

# **ADAPTIVE GROUND FAULT PROTECTION FOR A DISTRIBUTION NETWORK**

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**By**

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**ADAPTIVE GROUND FAULT PROTECTION FOR A  
DISTRIBUTION NETWORK**

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**Supervisor: Dr. M.S. Sachdev**

**M.Sc. Thesis submitted to the  
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**ABSTRACT**

Due to advances in the field of digital protection, more and more utilities are using microprocessor based relays for protection of power system equipment. However, the protection philosophy remains the same. The present practice of protection is to analyze all the abnormal operating conditions in advance and to set the relays considering all the contingencies. But it is not possible to identify all operating conditions in advance and to determine a set of relay settings that are optimum for all those conditions. Compromises are, therefore, made which result in the relays exhibiting poor selectivity, slow response and even failure to operate.

Many researchers are now investigating the capability of microprocessor based relays to adapt to the changing state of the power system. This approach, known as adaptive protection, will allow the relay settings to be changed dynamically to suit the prevailing operating condition of the power system.

The work reported in this thesis includes the investigations for setting ground overcurrent relays in a distribution system and changing the settings to adapt to the operating conditions of the system. Four software modules were developed for this purpose and were tested using a personal computer.

The results reported in the thesis demonstrate that improvements in coordination of ground overcurrent relays can be achieved when the adaptive approach of protection is implemented.

## TABLE OF CONTENTS

COPYRIGHT	i
ACKNOWLEDGMENTS	ii
ABSTRACT	iii
TABLE OF CONTENTS	v
LIST OF FIGURES	vii
LIST OF TABLES	ix
1. INTRODUCTION	1
1.1. Protection of a Power System	1
1.2. Objectives of the Thesis	3
1.3. Outline of the Thesis	3
2. PROTECTION OF A DISTRIBUTION SYSTEM	5
2.1. Introduction	5
2.2. Elements of a Distribution System	5
2.3. A Distribution Network of the Future	7
2.4. Protection of Distribution Systems	7
2.4.1. Protective relays	9
2.4.1.1. Overcurrent protection	9
2.4.1.2. Instantaneous overcurrent protection	12
2.4.1.3. Directional overcurrent protection	12
2.5. Digital Relays	14
2.5.1. Architecture	14
2.5.2. Algorithms	16
2.5.2.1. Trigonometric algorithms	16
2.5.2.2. Fourier algorithm	19
2.5.2.3. Rectangular wave algorithm	19
2.5.2.4. Least error squares algorithm	21
2.6. Summary	23
3. ADAPTIVE PROTECTION OF A DISTRIBUTION SYSTEM	24
3.1. Introduction	24
3.2. Adaptive Relaying Scheme	24
3.2.1. Basic adaptive relaying concept	25
3.3. Purpose of the adaptive distribution protection	27
3.4. Functions of an adaptive protection system	27
3.5. Software for an adaptive protection scheme	28
3.6. Summary	29
4. NETWORK TOPOLOGY DETECTION	30
4.1. Introduction	30
4.2. Substation Configuration	31
4.3. The Topology Detection Technique	35
4.3.1. Basic functions	35
4.3.2. Definitions	37
4.3.3. The technique	37
4.4. Testing	45
4.5. Summary	47

<b>5. STATE ESTIMATION, FAULT ANALYSIS AND RELAY SETTING TECHNIQUES</b>	<b>48</b>
5.1. Introduction.	48
5.2. State Estimation	48
5.2.1. Newton Raphson load flow technique	50
5.2.2. The Decoupled load flow technique	51
5.2.2.1. Decoupled load flow algorithm	52
5.3. Fault Analysis	53
5.3.1. Balanced faults	53
5.3.2. Unbalanced faults	55
5.3.2.1. Single line to ground faults	56
5.4. Relay Settings	57
5.4.1. Relay-setting and optimization algorithm	57
5.5. Summary	63
<b>6. SYSTEM STUDIES AND RESULTS</b>	<b>66</b>
6.1. Introduction	66
6.2. Model Distribution Network	66
6.3. Operating Conditions	68
6.4. Testing of Coordination Software for L-G Fault	69
6.4.1. Network topology detection	69
6.4.2. State estimation	69
6.4.3. Fault analysis	69
6.4.4. Relay settings and optimization	78
6.4.4.1. Criteria for settings of ground overcurrent relays	78
6.4.4.2. Ground overcurrent relay settings	78
6.4.4.3. Instantaneous relay settings	79
6.4.4.4. Time multipliers settings	79
6.4.4.5. Optimization	79
6.5 Results and Discussions	86
6.6 Summary	87
<b>7. SUMMARY AND CONCLUSIONS</b>	<b>106</b>
7.1. Summary	106
7.2. Conclusions	107
7.3. Suggestions for Future Work	107
<b>REFERENCES</b>	<b>108</b>
<b>Appendix A: Input Data and Test Results of Topology Detection Techniques for Selected Networks</b>	<b>111</b>
A.1. System 1: 6-Bus model system	111
A.2. System 2: City of Saskatoon distribution model network	111
A.3. System 3: 24-Bus IEEE RTS	111
A.4. Results and discussion	125
<b>Appendix B: Load Flow Technique (Derivation of Jacobian Matrix)</b>	<b>135</b>
<b>Appendix C: Derivation of Fault Admittance Matrix for L-G Fault</b>	<b>140</b>
<b>Appendix D: Additional Test Results</b>	<b>142</b>

## LIST OF FIGURES

<b>Figure 1.1:</b>	Typical zones of primary protection	2
<b>Figure 2.1:</b>	One-line diagram of a typical distribution system	6
<b>Figure 2.2:</b>	A future distribution network with DSG units connected to customers premises, power distribution feeders and a substation	8
<b>Figure 2.3:</b>	Time and current graded overcurrent relaying scheme for protecting radial distribution circuits	10
<b>Figure 2.4:</b>	Typical time-current characteristics of an overcurrent relay	11
<b>Figure 2.5:</b>	Inverse time and definite time relay characteristics for protecting radial feeders	11
<b>Figure 2.6:</b>	Reduction of operating time by using instantaneous relays	12
<b>Figure 2.7:</b>	Operating characteristics of a directional relay	13
<b>Figure 2.8:</b>	A functional block diagram of a stand alone digital relay	15
<b>Figure 2.9:</b>	Sampling of a voltage wave	17
<b>Figure 2.10:</b>	Correlation process using the Fourier Algorithm	20
<b>Figure 2.11:</b>	Orthogonal rectangular waveforms of the fundamental frequency	21
<b>Figure 3.1:</b>	An adaptive distribution protection scheme	25
<b>Figure 3.2:</b>	Typical time-current characteristics of overcurrent relays	26
<b>Figure 3.3:</b>	A functional block diagram for a of relay coordination software	28
<b>Figure 4.1:</b>	A typical single bus, single breaker scheme	32
<b>Figure 4.2:</b>	A typical double bus, double breaker scheme	32
<b>Figure 4.3:</b>	A typical main and transfer bus scheme	33
<b>Figure 4.4:</b>	A typical double bus single breaker scheme	33
<b>Figure 4.5:</b>	A typical breaker-and-a-half scheme	34
<b>Figure 4.6:</b>	A typical ring bus scheme	34
<b>Figure 4.7:</b>	Structure of topology detection module	36
<b>Figure 4.8:</b>	A model network for illustration of topology input data	38
<b>Figure 4.9:</b>	Flow diagram representing configuration update	44
<b>Figure 4.10:</b>	Program flow diagram for topology detection module	46
<b>Figure 5.1:</b>	Three phase representation of power system for fault at bus p	54
<b>Figure 5.2:</b>	Flow chart of the fault analysis module	58
<b>Figure 5.3:</b>	Flow diagram showing the two phase simplex method for optimization	64
<b>Figure 6.1:</b>	Single line diagram of the selected distribution network	67
<b>Figure 6.2:</b>	Operating characteristics of relay 1-1	90
<b>Figure 6.3:</b>	Operating characteristics of relay 1-2	91
<b>Figure 6.4:</b>	Operating characteristics of relay 1-3	92
<b>Figure 6.5:</b>	Operating characteristics of relay 2-1	93
<b>Figure 6.6:</b>	Operating characteristics of relay 2-2	94
<b>Figure 6.7:</b>	Operating characteristics of relay 3-1	95
<b>Figure 6.8:</b>	Operating characteristics of relay 3-2	96
<b>Figure 6.9:</b>	Operating characteristics of relay 4-1	97
<b>Figure 6.10:</b>	Operating characteristics of relay 4-2	98
<b>Figure 6.11:</b>	Operating characteristics of relay 5-1	99
<b>Figure 6.12:</b>	Operating characteristics of relay 5-2	100
<b>Figure 6.13:</b>	Primary relay operating times with and without adaptive settings for fault at far end ( operating condition MN-1)	101



<b>Figure 6.14:</b>	Primary relay operating times with and without adaptive settings for fault at far end ( operating condition MN-2)	102
<b>Figure 6.15:</b>	Backup relay operating times with and without adaptive settings for fault at far end ( operating condition MN-1)	103
<b>Figure 6.16:</b>	Backup relay operating times with and without adaptive settings for fault at far end ( operating condition MN-2)	104
<b>Figure A.1:</b>	Extended single line diagram of a 6-bus model system	112
<b>Figure A.2:</b>	Extended single line diagram of city of Saskatoon distribution model network	113
<b>Figure A.3:</b>	Single line diagram of IEEE RTS	114
<b>Figure A.4:</b>	Extended diagram showing circuit breakers, isolators and circuit elements at Substation 2 of System 3	115
<b>Figure A.5:</b>	Extended diagram showing circuit breakers, isolators and circuit elements at Substation 15 of System 3	116
<b>Figure A.6:</b>	Execution time for the topology detection module	134
<b>Figure C.1:</b>	L-G fault at phase a	140

## LIST OF TABLES

<b>Table 4.1:</b>	<b>Circuit breaker and isolator data for the system</b>	<b>39</b>
<b>Table 4.2:</b>	<b>Line flow status data for the system</b>	<b>40</b>
<b>Table 4.3:</b>	<b>Index data for the system</b>	<b>40</b>
<b>Table 5.1:</b>	<b>Starting Tableau for Phase I of the simplex method</b>	<b>61</b>
<b>Table 5.2:</b>	<b>Tableau after completion of Phase I of the simplex method</b>	<b>62</b>
<b>Table 6.1:</b>	<b>Output of the topology detection module</b>	<b>70</b>
<b>Table 6.2:</b>	<b>Input to the state estimation module (Load and generation at different buses)</b>	<b>71</b>
<b>Table 6.3:</b>	<b>Output of the state estimation module (prefault voltages)</b>	<b>72</b>
<b>Table 6.4:</b>	<b>Output of the state estimation module (prefault currents)</b>	<b>73</b>
<b>Table 6.5:</b>	<b>Line and source parameter for input to the fault analysis module</b>	<b>74</b>
<b>Table 6.6:</b>	<b>Postfault currents at the relay locations for input to the optimization module</b>	<b>76</b>
<b>Table 6.7:</b>	<b>Starting tableau of phase I for operating condition MX-2</b>	<b>84</b>
<b>Table 6.8:</b>	<b>Tableau after first iteration of phase I for the operating condition MX-2</b>	<b>85</b>
<b>Table 6.9:</b>	<b>Settings of the relays of the selected network determined without using the adaptive approach</b>	<b>88</b>
<b>Table 6.10:</b>	<b>Selected relay pickup settings and calculated TMS values for adaptive protection of the selected distribution network</b>	<b>89</b>
<b>Table 6.11:</b>	<b>Operating time of primary and backup relays for selected fault locations for operating condition MN-2</b>	<b>105</b>
<b>Table A.1:</b>	<b>Circuit breaker and isolator data for System 1</b>	<b>117</b>
<b>Table A.2:</b>	<b>Line flow status data for System 1</b>	<b>118</b>
<b>Table A.3:</b>	<b>Index data for System 1</b>	<b>119</b>
<b>Table A.4:</b>	<b>Circuit breaker and isolator data for system 2</b>	<b>119</b>
<b>Table A.5:</b>	<b>Line flow status data for System 2</b>	<b>121</b>
<b>Table A.6:</b>	<b>Index data for System 2</b>	<b>122</b>
<b>Table A.7:</b>	<b>Circuit breaker and isolator data for Substation 2 of System 3</b>	<b>123</b>
<b>Table A.8:</b>	<b>Circuit breaker and isolator data for Substation 15 of System 3</b>	<b>124</b>
<b>Table A.9:</b>	<b>Output of topology detection module for state I of System 1</b>	<b>126</b>
<b>Table A.10:</b>	<b>Output of topology detection module for state II of System 1</b>	<b>127</b>
<b>Table A.11:</b>	<b>Output of topology detection module for state III System 1</b>	<b>127</b>
<b>Table A.12:</b>	<b>Output of topology detection module for state I System 2</b>	<b>128</b>
<b>Table A.13:</b>	<b>Output of topology detection module for state II System 2</b>	<b>129</b>
<b>Table A.14:</b>	<b>Output of topology detection module for state III System 2</b>	<b>130</b>
<b>Table A.15:</b>	<b>Output of topology detection module for state I System 3</b>	<b>131</b>
<b>Table A.16:</b>	<b>Output of topology detection module for state II System 3</b>	<b>132</b>
<b>Table A.17:</b>	<b>Output of topology detection module for state III System 3</b>	<b>133</b>
<b>Table D.1:</b>	<b>Output of state estimation module for operating condition MX-1</b>	<b>142</b>
<b>Table D.2:</b>	<b>Output of state estimation module for operating condition MN-1</b>	<b>143</b>
<b>Table D.3:</b>	<b>Output of state estimation module for operating condition MN-2</b>	<b>144</b>
<b>Table D.4:</b>	<b>Output of fault analysis module for operating condition MX-1</b>	<b>145</b>
<b>Table D.5:</b>	<b>Output of fault analysis module for operating condition MN-1</b>	<b>147</b>
<b>Table D.6:</b>	<b>Output of fault analysis module for operating condition MN-2</b>	<b>149</b>
<b>Table D.7:</b>	<b>Relay operating time for operating condition MX-1</b>	<b>151</b>
<b>Table D.8:</b>	<b>Relay operating time for operating condition MN-1</b>	<b>152</b>
<b>Table D.9:</b>	<b>Relay operating time for operating condition MX-2</b>	<b>153</b>
<b>Table D.10:</b>	<b>Relay operating time for operating condition MN-2</b>	<b>154</b>

# **1. INTRODUCTION**

An electric power system includes three basic components: generation, transmission and distribution [1]. Bulk power is generated at generating stations and is transmitted to load centers by transmission lines. Power is distributed to the consumers by distribution networks. Equipment used in power systems includes relays, isolators and circuit breakers. The relays monitor the system and protect it from abnormal operating conditions which can severely damage the system equipment, reduce the operating voltage and cause prolonged outages. The protection systems isolate the faulty equipment from the rest of the system, keep the system in operation and reduce the loss of revenue.

## **1.1. Protection of a Power System**

The basic elements of a protection scheme are the relays and circuit breakers. If an abnormal operating condition arises, the relays recognize it, determine the cause and initiate the opening of appropriate circuit breakers to isolate the faulty equipment. The relays must respond as quickly as possible to abnormal conditions but must not operate during normal operation of the system. The success of a protection scheme depends on the speed, sensitivity and selectivity of the relays used.

A relay primarily protects an equipment, such as a generator, a transformer or a line, and provides some backup protection for the adjoining equipment. Figure 1.1 shows the basic protection zones of a power system [2, 3]. Each zone covers one or more components of the system. Some zones may overlap others so that no part of the system is left unprotected.

Two sets of relays, primary and backup, are usually provided for each zone of protection [2, 4]. The purpose of a primary relay is to initiate control action for a fault in its zone of protection. The backup relays operate after a certain time delay if a primary relay fails to operate. It is important that backup relays be arranged in such a manner that anything that might cause a primary relay to fail will not affect the operation of the backup relays [2].

The complexity of modern power systems requires that protective relays be reliable,

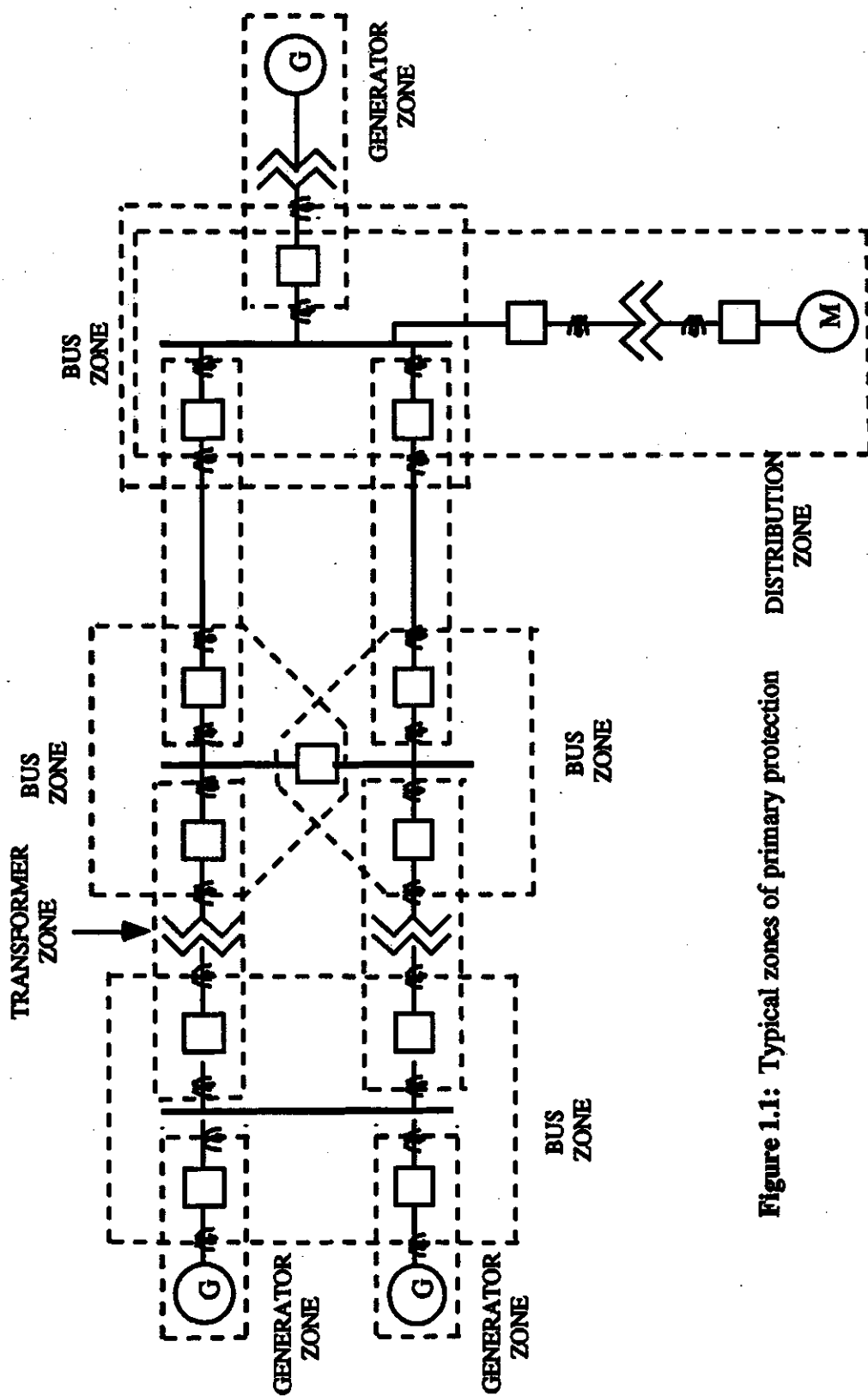


Figure 1.1: Typical zones of primary protection

fast and accurate. The electromechanical and solid state devices require high precision analog components to achieve these characteristics. With the advent of microprocessor technology and its ability to communicate [5], it is now possible to design protection schemes to provide faster and improved protection to a complex power system. The state of the power system can be continuously monitored and analyzed in real time to obtain optimum relay settings for the prevailing operating conditions. This approach of protection, classified as adaptive protection [6,7], has been used in this project for designing a protection system for a distribution network.

## **1.2. Objectives of the Thesis**

The objectives of this thesis were:

- To develop a general purpose network topology detection program, which is an essential module for coordinating relays for adaptive protection.
- To make necessary modifications and additions to the software previously developed at the University of Saskatchewan [8], so that the program is able to coordinate ground overcurrent relays in addition to phase overcurrent relays.
- To test the modules for adaptively coordinating relays for a reduced version of the City of Saskatoon Distribution Network.

The work reported in this thesis is a continuation of the work reported by Chattopadhyay [8]. That work included (a) the development of software modules for adaptive protection of a distribution network using phase overcurrent relays and (b) the testing of the developed software on a small scale prototype system.

## **1.3. Outline of the Thesis**

The thesis is organized in seven chapters and four appendices. The first chapter introduces the subject and objectives of the thesis. It also describes the organization of the thesis. The present protection philosophy of a power system is briefly reviewed.

Typical present and future distribution systems are described in Chapter 2. A protection scheme applied to a distribution system is reviewed. Different types of overcurrent relays are briefly discussed. This chapter also describes a typical microprocessor based relay and algorithms used in these relays.

Details of adaptive protection of a distribution network are outlined in Chapter 3. The purpose and functions of adaptive protection are then discussed and a brief description of the software necessary for implementing adaptive protection is presented.

The knowledge of network topology is vital for implementing an adaptive protection scheme. Therefore, a general purpose network topology detection program was developed. The constraints and procedure for developing the topology detection program are described in Chapter 4. Three systems were used to test the developed program. The descriptions of the systems and results obtained from the tests are presented in Appendix A.

The coordination software consists of four modules: network topology detection, state estimation, fault analysis, and relay setting modules. The mathematical developments of the state estimation, fault analysis and relay setting modules are discussed in Chapter 5. The modifications and additions made to the software modules previously developed and reported in [8], are outlined.

Chapter 6 reports the testing of the developed software. It also outlines and discusses the results of the coordination of ground overcurrent relays when applied to a model of the City of Saskatoon Distribution Network.

Chapter 7 provides a summary and conclusions drawn from the work reported in this thesis. Possible future work on the subject is also described. A list of references is then included.

This thesis also contains four appendices. Appendix A presents the network diagram, input data and the test results of topology detection technique for three selected networks. Appendix B describes the derivation of the Jacobian matrices for the Newton Raphson and decoupled load flow techniques for state estimation. Appendix C presents the derivation of the fault admittance matrix for a single line to ground fault in a network. Additional test results are included in Appendix D.

## **2. PROTECTION OF A DISTRIBUTION SYSTEM**

### **2.1. Introduction**

Distribution networks are links between bulk power sources of energy and customers' facilities. To transfer energy to the customers, a distribution system includes lines, transformers, reactors, capacitors and numerous protective devices. Most power outages are due to failures in the distribution systems, and, unfortunately, the outage of a distribution circuit has direct impact on the customers. Therefore, it is important to have a reliable and secure system to protect distribution networks.

This chapter presents the essential elements of a distribution network, and describes the existing schemes used to protect these networks. This chapter also describes a future distribution network and its protection.

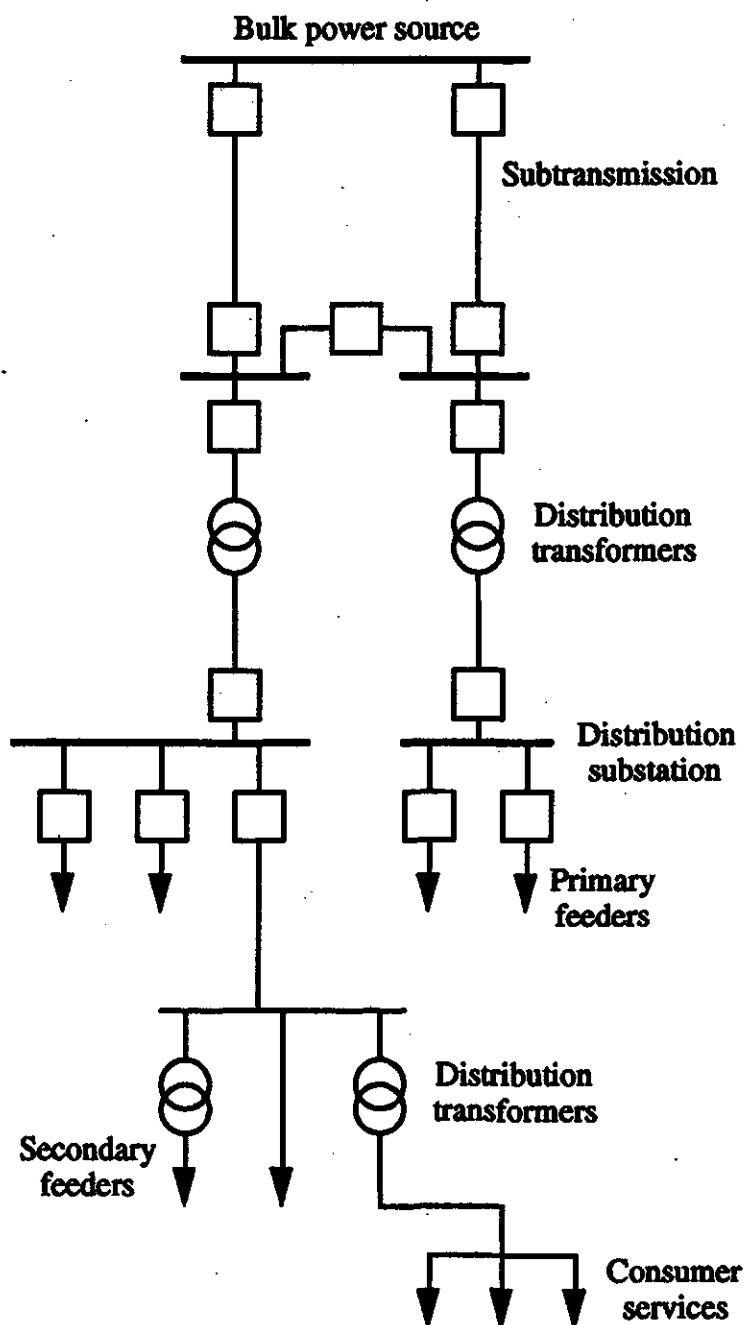
### **2.2. Elements of a Distribution System**

Figure 2.1 shows a single line diagram of a typical distribution network. As seen from the figure, the network includes the following basic elements.

1. Subtransmission circuits
2. Distribution substations
3. Distribution transformers
4. Secondary feeders.

Energy is delivered from a bulk power source to distribution substations by subtransmission circuits. The rated voltage of subtransmission circuits ranges from 34.5 to 138 kV. These circuits could be radial or could form interconnected networks. The major considerations for designing subtransmission circuits are cost and reliability.

In a distribution substation, voltage is stepped down from the subtransmission level to the primary distribution levels which range from 4.16 to 34.5 kV. Several bus configurations are used in distribution substations; these are presented in Section 4.3.



**Figure 2.1:** One-line diagram of a typical distribution system.

Capacitors, reactors and tap-changing transformers are used to regulate voltage in a distribution substation which supplies energy to primary feeders. These feeders are connected to distribution transformers which provide energy to customers via secondary



feeders and laterals. Distribution transformers of 10 to 500 kVA ratings are normally used.

### **2.3. A Distribution Network of the Future**

To reduce the use of conventional natural resources, efforts have been and are being made to develop alternative renewable resources to generate electrical power. Some predictions indicate that the number of dispersed-electricity-storage-and-generation (DSG) units in distribution systems will increase substantially during the next few years [9]. A DSG unit is defined as a generator typically of 100 kW to 10 MW capacity [1]. Typical examples of DSGs are hydroelectric generators, diesel generators, wind electric systems, solar electric systems, batteries, space and water heaters and photovoltaic and fuel cells. A typical distribution network of the future is shown in Figure 2.2 [9]. Some estimates indicate that the United States will have about 1200 GW generation in the year 2000 AD and 4 to 10 percent of that will be in the form of DSGs [1].

Also, distribution networks will become increasingly complex. Their operation will be automated for maintaining a reliable service and for reducing the operating costs. They will also be equipped with advanced protection schemes.

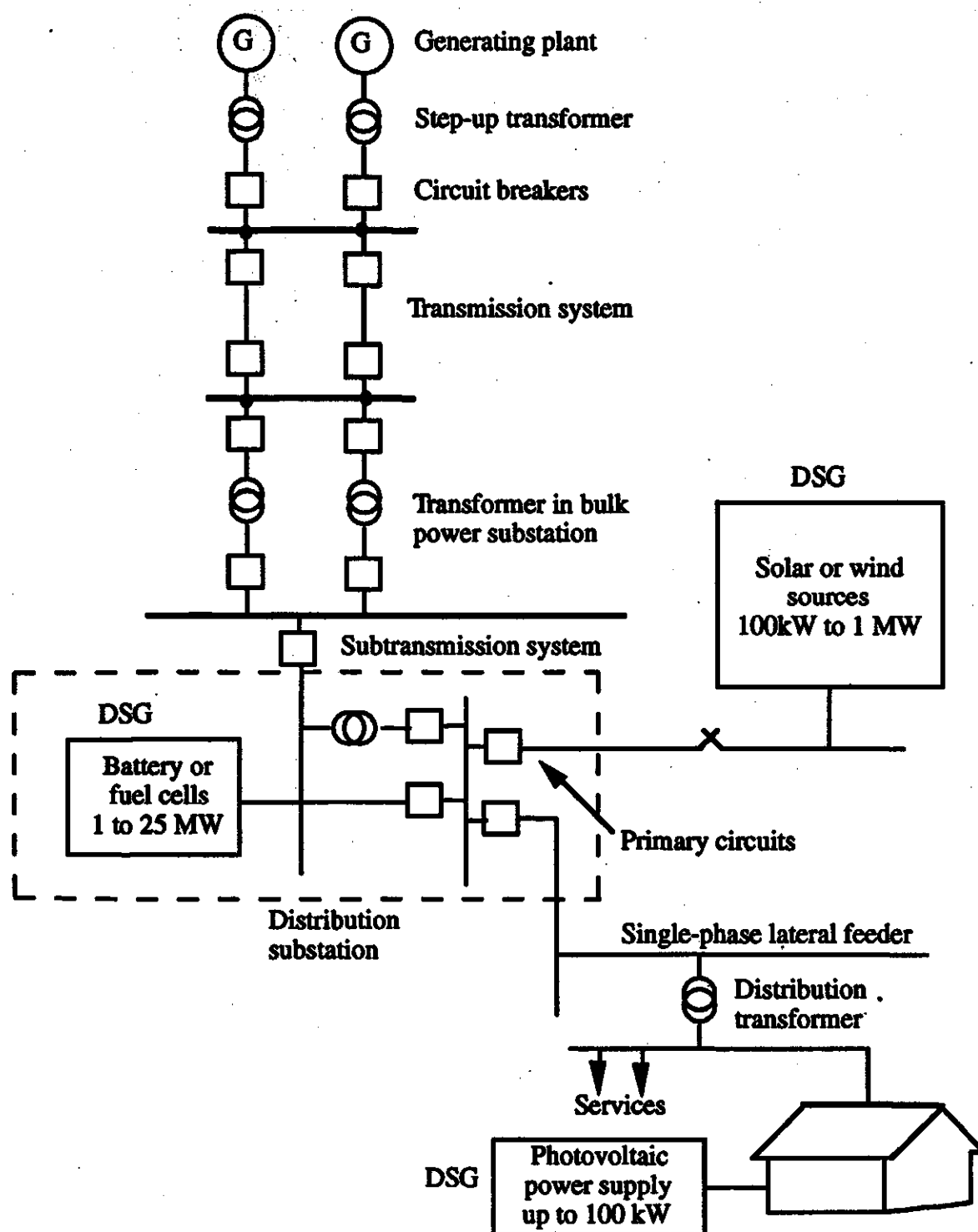
### **2.4. Protection of Distribution Systems**

Protection of distribution networks with radial and loop feeders is briefly discussed in this section. Distribution protection systems are designed:

1. to minimize the duration of fault,
2. to reduce the damage to the equipment, and
3. to minimize the number of customers affected by a fault.

The secondary objectives include elimination of safety hazards as far as possible and protection of the consumers' apparatus.

Most faults in distribution systems are of a temporary nature; they are caused by a phase conductor momentarily making an electrical connection with other phase conductors or the ground. These transient faults are associated with an arc at the location of the fault and are usually cleared by interrupting the power for a sufficient duration to extinguish the arc. This is achieved by tripping the circuit breakers controlling the line



**Figure 2.2:** A future distribution network with DSG units connected to customers premises, power distribution feeders and a substation [9].

and reclosing them after about 0.5 s. If a line circuit breaker trips on reclosing, it is reclosed again. On the occurrence of a fault, a circuit is reclosed a maximum of three times. After three trips and unsuccessful reclosing attempts, it is concluded that the fault is of a permanent nature.

### **2.4.1. Protective relays**

Two types of relays are generally used for protecting distribution circuits; these are overcurrent relays and distance relays. To achieve high speed operation of the breakers at both ends of a line, communication facilities are also included.

#### **2.4.1.1. Overcurrent protection**

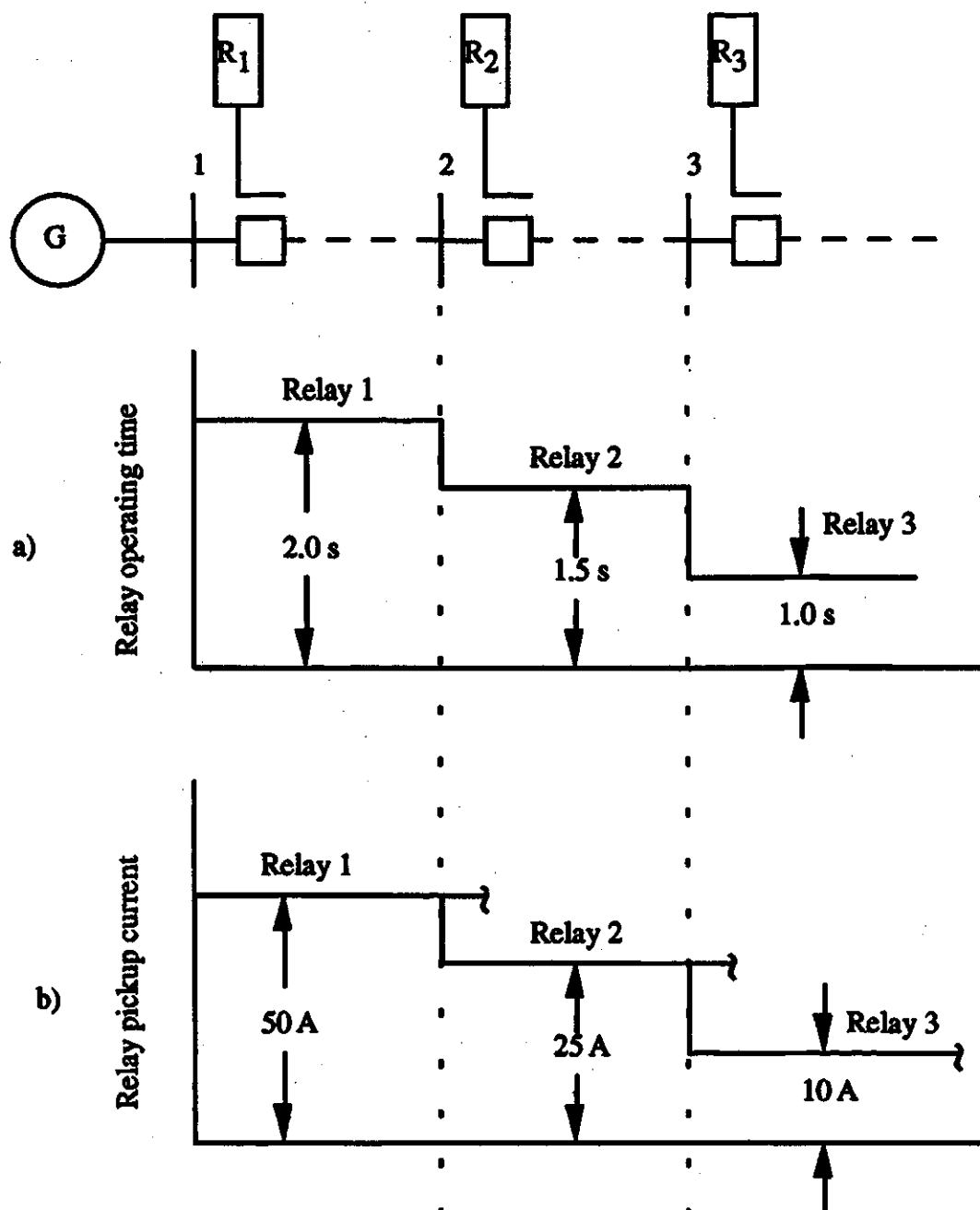
Overcurrent protection is based on the principle that current flowing in a line during a fault is more than the normal load current. This form of protection is applied in three forms, time graded, current graded and time-current graded overcurrent protection.

Figure 2.3(a) shows the operating characteristics of a time graded overcurrent protection scheme. It is applied to radial circuits which have a few line sections in series. The relay closest to the source is set to operate with maximum time delay; the delay is reduced gradually for relays which are progressively farther from the source. One disadvantage of this scheme is that the relay nearest to the source takes the most time to operate while the fault current is the largest. Time graded protection is preferred for systems where the level of fault current does not change substantially with the locations of the fault.

In a current graded protection scheme, relays closer to the source are set at progressively higher current levels. Figure 2.3(b) shows the settings of a current graded overcurrent scheme for a radial feeder. The relay  $R_1$  is set to operate for faults on the line between bus 1 and bus 2, the relay  $R_2$  is set to operate for faults on the line between bus 2 and bus 3, and so on. A disadvantage of the current graded overcurrent protection is that the relays cannot distinguish the faults on the line side of the remote bus, from the faults just beyond the remote bus.

Changes in the source impedance can adversely affect the operation of the time graded and current graded protection schemes. To achieve proper operation, inverse time overcurrent relays are used. The time-current characteristics of these relays are represented by a family of curves which specify the contact closing time of the relay

versus the relay current. Figure 2.4 shows the time-current characteristics of a typical inverse time overcurrent relay. To achieve a desired operating time for a specified current, an appropriate time dial setting (TDS) is selected. These relays have shorter operating times for faults near the source compared to the operating times for remote faults. Figure 2.5 shows a comparison of the relay operating times of inverse time and definite time overcurrent relays.



**Figure 2.3:** Time and current graded overcurrent relaying schemes for protecting radial distribution circuits, a) a time graded scheme and b) a current graded scheme.

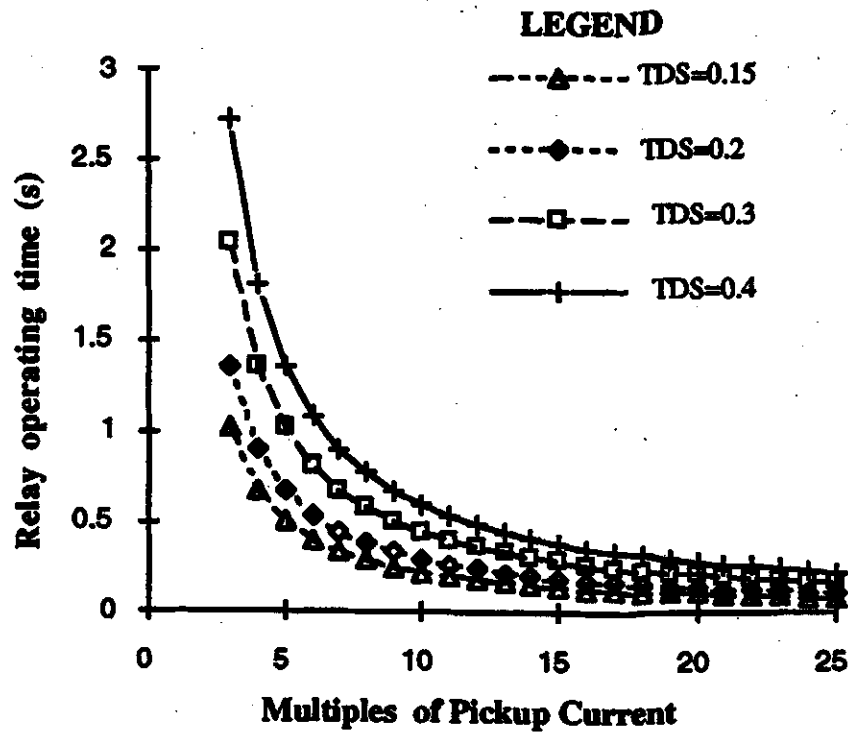


Figure 2.4: Typical time-current characteristics of an inverse time overcurrent relay.

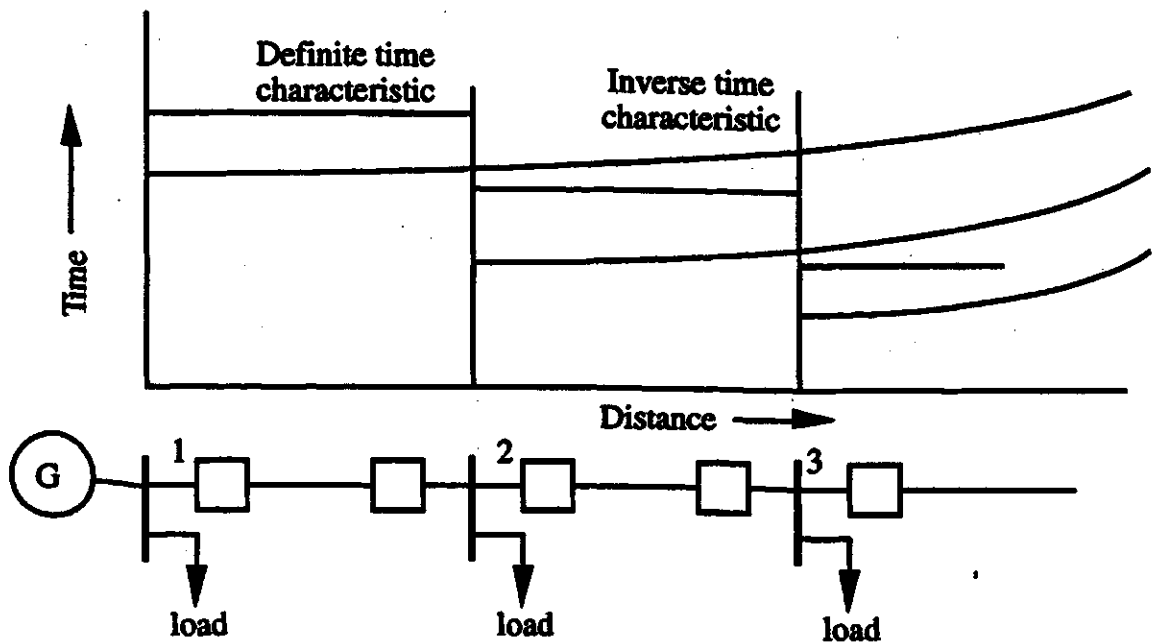


Figure 2.5: Inverse time and definite time relay characteristics for protecting radial feeders.

### 2.4.1.2. Instantaneous overcurrent protection

Instantaneous overcurrent relays are used if, under maximum generating conditions, current magnitude for a fault at the near end of a line is approximately three times the current for a fault at the far end of the line. An instantaneous relay, in conjunction with an overcurrent relay, provides faster operation for near end faults. In some cases, it may also permit relays in the adjacent sections to be set for faster operation. The shaded area of Figure 2.6 shows how much the use of instantaneous overcurrent relays reduces the overall operating times of the protection scheme.

