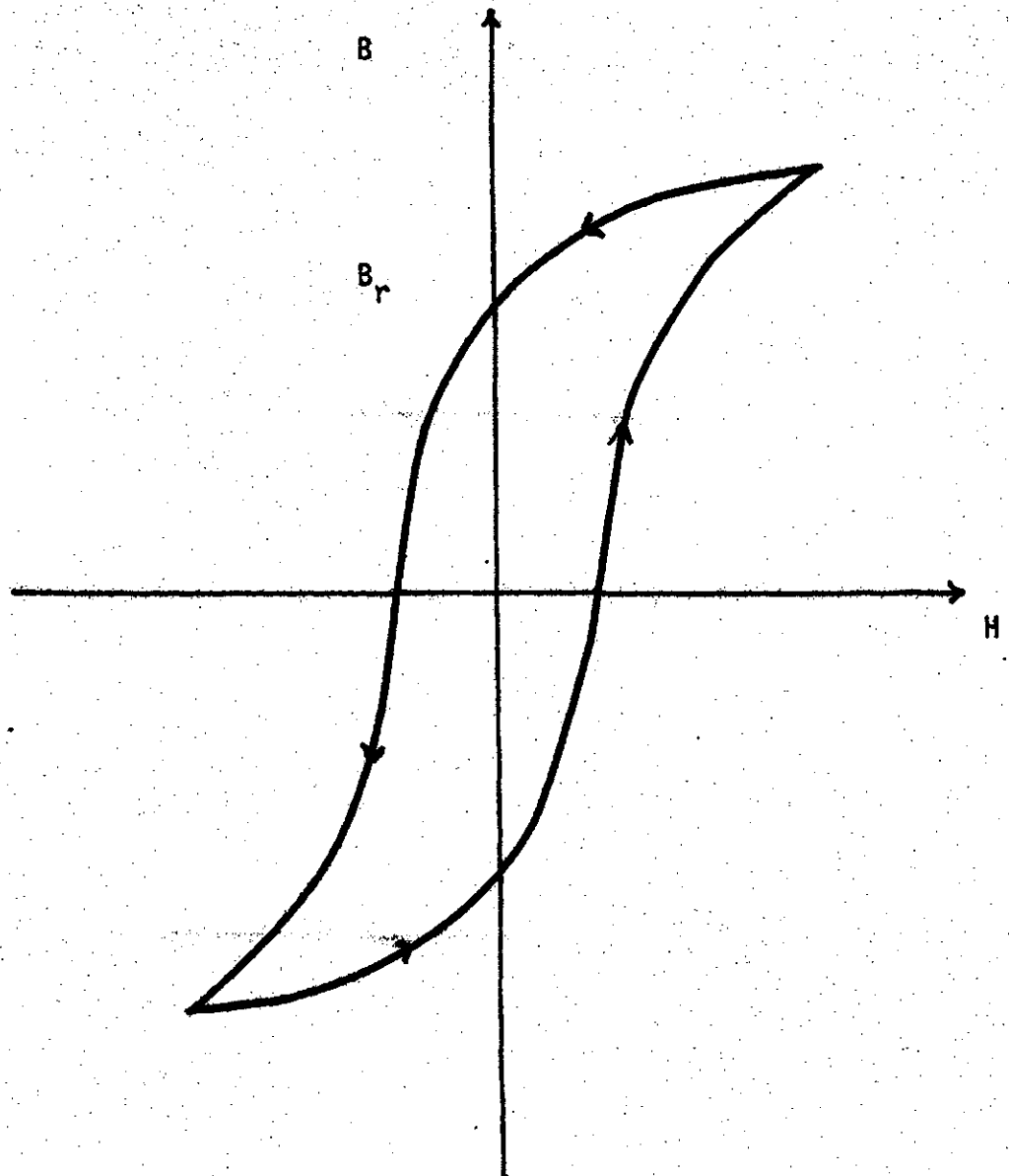


Figure 2.4 Hysteresis loop for transformer iron core.

When the power supply is switched off, the magnetizing current decreases to zero while the flux density follows the hysteresis loop. At current interruption the flux density in the core corresponds to B_r shown in Figure 2.4.

Figure 2.5 shows the waveforms of the magnetizing current and the flux density in the transformer core. In this case the current was interrupted at the instant of its natural negative going zero crossing. The core flux at current interruption had a value $+B_r$ - the residual flux. Since the transformer was considered to have been switched off and the magnetizing current was assumed to have been interrupted, the core flux remains at $+B_r$ and the current continues to be zero. If the transformer were not switched off, the current and flux density waveforms would have followed the dotted curves. Now assume that the transformer is re-energized at a later instant, t_2 , when the flux density would have been normally at its negative maximum value, $-B_{max}$. The magnetic flux which is in the transformer core cannot change instantaneously. The flux wave, therefore, starts from the residual value B_r and follows the curve B_3 . This curve is a sinusoid riding a d.c. offset. The transformer core saturates; it does not modify the flux. Instead, the magnetizing current required to produce the flux is drawn from the electrical source. This current is substantially greater than the normal magnetizing current I_1 shown in Figure 2.5.

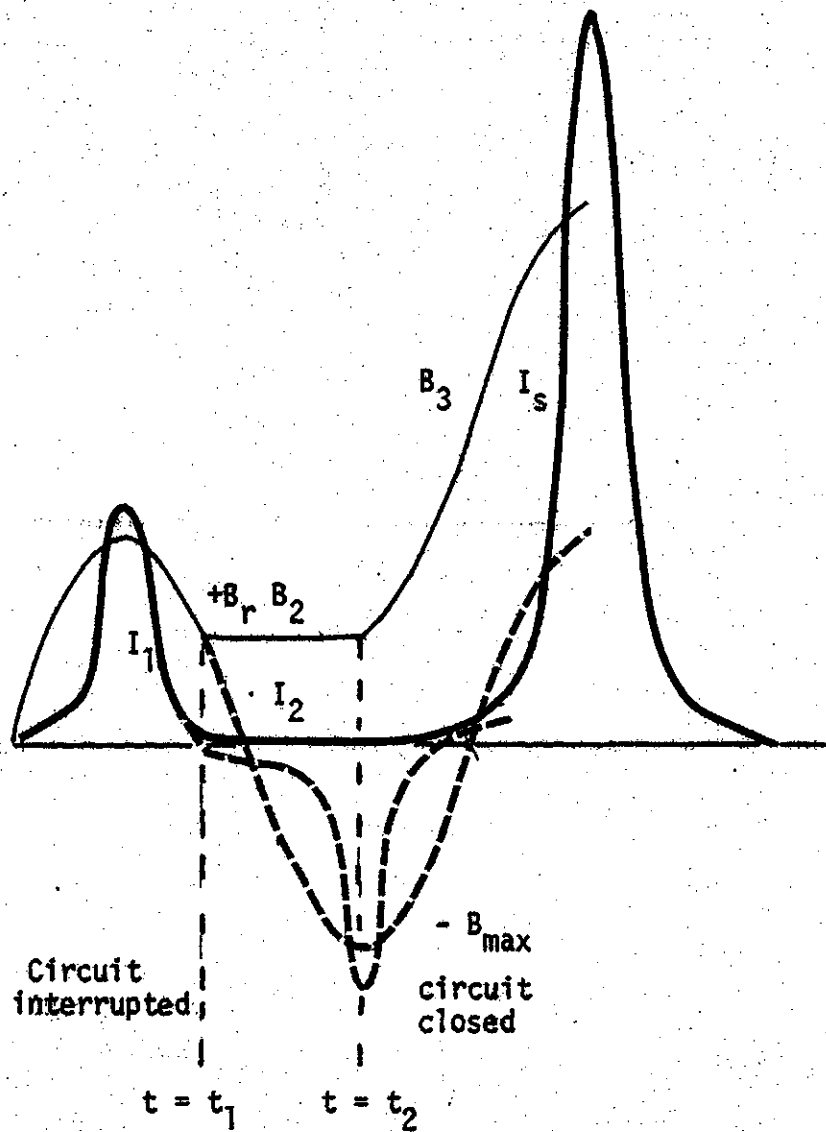


Figure 2.5 Residual flux and magnetizing inrush phenomena.

The peak value of the inrush current decays gradually due to the resistance in the primary circuit. The voltage impressed on the primary winding of the transformer is modified by the voltage drop in the ^{series} resistance. The rate of decay of the transient inrush phenomena is large during the first few cycles because of the shorter time constant of the circuit.

- Symmetrical inrush, clearing of a fault (external) gives inrush.

2.4.2 Mis-match of current transformers

Matching of the characteristics of the primary and secondary c.t.'s is difficult because of differences in the designs of c.t.'s for use on different voltage levels. The practice of utilizing materials to the limits of their capabilities has resulted in most commercially available current transformers having a tendency to saturate even at moderate overloads. Saturation manifests in the c.t. secondary currents which include harmonic components. The lengths of leads connecting the current transformers to the relay are usually unequal. The primary and secondary c.t.'s, therefore, feed loads of different VA burdens. This unbalance causes the c.t.'s to operate at different ratio error levels which results in currents flowing in the operating elements of the differential relay.

2.4.3 Transformer overexcitation

A transformer may be subjected to sustained overvoltage on load rejection or on clearing of an external fault. During these periods the flux density in the core increases resulting in the transformer drawing large magnetizing currents which include components of odd harmonics.

2.4.4 Ratio mismatch

Another source of unbalance is the mismatch of the ratios of c.t.'s and the power transformer which the c.t.'s are used to protect. Most power transformers are equipped with on-load tap changing equipment. The operation of tap changers, change the turns ratio between the primary and secondary sides of the power transformer. Since the c.t. ratios are usually selected to match at one tap setting only, the outputs of the primary and secondary side c.t.'s do not balance when the power transformer operates at a tap setting other than at which the c.t. ratios were initially matched.

Factors that affect the performance of the differential relay have been described in this section. In addition to these factors, the differential relay is limited in its ability to respond to earth faults. The operation of a differential relay in case of a single line to ground fault

fault on the wye side of a three phase transformer is discussed in the next section.

2.5 Limitations of Transformer Differential Protection Relays

The transformer differential protection is limited in its ability to respond effectively to an earth fault²⁵. The presense of an earthing resistor imposes further limitations on the sensitivity of transformer differential relays. The earth fault current in this case depends on the value of the earthing resistor and is proportional to the distance of the fault from the neutral end of the winding. Consider a delta/wye transformer with the neutral earthed through a resistor R as shown in Figure 2.6. For a 1:1 voltage ratio the primary to secondary turn ratio is $\sqrt{3}:1$. An earth fault at 100 % of the winding on the secondary side corresponds to a primary current whose magnitude is $1/\sqrt{3}$ times the fault current on the secondary side that can be represented as follows.

$$I_{fp} = \frac{1}{\sqrt{3}} I_{fs} \quad (2.1)$$

where:

I_{fp} is the fault current on the primary side

I_{fs} is the fault current on the secondary side

Since the position of the fault on the secondary side varies, the effective ratio of transformation between the primary and secondary windings also varies. For an earth fault at $x\%$ of the winding on the secondary side, the secondary current in terms of I_{fs} is as follows.

$$I_{sx} = I_{fs} \times \frac{x}{100} \quad (2.2)$$

The effective turns ratio of primary to secondary is now $\sqrt{3}:x/100$, and the corresponding current on the primary side is as follows.

$$I_{px} = I_{sx} \times \frac{x}{100} \times \frac{1}{\sqrt{3}} \quad (2.3)$$

where:

I_{px} is the primary current for an earth fault
at $x\%$ of the winding on the secondary side

Substituting in this equation the value of I_{sx} from Equation 2.2, the following equation is obtained.

$$I_{px} = (x/100)^2 I_{fs} / \sqrt{3} \quad (2.4)$$

From the above expression it is evident that the primary fault current is proportional to the square of the percentage of the shortcircuited winding. Figure 2.7 shows

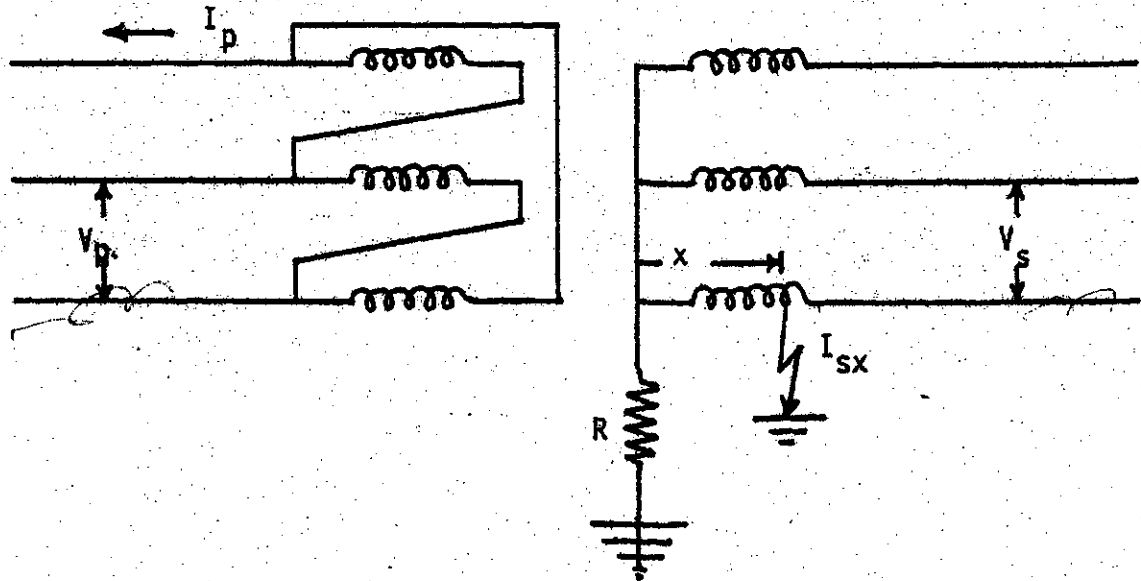


Figure 2.6 Earth fault on the wye winding of a delta/ye transformer.

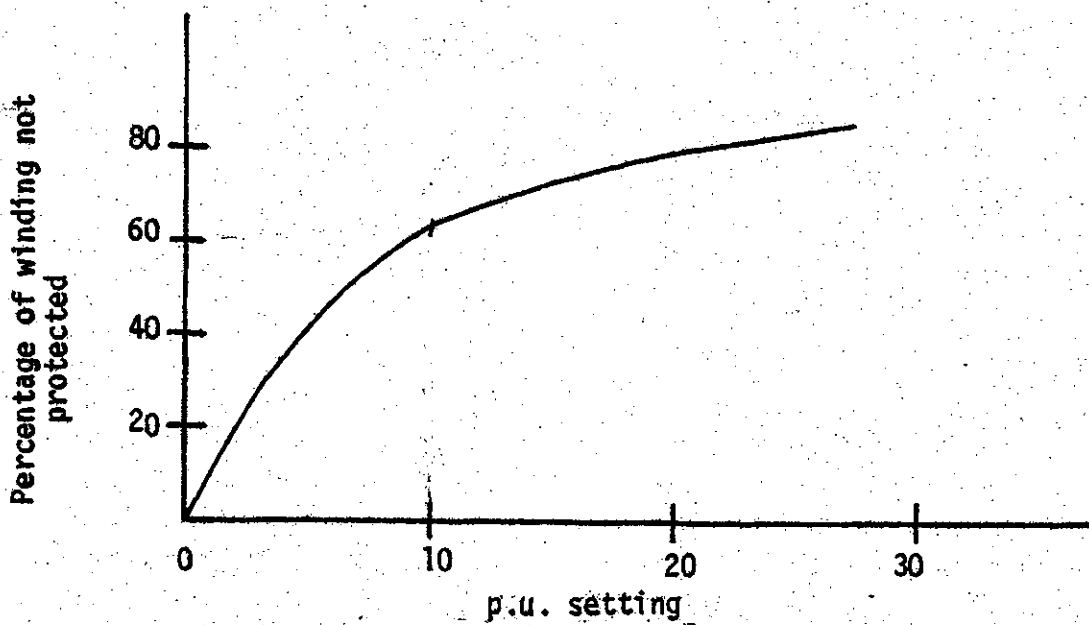


Figure 2.7 Amount of winding protected when transformer is resistance earthed.

fault currents in the primary winding for different locations of faults on the secondary winding. This figure indicates that, at practical relay settings, large portions of the windings are not protected. Restricted earth fault relays are therefore used in many cases. These relays provide earth fault protection to larger parts of the wye connected winding.

For a ground fault on the delta winding, the relation between the fault current and the fault location is very complex and depends on the nature of the power system to which the delta winding is connected.

2.6 Differential Relay with Percentage Bias and Harmonic Restraint

The principle of operation of a differential relay has been described in Section 2.3.5. The factors affecting the operation of a transformer differential relay have been reviewed in Sections 2.4 and 2.5. One of the most common type of transformer differential relay is the differential relay with percentage bias and harmonic restraint. A brief description of this relay is given in this section.

This relay is based on the differential principle described earlier in this chapter²⁵. The operating coil is provided with the vector sum of the primary and secondary

side currents and the restraining coil receives the through currents. The differential current required to operate the relay is expressed as a percentage of the through current. This ratio is referred to as percent slope of the relay characteristic. Figure 2.8 shows the variable percentage characteristic of a typical differential relay.

The differential relay with percentage bias and harmonic restraint can respond at a high speed to internal faults while restraining operation during external faults and magnetizing inrush conditions. Magnetizing inrush is recognized by its second and higher harmonic frequency components. Analyses of inrush currents indicate that the second harmonic component predominates. This component is, therefore, obtained using filters and is used to restrain the operation of the differential unit. The harmonic restraint unit makes the differential relay insensitive to magnetizing inrush currents.

Many differential relays are also provided with instantaneous over current units in the differential circuit. These units are set above the maximum inrush current but operate in less than one cycle if the differential current exceeds the high set value. In this way fast tripping is assured for heavy internal faults.

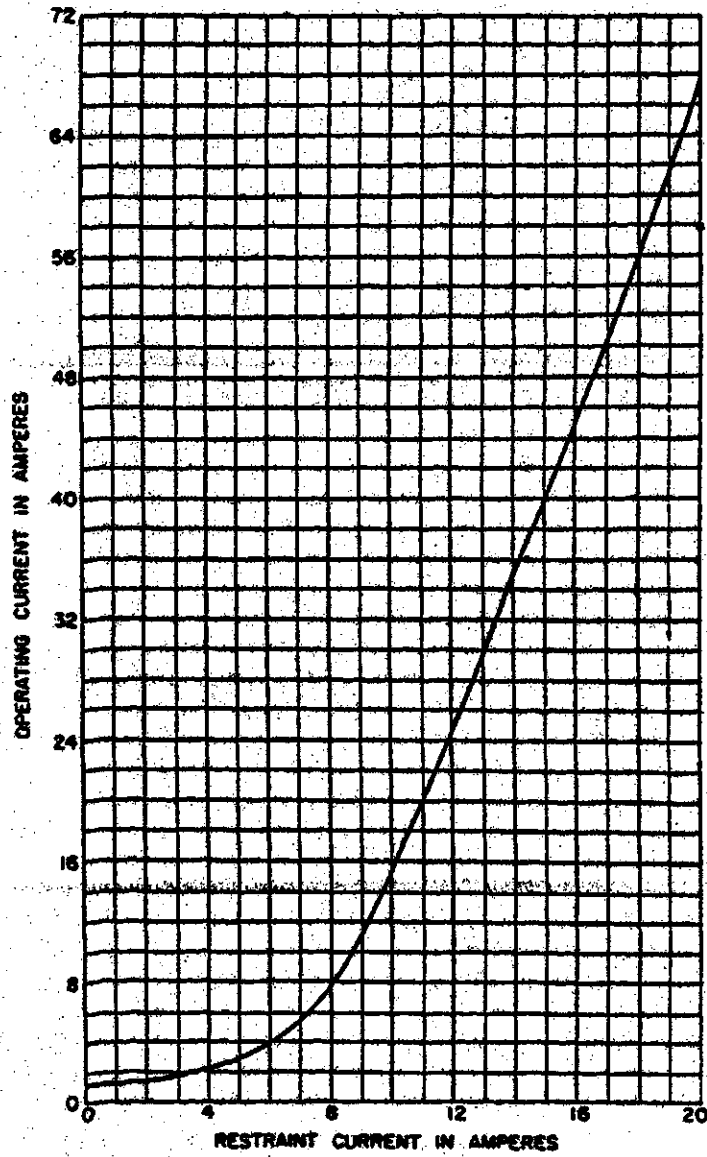


Figure 2.8 Variable percentage differential characteristic.

2.7 Simulation of Inrush and Fault Data

Techniques for simulating the magnetizing inrush, winding faults and simultaneous magnetizing inrush and winding faults are outlined in this section. Programs were developed to compute the current waveforms representing transformer magnetizing inrush, winding faults and simultaneous magnetizing inrush and winding faults. Appendix A lists the sampled and digitized values of the magnetizing inrush and internal fault currents. The simulated current waveforms were later used in the off-line testing of a digital transformer differential and restricted earth fault relay which is described in Chapter 5.

2.7.1 Magnetizing inrush

Simulated inrush current data can be obtained using the Specht's formula²¹. This formula determines the waveform of the magnetizing inrush current for a single phase transformer assuming that the system can be modelled by a series combination of an inductance and a resistance as shown in Figure 2.9. The transformer excitation current is assumed to be zero in the unsaturated state. In the saturated state the inductance is assumed to have a small value determined from the B/H curve shown in Figure 2.10.

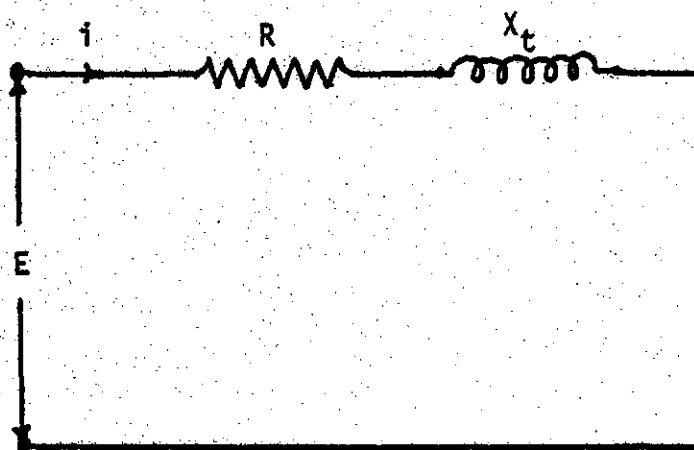


Figure 2.9 Equivalent circuit for modelling magnetizing inrush current.

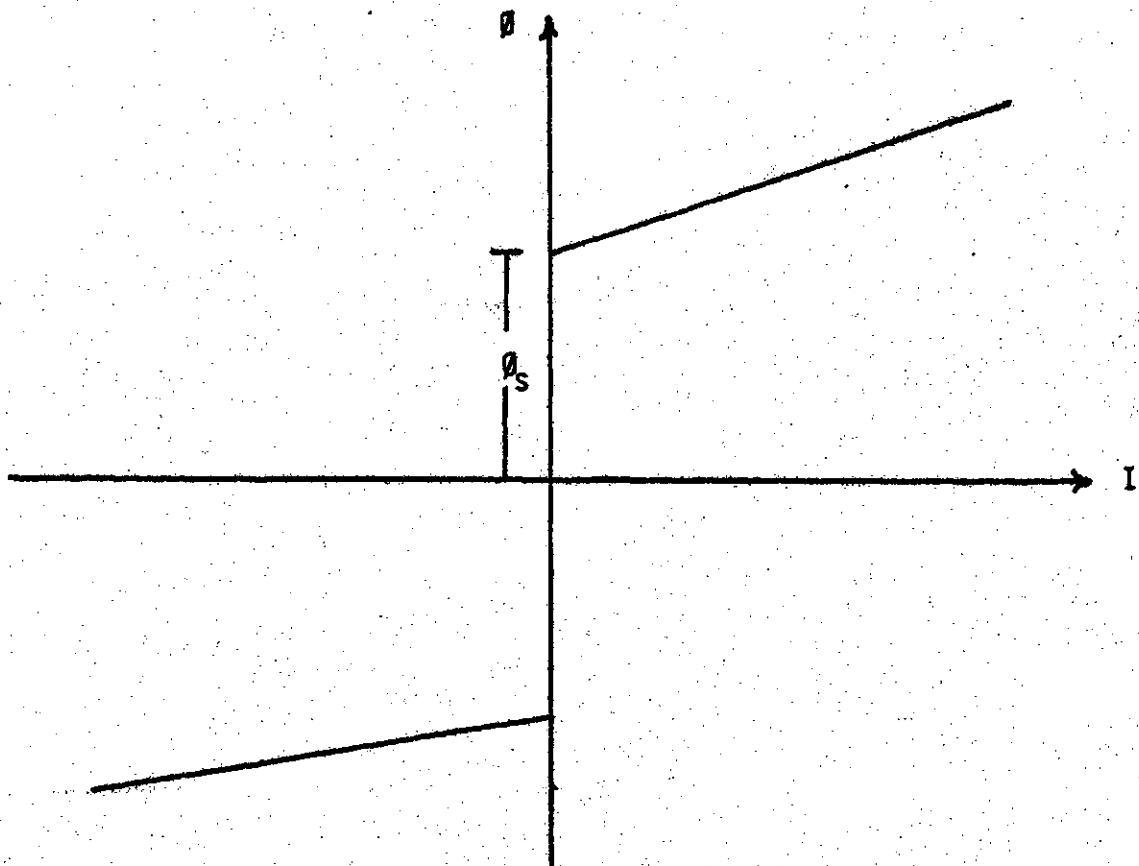


Figure 2.10 Idealized saturation curve for transformer iron core.

A detailed derivation of the equations modelling the transformer inrush phenomenon is given in Reference 21 and are outlined briefly in this section. Consider that the voltage applied to a transformer is:

$$e = \sqrt{2} E \sin(\omega t) \quad \text{volts} \quad (2.5)$$

The steadystate flux in the transformer core would be

$$\begin{aligned} \phi &= (\sqrt{2} E / N\omega) \cos(\omega t) + c1 \quad \text{Wb} \\ &= \phi_m \cos(\omega t) + c1 \quad \text{Wb} \end{aligned} \quad (2.6)$$

where:

N is the number of turns of the primary winding

ϕ_m is the peak value of magnetizing flux

The transformer current, $i_m(t)$, can be defined by

$$i_m(t) = 0, \quad \phi < \phi_s \quad (2.7a)$$

$$\begin{aligned} i_m(t) = \frac{\sqrt{2}E}{X_t} & \left[\left(\sin(\omega t - \tan^{-1} \frac{X_t}{R}) \right) \right. \\ & \left. + e^{-R(\omega t + \theta)/X_t} \sin(\theta + \tan^{-1} \frac{X_t}{R}) \right] \quad \phi > \phi_s \end{aligned} \quad (2.7b)$$

where:

E is the rms value of the applied voltage

- R is the total resistance in the circuit
 X is the saturated reactance of the transformer
 θ is the saturation angle of the transformer
 ω is the radian frequency

The angle used in Equation 2.7b is derived from the conditions of the transformer core at the instant of switching on and is given by the equation

$$\begin{aligned}
 \theta &= \cos^{-1} \left(\frac{\phi_s - \phi_m - \phi_r}{B_m} \right) \\
 &= \cos^{-1} \left(\frac{B_s - B_m - B_r}{B_m} \right) \quad (2.8)
 \end{aligned}$$

B_s , B_m and B_r are the flux densities corresponding to the fluxes ϕ_s , ϕ_m and ϕ_r .

Equation 2.7b is valid until the current $i_m(t)$ returns to zero for that cycle. At that instant, the flux density is again B_s . If $i_m(t)$ returns to zero at an angle $\omega t = \alpha$, the transformer core goes back in to saturation at $2\pi - \alpha$ radians for the next cycle. The saturation angle θ for the next cycle is therefore equal to α calculated for the previous cycle. In general, for the n th cycle, the saturation angle is expressed as follows:

$$\theta_n = \alpha_{n-1} \quad (2.9)$$

Figure 2.11 shows a typical plot of the inrush current for a single phase transformer obtained from Equation 2.7. The primary voltage of the transformer was assumed to be 1500 volts. The saturated reactance, the ratio of resistance to saturated reactance and the saturation angle were assumed to be 20 ohms, 0.1 and 100 degrees respectively. It was also assumed that ωt is equal to -180 degrees when the transformer is energized.

2.7.2 Internal faults

A mathematical model defining the current $i_f(t)$ due to a winding fault in a transformer is

$$i_f(t) = \frac{\sqrt{2}E}{X_f} \left[\sin(\omega t - \tan^{-1} \left\{ \frac{X_f}{R_f} \right\}) - e^{-R_f \omega t / X_f} \sin(X_f / R_f) \right] \quad (2.10)$$

where:

X_f is the reactance in the fault path

R_f is the resistance in the fault path

It is assumed in this case that the transformer is not supplying any load prior to the occurrence of the fault. The first term of Equation 2.10 is a sinusoid, whereas, the second term is an exponentially decaying d.c component which has a time constant of L_f / R_f . The initial value of this component depends on the instantaneous value of the voltage when the fault occurs. The extreme values of the decaying

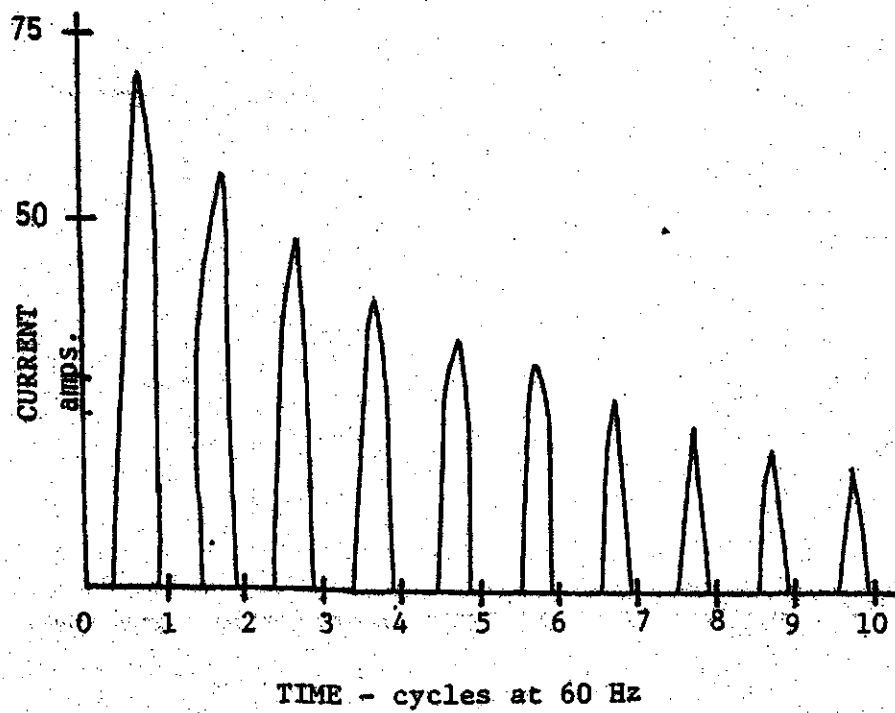


Figure 2.11 Magnetizing inrush current waveform.

d.c. component are zero and $\sqrt{2}E/X_f$.

2.7.3 Simultaneous internal fault and inrush condition

Some transformer faults result from the short circuiting of a few turns of one of the winding. These faults are difficult to detect because they generally manifest in small changes in the currents at the transformer terminals. In many instances transformer remain in service inspite of the winding fault. The heat generated by these faults gradually cause further damage to the winding and the fault is generally detected after substantial portion of the winding has been affected. In the mean time, the transformer may have to be switched on to the mains after having been removed from service for maintenance etc. This situation can lead to the system feeding simultaneously a fault and magnetizing inrush.

Figure 2.12 depicts a single phase transformer experiencing an internal fault. The fault represents some shorted primary turns. An equivalent circuit representing this condition is shown in Figure 2.13. In this figure N_1 is the number of primary winding turns not affected by the internal fault and N_2 is the number of primary turns that are shorted. The total primary current i_p is given by

$$i_p = i_m + i_f \quad (2.11)$$

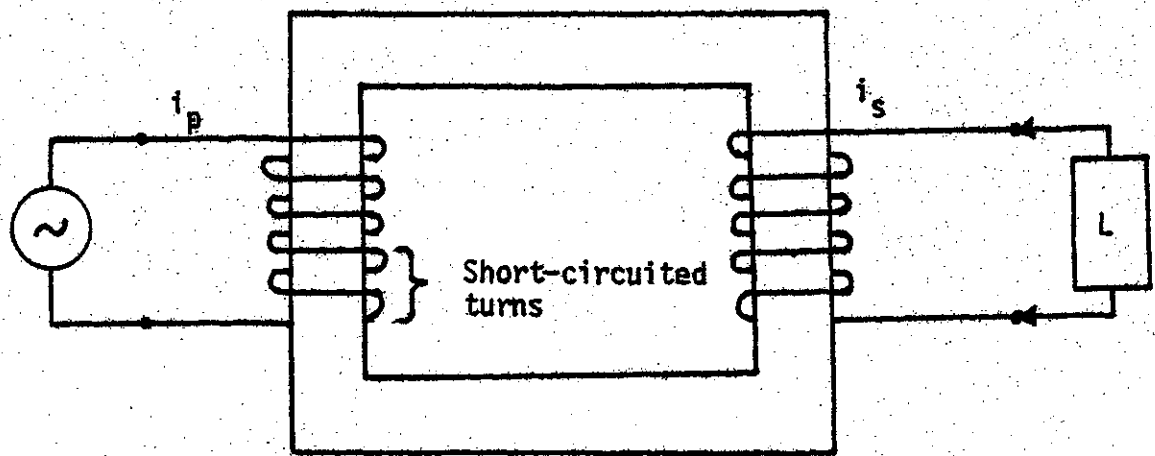


Figure 2.12 Interturn fault in a power transformer.

