EFFECT OF DISTRIBUTIONAL ASSUMPTIONS ON SPARE COMPONENT AVAILABILITY EVALUATION

A Thesis

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ABSTRACT

This thesis presents some basic techniques for availability modelling and quantitative evaluation in repairable systems. Two Monte Carlo simulation methods are developed for the evaluation of Markovian systems with spare components. The results of the simulation method are compared with those obtained by analytical methods. Markov techniques can not be applied directly if the distributions associated with the state residence times are not exponentially distributed. Availability evaluation of non-Markovian systems is usually much more complicated than that of a Markovian system. A practical non-Markovian simulation technique is discussed in this thesis and characteristics of important distributions are presented. A range of studies for selected repairable systems are conducted to study the effect on system unavailability of various distributions. The failure rate is assumed to be constant and distributional variation is applied to the repair and installation processes. In order to facilitate comparison, the mean values associated with the repair and installation times are held constant. The changes in unavailability are therefore due entirely to the changes in the shape of the distributions. The thesis provides a general appreciation of the effect of non-exponential residence times on basic system availability indices.

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1. INTRODUCTION

The importance of reliability as a parameter, which should be specified and paid for, has become apparent with the increasing cost and complexity of many modern industrial and defence systems. Reliability is an inherent property of the system. It is defined [1] as the probability of a device or system performing its purpose adequately for a period of time intended under the operating conditions encountered. Reliability assessment as such is not new. Some highly reliable products are produced by design and manufacturing teams who practise the traditional virtues of reliance on experience and maintenance of high quality. This method of assessing the reliability of a system is often referred as engineering judgement. In addition to this approach, quantitative reliability evaluation techniques are often used to supplement the knowledge of the decision maker in relating the quality of the system to economics and capital investment. In so doing, it can lead to better and more economic designs and a much improved knowledge of the operation and behaviour of a system.

The concept of reliability as a probability means that any attempt to quantify it must involve the use of statistical methods. Reliability evaluation techniques are concerned with the future behaviour of a component or a system. This future behaviour may be over a matter of seconds, such as in the case of ground to air missiles, or several decades, such as in the case of electrical generating units. In all cases, problems cannot be defined as deterministic but are stochastic in nature, i.e. varies randomly with time. The techniques utilized to compute reliability indices can be separated into two general categories [2]: Analytical methods and Monte Carlo simulation methods. Analytical methods use the rules of probability theory directly to compute system reliability indices while Monte Carlo simulation methods create histories of system operation by simulation on a digital computer and then compute reliability indices from these simulated histories of operation.

The term reliability [3] is generally used in the context of non-repairable systems. Reliability analysis is concerned with only one random variable, the failure time for a system. In the case of repairable systems, reliability can be traded off against other parameters such as maintainability and availability. Both maintainability and availability have the dimensions of a probability in the range zero to one, and are based upon time dependent phenomena. The difference is that maintainability is a measure of effectiveness of the performance only during the period of restoration to service, while availability is a measure of total performance effectiveness, i.e. the ratio of actual operating time to operating time plus repair time. As with reliability, maintainability analysis is concerned with only one random variable, the repair time for the failed system. Availability analysis, on the other hand, has to do with two events, both failure and repair. Availability is defined as the probability that an item will be available when required, or as the proportion of total time the item is available for use. The availability is a function of failure rate and repair rate for a simple repairable system. Unavailability is a complementary to availability and is defined as the probability that an item will not be available when required.

Availability [4] is an important consideration in relatively complex systems, such as power stations, satellites, chemical plants and radar systems. In such systems, high reliability by itself is not sufficient to ensure that the system will be available when needed. It is also necessary to ensure that it can be repaired quickly. Maintainability therefore is also an important aspect of design for maximum availability, and trade offs are often necessary between reliability and maintainability features. Availability is also affected by redundancy or standby units. If standby components can be installed in the system, overall availability can be greatly increased.

This thesis presents an evaluation of the availability indices for repairable systems with standby components. There have been many analytical techniques proposed to evaluate availability indices for various configurations of repairable systems [1, 5]. These techniques utilize probability theory as a basic mathematical tool. Most of the documented availability evaluation techniques are based on the assumption that systems are Markovian and transition rates from one state to another are constant.

In real life situations, the assumption of Markovian processes may not be valid for many repairable systems. In their useful life period, the failure times of components are exponentially distributed. The repair and installation times are often represented by other distributions such as normal, Weibull or Erlang. This gives rise to systems with non-Markovian characteristics. Determination of availability indices from a Markovian model is relatively straight-forward using existing techniques. Mathematical analysis of a non-Markovian model is often very difficult, if at all possible. Some techniques have been proposed, but are limited in application. Approximate methods of transforming a non-Markovian process into an equivalent Markov model is presented in Reference 5. Monte Carlo simulation methods [6, 7] find increasing applications in the evaluation of availability indices of both Markovian and non-Markovian systems.

An attempt has been made in this thesis to develop Monte Carlo simulation methodologies for the evaluation of both Markovian and non-Markovian systems. The main objective of the thesis is to study the effect of distributional assumptions on system unavailability. Monte Carlo simulation methods are used in this thesis to compute the system unavailabilities of non-Markovian systems.

The thesis is divided into six chapters. In the second chapter, two different Monte Carlo simulation methods to calculate availability indices of Markovian system are discussed. The basic analytical techniques used to evaluate availability indices of Markovian systems are described briefly and the frequency balance approach is used to compute system unavailabilities for different configurations of a Markovian system with a varying number of spares. The simulation results are then compared with the analytical results to determine the effectiveness of the simulation methods. In Chapter 3, the simulation methodology used to compute unavailability in the case of non-Markovian systems is illustrated. The application of the method of stages is discussed using two series stages. The results of both simulation and analytical techniques are compared. Chapter 4 is a core chapter. The effect of distributional assumptions on system unavailability is seen using a practical example. The failure time is exponentially distributed and the repair and installation times are considered to follow three distributions i. e. Erlang, normal and Weibull. A number of sensitivity studies are conducted and the results are shown in this chapter. Chapter 5 includes two methods for setting confidence interval limits on system unavailability. The effect of Weibull distributed state residence times is evaluated for a system with two spares and with an infinite number of spares. The simulation method has been modified to determine the frequency parameter for non-Markovian systems. The general conclusions arising from this work are given in the last chapter.

2. MARKOV MODELLING OF REPAIRABLE SYSTEMS

2.1 Introduction

A Markov model is characterized by a Markov process. This is a memory-less procedure in which the previous history of the process is not involved in determining the subsequent events. The outcome of any trial depends directly on the outcome of the previous trial. Systems in which the transition rates from one state to another are constant can be designated as Markovian systems.

A repairable component can be generally categorized into two states. These are the up or inservice state and the down or failed state. The associated transitions between the two states are the failure transition and the repair transition respectively. This implies that a component becomes operative and in service immediately following repair action.

In many systems, the restoration of a component to the up state may take place in two distinct stages, the first is concerned with the removal and repair of the failed component and the second is concerned with the re-installation of the repaired component.

This chapter deals with the modelling of repairable systems with standby components. The failure, repair and installation processes are modelled using a state-space diagram and the system state probabilities are determined by

Markov methods. The application of various existing Markov techniques i.e. analytical methods and Monte Carlo techniques, is illustrated by considering a set of examples in which the number of available spares is varied and their effect on system unavailabilty evaluated. The results obtained by analytical methods and Monte Carlo simulation techniques are compared to examine the effectiveness of simulation methods in reliability modelling.

2.2 Discrete And Continous Markov Processes

Consider [5] a stochastic process that is defined for a set of points which may be either integer co-ordinates ($n = 0,1,2,...$) or an interval of real time ($0 \leqslant t$ or $-\infty < t < +\infty$). At a particular point, the stochastic process is a simple random variable. Assuming k arbitrary points $t_1, t_m, t_n ... t_1 < t_m < t_n ...$, there are k random variables $Z(t_1), Z(t_m), Z(t_n)$.... In the discrete time case, these may be denoted by Z_1, Z_m, Z_n.... Consider the discrete time case. The stochastic process is said to be independent if

$$P(Z_n = x \,/\, Z_m = y,\ Z_1 = z, ...) = P(Z_n = x)$$

This means that the probability distribution of Z_n is independent of the present and the past history of the process. A slight weakening of this condition leads to a Markov process. In this case

$$P(Z_n = x \,/\, Z_m = y,\ Z_1 = z, ...) = P(Z_n = x \,/\, Z_m = y)$$

That is, the probability distribution of Z_n depends upon the latest of the time points and none prior to that. The Markov property essentially states that once the state occupied at the time point is known, the previous history of the process is not involved in determining the subsequent state probability distribution. If this probability is a function of time then the process is non-Markovian.

Reliability problems normally deal with systems that are discrete in space i.e. they can exist in one of a number of discrete and identifiable states and continous in time i.e. they exist continously in one of the system states until a transition occurs which takes them discretely to another state in which they can exist until another transition occurs. For $u < v < t$ the Markov chain would be

$$P(Z(t)=k \;/\; Z(v)=j,\; Z(u)=i) = P(Z(t)=k \;/\; Z(v)=j)$$

This property is basically of the form $(P(Z(t+x)=j \;/\; Z(t)=i)$ and is termed as the probability of transition from State i to State j during the time interval t to t+x. If this transition probability does not depend on the initial time t but only on the elapsed time x, the process is said to be time homogenous. This thesis is primarily concerned with this class of process in which state space is discrete but time is continous.

2.3 Definitions Of Availability, Unavailability And Frequency Of A State

Consider the case of a single repairable component for which the failure rate and repair rate are constant i.e. they are characterized by the exponential distribution. The state-transition diagram for this component is shown in Figure 2-1.

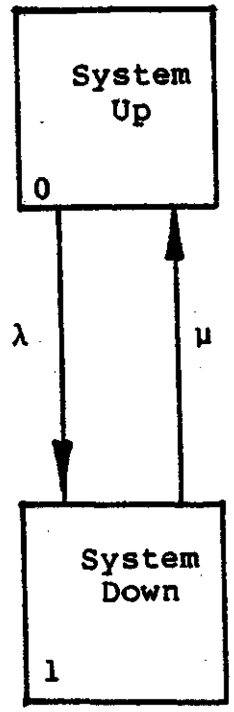

Figure 2-1: Basic Two State Model.

Let

$P_0(t)$ = Probability that the component is operable at time t

$P_1(t)$ = Probability that the component is failed at time t

λ = Failure rate = Failures/unit time

μ = Repair rate = Repairs/unit time

Solving the resultant Markov process [1] gives

$$P_0(t) = \frac{\mu}{\lambda+\mu}[(P_0(0)+P_1(0)] + \frac{e^{-(\lambda+\mu)t}}{\lambda+\mu}[\lambda P_0(0)-\mu P_1(0)] \quad (2.1)$$

$$P_1(t) = \frac{\lambda}{\lambda+\mu}[(P_0(0)+P_1(0)] + \frac{e^{-(\lambda+\mu)t}}{\lambda+\mu}[\mu P_1(0)-\lambda P_0(0)] \quad (2.2)$$

The term $P_0(0)+P_1(0) = 1$ for all initial conditions and therefore Equations 2.1 and 2.2 become

$$P_0(t) = \frac{\mu}{\lambda+\mu} + \frac{e^{-(\lambda+\mu)t}}{\lambda+\mu}[\lambda P_0(0)-\mu P_1(0)] \quad (2.3)$$

$$P_1(t) = \frac{\lambda}{\lambda+\mu} + \frac{e^{-(\mu+\lambda)t}}{\lambda+\mu}[\mu P_1(0)-\lambda P_0(0)] \quad (2.4)$$

The limiting or steady state probabilities when $t = \infty$ are

$$P_0 = P_0(\infty) = \frac{\mu}{\lambda+\mu}$$

$$P_1 = P_1(\infty) = \frac{\lambda}{\lambda+\mu}$$

The values P_0 and P_1 are called Steady State Availability and Unavailability respectively.

Availability[Unavailability] is basically a time dependent parameter and is the probability of the component being found in the up state[down state] at some time t in the future.

Time dependent availability[unavailability] is not under consideration in this thesis and therefore the term availability[unavailability] is used to designate the steady state or limiting value of availability[unavailability].

The frequency of encountering a particular state can be obtained as follows:

Frequency of encountering a state = (Probability of being in the state) x (The rate of departure from the state).

In the two state model shown in Figure 2-1

Frequency of the up state = $P_0 . \lambda$

Frequency of the down state = $P_1 . \mu$

2.4 Analytical Methods For Solving Markov Models

The emphasis in this thesis is on the utilization and solution of Markov models for engineering system availability evaluation. The solutions obtained by analytical methods are precise and may be in the form of explicit expressions or values obtained by numerical techniques. Various analytical solution methods [1] are available depending upon whether the systems are Markovian or non-Markovian.

Analytical methods used for solving Markovian models are:

1. Differential Equations Method.
2. Stochastic Transitional Probability Matrix Approach.
3. Frequency Balance Approach.

1.Differential Equations Method: This method uses the solution of the system differential equations to obtain time dependent and limiting state probabilities.

2.Stochastic Transitional Probability Matrix Approach : This approach is based upon the system transitional probability matrix. The principle of this method is that, once the limiting state probabilities have been reached by matrix multiplication, any further multiplication by the stochastic transitional matrix does not change the values of the limiting state probabilities.

3.Frequency Balance Approach: This approach is a very useful technique for solving relatively small systems which do not require matrix formulation and subsequent solution using a digital computer. The basic concept in the frequency balance approach is that for any state in a system the expected frequency of leaving a state must equal the expected frequency of entering the state. The frequency equations can be written for each state. The unknown parameters in these equations are the steady state probabilities. These probabilities must also sum to unity. This approach is used later in this chapter to obtain expressions for the Unavailabilities[Availabilities] and frequencies for a set of configurations.

The solution technique used in this thesis to obtain closed form expressions for system availability [unavailability] is the frequency balance approach. This can be illustrated by application to a set of examples which lead to the basic equipment configuration studied in this project.

2.4.1 Availability Evaluation Of A General Two State Transformer Model

Consider the situation in a two state system in which the system contains n components and if one fails, the system fails. The component is then repaired and the system continues to operate. The two state model is shown in Figure 2-2. The example chosen is of a bank of n single phase transformers. If one transformer fails, the system fails.

k = Failure rate of a bank

and $k = n\lambda = 3\lambda$ in the case of a 3 phase transfomer bank

λ = Failure rate of a transformer

μ = Repair rate of a transformer

State 1 represents the operating state in which the bank is up i.e. all transformers are operating. State 2 represents the condition in which the bank is down. The transition rates k and μ from State 1 to State 2 and State 2 to State 1 respectively are assumed to be constant.

