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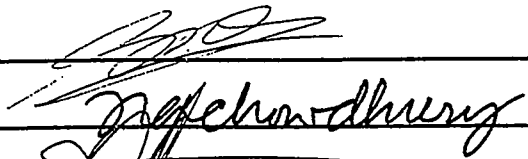
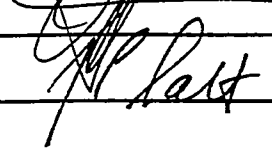
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May 17, 2001

**ANALYSIS OF NEW RIDGED CIRCULAR AND  
RECTANGULAR WAVEGUIDES USING THE  
FINITE ELEMENT METHOD**

A Thesis

Submitted to the College of Graduate Studies and Research

In Partial Fulfillment of the Requirements

For the Degree of Master of Science

In the Department of Electrical Engineering

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Saskatoon

By

Dai Qiu

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# ABSTRACT

Ridged waveguides have found many applications as microwave and millimeter-wave components. Their advantages include large single mode broadband operation and large fundamental mode cutoff wavelength. The study of ridged waveguides may be traced back to the 1940s. Since then, new ridged waveguide structures and new numerical computational techniques have been proposed and studied extensively. In this thesis, some new ridged waveguide structures, including a circular waveguide loaded with two T-shaped septa, a triple ridge loaded trough rectangular waveguide and a cross ridge loaded trough rectangular waveguide are proposed and theoretically analyzed by using the Finite Element Method (FEM). This analysis is verified by comparing the results with existing results from various sources. Numerical results are presented demonstrating that these structures offer a significant increase in fundamental mode cutoff wavelength and bandwidth when compared to various unloaded and loaded waveguides. These new structures can be used in the design of compact and wide-band microwave and millimeter-wave passive components and monolithic microwave integrated circuits (MMIC). An independent FEM analysis program for solving eigenvalue problems in 2-dimensional domain is developed to satisfy one of the objectives of this thesis. This program is the basis of a proposed 3-dimensional analysis program using the mode matching method.

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## **DEDICATION**

This thesis is dedicated to the author's parents, Chiu Xuxiang, Li Kuiluan, his wife Wang Qian, his lovely son William Chiu and all the author's family members for their love, care, patience and constant support.

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# LIST OF SYMBOLS AND ABBREVIATIONS

## 1. Greek and non-Latin Symbols

$\alpha_x, \alpha_y, \beta$	Known constants for the physical properties of a domain
$\Delta^e$	Area of the $e_{th}$ element
$\epsilon_0, \epsilon, \epsilon_r$	Air, media and the relative permittivity
$\delta_{ij}$	Delta function
$\varphi$	Unknown quantity; Tangential component in cylindrical coordinate system
$\varphi_i^e$	Value of $\varphi$ at node $i$ of the $e_{th}$ element, $i=1,2,3$
$\varphi^e(x, y)$	Unknown value in $e_{th}$ element
$\Gamma$	Boundary of unknown domain
$\Gamma_1$	Boundary 1
$\Gamma_2$	Boundary 2
$\lambda$	Wavelength
$\lambda_{cTE_{mn}}$	Cutoff wavelength of $TE_{mn}$ mode
$\lambda_{cTM_{mn}}$	Cutoff wavelength of $TM_{mn}$ mode
$\lambda_e$	Eigenvalues
$\mu_0, \mu, \mu_r$	Air, media and the relative permeability
$\rho$	Electric charge density; Radial component of cylindrical coordinate system
$\rho_s$	Surface charge density
$\sigma$	Conductance
$\omega$	Angular frequency
$\Omega$	Unknown domain
$\Omega^e$	The $e_{th}$ sub-domain
$\xi_{mn}$	The $n_{th}$ root of the $m_{th}$ order Bessel function
$\xi'_{mn}$	Roots of $J'_m(\cdot)$ such that $J'_m(\xi'_{mn}) = 0$
$\nabla$	Del operator
$\nabla_t$	Transverse Del operator