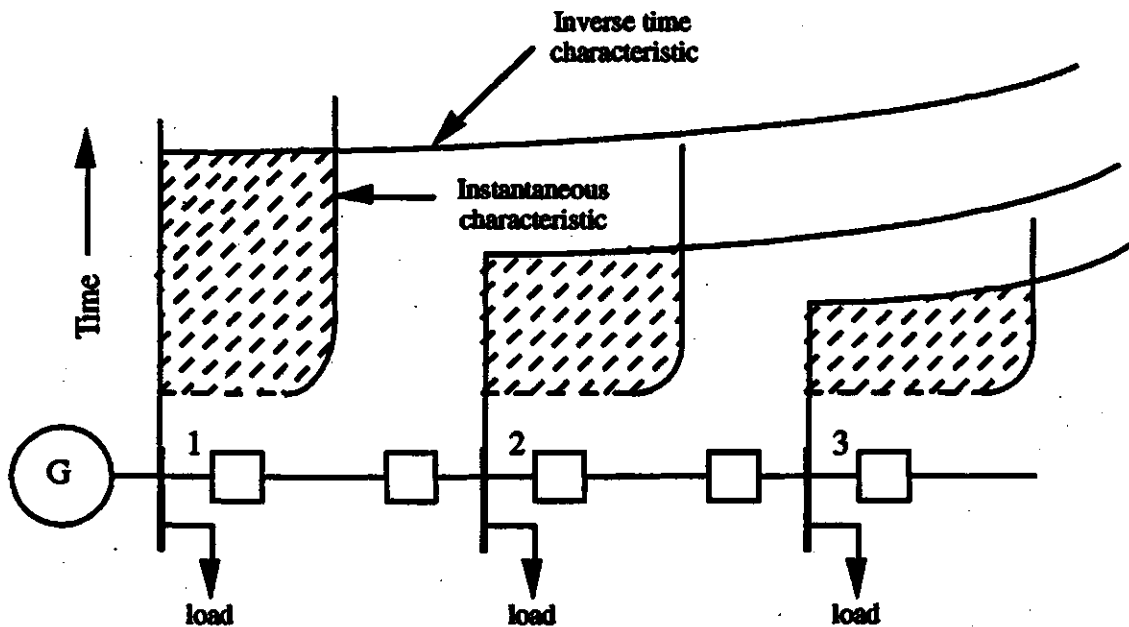


Figure 2.6: Reduction of operating time by using instantaneous relays.

### 2.4.1.3. Directional overcurrent protection

In interconnected networks, it is necessary to prevent a relay from operating if the fault is on the bus side of the relay. To achieve this, directional elements are included in overcurrent relays. The torque,  $T$ , developed by an electromechanical directional relay can be expressed as

$$T = k_1 |V| |I| \cos(\theta - \tau) - k_2, \quad (2.1)$$

where:

$V$  is the voltage phasor,

- $I$  is the current phasor,
- $\theta$  is the phase angle of  $I$  when phasor  $V$  is the reference,
- $\tau$  is the maximum torque angle,
- $k_1$  is the relay constant and
- $k_2$  is the torque of the restraining spring.

When the relay is on the verge of operation, the operating torque is zero. This leads to the following equation which expresses the operation of the relay in terms of the system and relay parameters.

$$|V| |I| \cos(\theta - \tau) > \frac{k_2}{k_1} \quad (2.2)$$

Figure 2.7 shows the torque diagram of a directional relay which operates when the tip of the current phasor is in the positive torque region. For a specified current magnitude, the torque is maximum when the current phasor is along the maximum torque line.

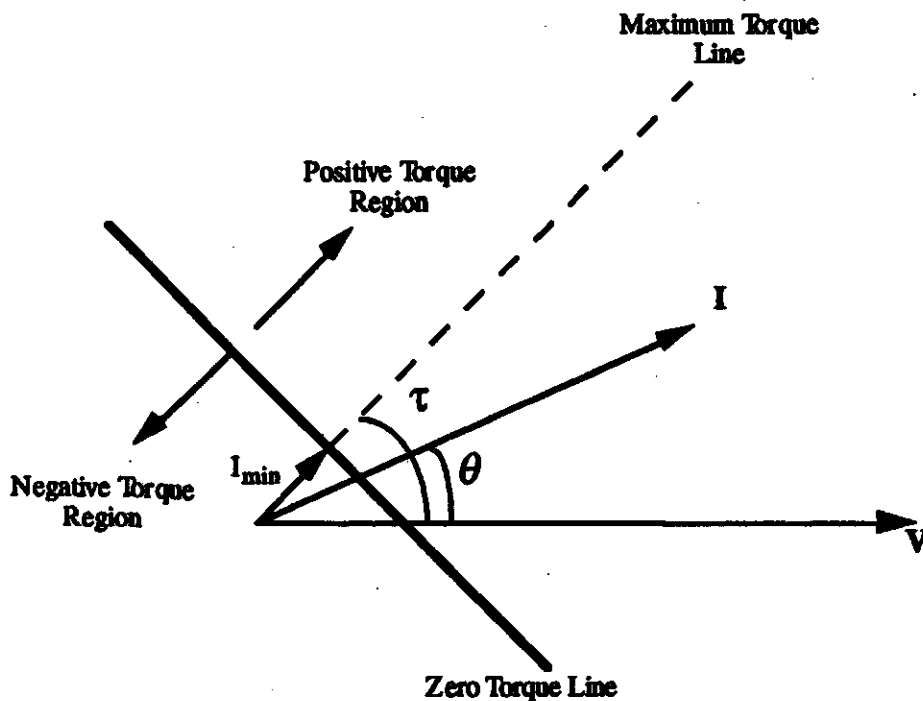


Figure 2.7: Operating characteristic of a directional relay.

## **2.5. Digital Relays**

In 1969, G.D. Rockefeller reported on the feasibility of protecting the major components of a substation with a single computer [10]. A few years later, the first digital relay was installed at the Tesla substation of the Pacific Gas and Electric Company [11]. Since then, work on designing microprocessor based relays for protecting power systems has continued at a steady pace [5,12]. Microprocessor relays have now become cheaper than their electromagnetic and solid state counterparts. They are reliable and flexible. The relay characteristics can be modified by altering the software only. Besides protection, microprocessor relays can also perform other background tasks, such as the storage and analysis of fault data, monitoring supervisory alarms etc. These are some of the reasons why many researchers prefer using microprocessors in protective relay designs.

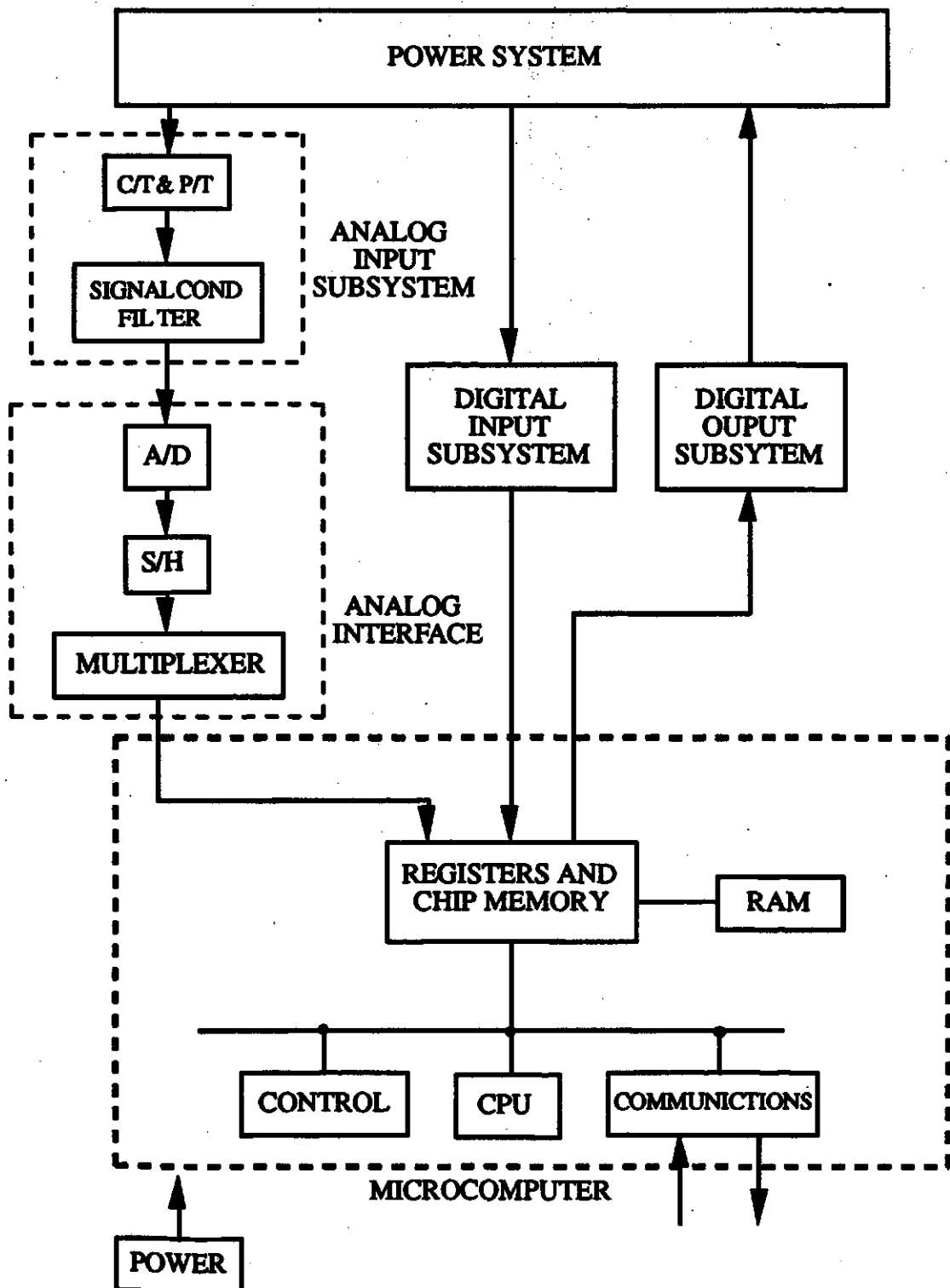
### **2.5.1. Architecture**

The major functional blocks of a typical microprocessor-based relay is depicted in Figure 2.8. An analog input subsystem, which consists of current and voltage transformers and antialiasing and signal conditioning filters, provides properly conditioned power system signals to the relay. It also isolates the relay from the power system, protecting the relay from transient overvoltages. The filters attenuate high frequency components to minimize the effects of aliasing. The output of the analog input subsystem is fed to the analog interface which includes analog-to-digital converters, sample and hold circuits and a multiplexer. Information concerning the status of circuit breakers and isolators is provided to the relay via the digital input subsystem. The output of the relay is transmitted to the system through the digital output subsystem. Suitable isolation from the power system is included.

The software resides in the read only memory (ROM) and the relay settings reside in the electronically erasable and programmable read only memory (EEPROM). The data and results are stored temporarily in the random access memory (RAM). These are moved to a secondary storage device, local or remote, soon after a disturbance is experienced.

The power supply to the relay must be uninterrupted. Therefore, a battery and a battery charger are included. The microprocessor communicates with the substation computer and other devices via a communication link.





**Figure 2.8:** A functional block diagram of a stand alone digital relay.

## 2.5.2. Algorithms

Digital relaying algorithms can be broadly classified into two categories, non-recursive and recursive algorithms. A non-recursive algorithm uses a finite number of data samples to obtain the estimate of a voltage or current phasor. The period in which the data samples are taken is called the data window. Trigonometric, correlation and least error-squares are examples of non-recursive algorithms. A recursive algorithm uses feedback by including the output as one of the inputs. The output of a recursive filter is, therefore, a function of the present as well as the previous inputs. Kalman filters have been used in the past to design recursive algorithms for protective relays.

### 2.5.2.1. Trigonometric algorithms

Two assumptions are made while designing these algorithms. The first assumption is that the waveform of the input signal is a sinusoid of a single frequency, and the second assumption is that the frequency of the input signal is invariant. The instantaneous value of an input, a voltage, is, therefore, expressed as

$$v = V_p \sin(\omega_0 t + \theta_v), \quad (2.3)$$

where:

- $v$  is the instantaneous value of the voltage,
- $V_p$  is the peak value of the voltage phasor,
- $\theta_v$  is the phase angle of the voltage phasor,
- $\omega_0$  is the fundamental radian frequency and
- $t$  is the time.

The first derivative of Equation 2.3 with respect to time is

$$v' = \omega_0 V_p \cos(\omega_0 t + \theta_v). \quad (2.4)$$

The peak value and phase angle of the voltage phasor can be obtained from Equations 2.3 and 2.4 as follows

$$V_p^2 = v^2 + (v'/\omega_0)^2, \quad (2.5)$$

$$\omega_0 t + \theta_v = \tan^{-1}(\omega_0 \frac{v}{v'}). \quad (2.6)$$

Input signals are sampled continuously at uniform time intervals, each interval of  $\Delta T$  seconds, as is shown in Figure 2.9. If the sample taken at  $k\Delta T$  seconds is the present

sample,  $k-1$ ,  $k-2$  are the previous samples, and  $k+1$ ,  $k+2$  are the future samples which are not available as yet. The values of the voltage samples are designated as  $v_{k-2}$ ,  $v_{k-1}$ ,  $v_k$ ,  $v_{k+1}$ ,  $v_{k+2}$ .

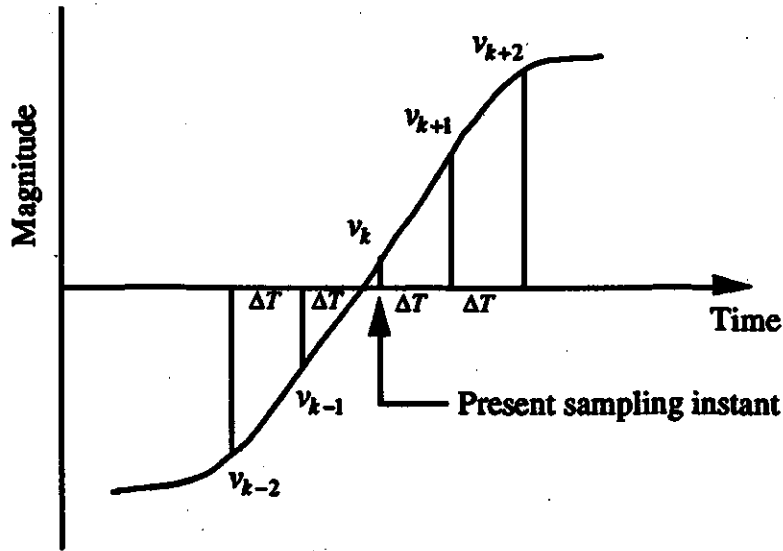


Figure 2.9: Sampling of a voltage wave.

Mann and Morrison [58] used a 3-sample window to calculate the peak value and phase angle of the phasor. They used the following equation to obtain the first derivative at time  $k\Delta T$ .

$$v'_k = \frac{v_{k+1} - v_{k-1}}{2\Delta T}. \quad (2.7)$$

An estimate of the peak value and phase angle of the voltage phasor can be obtained by substituting in Equations 2.5 and 2.6  $v_k$  for  $v$  and  $\frac{v_{k+1} - v_{k-1}}{2\Delta T}$  for  $v'$ . The resulting equations for the peak value and phase angle become

$$V_p = \left[ v_k^2 + \left( \frac{v_{k+1} - v_{k-1}}{2\omega_0 \Delta T} \right)^2 \right]^{1/2} \text{ and} \quad (2.8)$$

$$\theta_v = \tan^{-1} \left( \frac{2\omega_0 \Delta T v_k}{v_{k+1} - v_{k-1}} \right) - \omega_0 k \Delta T. \quad (2.9)$$

This algorithm is fast, is simple to implement and, if a proper sampling interval is selected, the computations are minimal. However, the algorithm amplifies noise and the

results obtained by this algorithm are adversely affected by the decaying dc components in the inputs.

Gilcrest *et al.* [11] used the first and second derivatives of the voltages and currents to estimate the peak values and phase angles of the phasors. This algorithm also uses a 3-sample window. The second derivative is used to suppress the decaying dc component.

The second derivative of the voltage, which is obtained by differentiating Equation 2.4 with respect to time, is given by

$$v'' = -\omega_0^2 V_p \sin(\omega_0 t + \theta_v). \quad (2.10)$$

Rearranging this equation provides

$$\frac{v''}{-\omega_0^2} = V_p \sin(\omega_0 t + \theta_v). \quad (2.11)$$

Equations 2.4 and 2.11 can be used to obtain the following equations which express the magnitude and phase angle of the voltage phasor as functions of the first and second derivatives of the voltage.

$$V_p^2 = \left(\frac{v'}{\omega_0}\right)^2 + \left(\frac{v''}{-\omega_0^2}\right)^2 \quad (2.12)$$

$$\omega_0 t + \theta_v = \tan^{-1}\left(\frac{-v''}{\omega_0 v'}\right). \quad (2.13)$$

The following equation can be used to obtain the second derivative of the voltage from its sampled values.

$$v_k'' = \frac{v_{k+1} - 2v_k + v_{k-1}}{\Delta T^2}. \quad (2.14)$$

Substituting  $\frac{v_{k+1} - v_{k-1}}{2\Delta T}$  for  $v'$  and  $\frac{v_{k+1} - 2v_k + v_{k-1}}{\Delta T^2}$  for  $v''$  into Equations 2.12 and 2.13 the peak value and phase angle of the voltage phasor is expressed as

$$V_p = \left[ \left( \frac{v_{k+1} - v_{k-1}}{2\omega_0 \Delta T} \right)^2 + \left( \frac{v_{k+1} - 2v_k + v_{k-1}}{-\omega_0^2 \Delta T^2} \right)^2 \right]^{1/2} \text{ and} \quad (2.15)$$

$$\theta_v = \tan^{-1} \left( \frac{-v_{k+1} + 2v_k - v_{k-1}}{0.5\omega_0\Delta T(v_{k+1} - v_{k-1})} \right) - \omega_0 k \Delta T. \quad (2.16)$$

The performance of this algorithm is similar to the Mann and Morrison algorithm except that this algorithm is more sensitive to the presence of high frequencies in the signal.

### 2.5.2.2. Fourier algorithm

Ramamoorthy [13] proposed that the fundamental frequency components of voltages and currents be extracted by correlating one cycle of data with sine and cosine waves of the fundamental frequency. Figure 2.10 illustrates the correlation process. This is commonly known as the Fourier technique which can be expressed mathematically as

$$V_r(k) = \frac{2}{m} \sum_{n=0}^{m-1} v_{k+n-m+1} \sin \frac{2\pi n}{m}, \quad (2.17)$$

$$V_i(k) = \frac{2}{m} \sum_{n=0}^{m-1} v_{k+n-m+1} \cos \frac{2\pi n}{m}, \quad (2.18)$$

where:

$V_r$  is the real part of the fundamental frequency phasor,

$V_i$  is the imaginary part of the fundamental frequency phasor and

$m$  is the number of samples per fundamental cycle.

The peak value and the phase angle of the voltage phasor can be determined from Equations 2.17 and 2.18 as follows.

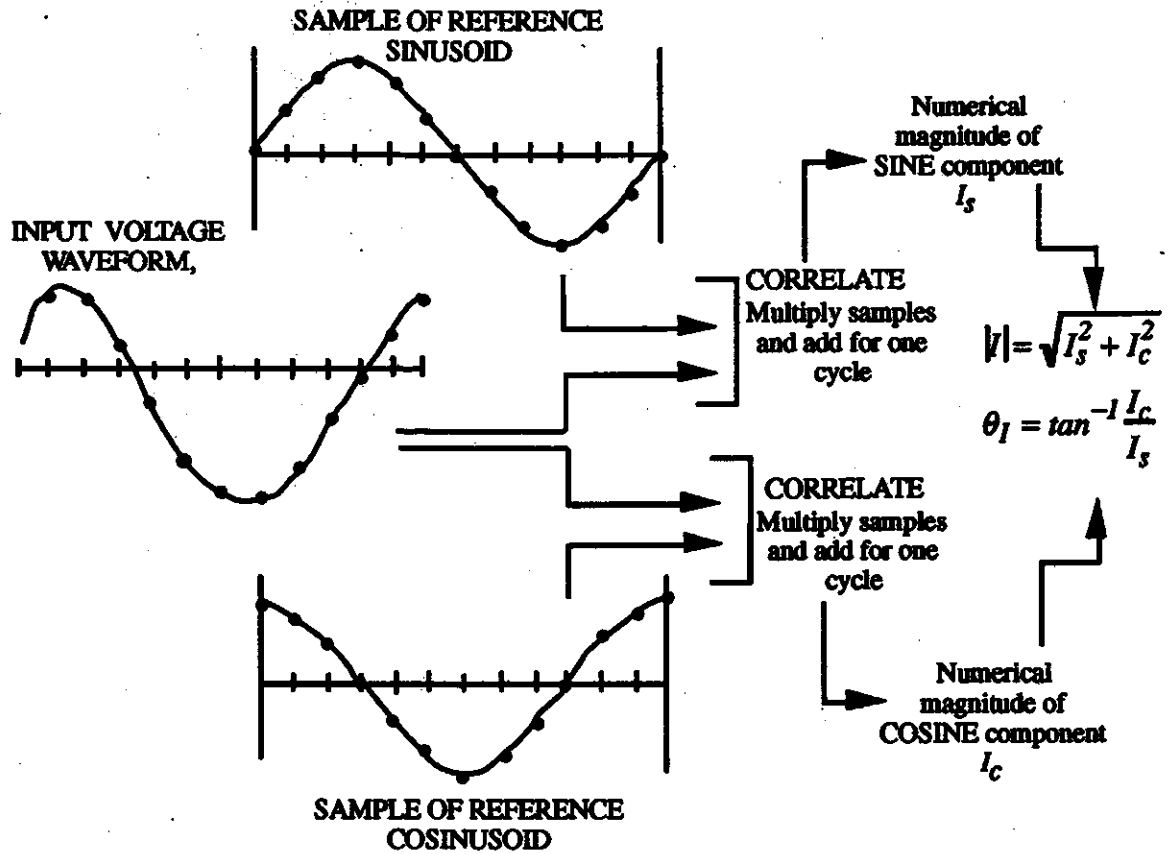
$$V_p = \sqrt{V_r^2 + V_i^2}, \quad (2.19)$$

$$\theta_v = \tan^{-1} \frac{V_i}{V_r}. \quad (2.20)$$

### 2.5.2.3. Rectangular wave algorithm

In this algorithm the signal is correlated with two orthogonal rectangular waveforms shown in Figure 2.11. The waveforms can be expressed by

$$W_r(n) = \text{Signum} \left[ \sin \left( \frac{2\pi n}{m} + \frac{\omega_0 \Delta T}{2} \right) \right], \quad (2.21)$$



**Figure 2.10: Correlation process using the Fourier algorithm [12].**

$$W_i(n) = \text{Signum}| \cos(\frac{2\pi n}{m} + \frac{\omega_0 \Delta T}{2}) |. \quad (2.22)$$

where:

$$\text{Signum}|x| = \begin{cases} 0 & \text{for } x = 0 \\ 1 & \text{for } x > 0 \\ -1 & \text{for } x < 0 \end{cases}$$

The real and imaginary parts of a voltage phasor are given by

$$V_r^{m-1} = \frac{1}{A} \sum_{k=0}^{m-1} v_k W_r(k), \quad (2.23)$$

$$V_i^{m-1} = \frac{1}{A} \sum_{k=0}^{m-1} v_k W_i(k), \quad (2.24)$$

where, A is a scaling factor.

Since the weights,  $W_r$  and  $W_i$ , are either +1, -1 or 0, the real and imaginary parts of the phasors are obtained by performing additions and subtractions only; no multiplications are required.

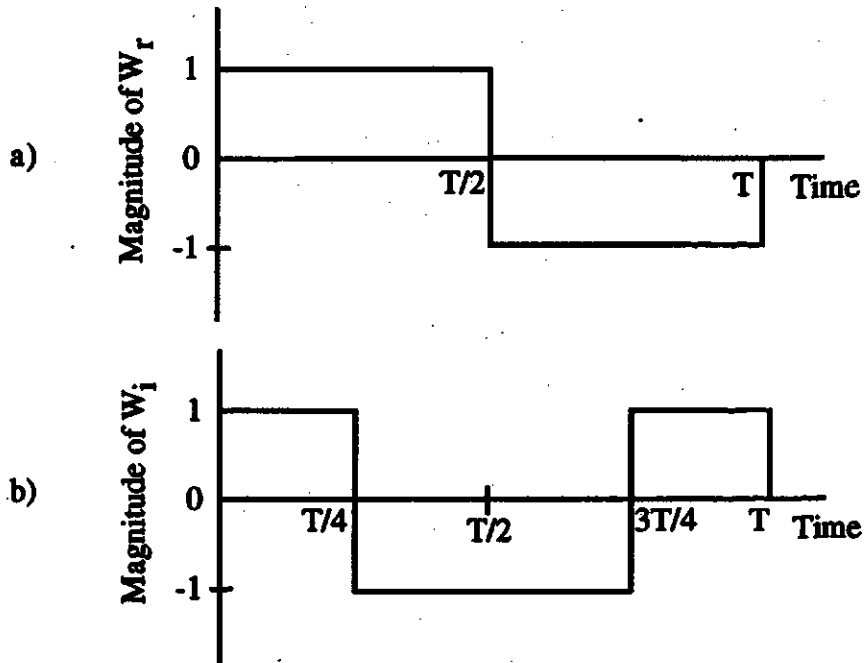


Figure 2.11: Orthogonal rectangular waveforms of the fundamental frequency.

#### 2.5.2.4. Least error squares algorithm

The decaying dc components, which are present in fault currents, adversely affect the performance of the algorithms discussed previously. Luckett *et al.* [14] proposed the use of least error squares (LES) curve fitting for estimating the peak value and phase angle of voltage and current phasors. Sachdev and Baribeau [15] further developed this method and suggested that most of the computations can be done off-line before acquiring the data. They also modeled the waveform as a combination of the fundamental frequency component, a decaying dc component and harmonics of specified orders. They expressed a voltage signal as follows

$$v(t) = V_0 e^{-t/\tau} + \sum_{n=1}^N V_n \sin(n\omega_0 t + \theta_n), \quad (2.25)$$

where:

- $v(t)$  is the instantaneous value of the voltage,
- $\tau$  is the time constant of the decaying dc component,

- N** is the highest order of harmonics present in the signal  
 **$\omega_0$**  is the fundamental frequency of the system,  
 **$V_0$**  is the magnitude of the dc. offset,  
 **$V_n$**  is the peak value of the  $n^{\text{th}}$  harmonic component and  
 **$\theta_n$**  is the phase angle of the  $n^{\text{th}}$  harmonic phasor.

Expanding the decaying dc component by the Taylor series expansion and retaining the first two terms, the following equation was obtained.

$$v(t) = V_0 - (V_0 / \tau)t + \sum_{n=1}^N V_n \sin(n\omega_0 t + \theta_n) \quad (2.26)$$

Assuming that the fourth and higher order harmonics have been removed by filters Equation 2.26 for  $t=t_1$  can be expressed as

$$v(t_1) = V_0 - (V_0 / \tau)t_1 + V_1 \sin(\omega_0 t_1 + \theta_1) + V_2 \sin(2\omega_0 t_1 + \theta_2) + V_3 \sin(3\omega_0 t_1 + \theta_3). \quad (2.27)$$

Using trigonometric identities, this equation can be written as

$$v(t_1) = V_0 - (V_0 / \tau)t_1 + (V_1 \cos \theta_1) \sin \omega_0 t_1 + (V_1 \sin \theta_1) \cos \omega_0 t_1 + (V_2 \cos \theta_2) \sin 2\omega_0 t_1 + (V_2 \sin \theta_2) \cos 2\omega_0 t_1 + (V_3 \cos \theta_3) \sin 3\omega_0 t_1 + (V_3 \sin \theta_3) \cos 3\omega_0 t_1. \quad (2.28)$$

This equation can be expressed in the form

$$v(t_1) = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 + a_{17}x_7 + a_{18}x_8, \quad (2.29)$$

where:

$x_1 = V_0,$	$a_{11} = 1,$
$x_2 = -V_0 / \tau,$	$a_{12} = t_1,$
$x_3 = V_1 \cos \theta_1,$	$a_{13} = \sin \omega_0 t_1,$
$x_4 = V_1 \sin \theta_1,$	$a_{14} = \cos \omega_0 t_1,$
$x_5 = V_2 \cos \theta_2,$	$a_{15} = \sin 2\omega_0 t_1,$
$x_6 = V_2 \sin \theta_2,$	$a_{16} = \cos 2\omega_0 t_1,$
$x_7 = V_3 \cos \theta_3,$	$a_{17} = \sin 3\omega_0 t_1,$
$x_8 = V_3 \sin \theta_3 \text{ and}$	$a_{18} = \cos 3\omega_0 t_1.$



Since the voltage is sampled at intervals of  $\Delta T$  seconds,  $t_1$  can be replaced by  $m\Delta T$ . Redefining the coefficients:

$$\begin{aligned} a_{m1} &= 1 & a_{m3} &= \sin(\omega_0 m\Delta T) & a_{m5} &= \sin(2\omega_0 m\Delta T) & a_{m7} &= \sin(3\omega_0 m\Delta T) \\ a_{m2} &= m\Delta T & a_{m4} &= \cos(\omega_0 m\Delta T) & a_{m6} &= \cos(2\omega_0 m\Delta T) & a_{m8} &= \cos(3\omega_0 m\Delta T). \end{aligned}$$

Therefore, Equation 2.21 can be written as

$$v(m\Delta T) = a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + a_{m4}x_4 + a_{m5}x_5 + a_{m6}x_6 + a_{m7}x_7 + a_{m8}x_8. \quad (2.30)$$

For a pre-selected time reference and sampling rate, the value of 'a' coefficients can be pre-calculated. Assuming that there are n samples and n is greater than 8, the following equation is obtained.

$$\begin{matrix} [A] & [x] & = & [v], \\ n \times 8 & 8 \times 1 & n \times 1 & \end{matrix} \quad (2.31)$$

It can be shown that the following provides the least squares estimate of  $[x]$ .

$$\begin{aligned} [x] &= [[A^T] [A]]^{-1} [A^T] [v]. \\ &= [A]^+ [v] \end{aligned} \quad (2.32)$$

where  $[A]^+$  is the left pseudo-inverse of  $[A]$ .

Sachdev and Baribeau [60] showed that the elements of  $[A]^+$  can be determined in the off-line mode.

## 2.6. Summary

A typical distribution system and systems needed for its protection have been described in this chapter. The limitations of time and current graded protection schemes have been discussed. Advantages of using inverse time overcurrent relays have been outlined. Directional and instantaneous overcurrent relays have also been briefly mentioned. The organization of a typical digital relay and digital algorithms used in them have also been presented.

### **3. ADAPTIVE PROTECTION OF A DISTRIBUTION SYSTEM**

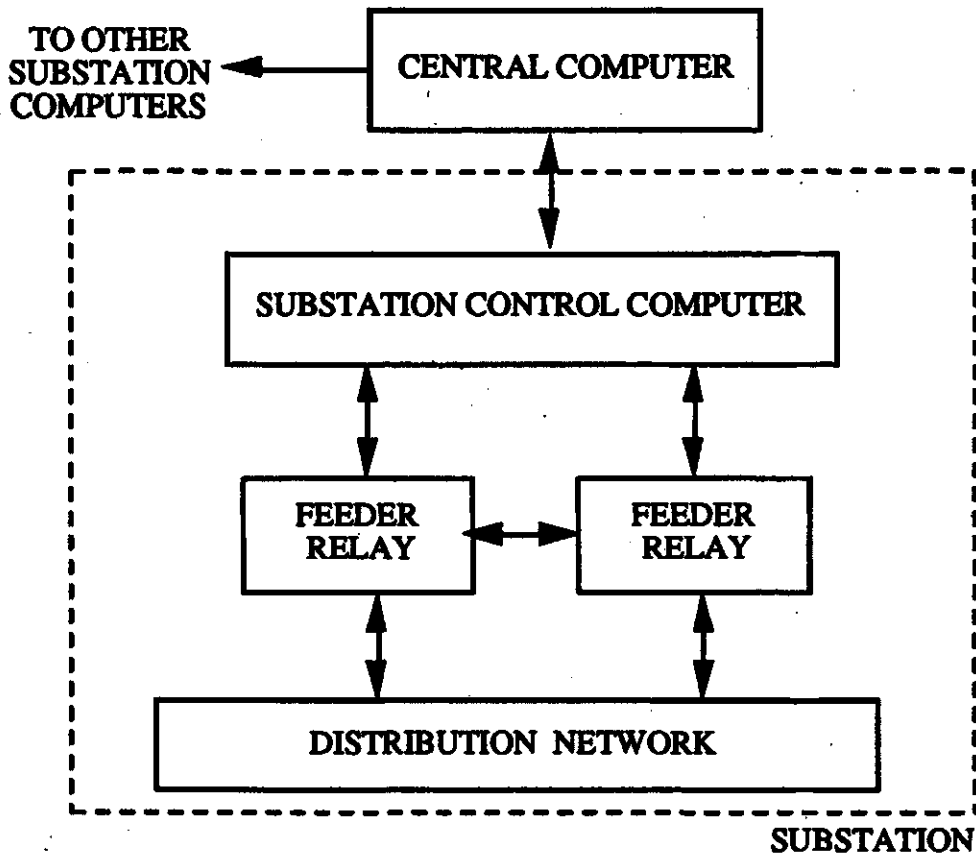
#### **3.1. Introduction**

The conventional approaches to protect distribution systems are presented in Chapter 2. Advantages and disadvantages of conventional approaches to protect distribution networks are also outlined. To overcome some of the problems associated with those approaches, an adaptive approach is proposed. This approach, called adaptive protection, is proposed to be an on-line activity. The objective is to determine the optimal settings for the relays in real time and implement them.

Sachdev and Wood [16] described the concept of adaptive protection of distribution networks using digital relays. Horowitz *et al.* [6] and Rockefeller *et al.* [7] introduced the concept of adaptive relaying for transmission systems. Shah *et al.* [17] described the technical feasibility and economic viability of an adaptive protection scheme for a distribution network. They also outlined other functions that an adaptive scheme can perform. Chan [9] described the automation for future distribution systems, which will eventually require more reliable protection schemes. Jampala *et al.* [18] described some software aspects of adaptive transmission protection. They demonstrated that the time required to determine coordinated relay settings is an important factor in the successful implementation of an adaptive protection scheme. Sachdev *et al.* [19, 20, 21, 22, 23, 24] investigated the feasibility of adaptive overcurrent protection for a distribution system and developed the hardware and software necessary to implement the scheme in the laboratory.

#### **3.2. Adaptive Relaying Scheme**

Figure 3.1 shows the functional block diagram of an adaptive overcurrent protection scheme for a distribution network. During normal operation, the relays monitor and analyze the currents and voltages and send the information to the substation control computer at regular intervals. The relays also provide the status of the isolators and circuit breakers to the station control computer. The substation control computers interface with the feeder relays on the one side and a central control computer on the other side.



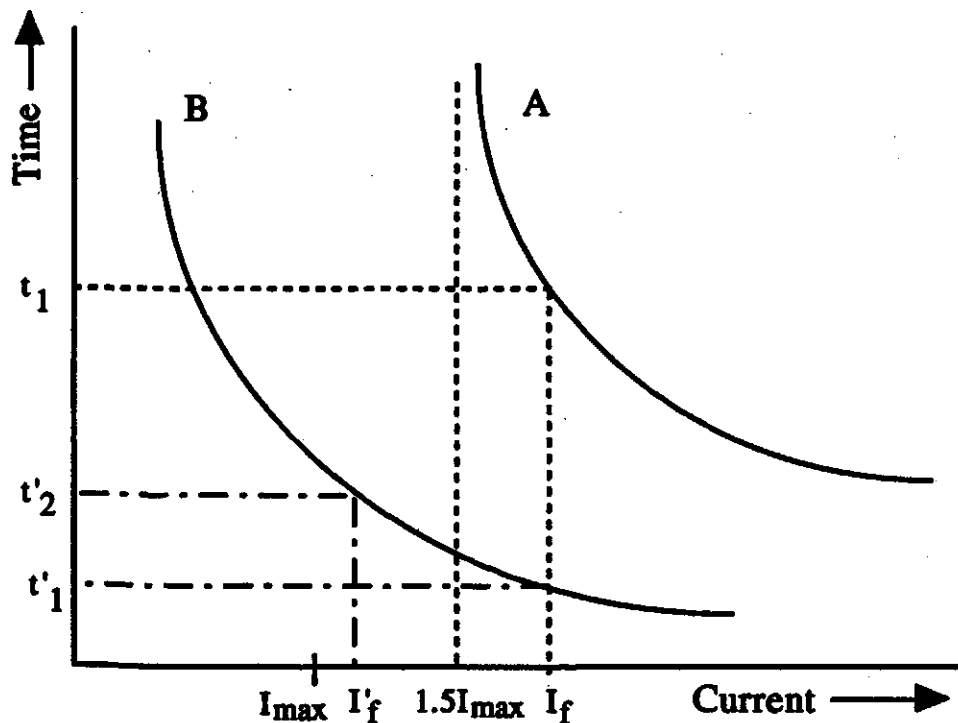
**Figure 3.1:** An adaptive distribution protection scheme.

The substation control computers pass on, at pre-specified intervals, the information received from the relays to the central control computer. The central control computer estimates the system state and determines if it is necessary to change the settings of the relays. In case it is found to be necessary to change the settings, the new settings are calculated and are conveyed to the relays via the substation computers. If the central control computer finds that there is no need to change the settings, the decision is conveyed to the relays for sharing the information as well as to confirm that the communication system is functioning properly.

### **3.2.1. Basic adaptive relaying concept**

The adaptive feature of the operation of a relay can be illustrated with the help of a typical time-current characteristic shown in Figure 3.2. In general, a phase relay is set at 150% to 200% of the maximum load current,  $I_{\max}$  in the figure. The relay must be set to operate as shown by the time current characteristic 'A'. For a fault current  $I_f$ , the relay

will operate after a time delay of  $t_1$  after the inception of the fault. Due to changes in the system operating condition or system topology, the magnitude of fault current can change. Let the changed magnitude of fault current be  $I'_f$  that is more than  $I_{max}$  but less than  $1.5I_{max}$ . As Figure 3.2 shows, the relay will not operate for this fault. An adaptive protection system, monitoring the feeder current, could change the relay setting so that its operating characteristics corresponds to 'B'. The relay will now operate after a time delay of  $t'_2$  after the inception of the fault. Also, for the larger fault current the relay will operate after the time delay of  $t'_1$  which is less than the time  $t_1$ . This example shows that optimum relay settings can be achieved by adaptively changing the settings as the system state changes.



**Figure 3.2:** Typical time-current characteristics of overcurrent relays.

### **3.3. Purpose of Adaptive Distribution Protection**

The primary purpose of an adaptive protection scheme is to provide reliable, fast and selective protection of all components of the distribution system. To achieve these objectives, the system voltages at all the buses and currents in the circuits are constantly monitored by the substation control computers. Information is exchanged between different devices via communication channels. The system state is then estimated by the central control computer. In addition to adaptive protection, the system could perform

1. optimal control of feeder loads, transformers, reactors and capacitors,
2. cold load pick up and
3. reclosing of circuit breakers and reclosers.

### **3.4. Functions of an Adaptive Protection System**

The principal components of an adaptive protection scheme for a distribution system are digital relays, substation control computers, central control computer and communication links. Besides protecting the distribution system components, an adaptive protection scheme can perform some other functions as well. Some of those functions are listed here.

1. Periodic and automatic self diagnosis.
2. Monitoring of
  - a) currents,
  - b) voltages,
  - c) status of protection equipment,
  - d) overload of transformers,
  - e) thermal conditions of distribution circuits and
  - f) status of reclosers and capacitors.
3. Control of
  - a) reclosing sequences,
  - b) load management,
  - c) feeder/lateral circuit load balancing and
  - d) capacitors.

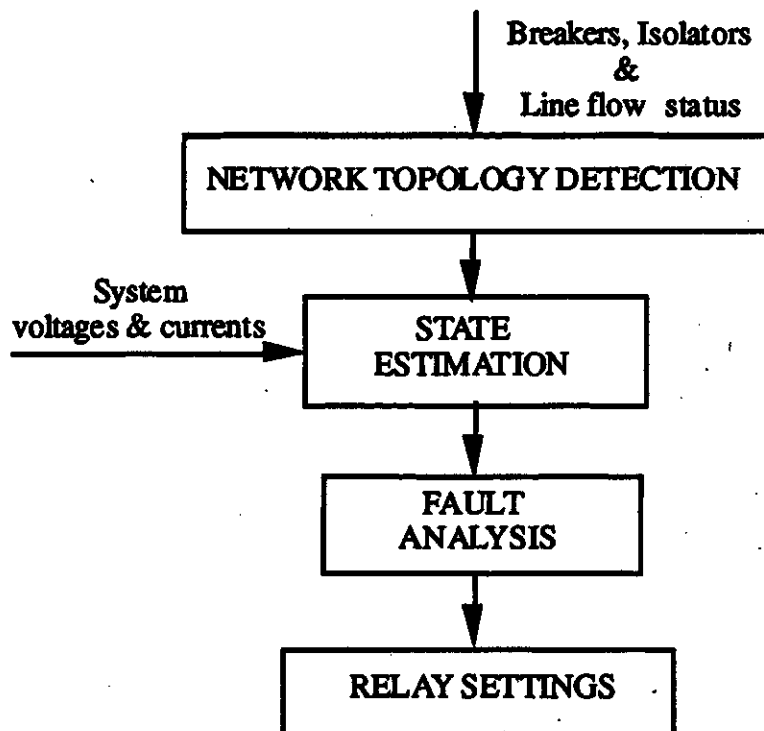
#### 4. Load shedding and system restoration.

### 3.5. Software for an Adaptive Protection Scheme

As mentioned previously, the objective of an adaptive protection scheme is to modify the relay settings necessitated by changes in expected fault currents and/or changes in system operating conditions. The changes in the system operating condition may be due to loss or addition of generation, loss or addition of loads or changes in the topology of the network. One of the objectives of an adaptive protection scheme is to keep the time delays of primary and backup relays to as low levels as might be possible.

To achieve these objectives three major software packages are included in an adaptive protection scheme. These are

1. relay modeling software,
2. relay setting and coordination software and
3. communication software.



**Figure 3.3:** A functional block diagram for a relay coordination software.

One of the objectives of this project, as mentioned in Chapter 1, was to study the feasibility of adaptive protection of ground fault directional overcurrent relays. Since the major task is the on-line coordination of relays, appropriate software modules are needed. The functional block diagram for the required software, outlined in Figure 3.3, includes

1. a network topology detection module,
2. a state estimation module,
3. a fault analysis module and
4. a relay setting and coordination module.

The development of the network topology detection module is discussed in the next chapter. The algorithm and software for state estimation, fault analysis and relay setting modules are discussed in Chapter 5.

### **3.6. Summary**

This chapter describes the basic concept of an adaptive protection scheme which can not only protect the power system equipment, but can also perform other important tasks. Software modules necessary to implement an adaptive protection scheme for a distribution network are also briefly outlined in this chapter.

## **4. NETWORK TOPOLOGY DETECTION**

### **4.1. Introduction**

One of the objectives of this research was to develop a topology detection technique and to use it for implementing an adaptive protection scheme.

