POWER SYSTEM STABILITY STUDIES USING THE DIRECT METHOD OF LIAPUNOV

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"POWER SYSTEM STABILITY STUDIES USING THE DIRECT METHOD OF LIAPUNOV"

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ABSTRACT

This thesis illustrates the principle of the application of the Direct Method of Liapunov to the assessment of stability of a multi-machine electric power system subjected to multiple sequential switching operations. A Liapunov function originally proposed by El-Abiad is modified to handle cases in which the power system frequency after a disturbance settles at a value which differs from that prior to the disturbance. This thesis also presents some extensions to numerical analysis techniques which were required during the course of the work; specifically, the application of the Steepest Descent Method of Minimization to the solution of multi-dimensional periodic non-linear equation problems, and the generalization of the Runge-Kutta Method of Integration to the solution of n-simultaneous second order non-linear differential equations.

In general, it was found that the principle of power system transient stability analysis using the Direct Method of Liapunov can be extended to solve multiple sequential switching problems. The use of this method for stability study in switching problems could result in considerable reduction in computing time in cases where the cut-and-try processes of conventional analysis methods would otherwise be required; however, a more suitable Liapunov function must be found before this method can be considered to be of great practical value as a general power system stability analytical tool.

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LIST OF PRINCIPAL SYMBOLS

AC	the acceleration factor of the load-flow calculation
a_T	the step-size in the minimization processes
BD	the boundary of the region of asymptotic stability
D_i	the damping constant of the i-th machine (pu)
E_i	the magnitude of the voltage behind the transient impedance of the i-th machine (pu)
I_i	the current flowing into the i-th node (pu)
M_i	the inertia of the i-th machine (pu)
P_i	the real power input at the i-th node (pu)
P_{mi}	the mechanical input power to the i-th machine (pu)
Q_i	the reactive power input at the i-th node (pu)
R_{ai}	the armature resistance of the i-th machine (pu)
T_{cr}	the critical line reclosing time (radians,where 377 radians = 1 second)
T_{cs}	the critical fault clearing time (radians)
T_{ij}	the tap ratio from node i to node j
T_s, T_{sw}	the fault clearing time (radians)
V	a Liapunov function
V_i	the phasor voltage at node i (pu)
Y	the admittance matrix
X'_{di}	the direct axis transient inductance of the i-th machine (pu)
Y_{ij}	the magnitude of the transfer admittance from the i-th machine to the j-th machine (pu)

δ_i	the phasor angle of E_i (radians)
δ^s_{ir}	the stable equilibrium of the i-th machine with respect to the reference machine (radians)
δ^{us}_{ir}	the unstable equilibrium of the i-th machine with respect to the reference machine (radians)
θ_{ij}	the phasor angle of Y_{ij} (radians)
τ	time (radians)
Φ	a positive definite function of the steepest descent method of minimization
ω_i	the first time derivative of δ_i

1. INTRODUCTION

1.1 Steady State and Transient State Stability

The satisfactory and economical delivery of electric energy to the customers is the basic requisite of a power utility. It is not possible to guarantee this ability absolutely; however, system studies may indicate means of reducing the system interruptions, the operating cost and the capital investment per unit of load to the lowest practical level. Of these system studies considerable attention has been placed upon system stability evaluation. This thesis is devoted to a particular aspect of stability evaluation.

Power system stability is, in general, divided into two categories: (a) steady (or dynamic) state and (b) transient state. Steady state is concerned with small or gradual system changes, whereas the transient state deals with large or sudden disturbances. The gradual changes in load, manual or automatic changes of excitation, irregularities in prime-mover input, and so forth are considered as small perturbations; while a fault occurrence, any major switching action in a network, etc. are classified as large disturbances. A power system is said to be stable if the synchronous machines possess the ability to retain synchronism with one another after a perturbation or disturbance.

The transient stability problem of a power system must be studied to provide a basis for actions which will lead to more reliable service. Techniques to improve the transient stability must be considered if the possibility of system transient instability is excessive.

There are several techniques [6,11,23] for improving the transient stability of a power system although not all of these are necessarily applicable or feasible in all practical cases. Of the available techniques these are most significant:

A. improvement of the generator characteristics through:

(1) reduction of the transient reactances of the generators,

(2) increase in the generator inertia,

(3) moderate rates of excitation response and introduction of speed signals to the excitation control,

(4) reduction of the amortisseur resistance of the salient pole machine,

(5) introduction of high-speed prime mover control with the stabilizing feedback, etc.

B. changes in the network through:

(1) the addition of transmission lines,

(2) generator dropping,

(3) load shedding,

(4) the use of fast fault-clearing and high-speed line reclosing breakers,

(5) switched series or shunt capacitors in the network,

(6) connection of braking resistances, etc.

In the latter group, techniques (2) to (6) involve complex multiple switching operations. The analytical methods for system stability studies provided to date have definite limitations with regard to giving precise and/or rapid solutions to system stability questions when these network switching techniques are used. A conceptual understanding of the available methods and their limitations is necessary before proceeding to the consideration of the selection of a specific method.

1.2 Available Methods for Transient Stability Analysis:

The analysis of power system transient stability is, in fact, a study of a set of nonlinear differential equations, known as the swing equations. These are discussed in detail in Chapter 2. Since there is no universally applicable method for solving general simultaneous non-linear differential equations, some special methods which are applicable especially to the power system stability problems have been developed. These are discussed briefly hereunder.

(1) Equal area criterion method[19]:

The equal area criterion method is a classical method used to study the simple two-machine problem. It is given in most power system analysis textbooks for illustrative purposes. Its value lies more in the conceptual insight it provides rather than its practical value for quantitative evaluation.

Figure 1.1 B, shows the transmitted power for different conditions in the system of Figure 1.1 A plotted against the power angle between the generator and the motor load. The postfault system is stable if the fault is cleared during the period for which area B is greater than or equal to area A. The particular fault-clearing angle, δ_{cs}, when B is equal to A is known as the "critical switching angle". The time required for δ to swing to δ_{cs} is the maximum allowable time for successful fault-clearing, and is known as the "critical switching time". This is denoted as t_{cs}.

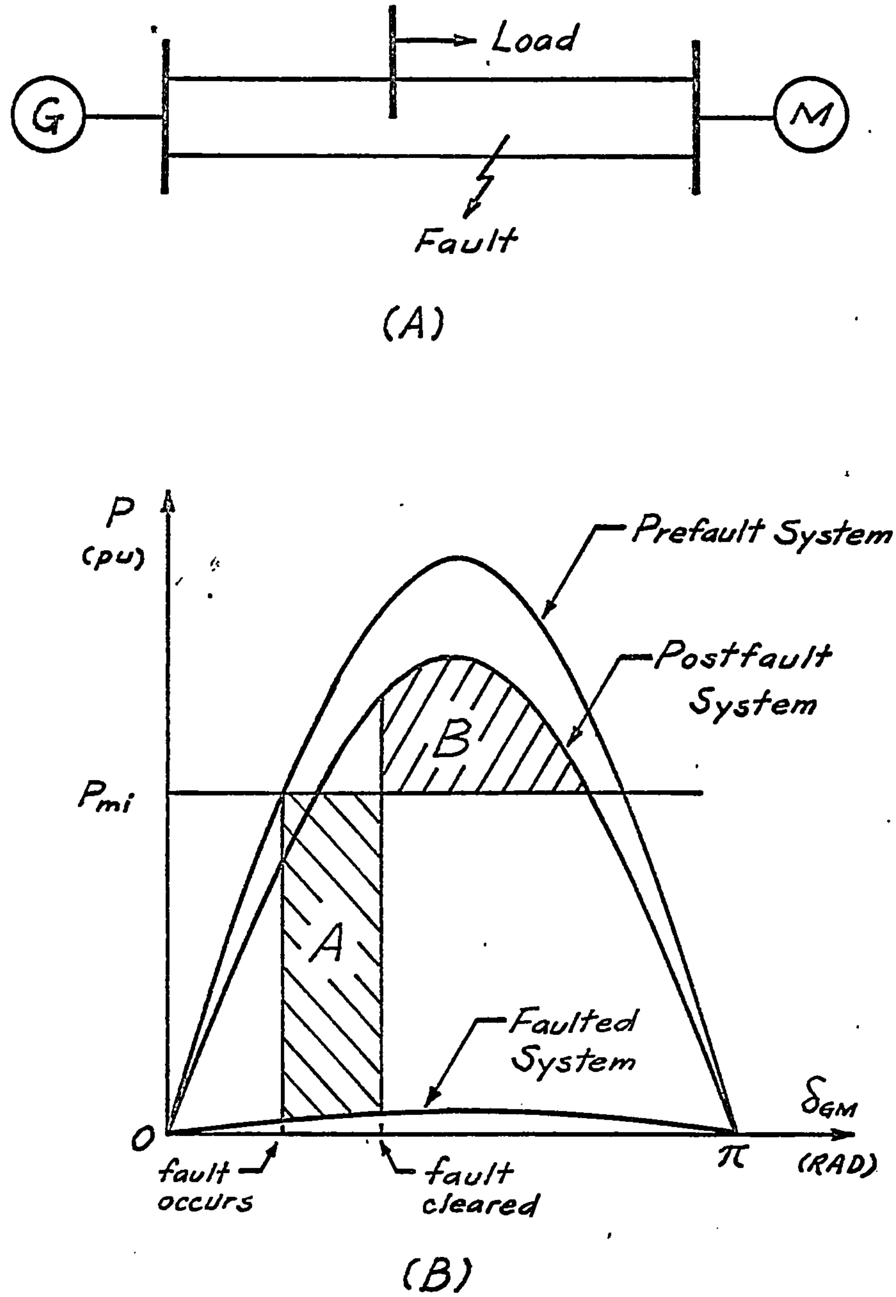

Figure 1.1 Equal area criterion method.
A -Single line representation of a power system.
B -Power transfer curves applied to equal area criterion method.

Since this method is capable of handling the simple two-machine problem only, it is of limited value for general use.

(2) Phase plane method[8,16]:

Graphical methods such as the phase plane techniques, e.g. isocline method, acceleration plane method, etc., could be applied to solve the power system transient stability problem; however, these techniques require very skilled and lengthy drafting work and so are impractical for general use. If the basic principles could be automated on a digital computer they might provide useful information. So far as the author is aware this has not been done to the practical multimachine system to date.

(3) Numerical integration methods:

A number of numerical integration methods have been developed to solve the nonlinear differential equation problem[4,18]. Of these the method which is most commonly used for power system work has been the so-called "step-by-step method". The step-by-step method is usually carried out with the aid of either a network analyser or a digital computer. The swing equations are solved by a step-by-step integration procedure, as shown in Figure 1.2, for a period after the fault occurrence. The accelerating power of each machine is assumed to be changed at the middle of the integrating step; while the angular velocity in each machine during the integrating step is assumed to be constant. The swing (or power angle) curves are calculated by directly integrating the velocity changes with respect to time. A step of 0.01 second is usually taken and an indication of transient stability is obtained if the machine angles of all machines are approaching each other at the end of the

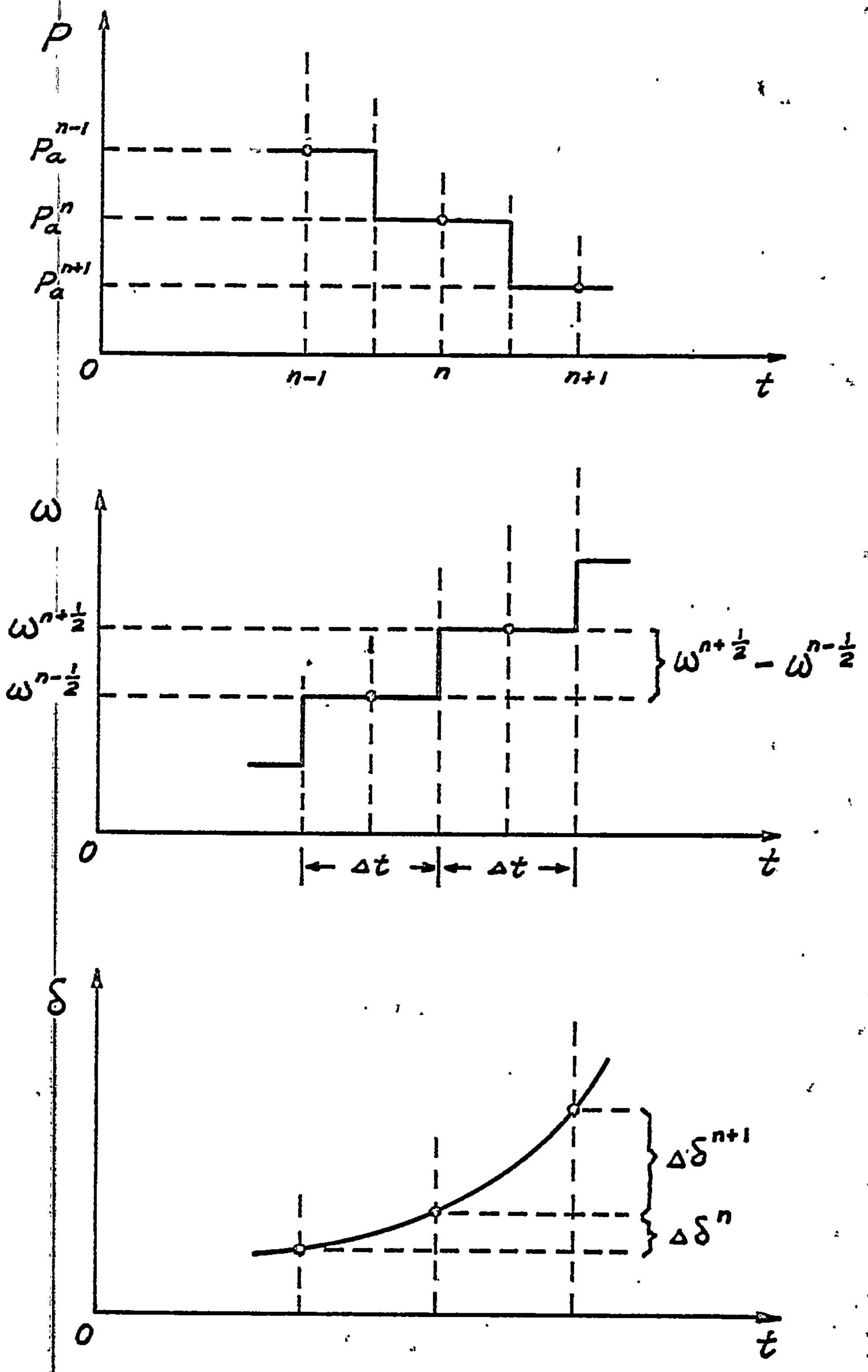

Figure 1.2 The step-by-step method of integration.

integration period. Since this method gives information of stability, up to the end of the integration period only, the system behaviour beyond the integration period is not predicted. Difficulties in giving any indication of system stability may arise if the rate of change of the angles at the end of integration are great. For critical condition investigations cut-and-try processes are required with these methods, and these are very time consuming.

(4) Energy integral method[1]:

The areas A and B in Figure 1.1 are proportional to the accelerative or the decelerative energies in a generator; this energy concept has been generalized to a multimachine system. Aylett in 1958 provided an outline of the basic development leading to this technique. The critical switching time of a multimachine power system is defined by the instant when the sign of the system energy integral, that is the kinetic energy minus the potential energy, changes from negative to positive. This method, in fact, is the simplest case of El-Abiad's approach to the Direct Method of Liapunov given in the next section.

(5) The Direct Method of Liapunov:

The Direct, or Second, Method of Liapunov[10,12,13,24,25] provides another approach to determine the stability behaviour of a system of simultaneous, first order, ordinary differential equations which does not require searching for the solution of the equations of motion. Stability information is obtained from the general properties of a function which describes the state of the system in the neighbourhood of the equilibrium. It is based on the generalized energy idea that if a system has an asymptotically stable equilibrium state, the stored energy of such a system displaced within the domain of attraction (or the region of asymptotic stability) decays naturally with increasing time until it finally reaches its minimum value at the equilibrium state.

This method requires the determination of a suitable energy function, called the Liapunov function. A Liapunov function, denoted as V, is a scalar function. Its first time derivative $\dot{V}$, or $\frac{dV}{dt}$, gives information on asymptotic stability, stability, or instability of the equilibrium state in the sense of Liapunov. Due to the lack of a systematic way of determining a suitable V function for a general non-linear system, the application of this method is not widespread.

The application of the Direct Method of Liapunov to the multi-machine power system transient stability problem was first proposed by Gless[9], El-Abiad and Nagappan[7] in 1966 for the critical fault-clearing time calculation. An energy integral function was chosen as the Liapunov function and the results were verified by the trajectories of the power angles of each machine due to the switching action at and after the critical fault-clearing time. Yu and Vongsuriya[21], in early 1967, reported the use of Zubov's proposal to construct the asymptotic stability region for the case of a single machine connected to an infinite bus including the transient saliency and variable damping effects. Unfortunately, it was found that the stability boundary was determined by a Liapunov function in a series form which was not monotonic. Later in 1967, Undrill[20] reported the application of the Direct Method of Liapunov to the power system optimization problem based upon the small signal equation at the rated operating condition. The extension of this work was found to be impractical due to the excessive amount of digital computer memory capacity required. In 1968, Rao[17] proposed a group of classical methods for the generation of Liapunov functions for a power system. In the same period, Siddiqee[24] proposed a new Liapunov function which included the effects of flux decay and first order representations of the speed governor and the voltage regulator for a one machine to infinite bus system. More recently, Willems[25] developed a Liapunov function with the inclusion of asynchronous damping torque effects on the same type of system. It was found to be very difficult to extend any of the results obtained by Rao, Siddiqee and Williams to a system with more than three machines, due to the same difficulties which were met by Gless. The proposal given by El-Abiad and Nagappan has been indicated to be the most suitable tool for a multimachine power system transient stability studies using the principle of the Second Method of Liapunov.

Of the methods stated above, the numerical integration method and the Direct Method of Liapunov are the two most suitable tools applicable to the multimachine power system transient stability problem. The combination of these two offers some significant advantages, since the Direct Method of Liapunov gives the information on stability at the equilibrium state which covers the whole time domain. The numerical integration process is required to provide the initial condition data necessary for the application of the Direct Method of Liapunov.

1.3 The Scope of This Thesis

The Direct Method of Liapunov was chosen as a tool for general multimachine power system transient stability studies in this thesis. The proposal given by El-Abiad and Nagappan was extended, modified and generalized to solve the general multiple switching problems which may arise in a power system following transient disturbances. The calculation of critical line-reclosing time and the effects on the transient stability of braking resistances are given as two specific multiple switching examples. The example given by Stevenson (Chapter 15 of (19)) is studied for the primary verification of the Liapunov Method to the critical switching angle calculation problem. Following this, a six machine system based on a simplified version of the Saskatchewan Power Corporation System was used as the numerical example for further illustrative purposes.

The theorem of Liapunov on asymptotic stability, the power system representation and the Liapunov function of the multimachine power system are illustrated in Chapter 2. Chapter 3 outlines some techniques such as the load-flow calculation procedure, equivalent Y-matrix calculations, the solution problem of system equilibria based on the Steepest Descent Method of Minimization and the Runge-Kutta Method of Numerical Integration, which are repeatedly applied in Chapter 4. Chapter 4 illustrates the application of the method and techniques developed in this thesis to the critical-fault-clearing time, the line-reclosing time calculation problems as well as the braking resistance effects. In Chapter 5 conclusions based on this series of investigations are given and suggestions are presented for further research in this field.

The most significant contributions arising from the research reported in this thesis can be enumerated as follows:

(1) the principle of application of the Direct Method of Liapunov has been extended to multiple sequential switching operation cases,

(2) El-Abiad's V-function has been modified to handle cases in which the power system frequency after a disturbance settles at a value which differs from that prior to the disturbance,

(3) El-Abiad's V-function and its modified form have been found to have limitations which severely restrict their application in practical problems and these limitations cannot be readily overcome,

(4) the Steepest Descent Method of Minimization has been adapted for the solution of multi-dimensional periodic function problems,

(5) the Runge-Kutta Method of Integration has been generalized and then simplified to solve the problem of n-simultaneous second order non-linear differential equations.

In general, it was found that the principle of stability analysis using the Direct Method of Liapunov can be extended to consider multiple switching sequence problems in a power system, but that a more suitable V-function must be found before this method can be considered to be of great practical value as a power system analytical tool.

2. POWER SYSTEM AND THE DIRECT METHOD OF LIAPUNOV

2.1 The Transient State Power System Description

From a mathematical viewpoint, any time-continuous system in the transient state can be expressed in terms of a set of nonlinear differential equations, and any switching actions performed in such a system can be analytically described by this set of equations with a replacement of the old parameters by a new set of parameters. The validity of the performance obtained from the equation set depends upon the completeness of the models of the system elements considered. It is ideally possible to describe a power system exactly under all conditions; however, this is impractical when the size of a system is at a practical level. The neglect of some second order effects is a necessity in actual studies especially in the analysis of transient stability of a multimachine power system.

2.1.1 Preliminary assumptions for power system transient stability studies.

A number of well recognized assumptions have been made [6,7,11,19] in power system transient stability studies, regardless of which analytical method mentioned in Section 1.3 was applied, for the purpose of simplifying the analytical problems. The more recent studies in power system analysis have been directed towards developing methods for which these assumptions need not be made. The more important of these assumptions are:

(1) the voltages behind the transient impedances are constant, (This implies constant flux linkage in the alternators and no automatic voltage regulator operation.)

(2) the saliency and the saturation effects in the alternators can be neglected,

(3) the machine damping coefficients are constant,

(4) loads other than synchronous motors can be represented as constant impedances,

(5) the speed governor effects are negligible,

(6) transformer voltages and speed voltage changes are negligible.

The main purpose of the first, the second and the sixth assumptions is to avoid the use of Park's equations[11] for generator representation which would require an excessive amount of digital computer memory. These assumptions really imply that the synchronous machines have cylindrical rotors, the constant flux linkage theorem applies, the voltage regulator action is slow and the speed changes are slow. The fourth assumption is based on the fact that the residential load dominates over industrial load; however, this is not true in certain areas, but may be acceptable from the overall system point of view. The fifth assumption may be accepted if the time constants of the governors are large and the effects of the feedback loops are small enough to be negligible. The machine damping is generally due to the characteristics of (a) the prime mover, and (b) the alternator. The prime mover damping is linearly proportional to the speed deviation. The alternator damping is mainly due to the effects of the field winding as well as the amortisseur winding performed as one type of asynchronous torque, and is a function of the transient performance of a machine during a disturbance; this type of damping is proportional to the speed deviation if it is not too large.

Lokay and Bolger[14] pointed out that the machine control features of importance for "first swing" stability studies ranked in order of importance are:

(a) machine damping,

(b) excitation control and resulting change of field flux linkage,

(c) saturation,

(d) system damping, and

(e) speed-governor action,

and that the stability limit obtained by the conventional method (i.e. the method with the previous assumptions as well as the assumption of constant shaft input power) agrees within 1 to 2 percent with the limits obtained when the assumptions are removed. In view of the fact that this study constitutes a preliminary investigation into the application of the Liapunov technique it was felt that these simplifying assumptions should be retained here to avoid unnecessary complication of the principles involved.

2.1.2 Power system equation[6,11] to be studied

Under the assumptions mentioned in Section 2.1.1, the equation which defines the performance of a power system in the transient state is given by Equation (2.1)

$$M_i \frac{d^2\delta_i}{d\tau^2} + D_i \frac{d\delta_i}{d\tau} + \sum_{K=1}^{n} E_i E_K Y_{iK} \operatorname{Cos}(\theta_{iK} - \delta_i + \delta_K) = P_{mi} \tag{2.1}$$

$$i = 1, 2, \ldots, n$$

where:

τ = the independent variable = $2\pi ft$ = time in radians

$$M_i = 4\pi f H_i = 4\pi f \times \frac{0.231 \times WR^2 \times (RPM)^2 \times 10^{-9}}{MVA\ (base)}$$

$= (4\pi f) \times$ (inertia constant of the i-th machine)

$D_i = 2\pi f d_i = (2\pi f) \times$ (damping coefficient for the i-th machine)

P_{mi} = the steady state mechanical input power to the i-th machine

$E_i \angle \delta_i$ = the voltage behind the transient impedance of the i-th machine

$Y_{iK} \angle \theta_{iK} = G_{iK} + j\, B_{iK}$

= the transfer admittance between the i-th and the k-th machines.

An alternative to Equation (2.1) is its state variable form given in Equation (2.2)

$$\left.\begin{aligned} \dot{\delta}_i &= \omega_i \\ \dot{\omega}_i &= \frac{1}{M_i}\left[P_{mi} - \sum_{K=1}^{n} E_i E_K Y_{iK} \cos(\theta_{iK} - \delta_i + \delta_K) - D_i \omega_i\right] \end{aligned}\right\} \quad (2.2)$$

$$i = 1, 2, \ldots, n$$

The state variable form of the system equations can be used to link with and apply the Direct Method of Liapunov which is presented in Section 2.2, and applied to the problem of power system stability analysis. It is to be noted that Equation (2.1) or (2.2) can be applied to describe a power system under any operation conditions such as

(a) the normal system (prefault system),

(b) a faulted system,

(c) a system with the fault isolated (post fault system),

(d) a system with the presence of artificial braking resistances,

(e) a system subjected to sequential relay operation,

(f) a system under unsymmetrical fault conditions, or

(g) the combination of any of the preceeding operations and others, by simply substituting the appropriate Y-parameter matrix elements and the proper initial conditions into the equation.

