MULTIVARIABLE TRANSFER FUNCTIONS OF A SYNCHRONOUS GENERATOR

by

Shi-yaw Chang

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A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of
Master of Science
in the Department of Electrical Engineering
University of Saskatchewan

by
Shi-yaw Chang
Saskatoon, Saskatchewan
August, 1969

ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to Dr. R.J. Fleming for the assistance and encouragement which he gave during the course of this work. Special mention is due to Professor K.E. Bollinger for his advice at various stages of the work.

Acknowledgement of financial support given by the National Research Council of Canada under Grant No. A2765 is made.

UNIVERSITY OF SASKATCHEWAN

Electrical Engineering Abstract 69A116

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Student: S. Chang   Supervisor: Dr. R.J. Fleming

M.Sc. Thesis presented to College of Graduate Studies
August 1969

ABSTRACT

It is a complex problem to find a faithful mathematical representation for a synchronous electric generator due to the large number of circuit parameters involved, the inherent non-linearities and the high degree of coupling between the variables. Such a mathematical model is required for analytical study of power systems. A multivariable mathematical model provides a convenient means of system identification as well as a significant simplification in analytical studies of control engineering problems.

This thesis presents a "black box" approach to the problems of evaluating the multivariable transfer functions of a generator unit using step input test signals. It is shown that the multivariable model can be deduced from the response curves of the terminal variables by curve fitting techniques which employ either digital computations or graphical solutions without using involved analytical techniques.

In general the results from actual machine tests were found to be satisfactory compared with those calculated directly from the small signal forms of Park's equations.

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1. INTRODUCTION

1.1 General

The mathematical representation of a complex system by a multivariable model presents on one hand a convenient means of plant identification in many industrial applications and on the other hand provides significant simplification in analytical control engineering problems involving linear systems.

It is interesting to review briefly how the question of multivariable representation of transfer functions has been treated so far with respect to synchronous generator units. It is quite a complex problem to find a faithful representation for a generator due to the large number of circuit parameters involved, the inherent non-linearities and the high degree of coupling existing between the variables.

Stanton[13] has shown that the turbogenerator transfer functions can be determined from measurements obtained during normal operating conditions. Michael[11] has developed specific instrumentation devices and computer programs which can be used to analyse the measured data from an operating system. Fleming and Bollinger[9] have devised an equivalent circuit for the synchronous machine obtained from the frequency response analysis of a simulated machine represented by a set of linearized Park's equations.

This thesis presents a "black box" approach toward evaluating the multivariable model of a generating unit from the characteristics of the step response curves. The system description can be deduced from the observation of the terminal variables of the system.

1.2 Arrangement of this thesis

The purpose of this thesis is to offer a practical approach to the problem of evaluating the multivariable transfer functions of a generator using step input test signals. It demonstrates that this multivariable model can be obtained by experiments performed on the actual system without using involved analytical techniques which would require that all the element parameters and the interelation phenomena between them be known.

To achieve this purpose, this thesis is presented in the following manner:

First, the multivariable representation for a generator unit is reviewed using matrix methods to establish the multivariable models for two cases of terminal connection — the isolated load case and the infinite bus case. These models suggest the terminal quantities required to identify the generator unit which is to be represented.

The methods used in measuring the terminal quantities on a laboratory synchronous generator are given. This is followed by the curve fitting techniques used to transform the time response functions into the frequency response functions eventually leading up to the transfer functions.

The pertinent theory of linear control systems has been reviewed with the emphasis on the characteristics of the step response curves which lead to the establishment of methods for obtaining the transfer functions by either the graphical methods or digital computer programs.

The experimental results under different loading conditions were compared with those derived directly from the small signal forms of Park's equations by the method used by Bollinger and Fleming.[9]

Several appendices are included as follows:

A. Digital computer program for fitting a transient response function having overshoots by the pure second order system.

B. Digital computer program for fitting a transient response function having overshoots by the second order system with a zero.

C. Digital computer program for fitting a transient response function having oscillations by an exponentially decaying sinusoidal function.

D. Machine dynamics of a generator connected to an infinite bus.

E. Machine dynamics of a generator connected to an isolated load.

F. Nomenclature used in Table 5.5

2. REPRESENTATION OF POWER GENERATING UNITS

2.1 Single-variable systems

In this section the pertinent concepts of linear single-variable systems are reviewed to provide background for the multivariable theory actually used in this thesis and described in Section 2.2.

