GENERATING CAPACITY RELIABILITY EVALUATION USING FREQUENCY AND DURATION METHODS

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ABSTRACT

In the planning of power system generation facilities, reliability considerations are as important as those of economics. The system must have sufficient reserve capacity to permit it to operate at an adequate level of reliability. The evaluation of generating capacity reliability can be considered to take two basic forms which can be designated as the static and the spinning reserve requirements. This thesis investigates the application of the frequency and duration concept to the static reserve problems.

Equations for fast computation of the cumulative availability and frequency of capacity outage states are developed. Relationships are also developed for the reliability evaluation of a system connected to one or more systems. A general computer programme for the reliability evaluation of single and two area problems has been developed. The concepts developed in this programme are illustrated by application to the Saskatchewan and Manitoba Systems.

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1. INTRODUCTION

In the planning of power system generation facilities, reliability considerations are as important as those of economics. The system must have sufficient reserve capacity to permit it to operate at an adequate level of reliability. The evaluation of generating capacity reliability can be considered to take two basic forms which can be designated as static and spinning reserve requirements.

Static Reserve[(1)] - The installed capacity which must be planned and constructed in advance of the system requirements. This reserve must be sufficient to provide for the maintenance of generating equipment, outages of generating units which are not planned and load growth requirements in excess of the estimates.

Spinning Reserve[(1)] - The capacity which must be available at all times to meet load changes on the system without impairing the system frequency and also capable of satisfying the loss of some portion of the system generating capacity.

At the planning level both of these areas must be investigated but once a decision has been reached, the spinning reserve requirement becomes an operating problem. The analysis of these two areas can be considered in two basic parts each of which requires a different approach and a different set of system statistics. The theoretical techniques presented in this thesis and the illustrated applications are limited to the static capacity problem.

The static reserve requirement in the past has normally been evaluated using rule of thumb methods. One popular technique is the percentage reserve approach. There are, however, some important objections to the use of this method.

It does not give adequate representation to the generating system composition and the load model configuration. According to this method, two systems with the same peak load over the same period will have the same reserve irrespective of the fact that the two systems may have units of different capabilities, different mean failure and repair rates and that the load characteristics of the two systems may be entirely different.

This method regards the generating capacity reserve as being independent of the system composition and is therefore not adequate for comparing alternate generation planning proposals.

It is not possible to assess the effect of interconnections with other systems when using the percentage reserve approach.

Generation system planners have long been concerned with the need for analytical techniques to solve these problems.

The interest in the application of probability techniques to generating capacity reliability evaluation became evident as early as 1933. Since then a number of papers have been published on this subject. The methods proposed in these papers fall into the following four broad classifications.

The Loss of Load Probability Method[2]

The adequacy of the proposed and available generating capacity is measured in terms of the expected time that the load will exceed the available capacity.

The Frequency and Duration Method

In this approach the frequency, the mean duration and the probability of encountering the various levels of generating capacity inadequacy are determined as the indices of reliability.

The Loss of Energy Probability Method[3]

This technique is similar to the Loss of Load Probability Approach and utilizes the ratio of the expected load energy curtailed due to generating unit forced outages to the total energy required by the system.

The Simulation Method[4]

This approach develops a capacity model from the historical data of generating unit up time and down time durations and a load model from the historical load data.

Using Monte Carlo techniques these two models are combined to give a probability distribution of the available reserve margins.

"A survey of the published literature indicates that the loss of load probability method using a daily peak load variation curve appears to be the most widely accepted technique and used more often than any other technique"[3]. Though relatively simple to appreciate and to apply, this approach has the following drawbacks.

It is difficult to relate the loss of load probability index LOLP with the actual physical performance of the system.

The LOLP index is not sufficiently sensitive to the mean failure and the mean repair rates of the generation units and the available interconnections. Two identical units with the same forced outage rate, according to this method, will have the same effect on the system reliability despite the fact that this forced outage rate can be obtained by an infinite number of combinations of the mean failure and the mean repair rates.

This approach is not consistent with the technique most frequently employed for transmission system reliability evaluation, i.e., the frequency and duration approach[5].

The frequency and duration method though a little more complicated, at least in appearance, overcomes these drawbacks. Equations for computing the frequency and the duration of various capacity outages were first given by Halperin and Adler[6]. The application was extended by the publication of four recent papers[7,8,9,10] which presented for the first time, a set of recursive relations for obtaining the frequency and the duration of various capacity outages. These papers also proposed suitable load models, gave techniques for combining the capacity outage data with the load model and discussed the inclusion of partial capacity states. The techniques given in these four papers are potentially very powerful and can be extended to cover many areas of power system reliability evaluation[11,12,13].

The frequency and duration method has been examined in detail in this thesis and particularly in regard to its application to a practical system such as that of the Saskatchewan Power Corporation. Steady state availability and unavailability have been defined as the relative durations in the up and the down states. The frequency of encountering these states can be obtained by multiplying these relative durations by the expected number of departures per unit time. Starting with the simple application of the principle of expectation, the various relationships have been developed using a slightly different approach to that previously presented[7,8]. The main emphasis in this approach is on the computation of the availability and the frequency of cumulative rather than individual states. In order to apply these to practical system configurations, fast computing techniques have been developed to evaluate the reliability measures of the cumulative states.

Relationships for evaluating the reliability indices of a generation system connected to one or more systems have been developed. These equations have been incorporated in a general computer programme and the effect of variations in the tie line parameters on the reliability measures has been examined.

Data from the Saskatchewan Power Corporation System for the period June 1968 to July 1969 has been analyzed using the digital computer. The exposure factor for various load levels has been determined and the peak load levels converted into a load model. The effects of variations in the exposure factor on the reliability measures have been examined in detail.

The effect of load forecast uncertainty on the reliability indices has also been examined.

The basic considerations are illustrated in this thesis by the development and application to a hypothetical system. The effects in a practical configuration of variations in the statistical parameters are illustrated by application to the Saskatchewan and Manitoba Systems.

2. THE FREQUENCY AND DURATION METHOD

Generating capacity reliability evaluation normally utilizes only two system parameters. The assessment is basically concerned with the relative behaviour of the system load and the system generation facilities. The application of the frequency and duration approach to this problem involves three basic steps.

1. A model describing the probabilistic behaviour of capacity outages is developed first. This is referred to as the "Generation System Model".
2. The probabilistic nature of the occurrence and the duration of selected peak loads is incorporated into a "Demand Model" or "Load Model".
3. The generation system model and the load model are then merged to give a "Generation Reserve Model" which depicts the expected occurrence of surplus capacity and capacity deficiencies.

2.1 The Generation System Model

2.1.1 Basic Principles

The Generation System Model (GSM) is similar to the capacity outage probability table developed in the loss of load probability approach. It offers, however, some additional information which is not provided by the capacity outage probability table. In addition to the steady state probabilities of exact or cumulative capacity outage levels as provided in the capacity outage probability table, the GSM includes the frequency of encountering these conditions.

The data required to construct the GSM are the mean durations at the different levels of generating capability for all the units in the system. A generating unit, especially a large thermal one, may have many different capability levels. The consideration of partial capacity states is not a major problem[10], however, in order to illustrate the basic approach each unit has been assumed to exist either in an up (full capability) or in a down (zero capability) state. The average behaviour of the binary model can be represented as in Figure 2.1 where

m = The mean up time of the unit

r = The mean down time of the unit

It may be noted that both "m" and "r" are mean durations and are assumed to be constant. The reciprocal of the mean up time is the average failure rate λ. This is the rate with which on the average a unit would transit to the down state given that it is in the up state. Similarly, the reciprocal of the mean down time is the average repair rate. This is the rate with which on the average a unit would transit to the up state given that it is in the down state. Expressed mathematically

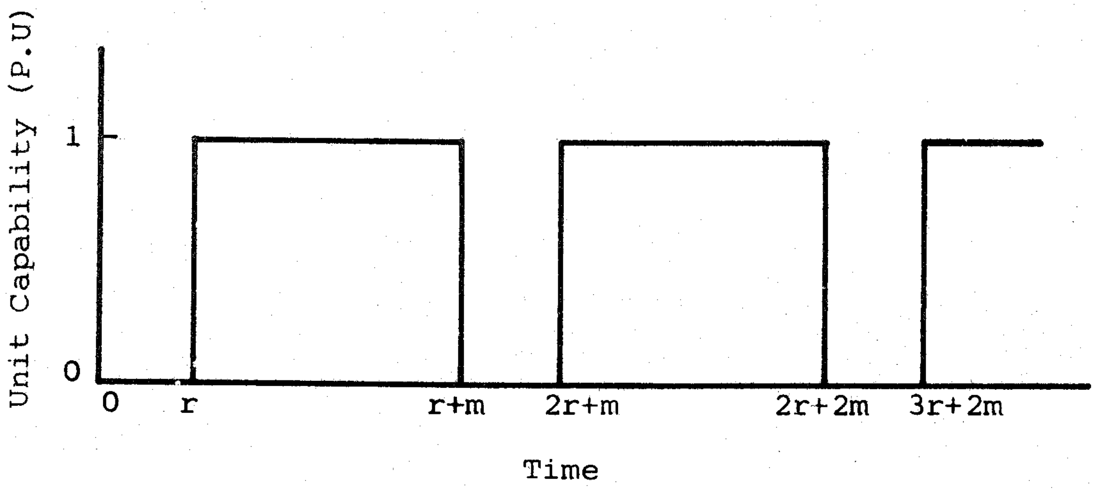


Figure 2.1: Average History of Unit Capability

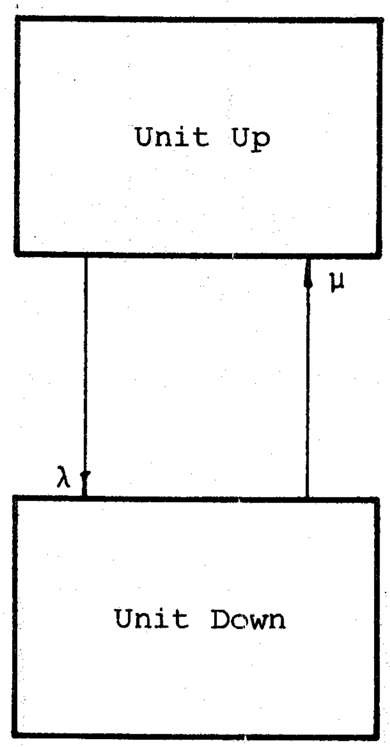


Figure 2.2: State Transition Diagram of the Binary Model of a Unit

$$\lambda = \text{Average failure rate}$$
$$= \frac{1}{m}$$

$$\mu = \text{Average repair rate}$$
$$= \frac{1}{r}$$

The state transition diagram of a binary model is shown in Figure 2.2. Two relations can be deduced directly from Figure 2.1

A = The availability of a unit, i.e., the steady state probability of finding the unit in the up state.

$$= \frac{m}{m+r}$$

$$= \frac{\mu}{\mu+\lambda} \qquad (2.1)$$

and $\bar{A}$ = The unavailability of a unit

$$= \frac{r}{m+r}$$

$$= \frac{\lambda}{\mu+\lambda} \qquad (2.2)$$

Two simple relations which form the basis of the frequency and duration approach can be stated as

f = The frequency of encountering a state
$= A.\lambda$
= (Steady state probability of being in the state) (Rate of departure from that state) (2.3)

Alternatively $f = \bar{A}.\mu$

= (Steady state probability of not being in the state) (Rate of entry into that state) (2.4)

It will be noted that these relations are a direct application of the principle of expectation in probability theory. Availability of a unit is simply the relative duration of the up state and when multiplied by the rate of transition from this state, gives the frequency with which the state is likely to be encountered. The same results can be obtained by assuming the up and down times to be exponentially distributed with mean values of m and r respectively and applying Markov chain theory.

For a system composed of a number of units, it is helpful to view these relations in the following way.

f = Expected transitions per unit time from state "i" to state "j" (E_{ij})
= Expected transitions per unit time from state "j" to state "i" (E_{ji}) (2.5)

The availabilities and frequencies of the exact states can be obtained using equations 2.1 - 2.4.

2.1.2 Grouping Identical Capacity Outage States

The generation system model may contain a number of identical capacity outage states. The only way the system can transit from one state to another within a group of identical capacity outage states is by the simultaneous failure and repair of two units with the same capability. The probability of such an occurrence is negligible compared with the probability of occurrence of a single event and therefore the identical capacity states can be regarded as separated in time. The equations for merging "N" identical capacity outage states can be stated as[7]

Capacity

$$c_k = c_1 = c_2 = \dots = c_i = \dots = c_N$$

Availability

$$A_k = \sum_{i=1}^{N} A_i \tag{2.6}$$

and frequency

$$f_k = \sum_{i=1}^{N} f_i \tag{2.7}$$

The subscript "i" refers to those states having the same capacity on outage and "k" denotes the merged state. The total rates of departure to greater and lesser capacity states are given by:

$$\lambda_{up,k} = \sum_{i=1}^{N} A_i \cdot \lambda_{+i} / A_k \tag{2.8}$$

and

$$\lambda_{dn,k} = \sum_{i=1}^{N} A_i \cdot \lambda_{-i} / A_k \tag{2.9}$$

where

λ_{+i} = Transition rate from state i to states with more capacity available

λ_{-i} = Transition rate from state i to states with less capacity available

$\lambda_{up,k}$ = Transition rate from state k to states with higher capacity

$\lambda_{dn,k}$ = Transition rate from state k to states with lower capacity

2.1.3 Cumulative Capacity Outage States

As shown above, the frequency of encountering exact capacity states can be calculated without too much difficulty. It is often of more interest, however, to determine the availability and frequency of the cumulative capacity outage states, i.e., those states with capacity on outage equal to or greater than a given value. The availability and frequency can be obtained by repeated application of the following relations[7] to the capacity outage states arranged in the ascending order of outage (see Figure 2.3)

Availability

$$A_{n-1} = A_n + A_k \tag{2.10}$$

and frequency

$$f_{n-1} = f_n - A_k \cdot \lambda_{-k} + A_k \cdot \lambda_{+k} \tag{2.11}$$

where

A_{n-1}, f_{n-1} = Availability and frequency of the (n-1)th cumulative capacity outage state

A_n, f_n = Availability and frequency of the nth cumulative capacity outage state

Less Capacity in	Cumulative Capacity Outage		Exact Capacity Outage	More Capacity Outage
	$\bar{C}_1$	$\geq$	$\bar{c}_1$	
	$\bar{C}_2$	$\geq$	$\bar{c}_2$	
	$\bar{C}_3$	$\geq$	$\bar{c}_3$	
	.		.	
	.		.	
	.		.	
	$\bar{C}_{n-1}$	$\geq$	$\bar{c}_k$	$(k = n-1)$
	$\bar{C}_n$	$\geq$	$\bar{c}_n$	
	.		.	
	.		.	
	.		.	
	$\bar{C}_N$	$\geq$	$\bar{c}_N$	

$\bar{c}_n = IC - c_n$

where IC = Installed Capacity

and c_n = Capacity in

Therefore $\bar{C}_n \geq IC - c_n$

or $C_n \leq c_n$

Figure 2.3 Relationship of Cumulative Capacity Outages To Exact Capacity Outages Arranged in Ascending Order of Magnitude

A_k = Availability of the state being added

λ_{+k} = Transition rate from state k to states in which more capacity is available

λ_{-k} = Transition rate from state k to states in which less capacity is available

Equation 2.11 is in fact an application of equation 2.5.

$$A_k \cdot \lambda_{-k} = \sum_{j=n}^{N} E_{kj}$$

$$= \sum_{j=n}^{N} E_{jk}$$

f_n = Expected transitions per unit time from the cumulative capacity outage state n to states with lesser capacity on outage

$$= \sum_{j=n}^{N} E_{jk} + \sum_{j=n}^{N} E_{j(<k)}$$

Also

$$A_k \cdot \lambda_{+k} = \sum_{j<k} E_{kj}$$

Therefore

$$f_n - A_k \cdot \lambda_{-k} + A_k \cdot \lambda_{+k} = \sum_{j=n}^{N} E_{jk} + \sum_{j=n}^{N} E_{j(<k)} - \sum_{j=n}^{N} E_{jk} + \sum_{j<k} E_{kj}$$

$$= \sum_{j=n}^{N} E_{j(<k)} + \sum_{j<k} E_{kj}$$

$$= f_{n-1}$$

2.1.4 Quick Computing Techniques

The generation system model can be constructed using the equations 2.1 - 2.11. As noted previously, the availability and frequency of the cumulative states are of prime interest and therefore relationships for the direct generation of these values from the unit data were developed. These equations have been implemented in the segment of computer programme which evolves the GSM by successive unit additions. The basis for these equations is explained below.

Consider that a GSM already exists and it is desired to add a new unit to it. Let

$\bar{C}_i$, A_i, f_i = Capacity outage, availability and frequency of the cumulative states, $i = 1,2,3 \ldots\ldots N$

and

C_k, λ_k, μ_k = Capacity, average failure rate and average repair rate of the unit being added.

Since the unit is assumed to exist either in the up state (capacity out = 0) or in the down state (Capacity out = C_k), the cumulative outage states obtained after the unit addition are

Unit k in			Unit k out
$\bar{C}_1 + 0$	$\xleftarrow{\mu_k}$	$\xrightarrow{\lambda_k}$	$\bar{C}_1 + C_k$
$\bar{C}_2 + 0$	$\xleftarrow{\mu_k}$	$\xrightarrow{\lambda_k}$	$\bar{C}_2 + C_k$
$\bar{C}_3 + 0$	$\xleftarrow{\mu_k}$	$\xrightarrow{\lambda_k}$	$\bar{C}_3 + C_k$
$\bar{C}_4 + 0$	$\xleftarrow{\mu_k}$	$\xrightarrow{\lambda_k}$	$\bar{C}_4 + C_k$
$\bar{C}_5 + 0$	$\xleftarrow{\mu_k}$	$\xrightarrow{\lambda_k}$	$\bar{C}_5 + C_k$
$\bar{C}_6 + 0$	$\xleftarrow{\mu_k}$	$\xrightarrow{\lambda_k}$	$\bar{C}_6 + C_k$
$\vdots$			$\vdots$
$\bar{C}_n + 0$	$\xleftarrow{\mu_k}$	$\xrightarrow{\lambda_k}$	$\bar{C}_n + C_k$
$\vdots$			$\vdots$
$\bar{C}_N + 0$	$\xleftarrow{\mu_k}$	$\xrightarrow{\lambda_k}$	$\bar{C}_N + C_k$

Availability and frequency of existing cumulative outage states:

Assume that it is desired to determine the frequency of encountering the cumulative capacity outage $\bar{C}_5$, given that $(\bar{c}_3 + C_k) \geq \bar{c}_5$ and $(\bar{c}_2 + C_k) < \bar{c}_5$. The boundary for such a cumulative state is shown by the dotted line in the above table. The frequency of this state is

$$= f_5 . A_k + f_3 . \bar{A}_k + (A_3 - A_5)\, A_k . \lambda_k$$

where

$$A_k = \text{The availability of the unit being added}$$

$$= \frac{\mu_k}{\mu_k+\lambda_k}$$

and

$$\bar{A}_k = \text{The unavailability of the unit being added}$$

$$= 1 - A_k$$

In general, the modified availability and frequency of a cumulative capacity outage state "i" in the existing GSM will be given by

$$\overset{\text{new}}{A_i} = \overset{\text{old}}{A_1} \cdot A_k + \overset{\text{old}}{A_j} \cdot \bar{A}_k \tag{2.12}$$

and

$$\overset{\text{new}}{f_i} = \overset{\text{old}}{f_i} \cdot A_k + \overset{\text{old}}{f_j} \cdot \bar{A}_k + (\overset{\text{old}}{A_j} - \overset{\text{old}}{A_i}) \cdot A_k \cdot \lambda_k \tag{2.13}$$

such that

$$\bar{c}_j \geq (\bar{c}_i - C_k)$$

Availability and frequency of generated capacity outage states:

$$A_{(i+k)} = \overset{\text{old}}{A_i} \cdot \bar{A}_k + \overset{\text{old}}{A_j} \cdot A_k \tag{2.14}$$

and

$$f_{(i+k)} = \overset{\text{old}}{f_i} \cdot \bar{A}_k + \overset{\text{old}}{f_j} \cdot A_k + (\overset{\text{old}}{A_i} - \overset{\text{old}}{A_j})\, A_k \cdot \lambda_k \tag{2.15}$$

such that

$$\bar{c}_j \geq (\bar{c}_i + C_k)$$

$A_{(i+k)}$, $f_{(i+k)}$ = The availability and frequency of capacity outage $\geq$ $(\bar{c}_i + C_k)$

The process is started by constructing the GSM for a single unit and each unit is then added in turn.

2.1.5 Hypothetical System Example

The system of 22 units suggested by Arnoff and Chambers(14) has been selected for illustration. The relevant data for the units in this system are listed below.

No. of Identical Units	Unit Size M.W	Mean Down Time (r) Years	Mean Up Time (m) Years
1	250	0.06	2.94
3	150	↓	↓
2	100		
4	75		
9	50		
3	25		

Total number of units = 22
Total installed capacity = 1725 MW

The GSM obtained from the developed computer programme is given in Table 2.1. Capacity outage states

TABLE 2.1

Generation System Model of the 22 Unit System

Capacity Outage Equal to or Greater than MW	Availability (A)	Frequency (f) Per Day	Cycle Time (1/f) Days	Mean Duration (A/f) Days
0.0	0.100000 x 10			
25.0	0.358329×10^{0}	0.131448×10^{-1}	0.760757×10^{2}	0.272981×10^{2}
50.0	0.319573×10^{0}	0.121205×10^{-1}	0.825046×10^{2}	0.263663×10^{2}
75.0	0.201006×10^{0}	0.898946×10^{-2}	0.111241×10^{3}	0.223602×10^{2}
100.0	0.141450×10^{0}	0.709902×10^{-2}	0.140864×10^{3}	0.199252×10^{2}
125.0	0.102314×10^{0}	0.546692×10^{-2}	0.182918×10^{3}	0.187151×10^{2}
150.0	0.904434×10^{-1}	0.457351×10^{-2}	0.218650×10^{3}	0.197755×10^{2}
175.0	0.436883×10^{-1}	0.295266×10^{-2}	0.338678×10^{3}	0.147963×10^{2}
200.0	0.379311×10^{-1}	0.247558×10^{-2}	0.403946×10^{3}	0.153221×10^{2}
225.0	0.295167×10^{-1}	0.181697×10^{-2}	0.550368×10^{3}	0.162450×10^{2}
250.0	0.253559×10^{-1}	0.146596×10^{-2}	0.682147×10^{3}	0.172965×10^{2}
275.0	0.968847×10^{-2}	0.887232×10^{-3}	0.112710×10^{4}	0.109199×10^{2}
300.0	0.809136×10^{-2}	0.729771×10^{-3}	0.137029×10^{4}	0.110875×10^{2}
325.0	0.438965×10^{-2}	0.433177×10^{-3}	0.230853×10^{4}	0.101336×10^{2}
350.0	0.291192×10^{-2}	0.302569×10^{-3}	0.330503×10^{4}	0.962397×10
375.0	0.189202×10^{-2}	0.202682×10^{-3}	0.493384×10^{4}	0.933492×10
400.0	0.154317×10^{-2}	0.158243×10^{-3}	0.631938×10^{4}	0.975185×10
425.0	0.528440×10^{-3}	0.719494×10^{-4}	0.138986×10^{5}	0.734460×10
450.0	0.391825×10^{-3}	0.533764×10^{-4}	0.187349×10^{5}	0.734079×10
475.0	0.203941×10^{-3}	0.293670×10^{-4}	0.340519×10^{5}	0.694457×10
500.0	0.114013×10^{-3}	0.173100×10^{-4}	0.577700×10^{5}	0.658653×10
525.0	0.587161×10^{-4}	0.951282×10^{-5}	0.105121×10^{6}	0.617232×10
550.0	0.412809×10^{-4}	0.649118×10^{-5}	0.154055×10^{6}	0.635955×10
575.0	0.145389×10^{-4}	0.270617×10^{-5}	0.369526×10^{6}	0.537251×10
600.0	0.899488×10^{-5}	0.169028×10^{-5}	0.591619×10^{6}	0.532154×10
625.0	0.438848×10^{-5}	0.859062×10^{-6}	0.116406×10^{7}	0.510845×10
650.0	0.217446×10^{-5}	0.443870×10^{-6}	0.225291×10^{7}	0.489887×10
675.0	0.926480×10^{-6}	0.204309×10^{-6}	0.489455×10^{7}	0.453470×10
700.0	0.531049×10^{-6}	0.116074×10^{-6}	0.861516×10^{7}	0.457507×10
725.0	0.199338×10^{-6}	0.477871×10^{-7}	0.209261×10^{8}	0.417138×10
750.0	0.963639×10^{-7}	0.236983×10^{-7}	0.421971×10^{8}	0.406627×10
775.0	0.431916×10^{-7}	0.109852×10^{-7}	0.910313×10^{8}	0.393179×10
800.0	0.188172×10^{-7}	0.493791×10^{-8}	0.202515×10^{9}	0.381076×10

with cumulative availability less than 0.1×10^{-8} are truncated after each unit addition.

It should be recognized that the principal addition to the LOLP capacity outage probability table is the frequency of encountering the cumulative outage states. The cycle time and the mean duration of a cumulative outage state can be simply calculated from the availability and frequency by the following equations.

$$\text{Cycle Time} = 1 / \text{frequency}$$

$$\text{Mean Duration} = \text{Availability} / \text{frequency}$$

2.2 Load Model

The basic load model suggested in reference[8] is shown in Figure 2.4. It consists of a random sequence of N load states, each of which is followed by a low load state. Each day contains a high load state and the low load level. All the load levels are assumed to have the same constant mean duration. The level of low load is assumed fixed and of constant average duration.

The description of the load model is given below.

Number of load levels	N
Description of load levels, MW	L_i, $i = 1,2,3....,N$ and $L_1 > L_2 > ...... > L_N$
Number of occurrences of L_i	n_i, $i = 1,2,3....,N$
Interval length, days	D

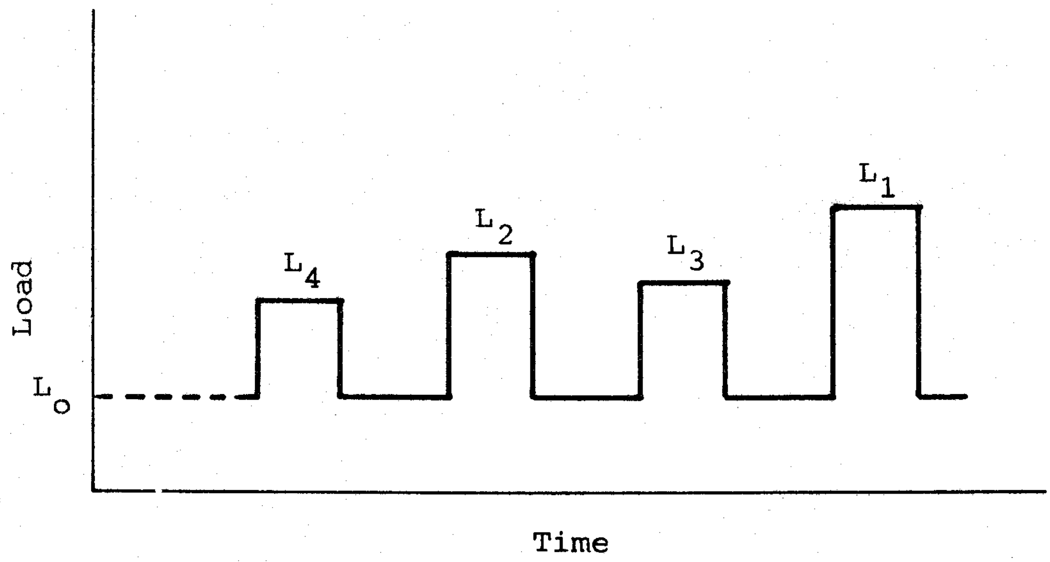


Figure 2.4: Basic Load Model

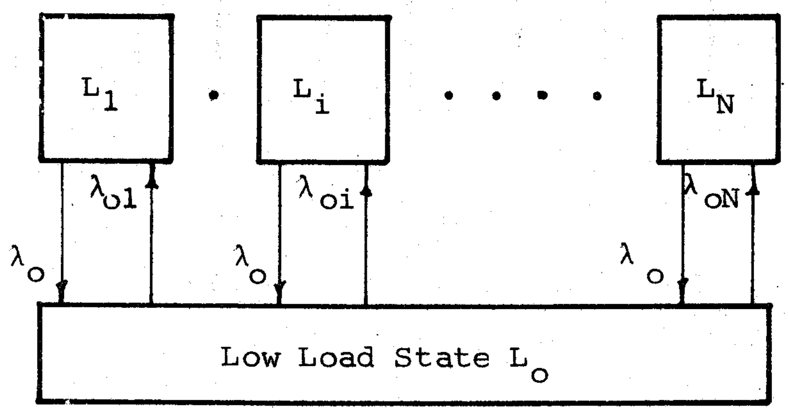


Figure 2.5: State Transition Diagram of the Basic Load Model

Mean duration of peak, days	$e < 1$
Mean duration of lowload, days	$1 - e$
Availability of L_i	A_i
Transition rate from L_i to a higher load level	λ_{+i}
Transition rate from L_i to a lower load level	λ_{-i}
Transition rate from lowload L_o to greater load	λ_{+o}
Transition rate from lowload to lesser load	λ_{-o}
Frequency of occurrence of L_i	f_i
Frequency of occurrence of L_o	f_o

The state transition diagram of this load model is shown in Figure 2.5. From the basic load model and its state transition diagram, the following relationships can be obtained.

1. The interval length

$$D = \sum_{i=1}^{N} n_i$$

2. A load level L_i of mean duration e is expected to occur n_i times during an interval of D days. The availability or relative duration is given by

$$A_i = \frac{n_i \cdot e}{D}$$

3. The system can transit from one peak load level to another only through an intervening low load state,

Therefore:

$$\lambda_{+i} = 0$$

4. The mean duration of a load level is e and therefore the transition rate to the low load state is

$$\lambda_{-i} = \lambda_o$$
$$= \frac{1}{e}$$

5. An expression for frequency of load level L_i can be written from

f_i = (Steady state probability of being in state L_i) x (Rate of departure from L_i)

$$= A_i\ (\lambda_{+i} + \lambda_{-i})$$
$$= A_i\ (0 + \lambda_o)$$
$$= \frac{n_i}{D}$$

6. The availability of low load state

$$A_o = 1 - e$$

7. A system can transit from the low load state only to one of the peak load levels

Therefore:

$$\lambda_{-o} = 0$$

8. The mean duration of the low load state is (1-e) and the transition rate to the complex of peak load levels is given by

$$\lambda_{+o} = \frac{1}{1-e}$$

9. The expected time spent in the low load state during a period of D days is

$$= (1-e)\ D$$

Considering only the state L_i, the mean duration of low load

$$= \frac{(1-e)\ D}{n_i}$$

and therefore

λ_{oi} = The transition rate from the low load state to the peak load state i

$$= \frac{n_i}{D}(1-e)$$

It should be pointed out that the same relationships are obtained if the load state durations are assumed to be exponentially distributed with a mean value e and Markov chain theory applied.

2.3 Capacity Reserve Model

2.3.1 Definition

Assuming stochastic independence of the generation system model and the load model, they may be combined to form a capacity reserve model. Capacity reserve or margin is an excess of capacity over the demand. Denoting capacity, demand and margin by c,L and m respectively,

$$m_k = c_i - L_j$$

Where "i" and "j" refer to the capacity and load states in the margin state matrix shown in Figure 2.6.

2.3.2 Exact Margin States

Availabilities and frequencies of exact margin states can be found from the following relationships.

Availability

Availability of the margin state $m_k = c_i - L_j$ is given by

$$A_k = A_i . A_j$$

Frequency

Denoting the up and down transitions by "+" and "-" respectively, the transition rates from margin state m_k are given by

$$\lambda_{+k} = \lambda_{+i} + \lambda_{-j}$$

and

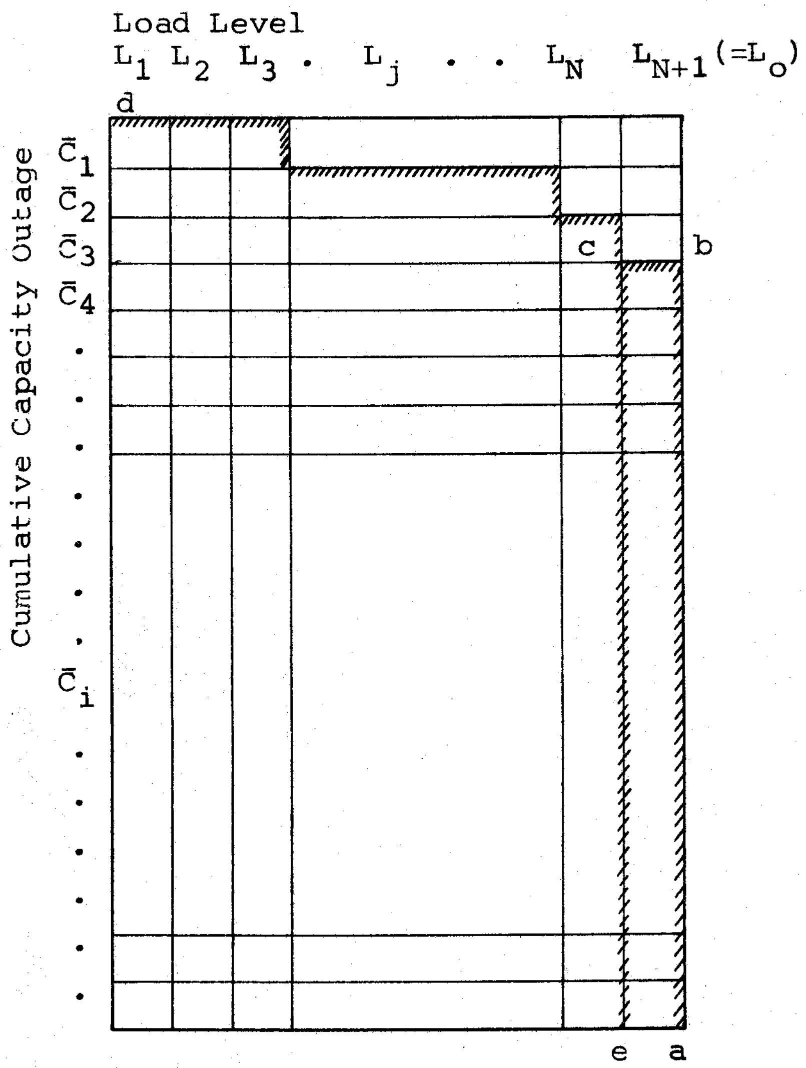
$$\lambda_{-k} = \lambda_{-i} + \lambda_{+j}$$

where

λ_{+i}, λ_{-i} = Transition rates from capacity state "i" to states with greater or lesser capacities

and

λ_{+j}, λ_{-j} = Transition rates from load state "j" to higher or lower load levels



$\bar{C}_i \geq \bar{c}_i$

$\bar{c}_i + c_i$ = Installed Capacity

Figure 2.6: Margin State Matrix

The frequency of the exact margin state m_k is

$$f_k = A_k \, (\lambda_{+k} + \lambda_{-k})$$

2.3.3 Identical Margin States

Assuming the simultaneous occurrence of two events to be stochastically impossible, there can be no direct transitions within identical margin states. Substituting capacity reserve for capacity, the identical margin states can be merged using equations 2.6 - 2.9.

