MODAL CONTROL
OF
MACHINE INTERACTIONS

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"MODAL CONTROL OF MACHINE INTERACTIONS"

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ABSTRACT

The interactions among the individual machines in a power plant can pose problems in maintaining the quality of power generated at the plant. The individual controllers on each machine are not designed to handle these interactions. Using modal control theory, one method of minimising the machine interactions has been arrived at. First, a detailed model for the plant is constructed and from this, a reduced-order model describing the rotor dynamics of the machines in the plant is formulated. Using the reduced-order model, modal control theory is applied to minimise the machine interactions. Network reduction techniques necessary for the simulation of multi-machine plants are also given.

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PRINCIPAL SYMBOLS

subscript i	refers to the i^{th} machine
v_{ti}	terminal voltage of machine
v_{di}, v_{qi}	d and q axis components of the terminal voltage
i_{di}, i_{qi}	d and q axis components of machine currents
E_i	excitation voltage
ψ_{fdi}, ψ_{di}, ψ_{kdi} ψ_{qi}, ψ_{kqi}	flux linkages of machine field, d-axis armature, d-axis damper, q-axis armature and q-axis damper windings
i_{fdi}, i_{kdi}, i_{kqi}	rotor circuit currents
x_{afdi}, x_{ffdi}, etc.	machine self and mutual reactances
ω_o	rated angular frequency
ω_i	instantaneous angular frequency
$n_i = (\omega_i - \omega_o)/\omega_o$	per unit speed deviation
δ_i	rotor angle
T_{gi}	air gap torque
$T_{mi} = 2H_i$	inertia time constant
T_{rg}, K_r	time constant and gain of excitation system
superscript $'$	denotes matrix transpose
[A]	system coefficient matrix
[U]	modal matrix of eigenvectors of [A]
[V]	modal matrix of eigenvectors of $[A]'$
λ, ρ	eigenvalues
superscript *	denotes conjugate quantities

PRINCIPAL SYMBOLS (CONT)

R_i	armature resistance of the machine
$r_{fdi}, r_{kdi}, r_{kqi}$	rotor circuit resistances of the machine
G_{ij}	real components of a network self or mutual admittance
B_{ij}	imaginary components of a network self or mutual admittance
D	mechanical damping coefficient
$M_i = 2H_i$	inertia constant of the machine
$[I_n]$	n x n unity matrix
[x]	state vector of the plant
[y]	output vector of the plant
[z]	input vector of the plant
[B]	input matrix
[C]	output matrix
prefix Δ	indicates small changes
subscript o	indicates operating point values
superscript $^-$	indicates phasor quantity
prefix p	time differential operator
P_{mi}	mechanical power of the machine
P_{ei}	electrical power of the machine

1. INTRODUCTION

1.1 Introduction

The responses of individual machines in a power station to external disturbances can differ due to differences among the generating units or from differences in the machine controllers and their settings. These differences in individual responses are a cause of the mechanical oscillations among the rotors of the individual generators which if uncorrected, may lead to undesirable operation of the plant as a whole. There are several reasons for concern about these mechanical oscillations. They could give rise to periodic variations in the electrical quantities at the bus or in the tie line connecting the plant to the system. They could also give rise to sustained hunting of machines and controllers within the plant. If these sustained oscillations are very large, they should be accounted for in large system studies and thus they pose difficulties with respect to methods to be employed to represent the multi-machine plant by reduced-order models for large system studies. Hence, there is a need for a comprehensive study of the problem and to devise techniques that will eliminate or minimise the machine interactions.

A number of studies dealing with different problems in the areas of dynamic and transient stability of power systems have been reported in the recent past. A very important development has been the use of additional signals in the automatic voltage regulator (AVR) to provide additional damping of speed oscillations. References 1, 2 and 3 are among some of the more important papers in this area. Another prominent field of study has been the problem of constructing reduced-order models. Undrill et al[4,5] have investigated the problem in considerable depth and have constructed reduced-order models for realistic size power systems. References 6, 7 and 8 represent alternative approaches to the problem.

A number of studies have been reported concerning the dynamic stability problem itself. Janischewskyj and Kundur[9] have simulated the non-linear dynamic response of interconnected synchronous machines. Undrill[10] has developed modelling techniques for dynamic stability calculations for an arbitrary number of interconnected synchronous machines.

There are other studies concerning the effect of excitation control. Using frequency domain techniques, deMello and Concordia[11] have investigated in detail the effect of excitation control in the synchronous machine stability. Baker and Krause[12,13] have investigated the effect of excitation systems on the mechanical oscillations of the system. Yet another group of studies deals with the application of optimal control theory and estimation techniques to power system stability problems. Yu and Siggers[14] and Laha[15] have attempted to provide stabilisation through the use of optimal control principles.

The problem of interactions between the machines in a single plant has not been dealt with extensively so far. Dineley and Morris[16] have investigated the effect of machine inertia on the transient stability of a system comprising similar machines. Fleming[17] has discussed the basic interaction phenomena within a multi-machine plant and the models required for analytic studies. He has also suggested the types of studies that are required in this area. The work reported in this thesis is a first step in this direction and has been a result of a natural development of some of the ideas enumerated by Fleming[17].