Over the last quarter-century a considerable body of analytical techniques has been built up for linear control systems, such as frequency response, transient response, stability criteria, pole-zero, root-locus and compensation methods. These tools are based on the concept of a transfer function which uniquely describes the real physical system under the following conditions:

1. Linearity: The system must be linear or approximately linear around its operating point so that it can be described by a linear, stationary differential equation which is Laplace transformable. In cases where the input consists of several components, the output can be obtained by the summation of the individual responses due to each component.
2. Unidirectional signal flow: The system must be operated in such a manner that a signal enters the system at the input terminal and leaves only at the output. The signal information flows through the system in only one direction.
3. Zero initial conditions: In deriving the transfer function all information about the effect of initial conditions which is inherent in the differential equation is discarded because they are zero or of no importance (as in the steady

state operating condition).

Consider a single-variable system with one input and one output. The response of this system to an impulse input signal has a form that is characteristic of the system. By dividing the actual impulse response by the magnitude of the input exciting impulse a normalized impulse response is obtained. This normalized impulse response is called the system weighting function and , for a linear system, is independent of the strength of the exciting impulse. By the principle of superposition the output quantity u(t) of a linear system can be found by the convolution of the input quantity v(t) and the weighting function w(t) as shown below:

$$u(t) = \int_{-\infty}^{\infty} v(t-\tau)\, w(\tau)\, d\tau \qquad (2.1)$$

or

$$u(t) = v(t) * w(t) \qquad (2.2)$$

where the symbol * denotes convolution.

In the frequency domain, there exists an alternate way of formulating the input-output relation of a linear system. Transforming both sides of Equation (2.1) by multiplying by e^{-st} and integrating with respect to time t, then the result is

$$\int_{-\infty}^{\infty} u(t)\, e^{-st}\, dt = \int_{-\infty}^{\infty} w(\tau)\, d\tau \int_{-\infty}^{\infty} e^{-st}\, v(t-\tau)\, dt$$

$$= \int_{-\infty}^{\infty} w(\tau)\, e^{-s\tau}\, d\tau \int_{-\infty}^{\infty} v(t-\tau)\, e^{-s(t-\tau)}\, d(t-\tau) \qquad (2.3)$$

The weighting functions for actual physical systems are always zero for negative time since it is physically impossible for an effect to preceed its cause in time. It can thus be concluded that the lower limit of the integration of Equation (2.3) should be zero and functions are defined only for the positive values of time.

If the Laplace transforms of the input, the output and the weighting function are denoted by V(s), U(s) and W(s) respectively, then Equation (2.3) can be written as

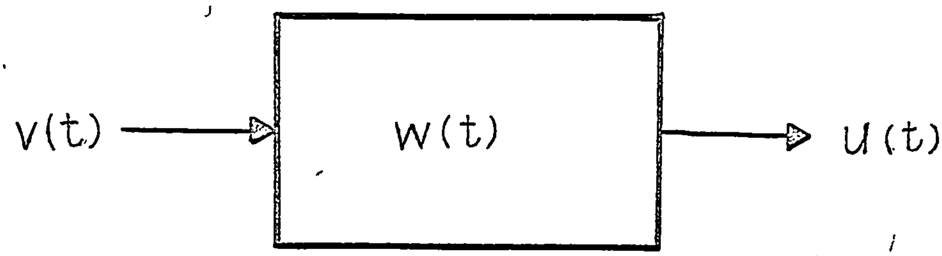
$$U(s) = W(s)\ V(s) \tag{2.4}$$

This is the frequency domain equivalent of the convolution integral of Equation (2.1). Figure 2.1 shows the block diagrams representing the input-output relationships for a single-variable system.

2.2 Multivariable systems

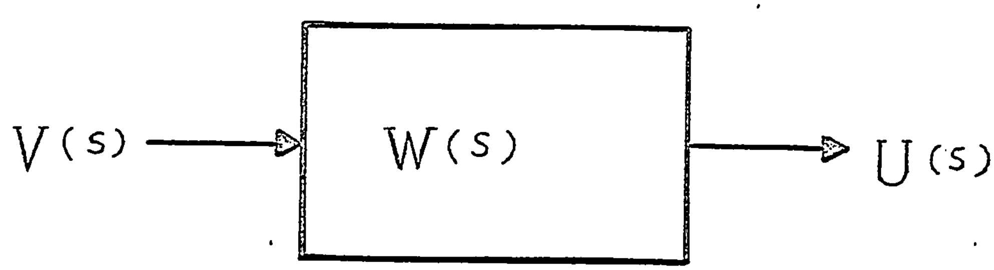
Systems with several inputs and outputs shall be called multivariable systems. Transfer function techniques described in the previous section have been extended to multivariable systems[12] by defining a vector V(s) consisting of the Laplace transforms of all the input variables in a given order, a vector U(s) consisting of the Laplace transforms of all the output variables similarily ordered and a transfer matrix W(s) consisting of the Laplace transforms of all the transfer functions relating all possible pairs of input-output combinations.