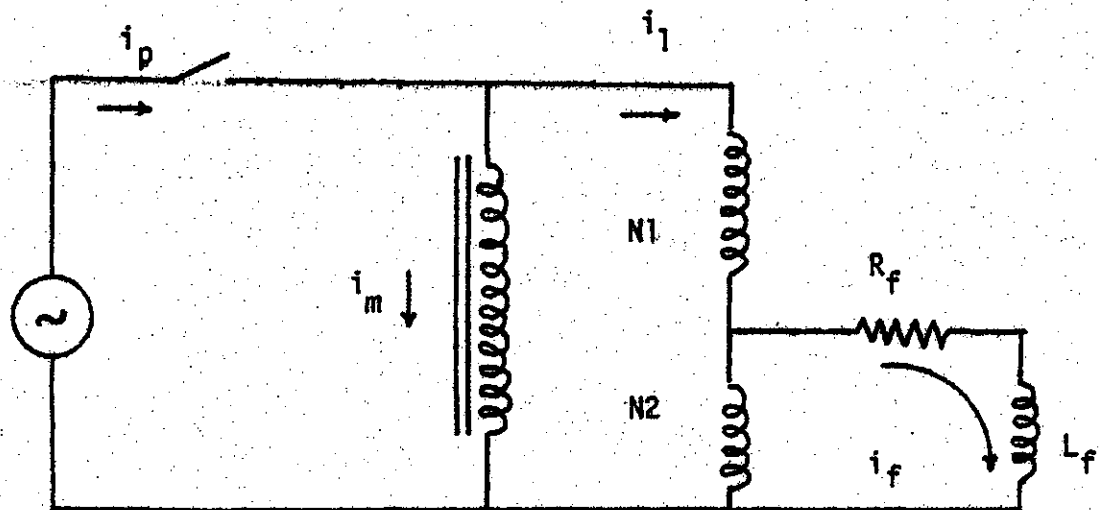


Figure 2.13 Equivalent circuit for modelling simultaneous internal fault and inrush currents.

where:

i_m is the magnetizing current of the transformer
 i_1 is the current in the unaffected portion of the primary winding due to the short circuit

The magnetizing current can be obtained using Equation 2.7. The current, i_f , in the short circuited portion of the winding can be calculated using Equation 2.10. The current i_1 can be expressed as

$$i_1 = i_f \left(\frac{N_2}{N_1 + N_2} \right) \quad (2.12)$$

where:

i_f is the current in the short circuited winding

When a few turns are short-circuited, the current in these turns is large but the corresponding primary current is relatively small.

Choosing suitable values for the inductance and resistance of the short circuited portion of the primary winding, current due to simultaneous inrush and short circuited turns can be determined using the procedure described above. Figure 2.14 shows the plot of simultaneous

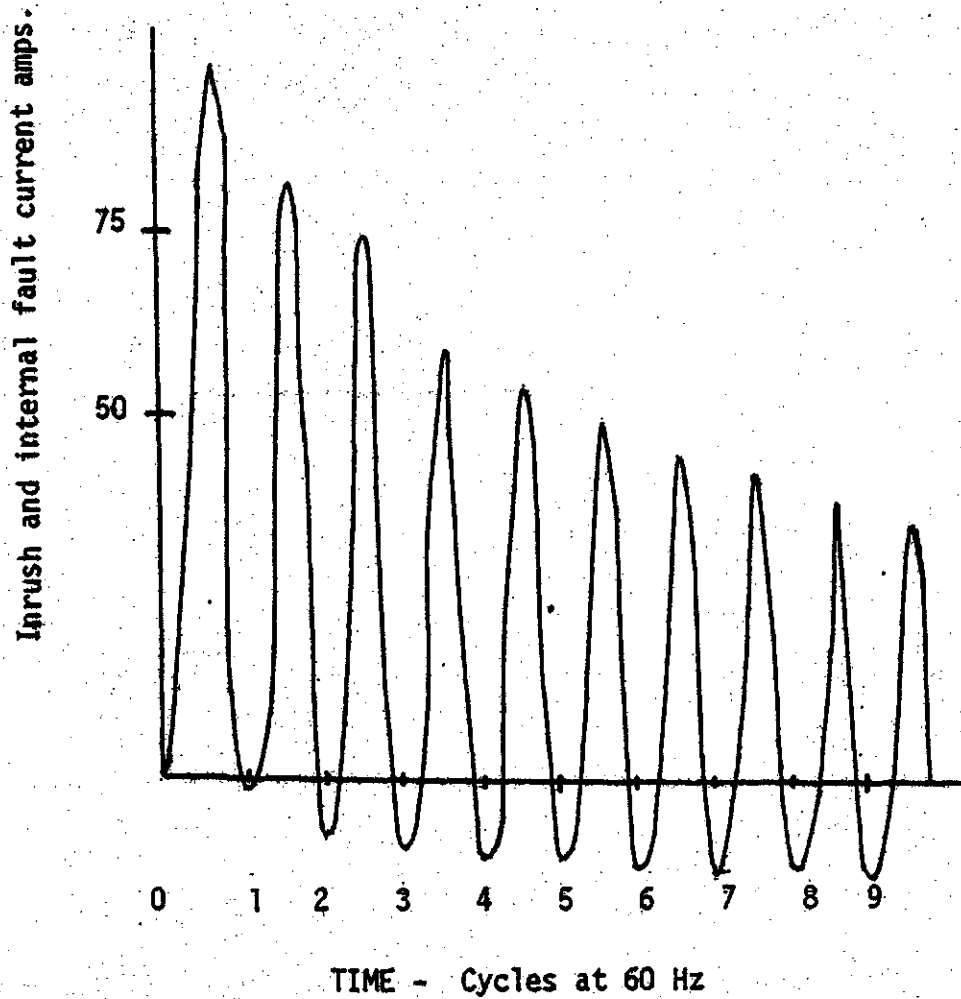


Figure 2.14 Simultaneous inrush and internal fault current waveform.

inrush and internal fault current in a transformer when 5 percent of the primary winding turns were considered to have been shortcircuited. The primary side rated voltage, the saturated reactance and the resistance to saturated reactance ratio were assumed to be 1500 volts, 20 ohms and 0.1 respectively. The impedance and resistance to reactance ratio of the shorted portion of the winding were assumed to be 0.2 ohms and 0.05 respectively. It was also assumed that ϕ is equal to -180 degrees when the transformer is energized.

2.8 Summary

A brief survey of the faults affecting power transformers have been presented in this chapter. Conventional relays used for transformer protection have been described with particular emphasis on differential relays. Problems associated with and the shortcomings of transformer differential protection have been outlined. Mathematical models for simulating the magnetizing inrush, winding faults and simultaneous magnetizing inrush and winding faults have also been established.

3. REVIEW OF PREVIOUSLY PUBLISHED ALGORITHMS

Faults affecting power transformers and methods of detecting these faults have been described briefly in the last chapter. Differential protection of transformers using programmable digital processors has received some attention during the last ten years. The methods used in the past for processing sampled data for transformer differential protection are reviewed in this chapter. Methods for identifying magnetizing inrush in transformers from internal faults are also examined.

Three approaches have been suggested previously for digital protection of transformers. The first approach consists of wave shape identification in a relatively simple form. The second and third methods use the cross-correlation and digital filtering techniques to determine components of various frequencies in the transformer currents. The cross-correlation and digital filtering techniques determine the second harmonic component of the differential current and if this component exceeds a threshold level, it is concluded that the transformer is experiencing magnetizing inrush.

3.1 Wave Shape Identification Technique

Substantial differential current is experienced when a fault occurs in the protected zone. Unfortunately, magnetizing inrush, unequal ratios of current transformers and unequal levels of c.t. saturation during external faults also manifest as excessive differential currents. In these circumstances presence of differential currents do not represent¹⁵ faults in the transformer zone. Rockefeller suggested a relatively simple technique which identifies differential current waveforms for distinguishing inrush currents from fault currents. He suggested that magnetising inrush current can be identified by monitoring the time between successive peaks of the differential current. Successive peaks of inrush current occur at intervals of about 4 or 16 msecs. whereas successive peaks of the fundamental frequency fault current occur 7.5 to 10 msecs apart. Also, the peak value of a fault current is within 75 % to 125 % of its previous peak value. Also, two successive peaks are of opposite sign.

3.2 Cross-Correlation Technique

The cross-correlation approach which was proposed and used by Malik, Dash and Hope^{10,12} is described briefly in this paragraph. In this approach, the fundamental and harmonic frequency components of the differential current are

determined by correlating the current with two orthogonal functions. If excessive second and third harmonic components are observed, it is concluded that the transformer is experiencing magnetizing inrush.

A cross-correlation function relating a signal $X(t)$ and a function $Y(t)$, both representing stationary continuous processes, may be defined as follows:

$$\phi_{XY}(\gamma) = \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} X(t) Y(t - \gamma) dt \quad (3.1)$$

where:

γ is a time shift imposed upon one of the signals

T_0 is the interval over which the function is calculated

Two orthogonal functions are correlated with a signal to obtain two cross-correlation functions. The usually selected orthogonal functions are the sine and cosine waveforms or even and odd square waves. The sinusoidal weighting functions, shown in Figure 3.1, can be represented mathematically as follows:

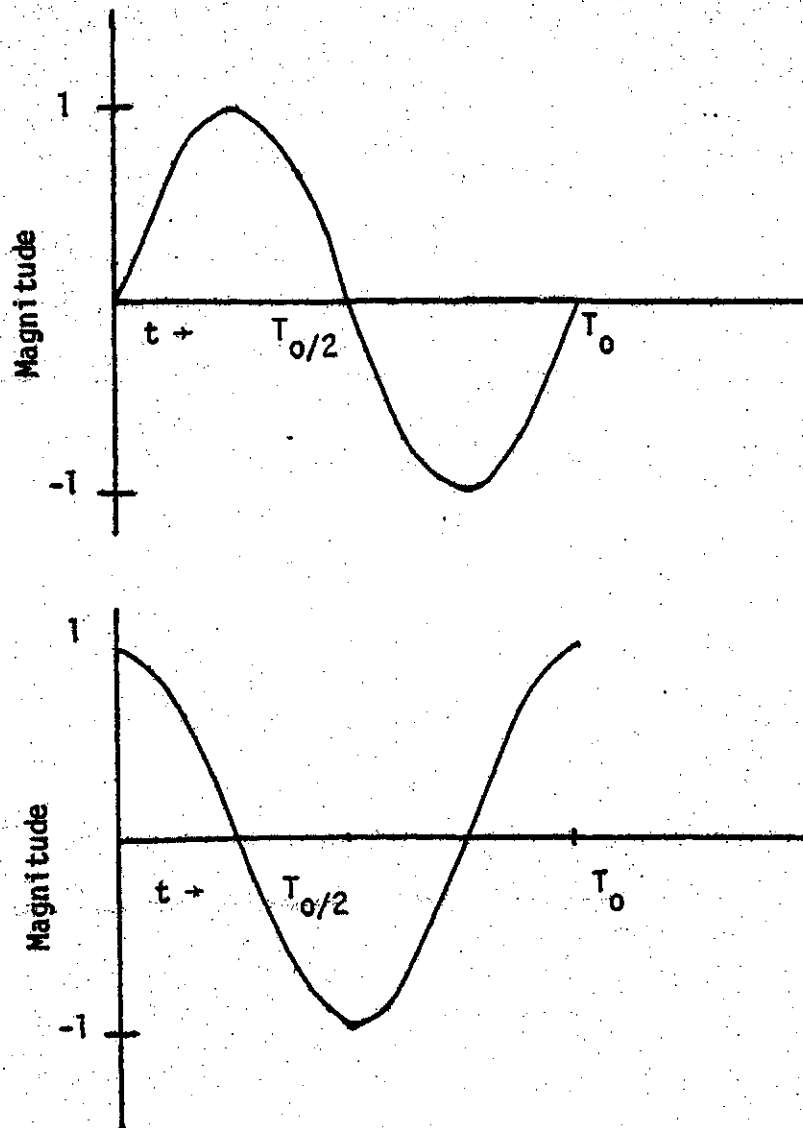


Figure 3.1 Sine and cosine time domain orthogonal functions.

$$Y(t) = \sin(n\omega t) \quad (3.2a)$$

$$Y(t) = \cos(n\omega t) \quad (3.2b)$$

The even and odd square wave functions shown in Figure 3.2 and can be represented mathematically as follows:

$$Y(t) = \begin{cases} 1 & 0 \leq t \leq T_0/4, 3T_0/4 \leq t \leq T_0 \\ -1 & T_0/4 \leq t \leq 3T_0/4 \end{cases} \quad (3.3a)$$

$$Y(t) = \begin{cases} 1 & 0 \leq t \leq T_0/2 \\ -1 & T_0/2 \leq t < T_0 \end{cases} \quad (3.3b)$$

The cross-correlation functions measure the similarities between the signal $X(t)$ and the two orthogonal functions $Y(t)$.

In References 11 and 12, magnitudes and phase angles of the fundamental and harmonic frequency components of $X(t)$ were extracted from the cross-correlation functions as demonstrated hereafter.

Assume that the waveform of a current experienced during an abnormal condition is of the form:

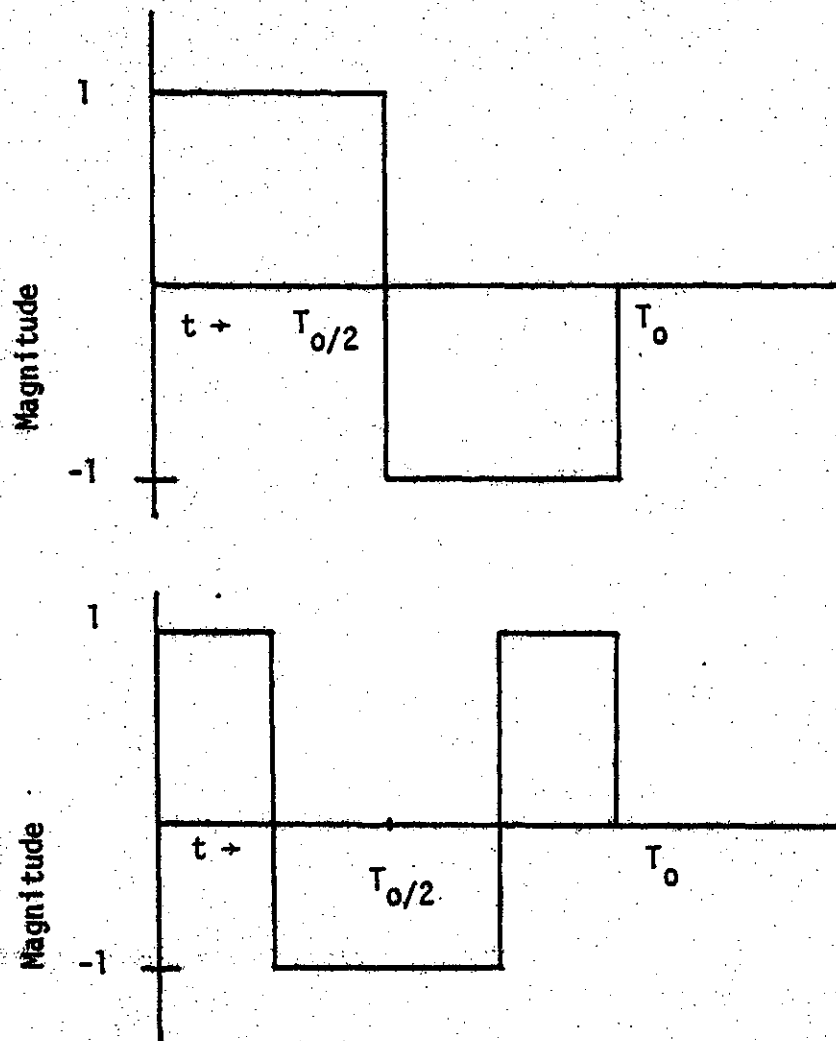


Figure 3.2 Time domain orthogonal square waves.

$$X(t) = I_0 e^{-t/\tau} + \sum_{n=1}^N I_n \sin(nw_1 t + \theta_n) \quad (3.4)$$

where:

N is the highest order of the harmonic component present in the waveform

w_1 is the fundamental frequency of the system

I_0 is the magnitude of the d.c. offset at $t=0$

I_n is the peak value of the n th harmonic component

θ_n is the phase angle of the n th harmonic component

τ is the decay time constant of the d.c. component

$I_0 e^{-t/\tau}$ is an exponentially decaying component. For simplicity, assume that this component is constant over the period of integration used for calculating the cross-correlation functions. The two cross-correlation functions can now be expressed as follows.

$$\phi_{xy1}(0) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \sin(nwt) \{ I_0 + \sum_{n=1}^N I_n \sin(nwt + \theta_n) \} dt \quad (3.5a)$$

$$\phi_{xy2}(0) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \cos(nwt) \{ I_0 + \sum_{n=1}^N I_n \sin(nwt + \theta_n) \} dt \quad (3.5b)$$

where:

T_0 is the period of the fundamental frequency
of the system

These equations simplify to Equations 3.6a and 3.6b if the integration interval is one cycle of the fundamental frequency.

$$\phi_{xy1}(0) = I_n \cos(\theta_n) \quad (3.6a)$$

$$\phi_{xy2}(0) = I_n \sin(\theta_n) \quad (3.6b)$$

The peak value and the phase angle of the nth harmonic can be obtained from these correlation functions as follows:

$$I_n = [(\phi_{xy1})^2 + (\phi_{xy2})^2]^{1/2} \quad (3.7)$$

$$\sigma = \tan^{-1} (\phi_{xy2} / \phi_{xy1}) \quad (3.8)$$

The averaging process represented by Equation 3.1 implies that the function $\phi_{xy}(\gamma)$ can be determined by either analog or digital methods. In a digital system it is more convenient to approximate the integration by sampling the signal every ΔT seconds, multiplying each sample with an appropriate weighting function and summing a finite number of weighted samples of the signal. This procedure can be

mathematically represented by Equation 3.9.

$$\phi_{xy}(k \Delta T, \gamma) = \frac{1}{N} \sum_{m=1}^N X\{(k+m) \Delta T\} Y\{(k+m) \Delta T - \gamma\} \quad (3.9)$$

where:

$Y\{(k+m) \Delta T - \gamma\}$ is the sampled value of a
preselected function, sampled at time
 $\{(k+m) \Delta T - \gamma\}$

$X\{(k+m) \Delta T\}$ is the value of the signal
sampled at time $\{(k+m) \Delta T\}$

The magnitudes and phase angles of the fundamental and harmonic components of a current can also be obtained by correlating the currents with even and odd square wave functions as follows:

$$\phi_{xy1}(0) = \frac{1}{T_0} \int_0^{T_0/4} X(t) dt + \frac{1}{T_0} \int_{3T_0/4}^{T_0} X(t) dt - \frac{1}{T_0} \int_{T_0/4}^{3T_0/4} X(t) dt \quad (3.10a)$$

$$\phi_{xy2}(0) = \frac{1}{T_0} \int_0^{T_0/2} X(t) dt - \frac{1}{T_0} \int_{T_0/2}^{T_0} X(t) dt \quad (3.10b)$$

Substituting for $X(t)$ from Equation 3.4, integrating and simplifying provides the following cross-correlation functions.

$$\phi_{xy1}(0) = \frac{4I_n}{\pi} \cos(\theta_n) \quad (3.11a)$$

$$\phi_{xy2}(0) = \frac{4I_n}{\pi} \sin(\theta_n) \quad (3.11b)$$

The peak value and the phase angle of the n th harmonic can now be obtained as follows:

$$I_n = \frac{\pi}{4} [(\phi_{xy1})^2 + (\phi_{xy2})^2]^{1/2} \quad (3.12)$$

$$\theta_n = \tan^{-1} [\phi_{xy2} / \phi_{xy1}] \quad (3.13)$$

The advantage of using even and odd square waves is that the computations of cross-correlation functions consist of additions and subtractions only.

In References 11 and 12, peak values of the fundamental and second harmonic components of the differential current were obtained by using the procedures described above. The magnitude of the second harmonic component of the differential current as a percentage of the fundamental frequency component was then used as the basis of identifying magnetizing inrush from faults.

3.3 Digital Filtering Technique

A digital filter may be defined as a device which accepts a sequence of numbers as input and processes them to

produce another sequence of numbers as output. A general form of a digital filter is as follows:

$$Y_k = \sum_{i=0}^r L_i X_{k-i} - \sum_{i=1}^m Q_i Y_{k-i} \quad (3.14)$$

where:

X_k is the k'th input sample

Y_k is the k'th output sample

L_i and Q_i are the digital filter coefficients

The procedure for designing a digital filter consists of finding the coefficients, L and Q , such that the specified filtering requirements are achieved. Digital filters can be classified into two categories; recursive and non-recursive filters. A recursive filter incorporates feedback by using a part of the output as input, whereas a non-recursive filter does not use any feedback. Both recursive and non-recursive filters have been used in transformer differential protection work previously reported in the literature.

3.3.1 Recursive filtering technique

Digital filtering technique was first applied to transformer differential protection by Sykes and Morrison²³. Sampled values of the differential current were processed to determine its fundamental frequency and

second harmonic components. If the magnitude of the second harmonic component exceeded a pre-specified percentage of the fundamental frequency component, it was concluded that the transformer was experiencing magnetizing inrush. The form of bandpass recursive filters used in Reference 23 is:

$$Y_k = L_0 X_k + L_1 X_{k-1} - Q_1 Y_{k-1} - Q_2 Y_{k-2} \quad (3.15)$$

where: L_0 , L_1 , Q_1 and Q_2 are the filter coefficients

X_k is the input sample at $t=k \Delta T$

Y_k is the output sample at $t=k \Delta T$

ΔT is the sampling interval

50 Hz and 100 Hz bandpass filters used in this work are:

$$Y_k = 0.096X_k - 0.096X_{k-1} + 1.810Y_{k-1} - 0.905Y_{k-2} \quad (3.16)$$

$$Y_k = 0.045X_k - 0.045X_{k-1} + 1.580Y_{k-1} - 0.953Y_{k-2} \quad (3.17)$$

The outputs of these filters are not suitable for direct point by point comparison because the outputs of the fundamental and second harmonic filters are phase dependent. The values of the outputs, Y_k , were therefore rectified and smoothed before comparison. A digital low pass filter described by Equation 3.18 was used for this purpose.

$$Y_k = 0.0087X_k + 1.904Y_{k-1} - 0.913Y_{k-2} \quad (3.18)$$

The authors of References 23 tested the proposed technique by off-line simulation of faults and magnetizing inrush in a single phase transformer. The tests revealed that the procedure can identify inrush conditions from internal faults. However, the frequency responses of the filters indicate that they do not completely suppress the decaying d.c. component present in the sampled waveform.

3.3.2 Nonrecursive filtering technique

Schweitzer, Larson and Flechsig²⁰ used finite impulse response digital filters in a digital transformer differential relay. This method is basically similar to the method of odd and even square wave cross-correlation technique suggested by Malick et. al. However, Larson's criterion for discriminating magnetizing inrush conditions from internal faults was quite different.

Four FIR filters were used in References 7 and 20; two each for the fundamental frequency and the second harmonic components. The weighting functions used in these filters are +1 or -1 which reduced the computations to additions and subtractions of the sampled inputs as follows.

$$S_1(k) = \sum_{m=k-N+1}^{k-N/2} [i_m - i_{m+N/2}] \quad (3.19)$$

$$C_1(k) = \sum_{m=k-N+1}^{k-3N/4} [i_m - (i_{m+N/4} + i_{m+N/2}) + i_{m+3N/4}] \quad (3.20)$$

$$S_2(k) = \sum_{m=k-N+1}^{k-3N/4} [i_m - i_{m+N/4} + i_{m+N/2} - i_{m+3N/4}] \quad (3.21)$$

$$C_2(k) = \sum_{m=k-N+1}^{k-7N/8} [i_m - (i_{m+N/8} + i_{m+N/4}) + i_{m+3N/8} - i_{m+N/4} - (i_{m+5N/8} + i_{m+3N/4}) + i_{m+7N/8}] \quad (3.22)$$

where:

i_m is the value of the sampled waveform

N is the number of samples over a period of one cycle

S_1 and S_2 are the outputs of the digital filters that have 'odd' weighting function

C_1 and C_2 are the outputs of the digital filters that have 'even' weighting function

The sampling rate used in this work was 480 Hz. and the criterion for discriminating magnetizing inrush from internal faults was based on ξ which is defined as follows.

$$\xi = \frac{\text{Max} [|S_2|, |C_2|]}{\text{Max} [|S_1|, |C_1|]} \quad (3.23)$$

The theoretical limits of the value of the ratio ξ , were investigated and were found to be between 0 and 0.146 for internal faults and between 0.334 and 0.586 during magnetizing inrush. These values were determined by using system X/R ratios ranging from zero to infinity.

The procedure described above was tested using simulated data representing inrush and internal fault conditions. Subsequently the authors reported tests carried out on a model transformer and observed that the procedure satisfactorily identifies internal faults from abnormal conditions such as magnetizing inrush.

3.4 Summary

Previously published digital algorithms for differential protection of transformers have been reviewed in this chapter. Waveshape identification, cross-correlation and digital filtering techniques have been described. The cross-correlation and digital filtering techniques were previously used to analyze the differential current waveforms. If the analysis indicated that the second harmonic component of the differential current exceeds a pre-specified percent of its fundamental component, it was concluded that the abnormal differential current was due to magnetizing inrush.

4. DESIGN OF NON-RECURSIVE FILTERS

Several digital methods for analyzing power system voltage and current waveforms have been proposed during the last twelve years. Three principal approaches proposed in the past for digital differential protection of transformers have been briefly presented in the last chapter. For this project, three non-recursive digital filter design techniques, other than those described in Chapter 3, were examined and used. The design techniques and filter designs are presented in this chapter.

One of the filter design techniques used in this work is the least error squares approach¹⁷. The other two approaches are the Fourier series and the frequency sampling methods^{2,13} which attempt to approximate the frequency response of an ideal filter. The latter two methods are quite well known in the field of communications, but have not been used in digital relaying.

4.1 Least Error Squares Approach

The concept of least error squares was first used in power system protection by Luckette et. al⁹. A process of curve fitting was used to extract the fundamental frequency

components from digitized sampled data. The inputs were assumed to be made up of a decaying d.c. component and sinusoidal components of the fundamental and harmonic frequencies. The frequency of the sinusoids and the time constant of the d.c. offset were assumed to be known. The waveform was modelled in Reference 9 as follows:

$$y = K_1 e^{-t/\tau} + \sum_{m=1}^N \{K_{2m} \sin(mwt) + K_{2m+1} \cos(mwt)\} \quad (4.1)$$

where:

$K_1, K_2, \dots, K_{2N+1}$ are the unknown parameters of an input signal

N is the number of harmonics being considered

τ is the decay constant of the dc offset

w is the angular frequency of the fundamental frequency component

The least squares fit involved minimizing the expression

$$E^2 = \int_0^T [i(t) - K_1 e^{-t/\tau} - \sum_{m=1}^N \{K_{2m} \sin(mwt) + K_{2m+1} \cos(mwt)\}]^2 dt \quad (4.2)$$

where:

$i(t)$ is the waveform to be analyzed

T is the data window in secs.

For the error squares to be minimum, the following conditions had to be satisfied.

$$\frac{\partial E^2}{\partial K_r} = 0 \quad \text{for } r = 1, 2, \dots, 2N + 1 \quad (4.3)$$

Substituting the value of E^2 in the above condition the following set of equations were obtained.

$$\int_0^T \frac{\partial}{\partial K_r} [i(t) - K_1 e^{-t/\tau} - \sum_{m=1}^N \{ K_{2m} \sin(m\omega t) + K_{2m+1} \cos(m\omega t) \}]^2 dt = 0$$

$$r = 1, 2, \dots, 2N + 1 \quad (4.4)$$

Reference 9 proposed that by using a numerical integration technique, these equations can be solved for the $(2N + 1)$ unknown parameters of the input signal.

4.1.1 Least error squares solution and the pseudoinverse of a rectangular matrix

Sachdev and Baribeau¹⁷ proposed an algorithm which used digital filters designed by the least error squares approach. The outputs of the filters provided real and imaginary components of the voltage and current phasors. These were then used to calculate apparent impedances. The decay rate of the d.c. component is affected by the

resistance of the arc at the fault and the system configuration. In Reference 17, the decay rate was, therefore, not assumed to be known in advance. The mathematical background of the least error squares and the concept of pseudoinverse have been reported in Reference 23 and are reproduced here for ready reference.

Assume that the differential current during a fault or an inrush condition can be represented by Equation 4.5.

$$v(t_1) = K_0 e^{-t/\tau} + K_1 \sin(\omega_1 t_1 + \theta_1) + \sum_{h=2}^N K_h \sin(\omega_h t_1 + \theta_h) \quad (4.5)$$

where:

- $v(t_1)$ is the instantaneous value of current at time t
- K_0 is the magnitude of the dc offset at time $t=0$
- K_1 is the peak value of the fundamental frequency component
- K_h is the peak value of the h th frequency component
- τ is the decay time constant of the dc component
- θ_1 is the phase angle of the fundamental frequency component
- θ_h is the phase angle of the higher frequency component

w_1 is the fundamental frequency of the system in radians per second

w_h is the frequency of the higher frequency component in radians per second

Fault currents generally contain sinusoidal components of the fundamental, odd harmonic and some non-harmonic frequencies. Transformer inrush currents contain a large component of the second harmonic frequency. If the relay system is designed in such a manner that the input waveforms are band limited by analog filters, the relay input signal may be assumed to be made up of the decaying d.c. component and sinusoids of the fundamental, second and third harmonic frequencies. Such a relay input waveform at $t=t_1$ may be represented as follows:

$$v(t_1) = K_0 e^{-t_1/\tau} + K_1 \sin(w_1 t_1 + \theta_1) + K_2 \sin(2w_1 t_1 + \theta_2) + K_3 \sin(3w_1 t_1 + \theta_3) \quad (4.6)$$

The decaying d.c. component in Equation 4.6 can be represented by the first two terms of its Taylor series expansion given in Equation 4.7.

$$K_0 e^{-t_1/\tau} = K_0 [1 - t_1/\tau + t_1^2/2\tau^2 - t_1^3/6\tau^3 + \dots] \quad (4.7)$$

The sinusoidal terms in Equation 4.6 can be expanded using the following trigonometric identity:

$$\sin(\omega t + \theta) = \sin(\omega t) \cos(\theta) + \cos(\omega t) \sin(\theta)$$

Incorporating this representation for the sinusoidal terms Equation 4.6 can be expressed as follows:

$$v(t_1) = a_{11} X_1 + a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7 + a_{18} X_8 \quad (4.8)$$

where:

$$a_{11} = 1$$

$$a_{12} = \sin(\omega_1 t_1)$$

$$a_{13} = \cos(\omega_1 t_1)$$

$$a_{14} = \sin(2\omega_1 t_1)$$

$$a_{15} = \cos(2\omega_1 t_1)$$

$$a_{16} = \sin(3\omega_1 t_1)$$

$$a_{17} = \cos(3\omega_1 t_1)$$

$$a_{18} = t_1$$

$$X_1 = K_0$$

$$X_2 = K_1 \cos(\theta_1)$$

$$X_3 = K_1 \sin(\theta_1)$$

$$X_4 = K_2 \cos(\theta_2)$$

$$X_5 = K_2 \sin(\theta_2)$$

$$X_6 = K_3 \cos(\theta_3)$$

$$X_7 = K_3 \sin(\theta_3)$$

$$X_8 = -K_0 / T$$

The next sample $v(t_2)$ is received ΔT seconds later ($t=t_1 + \Delta T$) and can be represented by an equation similar to Equation 4.8 as follows:

$$v(t_2) = a_{21} X_1 + a_{22} X_2 + a_{23} X_3 + a_{24} X_4 + a_{25} X_5 + a_{26} X_6 + a_{27} X_7 + a_{28} X_8 \quad (4.9)$$

where:

$$\Delta T = 2\pi / w_s$$

$$t_2 = t_1 + \Delta T$$

$$a_{21} = 1$$

$$a_{22} = \sin(w_1 t_2)$$

$$a_{24} = \sin(2w_1 t_2)$$

$$a_{26} = \sin(3w_1 t_2)$$

$$a_{28} = t_2$$

$$X_1 = K_0$$

$$X_2 = K_1 \cos(\theta_1)$$

$$X_4 = K_2 \cos(\theta_2)$$

$$X_6 = K_3 \cos(\theta_3)$$

$$X_8 = -K_0 / T$$

$$a_{23} = \cos(w_1 t_2)$$

$$a_{25} = \cos(2w_1 t_2)$$

$$a_{27} = \cos(3w_1 t_2)$$

$$X_3 = K_1 \sin(\theta_1)$$

$$X_5 = K_2 \sin(\theta_2)$$

$$X_7 = K_3 \sin(\theta_3)$$

The 'a' coefficients of Equation 4.8 and 4.9 can be precalculated as these values depend only on the fundamental and harmonic frequencies and time reference which may be selected arbitrarily. For instance if time is considered to be zero at the instant of the first sample, $t_1 = 0$, ΔT and w_1 are known, hence the values of 'a' coefficients of Equations 4.8 and 4.9 can be predetermined. Equations 4.8 and 4.9 have eight unknowns; at least eight sampled values are, therefore, required to determine the unknowns. The number of samples selected to find the unknown terms determines the window length. If m ($m > 8$) samples are used to determine the unknowns, the following m equations are established.

$$\begin{array}{ccc}
 [A] & [X] & = [V] \\
 (m \times 8) & (8 \times 1) & (m \times 1)
 \end{array} \quad (4.10)$$

The matrix $[A]$ is a rectangular matrix and its left pseudo-inverse $[A]^\dagger$ provides the least error squares solution as follows:

$$\begin{array}{ccc}
 [X] & = [A]^\dagger [V] \\
 (8 \times 1) & (8 \times m) & (m \times 1)
 \end{array} \quad (4.11)$$

where:

$$\begin{array}{ccc}
 [A]^\dagger & = \left[[A]^T [A] \right]^{-1} [A]^T \\
 (8 \times m) & (8 \times m) & (m \times 8) \quad (8 \times m)
 \end{array}$$

The second and third rows of the left pseudo-inverse of $[A]$ are required to compute the fundamental frequency components X_2 and X_3 in Equations 4.8 and 4.9. The peak value of the fundamental frequency component can be calculated as follows:

$$K_1 = \left[(X_2)^2 + (X_3)^2 \right]^{1/2} \quad (4.12)$$

Similarly fourth, fifth, sixth and seventh rows are required to compute X_4 , X_5 , X_6 and X_7 . The peak values of the second and third harmonic components can be computed from these components using a procedure similar to Equation 4.12.