1.2 Outline of the Research Reported in this Thesis

As stated in Section 1.1, the machine interactions are a result of the mechanical oscillations among the rotors. Hence, if the machines can be controlled so that the rotors move together, then the interactions are minimised. First, a reduced-order model representing the rotor dynamics is derived from a detailed model. Then, using the reduced-order model, modal control theory is applied to minimise the machine interaction.

In the following sections of this Chapter 1, a review of the basis phenomena of machine interactions is given.

The aspects of modal control thoery important to this particular application are discussed in Chapter 2. Basic familiarity with these topics facilitates an easy understanding of Chapters 3 and 4. The concepts presented in Chapter 2 are either recalled or are referred to later on in Chapters 3 and 4.

In Chapter 3, a reduced-order model is derived to represent the rotordynamics of a system. First, a detailed linear model is formulated following the procedure described by Undrill[4]. Then from a pair-wise eigenvalue analysis, the reduced-order model is arrived at.

Chapter 4 deals with the application of the modal control theory. The eigenvalue representing the interaction mode is changed to a more desirable value yielding improved responses. The results of several case studies are presented.

Conclusions and some suggestions are given in Chapter 5.

Network reduction techniques necessary for the simulation of multimachine systems are described in detail in the Appendix.

1.3 Basic Phenomena Involved

The generators within a multimachine plant are coupled together through the inherent synchronous connections of the generators to a common electrical bus. The artificial cross connections of the speed and excitation controllers for load and bar sharing introduce additional coupling.

Interactions among the machines within a plant can be initiated internally or externally. Internally, any adjustment to controller settings or disturbances which do not affect all units identically can initiate a sequence of interactions among the units. External disturbances which are reflected onto the common plant bus can also initiate internal interactions if the individual units do not respond in the same way in concert to the disturbance that they all see.

A simple mechanical analogy will help to illustrate the nature of these interactions. Figure 1.1 shows a system of four masses, each connected through a spring and a dashpot to a common bar and the bar is connected, in turn,

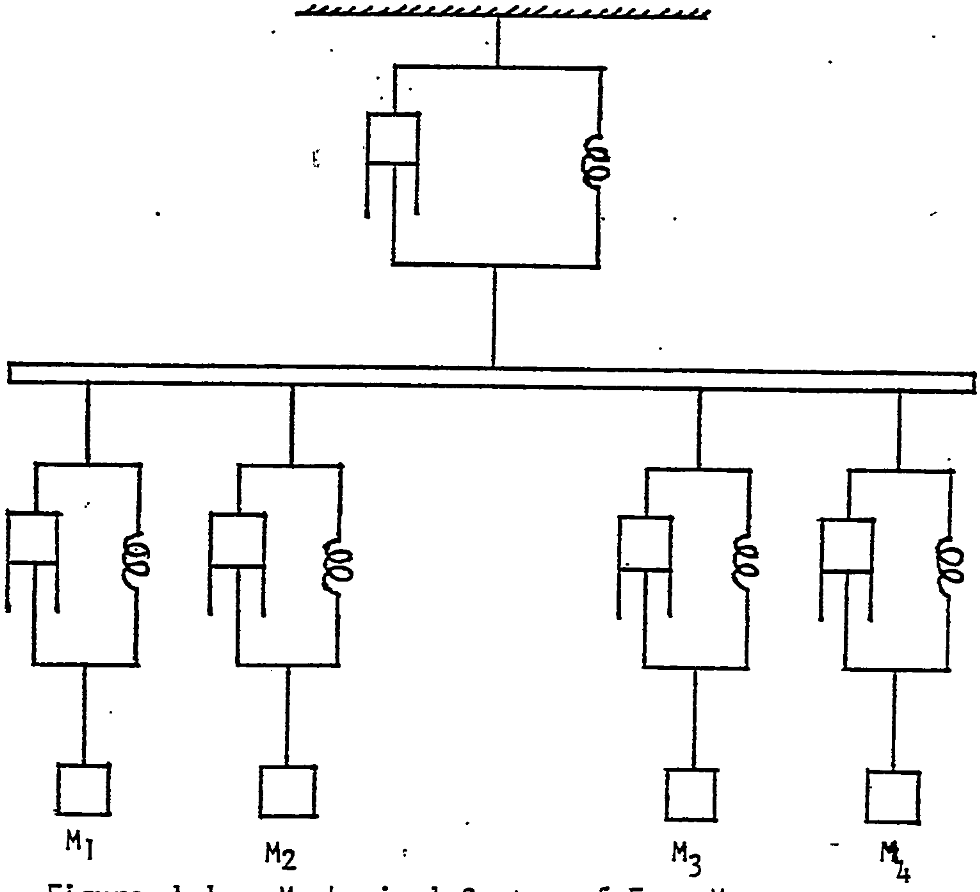

Figure 1.1 Mechanical System of Four Masses

through a spring and dashpot to a stationary frame. Now, if a small force is applied to the bar for an instant and withdrawn, the bar begins to oscillate. Each of the masses begins to oscillate with respect to each of the other masses and the system of masses and the bar as a whole oscillates with respect to the stationary frame. The situation in a multimachine plant is similar. The masses are analogs of machine inertias and the springs and dashpots are analogs of the synchronising and damping torque coefficients respectively. Each machine pair has a frequency of oscillation and the plant as a whole has a frequency of oscillation with respect to the rest of the system.