There are several publications which report the use of network topology in monitoring and control of power systems. Couch and Morrison [25] described a method for analyzing switch status data to determine the configuration of a transmission system and then to verify, from other data, that there had been no inconsistencies. Dy Liacco *et al.* [26] presented a method of network status analysis for use in a control center. They also discussed the underlying design concepts. Dy Liacco and Kraynak [27] described a technique which detects incorrect operations and failures of protective relays from the system configuration, and the history of circuit breaker and relay operations. A large part of the information available for detecting network topology is, however, redundant.

Paris and Bose [28] described a technique that uses circuit breaker status and network connectivity data to determine the system topology. This technique works in two phases, initialization and tracking. In the initialization phase, it processes the status of every breaker and determines the initial topology. In the tracking phase, it processes only those substations in which the status of circuit breakers changed since the topology was last checked. These processors are fast but require additional computer memory for storing the data pertaining to topology changes. They are, however, not in common use at this time [29].

Sasson *et al.* [30] described a scheme, which was implemented in the American Electric Power (AEP) Service Corporation, for determining the topology of the AEP's transmission network. The inputs to the program are the status of circuit breakers and current flows in the lines. This technique is used in the research reported in this thesis, except that some of the data is processed in substation control computers. This modification reduced the work load of the central control computer.



Some of the on-line functions, performed in power systems, for which it is necessary to know the network configuration are

1. automatic substation switching,
2. substation interlocking,
3. optimal sequence of switching operations,
4. switching after tripping of circuit breakers,
5. state estimation,
6. security analysis and
7. adaptive protection.

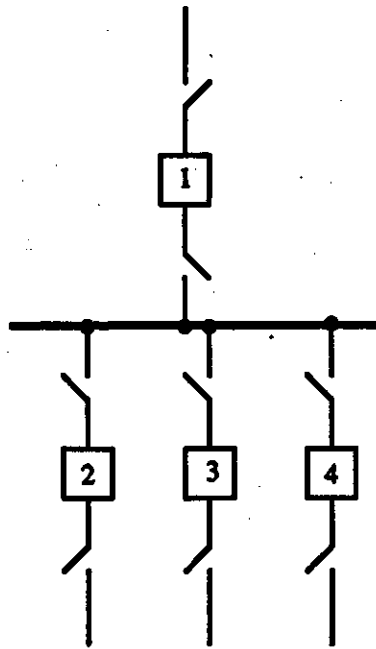
For the work reported in this thesis, a topology processing procedure was developed. It generated information that is needed to perform the functions listed in Section 3.4. Specifically, it was intended to determine the network topology for coordinating relays in adaptive protection for distribution networks.

## **4.2. Substation Configuration**

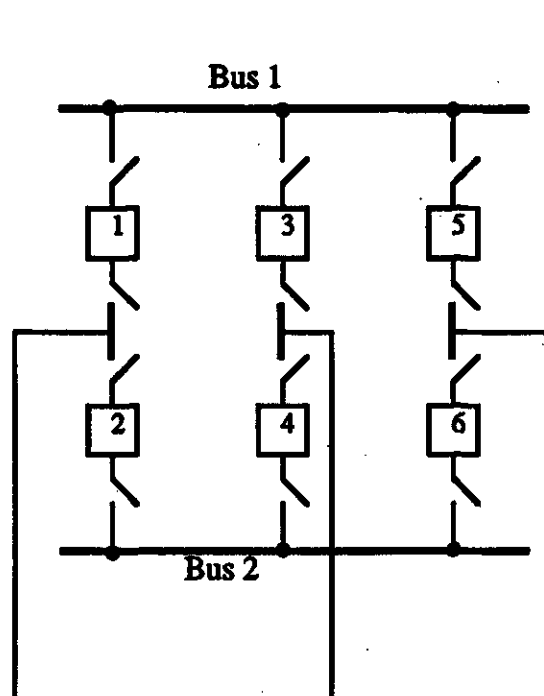
The complexity of the task of determining the configuration of a substation depends on the configuration of the substation. Also, if a line is protected by dedicated circuit breakers, the status of the circuit breakers provide the topology [22] but, unfortunately, lines are not always equipped with dedicated circuit breakers. The following are the most commonly used substation configurations.

1. Single bus, single breaker scheme
2. Double bus, double breaker scheme
3. Main and transfer bus scheme
4. Double bus, single breaker scheme
5. Breaker-and-a-half scheme and
6. Ring bus scheme.

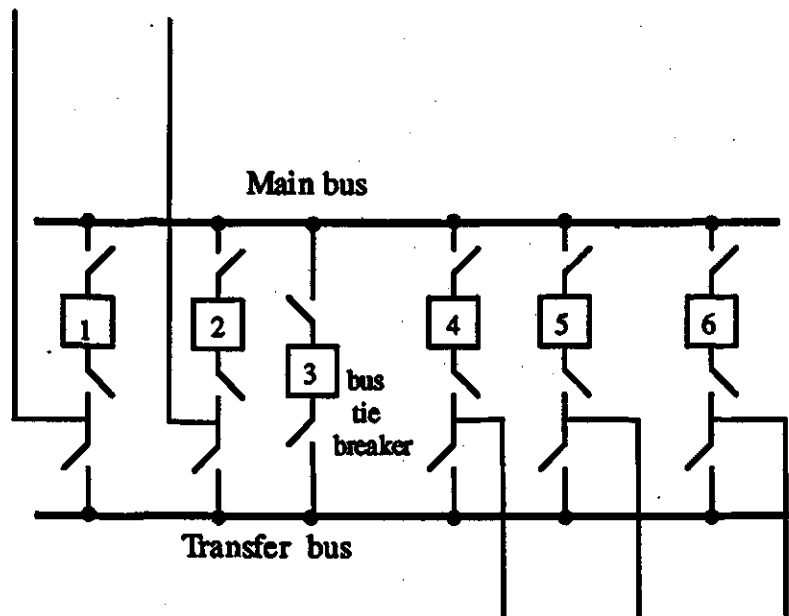
Typical single line diagrams for these schemes are shown in Figures 4.1 through 4.6.



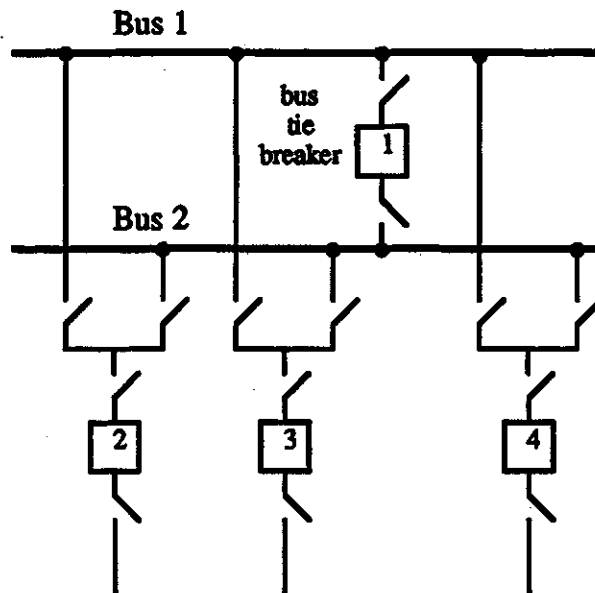
**Figure 4.1:** A typical single bus, single breaker scheme.



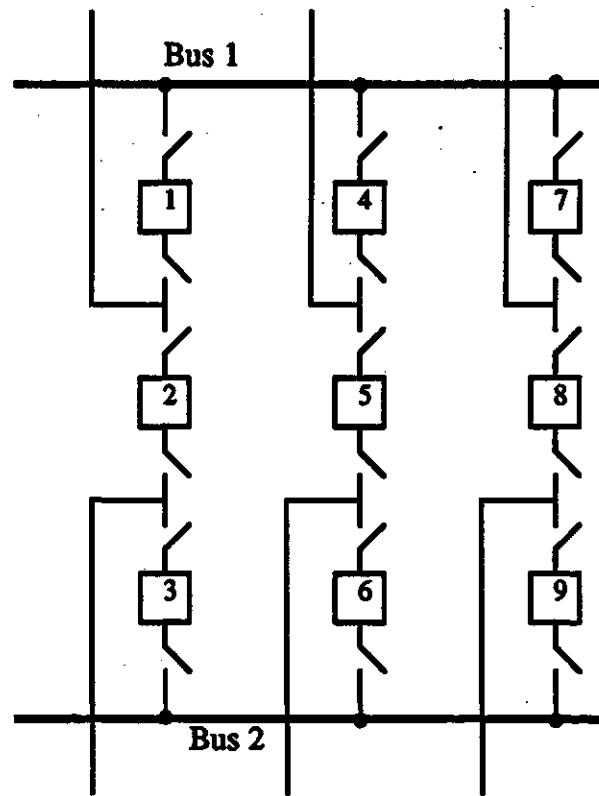
**Figure 4.2:** A typical double bus, double breaker scheme.



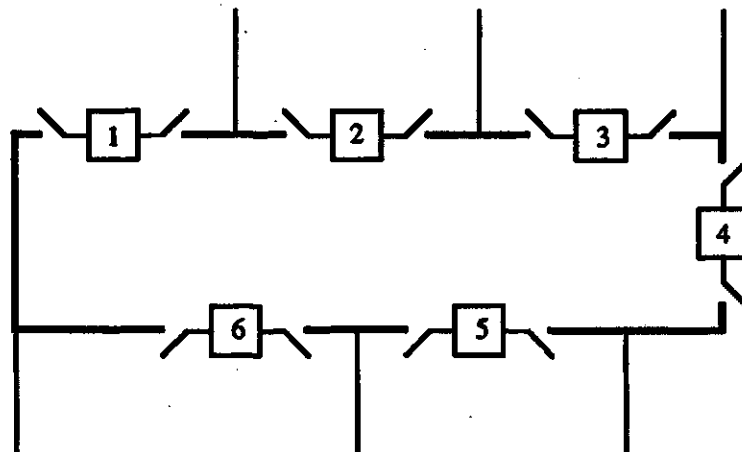
**Figure 4.3:** A typical main and transfer bus scheme.



**Figure 4.4:** A typical double bus, single breaker scheme.



**Figure 4.5:** A typical breaker-and-a-half scheme.



**Figure 4.6:** A typical ring bus scheme.

### **4.3. The Topology Detection Technique**

Since the proposed adaptive protection scheme has substation control computers, in addition to a central control computer, it was decided that the topology of each substation be determined by its substation control computer. It was also decided that the system-wide topology be then determined by the central control computer from the information generated by the substation control computers.

#### **4.3.1. Basic functions**

The topology analysis technique performs the following basic functions.

1. Develop the logic to process the status of circuit breakers and isolators independent of the substation configurations.
2. Identify the changes of the status of circuit breakers, isolators and line flows due to the opening or closing of switches.
3. Subdivide a substation into two or more nodes, to account for parts of the substation separated by tie breakers.
4. Subdivide the distribution network to find the line node connectivity.
5. Provide the line-node connectivity data, as well as load and generation data, to the load flow program.

The functional block representation of the topology detection technique is shown in Figure 4.7. During initialization, the technique reads the off-line input data that are described in a later section. When there is a change of status of a circuit breaker or an isolator, the substation control computer analyzes it and finds out if the substation configuration has changed or not. The substation computers then update the configuration and send the information along with the information on line flows and changes of load and generation to the central control computer. The central control computer processes the information and finds out the new line-node connectivities of the system and updates the inputs to a load flow program.

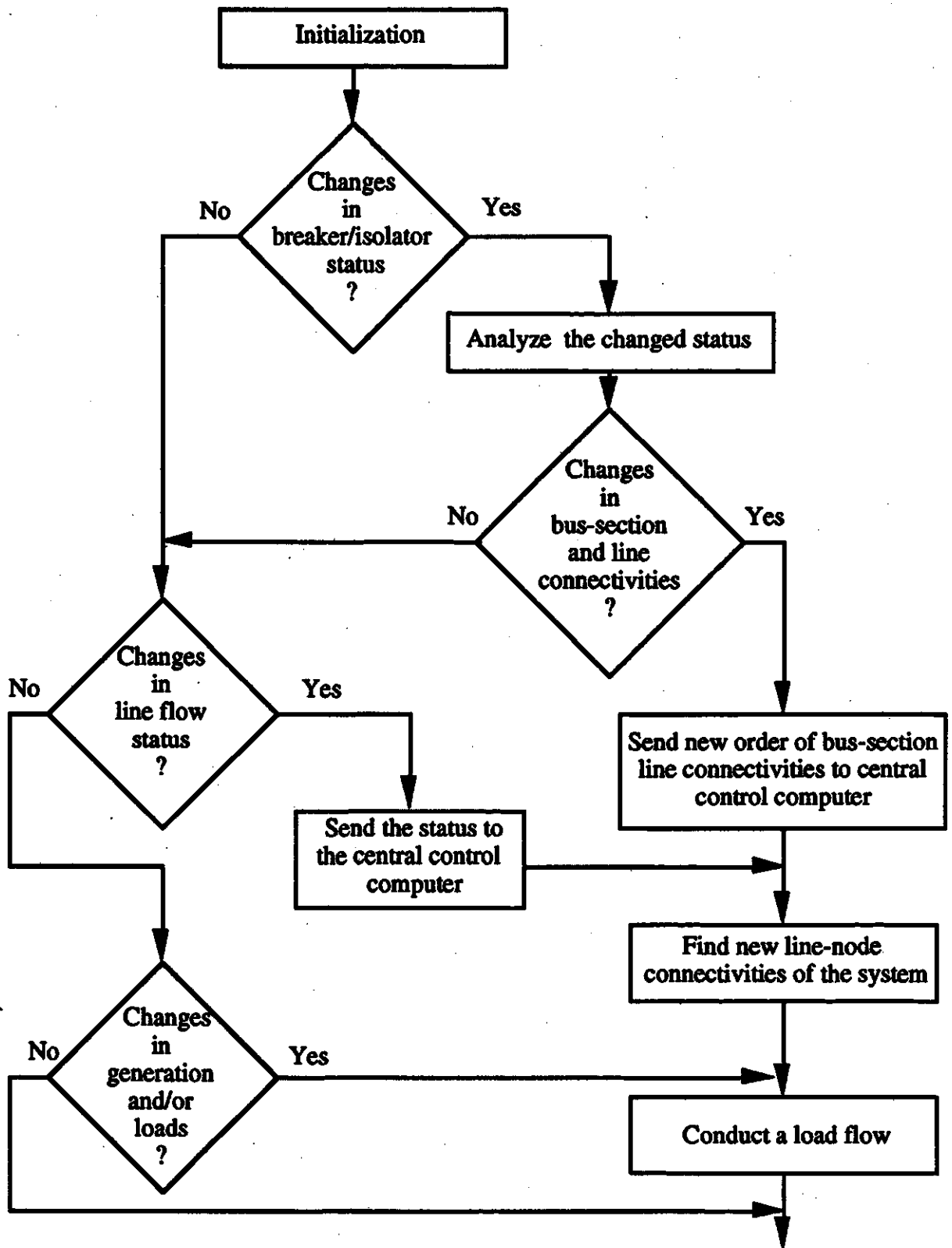


Figure 4.7: Structure of topology detection module.

### 4.3.2. Definitions

The following definitions of terms are adopted for describing the technique.

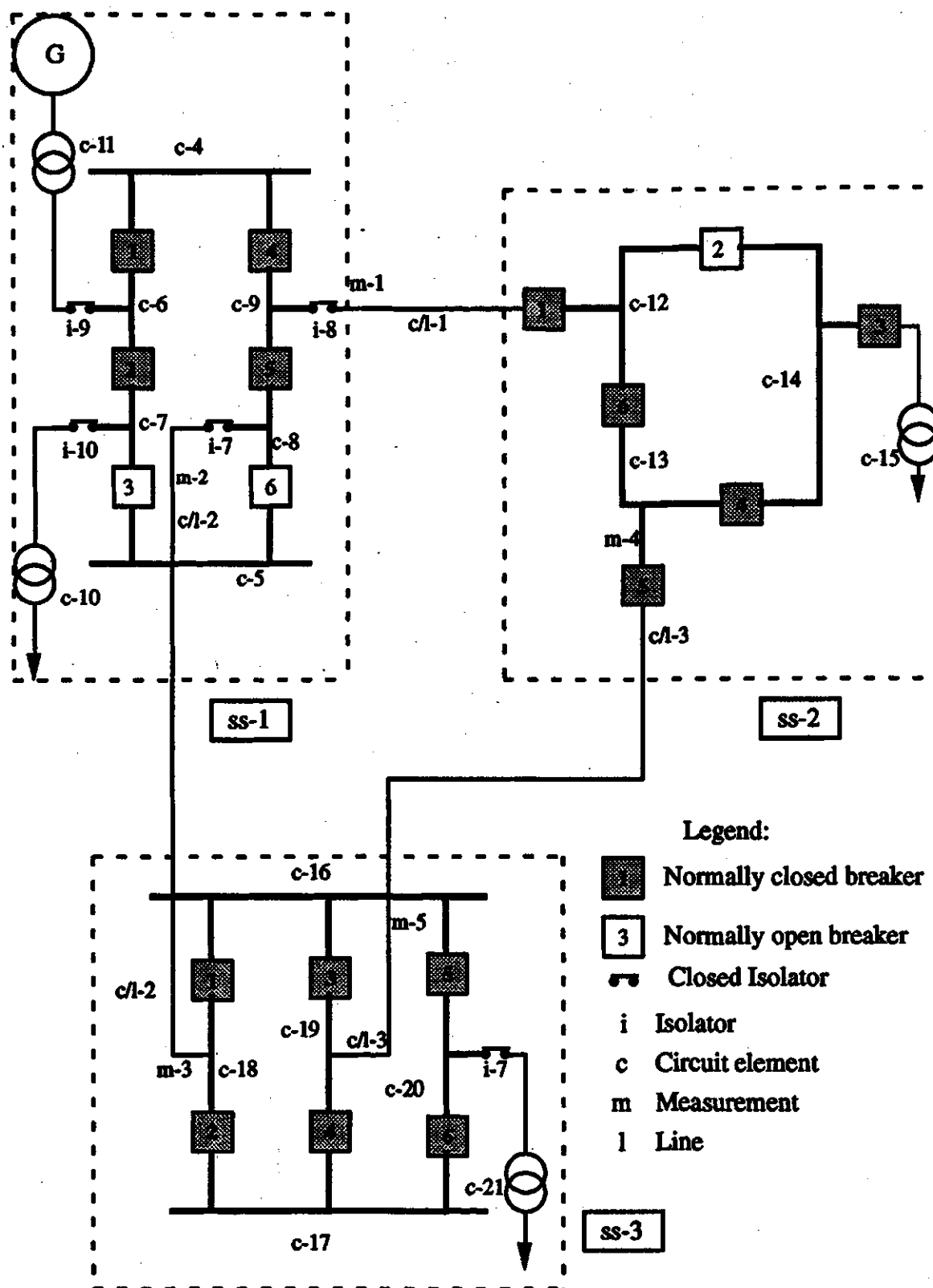
- Node:** A substation is considered as a node when all the buses or bus sections are connected by closed circuit breakers and isolators.
- Sub-node:** A sub-node is a junction, outside a substation where two or more lines meet.
- Circuit element:** A circuit element is any node, sub-node, bus, bus-section, line, transformer, or generator, that is electrically connected to a breaker or an isolator.
- Breaker status:** The circuit breaker status is '1' if it is open and '0' if it is closed.
- Isolator status:** An isolator status is '1' if it is open , and is '0' if it is closed.
- Line flow status:** The line flow status is '1' if there is no line flow, and '0' otherwise.

### 4.3.3. The technique

The topology detection technique is developed in the following five parts.

**I. Off-line data:** The information concerning all circuit breakers, isolators, buses and bus-sections in a substation are required. Information on circuit elements, and current flows in the circuit elements, is also required. The off-line data are placed in a single file. The details of preparing the off-line data are explained in the following paragraph by applying it to a 3-bus network shown in Figure 4.8.

**Circuit breaker data:** The circuit breaker data for the model system is shown in Table 4.1. There are four columns for writing the breaker data. Columns 1 & 2 identify the two circuit elements connected to a breaker/isolator; the third column identifies the substation where the breaker/isolator is located; the fourth column records the normal status of the breaker/isolator. The total number of circuit breakers and isolators for each substation is also provided as an input. In the model system, there are six circuit breakers and four isolators in Substation 1. Similarly, six circuit breakers and one isolator are in Substation 3 and six circuit breakers are in Substation 2.



**Figure 4.8:** A model network for illustration of topology input data



**Line flow data:** It is assumed that line flow status is recorded at one end, or at both ends of each line. The line flow status are included in the off-line data. The data, shown in Table 4.2 for the network of Figure 4.8, are arranged in three columns. Column 1 records the identity of the substation where the line flow is checked. Column 2 records the identity of the far end substation. The third column identifies the normal status of the line flow. Initially it is assumed that all the lines are available and have current flows and, therefore, their flow status are '0'.

**Table 4.1:** Circuit breaker and isolator data for the system.

From ckt. element no.	To ckt. element no.	At sub-stn. no.	Normal status
4	6	1	0
6	7	1	0
7	5	1	1
4	9	1	0
9	8	1	0
8	5	1	1
9	1	1	0
8	2	1	0
6	11	1	0
7	10	1	0
1	12	2	0
12	14	2	0
13	14	2	0
12	13	2	0
13	3	2	0
14	15	2	0
16	18	3	0
18	17	3	0
16	19	3	0
19	17	3	0
16	20	3	0
20	17	3	0
20	21	3	0

**Table 4.2: Line flow status data for the system.**

Line flow status no.	Near sub-stn. no.	Far sub-stn no.	Availability
1	1	2	0
2	1	3	0
3	3	1	0
4	2	3	0
5	3	2	0

**Index data:** For each line, two indices are required; they indicate the ends of the line where the flow status are checked. A '0' entry in the index table means that the line flow status at that end of the line is not considered. For the model network of Figure 4.8, the index data are presented in Table 4.3.

**Bus type:** The following notations for bus type are assigned: 1 for PV bus, 2 for PQ bus and 3 for slack bus.

**Table 4.3: Index data for the system.**

Line no.	Index I	Index II
1	1	0
2	2	3
3	4	5

**II. Circuit breaker/isolator status:** The changes in breaker status and isolator status are processed by the central control computer from the information supplied by the substation control computers. A change is first compared to the normal status of the breaker/isolator. If the status is different from the normal status, the change is processed. The procedure consists of the following steps.

1. Create a list of the substations where changes in breaker/isolator status have taken place.

2. For each substation in the list, compare the new status of each breaker/isolator with the normal status. If the normal status is the same as the new status, check the status of the next breaker/isolator.
3. If the new status is different from the normal status of the breaker/isolator, check the new status. If the status is '0' (closed), take the next breaker or isolator and go to step 2.
4. If the status is '1' (open) take the circuit element of one side of the open breaker/isolator and add to a list of circuit elements.
5. Check connectivity of all other circuit elements of the substation with this circuit element. Include in the list, the circuit elements that are electrically connected to the circuit element identified in Step 4.
6. If the circuit element at the other end of the breaker/isolator is included in the list, go to the next step. Otherwise include the element in a separate list, check connectivity with other elements and update the list.
7. If all the circuit breakers/isolators of the substation are in their normal states after the changes, remove the substation from the substation list.

In the model system of Figure 4.8, let us assume that, circuit breaker 5 in Substation 1, has changed from closed to open and circuit breaker 3 has changed from open to close. For these changes, the procedure used to update the topology is illustrated below.

A substation list is created. Let this list be called STN\_LIST. The status of circuit breaker number 3 is changed. The new status of the circuit breaker is different from the normal status; the status is '0', (closed). So, the next circuit breaker is taken for analysis, which is circuit breaker number 5. The new status of this circuit breaker is also different from the normal status; it is open. Substation 1 is added to STN\_LIST. The circuit element number 9 is at one side of the circuit breaker number 5. This circuit element is added to a list, say CKT\_LIST1. Circuit element number 4 is connected to circuit element number 9 by closed circuit breaker number 4. So circuit element number 4 is added to CKT\_LIST1. Similarly, circuit element number 6 is connected to circuit element number 4 by closed circuit breaker number 1. So circuit element number 6 is added to CKT\_LIST1. Thus, all the circuit elements that have connectivity with circuit element number 9 are added to CKT\_LIST1. Next, circuit element number 8, which is at the other side of the circuit breaker number 5, is taken for processing. This circuit element is not present in CKT\_LIST1. Therefore, this circuit element is added to another list, CKT\_LIST2. Circuit element number 2 is connected to circuit element number 8 by closed isolator number 7.

So circuit element number 2 is added to CKT\_LIST2. There are no other circuit elements that are connected to circuit element number 8. All circuit breaker/isolator status changes have been accounted for. Steps 3 through 6 for Substation 1 are now complete. Since the status of switches at all substations in STN\_LIST have been checked, the processing is complete. The processing has provided the following three lists, their contents are shown in parenthesis.

STN\_LIST=(1)  
 CKT\_LIST1=(1,9,4,6,11,7,10,5)  
 CKT\_LIST2=( 8,2)

**III. Line flow status:** The changes of line flow status are processed in this module. Initially, the status of each line flow in the system is assumed to be '0', i.e., available. The status are entered into the substation computer in real time and then are sent to the central control computer via communication links. If the line flow status at one end of a line is '1', i.e., unavailable, and line flow status is taken only at that end of the line, the status of the line is assumed to be open. If line flow status is taken at both ends of the line and status is unavailable only at one end of the line, the line is assumed to be closed for further analysis.

**IV. Updating configuration:** The previous modules provided lists of substations associated with changes of the status of circuit breakers/isolators and lists of circuit elements connected by closed circuit breakers/isolators and status of line flows. This information is used to update the configuration of the system. Three different possibilities can arise.

1. The lines, designated as disconnected have been actually disconnected by the operation of circuit breakers/isolator or have been so designated due to the unavailability of the status of their line flows.
2. A substation has been split into two or more distinct parts introducing one or more new nodes in the system.
3. New lines have been added since the configuration was previously updated .

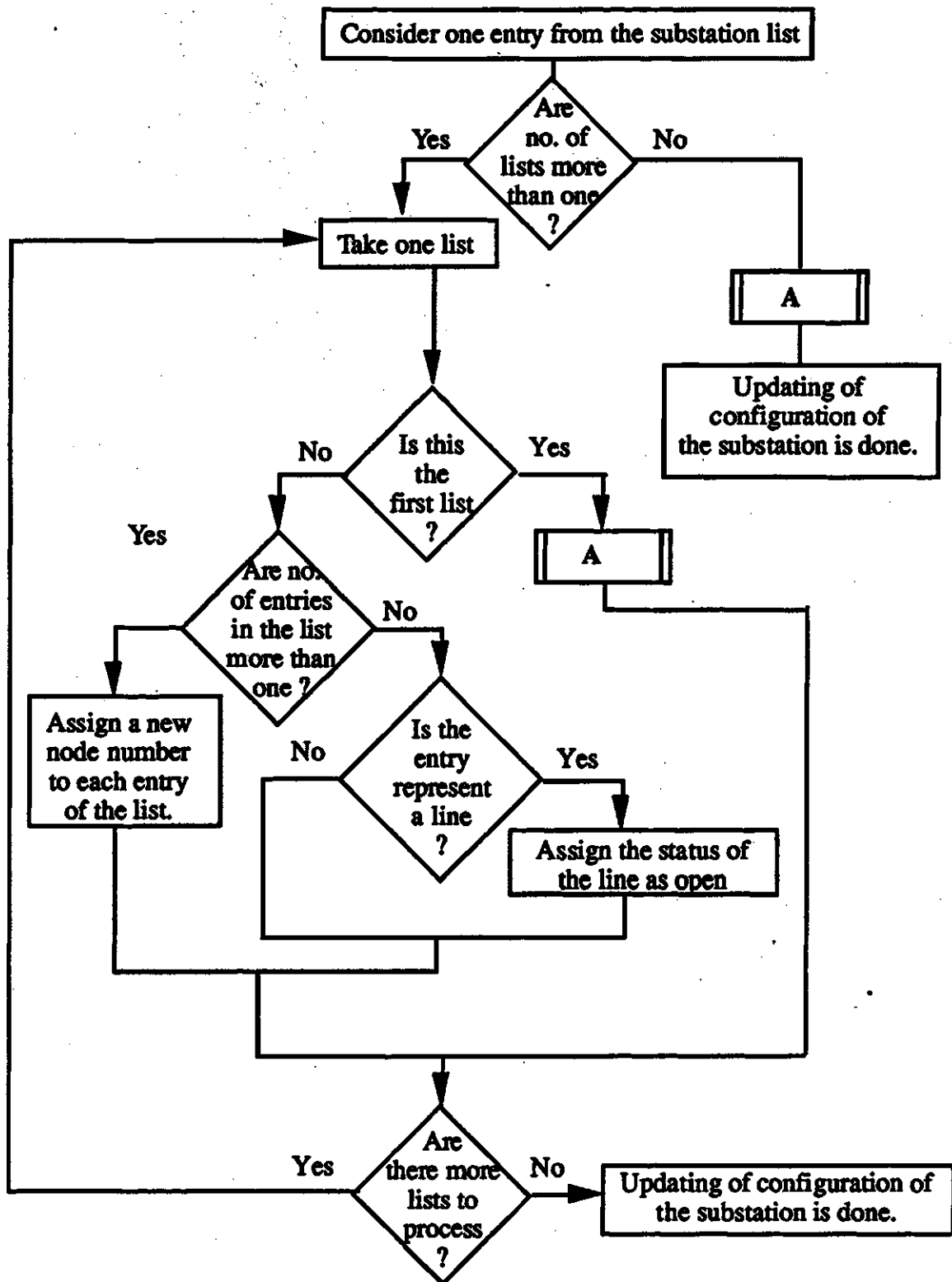
These possibilities are taken into consideration in the following procedure which determines the configuration of the system. The sequential steps of the procedure are shown in Figure 4.9.

1. Check the number of lists (of circuit elements) which corresponding to a substation. If there are two or more lists go to Step 2. If there is only one list, the substation has not been split. Perform the following steps to analyze the list.
  - a) Check the number of entries in the list of circuit elements. Only one line entry in the list indicates that the line has been opened. Assign the status of the line as open. If the entry is not a line, do not take any action.
  - b) If there are multiple entries in the list of circuit elements, do not take any action because the changes of the status of circuit breakers/isolator did not change the line node connectivities of the substation.
2. The presence of multiple lists of circuit elements for a substation, indicates that the substation may have been split into two or more parts. For the first list of circuit elements, Steps 1a and 1b are performed. For the remaining lists the following steps are carried out.
  - a) The number of entries of the list are checked. If there are multiple entries proceed to Step 2b. If there is only one entry and it is a line, assign the line as open. If the entry is not a line, take no action.
  - b) Introduce a new node number that will correspond to each entry of the circuit element list because the substation has been split into two or more distinct parts.

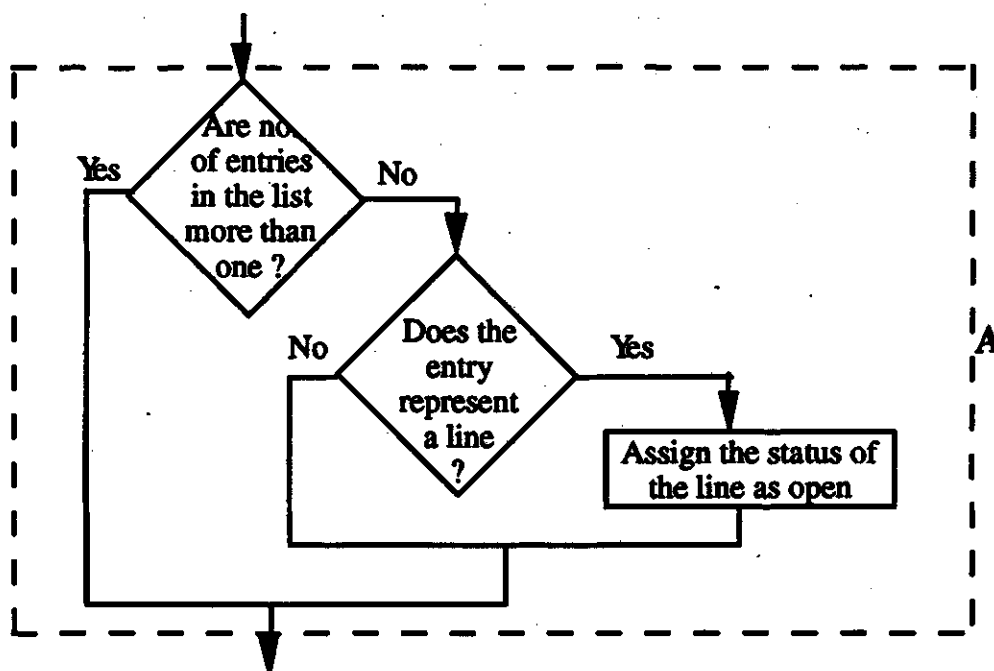
**V. System processing:** The output of Subsections III and IV are processed in this module. Following three possibilities are investigated.

1. Have some lines been opened ?
2. Have some substations been split into two or more parts introducing additional nodes in the system ?
3. Has the system been split into two or more subsystems ?

Situation 1 and 2 would have been handled by the procedure described in subsection IV. To check for the possibility of the system having been split, the entire system is considered as one substation. Lines are considered as circuit breakers and the substations or the nodes as buses or bus sections. The logic used to analyze the substation, is then applied to the system. The steps described in subsection II are carried out. The central control computer creates the lists by processing the system information. If there is only one list, it is inferred that the system has not been split but if there are multiple lists, it



**Figure 4.9:** Flow diagram representing the configuration update.



**Figure 4.9: Continued.**

is concluded that the system has been split into one or more subsystems.

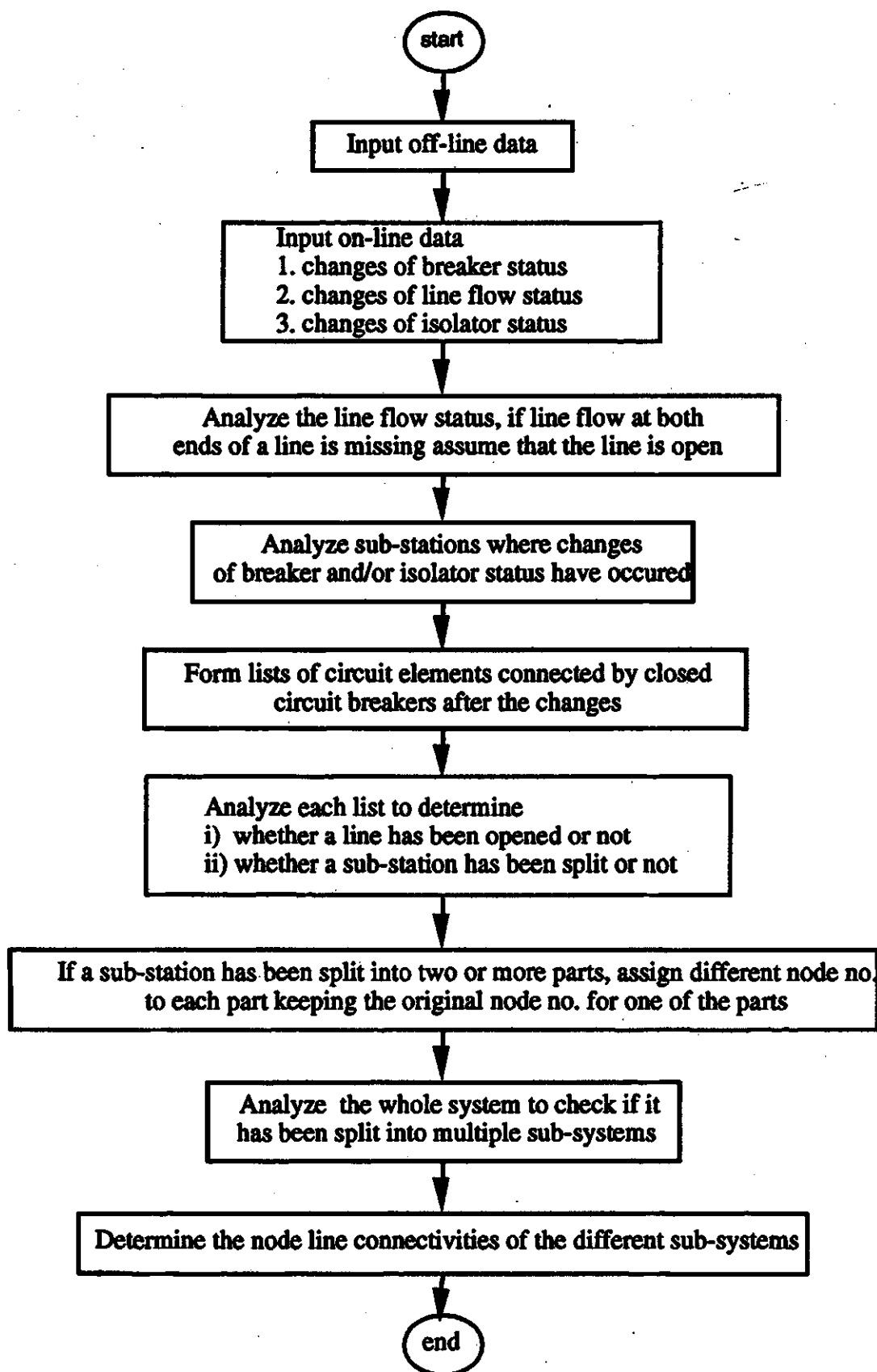
Line-node connectivities of the subsystems are obtained from each list. The lines, whose status is open, do not appear in this list. The output list also indicates the buses on which the voltage is measured. This information is used for on-line load flow analysis.

The flow diagram of the overall program is shown in Figure 4.10.

#### 4.4. Testing

To investigate the performance of the developed technique for topology detection, three test systems were considered. The first system has six substations and eleven lines. The second system is a model distribution network of the city of Saskatoon. The third system is the IEEE reliability test system. This system has 24 buses interconnected by 38 lines; it has 321 circuit breakers and isolators. The single line diagrams and the input data for the three systems are presented in Appendix A.

The developed technique was tested using the three systems. Three sets of operations of circuit breakers and/or isolators were considered for each system. The system configurations as well as the test results obtained for each case are also presented in Appendix A. The results indicate that the developed technique works properly.



**Figure 4.10:** Program flow diagram for topology detection module.



Adaptive protection is an on-line activity. It is, therefore, necessary that each software module should execute in minimum possible time. The execution times needed to identify the topology for the selected states were computed. These are also included in Appendix A.

#### **4.5. Summary**

The details of the topology detection software have been presented in this chapter. Substation configurations that have to be taken into account are discussed. The data that are provided to the software have been identified. The software was tested by applying it to three networks. Three operating states of each system were used for the tests. The results show that the software works properly. The input data and results are presented in Appendix A.

## **5. STATE ESTIMATION, FAULT ANALYSIS AND RELAY SETTING TECHNIQUES**

### **5.1. Introduction**

This chapter describes the development of the coordination software for updating relay settings as the operating state of a system changes. The software is divided in four modules; the first module determines the topology of the network; the second module estimates the operating state of the system; the third module conducts short circuit studies; and the fourth module determines the properly coordinated relay settings. The development of the topology detection module has been presented in the previous chapter; the development of the remaining three modules is presented in this chapter.

For state estimation of the system, the fast decoupled load flow technique [31] was used. Since, the decoupled load flow technique is a modification of the Newton Raphson method of load flow analysis, both techniques are presented. The faults that are usually experienced in a power system, are briefly discussed. The Z-bus matrix technique [31] used to conduct short circuit studies is presented. The coordination algorithm, which used an optimization technique, for determining appropriate settings of relays is also presented.

### **5.2. State Estimation**

The state estimation problem consists of calculating line flows and bus voltages in a network. For determining the relay settings, only line currents are needed but bus voltages are needed to conduct short circuit studies.

To perform a load flow study, the power system is assumed to be balanced. Six operating parameters are associated with each bus, the real power and reactive power generation and load, and the amplitude and phase angle of the bus voltage. Each system bus is assigned one of the three classifications for performing a load flow study; slack bus where amplitude and phase angle of the voltage are specified, voltage control bus where real power and magnitude of the voltage are specified, and load bus where real and reactive power loads are specified.

The mathematical formulation of the load flow problems results in algebraic non-linear equations, mostly using the bus frame of reference approach [31]. The solution of the algebraic equations is obtained using an iterative technique. The convergence is checked by comparing the real and reactive powers flowing into a bus with the real and reactive powers flowing out of the bus.

The network performance equation, is expressed as

$$I_{bus} = Y_{bus} V_{bus}, \quad (5.1)$$

where:

$I_{bus}$  is the vector of currents injected into the buses,  
 $V_{bus}$  is the vector of bus voltages and  
 $Y_{bus}$  is the bus admittance matrix.

The real and reactive powers at a bus, say p, can be expressed as

$$P_p - jQ_p = V_p^* I_p, \quad (5.2)$$

and the current injected into the bus,  $I_p$ , is

$$I_p = \frac{P_p - jQ_p}{V_p^*}. \quad (5.3)$$

After a solution is obtained, line flows are calculated using

$$I_{pq} = (V_p - V_q) y_{pq} + V_p \frac{y'_{pq}}{2}, \quad (5.4)$$

where:

$y_{pq}$  is the line admittance and  
 $y'_{pq}$  is the line charging admittance.

The power flow from bus p to bus q is

$$P_{pq} - jQ_{pq} = V_p^* (V_p - V_q) y_{pq} + V_p^* V_p \frac{y'_{pq}}{2}, \quad (5.5)$$

and power flow from bus q to bus p is

$$P_{qp} - jQ_{qp} = V_q^* (V_q - V_p) y_{pq} + V_q^* V_q \frac{y_{pq}}{2}. \quad (5.6)$$

### 5.2.1. Newton Raphson load flow technique

In this section, the Newton Raphson method for load flow studies is presented briefly. Equation 5.1 can be written for bus p of an n-bus system as

$$I_p = Y_{p1}V_1 + Y_{p2}V_2 + \dots + Y_{pp}V_p + \dots + Y_{pn}V_n,$$

$$\text{or} \quad I_p = \sum_{q=1}^n Y_{pq}V_q. \quad (5.7)$$

where:

$Y_{pp}$  is the self admittance and  
 $Y_{pq}$  is the transfer admittance.

Substituting Equation 5.7 in Equation 5.2, gives

$$P_p - jQ_p = V_p^* \sum_{q=1}^n Y_{pq}V_q. \quad (5.8)$$

The voltages at bus p and bus q can be expressed as

$$\begin{aligned} V_p &= |V_p|e^{j\delta_p} \quad \text{then} \quad V_p^* = |V_p|e^{-j\delta_p}, \\ V_q &= |V_q|e^{j\delta_q} \quad \text{and} \quad Y_{pq} = |Y_{pq}|e^{j\theta_{pq}}, \end{aligned} \quad (5.9)$$

where  $\delta$  is the phase angle of the bus voltage and  $\theta_{pq}$  is the angle of the admittance  $Y_{pq}$ .