2.3.4 Cumulative Margin States

The cumulative margin states, i.e., states with margin equal to or less than a specified value are of primary interest and their availability and frequency can be determined by the application of equations 2.10 and 2.11.

2.3.5 Computational Techniques

Availabilities and frequencies of cumulative margin states can be computed much more efficiently by combining cumulative capacity states with exact load states. This can be achieved using the following equations[8].

Availability of margin M,MW or less:

$$A_M = \sum_j A_j . A_G \qquad (2.16)$$

and frequency:

$$f_M = \sum_j A_j \left[f_G + A_G (\lambda_{-j} - \lambda_{+j})\right] \qquad (2.17)$$

where

$$\{G\} = \{C_i \leq L_j + M\}$$

These equations can be understood by reference to the margin state matrix of Figure 2.6. At each load level the boundary of margin equal to or less than M can be defined by a corresponding capacity level G (or capacity outage $\bar{G}$) such that

$$(G - L_j) \leq M$$

or

$$G \leq (L_j + M)$$

The frequency can be determined by computing the expected transitions per unit time out of this boundary. The system can transit out either vertically due to changes in generation system or horizontally due to load variation.

Contribution due to vertical transitions $= \sum_{j=1}^{N+1} A_j . f_G$

Contribution due to horizontal transitions =

$$\sum_{j=1}^{N+1} A_j . A_G \ (\lambda_{-j} - \lambda_{+j})$$

The contribution to the horizontal transitions of the portion $\sum_{j=1}^{N+1} A_j.A_G.\lambda_{-j}$ represent the transitions out of the boundary ecd in Figure 2.6. There are no down transitions from L_0. The remainder, i.e., $-\sum_{j=1}^{N+1} A_j.A_G.\lambda_{+j}$ cancels the transitions out of ec.

Adding up the contributions

$$f_M = \sum_{j=1}^{N+1} A_j \left[f_G + A_G\ (\lambda_{-j}-\lambda_{+j})\right]$$

The expressions 2.16 and 2.17 have been implemented in a subroutine of the computer programme to evaluate the availabilities and frequencies of cumulative margin states.

2.4 Load Forecast Uncertainty

The effect of uncertainty in load forecasting can be included using a conditional probability approach. The probability distribution for load forecast uncertainty can be divided into class intervals, the area of each class representing the probability of the load being the class interval mid value. Assuming "m" such class intervals with areas $A_p, p = 1,2,3\ldots\ldots m$, the load model can exist in m possible ways

$$(L_{ip}, i = 1,2,3\ldots\ldots N), p = 1,2,3\ldots\ldots m$$

where

L_{1p} = The forecast peak having a probability of A_p

The basic structure of the load model is assumed to be independent of the variation in the load forecast. Therefore, regardless of the forecast load, the number of load levels "N", the number of occurrences of state i, "n_i", the exposure factor "e" and the ratios $\frac{L_i}{L_1}$, i = 1,2,3....N stay the same. Given the load model (L_{ip}, i = 1,2,3....N), the availability of load state L_{ip} is

$$= \frac{n_i \cdot e}{D}$$

Therefore the availability of L_{ip} is

$$= \frac{n_i \cdot e}{D} \cdot A_p$$

$$A_{(M \wedge p)} = A_{(M|p)} \cdot A_p$$

and

$$f_{(M \wedge p)} = f_{(M|p)} \cdot A_p$$

The "m" class intervals being mutally exclusive

$$A_M = \sum_{p=1}^{m} A_{(M \wedge p)}$$

and

$$f_M = \sum_{p=1}^{m} f_{(M \wedge p)}$$

where

$A_{(M|p)}$, $f_{(M|p)}$ = The availability and frequency of margin M,MW or less given the forecast peak L_{1p}

$A_{(M \wedge p)}$, $f_{(M \wedge p)}$ = The availability and frequency of margin M,MW or less and the forecast peak L_{1p}

and

A_M, f_M = The availability and frequency of margin M,MW or less

A subroutine incorporating the effect of load forecast uncertainty on the availability and the frequency of the cumulative margin states has been added in the computer programme.

2.5 Effect of Maintenance on Generation System Model

During the period of a year, different units are on maintenance and therefore one generation system model cannot be used for the entire period. The year can be divided into a number of intervals during which the units on maintenance stay the same. For each interval a new generation system model can be developed either ab-initio or by removing the units on maintenance from the existing generation system model. Unit removal is the reverse of the process of unit addition described by equations 2.12 - 2.15. In order to reconstruct the old GSM, i.e., the one prior to the addition of C_k, equations 2.14 and 2.15 can be modified. Since $\bar{C}_j$ is just greater than or equal to $\bar{c}_i + C_k$,

$$\overset{old}{A}_{(i+k)} = \overset{old}{A}_j$$

and

$$\overset{old}{f}_{(i+k)} = \overset{old}{f}_j$$

Substituting these in equations 2.14 and 2.15, and replacing the indices (i+k), i by i and j respectively,

$$\overset{new}{f_i} = \overset{old}{f_i}.A_k + \overset{old}{f_j}.\bar{A}_k + (\overset{old}{A_j} - \overset{old}{A_i})\, A_k.\lambda_k$$

and

$$\overset{new}{A_i} = \overset{old}{A_i}.A_k + \overset{old}{A_j}.\bar{A}_k$$

such that

$$\bar{c}_i - \bar{c}_j = C_k$$

or

$$\bar{c}_j = \bar{c}_i - C_k$$

i.e.

$$\bar{c}_j \geq \bar{c}_i - C_k$$

These equations are the same as those of 2.12 and 2.13 with the same qualifying statement. Availabilities and frequencies of the old system are

$$\overset{old}{A_i} = (\overset{new}{A_i} - \overset{old}{A_j}.\bar{A}_k)/A_k \tag{2.18}$$

and

$$\overset{old}{f_i} = (\overset{new}{f_i} - \overset{old}{f_j}.\bar{A}_k + (\overset{old}{A_i} - \overset{old}{A_j})\, A_k.\lambda_k)/A_k \tag{2.19}$$

such that

$$\bar{C}_j \geq \bar{C}_i - C_k$$

After each unit removal, the generation system model may contain sets of states with different capacity outages but the same availability and frequency. In each such set all the states except the last one are redundant and may, therefore, be deleted to get the exact old generation system model. A subroutine has been added to the computer programme to accomplish unit removal.

2.6 Non-stationary Effect in Load Model

There is a considerable variation in daily peaks over the period of a year. Thus one load model cannot be employed over all maintenance intervals. A separate load model is constructed for each interval and combined with the corresponding generation system model to obtain the availability and frequency of cumulative margin states on annual basis.

The availability of load state L_i on an annual basis is given by $A_i \frac{D}{365}$ with its mean duration remaining unchanged. The intervals are separated in time and the availabilities and frequencies add to give the annual quantities. Assuming "I" intervals

$$A_M = \sum_{q=1}^{I} A_q$$

and

$$f_M = \sum_{q=1}^{I} f_q$$

2.7 Availability, Frequency and LOLP

It is interesting to note that these three measures of reliability are related by the following equation[9]

$$\begin{pmatrix}\text{Annual LOLP}\\ \text{days/year}\\ \end{pmatrix} \begin{pmatrix}\text{Average Peak}\\ \text{Load Duration}\\ e\end{pmatrix} = \frac{\text{Average duration of a capacity shortage}}{\text{Average Cycle Time in years}}$$

$$= 365 \text{ (Availability of capacity deficiency)}$$

or

$$\text{LOLP in days/year} = \frac{365}{e} . A_{M-} \qquad (2.20)$$

where

A_{M-} = Cumulative availability of the first negative margin.

This relationship implies that the same load model has been employed for both the frequency and duration and the loss of load probability studies.

2.8 Hypothetical System Studies

To study the characteristics of the capacity reserve model, studies were conducted on a hypothetical system designated SYS.RS, which was assumed to be composed of the 22 unit generation system outlined in the previous example and the load model shown below.

Exposure factor = 0.5 day
Period = 20 days

Load Level (L_i) MW	No. of Occurrences (n_i)
1450	8
1255	4
1155	4
1080	4

The low load level was assumed to be at zero MW.

2.8.1 LOLP and Availability

Availability of the failure state in SYS.RS, on an annual basis, for a twenty day period was computed to be

$$A = 0.8988137 \times 10^{-4}$$

Therefore

$$\text{LOLP} = \frac{365}{e} . A$$

$$= 0.0656134 \text{ days/year}$$

The load model of SYS.RS was then rearranged as a load duration curve as shown in Figure 2.7 and the LOLP calculated using the equation

$$LOLP = \sum p_k . t_k$$

where

p_k = Probability of capacity outage A_k

and

t_k = No. of time units for which such a capacity outage results in a loss of load.

LOLP obtained in this manner

= 0.0656134 days/year

As previously noted, the LOLP obtained by both the equations is the same. Availability in the frequency and duration approach is therefore an index of load loss in the LOLP approach provided that an equivalent load model is used.

2.8.2 Load Forecast Uncertainty

The computer programme can accept any distribution for uncertainty using a discrete step representation. The normal distribution shown in Figure 2.8[2] was selected for the examples considered in this thesis.

The variations in the availability of and the cycle time to the failure state in SYS.RS are shown in Figures 2.9 and 2.10 respectively. These values are for a twenty day

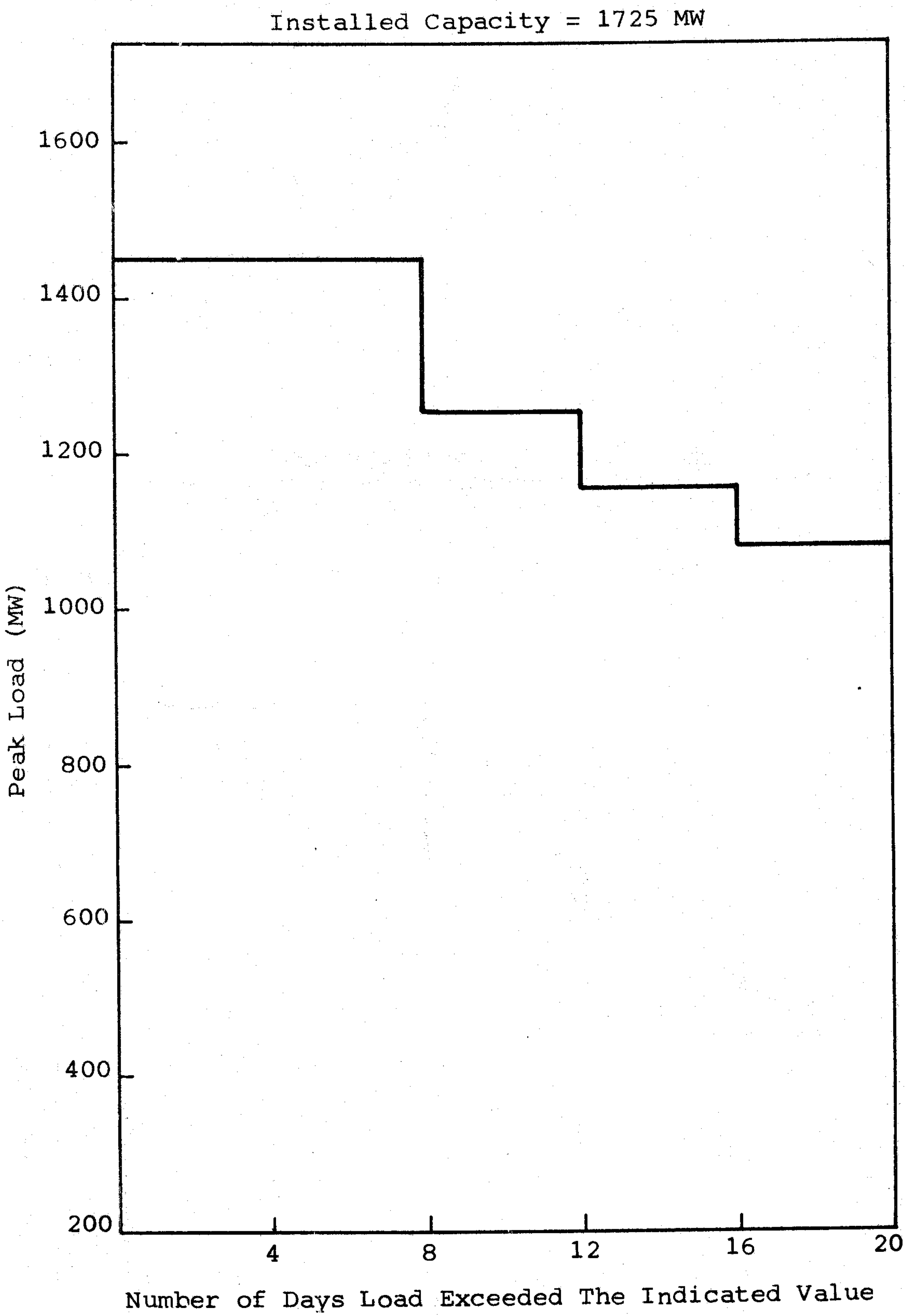


Figure 2.7: The Load Duration Curve

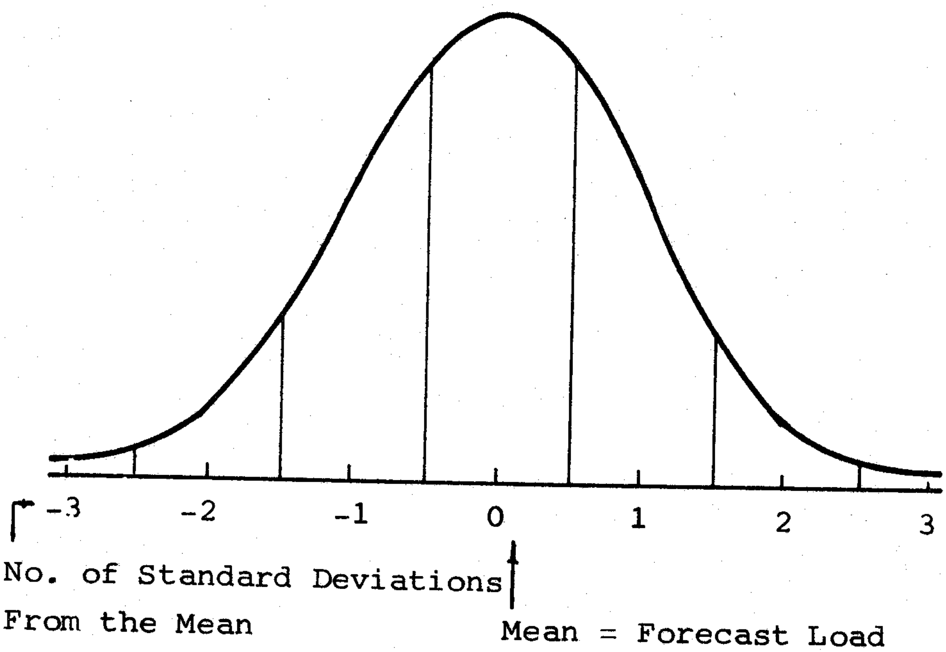


Figure 2.8: Seven Step Approximation of Normal Distribution For Uncertainty In Load Forecasting

$$1\ S.D = \frac{\%\ \text{Uncertainty x Forecast Load, MW}}{100}$$

No. of S.D from the Mean	Probability of Actual Load=Forecast Load + Number of Standard Deviation in Col. 1
-3	0.006
-2	0.061
-1	0.242
0	0.382
+1	0.242
+2	0.061
+3	0.006

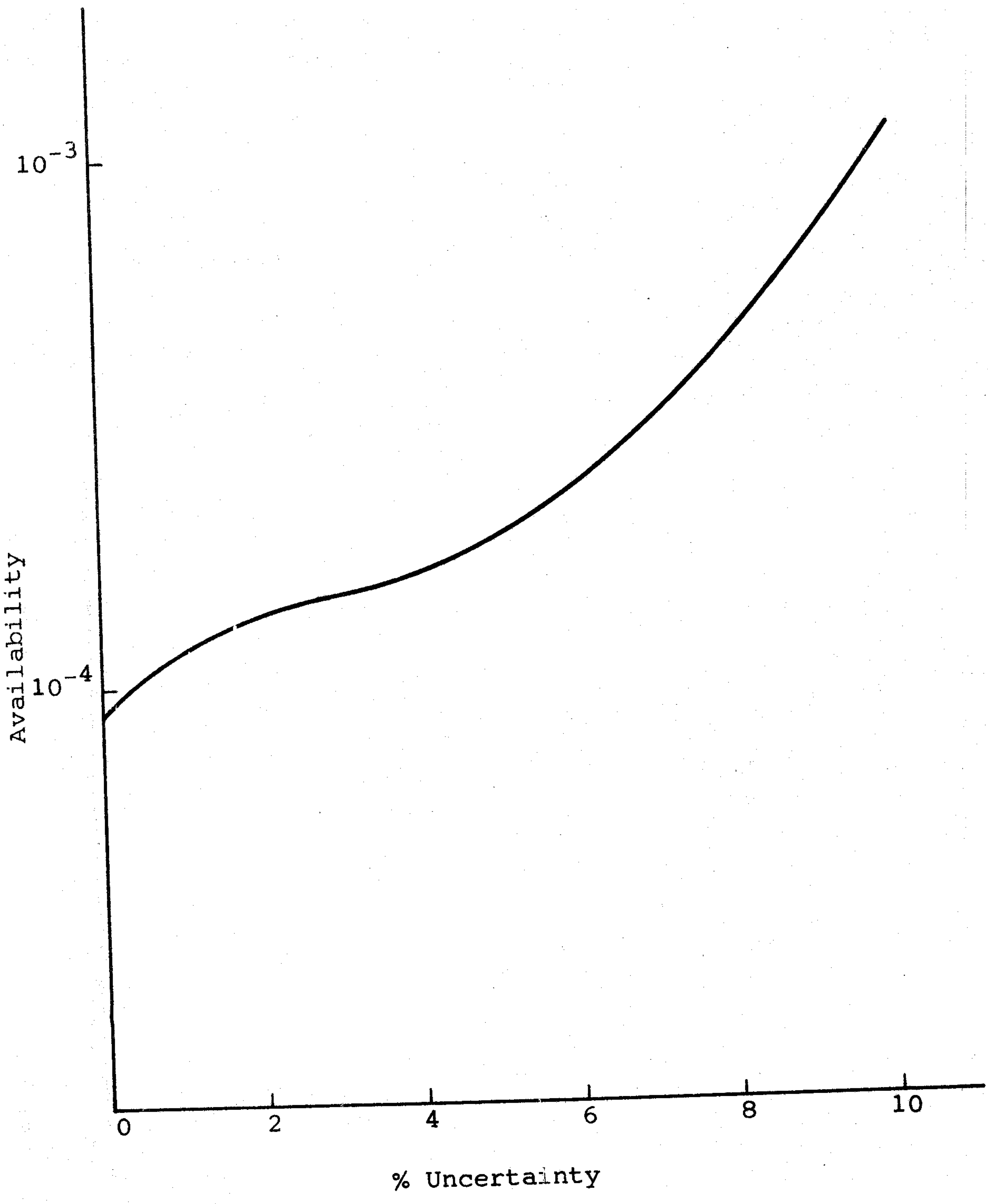


Figure 2.9: Variation of Risk Level (Availability) of SYS.RS with % Load Forecast Uncertainty

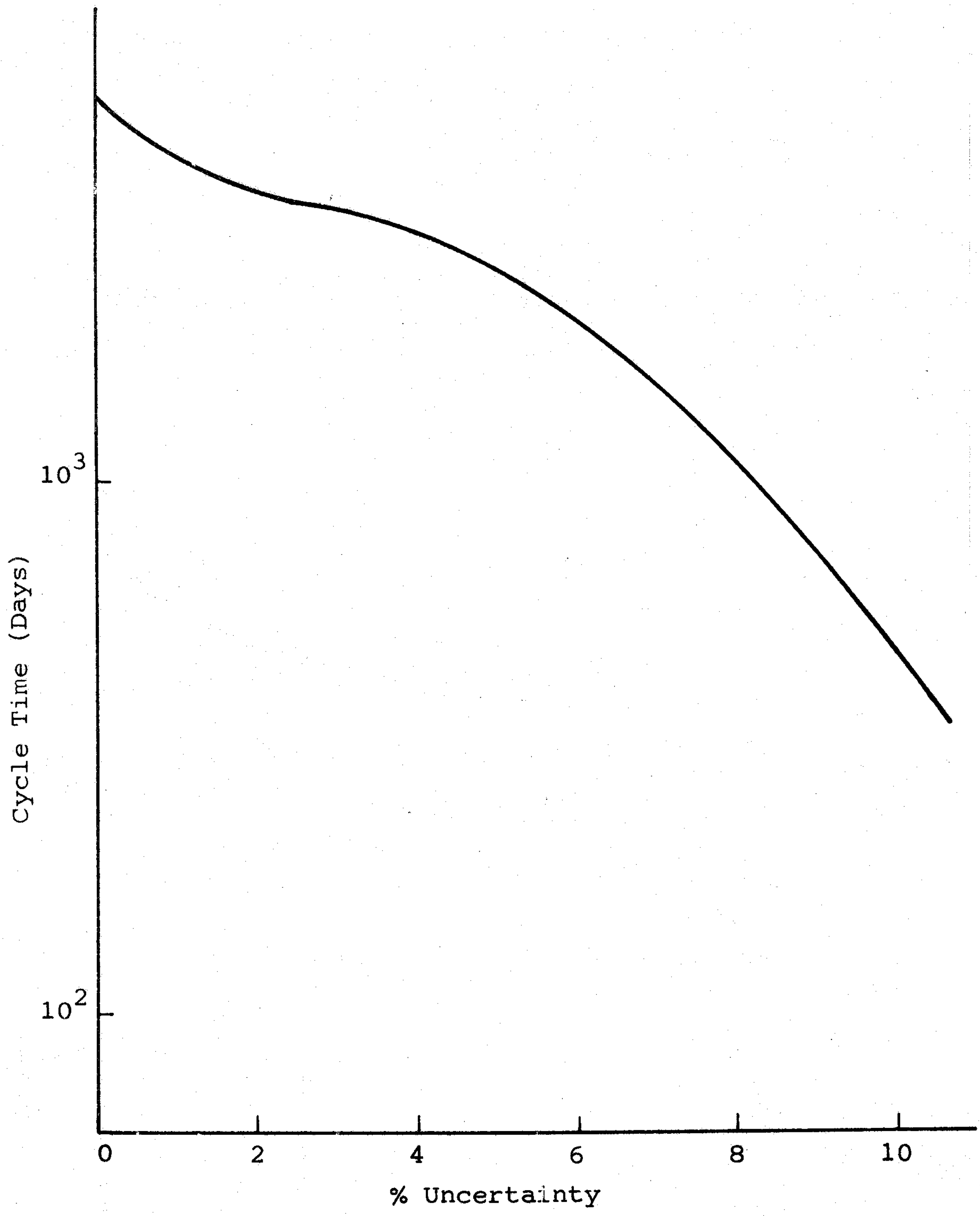


Figure 2.10: Variation of Risk Level (Cycle Time) of SYS.RS with % Load Forecast Uncertainty

interval on an annual basis. The variation in the reliability indices with uncertainty clearly indicates the need for accurate and comprehensive forecasting techniques. The critical uncertainty is that value which exists at the time when the expansion plan must be finalized in order to allow design and construction to begin.

2.8.3 An Expansion Study on SYS.RS

This study was conducted to compare the expansion schemes for SYS.RS for 1971 - 79, using the availability of failure (which as previously mentioned is an index of LOLP) and the cycle time to failure as the criteria.

The peak was assumed to increase by ten percent every year with the basic load model remaining unchanged. One load model was assumed to hold throughout and no maintenance was included. The variability in reliability measures results only from changes in the peak load and from unit additions.

Curves showing the availability and cycle time of the failure state versus peak load were obtained first with an installed capacity of 1725 MW, i.e., for 1970 and then for each subsequent addition of a 250 MW unit. These curves are shown in Figures 2.11 and 2.12 respectively. The unit additions in both cases are indicated by the dotted lines

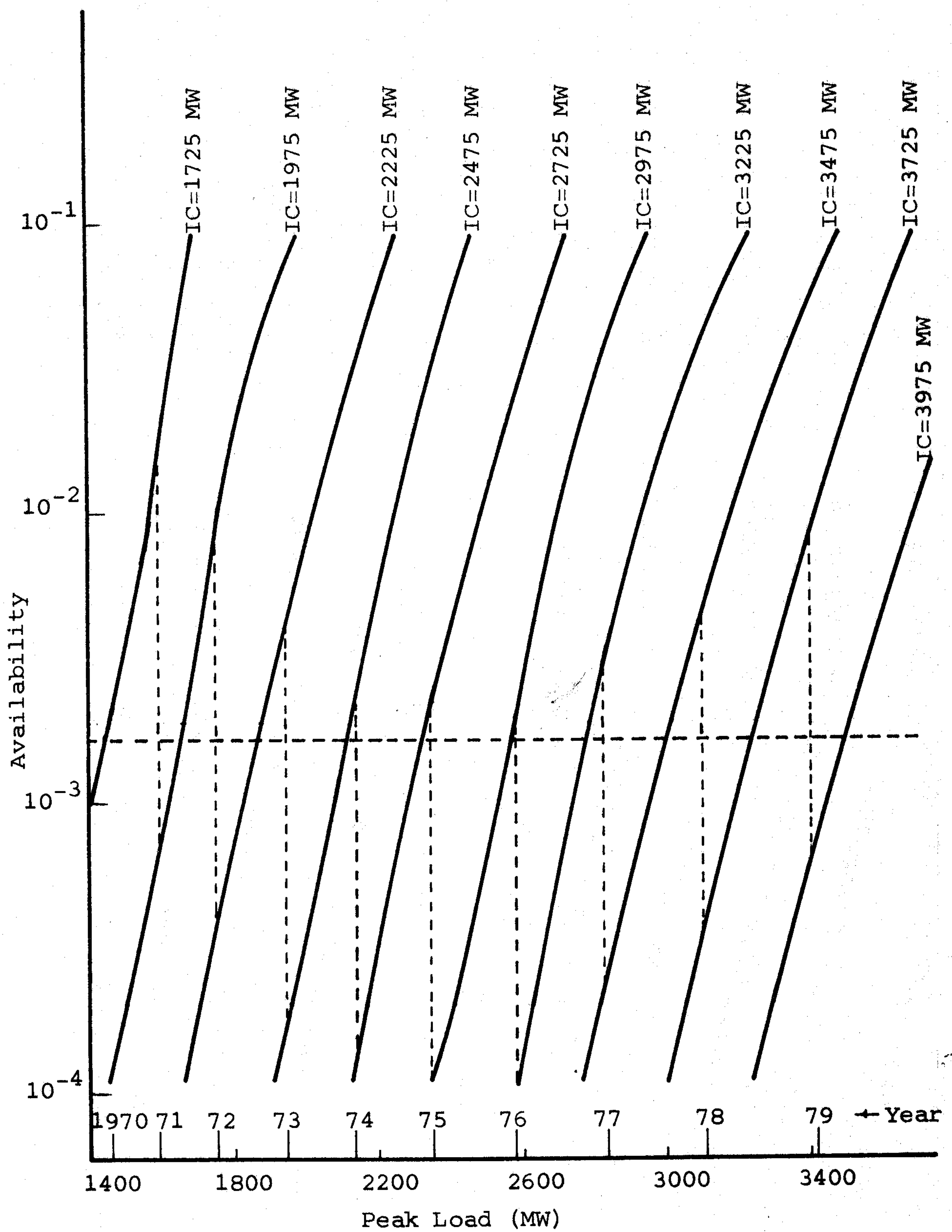


Figure 2.11: Expansion Study on SYS.RS - 1971-79, Using Availability as the Criterion

and in this particular study are virtually identical. Unit additions are required in 1971,72,73,75,77,78 and 79.

The effective load carrying capability of a unit is the increase in load carrying capability of the system at a given risk level after the unit addition. It is measured by the displacement of "Risk Level vs Peak Load" curve along a horizontal line passing through the standard risk level. It should be pointed out that the availability or cycle time versus peak load curves are not as smooth as shown in Figures 2.11 and 2.12. These are stepped curves, the average width of the step depending upon the structure of the load model and the capacity outage increment in the generation system model. Accurate assessment of the effective load carrying capability of the unit being added is not, therefore, possible from the availability or cycle time versus peak load characteristics. It can, however, be determined by successive iterations about the step change point. The effective load carrying capabilities of the units found by successive iterations are shown in Table 2.2. In this case, the results are the same when either the availability or the cycle time is used as the criterion of risk level. This is, however, a special case and in general the effective load carrying capabilities determined on the basis of availability are found to be slightly different from those obtained using cycle time (See 4.4).

TABLE 2.2

The Effective Load Carrying Capabilities
of the Units Added

Unit #	BOA	BOCT
1	200.00 MW	200.00 MW
2	225.00 MW	225.00 MW
3	225.00 MW	225.00 MW
4	225.00 MW	225.00 MW
5	225.00 MW	225.00 MW
6	175.00 MW	175.00 MW
7	250.00 MW	250.00 MW
8	250.00 MW	250.00 MW
9	250.00 MW	250.00 MW

BOA Based on the availability of failure as the criterion

BOCT Based on the cycle time to failure as the criterion

Capacity of the unit added is 250 MW.

All units have the same mean failure and the mean repair rates.

A subroutine has been added to the computer programme which can determine unit additions for a specified cycle time and print out the results in a tabular form. The output obtained for the above case is shown in Table 2.3. This removes the necessity of having to plot all the results.

TABLE 2.3

250 MW Unit Addition Schedule

Standard Risk = 291.56 days

Year of Study	Peak Load MW	Total Capacity MW	Risk Level Days	Whether Unit Addition Required
1970	1450.0000	1725.0	291.56	NO
1971	1595.0000	1725.0	26.48	YES
CAPACITY OF UNIT ADDED,MW. = 250.0		λ = .0009318795		μ = .04566210
1971	1595.0000	1975.0	709.43	NO
1972	1754.0000	1975.0	49.29	YES
CAPACITY OF UNIT ADDED,MW. = 250.0		λ = .0009318795		μ = .04566210
1972	1754.0000	2225.0	1345.24	NO
1973	1930.0000	2225.0	115.00	YES
CAPACITY OF UNIT ADDED,MW. = 250.0		λ = .0009318795		μ = .04566210
1973	1930.0000	2475.0	2624.68	NO
1974	2123.0000	2475.0	265.59	YES
CAPACITY OF UNIT ADDED,MW. = 250.0		λ = .0009318795		μ = .04566210
1974	2123.0000	2725.0	5582.01	NO
1975	2335.0000	2725.0	224.61	YES
CAPACITY OF UNIT ADDED,MW. = 250.0		λ = .009318795		μ = .04566210
1975	2335.0000	2975.0	4138.32	NO
1976	2569.0000	2975.0	283.60	YES
CAPACITY OF UNIT ADDED,MW. = 250.0		λ = .009318795		μ = .04566210
1976	2569.0000	3225.0	5116.20	NO
1977	2826.0000	3225.0	141.31	YES
CAPACITY OF UNIT ADDED,MW. = 250.0		λ = .009318795		μ = .04566210
1977	2826.0000	3475.0	2041.30	NO
1978	3108.0000	3475.0	107.29	YES
CAPACITY OF UNIT ADDED,MW. = 250.0		λ = .009318795		μ = .04566210
1978	3108.0000	3725.0	1374.17	NO
1979	3419.0000	3725.0	56.91	YES
CAPACITY OF UNIT ADDED,MW. = 250.0		λ = .009318795		μ = .04566210
1979	3419.0000	3975.0	614.36	NO

3. INTERCONNECTED SYSTEMS

3.1 General

The interconnection of a power system to one or more power systems provides a definite improvement in the level of generating capacity reliability. This effect is due to diversity in the occurrence of the load and the outages of capacity in the different systems. When the power system suffers a loss of load, help is generally available from the power systems to which it is connected, thus compensating partially or completely the capacity deficiency. For a given operating reserve, the level of generating capacity reliability is improved or conversely for a given risk level, the power system can operate at a lower operating reserve.

The evaluation of reliability of an interconnected system depends on its type of agreement with the other systems. Relationships have been derived assuming that one system helps the other as much as it can without curtailing its own load. The principles involved are, however, general and may be easily extended to cover other agreements.
The other assumptions utilized in the development are:

1. The load and generation models of the different systems are stochastically independent.

2. The simultaneous occurrence of two or more transitions is stochastically impossible.

Relationships have been developed for a system connected to one other system and subsequently extended to the case of a system connected to two or more systems.

