COMPLIANT MECHANISMS DESIGN WITH FATIGUE STRENGTH CONTROL: A COMPUTATIONAL FRAMEWORK

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ABSTRACT

A compliant mechanism gains its motion from the deflection of flexible members or the deformation of one portion of materials with respect to other portions. Design and operation of compliant mechanisms are very important, as most of the natural objects are made of compliant materials mixed with rigid materials, such as the bird wings. The most serious problem with compliant mechanisms is their fatigue problem due to repeating deformation of materials in compliant mechanisms. This thesis presents a study on the computational framework for designing a compliant mechanism under fatigue strength control. The framework is based on the topology optimization technique especially ground structure approach (GSA) together with the Genetic Algorithm (GA) technique.

The study presented in this thesis has led to the following conclusions: (1) It is feasible to incorporate fatigue strength control especially the stress-life method in the computational framework based on the GSA for designing compliant mechanisms and (2) The computer program can well implement the computational framework along with the general optimization model and the GA to solve the model.

There are two main contributions resulting from this thesis: First one is provision of a computational model to design compliant mechanisms under fatigue strength control. This model also results in a minimum number of elements of the compliant mechanism in design, which means the least weight of mechanisms and least amount of materials. Second one is an experiment for the feasibility of implementing the model in the MATLAB environment which is widely used for engineering computation, which implies a wide applicability of the design system developed in this thesis.

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Many thanks also go out to friends for being there with me through the good and the bad times, including Fang He, Gary Huang and to name but a few.

Specially, I would like to thank my Mom and Dad and all the family members for their deep love and helpful support, both morally and financially.

DEDICATION

To my dearest Mom and Dad

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LIST OF ACRONYMS

CM	Compliant Mechanism
CMFTO	Compliant Mechanisms design under Fatigue strength control using Topology Optimization
FEM	Finite Element Method
GA	Genetic Algorithm
GSA	Ground Structure Approach
MEMS	Micro-electromechanical Systems
MP	Mathematical Programming
OC	Optimality Criteria
PRBM	Pseudo-Rigid-Body-Model
SF	Safety Factor
SIMP	Solid Isotropic Material with Penalization
TO	Topology Optimization

LIST OF NOMENCLATURES

$S_e^{'}$	: theoretical endurance limit
S_e	: modified endurance limit
S_f'	: theoretical fatigue strength
S_f	: modified fatigue strength
$C_{surface}$	: surface modification factor
C_{size}	: size modification factor
C_{load}	: load modification factor
C_{reliab}	: reliability modification factor
C_{misc}	: miscellaneous modification factor
K_t	: Normal stress concentration factor
K_{ts}	: Shear stress concentration factor
E	: Modulus of elasticity
I	: Moment inertia
A	: Cross sectional area
L	: Length of element
θ	: Inclined angle
$U_{n,d}$	: Output displacement of the node n in a direction d
ne	: Number of elements
de	: Desirable number of elements
$\sigma_{constraint}$	: Stress constraint
σ_{max}	: Maximum stress
σ_m	: Mean stress
σ_a	: Alternating stress
N_{ind}	: Number of individuals
N_{gen}	: Number of generations

CHAPTER1 INTRODUCTION

1.1 Research Background and Motivation

A compliant mechanism gains “motion” from the deflection of flexible members. Figure 1.1 shows an example of a compliant crimping mechanism. The input motion and force are at “hand grips” while the output motion and force are at “output port”. The structure of the system involves the “bar” object and “simple flexural pivot” and “compound flexural pivot” (Figure 1.1).

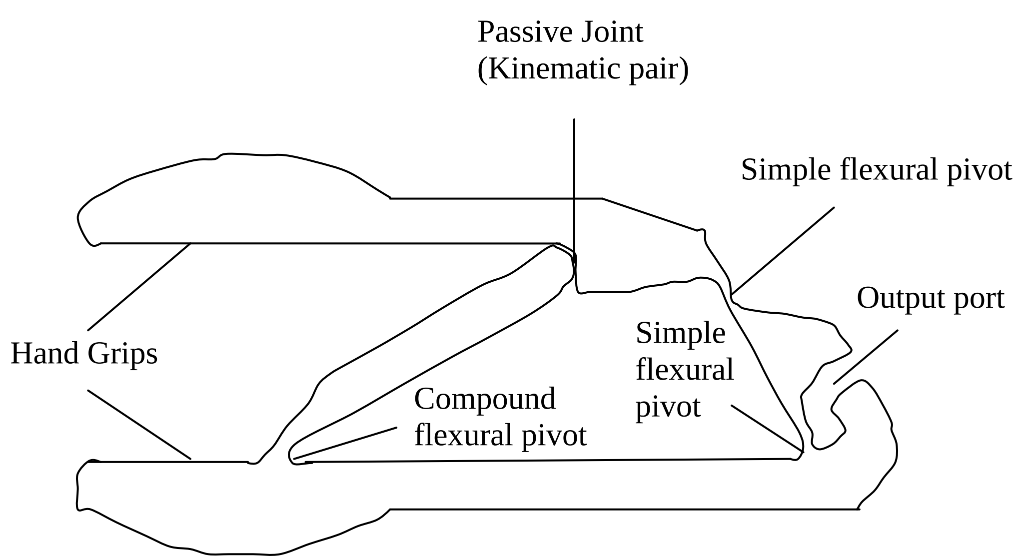


Figure 1.1 An example of a compliant crimping mechanism (adapted from Howell, 2001)

In the real world, it has been well recognized that most of the man-made objects are rigid body structures or mechanisms. On the other hand, natural objects are mostly made of compliant materials mixed with rigid materials such as bones and teeth. Flapping of birds wings to achieve flight is a good example. By mimicking the bird, a system was designed by integrating flexible components into rigid components to promote draft and lift, caused by the air, to rotate the wings throughout the cyclic process (Vogel, 1995). One of the salient points with this design is that the system can achieve a high cyclic flapping rate with little energy consumption. Another example is the trunk of the elephant. The compliance of the trunk enables the elephant to grasp objects, twist and coil them. It is known that the trunk can lift up to 770lb (Shoshani, 1998).This ability of the trunk tells us that an object can be compliant and strong as well. In April 2011, a German company FESTO created an autonomous ultra-light unmanned aerial vehicle, “SmartBird”. It is an avian robot that can take off, fly and land through the air by simply flapping its compliant wings.

Compliant mechanisms have many advantages over the traditional mechanisms that employ rigid joints and connections. Among many others, two are profound, that is, cost reduction and performance enhancement. The two are further achieved by the general characteristics of compliant mechanisms including (1) the absence of assembly in production methods, (2) reduction of weight, (3) reduction of wear between joints and less lubricant. Another advantage with compliant mechanisms is that they can facilitate the energy release and storage owing to its primary motion principle – i.e., deformation.

Finally, the concept of the compliant mechanism can readily be employed to make more complex Micro-electromechanical Systems (MEMS), as MEMSs are naturally a compliant piece of material. Kota *et al.* (2001) designed and fabricated a compliant based actuation system, which has a short stroke comb drive with stroke amplifier. Compared to the comb drives currently used, the compliant based actuation system is considerably smaller (Kota *et al.*, 2001).

Design of compliant mechanisms has two schools. The first school is to convert a compliant mechanism to a rigid equivalent mechanism and then apply design knowledge for the rigid body mechanism to the equivalent rigid body mechanism. The second school is topology optimization (TO). There is an agreement in literature that the second school is promising.

One of the most important problems with compliant mechanisms is fatigue failure due to the repeating deformation in materials and stress concentrations on elements. Unfortunately, there are only a few studies on the TO design of compliant mechanisms with consideration of fatigue strength control in the body of compliant mechanism design literature (Bahia *et al*. 2006, Dirksen *et al*. 2013). More details of the comment on literature to justify this observation will be provided later in Chapter 2. This thesis study was motivated by this observation and was aimed at developing the design technology for compliant mechanisms using the TO technique under fatigue strength control.

1.2 Research Objective and Scope

The overall objective of the work described in this thesis was to develop a computational framework for design of compliant mechanisms under fatigue strength control by means of topology optimization techniques. By computational framework it was meant that various specific design methods for fatigue failure control can be integrated with a common computational service such as model formulation and model solving. To achieve the overall objective, specific objectives are listed below.

Objective 1: To formulate a general computational model for design of compliant mechanisms under fatigue strength control using the topology optimization technique.

Objective 2: To implement the model in a general-purpose computational facility such as MATLAB and to demonstrate the application of the model or framework.

This study was limited to the so-called distributed compliant mechanism that has distributed compliance and members that have both bending and axial deformations. When a distributed compliant mechanism is loaded, almost all parts of compliant mechanisms will contribute to the deflection at the output ports. Further, this study was not in the pursuit of the optimal design of any specific compliant mechanism but rather in the pursuit of a general computational framework with extendibility (i.e., new specific design methods for fatigue failure control can be incorporated into the computational framework and its computer code).

1.3 General Research Idea and Methodology

The ground structure approach (GSA) in the TO technique was taken. In this approach, the beam element was used to account for the bending and axial deformation. The connection among beam elements results in the node at which the connecting elements share the same orientation and displacement or deflection. A design domain was meshed by beam elements.

In each iteration, the maximum stress was found among all beam elements, so is the endurance limit or fatigue strength of beam elements in the system per iteration depending on different types of materials. The constraint that the maximum stress is less than the endurance limit or fatigue strength and minimization of the total number of elements in the mechanism in design, and so on, are then evaluated to decide whether particular beam elements should remain or not. The problem is an optimization problem and, in particular, it is a constrained optimization problem. In this study, the maximum displacement was considered as a design requirement with loss of the generality of this research. The genetic algorithm was used to solve the optimization model due to its generality. At the implementation level, an open-source code for the ground structure approach with the genetic algorithm was modified for the purpose of this study.

1.4 Organization of Thesis

This thesis consists of 6 chapters. In Chapter 2, a literature review will be presented. In this chapter, background knowledge and the previous research on compliant mechanisms design with the TO technique are described. Further, a detailed discussion on the literature regarding compliant mechanisms design under fatigue strength control is presented with a further justification of the need of the proposed research objectives.

In Chapter 3, there will be a detailed illustration about the fatigue analysis theory. It gives the necessary background for understanding the method and procedure proposed in this study. In Chapter 4, the topology optimization of compliant mechanisms under fatigue strength control is presented, including the detailed procedure of computing endurance limit or fatigue strength of a compliant mechanism represented by a scheme of beam elements.

In Chapter 5, validation of the proposed design procedure described in Chapter 4 is carried out with two examples. As a reference computation, ANSYS software is employed to compute the maximum stress and displacement of a resulting compliant mechanism resulting from the computational procedure in Chapter 4.

In Chapter 6, a conclusion with discussion of future work is presented.

CHAPTER 2LITERATURE REVIEW

2.1 Introduction

The purpose of this chapter is to provide an outline of the origin of compliant mechanisms and the previous work in the area of topology optimization of compliant mechanisms with a focus on strength control. Only relevant developments with respect to the objectives of this study are reviewed and commented. The secondary purpose of this chapter is to confirm the need of the proposed research objectives as described in Chapter 1. The organization of the chapter is as follows. Section 2.2 discusses the design of compliant mechanisms using the topology optimization (TO) technique. Section 2.3 discusses design of compliant mechanisms under fatigue strength control using the TO technique. Section 2.4 discusses the algorithms to solve the TO problem. Section 2.5 presents the existent computer code in designing compliant mechanism. Section 2.6 gives a conclusion with revisiting the proposed objectives in Chapter 1.

2.2 Design of Compliant Mechanisms with the Topology Optimization Technique

Compliant mechanisms are divided into two classes: lumped compliant mechanism (Figure 2.1) and distributed compliant mechanism (Figure 2.2). The former is the mechanism that has distinct flexural joints and rigid members and only the joint contributes to the deformation of the mechanism. The latter is the mechanism that although there are distinct flexural joints, members also deform, and both the joint and member contribute to the deformation of the mechanism. The lumped compliant mechanism has the benefits including articulation in motion and reduction in material but it has poor strength situation – especially stress concentration in the joint. In this study, the distributed compliant mechanism was concerned.

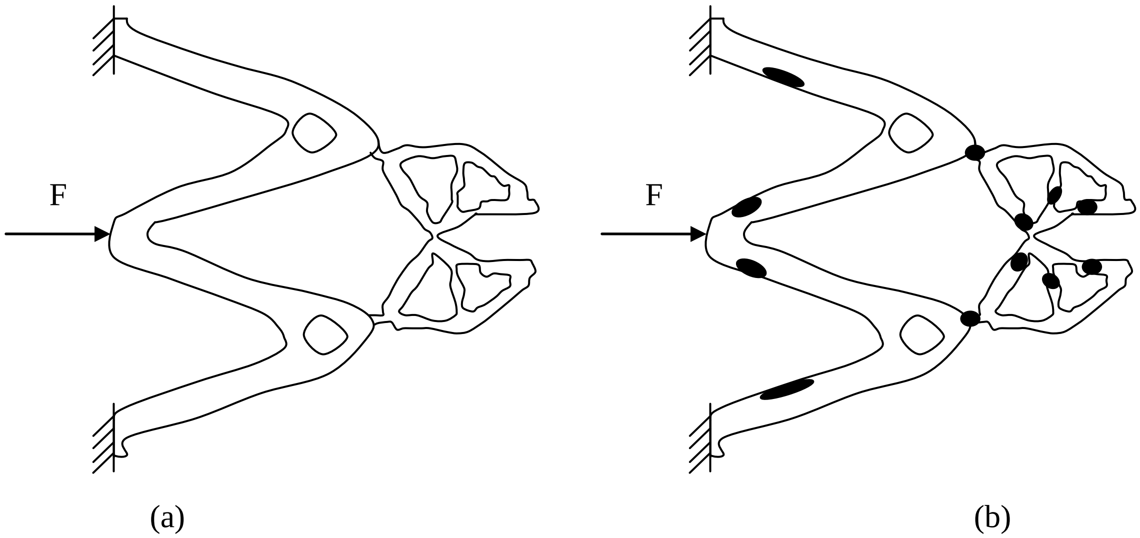


Figure 2.1 (a) A lumped compliant mechanism with boundary conditions (b) The stress distribution of a lumped compliant mechanism (stress concentration can be found in hinge areas with black dots) (adapted from Yin et al., 2003)

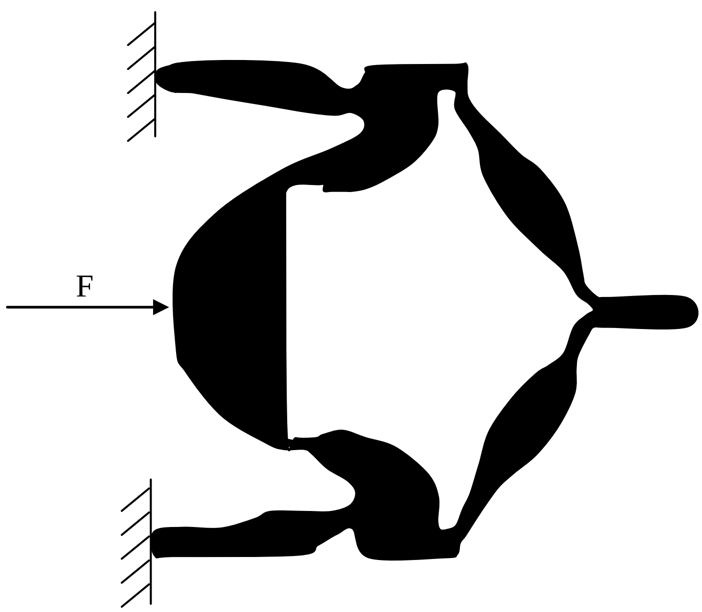


Figure 2.2 A distributed compliant mechanism with boundary conditions (adapted from Yin et al., 2003)

The earliest approach to designing compliant mechanisms is to view them as an equivalent rigid body mechanism with flexural joints and perhaps flexible members or links (Salamon, 1989). This approach has a benefit in that design knowledge for rigid body mechanisms can readily be applied with some modification (Howell, 2001). However, there is an inherent difficulty to have an accurate equivalence, as the process of making a compliant portion of material equivalent to a "rigid" joint goes along with an individual and independent procedure, while the compliant system or material always work as a whole. It is noted that the foregoing design methodology is called Pseudo-Rigid-Body Model (PRBM) approach (Salamon, 1989).

In this study, the topology optimization (TO) approach was used to design compliant mechanisms especially distributed compliant mechanisms. The general idea of the TO approach is as follows. (1) Start with a region of materials called the domain of a design. Design is viewed as distributing materials in this region or domain. Materials are considered as an assembly of elements. (2) Define a criterion or criteria for deciding where elements should stay. (3) Design a procedure that realizes this process.

The earliest work on the TO technique may refer to the work of Bendsøe and Kikuchi (1988) on a so-called "homogenization approach". In this approach, the domain is represented by a set of holes or voids, resulting in a porous medium object. The design in this case is to decide "dropping" of holes. If there is no hole, a solid material is present. In the homogenization approach, the number of holes is assumed to be so large that the mechanical behavior of the holes or solids in the neighbors of the holes is linear.

Rather than considering the domain as meshed by holes with the homogenization approach, material elements can be used to mesh the domain of a design. The computational representation of presence and absence of an element is through an integer 0 and 1, where 0 means the absence of material and 1 means the presence of material. Suppose that a domain is divided into $n \times m$ cells (where n: the number of columns and m: the number of rows). There will be the $n \times m$ variables which take either 0 or 1, and they are design variables. The design of a compliant mechanism is then to determine the values of these variables by making the behavior of the material to meet the required functions and constraints. Indeed, if these requirements mean some best, the result of the design is then the best design or optimal design. Computationally, the above problem becomes a 0-1 optimization problem, which is computational challenging.

One idea to overcome the computational challenge is to take the variable as a continuous variable in the region of 0 and 1 where 0 and 1 represent two crisp situations (0: absence of material; 1: presence of material). Since semantically, this variable represents the density of the material, the use of 0 to 1 to represent the density is thus called relative density. While mathematically, the relative density variable can take any value between 0 and 1, semantically or physically, the variable is expected to take values either close to 0 or to 1. Therefore, somewhat a plenty concept is applied to the value that is not close to 0 or 1, forcing the variable tends to take the value close to 0 or 1, and this approach is called the SIMP (Solid Isotropic Material with Penalization) approach, which was proposed by Bendsøe (1989), Zhou and Rozvany (1991), and Mlejnek (1992).The disadvantage of the foregoing treatment with the SIMP approach is that the solution depends on the value of penalization and it does not essentially converge to the optimal solution (Stolpe *et al*., 2001). Improvements of SIMP are referred to the work (Mario 2004; Bruns 2005).

A network of truss or beam elements was taken to mesh the domain of a design (Figure. 2.3), which is the so-called ground structure approach (GSA). The approach was first proposed by Dorn *et al*. (1964). In the GSA, the cross section areas A of the truss or beam elements are considered as design variables. A design variable of an existing element varies from a lower limit A_{min} to an upper limit A_{max}. However, if an element is absent, the value of its design variable is assigned with a relatively small value (close to zero) so that the influence of the element can be neglected. With truss element, different schemes are possible with the GSA approach (Figure. 2.3) according to Bendsøe (1995), and they may create different results. There is another approach with the GSA, where the connectivity of truss elements is represented as a code, and then Genetic Algorithm (GA) is applied. More details regarding this approach are given later in Section 2.5.

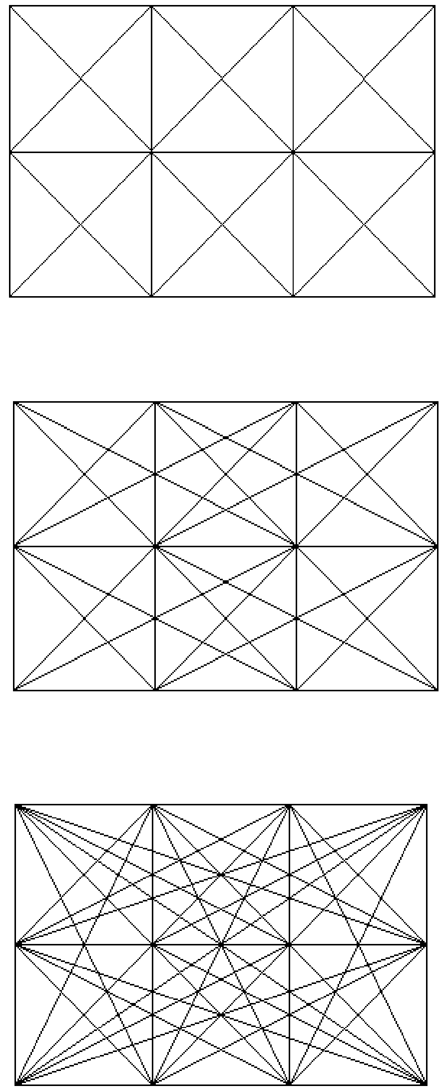


Figure 2.3 Ground structures of given sets of connections (adapted from Bendsøe, 1995)

Frecker *et al*. (1997) implemented topology optimization using truss element with the ground structure approach to design compliant mechanisms. A full ground structure, where every node is connected to every other node by a truss element, was used to mesh the design domain. Truss elements are limited to their natural way to represent the physical characteristics of the structure and mechanism. Beam elements are applied to mesh the structure and mechanism in literature. Hetric and Kota (1999) used the parametric finite element beam model and GSA to perform the shape and size optimization of compliant mechanisms. Stress constraints were employed to limit the maximum stress in the mechanism (Hetric *et al*., 1999). The problem of this design methodology is that the shapes of the optimal design are not smooth. As shown in Figure 2.4, cross-sectional areas of the beams critically change. Joo *et al*. (2001) proposed a tapered beam element model (Figure 2.5). This model provided a smooth change in cross sectional areas rather than the critical change in the parametric finite element beam model previously. A nonlinear FEM analysis was used in their work. The advantage of the GSA in designing compliant mechanisms with beam element is that the result is a distributed compliant mechanism. One of the purposes of the GSA is to avoid the appearance of flexure hinge.

