

POWER SYSTEM RELIABILITY EVALUATION

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the Faculty of Graduate Studies
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the Degree of Doctor of Philosophy
in the Department of
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by

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ABSTRACT

Reliability considerations are an important aspect of overall power system optimization in both the planning and operating areas. This thesis investigates the application of probability concepts to static and spinning generation reserve problems. The use of confidence levels in the basic component outage statistics are proposed as an additional degree of consistency in system reliability assessment.

Transmission system reliability is studied using published methods and compared with results obtained using a Markov process with a two state fluctuating failure environment covering normal and stormy weather periods. A method of evaluating the composite reliability at any load point and encompassing all the inherent system failure processes is proposed and illustrated. This approach will permit reliability evaluation to become an integral component in the economic appraisal of alternate facilities.

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INTRODUCTION

A basic requisite of a modern power system is the ability to satisfy the constantly changing system load requirement at all times. It is not possible to absolutely guarantee this ability and any attempt to do so is impractical and uneconomical. An important management function is to decide where the limited funds available to improve system security should be used to achieve the best overall result. In most power systems it becomes the responsibility of the System Planning Engineer to analytically determine the cost associated with a particular level of reliability and to provide management with quantitative assistance in making the final decision.

Reliability evaluation can take two distinct forms. It can serve as a prediction of average system performance over a relatively long period of time or it can be considered as a means of obtaining a consistent quantitative assessment of alternate proposals. In a continually changing system it is important to retain the concept that quantitative reliability evaluation is a basic process by which engineering personnel can advise management on the issue of security of customer supply. Reliability constraints should not be considered as separate issues but as an integral part of overall economic evaluation.

Until quite recently, nearly all power system reliability evaluation was limited to the area of total installed generating capacity requirements. Attention is now being given to other system component areas in attempts to optimize supply security to the actual consumer. Complete optimization entails the consideration of all system component areas from generation, trunk transmission and subtransmission facilities

down to the actual customer distribution system. The techniques in use at the present time are divided into the two completely separate areas of generating capacity reliability and subtransmission reliability. Very little emphasis has been placed on the evaluation of composite system reliability encompassing all component areas.

This thesis proposes and illustrates a method by which reliability assessment can become an integral part of system performance evaluation in both the planning and operating areas. A value for steady state adequacy can be obtained for any point within the system by utilizing a quality of service approach rather than an approach involving only continuity of service. A breach of continuity becomes only one form of violation of the service quality standard for the point in question. This approach requires a detailed knowledge of the conditions acceptable to the system in terms of voltage levels, permissible line loadings leading to relay action and stability implications. The ability to incorporate system parameters of this type has increased considerably within the past few years with the continued utilization of high speed digital computers in power system analysis.

Quantitative reliability analysis has not been applied to any great extent in the area of power system operation. This thesis illustrates the application of probability theory to the evaluation of operating capacity reliability and proposes the use of failure rate confidence levels as an additional aspect of consistent evaluation. The thesis also examines the bunching effect of storm associated transmission component failures using Markov processes and compares the results to those obtained by a published approximate method.

Power system reliability evaluation, with the exception of the static generating capacity area, has been generally limited to qualitative assessment. {The techniques proposed in this thesis should provide considerable assistance in developing quantitative incremental reliability costs for any point in a system.} The utilization of high speed digital equipment will permit complex composite systems to be studied in detail and therefore simplify the management problem of optimizing the reliability of customer supply.

2. REVIEW OF THE AVAILABLE METHODS IN POWER SYSTEM RELIABILITY EVALUATION

2.1 Generating Capacity Requirements

A considerable amount of work has been done in this area and some excellent papers published. A comprehensive survey of the available material has resulted in a bibliography⁽¹⁾ containing ninety six publications on the subject. The development of the techniques used at the present time is extremely interesting and although it is rather difficult to determine just when the first published material appeared, it was at least thirty years ago. Interest in the application of probability methods to the evaluation of capacity requirements became evident about 1933. There is, however, very little published material available for the period 1933 to 1947 at which time the first large group of papers appeared. This group of papers by Calabrese⁽¹⁻¹¹⁾, Lyman⁽¹⁻¹²⁾, Seelye⁽¹⁻¹³⁾, Loane and Watchorn⁽¹⁻¹⁴⁾ proposed the basic concepts upon which some of the methods in use at the present time are based. Shortly after this in 1948 the first A.I.E.E. Subcommittee on the Application of Probability Methods was organized. This Subcommittee chaired by Calabrese produced an important report in 1949 containing comprehensive definitions of equipment outage classifications and some statistical data on equipment outage expectancies⁽¹⁻²⁰⁾. Later reports on this subject by this Subcommittee appeared in 1954⁽¹⁻³⁰⁾ and 1957⁽¹⁻³⁶⁾. The 1947 group of papers proposed the methods which with some modifications are now generally known as the "Loss of Load Approach", "Loss of Energy Approach"

The Reference designation (1-11) refers to Paper Number 11 in Reference Number 1 "Bibliography On The Application Of Probability Methods In The Evaluation Of Generating Capacity Requirements". This Bibliography is included in the Appendix.

and the "Frequency and Duration of Outage Approach". They are described in considerable detail in a 1960 A.I.E.E. Committee Report⁽¹⁻⁷²⁾. The "Loss of Load Approach" is sometimes referred to as the "Calabrese Method". The effect of interconnections and the determination and allocation of capacity benefits resulting from interconnections were discussed by Watchorn⁽¹⁻²²⁾ and Calabrese⁽¹⁻²⁶⁾ in 1950 and 1953 respectively. Until 1954 most probability studies had been done either by hand or using conventional desk calculators. The benefits associated with using digital computers to reduce the tedious arithmetic required in these investigations were noted by Watchorn⁽¹⁻³¹⁾ in 1954 and illustrated in 1955 by Kirchmayer and his associates⁽¹⁻³²⁾ in the evaluation of economic unit additions in system expansion studies.

Several excellent papers appeared each year until in 1958 a second large group of papers was published. This group of papers modified and extended the methods proposed by the 1947 group and also introduced a more sophisticated approach to the problem using "Game Theory" or "Simulation" techniques^(1-59 to 70). Additional material in this area appeared in 1961 and 1962 but since that time interest in this approach appears to have declined. A recent Federal Power Commission Report⁽²⁾ stated that former users of the Monte Carlo approach "are currently substituting the loss of load probability type computations for the gaming approach". The A.I.E.E. Subcommittee published in 1961 the previously noted report⁽¹⁻⁷²⁾, which apart from the simulation approach, provided an extremely comprehensive summary of the earlier papers and available methods. The three A.I.E.E. Committee Reports on equipment forced outage experience published in 1949, 1954 and 1957 were generally restricted to

thermal unit equipment information with the exception of a short section on hydraulic equipment in the 1949 report. Brown, Dean and Caprez⁽¹⁻⁷⁴⁾ in 1960 published the results of a statistical study of five years of data on 387 hydro-electric generating units using punched cards for the initial collection and sequential processing of the data. Shortly after this in 1961 the A.I.E.E. Subcommittee produced a manual⁽¹⁻⁷⁹⁾ outlining reporting procedures and methods of analyzing forced outage data using digital equipment. In spite of the many excellent publications available there is still considerable reluctance among many power system engineers to accept the application of probability methods to this problem. This is particularly true in Canada, where out of eleven major companies replying to a questionnaire, only one company stated that its capacity requirement criterion was based upon probability concepts⁽⁷⁾. The remaining ten organizations used relatively inflexible rule of thumb techniques such as fixed percent margins⁽³⁾.

Until 1963 almost all publications on generating capacity requirements were confined to the area of total installed capacity requirements.* An important paper based entirely on the application of probability theory to the area of system spinning requirements was published in 1963 by Anstine and his associates⁽¹⁻⁸⁸⁾. Additional work in this area has since been done in Canada^(4,5,6), and is discussed in Chapter 3.

As previously noted, there are several accepted methods of evaluating the reliability of a given generating capacity condition. Neglect-

*The total installed generating capacity requirement is often designated as the static requirement⁽³⁾ and considered a system planning problem. The spinning capacity requirement⁽³⁾ is a system operating problem and concerns the capacity actually rotating or capable of supplying load within a predetermined minimum time.

ing the simulation approach, the most flexible of these techniques is the "Loss of Load Probability Approach". This method involves the construction of a capacity outage probability table for the available system generating capacity to which the probabilities of coincident load levels can be combined to produce a mathematical expectation or risk level for a specified period. The capacity outage probability table lists the probabilities of having various amounts of capacity on forced outage at any time in the future. The basic statistic is the probability of finding the generating unit on outage at some future time. This probability value is known as the generating unit forced outage rate (F.O.R.). For a particular unit and over some previously defined time period this value is obtained from past performance records.

$$\text{Generating Unit F.O.R.} = \frac{\text{Time on Forced Outage}}{\text{Exposure Time}} \quad (2.1)$$

where: Exposure Time = Operating Time + Forced Outage Time. The best estimate of the probability of the generating unit being on forced outage during future operating periods is obtained by using cumulative values of forced outage and exposure times. It may be necessary to modify the cumulative value due to aging effects or variations in maintenance practices or operating roles. The definitions and classifications⁽²⁴⁾ covering equipment outages must be quite explicit for the resulting statistics to be of use in the prediction of future outage occurrence. System experience has shown that the existence of generating unit forced outages can be considered as random independent events. The probability of existence of simultaneous outages of two or more units, is, therefore, the product of the individual unit outage existence probabilities. In a study involving identical units the Binomial Expansion can be used to

obtain the probability of existence for each capacity outage level. For a system containing generating units with different capacities and forced outage rates, the capacity outage probability table is developed by correctly combining the probabilities of the independent events. As the probabilities of having several units on outage at the same time can be quite small, it has been found convenient to curtail the capacity outage probability table by omitting probability values smaller than 10^{-8} .

When many units of different capacities are combined, the table will contain an extremely large number of discrete capacity levels. The number of levels can be reduced by proportionally summing the discrete probability values at selected capacity levels, thus limiting computer storage requirements and facilitating later combination with the load distribution⁽³⁾. A detailed study⁽²⁵⁾ using mathematical models of both the Saskatchewan and Manitoba Systems has shown that there is negligible overall error when a 5MW capacity increment is used. The magnitude of the overall error is dependent upon the capacity increment and the slope of the system load characteristic. The error decreases with increasing slope and is a maximum for a system with one hundred percent load factor.

A failure to carry the system load for any given time occurs when the load for that period exceeds the available generating capacity. A single system capacity outage probability table is required if the assumption can be made that units will not be removed from service for planned or scheduled maintenance. If maintenance is necessary, a theoretically accurate solution involves a continually changing, applicable capacity outage probability table applied to the corresponding system load model. Several approximate methods have been developed to avoid modifying the

capacity outage probability table. They can, however, be quite inaccurate in certain cases.

System loss of load can occur at times other than at the daily system peak. The conventional System Load Duration Curve is a suitable hourly load model and can be used on an annual, monthly, weekly or daily basis depending upon the variation in available system generating capacity. If almost all planned maintenance is performed during the light load months which add relatively small values of loss of load expectancy to the annual total, then the assumption of a constant capacity model may be quite valid and used in conjunction with an annual load distribution.

The system load level expected to occur at some time in the future cannot be forecast exactly. It may be possible, however, to define the load in terms of a probability distribution. This condition can be included in the loss of load probability computations without too much difficulty and the capacity required to satisfy a future uncertain load requirement at a specified risk level evaluated. The inclusion of probability of load forecast uncertainty in reserve generating capacity studies dictates a larger required installed capacity margin to serve the future uncertain peak load than that required to meet an equivalent exact value. These concepts have been applied in detail to mathematical models of the Saskatchewan and Manitoba Systems operating as single systems and as an interconnected system with finite and infinite interconnecting capacity. The results of this study⁽²⁵⁾ are extremely interesting and illustrate the application of probability theory to the determination and allocation of interconnection benefits.

The relative merits of the different methods of evaluating the reliability of generating capacity requirements are not discussed in this thesis. The "Loss of Load Probability Approach" is, however, the basis of several highly automated system expansion programs used by the industry at the present time⁽²⁶⁾. This method is extended in Chapter 5 to the evaluation of composite system reliability encompassing generation and transmission facilities.

2.2 Transmission System Reliability Evaluation

In comparison to the numerous publications in the area of generating capacity reliability there is relatively little published material on the subject of transmission system reliability evaluation. A group of excellent papers were presented at the 1965 Eastern Zone Meeting of the Canadian Electrical Association^(8 to 17) but with a single exception⁽¹⁵⁾ they discussed reliability in a qualitative rather than a quantitative sense. Apart from several earlier papers in the field^(18,19,20), the techniques available at the present time are summed up in two 1964 publications^(21,22). The two methods are considerably different in concept and in the extent to which probability theory is applied.

The first of these methods⁽²¹⁾ deals with the simultaneous conditions that must exist for power flow in series and parallel combinations of system components. The application is quite straightforward and is based upon four relatively simple principles.

- (a) A component operates in only two states, available and unavailable. Maintenance is not considered and the probability of a component being unavailable is given by its forced outage rate "p". If "q" is the availability rate then $p + q = 1.0$.

- (b) Component failures are assumed to be independent and therefore the probability of simultaneous failures is given by the product of the respective probabilities.
- (c) In a series system all components must be available for power flow into a receiving point. The probability of success is the product of the availability probabilities. For a two unit system with outage rates p_1 and p_2 and availability rates q_1 and q_2 .

$$q_s = q_1 q_2$$

$$p_s = 1 - q_1 q_2 = p_1 + p_2 - p_1 p_2$$

If p_1 and p_2 are much less than unity the $p_1 p_2$ product can be neglected. The failure probability of a series system in this case is the sum of the element failure probabilities.

- (d) In a parallel system all paths must fail if no power is to flow into the receiving point. For a two element system the failure probability is the product of the two component failure values.

Forced outage rate is usually defined as the total component outage time divided by the total component exposure time and is the probability of component outage existence. To provide an indication of both outage frequency and outage duration the definition can be modified to indicate the probability of outage occurrence rather than outage existence.

Define the forced outage rate as:

$$p = \frac{\text{Sum of the days upon which an outage of specified minimum duration occurred}}{\text{Sum of unit days}} \quad (2.2)$$

If component forced outage rates for different minimum specified durations are compiled, it is possible to predict the frequency of occurrence of this condition at any particular point in the system. It should be realized that outages can occur on the same day but not simultaneously. This approach assumes that all outages occurring during one day are

simultaneous outages thus giving a pessimistic result. This can be partially reconciled by considering that if two components within an area are forced out of service during the same day the probability of simultaneous occurrence is somewhat higher than implied by absolute outage independence. This method is restricted to the evaluation of continuity at a particular point and cannot be extended to systems that are not fully redundant.

This approach has been applied⁽²⁷⁾ to the small hypothetical system shown in Figure 2.1.

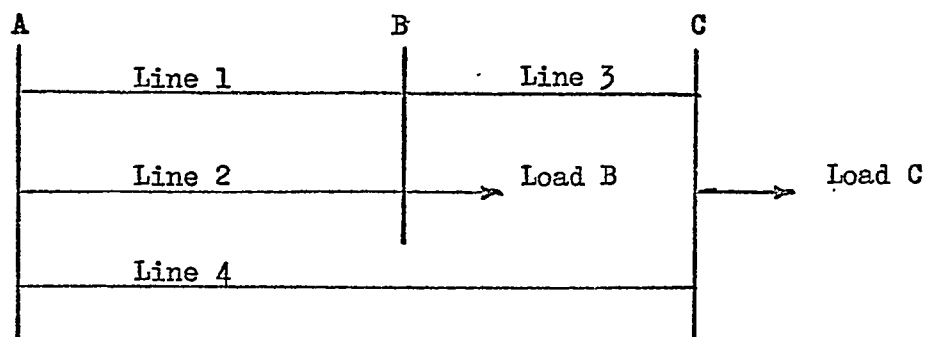


Figure 2.1. Simple Series Parallel System

The failure rates for each line section are:

TABLE 2.1

System Parameters For Method 1

<u>Line Section</u>	<u>Failures/Year</u>
1	0.5
2	0.5
3	0.1
4	0.6

Considering the failures per year as the number of days upon which failures occur within the year, the probability of an outage occurring on lines 1

or 2 is given by

$$p_1 = p_2 = \frac{0.5}{365} = 1.37 \times 10^{-3}$$

Similarly

$$p_3 = 0.274 \times 10^{-3}$$

$$p_4 = 1.644 \times 10^{-3}$$

Define Average Annual Customer Interruption Rate as the expected number of days in a year that the specified outage condition for the load bus will occur. Assuming that the system is first composed of Lines 1, 2 and 3 and then of Lines 1, 2, 3 and 4 the results are shown in Table 2.2.

TABLE 2.2

System Reliability Using Method 1

	<u>Average Annual Customer Interruption Rate</u>
Load B. Lines 1, 2 and 3	6.85×10^{-4}
Load B. Lines 1, 2, 3 and 4	1.31×10^{-6}
Load C. Lines 1, 2 and 3	0.1006
Load C. Lines 1, 2, 3 and 4	0.165×10^{-3}

The reliability indices are based entirely on the continuity of supply to the respective load points therefore assuming a completely redundant system. In an actual system the failure rates for each line section can be obtained by correctly combining the failure rates of the series or parallel equipment configurations within each section.

The second available method^(22,23) again deals with series and parallel systems but predicts both outage duration and outage frequency by making certain specific assumptions regarding the probability distributions of component repair and failure times. An extremely important

aspect of this approach is the introduction of a varying environmental condition associated with the operating component. Two states, normal weather and stormy weather describe the component environment. Each condition has an associated component failure rate in terms of failures per year of operation within that environment. Assuming that component failure times, component repair times, storm durations and normal weather durations are characterized by exponential probability distributions, it is possible to develop outage rates for parallel facilities that include the bunching effect of storm associated failures.

The method⁽²²⁾ uses an approximate expression for the overall outage rate of two parallel facilities under these conditions. A theoretically accurate solution for this system is developed in Chapter 4 using Markov processes. The simple four line system used to illustrate the first method and shown in Figure 2.1 is again considered using the overall annual failure rates and expected repair durations shown in Table 2.3.

TABLE 2.3

System Parameters for Method 2

<u>Line Section</u>	<u>Failure Rate Failures/Year</u>	<u>Expected Repair Time Hours</u>
1	0.5	7.5
2	0.5	7.5
3	0.1	7.5
4	0.6	7.5

Using these values the reliability indices for Loads B and C with and without Line 4 in-service are shown in Table 2.4. These values were obtained using the overall annual failure rates. If the component failure rates during stormy and normal weather conditions are not equal

then the predicted values shown in Table 2.4 may be considerably in error. The magnitude of the error depends upon the expected duration of stormy and normal weather periods and the percentage of component failures that occur under each condition.

TABLE 2.4
System Reliability Using Method 2

	<u>Failures per year</u>	<u>Average Total Outage time, hours</u>
Load B. Lines 1, 2 and 3	0.4281×10^{-3}	0.1605×10^{-2}
Load B. Lines 1, 2, 3 and 4	0.3848×10^{-6}	0.9621×10^{-6}
Load C. Lines 1, 2 and 3	0.1004	0.7515
Load C. Lines 1, 2, 3 and 4	0.1031×10^{-3}	0.3681×10^{-3}

Assume that for the system shown in Figure 2.1 the expected values of stormy weather and normal weather durations are 1.5 hours and 200.0 hours respectively. The predicted failure rate and average outage duration for each load point are shown as a function of the percentage of component failures occurring during the stormy periods in Figures 2.2 and 2.3. As the stormy component percentage failure increases, the differences between the values calculated using overall annual values and those calculated considering stormy periods increase quite rapidly. The magnitude of possible difference in reliability prediction is clearly shown in Figure 2.4 in which the load bus failure rate is shown in per unit of the value calculated using average annual failure rates. Figure 2.4 also shows the effect of longer storm durations occurring less frequently. In each case the error in the predicted value for Load C with line 4 unavailable is quite small as the reliability indices are almost entirely

dependent upon the values for Line 3, the series component. With Line 4 included, the error in the predicted failure rate for Load C again becomes evident due to the bunching effect of storm associated failures on parallel facilities.

It should be quite clear that the reliability indices calculated by the two methods outlined cannot be compared directly. The first method is relatively simple to apply even to a rather complicated system and if component failure is not a function of the environment but occurs purely by chance then this method can give quite useful results.

If the failure rate is a function of the environment then the utilization of overall annual component failure rates in the second method can lead to incorrect reliability predictions for paralleled or networked facilities. In its complete form this method is rather difficult to apply in a complicated system and becomes more of an approximation as additional parallel elements are combined. This is illustrated in Chapter 4 where results obtained by Markov processes are compared with those obtained using this method. The reliability indices calculated for the hypothetical system in the second case were again based only upon the criterion of continuity of supply. The system is assumed to be fully redundant.

At the present time there are very few organizations utilizing probability concepts in the evaluation of transmission system reliability. This is due to a large extent to the absence of comprehensive reporting procedures for transmission facilities. Committees in the United States and Canada are actively engaged in this area and procedures should be available in the near future.

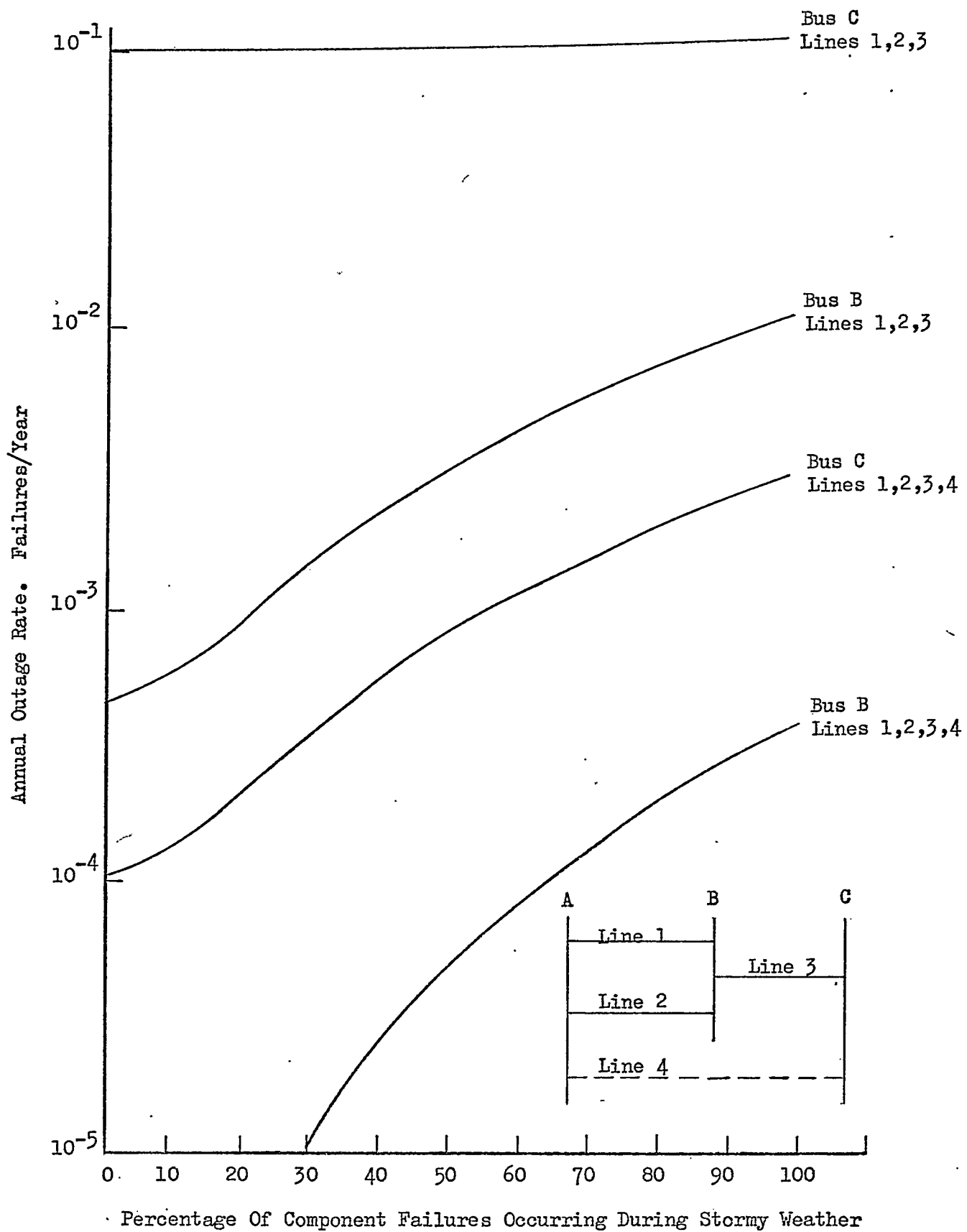


Figure 2.2 Variation in System Annual Outage Rate With Stormy Weather Component Failures

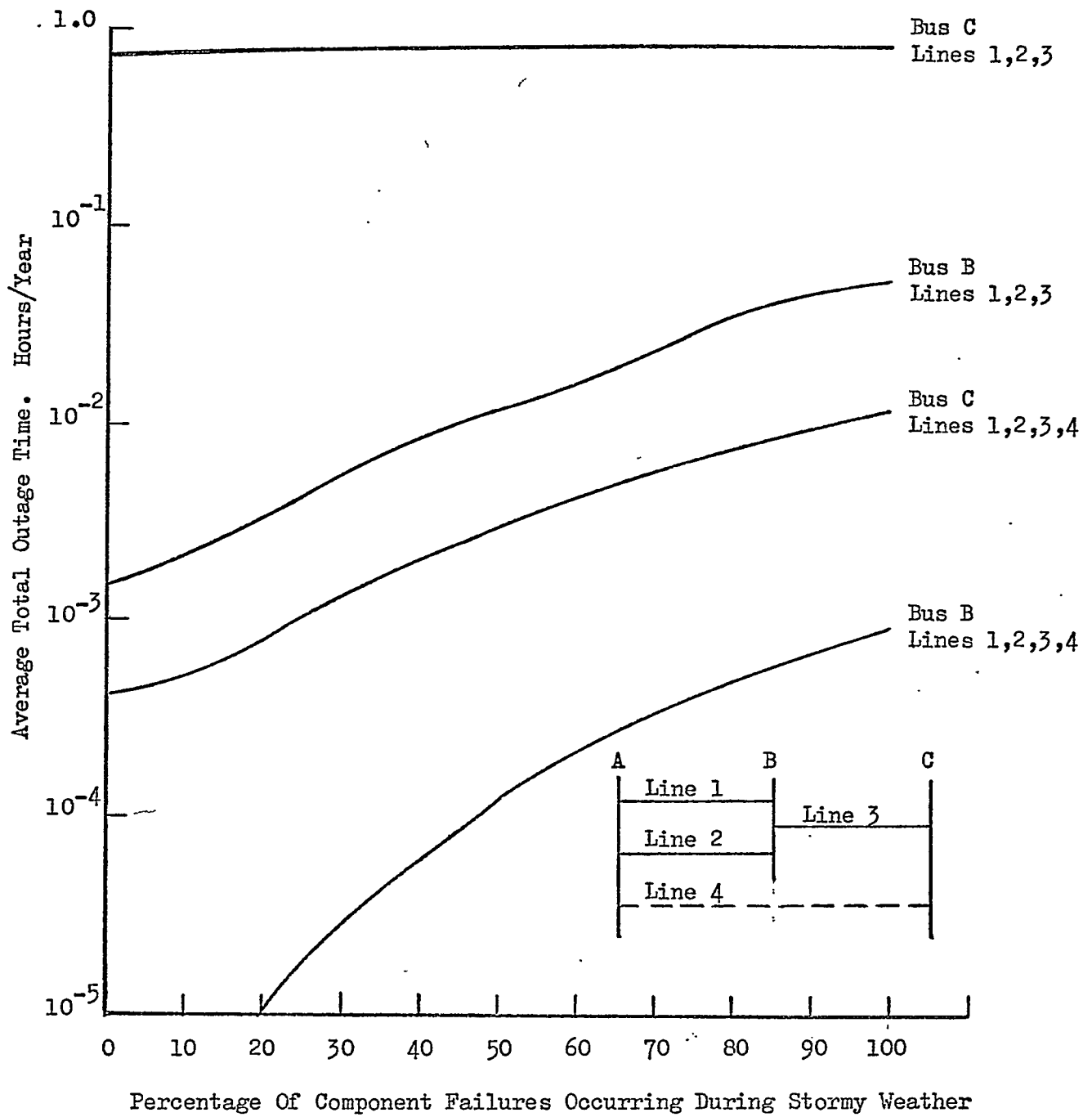
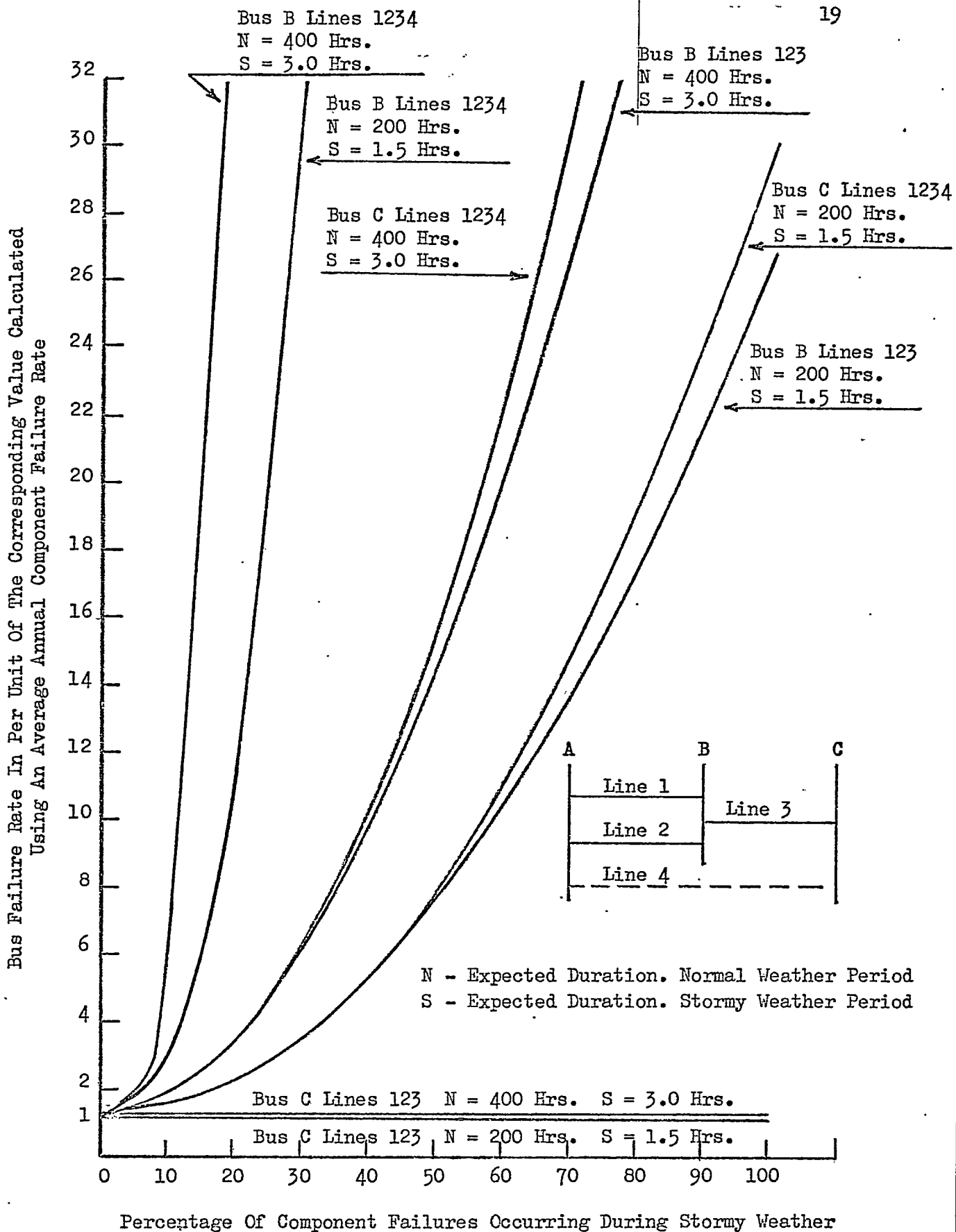


Figure 2.3 Variation In System Average Total Outage Time With Stormy Weather Component Failures



Percentage Of Component Failures Occurring During Stormy Weather

Figure 2.4 Variation In System Outage Rate With Weather Parameters

3. GENERATING CAPACITY RELIABILITY

3.1 Static Capacity Requirements

As noted previously, generating capacity reliability evaluation can be divided into the two basic areas of static and spinning requirements. Both of these areas must be examined at the planning level in evaluating alternate facilities. After the decision has been made, however, the spinning requirement becomes a system operating problem. This aspect is discussed in detail in Section 3.2. The static requirement can be considered as the installed capacity that must be planned and constructed in advance of the system requirements. The static reserve margin must be sufficient to provide for the overhaul of generating equipment, outages that are not planned or scheduled and load growth requirements in excess of the ^sestimates. A practice that has developed over many years is to measure the adequacy of both the planned and installed capacity in terms of a percentage reserve. An important objection to the use of the percentage reserve requirement criterion is the tendency to compare the relative adequacy of capacity requirements provided for totally different systems on the basis of peak loads experienced over the same time period for each system. Large differences in capacity requirements to provide the same assurance of service continuity may be required in two different systems with peak loads of the same magnitude. This situation arises when the two systems being compared have different load characteristics and different types and sizes of installed or planned generating capacity.