## 2. Latin Symbols and Abbreviations

1-D, 2-D, 3-D	One, two and three dimensional
$a$	Width of rectangular waveguide; radius of circular waveguide
$a^e, b^e, c^e$	Constant coefficient for the $e_{th}$ element
$a_j^e, b_j^e, c_j^e$	Constant coefficient for interpolation functions
$A$	Amplitude of wave
$A_1, B_1, C_1$	Constant coefficients for vector $\vec{E}_0$
$A_{ij}^e, B_{ij}^e$	Elements of the $e_{th}$ elemental matrix
$[A], [B]$	System matrix
$b$	Height of rectangular waveguide
$\{b\}$	Source or excitation matrix
$\vec{B}$	Magnetic flux density
BW	Band-width
$c$	Velocity of light
$C$	Capacitance
$C_d$	Fringe capacitance
$C_s$	Parallel plate capacitance
CPW	Coplanar waveguide
$\vec{D}$	Electric displacement or electric flux density
DRCW	Double ridge circular waveguide
DRRW	Double ridge rectangular waveguide
DTRCW	Double triangular ridge circular waveguide
DTSCW	Double T-septa ridge circular waveguide
DTSRW	Double T-septa ridge rectangular waveguide
$\vec{E}$	Electric field strength
$\vec{E}_0$	Constant vector
$E_x, E_y, E_z$	Components of $\vec{E}$ in rectangular coordinate system
$E_\rho, E_\phi$	Components of $\vec{E}$ in cylindrical coordinate system

EM	Electromagnetic
FDM	Finite difference method
FEM	Finite element method
$f$	Frequency
$f_c$	Cutoff frequency
$f_{cTE_{mn}}$	Cutoff frequency of $TE_{mn}$ mode
$f_s$	Source or excitation function
$F(\varphi)$	Functional
$F^e(\varphi^e)$	Functional of the $e_{th}$ element
$G(z,t)$	One dimensional wave function
GHz	$1 \times 10^9$ Hz
GiD	Pre-post-processing software package for numerical calculation
$\vec{H}$	Magnetic field strength
$H_x, H_y, H_z$	Components of $\vec{H}$ in rectangular coordinate system
$H_\varphi, H_\rho$	Components of $\vec{H}$ in cylindrical coordinate system
$i, j$	Integers; indices
$j$	$\sqrt{-1}$
$\vec{J}$	Electric current density
$J_m(\ )$	The $m_{th}$ order Bessel function
$J'_m(\ )$	The derivative of Bessel function
$k$	Wavenumber or propagation constant
$\vec{k}$	The $k$ vector
$k_c$	Cutoff wavenumber
$k_{cmn}$	Cutoff wavenumber with indices $m$ and $n$
$k_x, k_y, k_z$	Cutoff wavenumbers in $x, y, z$ directions
$k_\rho$	Cutoff wavenumber in radial direction
[K]	System matrix
$K_{ij}^e$	Matrix element of the $e_{th}$ element
$l, m, n$	Integers; indices
L	Inductance

$L, M$	Linear operator
$M, N$	Integers
MEFiSTo-2D	TLM software package
MHz	$1 \times 10^6$ Hz
$\hat{n}$	Normal unit vector
$n(1, e), n(2, e), n(3, e)$	Connectivity array of the $e_{th}$ triangular element
$nn$	Total number of nodes
$ne$	Total number of elements
$nl$	Number of nodes on $\Gamma$
$nd(i)$	Global node numbers on $\Gamma$
$n(i, j)$	Local node number of each element
$N_j^e(x, y)$	The $j_{th}$ shape function of the $e_{th}$ element
$p$	Prescribed boundary value
PDE	Partial differential equation
PDEase2	FEM analysis software package
PHz	$1 \times 10^{15}$ Hz
Q	Quality factor
$\bar{r}$	Space position vector
SCRTRW	Single cross ridge trough rectangular waveguide
SRRW	Single ridge rectangular waveguide
SRTRW	Single ridge trough rectangular waveguide
$t$	Time
$T$	Period
TE	Transverse Electric
TE <sub>mn</sub>	TE mode with index $m, n$
THz	$1 \times 10^{12}$ Hz
TM	Transverse Magnetic
TM <sub>mn</sub>	TM mode with index $m, n$
TEM	Transverse Electric and Magnetic
TLM	Transmission Line matrix method
UHF	Ultra high frequency

