

# **Assessment of Spinning Reserve Requirements in a Deregulated System**

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A Thesis Submitted to the College of Graduate Studies and Research  
in Partial Fulfillment of the Requirements for the Degree of

## **Master of Science**

in the Department of Electrical and Computer Engineering  
University of Saskatchewan  
Saskatoon, Saskatchewan  
Canada

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## **Declaration**

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification for another degree or qualification of this or any other university or any other institute of learning.



## Acknowledgements

To God be all glory for the life and opportunity He gave me to successfully complete this work at His own time. You are the love of my life and the hope that I will always cling to and I will always be indebted to you all my life.

A very big thank you to my supervisor, Dr. Nurul Chowdhury who was very considerate and supportive of me during the good and tough times in this my academic journey.

I wish to acknowledge my uncle, Zik and his wife Joanna for their unlimited support and the home that they gave me during this academic study. To my two special boys Dozie and Yagazie, I owe it to you because being around you helped me ease off the stress. To my parents who supported me all these years in a lot of ways that are innumerable to mention, may God bless and keep you alive to reap the fruits of your labour. A big thank you to Dr. & Mrs. Onwudiwe and Prof. & Mrs. Arinze for your advice and support that enabled me to finish this study.

Tommy, I owe you a lot for the success of this work. Your time and sacrifice for me in developing the computer program were enormous and appreciated. To Nini, a special friend who is more than a sister, I know my stay in Canada would not have been this blessed if not for you. To all my colleagues, all I can say is thank you very much for what has a beginning must surely come to an end someday.

To my best friend Nkem, thank you for your prayers, support, care, understanding and love. You stood by me and encouraged me even when I could not continue. I am also grateful to Mark Grant for accommodating me during my last days of school here. I know there are still many people who helped me in one way or the other to pull this stunt and I humbly say a BIG THANK YOU for being there for me.



**Dedication**

**To the Families of  
Odinakaeze, Obianwa, and  
Nkamuke**



# **Abstract**

A spinning reserve assessment technique for a deregulated system has been developed and presented in this thesis. The technique is based on direct search optimization approach. Computer programs have been developed to implement the optimization processes both for transmission loss and without transmission loss.

A system commits adequate generation to satisfy its load and export/import commitment. Additional generation known as spinning reserve is also required to satisfy unforeseen load changes or withstand sudden generation loss. In a vertically integrated system, a single entity generates, transmits and distributes electrical energy. As a part of its operational planning, the single entity decides the level of spinning reserve. The cost associated with generation, transmission, distribution including the spinning reserve is then passed on to the customers.

In a deregulated system, generation, transmission and distribution are three businesses. Generators compete with each other to sell their energy to the Independent System Operators (ISO). ISO coordinates the bids from the generation as well as the bids from the bulk customers. In order to ensure a reliable operation, ISO must also ensure that the system has adequate spinning reserve. ISO must buy spinning reserve from the spinning reserve market. A probabilistic method called the load forecast uncertainty (LFU)-based spinning reserve assessment (LSRA) is proposed to assess the spinning reserve requirements in a deregulated power system.

The LSRA is an energy cost- based approach that incorporates the load forecast uncertainty of the day-ahead market (DAM) and the energy prices within the system in the assessment process. The LSRA technique analyzes every load step of the 49-step LFU model and the probability that the hourly DAM load will be within that load step on the actual day. Economic and reliability decisions are made based on the analysis to determine and minimize the total energy cost for each hour subject to certain system constraints in order to assess the spinning reserve requirements. The direct search optimization approach is easily implemented in the determination of the optimal SR requirements since the objective function is a combination of



linear and non-linear functions. This approach involves varying the amount of SR within the system from zero to the maximum available capacity. By varying the amount of SR within the system, the optimal SR for which the hourly total operating cost is minimum and all operating constraints are satisfied is evaluated.

One major advantage of the LSRA technique is the inclusion of all the major system variables like DAM hourly loads and energy prices and the utilization of the stochastic nature of the system components in its computation. The setback in this technique is the need to have access to historical load data and spot market energy prices during all seasons. The availability and reliability of these historical data has a huge effect on the LSRA technique to adequately assess the spinning reserve requirements in a deregulated system.

The technique, along with the effects of load forecast uncertainty, energy prices of spinning reserve and spot market and the reloading up and down limits of the generating zones on the spinning reserve requirements are illustrated in detail in this thesis work. The effects of the above stochastic components of the power system on the spinning reserve requirements are illustrated numerically by different graphs using a computer simulation of the technique incorporating test systems with and without transmission loss.



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# List of Symbols

## Indices

$j$	index of generating zones running from 1 to $N$
$s$	index of LFU intervals running from 1 to $S$
$t$	index of time periods running from 1 to $T$ , hour

## Sets

$N$	set of generating zones
$T$	set of time periods
$S$	set of intervals in the LFU

## Parameters

$\frac{dF_j}{dP_j}$	fuel cost of zone $j$ , \$/MW
$DR_j$	ramp-down rate of zone $j$ , MW/h
$D_s^t$	load level of interval $s$ , during period $t$ , MW
$D_p$	change in demand during period $t$ , for interval $s$ , MW
$MCP^t$	market clearing price during period $t$ , \$/MWh
$MF^t$	maintenance fee during period $t$ , \$/MWh
$P_j^{max}$	maximum capacity of generating zone $j$ , MW
$P_j^{min}$	minimum capacity of generating zone $j$ , MW
$P_{Load}^t$	forecast load during period $t$ , MW
$P_{Loss}^t$	transmission loss during period $t$ , MW
$PF_j$	penalty factor of zone $j$
$RDP_j^t$	ramp-down price of zone $j$ , during period $t$ , \$/MWh
$RUP_j^t$	ramp-up price of zone $j$ , during period $t$ , \$/MWh
$SRP^t$	spinning reserve price during period $t$ , \$/MWh
$SMP^t$	spot market price during period $t$ , \$/MWh



$T_s^A$	operating cost of interval $s$ , in scenario A, \$
$T_s^B$	operating cost of interval $s$ , in scenario B, \$
$T_s^C$	operating cost of interval $s$ , in scenario C, \$
$T_t^D$	total operating cost of during period $t$ , \$
$UR_j$	ramp-up rate of zone $j$ , MW/hr
$w_s^t$	amount of power from the spinning reserve market for interval $s$ , during period $t$ , MW
$x_j^s$	amount of power ramped-down by zone $j$ , MW
$y_j^s$	amount of power ramped-up by zone $j$ , MW
$z_s^t$	amount of power from the spot market for interval $s$ , during period $t$ , MW
$\beta_t$	spinning reserve factor during period $t$
$\gamma_j$	incremental cost of zone $j$ , \$/MW

## Continuous Variables

$P_j^t$	power generated by zone $j$ , during period $t$ , MW
$SR^t$	system spinning reserve requirements at period $t$ , MW



# List of Acronyms

<i>CAISO</i>	California Independent System Operator
<i>COPT</i>	Capacity on Outage Probability Table
<i>ED</i>	Economic Dispatch
<i>EENS</i>	Expected Energy Not Served
<i>ELNS</i>	Expected Load Not Served
<i>IESO</i>	Independent Electricity System Operator
<i>LFU</i>	Load Forecast Uncertainty
<i>LOLP</i>	Loss-Of-Load-Probability
<i>LSRA</i>	Load Forecast-based Spinning Reserve Assessment
<i>NYISO</i>	New York Independent System Operator
<i>ORR</i>	Outage Replacement Rate
<i>PJM</i>	Pennsylvania-New Jersey-Maryland
<i>SR</i>	Spinning Reserve
<i>SRR</i>	Spinning Reserve Requirements
<i>TS</i>	Test System
<i>UC</i>	Unit Commitment
<i>UCR</i>	Unit Commitment Risk



# Chapter 1

## Introduction

### 1.1 Power systems

A power system is a complex interconnected network of generating units, transmission lines and substations that supply electrical energy to its customers. The underlying requirement of any power system is to meet the need of electrical energy with minimum cost while maintaining reliability. The electrical energy is the most popular form of energy, because it can be transported easily at high efficiency and reasonable cost [40]. A characteristic of electrical energy is that it cannot be easily stored and thus must be utilized as it is being produced.

There are four major components of a modern electric power system. They are:

- Generation
- Transmission
- Distribution and
- Load

#### 1.1.1 Generation

Generation is the first stage in the process of delivering electrical energy to customers. Based on economics, the generating unit with the least cost of operation is usually chosen first unless the reliability of such a unit is below acceptable standards. More expensive units are gradually brought in line as the load increases. There are different methods of generating electricity



through the conversion of other forms of energy. Thermal and hydro-power are the most common methods of electricity generation. Nuclear power generation is also used in developed and in many developing countries. Renewable energy sources like wind and solar are though more expensive on per KW basis are becoming more popular due to their zero emission.

### **1.1.2 Transmission**

Electrical energy is transferred from the generation points to various distribution points with the help of transmission lines. Transmission lines are used in interconnecting generating power stations with substations as well as interconnecting neighbouring regions to form a transmission grid. Transmission lines are constructed with light-weight conductors that offer lesser heat loss based on  $I^2Rt$ . A transmission grid must be reliable and robust to carry the capacity of the electric power being transmitted through the lines with minimal loss. When the economics of system reliability, transmission loss, cost of power and transmission are put into consideration during network designs, the ring transmission network becomes more reliable than the radial system since the outage of any transmission line segment does not necessarily stop transmission of power to needed locations.

At the substations, transformers are used to step-down transmission line voltages to lower voltages for distribution to different customers. The part of the transmission system that handles this step-down process is called the sub-transmission system. Sub-transmission voltage levels are usually lower than the transmission line voltages.

### **1.1.3 Distribution**

A distribution system is the final stage of a power system which connects the customers to the distribution substation. Distribution system is an important portion of a power system due to its high investment and its direct effect on customers. A typical distribution network consists of medium-voltage (less than 50 kV) power lines, electrical substations, and pole-mounted transformers, low-voltage (less than 1000 V) distribution wiring and sometimes electricity meters. The distribution of energy is carried out at low voltage and high current since the



distance covered by the lines are very much shorter than that of the transmission lines. Also heavier conductors are used in the construction of the distribution lines. The medium-voltage power lines are the primary distribution lines that serve loads in a well-defined geographical location as well as small industrial customers. In order to serve commercial and residential loads, the secondary distribution network voltage is reduced and power is transmitted via short distance lines and cables.

Distribution networks can either be configured as a radial or as an interconnected system. The interconnected system is more prevalent in urban areas and offers more security and reliability in times of line failures or maintenance. Distribution lines can either be overhead or underground. Each type of layout has its advantages and disadvantages. Factors like cost, location, environment and reliability determine the type of line system to adopt.

#### **1.1.4 Loads**

Industrial, commercial and residential are the different categories of power system loads. Very large industrial loads are usually served by transmission networks while sub-transmission networks supply power to large industrial loads. Primary distribution networks serve small industrial loads. Industrial loads are composite loads, in which induction motors form a high proportion. Composite loads are functions of voltage and frequency and form a major part of a system load. The composite loads are major consumers of reactive power and account for frequency fluctuations within the system when not operated in the right state. Commercial and residential loads consist mainly of lighting, heating and cooling. They are independent of frequency and consume negligibly small reactive power.

The magnitude of load varies throughout the day. This makes it necessary to maintain a continuous and almost instantaneous balance between the production and consumption of electricity in power systems. Power must be available to meet consumers' demands whatever they may be. Also the voltage level at the customers' end must be maintained at or near nominal rated value. In order to ensure energy balance, some margin of generation above expected load demand must be kept so that the system can deal with unexpected imbalance between supply and



demand that can lead to load shedding. This extra generating capacity, called reserve, must be readily available when needed and is therefore included in the system daily load forecast and scheduling.

## **1.2 Natural Monopoly**

An industry is said to be a natural monopoly if one firm can produce a desired output at a lower social cost than two or more firms. This monopoly exists because the cost of producing the product, either goods or services, is lower due to economies of scale if there is just a single producer than if there are several competing producers. Economies of scale is the situation in which the cost to a company of producing or supplying each additional unit of a product (referred to by economists as marginal cost) decreases as the volume of output increases. Under the natural monopoly cost structure, the fixed cost of the capital goods within the industry is very high while the marginal cost is extremely low making it unprofitable for a second firm to enter and compete.

The power industry used to be one of the natural monopolies prior to restructuring. For a long time, local or government established firms owned and managed the industry within a region, thereby making it difficult for competitors to emerge because of the very high capital costs needed to generate, transmit and distribute electrical energy. The high fixed cost of infrastructure in the power industry makes it almost impossible to have new entrants invest in building their own transmission and distribution networks. It is more economical for the available transmission and distribution networks to serve all customers since the cost per customer is a lot lower than several individual networks that each serves only some customers. And, from a practical point of view, city governments will not accept many companies digging up the streets or covering the sky with wires [18]. On the generator function, the larger the facility, the lower the cost per unit of output. Therefore, economically it was better to increase the capacity of existing generating facilities than to build new ones. Having more than one firm in a region becomes inefficient since duplication of facilities like transmission and distribution lines is not economical in any way. Also the uncertainty in the ability of the new entrants to compete successfully with the already existing service provider, who has the advantage of a low marginal cost, eliminates any



possible competition. These factors were barriers to competition and they helped in sustaining natural monopoly of the power sector for a very long time.

Some of the problems with traditional monopolies of the power industry included power resource location, low administration efficiency, uncontrolled increases in cost of services, poor quality of services, non-advancements in technology, the inability of consumers to make choices and abuse of market position. To solve these and other problems, regulations of different forms have been imposed over a long period of time. These regulatory measures included joint use of existing infrastructure, public ownership, denationalization, outsourcing and the introduction of competition. The joint use of infrastructure by different service providers seemed a better competitive solution to these problems until deregulation opened up opportunities for competitors to offer better electrical supply services to consumers at competing prices even though there are additional risks with deregulation.

### **1.3 Traditional power system operation**

In the past, a traditional power system operated as a vertically integrated system and was developed to own and operate all functions associated with generation, transmission and distribution of electrical energy. This type of traditional systems allowed very large power stations to be built and operated based on economies of scale and efficiency. The national grid was an integral part of this power system and it ensured security of electricity supply to consumers through a centralized control and supervision system.

A traditional power system is generally subdivided according to geographical size into utilities, control areas, pools and coordinating councils [18]. A single entity (utility) generates, transmits and distributes electrical energy over a large geographical area and also handles the sale of electricity to all consumers. It must commit adequate generation to satisfy its load and export/import commitment. As a part of its planning (Operational planning), the single entity decides the level of spinning reserve. The cost associated with generation, transmission and distribution including the spinning reserve is then passed on to customers. In a traditional power system generation, transmission, distribution and marketing functions are all natural monopolies.



The natural monopoly of a traditional power system eliminated competition within all its functions. This non-competitive nature resulted in a system that is neither reliable nor cost-effective in its operation. Also, energy prices were not driven by competition through supply and demand but rather by the owners of these utility functions. The poor services within traditional systems of power operation necessitated the need to re-regulate the power system and that gave birth to the deregulation of the power industry.

## **1.4 Deregulation**

Deregulation was one of the solutions preferred by economists to the problems with the natural monopoly of the power industry. Over the years, the electricity industry throughout the world which has long been dominated by vertically integrated, highly regulated and monopolistic utilities is undergoing dramatic changes. It is evolving into a system that is horizontally integrated with generation, transmission and distribution facilities unbundled [51]. This evolution has resulted in a distributed and competitive industry driven by market forces and increased competition. The re-organization of the electric sector allows for competition among generators and to create market condition in the sector, seen as necessary conditions for increasing the efficiency of electric energy production and distribution, offering a lower price, higher quality and secure products and services [41].

The decomposition of the three components of the electric power industry through deregulation gives consumers the freedom to buy power from any of the competing providers of electrical energy. In principle, everyone has access to the main grid and to the organized power market [14]. Each of these components of the power system is a business function on its own. There exists competition for each of these functions and different market participants compete to render any of the services available in any of these functions as long as they meet the requirements needed to be a market participant as set by the regulatory body overseeing the industry. All competitors with the ability to produce any of the power system services can utilize already existing transmission and distribution networks on equal terms without having to invest in its own network. This results in a reduction in the net cost through competition and these savings can be passed to consumers. There is, therefore, a levelling out of energy prices to the consumers



driven by competition through supply and demand coupled with increased economic efficiency and higher productivity of the power sector.

## **1.5 Deregulated Electrical System Operation**

In a deregulated system, energy suppliers compete with each other to sell their energy to buyers within a predefined sector. This restructuring process needed the separation of different power system components and their controls such that no function has a monopolistic influence on the other. The ownership of the transmission function, an integral part of the industry and its control, needed to be separate in order to avoid this monopoly. To achieve this, there was a need for an independent operational control of the transmission grid in the restructured industry to facilitate an unbiased competitive market for power generation and direct retail access. However, the independent operation of the grid to meet this expectation cannot be guaranteed without independent entities such as the Independent Market Operator (IMO) and the Independent System Operator (ISO).

The IMO and ISO as major key players in this industry are required to be independent of the individual market participants such as transmission owners, generators, distribution companies, retail companies and end-users. In order to operate the competitive market both efficiently and economically while ensuring the reliability of the power system, the IMO as a market operator, must establish sound rules on energy and ancillary services markets. On the other hand, the ISO must coordinate and manage the transmission system in a fair and non-discriminatory manner acceptable to all market participants.

Within a deregulated system, electricity market is unbundled. Energy and ancillary services are offered as unbundled services and generating companies compete to sell energy and ancillary services to customers by submitting competitive bids to the IMO. In this system, generating companies and other market participants are no longer controlled by entities that control the transmission and distribution systems. Deregulation allows these market participants to acquire computational tools, such as price and load forecasting, unit and demand commitment, arbitrage and risk management to make sound decisions in this competitive market [41]. In summary, the



role of the IMO is to provide and maintain an effective infrastructure for the efficient operation of the wholesale electricity market within their jurisdiction.

Maintaining the reliability and security of the deregulated power system is a challenge. The ISO must at all times maintain the system real-time load balancing, congestion management and provision of ancillary services to various participants including providers and purchasers in a fair, equitable and economical way[9, 53]. In most deregulated utilities, the responsibilities of the IMO and ISO are similar and will be discussed in details in the next chapter.

## **1.6 Load Forecasting**

Load forecasting can be defined as a thorough study of the consumers' load demands and factors affecting those loads in order to determine the consumers' future requirements for energy and capacity. Load forecasting plays an important role in the scheduling and secure operation of a power system.

In order to develop an accurate load forecast, historical analysis of different data over a period of time is required. Some of the data sources include weather, demography and economics. The location and population of the consumers play a huge role in the energy market load forecast. Weather is certainly a major driver of the day-ahead and week-ahead load forecast and adequate weather data is important in load forecasting. A load forecast model is usually developed after this comprehensive study of different factors affecting load demands. The load forecast model (LFM) is used for operations, analysis and planning purpose by the ISO. While a load forecast study and model design can be comprehensive and thorough, it is almost impossible to forecast the energy market load accurately for every hour. This inaccuracy is a result of the complex nature of loads and the numerous factors affecting them.

## **1.7 Load Forecast Uncertainty**

Load forecast uncertainty affects the amount of power that must be scheduled during unit commitment. Load forecast uncertainty also causes increase in operating cost of a power system due to the spare generation capacity (spinning reserve) that needs to be scheduled in order to



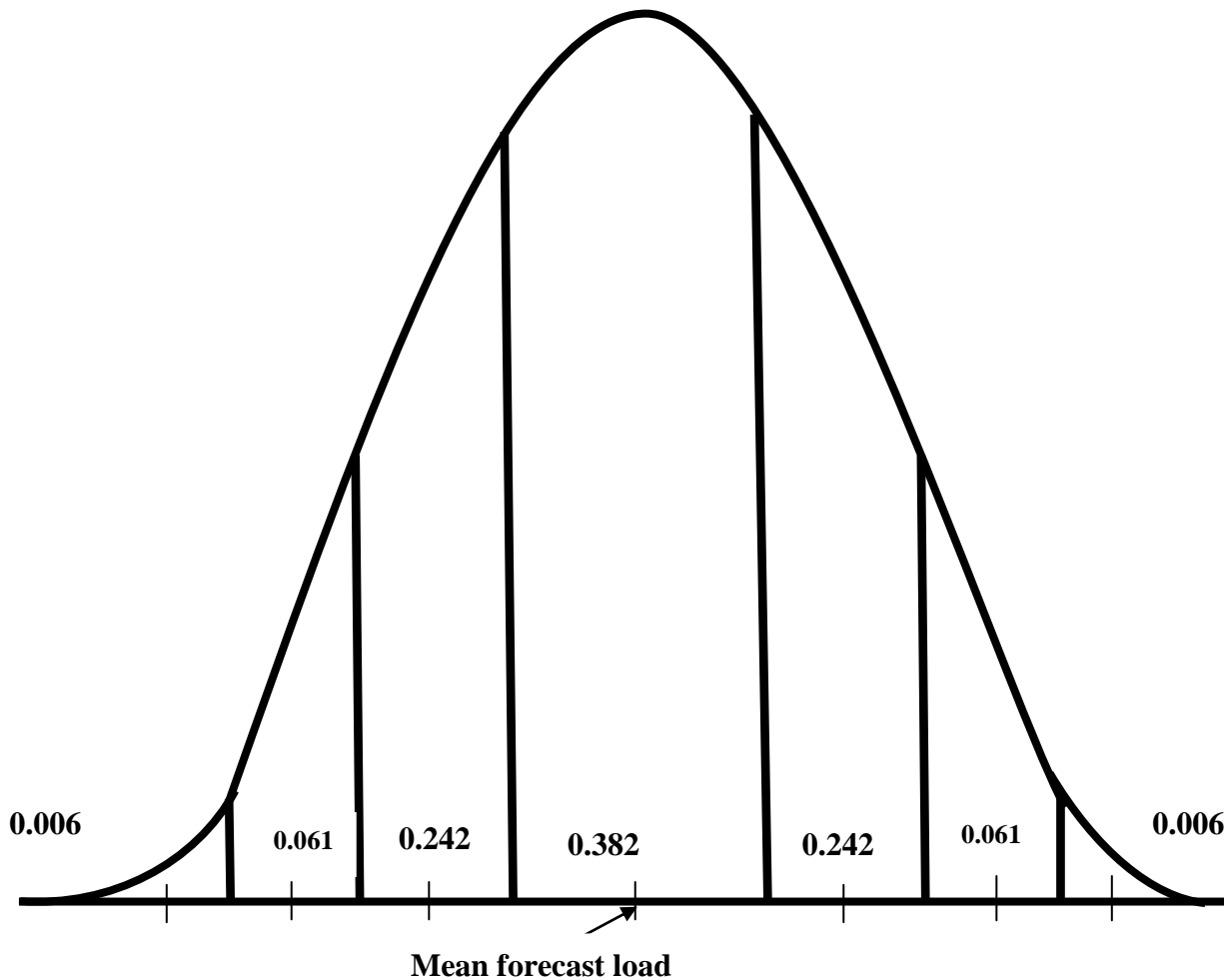
protect the system from the effects of contingencies like sudden increase in load demand. An accurate load forecast usually minimizes the cost of operation since very little spinning reserve has to be maintained for system security and reliability.

Load forecast uncertainty is a significant factor in the determination of the amount of power to be scheduled during unit commitment in any energy market. It plays an important role during unit commitment and scheduling and hence affects the reliability of the generating and transmission system. It is extremely difficult to obtain sufficient historical data to determine the distribution describing the load forecast uncertainty. Published articles [16, 25] suggested that the load forecast uncertainty can be reasonably described by a normal distribution whose parameters can be estimated from past experience and future considerations. The load forecast uncertainty can be described as a probability distribution with the forecast peak load as the distribution mean and the standard deviation,  $\sigma$  as the level of uncertainty. This probability distribution can be divided into discrete number of class intervals. The area of each class interval can then represent the probability that the load is equal to the class interval mid value. Figure 1.1 shows a load forecast uncertainty with its distribution divided into seven discrete class intervals. It has been found that there is little difference in the end result between representing the distribution of the load forecast uncertainty by seven steps or forty-nine steps. In this work, the load forecast uncertainty model is described as a probability distribution divided into 49 discrete class intervals.

## **1.8 Unit Commitment**

Unit commitment (UC) is the process of deciding when and which generating units at each power station to start-up and shut-down, while deciding the individual power outputs of the scheduled units and maintaining a given level of spinning reserve at each time period [29]. Unit commitment attempts to answer the question, “Given all possible combinations of the generating units that can meet scheduled (expected) demand, which one offers the least operating cost when used over a given time horizon?” Unit commitment schedules, among other things, depend on system load, firm transaction and spinning reserve requirement.





**Fig. 1.1      7-Step Normal Probability Distribution of the Load Forecast Uncertainty**

In a vertically integrated system, it is relatively easy to obtain unit commitment schedules. This is due to the fact that a single entity owns and operates all sectors of a vertically integrated system. Spinning reserve is considered as part of the unit commitment solution in a traditional power system. In a deregulated system, generation, transmission and distribution are three separate businesses. This adds to the complexity of unit commitment schedules as spinning reserve and regulating requirements have to be procured from markets different from bulk-energy markets. Spinning reserve is not embedded into the unit commitment solution for a deregulated system.



A typical solution to unit commitment includes generation schedules, reserve and regulation market schedules, and firm transactions schedules for a particular time.

## **1.9 Spinning Reserve**

Spinning reserve (SR) is the on-line reserve capacity that is synchronized to the grid and ready to meet electrical demand within 5 to 10 minutes of a dispatch instruction by the ISO. Spinning reserve can be provided by any generating unit that is connected to the grid and electrically close enough to the control area provided that transmission limitations do not prevent the importation of the power. Spinning reserve is usually called up during such contingencies in order to maintain system frequency, stability and avoid loss of load. SR is one of the most important ancillary services used by power system operators because it mitigates the considerable social and economic costs of occasional outages [30]. It is recognized that the maintenance of the spinning reserve adds value to the service of electric energy supply since the operation of the system has adequate level of security [41].

## **1.10 Objectives and Outline of the Thesis**

This research work deals with the assessment of spinning reserve (SR) requirements in a deregulated system. Day-ahead schedules and spot energy availability are considered during the assessment of SR. The objectives of this work were:

1. To determine the hours in a day-ahead market schedule that require SR.
2. To determine the amount of SR that should be scheduled during these hours.
3. To study the effects of load forecast uncertainty on the assessment of the spinning reserve requirements.
4. To study the effects of SR Price on the assessment of spinning reserve requirements.
5. To study the effects of real-time (spot market) energy price on the assessment of spinning reserve requirements.
6. To study the effects of reloading limits (ramping rates) on the assessment of spinning reserve requirements.



This thesis is divided into five chapters. In Chapter 2, the different types of energy markets and the roles and responsibilities of the Independent System and Market Operators in a deregulated system are described. Different deterministic criteria for the assessment of the spinning reserve requirements as practiced by different independent system operators are discussed. A literature review of different probabilistic techniques to assess spinning reserve requirements in a deregulated system is also presented in this chapter.

A technique to optimize SR requirements based on a direct search approach has been developed and presented in Chapter 3. The load forecast for each period is modeled using a probability distribution of 49 discrete class intervals. The total operating cost including the cost of scheduling the SR requirements is developed. The optimization technique determines the optimal SR required for each period by varying the amount of SR from zero to the maximum available capacity and determining that amount for which total operating cost is minimum and operating constraints are satisfied. A C++ computer program has been developed to implement the optimization of SR requirements in a test system without transmission loss. The results and the effects of the spinning reserve and spot market prices on the SR requirements are discussed.

Chapter 4 presents a direct-search SR optimization technique with the inclusion of transmission loss. Incremental loss and energy prices are considered in the development of the optimization cost function for a 49-step load forecast uncertainty model. The total operating cost is minimized with respect to certain constraints to determine the optimized SR requirements for each period within the optimization horizon for the test system. The optimization technique involves varying the amount of SR in the system from zero to the total scheduled generating capacity for each hour. The amount of SR that offers the least hourly total operating cost and also satisfies all operating constraints is outputted as the optimal SR requirement for that hour. A C++ computer program has been developed to optimize the SR requirements in a test system with transmission loss. The results and the effects of the spinning reserve and spot market prices as well as reloading limits on the SR requirements are presented and discussed.

Chapter 5 summarizes the main achievements of this proposed technique. Further work is also suggested in this chapter. This thesis is complemented by a number of appendices. Appendix A.1



presents the data of a test system without transmission loss while appendix A.2 presents the data of a test system with the inclusion of transmission loss. Appendix B presents the computer codes for the optimization simulation while appendix C shows the charts of the test results. Appendix D presents the derivation of the transmission loss equation used in Chapter 4.



# Chapter 2

## Various Markets in a Deregulated System

### 2.1 Introduction

In the context of power system deregulation, secure and reliable operation of a power system is a challenging task for an Independent System Operator [54]. In order to ensure secure and reliable operation, the ISO must ensure an adequate supply of energy and ancillary services from various markets. A generator or a bulk energy supplier may take part in both energy and ancillary services markets within a deregulated power system. The scheduling of energy and ancillary services within these markets is a complex process and varies from the type of energy to the type of ancillary service. This complex process of economic scheduling of energy and ancillary services within a system starts with scheduling of generating units, commonly known as unit commitment for any energy market.

### 2.2 Energy Markets

Generally, the energy market operates much like a stock exchange, with market participants establishing a price for electricity by matching supply and demand. There are three types of energy markets:

- Day-Ahead
- Hourly and



- Real-Time

The independent market and system operators play major roles in the efficient, effective, reliable and economical operation of the above energy markets. Their roles are discussed in details in the following section.

### **2.2.1 Independent Market Operator (IMO)**

The IMO promote the ongoing development of the energy market with the objective of continually improving its performance to ensure that the market is efficient and effective. The IMO is responsible for the:

- administration of the market rules,
- operation and regulation of the wholesale electricity system and the wholesale electricity market place by linking buyers and sellers while directing the flow of electricity through the existing transmission system from generators and suppliers to local distribution companies and wholesale buyers,
- facilitation of the provision of sufficient generation capacity and demand side management to meet expected demand and
- facilitation of the provision of reliable and competitively priced electricity.

Over time, the roles and responsibilities of the IMO have been integrated into that of the Independent System Operator (ISO) and very few deregulated utility systems have both IMO and ISO in their organizational structure.

### **2.2.2 Independent System Operator (ISO)**

The Independent System Operator (ISO) coordinates the continuous buying, selling and delivery of wholesale electricity through the energy market. In its role as market operator, the ISO balances the needs of energy suppliers, wholesale customers and other market participants and monitors market activities to ensure open, fair and equitable access [35]. It is the responsibility of the ISO to maintain reliability and run both an effective and equitable market for all market



participants. The ISO must be equipped with powerful computational tools, involving market monitoring, ancillary services auctions and congestion management, for example, in order to fulfil its responsibilities [41].

Based on the CAISO, ISO New England, NYISO, and PJM ISO energy markets, the ISO responsibilities include the:

- development of a financially binding Day-Ahead energy market schedules based upon market participant-supplied demand bids, decrement bids, supply offers, increment offers, and priced and self-scheduled external transactions utilizing least-cost security-constrained unit commitment and dispatch analysis tools;
- posting of the following information after the clearing time (clearing time varies with ISO) of the Day-Ahead energy market on the day before the operating day:
  - (a) schedules for the next operating day by market participant (generation & demand),
  - (b) external transaction schedules (which may be in partial MW quantities),
  - (c) Day-Ahead Locational-based marginal prices (LMPs),
  - (d) Day-Ahead binding transmission constraints including reactive constraints,
  - (e) Day-Ahead net tie schedules,
  - (f) ISO load forecast for the next Operating Day,
  - (g) Aggregate demand bids,
  - (h) ISO Real-Time Operating Reserve objective,
  - (i) ISO Real-Time Replacement Reserve objective.
- performing of Reserve Adequacy Analysis (RAA) for the Real-Time energy market based upon the ISO forecast load and Real-Time operating reserve and replacement reserve requirements. Real-time load requirements, operating reserve, and replacement reserve requirements not covered through scheduling in the Day-Ahead energy market are accounted for by the ISO in the real-time energy market. This is done by adding to or modifying the resource schedule created in the day-ahead energy market based upon day-ahead bids and offers supplemented by bids (for Dispatchable Asset Related Demands only) and offers made during the re-offer period. Scheduling activities by the



ISO for the real-time energy market start prior to and continue throughout the operating day;

- posting of pre-dispatch schedule report containing the forecast output/consumption of resources (subject to appropriate confidentiality restrictions);
- clearing of the regulation market and posting of the preliminary real-time regulation clearing price (RCP) for each applicable interval consistent with the posting of preliminary LMPs for each hour of the operating day based upon regulation offers received prior to the close of the re-offer period;
- maintaining data and information which is related to generation facilities in the ISO control area, as may be necessary to conduct the scheduling and dispatch of the markets and control area;
- revision of current operating plan to reflect updated projections of load, changing electric system conditions and changing availability of resources;
- direction of market participants in the adjustment of the output of any resource including canceling the selection of resources and the dispatch of resource increments above amounts that were self-scheduled;
- provision of emergency external transaction sales to other control areas in accordance with the terms and conditions of the applicable interconnection agreements;
- operation of the ISO control area in accordance with NERC, NPCC and the ISO reliability criteria;
- obtaining of the most cost efficient and reliable regulation service available;
- designation of resources for the provision of operating reserve as part of the real-time linear optimization of resource dispatch to minimize the energy, congestion, and transmission loss costs, given system conditions and constraints;
- posting of preliminary values of real-time LMPs, real-time reserve clearing prices and regulation clearing prices during each hour of the operating day;



- posting of final values of real-time LMP, real-time reserve clearing prices and regulation clearing prices no later than five business days following the operating day;
- initiation and completion of energy settlement between market participants.

Generally, the Independent System Operator acts as the interface between the buyers and sellers of electrical energy and ensures that both parties are always satisfied with operational services offered to them.

### **2.2.3 Day-Ahead Market (DAM)**

This is the forward energy market for the following day or more specifically, the market for energy 24 hours in advance of a given time in any day. A day in this context may be more or less than 24 hours [12]. For example, a utility may purchase the next morning's energy in the afternoon (less than 24 hours ahead) or purchase the next afternoon's energy the previous morning (more than 24 hours ahead). Energy producers offer energy on this market based on their ability to produce energy for a specific period on the following day. Energy schedules (generation capacity and load demand) and ancillary service bids are submitted to the ISO a day prior to the operating day that the energy is needed. The hourly location-based marginal prices at the pre-determined locations are calculated for each hour of the next operating day based on supply offers, demand bids, increment offers, decrement bids and external transaction schedules submitted into the day-ahead energy market all of which may clear in partial MW quantities. In the day-ahead energy market, the ISO determines the least-cost means of satisfying the cleared demand bids, cleared decrement bids, operating reserve, replacement reserve, local second contingency protection resource requirements and other applicable ancillary services requirements of market participants, including the reliability requirements of whole ISO control area [50].

Market participants are referred to all energy and service providers involved in the generation, transmission, and distribution of power and provision of any of the services required in maintaining the reliability of the power system at all times. Market participants can be involved in buying energy from the energy market or selling into the energy market. Others can be



involved in wheeling energy through the Real-Time energy market or participating in load response programs as can be seen with the ISO of New England Inc.

The ISO evaluates the energy schedules and ensures that it is “balanced” that is the total capacity from all generating sources must be greater than or equal to all demands every hour by a margin set by the ISO. The California Independent System Operator for example assumes balanced energy schedules if the sum of all resources (generation, imports and purchasing trades) is within 2 MWs of all demands (loads, exports and selling trades) for each hour [7]. After confirmation of the balance of energy schedules, the ISO processes the schedules and bids through the security constrained unit commitment scheduling software which co-optimizes energy and ancillary services (reserve and regulation) and takes into consideration transmission constraints. After the scheduling process has been completed, the schedule result is sent back to the Day-ahead operator who reviews it and sends it out to scheduling coordinators. The scheduling coordinators review the schedule and make changes if necessary considering congestion and other changes that may arise due to their submitted energy schedules and bids. If a change is made, the scheduling process is conducted using the security constrained unit commitment scheduling software. The new schedule is reviewed, published and sent to all scheduling operators including suppliers and loads who sign off the contract confirming acceptance of the Day-Ahead final schedule result. At this point all financial results are final and binding to all market participants. For the California Independent System Operator, the final day-ahead market is converted to the “Default Hourly schedules” to be processed through the hourly market if no other modifications are made to them by the scheduling coordinators prior to the hourly market closing [7].

The day-ahead energy market settlement based on day-ahead scheduled hourly quantities and on day-ahead LMPs is generally given by

$$DAM\ Energy\ Settlement = DAM\ Schedule\ MWh * DAM\ LMP \quad (2.2)$$



#### 2.2.4 Hourly Market (HM)

The hourly market (HM) is the market for the electrical energy that can be delivered to the customer for use in the next hour. It is designed to be a way for the scheduling coordinators to make incremental changes to their day-ahead finals and/or add any new schedules to their portfolios [7]. The hourly market is developed just like the day-ahead market and its review is done concurrently with the Day-ahead final result. If any of the hourly schedules is different from the day-ahead final, it overwrites that particular hour schedule in the day-ahead final and if there is no change, the day-ahead final remains the hourly schedule for the hour being considered.

The hourly market is usually not a market adopted by most ISOs since it can be incorporated into the real-time energy market without any major difference or setback in economic dispatch. The hourly energy market settlement given below is similar to that of the day-ahead market settlement

$$HM \text{ Energy Settlement} = HM \text{ Schedule MW} * HM \text{ LMP} \quad (2.3)$$

#### 2.2.5 Real-time market (RTM)

The real-time energy market is a balancing market for energy scheduled and energy demanded on the actual consumption day. The LMPs at pre-determined locations are calculated every five minutes (for example) based on the actual system operations security-constrained economic dispatch. Real-time market exists for electricity, where the time scale can be as small as a few minutes.

The ISO usually monitors and controls the Control Area such that the least-cost means of satisfying the projected hourly energy, regulation, operating reserve, replacement reserve and other ancillary services requirements (as required) as well as system reliability requirements are always met. The ISO procures power from the real-time market during contingencies that cannot be met by the already scheduled capacity of the day-ahead market. It is usually one of the most



expensive sources of power. The conventional bidding and unit commitment process that take place in the DAM and HM are also conducted in the real-time market except that the supply and demand bidding as well as real-time unit commitment and dispatch and actual power delivery are conducted within a shorter time interval.

During real-time operation, changes in operating conditions, the influence of additional real-time supply bids, and variations in actual load will cause the real-time schedules and prices to be different from the day-ahead schedules and prices. Difference in generation levels and in load consumption as compared to the first settlement (DAM and HM) values are settled at the second settlement, or real-time price.

The real-time energy market settlement is based on actual hourly (integrated) quantity deviations from day-ahead scheduled quantities and real-time LMPs integrated over the hour. Generally for a particular hour, the RTM energy settlement is given as:

$$RTM \text{ Energy Settlement} = (RT \text{ MW} - DAM \text{ Schedule MW}) * RT \text{ LMP} \quad (2.4)$$

### **2.3 Ancillary Services**

In order to ensure that electricity is delivered reliably and the system is operated securely, various ancillary services are needed. Ancillary services are additional services necessary to support the transmission of capacity and energy from generation resources to consumers, while maintaining a secure and reliable operation of the power system in accordance with accepted electric industry practice. There are different types of ancillary services that must be optimally priced and provided. Some ancillary services like Scheduling, System Control and Dispatch (S, SC, D); Voltage Support; and Black Start Capability are provided at Cost-Based Prices while services like Regulation and Frequency Response; Operating Reserves; and Energy Imbalance are provided at Market-Based Prices [28].

The cost associated with the operation and administration of the Independent System Operators (ISO) falls under Scheduling, System Control and Dispatch while the cost that deals with maintaining proper balance in the delivery of electrical energy within the ISO's Control Area is



in the Voltage Support category. The Black-Start services deal with the cost of providing system restoration in the event system-wide black out [28].

### 2.3.1 Market-Based Ancillary Services

The ancillary services market helps adjust the flow of electricity when the unexpected happens, such as a power plant failure or a sharp rise in demand for power. The capacity that is bought and sold can be dispatched within seconds, minutes or hours [7]. Four types of energy are for sale in the ancillary services market and they make up the category of ancillary services called operating reserves. The sale of the following types of operating reserves can be conducted both in the day-ahead, hourly and real-time markets of when electricity is used:

- **Regulation and Frequency Response** – This refers to generation that is already up and running (synchronized with the power grid) and can be increased or decreased instantly to keep energy supply and energy use in balance. It is necessary for the continuous balancing of resources with load and assists in maintaining scheduled Interconnection Frequency at 60 Hz.
- **Spinning Reserve** -- Generation that is running, with additional capacity that can be dispatched within minutes.
- **Non-Spinning Reserves** -- Generation that is not running, but can be brought up to speed, within ten minutes.
- **Replacement Reserves** -- Generation that can begin contributing to the Grid within an hour.

### 2.4 Spinning Reserve in a Deregulated System

Operating reserves make up the real power generating capacity related to market-based ancillary services. It is the backup generation scheduled to be available for specified periods of an operating day and called up during contingencies like generator failure or escalation of the load demand to maintain the security and reliability of the system.



Operating reserves energy settlement is based on Locational Marginal Pricing (LMP) system which includes the actual cost of providing the next MW of reserve for a particular reserve type and at a particular reserve location. Energy settlement for operating reserves can either be in the day-ahead or real-time markets. In day-ahead market (DAM), DAM operating reserve settlement is given as:

$$DAM Reserve Settlement = DAM Schedule Reserve MWh * DAM Reserve LMP \quad (2.5)$$

While in real-time market (RTM), RTM operating reserve energy settlement is given as:

$$RTM Reserve Settlement = (RT Reserve MW - DAM Schedule Reserve MW) * RT Reserve LMP \quad (2.6)$$

Operating reserves can be classified into spinning reserve and non-spinning reserve, based on units' operating status [54]. Non-spinning reserve is an off-line generation capacity that can be started, ramped to capacity and synchronized to the grid within minutes of a dispatch instruction by the ISO, and is capable of maintaining that output for at least two hours [7]. Non-spinning reserve response time to synchronization and loading depends on the type of non-spinning reserve. The Response Time of the non-spinning reserve defines the type of non-spinning reserve. The NYISO for example has 10 minutes and 30 minutes NSR. Non-spinning reserve is needed to maintain system frequency stability during emergency conditions. Non-spinning reserve differs from the spinning reserve (SR) in that it is not synchronized to the system.

Spinning reserve (SR) as defined in the previous chapter is one of the most important operating reserves used by power system operators to respond to contingencies like sudden generation outages and increases in load demand. The ISO needs to maintain an acceptable level of spinning reserve at all times in order to withstand possible load demand increases or contingencies. The major challenge associated with spinning reserve is the determination of the quantity needed in the day-ahead market on hourly basis. It is important that the procurement, pricing and allocation of spinning reserve are done in an economical way that ensures that the system remains reliable and secure.



A system with inadequate SR is exposed to load shedding and system imbalance as a result of its inability to meet any unforeseen increase in load demand. However, increasing the SR may mitigate any unforeseen increase in load demand and maintain security but becomes sub-optimal from an economic point of view. The amount of spinning reserve that a system desires to carry usually is based on economy and its tolerance to risk [2]. The cost of operation is directly related to the level of spinning reserve. As a result, most utility systems are always encouraged to operate “closer to edge”, that is minimizing the SR requirement while attaining a target level of risk [29]. In order to determine the most economic generation schedule for daily operation, the evaluation of the optimum spinning reserve for maintaining adequate level of service reliability is required. It is important for an Independent System Operator to define the amount of SR needed, its allocation and the total cost of the service [24] while maintaining reliability and security of the system.

In a traditional regulated utility structure, spinning reserve is determined and allocated during the unit commitment process. During this process, the amount of generation capacity scheduled is usually greater than the load for that period. This additional capacity accounts for the SR and is scheduled to satisfy any unforeseen contingencies in the system. Deterministic (or fixed) criteria are used in determining this additional capacity in a traditional structure. The objective of the fixed criteria approach utilized by power system operators is to minimize operating cost up to a particular level of risk in the system throughout the operating period. It can be said that as the amount of SR provided in the system increases, the system risk of possible load shedding reduces. This means that by maximizing the SR requirements, the system risk is minimized with a corresponding increase in operating cost. The fixed criterion for SR requirement minimizes the running cost by dividing this requirement among the various generating units to get the minimum total start-up/back-down and operating costs [29]. While it is a lot easier to implement the fixed criteria SR requirements, it does not offer an optimal way of maximizing the SR while minimizing cost. During the times when scheduled load demand is lower than the actual load, the fixed SR requirements scheduled become unnecessary and therefore uneconomical for the system.



Generally, the deterministic approach used by various system operators involves one or a combination of the following:

- a) a fixed capacity margin
- b) a fixed percentage of the system load
- c) a fixed percentage of the online operating capacity and
- d) largest online generating unit

The deterministic criteria do not utilize the stochastic nature of the power system components and therefore, is sub-optimal for today's competitive electricity market environment.