Substituting Equation 5.9 in Equation 5.8 provides

$$P_p - jQ_p = \sum_{q=1}^n |V_p V_q Y_{pq}| e^{j(\theta_{pq} - \delta_p + \delta_q)}. \quad (5.10)$$

The real and reactive power injections can, therefore, be expressed as

$$P_p = \sum_{q=1}^n |V_p V_q Y_{pq}| \cos(\theta_{pq} - \delta_p + \delta_q), \quad (5.11)$$

$$\text{and} \quad Q_p = - \sum_{q=1}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} - \delta_p + \delta_q). \quad (5.12)$$

This formulation results in a system of non-linear equations. For a load bus the real and reactive power are specified and for a voltage controlled bus real power and magnitude of the voltage are specified. For the slack bus the magnitude and phase angle of the voltage are specified and are kept fixed throughout the iteration process. If there are  $m$  number of load buses for an  $n$  bus system, the total number of equations to be solved for a load flow solution are  $(n+m-2)$ . The Newton Raphson method requires that a set of linear equations be formed expressing the relationships between the changes in real and reactive powers and the components of bus voltages. The linear set of equations are expressed as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V|/|V| \end{bmatrix}, \quad (5.13)$$

where:

$J$ 's are the Jacobian submatrices,

$\Delta \delta$  is the vector of increments in phase angle and

$\Delta |V|$  is the vector of increments in the voltage magnitudes.

An iterative solution of the equations is initialized by estimating an initial solution vector  $[\delta^0 \ |V^0|]^T$  and expanding each function in the neighborhood of this estimate using Taylor series as illustrated in Appendix B.

The mismatches of the real and reactive powers at bus  $p$  are defined as

$$\Delta P_p = P_p(\text{specified}) - P_p(\text{calculated}), \quad (5.14)$$

$$\Delta Q_p = Q_p(\text{specified}) - Q_p(\text{calculated}). \quad (5.15)$$

### 5.2.2. The decoupled load flow technique

The decoupled load flow technique is much faster than other load flow techniques and is, therefore, suitable for use in a real time application, such as adaptive protection of a distribution network. This technique recognizes that the changes in real power flows are less sensitive to the changes in voltage magnitudes, and changes in reactive power flows are less sensitive to the changes of phase angles of bus voltages. The decoupling is achieved by neglecting the submatrices  $J_2$  and  $J_3$  in Equation 5.13. The procedure provides the following equation replacing  $J_1$  and  $J_4$  by  $H$  and  $L$  respectively.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V|/|V| \end{bmatrix} \quad (5.16)$$

The following approximations are also made.

$$\begin{aligned} \cos(\delta_p - \delta_q) &\cong 1 \\ G_{pq} \sin(\delta_p - \delta_q) &\ll B_{pq} \\ Q_{pq} &\ll B_p |V_p^2| \end{aligned} \quad (5.17)$$

With these approximations, the elements of Jacobian matrices become

$$\begin{aligned} H_{pq} = L_{pq} &= -|V_p| |V_q| B_{pq} \text{ for } q \neq p \text{ and} \\ H_{pp} = L_{pp} &= -B_{pp} |V_p^2|. \end{aligned} \quad (5.18)$$

Equation 5.16 takes the form

$$[\Delta P] = [V_p] [V_q] [B'_{pq}] [\Delta \delta] \text{ and} \quad (5.19)$$

$$[\Delta Q] = [V_p] [V_q] [B''_{pq}] \frac{\Delta |V|}{|V|}. \quad (5.20)$$

$B'_{pq}$  and  $B''_{pq}$  are elements of  $[-B]$  matrix.

### 5.2.2.1. Decoupled load flow algorithm

The following steps are performed to implement the decoupled load flow technique.

1. For each load bus, where P & Q are specified, a bus voltage magnitude and a phase angle are assumed.  $|V|$  and  $\delta$  for the slack bus are specified. Voltage magnitudes at the voltage controlled buses are also specified and phase angles are assumed to be  $0^\circ$ . Flat voltage profile, i.e. 1.0 voltage magnitudes and  $0^\circ$  phase angles, are assumed for all load buses.
2. P and Q for each bus are calculated using Equation 5.11 and 5.12.
3. The mismatches for each bus using,  $\Delta P^k$  and  $\Delta Q^k$ , are calculated as follows.

$$\Delta P^k = P(\text{scheduled}) - P^k,$$

$$\Delta Q^k = Q(\text{scheduled}) - Q^k.$$

where  $k$  is the iteration counter.

4. The representation of the network elements that affect MVAR flows are omitted from  $B'$ .
5. The phase angle shifts due to the phase shifters are omitted from  $B''$ .
6. The elements of the Jacobian submatrices  $H$  and  $L$  are calculated using the latest estimates of bus voltages and phase angles. Power flows in all lines and transformers are calculated.
7. Equation 5.19 and 5.20 are solved to determine the voltage corrections,  $\Delta|V|$ 's and  $\Delta\delta$ 's.  $[L]-[U]$  factorization and backward and forward substitution procedures are used for this purpose.
8. Bus voltages are updated.
9. The new estimates of bus voltages are used to calculate power flows and mismatches.
10. If  $[\Delta P / V]$  and  $[\Delta Q / V]$  for each bus are within a specified limit, the load flow is done otherwise steps 6 to 10 are repeated.
11. The line flows, generator outputs and line losses are calculated.

### 5.3. Fault Analysis

Faults at critical locations of the system are considered. The analysis provides voltages at the system buses and currents in the lines, transformers and generators. This information is used to determine the relay settings. A brief description of the types of faults experienced in power systems is presented in this section. Since objective of this thesis is to coordinate ground relays, mathematical formulation, criteria and assumptions used for analyzing single phase to ground faults are presented.

#### 5.3.1. Balanced faults

Power system faults are divided into two broad categories: balanced faults and unbalanced faults. Three phase and three phase to ground faults are balanced faults. Three phase faults are the most severe faults; they are studied to determine the ratings of circuit interrupting devices and for setting overcurrent relays. The method of analyzing three phase faults is described briefly.

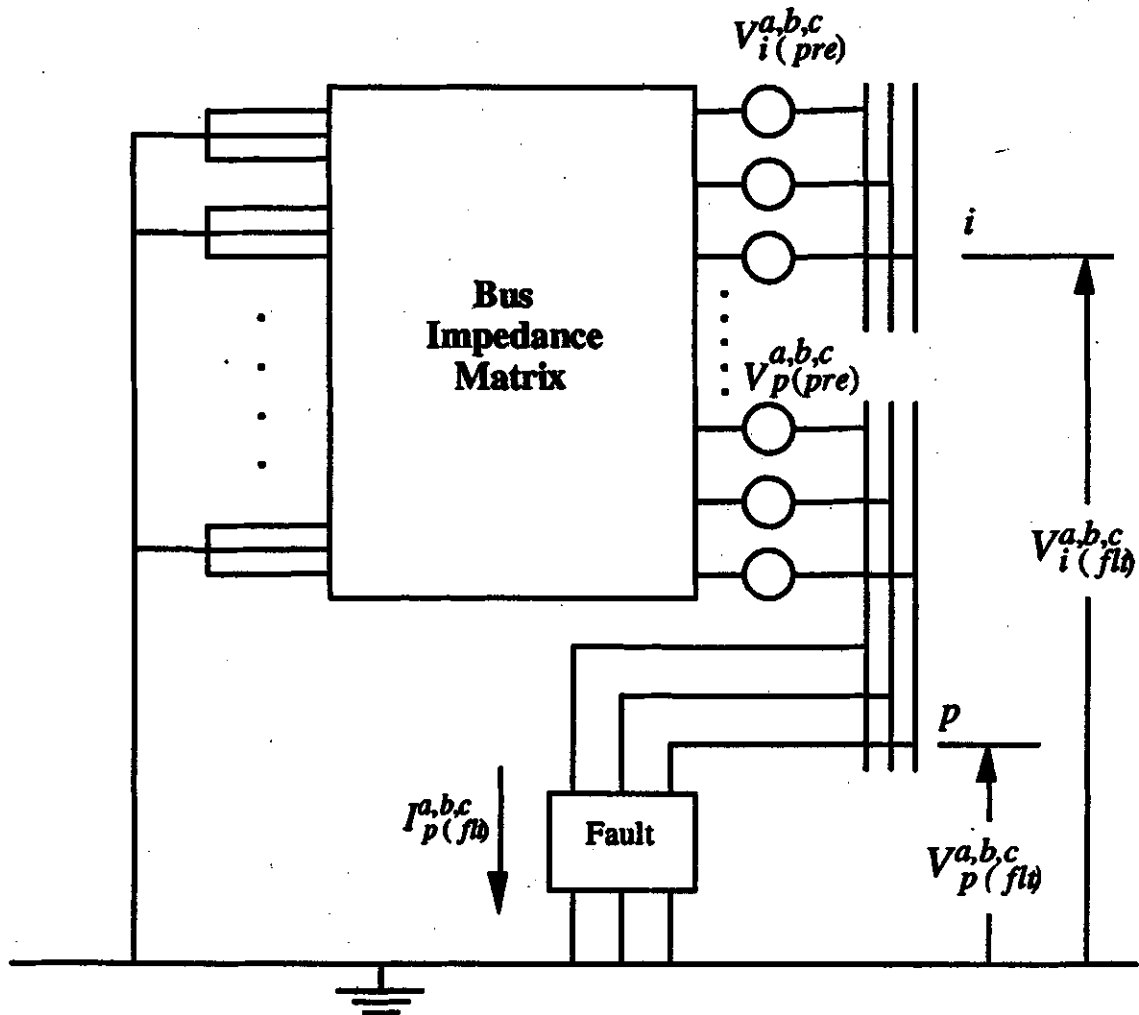


Figure 5.1: Three phase representation of power system for fault at bus p.

Figure 5.1 shows the Thevenin equivalent of a power system experiencing a fault at bus p. The Z-bus-matrix algorithm [31] is used to form the bus impedance matrix. The impedances of the system elements are placed at appropriate locations of the bus impedance matrix. The pre-fault voltage at bus p is the voltage that drives the current through the fault.

The circuit equations of a power system during a fault can be expressed as

$$V_{bus}^{a,b,c}(flt) = V_{bus}^{a,b,c}(pre) - Z_{bus}^{a,b,c} I_{bus}^{a,b,c}(flt), \quad (5.21)$$

where:

$V_{bus}^{a,b,c}(flt)$  is the vector of voltages after the inception of the fault,  
 $V_{bus}^{a,b,c}(pre)$  is the vector of prefault bus voltages,



$I_{bus}^{a,b,c}(ft)$  is the vector of fault currents; all elements except  $p^{th}$  elements of this vector are zero and  
 $Z_{bus}^{a,b,c}$  is the three phase bus impedance matrix.

From Equation 5.21, the voltage at the fault is determined using the equation

$$V_p^{a,b,c}(ft) = V_p^{a,b,c}(pre) - Z_{pp}^{a,b,c} I_p^{a,b,c}(ft). \quad (5.22)$$

The fault current is obtained by

$$I_p^{a,b,c}(ft) = Y_{ft}^{a,b,c} V_p^{a,b,c}(ft), \quad (5.23)$$

where  $Y_{ft}^{a,b,c}$  is the three phase admittance matrix for the faulted system.

From Equations 5.22 and 5.23, the voltage at bus  $p$  after the occurrence of the fault is obtained by

$$V_p^{a,b,c}(ft) = V_p^{a,b,c}(pre) - Z_{pp}^{a,b,c} Y_{ft}^{a,b,c} V_p^{a,b,c}(ft). \quad (5.24)$$

Equations 5.23 and 5.24 can be rewritten as

$$I_p^{a,b,c}(ft) = Y_{ft}^{a,b,c} [ U + Z_{pp}^{a,b,c} Y_{ft}^{a,b,c} ]^{-1} V_p^{a,b,c}(pre). \quad (5.25)$$

$$V_p^{a,b,c}(ft) = [ U + Z_{pp}^{a,b,c} Y_{ft}^{a,b,c} ]^{-1} V_p^{a,b,c}(pre). \quad (5.26)$$

Voltages at the system buses are obtained by

$$V_i^{a,b,c}(ft) = V_i^{a,b,c}(pre) - Z_{ip}^{a,b,c} Y_{ft}^{a,b,c} [ U + Z_{pp}^{a,b,c} Y_{ft}^{a,b,c} ]^{-1} V_p^{a,b,c}(pre) \quad i \neq p. \quad (5.27)$$

### 5.3.2. Unbalanced faults

Most faults in a power system are unbalanced, either bolted or with extended arcs. They can be classified as

1. single line to ground (L-G) faults,
2. line to line (L-L) faults and
3. double line to ground (L-L-G) faults.

As unbalanced faults make the system unbalanced, the method of symmetrical components is used to analyze them. Transforming Equations 5.25 through 5.27 from phase quantities to their symmetrical components provides

$$I_p^{0,1,2}(ft) = Y_{ft}^{0,1,2} [U + Z_{pp}^{0,1,2} Y_{ft}^{0,1,2}]^{-1} V_p^{0,1,2}(pre), \quad (5.28)$$

$$V_p^{0,1,2}(ft) = [U + Z_{pp}^{0,1,2} Y_{ft}^{0,1,2}]^{-1} V_p^{0,1,2}(pre), \text{ and} \quad (5.29)$$

$$V_i^{0,1,2}(ft) = V_i^{0,1,2}(pre) - Z_{ip}^{0,1,2} [U + Z_{pp}^{0,1,2} Y_{ft}^{0,1,2}]^{-1} V_p^{0,1,2}(pre). \quad (5.30)$$

The most frequently occurring unbalanced faults are of the single line to ground (L-G) type [3]. The mathematical formulation and algorithm for analyzing these faults follow.

### 5.3.2.1. Single line to ground faults

Equations 5.28 through 5.30 are modified to calculate the fault currents in the lines and voltages at the system buses. The fault admittance matrix for a single phase to ground fault is expressed as follows.

$$Y_{ft}^{0,1,2} = \frac{y_F}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \quad (5.31)$$

where:

$$y_F = \frac{1}{z_F} \text{ and}$$

$z_F$  is fault impedance from phase to ground.

The derivation of the fault admittance matrix is presented in Appendix C.

Substituting Equation 5.31 in Equations 5.28 through 5.30 the following equations for currents and voltages during a fault are obtained.

$$I_p^0(ft) = I_p^1(ft) = I_p^2(ft) = \frac{V_p(pre)}{Z_{pp}^0 + 2Z_{pp}^1 + 3z_F}, \quad (5.32)$$

$$\begin{bmatrix} V_p^0(ft) \\ V_p^1(ft) \\ V_p^2(ft) \end{bmatrix} = \frac{V_p(pre)}{Z_{pp}^0 + 2Z_{pp}^1 + 3z_F} \begin{bmatrix} -Z_{pp}^0 \\ Z_{pp}^0 + 2Z_{pp}^1 + 3z_F \\ -Z_{pp}^1 \end{bmatrix}, \quad (5.33)$$

$$\begin{bmatrix} V_i^0(fault) \\ V_i^1(fault) \\ V_i^2(fault) \end{bmatrix} = \begin{bmatrix} 0 \\ V_i(pre) \\ 0 \end{bmatrix} - \frac{V_P(pre)}{Z_{PP}^0 + 2Z_{PP}^1 + 3Z_F} \begin{bmatrix} Z_{ip}^0 \\ Z_{ip}^1 \\ Z_{ip}^1 \end{bmatrix} \quad (5.34)$$

The postfault branch currents are obtained using the following equation.

$$\begin{bmatrix} I_{ij}^0(fault) \\ I_{ij}^1(fault) \\ I_{ij}^2(fault) \end{bmatrix} = \begin{bmatrix} y_{ij}^0 (V_i^0(fault) - V_j^0(fault)) \\ y_{ij}^1 (V_i^1(fault) - V_j^1(fault)) \\ y_{ij}^2 (V_i^2(fault) - V_j^2(fault)) \end{bmatrix} \quad (5.35)$$

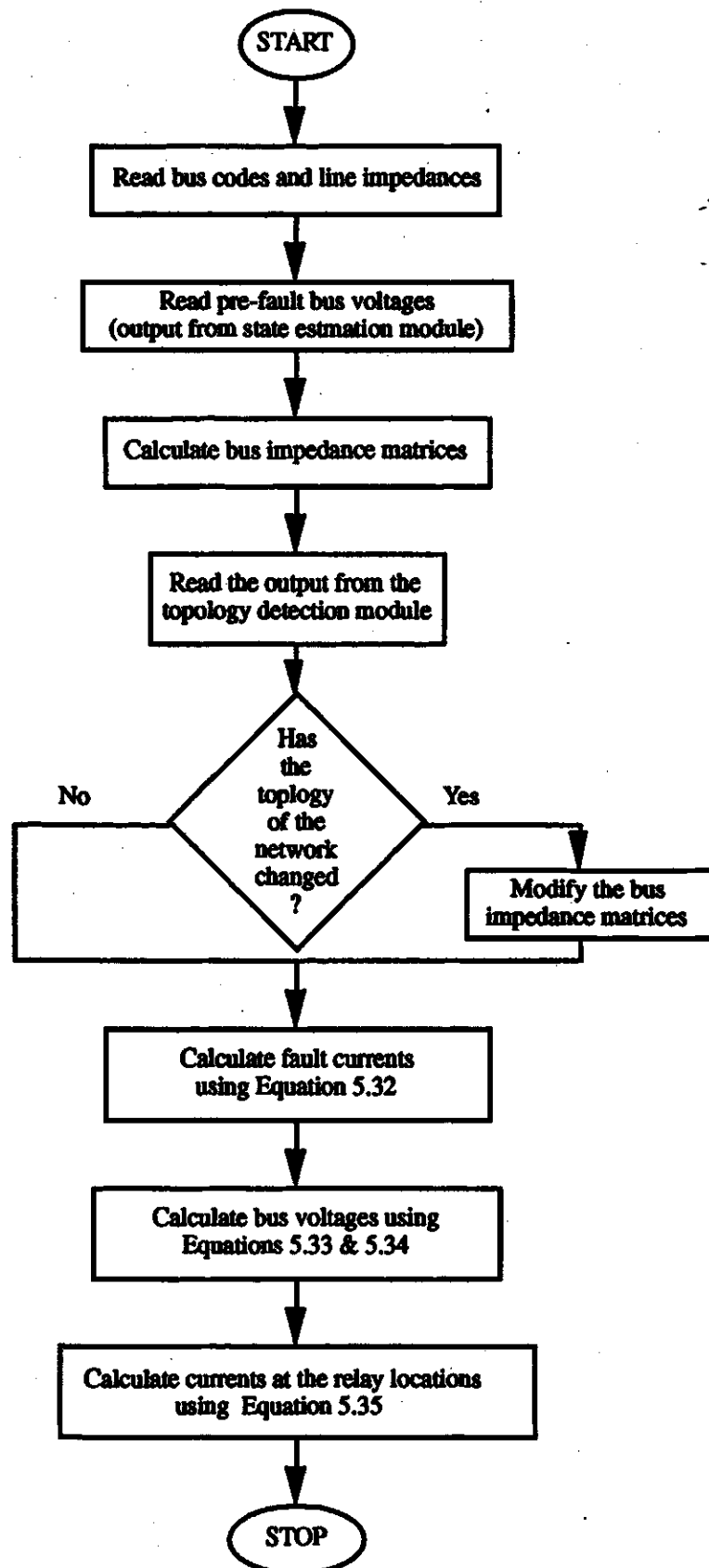
Equations 5.32 through 5.35 are used in the fault analysis program to determine fault currents and bus voltages for single line to ground faults. Figure 5.2 shows the steps incorporated in the fault analysis module.

## 5.4. Relay Settings

One of the objectives of protective relaying is to achieve minimum possible relay operating times, without loosing coordination among them. Conventionally, a trial and error approach is used for setting relays in multiloop networks. Recently, a relay coordination algorithm, which used an optimization technique, was developed [8]. The proposed method was successful in coordinating phase fault relays in a distribution network. The technique has now been used in this project for coordinating ground fault relays. The algorithm is briefly discussed in the next section.

### 5.4.1. Relay setting and optimization algorithm

A linear programming approach [32, 33] was previously used to determine the feasible settings of relays. The two phase simplex method [32] was used for solving the problem. The method detects redundant and unacceptable constraints in the problem and identifies instances when the objective value is unbounded over the feasible region. Phase I of the two phase simplex method finds a feasible solution, and Phase II finds the optimal solution. In Phase I, the initial extreme point is moved from the origin to the feasible region. In Phase II, pivoting is done from the initial extreme point to an optimal extreme point. The problem has an unbounded solution if the method fails to find an optimal extreme point.



**Figure 5.2:** Flow chart of the fault analysis module.

The linear programming problem is started by formulating an objective function

$$z = \sum_{j=1}^n c_j x_j \quad (5.36)$$

which is minimized subject to

$$x_j \geq 0 \quad j = 1, 2, \dots, n \quad (5.37)$$

$$\text{and} \quad \sum_{j=1}^n a_{ij} x_j \geq b_i \quad i = 1, 2, \dots, m; \quad m < n, \quad (5.38)$$

where:

- $c_j$  are the coefficients of the objective function,
- $x_j$  are the decision variables,
- $a_{ij}$  are the constraint coefficients and
- $b_i$  are the values of the constraints.

Initially, a canonical form of this equation is developed from the general linear programming problem. The transformation to canonical form is achieved as follows.

1. The decision variables, that are unconstrained, are replaced by a difference between two non-negative variables. The procedure is applied to all equations including the objective function.
2. The inequalities are changed to equalities by introducing slack and surplus variables. For ' $\geq$ ' inequalities, the non-negative surplus variables represents the margin by which the left hand side exceeds the right hand side. For ' $\leq$ ' inequalities, the non-negative slack variables represent the margin by which the right hand side exceeds the left hand side.
3. If an equation has a negative right hand side, it is multiplied by -1 to make the right hand side positive.
4. A non-negative artificial variable has to be added to an equation in which an isolated variables does not exist.

After the transformation, Equation 5.38 can be written in the following canonical form, in which  $x_{11}, x_{12}, x_{13}, \dots, x_{1m}$  are artificial non-negative variables and  $x_{s1}, x_{s2}, x_{s3}, \dots, x_{sm}$  are slack variables.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n - x_{s1} + x_{11} &= b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n - x_{s2} + x_{12} &= b_2, \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n - x_{sm} + x_{1m} &= b_m. \end{aligned} \quad (5.39)$$

The feasible solution to the optimization problem is achieved when all the artificial variables are zero. Since artificial variables are non-negative, they are all zero when their sum is zero.

An attempt is made to eliminate the artificial variables by minimizing  $w$ , such that  $w = x_{11} + x_{12} + \dots + x_{1m}$ . During the process of minimization, the objective function  $z$  is also updated. The process of minimizing  $w$  is Phase I of the simplex method which is implemented in the form of a table. Samples of starting and final table for Phase I are shown in Tables 5.1 and 5.2 respectively.

The procedure for minimizing the function  $w$  is as follows. The quantities with a bar represent the most recent values of those quantities during the computation process.

**Step 1:** If  $\bar{w}_j \leq 0$  for  $j = 1, 2, \dots, n + 2m$ , a feasible solution is obtained. Stop the process and Phase I is complete. Here  $\bar{w}_j$  is the current coefficient of column  $j$  in the  $w$  row. If there exists some  $\bar{w}_j > 0$ , continue.

**Step 2:** Choose the column to pivot in i.e., variable to introduce to the basis, by

$$\bar{c}_1 = \max_j \{\bar{w}_j\} \text{ such that } \bar{w}_j > 0.$$

If  $a_{i1} \leq 0$  for  $i = 1, 2, \dots, n$  then stop. The problem is unbounded, continue if some  $a_{i1} > 0$  for  $i = 1, 2, \dots, m$ .

**Step 3:** Choose row  $r$  to pivot in, i.e., the variable to drop from the basis by the ratio test,

$$\frac{\bar{b}_r}{\bar{a}_{r1}} = \min_i \left\{ \frac{\bar{b}_i}{\bar{a}_{i1}} \right\}, \text{ such that } \bar{a}_{i1} > 0$$

where the variables with bar represent the most recent values of that variable during the computation process.

Step 4: Replace the basic variable in the row  $r$  with variable  $l$  and reestablish the canonical form as follows,

$$\bar{b}_{r(new)} = \frac{\bar{b}_r}{\bar{a}_{rl}},$$

$$\bar{b}_{i(new)} = \bar{b}_i - \frac{\bar{b}_r}{\bar{a}_{rl}}, i \neq r.$$

Step 5: Go to Step 1

At the end of Phase I if the current value of  $w$  reduces to zero, current values of all artificial variables become zero. If  $w$  does not reduce to zero in Phase I, no feasible solution exists to the original problem. One or more artificial variables at this stage will still be in the basis with positive values. In such case, at the end of Phase I, infeasible constraints corresponding to these positive artificial variables are withdrawn from the original problem. Phase I is again conducted to obtain the feasible solution. Phase II of the simplex method is then followed for the  $z$  function. The procedure of minimizing the object function  $z$  is similar to minimizing the  $w$  function. Figure 5.3 summarizes the simplex method. It illustrates both the computational steps of the algorithm and the transition from Phase I to Phase II. It also explains the steps to remove the infeasible constraints, if any, existed in the original problem.

## 5.5. Summary

This chapter discusses the development of three of the four software modules necessary for relay coordination for adaptive protection. The mathematical development of each of the software modules, as well as the sequential steps for developing the software modules, is presented. Microsoft FORTRAN was used to develop the software modules. The fast decoupled load flow technique, which was used for state estimation of the system is described. As the decoupled load flow is a modification of the Newton Raphson method, the latter is also discussed briefly. A discussion of different types of faults that may occur in a power system is presented, followed by details of mathematical derivation for single phase to ground fault. Finally, the relay setting and coordination algorithms using an optimization technique are discussed.

Table 5.1: Starting tableau for Phase I of the simplex method.

Basis	Current Values	$x_1$	$x_2$	.....	$x_n$	$x_{s1}$	$x_{s2}$	.....	$x_{sm}$	$x_{t1}$	$x_{t2}$	.....	$x_{tm}$
$x_1$	$b_1$	$a_{11}$	$a_{21}$		$a_{1n}$	$-1$	$0$	.....	$0$	$1$	$0$	.....	$0$
$x_2$	$b_2$	$a_{21}$	$a_{22}$		$a_{2n}$	$0$	$-1$	.....	$0$	$0$	$1$	.....	$0$
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.	.												
.	.												
.	.												
.	.												
.	.												
$x_{tm}$	$b_m$	$a_{m1}$	$a_{m2}$	.....	$a_{mn}$	$0$	$0$	.....	$-1$	$0$	$0$	.....	$1$
$z$	$0$	$-c_1$	$-c_2$	.....	$-c_n$	$0$	$0$	.....	$0$	$0$	$0$	$0$	$0$
$w$	$d_0$	$d_1$	$d_2$	.....	$d_n$	$-1$	$-1$	.....	$-1$	$-1$	$-1$	$-1$	$-1$

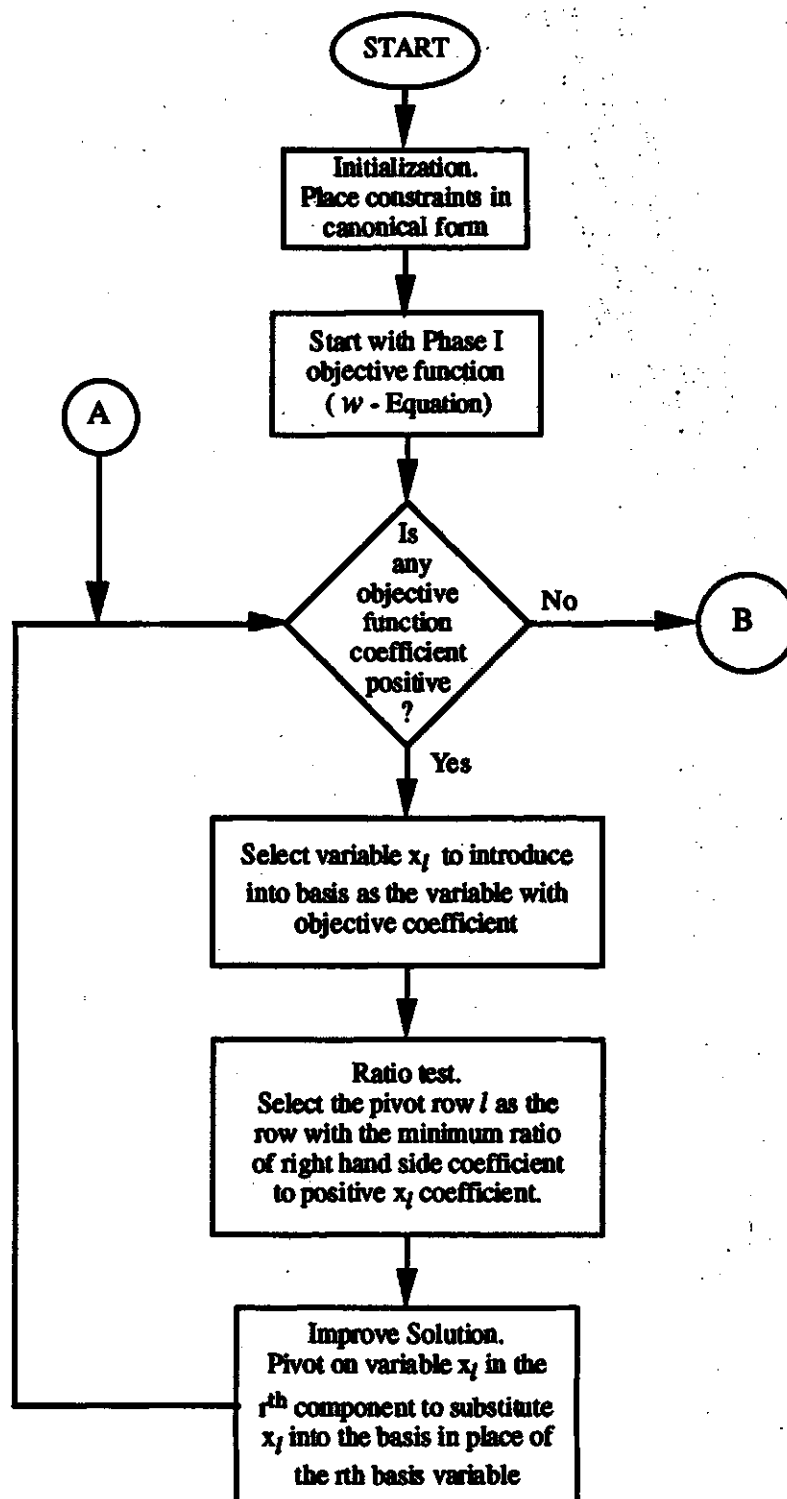
$$d_0 = \sum b_m, d_1 = \sum a_{m1}, \dots, d_n = \sum a_{mn}$$



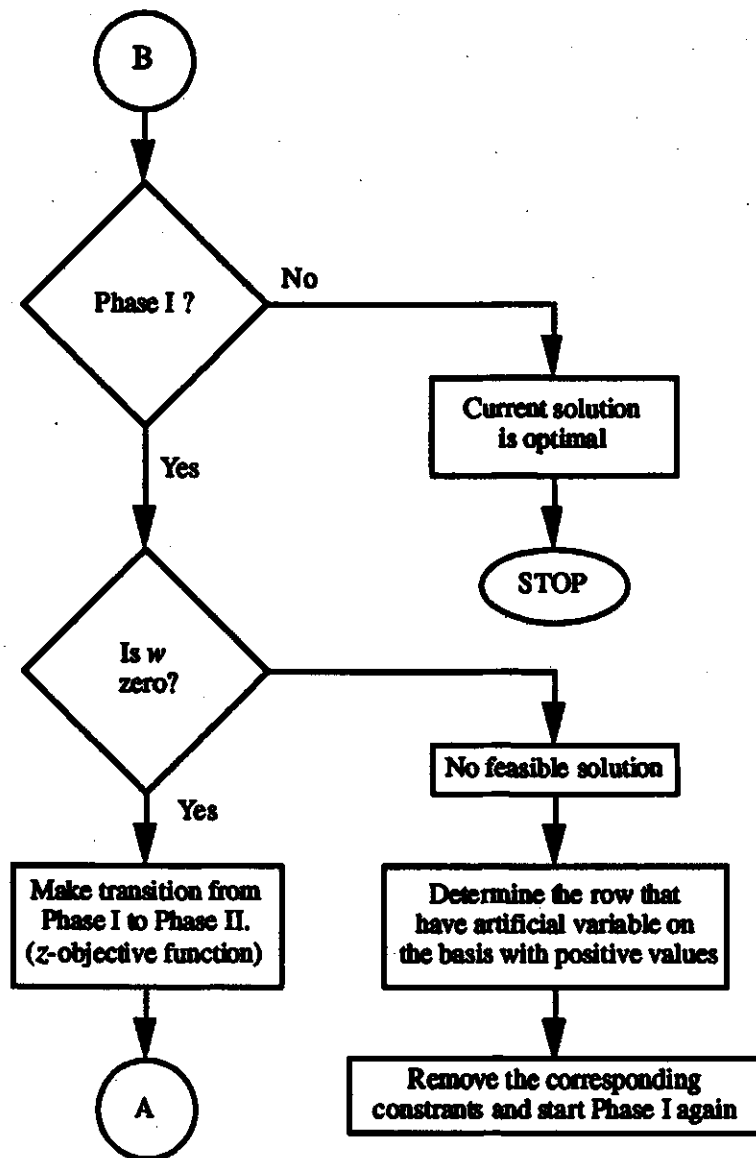
Table 5.2: Tableau after completion of Phase I of the simplex method.

Basis	Current Values	$x_1$	$x_2$	.....	$x_n$	$x_{s1}$	$x_{s2}$	.....	$x_{sm}$	$x_{t1}$	$x_{t2}$	.....	$x_{tm}$
$x_g$	$b_g$	$a_{g1}$	$a_{g1}$		$a_{gn}$	$a_{gs1}$	$a_{gs2}$	.....	$a_{gsm}$	$a_{gt1}$	$a_{gt2}$	.....	$a_{gtm}$
$x_h$	$b_h$	$a_{h1}$	$a_{h2}$		$a_{hn}$	$a_{hs1}$	$a_{hs2}$	.....	$a_{hsm}$	$a_{ht1}$	$a_{ht2}$	.....	$a_{htm}$
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.	.												
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.	.												
$x_k$	$b_k$	$a_{k1}$	$a_{k2}$	.....	$a_{kn}$	$a_{ks1}$	$a_{ks2}$	.....	$a_{ksm}$	$a_{kt1}$	$a_{kt2}$	.....	$a_{ktm}$
$z$	$\bar{b}$	$\bar{c}_1$	$\bar{c}_2$	.....	$\bar{c}_n$	$\bar{c}_{s1}$	$\bar{c}_{s2}$	.....	$\bar{c}_{sm}$	$\bar{c}_{t1}$	$\bar{c}_{t2}$	.....	$\bar{c}_{tm}$
$w$	0	$\bar{d}_1$	$\bar{d}_2$	.....	$\bar{d}_n$	$\bar{d}_{s1}$	$\bar{d}_{s2}$	.....	$\bar{d}_{sm}$	$\bar{d}_{t1}$	$\bar{d}_{t2}$	.....	$\bar{d}_{tm}$

$\bar{d}_1, \bar{d}_2, \dots, \bar{d}_n, \bar{d}_{s1}, \bar{d}_{s2}, \dots, \bar{d}_{sm}, \bar{d}_{t1}, \bar{d}_{t2}, \dots, \bar{d}_{tm}$  in the  $w$  row are either 0 or negative.



**Figure 5.3:** Flow diagram showing the two phase simplex method for optimization.



**Figure 5.3 Continued.**

## **6. SYSTEM STUDIES AND RESULTS**

### **6.1. Introduction**

In the previous chapter, the algorithms for the software modules needed for coordinating relays were discussed. In this chapter, results of tests performed on each module, using the City of Saskatoon Distribution Network as a model system, are reported. The benefits that can be derived from adaptive protection of the system are also reported in this chapter.

The criteria used for the operation of ground overcurrent and ground instantaneous relays are outlined. The estimated Time Multiplier Settings (TMS) for the ground overcurrent relays, obtained from the optimization module, are also reported. The relay operating times, for faults at different locations of the system, are then presented. A discussion on the results obtained from the adaptive protection approach then follow. The operating times of the ground overcurrent relays using adaptive and non-adaptive approaches are also compared in this chapter.

### **6.2. Model Distribution Network**

A model distribution network was used during the course of the work presented in this thesis. The network is a reduced version of the City of Saskatoon Distribution Network and is shown in Figure 6.1. The network consists of five substations: Avenue C, Taylor, Friebe, Cowley and Pleasant Hill substations. Two 138 kV lines QE-1A and QE-2A connect the Queen Elizabeth (QE) generating station, of the Saskatchewan Power Corporation, to the Avenue C substation. As shown in the diagram, three transformers at the Avenue C substation step the voltage down to the 14.4 kV level. The Pleasant Hill substation is fed by a 72 kV line (QE-5) from the QE station. Another 72 kV line, QE-18, connects the Cowley, Friebe and Taylor substations via the Bunn substation (not shown in the diagram).

The voltage is stepped down to 14.4 kV at each substation. The buses of the substations are interconnected by 14.4 kV lines to form the distribution network. Each line connects two substations and is protected by a circuit breaker at each end. For

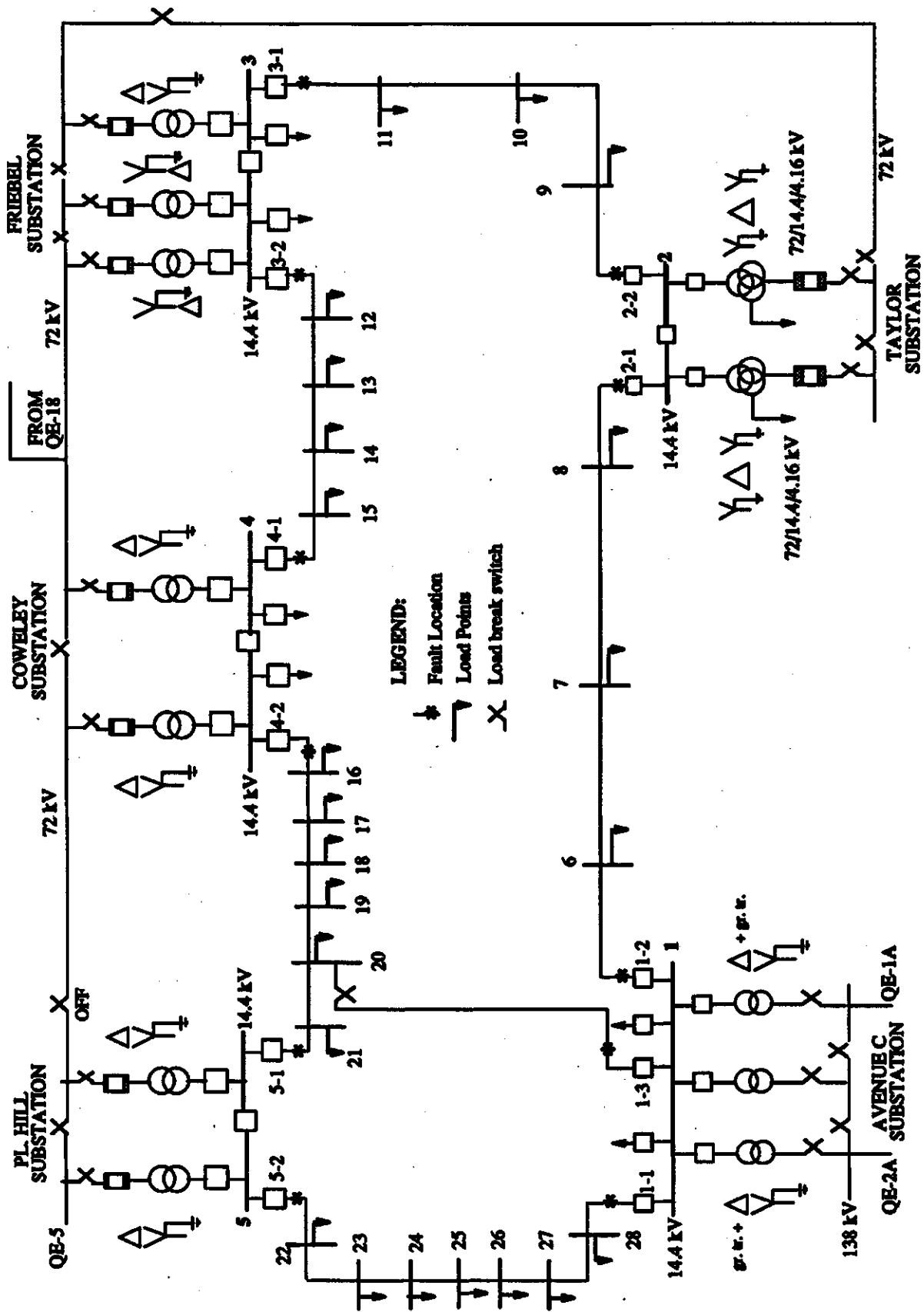


Figure 6.1: Single line diagram of the selected distribution network

ground fault protection, each breaker is equipped with a ground directional overcurrent relay and ground instantaneous relay. The overcurrent relays are residually connected. The work reported in [8] used two sets of current transformers (cts), one set for phase overcurrent and instantaneous relays and the other set for overload relays. As the expected levels of ground fault currents are low, due to the presence of fault resistances, the ground overcurrent relays are connected to the lower ratio cts and ground instantaneous relays are connected to the higher ratio cts. The reason for providing instantaneous relays is that the ratios of the near end fault currents to the far end fault currents are high.

The present practice of the City of Saskatoon Distribution system is to operate each line as a radial circuit while each substation serves its local loads. When there is loss of power to a substation, a different line, connected to the substation, is energized from the other end. For the studies reported in this thesis, the lines are assumed to be connected at both ends. All the 14.4 kV breakers, shown in Figure 6.1, are assumed to be closed forming a looped network. The details of the loads and line parameters are listed in Tables 6.2 and 6.5 respectively.

### **6.3. Operating Conditions**

The following four operating conditions were considered for the coordination of ground overcurrent relays.

- a) Maximum system generation and maximum system load when line 1-20 is closed (MX-1).
- b) Minimum system generation and minimum system load when line 1-20 is closed (MN-1).
- c) Maximum system generation and maximum system load when line 1-20 is open (MX-2).
- d) Minimum system generation and minimum system load when line 1-20 is open (MN-2).

Minimum system load is considered as half the maximum system load. Similarly, minimum system generation is considered to be half of the maximum system generation.

## **6.4. Testing of Coordination Software for L-G Fault**

The developed software modules were tested using the model distribution system described in Section 6.2 and the operating conditions listed in Section 6.3. The data used for this purpose were obtained from the Electrical Department of the City of Saskatoon. Test results for the maximum generating and maximum load condition when line 1-20 is open (MX-2) are reported in this section. The results of the remaining cases are reported in Appendix D.

### **6.4.1. Network topology detection**

The development and testing of this module has been discussed in detail in Chapter 4. The software module was tested using three systems including the model system described in Section 6.3. Several operating states of the systems were considered. The test results are reported in Appendix A. The results revealed that the developed technique worked properly for all the states considered. It was, therefore, decided to use this module for adaptive protection. The output of the topology detection module for the model network is shown in Table 6.1. This output file provided inputs to the state estimation and the fault analysis modules.

### **6.4.2. State estimation**

The state estimation algorithm which used the decoupled load flow method is described in Section 5.2. The topology detection module provided the line node connectivity status of the system, as an input file, to this module. The system generation and load data were provided by another file which is of the form shown in Table 6.2. After executing the program, it created two files in the form of Tables 6.3 and 6.4. One of the files contained the data for prefault bus voltages which provided input to the fault analysis module whereas the other file, which contained the information of prefault currents, provided input to the optimization module.