$v$	Velocity
$V$	Electric potential
VHF	Very high frequency
$x, y, z$	Space coordinates in rectangular coordinate system
$\hat{x}, \hat{y}, \hat{z}$	Unit vector in $x, y, z$ direction
$x_i, y_i$	Coordinates of each nodes in an element, $i = 1, 2, 3$

# Chapter 1

## INTRODUCTION

### 1.1 Microwave and Its Applications

#### 1.1.1 Characteristics of Microwave

Waves are very common in nature. A wave can be described as a function of time ( $t$ ) and space position ( $x, y, z$ ) in a rectangular coordinate system. For example, a one-dimensional (1-D) sinusoidal wave propagating in the  $z$  direction can be expressed as

$$G(z, t) = A \cos[2\pi(f t - z/\lambda)] \quad (1.1)$$

and characterized by three parameters, the amplitude of the wave ( $A$ ), the frequency ( $f$ ), and the wavelength ( $\lambda$ ). Frequency ( $f$ ) is the rate of wave oscillation in cycles per second in (Hz) and the wavelength ( $\lambda$ ) is the distance in meters between two identical points on the wave. An example of a 1-D sinusoidal wave is shown in Figure 1.1.

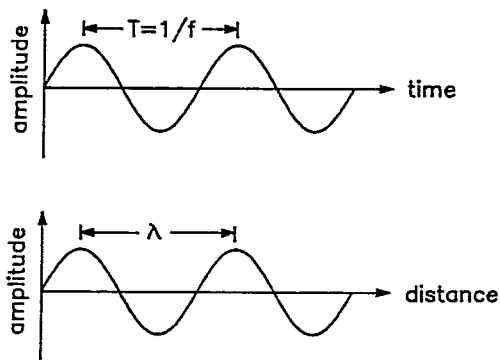


Figure 1.1 Amplitude of waves versus time and distance.

The velocity of the wave ( $v$ ) equals the product of its frequency ( $f$ ) and wavelength ( $\lambda$ ), i.e.

$$v = f\lambda \quad (1.2)$$

and the period of the wave ( $T$ ) is defined as

$$T = 1/f . \quad (1.3)$$

Microwaves are electromagnetic wave. Usually this includes the frequency range from 300 MHz to 300 GHz where the wavelength ranges from 1 m to 1 mm. The range of usable frequencies is called the electromagnetic spectrum and is shown in Figure 1.2. In theory, it is possible to have electromagnetic waves with frequencies from zero through to infinity. In practice the range is more limited but still extends over many tens of decades. From this figure, it is seen that microwaves are electromagnetic waves with frequencies just higher than AM/FM radio and lower than infrared. The frequency used in the microwave oven is about 2.45 GHz for instance.

<u>Frequency</u>	<u>Wavelength</u>	<u>Name of band</u>
$10^6$ (1MHz)	1km	radio
$10^9$ (1GHz)	1m	microwave
$10^{12}$ (1THz)	1mm	infrared
$10^{15}$ (1PHz)	$1\mu\text{m}$	optical
		ultraviolet

Figure 1.2 The electromagnetic spectrum

Classical electromagnetic theory [1] tells us that electrostatic fields are usually produced by static electric charge while magnetostatic fields are due to the motion of electric charges with uniform velocity (direct current) or static magnetic charges (magnetic poles). Time varying fields or electromagnetic waves are produced as a result of time-varying currents or accelerated charges.