3.2 System A Connected to System B

3.2.1 The Direct Approach

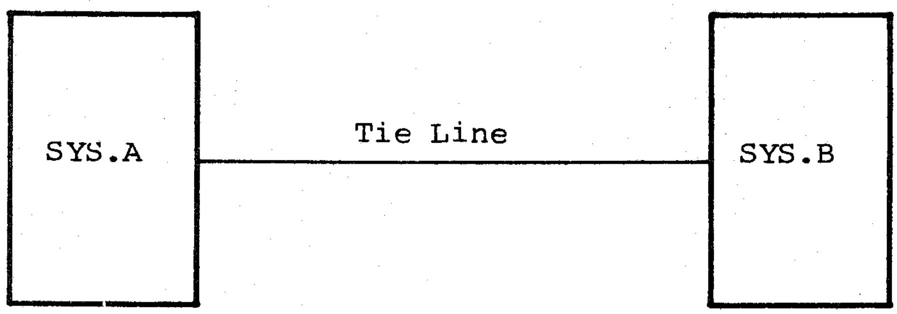
General relations for determining the availability and frequency of positive or negative cumulative capacity margins in System A (Figure 3.1) have been developed and simplified for application to the negative states only.

Since the capacity and the load in each system are assumed to exist at a discrete number of levels, the margin state, which is the capacity available less the load on the system, would also exist at a discrete number of levels in each system. In Figure 3.2 M_a is a column vector containing the unaffected margin states in System A and M_b is similarly a row vector containing the unaffected margin states in System B. These states are arranged in the order of decreasing reserve.

Therefore

$$m_{a1} > m_{a2} > \ldots\ldots > m_{ai} > \ldots\ldots m_{aNA}$$

and

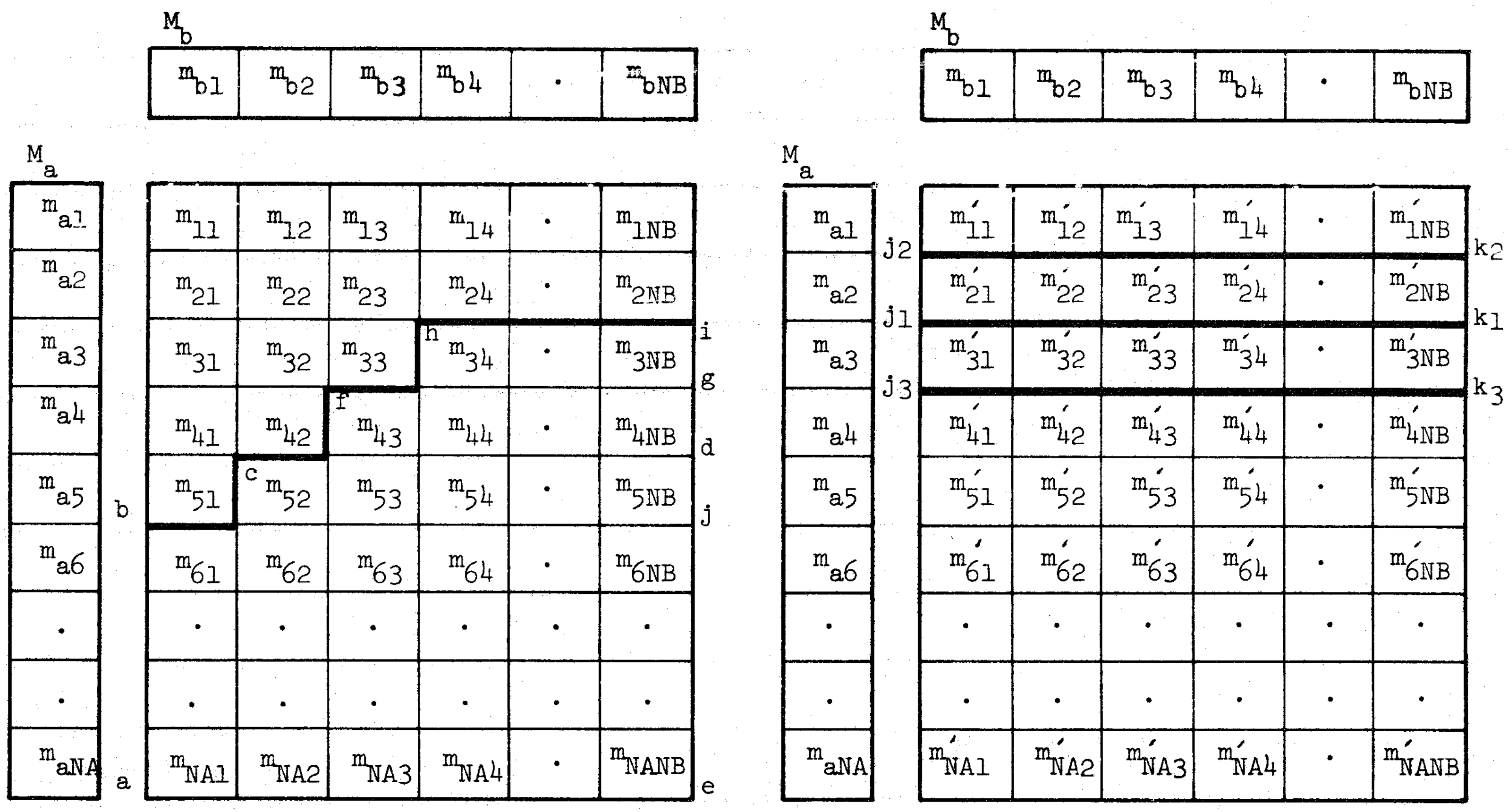


T_{ab} = The Capability of the Tie Line

λ_{ab} = The Average Failure Rate of the Tie Line

μ_{ab} = The Average Repair Rate of the Tie Line

Figure 3.1: System A Connected to System B



Interconnection Up

Interconnection Down

Figure 3.2: Effective Margin States Matrices for System A Connected to System B

$$m_{b1} > m_{b2} > \ldots\ldots > m_{bj} > \ldots\ldots m_{bNB}$$

The term unaffected denotes that the interconnection is not effective. The effective margin states in System A, i.e., when the tie line is in operation, are given by the elements of the matrix M,

$$m_{ij} = m_{ai} + h_{ij} \tag{3.1}$$

where h_{ij} is either the help available to System A from System B or it is the help required by System B from System A. In the latter case h_{ij} has a negative sign. If no help can be rendered by one system to the other $h_{ij} = 0$. The maximum value of h_{ij} is limited by the tie line capability.

The effective margin states in System A, while the tie line is on forced outage are given by the elements of matrix M',

$$m'_{ij} = m_{ai} \tag{3.2}$$

Equations 3.1 and 3.2 define the boundaries in M and M' respectively of any effective cumulative margin state. In this presentation, m with proper subscript represents an exact margin and M with the same subscript denotes the corresponding cumulative margin, e.g., M_{ij} means a margin equal to or less than m_{ij}.

The principles involved in the determination of the availability and frequency of a particular state, say

M_{34}, will be examined and then these will be condensed into generalized equations. It is assumed that in the matrix M, the thick line bcfhi demarcates the effective reserve equal to or less than m_{34}. The boundary for this margin state in M' may be theoretically anywhere - in line with hi as j_1k_1, above hi as j_2k_2 or below hi as j_3k_3. In practice the boundary in M' can be:

1. Above hi only if there are no zero or negative margins in System B. Such a system is highly improbable in actual practice.
2. Below hi only if m_{34} is a positive margin.
3. The boundary in M' will be always in line with hi for negative m_{34}.

The matrices M and M' can be represented by the state transition diagram shown in Figure 3.3.

Assuming $m_{34} < m_{a3}$ and that all other corner states are greater than the corresponding state in M_a, the boundary for M_{34} is shown by the dotted line in the state transition diagram. If m_{34} were equal to or greater than m_{a3}, the portion yz of the boundary would be modified as shown by arrows.

Availability:

The effective margin states are separated in time and the contribution to the availability of M_{34} by the rows

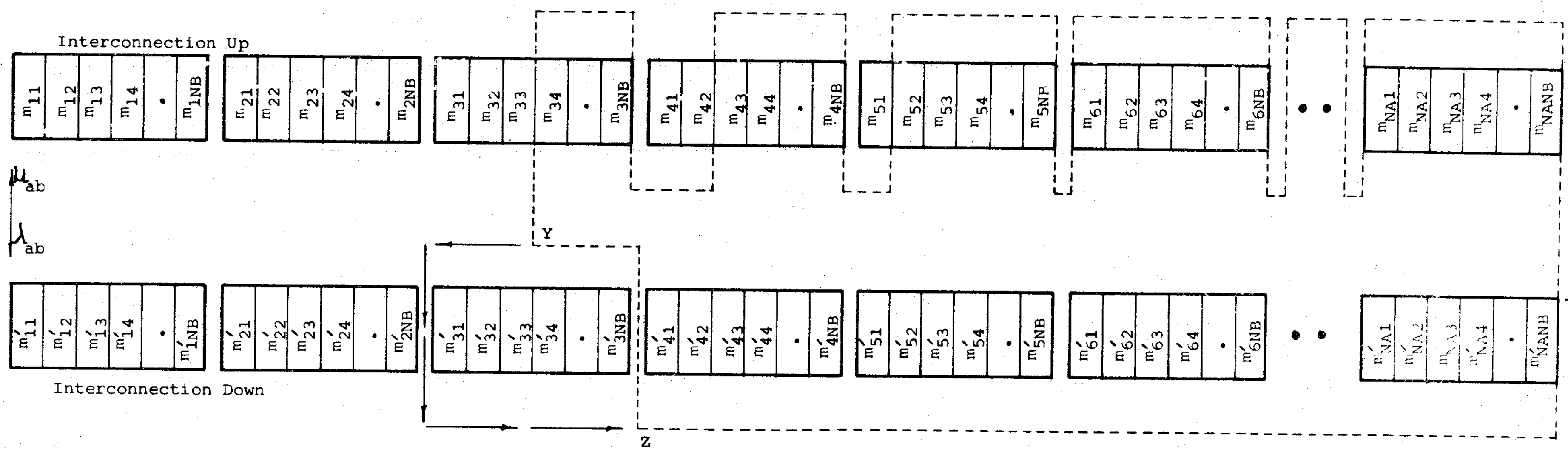


Figure 3.3: The State Transition Diagram of System A Connected to System B

corresponding to m_{a3} for the condition $\{m_{34} \geq m_{a3}\}$ will be given as

$$= (A_{a(3)} - A_{a(3+1)}) \; (A_{ab} \sum_{j=4}^{NB} (A_{b(j)} - A_{b(j+1)}))$$

$$+ (A_{a(3)} - A_{a(3+1)}) \; (\bar{A}_{ab} \sum_{j=1}^{NB} (A_{b(j)} - A_{b(j+1)}))$$

$$= (A_{a(3)} - A_{a(3+1)}) \; (A_{ab} \cdot A_{b(4)} + \bar{A}_{ab})$$

If m_{34} were less than m_{a3}, the row in M' corresponding to m_{a3} will not contribute to the availability of M_{34} since those states being equal to m_{a3} would not be included in M_{34}. Contribution under this condition would be given as

$$= (A_{a(3)} - A_{a(3+1)}) \; (A_{ab} \cdot A_{b(4)})$$

Generalizing from the above

$$A_N = \sum_{L,K} (A_{a(l)} - A_{a(l+1)}) \; (A_{ab} \cdot A_{b(k)} + \beta \bar{A}_{ab}) \qquad (3.3)$$

where

$A_{a(l)}, A_{b(k)}$ = The availabilities of the cumulative capacity margins $M_{a(l)}$ and $M_{b(k)}$ respectively

A_{ab} = The availability of the tie line between A and B

A_N = The availability of an effective margin state $\leq N$.

L,K = The indices defining the corner states of the boundary of N. For example, the indices for N = m_{34} are (NA,1)--(6,1), (5,2) (4,3) and (3,4).

and

$$\beta = 1 \text{ for } N \geq m_{a(L)}$$

$$= 0 \text{ for } N < m_{a(L)}$$

Frequency:

Let

$f_{a(l)}$, $f_{b(k)}$ = The frequencies of encountering the cumulative margin states $M_{a(l)}$ and $M_{b(k)}$ respectively.

λ_{ab}, μ_{ab} = The mean failure and the mean repair rates of the tie line

and

f_N = The frequency of encountering an effective margin $\leq N$.

System A can transit from one effective margin state to another in any of the following ways.

1. The capacity or load transitions in System A itself

System A will shift vertically in M when the interconnection is operative and in M' when the interconnection is on forced outage. In the state transition diagram, the system will transit correspondingly from a state in one composite block to the corresponding state in some other composite block. In terms of the matrix notation:

When the interconnection is in the up state, given that System A is in the effective state m_{ij}, it can transit to m_{kj}, $k \neq i$

and

When the interconnection is in the down state, given that the system is in the effective state m'_{ij}, it can transit to m'_{kj}, $k \neq i$.

2. The capacity or load transitions in System B.

Due to the transitions in System B, System A will shift horizontally from one effective state to another in the matrix M, when the interconnection is in the up state. With the interconnection in the down state, System A will shift horizontally from one effective state to another in the matrix M'. As will be seen later, such transitions do not ultimately reflect into the effective operation of System A. Again in terms of the matrix notation

When the interconnection is in the up state, given that System A is in the effective state m_{ij}, it can transit to

m_{ik}, $k \neq j$

and

When the interconnection is in the down state, given that System A is in the effective state m'_{ij}, it can transit to m'_{ik}, $k \neq j$.

3. The failure or the repair of the tie line

When the tie line fails or is repaired, System A will transit from a state in M' to the corresponding state in M and vice versa. In the state transition diagram transition is between two vertically opposite states and in terms of the matrix notation

Transition due to the failure of the interconnection will be from m_{ij} to m'_{ij}

and

Transition due to the repair of the interconnection will be from m'_{ij} to m_{ij}.

The frequency of encountering an effective cumulative margin state in System A equals the "expected transitions per unit time" across the boundary defining that state plus the "expected transitions per unit time" associated with the deserting states. The deserting states are defined as the states which leave the domain of the cumulative margin state as a result of the failure or the repair of the tie line. As an example, if $m_{34} > m_{a3}$, the states m'_{31}, m'_{32} and m'_{33} desert M_{34} as a result of the repair of the tie line. Similarly, if $m_{34} < m_{a3}$, the states m_{34}, $m_{35}, \ldots\ldots, m_{3NB}$ desert M_{34} as a result of the failure of the tie line.

In determining the contributions to f_N due to the three different modes of system transition, it is helpful to bear in mind the following equations.

$$f_{a(l)} = \sum_{i=1}^{NA} E^{a}_{i(<l)}$$

$$f_{b(k)} = \sum_{j=k}^{NB} E^{b}_{j(<k)}$$

$$f_{a(l)} - f_{a(l+1)} = \sum_{i=1}^{l-1} E^{a}_{li} - \sum_{i=l+1}^{NA} E^{a}_{li}$$

and

$$f_{b(k)} - f_{b(k+1)} = \sum_{j=1}^{k-1} E^{b}_{kj} - \sum_{j=k+1}^{NB} E^{b}_{kj}$$

where

NA,NB = The total number of discrete levels in the capacity reserve models of Systems A and B respectively

and

E^x_{ij} = The expected number of "transitions per unit time" from an exact state "i" to an exact state "j" in the system "x".

As in the case of the availability, the contribution of the rows corresponding to m_{a5} to the frequency of M_{34} will be determined first and then out of this the generalized formulation will be evolved.

The total "E" (expected transitions per unit time) out of the boundary b_j in M and the corresponding boundary in M' is

$$= \sum_{j=1}^{NB} A_{ab}\cdot A'_{b(j)} \sum_{i=6}^{NA} E^a_{i(<6)} + \sum_{j=1}^{NB} \bar{A}_{ab}\cdot A'_{b(j)} \sum_{i=6}^{NA} E^a_{i(<6)}$$

The total E out of the boundary abcde in M and the corresponding boundary in M' is

$$= \sum_{j=1}^{NB} A_{ab}\cdot A'_{b(j)} \sum_{i=6}^{NA} E^a_{i(<6)} + \sum_{j=1}^{NB} \bar{A}_{ab}\cdot A'_{b(j)} \sum_{i=6}^{NA} E^a_{i(<6)}$$

$$+ \sum_{j=2}^{NB} A_{ab}\cdot A'_{b(j)} \sum_{i=1}^{4} E^a_{5i} + \beta \sum_{j=1}^{NB} \bar{A}_{ab}\cdot A'_{b(j)} \sum_{i=1}^{4} E^a_{5i}$$

$$- \sum_{j=2}^{NB} A_{ab}\cdot A'_{b(j)} \sum_{i=6}^{NA} E^a_{5i} - \beta \sum_{j=1}^{NB} \bar{A}_{ab}\cdot A'_{b(j)} \sum_{i=6}^{NA} E^a_{5i}$$

$$+ A_{ab} \cdot A'_{a(5)} \sum_{j=2}^{NB} E^{b}_{j(<2)}$$

Here $\beta = 1$, if $m_{34} \geq m_{a5}$ since then the corresponding row in M' will be included in the boundary,

$= 0$, if $m_{34} < m_{a5}$ since then the corresponding row in M' will not be included in the boundary.

The primed designation has been used to denote the availabilities of the exact states.

Rearranging the expression for total E out of abcde

$$= \sum_{j=1}^{NB} A_{ab} \cdot A'_{b(j)} \sum_{i=6}^{NA} E^{a}_{i(<6)} + \sum_{j=1}^{NB} \bar{A}_{ab} \cdot A'_{b(j)} \sum_{i=6}^{NA} E^{a}_{i(<6)}$$

$$+ \sum_{j=2}^{NB} A_{ab} \cdot A'_{b(j)} \left(\sum_{i=1}^{4} E^{a}_{5i} - \sum_{i=6}^{NA} E^{a}_{5i}\right) +$$

$$\beta \sum_{j=1}^{NB} \bar{A}_{ab} \cdot A'_{b(j)} \left(\sum_{i=1}^{4} E^{a}_{5i} - \sum_{i=6}^{NA} E^{a}_{5i}\right)$$

$$+ A_{ab} \cdot A'_{a(5)} \sum_{j=2}^{NB} E^{b}_{j(<2)}$$

$$= f_{a(6)}$$

$$+ A_{ab} \cdot A_{b(2)} \left(f_{a(5)} - f_{a(5+1)}\right) + \beta \cdot \bar{A}_{ab} \left(f_{a(5)} - f_{a(5+1)}\right)$$

$$+ A_{ab}.A'_{a(5)}.f_{b(2)}$$

$$= f_{a(6)}$$

$$+ (f_{a(5)} - f_{a(5+1)})\ (A_{ab}.A_{b(2)} + \beta.\bar{A}_{ab})$$

$$+ (A_{a(5)} - A_{a(5+1)})\ .A_{ab}.f_{b(2)} \qquad (3.4)$$

Let

$E^{ij}_{mm'}$ = The expected transitions per unit time from m_{ij} to m'_{ij}.

The contribution due to deserting states

If $m_{34} \geq m_{a5}$ is

$$= \sum_{j=1} E^{5j}_{m'm}$$

and if $m_{34} < m_{a5}$ is

$$= \sum_{j=2}^{NB} E^{5j}_{mm'}$$

Rearranging, the contribution due to deserting states

$$= \beta \left[(1 - A_{b(2)})\ (A_{a(5)} - A_{a(5+1)})\ \bar{A}_{ab}.\ \mu_{ab} \right]$$

$$+ \gamma \left[A_{b(2)}\ (A_{a(5)} - A_{a(5+1)})\ A_{ab}.\ \lambda_{ab} \right] \qquad (3.5)$$

where

$$\gamma = 1 \quad , \text{ for } m_{34} < m_{a5}$$

$$= 0 \quad , \text{ for } m_{34} \geq m_{a5}$$

Adding up 3.4 and 3.5 and substituting L and K for 5 and 2 respectively, the general expression for f_N becomes

$$f_N = \sum_{L,K} \Big[(f_{a(l)} - f_{a(l+1)}) \; (A_{ab} \cdot A_{b(k)} + \beta \cdot \bar{A}_{ab})$$

$$+ (A_{a(l)} - A_{a(l+1)}) \; (A_{ab} \cdot f_{b(k)})$$

$$+ \beta \cdot \bar{A}_{ab} \cdot \mu_{ab} \; (1 - A_{b(k)}) \; (A_{a(l)} - A_{a(l+1)})$$

$$+ \gamma \cdot \bar{A}_{ab} \cdot \mu_{ab} \cdot A_{b(k)} \; (A_{a(l)} - A_{a(l+1)}) \Big]$$

Where L,K are, as before, the indices defining the corners of the boundary of the effective cumulative margin N, in the matrix M.

Manipulating the above expression,

$$f_N = \sum_{L,K} \Big[(f_{a(l)} - f_{a(l+1)}) \; (A_{ab} \cdot A_{b(k)} + \beta \bar{A}_{ab})$$

$$+ (A_{a(l)} - A_{a(l+1)}) \; (f_{b(k)} + \beta (1 - A_{b(k)}) \lambda_{ab}$$

$$+ \gamma . A_{b(k)} \cdot \lambda_{ab}) \; A_{ab} \Big] \qquad (3.6)$$

where

$$\beta = 1 \quad \text{for } N \geq m_{aL}$$
$$= 0 \quad \text{for } N < m_{aL}$$

and

$$\gamma = 1 \quad \text{for } N < m_{aL}$$
$$= 0 \quad \text{for } N \geq m_{aL}$$

Equations 3.3 and 3.6 describe the general relations for the availability and the frequency of any effective margin equal to or less than N, in System A. These equations are conveniently adoptable to the computer. As will be seen later, the effective positive margins come into the picture in the case of multiarea problems. For the two area problem, the main concern is with the deficiency states. In the case of the deficiency states, N is always equal to or greater than m_{aL} and equations 3.3 and 3.6 can be simplified as follows:

$$A_N = \sum_{L,K} (A_{a(l)} - A_{a(l+1)}) \; (A_{ab} \cdot A_{b(k)} + \bar{A}_{ab}) \qquad (3.7)$$

and

$$f_N = \sum_{L,K} \Big[(f_{a(l)} - f_{a(l+1)}) \; (A_{ab} \cdot A_{b(k)} + \bar{A}_{ab})$$
$$+ (A_{a(l)} - A_{a(l+1)}) \; (f_{b(k)} + (1 - A_{b(k)})\lambda_{ab}) \; A_{ab} \Big]$$
$$(3.8)$$

3.2.2 The Indirect Approach

In this approach, the effective generation system model for System A is developed by combining the generation system model of System B with that of System A, under the constraints of the load model of System B. The load model of System A is then combined with this generations system model to generate the effective capacity reserve model.

The development of the effective generation system model is analogous to the development of the effective margin state model. Referring to Figure 3.4,

with the interconnection in the up state

$$\bar{c}_{ij} = \bar{c}_{ai} - h_{ij} \tag{3.9}$$

where

$\bar{c}_{ij}$ = The effective outage of capacity in System A with $\bar{c}_{bj}$ as the capacity outage in System B and $\bar{c}_{ai}$ as the unaffected capacity outage in System A

h_{ij} = The help available to System A from System B or the negative of help required by System B

$= \min\ (R_b - \bar{c}_{bj},\ T_{ab})$

or $= -\min\ (R_a - \bar{c}_{ai},\ \bar{c}_{bj} - R_b,\ T_{ab})$ in the latter case

or $= 0$, if no help can be rendered by one system to the other

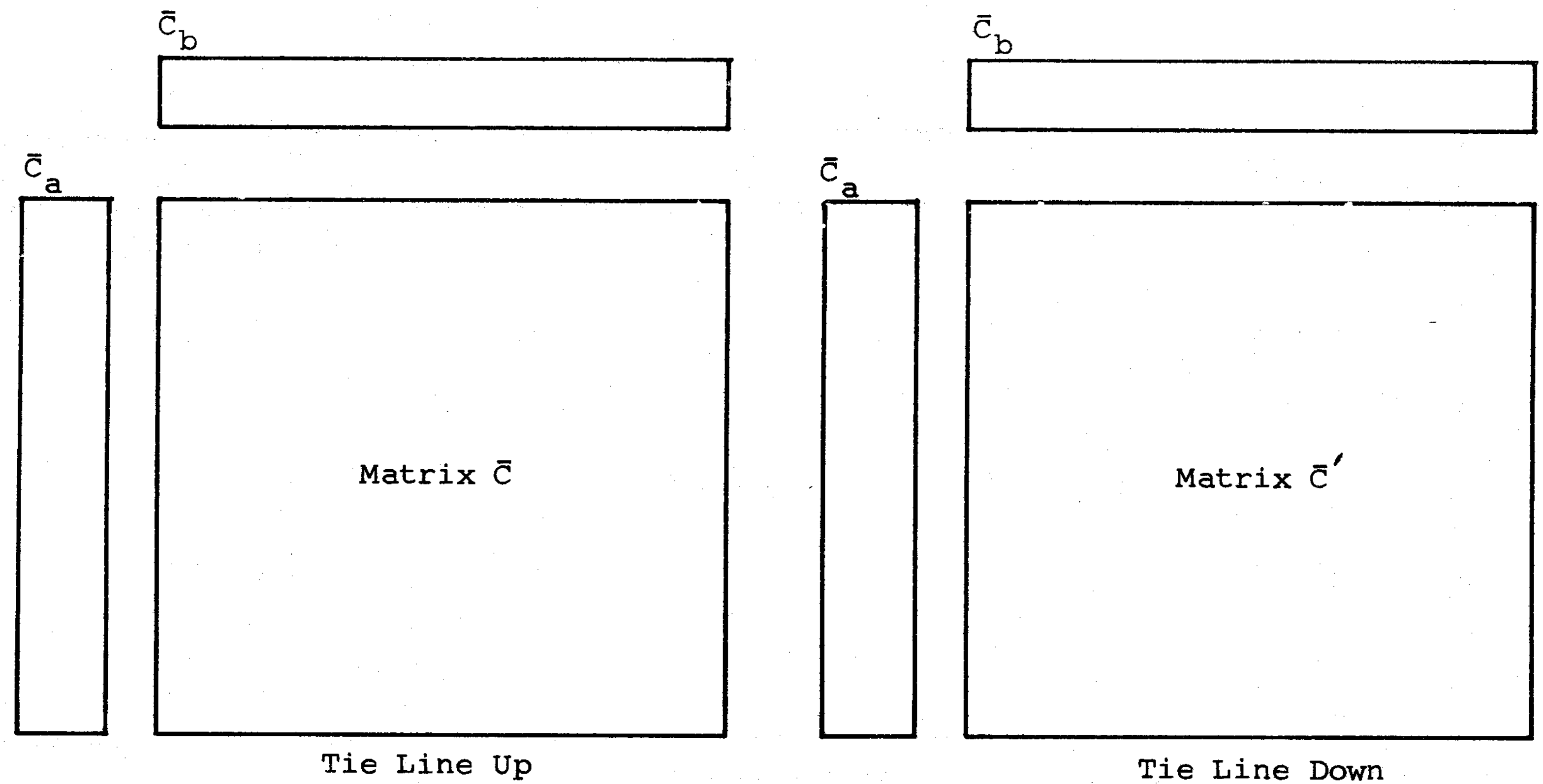


Figure 3.4: The Effective Generation System Model of System A Connected to System B

R_a, R_b = The operating reserves in Systems A and B respectively

= The installed capacity - The load on the system.

T_{ab} = The capability of the interconnection between System A and System B.

With the interconnection in the down state

$$\bar{c}'_{ij} = \bar{c}_{ai} \qquad (3.10)$$

Equations 3.9 and 3.10 define the boundaries of the cumulative capacity outages in the matrices $\bar{C}$ and $\bar{C}'$. Whereas $\bar{c}$ with proper subscript denotes an exact outage of capacity, the corresponding cumulative outage of capacity is represented by $\bar{C}$ with the same subscript, e.g. $\bar{C}_{ai}$ means a capacity outage equal to or greater than $\bar{c}_{ai}$.

Equations 3.3 and 3.6 apply to the availability and the frequency of the effective cumulative capacity outage states with some modification of the qualifying statements. These are restated below:

$$A_O = \sum_{L,K} (A_{a(l)} - A_{a(l+1)}) \ (A_{ab} \cdot A_{b(k)} + \beta . \bar{A}_{ab}) \qquad (3.11)$$

and

$$f_O = \sum_{L,K} \Big[(f_{a(l)} - f_{a(l+1)}) \ (A_{ab} \cdot A_{b(k)} + \beta . \bar{A}_{ab})$$

$$+ (A_{a(l)} - A_{a(l+1)}) \ (f_{b(k)} + \beta (1 - A_{b(k)}) \lambda_{ab}$$

$$+ \gamma . A_{b(k)} . \lambda_{ab}) \; A_{ab} \Big] \tag{3.12}$$

where

A_O, f_O = The availability and the frequency of a capacity outage equal to or greater than "O".

L, K = The indices defining the boundary of this cumulative capacity outage

$A_{a(l)}, f_{a(l)}$ = The availability and the frequency of the cumulative capacity outage $\bar{C}_{al}$

$A_{b(k)}, f_{b(k)}$ = The availability and the frequency of the cumulative capacity outage $\bar{C}_{bk}$

$$\beta = 1 \qquad \text{for } O \leq \bar{c}_{ai}$$

$$= 0 \qquad \text{for } O > \bar{c}_{ai}$$

and

$$\gamma = 1 \qquad \text{for } O > \bar{c}_{ai}$$

$$= 0 \qquad \text{for } O \leq \bar{c}_{ai}$$

If this effective generation system model of System A is required to generate the availability and frequency of only the negative margins in System A, only those capacity outages in excess of the operating reserve in System A will be required. For this "O" is always less than or equal to $\bar{c}_{ai}$ and equations 3.11 and 3.12 simplify to

$$A_O = \sum_{L,K} (A_{a(l)} - A_{a(l+1)})\ (A_{ab} \cdot A_{b(k)} + \bar{A}_{ab}) \qquad (3.13)$$

and

$$f_O = \sum_{L,K} \Big[(f_{a(l)} - f_{a(l+1)})\ (A_{ab} \cdot A_{b(k)} + \bar{A}_{ab})$$

$$+ (A_{a(l)} - A_{a(l+1)})\ (f_{b(k)} + \lambda_{ab}\ (1 - A_{b(k)}))\ A_{ab} \Big] \qquad (3.14)$$

Only a limited portion of the effective generation system model is required to determine the availability and the frequency of any particular cumulative negative margin state. The steps involved in determining the availability and the frequency of the failure state are outlined below.

Let

L_{ap} be the load levels in System A, $p = 1,2,3.....na$,

$L_{a1} > L_{a2} > .... > L_{ana}$

L_{bq} be the load levels in System B, $q = 1,2,3...nb$,

$L_{b1} > L_{b2} > .... > L_{bnb}$

v be a small positive value (say 0.0000001)

IC_a, IC_b be the installed capacities on systems A and B respectively

and

L_{ao}, L_{bo} be the low load levels in systems A and B respectively

Step # 1

$$R_{bo} = IC_b - L_{bo}$$

$$R_a = IC_a - L_{a1}$$

With these values of the operating reserves, the availability and the frequency of the effective capacity outages equal to or greater than $\{(IC_a - L_{ap} + v),\ p = 1, 2, 3 \ldots na\}$ are calculated.

Step # 2

$$R_{bq} = IC_b - L_{bq} \ , \ q = 1,2,3 \ldots nb$$

For each of these operating reserves in System B, the availability and frequency of the effective capacity outages equal to or greater than $\{(IC_a - L_{ap} + v),\ p = 1,2, 3 \ldots na\}$ are computed. The availabilities can be calculated directly using equation 3.13 but to compute the frequencies a term, to take care of the load transitions in System B must be added to equation 3.14. The complete expression for frequency is given by

$$f_{\bar{c}p(q)} = \sum_{L,K} \Big[(f_{a(l)} - f_{a(l+1)}) \ (A_{ab} \cdot A_{b(k)} + \bar{A}_{ab})$$

$$+ (A_{a(l)} - A_{a(l+1)}) \ (f_{b(k)} + \lambda_{ab} \ (1 - A_{b(k)})) \ A_{ab} \Big]$$

$$+ (A_{\bar{c}p(o)} - A_{\bar{c}p(q)})/e$$

where

$A_{\bar{c}p(o)}$ = The availability of the effective capacity outage equal to or greater than $(IC_a - L_{ap} + v)$ given the low load state in System B.

$A_{\bar{c}p(q)}$ = The availability of the effective capacity outage equal to or greater than $(IC_a - L_{ap} + v)$ given that the load in System B is equal to L_{bq}.

A_{Lbq} = The availability of the load level L_{bq}

$f_{\bar{c}p(q)}$ = The frequency of encountering an effective capacity outage equal to or greater than $(IC_a - L_{ap} + v)$ given only two load levels in System B, i.e., L_{bq} and L_{bo}.

Then

$$f_{(N \leq -v)q} = \sum_p A_{Lap} \left[f_{\bar{c}p(q)} + A_{\bar{c}p(q)} (\lambda_{-L} - \lambda_{+L}) \right]$$

where

$f_{(N \leq -v)q}$ = The frequency of encountering an effective margin equal to or less than "-v" given the load level in System B as L_{bq}

A_{Lap} = The availability of the load level L_{ap}

and

λ_{+L}, λ_{-L} = The transitions to higher and lower load levels respectively in System A

Finally the frequency of encountering an effective margin equal to or less than "-v" is given by

$$f_{(N\leq -v)} = \sum_{q=1}^{nb} A_{Lbq} \cdot f_{(N\leq -v)q}$$

and

$$A_{(N\leq -v)} = \sum_{q=1}^{nb} A_{Lbq} \cdot A_{(N\leq -v)q}$$

where

$$A_{(N\leq -v)q} = \sum_{p=1}^{na} A_{Lap} \cdot A_{\bar{c}p(q)}$$

3.3 System A connected to two or more than two systems

The reliability evaluation of System A connected to Systems B and C as shown in Figure 3.5 is illustrated first and later it is shown that this can be extended to the case where System A is connected to more than two systems. The techniques for determining the availability and the frequency of the failure state, i.e., the first negative cumulative margin in System A is outlined. The techniques are quite general and may be applied to any negative cumulative margin.