In the new competitive electricity market environment, spinning reserve is procured from the spinning reserve market when needed. Assuming there are no outages of generators, the major reason system operators schedule SR is due to the uncertainty associated with forecasting hourly load demands in the Day-ahead market. Due to the imbalance between load demand and generation schedule, Day-ahead energy settlement may differ from real time needs. System operators try to forecast the load demand as accurately as possible but accurate forecasting is extremely difficult. Usually, the load and DAM schedule imbalance is found to be either positive or negative. When there is an unexpectedly high demand, scheduled SR becomes inadequate to meet the increasing load demand. To meet this deficit, the Independent System Operator procures extra power from the spot market at the spot market price. This also results in an increased operating cost which is a lot higher than the cost during periods when actual load is less than forecasted load.

Probabilistic approach incorporates the stochastic nature of system components in the assessment of the spinning reserve requirements. Stochastic components of the power system include load demand and the failure and repair rates of generation and transmission. Different levels of deviations that arise from the Day-ahead load forecasting influence the spinning reserve requirements. This raises the question of what amount of SR is required such that the operating cost is minimized while ensuring that there is no load shedding during any hour. The optimal spinning reserve is that amount which minimizes the overall cost of running the system up to the point where an extra MW of spinning reserve is not economically justified [29]. This point is defined in the form of a probability index or risk. In the hourly operation of the electric utility



system, the objective is to operate at any time during the day only such capacity as is required to supply the load and to provide reasonable protection against load forecasting deviations and generating equipment failure, except where it may be more economical to operate additional equipment while meeting the reliability index of that system [1].

#### 2.4.1 Deterministic (Fixed) Criteria for Spinning Reserve Requirements

Most ISOs utilize deterministic criteria to determine the required amount of SR for overcoming unscheduled generator outages and load forecast uncertainty. Different deterministic approaches to assess spinning reserve requirements have been implemented by system operators. Some of those approaches are discussed below.

A widely used technique named as largest contingency approach requires the spinning reserve to be greater than the capacity of the largest online generating unit (Wood and Wollenberg, 1996). While this criterion ensures that scheduled load demand is always met as long as the loss of generation is limited to one generating unit or equal to the amount of increase in the scheduled load demand. Load shedding will result if there is an outage of more than one generating unit assuming that the sum of the capacities of the failed units is greater than the scheduled SR plus the amount of increase in the scheduled load demand. This criterion is used by systems like Southern zone of PJM [34] in managing their hourly spinning reserve requirements.

The Australian, Ontario, and New Zealand linear programming based dispatch models utilize a form of the deterministic criteria shown below in determining the hourly SR requirements [8]:

$$\forall i, t, \quad SR_t \geq \max(p_i^t) \quad (2.7)$$

where,

$i$  indexes on – line units  $1, \dots, n$

$t$  indexes the dispatch period

$p_i^t$  is the MW dispatch of unit  $i$  at time  $t$ .

The above criterion assumes that simultaneous outage of generating units is unlikely. It also ensures that unnecessary reserve is not scheduled since the SR requirement is forced to be at



least as large as the maximum on-line generation. Even though the SR requirement is equal to the output of the most heavily loaded generating unit, it has been shown that this criterion does not always ensure that the entire load would be served in case of a single outage [29].

Another criterion for SR requirement used in the area of small isolated power system (SIPS) by the Manitoba hydro is a combination of a percentage of the largest online generating unit and a percentage of the installed capacity [4]:

$$\forall i, t \quad SR_t = 80\% \max(u_i^t P_i^{max}) + 20\% \left( \sum_{i=1}^N P_i^{max} \right) \quad (2.8)$$

where,  $P_i^{max}$  is the capacity of the online unit,  $i$  at time,  $t$

The merit of this criterion is that it can sustain security of the system against larger contingencies that are greater than the outage of a single generating unit as well as unpredictable load demand swings. This security comes at a price of an increased operating cost thereby making it a suboptimal system.

Other fixed deterministic criteria that are still been used by different system operators are listed in Table 2.1 [4, 29]. The deterministic criterion varies from system to system based on the acceptable level of security risk desired [29] by the system operators.

Each criterion is characterised by its merits and limitations like inadequate security against contingencies as well as possibility of an increased cost of operation during suboptimal allocation of SR. Deterministic criteria are easy to implement but they do not reflect the stochastic nature of load forecast and outage of generating units [15]. They do not consistently define the true risk of generation shortages and are also hard to combine with economic criteria. Most times, deterministic criteria may provide a misleading sense of confidence in the adequacy of system generation on the basis of rule of thumb [37].



**Table 2.1: Deterministic Criteria for Spinning Reserve Requirements of different System Operators**

System	Criterion, ( $SR_t$ )
Australia and New Zealand	$\max(u_i^t p_i^t)$
BC Hydro	$\max(u_i^t P_i^{max})$
Belgium	<i>UCTE rules, currently at least 460MW</i>
California	$50\% \max(5\% P_{hydro}$ $+ 7\% P_{other\ generation}, P_{largest\ contingency})$ $+ P_{non-firm\ import}$
France	<i>UCTE rules, currently at least 500MW</i>
Manitoba Hydro	$80\% \max(u_i^t P_i^{max}) + 20\% \left( \sum_{i=1}^N P_i^{max} \right)$
PJM(Southern)	$\max(u_i^t P_i^{max})$
PJM(Western)	$1.5\% p_d^{max}$
PJM(Other)	1.1% of the peak + probabilistic calculation on typical days and hours
Spain	Between $3(p_d^{max})^{\frac{1}{2}}$ and $6(p_d^{max})^{\frac{1}{2}}$
The Netherlands	<i>UCTE rules, currently at least 300MW</i>
UCTE	No specific recommendation. The recommended maximum is: $(10p_{d,zone}^{max} + 150^2)^{\frac{1}{2}} - 150$
Yukon Electrical	$\max(u_i^t P_i^{max}) + 10\% p_d^{max}$
Newfoundland Hydro	$\max(u_i^t P_i^{max})$



Table 2.1 Continued

Hydro Quebec	$90\% \max(u_i^t P_i^{max})$ $+ 10\% \left( \sum_{i=1}^N P_i^{max} \right) \text{ for plants } < 5 \text{ engines}$ <p style="text-align: center;"><i>or</i></p> $90\% \max(u_i^t P_i^{max}) + 90\% \min(u_i^t P_i^{max})$ $+ 10\% \left( \sum_{i=1}^N P_i^{max} \right) \text{ for plants } > 6 \text{ engines}$
Ontario Hydro	$\max(u_i^t P_i^{max})$
NWT Power Corporation	$\max(u_i^t P_i^{max}) + 10\%(\text{peak load}) \text{ for peak load } < 3\text{MW}$ <p style="text-align: center;"><i>or</i></p> $\max(u_i^t P_i^{max}) + 5\%(\text{peak load}) \text{ for peak load } > 3\text{MW}$
Alberta Power Ltd	$\max(u_i^t P_i^{max}) \text{ or}$ $90\% \max(u_i^t P_i^{max}) + 10\% \left( \sum_{i=1}^N P_i^{max} \right) \text{ for remote sites}$

#### 2.4.2 Probabilistic Criteria for Spinning Reserve Requirements

The consideration of the probabilistic nature of load forecast in the evaluation of spinning reserve requirements was first analyzed by Anstine *et al.*, [1]. The authors suggested that the spinning reserve required to provide a given degree of reliability of service for a given period is a function of the size of the individual units, the number of units and the reliability of the units scheduled to operate during the period as well as the time required to start up marginal equipment and load forecast uncertainty.

They also established that with a satisfactory design level of risk, probability methods provide a convenient means of varying the spinning reserve from period to period so as to maintain a



uniform level of reliability. This satisfactory design level of risk is solely based on choice rather than an intuitive interpretation and therefore does not *per se* tell how much SR should be scheduled [29]. As a result of this, the amount of SR required can either be increased or decreased depending on the level of risk chosen. This technique does not optimize the SR provision itself but instead, it just increases the committed capacity until the target design risk level is attained since the risk level is a factor of number of generating units, and the capacity, loading and reliability of these units as well load forecast uncertainty [29]. Also, this technique produces suboptimal solutions since it ignores the individual start-up and production costs of the generating units. Another deficiency of this technique is the use of a uniform design level of risk for all periods. This may not be economical since the commitment of extra generating capacity during any of the periods may not necessarily be economically efficient for the system and may not be beneficial.

Another technique proposed by Thapar and Chauhan suggested that unforeseen changes in the load can be reduced considerably by accurate daily and hourly load forecasting and by constant and adequate generation dispatching [45]. This technique evaluated the actual SR requirement as a factor of the desired level of reliability defined as that level in which the load will exceed the probable available capacity. They considered the deviations of the actual load from the forecast load as a normal probability distribution. An assumption was made by the authors that sufficient capacity is available to meet any possible outage of generation which can be replaced in a matter of time. A deficiency in this technique is having a uniform level of risk for all periods which can result in scheduling extra capacity even though it may not be economically efficient. This extra capacity increases operating cost and makes the SR requirements suboptimal for a deregulated system.

S. Porkar *et al*, proposed a new spinning reserve market structure that best utilizes the available resources to meet load and spinning reserve requirements with transmission constraints while maintaining system security in the context of an energy market [36]. Their probabilistic technique generates a risk index, defined as the probability that the system will fail to meet the load or be able to just meet the load during a specified time in the future and the failure to meet



this index results in an increased system cost. The optimization problem formulated is based on a model of joint energy and spinning reserve dispatching in a competitive pool and it is given as:

$$\text{Min}_{P^g, P^S} \left\{ \sum_{i=1}^n C_i^g P_i^g + \sum_{k=1}^K Q(k) * \left[ \sum_{i=1}^n C_i^S P_i^S + EENS(k) * VOLL(k) \right] \right\} \quad (2.9)$$

Where  $n$  is the number of generation buses and  $k$  is a list of credible contingencies with their associated probabilities. These contingencies may include sudden increase of system demand, a generator unit and/or a transmission line forced outage. The term  $EENS(k)*VOLL(k)$  is the value of lost load if demand has to be curtailed in condition  $k$ , or the cost of the expected energy not served in condition  $k$ . Assuming that there are no outages of generators, it can be seen from the model that the authors were able to consider the probabilistic nature of load forecast since there is always a possibility that the system demand will be different from DAM schedule. The proposed technique considers the total cost in only one particular situation that is when system demand is greater than DAM schedule. Ignoring the other situation where there is more power scheduled than needed and the corresponding cost makes the minimization problem model incomplete. Also the probability of the occurrence of a contingency  $k$ , due to increased demand is a random value which has no direct link with the load forecast. If this random choice of probability value describing the increase in system demand is inaccurate, the SR requirements evaluated by minimizing the total cost can be sub-optimal.

Gooi *et al* proposed another probabilistic technique for computing the spinning reserve requirement (SRR) based on the risk index [15]. The technique takes a step further in determining the optimal risk index instead of choosing an arbitrary value for the system risk index. This optimal risk index is the probability value which balances the cost of providing the SR and the expected cost of energy not served. The SRR is evaluated considering the reliability of the individual scheduled units and the accuracy of the load forecast. Considering a seven-step normal probability distribution for the load forecast uncertainty, the final system risk index is the sum of the seven products of the COPT risk and demand probability. The main weakness of the risk index approach to probabilistic spinning reserve assessment is the lack of an intuitively quantifiable interpretation of this index. The authors proposed that in order to determine an



accurate system risk index, the marginal cost of providing the spinning reserve at that index must equal the cost of energy not served. If a risk index does not provide a SR with a marginal cost greater than the expected cost of energy not served, the risk index is reduced and the SRR re-evaluated and its cost compared to the cost of energy not supplied. The inefficiency in the evaluation of the SRR based on risk index is a set-back for this technique.

Another probabilistic technique proposed by A.M. Leite da Silva and G.P. Alvarez [9] had the advantage of not utilizing a risk index set a priori since setting such risk index subjectively makes it impractical to know if the SRR associated with this risk index is actually appropriate and meets the optimal balance between system reliability and total operating cost. The authors evaluated all the different operating reserves requirements from a cost/benefit analysis point of view utilizing an “Interruption Cost as Moderator agent” called the loss of load cost, LOLC. The LOLC of the system is used as a reference value in determining which reserve resources should or should not be selected (committed). The lower the LOLC is, the higher the reliability since more reserves are selected to be utilized. While this technique balances cost and reliability, the authors only incorporated generator outage contingency in the form of a probability called outage replacement rate (ORR) while ignoring the probabilistic nature of load forecast as it affects system demand, a major contributor to the need for operating reserves, in this case spinning reserve.

In the paper, an Electricity Market with a probabilistic spinning reserve criterion by Bouffard and Galiana [6], the authors proposed a reliability-constrained market clearing technique in a pool-based electricity market with unit commitment. The spinning reserve requirements are evaluated using two probabilistic reliability criteria metrics namely the Loss-of-Load Probability (*LOLP*) and the Expected Load Not Served (*ELNS*). They proposed that the probabilistic spinning reserve requirement should be set by imposing a ceiling on the *ELNS* and the *LOLP* such that at each period in the optimization horizon:

$$LOLP^t \leq LOLP_{target} \quad (2.10)$$

$$ELNS^t \leq ELNS_{target} \quad (2.11)$$



Setting these two metrics to be less than or equal to an acceptable level ( $LOLP_{target}$  and  $ELNS_{target}$ ) ensures that the assessment of spinning reserve requirement is based on statistical availabilities of the various generating units. However, using these arbitrary targets makes the SR requirements suboptimal. Both the LOLP and the ELNS are expressed explicitly in terms of the unit commitment variables. The demerit of this technique is that it is developed for only simultaneous outage events in order to reduce the high combinatorial burden and computational time due to the non-linearity of the functions.

Z. Song *et al* proposed a spinning reserve allocation method that is based on minimizing the spinning reserve cost (SRC) and the unit commitment risk (UCR) as shown in Equation 2.12 [43, 44]. A lower UCR indicates a reduction in the cost for loss of load while increasing the cost for reliability. The UCR is evaluated based on the outage replacement rate ( $ORR$ ) of each system component (generating units and transmission lines).

$$\min TC_i = RC(UCR_i) + SRC_i \quad (2.12)$$

In order to evaluate optimal spinning reserve requirement, a trade off between cost and reliability must be made in an effective and efficient way. The reliability cost as a function of the UCR is the cost of consumers' loss of load. This cost varies with different ISOs since the value that a consumer places on a loss of load varies with different consumers. The deficiency in this approach is that it utilizes a fixed criterion in evaluating first the system SR requirements and then determines the minimum periodic allocations based on cost and risk. This makes the SR requirements suboptimal when contingencies like increased load demand and generator outage exceeds that fixed requirement or when the fixed requirement is greater than the total contingency.

Chattopadhyay and Baldick proposed a technique of integrating the probabilistic criterion based on the full capacity outage probability distribution (like the LOLP) into the unit commitment (UC) optimization using simple statistical approximation [8]. The capacity outage probability table (COPT) is approximated using an exponential function whose parameters  $A_0$  and  $M$  are system-dependent and can be statistically determined. In order to determine the level of SR



required to attain a predetermined reliability risk level (for example  $LOLP$ ), the SR requirement for any period is expressed as a linear constraint:

$$SR_t \geq M [A_0 - \ln(LOLP_{target})] \quad (2.13)$$

This linear constraint approximation is incorporated into the unit commitment optimization problem thereby eliminating post-processing of the unit commitment schedule as proposed by Gooi *et al* [15]. Chattopadhyay and Baldick[8] proposed that the  $LOLP_{target}$  can be based on some pre-determined criterion which can give sub-optimal solution if not accurately estimated for the system. They also proposed that the limit can also be implicitly determined by cost/benefit analysis. The determination of this limit by cost/benefit analysis can be difficult because the reserve and interruption costs depend on the generating units that are scheduled and thus change at each period. Secondly, the LOLP is not particularly suited to the computation of the societal cost of outages because it measures only the probability that the load exceeds the generating capacity but does not quantify the extent of the disconnections that might result from such deficits [29]. This technique is dependent on the accuracy of the approximation and the assumptions made to extend the risk approximation for different unit commitment patterns.

A probabilistic approach called load forecast uncertainty (LFU)–based spinning reserve assessment (LSRA) which assumes the load forecast uncertainty as the only stochastic component of the system to assess the spinning reserve requirements in a deregulated system is developed and discussed in details in the next chapters.

## 2.5 Summary

Definitions and terms associated with a deregulated power system are presented in this chapter. Also covered in this chapter are the roles and responsibilities of the Independent Market Operator (IMO) and the Independent System Operator (ISO) and the different energy markets. Different deterministic techniques for the assessment of the spinning reserve requirements and their merits/demerits are discussed. The final part of this chapter presents different probabilistic



techniques that have been proposed by some authors for the assessment of spinning reserve requirements in a deregulated system and their drawbacks.



## **Chapter 3**

# **Assessment of Spinning Reserve Requirements without Transmission Loss**

### **3.1 Introduction**

Adequate spinning reserve is essential in maintaining the reliability and security of a power system. As discussed in the previous chapter, deterministic techniques do not consider the stochastic nature of system contingencies in assessing spinning reserve requirements.

A probabilistic technique for spinning reserve assessment in a deregulated system has been developed based on the direct search approach. The technique optimizes the procurement of energy from the day-ahead market, spinning reserve from spinning reserve market and any energy deficiency due to real time imbalance from the spot energy market. The proposed model assumes that stochastic nature of the load forecast is the only source of uncertainty. Generators commit to supply energy through a bidding process and hence the random outages of generators are not taken into account. A 49-step discretized normal probability distribution with the forecast load as the mean,  $\mu$  and a known percentage standard deviation,  $\sigma$  is used in describing the load forecast uncertainty. The model is incorporated into the development of the optimization function for calculating the total operating cost for a given period. The optimization cost function is formulated without transmission loss. During each period within the optimization interval, the



day-ahead market LMPs, ramp-rates, reloading limits and real-time market LMPs for both energy and spinning reserve are considered in assessing the spinning reserve requirements during that period. Therefore, all available data in the day-ahead market are utilized in the assessment of the spinning reserve requirements. This method is described in more details in the following section.

### 3.2 Optimal Spinning Reserve Considering Load Forecast Uncertainty

There are two stages involved in the determination of the optimum level of spinning reserve. Both stages are implemented after unit commitment has been done for the forecast load without the incorporation of spinning reserve. The unit commitment process for generators or suppliers forms the initialization part of this approach. Also in the initialization section, relevant information of the power system is gathered (like market clearing price for energy from generating zones, zones' ramp rates, ramping cost, spinning reserve price, spot market price, zones' minimum and maximum generating capacity limits, standard deviation of the load forecast uncertainty model, etc). The first stage deals with the probability distribution and deviation of the load forecast within the load forecast uncertainty (LFU) model. Each of the 49 steps of the load forecast uncertainty model is described by a load demand,  $D_s^t$ , the forecast load,  $P_{Load}$ , the standard deviation of the LFU model  $\sigma$  and the probability  $p(L = D_s^t)$  that, for a given estimated load  $P_{Load}$ , the actual load will be an amount  $D_s^t$  in real time [1].

The second stage deals with the analysis and decision involving each of the steps of the load forecast uncertainty model. This stage entails deciding which of the energy options available can be utilized to meet demand and procure spinning reserve in the most economical way and by what amount based on the prices of energy and the ramping rates of the generating zones for every step within that particular period and over the time horizon being optimized. Figure 3.1 shows the simplified flow diagram of the proposed approach.



### 3.2.1 Load Model Formulation

In Chapter 2, it was pointed out that the uncertainty in the load forecast of the day-ahead market can be described by a normal probability distribution divided into class intervals (steps), the number of which depends upon the accuracy desired. The area of each class interval represents the probability of the load being the class interval mid-value [45]. Figure 1.1 describes this distribution which is defined by the mean as the forecast peak load and the standard deviation,  $\sigma$  of the uncertainty. In this approach, a 49-step normal probability distribution is considered. Table 3.1 gives the load forecast uncertainty versus magnitude of uncertainty for each of the forty-nine (49) class intervals or steps [1]. It is important to note that the sum of the all the probabilities is equal to 1.

### 3.2.2 Description of the Test System without Transmission Loss

A test system is used for the demonstration of the proposed approach and it consists of three (3) generating zones of total installed capacity of 9900MW. Each zone consists of generators of various sizes. The total system peak load and base load are 5101MW and 3297 MW respectively. The load demand is spread across two of the seven system buses. In this test system, it is assumed that there are no transmission losses and therefore energy from any of the zones (Z1, Z2, and Z3), the spot market (SM) or the spinning reserve market (SRM) is available to any of the load buses (buses 2 and 5) at any point in time. Fig. 3.2 shows the single line diagram of the test system. The zone information including hourly zone commitment, market clearing price, reloading up and down prices, spinning reserve price, spot market price, reloading limits and generating zone limits are shown in Appendix A. The zone commitment is based on the DAM hourly loads and since transmission loss is excluded, the total hourly zone commitment is equal to hourly demand.



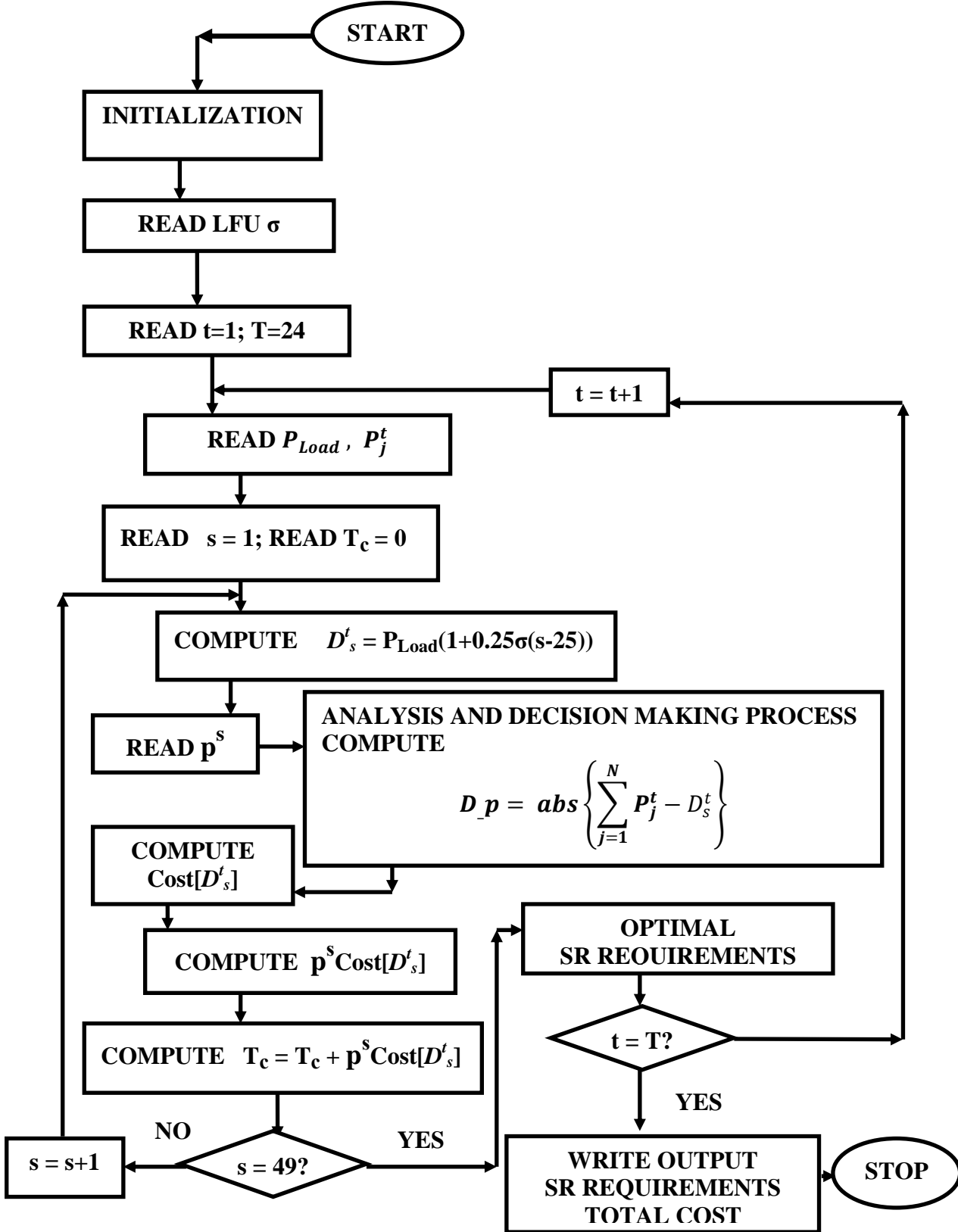


Figure 3.1 Simplified Flow diagram of Proposed Approach without Transmission Loss



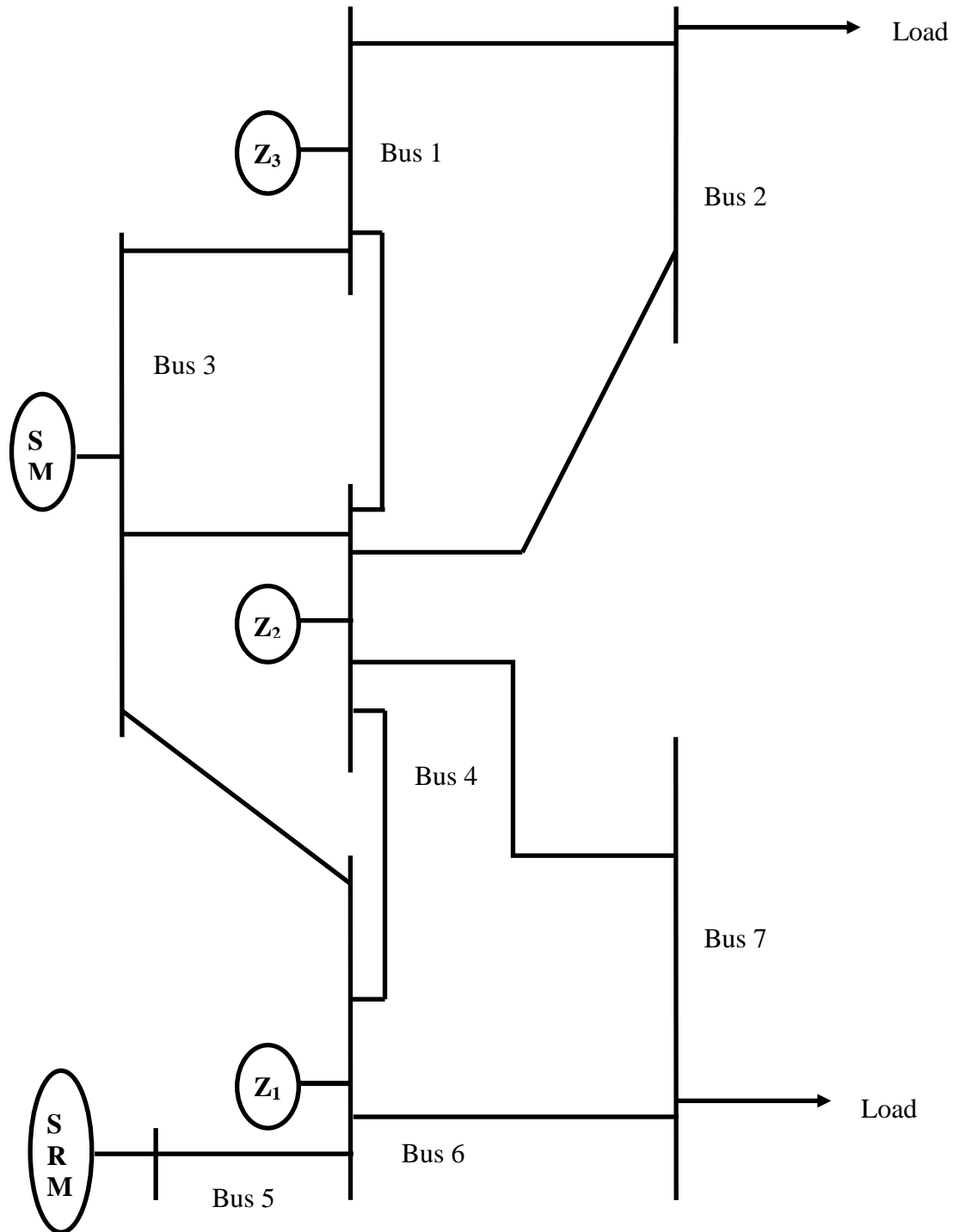
**Table 3.1 Load Forecast Uncertainty Probability Versus Magnitude of Uncertainty for all Class Intervals**

Sub-scenarios	Load Forecast Uncertainty in $\sigma$ 's	Probability of Load Equal to Amount Shown	Sub-scenarios	Load Forecast Uncertainty in $\sigma$ 's	Probability of Load Equal to Amount Shown
1	-6	0.000000002	26	0.25	0.096431542
2	-5.75	0.000000007	27	0.5	0.087844705
3	-5.5	0.000000029	28	0.75	0.075198576
4	-5.25	0.000000110	29	1	0.060492436
5	-5	0.000000395	30	1.25	0.045728795
6	-4.75	0.000001329	31	1.5	0.032484443
7	-4.5	0.000004199	32	1.75	0.021339370
8	-4.25	0.000012465	33	2	0.013948602
9	-4	0.000034776	34	2.25	0.008018831
10	-3.75	0.000091168	35	2.5	0.004442027
11	-3.5	0.000224598	36	2.75	0.000000023
12	-3.25	0.000519947	37	3	0.001131112
13	-3	0.001131112	38	3.25	0.000519947
14	-2.75	0.000000023	39	3.5	0.000224598
15	-2.5	0.004442027	40	3.75	0.000091168
16	-2.25	0.008018831	41	4	0.000034776
17	-2	0.013948602	42	4.25	0.000012465
18	-1.75	0.021339370	43	4.5	0.000004199
19	-1.5	0.032484443	44	4.75	0.000001329
20	-1.25	0.045728795	45	5	0.000000395
21	-1	0.060492436	46	5.25	0.000000110
22	-0.75	0.075198576	47	5.5	0.000000029
23	-0.5	0.087844705	48	5.75	0.000000007



Table 3.1 Continued

24	-0.25	0.096431542	49	6	0.000000002
25	0	0.099476450			



**Fig. 3.2 Single Line Diagram of the Test System without Transmission Loss**



### 3.2.3 Formulation of the Operating Cost Function

The cost function is developed at the Analysis and Decision Making stage as can be seen from Figure 3.1. For each hour in the load curve of a 24-hr period, economic and reliability-based decisions are made with respect to expected actual load, the total zone commitment and energy prices for that hour. Consider Figure 1.1 which describes a 7-step normal distribution of the day-ahead load forecast uncertainty defined by the mean as the forecast load and the standard deviation of the forecast. For the purpose of accuracy, a 49-step normal distribution of the load forecast uncertainty is used in the formulation of the cost function.

The forty-nine (49) class intervals define 49 possible values of what the actual load can be during each of the hours within the time horizon of a day-ahead market. During hour  $t$ , each class interval,  $s$  of the 49 steps is characterized by:

- $p(L = D_s^t)$  - probability that for a given estimated hourly load ( $P_{Load}$ ) the actual load will be an amount “ $D_s^t$ ”. This forms the load model;
- $D_s^t$  - actual load during hour  $t$  for a class interval  $s$  and
- $P_j^t$  - scheduled generating capacity of generating zone,  $j$  at time,  $t$ .

Mathematically,  $D_s^t$  can be computed using Equation 3.1

$$D_s^t = P_{Load}[1 + 0.25\sigma(s - 25)] \quad (3.1)$$

It can be deduced from Figure 1.1 that there are three major scenarios during each hour in the time horizon for all the 49-steps in the load model. Each of these three scenarios defines the type of economic and reliability decisions that can be made during that particular hour by the system operator in order to meet demand and minimize operating cost during that period. Scenario A describes any value of  $s$ :  $s = 1, \dots, 24$  for which the total hourly zone commitment exceeds the actual load during that hour while scenario B defined as  $s = 25$  describes the state in which the actual load is equal to the sum of all zone commitment for that hour. Similarly, scenario C describes all values of  $s$ :  $s = 26, \dots, 49$  for which the total zone commitment is less than the actual demand for that hour.



Generally, the total operating cost for the period,  $t$  given by Equation 3.2 is evaluated as the sum of the product of the probability,  $p(L = D_s^t)$  and the cost for meeting the demand,  $D_s^t$  for every value of  $s$ .

$$Total\ Cost = \sum_{s=1}^{49} p(L = D_s^t) cost(D_s^t) \quad (3.2)$$

There are different costs for each value of  $D_s^t$  which depends on which of the categories that the value of  $s$  falls into. The cost function is dependent on these three categories as described below:

### 3.2.3.1 Scenario A

For all values of  $s$  within scenario A, the actual demand given as  $D_s^t$  is less than the total hourly scheduled capacity during that hour. Under this scenario, there is more power committed than needed. This results in a situation whereby generating zones are decommitted in the most economical way to the required demand level for that hour while still maintaining reliability at a minimized operating cost. The analysis and decision process of the decommitment starts with the determination of the cheapest path. A path describes the decommitment route to follow based on which generating zone capacity should be ramped down first and the next zones to follow in an increasing order of their respective reloading-down price,  $RDP_j^t$ . Based on this approach, the zone with the least incremental cost given as  $abs(RDP_j - MCP)$  is decommitted first followed by the next zone with second lowest incremental cost and so forth while meeting the capacity limits of those zones. The amount of power decommitted from a zone is dependent on three factors, namely:- the lower limit of that generating zone given as  $P_j^{min}$ , its reloading-down limit,  $DR_j$  and  $D_p$  given by Equation 3.3.

$$D_p = \sum_{j=1}^N P_j^t - D_s^t \quad (3.3)$$



$D_p$  is the difference between the sum of all the zone commitments and the actual load based on the value of  $s$  for that hour. It is the surplus power above the amount required to meet demand during a particular period. Let the term,  $x_j^s$  be defined as the amount by which the generating zone  $j$ , is decommitted for that value of  $s$  during that period in the optimization time horizon. After the decommitment process is completed with all constraints satisfied, energy settlement is conducted based on the economic and reliability decisions and the corresponding cost for each value of  $s$  within scenario  $A$  for that hour,  $t$  as shown by Equation 3.4.

$$T_s^A = p^s \left\{ \sum_{j=1}^N MCP_t (P_j^t - x_j^s) + \sum_{j=1}^N RDP_j^t x_j^s \right\} \quad (3.4)$$

where  $p^s = p(L = D_s^t)$  as can be seen from Table 3.1.

From Equation 3.4, it can be seen that each of the zone's commitment is reduced by an amount,  $x_j^s$ . A penalty fee called reloading-down price  $RDP_j^t$  is paid to the generating zones for per unit power decommitted. Also, only the amount of energy needed to meet the demand for that hour and any extra power available that could not be decommitted are paid for at the hourly market clearing price,  $MCP_t$  by the market operator.

### 3.2.3.2 Scenario B

For scenario B when the value of  $s$  is 25, the actual load is equal to the sum of all the zone commitments for that hour. Therefore,  $D_p$  is equal to zero when scenario B occurs in the load model. When this happens, there is no decommitment or recommitment necessary within the system. Energy settlement is equal to expected cost in the DAM. The corresponding cost when  $s = 25$ , is therefore given by Equation 3.5 as:

$$T_s^B = p^s \left\{ \sum_{j=1}^N MCP_t P_j^t \right\} \quad (3.5)$$



### 3.2.3.3 Scenario C

Scenario C forms the part of the load model in which the value of  $s$  is in the range of  $s = 26, \dots, 49$ . Scenario C is characterised by demand exceeding supply as a result of the increase in demand above scheduled commitment. Recall that when scenario A occurs, there is more power scheduled than required. But this is not the case in Scenario C. Simply put, Scenario C is the opposite of scenario A in a DAM. When demand is greater than supply, more power is required to meet demand than is committed. There are three sources available to meet this increased demand in any combination as necessary. They are:

- Spinning reserve market
- Generators in the form of reloading up
- Spot energy market

The choice of which of these sources to utilize in meeting demand depends on the incremental cost of that source. The incremental cost for any of the generators is generally given as  $(RUP_j^t + MCP_t)$  while that of the spinning reserve and spot market are given by  $SRP_t$  and  $SMP_t$  respectively. It is assumed that the spinning reserve market is independent of the energy market and there is no power limit in the spot market. This means that the spinning reserve market and the energy market can be operated in parallel.

In this work, the amount of spinning reserve available during any hour in the optimization time horizon has been expressed as a function of the sum of the scheduled generator commitments. It is given by:

$$SR = \beta_t \sum_{j=1}^N P_j^t \quad (3.6)$$

where  $0 \leq \beta_t \leq 1$

From Equation 3.6, it can be said that the minimum amount of SR available every hour is equal to zero when  $\beta_t = 0$  and maximum when  $\beta_t = 1$ . A fixed fee,  $MF_t$  in \$/MW called maintenance



fee is paid each time SR is scheduled. However, if it is eventually used during the hour that it is scheduled,  $SRP_t$  in \$/MW is paid for the amount of SR used while still paying the maintenance fee for having the SR on stand-by. This means that the market operator pays  $MF_t$  whether the SR is used or not.

In order to develop this part of the cost function, the following terms are defined:

- $y_j^s$  - amount of power by which zone  $j$  is increased, MW
- $w_s^t$  - amount of SR utilized during any period within the time horizon, MW and
- $z_s^t$  - amount of power procured from the Spot market during hour,  $t$ , MW

The extra power,  $D_p$  required under scenario C can be expressed as:

$$D_p = D_s^t - \sum_{j=1}^N P_j^t \quad (3.7)$$

The source with the least incremental cost is increased up to the amount that satisfies all constraints pertaining to that source. After that, the second source with the least cost is increased. This process is continued until the demand during that hour is met with all constraints satisfied. At the end of all these economic and reliability decisions, energy settlement is conducted. The cost of satisfying this demand for each of the values of  $s$  in Scenario C during any period,  $t$  is therefore given by the equation below:

$$T_s^C = p^s \left\{ \sum_{j=1}^N MCP_t(P_j^t + y_j^s) + \sum_{j=1}^N RUP_j^t y_j^s + SRP_t w_s^t + SMP_t z_s^t + MF_t \beta_t \sum_{j=1}^N P_j^t \right\} \quad (3.8)$$

The first term in Equation 3.8 is the probability that the actual load will be  $D_s^t$  for that value of  $s$ . The 1st term within the bracket in Equation 3.8 gives the total cost of energy supplied from all the generating zones while the 2nd term is the sum of the penalty fee paid to all the generating zones for the increase in already scheduled generating capacity. Similarly, the 3rd and 4th terms



within the bracket are the cost of procuring extra energy from the spinning reserve and spot markets respectively in order to meet the increase in demand. The last term within the bracket is the cost paid to the provider of the spinning reserve for having this reserve on stand-by whether it is used or not.

The objective function is derived by combining all costs for different values of  $s: s = 1, \dots, 49$  for each period,  $t$  within the optimization horizon. The optimization horizon is given as 24 hours which makes up the DAM being studied. Therefore, the cost model can be generally described as:

$$\text{Min } T_t^D = \sum_{t=1}^{24} \left\{ \sum_{s=1}^{24} T_s^A + T_s^B + \sum_{s=26}^{49} T_s^C \right\} \quad (3.9)$$

The above objective function is minimized subject to operating constraints as described in the following subsection.

### 3.2.4 Constraints of the Optimization/Objective Function

For a system to be operated reliably and economically certain constraints must be satisfied during any time of operation. The objective of this proposed approach is to minimize the total energy cost during any period subject to the following constraints.

#### 3.2.4.1 Generating Zone Limits

In order to maintain a reliable operation of any generator during any period, it must be operated within its design or operating limits. Therefore for any of the generating zones, the amount of power scheduled from it must be within its limit. Generally, this constraint is expressed as:

$$P_j^{\min} \leq P_j^t \leq P_j^{\max} \quad (3.10)$$



The operating limits vary from generator to generator and are designed in such a way to reduce forced outages and improve the lifespan of the units.

### 3.2.4.2 Power Balance

An important constraint that must be satisfied in the reliable operation of any power system is the energy balance between supply and demand of power. A system is only said to be reliable if the demand during any particular period is completely satisfied by the available supply without load shedding. Considering the terms used in the development of the objective function in Section 3.2.3.3, the power balance constraint is generally given as:

$$\sum_{j=1}^N P_j^t + w_s^t + z_s^t \geq P_{load}^t \quad (3.11)$$

The power balance constraint states that the total scheduled generating capacity of all generating zones plus the power from the spinning reserve and spot markets must always be greater than or equal to the demand during that particular time in the optimization time horizon.

### 3.2.4.3 Reloading-Down Rate

During the decommitment of a generator, its scheduled capacity can only decreased at a rate that is permissible by the generator owner. This limitation is inherent to the operation of any generating unit. A generator's decommitment rate must be maintained in order to ensure system security and maintain system frequency. This also helps in improving the lifespan of the generating unit while eliminating forced outages. The rate at which a generator is decommitted is called the ramping down rate (or reloading down rate). It refers to the maximum amount of power per hour that the generator can be decreased. The unit of the reloading down rate is *MW/hr*. In this proposed approach, the reloading-down rate constraint can be expressed as:

$$x_j^s \leq DR_j \quad (3.12)$$



Equation 3.12 shows that for a generating zone to be decommitted, the decommitted amount within a time-step must not be greater than the reloading-down limit,  $DR_j$  of that zone. It is assumed that the reloading-down limit is constant throughout the optimization time horizon.

#### 3.2.4.4 Reloading-Up Rate

Equation 3.13 expresses the constraint for increasing the capacity commitment of a zone above scheduled capacity during any hour within the optimization time horizon. The rate at which a generating unit can increase its output is limited by its overall energy conversion process. For example, a large thermal unit can increase its output by a 1% of its rated capacity per minute. Hydro units can increase its output from zero to 100% of its capacity in the order of 5 to 10 minutes. If the capacity output of a generating zone  $j$  is to be increased during a particular hour, the ramping-up of the generating zone must not exceed the reloading-up limit of that zone. Generator owners supply this data to system operators to ensure that it is included in the security constrained unit commitment and economic dispatch of the system.

$$y_j^s \leq UR_j \quad (3.13)$$

For the purpose of this work, the reloading-up limit,  $UR_j$  is assumed not to be time dependent and therefore is fixed during the entire optimization process. This is not to say that this limit is always fixed. It can vary from hour to hour and depends on the type of generating unit and its operating condition.

#### 3.2.4.5 Decommitment Limits

This constraint is similar to the constraint in Section 3.2.4.1. During the reduction in the zone commitment, the difference between the initial zone commitment,  $P_j^t$  and the amount by which the zone is decommitted,  $x_j^s$  must be less than or equal to the minimum limit of that generating zone. This constraint can be expressed mathematically as:



$$P_j^t - x_j^s \leq P_j^{min} \quad (3.14)$$

Equation 3.14 shows that irrespective of the value of the difference between the total hourly commitment and actual demand, none of the zones can be decreased below its limit,  $P_j^{min}$ . With this in mind, the maximum reduction in the surplus power available during any hour is the sum of the reloading-down limits of all the zones provided the minimum generating limit of any of the zones is not exceeded.

#### 3.2.4.6 Recommitment Limits

To maintain balance between the maximum capacity of any of the generating zones and any increase above scheduled output of that zone, a constraint that limits this increase is included in the proposed approach. Whenever the demand is greater than the total scheduled generating capacity, there is the option of increasing the capacity of the already scheduled generating zones to meet this increase. To maintain balance between demand and supply, the output capacity of the zones can be increased up to their reloading-up limits. This increase is also constrained by the maximum generation capacity of any zone. This constraint has been factored into this proposed approach by including Equation 3.15 in the development of the optimization cost model. Taking a look at Equation 3.15, it can be seen that the deficit in supply can be reduced by increasing the capacity of the generating zones up to the sum of their reloading-up limits as long as the constraint of maximum generating capacity is not violated. If after utilizing the recommitment limits of all generating zones more power is still needed to meet demand, it can be sourced from either the spinning reserve market or the real-time market as long as it is economical to do so. The choice of which of the sources to procure extra power from depends on the energy cost of that source and other operating factors.

$$P_j^t + y_j^s \leq P_j^{max} \quad (3.15)$$



### 3.3 Implementation of the Proposed Optimization Approach

A C++ program has been developed to carry out the proposed optimization. The details of this computer code and the formulation can be found in Appendix B.1. All the constraints discussed in Section 3.2.4 were taken into account in the assessment of the SR requirements for each hour in the optimization time horizon. The minimization of the cost is done for each period by comparing different values of  $\beta_t$  from 0 to 1 for which the total operating cost is minimum given the different energy prices and penalty fees for all possible decisions made for each value of  $s$  during that time period.

For a given unit commitment of zones at a period  $t$  and load forecast for that period, computational analysis is performed starting with the development of the load model based on the 49-step normal distribution of the LFU. The value of  $s$  and its corresponding probability are read from Table 3.1. Also the actual load  $D_s^t$  for that value of  $s$  is computed. The value of  $D_s^t$  is compared with the total zone commitment for that period. Decisions are made by the computer program based on prices and constraints to determine the expected total operating cost for that period given by Equation 3.9. Using the zero-order method (direct search approach),  $\beta_t$  is varied from 0 to 1 in step-size to determine the optimal value of  $\beta_t$ . This value is optimal if and only if the total operating cost is minimum and all operating constraints are satisfied. However, the ISO must have a prior knowledge of what the system SR requirement should be. This is to avoid a situation whereby the optimization technique does not give any minimum as desired. Using Equation 3.6, the corresponding optimal SR requirement for that period is evaluated.