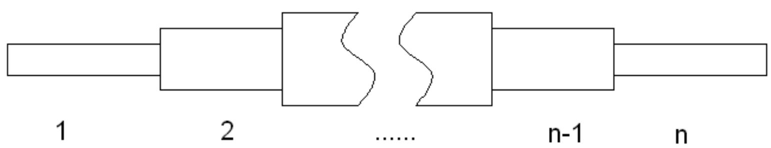


Figure 2.4 A parametric finite element beam model with sudden changes in the cross-sectional areas (adapted from Joo et al., 2001)

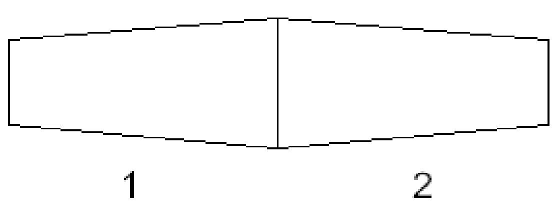


Figure 2.5 A tapered beam element model with smooth cross-sectional areas (adapted from Joo et al., 2001)

There is a slight difference between the structure and mechanism. The function of the structure is to support the load; while the function of the mechanism has a variety of purposes. Ananthasuresh *et al*. (1994) was a pioneer to apply the TO technique to compliant mechanisms design. Three models were developed to formulate the design problems of compliant mechanisms through the TO technique. These models are (1) force-deflection model, (2) spring model, (3) multi-criteria model (Ananthasuresh, 1994). The force-deflection model, specifying the input forces, was aimed to obtain compliant mechanisms for maximum output displacements. However, the results tended to be infinitely flexible to bear any loads. In the spring model, a spring with a given stiffness was attached to the output port to model the work-piece. The advantage of this model is the implicit inclusion of the output force requirement by relating the output force and displacement in a realistic manner (Ananthasuresh, 1994).

In the multi-criteria model, a compliant mechanism is viewed in a slightly different way. Specifically, the output displacement and load-bearing strength requirements are regarded as two opposing objectives. One case is to maximize the output displacement (flexibility requirement), while the other case is to maximize the load-bearing strength (stiffness requirement), that is to minimize the compliance. In order to perform the function of compliant mechanisms, both flexibility and stiffness are required simultaneously. The flexibility requirement meets the kinematic (motion) requirements and the stiffness requirement meets the structural (loading) requirements, as shown in Figure 2.6. To solve the conflicting design problems, multi-criteria model, which incorporates both requirements, can provide us an optimization scheme. The first objective was to maximize the flexibility, that is, maximize the deflection at the output port. The measurement of the deflection is equivalent to measure Mutual Potential Energy (MPE), which was proposed by Shield and Prager (1970). The second objective was to maximize the stiffness of the compliant mechanisms. Strain energy (SE) was applied as a measurement of stiffness. By minimizing SE, stiffness is maximized. Consequently, these two objectives, minimizing SE and maximizing MPE, were developed into a multi-criteria model.

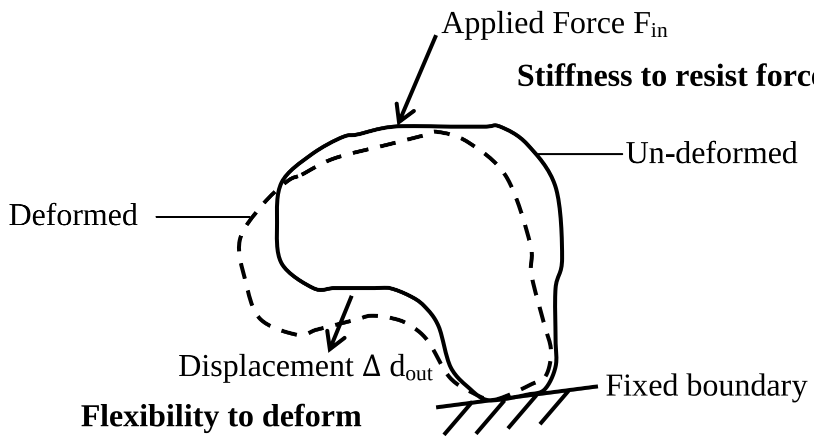


Figure 2.6 A comparison of an un-deformed compliant mechanism and a deformed compliant mechanism with input force, output displacement and boundary conditions

2.3 Design of Compliant Mechanisms under Fatigue Strength Control

As discussed in Chapter 1, fatigue failure is an essential problem in the compliant mechanisms design. It may cause the compliant mechanisms have insufficient fatigue life to perform their prescribed functions. Until now, however, there are only a few studies on design of compliant mechanisms under fatigue strength control. Bahia *et al*. (2006) incorporated the Modified Goodman fatigue strength theory with the topology optimization technique for the design of CMs. Optimality criteria algorithm was applied in the topology optimization process. However, the design process is of high computational cost, taking almost ten days to get an optimal design. Moreover, as stated in their paper, the design could not guarantee CMs of infinite life and the violation of the fatigue strength constraint may still occur in some elements.

Recently, Dirksen *et al*. (2013) presented an approach to consider fatigue strength in a post-design manner. That is to say, they first completed the design of a configuration of the mechanism or structure and then design flexural hinges. In their work, the members that connect to flexural hinges do not contribute to the behavior of the mechanism or structure at the same time the flexural hinges do. Besides, three different kinds of flexure hinges, i.e. rectangular, circular and parabolic geometrical flexure hinges (Figure 2.7) were taken into consideration.

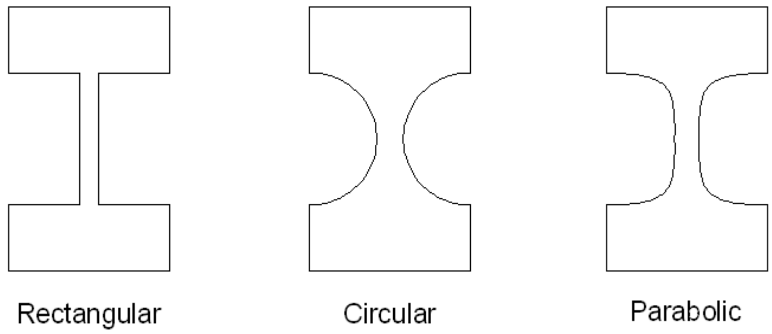


Figure 2.7 Three different kinds of flexure hinges in the compliant mechanisms design (adapted from Dirksen et al., 2013)

2.4 Algorithms for Optimization Problems

There are three algorithms commonly used for the TO problem, namely Mathematical Programming (MP), Optimality Criteria (OC) and Genetic Algorithm (GA). The MP is suitable to the problems with multiple constraints for the TO problem. The MP demands high computational cost especially with the increase of the number of variables in the context of TO (Rozvany *et al*., 1991). In fact, the MP is a general notion to solve a constrained optimization problem. There are two general methods in the MP: calculus-based and iteration-based. The variable in the MP program can be discrete variable or continuous variable.

The OC can be viewed a special type of the MP. It adds to the MP in that among a set of solutions generated by an MP algorithm, the criteria are set up for the solutions to lead to the best one without a need to try out all the solutions.

Note that in any optimization algorithm that follows an iteration scheme, there is a need to have a scheme to update the solution or solutions starting from an initial solution or solutions. In the above algorithms, the updating equation is always based on a deterministic method. If the updating equation is based on a non-deterministic method, algorithms to the optimization problem are called intelligent or evolutionary. Among many others, the GA is the most well-known one.

In the GA, the optimal variable needs to be coded into the bit format, e.g., 111011. The updating equation to generate more codes follows the method in genetic engineering, including crossover, mutation. The updating of solutions is also affected by the concept of generation and population. The benefits of using the GA include: (1) conducive to local minima with the MP and OC algorithms and (2) relaxation on the characteristics of both objective functions and constraints. The GA is most suitable to the discrete variable optimization problem. In the area of TO for design of mechanisms and structures, the application of the GA includes the works of Parsons and Canfield (2002), Lu and Kota (2003), Saxena (2005).

2.5 Computer Code

Larsen and Sindholt (2003) wrote a MATLAB code for the topology optimization of compliant mechanisms using the genetic algorithm (GA). In their code, truss elements were adopted and ground structure approach was applied. In their code, a 6-node structure was exemplified, as shown in Figure 2.8. The functional requirement of the mechanism to be designed is: maximizing the output displacement at one node under the constraints on strain and the number of truss elements.

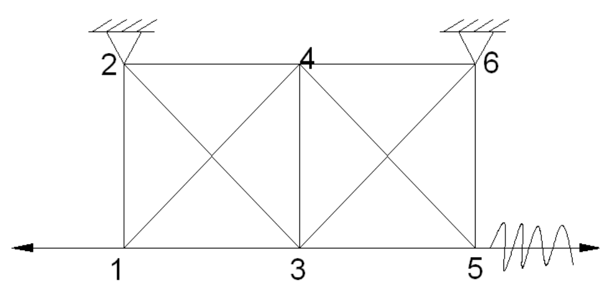


Figure 2.8 The ground structure of 6 node design domain (adapted from Larsen et al., 2003)

With input force at node 1, the requirement is in particular such that the displacement (horizontal) at node 5 should be a maximum under the constraints of a spring at node 5 and restriction of motion in translation at node 2 and node 6 (Figure 2.8). The design domain is meshed by a set of truss or bar elements. Finite element analysis was used to calculate the output displacement. The optimization problem can be expressed as follows:

Maximize: $\mathrm{U}_{\mathrm{n,d}}$

Subject to: $\varepsilon < \varepsilon_{\mathrm{max}}$

Where,$U_{n,d}$ is the output displacement at the node *n* in the direction *d*, and$\varepsilon$is the actual strain of trusses remaining in the design domain and $\varepsilon_{\mathrm{max}}$ is the maximum acceptable strain. It is noted that the strain constraint introduced by them is to prevent the situation where one or more trusses may elongate enormously. But how to determine the maximum strain is not given by them, which looks like that this should be determined by the user of the software (i.e., designer).

The design variable in their code is the representation of the connectivity of nodes. For the configuration as shown in Figure 2.8, the code for the design variable is in the bit form with 0 and 1, where 1: there is an element between two nodes and 0: there is no connection between two nodes. For the example system in Figure 2.8, the code representation is illustrated as follows:

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 4 & 1 \\ 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \\ 3 & 6 & 1 \\ 3 & 4 & 1 \\ 4 & 5 & 1 \\ 4 & 6 & 1 \\ 5 & 6 & 1 \end{bmatrix}$$

In the above expression, the first column represents the node with smaller number and the second column the node with larger number. The third column represents the connection status between the two nodes on the first and second column, respectively.

Figure 2.9 is the final solution, and its code representation is illustrated in the following:

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 4 & 1 \\ 4 & 5 & 1 \\ 4 & 6 & 1 \\ 5 & 6 & 1 \end{bmatrix}$$

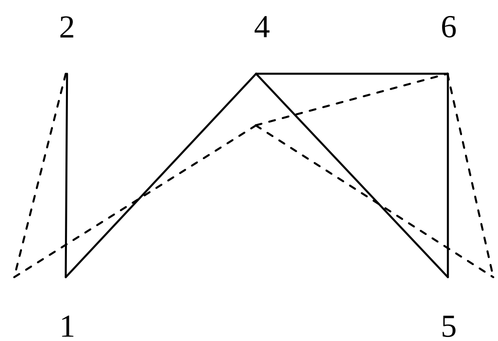


Figure 2.9 An example of topology optimization result of 6 nodes truss structure (adapted from Larsen, et al., 2003)

2.6 Conclusion

From the above review of the earlier work, none of the previous studies, to the author's best knowledge, has considered fatigue failure control in design of compliant mechanisms with the TO technique. This then confirms the need and novelty of the proposed work especially with the specific research objectives as defined in Chapter 1. Further, the main challenge with the first specific objective is computation of endurance limit or fatigue strength and to incorporate it in the TO process. The second specific objective was fulfilled by the modification and extension of the computer code of Larsen and Sindholt (2003).

CHAPTER 3 THEORETICAL FOUNDATIONS OF FATIGUE

3.1 Introduction

In this chapter, a detailed description of fatigue strength control in design pertinent to this study is presented. This includes the concept of fatigue failure, the method for analysis and design of systems for no fatigue failure. Section 3.2 discusses the basic concept of fatigue failure and one of the most powerful fatigue strength control methods called stress-life method. Section 3.3 discusses the concept of stress concentration, which affects fatigue failure significantly. Section 3.4 describes the method to calculate endurance limit or fatigue strength. Since this study considered beam element to mesh the domain of a design, endurance limit and fatigue strength of beam element are discussed in section 3.5.

3.2 Fatigue failure and Stress-Life Method

When a structure is subject to time-varying loadings, sometimes, the structure may fail despite the fact that the stress in the structures is lower than the ultimate strength of the structure, and quite frequently even lower than the yield strength (Budynas *et al*., 2011). Apparently, such failures are strongly related to repeated or fluctuating loading, and are called fatigue failure. Fatigue failures are very common in compliant mechanisms, as they operate in a cycle of loading.

Fatigue life is defined closely related to repeating loadings. Fatigue life is divided into three categories: low cycle fatigue (fatigue failure occurs between 1 to 10^3 cycles), high cycle fatigue (fatigue failure occurs more than 10^3cycles), and infinite life (no fatigue failure at a given load). For the material with infinite life, the strength at a known number of cycles is endurance limit, S_e. For the material without infinite life, like Al or Cu, it has the high-cycle fatigue life. When it goes into a point of known fatigue strength for a known number of cycles (usually5×10^8cycles), the strength at this point will be defined as the fatigue strength, S_f, which is regarded as the same function of the endurance limit.

There are many factors that affect the fatigue life of a material, such as the temperature, a corrosive environment, surface finish, and geometry. In this study, only bending fatigue failure was considered because the bending is a primary type of deformation in compliant mechanisms.

There are three basic approaches for fatigue failure analysis, and they are stress-life method, strain-life method and linear-elastic fracture mechanics method (Budynas *et al.*, 2011). Among the three methods, the stress-life method is the most traditional one. In this study, only the stress-life method was applied for its simplicity and suitability.

Figure 3.1 is an S-N curve of steel, which shows the relationship between a given stress (S) and the number of cycles (N) at failure. Note that this is a log-log plot.

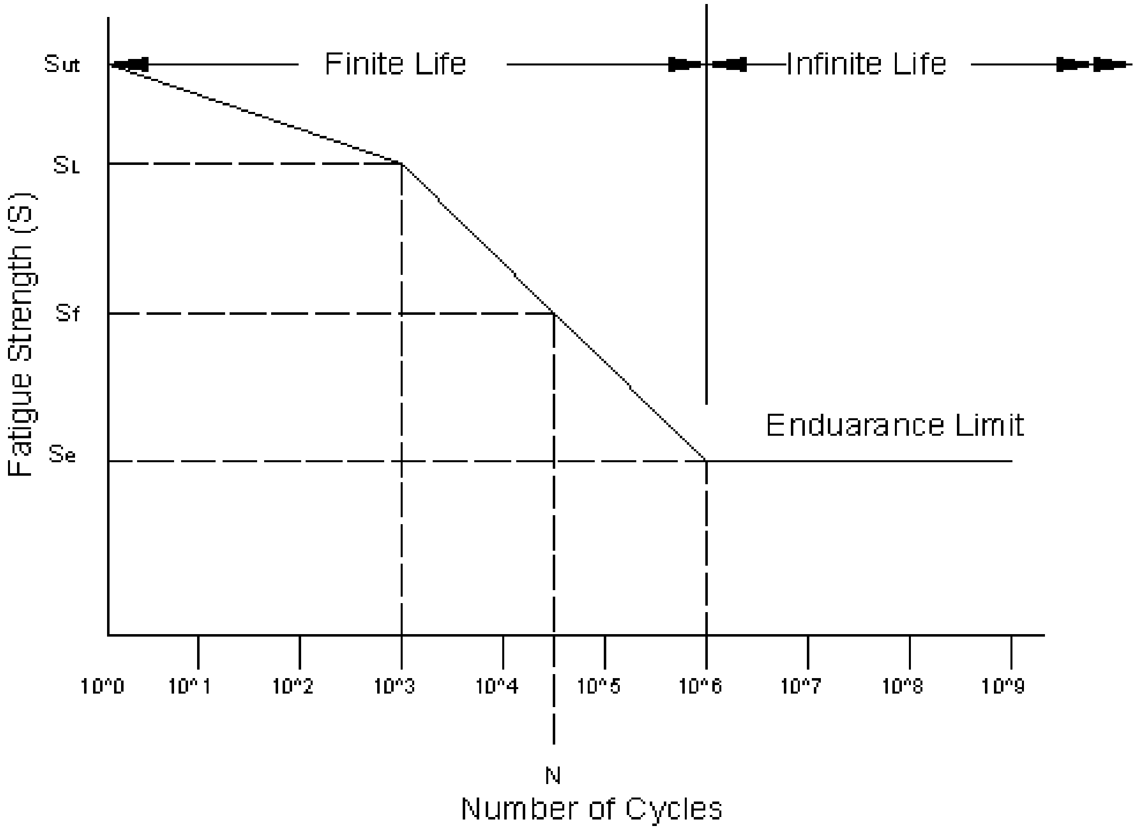


Figure 3.1 An S-N curve for the steel (log-log plot) (adapted from Howell, 2001)

Figure 3.1 shows that for a given stress (S), there is a corresponding number of cycles (N), which is the maximum number of cycles before fatigue failure happens. The fatigue strength,S_f, is the maximum totally reversed stress that a fatigue specimen can endure for N cycles (Howell, 2001). The endurance limit, S_e, is the stress below which failure will never occurs, no matter whatever the number of cycles is. So, if the given stress is maintained below the endurance limit, this specimen will have infinite life. However, not every material has the endurance limit, e.g. aluminum, fatigue failure will eventually occur no matter how small the given stress is (Howell, 2001). So aluminum only has the fatigue strength and its S-N curve will extend to a point of known fatigue strength at a known number of cycles, as shown in Figure 3.2.

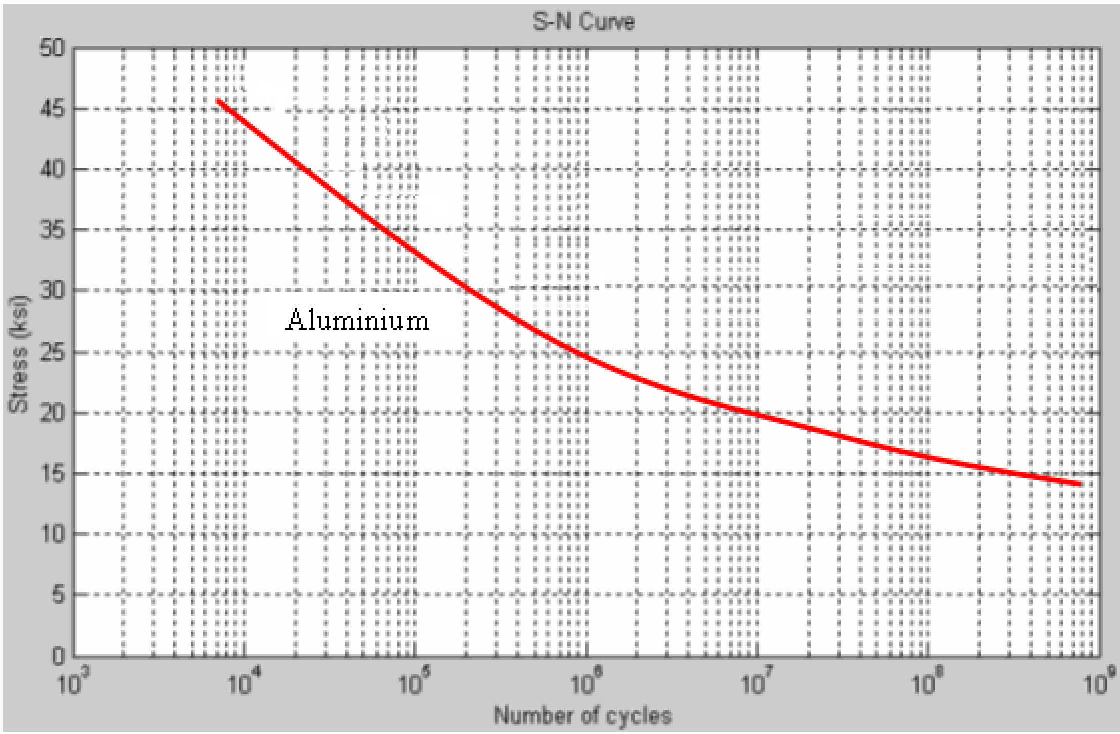


Figure 3.2 An S-N curve for aluminum (http://commons.wikimedia.org/wiki/File:S-N_curves.PNG)

It is noted that the S-N curves can be obtained from a traditional fatigue-testing device, R.R.Moore high-speed rotating-beam machine (Figure 3.3) based on a standard specimen. For a component in a particular system, modification has to be done, as the fatigue strength or endurance limit is dependent on the actual dimension and state of the component. That is to say, fatigue strength or endurance limit is not only dependent on the material. In this study, the stress-life method was taken due to its simplicity and suitability to the high cycle of loading (Budynas *et al.*, 2011). In the case of mechanisms, the cycle of loading seems to be high.