2.2 The Direct Method of Liapunov

Two methods for nonlinear system stability studies have been proposed by Liapunov[21]: (a) the Indirect (or First) Method, and (b) the Direct (or Second) Method.

The Indirect Method of Liapunov[15] treats the stability of a nonlinear system in terms of its approximate linearized equation. The stability information is obtained from the real parts of the roots of the linearized system equation in the same fashion as with the conventional Routh - Hurwitz Method[8] for linear systems. The terminology of the Liapunov First Method is not well known in the Western World and is usually overshadowed by the more common terms of linearization and Routh - Hurwitz Criterion. Since this method gives no indication of how small the neighborhood must be for the validity of the stability information of the linearized system compared to the original nonlinear system, it is merely applied to cases with slight nonlinearity or for the preliminary estimation of stability behaviour.

The Direct Method of Liapunov is a method which directly investigates the stability behaviour of a system without any approximation and gives the stability information directly without solving the system equations. It is applicable to both linear and nonlinear, forced or unforced, autonomous or nonautonomous systems. The key factor in the application of the Liapunov Direct Method is the construction or discovery of a so-called "Liapunov Function"; usually denoted as a "V-function". There is at present no systematic means of determining a suitable V-function for a generalized nonlinear system and this is the main handicap of the method and hinders its universal adoption.

Since all the theorems in the Direct Method are based on the stability definitions of Liapunov, some basic and useful definitions must be stated clearly before the theorems can be applied.

2.2.1 Definition of stability in the sense of Liapunov

The concept of stability of a linear system is clearly defined in terms of the bounded output response due to a bounded input. Hence, the stability is independent of the size of the input, and the region of stability is not in question. It is not the same, however, in the case of a nonlinear system since the stability in this case is generally a local phenomenon which is related closely to the initial condition of the system at the time a disturbance occurs.

There are basically three categories of stability concepts for a nonlinear system: these are commonly attributed to Laplace, Poincaré and Liapunov. Laplace stability is a boundedness concept, and Poincaré stability possesses orbital stability. The definition of stability in the sense of Liapunov is much more stringent and complex. For instance, in the Liapunov sense, Kalman and Bertran(12) define eight types of stability, Antosiewicz(2) nine types, etc. For the system of interest in this study, not all the definitions are required, so only the applicable aspects are given here.

A system such as that defined by Equation (2.2) can be expressed in a more general state variable form as

$$\dot{X} = F(X) \tag{2.3}$$

with origin $\dot{X} = F(0) = 0$,

$$\dot{X} = [\dot{x}_1 \; \dot{x}_2 \; . \; . \; . \; \dot{x}_n]^T \tag{2.4}$$

and

$$F(X) = [f_1(X) \; f_2(X) \; . \; . \; . \; f_n(X)]^T. \tag{2.5}$$

In the state space if $S(R)$ is a spherical region of radius $R > 0$ around the origin, and $S(R)$ contains points X satisfying $||X|| < R$, then the required stability definitions (after La Salle[13]) are as follows:

DEFINITION 1: Stability in the Sense of Liapunov:

The origin $F(0)$ is said to be stable in the sense of Liapunov, or simply stable, if, whenever for each $R < A$ there is an $r \leq R$ such that if a motion initiated at a point X_o of the spherical region $S(r)$ then it remains in the spherical region $S(R)$ even after; that is, a motion starting in $S(r)$ never reaches the boundary sphere $H(R)$ of $S(R)$ (Figure 2.1).

DEFINITION 2: Asymptotic Stability:

If the origin $F(O)$ is stable and in addition every motion starting inside some $S(R_o)$, $R_o > 0$, tends to the origin as $t \to \infty$ (Figure 2.1), then the system is asymptotically stable.

DEFINITION 3: Instability:

Whenever for some R and any r, no matter how small, there is always in the spherical region $S(r)$ a point X such that the motion through X reaches the boundary sphere $H(R)$ the system is unstable.

It is to be noted again that the preceeding definitions emphasize the local character of stability for nonlinear systems, since the region of initial conditions $S(r)$ for which a system is stable is bounded and arbitrarily fixed. As a consequence, for a linear system if the characteristic equation of the linear system has poles on the $j\omega$ axis it is stable in the sense of Liapunov. The asumptotic stability behaviour in linear system is a global asymptotic stability in the sense of Liapunov.

2.2.2 Liapunov asymptotic stability theorem

A large number of theorems exist which are related to the Direct Method of Liapunov; for example La Salle[13] lists nineteen, Hahn[10] lists more than seventy. For the power system stability problem, the local asymptotic stability theorem is applicable and is

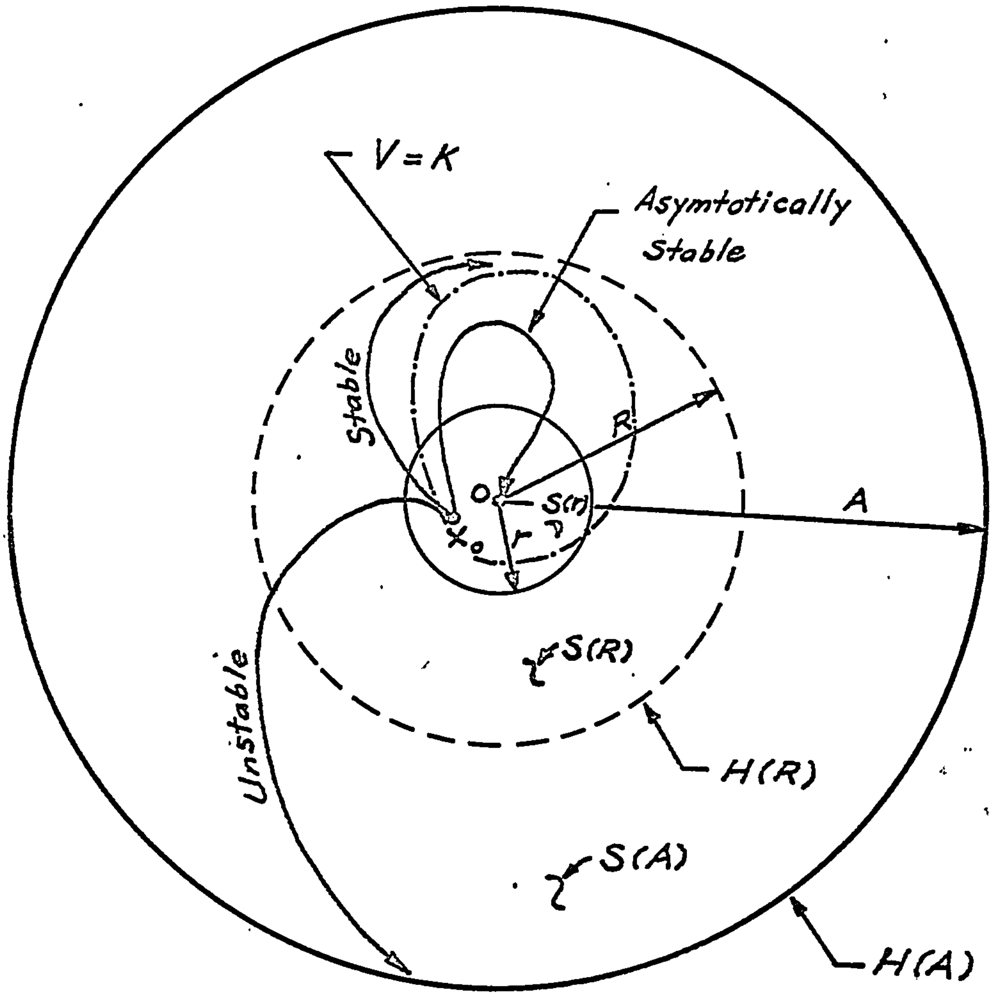

Figure 2.1 The definition of stability in the sense of Liapunov.

of primary importance. This is because only the first period of the power transfer curve, which lies in the region of $\pm\pi$ approximately, is of interest; moreover, the ability of each machine to maintain synchronism with the system as a whole is an asymptotic phenomenon. The Local Asymptotic Stability Theorem based on Liapunov[10,13] can be stated as follows:

(1) if Ω is a neighborhood which includes the origin of Equation (2.3) [which is $\dot{X} = F(X)$], and

(2) if in region Ω, there exists a scalar function, namely a "Liapunov function V" which is differentiable, bounded and positive definite,

(3) and the total time derivative of V is negative definite in Ω,

then the origin of Equation (2.3) is asymptotically stable in region Ω. This theorem can be understood by referring to Figure 2.1 and considering V as an equipotential parametric surface. Since V is differentiable, the first partial derivatives with respect to the state variables exist, therefore the total time derivative of V, which is

$$\frac{dV}{dt} = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \cdot \frac{dx_i}{dt}$$

exists. This is actually implied by Equation (2.3). V is positive definite, i.e. $V(x_1, x_2, \ldots, x_n) \geq 0$ when at least one of the state variables is not zero and $V(0, 0, \ldots, 0) = 0$ at the origin (or the stable equilibrium). The value of V will decrease as time increases if $\dot{V}$ is negative definite for any initial condition X_o. When V decreases as time increases, the trajectory (the motion or the solution) will move towards the origin; and, based on Definition 2, the origin is asymptotically stable. Since both Equation (2.3) and the Liapunov function V are defined only in region Ω, the stability in this sense is a local concept.

If both the Liapunov function of Equation (2.3) and the largest region of Ω can be found, any motion initiated inside this region can be guaranteed to be asymptotically stable; thereby, the stability problem of Equation (2.1) or (2.2) can be likewise solved.

2.3 El-Abiad's Function

For a system of the type described by Equation (2.2), El-Abiad and Nagappan[7] have verified that the system energy function with the form of Equation (2.6)

$$V = \frac{1}{2}\sum_{i=1}^{n} M_i\omega_i^2 + \sum_{i=1}^{n}(E_i^2G_{ii} - P_{mi})(\delta_i - \delta_i^s) + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} E_iE_jY_{ij}$$

$$[\mathrm{Sin}(\theta_{ij} + \delta_i^s - \delta_j^s) - \mathrm{Sin}(\theta_{ij} + \delta_i - \delta_j)] \tag{2.6}$$

is a suitable Liapunov function; where δ^s is the stable equilibrium power angle of the i-th machine measured with respect to a synchronous reference. The first summation term in Equation (2.6) is the kinetic energy of the system, and is equal to $\sum_{i=1}^{n} M_i\int_o^{\omega_i}\omega_i d\omega_i$, where $\omega_i = \frac{d\delta_i}{d\tau}$; the last two summation terms on the right-hand side of Equation (2.6) are derived from

$$\sum_{i=1}^{n}\int_{\delta_i^s}^{\delta_i}(E_i^2G_{ii} - P_{mi})\,d\delta_i + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} E_iE_jY_{ij}\int_{\delta_{ij}^s}^{\delta_{ij}}$$

$$\mathrm{Cos}(\theta_{ij} - \delta_{ij})\,d(\delta_{ij}).$$

This is the potential energy of the system measured with reference to the equilibrium state $((\delta^s)\ (\omega^s))$, where (ω^s) is, in fact, equal to (0). Therefore, the function in Equation (2.6) represents the total energy of the system described by Equation (2.2).

From Equation (2.6) the first time derivative of V can be shown to be (see Appendix 7.1)

$$\dot{V} = -\sum_{i=1}^{n} D_i\omega_i^2 - 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} E_iE_jG_{ij}\omega_i \text{Cos}(\delta_i - \delta_j) \tag{2.7}$$

According to Theorem 3 mentioned previously, the original system of Equation (2.2) is asymptotically stable if and only if $\dot{V}$ in Equation (2.7) is negative definite. This is the case when the transfer conductances (G_{ij}) are all zero. In practical cases the values of G_{ij} are small so that the second part on the right hand side of Equation (2.7) can be neglected; therefore, Equation (2.6) can be accepted as a Liapunov function for the power system.

2.4 The Region of Asymptotic Stability Based on El-Abiad's Function

The region Ω as stated in Section 2.2.2 is the region of asymptotic stability. The maximum limits of this region are defined by the boundary BD. This domain contains only one stable equilibrium which is the origin of the system equation. If more than one equilibrium state were included in this region, it would not be guaranteed that the trajectory would tend to the desired equilibrium as time tended toward infinity. For this reason the boundary BD is defined by the unstable equilibrium (δ^{us}) closest to the stable equilibrium. Consequently, the substitution of (δ^{us}) for (δ) in the Liapunov function gives the boundary for asymptotically stable behaviour.

Since (ω) is a null matrix at the equilibrium state, the boundary (BD) for power system asymptotic stability determined from Equation (2.6) is

$$BD = \sum_{i=1}^{n} (E_i^2 G_{ii} - P_{mi})(\delta_i^{us} - \delta_i^{s})_{\text{final desired state}}$$

$$+ \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j Y_{ij} \text{ (final desired state)}$$

$$\times [\sin(\theta_{ij} + \delta_i^{s} - \delta_j^{s})$$

$$- \sin(\theta_{ij} + \delta_i^{us} - \delta_j^{us})]_{\text{final desired state}}$$

(2.8)

The region of asymptotic stability defined by Equation (2.8) depends upon the system conditions in the final state (called 'final desired state' or 'f.d.s.' in Equation (2.8)). For a simple fault clearing problem, the postfault system is the final desired state; therefore $(Y)_{\text{postfault}}$ [or $(Y)_{pf}$] with its associated $(\delta^{us})_{pf}$ and $(\delta^{s})_{pf}$ must be applied in Equation (2.8) to evaluate the boundary. The initial condition for the postfault system is the system condition immediately following the clearance of the fault. The postfault system is said to be stable in the sense of Liapunov if this initial condition is in the region Ω inside the boundary BD. In the simple line reclosing problem, the prefault system is the final desired state, the system condition immediately following the line-reclosing instant determines the required initial condition, and BD is obtained by the use of $(Y)_{\text{prefault}}$ [or $(Y)_{\text{pref}}$], $(\delta^{us})_{\text{pref}}$ and $(\delta^{s})_{\text{pref}}$ in Equation (2.8). In general any final state will be asymptotically stable provided its initial condition lies inside the boundary BD, and it will tend toward its stable equilibrium as time tends to infinity.

2.5 The Liapunov Function for Power System

If a system of Equation (2.2) is investigated carefully, it will be found that three types of performance (shown as group (A), group (B) and group (C) in Figure 2.2) can be classified. Group (A) is the case in which the frequency level of the final desired state of the power system is retained equal to the level before the fault occurrence. Group (B) represents the case of a higher frequency level in the final desired state, and group (C) shows a lower frequency level with respect to the level before the fault occurrence. Mathematically, both groups (B) and (C) are unstable because the solution increases or decreases monotonically as time increases. However, all are defined as stable operation by electric power engineers. Further, it is to be understood that the system of Equation (2.2) does not have any control giving the fixed frequency type (or Group(A)) of operation. Therefore, equations applied for stability studies, as shown in Equations (2.2), (2.6) and (2.8), must be modified to include groups (B) and (C) cases.

The modification of group (B) can be done by integrating Equation (2.2), setting $\delta_{iR} = \delta_i - \delta_R$, and replotting the motion as δ_{iR} v.s. time instead of δ_i v.s. time. The power angle of any machine can be chosen as δ_R; however, it is advisable to select the most stable one for easier curve plotting. The result of the modification process is shown in Figure 2.3, and the new result is similar to group (A) in Figure 2.2. Analytically, this type of manipulation is equivalent to the transformation of Equation (2.2) to Equation (2.9)

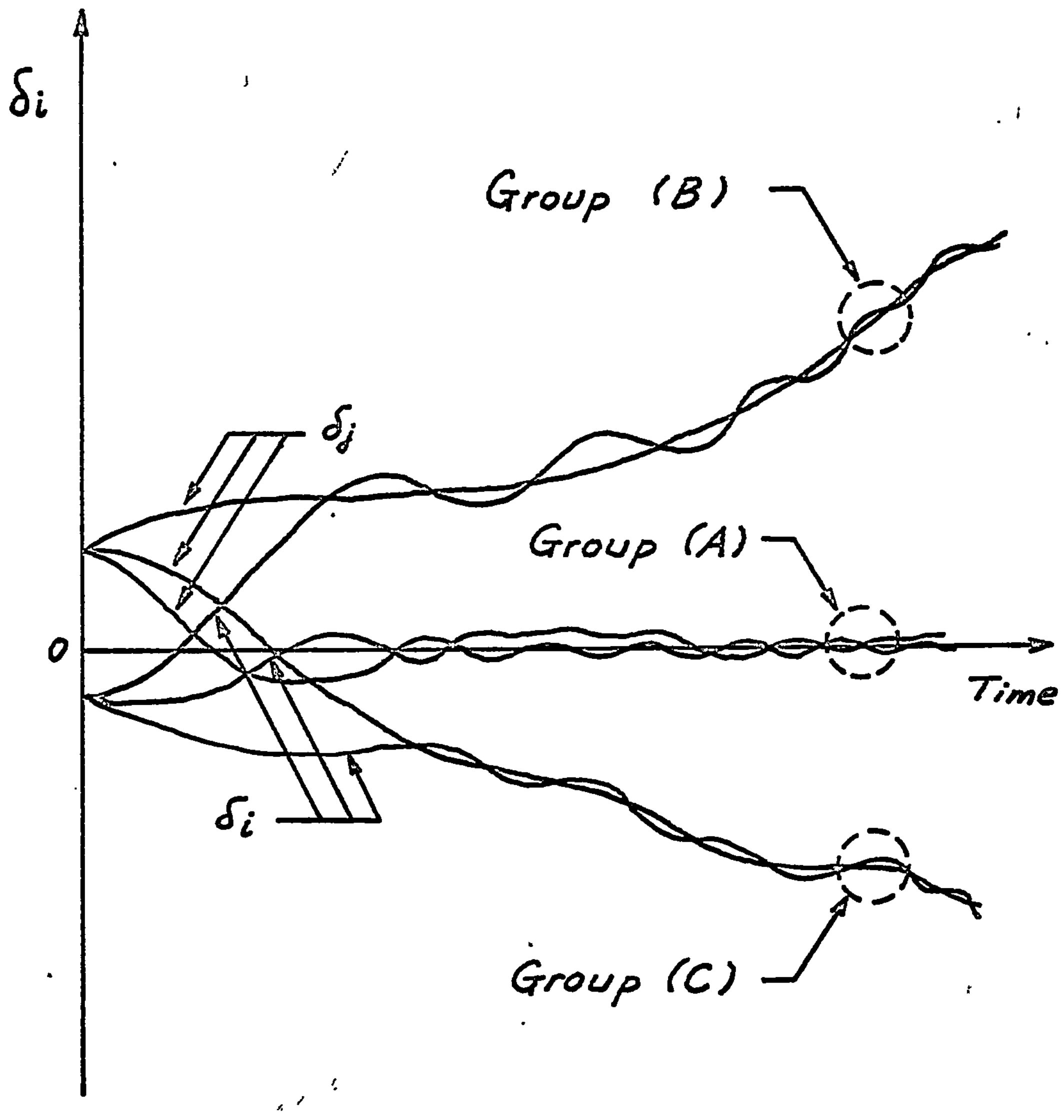

Figure 2.2 The types of swing curves expected from Equation (2.2).
A - frequency level after switching operations is equal to the level before switching.
B - frequency level after switching operations is higher than the level before switching.
C - frequency level after switching operations is lower than the level before switching.

$$\left.\begin{aligned}
\dot{\delta}_{iR} &= \omega_{iR} = \omega_i - \omega_R \\
\dot{\omega}_{iR} &= -\frac{D_i}{M_i}\omega_i + \frac{D_R}{M_R}\omega_R + \frac{P_{mi}}{M_i} - \frac{P_{mR}}{M_R} \\
&\quad - \sum_{j=1}^{n} E_j [E_i Y_{ij} \operatorname{Cos}(\theta_{ij} - \delta_{iR} + \delta_{jR}) \cdot \frac{1}{M_i} \\
&\quad - E_R Y_{Rj} \operatorname{Cos}(\theta_{Rj} + \delta_{jR}) \cdot \frac{1}{M_R}] \\
& i = 1, 2, \ldots, R-1, R+1, \ldots, n
\end{aligned}\right\} \tag{2.9}$$

Equation (2.9) represents a system with 2(n-1) equations and 2n unknowns. Since the number of variables is greater than the number of equations, the number of solution sets is infinite. If δ_R is considered as a dummy variable (or parameter) and set equal to δ_R^s, then Equation (2.9) has an unique solution. By the investigation of Figure 2.3(B), it is understood that δ_R can be set to δ_R^s because $\dot{\delta}_{RR}$ is always equal to zero. Similarly, this analysis can also be applied to group (C).

The Direct Method of Liapunov can be applied directly to include problems of the type shown in groups (B) or (C) by a simple modification of El-Abiad's V-function. The modified function is given in Equation (2.10)

$$\begin{aligned}
V = \frac{1}{2}\sum_{i=1}^{n} M_i \omega_{iR}^2 &+ \sum_{i=1}^{n} (E_i^2 G_{ii} - P_{mi})(\delta_{iR} - \delta_{iR}^s) \\
&+ \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j Y_{ij} [\operatorname{Sin}(\theta_{ij} + \delta_{iR}^s - \delta_{jR}^s) \\
&- \operatorname{Sin}(\theta_{ij} + \delta_{iR} - \delta_{jR})]
\end{aligned} \tag{2.10}$$

The validity of the application of Equation (2.10) depends upon whether or not the first time derivative $\dot{V}$ is negative definite. $\dot{V}$ can be obtained using the procedure previously used for Equation (2.7) with the additional condition that $\dot{\omega}_R = 0$. (Since $\delta_R = \delta_R^s$ = constant, hence $\omega_R = \dot{\omega}_R = 0$). This results in Equation (2.11)

$$\dot{V} = -\sum_{i=1}^{n} D_i \omega_{iR}^2 \tag{2.11}$$

in which $\dot{V}$ is negative definite. Therefore Equation (2.10) can be accepted as a generalized Liapunov function for power system transient stability studies including cases with frequency level shifting effects.

The boundary of local asymptotic stability is thus equal to

$$\begin{aligned} BD = & \sum_{i=1}^{n} (E_i^2 G_{ii} - P_{mi})(\delta_{iR}^{us} - \delta_{iR}^{s})_{f.d.s.} \\ & + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j Y_{ij(f.d.s.)} [\sin(\theta_{ij} + \delta_{iR}^{s} - \delta_{jR}^{s}) \\ & - \sin(\theta_{ij} + \delta_{iR}^{us} - \delta_{jR}^{us})]_{f.d.s.} \end{aligned} \tag{2.12}$$

The solution of Equation (2.9) is exactly equivalent to the solution of Equation (2.2) with the additional constrains that

$$\left.\begin{aligned} \delta_{iR} &= \delta_i - \delta_R \\ \omega_{iR} &= \omega_i - \omega_R \end{aligned}\right\} \quad i = 1, 2, \ldots, n \tag{2.13}$$

and is plotted as δ_{iR} v.s. time (shown in Figure 2.3(B)). In conclusion, it can be stated that Equation (2.2) associated with Equation (2.13), Equation (2.10) and Equation (2.12) are suitable tools for the study of power system transient stability using the principles of Liapunov. The analysis described in the following chapters of this thesis is based on these four equations.