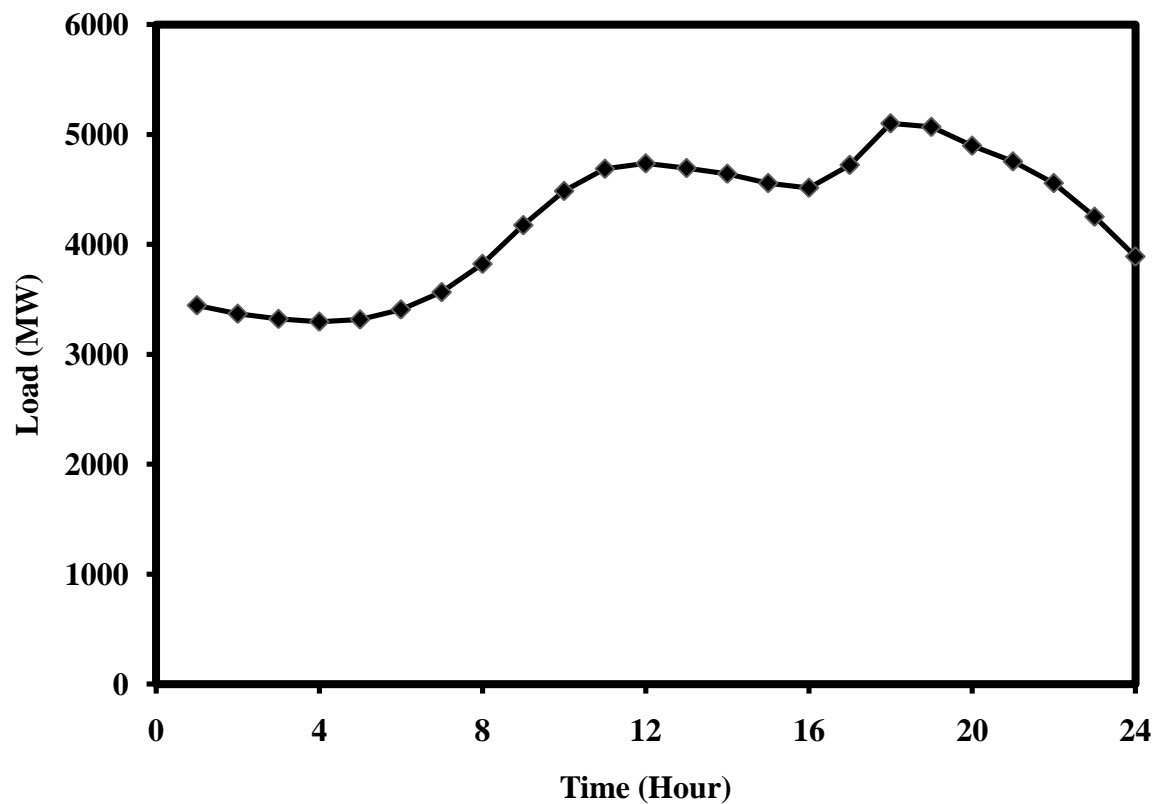
This process continues until the 24-hour optimization time horizon is completed. The spinning reserve requirements for each time period as well as the total operating cost are derived for analysis and discussion.

### 3.4 Results on System without Transmission Loss

The proposed approach for the assessment of the spinning reserve requirements for a day-ahead market was tested on a test system without transmission loss. The details of the test system data



can be found in Appendix A.1. The unit commitment based on load forecast, market clearing price, spinning reserve price, spot market price as well as zonal reloading up and down prices in \$/MWh for each period, the operating constraints and other important information required for this approach implementation can be found in this Appendix. The load profile used for this simulation is shown in Figure 3.3.



**Fig. 3.3 Test System load curve for a 24-hr period**

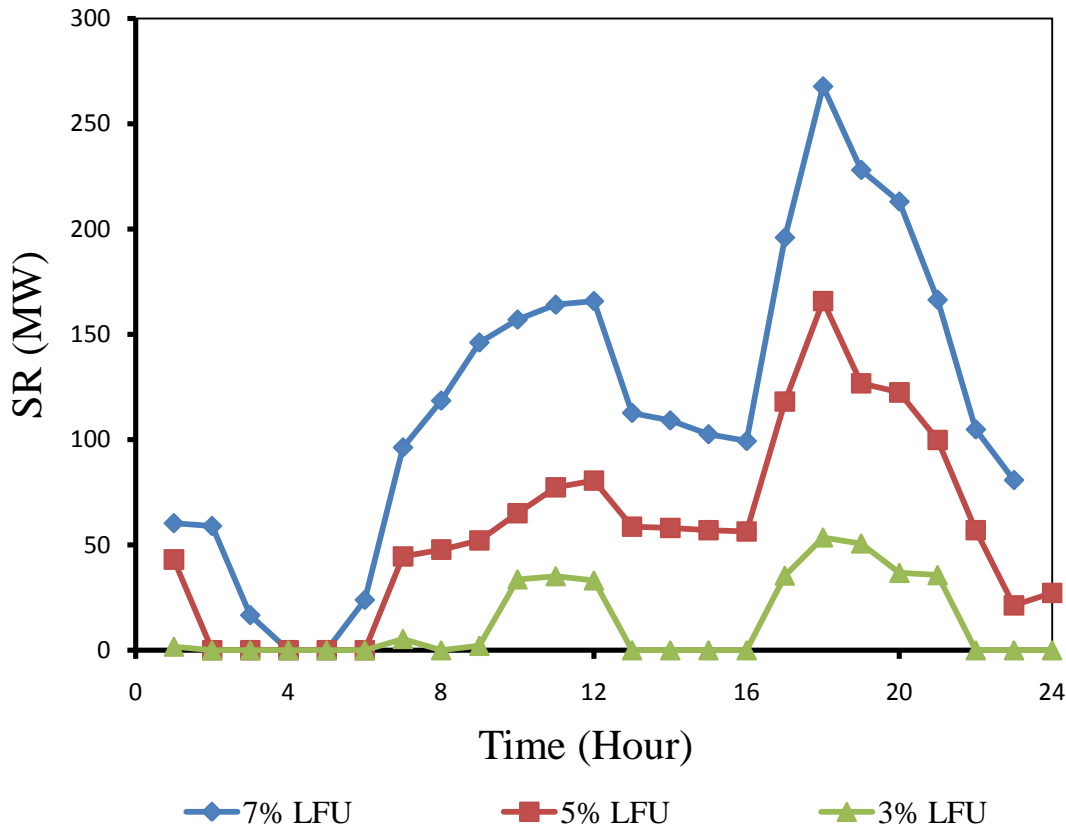
### **3.4.1 Effect of Load Forecast Uncertainty (LFU)**

Spinning reserve requirements have been assessed with the proposed optimization for load forecast uncertainty of 3%, 4% and 5% of the forecast mean. Figure 3.4 compares the amount of SR requirements under these percentage load forecast uncertainty (LFU). It can be seen from



Figure 3.4 that the amount of SR required increases with an increase in the load forecast uncertainty. A 3% LFU requires less amount of SR since the deviation from the forecast mean is relatively small compared to the deviation from the forecast mean for either a 5% or 7% LFU. It can be inferred that the more accurate a load forecast is, the lesser the amount of SR required for any period. Furthermore, an accurate load forecast in a Day-Ahead market can minimize system operating cost since the SR requirement will be minimal during any period. The SR requirements are therefore dependent on the LFU.

A close look at the profiles in Figure 3.3 and Figure 3.4 shows that the SR requirement changes with load as well. During the peak load hour (as seen from Figure 3.3), the SR requirement is the highest when compared to other periods. Figures 3.3 and 3.4 also show that during the non-peak periods in the load profile, there is little or no SR required. Therefore it can be concluded that as demand increases, the SR requirement also increases irrespective of the LFU.



**Fig. 3.4 Effect of Load Forecast Uncertainty (LFU) on SR Requirements**



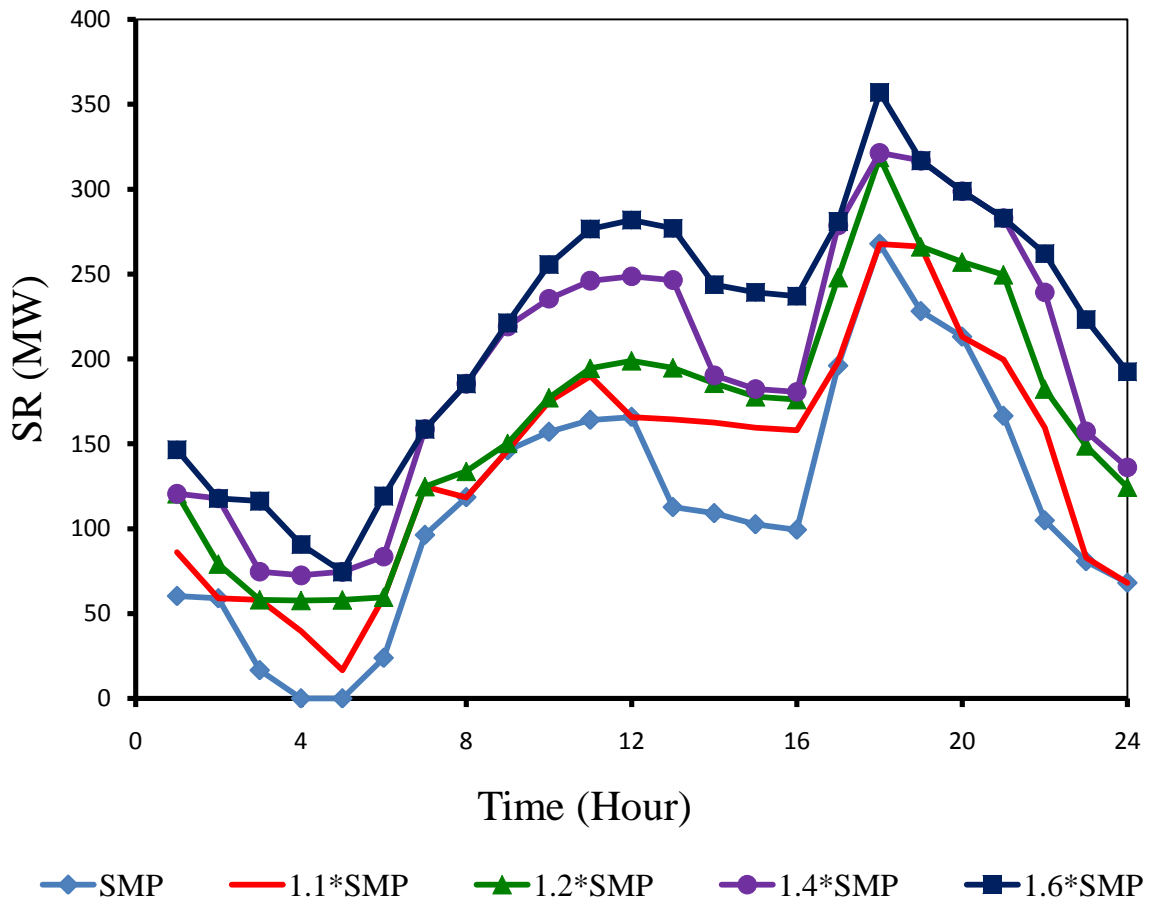
### 3.4.2 Effect of Spot Market Price (SMP)

The decision to buy energy from the spinning reserve market or from the spot energy market would depend on the predicted price of energy in these markets in \$/MW. Figure 3.5 shows the effect of varying SMP on the assessment of the SR requirements in a day-ahead market. Five curves corresponding to five different SMP have been plotted. In the test system data, it is assumed that the nominal SMP (100%) is at least twice the nominal SRP (100%) for any period. As can be seen, the amount of SR requirement is at its lowest level when the SMP is at its nominal level (100%).

Figure 3.5 shows that when real time energy is priced above the nominal value of the SMP, it is economical to schedule more SR than the periods when energy price is at the nominal value. When demand increases beyond scheduled generation capacity, system operators must source power from either the spot market or from ramping up of already scheduled generators based on their ramp-up rates and generation limits. When the source is the spot market, the energy price in real time must be considered when assessing the amount needed to reduce the deficit. When energy price in the spot market is high, the operating cost can be maintained at a lower level by procuring more energy from the spinning reserve market. Conversely when the spot market price is relatively low, the operating cost can be lowered by maintaining a reduced level of SR.

Minimizing the total operating cost, however, requires a prior knowledge of what the energy price is in the real-time market. This is very essential to the market operators if they are to economically and reliably assess the SR requirements during any period. Market operators must be able to model the expected price of energy in real time in the assessment of the periodic SR requirements. It is, however, difficult to accurately determine the real time energy price that will exist the following day. However, based on historical data, this information can be readily estimated and used in this proposed approach to assess the effects of spot market price in the SR requirements.

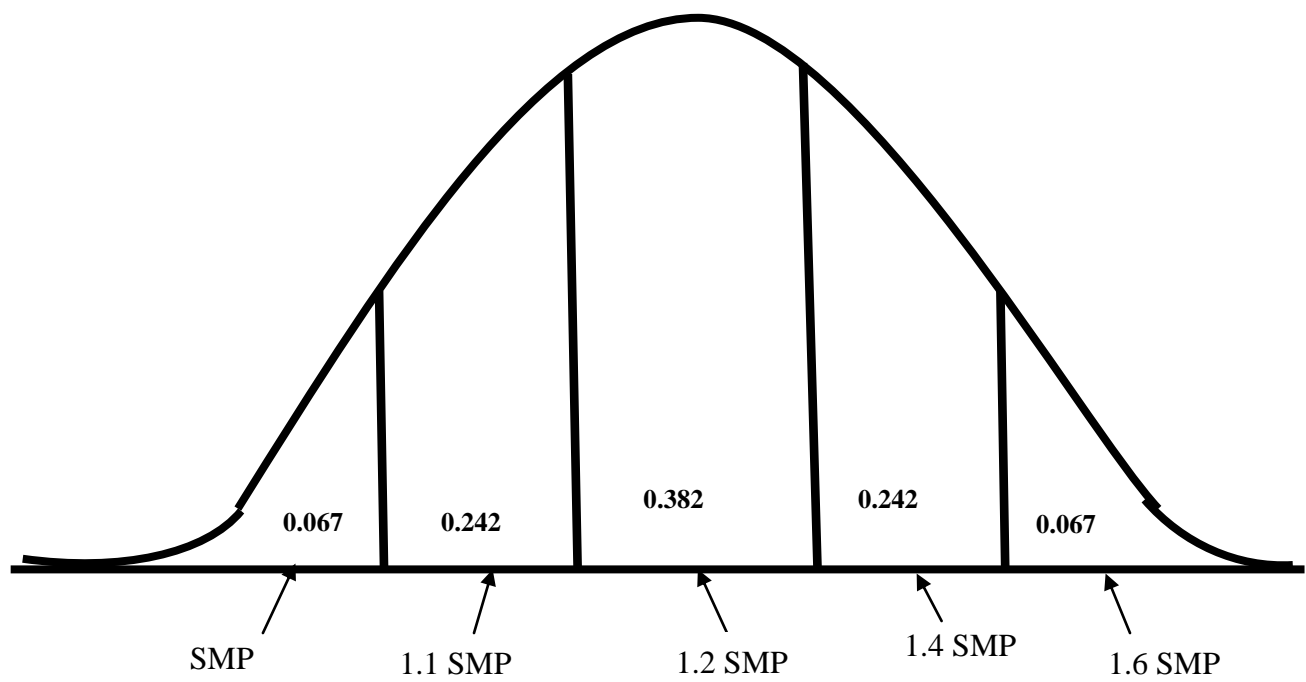




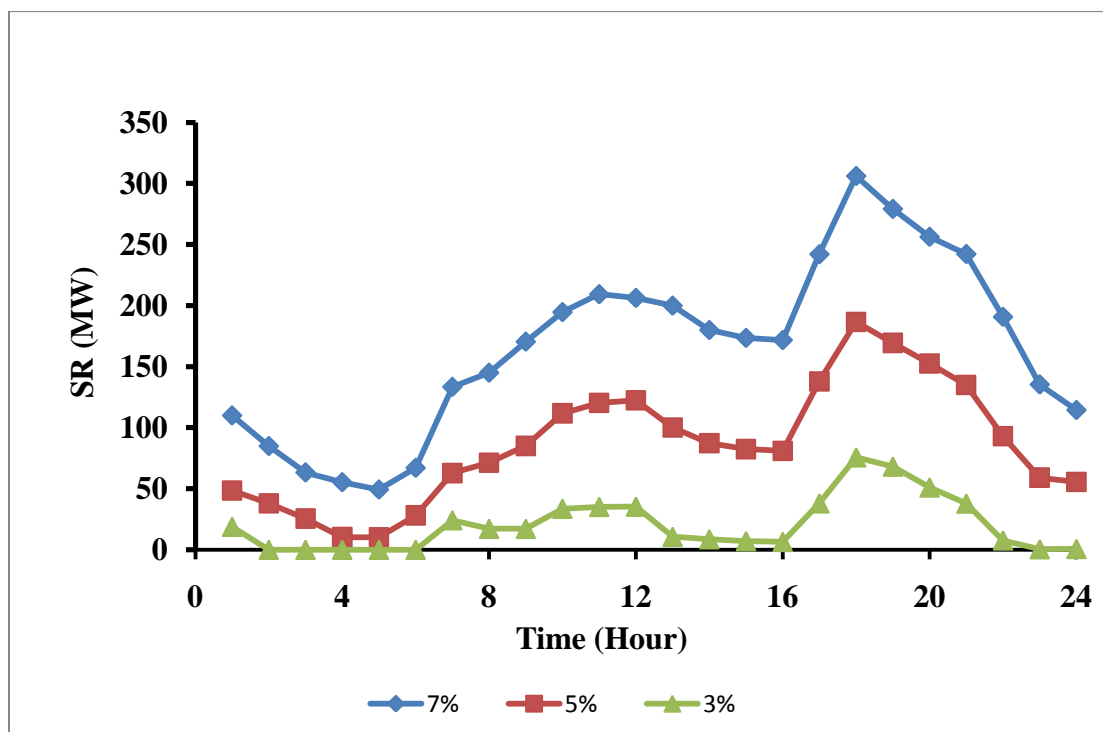
**Fig. 3.5 Effect of SMP at Nominal SRP (100%) on SR requirements for 7% LFU**

If we assume a 5-step normal distribution of the SMP with 1.2 SMP as the mean of the distribution as shown in Figure 3.6, then we can estimate an aggregate value of the SR requirements for the following day by obtaining the weighted sum of the spinning reserve requirements for five SMPs. Figure 3.7 shows the aggregate SR requirements for different LFUs at the nominal SRP (100%).





**Fig. 3.6 5-step Normal Distribution of SMP**



**Fig. 3.7 Aggregate SR Requirements for a Nominal SRP (100%)**



### 3.4.3 Effect of Spinning Reserve Price (SRP)

Figure 3.8 shows that, for a change in the SRP in a DAM, there is a corresponding change in the SR requirements. The SR requirement decreases with an increase in the SRP. As the price of the SR approaches the energy price in real-time market, it becomes less economical to schedule SR for any period. The maintenance fee paid to SR providers increases the operating cost especially when scheduled but not used. Therefore, when the SRP is higher or equal to the SMP, it is economical for the market and system operators to procure energy directly from the spot market in real time since there is no maintenance fee.

In Figure 3.8, when the nominal SRP is 40% higher, it can be seen that only the peak hour requires SR even though the spinning reserve price approaches the price of energy in real time. Section 3.4.1.1. explains why SR is scheduled during such peak load periods. Similarly, when the nominal SRP increases by 60%, there is no SR scheduled during any periods. This means that  $1.6 \times \text{SRP}$  is greater than or equal to the SMP. It also means that any economic or reliability benefit that would have been derived from scheduling SR priori is negligible. On the other hand, when the spinning reserve is priced at either SRP or  $1.1 \times \text{SRP}$  or  $1.2 \times \text{SRP}$  as seen from Figure 3.8, it still makes economic and reliability sense to schedule SR in the DAM even though maintenance fee is charged for the amount of SR scheduled. It can be inferred from the plot of SR requirement for different values of the SRP at hourly nominal SMP that the SR requirement is inversely proportional to the SRP.

If we assume a 5-step normal distribution of the SRP with 1.2 SRP as the mean of the distribution similar to the distribution of the SMP shown in Figure 3.6, then an aggregate value of the SR requirements for the following day can be evaluated by obtaining the weighted sum of the spinning reserve requirements for five SRPs. Figure 3.9 shows the aggregate SR requirements for different LFUs at nominal SMP (100%).



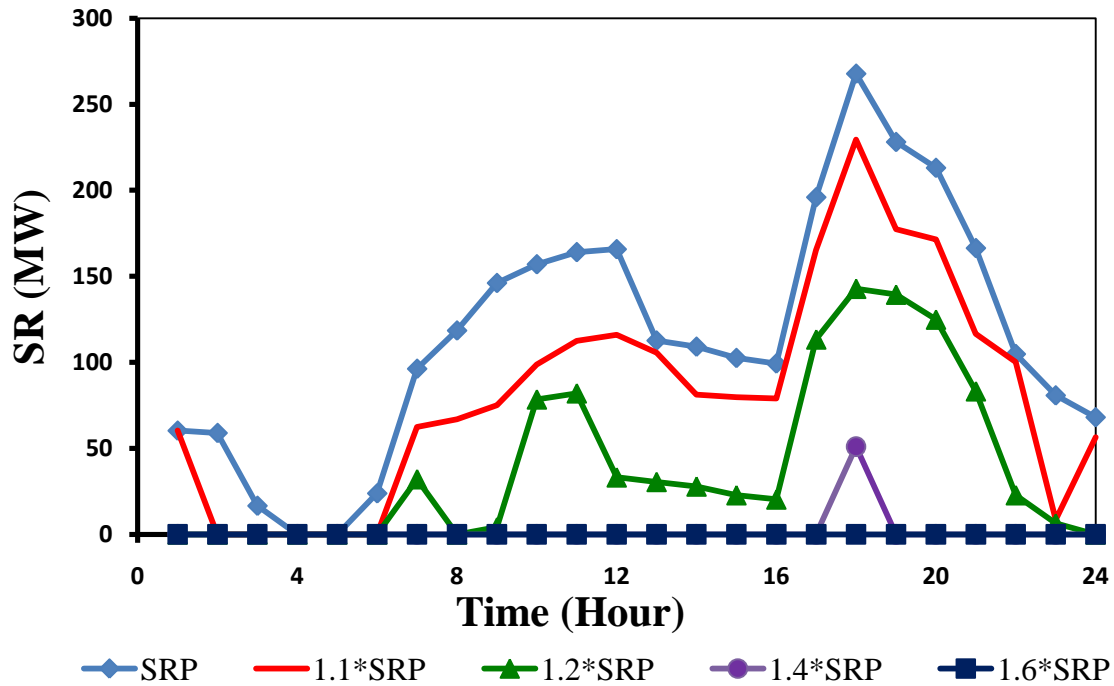


Fig. 3.8 Effect of SRP at Nominal SMP (100%) on SR requirements for 7% LFU

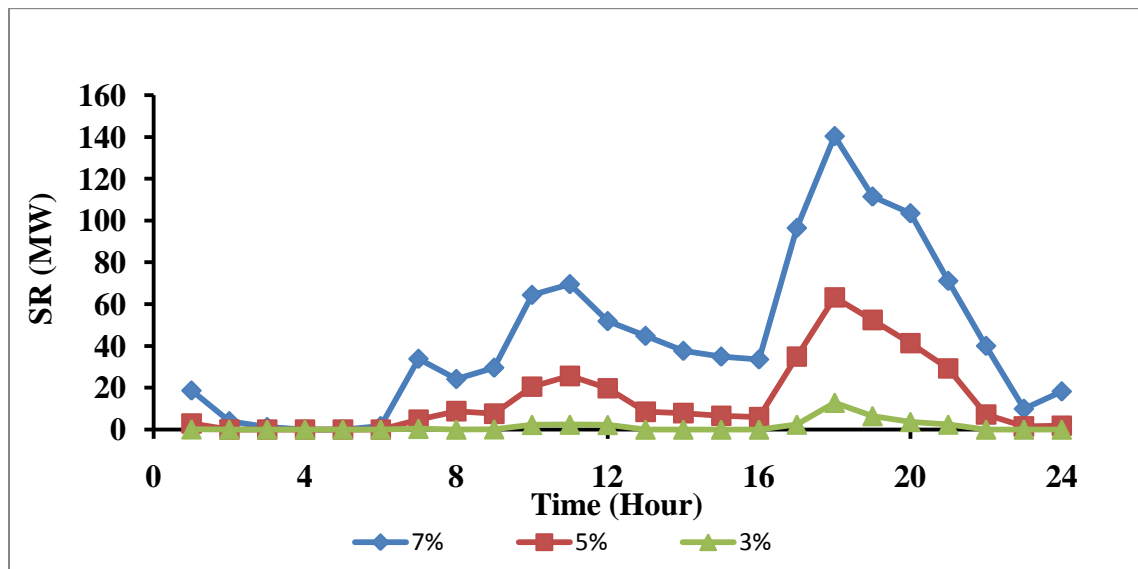
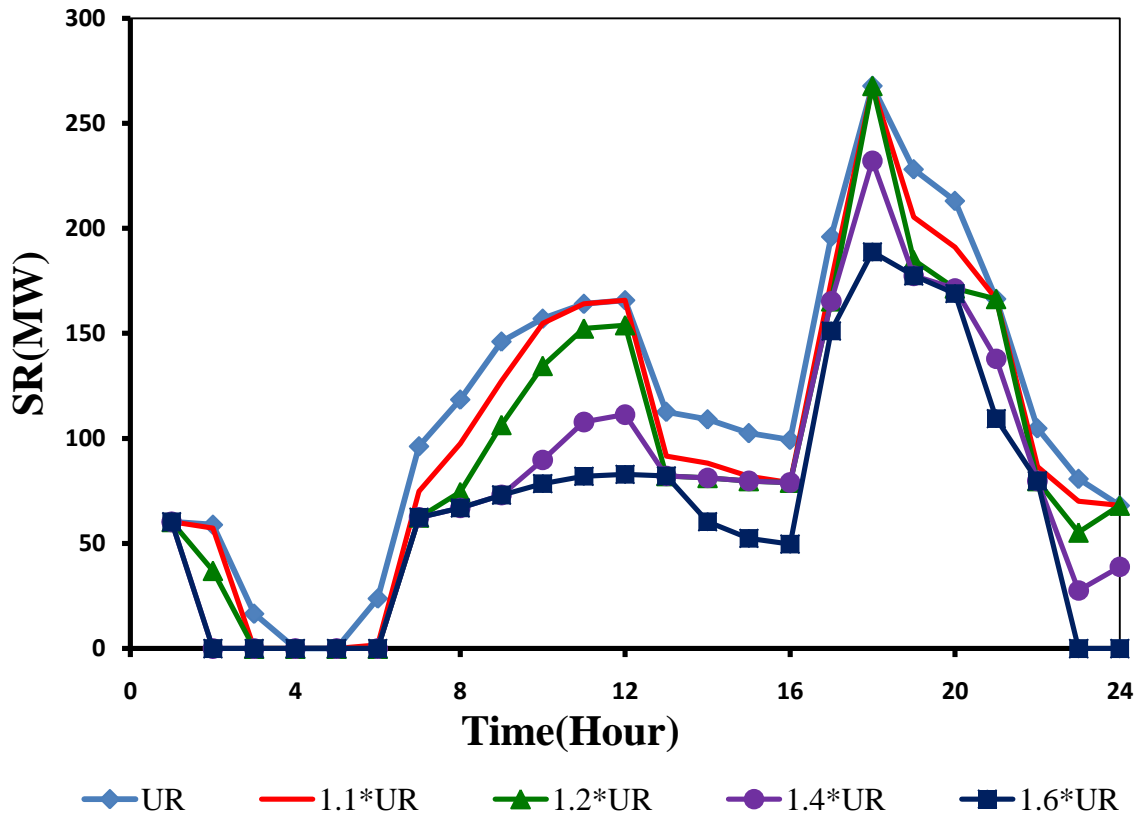


Fig. 3.9 Aggregate SR Requirements for a Nominal SMP (100%)



### 3.4.4 Effect of Reloading Limits of Generating Zones

An understanding of the effects of a generator's reloading limits on the SR requirements in a deregulated system is very crucial in minimizing operating cost of a system. Power suppliers provide their generators' ramp limits to the system operator for the purposes of security constrained unit commitment and economic dispatch. A look at Figure 3.10 shows that an increase in the reloading-up (ramp-up) limit results in a decrease in the periodic SR requirements.



**Fig. 3.10 Effect of Reloading-Up Limit on SR requirements for 7% LFU at Nominal SMP and SRP**

Usually, there are three sources from which any increase in demand can be met. They are either the spot market, spinning reserve market or the scheduled generators. This proposed approach makes economic decisions based on which of the three sources is most economical. Assuming that the nominal SMP and SRP remain unchanged during the 24-hour period but the reloading up

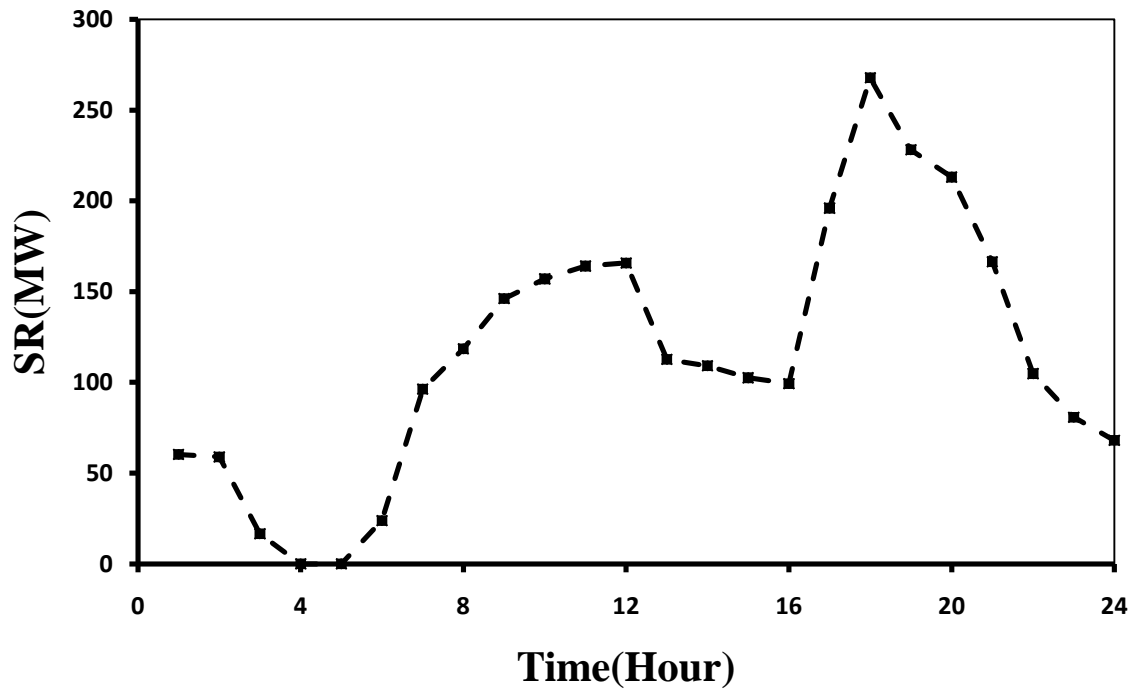


limit is increased say by 10%, the SR requirement during certain periods is reduced by more than 100% as can be seen in hour 3 while the requirement during peak hours 11, 12, 18, 21 and 24 remain unchanged. During other periods, the SR requirement is reduced by different amounts. The reduction factor is dependent on the energy prices, the amount by which the forecast load deviates from the actual load and the corresponding probability. Increments of 20%, 40% and 60% in the reloading-up limits of the generating zones also exhibit similar effects on the hourly SR requirements.

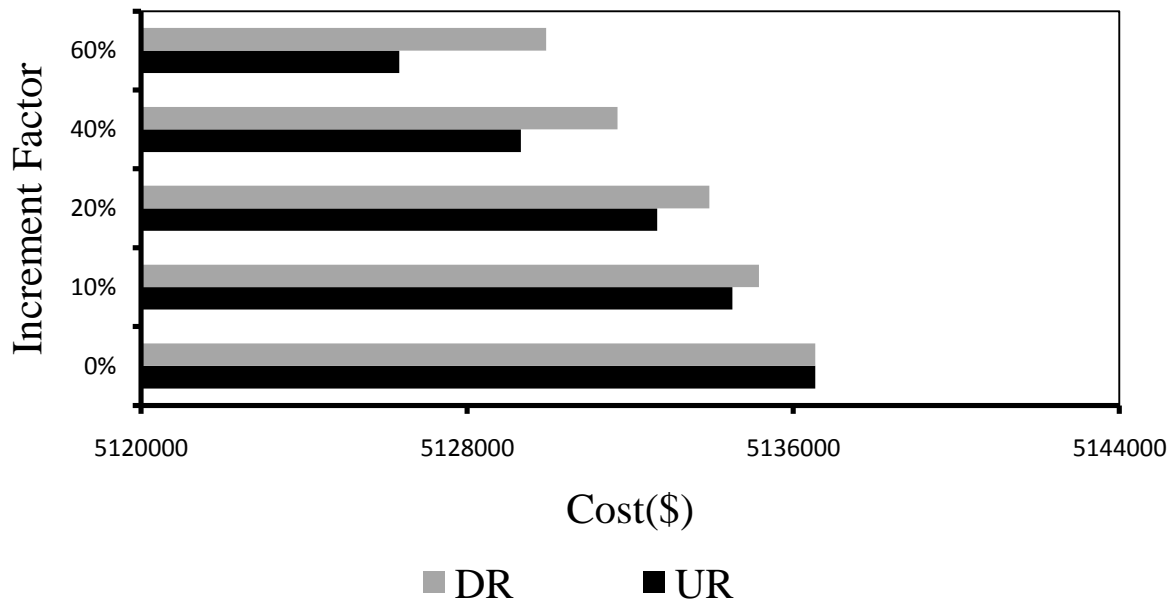
On the other hand, when the reloading-down limits of the generating zones are changed, the effect on the SR requirements is not the same when the ramp-up limits are changed. Figure 3.11 shows that the SR requirements remain unchanged for any incremental change in the reloading-down limit. One can also see that when the nominal reloading-down limit is increased by 10%, 20%, 40% and 60%, that the SR requirements remained same as the SR requirements when the reloading-down limit is equal to  $DR_j$ . Even though there is no change in the SR requirements, an interesting effect was identified. For each of the changes in the reloading-down limit, there is a reduction in the total operating cost for the optimization time horizon. Figure 3.12 shows that when there is a change in the limits for reloading the generating zones, there is a decrease in the total operating cost. The reduction in the SR requirements when the reloading-up limit is increased explains the decrease in the operating cost for such a change. But this does not explain why an increase in the reloading-down limit results in a decrease in the total operating cost even though the SR requirements remain the same.

A look at the proposed approach from Figure 3.1 and Section 3.2.3 shows that there are 24 out of 49 possibilities that the actual load will be less than the total zone commitment for that period. For each of these 24 possibilities, the initial zone commitment is reduced based on the difference between the total scheduled capacity and the actual load and the reloading-down limits of the generating zones while considering the penalty fee for such reductions. When generating zones are decommitted, there is no need for SR which means that any change however small or large in the ramp-down limits of the zones does not imply a change in the SR requirements.





**Fig. 3.11 Effect of Reloading-down Limit on SR requirements for 7% LFU at Nominal SMP and SRP**



**Fig. 3.12 Effect of Reloading limits on Total Operating Cost for a 7% LFU at Nominal SMP and SRP**



However, for zone recommitment, more power is needed than scheduled and therefore SR is required to meet the demand deficit. This means that for any change in the reloading-up limit of any zone, there is a corresponding change in the SR requirements. It can be deduced from Figures 3.10 and 3.12 that increasing the ramp-up limit of generators decreases the SR requirements and also minimizes the total operating cost for that system. The operating cost can also be minimized by increasing the reloading-down limits of the generating zones without any change in the SR requirements.

### **3.4.5 Computation Time**

All simulations were performed in a PC with an Intel Duo Core processor T5800, 2 GHz with a 4GB RAM. The time required to complete one simulation and save result was approximately 10 seconds.

## **3.5 Summary**

A new technique to determine the SR requirements at each period of the optimization horizon in DAM considering load forecast uncertainty is presented in this chapter. It was assumed in this approach that there is no transmission loss in the system. This technique assesses the amount of SR that minimizes the total operating cost of the system. It utilizes the initial unit commitment conducted for the DAM and the DAM load forecast and makes decisions based on a load model. The load model is developed using a normal distribution of the load forecast uncertainty. For accuracy purpose, a 49-step LFU normal probability distribution was used. It was assumed in this approach that there are no generator outages. The only uncertainty considered is the load forecast uncertainty. The effects of the LFU, SRP, SMP and the reloading limits of the generating zones are studied in determining their relationships and impacts on the SR requirements for a DAM.



# Chapter 4

## Spinning Reserve Assessment with Transmission Loss

### 4.1 Introduction

Every power system is characterized by energy losses along power lines and congestion limits. System operators must plan and schedule SR in such a way that transmission losses and congestion are minimized. The role that transmission loss plays in the economic dispatch of any power system is very important. In order to maintain reliability and security of any system, adequate Spinning Reserve must be available to meet changes in system load plus losses. The previous chapter discussed the assessment of SR without the inclusion of transmission loss. In Chapter 3, the total scheduled generation capacity was set equal to the forecast load. But in this chapter, the total scheduled generation capacity is equal to the sum of the forecast load and transmission loss.

During unit commitment, the generating zones are committed to meet both forecast load and transmission loss including any congestion constraints. In order to determine the optimal amount of SR required in a DAM, a transmission loss equation is included in the development of the cost function. With transmission loss included, the amount of SR must be adequate to meet the changes in the demand and the resultant loss for that increase in the demand.



This chapter presents an assessment technique of SR requirements for a system with transmission loss. The proposed model assumes that stochastic changes in the load forecast are the only source of uncertainty. Random outages of generators are internal to the operation of a supplier zone and are ignored in this approach. A 49-step normal probability distribution with the forecast load as the mean and a known percentage standard deviation is used in describing the load forecast uncertainty model. The model is incorporated into the development of the optimization cost function for the test system with transmission loss. All available data in the day-ahead market is utilized in the assessment of the spinning reserve requirements. This approach is described in more details in the following section.

## 4.2 Optimal Spinning Reserve Considering Load Forecast Uncertainty

There are two stages involved in this approach. Both stages are implemented after unit commitment has been done for the forecast load without the incorporation of spinning reserve. The unit commitment process for generating zones forms the initialization part of this approach. Also in the initialization part, relevant information of the power system for a DAM is gathered (like market clearing price for energy from generating zones, zones' ramp rates, ramping cost, Spinning Reserve price, Spot Market Price, zones' minimum and maximum limits, standard deviation of the load forecast uncertainty model, etc). The first stage deals with the probability distribution and deviation of the Load forecast within the Load forecast Uncertainty (LFU) model. Each of the 49 steps of the load forecast uncertainty model is described by a load demand,  $D_s^t$ , the forecast load,  $P_{Load}$ , the standard deviation of the LFU model and the probability  $p(L = D_s^t)$  that, for a given estimated load  $P_{Load}$ , the actual load will be an amount  $D_s^t$  in real time [1].

The second stage deals with the decision making involving each of these steps. The analysis and decision process entails deciding which of the energy options available to procure Spinning Reserve is the most economical and by what amount based on the prices of energy from these sources and the ramping rates of the generating zones for every step within that particular time and over the time period being optimized. Figure 4.1 shows the simplified flow diagram of the proposed approach.



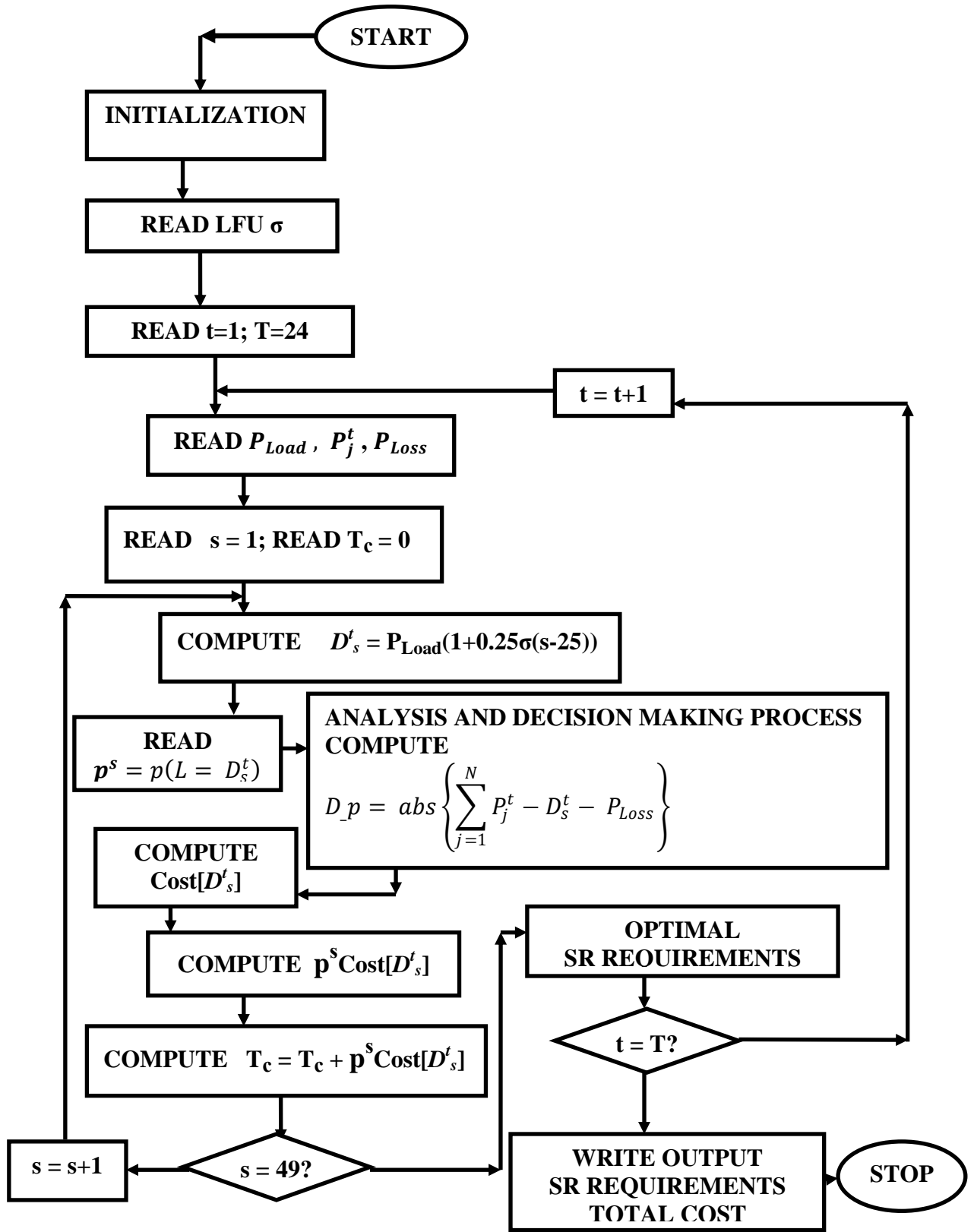


Fig. 4.1 Simplified Flow diagram of Proposed Approach with Transmission loss



#### 4.2.1 Description of the Test System with Transmission Loss

A test system has been utilized for the demonstration of the proposed approach. Figure 4.2 shows the single line diagram of the test system. The zone information including hourly zone commitment, market clearing price, reloading up and down prices, spinning reserve price, spot market price, reloading limits and generating zone limits are shown in the appendix. It consists of three (3) generating zones of total installed capacity of 9900MW. Each zone consists of generators of various sizes. The total system peak load and base load are 5101MW and 3297 MW respectively. The load demand is spread across two of the seven system buses. In this test system, there is transmission loss along the lines. The zone commitment is based on the DAM hourly loads and the total transmission loss within the system. The total zone commitment for any period is equal to the sum of the forecast load and the system loss for that period.

#### 4.2.2 Formulation of the Operating Cost Function

The cost function is developed at the Analysis and Decision Making stage as can be seen from Figure 4.1. For each hour in the 24-hour period load curve, economic and reliability-based decisions are made with respect to expected load, the total zone commitment, transmission loss and energy prices for that hour.