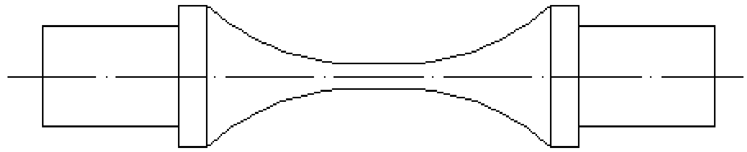
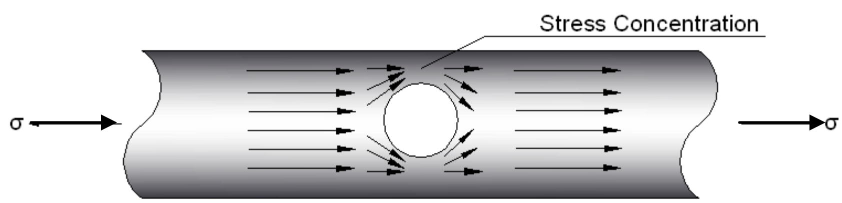


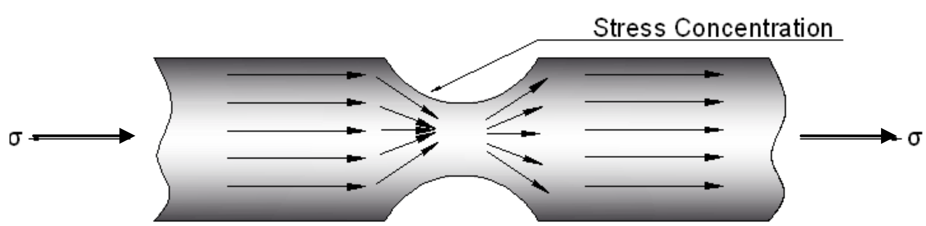
Figure 3.3 A specimen used in the R.R, Moore high-speed rotating-beam machine (adapted from Budynas *et al*., 2011)

3.3 Stress Concentration

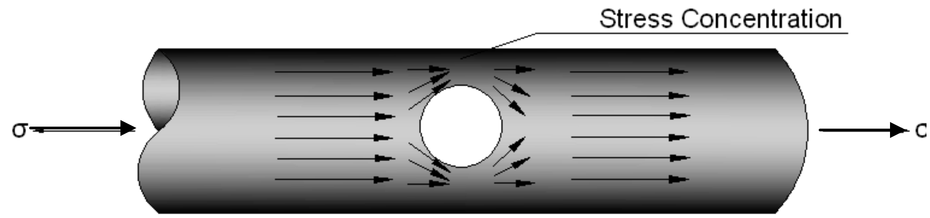
Fatigue strength or endurance limit is dependent on the structure of a member under design especially irregular structure such as grooves, holes, and notches. The effect of these structures on the fatigue strength is that they increase the stress in these areas. Such a phenomenon is called stress concentration (Budynas *et al*., 2011) (Figure 3.4).



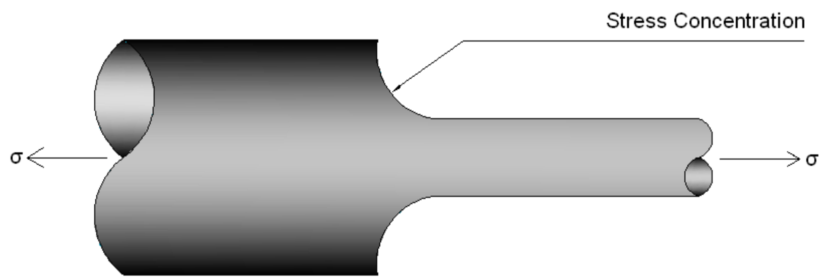
(a) Bar in tension or simple compression with a transverse hole



(b) Notches rectangular bar in tension or simple compression



(c) Round shaft in bending with a transverse hole in tension or simple compression



(d) Round shaft with shoulder fillet in tension or simple compression

Figure 3.4 Stress concentrations of some components with discontinuities or shape changes

A coefficient is defined to modify the stress in these areas, and this coefficient is called stress concentration factor, denoted by K_t or K_{ts}, respectively, where K_t is used for normal stresses and K_{ts} is used for shear stresses (Budynas *et al.*, 2011). The values for the coefficients are determined by experiments. For certain structures, the coefficients can be found from Peterson's Stress Concentration Factors (Pilkey *et al*., 2008). The next section shows how the various factors are considered to compute endurance limit or fatigue strength.

3.4 Calculation of Endurance Limit and Fatigue Strength

The theoretical endurance limit and fatigue strength are related to the rotating-beam specimen, which is carefully prepared, polished and tested under closely controlled conditions, its value varies from different kinds of materials and can be known by the reference books (Budynas *et al*., 2011). For example, the theoretical endurance limit for steel is:

$$S'_e = \begin{cases} 0.5S_{ut} & \text{for } S_{ut} < 200kpsi\ (1400MPa) \\ 100\text{kpsi (700MPa)} & \text{for } S_{ut} \geq 200\text{kpsi (1400MPa)} \end{cases} \quad (3\text{-}1)$$

whereS_{ut} is the ultimate strength ofthe steel.

The endurance limit and fatigue strength of the structure or mechanism in design are computed with two systems: theoretical value and modification. Modification of the theoretical value of endurance limit and fatigue strength considers surface conditions, stress concentration, temperature, size, shape, loading conditions. The modification is made via Marin correction factors (Marin, 1962).

Endurance Limit:

$$S_e = C_{surface} \times C_{size} \times C_{load} \times C_{reliab} \times C_{misc} \times S'_e \quad (3\text{-}2)$$

S'_e : theoretical endurance limit,

S_e : modified endurance limit,

$C_{surface}$: surface modification factor,

C_{size} : size modification factor,

C_{load} : load modification factor,

C_{reliab} : reliability modification factor, and

C_{misc} : miscellaneous modification factor.

Fatigue Strength:

$$S_f = C_{surface} \times C_{size} \times C_{load} \times C_{reliab} \times C_{misc} \times S'_f \quad (3\text{-}3)$$

S'_f : theoretical fatigue strength,

S_f : modified fatigue strength,

$C_{surface}$: surface modification factor,

C_{size} : size modification factor,

C_{load} : load modification factor,

C_{reliab} : reliability modification factor, and

C_{misc} : miscellaneous modification factor.

Therefore, the endurance limit or fatigue strength for various kinds of materials can be obtained from Equations (3-2) and (3-3), respectively. In this study, the miscellaneous modification factor will only consider stress concentration effect K_t, as the system in design is supposed to be used in the same condition as the supposed (K_t takes 1.6 with a detailed reason given later). Further, load modification factor takes 0.85 for the axial load and 1.0 for the bending load according to Shigley (2011). Reliability modification factor takes 99.99% according to Howell (2001). Size modification factor and surface modification factor were found according to the procedure provided by Howell (2001).

3.5 Stress Analysis in Truss and Beam

Two elements were used in the design of compliant mechanisms with topology optimization, and they are truss and beam elements. They have different functions and their stress conditions are different.

Truss element has two nodes. Each node has two degrees of freedom in translations: *x* and *y* (considering the plane mechanism). Therefore, the truss can only sustain the compression or tension deformation, as shown in figure 3.5.

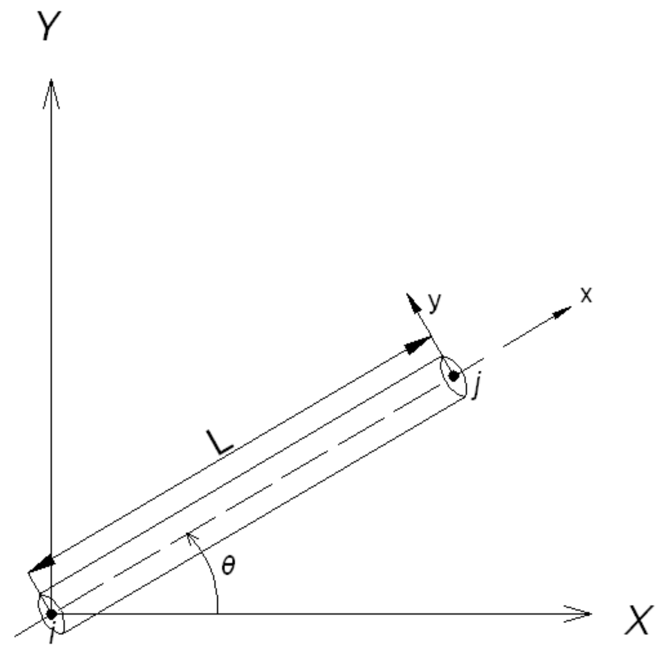


Figure 3.5 Truss element (adapted from Kattan, 2007)

Let E for the modulus of elasticity of a truss, A for cross-sectional area, and L for length. Further, each truss is inclined with an angle θ, measured counterclockwise from the positive *x* axis in the *x*-*y* plane(C= cos θ, S=sin θ).Further assume that the shape function of truss element is linear. The stiffness matrix for the truss element is (Kattan, 2007):

$$k = \frac{EA}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \tag{3-4}$$

Beam element has two nodes. Each node has three degrees of freedom in the x-y plane: two translations and one rotation, as shown in Figure 3.6.

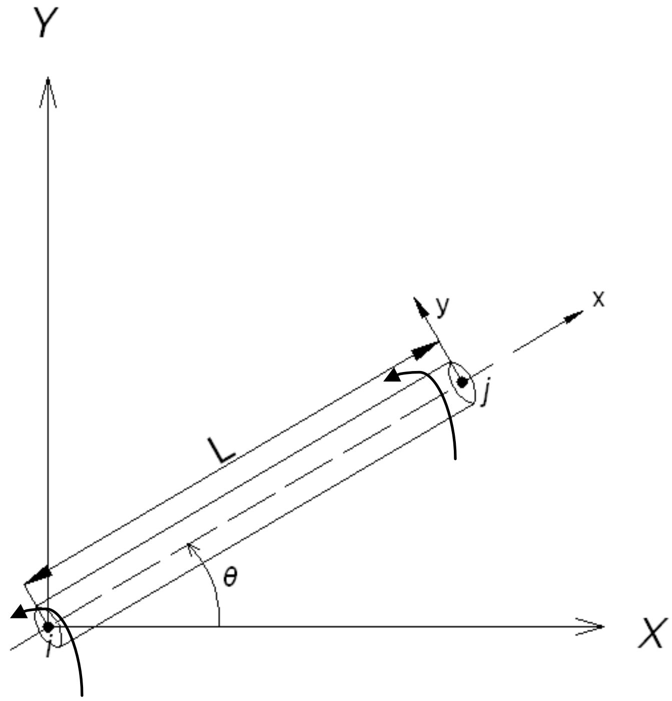


Figure 3.6 Beam element (adapted from Kattan, 2007)

Let E for the modulus of elasticity, I for moment inertia, A for cross-sectional area and L for length. Each beam element is inclined with an angle θ, measured counterclockwise from the positive *x* axis (C=cos θ, S=sin θ). The stiffness matrix for the beam element can be expressed as (Kattan, 2007):

$$k = \frac{E}{L}\begin{bmatrix} AC^2+\frac{12I}{L^2}S^2 & (A-\frac{12I}{L^2})CS & -\frac{6I}{L}S & -(AC^2+\frac{12I}{L^2}S^2) & -(A-\frac{12I}{L^2})CS & -\frac{6I}{L}S \\ (A-\frac{12I}{L^2})CS & AS^2+\frac{12I}{L^2}C^2 & \frac{6I}{L}C & -(A-\frac{12I}{L^2})CS & -(AS^2+\frac{12I}{L^2}C^2) & \frac{6I}{L}C \\ -\frac{6I}{L}S & \frac{6I}{L}C & 4I & \frac{6I}{L}S & -\frac{6I}{L}C & 2I \\ -(AC^2+\frac{12I}{L^2}S^2) & -(A-\frac{12I}{L^2})CS & \frac{6I}{L}S & AC^2+\frac{12I}{L^2}S^2 & (A-\frac{12I}{L^2})CS & \frac{6I}{L}S \\ -(A-\frac{12I}{L^2})CS & -(AS^2+\frac{12I}{L^2}C^2) & -\frac{6I}{L}C & (A-\frac{12I}{L^2})CS & AS^2+\frac{12I}{L^2}C^2 & -\frac{6I}{L}C \\ -\frac{6I}{L}S & \frac{6I}{L}C & 2I & \frac{6I}{L}S & -\frac{6I}{L}C & 4I \end{bmatrix}$$

(3-5)

CHAPTER 4 THE COMPUTATIONAL MODEL

4.1 Introduction

In this chapter, a computational model developed this study for compliant mechanisms design under fatigue strength control is presented. The model was primarily an optimization model that was expected to capture (1) the functional requirement of a compliant mechanism in design, (2) the constraint requirement of a compliant mechanism in design, and (3) the evaluation of fatigue strength of a compliant mechanism under evolution with design iteration.

The main challenge in formulating the model lies in the evaluation of fatigue strength. According to the discussion in Chapter 3, fatigue strength is much dependent on the structure of a system in analysis, and the type of loading and knowledge for fatigue is not available for general structure but some special structures in the literature. Therefore, an approximate mapping of the structure, along with its loading condition to that available knowledge (procedure, table, and chart), has to be done and this modeling strategy was taken in this thesis study.

This chapter is organized in the following way. Section 4.2 (next section) describes the general scheme of the model which is an optimization problem model. Section 4.3 presents details to compute the objective function and constraint in the optimization model. Section 4.4 illustrates two assumptions of loading conditions. Section 4.5illustrates the computer program for the model. In the last section, a summary with some discussions is given.

4.2 The General Scheme of the Model

To facilitate the discussion, a sample design case of Larsen and Sindholt (2003) is employed in the following discussion. It is a 12-node design domain in a two-dimensional coordinate-system (Figure 4.1). The connection between nodes is full in the sense that each node has one connection with all its neighboring nodes (Figure. 4.1). It is noted that such a design domain implies that the particular TO technique employed herein is the so-called GSA.

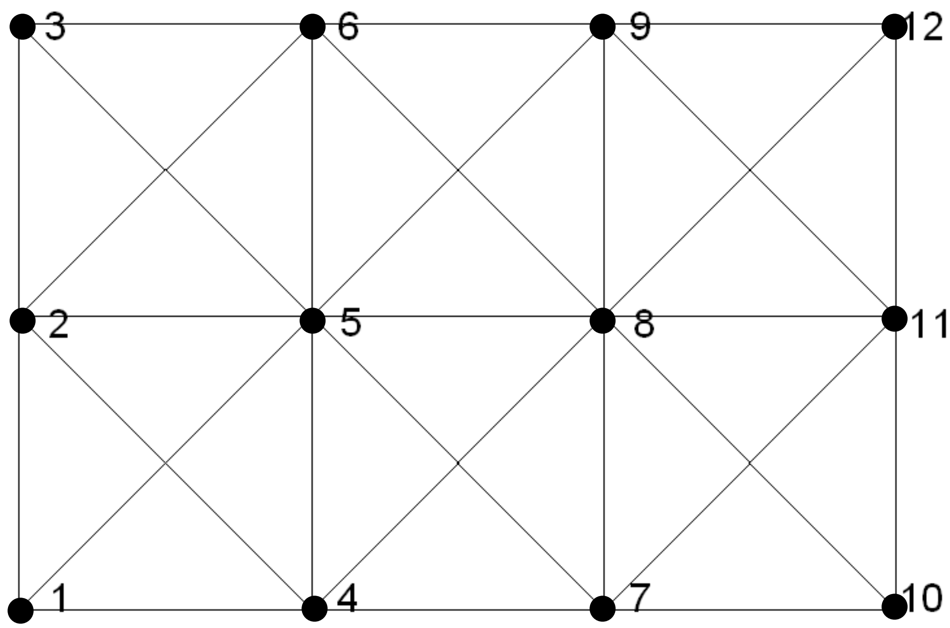


Figure 4.1 The 12 node design domain

The function of the structure under design is to produce a displacement at certain points in the domain under external or internal forces. For example, the horizontal displacement at Node 10 serves as an output and a force is applied on Node 1 horizontally (Figure 4.2). It is further required that the horizontal displacement at Node 10 be as large as possible and the number of elements be the same as prescribed. There is also a spring put on Node 10 and its orientation is horizontal, which is in agreement of the horizontal displacement at Node 10. The constraint imposed to this structure is as follows: (1) the displacement of

Node 3 is completely restrained in both horizontal and vertical directions and (2) the vertical displacements of Nodes 1,4,7,0 are vertically restrained (Figure 4.2).

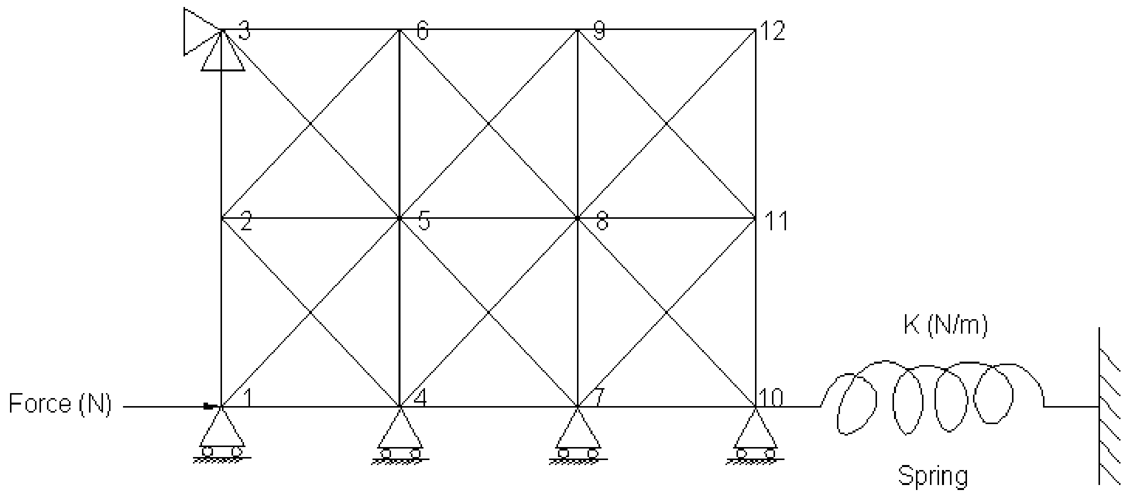


Figure 4.2 Required functions and constraints of the structure of Figure 4.1

The model can then be expressed as follows:

Minimize: $-U_{n,d}$ (4-1a)

Minimize: $|\, ne - de|$ (4-1b)

Subject to: $\sigma_{max}<\sigma_{constraint}$ (4-2)

where n stands for a node number; d stands for the direction of a displacement;$U_{n,d}$ represents the displacement at Node n in Direction d; ne is the number of elements;$\sigma_{constraint}$ is the fatigue strength for a specified of cycles or endurance limit for infinite life and$\sigma_{max}$is the maximum stress.

In order to take into account fatigue failure in the design of compliant mechanisms, it is straightforward to write the fatigue failure criterion in replacement of the above Equation (4-2). The model is a constrained optimization problem. To solve the model, a strategy based on the penalty concept to convert it to an unconstrained optimization problem is taken, namely a generalized objective function is defined as follows:

$$\text{Maximize} f = [w_1 \times f(U)] + [w_2 \times f(\sigma)] + [w_3 \times f(ne)] \qquad (4\text{-}3)$$

In Equation (4-3), f is a generalized function which is also called the fitness function when the genetic algorithm (GA) is employed to solve this optimization problem; f(U) is the fitness function corresponding to the displacement in a specific direction at a concerned node; f(σ) is the fitness function corresponding to the stress constraint; f(ne) is the fitness function corresponding to the number of elements; w_1 (w_2, w_3) are weights corresponding to the foregoing three fitness functions and they reflect the importance of the three fitness functions, respectively, from a designer's point of view. Several remarks on Equation (4-3) are made as follows:

Remark 1: In Equation (4-3) *f(U)* is further defined to be 10× ($U_{n,d}$).Multiplying 10 in the expression is to normalize the ($U_{n,d}$) to make it bounded between 0 and 1.This is further associated with an implicit constraint in the program, which restricts the maximum displacement at the output node – in particular the maximum displacement should be less than 0.1 m (details can be found in Appendix C).

Remark 2: *f(σ)* is the function that describes the strength control and it is defined as follows:

$$f(\sigma)=\begin{cases} 1 & \sigma_{\max} < \sigma_{\text{constraint}} \\ 0 & \sigma_{\max} \geq \sigma_{\text{constraint}} \end{cases} \qquad (4\text{-}4)$$

Note that in this study, endurance limit or fatigue strength of the compliant mechanism in design has considered the stress concentration effect.

Remark 3: *f(ne)* is defined as follows:

$$f(ne) = \frac{1}{e^{\left\{\frac{|ne-de|}{p3}\right\}}} \qquad (4\text{-}5)$$

where *de* is the desired number of elements and *ne* is the actual number of elements. It is noted that *f(ne)*is normalized to be in the range of 0 to 1. Further, the parameter *p3* is a parameter that is determined empirically with reference to the overall fitness function. The function of (4-5) is plotted as shown in Figure 4.3. In this study, *p3* was taken as 10 (a detailed reason is seen from Appendix C).

Remark 4: The parameters w_1, w_2 and w_3 are the weighting factors for *f(U)*, *f(σ)* and *f(ne)*, respectively. They are determined by the user based on the experience or by more sophisticated procedures such as the Pareto set theory.