It is worth noting [2] for microwave in general, that the decrease in wavelength as frequency increases corresponds to a reduction in component size, resulting in more compact systems. The features of high frequency and short wavelength of microwave are commonly used in communication and radar systems.



### **1.1.2 Microwave History and Application**

The forerunner of microwave was the development of the theories for static electric and magnetic fields. Particularly important for the background of electromagnetic waves is the work of Charles Coulomb who demonstrated the law of electrostatic force, Karl Friedrich Gauss who discovered the divergence properties of electric field and Andre-Marie Ampere who discovered the relation between a steady current in a wire and the associated magnetic force. The foundation for time-dependent electromagnetics was laid by Michael Faraday who found that a changing magnetic field induced a current in a wire, and vice versa. This led the theoretician James Clerk Maxwell in 1873 to summarize and extend all the empirical knowledge on electromagnetics into a single set of mathematical equations that have remained the basis for the theoretical study of microwaves to the present day.

In 1901, Guglielmo Marconi demonstrated that transmission across the Atlantic was possible by using lower frequency microwaves. This led to the rapid development of microwave communications, first at low frequencies, then at high frequencies. The 1930s saw a number of microwave developments. Microwave point-to-point communications was born and microwave waveguides were developed. A microwave radio link across the English Channel was built in 1931. Radar was developed in a number of countries and had an important impact on the Second World War. This was at very high frequencies (VHF) and ultra high frequencies (UHF). After the invention of the magnetron, microwave radar became the main type of radar because the higher

frequencies offered greater angular resolution. The concept of satellite communications was described by Arthur C. Clarke in 1945. The realization of the ideas awaited the availability of suitable rockets, but since the 1960s, satellite communications has formed one of the most important uses of microwaves for communications. It has led to the development of high performance antennas and microwave solid state devices. In the early 1970s, the use of microwaves was largely confined to radar, simple satellite systems and microwave radio links. Since then the potential and actual uses of microwaves has expanded rapidly. These include complicated terrestrial and satellite fixed and mobile communication systems, remote sensing, and heating applications.

## **1.2 Microwave Transmission Line**

Microwaves propagate in all directions in free space. Devices that guide microwave energy are called microwave transmission lines. There are three common types of transmission lines shown in Figure 1.3: coaxial cable, waveguide, and planar circuits (microstrip shown).

Coaxial cable consists of an inner conductor surrounded by an outer conductor. Microwave energy travels through the media filled between the two conductors. Microstrip, which consists of a conductor, a substrate and a ground plane, is very suitable for making microwave integrated circuits. Coaxial cable and microstrip have

large bandwidth and small size but have limited power-handling capability and relatively high attenuation at high frequencies.

Waveguide is another type of microwave transmission line. It is a hollow metal pipe with various shapes of cross-section. Usually, it has a rectangular or a circular cross-section. Waveguide offers the advantage of high power handling and low attenuation, but has the disadvantages of large size and narrow operating frequency range.

Ridged waveguide is a type of waveguide with one or more metal ridges connected to the walls of a hollow waveguide. Figure 1.4 shows some common structures of ridged waveguides. Ridged waveguide has wider operation bandwidth, lower characteristic impedance and more compact size compared to its counterpart of conventional hollow waveguide.

This thesis will concentrate on the study of ridged waveguides. Since one of the objectives of this thesis is to propose some new ridged waveguide structures with improved characteristics, an overview of the history of ridged waveguide will be presented in the next section.