The transmission loss formula for this test system is modeled as a polynomial function in terms of generator outputs and contains  $B_{ij}$  coefficients and some loss components in product forms for each of the energy sources. It is based on the  $B$ -matrix approach shown in Equation 4.1.

$$P_{Loss} = P^T[B]P + B_0^T P + B_{00} \quad (4.1)$$

where,

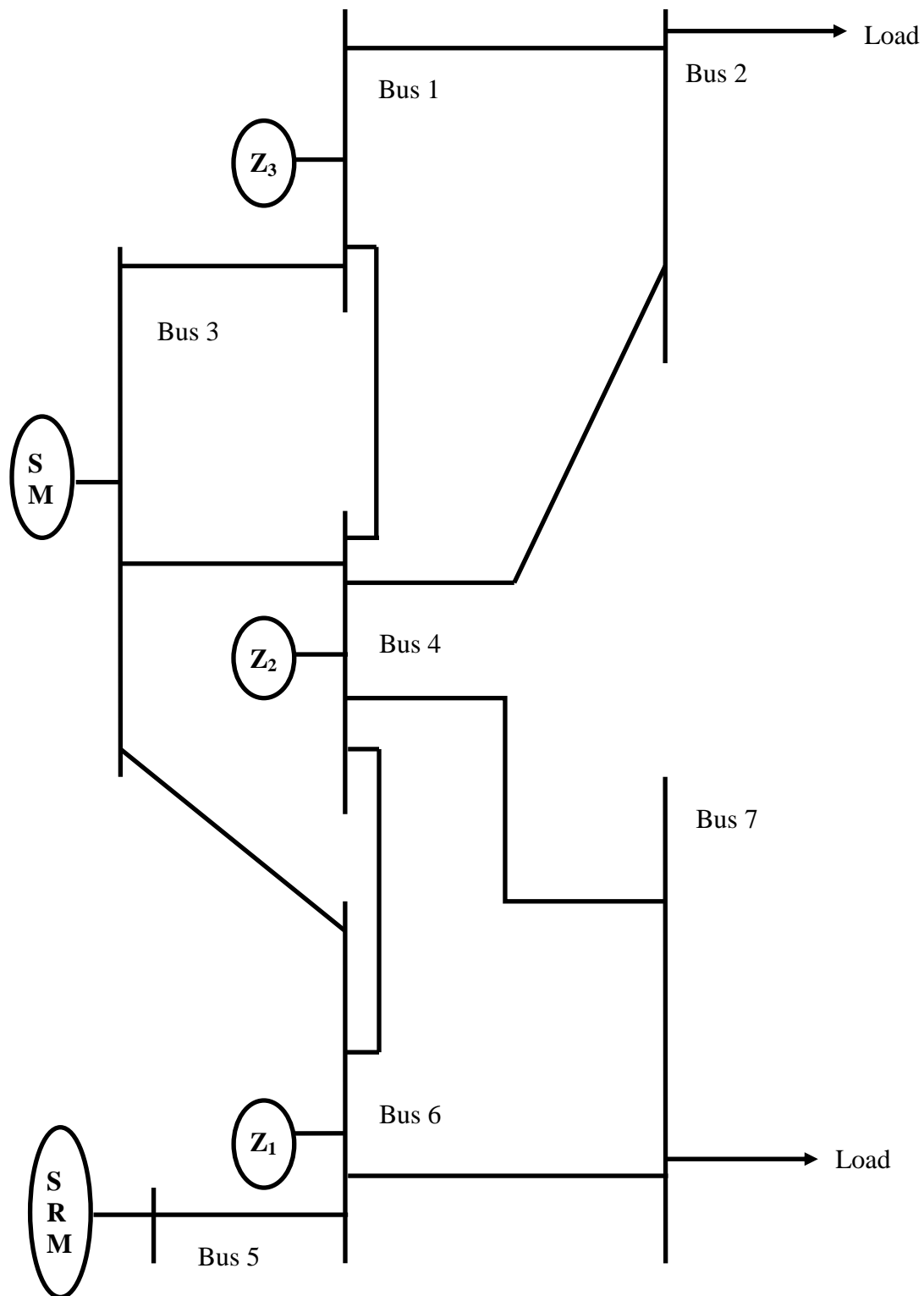
$P$  - vector of net power injections at buses MW

$[B]$  – square matrix of the same dimension as  $P$  and is the non-linear part of loss coefficient

$B_0^T$  – vector of the same length as  $P$  and is the linear part of loss coefficient

$B_{00}$  – constant term





**Fig. 4.2 Single Line Diagram of the Test System with Transmission Loss**



The first product in Equation 4.1 is the non-linear term while the second is the linear term and  $B_{00}$  is the constant term of the B-matrix loss equation. The B-matrix loss formula is developed using a series of transformations on the full-impedance matrix of the transmission system network. The derivation of the B-matrix loss formula is based on a paper by Meyer [26] and is related to much earlier pioneering work by Kron and Early and Watson [11, 22]. This derivation has been referenced in the book, Power Generation Operation and Control by Allen J. Wood and Bruce F. Wollenberg [49].

Equation 4.2 is an approximate expression of the system transmission loss since there is no single  $P_{Loss}$  formula that is applicable to all operating points for any period in the optimization horizon. It is expressed as a non-linear function for a 3-generating zone system including the SM and SR given as  $w$  and  $z$  respectively.

$$\begin{aligned}
P_{Loss} = & 0.00006280P_1^2 + 0.00003989P_2^2 + 0.00005519P_3^2 + 0.00005103w^2 \\
& + 0.00006181z^2 + 0.000076P_1P_2 + 0.000081P_1P_3 + 0.000084P_1w \\
& + 0.000099P_1z + 0.000077P_2P_3 + 0.000074P_2w + 0.00006P_2z \\
& + 0.000087P_3w + 0.000063P_3z + 0.000068wz - 0.1155P_1 - 0.1208P_2 \\
& - 0.1421P_3 - 0.1213w - 0.0943z + 153.8 \text{ MW}
\end{aligned} \tag{4.2}$$

In order to derive Equation 4.2 based on Equation 4.1, certain assumptions and base case operating points are made as outlined in Appendix D.

As mentioned earlier, a 49-step normal distribution of the load forecast uncertainty is used in the formulation of the cost function for the purpose of accuracy. The forty-nine (49) class intervals define 49 possible values of what the actual load can be during each of the hours within the time horizon of the day-ahead market.

During hour  $t$ , each class interval,  $s$  of the 49 steps is characterised by:

- $p^s = p(L = D_s^t)$ . This forms the load model.
- $D_s^t$ : – Given as the actual load during hour  $t$  for a class interval  $s$ .



- $P_j^t$ : – Scheduled generating capacity of generating zone,  $j$  at time,  $t$ .

Mathematically,  $D_s^t$  can be computed using Equation 4.3

$$D_s^t = P_{Load}[1 + 0.25\sigma(s - 25)] \quad (4.3)$$

The 49-steps in the load model can be divided into three categories. Each of these three categories or scenarios is characterised by varying economic and reliability decisions that can be made during that period for that value of  $s$ . Scenario A describes any value of  $s$ :  $s = 1, \dots, 24$  for which the total hourly zone commitment exceeds the sum of the actual load,  $D_s^t$  and the transmission loss for that period  $P_{Loss}$ . In scenario B defined as  $s = 25$ , the sum of the actual load and the transmission loss is equal the total zonal commitment for that period. Similarly, scenario C describes all values of  $s$ :  $s = 26, \dots, 49$  for which the total zone commitment is less than actual demand plus transmission loss for that hour.

In order to compute the total operating cost for any period, the cost of each of the scenarios is added together to give Equation 4.4. As one can see from Equation 4.4, the periodic operating cost is given by the sum of the product of the probability,  $p^s$  and the cost of meeting the demand,  $D_s^t$  for every value of  $s$ .

$$Total\ Cost = \sum_{s=1}^{49} p^s \text{ cost}(D_s^t) \quad (4.4)$$

The objective cost function is dependent on the three scenarios described above. The detailed description of the cost function formulation is given in the following section.

#### 4.2.2.1 Scenario A

Whenever Scenario A occurs in real time, the real-time demand given as  $D_s^t$  is less than the total scheduled generating capacity of all zones during that period. Under this scenario, there is more power committed than needed to meet the demand for that period. In order to minimize energy



cost under this scenario, generating zones are decommitted or simply put ramped down to allowable capacity based on certain constraints. The decommitment must be carried out in such a way that the system remains reliable and still meets the demand for that period at an optimal cost. In the decommitment process, the most economical path is determined by following which generating zone should be reduced first and by what amount in an increasing order of their respective incremental cost in  $\$/MWhr$  starting with the least expensive zone.

Equation 4.5 gives the incremental cost of delivered power for source  $j$ , including losses as:

$$\gamma_j = \frac{dF_j}{dP_j} * PF_j \quad (4.5)$$

Where,

$\frac{dF_j}{dP_j}$  is the fuel cost for a  $MW$  change in the capacity of source,  $j$ , in  $\$/MW$  and

$PF_j$  is the penalty factor in per MW for the  $jth$  unit given in Equation 4.6 as:

$$PF_j = \frac{1}{\left(1 - \frac{\partial P_{Loss}}{\partial P_j}\right)} \quad (4.6)$$

where  $P_{Loss}$  is the system transmission loss.

Under Scenario A, the fuel cost for any of the generating zones  $j$ , is generally given by equation 4.7 as:

$$\frac{dF_j}{dP_j} = RDP_j - MCP^t \quad (4.7)$$

The fuel cost of each generator in this system is known by the ISO since the variables in the incremental cost,  $RDP_j$  and  $MCP^t$ , are usually available to the system operator by the supplier as part of day-ahead market data. In this approach, the incremental cost for each of the generating zones is computed prior to the analysis and decision making stage. After this computation, the zone with the least incremental cost is ramped down followed by next zone with a lower incremental cost. The zones are ramped down based on their ramp-down limit and other constraints that must be satisfied. Also, it is dependent on the excess power that must be reduced as given by Equation 4.8 below.



$$D_p = \sum_{j=1}^N P_j^t - D_s^t - P_{Loss} \quad (4.8)$$

$D_p$  is the absolute difference between the sum of all zone commitments and the sum of the actual load and the transmission loss based on the value of  $s$  for that period. It is the surplus power above the amount required to meet demand during a particular period. Let the term,  $x_j^s$  be defined as the amount by which the generating zone  $j$ , is decommitted for that value of  $s$  during that period in the optimization horizon.

In real time after decommitment, the market operator conducts the energy settlement for that period using the following cost function shown by Equation 4.9 for that value of  $s$  in scenario A.

$$T_s^A = p^s \left\{ \sum_{j=1}^N MCP_t (P_j^t - x_j^s) + \sum_{j=1}^N RDP_j^t x_j^s \right\} \quad (4.9)$$

where  $p^s$  is read from Table 3.1.

$RDP_j^t$  is paid to the power supplier as a penalty fee for not using an already scheduled power for that period. It is the reloading-down price per unit power decommitted from a particular generating zone. The reloading price usually varies from zone to zone but it may be same across all power suppliers. It can be seen from Equation 4.9 that only the amount of power required to meet the demand for that hour and any extra power available that could not be decommitted due to the reloading limits of the zones is settled at the market clearing price,  $MCP_t$  by the market operator.

#### 4.2.2.2 Scenario B

When the value of  $s$  is 25, the actual load plus loss is equal to the sum of all the zone commitments for that hour. Therefore,  $D_p$  is equal to zero when Scenario B occurs in any power system in real time. When the above takes place in real time, market operator conducts energy settlement based on the DAM load forecast for that hour. The cost expression for this scenario is



given in Equation 4.10. Since there is no change in the load forecast, the cost of energy used in real time is equal to forecasted cost based on the DAM.

$$T_s^B = p^s \left\{ \sum_{j=1}^N MCP_t P_j^t \right\} \quad (4.10)$$

#### 4.2.2.3 Scenario C

Scenario C forms the part of the load model in which the value of  $s$  is in the range of  $s = 26, \dots, 49$ . There is an energy deficit when this happens in real time unlike in Scenario A where there is surplus power than is required. This energy deficit is the difference between the sum of the zone commitments and the sum of the expected actual load and the transmission loss within the system during that period. Under this scenario, demand is greater than total scheduled capacity and therefore more power must be injected into the system to avoid load shedding and possible generator failure due to imbalance in the system frequency. In order to meet this increased demand, power must be sourced from either the SR market or from the spot market or by increasing the capacity of scheduled generators.

The choice of which of these sources to purchase this extra power from depends on the incremental cost of that source. The fuel cost for any of the zones when Scenario C occurs is  $(RUP_j^t + MCP_t)$  while that of the spinning reserve and spot markets are given by  $SRP_t$  and  $SMP_t$  respectively. The incremental cost can be evaluated using Equation 4.5 once the penalty factor for that source is determined through the partial differentiation of the  $P_{Loss}$  equation with respect to that energy source. It is assumed in this approach that there is no limit to the amount of power that can be procured from the spot market. In this work, the amount of spinning reserve available during any hour in the DAM is expressed as a function of the sum of the zone commitments during that period.

It is given by equation 4.11 as:

$$SR = \beta_t \sum_{j=1}^N P_j^t \quad (4.11)$$



where  $0 \leq \beta_t \leq 1$

The amount of SR available for the system is minimum when  $\beta_t = 0$  and maximum when  $\beta_t = 1$ . The cost model is developed based on the relationship in Equation 4.11. A computer program is used to vary  $\beta_t$  from 0 to 1 to determine the value of  $\beta_t$  for which the SR requirement is optimal for that period. The maintenance fee,  $MF_t$  in \$/MW is paid to the SR provider each time SR is scheduled. However, if SR is eventually used during the hour it is scheduled, the provider is settled at  $SRP_t$  based on the amount of SR used including the maintenance fee for scheduled SR.

In order to formulate the cost function of Scenario C, the following terms are defined for each step  $s$  in scenario C:

- $y_j^s$  – The amount of power by which zone  $j$  is increased
- $w_s^t$  – The amount of SR utilized during hour,  $t$  within the optimization horizon and
- $z_s^t$  – The amount of power procured from the Spot market during hour,  $t$ .

The increase in demand,  $D_p$  required under Scenario C can be expressed by equation 4.12 as:

$$D_p = D_s^t + P_{Loss} - \sum_{j=1}^N P_j^t \quad (4.12)$$

The source with the least incremental cost is first increased up to the amount that satisfies all constraints pertaining to that source followed by other sources ranked in an increasing order of their incremental cost. This process is continued until the demand during hour  $t$  is met with all constraints satisfied. Energy settlement is conducted by the market operator after the demand has been met. The cost of satisfying the demand for each of the values of  $s$  in scenario C during hour,  $t$  is given by Equation 4.13 below:

$$T_s^C = p^s \left\{ \sum_{j=1}^N MCP_t (P_j^t + y_j^s) + \sum_{j=1}^N RUP_j^t y_j^s + SRP_t w_s^t + SMP_t z_s^t + MF_t \beta_t \sum_{j=1}^N P_j^t \right\} \quad (4.13)$$



The first term in Equation 4.13 is the probability that the actual load will be  $D_s^t$  for that value of  $s$  during that period. The 1st term within the bracket gives the total cost of energy from all the generating zones including the increase in capacity while the 2nd term is the sum of the penalty fee paid to all the generating zones for the increase in scheduled generating capacity. Similarly, the 3rd and 4th terms within the bracket are the costs of procuring extra energy from the spinning reserve market and spot market respectively to meet the increase in demand. The last term is the sum paid to the provider of the spinning reserve for having this reserve on stand-by whether it is used or not.

The objective function in the optimization cost model is formulated by the combination of all costs for different values of  $s$ :  $s = 1, \dots, 49$  for each time period,  $t$  within the optimization horizon. The optimization horizon is given as 24 hours which makes up the DAM being studied. Therefore, the cost model can be generally described as the cost of Scenarios A, B and C combined.

$$\text{Min } T_t^D = \sum_{t=1}^{24} \left\{ \sum_{s=1}^{24} T_s^A + T_s^B + \sum_{s=26}^{49} T_s^C \right\} \quad (4.14)$$

The above objective function is minimized subject to operating constraints discussed below.

### 4.2.3 Constraints of the Optimization/Objective Function

The objective of this proposed approach is to minimize the total energy cost during any period subject to certain constraints that must be satisfied during any period in the optimization horizon. The following section describes the constraints in detail.

#### 4.2.3.1 Generating Zone Limits

In order to maintain a reliable operation of any generator during any period, it must be operated within its design or operating limits. Therefore, the amount of power scheduled from any of the



generating zones must be within its capacity limits. Generally, this constraint is expressed in the form:

$$P_j^{min} \leq P_j^t \leq P_j^{max} \quad (4.15)$$

Equation 4.15 shows that the power output from any generator at hour  $t$  must be between the minimum and maximum capacity of that generator. This constraint ensures that any change in the scheduled generating capacity of a generator either upwards or downwards must be within limits irrespective of that generator's ramp rate. The operating limits vary from generator to generator and are designed in such a way to reduce forced outages and improve the lifespan of the generating unit.

#### 4.2.3.2 Power Balance

A power system is said to be reliable if and only if the demand at any time can be satisfied by the scheduled generating capacity without load shedding or forced outage of generators. That is to say that the total available generation capacity must always exceed the demand during any period. In this proposed approach, the security and reliability of the system is considered by the inclusion of the power balance constraint. This constraint ensures that the energy balance between supply and demand of power is always positive. That means the sum of the hourly demand and loss must never exceed the available power that the system can generate and transmit at any time. The security of any power system is paramount to the system operator and it is the responsibility of the operator to ensure that there is balance in the power supply and demand within the system. With reference to the terms used in the development of the objective function in Section 4.2.2.3, the power balance constraint is generally given in the form of Equation 4.16 as:

$$\sum_{j=1}^N P_j^t + w_s^t + z_s^t \geq P_{load}^t + P_{Loss} \quad (4.16)$$



The total scheduled generating capacity of all the zones plus the power from the spinning reserve market and the spot market must be greater than or equal to the energy demand during that particular time in the optimization horizon.

#### 4.2.3.3 Reloading -Down Rate

During decommitment of a generator, its scheduled capacity can only be decreased at a rate based on the design and operability of that generator. Generators are designed and built with certain capabilities and limitations. A generator's decommitment rate must be maintained in order to ensure that the generator is economically efficient and reliable. This also helps in improving the lifespan of the generating unit while eliminating forced outages. The rate at which a generator is decommitted is called the ramping down rate (or reloading down rate). It refers to the maximum amount of power per hour that the generator can be decreased. The unit of the reloading-down rate is  $MW/hr$ . In this proposed approach, the reloading-down rate constraint can be expressed in the form:

$$x_j^s \leq DR_j \quad (4.17)$$

Equation 4.17 shows that for a generating zone to be decommitted, the decommitted amount must not be greater than the reloading-down limit of that zone at any time. It is assumed that the reloading-down limit is constant throughout the optimization horizon in this proposed approach. Nevertheless, generating units can have different ramp-down rates at different periods in the DAM.

#### 4.2.3.4 Reloading -Up Rate

Equation 4.18 expresses the constraint for increasing the unit commitment of a zone above scheduled generating capacity during any hour within the optimization horizon. Just like the reloading-down limit ensures that a decrease in the output of a zone does not affect the long term operation of that generating zone, the recommitment rate also ensures this. Take for instance, if generating zone  $j$  is to be increased during a particular hour in order to meet the increase in demand, the ramping up of the generating zone must not be above the reloading-up limit of that zone. An increase above this limit can result in the forced outage of that generating zone. This is



the reason why generator owners provide this data to system and market operators to ensure that it is considered during the DAM security constrained unit commitment and economic dispatch.

$$y_j^s \leq UR_j \quad (4.18)$$

For the purpose of this work, the reloading-up limit,  $UR_j$  is assumed not to be time dependent and therefore is fixed throughout the optimization time. This does not mean that in a real power system the ramp-up rate is always fixed for all periods. It can vary from hour to hour and depends on the owner of the generator and the operating conditions of that generator like start-up and shut down capabilities.

#### 4.2.3.5 Decommitment Limit

This constraint is similar to the constraint in Section 4.2.3.1. Recall that if in real time, the total schedule generating capacity is higher than the real-time demand, the capacities of the generating zones are reduced in the increasing order of their respective incremental cost. To accomplish this reduction while still maintaining a secure and reliable network, the difference between the initial zone commitment,  $P_j^t$  and the amount by which the zone is decommitted,  $x_j^s$  must be less than or equal to the minimum limit of that generating zone. This constraint can be expressed mathematically in the form:

$$P_j^t - x_j^s \leq P_j^{min} \quad (4.19)$$

Equation 4.19 shows that irrespective of the surplus level in the total hourly commitment, none of the zones can be decreased below its minimum generation limit,  $P_j^{min}$ . Based on Equations 4.17 and 4.18, one can say that the maximum reduction during surplus power is the sum of the reloading-down limits of all the zones provided the minimum generating limit of any the zones is not exceeded. Mathematically,

$$\text{Reduction in Surplus Power} = \sum_{j=1}^N DR_j \quad (4.20)$$



Equation 4.20 shows that amount of surplus power that can be reduced is dependent on the reloading-down limit of the zones. Hence, the total operating cost of a system is also dependent on this limit since reducing the excess power minimizes the system cost.

#### 4.2.3.6 Recommitment Limit

To maintain balance between the maximum capacity of any of the generating zones and any increase above scheduled output of a generating zone, a constraint that limits this increase is included in the proposed approach. Whenever the demand is greater than the total scheduled generating capacity, there is the need to source this extra power from the already scheduled generating zones. To meet this increase in demand, the output capacity of the generating zones must be increased up to their reloading-up limits. This increase is also constrained by the maximum generation capacity of that zone. This constraint was incorporated into this proposed approach by including Equation 4.21 in the development of the optimization cost model. Taking a look at Equation 4.21, it can be seen that one of the ways that the deficit in power supply with respect to the real-time demand can be reduced is by increasing the capacity of the generating zones up to the maximum.

$$P_j^t + y_j^s \leq P_j^{max} \quad (4.21)$$

This maximum given in Equation 4.22 is the sum of the reloading-up limits of the generating zones provided the constraint of maximum generating capacity is not violated. Power from the SR and spot markets can also be used in meeting the increase in demand depending on the incremental cost of power from any of these sources and other operating system factors that must be taken into consideration by the system and market operators in the energy market planning and operation.

$$\max(\text{Reduction in Deficit Power}) = \sum_{j=1}^N UR_j \quad (4.22)$$

There are other power system constraints that have not been included in this approach but can be included in the optimization model with little difficulty.



### 4.3 Implementation of the Proposed Optimization Approach

A computer program has been developed in C++ to implement the proposed approach. The details of this computer code and the formulation can be found in Appendix B.2. All the constraints discussed in Section 4.2.3 were taken into account in the assessment of the SR requirements for each hour in the optimization horizon. The minimization of the cost is done for each period in the DAM by comparing the different values of  $\beta_t$  from 0 to 1 for which the total operating cost is minimum given the different energy prices, the incremental cost, and zone commitment available for each value of  $s$  during that period.

For a given unit commitment of the generating zones at a period  $t$  and load forecast for that period, computational analysis is made starting with the development of the load model based on the 49-step normal distribution of the LFU. The value of  $s$  and its corresponding probability are read from Table 3.1. Also the actual load  $D_s^t$  for that value of  $s$  is computed. The sum of  $D_s^t$  and the system transmission loss is compared with the total zone commitment for that period. Certain economic and reliability decisions are made by the computer program based on prices and constraints to determine the total operating cost that period given by Equation 4.14. Due to the complexity of Equation 4.14 which is a combination of linear and non-linear functions, a direct search technique is used. This involves varying  $\beta_t$  from 0 to 1 by a small step-size and the value of  $\beta_t$  for which the operating cost is minimum and all operating constraints satisfied is determined. The corresponding SR requirement is evaluated based on the relationship between this optimal  $\beta_t$  and the total system zone commitment for that period. The computer program executes this cycle until the 24-hour DAM optimization horizon is reached.

The spinning reserve requirements for each period and the total operating cost are outputted to an excel file for analysis and discussion.

### 4.4 Results and Discussion

The test system was utilized to obtain numerical results of the spinning reserve requirements for a day-ahead market with transmission loss.



#### **4.4.1 Results on System with Transmission Loss**

The details of the test system data used for the implementation of the proposed approach can be found in Appendix A.2. The unit commitment based on load forecast, the hourly system transmission loss, the Market clearing price, spinning reserve price, spot market price, the zonal reloading up and down prices in \$/MWh for each period, the operating constraints and other important information required for this approach implementation can be found in this appendix. The computer program was developed to output results with a tolerance (error) of  $1 \times 10^{-3}$ . Figure 3.3 shows the load profile used for this assessment. It is the same as that of the system without transmission loss used in Chapter 3.

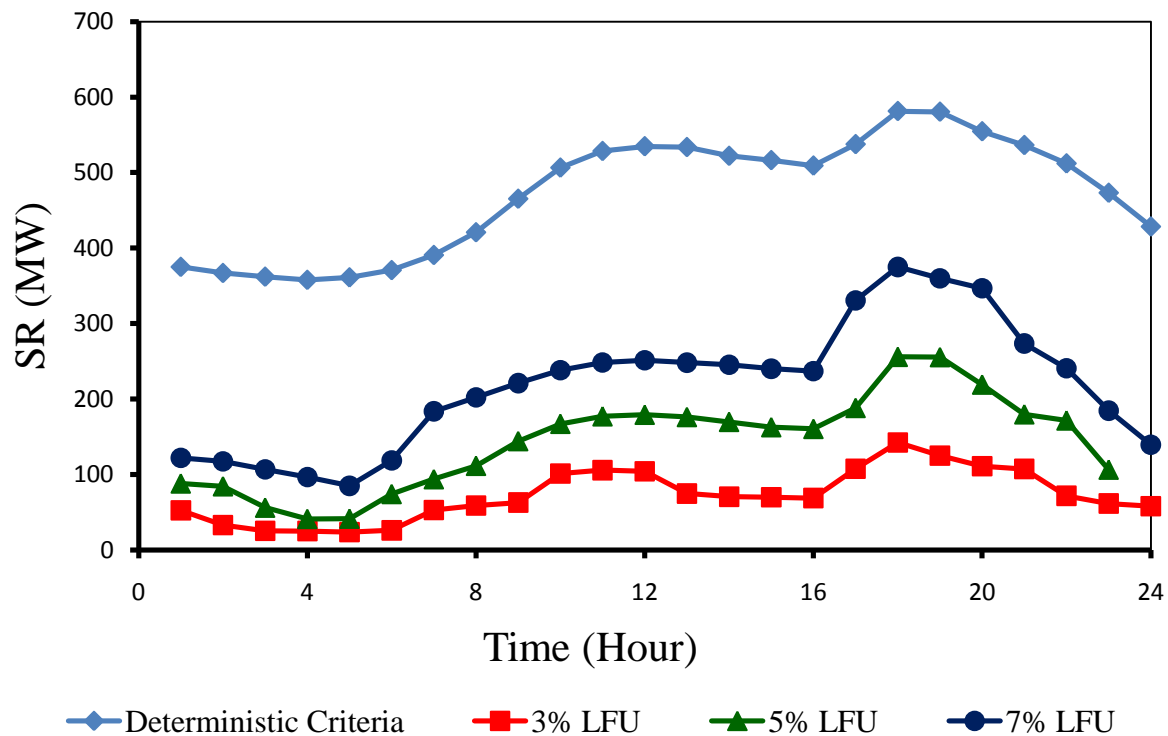
##### **4.4.1.1 Effect of Load Forecast Uncertainty (LFU)**

The proposed approach was simulated for a test system with a forecast load profile of Figure 3.3. The variability in the forecast load was considered by assuming that the standard deviation describing the uncertainty is 3%, 5% and 7% of the forecast load. The spinning reserve requirements when the standard deviation is 3%, 5%, and 7% and the deterministic criterion are shown in Figure 4.3. It can be seen from Figure 4.3 that the amount of SR in a DAM increases with increase in the load forecast uncertainty. The SR requirement increases significantly when the LFU uncertainty increases from 3% to 7%. One will also expect that at a 10% LFU, the SR will almost be double the requirement at 3%.

When the proposed approach is compared to the deterministic criterion in the assessment of the SR requirement for a DAM, it can be seen that the deterministic criterion method has more SR scheduled than might be needed. The deterministic criterion assumed here is given as 10% of the total scheduled generation capacity for any period. From Figure 4.3, the SR requirement for a 7% LFU is almost half of the SR requirement for the deterministic criterion during all periods. This excess SR ensures the security of the system in the event that there is an increase demand but this may not be optimal for the system as can be seen in Figure 4.4. In Figure 4.4, the total operating costs for the two techniques are shown. As one can see, the cost of operating the system for the deterministic criterion is greater than the cost for the proposed approach irrespective of the



percentage load forecast uncertainty. In the deterministic criterion, the percentage load forecast uncertainty is not put into

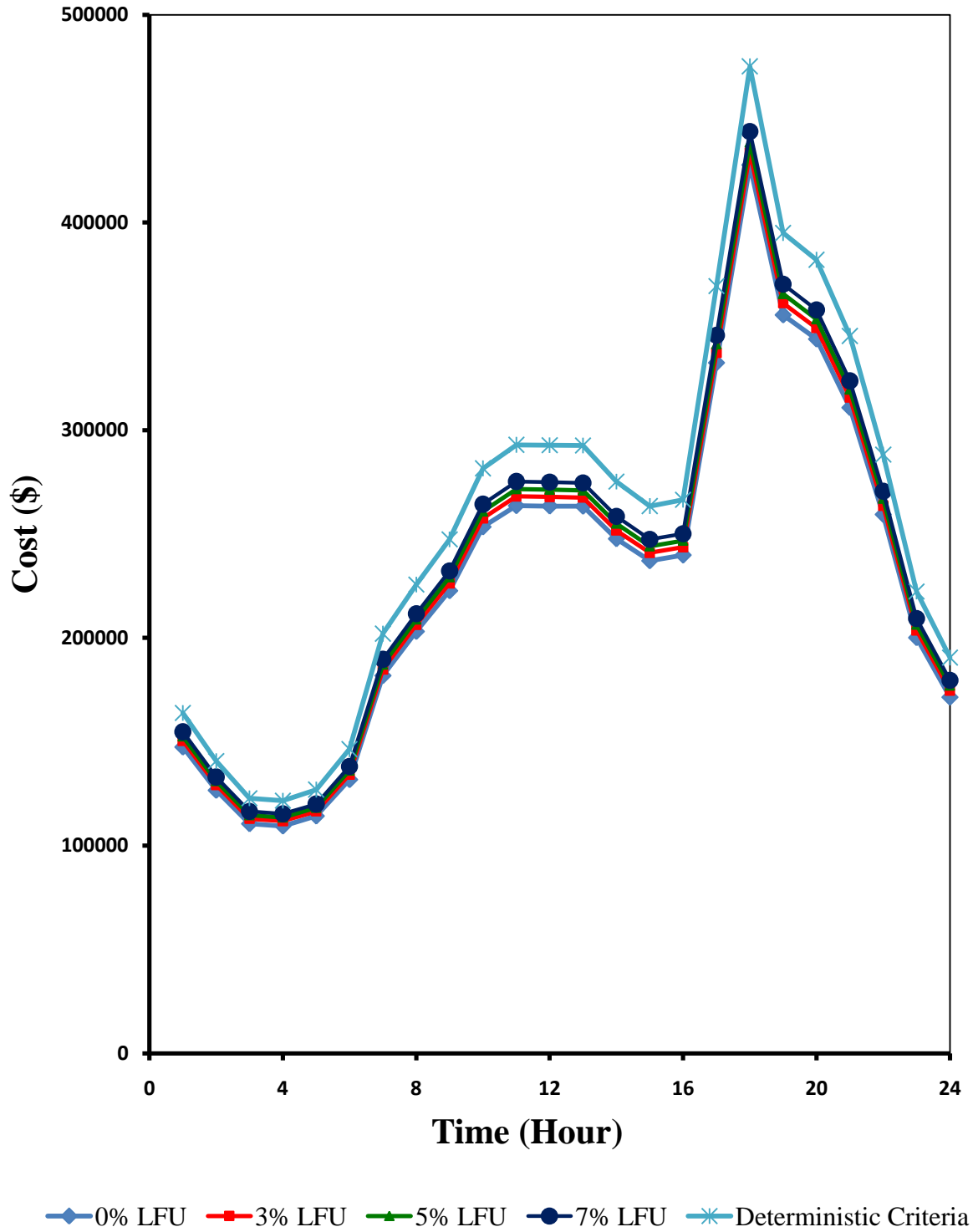


**Fig. 4.3 SR Requirements for 3%, 5%, 7% LFU and Deterministic Criterion**

consideration when determining the amount of SR to schedule for any period. This makes this technique sub-optimal and results in an increase in cost because more SR than is required is scheduled.

When the SR requirement is assessed using the probabilistic technique, the total operating cost is reduced. Table 4.1 shows the percentage decrease in the total cost when this approach is implemented and compared with the deterministic criterion technique. At 3% LFU, the total cost of operation when the proposed approach is used in the assessment of the SR decreases by more than 8% when compared to the cost of operating the system with the deterministic criterion.





**Fig. 4.4 Total Operating Cost for Deterministic Criterion versus Total Operating Cost for the LSRA Technique**



Similarly, at a 5% and 7% LFU, the decrease in cost is respectively more than 6% and 5% of the cost with the deterministic criterion approach. The largest decrease in the total system cost is for a 3% load forecast uncertainty. A 3% LFU requires less amount of SR since the deviation from the forecast load is relatively small compared to the deviation from the forecast mean for either a 5% or 7% LFU.

Figures 4.5, 4.6 and 4.7 compare the SR requirements of a system without transmission loss and another system with transmission loss for a 3%, 5% and 7% LFU. It can be seen that the transmission loss has an impact in the SR requirement. The SR requirement for the system with transmission loss is greater than that of the system without transmission loss even though the forecast loads of both systems are the same. It is therefore critical for system operators to consider transmission loss in the scheduling of SR since a change in the demand can either increase or decrease the transmission loss within the system which can result in a change in the SR requirements for that period either up or downwards.

#### **4.4.1.2 Effect of Spot Market Price (SMP)**

Figures 4.8, 4.9 and 4.10 show the effect of the SMP on the assessment of the SR requirements for a 3%, 5% and 7% LFU respectively. As can be seen, the amount of SR changes as the SMP changes with respect to its nominal value (100%) at a fixed SRP. When the real time energy cost is at the nominal SMP, the SR requirement is least. As the nominal SMP is increased by 10%, 20%, 40%, and 60%, there is a corresponding increase in the SR requirements. This change in the SR requirement can be attributed to the economic limitation in the amount of power that can be procured from the spot market in the event of an increase in demand.

As the SMP increases, the incremental cost per MW from the spot market increases. This increase in the incremental cost of power from the spot market results in a decrease in the amount of power that can be sourced from the spot market. Considering this limitation, the proposed approach determines the amount of SR that is optimal for the system.



**Table 4.1 Percentage Decrease in Total Operating Cost of the LSRA technique for different LFU compared to the Total Operating Cost utilizing the Deterministic Criterion**

<b>% Decrease in Total Operating Cost of the LSRA Technique compared to the Deterministic Criterion</b>			
<b>Time (Hour)</b>	<b>3% LFU</b>	<b>5% LFU</b>	<b>7% LFU</b>
1	8.18	6.88	5.53
2	8.17	6.84	5.46
3	8.06	6.65	5.20
4	8.04	6.63	5.16
5	8.23	6.89	5.47
6	8.30	7.02	5.67
7	8.52	7.37	6.13
8	8.57	7.44	6.22
9	8.53	7.36	6.12
10	8.52	7.35	6.12
11	8.45	7.26	6.02
12	8.49	7.31	6.06
13	8.56	7.39	6.15
14	8.54	7.33	6.08
15	8.55	7.35	6.09
16	8.57	7.38	6.13
17	8.67	7.57	6.42
18	8.74	7.70	6.59
19	8.56	7.43	6.26
20	8.60	7.48	6.32
21	8.59	7.46	6.27
22	8.53	7.36	6.12
23	8.43	7.18	5.86
24	8.32	7.06	5.73



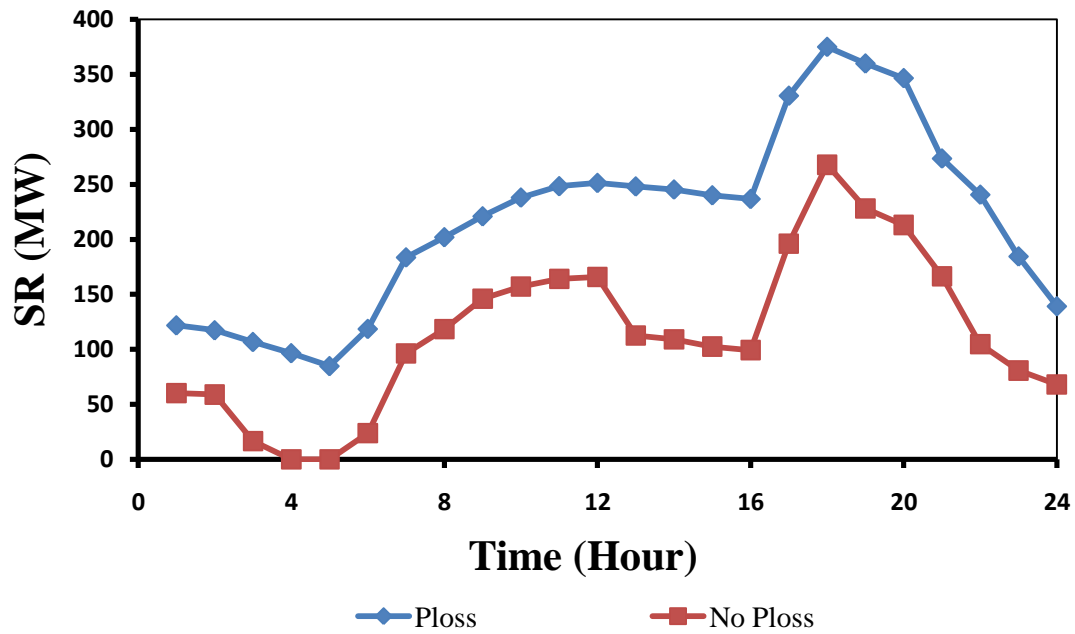


Fig. 4.5 SR Requirement for 7% LFU at Nominal SMP and SRP

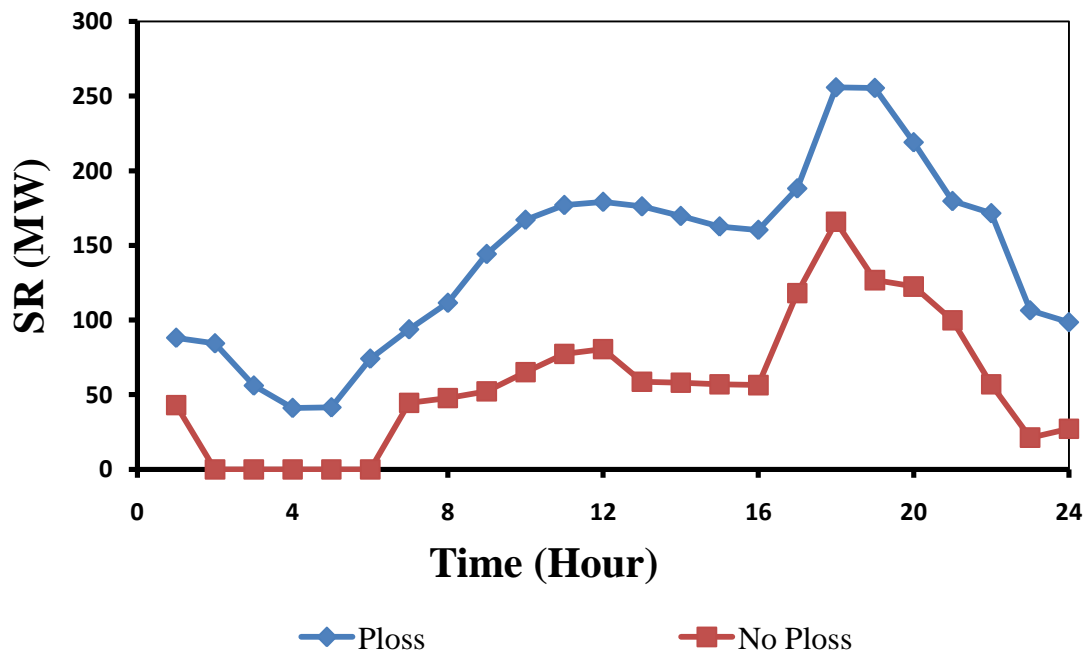
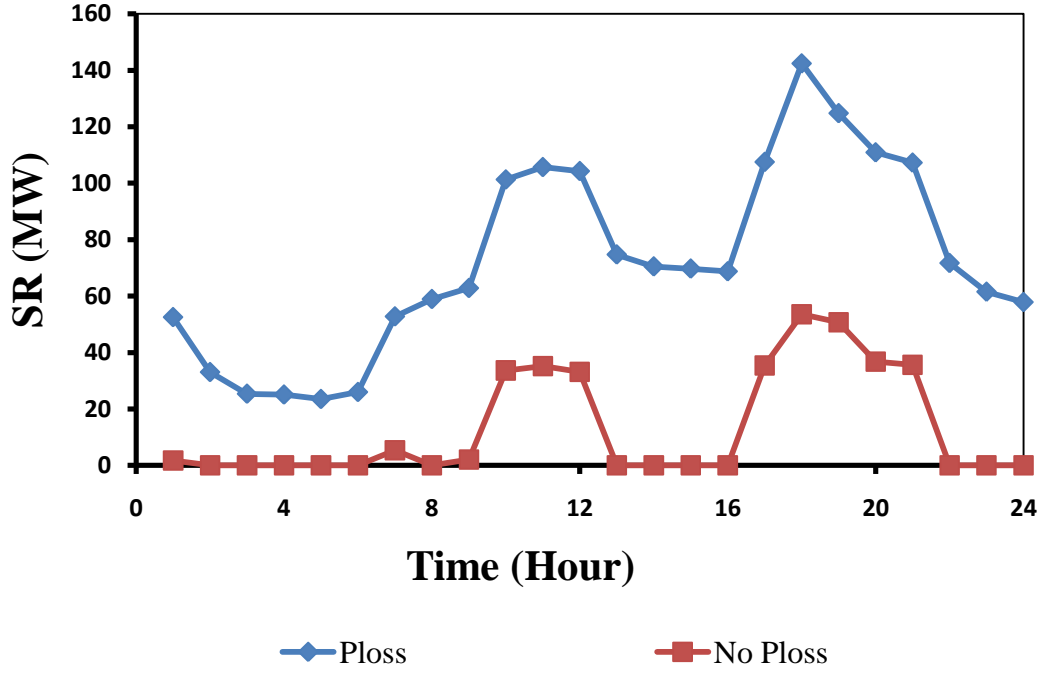


Fig. 4.6 SR Requirements for 5% LFU at Nominal SMP and SRP





**Fig. 4.7 SR Requirements for 3% LFU at Nominal SMP and SRP**

Recall that when there is an increase in demand, the system operator must schedule more power in the most economical way to meet this increase either by using the scheduled SR or increasing the capacity of scheduled generators or by direct energy purchase from the spot market. To achieve this in the proposed approach, the proposed method compares the incremental cost of the three sources and determines which amount from each of the sources is optimal for the system in meeting the demand increase. The focus in this technique is the SR requirement for different percentage LFU. Therefore, only the optimal SR is evaluated even though the pricing data of the other two sources are considered in this approach.