The percentage reserve criterion also attaches no penalty to a unit because of size unless this quantity exceeds the total capacity

reserve. The concept of maintaining a reserve equivalent to the capacity of the largest unit on the system plus a fixed percentage of the total system capacity is a more valid adequacy criterion and calls for larger reserve requirements with the addition of larger units to the system. This characteristic usually occurs when probability techniques are applied. The application of probability methods to the static capacity problem provides an analytical basis for capacity planning which can be extended to cover partial or complete integration of systems, capacity of interconnections, effects of unit size and design, effects of maintenance schedules and other system parameters. The economic aspects associated with different standards of reliability can only be compared using probability techniques.

The effects of adding relatively large units to an actual system can be clearly shown using the Manitoba System⁽³⁾. In 1963 the system generating capacity was 849MW, consisting of 50 hydraulic units with capacities ranging from 3.2 to 25MW and 10 thermal units with capacities ranging from 5 to 66MW. In 1965 three 110MW units were added at Grand Rapids. Prior to the addition of the Grand Rapids units to the system a reserve capacity criterion of 12 percent had been utilized. Using the loss of load probability approach on a whole day basis this corresponds to a risk level of 106.7 years/day. Using this as the basic criterion, the percent reserve margins with the sequential addition of the Grand Rapids units are shown in Table 3.1. The increase in load carrying capability of the system is clearly illustrated in Figure 3.1. The penalty associated with the addition of a large unit to a system is gradually reduced as more units of the same size are added. This is not true in

the spinning reserve case which is conceptually different and is discussed later in Section 3.2.

TABLE 3.1

Variation In Percentage Reserve Margins
With Grand Rapids Unit Additions

<u>System</u>	<u>Installed Capacity MW</u>	<u>Percent Reserve Margin</u>
C.S.M.S.	849	12.0
C.S.M.S. + 1 unit G.R.	959	14.8
C.S.M.S. + 2 units G.R.	1069	14.8
C.S.M.S. + 3 units G.R.	1179	14.7
C.S.M.S. - Combined Southern Manitoba System		
G.R. - Grand Rapids.		

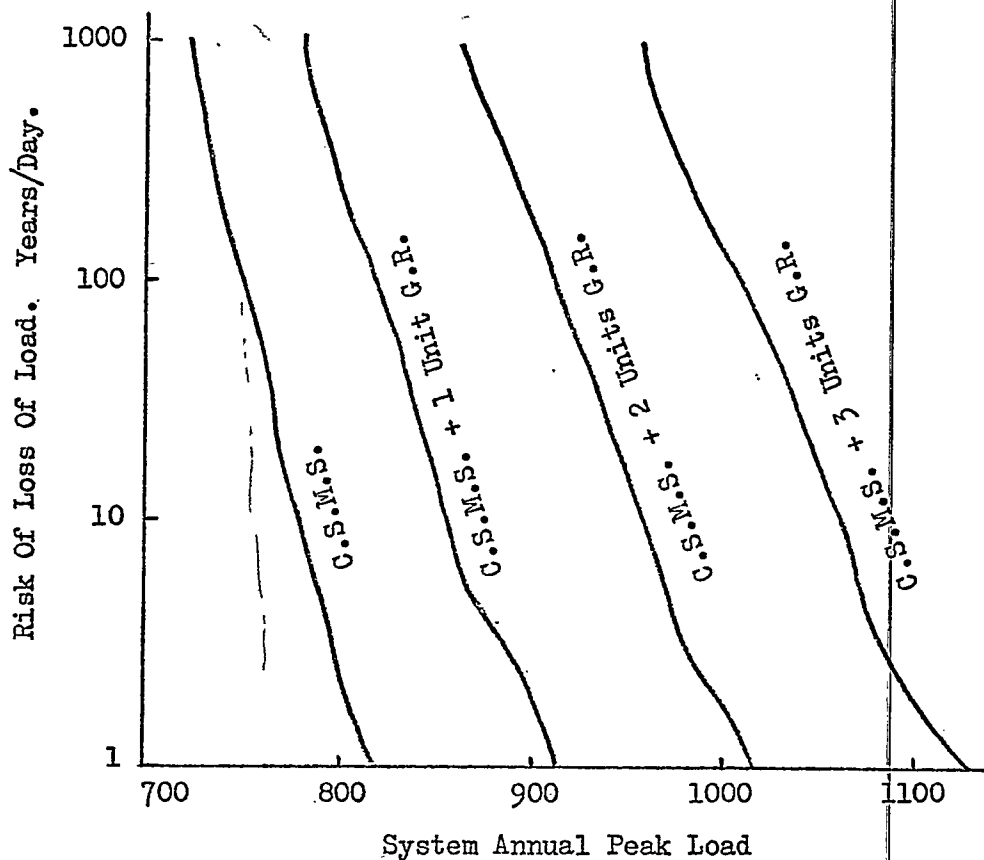


Figure 3.1 Variation In Risk Of Loss Of Load
With Annual System Peak Load

A recent I.E.E.E. paper⁽²⁸⁾ illustrated a technique by which load capability curves such as those shown in Figure 3.1 can be extended to include additional units without actually performing the required probability computations.

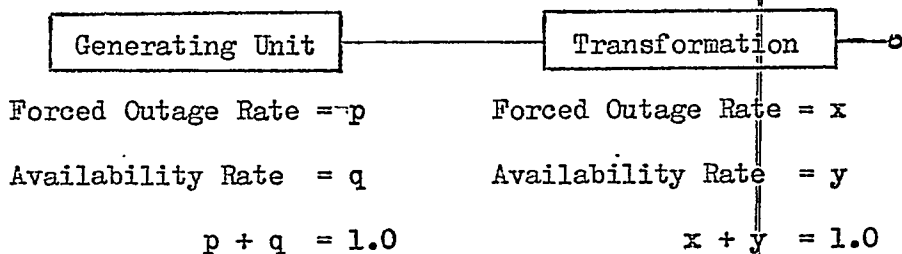
In nearly all generating capacity reliability studies, the generating unit is considered as a single component for the purpose of compiling forced outage statistics. Increased awareness of individual component reliabilities has, however, resulted in the development of more comprehensive reporting procedures⁽²⁴⁾. Continued collection of component outage statistics using these procedures will permit reliability optimization in many areas not previously considered. The transformation equipment used to connect the generating unit to the system is normally considered as part of the composite generating unit. To assess the effect of generating unit transformer reliability on overall system capacity reliability it is necessary to obtain adequate outage statistics for this equipment. A recent investigation of the forced outage statistics for the hydraulic generating units located on the Winnipeg River within the Southern Manitoba System produced the values shown in Table 3.2.

TABLE 3.2

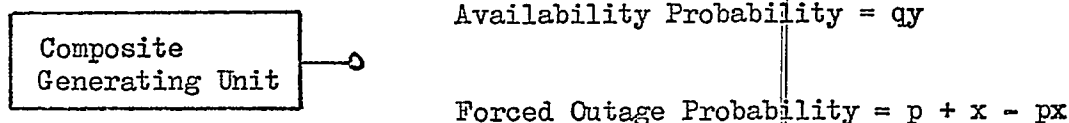
Hydraulic Generating Unit Forced Outage Rates	
	<u>Forced Outage Probability</u>
Generating Units (excluding Transformation)	0.0024
Generating Unit Transformers	0.0001

These statistics are cumulative values obtained for the five year period April 1959 to March 1964 assuming a homogeneous group of units. The forced outage probability of the transformation equipment is approximately one twenty-fifth of the outage probability of the composite generating unit. This relatively small component probability can have a considerable effect on the overall reliability of the system in cases where multiple generating units are connected to a single transformer bank. Utilizing the forced outage statistics in Table 3.2 obtained for the Manitoba System the effects of this configuration on the reliability of a simple hypothetical system and of a practical system have been investigated⁽²⁹⁾. The economic benefits of utilizing a single transformer with more than one generating unit can be readily evaluated. The overall effect of this configuration on the system reliability is not obvious and can only be determined using probability mathematics.

Consider a simple series system.



Assuming event independence, the reliability of the composite unit is as follows



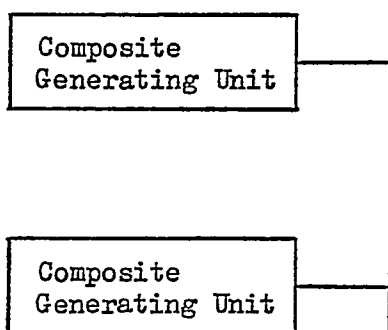
Using the statistics obtained for the Manitoba System

$p = 0.0024$	$x = 0.0001$
$q = 0.9976$	$y = 0.9999$

Composite Unit
Forced Outage Probability = 0.00249976 0.0025

Composite Unit
Availability Probability = 0.99750024 0.9975
1.00000000 1.0000

Consider the case of two parallel generating units each with individual transformers



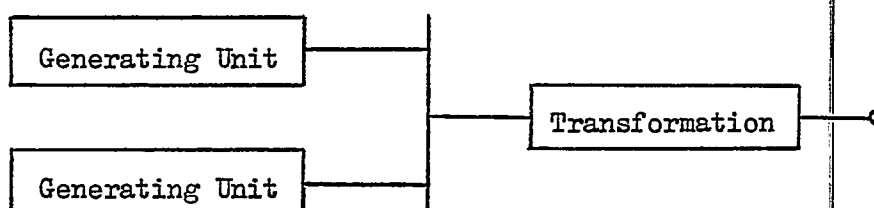
Forced Outage Rate = $p = 0.0025$

Availability Rate = $q = 0.9975$

The capacity outage probability table for this case is

<u>Number of Units on Outage</u>	<u>Probability</u>
0	$q^2 = 0.99500625$
1	$2pq = 0.00498750$
2	$p^2 = \underline{0.00000625}$
	<u>1.00000000</u>

Consider the case of two parallel generating units connected in series with a single transformer bank.



The capacity outage probability for this case is shown on the following page.

<u>Number of Units on Outage</u>	<u>Probability</u>
0	$q^2y = 0.99510624$
1	$2pqy = 0.00478800$
2	$p^2 + x - p^2x = \underline{0.00010576}$
	<u>1.00000000</u>

By comparing the probability of outage existence for two generating units in both the individual unit case and the two unit single transformer case it can clearly be seen that the transformer forced outage probability is a definite factor to be considered.

In order to illustrate this effect in a power system, the following hypothetical situations given in Table 3.3 have been studied⁽²⁹⁾.

TABLE 3.3

Hypothetical System Parameters

System A	12 - 20 MW generating units with individual unit transformers
System B	10 - 20 MW generating units with individual unit transformers. 2 - 20 MW generating units with a single transformer.
System C	12 - 20 MW generating units. Each pair of units connected to a single transformer bank.

In these systems

	<u>Availability Probability</u>	<u>Forced Outage Probability</u>
Composite Unit	0.9975	0.0025
Generating Unit	0.9976	0.0024
Transformer	0.9999	0.0001

The resulting capacity outage probability tables were combined with an annual Daily Peak Load Variation Curve represented by a straight line from the 100 percent to the 30 percent points using the loss of load probability approach. The risk levels for the three systems at selected yearly system peak loads are shown in Table 3.4.

TABLE 3.4
Hypothetical System Risk Levels

Peak Load <u>MW</u>	<u>Risk of Loss of Load. Years/Day</u>		
	<u>System A</u>	<u>System B</u>	<u>System C</u>
225	2.8	2.7	2.6
220	51.6	41.7	21.3
215	67.0	54.1	27.6
210	97.6	78.7	39.9
205	187.4	150.1	75.3

It is interesting to compare each system in terms of its load carrying ability at the same level of reliability. Using System A as the base, the penalties which must be attached to Systems B and C are shown in Figure 3.2. The maximum penalty occurs at a peak load point equal to the installed capacity minus the capacity of a single unit. The maximum penalty attached to System C is approximately 12.5 MW. This is a penalty of 62.5 percent of the capacity of a single unit and due entirely to the use of two generating units with a single transformer bank.

These hypothetical systems illustrate the configuration effect and are relatively simple to evaluate using only a conventional desk calculator. Practical systems, however, require the use of a digital computer to evaluate similar situations due to the large number of units involved. The Southern Manitoba System was selected as a practical example to illustrate this condition. The system generating capacity is

shown in Table 3.5.

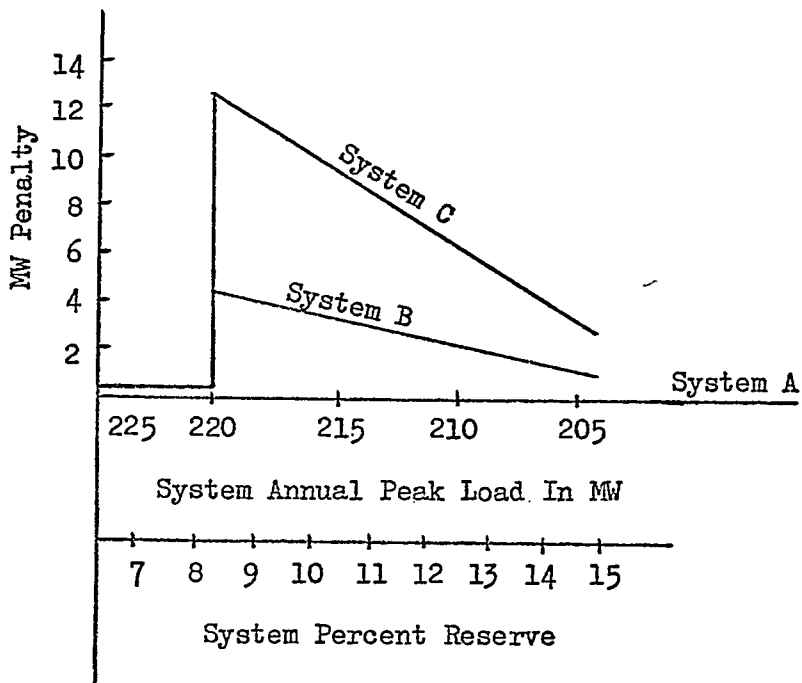


Figure 3.2 System Penalty Variation With Peak Load

TABLE 3.5

Southern Manitoba System Parameters

<u>Plant</u>	<u>No. of Units</u>	<u>Total Installed Capacity (MW)</u>	<u>Unit Forced Outage Rate</u>
Winnipeg River			
Hydraulic Generation	50	559	0.0025
Thermal Generation	10	290	0.0150
Grand Rapids			
Hydraulic Generation	4	<u>440</u>	0.0050
		<u>1289</u>	

It was postulated that to this system would be added 420 MW of additional capacity in the form of 6 - 70 MW hydraulic generating units located on

the Nelson River. Economic considerations dictate the connection of each pair of generating units to a single transformer bank. The forced outage probabilities for this additional generation are as follows;

	<u>Availability Probability</u>	<u>Forced Outage Probability</u>
Generating Unit	0.9951	0.0049
Transformer	0.9999	0.0001

The entire system generating capacity of 1709 MW was combined to form a single capacity outage probability table and applied to the same load characteristic as in the previous system example. To provide a basis for comparison, the procedure was repeated with each of the six additional generating units connected to an individual transformer using a composite generating unit outage probability value of 0.005. The comparative risk levels at selected system annual peak loads are shown in Table 3.6.

TABLE 3.6

Southern Manitoba System Risk Levels

<u>Peak Load MW</u>	<u>Risk of Loss of Load. Years/Day</u>	
	<u>System A Additional Generation With Individual Transformers.</u>	<u>System B Additional Generation With Two Generating Units/Transformer.</u>
1600	13.1	12.6
1580	28.5	27.2
1560	56.5	54.4
1540	127.4	122.5
1529	225.2	212.4
1520	300.7	282.7

Using System A as a base, the penalty to be applied to System B

is shown in Figure 3.3.

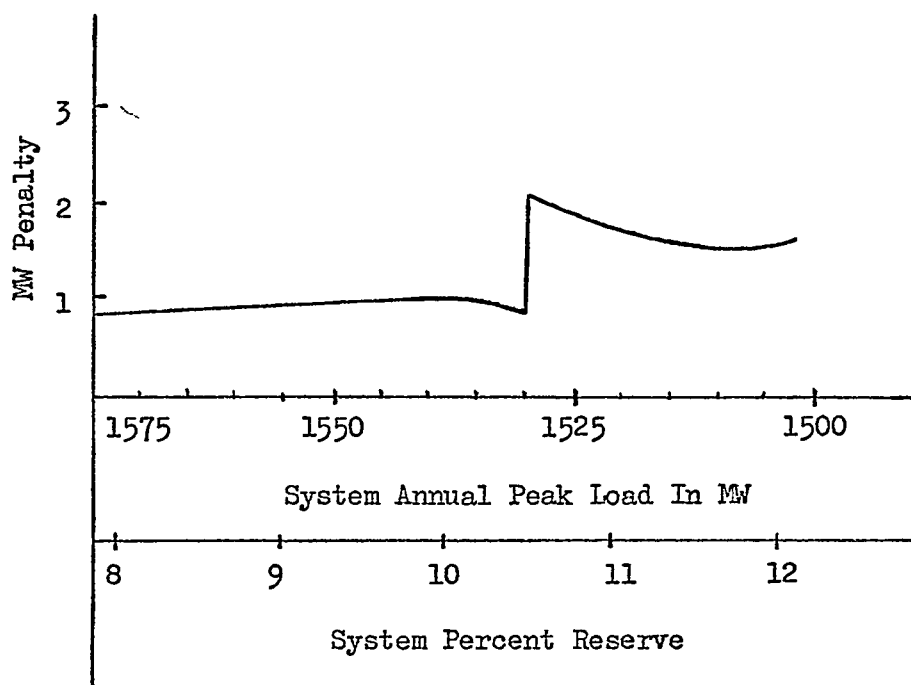


Figure 3.3 Southern Manitoba System Penalty Variation With Peak Load

The maximum penalty occurs at a system peak load of 1529 MW. This is a reserve of 180 MW and equal to the capacity of one unit at Grand Rapids (110 MW) plus the capacity of one of the additional units considered (70 MW). The maximum penalty is approximately 2.1 MW. This appears to be a rather small amount but if assessed at \$300/(KW installed), represents a penalty of approximately \$630,000. The capacity of the units at Grand Rapids produce a swamping effect that tends to reduce the penalty attached to the paired generating unit - single transformer additions. If, however, further additional generation in the paired unit configuration is added to the system, the penalty will increase.

The results shown are based upon the indicated values of gene-

rating unit and transformer forced outage probabilities. Any changes in these quantities would affect the penalty magnitudes shown. It is interesting to note that the penalty attached to a capacity addition due to the utilization of multiple units with a single transformer is not a constant value but is dependent upon the desired risk level for the system. The magnitude and variability of the penalty can only be evaluated using probability methods. Each system or capacity addition must be analyzed on its own merits and it is not possible to arrive at any specific conclusions that apply to all systems. If the effects of large differences in forced outage probabilities are neglected, several general observations can be made.

The penalty function attached to multiple unit - single transformer configurations is largely dependent upon the capacity of the unit additions relative to the units existing within the system. If the additional units are larger or equal to any already in the system the maximum penalty can be quite high. This maximum penalty can, however, occur at some load point or reliability level, higher or lower than the system criterion. The effect at the actual desired risk level may be considerably less than the maximum penalty. Regardless of the generating unit relative size or its forced outage probability, a penalty does exist for the multiple unit - single transformer configurations. A similar application of basic probability concepts can be used to evaluate the effect on overall system reliability of common header systems in thermal plants⁽³⁰⁾.

The cumulative generating unit forced outage rate based upon past history is the best available estimate of the probability of future unit

outage existence. It can be replaced, however, with a probability distribution of the forced outage rate for some predetermined time period. Power system load cycles, planning and economic evaluation are normally concerned with an annual time base. It is therefore reasonable to consider the probability of outage existence over any one year period as a probability distribution. It has been shown that the utilization of a single value for the unit outage probability in the development of a capacity outage probability table provides the same result as using a probability distribution of the forced outage value distributed symmetrically with respect to the single forced outage value⁽⁴⁾.

An examination⁽³⁶⁾ of hydraulic unit forced outage rates for the Manitoba System revealed that for the six year period of record, slightly less than half the unit years contained no forced outage. The unit years of record containing at least one outage indicated a log normal distribution of the annual forced outage rate. Further investigation⁽²⁵⁾ indicated that the overall cumulative forced outage rate of 0.0025 for these hydraulic units would only be exceeded on an annual basis 13 percent of the time. The cumulative forced outage rate therefore corresponds to an upper confidence bound of 87 percent. Capacity outage probability tables have been developed using other confidence levels and combined with the load model using the loss of load probability approach⁽²⁵⁾. As additional unit forced outage data becomes available, this approach should add additional consistency to the evaluation of capacity reliability by the introduction of confidence to the basic outage statistic.

3.2 Spinning Capacity Requirements

An important requisite in the daily operation of a complex power system is the ability to determine the most economic generating schedule while maintaining an adequate level of service reliability. The level of reliability in regard to the operating generating capacity is largely dependent upon the quantity of rotating or spinning capacity held in reserve above the actual system load requirement. This reserve must be sufficient to satisfy unforeseen changes in the system load without impairing the system frequency and tie line regulation and also protect against probable loss of operating capacity.

The actual spinning reserve capacity margin can be modified by the use of such methods as, automatic or manual load shedding, load reduction by voltage variation and the availability of assistance from neighbouring systems. The use of any of these methods will result in a lower level of reliability to be directly assigned to the actual operating capacity. A consistent approach to the evaluation of the two basic elements in the spinning reserve requirement is the determination of a figure of chance for contingent unforeseen load conditions and operating capacity deficiencies. Once a desirable risk level has been evaluated, it can be used as a basic reference in the economic evaluation of possible changes in operating conditions and in the planning of future generating capacity additions to the system.

A basic difference exists between the generating unit probability statistic used in a static reserve study and that used in a spinning reserve study. The statistic in the latter case is the probability that a generating unit will have been removed from service due to forced

outage and not yet replaced by another unit. In the static reserve case the available generating capacity under normal conditions is the entire system installed capacity and the criterion is some risk level for which the load will exceed the probable available capacity. In a spinning capacity study the assumption is made that there is sufficient installed capacity available to the system that if a unit is forced out of service it is only a matter of time before another unit is placed into service to meet the requirement. The time delay is dependent upon the equipment available for service and to some extent upon the time since this equipment last operated. In order to differentiate between the forced outage rate (F.O.R.) used in a static reserve study, the outage probability value used in a spinning reserve study is designated as the outage replacement rate (O.R.R.)⁽⁴⁾.

For a particular unit and over some previously defined time period the O.R.R. is given by:

$$\text{O.R.R.} = (\text{Failure Rate})(\text{Delay Time For Additional Equipment})$$

This expression is an approximation of the actual value and is valid if

$$(\text{Failure Rate})(\text{Delay Time}) \ll 1$$

If the assumption can be made that the generating unit is operating within its useful life period in which failures occur purely at random then the failure rate has a constant value. The probability that the unit will be operating at time t since last placed into service is called $R(t)$.

$$R(t) = e^{-\lambda t}$$

Where λ = useful life failure rate.

The failure density function for the useful life period is $f(t)$

$$f(t) = \lambda e^{-\lambda t}$$

The cumulative probability of failure is $Q(t)$

$$\begin{aligned} Q(t) &= \int_0^t \lambda e^{-\lambda t} dt \\ &= 1 - e^{-\lambda t} = 1 - R(t) \end{aligned}$$

The probability that the unit will fail in a particular operating period of length t is conditional upon the fact that it did not fail prior to the start of that period, at time T .

$$\begin{aligned} Q(t) &= \int_T^{T+t} \lambda e^{-\lambda t} dt \\ &= e^{-\lambda T} - e^{-\lambda T} \cdot e^{-\lambda t} \end{aligned}$$

The probability of survival up to time T is $e^{-\lambda T}$. The conditional probability of failure during t is $F(t)$

$$\begin{aligned} F(t) &= \frac{e^{-\lambda T} - e^{-\lambda T} \cdot e^{-\lambda t}}{e^{-\lambda T}} \\ &= 1 - e^{-\lambda t} = Q(t) \end{aligned}$$

The probability of failure in an arbitrary time interval t is independent of the operating age T of the component during the useful life period.

$$\begin{aligned} F(t) &= 1 - e^{-\lambda t} \\ &= 1 - \left(1 - (\lambda t) + \frac{(\lambda t)^2}{2!} - \frac{(\lambda t)^3}{3!} + \dots \right) \end{aligned}$$

If $(\lambda t) \ll 1$

$$F(t) \approx \lambda t$$

$$\approx (\text{Failure Rate})(\text{Operating Time Period})$$

If the operating time period is considered as the time required to place

additional capacity into service then $F(t)$ is the probability that the unit has failed and not yet been replaced.

Using the outage replacement rate values, a capacity outage probability table can be created similar to the one obtained using the forced outage rate values in a static reserve study. This table gives the probabilities of having various quantities of capacity out of service and not yet replaced. The time delay for additional equipment may vary at each operating capacity level as different types of capacity are placed in service. As a result, a completely different capacity outage probability table using different outage replacement rate values may be required at these levels.

If it can be assumed that there is no uncertainty associated with the load forecast, the capacity outage probability table can be used directly to evaluate a spinning reserve criterion. If each capacity outage element in the table is subtracted from the operating capacity level for that table, a table of available capacity levels and associated probability values is obtained. For a forecast load equal to the available capacity value the associated cumulative probability value is the probability that the system will just carry or fail to carry the load level.

If there is some uncertainty associated with the load forecast, the system load can be represented by a probability distribution. Studies of this condition in the Manitoba System have indicated that this uncertainty can be reasonably described by a normal distribution. The continuous normal distribution was approximated by a 49 step representation for digital computer use. A 7 step representation was used for

preliminary manual work. The 49 step representation approximates ± 6 standard deviations. For any forecast load condition there exists 49 actual system load values with their attendant probabilities of being realized. The total risk for a particular forecast load and generating condition is the summation of the products of the probability that the available capacity will just carry or fail to carry the system load and the probability that the load will be that particular value. A pictorial representation of this situation is shown in Figure 3.4.

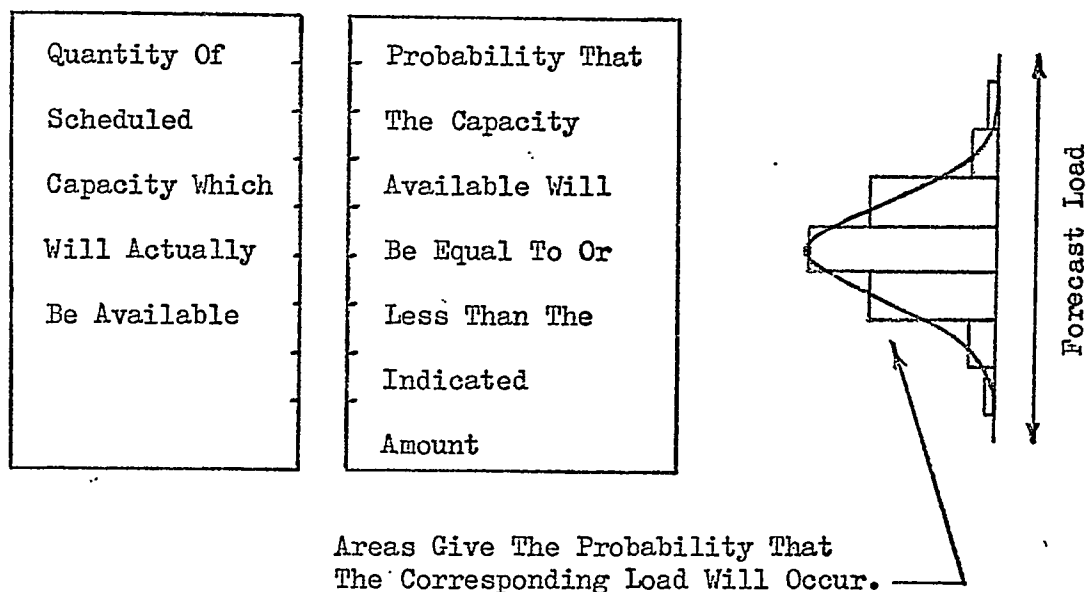


Figure 3.4 Combination Of The Capacity Outage And Load Models In Spinning Reserve Evaluation.