Many combinations of sampling rate, time reference, data window size and waveform model can be used to obtain coefficients of digital filters. Different time references result in different numbers in the matrix $[A]$. Consequently, the elements of the pseudoinverse are different. Selecting the time reference to be zero at the centre of the sampling window provides symmetrical values for the elements in the rows of $[A]^{\dagger}$. The effects of varying the sampling rate, time reference and data window size were briefly examined in References 3 and 17.

4.1.2 Filter designs

Many filters were designed using the least error squares approach described in the last section. The combinations of the parameters selected for these designs are listed in Table 4.1. In these designs, the exponential d.c. term was represented by the first two or the first three terms of the Taylor series expansion of $e^{-t/\tau}$. A sinusoid of the fundamental frequency was, of course, included. Various combinations of the sinusoids of higher frequencies were also included in these designs. The designs were examined to determine their suitability for differential protection application.

Table 4.2 shows the elements of the second, third, fourth and fifth rows of $[A]^{\dagger}$. In this case the selected

Table 4.1 Combination of variable parameters used in the design of filters by the least error squares approach.

Item No.	d.c. turns	sinusoidal terms	No. of samples	sampling frequency	window length in msec.
1	2	1,2	11	720	15.28
2	2	1,2,3	11	720	15.28
3	2	1,2,3,4	11	720	15.28
4	3	1,2	11	720	15.28
5	3	1,2,3	11	720	15.28
6	3	1,2,3,4	11	720	15.28
7	2	1	12	720	16.66
8	2	1,2	12	720	16.66
9	2	1,2,3	12	720	16.66
10	2	1,2,3,4	12	720	16.66
11	3	1,2	12	720	16.66
12	3	1,2,3	12	720	16.66
13	2	1	13	720	18.05
14	2	1,2	13	720	18.05
15	2	1,2,3	13	720	18.05
16	2	1,2,3,4	13	720	18.05
17	3	1,2	13	720	18.05
18	3	1,2,3	13	720	18.05
19	3	1,2,3,4	13	720	18.05
20	2	1,2,3	15	960	15.63
21	2	1,2,3,4	15	960	15.63
22	2	1,2,3,4,5	15	960	15.63
23	3	1,2,3	15	960	15.63
24	3	1,2,3,4	15	960	15.63
25	2	1,2,3	16	960	16.66
26	2	1,2,3,4	16	960	16.66
27	2	1,2,3,4,5	16	960	16.66
28	3	1,2,3	16	960	16.66
29	3	1,2,3,4	16	960	16.66
30	2	1,2,3,4,5,6	24	1440	16.66
31	3	1,2,3,4,5,6	24	1440	16.66

design parameters were:

Sampling rate	720 Hz.
Window size	12 samples
Terms representing the decaying d.c. component	2
Frequencies of sinusoidal terms	w_0 , $2w_0$ and $3w_0$

Similarly Table 4.3 shows the elements of the second, third, fourth and fifth rows of $[A]^{\dagger}$ when the following design parameters were used.

Sampling rate	720 Hz.
Window size	13 samples
Terms representing the decaying d.c. component	2
Frequencies of sinusoidal terms	w_0 , $2w_0$, $3w_0$ and $4w_0$

Appendix A lists rows of $[A]^{\dagger}$, which are the coefficients of the filters designed by selecting different combinations of sampling rate, data window size, number of terms of the Taylor series expansion and the harmonics used in the model for the waveforms.

Table 4.2 The elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental, second and third harmonic frequencies.

Second row	Third row	Fourth row	Fifth row
0.5110	-0.1610	-0.2035	0.1443
-0.3909	-0.1179	0.3080	0.0000
-0.2124	-0.0431	0.1100	-0.1443
0.0092	0.0431	-0.1714	-0.1443
-0.1693	0.1179	-0.1400	0.0000
-0.1540	0.1610	-0.0260	0.1443
0.1540	0.1610	0.0260	0.1443
0.1693	0.1179	0.1400	0.0000
-0.0092	0.0431	0.1714	-0.1443
0.2124	-0.0431	-0.1100	-0.1443
0.3909	-0.1179	-0.3080	0.0000
-0.5110	-0.1610	0.2035	0.1443

Table 4.3 The elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of fundamental, second, third and fourth harmonic frequencies.

Second row	Third row	Fourth row	Fifth row
0.3092	-0.0952	-0.1435	0.0952
-0.0764	-0.1227	0.1411	0.0617
-0.1563	-0.0992	0.1499	-0.0675
-0.1529	0.0079	-0.0064	-0.1746
-0.1563	0.0833	-0.1388	-0.0833
-0.0764	0.1385	-0.1475	0.0891
0.0000	0.1746	0.0000	0.1587
0.0764	0.1385	0.1475	0.0891
0.1563	0.0833	0.1388	-0.0833
0.1529	0.0079	0.0064	-0.1746
0.1563	-0.0992	-0.1499	-0.0675
0.0764	-0.1227	-0.1411	-0.0617
-0.3092	-0.0952	0.1435	0.0952

4.1.3 Frequency response of a digital filter

The suitability of a digital filter can be determined by examining its frequency response. One of the methods used for this purpose is the z -transform technique. This technique and the frequency response of the designed filters are presented in this section.

The transfer function of a non-recursive digital filter can be represented as follows.

$$H(z) = \sum_{n=0}^m c_n z^{-n} \quad (4.13)$$

where:

c_n are the filter coefficients

The $(m+1)$ filter coefficients are generally referred to as the impulse response of a digital filter. A useful feature of the transfer function is its interpretation as a frequency function by evaluating $H(z)$ for $z = e^{j\omega\Delta T}$. This substitution provides the means for determining the response characteristics of a filter. Substituting $e^{j\omega\Delta T}$ for z in Equation 4.13 leads to the following equation.

$$H(e^{j\omega\Delta T}) = \sum_{n=0}^m c_n e^{-jn\omega\Delta T} \quad (4.14)$$

The left hand side of Equation 4.14 may be expressed as follows.

$$H(e^{j\omega\Delta T}) = H_R(e^{j\omega\Delta T}) + j H_I(e^{j\omega\Delta T}) \quad (4.15)$$

$$H(e^{j\omega\Delta T}) = |H(e^{j\omega\Delta T})| e^{j \arg[H(e^{j\omega\Delta T})]} \quad (4.16)$$

Using these equations and varying ω from 0 to $\omega_s/2$, the Nyquist frequency, the magnitude and phase response of a non-recursive digital filter can be calculated.

Figures 4.1 and 4.2 depict the frequency responses of orthogonal filters whose coefficients are listed in columns 1 and 2 of Table 4.2. Similarly Figure 4.3 and 4.4 show the frequency response of the second harmonic orthogonal filters whose coefficients are listed in the last two columns of Table 4.2. Recollect that:

- (i) these filters use a data window of 12 samples taken at 720 Hz.
- (ii) the filter coefficients were determined assuming that the input waveform consists of a decaying d.c. component and components of the fundamental frequency and the second and the third harmonic frequencies.

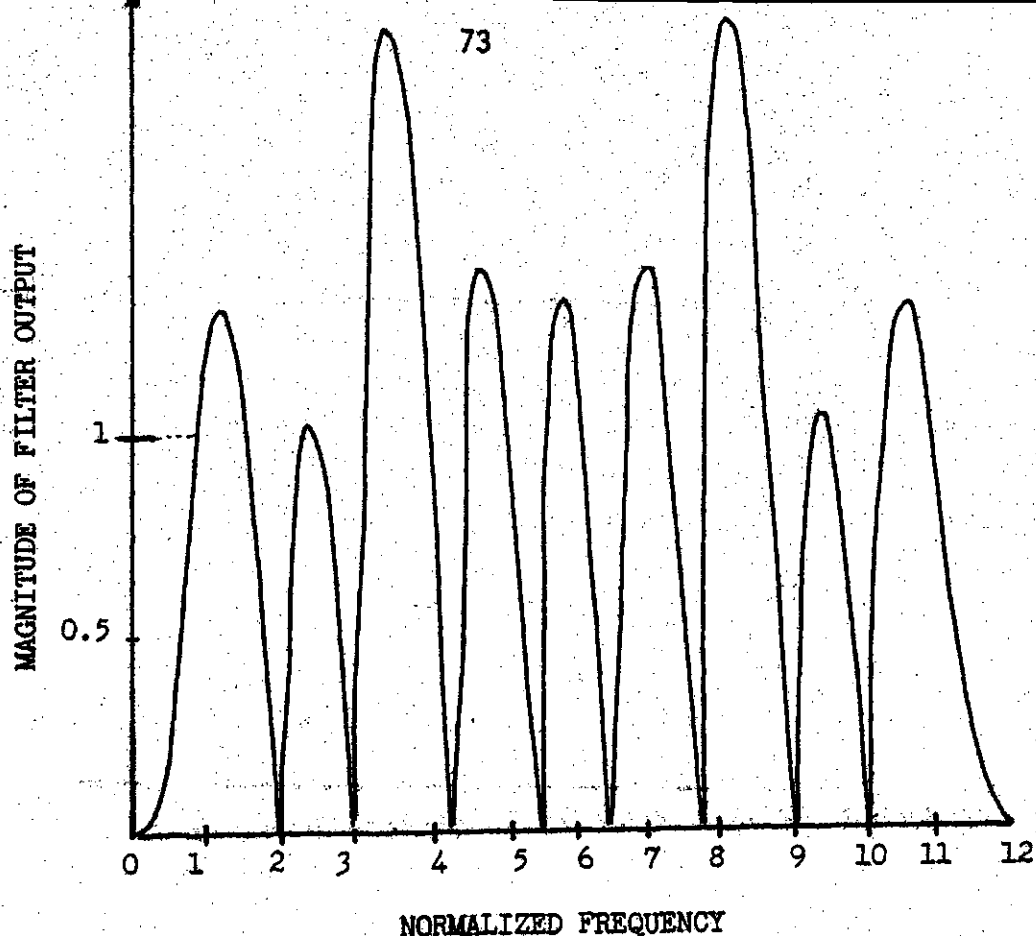


Figure 4.1 Frequency response of the 60 Hz digital filter designed by the least error squares approach whose coefficients are listed in column one of Table 4.2.

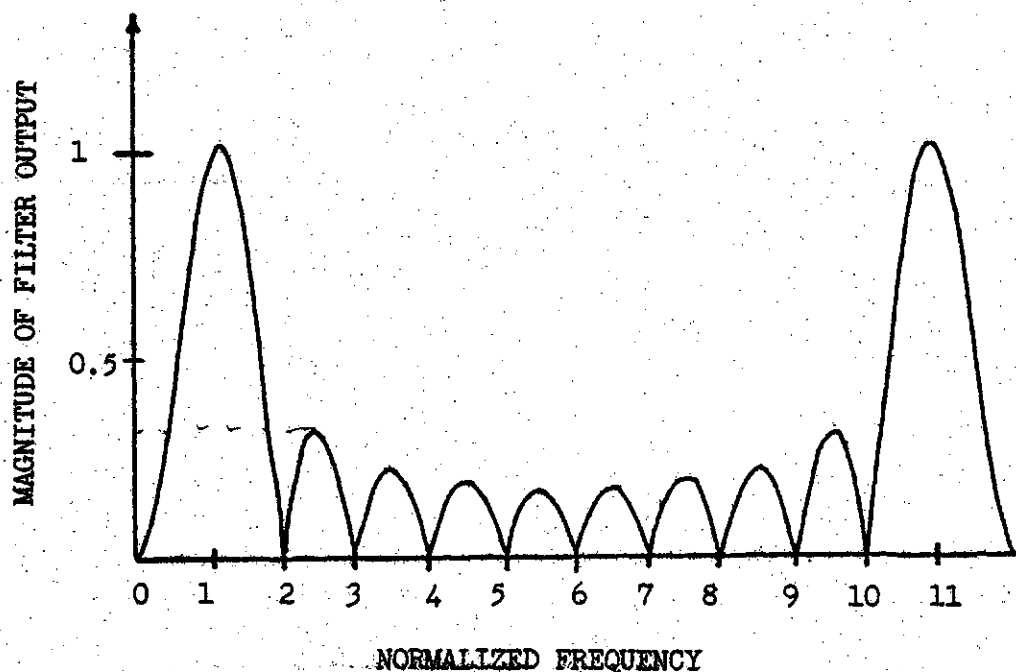


Figure 4.2 Frequency response of the 60 Hz digital filter designed by the least error squares approach whose coefficients are listed in column two of Table 4.2.

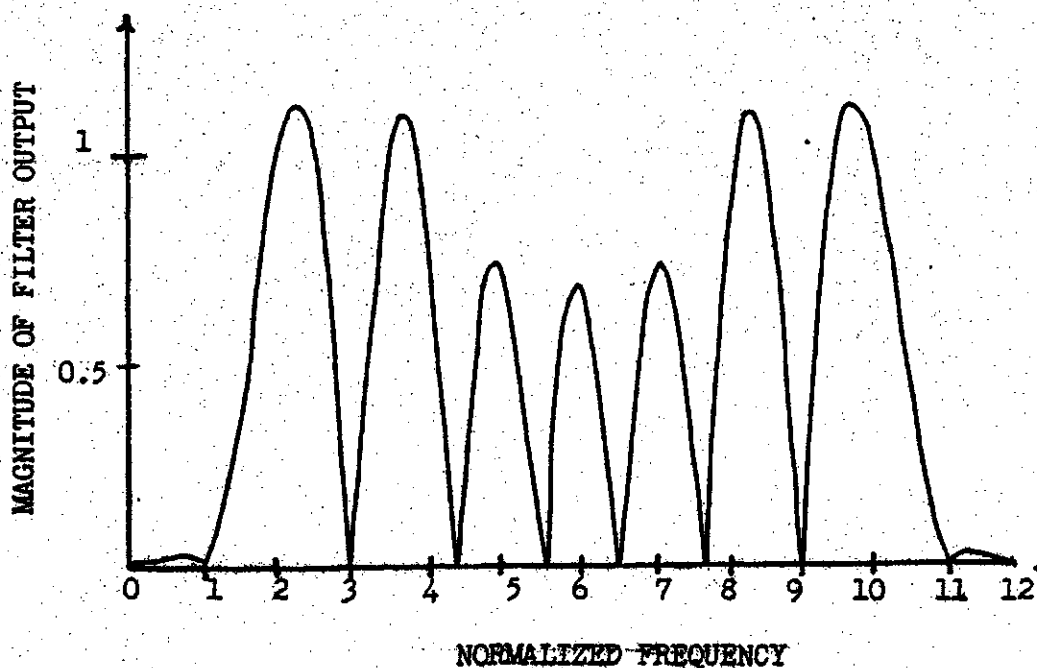


Figure 4.3 Frequency response of the 120 Hz digital filter designed by the least error squares approach whose coefficients are listed in column three of Table 4.2.

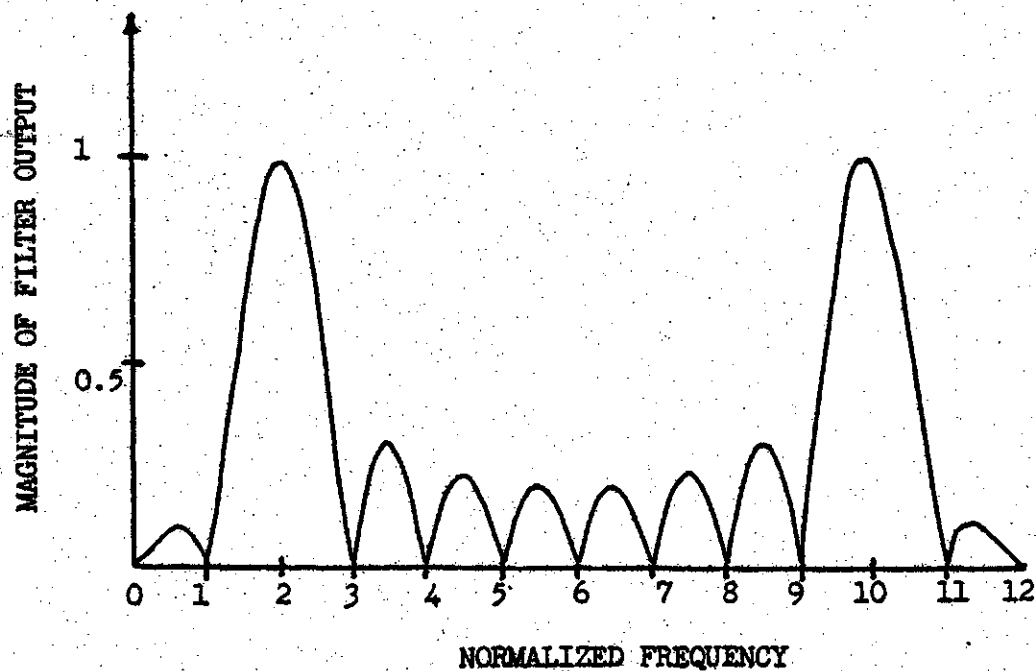


Figure 4.4 Frequency response of the 120 Hz digital filter designed by the least error squares approach whose coefficients are listed in column four of Table 4.2.

Figures 4.5 to 4.8 show the frequency responses of the fundamental and second harmonic orthogonal filters whose coefficients are given in Table 4.3. Recollect that in this case the filters were designed:

- (i) using a data window of 13 samples taken at 720 Hz.
- (ii) assuming that the input waveforms are made up of a decaying d.c. component and components of the fundamental frequency and the second, third and fourth harmonic frequencies.

An examination of Figures 4.1 to 4.8 show that the digital filters whose responses are shown in Figures 4.5 to 4.8 have more effective filtering characteristics.

The frequency response of an orthogonal pair of filters can also be combined to obtain a composite response. This provides the average and variance of the filter output to input ratio. It can also provide the expected maximum and minimum values of the output to input ratios.

Consider that input to a pair of orthogonal digital filters is $V_p \sin(\omega t + \theta)$. The outputs of the filters will be:

$$o_1 = k_1 V_p \sin(\omega t + \theta - \theta_1) \quad (4.17a)$$

$$o_2 = k_2 V_p \sin(\omega t + \theta - \theta_2) \quad (4.17b)$$

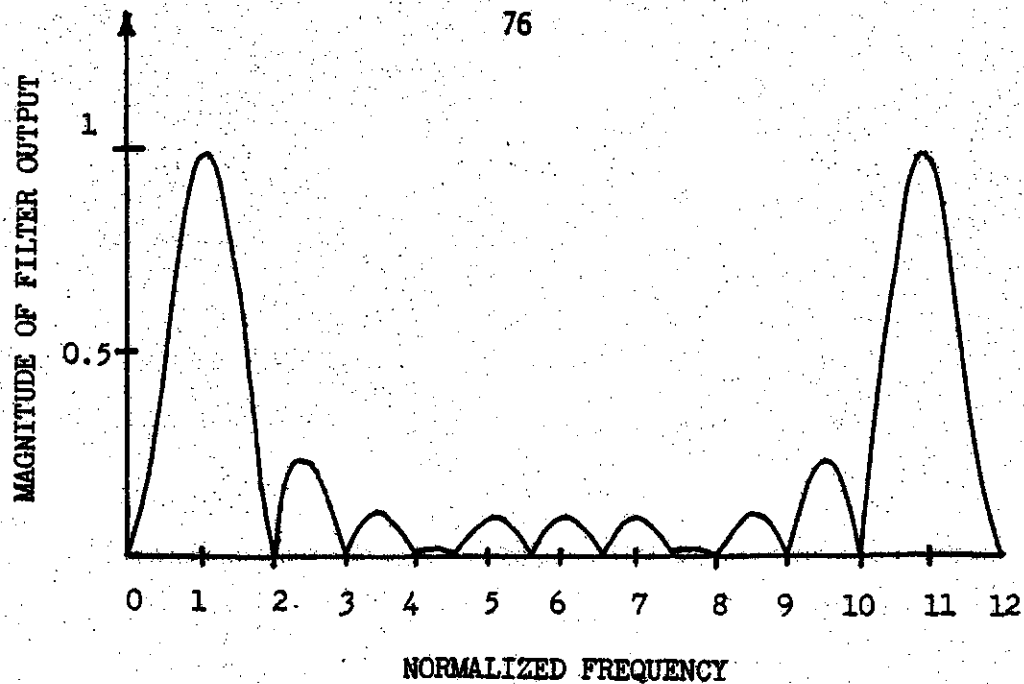


Figure 4.5 Frequency response of the 60 Hz digital filter designed by the least error squares approach whose coefficients are listed in column one of Table 4.3.

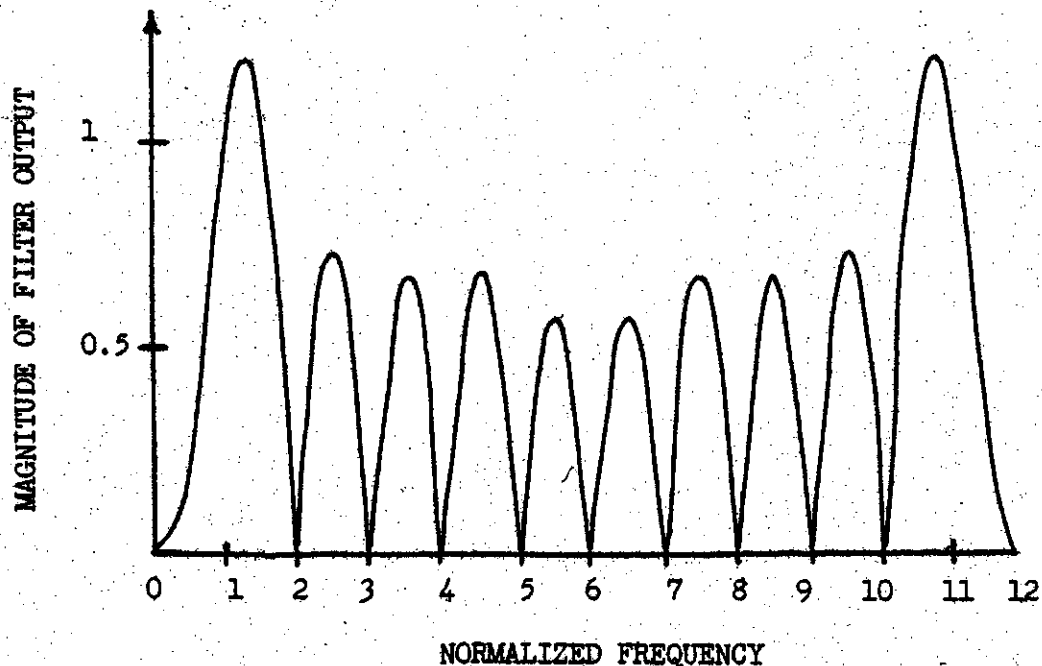


Figure 4.6 Frequency response of the 60 Hz digital filter designed by the least error squares approach whose coefficients are listed in column two of Table 4.3.

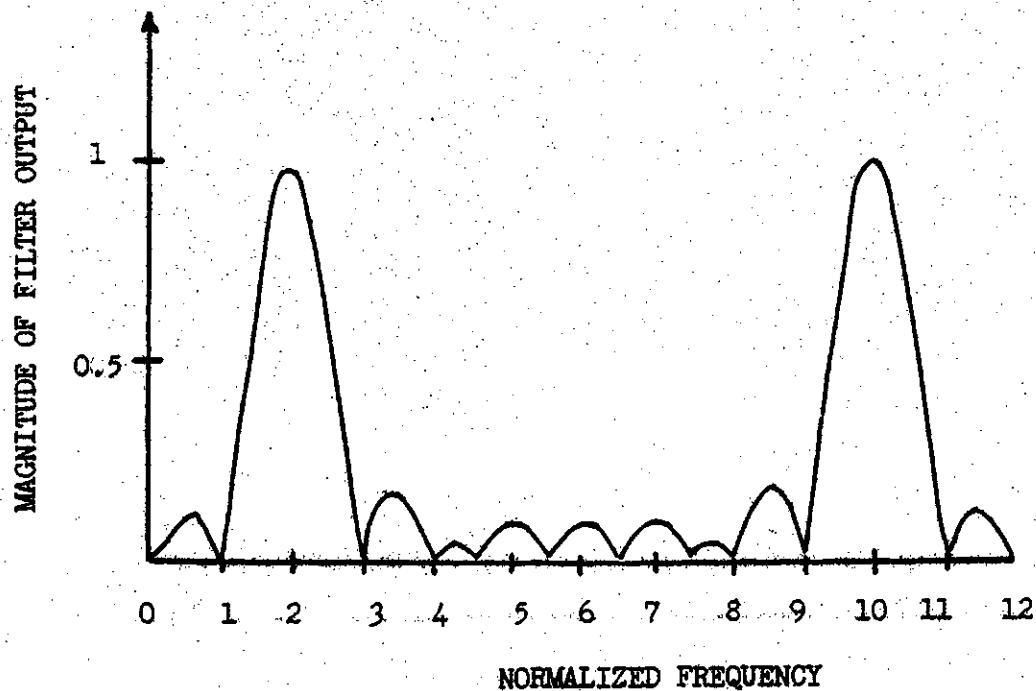


Figure 4.7 Frequency response of the 120 Hz digital filter designed by the least error squares approach whose coefficients are listed in column three of Table 4.3.

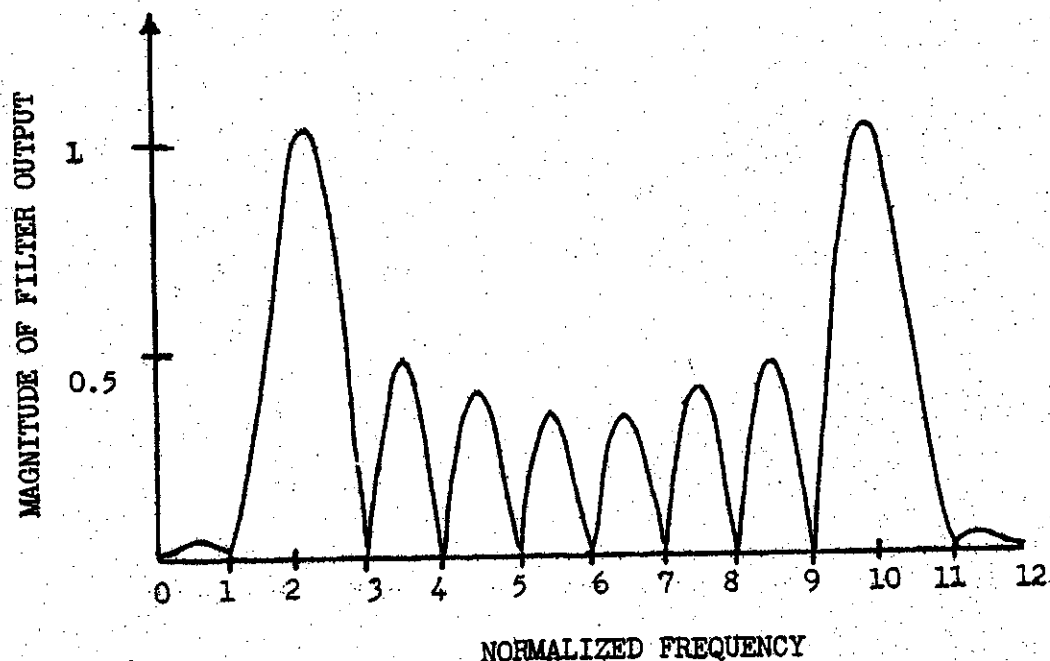


Figure 4.8 Frequency response of the 120 Hz digital filter designed by the least error squares approach whose coefficients are listed in column four of Table 4.3.

where:

k_1 and k_2 are the gains of the filters
at the selected frequency

θ_1 and θ_2 are the phase delays of the two
filters at the selected frequency

θ is the phase angle of the input signal

The outputs of the orthogonal filters are displaced in phase
by $\pi/2$ radians at all frequencies and can, therefore, be
represented as follows:

$$o_1 = A \sin(\omega t + \varphi) \quad (4.18a)$$

$$o_2 = B \cos(\omega t + \varphi) \quad (4.18b)$$

where:

$$A = k_1 V_p$$

$$B = k_2 V_p$$

$$\varphi = \theta - \theta_1$$

To find the maximum and minimum value of the composite
output, the outputs from the filters can be combined as
follows:

$$\begin{aligned} o^2 &= o_1^2 + o_2^2 \\ &= [A \sin(\omega t + \varphi)]^2 + [B \cos(\omega t + \varphi)]^2 \end{aligned}$$

$$= B^2 + \frac{1}{2}(A^2 - B^2) - \frac{1}{2}(A^2 - B^2) \cos(2\omega t + 2\phi) \quad (4.19)$$

In this equation ϕ is time dependent; however, the outputs will be time invariant if A and B are equal. The standard deviation of the composite output over a period of one cycle at a selected frequency is represented as follows:

$$\sigma = \frac{1}{2}(A^2 - B^2)/\sqrt{2} \quad (4.20)$$

where:

σ is the standard deviation of the composite output at a selected frequency

Figures 4.9 and 4.10 show the average value and the standard deviation of the composite filter response of the digital filter pair whose coefficients are listed at item 7 of Table 4.1.

The design and assesment of digital filters has revealed that the use of small data windows (3/4 cycle and less) result in filters whose frequency response is usually unacceptable. For a selected data window length, increasing the sampling rate does not improve the frequency response. Improvement in the filter response can, however, be achieved by increasing the size of the sampling window which increases operating time of the relay. Increasing the number of harmonic components in the waveform model

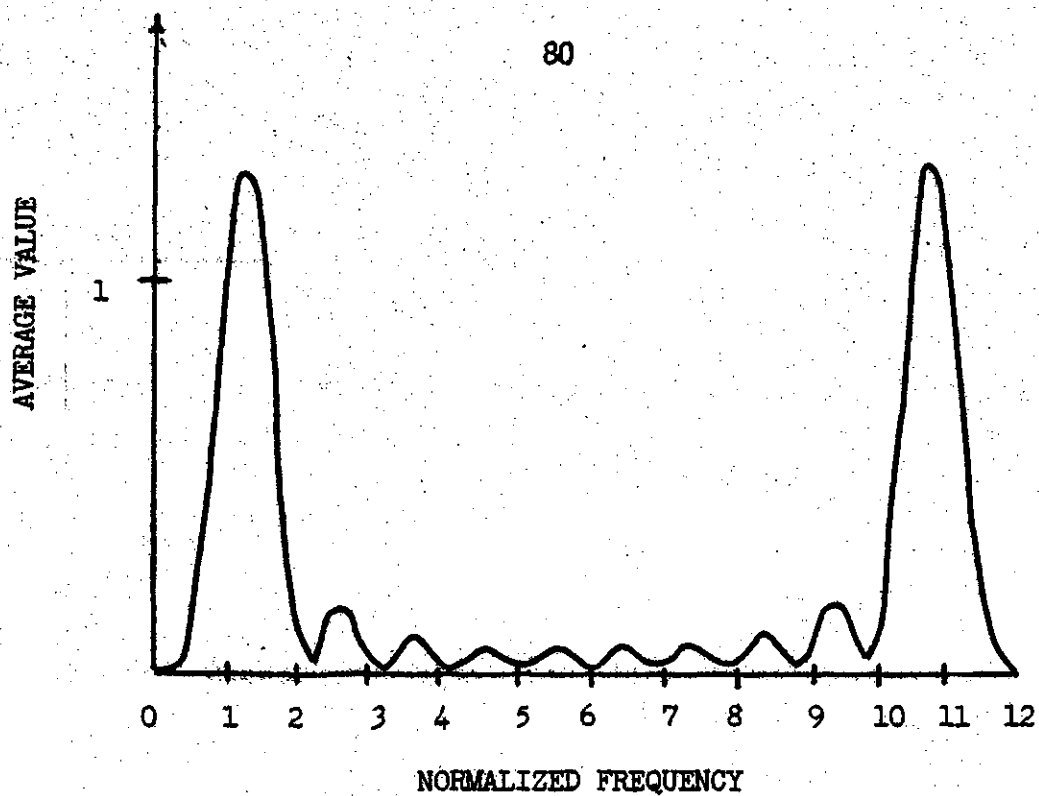


Figure 4.9 Average value of the composite filter response.