Figure 4.8 shows that as the real time energy price SMP increases from its nominal value to 1.6 times this value while keeping SRP fixed, the SR requirement also increases. The change in the SR requirement becomes significant at 5% and 7% LFU. One can see that at 1.6 times the SMP, the SR is almost 40% higher than the SR at the nominal SMP. This shows that the SMP has a significant impact in the determination of SR requirement. The problem with using the SMP in the assessment of the SR



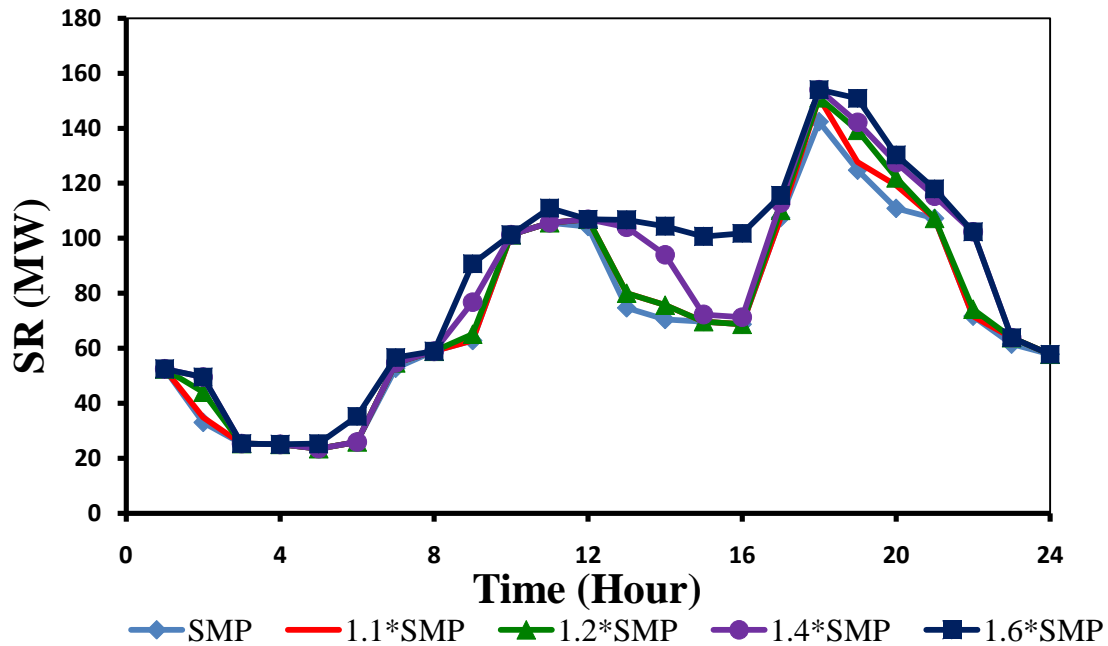


Fig. 4.8 Effect of SMP at Nominal SRP on SR requirements for 3% LFU

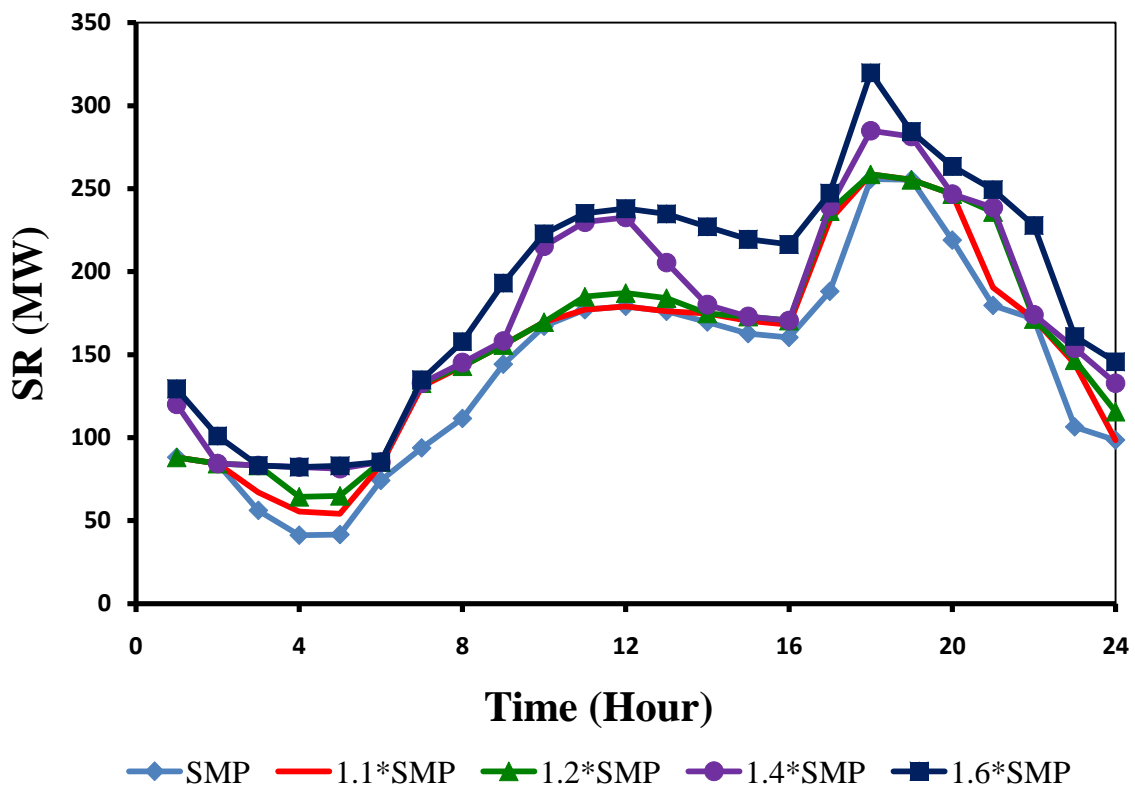
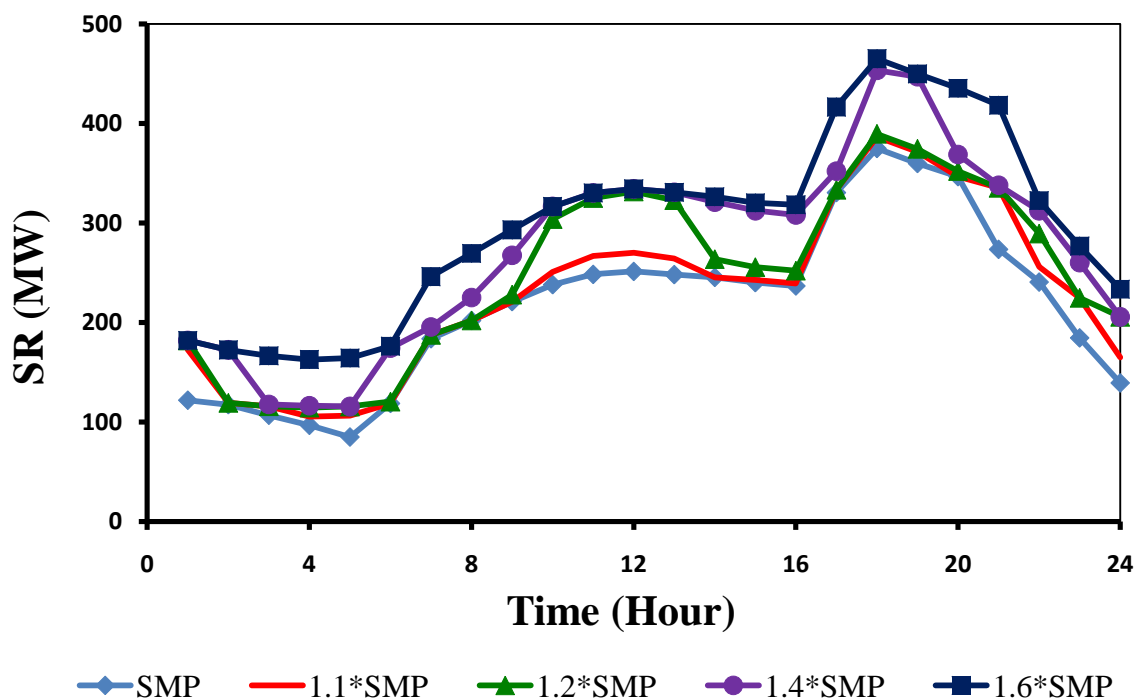


Fig. 4.9 Effect of SMP at Nominal SRP on SR requirements for 5% LFU



requirement is that it is difficult to determine what the energy price will be in real time for a DAM. To solve this problem, system and market operators must rely on historical data of energy prices in real time. Just as the load forecast is developed based on historical study of load profiles over a period of time, the market operator must be able to forecast to a level of accuracy the energy price in real time for all periods in the DAM. With this data available, system operators can use this approach in determining optimal SR required in a DAM thereby minimizing total operating cost. If the SMP is modeled as a 5-step normal distribution as shown in Figure 3.6, the aggregate SR requirements for different LFUs are shown in Figure 4.11 for a fixed value of the SRP (set at nominal value of 100%).

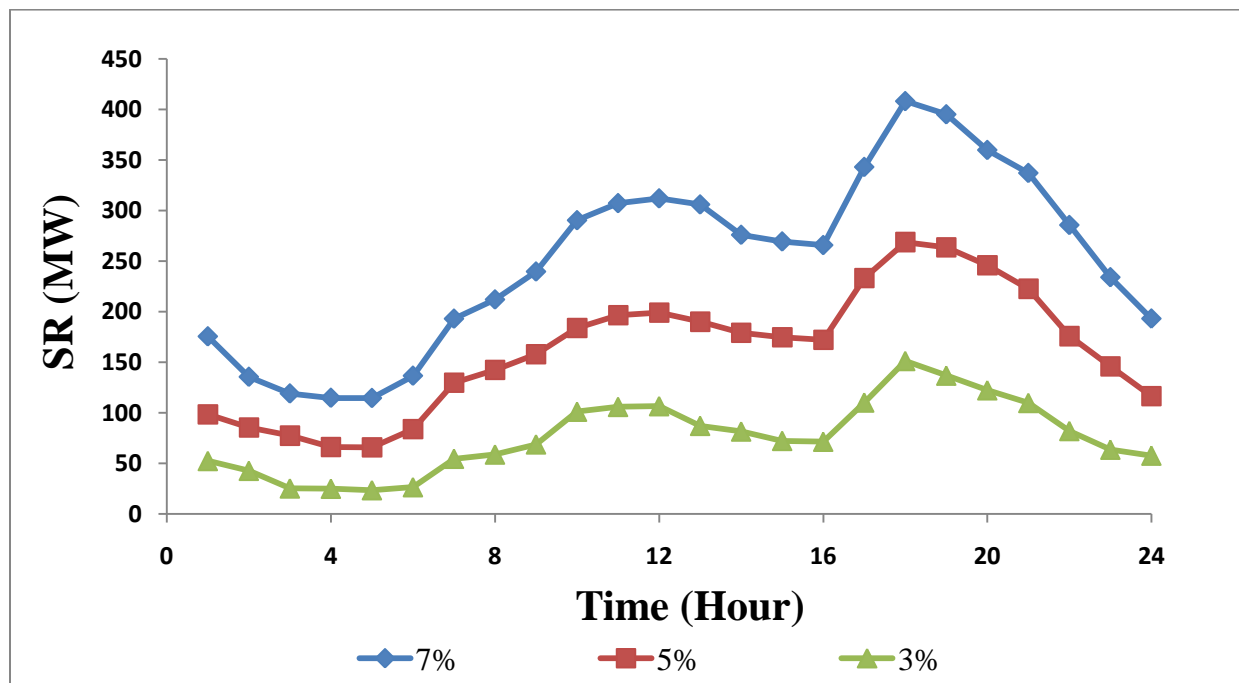


**Fig. 4.10 Effect of SMP at Nominal SRP on SR requirements for 7% LFU**

The change in the SR requirements for a change in the SMP is also dependent on the LFU's standard deviation. Figures 4.8, 4.9 and 4.10 show that the SR requirement significantly increases when the standard deviation of the LFU increases from 3% to a 7% LFU. In Figure 4.8, the SR requirement is relatively the same for all changes in the SMP except for the peak load



hour when the SR requirement increases relatively above other periods. The 3% standard deviation of the load forecast from the actual demand is very small and as a result the required amount of SR for this system is not high. A look at Figure 4.9 shows that for a 5% standard deviation in the load forecast uncertainty, the SR requirement for different values of the SMP is greater than that of a 3% LFU. Similarly, with a standard deviation of 7%, there is a significant amount in the SR scheduled for all periods.



**Fig. 4.11 Aggregate SR Requirements for a Nominal SRP (100%)**

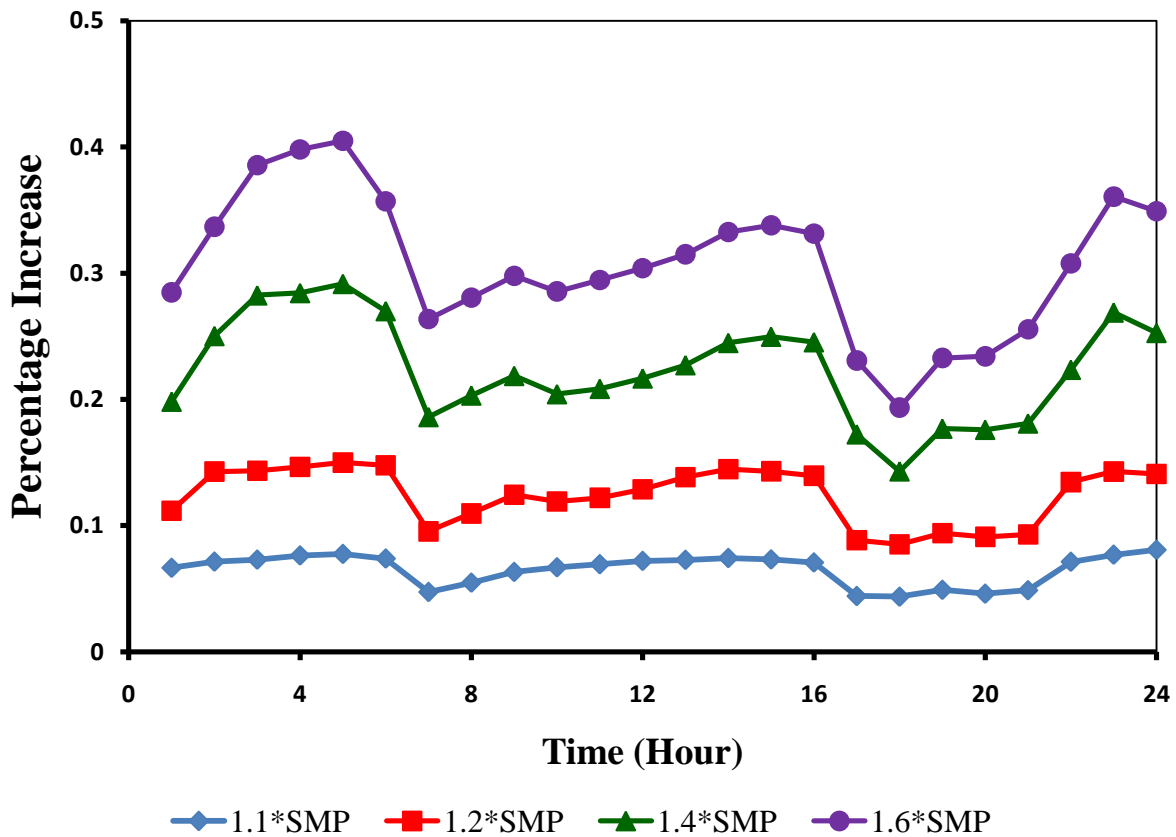
Figure 4.12 shows the percentage change in the total operating cost as the SMP changes. For a 10% and 20% increase in the nominal value of SMP, the increases in the system operating cost are less pronounced when compared to the operating cost for 40% and 60% increases. Higher energy prices in real time increases the amount of SR during such periods. Similarly, when the energy price in real time during any period is low, the SR requirement for that period is also expected to be low. This change in cost is as a result of the change in SR due to change in the SMP for such periods when compared to the fixed value of the SRP. In order to economically



operate a deregulated power system, the effect of real-time energy prices must be considered in the assessment of the SR requirements of any period in the DAM.

#### 4.4.1.3 Effect of Spinning Reserve Price (SRP)

In order to determine the impact of the SR price on the SR requirements of a DAM, the value of the SRP is varied while keeping other prices like the MCP and SMP fixed. This variation in the SRP and its effects were analyzed for a 3%, 5% and 7% standard deviation of the load forecast uncertainty. Figure 4.13 shows that as the SRP changes, the SR requirement also changes.



**Fig. 4.12 Percentage increase in Total Operating Cost at Nominal SRP with varying SMP**

When the nominal SRP is increased by 10%, 20%, 40% and 60%, one can see that the SR requirements for any period during this change decreases. This is to say that the SRP is inversely proportional to the SR requirements. In economics, as the price of a commodity increases, the demand for that commodity decreases provided other factors remain constant. This theory is



applicable to the economics of SR. The SR requirements decrease with an increase in the SRP since the price of energy from other sources is fixed. When the SRP increases in such a way that incremental cost of scheduling a MW of power from the SR market is higher than the incremental cost of procuring the same amount of power from either the spot market or from scheduled generating zones, the amount that can be sourced from the SR market decreases. The SR decreases up to that amount in which the total operating cost is minimum for that SRP.

Figures 4.13, 4.14 and 4.15 show this optimal SR for 3%, 5% and 7% standard deviation respectively. For a 40% and 60% increase in the nominal SRP, one can see in Figures 4.13 and 4.14 that the SR for most of the periods in the optimization horizon is zero. This does not mean that there is no need for SR during that period. Rather, it means that it is most economical for the market operator to source power from spot market to meet this increase in demand. It is known that maintenance fee is paid each time SR is scheduled (used or not). To eliminate this cost and minimize the total operating cost while maintaining reliability of the system, the system operators have the option of waiting until the demand increases above scheduled capacity in real time. At that point, they can either buy energy directly from the spot market or instruct generators to ramp up their capacities at a fee to meet the increased demand.

Figure 4.15 shows that there is SR scheduled for most of the period when the SRP is 60% higher unlike Figures 4.13 and 4.14. This is caused by the increase in the standard deviation of the load forecast uncertainty. Not only is the deviation of the forecast load high, the transmission loss in the system also increases as the demand increases. Therefore, SR must be scheduled to meet this increase in demand and change in system loss irrespective of the cost of the SR requirements for that hour. One common characteristic of Figures 4.13, 4.14 and 4.15 is the SR scheduled during the peak load hours of 17 and 18. One can see that irrespective of the SRP, there is SR scheduled during these peak hours even though the amount for each % LFU decreases with an increase in the SRP. This means that during peak hours, it is critical for the system operator to synchronize an extra capacity into the system for system security. The Figures also show that the reduction in the SR requirements as the SRP increases is significant for the 3% LFU than in the 5% and 7% LFU. This is because it may be economical to schedule little or no SR when the standard deviation is 3% than when the standard deviation of LFU is 5% and 7%.



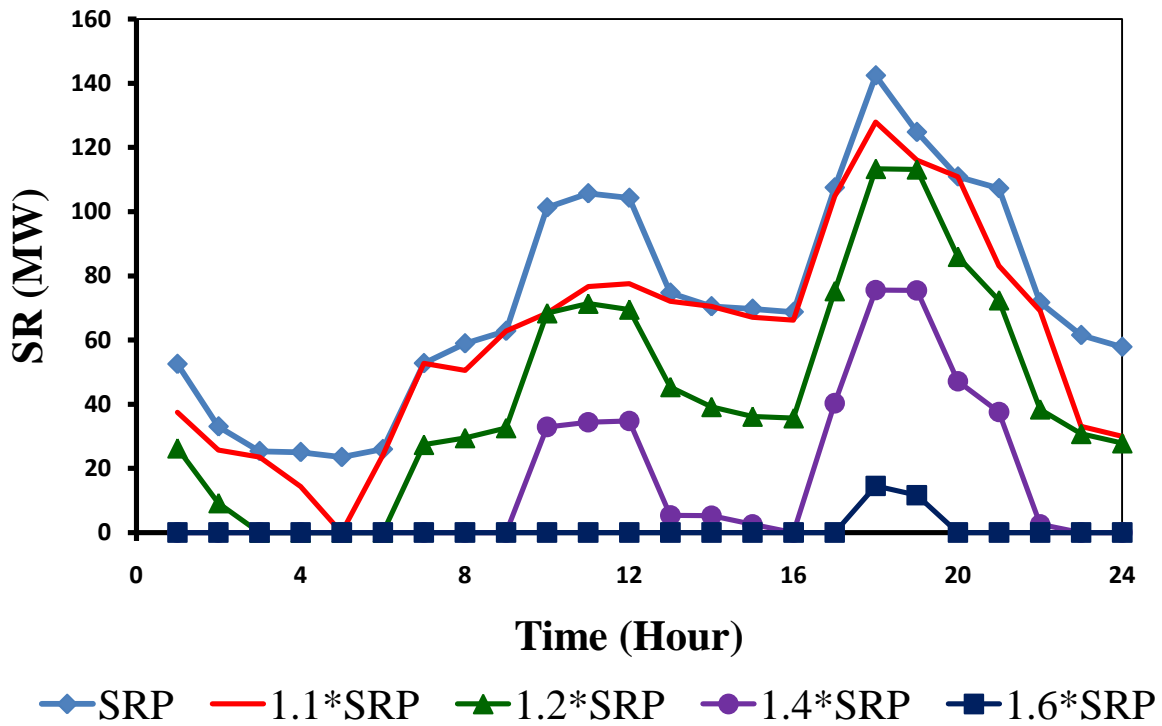


Fig. 4.13 Effect of SRP on SR requirements for a 3% LFU at Nominal SMP

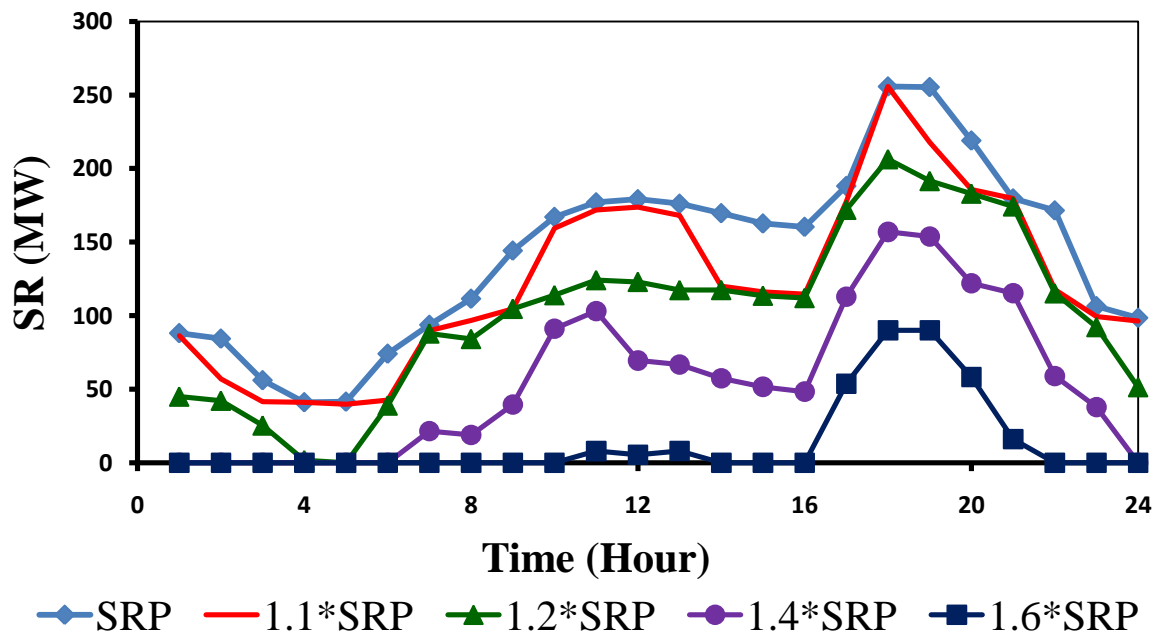
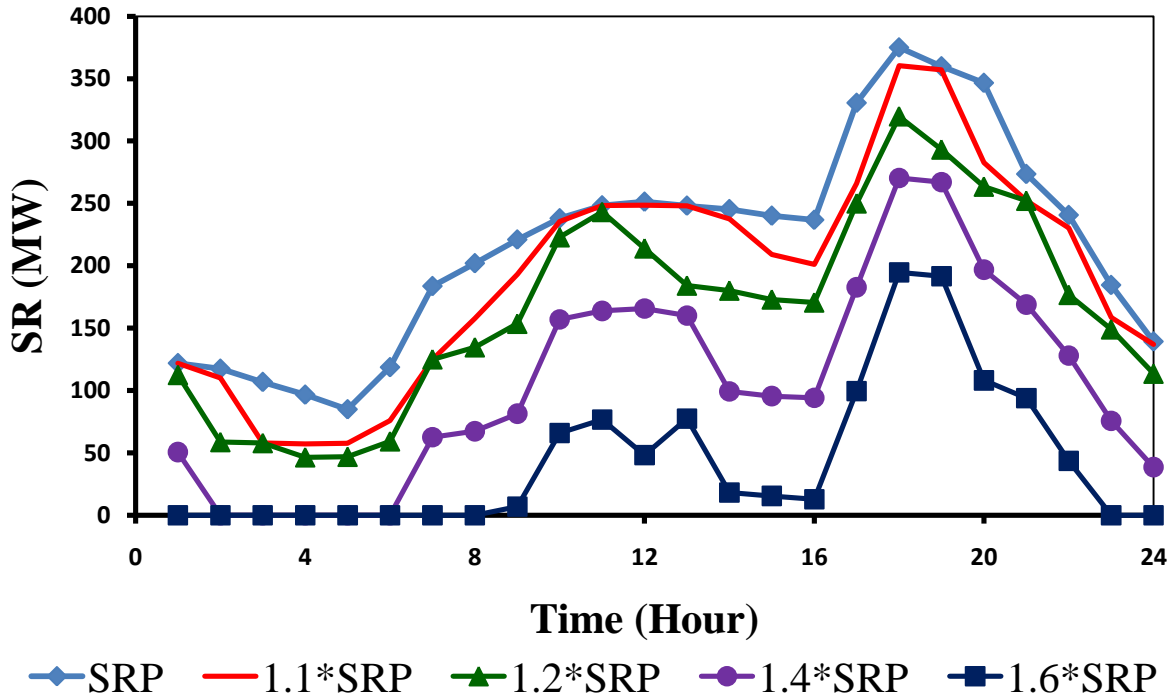


Fig. 4.14 Effect of SRP on SR requirements for a 5% LFU at Nominal SMP





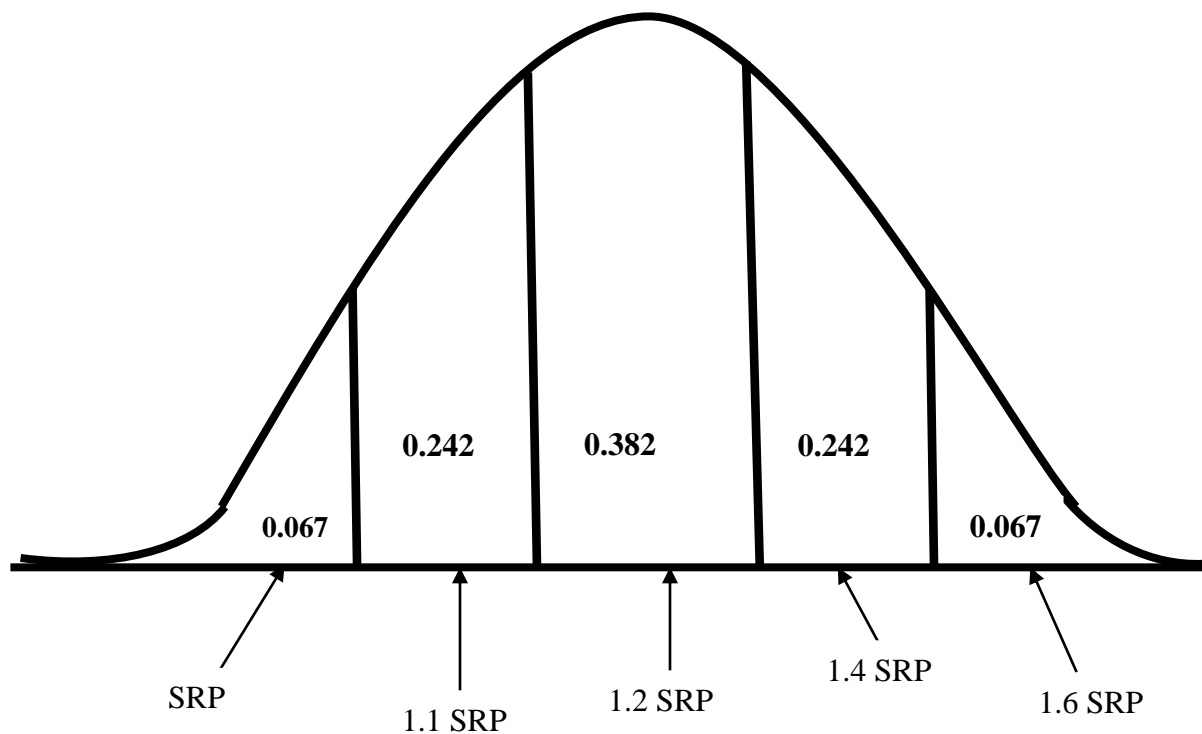
**Fig. 4.15 Effect of SRP on SR requirements for a 7% LFU at Nominal SMP**

The variations in the SRP can be modeled as a 5-step normal distribution with a mean value of  $1.2*SRP$  for expected values as shown in Figure 4.16. Using the SRP distribution, the aggregate spinning reserve requirements for a 3%, 5% and 7% LFU can be obtained by the weighted sum of the spinning reserve requirements for five SRPs in each of Figures 4.13, 4.14 and 4.15 respectively. This is shown in Figure 4.17.

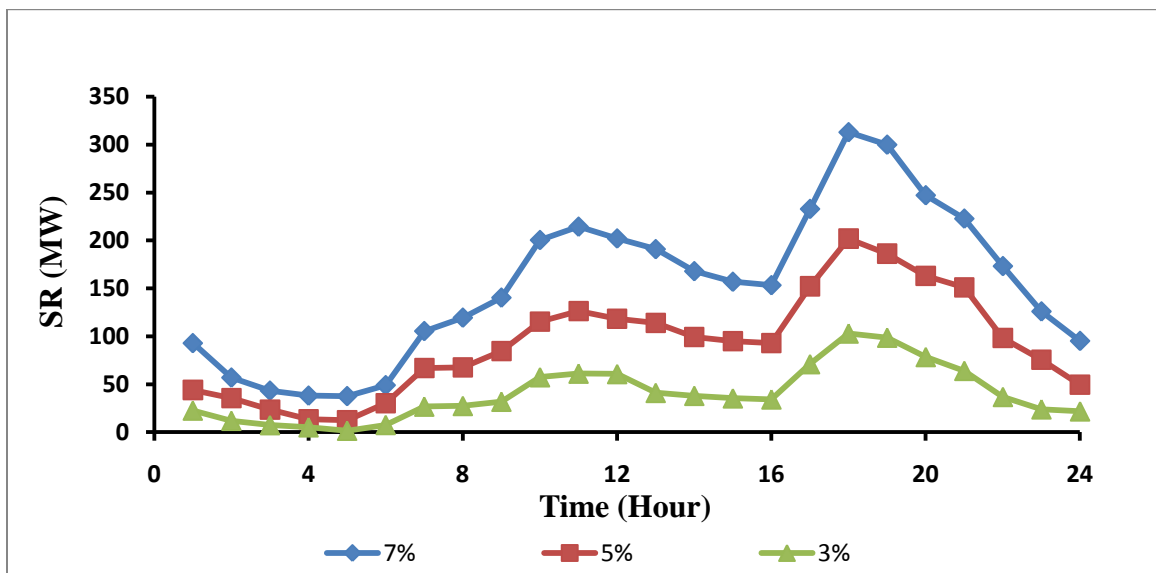
#### 4.4.1.4 Effect of Reloading Limits of Generating Zones

In order to assess the SR requirements of the system, the reloading limits of the generating zones were varied keeping other system factors fixed. This was done to determine how a change in the reloading limits of generators affects the system SR requirements. A load forecast uncertainty with a 7% standard deviation was used for this analysis. Power suppliers provide their generators' ramp limits to the system operator prior to the system security constrained unit commitment and economic dispatch process for the DAM. A look at Figure 4.18 shows that a change in the reloading-up (ramp-up) limit changes the periodic SR requirements.



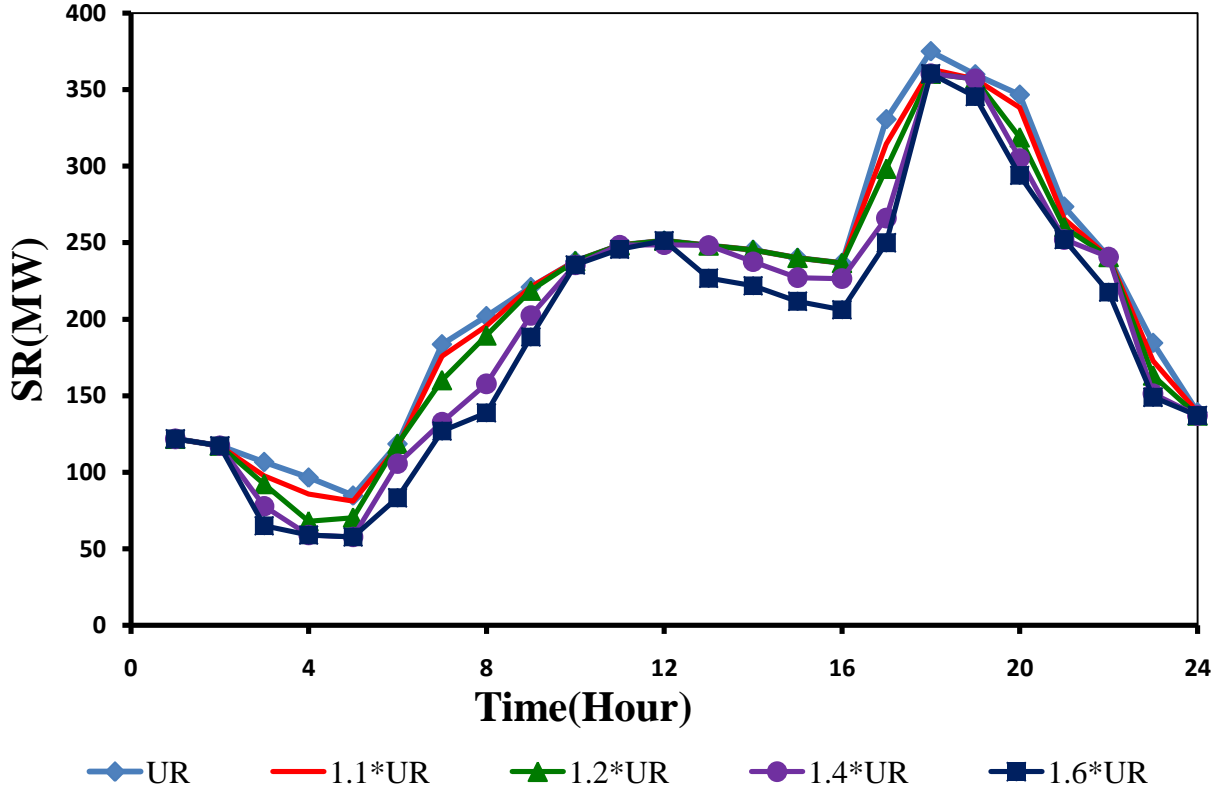


**Fig. 4.16 5-step Normal Distribution of SRP**



**Fig. 4.17 Aggregate SR Requirements for a Nominal SMP (100%)**





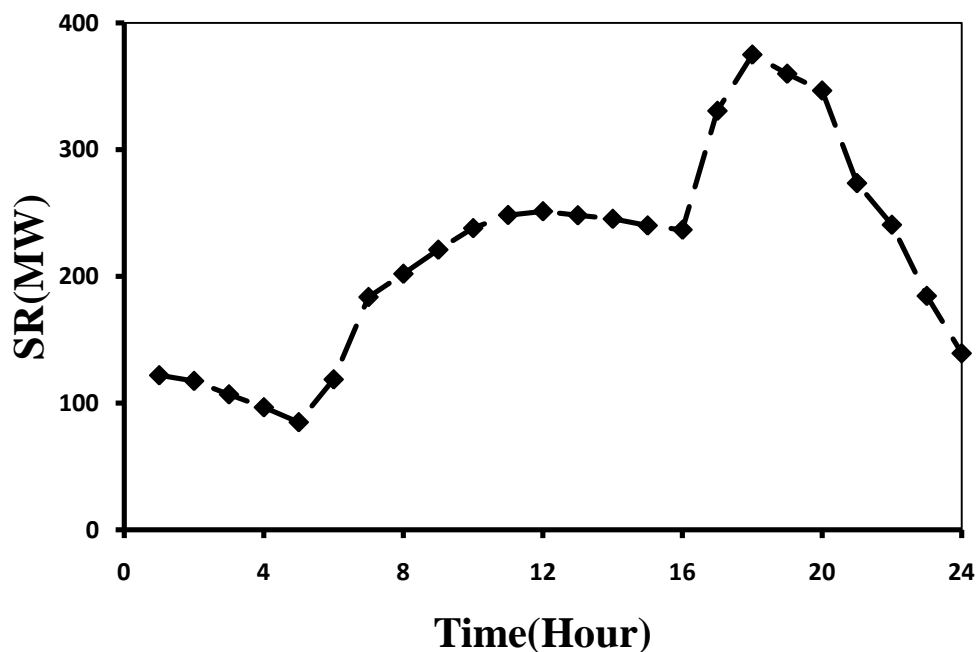
**Fig. 4.18 Effect of Reloading-Up Limit on SR requirements for 7% LFU**

With the assumption that the SRP and SMP are unchanged during any period in the optimization horizon, let us say that the nominal ramp-up limits of all the zones are increased by 10%, 20%, 40% and 60%. One can see from Figure 4.18 that for every increase in the reloading up limit, there is a decrease in the SR requirement for that period. This decrease is not proportional to the incremental factor of the reloading up limit. This is because a change in the SR requirement does not only depend on the ramp-up limit but also on other variables discussed in Sections 4.4.1.1 to 4.4.1.3. It can also be seen that the change in the SR requirement is not that significant for a change in the reloading up limit due to the transmission loss within the system.

On the other hand, when the reloading-down limits of the generating zones are changed, the effect on the SR requirements is not the same with changing the ramp-up limit. Figure 4.19 shows that the SR requirements remain unchanged for any incremental change in the reloading-down limit. When the nominal reloading-down limit was increased by 10%, 20%, 40% and 60% while keeping other system factors constant, it was found that the SR requirements remained the



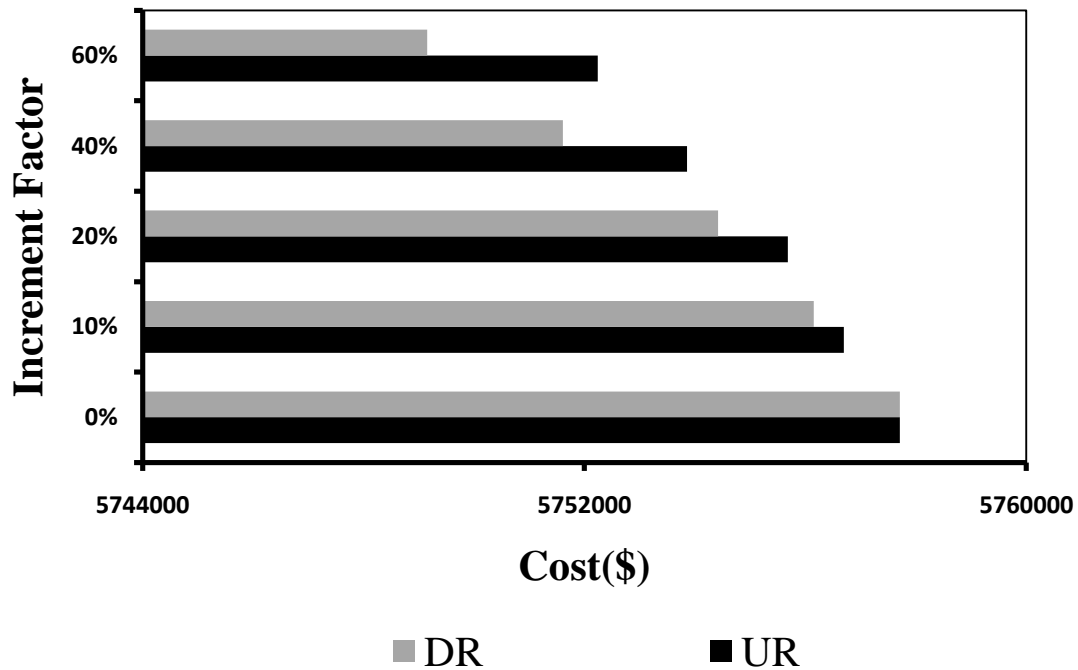
same irrespective of the change. Even though there was no change in the SR requirements, an interesting effect was identified. For each of the changes in the reloading-down limit, there was a reduction in the total operating cost for the optimization time horizon. Figure 4.20 shows that when there is a change in the ramp limits of the generating zones, there is a change in the total operating cost. It can be seen that the total operating cost decreases as the ramp limits of the generating zones increase. The reduction in the SR requirements when the reloading-up limit is increased explains the decrease in the operating cost for such a change but does not explain why an increase in the reloading-down limits results in a decrease in the total operating cost even though the SR requirements remain unchanged. A look at the proposed approach from Figure 4.1 and Section 4.2.2 shows that there are 24 out of 49 possibilities that the forecast load will be less than the actual load. For each of these 24 possibilities, the initial zone



**Fig. 4.19 Effect of Reloading-down Limit on SR requirements for 7% LFU**

commitment is reduced based on the difference between the total scheduled generating capacity for that period and the sum of the actual load and the transmission loss within the system, the reloading limits of the generating zones and the incremental cost for any MW change in the zone commitment.





**Fig. 4.20 Effect of the Reloading limits on Total Operating Cost for a 7% LFU**

When generating zones are decommitted, there is no need for SR which means that any change however small or large in the ramp-down limits of the zones does not directly imply a change in the SR requirements. However, for zone recommitment, more power is needed than scheduled and therefore SR is required to meet the increase in demand for that period. This means that for any change in the reloading-up limit of any zone, there is a direct effect on the system SR requirements. It can be deduced from Figures 4.18 and 4.20 that increasing the ramp-up limit of generators decreases the SR requirements and also minimizes the total operating cost for that system. The operating cost can also be minimized by increasing the reloading-down limits of the generating zones without any change in the SR requirements. Comparing both limits in Figure 4.20, one can say that in order to minimize total system cost, it is more economical to increase the reloading down limit of any generator than it is to increase its ramp-up limit. However, an increase in the ramp-up limit of the generating zone can reduce the SR requirements thereby eliminating the cost of having more SR than required on stand-by. Therefore, system and market operators can minimize cost by requesting increases in the limits of the generators scheduled in the DAM.



#### **4.4.1.5 Computation Time**

All simulations were performed in a PC with an Intel Duo Core processor T5800, 2 GHz with a 4GB RAM. The proposed approach is iterative with a tolerance of  $1 \times 10^{-3}$ . The time required to complete one simulation and save result was approximately 600 seconds.

### **4.5 Summary**

An approach is considered for the assessment of the SR requirements for a deregulated system incorporating transmission loss. The basic idea in this proposed approach is to utilize the zone commitment conducted in the DAM for the sum of the forecast load plus transmission loss without including any spinning reserve and the DAM forecast load to determine the SR requirements for the system. A load model is developed and is described by a 49-step normal probability distribution load forecast uncertainty with the forecast load as the mean and a known standard deviation. In order to assess the SR for each period, different DAM data like energy prices and zone commitments are used in making economic and reliability based decisions for each step in the 49-step LFU load model. A computer program was developed to make these economic and reliability based decisions based on the incremental cost of each energy source and certain system constraints that must be satisfied for any period. The optimal SR requirement is determined by minimizing the total system operating cost formulated from the load model subject to certain constraints for each period. A test system of three generating zones is used to simulate this proposal and assess the SR requirement for each period in the DAM. The results and effects of certain system variables (like the standard deviation of the LFU, SMP, SRP and the reloading limits of generating zones) on the SR requirements were studied and discussed with the view of understanding how these variables impact SR in a DAM.



# Chapter 5

## Conclusions

### 5.1 Conclusions

Generation and spinning reserve are scheduled usually 24 hour ahead of time based on the load forecast. Utilities include spinning reserve to protect the system against contingencies and load variations. Most utilities utilize same form of deterministic criterion to assess spinning reserve. While the fixed criterion for determining SR requirements may offer security of the system and simplicity in application by scheduling large amount of SR, there are disadvantages of utilizing this technique in the assessment of the SR requirements in a deregulated system. These demerits are:

- It produces a sub-optimal solution
- The system operating cost increases when SR requirement is over-forecasted
- It does not utilize the probability-based system parameters that have direct impact on the SR requirements

This thesis proposes a new application of the zero-order optimization technique that considers the load forecast uncertainty to assess the spinning reserve requirements of a deregulated system. The proposed probabilistic approach named “LFU-based Spinning Reserve Assessment (LSRA)” has advantages over the deterministic criterion for SR assessment and they include:

- There is no traditional unit commitment constraint that fixes the amount of spinning reserve for any period
- There is no need to set any risk target in the optimization model



- The required spinning reserve level is determined directly by minimizing the total cost of operation by considering the load forecast uncertainty, different energy prices in the system and system constraints
- There is no spinning reserve over-forecasting or under-forecasting since the assessment is directly based on the probabilistic nature of load forecast

The LSRA technique is based on the load forecast uncertainty model. The model in the LSRA technique is a 49-step normal probability distribution with a known standard deviation and the mean as the forecast load of that period in the DAM. The LSRA utilizes the 49-step LFU model instead of a 7-step for the purpose of accuracy. The LSRA develops the load model based on the periodic forecast load and the scheduled generating capacity of the zones. The unit commitment of the DAM is the initial stage in this approach and is conducted offline. The result of the unit commitment is fed into the optimization process as system data. Each of the steps in the 49-step LFU is characterised by a probability and a load for that step. The load at each step is computed prior to comparing it with the total scheduled generation. If the value of the load is less than the total scheduled capacity, the units are decommitted. Similarly, if the load for any step is greater than the total scheduled generation for the period, either SR is called up or the units are recommitted or power is directly procured from the spot market. The decision on how to meet the increase in demand is based on the incremental cost of a MW change in the output of any of the sources. The total system operating cost for a given period is evaluated as the sum of the cost of all the 49 steps. This cost is minimized to determine the optimal SR for that period. The concept and application of the LSRA have been discussed in detail in Chapter 3 with transmission loss ignored within the system. Chapter 4 implements the LSRA technique with the inclusion of transmission loss as a non-linear function. The details and concepts are also covered in Chapter 4.

Spinning reserve requirements were assessed for the test system with LFU of 3%, 5% and 7% of standard deviation. The SR requirement changes with change in the LFU. The change in the SR requirement is directly proportional to the magnitude of the LFU. Comparing both test systems, one can say that the transmission loss in any system increases the SR requirements of that system. Another inference that was drawn from this technique is that the SR requirement



increases as the spot market price for energy increases and vice versa. On the other hand, the spinning reserve price is inversely proportional to the SR requirements during any period in DAM. One other variable in the DAM that impacts the SR requirements is the reloading (or ramp) limits of the generators. An increase in the ramp-up limit of the scheduled generators decreases the SR requirements as well as the total system operating cost. However, there is no significant effect on the SR requirements for any increase in the ramp down limit of the generators but there is considerable reduction in the operating cost.