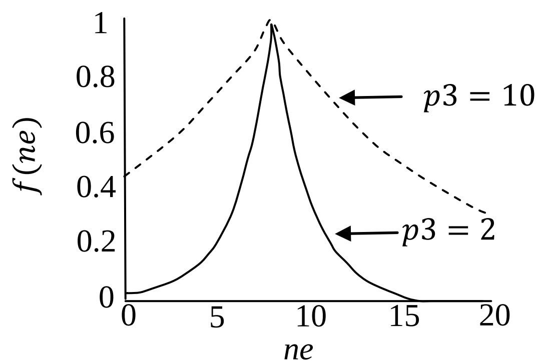
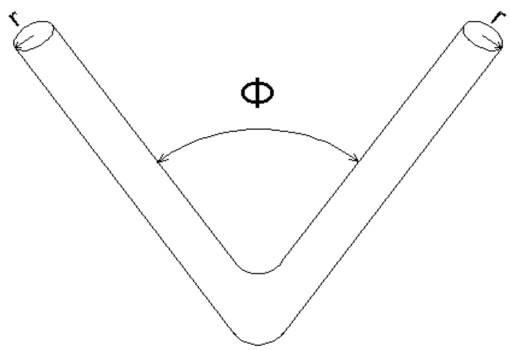


Figure 4.3 The relationship between number of elements in the compliant mechanisms and *f(ne)* (adapted from Larsen and Sindholt, 2003)

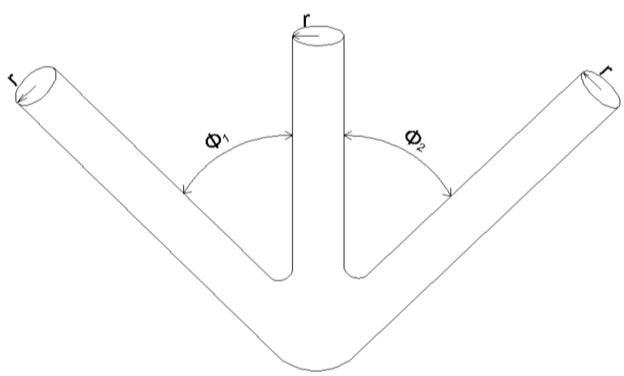
4.3 Detailed Calculation of the Fitness Function: Calculation of f(σ)

Endurance limit or fatigue strength, and maximum stress of each element change with cross sectional area and its connection situation with other nodes. In this computational model, cross sectional area is prescribed, that is, it is not a variable to be optimized.

Figure 4.4 and Figure 4.5 show several situations at the end node of an element, which are characterized by the number of other elements, the angles between any two elements, and cross section areas of the connecting elements. In this study, the different situations at the end node of an element were not taken into account when endurance limit or fatigue strength and maximum stress of each element are calculated. Each element was considered in the context where the one end of the element is fixed and free at the other end; i.e., the situation as shown in Figure 4.6.

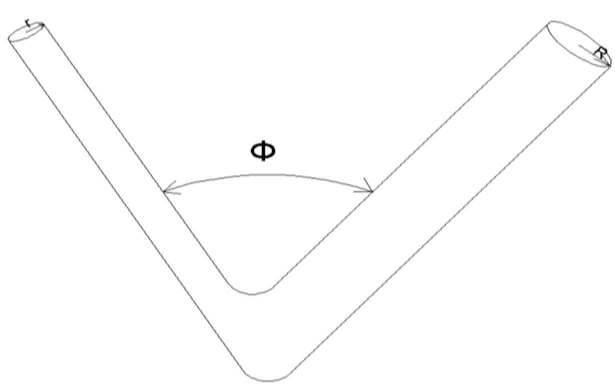


(a)

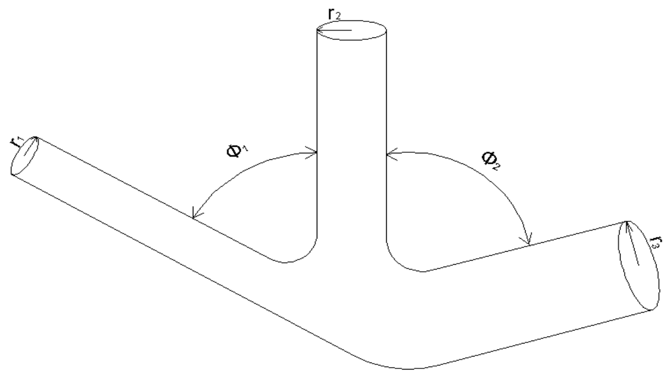


(b)$\varphi_1 \neq \varphi_2$

Figure 4.4 Different numbers of elements at one node with the same cross sectional area



(a) $r \neq R$



(b) $r_1 \neq r_2 \neq r_3$, $\varphi_1 \neq \varphi_2$

Figure 4.5 Different numbers of elements at one node with different cross sectional areas

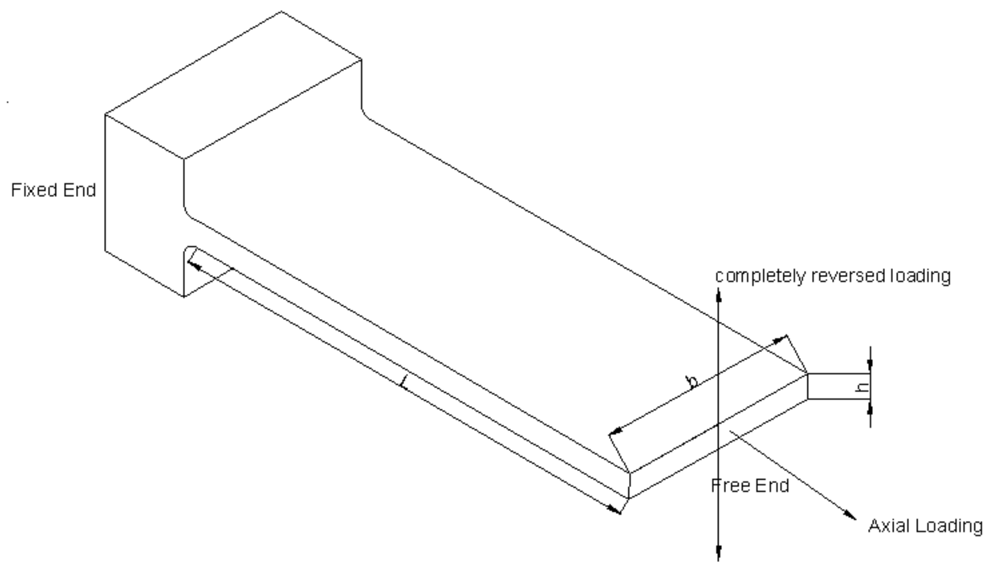


Figure 4.6 The assumption of an element in the compliant mechanism

In this assumption, the free end is subjected to a completely reversed loading and an axial loading at the same time. For the details of internal stress analysis of the beam element, the interested reader refers to Appendix B. In this case, the maximum stress will happen on the outer surface of the fixed end, as shown in Figure 4.7 (ANSYS analysis).

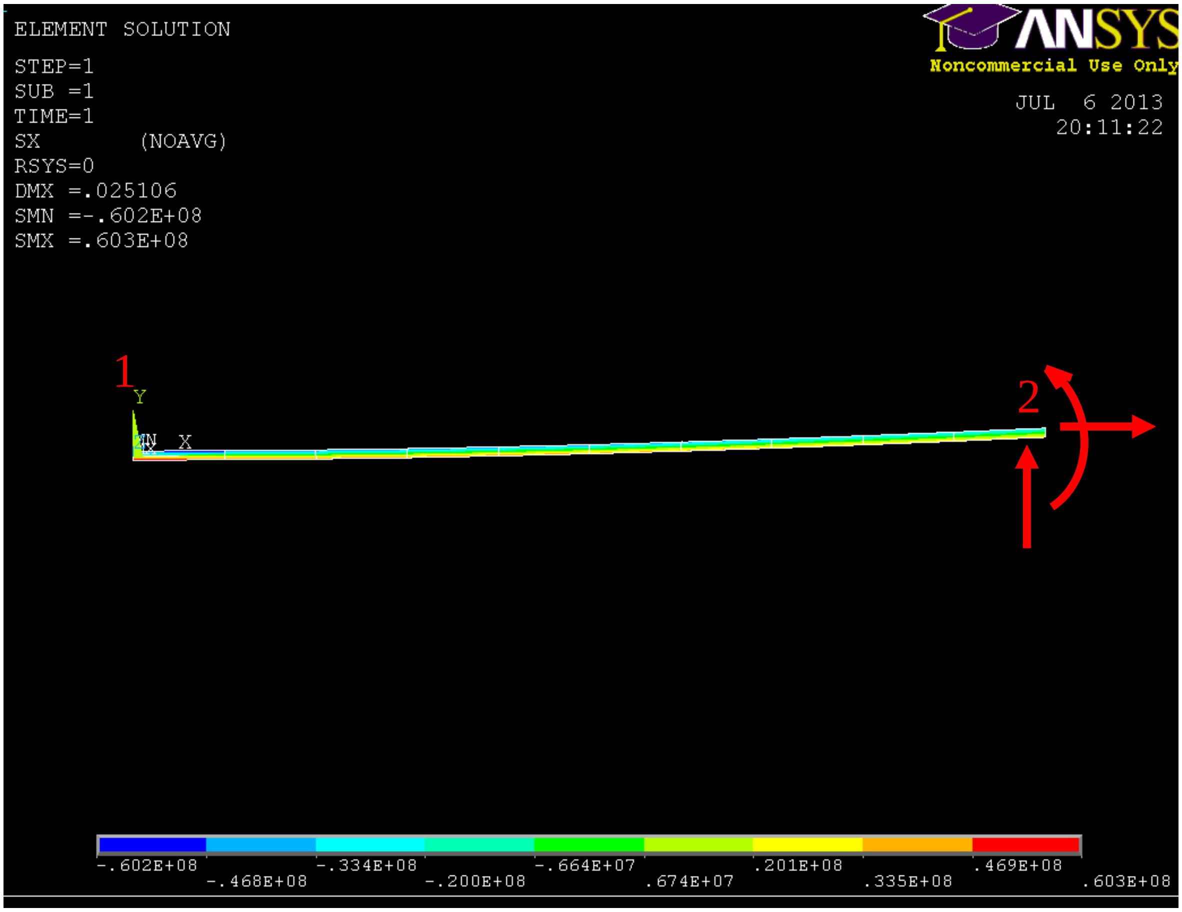


Figure 4.7 Stress distribution of a beam element

In Figure 4.7, node 1 is the fixed end and node 2 is free end. A force of 5000 N is put on the free end node 2. From finite element analysis, we can see that the maximum stress is placed on the outside surface of the fixed end node 1. The ANSYS result was further validated by a manual calculation (see Appendix D).Furthermore, in Appendix D, the calculation of the endurance limit for the beam in Figure 4.6 is also included.

With the aforementioned assumption, the stress concentration K_t has not considered the effect of different connection situations in Figure 4.4 and Figure 4.5. That is K_t is assumed constant for all these situations. Further, according to Howell (2001), K_t is chosen to be 1.6 for compliant mechanisms.

4.4 Loading Condition Assumption

Fatigue failure is caused by repetitive loading conditions. The repetitive loadings can be divided into two categories: completely reversed and fluctuating. In the completely reversed loading condition (Figure 4.8), safety factor (SF) is computed by

$$SF=\frac{S_e}{\sigma_{max}}, \tag{4-6}$$

or

$$SF=\frac{S_f}{\sigma_{max}}. \tag{4-7}$$

Where, S_e and S_f are endurance limit or fatigue strength, respectively, and σ_{max} is the maximum stress in the system. If SF > 1, the system has an infinite life; otherwise a finite life.

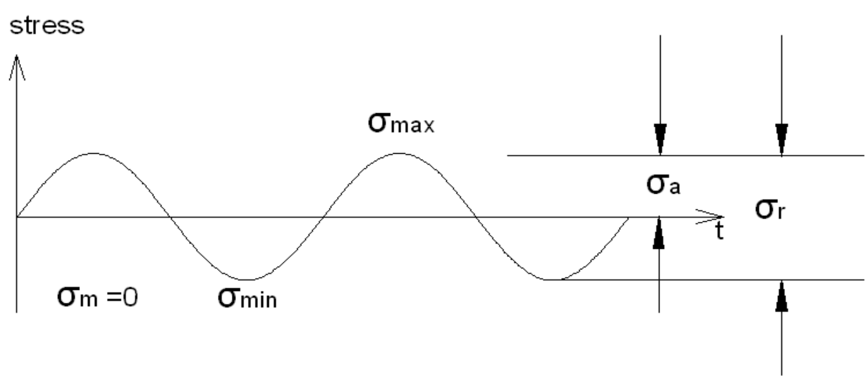


Figure 4.8 Completely Reversed Loading

In the fluctuating stress condition (Figure 4.9), mean stress σ_m and alternating stress σ_a are used to describe the characteristics of the stress, and defined as follows:

$$\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2} \tag{4-8}$$

$$\sigma_a = \frac{\sigma_{max}-\sigma_{min}}{2} \tag{4-9}$$

SF can be found from the so-called, modified Goodman, which is:

$$\frac{1}{SF} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \tag{4-10}$$

or

$$\frac{1}{SF} = \frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_{ut}} \tag{4-11}$$

If $\frac{1}{\mathrm{SF}} > 1$, the system has a finite life and otherwise an infinite life.

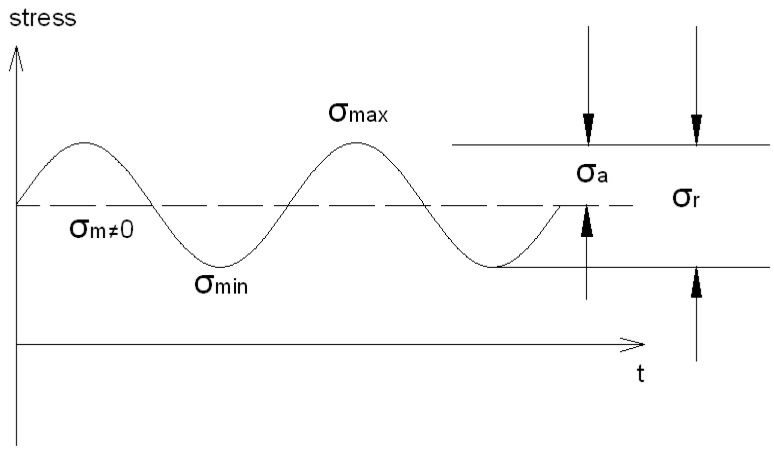


Figure 4.9 Fluctuating Stress

In this study, only the completely reversed loading condition was considered without loss of generality.

4.5 Flowchart of the Computer Program for the Model

To realize this computational model, a MATLAB code was written. The flowchart of the code is illustrated in Figure 4.10. The design variable in this model is the presence or absence of element. In MATLAB, the existence of element is represented by bit code that consists 0 and 1, where 0 is the absence of element and 1 is the presence of element (see the previous discussion in Chapter 2).The design variables or GA codes were updated in every generation of the GA codes (see Appendix A.1). The GA algorithm selected parents based on the fitness value and parents will reproduce, crossover and mutate the next child generation. The convergence criterion in the GA iteration is the number of generations.

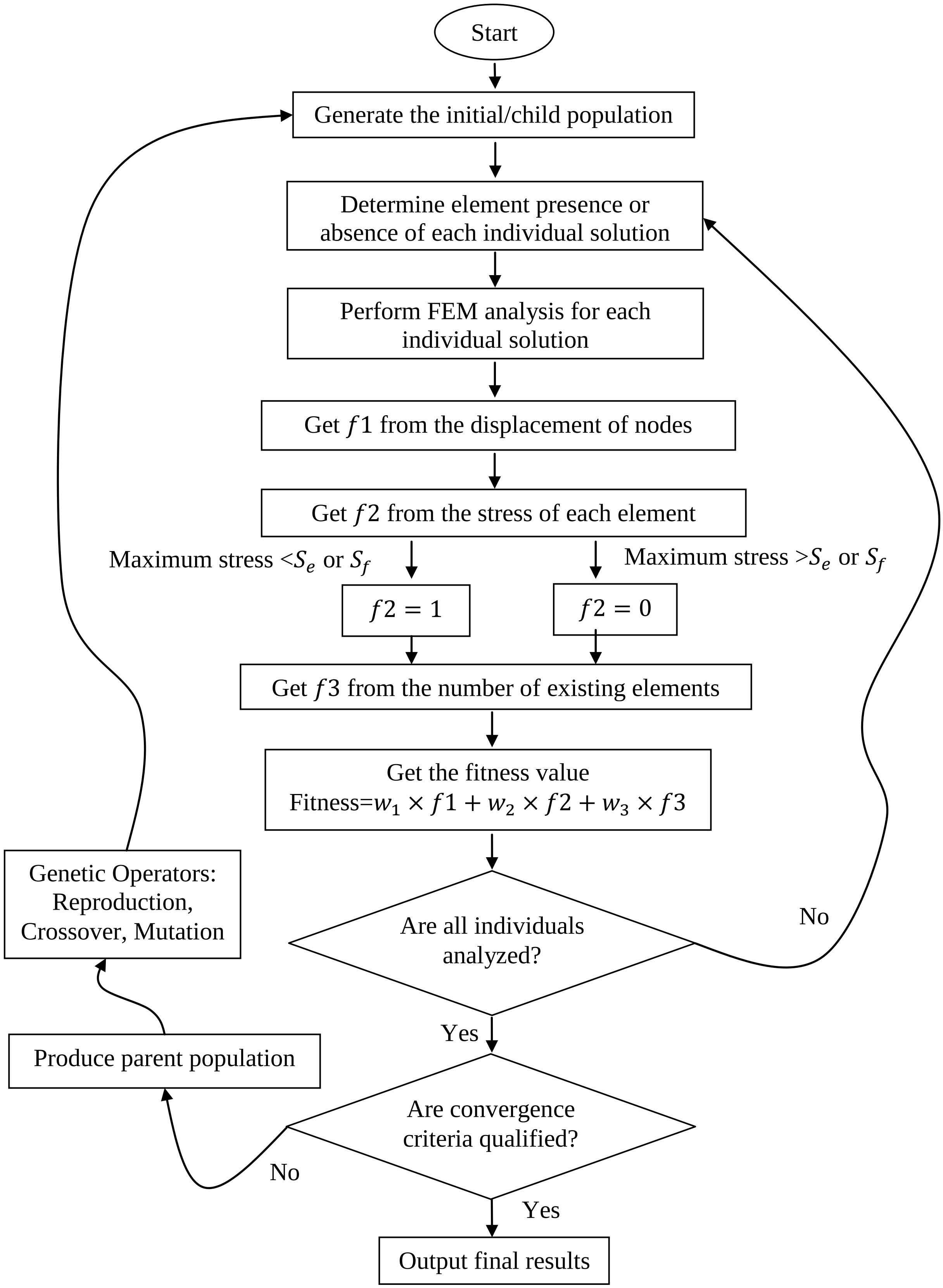


Figure 4.10 The flowchart of the computer code

4.6 Summary and Discussion

This chapter presented a computational model for designing compliant mechanisms with consideration of fatigue strength control. The model was based on the GSA. A design domain was meshed by beam elements. The objective functions included three parts: the displacement at a particular place, the fatigue strength control, and the total number of elements. The “best” design was expressed by the minimum number of beam elements, the maximum displacement at the place and the fatigue strength control. The implementation of the model was done in the MATLAB environment (see Appendix A).

The model developed in this thesis is different from the model developed by Larsen and Sindholt (2003) in several areas. First, the element in this study is beam element as opposed to the truss element in Larsen and Sindholt (2003); the beam element is more accurate in representing the physics of the structure or mechanism. Second, fatigue strength control is realized in this thesis; in their model, the strain constraint was included for however a reason other than stress failure control but geometrical integrity control (see also the respective discussion in Appendix C).

CHAPTER 5RESULTS AND DISCUSSION

5.1 Introduction

In this chapter, design examples are discussed to validate if the computer code developed for designing compliant mechanisms under fatigue strength control is an effective tool. The second purpose of this chapter is to study the implication of assumptions behind the method and code developed in this thesis. The general procedure of validation is as follows. First, the requirement for the case design is described. Second, the method and code developed in this thesis is applied to the design problem, resulting in the design. Finally, a general-purpose computer code ANSYS is used to compute the maximum stress and displacement to compare them with the ones obtained by the design software developed in this thesis. For convenience of the following discussion, the software that implements the model as described in Chapter 4 is called the **CMFTO** (Compliant Mechanisms design under Fatigue strength control using Topology Optimization).

5.2 Design Case I

The material used in this model is 1010 HR steel. Its Young’s Modulus is 200GPa, Poisson ratio is 0.29, and Ultimate strength is 320MPa. The surface of the material is made as hot rolled finish. Stress concentration factor is 1.6 according to Howell (2001), and the reliability is supposed to be 99.99% according to Howell (2001).The length of each element is 1m or 1.44m in the model. Cross sectional area is a constant: 1E-3 m^2.

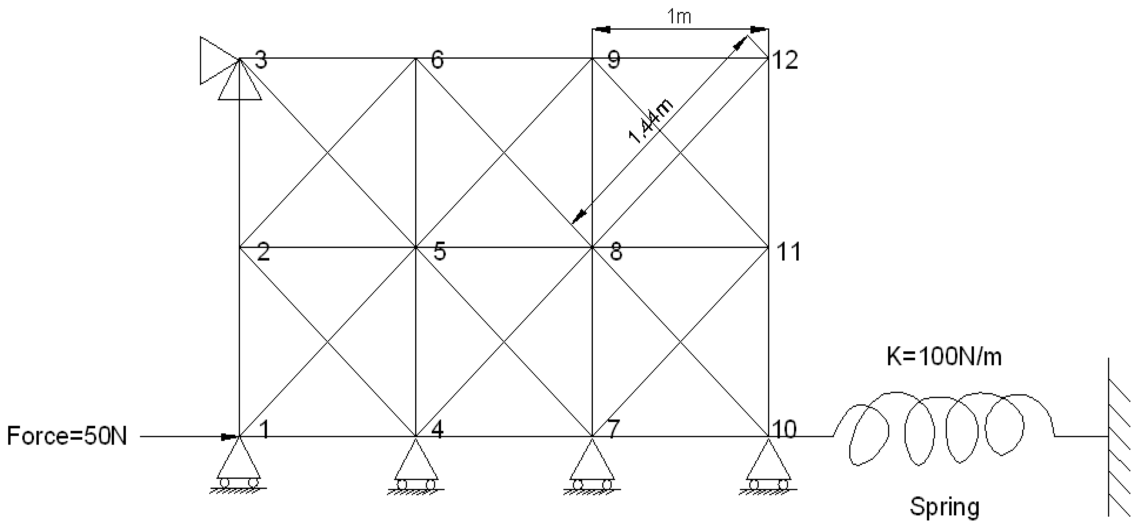


Figure 5.1 The design domain with initial boundary condition

5.2.1 Design requirement

Requirement:

Minimize: $-U_{n,d}$ (5-1a)

Minimize: $|ne - de|$ (5-1b)

Subject to:

$\sigma_{max} < \sigma_{constraint}$ (5-2a)

de=15 (de: the desired number of elements) (5-2b)

Fitness function:

$$f = w_1 \times f(U) + w_2 \times f(\sigma) + w_3 \times f(ne) \quad (5\text{-}3)$$

Let (depending on the designer or user)

$$w_1 = 1$$

$$w_2 = 5$$

$$w_3 = 1$$

Since the value of $f(U), f(\sigma)$ and $f(ne)$fall to the range of [0,1], the maximum fitness value should be $1\times 1 + 1 \times 5 + 1 \times 1 = 7$.