Consider a linear multivariable system with n inputs and m outputs as shown in Figure 2.2. The input and output



v(t) * w(t) = u(t)

(a)



V(s) W(s) = U(s)

(b)

Figure 2.1 Block diagrams for a single variable system

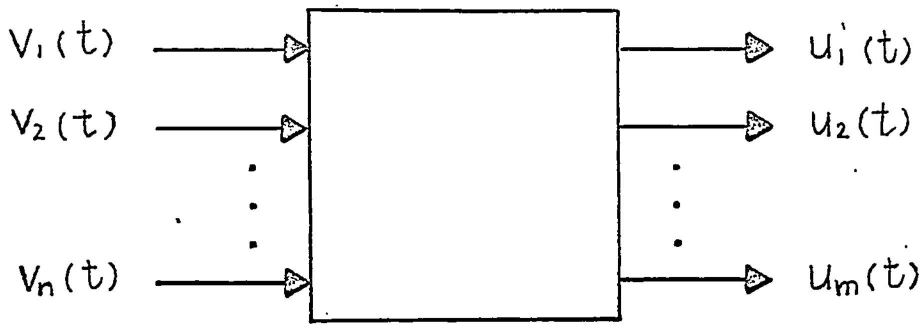


Figure 2.2 Multivariable system with n inputs and m outputs

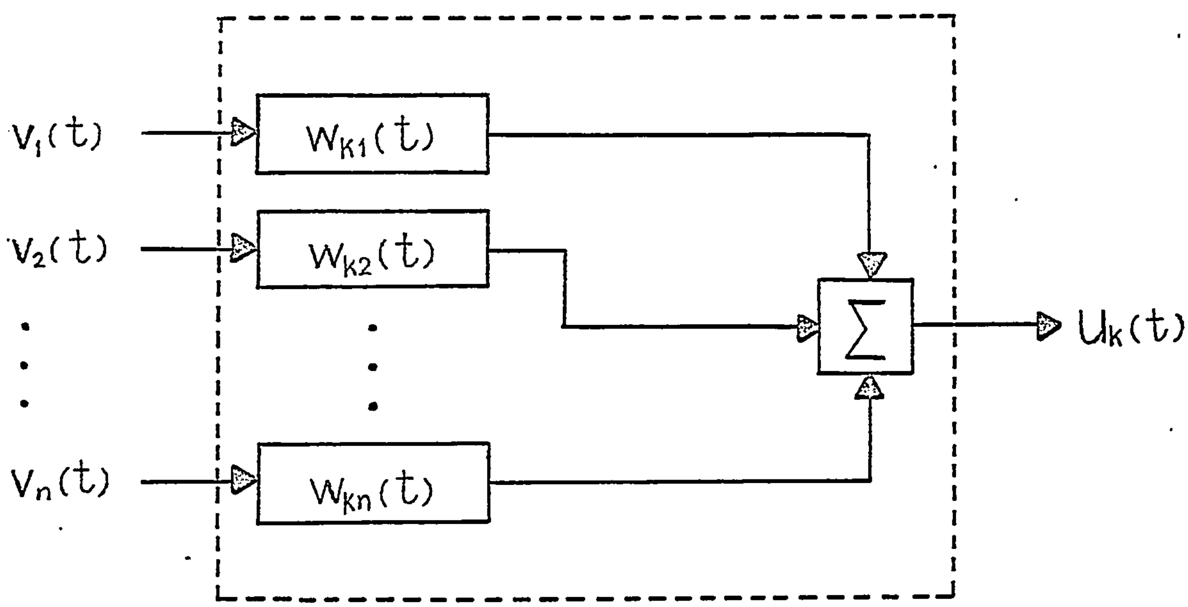


Figure 2.3 k-th output of a multivariable system represented by the combination of single variable subsystems

quantities have the vector forms

$$\left[v(t)\right] = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \cdot \\ \cdot \\ \cdot \\ v_n(t) \end{bmatrix} \quad \text{and} \quad \left[u(t)\right] = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \cdot \\ \cdot \\ \cdot \\ u_m(t) \end{bmatrix} \qquad (2.5)$$

Figure (2.3) shows the portion of the system corresponding to the k-th output $u_k(t)$, where $w_{ki}(t)$ is the weighting function relating $v_i(t)$ and $u_k(t)$. The matrix consisting of weighting functions in the form of $w_{ki}(t)$ is shown in Equation (2.6).

$$\left[w(t)\right] = \begin{bmatrix} w_{11}(t) & w_{12}(t) & \cdots & w_{1n}(t) \\ w_{21}(t) & w_{22}(t) & \cdots & w_{2n}(t) \\ \vdots & \vdots & \vdots & \vdots \\ w_{m1}(t) & w_{m2}(t) & \cdots & w_{mn}(t) \end{bmatrix} \qquad (2.6)$$

The problem is to derive the matrix relationship that describes the behavior of the multivariable system and is a generalization of Equations (2.1) and (2.4) which describe the behavior of single-variable systems. The derived results demonstrate that matrix algebra is a very useful means for investigating the multivariable systems.