Although it is obvious that the SR requirements would change with a corresponding change in LFU and a change in transmission loss, the proposed technique provides a quantitative means to assess the SR requirements in a deregulated environment. The technique is based on optimization of the operating cost in the presence of the market variables found in a deregulated system. The technique is flexible enough to include other market variables with little difficulty. The development of this new technique for the assessment of SR requirements in a deregulated system considering load forecast uncertainty has been illustrated in this thesis. Although a simple test system has been used in implementing this approach, the LSRA technique can be implemented for a larger system with the inclusion of more operating constraints that describe a practical deregulated power system.

## **5.2 Suggestions for Further Work**

The implementation of the LSRA technique presented in this thesis can be used to economically determine the SR requirements of a DAM such that the operating cost is minimized over all periods. To implement this technique in a practical system, the following can be included:

- i. The generator failure rate can be incorporated into the cost model using the COPT approach
- ii. The objective function can be developed to include expected cost of energy not served (EENS) due to capacity deficit
- iii. The cost/benefit analysis can be implemented with LSRA technique for the assessment of the SR requirements



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# APPENDICES

## APPENDIX A: SYSTEM DATA

### A.1 SYSTEM DATA WITHOUT TRANSMISSION LOSS

The LSRA technique was implemented on a test system without transmission loss. The test system data is given in Table A.1.1.

**Table A.1.1 Data for a Test System without Transmission Loss**

HOUR	ZONE 1			ZONE 2			ZONE 3			ENERGY PRICES				$P_{Load}$ (MW)
	RDP	RUP	P (MW)	RDP	RUP	P (MW)	RDP	RUP	P(MW)	MF	MCP	SRP	SMP	
1	37.2	29.4	1022	34.4	31.6	1500	34.6	31.9	924	12	39.3	43.7	87.4	3446
2	31.3	28.9	998	29.3	27.1	1479	29.8	27.1	892	12	34.5	38.4	76.7	3369
3	29.7	25.5	982	27.8	25.7	1470	28.2	25.8	870	12	30.5	33.9	67.9	3322
4	32.0	23.6	971	29.4	24.8	1461	28.5	25.1	865	12	30.6	34.0	68.0	3297
5	29.7	23.7	978	27.4	23.0	1473	26.8	22.9	866	12	31.6	35.2	70.3	3317
6	30.7	24.2	1005	28.6	25.8	1520	28.8	25.8	883	12	35.6	39.5	79.0	3408
7	34.9	29.7	1058	32.8	32.4	1588	33.3	32.8	920	12	46.5	51.7	103.4	3566
8	34.9	29.1	1143	32.9	29.9	1700	33.0	29.9	980	12	48.3	53.6	107.3	3823
9	37.0	33.3	1264	34.5	27.4	1849	34.0	27.3	1062	12	47.9	53.2	106.4	4175
10	39.5	33.9	1370	36.8	30.1	1964	36.3	30.0	1152	12	50.0	55.6	111.2	4486



Table A.1.1 Continued

11	41.5	33.6	1429	38.8	29.9	2049	38.4	29.8	1210	12	49.9	55.4	110.9	4688
12	39.5	30.8	1440	36.9	29.4	2068	36.1	29.2	1229	12	49.3	54.8	109.5	4737
13	39.5	30.5	1424	35.6	24.3	2050	35.0	24.1	1221	12	49.4	54.8	109.7	4695
14	39.8	30.1	1415	34.3	22.7	2016	33.5	22.5	1212	12	47.5	52.7	105.5	4643
15	36.0	28.6	1394	33.3	22.8	1970	32.7	22.6	1194	12	45.9	51.0	102.1	4558
16	36.6	29.5	1384	33.9	22.3	1950	33.3	22.1	1181	12	47.1	52.4	104.7	4515
17	46.8	37.8	1454	43.4	30.5	2030	42.9	30.4	1239	12	61.8	68.7	137.4	4723
18	55.3	44.5	1573	48.9	35.4	2174	48.4	35.1	1354	12	73.6	81.7	163.5	5101
19	50.0	40.5	1573	46.8	34.5	2159	46.0	34.3	1337	12	61.2	68.1	136.1	5069
20	48.7	38.3	1518	45.7	32.1	2092	44.9	31.9	1288	12	62.0	68.9	137.8	4898
21	45.2	35.8	1472	42.2	29.6	2050	41.6	29.5	1233	12	58.0	64.4	128.8	4755
22	41.2	31.9	1404	38.5	23.9	1992	38.1	23.7	1162	12	50.7	56.3	112.6	4558
23	35.9	31.3	1305	33.6	20.9	1865	33.9	20.4	1082	12	42.3	47.0	94.0	4252
24	35.5	27.8	1183	32.9	25.7	1734	33.0	25.6	973	12	40.0	44.5	88.9	3890

Table A.1.2 Ramp and Zone Limits for a Test System without Transmission Loss

ZONE	$P_{min}$	$P_{max}$	DR	UR
1	136	3200	65	76
2	154	3700	80	65
3	169	3000	90	75



## A.2 SYSTEM DATA WITH TRANSMISSION LOSS

Table A.2.1 shows the data for the test system with transmission loss included. The zonal generation and reloading limits for both systems are given in Table A.2.2

**Table A.2.1 Data for a Test System with Transmission Loss**

HOUR	ZONE 1			ZONE 2			ZONE 3			ENERGY PRICES				$P_{Load}$ (MW)
	RDP	RUP	P(MW)	RDP	RUP	P(MW)	RDP	RUP	P(MW)	MF	MCP	SRP	SMP	
1	37.2	29.4	1080	34.4	31.6	1606	34.6	31.9	1064	12	39.3	43.7	87.4	3446
2	31.3	28.9	1057	29.3	27.1	1650	29.8	27.1	962	12	34.5	38.4	76.7	3369
3	29.7	25.5	1040	27.8	25.7	1680	28.2	25.8	900	12	30.5	33.9	67.9	3322
4	32	23.6	1000	29.4	24.8	1586	28.5	25.1	992	12	30.6	34	68	3297
5	29.7	23.7	1100	27.4	23	1607	26.8	22.9	904	12	31.6	35.2	70.3	3317
6	30.7	24.2	1100	28.6	25.8	1577	28.8	25.8	1030	12	35.6	39.5	79	3408
7	34.9	29.7	1235	32.8	32.4	1686	33.3	32.8	986	12	46.5	51.7	103	3566
8	34.9	29.1	1206	32.9	29.9	1822	33	29.9	1180	12	48.3	53.6	107	3823
9	37	33.3	1272	34.5	27.4	2098	34	27.3	1282	12	47.9	53.2	106	4175
10	39.5	33.9	1586	36.8	30.1	2227	36.3	30	1251	12	50	55.6	111	4486
11	41.5	33.6	1510	38.8	29.9	2204	38.4	29.8	1570	12	49.9	55.4	111	4688
12	39.5	30.8	1604	36.9	29.4	2180	36.1	29.2	1563	12	49.3	54.8	110	4737
13	39.5	30.5	1794	35.6	24.3	2253	35	24.1	1289	12	49.4	54.8	110	4695
14	39.8	30.1	1673	34.3	22.7	2028	33.5	22.5	1519	12	47.5	52.7	105	4643
15	36	28.6	1730	33.3	22.8	2213	32.7	22.6	1220	12	45.9	51	102	4558
16	36.6	29.5	1443	33.9	22.3	2292	33.3	22.1	1356	12	47.1	52.4	105	4515
17	46.8	37.8	1823	43.4	30.5	2264	42.9	30.4	1288	12	61.8	68.7	137	4723
18	55.3	44.5	1608	48.9	35.4	2390	48.4	35.1	1815	12	73.6	81.7	163	5101
19	50	40.5	2001	46.8	34.5	2286	46	34.3	1517	12	61.2	68.1	136	5069



Table A.2.1 Continued

20	48.7	38.3	1612	45.7	32.1	2233	44.9	31.9	1700	12	62	68.9	138	4898
21	45.2	35.8	1602	42.2	29.6	2154	41.6	29.5	1607	12	58	64.4	129	4755
22	41.2	31.9	1542	38.5	23.9	2107	38.1	23.7	1471	12	50.7	56.3	113	4558
23	35.9	31.3	1502	33.6	20.9	1892	33.9	20.4	1337	12	42.3	47	94	4252
24	35.5	27.8	1202	32.9	25.7	1828	33	25.6	1254	12	40	44.5	88.9	3890

**Table A.2.2 Ramp and Zone Limits for a Test System without Transmission Loss**

<b>ZONE</b>	<b><math>P_{min}</math></b>	<b><math>P_{max}</math></b>	<b>DR</b>	<b>UR</b>
1	136	3200	65	76
2	154	3700	80	65
3	169	3000	90	75



# APPENDIX B: COMPUTER CODE FOR OPTIMIZATION

## B.1 COMPUTER CODE FOR SYSTEM WITHOUT TRANSMISSION LOSS

The computer code used in the implementation of the LSRA technique for a system without transmission loss was developed using the C++ program. There are three sections in the code and are given below:

### Minimize.cpp

```
#include <string>
#include <iostream>
#include <fstream>
#include <cmath>
#include "minimize.h"

minimize::minimize(const char *filename) {
    ifstream fin;
    fin.open(filename);
    if (! fin.is_open())
    {
        cout << "Could not open "<<filename<<". Quitting\n";
        exit(1);
    }
    char dummy;
    int fields=0;
    Hours=0;
    do {
```



```

        fin >> dummy;
        if (dummy=='') fields++;
    }
    while (dummy != 'm');
    cout << fields << endl;
    fin.seekg(0);
    sources.resize((fields-4)/3);

    for (int tmp=0;tmp<(fields-4)/3;tmp++) {
        sources[tmp]=new Generator;
        sources[tmp]->gen_num=tmp;
        fin >> sources[tmp]->Pmin>>dummy>>sources[tmp]-
>Pmax>>dummy>>dummy;
        if (dummy !='') cout <<"Error!! dummy != , \n";
    }
    while (dummy!='m') fin >> dummy;

    for (int tmp=0;tmp<(fields-4)/3;tmp++) {
        fin >> sources[tmp]->DR>>dummy>>sources[tmp]->UR>>dummy>>dummy;
        if (dummy !='') cout <<"Error!! dummy != , (2)\n";
    }

    while (dummy!='m') fin >> dummy;
    int size=24;
    for (int tmp=0;tmp<(fields-4)/3;tmp++)
    {
        sources[tmp]->P.resize(24);
        sources[tmp]->RDP.resize(24);
        sources[tmp]->RUP.resize(24);
    }
    MCP.resize(24);
    MF.resize(24);
    SRP.resize(24);
    SMP.resize(24);

```



```

while (! fin.eof())
{
    fin >> dummy;
    if (fin.eof()) continue;
    else fin.putback(dummy);
    if (Hours==size)
    {
        size+=24;
        MCP.resize(size);
        MF.resize(size);
        SRP.resize(size);
        SMP.resize(size);
        for (int tmp=0;tmp<(fields-4)/3;tmp++)
        {
            sources[tmp]->P.resize(size);
            sources[tmp]->RUP.resize(size);
            sources[tmp]->RDP.resize(size);
        }
    }
    for (int tmp=0;tmp<(fields-4)/3;tmp++)
        fin >> sources[tmp]->RDP[Hours]>>dummy>>sources[tmp]-
>RUP[Hours]>>dummy
        >>sources[tmp]->P[Hours]>>dummy;
    fin >>
MF[Hours]>>dummy>>MCP[Hours]>>dummy>>SRP[Hours]>>dummy>>SMP[Hours];
    while (dummy!='m') fin >> dummy;
    Hours++;
}
MF.resize(Hours);
MCP.resize(Hours);

```



```

SMP.resize(Hours);
SRP.resize(Hours);
for (int tmp=0;tmp<(fields-4)/3;tmp++)
{
    sources[tmp]->RDP.resize(Hours);
    sources[tmp]->RUP.resize(Hours);
    sources[tmp]->P.resize(Hours);
}
p[0]=2.1142160e-9;
p[1]=7.1611820e-9;
p[2]=2.9015942e-8;
p[3]=1.1047753e-7;
p[4]=3.9527343e-7;
p[5]=1.3289497e-6;
p[6]=4.1986320e-6;
p[7]=1.2465114e-5;
p[8]=3.4775612e-5;
p[9]=9.1168370e-5;
p[10]=2.2459773e-4;
p[11]=5.1994685e-4;
p[12]=1.1311122e-3;
p[13]=2.3123108e-3;
p[14]=4.4420267e-3;
p[15]=8.0188310e-3;
p[16]=1.3948602e-2;
p[17]=2.1339370e-2;
p[18]=3.2484443e-2;
p[19]=4.5728795e-2;
p[20]=6.0492436e-2;
p[21]=7.5198576e-2;
p[22]=8.7844705e-2;

```



```

    p[23]=9.6431542e-2;
    p[24]=9.9476450e-2;
    p[25]=9.6431542e-2;
    p[26]=8.7844705e-2;
    p[27]=7.5198576e-2;
    p[28]=6.0492436e-2;
    p[29]=4.5728795e-2;
    p[30]=3.2484443e-2;
    p[31]=2.1339370e-2;
    p[32]=1.3948602e-2;
    p[33]=8.0188310e-3;
    p[34]=4.4420267e-3;
    p[35]=2.3123108e-3;
    p[36]=1.1311122e-3;
    p[37]=5.1994685e-4;
    p[38]=2.2459773e-4;
    p[39]=9.1168370e-5;
    p[40]=3.4775612e-5;
    p[41]=1.2465114e-5;
    p[42]=4.1986320e-6;
    p[43]=1.3289497e-6;
    p[44]=3.9527343e-7;
    p[45]=1.1047753e-7;
    p[46]=2.9015942e-8;
    p[47]=7.1611820e-9;
    p[48]=2.1142160e-9;

}

vector<Generator*> *minimize::SRDP(int hour)

```



```

{
    vector<Generator*>*result=new vector<Generator*>;
    result->resize(sources.size());
    double RDPmin=sources[0]->RDP[hour];
    size_t pos_min=0,position=0;
    for (size_t tmp=1;tmp<sources.size();tmp++)
        if (RDPmin>sources[tmp]->RDP[hour])
        {
            RDPmin=sources[tmp]->RDP[hour];
            pos_min=tmp;
        }
    for (size_t tmp=pos_min;tmp<sources.size();tmp++)
        if (sources[tmp]->RDP[hour]==RDPmin)
            (*result)[position++] = sources[tmp];
    double RDPcur;
    do {
        for (size_t tmp=0;tmp<sources.size();tmp++)
            if (sources[tmp]->RDP[hour]>RDPmin)
            {
                RDPcur = sources[tmp]->RDP[hour];
                pos_min=tmp;
                tmp=sources.size();
            }
        for (size_t tmp=0;tmp<sources.size();tmp++)
            if ((sources[tmp]->RDP[hour]<RDPcur)&&(sources[tmp]-
>RDP[hour]>RDPmin)) {
                pos_min = tmp;
                RDPcur=sources[tmp]->RDP[hour];
            }
        for (size_t tmp=pos_min;tmp<sources.size();tmp++)
            if (sources[tmp]->RDP[hour]==RDPcur)

```



```

        (*result)[position++]=sources[tmp];
        RDPmin=RDPcur;
    }
    while (position < sources.size());
    return result;
}

vector<Generator*> *minimize::SRUP(int hour)
{
    vector<Generator*> *result=new vector<Generator*>;
    result->resize(sources.size());
    size_t position=0,pos_min=0;
    double RUPmin=sources[0]->RUP[hour];
    for (size_t tmp=1;tmp<sources.size();tmp++)
        if (RUPmin>sources[tmp]->RUP[hour]){
            RUPmin=sources[tmp]->RUP[hour];
            pos_min=tmp;
        }
    for (size_t tmp=pos_min;tmp<sources.size();tmp++)
        if (sources[tmp]->RUP[hour]==RUPmin)
            (*result)[position++]=sources[tmp];
    double RUPcur;
    do {
        for (size_t tmp=0;tmp<sources.size();tmp++)
            if (sources[tmp]->RUP[hour]>RUPmin){
                RUPcur=sources[tmp]->RUP[hour];
                pos_min=tmp;
                tmp=sources.size();
            }
        for (size_t tmp=pos_min+1;tmp<sources.size();tmp++)

```



```

        if ((sources[tmp]->RUP[hour]<RUPcur)&&(sources[tmp]-
>RUP[hour]>RUPmin)){
            pos_min=tmp;
            RUPcur=sources[tmp]->RUP[hour];
        }
        for (size_t tmp=pos_min;tmp<sources.size();tmp++)
            if (sources[tmp]->RUP[hour]==RUPcur)
                (*result)[position++]=sources[tmp];
        RUPmin=RUPcur;
    }
    while (position<sources.size());
    return result;
}

double minimize::cost(int hour, double beta)
{
    vector<Generator*> *SDRDP=SRDP(hour),*SDRUP=SRUP(hour);
    double sum=0;
    for (size_t tmp=0;tmp<sources.size();tmp++)
        sum+= sources[tmp]->P[hour];
    double TC=0;
    for (int s=1;s<50;s++)
    {
        double D,D_p,cost=0;
        D=sum*(1+(s-25)*0.25*sigma);
        if (s<25)
        {
            D_p=sum-D;
            for (size_t gen_num=0;gen_num<sources.size();gen_num++)
            {

```



```

        double P_avail=((*SDRDP)[gen_num]-
>DR<((*SDRDP)[gen_num]->P[hour]-(*SDRDP)[gen_num]->Pmin))?(*SDRDP)[gen_num]-
>DR:((*SDRDP)[gen_num]->P[hour]-(*SDRDP)[gen_num]->Pmin);
        if (P_avail < D_p)
        {
            D_p=P_avail;
            cost+=(*SDRDP)[gen_num]-
>RDP[hour]*P_avail+MCP[hour]*((*SDRDP)[gen_num]->P[hour]-P_avail);
        }
        else
        {
            cost +=(*SDRDP)[gen_num]-
>RDP[hour]*D_p+MCP[hour]*((*SDRDP)[gen_num]->P[hour]-D_p);
            D_p=0;
        }
    }
    cost +=MF[hour]*beta*sum;
}
else if (s==25) cost=MCP[hour]*sum+MF[hour]*beta*sum;
else
{
    D_p=D-sum;
    bool met=false, USR=false;
    double SR=0;
    size_t gen_num;
    for (gen_num=0;((gen_num<sources.size())&&(met==false));)
    {
        if
((SMP[hour]>SRP[hour])&&((SRP[hour])<((*SDRUP)[gen_num]-
>RUP[hour]+MCP[hour]))&&(USR==false))
        {

```



```

        if (D_p<=beta*sum)
        {
            SR=D_p;
            met=true;
            cost+=SRP[hour]*SR;
        }
        else
        {
            SR=beta*sum;
            D_p=SR;
            cost+=SRP[hour]*SR;
        }
        USR=true;
    }
    else if (SMP[hour]<((*SDRUP)[gen_num]-
>RUP[hour]+MCP[hour]))
    {
        met=true;
        cost+=SMP[hour]*D_p;
    }
    else
    {
        double P_avail=((*SDRUP)[gen_num]-
>UR<((*SDRUP)[gen_num]->Pmax-(*SDRUP)[gen_num]->P[hour]))?(*SDRUP)[gen_num]-
>UR:((*SDRUP)[gen_num]->Pmax-(*SDRUP)[gen_num]->P[hour]);
        if (P_avail>D_p)
        {
            met=true;
            cost += (*SDRUP)[gen_num]-
>RUP[hour]*D_p+MCP[hour]*(D_p+(*SDRUP)[gen_num]->P[hour]);
        }
    }

```



```

        else
        {
            D_p=P_avail;
            cost+=(*SDRUP)[gen_num]-
>RUP[hour]*P_avail+MCP[hour]*(P_avail+(*SDRUP)[gen_num]->P[hour]);
        }
        gen_num++;
    }
}
for (;gen_num<sources.size();gen_num++)
    cost+=MCP[hour]*(*SDRUP)[gen_num]->P[hour];
cost+=MF[hour]*beta*sum;
if (met==false)
    cost += SMP[hour]*D_p;
}
TC+=cost*p[s-1];
}

delete SDRDP;
delete SDRUP;
return TC;
}

```

### **Main.cpp**

```

#include <iostream>
#include <fstream>
#include <string>
#include "minimize.h"
using namespace std;
int main(){

```



```

string filename;
cout <<"Please enter filename: ";
cin >> filename;
minimize Gen_set(filename.c_str());
cout << "What!"<<endl;
cout <<"Please enter output filename: ";
cin >> filename;
ofstream fout;
fout.open(filename.c_str());
size_t i;
double beta;
double sum;
for(i=0;i<24;i++)
{
    cout<<i<<endl;
    sum=0;
    for (size_t tmp=0;tmp<Gen_set.sources.size();tmp++)
        sum+= Gen_set.sources[tmp]->P[i];
    double beta_best=0,cost_best=Gen_set.cost(i,0),cost;
    for(beta=0.0005;beta<1.001;beta+=0.0005)
    {
        cost=Gen_set.cost(i,beta);
        if (cost<cost_best){cost_best=cost; beta_best=beta;}
    }

    fout <<i+1<<","<< beta_best*sum << "," << cost_best<<endl;

}
fout.close();

return 0;

```



```
}
```

### **Minimize.h**

```
#include <vector>
```

```
#include <iostream>
```

```
#include <fstream>
```

```
using namespace std;
```

```
class Generator{
```

```
public:
```

```
    vector<double> RDP,RUP;
```

```
    int gen_num;
```

```
    double Pmin,Pmax,UR,DR;
```

```
    vector<double> P;
```

```
};
```

```
class minimize {
```

```
public:
```

```
    minimize(const char*);
```

```
    double cost(int, double);
```

```
    vector<Generator*> sources;
```

```
private:
```

```
    vector<Generator*> *SRDP(int), *SRUP(int);
```

```
    vector<double> MCP,SRP,MF,SMP,Pload,*beta_list(int);
```

```
    int Hours;
```

```
    double p[49];
```

```
};
```

```
#define sigma 0.07
```



The LSRA technique was implemented on the system with transmission loss using the following code in C++:

### **Minimize.cpp**

```
#include <string>
#include <iostream>
#include <fstream>
#include <cmath>
#include "minimize.h"
#define Ploss
(0.00006280*pow(curr[0],2)+0.00003989*pow(curr[1],2)+0.00005519*pow(curr[2],2)+0.00005
103*pow(curr[3],2)+0.00006181*pow(curr[4],2)+0.000076*curr[0]*curr[1]+0.000081*curr[0]*
curr[2] +0.000084*curr[0]*curr[3]+0.000099*curr[0]*curr[4]+0.000077*curr[1]*curr[2]
+0.000074*curr[1]*curr[3]+0.00006*curr[1]*curr[4]+0.000087*curr[2]*curr[3]
+0.000063*curr[2]*curr[4]+0.000068*curr[3]*curr[4]-0.1155*curr[0]-0.1208*curr[1]-
0.1421*curr[2]
-0.1213*curr[3]-0.0943*curr[4] +153.8)
#define cost_P1 ((sources[0]->RUP[hour]+MCP[hour])/(1.1155-0.000126*curr[0]-
0.000076*curr[1]
-0.000081*curr[2]-0.000084*curr[3]-0.000099*curr[4]))
#define cost_P2 ((sources[1]->RUP[hour]+MCP[hour])/(1.1208-0.000076*curr[0]-
0.00008*curr[1]
-0.000077*curr[2]-0.000074*curr[3]-0.00006*curr[4]))
#define cost_P3 ((sources[2]->RUP[hour]+MCP[hour])/(1.1421-0.00008*curr[0]-
0.000077*curr[1]
-0.00011*curr[2]-0.000087*curr[3]-0.000063*curr[4]))
#define cost_SM ((SMP[hour])/(1.1213-0.000084*curr[0]-0.000074*curr[1]
-0.000087*curr[2]-0.000102*curr[3]-0.000068*curr[4]))
```



```

#define cost_SR (SRP[hour]/(1.0943-0.000099*curr[0]-0.00006*curr[1]
-0.000063*curr[2]-0.000068*curr[3]-0.000124*curr[4]))
#define cost_P1d ((sources[0]->RDP[hour]-MCP[hour])/(1.1155-0.000126*curr[0]-
0.000076*curr[1]
-0.000081*curr[2]-0.000084*curr[3]-0.000099*curr[4]))
#define cost_P2d ((sources[1]->RDP[hour]-MCP[hour])/(1.1208-0.000076*curr[0]-
0.00008*curr[1]
-0.000077*curr[2]-0.000074*curr[3]-0.00006*curr[4]))
#define cost_P3d ((sources[2]->RDP[hour]-MCP[hour])/ (1.1421-0.00008*curr[0]-
0.000077*curr[1]
-0.00011*curr[2]-0.000087*curr[3]-0.000063*curr[4]))
#define err 1e-3

minimize::minimize(const char *filename) {
    ifstream fin;
    fin.open(filename);
    if (! fin.is_open())
    {
        cout << "Could not open " << filename << ". Quitting\n";
        exit(1);
    }
    char dummy;
    int fields=0;
    Hours=0;
    do {
        fin >> dummy;
        if (dummy=='.') fields++;
    }
    while (dummy != 'm');
    cout << fields << endl;
    fin.seekg(0);

```



```

sources.resize((fields-5)/3);
    for (int tmp=0;tmp<(fields-5)/3;tmp++) {
        sources[tmp]=new Generator;
        sources[tmp]->gen_num=tmp;
        fin >> sources[tmp]->Pmin>>dummy>>sources[tmp]-
>Pmax>>dummy>>dummy;
        if (dummy !=',') cout <<"Error!! dummy != , \n";
    }
    while (dummy!='m') fin >> dummy;
    for (int tmp=0;tmp<(fields-5)/3;tmp++) {
        fin >> sources[tmp]->DR>>dummy>>sources[tmp]->UR>>dummy>>dummy;
        if (dummy !=',') cout <<"Error!! dummy != , (2)\n";
    }
    while (dummy!='m') fin >> dummy;
    int size=24;
    for (int tmp=0;tmp<(fields-5)/3;tmp++)
    {
        sources[tmp]->P.resize(24);
        sources[tmp]->RDP.resize(24);
        sources[tmp]->RUP.resize(24);
    }
    MCP.resize(24);
    MF.resize(24);
    SRP.resize(24);
    SMP.resize(24);
    Pload.resize(24);

    while (! fin.eof())
    {
        fin >> dummy;
        if (fin.eof()) continue;

```



```

else fin.putback(dummy);
if (Hours==size)
{
    size+=24;
    MCP.resize(size);
    MF.resize(size);
    SRP.resize(size);
    SMP.resize(size);
    Pload.resize(size);
    for (int tmp=0;tmp<(fields-5)/3;tmp++)
    {
        sources[tmp]->P.resize(size);
        sources[tmp]->RUP.resize(size);
        sources[tmp]->RDP.resize(size);
    }
}
for (int tmp=0;tmp<(fields-5)/3;tmp++)
    fin >> sources[tmp]->RDP[Hours]>>dummy>>sources[tmp]-
>RUP[Hours]>>dummy
    >>sources[tmp]->P[Hours]>>dummy;
    fin >>
MF[Hours]>>dummy>>MCP[Hours]>>dummy>>SRP[Hours]>>dummy>>SMP[Hours]>>dum
my>>Pload[Hours];
    while (dummy!='m') fin >> dummy;
    Hours++;

}
MF.resize(Hours);
MCP.resize(Hours);
SMP.resize(Hours);
SRP.resize(Hours);

```



```

Pload.resize(Hours);
for (int tmp=0;tmp<(fields-5)/3;tmp++)
{
    sources[tmp]->RDP.resize(Hours);
    sources[tmp]->RUP.resize(Hours);
    sources[tmp]->P.resize(Hours);
}
p[0]=2.1142160e-9;
p[1]=7.1611820e-9;
p[2]=2.9015942e-8;
p[3]=1.1047753e-7;
p[4]=3.9527343e-7;
p[5]=1.3289497e-6;
p[6]=4.1986320e-6;
p[7]=1.2465114e-5;
p[8]=3.4775612e-5;
p[9]=9.1168370e-5;
p[10]=2.2459773e-4;
p[11]=5.1994685e-4;
p[12]=1.1311122e-3;
p[13]=2.3123108e-3;
p[14]=4.4420267e-3;
p[15]=8.0188310e-3;
p[16]=1.3948602e-2;
p[17]=2.1339370e-2;
p[18]=3.2484443e-2;
p[19]=4.5728795e-2;
p[20]=6.0492436e-2;
p[21]=7.5198576e-2;
p[22]=8.7844705e-2;
p[23]=9.6431542e-2;

```



```

    p[24]=9.9476450e-2;
    p[25]=9.6431542e-2;
    p[26]=8.7844705e-2;
    p[27]=7.5198576e-2;
    p[28]=6.0492436e-2;
    p[29]=4.5728795e-2;
    p[30]=3.2484443e-2;
    p[31]=2.1339370e-2;
    p[32]=1.3948602e-2;
    p[33]=8.0188310e-3;
    p[34]=4.4420267e-3;
    p[35]=2.3123108e-3;
    p[36]=1.1311122e-3;
    p[37]=5.1994685e-4;
    p[38]=2.2459773e-4;
    p[39]=9.1168370e-5;
    p[40]=3.4775612e-5;
    p[41]=1.2465114e-5;
    p[42]=4.1986320e-6;
    p[43]=1.3289497e-6;
    p[44]=3.9527343e-7;
    p[45]=1.1047753e-7;
    p[46]=2.9015942e-8;
    p[47]=7.1611820e-9;
    p[48]=2.1142160e-9;

}

double minimize::cost(int hour, double beta)
{

```



```

double sum=0;
for (size_t tmp=0;tmp<sources.size();tmp++)
    sum+= sources[tmp]->P[hour];
double TC=0;
for (int s=1;s<50;s++)
{
    vector<double> curr;
    curr.resize(5);
    curr[0]=sources[0]->P[hour];
    curr[1]=sources[1]->P[hour];
    curr[2]=sources[2]->P[hour];
    curr[3]=0;
    curr[4]=0;
    double altcheap,altstep;
    size_t altsource;
    double D,D_p,cost=0;
    D=Pload[hour]*(1+(s-25)*0.25*sigma);
    if (s<25)
    {
        D_p=sum-D-Ploss;
        int last=0,cheapest_source;
        double last_step=32,step=32,cheapest,y=0,SR=0;
        while(fabs(D_p)>err)
        {
            if (D_p < 0)
            {
                curr[last]+=last_step;
                step/=2;
            }
            else
            {

```



```

        cheapest_source=3;
        altcheap=fabs(cost_P1d)+fabs(cost_P2d)+fabs(cost_P3d);
        altsource=3;
        cheapest=fabs(cost_P1d)+fabs(cost_P2d)+fabs(cost_P3d);
if ((cost_P1d<cheapest)&&(curr[0]-sources[0]->Pmin>step)&&((sources[0]->P[hour]-
curr[0])+step<=sources[0]->DR))
    {
        cheapest_source=0;
        cheapest=cost_P1d;
    }
else if ((cost_P1d<altcheap)&&(curr[0]-sources[0]->Pmin>err)&&(sources[0]->P[hour]-curr[0]-
sources[0]->DR<=0))
    {
        altsource=0;
        altcheap=cost_P1d;
        if ((curr[0]-sources[0]->Pmin)<(-sources[0]->P[hour]+curr[0]+sources[0]->DR))
            altstep=(curr[0]-sources[0]->Pmin);
        else altstep=(-sources[0]->P[hour]+curr[0]+sources[0]-
>DR);
    }
if ((cost_P2d<cheapest)&&(curr[1]-sources[1]->Pmin>step)&&((sources[1]->P[hour]-
curr[1])+step<=sources[1]->DR))
    {
        cheapest_source=1;
        cheapest=cost_P2d;
    }
else if ((cost_P2d<altcheap)&&(curr[1]-sources[1]->Pmin>err)&&(sources[1]->P[hour]-curr[1]-
sources[1]->DR<=0))
    {
        altsource=1;
        altcheap=cost_P1d;

```



```

        if ((curr[1]-sources[1]->Pmin)<(-sources[1]->P[hour]+curr[1]+sources[1]->DR))
            altstep=(curr[1]-sources[1]->Pmin);
        else altstep=(-sources[1]->P[hour]+curr[1]+sources[1]-
>DR);
    }
    if ((cost_P3d<cheapest)&&(curr[2]-sources[2]->Pmin>step)&&((sources[2]->P[hour]-
curr[2])+step<=sources[2]->DR))
    {
        cheapest_source=2;
        cheapest=cost_P3d;
    }
    else if ((cost_P3d<altcheap)&&(curr[2]-sources[2]->Pmin>err)&&(sources[2]->P[hour]-curr[2]-
sources[2]->DR<=0))
    {
        altsource=2;
        altcheap=cost_P1d;
        if ((curr[2]-sources[2]->Pmin)<(-sources[2]->P[hour]+curr[2]+sources[2]->DR))
            altstep=(curr[2]-sources[2]->Pmin);
        else altstep=(-sources[2]->P[hour]+curr[2]+sources[2]-
>DR);
    }
    if
((altcheap==fabs(cost_P1d)+fabs(cost_P2d)+fabs(cost_P3d))||((cheapest<altcheap)||((altstep<err))
        if(cheapest_source<3)
        {
            curr[cheapest_source]-=step;
            last=cheapest_source;
            last_step=step;
        }
        else
        {

```



```

D_p=(curr[0]+curr[1]+curr[2])-D-Ploss;
if (D_p>0) step/=2;
}
else
{
double altstepmin=0;
double altstepmax=altstep;
curr[altsource]-=altstep;
D_p=(curr[0]+curr[1]+curr[2])-D-Ploss;
if (D_p<0)
{
curr[altsource]+=altstep;
altstep/=2;
do
{
curr[altsource]-=altstep;
D_p=(curr[0]+curr[1]+curr[2])-D-
Ploss;

if (D_p<0)
{

curr[altsource]+=altstep;
altstepmax=altstep;
altstep=(altstepmin+altstepmax)/2;
}

else if (D_p>err)
{

curr[altsource]+=altstep;
altstepmin=altstep;
altstep=(altstepmin+altstepmax)/2;

```



```

        }
        else curr[altsource]+=altstep;
    }
    while (fabs(D_p)>err);
    curr[altsource]-=altstep;
}

}

}
D_p=(curr[0]+curr[1]+curr[2])-D-Ploss;
if (step<err) break;
}

cost=MCP[hour]*(curr[0]+curr[1]+curr[2])-(sources[0]->RDP[hour])*(curr[0]-sources[0]-
>P[hour])-
(sources[2]->RDP[hour])*(curr[2]-sources[2]->P[hour])-(sources[1]->RDP[hour])*(curr[1]-
sources[1]->P[hour])+ MF[hour]*beta*sum;

}
else if (s==25) cost=MCP[hour]*sum+MF[hour]*beta*sum;
else
{
    D_p=D-sum+Ploss;
    bool USR=false;
    int last=0,cheapest_source;
    double last_step=32,step=32,cheapest,y=0,SR=0;
    size_t altsource;
    double altcheap,altstep;
    while(fabs(D_p)>err)

```



```

{
    if (D_p < 0)
    {
        curr[last]-=last_step;
        D_p=D-(curr[0]+curr[1]+curr[2])-y-SR+Ploss;
        step/=2;
    }
    else
    {
        cheapest_source=3;
cheapest=cost_SM;
        altsource=3;
        altcheap=cost_SM;
        if ((!USR)&&(cost_SR<cost_SM)&&(beta*sum-
curr[4]>=step))
        {
            cheapest_source=4;
            cheapest=cost_SR;
        }
        else if ((!USR)&&(cost_SR<cost_SM)&&(beta*sum-
curr[4]>=err))
        {
            altcheap=cost_SR;
            altsource=4;
            altstep=beta*sum;
            altstep=beta*sum-curr[4];
        }
    }
    if ((cost_P1<cheapest)&&(sources[0]->Pmax-curr[0]>step)&&((curr[0]-sources[0]-
>P[hour]+step)<=sources[0]->UR))
    {
        cheapest_source=0;
    }
}

```



```

        cheapest=cost_P1;
    }
else if ((cost_P1<altcheap)&&(fabs(sources[0]->Pmax-curr[0])>err)&&(curr[0]-sources[0]-
>P[hour]-sources[0]->UR<0))
    {
        altcheap=cost_P1;
        altsource=0;
        if ((sources[0]->Pmax-curr[0])<(-curr[0]+sources[0]->P[hour]+sources[0]->UR))
            altstep=sources[0]->Pmax-curr[0];
        else altstep=-curr[0]+sources[0]->P[hour]+sources[0]->UR;
    }
if ((cost_P2<cheapest)&&(sources[1]->Pmax-curr[1]>step)&&((curr[1]-sources[1]-
>P[hour]+step) <=sources[1]->UR))
    {
        cheapest_source=1;
        cheapest=cost_P2;
    }
else if ((cost_P2<altcheap)&&(fabs(sources[1]->Pmax-
curr[1])>err)&&(curr[1]-sources[1]->P[hour]-sources[1]->UR<0))
    {
        altcheap=cost_P2;
        altsource=1;
        if ((sources[1]->Pmax-curr[1])<(-curr[1]+sources[1]->P[hour]+sources[1]->UR))
            altstep=sources[1]->Pmax-curr[1];
        else altstep=-curr[1]+sources[1]->P[hour]+sources[1]-
>UR;
    }
if ((cost_P3<cheapest)&&(sources[2]->Pmax-curr[2]>step)&&((curr[2]-sources[2]-
>P[hour]+step)<=sources[2]->UR))
    {
        cheapest_source=2;

```



```

        cheapest=cost_P3;
    }
else if ((cost_P3<altcheap)&&(fabs(sources[2]->Pmax-curr[2])>err)&&(curr[2]-sources[2]-
>P[hour]-sources[2]->UR<0))
    {
        altcheap=cost_P3;
        altsource=2;
        if ((sources[2]->Pmax-curr[2])<(-curr[2]+sources[2]->P[hour]+sources[2]->UR))
            altstep=sources[2]->Pmax-curr[2];
        else altstep=-curr[2]+sources[2]->P[hour]+sources[2]-
>UR;
    }
    if (cost_SM<cheapest)
    {
        cheapest_source=3;
        cheapest=cost_SM;
    }
    if ((altsource==3)||((cheapest<altcheap)||((altstep<err))
        if(cheapest_source<4)
        {
            curr[cheapest_source]+=step;
            D_p=D-(curr[0]+curr[1]+curr[2])-curr[3]-
curr[4]+Ploss;

            last=cheapest_source;
            last_step=step;
        }
        else
        {
            curr[4]+=step;
            D_p=D-(curr[0]+curr[1]+curr[2])-curr[3]-
curr[4]+Ploss;

```



```

        if (curr[4]>=beta*sum) USR=true;
        last=cheapest_source;
        last_step=step;
    }
else
{
    double altstepmin=0;
    double altstepmax=altstep;
    curr[altsource]+=altstep;
    D_p=D-(curr[0]+curr[1]+curr[2])-curr[3]-
curr[4]+Ploss;

    if (D_p<0)
    {
        curr[altsource]-=altstep;
        altstep/=2;
        do
        {
            curr[altsource]+=altstep;
            D_p=D-(curr[0]+curr[1]+curr[2])-
curr[3]-curr[4]+Ploss;

            if (D_p<0)
            {
                curr[altsource]-=altstep;

altstepmax=altstep;

altstep=(altstepmin+altstepmax)/2;

D_p=D-(curr[0]+curr[1]+curr[2])-curr[3]-curr[4]+Ploss;
            }
        } while (D_p>err)
    }
}

```



```

        {
            curr[altsource]-=altstep;
            altstepmin=altstep;
            altstep=(altstepmin+altstepmax)/2;

        }
        else curr[altsource]-=altstep;
    }
    while (fabs(D_p)>err);
    curr[altsource]+=altstep;
}
else if (altsource==4) USR=true;
}

}

}

cost=MCP[hour]*(curr[0]+curr[1]+curr[2])+sources[0]-
>RUP[hour]*(curr[0]-sources[0]->P[hour])+
sources[2]->RUP[hour]*(curr[2]-sources[2]->P[hour])+sources[1]-
>RUP[hour]*(curr[1]-sources[1]->P[hour])+
SMP[hour]*curr[3]+curr[4]*SRP[hour]+MF[hour]*beta*sum;

}
TC+=cost*p[s-1];
}

return TC;

}

```



## Main.cpp

```
#include <iostream>
#include <fstream>
#include <string>
#include "minimize.h"
using namespace std;
int main(){
    string filename;
    cout << "Please enter filename: ";
    cin >> filename;
    minimize Gen_set(filename.c_str());
    cout << "What!" << endl;
    cout << "Please enter output filename: ";
    cin >> filename;
    ofstream fout;
    fout.open(filename.c_str());
    size_t i;
    double beta;
    double sum;
    for(i=0; i<24; i++)
    {
        cout << i << endl;
        sum=0;
        for (size_t tmp=0; tmp<Gen_set.sources.size(); tmp++)
            sum+= Gen_set.sources[tmp]->P[i];
        double beta_best=0, cost_best=Gen_set.cost(i,0), cost;
        for(beta=0.0005; beta<1.001; beta+=0.0005)
        {
            cost=Gen_set.cost(i,beta);
            if (cost<cost_best){cost_best=cost; beta_best=beta;}
```



```

    }

    fout << i+1 << ", " << beta_best*sum << ", " << cost_best << endl;

}

fout.close();

return 0;
}

```

### **Minimize.h**

```

#include <vector>
#include <iostream>
#include <fstream>

using namespace std;

class Generator{
public:
    vector<double> RDP,RUP;
    int gen_num;
    double Pmin,Pmax,UR,DR;
    vector<double> P;
};

class minimize {
public:
    minimize(const char*);
    double cost(int, double);
    vector<Generator*> sources;
private:

```



```
vector<Generator*> *SRDP(int), *SRUP(int);  
vector<double> MCP,SRP,MF,SMP,Pload,*beta_list(int);  
int Hours;  
double p[49];  
  
};  
#define sigma 0.07
```



## APPENDIX C: RESULTS

### C.1 VARIATION OF SRP AND SMP ON THE SR REQUIREMENTS FOR A 3% LFU WITHOUT TRANSMISSION LOSS

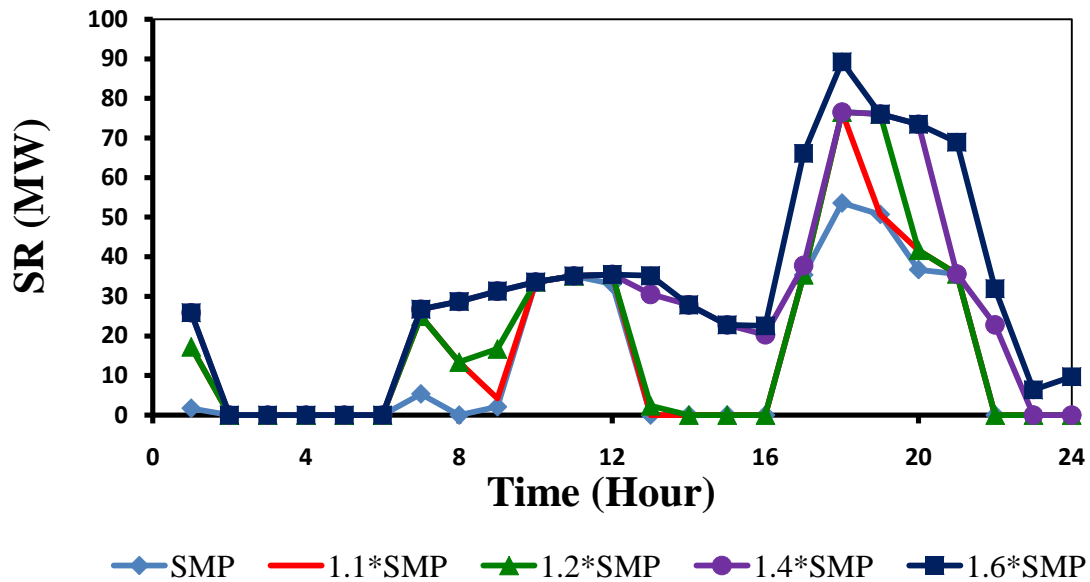


Fig. C.1.1 LSRA at fixed SRP and varying SMP for 3% LFU without Transmission Loss