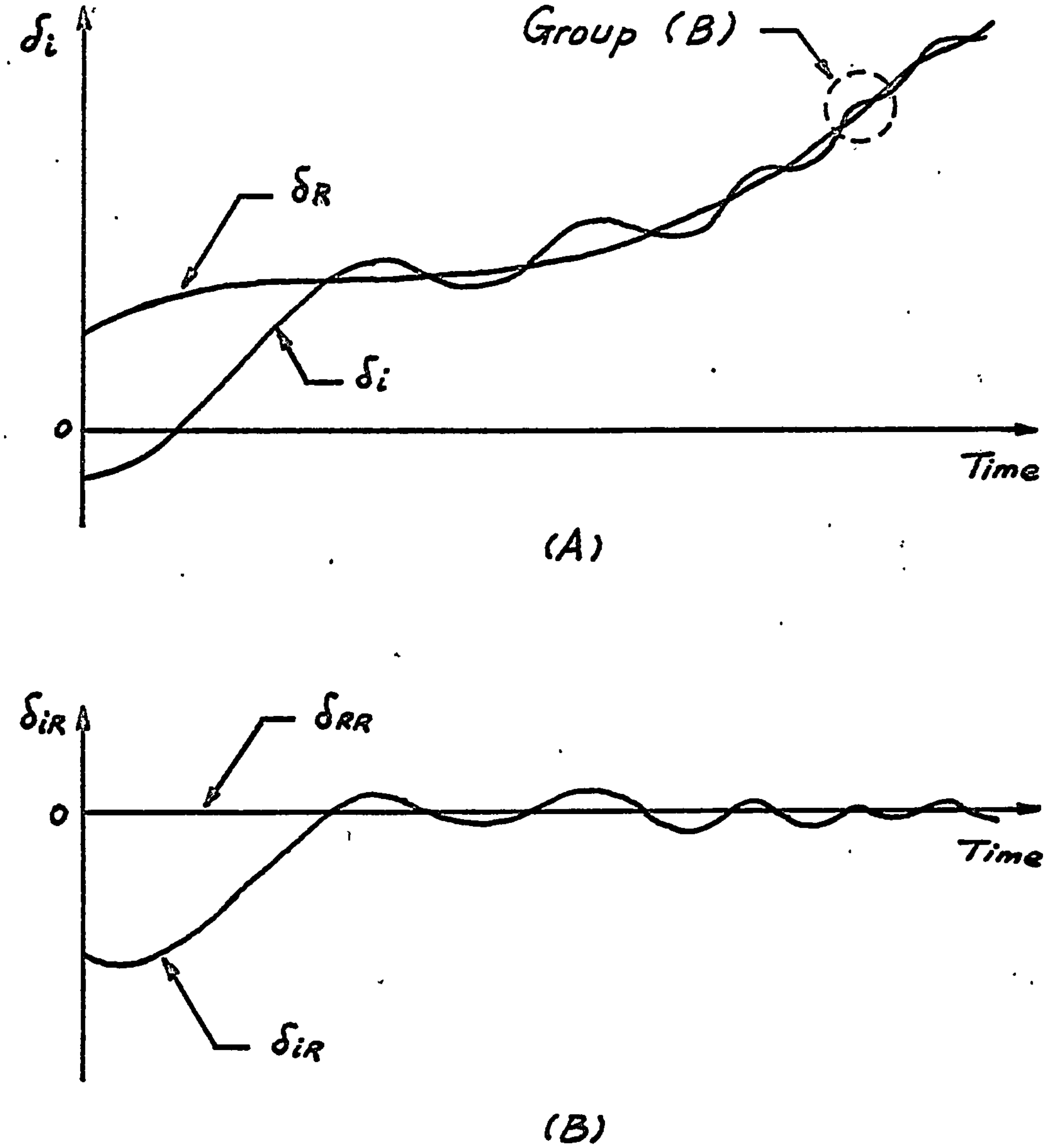

Figure 2.3 The swing curve transformation technique;
A - before transformation,
B - after transformation.

3. POWER SYSTEM ANALYSIS TECHNIQUES

3.1 Introduction

Before the Liapunov Direct Method can be applied in power system stability studies, the closely associated analytical techniques which give the initial conditions must be clearly understood. These are:

(a) load flow calculations,

(b) Y-matrix calculations,

(c) steepest descent method of minimization, and

(d) Runge-Kutta method of integration.

The purpose of this chapter is to discuss the techniques which were employed in this study to carry out these fundamental operations. An understanding of these fundamental tools is necessary before consideration of the application of the Liapunov theorem itself.

3.2 Load-Flow Calculations

The complete solution of the power system differential equations for a faulted system is obtainable only if the initial condition for the faulted system is known. Load-flow studies [19] are done on the prefault system to obtain this initial condition. For a specified system under specific operating conditions the voltage, the current, and the power factor or the reactive power at various nodes in the network under steady state operating conditions can be obtained.

In order to carry out a load flow study, a number of variables must be specified. These are:

(a) the series impedances and the shunt admittances of the transmission lines,

(b) the impedances and the tap ratios of the transformers,

(c) the voltage and power (both real and reactive) magnitudes at all generator buses except the swing generator bus,

(d) the voltage (both magnitude and angle) of the swing generator bus, and

(e) the real and reactive power levels at the load buses.

The load flow study is in fact a matter of solving a set of simultaneous non-linear algebraic equations. A large number of methods are available to solve the load flow problem[5]; the Gauss-Seidel method of iteration (Appendix 7.2) was applied in this study.

The nodal equation for a general passive electric network

$$I = YV \tag{3.1}$$

or

$$I_i = Y_{i1}V_1 + Y_{i2}V_2 + \ldots + Y_{ii}V_i + \ldots + Y_{in}V_n$$

$$i = 1, 2, \ldots, n \tag{3.2}$$

is used for system representation. Y_{ii} and Y_{ij} are the self- and the mutual admittances at the i-th node with respect to the j-th node. A sign of convention of

$$V_i^* I_i = P_i - jQ_i \tag{3.3}$$

for a generator output power with lagging power factor is used; where $V_i{}^*$ is the conjugate of V_i. The known (K) and the unknown (U) quantities for load-flow studies are listed in tabular form as follows:

	$\|V\|$	V	P	Q
Generator Bus	K	U	K	U
Swing Bus	K	K	U	U
Load Bus	U	U	K	K

Following the Gauss-Seidel method of iteration (shown in Appendix 7.2) the equations for solving the unknown quantities are as follows:

(1) For the generator bus:

$$Q_i = -\mathrm{Im}\,\{(Y_{ii}V_i + \sum_{\substack{j=1\\ j\neq i}}^{n} Y_{ij}V_j)V_i{}^*\} \tag{3.4}$$

$$VN_i = AC \cdot [\{(P_i - jQ_i)/V_i{}^* - \sum_{\substack{j=1\\ j\neq i}}^{n} Y_{ij}V_j\}/Y_{ii} - V_i] + V_i \tag{3.5}$$

$$V_i' = VN_i \cdot |V_i|/|VN_i| \tag{3.6}$$

Where AC = the acceleration factor (in the range of 1.0 to 1.6),

VN_i = the intermediate voltage at the i-th bus,

$|VN_i|$ = the absolute value of VN_i,

V_i' = the new voltage for the i-th bus.

(2) For the swing bus:

$$P_i - jQ_i = (Y_{ii}V_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} Y_{ij}V_j)\ V_i - PL_i + jQL_i \tag{3.7}$$

where PL_i = the real power of the load at the i-th bus

QL_i = the reactive power of the load at the i-th bus.

(3) For the load bus (buses without synchronous machines connected):

$$VN_i = \{-P_i + jQ_i)/V_i^* - \sum_{\substack{j=1 \\ j\neq i}}^{n} Y_{ij}V_j\} /Y_{ii} \tag{3.8}$$

$$V_i' = AC \cdot (VN_i - V_i) + V_i \tag{3.9}$$

It is to be noted that:

(a) all the synchronous motor buses are treated in the same fashion as generator buses except -P + jQ for lagging power factor is used;

(b) loads other than synchronous motors are treated as constant impedance loads;

(c) when more than one generator is connected to the same bus, the real power levels of all generators are added together before Equations (3.4) to (3.9) are evaluated;

(d) the acceleration factor AC is applied to speed up the rate of convergence. The problem of selecting a proper value of AC has been discussed by Chan[5].

After completion of the load-flow studies, the voltage behind the transient impedance for each generator E_i can be obtained by Equations (3.10) and (3.11).

$$P_i - jQ_i = P_i - jQ_{it} \left|P_i/P_{it}\right| \tag{3.10}$$

$$E_i = V_i - (P_i - jQ_i)/V_i^*/(R_{ai} + jX'_{di}) \tag{3.11}$$

where Q_{it} and P_{it} are the total value of Q and P at bus i. Equations (3.10) and (3.11) are based on the assumptions that all generators connected at the same bus are operated at the same power factor and no synchronous motor is connected to the generator group. The results obtained from Equation (3.11) give the initial value for Equation (2.2) at the beginning of the faulted period.

It is to be noted that Y in Equation (3.1) is a symmetric matrix if there are no off-nominal taps at any of the transformers; however, it will be unsymmetric if at least one of the transformer taps is off-nominal. In the case of an off-nominal transformer, the following manipulations must be made before Equations (3.4) to (3.9) can be used:

$$Y_{ij} = (\text{original } Y_{ij})/T_{ij} \tag{3.12}$$

$$Y_{ij} = Y_{ji} \tag{3.13}$$

$$Y_{ii} = \sum_{\substack{j=1 \\ j\neq i}}^{n} (Y_{ij}/T_{ij}) + \sum_{\substack{j=1 \\ j\neq i}}^{n} Y_{ij}(\text{shunt}) \tag{3.14}$$

where $T_{ij} = N_i/N_j$ = tap ratio.

The power flow between buses for the output printing by a digital computer is simply

$$P_{ij} + jQ_{ij} = V_i^*[(V_i/T_{ij} - V_j)\, Y_{ij} + V_i Y_{ij(shunt)}] \qquad (3.15)$$

3.3 Y-Matrix Calculation - Network Reduction

The purpose of the Y-matrix calculation is to reduce the passive electric network to the smallest possible size still retaining the synchronous machine buses. Due to the reduction of the original system, the calculation of the transient stability problem is much simplified and the digital computer memory requirement is much reduced.

The Gauss-Seidel method of iteration can be readily applied to the Y-matrix calculation; however, due to the convergency problem, a large amount of computation time might be required. For this reason matrix operation techniques were used in this research. These techniques are outlined briefly hereunder. Equation (3.1) can be rewritten as

$$\begin{bmatrix} I_G \\ I_B \end{bmatrix} = \begin{bmatrix} I_G \\ 0 \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \begin{bmatrix} V_G \\ V_B \end{bmatrix} \qquad (3.16)$$

where the subscript G denotes the buses with synchronous machines connected and the subscript B denotes the buses with no synchronous machine connected. I_B is a null matrix because there is no active source feeding current to B-type buses. Y_1, Y_2, Y_3 and Y_4 represent the complete information of a passive electric network in any operating condition. The equivalent generator matrix, (Y), can be obtained by simple matrix operation of

Equation (3.16). From Equation (3.16), Equation (3.17) can be derived

$$(I_G) = [(Y_1) - (Y_2)\ (Y_4)^{-1}\ (Y_3)]\ \ (V_G) \tag{3.17}$$

by definition,

$$(Y) = (I_G)\ (V_G)^{-1} \tag{3.18}$$

therefore,

$$(Y) = (Y_1) - (Y_2)\ (Y_4)^{-1}\ (Y_3) \tag{3.19}$$

This equation is applicable to any reduced (Y) calculation for any type of switching action in a passive network.

3.4 The Most Critical Machine, the Reference Machine and the Steepest Descent Method of Minimization

Conceptual power transfer curves drawn with reference to system synchronous speed for a multimachine power system are shown in Figure 3.1. The points at the intersections, between mechanical and electrical power curves denoted as a_i and b_i for the i-th machine, are the stable and the unstable equilibria respectively in this specific operating condition. A power flow following the fault occurrence will cause the power angle δ to be forced to move from a_i toward b_i, and the machine whose angle reaches the b_i state first is called "The Most Critical Machine". The machine with the lowest rate of δ angle increase is called "The Reference Machine". Since $\dot{\omega}_i$ in the second part of Equation (2.2) denotes the angular acceleration of the power angle, therefore, the most critical machine as well as

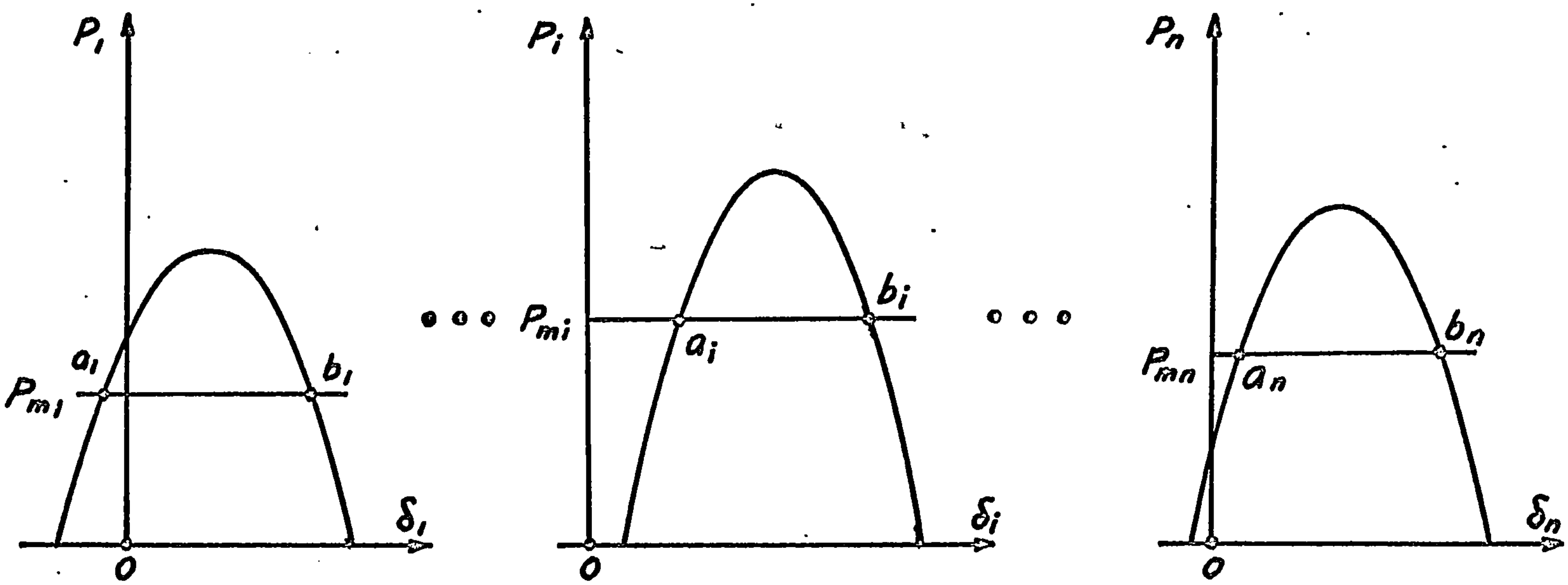

Figure 3.1 The possible equilibria of Equation (3.20) which are of interest to power system engineers.

the reference machine can be determined by the application of that equation. The one with the highest absolute value of $\dot{\omega}_i$ at the switching instant which immediately preceeds the desired final state is the most critical machine; and the one with the lowest absolute value of $\dot{\omega}_i$ at the instant of fault occurance is the reference machine.

It is understood, from the observation of Figure 3.1, that there exist n x n combinational sets of the system equilibria. The special set which contains only a_i values is the "stable equilibrium" set and is denoted by (δ^s). The set which contains a single b_i, where 'i' is the most critical machine, the remainder of values being a_j, is the system "unstable equilibrium", denoted as (δ^{us}).

The calculation of (δ^s) and (δ^{us}) is required to find the region of asymptotic stability, BD, shown in Equation (2.12). The problem of solving for (δ^s) and (δ^{us}) is in fact a problem of solving the steady-state portion of Equation (2.2). This equation can be rewritten in the form of Equation (3.20).

$$\sum_{k=1}^{n}{}' E_i E_K Y_{iK} \operatorname{Cos}(\theta_{iK} - \delta_i + \delta_K) = P_{mi}$$

$$i = 1, 2, \ldots, n \qquad (3.20)$$

Equation (3.20) has an infinite set of solutions; the desired solution set of (δ^s) and (δ^{us}) can be obtained quantitatively by the iterative process if a proper initial guess is chosen. The angles of the voltages behind the transient impedances provide the stable equilibrium set for the prefault system. The angles of this stable equilibrium can be taken as the initial guess for the unstable equilibrium calculation except for the angle of the most critical machine. The initial guess for the most critical machine can be chosen as $\pi - |\delta^s|$ for a generator, or $-\pi + |\delta^s|$ for a motor. After the proper initial guesses have been made, the equilibrium can be solved iteratively by the application of the 'Steepest Descent Method of Minimization' (Appendix 7.3).

For the steepest descent method, Equation (3.20) can be alternatively represented as

$$f_i = \sum_{j=1}^{n} E_i E_j Y_{ij} \operatorname{Cos}(\theta_{ij} - \delta_i + \delta_j) - P_{mi}$$

$$= E_i^2 G_{ii} - P_{mi} + \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j Y_{ij} \operatorname{Cos}(\theta_{ij} - \delta_i + \delta_j)$$

$$i = 1, 2, \ldots, n \qquad (3.21)$$

It is shown in Appendix 7.3 that if a function Φ of the form given by Equation (3.22) is defined

$$\Phi = \frac{1}{2} \sum_{i=1}^{n} (f_i - P_{mi})^2 \qquad (3.22)$$

and its partial derivative with respect to δ_i is evaluated as in Equation (3.23)

$$\frac{\partial\Phi}{\partial\delta_i}\bigg|_{[\delta]=[\delta]^\circ} = \left\{(f_i - P_{mi})\frac{\partial f_i}{\partial\delta_i} + \sum_{\substack{j=1\\ j\neq i}}^{n}(f_j - P_{mj})\frac{\partial f_j}{\partial\delta_i}\right\}_{[\delta]=[\delta]^\circ}$$

$$= \Big\{(f_i - P_{mi})\sum_{\substack{j=1\\ j\neq i}}^{n} E_iE_jY_{ij}\,\text{Sin}(\theta_{ij} - \delta_i + \delta_j)$$

$$-\sum_{\substack{j=1\\ j\neq i}}^{n}(f_j - P_{mj})\,E_jE_iY_{ji}\,\text{Sin}(\theta_{ji} - \delta_j + \delta_i)\Big\}_{[\delta]=[\delta]^\circ}$$

(3.23)

where $[\delta]^\circ$ is the initial guess, then the value of the increment of the dependent variable a_T° can be calculated from Equation (3.24)

$$a_T^\circ = \frac{-\Phi\,([\delta]^\circ)}{\sum_{i=1}^{n}\left(\frac{\partial\Phi}{\partial\delta_i}\right)^2\bigg|_{[\delta]=[\delta]^\circ}} \tag{3.24}$$

Further it is shown in Appendix 7.3 that the value of δ_i for the second iteration is given by the relationship

$$\delta_i' = \delta_i^\circ + a_T^\circ\,\frac{\partial\Phi}{\partial\delta_i}\bigg|_{[\delta]=[\delta]^\circ} \tag{3.25}$$

Values of δ_i' for successive iterations are calculated in the same way. The geometrical explanation of the iteration procedure is shown in Figure

7.2. The calculation is terminated when the assigned tolerable error requirement is met. This is generally called "The Steepest Descent Method".

In this research work, "The Varient Steepest Descent Method"[4] was first applied. However, because the surfaces of Equation (3.21) are not simple hyper-ellipsoids, the quadratic approximation of Φ does not fit the curve well, this causes the valley shifting pheomenon during the minimization process and is found to be inapplicable for the solution of Equation (3.21).

Furthermore, it was found that the application of the Steepest Descent Method to the problem of Equation (3.21) converged rapidly during the first three iterations; but the rate of convergence decreased rapidly thereafter. This was due to the fact that both $\frac{\partial f_i}{\partial \delta_i}$ and $\frac{\partial f_i}{\partial \delta_j}$ terms in Equation (3.23) became very small as the minimum was approached. The minimization processes became almost idle when Φ was reduced to approximately 5 per cent of the maximum value of Φ. The limit of the step-size (a_T in Equation (3.24)) in the range of -0.01 to -0.1 was found to be the best convergent region for the problem of Equation (3.21) for the power system considered. One set of data shown in Table I is listed to illustrate the convergence and the step-size effects in the application of this minimization technique to the power system problem.

3.5 Runge-Kutta Method of Integration

The purpose of the Runge-Kutta method of integration is to obtain an approximate solution of a set of differential equations with given initial values. In this research work, it was used to solve for the initial condition of the final desired state. The equations to be integrated are given by Equation (2.1) or (2.2) accompanied by Equation (2.13).

TABLE I

The effect of step-size on the convergence of the steepest descent method
(abstracted from the postfault stable equilibrium calculation of the system shown in Figure 4.5)

Iteration	$a_t \geq -.02$		$a_t \geq -.04$		$a_t \geq -.06$		$a_t \geq -.08$		$a_t \geq -.11$	
	$+a_t$	Φ	$+a_t$	Φ	$+a_t$	Φ	$+a_t$	Φ	$+a_t$	Φ
0	-.0193	.1923	-.0193	.1923	-.0193	.1923	-.0193	.1923	-.0193	.1923
1	-.0200	.0698	-.0400	.0698	-.0469	.0698	-.0469	.0698	-.0469	.0698
2	-.0200	.0448	-.0400	.0292	-.0600	.0259	-.0800	.0259	-.0856	.0259
3	-.0200	.0322	-.0400	.0193	-.0534	.0180	-.0298	.0199	-.0264	.0209
4	-.0200	.0250	-.0400	.0147	-.0314	.0164	-.0800	.0119	-.1100	.0112
5	-.0200	.0205	-.0400	.0116	-.0600	.0102	-.0278	.0118	-.0290	.0096
6	-.0200	.0175	-.0400	.0094	-.0431	.0092	-.0800	.0067	-.1100	.0057
7	-.0200	.0152	-.0400	.0077	-.0450	.0075	-.0438	.0061	-.0219	.0075
8	-.0200	.0134	-.0400	.0065	-.0426	.0066	-.0457	.0053	-.1100	.0035
9	-.0200	.0120	-.0400	.0055	-.0480	.0055	-.0429	.0048	-.1100	.0029
10	-.0200	.0108	-.0400	.0048	-.0393	.0052	-.0493	.0042	-.0305	.0040
11	-.0200	.0097	-.0400	.0042	-.0595	.0042	-.0386	.0042	-.1100	.0026
12	-.0200	.0088	-.0400	.0038	-.0299	.0049	-.0651	.0034	-.0233	.0045
13	-.0200	.0080	-.0400	.0034	-.0600	.0031	-.0271	.0045	-.1100	.0023
14	-.0200	.0073	-.0400	.0032	-.0600	.0031	-.0800	.0026	-.1100	.0022
15	-.0200	.0067	-.0400	.0030	-.0378	.0036	-.0800	.0028	-.1100	.0023
16	-.0200	.0062	-.0400	.0028	-.0600	.0029	-.0231	.0046	-.0422	.0030
17	-.0200	.0057	-.0400	.0027	-.0350	.0035	-.0800	.0023	-.0563	.0028
18	-.0200	.0053	-.0400	.0026	-.0600	.0037	-.0800	.0023	-.0332	.0034

Since there are no methods available for solving general simultaneous differential equations, the Runge-Kutta methods given by Scarborough[18] for a single second order equation and for two simultaneous equations of the first order (Appendix 7.4) were generalized to solve the set of second order differential equations given by Equation (2.1) or (2.2). The method which was developed could be used to solve a general set of second order non-linear differential equations. A brief description of this method follows.