If the symbol $\left[x\right] * \left[y\right]$ is used to denote the matrix

$$\left[z\right] = \left[x\right] * \left[y\right] \qquad (2.7)$$

whose elements are determined by the relationship

$$z_{ki} = \sum (x_{kj} * y_{ji}) \qquad (2.8)$$

where x_{kj} and y_{ji} are elements of matrices $\left[x\right]$ and $\left[y\right]$, then the matrix $\left[z\right]$ is called the "product" of two matrices $\left[x\right]$ and $\left[y\right]$ where the product of two elments denotes their convolution.

The output $u_k(t)$ can be written directly from Figure 2.3 in the form of a summation of convolution integrals which are contributed by each signal $v_i(t)$ and the corresponding weighting function $w_{ki}(t)$.

$$u_k(t) = \sum_{i=1}^{n} \left\{ w_{ki}(t) * v_i(t) \right\} \qquad (2.9)$$

where k = 1, 2, ..., m, m is the number of the output variables and n is the number of the input variables.

In accordance with the definition given by Equations (2.7) and (2.8), Equation (2.9) can be expanded to include the entire system and written in matrix form by utilizing Equations (2.5) and (2.6) as in Equation (2.10).

$$\left[u(t)\right] = \left[w(t)\right] * \left[v(t)\right] \qquad (2.10)$$

The Laplace transform of $u_k(t)$ can be obtained by successive use of the transform technique applied in deriving Equation (2.4) from (2.1). From Equation (2.9)

$$U_k(s) = \sum_{i=1}^{n} \left\{ W_{ki}(s)\ V_i(s) \right\} \qquad (2.11)$$

where k = 1, 2, ..., m. These are the elements of $\left[U(s)\right]$ as given by

$$\left[U(s)\right] = \left[W(s)\right] \left[V(s)\right] \qquad (2.12)$$

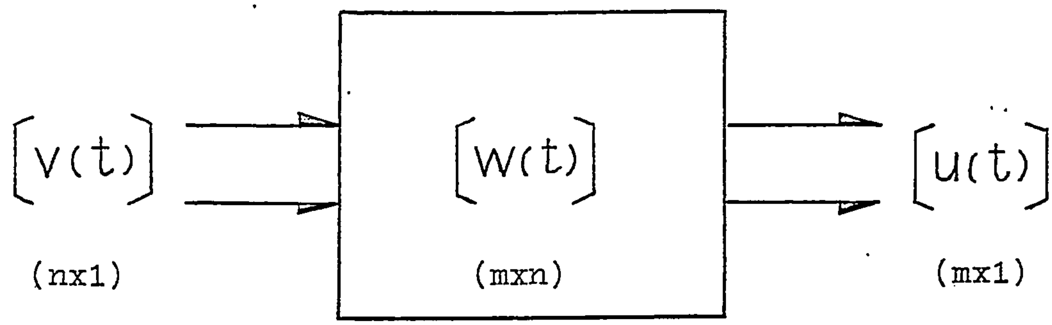
In this development use is again made of the concept that the Laplace transform of the convolution is the product of the transform of the functions entering into the convolution.

Equations (2.10) and (2.12), which describe the behavior of multivariable systems, are evidently analogous to Equations (2.2) and (2.4), which describe the behavior of single-variable systems.

In Equation (2.12), if the V_i's are the only variables directly affected by agencies outside the system and U_i's are the only variables observed, the transfer matrix $\left[W(s)\right]$ contains the same information about the multivariable system as the transfer function W(s) does for a single-variable system as illustrated in Equation (2.4).