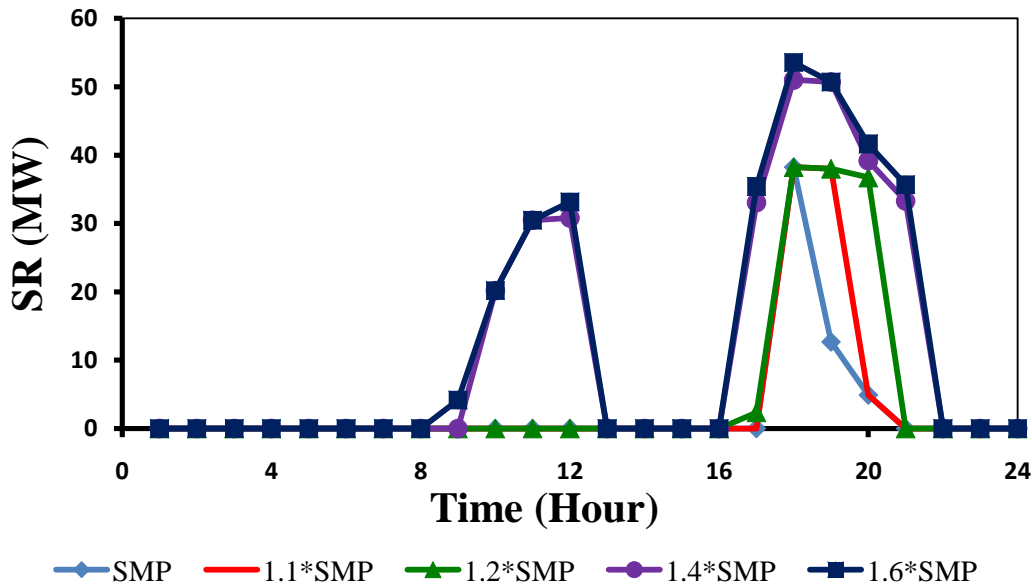
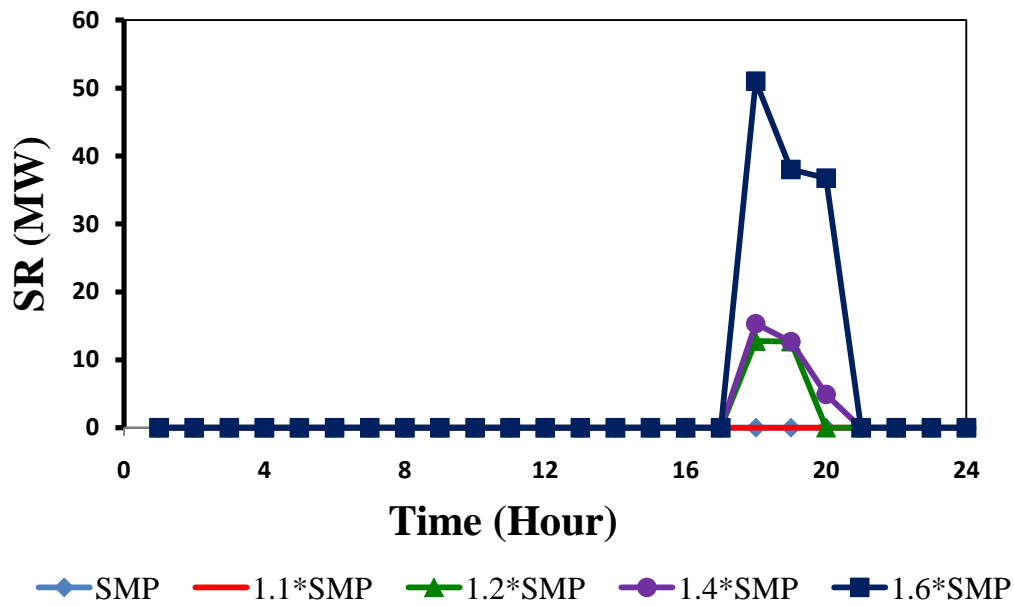
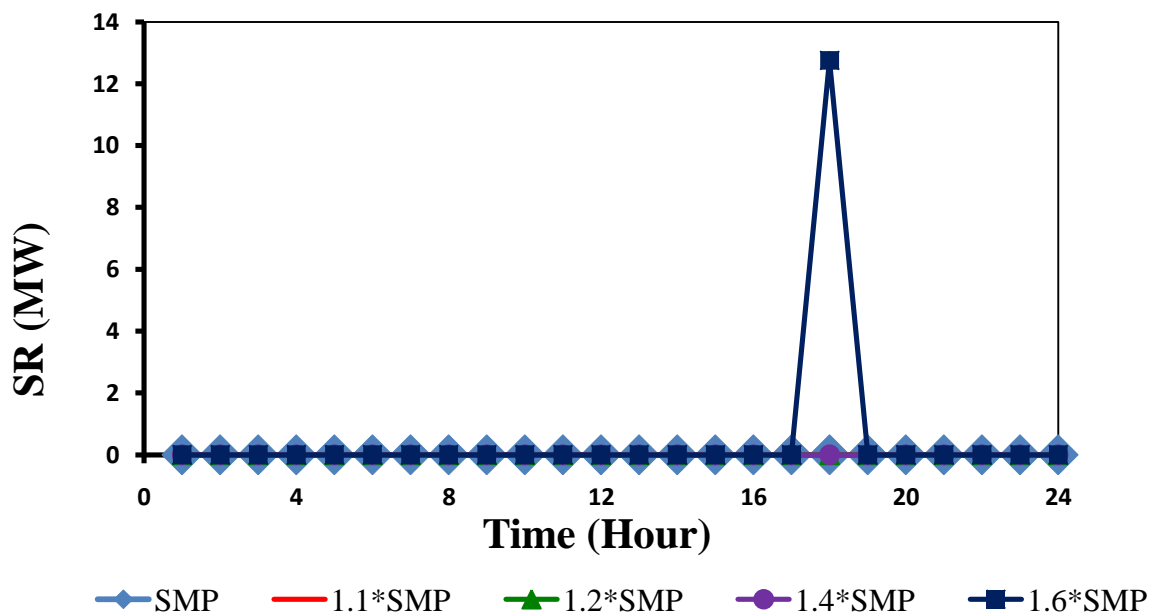


Fig. C.1.2 LSRA at fixed 1.1\*SRP and varying SMP for 3% LFU without Transmission Loss



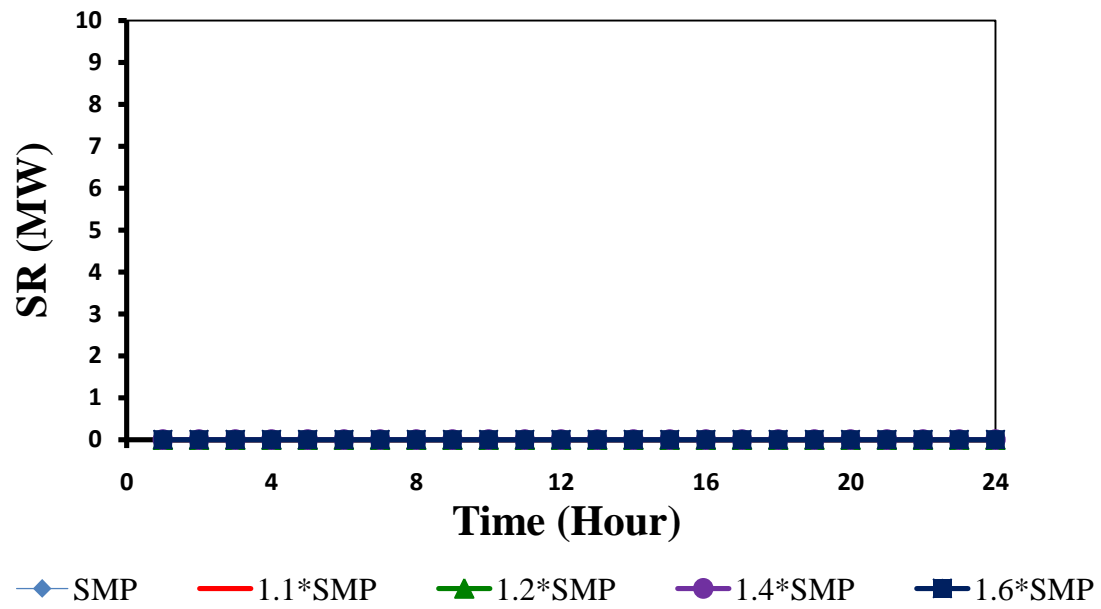


**Fig. C.1.3 LSRA at fixed 1.2\*SRP and varying SMP for 3% LFU without Transmission Loss**



**Fig. C.1.4 LSRA at fixed 1.4\*SRP and varying SMP for 3% LFU without Transmission Loss**





**Fig. C.1.5 LSRA at fixed 1.6\*SRP and varying SMP for 3% LFU without Transmission Loss**



## C.2 VARIATION OF SRP AND SMP ON THE SR REQUIREMENTS FOR A 3% LFU WITH TRANSMISSION LOSS

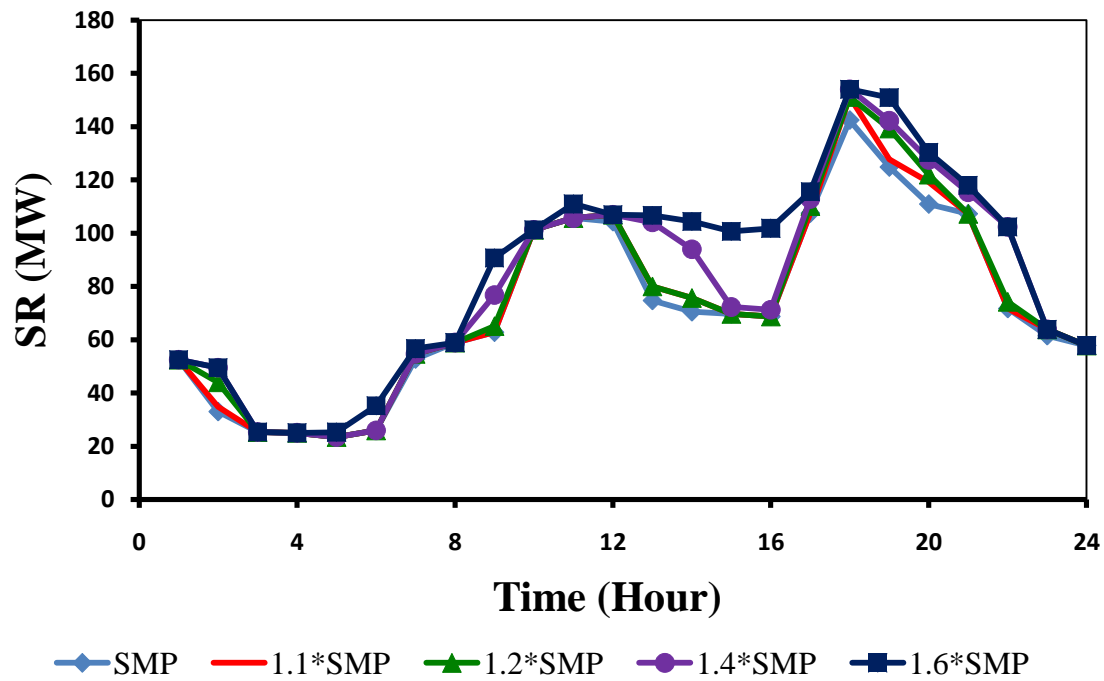


Fig. C.2.1 LSRA at fixed SRP and varying SMP for 3% LFU with Transmission Loss

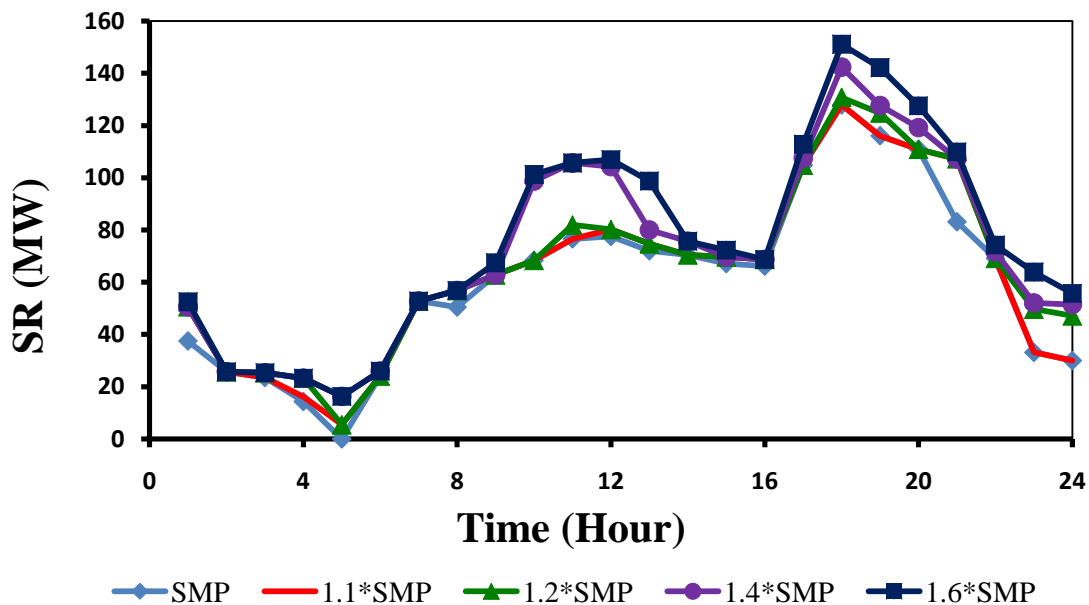
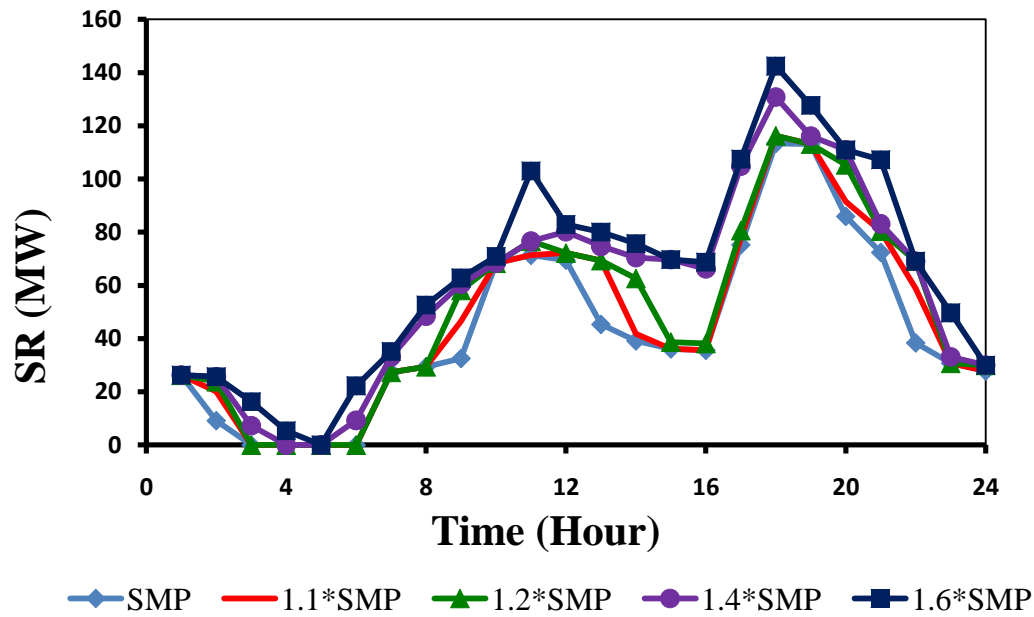
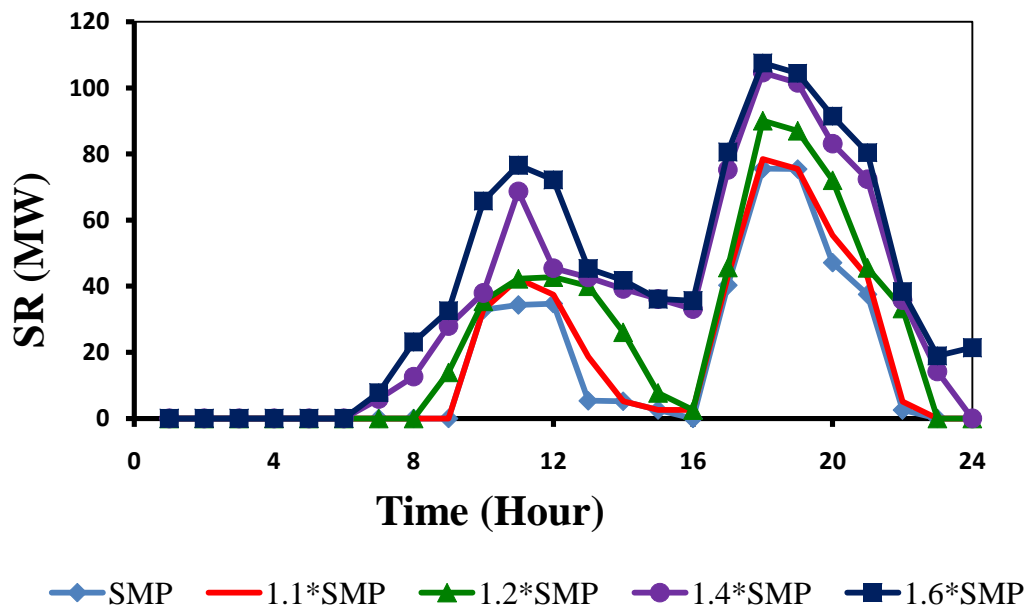


Fig. C.2.2 LSRA at fixed 1.1\*SRP and varying SMP for 3% LFU with Transmission Loss



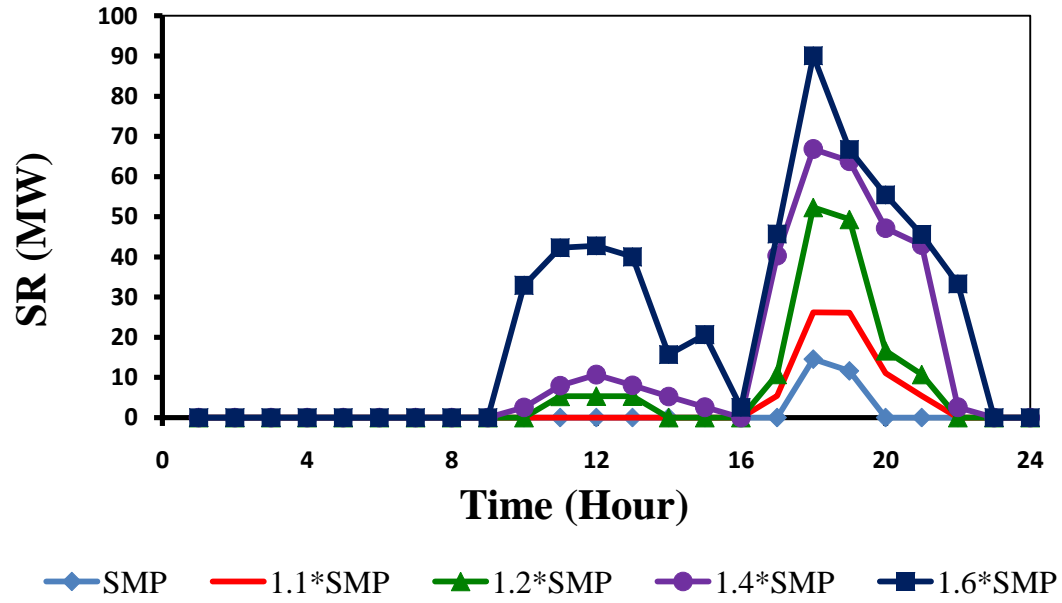


**Fig. C.2.3** LSRA at fixed 1.2\*SRP and varying SMP for 3% LFU with Transmission Loss



**Fig. C.2.4** LSRA at fixed 1.4\*SRP and varying SMP for 3% LFU with Transmission Loss

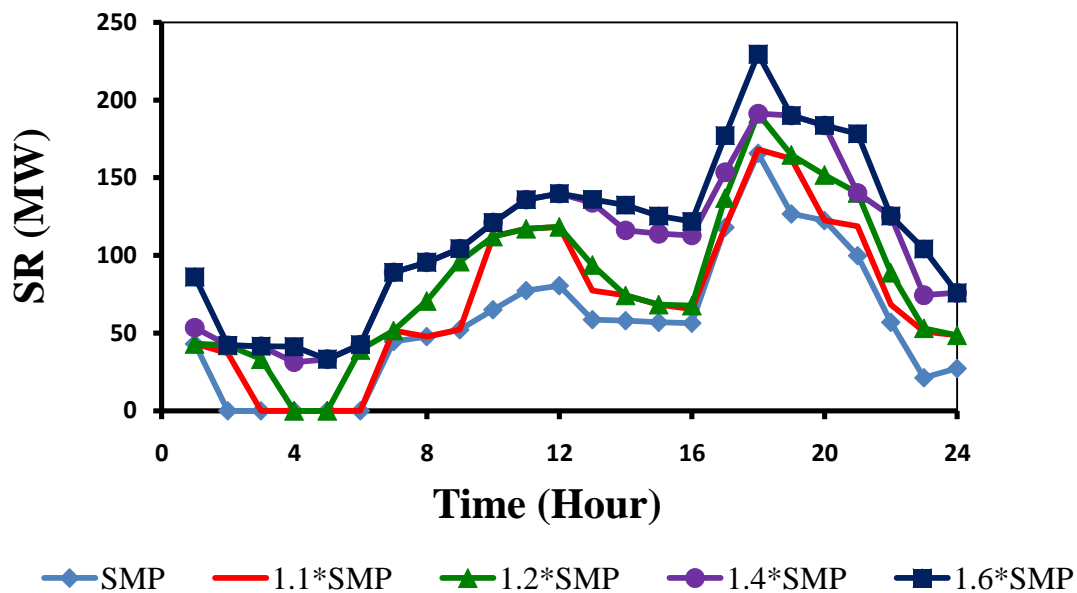




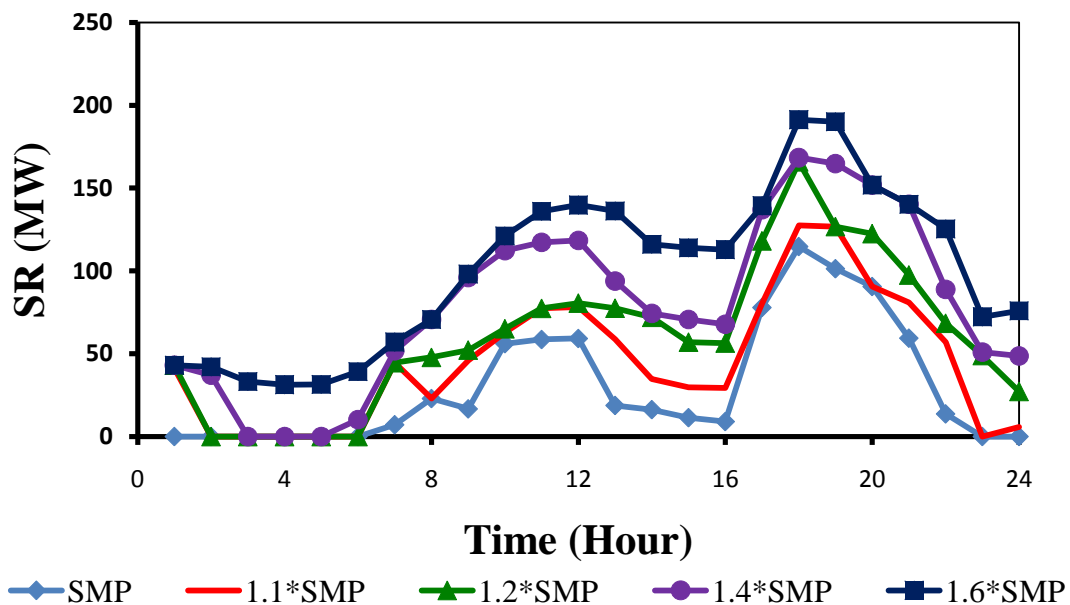
**Fig. C.2.5** LSRA at fixed 1.6\*SRP and varying SMP for 3% LFU with Transmission Loss



### C.3 VARIATION OF SRP AND SMP ON THE SR REQUIREMENTS FOR A 5% LFU WITHOUT TRANSMISSION LOSS

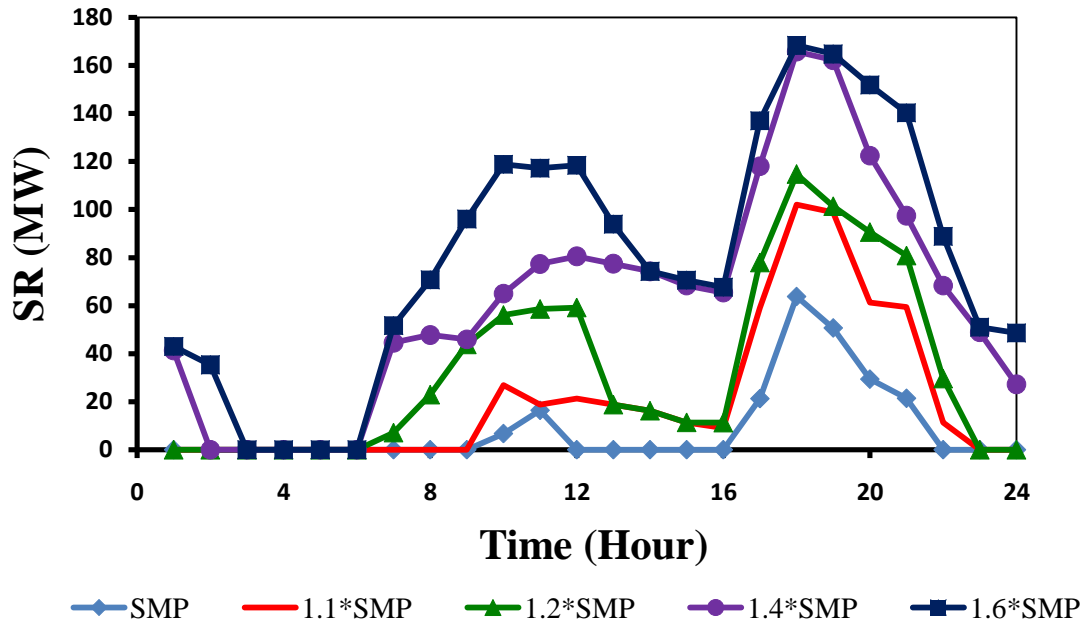


**Fig. C.3.1 LSRA at fixed SRP and varying SMP for 5% LFU without Transmission Loss**

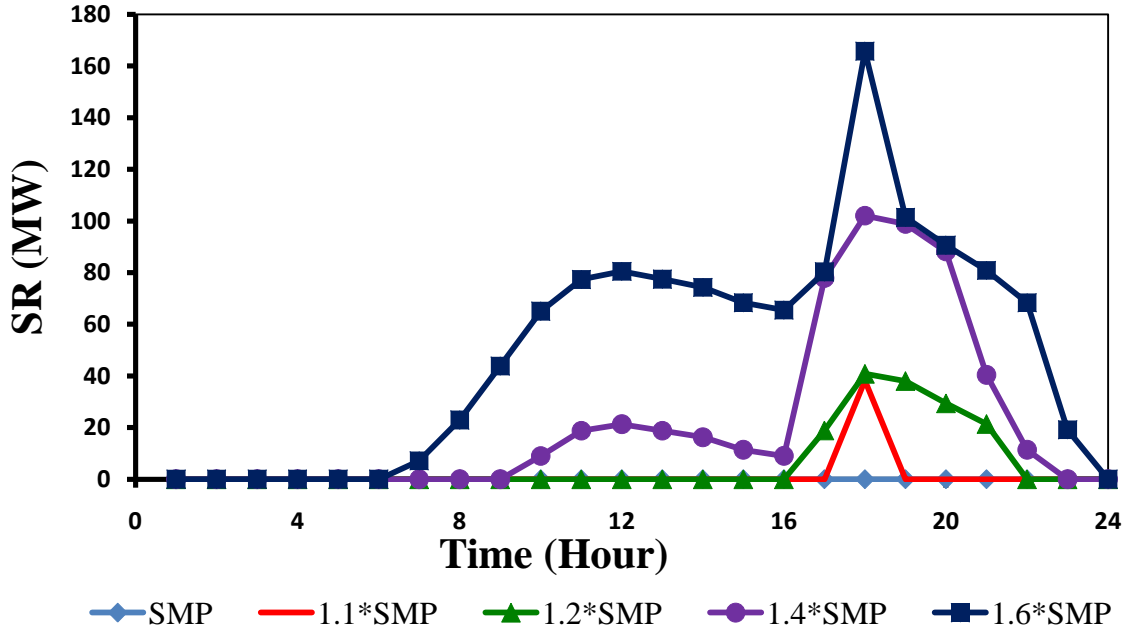


**Fig. C.3.2 LSRA at fixed 1.1\*SRP and varying SMP for 5% LFU without Transmission Loss**



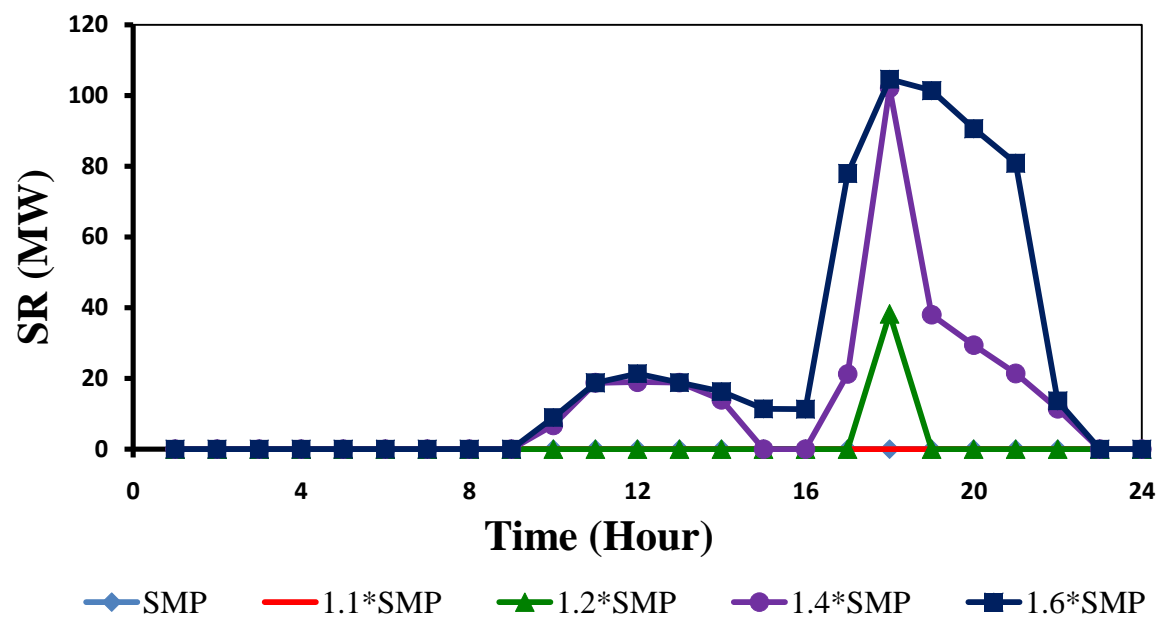


**Fig. C.3.3** LSRA at fixed 1.2\*SRP and varying SMP for 5% LFU without Transmission Loss



**Fig. C.3.4** LSRA at fixed 1.4\*SRP and varying SMP for 5% LFU without Transmission Loss





**Fig. C.3.5 LSRA at fixed 1.6\*SRP and varying SMP for 5% LFU without Transmission Loss**



C.4 VARIATION OF SRP AND SMP ON THE SR REQUIREMENTS FOR A 5% LFU  
WITH TRANSMISSION LOSS

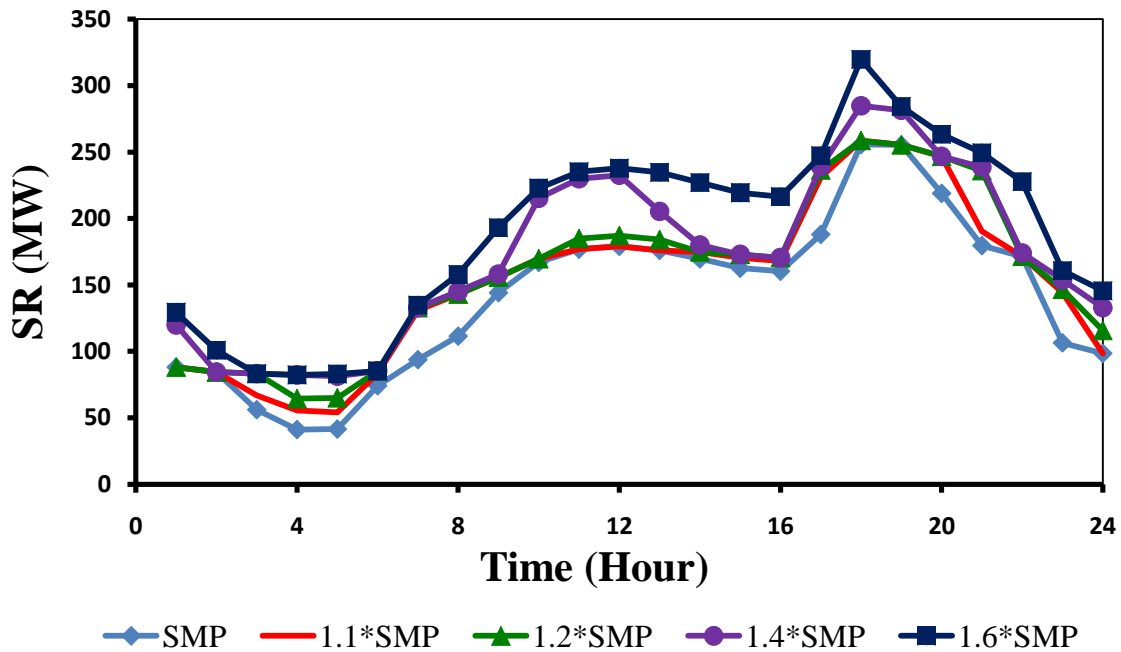


Fig. C.4.1 LSRA at fixed SRP and varying SMP for 5% LFU with Transmission Loss

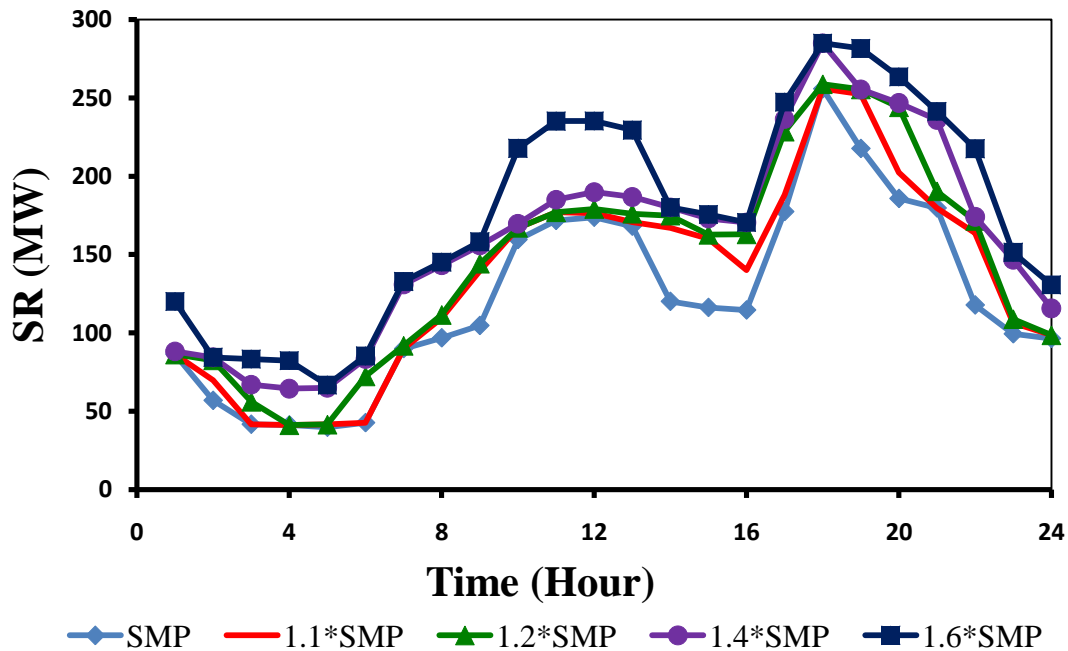
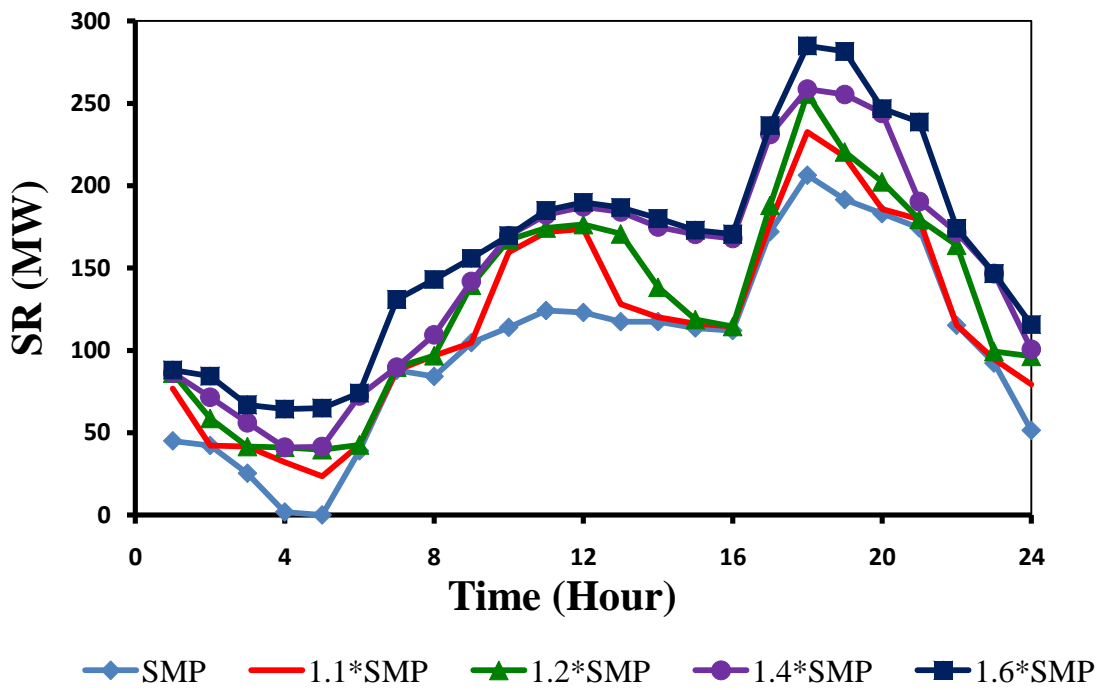
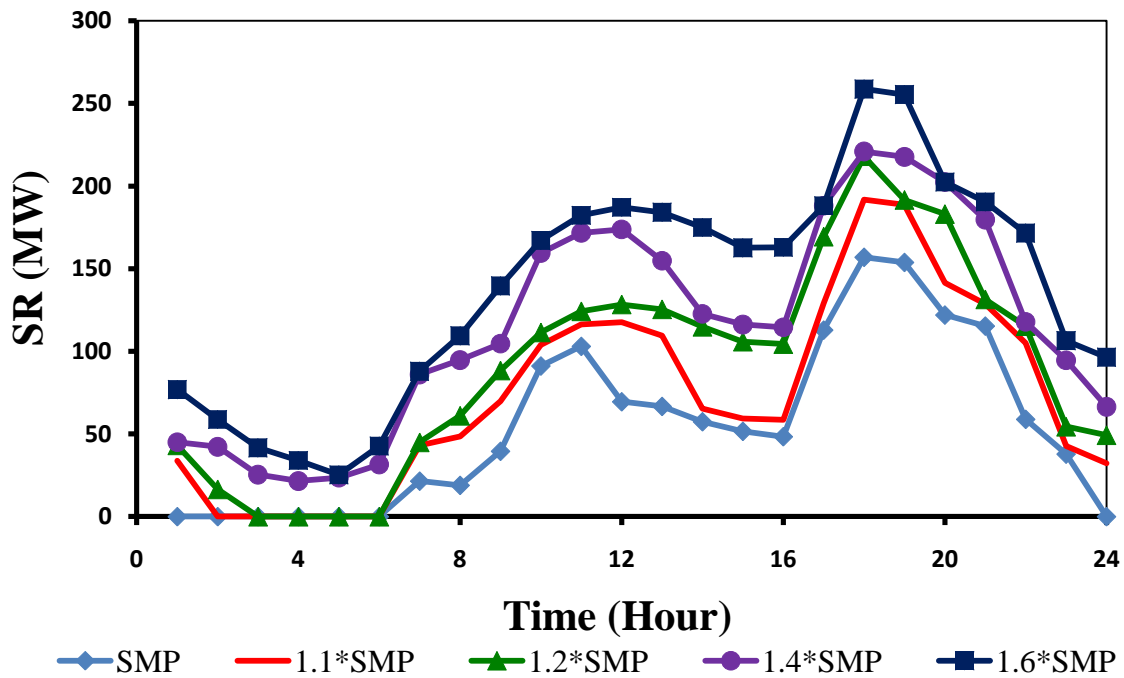


Fig. C.4.2 LSRA at fixed 1.1\*SRP and varying SMP for 5% LFU with Transmission Loss



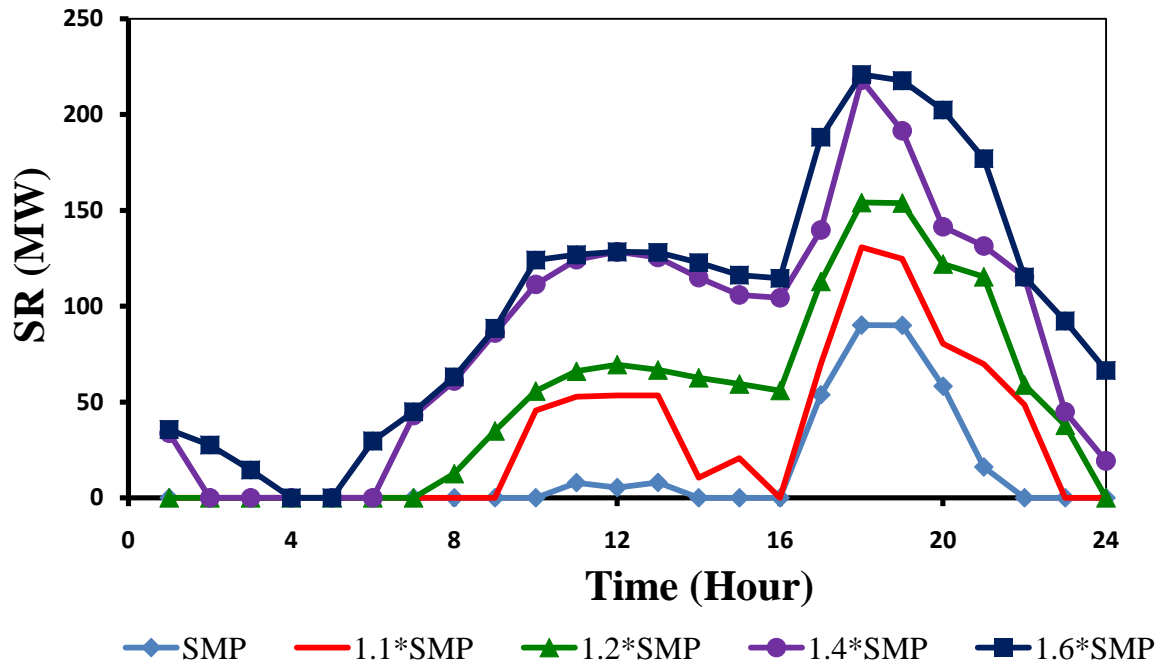


**Fig. C.4.3 LSRA at fixed 1.2\*SRP and varying SMP for 5% LFU with Transmission Loss**



**Fig. C.4.4 LSRA at fixed 1.4\*SRP and varying SMP for 5% LFU with Transmission Loss**





**Fig. C.4.5 LSRA at fixed 1.6\*SRP and varying SMP for 5% LFU with Transmission Loss**



C.5 VARIATION OF SRP AND SMP ON THE SR REQUIREMENTS FOR A 7% LFU  
WITHOUT TRANSMISSION LOSS

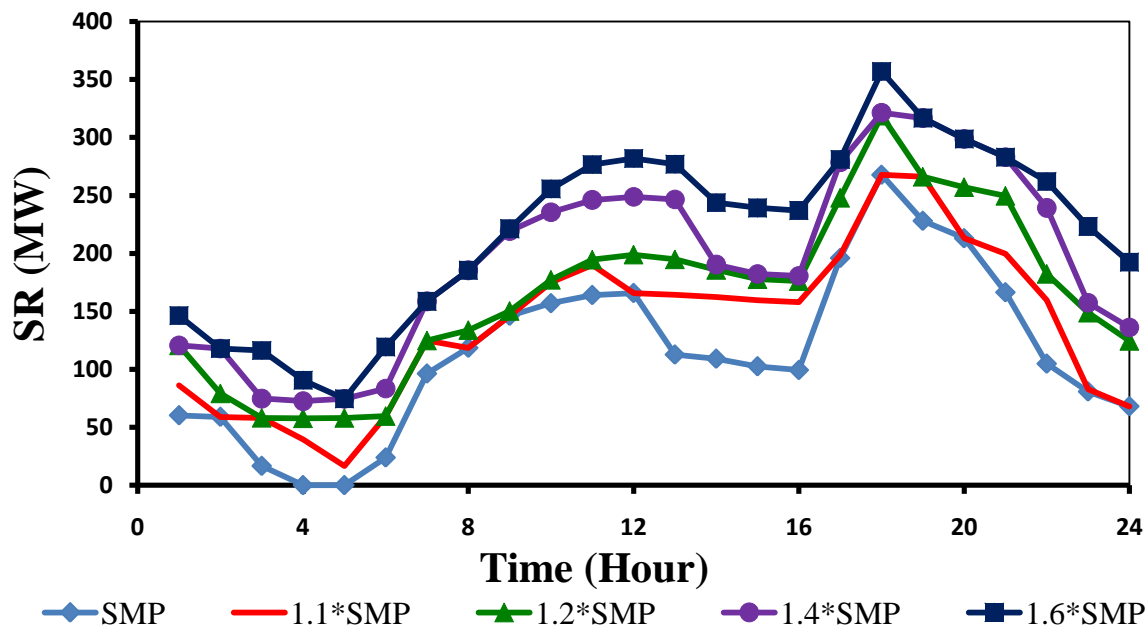


Fig. C.5.1 LSRA at fixed SRP and varying SMP for 7% LFU without Transmission Loss