GA parameters (details for these selections can be found in Appendix E):

Number of individuals (N_{ind}): 1000

Number of generations (N_{gen}): 50

Mutation rate: 0.1

Crossover rate: 1

Boundary conditions and input forces:

As shown in Figure 5.1, a force 50 N is put in at node 1, and a spring with 100 N/m stiffness is attached at node 10 on one side, and on the other side is fixed. Nodes 1, 4, 7 and 10 are vertically constrained. Node 3 is completely constrained. The horizontal output displacement is at node 10.

5.2.2 Result

Figure 5.2 shows the result of the design with CMFTO. The solid line is the un-deformed state and dash line is the deformed state. The output at node 10 has a small displacement, 0.0492 m. From the definition of compliant mechanisms, this displacement comes from the deformation of compliant mechanisms rather than rigid body motion. It can be seen that nodes 1,4,7 and 10 move in the *x* direction only and no displacement in the *y* direction and no rotation. Node 3 is fixed and has no displacement. The final results show that fitness value is 6.4920, and the maximum stress is 3.5456E7 Pa. The endurance limit S_e is 5.6205E7 Pa. Therefore, the maximum stress of the mechanism is less than endurance limit. The compliant mechanism has an infinite life.

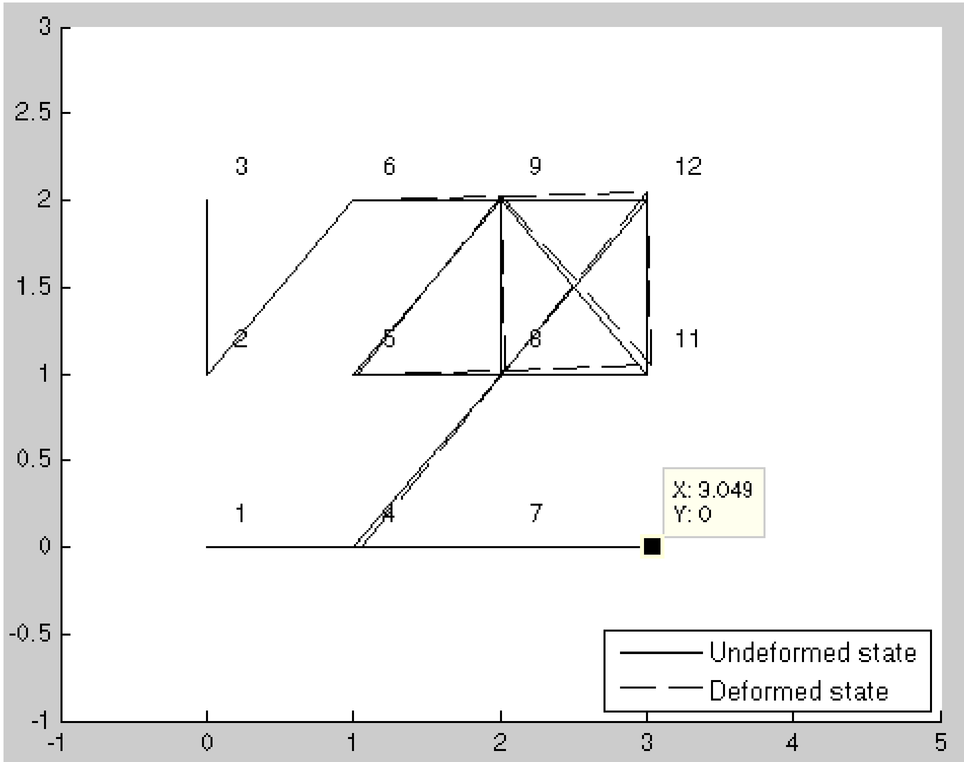


Figure 5.2 CASE I: Optimization result of compliant mechanism from MATLAB

5.2.3 ANSYS result

In order to verify the effectiveness of the CMFTO, ANSYS was used to compute the maximum stress, deformation and displacement of the mechanism generated by the design system. Figure 5.3 shows the result of the deformation of the compliant mechanism. The white solid line is the un-deformed state, and the dark blue line is the deformed state. It is seen that the ANSYS calculated deformation is very close to the deformation from the CMFTO. Table 5.2 shows nodes output displacement. In particular, the maximum displacement of the optimized compliant mechanism is 0.0566 m, the displacement at node 10 calculated with ANSYS is 0.049375 m which is very close to the displacement 0.04920 m calculated with the CMFTO.

Table 5.1 Nodes output displacement of Case I

Node	Output Displacement
1	0.49376e-1
2	0.31377e-1
3	0
4	0.49375e-1
5	0.32735e-2
6	0.31377e-1
7	0.49376e-1
8	0.22784e-2
9	0.22786e-2
10	0.49375e-1
11	0.31377e-1
12	0.22786e-2
13	0

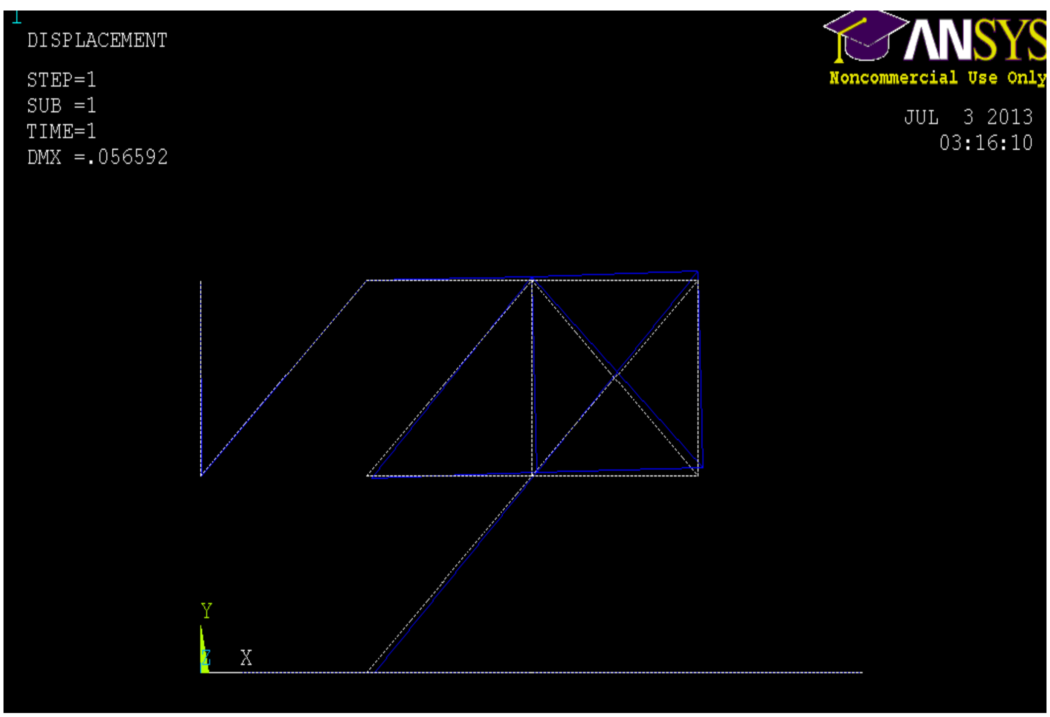


Figure 5.3 Deformation and displacement of the CASE I optimized compliant mechanism

Figure 5.4 shows the internal stress distribution of the compliant mechanism calculated with ANSYS. The maximum stress is 3.55E7 Pa, which is the very close to the maximum stress 3.5456E7 Pa calculated with the CMFTO. Additionally, from Figure 5.4, the maximum stress happens at node 4. Therefore, node 4 is the most vulnerable point of the compliant mechanism. More materials can be put on node 4 to strengthen it.

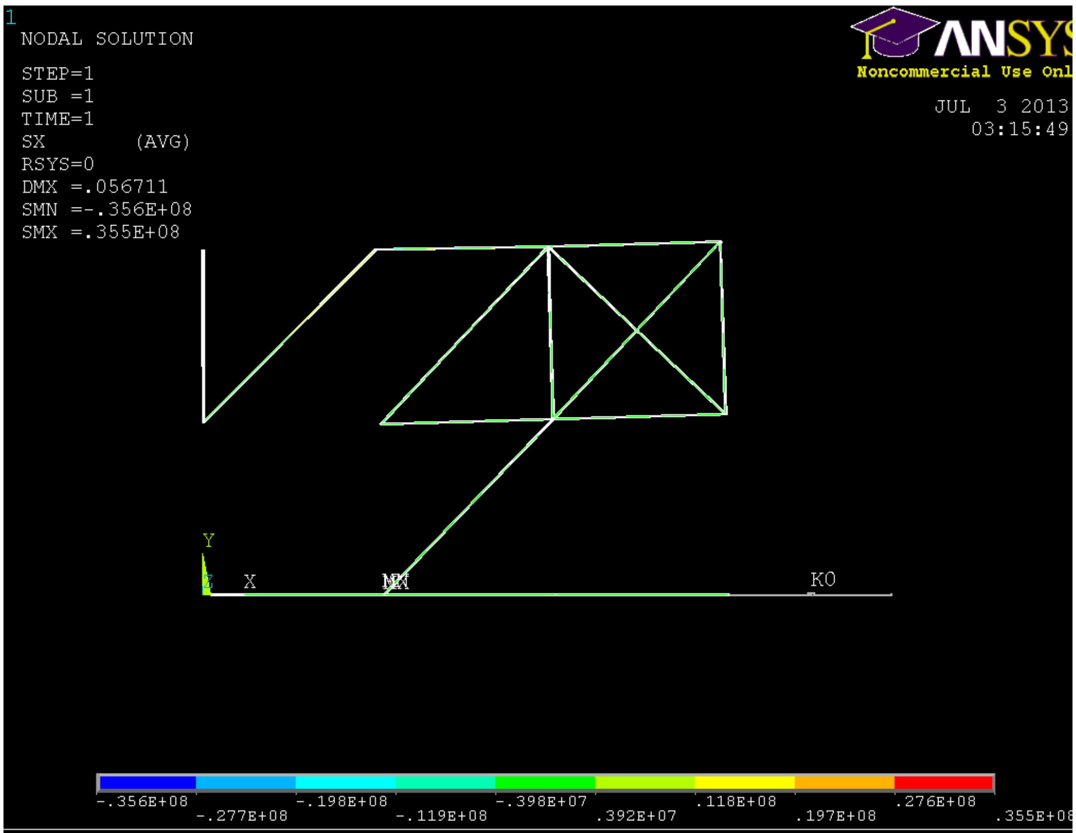


Figure 5.4 Stress distribution of the CASE I compliant mechanism

5.3 Design Case II

The material used in this model is 1010 HR steel. The properties of the material are the same as those of design case I.

5.3.1 Design Requirement

The functional requirement, fitness function and boundary conditions are the same as those of design case I.

5.3.2 Result

The result with the CMFTO is shown in Figure 5.5. The solid line is the un-deformed state and the dash line is the deformed state. With the input force of 50 N at node 1, the output at node 10 has a small displacement, 0.0455 m. From the definition of compliant mechanisms, this displacement comes from the deformation of compliant mechanisms rather than rigid body motion. It can be seen that nodes 1,4,7 and 10 move in the *x* direction only and no displacement on the *y* direction and no rotation. Node 3 is fixed and has no displacement. The final results show that fitness value is 6.4549, and the maximum stress is 3.443E7 Pa. The endurance limit or fatigue strength S_e is 5.6205E7 Pa. Therefore, the maximum stress of the compliant mechanism 3.443E7 Pa is less than endurance limit 5.6205E7 Pa. The compliant mechanism has an infinite life.

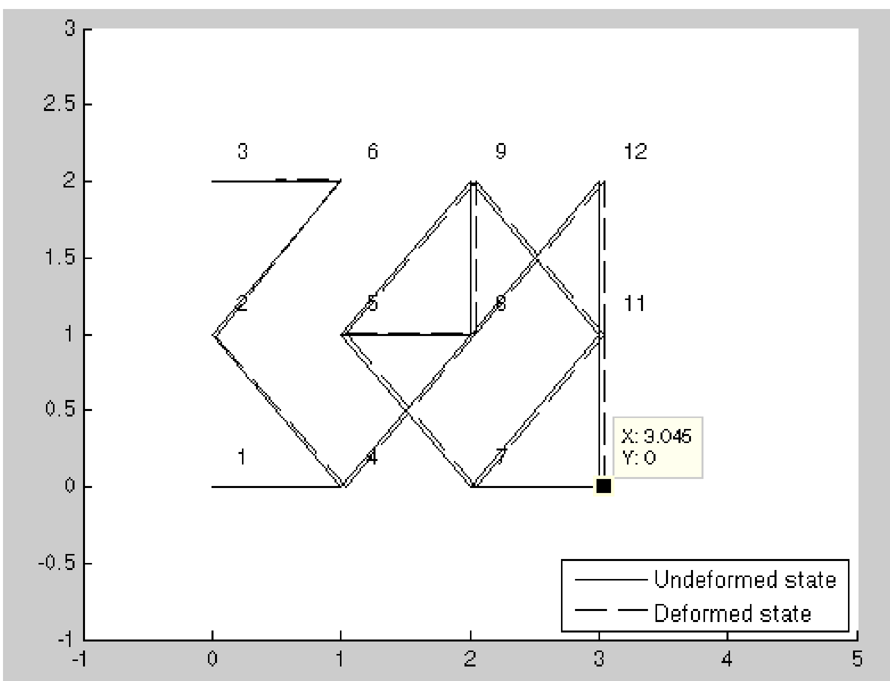


Figure 5.5 CASE II: optimization result of compliant mechanism from MATLAB

5.3.3 ANSYS result

Just the same as what was performed for the first experiment, verification was performed with ANSYS. Figure 5.6 shows the deformation and displacement of the compliant mechanism. The white solid line is the un-deformed state and the blue line is the deformed state. Table 5.2 shows nodes output displacement. It can be seen that at node 10, the deformation 0.045651 m is almost the same as the deformation0.0455 m resulting from the CMFTO.

Table 5.2 Nodes output displacement of Case II

Node	Output Displacement
1	0.45651e-1
2	0.22717e-6
3	0
4	0.45651e-1
5	0.45839e-1
6	0.45839e-1
7	0.45651e-1
8	0.46027e-1
9	0.45839e-1
10	0.45651e-1
11	0.46027e-1
12	0.28069e-2
13	0

Figure 5.7demonstrates the internal stress distribution of the compliant mechanism. The maximum stress is 3.46E7 Pa, which is very close to the maximum stress calculated via CMFTO. Moreover, from Figure 5.6, we can see that the maximum stress happened at node 2. Therefore, node 2 is the most vulnerable point of the compliant mechanism, and more materials can be put on node 2 to strengthen it.

Figure 5.6 Deformation and displacement of CASE II optimized compliant mechanism

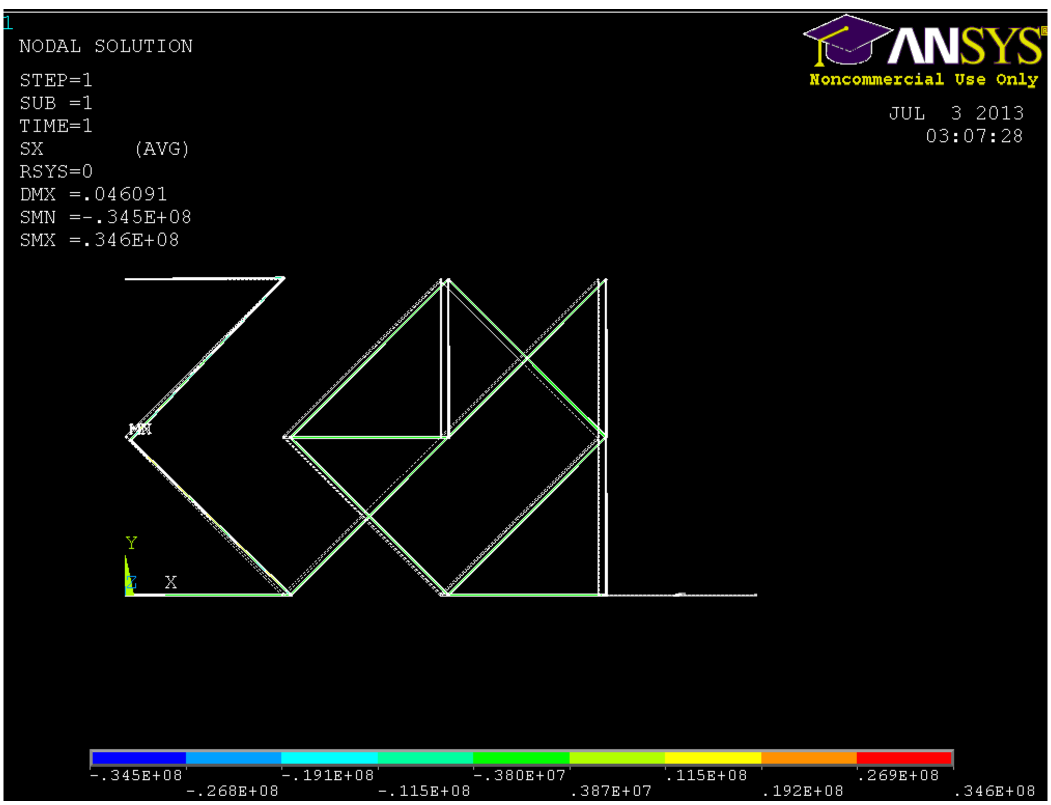


Figure 5.7 Stress distribution of the CASE II compliant mechanism

5.4 Design Case III with different cross sectional area

In order to see the influence of cross sectional area of beam element on the design of compliant mechanisms, another two cases with different cross sectional areas of beam element were made, and they are called Case IIIa and Case IIIb, respectively. The material used in these two cases is 1010 HR steel, and the properties of the material for the two cases are the same. The functional requirement, fitness function and boundary conditions, except cross sectional area of beam element, are the same as CASE I and II for these two cases.

5.4.1 Beam element with cross sectional area of 1E-5 m^2: Case IIIa

The result of this Case IIIa resulting from the CMFTO is shown in Figure 5.8. The solid line is the un-deformed state and the dash line is the deformed state. With the input force of 50 N at node 1, the output at node 10 has a small displacement, 0.001 m. From the definition of compliant mechanisms, this displacement comes from the deformation of compliant mechanisms rather than rigid body motion. It can be seen that nodes 1,4,7 and 10 all move in the *x* direction only and no displacement on the *y* direction and nor does the rotation. Node 3 is fixed and has no displacement. The final results show that fitness value is 6.0067, and the maximum stress is 1.459E7 Pa. The endurance limit S_e is 6.4463E7 Pa. The maximum stress of the compliant mechanism is 1.459E7 Pa, which is less than the endurance limit 6.4463E7 Pa. The compliant mechanism has an infinite life. Compared with case I and II, this compliant mechanism has a pretty small displacement, and besides, the maximum stress is much less than endurance limit. This implies that there are still rooms to improve this compliant mechanism.

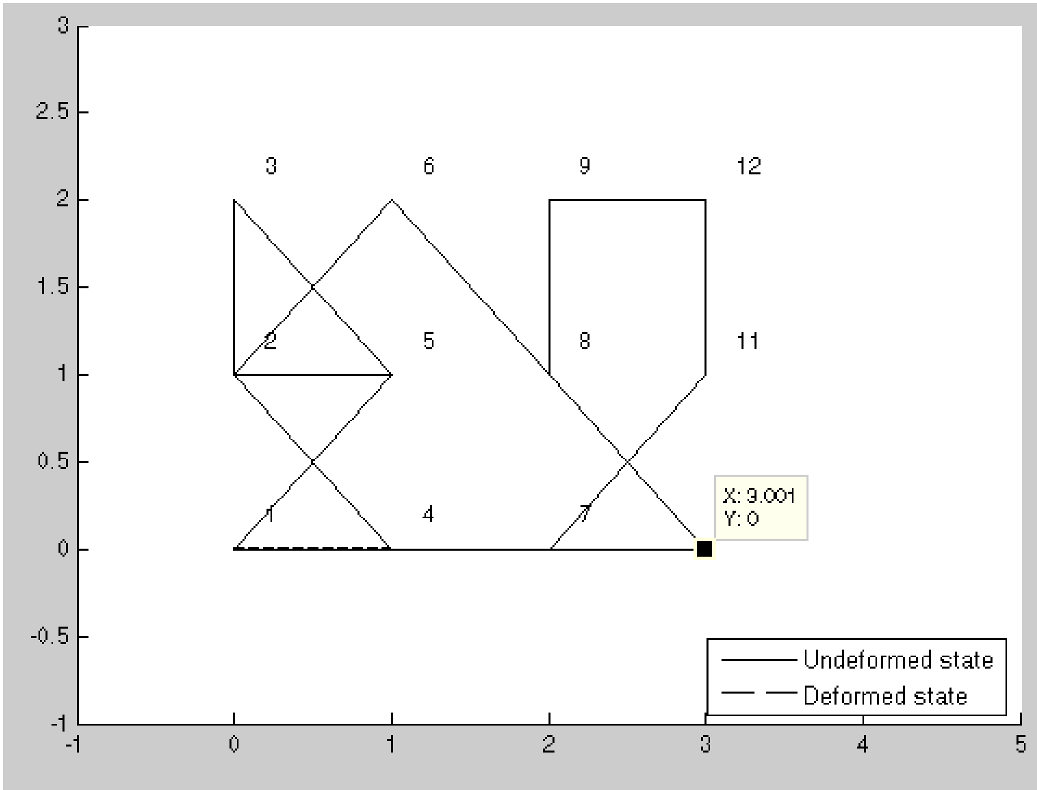


Figure 5.8 A compliant mechanism with beam element's cross sectional area of 1E-5 m^2

5.4.2 Beam element with cross sectional area of 1E-2 m^2: Case IIIb

The result of Case IIIb resulting from the CMFTO is shown in Figure 5.9.