A general second order simultaneous differential equation

$$\frac{d^2 y_i}{dx^2} = f_i(x;\ y_1,\ y_2,\ .\ .\ .\ ,\ y_n;\ \dot{y}_1,\ \dot{y}_2,\ .\ .\ .\ ,\ \dot{y}_n)$$

$$i = 1,\ 2,\ .\ .\ .,\ n \tag{3.26}$$

with initial condition $y_1(x^o) = y_1{}^o$, $y_2(x^o) = y_2{}^o$, . . ., $y_n(x^o) = y_n{}^o$; $\dot{y}_1(x^o) = \dot{y}_1{}^o$, $\dot{y}_2(x^o) = \dot{y}_2{}^o$, . . ., $\dot{y}_n(x^o) = \dot{y}_n{}^o$, and step size Δx, can be solved numerically by calculating the immediate values $K_{1,i}$, $K_{2,i}$, $K_{3,i}$, $K_{4,i}$ and then substituting them into a relevant formula. The equations for the intermediate value calculation are:

$$K_{1,i} = \Delta x \cdot f_i(x^o;\ y_1{}^o,\ y_2{}^o,\ .\ .\ .,\ y_n{}^o;\ \dot{y}_1{}^o,\ \dot{y}_2{}^o,\ .\ .\ .,\ \dot{y}_n{}^o)$$

$$K_{2,i} = \Delta X \cdot f_i (x^\circ + \frac{\Delta X}{2}; \; y_1^\circ + \frac{\Delta X}{2} \dot{y}_1^\circ + \frac{\Delta X}{8} K_{1,1},$$

$$y_2^\circ + \frac{\Delta X}{2} \dot{y}_2^\circ + \frac{\Delta X}{8} K_{1,2},$$

$$. \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \;,$$

$$y_n^\circ + \frac{\Delta X}{2} \dot{y}_n^\circ + \frac{\Delta X}{8} K_{1,n};$$

$$\dot{y}_1^\circ + \frac{K_{1,1}}{2},$$

$$\dot{y}_2^\circ + \frac{K_{1,2}}{2},$$

$$. \; . \; . \; . \; . \;,$$

$$\dot{y}_n^\circ + \frac{K_{1,n}}{2})$$

$$K_{3,i} = \Delta X \cdot f_i (x^\circ + \frac{\Delta X}{2}; \; y_1^\circ + \frac{\Delta X}{2} \dot{y}_1^\circ + \frac{\Delta X}{8} K_{2,1},$$

$$y_2^\circ + \frac{\Delta X}{2} \dot{y}_2^\circ + \frac{\Delta X}{8} K_{2,2},$$

$$. \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \;,$$

$$y_n^\circ + \frac{\Delta X}{2} \dot{y}_n^\circ + \frac{\Delta X}{8} K_{2,n}; \; \dot{y}_1^\circ + \frac{K_{2,1}}{2},$$

$$\dot{y}_2^\circ + \frac{K_{2,2}}{2},$$

$$. \; . \; . \; . \; . \; . \;,$$

$$\dot{y}_n^\circ + \frac{K_{2,n}}{2})$$

$$K_{4,i} = \Delta X \cdot f_i(X^\circ + \Delta X;\ y_1^\circ + \dot{y}_1^\circ \Delta X + \frac{\Delta X}{2} K_{3,1},$$

$$y_2^\circ + \dot{y}_2^\circ \Delta X + \frac{\Delta X}{2} K_{3,2};$$

$$\cdots\cdots\cdots,$$

$$y_n^\circ + \dot{y}_n^\circ \Delta X + \frac{\Delta X}{2} K_{3,n};$$

$$\dot{y}_1^\circ + K_{3,1},$$

$$\dot{y}_2^\circ + K_{3,2},$$

$$\cdots\cdots,$$

$$\dot{y}_n^\circ + K_{3,n})$$

where $i = 1, 2, \ldots, n$ (3.27)

and, the relevant formulae are

$$y_i' = y_i^\circ + \Delta X \cdot [\dot{y}_i^\circ + \frac{1}{6}(K_{1,i} + K_{2,i} + K_{3,i})]$$

$$\dot{y}_i' = \dot{y}_i^\circ + \frac{1}{6}(K_{1,i} + 2K_{2,i} + 2K_{3,i} + K_{4,i})$$

where $i = 1, 2, \ldots, n$ (3.28)

The application of this technique to the problem of Equation (2.1) or (2.2) requires too much calculating time. A simplified scheme represented by the following equations

$$K_{1,i} = \Delta X \cdot f_i(X^\circ;\ y_1^\circ, y_2^\circ, \ldots, y_n^\circ;\ \dot{y}_1^\circ, \dot{y}_2^\circ, \ldots, \dot{y}_n^\circ)$$

$$K_{2,i} = \Delta X \cdot f_i(X^\circ + \frac{\Delta X}{2};\ y_1{}^\circ, y_2{}^\circ, \ldots, y_n{}^\circ;$$

$$\dot{y}_1{}^\circ + \frac{K_{1,1}}{2}, \dot{y}_2{}^\circ + \frac{K_{1,2}}{2}, \ldots,$$

$$\dot{y}_n{}^\circ + \frac{K_{1,n}}{2})$$

$$K_{3,i} = \Delta X \cdot f_i(X^\circ + \frac{\Delta X}{2};\ y_1{}^\circ, y_2{}^\circ, \ldots, y_n{}^\circ;$$

$$\dot{y}_1{}^\circ + \frac{K_{2,1}}{2}, \dot{y}_2{}^\circ + \frac{K_{2,2}}{2}, \ldots,$$

$$\dot{y}_n{}^\circ + \frac{K_{2,n}}{2}).$$

$$K_{4,i} = \Delta X \cdot f_i(X^\circ + \Delta X;\ y_1{}^\circ, y_2{}^\circ, \ldots, y_n{}^\circ;$$

$$\dot{y}_1{}^\circ + K_{3,1}, \dot{y}_2{}^\circ + K_{3,2}, \ldots,$$

$$\dot{y}_n{}^\circ + K_{3,n})$$

$$\dot{y}_i' = \dot{y}_1{}^\circ + \frac{1}{6}(K_{1,i} + 2K_{2,i} + 2K_{3,i} + K_{4,i})$$

$$y_i' = y_i{}^\circ + \Delta X \cdot \dot{y}_i'$$

where $i = 1, 2, \ldots, n$ (3.29)

was found to be more convenient for the practical application required in this work. The computational speed by the application of Equation (3.29) is approximately six times faster than that of Equations (3.27) and (3.28).

The difference between the results obtained by these two schemes can hardly be distinguished in the plotted outputs.

When Equations (2.2) and (2.13) are rewritten in the form of Equation (3.30)

$$\dot{\delta}_i = \omega_i$$

$$\dot{\omega}_i = \frac{1}{M_i}\left[P_{mi} - D_i\omega_i - \sum_{j=1}^{n} E_iE_jY_{ij}\, \mathrm{Cos}(\theta_{ij} - \delta_i + \delta_j)\right] \tag{3.30}$$

$$\delta_{iR} = \delta_i - \delta_R$$

$$\omega_{iR} = \omega_i - \omega_R$$

and the techniques shown in Equation (3.29) are applied, the integration process at the K-th step of iteration carried out in terms of the set of equations numbered (3.31) hereunder:

$$K_{1,i} = \Delta\tau[P_{mi} - D_i\omega_i^K - \sum_{j=1}^{n} E_iE_jY_{ij}\, \mathrm{Cos}(\theta_{ij} - \delta_i^K + \delta_j^K)]/M_i$$

$$K_{2,i} = \Delta\tau[P_{mi} - D_i(\omega_i^K + \frac{K_{1,i}}{2}) - \sum_{j=1}^{n} E_iE_jY_{ij}\mathrm{Cos}(\theta_{ij} - \delta_i^K + \delta_j^K)]/M_i$$

$$K_{3,i} = \Delta\tau[P_{mi} - D_i(\omega_i^K + \frac{K_{2,i}}{2}) - \sum_{j=1}^{n} E_iE_jY_{ij}\mathrm{Cos}(\theta_{ij} - \delta_i^K + \delta_j^K)]/M_i$$

$$K_{4,i} = \Delta\tau[P_{mi} - D_i(\omega_i^K + K_{3,i}) - \sum_{j=1}^{n} E_iE_jY_{ij}\mathrm{Cos}(\theta_{ij} - \delta_i^K + \delta_j^K)]/M_i$$

$$\left.\begin{aligned}
\omega_i^{K+1} &= \omega_i^K + (K_{1,i} + 2K_{2,i} + 2K_{3,i} + K_{4,i})/6 \\
\delta_i^{K+1} &= \delta_i^K + \omega_i^{K+1} \times \Delta\tau \\
\omega_{iR}^{K+1} &= \omega_i^{K+1} - \omega_R^{K+1} \\
\delta_{iR}^{K+1} &= \delta_i^{K+1} - \delta_R^{K+1}
\end{aligned}\right\}$$

where $i = 1, 2, \ldots, n$ (3.31)

The value of $\Delta\tau$ taken in this study was 1 radian, or 1/377 second which gave satisfactory accuracy during the integration.

Since the numerical method of integration is applicable for continuous functions only, the 'average' angular acceleration $\ddot{\delta}_i$ must be taken at instants of switching (or, at instants of Y-parameter replacement).

4. THE NUMERICAL EXAMPLES AND APPLICATIONS

4.1 Introduction:

A number of numerical examples were calculated to study the principle of stability limit determination by the Direct Method of Liapunov described in Chapter 2. To provide numerical data for the working examples, specific engineering problems were hypothesized but no particular attempts were made to study any one problem in detail. These examples illustrate the general method of approach which should be followed to use the Liapunov Direct Method. Experience indicated that the particular V-function derived in Section 2.5 has serious limitations. These limitations are discussed in detail in Chapter 5.

The examples chosen fall into three categories: the first deals with the critical switching time calculation and is a repetition of the type of work reported by El-Abiad; the second category deals with the line-reclosing problem and is an extension of the first category into the area of multiple switching situations; the third category extends the multiple switching studies into the area of braking resistance problems.

If a fault occurs on a line in a power system, the system will tend to be unstable in the faulted state. If the postfault system can be made stable by removing the faulted portion quickly enough, then a critical switching time (T_{cs}) can be defined which is the maximum time that the fault can be allowed to remain in before its isolation from the system by switching action. If the fault were cleared before

T_{cs} were reached, no line-reclosing or any other of the techniques mentioned in Section 1.1 for improving the power system transient stability would be required. This is an important factor to consider in the relay setting problems for the somewhat less critical sections of line in a power system. With the advancement in the production of fast-opening and reclosing relays and breakers, this factor (T_{cs}) is losing its importance to electric power engineers; however, it would regain its value if techniques to distinguish between permanent and temporary faults could be developed.

If the system were subjected to a fault and if the fault were cleared after the critical switching time (T_{cs}), the postfault system would not be stable; however, a stable situation might exist if the faulted lines could be reclosed at a proper instant of time. The critical line reclosing time (or simply called 'the critical reclosing time'), T_{cr}, is defined as the maximum time period following the fault occurrence within which such a successful reclosure is possible. Alternatively, a power system is said to be transient state stable (or asymptotically stable in the sense of Liapunov), if the fault is cleared at $T_s > T_{cs}$ and the lines are reclosed before the end of the period defined by T_{cr}.

It has been mentioned in Section 1.1 that a number of techniques to improve power system transient stability are available in addition to fast fault clearing and line reclosure. The utilization of the braking (or damping) resistances was chosen as an example to illustrate the analysis of this type of switching problem. The connection of the braking resistances to improve power system transient stability was first proposed by R.C. Bergvall[3] in 1931. In 1947, S.B. Crary[6] gave more detailed information about the connection of the braking resistances for the cases of a generator connected to an isolated load and to an infinite bus. It is generally understood that a braking resistor can be applied to the most critical machine to dissipate the accelerating power thus the transient stability can be improved; however, to date, doubts still exist regarding the engineering feasibility of this technique. The only practical application of the braking resistor, known to the author in the western world, has been planned for the Portage Mountain plant of British Columbia Hydro and Power Authority[23] in Canada.

In the following sections, the analytical considerations and the computational details associated with these three categories of specific numerical examples are discussed. It is to be emphasized that these examples are intended to illustrate the principle of the application of the Direct Method of Liapunov to a number of situations. Unfortunately the specific Liapunov function used does not give results which are significant from a practical point of view. This should be interpretted as an indictment of the specific V-function chosen and not an indictment of the Direct Method of Liapunov itself.

4.2 Category One: - The Critical Switching Time Calculations;

In this section, the problem of the critical switching time calculation is solved by the method described in this thesis in two cases; the first example being a single machine infinite bus case and the second a six machine case. Section (A) discusses the analytical considerations based on the mathematical analysis given in Chapter 2.

The computational considerations in Section (B) outline the calculations involved using the techniques given in Chapter 3. Finally, the details of the studies on the two power systems are given and the results obtained are discussed in Section (C). The accuracy of the critical switching time calculation determined from the application of the Liapunov Direct Method can be deduced from a comparison of the power angle trajectories when the fault is cleared 'at' the critical switching time and the trajectories when the fault is isolated 'after' the critical switching time.

(A) Analytical considerations

The **example** developed in this section of critical switching time calculations are based on a PREFAULT - FAULT $\xrightarrow{T_{cs}}$ POSTFAULT type of operation. The sequence of these operations is illustrated in Figure 4.1A for the i-th machine. The postfault system (with the faulted portion isolated) is the final desired state for this problem, and $(Y)_f$ associated with Equation (2.2) gives the faulted trajectory shown in Figure 4.1B. The stable and the unstable equilibria of the postfault system specify the boundary of the region of asymptotic stability BD. The point of intersection where the faulted trajectory crosses the boundary BD is a critical point. If the system enters the final state with its variables outside this boundary, the postfault system will not be stable. At the instant of switching, the trajectory is detoured in a continuous fashion from the faulted to the postfault trajectory. The time which corresponds to the instant of intersection between the faulted trajectory and BD is the critical switching time T_{cs} and the angle (δ^{cs}) is the critical switching angle.

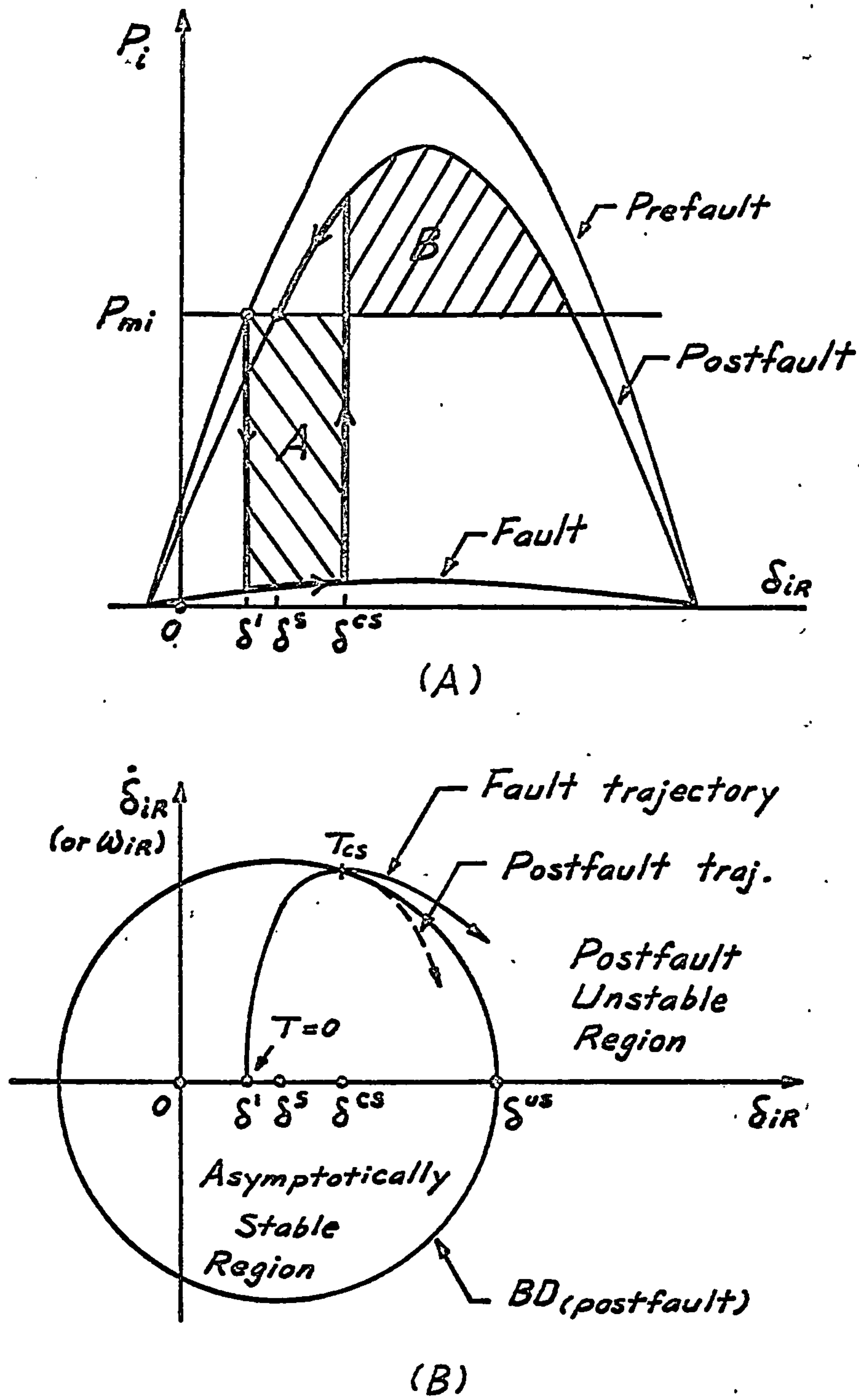

Figure 4.1 The conceptual explanation of the critical switching problem.
A - Equal area diagram.
B - Phase plane diagram.

It is to be noted that the steady-state stability of the postfault system must be investigated when the postfault stable equilibrium (point δ^S in Figure 4.1) is obtained. This is due to the fact that the study of the postfault transient stability is practically meaningless if the postfault system is steady-state unstable. A steady-state is stable if $\frac{\partial f_i}{\partial \delta_i}$, i = 1, 2, ..., n, at the stable equilibrium are all greater than or equal to zero; and it is unstable if at least one of them is less than zero[6]. From Equation (3.21) $\frac{\partial f_i}{\partial \delta_i}$ can be easily derived and shown to be

$$\frac{\partial f_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j) \Big|_{(\delta^S)} \qquad (4.1)$$

where $(Y) = (Y)_{\text{postfault}}$.

(B) Computational considerations

The critical switching time calculation by the application of the Direct Method of Liapunov consists of a number of steps. Briefly these are as follows:

(1) the determination of the boundary in the state space of the region of asymptotic stability for the postfault system,

(2) the calculation of the faulted system swing curves to provide the values of the state variables as functions of time,

(3) a comparison of the values of the Liapunov Function using the calculated state variables with respect to the postfault boundary at each step in the swing curve calculation. It has been established that the postfault system is stable if and only if the values of the state variables lie inside the stability boundary when the system is switched to the postfault state.

A number of specific computational procedures must be employed to implement these basic steps. In particular a load flow calculation is required to establish the system operating conditions (state variable δ_i) prior to the inception of the fault. This procedure includes a system matrix reduction to reduce the static network to a minimum equivalent for speed of calculation. A similar matrix operation is required also to provide the faulted system model upon which the swing curve calculation is based. The establishment of the boundary requires an evaluation of the postfault stable and unstable equilibria, this is accomplished using the steepest descent method.

Figure 4.2 is digital computer program flow chart which illustrates the sequence of calculation. A few detailed comments regarding the specific computational procedures are required.

(1) Load-flow calculations:

Following the techniques outlined in Section 3.2, the load flow studies can be done easily using the complete unreduced system admittance matrix for the calculation. From the load-flow studies, the voltages (both magnitudes and angles) behind the transient impedances can be obtained. These angles define the stable equilibrium of the prefault system, and also the initial values for the Runge-Kutta method of integration used to calculate the swing curves during the faulted period.

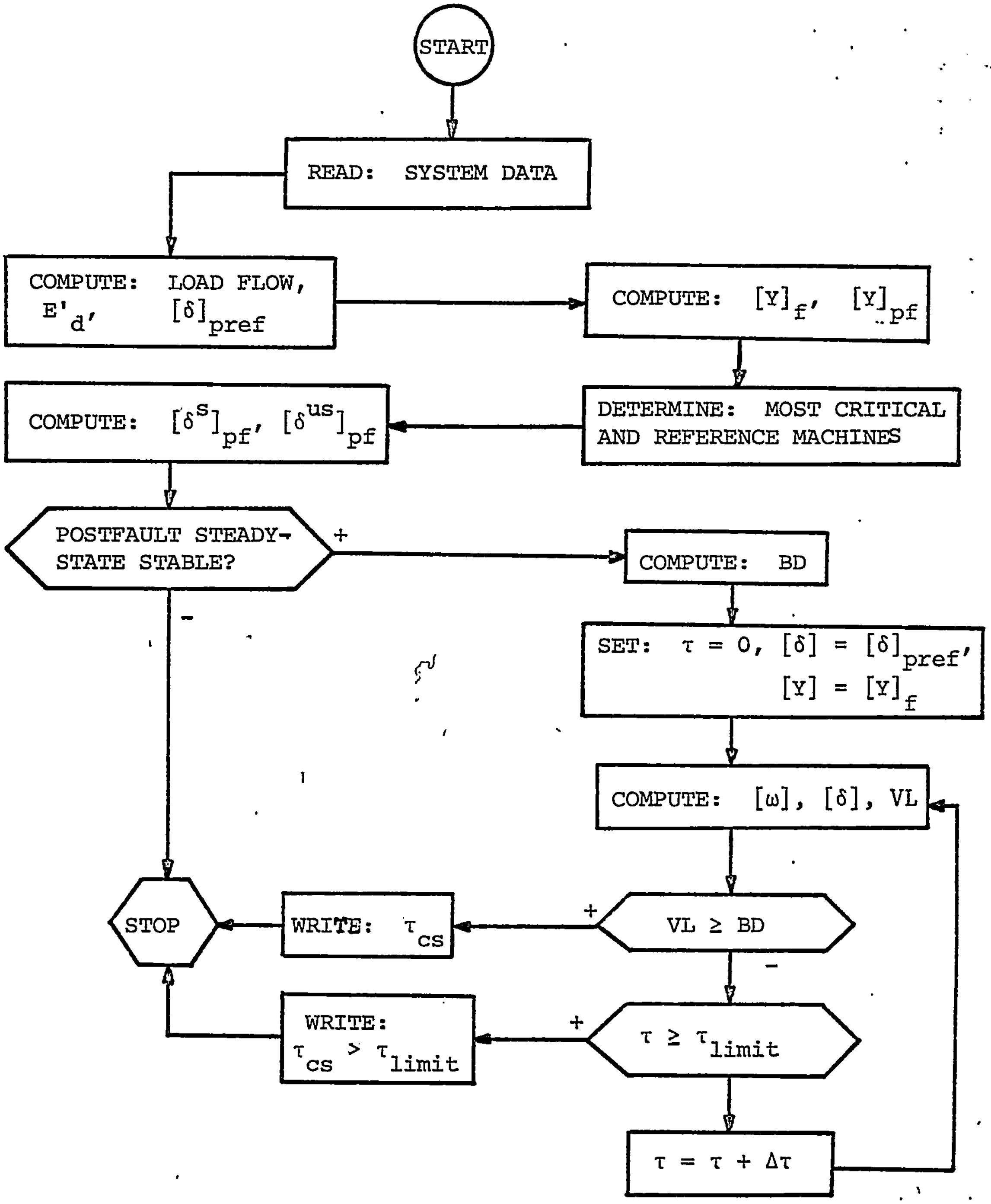

Figure 4.2 Computer program flow-chart for the critical switching time calculation.

(2) $(Y)_{fault}$ and $(Y)_{postfault}$ calculations:

The complete faulted system and postfault system admittance matrices can be simplified and reduced to the equivalent matrices $(Y)_{fault}$ (or $(Y)_f$) and $(Y)_{postfault}$ (or $(Y)_{pf}$), which are the admittance matrices with only the generator buses retained. The reduction processes can be done by following the technique given in Section 3.3. These provide the admittance parameters required for the equations given in Chapter 2 to represent different operating conditions.

(3) The postfault stable equilibrium, $(\delta_{iR}^{s})_{pf}$, and the postfault unstable equilibrium, $(\delta_{iR}^{us})_{pf}$, calculations:

The techniques given in Section 3.4 can be used to calculate the postfault stable and unstable equilibria using the $(Y)_{pf}$ in the method of steepest descent. For the determination of the most critical machine and the least critical machine, which is used as the reference machine, $(Y)_f$ is used.

(4) Check the postfault steady-state stability:

The steady-state stability of the postfault system can be checked by the application of Equation (4.1) given previously. The studies are continued only if a positive answer is obtained in this step.

(5) The boundary of the region of asymptotic stability, BD, calculations:

By the substitution of $(Y)_{pf}$, $(\delta_{iR}^{s})_{pf}$ and $(\delta_{iR}^{us})_{pf}$ into Equation (2.12), BD for the postfault system can be obtained.

(6) The critical switching time, T_{cs}, calculations:

The faulted trajectory is obtained by the application of the Runge-Kutta method of integration mentioned in Section 3.5 with the substitution of $(Y)_f$ into Equation (3.31). The value of V is obtained by the substitution of $(Y)_{pf}$, $(\delta_{iR}^{s})_{pf}$ and (δ_{iR}) (obtained at each integration step) into Equation (2.10), V is then compared with BD at each step and when $V \geq BD$ at time $T = \tau^{n+1}$, then the critical switching time T_{cs} is given by τ^{n}.

The analytical and computational procedures discussed above were applied to specific numerical problems to provide practical experience in the application of the method to realistic systems. Two specific examples are discussed in Section (C).

(C) Numerical examples and discussions:

Two example systems were studied for different purposes. The first example was the single machine - infinite bus system given by Stevenson (Chapter 15 of (19)), the second was a six machine system based on a simplified model of the Saskatchewan Power Corporation System. The first example was studied for the following reasons:

(1) to compare the critical switching time obtained by the application of the Direct Method of Liapunov to that obtained from the Equal Area Criterion Method, and

(2) to check the computer program written by the author.

The second example was studied to illustrate the application of the method to multimachine system transient stability evaluation. This is similar to the work carried out by El-Abiad and serves as a further check on the work of this author.

(i) Stevenson's example system

The system chosen by Stevenson is shown in Figure 4.3 with the numerical values of the parameters expressed in per unit on a 230KV, 100MVA base.

