

COMMUNICATION BY VHF FORWARD -  
SCATTER IN AND CLOSE TO THE  
AURORAL ZONE

A Thesis

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BY

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## ABSTRACT

Bailey et al (1955) have shown that VHF forward-scatter provides a very reliable method of communication over path lengths of several thousand kilometers. It has been suggested that this mode of propagation may be utilized to provide communications links in auroral latitudes using low power transmitters and very narrow bandwidth receivers. Such systems would only be feasible if the received signal doppler broadening was small. Previous investigation in auroral latitudes by back-scatter techniques had shown that scatterer velocities of up to several km/s are not uncommon.

A 50 watt transmitter was set up in Fort Smith, N.W.T. using a five-element Yagi antenna and operating on a frequency of 40.49 Mc/s. At Saskatoon, an identical antenna was used in conjunction with a receiver whose bandwidth could be varied from 5 c/s to 100 c/s. By changing the bandwidth and noting its effect on the received signal-to-noise ratio, it was possible to estimate that the received signal, under undisturbed and mildly disturbed conditions, had a bandwidth of the order of 10 c/s. This corresponds to a r.m.s. scatterer velocity of about 50 m/s.

There was some evidence to show that velocities greater than 50 m/s do occur during periods when the ionosphere is greatly disturbed. These periods, however, are always associated with an enhancement in the received signal amplitude.

The experiments indicate that, using a 100 watt transmitter, a receiver with a bandwidth of 10 c/s and antenna similar to those used, an information rate of at least 10 words/minute would be possible 99.9% of the time over a path length of 1000 km.

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## I. INTRODUCTION

### 1.0 General

NORMAL high frequency radio communication through the auroral zone is subject to interruptions because of the wide variations in, and the turbulent nature of, the ionization in the E region. In particular, periods of intense absorption occur which may last up to several days and which sever communications abruptly and often completely. Two ways of circumventing this problem are <sup>by</sup> the use of very low frequencies or ultra high frequency tropospheric scattering. Both these systems involve the use of large cumbersome antennae and the latter especially is extremely expensive.

Conventional communication techniques such as the use of radio-teletype are also feasible when the mode of propagation involves very high frequency forward-scatter. Normally however, fairly high transmitter powers are necessary and elaborate antennae are used. This type of system has proven to be very reliable and it has been suggested that low power transmitters and simple antennae may be used in conjunction with very narrow band receivers to provide slow but inexpensive and relatively portable communication links having the same reliability.

If the received signal were discrete in frequency, then any narrowing of the receiver bandwidth would increase the signal-to-noise ratio. The spectral spread of the signal will determine the limiting receiver bandwidth beyond which further narrowing of the bandwidth will not increase the signal-to-noise ratio. If this limiting signal-to-noise ratio is of the order of 8-10 db.

or greater, then communication is possible, without excessive error, at information rates determined solely by the bandwidth of the receiver.

### 1.1 Previous Work

Most of the work on VHF forward-scatter has been done in the last decade. Although abnormal, beyond-the-horizon VHF propagation had been noticed by radio amateurs and others, it was assumed to occur rarely, 'Sporadic-E' propagation as it was called was never seriously investigated until Bailey et al. (1952) noticed that some signal was propagated all the time provided sufficient transmitter power was used. They found that at least over their period of observation, periods of HF absorption were often characterized by an enhancement of signal strength on the VHF circuit. During these SID's (Sudden Ionospheric Disturbances), the level of background cosmic noise decreased on the VHF link but the signal level never did and about three-quarters of the time, the signal level increased. There was a midday signal level maximum which they suggested was due to extra ionization provided by solar ultra-violet radiation and, additionally, a diurnal variation which appeared to be caused by meteoric ionization. Because of the immunity to SID's and because of the meteoric control, they presumed that the region causing scattering was in the lower E-layer and possibly the D-layer. It was also noted that the received power level was considerably below that calculated on the assumption that scattering was produced by turbulent inhomogeneities in the E-layer.

A large number of observers began to investigate the size and motions of the large and small irregularities observed in

the E-region and two rival theories as to the cause of the small ones were propounded (Booker, 1957); meteor trails and turbulent mixing. It is now generally agreed (Ratcliffe and Weekes, 1960) that both mechanisms are probably present with their relative importance changing from time to time. No simple explanation has yet been postulated for the origin of the larger irregularities although several suggestions have been made (ibid, Ratcliffe and Weekes).

In a later paper, Bailey et al. (1955), on the basis of a longer period of observation on several VHF forward-scatter links, confirmed and enlarged their earlier results. Because of the high power background signal received all the time, the occurrence of a meteor was noticed as a weak audio beat tone due to the doppler shift caused by the meteor head. At high latitudes, the results conformed generally to the earlier results except that enhancements occurred more often near the auroral zone than further south, were of greater magnitude and showed greater and more rapid variations. There was also less of a diurnal variation and signals tended to be strongest during the summer. It was deduced that at least in southerly latitudes, scattering took place at a height of 75-80 km. during the day and about ten km. higher at night. A particular type of rapid fading during some enhancements, called sputter (with fading rates of the order of 200-300 c/s) was noticed less than 0.5% of the time. It was also noticed that the scattering took place in a relatively small volume (cf. meteor

trails) making directional antennae desirable. The paper also contained calculations as to the merit of this method of propagation as a means of communications on the basis of transmitter powers of the order of 50 kw. and rhombic antennae.

Some later work was done in Canada by other workers to obtain more information on the nature of forward-scattering nearer the auroral zone. Collins and Forsyth (1959), using several low power transmitters, classified the types of propagation normally encountered as follows: E - a sudden enhancement with very high signal levels, S- a gradual low level enhancement with slow deep fading which occurred most often during the day, and several types of A propagation. The A-type was differentiated from the other types by the presence of magnetic activity near the scattering region. In the absence of enhancements, because of the low transmitter powers used, the signal consisted almost solely of meteor trail propagated signals with a background level which sometimes slightly raised the background noise level. In a later paper, Forsyth et al. (1960), continuing the same line of inquiry, expanded the earlier results.

For the purposes of this thesis, it is irrelevant to summarize the results of these investigations except to note that the occurrence of forward-scatter enhancements depends on the solar and auroral activity at the time, and that it is a function of the orientation of the transmitter-receiver path of and its magnetic latitude. It was also found that there was  
^

less directivity in the scattered signal than was mentioned by Bailey et al.

Forsyth and Vogan, (1957) also mention occurrence of absorption in VHF forward-scatter. They noticed that sometimes the received signal level dropped about 20 db., accompanied by a drop in the background cosmic noise. Since the level of meteor 'spikes' (so called because of the appearance on a moving chart record of the signal propagated by meteor trails) also decreased by this amount, it was apparent that the level at which the absorption takes place must be below that of meteoric ionization. This agrees with the observations of other workers and the absorbing layer is generally agreed to be at a height of about 100 km. normally, although it does sometimes appear lower.

## 1.2 Propagation Conditions

It can be assumed that the signal received in forward-scatter will have a time-dependent gross frequency shift as well as being instantaneously spread in frequency.

The transmitter spectral bandwidth can be considered to be discrete in the absence of modulation. For the transmitter used in the experiment to be outlined, identification breaks will cause a negligibly small increase in this bandwidth. Any spectral dispersion will thus be caused solely by the doppler frequency shifts due to the velocities of the scattering bodies. The scattering observed at VHF is not due to the individual electrons; rather, it is scattering from macroscopic changes in the refractive index of the medium caused by inhomogeneities of the ion density.

Gross movement of these inhomogeneities will result in a gross frequency shift; the spectral spread will be caused by their disorderly motion.

Booker (loc. cit.) deduces a value of 10 m/s for turbulent velocities at heights of 80-100 km. Ratcliffe and Weekes (loc. cit.) give a figure of from 5 to 50 m/s for irregular movements in the E-region.

The actual doppler shift due to these motions will also be a function of their geometry. More specifically, it is a function of the half-angle  $\theta$  as shown in Figure 1.0. It is easily shown that the observed doppler shift will be approximately

Figure 1.0      Geometry of VHF forward-scatter propagation.

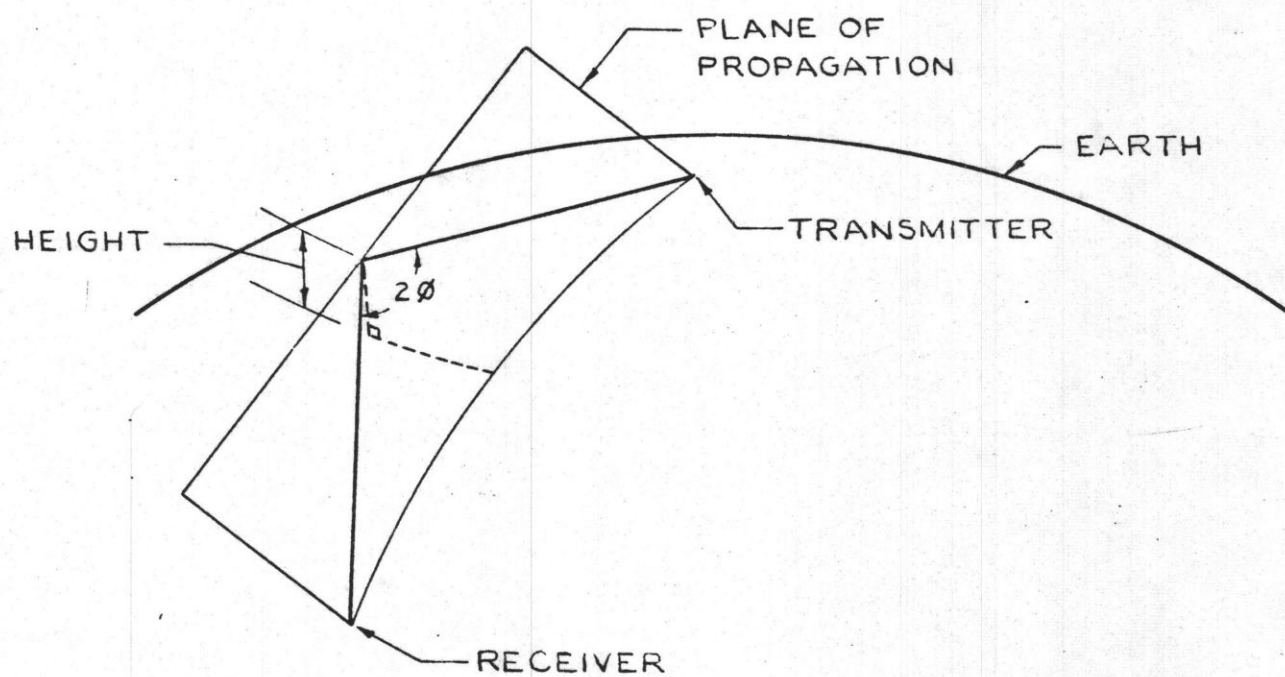


FIGURE 1.0

given by:

$$1) \text{ --- } f = \pm \frac{2 v_{\perp} \cos \theta}{\lambda}$$

where:

$v_{\perp}$  = the component of velocity perpendicular to the TR path in the plane of propagation.

$\lambda$  = the transmitter wavelength.

The maximum doppler shift will thus be caused by scattering points off to the side of the transmitter-receiver path. Because of antenna directivity however, the power received will decrease as  $\theta$  gets smaller. In Appendix I, these effects have been treated analytically and for the case outlined there, assuming that 90% of the scattering bodies have a velocity of less than 50 m/s, the half-power bandwidth of the received signal will be about 5 c/s.

This result however is only as valid as the assumptions. At least for sub-auroral latitudes the scattering region is small and the turbulent velocities are probably of the cited order of magnitude. There is some evidence that this is not the case in auroral latitudes (Bowles, 1952; McNamara, 1955). McNamara, using a pulsed double doppler radar, found some auroral echoes had spectral bandwidths of the order of 100 c/s at a frequency of 90.7 mc/s and he found that gross velocities of up to several km/s were not uncommon. Bowles obtained similar results at 50 mc/s using a continuous wave transmitter. However, this too was back-scatter.

Meteor trails also cause doppler broadening of the observed signal scattered from them. The two main causes of this broadening are diffusion and turbulence. Also, 'over-dense'\* meteor trails may cause a gross frequency shift due to the shrinking of the apparent diameter of the trail caused by diffusion. Vogan (1954), measured the power spectrum of many meteor trail propagated signals at a frequency of 49.8 mc/s and found for short duration signals a half-power bandwidth of about 20 c/s. For long duration signals it was slightly wider, indicating that the role played by turbulence in spectral widening only becomes important after several seconds.