As the standard deviation of load forecast uncertainty increases, the probability of having much higher or much lower load values increase. With a relatively high forecast load it is possible to have a sizeable probability that the load will exceed the total scheduled capacity. The risk of failing to carry the load will be relatively high in these cases.

This approach has been applied to the evaluation of spinning

reserve criteria in the Manitoba System. The generating unit operating experience over a six year period is shown in Table A-1 in the Appendix. This table also gives the point estimate of the unit failure rates together with the failure rates at selected confidence intervals. This last aspect is discussed later in this Chapter.

Prior to the addition to the system of the hydraulic generating units at Grand Rapids, economic scheduling of generation dictated the utilization of the Winnipeg River units followed by thermal generation at Brandon and Selkirk. This natural division of operating capacity occurring under normal system conditions at 559 MW provides an excellent application of these probability concepts. Using a four hour time delay for additional thermal equipment, the probability of just carrying or failing to carry the system load for standard deviations of load forecast uncertainty of 0, 1, 2 and 3 percent of the forecast load are shown in Figure 3.5. The increased discontinuities occurring with higher forecast load uncertainty values are due to the use of only 49 values to represent the uncertainty in system load. The Winnipeg River generating units are an ideal application as there are no relatively large capacity units in the 559 MW total. The effect of using a 5 MW rounding increment in the development of the capacity outage probability table on the overall results was examined by computing the risk levels at various system load levels using a complete outage table. The difference between the two sets of results was found to be negligible.

The system spinning criterion changes drastically as units at Grand Rapids are added to the system. These units are over four times as big as the largest capacity units on the Winnipeg River and under

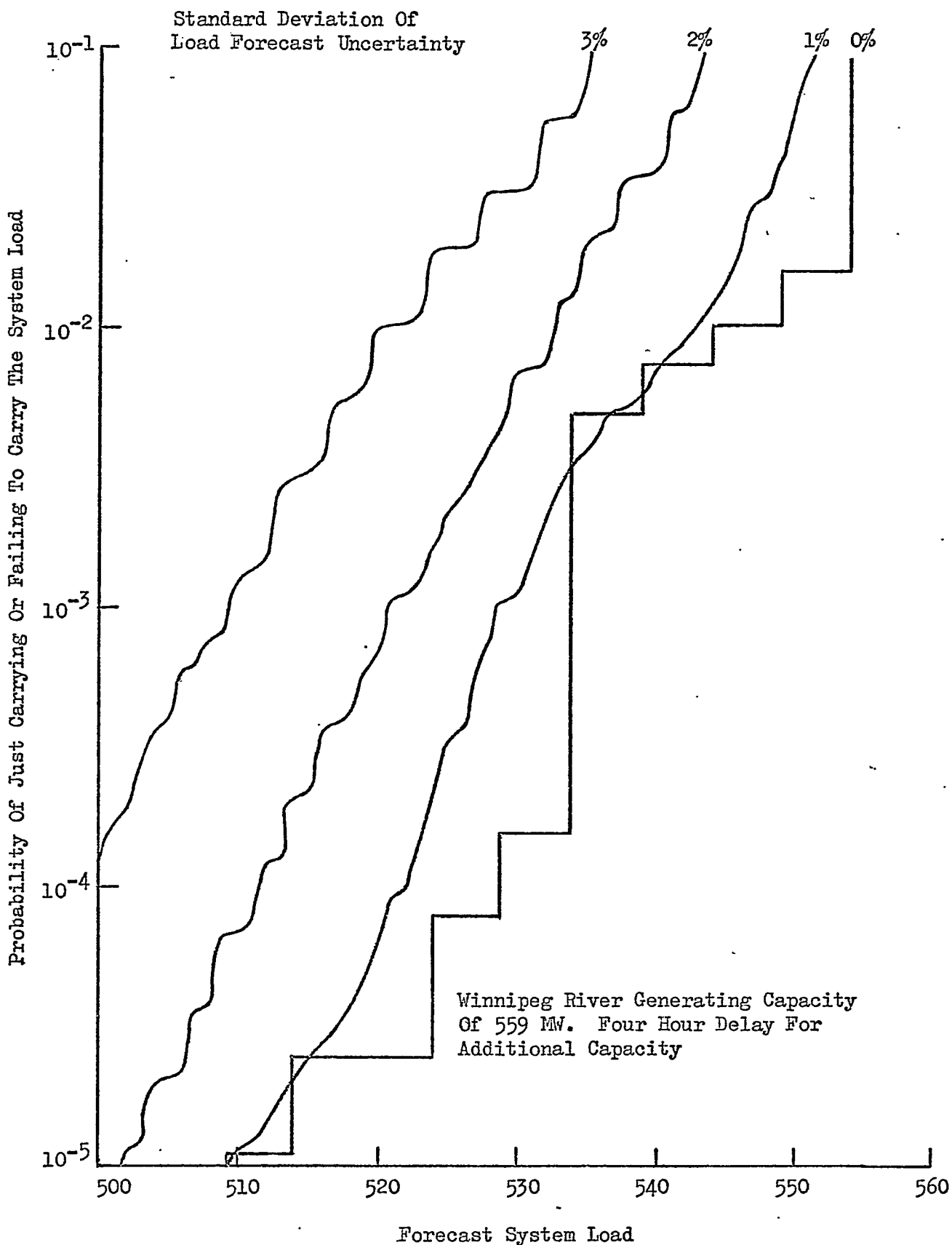


Figure 3.5 Variation In Load Carrying Capability With Load Forecast Uncertainty

normal conditions they would be scheduled prior to the utilization of thermal generation. Using an outage replacement rate of 0.001 for the Grand Rapids units, the effects of adding these units to the system are clearly shown in Figure 3.6. The load risk characteristic in Figure 3.6 is considerably different from that shown in Figure 3.5. Figure 3.7 illustrates the same phenomena in terms of the spinning reserve required at each risk level for the four system conditions.

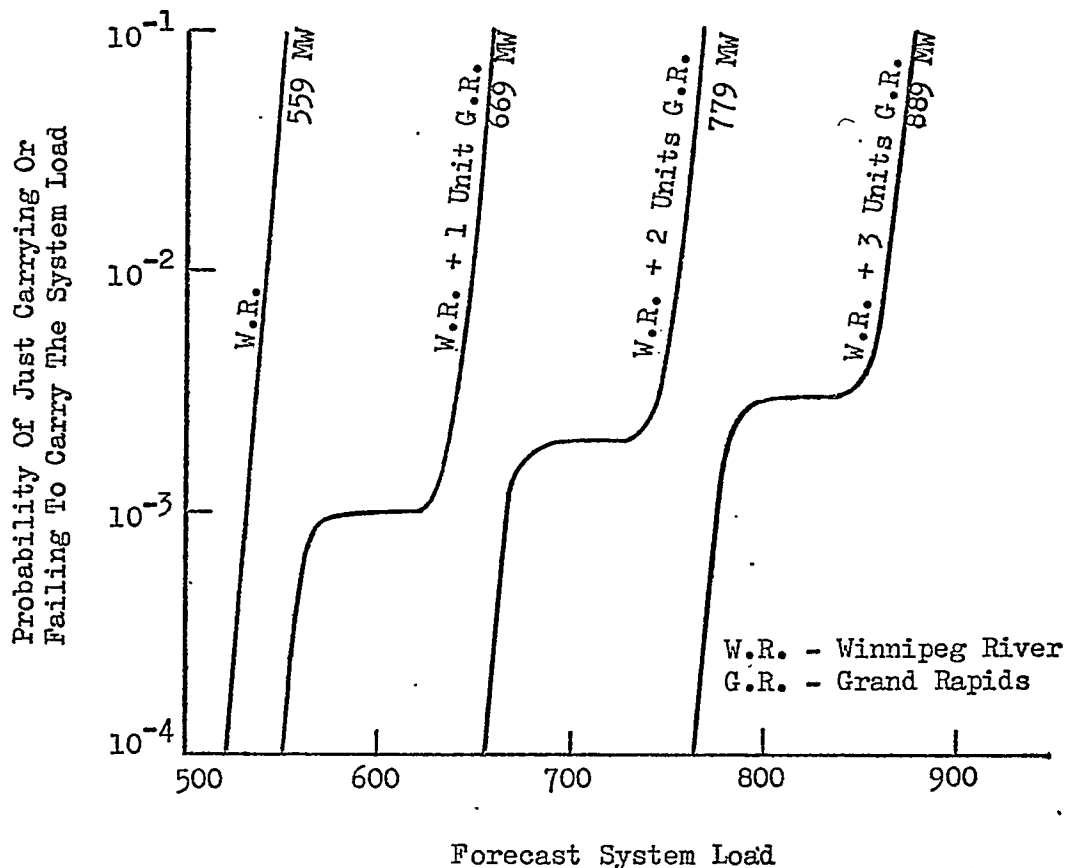


Figure 3.6 Load-Risk Characteristics With Unit Additions At Grand Rapids

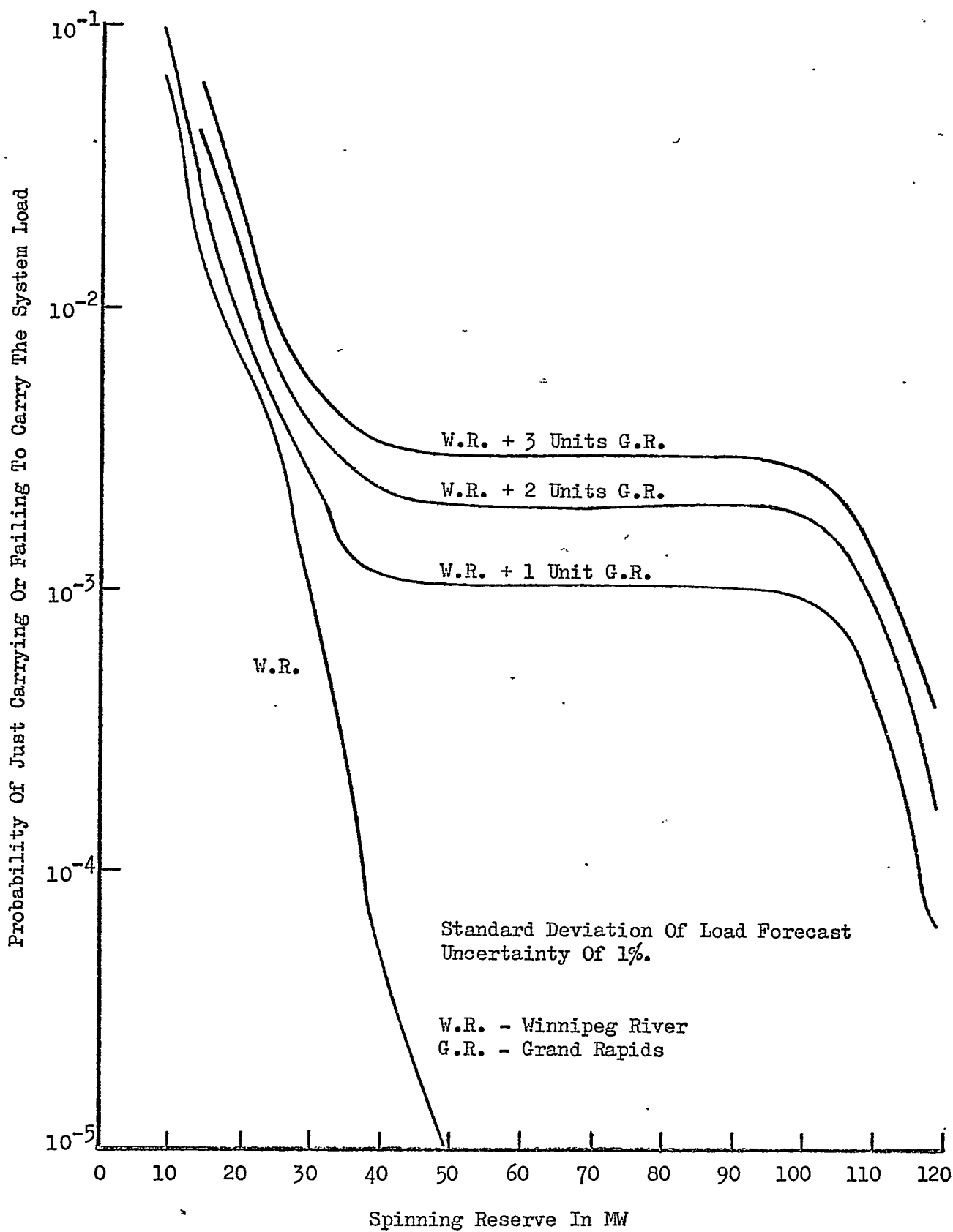


Figure 3.7 Spinning Reserve-Risk Characteristics
With Unit Additions At Grand Rapids

It should be realized that the curves for 1, 2 and 3 units at Grand Rapids apply only to exactly that number of units available to the system. The start-up and synchronizing time of a hydro unit could be less than a minute in certain cases and there would essentially be no spinning reserve problem if the system load was being supplied by the Winnipeg River generation and one or two Grand Rapids units with at least one other Grand Rapids unit available for immediate operation.

It is interesting to compare the phenomenon shown in Figure 3.7 with that arising from a conventional static capacity investigation of the same system. In the static capacity case, further addition of large units results in increased load carrying capability of the unit additions. In the spinning reserve case at each particular operating level the large unit additions require additional quantities of spinning reserve to maintain a constant risk.

The load-risk characteristics for the 889 MW hydraulic generating capacity condition are shown in Figure 3.8. The load forecast uncertainty in this case tends to round off the sharp characteristic obtained for zero uncertainty. Increased uncertainty produces an approximate straight line similar in form to those shown in Figure 3.5. The uncertainty in load forecasting would normally be greatest when forecasts are made considerably in advance. The critical uncertainty occurs at the latest time at which a decision must be made regarding the start-up of thermal equipment. This could be in the order of four to six hours before the load materializes, depending on prior operation of the thermal units. The actual magnitude of the standard deviation of load forecast uncertainty is dependent upon the attention given to load forecasting

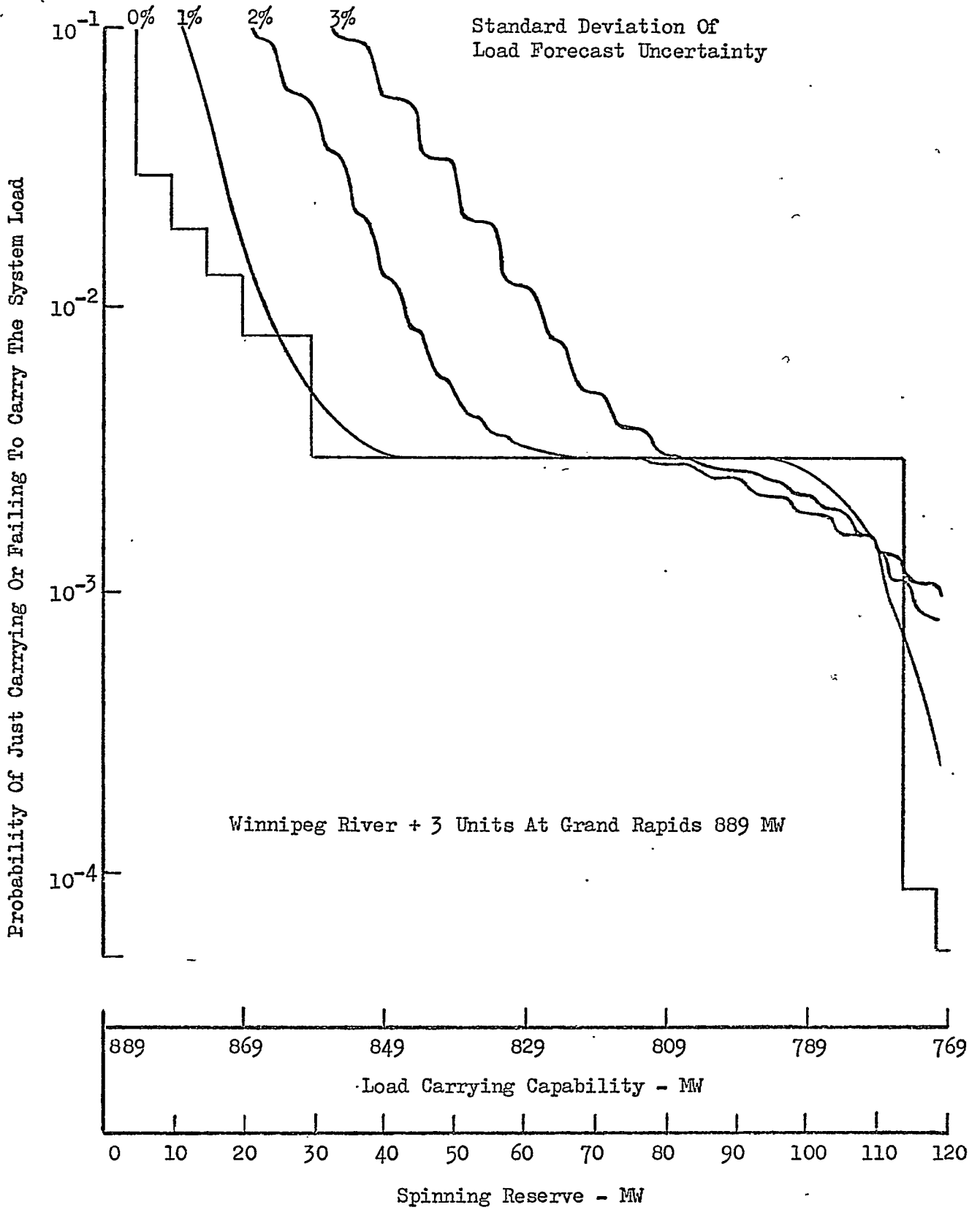


Figure 3.8 Variation In Load Carrying Capability With Load Forecast Uncertainty

methods down to some minimum value which is a function of the system and its weather pattern variability. The load forecasting method and the application of probability techniques to the evaluation of a spinning reserve criterion for the Manitoba System were discussed in detail in a recent publication⁽⁵⁾.

The effect of increasing or decreasing the operating capacity reliability level can be translated into a monetary quantity by a detailed examination of the daily load cycles. An analysis of this type would call for considerable generation dispatching and system operating experience. A relatively simple method of providing management with a monetary guide involves the comparison of thermal unit start-ups for different reliability levels. Load carrying capability curves similar to those shown in Figure 3.8 have been used in conjunction with the 1966-67 forecast Daily Peak Load Variation Curve for the Manitoba System to give the variation in thermal activity days with risk level and standard deviation of load forecast uncertainty shown in Figure 3.9⁽³¹⁾.

A frequent objection to the application of probability methods in this area is due to the lack of statistics for the system under study and an understandable reluctance to use published data for comparable facilities. System information must be consistently collected over a period of time if the resulting statistics are to be acceptable for utilization in future system predictions. System reliability can, however, be expressed in terms of confidence limits which take into account the amount of failure experience accumulated for the individual components. Conventional confidence limits are of the two tailed variety in which the statistic in question is estimated with a certain degree of

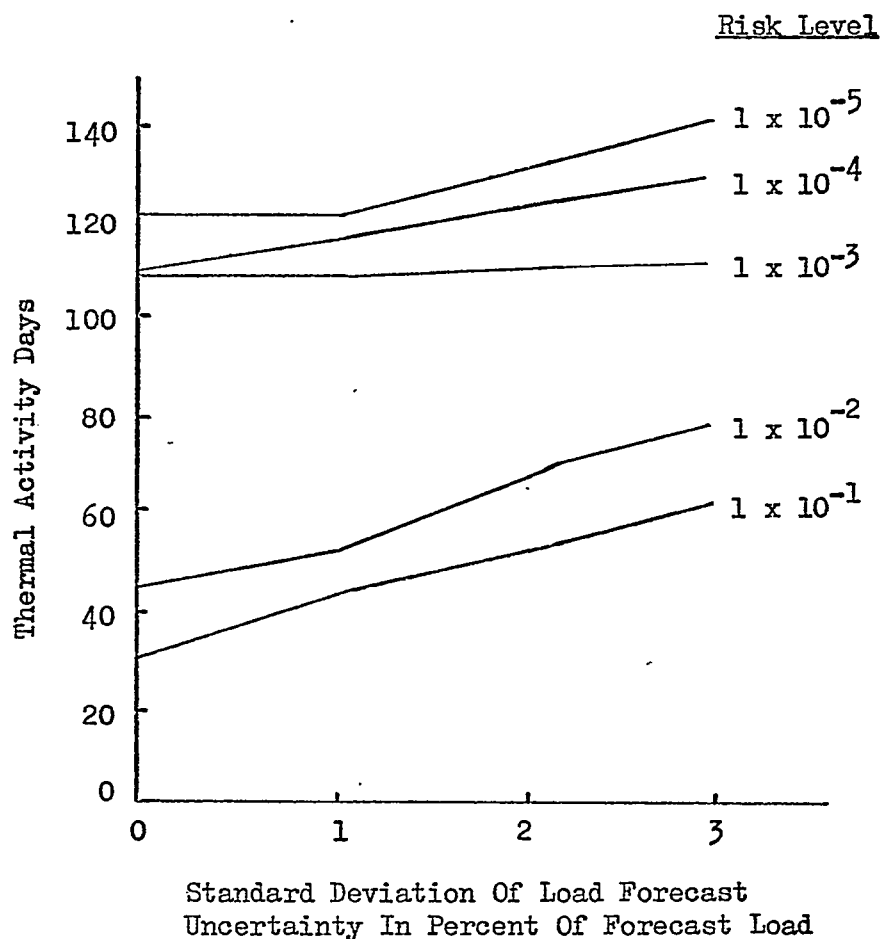


Figure 3.9 Thermal Activity Days At Selected Risk Levels

confidence to lie between an upper and lower bound. In a reliability investigation it is often useful to express the estimate in terms of only an upper or a lower bound depending on the statistic itself. Usual reliability failure rate prediction is based upon replacement or non-replacement tests on a group of similar devices. The same basic concepts apply to a generating unit using scheduled preventive maintenance if it is assumed that the unit is operating during its useful life period and that after each chance failure the unit is restored to a semi-original condition. This situation is therefore essentially the same as making individual tests on identical components.

For a unit operating in the useful life period, the mean time to failure MTTF (m) is the reciprocal of the failure rate (λ). If the total unit operating time is designated as T , the MTTF is given by: ⁽⁸⁾

$$m \geq \frac{2T}{\chi_{\alpha, 2r+2}^2}$$

Where

$$\chi^2 = \text{chi-squared}$$

$$\alpha = \text{confidence bound} = 1 - (\text{confidence interval})$$

$$r = \text{number of failures}$$

The true mean time to failure m is estimated to equal or exceed the value given by the above equation with a degree of confidence given by $(1 - \alpha)$. Identical results can be obtained by direct application of the Poisson Distribution. Confidence values for the MTTF are shown graphically in Figure 3.10 in terms of a multiplication factor applied to the total unit operating time. As the number of failures increase, the underlying distribution approaches a normal distribution. The 50 percent confidence value for k the multiplication factor, approaches the reciprocal of the number of failures. The lower confidence bound on the MTTF is an upper bound on the failure rate. The failure rate values are therefore continually modified by continuing system experience within the component useful life period. The selection of an acceptable confidence level is a management decision. Once selected, it permits a consistent approach to the estimation of equipment failure rates based upon the information available.

The effect on the system load carrying capability of placing confidence bounds on the Winnipeg River unit failure rates is shown in Figure 3.11.

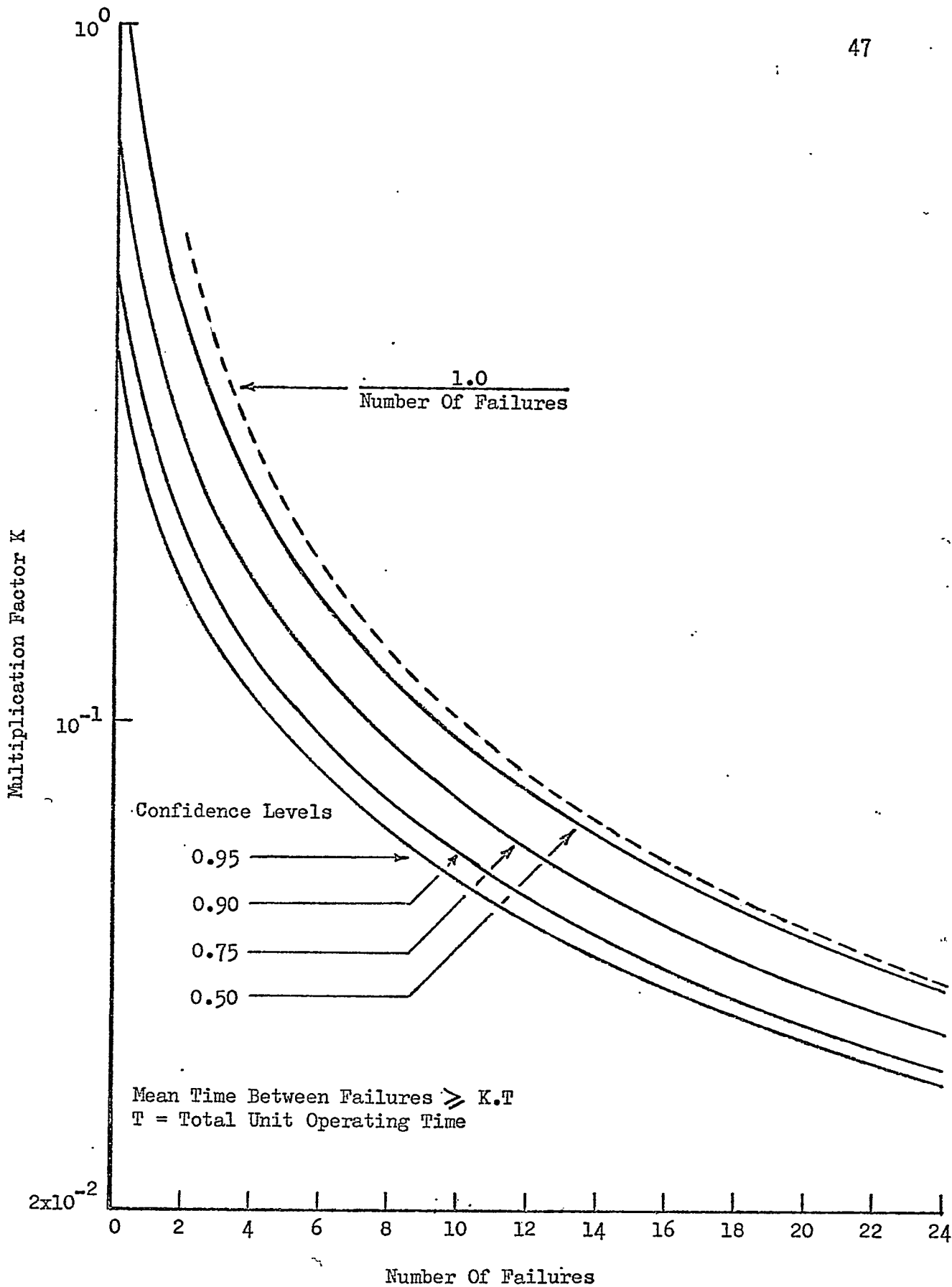


Figure 3.10 Confidence Level Multiplication Factors For MTF

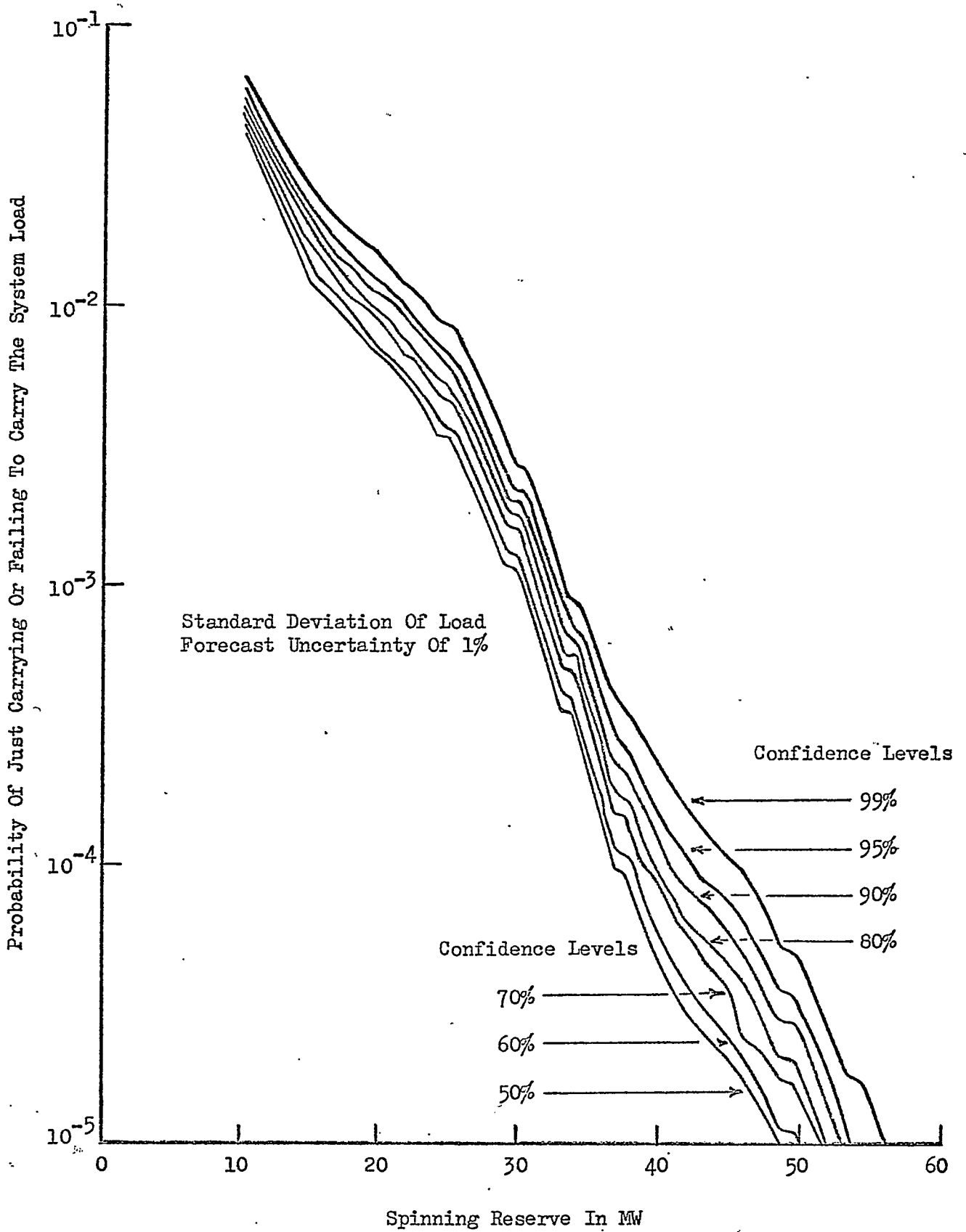


Figure 3.11 Load-Risk Characteristics At Selected Confidence Levels

Additional capacity was assumed to have a delay time of four hours and the standard deviation of load forecast uncertainty was taken as one percent. The individual unit failure rates at the selected confidence levels are shown in Table A-1 in the Appendix. The generation capability can now be expressed as the load carrying capability of the 559 MW operating capacity level at a risk of 0.0001 is 513 MW with 99 percent confidence. Using the point estimates of the unit failure rates, the load carrying capability at a risk level of 0.0001 is 522 MW.