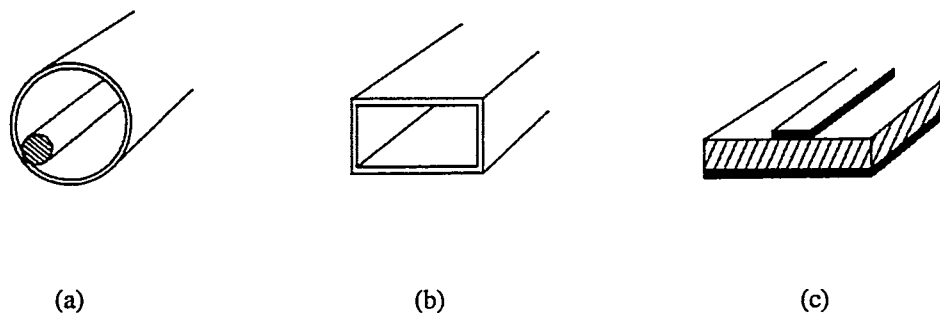


Figure 1.3 Structures of microwave transmission lines (a) coaxial cable, (b) rectangular waveguide, (c) microstrip.

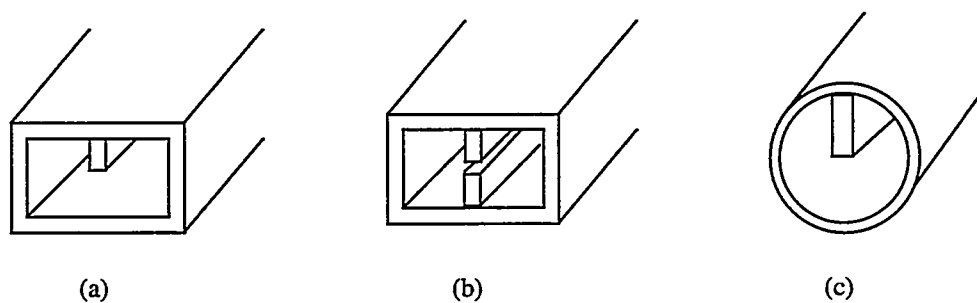


Figure 1.4 Ridged waveguide structures (a) single ridge rectangular waveguide (b) double ridge rectangular waveguide (c) single ridge circular waveguide.

### 1.3 Literature Survey

The role of ridges in waveguide is well known for enhancement of the bandwidth [3], [4]. Ridged waveguides have many applications in microwave and antenna systems because of their unique characteristics of low cutoff frequency, wide bandwidth, and low impedance [5],[6]. Early investigations were focused on properties

of single and double ridge rectangular waveguides by using the equivalent transmission line method [7]. Later, other structures were reported to be superior alternatives to the conventional single and double ridge waveguide. These structures include rectangular waveguide with one or two T-shaped septa [8], [9], two double ridge waveguide [10], antipodal ridge waveguides [11], double antipodal ridge waveguide [12] and quadruple ridge waveguide [13]-[18]. The T-septa waveguide was first introduced by Elliot [6] for creation of degenerate TE modes in a rectangular waveguide. In 1985, Mazumder and Saha [19], [20] proposed a novel rectangular waveguide structure with double T-septa. The structure is capable of offering considerable fundamental mode bandwidth improvement over the conventional rectangular waveguide with rectangular ridges. For several years since then, there was controversy over the bandwidth enhancement of such T-septa waveguide [19], [20], [21]. However, today the existence of enhanced bandwidth in a T-septa waveguide is well established.

Circular waveguides are counterparts of rectangular waveguides and they offer substantially higher unloaded Q-factor than that offered by rectangular waveguides. There have been several papers on the eigenmode analysis of circular waveguide with radial ridges and the use of such waveguides as filters [18], [22], [23], [24].

Ridged trough rectangular waveguide (SRTRW) was first used to build V-band (60-80 GHz) rectangular waveguide to CPW (coplanar waveguide) transition by Godshalk [25] in 1991. Further study of some characteristics of ridged trough

rectangular waveguides can be found in Guha and Saha [26], who show that a SRTRW can be a low impedance broadband structure.