Steady state probabilities and frequencies for the states can be calculated by solving the linear simultaneous equations associated with the system model. The equations can be

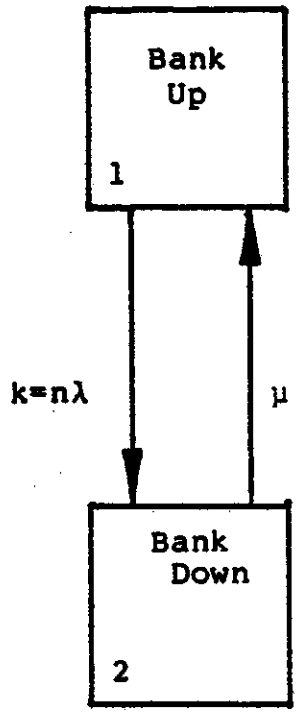

Figure 2-2: General Two State Model Of The Transformer Bank.

obtained by the frequency balance approach as follows:

$$P_1 k = P_2 \mu \quad (2.5)$$

$$P_1 + P_2 = 1 \quad (2.6)$$

giving

$$P_1 = \mu/(k+\mu)$$

$$P_2 = k/(k+\mu)$$

$$\text{Availability } A = P_1 = \mu/(k+\mu)$$

$$\text{Unavailability } U = P_2 = k/(k+\mu)$$

Frequency $F1 = P_1 k$

Frequency $F2 = P_2 \mu$

In the model of Figure 2-2, there are two transitions i.e. from the up state to the down state and from the down state to the up state. The system becomes operative immediately following a repair action.

In many practical situations such as a transformer bank, the restoration of the system to the operating state is a two step process. The first phase is the removal and repair of the failed transformer and second phase is the re-installation of a good transformer.

This two stage repair and installation process can be modelled using a state space diagram and solved by Markov techniques. The re-installation of a good transformer in the bank may occur after the repair of the failed transformer or due to the availability of a spare transformer. In the latter case, the repair of the failed transformer is independent of the spare installation and restoration of the bank to service. The effect on unavailability of the system of the number of spares can be evaluated by creating and solving the appropriate Markov models. This can be illustrated using a sequential series of models by deriving equations and then applying these equations to practical situations [1,8].

2.4.2 Transformer Bank - No Spare Available

Consider a bank of n transformers in which the repair process is divided into two distinct aspects of actual repair and installation. The state transition diagram is shown in Figure 2-3.

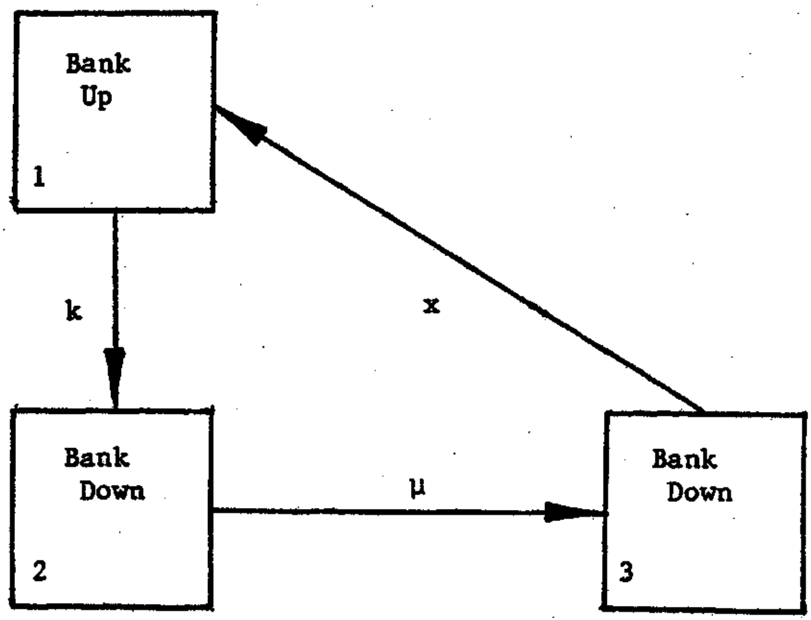

Figure 2-3: Basic Three State Model.

k = Failure rate of the bank

μ = Repair rate of a transformer

x = Installation rate of a transformer

The transition rates are assumed to be constant and therefore the associated state residence times are exponentially distributed.

In the state space diagram of Figure 2-3, State 1 represents the up state and State 2 and State 3 represent the down state of the system. State 3 represents the condition in which the transformer has been repaired but not yet installed.

Using the frequency balance approach

$$P_1 k = P_3 x \quad (2.7)$$

$$P_2 \mu = P_1 k \quad (2.8)$$

$$P_3 x = P_2 \mu \quad (2.9)$$

$$P_1 + P_2 + P_3 = 1 \quad (2.10)$$

Solving Equations 2.7, 2.8, 2.10 gives

$$P_1 = \mu x / D$$

$$P_2 = kx / D$$

$$P_3 = k\mu / D$$

where $D = k(\mu + x) + \mu x$

Availability $A = P_1 = \mu x / D$

Unavailability $U = P_2 + P_3 = k(\mu + x)/D$

The frequency of encountering the system states can be evaluated as follows:

Frequency $F1 = P_1 k = kx\mu / D$

$F2 = P_2 \mu = kx\mu / D$

$F3 = P_3 x = kx\mu / D$

$$F(UP) = F1 = kx\mu/D$$

$$F(DN) = F2 + F3 - P_2\mu = kx\mu/D$$

where

F(UP) is the frequency of encountering the up state

F(DN) is the frequency of encountering the down state

2.4.3 Transformer Bank - One Spare Available

Consider a situation in which one spare is normally available. As soon as a transformer fails, the entire bank is shut down until the failed unit has been replaced by the spare. The down time consists of two phases, the repair time and the installation period. The state transition diagram is shown in Figure 2-4.

States 1, 2 and 5 are similar to Figure 2-3. The system up state is given by States 1 and 3 and the system down state is given by States 2, 4 and 5. In State 4, the bank is down and the spare is also in the failed state and thus both require repair. When one repair is completed, the relevent transition takes the system into State 2. It is assumed that repair facilities are unrestricted i.e. as soon as the transformer fails, the repair is started irrespective of the fact that another unit may also be undergoing repair. Therefore the transition rate from State 4 to State 2 is 2μ. If repair is restricted i.e. resources are limited and only one repair is possible at any particular time, the transition rate will be reduced to μ.

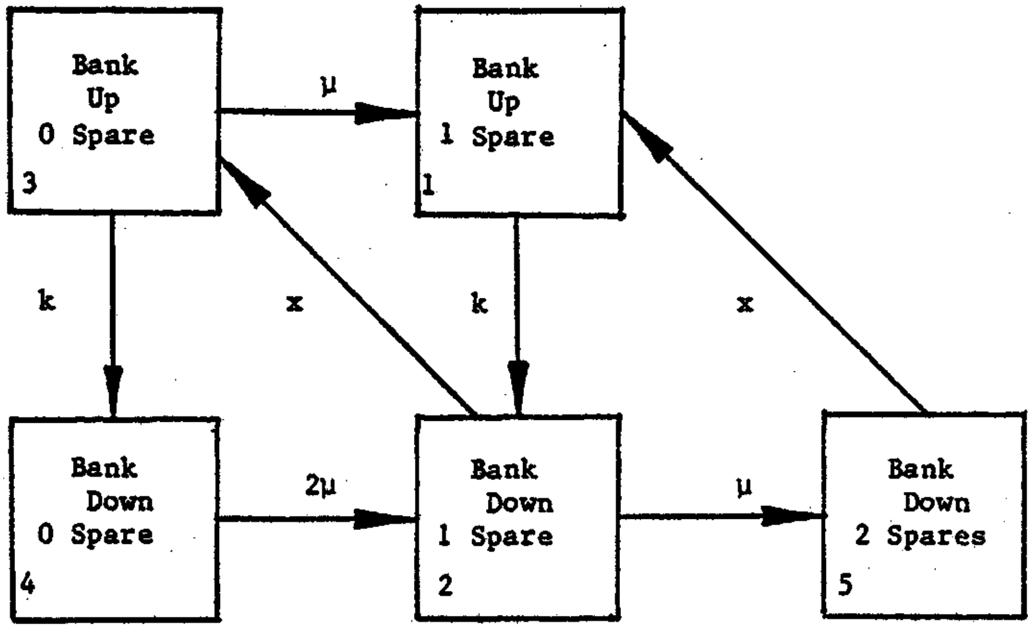

Figure 2-4: Transformer Bank With One Spare.

Assuming unrestricted repair resources and the model shown in Figure 2-4, the application of the frequency balance approach gives the following equations:

$$P_1 k = P_3 \mu + P_5 x \qquad (2.11)$$

$$P_2(\mu + x) = P_1 k + P_4 2\mu \qquad (2.12)$$

$$P_3(\mu + k) = P_2 x \qquad (2.13)$$

$$P_4 2\mu = P_3 k \qquad (2.14)$$

$$P_5 x = P_2 \mu \qquad (2.15)$$

$$P_1 + P_2 + P_3 + P_4 + P_5 = 1 \qquad (2.16)$$

Solving Equations 2.11, 2.12, 2.13, 2.14, 2.16

$$P_1 = (2x^2\mu^2 + 2x\mu^2(k+\mu))/D$$

$$P_2 = (2k\mu x(k+\mu))/D$$

$$P_3 = (2kx^2\mu))/D$$

$$P_4 = (k^2x^2)/D$$

$$P_5 = (2k\mu^2(k+\mu))/D$$

where

$$D = k^2x^2 + 2\mu((x+\mu)(k+x)(k+\mu))$$

$$\text{Availability } A = P_1 + P_3 = 2x\mu(k+\mu)(x+\mu)/D$$

$$\text{Unavailability } U = P_2 + P_4 + P_5 = (k^2x^2+2\mu k(k+\mu)(x+\mu))/D$$

The cumulative frequencies of encountering the system up state F(UP) and system down state F(DN) are

$$F(UP) = (P_1+P_3)k = (2kx\mu(k+\mu)(x+\mu))/D$$

$$F(DN) = (P_2+P_5)x = (2kx\mu(k+\mu)(x+\mu))/D$$

2.4.4 Transformer Bank - Two Spares Available

If there are two spares initially available, the state space diagram shown in Figure 2-4 can be extended to give the diagram shown in Figure 2-5.

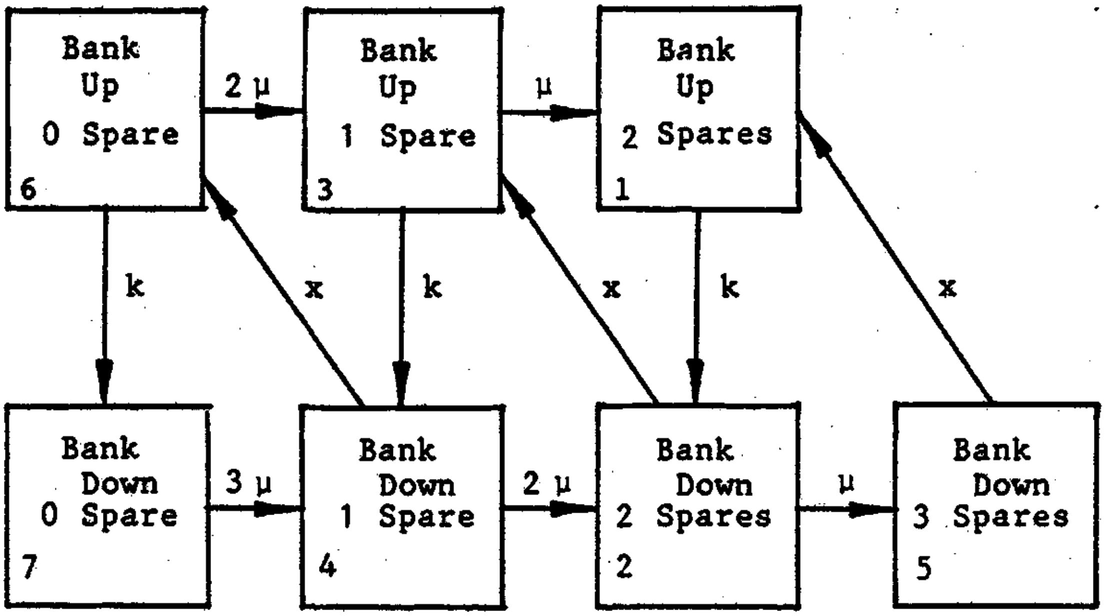

Figure 2-5: Transformer Bank With Two Spares.

The system equations are as follows:

$$P_1 k = P_3 \mu + P_5 x \tag{2.17}$$

$$P_2(\mu+x) = P_1 k + P_4 2\mu \tag{2.18}$$

$$P_3(\mu+k) = P_2 x + P_6 2\mu \tag{2.19}$$

$$P_4(2\mu+x) = P_3 k + P_7 3\mu \tag{2.20}$$

$$P_5 x = P_2 \mu \tag{2.21}$$

$$P_6(2\mu+k) = P_4 x \tag{2.22}$$

$$P_7 3\mu = P_6 k \tag{2.23}$$

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 = 1 \tag{2.24}$$

Solving Equations 2.17, 2.18, 2.19, 2.20, 2.21, 2.22 and 2.24 gives [9]

$$P_1 = (6\mu^3 x(x(x+k+3\mu)+(\mu+k)(2\mu+k))/D$$

$$P_2 = (6\mu^2 kx(\mu(2\mu+x)+k(3\mu+k))/D$$

$$P_3 = (6\mu^2 kx^2(2\mu+k+x))/D$$

$$P_4 = (3\mu k^2 x^2(2\mu+k))/D$$

$$P_5 = (6\mu^3 k(k(k+3\mu)+\mu(x+2\mu))/D$$

$$P_6 = (3\mu k^2 x^3)/D$$

$$P_7 = (k^3 x^3)/D$$

Availability A $= P_1 + P_3 + P_6$

Unavailability U $= P_2 + P_5 + P_4 + P_7$

where

$$D = k^3x^3+3\mu(4\mu^4 k + 4\mu^4 x + 6\mu^3 k^2 + 12\mu^3 kx + 6\mu^3 x^2 + 2\mu^2 k^3 + 2\mu k^3 x + 4\mu k^2 x^2 + 8\mu^2 k^2 x + 8\mu^2 kx^2 + 2\mu^2 x^3 + 2\mu kx^3 + k^3 x^2 + k^2 x^3)$$

The frequencies of encountering the system up state F(UP) and the system down state F(DN) are

$$F(UP) = (P_1+P_3+P_6)k$$

$$F(DN) = (P_4+P_2+P_5)x$$

2.4.5 Transformer Bank - Limiting Number Of Spares

As the number of spares increases, the availability of the system also increases and approaches a limiting value as the number of spares tends to infinity.

This limiting unavailability can be easily evaluated by realizing that the repair process becomes irrelevent as the number of spares increases and therefore can be ignored. The limiting case state space diagram therefore reduces to a two state model with two transition rates, system failure rate k and installation rate x. This diagram is shown in Figure 2-6.

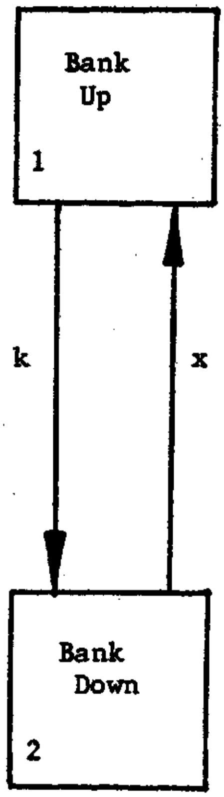

Figure 2-6: Reduced State Space Diagram With Infinite Spares.