### **6.4.3. Fault analysis**

The fault analysis algorithm is described in Section 5.3. The algorithm modifies the sequence bus impedance matrices when the topology of the system changes. Three input files are needed by the fault analysis module. The first file, which contains data concerning prefault bus voltages and line currents, is the output of the state estimation module. The second file, shown in Table 6.5, contains data on positive and zero

sequence impedances of the system. The negative sequence impedances were assumed to be identical to the positive sequence impedances. The third input file, which contained the information of changes of line node connectivity of the system, is the output of the topology detection module. The fault analysis module calculated currents for faults at critical locations of the system shown in Figure 6.1. The output of this module, which provided information concerning the fault currents at relay locations, is shown in Table 6.6.

**Table 6.1: Output of the topology detection module.**

Line no.	From bus no.	To bus no.	Line status
1	1	6	0
2	6	7	0
3	7	8	0
4	2	8	0
5	2	9	0
6	9	10	0
7	11	10	0
8	3	11	0
9	3	12	0
10	13	12	0
11	14	13	0
12	15	14	0
13	4	15	0
14	4	16	0
15	16	17	0
16	18	17	0
17	19	18	0
18	20	19	0
19	21	20	0
20	5	21	0
21	5	22	0
22	22	23	0
23	24	23	0
24	25	24	0
25	26	25	0
26	27	26	0
27	28	27	0
28	1	28	0
29	1	20	1



**Table 6.2:** Input to the state estimation module ( Load and generation at different buses).

Bus no.	Load		Generation	Voltage	
	P p.u.	Q p.u.	P p.u.	Magnitude p.u.	Phase rad.
1	0.0	0.0	0.0	1.02	0.0
2	1.3500	0.6530	1.768	1.02	0.0
3	1.1020	0.5340	1.862	1.02	0.0
4	1.3730	0.6650	2.075	1.02	0.0
5	0.0	0.0	1.626	1.02	0.0
6	0.0	-0.2400	0.0	1.00	0.0
7	1.0930	0.5290	0.0	1.00	0.0
8	0.1350	-0.0546	0.0	1.00	0.0
9	0.6300	0.3050	0.0	1.00	0.0
10	0.0	-0.1200	0.0	1.00	0.0
11	0.0315	0.0152	0.0	1.00	0.0
12	0.0180	0.0872	0.0	1.00	0.0
13	0.0810	0.3920	0.0	1.00	0.0
14	0.0520	0.2524	0.0	1.00	0.0
15	0.0	-0.1200	0.0	1.00	0.0
16	0.1800	0.0872	0.0	1.00	0.0
17	0.2700	0.1307	0.0	1.00	0.0
18	0.6367	0.3084	0.0	1.00	0.0
19	0.0	-0.1200	0.0	1.00	0.0
20	0.1035	0.0501	0.0	1.00	0.0
21	0.8704	0.2415	0.0	1.00	0.0
22	0.2780	0.1349	0.0	1.00	0.0
23	0.4500	0.0984	0.0	1.00	0.0
24	0.0280	0.0135	0.0	1.00	0.0
25	0.1620	0.0784	0.0	1.00	0.0
26	0.0900	0.0436	0.0	1.00	0.0
27	0.3303	0.3997	0.0	1.00	0.0
28	0.3900	0.1900	0.0	1.00	0.0

Base power = 10 MVA Base voltage = 14.4 kV

**Table 6.3:** Output of the state estimation module (prefault voltages).

Bus no.	Voltage	
	Magnitude p.u.	Phase rad.
1	1.0200	0.0
2	1.0200	-0.0266
3	1.0200	-0.0370
4	1.0200	-0.0645
5	1.0200	-0.0509
6	1.0162	-0.0063
7	1.0122	-0.0183
8	1.0140	-0.0206
9	1.0195	-0.0409
10	1.0151	-0.0406
11	1.0156	-0.0402
12	1.0158	-0.0467
13	1.0116	-0.0671
14	1.0107	-0.0649
15	1.0128	-0.0650
16	1.0082	-0.0725
17	0.9943	-0.0824
18	0.9927	-0.0826
19	0.9952	-0.0797
20	0.9969	-0.0765
21	1.0058	-0.0669
22	1.0104	-0.0475
23	1.0079	-0.0450
24	1.0067	-0.3542
25	1.0066	-0.0338
26	1.0060	-0.0306
27	1.0050	-0.0215
28	1.0093	-0.0099

**Table 6.4:** Output of the state estimation module (prefault currents).

Bus no		Line flow (magnitude) A
From	To	
1	6	506.449
6	7	494.741
7	8	238.715
2	8	209.406
2	9	171.625
9	10	129.854
11	10	96.513
3	11	110.139
3	12	207.947
13	12	200.969
14	13	170.709
15	14	133.642
4	15	89.175
4	16	279.689
16	17	200.655
18	17	82.016
19	18	209.038
20	19	198.469
21	20	242.499
5	21	601.967
5	22	332.591
22	23	280.049
24	23	332.591
25	24	337.019
26	25	369.679
27	26	392.446
28	27	483.588
1	28	641.823

**Table 6.5:** Line and source parameters for input to the fault analysis module.  
a) Line impedances

Line		Positive seq. impedance		Zero seq. impedance	
From bus no.	To bus no.	Real component	Imaginary component	Real component	Imaginary component
1	6	0.3995	0.4325	0.5325	0.1585
6	7	0.3650	0.9755	0.8090	4.6175
7	8	0.2900	0.4010	0.4620	1.8685
8	2	1.1423	1.1777	4.2663	3.7373
2	9	1.2710	3.5440	3.8720	9.7980
9	10	0.3440	0.5950	1.0180	2.2530
10	11	0.0770	0.2400	0.2790	0.9090
11	3	0.6330	1.8860	1.8240	6.9940
3	12	0.8450	1.8840	2.6940	6.0910
12	13	0.9920	2.7890	2.9760	8.7550
13	14	0.3630	0.9460	1.1980	3.6460
14	15	0.2280	0.5890	0.6320	2.9010
15	4	1.1460	3.0400	3.8830	8.5890
4	16	0.8345	1.8765	2.6970	7.6070
16	17	1.2220	3.1810	3.1890	13.569
17	18	0.4990	0.6195	0.7505	2.8565
18	19	0.2710	0.6840	0.5990	3.5030
19	20	0.2710	0.6840	0.5990	3.5030
20	21	1.1740	1.8120	2.7220	10.305
21	5	0.6565	1.2785	1.3150	4.2135
5	22	0.5930	1.0750	1.5660	3.4070
22	23	0.2850	0.4267	0.6080	1.1660
23	24	0.7250	0.9230	1.4680	2.5420
24	25	0.1170	0.1490	0.2380	0.4110
25	26	0.1245	0.3325	0.3205	1.4045
26	27	0.3300	0.8820	0.9740	2.0570
27	28	0.3810	0.9630	1.1135	3.9145
28	1	0.5965	0.6960	1.3655	0.3725
1	20	0.5283	1.0367	1.4220	3.9773

**Table 6.5: Continued.**  
**b) Source Impedances**

Source bus no.	Positive seq impedance		Zero seq impedance	
	Real component	Imaginary component	Real component	Imaginary component
1	0.0600	1.2318	0.1000	2.3800
2	0.1025	3.9022	0.1705	2.5031
3	0.0301	3.5336	0.0000	7.0400
4	0.0930	3.7010	0.2143	5.5536
5	0.0000	4.8450	0.0000	3.9050

**Note:** Impedances are in %, Base power =10 MVA, Base voltage = 14.4 kV.

**Table 6.6:** Postfault currents at the relay locations for input to the optimization module.

Relay no.	Fault location	Fault current (A)		
		Real component	Imaginary component	Magnitude
1-1	2-1/2-2	-237.24	224.74	326.79
1-1	3-1/3-2	-13.82	6.34	15.20
1-1	4-1/4-2	101.69	-168.79	197.05
1-1	5-1/5-2	1102.91	-2230.58	2488.36
1-1	1-1	3043.29	-29563.58	29719.81
1-1	1-2	-1135.44	2774.38	2997.73
1-2	2-1/2-2	1530.07	-2756.02	3152.26
1-2	3-1/3-2	247.27	-344.21	423.82
1-2	4-1/4-2	19.79	-19.08	27.49
1-2	5-1/5-2	-260.78	246.18	358.62
1-2	1-1	-1924.91	3880.05	4331.29
1-2	1-2	2253.81	-28457.92	28547.03
2-1	1-1/1-2	1924.92	-3880.05	4331.29
2-1	3-1/3-2	-247.27	344.21	423.82
2-1	4-1/4-2	-19.79	19.08	27.50
2-1	5-1/5-2	260.78	-246.18	358.62
2-1	2-1	2494.81	-19020.46	19183.38
2-1	2-2	-1530.07	2756.01	3152.26
2-2	1-1/1-2	-218.18	291.74	364.30
2-2	3-1/3-2	804.66	-2664.07	2782.94
2-2	4-1/4-2	184.01	-318.54	367.87
2-2	5-1/5-2	-6.94	-3.59	7.81
2-2	2-1	-514.07	1555.87	1683.60
2-2	2-2	3510.81	-20220.61	20523.13
3-1	1-1/1-2	-26.86	18.21	32.45
3-1	2-1/2-2	-144.38	218.85	262.18
3-1	4-1/4-2	440.30	-1514.63	1577.33
3-1	5-1/5-2	54.02	-101.81	115.26
3-1	3-1	581.07	-11852.68	11866.92
3-1	3-2	-514.30	1677.01	1754.10

**Table 6.6: Continued.**

3-2	1-1/1-2	218.18	-291.74	364.30
3-2	2-1/2-2	514.07	-1555.87	1638.60
3-2	4-1/4-2	184.01	318.54	367.87
3-2	5-1/5-2	6.94	3.59	7.81
3-2	3-1	-804.66	2664.07	2728.94
3-2	3-2	290.70	-10865.62	10869.50
4-1	1-1/1-2	115.22	-169.67	205.10
4-1	2-1/2-2	1.57	7.04	7.22
4-1	3-1/3-2	-78.16	153.67	172.40
4-1	5-1/5-2	244.30	-879.34	912.65
4-1	4-1	446.58	-12110.05	12118.28
4-1	4-2	-237.86	1145.92	1170.34
4-2	1-1/1-2	26.86	-18.21	32.45
4-2	2-1/2-2	144.38	-218.85	262.18
4-2	3-1/3-2	514.30	-1677.01	1754.10
4-2	5-1/5-2	-54.02	101.81	115.26
4-2	4-1	-440.30	1514.63	1577.33
4-2	4-2	244.14	-11741.34	11743.88
5-1	1-1/1-2	1135.43	-2774.38	2997.73
5-1	2-1/2-2	237.23	-224.74	326.79
5-1	3-1/3-2	13.82	-6.34	15.21
5-1	4-1/4-2	-101.69	168.79	197.05
5-1	5-1	649.04	-12996.96	13013.16
5-1	5-2	-1102.91	2230.58	2488.36
5-2	1-1/1-2	-115.23	169.67	205.10
5-2	2-1/2-2	-1.57	-7.05	7.22
5-2	3-1/3-2	78.16	-153.67	172.40
5-2	4-1/4-2	237.86	-1145.91	1170.34
5-2	5-1	-244.30	879.34	912.64
5-2	5-2	1507.66	-14348.20	14427.19

#### **6.4.4. Relay settings and optimization**

This module was applied in two stages. In the first stage, the pickup settings of the ground overcurrent relays and the instantaneous ground overcurrent relays were calculated using the criteria reported in the next few sections. The prefault current in each line and the prefault voltage at each bus were calculated using the state estimation module. From the fault currents, calculated by the fault analysis module, the operating times of ground overcurrent relays for near end faults were estimated. The relay setting and optimization modules were then executed to obtain the pickup setting and optimized TMS values.

##### **6.4.4.1. Criteria for setting of ground overcurrent relays**

The criteria used by utilities for setting ground overcurrent relays include considerations of [34]

1. phase overcurrent,
2. normal load unbalance and
3. normal line flows.

Most utilities try to keep the unbalance in the range of 25% to 50% of the normal load whereas some others try to keep the unbalance below 25% level.

The phase overcurrent relay settings equal to 200% of the normal load were chosen in [8]. In the proposed work, the ground overcurrent relay settings equal to 25% of the phase overcurrent relay settings were selected. This set the pickup values of ground overcurrent relays at 50% of the normal load.

##### **6.4.4.2. Ground overcurrent relay settings**

Using the criteria discussed in the previous section, the pickup settings for the ground overcurrent relays were calculated. In case of non-adaptive approach the pickup settings were based on the largest fault current that is likely to be experienced by the relay. This approach ensures that the operation of the relay remains coordinated for all operating states.



### 6.4.4.3. Instantaneous relay settings

As the ratios of the far end fault currents to the near end fault currents are quite high, it was decided to use ground instantaneous overcurrent relays in addition to the ground overcurrent relays. The tap settings of the instantaneous relays for ground fault protection were made equal to 1.3 times the far end maximum ground fault current. For example, the instantaneous element of relay 1-1 was set at

$$\frac{1.3 \times \text{ground fault current for fault at bus 5}}{\text{C.T. ratio}}$$

### 6.4.4.4. Time multiplier settings

The following equation was used to determine the operating time of a relay.

$$t = \frac{a \times \text{TMS}}{I_{mpu}^n - 1}, \quad (6.1)$$

where:

TMS is the time multiplier setting,

$I_{mpu}$  is the current in multiples of the pickup current,

a is a constant and

n is an index which determines the relay characteristic, this index is

1 for a very inverse characteristic and

2 for an extremely inverse characteristic.

### 6.4.4.5. Optimization

After calculating the different settings of the relays, the objective function was formed. The constraints which describe the coordinated operating times of the primary and backup relays were established. To calculate the optimized TMS values, the operating times of the primary relays for faults at the near end were minimized. The operating times of the backup relays formed the inequality constraints. The objective function was

$$z = c_{1-1}TMS_{1-1} + c_{1-2}TMS_{1-2} + c_{1-3}TMS_{1-3} + c_{2-1}TMS_{2-1} + c_{2-2}TMS_{2-2} + c_{3-1}TMS_{3-1} \\ + c_{3-2}TMS_{3-2} + c_{4-1}TMS_{4-1} + c_{4-2}TMS_{4-2} + c_{5-1}TMS_{5-1} + c_{5-2}TMS_{5-2}, \quad (6.2)$$

where:

$TMS_{m-n}$  is the time multiplier setting of relay m-n,

$$c_{m-n} = \frac{a}{I_{mpu}^n - 1},$$

$I_{mpu}$  is the current in multiples of the pickup current at the relay location m-n for near end fault,

$a$  is a constant (=13.5) and

$n$  is a constant (=1 for very inverse characteristics).

The inequality constraints were

$$c_{1-2,2}TMS_{1-2} \geq c_{2-2}TMS_{2-2} + 0.2,$$

$$c_{2-1,1}TMS_{2-1} \geq c_{1-1}TMS_{1-1} + 0.2,$$

$$\dots\dots\dots,$$

$$c_{1-1}TMS_{1-1} \geq 0.05,$$

$$c_{2-2}TMS_{2-2} \geq 0.05,$$

$$\dots\dots\dots,$$

(6.3)

where:

$$c_{m-n,l} = \frac{a}{I_{mpu}^n - 1}$$

$I_{mpu}$  is the current in multiples of the pickup current at the relay location m-n for fault at bus l,

$a$  is a constant (=13.5),

$n$  is a constant (=1 for very inverse characteristics),

0.2 is the relay coordination time interval (CTI) in seconds and

0.05 is the minimum operating time for a ground overcurrent relay in seconds.

and the  $w$  function was

$$w = TMS_{1-1}(c_{1-1} + \sum c_{1-1,l}) + TMS_{1-2}(c_{1-2} + \sum c_{1-2,l}) + \dots\dots\dots \\ \dots\dots\dots + TMS_{5-2}(c_{5-2} + \sum c_{5-2,l}) - x_1 - x_2 - \dots\dots + K,$$

(6.4)

where:

$x$ 's are the surplus variables and

$K$  is a constant equal to sum of the r.h.s of the constraints.

The operating condition MN-2 is used to illustrate the optimization technique. The function to be minimized was

$$\begin{aligned}
 z = & 1.301 TMS_{1-1} + 1.102 TMS_{1-2} + 0.587 TMS_{2-1} + 0.588 TMS_{2-2} \\
 & + 0.584 TMS_{3-1} + 0.573 TMS_{3-2} + 0.567 TMS_{4-1} + 0.570 TMS_{4-2} \\
 & + 1.254 TMS_{5-1} + 0.742 TMS_{5-2},
 \end{aligned} \tag{6.5}$$

subject to

$$\begin{aligned}
 & -1.301 TMS_{1-1} + 4.838 TMS_{2-1} \geq 0.2, \\
 & -0.742 TMS_{5-2} + 21.932 TMS_{4-2} \geq 0.2, \\
 & -0.587 TMS_{2-1} + 10.559 TMS_{3-1} \geq 0.2, \\
 & -1.102 TMS_{1-2} + 11.387 TMS_{5-2} \geq 0.2, \\
 & -0.584 TMS_{3-1} + 5.128 TMS_{4-1} \geq 0.2, \\
 & -0.588 TMS_{2-2} + 12.362 TMS_{1-2} \geq 0.2, \\
 & -0.567 TMS_{4-1} + 137.32 TMS_{5-1} \geq 0.2, \\
 & -0.573 TMS_{3-2} + 3.224 TMS_{2-2} \geq 0.2, \\
 & -1.254 TMS_{5-1} + 14.562 TMS_{1-1} \geq 0.2, \\
 & -0.570 TMS_{4-2} + 5.786 TMS_{3-2} \geq 0.2, \\
 & 1.301 TMS_{1-1} \geq 0.05, \\
 & 1.102 TMS_{1-2} \geq 0.05, \\
 & 0.587 TMS_{2-1} \geq 0.05, \\
 & 0.573 TMS_{2-2} \geq 0.05, \\
 & 0.584 TMS_{3-1} \geq 0.05, \\
 & 0.573 TMS_{3-2} \geq 0.05, \\
 & 0.567 TMS_{4-1} \geq 0.05, \\
 & 0.570 TMS_{4-2} \geq 0.05, \\
 & 1.254 TMS_{5-1} \geq 0.05, \\
 & 0.798 TMS_{5-2} \geq 0.05.
 \end{aligned} \tag{6.6}$$

The inequality constraints were changed to equality constraints by adding surplus variables. Since all the constraints are of ' $\geq$ ' type, a non-negative surplus variable was added to each of the constraints. After adding the surplus variables ( $x$ 's) and artificial variables ( $y$ 's) the constraint equations were

$$\begin{aligned}
 & -1.301 TMS_{1-1} + 4.838 TMS_{2-1} - x_1 + y_1 = 0.2, \\
 & -0.742 TMS_{5-2} + 21.932 TMS_{4-2} - x_2 + y_2 = 0.2, \\
 & -0.587 TMS_{2-1} + 10.559 TMS_{3-1} - x_3 + y_3 = 0.2, \\
 & -1.102 TMS_{1-2} + 11.387 TMS_{5-2} - x_4 + y_4 = 0.2,
 \end{aligned}$$

$$\begin{aligned}
& -0.584 TMS_{3-1} + 5.128 TMS_{4-1} - x_4 + y_4 = 0.2, \\
& -0.588 TMS_{2-2} + 12.362 TMS_{1-2} - x_6 + y_6 = 0.2, \\
& -0.567 TMS_{4-1} + 137.32 TMS_{5-1} - x_7 + y_7 = 0.2, \\
& -0.573 TMS_{3-2} + 3.224 TMS_{2-2} - x_8 + y_8 = 0.2, \\
& -1.254 TMS_{5-1} + 14.562 TMS_{1-1} - x_9 + y_9 = 0.2, \\
& -0.570 TMS_{4-2} + 5.786 TMS_{3-2} - x_{10} + y_{10} = 0.2, \\
& 1.301 TMS_{1-1} - x_{11} + y_{11} = 0.05, \\
& 1.102 TMS_{1-2} - x_{12} + y_{12} = 0.05, \\
& 0.587 TMS_{2-1} - x_{13} + y_{13} = 0.05, \\
& 0.573 TMS_{2-2} - x_{14} + y_{14} = 0.05, \\
& 0.584 TMS_{3-1} - x_{16} + y_{16} = 0.05, \\
& 0.573 TMS_{3-2} - x_{15} + y_{15} = 0.05, \\
& 0.567 TMS_{4-1} - x_{18} + y_{18} = 0.05, \\
& 0.570 TMS_{4-2} - x_{17} + y_{17} = 0.05, \\
& 1.254 TMS_{5-1} - x_{20} + y_{20} = 0.05, \\
& 0.798 TMS_{5-2} - x_{19} + y_{19} = 0.05.
\end{aligned} \tag{6.7}$$

Phase I of the optimization problem was started with minimizing the sum of the artificial variables. Since the artificial variables are non-negative, their sum was minimized. To achieve this the objective function  $w$  was formed by adding the artificial variables to the  $w$ -equation. That is, the function  $w$  and all the constraints were added to

$$(-w) - y_1 - y_2 - \dots - y_{20} = 0, \tag{6.8}$$

which was then expressed by substituting Equation 6.5 and the constraints in it.

$$\begin{aligned}
w = & 14.562 TMS_{1-1} + 12.362 TMS_{1-2} + 4.838 TMS_{2-1} + 3.224 TMS_{2-2} \\
& + 10.559 TMS_{3-1} + 5.786 TMS_{3-2} + 5.128 TMS_{4-1} + 21.932 TMS_{4-2} \\
& + 137.32 TMS_{5-1} + 11.387 TMS_{5-2} - x_1 - x_2 - \dots - x_{20} - 2.5.
\end{aligned} \tag{6.9}$$

The problem is documented in Table 6.7; this was the starting table. The first column of the table represents the artificial variables considered as basic variables to start with. The second column of the table represents the values on the right hand side of the constraint equations. Each row of the table represents a constraint equation. For example the first row of the table represents the constraint equation  $-1.301 TMS_{1-1} + 4.838 TMS_{2-1} - x_1 + y_1 = 0.2$ . Similarly, the  $z$ -row and  $w$ -row represent the Equations 6.5 and 6.9 respectively. The optimization of  $w$  was achieved by the following procedure.

1. Select the variable that corresponds to of the highest value in w-row; in the example, the  $TMS_{5-1}$ -column in the w-row has the highest value. This variable,  $TMS_{5-1}$ , is now a new basic variable which replaces one of the initial basic variables.
2. Perform the ratio test to select the row to pivot in. In the  $TMS_{5-1}$ -column there are positive values, one in the  $y_7$ -row and the other in the  $y_{20}$ -row. Consider these two rows for the ratio test. The ratio of the right hand side value to the value in the  $TMS_{5-1}$ -column are  $(0.20:137.315)$  and  $(0.05:1.254)$  respectively. As the ratio for  $y_7$ -row is less than the ratio for the  $y_{20}$ -row, the variable  $y_7$  was replaced by  $TMS_{5-1}$ . This is show in Table 6.8.
3. The table is now updated. The elements of the pivot row are modified by

$$a_{in}^{new} = a_{in} / a_{ij},$$

where  $a_{ij}$  is the element of the pivot row ( $y_7$ ) and pivot column ( $TMS_{5-1}$ ).

The elements of the other rows are modified by

$$a_{kn}^{new} = a_{kn} - a_{kj}(a_{in} / a_{ij}).$$

Therefore, the new value of the element of the  $TMS_{4-1}$ -column in the pivot row of the above problem is  $-0.567 / 137.32 = -0.004$  and the new value of  $x_7$ -column of the pivot row is  $-1 / 137.32 = -0.007$ .

The new values of the elements of  $TMS_{4-1}$ -column and  $x_7$ -column of the  $y_9$ -row respectively are  $-0.0 - (-1.254) \times (-0.567) / 137.32 = -0.005$  and  $-0.0 - (-1.254) \times (-1 / 137.32) = -0.009$ .

Similarly, the new values of the elements of  $TMS_{4-1}$ -column and  $x_7$ -column of the z-row respectively are  $-0.567 - (-1.254) \times (-0.567) / 137.32 = -0.572$ . and  $-0.0 - (-1.254) \times (-1 / 137.32) = -0.009$  and so on.

All the updated values after first iteration are shown in Table 6.8.

This completed the first iteration of the optimization process of phase I. Similar steps were followed in the consecutive iterations. For example in the selected problem the variable  $TMS_{4-2}$  replaced the variable  $y_2$  in the second iteration. The procedure was continued until the updated value of the r.h.s-column of w-row became negligibly small.



Table 6.8: Table after first iteration of phase I for the operating condition MX-2

basic var.	r.h.s. values	$TMS_{1-1}$	$TMS_{1-2}$	$TMS_{2-1}$	$TMS_{2-2}$	$TMS_{3-1}$	$TMS_{3-2}$	$TMS_{4-1}$	$TMS_{4-2}$	$TMS_{5-1}$	$TMS_{5-2}$	$x_7$	....	$x_{20}$	$y_1$	....	$y_{20}$
$y_1$	0.20	-1.301		4.838													
←	0.20			-0.587					(21.932)		-0.742						
.	0.20		-1.102		10.559						11.387						
.	0.20				-0.584			5.128									
.	0.20		12.362					-0.004		1.0		-0.007					
$TMS_{5-1}$	0.001				-0.588												
.	0.20				3.224			-0.005				-0.009					
.	0.202	14.562							-0.570								
.	0.20							5.786									
.	0.05	1.301															
.	0.05		1.102														
.	0.05			0.587													
.	0.05																
.	0.05																
.	0.05																
.	0.05																
.	0.05																
.	0.05																
$y_{20}$	0.048													-1			
z-row	0.0018	-1.301	-1.102	-0.587	-0.588			-0.572	-0.570		-0.742	-0.009					
w-row	2.30	14.562	12.362	4.838	3.224	10.559	5.786	5.695	21.932↑		11.387			-1			

After second iteration  $y_2$  will go out of the basis and  $TMS_{4-2}$  will enter into the basis

To start phase II, the w-row was dropped from the canonical form and the steps described above were carried out for the z function. Phase II of the optimization problem was stopped when no positive value was left in the z-row. While this is achieved we have all TMS become basic variables. The values of r.h.s-column corresponding to each TMS are the optimum values of the TMS.

## 6.5. Results and Discussion

Table 6.9 shows the settings that were needed for the relays when the non-adaptive approach was used. The table includes the desired relay settings for phase and ground fault relays. The settings corresponding to the maximum load currents were selected and are documented in the table. The pickup settings of the ground fault relays were calculated from the settings selected for the phase fault relays; these settings are also included in the table.

The pickup settings time, multiplier settings (TMS), and instantaneous tap settings for ground overcurrent relays were calculated using the adaptive approach. These settings are presented in Table 6.10. The TMS values were obtained after a successful completion of the optimization program. These values are considerably different for the different operating conditions. The table also includes load currents, fault currents and ct ratios. It shows that the pickup settings of the relays change with the change in system operating condition. The pickup settings reduce as the load current decreases. The settings are, however, kept above the maximum allowable unbalance for each operating state. As the ground fault currents change, due to changes in the system load and generation, and the system topology, the settings of the instantaneous relays are also changed.

Figures 6.2 through 6.12 show the relay operating characteristics for the selected operating conditions. It is seen that the relay characteristics that could be used changes as the system conditions change. The adaptive protection chose the most appropriate characteristics for each operating state. The figures also include the time-current characteristics of the relays when they are set using non-adaptive approach. The figures show that, in most cases, the relay characteristics selected by the adaptive approach for the maximum load and maximum generation state are approximately the same as the characteristics selected by the non-adaptive approach.

Figures 6.13 through 6.16 show the comparison of the operating times of the relays when they are set using the adaptive and non-adaptive approaches. Figure 6.13



shows the relay operating times for the primary relays for faults at the far ends of the lines during the operating condition MN-1. The relays operate in lesser time when the adaptive approach is used. The operating times of the adaptive relays 1-1, 1-3, 5-1 & 5-2 are reduced by 10 to 25% of the operating times of the non-adaptive relays. Figure 6.14 provides similar information for the operating condition MN-2. Here again, reduced operating times are achieved for the relays 5-1, 5-2, 1-1 and 1-2.

Operating times of the backup relays for selected fault locations were compared for the adaptive and non-adaptive approaches. This is shown in Figure 6.15 for operating conditions MN-1. The figure shows that the operating time reduces in most cases when the adaptive approach is used. It is also observed that relay 1-1 for fault at bus 21 takes very long time to operate when non-adaptive approach is used. Similarly the operating time of relay 5-1 for fault at bus 1 is very long for non-adaptive case. Moreover the operating time for relay 1-2 for fault at bus 9 is reduced to almost half of the operating time of the non-adaptive relays. Figure 6.16 provides similar information for operating condition MN-2. Here again relay 1-2 for fault at bus 9 takes a very long time to operate when the relay is set using the non-adaptive approach. Similarly relay 5-2 operates after long time delay for fault at bus 4 when non-adaptive approach is used to set the relays.

Relay operating times, for the primary and backup relays, were calculated for different fault locations to check if their operation remains properly coordinated. Table 6.11 shows the operating times for the operating state MN-2. It was observed that the primary and backup relays remain properly coordinated for the selected fault locations. The relay operating times for the other operating states are presented in Appendix D.

## 6.6. Summary

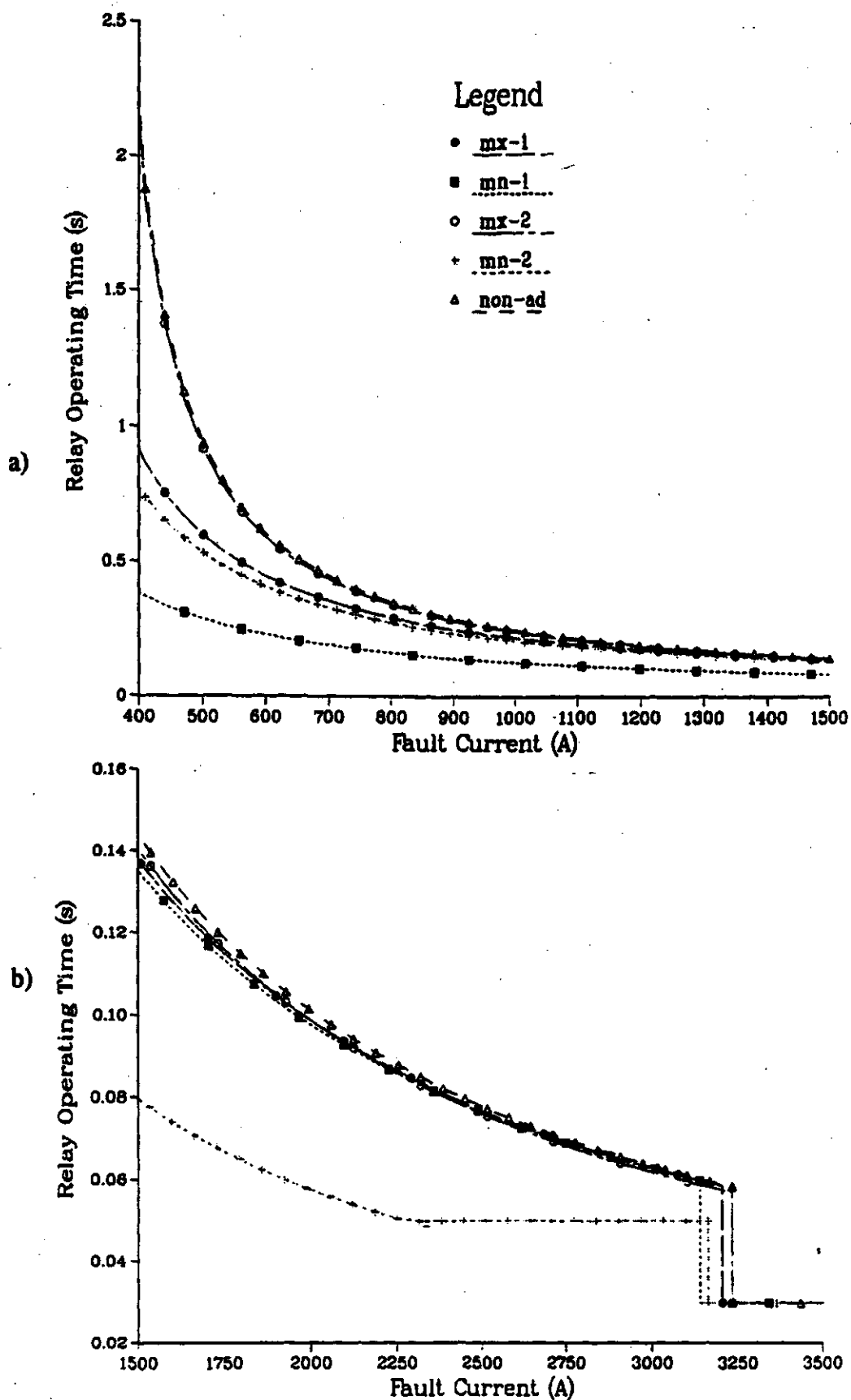
This chapter describes the results from the coordination of relays for ground faults. System studies conducted for the four operating states when protected by adaptive and non-adaptive relaying systems, are also presented. The criteria used for selecting the settings of the ground overcurrent relays are outlined. A comparison of the relay operating times using adaptive and non-adaptive approaches is presented. Operating times for the primary and backup relays for different fault locations are also presented in this chapter. It is observed that the relay coordination margins remain sufficient when the adaptive approach is used to coordinate the ground overcurrent relays.

**Table 6.9:** Settings of the relays of the selected network determined without using the adaptive approach.

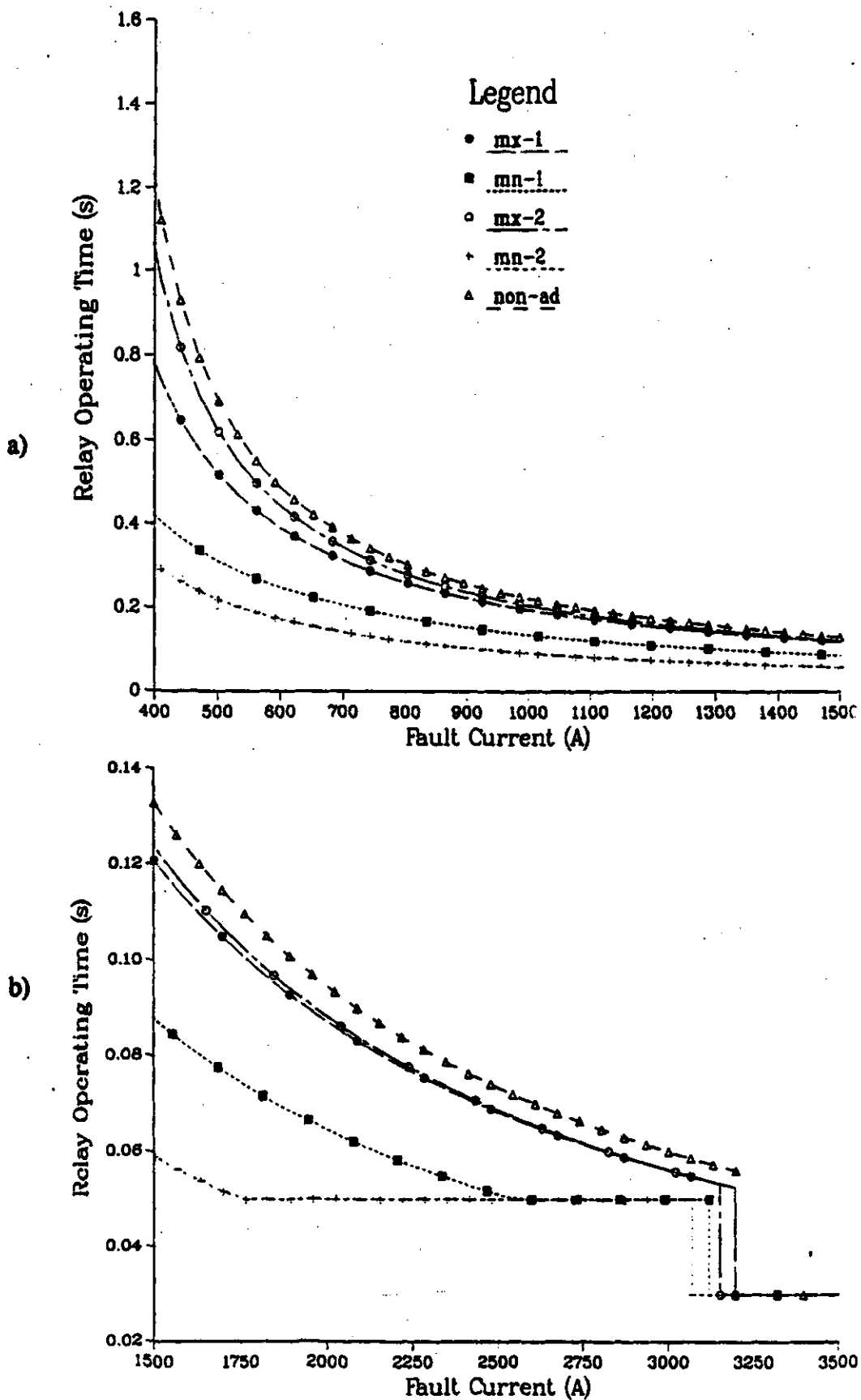
Relay	Operating condition	Load current (A)	Ct ratio for ground o/c relay	Max load/load	Desired relay setting in multiple of load current		Selected setting for ground relay in multiple of load current	Selected ground relay pickup (A)
					phase relay	ground relay		
1-1	MX-1	420	800:5	1.53	3.06	0.80	0.50	2.00
	MN-2	187		3.43	6.86	1.75		
	MX-2	642		1.00	2.00	0.50		
	MN-2	340		1.89	3.78	0.95		
1-2	MX-1	394	600:5	1.28	2.56	0.64	0.50	2.10
	MN-1	206		2.46	4.92	1.23		
	MX-2	506		1.00	2.00	0.50		
	MN-2	264		1.92	3.84	0.96		
1-3	MX-1	375	600:5	1.00	2.00	0.50	0.50	1.60
	MN-1	181		2.07	4.14	1.04		
	MX-2	---		---	---	---		
	MN-2	---		---	---	---		
2-1	MX-1	163	300:5	1.28	2.56	0.65	0.50	1.75
	MN-1	65		3.21	6.42	1.60		
	MX-2	209		1.00	2.00	0.50		
	MN-2	81		2.58	5.16	1.30		
2-2	MX-1	164	300:5	1.05	2.10	0.55	0.50	1.45
	MN-2	40		4.30	8.60	2.15		
	MX-2	172		1.00	2.00	0.50		
	MN-2	83		2.07	4.14	1.05		
3-1	MX-1	113	300:5	1.84	3.86	0.92	0.50	1.75
	MN-1	55		3.78	7.56	1.90		
	MX-2	208		1.00	2.00	0.50		
	MN-2	103		2.02	4.04	1.00		
3-2	MX-1	192	300:5	1.00	2.00	0.50	0.50	1.60
	MN-1	95		2.02	4.04	1.00		
	MX-2	110		1.89	3.78	0.95		
	MN-2	49		4.24	8.48	2.12		
4-1	MX-1	170	400:5	1.65	3.30	0.85	0.50	1.75
	MN-1	82		3.41	6.82	1.70		
	MX-2	280		1.00	2.00	0.50		
	MN-2	131		2.14	4.28	1.07		
4-2	MX-1	143	300:5	1.00	2.00	0.50	0.50	1.20
	MN-1	86		2.10	4.20	1.05		
	MX-2	89		1.61	3.22	0.80		
	MN-2	31		4.61	9.22	2.30		
5-1	MX-1	366	600:5	1.00	2.00	0.50	0.50	1.55
	MN-1	164		2.23	4.46	1.16		
	MX-2	333		1.10	2.20	0.55		
	MN-2	76		4.82	9.64	1.50		
5-2	MX-1	327	800:5	1.84	3.68	0.92	0.50	1.90
	MN-1	157		3.83	7.66	1.90		
	MX-2	602		1.00	2.00	0.50		
	MN-2	291		2.07	4.14	1.05		

**Table 6.10:** Selected relay pickup settings and calculated TMS values for adaptive protection of the selected distribution network.