At the time this study was prepared only one year's data were available for the Grand Rapids units. During this period the units experienced extremely high failure frequencies as shown in Table A-1. The effect on the risk-load characteristic using the actual outage data is shown in Figure 3.12 together with the results obtained using expected outage data obtained from published literature. As the equipment passes through the burn-in period into the useful life period, the point estimate characteristic and the curve obtained from published data should come together. The early wear out or burn-in period for large generating units is an extremely important area and it is unfortunate that there is so little published data available. Continued observation of the Grand Rapids units will provide extremely important information for future studies.

The theoretical concepts and practical examples shown, illustrate that given a selected risk criterion and confidence level it is possible to determine the required spinning reserve margin for a given operating capacity level and degree of load forecast uncertainty. No statement has been made, however, as to where this reserve should be located

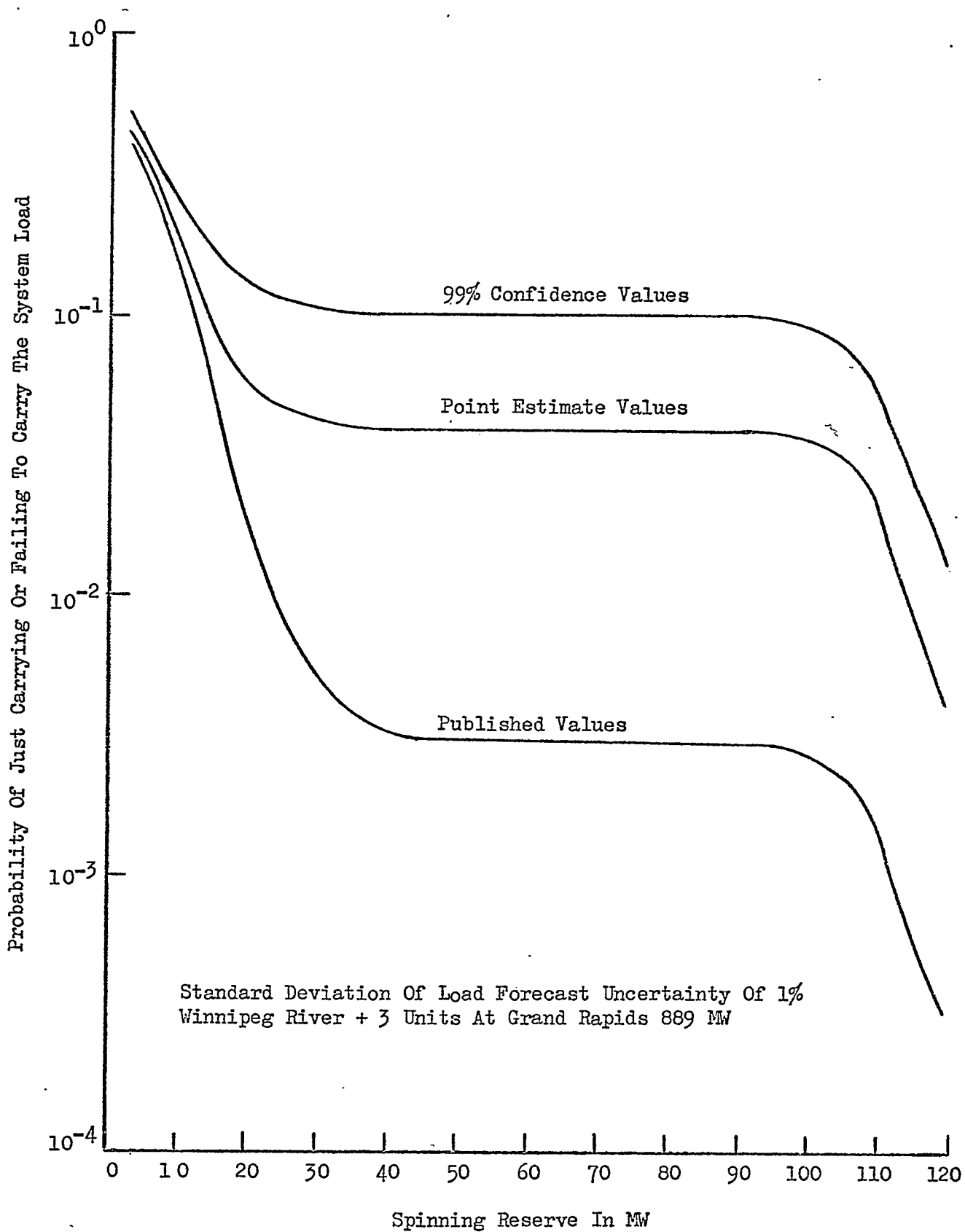


Figure 3.12 Load-Risk Characteristics Using Confidence Levels With Units At Grand Rapids

within the system. If the transmission is assumed to be completely reliable, the only constraints upon load distribution are those imposed by optimum system frequency regulation and economic operation. If the transmission cannot be assumed to be completely reliable, the problem is vastly different. These aspects are discussed in Chapter 5 of this thesis.

4. TRANSMISSION SYSTEM RELIABILITY

4.1 Markov Processes

In Chapter 2 of this thesis the application of two recently published methods^(21,22) of evaluating transmission system reliability was illustrated by applying them to a simple transmission configuration. The second method⁽²²⁾ introduced the concept of a two state fluctuating environment to describe the failure rate of system components and to evaluate the bunching effect of storm associated failures in parallel equipment. This method is an approximate approach. It has been stated that it will give results within a few percent of those obtained using more theoretical techniques such as Markov processes. Within the bounds of the necessary distributional assumptions the Markov technique is theoretically the most accurate approach to this aspect of reliability evaluation. The use of Markov chains was briefly discussed in a recent I.E.E.E. publication⁽³²⁾ but no attempt was made to apply these concepts in a practical problem. In this chapter, the basic application of Markov processes to power system reliability evaluation is illustrated and in particular the effect of a two state fluctuating environment on simple transmission configurations. The results are compared to those obtained using the previously noted published method⁽²²⁾. For comparison purposes this approach has been designated as the "Approximate Method".

The theoretical aspects of Markov processes are discussed in considerable detail in the published literature^(33,34) particularly in regard to processes that are discrete in time. The reliability problem normally deals with systems that are continuous in time, as component operating modes in the form of failures and repairs do not occur at

fixed intervals but can occur at any instant of time. In the spinning reserve application in the previous chapter, it was noted that for a device operating within its useful life, the failure rate is a constant. It was also noted that there is a lack of memory in the failure process as the conditional probability of failure during any fixed time interval is independent of the prior operating time.

If the component failure rate is designated as λ , the probability of a failure in a period $t, t + dt$, of length dt , given that the device was operating at time t is λdt . The repair process can also in certain cases⁽²²⁾ be described by an exponential distribution and characterized by a repair rate designated as μ .

A relatively simple Markov application to a practical condition is the case of a single continuously operable maintainable component.

Define:-

$P_0(t)$ = Probability of the component being operable at time t .

$P_1(t)$ = Probability of the component being inoperable at time t .

λ = Failure rate

μ = Repair rate

Considering an incremental time interval dt

$$P_0(t + dt) = P_0(t) (1 - \lambda dt) + P_1(t) (\mu dt) \quad (4.1)$$

$$P_1(t + dt) = P_0(t) (\lambda dt) + P_1(t) (1 - \mu dt) \quad (4.2)$$

The probability of two or more events occurring during the increment of time dt is considered to be negligible. The terms describing these conditions involve powers of dt .

The state space diagram for this simple system is shown in Figure 4.1.

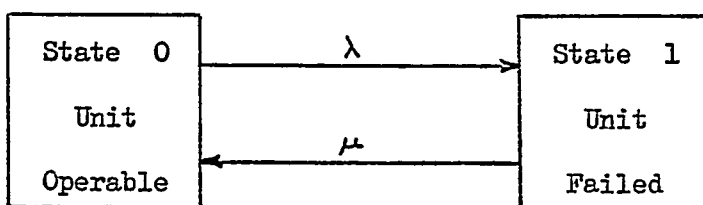


Figure 4.1 Single Unit State Space Diagram

Since

$$\left. \frac{P_0(t + dt) - P_0(t)}{dt} \right|_{dt \rightarrow 0} = \frac{dP_0(t)}{dt} = P_0'(t)$$

Equations 4.1 and 4.2 become

$$P_0'(t) = -\lambda P_0(t) + \mu P_1(t)$$

$$P_1'(t) = \lambda P_0(t) - \mu P_1(t)$$

In matrix form

$$\begin{bmatrix} P_0'(t) \\ P_1'(t) \end{bmatrix} = \begin{bmatrix} P_0(t) & P_1(t) \end{bmatrix} \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$$

Designating $P_0(0)$ and $P_1(0)$ as initial conditions at time 0, for which

$$P_0(0) + P_1(0) = 1.0$$

$$P_0(t) = \frac{\mu}{\lambda + \mu} [P_0(0) + P_1(0)] + \frac{e^{-(\lambda + \mu)t}}{\lambda + \mu} [\lambda P_0(0) - \mu P_1(0)] \quad (4.3)$$

$$P_1(t) = \frac{\lambda}{\lambda + \mu} [P_0(0) + P_1(0)] + \frac{e^{-(\lambda + \mu)t}}{\lambda + \mu} [\mu P_1(0) - \lambda P_0(0)] \quad (4.4)$$

If the assumption is made that the system is operable at time 0 then

$P_0(0) = 1.0$ and $P_1(0) = 0.0$. Equations 4.3 and 4.4 become

$$P_0(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} \quad (4.5)$$

$$P_1(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} \quad (4.6)$$

As $t \rightarrow \infty$

$$P_0(\infty) = \frac{\mu}{\lambda + \mu}$$

$$P_1(\infty) = \frac{\lambda}{\lambda + \mu}$$

As the repair and failure processes are represented by exponential distributions

$$\text{The Mean Time To Failure} = F = \frac{1}{\lambda}$$

$$\text{The Mean Time To Repair} = R = \frac{1}{\mu}$$

Then

$$P_0(\infty) = \frac{F}{F + R} = \text{Conventional Availability Rate}$$

$$P_1(\infty) = \frac{R}{F + R} = \text{Conventional Forced Outage Rate}$$

Designate $P_0(\infty)$ and $P_1(\infty)$ as P_0 and P_1 . The limiting state probabilities can also be obtained directly from the differential matrix. When $t \rightarrow \infty$

$$\frac{dP_0(t)}{dt} = 0 \quad \text{and} \quad \frac{dP_1(t)}{dt} = 0$$

The system equations then become

$$-\lambda P_0 + \mu P_1 = 0$$

$$\lambda P_0 - \mu P_1 = 0$$

$$P_0 + P_1 = 1.0$$

From which the limiting state values P_0 and P_1 can be easily obtained.

The limiting state probabilities can also be obtained by considering the system as a Markov process that is discrete in time. The stochastic matrix of transition probabilities that defines the process

from time t , to time $t + \Delta t$, ie. a discrete period Δt , is designated as P .

$$P = \begin{bmatrix} 1 - \lambda & \lambda \\ \mu & 1 - \mu \end{bmatrix}$$

The process is ergodic, as it is possible to move from one state to any other state in a finite number of steps and therefore the limiting state probabilities are constant. Using conventional Markov techniques, it is necessary to find the probability vector $\alpha = [P_0 \ P_1]$ such that

$$\alpha P = \alpha$$

This is obtained from

$$\begin{aligned} P_0 + P_1 &= 1.0 \\ (1 - \lambda) P_0 + \mu P_1 &= P_0 \\ \lambda P_0 + (1 - \mu) P_1 &= P_1 \end{aligned}$$

From which

$$P_0 = \frac{\mu}{\lambda + \mu} \qquad P_1 = \frac{\lambda}{\lambda + \mu} \qquad (4.7)$$

In more complicated systems it becomes increasingly difficult to obtain a general time dependent expression. The limiting state probabilities are much easier to obtain as the system simultaneous differential equations reduce to ordinary simultaneous equations. The transient component in all practical power system cases is negligible as the $\frac{\mu}{\lambda}$ ratio is normally very large.

Consider the case of two identical units in parallel. The differential equations for this system are as follows

$$P_0(t) = \text{Probability that both units are in an operable state at time } t.$$

$P_1(t)$ = Probability that only one unit is in an operable state at time t and that the other unit is in a failed state at time t .

$P_2(t)$ = Probability that both units are in the failed state at time t .

$$\begin{bmatrix} P_0'(t) \\ P_1'(t) \\ P_2'(t) \end{bmatrix} = \begin{bmatrix} P_0(t) & P_1(t) & P_2(t) \end{bmatrix} \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -(\lambda+\mu) & \lambda \\ 0 & 2\mu & -2\mu \end{bmatrix} \quad (4.8)$$

If $P_0(0) = 1.0$ and $P_1(0) = P_2(0) = 0$.

$$P_0(t) = \frac{\mu^2}{(\lambda+\mu)^2} + \frac{2\lambda\mu}{(\lambda+\mu)^2} e^{-(\lambda+\mu)t} + \frac{\lambda^2}{(\lambda+\mu)^2} e^{-2(\lambda+\mu)t} \quad (4.9)$$

$$P_1(t) = \frac{2\lambda\mu}{(\lambda+\mu)^2} + \frac{2\lambda(\lambda-\mu)}{(\lambda+\mu)^2} e^{-(\lambda+\mu)t} - \frac{2\lambda^2}{(\lambda+\mu)^2} e^{-2(\lambda+\mu)t} \quad (4.10)$$

$$P_2(t) = \frac{\lambda^2}{(\lambda+\mu)^2} - \frac{2\lambda^2}{(\lambda+\mu)^2} e^{-(\lambda+\mu)t} + \frac{\lambda^2}{(\lambda+\mu)^2} e^{-2(\lambda+\mu)t} \quad (4.11)$$

In this system, the behaviour of the individual units are independent. Equations 4.9, 4.10 and 4.11 could have been obtained using the Binomial Expansion from the results of the single unit case given in equations 4.5 and 4.6. The limiting state probabilities can be obtained directly from the differential matrices by allowing $\frac{dP}{dt} \rightarrow 0$. If the availability A is defined as the probability that at least one unit is available

$$\begin{aligned}
 A(t = \infty) &= P_0 + P_1 \\
 &= \frac{\mu^2 + 2\lambda\mu}{(\lambda + \mu)^2}
 \end{aligned}$$

For a completely redundant system, reliability can be defined as the probability that the system does not enter state 2, ie. both units failed. State 2 can be considered as an absorbing state, which once entered is not left until the process starts again. Equation 4.8 becomes

$$\begin{bmatrix} P_0'(t) \\ P_1'(t) \\ P_2'(t) \end{bmatrix} = \begin{bmatrix} P_0(t) & P_1(t) & P_2(t) \end{bmatrix} \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -(\lambda + \mu) & \lambda \\ 0 & 0 & 1 \end{bmatrix}$$

The solution for this case is ⁽³⁵⁾

$$R(t) = \frac{S_1 e^{-S_2 t} - S_2 e^{-S_1 t}}{S_1 - S_2}$$

where:

$$\begin{aligned}
 S_1 &= \frac{1}{2} (3\lambda + \mu + \sqrt{\lambda^2 + 6\lambda\mu + \mu^2}) \\
 S_2 &= \frac{1}{2} (3\lambda + \mu - \sqrt{\lambda^2 + 6\lambda\mu + \mu^2})
 \end{aligned}$$

The Mean Time To Failure for this system can be found by integrating the reliability function $R(t)$ over the range 0 to ∞ .

$$\text{MTTF} = \int_0^{\infty} R(t) dt = \frac{S_1 + S_2}{S_1 S_2} = \frac{3\lambda + \mu}{2\lambda^2} \quad (4.12)$$

As previously noted, it is virtually impossible to obtain a general time dependent expression for the reliability of more complicated systems.

The MPTF of the system can be obtained, however, using discrete Markov chain concepts⁽³⁴⁾.

The stochastic matrix of transition probabilities for the two unit case is

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 1 - 2\lambda & 2\lambda & 0 \\ \mu & 1 - (\lambda + \mu) & \lambda \\ 0 & 2\mu & 1 - 2\mu \end{bmatrix} \end{matrix}$$

State 2 is designated as an absorbing state and a new truncated matrix Q is obtained by eliminating the absorbed state.

$$Q = \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 1 - 2\lambda & 2\lambda \\ \mu & 1 - (\lambda + \mu) \end{bmatrix} \end{matrix}$$

Let I = Identity Matrix

$$[I - Q] = \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 2\lambda & -2\lambda \\ -\mu & \lambda + \mu \end{bmatrix} \end{matrix}$$

N = Fundamental matrix, where n_j is the time spent by the process in state S_j before being absorbed

$$N = [I - Q]^{-1} = \frac{1}{2\lambda^2} \begin{bmatrix} \lambda + \mu & 2\lambda \\ \mu & 2\lambda \end{bmatrix}$$

Starting in State 0 the time before entering the absorbed state (MPTF) of the process is $M_{0,2}$

$$M_{0,2} = \frac{1}{2\lambda^2} [\lambda + \mu + 2\lambda] = \frac{3\lambda + \mu}{2\lambda^2}$$

This is the result given in equation 4.12. The matrix inversion approach is extremely useful when dealing with more complicated systems. The MTTF is normally much longer than the time spent in the failed state and therefore the long term failure frequency can be expressed as the reciprocal of the MTTF and can be designated as the system outage rate. It should be realized, however, that the MTTF or outage rate of a two unit system cannot be associated with a single exponential function to give an expression for the reliability of the system.

The case of two units in series can be examined using the equations developed for the two identical unit redundant case by redefining the failed states. If it is assumed that it is possible for a unit to fail while the other unit is in a failed state.

$$\begin{aligned}
 \text{Availability } A(t) &= P_0(t) \\
 P(\text{System Failure}) &= P_1(t) + P_2(t) \\
 A(t = \infty) = P_0 &= \frac{\mu^2}{(\lambda + \mu)^2} \\
 \text{MTTF} &= \frac{1}{2\lambda} \qquad (4.13) \\
 \text{Failure Rate} &= 2\lambda .
 \end{aligned}$$

4.2 Two State Fluctuating Environment

The environment encountered by a generating unit or any component contained indoors is relatively constant. In these cases the failure rate can be considered constant within the useful life period. For a transmission line and other outdoor components the environment is not constant and for certain components can have a considerable effect upon their failure rates. The use of a two state fluctuating environment

covering normal and stormy weather has been proposed⁽²²⁾ in which weather duration distributions were assumed to be exponential. Using these distributional assumptions the Markov approach can be applied as follows to a single unit in a two state failure environment.

Define

λ, μ = Normal weather failure and repair rates

λ', μ' = Stormy weather failure and repair rates

$m = \frac{1}{S}$ where S = Expected duration of a stormy weather period

$n = \frac{1}{N}$ where N = Expected duration of a normal weather period

The state space diagram for this system is given in Figure 4.2.

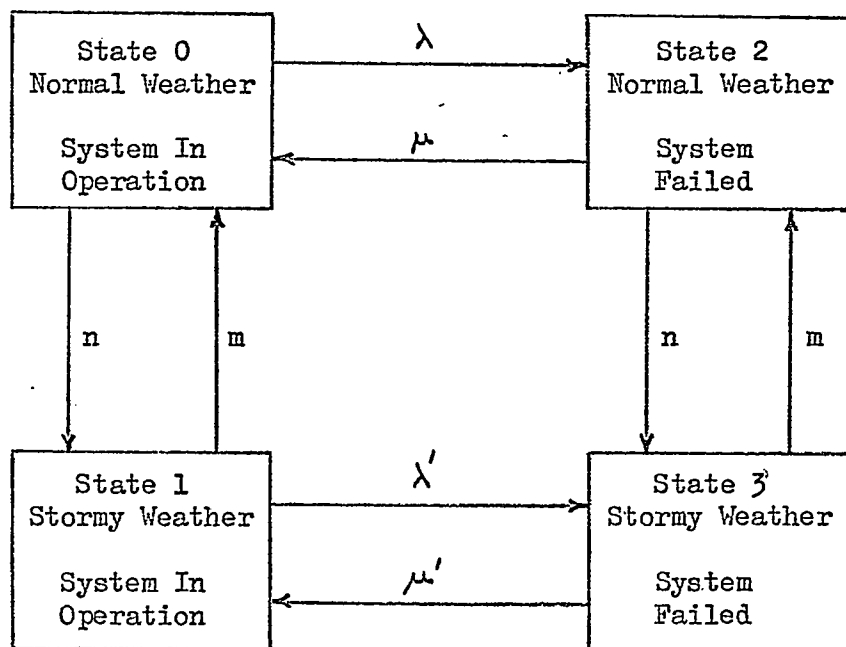


Figure 4.2 Single Unit State Space Diagram For A Two State Fluctuating Environment

Proceeding directly from this diagram the differential equations in matrix form are

$$\begin{bmatrix} P_0'(t) \\ P_1'(t) \\ P_2'(t) \\ P_3'(t) \end{bmatrix} = \begin{bmatrix} P_0(t) & P_1(t) & P_2(t) & P_3(t) \end{bmatrix} \begin{bmatrix} -(\lambda+n) & n & \lambda & 0 \\ m & -(m+\lambda') & 0 & \lambda' \\ \mu & 0 & -(\mu+n) & n \\ 0 & \mu' & m & -(\mu'+m) \end{bmatrix}$$

The steady state or limiting probabilities can be found from

$$\begin{aligned} -(\lambda+n)P_0 + mP_1 + \mu P_2 &= 0 \\ nP_0 - (m+\lambda')P_1 + \mu' P_3 &= 0 \\ \lambda P_0 - (\mu+n)P_2 + mP_3 &= 0 \\ \lambda' P_1 + nP_2 - (\mu'+m)P_3 &= 0 \\ P_0 + P_1 + P_2 + P_3 &= 1.0 \end{aligned}$$

For this system

$$P(\text{System Operating}) = P_0 + P_1$$

$$P(\text{System Failed}) = P_2 + P_3$$

If the repair rate is assumed to be independent of the environment

$$\mu = \mu'$$

$$P(\text{System Operating}) = \frac{\mu}{m+n} \left[\frac{(m+n)^2 + m(\mu+\lambda') + n(\mu+\lambda)}{(\mu+\lambda)(\mu+\lambda') + m(\mu+\lambda) + n(\mu+\lambda')} \right]$$

$$P(\text{System Failed}) = \frac{1}{m+n} \left[\frac{n\lambda'(n+\mu) + m\lambda(m+\mu) + nm(\lambda+\lambda') + \lambda\lambda'(m+n)}{(\mu+\lambda)(\mu+\lambda') + m(\mu+\lambda) + n(\mu+\lambda')} \right]$$

Considering the normal weather case only,

$$\text{let } \lambda' = 0, m = 1, n = 0$$

$$P(\text{System Operating}) = \frac{\mu}{\lambda + \mu}$$

$$P(\text{System Failed}) = \frac{\lambda}{\lambda + \mu}$$

This is the result given in equation 4.7

In obtaining the MTTF, both states 2 and 3 can be considered as absorbing states as it is not important which state the system failed in, only that it did fail. The stochastic transitional probability matrix P is

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1-(\lambda+n) & n & \lambda & 0 \\ m & 1-(m+\lambda') & 0 & \lambda' \\ \mu & 0 & 1-(\mu+n) & n \\ 0 & \mu' & m & 1-(\mu'+m) \end{bmatrix} \end{matrix}$$

The truncated Q matrix is formed by deleting states 2 and 3

$$Q = \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 1-(\lambda+n) & n \\ m & 1-(m+\lambda') \end{bmatrix} \end{matrix}$$

The fundamental matrix N is

$$N = [I - Q]^{-1} = \frac{1}{\lambda\lambda' + \lambda m + \lambda' n} \begin{bmatrix} m + \lambda' & n \\ m & \lambda + n \end{bmatrix}$$

Starting in State 0 (ie. the unit is operating in normal weather) the

MTTF $M_{0,23}$ is

$$M_{0,23} = \frac{m + \lambda' + n}{\lambda\lambda' + \lambda m + \lambda' n} \quad (4.14)$$

Considering normal weather only,

$$\text{let } \lambda' = 0, m = 1, n = 0$$

$$M_{0,23} = \frac{1}{\lambda}$$

The limiting state probabilities and mean time to failure for two and three unit systems can be evaluated in a similar manner using the same basic symbols as for the single unit case. The state space diagram becomes quite important as the system complexity increases. The diagram for the two unit case is shown in Figure 4.3. Proceeding directly from this diagram the system differential equations in matrix form are

$$\begin{bmatrix} P'(t) \end{bmatrix} = \begin{bmatrix} P(t) \end{bmatrix} \begin{bmatrix} D_1 & n [I] \\ m [I] & D_2 \end{bmatrix}$$

The D_1 and D_2 matrices are shown in Table 4.1. They apply to either two units in parallel or to two units in series. In the case of two redundant units in parallel, the probability of the system occupying the failed state is obtained by summing the limiting state probabilities for states 3 and 7 given in Figure 4.3. The MTTF is obtained by truncating the stochastic transitional probability matrix by eliminating states 3 and 7. It is extremely laborious to attempt to obtain a general expression for the probability of occupying the failed states and for the system MTTF. A digital computer program was developed to obtain a numerical solution for these parameters using specific values of failure, repair, stormy weather and normal weather rates. Some results are shown in Figures 4.5 and 4.6.

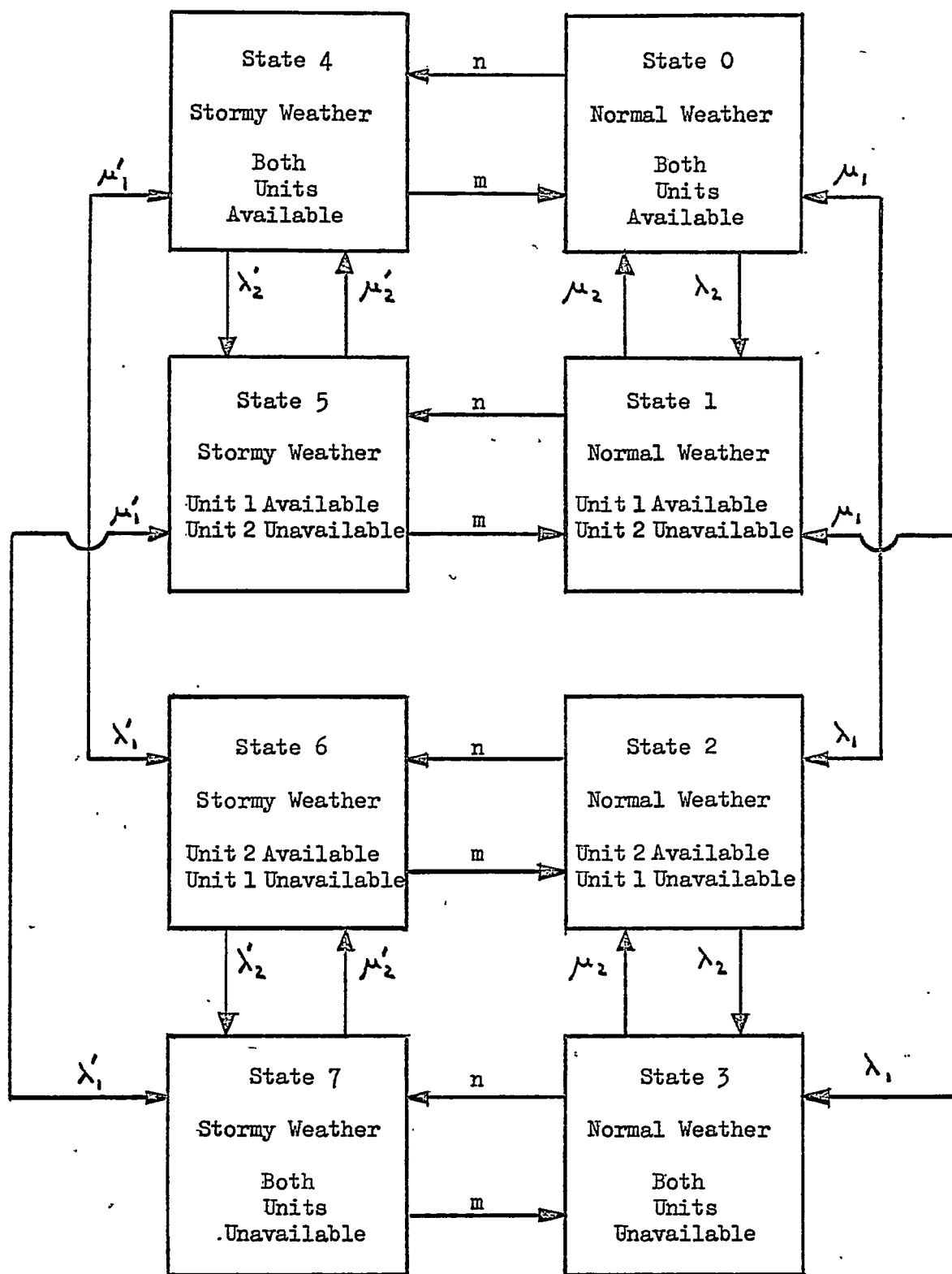


Figure 4.3 Two Unit State Space Diagram For A Two State Fluctuating Environment

TABLE 4.1

Partitioned Matrices For The Two Unit Case

$$\left[\begin{array}{c} D_1 \end{array} \right] = \left[\begin{array}{cccc} (-\lambda_1 - \lambda_2 - n) & \lambda_2 & \lambda_1 & 0 \\ \mu_2 & (-\lambda_1 - \mu_2 - n) & 0 & \lambda_1 \\ \mu_1 & 0 & (-\mu_1 - \lambda_2 - n) & \lambda_2 \\ 0 & \mu_1 & \mu_2 & (-\mu_1 - \mu_2 - n) \end{array} \right]$$

$$\left[\begin{array}{c} D_2 \end{array} \right] = \left[\begin{array}{cccc} (-\lambda'_1 - \lambda'_2 - m) & \lambda'_2 & \lambda'_1 & 0 \\ \mu'_2 & (-\lambda'_1 - \mu'_2 - m) & 0 & \lambda'_1 \\ \mu'_1 & 0 & (-\lambda'_2 - \mu'_1 - m) & \lambda'_2 \\ 0 & \mu'_1 & \mu'_2 & (-\mu'_1 - \mu'_2 - m) \end{array} \right]$$

For two components in series, the assumption is made that it is possible for a component to fail while the other component is being repaired. This is a reasonable assumption, as many failures particularly storm associated ones are not contingent upon the component being energized. In this case, the probability of the system occupying the failed state is obtained by summing the limiting state probabilities for states 1, 2, 3, 5, 6, 7. The MTF is obtained by truncating the stochastic transitional probability matrix by eliminating these states.