Using frequency balance approach, it gives

$$P_1 k = P_2 x \qquad (2.25)$$

$$P_1 + P_2 = 1 \qquad (2.26)$$

Solving Equations 2.25 and 2.26 gives

Availability A $= P_1 = x/(k+x)$

Unavailability U $= P_2 = k/(k+x)$

The frequencies of encountering the up state F(UP) and the down state F(DN) are given by:

$F(UP) = P_1 k = kx/(k+x)$

$F(DN) = P_2 x = kx/(k+x)$

2.5 Monte Carlo Simulation Methods For Solving Markov Models

These are numerical methods [6, 7] for solving mathematical problems by means of random sampling. Monte Carlo simulation methods create histories of system operation by simulation using a digital computer. Availability indices can be computed from these simulated histories of the operation. The advantage of these methods is their easy use. The primary disadvantage is that it usually requires a large amount of computer time to simulate sufficient state histories to compute values for availability indices. A second disadvantage is that the calculated indices are not as precise as those obtained by analytical techniques. Analytical methods are generally preferable to simulation techniques whenever possible.

In some cases, the simulation method is the only technique available for obtaining a solution. In some cases, simulation results are evaluated to study the approximate behaviour of the system before applying sophisticated mathematical models. There is no general simulation model that can be applied to all problems. A different model may have to be devised for each problem under consideration. Many simulation models are possible for a given problem.

Two simulation methods for solving Markov systems of the type encountered in reliability evaluation are as follows:

1. Simulation method based on transition rates.
2. Simulation method based on state residence times.

2.5.1 Simulation Method Based On Transition Rates

This Monte Carlo simulation method is a specific one designed for Markov models. It uses the concept of constant transition rates from one state to another in estabilishing various availability indices. The state residence time or time duration in a state, which denotes the elapsed time that system spends in a state between entry and subsequent exit, is exponentially distributed.

The methodology used is illustrated using the example of a bank of three phase transformers with one spare. The flow chart of the simulation procedure is shown in Figure 2-7. The simulation is carried out in the following four steps:

1. Generation of random variables to determine the residence time in the present state.
2. Determination of the next state.
3. Advancing the simulation time.

4. Estimation of availability indices.

1. Generation of random variables: The time duration in a particular state is exponentially distributed. The random variables associated with the exponential distribution are generated using the following procedure:

The system subroutine RANDU was used in these studies to generate uniform random numbers between 0 and 1. The random numbers generated in this manner are called pseudorandom numbers. These are random numbers calculated by a specified rule, which is so devised that no reasonable statistical test will detect any significant departure from randomness. Given a particular uniformly distributed random number z, the variable x associated with a specified distribution F(x) (cumulative probability distribution)[10] is obtained by solving an equation

$$z = F(x).$$

The exponential distribution probability density function is

$$f(x) = \lambda e^{-\lambda x}$$

and $F(x) = 1 - e^{-\lambda x}$

$\lambda = \sum_{i=1}^{n} \lambda_i$ where n is the number of possible next states and λ_i are transition rates from the present state to i_{th} state.

Now $z = F(x)$

$$z = 1 - e^{-\lambda x}$$

$$1 - z = e^{-\lambda x}$$

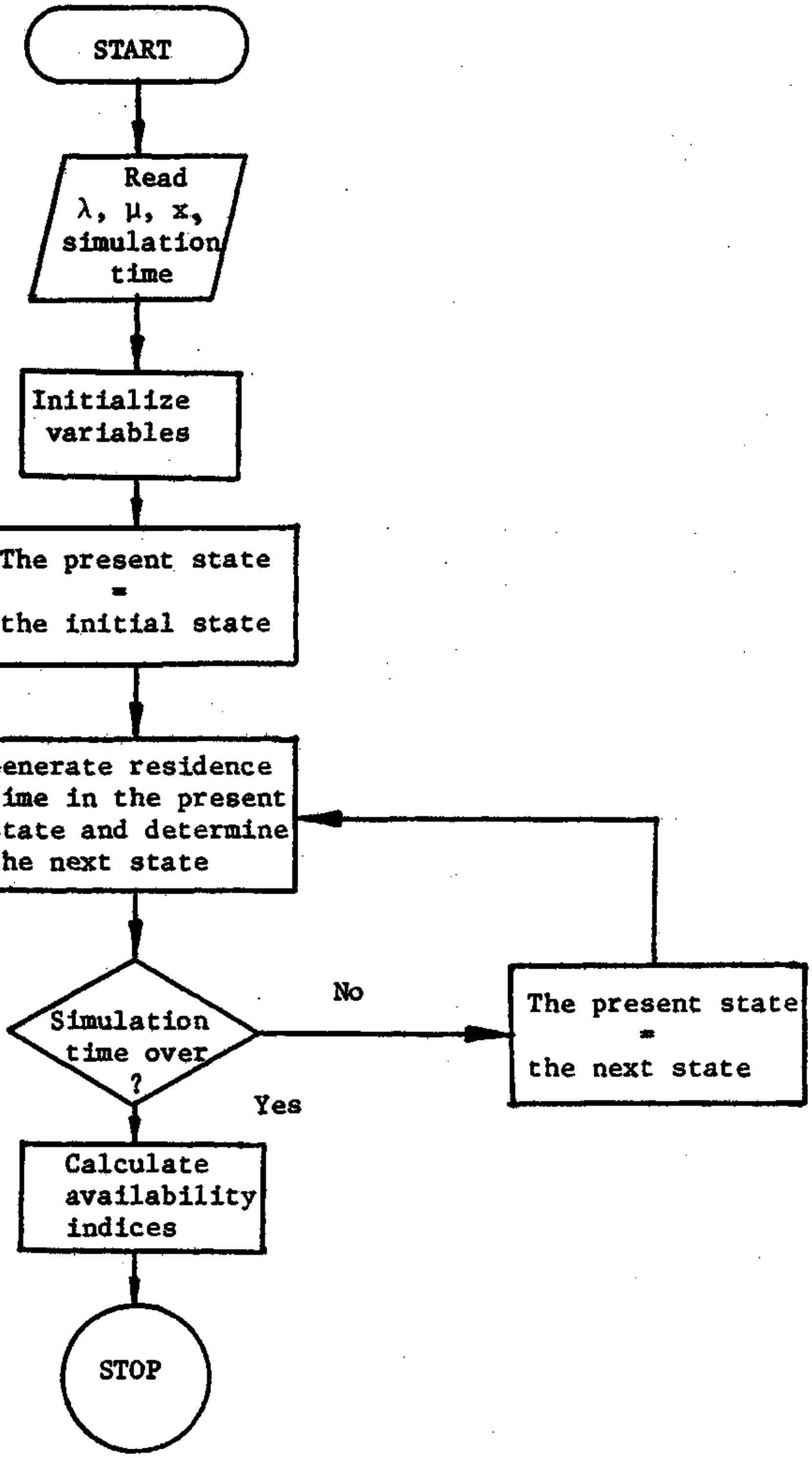

Figure 2-7: Flow Chart Of The Simulation Program Using The Transition Rate Approach.

$$z' = e^{-\lambda x} \qquad \text{where} \quad z' = 1-z$$

$$x = -(1/\lambda)(\ln z')$$

Since z is a random number,(1-z) is also a random number and x is an exponentially distributed random variable(time duration in a particular state).

2. Generation of the next state [11]: Given the system is in one state, all possible states appear with probability $\lambda_i / \sum_{i=1}^{m} \lambda_i$ where $\lambda_1, \ldots\ldots, \lambda_m$ are the transition rates out of the present state and into corresponding States $A_1, \ldots\ldots, A_m$. It is easy to generate a random number which will show which of A_j is to become the next present state in this trial. On the open interval (0,1), layout m spaces as shown in Figure 2-8. Generate a random number in (0,1) interval. If it falls in the interval labelled λ_i', choose the corresponding state as the present state. Consider the case of the three state model shown in Figure 2-8. Assume that the component is in State 1. The next state will be decided by λ_1 and λ_2 as described in the following steps:

1. Generate a uniform random number z between 0 and 1.
2. Calculate $\lambda_1 / (\lambda_1 + \lambda_2)$.
3. If $z \leqslant \lambda_1/(\lambda_1 + \lambda_2)$, the next state will be the State 2, else the next state will be the State 3.

The generation of only one random number is required to determine the next present state using this method.

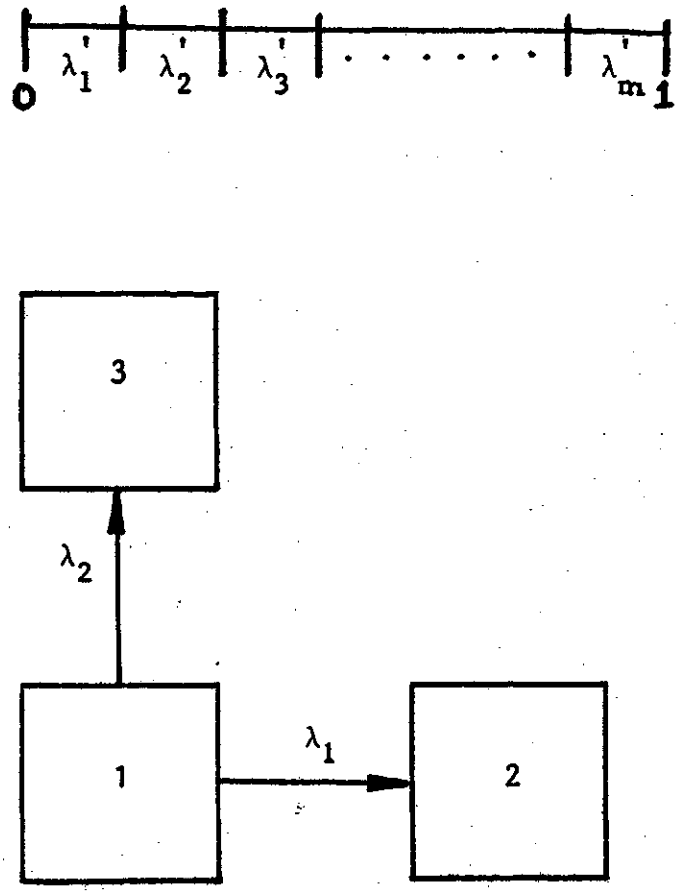

Figure 2-8: Selection Of The Next State.

3. After the next state is chosen, the current simulation time is increased by the time duration in the present state. The next state then becomes the present state. This procedure is repeated until the total simulation time is reached.

4. Various availability indices can be estimated from the data collected.

The steady state probability and frequency of a state can be estimated from the total time duration in the state, the total number of occurrences of the state and the total simulation time are as follows:

$$\text{Probability of a state} = \frac{\text{Total time duration in a state}}{\text{Total simulation time}}$$

$$\text{Frequency of a state} = \frac{\text{Total number of occurrences in a state}}{\text{Total simulation time}}$$

Unavailability = Sum of the probabilities of the appropriate down states

Frequency of encountering cumulative up(down) states =

$$\text{Sum of frequencies of individual states} - \frac{\text{Sum of number of transitions among up(down) states}}{\text{Total simulation time}}$$

This simulation methodology was used to obtain the state probabilities for the system shown in Figure 2-4 for the following set of data.

Failure rate = 0.75 f/yr

Repair rate = 12 r/yr

Installation rate = 182.5 inst/yr

The simulated and analytical results are shown in Tables 2-1 and 2-2. It can be seen that the simulation results compare favourably with the analytical results. This emphasizes that the simulation method can be applied to evaluate state probabilities without introducing significant error.

Table 2-1: State Probabilities For The System Shown In Figure 2-4 By Simulation Using The State Transition Rate Approach.

Simulation time = 200,000 years
Failure rate = 0.75 f/yr
Repair rate = 12 r/yr
Intallation rate = 182.5 inst/yr

State Values

State	Probability	Frequency
1	0.939489	0.702029
2	0.003824	0.742764
3	0.054745	0.698409
4	0.001705	0.040740
5	0.000238	0.044355

Capacity Values

State	Probability	Frequency
UP	0.994234	0.742769
DOWN	0.005766	0.742764

Table 2-2: State Probabilities For The System Shown In Figure 2-4 Using The Analytical Approach.

State Values

State	Probability	Frequency
1	0.939325	0.704494
2	0.003834	0.745650
3	0.054874	0.699645
4	0.001715	0.041156
5	0.000252	0.046004

Capacity Values

State	Probability	Frequency
UP	0.994199	0.745650
DOWN	0.005801	0.745650

2.5.2 Monte Carlo Simulation Method Based On State Residence Times

This simulation method is a general method of simulating repairable systems. The same methodology can be applied to evaluate non-Markovian systems.

The methodology is illustrated using the example of a three phase transformer bank with one spare. The flow chart of the simulation procedure is shown in Figure 2-9.

The simulation in this method is carried out in four steps:

1. Generation of up and down times.
2. Determination of the next state.
3. Advancing the simulation time and checking to see if the simulation time is over.
4. Evaluating availability indices.

1. Generation of up and down times: The IMSL subroutine [12] GGEXN is used to generate a random variate for the up times and down times. Mean failure, repair or installation times are given by 1/k, 1/μ,1/x respectively.

k = Failure rate per unit time

μ = Repair rate per unit time

x = Installation rate per unit time

2. Determination of the next state: In this method, the times of occurrences of all possible future events are generated and the event with the smallest time of occurrence

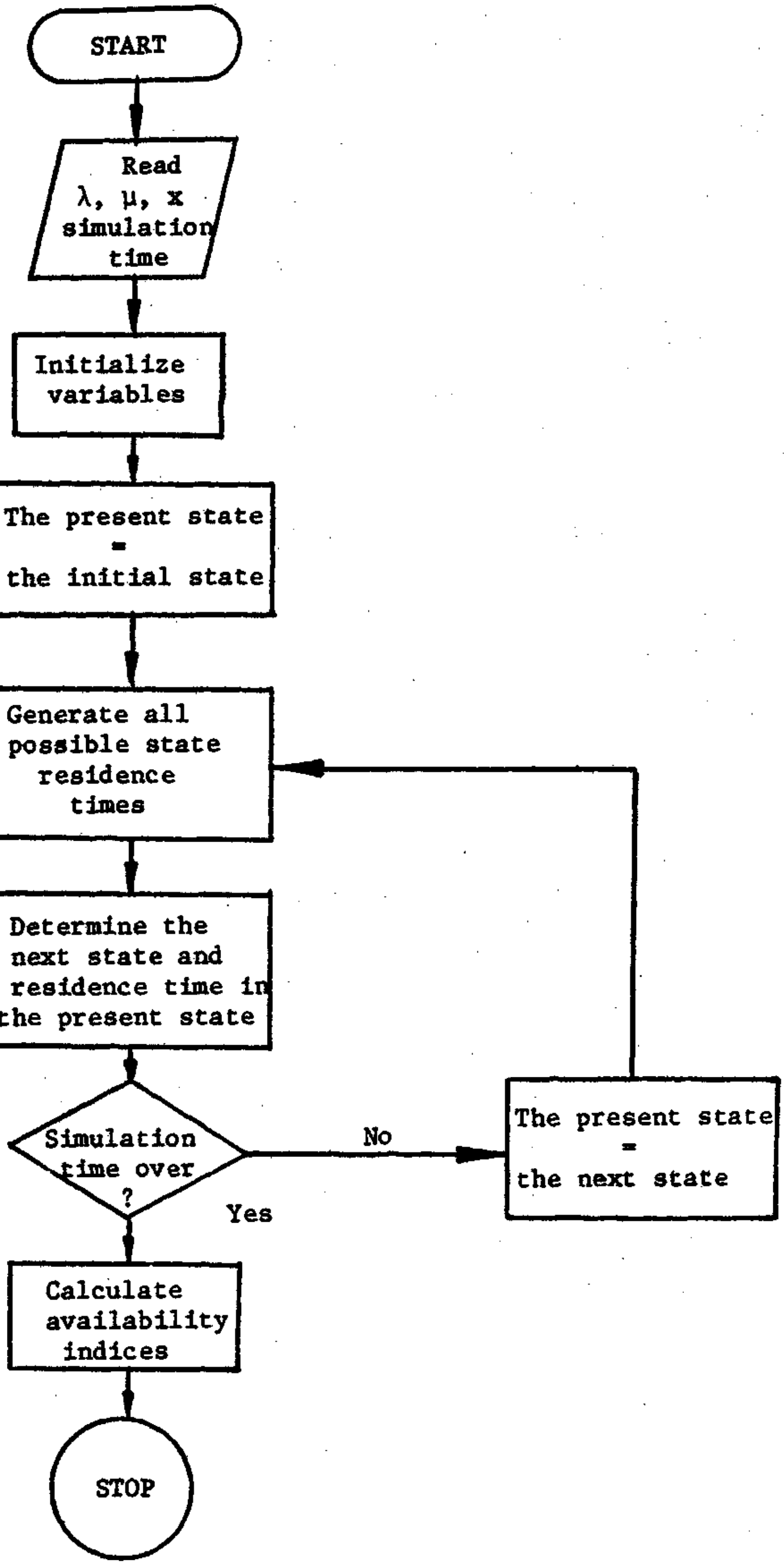

Figure 2-9: Flow Chart Of The Simulation Program Using The State Residence Time Method.

becomes the next event and the resulting state becomes the next state.