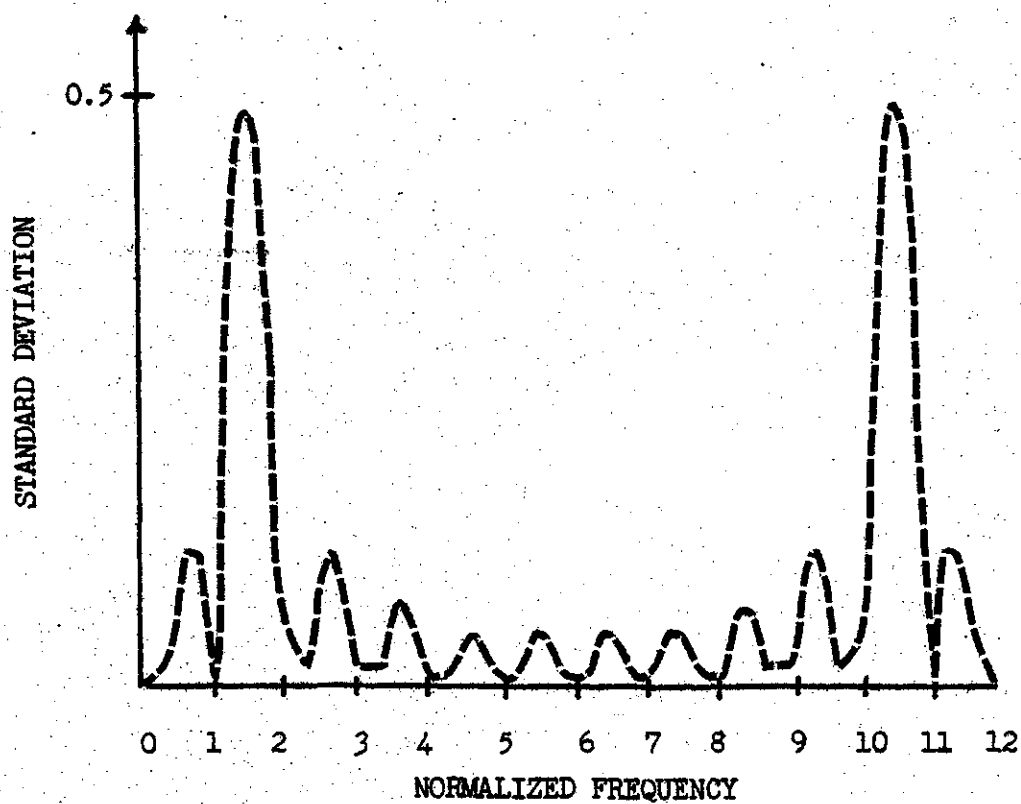


Figure 4.10 Standard deviation of the composite filter response.

increases the number of unknowns which in turn require longer data windows. With a judicious selection of the data window size, sampling rate, and the complexity of the waveform model, appropriate digital filters can be designed using the least error squares curve fitting approach.

4.2 Fourier Series Approach

The least error squares approach for designing non-recursive digital filters has been discussed in the last section. Another possible approach which can be used for this purpose is the Fourier series method which is described in this section.

The ideal frequency response of a band-pass filter is shown in Figure 4.11. The cut off frequencies of this filter are w_{c1} and w_{c2} . The transfer function of this filter can be mathematically represented as

$$H(w) = \begin{cases} 1 & w_{c1} < w < w_{c2} \\ 0 & \text{otherwise} \end{cases} \quad (4.21)$$

This response can not be physically realized but filters whose response is quite similar to the ideal response can be designed and built.

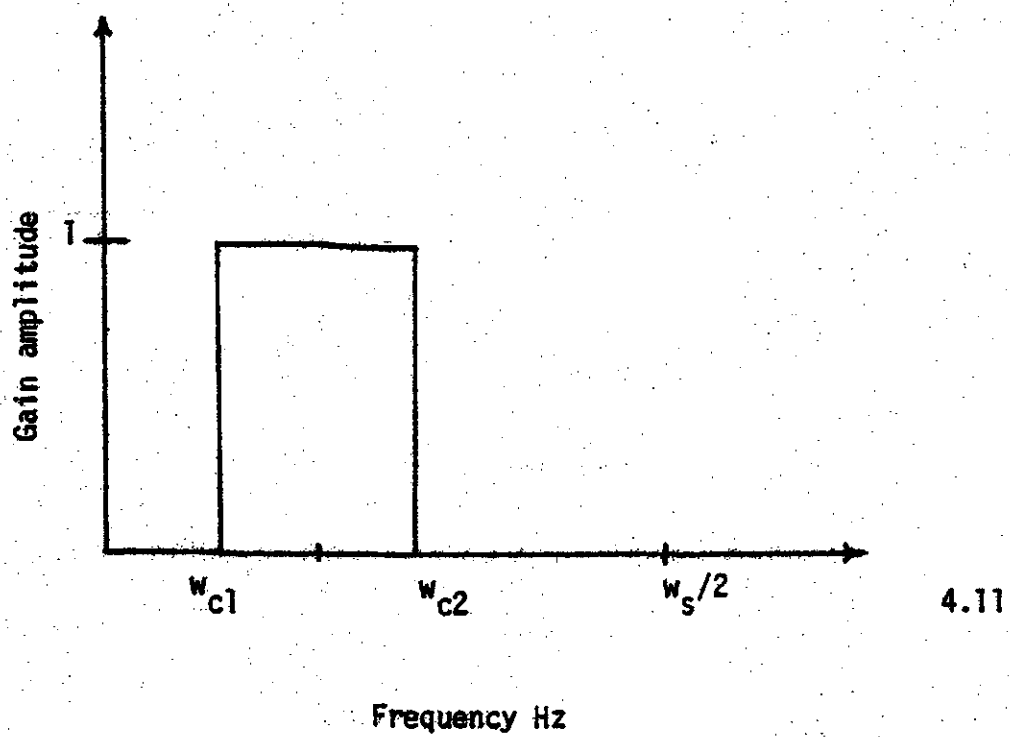


Figure 4.11 Ideal frequency response of a band-pass filter.

The transfer function of a non-recursive digital filter was expressed as Equation 4.13 which is reproduced below as Equation 4.22.

$$\begin{aligned}
 H(z) &= \sum_{n=0}^m c_n z^{-n} \\
 &= c_0 + c_1 z^{-1} + \dots + c_m z^{-m}
 \end{aligned} \tag{4.22}$$

Since the frequency response of a digital system is periodic, it can be expanded in the form of the Fourier series. The periodic frequency response function, $G(w)$, of a non-recursive filter of period w_s (sampling frequency) is represented by the following Fourier series.

$$G(w) = A_0 + \sum_{k=1}^{\infty} A_k \cos(kw \Delta T) + \sum_{k=1}^{\infty} B_k \sin(kw \Delta T) \tag{4.23}$$

In this equation w is the radian frequency at which the response is calculated and ΔT is the selected sampling interval. The odd or even Fourier series can be used in case of the desired frequency response to be odd or even. These series can be mathematically expressed as follows.

$$G(w)_{\text{even}} = A_0 + \sum_{k=1}^{\infty} A_k \cos(kw \Delta T) \tag{4.24}$$

$$G(w)_{\text{odd}} = \sum_{k=1}^{\infty} B_k \sin(kw \Delta T) \tag{4.26}$$

Equation 4.23 can also be written in the complex form as:

$$G(w) = \sum_{k=-\infty}^{\infty} a_k e^{jkw\Delta T} \quad (4.26)$$

where:

$$a_0 = A_0$$

$$a_k = \frac{1}{2} (A_k - jB_k) \quad \text{When } k \text{ is } +ve$$

$$a_k = \frac{1}{2} (A_k + jB_k) \quad \text{When } k \text{ is } -ve$$

The coefficients of this equation are complex conjugate pairs.

The z transform $X(z)$ of a sequence of digitized samples, $x(k)$, can be defined as follows:

$$X(z) = \sum_{k=-\infty}^{\infty} x(k) z^k \quad (4.27)$$

A comparison of Equations 4.26 and 4.27, indicates that the function $G(z)$ can be represented by:

$$G(z) = \sum_{k=-\infty}^{\infty} a_k z^k \quad (4.28)$$

where:

$$z = e^{jw\Delta T}$$

This equation indicates that a digital filter to realize $G(z)$ requires an infinite number of coefficients. A reasonable compromise is to truncate Equation 4.28 to the following form.

$$G_T(z) = \sum_{k=-M}^M a_k z^k$$

$$= z^M [a_M + a_{M-1} z^{-1} + \dots + a_{-M} z^{-2M}] \quad (4.29)$$

This equation has $(2M+1)$ coefficients. If Equations 4.22 and 4.29 are to have the same number of coefficients, m must be equal to $2M$. The term z is a phase shift operator and does not affect the magnitude response of the filter. The relationship between the coefficients of the digital filter and the Fourier series is obtained by equating the coefficients of like powers of z in Equation 4.22 and 4.29; this process indicates that:

$$\begin{aligned} c_0 &= a_M \\ c_1 &= a_{M-1} \\ &\cdot \\ &\cdot \\ c_{m/2} &= a_0 \\ &\cdot \\ &\cdot \\ c_{m-1} &= a_{-M+1} \\ c_m &= a_{-M} \end{aligned} \quad (4.30)$$

Orthogonal filters can also be designed by approximating the even and odd frequency characteristics by the Fourier series coefficients. Consider that the ideal magnitude response required of an orthogonal pair of bandpass digital filters is as shown in Figures 4.12 and 4.13. Designate the even and odd responses as $f(w)_e$ and $f(w)_o$ respectively and the band pass as $2w_g$. Each of the two filters pass signals of frequencies in a specified band around a frequency, w_1 . If the sampling rate is twelve times w_1 , the high frequency 'passband' will be around the eleventh harmonic. The coefficients of the digital filters approximating the ideal response can be computed as follows.

The 'even' and 'odd' frequency responses shown in Figure 4.12 and 4.13 can be approximated by the Fourier series of Equation 4.24 and 4.25. The Fourier series coefficients, A_k 's and B_k 's, of the 'even' frequency response can be calculated as follows.

$$\begin{aligned}
 A_0 &= \frac{1}{w_s} \int_0^{w_s} f(w)_e dw \\
 &= \frac{1}{w_s} \int_{w_1 - w_g}^{w_1 + w_g} dw + \frac{1}{w_s} \int_{w_{11} - w_g}^{w_{11} + w_g} dw \\
 &= 4w_g/w_s
 \end{aligned}$$

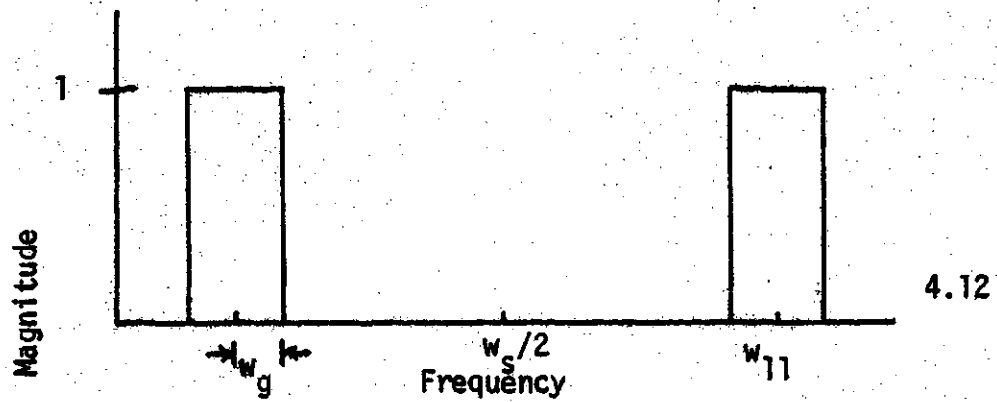


Figure 4.12 The 'even' frequency response characteristics of an ideal band-pass filter.

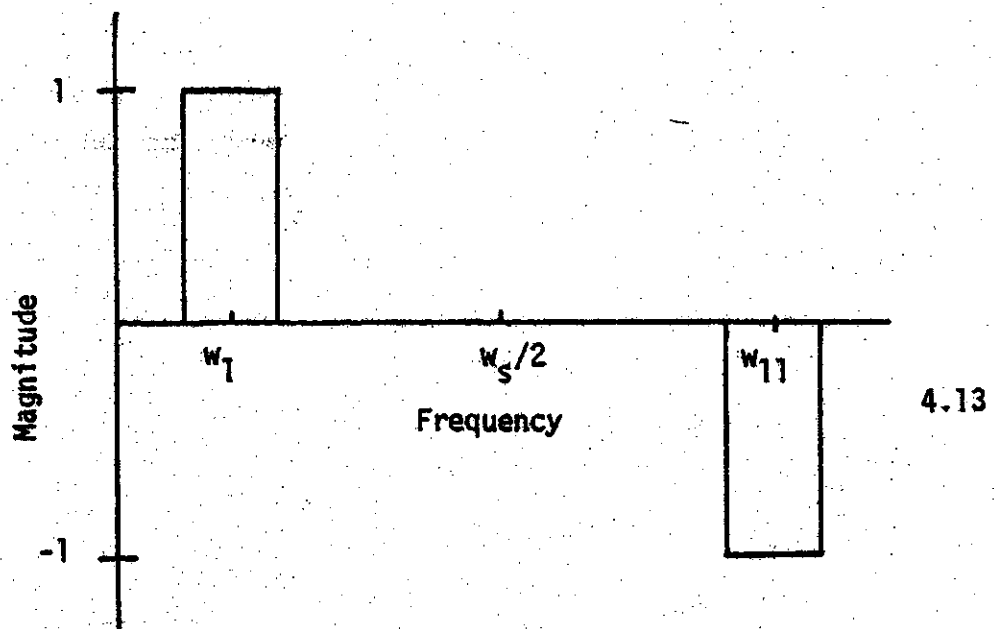


Figure 4.13 The 'odd' frequency response characteristics of an ideal band-pass filter.

$$\begin{aligned}
A_k &= \frac{2}{w_s} \int_0^{w_s} f(w)_e \cos\left(\frac{2\pi k w}{w_s}\right) dw \\
&= \frac{2}{w_s} \int_{w_1 - w_g}^{w_1 + w_g} \cos\left(\frac{2\pi k w}{w_s}\right) dw + \frac{2}{w_s} \int_{w_{11} - w_g}^{w_{11} + w_g} \cos\left(\frac{2\pi k w}{w_s}\right) dw \\
&= \frac{1}{\pi k} \left\{ \sin\left[2\pi k \left(\frac{w_1 + w_g}{w_s}\right)\right] - \sin\left[2\pi k \left(\frac{w_1 - w_g}{w_s}\right)\right] \right\} + \\
&\quad \frac{1}{\pi k} \left\{ \sin\left[2\pi k \left(\frac{w_{11} + w_g}{w_s}\right)\right] - \sin\left[2\pi k \left(\frac{w_{11} - w_g}{w_s}\right)\right] \right\} \quad (4.31a)
\end{aligned}$$

$$B_k = 0 \quad (4.31b)$$

Similarly the Fourier series coefficients A_k 's and B_k 's of the 'odd' frequency response can be calculated using the following equations.

$$A_k = 0 \quad (4.32a)$$

$$\begin{aligned}
B_k &= \frac{2}{w_s} \int_0^{w_s} f(w)_o \sin\left(\frac{2\pi k w}{w_s}\right) dw \\
&= \frac{2}{w_s} \int_{w_1 - w_g}^{w_1 + w_g} \sin\left(\frac{2\pi k w}{w_s}\right) dw - \frac{2}{w_s} \int_{w_{11} - w_g}^{w_{11} + w_g} \sin\left(\frac{2\pi k w}{w_s}\right) dw
\end{aligned}$$

$$= \frac{1}{\pi k} \left\{ \cos \left[2\pi k \left(\frac{w_l}{w_s} - \frac{w_g}{w_s} \right) \right] - \cos \left[2\pi k \left(\frac{w_l}{w_s} + \frac{w_g}{w_s} \right) \right] \right\} -$$

$$\frac{1}{\pi k} \left\{ \cos \left[2\pi k \left(\frac{w_{l1}}{w_s} - \frac{w_g}{w_s} \right) \right] - \cos \left[2\pi k \left(\frac{w_{l1}}{w_s} + \frac{w_g}{w_s} \right) \right] \right\} \quad (4.32b)$$

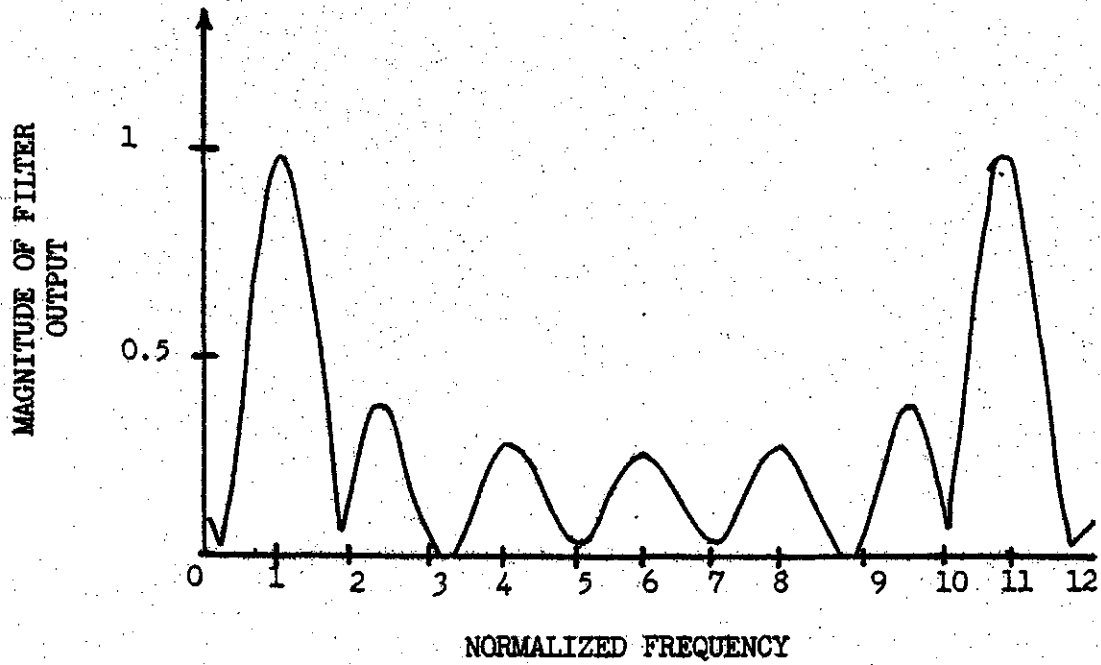
After calculating the A_k and B_k coefficients, a k 's defined in Equation 4.26 can be calculated for the two orthogonal filters. The filter coefficients, c_k 's can be then calculated using Equation 4.30.

4.2.1 Filters designed by the Fourier series method

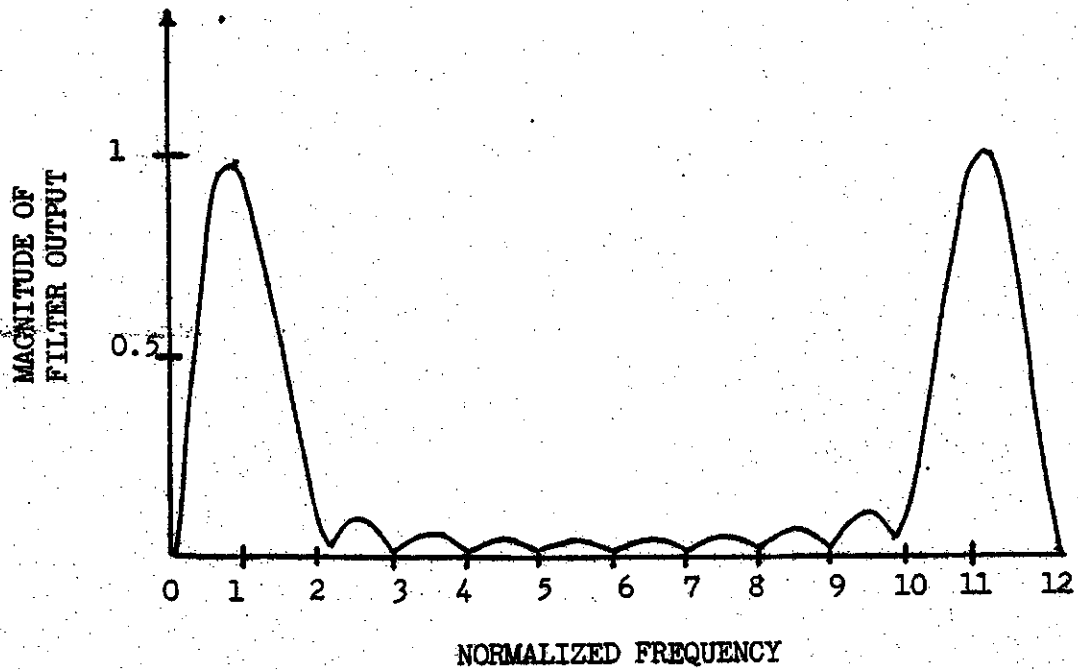
The Fourier series approach described in the last section is suitable for designing non-recursive filters whose response approximates an ideal response. Many filters were designed using this technique. In one of the filter designs, the band width was selected to be 60 Hz and the data window $(2M+1)$ was selected to be thirteen samples at a sampling rate of 720 Hz. The filter coefficients for the fundamental and second harmonic digital filters were calculated. The calculated coefficients are listed in Table 4.4. Figures 4.14a and 4.14b show the frequency response of the fundamental frequency filters and Figures 4.15a and 4.15b depict the frequency response of the second harmonic frequency filters. A study of these figures reveals that the filters effectively eliminate the unwanted frequencies

Table 4.4 Filter coefficients of the fundamental and second harmonic digital filters designed by the Fourier series method.

Fundamental		Second harmonic	
'odd'	'even'	'odd'	'even'
0.0000	-0.1115	0.0000	0.1101
0.0691	-0.1119	-0.1214	0.0638
0.1342	-0.0724	-0.1361	-0.0715
0.1687	0.0000	0.0000	-0.1557
0.1550	0.0836	0.1571	-0.0826
0.0926	0.1500	0.1627	0.0855
0.0000	0.1673	0.0000	0.1651
-0.0926	0.1500	-0.1627	0.0855
-0.1550	0.0836	-0.1571	-0.0826
-0.1687	0.0000	0.0000	-0.1557
-0.1342	-0.0724	0.1361	-0.0715
-0.0691	-0.1119	0.1214	0.0638
-0.0000	-0.1115	0.0000	0.1101

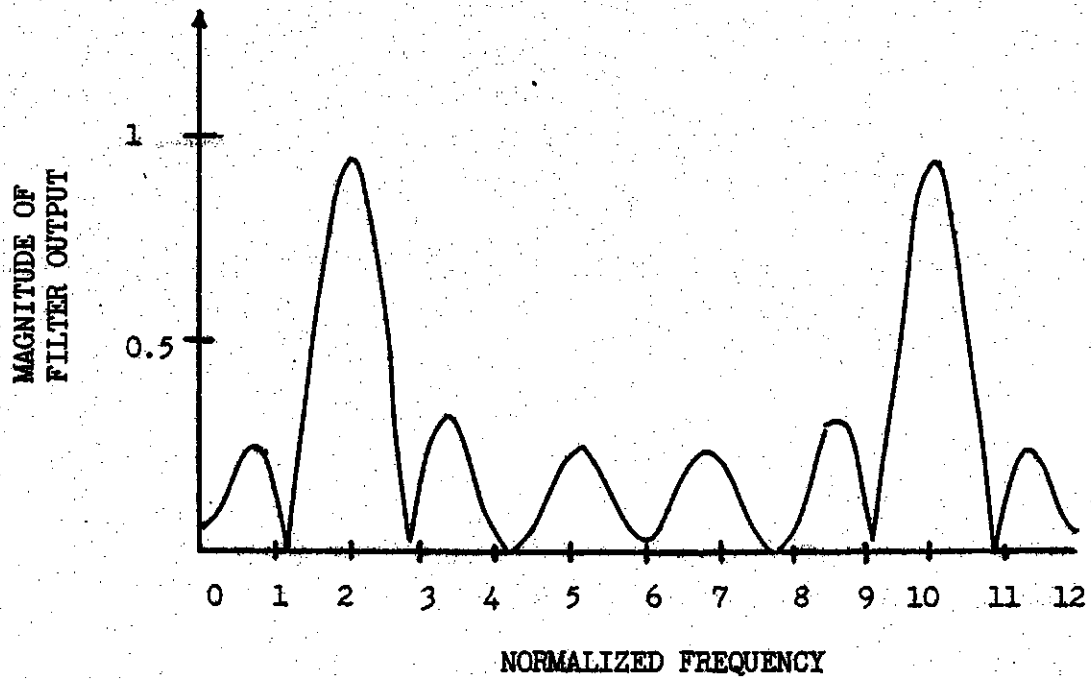


(a) 'even'

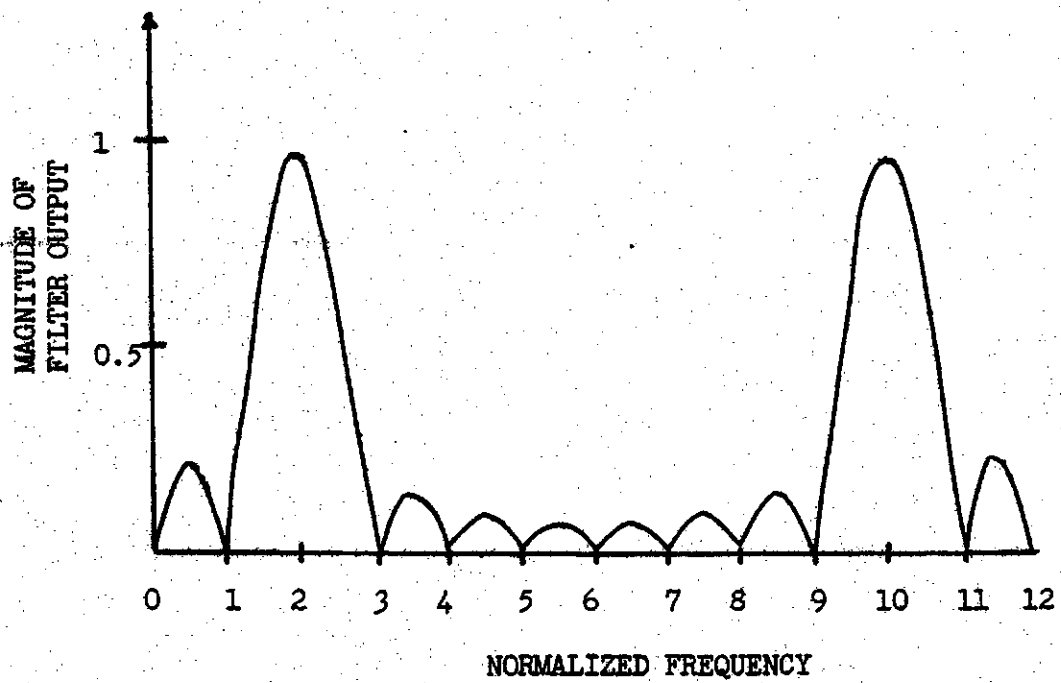


(b) 'odd'

Figure 4.14 Frequency responses of the fundamental frequency 'even' and 'odd' band-pass filters designed by the Fourier series method.



(a) 'even'



(b) 'odd'

Figure 4.15 Frequency response of the second harmonic frequency 'even' and 'odd' band-pass filters designed by the Fourier series method.

but fail to completely eliminate the d.c. component.

4.3 Frequency Sampling Approach

Least error squares and Fourier series techniques for designing digital filters have been presented in Sections 4.1 and 4.2 respectively. In this section, a third approach called the frequency sampling method^{1,3} is outlined. This technique approximates the desired filter response like the Fourier series approach does. The specified frequency response characteristic is sampled at N equispaced frequencies. The sampled responses are equated with the discrete Fourier transforms of the digital filter coefficients. This procedure provides simultaneous equations from which the filter coefficients can be determined. The mathematical basis of this approach follows immediately.

The frequency response of an ideal digital filter is shown in Figure 4.16. The frequencies at which the filter response is sampled are also shown in the figure. This filter has zero phase shift and either zero or unity gain.

The Discrete Fourier Transform (D.F.T)¹ of an array of N complex numbers, $(x_0, x_1, \dots, x_{N-1})$, is another array of the same dimension, $(X_0, X_1, \dots, X_{N-1})$. The elements of the array $\{X_k\}$ are defined as follows.

$$X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \quad (4.33)$$

The inverse of the Discrete Fourier Transform provides the array $[x_n]^T$ from the elements of $[X_k]^T$ as follows.

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \quad n = 0, 1, \dots, N-1 \quad (4.34)$$

The transfer function of a non-recursive digital filter and the frequency response characteristics of the filter have already been established in Equations 4.13 and 4.14. From those equations, the frequency response at N equispaced frequencies can be defined as follows:

$$H_k = \sum_{n=0}^{N-1} c_n e^{-jnk\Delta\omega\Delta T}, \quad k = 0, 1, \dots, N-1 \quad (4.35)$$

where:

$$\begin{aligned} H_k & \text{ are the frequency response samples} \\ \Delta T & = 2\pi/\omega_s \\ \Delta\omega & = \omega_s/N \\ \omega_s & = \text{sampling frequency} \end{aligned}$$

Equation 4.35 reduces to the following form by replacing ΔT by $2\pi/\omega_s$ and $\Delta\omega$ by ω_s/N .

$$H_k = \sum_{n=0}^{N-1} c_n e^{-j2\pi nk/N}, \quad k = 0, 1, \dots, N-1 \quad (4.36)$$

A comparison of this equation with Equation 4.13 indicates that H_k can be determined by evaluating $H(z)$ for $z = e^{j\omega\Delta T}$, as follows:

$$H_k = H(z) \Big|_{z = e^{j2\pi nk/N}}, \quad k = 0, 1, \dots, N-1 \quad (4.37)$$

This equation is similar to Equation 4.33, the Discrete Fourier Transform of a sequence of complex numbers. H_k in Equation 4.37 is, therefore, the D.F.T. of the digital filter coefficients. Since the frequency response is already defined, the filter coefficients can be determined using the inverse D.F.T. as follows:

$$c_n = \frac{1}{N} \sum_{k=0}^{N-1} H_k e^{j2\pi nk/N}, \quad n = 0, 1, \dots, N-1 \quad (4.38)$$

These coefficients can then be used to realize the required non-recursive digital filter.

4.3.1 Filters designed by the frequency sampling method

The frequency sampling method for designing non-recursive filters has been outlined in the last section. Using this method, many digital filters were designed. The data window of twelve samples and a sampling rate of 720 Hz. were used. Particulars of some of the designed filters are reported in this section.

The desired filter response characteristics for the 'even' and 'odd' filters were first selected. These characteristics are shown in Figures 4.16 and 4.17. Twelve discrete frequencies over the range of the characteristics were also selected as indicated in these figures. The frequency response samples, (H_k), for the fundamental frequency and second harmonic filters were obtained and are listed in Table 4.5. Coefficients of the filters were calculated using Equation 4.38 are listed in Table 4.6.

Figures 4.18 to 4.21 depict the magnitude response of the digital filters designed by the frequency sampling method. Figures 4.18 and 4.19 depict the frequency response of the fundamental frequency filters and Figures 4.20 and 4.21 depict the frequency response of the second harmonic filters.

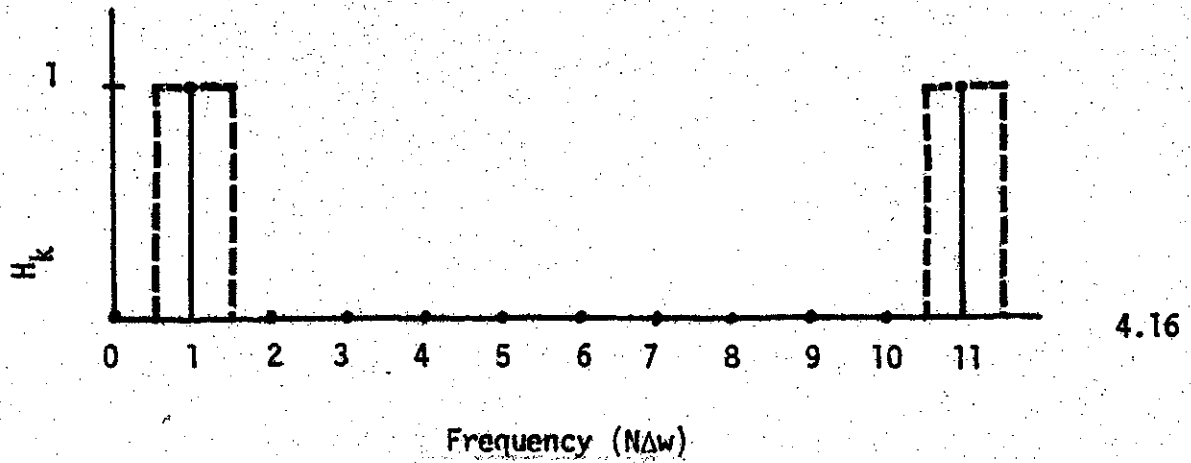


Figure 4.16 Frequency samples of the 'even' frequency response characteristics of an ideal band-pass filter.