3.3.1 Technique 1

The state transition diagram of System A is shown in Figure 3.6, where "i", "j" and "n" are the indices for

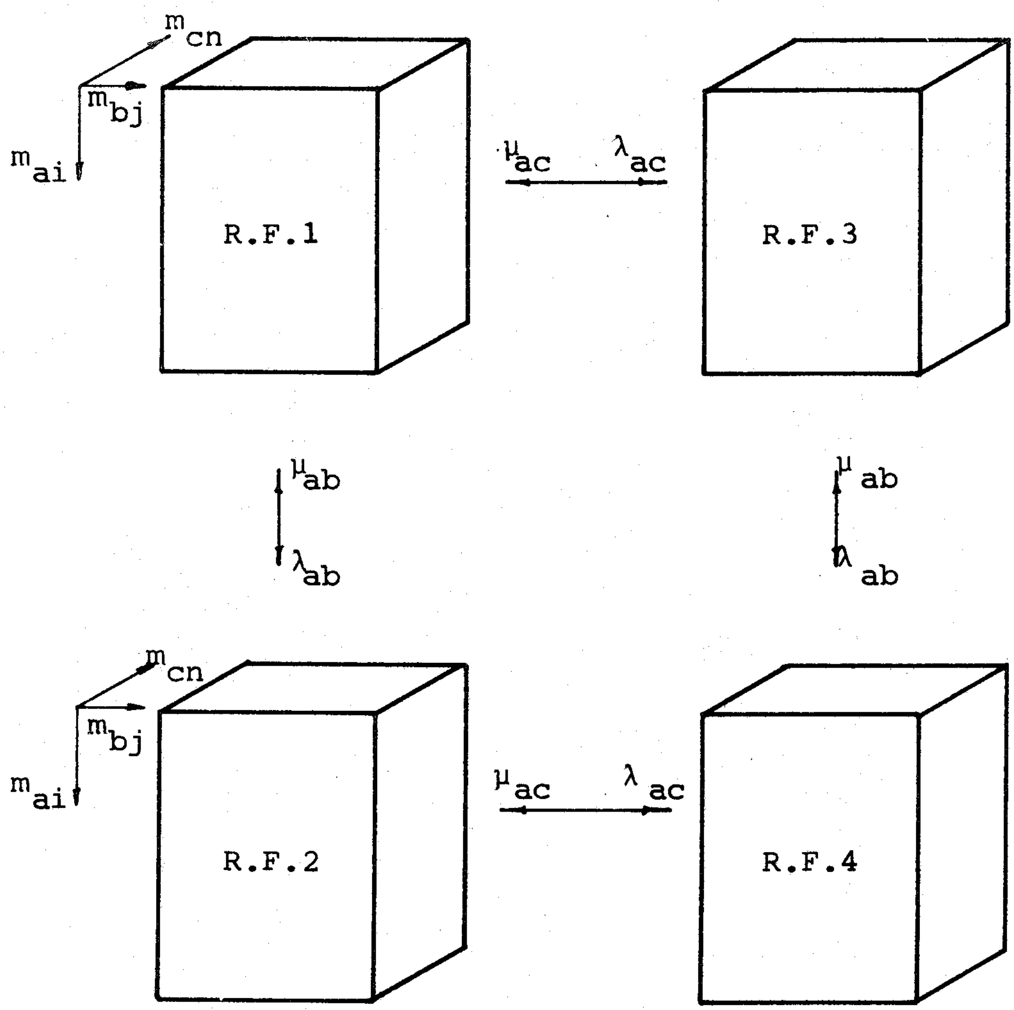


Figure 3.6: The State Transition Diagram of System A Connected to Systems B and C

the margin states in Systems A, B and C respectively, arranged in decreasing order of magnitude.

Let

$\lambda_{ab}, \lambda_{ac}$ = The mean failure rates of the tie lines AB and AC respectively

and

μ_{ab}, μ_{ac} = The mean repair rates of the tie lines AB and AC respectively

Reference frames 1 - 4 are the three dimensional arrays representing the effective margin states in System A under the following conditions:

The Reference Frame #	Tie Line A - B	Tie Line A - C
1	UP	UP
2	DN	UP
3	UP	DN
4	DN	DN

For the purpose of the derivation, it is convenient to view each three dimensional array as a series of two dimensional matrices projected in the third dimension. The reference frame 1 may, for example, be regarded as a series of two dimensional matrices M (obtained by combining M_a and M_b), there being one such two dimensional matrix for each margin state in M_c. The boundary of any effective cumulative margin state can be defined by the equations described below.

Reference Frame 1

$$m_{ijn} = M_{ai} + h_{ij} + h_{cn}$$

$$= m_{ij} + h_{cn}$$

where

m_{ijn} = The effective margin state in System A with m_{ai}, m_{bj} and m_{cn} as the unaffected margin states in the Systems A, B and C respectively

h_{cn} = The help available from System C at its nth margin state

$$= \min\ (m_{cn}, T_{ac})$$

$$= 0 \text{ if } m_{cn} \text{ is negative}$$

and

T_{ac} = The capability of the tie line A - C

Reference Frame 2

$$m_{ijn} = m_{ai} + h_{cn}$$

Reference Frame 3

$$m_{ijn} = m_{ai} + h_{ij}$$

Reference Frame 4

$$m_{ijn} = m_{ai}$$

These equations can be condensed into the form:

$$m_{ijn} = m_{ai} + b.h_{ij} + c.h_{cn} \tag{3.15}$$

where

$b = 1$ for the tie line A - B up

$= 0$ for the tie line A - B down

and

$c = 1$ for the tie line A - C up

$= 0$ for the tie line A - C down

The Availability:

Let

A_{xn} = The availability of the failure state, given the reference frame x and margin m_{cn} in System C

$$= \sum_{L_x, K_x} (A_{a(l)} - A_{a(l+1)}) \; A_{b(k)}$$

L_x, K_x = The indices defining in the reference frame x, the corner states of the boundary of the failure state

$A_{a(l)}$ = The availability of M_{al}, i.e., a margin equal to or less than m_{al} in System A

$A_{b(k)}$ = The availability of M_{bk}, i.e. a margin equal to or less than m_{bk} in System B

$A'_{c(n)}$ = The availability of m_{cn}

and

A_{ac} = The availability of the tie line A - C.

The expression for the availability is a straight forward summation:

$$A_{-} = \sum_{n=1}^{NC} A_{1n} \cdot A'_{c(n)} \cdot A_{ab} \cdot A_{ac}$$ The contribution of the reference frame 1

$$+ \sum_{n=1}^{NC} A_{2n} \cdot A'_{c(n)} \cdot \bar{A}_{ab} \cdot A_{ac}$$ The contribution of the reference frame 2

$$+ \sum_{n=1}^{NC} A_{3n} \cdot A'_{c(n)} \cdot A_{ab} \cdot \bar{A}_{ac}$$ The contribution of the reference frame 3

$$+ \sum_{n=1}^{NC} A_{4n} \cdot A'_{c(n)} \cdot A_{ab} \cdot \bar{A}_{ac}$$ The contribution of the reference frame 4

where

NC = The total number of discrete levels in the margin state matrix of System C

Some of the terms in the above expression can be further simplified:

$$A_{-} = \sum_{n=1}^{NC} A_{1n} \cdot A'_{c(n)} \cdot A_{ab} \cdot A_{ac} + \sum_{n=1}^{NC} A_{a(U)} \cdot A'_{c(n)} \cdot \bar{A}_{ab} \cdot A_{ac}$$

$$+ A_{31} \cdot A_{ab} \cdot \bar{A}_{ac} + A_{a(FM)} \cdot \bar{A}_{ab} \cdot \bar{A}_{ac} \qquad (3.16)$$

where

$A_{a(U)}$ = The cumulative availability of the margin state in M_a corresponding to the uppermost corner of the boundary of the failure state in the reference frame 1

and

$A_{a(FM)}$ = The cumulative availability of the first negative margin in M_a

The frequency:

System A can transit from one effective margin state to another in any of the following modes:

1. The capacity or the load transitions in System A
2. The capacity or the load transitions in System B
3. The capacity or the load transitions in System C
4. The failure or the repair of the tie line A - B
5. The failure or the repair of the tie line A - C

The contributions to the frequency of failure by the modes listed above are evaluated as follows.

(a) The contribution due to modes "1" and "2"

Reference Frame 1

$$f_{c10} = \sum_{n=1}^{NC} f_{1n} \cdot A'_{c(n)} \cdot A_{ab} \cdot A_{ac}$$

where

f_{cx0} = The contribution to the frequency of encountering the failure state in System A by the capacity or load transitions in System A or System B in reference frame x

and

f_{1n} = The frequency of encountering the failure state in System A by modes 1 and 2, given the reference frame 1 and the margin m_{cn} in System C

$$= \sum_{\substack{L1,K1 \\ \text{at } n}} \left[(f_{a(l)} - f_{a(l+1)}) A_{b(k)} + (A_{a(l)} - A_{a(l+1)}) f_{b(k)} \right] \tag{3.17}$$

As indicated earlier (L1, L2 at n) are the indices of the corners of the boundary of the failure state in the reference frame 1, with the margin in System C as m_{cn}.

Reference Frame 2

$$f_{c20} = \sum_{n=1}^{NC} f_{2n} \cdot A'_{c(n)} \cdot \bar{A}_{ab} \cdot A_{ac}$$

where

f_{2n} = The frequency of encountering the failure state in System A by the modes 1 and 2, given the reference frame 2 and the margin m_{cn} in System C

$$= \sum_{\substack{L2,K2 \\ \text{at } n}} \left[(f_{a(l)} - f_{a(l+1)}) A_{b(k)} + (A_{a(l)} - A_{a(l+1)}) f_{b(k)} \right]$$

$$= \sum_{\substack{L2,K2 \\ \text{at } n}} (f_{a(l)} - f_{a(l+1)}) \tag{3.18}$$

$$= f_{a(U)}$$

where

$f_{a(U)}$ = The frequency of the cumulative margin state in M_a corresponding to the uppermost corner of the failure state in the reference frame 1.

Reference Frame 3

$$f_{c30} = \sum_{n=1}^{NC} f_{3n} \cdot A'_{c(n)} \cdot A_{ab} \cdot \bar{A}_{ac}$$

where

f_{3n} = The frequency of encountering the failure state in System A by the modes 1 and 2, given reference frame 3 and the margin m_{cn}

$$= \sum_{\substack{L3,K3 \\ \text{at } n}} \left[(f_{a(l)} - f_{a(l+1)}) A_{b(k)} + (A_{a(l)} - A_{a(l+1)}) f_{b(k)} \right] \tag{3.19}$$

= The same at all values of n as help from System C is rendered ineffective due to the failure of the tie line A-C.

Therefore

$$f_{c30} = f_{3n} \cdot A_{ab} \cdot \bar{A}_{ac}$$

where n may be any margin state in System C

Reference Frame 4

$$f_{c40} = \sum_{n=1}^{NC} f_{4n} \cdot A'_{c(n)} \cdot \bar{A}_{ab} \cdot \bar{A}_{ac}$$

where

f_{4n} = The frequency of encountering the failure state in System A by the modes 1 and 2, given reference frame 4 and the margin m_{cn}

$$= \sum_{\substack{L4,K4 \\ \text{at } n}} \left[(f_{a(l)} - f_{a(l+1)}) A_{b(k)} + (A_{a(l)} - A_{a(l+1)}) f_{b(k)} \right]$$

$$= \sum_{\substack{L4,K4 \\ \text{at } n}} (f_{a(l)} - f_{a(l+1)}) \qquad (3.20)$$

= The same at all values of n since no help is available from System C

$= f_{a(FM)}$

= The frequency of encountering the first cumulative negative margin in System A as no help is available from System B

(b) The contribution due to mode 3

Reference Frame 1

$$f_{c11} = \sum_{n=1}^{NC} A_{1n} (f_{c(n)} - f_{c(n+1)}) A_{ab} \cdot A_{ac} \qquad (3.21)$$

where

f_{cx1} = The contribution to the frequency of encountering the failure state by mode 3 in reference frame x

Reference Frame 2

$$f_{c21} = \sum_{n=1}^{NC} A_{2n}(f_{c(n)} - f_{c(n+1)})\bar{A}_{ab}.A_{ac} \qquad (3.22)$$

where

$$A_{2n} = A_{a(U)}$$

Reference Frame 3

$$f_{c31} = \sum_{n=1}^{NC} A_{3n}(f_{c(n)} - f_{c(n+1)})A_{ab}.\bar{A}_{ac}$$

$$= A_{3n}.A_{ab}.\bar{A}_{ac} \sum_{n=1}^{NC} (f_{c(n)} - f_{c(n+1)})$$

$$= 0$$

Reference Frame 4

$$f_{c41} = \sum_{n=1}^{NC} A_{4n}(f_{c(n)} - f_{c(n+1)})\bar{A}_{ab}.\bar{A}_{ac}$$

$$= A_{4n}.\bar{A}_{ab}.\bar{A}_{ac} \sum_{n=1}^{NC} (f_{c(n)} - f_{c(n+1)})$$

$$= 0$$

(c) The contribution due to mode 4

Reference frame 1 to reference frame 2

The contribution associated with the deserting states is given by

$$f_{c1-2} = \sum_{n=1}^{NC} A'_{c(n)} (A_{2n}-A_{1n}) A_{ac} \cdot A_{ab} \cdot \lambda_{ab} \tag{3.23}$$

Reference frame 3 to reference frame 4

$$f_{c3-4} = \sum_{n=1}^{NC} A'_{c(n)} (A_{4n}-A_{3n}) \bar{A}_{ac} \cdot A_{ab} \cdot \lambda_{ab} \tag{3.24}$$

(d) The contribution due to mode 5

Reference frame 1 to reference frame 3

$$f_{c1-3} = \sum_{n=1}^{NC} A'_{c(n)} (A_{3n}-A_{1n}) A_{ab} \cdot A_{ac} \cdot \lambda_{ac} \tag{3.25}$$

Reference frame 2 to reference frame 4

$$f_{c2-4} = \sum_{n=1}^{NC} A'_{c(n)} (A_{4n}-A_{2n}) \bar{A}_{ab} \cdot A_{ac} \cdot \lambda_{ac} \tag{3.26}$$

The frequency of encountering the failure state in System A is given by the summation of all these contributions:

$$f_{-} = \sum_{i=1}^{4} (f_{ci0}+f_{ci1}) + f_{c1-2} + f_{c1-3} + f_{c2-4} + f_{c3-4} \tag{3.27}$$

The indirect approach employed in the two area problem can be extended to this case in a similar manner. It may be noted that the above procedure can be conveniently adopted to a digital computer.

3.3.2 Technique 2

This is a very convenient technique for the evaluation of the availability and the frequency of the effective capacity reserve margins in System A connected to two (as shown in Figure 3.5) or more than two systems. The effective capacity reserve model of System A connected to System B is developed first and this is then modified by successive reactions with the other systems using equations 3.3 and 3.6. The evaluation of the availability and the frequency of the failure state by this technique is as follows.

The effective margin state matrices for System A connected to Systems B and C are shown in Figure 3.7. The column vector M_{ab} represents the effective negative margin states in System A connected to System B. The boundary of the failure state in matrices MM and MM′ can be determined using the equations

$$mm_{ij} = m_{abi} + hc_{ij}$$

and

$$mm'_{ij} = m_{abi}$$

where

hc_{ij} = The help available from System C at (i,j)

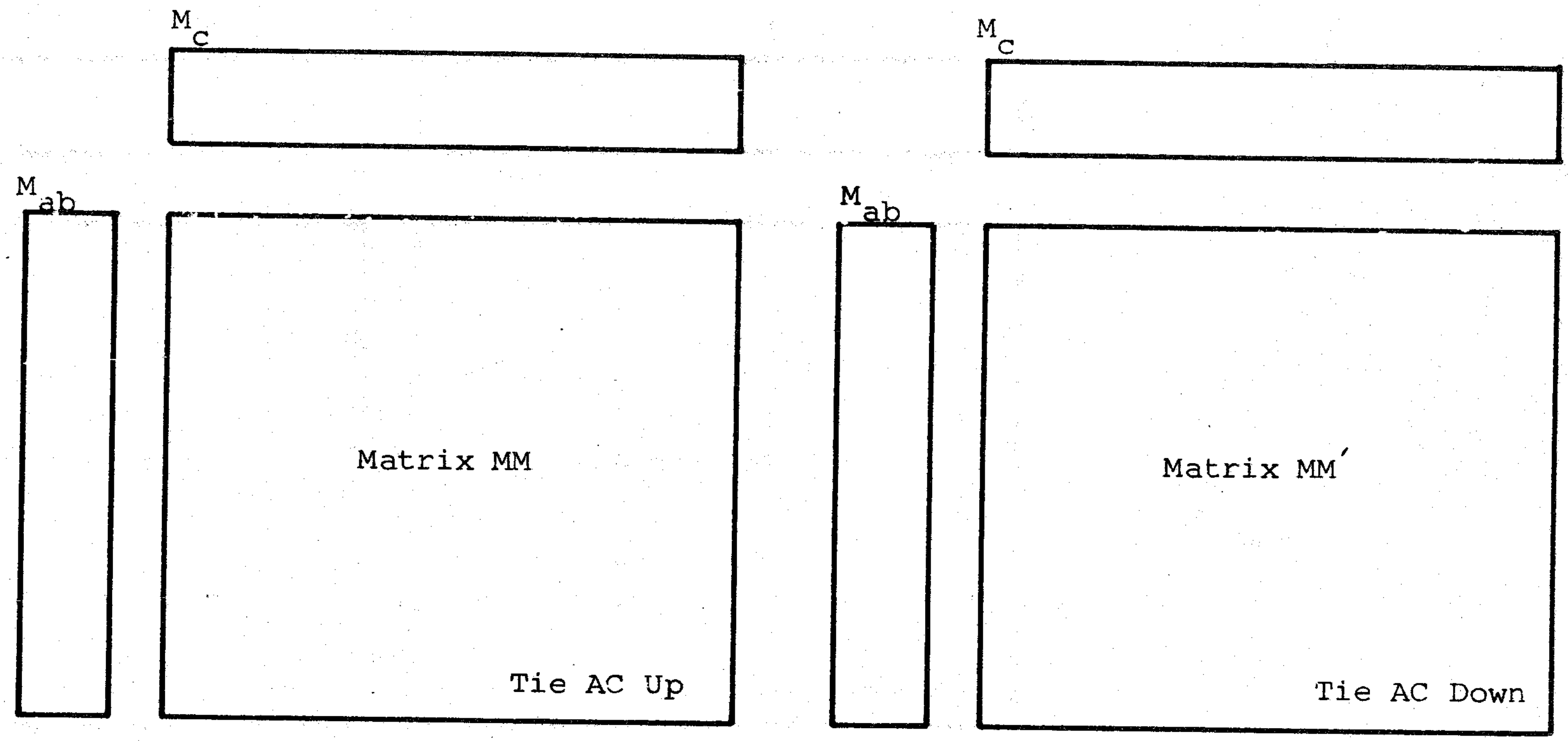


Figure 3.7: The Effective Margin States Matrices of System A Connected to Systems B and C

$$= \min (m_{cj}, T_{ac})$$

$$= 0 \quad \text{if } m_{cj} \text{ is negative}$$

The equations for the availability and the frequency of the failure state are given below:

$$A_- = \sum_{P,N} (A_{ab(p)} - A_{ab(p+1)})(A_{ac} \cdot A_{c(n)} + \bar{A}_{ac}) \tag{3.28}$$

and

$$f_- = \sum_{P,N} \Big[(f_{ab(p)} - f_{ab(p+1)})(A_{ac} \cdot A_{c(n)} + \bar{A}_{ac}) + (A_{ab(p)} - A_{ab(p+1)})(f_{c(n)} + (1 - A_{c(n)})\lambda_{ac}) A_{ac} \Big] \tag{3.29}$$

where

$A_{ab(p)}, f_{ab(p)}$ = The availability and the frequency of a margin state equal to or less than m_{abp}

$A_{c(n)}, f_{c(n)}$ = The availability and the frequency of a margin state equal to or less than m_{cn}

λ_{ac}, A_{ac} = The mean failure rate and the availability of tie line A-C

and

P,N define the indices p,n of the corners of the boundary of the failure state in matrix MM.

By suitable manipulation it can be shown that equations 3.28, 3.29 and 3.16, 3.27 are equivalent.

Equation 3.29 can be rewritten as

$$f_{-} = f_{1-}+f_{2-}+f_{3-}+f_{4-}$$

where

$$f_{1-} = \sum_{P,N} (f_{ab(p)}-f_{ab(p+1)})A_{ac} \cdot A_{c(n)}$$

$$f_{2-} = \sum_{P,N} (f_{ab(p)}-f_{ab(p+1)})\bar{A}_{ac}$$

$$f_{3-} = \sum_{P,N} (A_{ab(p)}-A_{ab(p+1)})A_{ac} \cdot f_{c(n)}$$

$$f_{4-} = \sum_{P,N} (A_{ab(p)}-A_{ab(p+1)})A_{ac} \cdot \lambda_{ac}(1-A_{c(n)})$$

The expression for f_{1-} can be re-written as

$$f_{1-} = \sum_{n=1}^{NC} f_{ab(P)} \cdot A'_{c(n)} \cdot A_{ac}$$

The summation is over all the values of n in M_c and P defines the corresponding state in M_{ab} on the boundary of the failure state in matrix MM.

$$f_{ab(P)} = \sum_{L,K} \Big[(f_{a(l)}-f_{a(l+1)})(A_{ab} \cdot A_{b(k)}+\bar{A}_{ab})$$

$$+ (A_{a(l)}-A_{a(l+1)})(f_{b(k)}+\lambda_{ab}(1-A_{b(k)}))A_{ab}\Big]$$

L,K define the indices l,k of the corner states of the boundary of M_{abP} ($\leq m_{abP}$) in the matrix M. It can be visualized that the boundary of M_{abP} in M and M′ is the same as that of the failure state in the two dimensional matrices in reference frames 1 and 2 at the margin m_{cn}. The expression for $f_{ab(P)}$ can be rewritten as

$$f_{ab(P)} = f^1_{ab(P)} + f^2_{ab(P)} + f^3_{ab(P)}$$

where

$$f^1_{ab(P)} = \sum_{L,K} \left[(f_{a(l)} - f_{a(l+1)}) A_{b(k)} + (A_{a(l)} - A_{a(l+1)}) f_{b(k)} \right] A_{ab}$$

$$f^2_{ab(P)} = \sum_{L,K} (f_{a(l)} - f_{a(l+1)}) \bar{A}_{ab}$$

and

$$f^3_{ab(P)} = \sum_{L,K} (A_{a(l)} - A_{a(l+1)}) (1 - A_{b(k)}) \lambda_{ab} \cdot A_{ab}$$

From equation 3.17

$$f^1_{ab(P)} = f_{1n} \cdot A_{ab}$$

From equation 3.18

$$f^2_{ab(P)} = f_{2n} \cdot \bar{A}_{ab}$$

and from equation 3.24

$$f^3_{ab(P)} = (A_{2n}-A_{1n})\,\lambda_{ab}\cdot A_{ab}$$

Substituting these values in the expression for f_{1-}

$$f_{1-} = \sum_{n=1}^{NC}\left[f_{1n}\cdot A_{ab}+f_{2n}\cdot \bar{A}_{ab}+(A_{2n}-A_{1n})\,\lambda_{ab}\cdot A_{ab}\right]A'_{c(n)}\cdot A_{ac}$$

$$= \sum_{n=1}^{NC} f_{1n}\cdot A_{ab}\cdot A_{ac}\cdot A'_{c(n)} + \sum_{n=1}^{NC} f_{2n}\cdot \bar{A}_{ab}\cdot A'_{c(n)}\cdot A_{ac}$$

$$+ \sum_{n=1}^{NC}(A_{2n}-A_{1n})A'_{c(n)}\cdot A_{ac}\cdot A_{ab}\cdot \lambda_{ab}$$

$$= f_{c10} + f_{c20} + f_{c1-2}$$

The expression for f_{2-} can be rewritten as

$$f_{2-} = \sum_{n=1}^{NC} f_{ab(P)}\cdot A'_{c(n)}\cdot \bar{A}_{ac}$$

The summation is over all the values of n in M_c and P defines the corresponding margin state in M_{ab} on the boundary of the failure state in matrix MM.

$$f_{ab(P)} = f^4_{ab(P)} + f^5_{ab(P)} + f^6_{ab(P)}$$

where

$$f^4_{ab(P)} = \sum_{L,K}\left[(f_{a(l)}-f_{a(l+1)})A_{b(k)}+(A_{a(l)}-A_{a(l+1)})f_{b(k)}\right]A_{ab}$$

$$f^{5}_{ab(P)} = \sum_{L,K} (f_{a(l)} - f_{a(l+1)}) \bar{A}_{ab}$$

and

$$f^{6}_{ab(P)} = \sum_{L,K} (A_{a(l)} - A_{a(l+1)})(1 - A_{b(k)}) \lambda_{ab} \cdot A_{ab}$$

The summation is over the boundary of M_{abP} in the matrix M. The boundary of M_{abP} in M and M' is the same as that of the failure state in the two dimensional matrices in the reference frames 3 and 4 at m_{cn}.

Therefore,

From equation 3.19

$$f^{4}_{ab(P)} = f_{3n} \cdot A_{ab}$$

From equation 3.20

$$f^{5}_{ab(P)} = f_{4n} \cdot \bar{A}_{ab}$$

and

$$f^{6}_{ab(P)} = (A_{4n} - A_{3n}) \lambda_{ac} \cdot A_{ab}$$

Substituting in f_{2-}

$$f_{2-} = \sum_{n=1}^{NC} f_{3n} \cdot A_{ab} \cdot A'_{c(n)} \cdot \bar{A}_{ac} + \sum_{n=1}^{NC} f_{4n} \cdot \bar{A}_{ab} \cdot A'_{c(n)} \cdot \bar{A}_{ac}$$

$$+ \sum_{n=1}^{NC} (A_{4n} - A_{3n}) A_{ab} \cdot \lambda_{ab} \cdot A'_{c(n)} \cdot \bar{A}_{ac}$$

$$= f_{c30} + f_{c40} + f_{c3-4}$$

The expression for f_{3-} can be rewritten as

$$f_{3-} = \sum_{P,N} A_{ab(p)}(f_{c(n)} - f_{c(n+1)})A_{ac}$$

$$A_{ab(P)} = \sum_{L,K} (A_{a(l)} - A_{a(l+1)})A_{b(k)} \cdot A_{ab} \qquad \text{when AB is up}$$

$$+ \sum_{L,K} (A_{a(l)} - A_{a(l+1)})\bar{A}_{ab} \qquad \text{when AB is down}$$

$$= A_{1n} \cdot A_{ab} + A_{2n} \cdot \bar{A}_{ab}$$

Substituting back in the expression for f_{3-}

$$f_{3-} = \sum_{n=1}^{NC} A_{1n}(f_{c(n)} - f_{c(n+1)})A_{ab} \cdot A_{ac}$$

$$+ \sum_{n=1}^{NC} A_{2n}(f_{c(n)} - f_{c(n+1)})\bar{A}_{ab} \cdot A_{ac}$$

$$= f_{c11} + f_{c21}$$

And finally

$$f_{4-} = A_{ac} \cdot \lambda_{ac} \sum_{P,N} (A_{ab(p)} - A_{ab(p+1)})(1 - A_{c(n)})$$

$$= A_{ac} \cdot \lambda_{ac} \sum_{n=1}^{NC} A'_{c(n)}(A_{ab(P')} - A_{ab(P)})$$

where P and P$'$ define the boundary of the failure state in MM and MM$'$ respectively, corresponding to each m_{cn}.

Therefore

$$f_{4-} = A_{ac} \cdot \lambda_{ac} \sum_{n=1}^{NC} A'_{c(n)} \Big[\sum_{L3,K3} (A_{a(l)} - A_{a(l+1)}) A_{b(k)} \cdot A_{ab}$$

$$+ \sum_{L4,K4} (A_{a(l)} - A_{a(l+1)}) A_{b(k)} \cdot \bar{A}_{ab} - \sum_{L1,K1} (A_{a(l)} - A_{a(l+1)})$$

$$A_{b(k)} \cdot A_{ab} - \sum_{L2,K2} (A_{a(l)} - A_{a(l+1)}) A_{b(k)} \cdot \bar{A}_{ab} \Big]$$

$$= A_{ac} \cdot \lambda_{ac} \cdot A_{ab} \sum_{n=1}^{NC} A'_{c(n)} \Big[\sum_{L3,K3} (A_{a(l)} - A_{a(l+1)}) A_{b(k)}$$

$$- \sum_{L1,K1} (A_{a(l)} - A_{a(l+1)}) A_{b(k)} \Big] + A_{ac} \cdot \lambda_{ac} \cdot \bar{A}_{ab} \sum_{n=1}^{NC} A'_{c(n)}$$

$$\Big[\sum_{L4,K4} (A_{a(l)} - A_{a(l+1)}) A_{b(k)} - \sum_{L2,K2} (A_{a(l)} - A_{a(l+1)}) A_{b(k)} \Big]$$

$$= A_{ac} \cdot \lambda_{ac} \cdot A_{ab} \sum_{n=1}^{NC} (A_{3n} - A_{1n}) + A_{ac} \cdot \lambda_{ac} \cdot \bar{A}_{ab} \sum_{n=1}^{NC} (A_{4n} - A_{2n})$$

$$= f_{c1-3} + f_{c2-4}$$

Substituting the values of f_{1-}, f_{2-}, f_{3-} and f_{4-}

into the expression for f_-

$$f_- = \sum_{i=1}^{4} (f_{ci0} + f_{ci0}) + f_{c1-2} + f_{c1-3} + f_{c2-4} + f_{c3-4}$$

which is the same as expression 3.27

Equivalence of the availability equations:

$$A_- = \sum_{P,N} (A_{ab(p)} - A_{ab(p+1)})(A_{ac} \cdot A_{c(n)} + \bar{A}_{ac})$$

$$= \sum_{n=1}^{NC} A_{ab(P)} \cdot A'_{c(n)} \cdot A_{ac} + \sum_{n=1}^{NC} A_{ab(P)} \cdot A'_{c(n)} \cdot \bar{A}_{ac}$$

$$= \sum_{n=1}^{NC} A'_{c(n)} \cdot A_{ac} \Big[\sum_{L1,K1} (A_{a(l)} - A_{a(l+1)}) A_{b(k)} \cdot A_{ab}$$

$$+ \sum_{L2,K2} (A_{a(l)} - A_{a(l+1)}) \bar{A}_{ab} \cdot A_{b(k)} \Big]$$

$$+ \sum_{n=1}^{NC} A'_{c(n)} \cdot \bar{A}_{ac} \Big[\sum_{L3,K3} (A_{a(l)} - A_{a(l+1)}) A_{b(k)} \cdot A_{ab}$$

$$+ \sum_{L4,K4} (A_{a(l)} - A_{a(l+1)}) A_{b(k)} \cdot \bar{A}_{ab} \Big]$$

$$= \sum_{n=1}^{NC} A_{1n} \cdot A'_{c(n)} \cdot A_{ab} \cdot A_{ac} + \sum_{n=1}^{NC} A_{2n} \cdot A'_{c(n)} \cdot A_{ac} \cdot \bar{A}_{ab}$$

$$+ \sum_{n=1}^{NC} A_{3n} \cdot A'_{c(n)} \cdot A_{ab} \cdot \bar{A}_{ac} + \sum_{n=1}^{NC} A_{4n} \cdot \bar{A}_{ac} \cdot \bar{A}_{ab} \cdot A'_{cn}$$

which is the same as expression 3.16

Thus either of the techniques may be employed for the reliability evaluation of System A connected to Systems B and C. In the case of System A connected to more than two systems, Technique 1 becomes unwieldy and Technique 2 can be conveniently employed.

3.3.3 Example

In this example, the frequency of encountering failure in System A connected to two identical systems B and C has been calculated by both the techniques. The description of each system and the interconnection is:

The Generation System[7]:

Unit #	Capacity MW	Mean Repair Rate Per Day	Mean Failure Rate Per Day
1	20	0.49	0.01
2	30	0.49	0.01

The Load Model[8]

State #	Load Level M.W	No. of Occurrences
1	40	2
2	25	5

continued:

State #	Load Level M.W	No. of Occurrences
3	20	8
4	15	5
0	0	20

Exposure factor = 0.5 day

The Tie Line Data:

Tie From	Line To	Capability MW	Mean Failure Rate Per day	Mean Repair Rate Per day
A	B	15.0	0.01	0.49
A	C	10.0	0.01	0.49

The required segment of capacity reserve model of each system is shown in Table 3.1. The contribution due to the low load level (assumed = zero MW) has been taken into account.

Technique 1

The help from System C is limited by the capability of the tie line A-C and therefore there will be three basic configurations of the two dimensional matrix, in each of the reference frames 1 and 2 corresponding to the following ranges of the margin states in System C:

1. 50 MW to 10 MW
2. 5 MW
3. 0 MW or less.

TABLE 3.1

The Required Segment of the Capacity Reserve

Model of Identical Systems A, B and C

Margin MW	Availability of Cumulative State	Frequency of Cumulative State Per day
50	1.0000000×10^{0}	0.0000000×10^{0}
15	0.6801993×10^{-1}	0.1440794×10^{0}
10	0.6556994×10^{-1}	0.1380035×10^{0}
5	0.1362999×10^{-1}	0.3320238×10^{-1}
0	0.8729991×10^{-2}	0.2105038×10^{-1}
-5	0.4609991×10^{-2}	0.1153278×10^{-1}
-10	0.2159998×10^{-2}	0.5456783×10^{-2}
-15	0.1179999×10^{-2}	0.3026397×10^{-2}
-20	0.1129999×10^{-2}	0.2877398×10^{-2}
-25	0.6999995×10^{-4}	0.2085998×10^{-3}
-40	0.1999998×10^{-4}	0.5959993×10^{-4}

With no help available from System C, there will be only one type of configuration in each of the reference frames 3 and 4. These different configurations are shown in Figures 3.8a to 3.8c.