3. After the next state is chosen, the current simulation time is increased by the amount of time duration in the present state. The next state then becomes the present state for the next step. The above procedure is repeated until the total simulation time is reached.

4. The availability indices are evaluated using an identical procedure to that used in the state transition rate approach.

The simulation method was used to evaluate the state probabilities for the system shown in Figure 2-4. The following data were used to compare the simulated and analytical state probabilities.

Failure rate = 0.75 f/yr

Repair rate = 12 r/yr

Installation rate = 182.5 inst/yr

The results are shown in Tables 2-3 and 2-4. It can be seen from the results that there is very little difference between the simulated and analytical state probabilities. The simulation method can be used interchangeably with analytical technique.

Table 2-3: State Probabilities For The System Shown In Figure 2-4 Using The State Residence Time Approach.

Simulation time = 200,000 years
Failure rate = 0.75 f/yr
Repair rate = 12 r/yr
Installation rate = 182.5 inst/yr

State Values

State	Probability	Frequency
1	0.939219	0.702069
2	0.003824	0.743094
3	0.055001	0.697084
4	0.001705	0.041029
5	0.000252	0.046009

Capacity Values

State	Probability	Frequency
UP	0.994220	0.743099
DOWN	0.005780	0.743094

Table 2-4: State Probabilities For The System Shown In Figure 2-4 Using The Analytical Approach.

State Values

State	Probability	Frequency
1	0.939325	0.704494
2	0.003834	0.745650
3	0.054874	0.699645
4	0.001715	0.041156
5	0.000252	0.046004

Capacity Values

State	Probability	Frequency
UP	0.994199	0.745650
DOWN	0.005801	0.745650

2.6 Length of The Simulation Time- A Comparison Of Analytical And Simulation Results

The desired length of the simulation period is a difficult question to answer. Sufficient state histories have to be produced in order to obtain reasonable estimates of the availability indices.

Simulation was carried out for the following values of failure rate, repair rate and installation rate.

Failure rate = 0.25 ,0.5 f/yr

Repair rate = 12 ,24 ,36 ,48 r/yr

Installation rate = 182.5 ,365 inst/yr

The example chosen is the one spare model shown in Figure 2-4. Simulation intervals of 1000, 10000, 50000, 100000 and 200000 years were selected. It can be observed from the results that the precision of the simulation method increases as the simulation time increases. It can be seen that there is no significant deviation in the results for 100000 years and 200000 years simulations. In Table 2-5, the simulation results using the transition rate approach for 200000 years are close to the analytical results. In Table 2-5, the simulation results using the state residence times method approach the analytical results at a simulation time of 100000 years.

Table 2-5: Comparison Of Analytical Unavailability With Simulated Unavailability For The system Shown In Figure 2-4 Using The Transition And State Residence Time Approaches For Several Simulation Times.

Failure rate f/yr	Repair rate r/yr	Inst. rate inst/yr	Analytical Unav.	Simulation time In Years: 1000 Unav.	10,000 Unav.	50,000 Unav.	100,000 Unav.	200,000 Unav.
Transition Rate Approach								
0.25	12.0	182.5	0.001567	0.001916	0.001680	0.001549	0.001542	0.001548
0.25	24.0	182.5	0.001415	0.001517	0.001336	0.001394	0.001408	0.001412
0.25	36.0	182.5	0.001388	0.001348	0.001283	0.001334	0.001365	0.001378
0.25	48.0	182.5	0.001379	0.001540	0.001296	0.001333	0.001365	0.001371
0.50	12.0	182.5	0.003509	0.003906	0.003458	0.003456	0.003484	0.003470
0.25	12.0	365.0	0.000890	0.000981	0.000916	0.000872	0.000882	0.000889
State Residence Time Approach								
0.25	12.0	182.5	0.001567	0.001186	0.001494	0.001535	0.001558	0.001554
0.25	24.0	182.5	0.001415	0.001535	0.001419	0.001411	0.001408	0.001414
0.25	36.0	182.5	0.001388	0.001263	0.001420	0.001404	0.001385	0.001394
0.25	48.0	182.5	0.001379	0.001149	0.001391	0.001386	0.001381	0.001373
0.50	12.0	182.5	0.003509	0.002924	0.003358	0.003490	0.003501	0.003486
0.25	12.0	365.0	0.000890	0.000887	0.000983	0.000903	0.000903	0.000883

2.7 The Effect Of Spares On Unavailability

In order to study the effect of spares on unavailability, the transformer bank example used earlier was chosen.

The results in this study were obtained by the Monte Carlo simulation method based on transition rates and also by analytical techniques. The results obtained by simulation method were quite close to the results obtained by the analytical method.

The analytical approach was used to analyze a system with up to two spares and the limiting case for an infinite number of spares. It can be seen from the equations for the two spares case, that it is not practical to derive general expressions for more complex systems.

The simulation results are in close agreement with the analytical results for the one spare model shown in Table 2-2. A similar comparison of unavailabilities for an increasing number of spares is shown in Tables 2-6, 2-7 and 2-8. The effect of further increase in spares was studied using analytical and simulation approaches. It can be seen from the Tables 2-6, 2-7 and 2-8 that the system unavailability approaches a limiting value as the number of spares increases. The results shown in Tables 2-6, 2-7 and 2-8 for unavailability analysis of a three phase transformer bank were predicated on the following basic data.

Case 1 - Table 2-6

System unavailability as a function of number of spares with variable failure rate.

Repair rate = 12 r/yr

Installation rate = 182.5 inst/yr

Failure rate = 0.25, 0.5, 0.75, 1.0 f/yr

The results are shown graphically in Figure 2-10.

Case 2 - Table 2-7

System unavailability as a function of number of spares with variable repair rate.

Failure rate = 0.25 f/yr

Installation rate = 182.5 inst/yr

Repair rate = 12, 24, 36, 48 r/yr

The results are shown graphically in Figure 2-11.

Case 3 - Table 2-8

System unavailability as a function of the number of the spares with variable installation rate.

Failure rate = 0.25 f/yr

Repair rate = 12 r/yr

Installation rate = 365, 182.5, 91.25, 45.625 inst/yr

The results are shown graphically in Figure 2-12.

Figures 2-10, 2-11 and 2-12 show the results obtained by both analytical and simulation methods. Analytical and simulation results are shown up to two spares and for the limiting case. It can be seen that for this system and for this set of basic data, the system unavailability approaches the limiting value after two spares. The simulation results are close to those obtained using the analytical approach.

The simulation technique is quite flexible and provides a general tool for the examination of the system unavailability. This approach is used further in this thesis for the examination of more complex situations.

2.8 Conclusion

This chapter has presented some basic techniques used for Markov system modelling of repairable systems with spare components. The emphasis is on the application of these techniques to practical system unavailability problems. The analytical approach provides precise solutions and was used to evaluate availability indices of a system with up to two spares and the limiting case for an infinite number of spares. The derivation of the required expressions becomes increasingly difficult as the number of states in the model increases. Two simulation methods based on transition rates and state residence times have been presented by illustration. The results obtained by both techniques are in close agreement with those obtained by the analytical approach.

Simulation techniques are not as precise as analytical methods. They do, however, provide a general framework to evaluate system reliability parameters. The simulation method based on the transition rates assumes constant transition rates from one state to another state and this can be applied only to Markovian models. The technique based on state residence times is a general approach for solving Markovian as well as non-Markovian systems. This technique is used in the next chapter for the solution of systems with non-exponential state residence times.

Table 2-6: Comparison Of Unavailabilities For Increasing Number Of Spares As A Function Of The Failure Rate Using Simulation And Analytical Methods.

Repair rate = 12 r/yr
Installation rate = 182.5 inst/yr
Simulation time = 200,000 years

Number of Spares	Simulated Unavailability	Number of Spares	Analytical Unavailability
Failure rate = 0.25 f/yr			
0	0.021872	0	0.021721
1	0.001548	1	0.001567
2	0.001354	2	0.001369
Infinity	0.001366	Infinity	0.001368
Failure rate = 0.5 f/yr			
0	0.042726	0	0.042518
1	0.003470	1	0.003509
2	0.002730	2	0.002742
Infinity	0.002736	Infinity	0.002732
Failure rate = 0.75 f/yr			
0	0.062587	0	0.062450
1	0.005766	1	0.005801
2	0.004100	2	0.004124
Infinity	0.004087	Infinity	0.004093
Failure rate = 1.0 f/yr			
0	0.081912	0	0.081568
1	0.008401	1	0.008415
2	0.005492	2	0.005522
Infinity	0.005439	Infinity	0.005450

Table 2-7: Comparison Of Unavailabilities For Increasing Number Of Spares As A Function Of The Repair Rate Using Simulation And Analytical Methods.

Failure rate = 0.25 f/yr
Installation rate = 182.5 inst/yr
Simulation time = 200,000 years

Number of Spares	Simulated Unavailability	Number of Spares	Analytical Unavailability
	Repair rate = 12 r/yr		
0	0.021872	0	0.021721
1	0.001548	1	0.001567
2	0.001354	2	0.001369
Infinity	0.001366	Infinity	0.001368
	Repair rate = 24 r/yr		
0	0.011732	0	0.011649
1	0.001412	1	0.001415
2	0.001376	2	0.001368
Infinity	0.001366	Infinity	0.001368
	Repair rate = 36 r/yr		
0	0.008309	0	0.008246
1	0.001378	1	0.001388
2	0.001360	2	0.001368
Infinity	0.001366	Infinity	0.001368
	Repair rate = 48 r/yr		
0	0.006585	0	0.006535
1	0.001371	1	0.001379
2	0.001360	2	0.001368
Infinity	0.001366	Infinity	0.001368

Table 2-8: Comparison Of Unavailabilities For Increasing Number Of Spares As A Function Of The Installation Rate Using Simulation And Analytical Methods.

Failure rate = 0.25 f/yr
Repair rate = 12 r/yr
Simulation time = 200,000 years

Number of Spares	Simulated Unavailability	Number of Spares	Analytical Unavailability
Installation rate = 91.25 inst/yr			
0	0.023193	0	0.023030
1	0.002911	1	0.002919
2	0.002728	2	0.002733
Infinity	0.002728	Infinity	0.002732
Installation rate = 182.5 inst/yr			
0	0.021872	0	0.021721
1	0.001548	1	0.001567
2	0.001354	2	0.001369
Infinity	0.001366	Infinity	0.001368
Installation rate = 273.75 inst/yr			
0	0.021431	0	0.021284
1	0.001104	1	0.001116
2	0.000910	2	0.000914
Infinity	0.000911	Infinity	0.000912
Installation rate = 365 inst/yr			
0	0.021210	0	0.021065
1	0.000889	1	0.000890
2	0.000683	2	0.000686
Infinity	0.000683	Infinity	0.000684

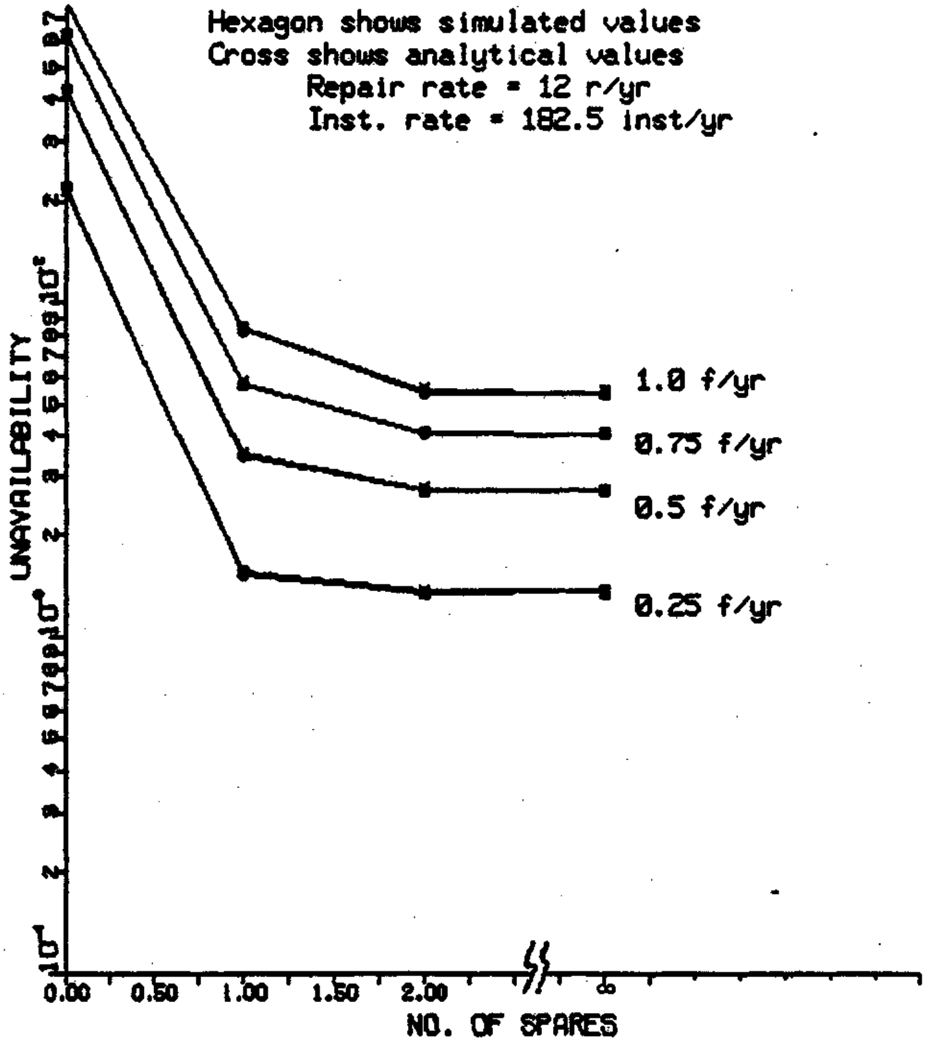

Figure 2-10: System Unavailability As A Function Of The Number Of Spares With Variable Failure Rate.