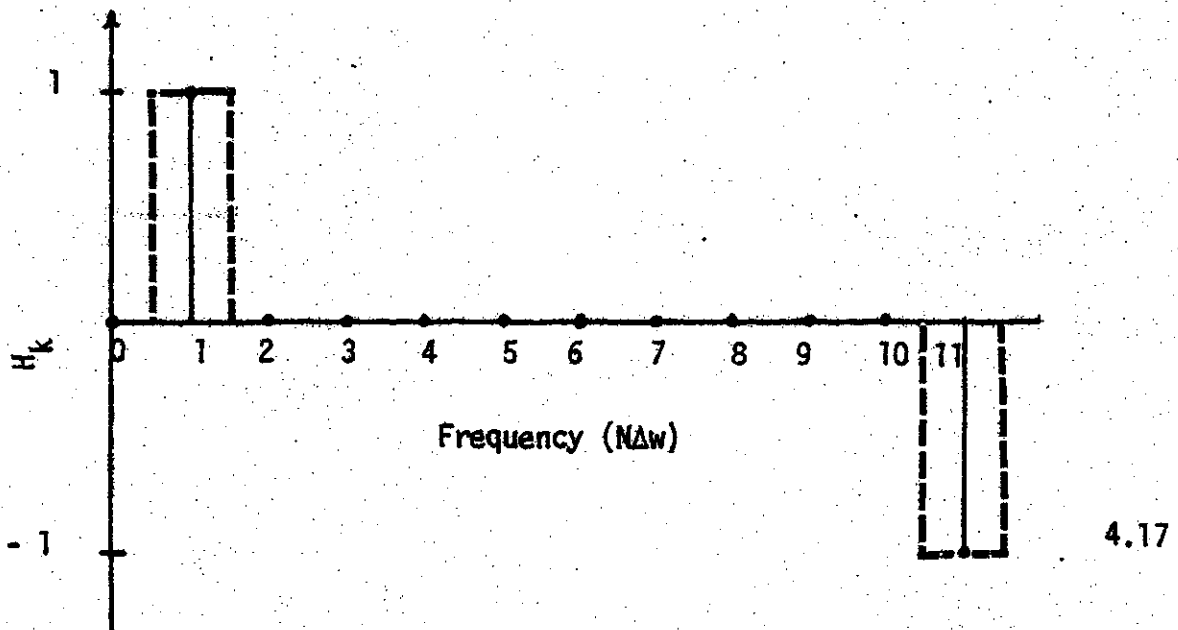


Figure 4.17 Frequency samples of the 'odd' frequency response characteristics of an ideal band-pass filter.

Table 4.5 Design specifications for the fundamental and second harmonic digital filters.

	Frequency		Samples		H_k
Multiple of fundamental frequency	Fundamental		Second harmonic		
	'odd'	'even'	'odd'	'even'	
0	0	0	0		0
1	1	1	0		0
2	0	0	1		1
3	0	0	0		0
4	0	0	0		0
5	0	0	0		0
6	0	0	0		0
7	0	0	0		0
8	0	0	0		0
9	0	0	0		0
10	0	0	-1		1
11	-1	1	0		0

Table 4.6 Filter coefficients of the fundamental and second harmonic digital filter designed by the frequency sampling method.

n	Filter Coefficients		h(n)	
	Fundamental		Second harmonic	
	'odd'	'even'	'odd'	'even'
0	0.0000	0.1667	0.0000	0.1667
1	0.0833	0.1443	0.1443	0.0833
2	0.1443	0.0833	0.1443	-0.0833
3	0.1667	0.0000	0.0000	-0.1667
4	0.1443	-0.0833	-0.1443	-0.0833
5	0.0833	-0.1443	-0.1443	0.0833
6	0.0000	-0.1667	0.0000	0.1667
7	-0.0833	-0.1443	0.1443	0.0833
8	-0.1443	-0.0833	0.1443	-0.0833
9	-0.1667	0.0000	0.0000	-0.1667
10	-0.1443	0.0833	-0.1443	-0.0833
11	-0.0833	0.1443	-0.1443	0.0833

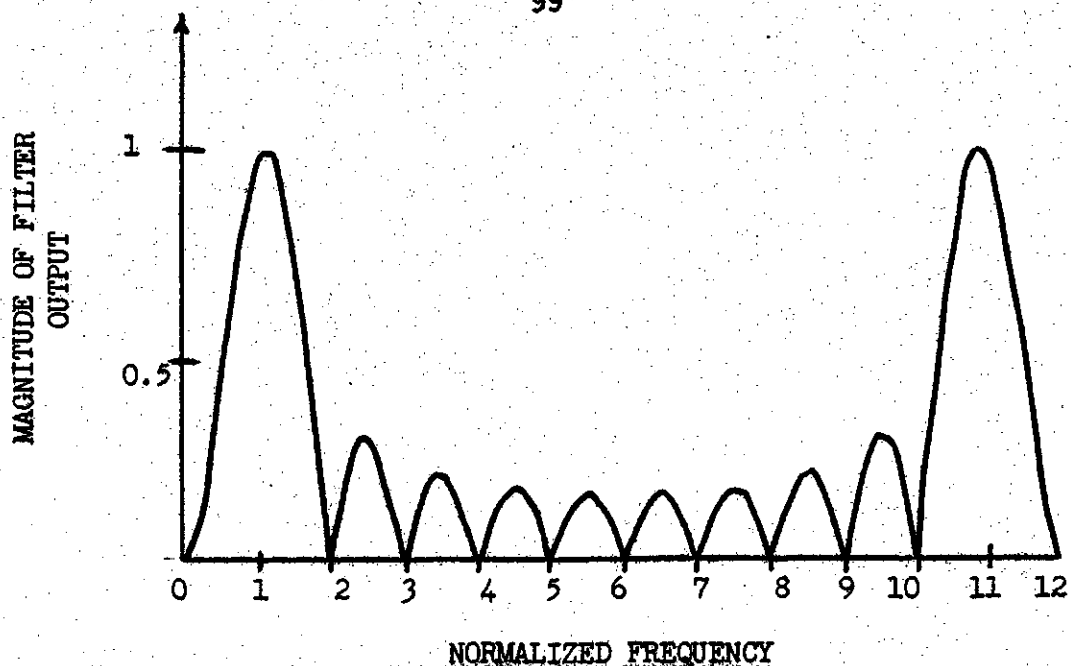


Figure 4.18 Frequency response of the 60 Hz digital filter designed by the frequency sampling method whose coefficients are listed in column one of Table 4.6.

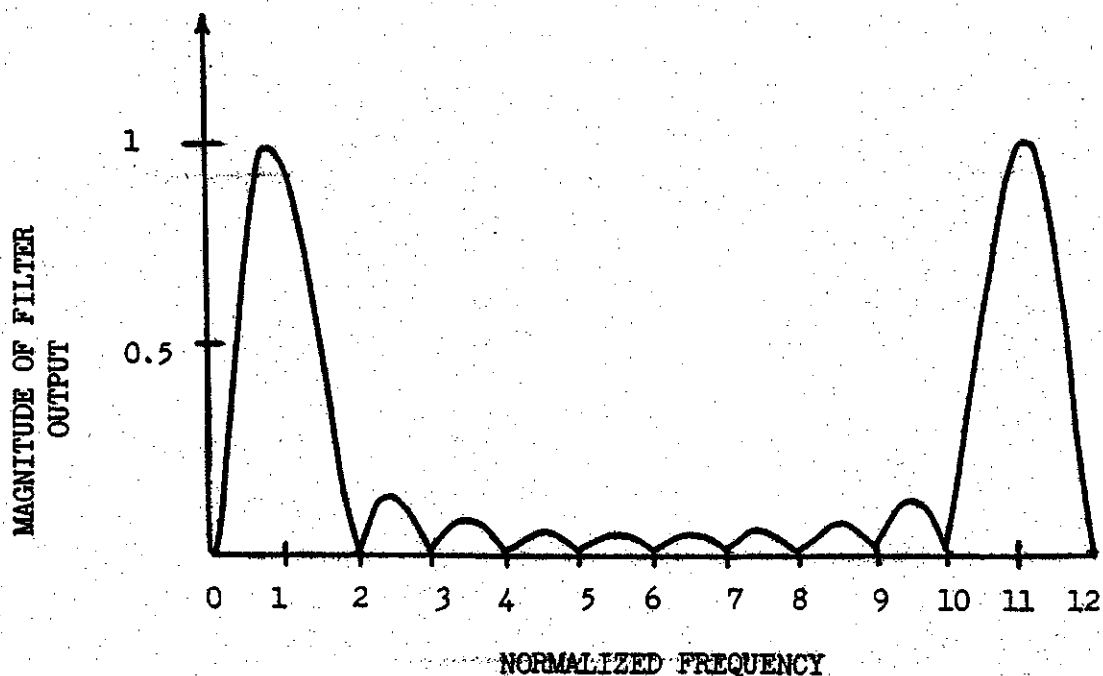


Figure 4.19 Frequency response of the 60 Hz digital filter designed by the frequency sampling method whose coefficients are listed in column two of Table 4.6.

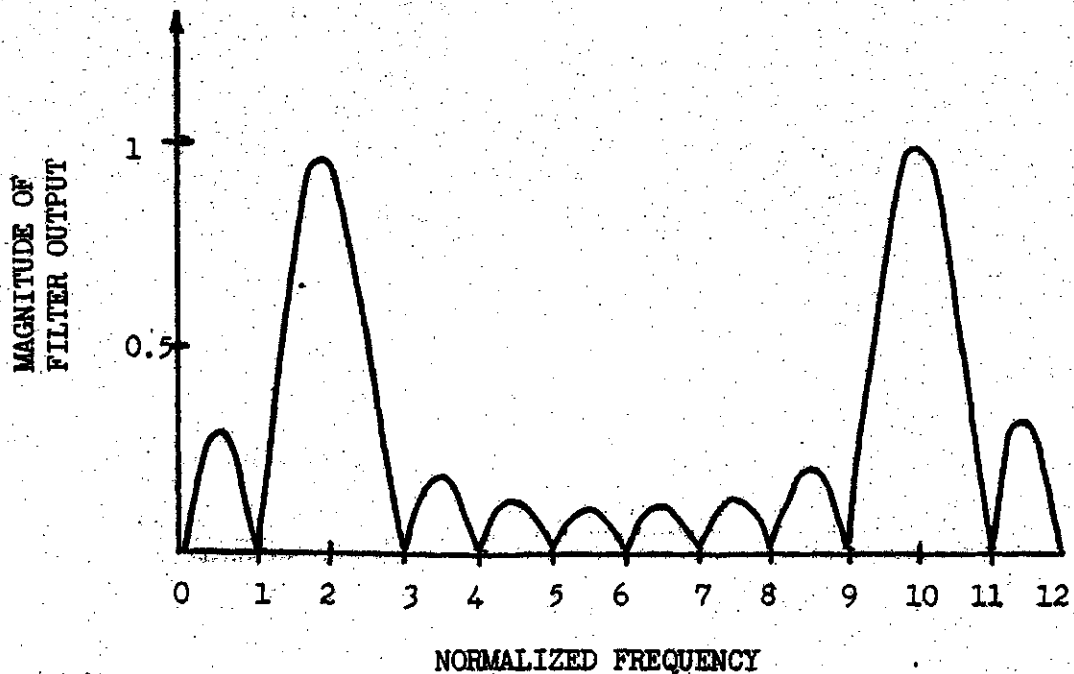


Figure 4.20 Frequency response of the 120 Hz digital filter designed by the frequency sampling method whose coefficients are listed in column three of Table 4.6.

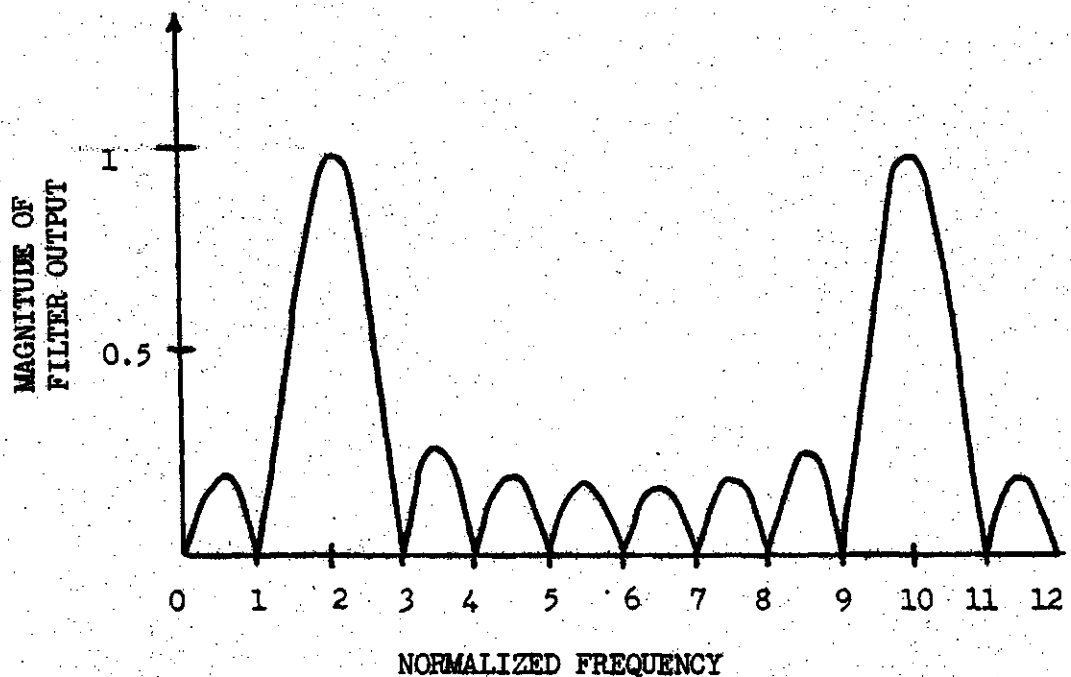


Figure 4.21 Frequency response of the 120 Hz digital filter designed by the frequency sampling method whose coefficients are listed in column four of Table 4.6.

From an examination of the frequency responses of filters designed by the frequency sampling method, it is observed that these filters can effectively eliminate d.c. from the input signals. The energy in the sidelobes is small compared to the energy in the mainlobe and the zeroes of the frequency response are placed where the sampled response was designated to be zero. The frequency response can be further improved if the filters are designed for longer data windows.

4.4 Window Functions

The effect of window functions¹ on the digital filter responses was also examined. Appendix B describes the purpose of the window function along with its effects on the response of some of the designed filters.

4.5 Filter Evaluation

Three methods for designing non-recursive digital filters have so far been presented in this chapter. As is the common practice, strengths and weaknesses of digital filters have been examined by comparing their frequency responses. Three methods of evaluating the filter responses are described in this section.

In an ideal band-pass filter, the entire output energy is in the pass-band. In practice leakage is present because the actual frequency response deviates from the ideal response. The effectiveness of a filter can therefore be measured as the ratio of the output energy in the pass-band to the total output energy². For a digital filter, the total energy stored can be expressed as follows.

$$E = \frac{1}{w_s} \int_0^{w_s} |H(e^{j\omega T})|^2 d\omega \quad (4.39)$$

Another index which measures the quality of filter performance is the noise transmission index² which can be expressed as follows.

$$\text{Effective noise transmission} = \sum_{j=0}^{N-1} (a_j)^2 \quad (4.40)$$

For non-recursive filters smaller the value of the sum of the squares of the filter coefficients, lower is the variance of noise in the output.

Another index which can be considered for evaluating a digital filter is the ripple ratio² defined as follows.

$$\text{Ripple ratio} = \frac{\text{Maximum side-lobe amplitude}}{\text{Maximum main-lobe amplitude}} \quad (4.41)$$

Low values of ripple ratio are desirable in good quality filters.

A well designed digital filter should therefore have:

1. a substantial portion of the output energy in the desired pass-band.
2. low noise transmission characteristics.
3. low ripple ratios.

These indices were calculated for the digital filters whose design has been reported in this chapter. The calculated indices are given in Table 4.7. An examination of this table shows that digital filters designed by the Fourier series approach have the 'best' filtering characteristics except that it does not attenuate the decaying d.c. components sufficiently which is an important requirement for power system control and protection applications. An overall consideration indicates that both the least error squares and the frequency sampling designs are suitable. Further off-line and on-line testing of the filters in a relay algorithm is, however essential to determine the suitability of the designed filters.

Table 4.7 Digital filter performance indices for the designed filters.

Filter design method	Type of filter	Energy index %	Ripple ratio %	Noise transmission index
----------------------	----------------	----------------	----------------	--------------------------

Fundamental frequency band-pass filter

Least error squares	odd	80	41.50	0.3081
	even	91	27.70	0.1444
Fourier series method	odd	99	9.94	0.1676
	even	95	21.54	0.1472
Frequency sampling method	odd	98	13.70	0.1660
	even	89	31.32	0.1660

Second harmonic frequency band-pass filter

Least error squares	odd	81	43.50	0.3025
	even	90	30.18	0.1666
Fourier series method	odd	96	22.89	0.1690
	even	96	18.24	0.1465
Frequency sampling method	odd	91	30.98	0.1660
	even	91	26.98	0.1660

4.6 Summary

Three non-recursive digital filter design techniques have been presented in this chapter. Non-recursive filters were designed using these three approaches. The coefficients of the digital filters have been reported in this chapter. The frequency response of the digital filters have been presented. Methods of evaluating the filter response of different digital filter designs have also been described.

5. DIFFERENTIAL AND RESTRICTED EARTH FAULT RELAY DESIGN

Procedures for designing non-recursive digital filters have been presented in the last chapter. Coefficients of some digital filters designed for this project have also been reported. The selected filters were used in digital processor based transformer differential and restricted earth fault relays. The design of the relays is then presented in this chapter. Some test results are also included. Before outlining the software, the underlying principles of differential and restricted earth fault relays are described.

5.1 Transformer Differential Relay

The principle of differential protection of transformers has been described in Chapter 2 and is outlined briefly in this chapter for ready reference. A transformer differential relay compares the currents in the windings of the protected transformer. Before comparison the levels of the transformer winding currents are reduced by using c.t.'s. The ratio of the c.t.'s and their connections are selected to ensure that a direct comparison is possible. The magnetizing currents of power transformers are relatively small and are, therefore, neglected in this comparison. Figure 5.1 shows a typical transformer

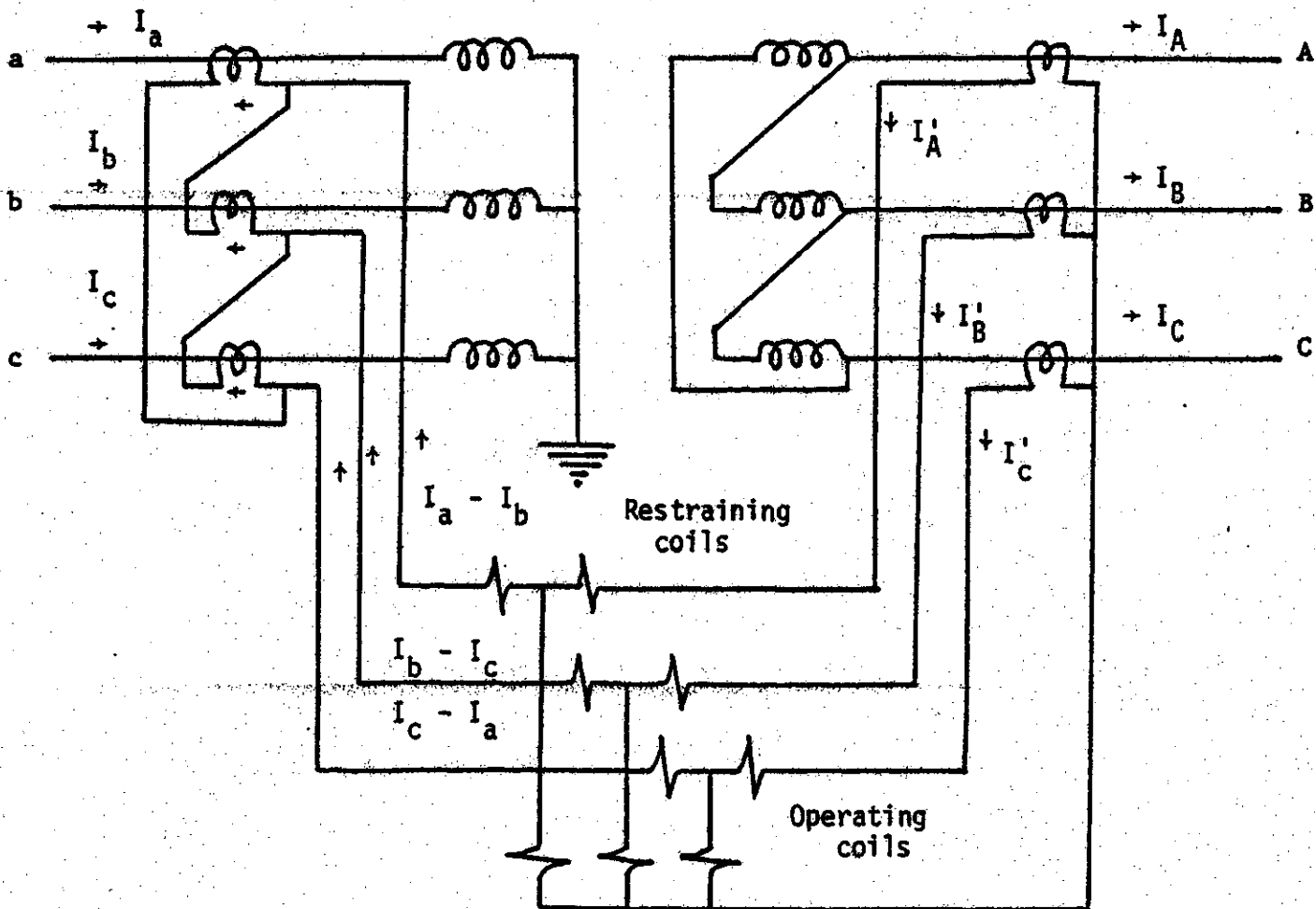


Figure 5.1 Connection diagram for differential protection of a delta/ye transformer.

differential arrangement for a three phase two winding transformer. As already mentioned, the selected ratio of the c.t.'s and their connections ensure that the c.t. secondary currents circulate in the restraint elements during normal loading of the transformer and during external faults. However, a fault within the transformer disturbs the balance between the secondary currents of the c.t.'s installed on the primary and secondary sides of the transformer resulting in flow of currents in the operating windings of the relay. In practice, several differential protection arrangements are used for transformers. One of the arrangements includes percentage and harmonic restraint features. The relays of this type usually consist of four relay elements; the differential current element, the through current element, the harmonic current element and the high set current element. The differential current element provides operating torque while the through current and harmonic current elements provide restraining torques. The high set current element is designed to take the transformer out of service if the differential current in any phase exceeds a set value, usually ten to twenty times²⁵ the full load current.

The differential unit may have a variable percentage ratio characteristic to provide high sensitivity at small through current magnitudes and comparatively low sensitivity at large through currents. This approach enables the relay

to detect low level internal faults in the protected zone and prevents false trippings on heavy external faults when differential currents might flow in the operating coil due to mismatch of c.t.'s

The harmonic restraint element is designed to prevent tripping due to magnetizing inrush currents which appear to the relay as internal fault currents. Inrush currents contain large magnitudes of harmonic components; the second and third harmonics predominate. The operation of this unit usually depends on the magnitudes of one or more harmonic components of the differential current.

The high set current element is an instantaneous overcurrent relay which operates within one cycle. In case of a heavy internal fault, the instantaneous overcurrent unit picks up and trips the circuit breakers for isolating the transformer from the system. This element is set to override the maximum differential current which may be experienced because of c.t. mismatch, off-normal tap settings and magnetizing inrush. The pick up setting of the instantaneous unit is usually ten to twenty times the transformer full-load current.

5.2 Restricted Earth Fault Protection

As discussed in Chapter 2, restricted earth fault protection is required in addition to differential protection to increase the section of the winding which is effectively protected in case of single phase to ground faults. In some cases, the use of the restricted earth fault protection is essential because single phase to ground faults cause relatively small differential currents.

Figure 5.2 shows a typical arrangement of the restricted earth fault protection scheme. This arrangement compares the magnitudes and relative directions of the zero sequence current and the transformer neutral current. The neutral current I_n will normally be equal to the sum of the currents in the three phases of the wye side windings. For a line to ground fault outside the zone of the restricted earth fault relay, the neutral current is the same as $3I_0$ at the transformer terminals. Current in the operating coil of the restricted earth fault relay is, therefore, zero. However, in case of a phase to ground fault in the protected zone, the neutral current will not be balanced by the $3I_0$ current at the transformer terminals. Current would, therefore, flow in the restricted earth fault relay which would operate. This restricted earth fault protection will, however, not respond to two phase and three phase faults on the wye side which may be inside or outside the protected

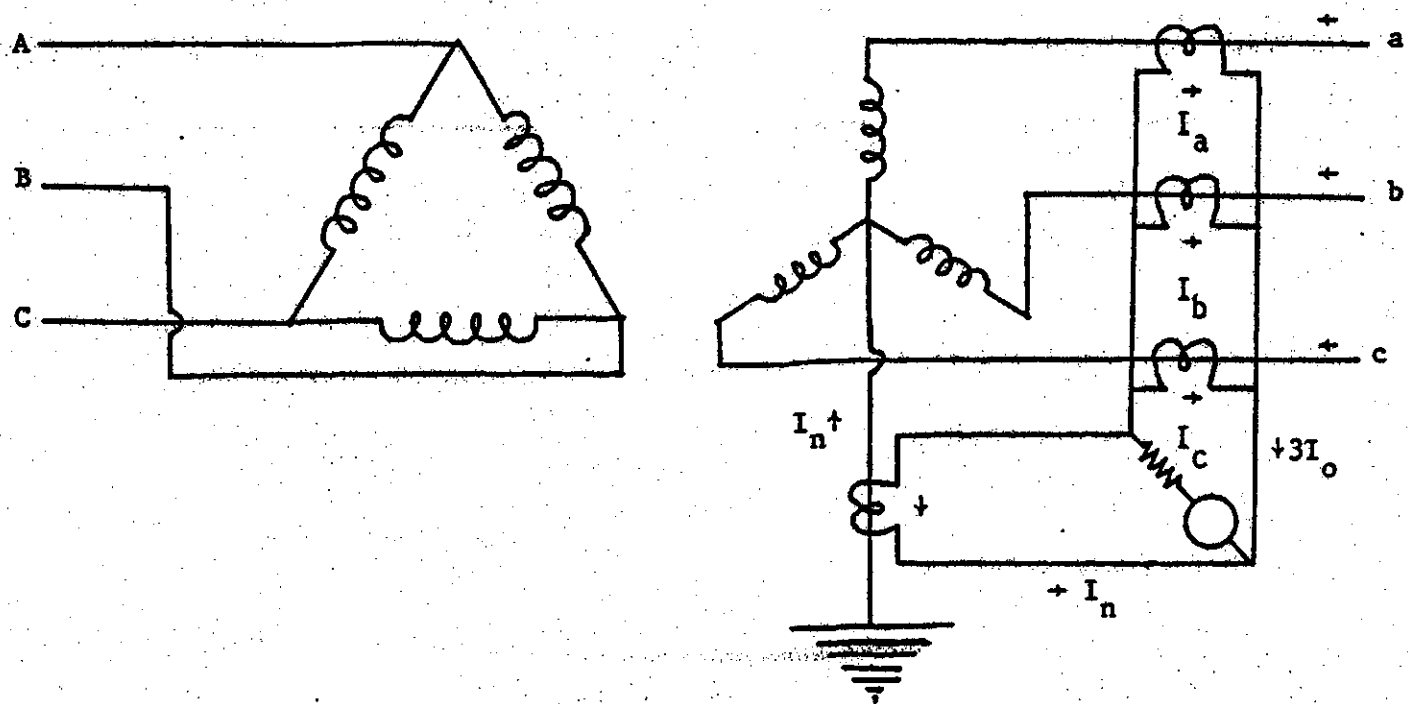


Figure 5.2 Current balancing for restricted earth fault protection of the wye winding of a delta-wye transformer.

zone of the restricted earth fault relay.

5.3 Digital Relay Design

The principles of operation of a conventional transformer differential relay and a restricted earth fault relay have been described in Sections 5.1 and 5.2. One of the objectives of this project was to design a transformer differential and restricted earth fault relay system for implementation on a programmable digital processor. The finalized relay design is described briefly in this section. Implementation of the design is then outlined.

When designing a digital relay two options are available. One option is to calculate the primary and secondary side line currents and the neutral current continuously, and monitor these currents for possible transformer faults. The second option is to use a starting element which would initiate execution of either the differential protection or the restricted earth fault algorithm as soon as the inception of a fault is suspected. The advantages of using the first option are³ that:

- (i) no starting algorithm is required
- (ii) changes in the fundamental and second harmonic frequency components can be monitored continuously.

The disadvantage of using this approach is that the magnitudes of the differential and through currents and the harmonic components of the differential currents have to be computed and the differential characteristic has to be applied in every sampling interval. The use of a starting element eliminates most of these computations during the normal operation of the transformer.

As discussed in the last paragraph, the transformer differential and restricted earth fault relay algorithms require many computations in each pass. Most of these computations, if performed during normal operating conditions, would constitute unnecessary computational effort. It was, therefore, decided that the relay design be equipped with a starting algorithm. The relay system design was therefore developed in the following three parts.

- (i) Starting relay
- (ii) Restricted earth fault relay
- (iii) Differential protection relay

5.3.1 Starting relay design

The objective of this relay is to monitor the currents on the primary and secondary sides of the protected transformer and decide if there is a fair chance that the transformer is experiencing a fault.

In an application, c.t.'s for the primary and secondary side of the transformer and the transformer neutral are first selected considering the transformer voltage and mega-volt-ampere ratings. The outputs of the selected c.t.'s are then matched by using auxiliary transformers. For a digital relay the line and neutral currents are passed through small resistances to obtain voltages which are proportional to the currents. The voltages are sampled at a pre-selected rate, are digitized and then placed in the processor memory for subsequent use. The first task after receiving a set of samples is to decide if there are reasons to believe that the transformer is experiencing a fault. For this purpose, instantaneous values of the three differential currents are computed from the sampled values. The magnitudes of the differential currents and the neutral current are then compared with pre-specified threshold values. If none of the currents exceeds the threshold, the program reverts to the wait mode until next set of samples is received. On receipt of new samples of currents, the differential currents are again calculated. The latest values of the differential currents and the neutral currents are then compared with the thresholds. If any of the currents is larger than the threshold, the phase, phases or the neutral whose current exceeded the threshold are identified. The processor then waits until the next set of samples is received. The differential currents and the neutral current are again calculated and compared with the

threshold. Two possibilities now exist; one that none of the currents exceeds the threshold value. If so, the starting logic is reset and the processor receives another set of samples. The second possibility is that either a differential current or the neutral current again exceeds the threshold. If the differential current of a phase is observed to have exceeded the threshold in two successive passes, the differential relay logic is activated. Similarly if the neutral current is observed to have exceeded the threshold for two successive samples, the restricted earth fault logic is activated.

5.3.2 Restricted earth fault logic

On activation by the starting relay logic, the restricted earth fault relay continues to sample all the currents at the specified rate. On receipt of a set of samples, the digitized values are stored in circular tables which are set up in the processor memory. Instantaneous value of three times the line side zero sequence current ($3I_0$) is calculated by adding the instantaneous values of the line currents. The value of $3I_0$ is then subtracted from the digitized values of I_n . The difference ($I_n - 3I_0$) and $3I_0$ are also stored in the processor memory. As soon as enough samples to fill a data window become available, the phasor of the 60 Hz component of the difference current ($I_n - 3I_0$) is determined. The magnitude of the calculated

of the calculated phasor is compared with a pre-specified threshold. If the magnitude of the phasor is less than the threshold, it is concluded that the fault is outside the transformer zone. However, if the magnitude of the phasor is greater than the threshold it is concluded that the transformer is experiencing a fault.

The 60 Hz component of the difference current ($I_n - 3I_0$) is the quantity on which operation of the restricted earth fault relay depends. The procedure used to compute the magnitude of the phasor representing the difference current is as follows.

The instantaneous value of the residual current is calculated using Equation 5.1 and the instantaneous value of the relay current is calculated using Equation 5.2.

$$3i_0 = i_a + i_b + i_c \quad (5.1)$$

$$i_{rx} = i_n - 3i_0 \quad (5.2)$$

where:

i_a , i_b and i_c are the sampled values of the currents in the a, b and c phases

Digitized values of i_n are used as inputs for the selected digital filter. The outputs of the fundamental frequency

orthogonal filters, $S_{rx}(n)$ and $C_{rx}(n)$, are obtained as follows.

$$S_{rx}(n) = \sum_{j=0}^N a_{1j} i_{rx}(n-j) \quad (5.3)$$

$$C_{rx}(n) = \sum_{j=0}^N b_{1j} i_{rx}(n-j) \quad (5.4)$$

In this equations, a_{1j} and b_{1j} are the coefficients of the bandpass fundamental frequency filters and n is the number of the latest set of data samples received. The peak value of the fundamental frequency component of i_{rx} is then obtained as follows.

$$I_{rx}^2(n) = S_{rx}^2(n) + C_{rx}^2(n) \quad (5.5)$$

The computations detailed in this paragraph are performed every time a new set of samples is received and the restricted earth fault relay is active.

5.3.3 Differential protection logic

On activation by the starting element, the differential relay continues to sample and digitize all the currents. Instantaneous values of the through and differential currents are computed and stored in circular tables. As

soon as sufficient set of samples are available, real and imaginary components of the fundamental frequency phasors of the differential currents are computed. The magnitudes of the calculated phasors are compared with a pre-specified high set value. If the computed magnitude of one of the phasors exceeds the pre-specified high set value, a trip command is issued to isolate the transformer from the system. Should all the computed magnitudes of the fundamental frequency phasors representing the differential currents be less than the high set value, magnitudes of the phasors representing the second harmonic components of the differential currents are calculated. The magnitudes of the fundamental frequency phasors representing the through currents are also computed. The magnitudes of the phasors representing the second harmonic and fundamental frequency components of the differential currents are compared. If the second harmonic component of a phase current exceeds 33% of the magnitude of its fundamental frequency component, the condition is classified as magnetizing inrush; otherwise, the differential relay algorithm is implemented.