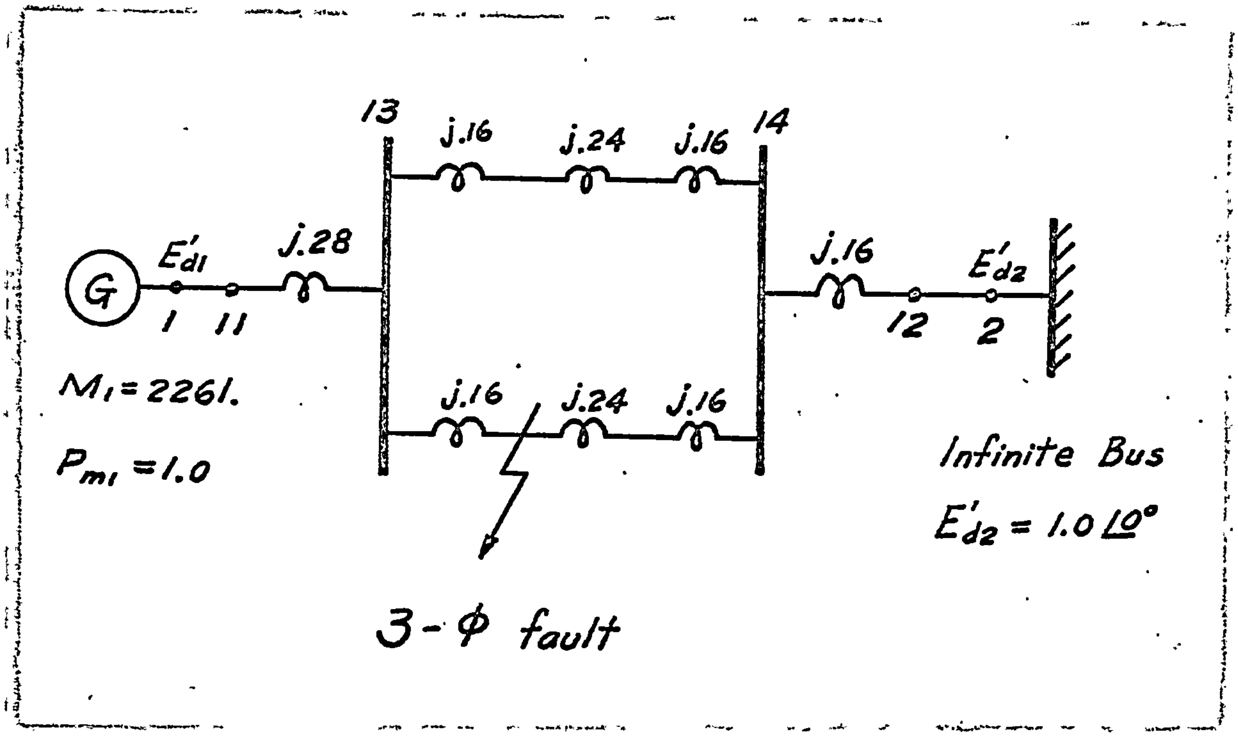

Figure 4.3 Stevenson's example system.

The results obtained by the Direct Method of Liapunov and the Equal Area Criterion Method used by Stevenson are given in the following table:

Item		Author's Results	Stevenson's Results
E'_{d1}	(pu)	$1.25 \angle 35.20^\circ$	$1.25 \angle 35.20^\circ$
Y_{12f}	(pu)	0.355	1/2.98 = 0.355
Y_{12pf}	(pu)	1.000	1/1.000 = 1.000
δ^s_{12}	(rad)	0.76856 + 0.15475 = 0.92331	3.14 - 2.22 = 0.92
δ^{us}_{12}	(rad)	2.37301 - 0.15473 = 2.21828	2.22
δ^{cs}		0.910 (rad) or 52°	51.6°
T_{cs}		42(rad) 6.685(cyc) or .11141 (sec)	between 0.1 to 0.15 sec.

The results obtained by these two methods agree. The swing curves when the fault is cleared 'at' the critical switching time (42 radians), and when the fault is cleared 1.0 radian after the critical switching time, are shown in Figure 4.4. From this figure, it can be seen that the calculation by the Direct Method of Liapunov is quite accurate for this case. By experience in other test runs on the same system, it was found that the critical switching time obtained by the Direct Method of Liapunov using Equation (2.10) was pessimistic if the machine damping constants were included. For example, the result based on the Liapunov criterion was 43 radians, but the actual critical switching time by the swing curve method was found to be 46 radians for one particular case. This results because a damping energy term was not included in the V-function given by Equation (2.10), but system damping was included in the swing curve calculations.

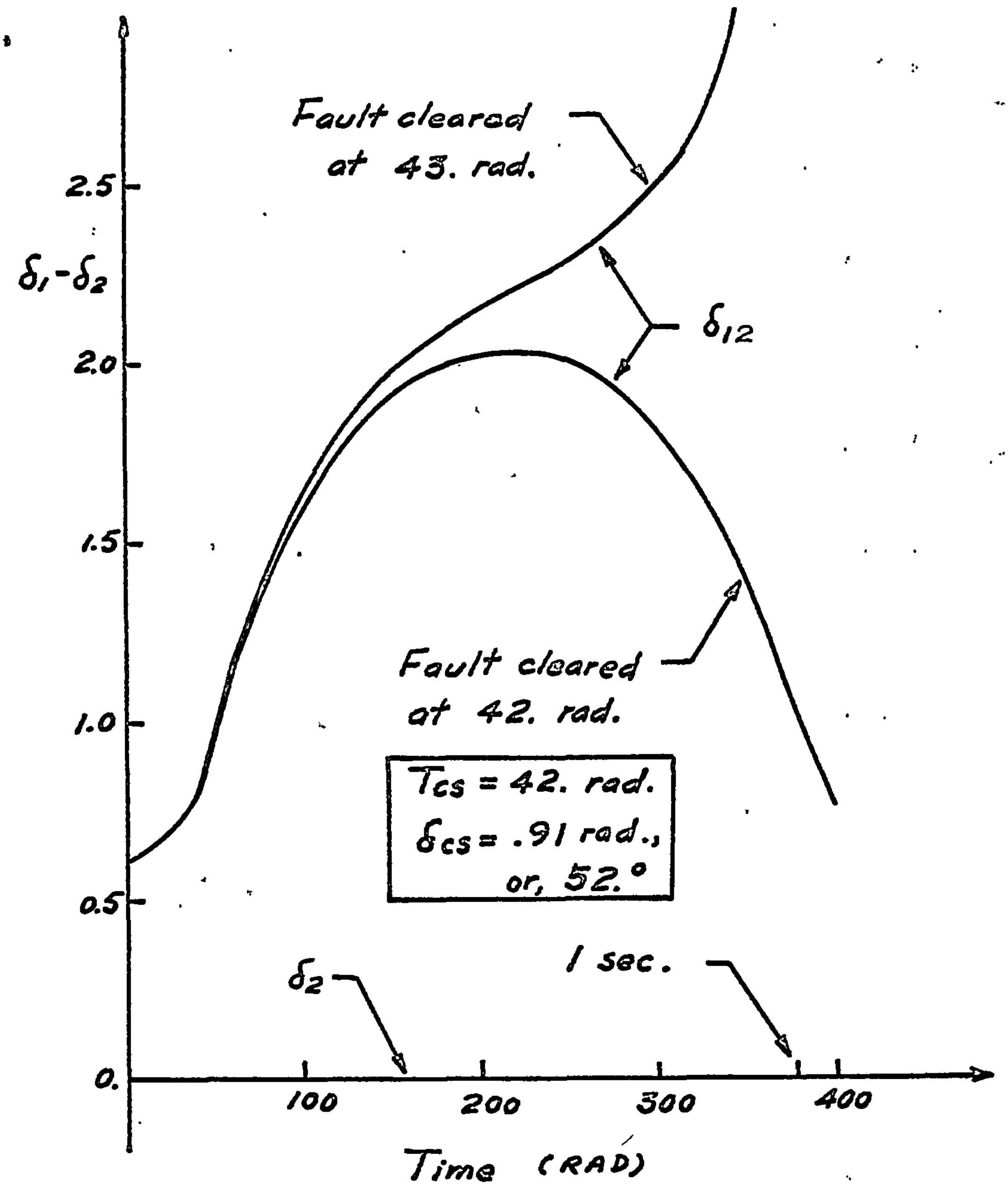

Figure 4.4. The Swing curves of Stevenson's example system.

(ii) Multimachine example system:

A six machine system, which is shown schematically in Figure 4.5, was chosen as the second numerical example. The detailed data for Figure 4.5 are given in Table I and II. To simplify the calculation, three phase faults were considered and the constraints listed in Section 2.1.1 were retained.

The intermediate results such as the system load flow, Y-parameters of different operating conditions, etc. are omitted. The conclusive and important results are shown in Figure 4.6 and 4.7 for the fault near bus 14 in which No. 3 generator is the reference machine. The critical switching time found by the Direct Method of Liapunov was 12 radians while by the swing curve method it was found to be 15 radians (0.0418 second or 2.398 cycles). The difference is attributed to the effects of machine damping and system line resistances. Figure 4.6 shows the swing curves which result if the fault is cleared 1 radian after the critical switching instant. The time trajectory of the power angle for generator No. 1 diverges rapidly with respect to the rest of the generators; therefore, the system in the postfault state is unstable. Figure 4.7 shows that the power angles for all the machines move together, and a conclusion of stability can be drawn.

Similar calculations were run for faults at other points within the network. In general, it was found that the Liapunov test indicated critical switching times which were shorter than those indicated by the swing curve method alone. For example, with a fault in the line between buses 13 and 22, 1% of the line length from bus 13, it was indicated that the critical clearing time should be 92 radians by the Liapunov Method and 121 radians by the swing curve method. For this case the most critical machine was still generator no. 1 as for the case with the fault near bus 14. No

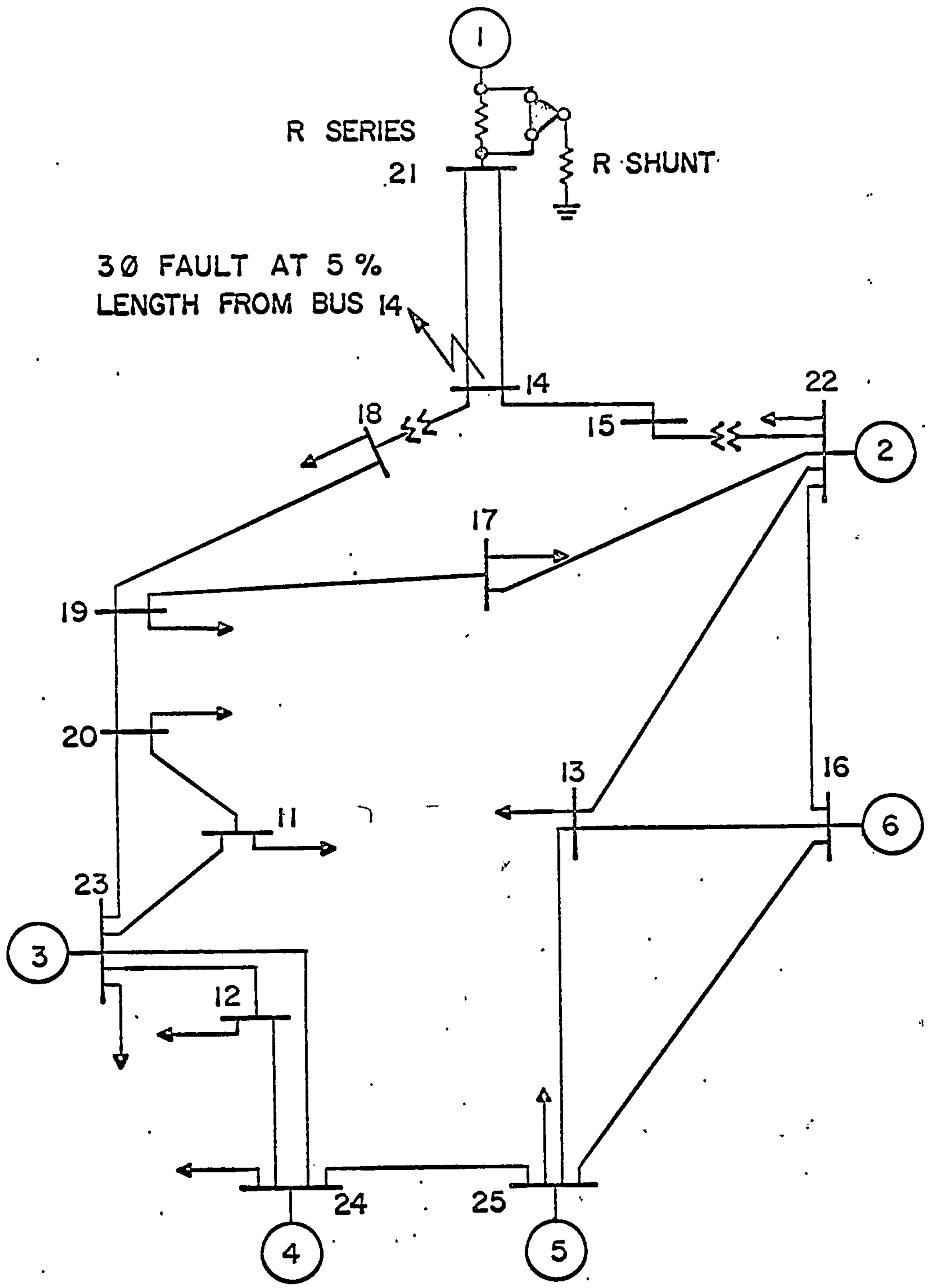

Figure 4.5 Six machine example system.

TABLE II

Bus data after prefault load-flow studies:
100 MVA, 230 kV base

BUS	VOLTAGE		GENERATION		LOAD		R_a	X'_d	D_i	M_i
	pu	radians	P(pu)	Q(pu)	P(pu)	Q(pu)	pu	pu	pu	pu
11	.967	-.399			-0.771	.040				
12	1.022	-.242			-0.453	.140				
13	1.046	-.025			.443	.000				
14	1.083	.104			.000	.000				
15	1.100	.017			.000	.000				
16	1.050	.029	1.650	-0.318	.000	.000	001	1144	6.0	1044.
17	.966	-.084			-0.387	.135				
18	1.031	.085			-0.640	.000				
19	.953	-.148			-0.720	.035				
20	.969	-.346			-0.407	0.71				
21	1.055	.255	2.800	0.878	.000	.000	.001	.094	2.5	516.
22	1.050	.000	1.501	-0.573	-2.022	.265	.001	.105	5.0	5920.
23	1.050	-.138	2.250	-0.165	-0.610	.190	.001	.080	10.0	4440.
24	1.045	-.250	.900	-0.883	-1.805	.669	.002	.250	2.5	629.
25	1.045	-.202	.220	-0.211	-0.699	.110	.005	.500	2.5	2200.

Total generation = 9.321 - j .635 (pu)
Total load = -8.957 + j 165.5 (pu)
Total line comsumption = .364 + j 1.020 (pu)

TABLE III

Line data: same base as Table I

BUS	BUS	R(pu)	X(pu)	BC/2(pu)	TAP
14	15	0.009	0.071	0.302	1.000
14	18	0.0	0.015	0.0	1.050
15	22	0.0	0.013	0.0	1.050
16	13	0.017	0.070	0.007	1.000
17	19	0.034	0.140	0.014	1.000
18	19	0.146	0.331	0.040	1.000
19	20	0.137	0.530	0.071	1.000
20	11	0.051	0.200	0.026	1.000
21	14	0.007	0.060	0.254	1.000
22	13	0.051	0.199	0.027	1.000
22	16	0.061	0.238	0.031	1.000
22	17	0.080	0.116	0.016	1.000
23	11	0.074	0.491	0.062	1.000
23	12	0.061	0.236	0.031	1.000
23	20	0.161	0.622	0.083	1.000
23	24	0.117	0.482	0.057	1.000
24	12	0.062	0.240	0.032	1.000
24	25	0.020	0.075	0.040	1.000
25	13	0.092	0.357	0.048	1.000
25	16	0.091	0.353	0.046	1.000

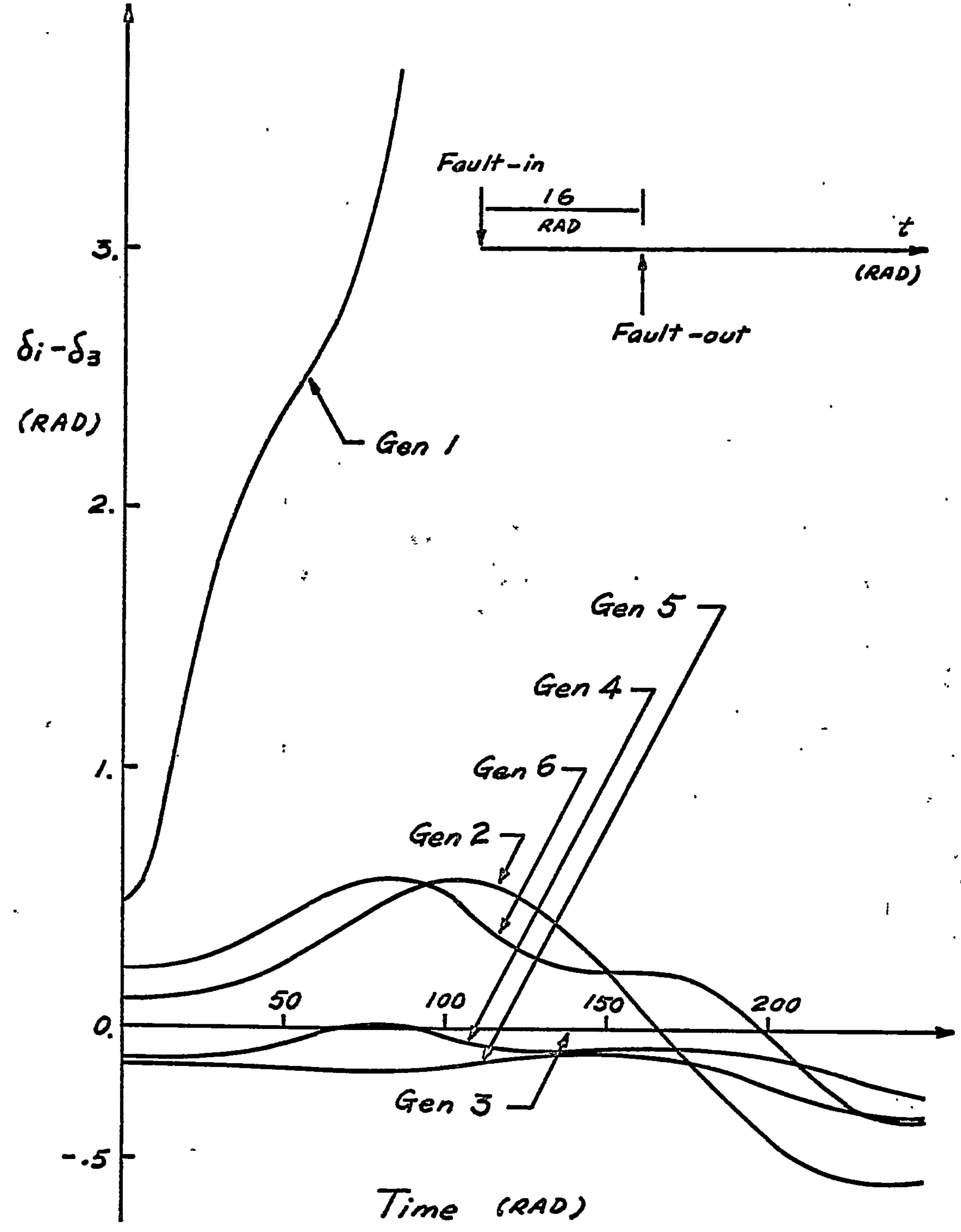

Figure 4.6 The swing curves of six machine system when the fault is cleared at 16 radians.

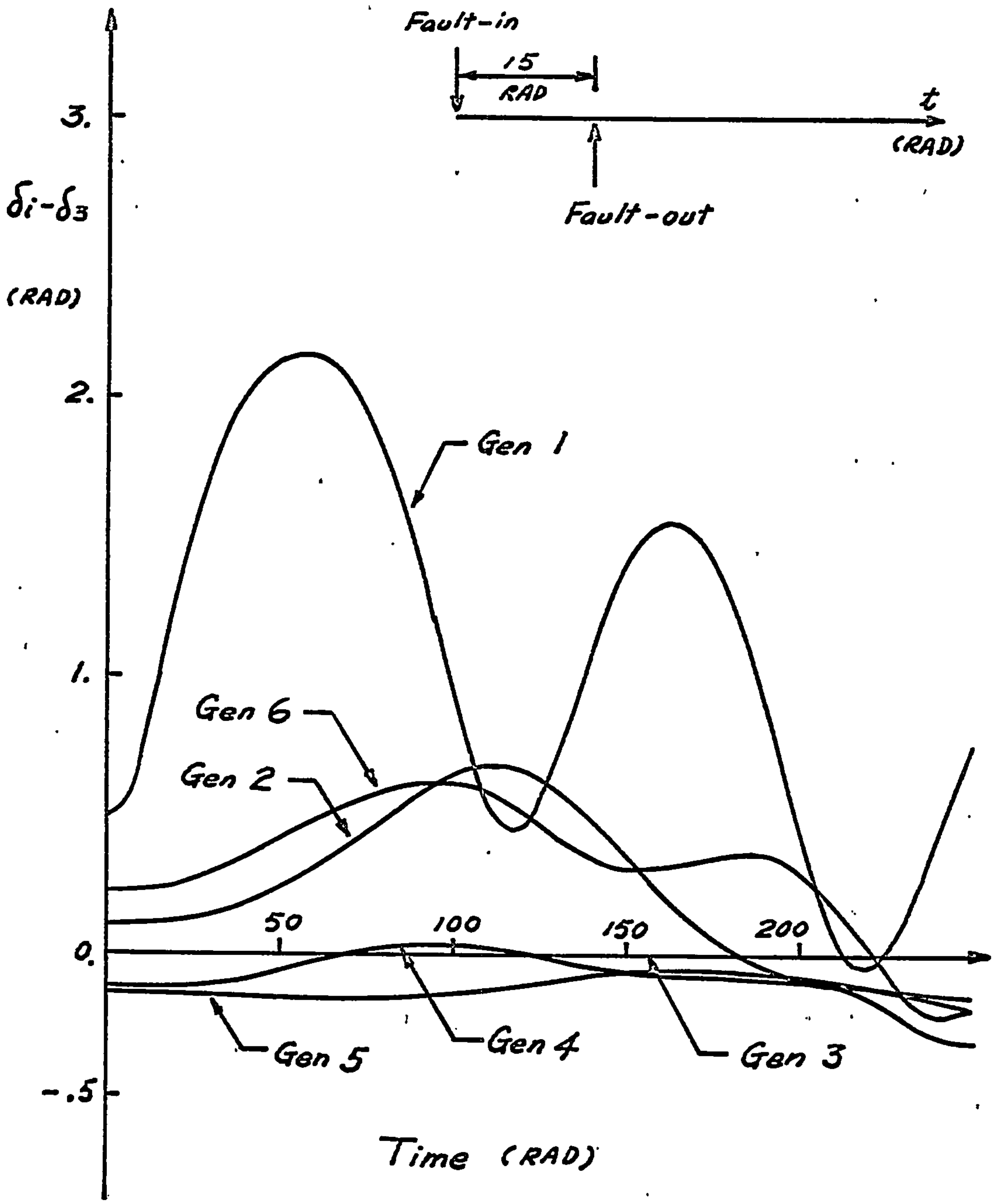

Figure 4.7 The swing curves of six machine system when the fault is cleared at the critical switching time, T_{cs} = 15 radians.

problems were encountered in applying the principle of the Direct Method of Liapunov in any of these cases.

From the results, it can be concluded that the critical switching time can be calculated by the application of the Direct Method of Liapunov for this type of problem provided the constraints listed in Section 2.1.1 are applied. The assessment of the accuracy of the critical switching time calculation involves the consideration of the system resistances, machine damping and the sensitivity of the stability boundary calculation. The latter may be the most significant; its consideration is beyond the scope of this thesis.

It is to be noted that there would be a considerable saving in computation time in establishing the critical switching time calculation in cases of this type if a more suitable Liapunov function were employed. If the swing curve calculations alone are used, the critical switching time would have to be determined by a number of calculating runs in a cut-and-try process. If a criterion based on the Direct Method of Liapunov were to be combined with the swing curve calculation, it would be necessary only to carry out one set of swing curve calculations on the faulted system up to the time instant which corresponded to the critical switching time.

4.3 Category Two: - The Line Reclosing Problems

It is the purpose of this section to illustrate that the Direct Method of Liapunov can be extended to multiple switching problems; specifically to the investigation of line reclosing phenomena. The analytical considerations in Section (A) and the computational considerations in Section (B) serve the same aim as in Section 4.2.

(A) Analytical considerations

The critical reclosing time calculation is based on a PREFAULT $\rightarrow$ FAULT $\xrightarrow{T_{sw}}$ POSTFAULT $\xrightarrow{T_{cr} - T_{sw}}$ PREFAULT type of operation as illustrated in Figure 4.8A. Because of the line reclosing operation, the prefault system is also the final desired state of the system. The phase plane response for the i-th machine is shown in Figure 4.8B. The faulted trajectory given by Equations (2.2) and (2.13) using the faulted system admittance matrix $(Y)_f$ is detoured at time instant T_{sw} in a continuous fashion. The trajectory after T_{sw}, is obtained by the same equation except that $(Y)_f$ is replaced by $(Y)_{pf}$. In this case, the boundary of the region of asymptotic stability BD is defined by the stable and the unstable equilibria of the prefault system. The critical situation which specifies the critical reclosing time exists when the postfault trajectory intersects with BD. The time which corresponds to the instant of intersection is the critical reclosing time T_{cr}, and the angle (δ^{cr}) is the critical reclosing angle. Finally, the postfault trajectory is detoured to the prefault trajectory when the line reclosure is made.