By calculation:

$$f_{c10} + f_{c20} + f_{c30} + f_{c40} = 0.29636059 \times 10^{-3}$$

$$f_{c11} = 0.24132682 \times 10^{-4}$$

$$f_{c12} = 0.16485864 \times 10^{-5}$$

$$f_{c1-2} = 0.11199958 \times 10^{-4}$$

$$f_{c3-4} = 0.67462575 \times 10^{-6}$$

$$f_{c1-3} = 0.10751249 \times 10^{-4}$$

$$f_{c2-4} = \underline{0.66546818 \times 10^{-6}}$$

$$f_{-} = 0.34543316 \times 10^{-3} \quad \text{Per day}$$

Similarly for availability:

Contribution of the reference frame 1 = $0.46644936 \times 10^{-4}$

Contribution of the reference frame 2 = $0.23808985 \times 10^{-4}$

Contribution of the reference frame 3 = $0.22893253 \times 10^{-4}$

Contribution of the reference frame 4 = $\underline{0.18439951 \times 10^{-5}}$

A_{-} = $0.95191169 \times 10^{-4}$

M_a \ M_b	50	35 To 20	15	10	5	0
-5						
-10						0
-15					0	-5
-20				0	-5	
-25	0	0	0	-5		
-40	-15	-15	-15			

R.F.1

M_a \ M_b	50	35 To 20	15	10	5	0
-5					0	-5
-10				0	-5	
-15	0	0	0	-5		
-20	-5	-5	-5			
-25						
-40						

R.F.3

M_a \ M_b	50	35 To 20	15	10	5	0
-5						
-10	0	0	0	0	0	0
-15	-5	-5	-5	-5	-5	-5
-20						
-25						
-40						

R.F.2

M_a \ M_b	50	35 To 20	15	10	5	0
-5	-5	-5	-5	-5	-5	-5
-10						
-15						
-20						
-25						
-40						

R.F.4

Figure 3.8a: The Failure State Boundary in System A, m_c Varying from 50 MW to 10 MW

M_a \ M_b	50	35 To 20	15	10	5	0
-5						0
-10					0	-5
-15				0	-5	
-20	0	0	0	-5		
-25	-5	-5	-5			
-40						

R.F.1

M_a \ M_b	50	35 To 20	15	10	5	0
-5	0	0	0	0	0	0
-10	-5	-5	-5	-5	-5	-5
-15						
-20						
-25						
-40						

R.F.2

The Same As In
Figure 3.8a

R.F.3

The Same As In
Figure 3.8a

R.F.4

Figure 3.8b: The Failure State Boundary in System A, m_c Equals 5 MW

M_a \ M_b	50	35 To 20	15	10	5	0
-5					0	-5
-10				0	-5	
-15	0	0	0	-5		
-20	-5	-5	-5			
-25						
-40						

R.F.1

M_a \ M_b	50	35 To 20	15	10	5	0
-5	-5	-5	-5	-5	-5	-5
-10						
-15						
-20						
-25						
-40						

R.F.2

The Same As In
Figure 3.8a

R.F.3

The Same As In
Figure 3.8a

R.F.4

Figure 3.8c: The Failure State Boundary in System A, m_c Equals Zero MW or is Less

Technique 2

The two dimensional matrix MM is shown in Figure 3.9. The availabilities and the frequencies of the required effective cumulative negative margins in M_{ab} are given below.

Effective Margin in M_{ab} Equal to or Less Than	Availability	Frequency Per Day
-5 MW	$0.12368627 \times 10^{-2}$	$0.32674489 \times 10^{-2}$
-10 MW	$0.18896602 \times 10^{-3}$	$0.69275155 \times 10^{-3}$
-15 MW	$0.60999454 \times 10^{-4}$	$0.21890749 \times 10^{-3}$

The availability and frequency by this technique are calculated to be

$$A_{-} = 0.95191182 \times 10^{-4}$$

and

$$f_{-} = 0.34543316 \times 10^{-3} \quad \text{Per day}$$

These values agree with those calculated using technique 1.

3.4 System A Connected to Other Interconnected Systems

When the system whose reliability is being evaluated is connected to several interconnected systems, the problem requires careful analysis, which includes the establishment of priorities for emergency help. Some simplifying

M_{ab} \ M_c	50	35 .. 20	15	10	5	0
-5					0	-5
-10	0	0	0	0	-5	
-15	-5	-5	-5	-5		
-20						
-25						
-30						
-35						
-40						

Matrix MM′

$T_{ac} = 10$ MW

Figure 3.9: The Failure State Boundary in System A Using Technique 2

assumptions may also be required. To illustrate this situation, the configuration shown in Figure 3.10 has been selected. The following priority constraints have been assumed.

Constraints:

1. When both System A and System B need emergency help from System C, System B will get preference. Similarly when both System A and System C need help from System B, System C will get preference.
2. When the tie line AB is on forced outage, emergency help by System B will be supplied to System A via BC-CA. Similarly when tie line AC is on forced outage, emergency help by System C will be supplied to System A via CB-BA.
3. When the tie line AB is in, emergency help from System B will only be supplied via BA and similarly when AC is in, emergency help from System C will only be supplied via CA.

The reliability evaluation of System A is made in the following steps.

Step 1.

The availabilities and frequencies of the effective positive margins in System B, from the point of view of emergency help to System A, are determined. The effective margin

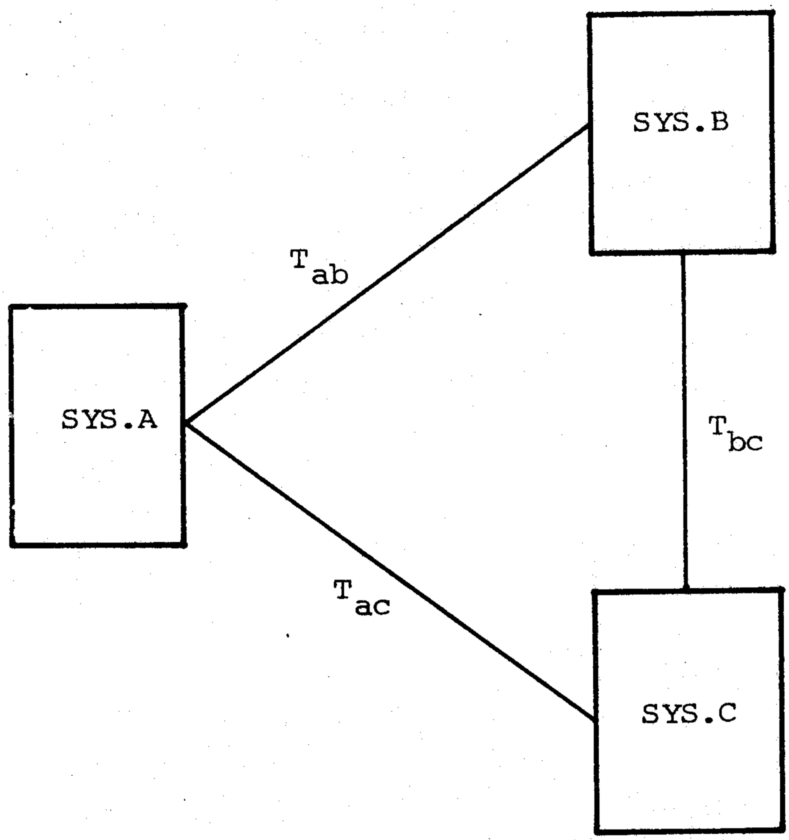


Figure 3.10: System A Connected to Two Other Interconnected Systems

state matrices for System B are shown in Figure 3.11. The boundaries of positive margins in matrices M and M' are defined using equations 3.1 and 3.2 respectively. The same equations are used to define the boundaries in matrices MX and MX' except that $h_{ij} = 0$ when no help is needed by System B, i.e., when m_{bi} is zero or more.

The equation for the frequency of a cumulative effective margin N, in System B is given as

$$f_N = \bar{A}_{ac} \sum_{L,K} f_Y + A_{ac} \sum_{LX,KX} f_Y + \sum_{LX,KX} (A_{b(l)} - A_{b(l+1)})(A_{c(k)} - A_{c(KK)})$$

$$A_{bc} \cdot A_{ac} \cdot \lambda_{ac}$$

where

L,K = The indices defining the corners of the boundary of the effective margin N in the matrix M

LX,KX = The indices defining the corners of the boundary of the effective margin N in the matrix MX

KK = The index defining the corner of the boundary of the effective margin N in the matrix M corresponding to LX

$A_{ac}, \bar{A}_{ac}$ = The availability and the unavailability of the tie line AC

λ_{ac} = The mean failure rate of the tie line AC

$$f_Y = (f_{b(l)} - f_{b(l+1)})(A_{bc} \cdot A_{c(k)} + \bar{A}_{bc}) + (A_{b(l)} - A_{b(l+1)})$$

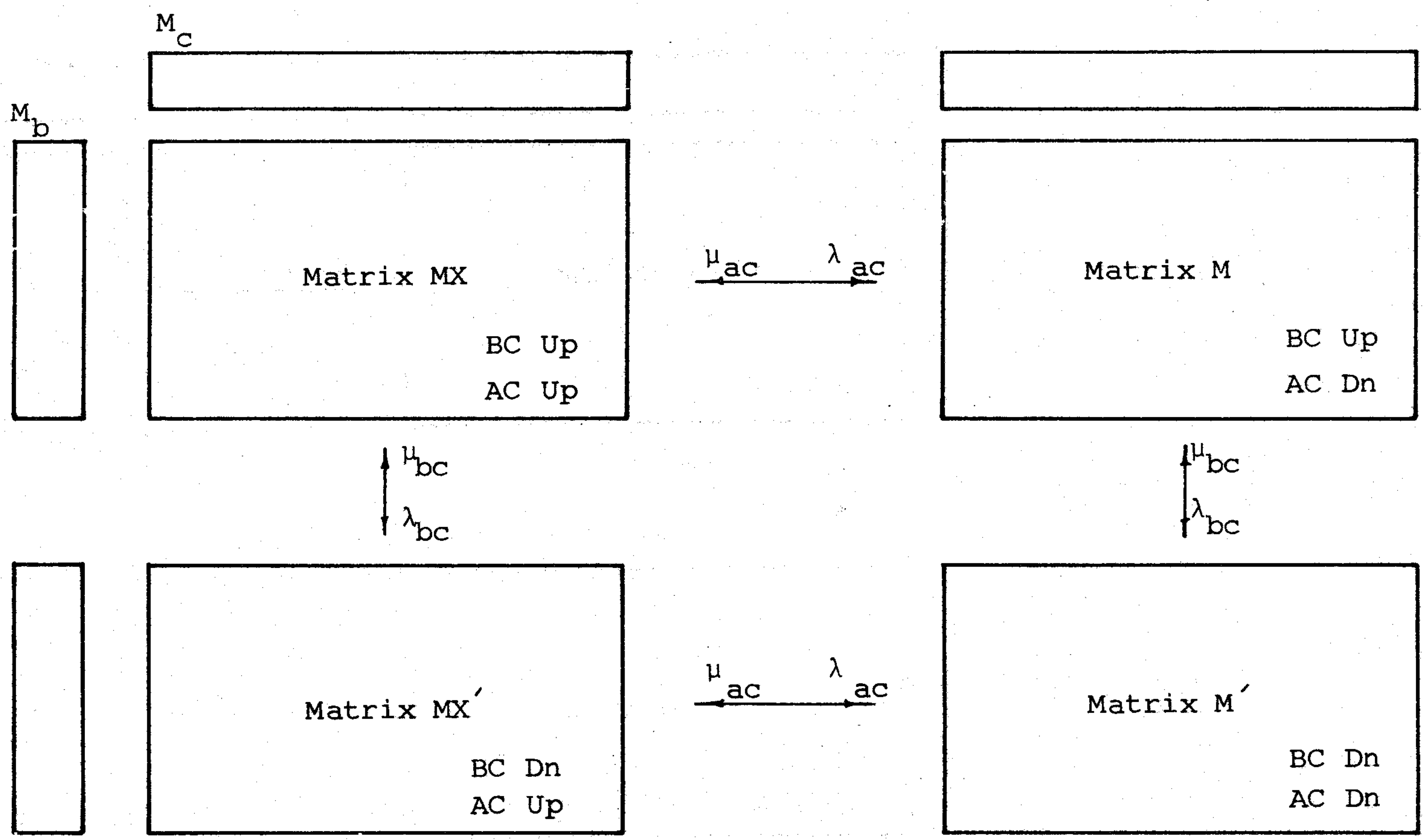


Figure 3.11: The Effective Margin States Matrices for System B

$$(f_{c(k)} + \beta(1-A_{c(k)})_{bc} + \gamma \cdot A_{c(k)} \cdot \lambda_{bc}) A_{bc}$$

This is equation 3.6 as applied to System B and System C.

Similarly the expression for the availability of a cumulative effective margin N, in System B is given as

$$A_N = \bar{A}_{ac} \sum_{L,K} A_Y + A_{ac} \sum_{LX,KX} A_Y$$

where

$$A_Y = (A_{b(1)} - A_{b(1+1)})(A_{bc} \cdot A_{c(k)} + \beta . \bar{A}_{bc})$$

This is equation 3.3 as applied to Systems B and C.

Step 2.

The availabilities and frequencies of the effective positive margins in System C from the point of view of emergency help to System A are evaluated in exactly the same manner.

Step 3.

The effective positive margins of System B are combined with the negative margins of System A to obtain the availabilities and frequencies of the effective negative margins of System A. The effective positive margins of System C are then combined with the effective negative margins of System A to obtain the availability and frequency of the failure state in System A.

3.5 The Interconnection System Studies

3.5.1 General

These studies were conducted with the intention of studying the characteristics of the Capacity Reserve Model of the hypothetical system SYS.RS connected to an identical system, under different sets of interconnection and system parameters. The base values of the mean failure rate and the mean repair rate of the tie line were assumed to be 0.01 and 2.5 per day respectively. The two identical systems have been designated as "A" and "B".

3.5.2 The Effect of Tie Capacity on the Risk Level in SYS.A

As the tie capacity is varied from zero, the two dimensional matrix M representing the effective capacity reserve model of System A, will continue to be modified until the tie capacity reaches the limiting value

$$LT_{ab} = \mathrm{Min}\ (M_{bm+}, M_{am-})$$

where

M_{bm+} = The largest positive margin in System B

$= IC_b - L_{bo}$

and M_{am-} = The largest of the absolute values of the negative margins in System A.

The availability and the frequency of the failure state will continue to decrease and the cycle time to increase until this limit is reached. Now

$$M_{am-} = \left| IC_a - \bar{c}_{amax} - L_{a1} \right|$$

where

$\bar{c}_{amax}$ = The maximum possible outage of capacity in System A.

Theoretically all the units in System A may be on forced outage and therefore,

$$M_{am-} = L_{a1}$$

In practice, however, the generation system model may be curtailed when the availability of capacity outage is less than a minimum specified value. In such a case

$\bar{c}_{amax}$ = The last significant outage of capacity.

In certain cases, the margin states having availabilities less than a minimum specified value may be neglected. In these cases

M_{am-} = |The last significant cumulative negative reserve margin|

The installed capacity of SYS.RS is 1725 MW and the peak level in both the systems was maintained at 1450 MW.

Thus

$$M_{bm+} = 1725-0$$
$$= 1725 \text{ MW}$$

and

$$M_{am-} = |1725-800-1450|$$
$$= 525 \text{ MW}$$

as the generation system model was curtailed at the capacity outage of 800 MW. Therefore,

$$LT_{ab} = \text{Min } (1725,525)$$
$$= 525 \text{ MW}$$

The study was carried out by varying the tie capacity from 25 MW to 625 MW. The mean failure and the mean repair rates were maintained at 1.0 p.u. each. The results of this study are portrayed-Figures 3.12 and 3.13. The last segment of these graphs is shown in tabular form below:

TABLE 3.2

The Effect of Tie Capacity on the Reliability Measures of System A

Tie Capacity MW	Availability	Cycle Time Days
475	0.1680375×10^{-4}	0.1375250×10^{5}
500	0.1680049×10^{-4}	0.1375328×10^{5}
525	0.1679803×10^{-4}	0.1375384×10^{5}

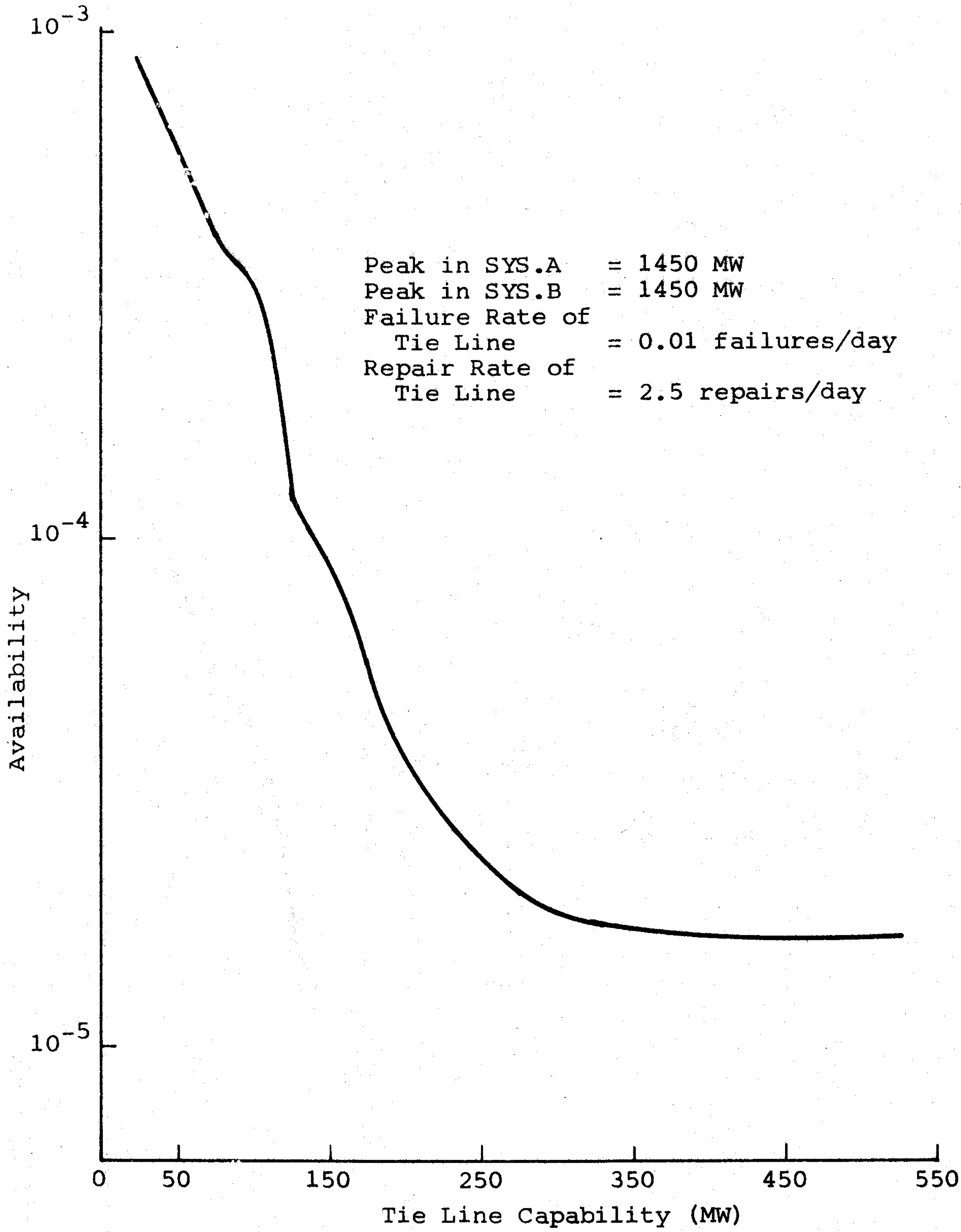


Figure 3.12: Variation of Risk Level (Availability) in SYS.A with the Variation of Tie Line Capability

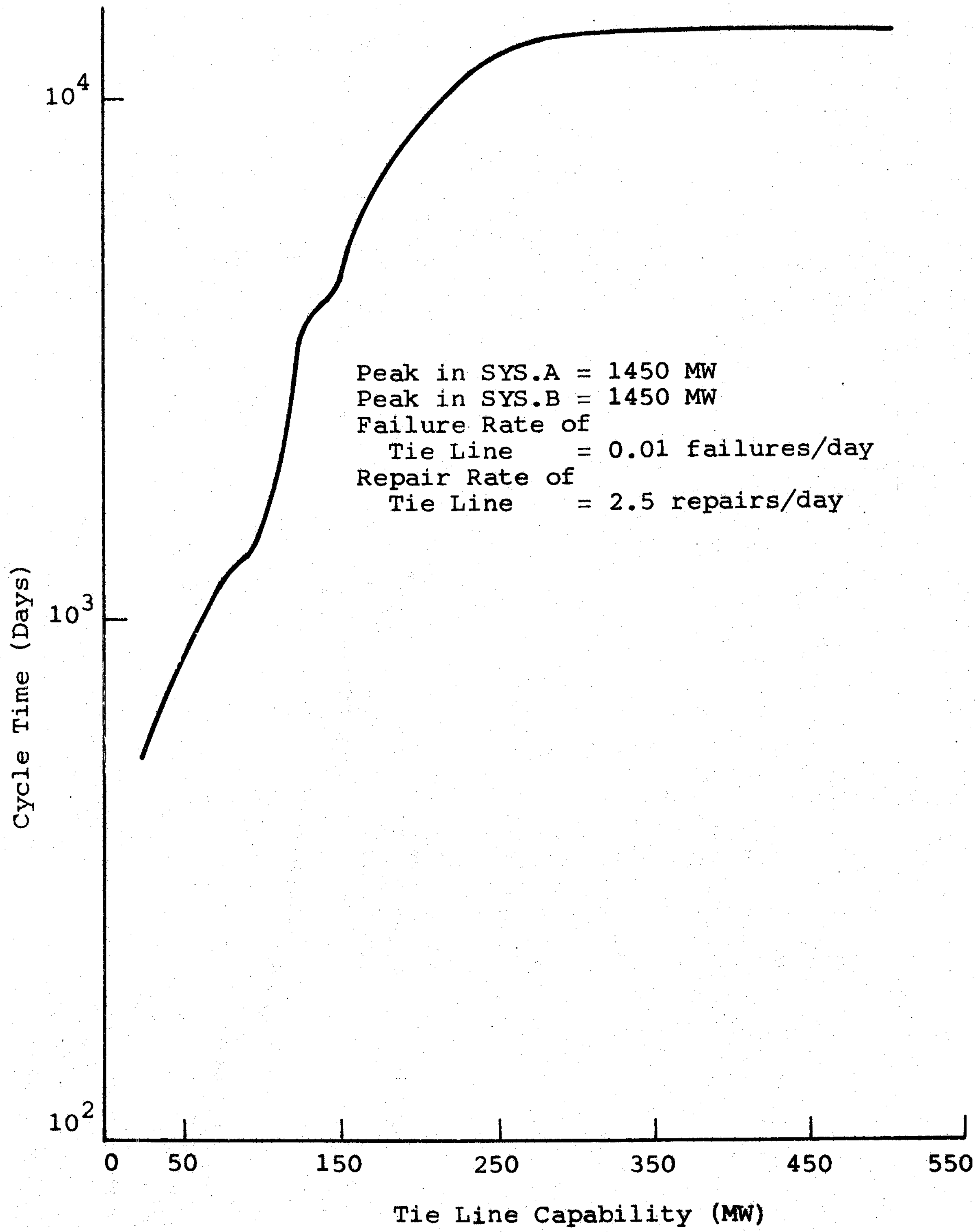


Figure 3.13: Variation of Risk Level (Cycle Time) in SYS.A with the Variation of Tie Line Capacity

550	0.1679803×10^{-4}	0.1375384×10^{5}
575	0.1679803×10^{-4}	0.1375384×10^{5}
600	0.1679803×10^{-4}	0.1375384×10^{5}
625	0.1679803×10^{-4}	0.1375384×10^{5}

It can be noted that there is no improvement in reliability beyond the predicted limiting value for the tie capacity, i.e., 525 MW. Also it can be observed from the graph that no significant improvement in reliability takes place beyond a tie capacity of 275 MW. This sets the practical limit for tie capacity which may be defined as the point after which

$$\frac{d(CT)}{dT_{ab}} < D_1$$

and

$$\frac{d(A)}{dT_{ab}} < D_2$$

where D_1 and D_2 are arbitrary constants depending on the acceptable risk level in System A.

3.5.3 The Effect of the Tie Line Mean Failure Rate and Mean Repair Rate on the Risk Level in System A

The peak load in both systems was held at 1450 MW and the tie capacity at 200 MW. The availability and the frequency of the failure state were computed under the following conditions:

1. Keeping μ_{ab} fixed at 2.5 per day (i.e. 1.0 P.U) and varying λ_{ab} from 0.005 per day (i.e. 0.5 P.U) to 0.03 per day (i.e. 3 P.U)

2. Keeping λ_{ab} fixed at 1.0 p.u and varying μ_{ab} from o.5 p.u to 3.0 p.u.

and

3. Varying both λ_{ab} and μ_{ab} from 0.5 p.u to 3.0 p.u.

The results of these studies are shown as the ratios of the base availability and the base cycle time in Figures 3.14 and 3.15. The base availability and the base cycle time were computed with λ_{ab} = 0.01 per day and μ_{ab} = 2.5 per day.

The variation in Availability:

If only λ_{ab} and μ_{ab} are varied, the expression for the availability of the failure state can be written as

$$A = C_1 \frac{\mu_{ab}}{\lambda_{ab}+\mu_{ab}} + C_2 \frac{\lambda_{ab}}{\lambda_{ab}+\mu_{ab}}$$

where C_1 and C_2 are constants corresponding to the contributions by the matrices M and M' respectively. Expressed in p.u of the base availability A_b,

$$A_{p.u} = \frac{C_1}{A_b} \cdot \frac{\mu_{ab}}{\lambda_{ab}+\mu_{ab}} + \frac{C_2}{A_b} \cdot \frac{\lambda_{ab}}{\lambda_{ab}+\mu_{ab}}$$

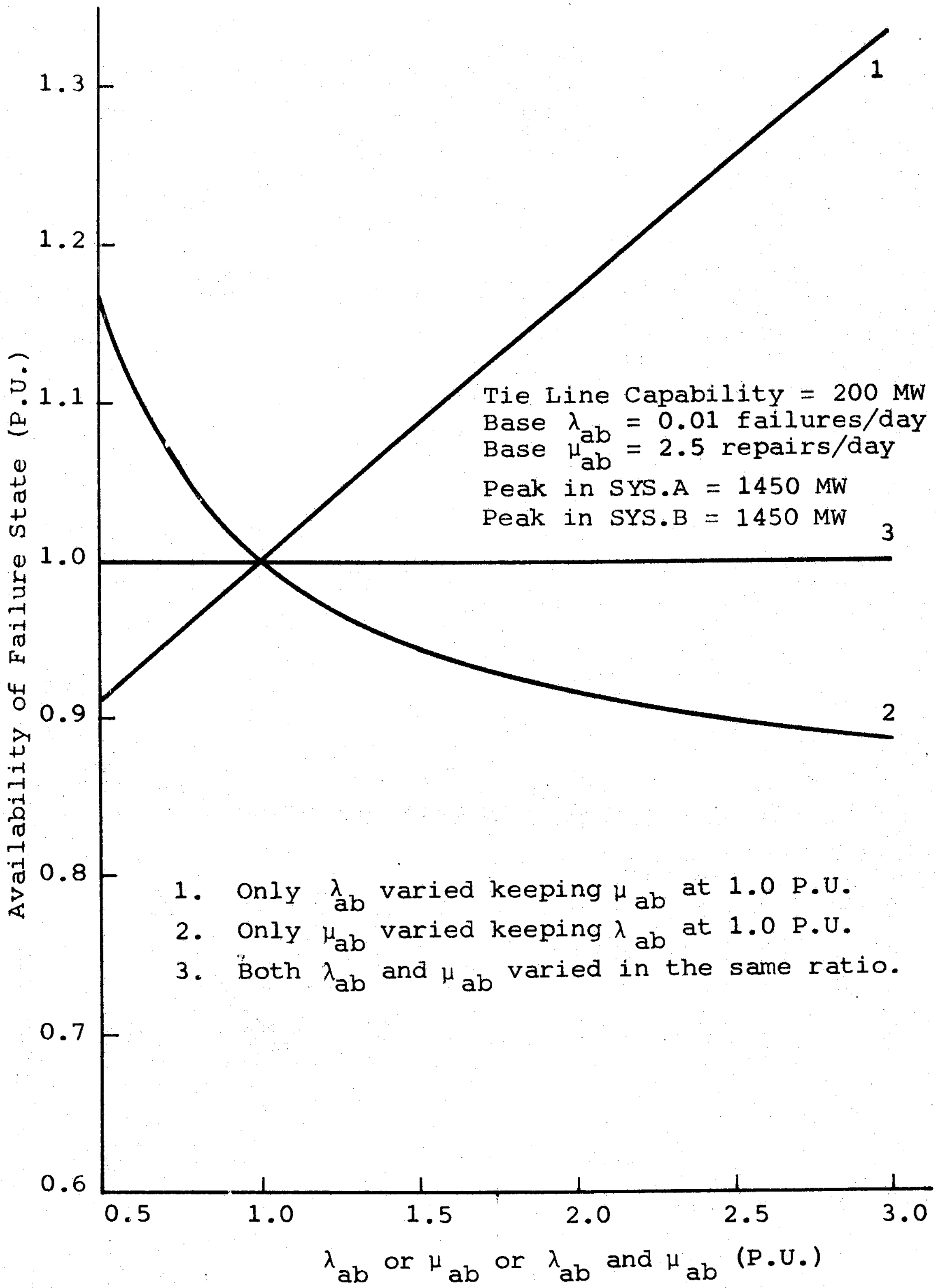


Figure 3.14: Variation in Risk Level (Availability) of SYS.A with Variations in Tie Line Mean Failure and Mean Repair Rates

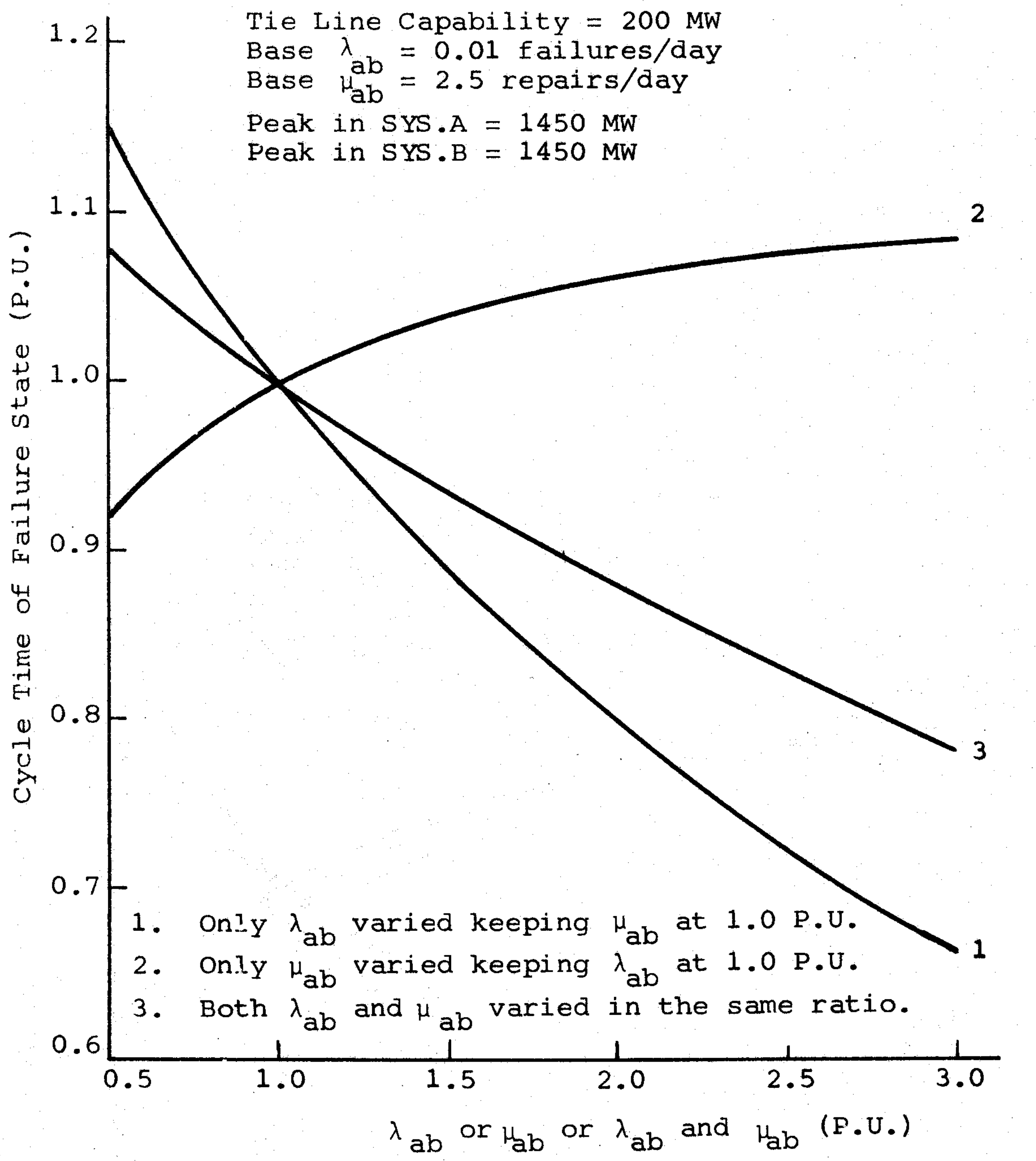


Figure 3.15: Variation in Risk Level (Cycle Time) of SYS.A with Variation in Tie Line Mean Failure and Mean Repair Rates

$$= \frac{K_1 \cdot \mu_{ab} + K_2 \cdot \lambda_{ab}}{\lambda_{ab} + \mu_{ab}}$$

where K_1 and K_2 is another set of constants.