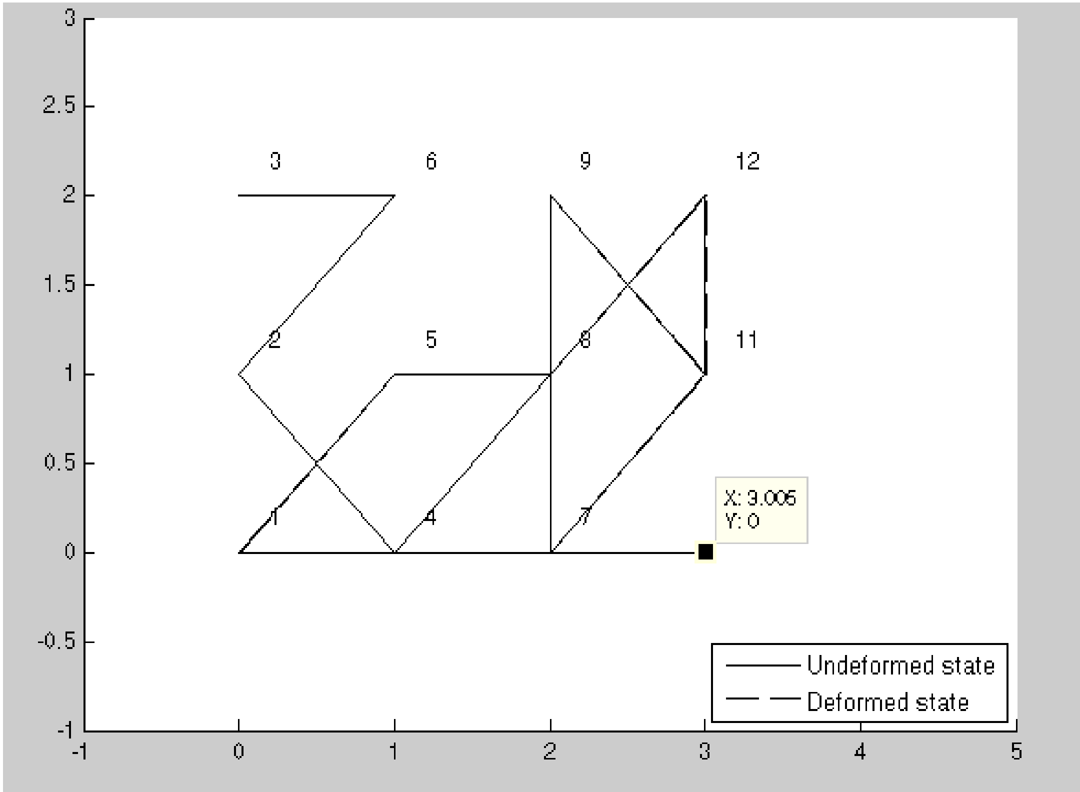


Figure 5.9 A compliant mechanism with beam element's cross sectional area of 1E-2 m^2

The solid line is the un-deformed state and the dash line is the deformed state. With the input force of 50 N at node 1, the output at node 10 has a small displacement, 0.005m. From the definition of compliant mechanisms, this displacement comes from the deformation of compliant mechanisms rather than rigid body motion. It can be seen that nodes 1,4,7 and 10 all move in the *x* direction only and no displacement in the *y* direction and does nor the rotation. The final results show that fitness value is 6.0499, and the maximum stress is 3.75E6 Pa. The endurance limit S_e is 3.8678E7 Pa. Therefore, the maximum stress of the compliant mechanism 3.75E6 Pa is less than endurance limit 3.8678E7 Pa. The compliant mechanism has an infinite life. Compared with case I and II, this compliant mechanism has a pretty small displacement. Besides, the endurance limit is 10 times larger than the maximum stress. This implies that there are still rooms to improve this compliant mechanism.

5.4 Conclusion

The CMFTO developed in this study is effective in designing compliant mechanisms under fatigue strength control based on the simulated verification with ANSYS. From the excellent agreement between the result from the CMFTO and ANSYS it can be concluded that the assumptions made in this study are reasonable.

Table 5.1 summarizes the result of all the cases. From this table it can be seen that the cross section area is an important variable for compliant mechanisms, and this variable is coupled with the configuration of the mechanism or structure. This further implies that the design can be further improved by taking the cross section area as a variable to be optimized.

Table 5.3 Summary of case study

Case	Cross section area (m^2)	Number of elements	Maximum displacement (m)	Maximum stress (Pa)	Endurance limit (Pa)
I	1E-3	15	0.0492	3.5456 E7	5.6205E7
II	1E-3	15	0.0455	3.4430 E7	5.6205E7
IIIa	1E-5	15	0.001	1.459 E7	6.4463 E7
IIIb	1E-2	15	0.005	3.750 E6	3.8678 E7

CHAPTER 6CONCLUSION AND FUTURE WORK

6.1 Overview and Conclusions

This thesis presented a comprehensive study on design of compliant mechanisms under fatigue strength control. The design technique employed in the thesis is topology optimization. The study was motivated by two observations. The first observation is that technology for design of compliant mechanisms using topology optimization (TO) to prevent fatigue failure was not available in literature. The second observation is that fatigue failure is highly likely in operations of compliant mechanisms. The general research question of the thesis was then: how can the fatigue strength control be incorporated in the TO design of compliant mechanisms?

The overall objective of the work described in this thesis was to develop a computational framework for design of compliant mechanisms under fatigue strength control by means of topology optimization techniques. By computational framework it was meant that various specific design methods for fatigue failure control can be integrated with a common computational service such as model formulation and model solving. To achieve the overall objective, specific objectives are listed below.

Objective 1: To formulate a general computational model for design of compliant mechanisms under fatigue strength control using the topology optimization technique.

Objective 2: To implement the model in a general-purpose computational facility such as MATLAB and to demonstrate the application of the model or framework.

Considering the difficulty of computing fatigue strength of a general compliant mechanism, this study considered the compliant mechanism which is composed of truss or beam elements. As such, the ground structure approach of the TO technique in literature was employed for designing compliant mechanisms.

These objectives have been achieved, and the detailed description of the work related to these objectives was documented in the preceding chapters. In particular, after a comprehensive literature review presented in Chapter 2, the originality of the study was confirmed. Chapter 3 has provided a discussion of fatigue strength control theory in a general mechanical system, which concludes that the stress-life method is suitable for being incorporated in compliant mechanism design under fatigue strength control. Further in this chapter, the calculation of endurance limit or fatigue strength and its modification factors were explained in detail, including comments on previous work in this area. Chapter 4 presented the model and its implementation. Chapter 5 discussed the examples to verify the effectiveness of the model and to demonstrate the design of compliant mechanisms under fatigue strength control using the ground structure approach of the TO technique.

The work leads to the following conclusions: (1) It is feasible to include the fatigue strength control in design of compliant mechanisms using the ground structure approach of the TO technique. (2) The computer program that realizes the ground structure approach of the TO is a viable tool for practical applications in designing a compliant mechanism under the fatigue strength control.

6.2 Contributions

There are two contributions out of this study, and they are discussed as follows. The first contribution is the provision of the computational framework for designing compliant mechanisms under fatigue strength control. The computational model in the framework also allows for achieving a pre-defined number of elements used in the system, which is in line with design practice in particular an interactive design process (some are controlled by designers). Indeed, to the author's best knowledge, there is nobody who has incorporated fatigue strength control in designing compliant mechanisms using the TO technique. The second contribution is the experiment of the feasibility to implement the model in the MATLAB environment which is widely used for engineering analysis.

6.3 Future Work

Several future works are expected to improve this study further. First, sensitivity of the initial scheme in terms of the number of connections needs to be examined. Currently, only a 12-node and 29-connection example structure was considered. However, there may be more connections such as full connections (Figure 6.1). A legitimate question may be whether a further optimal design can be achieved and how significant is with this design.

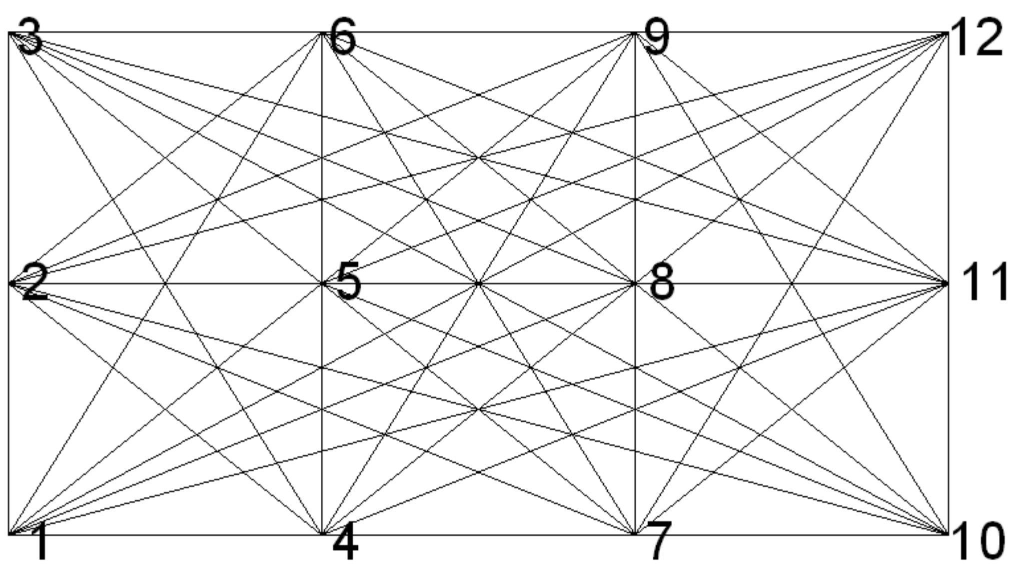


Figure 6.1 The 49 connections between 12 nodes

Second, if the number of nodes is increased, such as 24 nodes (Figure 6.2) or more, further optimal compliant mechanisms may be produced, and this problem is worth further study.

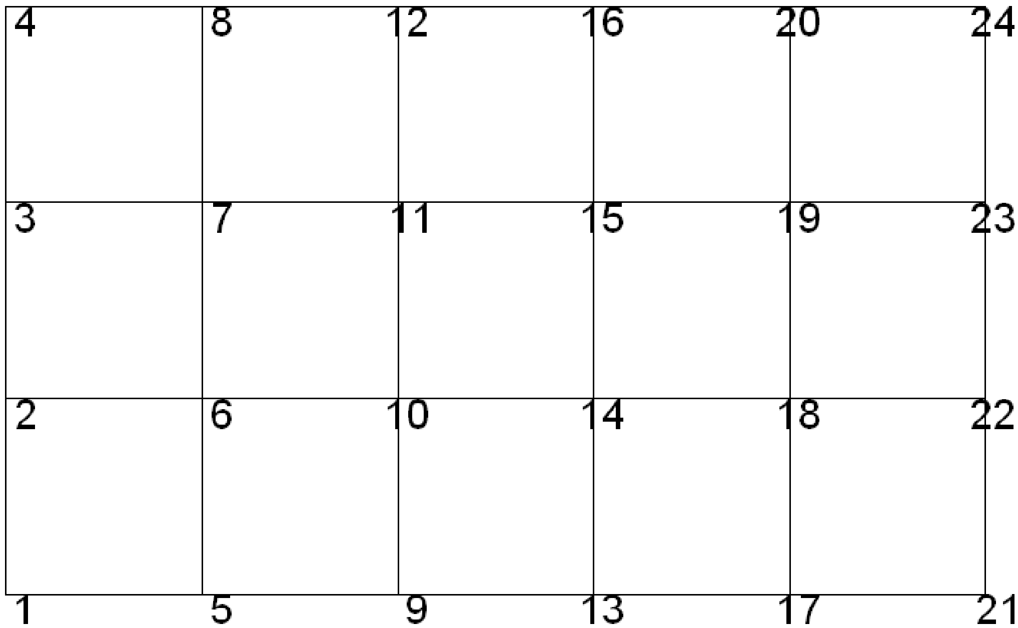


Figure 6.2 The design domain with 24 nodes

Third, there are some assumptions in this work related to the computation of fatigue stress. Among them, the one that assumes that each element is connected with one other element is most significant in that it seems to depart most away from the actual situation. A sort of "superposition" idea may improve the accuracy, which considers a modification coefficient for different situations of the end connection to an element. This will in particular affect stress concentration modification factor. At present, this factor is taken as a constant (1.6 in particular), which departs away from the real situation significantly. The future work is warranted to address this shortcoming. The general idea is to develop the relation between this factor and the node connection situations (see Figure 4.4 and Figure 4.5), and then in the iteration process, this factor takes different values with respect to different node connection situations.

Finally, inclusion of cross section area as a variable to be optimized is warranted for a further investigation. The general idea is that the optimization goes two layers. The first layer is to set cross section area as a variable and the second layer is to take the configuration as a variable.

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APPENDIX A

Computer Code for Compliant Mechanisms Design under Fatigue Strength Control

A.1 Genetic algorithm routine (adapted from Larsen et al., 2003)

```
Nind=1000 ; %number of individuals in population
Lind=size ( IXFull , 1 ) ; %length of individuals ( number of 'truss'  )
Base=2 ; %base o f the chromosome elements
Nsel=Nind ; %SubPopulation (Number of individuals from rws )
mutRate=0.1 ; %Mutation Rate
crossRate=1 ; %crossover Rate
gen=50; %Generations
Bars=15 ;
d=0;
keepBars=1 ; %The construction must contain ¡¯ Bars ¡¯ bars in the end . (boo lean )
symmetric=0 ; %Construction symmetric or not .
Sut=320;
Kf=1.6;
aa=58.1; %Hot-rolled steel surface modification factor in fatigue strength calculation
bb=-0.719; %Hot-rolled steel surface modification factor in fatigue strength calculation
p3=10 ; %Curvature of 1/ exp ( x/p3 ) for number of elements
%weights for fitness functions
w1=1 ; %for f1
w2=5 ; %for f2
w3=1 ; %for f3
%resetting variables
bestConstrFit=0 ;
bestConstr=[ ] ;
bestGen=[] ;
fitness=0 ;
plotMfit=[ ] ;
plotAfit=[ ] ;
plotd=[];
oldBest=[ ] ;
oldBestFit=0 ;
```

```
thetam=[];
h=0.01;
if symmetric
newPop=makePopBars(Nind,Lind/2,Bars/2) ;
else
newPop=makePopBars(Nind,Lind,Bars) ;
end ;
for a=1:(gen+1)
Pop=newPop ;
Chrom=newA;
FitnV=[ ] ;
thetamall=[];
FitnD=[ ] ;
maxtheta1=[ ];
maxtheta2=[ ];
Uall=[];
for p=1:Nind
%Make construcion from bitstring
if symmetric
[IX]=makeConstrSym(IXFull,Pop(p,:)) ;
else
[IX]=makeConstr(IXFull,Pop(p,:)) ;
end
IX=connectioncheck(X,IX,cmbound,symbound,S,P);
IX=connectioncheck(X,IX,cmbound,symbound,S,P);
% Check that DispNode is in construction
for i=1:size(boundednode,1)
    [row,col]=find(IX(:,1:2)==boundednode(i));
    if isempty(find(IX(row,3) == 1, 1) )
        boundError = 1 ;
    else
        boundError = 0 ;
    end
end
d=0;
[row,col] =find(IX(:,1:2)==DispNode) ;
if isempty(find(IX(row,3) == 1, 1) )
dispError = 1 ;
else
dispError = 0 ;
end
% Check that ForceNode is in construction
forceError =0 ;
for i=1:length(ForceNode)
[row,col]=find(IX(:,1:2)==ForceNode(i)) ;
   if isempty(find(IX(row,3)==1,1))
```

```
        forceError=1 ;
    else
        forceError=0 ;
    end;
end ;
%Reset Error
Error=0 ;

%f1  fitness from displacement

d= U(3*DispNode-3+DispDir,1);

if (dispError||forceError||boundError||d>1e-1)
f1 = 0 ;
else
f1 =10*d ;
end ;

%f2- fitness from stress

thetamtotal1=[];
thetamtotal2=[];
for e=1:size(IX,1)
if IX(e,3)==1
dx = X(IX(e,2),1)-X(IX(e,1),1) ;
dy = X(IX(e,2),2)-X(IX(e,1),2) ;
L=sqrt((dx^2+dy^2));
Sin=dy/L;
angle=(asin(Sin))*180/pi;
E=mprop(IX(e,3),1);
Se=ftgstr(Sut,aa,bb,A,Kf);
b=A/h;
I=b*h^3/12;
uu=[U(3*IX(e,1)-2); U(3*IX(e,1)-1); U(3*IX(e,1)); U(3*IX(e,2)-2); U(3*IX(e,2)-1);
U(3*IX(e,2))];
f=PlaneFrameElementForces(E,A,I,L,angle,uu);
m1=abs(f(3));
m2=abs(f(6));
axial1=abs(f(1));
axial2=abs(f(4));
thetam1=(6*m1/(b*h^2))/1;
thetam2=(6*m2/(b*h^2))/1;

axialm1=(axial1/A)/0.85;
axialm2=(axial2/A)/0.85;
thetamall1=thetam1+axialm1;
```

```
thetamall2=thetam2+axialm2;
thetamtotal1=[thetamtotal1;thetamall1];
thetamtotal2=[thetamtotal2;thetamall2];
if isempty(thetamall1||thetamall2)
    f2 = 0 ;
else
    if  (thetamall1/Se<1)&&(thetamall2/Se<1)
        f2=1;

    else
        f2=0;
        break;
    end
end
end
end
maxindtheta1=max(thetamtotal1);
maxindtheta2=max(thetamtotal2);
maxtheta1=[maxtheta1;maxindtheta1];
maxtheta2=[maxtheta2;maxindtheta2];


%f3 - fitness from number of bars
if keepBars
  if symmetric
nbars=calcBars(IXFull,Pop(p,:)) ;
f3=1/exp((abs(nbars-Bars))/p3 ) ;
  else
nbars=length(find(Pop(p,:)==1)) ;
f3 = 1/exp((abs(nbars-Bars))/p3 ) ;
  end
else
  f3=0;
end

fitness=w1*f1+w2*f2+w3*f3;
if Error || (fitness<0) || isempty(DispNode )
fitness = 0 ;
end ;
FitnV=[FitnV;fitness ] ;
FitnD=[FitnD;d ] ;
if fitness>(bestConstrFit+1e-5)
bestConstr=Pop(p,:) ;
bestConstrFit = fitness ;
bestGen=[bestGen;a] ;
end ;
```

```
end;


%Elitist model
[currentBestFit,currentBestIndex]=max(FitnV) ;
currentBest=Pop(currentBestIndex,:) ;
currentBestD=FitnD(currentBestIndex,:);
currentBestmaxtheta1=maxtheta1(currentBestIndex,:);
currentBestmaxtheta2=maxtheta2(currentBestIndex,:);
edoff=[3*12*(currentBestIndex-1)+1 3*12*(currentBestIndex-1)+2
3*12*(currentBestIndex-1)+3  3*12*(currentBestIndex-1)+4 3*12*(currentBestIndex-
1)+5 3*12*(currentBestIndex-1)+6 3*12*(currentBestIndex-1)+7
3*12*(currentBestIndex-1)+8 3*12*(currentBestIndex-1)+9 3*12*(currentBestIndex-
1)+10 3*12*(currentBestIndex-1)+11   3*12*(currentBestIndex-1)+12
3*12*(currentBestIndex-1)+13    3*12*(currentBestIndex-1)+14
3*12*(currentBestIndex-1)+15   3*12*(currentBestIndex-1)+16
3*12*(currentBestIndex-1)+17    3*12*(currentBestIndex-1)+18
3*12*(currentBestIndex-1)+19   3*12*(currentBestIndex-1)+20
3*12*(currentBestIndex-1)+21    3*12*(currentBestIndex-1)+22
3*12*(currentBestIndex-1)+23   3*12*(currentBestIndex-1)+24
3*12*(currentBestIndex-1)+25    3*12*(currentBestIndex-1)+26
3*12*(currentBestIndex-1)+27   3*12*(currentBestIndex-1)+28
3*12*(currentBestIndex-1)+29    3*12*(currentBestIndex-1)+30
3*12*(currentBestIndex-1)+31   3*12*(currentBestIndex-1)+32
3*12*(currentBestIndex-1)+33    3*12*(currentBestIndex-1)+34
3*12*(currentBestIndex-1)+35   3*12*(currentBestIndex-1)+36];
currentBestUall=Uall(edoff(1:1:36),:);
if oldBestFit>currentBestFit
Pop(1,:)=oldBest ;

FitnV(1,:)=oldBestFit ;
FitnD(1,1)=oldBestD;
maxtheta1(1,:)=oldBestmaxtheta1;
maxtheta2(1,:)=oldBestmaxtheta2;
edofff=[1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17 18  19  20  21  22  23
24  25  26  27  28  29  30  31  32  33  34  35  36];
Uall(edofff(1:1:36),:)=oldBestUall;
else
oldBest=currentBest ;
oldBestFit=currentBestFit ;
oldBestD=currentBestD;
oldBestmaxtheta1=currentBestmaxtheta1;
oldBestmaxtheta2=currentBestmaxtheta2;
oldBestUall=currentBestUall;
end;
%Generate plot-vectors
```

```
plotMfit=[plotMfit,max(FitnV)] ; %MaxFit-vector
plotAfit=[plotAfit,mean(FitnV)] ; %MeanFit-vector
plotd=[plotd,max(FitnD)];

%Genetic operators
%Reproduction
if max( FitnV)~= 0
rwsPop = rws (FitnV , Nsel ) ;
reprPop = Pop( rwsPop , : ) ;

else
reprPop=Pop ;
end;

%Crossover
crosPop=xovsp(reprPop,crossRate) ;

%Mutation
 if rand<(0.09*a+0.19)/50
       mutRate=round(20*rand);
    end
mutPop=mut(crosPop,mutRate) ;
mutA=mut(crosA,mutRate) ;
newPop=mutPop ;
newA=mutA ;
[M,J]=max(FitnV);
bestthetam1=maxtheta1(J,1);
bestthetam2=maxtheta2(J,1);
disp([' It.: ' sprintf('%4i',a) ' Obj.: ' sprintf('%10.4f',max( FitnV))  ' u.: '
sprintf('%10.4f',FitnD(J,1)) ' st.1: ' sprintf('%10.4f', bestthetam1) ' st.2: ' sprintf('%10.4f',
bestthetam2)])

end ;

% Find the best construction in the last population.
[bestFitLast,bestIndexLast]=max(FitnV);
bestConstrLast=Pop(bestIndexLast,:);
maxtheta1last=maxtheta1(bestIndexLast,1);
maxtheta2last=maxtheta2(bestIndexLast,1);
disp([' best gen.: ' sprintf('%4i',bestIndexLast)  '  best Obj.: ' sprintf('%10.4f',bestFitLast) '
best u.: ' sprintf('%10.4f',FitnD(bestIndexLast,1))  ' st.1: ' sprintf('%10.4f',maxtheta1last) '
st.2: ' sprintf('%10.4f',maxtheta2last) ])
% Generate construction-vectors from bit-pattern
if symmetric
[IX]=makeConstrSym(IXFull,bestConstrLast) ;
else
```

```
[IX]=makeConstr(IXFull,bestConstrLast) ;
end ;
%Plot Un-deformed and Deformed Compliant Mechanism
figure(1) ;
clf ;
hold on

for e=1:ne
if IX(e,3)==1
xx = X(IX(e,1:2),1) ;
yy = X(IX(e,1:2),2) ;
p1Lbl=IX(e,1);
x1Cor=X(IX(e,1),1)+0.2 ;
y1Cor=X(IX(e,1),2)+0.2 ;
p2Lbl=IX(e,2);
x2Cor=X(IX(e,2),1)+0.2;
y2Cor=X(IX(e,2),2)+0.2 ;
plot(xx,yy,'k-','LineWidth',1)
edof=[3*IX(e,1)-2 3*IX(e,1)-1 3*IX(e,1) 3*IX(e,2)-2 3*IX(e,2)-1 3*IX(e,2)] ;
xx=xx+Uall(edof(1:3:6)) ;
yy=yy+Uall(edof(2:3:6)) ;
plot(xx,yy,'k--','LineWidth',1)
t1=text(x1Cor,y1Cor,num2str(p1Lbl)) ;
t2=text(x2Cor,y2Cor,num2str(p2Lbl)) ;
end
end


legend( 'Undeformed state' , 'Deformed state' , 4 )
axis([-1 5 -1 3]) ;
hold off

%Plot Un-deformed Compliant Mechanism

figure(2) ;
clf ;
hold on


for e=1:ne
if IX(e,3)==1
xx = X(IX(e,1:2),1) ;
yy = X(IX(e,1:2),2) ;
p1Lbl=IX(e,1);
x1Cor=X(IX(e,1),1)+0.2 ;
y1Cor=X(IX(e,1),2)+0.2 ;
```

```
p2Lbl=IX(e,2);
x2Cor=X(IX(e,2),1)+0.2;
y2Cor=X(IX(e,2),2)+0.2 ;
plot(xx,yy,'k-','LineWidth',1)

end
end

axis([-1 5 -1 3]) ;
hold off

%Plot Deformed Compliant Mechanism

figure (3);
clf;
hold on


for e=1:ne
if IX(e,3)==1
xx = X(IX(e,1:2),1) ;
yy = X(IX(e,1:2),2) ;
p1Lbl=IX(e,1);
x1Cor=X(IX(e,1),1)+0.2 ;
y1Cor=X(IX(e,1),2)+0.2 ;
p2Lbl=IX(e,2);
x2Cor=X(IX(e,2),1)+0.2;
y2Cor=X(IX(e,2),2)+0.2 ;

edof=[3*IX(e,1)-2 3*IX(e,1)-1 3*IX(e,1) 3*IX(e,2)-2 3*IX(e,2)-1 3*IX(e,2)] ;
xx=xx+Uall(edof(1:3:6)) ;
yy=yy+Uall(edof(2:3:6)) ;
plot(xx,yy,'k--','LineWidth',1)
t1=text(x1Cor,y1Cor,num2str(p1Lbl)) ;
t2=text(x2Cor,y2Cor,num2str(p2Lbl)) ;
end
end

axis([-1 5 -1 3]);
hold off
```