For this case

$$\begin{array}{c}
 \begin{array}{cc} & \begin{array}{c} 0 \\ 4 \end{array} \\ \begin{array}{c} 0 \\ 4 \end{array} & \begin{array}{c} \begin{bmatrix} \lambda_1 + \lambda_2 + n & -n \\ -m & \lambda'_1 + \lambda'_2 + m \end{bmatrix} \\ \begin{bmatrix} \lambda'_1 + \lambda'_2 + m & n \\ m & \lambda_1 + \lambda_2 + n \end{bmatrix} \end{array} \\ \hline
 \lambda_1 \lambda'_2 + \lambda_1 \lambda'_1 + \lambda_1 m + \lambda_2 \lambda'_2 + \lambda_2 \lambda'_1 + \lambda_2 m + \lambda'_2 n + \lambda'_1 n
 \end{array}
 \end{array}$$

Starting in State 0 and letting

$$\lambda_1 = \lambda_2 = \lambda$$

$$\lambda'_1 = \lambda'_2 = \lambda'$$

$$M_0 = \frac{2 \lambda' + m + n}{2(2 \lambda \lambda' + \lambda m + \lambda' n)}$$

Considering normal weather only let

$$\lambda' = 0, m = 1, n = 0$$

$$M_0 = \frac{1}{2\lambda}$$

This is the result given in equation 4.13

The state space diagram for the two unit case could be extended to include maintenance conditions by assuming that no maintenance would be performed if there was any possibility of the system entering a stormy period. The state space diagram in Figure 4.3 would then be extended only on the normal weather side to include maintenance probabilities.

The state space diagram for the three unit case is shown in Figure 4.4. The differential equations in matrix form are similar to those for the two unit case.

$$\begin{bmatrix} P'(t) \end{bmatrix} = \begin{bmatrix} P(t) \end{bmatrix} \begin{bmatrix} D_3 & | & n[I] \\ \hline m[I] & | & D_4 \end{bmatrix}$$

The D_3 and D_4 matrices are shown in Table 4.2. As in the two unit case, solutions for specific values of failure, repair, stormy weather and normal weather rates have been obtained and are shown in Figures 4.5 and 4.6.

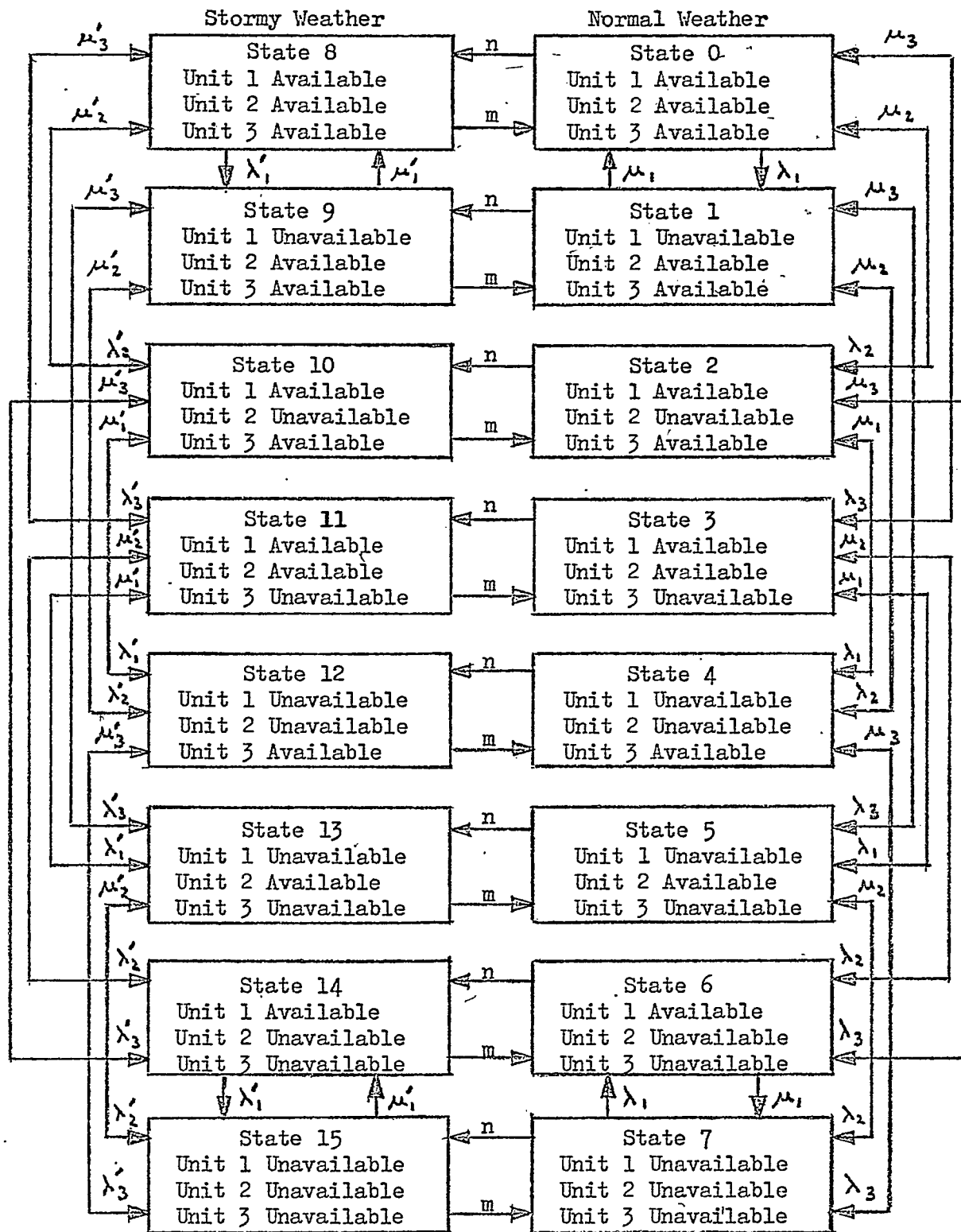


Figure 4.4 Three Unit State Space Diagram For A Two State Fluctuating Environment

TABLE 4.2.

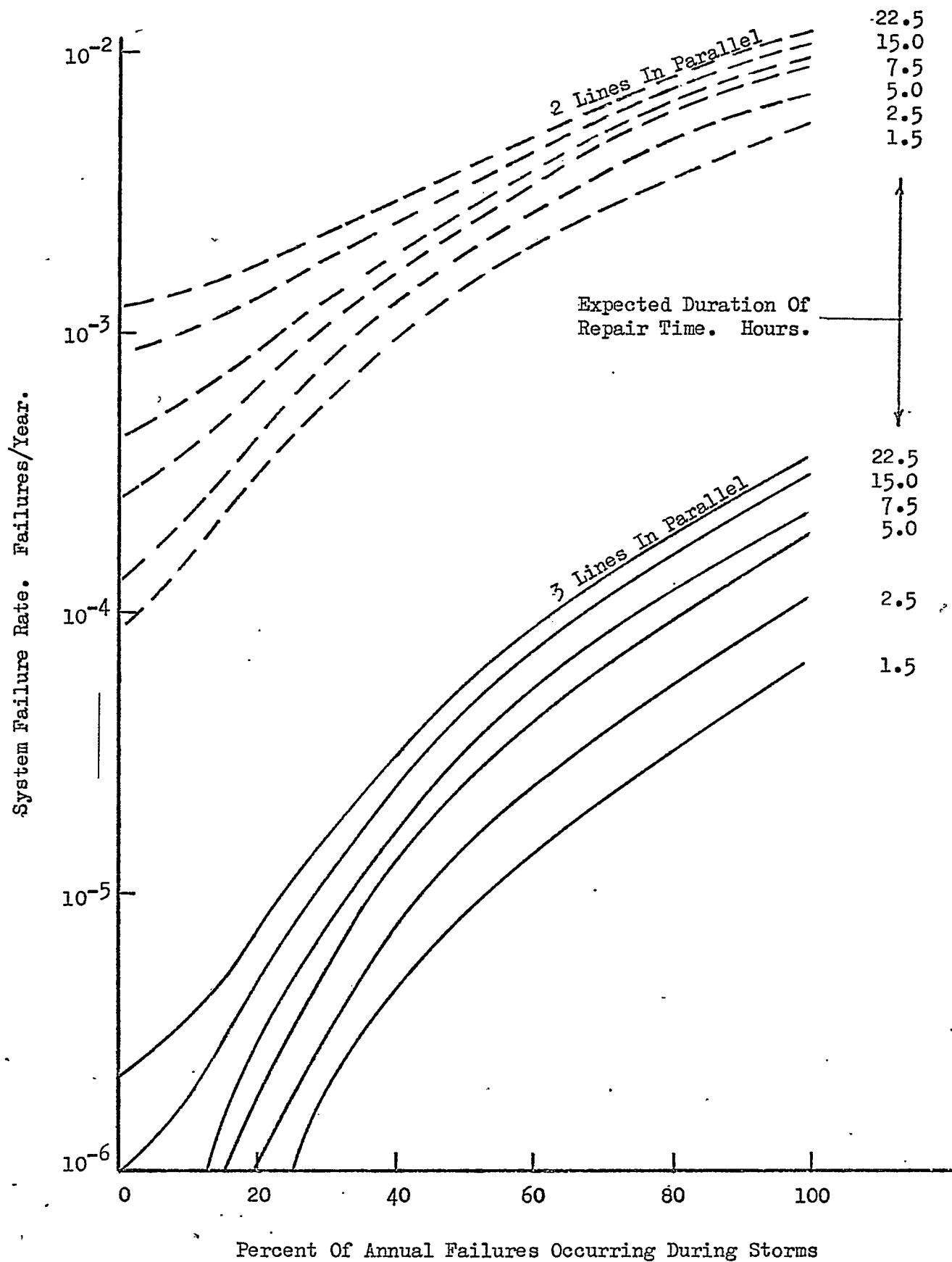
Partitioned Matrices For The Three Unit Case

$$\left[D_3 \right] = \begin{bmatrix}
 (-\lambda_1 - \lambda_2 - \lambda_3 - \pi) & \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 & 0 \\
 \mu_1 & (-\mu_1 - \lambda_2 - \lambda_3 - \pi) & 0 & 0 & \lambda_2 & \lambda_3 & 0 & 0 \\
 \mu_2 & 0 & (-\mu_2 - \lambda_1 - \lambda_3 - \pi) & 0 & \lambda_1 & 0 & \lambda_3 & 0 \\
 \mu_3 & 0 & 0 & (-\mu_3 - \lambda_1 - \lambda_2 - \pi) & 0 & \lambda_1 & \lambda_2 & 0 \\
 0 & \mu_2 & \mu_1 & 0 & (-\mu_1 - \mu_2 - \lambda_3 - \pi) & 0 & 0 & \lambda_3 \\
 0 & \mu_3 & 0 & \mu_1 & 0 & (-\mu_1 - \mu_3 - \lambda_2 - \pi) & 0 & \lambda_2 \\
 0 & 0 & \mu_3 & \mu_2 & 0 & 0 & (-\mu_2 - \mu_3 - \lambda_1 - \pi) & \lambda_1 \\
 0 & 0 & 0 & 0 & \mu_3 & \mu_2 & \mu_1 & (-\mu_1 - \mu_2 - \mu_3 - \pi)
 \end{bmatrix}$$

TABLE 4.2 (Continued)

Partitioned Matrices For The Three Unit Case

$$\left[D_4 \right] = \begin{bmatrix}
 (-\lambda'_1 - \lambda'_2 - \lambda'_3 - m) & \lambda'_1 & \lambda'_2 & \lambda'_3 & 0 & 0 & 0 & 0 \\
 \mu'_1 & (-\mu'_1 - \lambda'_2 - \lambda'_3 - m) & 0 & 0 & \lambda'_2 & \lambda'_3 & 0 & 0 \\
 \mu'_2 & 0 & (-\mu'_2 - \lambda'_1 - \lambda'_3 - m) & 0 & \lambda'_1 & 0 & \lambda'_3 & 0 \\
 \mu'_3 & 0 & 0 & (-\mu'_3 - \lambda'_1 - \lambda'_2 - m) & 0 & \lambda'_1 & \lambda'_2 & 0 \\
 0 & \mu'_2 & \mu'_1 & 0 & (-\mu'_1 - \mu'_2 - \lambda'_3 - m) & 0 & 0 & \lambda'_3 \\
 0 & \mu'_3 & 0 & \mu'_1 & 0 & (-\mu'_1 - \mu'_3 - \lambda'_2 - m) & 0 & \lambda'_2 \\
 0 & 0 & \mu'_3 & \mu'_2 & 0 & 0 & (-\mu'_2 - \mu'_3 - \lambda'_1 - m) & \lambda'_1 \\
 0 & 0 & 0 & 0 & \mu'_3 & \mu'_2 & \mu'_1 & (-\mu'_1 - \mu'_2 - \mu'_3 - m)
 \end{bmatrix}$$



Percent Of Annual Failures Occurring During Storms

Figure 4.5 System Failure Rate Variation With Storm Associated Failures

Expected Duration Of
Repair Time. Hours.

22.5

15.0

7.5

5.0

2.5

1.5

22.5

15.0

7.5

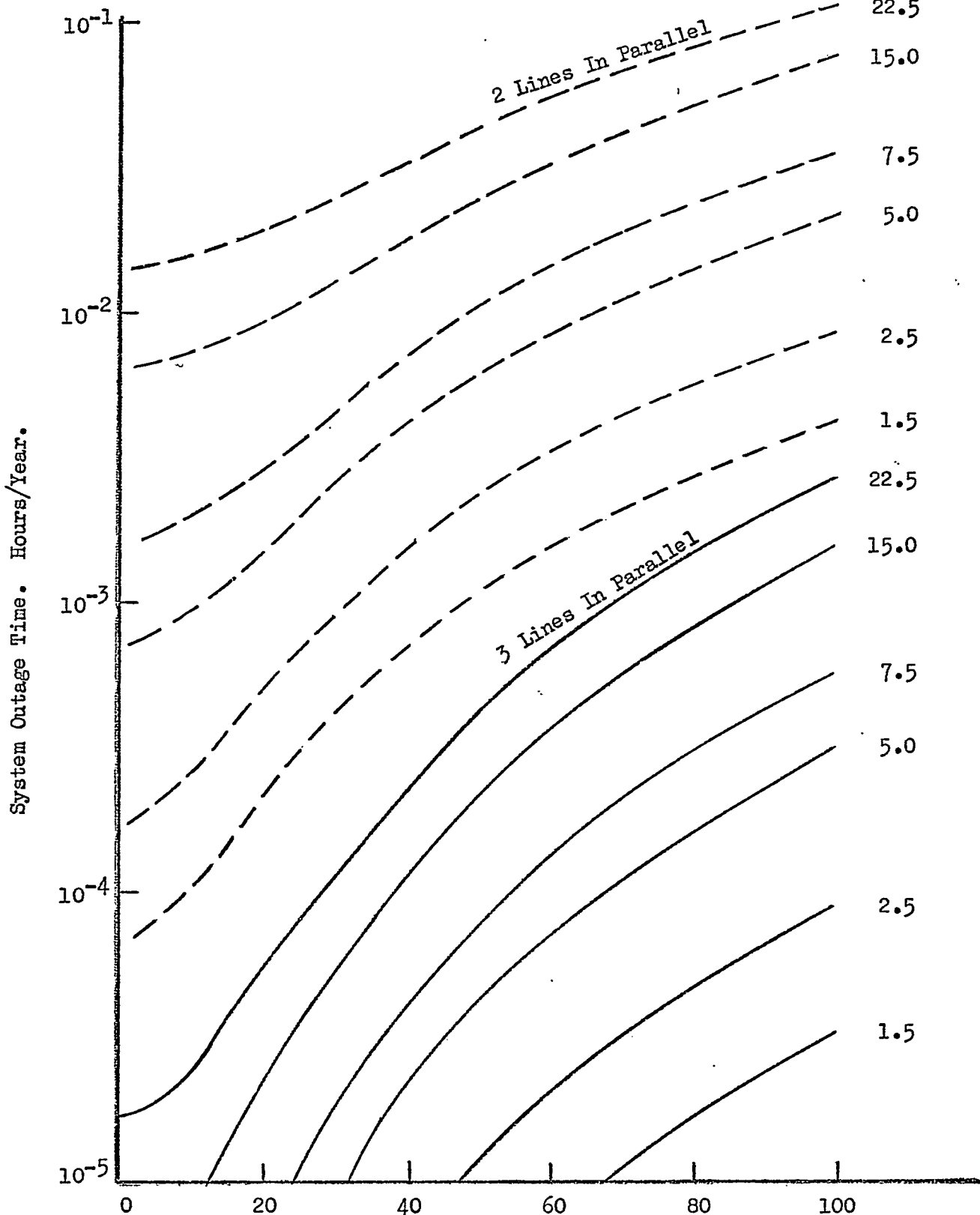
5.0

2.5

1.5

2 Lines In Parallel

3 Lines In Parallel



Percent Of Annual Failures Occurring During Storms

Figure 4.6 System Outage Time Variation With
Storm Associated Failures

4.3 System Studies

The MTF for a single unit with a two state fluctuating environment was given in equation 4.14 as

$$M_{0,23} = \frac{\lambda' + m + n}{\lambda \lambda' + \lambda m + \lambda' n}$$

Neglecting the short repair period compared to the MTF, the expected failure rate λ_{AV} is

$$\lambda_{AV} = \frac{\lambda \lambda' + \lambda m + \lambda' n}{\lambda' + m + n}$$

If $\lambda \lambda' \ll \lambda m + \lambda' n$

And $\lambda' \ll m + n$

Then
$$\lambda_{AV} \approx \frac{\lambda m}{m + n} + \frac{\lambda' n}{m + n}$$

$$\approx \lambda \left[\frac{N}{S + N} \right] + \lambda' \left[\frac{S}{S + N} \right] \quad (4.15)$$

This approximation is valid in nearly all practical cases. Equation 4.15 is identical to an expression used in the "Approximate Method"⁽²²⁾.

The effects of storm associated failures on two and three line redundant configurations were examined. The overall annual failure rate was held constant at 0.5 failures/year and the percentage of failures occurring during storms allowed to vary. The expected duration of normal and stormy weather periods was held constant at 200 hours and 1.5 hours respectively. The variation in expected system failure rate and average annual system outage duration due to variable expected repair duration

at different percentages of storm associated failures is shown in Figures 4.5 and 4.6.

The results obtained for average annual system outage duration by the Markov approach have been compared with those obtained using the "Approximate Method". The effects of changes in expected repair times are shown in Figure 4.7 for the two and three unit redundant systems. The results obtained by varying the expected duration of stormy and normal weather periods are shown in Figure 4.8. The effects of component failure rate variation are shown in Figure 4.9. The "Approximate Method" responds quite differently to changes in the system parameters and therefore no overall statement can be made regarding the actual differences except in specific cases. This is clearly shown in Figures 4.7, 4.8 and 4.9.

As the transmission voltage increases, storms will have a diminishing effect upon the overall annual failure rate. The stormy weather failure rate will decrease in this case. It may be more convenient from the viewpoint of data collection to regard this phenomena as an increase in normal weather duration for which the normal weather failure rate applies. Only severe storms occurring less frequently will result in storm associated failures. This condition is shown in Figure 4.10 in which the variation in configuration failure rate is shown for variable normal weather period durations. The overall annual component failure rate is shown for each of the calculated points. A similar situation is shown in Figure 4.11 in which the expected storm duration is varied.

The state space diagram shown in Figure 4.4 for the three unit system can be applied to the case of two parallel units in series with

a third unit. The system is shown in Figure 4.12. For this configuration the failure rates for lines 1 and 2 were held constant at 0.5 failures/year. The expected duration of normal weather, stormy weather and repair times for each component were held at 200 hours, 1.5 hours and 7.5 hours respectively. The failure rate for the series component was allowed to vary and the configuration failure rate calculated with the percentage of storm associated failures held at 50 percent for all components. The variation in configuration failure rate with respect to the failure rate of the series component is shown in Figure 4.12. The series component failure rate dominates the configuration value even for relatively small series component values. This situation would be modified somewhat if the parallel system was not assumed to be completely redundant.

The complete Markov approach can be applied without too much difficulty to relatively small systems. While it is extremely cumbersome to attempt to obtain general expressions even for very small systems, numerical solutions can be easily obtained using a digital computer. The number of system components that can be included depend only upon the capacity of the digital computer available. The Markov approach can be applied to segments of a system, such as parallel configurations that suffer the same environmental variations to obtain an exact solution. Certain system component failure rates are not greatly affected by the environment and therefore do not require the complete two state condition. These components will simplify the analysis considerably. In a high voltage system it is quite possible that a storm does not cover the entire system and therefore failure bunching for the entire system is unrealistic and only applies to local parallel facilities. In this case exact

solutions by Markov methods could be applied to the parallel configurations and these segments linked by independent probabilities.

The Markov approach does depend upon certain specific distributional assumptions. Other distributions resulting in non-stationary processes could be considered. They would, however, become extremely complicated even for simple system configurations. In this chapter it has been assumed that all parallel facilities are completely redundant. The reliability criterion in these cases is based upon continuity of supply. The state probabilities obtained from a Markov approach can easily be applied to systems that are not fully redundant by the use of a quality of service criterion. This criterion is discussed in detail in the next chapter of this thesis.

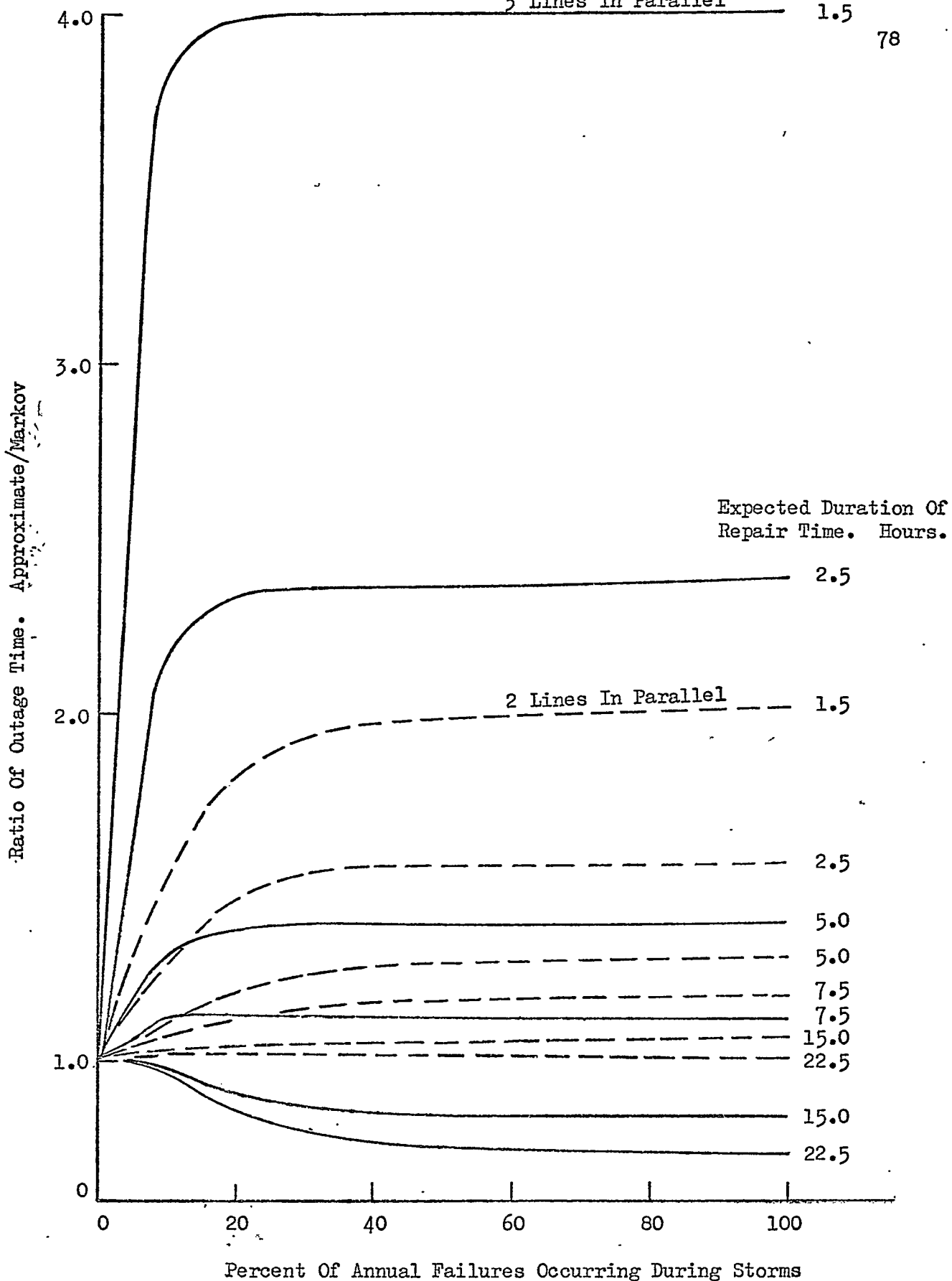


Figure 4.7 Comparison Of System Outage Time Using The Approximate And Markov Methods. Variable Repair Times.

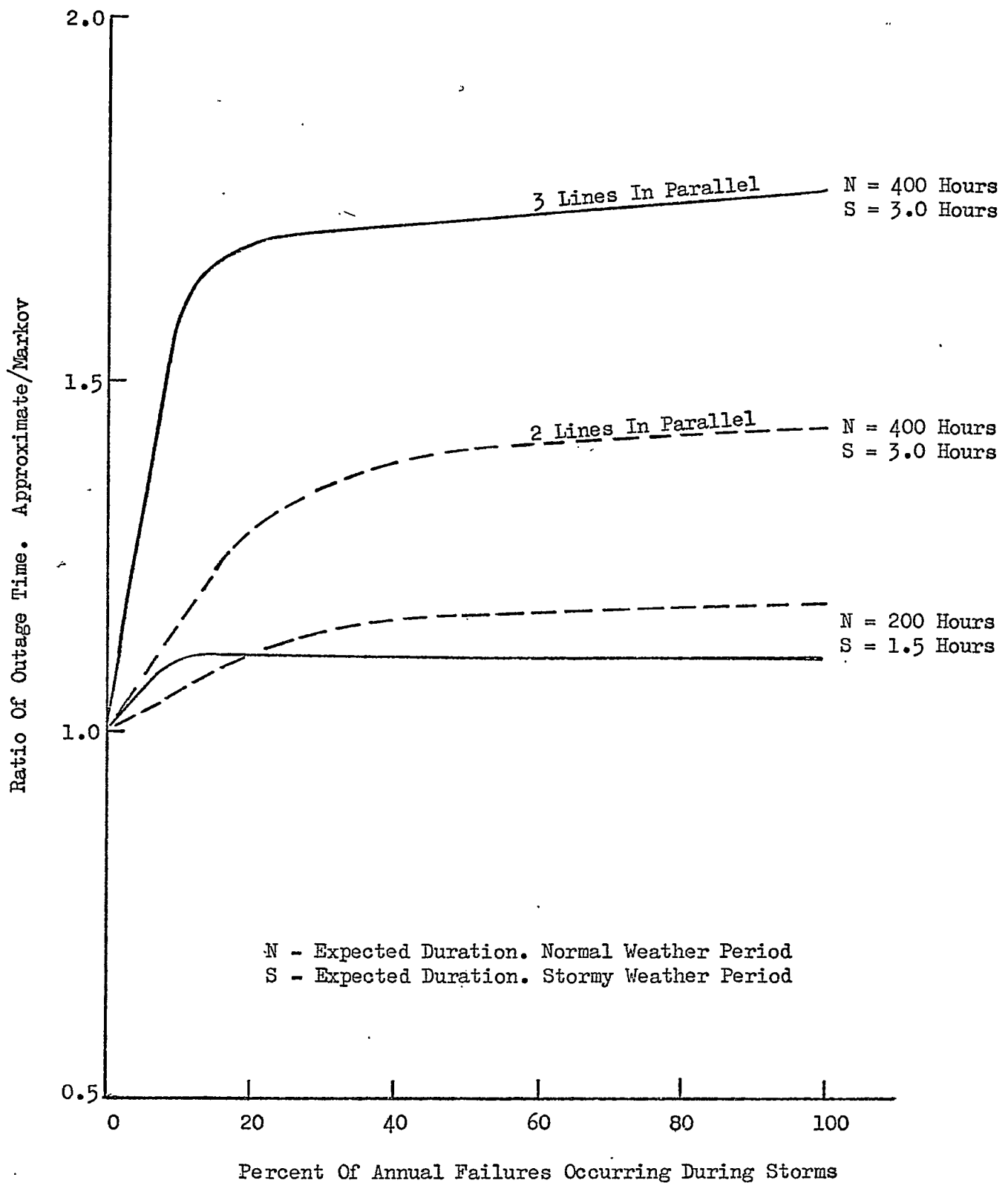


Figure 4.8 Comparison Of System Outage Time Using The Approximate And Markov Methods. Variable Weather Durations.