Simple generator models, like the constant voltage behind transient reactance model, cannot be used to produce reliable quantitative information because they do not recognise adequately the effects of the various generator rotor circuits and controllers.

In the next sections of this chapter, the types of analytical models that are required and the network reduction techniques that are necessary for the simulation of multimachine plants are discussed.

1.4 Analytical Models Required

There are several kinds of mathematical models of generators available from the literature ranging from the simple constant voltage behind transient reactance to the complex multiwinding rotor circuit representation. Schulz[19] and Dandeno, Hauth and Schulz[20] have demonstrated the effects of machine models on large system transient stability calculations and they have defined a range of models that can be used. These studies point to the conclusion that for an accurate representation of the dynamics of machines in a plant, it is necessary to represent the rotor electrical features by at least one damper winding in each axis.

The basic machine block in its linearised form, defined by Undrill[10], has these features and also the procedure of constructing the model for an arbitrary number of machines is generalised in the paper. Hence, this model has been adopted for use in the present work. The model makes provision for any type of representation of controllers.

1.5 Conclusions

So far in this Chapter, the problem area, namely the machine interactions, with which this research work is concerned, is clearly defined. The proposed approach to the solution is stated briefly. The topics covered in this Chapter form a basic preparatory material to what follows in the remaining Chapters.

2. FUNDAMENTALS OF MODAL CONTROL THEORY

2.1 Concepts of Modal Control

Some of the important results of modal control theory are discussed in this Chapter. Porter and Crossley[23] have dealt with the subject extensively. The results presented here and the terminology adopted are based on their work.

Let the system to be controlled be represented by the differential state equation

$$[\dot{x}(t)] = [A][x(t)] + [B][z(t)] \tag{2.1}$$

and the algebraic output equation

$$[y(t)] = [C]' \; [x(t)] \tag{2.2}$$

where

[A]: n x n is the plant matrix or coefficient matrix,

[B]: n x r is the input matrix, and

[C]: n x q is the output matrix

The main concept behind modal control is that of generating the input vector of a system by linear feedback of the state vector in such a way that the resulting closed-loop system has prescribed eigenvalues. This means that the resultant closed-loop system has dynamical response modes which are predetermined.

To illustrate the idea, consider the following first order scalar state equation

$$\dot{x}(t) = a\,x(t) + b\,z(t) \tag{2.3}$$

where a and b are real constants.

In the absence of control, i.e., when $z = 0$, the state of the system at any time t is given by the equation

$$x(t) = x(0) \exp(at) \tag{2.4}$$

where exp (at) defines the single dynamical mode of the system (2.3). If a is positive, the uncontrolled system will be unstable and if a is negative, it will be stable. Let the system be stable but the decay $x(t) \rightarrow 0$ not sufficiently rapid. Now if linear feedback of state according to the control law,

$$z(t) = g\,x(t) \tag{2.5}$$

is introduced, equation (2.3) assumes the closed loop form,

$$\dot{x}(t) = (a + bg)\,x(t) \tag{2.6}$$

where g is an arbitrary real constant.

Figure 2.1 shows the introduction of this additional feedback.

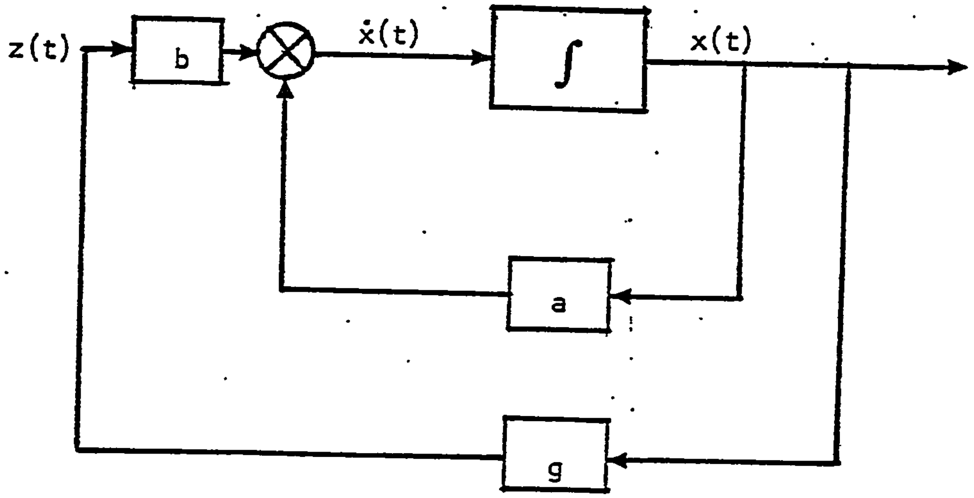

Figure 2.1 Example System with State Feedback

Since the solution of equation (2.6) may be written in the form

$$x(t) = x(0)\exp(\rho t) \tag{2.7}$$

where $\rho = a + bg$, it is obvious that ρ may be assigned any arbitrary real negative value simply by choosing g in the feedback law (2.5), according to the formula

$$g = (\rho - a) / b \tag{2.8}$$

A generalisation of this procedure of modal eigenvalue reassignment constitutes the main part of modal control theory. Some of the important results of modal control theory which are relevent to this work are discussed in the following sections.