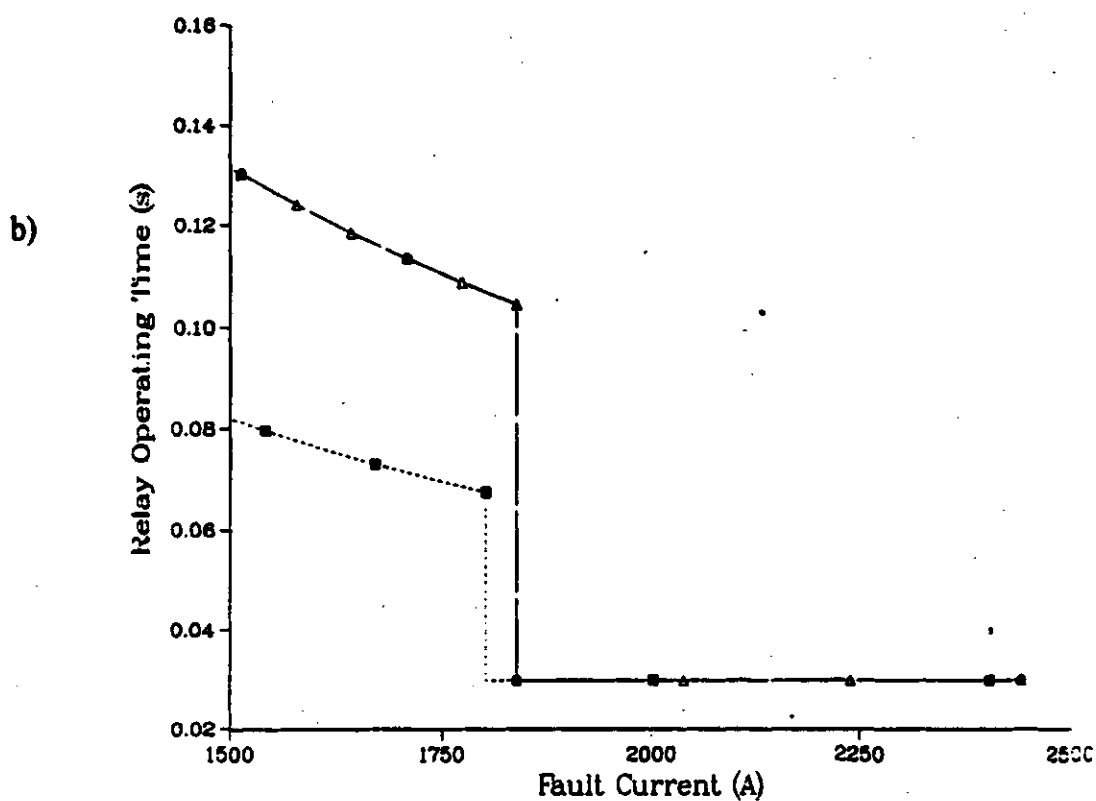
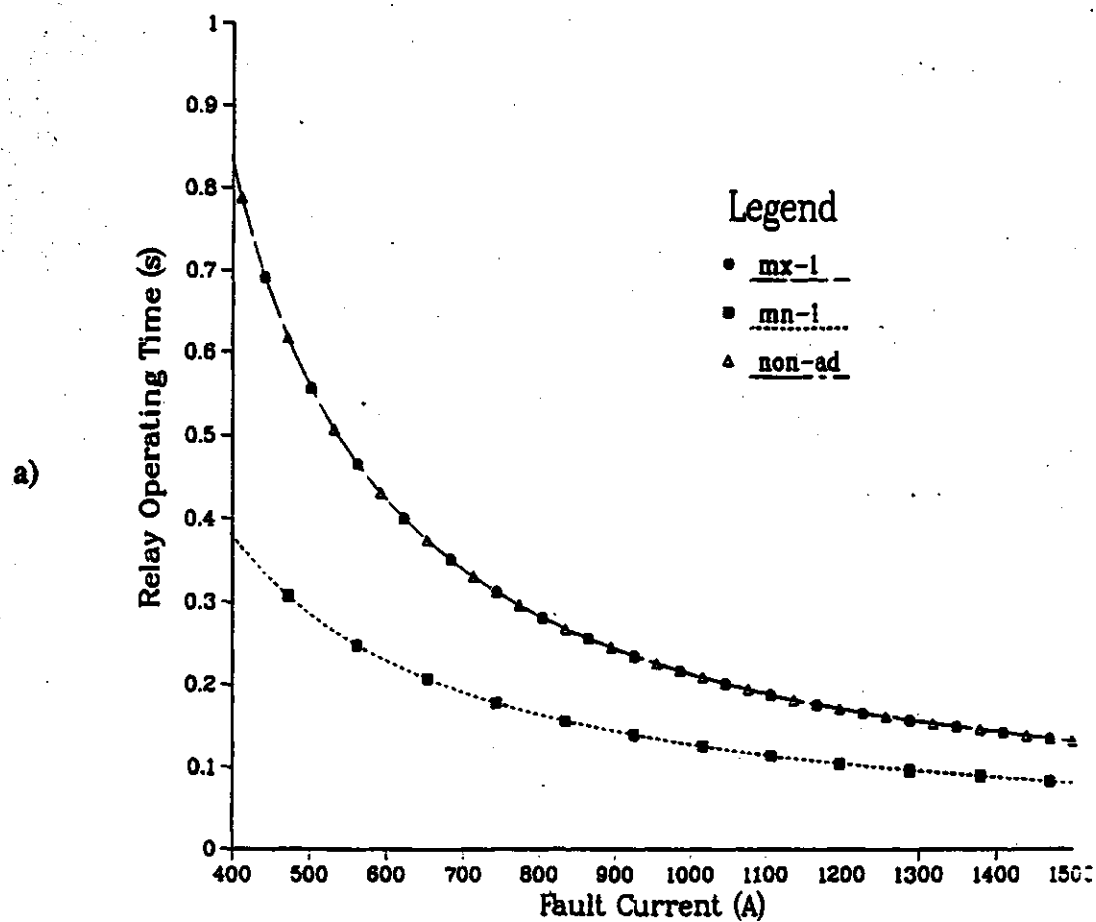
Relay	Operating condition	Load current (A)	Fault current for inst. relay setting (A)	Cts for ground o/c relay	Cts for inst. relay	Relay pickup setting (A)		TMS for ground o/c relay
						ground o/c	Inst.	
1-1	MX-1	420	3206	800:5	2000:5	1.30	8.00	0.06255
	MN-1	187	3140			0.60	7.50	0.09000
	MX-2	642	3234			2.00	8.25	0.03844
	MN-2	340	3164			1.10	8.00	0.07285
1-2	MX-1	394	4156	600:5	2000:5	1.65	10.25	0.06264
	MN-1	206	4056			0.90	10.00	0.16921
	MX-2	506	4098			2.10	10.25	0.04770
	MN-2	264	3996			1.10	10.00	0.09431
1-3	MX-1	375	2391	600:5	2000:5	1.60	6.00	0.06641
	MN-1	181	2343			0.80	6.00	0.08889
	MX-2	-----	-----			-----	-----	-----
	MN-2	-----	-----			-----	-----	-----
2-1	MX-1	163	5647	300:5	1000:5	1.40	28.00	0.08904
	MN-1	65	5476			0.60	27.50	0.23226
	MX-2	209	5630			1.75	28.00	0.08889
	MN-2	81	5456			0.70	27.30	0.21525
2-2	MX-1	164	3616	300:5	1000:5	1.40	18.00	0.16474
	MN-1	40	3531			0.50	17.50	0.62126
	MX-2	172	3618			1.45	18.00	0.14707
	MN-2	83	3531			0.70	17.50	0.35710
3-1	MX-1	113	2098	300:5	1000:5	0.95	10.50	0.12347
	MN-1	55	2046			0.50	10.25	0.26571
	MX-2	208	2050			1.75	10.25	0.08889
	MN-2	103	1994			0.90	10.00	0.13523
3-2	MX-1	192	2132	300:5	1000:5	1.60	10.50	0.08889
	MN-1	95	2080			0.80	10.25	0.12129
	MX-2	110	2129			0.95	10.50	0.08889
	MN-2	49	2077			0.50	10.25	0.20339
4-1	MX-1	170	1676	400:5	1000:5	1.10	8.50	0.08889
	MN-1	82	1652			0.55	8.25	0.11607
	MX-2	280	3525			1.75	17.75	0.08776
	MN-2	131	3412			0.85	17.00	0.09000
4-2	MX-1	143	2289	300:5	1000:5	1.20	11.25	0.10799
	MN-1	68	2328			0.60	11.00	0.25072
	MX-2	89	2280			0.75	11.25	0.18327
	MN-2	31	2225			0.50	11.00	0.35614
5-1	MX-1	366	3382	600:5	1000:5	1.55	17.00	0.06254
	MN-1	164	3278			0.70	16.25	0.08889
	MX-2	333	3897			1.40	19.50	0.06739
	MN-2	76	3774			0.50	19.00	0.09589
5-2	MX-1	327	2629	800:5	1000:5	1.05	13.25	0.06994
	MN-1	157	2549			0.50	12.75	0.08889
	MX-2	602	3445			1.90	17.25	0.03986
	MN-2	291	3398			0.95	17.00	0.08316



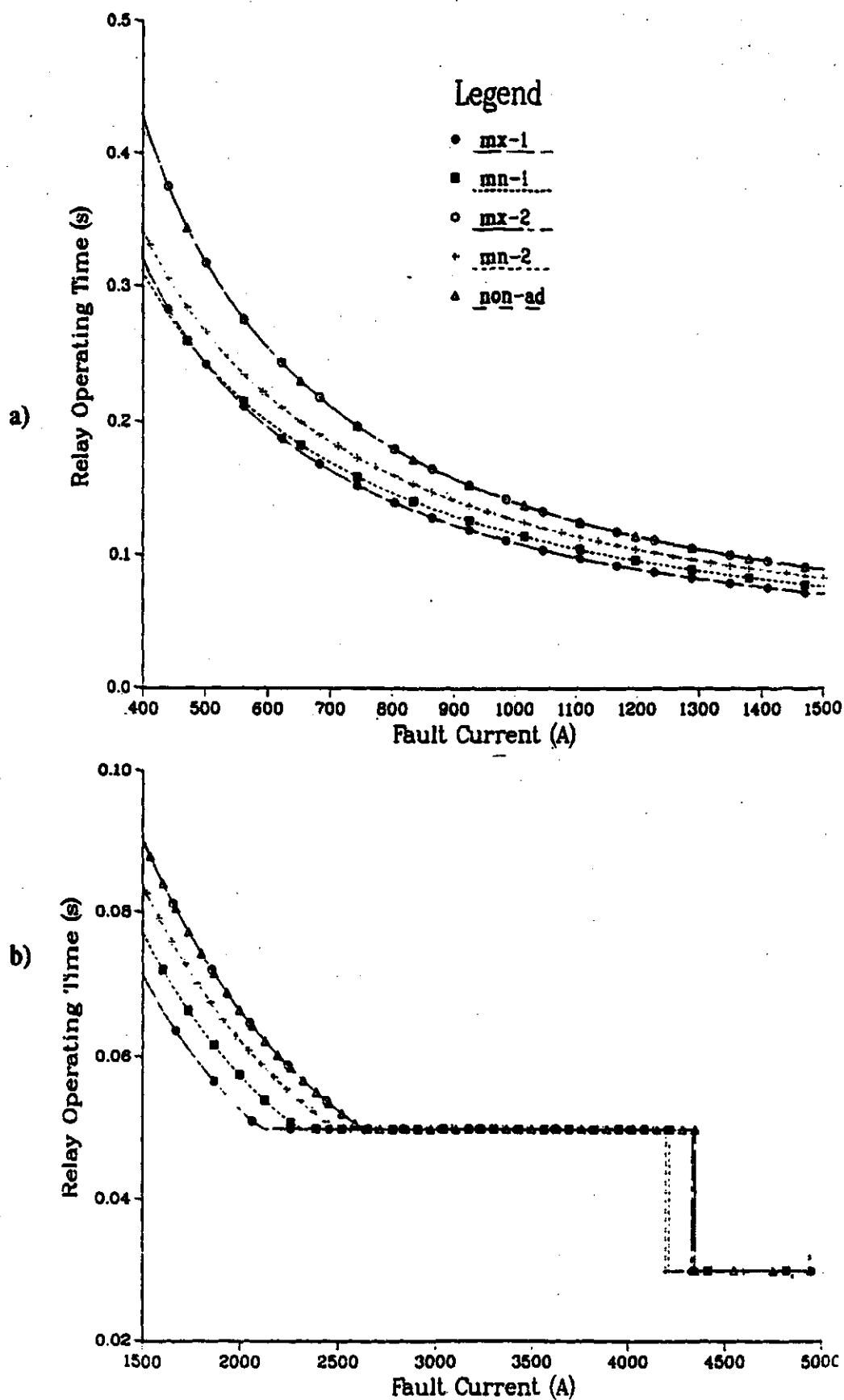
**Figure 6.2:** Operating Characteristics of relay 1-1: a) for fault currents 400-1500 A b) for fault currents higher than 1500 A.



**Figure 6.3:** Operating Characteristics of relay 1-2: a) for fault currents 400-1500 A  
b) for fault currents higher than 1500 A.



**Figure 6.4:** Operating Characteristics of relay 1-3: a) for fault currents 400-1500 A. b) for fault currents higher than 1500 A.



**Figure 6.5:** Operating Characteristics of relay 2-1: a) for fault currents 400-1500 A  
b) for fault currents higher than 1500 A.

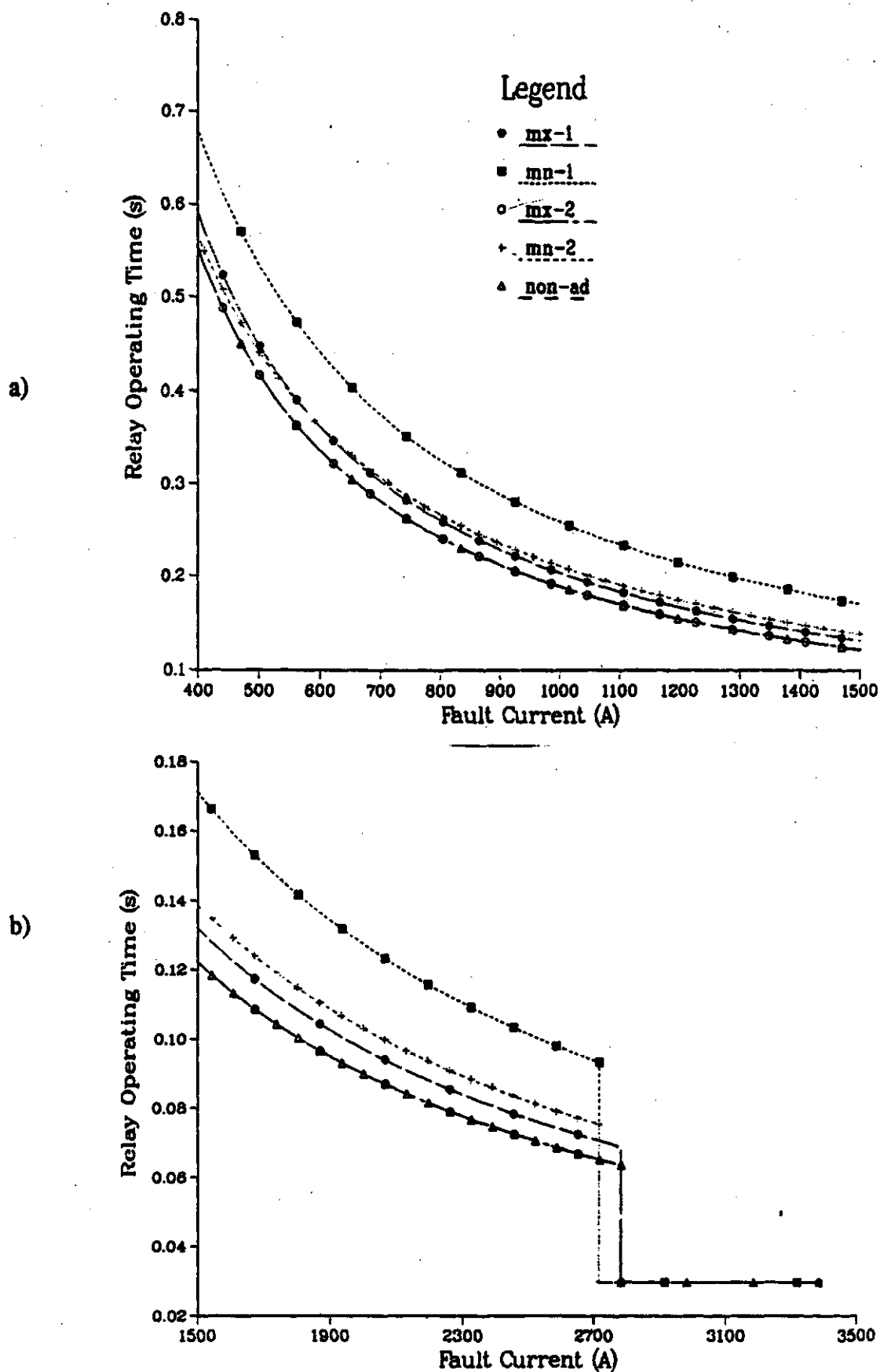


Figure 6.6: Operating Characteristics of relay 2-2: a) for fault currents 400-1500 A  
b) for fault currents higher than 1500 A.



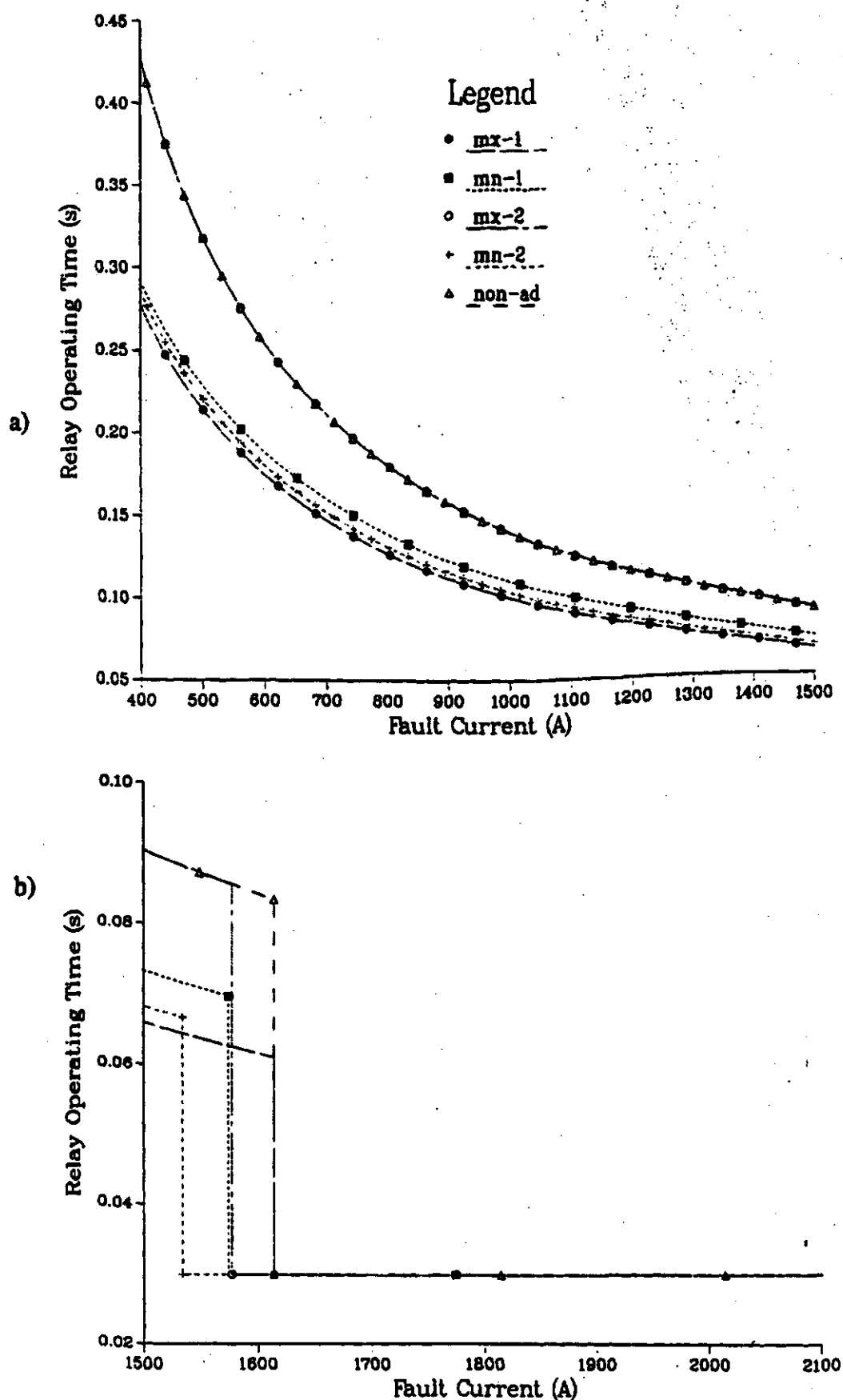


Figure 6.7: Operating Characteristics of relay 3-1: a) for fault currents 400-1500 A  
b) for fault currents higher than 1500 A.

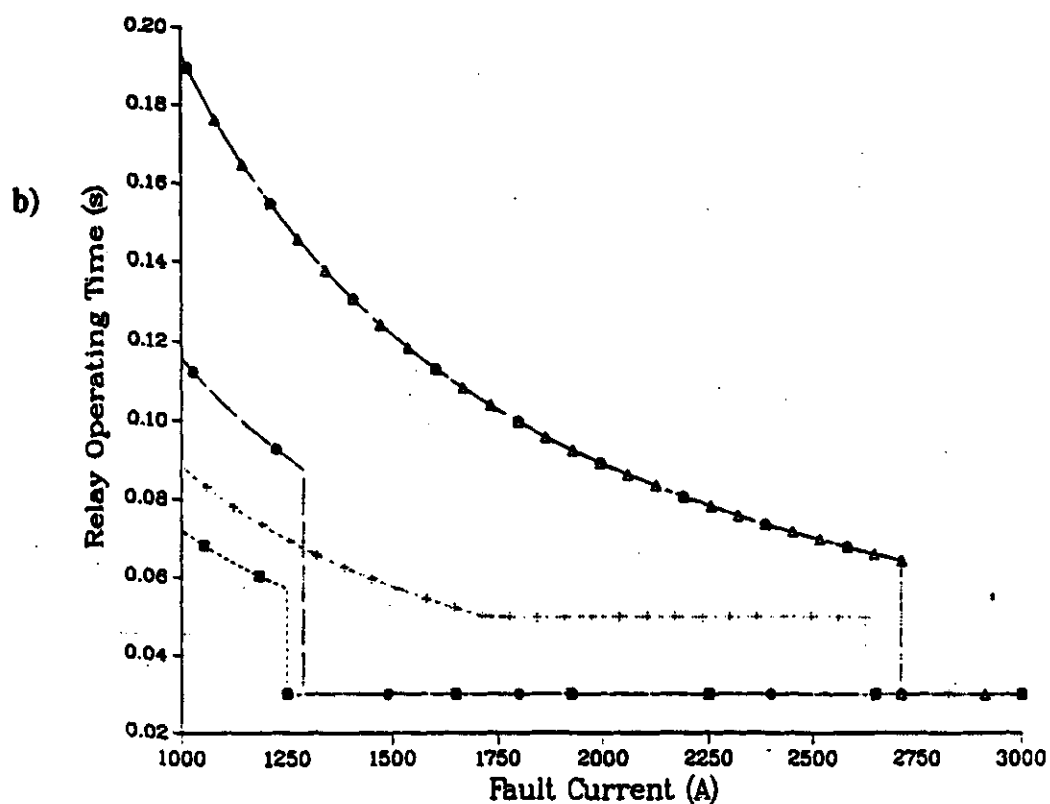
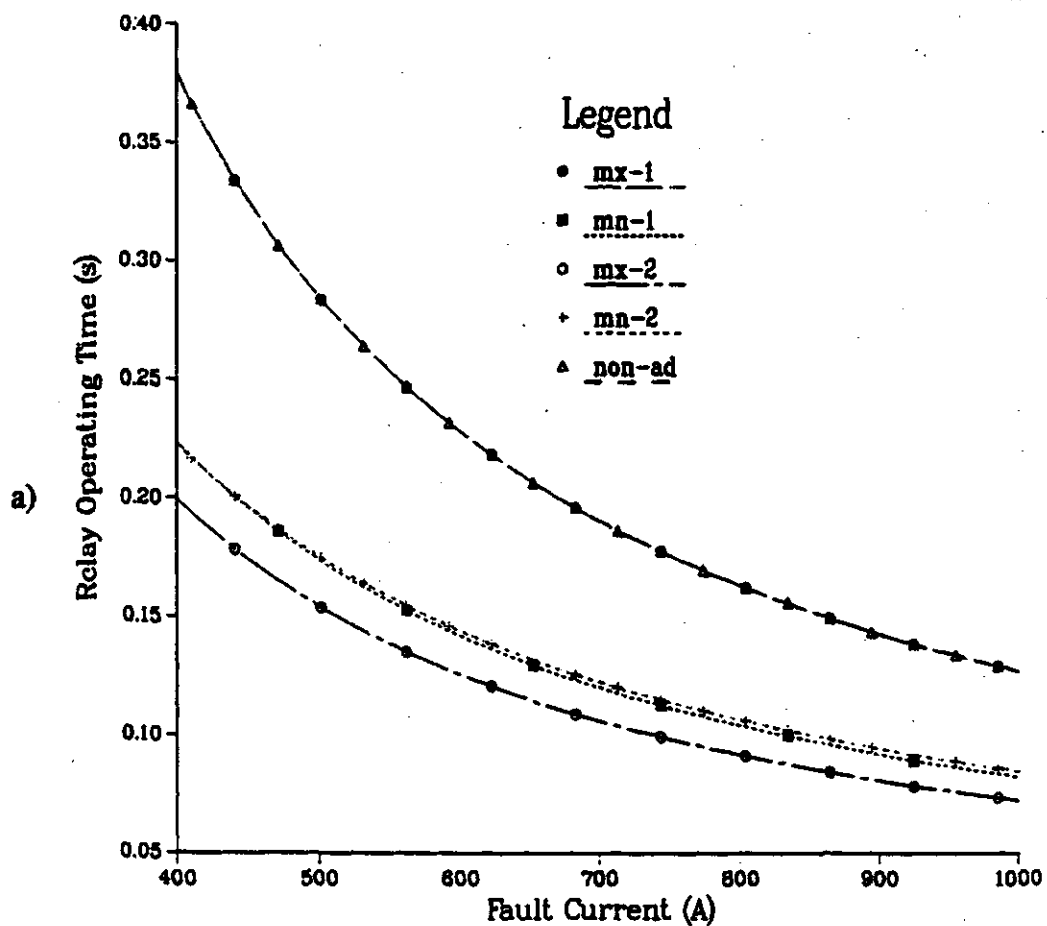


Figure 6.8: Operating Characteristics of relay 3-2: a) for fault currents 400-1000 A.  
b) for fault currents higher than 1000 A.

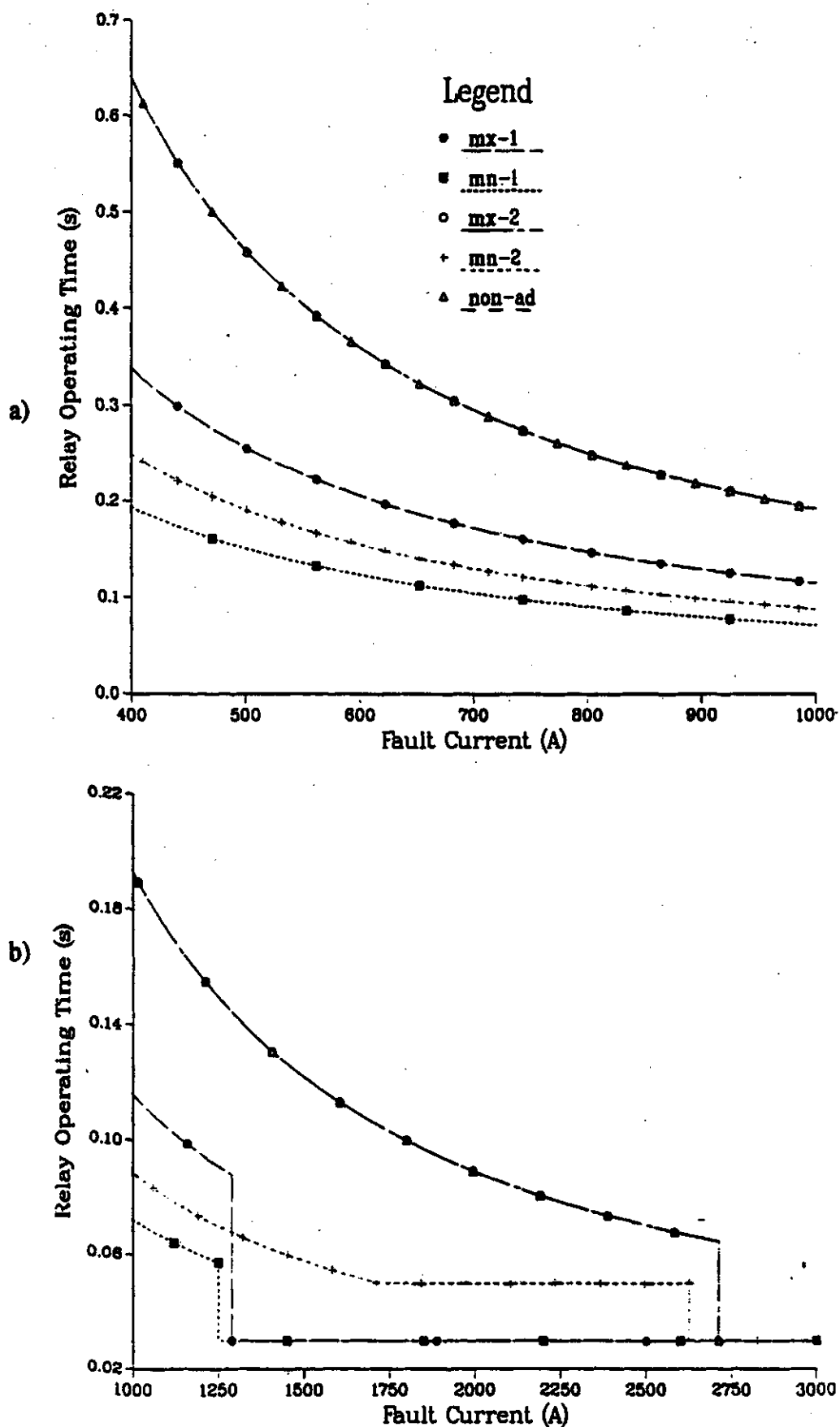


Figure 6.9: Operating Characteristics of relay 4-1: a) for fault currents 400-1000 A.  
b) for fault currents higher than 1000 A.

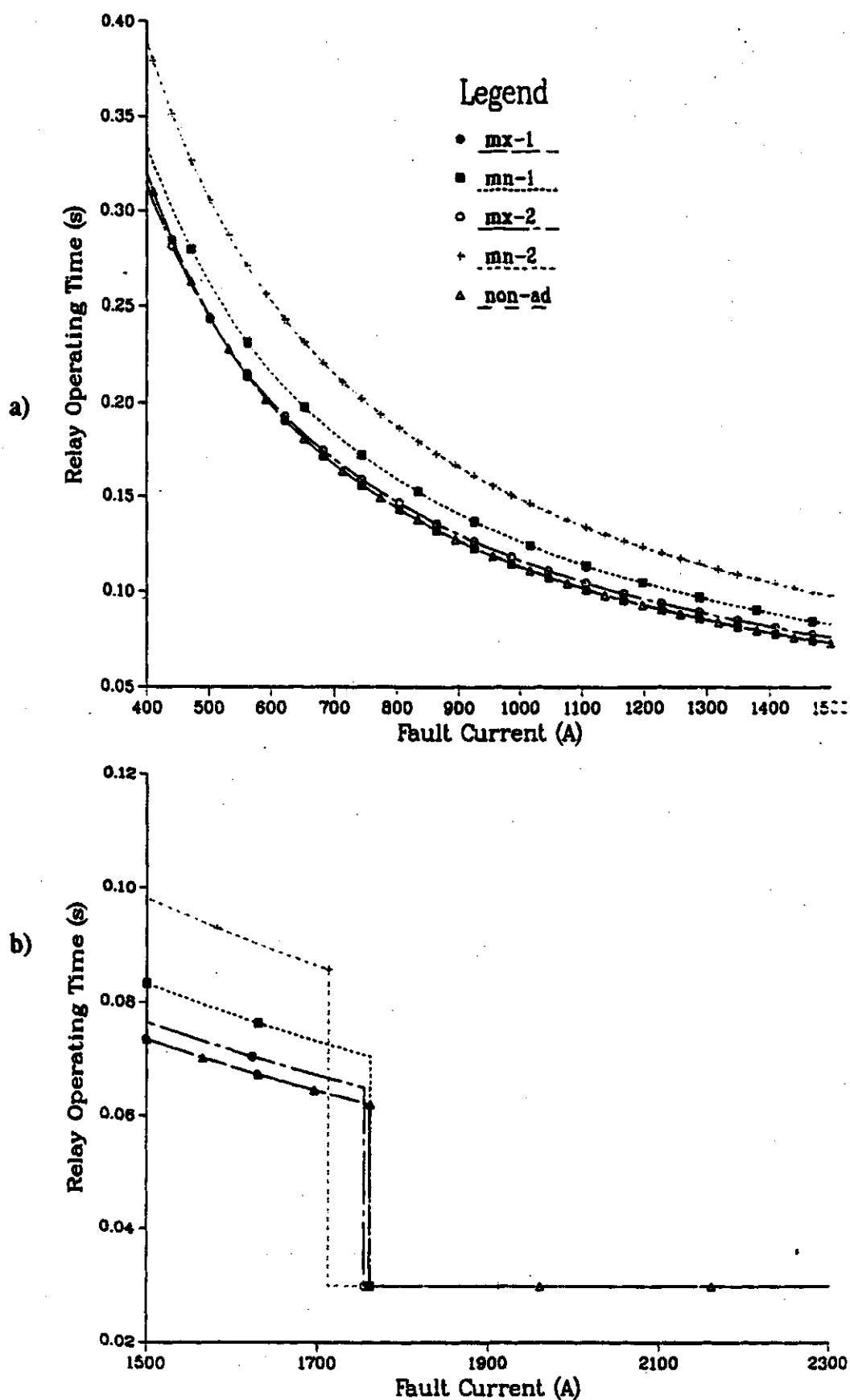


Figure 6.10: Operating Characteristics of relay 4-2: a) for fault currents 400-1500 A.  
b) for fault currents higher than 1500 A.

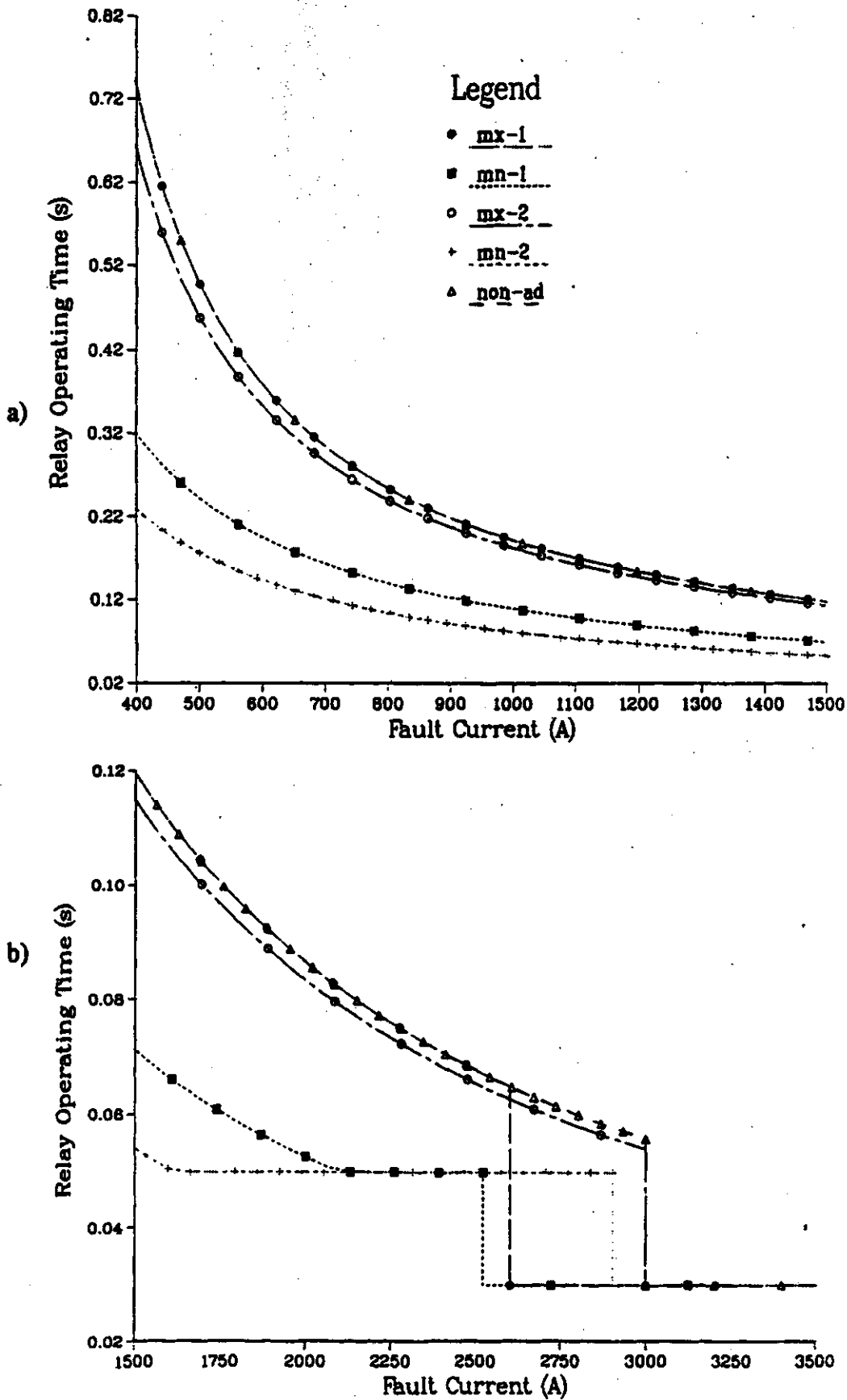
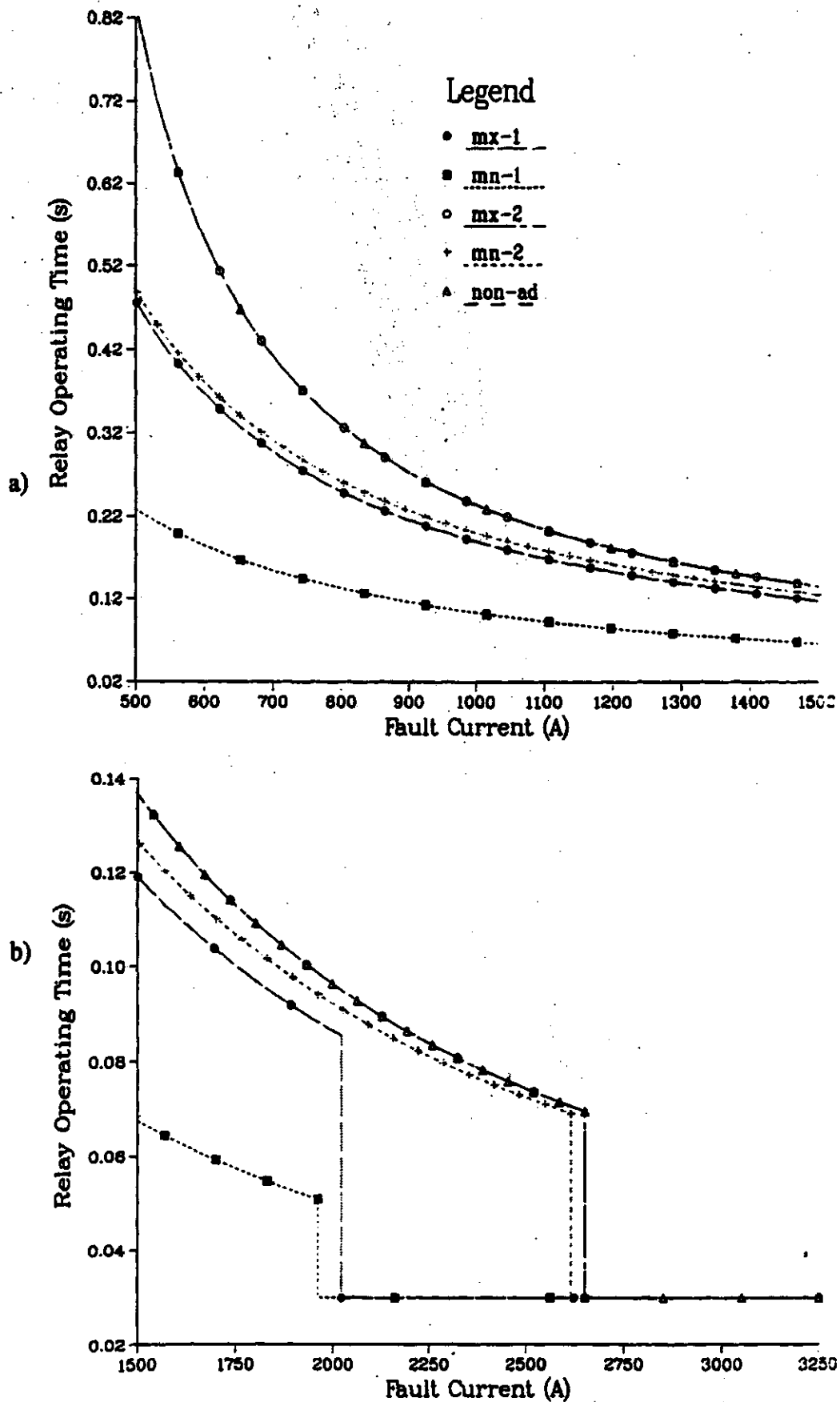
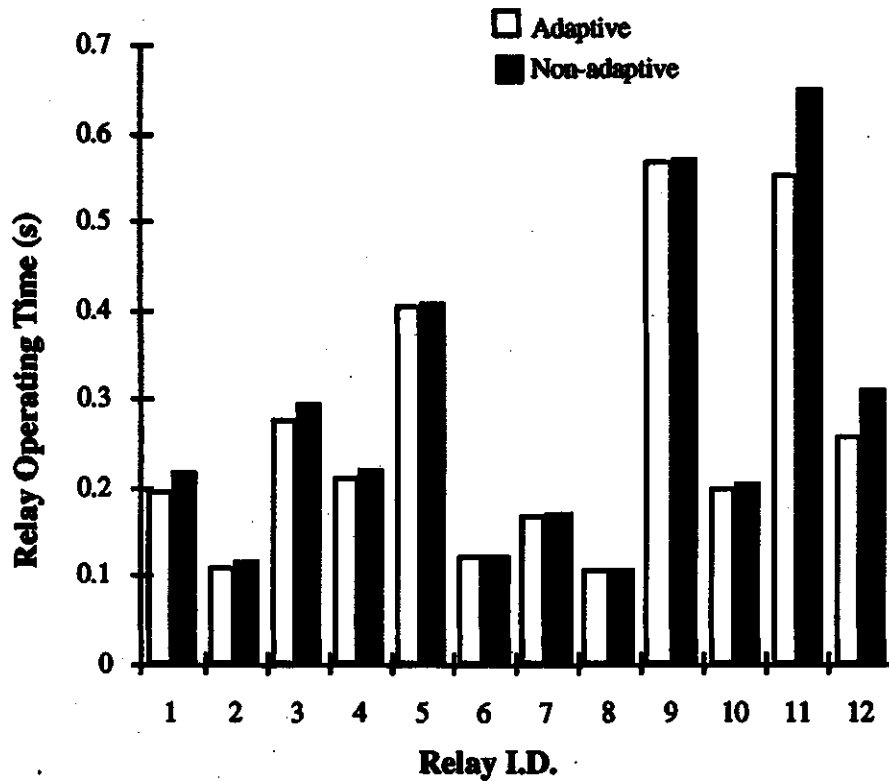


Figure 6.11: Operating Characteristics of relay 5-1: a) for fault currents 400-1500 A. b) for fault currents higher than 1500 A.

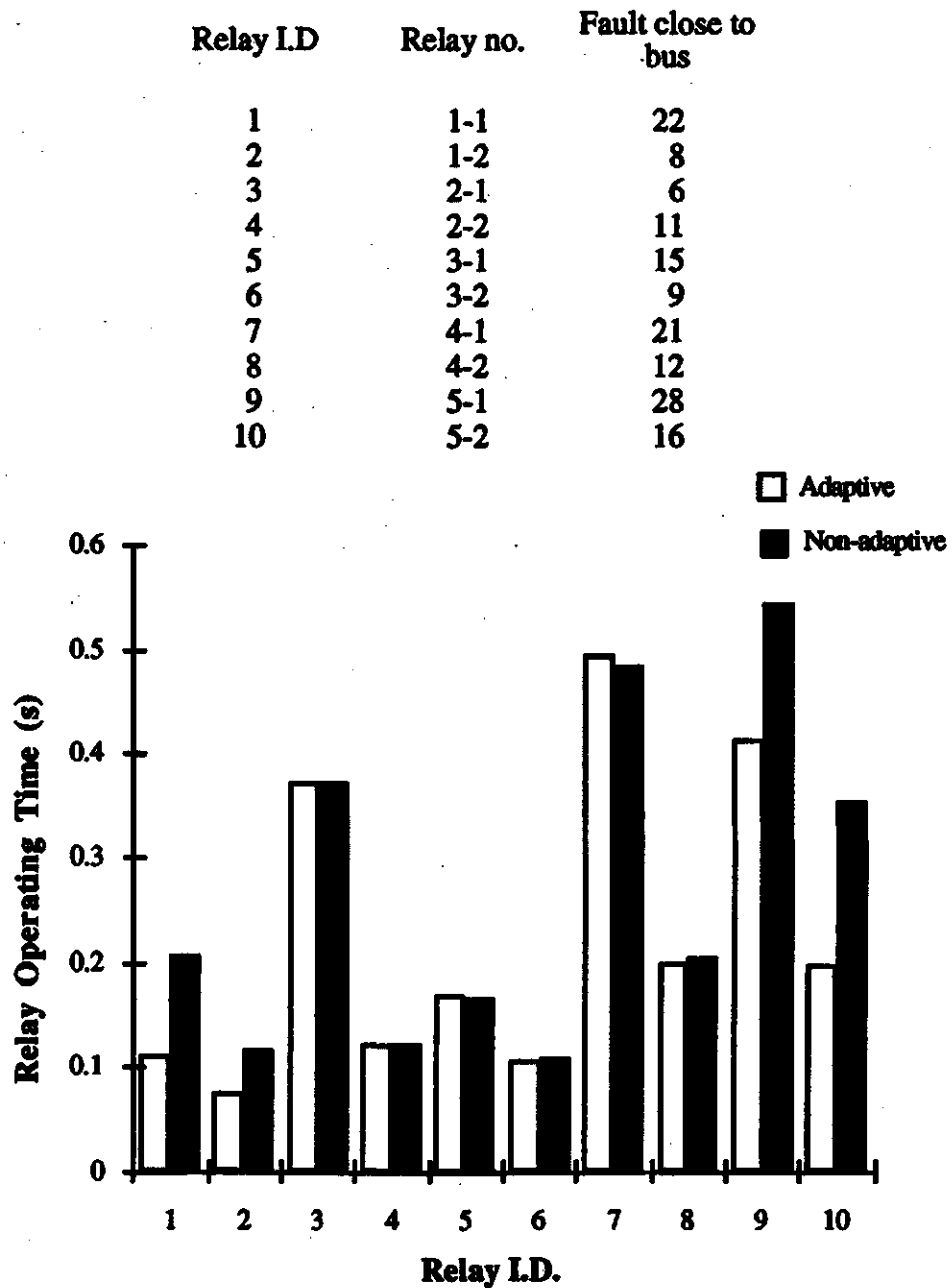


**Figure 6.12:** Operating Characteristics of relay 5-2: a) for fault currents 500-1500 A. b) for fault currents higher than 1500 A.