The effect of meteor trails in the type of communications link envisaged may be more important than would at first appear. Although most meteor trail signals last less than a second in a conventional receiver, due to the lowering of noise in a narrow band receiver the duration of even these signals will be of the order of seconds. Also, the rate at which meteor trail signals occur increases rapidly with their increasing optical magnitude. At very low signal levels, meteor trail signals will be seen continuously (McKinley and Millman, 1949).

The time-dependent gross movement of the inhomogeneities will also contribute to the received signal's spectral broadening because of frequency modulation.

\*meteor trails with an ion density of greater than about  $10^{14}$  ions/m are called over-dense. The electron density is sufficiently high for the trail to appear as a metallic cylinder.

### 1.3 Scope of Thesis

The object of the experimental work reported herein is to find out whether a low power, narrow bandwidth communications link is possible through the auroral zone. The feasibility of such a system depends upon the narrowness of the received signal power spectrum bandwidth. Propagated near the auroral zone, this bandwidth is not accurately known; back-scatter measurements indicate wide bandwidths, whereas forward-scatter measurements at low latitudes exhibit narrow bandwidths.

With the use of a receiver with a very narrow, variable bandwidth, it is possible to study the signal bandwidth by observing the effect of decreasing receiver bandwidth on the output signal-to-noise ratio.

## II. EQUIPMENT

### 2.0 General

THE equipment consisted of a transmitter at Fort Smith N.W.T., and a receiver at Saskatoon, Sask. The transmitter was part of a previous project (Green, 1961); it and its operation were not in any way changed. The frequency used was 40.49 Mc/s and the transmitter output power was 50 watts into a five-element Yagi pointed toward Saskatoon.

The receiving antenna was identical to the transmitting one and was pointed toward Fort Smith. The receiver was followed by a phase-lock detector.

Two recording milliammeters (Esterline-Angus) were used to record the frequency and amplitude of the received signal.

## 2.1 Receiver

The receiving system was made up of two conventional receivers. It consisted of a Ferranti-Packard F-3 type 155 VHF crystal-controlled receiver followed by a modified military BC-453 'command' receiver. The BC-453 was modified by connecting its intermediate frequency output directly to the phase-lock detector, removing all audio and associated stages and replacing the variable local oscillator with a crystal oscillator so that it was tuned to the intermediate frequency of the F-3 receiver.

The phase-lock detector was constructed to operate at the intermediate frequency of the BC-453. The circuit used only semi-conductors as active elements.

A block diagram of the receiving system is shown in Figure 2.0 and Plate I is a photograph of it (not showing the F-3 receiver).

Plate I      Photograph of the receiving system

Figure 2.0      Block diagram of the receiving system

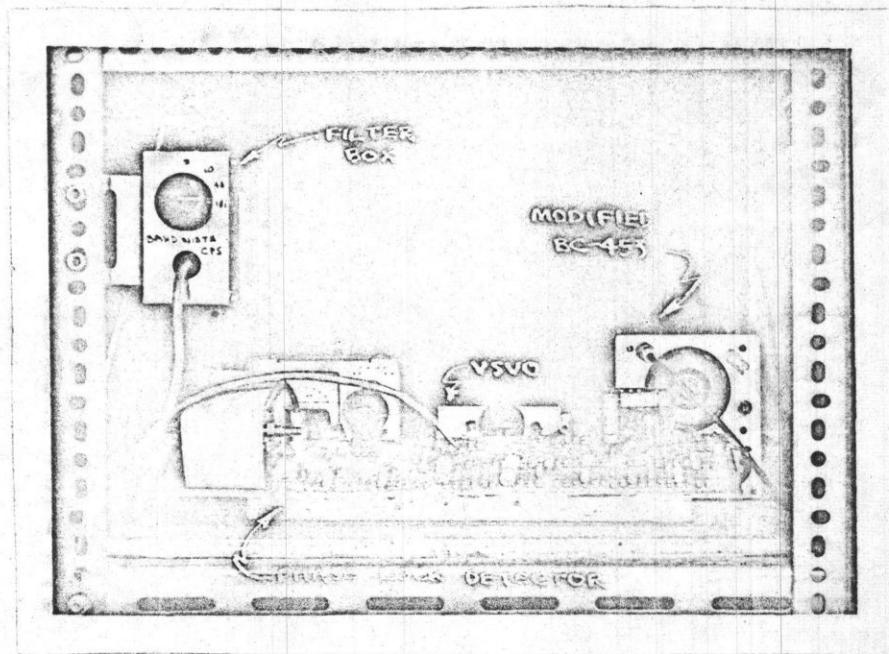


PLATE I

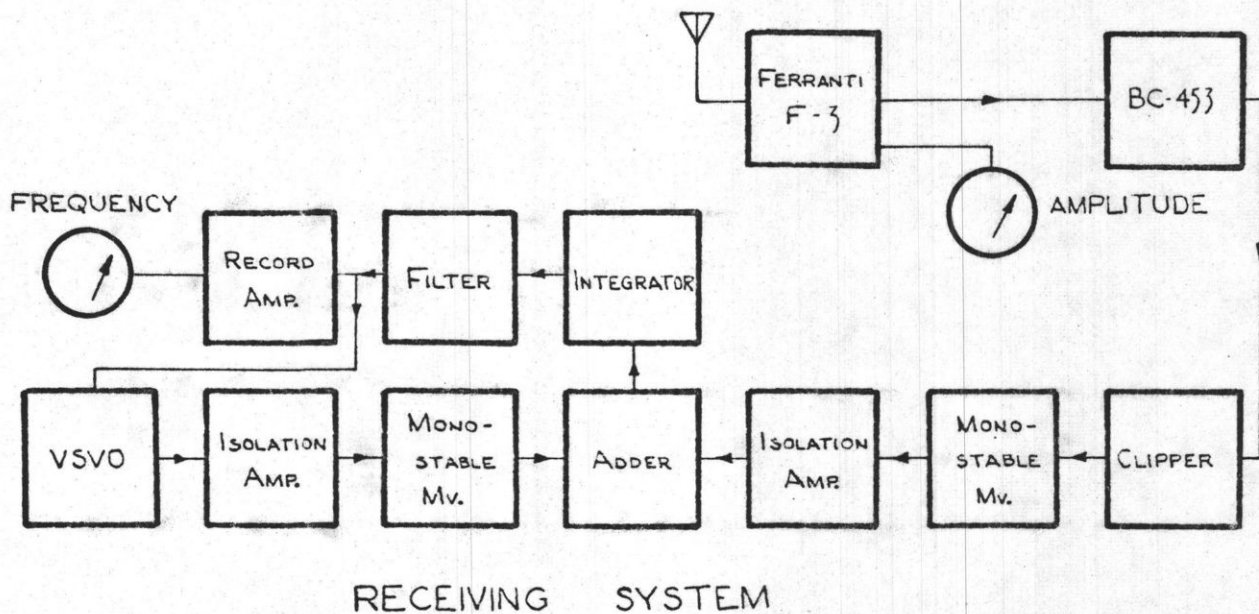


FIGURE 2.0

## 2.2 Phase-lock Detector

### a) Detector Theory

The phase-lock detector consisted of a voltage-sensitive variable oscillator (VSVO). The output of this oscillator along with the received signal is fed into a phase-sensitive detector giving an output proportional to the difference in the phases of the two inputs. This output is filtered and connected to the frequency sensitive element in the VSVO in such a manner that the VSVO frequency will follow the frequency of the received signal.

This type of servo-mechanism has zero frequency error; the error being in the integral of the frequency, i.e. the phase. The phase error is a measure of how great a frequency change has occurred in the VSVO from its centre frequency. Thus measurement of the phase error is measurement of the instantaneous frequency of the received signal.

The low-pass filter between the phase-sensitive detector and the VSVO is a very important part of the system since it determines the effective bandwidth of the receiver. It also has an effect on the 'lock-on' range of the system; that is, the frequency range over which the VSVO will stay locked-on to the received signal, and the 'capture' range; that is, the frequency range over which the VSVO will lock-on to the received signal having once been perturbed from it.

The phase-lock detector has been extensively analyzed in the literature (Preston and Tellier, 1953; Gruen, 1953; and McAleer, 1959) along with the effect of the low-pass filter. It can be

shown that with no filter, the bandwidth is solely dependent on the lock-on range which will be identical with the capture range. Introducing a filter reduces the bandwidth, has no effect on the lock-on range but also reduces the capture range. For an RC filter, the capture range is reduced in the same proportion as that of the bandwidth. For McAleer's optimum filter, the capture range is reduced as the square-root of the bandwidth reduction.

#### b) Phase-sensitive Detector

The phase-sensitive detector used produces an output which is independent of the input amplitudes and which is linear with phase difference. In this latter respect, it should be noted that it conforms with the simplifying assumption in Gruen's theory. Figure 2.1 shows a block diagram of the phase-sensitive detector along with the waveforms observed at various points. The output sawtooth waveform has a magnitude which is directly proportional to the phase difference between the inputs. Strictly speaking, this operation is unnecessary since waveform A when filtered produces a voltage with all the necessary information required to control the VSVO. However, it was considerably easier in practice to operate the equipment by observing the sawtooth waveform on an oscilloscope.

The actual circuitry used is simple and is shown in Figure 2.2. Only the integrator is unconventional; it takes advantage of the extremely high collector impedance of a grounded-base

Figure 2.1      Block diagram of phase-sensitive detector and  
a drawing of associated waveforms.

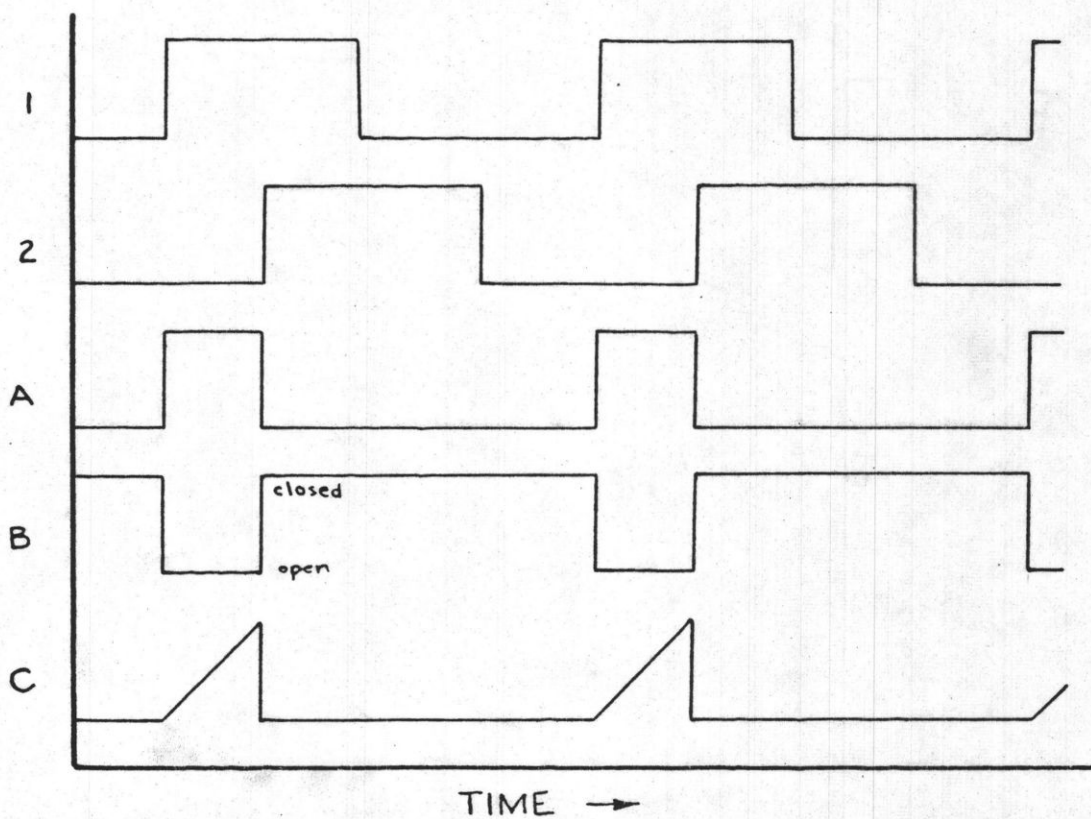
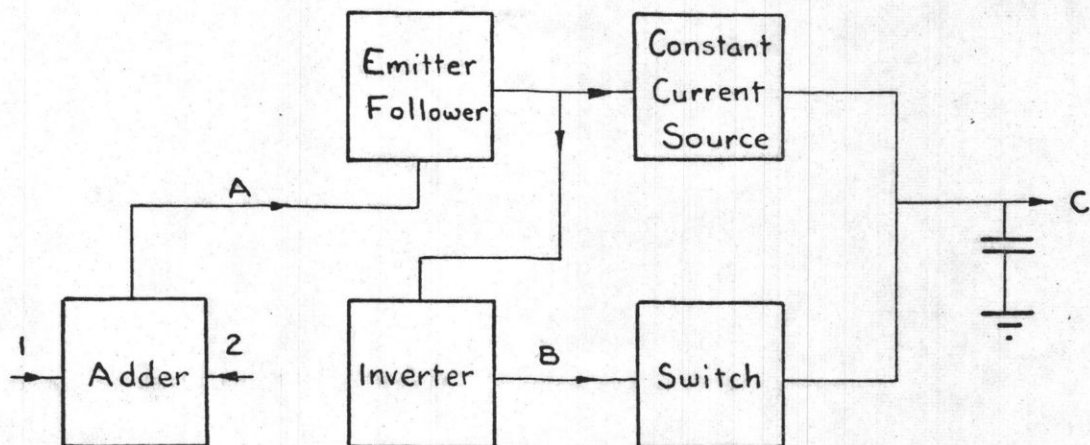