2.2 Dynamical Characteristics of Linear Systems

2.2.1 Free-response characteristics

In the absence of any input, the state equation of the system governed by equation (2.1) assumes the form,

$$[\dot{x}(t)] = [A][x(t)] \tag{2.9}$$

where $[x(t)]$ is an $n \times 1$ state vector whose variation with time defines the free motion of the system. The eigenvalues and eigenvectors of the plant matrix [A] describe the precise nature of the free motion of the system following any disturbance.

If [A] has n distinct eigenvalues λ_i $(i = 1,2, \dots n)$ then it will also have n corresponding linearly independent $n \times 1$ eigenvectors $[u_i]$ $(i = 1,2, \dots n)$ which are related by the equations,

$$[A][u_i] = \lambda_i [u_i] \qquad (i = 1,2, \dots n) \tag{2.10}$$

The n eigenvalues of [A] are given by the n roots of the characteristic equation

$$\left| [A] - \lambda [I_n] \right| = 0 \tag{2.11}$$

where $[I_n]$ is the $n \times n$ unit matrix.

The eigenvectors of [A] are obtained by solving equations (2.10) for each of the $[u_i]$ after substitution of the corresponding eigenvalue λ_i into the appropriate equation. $[u_i]$ are determined only to within a scalar multiplier.

In addition to the eigenproperties of [A], the corresponding properties of the transposed plant matrix $[A]'$ play an important role in the modal analysis of linear systems. If $[A]'$ has n distinct eigenvalues μ_j $(j = 1,2, \ldots n)$ and n corresponding $n \times 1$ eigenvectors $[v_j]$ $(j = 1,2, \ldots n)$, these quantities are related by the equations,

$$[A]' \, [v_j] \quad = \quad \mu_j [v_j] \qquad (j = 1,2, \ldots n) \tag{2.12}$$

which are analogous to equation (2.10)

The n eigenvalues of $[A]'$ are given by the n roots of the characteristic equation

$$\left| [A]' - \mu [I_n] \right| \quad = \quad 0 \tag{2.13}$$

Since $[I_n]$ is symmetric and since

$$\left| [M]' \right| \quad = \quad \left| [M] \right|$$

for any square matrix [M], equation (2.13) implies that,

$$\left| [A] - \mu [I_n] \right| \quad = \quad 0 \tag{2.14}$$

By comparing equations (2.11) and (2.14), it is clear that [A] and $[A]'$ have the same eigenvalues

$$\mu_j \quad = \quad \lambda_j \qquad (j = 1,2, \ldots n) \tag{2.15}$$

However, the corresponding sets of eigenvectors will not in general be equal. But, since $\mu_j \neq \mu_k$ $(j \neq k,\ j,k = 1,2, \ldots n)$, the n corresponding

$n \times 1$ eigenvectors $[v_j]$ $(j = 1,2, \ldots n)$ given by equation (2.12) will be linearly independent as are the $[u_i]$ $(i = 1,2, \ldots n)$.

Now, in view of equation (2.15), equation (2.12) can be rewritten in the form

$$[A]' \; [v_j] \;\; = \;\; \lambda_j \; [v_j] \qquad (j = 1,2, \ldots n) \qquad (2.16)$$

Transposing equation (2.16) and post-multiplying by an eigenvector $[u_i]$ $(i \neq j)$ of $[A]$, it follows that

$$[v_j]' \; [A] \; [u_i] \;\; = \;\; \lambda_j \; [v_j]' \; [u_i] \quad (i \neq j;\; j = 1,2, \ldots n) \qquad (2.17)$$

Similarly, pre-multiplying equation (2.10) by a transposed eigenvector $[v_j]'$ $(j \neq i)$ of $[A]'$, it follows that,

$$[v_j]' \; [A] \; [u_i] \;\; = \;\; \lambda_i \; [v_j]' \; [u_i] \quad (i \neq j;\; j = 1,2, \ldots n) \qquad (2.18)$$

Subtraction of equation (2.17) from equation (2.18) yields

$$(\lambda_i - \lambda_j)[v_j]' \; [u_i] \;\; = \;\; 0 \qquad (i \neq j;\; j = 1,2, \ldots n) \qquad (2.19)$$

which in turn implies that

$$[v_j]' \; [u_i] \;\; = \;\; 0 \qquad (i \neq j;\; j = 1,2, \ldots n) \qquad (2.20)$$

This shows that eigenvectors of $[A]$ and $[A]'$ corresponding to different eigenvalues are orthogonal. In the case of eigenvectors corresponding to the same eigenvalue, it is clear that,

$$[v_i]' \; [u_i] \;\; = \;\; c_i \qquad (i = 1,2, \ldots n) \qquad (2.21)$$

where c_i are non-zero constants. Normalising the eigenvectors transforms equation (2.21) to

$$[v_i]' \; [u_i] \; = \; 1 \quad (i = 1,2, \ldots n) \tag{2.22}$$

Euqations (2.20) and (2.22) may be combined to form the equations,

$$\begin{aligned} [v_j]' \; [u_i] \; = \; [u_i]' \; [v_j] \; &= \; 1 \quad \text{if} \quad i = j \\ &= \; 0 \quad \text{if} \quad i \neq j \end{aligned} \tag{2.23}$$