The new ridged waveguide structures proposed in this thesis are an extension of those mentioned above.

## **1.4 Research Objectives**

Ridged waveguide has attracted much attention in the literature. One objective of this thesis is to propose new types of ridged waveguide structures with improved characteristics such as fundamental mode bandwidth and cutoff wavelength which will be explained in detail in Chapter 2.

In order to find the waveguide characteristics, such as bandwidth and cutoff wavelength, it is necessary to solve Maxwell's Equations for specific boundary conditions. For simple waveguide structures, such as rectangular or circular waveguide, analytical results may be obtained, but that is not always the case for more complicated structures. Numerical techniques are commonly used to find approximate results for complex structures. The finite element method (FEM) [27] is one of the common numerical techniques and was used to analyze the proposed ridged

waveguide structures in this thesis. Commercial software PDEase2 [28] based on 2-D FEM was used and also verified by comparing the results to those obtained from a recent published paper [29] and also to results obtained using a time domain simulator (MEFiSTo-2D) [30].

Numerical techniques, such as the moment method (MOM), finite element method (FEM), finite difference method (FDM) and mode matching method [27] have been widely used in solving EM problems. Many commercial software packages are available in the market which can give quite accurate results for complicated EM problems. Unfortunately, the computational time required usually makes using them prohibitive for engineering applications, especially in solving 3-D problems. The combination of the mode matching method, finite element method, and modern microwave network theory [27] is quite promising for overcoming this difficulty. If the 2-D modal solutions using FEM at each discontinuity inside a waveguide structure can be found, a 3-D waveguide structure can be analyzed by matching the fields at each discontinuity using the mode matching method. Then, the overall network parameters can be extracted using microwave network theory. This will convert a 3-D problem to a 2-D problem that will save lots of computational time. For the purpose of solving complex 3-D passive EM problems, an independent 2-D FEM program was developed as part of this research to analyze modal characteristics for different waveguide structures.

Actually, 2-D solutions can be also obtained from some commercial software packages like PDEase2. Unfortunately, the source codes typically are unavailable for inclusion in one's own 3-D analysis program. The 2-D results from the commercial software packages must be transferred manually to another program for further analysis. This makes the automatic analysis impossible. Therefore, an independent program to solve 2-D problems was developed for use in a future 3-D analysis program.

In summary, the objectives of this research include:

- Proposing new ridged waveguide structures with potential improvement in fundamental mode cutoff wavelength and bandwidth characteristics.
- Analysis of the modal characteristics of the first few modes (including cutoff wavelength, bandwidth, modal nomenclature, and field distribution) of these new ridged waveguide structures using PDEase2.
- Validation of the proposed analysis method by comparing the results obtained from PDEase2 with published results and also with results obtained using an alternative time domain analysis method (MEFiSTo-2D).
- Development and validation of an independent finite element method (FEM) program for future use in 3-D analysis.



## **1.5 Thesis Organization**

This thesis is divided into five chapters. The first chapter introduces the objectives of this thesis and describes its organization. A brief introduction to microwave history and applications as well as microwave transmission devices is presented. The survey of ridged waveguide history is also included. Chapter 2 presents the basic electromagnetic theory and the solutions of some simple waveguide structures on which some important concepts are established. Numerical techniques are also introduced. New ridged waveguide structures are also proposed in this chapter. Chapter 3 describes in detail the finite element method (FEM) and its application in solving 2-D EM problems. The procedure of programming the FEM is also introduced. Chapter 4 details the numerical results obtained and also the validation of PDEase2 and the developed computer program. The thesis is concluded in Chapter 5. Future work is also presented in the last chapter.

## **1.6 Summary**

This chapter presents an introduction to microwave, microwave transmission lines and various applications. A literature survey on ridged waveguide is included and the performance improvements offered by ridges is discussed. A potentially powerful numerical technique is proposed for analyzing complex 3-D structures including new ridged waveguide structures which may offer further performance improvement. This

technique necessitates development of an independent FEM program for future use in 3-D analysis. This chapter ends with a detailed description of the objectives and the organization of the thesis.