Relay I.D.	Relay no.	Fault close to bus
1	1- 1	22
2	1- 2	8
3	1-3	16
4	1-3	21
5	2-1	6
6	2-2	11
7	3-1	15
8	3-2	9
9	4-1	20
10	4-2	12
11	5-1	28
12	5-2	20



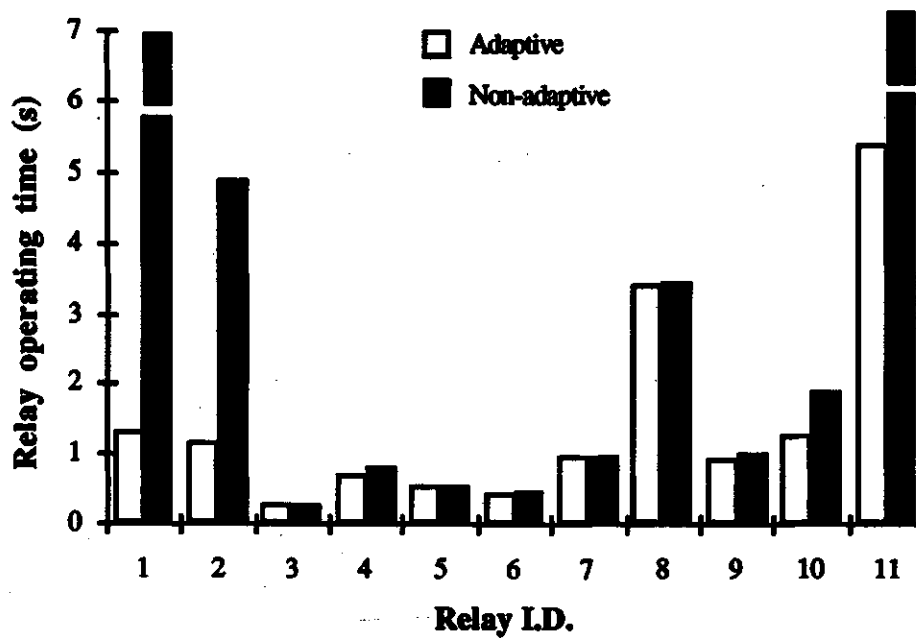
**Figure 6.13:** Primary relay operating times with and without adaptive settings for fault at far end (operating condition MN-1).



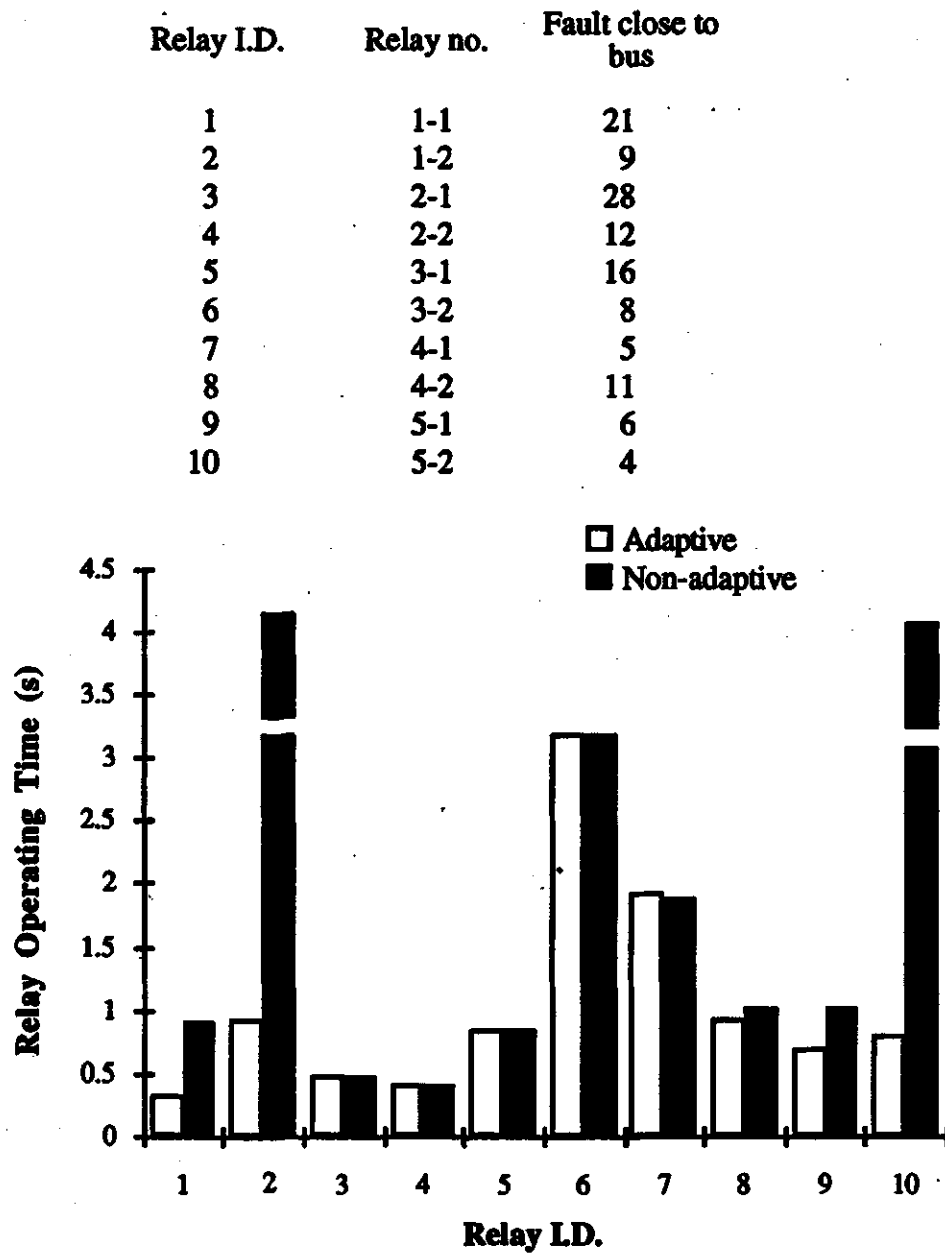
**Figure 6.14:** Primary relay operating times with and without adaptive settings for fault at far end (operating condition MN-2).



Relay I.D.	Relay no.	Fault close to bus
1	1-1	21
2	1-2	9
3	1-3	16
4	1-3	5
5	2-1	28
6	2-2	12
7	3-1	16
8	3-2	8
9	4-2	11
10	5-1	6
11	5-2	1



**Figure 6.15:** Backup relay operating times with and without adaptive settings for fault at far end (operating condition MN-1).



**Figure 6.16:** Backup relay operating times with and without adaptive settings for fault at far end (operating condition MN-2).

**Table 6.11:** Operating times of primary and backup relays for selected fault locations for operating condition MN-2.

Fault at bus	Relay Location	Primary Protection			Backup Protection		
		Operating Relay	Type of Protection	Operating Time(s)	Operating Relay	Type of Protection	Operating Time(s)
6	Near-end	1-2	inst.	0.033	4-1	o/c	0.652
	Far-end	2-1	o/c	0.118	3-2	o/c	5.620
8	Near-end	2-1	inst.	0.033	3-2	o/c	0.386
	Far-end	1-2	o/c	0.050	5-1	o/c	0.439
9	Near-end	2-2	inst.	0.033	1-2	o/c	0.374
	Far-end	3-2	o/c	0.050	4-2	o/c	0.275
11	Near-end	3-2	inst.	0.033	4-2	o/c	0.265
	Far-end	2-2	inst.	0.033	1-2	o/c	0.541
12	Near-end	3-1	inst.	0.033	2-2	o/c	0.071
	Far-end	4-2	o/c	0.050	5-2	o/c	-
15	Near-end	4-2	inst.	0.033	5-2	o/c	0.923
	Far-end	3-1	o/c	0.057	2-2	o/c	0.284
16	Near-end	4-1	inst.	0.033	3-1	o/c	0.217
	Far-end	5-2	o/c	0.116	2-2	o/c	0.943
21	Near-end	5-2	inst.	0.033	1-1	o/c	0.479
	Far-end	4-1	o/c	0.159	3-1	o/c	3.907
22	Near-end	5-1	inst.	0.033	4-1	o/c	0.551
	Far-end	1-1	o/c	0.061	2-1	o/c	0.648
28	Near-end	1-1	inst.	0.033	2-1	o/c	0.220
	Far-end	5-1	o/c	0.118	4-1	o/c	0.712

## **7. SUMMARY AND CONCLUSIONS**

### **7.1. Summary**

The purpose of protecting a power system is to limit damage to the system equipment due to faults or abnormal operating conditions. Relays detect abnormal operating conditions and faults and initiate opening of circuit breakers to isolate the faulted section of the system.

The concepts of protecting power systems from faults and abnormal operating conditions are described in Chapter 1. The concept of primary and backup protection is also briefly discussed. Protection devices used for protecting a distribution system are described in Chapter 2. The commonly used relays for protecting distribution systems include directional overcurrent and instantaneous overcurrent type relays. The use of directional inverse time overcurrent relays is generally necessary to protect looped distribution circuits. Instantaneous overcurrent relays are used to reduce the tripping times. Overcurrent digital relaying algorithm, proposed in the past have also been reviewed in this chapter. The probable nature of the distribution systems of the future, which will include increased automation and advanced protection schemes, is also briefly described.

Adaptive approaches are likely to be used for improved protection of distribution systems. The basic concept, purpose and functions of an adaptive protective scheme are presented in Chapter 3. The work already done and reported in the literature is reviewed. Software modules, that are necessary to implement an adaptive protection scheme, for a distribution network are also briefly outlined in this chapter.

A software module to detect the topology of a network was developed. The details of the topology detection software have been presented in Chapter 4. The data that were provided to the software module have been identified. The developed software was tested for three selected networks. It was demonstrated that the software works adequately for all the three selected networks. So it can be concluded that the software will work for networks of all size and complexity.

Besides topology detection, three other software modules are needed for coordinating the relays. The mathematical developments and the algorithms used in these modules are described in Chapter 5. The modules were tested for the selected network of the City of Saskatoon distribution system and the test results are presented in Chapter 6. The test results indicate that the software modules provide the intended performance. Chapter 6 then presents the results of the coordination studies of ground overcurrent relays for adaptive protection. The relay setting criteria are also illustrated in this chapter. Several operating conditions were considered and the performance of primary and backup relays using adaptive settings were compared to the performance of non-adaptive relays.

## **7.2. Conclusions**

In conclusion, the coordination of ground overcurrent relays for a distribution network using adaptive approach of protection has been described in this thesis. The studies demonstrated that the adaptive approach provides better protection compared to non-adaptive approach. The results show that the sensitivities of adaptive relays are better than those of non-adaptive relays. The results also show that the ground overcurrent relays remain coordinated and proper coordination margins are achieved for the selected operating conditions. Since the selected operating conditions represent the extreme state of the network, it is concluded that the relays will exhibit similar performance for all other operating conditions.

## **7.3. Suggestions for Future Work**

To implement adaptive protection in a distribution network there are certain aspects that need to be addressed further. Ground relays are coordinated by considering single phase to ground fault currents. Studies on relay coordination need to be carried out for other unbalanced faults. Cold-load inrush is frequently experienced in distribution systems. Relays should be adaptive to the cold-load inrush to prevent undesired trippings. Future work could address this problem. To isolate transient faults, auto reclosing of breakers are widely used in a distribution system. This feature could be included in future work.

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## **A. INPUT DATA AND TEST RESULTS OF TOPOLOGY DETECTION TECHNIQUE FOR SELECTED NETWORKS**

This appendix describes the three test systems used for studying the performance of the developed topology detection software. Each system is described in the subsections that follow. Substations, lines, circuit breakers, isolators and circuit elements are shown in the single line diagrams of the systems.

### **A.1. System 1: 6-Bus model system**

The single line diagram of System 1 [30] including circuit breakers, isolators and lines is shown in Figure A.1. This system has six substations interconnected by eleven lines. The assigned circuit element numbers and circuit breaker/isolator numbers are also included in the diagram. The off-line input data of the system are presented in Tables A.1 through A.3.

### **A.2. System 2: City of Saskatoon distribution model network**

A single line diagram for the reduced version of the City of Saskatoon Distribution system is shown in Figure A.2. This network was selected for implementing in the laboratory the adaptive protection scheme developed at the University of Saskatchewan [8]. The system contains six lines and five switching substations. However, loads are tapped at the substations and at different locations on the lines. Each load point is considered to be a bus, and the line section between two load points is considered as a distinct line. This is necessary for conveniently detecting topology and estimating the state of the system. Circuit breakers, isolators, buses, bus sections, lines, circuit elements, are assigned identifying numbers as discussed in Section 4.4. The off-line input data for the system is presented in Tables A.4 through A.6.

### **A.3. System 3: 24-Bus IEEE RTS**

A single line diagram of the IEEE Reliability Test System [34] is presented in Figure A.3. The system has 24 buses interconnected by 38 lines. The system also has 321 circuit breakers/isolators. The extended diagram of Substations 2 and 15 of the system are shown in Figures A.4 and A.5 respectively. The input data for circuit breakers and isolators installed at these substations are presented in Tables A.7 and A.8.

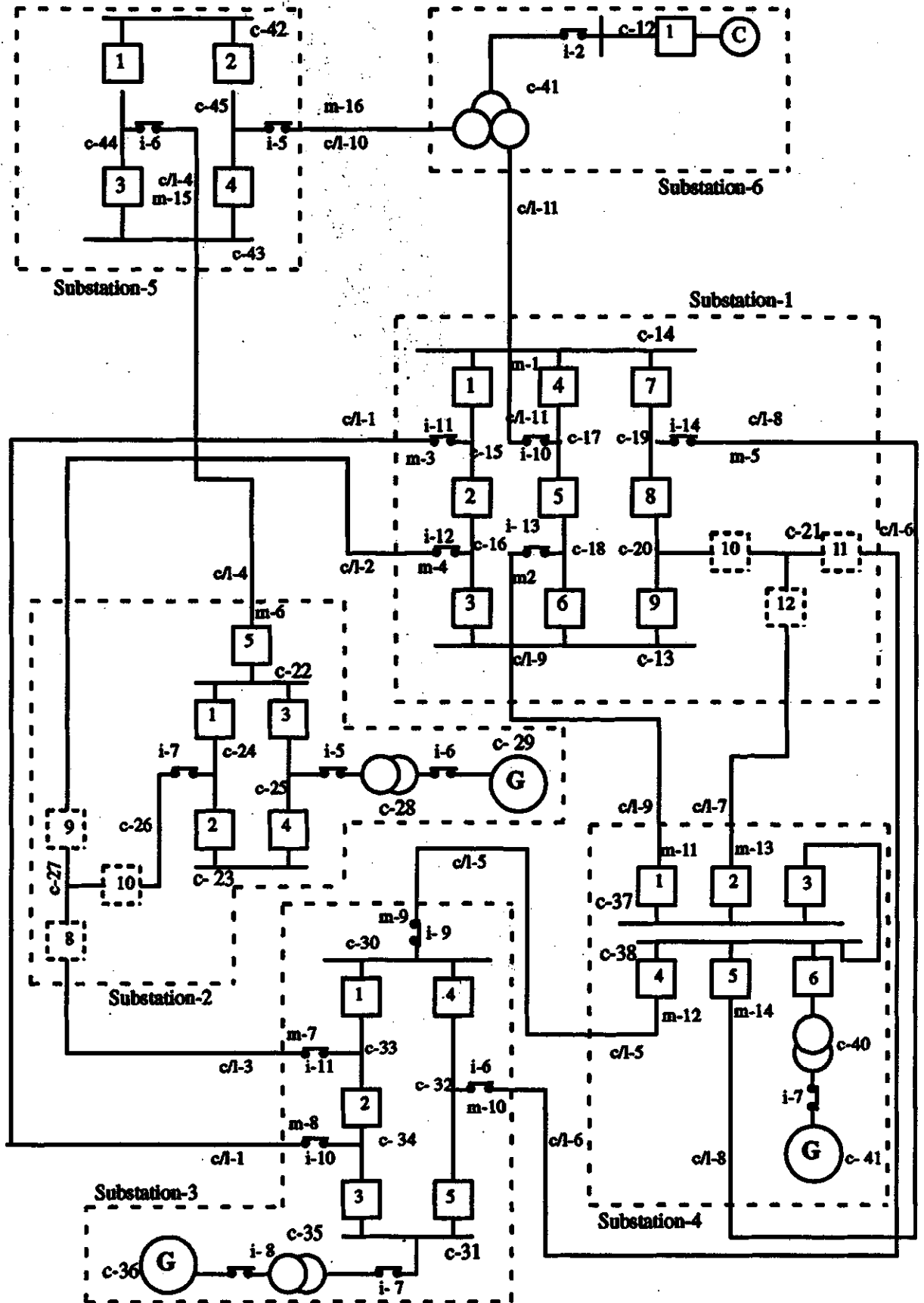
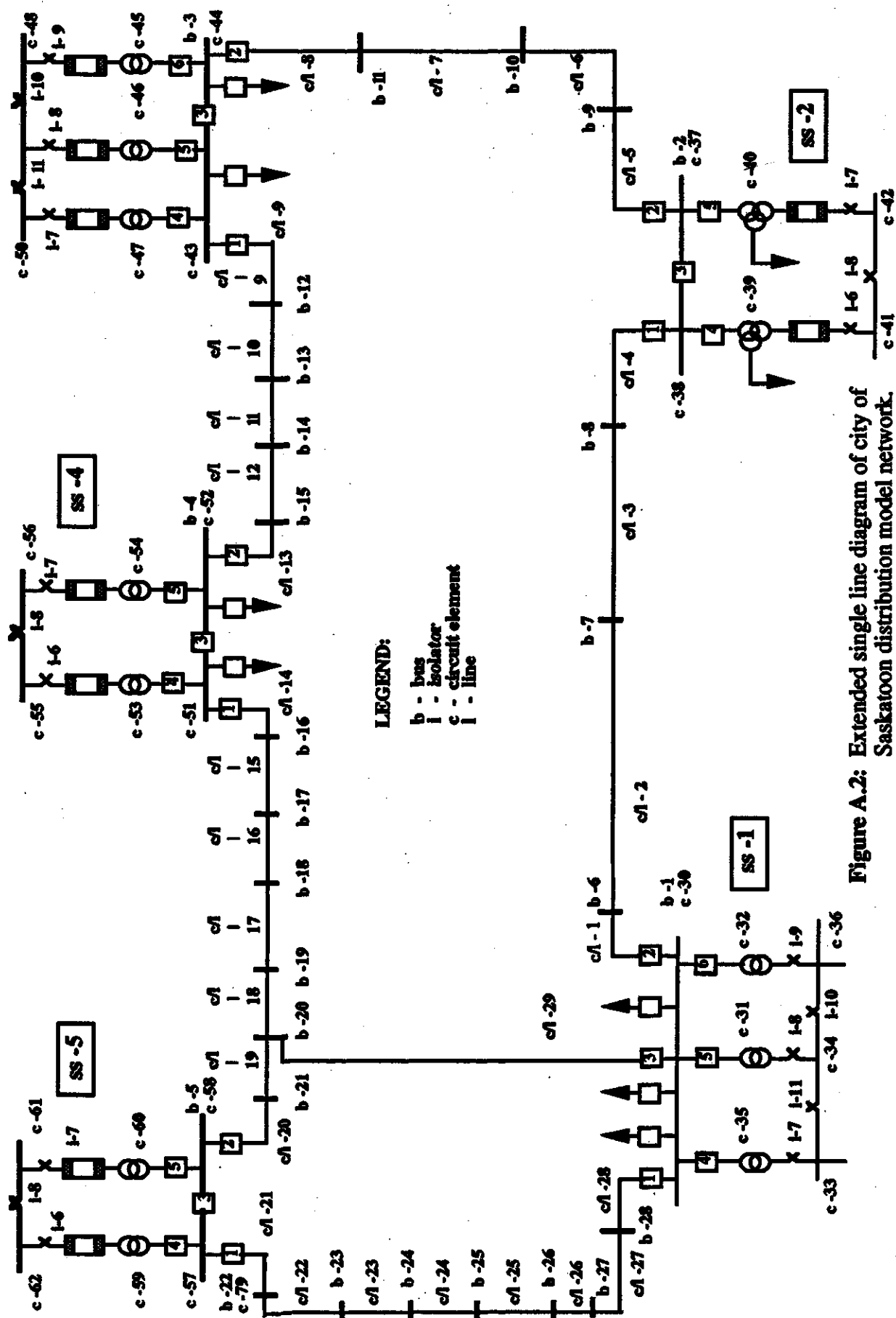


Figure A.1: Extended single line diagram of a 6-bus model system.



**Figure A.2: Extended single line diagram of city of Saskatoon distribution model network.**

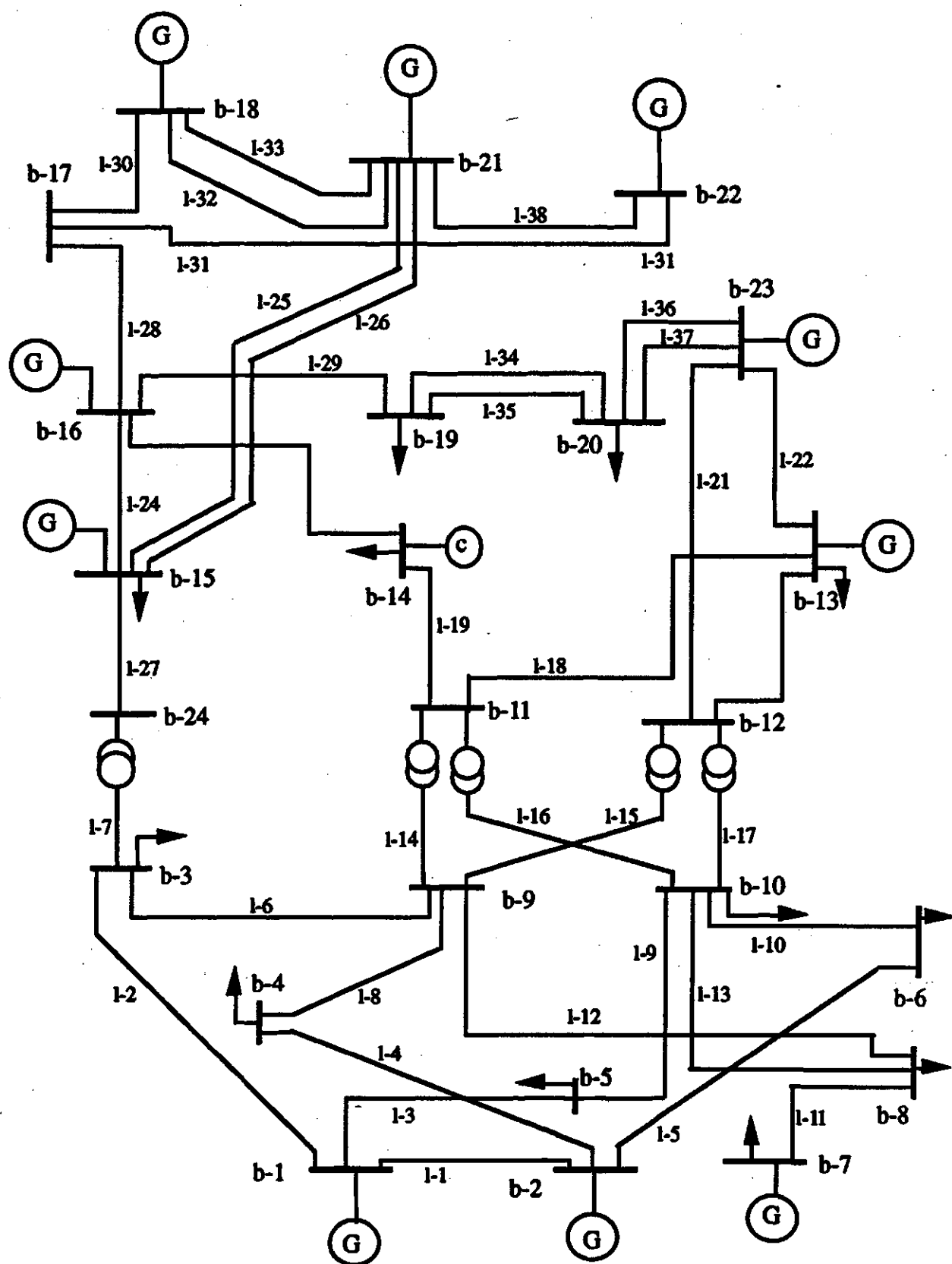
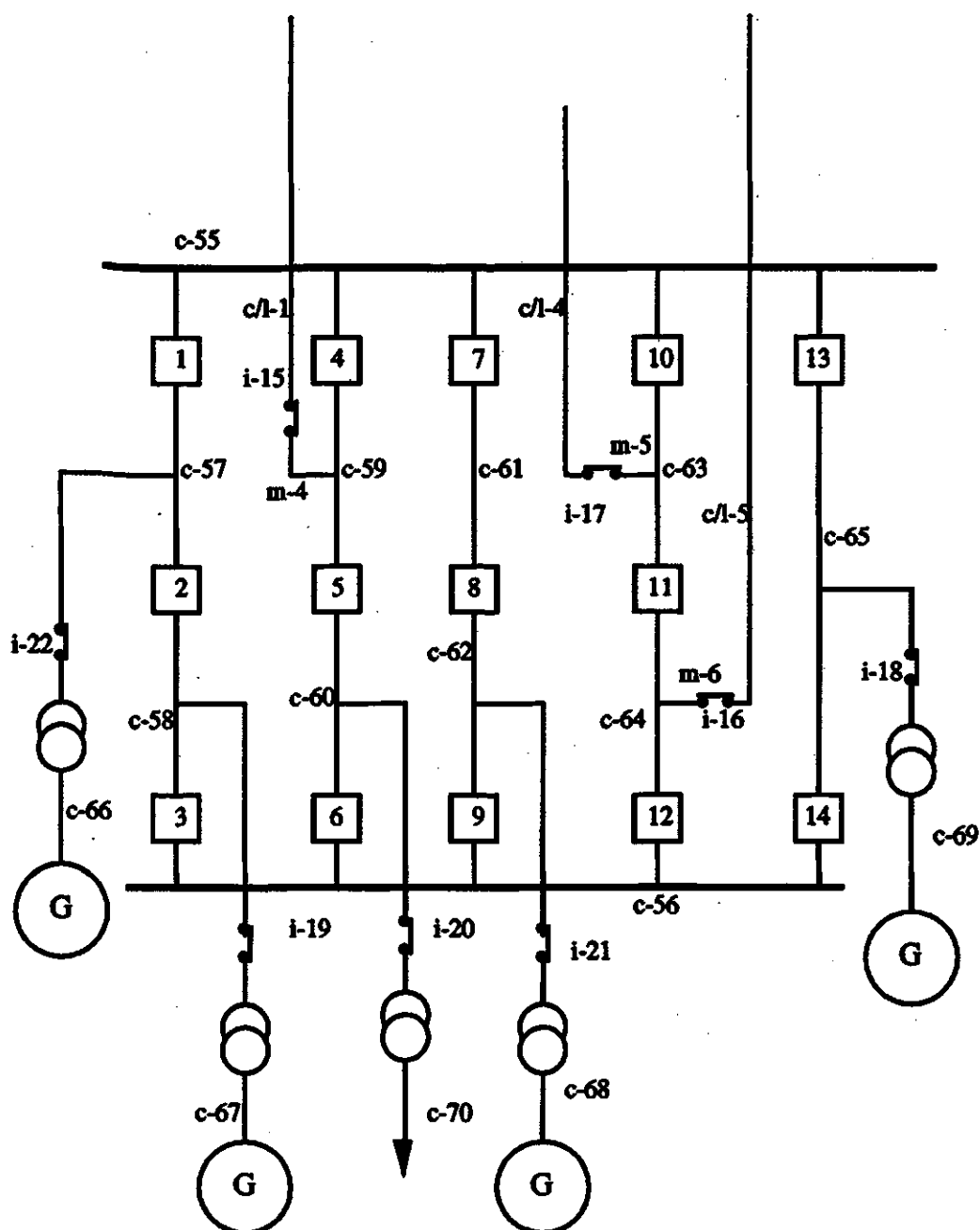
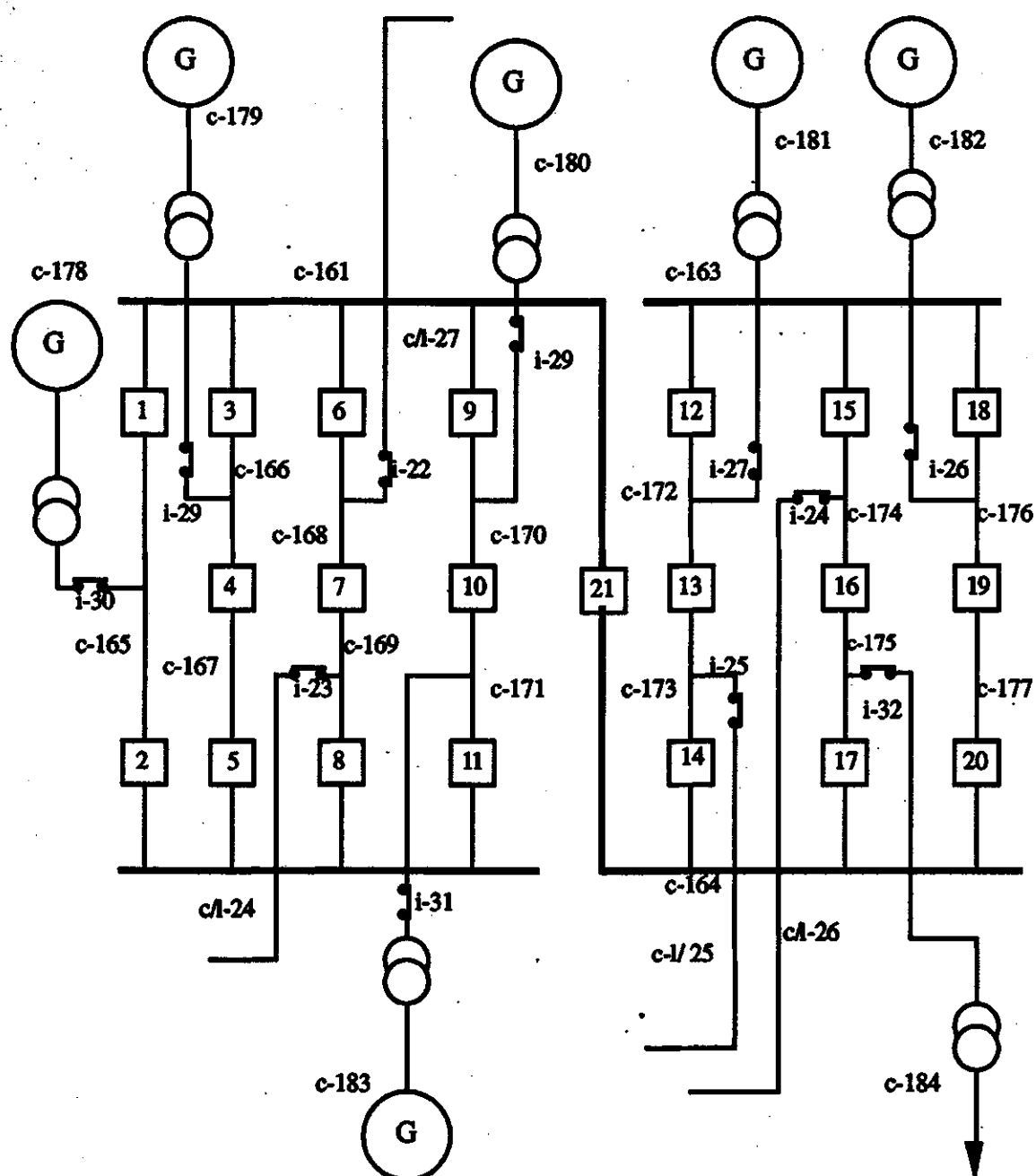


Figure A.3: Single line diagram of IEEE RTS



**Figure A.4:** Extended diagram showing circuit breakers, isolators and circuit elements at Substation 2 of System 3.



**Figure A.5:** Extended diagram showing circuit breakers, isolators and circuit elements at Substation 15 of System 3.

**Table A.1: Circuit breaker and isolator data for System 1.**

From ckt. element no.	To ckt element no.	At sub-stn. no	Normal bkr. status
14	15	1	0
15	16	1	0
16	13	1	0
14	17	1	0
17	18	1	0
18	13	1	0
14	19	1	0
19	20	1	0
20	13	1	0
20	21	1	0
21	6	1	0
21	7	1	0
9	18	1	0
8	19	1	0
11	17	1	0
1	15	1	0
2	16	1	0
22	24	2	0
24	23	2	0
22	25	2	0
25	23	2	0
22	4	2	0
29	28	2	0
24	26	2	0
27	3	2	0
2	27	2	0
27	26	2	0
25	28	2	0
30	33	3	0
33	34	3	0
34	31	3	0
30	32	3	0
32	31	3	0
32	6	3	0

**Table A.1: Continued.**

From ckt. element no.	To ckt element no.	At sub-stn. no	Normal bkr. status
31	35	3	0
35	36	3	0
5	30	3	0
1	34	3	0
3	33	3	0
37	9	4	0
37	7	4	0
37	38	4	0
38	5	4	0
38	8	4	0
38	40	4	0
39	40	4	0
42	44	5	0
42	45	5	0
44	43	5	0
43	45	5	0
45	10	5	0
44	4	5	0
12	41	6	0
12	46	6	0

**Table A.2: Line flow status data for System 1.**

Status no.	Near end sub- stn. no.	Far end sub- stn no.	Availability
1	1	6	0
2	1	4	0
3	1	3	0
4	1	2	0
5	1	4	0
6	2	5	0
7	3	2	0
8	3	1	0
9	3	4	0
10	3	1	0
11	4	1	0
12	4	3	0
13	4	1	0
14	4	1	0
15	5	2	0
16	5	6	0



**Table A.3: Index data for System 1.**

Line no.	Index I	Index II
1	3	8
2	4	0
3	7	0
4	6	15
5	9	12
6	10	0
7	13	0
8	14	5
9	11	2
10	16	0
11	1	0

**Table A.4: Circuit breaker and isolator data for System 2.**

From ckt. element no.	To ckt element no.	At sub-stn no.	Normal status
30	28	1	0
30	1	1	0
30	29	1	0
30	35	1	0
30	31	1	0
30	32	1	0
33	35	1	0
31	34	1	0
32	36	1	0
34	36	1	0
33	34	1	1
4	37	2	0
5	38	2	0
37	38	2	0
37	39	2	0
38	40	2	0

Table A.4: Continued.

From ckt. element no.	To ckt element no.	At sub-stn no.	Normal status
39	41	2	0
40	42	2	0
41	42	2	0
9	43	3	0
8	44	3	0
43	44	3	0
43	47	3	0
43	46	3	0
44	45	3	0
47	50	3	0
46	49	3	0
45	48	3	0
50	49	3	0
49	48	3	0
14	51	4	0
52	23	4	0
51	52	4	0
51	53	4	0
52	54	4	0
55	53	4	0
54	56	4	0
55	56	4	0
57	21	5	0
58	20	5	0
57	58	5	0
57	59	5	0
58	60	5	0
60	61	5	0
59	62	5	0
61	62	5	0

**Table A.5: Line flow status data for System 2.**

Status no.	Near end ckt. no.	Far end ckt. no.	Availability
1	1	28	0
2	28	1	0
3	1	6	0
4	6	1	0
5	2	8	0
6	8	2	0
7	2	9	0
8	9	2	0
9	3	12	0
10	12	3	0
11	3	11	0
12	11	3	0
13	4	16	0
14	16	4	0
15	4	15	0
16	15	4	0
17	5	22	0
18	22	5	0
19	5	21	0
20	21	5	0
21	6	7	0
22	7	8	0
23	9	10	0
24	11	10	0
25	13	12	0
26	14	13	0
27	15	14	0
28	16	17	0
29	18	17	0
30	19	18	0
31	21	20	0
32	22	23	0
33	24	23	0
34	25	24	0
35	26	25	0
36	27	26	0
37	28	27	0
38	1	20	0
39	20	19	0
40	20	1	0

**Table A.6: Index data for System 2.**

Line no.	Index I	Index II
1	3	4
2	21	0
3	22	0
4	5	6
5	7	8
6	23	0
7	24	0
8	11	12
9	9	10
10	25	0
11	26	0
12	27	0
13	15	16
14	13	14
15	28	0
16	29	0
17	30	0
18	39	0
19	31	0
20	19	20
21	17	18
22	32	0
23	33	0
24	34	0
25	35	0
26	36	0
27	37	0
28	1	2
29	38	40

**Table A.7: Circuit breaker and isolator data for Substation 2 of System 3.**

From ckt. no	To ckt. no.	At sub-stn.	Normal status
55	57	2	0
57	58	2	0
58	56	2	0
55	59	2	0
59	60	2	0
60	56	2	0
55	61	2	1
61	62	2	1
62	56	2	0
55	63	2	0
63	64	2	0
64	56	2	0
55	65	2	0
65	56	2	0
59	1	2	0
64	5	2	0
63	4	2	0
65	69	2	0
68	67	2	0
60	70	2	0
62	68	2	0
57	66	2	0

**Table A.8: Circuit breaker and isolator data for Substation 15 of System 3.**

From Ckt. no	To ckt. no.	At sub-stn.	Normal status
161	165	15	0
162	165	15	0
161	166	15	0
166	167	15	1
162	167	15	1
161	168	15	0
168	169	15	0
162	169	15	0
161	170	15	0
170	171	15	0
162	171	15	0
163	172	15	0
172	173	15	0
164	173	15	0
163	174	15	0
174	175	15	0
164	175	15	0
163	176	15	0
176	177	15	0
164	177	15	0
161	164	15	0
27	168	15	0
24	169	15	0
26	174	15	0
25	173	15	0
176	182	15	0
172	181	15	0
170	180	15	0
166	179	15	0
165	178	15	0
171	183	15	0
175	184	15	0

#### A.4. Results and discussion

The topology detection software was tested using the three systems described in the previous section. Three operating states of each system were considered. In the first operating state, all circuit breakers and line isolators are closed and all line flows are available (State I). The second operating state is obtained by opening circuit breakers and/or isolators which disconnect a line from the rest of the system (State II). The third operating state is obtained by opening circuit breakers and/or isolators in a station which split the substation into two parts or divide the system into two subsystems (State III). The results of these studies follow.

Table A.9 shows the output of the program for State I of System 1. Since there are no changes in the system, the table shows the normal line node connectivity of the network. Table A.10 shows the output for State II of this system. Circuit breakers 1 and 2 and isolator 11 in substation 1 are opened. As a result, line 1 is disconnected from the system. The program removes the line from the line node connectivity as is shown in the table. Table A.11 shows the output of for State III. In this case, closed circuit breaker 3 at Substation 4 is opened. As a consequence, the substation has been split into two parts. A new node, '# 7', is added with lines 5 and 8 connected to this node. These changes are shown in Table A.11 and are marked with asterisks.

The output for State I of System 2 is presented in Table A.12. The table shows all the lines that are present in the system. This is true since there are no changes of the normal status of circuit breakers and/or isolators. Table A.13 shows the output for State II of this system. Circuit breaker 3 at Substation 1 is opened. As a result, status of line 1-20 has changed from 'close' to 'open'. The program deletes this line from the output list of line node connectivity. For State III, circuit breaker 2 at Substation 1 and circuit breaker 1 at Substation 2 are opened. As seen from Figure A.2 these will cause the lines 1 through 4 to be separated from the system. The program will create two lists as shown in Table A.14. List B for the separated lines and List A for the rest of the lines. In this situation the lines of List A will be considered for further analysis for adaptive protection.

Table A.15 through Table A.17 show the output for System 3. Table A.15 shows the output when there are no changes of normal status of circuit breakers and/or isolators. All the lines of the system are appeared in the table. Table A.16 shows the output when circuit breaker 4 and 5 and isolator 15 at Substation 2 are opened. As a result line 1 is disconnected at this end and removed from the output table. For State III, circuit breaker

21 at Substation 15 has changed its 'closed' state to 'open' state. As seen from the Figure A.6, this operation will cause the station to split into two parts. One part of the station is provided with a new node, '# 25'. Lines 25 and 26 are connected to this new node after the analysis as marked with asterisks in the table.

**Table A.9:** Output of topology detection module for State I of System 1.

No changes of status of circuit breakers and/or isolators.