2.2.2 Modal properties of [A] and [A]$'$

Next, the following three matrices are introduced in terms of which the modal properties of [A] and [A]$'$ can be expressed. These are:

the n x n modal matrix of [A]

$$[U] \; = \; \Big[[u_1],[u_2], \ldots [u_n]\Big] \tag{2.24}$$

the n x n modal matrix of [A]$'$

$$[V] \; = \; \Big[[v_1],[v_2], \ldots [v_n]\Big] \tag{2.25}$$

and the n x n eigenvalue matrix of [A] and [A]$'$

$$[\Lambda] \; = \; \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ & & \\ 0 & & \lambda_n \end{bmatrix} \; = \; \text{diag} \;\; \lambda_1, \lambda_2 \ldots \lambda_n \tag{2.26}$$

In terms of these matrices, equations (2.10), (2.12) and (2.23) can be written in the respective forms

$$[A] \; [U] \; = \; [U] \; [\Lambda] \tag{2.27}$$

$$[A]' [V] \; = \; [V] \; [\Lambda] \tag{2.28}$$

and $$[V]' [U] \; = \; [I_n] \tag{2.29}$$

It follows readily from these equations that,

$$[V]' = [U]^{-1} \quad (2.30)$$

$$[V] = \left[[U]^{-1}\right]' \quad (2.31)$$

$$[U]^{-1}[A][U] = [\Lambda] \quad (2.32)$$

and

$$[U]\ [\Lambda][U]^{-1} = [A] \quad (2.33)$$

Now, using these results, the free-motion of the system described by euqation (2.9) can readily be analysed.

If a new state vector $[\xi(t)]$ is introduced into equation (2.9) by the transformation

$$[x(t)] = [U][\xi(t)] \quad (2.34)$$

where $[U]$ is the modal matrix of $[A]$, the new state equation has the form

$$[U][\xi(t)] = [A][U][\xi(t)] \quad (2.35)$$

It follows from equation (2.35) that

$$[\xi(t)] = [U]^{-1}[A][U][\xi(t)]$$

and therefore that,

$$[\xi(t)] = [\Lambda][\xi(t)] \quad (2.36)$$

in view of equation (2.32). The significance of equation (2.36) compared to equation (2.9) is that $[\Lambda]$ is a diagonal matrix whereas $[A]$ is not, which means that the new states $[\xi(t)]$ are completely decoupled whereas the states $[x(t)]$ are not.

Equation (2.36) implies that

$$\xi_i(t) = \xi_i(0) \exp (\lambda_i t) \qquad (i = 1,2, \dots n) \quad (2.37)$$

where $\xi_i(0)$ $(i = 1,2, \dots n)$ are the initial values of the components of $[\xi(t)]$. Now from equation (2.34) the original state vector $[x(t)]$ can be obtained as

$$[x(t)] = [U][\xi(t)] = [u_1, u_2, \dots u_n] \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_n(t) \end{bmatrix}$$

which gives

$$[x(t)] = [u_1]\xi_1(0) \exp(\lambda_1 t) + [u_2]\xi_2(0) \exp(\lambda_2 t) + \dots$$
$$+ [u_n]\xi_n(0) \exp(\lambda_n t) \qquad (2.38)$$

At $t = 0$, according to equation (2.38)

$$[x(0)] = [u_1]\xi_1(0) + [u_2]\xi_2(0) + \dots + [u_n]\xi_n(0) \qquad (2.39)$$

which together with equation (2.23) yields the formulae

$$[\xi_i(0)] = [v_i]' [x(0)] \quad (i = 1,2, \dots n) \qquad (2.40)$$

Now, equation (2.38) can be expressed in the form,

$$[x(t)] = [u_1][v_1]'[x(0)] \exp(\lambda_1 t) + [u_2][v_2]'[x(0)] \exp(\lambda_2 t) +$$
$$+ \dots + [u_n][v_n]'[x(0)] \exp(\lambda_n t)$$

or more compactly,

$$[x(t)] = \sum_{i=1}^{n} \{\exp(\lambda_i t)\} [u_i][v_i]'[x(0)] \qquad (2.41)$$

This equation (2.41) shows that the free motion of the continuous-time linear system given by equation (2.9) is a linear combination of n functions of the form $\{\exp(\lambda_i t)\}[u_i]$ $(i = 1,2, \ldots n)$ which describe the n dynamical modes of the system. Thus the shape of a mode is described by its associated eigenvector $[u_i]$ and its time-domain characteristics by its associated eigenvalue λ_i.

2.3 Mode Controllability and Observability Conditions

2.3.1 Introduction

The state-vector transformation

$$[x(t)] = [U][\xi(t)] \quad (2.34)$$

which was used in the previous section in connection with the free-response of the system given by equation (2.9) is also of significance in the study of the forced modal response of the systems governed by state equation (2.1) and output equation (2.2)

$$[\dot{x}(t)] = [A][x(t)] + [B][z(t)] \quad (2.1)$$

$$[y(t)] = [C]'[x(t)] \quad (2.2)$$

Under the transformation given by equation (2.34), these equations (2.1) and (2.2) assume the form,

$$[\dot{\xi}(t)] = [J][\xi(t)] + [U]^{-1}[B][z(t)] \quad (2.42)$$

$$[y(t)] = [C]'[U][\xi(t)] \quad (2.43)$$

where

$[J] = [U]^{-1}[A][U]$ is the Jordan cannonical form of $[A]$

$[B] = n \times r$ input matrix

$[z(t)] = r \times 1$ input vector

$[y(t)] = q \times 1$ output vector, and

$[C] = n \times q$ output matrix

Recalling equation (2.30) the modal matrix [V] of [A]' is related to the modal matrix [U] of [A] by