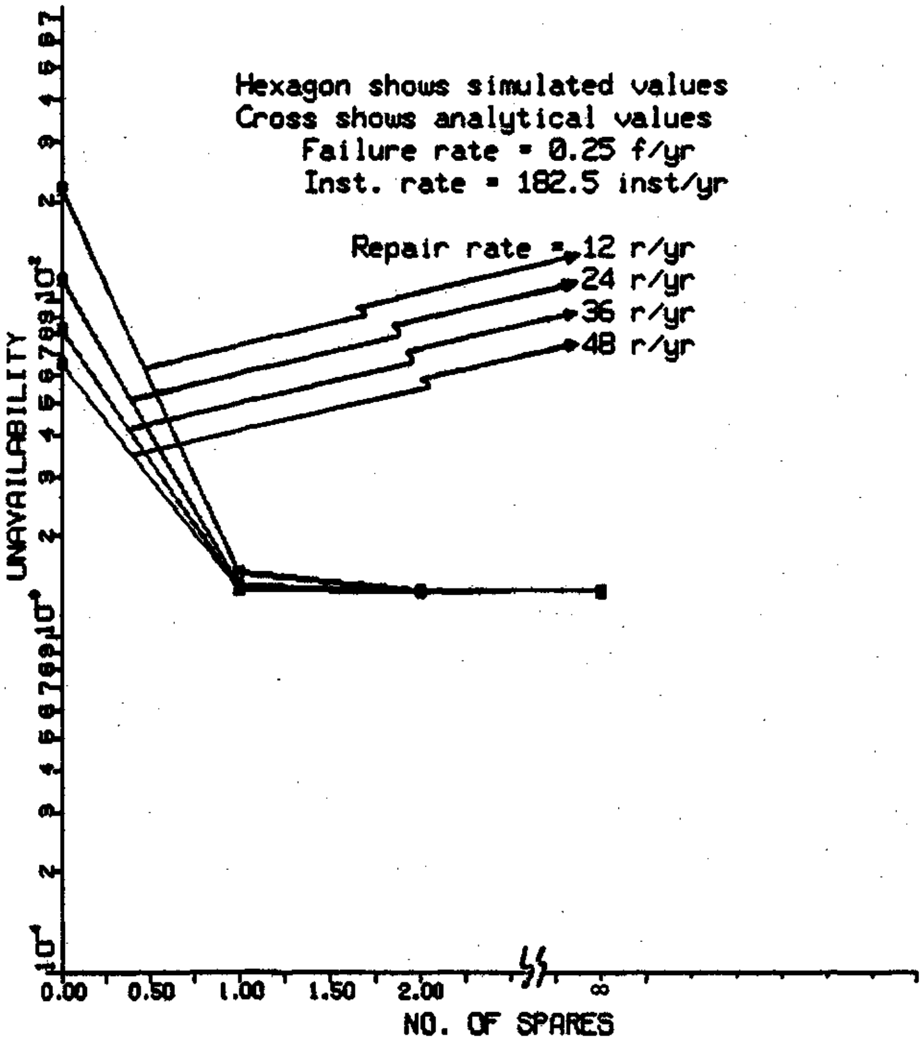

Figure 2-11: System Unavailability As A Function Of The Number Of Spares With Variable Repair Rate.

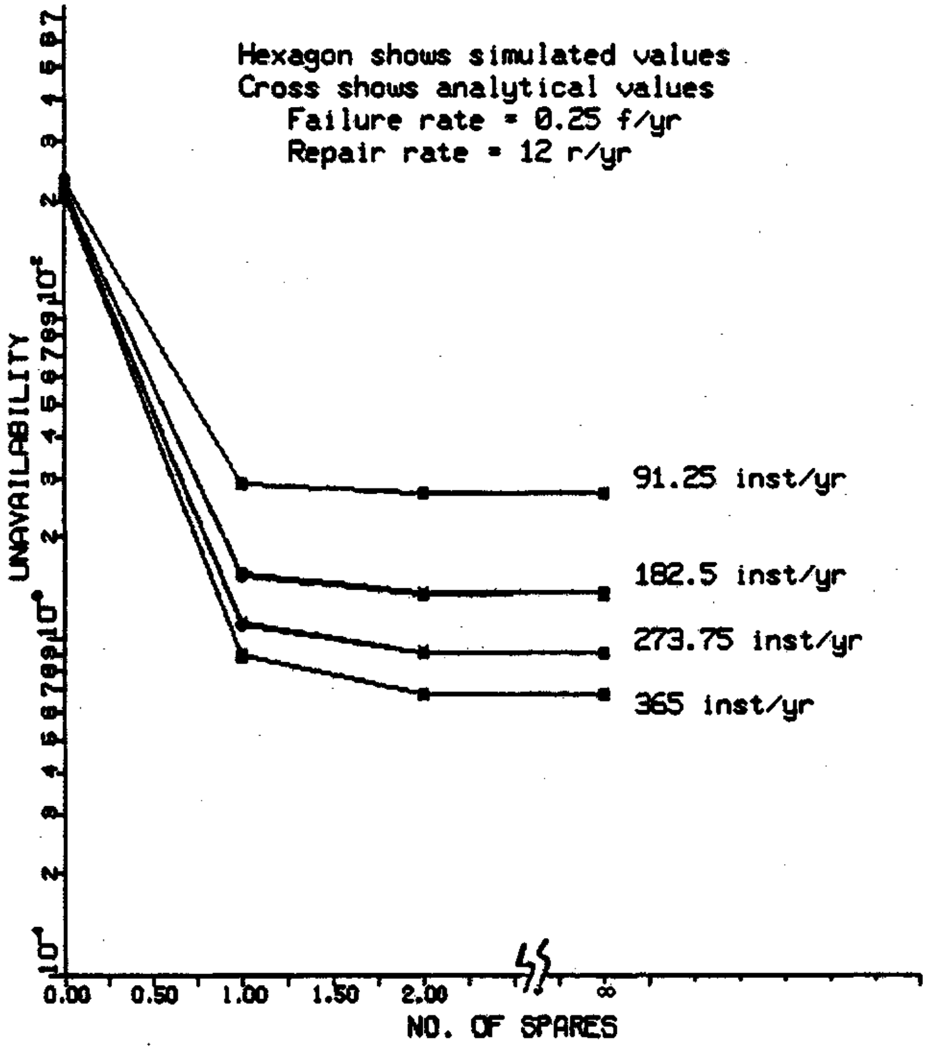

Figure 2-12: System Unavailability As A Function Of The Number Of Spares With Variable Installation Rate.

3. NON-MARKOV MODELLING OF REPAIRABLE SYSTEMS

3.1 Introduction

In most reliability evaluation studies, it is assumed that the underlying distributions of the component state residence times are exponential. In these systems, the transition rate from one state to another state of the system is constant and does not depend on how long the system spends in the given state nor does it depend on how it arrived at the particular state. This assumption is valid for the failure events of engineering systems in which the components are in their useful life periods. In many practical applications, however, the assumption of an exponential distribution associated with repair and installation times may not be valid. Such systems are termed non-Markovian processes.

The techniques that are used to evaluate Markovian systems cannot, in general, be easily applied to solve non-Markovian systems. The analysis of non-Markovian systems, in general, is much more difficult. There are three basic analytical techniques used for reliability modelling of systems with non-Markovian characteristics. These techniques are: the method of stages, the method of supplementary variables and the method of semi-Markov processes.

The available analytical techniques are usually of limited application in practical problems. In these situations, Monte Carlo simulation methods are quite popular for reliability modelling. Two techniques for solving non-Markovian systems, the method of stages and the Monte Carlo simulation method based on state residence times are discussed in this chapter. The results of the analytical technique i.e. the method of stages are used as a reference to evaluate the effectiveness of the simulation method which is then used as the main methodology in solving non-Markovian systems.

3.2 Discussion Of A Non-Markovian Model For A Transformer Bank With One Spare

The non-Markovian model of a transformer bank with one spare is shown in Figure 3-1. It is assumed that if one transformer fails, the entire bank is shut down. A similar model was discussed in the previous chapter where it was assumed that the system is Markovian, i.e. all processes (failure, repair and installation) follow exponential distributions. The transition rates k,μ and x were assumed to be constant with respect to time. In this case, the transition rates from one state to another do not depend upon the time spent in a particular state. In the non-Markovian model shown in Figure 3-1, it is assumed that the failure process can be described by an exponential distribution. The repair and installation times are now represented by non-exponential distributions. These times are assumed to follow other distributions i.e. Weibull, normal, Erlang distributions etc. The interstate transition rates are no

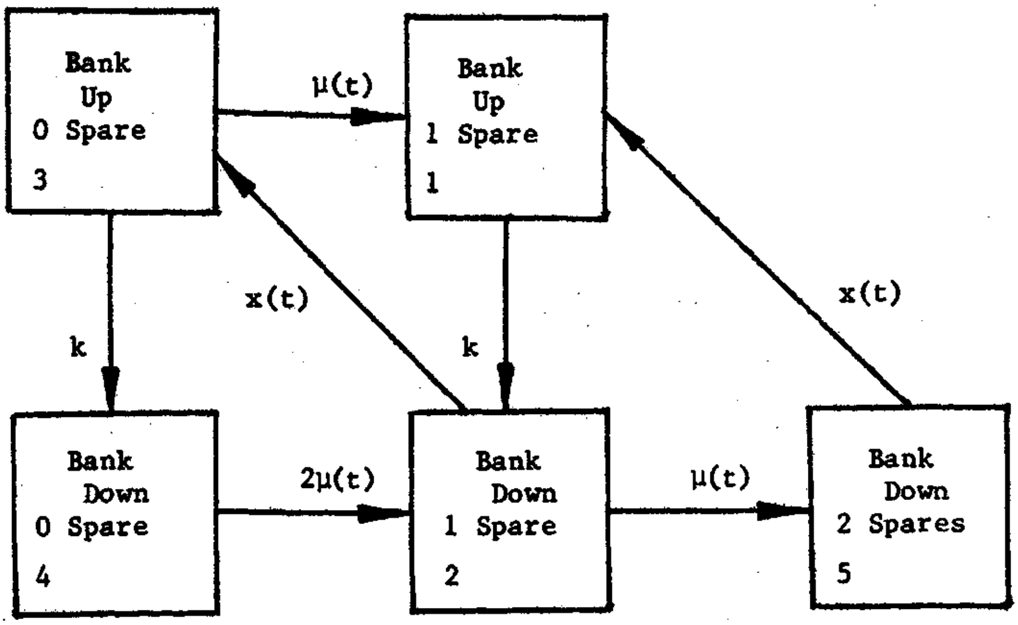

Figure 3-1: Non-Markovian Model Of Transformer Bank With One Spare Unit

longer constant in the case of the repair and installation processes. The transition rates become a function of the time spent in the given state. The analysis of this model is difficult as the associated stochastic processes become non-Markovian.

3.3 Main Distributions Used In Reliability Studies

The main distributions [1, 13] commonly used to describe non-exponential state residence times are as follows:

1. Weibull distribution
2. Normal distribution
3. Erlang distribution

Many authors have proposed the Weibull distribution as being the most suitable distribution in reliability evaluation because it is flexible and is capable of expressing many other distributions. The Erlang distribution is a special case of the Gamma distribution. The normal distribution is a symmetrical distribution often used to represent down times. A main objective of this thesis is to observe the effect of various distributions on the total system unavailability. This effect is illustrated by the practical example of the one spare model shown in Figure 3-1.

3.3.1 Weibull Distribution

This distribution was created by W.Weibull in 1951. The distribution has no specified characterstic shape so it can be widely applied to many different physical phenomena. Depending upon the values of the parameters involved, the density function can be shaped to represent many distributions.

The failure density function of the Weibull distribution is as follows:

$$f(t) = \frac{\beta\, t^{\beta-1}}{\alpha^{\beta}}\, e^{-(t/\alpha)^{\beta}}$$

where $t \geqslant 0, \alpha > 0$, and $\beta > 0$

t = time, β = the shape parameter(dimensionless)

α = the scale parameter(unit of time)

The survival function is

$$R(t) = \int_t^\infty f(t).dt = \exp[-(t/\alpha)^{\beta}].$$

The cumulative failure distribution is

$$Q(t) = 1 - R(t) = 1 - \exp[-(t/\alpha)^{\beta}].$$

The hazard rate is

$$h(t) = \frac{f(t)}{R(t)} = \frac{\beta\, t^{\beta-1}}{\alpha^{\beta}}.$$

i.e h(t) is a function of time and depending on the value of β, it can take many shapes.

Effect of the shape parameter on reliability functions.

For $\beta = 0.5$

$$f(t) = \frac{0.5\, t^{0.5-1}}{\alpha^{0.5}}\, e^{-(t/\alpha)^{0.5}}$$

$$f(t) = \frac{1}{2\sqrt{\alpha\, t}}\, e^{-(t/\alpha)^{0.5}}$$

$$R(t) = e^{-(t/\alpha)^{0.5}}$$

$$h(t) = \frac{1}{2\sqrt{\alpha t}}$$

In the case of $\beta = 0.5$, the Weibull distribution takes a hyperexponential shape and h(t) is decreasing with time as shown in Figure 3-2.