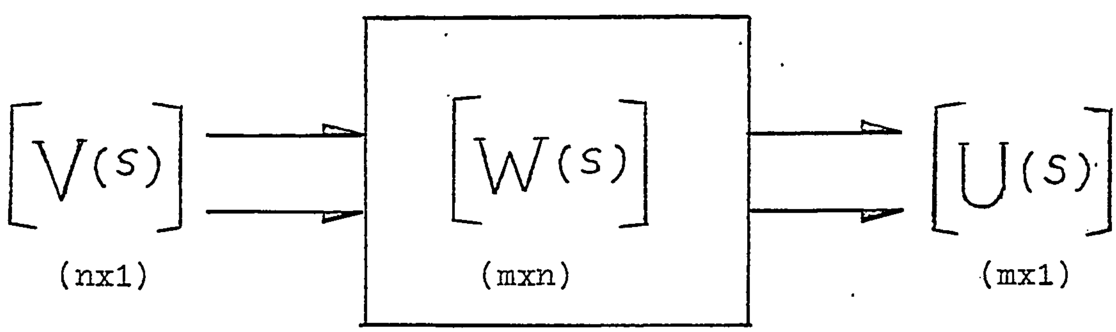
Similarily, it can be proved that analogy exists between the single-variable and the multivariable systems in the expression of correlation functions and spectral density functions. The block diagrams of a multivariable system are given in Figure 2.4 for time- and frequency- domain representation where (mxn) denotes the order of a matrix which has m rows and n colums, and * the convolution integral.

Since an analogy exists between the single variable and the multivariable systems in their mathematical expressions, it is reasonable to believe that the analytical and measuring techniques applicable to the one are equally applicable to the other. Stanton[13] has shown that this is, in fact, the case.



$$\left[w(t)\right] * \left[v(t)\right] = \left[u(t)\right]$$

(a)



$$\left[W(s)\right]\left[V(s)\right] = \left[U(s)\right]$$

(b)

Figure 2.4 Block diagrams for a multi-variable system

2.3 Parameters of the multivariable systems

There exists at the present time, in the various branches of industry, a large class of complex systems which are characterized by a multiplicity of interacting parameters. The parameters acting upon such a complex system can be divided into a series of groups depending upon their character and the part which they play in the system. In Figure 2.5 the parameters have been divided into four groups[4] which possess the following characteristics:

1. Group X are reference input parameters.
   By reference input parameters is meant the totality of indices characterizing the quantity of the command. The quantitative values of the input parameters are usually known and are independent variables which can be manipulated by some external means.
2. Group Y are controlled output parameters.
   The output parameters contain information regarding the characteristics of the end products and other generalized indices which evaluate the accuracy and effectiveness of the system operation. Usually the output parameters are variables which can not be directly manipulated externally, but whose values are of interest.
3. Group D are disturbance parameters.
   The presence of disturbance parameters vastly complicates the general picture of the system. In this group are included the uncontrollable properties of the Group X, the action of random external factors, changes in the characteristics of technological equipment. etc.

4. Group W are control parameters.

These are available to the operator to control the state of action in order to neutralize the effect of Group D and to achieve optimum operating conditions. These include the control equipment , regulating devices etc.

The model described above may be applied to many technological objects in electric power systems, chemical processes, metallurgical plants and even an entire enterprise or industry. A multivariable model representing usual system elements and signal variables is shown as a block diagram illustrated in Figure 2.6, where G is the disturbance elements, H the controlled system elements, K the feedback control elements and C the series control elements. Usually control parameters W's are obtained by passing the output parameters through the feedback elements. Actuating parameters Z's are so called error signals resulting from the difference between the reference input and controlled output.

2.4 Multivariable models of power generating units

If the small range of signal variations around the steady state operating condition are considered, generating units, though non-linear in nature, can be considered as linear. As mentioned in the later part of this section, the models which are two dimentional are of prime importance for case studies of the generator unit in the laboratory.

One of two semi-idealized models of electric generating units can be used to represent a large number of actual situations. One is a generator or a group of generators with