$$[V]' = [U]^{-1} \qquad (2.30)$$

Substituting this into equation (2.42) gives

$$[\dot{\xi}(t)] = [J][\xi(t)] + [V]'[B][z(t)] \qquad (2.44)$$

$$[y(t)] = [C]'[U][\xi(t)] \qquad (2.43)$$

An n x r matrix [P] and an n x q matrix [R] can be defined where

$$[P] = [V]'[B] \qquad (2.45)$$

$$[R] = [U]'[C] \qquad (2.46)$$

Now equations (2.44) and (2.43) can be written as

$$[\dot{\xi}(t)] = [J][\xi(t)] + [P][z(t)] \qquad (2.47)$$

$$[y(t)] = [R]'[\xi(t)] \qquad (2.48)$$

Matrix [P] is called the mode-controllability matrix of the system and [R] is called the mode-observability matrix of the system.

2.3.2 Mode-controllability matrix

In the case of distinct eigenvalues, the vector-matrix equation (2.47) can be written as the n uncoupled scalar equations

$$\dot{\xi}_i(t) = \lambda_i \xi_i(t) + \sum_{j=1}^{r} p_{ij} z_j(t) \quad (i = 1,2, \ldots n) \qquad (2.49)$$

where λ_i's are the elements of the matrix J given by

$$[J] = \text{diag}\,\{\lambda_1, \lambda_2, \ldots \lambda_n\} \qquad (2.50)$$

From equation (2.49), it is clear that the j^{th} input variable $z_j(t)$ can influence the element $\xi_i(t)$ of the state vector $[\xi(t)]$ only if

$$p_{ij} \quad = \quad [v_i]'[b_j] \quad \neq \quad 0 \qquad (2.51)$$

where $[v_i]$ is the i^{th} column of the modal matrix [V] and $[b_j]$ is the j^{th} column of the $n \times r$ input matrix [B]. The i^{th} mode of the system (2.49) is controllable by the j^{th} input variable if $p_{ij} \neq 0$ and is uncontrollable by the j^{th} input variable if $p_{ij} = 0$. Also, the i^{th} mode is said to be controllable if and only if it is controllable by at least one input variable and the system (2.49) is controllable if each of the n modes is controllable. Since p_{ij}'s are elements of the matrix [P], the matrix [P] is called the mode-controllability matrix.

2.3.3 Mode-observability matrix

Equation (2.48) can be written as

$$y_j(t) \quad = \quad \sum_{i=1}^{n} r_{ij}\,\xi_i(t) \qquad (j = 1,2, \ldots q) \qquad (2.52)$$

It is evident from this equation that the i^{th} element, $\xi_i(t)$ of the modal state vector will be present in the j^{th} output variable, $y_j(t)$, if

$$r_{ij} \quad = \quad [u_i]' \; [c_j] \quad \neq \quad 0$$

where $[u_i]$ is the i^{th} column of the $n \times n$ modal matrix [U] and $[c_j]$ is the j^{th} column of $n \times q$ output matrix [C]. Thus, the i^{th} mode of the system (2.47) is observable in the j^{th} output variable if $r_{ij} \neq 0$ and is unobservable in the j^{th} output variable if $r_{ij} = 0$. The i^{th} mode is said to be observable if it is observable in at least one output variable. The system

(2.48) and (2.52) is observable if each of the n modes is observable. Since r_{ij}'s are elements of the matrix [R], the matrix [R] is called the mode-observability matrix.

2.4 Single and Multi-Mode Control

2.4.1 Introduction

It was shown in Section 2.2 that the free response of an uncontrolled system is given by a linear combination of the dynamical modes of the system where the time-domain characteristics are determined by the eigenvalues of the plant matrix [A].

Now, if control loops are introduced which generate the input vector by linear feedback of the state vector of the plant, then the response characteristics of the resulting closed-loop system will no longer be determined by the eigenproperties of [A], but by the eigenproperties of some new closed-loop plant matrix whose eigenvectors and eigenvalues depend upon the precise nature of the feedback loops. By introducing appropriate feedback loops, it is possible to design a closed-loop system whose plant matrix is such that those of its eigenvalues which correspond to the controllable modes of the uncontrolled system can be assigned new values. Such a reassignment can be made to obtain closed-loop response characteristics that are superior to the corresponding characteristics of the original uncontrolled plant. This concept was introduced in Section 2.1. The following sub-section explains how to obtain the required control law.

2.4.2 Single and multi-mode control

If all the elements of the state vector [x(t)] are measurable by appropriate transducers, it will be possible to combine these transducer outputs to generate a signal given by the equation

$$s(t) = \sum_{k=1}^{n} \mu_k x_k(t) = [\mu]'[x(t)] \qquad (2.53)$$

where $[\mu]$ is a measurement vector. This signal $s(t)$ can then be amplified by a single proportional controller having a gain K, yielding an input variable $z(t)$ given by

$$z(t) = Ks(t) = K[\mu]'[x(t)] \qquad (2.54)$$

The original system equation is, from Section 2.1

$$[\dot{x}(t)] = [A][x(t)] + [B][z(t)] \qquad (2.1)$$

Assuming for simplicity, a single-input system, equation (2.1) can be written as

$$[\dot{x}(t)] = [A][x(t)] + [b]z(t) \qquad (2.55)$$

where $[b]$ is $n \times 1$ input vector.