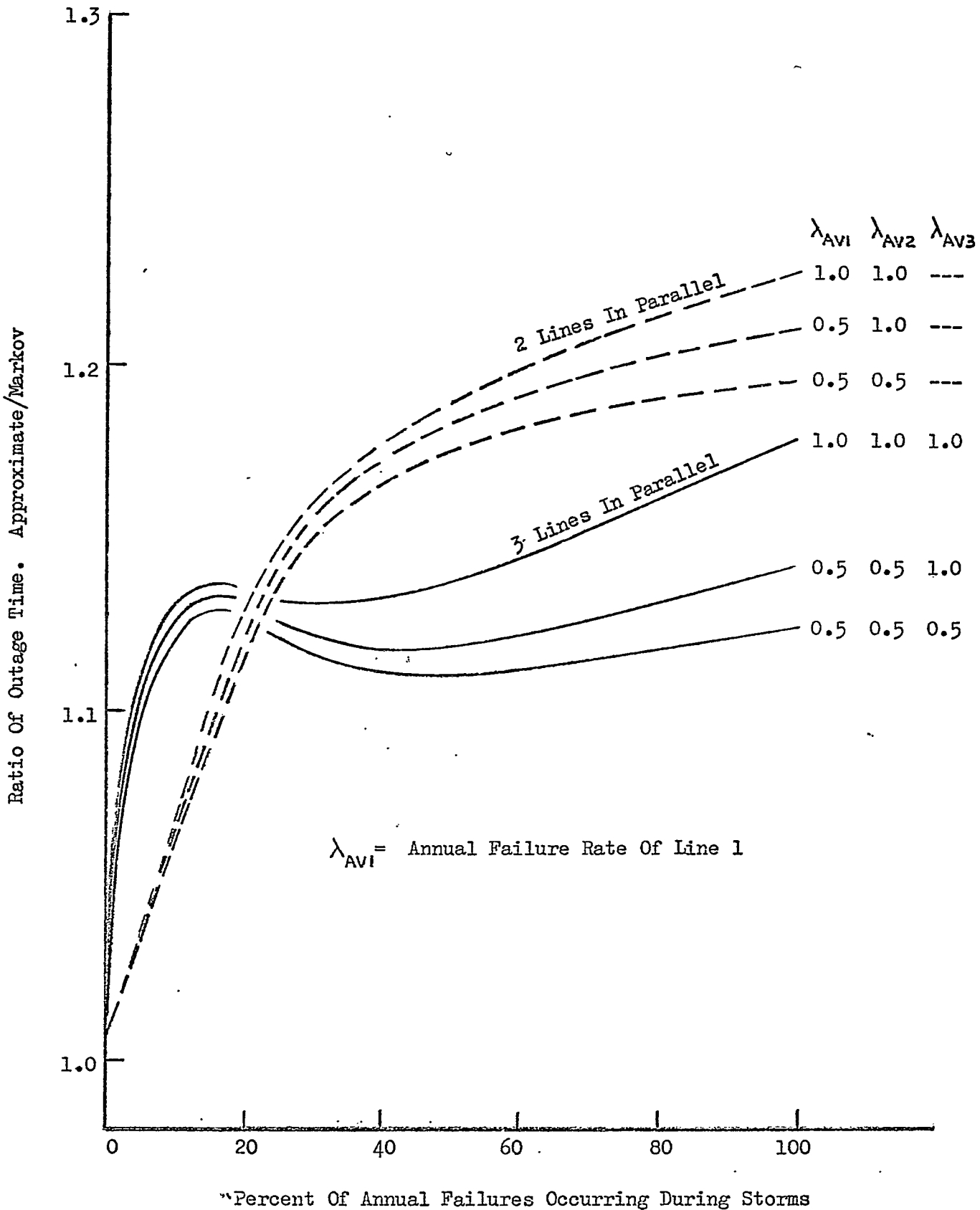


Figure 4.9 Comparison Of System Outage Time Using The Approximate And Markov Methods. Variable Component Failure Rates.

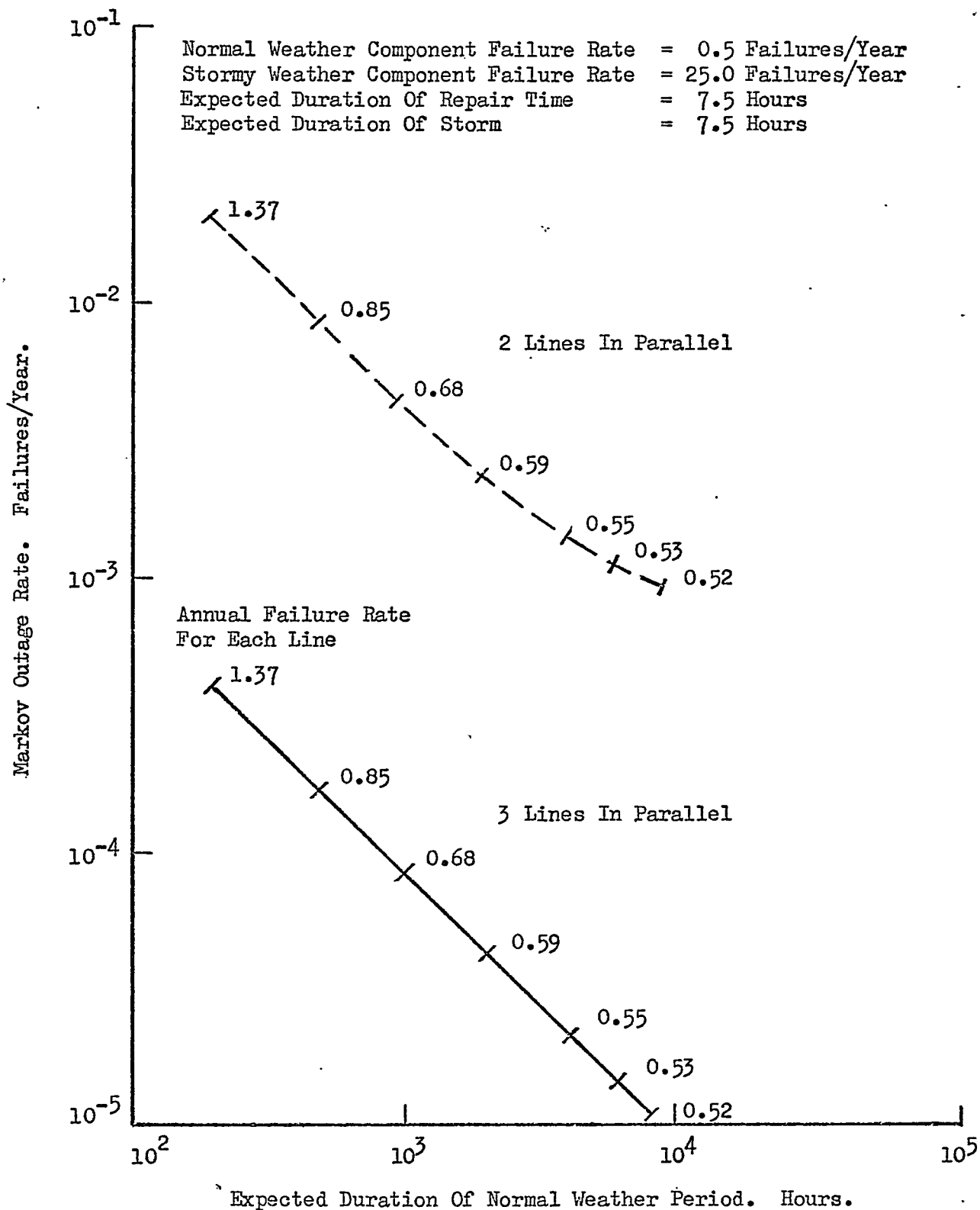


Figure 4.10 System Failure Rate For Variable Normal Weather Durations

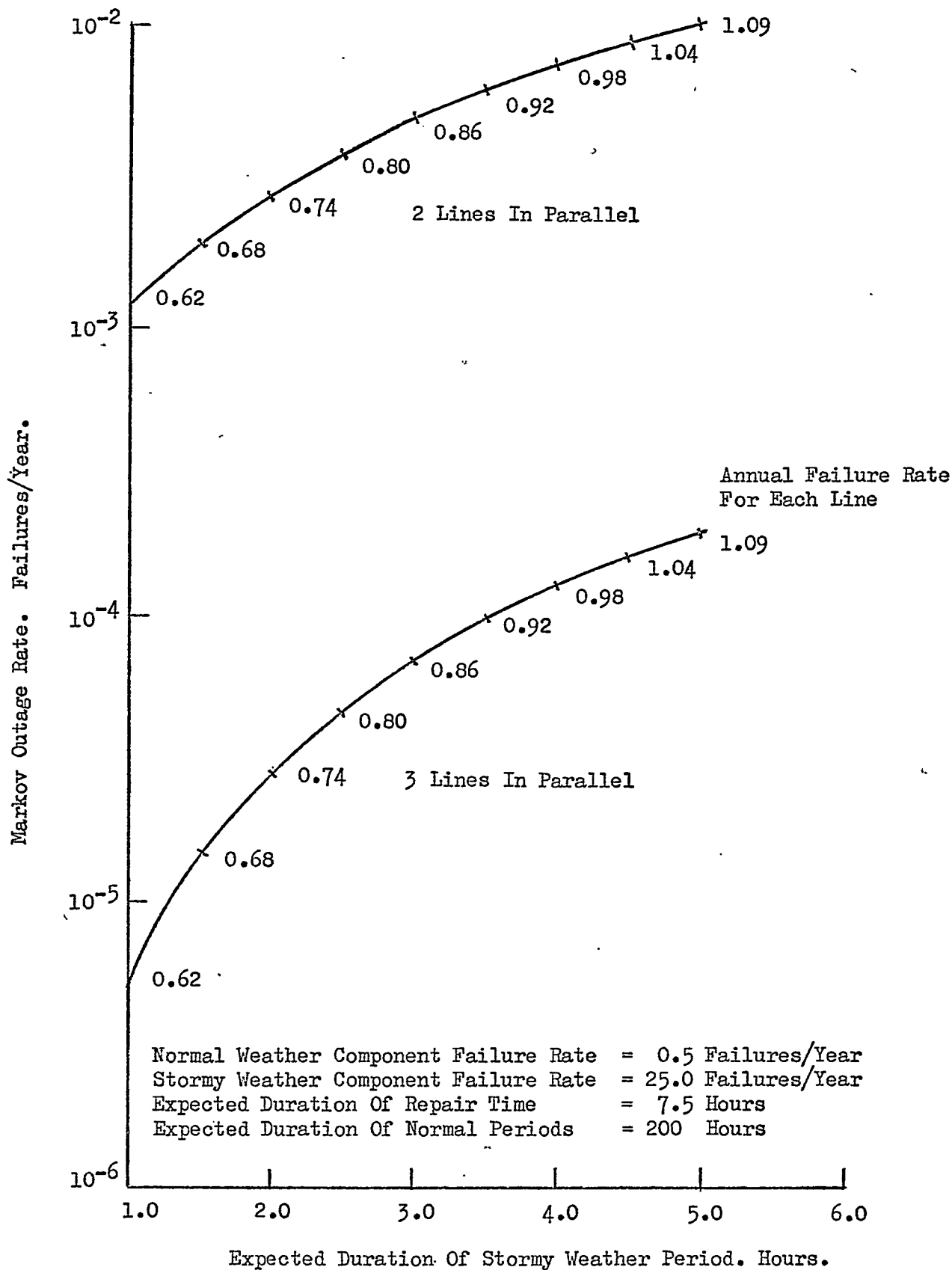


Figure 4.11 System Failure Rate For Variable Stormy Weather Durations

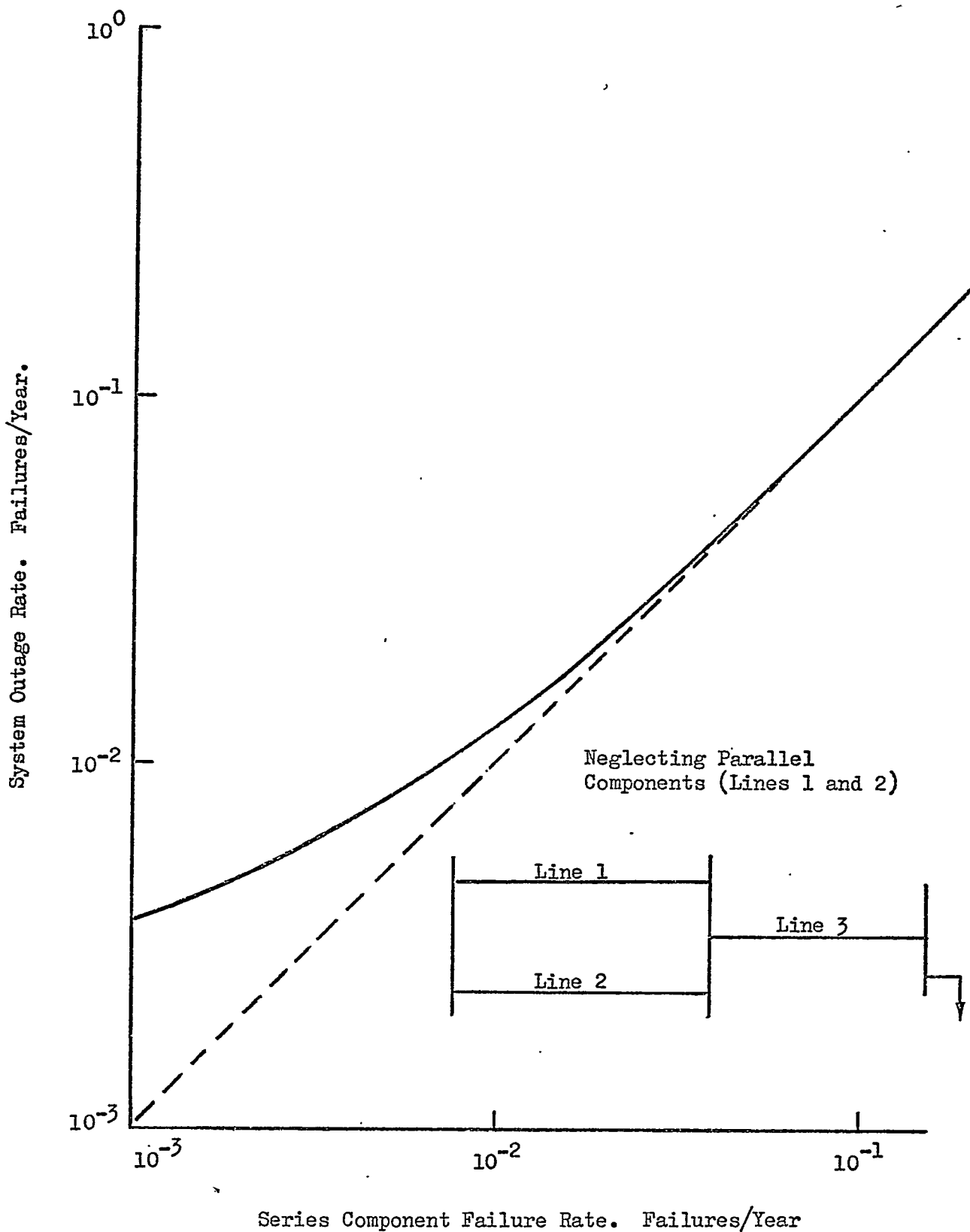


Figure 4.12 Effect Of The Series Component Failure Rate In A Series Parallel System

5. COMPOSITE SYSTEM RELIABILITY

5.1 Service Quality Criterion

It has been previously noted that compared with the publications available in the area of generating capacity reliability evaluation there are relatively few publications dealing with the reliability of transmission systems. There are virtually no publications dealing with the overall evaluation of generation and transmission reliabilities leading to a composite reliability index for every point in a system. If all components in a transmission system are fully redundant then reliability is only a matter of continuity with success and failure easily defined. In this regard it has been suggested⁽³²⁾ that a possible approach is to write a Boolean description of the network configuration, then reduce it to a success or failure conclusion with respect to a particular node. This approach is readily adaptable to simple series-parallel configurations but becomes complicated and virtually unworkable when applied to a networked system with several supply points.

Continuity is not an acceptable single criterion, as complete component redundancy is not economically feasible in a modern power system. The definition of a breach of continuity can be extended to include a breach of quality (i.e. if due to line outages a low voltage condition exists at a load point this is not an actual breach of continuity though the voltage level may be considerably lower than a desired minimum). If bounds are placed upon desirable voltage levels at each point in the system, any departures from these ranges can be classed as a breach of continuity. This would not include voltage transients

caused by system disturbances unless the voltage remained for a defined period of time in the unacceptable region. Customers can be served at reduced voltage levels but this should not be considered as a design criterion but as a last resort.

A clear definition of quality of service at each point must be made and any departure from these requirements classed as a breach of continuity of that service. The reason often given for the construction of an additional infeed to an area is increased transmission reliability. It is obvious that the addition of a line to supply a station will increase the reliability of that station, as any duplication of facilities will do this to some extent. The construction of a line entirely on reliability grounds implies extremely high outage rates for the remaining lines supplying the station. A general design criterion could be as follows:

- (a) If with all transmission and generation facilities in service, station voltage levels are outside the defined limits then new facilities are required to meet the quality of service standards.
- (b) If with the various possible combinations of system components out of service, the reliability index for the station is below an acceptable minimum, then additional facilities are required to meet the reliability standards.

A station could still meet quality of service standards for several years of load growth but not meet present reliability standards if a minimum number of system components with high outage rates were installed.

It can be clearly seen that the determination of a reliability index based upon service quality standards involves considerably more effort than the determination of a success or failure probability based only on continuity. It will, however, provide a valuable tool in assessing the adequacy of proposed system alternatives from a planning, design and operating viewpoint.

4.2 Conditional Probabilities Of System Failure

In almost all probability applications in reliability evaluation, component failures within a fixed environment are assumed to be independent events. It is entirely possible that component failure can result in system failure in a conditional sense. This can occur in parallel facilities that are not completely redundant. If the load can be considered as a random variable and described by a probability distribution then failure at any station due to component failure is conditional upon the load exceeding some value at which a satisfactory voltage level at the load point can be maintained.

If two events designated A and B are considered to be independent,

$$P(A \cap B) = P(A) \cdot P(B)$$

If the events are not independent and $P(B|A)$ denotes the conditional probability that B occurs given that A has occurred.

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Also

$$P(A \cap B) = P(B) \cdot P(A|B)$$

If the occurrence of A is dependent upon a number of events B_j which are mutually exclusive.

$$P(A) = \sum_{i=1}^j P(A|B_i) \cdot P(B_i)$$

If the occurrence of A is dependent upon only two mutually exclusive events for component B, success and failure, designated B_x and B_y respectively

$$P(A) = P(A|B_x) \cdot P(B_x) + P(A|B_y) \cdot P(B_y)$$

With respect to reliability this can be expressed in a simpler form. ⁽³⁷⁾

$$\begin{aligned} P(\text{System Failure}) &= P(\text{System Failure if B is good}) \cdot P(B_x) \\ &\quad + P(\text{System Failure if B is bad}) \cdot P(B_y) \end{aligned} \quad (5.1)$$

The complementary situation is similar in form

$$\begin{aligned} P(\text{System Success}) &= P(\text{System Success if B is good}) \cdot P(B_x) \\ &\quad + P(\text{System Success if B is bad}) \cdot P(B_y) \end{aligned}$$

A simple application to two redundant components A and B in parallel is as follows:

R_A, R_B - Respective probabilities of A and B being available

Q_A, Q_B - Respective probabilities of A and B being unavailable

$$\begin{aligned} P(\text{System Failed}) &= P(\text{Both Components Fail}) \\ &= P(\text{System fails if A is good}) \cdot R_A \\ &\quad + P(\text{System fails if A is bad}) \cdot Q_A \\ &= 0 \cdot R_A + Q_B \cdot Q_A \\ &= Q_A \cdot Q_B = \text{Probability that both components fail.} \end{aligned}$$

4.3 Simple System Application

Consider a simple system consisting of a generating station with two parallel transmission lines feeding a single load as shown in Figure 5.1.

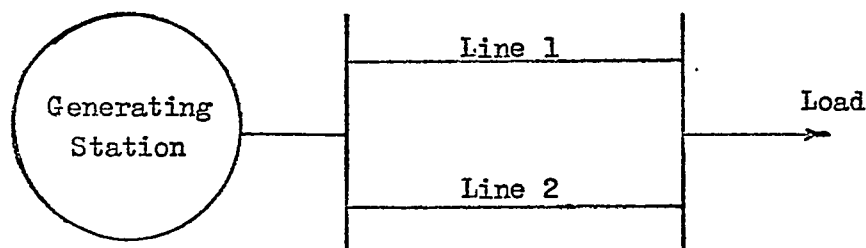


Figure 5.1 System Configuration

Define:

P_g = Probability of generation inadequacy as determined by the loss of load probability approach.

P_c = Probability of transmission inadequacy, i.e. $P_c(1)$ = Probability that the load will exceed the carrying capability of Line 1.

Q_s = Probability of system failure.

Starting with

$$Q_s = Q_s(L1 - IN) \cdot R_{L1} + Q_s(L1 - OUF) \cdot Q_{L1}$$

For L1 - IN

$$Q_s = Q_s(L2 - IN) \cdot R_{L2} + Q_s(L2 - OUF) \cdot Q_{L2}$$

For L1 - IN and L2 - IN

$$Q_s = P_g + P_c(1,2) - P_g \cdot P_c(1,2)$$

The probabilities of capacity deficiencies and transmission inadequacies are independent.

For L1 - IN and L2 - OUF

$$Q_s = P_g + P_c(1) - P_g \cdot P_c(1)$$

Therefore for L1 - IN

$$Q_s = R_{L2}(P_g + P_c(1,2) - P_g \cdot P_c(1,2)) + Q_{L2}(P_g + P_c(1) - P_g \cdot P_c(1))$$

For L1 - OUT

$$\begin{aligned} Q_s &= Q_s(L2 - IN)R_{L2} + Q_s(L2 - OUT)Q_{L2} \\ &= R_{L2}(P_g + P_c(2) - P_g \cdot P_c(2)) + Q_{L2} \end{aligned}$$

For the complete system,

$$\begin{aligned} Q_s &= R_{L1} \left[R_{L2}(P_g + P_c(1,2) - P_g \cdot P_c(1,2)) + Q_{L2}(P_g + P_c(1) - P_g \cdot P_c(1)) \right] \\ &\quad + Q_{L1} \left[R_{L2}(P_g + P_c(2) - P_g \cdot P_c(2)) + Q_{L2} \right] \end{aligned}$$

If the two lines are identical this reduces to

$$Q_s = R_L^2 \left[P_g + P_c(1,2) - P_g \cdot P_c(1,2) \right] + 2R_L Q_L \left[P_g + P_c(1) - P_g \cdot P_c(1) \right] + Q_L^2 \quad (5.2)$$

where

$$R_{L1} = R_{L2} = R_L$$

$$Q_{L1} = Q_{L2} = Q_L$$

The solution for this simple system could have been obtained directly from the Binomial Expansion of $(R_L + Q_L)^2$. Equation 5.2 expresses the reliability at the load in terms of the probabilities of adequacy of the generation and transmission facilities.

To illustrate the effect of various system parameters the following system has been analyzed. A hydraulic generating station contains 6 - 40 MW generating units. It is connected to a load 150 miles away by 2 - 230 KV transmission lines. Each line has a single 605,000 CM ACSR (26/7) conductor per phase. The line parameters are given in Table A-5 in the Appendix. The voltage at the receiving end of the line was

assumed to be fixed at 230 KV and any deviation from this, other than transient was considered to be a breach of quality. The sending end voltage of the 230 KV line was allowed to vary within limits of 5 percent, i.e. 218.5 - 241.5 KV.

The load carrying ability of the transmission system can be shown graphically using the conventional circle diagram. A modified graph is shown in Figure 5.2, obtained from a digital computer solution using the conventional long line equations. This graph shows the carrying capacity of a single line with the assigned voltage restrictions. In addition to the 230 KV receiving end voltage, 225 KV and 220 KV are also shown. Assuming a unity power factor load the line capabilities are shown in Table 5.1.

TABLE 5.1

Transmission System Load Carrying Capability	
<u>Receiving End Voltage KV</u>	<u>Maximum Load Carrying Capability MW</u>
230	120
225	133
220	146

For two identical lines the load carrying capability is double the single line values. The system load was represented by a straight line Load Duration Curve with a 75 percent load factor. Using this load distribution, the loss of load probabilities at various peak loads for a plant containing 6 - 40 MW hydraulic generating units and for a plant containing 7 - 40 MW units are shown in Table 5.2. In each case the unit forced outage rate was assumed to be 0.005.

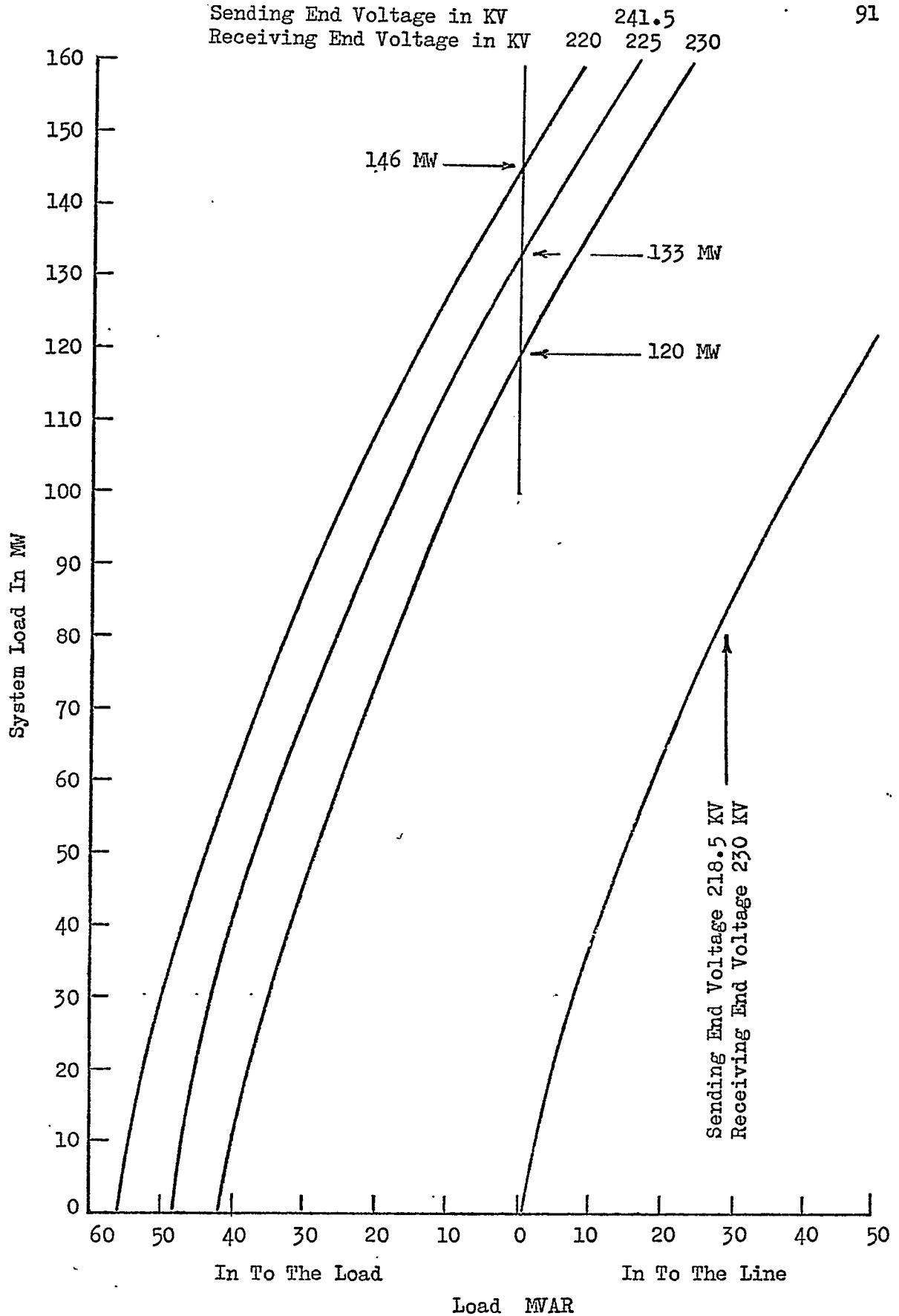


Figure 5.2 Transmission System Load Carrying Capability

TABLE 5.2

Generating System Loss Of Load Probabilities

240 MW Capacity 6 - 40 MW Units

<u>Peak Load MW</u>	<u>Loss Of Load Probability</u>
160	1×10^{-6}
170	44.6×10^{-6}
180	83.0×10^{-6}
190	117.7×10^{-6}
200	148.8×10^{-6}

280 MW Capacity 7 - 40 MW Units

<u>Peak Load MW</u>	<u>Loss Of Load Probability</u>
200	2×10^{-6}
210	50×10^{-6}
220	95.4×10^{-6}
230	136.5×10^{-6}
240	173.7×10^{-6}

Neglecting the system transmission losses and the effect of transmission reliability, the figures shown in Table 5.2 represent the conventional reliability assessment for the system. The probability of system failure was given in Equation 5.2

$$Q_s = R_L^2 [P_g + P_c(1,2) - P_g \cdot P_c(1,2)] + 2R_L Q_L [P_g + P_c(1) - P_g \cdot P_c(1)] + Q_L^2$$

where

R_L^2 = Probability of both lines being available

$2R_L Q_L$ = Probability of one line being available

Q_L^2 = Probability of no lines being available

Using the results obtained for two lines in parallel from a Markov analysis as shown in the previous chapter, the line outage existence probabilities can be obtained by summing the state probability values.