If λ_{ab} is varied keeping μ_{ab} constant, the slope of the plot representing $A_{p.u}$ vs λ_{ab} will be given by

$$\frac{\partial A_{p.u}}{\partial \lambda_{ab}} = \frac{(K_2 - K_1)\mu_{ab}}{(\mu_{ab} + \lambda_{ab})^2}$$

$$= \frac{K_2 - K_1}{\mu_{ab}} \quad (1 - \frac{2\lambda_{ab}}{\mu_{ab}}) \qquad \text{if } \lambda_{ab} << \mu_{ab}$$

In actual practice λ_{ab} is usually very small compared with μ_{ab} and the slope is almost constant, i.e., in this range the availability of the failure state varies approximately linearly with the change in the mean failure rate of the tie line. The value of C_2 is independent of the tie capacity but C_1 decreases with an increase of tie capacity until the limiting value is reached. Therefore the greater the tie capacity, the greater is the sensitivity of "the availability of the failure state" to the mean failure rate of the tie, the limit being $\frac{\partial A}{\partial \lambda_{ab}}$ corresponding to the LT_{ab}.

In the second case where μ_{ab} is varied keeping λ_{ab} constant, the slope of the characteristic 2 is given by

$$\frac{\partial A_{p.u}}{\partial \mu_{ab}} = -\frac{K_2-K_1}{\mu_{ab}} \left(1 - 2\frac{\lambda_{ab}}{\mu_{ab}}\right)$$

Since K_2 is always greater than or equal to K_1, the slope is negative and decreases with an increase in the mean repair rate. Therefore, the availability of the failure state is inversely sensitive to the mean repair rate.

If, however, both λ_{ab} and μ_{ab} are varied in the same ratio, the availability of the failure state will not be affected as indicated by the characteristic 3 in Figure 3.14.

The Variation in the Frequency:

If only λ_{ab} and μ_{ab} are varied, the expression for the frequency of the failure state can be written as follows:

$$f_{-} = C_3 \frac{\mu_{ab}}{\mu_{ab}+\lambda_{ab}} + C_4 \frac{\lambda_{ab}}{\mu_{ab}+\lambda_{ab}} + (C_2-C_1)\frac{\mu_{ab}\cdot\lambda_{ab}}{\mu_{ab}+\lambda_{ab}}$$

Expressed in p.u of the base frequency

$$f_{-(p.u)} = \frac{K_3\cdot\mu_{ab}+K_4\cdot\lambda_{ab}+(K_2-K_1)\mu_{ab}\cdot\lambda_{ab}}{\mu_{ab}+\lambda_{ab}}$$

Since $\lambda_{ab} << \mu_{ab}$

$$f_{-(p.u)} = K_3 + \frac{K_4}{\mu_{ab}}\cdot\lambda_{ab} + (K_2-K_1)\lambda_{ab}$$

$$= K_3 + (\frac{K_4}{\mu_{ab}} + K_2 - K_1)\lambda_{ab}$$

If λ_{ab} is varied while keeping μ_{ab} unchanged, the frequency will vary almost linearly and the expression for frequency can be written in the form

$$f_{-(p.u.)} = m_1 \cdot \lambda_{ab} + c_1$$

where

$$c_1 = K_3$$

and

$$m_1 = \frac{K_4}{\mu_{ab}} + K_2 - K_1$$

If both λ_{ab} and μ_{ab} are varied in the same proportion, the frequency will again vary linearly but the coefficient of λ_{ab} is given by

$$m_2 = K_2 - K_1$$

Since m_2 is less than m_1, the frequency of the failure state is more sensitive to λ_{ab} than λ_{ab} and μ_{ab} both varied together. This is, of course, indicated by Figure 3.15.

If μ_{ab} alone is varied

$$\frac{\partial f_-}{\partial \mu_{ab}} = - \frac{K_4}{\mu_{ab}^2} \cdot \lambda_{ab}$$

and if λ_{ab} alone is varied

$$\frac{\partial f_-}{\partial \lambda_{ab}} = \frac{K_4}{\mu_{ab}} + K_2 - K_1$$

Thus $$\frac{\partial f_-}{\partial \lambda_{ab}} > \frac{\partial f_-}{\partial \mu_{ab}}$$

Thus the frequency or cycle time of the failure state is more sensitive to the λ_{ab} variation than μ_{ab} variation. This is shown by the characteristics of Figure 3.15. These studies are important in that they show how the reliability may be improved by controlling the failure rate of the tie line or improving its repair rate. These parameters depend both upon the design and the operation of the line.

3.5.4 The Effect of the Peak Load in System B on the Risk Level in System A

Assuming the basic configuration of System B Load Model to stay unchanged, the peak load was varied from 1200 MW to 1725 MW and the results are shown in Figures 3.16 and 3.17.

The complete generation system model of any system contains capacity outage states varying from zero to the installed capacity. If the peak in System B was quite low virtually all the capacity in System B would be available to System A depending upon the capacity of the tie line. As the peak in System B increases, the positive margins decrease and consequently the improvement in reliability of

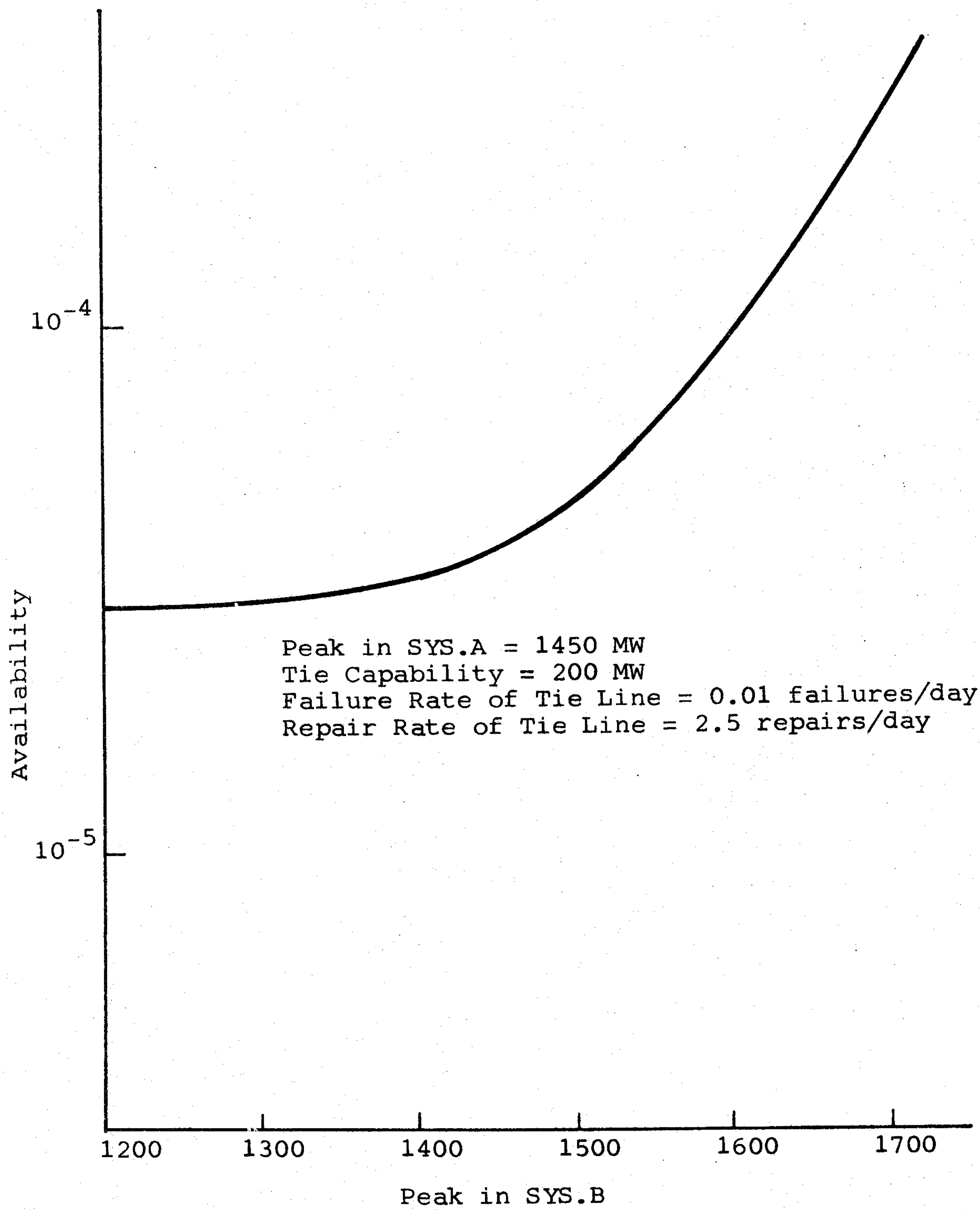


Figure 3.16: Variation of Risk Level (Availability) in SYS.A with Variation of Peak in SYS.B

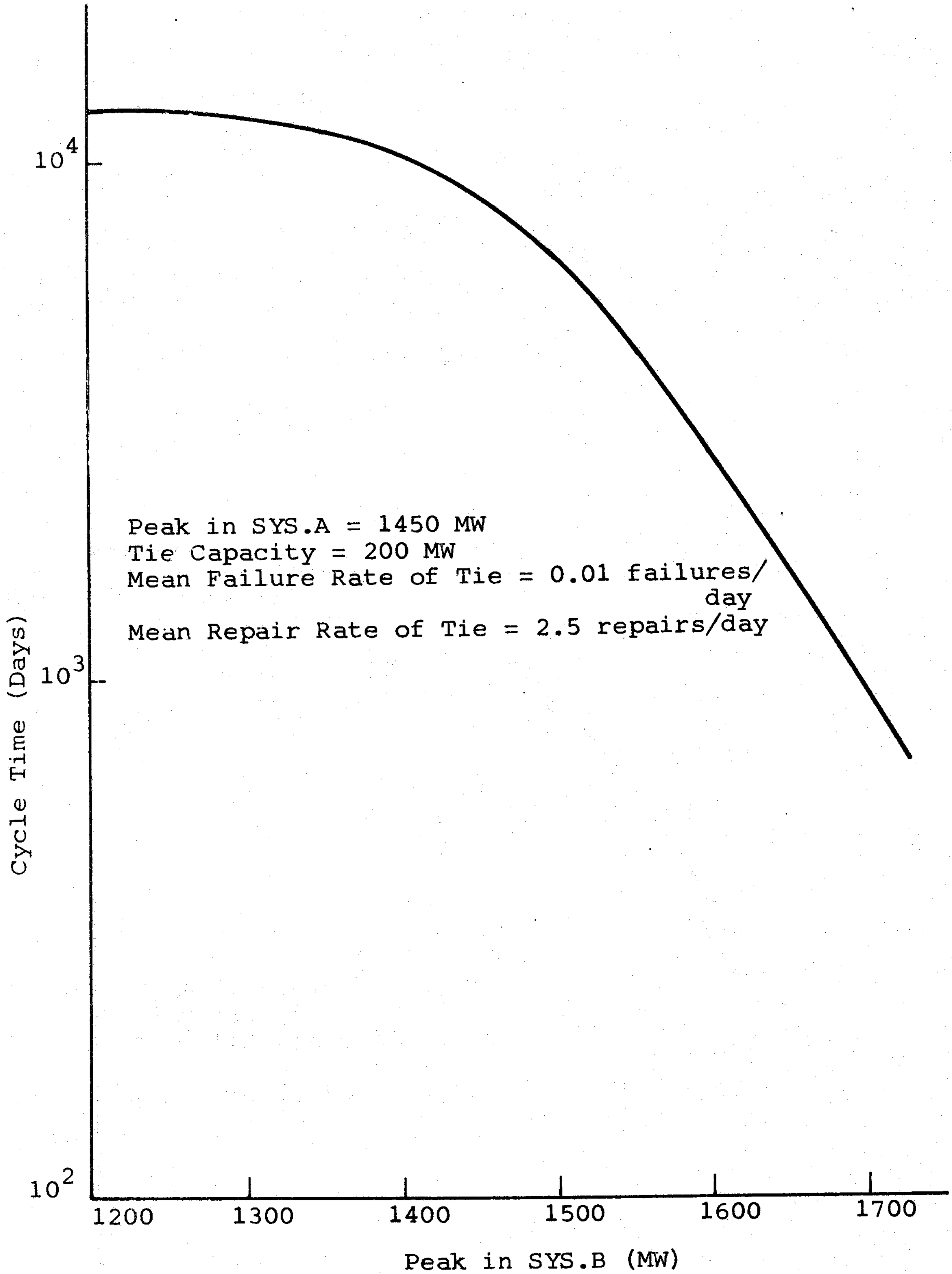


Figure 3.17: Variation of Risk Level (Cycle Time) in SYS.A with the Variation of Peak in SYS.B

System A due to System B also reduces. In practice, System B generation system model is curtailed beyond a certain level of capacity outage and therefore there is no discernable improvement in the reliability of System A by reduction in the peak of System B beyond

Installed capacity - Last value in the capacity outage table

= 1725 - 800

= 925 MW.

The interconnection will be of benefit to System A as long as there is a single positive margin in System B. The improvement in reliability will, therefore, continue as long as the following limit is not reached

$$L_{bo} = IC_b$$

At this limit,

The Risk level in the System A with interconnection

= The Risk level in the System A without interconnection

Both of these limits are improbable in actual practice and the steepest range in the improvement of reliability lies somewhere in between. In this study there is a very sharp improvement in the reliability of System A as the peak load in System B drops down to 1500 MW. The improvement diminishes rapidly below this point.

3.5.5 The Effect of the Peak Load in System A on the Risk Level in System A

The peak in System A was varied from 1250 MW to 1725 MW while the peak in the System B was maintained at 1450 MW. The availability of and the cycle time to failure versus peak load are indicated by curve "1" in Figures 3.18 and 3.19 respectively. Curve "2" indicates the risk indices versus the peak load with no help received from System B. The dotted curve shows the risk indices with no help received from System B but with an addition of 250 MW unit to System A, raising its installed capacity from 1725 MW to 1975 MW. It can be seen that the addition of this 250 MW unit has approximately the same effect as the help rendered by System B with its minimum operating reserve of 275 MW. The reliability of a system may thus be greatly improved by interconnection.

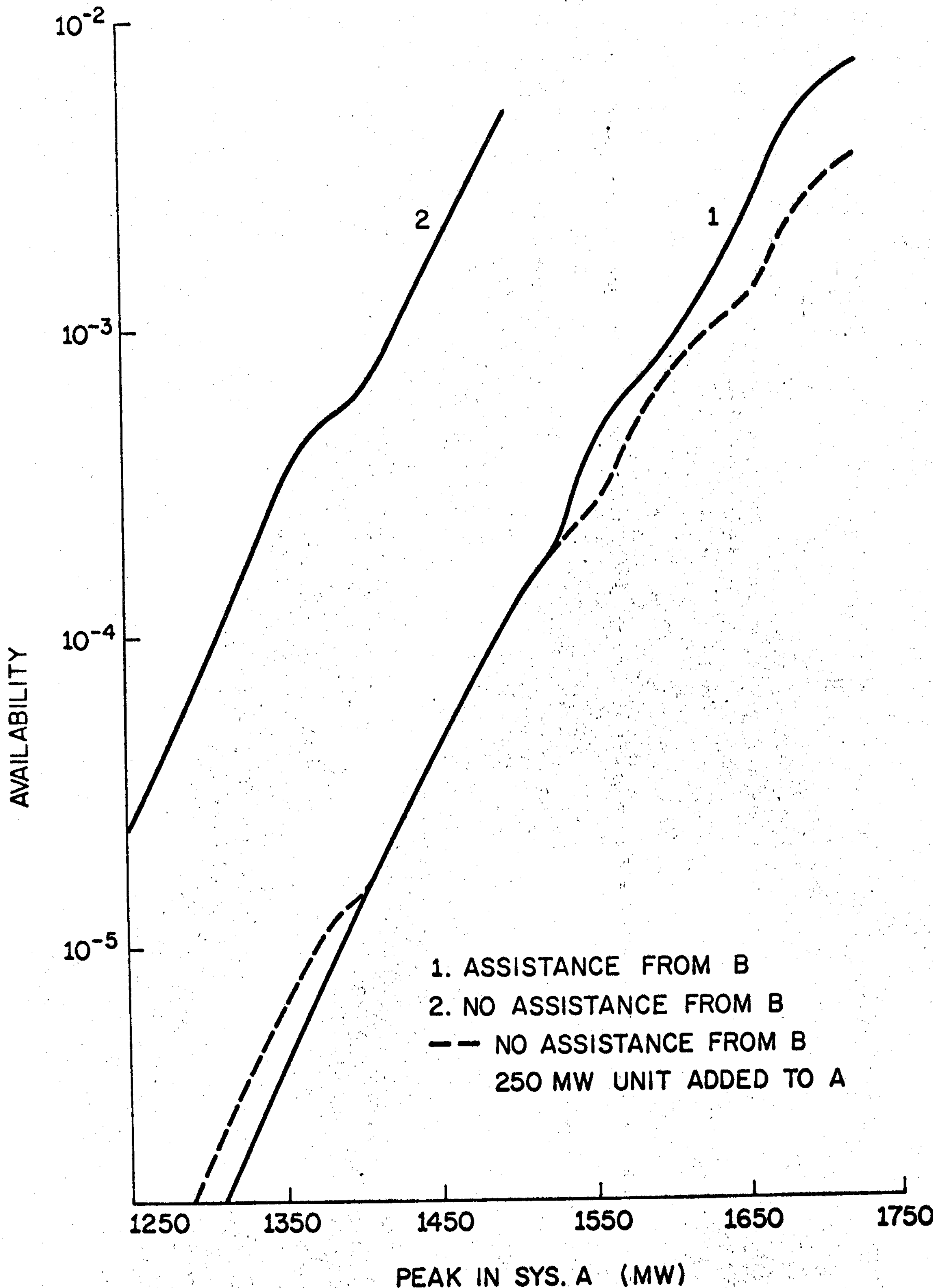


Figure 3.18: Variation of Risk Level (Availability) of SYS.A with Variation in Peak in SYS.A

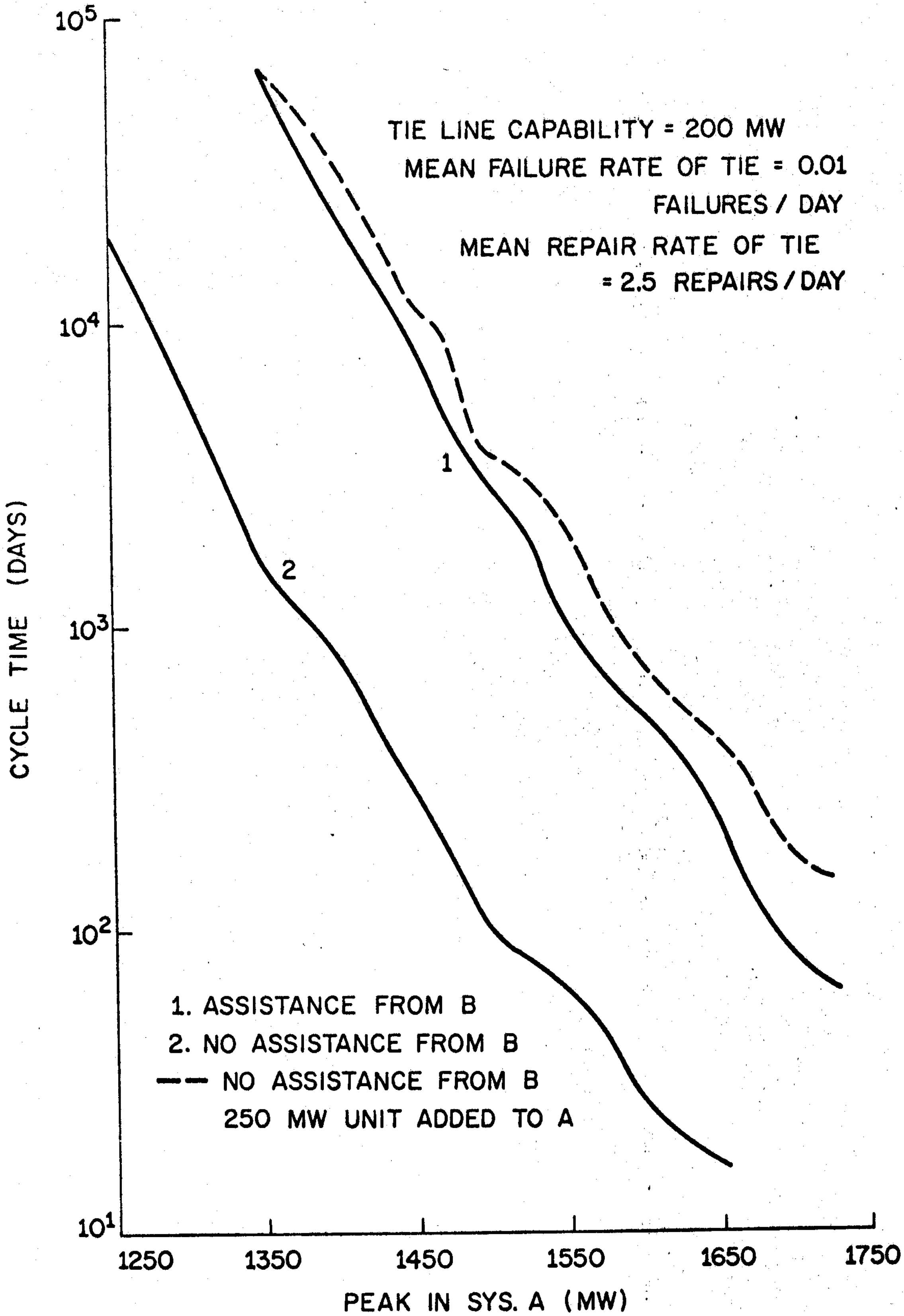


Figure 3.19: Variation of Risk Level (Cycle Time) in SYS.A with Variation in Peak Load of SYS.A

4. STUDIES ON SASKATCHEWAN POWER CORPORATION SYSTEM

4.1 GSM(SPC), 1971-1972

The data required for constructing the GSM(SPC) for the year 1971-1972 is shown in Appendix A. Only the capability and the FOR of each unit were known. The mean failure rate of each unit was assumed to be 0.01 failures per day and the corresponding mean repair rate was calculated using the equation

$$\mu = \left(\frac{1}{FOR} - 1\right) \lambda$$

The availabilities of the cumulative capacity outage states are not affected by the variation in λ or μ when the FOR remains unchanged. The frequencies are, however, quite sensitive to λ and μ variations even if the FOR is constant. The studies shown on the following pages illustrate the general probablistic nature of the capacity reserve model and also show how the frequency and duration technique may be applied to the reliability evaluation of practical generation systems.

4.2 The Demand Model

4.2.1 The Exposure Factor

To determine the exposure factor for the SPC System, the log data for the period July 1968 - June 1969 was analyzed on the computer. The mean durations of the peak load for the range 97 percent to 70 percent of the daily peak were computed for weekdays, weekends and the overall period. The exposure factor as a function of the load level is shown in Figure 4.1. For a given load level, the exposure factor for weekends is higher than that for weekdays. This is due to the fact that the weekday load curves are more peaked than the weekend curves. The exposure factor characteristic, with no distinction made between the weekdays and the weekends is closer to the weekday characteristic since the latter is representative of the 5/7 of the total days.

Figure 4.2 shows the probability distribution of the peak load duration with a mean value of eight hours. An exponential distribution with the same mean value is also shown.

4.2.2 The Frequency Distribution of the Load Levels

The daily peaks were arranged in the rank order and then grouped in class intervals of 400-500, 500-600, 600-700, 700-800, 800 and above. The median of each class was taken as the load level and the class frequency as the frequency of occurrence of that load level. The load model obtained after suitable rounding off is

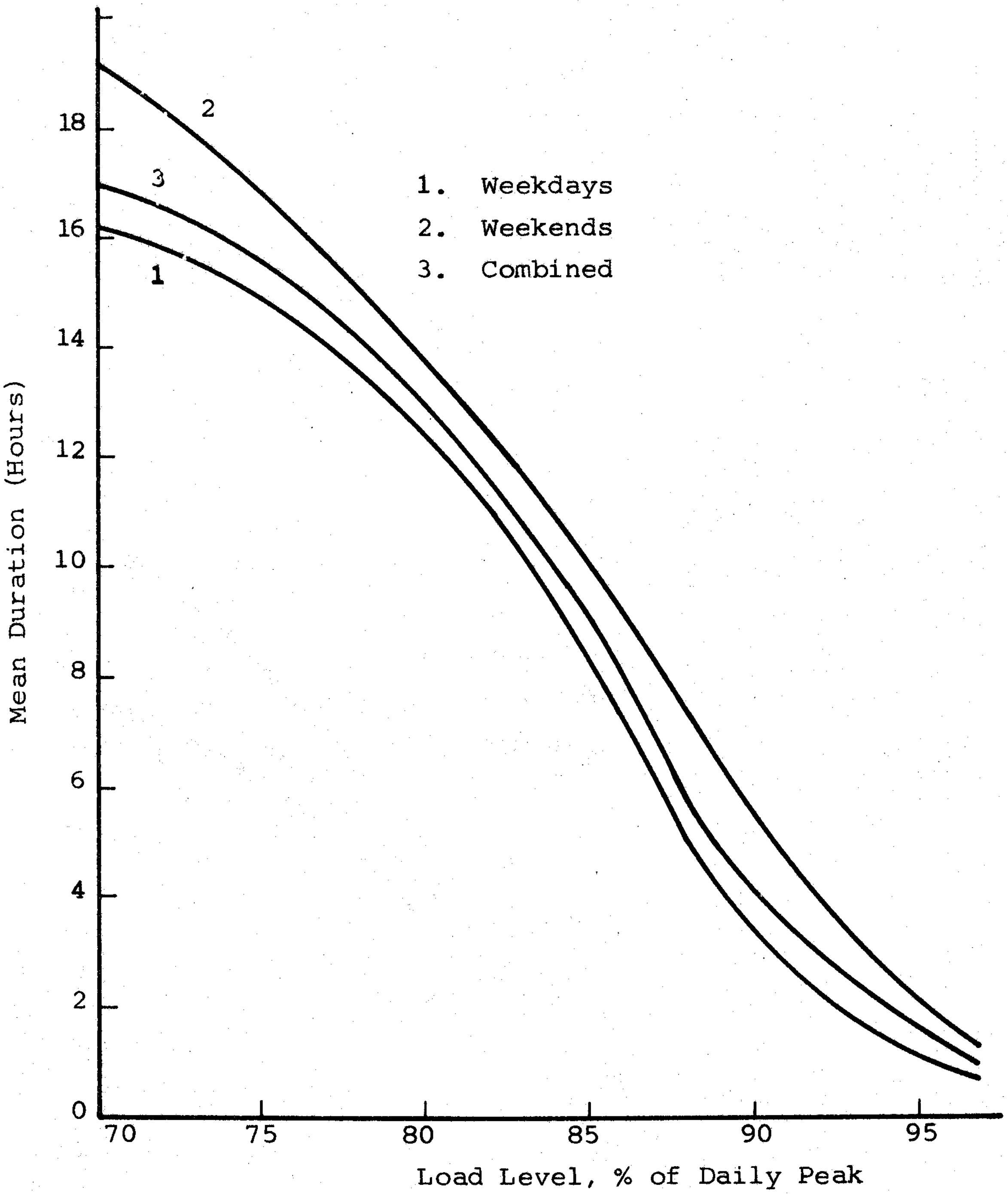


Figure 4.1: Mean Duration of the Peak vs Load Level - Saskatchewan Power Corporation July 1968-June 1969

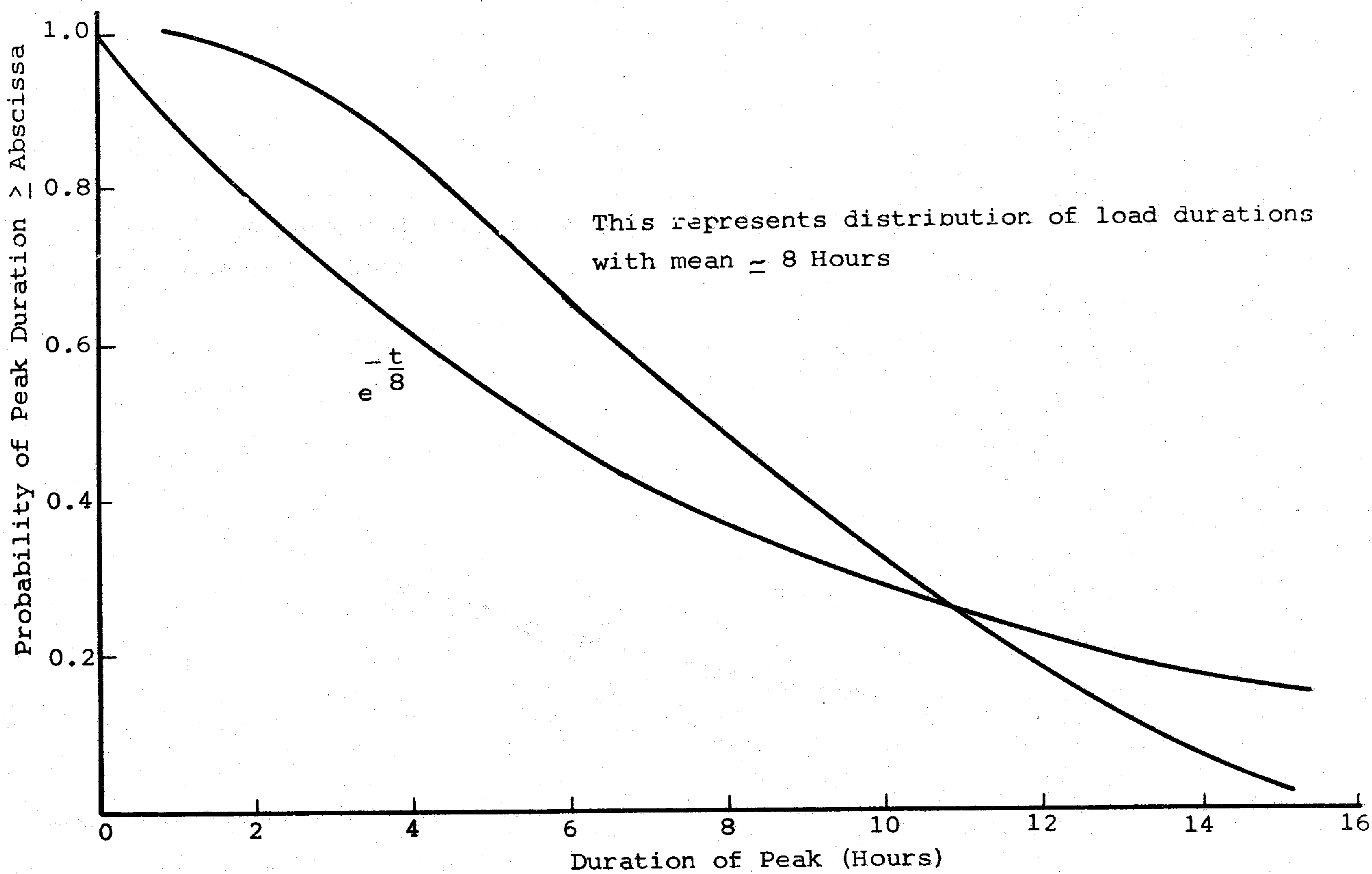


Figure 4.2: Distribution of Load Durations (Weekdays) for Saskatchewan Power Corporation System

Load Level MW	Number of Occurrences
850	51
735	95
655	145
560	60
475	14

The period length = 365 days
The Annual Peak = 918 MW

Assuming 1245 MW as the expected peak for 1971-1972, the load levels were multiplied by (1245/918) and suitably rounded off. The demand model for 1971-1972 is

Load Level MW	Number of Occurrences
1150	51
1000	95
890	145
765	60
645	14

The most accurate approach is to divide the year into a number of intervals and develop a load model for each interval.

4.3 The Effect of the Exposure Factor on the Availability and the Frequency of a Cumulative Margin State

The availability and the frequency of the zero MW or less margin were computed as a function of the exposure factor for different peak loads. The exposure factors used were 0.2, 0.35 and 0.7 days which correspond to 88%, 85% and 70% of the load levels respectively. The results are indicated in Figures 4.3 and 4.4. The availability curves are parallel to each other but the cycle time curves deviate slightly. This is explained in the next section.

4.4 The Effect of the Exposure Factor on the Unit Addition Programme

The exposure factor affects the availability of and the cycle time to the occurrence of the cumulative margin states. This study was conducted to investigate the effects of the exposure factor on the unit addition programme and the effective load carrying capabilities of the units added.

Figure 4.5A shows the availability of the failure state as a function of the peak load with the exposure factor equal to 0.2 days. The first curve was developed using the installed capacity expected in 1971-1972, i.e., 1442 MW. A set of four more similar curves were obtained by adding a unit of 200 MW each time. Figures 4.5B and 4.5C show similar curves for exposure factors of 0.35 and 0.70 days respectively.