## **Chapter 2**

# **BACKGROUND**

Basic electromagnetic field theory is presented in this chapter as a foundation for the waveguide analysis. Some simple waveguide structures are analyzed using traditional analytical technique. The complexity of this analysis illustrates the difficulty in extending pure analytical techniques to complicated waveguide problems. The proposed new waveguide structures are introduced and the basic numerical technique to be used in analyzing these structures is presented.

### **2.1 Basic Electromagnetic Field Theory**

#### **2.1.1 Maxwell's Equations**

Maxwell's Equations of electromagnetic field theory were established by James Clerk Maxwell in 1873 and experimentally verified by Heinrich Hertz in 1888. Since then electromagnetic field theory has played an important role in the research and development of radio, television, telecommunications, radar, microwave heating, remote sensing, and numerous other practical applications.

The differential forms of Maxwell's Equations [31] in three-dimensional space

are

$$\nabla \times \vec{E}(\vec{r}, t) + \frac{\partial}{\partial t} \vec{B}(\vec{r}, t) = 0 \quad (2.1)$$

$$\nabla \times \vec{H}(\vec{r}, t) - \frac{\partial}{\partial t} \vec{D}(\vec{r}, t) = \vec{J}(\vec{r}, t) \quad (2.2)$$

$$\nabla \cdot \vec{B}(\vec{r}, t) = 0 \quad (2.3)$$

$$\nabla \cdot \vec{D}(\vec{r}, t) = \rho(\vec{r}, t), \quad (2.4)$$

where  $\vec{E}, \vec{B}, \vec{H}, \vec{D}, \vec{J}$ , and  $\rho$  are all functions of position and time. The parameters are defined as:

$\vec{E}(\vec{r}, t)$	= electric field strength	(volts/m)
$\vec{B}(\vec{r}, t)$	= magnetic flux density	(webers/m <sup>2</sup> )
$\vec{H}(\vec{r}, t)$	= magnetic field strength	(amperes/m)
$\vec{D}(\vec{r}, t)$	= electric displacement	(coulombs/m <sup>2</sup> )
$\vec{J}(\vec{r}, t)$	= electric current density	(amperes/m <sup>2</sup> )
$\rho(\vec{r}, t)$	= electric charge density	(coulombs/m <sup>3</sup> )

Equation (2.1) is Faraday's induction law based on the experimental fact that time-changing magnetic flux induces electromotive force. Equation (2.2) is the generalized Ampere's circuit law, which relates the magnetic field to current. Equation (2.3) and (2.4) are Gauss's laws for magnetic and electric fields which states that the magnetic field always form closed loops and the electric flux flowing out of a closed surface is equal to the charge enclosed [1].

## 2.1.2 Constitutive Relations

Maxwell's Equations are fundamental laws governing the behavior of electromagnetic fields in free space. It is also important to understand how electromagnetic fields behave in the presence of media. Material media is characterized by the constitutive relations.

The constitutive relations for an isotropic medium can be written as

$$\vec{D} = \epsilon \vec{E}, \quad \text{where } \epsilon = \text{permittivity} \quad (2.5)$$

and

$$\vec{B} = \mu \vec{H}, \quad \text{where } \mu = \text{permeability.} \quad (2.6)$$

Isotropy means that the field vector  $\vec{E}$  is parallel to  $\vec{D}$  and the field vector  $\vec{H}$  is parallel to  $\vec{B}$ . In free space without any matter,  $\mu = \mu_0$  and  $\epsilon = \epsilon_0$ , where

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henry/meter}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ Farad/meter.}$$

Inside a material medium, the permittivity  $\epsilon$  is determined by the electric properties of the medium and the permeability  $\mu$  by the magnetic properties of the medium.