The magnitude of the phasors representing the fundamental and high frequency components of the differential and through currents are obtained from the instantaneous values representing the primary and secondary currents. To compensate for the phase shift between the primary and secondary currents in a wye/delta transformer,

the instantaneous values of the equivalent primary and secondary currents for the transformer shown in Figure 5.1 are calculated as follows.

$$i_{pa} = i_a - i_c \quad (5.6a)$$

$$i_{pb} = i_b - i_c \quad (5.6b)$$

$$i_{pc} = i_c - i_a \quad (5.6c)$$

and

$$i_{sA} = i_A \quad (5.7a)$$

$$i_{sB} = i_B \quad (5.7b)$$

$$i_{sC} = i_C \quad (5.7c)$$

where:

i_{pa} , i_{pb} and i_{pc} are the equivalent primary currents in the a, b and c phases

i_{sA} , i_{sB} and i_{sC} are the equivalent secondary currents in the A, B and C phases

The instantaneous value of the differential and through currents for each phase are then obtained using Equations 5.8 and 5.9.

$$i_{Dj} = i_{pj} - i_{sj} \quad (5.8)$$

$$i_{\Sigma j} = i_{pj} + i_{sj} \quad (5.9)$$

where:

i_{Dj} is the instantaneous value of the differential current for the phase j

$i_{\Sigma j}$ is the instantaneous value of the through current for the phase j

i_{pj} is the instantaneous value of the compensated primary side current for the phase j

i_{sj} is the instantaneous value of the compensated secondary side current for the phase J

j and J represent the phases being considered;
a, b or c for j and A, B and C for J

In this manner six instantaneous values, three differential currents and three through currents, are computed on receiving a set of samples. The computed values of the differential and through currents are stored in circular tables set up in the processor memory.

The magnitude of the phasors representing the fundamental and high frequency components of the differential and through currents are obtained from the digitized samples. The digitized values of the currents are used as inputs to digital filters. The outputs of these filters are determined as follows.

$$S_{kD}(n) = \sum_{j=0}^N a_{kj} i_D(n-j) \quad (5.10)$$

$$C_{kD}(n) = \sum_{j=0}^N b_{kj} i_D(n-j) \quad (5.11)$$

$$S_{k\Sigma}(n) = \sum_{j=0}^N a_{kj} i_{\Sigma}(n-j) \quad (5.12)$$

$$C_{k\Sigma}(n) = \sum_{j=0}^N b_{kj} i_{\Sigma}(n-j) \quad (5.13)$$

$$n > (N + 1)$$

where:

k is a multiple of the fundamental frequency
 $S_{kD}(n)$ and $C_{kD}(n)$ are the outputs of the k th
 harmonic bandpass digital filters
 processing the differential currents

$S_{k\Sigma}(n)$ and $C_{k\Sigma}(n)$ are the outputs of the k th
 harmonic bandpass digital filters
 processing the through currents

a_{kj} and b_{kj} are the coefficients of the
 k th harmonic bandpass filter pair

n is the number of the latest set of data
 samples received

For each phase, the peak values of the k th harmonic components of the differential current and the through current are computed from the filter outputs as follows:

$$I_{kD}^2(n) = S_{kD}^2(n) + C_{kD}^2(n) \quad (5.14)$$

$$I_{k\Sigma}^2(n) = S_{k\Sigma}^2(n) + C_{k\Sigma}^2(n) \quad (5.15)$$

$$k = 1 \text{ to } 3$$

The magnitudes of the fundamental, second and third harmonic frequency components of the differential and through currents are then used to calculate the rms values of the differential and through currents as follows.

$$I_D^2(n) = \frac{I_{1D}^2(n) + I_{2D}^2(n) + I_{3D}^2(n)}{2} \quad (5.16)$$

$$I_{\Sigma}^2(n) = \frac{I_{1\Sigma}^2(n) + I_{2\Sigma}^2(n) + I_{3\Sigma}^2(n)}{2} \quad (5.17)$$

where:

$I_D(n)$ is the rms value of the differential current

$I_{\Sigma}(n)$ is the rms value of the through current

The d.c. components of the magnetizing inrush and fault current waveforms are partially blocked by the current

transformers and are further attenuated by the filters. These components are therefore not included in Equations 5.16 and 5.17.

5.3.4 Relay characteristic

The variable percentage differential algorithm first proposed by Rockefeller¹⁵ is used to prevent tripping on differential currents resulting from magnetization inrush, current transformer dissimilarities or differences in the c.t. ratios in auto-tapchanging transformers. This algorithm compares the differential current(s) of the faulted phase(s) to a function of the primary and secondary currents of the phase(s). This criteria is depicted in Figure 5.3. If the differential current exceeds the threshold defined in that figure, the possibility is that the transformer is experiencing an internal fault. On the other hand, if the differential current is less than the threshold, it can be concluded that the transformer is operating normally. At full load, the magnetization current is approximately 3% of the primary current and therefore contributes very little to the ratio of the differential current to the through current. However, if the transformer is unloaded, the differential current equals the through current and the ratio is therefore 100 percent. The magnetizing current establishes the minimum value of the threshold with which the differential current is first

compared. If the value of the differential current exceeds this minimum value, I_D is compared with I_Σ according to the piecewise linear threshold characteristic shown in Figure 5.3. The threshold characteristic is selected such that the differential currents due to current transformer dissimilarities and transformation ratio changes due to tapchanger operation do not cause the relay to operate.

Two versions of the variable percentage characteristic were used in this project. These characteristics are shown in Figure 5.3. The criteria of Figure 5.3a uses the rms values to determine the differential and through currents as follows:

$$I_\Sigma = |I_p| + |I_s| \quad (5.18)$$

$$I_D = |I_p - I_s| \quad (5.19)$$

where:

I_p is the rms value of the primary current
in a phase

I_s is the rms value of the secondary current
in a phase

This characteristic is bounded by a straight line whose slope equals one because the ratio of the differential current to the through current can never exceed 1.0. The

criteria of Figure 5.3b uses the phasor representation of the primary and secondary currents to compute the differential and through currents. The criteria is applied using the following procedure.

- (i) If I_D is larger than I_{min} , continue to step (ii); otherwise proceed to step (iv).
- (ii) If I_D is larger than $(M1 I_\Sigma + C1)$, continue to step (iii); Otherwise proceed to step (iv).
- (iii) If I_D is larger than $(M2 I_\Sigma + C2)$, conclude that the transformer is experiencing an internal fault.
- (iv) The fault is not in the transformer zone.

$M1$, $M2$, $C1$, $C2$ and I_{min} are indicated in Figure 5.4.

Appendix C describes how the equations modelling the variable percentage differential characteristic can be modified to directly use the squared values of the differential and through currents. This feature considerably reduces the computations required in the implementation of the transformer differential relay.

The procedure outlined in this section is applied to all three phases so long as the differential relay algorithm remains active.

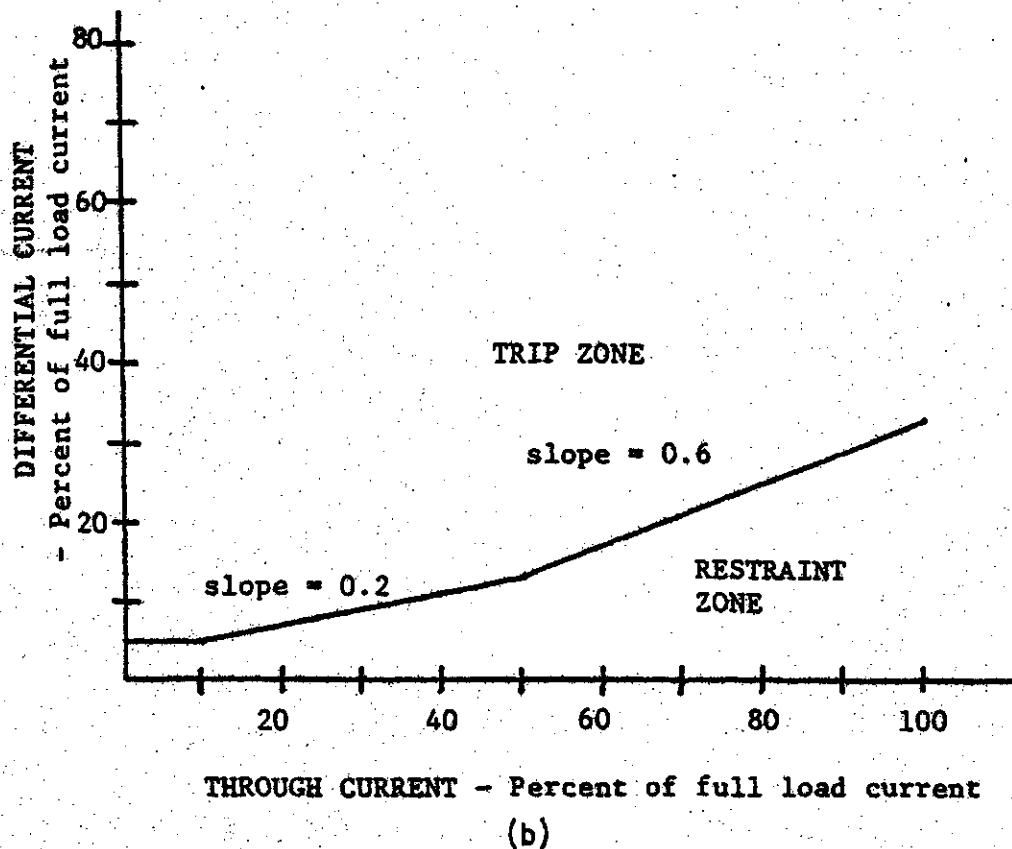
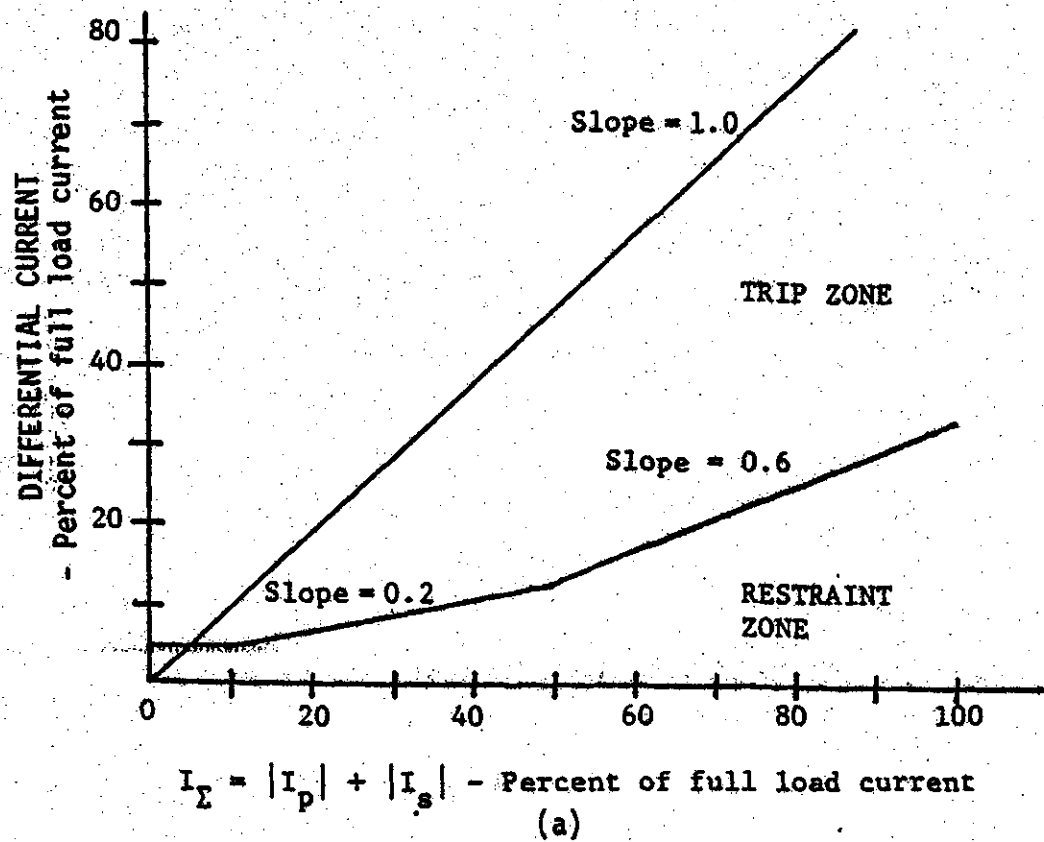


Figure 5.3 Variable percentage differential characteristics.

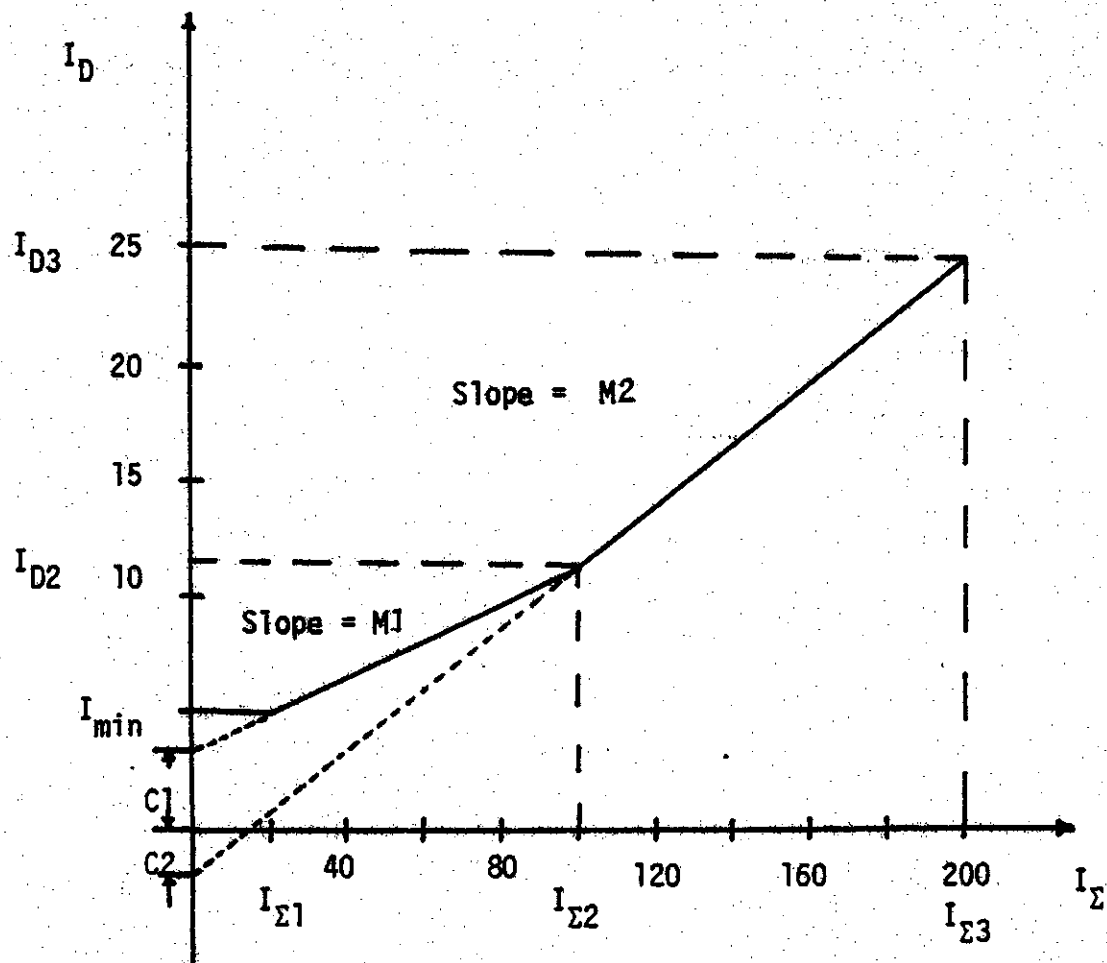


Figure 5.4 Parameters of the variable percentage differential characteristic.

5.4 Digital Relay Algorithm

The differential and restricted earth fault relay design described in the previous section was used to program a digital relay which is outlined in this section. The flow chart given in Figure 5.5 describes the major steps of the logic used in the relay. Figures 5.6, 5.7, 5.8 and 5.9 show the essential details of the various segments of the restricted earth fault and differential protection relays.

5.4.1 Starting element

One of the segments of the digital relay is the starting unit which consists of the following steps.

- (i) Activate relay system by initializing variables and counters for the relay logic and setting up clock for data sampling.
- (ii) Sample the six line currents and the neutral current and place them in circular tables in the processor memory.
- (iii) Compute delta currents using the appropriate current samples ($i_a - i_b$ etc.) of the wye connected windings.
- (iv) Compute the instantaneous values of the differential currents.
- (v) Compare the instantaneous values of the differential currents and the neutral current with the

threshold values and identify the phase(s) and/or neutral whose current(s) has (have) exceeded the threshold.

(vi) Wait for the next set of samples to arrive. On receipt of the new set of samples repeat steps (iii) to (v).

(vii) If the instantaneous values of the neutral current have exceeded the threshold in the last two checks activate the earth fault relay logic. Otherwise, check if the differential currents of any phase or phases have exceeded the threshold in the last two checks. If yes, activate the differential relay logic; otherwise revert to step (vi).

5.4.2 Restricted earth fault relay

The second segment of the digital relay is the restricted earth fault unit which was realized by the logic described in the following steps.

(i) Sample the line currents and the neutral current. Compute the instantaneous values of the difference and through currents and the sum of the phase currents ($3I_0$).

(ii) Compute the difference between the neutral current and the sum of the phase currents calculated in the previous step. This difference represents the relay operating current.

(iii) If sufficient sampled values of the relay current to fill the data window have been received, proceed to step (iv); otherwise revert to step (i).

(iv) Compute the real and imaginary components of the

phasor representing the fundamental frequency component of the relay current.

(iv) Compare the magnitude of the fundamental frequency phasor representing the relay current with the relay setting.

(v) If the relay current exceeds the set value, advance the increment counter by one. If the counter exceeds the threshold, issue a trip signal; otherwise revert to step (i). On the other hand if the magnitude of the relay current is less than the allowable setting, advance the decrement counter by one. If the decrement counter reaches the threshold, reset the increment and decrement counters and revert to the starting algorithm; otherwise revert to step (i).

5.4.3 Differential relay

The third segment of the digital relay is the differential protection unit which was realized as follows:

(i) Sample the line currents and the neutral current and check if the data windows are full. If yes, proceed to step (ii); otherwise repeat step (i).

(ii) Compute the real and imaginary components of the phasors representing the fundamental frequency components of the differential currents.

(iii) Compare the magnitudes of the phasors with the high current setting. If the magnitude of any differential

current phasor exceeds the high setting, issue a trip command. If not, proceed to step (iv).

(iv) Compute the magnitudes of the phasors representing the second harmonic component of the differential currents.

(v) Check for each phase if the magnitude of the second harmonic component of the differential current is greater than one third of the magnitude of the fundamental frequency component of the differential current. If the second harmonic component of the differential current exceeds one third of the magnitude of its fundamental frequency component, check further if the fundamental component of the differential current exceeds the normal magnetizing current setting. If it exceeds this value, revert to step (i); otherwise exit to the starting algorithm. However, if the magnitude of the second harmonic component of the differential current is less than one third of the magnitude of its fundamental frequency component, proceed to step (vi).

(vi) Implement the variable percentage differential characteristic by comparing the differential currents with the through currents. The variable percentage differential characteristics was sequentially implemented as shown in Figure 5.9. If the values of any through current and the corresponding differential current remain within the trip zone of the characteristic, advance the increment counter by one. If the increment counter exceeds the 'fault' threshold, issue a trip signal but if the increment counter

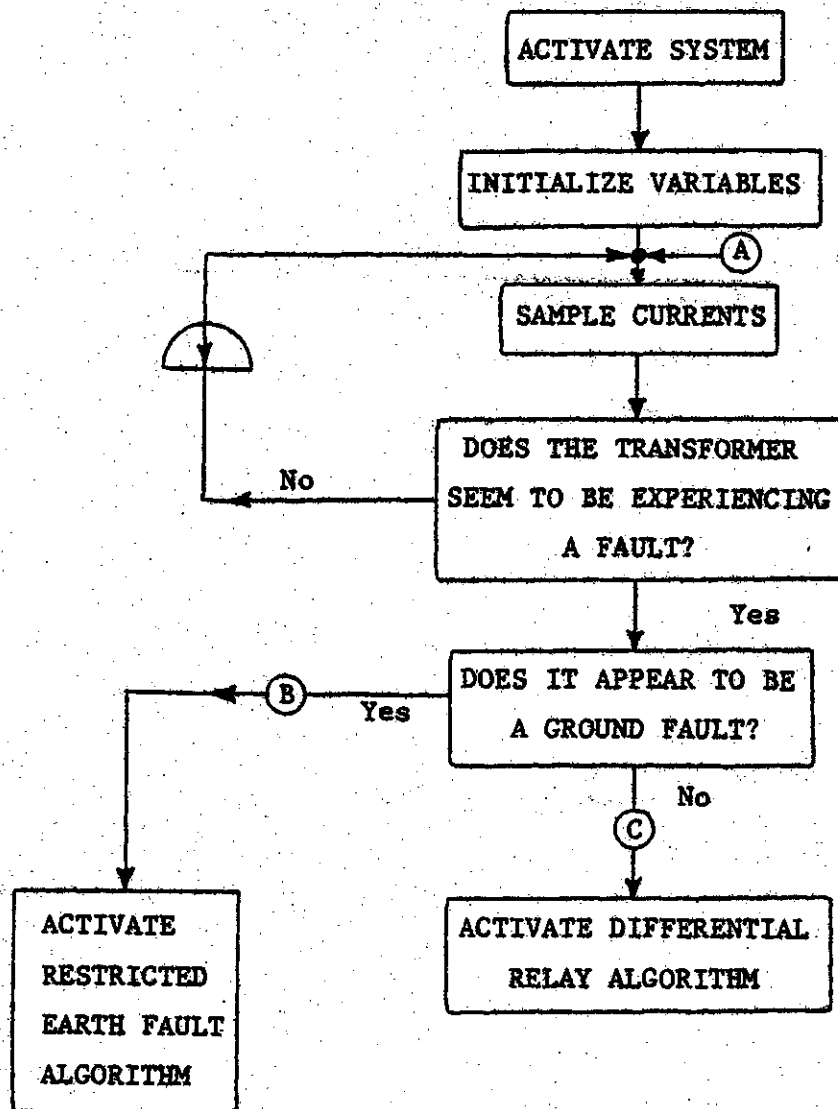


Figure 5.5 Overview of the transformer differential and restricted earth fault protection.

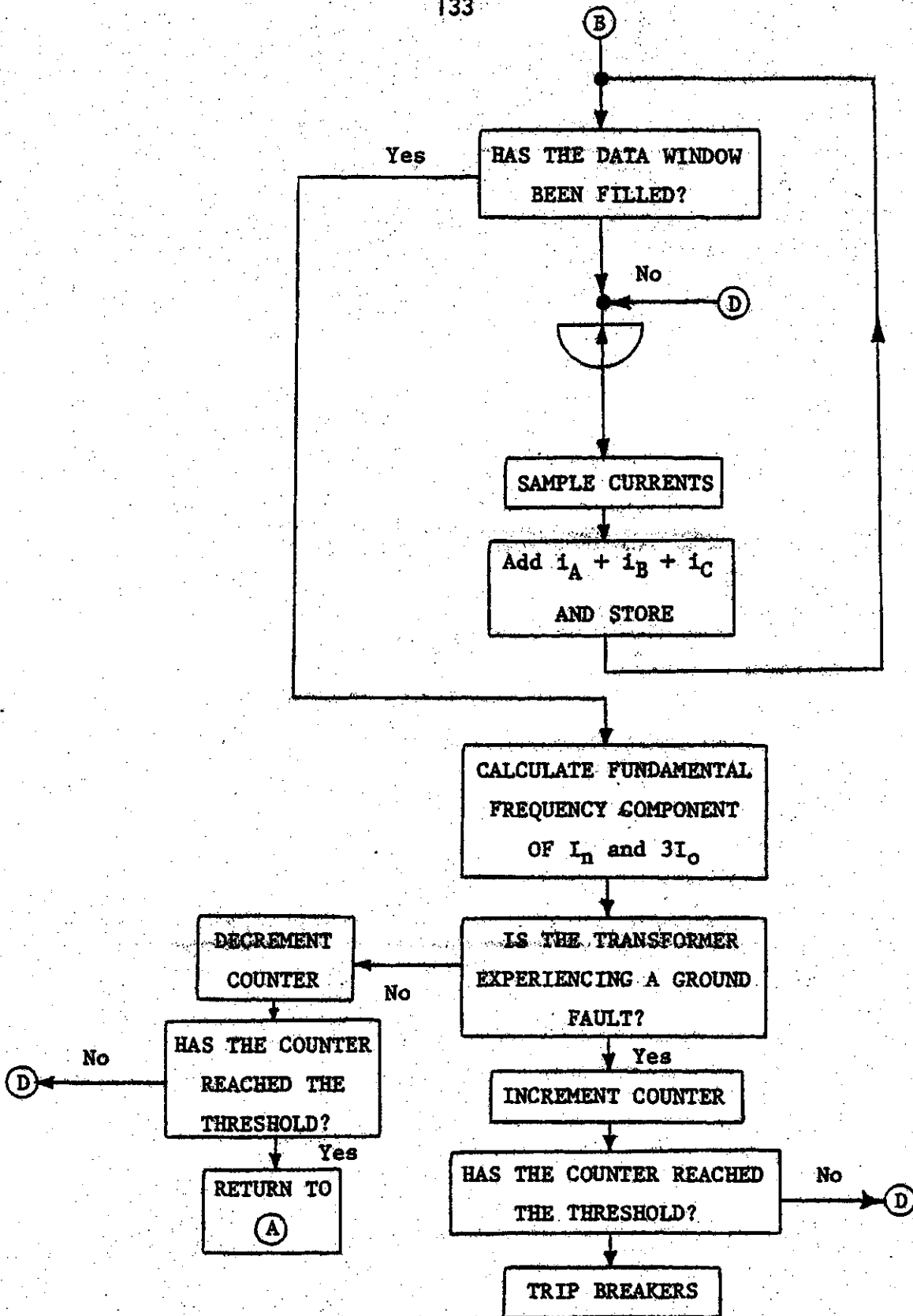


Figure 5.6 Flowchart used for implementing the restricted earth fault protection.

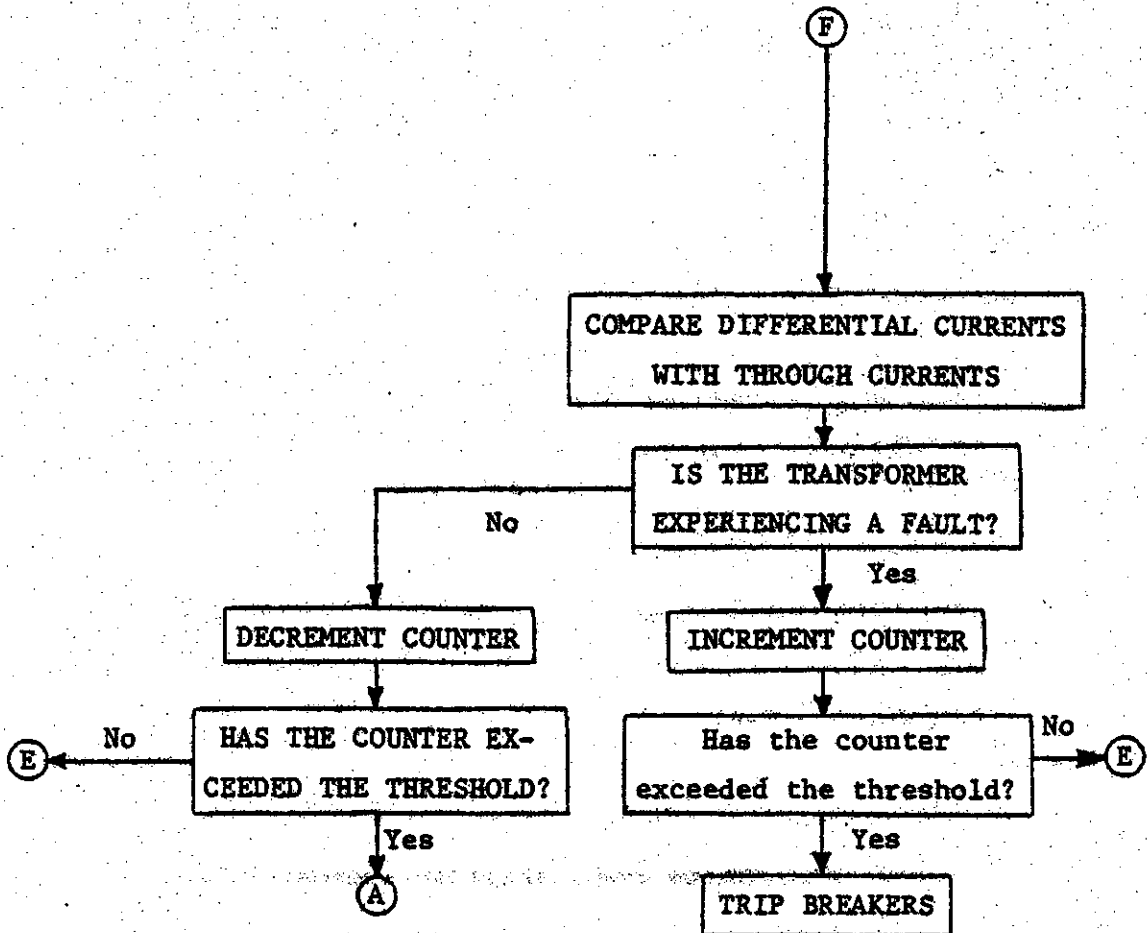


Figure 5.8 Flowchart used to detect a possible fault condition within the zone of the transformer.

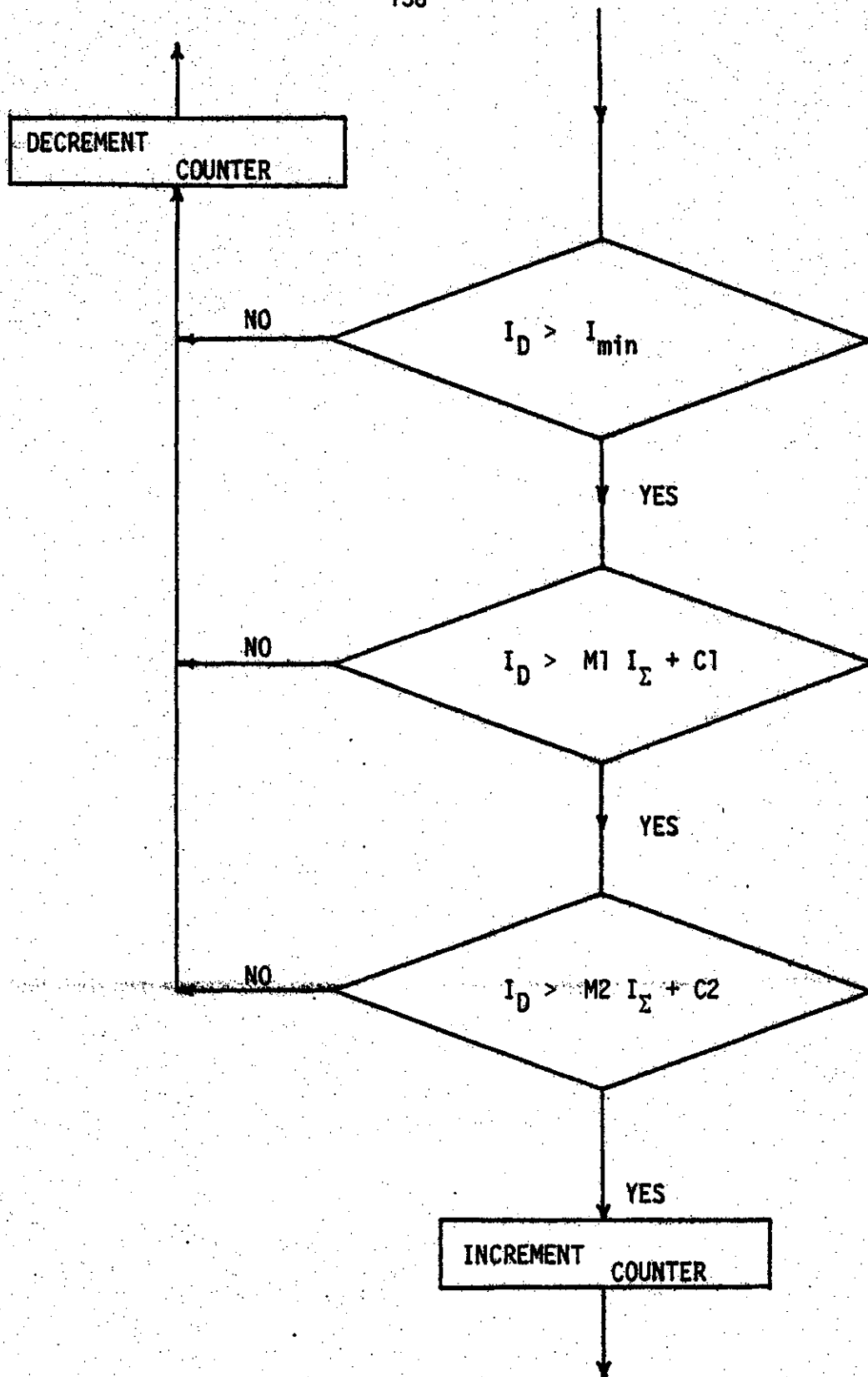


Figure 5.9 Flowchart used to implement the variable percentage characteristics.

has not exceeded the 'fault' threshold revert to step (1). On the other hand if the values of the through currents and the differential current are outside the trip zone of the relay, advance the decrement counter by one. If the decrement counter exceeds the 'no-fault' threshold exit to the starting algorithm; otherwise revert to step (1).

The main steps involved in the implementation of a differential and restricted earth fault relay have been outlined in this section. Implementation of the digital relay involves the execution of the starting algorithm and either the differential or the restricted earth fault relay logic.

5.5 Off-line Testing

A software based transformer differential and restricted earth fault relay was designed and was implemented using the algorithm described in Sections 5.3 and 5.4. This relay program was written in FORTRAN and was tested using simulated transformer magnetizing inrush and fault conditions.