The steady-state stability of the final desired state does not require testing in this case since it is, in fact, the prefault or unfaulted system.

(B) Computational considerations

The critical reclosing time calculation by the application of the Direct Method of Liapunov consists of a number of steps. Briefly these are as follows:

(1) the determination of the boundary BD in the state space in the region of asymptotic stability for the prefault system (the line-reclosed state),

(2) the calculation of the faulted system swing curves up to T_s, then the calculation of the postfault system swing curves to provide the value of the state variables as functions of time,

(3) a comparison of the values of the Liapunov Function using the calculated state variables with respect to the prefault boundary at each step in the postfault swing curve calculation. T_{cr} is reached when the value of Liapunov Function at the following step in the calculation is greater than BD,

(4) the increase of T_s by 1 radian and repetition of steps (2) and (3) until T_{cr} is equal or less than T_s.

The computational procedures are similar to those stated in the preceding example. The sequence of calculations is illustrated in Figure 4.9. A few detailed comments regarding the specific computational procedures are required:

(1) Load-flow calculations:

The load flow calculations are carried out in the same fashion as described in Section 4.2(B) for the previous illustrative example. $(\delta^s)_{pref}$ is also obtained in these calculations.

(2) $(Y)_f$, $(Y)_{pf}$ and $(Y)_{prefault}$ (or $(Y)_{pref}$) calculations:

The admittance matrix calculations are performed also in the same way as given in Section 4.2(B) except that one more Y-matrix, $(Y)_{pref}$, is required.

(3) The faulted trajectory calculation:

In the swing curve calculations $(Y)_f$ is applied in Equation (3.31) and the integration is carried out by the Runge-Kutta method described in Section 3.4 up to the fault clearing time T_s. The reference machine is determined before the integration is started.

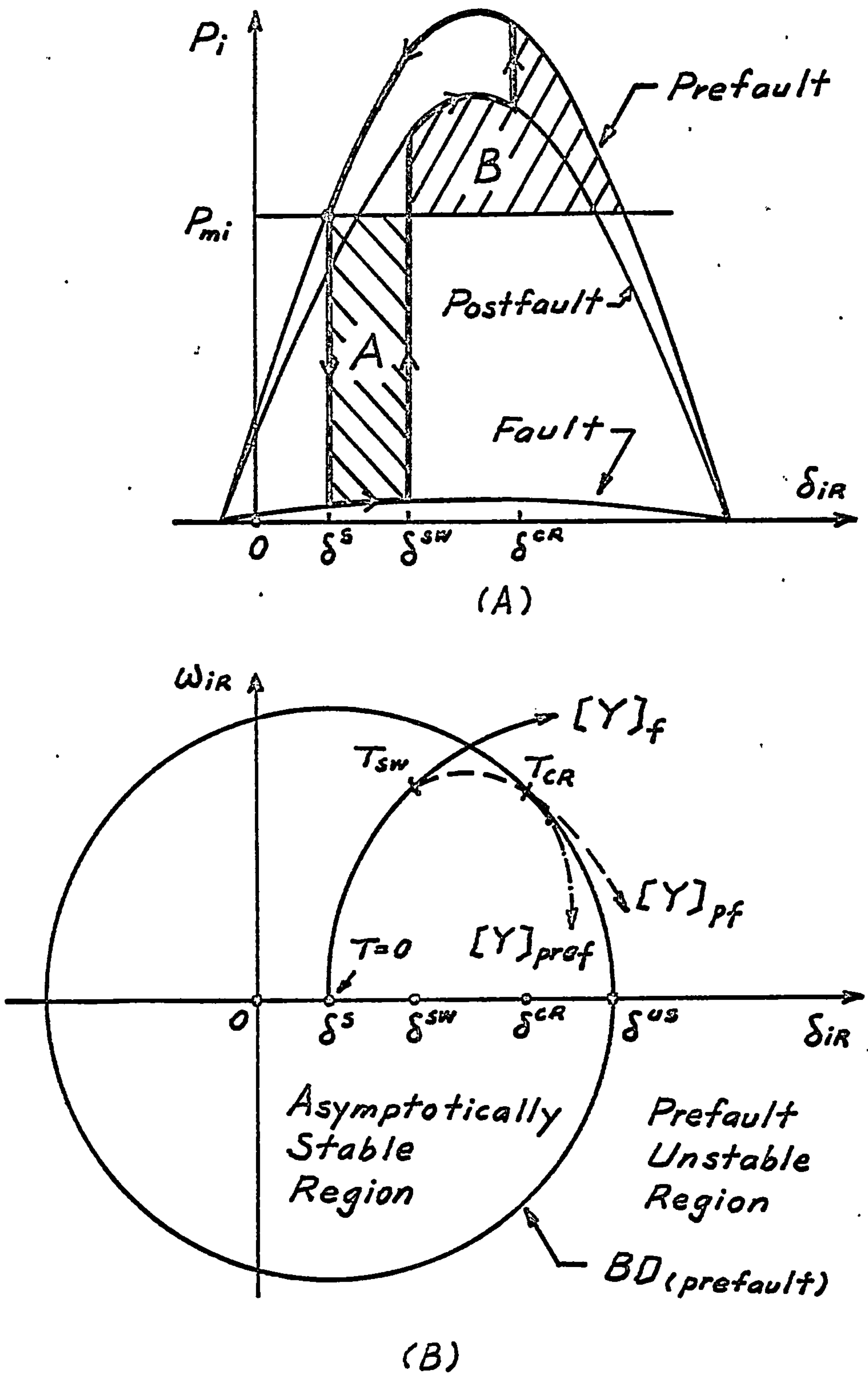

Figure 4.8 The conceptual explanation of the critical line reclsoing problems.
A - Equal area diagram.
B - Phase plane diagram.

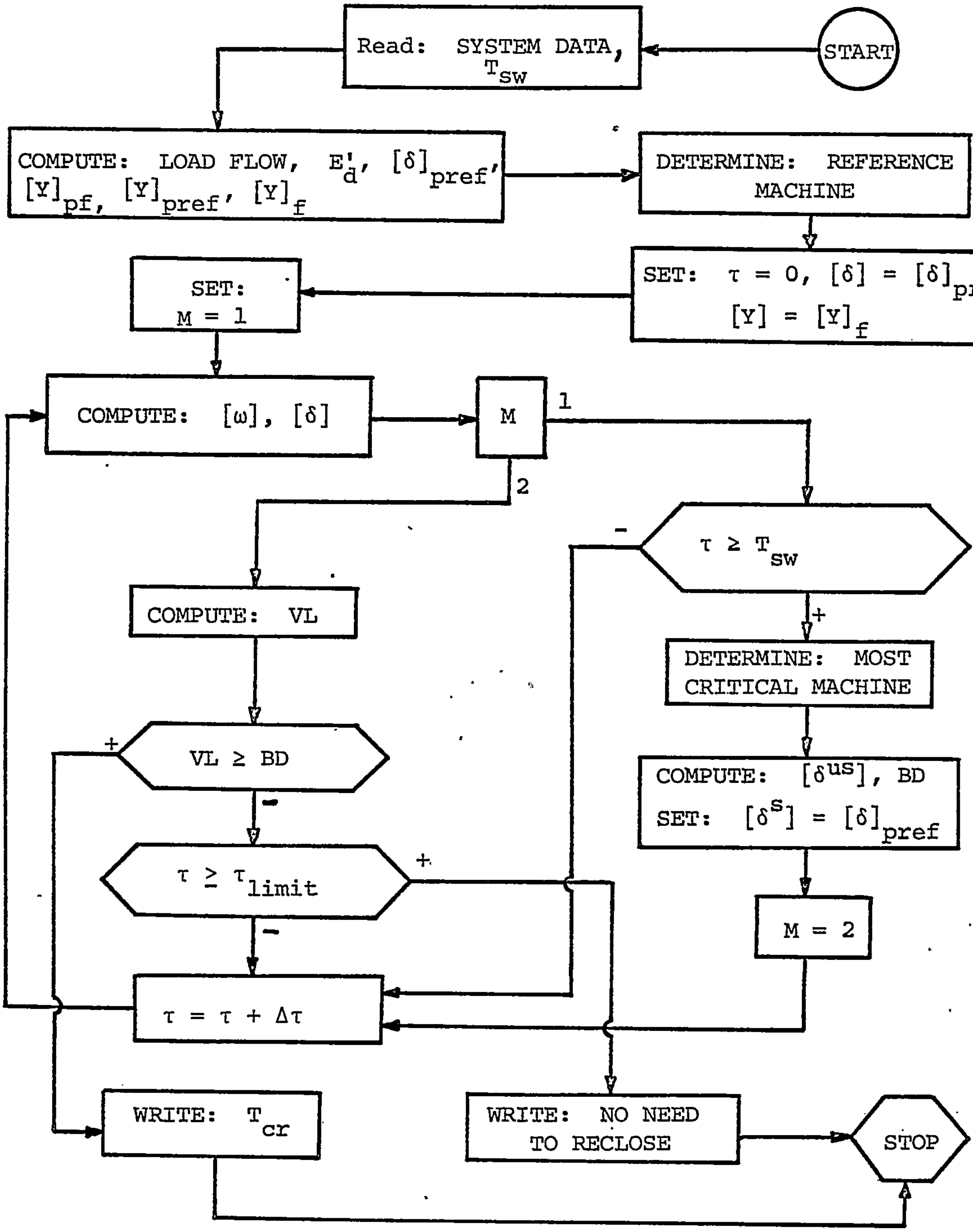

Figure 4.9 The computer program flow-chart for the critical reclosing time calculation

(4) The prefault unstable equilibrium $(\delta^{us})_{pref}$, calcultation:

The most critical machine is determined at the instant T_s by the substitution of $(Y)_{pf}$ into the second equation of Equation (2.2) as shown in Section 2.1.2. The admittance matrix $(Y)_{pref}$ is applied to the equilibrium calculation.

(5) The boundary of the region of asymptotic stability, BD, calculation:

By the substitution of $(Y)_{pref}$, $(\delta^{s})_{pref}$ and $(\delta^{us})_{pref}$ into Equation (2.12), BD for the prefault system can be obtained.

(6) The critical reclosing time T_{cr} calculations:

After T_s the postfault trajectory is obtained by the Runge-Kutta method with the replacement of $(Y)_f$ by $(Y)_{pf}$. The value of V, given by Equation (2.10), is calculated at each integration step using $(Y)_{pref}$, $(\delta^{s})_{pref}$ and (δ) obtained at each step. The value of V is compared to BD. If V is greater than or equal to BD at the n + 1 step at time τ^{n+1}, the critical reclosing time T_{cr} is given by τ^{n}.

(7) Successive switching effect calculations:

T_s is changed to $T_s + \Delta T_s$, and the new value of T_{cr} is obtained in the same fashion as in procedure (6). The study is terminated when the line reclosing can no longer make the prefault system stable.

The analytical and computational procedures described above were applied to a specific numerical problem to provide practical experience in the application of the method to a realistic system.

This example is given in the next section.

(C) Numerical example and discussions:

The system and the fault condition given in Figure 4.4 were again taken as an example for the line reclosing studies.

Figures 4.10 and 4.11 show that if the fault is cleared at 16 radians the line must be reclosed before 56 radians to maintain stable operation. More generally, the region for successful prefault operation is shown in Figure 4.12 as a semi-bounded area A, while area B is the region for unstable operation and area C is the region which is unrealizable in the physical system. The trajectory of motion for T_s = 16 radians and T_r = 56 radians, which lies in region B, is shown in Figure 4.10, while the case for T_s = 16 radians and $T_r = T_{cr}$ = 55 radians which lies in region A, is given in Figure 4.11. The cross hatched region in area A of Figure 4.12 is the net gain due to the line reclosing operation.

The advantage of using the Liapunov Direct Method in problems of this type is that a single numerical integration is required only up to the critical reclosing time in order to establish each point on the boundary (between regions A and B in Figure 4.12 for this specific example). If the numerical integration technique were used alone for this purpose a cut-and-try process would be required to establish each point on the boundary line.

As in the previous examples the validity of the results obtained with the Liapunov Method are dependent upon the limitations imposed by the specific V-function employed.

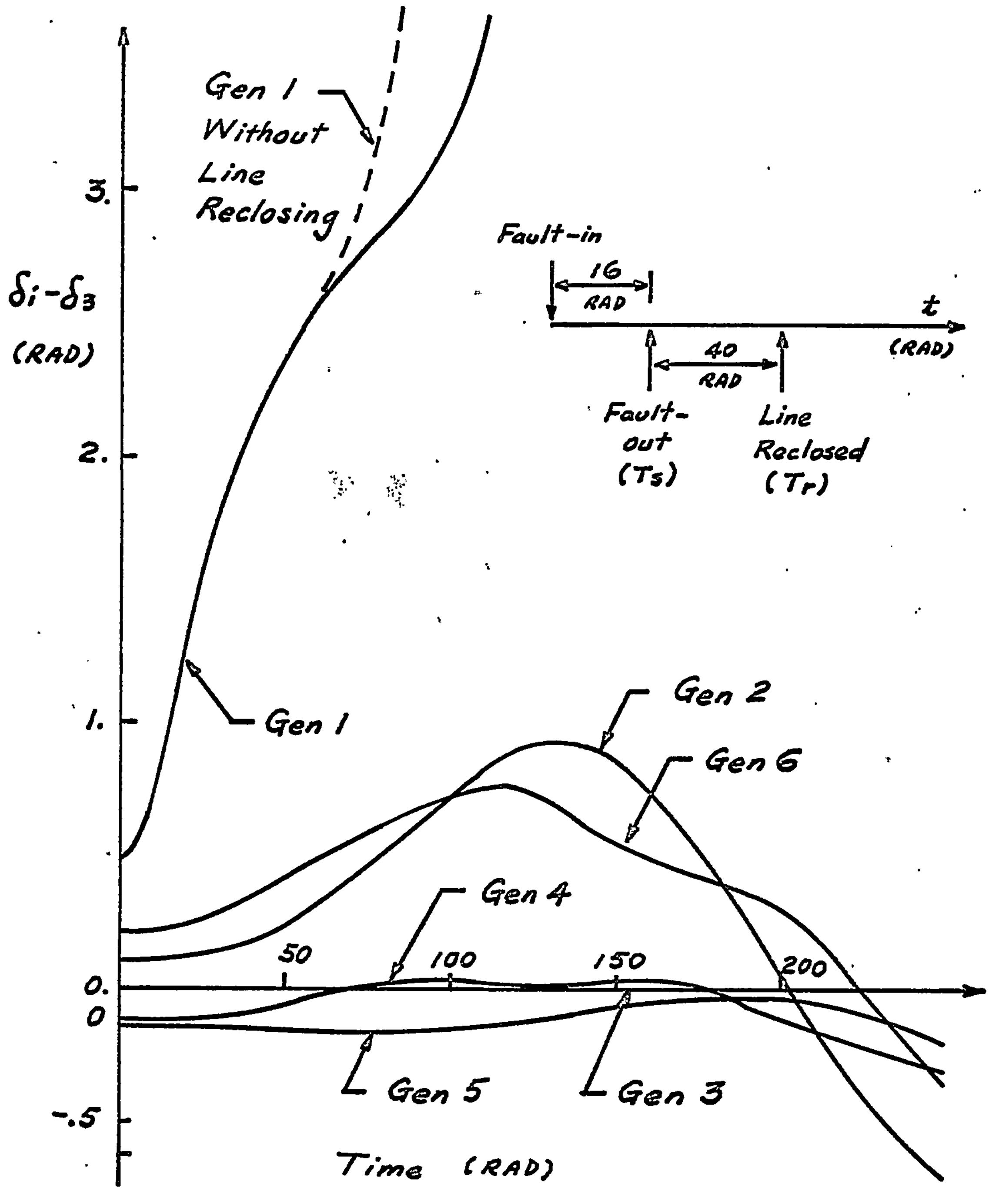

Figure 4.10 The swing curves of six machine system when the faulted lines are reclosed too late.

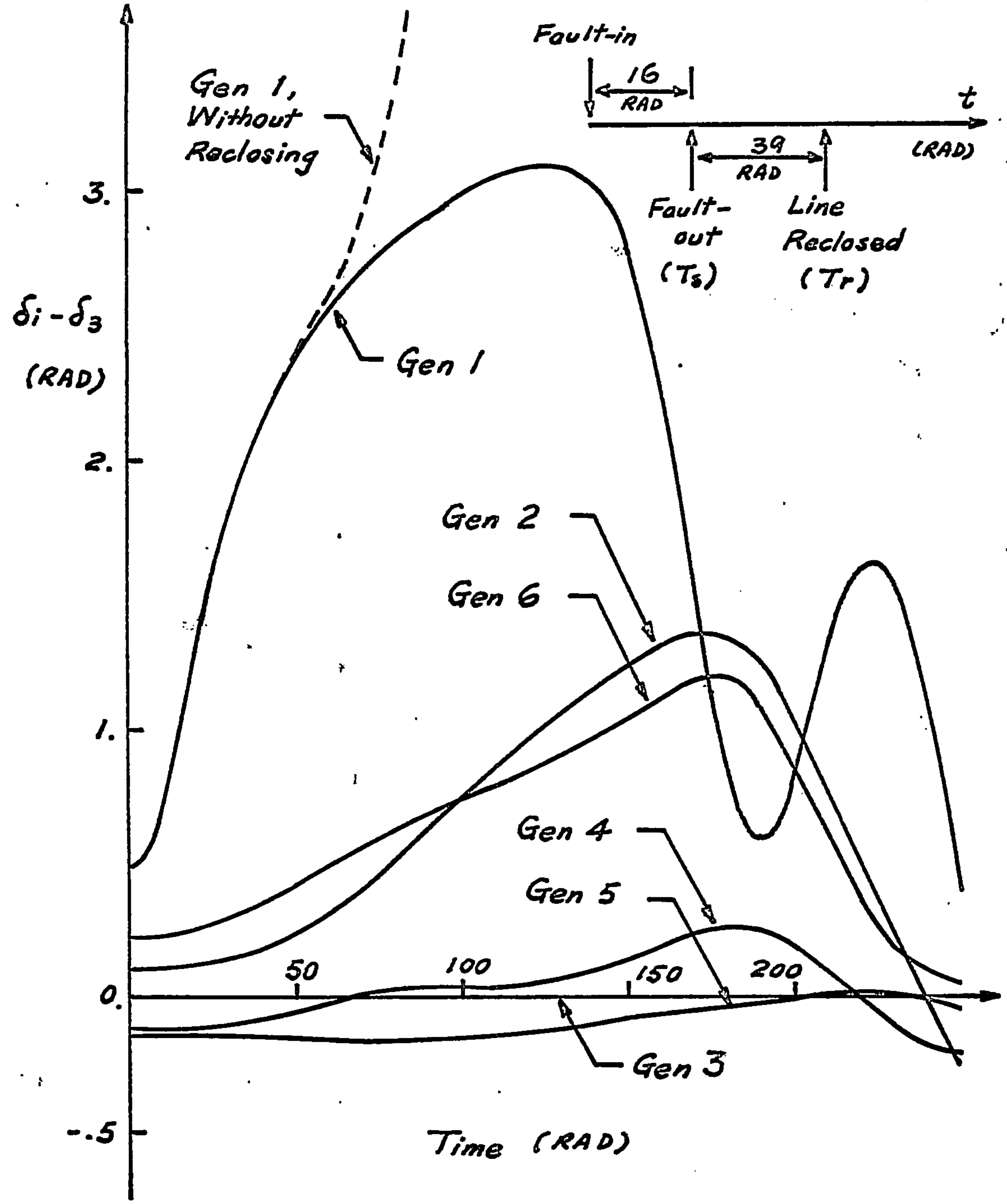

Figure 4.11 The swing curves of six machine system when the faulted lines are reclosed at the critical reclosing time, T_{cr} = 55 radians.

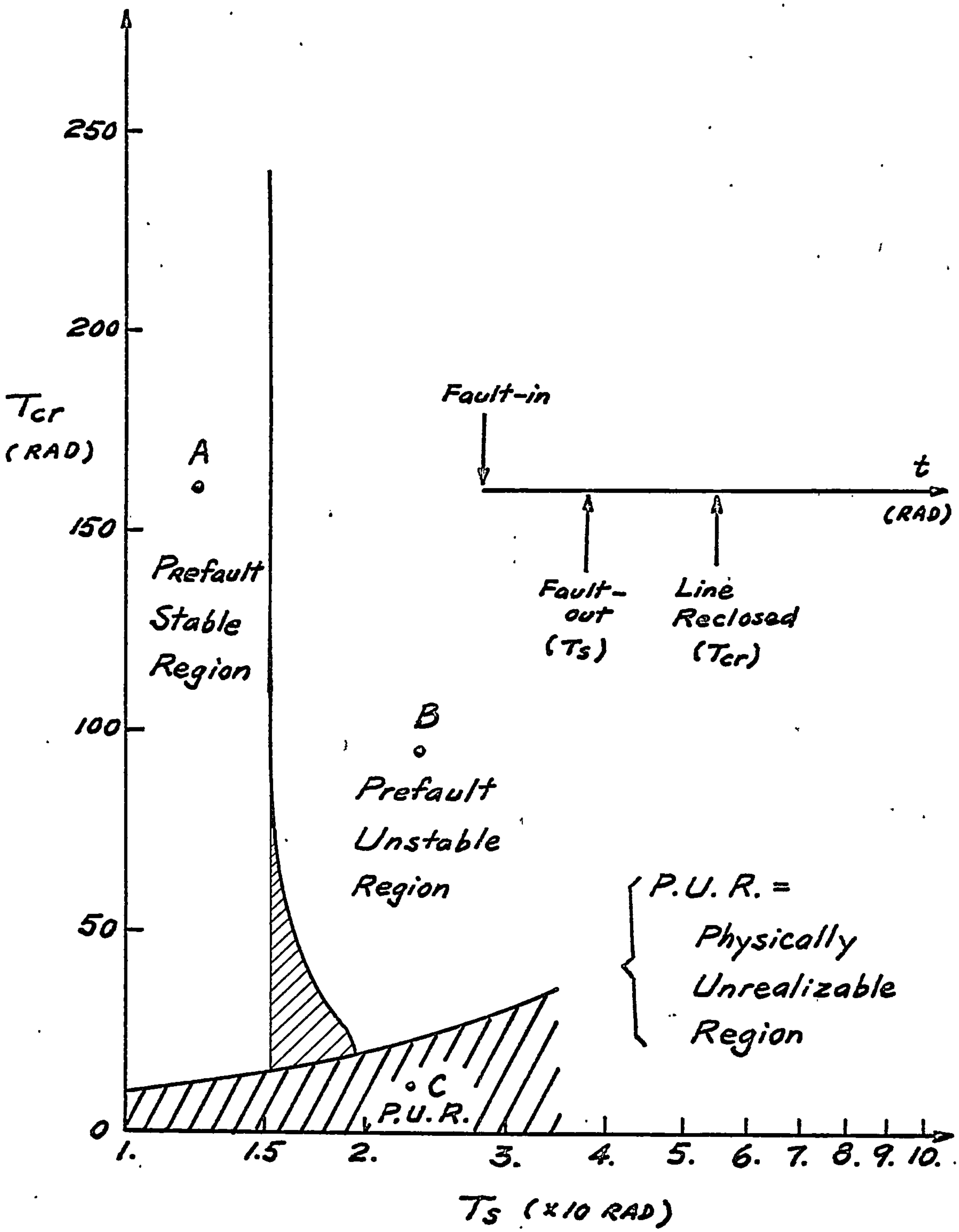

Figure 4.12 The regions of stability related to line reclosing operation.

4.4 Category Three - The Braking Resistance Effects:

The extension of the Direct Method of Liapunov to the study of problems of braking resistance effects was done to further illustrate the principle for stability analysis in situations involving an even more complex sequence of switching operations.

For the maximum effect the braking resistances should be connected either in series or shunt to the most critical machine terminals in the shortest possible time after the fault occurrence. Since the most critical machine possesses the highest acceleration, the optimized effect to the overall system stability under the transient disturbance can be obtained. The instant of time that the resistances can be connected is dependent upon the response time of the protective system. In general, due to the cost of the braking resistance, a short-time rating type instead of continuous-duty type is recommended[(23)]. The practical range of connected time-duration is about 0.5 to 1.0 second. The basis for the size and the time duration decisions is the fact that the absorbed energy should be slightly greater than the excess kinetic energy of the most critical machine due to the fault. The time of resistance disconnection is determined either by (1) the most critical machine reaching its first minimum kinetic energy point, or (2) the braking resistor reaching its temperature limit. More information on this subject can be obtained by referring to Crary[(6)] and Kimbark[(23)].