A.2 Boundary conditions (adapted from Larsen et al., 2003)

```
clear all

% Nodal Coordinates X( x , y )
```

```
X =[0.0 0.0;0.0 1.0;0.0 2.0;1.0 0.0;1.0 1.0;1.0 2.0;2.0 0.0;2.0 1.0;2.0 2.0;3.0 0.0;3.0
1.0;3.0 2.0] ;
% Topology matrix IX( node1 , node2 , propno )
IXFull = [1 2 1 ;2 3 1 ;1 4 1 ;3 6 1 ;1 5 1 ;3 5 1;2 4 1;2 6 1;2 5 1;4 5 1;5 6 1;4 7 1 ;6 9 1;4
8 1;6 8 1;5 7 1;5 9 1;5 8 1;7 8 1;8 9 1;7 10 1;9 12 1;7 11 1;9 11 1;8 10 1;8 12 1;8 11 1;10
11 1;11 12 1] ;
% Element property matrix mprop = [ E A ] ,
mprop = [200.0e9 10e-4;1.0 0.10e-9] ;
% Boundary conditions bound (node,dof,disp) ,
cmbound =[3 1 0;3 2 0;3 3 0];
symbound=[1 2 0;1 3 0;4 2 0;4 3 0;7 2 0;7 3 0;10 2 0;10 3 0];
bound=[cmbound;symbound];
% Actuation forces P(node,dof,force) ,
P = [1 1 50];
% Extraspring S(node,dof,stiffness) ,
S = [10 1 100];
% Optimization node and direction
DispNode=S(1,1);%Optimized displacement node
DispDir=S(1,2); %Optimized displacement direction
ForceNode=[1];
ForceDir=[1];
boundednode=[3];
```

A.3 Beam element analysis(adapted from Larsen et al., 2003)

```
% Linear analysis for beam structure
ndof=size(X,1)*3; % Number of degrees of freedom
ne=size(IX,1);% Number o f elements
nb=size(bound,1); % Number o f boundaries
ns=size(S,1); % Number of extra spring
sprintf('Number of DOF %d, Number of elements %d',ndof,ne);
% Initialize Global Variables
K=zeros(ndof,ndof);
F(ndof,1)=0; % Force vector
U(ndof,1)=0; % Displacement vector
R(ndof,1)=0; % Residual vector
%Added resetting of F, U, R.
F=zeros(ndof,1);
U=zeros(ndof,1);
R=zeros(ndof,1);
% Initialize Stiffness Matrix
K=zeros(ndof,ndof) ;
% Assemble global stiffness matrix
ke=zeros(6,6);
for e=1:ne
dx=X(IX(e,2),1)-X(IX(e,1),1) ;
```

```
dy=X(IX(e,2),2)-X(IX(e,1),2) ;
E=mprop(IX(e,3),1) ;
A=mprop(IX(e,3),2);

edof=[3*IX(e,1)-2 3*IX(e,1)-1 3*IX(e,1) 3*IX(e,2)-2 3*IX(e,2)-1 3*IX(e,2)] ;
L=sqrt((dx^2+dy^2)) ;
C=dx/L;
Sin=dy/L;
b=A/h;

I=b*h^3/12;
ke=E/L*[A*C*C+12*I*Sin*Sin/(L^2) (A-12*I/(L^2))*C*Sin -6*I*Sin/L -
(A*C*C+12*I*Sin*Sin/(L^2)) -(A-12*I/(L^2))*C*Sin -6*I*Sin/L;
   (A-12*I/(L^2))*C*Sin A*Sin*Sin+12*I*C*C/(L^2) 6*I*C/L -(A-12*I/(L^2))*C*Sin -
(A*Sin*Sin+12*I*C*C/(L^2)) 6*I*C/L;
   -6*I*Sin/L 6*I*C/L 4*I 6*I*Sin/L -6*I*C/L 2*I;
   -(A*C*C+12*I*Sin*Sin/(L^2)) -(A-12*I/(L^2))*C*Sin 6*I*Sin/L
A*C*C+12*I*Sin*Sin/(L^2) (A-12*I/(L^2))*C*Sin 6*I*Sin/L;
   -(A-12*I/(L^2))*C*Sin -(A*Sin*Sin+12*I*C*C/(L^2)) -6*I*C/L (A-
12*I/(L^2))*C*Sin A*Sin*Sin+12*I*C*C/(L^2) -6*I*C/L;
   -6*I*Sin/L 6*I*C/L 2*I 6*I*Sin/L -6*I*C/L 4*I];


K(edof,edof)=K(edof,edof)+ke ;
end
for j=1:nb
K(3*bound(j,1)+bound(j,2)-3,:)=0 ;
K(:,3*bound(j,1)+bound(j,2)-3)=0 ;
K(3*bound(j,1)+bound(j,2)-3,3*bound(j,1)+bound(j,2)-3)=1;
end
for i=1:ns
j=3*S(i,1)+S(i,2)-3;
K(j,j)=K(j,j)+S(i,3) ;
end
if isempty(P)
Error=1;
else
F(3*P(1:size(P,1),1)+P(1:size(P,1),2)-3)=P(1:size(P,1),3) ;
% FE-ANALYSIS
lastwarn ( '') ;
warning off ;
U=K\F;
Uall=[Uall;U];
if isempty( lastwarn )
Error = 0 ;
else
```

```
Error = 1 ;
end;
warning on ;
warning backtrace ;
end
```

A.4 Fatigue strength or endurance limit calculation

```
function Se=ftgstr(Sut,a,b,crossarea,Kf)   % only for steel

if Sut<1400
   St=0.5*Sut;
else
   St=700;
end

Creliab=0.702;   %Suppose the reliability is 99.9%

if a*Sut^b<1
   Csurf=a*(Sut^b);
else
   Csurf=1;
end

d=0.808*((crossarea*10^6)^(1/2));
if d<2.79
   Csize=1;
else
   if d>51
      Csize=0.6;
   else
      Csize=(d/(7.62))^(-0.1133);
   end;
end

Cmisc=1/Kf;
Se=(Csurf*Csize*Creliab*Cmisc*St)*(10^6);
```

A.5 Population generation(Larsen et al., 2003)

```
function [P]= makePopBars(Individuals, Bits, Bars)

   if Bars==0
      Bars=1;
   end
```

```
   onePropr=Bars/Bits;
```

A.6 Renumbering routine (Larsen et al., 2003)

```
function [IXnew]=makeConstr(IXall,Bitstr)

   IXnew=IXall;
   for i=1:length(Bitstr)
      if Bitstr(i)==0
         IXnew(i,3)=2;
      else
         IXnew(i,3)=1;
      end
   end
```

A.7 Symmetric renumbering routine (Larsen et al., 2003)

```
function [IXnew] = makeConstrSym(IXall, Bitstr)
   % Generates sym. IXnew from Bitstr
   IXnew = IXall ;
   for i = 1 : length (Bitstr)
      if Bitstr(i) == 0
         IXnew(2*i-1:2*i,3) = 2;
      else
         IXnew(2*i-1:2*i,3) = 1;
      end
   end
```

A.8 Element calculation(Larsen et al., 2003)

```
function [ bars ] = calcBars(IXFull,ind)
   % calcBars calculates the number of elements present in IND
   % given IXFULL for a symmetric design.
```

```
bars = 0 ;
for i = 1:length(ind)
   if ind (i) == 1
      if isempty(find((IXFull(i*2-1,:) == IXFull(i* 2,:))==0))
         bars=bars+1 ;
      else
         bars = bars + 2 ;
      end ;
   end;
end ;
```

APPENDIX B

Internal Stress Analysis of Compliant Mechanisms

The standard scheme used is shown in Figure B.1. The system is subject to two kinds of loadings. One is axial loading and the other is bending loading. In order to get the idea of internal stress conditions, a small portion of the system is taken out to get stress analysis (Figure B.1).

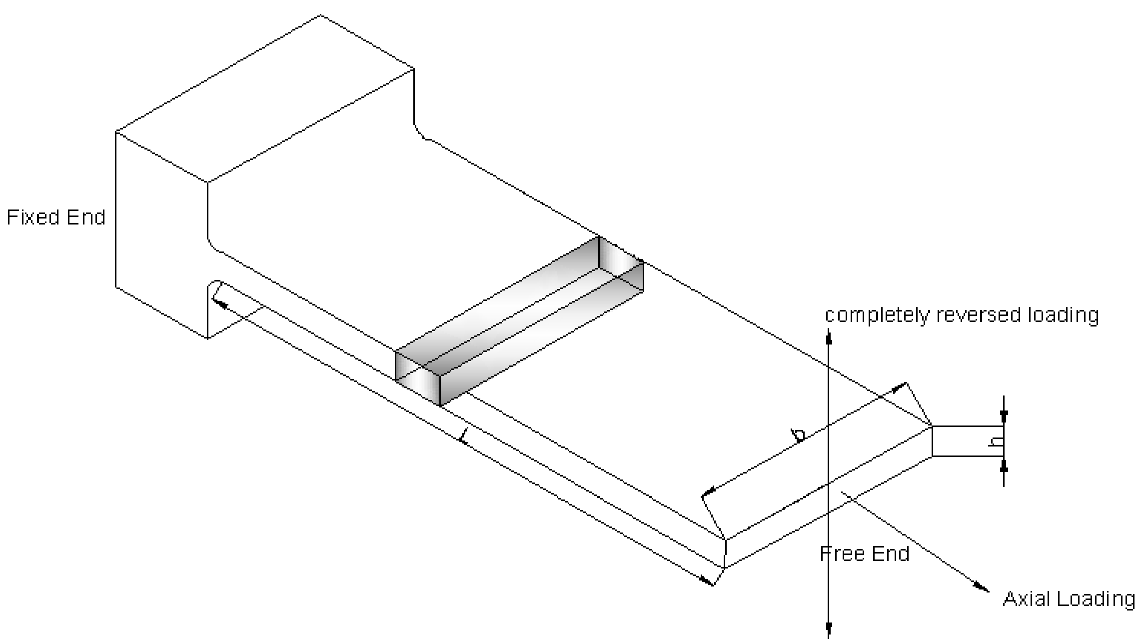
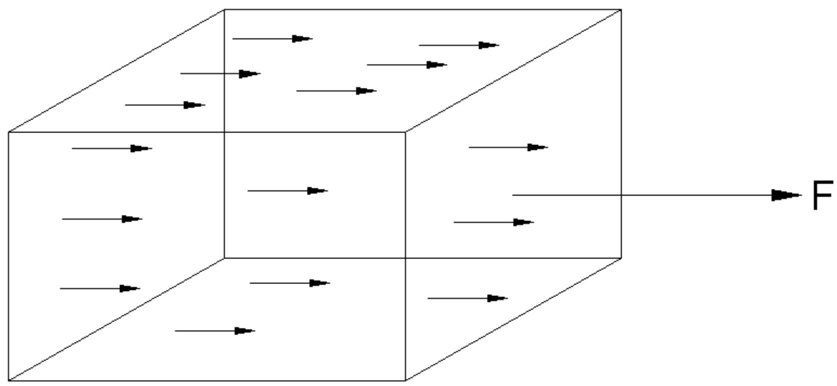
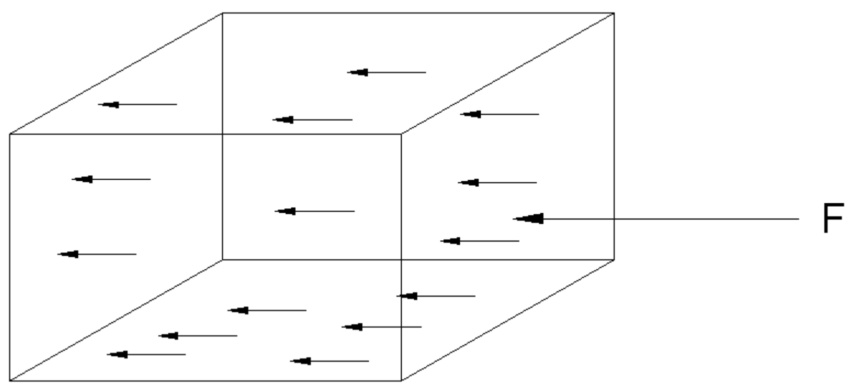


Figure B.1 A small portion of the standard sample

If the system is subject to the pure axial loading (tension and compression), the internal stress distribution is shown in Figure B.2



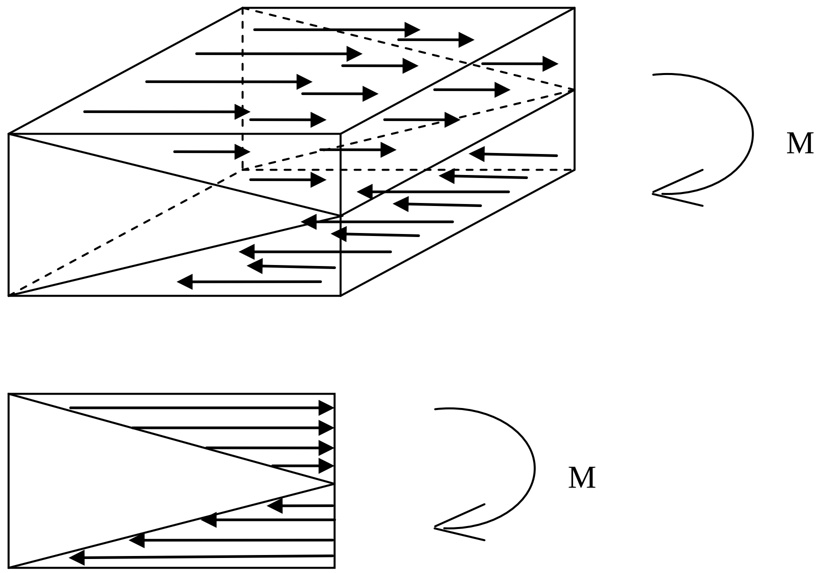
(a) Tension



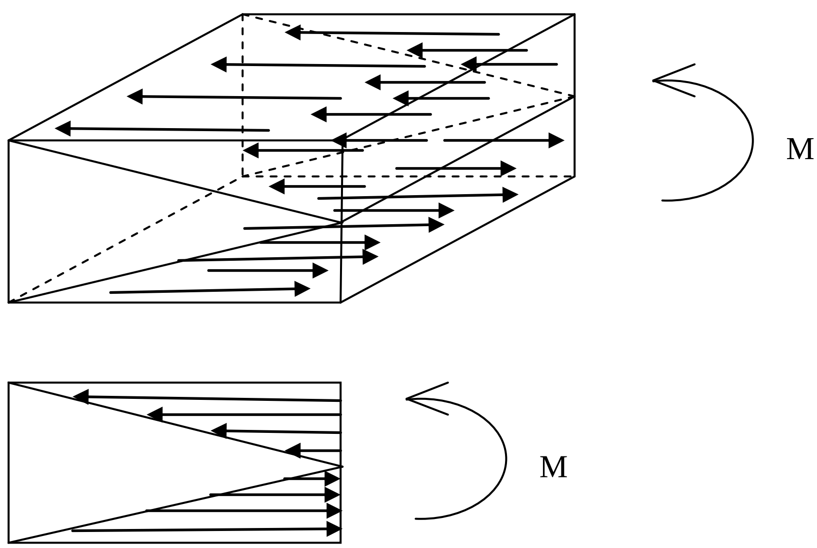
(b) Compression

Figure B.2 The system sample is subject to axial loading

If the system is subjected to bending, the internal stress distribution is shown in Figure B.3.



(a) Clockwise direction



(b) Counter-clockwise direction

Figure B.3 The system sample is subject to bending loading

Shear stress was not considered in this study.

In the axial loading condition, the internal stress is computed by $\sigma = \frac{F}{A}$, where F is the axial force and A is the cross sectional area. In the bending condition, the bending stress is computed by

$$\sigma = \frac{My}{I}, \tag{B-1}$$

where M is the moment, y is the distance from the axis to the point of interest, and I is the moment of inertia of the cross section. The maximum bending stress on the top and bottom surface are computed by

$$\sigma_{max} = \frac{Mc}{I} \tag{B-2}$$

where c is the distance from neutral axis to the outside surface.