Digital filter designs developed by the least error squares approach and the Fourier series and frequency sampling methods have been presented in Chapter 4. The frequency response of those filters have also been examined

and filters whose responses are close to the ideal response have been identified. Using these filter designs, a digital restricted earth fault and differential relay was designed. The philosophy outlined in Sections 5.3 and 5.4 was used in the design. The designed relay was tested in an off-line mode. The testing was in two parts. The digital filters were tested first using simulated data representing magnetizing inrush, transformer winding faults and simultaneous magnetizing inrush and winding faults. The operation of the filters was examined by observing their outputs. Suitable filters were then selected for use in the digital relay. The second part consisted of testing the digital relay to examine its operation during magnetizing inrush, transformer winding faults and simultaneous magnetizing inrush and winding fault conditions.

A wide range of simulated data was generated using the techniques presented in Section 2.7. The generated data was used as input for the filters designed by the Fourier series, frequency sampling and least error squares methods. The outputs were examined and the filters which provide consistent outputs were then selected for use in the relay. The outputs of the filters were also used to determine the fundamental, second and third harmonic components of the current.

Figures 5.10 to 5.15 illustrate some of the typical test results. The inrush current data in this case was generated using Equation 2.7. The saturation angle, resistance to saturated reactance ratio and the ratio of the primary side line voltage to saturated reactance were 101 degree, 0.1 and 1 p.u. Simultaneous internal fault and magnetizing inrush current data generated by using Equation 2.11 was used for this purpose. It was assumed that 5 percent of the primary turns were short circuited. Saturation angle for the magnetizing inrush component of the current and ratio of resistance to saturated reactance were assumed to be 101 degrees and 0.1 respectively. The resistance to reactance ratio for the shorted turns of the winding was assumed to be 0.05.

Figure 5.10 depicts the second and third harmonic components of the inrush current as fractions of its fundamental frequency component. The digital filters used in this case were designed by the Fourier series method; the frequency response of the filters are shown in Figures 4.14 and 4.15. Figure 5.11 shows the second harmonic and the third harmonic components of the current during simultaneous inrush and internal fault conditions. An examination of Figure 5.10 and 5.11 shows that the ratios of the second and third harmonic components to the fundamental frequency component vary considerably as time passes. Some of the values of the ratio are much lower than that required

to identify magnetizing inrush conditions.

Figure 5.12 and 5.13 show the ratios of the magnitudes of the second and third harmonic components to the fundamental frequency component of the cases described in the last paragraph. The digital filters used in these analyses were designed by the frequency sampling method; the frequency response of the filters are shown in Figures 4.18, 4.19, 4.20 and 4.21. An examination of Figures 5.12 and 5.13 shows a definite trend for both the inrush and simultaneous inrush and internal fault conditions. The second harmonic to the fundamental frequency component ratio varies from 38 percent to 77 percent for magnetizing inrush and from 27 to 29 percent for simultaneous magnetizing and winding fault. Since the values of the filtered outputs lie within bounds that can be defined distinctly, these filters can be successfully used in a digital transformer differential relay.

The performance of the least error squares filters was investigated in a manner similar to that described in the last two paragraphs. Figure 5.14 and 5.15 show the ratio of the second and third harmonic frequency components to the fundamental frequency components when the least error squares filters were used. The frequency response of the filters used in this analyses have been depicted in Figures 4.5, 4.6, 4.7 and 4.8. A study of Figures 5.14 and 5.15

reveals that the second and third harmonic components to the fundamental frequency component ratio have a definite trend in both the magnetizing inrush and simultaneous inrush and internal fault conditions. The second harmonic component to the fundamental frequency component ratio varies from a minimum of 38 percent to 77 percent for magnetizing inrush. This ratio varies from 27 to 30 percent for simultaneous magnetizing inrush and winding fault.

A comparison of the performance of the three types of digital filters indicates that the filters designed by the least error squares and frequency sampling methods provide reliable and consistent results. The digital filters designed by the least error squares and frequency sampling methods were therefore selected for use in the software based transformer differential and earth fault relay.

A software based relay was developed using the algorithm presented in Sections 5.3 and 5.4. Non-recursive filters designed by the least error squares and frequency sampling methods were used. The relay was also tested in the off-line mode using the simulated data representing magnetizing inrush, internal fault and simultaneous internal fault and inrush currents. The object of testing the relay logic was to determine its ability to correctly recognize these conditions. While testing the digital relay the following simplifying assumptions are made.

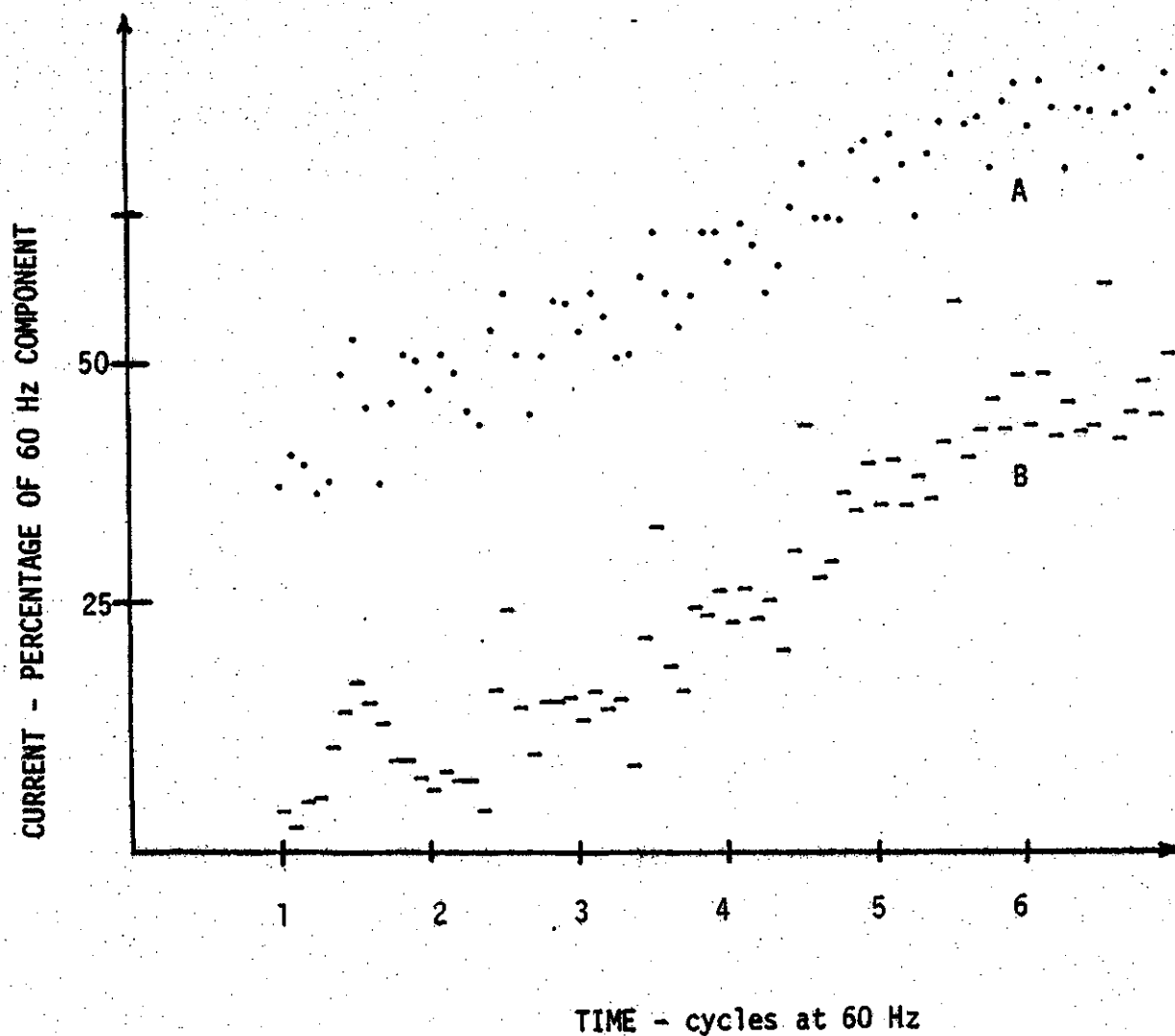


Figure 5.10 Second harmonic (plot A) and third harmonic (plot B) components of the magnetizing inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the Fourier series method.

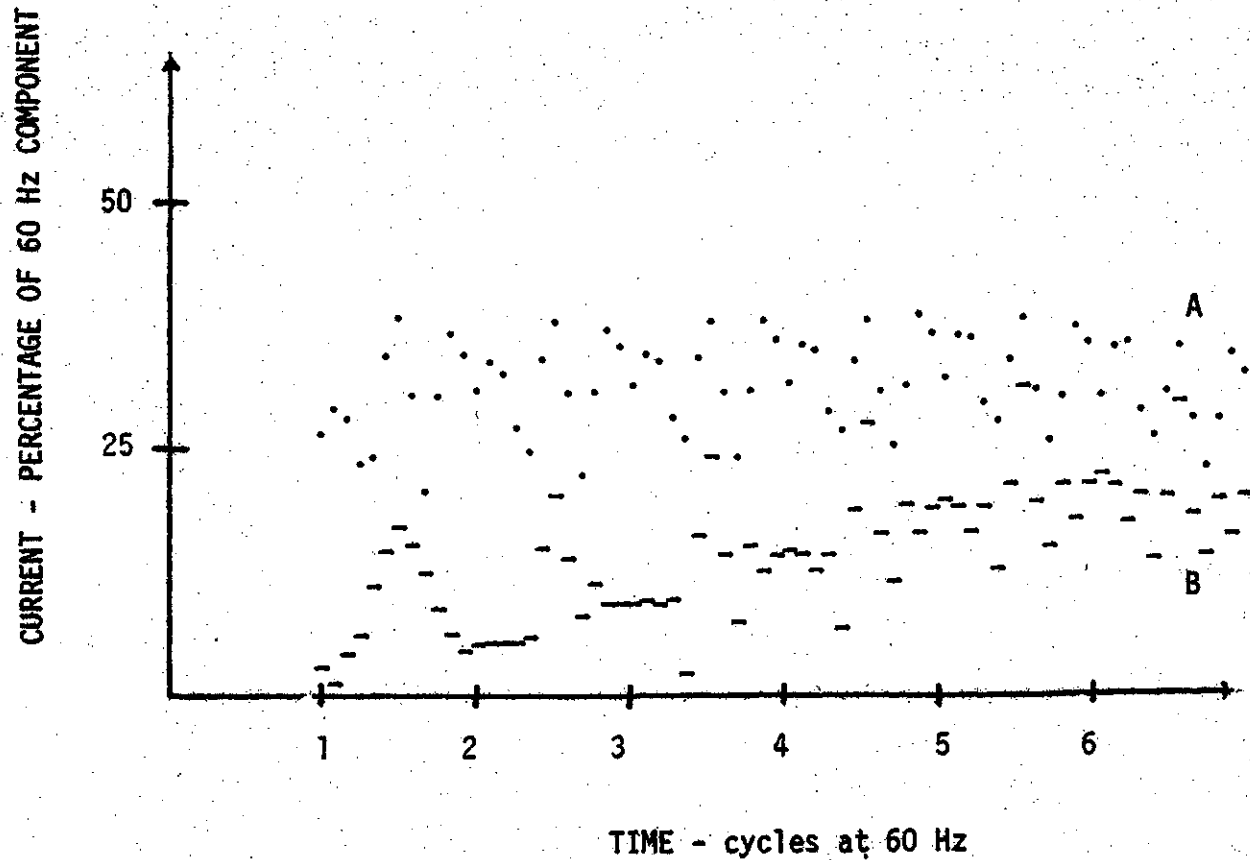


Figure 5.11 Second harmonic (plot A) and third harmonic (plot B) components of the simultaneous internal fault and inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the Fourier series method.

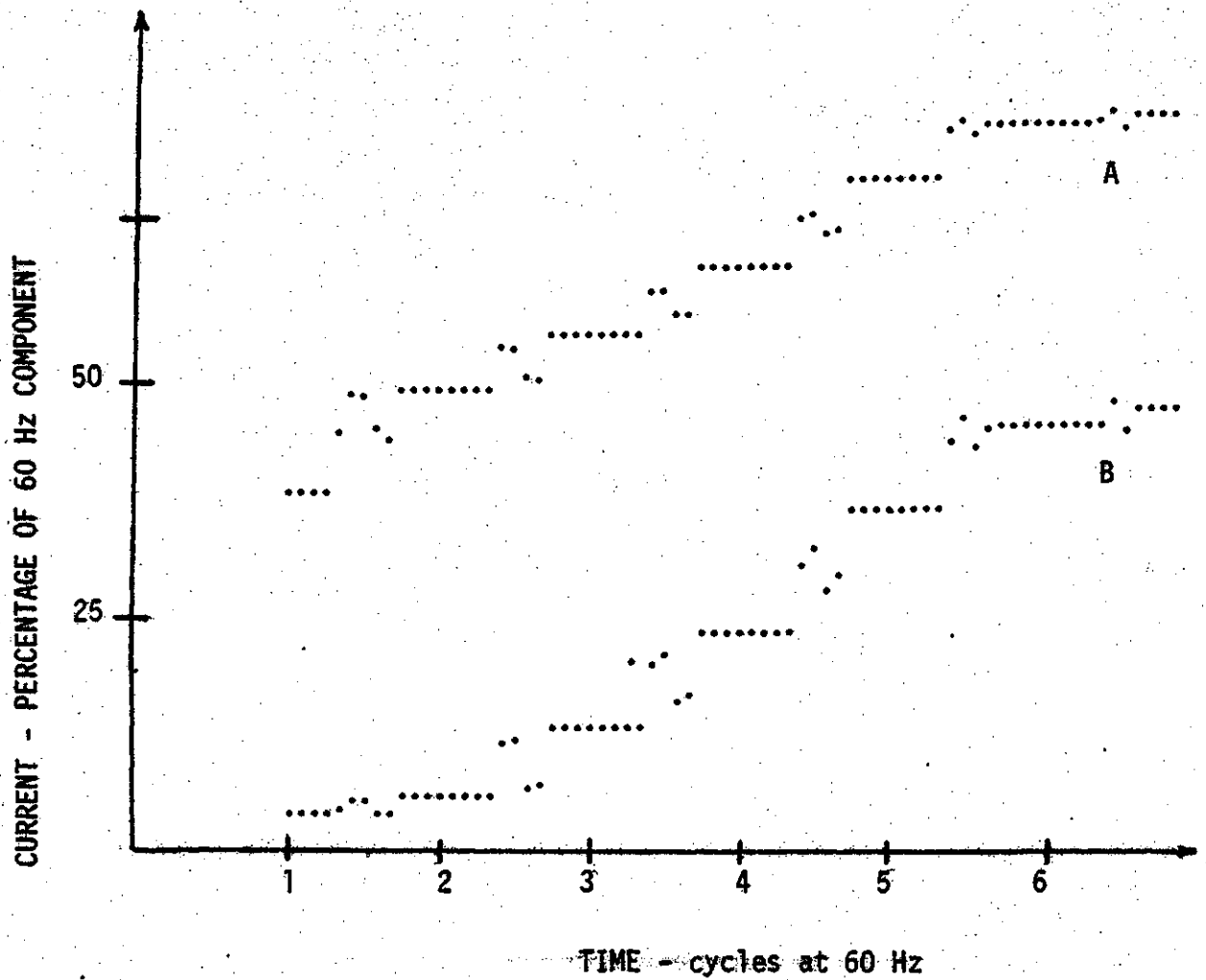


Figure 5.12 Second harmonic (plot A) and third harmonic (plot B) components of the magnetizing inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the frequency sampling method.

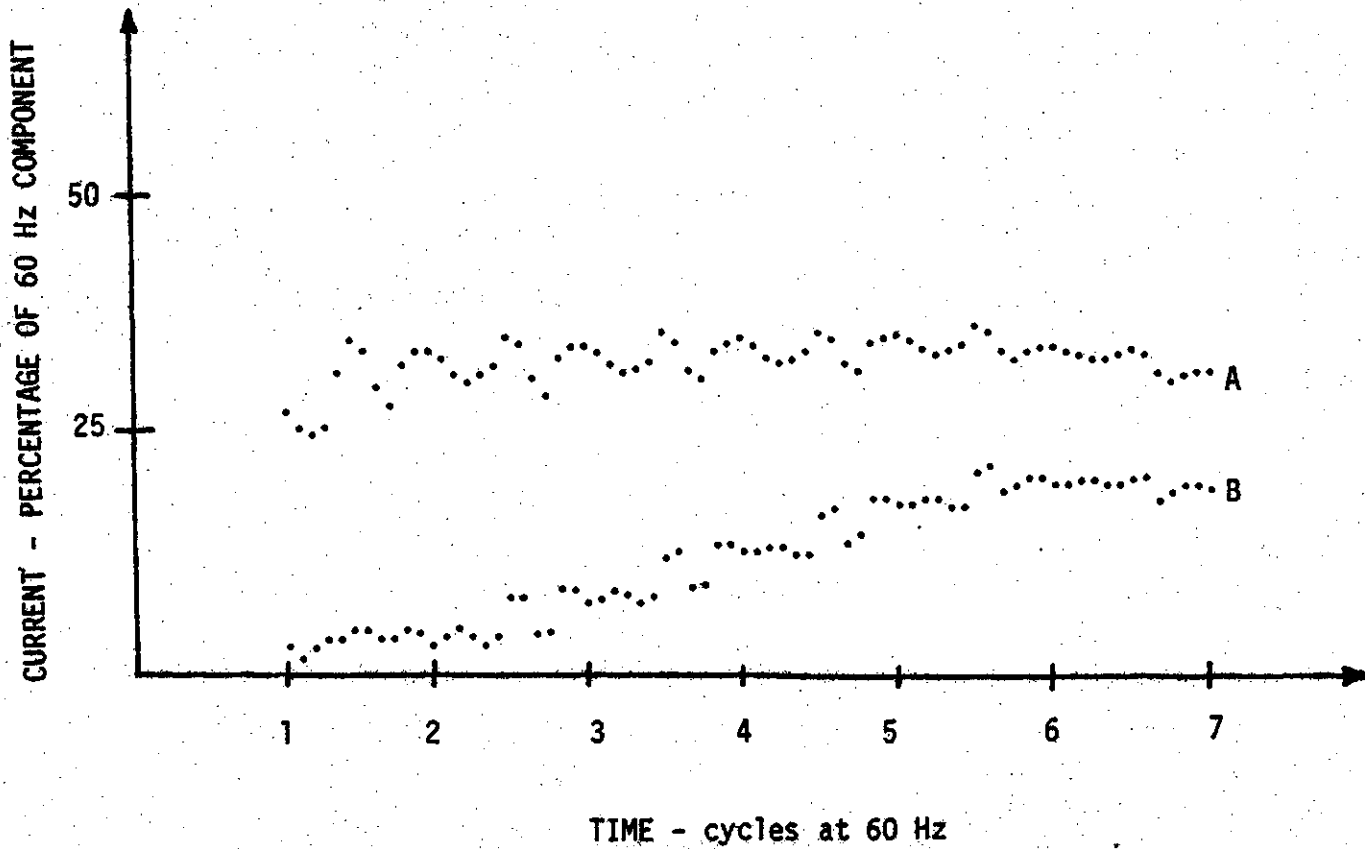


Figure 5.13 Second harmonic (plot A) and third harmonic (plot B) components of the simultaneous internal fault and inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the frequency sampling method.

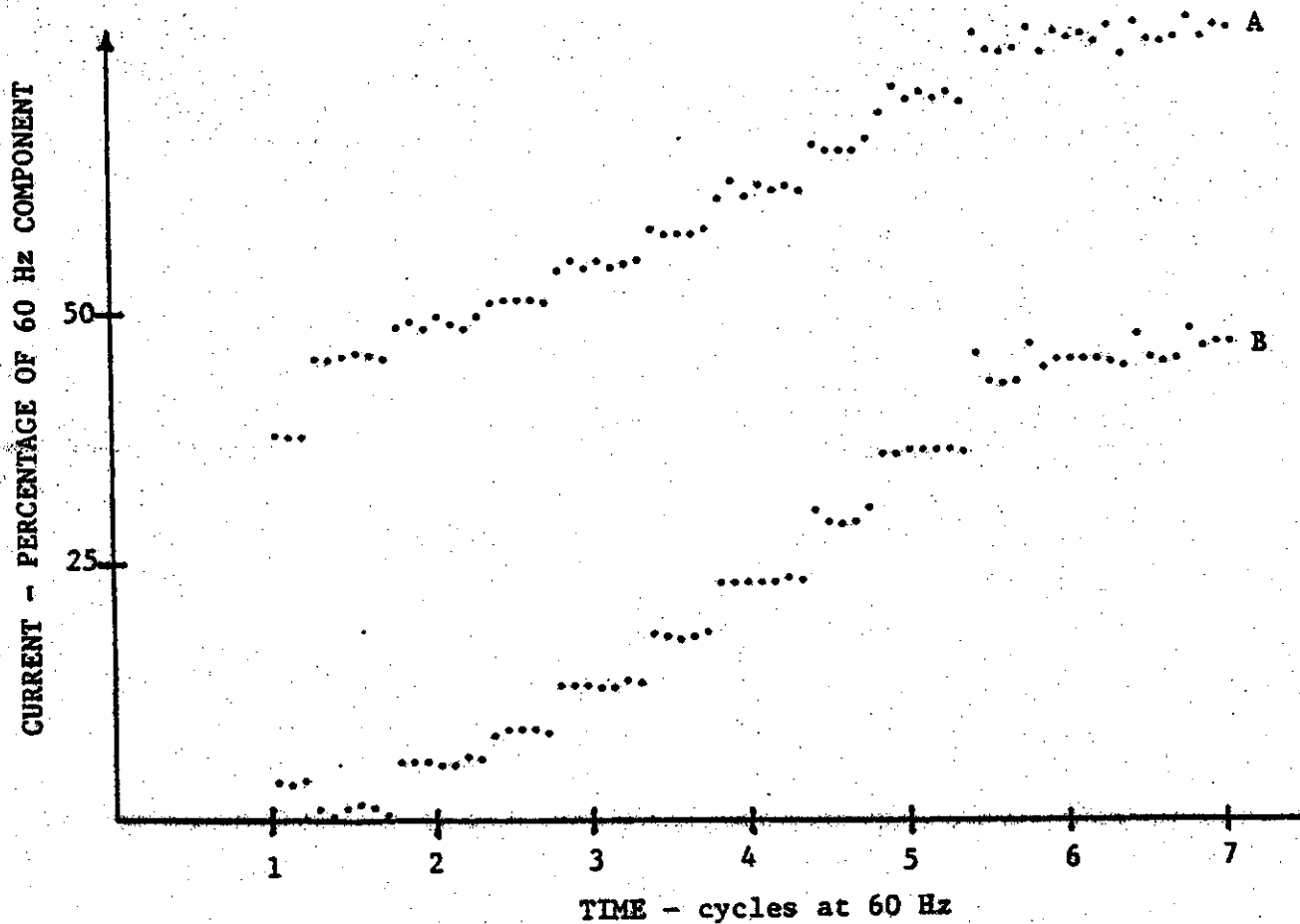


Figure 5.14 Second harmonic (plot A) and third harmonic (plot B) components of the magnetizing inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the least error squares approach.

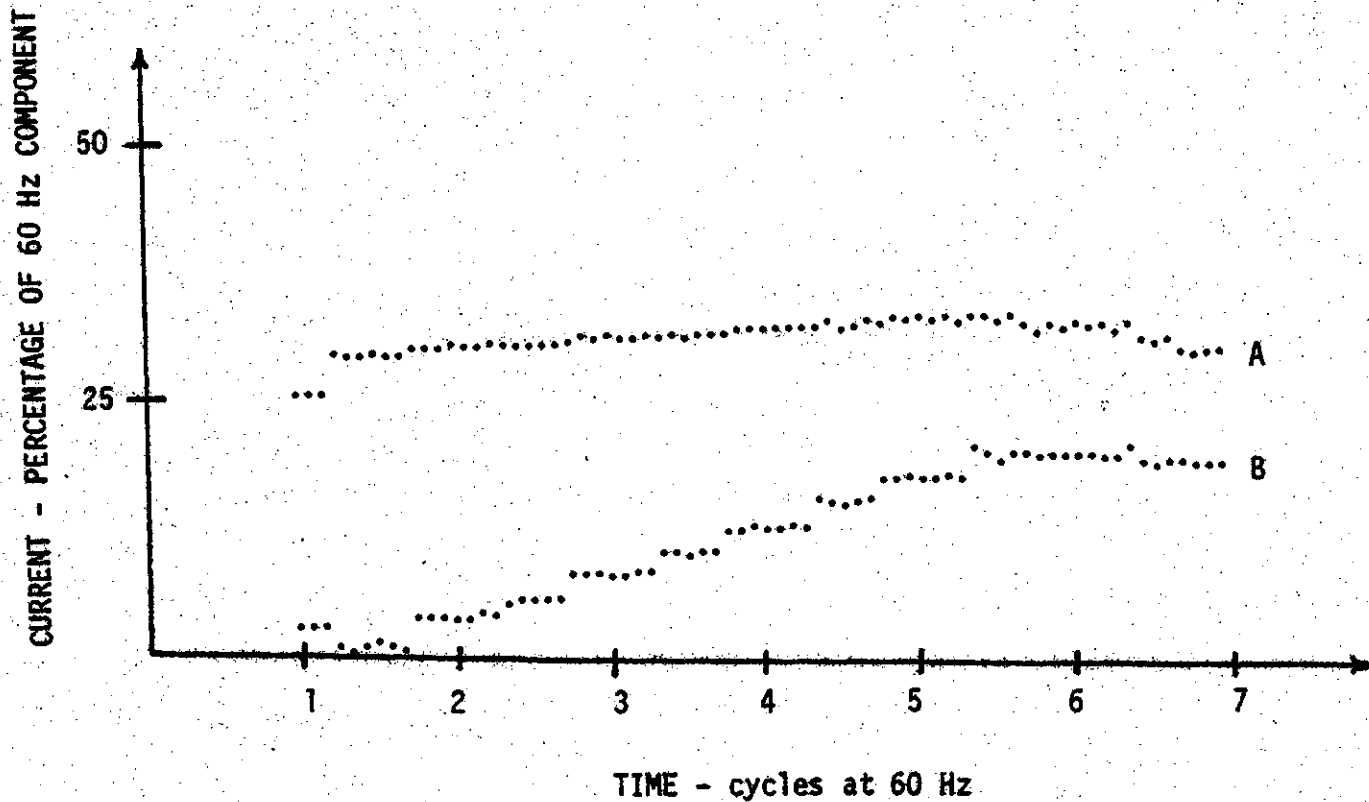


Figure 5.15 Second harmonic (plot A) and third harmonic (plot B) components of the simultaneous internal fault and inrush current as a percentage of the fundamental frequency component obtained by the digital filters designed by the least error squares approach.

- (i) the primary and secondary side c.t.'s are perfectly matched.
- (ii) the c.t.'s do not saturate.
- (iii) load currents are negligible compared to the inrush and fault currents.
- (iv) the inrush phenomenon in a three phase transformer is identical to the inrush phenomenon in a single phase transformer.

Transformer currents representing the magnetizing inrush, winding faults and simultaneous magnetizing inrush and winding faults were used for testing the relay. The primary winding was assumed to be rated at 10 amps at 1500 volts. The saturation angle and saturated reactance were assumed to be 78 degrees and 20 ohm respectively. For simultaneous magnetizing inrush and fault conditions, the resistance to reactance ratio of the shortcircuited turns was considered to be 0.05. Five percent of the primary winding turns were considered to have been shorted. The internal fault was considered to short 10 percent of the primary winding.

Table 5.1 shows the successive calculated values of the differential current, its second and third harmonic components and the threshold differential current corresponding to the through current observed over a period of one cycle after the data window had been filled with the

samples representing the magnetizing inrush or fault conditions. The calculated values listed in this table were obtained when the relay was supplied with currents representing three operating conditions; a magnetizing inrush, a winding fault and a simultaneous magnetizing and winding fault. Non-recursive digital filters designed by the frequency sampling method were used in these cases. The differential relay implemented the two piece linear differential characteristic; the slope $M1=0.1$ for through currents between one and two times the rated full load current and the slope $M2=0.15$ for through currents greater than two times the full load current. The minimum differential current, I_{min} , used in the characteristic was 5 percent of the the rated current. An examination of the Table 5.1 shows that the differential current exceeds the variable percentage threshold for all three operating conditions. In the case of magnetizing inrush, the second harmonic is more than the 33 percent setting used for identifying such conditions. In case of simultaneous magnetizing inrush and winding fault, the second harmonic component is close to the selected threshold. The relay operation is delayed due to the uncertainty associated with this condition. The second harmonic content is negligible for the internal fault case. The relay operates without unnecessary delay because the differential current in the relay has exceeded the threshold corresponding to the through current observed.

Table 5.1 Analysis of the differential current and the corresponding threshold values of the variable percentage differential characteristic for a digital relay that uses non-recursive filters designed by the frequency sampling method.

Time msec.	Threshold value amps.	Differential current amps.	% 2nd harmonic	% 3rd harmonic
---------------	-----------------------------	----------------------------------	-------------------	-------------------

Magnetizing inrush

16.66	3.2	35.5	60.5	21.0
18.05	3.2	35.5	60.5	21.0
19.44	3.2	35.5	60.5	21.0
20.83	3.2	35.5	60.5	21.0
22.22	3.2	35.5	60.5	21.0
23.60	3.2	35.5	65.1	28.0
25.00	3.0	33.8	65.1	29.3
26.38	2.6	31.2	63.3	24.8
27.77	2.3	29.7	63.7	26.3
29.16	2.4	30.0	69.0	33.2
30.55	2.4	30.0	69.0	33.2
31.94	2.4	30.0	69.0	33.2

Simultaneous inrush and winding fault

16.66	6.8	59.0	33.3	10.5
18.05	6.9	60.0	30.5	12.2
19.44	7.0	60.8	29.2	11.7
20.83	7.1	61.4	30.2	9.7
22.22	7.1	61.6	32.1	10.0
23.60	7.0	61.0	35.0	15.3
25.00	6.6	58.2	34.0	15.8
26.38	6.0	54.6	30.7	11.5
27.77	5.7	52.2	29.4	12.2
29.16	5.6	51.8	33.1	17.4
30.55	5.7	52.1	35.0	17.2
31.94	5.7	52.6	35.3	15.1

Winding fault

16.66	13.5	104.0	4.6	3.2
18.05	14.2	109.0	4.3	3.1
19.44	14.8	112.5	4.1	2.9
20.83	15.0	114.4	3.9	2.8
22.22	15.0	114.2	3.8	2.7
23.60	14.7	112.0	3.8	2.7
25.00	14.1	108.3	3.8	2.7
26.38	13.5	104.3	3.8	2.7
27.77	13.0	100.8	3.9	2.7
29.16	12.7	99.0	3.8	2.7
30.55	12.7	99.1	3.6	2.6
31.94	13.0	101.2	3.5	2.5

Table 5.2 shows the analysis of the differential current for the three cases when non-recursive filters designed by the least error squares method were used. An examination of this table shows that the differential current exceeds the variable percentage threshold for all three operating conditions. The second harmonic content of the differential current helps in identifying the condition. A comparison of Tables 5.1 and 5.2 show that the filter outputs in both the cases match closely.

The restricted earth fault segment of the relay logic was also tested using the simulated data. A line to ground fault was assumed on the primary winding of the transformer. Ten percent of the primary winding turns were assumed to have been short circuited and the ground current was assumed to be limited by a 10 ohm reactance installed between the neutral and the ground. Table 5.3 shows the relay operating current when digital filters designed by the frequency sampling and least error squares methods were used. The relay was set at 5.0 amps. A study of Table 5.3 reveals that the earth fault is detected promptly.

The relay could not be tested with power system data because of its non-availability.

Table 5.2 Analysis of the differential current and the corresponding threshold values of the variable percentage differential characteristic for a digital relay that uses non-recursive filters designed by the least error squares method.

Time msec.	Threshold value amps.	Differential current amps.	% 2nd harmonic	% 3rd harmonic
---------------	-----------------------------	----------------------------------	-------------------	-------------------

Magnetizing inrush

18.05	3.2	35.6	60.1	21.2
19.44	3.2	35.5	60.6	21.1
20.83	3.2	35.6	60.5	20.8
22.22	3.2	35.5	60.2	21.0
23.60	2.7	32.4	64.3	27.1
25.00	2.8	32.7	64.0	26.7
26.38	2.7	32.4	64.0	26.6
27.77	2.8	32.6	64.1	26.7
29.16	2.7	32.4	64.4	27.2
30.55	2.3	30.0	68.5	33.3
31.94	2.4	30.0	69.0	32.6
33.33	2.4	30.0	69.8	33.3

Simultaneous inrush and winding fault

18.05	6.8	59.5	32.0	11.2
19.44	6.8	59.4	32.2	11.2
20.83	6.8	59.5	32.1	11.1
22.22	6.8	59.5	31.9	11.1
23.60	6.3	56.2	32.1	13.5
25.00	6.3	56.5	32.2	13.4
26.38	6.3	56.4	32.1	13.3
27.77	6.3	56.5	32.2	13.4
29.16	6.3	56.2	32.2	13.6
30.55	5.8	53.4	32.3	15.8
31.94	5.9	53.5	32.7	15.5
33.33	5.9	53.3	32.9	15.7

Winding fault

18.05	13.7	105.5	0.1	0.0
19.44	13.7	105.5	0.1	0.0
20.83	13.7	105.7	0.1	0.0
22.22	13.8	106.0	0.1	0.0
23.60	13.8	106.1	0.1	0.0
25.00	13.8	106.3	0.1	0.0
26.38	13.8	106.3	0.1	0.0
27.77	13.8	106.3	0.1	0.0
29.16	13.8	106.1	0.1	0.0
30.55	13.8	106.0	0.1	0.0
31.94	13.7	105.7	0.1	0.0
33.33	13.7	106.6	0.1	0.0

Table 5.3 The restricted earth fault setting and the restricted earth fault relay operating current ($I_n - 3I_0$) obtained by non-recursive digital filters designed by the frequency sampling method and the least error squares method.