Substituting equation (2.54) into equation (2.55), gives

$$[\dot{x}(t)] = [A][x(t)] + K[b][\mu]'[x(t)] = [C][x(t)] \qquad (2.56)$$

Equation (2.56) indicates that the effect of the input variable defined by equation (2.54) is to change the plant matrix $[A]$ to a new matrix $[C]$ given by

$$[C] = [A] + K[b][\mu]' \qquad (2.57)$$

If $[\mu]$ is chosen to be equal to $[v_j]$, the j^{th} eigenvector of the matrix $[A]'$, then the plant matrix of the controlled system given by equation (2.57) will have the form

$$[C] = [A] + K[b][v_j]' \qquad (2.58)$$

Post-multiplying equation (2.58) by $[u_k]$, the k^{th} eigenvector of [A], $k \neq j$, and using the result of equation (2.23) and (2.10)

$$[C][u_k] = [A][u_k] = \lambda_k[u_k] \tag{2.59}$$

indicating that $[u_k]$ and λ_k ($k \neq j$; $k = 1,2, \ldots n$) are eigenvectors and eigenvalues of [C] as well as of [A]. It is also evident from equation (2.58) and (2.23) that,

$$[C][u_j] = [A][u_j] + K[b] = \lambda_j[u_j] + K[b] \tag{2.60}$$

which implies that λ_j is not an eigenvalue of [C] and $[u_j]$ is no longer the corresponding eigenvector if $K \neq 0$. The effect of using the measurement vector $[v_j]$ is thus to change the eigenvalue λ_j to some new value ρ_j and the eigenvector $[u_j]$ to some corresponding new vector $[w_j]$ leaving the remaining $(n-1)$ pairs of eigenvalues and eigenvectors of the plant matrix of the uncontrolled system unchanged.

For the single input system under consideration the mode controllability matrix will be a $n \times 1$ vector given by

$$[P] = [V]' [b] \tag{2.61}$$

where [V] is the $n \times n$ modal matrix of $[A]'$ and [b] is the $n \times 1$ input vector.

The input vector [b] can be written in the form

$$[b] = \sum_{k=1}^{n} p_k[u_k] \tag{2.62}$$

where p_k ($k = 1,2, \ldots n$) are the elements of [P]. This is true because in view of equation (2.61), p_k can be written as

$$p_k = [v_k]'[b] \quad (k = 1,2, \ldots n) \tag{2.63}$$

and substituting this in equation (2.62) and using the results of equation (2.23) prove the validity of equation (2.62).

Now, transposing equation (2.58) and post-multiplying it by $[v_k]$ gives

$$[C]'[v_k] = [A]'[v_k] + K[v_j][b]'[v_k] \quad (k \neq j;\ k = 1,2, \dots n)$$

$$= \lambda_k[v_k] + K[v_j][b]'[v_k] \quad (k \neq j;\ k = 1,2, \dots n) \qquad (2.64)$$

using the results of equation (2.23). From equation (2.62)

$$[b] = \sum_{k=1}^{n} p_k [u_k] \quad (k = 1,2, \dots n) \qquad (2.62)$$

Therefore,

$$[b]' = \sum_{k=1}^{n} p_k [u_k]' \quad (k = 1,2, \dots n) \qquad (2.65)$$

Substituting equation (2.65) in equation (2.64) gives

$$[C]'[v_k] = \lambda_k[v_k] + K[v_j] \sum_{k=1}^{n} p_k[u_k]'[v_k] \quad (k \neq j;\ k = 1,2, \dots n)$$

$$= \lambda_k[v_k] + K[v_j] \sum_{k=1}^{n} p_k[v_k]'[u_k] \quad (k \neq j;\ k = 1,2, \dots n) \qquad (2.66)$$

Again, from equation (2.23), equation (2.66) can be simplified to

$$[C]'[v_k] = \lambda_k[v_k] + K p_k [v_j] \quad (k \neq j;\ k = 1,2, \dots n) \qquad (2.66)$$

and

$$[C]'[v_j] = (\lambda_j + Kp_j)[v_j] \tag{2.67}$$

which indicates that the new eigenvalue ρ_j is

$$\rho_j = \lambda_j + Kp_j \tag{2.68}$$

Hence, the proportional-controller gain necessary to alter the j^{th} eigenvalue λ_j to any desired real value is given by

$$K = (\rho_j - \lambda_j)/p_j \tag{2.69}$$

if $p_j \neq 0$, i.e., the j^{th} mode is controllable.

If λ_j is real, then the vector $[v_j]$ and the scalar p_j will also be real and the feedback loop incorporating $[v_j]$ as a measurement vector will therefore be physically realisable.

If λ_j is complex, then both λ_j and its conjugate will have to be altered simultaneously and the problem changes to the one of multimode control.