FIGURE 2.1

**Figure 2.2**      **Circuit diagram of the phase-sensitive detector**

**Figure 2.4**      **Circuit diagram of the voltage-sensitive variable  
oscillator**

# ADDER, INTEGRATOR

FIGURE 2.2

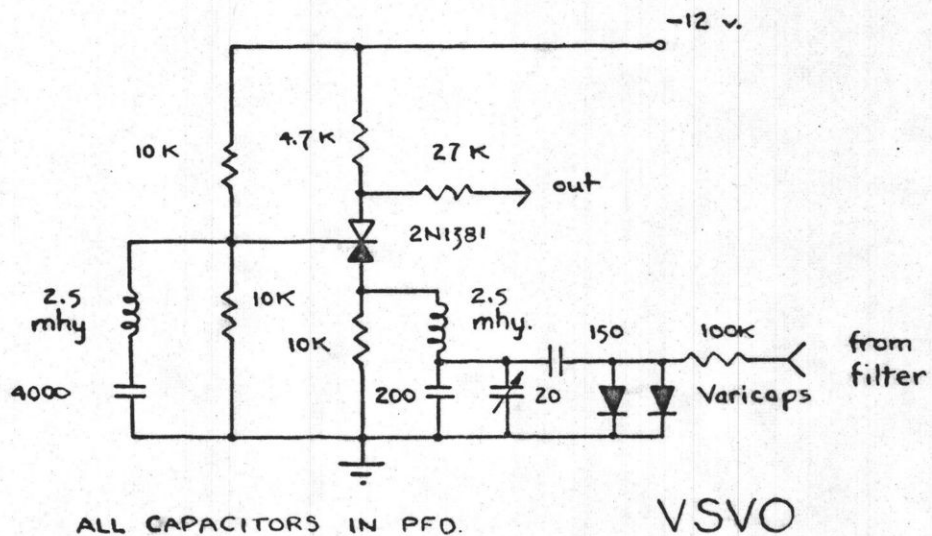


FIGURE 2.4

transistor to produce good linearity in the output.

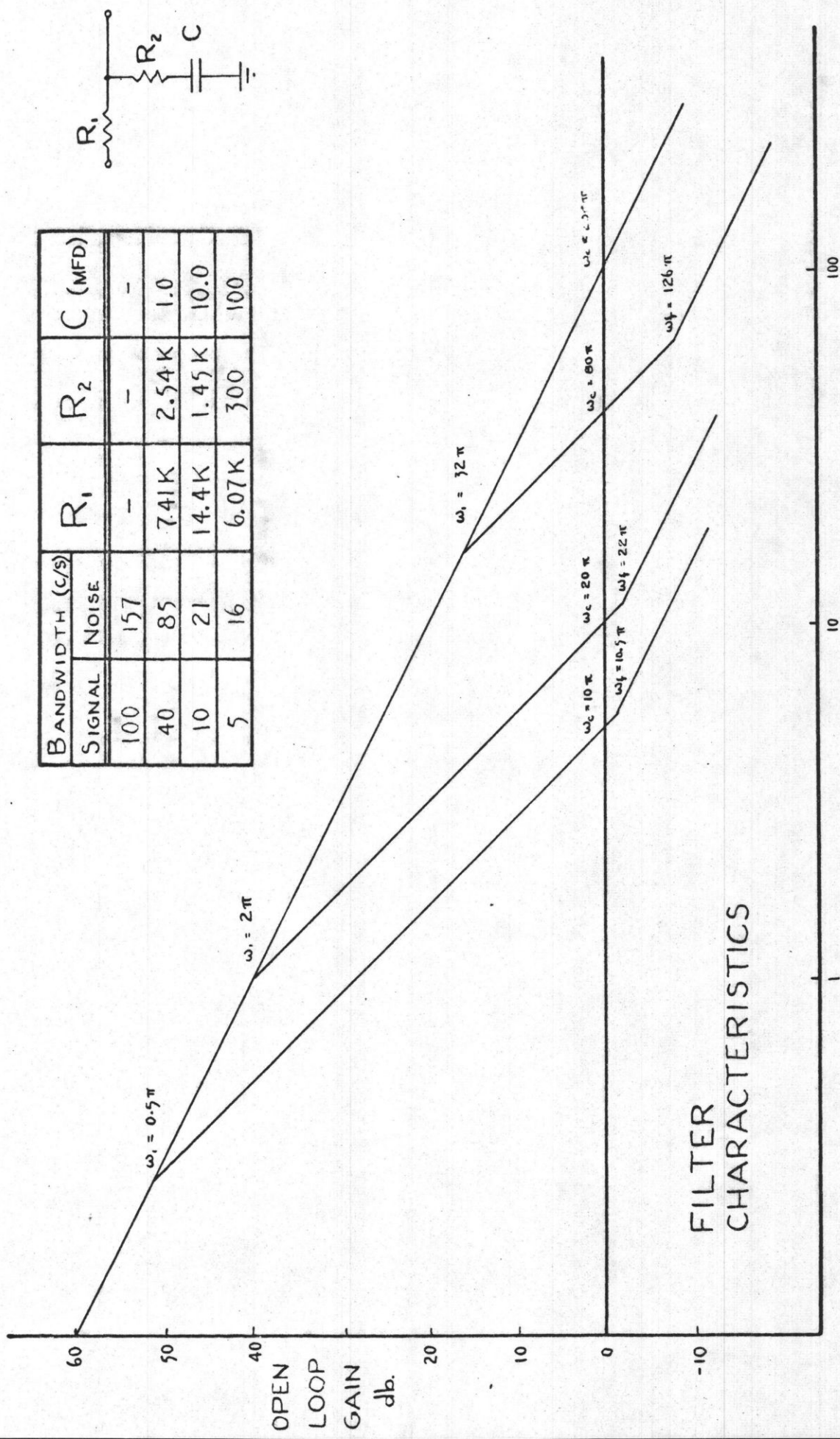
#### c) Filters

The filters were constructed according to McAleer's design criteria for an optimum lag network as shown in Figure 2.3. The fundamentals of the theory involved are outlined in Appendix II.

#### d) Voltage-Sensitive Variable Oscillator

This is merely an oscillator which has a voltage-sensitive capacity in the tuned circuit. The oscillator circuit itself was taken from the "Handbook of Selected Semi-conductor Circuits"\*. The voltage-sensitive capacitance is a 'vari-cap'; a small junction diode with a large back-bias capacity which is a non-linear function of the biasing voltage. The circuit diagram of this oscillator is shown in Figure 2.4.

\*United States Navy Publication; Navships #93484



BANDWIDTH (c/s)		$R_1$	$R_2$	$C$ (MFD)
SIGNAL	NOISE			
100	157	-	-	-
40	85	7.41K	2.54K	1.0
10	21	14.4K	1.45K	10.0
5	16	6.07K	300	100

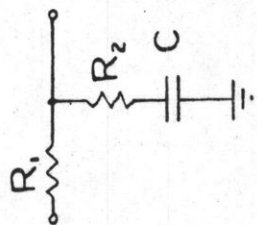


FIGURE 2.3

## 2.3 Comments and Experimental Details

### a) Circuit

The complete circuit diagram of the phase-lock detector is shown in Figure 2.5. It is essentially digital in nature to remove as much as possible any amplitude sensitivity. The signal from the receiver is 'clipped' top and bottom and used to trigger a monostable multi-vibrator. The VSVO output also triggers a monostable multi-vibrator.

The actual circuitry used could undoubtedly be improved. The circuit 'grew' with the project and no overall consideration was given to rigorous design.

Modification of the BC-453 was simple. A Pierce oscillator using a 540 kc/s crystal was built using one of the tube sockets formerly used in the audio section. The existing local oscillator was disabled and the 540 kc/s signal was injected at the grid of the mixer. The radio frequency amplifier stage was tuned to 455 kc/s and the tuning knob removed. The intermediate frequency amplifier section was 'broad-banded' using a sweep frequency signal generator.

The VSVO operated at a frequency of 170 kc/s which was halved by the monostable multivibrator following it, producing a signal at the intermediate frequency of the BC-453 receiver. This was done for simplicity since the desired tuning range could then be obtained using standard 2.5 mhy. RF chokes in the tuned circuit of the VSVO. A previous attempt to construct a

Figure 2.5      Circuit diagram of phase-lock detector



similar oscillator at a frequency of 85 kc/s resulted in a poor oscillator when using readily available components.

The low-pass filters were constructed in a 'mini-box' separate from the rest of the circuit and connected to it by a cable. This was done to isolate the circuitry from the mechanical shocks resulting from switching the filters.

By varying the size of the coupling condensers from the 'vari-caps' to the tuned circuit in the VSVO, the lock-on range of the VSVO was adjusted to about 300 c/s.

The filter bandwidths were probably only within  $\pm 100\%$  of the  $-50\%$  stated value for the two narrowest bandwidths due to the use of electrolytic condensers in the filters and to the uncertainty in the lock-on range of the VSVO.

The VSVO frequency variation was approximately linear with the voltage applied to the 'vari-caps' over the range used. Figure 2.6 is a graph of the frequency vs. the indicated output current of the chart recorder.

The major problem encountered in the experiment was that of temperature stability. The environment at the receiving site was far from ideal since all the windows face south. The daily temperature variation was of the order of  $40^{\circ}\text{F}$ . In converting from the transmitting frequency to the phase-lock detector frequency, three separate crystal oscillators were used and as well, a 'mechanical filter' was used in the F-3 receiver. All these controlling elements show some frequency