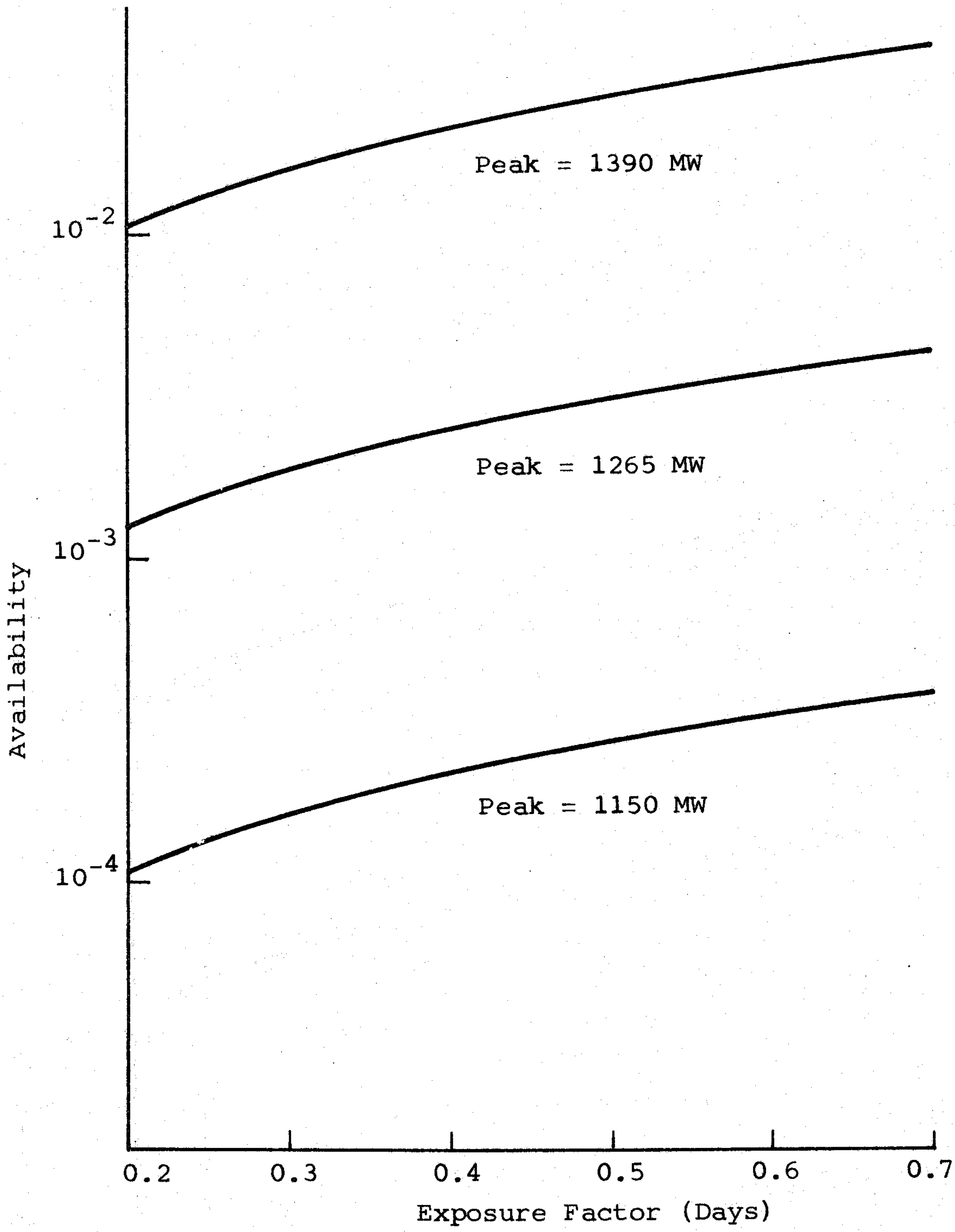


Figure 4.3: Variation of Availability of Failure State in S.P.C. System with Variation in Exposure Factor

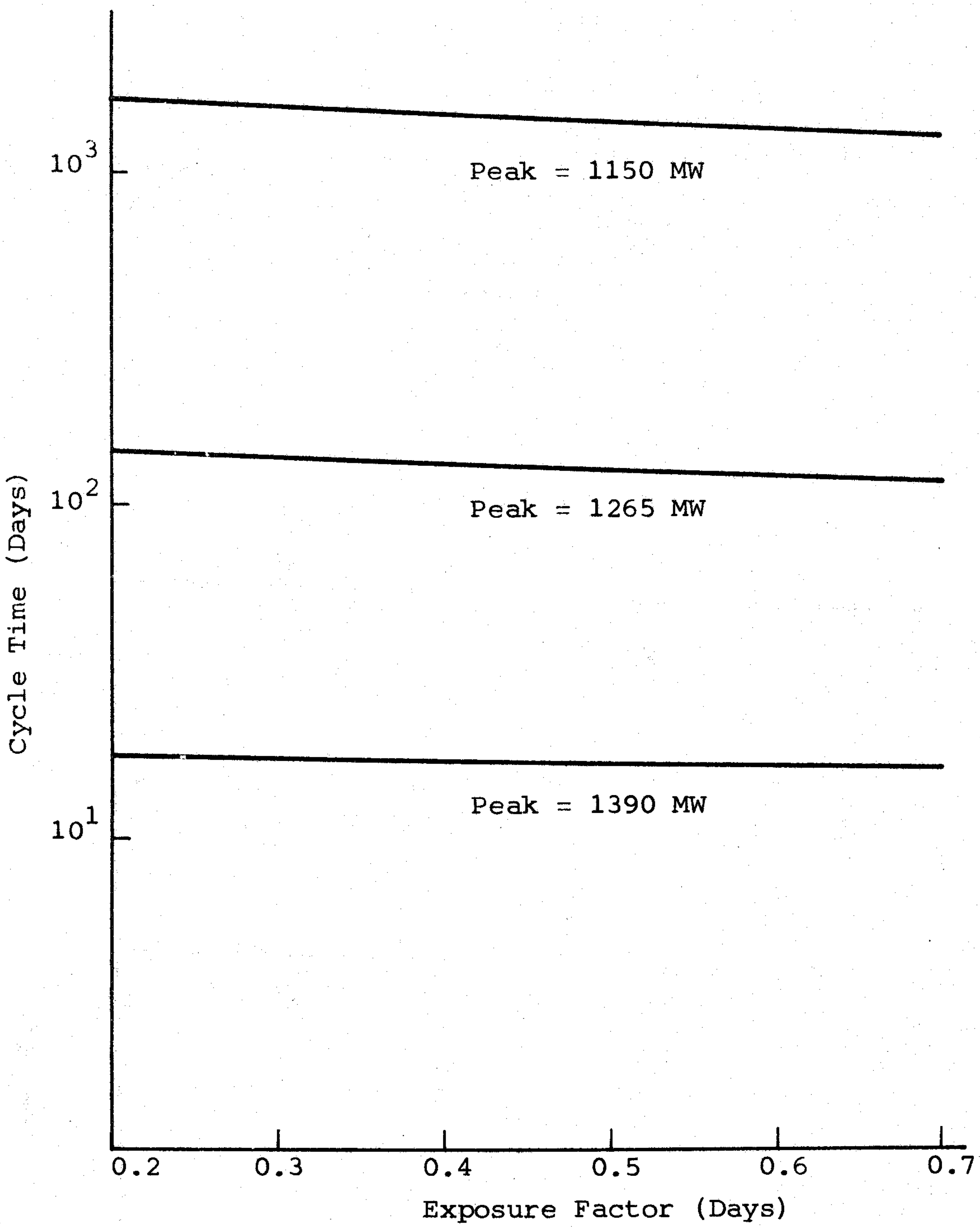


Figure 4.4: Variation of Cycle Time to Failure in S.P.C. System with Variation in Exposure Factor

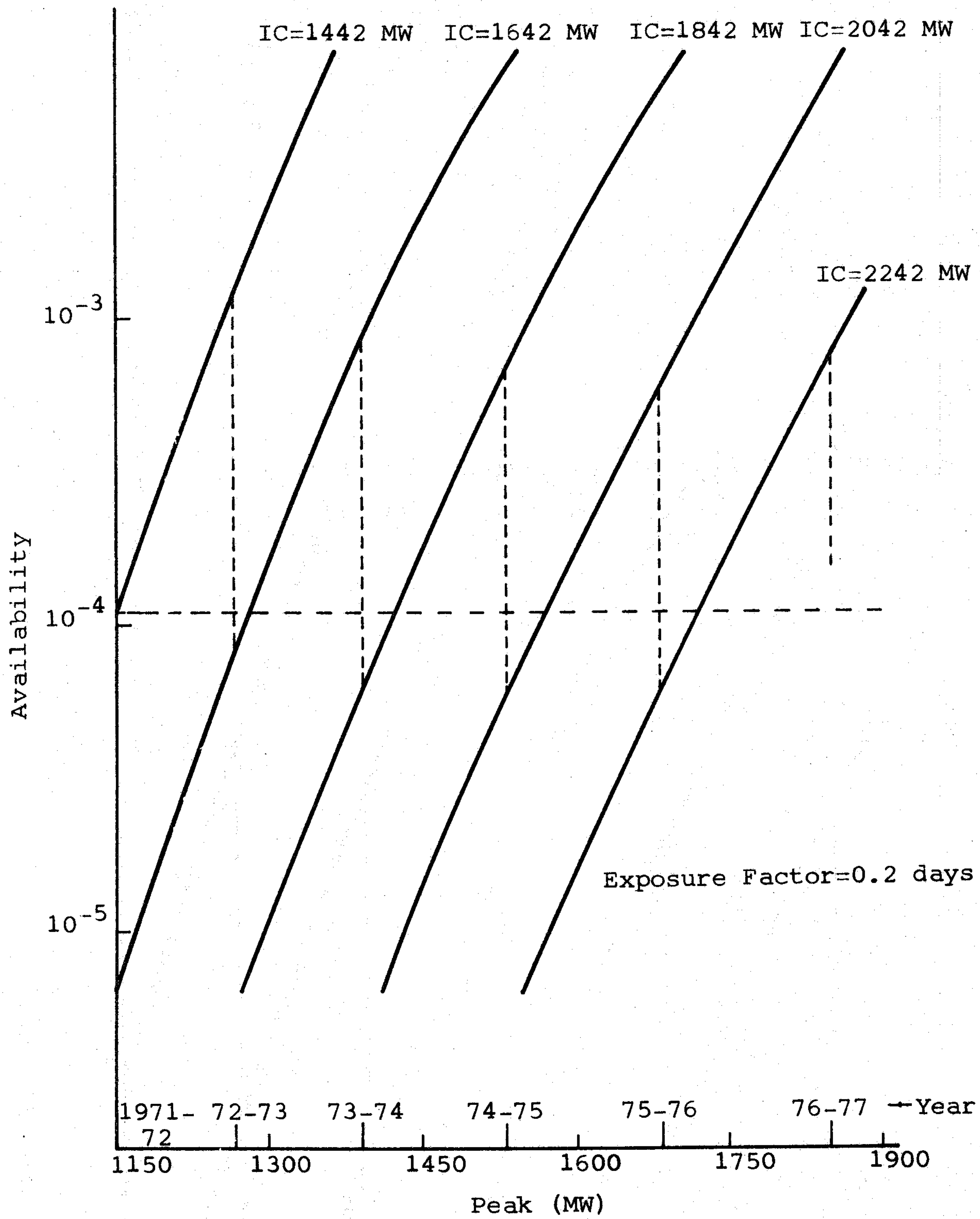


Figure 4.5A: Variation in Availability of Failure State with Variation in Peak Load in S.P.C. System

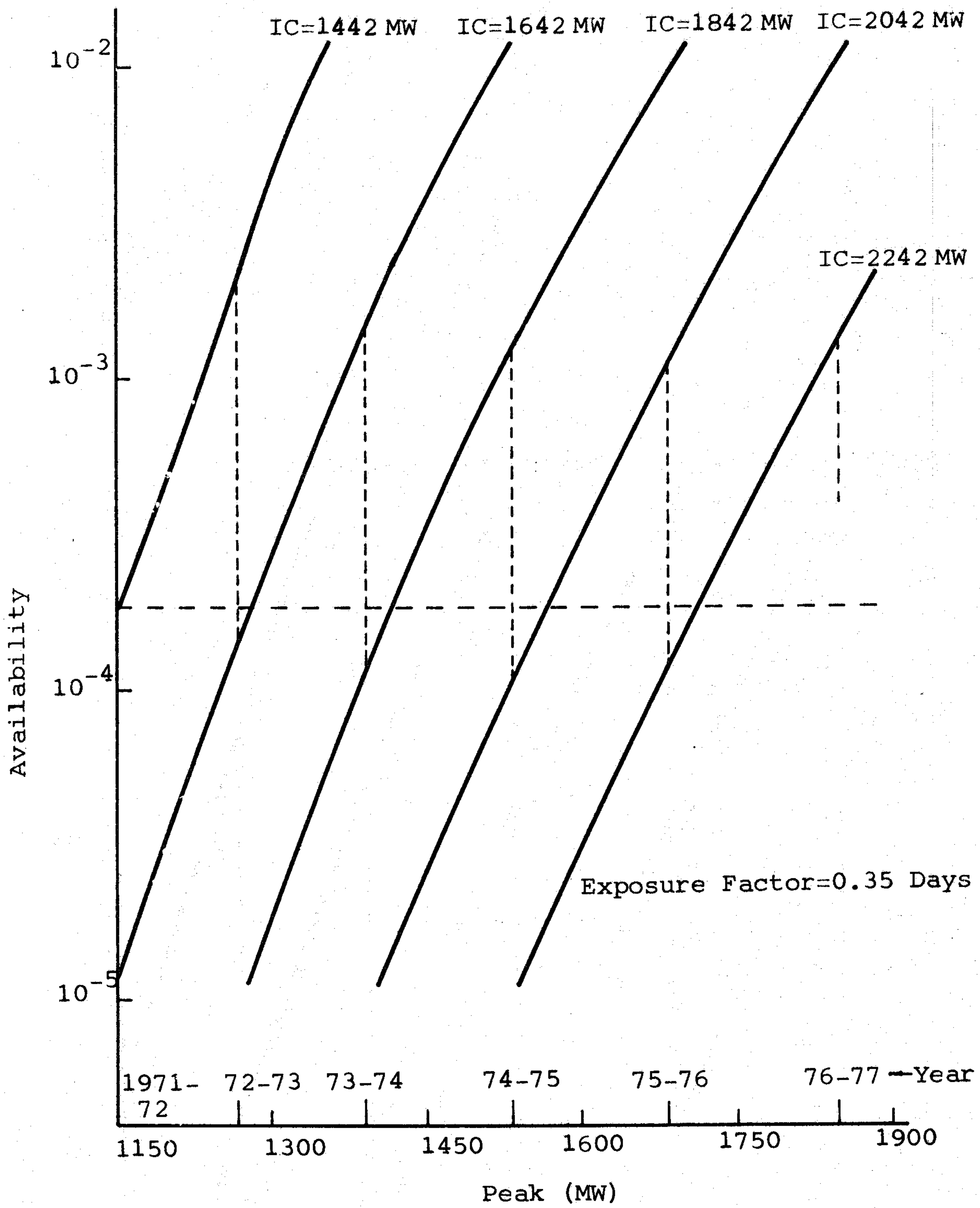


Figure 4.5B: Variation in Availability of Failure State with Variation in Peak Load in S.P.C. System

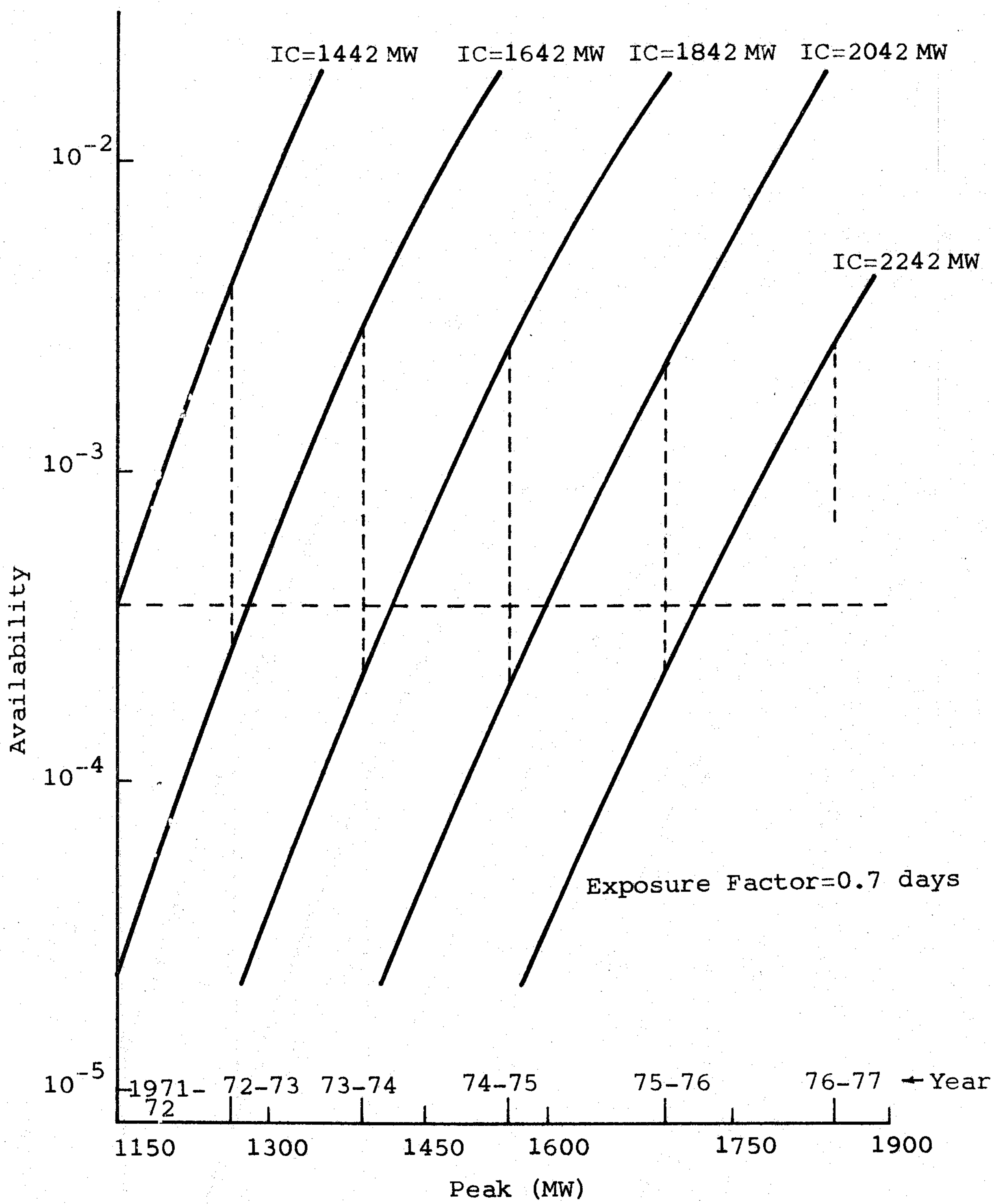


Figure 4.5C: Variation in Availability of Failure State with Variation in Peak Load in S.P.C. System

The availability at a peak load of 1150 MW and the installed capacity of 1442 MW, i.e., corresponding to the year 1971-1972 was accepted as the standard risk level. The unit additions in each case are shown by the dotted line. The effective load carrying capabilities of the various units based on the selected availability criterion are shown in Table 4.1. These values were found by successive iterations correct to third decimal place. It can be noted that the unit additions and the effective load carrying capability of the unit being added are independent of the value of the exposure factor. This is, of course, as expected. Everything else remaining unchanged, the expression for the availability of any cumulative margin state can be written as

$$A_M = E.K$$

where

$E =$ The exposure factor

$K =$ Constant

when plotted on semilog paper

$$\log A_M = \log E + \log K$$

Thus the effect of the exposure factor is an equal vertical shift to all the "Availability vs Peak" curves. Since the standard risk will also shift vertically by the

TABLE 4.1

The Effective Load Carrying Capabilities

of the Units Added

Unit #	Exposure Factor= 0.7 days		Exposure Factor= 0.35 days		Exposure Factor= 0.2 days	
	BOA	BOCT	BOA	BOCT	BOA	BOCT
1	128.001	127.001	128.001	127.001	128.001	127.001
2	138.999	140.999	138.999	140.999	138.999	139.999
3	148.001	151.999	148.001	149.000	148.001	149.300
4	138.000	149.000	138.000	147.001	138.000	144.700

BOA = Based on Availability

BOCT = Based on cycle time

The capability of the unit being added = 200.00 MW same amount, the variation in the exposure factor will not affect the unit addition or the effective load carrying capabilities of the units being added.

Figures 4.6A - 4.6C are the corresponding curves for the cycle time. The cycle time at the 1150 MW peak load point for the 1971-1972 conditions was considered to be the standard risk. The load carrying capabilities of the various units based on the cycle time criterion are also shown in Table 4.1. The unit addition pattern is the same as for the availability criterion, however, the effective capabilities of the units are not exactly the same--though quite close, for the different values of exposure factor.

Everything else remaining unchanged,

$$CT_M = \frac{1}{E.K_1+K_2}$$

where

K_1, K_2 = constants for a particular peak load and the particular GSM

$$\log CT_M = -\log (E.K_1+K_2)$$

$$= -(\log E + \log K_1 + \log K_2 + \frac{2K_2}{2.E.K_1+K_2})$$

neglecting the higher powers

The effect of log E is to shift all the curves

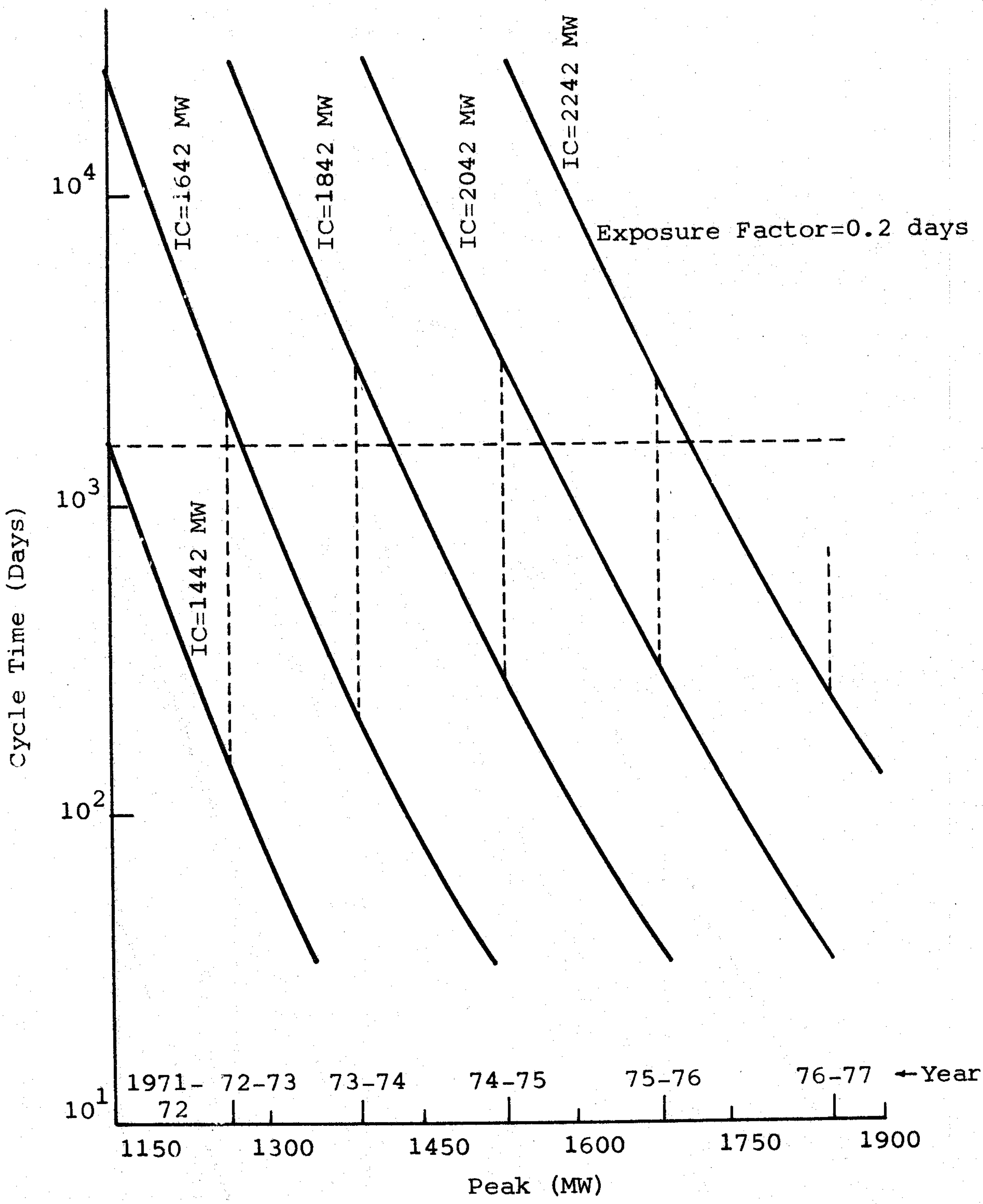


Figure 4.6A: Variation in Cycle Time to Failure with Variation in Peak Load in S.P.C. System

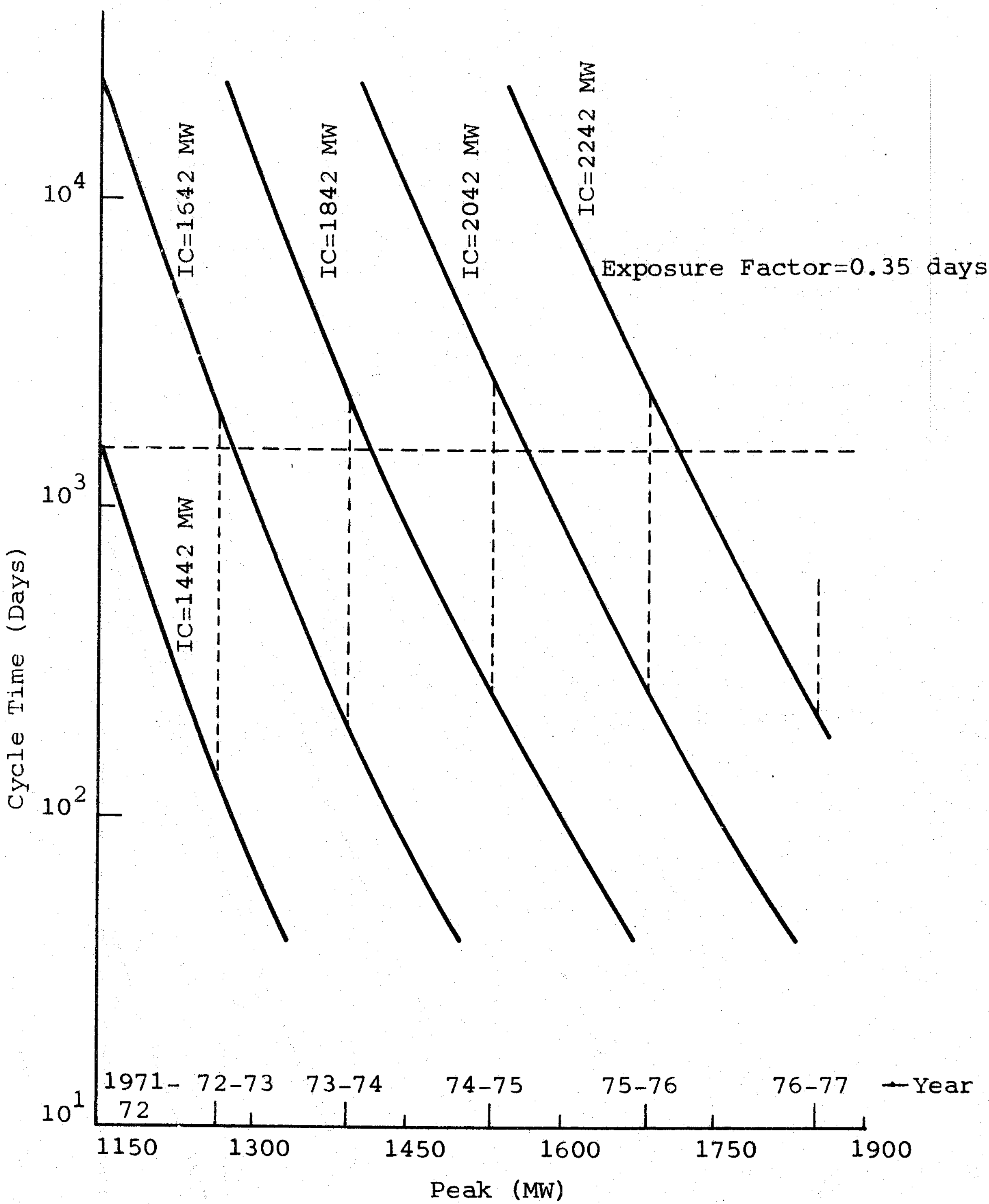


Figure 4.6B: Variation in Cycle Time to Failure with Variation in Peak Load in S.P.C. System

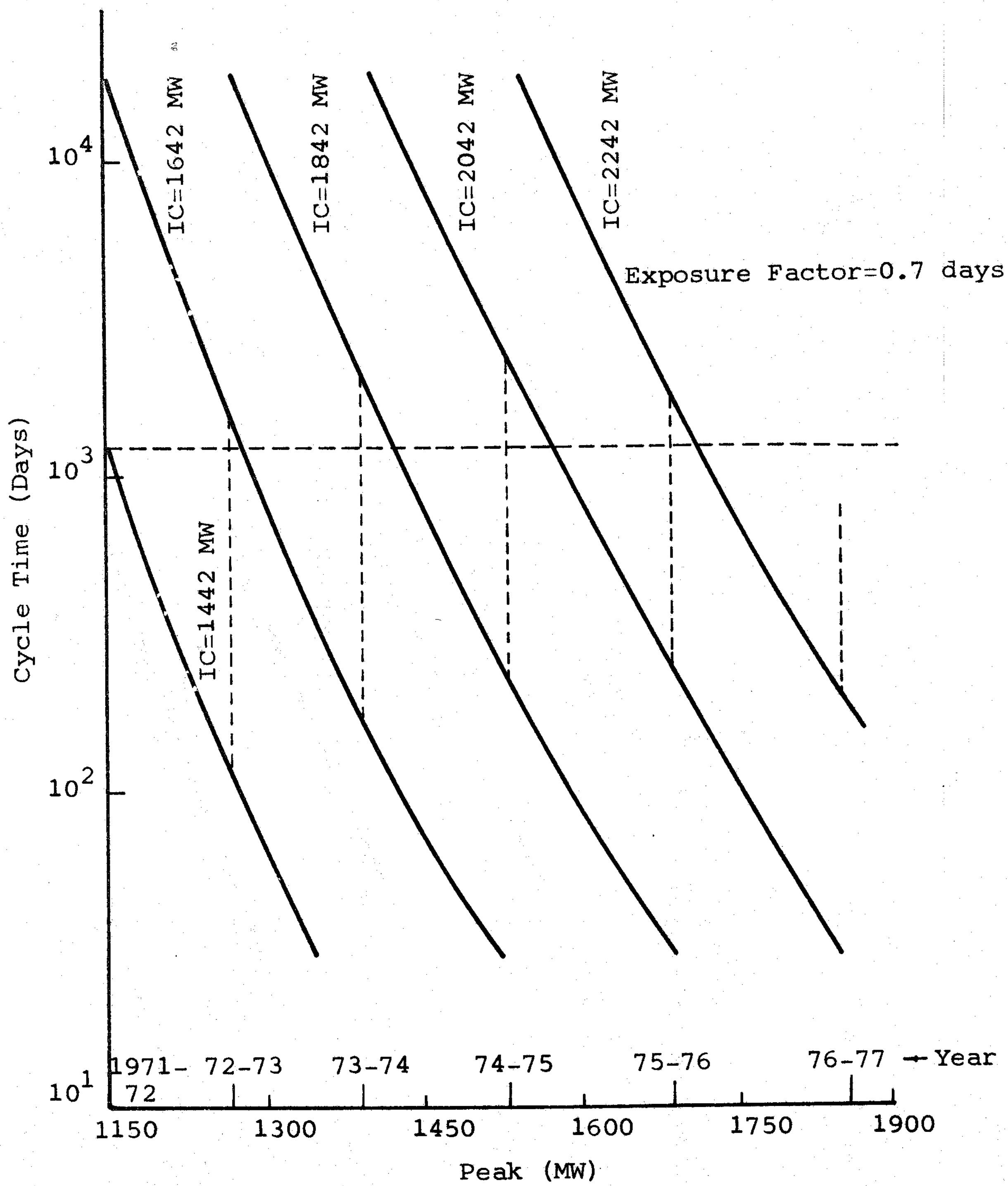


Figure 4.6C: Variation in Cycle Time to Failure with Variation in Peak Load in S.P.C. System

vertically by the same amount. Now

$$\frac{2.K_2}{2.E.K_1+K_2} = \frac{2}{1+2.E.\frac{K_1}{K_2}}$$

Therefore, if the ratio $\frac{K_1}{K_2}$ is the same for all peak loads and is not affected by the addition of a unit to the GSM, the curves will shift simply vertically by an equal amount. In a practical system, this ratio does not vary widely within a normal peak load range and therefore, the effect of the exposure factor on the unit addition programme and the effective load carrying capabilities is not very significant.

4.5 Interconnection with Manitoba Hydro (MH) System

4.5.1 General

The data required to construct the GSM (MH) for the period 1971-1972 is given in Appendix B. The basic configuration of the load model was assumed to be the same as that for the S.P.C. with the load levels multiplied by the ratio of the respective system peaks. The exposure factor for both the systems was assumed to be 0.35 days.

The annual peak for 1971-1972 in the MH has been taken as 1710 MW. This gives an equivalent peak of 1580 MW, after suitable rounding off.

4.5.2 The Effect of Tie Capacity on the Risk Level in S.P.C.

Assuming 0.015 failures per day and 3.75 repairs per day as the mean failure and the mean repair rates of the interconnection, the tie capacity was varied from 100 MW to 400 MW. The availability and the cycle time of the failure state as a function of the tie capacity are indicated in Figures 4.7 and 4.8 and the last segment of these characteristics is shown in Table 4.2.

TABLE 4.2

The Last Segment of the Availability and the Cycle Time Versus Tie Capacity Characteristics for S.P.C. System

Tie Capability MW	Availability	Cycle Time Days
250	0.8290760×10^{-7}	0.1638851×10^{7}
273	0.8133878×10^{-7}	0.1659399×10^{7}
300	0.8133878×10^{-7}	0.1659399×10^{7}
350	0.8133878×10^{-7}	0.1659399×10^{7}
400	0.8133878×10^{-7}	0.1659399×10^{7}

The GSMs of both the systems were truncated beyond capacity outages with cumulative availabilities less than 0.1×10^{-6}. This gives

The last significant capacity outage in S.P.C. System = 565 MW.