Assuming that for each line:

$$\lambda_{av} = \text{annual failure rate} = 0.5 \text{ failures/year}$$

$$N = \text{expected duration of normal weather periods} = 200 \text{ hours}$$

$$S = \text{expected duration of stormy weather periods} = 1.5 \text{ hours}$$

$$R = \text{expected repair time} = 7.5 \text{ hours}$$

The probabilities of occupying the three states are tabulated in Table A-2 in the Appendix. It is interesting to note that while the probability of having both lines out of service increases rapidly as the percentage of failures during storms increases, the probability of one line being available actually decreases slightly over this range. It must be remembered that the annual failure rate is constant and therefore while the stormy failure rate is increasing, the normal weather failure rate is decreasing. With regard to the probability of occupying a given state, the inclusion of the storm condition materially affects only the catastrophic condition of both lines being unavailable. The probability of system failure for various percentages of storm associated failures are tabulated in Table A-3 in the Appendix for 240 MW of capacity and peak loads of 200 MW to 120 MW in 10 MW increments. The effect of storm associated failures on the probability of system failure is virtually negligible in those cases in which the system is not fully redundant. In the 120 MW peak load case, the probability of system failure is the probability of losing both lines and is governed by the degree of storm associated failures.

The assumption was made that 50 percent of the transmission failures for this system occurred during stormy periods. The probability of system failure for 240 MW of capacity as a function of peak load is shown in Figure 5.3. As previously noted, system transmission losses have been neglected in evaluating the generating capacity failure probability component. This graph illustrates the change in system failure probability due to the ability to relax the receiving end conditions from 230 KV to 225 KV and 220 KV. Reliability considerations then become part of the economic evaluation of on-load tap changing facilities at the receiving end. These results were obtained assuming a unity power factor load. Similar results at any other power factor could be obtained by finding the correct power carrying capabilities of a single line using Figure 5.2.

If a generating capacity reserve margin of at least one unit is maintained, then neglecting transmission losses, the maximum peak load is 200 MW in the 240 MW capacity case. Figure 5.4 shows the variation in the probability of system failure with peak load for the 240 MW and 280 MW installed capacity conditions. It can be clearly seen that for the failure values chosen, the probability of system failure is clearly dominated by the probability of transmission inadequacy.

The load carrying capability of a line with fixed receiving and sending end voltages can be increased by the addition of shunt capacitive compensation at the receiving end. Depending of course on the system, this can have a considerable effect on the probability of system failure. This condition is shown in Figure 5.5 where a maximum of 50 MVAR of capacitance is available at the receiving end. The capacitor installation

was assumed to be 100 percent reliable. The load was considered to be at unity power factor. The loading limitations were obtained from Figure 5.2. As in the case of on-load tap changing transformers, reliability assessment can be an integral part of the economic evaluation of shunt compensation requirements.

The results obtained from a Markov solution for three parallel facilities were used to assess the reliability of three 230 KV lines. For a peak load of 240 MW there must be at least two lines out of service for there to be any possible transmission curtailment. The probabilities of occupying the various states as a function of the percentage of storm associated failures are shown in Table A-4 in the Appendix. In this case the probability of system failure is governed almost entirely by the probability of generating capacity inadequacy.

The system considered is an extremely simple one. It illustrates, however, that reliability evaluation can be an integral part of system planning, design and operation. The probability of system failure is the probability that the generation and transmission facilities will not satisfy the required load condition. No time period has been considered and as previously illustrated this could be any consistent period for which the load probability distribution is applicable. The generating capacity failure probability was calculated on an installed capacity basis using the loss of load approach. A similar study could have been performed to evaluate spinning requirements though it would be trivial in this simplified case.

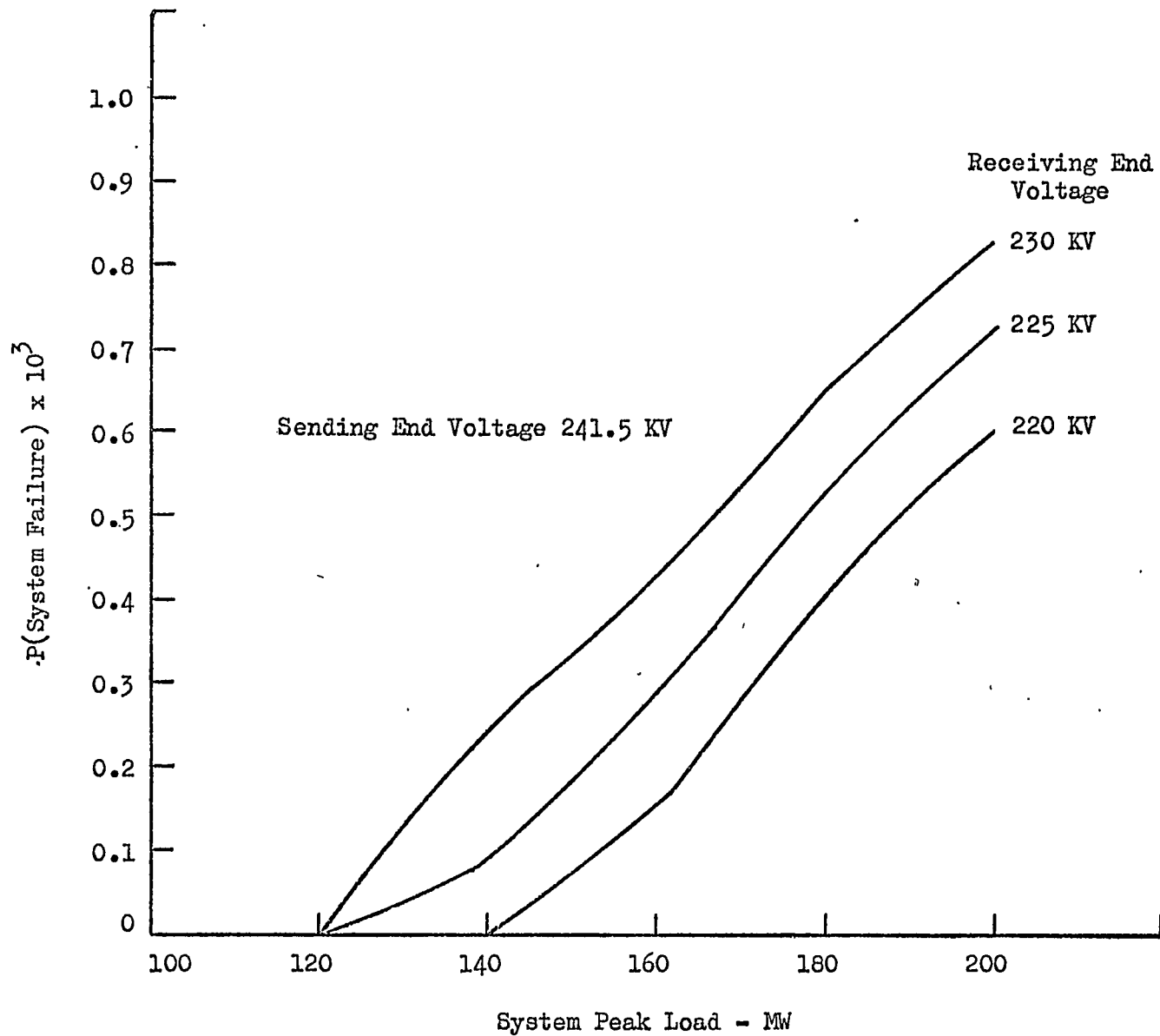


Figure 5.3 Probability Of System Failure As A Function Of System Peak Load, For Various Receiving End Voltages.

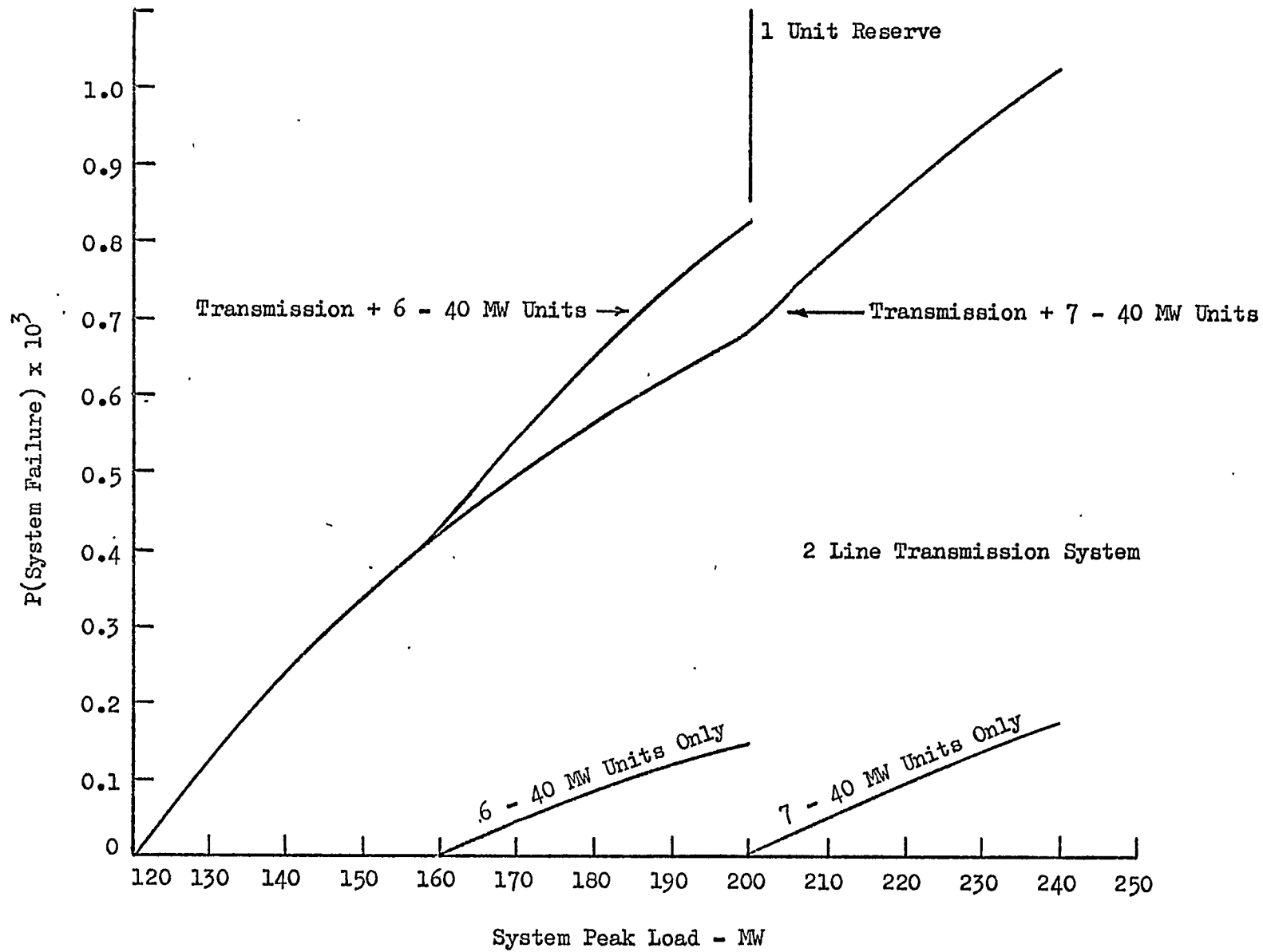


Figure 5.4 Probability Of System Failure As A Function Of System Peak Load

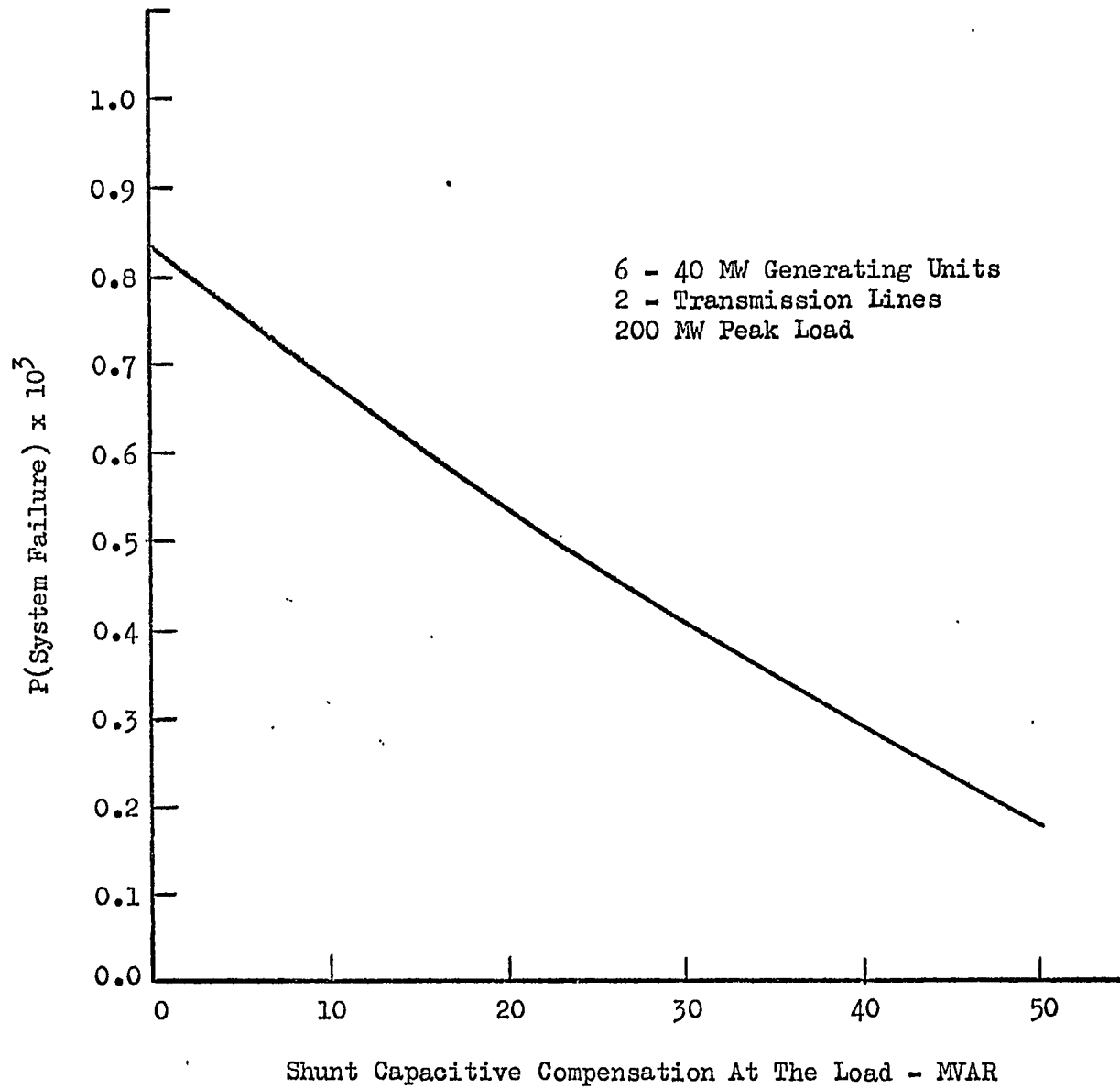


Figure 5.5 Probability Of System Failure As A Function Of Shunt Compensation At The Load Point

5.4 Two Plant Single Load System

The same basic approach illustrated in the simple series system case can be applied to any system. Consider the system shown in Figure 5.6.

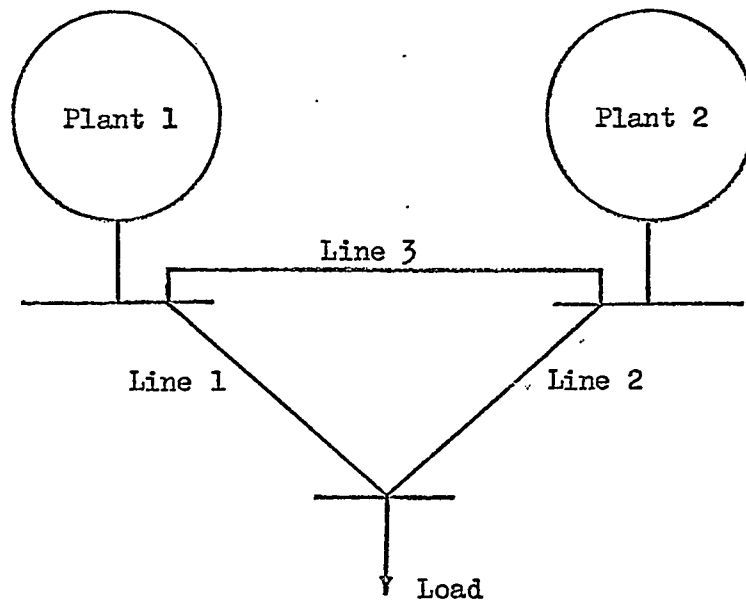


Figure 5.6 Two Plant Single Load System Configuration

Starting with

$$Q_s = Q_s(L1 - IN)R_{L1} + Q_s(L1 - OUT)Q_{L1}$$

For L1 - IN

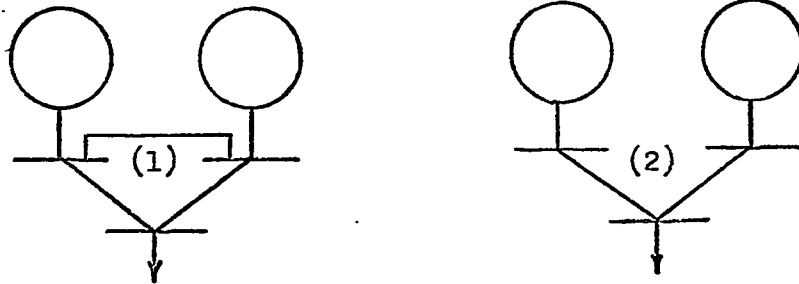
$$Q_s = Q_s(L2 - IN)R_{L2} + Q_s(L2 - OUT)Q_{L2}$$

For L1 - IN and L2 - IN

$$\begin{aligned} Q_s &= Q_s(L3 - IN)R_{L3} + Q_s(L3 - OUT)Q_{L3} \\ &= (P_g(1,2) + P_c(1) - P_g(1,2) \cdot P_c(1))R_{L3} \\ &\quad + (P_g(1,2) + P_c(2) - P_g(1,2) \cdot P_c(2))Q_{L3} \end{aligned}$$

$P_g(1,2)$ is the probability of load curtailment for both generating plants.

$P_c(1)$ and $P_c(2)$ are the curtailment probabilities for the transmission configurations shown below



In many practical cases $P_c(1) = P_c(2) = 0$ and therefore $Q_s = P_g(1,2)$.

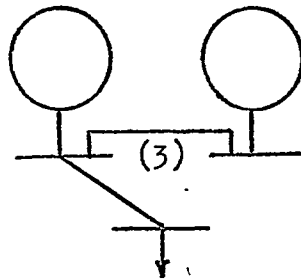
For L1 - IN and L2 - OUT

$$Q_s = Q_s(L3 - IN)R_{L3} + Q_s(L3 - OUT)Q_{L3}$$

For L1 - IN and L2 OUT and L3 - IN

$$Q_s = P_g(1,2) + P_c(3) - P_g(1,2) \cdot P_c(3)$$

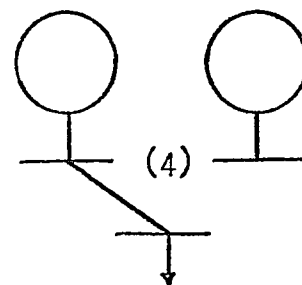
$P_c(3)$ is the curtailment probability for the system shown below



For L1 - IN and L2 - OUT and L3 - OUT

$$Q_s = P_g(1) + P_c(4) - P_g(1) \cdot P_c(4)$$

$P_c(4)$ is obtained from



$P_g(1)$ represents the curtailment probability due to plant 1 supplying the entire system load.

For L1 - IN

$$Q_s = R_{L2} \left[(P_g(1,2) + P(1) - P_g(1,2) \cdot P(1)) R_{L3} + (P_g(1,2) + P_c(2) - P_g(1,2) \cdot P_c(2)) Q_{L3} \right] \\ + Q_{L2} \left[R_{L3} (P_g(1,2) + P_c(3) - P_g(1,2) \cdot P_c(3)) + Q_{L3} (P_g(1) + P_c(4) - P_g(1) P_c(4)) \right]$$

For L1 - OUT

$$Q_s = Q_s(L2 - IN) R_{L2} + Q_s(L2 - OUT) Q_{L2}$$

For L1 - OUT and L2 - IN

$$Q_s = Q_s(L3 - IN) R_{L3} + Q_s(L3 - OUT) Q_{L3}$$

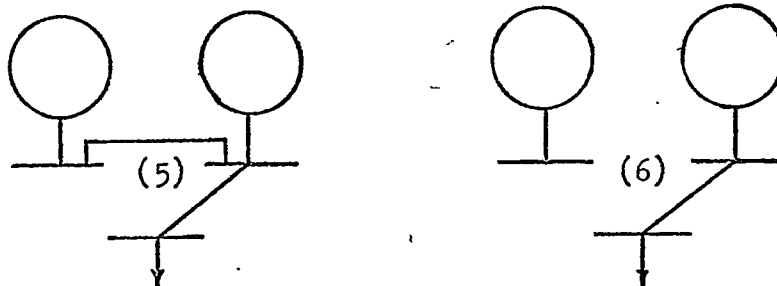
For L1 - OUT and L2 - IN and L3 - IN

$$Q_s = P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5)$$

For L1 - OUT and L2 - IN and L3 - OUT

$$Q_s = P_g(2) + P_c(6) - P_g(2) \cdot P_c(6)$$

Configurations 5 and 6 are shown below



For L1 - OUT and L2 - IN

$$Q_s = R_{L3} \left[P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5) \right] + Q_{L3} \left[P_g(2) + P_c(6) - P_g(2) \cdot P_c(6) \right]$$

For L1 - OUT and L2 - OUT

$$Q_s = 1.0$$

For L1 - OUT

$$Q_s = R_{L2} \left[R_{L3} (P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5)) \right. \\ \left. + Q_{L3} (P_g(2) + P_c(6) - P_g(2) \cdot P_c(6)) \right] + Q_{L2}$$

The complete expression for the system becomes

$$Q_s = R_{L1} \left[R_{L2} \left[\left[P_g(1,2) + P_c(1) - P_g(1,2) \cdot P_c(1) \right] R_{L3} \right. \right. \\ \left. \left. + \left[P_g(1,2) + P_c(2) - P_g(1,2) \cdot P_c(2) \right] Q_{L3} \right] \right. \\ \left. + Q_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(3) - P_g(1,2) \cdot P_c(3) \right] \right. \right. \\ \left. \left. + Q_{L3} \left[P_g(1) + P_c(4) - P_g(1) \cdot P_c(4) \right] \right] \right] \\ + Q_{L1} \left[R_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5) \right] \right. \right. \\ \left. \left. + Q_{L3} \left[P_g(2) + P_c(6) - P_g(2) \cdot P_c(6) \right] \right] + Q_{L2} \right]$$

In a practical system many of the transmission curtailment probabilities would be unity, thus eliminating many terms. The system chosen is still an extremely simple one compared to a practical power system. The form of analysis is, however, logical and sequential and therefore amenable to digital computer application. The probabilities of transmission curtailment are extremely important as was illustrated in the simple series system analysis. These probabilities can be obtained by an automated load flow program on a digital computer or by load flow studies on an a.c. network analyzer.

It should be realized at this time that the quality of service may not be a matter of voltage only and that tripping due to overload or to steady state instability can be incorporated into the probabilities of transmission curtailment as the different configurations are analyzed.

In an installed capacity investigation the entire system capacity is used to evaluate the generation component of system failure probability. In a spinning capacity study, the capacity in operation is dependent upon the load level and the failure probabilities are dependent upon the time required to put additional capacity into service. It was previously noted in Chapter 3 that if the transmission is assumed to be completely reliable, it is immaterial where the spinning reserve is located within the system. This is not true, however, if the transmission system is not fully reliable. A transmission adequacy analysis in this case may involve the utilization of several generation schedules for each load condition. In many cases the best schedule is reasonably obvious due to economic constraints, however, it may be offset by reliability benefits.

5.5 Two Plant Two Load System

The simple two plant, two load system shown in Figure 5.7 was analysed.

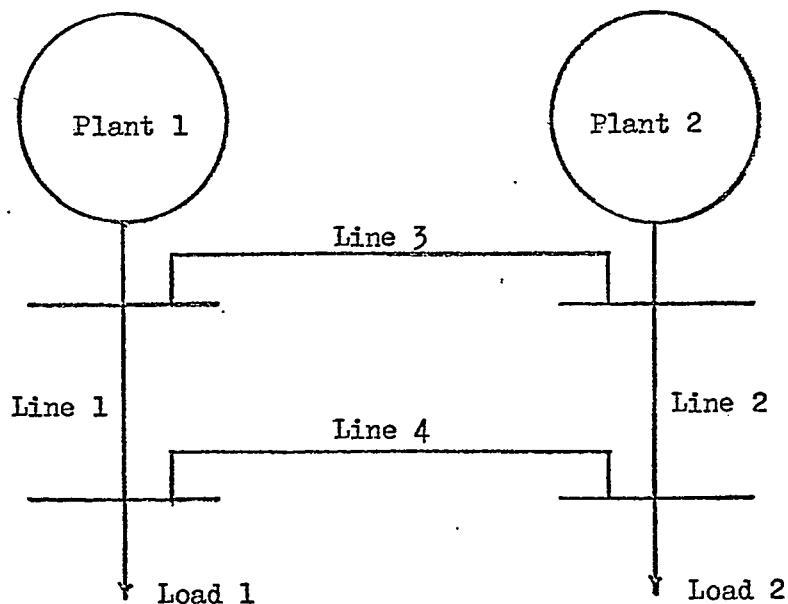


Figure 5.7 Two Plant Two Load System Configuration

The manipulations required to arrive at an expression for the probability of system failure for Load 1 are shown in Appendix B. The complete expression is as follows

$$\begin{aligned}
 Q_S = & R_{L1} \left[R_{L4} \left[R_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(ALL) - P_g(1,2) \cdot P_c(ALL) \right] \right. \right. \right. \\
 & \quad \left. \left. + Q_{L3} \left[P_g(1,2) + P_c(1) - P_g(1,2) \cdot P_c(1) \right] \right] \right. \\
 & \quad \left. + Q_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(2) - P_g(1,2) \cdot P_c(2) \right] \right. \right. \\
 & \quad \quad \left. \left. + Q_{L3} \left[P_g(1) + P_c(3) - P_g(1) \cdot P_c(3) \right] \right] \right] \\
 & \quad + Q_{L4} \left[R_{L3} \left[R_{L2} \left[P_g(1,2) + P_c(4) - P_g(1,2) \cdot P_c(4) \right] \right. \right. \\
 & \quad \quad \left. \left. + Q_{L2} \left[P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5) \right] \right] \right. \\
 & \quad \quad \left. + Q_{L3} \left[P_g(1) + P_c(6) - P_g(1) \cdot P_c(6) \right] \right] \\
 & \quad + Q_{L1} \left[R_{L4} \left[R_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(7) - P_g(1,2) \cdot P_c(7) \right] \right. \right. \right. \\
 & \quad \quad \left. \left. + Q_{L3} \left[P_g(2) + P_c(8) - P_g(2) \cdot P_c(8) \right] \right] + Q_{L2} \right] + Q_{L4} \right]
 \end{aligned}$$

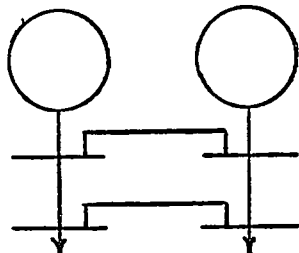
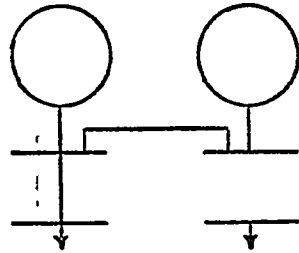
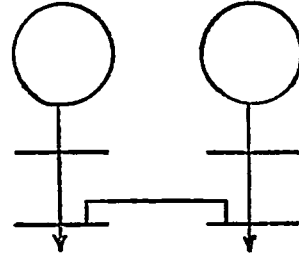
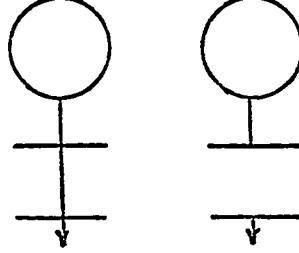
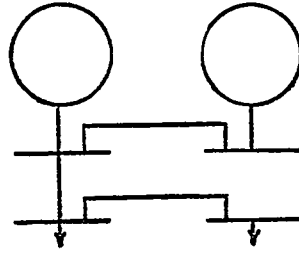
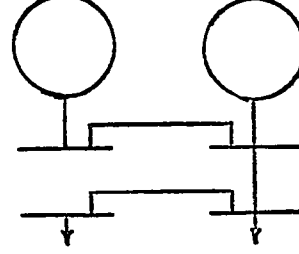
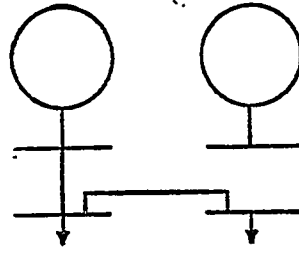
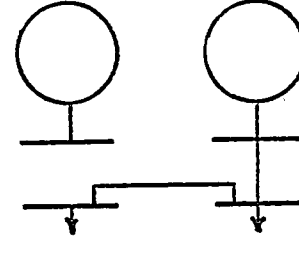
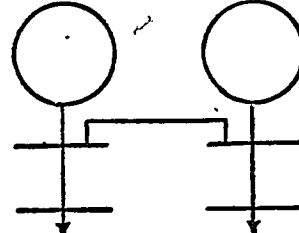
A similar expression can be obtained for Load 2. The transmission curtailment configurations are shown in Table 5.3. An actual system with this configuration would not normally be symmetrical and therefore the probability of system failure at the two load points would not be identical. The minimum reliability acceptable to the system is a management decision.

Operating considerations can be easily included in the system adequacy analysis. In certain cases loads may be tripped off the system rather than allow other load points to experience undesirable voltage levels. These conditions can all be incorporated into the analysis of any particular configuration. It should be realized that the addition of shunt compensation or on-load tap changing transformers in general

affects the reliability of the entire networked system rather than just the point of application. The effect of course may be negligible at points removed from the immediate locale.