For $\beta = 1$

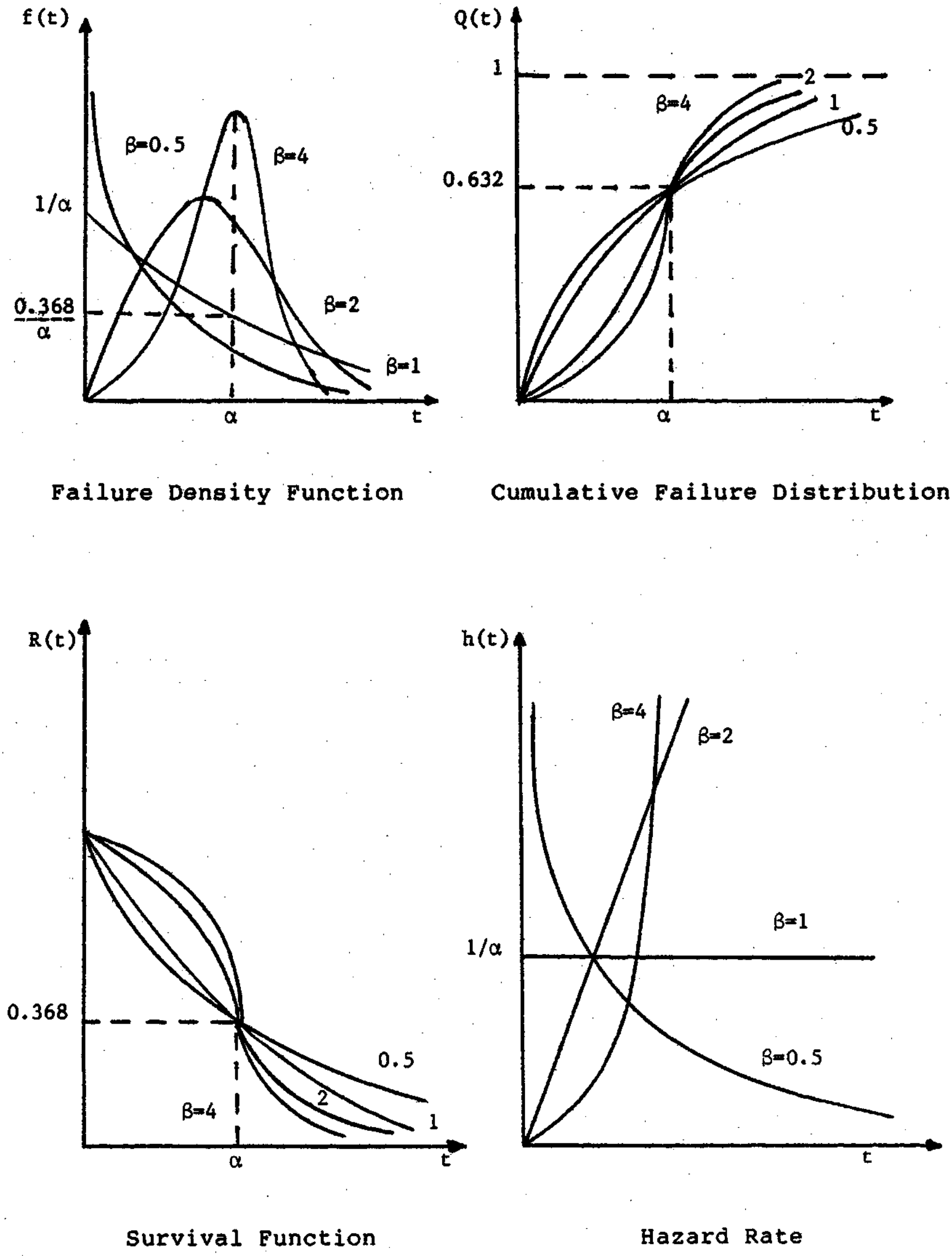

Figure 3-2: Weibull Reliability Functions

$$f(t) = (1/\alpha) \cdot e^{-t/\alpha}$$

$$f(t) = \lambda e^{-\lambda t} \quad \text{if } \alpha = 1/\lambda$$

$$R(t) = e^{-t/\alpha} = e^{-\lambda t}$$

$$h(t) = 1/\alpha$$

i.e. the hazard rate is constant. In the case of $\beta = 1$, the Weibull distribution is exponential and its shape is shown in Figure 3-2.

For $\beta = 2$

$$f(t) = \frac{2t}{\alpha^2} e^{-(t^2/\alpha^2)}$$

$$R(t) = e^{-t^2/\alpha^2}$$

$$h(t) = \frac{2t}{\alpha^2} = kt \quad \text{if } k = 2/\alpha^2$$

i.e. hazard rate is increasing linearly with time at a slope given by k.

Thus for $\beta = 2$, the Weibull distribution reduces to the Rayleigh distribution. The shape is shown in Figure 3-2.

For $\beta = 4$

$$f(t) = \frac{4t^3}{\alpha^4} e^{-t^4/\alpha^4}$$

$$R(t) = e^{-t^4/\alpha^4}$$

$$h(t) = \frac{4t^3}{\alpha^4}$$

In this case, the hazard rate takes a parabolic shape as shown in Figure 3-2.

It can be concluded from the Figure 3-2 that if $0 < \beta < 1$, the hazard rate is a decreasing function of time and the distribution takes a hyperexponential shape. When $\beta = 1$, the hazard rate is constant and the distribution becomes exponential. When $\beta > 1$, the hazard rate is increasing with time. The failure density function becomes bell-shaped like the normal distribution at $\beta \simeq 3.5$. For $\beta > 3.5$, the distribution becomes negatively skewed.

Mean $m = \alpha\Gamma(1+1/\beta)$

where $\Gamma(n) = \int_0^\infty x^{n-1}.e^{-x}.dx$, called the Gamma function.

Variance $\sigma^2 = \alpha^2[\,\Gamma(1+2/\beta) - (\Gamma(1+1/\beta))^2]$

3.3.2 Normal Distribution

The normal distribution was introduced in 1773 by De Moivre, during the study of the pattern in errors of measurements which were observed to follow a symmetrical bell-shaped distribution. In 1809, Gauss first published reference to it and therefore it is also called the Gaussian distribution. The normal distribution is often used in reliability studies.

The probability density function of the normal distribution is given by

$$f(t) = f(t;m,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-((t-m)^2/2\sigma^2)}$$

where $-\infty < t < +\infty$, $-\infty < m < +\infty$, $\sigma > 0$

m = location parameter, σ = scale parameter

Mean = m, Variance = σ^2

Figure 3-3 shows f(t) for various values of σ. All plots are symmetrical about m and have the bell shape i.e. the distribution has no shape parameter.

The cumulative probability distribution is

$$Q(t) = F(t;m,\sigma) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}\sigma} e^{-(t-m)^2/2\sigma^2}\, dt$$

This integral cannot be expressed in a simple functional form and is not amenable to simple integration techniques. In order to make the cumulative probability distribution available for use in tabular form a random variable z is defined as $z=(t-m)/\sigma$ for which $m = 0$ and $\sigma = 1$.

$$\text{So } Q(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{(z^2/2)}\, dz$$

z is known as the standardized or standard normal variable. Now if Q(t) is a failure distribution function then

$$R(t) = 1 - Q(t)$$

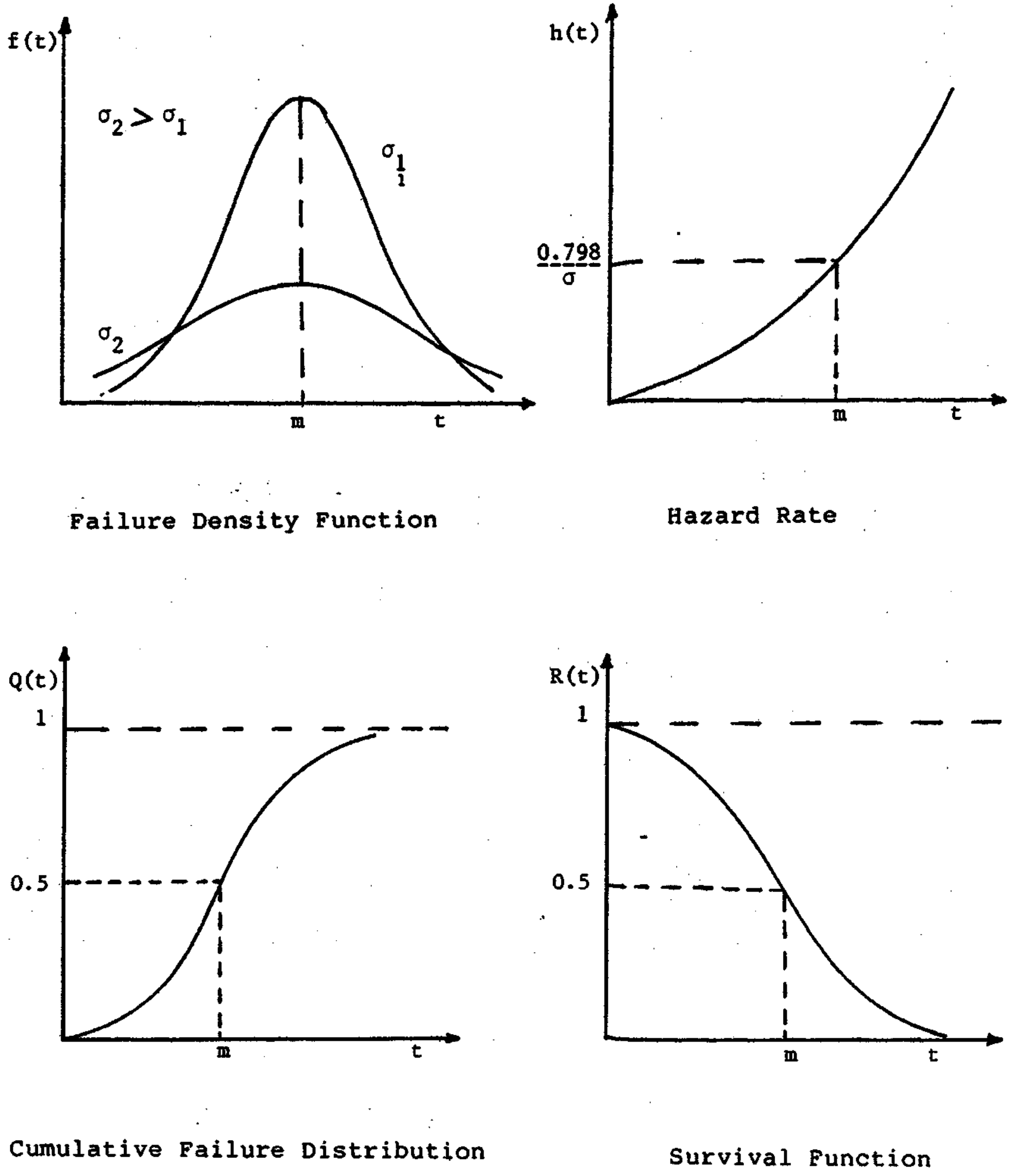

Figure 3-3: Normal Reliability Functions.

$$h(t) = \frac{f(t)}{R(t)}$$

The plots of f(t), Q(t), R(t) and h(t) are shown in Figure 3-3.

3.3.3 Erlang Distribution

Another two parameter distribution used in reliability studies is the Erlang distribution which is a special case of the Gamma distribution.

The failure density function for the Gamma distribution is given by

$$f(t) = \frac{t^{\beta-1}}{\alpha^{\beta}\,\Gamma(\beta)} e^{-t/\alpha}$$

or

$$f(t) = \frac{t^{\beta-1}\ \lambda^{\beta}}{\Gamma(\beta)} e^{-\lambda t} \qquad \text{if } \alpha = 1/\lambda$$

where $t > 0$, $\alpha > 0$ (or $\lambda > 0$) and $\beta > 0$

t = time, β = the shape parameter(dimensionless)

α = the scale parameter(unit of time)

The Gamma function given by $= \int_0^{\infty} t^{\beta-1} e^{-t} dt$.

For β = positive integer $\Gamma(\beta) = (\beta-1)!$

When β = positive integer

$$f(t) = \frac{\lambda(\lambda t)^{\beta-1}}{(\beta-1)!} e^{-\lambda t} \quad \text{where } \lambda = 1/\alpha \ .$$

This is the Erlang distribution for which

$$R(t) = e^{-\lambda t} \sum_{i=1}^{\beta-1} \frac{(\lambda t)^{i}}{(i)!}$$

$$h(t) = f(t) / R(t)$$

$$Q(t) = 1 - R(t)$$

The f(t), Q(t), R(t) and h(t) are shown in Figure 3-4.

Expected Value = Mean = m $= \beta/\lambda = \alpha\,\beta$

Variance = $\sigma^2 = \alpha^2\beta$

This section has illustrated the important distributions which are used extensively in reliability studies.

In many practical situations, the available data suggests that hazard rates are not constant. These functions vary with time and can assume many shapes. The previous noted distributions can be used to model most non-Markovian systems depending upon the variability of the hazard rate function.

The following sections give a brief description of two basic techniques. These two methods used to analyze non-Markovian systems are the Monte Carlo simulation method based upon the state residence times and an analytical technique based upon the method of stages.

3.4 Monte Carlo Simulation Method For Non-Markovian System Reliability Evaluation

This simulation method is based on the state residence times. The simulation is carried out in four basic steps:

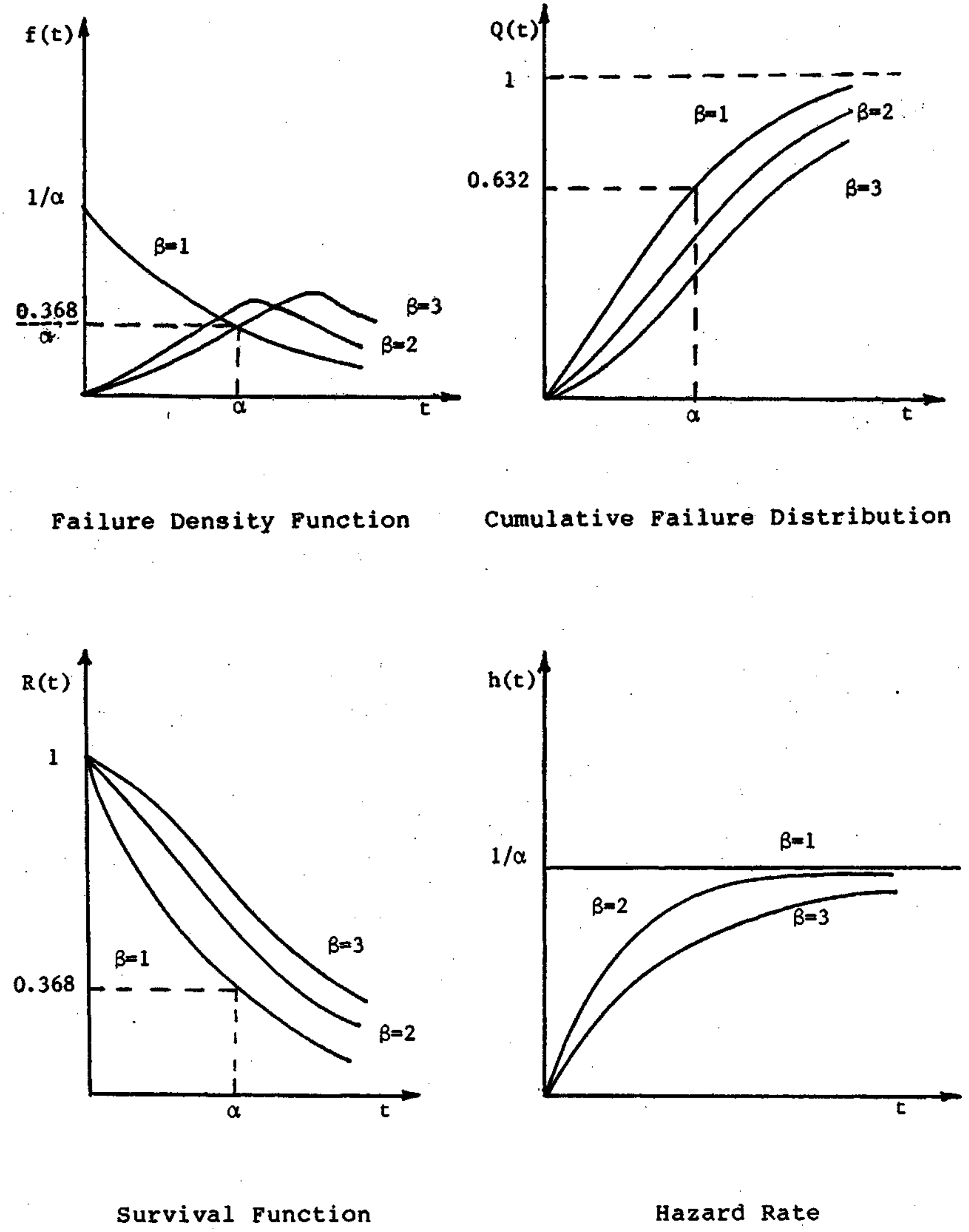

Figure 3-4: Erlang Reliability Functions.