**Figure 2.6**

**Graph of the VSVO frequency versus the output  
chart recorder current**

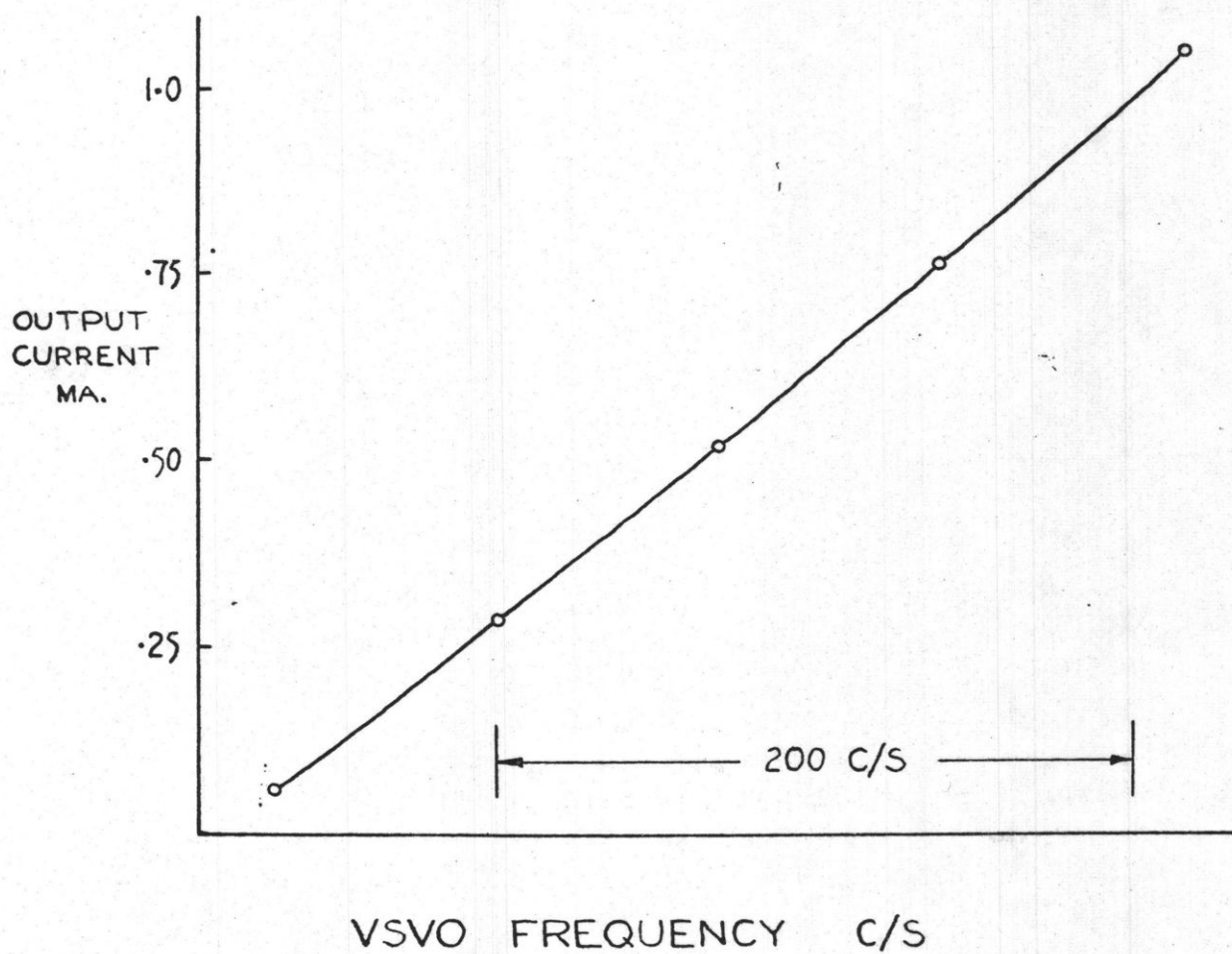


FIGURE 2.6

drift. The environment at the transmitter was not known but it is likely that the transmitter frequency also drifted. The major source of frequency instability however was the VSVO. Basically, in any oscillator ease of frequency change must be traded for stability and this coupled with the poor temperature characteristics of the 'vari-caps', resulted in an oscillator which was not stable with temperature changes.

It was originally thought that it would be worthwhile to bury the VSVO about ten feet in the earth to reduce the instability and to this end the VSVO was constructed in a 'mini-box' at the end of thirty-five feet of cable. However, it soon became apparent that the other frequency drifts were not as insignificant as originally supposed. It was obvious that continuous records could not be made without major modifications to the receivers and the transmitter. The VSVO was left unburied and a manual frequency control added so that the VSVO could be retuned to the signal frequency when an observation was being made.

#### b) Procedure

The procedure for aligning the equipment was fairly straight forward. An auxiliary, highly stable oscillator (BC-221) was tuned to the centre of the band-pass of the 455 kc/s output of the F-3 receiver. This output was temporarily disconnected from the input of the modified BC-453 receiver, to which a small piece of wire was connected to act as an antenna, so that

the BC-453 received the output signal from the BC-221 signal generator. The VSVO was then tuned so that the BC-221 signal was at the VSVO centre frequency. The short piece of wire was removed from the BC-453 and the F-3 output was reconnected. The fine tuning control of the F-3 was then varied until the signal was detected on the 'frequency' Esterline-Angus meter.

The VSVO centre frequency was regarded as being that frequency to which the VSVO would go when the input to the F-3 was white noise; or when the antenna was disconnected.

### III. RESULTS

It was found that the transmitter varied in frequency in a characteristic manner; during the thirteen minutes that the transmitter was keyed, the frequency increased approximately 30 c/s in a roughly exponential manner, then returned to the starting frequency during the two minutes of unkeyed time. This typical behaviour is illustrated in Figure 3.0 which is a record of the frequency during an enhancement of the forward-scattered signal. It is very unlikely that the small irregularities in the frequency are caused by the transmitter because they do not recur in every cycle.

Continuous records were made from the beginning of July to the middle of September 1961, but these records are largely unuseable because of the temperature instability of the equipment. However, up to a few hours useful information could be gleaned from a day's records although most of the data was accumulated during periods when the author was at the receiving site and could manually reset the equipment.

The bandwidth of the receiver was varied from time to time and the equipment allowed to run. It would be expected that at narrow bandwidths the receiver would lock-on over greater periods of time than at wider bandwidths if the received signal had a spectral bandwidth narrower than the narrowest receiver bandwidth. With the equipment used however, this would not be the case since with narrower receiver bandwidths, there is a corresponding reduction in the capture range of the

**Figure 3.0**      **Tracing of frequency record during a period of  
high level enhancement showing typical  
transmitter frequency characteristic**

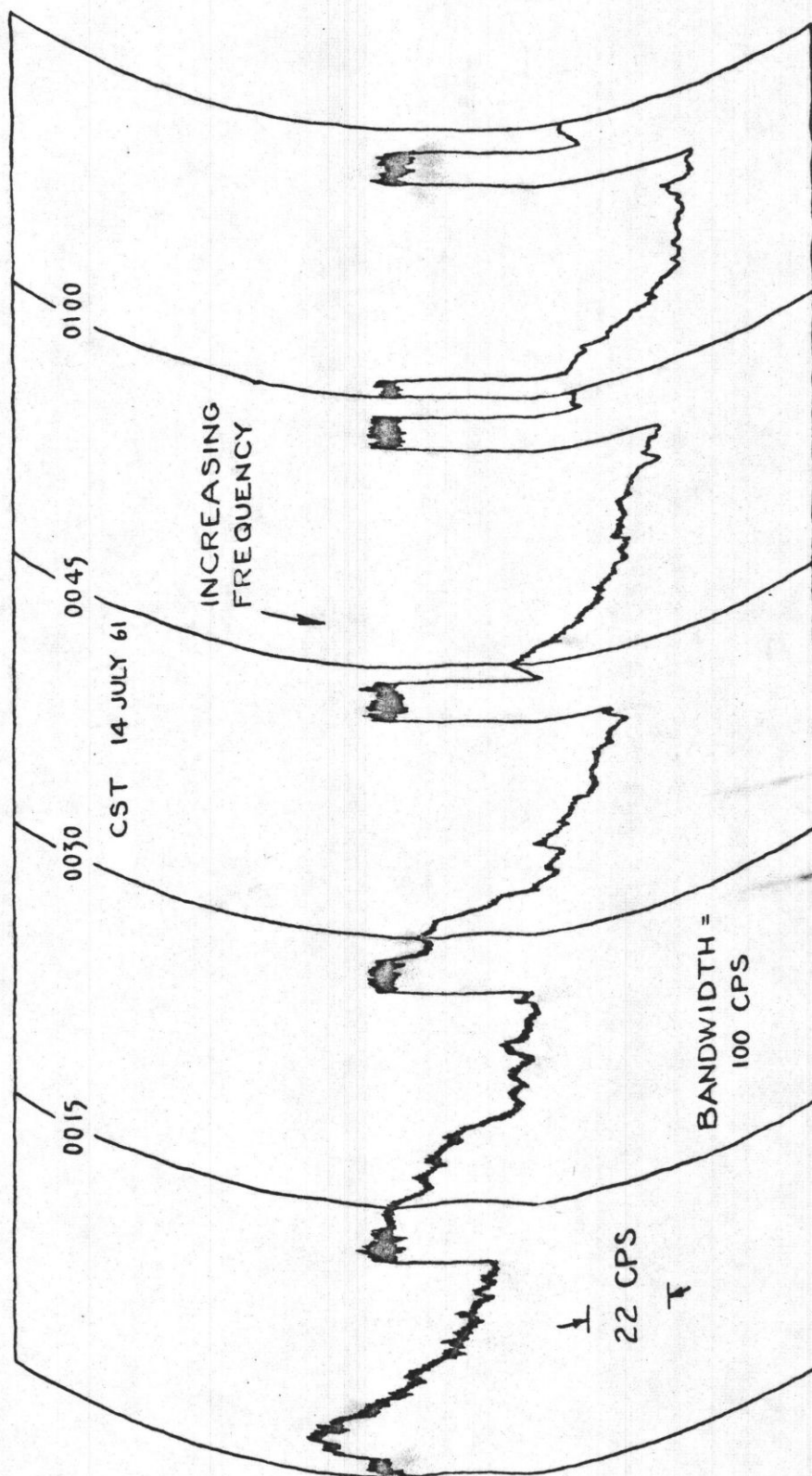


FIGURE 3.0

detector. Consequently, most of the continuous records obtained were of little interest except when the received signal frequency was within the capture range of the detector during an interesting event.

Perhaps the best expression of the results can be obtained by commenting on the different forward-scattered signal amplitude events encountered.

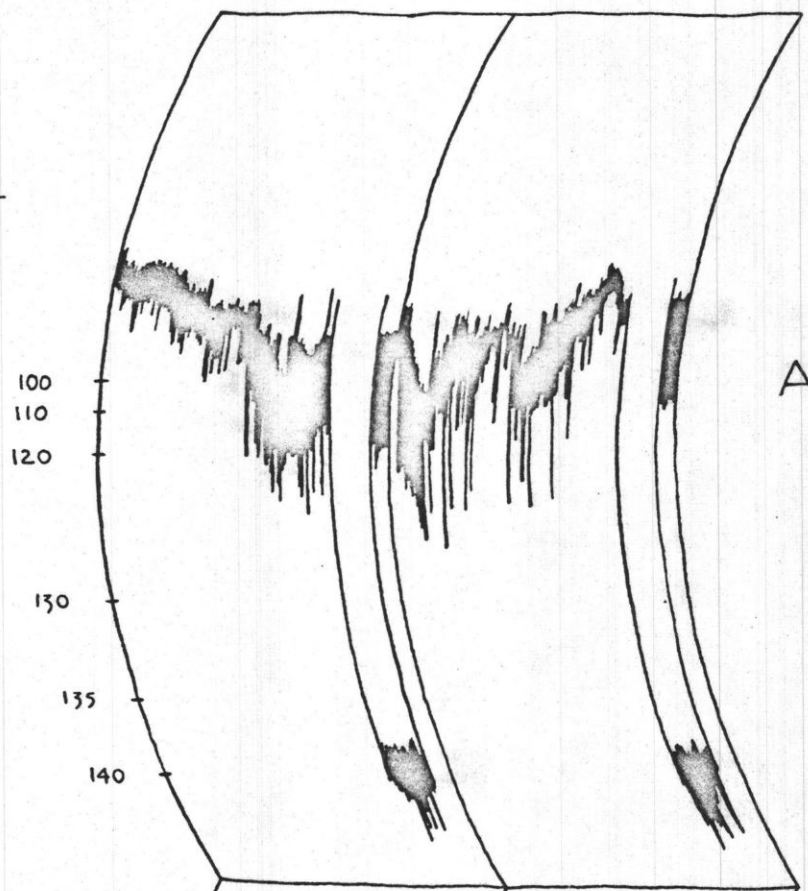
1) Enhancements - A or E type; characterized by an absence of meteor 'spikes' and a signal level 30 - 60 db. greater than the background noise with low amplitude, slow fading.

During these periods, the signal could be locked-on on all receiver bandwidths although sometimes, some difficulty was had in using the narrowest bandwidths (five or ten c/s). Rapid frequency excursions of the order of 100 c/s, which lasted only milliseconds were sometimes seen on the oscilloscope and were too fast to be reproduced on the moving chart records. These excursions were noticed to be particularly frequent during times when difficulty was experienced in locking-on the narrow bandwidths, and are undoubtedly the reason for the difficulty.

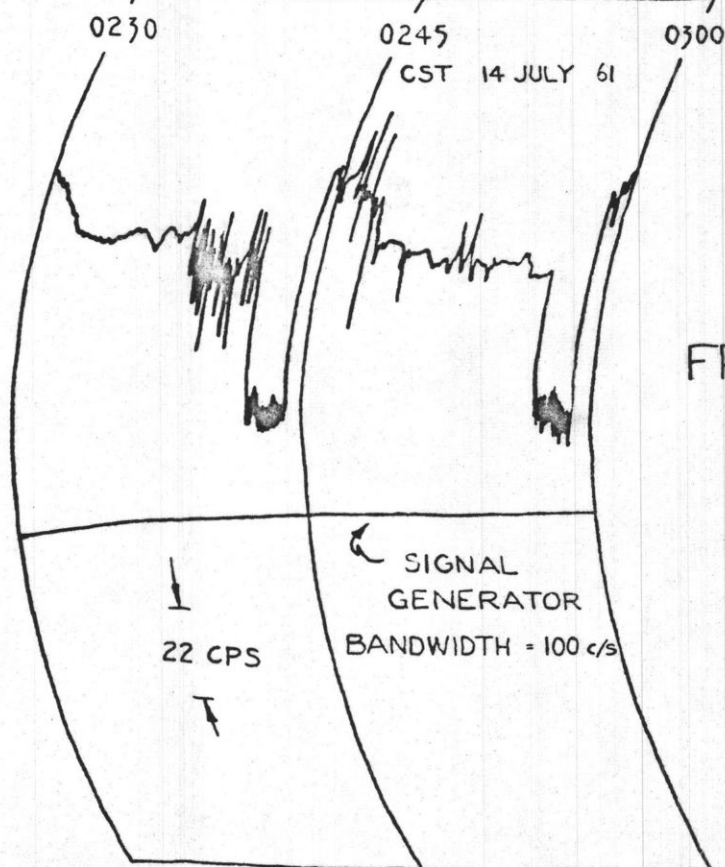
A typical frequency record of this type is reproduced in Figure 3.1 along with the corresponding amplitude record. As well, plotted on the same record is the trace obtained when a signal generator was connected to the receiver input. The amplitude of the signal generator output was roughly the same as that of the received forward-scattered signal.