TABLE 5.3

Transmission Curtailment Configurations

<u>Designation</u>	<u>Configuration</u>	<u>Designation</u>	<u>Configuration</u>
All		(5)	
(1)		(6)	
(2)		(7)	
(3)		(8)	
(4)			

Maintenance considerations can easily be included in the analysis by placing the probability of component availability to zero and the probability of component unavailability to unity.

As previously noted in the analysis of the simple series system configuration, the effect of storm associated failures on the system failure probabilities depends almost entirely on the degree of redundancy. This effect will be further diminished in the evaluation of major transmission reliability over relatively long distances as storms become more local in nature.

Using the techniques described in this chapter, it should be possible to arrive at a measure of steady state adequacy for any bus in the system, particularly those points at which major transmission terminates and subtransmission begins. For an actual system involving many components it may not be necessary to consider all the possible cases. The digital computer program can be instructed to consider up to a fixed number of components out of service at any one time. The maximum number will depend upon the component probabilities.

6. CONCLUSIONS

The investigations resulting in this thesis have clearly illustrated that quantitative reliability assessment can be applied to the planning and operation of a modern power system. The design level of load point reliability is a management decision and is dependent to a large extent on the cost of attaining and maintaining that level. Engineering personnel can provide management with considerable assistance in making this decision by developing incremental reliability costs for the system load points. Consistent quantitative evaluation of load point reliability is not a purely mathematical problem. It requires a detailed knowledge of the power system and of the inherent system failure processes.

The adequacy of system generating capacity requirements are normally assessed using a single contingency or percent reserve criterion. These methods do not reflect basic changes in system and component design parameters. The criterion of constant risk involving all the available parameters provides a basic reference for the economic evaluation of system changes. The effect on capacity requirements of individual component reliabilities in series-parallel systems such as multiple generating unit-single transformer configurations can be readily evaluated. The maximum penalty to the system due to such a configuration can be quite large under certain conditions, however, it may occur at a load point or reliability level higher or lower than the system criterion. The penalty magnitude will increase with further system additions using the particular configuration but may disappear almost entirely in other cases. The criterion of constant risk can also be applied to the daily scheduling

of generation. Given the acceptable level of operating capacity reliability, short term economic generation scheduling is relatively straightforward.

Within the bounds of the necessary distributional assumptions, a theoretically accurate solution for transmission system reliabilities can be obtained using Markov processes. The results obtained for simple system configurations and compared with those obtained using a previously published method⁽²²⁾ indicate that this latter approach responds quite differently to changes in system parameters and can be quite incorrect in certain cases. It is extremely difficult to obtain a general expression for the reliability of relatively simple systems using Markov processes, especially if a two state fluctuating failure environment is considered. It is not at all difficult, however, to obtain a limiting state solution using a digital computer for specific system component parameters. The limit to the size of the system that can be handled depends upon the digital equipment available. This approach could be easily applied to give a theoretically accurate solution for the important case of a major high voltage transmission system used to link a collection of remote generating stations to a system. The additional complexity of the two state fluctuating environment can be completely neglected in the evaluation of the state existence probabilities if the system is not fully redundant. Conditional system failure probabilities can greatly exceed the catastrophic failure probabilities produced by considering storm associated failures.

An additional degree of consistency can be incorporated into system reliability studies by applying confidence levels to the basic

component failure statistics. This is particularly important in those cases in which a minimum of component data is available. The degree of confidence required in the basic statistic becomes a part of the system reliability criterion. It should be realized that reliability evaluation in this case is not an attempt to predict the average performance of a changing system over a relatively long period of time but a means of obtaining a consistent quantitative assessment of alternate situations.

The most important single aspect of this study is the realization that composite system reliability indices can be obtained for any load point within the system. | These indices can include in a conditional sense all the inherent system processes that lead to a load point failure. | A detailed knowledge of the behaviour of the system is required, especially if remedial measures such as load shedding and backfeeding on low voltage circuits are considered. It must be realized that economics and reliability are not independent factors. Reliability implications of such factors as load voltage tolerances, tap changing transformers and shunt compensation facilities have a considerable effect on their economic justification. Automatic load flow studies using high speed digital computers can be used to evaluate system configuration inadequacies to be used in conjunction with the existence probabilities for these configurations.

As previously noted, generating capacity adequacy has often been evaluated on a single contingency worst case basis. This approach is also often used in the study of system transient instability. In general, instability can be said to be dependent upon the system load level, the available generation, the system transmission configuration and the

disturbance to the system. A sequential form of analysis, involving conditional probabilities similar to that used in establishing a value of steady state adequacy will give a far more meaningful assessment of the system design relative to instability than that of a worst case analysis. The probability of system instability coupled with the probability of steady state adequacy are two basic system parameters that would be extremely useful in attempting to obtain a consistent evaluation of alternate facilities. The problem of composite system reliability in both a transient instability and a steady state adequacy sense is an interesting and challenging one. It is expected that this is an area in which considerable work will be done in the near future.

The single major obstacle to the use of probability methods in power system reliability evaluation is the lack of consistent and meaningful statistics for the individual system components. This obstacle is rapidly disappearing as committees in Canada and the United States are actively working on this problem.

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Conference Paper



BIBLIOGRAPHY ON APPLICATION OF PROBABILITY METHODS IN THE EVALUATION OF GENERATING CAPACITY REQUIREMENTS

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TABLE A-1

COMBINED SOUTHERN MANITOBA SYSTEM HYDRAULIC GENERATING UNIT OPERATING DATA FOR THE SIX YEAR PERIOD

APRIL 1, 1959 TO MARCH 31, 1965

PLANT	UNIT NO.	UNIT CAPACITY MW	FAILURES	OPERATING HOURS	FAILURE RATES AT VARIOUS CONFIDENCE LEVELS x 1000						
					POINT ESTIMATE	50%	70%	80%	90%	95%	99%
POINTE DU BOIS	1	3.2	5	49,174	0.1017	0.1153	0.1425	0.1608	0.1886	0.2135	0.2666
	2	3.2	7	48,809	0.1434	0.1571	0.1887	0.2097	0.2411	0.2694	0.3278
	3	3.2	7	49,323	0.1419	0.1555	0.1867	0.2075	0.2386	0.2666	0.3244
	4	3.2	3	48,915	0.0613	0.0750	0.0973	0.1127	0.1366	0.1584	0.2055
	5	3.2	5	45,688	0.1094	0.1241	0.1533	0.1730	0.2030	0.2298	0.2869
	6	4.0	6	45,768	0.1311	0.1457	0.1772	0.1983	0.2301	0.2589	0.3183
	7	4.0	3	49,208	0.0610	0.0746	0.0967	0.1121	0.1358	0.1575	0.2042
	8	4.0	7	46,250	0.1514	0.1658	0.1991	0.2213	0.2545	0.2843	0.3459
	9	5.5	6	47,230	0.1270	0.1412	0.1717	0.1921	0.2230	0.2509	0.3085
	10	5.5	6	47,497	0.1263	0.1404	0.1707	0.1911	0.2217	0.2495	0.3068
	11	5.5	8	46,945	0.1704	0.1847	0.2194	0.2424	0.2768	0.3078	0.3708
	12	5.5	12	47,176	0.2544	0.2686	0.3100	0.3370	0.3769	0.4123	0.4837
	13	5.5	7	47,085	0.1487	0.1629	0.1956	0.2174	0.2500	0.2793	0.3398
	14	5.5	9	47,916	0.1878	0.2022	0.2377	0.2613	0.2965	0.3277	0.3920
	15	5.5	1	47,498	0.0210	0.0354	0.0514	0.0631	0.0819	0.0999	0.1398
	16	5.5	6	48,031	0.1249	0.1389	0.1688	0.1889	0.2192	0.2467	0.3033
SLAVE FALLS	1	8.5	3	45,251	0.0663	0.0811	0.1052	0.1219	0.1476	0.1713	0.2221
	2	8.5	9	39,043	0.2305	0.2482	0.2917	0.3207	0.3638	0.4021	0.4811
	3	8.5	3	45,988	0.0652	0.0798	0.1035	0.1199	0.1453	0.1685	0.2185
	4	8.5	1	44,756	0.0223	0.0375	0.0545	0.0669	0.0869	0.1060	0.1484
	5	8.5	1	45,971	0.0217	0.0365	0.0531	0.0651	0.0846	0.1032	0.1444
	6	8.5	1	44,291	0.0226	0.0379	0.0551	0.0676	0.0878	0.1071	0.1499
	7	8.5	2	45,895	0.0436	0.0583	0.0788	0.0933	0.1160	0.1373	0.1831
	8	8.5	6	43,852	0.1368	0.1521	0.1849	0.2069	0.2401	0.2702	0.3323

TABLE A-1 (continued)

	UNIT NO.	UNIT CAPACITY MW	FAILURES	OPERATING HOURS	FAILURE RATES AT VARIOUS CONFIDENCE LEVELS x 1000						
					POINT ESTIMATE	50%	70%	80%	90%	95%	99%
SEVEN SISTERS	1	25.0	5	43,048	0.1161	0.1317	0.1627	0.1836	0.2155	0.2439	0.3045
	2	25.9	5	41,312	0.1210	0.1372	0.1696	0.1913	0.2245	0.2542	0.3173
	3	25.0	3	43,787	0.0685	0.0838	0.1087	0.1260	0.1526	0.1770	0.2295
	4	25.0	10	49,036	0.2039	0.2176	0.2543	0.2784	0.3142	0.3457	0.4108
	5	25.0	8	48,827	0.1638	0.1776	0.2109	0.2331	0.2661	0.2959	0.3565
	6	25.0	7	43,426	0.1612	0.1766	0.2121	0.2357	0.2710	0.3028	0.3684
McARTHUR FALLS	1	7.0	16	33,844	0.4728	0.4949	0.5604	0.6020	0.6619	0.7138	0.8162
	2	7.0	10	37,234	0.2686	0.2866	0.3349	0.3666	0.4137	0.4552	0.5410
	3	7.0	17	41,687	0.4078	0.4258	0.4805	0.5155	0.5651	0.6083	0.6933
	4	7.0	12	44,268	0.2711	0.2862	0.3304	0.3592	0.4016	0.4394	0.5155
	5	7.0	8	42,351	0.1889	0.2047	0.2342	0.2687	0.3068	0.3412	0.4110
	6	7.0	6	40,772	0.1472	0.1636	0.1989	0.2226	0.2583	0.2906	0.3574
	7	7.0	5	40,068	0.1248	0.1415	0.1748	0.1973	0.2315	0.2621	0.3272
	8	7.0	6	40,219	0.1492	0.1658	0.2016	0.2256	0.2618	0.2946	0.3623
GREAT FALLS	1	22.0	15	42,081	0.3565	0.3743	0.4254	0.4579	0.5048	0.5455	0.6258
	2	22.0	3	42,558	0.0705	0.0862	0.1118	0.1296	0.1570	0.1821	0.2361
	3	22.0	3	36,329	0.0826	0.1010	0.1310	0.1518	0.1839	0.2133	0.2766
	4	22.0	9	39,535	0.2276	0.2451	0.2881	0.3167	0.3593	0.3971	0.4751
	5	22.0	6	46,758	0.1283	0.1426	0.1734	0.1941	0.2252	0.2534	0.3116
	6	22.0	8	39,309	0.2035	0.2206	0.2620	0.2895	0.3306	0.3676	0.4428
PINE FALLS	1	13.5	6	36,377	0.1649	0.1834	0.2229	0.2495	0.2895	0.3258	0.4005
	2	13.5	7	40,775	0.1717	0.1881	0.2259	0.2510	0.2887	0.3225	0.3924
	3	13.5	6	50,573	0.1186	0.1319	0.1604	0.1794	0.2082	0.2343	0.2881
	4	13.5	6	50,399	0.1190	0.1323	0.1609	0.1801	0.2089	0.2351	0.2891
	5	13.5	8	39,883	0.2006	0.2174	0.2583	0.2853	0.3258	0.3623	0.4364
	6	13.5	6	37,822	0.1586	0.1764	0.2144	0.2399	0.2784	0.3133	0.3852
GRAND RAPIDS	1	110.0	6	944	6.356	7.066	8.591	9.613	11.15	12.55	15.43
	2	110.0	4	2,184	1.832	2.138	2.697	3.077	3.661	4.190	5.314
	3	110.0	4	2,146	1.863	2.175	2.743	3.130	3.724	4.262	5.405

TABLE A-2

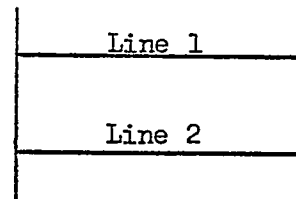
Probabilities Of Occupying The Various States For Two Lines In Parallel

$$\lambda_1 = \lambda_2 = 0.5 \text{ failures/year}$$

$$N = 200 \text{ hours}$$

$$S = 1.5 \text{ hours}$$

$$R = 7.5 \text{ hours}$$



$P_c(1,2)$ = Probability of both lines being available.
Markov States 0 and 4.

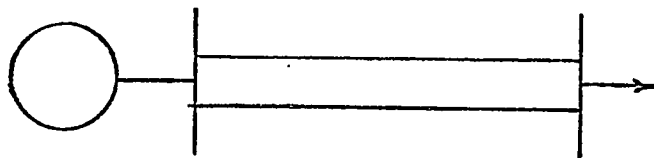
$P_c(1 \text{ or } 2)$ = Probability of one line being available and
one line unavailable. Markov States 1, 2,
5 and 6.

$P_c(0)$ = Probability of both lines being unavailable.
Markov States 3 and 7.

<u>Percent Failures During Storms</u>	<u>$P_c(1,2)$</u>	<u>$P_c(1 \text{ or } 2)$</u>	<u>$P_c(0)$</u>
0	0.9991	0.8554×10^{-3}	0.1831×10^{-6}
10	0.9991	0.8552×10^{-3}	0.2182×10^{-6}
20	0.9991	0.8549×10^{-3}	0.3343×10^{-6}
30	0.9991	0.8541×10^{-3}	0.5313×10^{-6}
40	0.9991	0.8530×10^{-3}	0.8084×10^{-6}
50	0.9991	0.8514×10^{-3}	0.1165×10^{-5}
60	0.9991	0.8498×10^{-3}	0.1601×10^{-5}
70	0.9991	0.8477×10^{-3}	0.2115×10^{-5}
80	0.9991	0.8453×10^{-3}	0.2707×10^{-5}
90	0.9991	0.8426×10^{-3}	0.3375×10^{-5}
100	0.9991	0.8415×10^{-3}	0.4120×10^{-5}

TABLE A-3

System Failure Probabilities For The 240 MW Capacity System



60 - 40 MW generating units, forced outage rate = 0.005

2 - 230 KV transmission lines

$\lambda_1 = \lambda_2 = 0.5$ failures/year

N = 200 hours

S = 1.5 hours

R = 7.5 hours

Straight line load duration curve. 75 percent load factor.

Probability Of System Failure x 1000
For Selected Peak Loads

<u>Percent Failures During Storms</u>	<u>120 MW</u>	<u>140 MW</u>	<u>160 MW</u>	<u>180 MW</u>	<u>200 MW</u>
0	0.0002	0.2448	0.4289	0.6534	0.8332
10	0.0002	0.2448	0.4288	0.6532	0.8331
20	0.0003	0.2448	0.4287	0.6531	0.8329
30	0.0005	0.2448	0.4285	0.6528	0.8325
40	0.0008	0.2448	0.4283	0.6524	0.8314
50	0.0012	0.2447	0.4279	0.6517	0.8310
60	0.0016	0.2446	0.4275	0.6510	0.8301
70	0.0022	0.2445	0.4270	0.6502	0.8291
80	0.0027	0.2445	0.4263	0.6481	0.8276
90	0.0034	0.2444	0.4257	0.6480	0.8262
100	0.0041	0.2448	0.4258	0.6480	0.8260

TABLE A-4

Probabilities Of Occupying The Various States For Three Lines In Parallel

$$\lambda_1 = \lambda_2 = \lambda_3 = 0.5 \text{ failures/year}$$

$$N = 200 \text{ hours}$$

$$S = 1.5 \text{ hours}$$

$$R = 7.5 \text{ hours}$$

Line 1
Line 2
Line 3

$P_c(\text{ALL})$ = Probability of all lines being available.
Markov States 0 and 8.

$P_c(2)$ = Probability of two lines being available and
one line unavailable. Markov States 1, 2, 3,
9, 10 and 11.

$P_c(1)$ = Probability of one line being available and
two lines unavailable. Markov States 4, 5, 6,
12, 13 and 14.

$P_c(0)$ = Probability of all lines being unavailable.
Markov States 7 and 15.

Percent Failures
During Storms

	$P_c(\text{ALL})$	$P_c(2)$	$P_c(1)$	$P_c(0)$
0	0.9991	0.8554×10^{-3}	0.2446×10^{-6}	0.2333×10^{-10}
10	0.9991	0.8552×10^{-3}	0.3088×10^{-6}	0.7004×10^{-10}
20	0.9991	0.8545×10^{-3}	0.5227×10^{-6}	0.3571×10^{-9}
30	0.9991	0.8536×10^{-3}	0.8997×10^{-6}	0.1094×10^{-8}
40	0.9991	0.8520×10^{-3}	0.1394×10^{-5}	0.2488×10^{-8}
50	0.9991	0.8500×10^{-3}	0.2048×10^{-5}	0.4741×10^{-8}
60	0.9991	0.8475×10^{-3}	0.2844×10^{-5}	0.8054×10^{-8}
70	0.9991	0.8446×10^{-3}	0.3783×10^{-5}	0.1262×10^{-7}
80	0.9991	0.8415×10^{-3}	0.4862×10^{-5}	0.1864×10^{-7}
90	0.9991	0.8377×10^{-3}	0.6081×10^{-5}	0.2629×10^{-7}
100	0.9991	0.8335×10^{-3}	0.7434×10^{-5}	0.3577×10^{-7}

0.0012
 $P_c(2)$ results
 via correct

TABLE A-5

Transmission Line Parameters

Single circuit 230 KV Line

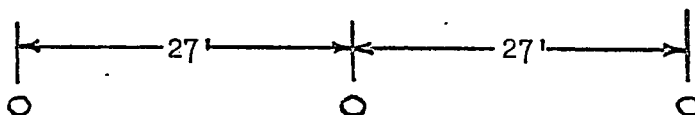
Single conductor per phase

605,000 CM A.C.S.R. (7/0.1186 + 26/0.1525)

Conductor resistance at 50°C and 75 percent

capacity = 0.1720/mile

Horizontal configuration



Geometric Mean Radius = 0.0327 feet

Geometric Mean Distance = 34.02 feet

Outside diameter = 0.0805 feet

Series reactance at 60 cps = 0.8429 ohms/phase/mile

Shunt susceptance at 60 cps = 5 micromhos/phase/mile

Surge Impedance $Z_0 = 414.5 \angle -5.8^\circ$ ohms

For a length of 150 miles

Propagation Constant = $0.311 \angle 84.2^\circ$

Line Coefficients:

$$A = 0.953 \angle 0.57^\circ$$

$$B = 127.02 \angle 78.65^\circ \text{ ohms}$$

$$C = 0.7378 \times 10^{-3} \angle 90.2^\circ \text{ mhos}$$

APPENDIX B

Two Plant Two Load System

Consider the system shown in Figure 5.7. The probability of failure for Load 1 can be evaluated as follows

Starting with

$$Q_s = Q_s(L1 - IN)R_{L1} + Q_s(L1 - OUT)Q_{L1}$$

For L1 - IN

$$Q_s = Q_s(L4 - IN)R_{L4} + Q_s(L4 - OUT)Q_{L4}$$

For L1 - IN and L4 - IN

$$Q_s = Q_s(L2 - IN)R_{L2} + Q_s(L2 - OUT)Q_{L2}$$

For L1 - IN and L4 - IN and L2 - IN

$$Q_s = Q_s(L3 - IN)R_{L3} + Q_s(L3 - OUT)Q_{L3}$$

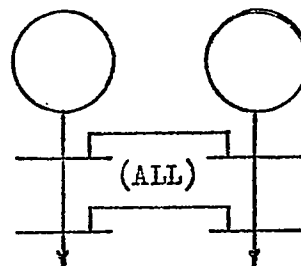
For L1 - IN and L4 - IN and L2 - IN and L3 - IN

$$Q_s = P_g(1,2) + P_c(ALL) - P_g(1,2) \cdot P_c(ALL)$$

Where $P_c(ALL)$ refers to the configuration shown

Normally $P_c(ALL) = 0$

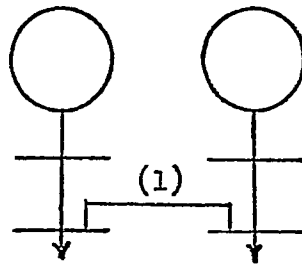
and $Q_s = P_g(1,2)$



For L1 - IN and L4 - IN and L2 - IN and L3 - OUT

$$Q_s = P_g(1,2) + P_c(1) - P_g(1,2) \cdot P_c(1)$$

The configuration for $P_c(1)$ is shown on the following page.



For L1 - IN and L4 - IN and L2 - IN

$$Q_s = R_{L3} \left[P_g(1,2) + P_c(ALL) - P_g(1,2) \cdot P_c(ALL) \right] \\ + Q_{L3} \left[P_g(1,2) + P_c(1) - P_g(1,2) \cdot P_c(1) \right]$$

For L1 - IN and L4 - IN and L2 - OUF

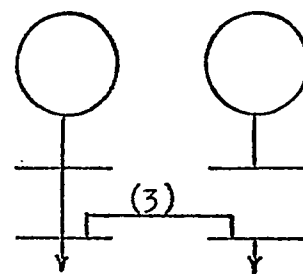
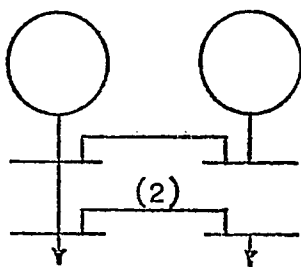
$$Q_s = Q_s(L3 - IN)R_{L3} + Q_s(L3 - OUF)Q_{L3}$$

For L1 - IN and L4 - IN and L2 - OUF and L3 - IN

$$Q_s = P_g(1,2) + P_c(2) - P_g(1,2) \cdot P_c(2)$$

For L1 - IN and L4 - IN and L2 - OUF and L3 - OUF

$$Q_s = P_g(1) + P_c(3) - P_g(1) \cdot P_c(3)$$



For L1 - IN and L4 - IN and L2 - OUF

$$Q_s = R_{L3} \left[P_g(1,2) + P_c(2) - P_g(1,2) \cdot P_c(2) \right] \\ + Q_{L3} \left[P_g(1) + P_c(3) - P_g(1) \cdot P_c(3) \right]$$

For L1 - IN and L4 - IN

$$Q_s = R_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(\text{ALL}) - P_g(1,2) \cdot P_c(\text{ALL}) \right] \right. \\ \left. + Q_{L3} \left[P_g(1,2) + P_c(1) - P_g(1,2) \cdot P_c(1) \right] \right] \\ + Q_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(2) - P_g(1,2) \cdot P_c(2) \right] \right. \\ \left. + Q_{L3} \left[P_g(1) + P_c(3) - P_g(1) \cdot P_c(3) \right] \right]$$

For L1 - IN and L4 - OUT

$$Q_s = Q_s(L3 - \text{IN})R_{L3} + Q_s(L3 - \text{OUT})Q_{L3}$$

For L1 - IN and L4 - OUT and L3 - IN

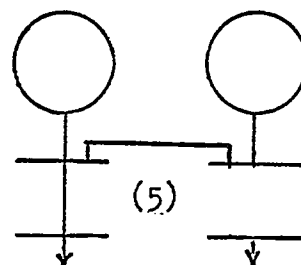
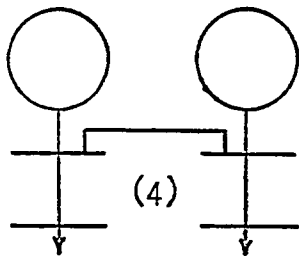
$$Q_s = Q_s(L2 - \text{IN})R_{L2} + Q_s(L2 - \text{OUT})Q_{L2}$$

For L1 - IN and L4 - OUT and L3 - IN and L2 - IN

$$Q_s = P_g(1,2) + P_c(4) - P_g(1,2) \cdot P_c(4)$$

For L1 - IN and L4 - OUT and L3 - IN and L2 - OUT

$$Q_s = P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5)$$

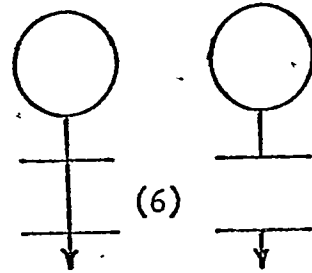


For L1 - IN and L4 - OUT and L3 - IN

$$Q_s = R_{L2} \left[P_g(1,2) + P_c(4) - P_g(1,2) \cdot P_c(4) \right] \\ + Q_{L2} \left[P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5) \right]$$

For L1 - IN and L4 - OUT and L3 - OUT

$$Q_s = P_g(1) + P_c(6) - P_g(1) \cdot P_c(6)$$



For L1 - IN and L4 - OUT

$$Q_s = R_{L3} \left[R_{L2} \left[P_g(1,2) + P_c(4) - P_g(1,2) \cdot P_c(4) \right] \right. \\ \left. + Q_{L2} \left[P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5) \right] \right] \\ + Q_{L3} \left[P_g(1) + P_c(6) - P_g(1) \cdot P_c(6) \right]$$

For L1 - IN

$$Q_s = R_{L4} \left[R_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(\text{ALL}) - P_g(1,2) \cdot P_c(\text{ALL}) \right] \right. \right. \\ \left. \left. + Q_{L3} \left[P_g(1,2) + P_c(1) - P_g(1,2) \cdot P_c(1) \right] \right] \right. \\ \left. + Q_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(2) - P_g(1,2) \cdot P_c(2) \right] \right. \right. \\ \left. \left. + Q_{L3} \left[P_g(1) + P_c(3) - P_g(1) \cdot P_c(3) \right] \right] \right] \\ + Q_{L4} \left[R_{L3} \left[R_{L2} \left[P_g(1,2) + P_c(4) - P_g(1,2) \cdot P_c(4) \right] \right. \right. \\ \left. \left. + Q_{L2} \left[P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5) \right] \right] \right. \\ \left. + Q_{L3} \left[P_g(1) + P_c(6) - P_g(1) \cdot P_c(6) \right] \right]$$

For L1 - OUT

$$Q_s = Q_s(L4 - \text{IN}) R_{L4} + Q_s(L4 - \text{OUT}) Q_{L4}$$

For L1 - OUT and L4 - IN

$$Q_s = Q_s(L2 - \text{IN}) R_{L2} + Q_s(L2 - \text{OUT}) Q_{L2}$$

For L1 - OUT and L4 - IN and L2 - IN

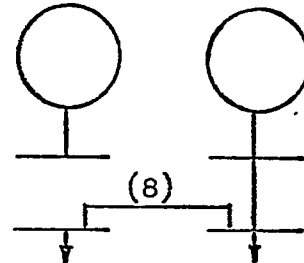
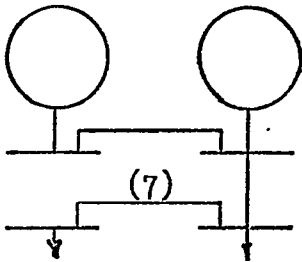
$$Q_s = Q_s(L3 - IN)R_{L3} + Q_s(L3 - OUT)Q_{L3}$$

For L1 - OUT and L4 - IN and L2 - IN and L3 - IN

$$Q_s = P_g(1,2) + P_c(7) - P_g(1,2) \cdot P_c(7)$$

For L1 - OUT and L4 - IN and L2 - IN and L3 - OUT

$$Q_s = P_g(2) + P_c(8) - P_g(2) \cdot P_c(8)$$



For L1 - OUT and L4 - IN and L2 - IN

$$Q_s = R_{L3} \left[P_g(1,2) + P_c(7) - P_g(1,2) \cdot P_c(7) \right] \\ + Q_{L3} \left[P_g(2) + P_c(8) - P_g(2) \cdot P_c(8) \right]$$

For L1 - OUT and L4 - IN and L2 - OUT

$$Q_s = 1$$

For L1 - OUT and L4 - IN

$$Q_s = R_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(7) - P_g(1,2) \cdot P_c(7) \right] \right. \\ \left. + Q_{L3} \left[P_g(2) + P_c(8) - P_g(2) \cdot P_c(8) \right] \right] + Q_{L2}$$

For L1 - OUT and L4 - OUT

$$Q_s = 1$$

For L1 - OUT

$$Q_s = R_{L4} \left[R_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(7) - P_g(1,2) \cdot P_c(7) \right] \right. \right. \\ \left. \left. + Q_{L3} \left[P_g(2) + P_c(8) - P_g(2) \cdot P_c(8) \right] \right] + Q_{L2} \right] + Q_{L4}$$

The complete expression is:

$$Q_s = R_{L1} \left[R_{L4} \left[R_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(ALL) - P_g(1,2) \cdot P_c(ALL) \right] \right. \right. \right. \\ \left. \left. + Q_{L3} \left[P_g(1,2) + P_c(1) - P_g(1,2) \cdot P_c(1) \right] \right] \right. \\ \left. + Q_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(2) - P_g(1,2) \cdot P_c(2) \right] \right. \right. \\ \left. \left. + Q_{L3} \left[P_g(1) + P_c(3) - P_g(1) \cdot P_c(3) \right] \right] \right] \\ \left. + Q_{L4} \left[R_{L3} \left[R_{L2} \left[P_g(1,2) + P_c(4) - P_g(1,2) \cdot P_c(4) \right] \right. \right. \right. \\ \left. \left. + Q_{L2} \left[P_g(1,2) + P_c(5) - P_g(1,2) \cdot P_c(5) \right] \right] \right. \\ \left. + Q_{L3} \left[P_g(1) + P_c(6) - P_g(1) \cdot P_c(6) \right] \right] \\ \left. + Q_{L1} \left[R_{L4} \left[R_{L2} \left[R_{L3} \left[P_g(1,2) + P_c(7) - P_g(1,2) \cdot P_c(7) \right] \right. \right. \right. \right. \\ \left. \left. + Q_{L3} \left[P_g(2) + P_c(8) - P_g(2) \cdot P_c(8) \right] \right] + Q_{L2} \right] + Q_{L4} \right]$$