In this study, the cross section area was assumed to be rectangular, so the moment of inertia $I=\frac{bh^3}{12}$, where b is the width and h is the height. The distance from neutral axis to the outside surface $c = \frac{h}{2}$, the maximum moment at the fixed end $M=F\times L$, where L is the length of specimen. The maximum bending stress is computed by

$$\sigma_{max}= \frac{6FL}{bh^2} \tag{B-3}$$

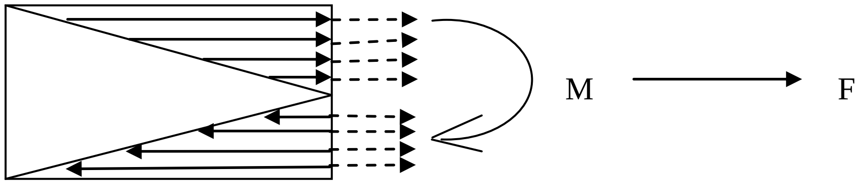
For the beam element in this study, each beam element is subjected to the bending and axial loading conditions. There are four kinds of possible loading conditions for each element:

Axial loading (tension) + clockwise bending;

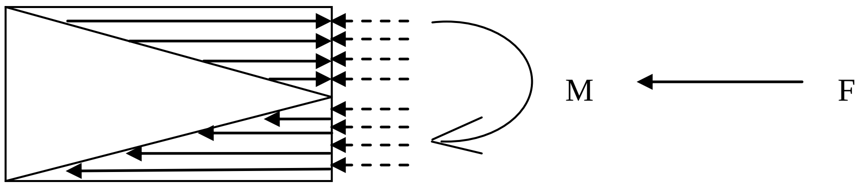
Axial loading (compression) + clockwise bending;

Axial loading (tension) + counter-clockwise bending;

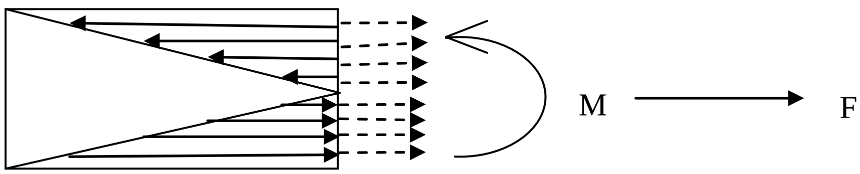
Axial loading (compression) + counter-clockwise bending (Figure B.4).



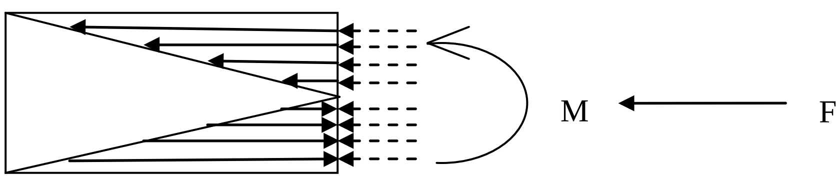
(a) Axial loading (tension) + clockwise bending



(b) Axial loading (compression) + clockwise bending



(c) Axial loading (tension) + counter-clockwise bending



(d) Axial loading (compression) + counter-clockwise bending

Figure B.4 Four loading conditions in compliant mechanisms in this study

For a full cycle, it can be axial loading (tension) + clockwise bending and axial loading (compression) + counter-clockwise bending, or axial loading (compression) + clockwise bending and axial loading (tension) + counter-clockwise bending. Therefore, the maximum internal stress for the system is the maximum bending stress $\sigma_{max\ \_b}+\sigma_{axial}$. The magnitude of maximum stress is the same as the magnitude of minimum stress. The maximum stress in this study was computed by$\sigma_{max} = |\sigma_{max\ _b}|+|\sigma_{axial}|$.

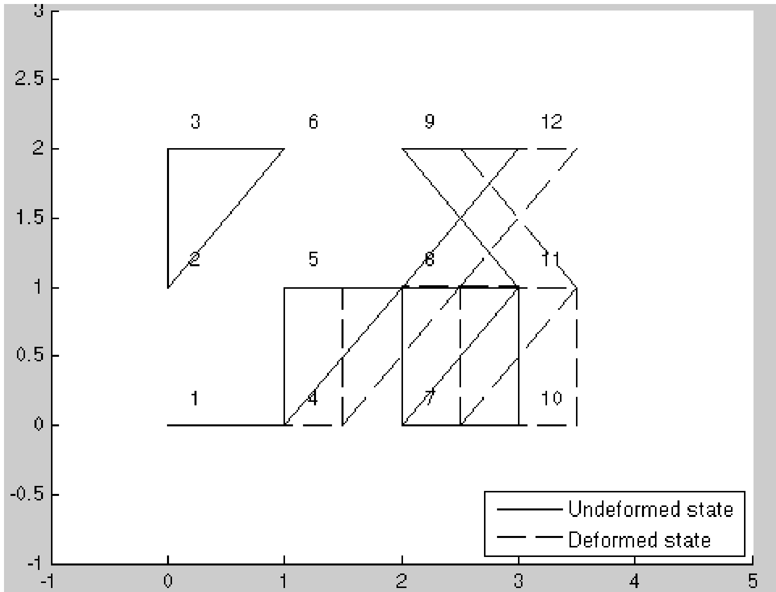
APPENDIX C

Notes for the selection of several parameters in the optimization model

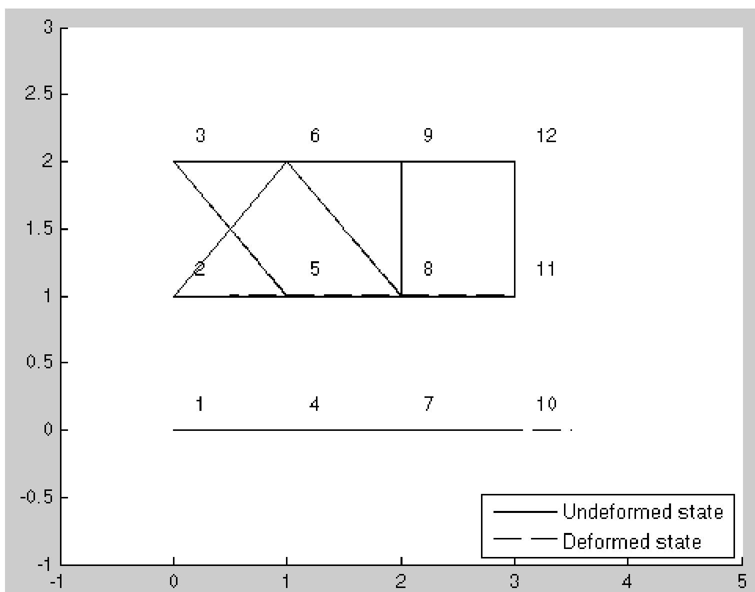
C.1 The constraint on the maximum displacement at the output node

The output displacement is desired to be a maximum, which however needs a constraint (i.e., the maximum displacement is less than a certain value, say 0.1). Conceptually, if there is no such constraint, the displacement tends to go as large as possible with the element deforms as large as possible (due to no constraint on the element in its deformation). Figure C.1 shows the result of design when there is no such constraint imposed. It can be seen from this figure that the mechanism is composed of several disconnected elements, which absolutely violates the integrity of the material continuity of a mechanism. A proper value of that constraint is however not straightforward in this case and it demands a trial-and-error process. To the problem dealt with in this thesis, it is found that 0.1 m is the proper constraint value. Due to this constraint, the fitness function corresponding to the output displacement is in the range of greater than 0.0 but less than 0.1. To normalize it to the range of 0 to 1, a factor 10 is multiplied to the output displacement. There would be of course some more sophisticated way to normalize this fitness function, which is seen beyond the scope of this thesis.

In fact, if there is no constraint on the deformation of the node or the element, the assumed linear finite element formulation in the current approach and program may no longer be valid, as some non-linear behavior of the elements may occur. To prevent this problem from occurring may be a reason for introducing the strain constraint in (Larsen and Sindholt, 2003).



(a)



(b)

Figure C.1 Optimized compliant mechanisms with no output displacement constraint. (a) and (b) are the results of two runs of the computer program

C.2 Explanation of P3=10

The choice of P3 is empirical. In this thesis, the evaluation of the fitness function for different p3 was made, and the result is shown in Table C.1. The average fitness function is calculated from the fitness function values from 10 times of runs of the program. From Table C.1, the highest fitness value is reached when p3=10. Therefore, p3=10.

Table C.1 The average fitness function for different p3 values

P3	Average fitness value
2	6.30835
3	6.44159
4	6.4373
5	6.38419
6	6.40184
7	6.45822
8	6.45771
9	6.39998
10	6.46722
11	6.42611
12	6.41018

APPENDIX D

Manual calculation of the maximum stress of a cantilever beam

Figure 4.6 is revised here. The ANSYS result has been shown in Figure 4.7. The following is presented manual calculation.

Given: Cross sectional area *A* of a cantilever beam is 0.001 m^2, height *h* of it is 0.01 m, Length *L* is 1 m. Node 1 is completely fixed and node 2 is free. Node 2 is subjected to an axial force 50 N, bending moment 50 Nm, vertical force 50 N. Material: HR steel.

Calculate: the maximum stress of this cantilever beam.

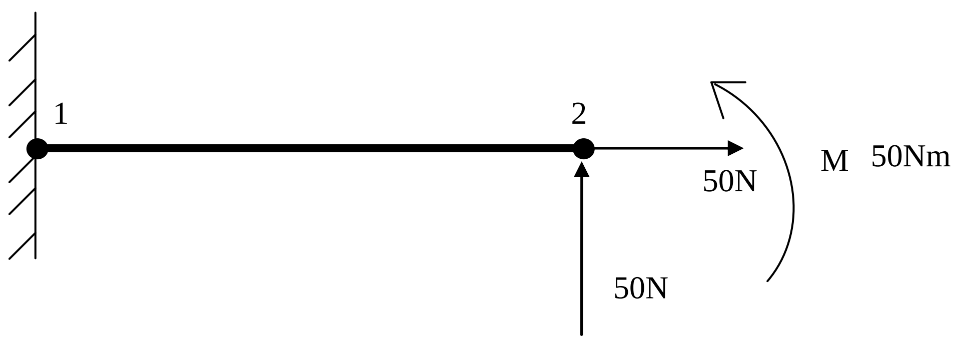


Figure D.1 Schematic of the cantilever beam with the loads at the free end

The manual calculation is as follows:

C=h/2=0.01/2=0.005 m,

$I=bh^3/12=0.1\times(0.01)^3/12=0.083\times10^{-7}\ m^4$

$\sigma_{axial} = \frac{F}{A} = \frac{50}{0.001} = 5 \times 10^4$ Pa

$\sigma_{bending} = \frac{Mc}{I} = \frac{50\times0.005}{0.083\times10^{-7}}=3.01205\times10^7$ Pa

$\sigma_{force} = \frac{F\times L\times c}{I} = \frac{50\times1\times0.005}{0.083\times10^{-7}}=3.01205\times10^7$ Pa

σ_{maxall}=$\sigma_{axial} + \sigma_{bending} + \sigma_{force}$

=$5 \times 10^4 Pa$ +3.01205×10^7Pa+3.01205×10^7 Pa

=6.0291×10^7 Pa

In summary, the manual calculation is nearly the same as the result from ANSYS.

If this cantilever beam is considered for fatigue, then the maximum stress is as follows:

$\frac{\sigma_{axial}}{C_{axial}} + \frac{\sigma_{bending}+\sigma_{force}}{C_{bending}}$

$= \frac{5\times10^4}{0.85} + \frac{3.01205\times10^7+3.01205\times10^7}{1}$

$= 58823.53 + 6.0241 \times 10^7$

$= 6.02998 \times 10^7$Pa

$S_e = C_{surface} \times C_{size} \times C_{reliability} \times C_{misc} \times S_e'$

$C_{surface}$= 58.1× $320^{-0.719}$=0.9183<1;

d_e=0.808$\sqrt{bh}$=0.808$\sqrt{0.1 \times 0.01}$=0.02555m=25.55mm

C_{size}= $(\frac{d_e}{7.62})^{-0.1133} = (\frac{25.55}{7.62})^{-0.1133}$=0.8719

$C_{reliability}$=0.702 (Suppose reliability is 99.99%)

C_{misc}=$\frac{1}{1.6}$=0.625

S_e'=0.5×S_{ut}=0.5×320=160 MPa

S_e=0.9183×0.8719×0.702×0.625×160=56.207 MPa=5.6207×10^7 Pa

So, $S_e < \sigma_{maxall}$, this cantilever has finite life.

APPENDIX E

Notes for the selection of several parameters in the GA

E.1 Mutation rate (MR) =0.1

It is noted that the value for MR is empirical. Usually, MR is in the range of 0.01 to 0.1. Semantically, if the MR is larger than 0.1, the GA becomes randomized search. In this thesis, three values were chosen to examine their fitness function (the higher the better). Table E.1 shows the result of this testing. From Table E.1, the value 0.1 was chosen for MR.

Table E.1 Relationship between mutation rate and average fitness value

MR	Average fitness value
0.01	6.31512
0.05	6.38748
0.1	6.46216

E.2 Cross over rate (CR) =1

It is noted that cross over rate is chosen empirically. Usually, it takes from 0.8 to 1.0. In this thesis, three values in the range were selected, and their fitness functions are shown in Table E.2. From this table, the largest fitness function is when CR is equal to 1. Therefore, CR=1.

Table E.2 Relationship between crossover rate and average fitness value

CR	Average fitness value
0.8	6.35774
0.9	6.38999
1.0	6.46541

E.3 Individual (IND) =1000

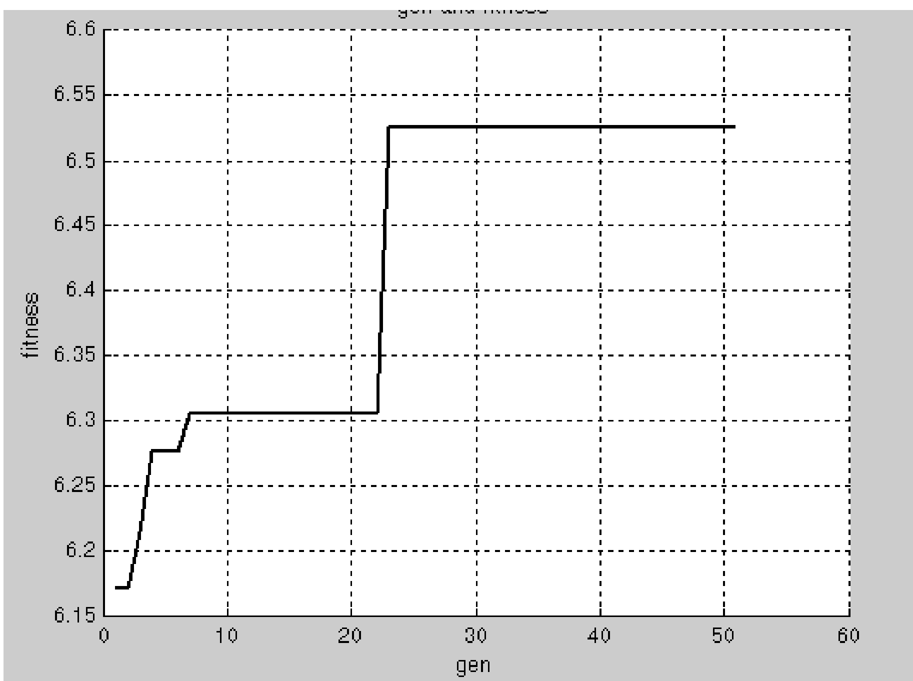
It is noted that individual is chosen empirically. A larger IND means a long computational time. A couple of INDs were shown and their results are listed in Table E.3. From this table, it can be seen that IND=1000 is adequate (as after that, the fitness function value tends to increase a very little).

Table E.3 Relationship between individual number of each generation and average fitness value

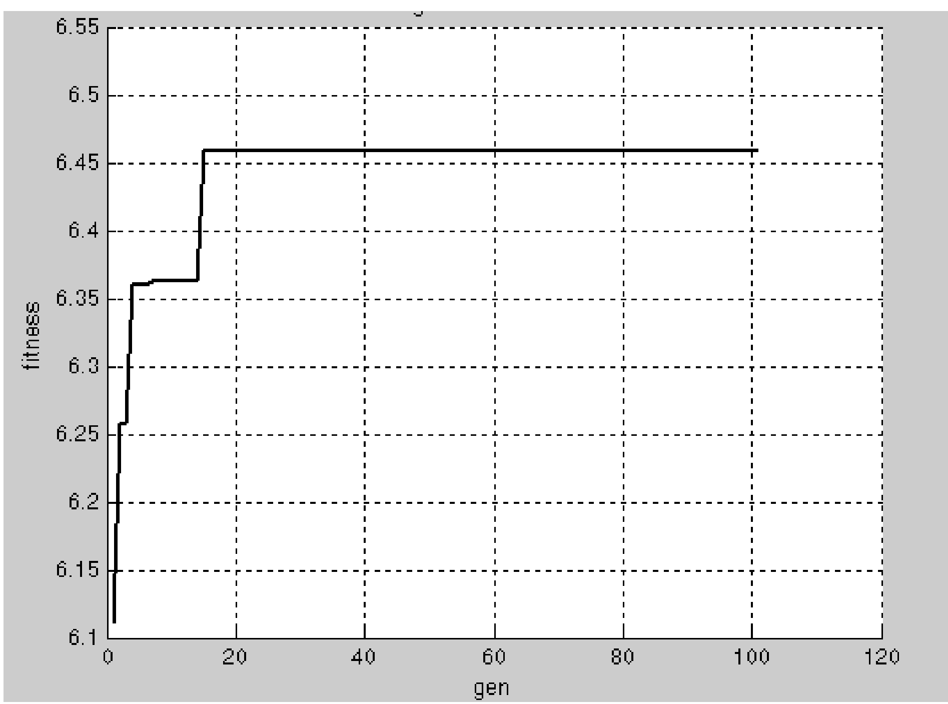
IND	Average fitness value
100	6.2399
500	6.4228
1000	6.46788
1500	6.47106

E.4 Generation (Gen) = 50

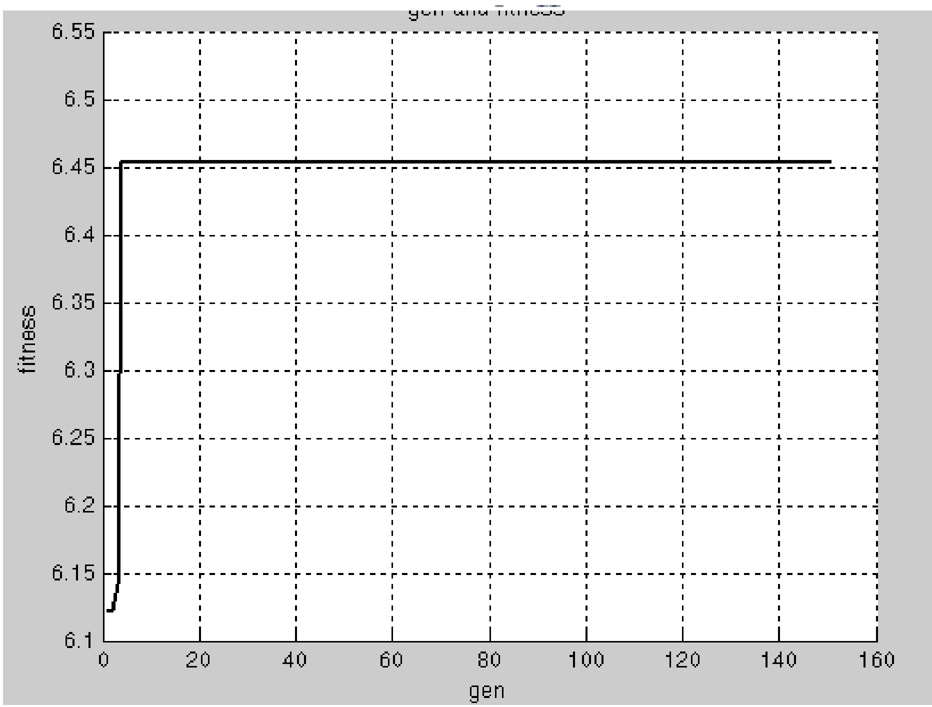
Generation is an empirical number. In this thesis, several generations were chosen to examine their fitness value; see Figure E.1. From this figure, it can be seen that Gen=50 is adequate since optimization with GA always converges within 50 generations.



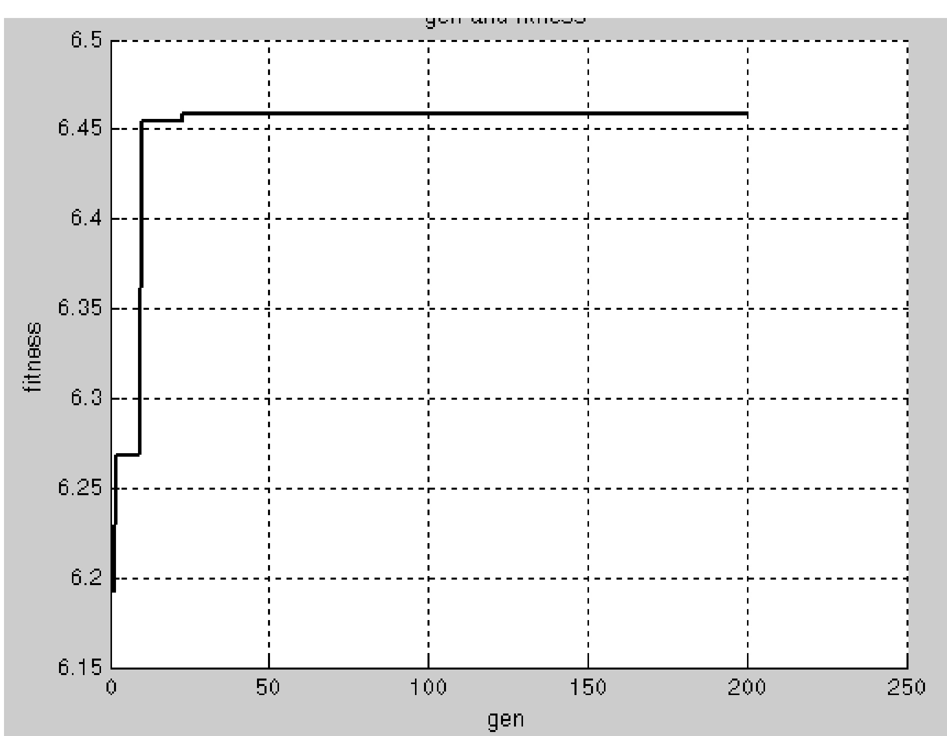
Gen=50, converged at 23, fitness=6.5262



Gen =100, converged at 15 fitness=6.4583



Gen=150, converged at 4, fitness=6.4538



Gen=200, converged at 23, fitness=6.4582

Figure E.1 Selection of Generation: Generation, convergence, and fitness function. (a) Gen=50, (b) Gen=100, (c) Gen=150, (d) Gen=200.