**Figure 3.1**      **Tracings of frequency and amplitude records  
during a period of high level enhancement**

AMPLITUDE  
SCALE  
DBM.



AMPLITUDE



FREQUENCY

FIGURE 3.1

2) Enhancement - S or A<sub>3</sub> type; a lower level enhancement than that described above but characterized by a slow, deep fading.

This type of signal could be locked-on on all bandwidths. The frequency and amplitude records of a typical example of this type of signal are shown in Figure 3.2.

3) No enhancement - the received signal consisted solely of meteor 'spikes' with no or very little increase in the level of the background noise.

This signal could be locked-on on the very narrowest bandwidths (five or ten c/s), could be detected on the bandwidth of 40 c/s, but could not be detected at all or very erratically on the bandwidth of 100 c/s. The amplitude and frequency records of a typical example of this type of propagation are shown in Figure 3.3.

4) Absorption - the signal received appears to be the same as the above type except that it is lower in amplitude. There is a decrease in the background noise level and the number and amplitude of meteor 'spikes' is less than normal.

This type of signal could only be locked-on on the narrowest bandwidths (five or ten c/s) but not at all on the others except during a meteor 'spike'. Figure 3.4 shows the frequency and amplitude records of an event of this type. From meteor counts on the nights of July 10 and 11, it is possible to estimate that the signal amplitude is more than 10 db. below its normal level under conditions of no enhancement. This represents a decrease

**Figure 3.2**

**Tracings of frequency and amplitude records  
during a period of low level enhancement**

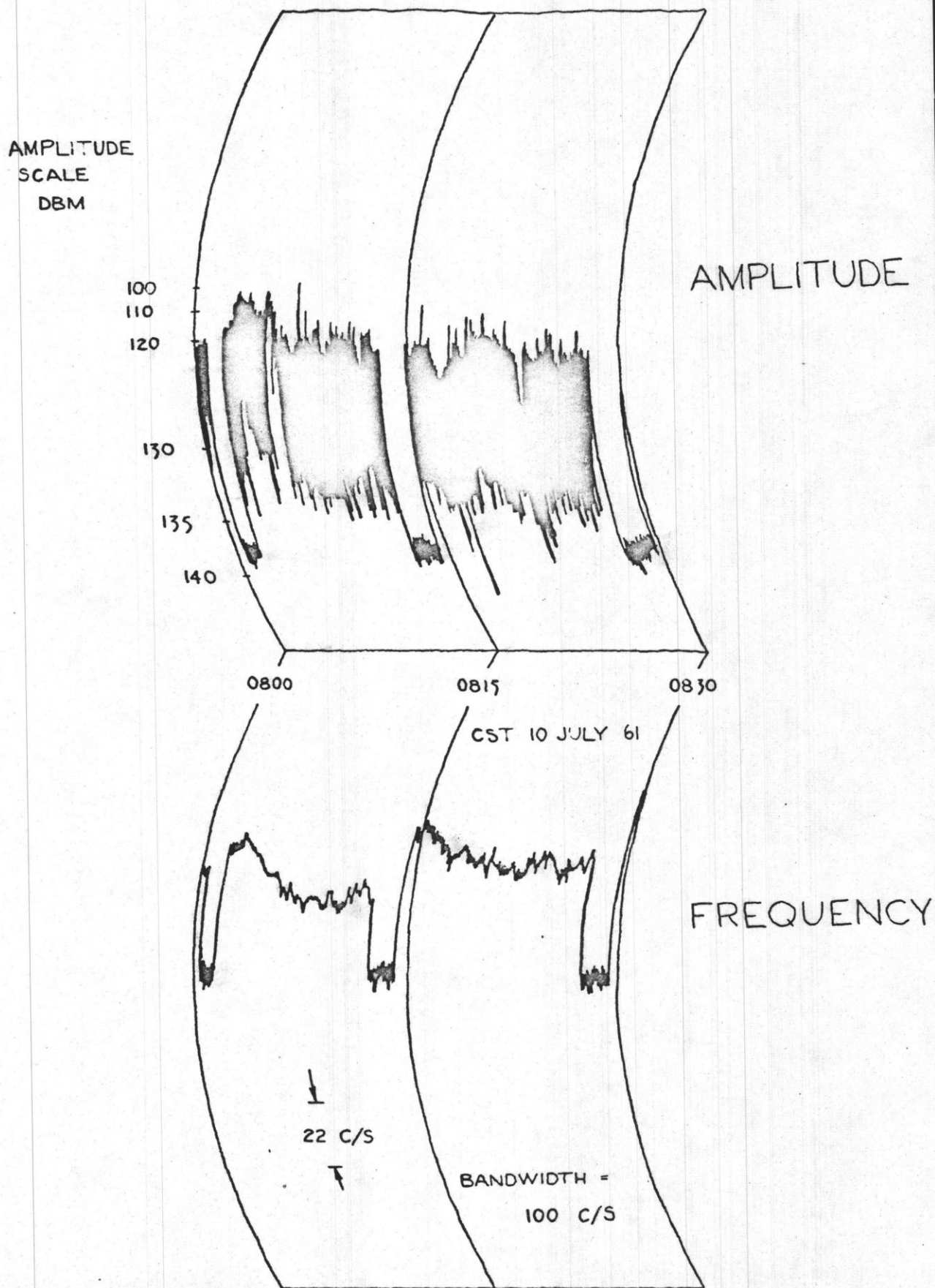


FIGURE 3.2

Figure 3.3

Tracing of frequency and amplitude records  
during a period of no enhancement

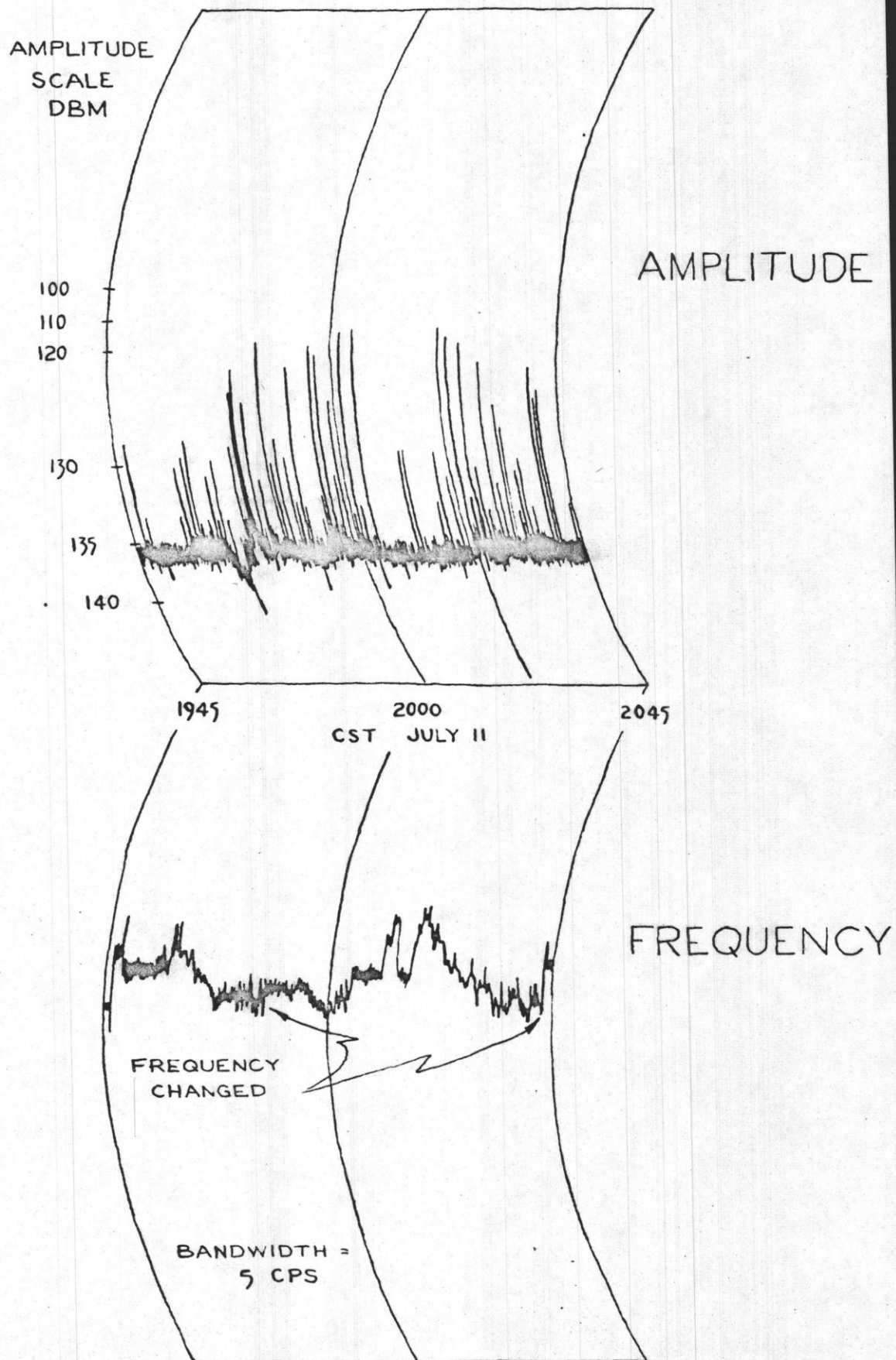


FIGURE 3.3

**Figure 3.4**      Tracings of frequency and amplitude records  
during a period of absorption

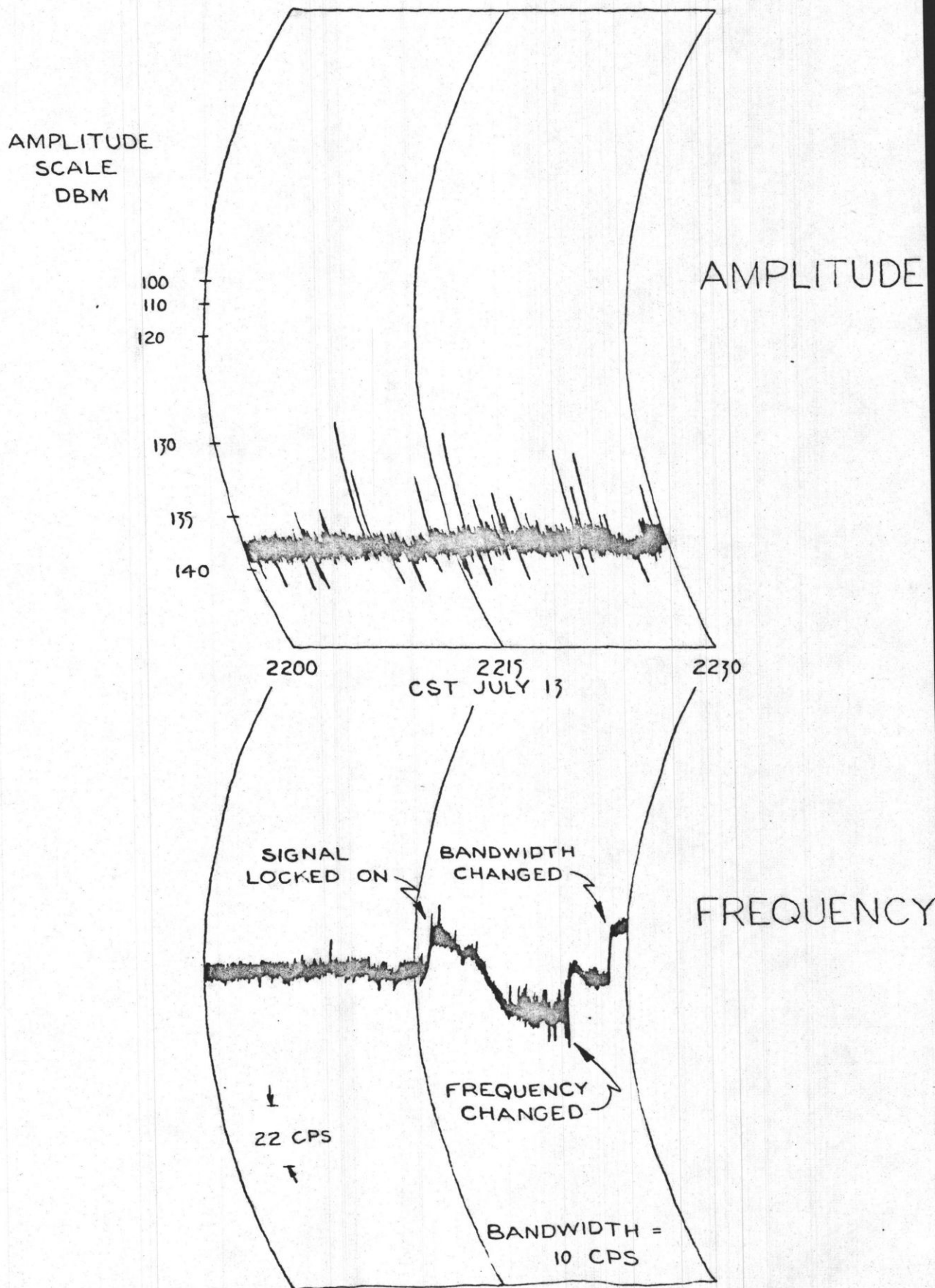


FIGURE 3.4

in the input signal-to-noise ratio of at least 6 db.