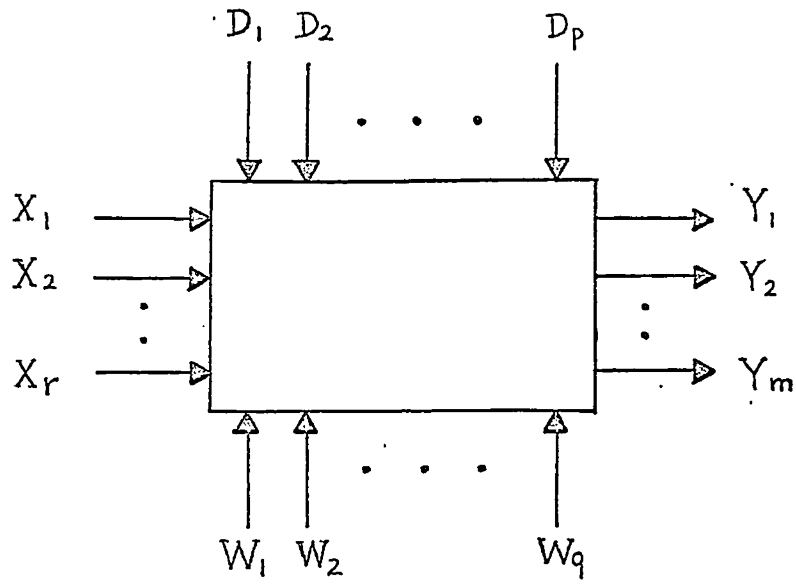


Figure 2.5 Parameters acting upon a complex system

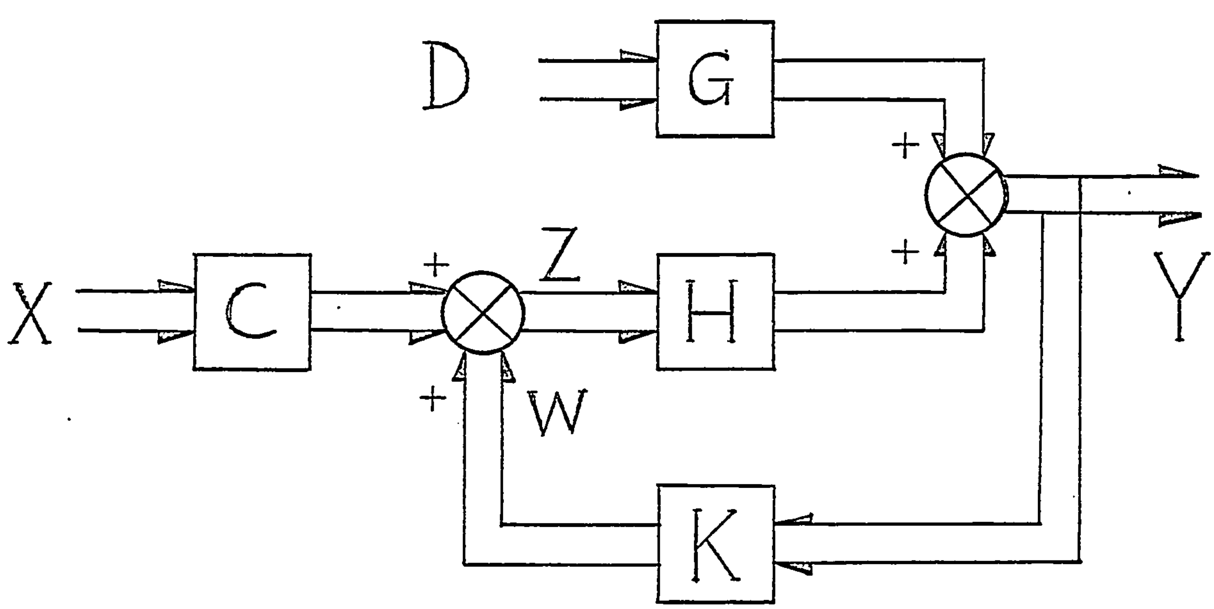


Figure 2.6 Multivariable model representing system elements and signal variables

close electrical coupling supplying energy to an infinite bus or a power pool. The other is a generator or a group of generators with close electrical coupling supplying energy to an isolated electrical load. The relationship between these models and the general multivariable configuration as well as the signal flow graph are described in Chapter 2 of Bibliography 7. For a generating unit, if only the disturbance elements and the controlled elements are considered, the block diagram and the signal flow graph can be simplified as illustrated in Figure 2.7.

1. The infinite bus case

   The case of a generating unit feeding an infinite bus is illustrated in Figure 2.8. The reference inputs are field voltage setting (V_f) and the gate opening setting (P_m). The disturbances are the frequency (ω_b) and the magnitude (V_b) of the infinite bus voltage. The outputs are the generator speed (ω_m) and the terminal voltage (V_t).

2. The isolated load case

   For a generating unit feeding an isolated load, the reference inputs are again V_f and P_m and the outputs again ω_m and V_t. The disturbances are variations of power loadings, i.e. the active load power (P) and the reactive load power (Q). The flow graph is essentially the same as that of the infinite bus case except that V_b and ω_b have been replaced by P and Q as shown in Figure 2.9.

In this chapter the basic principles of multivariable systems as they apply to the electric generator case have been briefly reviewed. In the following chapter, the methods employed to measure the multivariable parameters of a specific small machine are discussed.