1. Generation of Up and Down times.
2. Determination of the next state.
3. Advancing the simulation time.
4. Evaluating the system unavailability.

All four steps enumerated are illustrated using the transformer bank example shown in Figure 3-1. The flowchart of the simulation procedure is given in Figure 3-5.

1. Generation of Up and Down times: In the transformer bank model shown in Figure 3-1, the up time is assumed to be exponentially distributed. The repair and installation times can follow any distribution. The generation of random variables from three distributions, Weibull, normal and Erlang are considered in connection with this case. Additional information regarding each simulation subroutine [12] is provided in Appendices.

Generation of Up time:

The IMSL subroutine GGEXN is used to generate random variables for the up time. The mean failure time is calculated from the failure rate i.e. $1/\lambda$.

Generation of repair and installation times:

a. Weibull distribution: The IMSL subroutine GGWIB is used to generate random variables for the repair and installation times. The mean m is calculated from the formula

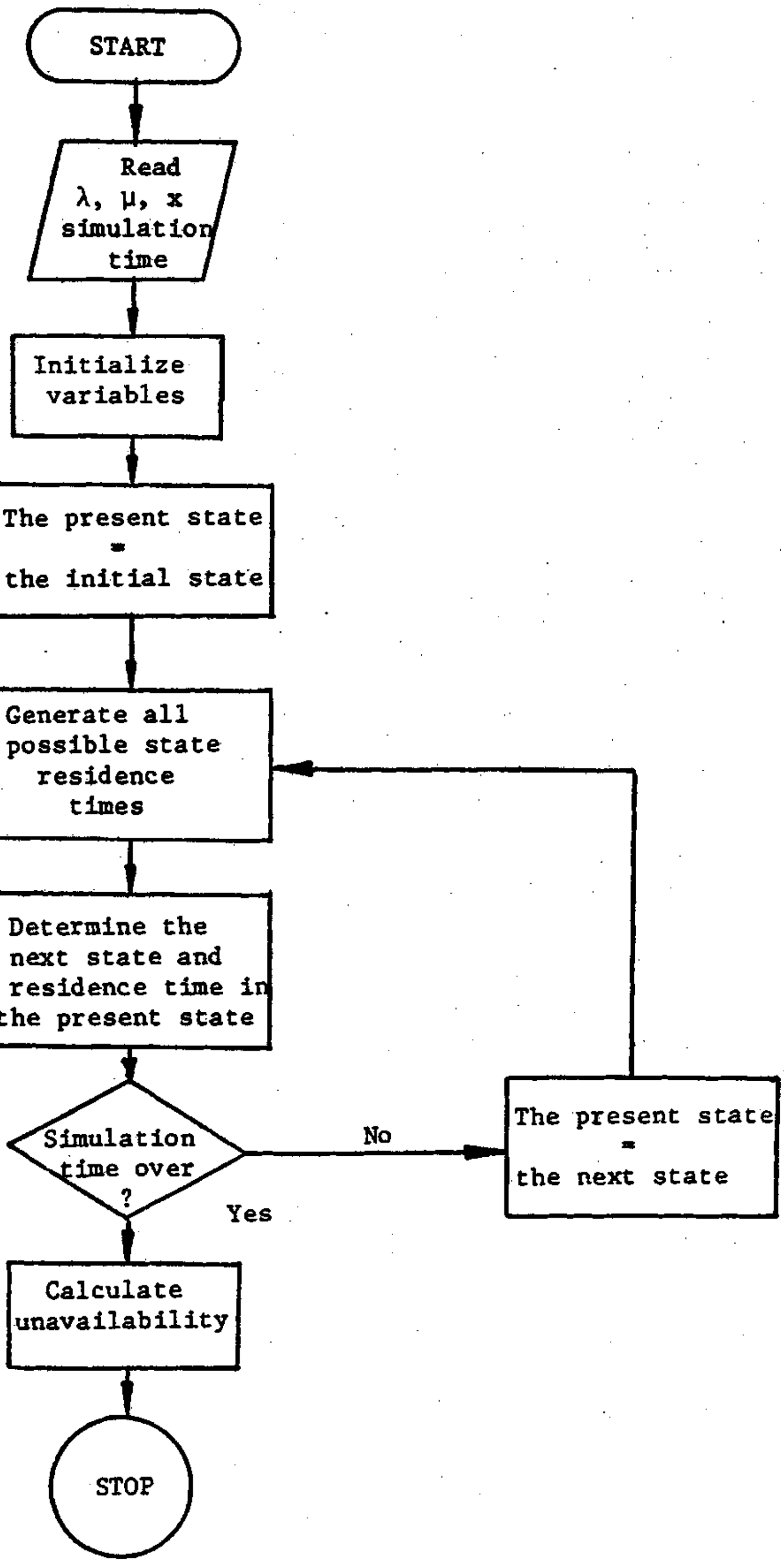

Figure 3-5: Flowchart Of Simulation Procedure For Non-Markovian Systems Using The State Residence Time Approach.

$m = \alpha\,\Gamma(1+1/\beta)$

α = scale parameter, β = shape parameter

α and β are varied to generate various random variables.

b. Normal distribution: The IMSL subroutine GGNML is used to generate a random normal variate between 0 and 1. The mean m and standard deviation σ are used to find the normal random variable

$x = \sigma z + m$ where z is a normal variate between 0 and 1.

x = normal variate with mean m and variance σ^2.

c. Erlang distribution: If the number of stages are p and n respectively for the repair and installation processes, then repair and installation times are obtained as follows:

$$\text{Repair time } x = \sum_{i=1}^{p} AT1$$
$$\text{Installation time } y = \sum_{i=1}^{n} AT2$$

where AT1 and AT2 are exponentially distributed variables. The mean repair and installation times are calculated from the repair and installation rates as $1/p\mu$, $1/nx$.

2. Determination of the next state: The process is non-Markovian. The transition from one state to another depends on the past history of the process.

In this method, the times of occurrences of all possible future events are generated and the event with the smallest time of occurrence becomes the next event and the corresponding state becomes the next state. After the next state has been selected, the current simulation time is increased by the time duration in the present state. Then, the next state becomes the present state for the next step.

3. The above procedure is repeated until the total simulation time is reached.

4. The system unavailability can be evaluated from the data collected. The steady state probability can be estimated from the total time duration in the state, the total number of occurrences of the state and the total simulation time as follows:

$$\text{Probability of a state} = \frac{\text{Total time duration in a state}}{\text{Total simulation time}}$$

Unavailability = Sum of the probabilities of the appropriate down states

3.5 Method Of Stages

Techniques [5, 11] have been developed in recent years for solving non-Markovian models using the method of stages. It has been found that if two or more exponentially distributed states are combined, the resulting state will not be exponentially distributed. The actual shape of the resulting distribution depends upon the number of states being combined and whether they are in series, in parallel or in series/parallel. It follows that the reverse process is also true i.e. if a state is not exponentially distributed, then it can be divided into a number of substates each of which is exponentially distributed.

The process of dividing a state into substates, each substate being defined as a stage, is known as the method of stages. In this method a non-exponentially distributed state is represented by a combination of stages each of which is exponentially distributed. Application of the method of stages involves the following procedure:

1. Determination of a stage combination that will reasonably approximate the behaviour of a given distribution or fit the available data.
2. Determination of the parameters of the stage device.

The selection of a proper stage combination will depend on the characteristic behaviour of the distribution simulated, the required degree of accuracy and the simplicity of the model. Since the transition rate is an important characteristic of a distribution, an awareness of its variability can be very useful in selecting a proper combination. After selecting a proper combination, the parameters can be derived using the appropriate techniques. The total number of stages in one state represents the given process. The stages may not have any physical significance when taken individually.

Consider a model in which the repair distribution can be represented by a series of stages. The stages are traversed in a sequential order as shown in Figure 3-6. The continous random variable is the sum of n independently exponentially

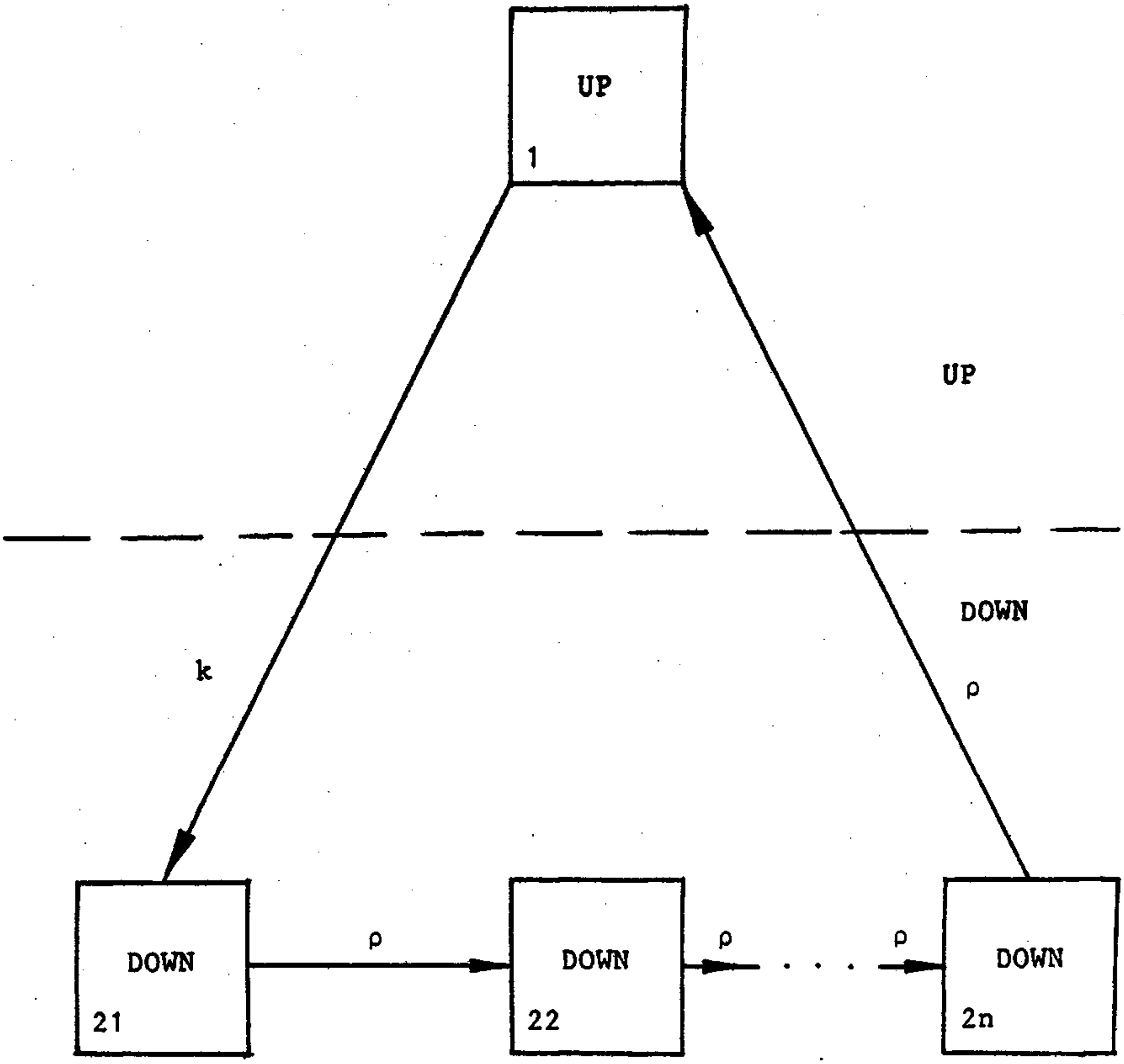

Figure 3-6: A Two State Model With Repair Process Represented By n Identical Stages.

distributed random variables and the probability distribution function is given by

$$f(t) = \frac{\rho^{n} e^{-\rho t} t^{n-1}}{n-1\ !}$$

where $t \geqslant 0$, $\rho, n > 0$

Here all stages are assumed to be identical with parameter ρ. If n is an integer, the distribution reduces to the Erlang distribution with two parameters ρ and n. By changing these two parameters, different distributions can be generated.

n=1 is a special case of the exponential distribution.

n=2 and n=3 represent different sets of distributions.

As n increases, the distribution approaches the normal distribution. The relationship between the original exponential repair rate μ and ρ is $\rho = n\mu$ where n = number of stages.

This approach is used in the next section to model assumed Erlang distributions for the repair and installation processes.

3.6 A Comparison Study Of The Analytical And Simulation Methods

The following section provides a comparison of unavailabilities obtained by the method of stages and the Monte Carlo method.

As discussed in the previous section, the method of stages provides an analytical tool to evaluate non-Markovian systems. The repair and installation processes are assumed to follow special Erlang distributions. The number of series stages used to represent this distribution are 2 in these studies. The following three cases are presented by illustration using the basic one spare transformer bank model shown in Figure 3-1.

1. The failure and repair processes are exponential and the installation process is Erlangian.
2. The failure and installation processes are exponential and repair process is Erlangian.
3. The failure process is exponential and the repair and installation processes are Erlangian.

Case 1: The failure and repair processes are exponential and the installation process is Erlangian.