In all of the above results, a signal was considered to be locked-on if the full transmitter frequency deviation could be seen. If the presence of a signal could be detected, but the full deviation was not noticed, the signal was considered to be detected.

The frequency records for the months of July and August were examined and those hours for which the signal was detected or locked-on for at least a continuous hour were counted. The amplitude records from the F-3 receiver (which had a bandwidth of 1 kc/s) were also examined for these same periods and an estimate of the signal-to-noise ratios were made. These signal-to-noise ratios were divided into four categories and plotted against the number of hours in that category for the various bandwidths. Due to the rigorous selection, records of only 249 hours were used and these are shown in Figure 3.5 and Table I.

For several days, an attenuator was connected between the antenna and the receiver and set to 10 db. attenuation. This attenuation can be considered to have lowered the input signal-to-noise ratio by at least 6 db. during periods of no enhancement. It made no noticeable difference in the ease with which the signal could be locked-on on the narrower bandwidths during these periods.

During one of these afternoons when there was no

**Table I**      Table showing number of hours counted in each signal-to-noise ratio division for the various receiver bandwidths

**Figure 3.5**      The values in Table I shown graphically

S/N RATIOS (DB.)	HOURS IN EACH BANDWIDTH (C/S)			
	100	40	10	5
>25	19	8	1	3
10-20	45	10	7	10
3-9	31	7	21	6
<2	10	13	32	26

TABLE I

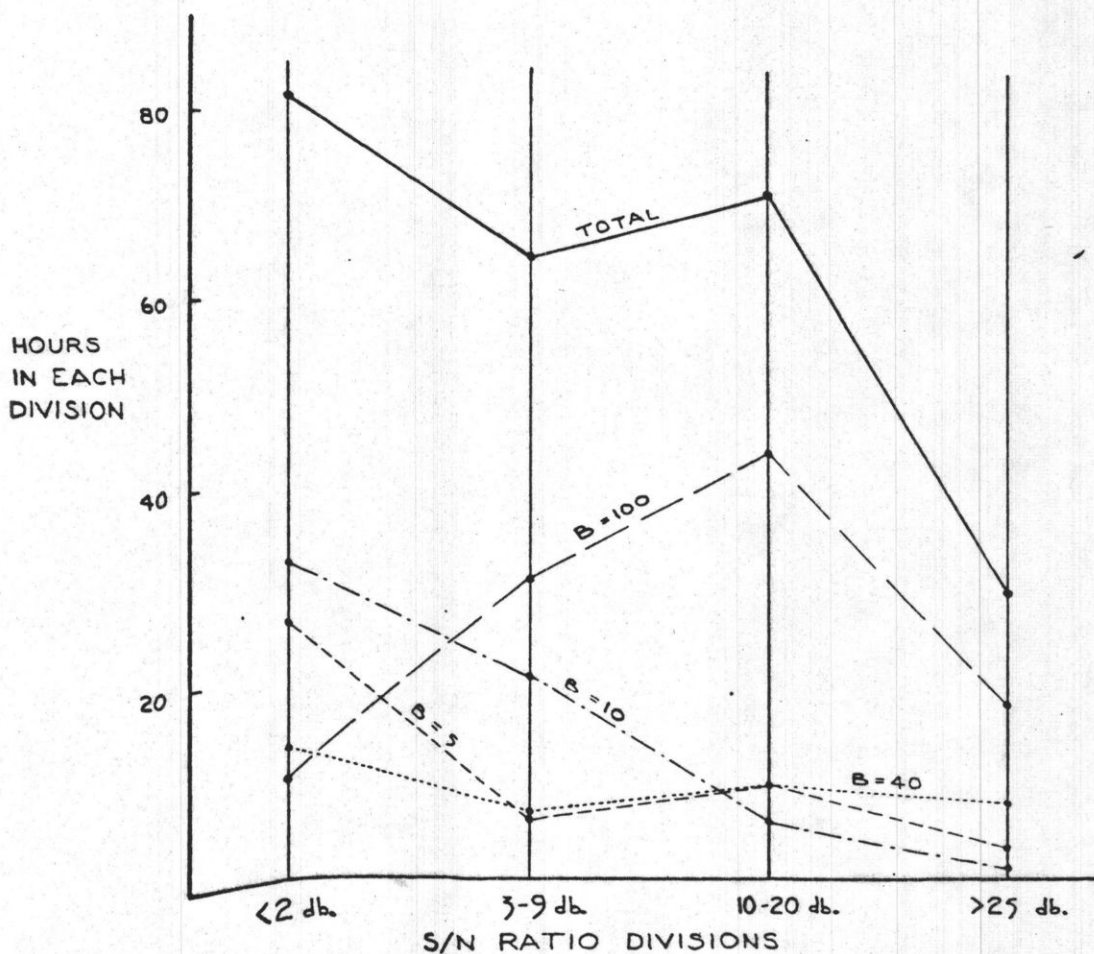


FIGURE 3.5

enhancement, the attenuator was varied to find the maximum value of attenuation which could be inserted before no signal could be detected on the various bandwidths. This experiment was necessarily rough since there are always slow frequency variations and it was impossible to tell whether a specific observed movement on the 'frequency' meter was due to such a variation or whether the signal had just been 'lost'. At any rate, it seemed that there was no difference in the amount of attenuation which could be added for the two narrower bandwidths and that this amount of attenuation was about five db. more than could be added for a bandwidth of 100 c/s. Although these values are not precise there was clearly an improvement in the system sensitivity at the narrow bandwidths over the sensitivity obtained with the bandwidth of 40 c/s.

The frequency excursions of the received signal can be divided into two broad categories; those which appeared on the chart record and those which only lasted milliseconds. Both types occurred only rarely and not much can be said about the latter type since they were only noticed occasionally and no effort was made to examine them any further. The slow frequency excursions (which lasted up to minutes) were only recorded around the period of July 12-14. Generally, these excursions were evenly distributed between those that caused a frequency increase and those which caused a decrease. Their magnitude seemed to have a roughly Gaussian distribution (as well as can be determined on so

little data). Several of these frequency excursions are seen in Figure 3.1.

The results may be summarized as follows; it was possible to detect a signal on the narrowest bandwidths except on rare occasions. These rare occasions occurred solely during enhancement and not during all enhancements, and it was then possible to lock-on with the broad bandwidths. During the periods when the signal level was low, the narrow bandwidths provided a better and more constant output than did the larger bandwidths.

#### IV. DISCUSSION

The receiver signal and noise bandwidths are not the same and do not vary in a proportional manner. Thus, reducing the receiver bandwidth about a discrete signal in white noise will not necessarily increase the output signal-to-noise ratio in the same proportion. This effect, however, is only significant for the two narrowest bandwidths used; five and ten c/s. Although the bandwidth is here reduced by a factor of two, the maximum possible increase of the signal-to-noise ratio is only about 1.3.

At all times that the receiver was being operated manually, it was possible to detect a signal when using the narrowest receiver bandwidths of five and ten c/s, even at times when no signal could be detected on the other bandwidths. This implies a signal spectral spread of less than the next larger bandwidth used; 40 c/s.

Figure 3.5 confirms this inference. The proportion of time that the signal was detected on the receiver bandwidth of 100 c/s is less for low amplitude signals than for the other bandwidths used. From this graph, it is possible to say that a bandwidth of 40 c/s is sufficient to always detect a signal during times of no enhancement. However, the actual frequency records during these times on a bandwidth of 40 c/s compared to similar times on the narrower bandwidths show that the final signal-to-noise ratio on the narrower bandwidths is higher. This implies that the received signal spectral spread is less than 40 c/s but that

the transmitter power is sufficiently high to permit detection even with a receiver bandwidth of 40 c/s. Supporting this contention is the fact that the addition of an attenuator between the antenna and the receiver made no appreciable difference in the ability to detect a signal on the narrow bandwidths.

Figure 3.5 also tends to show another characteristic of the signal during enhancements that was noticed before; that during an enhancement, the received signal undergoes frequency excursions too fast for the detector to follow. This is shown by the disproportionately small number of times that a signal was detected during enhancements on the narrower bandwidths.

From these facts, it is possible to state generally that, in the absence of high level enhancements, the signal spectral spread is smaller than 40 c/s and is probably about 10-20 c/s. During high level enhancements, the spectral spread is not known but whatever it is, there are sometimes gross frequency shifts present at a rate greater than ten c/s.

There seems to be little doubt that during periods of absorption, if the present observations are typical, the spectral spread is about ten c/s.

It is not inconceivable that the slow frequency excursions seen are really the result of interference, since during an enhancement, conditions are particularly favorable for the reception of signals from distant transmitters. However, most of

these excursions were noticed in a period which was the occasion of two magnitude 3 solar flares and the ionosphere was undoubtedly disturbed.

McNamara (loc. cit.) often saw 'flat' spectra and suggested that they may be due to a large gross doppler frequency shift of the signal with a long frequency tail which extended into the bandpass of the receiver. Some of the frequency excursions lasted up to several minutes and may have produced spectra of this type. On the basis of the seventy-odd instances of frequency excursions which appeared on the records, it is possible to estimate that their average (rms) magnitude was about 20 c/s. Because of the  $\cos \theta$  factor, this implies a velocity of about 250 m/s. A signal with a doppler shift corresponding to this velocity would certainly have been within the bandpass of McNamara's receiver. Of the very few really large frequency excursions seen (60-70 c/s), none appeared to last more than a few seconds.

Although McNamara did find many instances of wide bandwidths in his pulsed radar echoes corresponding to high rms scatterer velocities, he often found echo bandwidths corresponding to an rms velocity of about 50 m/s. For this value, according to Appendix I, the forward-scatter spectral spread would be ten c/s or less. It is interesting to note that later studies made at the Institute of Upper Atmospheric Physics, University of Saskatchewan, by G. Lyon (private communication) show an extremely good correlation between times of forward-scatter enhancement and the occurrence of

pulsed radar echoes at nearly the same frequency. It is not unreasonable to suppose that the extremely high velocities found by McNamara occur only during times of forward-scatter enhancement. This hypothesis is supported by the large velocities deduced from the magnitude of the frequency excursions recorded during one of these enhancements.

A possible explanation for the 'flat' spectra seen by Bowles (loc. cit.) and McNamara is that there are several frequency peaks separated from each other in frequency because of different gross velocities of bodies of scatterers in different regions of the scattering volume. If the relative importance of these several peaks changed from time to time, this would also be a reasonable explanation for the observed slow frequency variations of the received signal.

In Table II, a comparison is made of the received signal amplitudes observed by Bailey et al (1955) on their VHF link between Churchill, Manitoba and Fargo, North Dakota, and this experiment. The value cited for the 'signal-to-noise ratio exceeded 99.9% of the time' for this experiment was that obtained during a period of absorption. It is only estimated; the signal could be detected but not well (see Figure 3.4).

Since the amplitudes received in both experiments are comparable and since Bailey et al used more directive antennae, it seems likely that scattering takes place over a relatively small volume. Because of this directional effect, it does not seem likely that meteor trails are the major scattering bodies.