As in the case of single mode control, it can be shown that to change m eigenvalues $\lambda_1, \lambda_2, \ldots \lambda_m$ to $\rho_1, \rho_2, \ldots \rho_m$, the necessary controller gains K_j are given by

$$K_j = \prod_{k=1}^{m} (\rho_k - \lambda_j) \Big/ \left(p_j \prod_{\substack{k=1 \\ k \neq j}}^{m} (\lambda_k - \lambda_j) \right) \qquad (j = 1,2, \ldots m) \tag{2.70}$$

and the input signal z(t) is given by

$$z(t) = \sum_{j=1}^{m} K_j [v_j]' [x(t)]$$

$$= \sum_{j=1}^{m} \left[\frac{\prod_{k=1}^{m} (\rho_k - \lambda_j)[v_j]'}{p_j \prod_{\substack{k=1 \\ k \neq j}}^{m} (\lambda_k - \lambda_j)} \right] [x(t)] \tag{2.71}$$

$$= \quad [g]' \; [x(t)] \tag{2.72}$$

This control law will alter the eigenvalues $\lambda_1, \lambda_2, \dots \lambda_m$ of the plant matrix [A] of the uncontrolled system to prescribed new values $\rho_1, \rho_2, \dots \rho_m$, leaving the (n-m) eigenvalues $\lambda_{m+1}, \lambda_{m+2}, \dots \lambda_n$ unchanged. The control law defined in equation (2.71) will be real even in the case of complex eigenvelues. This follows from the fact that if $\lambda_k = \lambda_j^*$ then $[v_k] = [v_j]^*$ and $p_j = p_j^*$ and also the fact that the required new eigenvalues, ρ_k, will either occur in conjugate pairs or be real.

Thus, for example, in the case of a 5th order system, if it is desired to change the first three eigenvalues $\lambda_1, \lambda_2, \lambda_3$ to ρ_1, ρ_2, ρ_3 leaving λ_4 and λ_5 unchanged, the necessary proportional-controller gains are given by

$$K_1 = \frac{(\rho_1 - \lambda_1)(\rho_2 - \lambda_1)(\rho_3 - \lambda_1)}{p_1 \; (\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)}$$

$$K_2 = \frac{(\rho_1 - \lambda_2)(\rho_2 - \lambda_2)(\rho_3 - \lambda_2)}{p_2 \; (\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)}$$

$$K_3 = \frac{(\rho_1 - \lambda_3)(\rho_2 - \lambda_3)(\rho_3 - \lambda_3)}{p_3 \; (\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)}$$

and these gains will be associated with the measurement vectors $[v_1]'$, $[v_2]'$, and $[v_3]'$ to give the input signal,

$$z(t) = K_1[v_1]'[x(t)] + K_2[v_2]'[x(t)] + K_3[v_3]'[x(t)] \tag{2.73}$$

The resulting closed-loop system will have eigenvalues ρ_1, ρ_2, ρ_3, λ_4 and λ_5.

2.5 Some Implications of Modal Control Theory

The results presented in the foregoing sections are very useful from the viewpoint of this project work in two respects. First, a reduced-order model is derived from an eigenvalue analysis of a detailed model and secondly the eigenvalues of the reduced-order model are altered so as to obtain better response characteristics.

In deriving the rotor-dynamical reduced-order model, to be described in the next Chapter, it is necessary to identify the critical eigenvalue* representing rotor oscillations. Modal control theory implies that i^{th} element of the eigenvector $[v_j]$ represents the effect of the i^{th} state variable on the j^{th} eigenvalue. In a normalised eigenvector $[v_j]$, each element gives the effect of each state variable as a fraction of the total effect of all the state variables. For the critical eigenvalue representing rotor oscillations, the state variables which have dominant effect would be the rotor speed deviations. Checking for this property of dominant magnitudes of elements corresponding to the variables representing the rotor speeds in the eigenvector $[v_j]$ would help to determine whether or not λ_j is the critical eigenvalue.

To obtain the control law to control the machine interaction modes of response, direct use of the results of Section 2.4 are made.

Thus the results of this Chapter form a basic foundation in the development of Chapters 3 and 4.

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* This constitutes a definition of the term <u>critical eigenvalue</u> as it is used in this thesis.

3. REDUCED-ORDER MODEL BY PAIRWISE ANALYSIS

3.1 Necessity for Reduced-Order Model

A true representation of the dynamics of a single synchronous generator involves a large number of variables. The number of these variables increases if the AVR and governor dynamics are included. A typical model, representing a generator and its associated controllers, may have ten to twelve state variables. In a multimachine power plant, with ten machines for example, such a complete representation requires about 100-120 state variables. In such a case it is easy to lose the physical significance of each variable. Also, all the variables are not of the same importance. In most studies, attention is focused on one aspect of the system dynamics rather than on the complete dynamics of the system. For example, this study is concerned with the rotor dynamics of the plant. Other examples may be the study of effects of parameter variations of the AVR and governor on the system or the study of the effects of tie-line power or stabilising signals. In each of these cases a suitable reduced-order model could be developed to represent the most significant aspects of the system under study.

It is true that any one particular response cannot be isolated from the rest of the system responses because of the highly coupled nature of the state variables. However, if the reduced-order model can take into account the effect of all the state variables and represent a particular response faithfully, then the reduced-order model would be a valid model as far as that particular response is concerned. Such a model besides reducing the order of the system, gives a good physical understanding of the phenomenon involved. There are also computational advantages, since working with such models, where the number