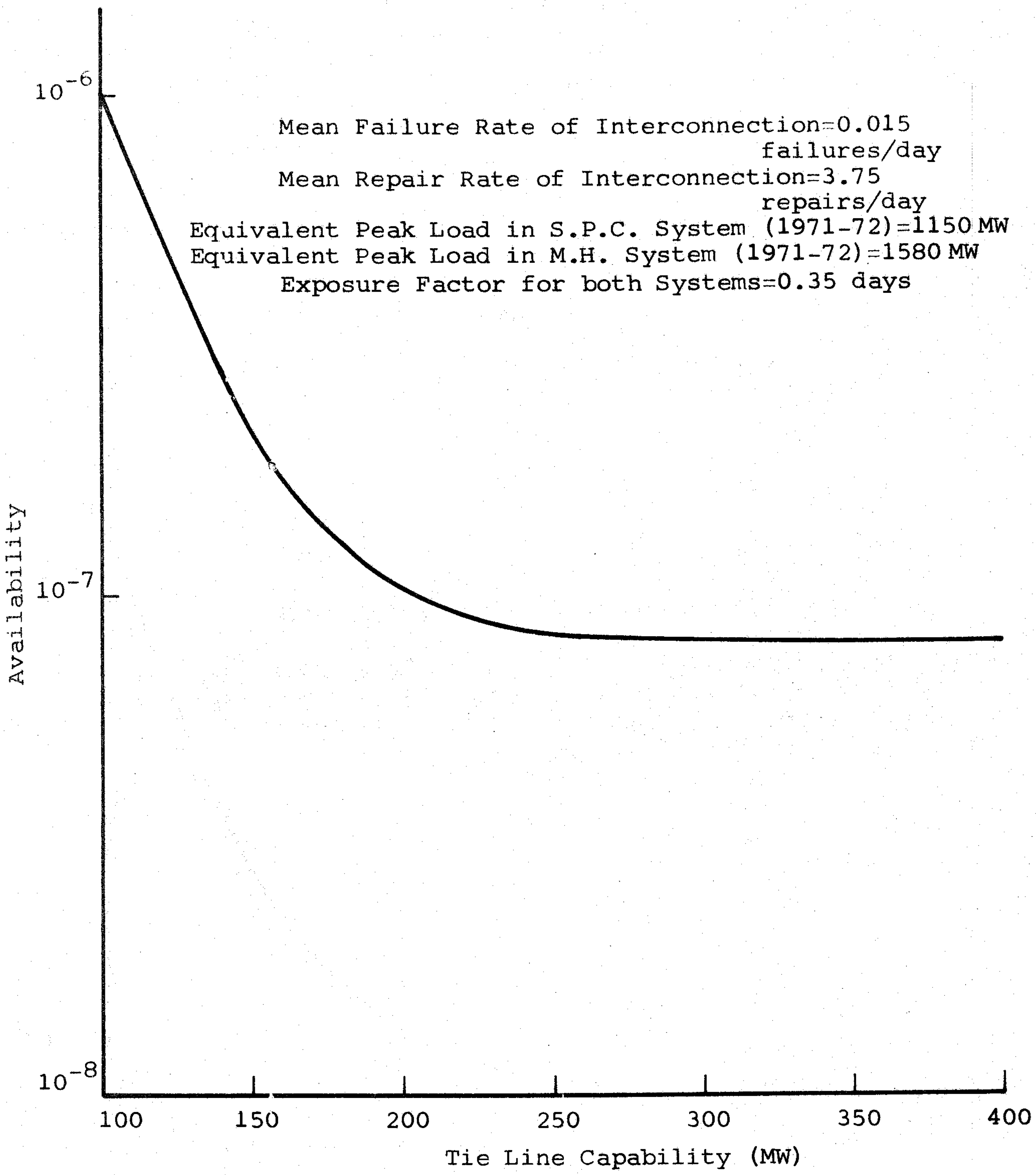


Figure 4.7: Variation in Availability of Failure State in S.P.C. System with Variation in the Capability of Interconnection Between S.P.C. and M.H.

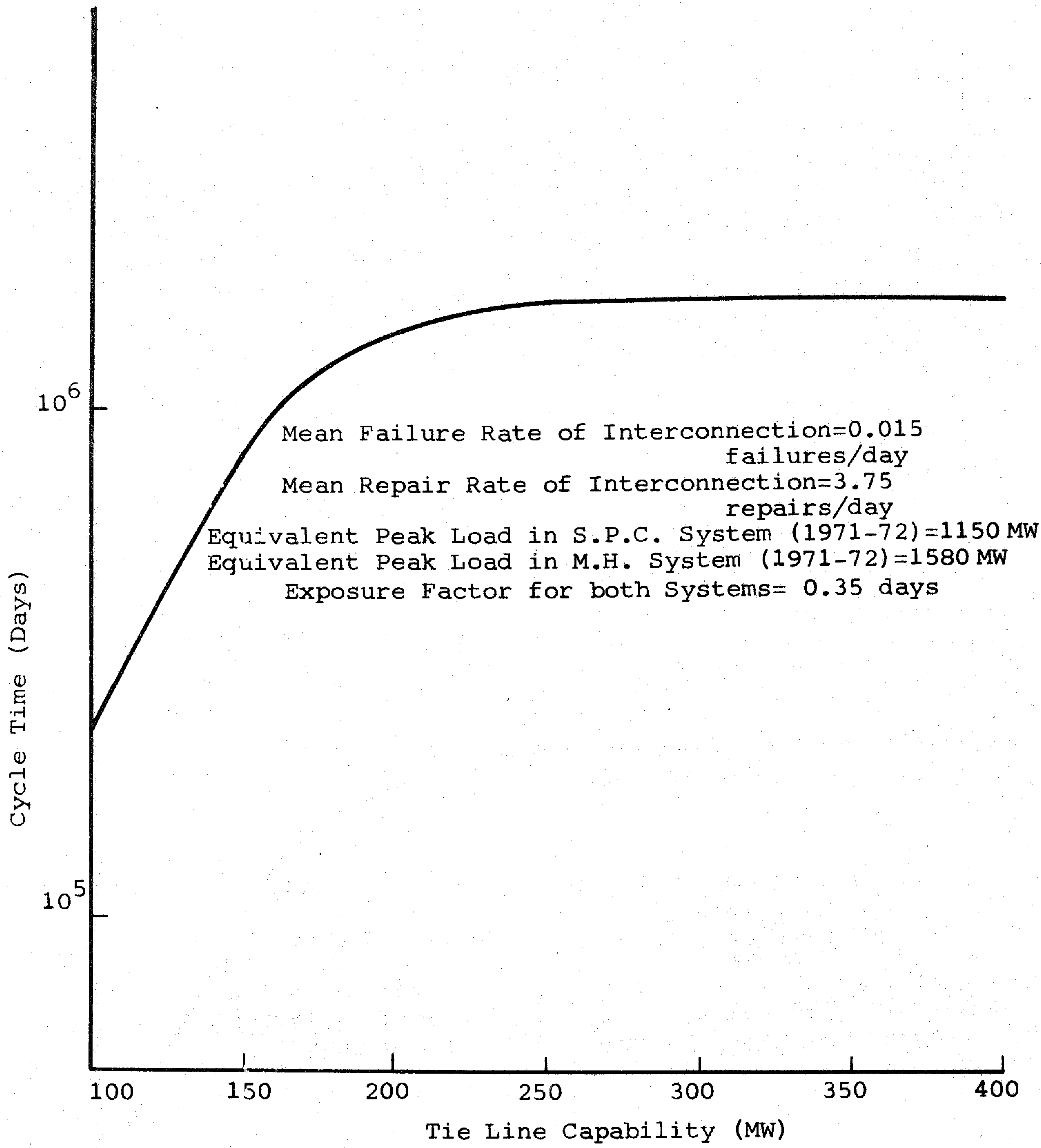


Figure 4.8: Variation in Cycle Time to Failure in S.P.C. System with Variation in the Capability of Interconnection Between S.P.C. and M.H. Systems

The last significant capacity outage in MH System $= 525$ MW.

Therefore,

$$M_{SPCm-} = \left| IC_{SPC} - 565 - 1150 \right|$$

$$= 273 \text{ MW}$$

$$M_{MHm+} = IC_{MH} - \text{Low Load on MH System}$$

$$= 2099 \text{ MW}$$

and

$$LT_{SM} = \text{Min } (2099, 273)$$

$$= 273 \text{ MW}$$

where

LT_{SM} = The limiting capability of the tie between the S.P.C. and MH systems.

It can be seen from Table 4.2 that there is no improvement in reliability beyond a tie capability of 273 MW. If, however, the GSMs were not curtailed at all, the absolute limiting value of the tie capacity would be 1441 MW. From Figure 4.7 it can be seen that there is no significant improvement in reliability beyond a tie capacity of 250 MW. This sets a practical limit on the maximum tie capacity.

4.5.3 The Effect of the Tie Line Mean Failure Rate and Mean Repair Rate on the Risk Level in S.P.C.

The peak loads in SPC and MH systems were maintained at 1150 MW and 1580 MW respectively and the availability and the frequency of failure were computed as a function of the M.F.R. and the M.R.R. of the tie line. The following three studies were done:

1. λ_{sm} was varied from .005-.03 failures/day, keeping μ_{sm} constant at 2.5 repairs/day.
2. μ_{sm} was varied from 1.25-7.5 repairs/day, keeping λ_{sm} constant at 0.01 failures/day.
3. λ_{sm} and μ_{sm} were varied together from (.005,1.25) to (.03,7.5) per day.

The results are plotted in the Figures 4.9,4.10 in p.u. The base values are

$$\mu_{sm} = 2.5 \text{ repairs/day}$$

$$\lambda_{sm} = .01 \text{ failures/day}$$

A_{base} = The availability of the failure state with the base λ_{sm}, μ_{sm} and T_{sm}=175 MW

CT_{base} = The cycle time to the failure state with the base λ_{sm}, μ_{sm} and T_{sm}=175 MW

These curves are similar to the ones obtained in the hypothetical case. It is interesting to see the relative magnitudes of the constants C_1 and C_2.

$$A_- = C_1\frac{\mu_{sm}}{\lambda_{sm}+\mu_{sm}} + C_2\frac{\lambda_{sm}}{\lambda_{sm}+\mu_{sm}}$$

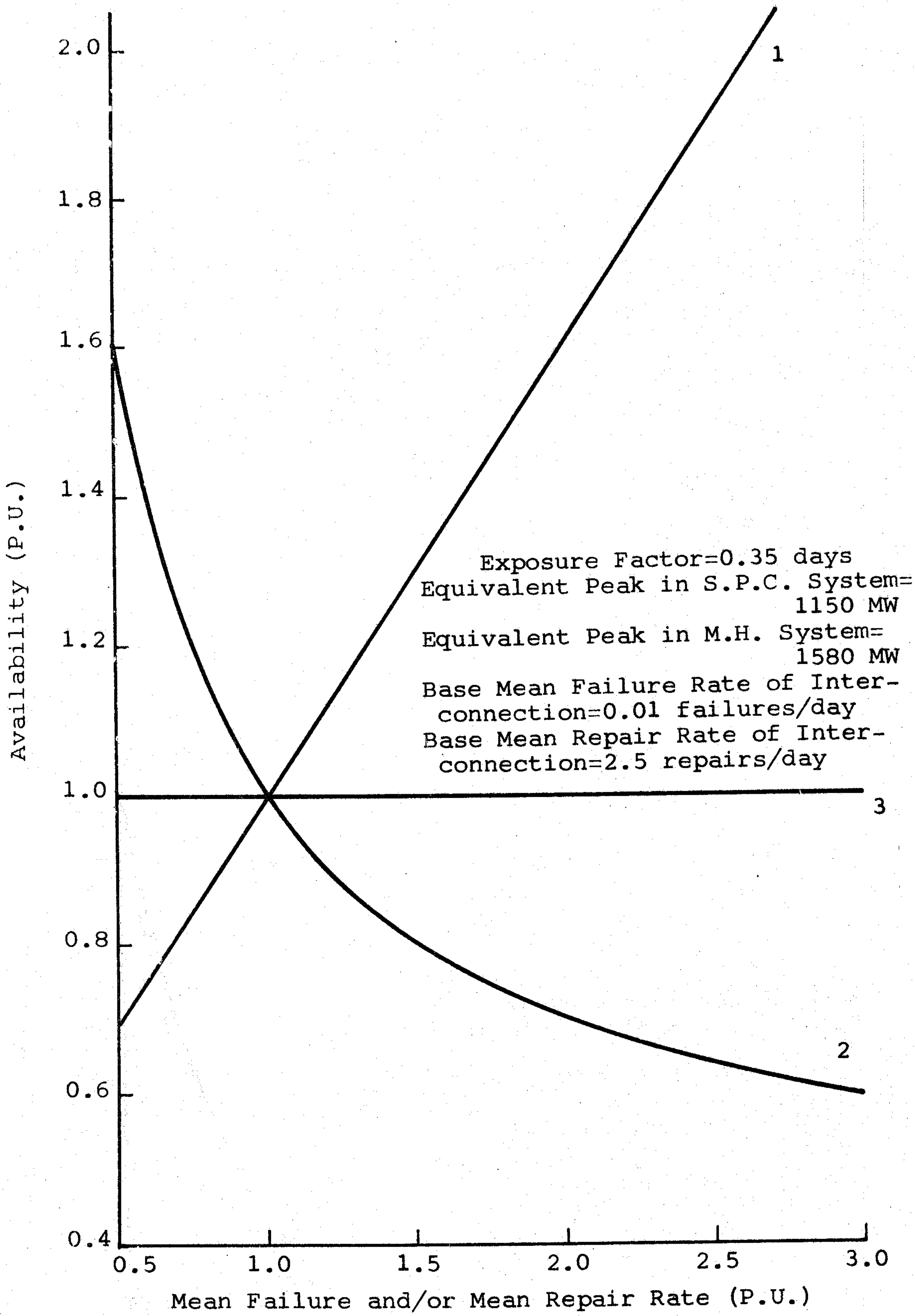


Figure 4.9: Variation in Availability of Failure State in S.P.C. System with Variation in Mean Failure and Mean Repair Rates of Interconnection Between S.P.C. and M.H. Systems

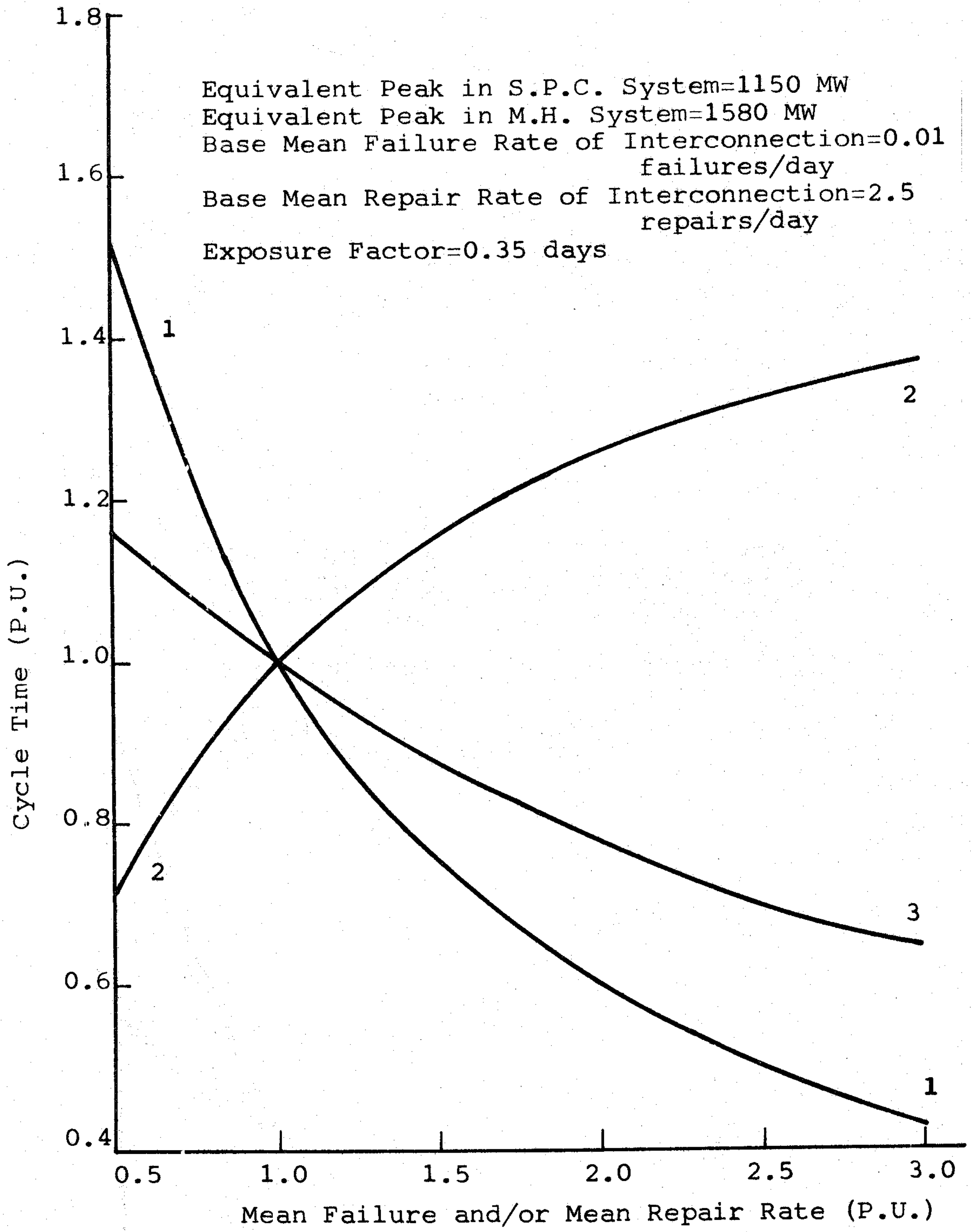


Figure 4.10: Variation in Cycle Time to Failure in S.P.C. System with Variation in Mean Failure and Mean Repair Rates of Interconnection Between S.P.C. and M.H. Systems

Selecting the first and the second values from Table 4.3(A)

$$C_1 \frac{2.5}{.005+2.5} + C_2 \frac{.005}{.005+2.5} = .9257553 \times 10^{-7}$$

and

$$C_1 \frac{2.5}{.01+2.5} + C_2 \frac{.01}{.01+2.5} = .1330631 \times 10^{-6}$$

From these two equations

$$C_1 = .51926 \times 10^{-7}$$

and

$$C_2 = .2041734 \times 10^{-4}$$

It should be appreciated that $C_2 >> C_1$ and that C_2 is in fact the availability of the first negative margin in the unaffected capacity reserve model of SPC System. The value of C_2 as calculated above agrees with the availability of the first negative margin as calculated directly.

TABLE 4.3

The Availability and the Cycle Time of Failure in SPC System as Functions of the Mean Failure Rate (λ_{sm}) and the Mean Repair Rate (μ_{sm}) of the Interconnection.

(A) The Mean Failure Rate is Varied Keeping the Mean Repair Rate at 2.5 Repairs/Day

λ_{sm} Per Day	λ_{sm} Per Unit	Availability	Availability Per Unit	Cycle Time Days	Cycle Time Per Unit
0.005	0.5	0.9257553×10^{-7}	0.69572	0.2081762×10^{7}	1.5189
0.010	1.0	0.1330631×10^{-6}	1.0	0.1370538×10^{7}	1.0000
0.015	1.5	0.1733911×10^{-6}	1.30307	0.1022575×10^{7}	0.7461
0.020	2.0	0.2135609×10^{-6}	1.6049	0.8162161×10^{6}	0.59554
0.025	2.5	0.2535708×10^{-6}	1.9056	0.6796053×10^{6}	0.49586
0.030	3.0	0.2934228×10^{-6}	2.2051	0.5824888×10^{6}	0.425

(B) The Mean Repair Rate is Varied Keeping the Mean Failure Rate at 0.01 Failures/Day

μ_{sm} Per Day	μ_{sm} Per Unit	Availability	Availability Per Unit	Cycle Time Days	Cycle Time Per Unit
1.25	0.5	0.2135522×10^{-6}	1.6048	0.9773801×10^{6}	0.71313
2.50	1.0	0.1330631×10^{-6}	1.0	0.1370538×10^{7}	1.000
3.75	1.5	0.1060938×10^{-6}	0.7973	0.1584056×10^{7}	1.1557
5.00	2.0	0.9258036×10^{-7}	0.69576	0.1718165×10^{7}	1.2536
6.25	2.5	0.8446278×10^{-7}	0.63475	0.1810242×10^{7}	1.3208
7.50	3.0	0.7904868×10^{-7}	0.5940	0.1877317×10^{7}	1.3697

(C) Both the Mean Failure Rate and the Mean Repair Rate are Varied

(λ_{sm}, μ_{sm}) Per Day	(λ_{sm}, μ_{sm}) Per Unit	Availability	Availability Per Unit	Cycle Time Days	Cycle Time Per Unit
(0.005,1.25)	0.5	0.1330557×10^{-6}	0.9999	0.1591887×10^{7}	1.1615
(0.010,2.50)	1.0	0.1330631×10^{-6}	1.0000	0.1370538×10^{7}	1.0
(0.015,3.75)	1.5	0.1330652×10^{-6}	1.0000	0.1203258×10^{7}	0.8779
(0.020,5.00)	2.0	0.1330664×10^{-6}	1.0000	0.1072375×10^{7}	0.7824
(0.025,6.25)	2.5	0.1330675×10^{-6}	1.0000	0.9671690×10^{6}	0.70568
(0.030,7.50)	3.0	0.1330675×10^{-6}	1.0000	0.8807698×10^{6}	0.64264

5. CONCLUSIONS

The generation system model for a conventional static capacity reliability study normally consists of the cumulative probabilities associated with designated capacity deficiencies. The frequency and duration method adds an additional parameter to this model in terms of the cumulative frequency of encountering a designated capacity deficiency. The frequency aspect of the computation requires a knowledge of the individual generating unit failure and repair rates rather than just the composite forced outage probabilities used in availability calculations. This information, however, should be available in any comprehensive outage reporting procedure.

The availabilities and the frequencies of exact capacity outages can be obtained and combined recursively to generate the availabilities and frequencies of cumulative capacity outage states. The cumulative values can, however, be obtained in a more efficient manner directly from the unit data as illustrated in Chapter 2 of this thesis.

The load data of a system for a sufficiently long period can be analyzed and by including the factors that can affect the future system load, a suitable load model can be developed. This load model differs from the conventional daily peak load variation curve used in loss of load probability studies in that the frequency of encountering the various peak load levels and the exposure time at each peak load level is included. The exact load states can be combined with the exact capacity outage states to obtain the availabilities and the frequencies of the exact margin states. These can be recursively combined to yield the frequencies and availabilities of cumulative margin states. The availabilities and frequencies of cumulative margin states can, however, be obtained directly by combining the exact load states with the cumulative capacity outage states as illustrated in this thesis.

The Saskatchewan Power Corporation data for the period July 1968-June 1969 indicates that the durations of peak load may not be exponentially distributed. This cannot, however, be regarded as a definite conclusion as the length of the study period is relatively short. This does not introduce any difficulty in regard to a long term static capacity study provided that a finite mean duration does exist. The actual value of the exposure factor depends upon the percentage of the daily peak at which it is determined. The variation in the exposure factor affects both the availability and the frequency of margin states. The load carrying capability of a unit determined on the basis of availability of failure is not affected by the choice of exposure factor.

If cycle time to failure is, however, taken as the criterion, the load carrying capability is slightly affected by the exposure factor. In a single system study, the exposure factor appears to be quite arbitrary provided that the system load model retains its basic shape. In reliability evaluation of interconnected systems, exposure factors at the same percentage of peak should be utilized.

The reliability study of a system connected to one or more other isolated systems can be conducted using the relationships developed in this thesis. For a system connected to other interconnected systems, careful analysis should be made before extending these equations. This situation has been illustrated by application to a simple triangular configuration.

The reliability of a power system is improved by interconnection to another power system. This improvement depends on the capability of the interconnection, the peak load on the system to which it is interconnected and the mean failure and the mean repair rate of the tie. The optimum tie capacity can be evaluated for any given set of conditions in the two systems and on the tie line.

It can clearly be seen in this thesis that the computational effort required to determine frequency and duration indices in single and multiple systems is considerably in excess of that required in a conventional loss of load approach. The level of application of probability mathematics is also much higher. In terms of capacity planning and unit commitment it is doubtful if one method can be stated to be superior under all conditions. The frequency and duration approach does possess a certain physical significance which is lacking in the loss of load method. It also utilizes more individual component reliability parameters and is, therefore, more suitable for sensitivity assessment arising from changes in maintenance, operating and planning policies.

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7. APPENDICES

7.1 Appendix A

Data for the Generation System Model of Saskatchewan Power Corporation System (1971-1972)

Name of Plant	Number of Units	Capacity Per Unit MW	Mean Failure Rate Per Day	Mean Repair Rate Per Day
Squaw Rapids	3	35.0	0.01	1.32333
	3	34.0	0.01	1.32333
	2	39.0	0.01	1.32333
Coteau Creak	3	63.0	0.01	1.32333
Boundary Dam	1	62.0	0.01	0.234499
	1	61.0	0.01	0.186078
Queen Elizabeth	1	62.0	0.01	0.283255
	1	61.0	0.01	0.138588
A.L. Cole	1	10.0	0.01	0.656666
	1	14.0	0.01	0.656666
	1	23.0	0.01	0.1045475
	1	23.0	0.01	0.656666
	1	32.0	0.01	0.656666
Regina	1	14.0	0.01	0.656666
	1	19.0	0.01	0.656666
	1	5.0	0.01	0.656666
	1	28.0	0.01	0.656666
	1	23.0	0.01	0.112249

Name of Plant	Number of Units	Capacity Per Unit MW	Mean Failure Rate Per Day	Mean Repair Rate Per Day
Estevan	1	5.0	0.01	0.656666
	1	14.0	0.01	0.214215
	1	19.0	0.01	0.52476
	1	28.0	0.01	0.603497
Kindersley	3	3.0	0.01	0.41735
	2	10.0	0.01	0.65666
Swift Current	1	2.0	0.01	0.226967
	1	1.0	0.01	0.226967
	4	3.0	0.01	0.656666
Success	3	15.0	0.01	0.49
Boundary Dam	2	140.0	0.01	0.156666
Queen Elizabeth	1	96.0	0.01	0.156666

7.2 Appendix B

Data for the Generation System Model of Manitoba Hydro System (1971-1972)

Name of Plant	Number of Units	Capacity Per Unit MW	Mean Failure Rate Per Day	Mean Repair Rate Per Day
Seven Sisters	1	25.0	0.00504	2.40000
	1	25.0	0.00480	6.85714
	1	25.0	0.00740	9.6
	1	25.0	0.00288	26.66666
	1	25.0	0.00192	120.00
	1	25.0	0.00288	26.66666
McArthur	1	8.0	0.00216	1.83206
	1	8.0	0.006	3.93442
	1	8.0	0.00528	3.75
	1	8.0	0.00552	2.40
	1	8.0	0.00552	3.42857
	1	7.0	0.00528	4.8
	1	7.0	0.00408	4.28571
	1	7.0	0.00408	3.52941
Great Falls	1	22.0	0.0024	18.46154
	1	22.0	0.00192	4.28571
	1	22.0	0.0072	3.58208
	1	22.0	0.00552	0.97560
	1	22.0	0.00576	0.73170
	1	22.0	0.0036	5.33333
Selkirk	1	66.0	0.08376	8.0
	1	66.0	0.02424	1.07142

Name of Plant	Number of Units	Capacity Per Unit MW	Mean Failure Rate Per Day	Mean Repair Rate Per Day
Pine Falls	1	14.0	0.0084	3.28767
	1	14.0	0.0120	4.13793
	1	14.0	0.00552	1.311475
	1	14.0	0.00648	2.016806
	1	14.0	0.01128	3.42857
	1	14.0	0.0084	4.44444
Grand Rapids	1	118.0	0.04032	0.48979
	1	118.0	0.00864	0.461538
	1	118.0	0.01992	0.474308
Kelsey	1	32.0	0.00624	0.88888
	1	32.0	0.006	4.44444
	1	32.0	0.00624	0.707964
	1	32.0	0.01008	2.926683
	1	32.0	0.01200	1.182266
Brandon	1	33.0	0.0384	0.857142
	1	33.0	0.0264	0.857142
	1	33.0	0.04416	0.857142
	1	33.0	0.06312	0.857142
Pointe duBois	1	3.0	0.00792	11.42857
	1	3.0	0.00288	6.85714
	1	3.0	0.00288	6.85714
	1	3.0	0.00504	1.46341
	1	4.0	0.00312	14.11765
	1	4.0	0.00504	1.96721
	1	4.0	0.00576	12.63157
	1	4.0	0.00816	2.376237
	1	6.0	0.01704	1.481481

Name of Plant	Number of Units	Capacity Per Unit MW	Mean Failure Rate Per Day	Mean Repair Rate Per Day
(Pointe duBois)	1	6.0	0.01464	0.275862
	1	5.0	0.00936	0.275862
	1	5.0	0.02112	0.461538
	1	5.0	0.00888	7.058823
	1	5.0	0.01392	3.287671
	1	5.0	0.01080	4.8
	1	5.0	0.0132	5.581395
Slave Falls	1	9.0	0.00504	0.253164
	1	9.0	0.00408	0.172662
	1	9.0	0.00288	13.33333
	1	9.0	0.00288	13.33333
	1	9.0	0.00384	17.14286
	1	9.0	0.00600	17.14286
	1	9.0	0.00288	16.00
	1	9.0	0.00192	1.73913
Amy St.	1	5.0	0.01	0.33333
	1	5.0	0.01	0.33333
	1	15.0	0.01	0.33333
	1	25.0	0.01	0.33333
Kettle Rapids	1	102.0	0.01	1.994999886
	1	102.0	0.01	1.994999886
	1	102.0	0.01	1.994999886
	1	102.0	0.01	1.994999886
Laurie River	1	3.0	0.01	0.33333
	1	3.0	0.01	0.33333
	1	4.0	0.01	0.33333

7.3 The Computer Programme

The principle paths in the computer programme for generating capacity reliability evaluation are indicated in Figure 7.1. The programme is suitable for continuous studies, the processes at each stage being decided by a control card. The description of various subroutines is given below.

Subroutine		Description
1.	WPGSM	Print or punch the GSM
2.	ROUND1	Round off the GSM
3.	RMC	Evaluate the availability and the frequency of cumulative margin states without taking load forecast uncertainty into account
4.	RMU	Evaluate the availability and the frequency of cumulative margin states, taking load forecast uncertainty into account
5.	EXPANU	Obtain unit addition scheme for generation planning, load forecast uncertainty being taken into account
6.	EXPANC	Obtain unit addition scheme for generation planning, load forecast uncertainty not being taken into account
7.	UR	Remove a unit from GSM
8.	GSMIC	Obtain the effective GSM of SYS.A for a given operating reserve in SYS.B

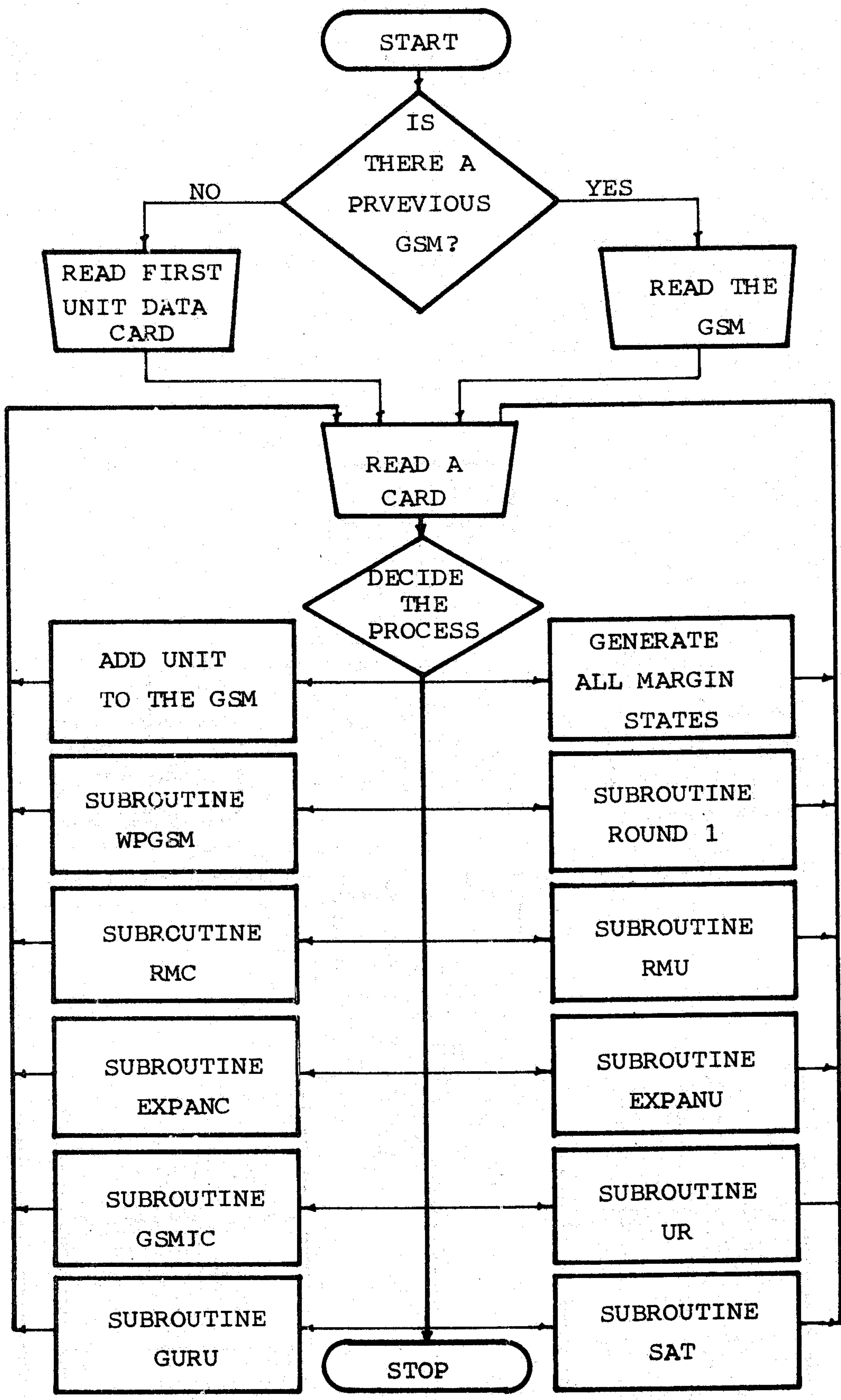

Figure 7.1: The Flow Diagram of the Computer Programme

9.	GURU	Evaluate the availability and the frequency of failure state in a system connected to one other system.
10.	SAT	Evaluate the effective load carrying capability of a unit added to a GSM.