Item	Bailey et al	This Experiment	Comparison
Transmitter power	30 kw.	50 w.	+ 28 db.
Antennae gain	36 db.	24 db.	+ 12 db.
Path length	1243 km.	940 km.	- 2 db.
Bandwidth	2 kc/s	10 c/s	- 23 db.
S/N ratio exceeded 99.9% of the time	15 db.	~ 3 db.	- 12 db.
TOTAL			+ 3 db.

TABLE II. Comparison of Results of Bailey et al (1955)  
and This Experiment at Comparable Latitudes

S/N ratio (db.)	0	3	7	10	13	17	20	23
Error Probability	25%	17%	7.5%	4.5%	2.3%	1.7%	0.5%	0.3%

TABLE III. Comparison of S/N Ratio and Probability of  
Error For a Binary Coded, Narrow Bandwidth  
Frequency Modulated Communications System  
in Which the Carrier is Subject to Rayleigh  
Fading

The inhomogeneities which do cause scattering may however be largely meteoric in origin.

The evidence of a small scattering volume tends to support the assumptions made in Appendix I. From Appendix I, a received signal bandwidth of less than about 10 c/s (as was found in conditions of no enhancement or absorption) implies an rms velocity of scatterers of less than about 50 m/s. It may be that in auroral latitudes, during 'quiet' periods, the ionosphere is no more turbulent than at lower latitudes.

It is interesting to speculate that the inhomogeneities are stratified in height. This would explain both the small scattering volume and also would result in a forward-scattered signal of very narrow bandwidth even if the rms velocity of the scatterers was very high.

Assuming that some type of frequency modulation would be used in a low power VHF forward-scatter communications system, the type of detector used in this experiment seems to be ideal. It provides a narrow bandwidth which will 'follow' the transmitter frequency over frequency variations much wider than the bandwidth provided these variations are at a frequency which is inside the detector signal bandwidth.

It is possible to evaluate the performance of a communications link of this type. Montgomery (1954) has calculated the effect of signal-to-noise ratio on the probability of error in a binary coded, narrow bandwidth, frequency modulated communications system in the presence of Rayleigh fading of the carrier. The results of these

calculations are shown in Table III. Over a 1000 km. communications link through the auroral zone, using a 100 watt transmitter and a receiver with a bandwidth of 10 c/s, and antennae similar to those used in this experiment, the received signal-to-noise ratio would be 7 db. or more 99.9% of the time. The error probability of a binary bit would thus be about 8%. The theoretical capacity of the channel would be 20 bits/sec.; in practice, it is difficult to achieve half this rate. It would be safe to assume an operating capacity of 5 bits/sec. This corresponds to an English language text information rate of about 10 words/min., assuming a five letter average word, a five bit letter, and a one bit space mark between letters. Because of the natural redundancy of the English language, an error rate of 8% would probably not be too large for satisfactory communications. Also, the use of an information rate only one-quarter that of the channel's theoretical maximum will tend to reduce the incidence of errors.

In this evaluation of the system, it was assumed that one bit would suffice to mark a space between letters. To do so, it would have to be at a different level from the other bits and the system would not truly be binary. The number of different levels which can be distinguished is also a function of the signal-to-noise ratio. Shannon (1949) has shown that the number of levels will be slightly higher than  $\sqrt{1 + S/N}$ . Taking into account the slower information rate compared to the theoretical maximum, it appears that the use of three levels will just be possible.

This evaluation was made on the basis of the lowest signal

amplitude seen during two months of observation. Most of the time, the signal-to-noise ratio would be considerably higher.

In such a communications system, it would be necessary to make provision for the disruptions caused by some enhancements. For this purpose, an auxiliary low power AM or wide band FM transmitter (which could be the same transmitter as used in the narrow bandwidth system) and a conventional receiver would be sufficient due to the extremely high received power levels which would be present. The receiver could be the one used in the narrow bandwidth system since a phase-lock detector connected to any receiver with arbitrarily wide bandwidth still provides a narrow bandwidth.

There is no doubt that this experiment would have yielded considerably more information if the frequencies employed had been stable to temperature change. The statistical results obtained are not reliable and can give only an indication of the signal behavior. Since the phase-lock detector was constructed wholly of transistors, it might be buried and a buried oscillator could be used at the transmitter.

A more accurate determination of the output signal-to-noise ratio could be obtained with a transmitter which is frequency modulated over a narrow bandwidth at some modulation frequency lower than the narrowest receiver bandwidth. With the use of suitable high-Q filters at the phase-lock detector output, the output signal-to-noise ratio could then be determined directly.

## V. SUMMARY

During periods of absorption, no enhancement and low level enhancement, the forward-scattered VHF signal from Fort Smith, N.W.T. to Saskatoon, Sask., had spectral bandwidths not greater than about 10 c/s.

During periods of high level enhancement, the signal bandwidth was not determined but there was some evidence to suggest a wider bandwidth than mentioned above. As well, gross frequency shifts of the order of 30 c/s were observed in the received signal during some of these enhancements. These frequency shifts lasted from milliseconds up to minutes and occurred rarely.

A low power, narrow bandwidth VHF communications system of extreme reliability for use in auroral latitudes appears to be feasible.

## APPENDIX I

The following assumptions were made to facilitate the calculation of the spectral distribution of the received signal.

- 1) The inhomogeneities causing scattering are moving randomly with a Gaussian velocity distribution.
- 2) The entire volume in which scattering takes place is evenly illuminated by the transmitter.
- 3) The decrease in signal strength due to longer path lengths off to the side of the transmitter-receiver path is neglected.
- 4) The scattering bodies scatter isotropically.
- 5) Scattering takes place only on a line 100 km. above the earth which bisects the Fort Smith - Saskatoon path.
- 6) All points beyond about  $\pm 40^\circ$  from the direction to the transmitter as seen from the receiver site on the above line make a negligible contribution to the received signal.

Assume the following velocity distribution:

$$1) \quad N(v)dv = \frac{1}{2\sqrt{2\pi}v_0} \exp\left\{\frac{-v^2}{2v_0^2}\right\}$$

where  $v_0$  = rms velocity

The received signal doppler shift from any scattering body is given by:

$$2) \quad f = f_0 \left(1 + \frac{2v \cos \theta}{c}\right)$$

Since the signal received from the individual scatterers will be of random phase, the resultant received power may be obtained by adding the individual contributions resulting in (Ratcliffe, 1948):

$$3) \quad W(f)df = \frac{\gamma \lambda}{2\sqrt{2\pi} v_0 \cos \theta} \exp \left\{ \frac{-\lambda^2(f - f_0)^2}{8v_0^2 \cos^2 \theta} \right\} df$$

$$\text{where } \gamma = \frac{1}{\int W(f)df}$$

Assuming an antenna gain pattern  $G(\theta)$ , then the contribution from any element  $d\theta$  will be:

$$4) \quad W(f)d\theta = \frac{A}{\cos \theta} \exp \left\{ \frac{-\lambda^2(f - f_0)^2}{8v_0^2 \cos^2 \theta} \right\} df \cdot G(\theta)d\theta$$

$$\text{where } A = \frac{\gamma \lambda}{4\sqrt{2\pi} v_0 \int G(\theta)d\theta}$$

Equation 4) can be integrated directly over  $\theta$  if:

$$5) \quad G(\theta) = \frac{k \sin \theta}{\cos^2 \theta}$$

Figure 5.0 compares the actual  $G(\theta)$  obtained on the basis of assumptions 3, 5 and 6, and a five-element Yagi receiving antenna, with the equation 5). It may be seen that at least over the range of  $\theta$  shown, the approximation is adequate. Integrating 4) thus produces:

$$6) \quad W(f)df = \left[ \frac{-2Ak}{C^2} \exp \left\{ \frac{-C^2}{\cos^2 \theta} \right\} \right]_{\theta_2}^{\theta_1}$$

$$\text{where } C = \frac{-\lambda^2(f - f_0)^2}{8v_0 \cos^2 \theta}$$

$\theta_1$  =  $\theta$  directly over the transmitter-receiver path

$\theta_2$  = some limiting  $\theta$  as stated in assumption 6.

Evaluating this integral by means of the values in Table I

Table IV      The relationship between the relative antenna gain and  $\theta$ , the scattering half-angle,  $\beta$ , the antenna beam angle and  $\frac{k \sin \theta}{\cos^2 \theta}$

Figure 5.0      A graph showing the actual relative antenna gain as a function of  $\theta$ , the scattering half-angle and  $\frac{k \sin \theta}{\cos^2 \theta}$

Antenna Power Gain	1.0	0.96	0.81	0.69	0.56	0.44	0.30	0.22	0.14
$\beta$	0	5°	10°	15°	20°	25°	30°	35°	40°
$\emptyset$	76°	75°	74°	70°	65°	61°	58°	51°	49°
$k \frac{\sin \emptyset}{\cos^2 \emptyset}$	1.0	0.87	0.76	0.48	0.31	0.22	0.19	0.12	0.11

TABLE IV

RELATION BETWEEN RELATIVE ANTENNA POWER GAIN  
AND  $\emptyset$ ,  $\beta$ ,  $k \frac{\sin \emptyset}{\cos^2 \emptyset}$

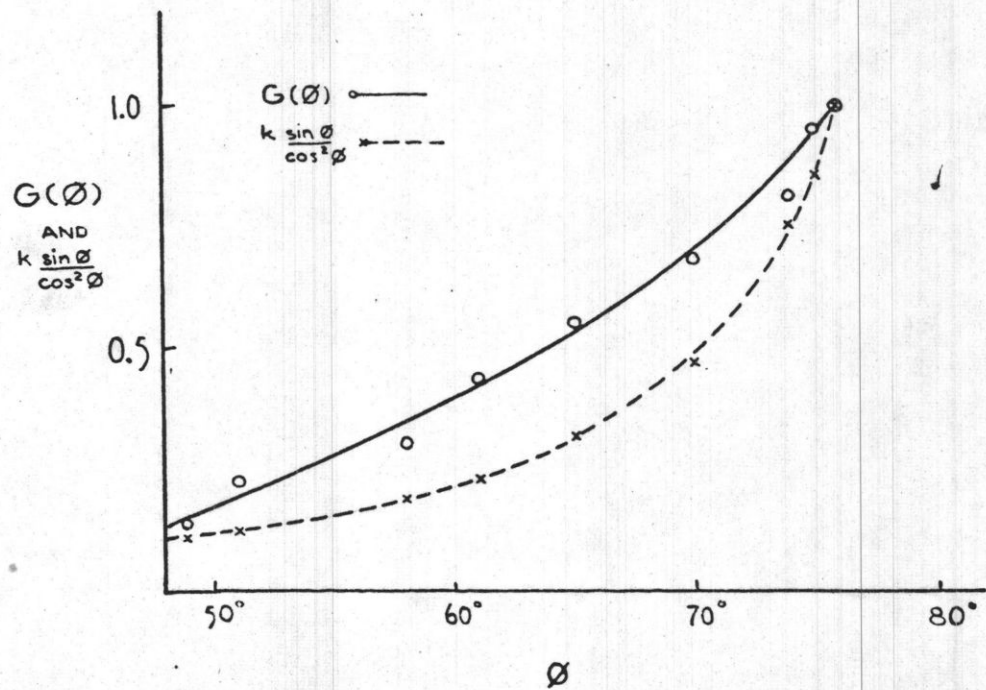


FIGURE 5.0

produces finally:

$$7) \quad W(f)df = \frac{-2B}{C^2} \left\{ \exp(-17C^2) - \exp(-2.3C^2) \right\}$$

where  $B = Ak$

This function is plotted in Figure 5.1. The half power point occurs at about  $C = 0.28$ . Thus, for the received signal at a frequency of 40 mc/s:

$$8) \quad \frac{\Delta f}{\nu_0} = 0.22$$

where  $\Delta f =$  half power bandwidth

The extent to which this result represents the physical situation will depend upon the validity of the assumptions. Assumptions 2, 3 and 4 tend to make the bandwidth broader than it is likely to be in reality; assumption 6, however, will tend to make it narrower. Assumption 5 will probably affect the bandwidth slightly but its main effect will be on the received power magnitude. From this it seems likely that the expression predicts bandwidths which are equal to, or greater than, those to be expected in the experimental situation.