Since it is the purpose of this thesis to illustrate the general applications of the Direct Method of Liapunov to the multimachine power system transient stability studies, the investigation of the effects caused by the connection of the braking resistances is intended to serve merely as an illustrative example; no intention has been made to consider the practical technical difficulties which may arise due to the detailed switching actions specified by the connection of the braking resistances. Two different types of problems as illustrated as follows:

(1) the effect on the critical switching time due to the connection of the braking resistances during the faulted period, and

(2) the effect on the critical reclosing time due to the connection of the braking resistances during the postfault period.

In the first case, the connection of the braking resistances is assumed to take place 10 radians after the fault occurrence and is maintained until the fault is cleared. In the second case, the resistances are connected at the fault clearing instant and removed 10 radians thereafter. The analytical consideration, the detailed computational processes, and finally, the numerical details of the examples are given in this section.

(A) Problem one:

For the case where the braking resistances are connected during the faulted period, the problem is a PREFAULT - FAULT $\xrightarrow{\text{10 rad}}$ FAULT (with R) $\xrightarrow{T_1}$ POSTFAULT (without R) type of operation. It is exactly the same as in the problems in Section 4.2 except that an additional admittance matrix, i.e. the matrix of the faulted system with the braking resistance connected, must be considered. At 10 radians after the fault occurrence, the faulted admittance matrix is replaced by the faulted admittance matrix with the braking resistance connected, and the most critical machine is determined. The numerical integration is continued until the trajectory intersects with the boundary of post-fault asymptotic stability. The computational processes for this problem are similar to those specified for the examples in Section 4.2.

The assumptions and breaker operating conditions made in Section 4.2 for the six-machine system are retained. Since generator number 1 is the most critical, the braking resistances are connected to the terminal of this generator either in series or in parallel. The types of system operation as well as the results of the study are shown in Figure 4.13.

Figure 4.13 illustrates the effects of the braking resistances on the system critical switching time. For this specific system, the connection of the series braking resistances with values in the range of 0.1 to 0.2 pu to generator number 1 can greatly improve the system transient stability for a fault at the location indicated. This agrees with the condition for the maximum power transfer from generator number 1 to the system. (A value of 0.146pu series resitance is the condition for maximum power transfer from generator number 1, if the line resistances and the shunt capacitances of the network are neglected, and the three phase fault is assumed to occur at Bus 14 for easier calculation). For the shunt braking resistance case, no optimum value can be obtained for this fault location. The rising side of the curve for the series braking resistance effect indicates the region that the braking resistance over-dissipates the accelerating power in generator number 1; while

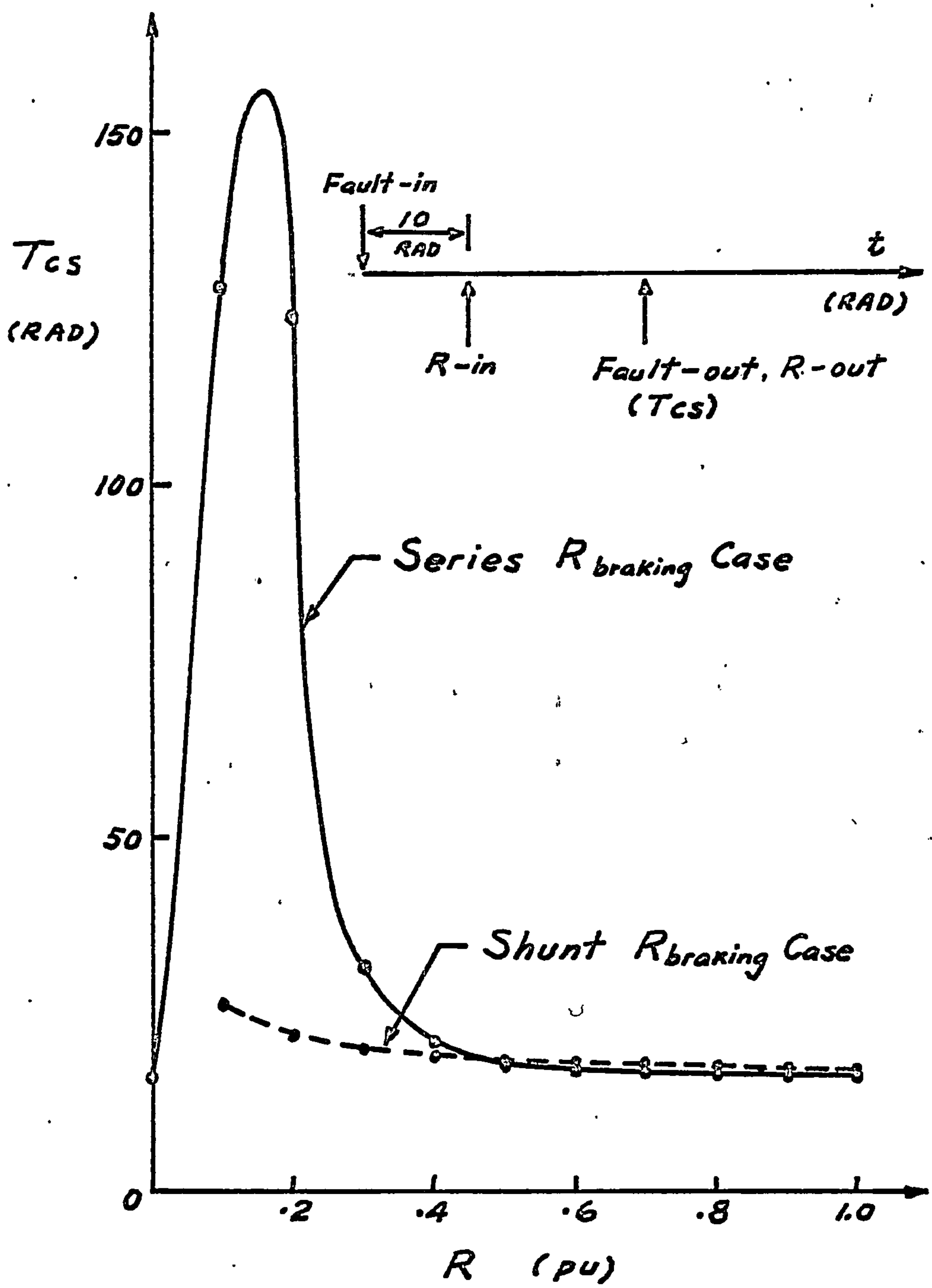

Figure 4.13 The effects of braking resistances on critical switching time.

the descending side of the curves (for both series and shunt braking resistance cases) illustrates that the accelerating power dissipation by the braking resistances is not enough. The comparison of the series and shunt braking resistance effects indicates that the series braking resistance is superior to the shunt one for this specific system with the operating conditions and fault location as defined.

(B) Problem two:

For the case where the braking resistances are connected for 10 radians during the postfault period, a PREFAULT - FAULT $\xrightarrow{T_1}$ POSTFAULT (with R) $\xrightarrow{T_2}$ POSTFAULT (without R) $\xrightarrow{T_3}$ PREFAULT type of operation is considered, when T_2 is 10 radians. The critical line reclosing can occur either in T_2 or T_3 depending upon the practical situation. If the critical reclosing time occurs in period T_2, a simultaneous operation of reclosing and braking resistance removal is assumed; therefore, the critical reclosing time T_{cr} can be equal to either $T_1 + T_2$ (part or whole) or $T_1 + T_2 + T_3$. A conceptual diagram for a case of the later reclosing operation is shown in Figure 4.14, where $T_s = T_1$. The trajectory, which denotes the operation for this case, is defined by Equations (2.2) and (2.13); and it is detoured three times due to the switching actions. The boundary of the region of asymptotic stability BD is defined in the same way as in Section 4.3. It is to be noted that the most critical machine at the fault clearing instant may not be the same as that at the instant of damping resistance removal, because $\dot{\omega}_i$ is not a function of (δ) only, but also a function of (ω) and (D). Therefore, the boundaries (BD) may be

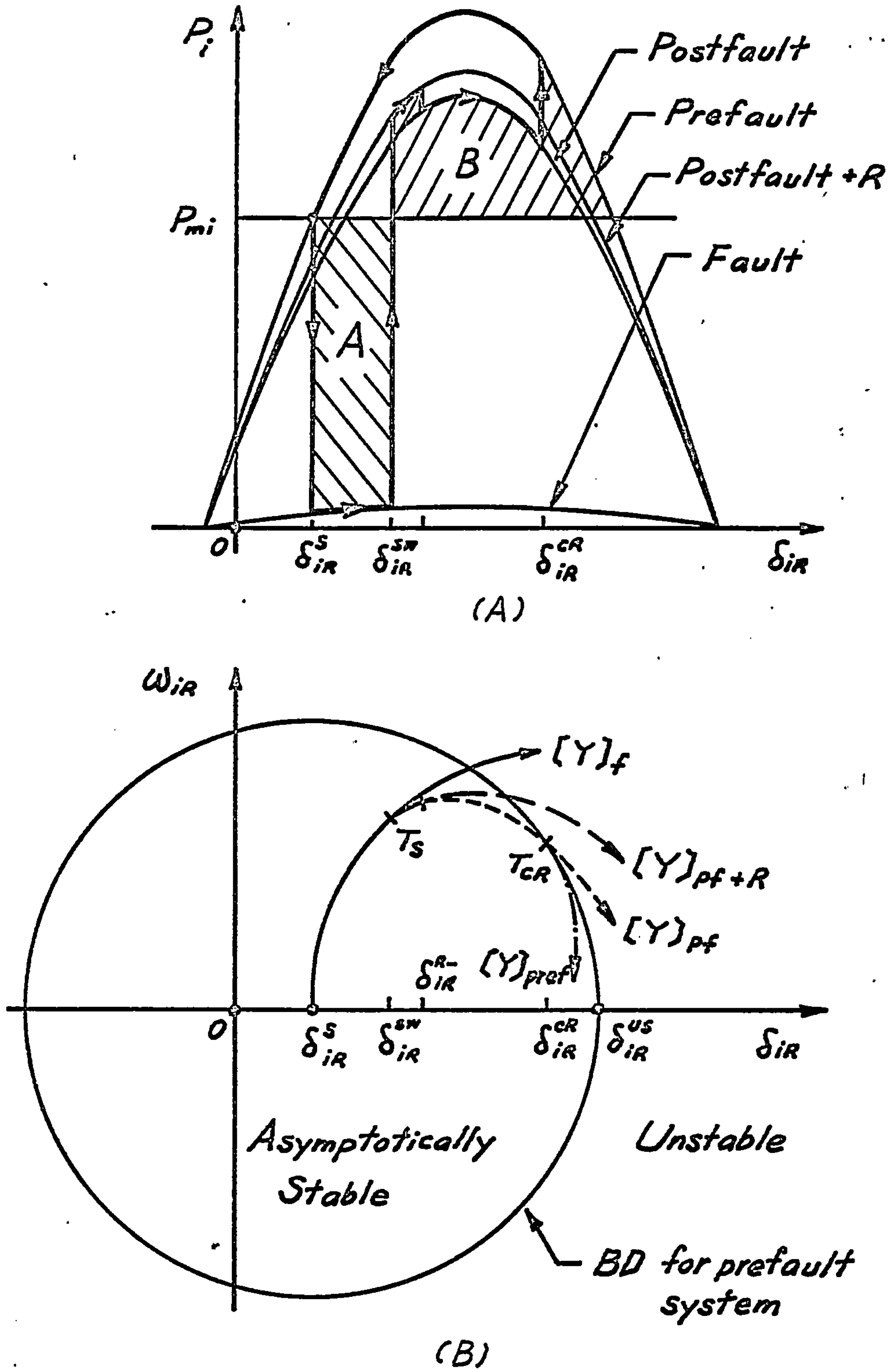

Figure 4.14 The conceptual explanation of the braking resistance connection problem.
A - Equal area diagram.
B - Phase plane diagram.

different in periods T_2 and T_3 even though the same prefault Y-matrix is used for the calculation of BD in both periods. Since it is not possible to predict whether the critical reclosing instant lies in T_2 or T_3, the most critical machine at the instant of fault-clearing must be determined in order to define the boundary; furthermore, at the end of T_2, the new most critical machine must be found again if the critical reclosing time goes beyond T_2.

The computational processes for this problem can be classified into specific steps as follows:

(1) the load-flow calculations which are exactly the same as those given in Section 4.2(B),

(2) the $(Y)_f$, $(Y)_{pf+R}$, $(Y)_{pf}$ and $(Y)_{pref}$ calculations which can be done in the same way as in Section 4.2(B),

(3) the faulted trajectory calculations in which $(Y)_f$ is applied to Equation (3.31) up to the instant T_1,

(4) the prefault unstable equilibrium where $(\delta^{us})_{pref}$ is calculated in the same way as in Section 4.2(B),

(5) the calculation of the boundary of the region of asymptotic stability BD which is done also in the same fashion as in Section 4.2(B),

(6) the T_{cr} determination during the T_2 period where in $(Y)_f$ is replaced by $(Y)_{pf+R}$ in Equation (3.31). The value of V, given by Equation (2.10), is calculated and compared with BD at each integration step. When $V \geq BD$, the critical reclosing time T_{cr} is obtained where $T_{cr} = T_1 + T_2$, and

(7) if V is less than BD at the end of T_2, the most critical machine and the new prefault unstable equilibrium are determined. This gives a new prefault stability boundary. The trajectory is detoured from the 'postfault + R' case to the case of postfault by the replacement of $(Y)_{pf+R}$ by $(Y)_{pf}$ in Equation (3.31). The processes of comparing V and BD are continued until V is greater than or equal to BD. At the instant when $V \geq BD$, the critical reclosing time T_{cr} is determined where $T_{cr} = T_1 + T_2 + T_3$.

For the successive line reclosing and different braking resistance effect studies, the steps are repeated as shown in the computer flow chart in Figure 4.15.

The results of this study are shown in Figures 4.16 and 4.17. From these two figures, it is understood that the shunt braking resistances are superior to the series for this specific system. This agrees with accepted practice[23]. In Figure 4.16, the small hill on the curve for R = 0.6 pu is caused by the change of the most critical machine from number 1 to number 6 at the instant of fault clearing and resistances removal. The net gain due to the connection of the braking resistances is not too significant, and this is especially true in the case of the series braking resistance in this specific system.

The numerical examples given in this section illustrate that in principle, the Liapunov Direct Method can be extended to multiple switching problems. Due to the severe limitations (Section 2.1.1) which must be imposed in order to use the specific V-function employed, the numerical results obtained in these examples do not necessarily have practical significance.

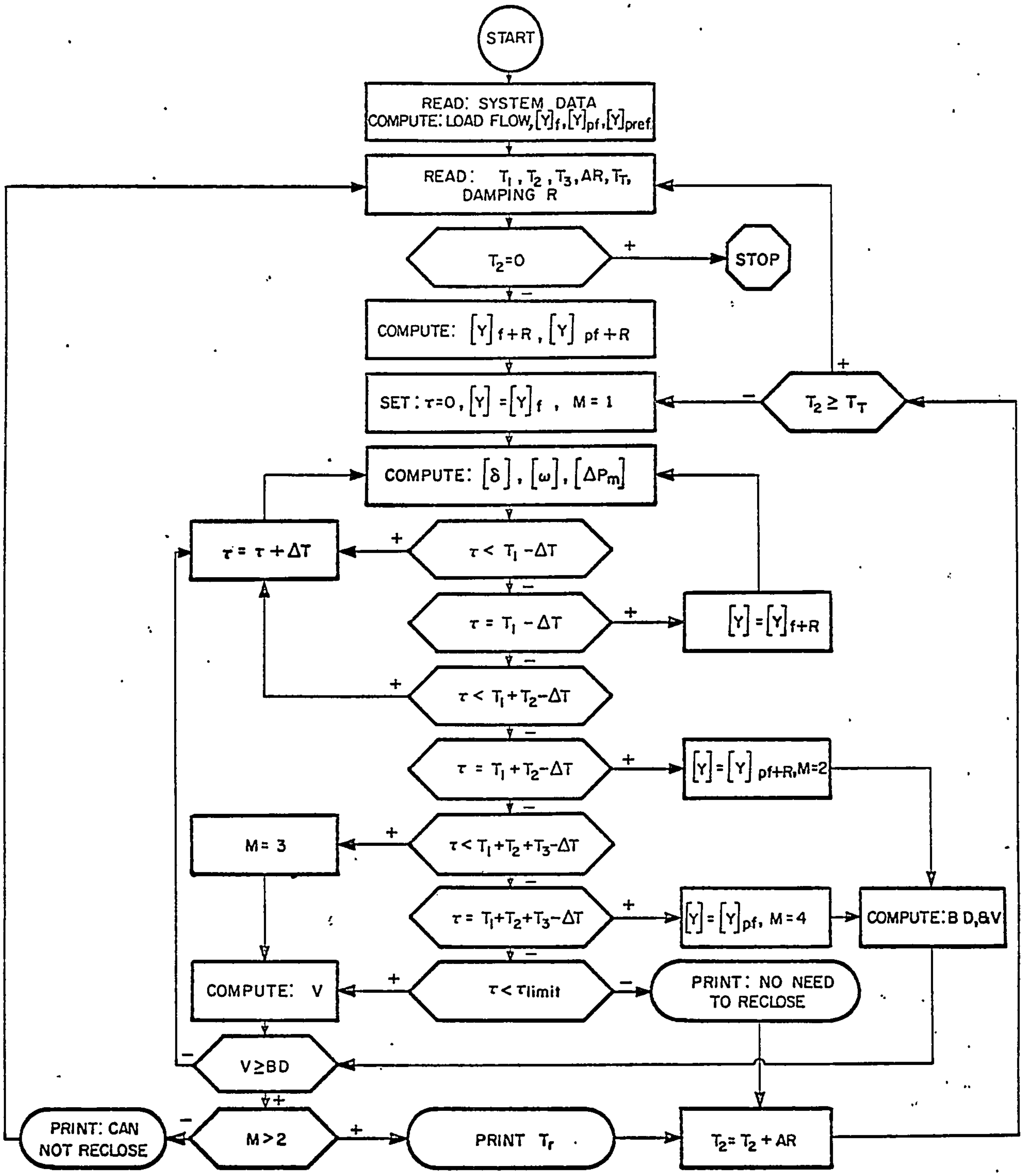

Figure 4.15 The computer program flow-chart for the breaking resistance and the successive line reclosing effects studies.

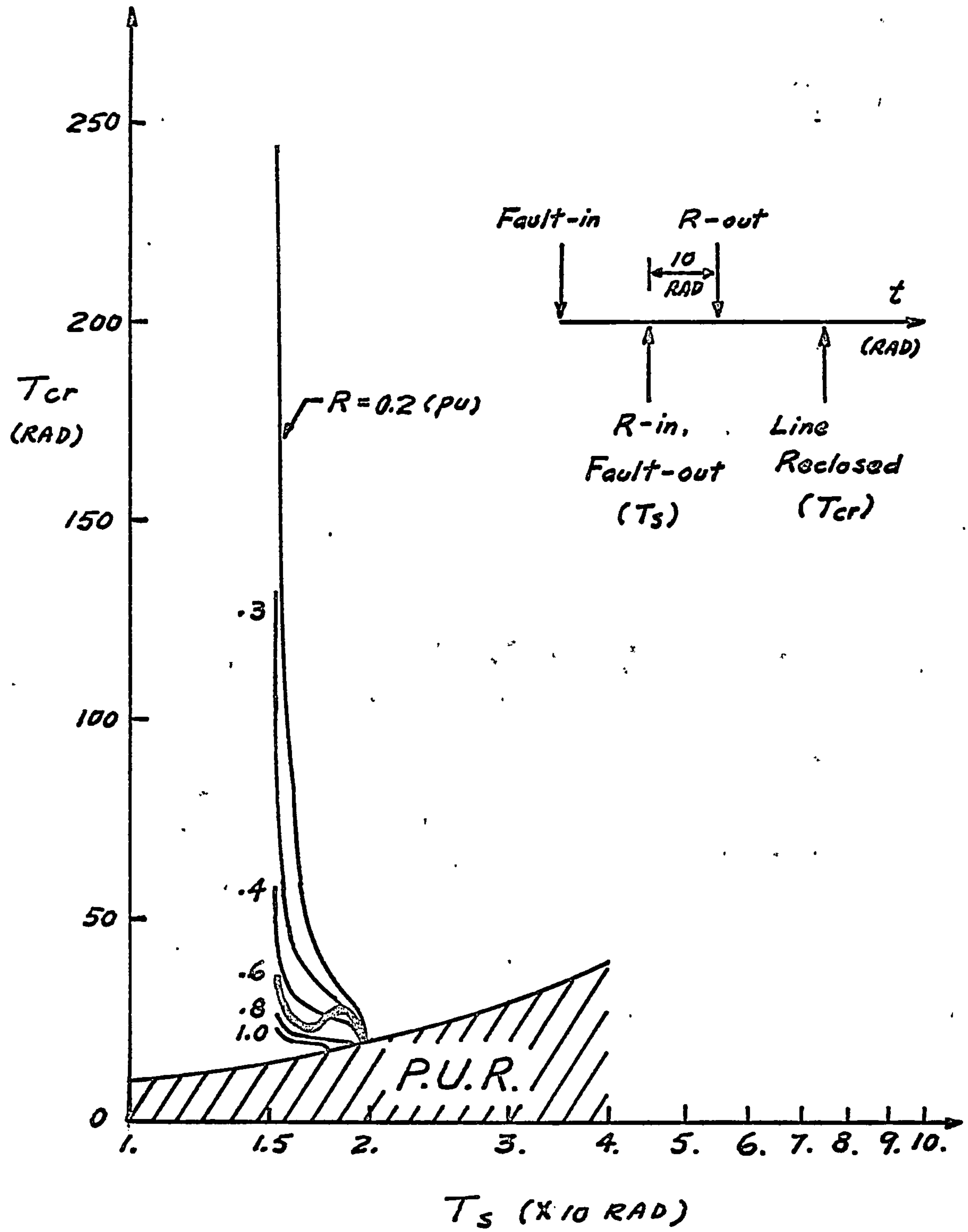

Figure 4.16 The effect of series braking resistances on the critical line reclosing problem.

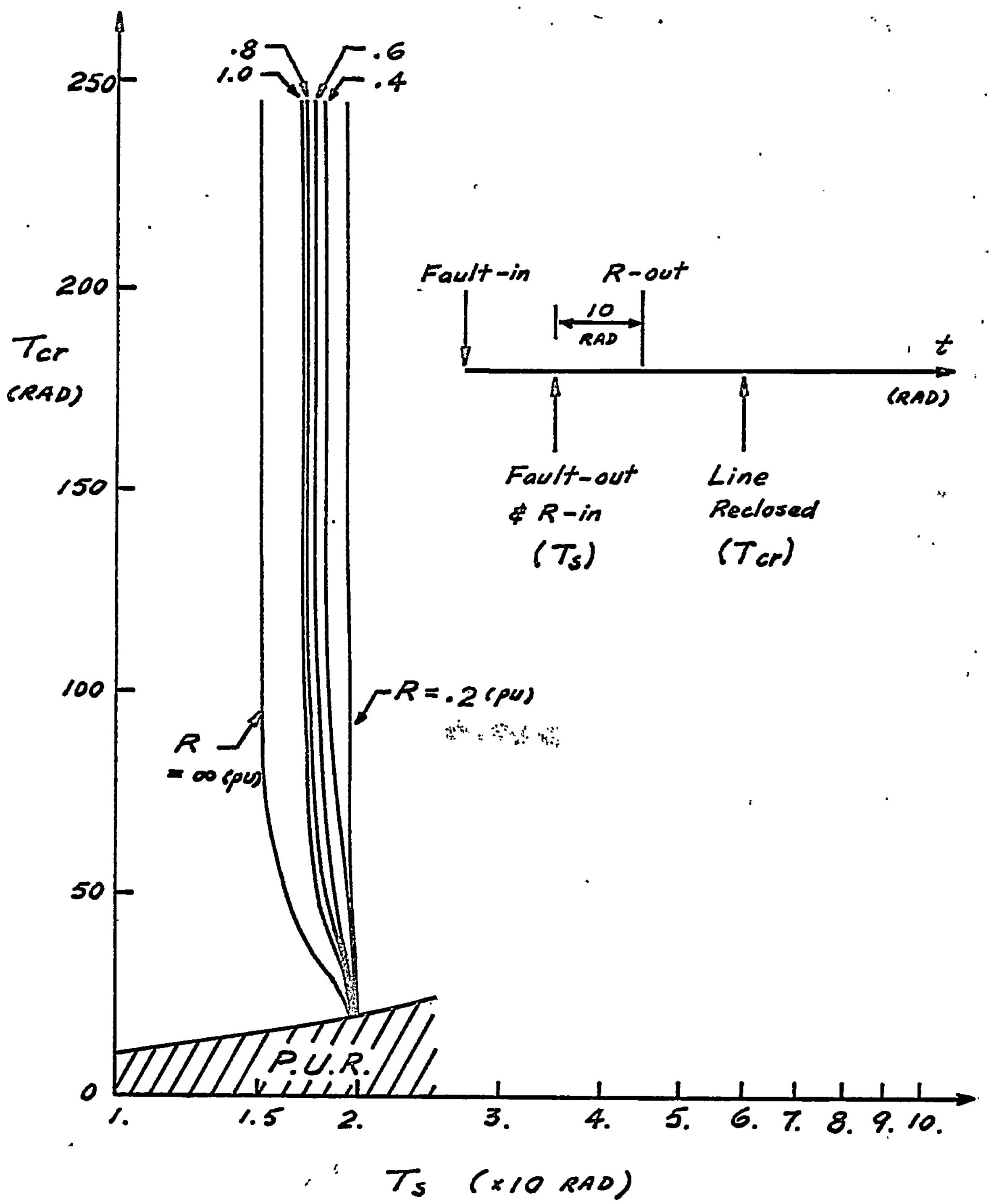

Figure 4.17 The effect of shunt braking resistances on the critical line reclosing problem.