Time msec.	Relay setting amps.	Restricted earth fault relay current ($I_n - 3I_0$) amps.	
		Frequency sampling method	Least error squares method
18.05	5.0	10.9	10.6
19.44	5.0	11.2	10.6
20.83	5.0	11.4	10.6
22.22	5.0	11.4	10.6
23.60	5.0	11.2	10.6
25.00	5.0	10.8	10.6
26.68	5.0	10.4	10.6
27.77	5.0	10.1	10.6
29.16	5.0	9.9	10.6
30.55	5.0	9.9	10.6
31.94	5.0	10.1	10.6

5.6 Summary

The design of a digital differential and restricted earth fault relay has been presented in this chapter. A software based relay has been designed and has been tested in the off-line mode using the simulated data. Some typical test results have been included in this chapter. The results reveal that the relay segments operate properly and are able to distinguish magnetizing inrush conditions from faults.

6. CONCLUSIONS

The object of this thesis was to investigate the feasibility of designing a digital transformer differential and restricted earth fault relay. Before starting the design, types of faults experienced by power transformers and relays used for transformer protection were reviewed. The problems associated with the systems used for transformer protection were examined. Mathematical models for simulating magnetizing inrush and fault waveforms were established. An examination of the shortcomings of transformer differential relays revealed that the problems can be alleviated considerably by using restricted earth fault relays in addition to differential relays. Computer algorithms proposed in the past for transformer differential protection were reviewed and are reported briefly in Chapter 3. Their study showed that a simple waveshape identification approach and both recursive and non-recursive filters have been tried for determining the fundamental frequency and harmonic components of the transformer currents. The developments reported in the literature used the criteria of the second harmonic component of the differential current exceeding its fundamental frequency component for distinguishing internal faults from magnetizing inrush currents.

Three other methods for designing orthogonal non-recursive digital filters were also investigated. These methods use the least error squares, Fourier integral and the frequency sampling techniques. The fundamental frequency and the second harmonic frequency filters were designed using these techniques. The techniques and the coefficients of the filters have been reported in Chapter 4. Frequency responses of the digital filters and their suitability were also examined. The results of these studies are also reported briefly in Chapter 4. The evaluation of the different designs show that the filter designed by the least error squares and the frequency sampling techniques are suitable for relaying applications.

An algorithm for a digital differential protection and restricted earth fault relay was developed. The algorithm was then transcribed into a computer program. Both the variable percentage characteristic and harmonic restraint features were implemented in the program. The switched design philosophy was adopted to avoid unnecessary computations. The use of switched design concept required that the starting algorithm detect a potentially hazardous condition and transfer control to either the differential relay logic or the restricted earth fault relay logic. The digital relay was tested in the off-line mode to evaluate its performance. Brief details of the algorithm, relay logic and test results have been reported in Chapter 5. The

off-line testing revealed that the digital relays using filters designed by the frequency sampling and least error squares methods successfully identify magnetizing inrush conditions from internal faults.

The testing could not duplicate the actual phenomena on several counts. The dissimilarities between the primary and secondary side c.t.'s were not taken into consideration. Distortions introduced by c.t. saturation were also ignored. The data was generated using models of a single phase transformer. These models also need to be upgraded to model for three phase transformers including the power transformer and c.t. connections.

In conclusion, the studies presented in this thesis demonstrates that a digital transformer differential and restricted earth fault relay can be designed using suitable non-recursive digital filters. The software based digital relay can include adjustable variable percentage characteristic to suite a wide range of operating conditions.

7. REFERENCES

- 1) Ackroyd, M. H., Digital Filters, Butterworths, London, 1973.
- 2) Antoniou, A., Digital Filters: Analysis and Design, Mc Graw-Hill Book Company, New York, 1979.
- 3) Baribeau, M. A., "The Design of a Digital Impedance Relay", M. Sc. thesis, University of Saskatchewan, Canada, 1978.
- 4) Blinchikoff, H. J. and Zverev, A. I., Filtering in the Time and Frequency Domain, John Wiley and Sons, New York, 1976.
- 5) Einvall, C. H. and Linders, J. R., "A Three Phase Differential Relay for Transformer Protection", IEEE Transactions on Power Apparatus and Systems, Vol. PAS - 94, No. 6, Nov/Dec 1975, pp. 1971 - 1980.
- 6) Finnestad, S. A., "Optimum Detection of Trip Signals in Noise", Developments in Power System Protection, IEE Conference Publication No. 185, London, June 1980, pp. 211 - 214.
- 7) Larson, R. R., Flechsig, A. J. and Schweitzer, E. O., "The Design and Test of a Digital Relay for Transformer Protection", IEEE Transactions on Power Apparatus and Systems, Vol. PAS - 98, No. 3, May/June 1979, pp. 795 - 804.
- 8) Last, F. H. and Stalewski, A., "Protective Gear as a Part of Automatic Power System Control", Symp. on Automatic Control in Electricity Supply, Manchester, Mar., 1966, IEE conf. Publ. 16, Part I.
- 9) Luckett, R. G., Munday, P. J. and Murray, B. E., "A Substation Based Computer for Control and Protection", Developments in Power System Protection, IEE Conference Publication No. 125, London, March 1975, pp. 252 - 260.
- 10) Makino, J. and Miki, Y., "Study of Operating Principles and Digital Filters for Protective Relays with Digital Computer", IEEE Publication No. 75 CH0990 - 2 PWR, Paper No. C75 1979, IEEE PES Winter Power Meeting, New York, January 1975, pp. 1 - 8.

- 11) Malik, O. P., Dash, P. K. and Hope, G. S., "Digital Protection of a Power Transformer", IEEE Publication No. 76 CH1075 - 1 PWR, Paper No. A 76 191 - 7, IEEE PES Winter Meeting, New York, January 1976, pp. 1 - 7.
- 12) Malik, O. P., et al., "A Unified Approach to Differential and Impedance Protection," IEEE PES Summer Meeting, Paper No. A79 571 - 1, July, 1979.
- 13) Rabiner, L. R., "Techniques for Designing Finite Duration Impulse Response Digital Filters", IEEE Trans. Commun. Technol., Vol. COM - 19, April 1971, pp. 188 - 195.
- 14) Ranjbar, A. M. and Cory, B. J., "Filters for Digital Protection of Long Transmission Lines", IEEE Publication No. CH1458 - 9 PWR, Paper No. A 79 416 - 9, IEEE PES Summer Meeting, Vancouver, July 1979, pp. 1 - 9.
- 15) Rockefeller, G. D., "Fault Protection with a Digital Computer", IEEE Transactions on Power Apparatus and Systems, Vol. PAS - 88, No. 4, April 1969, pp. 438 - 464.
- 16) Sachdev, M. S. (Coordinator), Baird, T. C., Breingan, W. D., Harbourt, C. O., Phadke, A. G., Russel, B. D., Udren, E. A. and Walker, L. N., "Computer Relaying Tutorial Text", IEEE Power Engineering Society Special Publication No. 97 EH0148 - 7 - PWR, 1979, pp. 1 - 79.
- 17) Sachdev, M. S. and Baribeau, M. A., "A new Algorithm for Digital Impedance Relays", IEEE Transactions on Power Apparatus and Systems, Vol. 98, No. 6, Nov./Dec., 1979, pp. 2232 - 2240.
- 18) Sachdev, M. S., Baribeau, M. A. and Shah D. V., "A Digital Computer Based Directional Distance Relay for Detecting Phase - Phase and Ground Faults", Developments in Power System Protection, IEE Conference Publication No. 185, London, June 1980, pp. 112 - 116.
- 19) Sachdev, M. S. and Shah, D. V., "Transformer Differential and Restricted Earth Fault Protection using a Digital Processor", CEA Power System Planning and Operation Section, Spring Meeting, March, 1981.

- 20) Schweitzer, E. O., Larson, R. R. and Flechsig, A. J. Jr., "An Efficient Inrush Current Detection Algorithm for Digital Computer Relay Protection of Transformers", Paper A 77 510 - 1, IEEE Power Engineering Society Summer Meeting, Mexico City 1977.
- 21) Specht, T. R., "Transformer Inrush and Rectifier Transient Currents", IEEE Transactions on Power Apparatus and Systems, Vol. PAS - 88, No. 4, April 1969, pp. 269 - 276.
- 22) Stigant, S. A. and Lacey, H. M., The J. and P. Transformer Book, Johnson and Phillips Ltd., London, 1941.
- 23) Strang, G., Linear Algebra and Its Application, Academic Press, New York, 1976, Ch. 3.
- 24) Sykes, J. A. and Morrison, I. F., "A proposed Method of Harmonic Restraint Differential Protection of Transformers by Digital Computers", IEEE Transactions on Power Apparatus and Systems, Vol. PAS - 91, No. 3, May/June 1972, pp. 1266 - 72.
- 25) Warrington, A. R. van C., Protective Relays, Their Theory and Practice, Vol. I, Chapman and Hall, London, 1962.
- 26) Westinghouse Electric Corporation, Relay Instrument Division, Newark, New Jersey, Applied Protective Relaying.

8.APPENDIX**A.1 Simulated Magnetizing and Fault Current Waveforms**

Data representing magnetizing inrush and fault current waveforms was generated using the techniques presented in Section 2.7. Table A.1 lists the instantaneous values of the currents representing magnetizing inrush, winding fault and simultaneous magnetizing inrush and winding fault conditions. The data was considered to have been sampled at 720 Hz. The parameters used in computing the waveforms have been given in Section 5.5.2. The data given in Table A.1 was used for off-line testing the digital transformer differential and restricted earth fault relay described in Chapter 5.

A.2 Elements of the Left Pseudoinverse of Rectangular Matrix

This appendix lists the elements of the left pseudo-inverse of $[A]^{\dagger}$ when different combinations of sampling rate, data window and model of the waveform are used. The method of computing the elements of $[A]^{\dagger}$ has been described in Chapter 4. Combinations of different parameters of the least error squares filters designed for this project have been listed in Table 4.1. Coefficients of some of the least error squares filters are listed in this

Table A.1 Instantaneous values of the simulated magnetizing inrush, simultaneous magnetizing inrush and winding fault and winding fault current waveforms sampled at 720 Hz.

Time msec.	Inrush amps.	Inrush and internal fault amps.	Internal fault amps.
0.00	0.0	0.0	0.0
1.39	0.0	3.5	14.1
2.78	0.0	13.0	52.1
4.17	0.0	25.8	103.1
5.56	21.1	59.3	152.8
6.94	58.0	104.7	187.1
8.33	68.8	117.8	196.2
9.72	51.2	95.5	177.1
11.11	10.5	44.0	134.1
12.50	0.0	19.6	78.3
13.89	0.0	6.0	23.9
15.28	0.0	-3.7	-15.0
16.66	0.0	-7.1	-28.5
18.05	0.0	-3.4	-13.7
19.44	0.0	6.3	25.0
20.83	0.0	19.2	76.8
22.22	10.2	42.0	127.1
23.60	47.6	88.1	162.1
25.00	59.0	102.0	171.8
26.38	42.0	80.2	153.3
27.77	1.7	29.4	111.0
29.16	0.0	14.0	55.7
30.55	0.0	-9.1	-36.3

appendix. The effect of changing the design parameters on the digital filter coefficients is also demonstrated.

Table A.2 lists the coefficients of the second, third, fourth and fifth rows of the $[A]^{\dagger}$ when the parameters listed at item 1 of Table 4.1 were used. In this case the data was assumed to belong to a waveform which can be represented by the first two terms of the Taylor series expansion of the decaying d.c. component and components of the fundamental and second harmonic frequencies. The data window in this case was considered to be 11 samples long and sampling frequency of 720 Hz. was assumed. Table A.3 lists the elements of the second to fifth rows of $[A]^{\dagger}$ when the parameters listed at item 14 of Table 4.1 were used. The input waveform and sampling rate were assumed to be identical to those used for the previous case; but the data window was increased to 13 samples. Examination of the elements of the matrices listed in Table A.2 and A.3 show that the values of the elements decrease as the window size is increased. This phenomenon indicates the increase of noise suppression quality of the filters when the data window is elongated. Noise transmission indices for the filters whose elements are given in Tables A.2 and A.3 are also given in these tables.

Table A.4 lists the coefficients of the second to fifth rows of the $[A]^{\dagger}$ whose design parameters are listed at item

16 of Table 4.1. The model of the waveform selected in this case consisted of the first two terms of the Taylor series expansion of the decaying d.c. component and terms for the fundamental, second, third and fourth harmonic components. The data was assumed to be sampled at 720 Hz and the data window of 13 samples was used. Table A.5 lists the coefficients of the second to fifth rows of $[A]^{\dagger}$ whose variable parameters are listed at item 19 of Table 4.1. In this case, the waveform was modelled by the first three terms of the Taylor series expansion of the decaying d.c. component and components of the fundamental, second, third and fourth harmonics. A study of Tables A.4 and A.5 shows that the elements of the first and third columns of these tables are the same. However, the values of the elements of the second and fourth columns are larger when the decaying d.c. is represented by the first three terms instead of the first two terms of the Taylor series expansion.

Table A.6 lists the elements of the second to fifth rows of the $[A]^{\dagger}$ when the selected waveform model and the sampling rate were the same as those used in the case of Tables 4.3 and 4.4 but the data window was increased to sixteen samples. Table A.7 lists the elements of the second to fifth rows of the $[A]^{\dagger}$ whose design parameters are listed at item 26 of Table 4.1. The design parameters used in this case were the same as the parameters used in the previous design except that the sampling frequency was increased to

960 Hz. An examination of Tables A.6 and A.7 shows that increasing the sampling frequency increases the magnitudes of the elements of $[A]^{\dagger}$. This phenomenon is also reflected by the increase of the filter's noise transmission index.

Tables A.2 to A.7 represent coefficients of some of the filters designed by the least error squares approach. Noise transmission indices of these filters are also given in these tables. The digital filter designs reported in this thesis indicate the manner in which the quality of the filters designed by the least error squares approach is affected when waveform models, window lengths and sampling frequencies are changed.

Table A.2 Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental and second harmonic frequencies.

Element No.	Digital filter coefficients			
1	0.6400	-0.2332	-0.1914	0.1722
2	-0.4684	-0.1071	0.2947	-0.0595
3	-0.4907	0.0238	0.1504	-0.1905
4	-0.0257	0.1071	-0.1994	-0.1071
5	0.1973	0.1380	-0.2746	0.0897
6	0.0000	0.1429	0.0000	0.1905
7	-0.1973	0.1380	0.2746	0.0897
8	0.0257	0.1071	0.1994	-0.1071
9	0.4907	0.0238	-0.1504	-0.1905
10	0.4684	-0.1071	-0.2947	-0.0595
11	-0.6400	-0.2332	0.1914	0.1722
Noise index	1.8187	0.2143	0.5225	0.2143

Table A.3 Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental and second harmonic frequencies.

Element No.	Digital filter coefficients			
1	0.2784	-0.1176	-0.1292	0.1176
2	-0.0075	-0.1077	0.1092	0.0467
3	-0.1783	-0.0735	0.1601	-0.0931
4	-0.2006	-0.0098	0.0158	-0.1569
5	-0.1319	0.0735	-0.1501	-0.0735
6	-0.0539	0.1470	-0.1580	0.0807
7	0.0000	0.1765	0.0000	0.1569
8	0.0539	0.1470	0.1580	0.0807
9	0.1319	0.0735	0.1501	-0.0735
10	0.2006	-0.0098	-0.0158	-0.1569
11	0.1783	-0.0735	-0.1601	-0.0931
12	0.0075	-0.1077	-0.1092	0.0467
13	0.2784	-0.1176	0.1292	0.1176
Noise index	0.3398	0.1471	0.2040	0.1471

Table A.4 Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental, second, third and fourth harmonic frequencies.

Element No.	Digital filter coefficients			
1	0.3092	-0.0952	-0.1435	0.0952
2	-0.0764	-0.1227	0.1411	0.0617
3	-0.1563	-0.0992	0.1499	-0.0675
4	-0.1529	0.0079	-0.0064	-0.1746
5	-0.1563	0.0833	-0.1388	-0.0833
6	-0.0764	0.1385	-0.1475	0.0891
7	0.0000	0.1746	0.0000	0.1587
8	0.0764	0.1385	0.1475	0.0891
9	0.1563	0.0833	0.1388	-0.0833
10	0.1529	0.0079	0.0064	-0.1746
11	0.1563	-0.0992	-0.1499	-0.0675
12	0.0764	-0.1227	-0.1411	0.0617
13	-0.3092	-0.0952	0.1435	0.0952
Noise index	0.3590	0.1508	0.2081	0.1508

Table A.5 Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated three terms representing the decaying d.c. component and components of the fundamental, second, third and fourth harmonic frequencies.

Element No.	Digital filter coefficients			
1	0.3092	1.1027	-0.1435	-0.2345
2	-0.0764	-2.3004	0.1411	0.6610
3	-0.1563	1.4821	0.1499	-0.5027
4	-0.1529	-0.7588	-0.0064	0.0364
5	-0.1563	0.0354	-0.1388	-0.0702
6	-0.0764	0.7828	-0.1475	-0.0882
7	0.0000	-0.6879	0.0000	0.3961
8	0.0764	0.7828	0.1475	-0.0882
9	0.1563	0.0355	0.1388	-0.0702
10	0.1529	-0.7588	0.0064	0.0364
11	0.1563	1.4821	-0.1499	-0.5027
12	0.0764	-2.3004	-0.1411	0.6610
13	-0.3092	1.1027	0.1435	-0.2345
Noise index	0.3590	20.2616	0.2081	1.6741

Table A.6 Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental, second, third and fourth harmonic frequencies.

Element No.	Digital filter coefficients			
1	0.1099	-0.0616	-0.0677	-0.0024
2	0.0427	-0.0795	-0.0115	0.0731
3	0.0318	-0.0795	0.0562	0.0731
4	-0.0354	-0.0616	0.1124	-0.0024
5	-0.1419	-0.0357	0.0737	-0.1377
6	-0.1705	0.0357	-0.0780	-0.1510
7	-0.1167	0.1233	-0.1682	0.0049
8	-0.0395	0.1590	-0.0839	0.1426
9	0.0395	0.1590	0.0839	0.1426
10	0.1167	0.1233	0.1682	0.0049
11	0.1705	0.0357	0.0780	-0.1510
12	0.1419	-0.0357	-0.0737	-0.1377
13	0.0354	-0.0616	-0.1124	-0.0024
14	-0.0318	-0.0795	-0.0562	0.0731
15	0.0427	-0.0795	0.0115	0.0731
16	-0.1099	-0.0616	0.0677	-0.0024
Noise index	0.1611	0.1265	0.1343	0.1456

Table A.7 Elements of the second, third, fourth and fifth rows of $[A]^T$. The waveform model incorporated two terms representing the decaying d.c. component and components of the fundamental, second, third and fourth harmonic frequencies.

Element No.	Digital filter coefficients			
1	0.5150	-0.1226	-0.2271	0.1155
2	-0.3048	-0.1039	0.2355	0.0478
3	-0.1852	-0.0694	0.1569	-0.0478
4	0.0268	-0.0244	-0.0283	-0.1155
5	-0.1273	0.0244	-0.0454	-0.1155
6	-0.2116	0.0694	-0.0606	-0.0478
7	-0.0230	0.1039	-0.1391	0.0478
8	0.0527	0.1226	-0.0871	0.1155
9	-0.0527	0.1226	0.0871	0.1155
10	0.0230	0.1039	0.1391	0.0478
11	0.2116	0.0694	0.0606	-0.0478
12	0.1273	0.0244	0.0454	-0.1155
13	-0.0268	-0.0244	0.0283	-0.1155
14	0.1852	-0.0694	-0.1569	-0.0478
15	0.3048	-0.1039	-0.2355	0.0478
16	-0.5150	-0.1226	0.2271	0.1155
Noise index	0.9149	0.1250	0.3303	0.1250

APPENDIX B

The use of weighting functions, also known as window functions, in conjunction with digital filters is examined in this appendix. The window functions are employed to concentrate the energy of the filter in its pass-band and to suppress leakage outside the band. Window functions are also used to overcome the Gibbs phenomenon^{2,4}. This phenomenon is caused by discontinuities in the filter impulse response and is characterized by a fixed percentage overshoot and ripple in the frequency response.

Windowing is accomplished by multiplying, a_n 's, the coefficients of a filter by a time limited even weighting function w_n . Multiplication of an impulse response by a window function corresponds to convolution of the frequency response of the filter with the Fourier transform of the window. If the weighting function, w_n , is chosen such that its Fourier transform has a main lobe which contains a large part of the total energy, the resulting frequency characteristics will be smooth - sharp transitions in the frequency response would be eliminated. This, however, is achieved at the expense of a wider main lobe and smoother transitions at the discontinuities.

Many window functions have been proposed in the literature. One of the commonly used window function is the Hamming's window¹ which is defined as follows.

$$w_n = 0.54 - 0.46 \cos(2\pi n/m), \quad n = 0, \dots, m \quad (\text{B.1a})$$

if m is even

$$w_n = 0.54 - 0.46 \cos\left(\frac{2\pi n + \pi}{m+1}\right), \quad n = 0, \dots, m \quad (\text{B.1b})$$

if m is odd

The transfer function of a non-recursive digital filter incorporating the Hamming's window function can be represented as follows:

$$H(z) = \sum_{n=0}^m a_n w_n z^{-n} \quad (\text{B.2})$$

The coefficients of a pair of non-recursive filters designed by the least error squares approach, and already reported in Chapter 4, are listed in columns one and two of Table B.1. Application of the Hamming's window function to these coefficients result in a new filter whose coefficients are listed in columns three and four of Table B.1. The frequency response of the original 'even' digital filter whose coefficients are listed in the second column of Table B.1 has been previously depicted in Figure 4.5. The

frequency response of this filter modified by the application of the Hamming's window is shown in Figure B.1. Comparing the two filter responses, it is observed that the application of the window function results in suppressing the sidelobes and, simultaneously, widening the mainlobe.

Filters for use in digital relays have short impulse responses and require zeroes at some specified frequencies. Widening of the main lobe is also not desirable. Window functions, therefore,, do not offer significant advantages in digital relaying applications. Application of window functions may prove useful if data windows larger than 32 msec. are used.

Table B.1 Coefficients of the fundamental frequency band-pass filters before and after the application of the Hamming window function

'odd'	'even'	'odd'	'even'
0.3092	-0.0952	0.0247	-0.0076
-0.0764	-0.1227	-0.0108	-0.0174
-0.1563	-0.0992	-0.0485	-0.0308
-0.1529	0.0079	-0.0825	0.0043
-0.1563	0.0833	-0.1203	0.0642
-0.0764	0.1385	-0.0717	0.1300
0.0000	0.1746	0.0000	0.1746
0.0764	0.1385	0.0717	0.1300
0.1563	0.0833	0.1203	0.0642
0.1529	0.0079	0.0825	0.0043
0.1563	-0.0992	0.0485	-0.0308
0.0764	-0.1227	0.0108	-0.0174
-0.3092	-0.0952	-0.0247	-0.0076

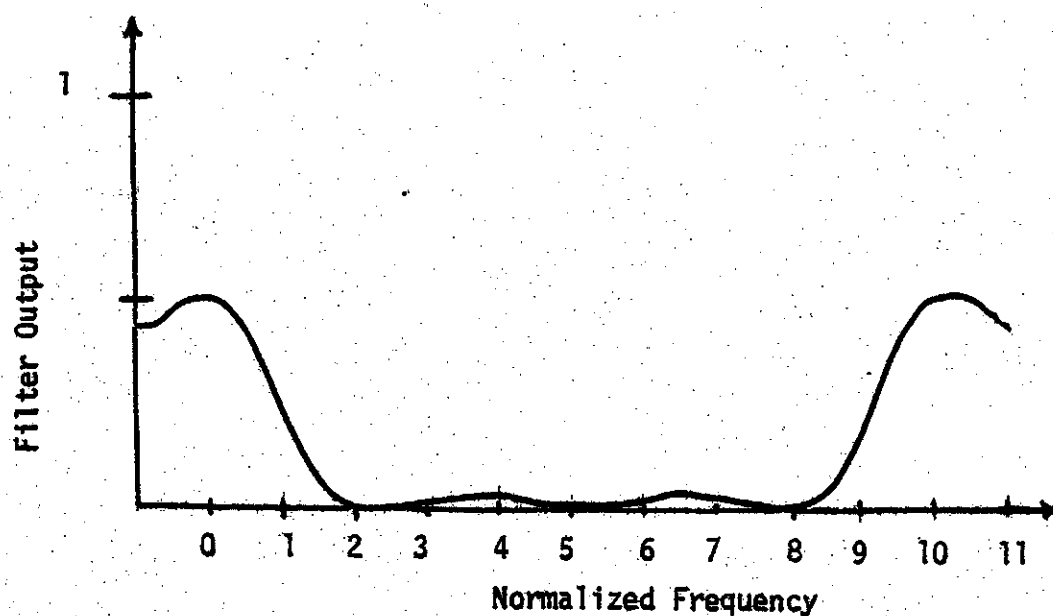


Figure B.1 Frequency response of the 60 Hz digital filter with the Hamming's weighting applied to the filter coefficients.

APPENDIX C

This appendix shows how a variable percentage differential characteristic can be implemented without taking the square roots of I_D^2 or I_Σ^2 . A typical characteristic of a variable percentage differential relay is shown in Figure C.1. This characteristic is piecewise linear.

Equations representing the straight lines in segments 2 and 3 of Figure C.1 are:

$$I_D = M1(I_\Sigma) + C1 \quad I_{\Sigma1} < I_\Sigma < I_{\Sigma2} \quad (C.1)$$

$$I_D = M2(I_\Sigma) + C2 \quad I_{\Sigma2} < I_\Sigma < I_{\Sigma3} \quad (C.2)$$

where:

M1 and M2 are the slopes of the straight line segments 2 and 3 respectively

These equations can also be written as:

$$(I_D)^2 = [M1(I_\Sigma) + C1]^2 \quad I_{\Sigma1} < I_\Sigma < I_{\Sigma2} \quad (C.3)$$

$$(I_D)^2 = [M2(I_\Sigma) + C2]^2 \quad I_{\Sigma2} < I_\Sigma < I_{\Sigma3} \quad (C.4)$$

Equations C.3 and C.4 can be approximated by the following pair of equations.

$$(I'_D)^2 = M11(I_{\Sigma})^2 + C11 \quad I_{\Sigma} < I_{\Sigma} < I_{\Sigma 2} \quad (C.5)$$

$$(I'_D)^2 = M22(I_{\Sigma})^2 + C22 \quad I_{\Sigma} < I_{\Sigma} < I_{\Sigma 3} \quad (C.6)$$

where:

I'_D is the value of the ordinates for the same value of I_{Σ} obtained by approximate equations.

The salient points on the characteristics can be used to obtain the values of M11, M22, C11 and C22 as follows.

$$M11 = \frac{(I_{D2})^2 - (I_{min})^2}{(I_{\Sigma 2})^2 - (I_{\Sigma 1})^2} \quad (C.7)$$

$$C11 = (I_{min})^2 - M11(I_{\Sigma 1})^2 \quad (C.8)$$

$$M22 = \frac{(I_{D3})^2 - (I_{D2})^2}{(I_{\Sigma 3})^2 - (I_{\Sigma 2})^2} \quad (C.9)$$

$$C22 = (I_{D2})^2 - M22(I_{\Sigma 2})^2 \quad (C.10)$$

The values of I'_D calculated using Equations C.5 and C.6 are shown by dotted lines in Figure C.2. The true values of

I_D are depicted as solid lines. A study of this figure shows that the differences between the exact and the approximate values of I_D are minimal. In this case the values used for I_{min} , I_{D2} , I_{D3} , $I_{\Sigma 1}$, $I_{\Sigma 2}$ and $I_{\Sigma 3}$ were 0.05, 0.8, 2.24, 1.0, 6.0 and 12.0 p.u. The values used for C_{11} , C_{22} , M_{11} and M_{22} were computed to be -0.02, -0.82, 0.02 and 0.04 p.u. respectively.

To reduce computation time, the relay characteristic was modelled in the digital relay by Equations C.5 and C.6.

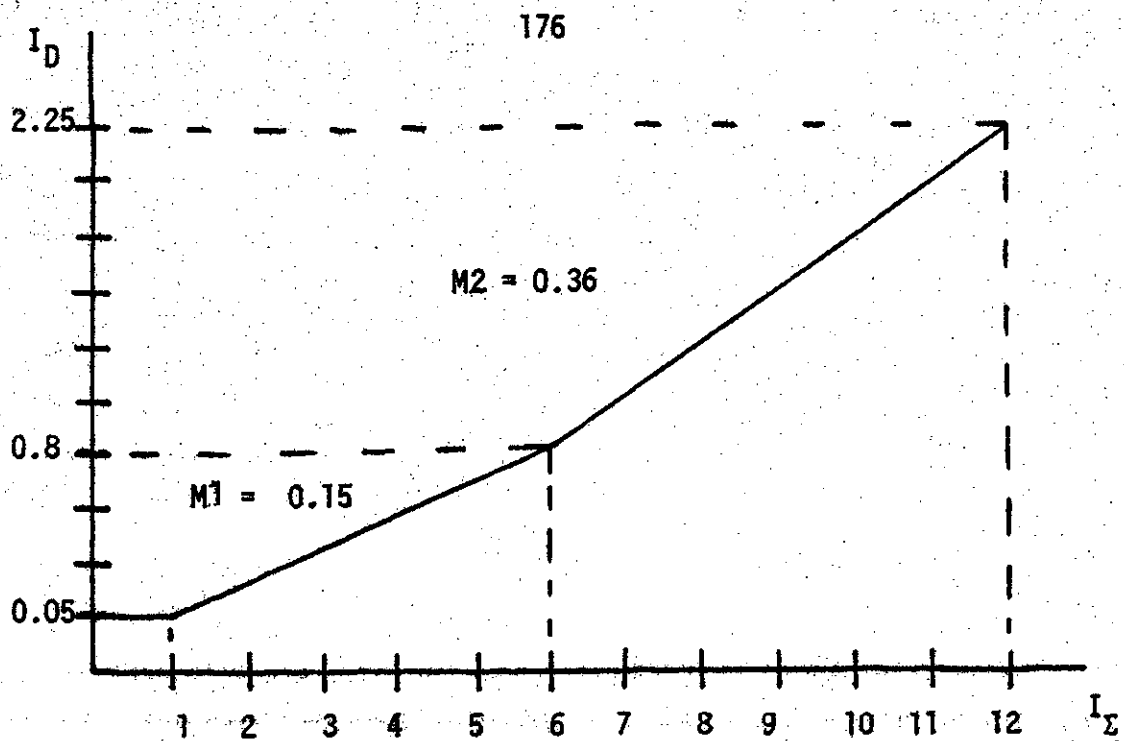


Figure C.1 Two-piece piecewise linear differential relay characteristics.

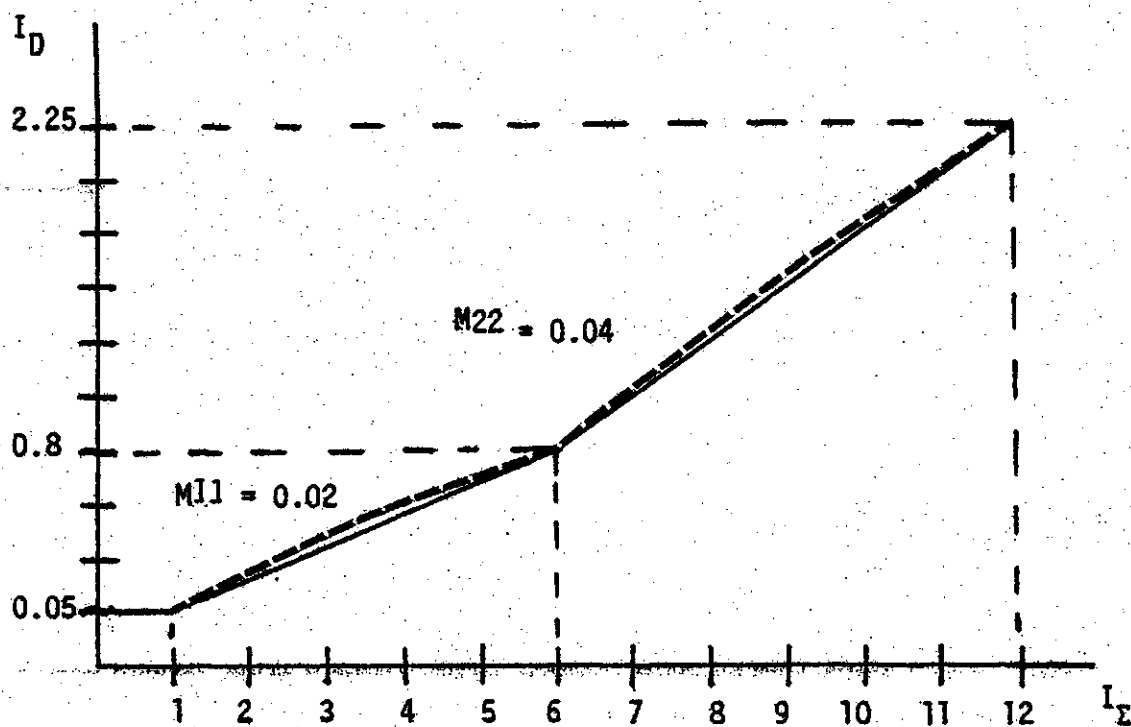


Figure C.2 The true and approximate differential protection characteristics.