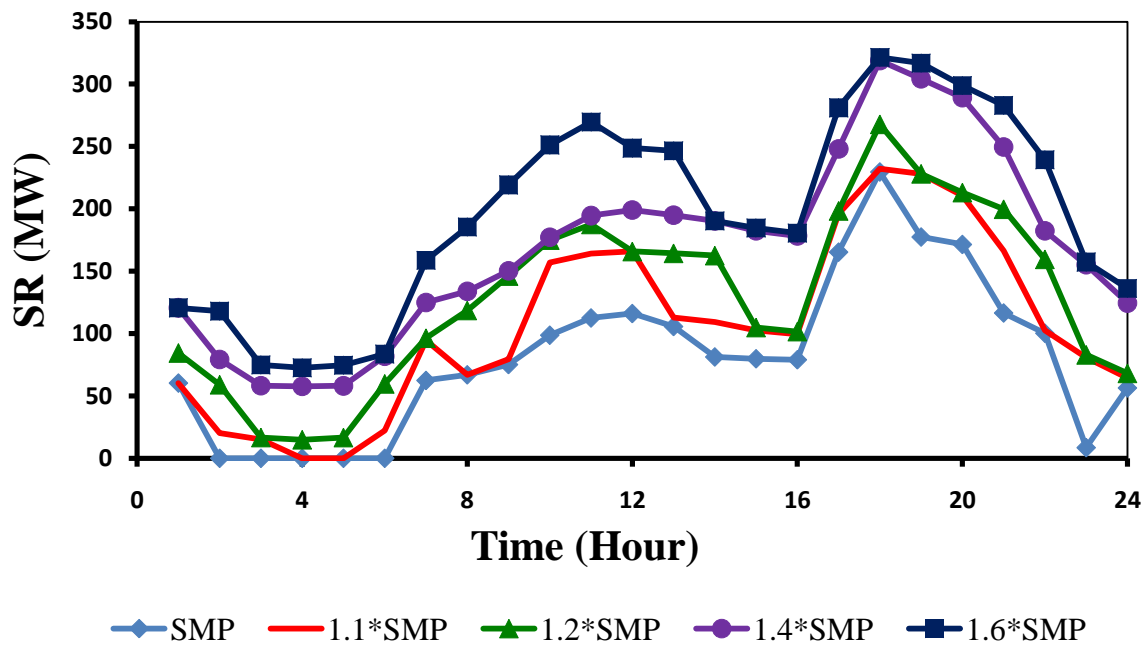
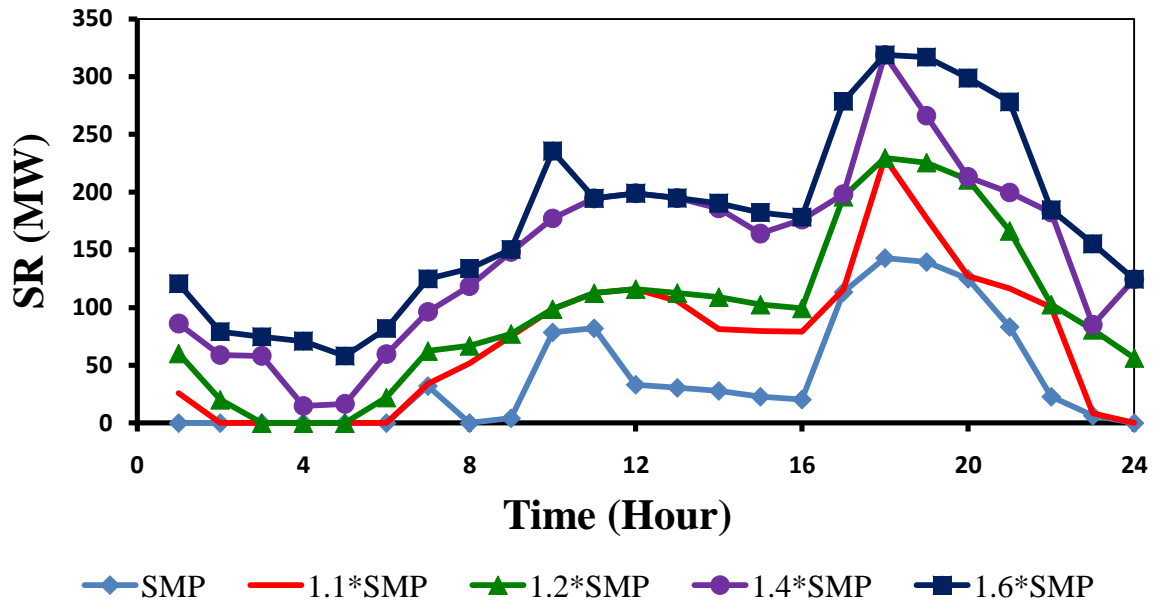
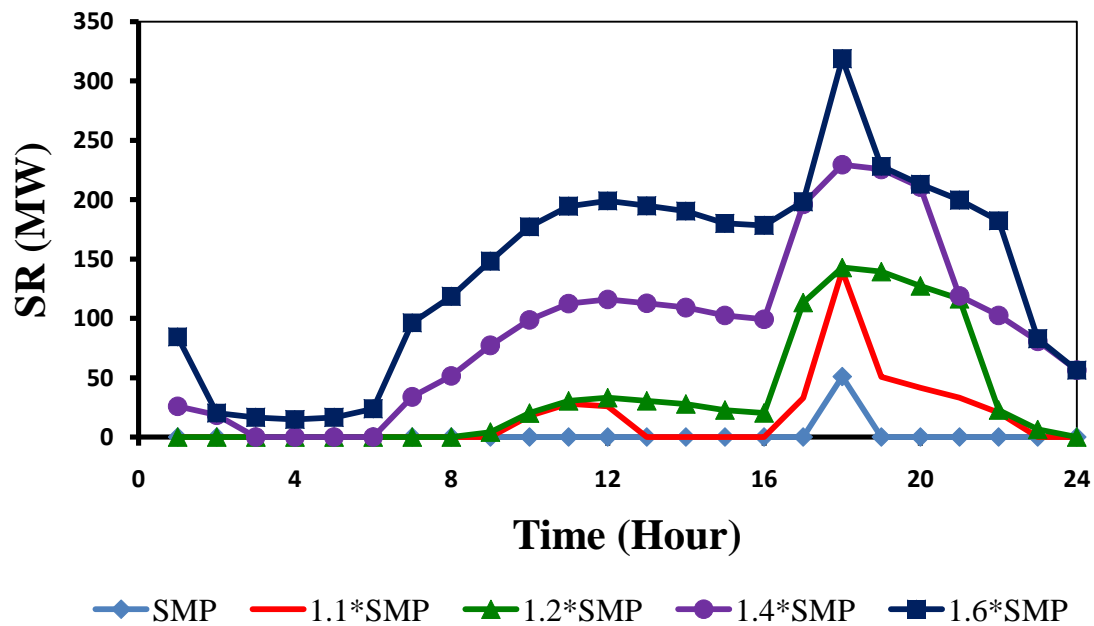


Fig. C.5.2 LSRA at fixed 1.1\*SRP and varying SMP for 7% LFU without Transmission Loss



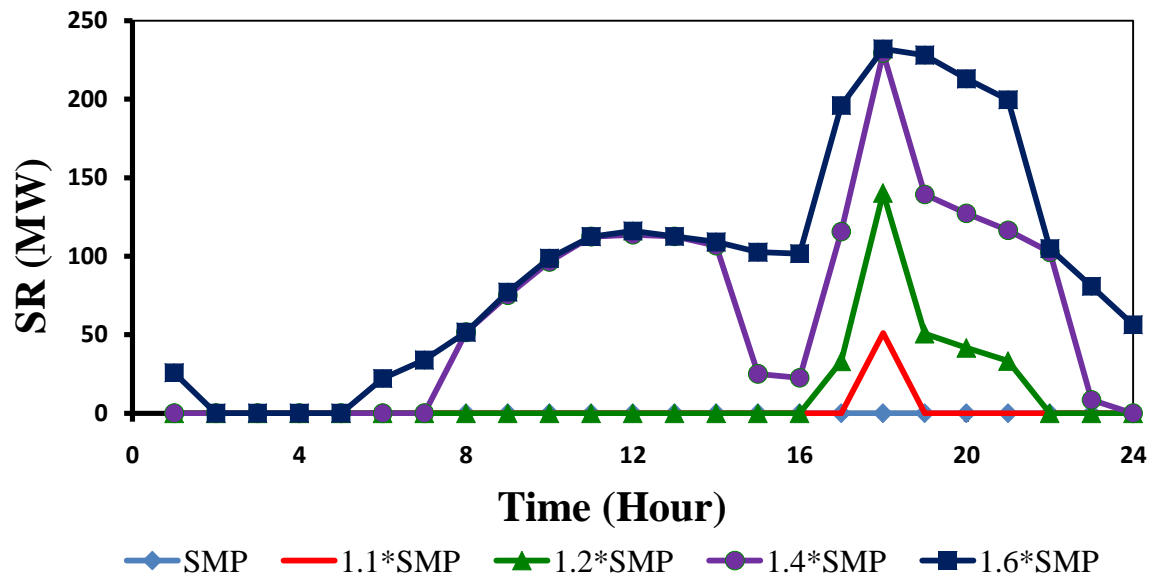


**Fig. C.5.3** LSRA at fixed 1.2\*SRP and varying SMP for 7% LFU without Transmission Loss



**Fig. C.5.4** LSRA at fixed 1.4\*SRP and varying SMP for 7% LFU without Transmission Loss





**Fig. C.5.5 LSRA at fixed 1.6\*SRP and varying SMP for 7% LFU without Transmission Loss**



C.6 VARIATION OF SRP AND SMP ON THE SR REQUIREMENTS FOR A 7% LFU  
WITH TRANSMISSION LOSS

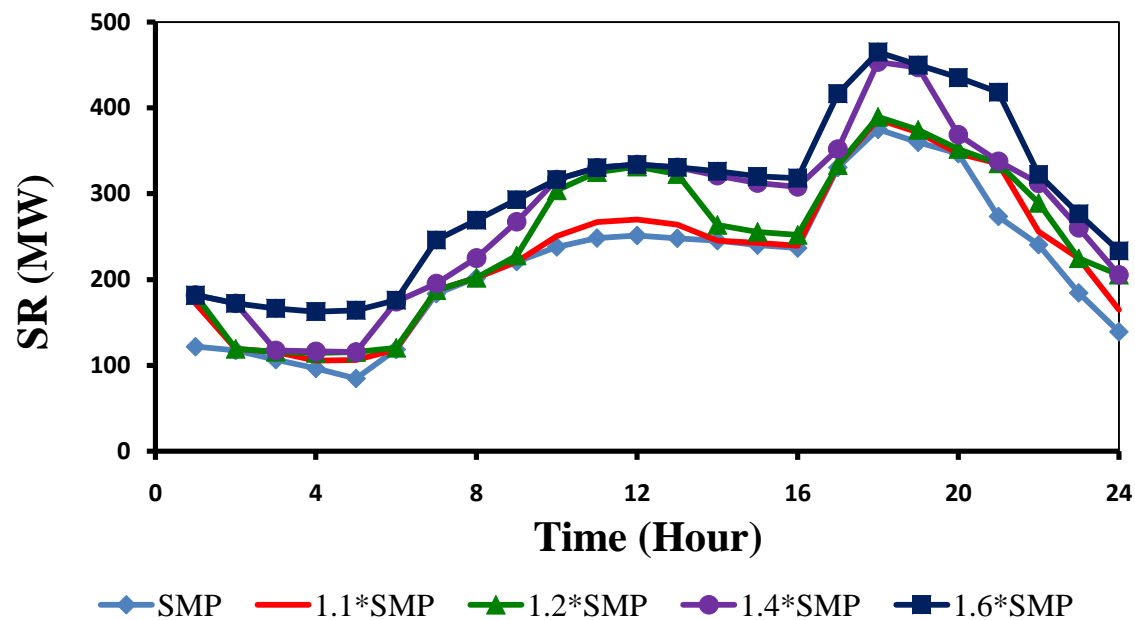


Fig. C.6.1 LSRA at fixed SRP and varying SMP for 7% LFU with Transmission Loss

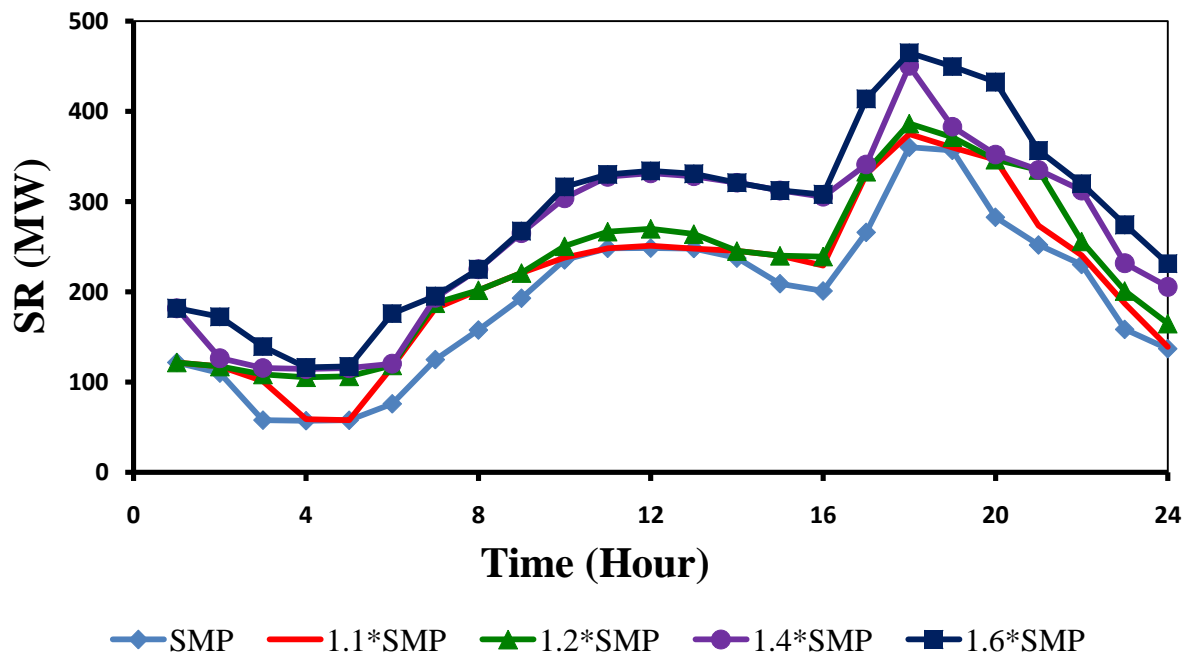
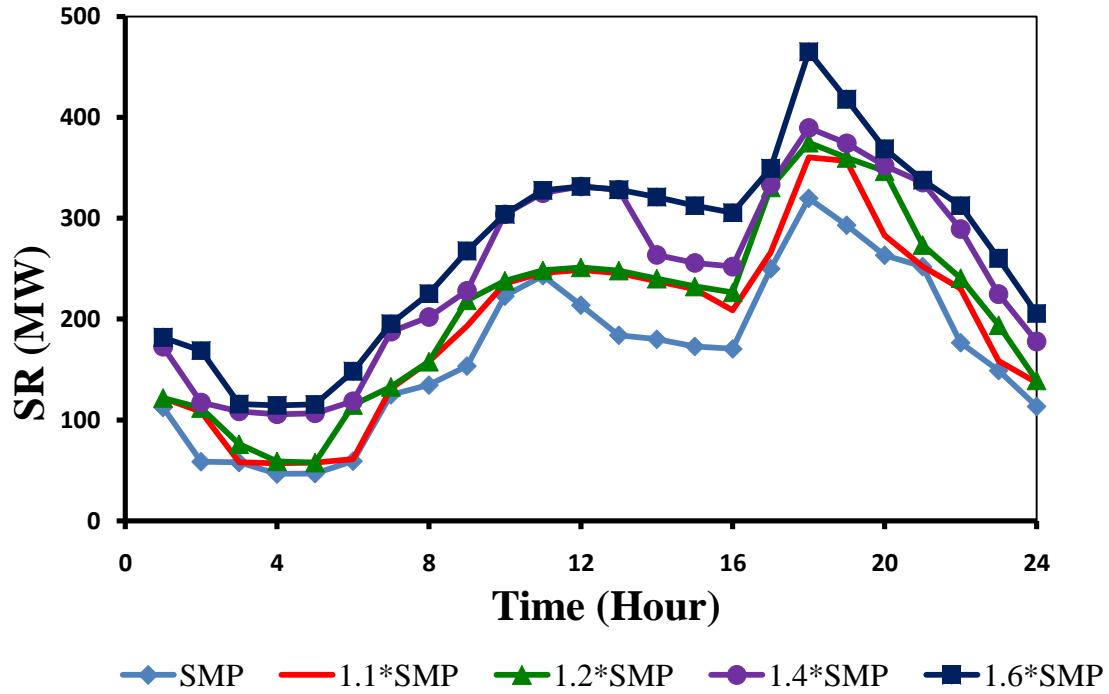
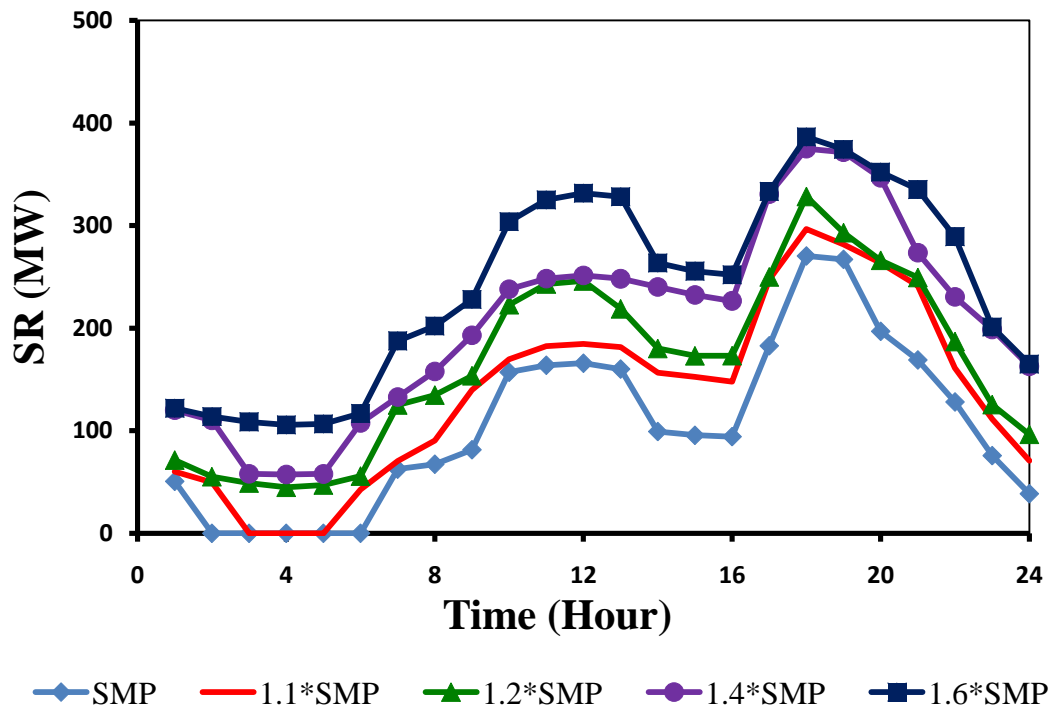


Fig. C.6.2 LSRA at fixed 1.1\*SRP and varying SMP for 7% LFU with Transmission Loss



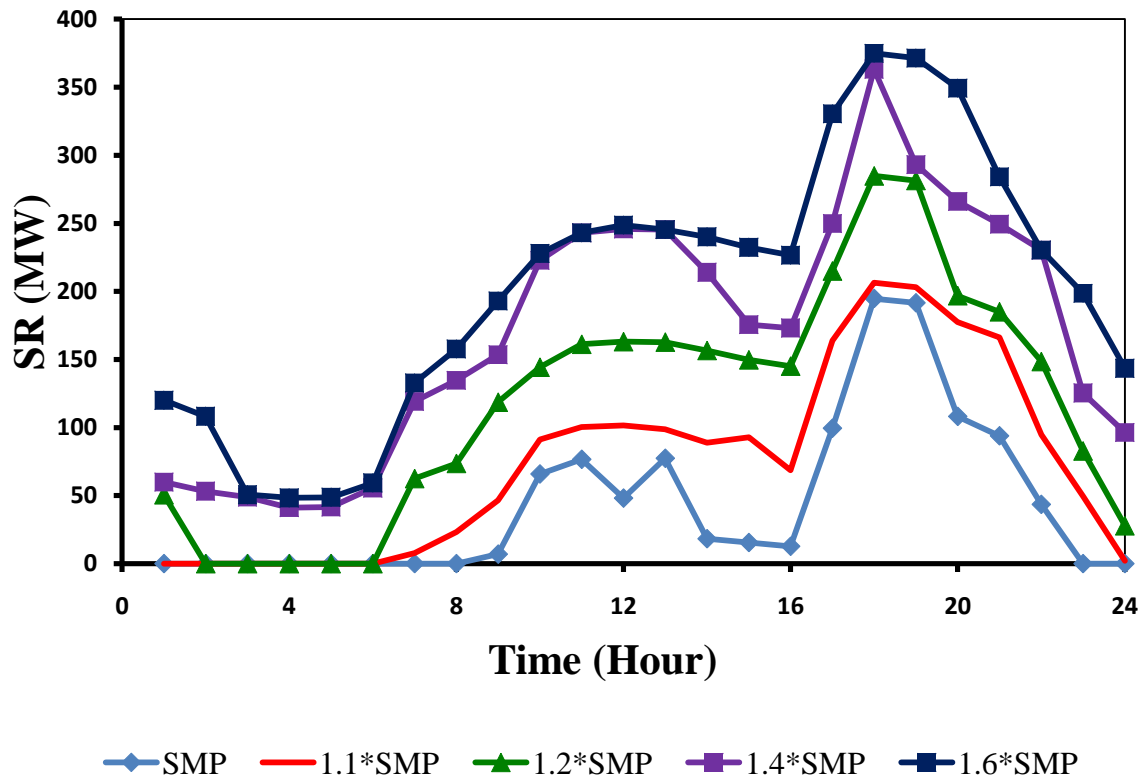


**Fig. C.6.3 LSRA at fixed 1.2\*SRP and varying SMP for 7% LFU with Transmission Loss**



**Fig. C.6.4 LSRA at fixed 1.4\*SRP and varying SMP for 7% LFU with Transmission Loss**





**Fig. C.6.5 LSRA at fixed 1.6\*SRP and varying SMP for 7% LFU with Transmission Loss**



# APPENDIX D: DERIVATION OF $P_{Loss}$ FORMULA

## D.1 Derivation of the Transmission Loss ( $P_{Loss}$ ) Formula

The transmission loss ( $P_{Loss}$ ) formula is derived by making the following assumptions:

- Each generator's reactive power is a linear function of its real power out as shown in Equation D.1 below:

$$Q_{Gi} = Q_{Gi0} + f_i P_{Gi} \dots \dots \dots (D.1)$$

where  $Q_{Gi0}$  is a constant and  $f_i$  is the percentage factor.

- Constant generator angular positions,  $\delta_i$
- Constant generator-bus voltage magnitudes
- A fixed demand pattern

The following data are used in the derivation of the transmission loss formula. The system base operating points are a base apparent power of 1000MVA and a base voltage of 138KV. With reference to Figure 4.2, Tables D.1 and D.2 show the line and bus data used in the derivation.

**Table D.1: Transmission Line Impedances**

Line, Bus to Bus	Resistance ( $\Omega$ )	Inductive Reactance ( $\Omega$ )
1-2	0.5	1
1-3	0.3	1.5
1-4	0.3	2
2-4	0.3	0.28
3-4	0.4	1.2
3-6	0.5	1.4
4-6	0.5	1.5
4-7	0.5	1.1
5-6	0.3	1.5
6-7	5	8



**Table D.2: System Load**

Bus	P(MW)	Q(MVAR)
2	1600	1350
7	1846	1250

The first stage in the transmission loss derivation is the Load flow Analysis used in the estimation of the bus voltages and angles.

### [1] Load Flow Analysis

The following assumptions are made in the load flow analysis:

- Bus 1 is chosen as the slack bus where  $|V_1| = 1pu, \delta_1 = 0$
- Buses 2 and 5 are the load buses
- Buses 3,4, 6 and 7 make up the voltage-controlled (PV) buses

Given the base case operating points, the base impedance is given by:

$$Z_b = \frac{kV^2}{MVA} = \frac{138^2}{1000} = 19.044\Omega$$

Using the base impedance, the transmission lines' impedances are converted to p.u as shown in Table D.3.

**Table D.3: Transmission Line data**

Line	Impedance $Z(\Omega)$	Impedance $Z/Z_b(\text{pu})$	Admittance $y=1/Z(\text{pu})$
1-2	$0.5+1j$	$0.026255+0.0525j$	$7.61992-15.2369j$
1-3	$0.3+1.5j$	$0.015753+0.078765j$	$2.44154-12.2077j$
1-4	$0.3+2j$	$0.015753+0.10502j$	$1.39687-9.31247j$
2-4	$0.3+0.28j$	$0.015753+0.0147028j$	$33.9264-31.6646j$
3-4	$0.4+1.2j$	$0.021004+0.063012j$	$4.761-14.283j$
3-6	$0.5+0.4j$	$0.026255+0.021004j$	$23.2244-18.5795j$
4-6	$0.5+1.5j$	$0.026255+0.078765j$	$3.8088-11.4264j$
4-7	$0.5+1.1j$	$0.026255+0.057761j$	$6.52191-14.3482j$
5-6	$0.3+1.5j$	$0.015753+0.078765j$	$2.44154-12.2077j$
6-7	$5+8j$	$0.26255+0.42008j$	$1.06989-1.71182j$



### Admittance matrix [Y]

Admittance matrix elements are calculated as follows:

$$Y_{ii} = \text{sum of all admittance connected to bus } i = \sum_j y(i, j)$$

$$Y_{ij} = \text{negative of admittance between bus } i \text{ \& } j = -y(i, j) = Y_{ji}$$

[Y] Matrix calculation:

$$\begin{aligned} Y_{11} &= (y_{12} + y_{13} + y_{14}) = (7.61992 - 15.2369j) + (2.44154 - 12.2077j) + (1.39687 - 9.31247j) \\ &= 29.8602 - 98.5813j \end{aligned}$$

$$Y_{12} = -y_{12} = -7.61992 + 15.2369j = Y_{21}$$

Using a Matlab program, the other elements in the Y-matrix are determined and outputted as follows:

[Y] =

	1	2	3	4	5	6	7
1	11.4560 - 36.7554j	-7.6176 +15.2352j	-2.4415 +12.2077j	-1.3969 + 9.3125j	0	0	0
2	-7.6176 +15.2352j	41.5440 - 46.8998j	0	-33.9264 +31.6646j	0	0	0
3	-2.4415 +12.2077j	0	30.4269 - 45.0702j	-4.7610 +14.2830j	0	-23.2244 +18.5795j	0
4	-1.3969 + 9.3125j	-33.9264 +31.6646j	-4.7610 +14.2830j	50.4150 - 81.0347	-1.874 + 39j	-3.8088 +11.4264j	-6.5219 +14.3482j
5	0	0	0	0	2.4415 - 12.2077j	-2.4415 +12.2077	0
6	0	0	-23.2244 +18.5795j	-3.8088 +11.4264j	-2.4415 +12.2077j	30.5446 - 43.9254j	-1.0699 + 1.7118j
7	0	0	0	-6.5219 +14.3482	0	-1.0699 + 1.7118j	7.5918 - 16.0600j



**Table D.4: Bus Parameters for Load flow Analysis**

Bus	Bus Code	Voltage ( p.u.) Initial values	Generation*		Load*	
			Real	Reactive	Real	Reactive
1	1(Slack)	1.0	0	0		
2	0 (load)	1.0			1600	1350
3	2 (Voltage Controlled bus)	1.0	700	100		
4	2 (Voltage Controlled bus)	1.0	1106	200		
5	2 (Voltage Controlled bus)	1.0	600	100		
6	2 (Voltage Controlled bus)	1.0	1040	300		
7	0 (load)	1.0			1846	1250

\*active power in MW & reactive power in MVAR

With reference to Table D.4, the Gauss-Seidel method is used in the load flow analysis to determine the bus voltages and angles. Bus 1 is the reference bus for this calculation. The Gauss-Seidel method uses the power flow equation expressed in terms of the bus admittance matrix as shown below:

$$V_i^{(k+1)} = \frac{\frac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} - \sum_{j \neq i} Y_{ij} V_j^{(k)}}{Y_{i1}} \quad (D.2)$$

where  $P_i^{sch}$  and  $jQ_i^{sch}$  are the net real and reactive powers expressed in per unit.

If Equation D.2 is solved, we have the following solutions:

$$P_i^{(k+1)} = \Re \left\{ V_i^{*(k)} \left[ V_i^{(k)} Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij} V_j^{(k)} \right] \right\} \quad j \neq i \quad (D.3)$$



$$Q_i^{(k+1)} = -\Im \left\{ V_i^{*(k)} \left[ V_i^{(k)} Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij} V_j^{(k)} \right] \right\} \quad j \neq i \quad (D.4)$$

Using the Gauss-Seidel method based on the above equations, the following results were obtained:

Power Flow Solution by Gauss-Seidel Method on Matlab

Maximum Power Mismatch = 9.48693e-007

No. of Iterations = 248

Bus No.	Voltage Mag.	Angle Degree	-----Load----- MW	Mvar	---Generation--- MW	Mvar	Injected Mvar
1	1.000	0.000	0.000	0.000	302.698	1797.627	0.000
2	0.927	-0.515	1600.000	1350.000	0.000	0.000	0.000
3	0.985	2.311	0.000	0.000	700.000	400.762	0.000
4	0.950	0.005	0.000	0.000	1106.000	925.638	0.000
5	1.000	6.122	0.000	0.000	600.000	21.088	0.000
6	0.990	3.405	0.000	0.000	1040.000	116.567	0.000
7	0.818	-4.262	1846.000	1250.000	0.000	0.000	0.000
Total			3446.000	2600.000	3748.698	3261.682	0.000

The load flow analysis is used to determine the relative angle difference between a reference bus and the other buses for a reasonable load. Taking bus 7 as the reference bus, the Load Flow (LF) bus angles with reference to bus 7 can be generally evaluated using the equation below:

$$\begin{aligned} \delta_i^{Ref \ 7} \\ = \delta_i^{LF} - \delta_7^{LF} \end{aligned} \quad (D.5)$$

The bus voltages and their respective angles with reference to bus 7 of the load flow solution is given below and will be used in the determination of the loss coefficients of the transmission loss formula:

$$V = [1 \ 0.927 \ 0.985 \ 0.950 \ 1 \ 0.99 \ 0.818] \text{ in p.u}$$



$$\delta = [4.262 \ 3.747 \ 6.573 \ 4.267 \ 10.384 \ 7.667 \ 0] \text{ in degrees}$$

The Z-bus matrix referenced to bus 7 is determined by eliminating the 7th row and 7th column of the admittance matrix and finding the inverse of the modified matrix,  $\hat{Y}_{bus \ 7}$ . Therefore,

$$Z_{bus \ 7} = (\hat{Y}_{bus \ 7})^{-1} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} & Y_{16} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} & Y_{26} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} & Y_{36} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} & Y_{46} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} & Y_{56} \\ Y_{61} & Y_{62} & Y_{63} & Y_{64} & Y_{65} & Y_{66} \end{bmatrix} \quad (D.6)$$

Using Matlab, the impedance matrix is given as:

$$Z_{bus \ 7} = \begin{bmatrix} 0.0353+0.0810j & 0.0291+0.0577j & 0.0291+0.0580j & 0.0240+0.0505j & 0.0251+0.0541j & 0.0251+0.0541j \\ 0.0291+0.0577j & 0.0364+0.0645j & 0.0259+0.0509j & 0.0241+0.0511j & 0.0238+0.049j & 0.0238+0.049j \\ 0.0291+0.0580j & 0.0259+0.0509j & 0.0348+0.0751j & 0.0240+0.0489j & 0.0268+0.0662j & 0.0268+0.0662j \\ 0.0240+0.0505j & 0.0241+0.0511j & 0.0240+0.0489j & 0.0242+0.0514j & 0.0228+0.0474j & 0.0228+0.0474j \\ 0.0251+0.0541j & 0.0238+0.0490j & 0.0268+0.0662j & 0.0228+0.0474j & 0.0533+0.1553j & 0.0375+0.0765j \\ 0.0251+0.0541j & 0.0238+0.0490j & 0.0268+0.0662j & 0.0228+0.0474j & 0.0375+0.0765j & 0.0375+0.0765j \end{bmatrix}$$

Recall that the transmission loss formula can be expressed in the form of Equation 4.2:

$$P_{Loss} = P^T [B] P + B_0^T P + B_{00} \quad (4.2)$$

The loss coefficients can be evaluated using the following relationships:

$$B = A_p - B_p F + F^T B_p + F^T A_p F \quad (D.7)$$

$$B_0^T = E_p + E_q F + 2Q_{G0}^T (A_p F + B_p) \quad (D.8)$$

$$B_{00} = [P_D^T \ Q_D^T] \begin{bmatrix} A_p & -B_p \\ B_p & A_p \end{bmatrix} \begin{bmatrix} P_D \\ Q_D \end{bmatrix} + Q_{G0}^T A_p Q_{G0} + E_q Q_{G0} \quad (D.9)$$

On a base MVA of 1000, with reference to bus 7 and using the result from the load flow analysis, Table D.5 shows the data in p.u for the evaluation of the loss coefficients above.

The reactive characteristics of the generating units including the spot market and spinning reserve market are:

$$\text{at Bus 1: } Q_3 = 120 + 0.75P_3$$



**Table D.5: Evaluated Bus Data in p.u**

Bus	$P_D$	$Q_D$	$P_G$	$Q_G$	$Q_{0D}$
1	0	0	0.303	1.798	120
2	1.6	1.35	0	0	0
3	0	0	0.7	0.401	90
4	0	0	1.106	0.926	100
5	0	0	0.6	0.0211	80
6	0	0	1.04	0.1166	100

at Bus 3:  $Q_w = 90 + 0.65w$

at Bus 4:  $Q_2 = 100 + 0.7P_2$

at Bus 5:  $Q_z = 80 + 0.4z$

at Bus 6:  $Q_1 = 100 + 0.8P_1$

The linear factor,  $f$  is given as:

$$f = [0.75 \ 0 \ 0.65 \ 0.7 \ 0.4 \ 0.8]$$

and the matrix  $F$  given as the diagonal of  $f$  is shown below

$$F = \begin{bmatrix} 0.75 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.65 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 \end{bmatrix}$$

The elements of  $A_p$  and  $B_p$  are evaluated as follows:-

Generally, each element of the matrix  $A_p$  and  $B_p$  is evaluated using Equations D.10 and D.11 respectively.

$$a_{ij} = \frac{r_{ij}}{|V_i||V_j|} \cos(\delta_i - \delta_j) \quad (D.10)$$



$$b_{ij} = \frac{r_{ij}}{|V_i||V_j|} \sin(\delta_i - \delta_j) \quad (D.11)$$

Where,

voltage angles,  $\delta_i$  is in radians;  $r_{ij}$  is the real part of the  $Z_{bus}$  matrix element  $Z_{ij}$  and  $|V_i|$  is the absolute value of the bus voltage,  $i$  in p.u gotten from the power flow solution by Gauss Seidel method.

A sample calculation is shown below.

$$a_{11} = \frac{r_{11}}{|V_1||V_1|} \cos(\delta_1 - \delta_1) = \frac{0.0353}{1 * 1} \cos(0.0744 - 0.0744) = 0.0353$$

$$b_{11} = \frac{r_{11}}{|V_1||V_1|} \sin(\delta_1 - \delta_1) = \frac{0.0353}{1 * 1} \sin(0.0744 - 0.0744) = 0$$

The other elements of  $A_p$  and  $B_p$  are calculated in a similar way using Matlab to give the output below.

$$A_p = \begin{bmatrix} 0.0353 & 0.0314 & 0.0295 & 0.0252 & 0.0249 & 0.0253 \\ 0.0314 & 0.0424 & 0.0284 & 0.0273 & 0.0256 & 0.0259 \\ 0.0295 & 0.0284 & 0.0359 & 0.0256 & 0.0272 & 0.0275 \\ 0.0252 & 0.0273 & 0.0256 & 0.0268 & 0.0238 & 0.0242 \\ 0.0249 & 0.0256 & 0.0272 & 0.0238 & 0.0533 & 0.0379 \\ 0.0253 & 0.0259 & 0.0275 & 0.0242 & 0.0379 & 0.0383 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 & 0.0003 & -0.0012 & 0 & -0.0027 & -0.0015 \\ -0.0003 & 0 & -0.0014 & -0.0002 & -0.003 & -0.0018 \\ 0.0012 & 0.0014 & 0 & 0.001 & -0.0018 & -0.0005 \\ 0 & 0.0002 & -0.001 & 0 & -0.0026 & -0.0014 \\ 0.0027 & 0.003 & 0.0018 & 0.0026 & 0 & 0.0018 \\ 0.0015 & 0.0018 & 0.0005 & 0.0014 & -0.0018 & 0 \end{bmatrix}$$

The matrix,  $E_p$  and  $E_q$  are calculated using Equations D.12 and D.13 respectively.

$$E_p = 2(-P_D^T \cdot A_p - Q_D^T \cdot B_p) \quad (D.12)$$

$$E_q = 2(P_D^T \cdot B_p - Q_D^T \cdot A_p) \quad (D.13)$$



Using the Matlab,

$$E_p = [-0.0997 \quad -0.1356 \quad -0.0870 \quad -0.0867 \quad -0.0737 \quad -0.0782] \text{ and}$$

$$E_q = [-0.0857 \quad -0.1144 \quad -0.0811 \quad -0.0745 \quad -0.0785 \quad -0.0757]$$

Using Equations D.7, D.8 and D.9 and the values of the matrices  $F, A_p, B_p, E_p$  and  $E_q$ , the loss coefficients of the transmission loss formula are obtained. A Matlab program in Appendix D.2 has been used to obtain these coefficients. They are given below as:

$$B = \begin{bmatrix} 0.0552 & 0.0316 & 0.0437 & 0.0385 & 0.0315 & 0.0405 \\ 0.0316 & 0.0424 & 0.0293 & 0.0275 & 0.0267 & 0.0273 \\ 0.0437 & 0.0293 & 0.051 & 0.0372 & 0.0338 & 0.0419 \\ 0.0385 & 0.0275 & 0.0372 & 0.0399 & 0.0298 & 0.0379 \\ 0.0315 & 0.0267 & 0.0338 & 0.0298 & 0.0618 & 0.0493 \\ 0.0405 & 0.0273 & 0.0419 & 0.0379 & 0.0493 & 0.0628 \end{bmatrix} p.u$$

$$B_0 = \begin{bmatrix} -0.1421 \\ -0.1344 \\ -0.1213 \\ -0.1208 \\ -0.0943 \\ -0.1155 \end{bmatrix} MW$$

$$B_{00} = 0.1538 \text{ p.u}$$

Converting the  $B$  and  $B_{00}$  matrix from p.u to MW and eliminating the 2<sup>nd</sup> row and 2<sup>nd</sup> column of matrix  $B$  and the 2<sup>nd</sup> row of matrix  $B_0$  which is a load bus, we have:

$$B = \begin{bmatrix} 0.00005519 & 0.00004372 & 0.0000385 & 0.00003147 & 0.00004052 \\ 0.00004372 & 0.00005103 & 0.00003718 & 0.00003378 & 0.00004187 \\ 0.00003850 & 0.00003718 & 0.00003989 & 0.00002975 & 0.00003786 \\ 0.00003147 & 0.00003378 & 0.00002975 & 0.00006181 & 0.00004927 \\ 0.00004052 & 0.00004187 & 0.00003786 & 0.00004927 & 0.0000628 \end{bmatrix} MW$$



$$B_0 = \begin{bmatrix} -0.1421 \\ -0.1344 \\ -0.1213 \\ -0.1208 \\ -0.0943 \\ -0.1155 \end{bmatrix} MW$$

$$B_{00} = 153.8 MW$$

Substituting these terms into Equation 4.2, the transmission loss formula is obtained as:

$$\begin{aligned} P_{Loss} = & 0.00006280P_1^2 + 0.00003989P_2^2 + 0.00005519P_3^2 + 0.00005103w^2 \\ & + 0.00006181z^2 + 0.000076P_1P_2 + 0.000081P_1P_3 + 0.000084P_1w \\ & + 0.000099P_1z + 0.000077P_2P_3 + 0.000074P_2w + 0.00006P_2z \\ & + 0.000087P_3w + 0.000063P_3z + 0.000068wz - 0.1155P_1 - 0.1208P_2 \\ & - 0.1421P_3 - 0.1213w - 0.0943z + 153.8 MW \end{aligned}$$



## D.2 Matlab Code for the Determination of the Loss Coefficients

The loss coefficients were determined after load flow analysis by Gauss-Seidel method using the Matlab code shown below.

```
%save variable j for complex
clear all;
clc;
MVA=1000;
KV=138;
PG=[]';
QG=[]';
PD=[0 1600/MVA 0 0 0 0 1846/MVA]';
QD=[0 1350/MVA 0 0 0 0 1250/MVA]';
V=[1 0.927 0.985 0.95 1 0.99 0.818]; % voltage in pu
del=[4.262 3.747 6.573 4.267 10.384 7.667 0]; %angle in degree with
reference to bus 7
QGo=[120/MVA 0 90/MVA 100/MVA 80/MVA 100/MVA]';
fi=[0.75 0 0.65 0.7 0.4 0.8];
F=diag(fi);
zbase=KV^2/MVA;
rr=zeros(7,7);
rr(1,2)=0.5+1j;rr(1,3)=0.3+1.5j;rr(1,4)=0.3+2j;
rr(4,6)=0.5+1.5j;rr(4,7)=0.5+1.1j;
rr(2,4)=0.3+0.28j;
rr(3,4)=0.4+1.2j;rr(3,6)=0.5+0.4j;
rr(5,6)=0.3+1.5j;rr(6,7)=5+8j;
rr=rr./zbase;
yy=zeros(7,7);
for m=1:7
    for n=1:7
        rr(n,m)=rr(m,n);
        if rr(m,n)~=0
            yy(m,n)=1./rr(m,n);
        else
            yy(m,n)=0;
        end
    end
end
```



```

        end
    end
    Y=zeros(7,7);
    for m=1:7
        for n=m:7
            sum=0;
            if m==n
                for l=1:7
                    sum=sum+yy(m,l);
                end
                Y(m,n)=sum;
            else
                Y(m,n)=-yy(m,n);
            end
            Y(n,m)=Y(m,n);
        end
    end
    tmpY=Y([1:6],[1:6]);    % taking 7th bus as reference bus, eliminating 7th
    row & 7th col
    Z=inv(tmpY);    % inverse
    R=real(Z);
    % for m=1:4
    %     for n=1:4
    %         ta(m,n)=R(m,n)*cos(del(m)*pi/180-del(n)*pi/180)/(V(m)*V(n));
    %     end
    % end
    % tmpC=ta;
    disp('Y');
    disp(Y);
    disp('tmpY');
    disp(tmpY);
    disp('Z');
    disp(Z)
    C=diag([cos(del(1:6).*pi./180)./V(1:6)]);
    D=diag([sin(del(1:6).*pi./180)./V(1:6)]);
    % Ap & Bp calculation
    Ap=C'*R*C+D'*R*D
    Bp=D'*R*C-C'*R*D

```



```

% Eq, Ep Calculation
K=F'*Ap*F
Ep=2.*(-PD(1:6) '*Ap-QD(1:6) '*Bp)
Eq=2.*(PD(1:6) '*Bp-QD(1:6) '*Ap)
% BL calculation
BL=Ap-Bp*F+F'*Bp+F'*Ap*F;
BLo=Ep+Eq*F+2*QGo'*(Ap*F+Bp);
%BLo=BLo';
tmpKLo=PD(1:6) '* (Ap*PD(1:6)-Bp*QD(1:6))+QD(1:6) '* (Bp*PD(1:6)+Ap*QD(1:6));
KLo=tmpKLo+QGo'*Ap*QGo+Eq*QGo;
disp('BL');
disp(BL);
disp('BLo');
disp(BLo);
disp('KLo');
disp(KLo);

```