**Figure 5.1**    A graph showing the received signal power spectrum  
on the basis of the calculations in Appendix I.

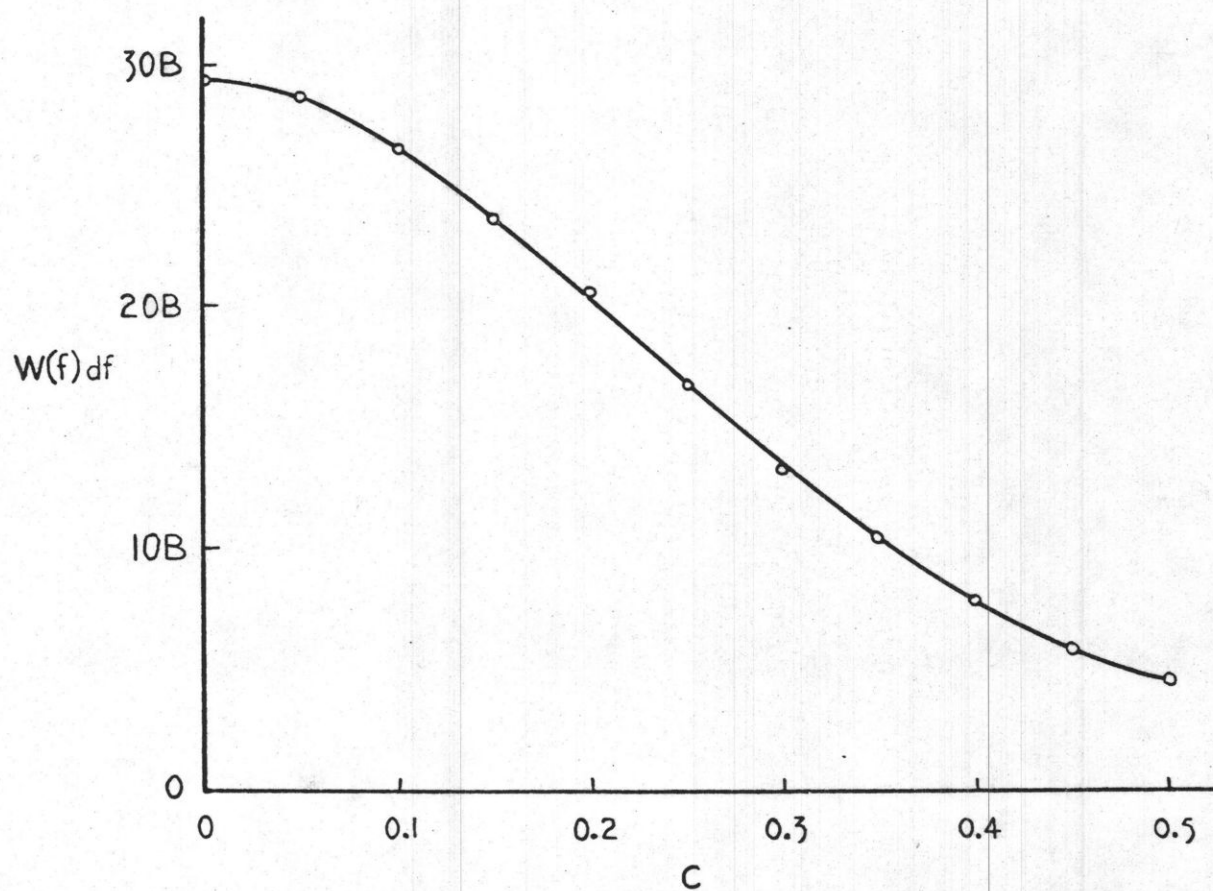


FIGURE 5.1

## APPENDIX II\*

A servo block diagram of the phase-locked oscillator is shown in Figure 6.0.

The frequency noise contributed by the source is represented by  $N_1$ ; internal noise contributed by phase detector circuit is represented by  $N_2$ . The  $1/S$  term represents the integration of frequency difference to phase difference which occurs in the phase detector. The gain of the phase detector is represented by  $K_1$  volts per radian. The frequency characteristic of the low-pass filter is indicated by  $F(S)$ , and the gain of the voltage sensitive capacitance in the oscillator is represented by  $K_2$  radians per second per volt.  $\Omega$  includes both the detuning of the oscillator and frequency noise in its output.

For a linear reactance modulator and a linear phase detector operating over a range of  $\pi$  radians, the lock range would equal  $\pi K_1 K_2$  radians per second. For any given lock range, it is best to have a large  $K_1$  and a small  $K_2$  to reduce the effect of internal noise.

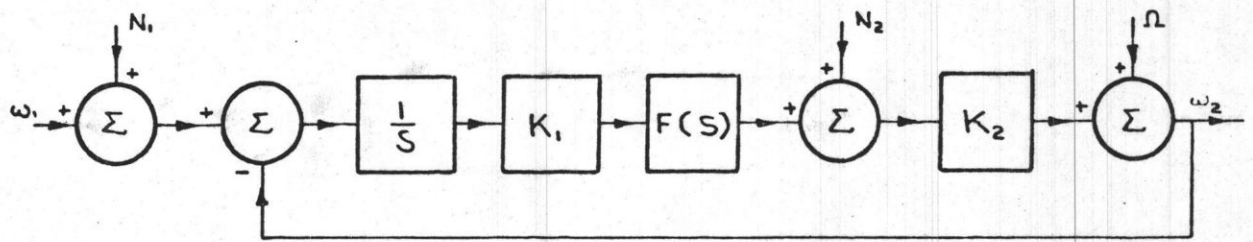
Figure 6.1 shows a plot of the open-loop gain of the system in decibels versus logarithmic frequency. Because of the  $1/S$  integration term, the loop gain decreases with frequency at a rate of 6 db. per octave, intersecting the unity gain line at a frequency of  $\omega_c = K_1 K_2$ . The low-pass filter is assumed to be a

\*This appendix is a precis of McAleer's text (1959). McAleer's paper is largely a summary of previous mathematical work done on the subject and is put in a more practical form than these other papers.

Figure 6.0    A servo block diagram of the phase-lock detector

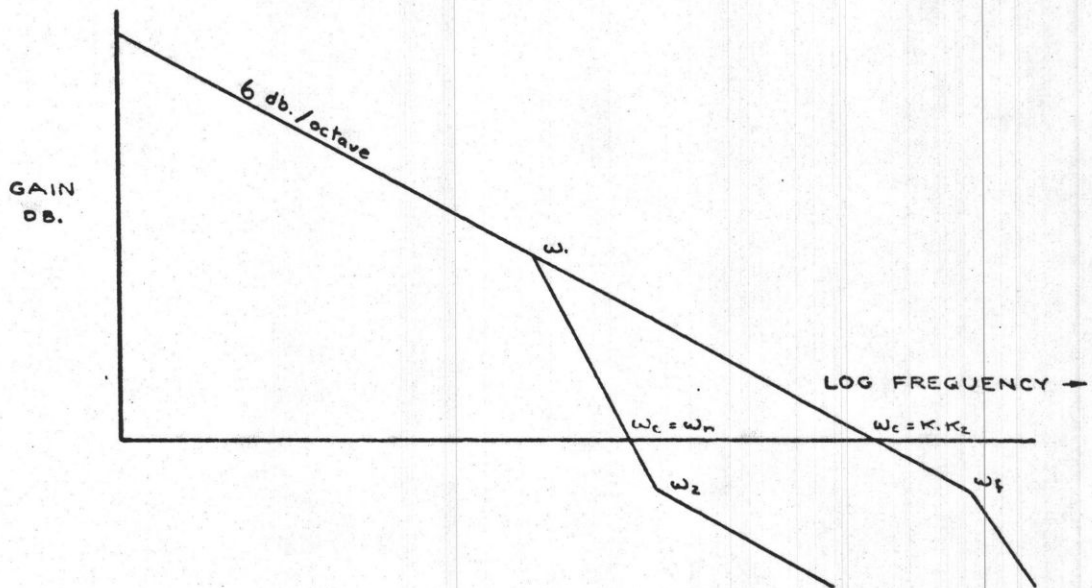
Figure 6.1    The frequency characteristic of the phase-lock  
detector with a low-pass filter

Figure 6.2    Circuit diagram of a lag network



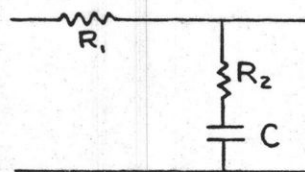
SERVO BLOCK DIAGRAM

FIGURE 6.0



OPEN LOOP GAIN

FIGURE 6.1



LAG NETWORK

FIGURE 6.2

simple RC section with a cut-off frequency of  $w_c$  (shown greater than  $w_c$ ).

The lock range is equal to  $\pi K_1 K_2$ , the capture range is equal to the lock range, and the cut-off frequency  $w_c$  is equal to  $K_1 K_2$ . With a simple RC filter reducing  $w_c$ , the capture range is reduced by approximately the same factor. With an optimized 'lag' network reducing  $w_c$ , the capture range is reduced by approximately the square root of the bandwidth reduction.

With RC Filter

#### Analysis of the Low-Pass Filter Effect

In Figure 6.0, let the transmission of the low-pass filter be:

$$F(s) = \frac{1 + t_2 s}{1 + t_1 s}$$

This corresponds to the use of a lag network shown in Figure 6.2, where:

$t_1 = (R_1 + R_2)C$  = the time constant of the lag break

$t_2 = R_2 C$  = the time constant of the lead break

The transfer function of the system can be calculated to be:

$$\frac{w_2(s)}{w_1(s)} = \frac{K(1 + t_2 s)}{t_1 [s^2 + s(1/t_1 + Kt_2/t_1) + (K/t_1)]}$$

where  $K = K_1 K_2$

The denominator is of the general form  $s^2 + 2\gamma w_n s + w_n^2$ , where

$w_n$  = the natural resonant frequency =  $\sqrt{K/t_1}$

$\gamma$  = the damping ratio =  $\frac{1 + Kt_2}{2w_n t_1}$

The noise bandwidth of the system can be shown to be:

$$F_n = \frac{\pi K}{2} \frac{\alpha^2 + Kt_1}{\alpha^2 + \alpha Kt_1}$$

where  $\alpha = t_1/t_2$ .

For the minimum noise bandwidth, it can be shown that the following conditions should be satisfied:

$$\gamma = \frac{1}{2} \sqrt{1 + (K/\omega_n)^2}$$

or, in terms of  $\alpha$ ,

$$\alpha = 1 + \sqrt{1 + Kt_1}$$

These equations may be manipulated to show that, in a system with minimum noise bandwidth and for the usual case  $K > (1/t_1)$ , the resonant frequency  $\omega_n = K/t_1$  equals  $\omega_c$ , the open-loop crossover frequency.

## BIBLIOGRAPHY

- |   |      |  |
|---|------|--|
| Bailey, D.K., Bateman, R.,<br>Berkner, L.V., Booker, H.G.,<br>Montgomery, G.F., Purcell, E.M.,<br>Salisbury, W.W. and Weisner, J.B. | 1952 | Phys. Rev., <u>86</u> , 141.                                       |
| Bailey, D.K., Bateman, R. and<br>Kirby, R.C.  | 1955 | Proc. IRE, <u>43</u> , 1181.                                       |
| Booker, H.G.  | 1957 | J. Atmosph. Terr. Phys.,<br>Special Supplement, p. 52.             |
| Bowles, K.  | 1952 | J. Geophys. Res., <u>57</u> , 191.                                 |
| Collins, C. and Forsyth, P.A.   | 1959 | J. Atmosph. Terr. Phys.,<br><u>13</u> , 315.                       |
| Forsyth, P.A., Green, F.D. and Mah, W.  | 1960 | Can. J. Phys., <u>38</u> , 770.                                    |
| Forsyth, P.A. and Vogan, E.L.   | 1957 | J. Atmosph. Terr. Phys.,<br><u>10</u> , 215.                       |
| Green, F.D.   | 1961 | Ph.D. Thesis, University<br>of Saskatchewan.                       |
| Gruen, W.J.   | 1953 | Proc. IRE, <u>41</u> , 1043.                                       |
| McAleer, H.T.   | 1959 | Proc. IRE, <u>47</u> , 1137.                                       |
| McKinley, D.W.R. and Millman, P.M.  | 1949 | Proc. IRE, <u>37</u> , 364.  |
| McNamara, A.G.  | 1955 | J. Geophys. Res., <u>60</u> , 257.                                 |
| Montgomery, G.F.  | 1954 | Proc. IRE, <u>42</u> , 447.  |
| Preston, G.W. and Tellier, J.C.   | 1953 | Proc. IRE, <u>41</u> , 249.  |
| Ratcliffe, J.A.   | 1948 | Nature, Lond., <u>162</u> , 9.                                     |
| Ratcliffe, J.A. and Weekes, K.  | 1960 | "Physics of the Upper<br>Atmosphere", Academic<br>Press, Lond.     |
| Rideout, V.C.   | 1954 | "Active Networks",<br>Prentice-Hall.                               |
| Shannon, C.E. and Weaver, W.  | 1949 | "The Mathematical Theory<br>of Communications",<br>Illinois Press. |
| Vogan, E.L.   | 1954 | Radio Physics Lab. Report<br>No. 12-1-6, D.R.B., Ottawa.           |