Serial no.	Line no.	From bus no.	To bus no.
1	1	1	3
2	2	1	2
3	3	3	2
4	4	2	5
5	5	3	4
6	6	3	1
7	7	4	1
8	8	4	1
9	9	4	1
10	10	5	6
11	11	1	6



**Table A.10: Output of topology detection module for State II of System 1.**

Changes of status of circuit breakers and/or isolators:

Station no. : 1 :: Breaker no. : 1 open

Station no. : 1 :: Breaker no. : 2 open

Station no. : 1 :: Isolator no. : 11 open

Serial no.	Line no.	From bus no.	To bus no.
1	2	1	2
2	3	3	2
3	4	2	5
4	5	3	4
5	6	3	1
6	7	4	1
7	8	4	1
8	9	4	1
9	10	5	6
10	11	1	6

**Table A.11: Output of topology detection module for State III of System 1.**

Changes of status of circuit breakers and/or isolators:

Station no. : 4 :: Breaker no. 3 open

Serial no.	Line no.	From bus no.	To bus no.
1	1	1	3
2	2	1	2
3	3	3	2
4	4	2	5
5	5	3	7*
6	6	3	1
7	7	4	1
8	8	7*	1
9	9	4	1
10	10	5	6
11	11	1	6

**Table A.12: Output of topology detection module for State I of System 2.****No changes of status of circuit breakers and/or isolators.**

Serial no.	Line no.	From bus no.	To bus no.
1	1	1	6
2	2	6	7
3	3	7	8
4	4	2	8
5	5	2	9
6	6	9	10
7	7	11	10
8	8	3	11
9	9	3	12
10	10	13	12
11	11	14	13
12	12	15	14
13	13	4	15
14	14	4	16
15	15	16	17
16	16	18	17
17	17	19	18
18	18	20	19
19	19	21	20
20	20	5	21
21	21	5	22
22	22	22	23
23	23	24	23
24	24	25	24
25	25	26	25
26	26	27	26
27	27	28	27
28	28	1	28

**Table A.13: Output of topology detection module for State II of System 2.**

Changes of status of circuit breakers and/or isolators:

Station no. : 1 :: Breaker no. : 3 open

Serial no.	Line no.	From bus no.	To bus no.
1	1	1	6
2	2	6	7
3	3	7	8
4	4	2	8
5	5	2	9
6	6	9	10
7	7	11	10
8	8	3	11
9	9	3	12
10	10	13	12
11	11	14	13
12	12	15	14
13	13	4	15
14	14	4	16
15	15	16	17
16	16	18	17
17	17	19	18
18	18	20	19
19	19	21	20
20	20	5	21
21	21	5	22
22	22	22	23
23	23	24	23
24	24	25	24
25	25	26	25
26	26	27	26
27	27	28	27
28	28	1	28

**Table A.14: Output of topology detection module for State III of System 2.**

Changes of status of breakers and/or isolators,

Station no. : 1 :: Breaker no. : 2 open

Station no. : 2 :: Breaker no. : 1 open

**List A:**

Serial no.	Line no.	From bus no.	To bus no.
1	5	2	9
2	6	9	10
3	7	11	10
4	8	3	11
5	9	3	12
6	10	13	12
7	11	14	13
8	12	15	14
9	13	4	15
10	14	4	16
11	15	16	17
12	16	18	17
13	17	19	18
14	18	20	19
15	19	21	20
16	20	5	21
17	21	5	22
18	22	22	23
19	23	24	23
20	24	25	24
21	25	26	25
22	26	27	26
23	27	28	27
24	28	1	28
25	29	1	20

**List B:**

Serial no.	Line no.	From bus no.	To bus no.
1	1	1	6
2	2	6	7
3	3	7	8
4	4	8	2

**Table A.15: Output of topology detection module for State I of System 3.**

No changes of status of circuit breakers and/or isolators.

Serial no.	Line no.	From bus no.	To bus no.
1	1	1	2
2	2	1	3
3	3	1	5
4	4	2	4
5	5	2	6
6	6	3	9
7	7	3	24
8	8	4	9
9	9	10	5
10	10	10	6
11	11	7	8
12	12	8	9
13	13	8	10
14	14	9	11
15	15	9	12
16	16	10	11
17	17	10	12
18	18	13	11
19	19	14	11
20	20	13	12
21	21	23	12
22	22	13	23
23	23	14	16
24	24	15	16
25	25	15	21
26	26	15	21
27	27	15	24
28	28	16	17
29	29	16	19
30	30	18	17
31	31	22	17
32	32	18	21
33	33	18	21
34	34	19	20
35	35	19	20
36	36	20	23
37	37	20	23
38	38	21	22

**Table A.16:** Output of topology detection module for State II of System 3.

Changes of status of circuit breakers and/or isolators:

Station no. : 2 :: breaker no. : 4 open

Station no. : 2 :: breaker no. : 5 open

Station no. : 2 :: isolator no. : 15 open

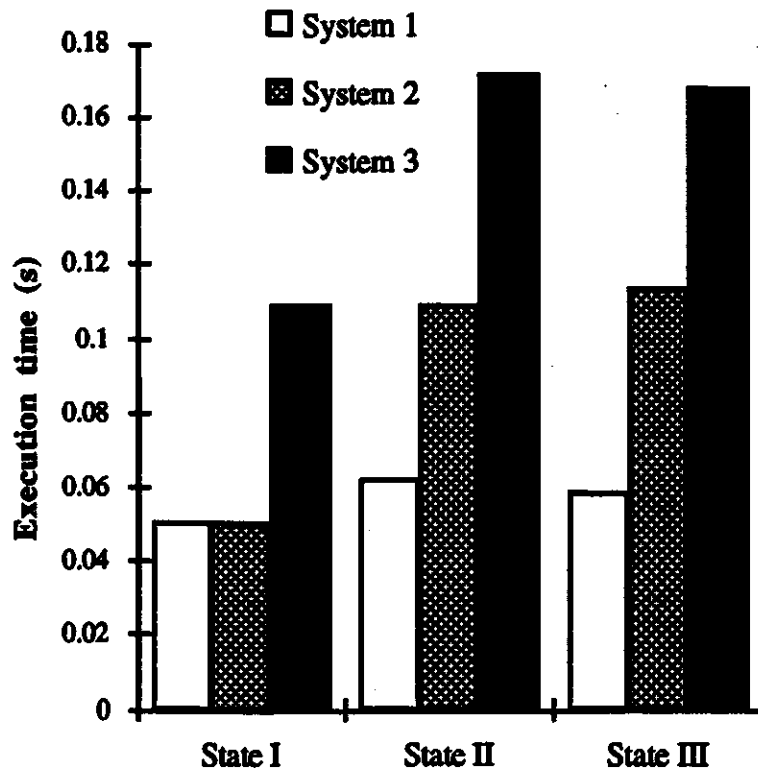
Serial no.	Line no.	From bus no.	To bus no.
1	2	1	3
2	3	1	5
3	4	2	4
4	5	2	6
5	6	3	9
6	7	3	24
7	8	4	9
8	9	10	5
9	10	10	6
10	11	7	8
11	12	8	9
12	13	8	10
13	14	9	11
14	15	9	12
15	16	10	11
16	17	10	12
17	18	13	11
18	19	14	11
19	20	13	12
20	21	23	12
21	22	13	23
22	23	14	16
23	24	15	16
24	25	15	21
25	26	15	21
26	27	15	24
27	28	16	17
28	29	16	19
29	30	18	17
30	31	22	17
31	32	18	21
32	33	18	21
33	34	19	20
34	35	19	20
35	36	20	23
36	37	20	23
37	38	21	22

**Table A.17: Output of topology detection module for State III of System 3.**

Changes of status of breakers and/or isolators:  
 Station no. : 15 :: breaker no. : 21 open

Serial no.	Line no.	From bus no.	To bus no.
1	1	1	2
2	2	1	3
3	3	1	5
4	4	2	4
5	5	2	6
6	6	3	9
7	7	3	24
8	8	4	9
9	9	10	5
10	10	10	6
11	11	7	8
12	12	8	9
13	13	8	10
14	14	9	11
15	15	9	12
16	16	10	11
17	17	10	12
18	18	13	11
19	19	14	11
20	20	13	12
21	21	23	12
22	22	13	23
23	23	14	16
24	24	15	16
25	25	25*	21
26	26	25*	21
27	27	15	24
28	28	16	17
29	29	16	19
30	30	18	17
31	31	22	17
32	32	18	21
33	33	18	21
34	34	19	20
35	35	19	20
36	36	20	23
37	37	20	23
38	38	21	22

Execution times the module took to process different states were checked and are shown in Figure A.6. It shows that the processing time increases with the increase of circuit breakers, isolators and lines. As mentioned earlier, the topology detection is a part of the on-line coordination process for adaptive protection. The execution times of 55 to 180 ms achieved by the topology detection technique are acceptable for implementing in adaptive protection schemes.



**Figure A.6:** Execution time for the topology detection module.



## B. LOAD FLOW TECHNIQUES (Derivation of Jacobian Matrix)

The calculation of Jacobian matrix for Newton Raphson method and decoupled load flow technique as well as Taylor series expansion of the power flow equations are explained in this appendix.

The active and reactive power flowing to a bus p of an n bus system are

$$P_p = \sum_{q=1}^n |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q), \quad (\text{B.1})$$

$$Q_p = - \sum_{q=1}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \quad (\text{B.1})$$

where:

$Y_{pq}$  is the (p, q) th element of the bus admittance matrix,  
 $\delta_p$  is the phase angle of voltage at bus p,  
 $\delta_q$  is the phase angle of voltage at bus q,  
 $V_p$  is the bus voltage at bus p and  
 $V_q$  is the bus voltage at bus q.

Rearrangement of Equation B.1 and B.2 provides

$$P_p = |V_p V_p Y_{pp}| \cos \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q), \quad (\text{B.3})$$

$$Q_p = -|V_p V_p Y_{pp}| \sin \theta_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q). \quad (\text{B.4})$$

An iterative solution of the equation is initiated by estimating the solution vectors  $[\delta^0 \ |V^0]^T$  and expanding each function in the neighborhood of this estimate using Taylor series of Equation B.3 and B.4. is

$$P_p(\delta_1^0 + \Delta \delta_1^0, \delta_2^0 + \Delta \delta_2^0, \dots, \delta_n^0 + \Delta \delta_n^0, |V_1^0| + \Delta |V_1^0|, |V_2^0| + \Delta |V_2^0|,$$

$$\begin{aligned}
 & \dots\dots\dots, |V_n^q| + \Delta |V_n^q| = P_p(\delta_1^0, \delta_2^0, \dots\dots\dots, \delta_n^0, |V_1^q|, |V_2^q| \dots\dots\dots \\
 & \dots\dots\dots, |V_n^q| + \sum_{q=1}^n \frac{\partial P_p}{\partial \delta_q} \Delta \delta_q + \sum_{q=1}^n \frac{\partial P_p}{\partial |V_q|} \Delta V_q,
 \end{aligned} \tag{B.5}$$

$$\begin{aligned}
 & Q_p(\delta_1^0 + \Delta \delta_1^0, \delta_2^0 + \Delta \delta_2^0, \dots\dots\dots, \delta_n^0 + \Delta \delta_n^0, |V_1^q| + \Delta |V_1^q|, |V_2^q| + \Delta |V_2^q| \\
 & \dots\dots\dots, |V_n^q| + \Delta |V_n^q|) = Q_p(\delta_1^0, \delta_2^0, \dots\dots\dots, \delta_n^0, |V_1^q|, |V_2^q| \dots\dots\dots \\
 & \dots\dots\dots, |V_n^q| + \sum_{q=1}^n \frac{\partial Q_p}{\partial \delta_q} \Delta \delta_q + \sum_{q=1}^n \frac{\partial Q_p}{\partial |V_q|} \Delta V_q.
 \end{aligned} \tag{B.6}$$

The left hand side of Equation B.5 is the specified active power injection into bus p and the first term of right hand side is the active power injection calculated using the estimated voltages. So the mismatch i.e. the difference between scheduled and calculated value is

$$\Delta P_p = \sum_{q=1}^n \frac{\partial P_p}{\partial \delta_q} \Delta \delta_q + \sum_{q=1}^n \frac{\partial P_p}{\partial |V_q|} \Delta V_q. \tag{B.7}$$

Similarly from Equation B.6

$$\Delta Q_p = \sum_{q=1}^n \frac{\partial Q_p}{\partial \delta_q} \Delta \delta_q + \sum_{q=1}^n \frac{\partial Q_p}{\partial |V_q|} \Delta V_q. \tag{B.8}$$

Expressing Equation B.7 for all but the slack bus, and B.8 for all load bus, the following set of equation is obtained:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V/V \end{bmatrix}, \tag{B.9}$$

where:

$J_1, J_2, J_3$  &  $J_4$  are the submatrices of the Jacobian,  
 $\Delta \delta$  is the vector increase in phase angle and  
 $\Delta V$  is the increment in voltage magnitude vector.

The elements of Jacobian are

for  $J_1$ :

$$\begin{aligned}\frac{\partial P_p}{\partial \delta_q} &= |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad q \neq p, \\ \frac{\partial P_p}{\partial \delta_p} &= - \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q).\end{aligned}\tag{B.10}$$

for  $J_2$ :

$$\begin{aligned}\frac{\partial P_p}{\partial |V_q|} &= |V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad q \neq p, \\ \frac{\partial P_p}{\partial |V_p|} &= 2|V_p Y_{pp}| \cos \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_p Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q),\end{aligned}\tag{B.11}$$

for  $J_3$ :

$$\begin{aligned}\frac{\partial Q_p}{\partial \delta_q} &= -|V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad q \neq p, \\ \frac{\partial Q_p}{\partial \delta_p} &= \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q),\end{aligned}\tag{B.12}$$

and for  $J_4$ :

$$\begin{aligned}\frac{\partial Q_p}{\partial |V_q|} &= |V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad q \neq p, \\ \frac{\partial Q_p}{\partial |V_p|} &= 2 \sin \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_p Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q).\end{aligned}\tag{B.13}$$

For first decoupled load flow the first step of decoupling is to neglect  $J_2$ ,  $J_3$  in Equation B.9. So, Equation B.9 can be written for decoupled load flow technique as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V/V \end{bmatrix}.\tag{B.14}$$

or

$$\begin{aligned}\Delta P &= H \Delta \delta \\ \Delta Q &= L \frac{\Delta |V|}{|V|},\end{aligned}\tag{B.15}$$

where H & L equivalent to  $J_1$ ,  $J_4$  respectively.

The elements of Jacobian (i.e. H & L) can be calculated using Equation B.3 and B.4. The off-diagonal elements of H are

$$\begin{aligned}
 H_{pq} &= |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \\
 \text{or } H_{pq} &= |V_p V_q| [Y_{pq} \sin \theta_{pq} \cos(\delta_p - \delta_q) + Y_{pq} \cos \theta_{pq} \sin(\delta_p - \delta_q)] \\
 H_{pq} &= |V_p V_q| [G_{pq} \cos(\delta_p - \delta_q) + B_{pq} \sin(\delta_p - \delta_q)]. \quad (B.16)
 \end{aligned}$$

Off-diagonal elements of L are

$$\begin{aligned}
 \text{or } L_{pq} &= |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \\
 L_{pq} &= |V_p V_q| [G_{pq} \sin(\delta_p - \delta_q) - B_{pq} \cos(\delta_p - \delta_q)]. \quad (B.17)
 \end{aligned}$$

Diagonal elements of H are

$$\begin{aligned}
 H_{pp} &= - \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \\
 \text{or } H_{pp} &= -Q_p - V_p^2 B_{pp}. \quad (B.18)
 \end{aligned}$$

and diagonal elements of L are

$$\begin{aligned}
 L_{pp} &= |2V_p^2 Y_{pp}| \sin \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \\
 L_{pp} &= Q_p + |V_p^2 Y_{pp}| \sin \theta_{pp} \\
 L_{pp} &= Q_p - V_p^2 B_{pp}. \quad (B.19)
 \end{aligned}$$

In case of fast decoupled load flow problem, the following assumptions are made:

$$\cos(\delta_p - \delta_q) \cong 1 ; G_{pq} \sin(\delta_p - \delta_q) \ll B_{pq} ; \text{ and } Q_p \ll |V_p^2| B_{pp}$$

So, the elements of Jacobian reduces to

$$H_{pq} = L_{pq} = -|V_p| |V_q| B_{pq} \quad (B.20)$$

$$H_{pp} = L_{pp} = -B_{pp} V_p^2. \quad (B.21)$$

and Equation B.15 becomes

$$[\Delta P] = [B'_{pq} |V_p| |V_q|] [\Delta \delta] \quad (\text{B.22})$$

$$[\Delta Q] = [B''_{pq} |V_p| |V_q|] [\Delta V / |V|], \quad (\text{B.23})$$

where,  $B'_{pq}$  and  $B''_{pq}$  are elements of  $[-B]$  matrix.

In the above equations both  $[B']$  and  $[B'']$  are real and sparse and have structure of H and L respectively. Since they contain only network admittance, they are constant and need to be triangularized only once at the beginning of the iteration. The matrix  $[B']$  is symmetrical upper triangular. Matrix  $[B'']$  is also symmetrical, if there is no phase shifter in the network

### C. DERIVATION OF FAULT ADMITTANCE MATRIX FOR L-G FAULT

Figure C.1 shows the fault in phase a through a fault impedance  $z_F$ . To calculate the diagonal element  $z_{fa}^{aa}$  of the fault impedance matrix  $z_{fa}^{a,b,c}$  a current of 1 p.u. is injected at phase a and voltage at phase a is measured, keeping the other phases open.

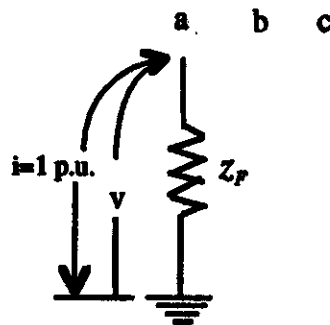


Figure C.1 L-G fault at phase a.

The voltage relation for phase a from Figure c.1 is

$$v = z_F i$$

$$v / i = z_F$$

$$z_F = v = z_{fa}^{aa} \text{ since, } i = 1 \text{ p.u.}$$

Similarly the impedances of phase b & c are

$$z_{fb}^{bb} = z_{fc}^{cc} = \alpha.$$

All the off diagonal element calculated by open circuit test will be

$$z_{fa}^{ab} = z_{fa}^{bc} = z_{fb}^{ca} = 0.$$

So, the fault impedance matrix can be represented as

$$Z_{ft}^{a,b,c} = \begin{bmatrix} Z_F & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix},$$

then

$$Y_{ft}^{a,b,c} = [Z_{ft}^{a,b,c}]^{-1} = \begin{bmatrix} y_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

where,  $y_F = 1 / Z_F$ .

Transformation of the fault admittance matrix from phase component to symmetrical component with the transformation matrix T is given by

$$Y_{ft}^{0,1,2} = T Y_{ft}^{a,b,c} T^*,$$

where:

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}.$$

Therefore,

$$Y_{ft}^{0,1,2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} y_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix},$$

$$Y_{ft}^{0,1,2} = \frac{y_F}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

## D. ADDITIONAL TEST RESULTS

The test results of the software modules for coordination study of ground relays for the city of Saskatoon Distribution model network were reported in Chapter 6 for the operating condition MX-2. The test results for the other three operating conditions mentioned in Chapter 6 are presented in Table D.1 through Table D.6.

**Table D.1:** Output of State Estimation Module for operating condition MX-1  
a) Prefault current, b) Prefault voltage.

(a)

Bus no.		Line flow (magnitude) (amp)
From	To	
1	6	394.247
6	7	396.801
7	8	160.777
2	8	163.505
2	9	89.959
9	10	190.151
11	10	179.413
3	11	192.454
3	12	112.745
13	12	104.974
14	13	70.221
15	14	163.275
4	15	143.263
4	16	169.967
16	17	92.596
18	17	40.634
19	18	313.992
20	19	300.028
21	20	32.054
5	21	326.719
5	22	366.486
22	23	243.919
24	23	80.410

(b)

Bus no.	Voltage	
	Magnitude <i>p.u.</i>	Phase <i>rad.</i>
1	1.0200	0.0
2	1.0200	-1.52E-02
3	1.0200	-8.11E-03
4	1.0200	-9.64E-03
5	1.0200	-1.22E-03
6	1.0165	-4.51E-03
7	1.0115	-1.34E-02
8	1.0134	-1.42E-02
9	1.0128	-1.96E-02
10	1.0153	-1.75E-02
11	1.0158	-1.65E-02
12	1.0162	-1.24E-02
13	1.0118	-1.87E-02
14	1.0109	-2.02E-02
15	1.0129	-1.87E-02
16	1.0125	-1.41E-02
17	1.0055	-1.75E-02
18	1.0058	-1.68E-02
19	1.0097	-1.26E-02
20	1.0128	-8.11E-03
21	1.0118	-9.49E-03
22	1.0107	-7.48E-03
23	1.0080	-9.00E-03



Table D.1: Continued.

25	24	70.656
26	25	49.869
27	26	74.492
28	27	249.628
1	28	420.349
1	20	375.996

24	1.0057	-8.63E-03
25	1.0054	-8.55E-03
26	1.0052	-8.14E-03
27	1.0055	-6.42E-03
28	1.0108	-2.90E-03

Table D.2: Output of State Estimation Module for operating condition MN-1  
a) Prefault current, b) Prefault voltage.

(a)

Bus no.		Line flow
From	To	(magnitude) (amp)
1	6	206.339
6	7	196.535
7	8	81.828
2	8	65.073
2	9	40.060
9	10	97.999
11	10	89.472
3	11	95.495
3	12	54.508
13	12	50.772
14	13	34.501
15	14	85.589
4	15	68.394
4	16	81.508
16	17	42.379
18	17	19.050
19	18	157.656
20	19	145.560
21	20	16.231
5	21	157.176
5	22	163.624
22	23	114.188
24	23	36.824
25	24	36.578
26	25	52.139
27	26	67.609
28	27	141.203
1	28	187.713
1	20	181.634

(b)

Bus no.	Voltage	
	Magnitude p.u.	Phase rad
1	1.0200	0.00E+00
2	1.0200	-7.41E-03
3	1.0200	-3.86E-03
4	1.0200	-4.65E-03
5	1.0200	-5.45E-04
6	1.0188	-2.73E-03
7	1.0164	-7.16E-03
8	1.0173	-7.52E-03
9	1.0172	-9.82E-03
10	1.0185	-8.86E-03
11	1.0186	-8.32E-03
12	1.0185	-6.19E-03
13	1.0168	-9.49E-03
14	1.0165	-1.03E-02
15	1.0177	-9.61E-03
16	1.0166	-7.00E-03
17	1.0136	-8.94E-03
18	1.0138	-8.67E-03
19	1.0158	-6.65E-03
20	1.0171	-4.32E-03
21	1.0169	-5.15E-03
22	1.0178	-4.96E-03
23	1.0174	-6.35E-03
24	1.0177	-7.35E-03
25	1.0178	-7.50E-03
26	1.0183	-7.50E-03
27	1.0198	-7.21E-03
28	1.0184	-3.92E-03

**Table D.3:** Output of State Estimation Module for operating condition MN-2  
a) Prefault current, b) Prefault voltage.

(a)

Bus no.		Line flow
From	To	(magnitude) (amp)
1	6	264.819
6	7	243.906
7	8	118.589
2	8	80.845
2	9	83.426
9	10	71.485
11	10	42.326
3	11	49.121
3	12	102.965
13	12	99.639
14	13	85.372
15	14	73.437
4	15	30.773
4	16	131.684
16	17	92.175
18	17	32.570
19	18	108.060
20	19	97.239
21	20	118.161
5	21	290.580
5	22	75.994
22	23	49.793
24	23	122.983
25	24	127.141
26	25	153.768
27	26	169.966
28	27	293.134
1	28	340.915

(b)

Bus no.	Voltage	
	Magnitude p.u.	Phase rad
1	1.0200	0.00E+00
2	1.0200	-1.29E-02
3	1.0200	-1.78E-02
4	1.0200	-3.12E-02
5	1.0200	-2.44E-02
6	1.0187	-3.59E-03
7	1.0167	-9.51E-03
8	1.0177	-1.06E-02
9	1.0172	-2.01E-02
10	1.0184	-2.00E-02
11	1.0186	-1.98E-02
12	1.0183	-2.28E-02
13	1.0168	-2.98E-02
14	1.0165	-3.19E-02
15	1.0176	-3.20E-02
16	1.0148	-3.55E-02
17	1.0090	-4.07E-02
18	1.0085	-4.10E-02
19	1.0099	-3.97E-02
20	1.0106	-3.80E-02
21	1.0145	-3.31E-02
22	1.0177	-2.41E-02
23	1.0174	-2.36E-02
24	1.0184	-2.02E-02
25	1.0185	-1.96E-02
26	1.0188	-1.83E-02
27	1.0197	-1.45E-02
28	1.0177	-7.32E-03

**Table D.4:** Output of fault analysis module for operating condition MX-1.

Relay no.	Fault location	Fault Current (Amp)		
		Real component	Imaginary component	Magnitude
1-1	2-1/2-2	-189.27	184.92	264.61
1-1	3-1/3-2	-32.77	25.98	41.81
1-1	4-1/4-2	-25.08	41.32	48.3
1-1	5-1/5-2	1198.29	-2155.57	2466.25
1-1	1-1	3693.58	-32927.48	33133.99
1-1	1-2	-903.48	2440.12	2602.01
1-1	1-3	-903.48	2440.12	2602.01
1-2	2-1/2-2	1605.18	-2765.80	3197.86
1-2	3-1/3-2	246.97	-325.28	408.41
1-2	4-1/4-2	-56.92	105.58	119.95
1-2	5-1/5-2	-439.32	441.58	622.89
1-2	1-1	-1922.90	3895.97	4344.66
1-2	1-2	2674.15	-31471.64	31585.04
1-2	1-3	-1922.90	3895.97	4344.66
1-3	2-1/2-2	-184.12	248.78	309.50
1-3	3-1/3-2	56.31	-120.18	132.72
1-3	4-1/4-2	344.79	-1192.12	1240.98
1-3	5-1/5-2	573.17	-1747.72	1839.31
1-3	1-1	-680.19	3238.78	3309.43
1-3	1-2	-680.19	3238.78	3309.43
1-3	1-3	3916.87	-32128.82	32366.69
2-1	1-1/1-2/1-3	1922.91	-3895.97	4344.67
2-1	3-1/3-2	-246.97	325.28	408.41
2-1	4-1/4-2	56.92	-105.58	119.95
2-1	5-1/5-2	439.33	-441.58	622.89
2-1	2-1	2752.59	-19030.47	19228.51
2-1	2-2	-1605.18	2765.80	3197.86
2-2	1-1/1-2/1-3	-197.38	267.38	332.34
2-2	3-1/3-2	880.64	-2639.79	2782.80
2-2	4-1/4-2	199.11	-305.36	364.54
2-2	5-1/5-2	-33.72	21.34	39.90
2-2	2-1	-533.13	1551.21	1640.27
2-2	2-2	3824.64	-20245.07	20603.17

Table D.4: Continued.

3-1	2-1/2-2	-137.58	208.92	250.15
3-1	4-1/4-2	541.21	-1520.96	1614.39
3-1	5-1/5-2	25.93	-43.90	50.99
3-1	3-1	923.81	-11834.98	11870.98
3-1	3-2	-567.14	1667.03	1760.86
3-2	1-1/1-2/1-3	197.38	-267.38	332.34
3-2	2-1/2-2	533.13	-1551.21	1640.27
3-2	4-1/4-2	-199.10	305.36	364.54
3-2	5-1/5-2	33.72	-21.34	39.91
3-2	3-1	-880.64	2639.79	2782.81
3-2	3-2	610.31	-10862.23	10879.36
4-1	1-1/1-2/1-3	331.07	-1246.28	1289.50
4-1	2-1/2-2	60.26	-75.08	96.27
4-1	3-1/3-2	-100.42	190.96	215.75
4-1	5-1/5-2	165.73	-411.39	443.52
4-1	4-1	1197.37	-12357.09	12414.97
4-1	4-2	-436.01	1583.79	1642.71
4-2	1-1/1-2/1-3	-34.41	110.64	115.87
4-2	2-1/2-2	137.58	-208.92	250.15
4-2	3-1/3-2	567.14	-1667.03	1760.86
4-2	5-1/5-2	-25.93	43.90	50.99
4-2	4-1	-541.21	1520.97	1614.39
4-2	4-2	1092.17	-12419.92	12467.85
5-1	1-1/1-2/1-3	903.48	-2440.12	2602.01
5-1	2-1/2-2	189.27	-184.92	264.61
5-1	3-1/3-2	32.77	-25.98	41.81
5-1	4-1/4-2	25.08	-41.32	48.33
5-1	5-1	1924.57	-15083.18	15205.47
5-1	5-2	-1198.29	2155.57	2466.25
5-2	1-1/1-2/1-3	349.12	-1992.50	2022.85
5-2	2-1/2-2	123.86	-173.69	213.34
5-2	3-1/3-2	44.11	-70.78	83.39
5-2	4-1/4-2	91.21	-391.67	402.15
5-2	5-1	-738.89	2159.12	2282.05
5-2	5-2	2383.96	-15079.63	15266.91

**Table D.5: Output of fault analysis module for operating condition MN-1.**

Relay no.	Fault location	Fault Current (Amp)		
		Real component	Imaginary component	Magnitude
1-1	2-1/2-2	-186.38	178.66	258.18
1-1	3-1/3-2	-32.14	25.15	40.81
1-1	4-1/4-2	-24.75	40.12	47.14
1-1	5-1/5-2	1180.19	-2109.18	2416.92
1-1	1-1	3548.09	-31924.41	32120.97
1-1	1-2	-873.44	2366.40	2522.45
1-1	1-3	-873.44	2366.40	2522.45
1-2	2-1/2-2	1591.81	-2683.61	3120.20
1-2	3-1/3-2	243.08	-315.96	398.65
1-2	4-1/4-2	-56.27	102.58	117.01
1-2	5-1/5-2	-431.74	431.55	610.44
1-2	1-1	-1860.26	3778.75	4211.83
1-2	1-2	2561.27	-30512.06	30619.37
1-2	1-3	-1860.26	3778.75	4211.83
1-3	2-1/2-2	-181.95	241.01	301.98
1-3	3-1/3-2	55.70	-116.96	129.54
1-3	4-1/4-2	344.75	-1160.44	1210.57
1-3	5-1/5-2	566.45	-1711.20	1802.52
1-3	1-1	-656.19	3140.43	3208.25
1-3	1-2	-656.19	3140.43	3208.25
1-3	1-3	3765.34	-31150.38	31377.13
2-1	1-1/1-2/1-3	1860.26	-3778.75	4211.83
2-1	3-1/3-2	-243.08	315.96	398.65
2-1	4-1/4-2	56.27	-102.58	117.01
2-1	5-1/5-2	431.74	-431.55	610.43
2-1	2-1	2862.32	-18541.93	18761.56
2-1	2-2	-1591.81	2683.61	3120.19
2-2	1-1/1-2/1-3	-191.07	259.40	322.18
2-2	3-1/3-2	875.99	-2571.16	2716.29
2-2	4-1/4-2	196.37	-296.46	355.60
2-2	5-1/5-2	-33.10	20.82	39.11
2-2	2-1	-534.56	1508.52	1600.43
2-2	2-2	3919.56	-19717.02	20102.83

Table D.5: Continued.

3-1	1-1/1-2/1-3	33.25	-107.29	112.33
3-1	2-1/2-2	-136.17	202.56	244.08
3-1	4-1/4-2	538.66	-1479.84	1574.83
3-1	5-1/5-2	25.53	-42.95	49.97
3-1	3-1	975.31	-11546.11	11587.23
3-1	3-2	-563.94	1623.62	1718.77
3-2	1-1/1-2/1-3	191.07	-259.40	322.18
3-2	2-1/2-2	534.56	-1508.52	1600.43
3-2	4-1/4-2	-196.37	296.46	355.60
3-2	5-1/5-2	33.10	-20.82	39.11
3-2	3-1	-875.99	2571.16	2716.29
3-2	3-2	663.25	-10598.57	10619.30
4-1	1-1/1-2/1-3	319.72	-1208.50	1250.08
4-1	2-1/2-2	59.48	-72.69	93.93
4-1	3-1/3-2	-99.20	185.77	210.60
4-1	5-1/5-2	163.53	-402.71	434.6
4-1	4-1	1255.20	-12045.53	12110.75
4-1	4-2	-436.48	1541.86	1602.46
4-2	1-1/1-2/1-3	-33.25	107.29	112.33
4-2	2-1/2-2	136.17	-202.56	244.08
4-2	3-1/3-2	563.94	-1623.62	1718.77
4-2	5-1/5-2	-25.53	42.95	49.97
4-2	4-1	-538.67	1479.84	1574.83
4-2	4-2	1153.02	-12107.56	12162.34
5-1	1-1/1-2/1-3	873.44	-2366.40	2522.45
5-1	2-1/2-2	186.38	-178.66	258.18
5-1	3-1/3-2	32.14	-25.15	40.81
5-1	4-1/4-2	24.75	-40.12	47.14
5-1	5-1	1927.14	-14776.20	14901.34
5-1	5-2	-1180.19	2109.18	2416.92
5-2	1-1/1-2/1-3	336.47	-1931.92	1961.00
5-2	2-1/2-2	122.46	-168.32	208.15
5-2	3-1/3-2	43.49	-68.81	81.40
5-2	4-1/4-2	91.73	-381.42	392.30
5-2	5-1	-729.99	2113.91	2236.40
5-2	5-2	2377.34	-14771.47	14961.55

**Table D.6:** Output of fault analysis module for operating condition MN-2.

Relay no.	Fault location	Fault Current (Amp)		
		Real component	Imaginary component	Magnitude
1-1	2-1/2-2	-234.70	215.64	318.73
1-1	3-1/3-2	-13.62	5.89	14.84
1-1	4-1/4-2	104.70	-160.61	191.72
1-1	5-1/5-2	1143.73	-2149.22	2434.60
1-1	1-1	2933.92	-28631.83	28781.76
1-1	1 2	-1098.34	2687.32	2903.11
1-2	2-1/2-2	1533.18	-2664.97	3074.53
1-2	3-1/3-2	248.43	-330.73	413.65
1-2	4-1/4-2	19.90	-17.87	26.75
1-2	5-1/5-2	-262.21	233.13	350.87
1-2	1-1	-1862.40	3758.44	4194.58
1-2	1 2	2169.86	-27560.70	27645.99
2-1	1-1/1-2	1862.41	-3758.44	4194.58
2-1	3-1/3-2	-248.43	330.73	413.65
2-1	4-1/4-2	-19.90	17.87	26.75
2-1	5-1/5-2	262.21	-233.13	350.87
2-1	2-1	2716.07	-18512.19	18710.38
2-1	2-2	-1533.18	2664.97	3074.53
2-2	1-1/1-2	-211.16	282.63	352.80
2-2	3-1/3-2	840.54	-2582.86	2716.18
2-2	4-1/4-2	189.92	-303.38	357.93
2-2	5-1/5-2	-6.68	-3.71	7.64
2-2	2-1	-524.49	1509.67	1598.19
2-2	2-2	3724.76	-19667.49	20017.09
3-1	1-1/1-2	-26.01	17.64	31.42
3-1	2-1/2-2	-144.05	211.28	255.71
3-1	4-1/4-2	480.42	-1457.57	1534.71
3-1	5-1/5-2	55.80	-97.98	112.76
3-1	3-1	813.34	-11553.68	11582.27
3-1	3-2	-536.70	1625.72	1712.02
3-2	1-1/1-2	211.16	-282.63	352.80
3-2	2-1/2-2	524.49	-1509.68	1598.19
3-2	4-1/4-2	-189.92	303.38	357.93
3-2	5-1/5-2	6.68	3.71	7.64
3-2	3-1	-840.54	2582.85	2716.18
3-2	3-2	509.49	-10596.54	10608.78

Table D.6: Continued.

4-1	1-1/1-2	111.51	-164.36	198.62
4-1	2-1/2-2	1.42	6.89	7.04
4-1	3-1/3-2	-79.46	148.32	168.27
4-1	5-1/5-2	264.59	-852.83	892.93
4-1	4-1	852.35	-11760.02	11790.87
4-1	4-2	-270.85	1106.04	1138.72
4-2	1-1/1-2	26.00	-17.64	31.429
4-2	2-1/2-2	144.05	-211.28	255.72
4-2	3-1/3-2	536.70	-1625.72	1712.02
4-2	5-1/5-2	-55.80	97.98	112.76
4-2	4-1	-480.42	1457.57	1534.71
4-2	4-2	642.78	-11408.49	11426.58
5-1	1-1/1-2	1098.34	-2687.32	2903.11
5-1	2-1/2-2	234.70	-215.64	318.73
5-1	3-1/3-2	13.62	-5.89	14.84
5-1	4-1/4-2	-104.70	160.61	191.72
5-1	5-1	1014.21	-12691.59	12732.05
5-1	5-2	-1143.73	2149.22	2434.60
5-2	1-1/1-2	-111.51	164.36	198.62
5-2	2-1/2-2	-1.42	-6.89	7.04
5-2	3-1/3-2	79.46	-148.32	168.27
5-2	4-1/4-2	270.85	-1106.04	1138.72
5-2	5-1	-264.58	852.82	892.92
5-2	5-2	1893.36	-13987.99	14115.55

The relay operating times for primary and backup relays for fault at the locations shown in Figure 6.13 are also calculated to check if their operation is properly coordinated. The operating times of the relays are presented in Table D.7 through Table D.10. It was observed that relay coordination between primary and backup relays is achieved for all the selected fault locations



**Table D.7: Relay operating time for operating condition MX-1.**

Relay no.	Fault location	Relay operating time (s)	Relay description
1-1	21	1.874	Backup for relay 5-2
1-1	22	0.202	Primary for far end fault
1-1	28	0.057	Primary for near end fault
1-2	6	0.053	Primary for near end fault
1-2	8	0.112	Primary for far end fault
1-2	9	1.9138	Backup for relay 2-2
1-2	11	5.900	Backup for relay 2-2
1-3	15	10.158	Backup for relay 4-2
1-3	16	0.293	Primary for far end fault
1-3	21	0.221	Primary for far end fault
1-3	22	6.584	Backup for relay 5-1
2-1	6	0.407	Primary for far end fault
2-1	8	0.087	Primary for near end fault
2-1	21	3.379	Backup for relay 1-3
2-1	22	3.105	Backup for relay 1-1
2-1	28	0.542	Backup for relay 1-1
2-2	9	0.096	Primary for near end fault
2-2	11	0.122	Primary for far end fault
2-2	12	0.423	Backup for relay 3-1
2-2	15	1.743	Backup for relay 3-1
3-1	12	0.071	Primary for near end fault
3-1	15	0.170	Primary for far end fault
3-1	16	0.973	Backup for relay 4-1
3-2	8	3.461	Backup for relay 2-1
3-2	9	0.107	Primary for far end fault
3-2	11	0.081	Primary for near end fault
4-1	16	0.074	Primary for near end fault
4-1	21	3.881	Primary for far end fault
4-2	9	1.808	Backup for relay 3-2
4-2	11	1.016	Backup for relay 3-2
4-2	12	0.203	Primary for far end fault
4-2	15	0.081	Primary for near end fault
5-1	6	1.902	Backup for relay 1-2
5-1	22	0.069	Primary for near end fault
5-1	28	0.648	Primary for far end fault
5-2	6	11.828	Backup for relay 1-2
5-2	16	3.117	Primary for far end fault
5-2	21	0.070	Primary for near end fault

**Table D.8: Relay operating time for operating condition MN-1.**

Relay no.	Fault location	Relay operating time(s)	Relay description
1-1	21	1.313	Backup for relay 5-2
1-1	22	0.194	Primary for far end fault
1-1	28	0.056	Primary for near end fault
1-2	6	0.053	Primary for near end fault
1-2	8	0.109	Primary for far end fault
1-2	9	1.149	Backup for relay 2-2
1-2	11	1.921	Backup for relay 2-2
1-3	15	2.930	Backup for relay 4-2
1-3	16	0.276	primary for far end fault
1-3	21	0.211	Primary for far end fault
1-3	22	2.513	Backup for relay 5-1
2-1	6	0.406	Primary for far end fault
2-1	8	0.086	Primary for near end fault
2-1	21	3.333	Backup for relay 1-3/1-1
2-1	22	3.046	Backup for relay 1-1
2-1	28	0.537	Backup for relay 1-1
2-2	9	0.096	Primary for near end fault
2-2	11	0.122	Primary for far end fault
2-2	12	0.424	Backup for relay 3-1
2-2	15	1.733	Backup for relay 3-1
3-1	12	0.071	Primary for near end fault
3-1	15	0.169	Primary for far end fault
3-1	16	0.969	Backup for relay 4-1
3-2	8	3.41928	Backup for relay 2-1
3-2	9	0.107	Primary for far end fault
3-2	11	0.081	Primary for near end fault
4-1	16	0.074	Primary for near end fault
4-1	21	3.823	Primary for far end fault
4-2	9	1.546	Backup for relay 3-2
4-2	11	0.931	Backup for relay 3-2
4-2	12	0.200	Primary for far end fault
4-2	15	0.081	Primary for near end fault
5-1	6	1.271	Backup for relay 1-2
5-1	22	0.068	Primary for near end fault
5-1	28	0.553	Primary for far end fault
5-2	6	11.512	Backup for relay 1-2
5-2	16	3.090	Primary for far end fault
5-2	21	0.069	Primary for near end fault

**Table D.9: Relay operating time for operating condition MX-2.**

Relay no.	Fault location	Relay operating time (s)	Relay Description
1-1	21	0.899	Backup for relay 5-2
1-1	22	0.203	Primary for far end fault
1-1	28	0.057	Primary for near end fault
1-2	6	0.053	Primary for near end fault
1-2	8	0.115	Primary for far end fault
1-2	9	4.985	Backup for relay 2-2
2-1	6	0.372	Primary for far end fault
2-1	8	0.086	Primary for near end fault
2-1	22	6.456	Backup for relay 1-1
2-1	28	0.486	Backup for relay 1-1
2-2	9	0.097	Primary for near end fault
2-2	11	0.122	Primary for far end fault
2-2	12	0.423	Backup for relay 3-1
2-2	15	1.693	Backup for relay 3-1
3-1	12	0.071	Primary for near end fault
3-1	15	0.169	Primary for far end fault
3-1	16	0.861	Backup for relay 4-1
3-2	8	3.211	Backup for relay 2-1
3-2	9	0.107	Primary for far end fault
3-2	11	0.082	Primary for near end fault
4-1	16	0.071	Primary for near end fault
4-1	21	0.501	Primary for far end fault
4-1	22	5.862	Backup for relay 5-1
4-2	9	1.544	Backup for relay 3-2
4-2	11	0.929	Backup for relay 3-2
4-2	12	0.200	Primary for far end fault
4-2	15	0.081	Primary for near end fault
5-1	6	0.864	Backup for relay 1-2
5-1	8	19.974	Backup for relay 1-2
5-1	22	0.068	Primary for near end fault
5-1	28	0.495	Primary for far end fault
5-2	16	0.371	Primary for far end fault
5-2	21	0.062	Primary for near end fault

**Table D.10: Relay operating time for operating condition MN-2.**

Relay no.	Fault location	Relay operating time	Relay description
1-1	21	0.337	Backup for relay 5-2
1-1	22	0.110	Primary for far end fault
1-1	28	0.050	Primary for near end fault
1-2	6	0.050	Primary for near end fault
1-2	8	0.073	Primary for far end fault
1-2	9	0.941	Backup for relay 2-2
1-2	11	1.862	Backup for relay 2-2
2-1	6	0.371	Primary for far end fault
2-1	8	0.086	Primary for near end fault
2-1	22	6.233	Backup for relay 1-1
2-1	28	0.480	Backup for relay 1-1
2-2	9	0.096	Primary for near end fault
2-2	11	0.122	Primary for far end fault
2-2	12	0.422	Backup for relay 3-1
2-2	15	1.680	Backup for relay 3-1
3-1	12	0.071	Primary for near end fault
3-1	15	0.168	Primary for far end fault
3-1	16	0.853	Backup for relay 4-1
3-2	8	3.169	Backup for relay 2-1
3-2	9	0.107	Primary for far end fault
3-2	11	0.081	Primary for near end fault
4-1	16	0.071	Primary for near end fault
4-1	21	0.494	Primary for far end fault
4-1	22	5.667	Backup for relay 5-1
4-2	9	1.530	Backup for relay 3-2
4-2	11	0.925	Backup for relay 3-2
4-2	12	0.199	Primary for far end fault
4-2	15	0.081	Primary for near end fault
5-1	6	0.696	Backup for relay 1-2
5-1	8	5.209	Backup for relay 1-2
5-1	22	0.060	Primary for near end fault
5-1	28	0.412	Primary for far end fault
5-2	15	9.802	Backup for relay 4-2
5-2	16	0.196	Primary for far end fault
5-2	21	0.050	Primary for near end fault