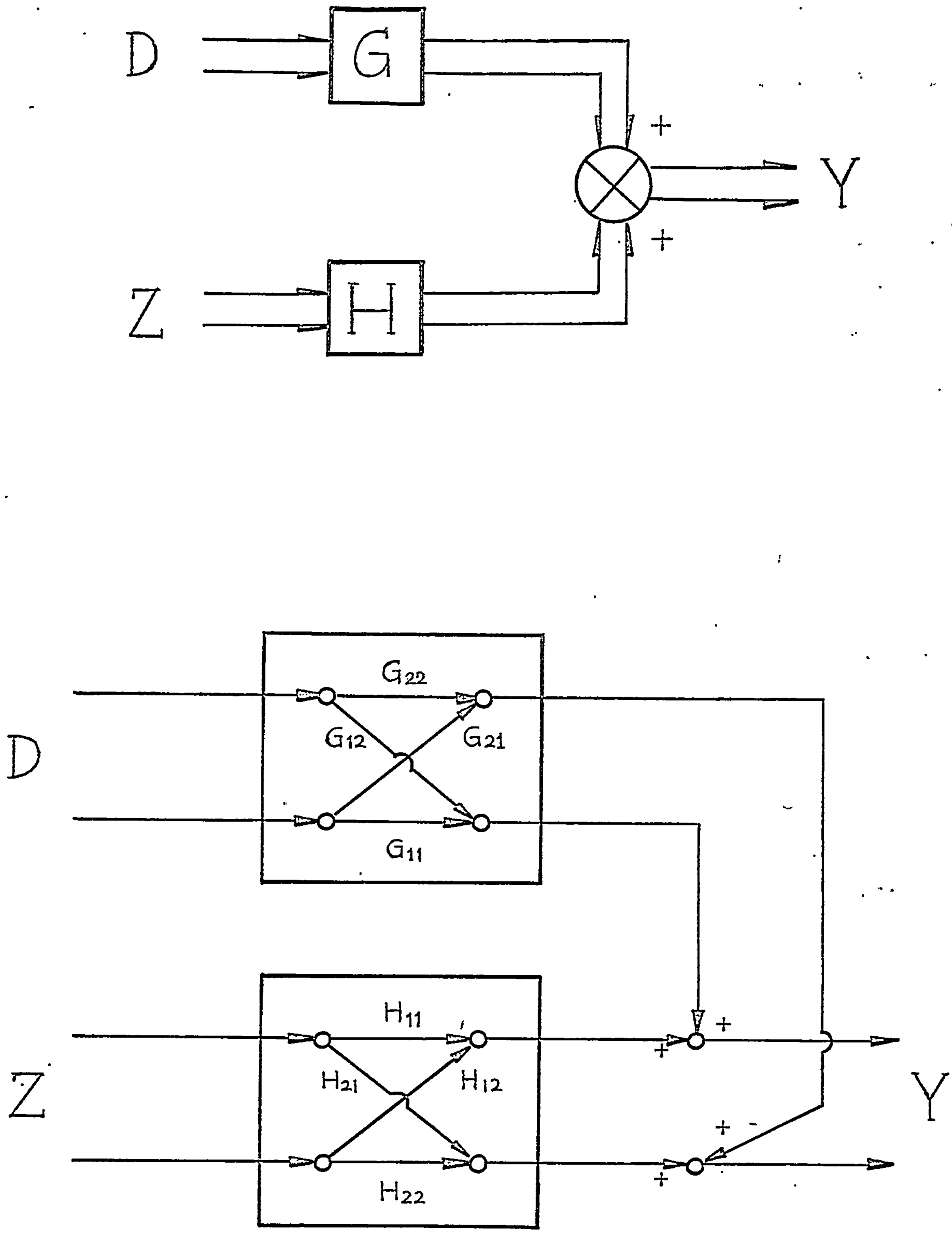


Figure 2.7 Block diagram and signal flow graph of a multivariable model

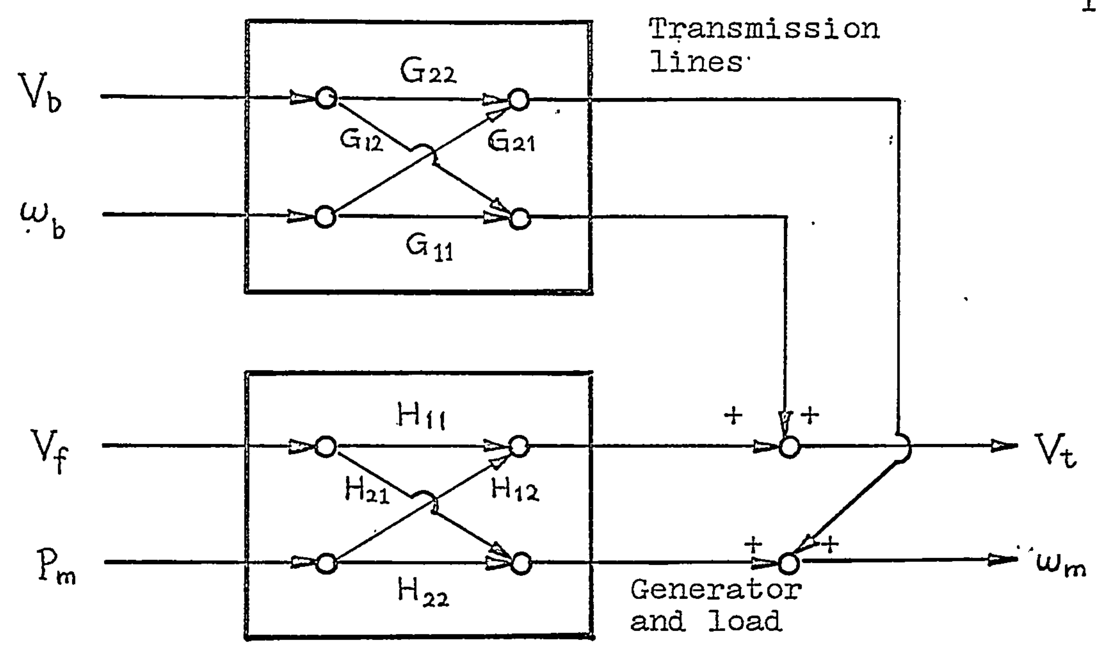


Figure 2.8 Multivariable model of a generator unit feeding an infinite bus

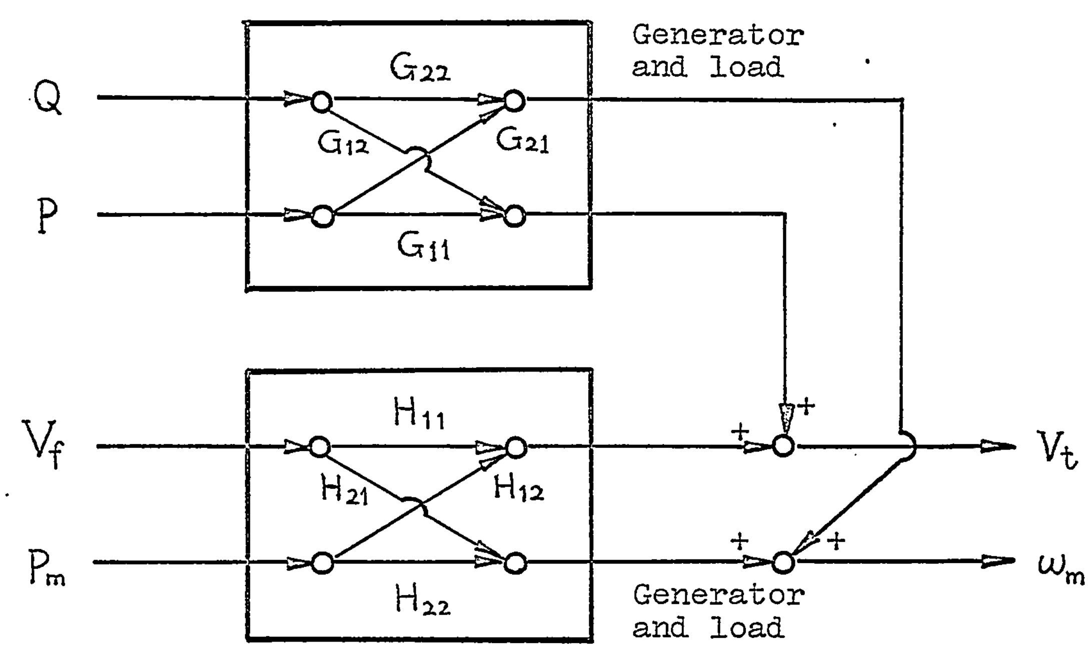


Figure 2.9 Multivariable model of a generator unit feeding an isolated load

3. TRANSIENT RESPONSE TESTS OF A GENERATOR

3.1 Introduction

All the different methods of dynamic testing for the purpose of system identification may be divided into two general classes: frequency domain tests and time domain tests. Frequency domain tests consist of measuring the output to input ratio of amplitude and phase angle as functions of the frequency of a sinusoidal driving signal. Time domain tests consist of determining system response to an input driving signal that is a function of time. All the testing methods applicable to any one system must inevitably yield the same information about the system. The selection of the appropriate test method for any particular system depends on the purpose of the test, configuration of the system and the available methods for data reduction, etc.

The input for time domain tests generally is an impulse function, a step function or a transient of arbitary wave form. Among those, when dealing with a generating unit, it is best to use step functions in performing actual machine tests because of the simplicity of the measuring instrumentation and the reduction of the test data to obtain the desired results.

In transient response testing, step changes are applied to the input terminals of the system and the resultant changes in measured output variables are recorded at the output terminals. The magnitude of the step input is not critical; the step change should be large enough to produce readily