5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

As a result of the research carried out and reported in this thesis, several conclusions have been drawn. These conclusions related specifically to (1) the general principle of the application of the Direct Method of Liapunov to power system stability; (2) the specific Liapunov function introduced by El-Abiad and its modified form as developed in this thesis; (3) the numerical techniques of Steepest Descent and Runge-Kutta integration used as tools associated with the stability analysis by the Liapunov Direct Method. Each of these is discussed hereunder.

(1) El-Abiad has shown that the Second Method of Liapunov can be applied to the stability evaluation of a multimachine power system subjected to a single switching operation. In this thesis, are illustrated the basic work of El-Abiad repeated on a larger power system and then an extension of the method to include multiple switching problems. The specific multiple switching problems solved deal with line reclosing operation and with switched braking resistances. It is demonstrated that the use of the Liapunov asymptotic stability criterion can provide a significant saving in digital computer time in stability evaluation for some types of switching operations; however it was also found that the advantages of this method can be realized only for cases which are trivial in nature with the specific Liapunov function chosen.

(2) As a result of a comprehensive review of the published literature in the field, it was initially decided that the Liapunov function developed by El-Abiad based on an energy-integral function was the most suitable for stability assessment in the case of a multimachine power system. The equations on which this V-function was based are related to a synchronous reference; therefore, they lead to an erroneous conclusion regarding stability of a power system which settles at a frequency which differs from the synchronous reference following a disturbance. In Chapter 2 of this thesis a modification of the El-Abiad V-function is given. This modification provides a V-function which gives a stability conclusion,

consistent with that commonly accepted by power system engineers wherein a power system is considered to be stable so long as the synchronous machines maintain synchronism with each other although not necessarily at synchronous speed.

It was found that the Liapunov function originally proposed by El-Abiad and its modified form both suffer from a number of restrictions which cannot be easily overcome. Specifically, this type of V-function is not adequate for universal application because:

(a) it does not include terms to account for machine damping, voltage regulator and governor effects, machine non-linearity such as magnetic saturation and saliency,

(b) non-linear loads cannot be considered,

(c) the stability boundary depends on the calculation of the stable and the unstable equilibria of the system which depend in turn on the calculation of the most critical machine. The criterion for selecting the most critical machine is not always adequate because of the non-linear character of the function involved.

(d) the stability boundary is somewhat indeterminant since it is calculated numerically and the accuracy problem is always present. This leads to a sensitivity problem in the vicinity of the point where the trajectory intersects with the boundary, such that the actual intersection point cannot be exactly defined. In the cases studied this led to a pessimistic indication of permissible switching time. It is suggested in the following section that these specific points require further study.

(3) The evaluation of the stability boundary requires a calculation of the equilibria. The Method of Steepest Descent was adapted for this purpose. Due to the fact that the equations considered have an infinite set of minima; restrictions were required to assure that the desired solution was obtained. This entailed the development of a step-size control routine which assured proper convergence. For a specific system a definite range of step-size is necessary, for the particular cases reported in Chapter 4 the optimized value of step-size was found to lie between -0.01 and -0.1.

The differential equations which describe the power system are non-linear and second order. The Runge-Kutta method of integration was generalized to solve these equations. This generalized numerical integration method was found to be extremely time consuming; therefore, a simplified scheme was developed as shown in Chapter 3. By means of this simplification, the computation speed was improved by a factor of about six times compared to the generalized one without simplification. The accuracy of the simplified scheme was adequate for this application.

In general, it was found that the principle of multimachine power system transient stability analysis based on the criterion of the Direct Method of Liapunov can be extended to more general types of multiple switching operations; however, a more suitable V-function must be found before this method can be of great practical value.

5.2 Recommendations for Further Work

In Section 5.1, it has been pointed out that some problems have been solved in this research; at the same time, several more problems have arisen. These problems must be solved, otherwise, the Direct Method of Liapunov cannot have great value for practical applications.

The first and most important problem is the need of a more general and universal V-function. This function should be general enough to include the effects of speed and voltage regulators, non-linear loads, system and machine dampings, machine non-linearities in multi-machine systems. The inclusion of most of these factors has been made by Siddiqee[24] and Willems[25] to a one machine - infinite bus type system; however, great difficulties still exist in the generalization of these types of V-functions to the multimachine system.

The second problem which requires further study is the criterion for determining the most critical machine which directly affects the problem of defining the boundary of the region of asymptotic stability of a power system. The criterion used in this thesis was found to be limited due to the non-linear characteristics in the criterion itself.

The third problem is the sensitivity of the intersection point between the trajectory and the boundary of the region of asymptotic stability. This problem demands further consideration of the solution of the stable and the unstable equilibria. Since a numerical method is applied to this problem, the equilibria obtained are approximate values only; therefore, inaccuracy in the critical condition calculation may arise if the sensitivity at the neighbourhood of intersection is too low.

It is suggested that these problems be considered as bases for further work extending from that reported in this thesis.

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APPENDIX 7

APPENDIX 7.1 THE DERIVATION OF $\dot{V}$

Since

$$V = \frac{1}{2}\sum_{i=1}^{n} M_i\omega_i^2 + \sum_{i=1}^{n} (E_i^2 G_{ii} - P_{mi})(\delta_i - \delta_i^s)$$

$$+ \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_iE_jY_{ij}[\sin(\theta_{ij} + \delta_i^s - \delta_j^s) - \sin(\theta_{ij} + \delta_i - \delta_j)]$$

(7.1.1)

is a function of ω_i, δ_i and δ_j; therefore, the first time (in τ) derivative of V is

$$\dot{V} = \frac{dV}{d\tau}$$

$$= \left(\frac{\partial V}{\partial \omega_i} \cdot \frac{d\omega_i}{d\tau} + \frac{\partial V}{\partial \delta_i} \cdot \frac{d\delta_i}{d\tau} + \frac{\partial V}{\partial \delta_j} \cdot \frac{d\delta_j}{d\tau}\Bigg|_{\substack{j=1,\cdots,n \\ j\neq i}}\right)_{i=1,\cdots,n}$$

$$= \left(\frac{\partial V}{\partial \omega_i}\dot{\omega}_i + \frac{\partial V}{\partial \delta_i}\omega_i + \frac{\partial V}{\partial \delta_j}\omega_j\Bigg|_{\substack{j=1,2,\cdots,n \\ j\neq i}}\right)_{i=1,\cdots,n}$$

$$= \sum_{i=1}^{n} M_i\omega_i\dot{\omega}_i + \sum_{i=1}^{n} (E_i^2G_{ii} - P_{mi})\omega_i$$

$$- \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_iE_jY_{ij}[\cos(\theta_{ij} + \delta_i - \delta_j)]\ (\omega_i - \omega_j) \qquad (7.1.2)$$

From Equation (2.2),

$$\dot{\delta}_i = \omega_i$$

$$\dot{\omega}_i = \frac{1}{M_i} [P_{mi} - D_i\omega_i - \sum_{j=1}^{n} E_iE_jY_{ij}\text{Cos}(\theta_{ij} - \delta_i + \delta_j)]$$

$$i = 1, 2, \ldots, n \qquad (7.1.3)$$

$\dot{V}$ thus can be rewritten as

$$\dot{V} = \sum_{i=1}^{n} \omega_i [P_{mi} - D_i\omega_i - \sum_{j=1}^{n} E_iE_jY_{ij}\text{ Cos}(\theta_{ij} - \delta_i + \delta_j)]$$

$$+ \sum_{i=1}^{n} (E_i^2 G_{ii} - P_{mi})\omega_i$$

$$- \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_iE_jY_{ij}(\omega_i - \omega_j)\text{Cos}(\theta_{ij} + \delta_i - \delta_j)$$

$$= - \sum_{i=1}^{n} D_i\omega_i^2 + \sum_{i=1}^{n} E_i^2G_{ii}\omega_i$$

$$-\sum_{i=1}^{n} \sum_{j=1}^{n} E_iE_jY_{ij}\omega_i\text{Cos}(\theta_{ij} - \delta_i + \delta_j)$$

$$-\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_iE_jY_{ij}\omega_i\text{Cos}(\theta_{ij} + \delta_i - \delta_j)$$

$$+\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_iE_jY_{ij}\omega_j\text{Cos}(\theta_{ij} + \delta_i - \delta_j)$$

$$= - \sum_{i=1}^{n} D_i\omega_i^2 + \sum_{i=1}^{n} E_i^2G_{ii}\omega_i$$

$$- \sum_{i=1}^{n} \sum_{j=1}^{n} E_iE_jY_{ij}\omega_i\text{Cos}(\theta_{ij} - \delta_i + \delta_j)$$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j Y_{ij} \omega_i \mathrm{Cos}(\theta_{ij} + \delta_i - \delta_j)$$

$$+ \sum_{j=1}^{n-1} \sum_{i=j+1}^{n} E_i E_j Y_{ij} \omega_i \mathrm{Cos}(\theta_{ij} - \delta_i + \delta_j) \qquad (7.1.4)$$

because $Y_{ij}\angle\theta_{ij} = Y_{ji}\angle\theta_{ji}$.

Referring to Figure 7.1, since the $\sum_{i=1}^{n-1} \sum_{j=1}^{n}$ term represents the sum of areas A,B and C, therefore, Equation (7.1.4) can be simplified to

$$\dot{V} = - \sum_{i=1}^{n} D_i \omega_i^{\,2}$$

$$- \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j Y_{ij} \omega_i \mathrm{Cos}(\theta_{ij} - \delta_i + \delta_j)$$

$$- \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j Y_{ij} \omega_i \mathrm{Cos}(\theta_{ij} + \delta_i - \delta_j)$$

$$= - \sum_{i=1}^{n} D_i \omega_i^{\,2}$$

$$- \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j Y_{ij} \omega_i [\mathrm{Cos}(\theta_{ij} - \delta_i + \delta_j) + \mathrm{Cos}(\theta_{ij} + \delta_i - \delta_j)]$$

$$= - \sum_{i=1}^{n} D_i \omega_i^{\,2}$$

$$-2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j Y_{ij} \omega_i \mathrm{Cos}\theta_{ij} \mathrm{Cos}(\delta_i - \delta_j) \qquad (7.1.5)$$

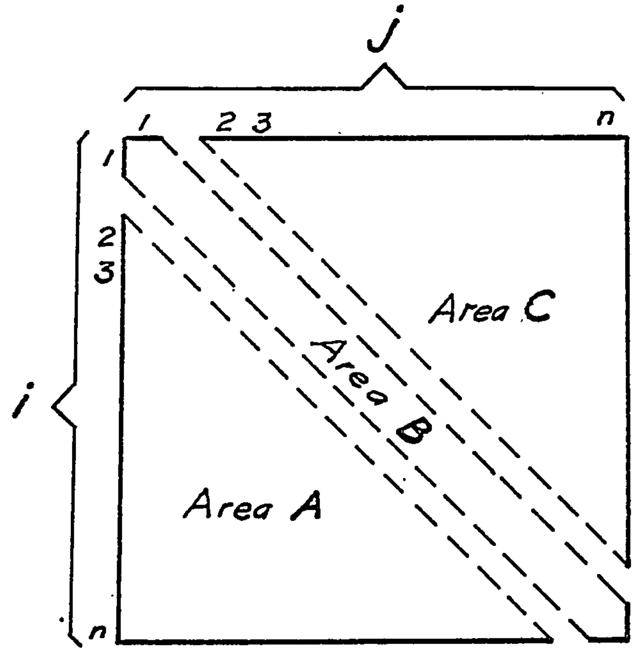

Figure 7.1 The geometrical representation of some of the terms in Equation (7.1.4)

(1) $$\text{area A} = \sum_{j=1}^{n-1} \sum_{i=j+1}^{n} E_i E_j Y_{ij} \cdot \text{Cos}(\theta_{ij} - \delta_i + \delta_j) \cdot \omega_i$$

(2) $$\text{area B} = \sum_{i=j=1}^{n} E_i E_j Y_{ij} \cdot \text{Cos}(\theta_{ij} - \delta_i + \delta_j) \cdot \omega_i$$

$$= \sum_{i=1}^{n} E_i^2 \, Y_{ii} \cdot \text{Cos}\theta_{ii} \cdot \omega_i$$

$$= \sum_{i=1}^{n} E_i^2 \, G_{ii} \cdot \omega_i$$

(3) $$\text{area C} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j Y_{ij} \cdot \text{Cos}(\theta_{ij} - \delta_i + \delta_j) \cdot \omega_i$$

But, $Y_{ij}\cos\theta_{ij} = G_{ij}$, hence

$$\dot{V} = -\sum_{i=1}^{n} D_i\omega_i^2 - 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} E_iE_jG_{ij}\ \omega_i\cos(\delta_i - \delta_j) \qquad (7.1.6)$$

APPENDIX 7.2 GAUSS-SEIDEL METHOD OF ITERATION

A general set of normal n-dimensional simultaneous algebraic equations

$$f_i(x_1, x_2, \ldots, x_n) = 0 \qquad i = 1, 2, \ldots, n \tag{7.2.1}$$

can be solved by a special method of iteration by the following processes:

$$x_i^{K+1} = g_i(x_i^{K+1}, x_2^{K+2}, \ldots, x_{i-1}^{K+1}, x_i^K, x_{i+1}^K, \ldots, x_n^K)$$

$$i = 1, 2, \ldots, n \tag{7.2.2}$$

This type of iterative process is called the "Gauss-Seidel Method of Iteration"[22].

The convergency of this method in the iterative process depends upon whether the Jacobian matrix of Equation (7.2.2) shown in Equation (7.2.3)

$$M(x_1, x_2, \ldots, x_n) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \cdots & \frac{\partial g_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial g_n}{\partial x_1} & \frac{\partial g_n}{\partial x_2} & \cdots & \frac{\partial g_n}{\partial x_n} \end{bmatrix} \tag{7.2.3}$$

satisfies the following inequalities

$$\sum_{j=1}^{n} \left. \frac{\partial g_i}{\partial x_j} \right|_{X=X^s} < 1 \qquad i = 1, 2, \ldots, n \qquad (7.2.4)$$

where X^s is the unknown solution denoted in matrix form.

The practical application of this type of iterative method of solving the power system load-flow problem is discussed by Chan[5].

APPENDIX 7.3 THE STEEPEST DESCENT METHOD OF MINIMIZATION

A simultaneous nonlinear algebraic equation can be generally expressed as

$$f_i(x_1, x_2, \ldots, x_n) = 0 \quad i = 1, 2, \ldots, n \tag{7.3.1}$$

A positive definite function Φ can be constructed from Equation (7.3.1) as follows

$$\Phi = \frac{1}{2} \sum_{i=1}^{n} f_i^2 \tag{7.3.2}$$

in which, the solution of Equation (7.3.1) satisfies Equation (7.3.2) and makes Φ a minimum. Therefore, the solution of Equation (7.3.1) can be expressed in terms of the minimization of Equation (7.3.2).

Let $X^o = (x_1{}^o, x_2{}^o, \ldots, x_n{}^o)^T$ be the initial guess of a solution of Equation (7.3.1). A Taylor expansion of Equation (7.3.2) gives

$$\Phi(X^o + aU) = \Phi(X^o) + \sum_{r=1}^{n} au_r \left.\frac{\partial\Phi}{\partial x_r}\right|_{X=X^o} + \frac{1}{2} \sum_{r=1}^{n}\sum_{s=1}^{n} a^2 u_r u_s \left.\frac{\partial^2\Phi}{\partial x_r \partial x_s}\right|_{X=X^o} + \ldots \tag{7.3.3}$$

where $U = (u_1, u_2, \ldots, u_n)^T$ = any vector. If u_r is taken as the component in the gradient direction of x_r, i.e. $u_r = \left.\frac{\partial \Phi}{\partial x_i}\right|_{X = X^\circ}$, the method is called the "The Steepest Descent Method of Minimization"[4].

The straight line equation $\overline{L_1L_2}$ which passes through points $(\Phi(X^\circ), 0)$ and $(0, a_T)$ as shown in Figure 7.2 can be obtained from Equation (7.3.3) by taking the first approximation. Hence

$$\overline{L_1L_2} = \Phi(X^\circ) + \sum_{r=1}^{n} a u_r \left.\frac{\partial \Phi}{\partial x_r}\right|_{X = X^\circ} \tag{7.3.4}$$

The value of a_T° in Figure 7.2 is solvable by setting $\overline{L_1L_2} = 0$, and is

$$a_T^\circ = \frac{-\Phi(X^\circ)}{\sum_{r=1}^{n} u_r \left.\frac{\partial \Phi}{\partial x_r}\right|_{X = X^\circ}}$$

$$= \frac{-\Phi(X^\circ)}{\sum_{r=1}^{n} \left.\left(\frac{\partial \Phi}{\partial x_r}\right)^2\right|_{X = X^\circ}} \tag{7.3.5}$$

Therefore, the next initial guess will be

$$x_i' = x_i^\circ + a_T^\circ \left.\frac{\partial \Phi}{\partial x_i}\right|_{X = X^\circ} \tag{7.3.6}$$

The iterative process is continued as shown in Figure 7.2 until the desired accuracy is attained.

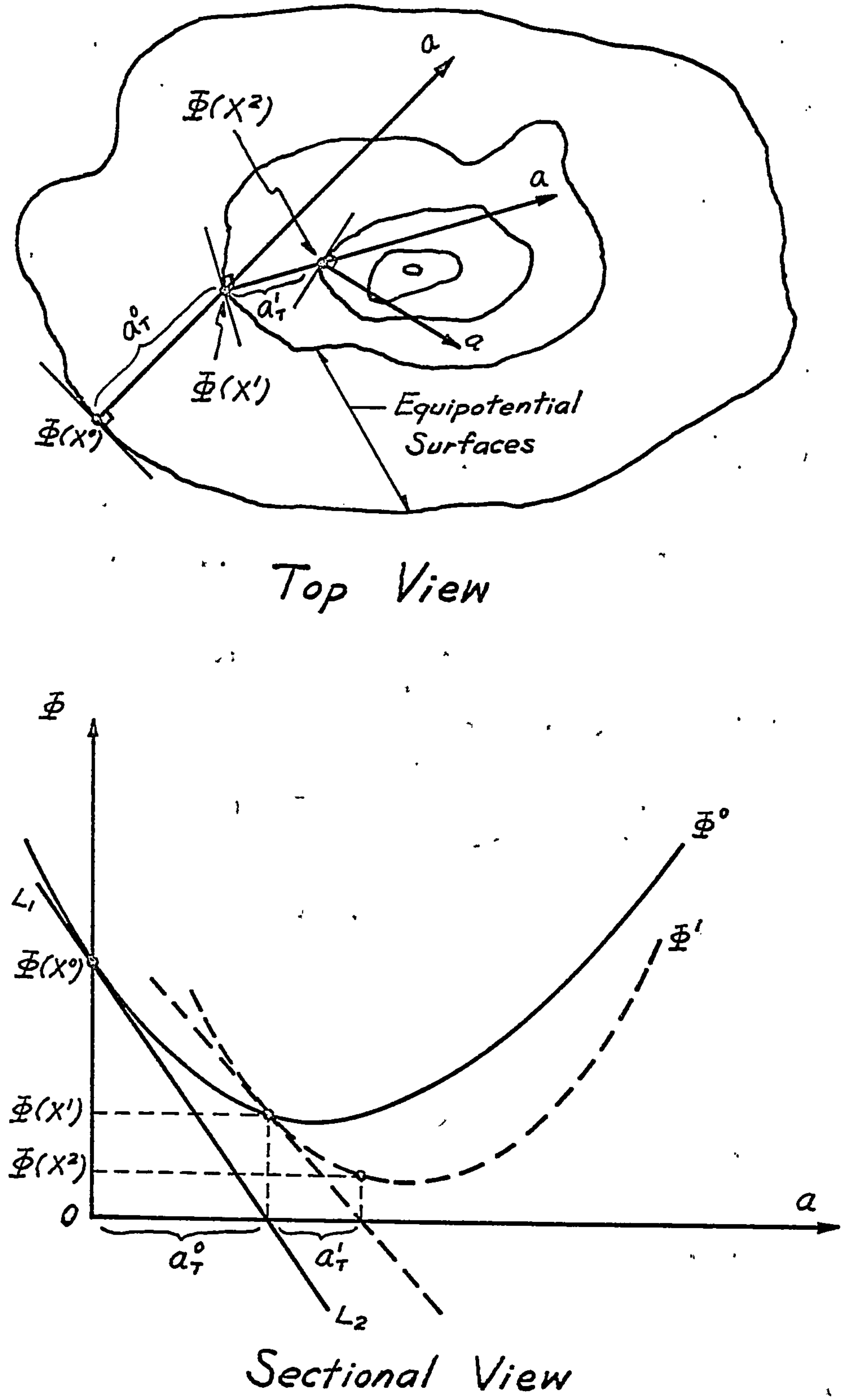

Figure 7.2 Geometrical explanation of the steepest descent method of minimization

APPENDIX 7.4 THE RUNGE-KUTTA METHOD OF INTEGRATION

The Runge-Kutta Methods of integration given by Scarborough[18] can be classified in the following two types:

(A) Second -order equations:

The general type second-order equation

$$\ddot{y} = f(x, y, \dot{y}) \tag{7.4.1}$$

can be integrated step by step by means of the following formulae, applied in the order given:

$$\left.\begin{aligned}
K_1 &= \Delta X \,.f(x_n, y_n, \dot{y}_n), \\
K_2 &= \Delta X \,.f(x_n + \frac{\Delta X}{2}, y_n + \frac{\Delta X}{2}\dot{y}_n + \frac{\Delta X}{8}K_1, \dot{y}_n + \frac{K_1}{2}), \\
K_3 &= \Delta X \,.f(x_n + \frac{\Delta X}{2}, y_n + \frac{\Delta X}{2}\dot{y}_n + \frac{\Delta X}{8}K_2, \dot{y}_n + \frac{K_2}{2}), \\
K_4 &= \Delta X \,.f(x_n + \Delta X, y_n + \Delta X.\dot{y}_n + \frac{\Delta X}{2}K_3, \dot{y}_n + K_3) \\
\Delta\dot{y} &= \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \\
\Delta y &= \Delta X \,. \left[\dot{y} + \frac{1}{6}(K_1 + K_2 + K_3)\right]
\end{aligned}\right\}$$

where $n = 0, 1, 2, \ldots\ldots$. (7.4.2)

(B) Simultaneous equations of the first order:

In a pair of simple simultaneous equations of the type

$$\left.\begin{aligned} \frac{dx}{dt} &= f_1(t, x, y), \\ \frac{dy}{dt} &= f_2(t, x, y) \end{aligned}\right\} \qquad (7.4.3)$$

the increments in x and y for the first interval are formed from the following formulae:

$$\left.\begin{aligned} K_1 &= \Delta t \,.\, f_1(t_o, x_o, y_o), \\ K_2 &= \Delta t \,.\, f_1(t_o + \frac{\Delta t}{2}, x_o + \frac{K_1}{2}, y_o + \frac{\ell_1}{2}), \\ K_3 &= \Delta t \,.\, f_1(t_o + \frac{\Delta t}{2}, x_o + \frac{K_2}{2}, y_o + \frac{\ell_2}{2}), \\ K_4 &= \Delta t \,.\, f_1(t_o + \Delta t, x_o + K_3, y_o + \ell_3), \\ \ell_1 &= \Delta t \,.\, f_2(t_o, x_o, y_o), \\ \ell_2 &= \Delta t \,.\, f_2(t_o + \frac{\Delta t}{2}, x_o \frac{K_1}{2}, y_o + \frac{\ell_1}{2}), \\ \ell_3 &= \Delta t \,.\, f_2(t_o + \frac{\Delta t}{2}, x_o + \frac{K_2}{2}, y_o + \frac{\ell_2}{2}), \\ \ell_4 &= \Delta t \,.\, f_2(t_o + \Delta t, x_o + K_3, y_o + \ell_3), \end{aligned}\right|$$

$$\left.\begin{aligned}\Delta x &= \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4),\\ \Delta y &= \frac{1}{6}(\ell_1 + 2\ell_2 + 2\ell_3 + \ell_4).\end{aligned}\right\} \qquad (7.4.4)$$

The increments for the succeeding intervals are computed in exactly the same way except that t_o, x_o, y_o are replaced by t_1, x_1, y_1, etc. as one proceeds.