The Markov model of the transformer bank with one spare when the installation rate of the transformer is in 2 series stages is shown in Figure 3-7. In this model, the failure rate and repair rate are the previous values k and μ. The installation process is Erlangian and therefore the installation rate x is segmented into an installation rate γ such that $\gamma = ax$ where a is the number of stages which is 2 in the present case. The first suffix of the segmented stage stands for the state and second suffix indicates what stage the non-exponential process is in. For example, the first suffix of the Stage 21 stands for state 2 and the second

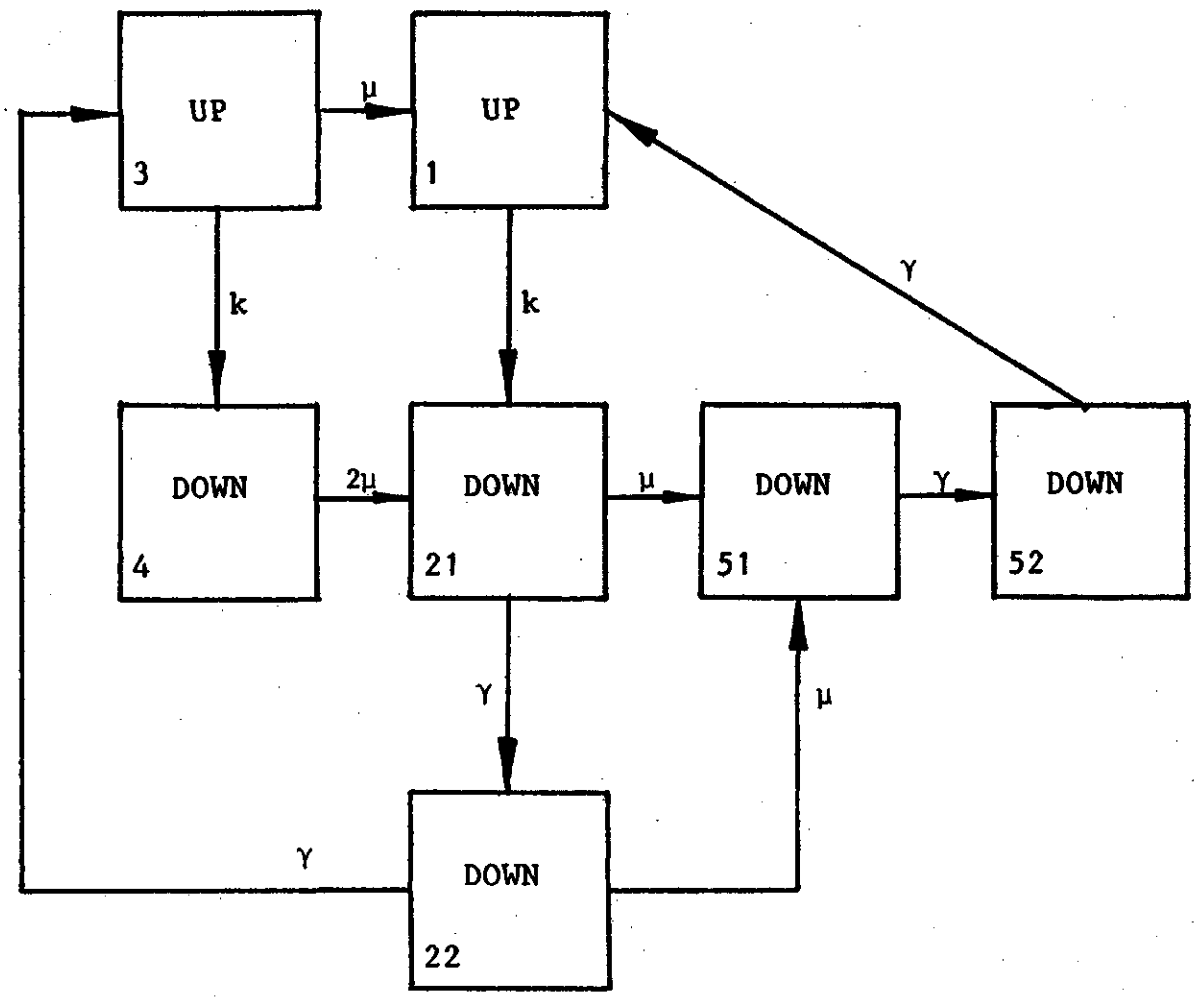

Figure 3-7: Markov Model Of The Transformer Bank With One Spare: Installation Rate Of The Transformer In 2 Series Stages.

suffix 1 shows that the installation is in stage 1. The same state probabilities are obtained by solving the Markovian model of Figure 3-7. The sum of the state probabilities of states 4, 21, 22, 51, 52 represents the unavailability.

The unavailabilities obtained by the method of stages and by the simulation method are given in Table 3-1.

Table 3-1: Comparison Of Simulation Unavailabilities With Analytical Unavailabilities For One Spare Model Shown In Figure 3-1 When Installation Process Is Erlangian.

Repair rate = 12 r/yr
Installation rate = 182.5 inst/yr
Simulation time = 100,000 years
Number of stages = 2

Failure rate	Analytical unavailability	Simulated unavailability
8.00	0.144927	0.145203
4.00	0.057770	0.057820
1.00	0.008495	0.008446
0.50	0.003550	0.003540
0.25	0.001588	0.001554

Case 2: The failure and installation processes are exponential and repair process is Erlangian.

The Markov model of the transformer bank with one spare when the repair rate of the transformer is in 2 series stages is shown in Figure 3-8. In this model the failure and installation rates are k and x respectively. The repair rate from a stage is given by ρ such that $\rho = a\mu$ where a is the

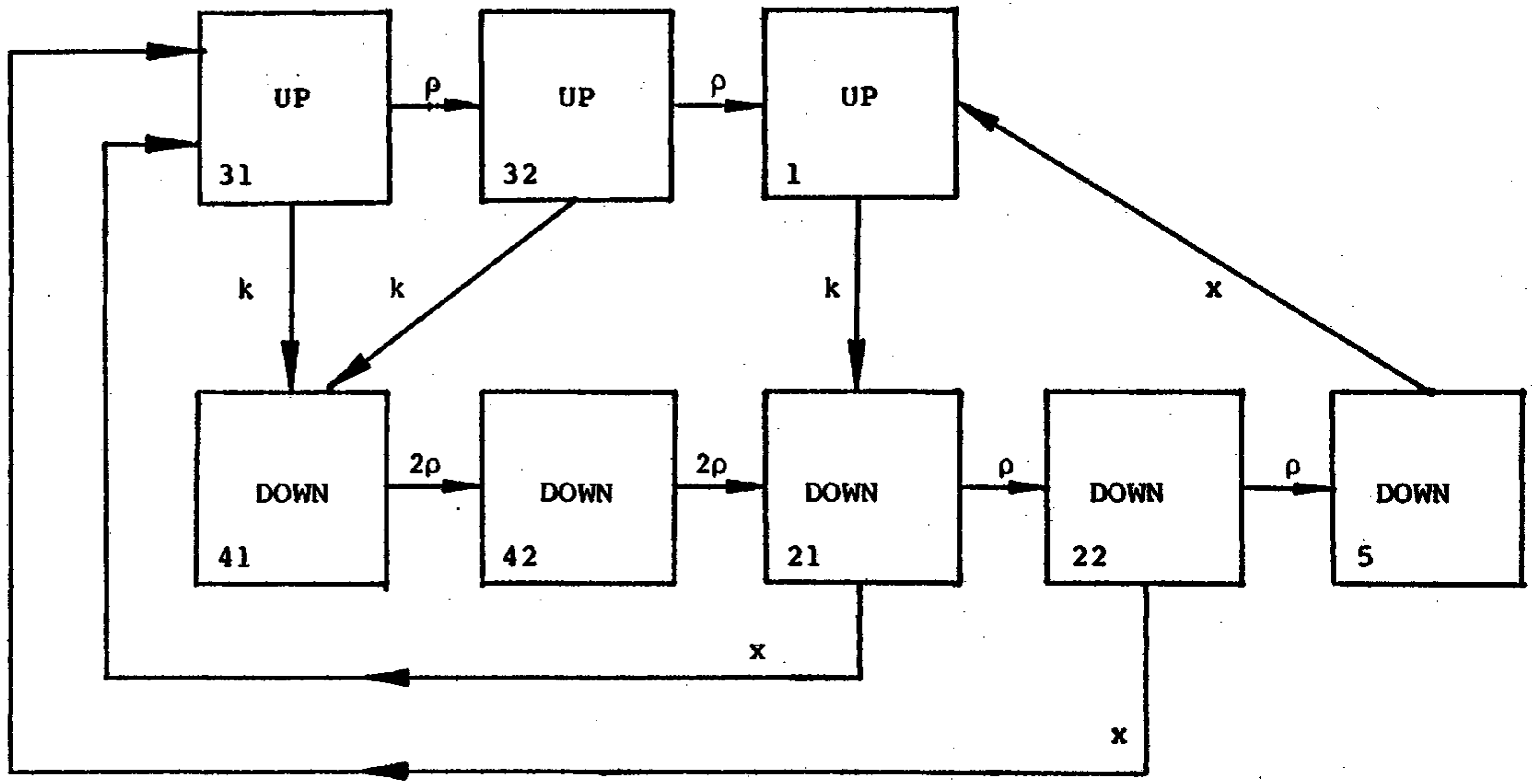

Figure 3-8: Markov Model Of The Transformer With One Spare: Repair Rate Of The Transformer In 2 Series Stages.

number of stages and is 2 in the present case. The first suffix of the stage stands for the state and the second suffix indicates which stage the repair process is in. For example, the first suffix of Stage 42 stands for state 4 and the second suffix 2 indicates that the repair process is in stage 2. The Markov model of Figure 3-8 was solved for various state probabilities. The sum of state probabilities of states 41, 42, 21, 22 and 5 represents the unavailability.

The unavailabilities obtained by the method of stages and by the simulation method are given in Table 3-2.

Table 3-2: Comparison of Simulated And Analytical Unavailabilities For One Spare Model Shown In Figure 3-1 When The Repair Process Is Erlangian.

Repair rate = 12 r/yr
Installation rate = 182.5 inst/yr
Simulation time = 100,000 years
Number of stages = 2

Failure rate	Analytical unavailability	Simulated unavailability
8.00	0.158036	0.158148
4.00	0.061507	0.061823
1.00	0.008627	0.008638
0.50	0.003557	0.003557
0.25	0.001578	0.001595

Case 3: The failure process is exponential and repair and installation processes are Erlangian.

Figure 3-9 shows the Markovian model of the transformer bank with one spare when both repair and installation rates of the transformer are in 2 series stages. In this case, both repair and installation processes are Erlangian. Transition rates $\rho = a\mu$, $\gamma = ax$ where a is the number of stages and is 2 in the present case. As in the previous cases

Stage 32 signifies that the state is 3 and the repair process is in the second stage.

In Stage 51, the first suffix stands for state 5 and the second suffix 1 indicates that the installation process is in the first stage.

Stage 212 stands for state 2, when the repair process is in the first stage and the installation process is in the second stage.

As in the previous cases, the state probabilities are determined by solving the Markov model of Figure 3-9. The unavailability is calculated by adding the state probabilities of states 41, 42, 211, 221, 51, 52, 212 and 222.

The unavailabilities calculated by this method and the simulation technique are given in Table 3-3.

An analysis of the results shown in Table 3-1, 3-2 and 3-3 shows that the unavailabilities obtained by simulation are quite close to the unavailabilities obtained by the analytical method. The method of stages is relatively easy to apply when the system is small and only a few of the transitions follow non-exponential distributions. As the system size increases,

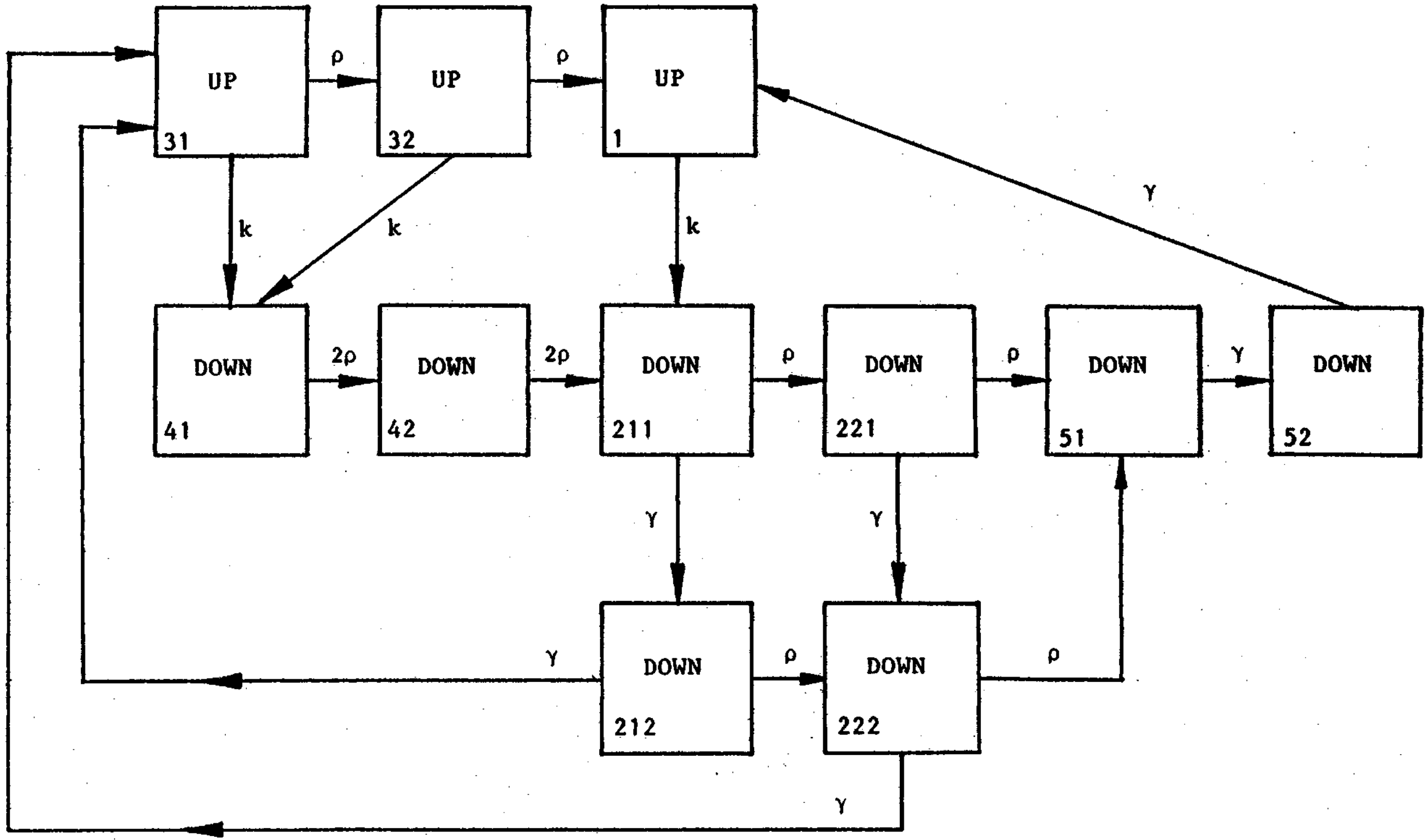

Figure 3-9:: Markov Model Of The Transformer Bank With One Spare: Both The Repair And The Installation Rate Of The Transformer In 2 Series Stages.

Table 3-3: Comparison Of Simulated And Analytical Unavailabilities For One Spare Model Shown In Figure 3-1 When Both Repair And Installation Processes Are Erlangian.

Repair rate = 12 r/yr
Installation rate = 182.5 inst/yr
Simulation time = 100,000 years
Number of stages = 2

Failure rate	Analytical unavailability	Simulated unavailability
8.00	0.158411	0.158321
4.00	0.061676	0.061878
1.00	0.008654	0.008753
0.50	0.003569	0.003631
0.25	0.001584	0.001575

the method becomes difficult to employ. If, in particular, the non-exponential distribution describing the repair and installation processes is not an Erlang distribution, the exact solution becomes more difficult as the number of required stages and the possible combinations increase.

The deviation of the simulation results when compared to the analytical results obtained by the method of stages is insignificant. The method of stages becomes more and more difficult to apply as the system size increases. The Monte Carlo simulation method described in section 3.4 is used in the next chapter to study the effect of various distributions on system unavailabilities.

3.7 Conclusion

This chapter has presented two basic techniques, the method of stages and the Monte Carlo simulation method. The method of stages is illustrated using stages in series. The basic model developed for the transformer bank with one spare has been extended to include non-exponential state residence times by utilization of both techniques.

Different cases have been presented and the unavailabilities determined. Stage combinations were used to represent the repair rate and installation rate of the transformer. The resultant unavailabilities were compared with those obtained by the simulation method.

It was observed that the unavailabilities values obtained by the method of stages were quite close to the values determined by the simulation method. The method of stages can be applied to relatively small models where a few of the transitions are non-exponential. The application of this method becomes difficult as the system size increases. The simulation method is a general methodology and is applied in the next chapter to evaluate non-Markovian systems.

4. THE EFFECT OF NON-EXPONENTIAL DISTRIBUTIONS ON SYSTEM UNAVAILABILITY

4.1 Introduction

As seen in the previous sections, the simulation method is a powerful tool in the modelling of non-Markovian systems. Using this approach, models can be analyzed in a relatively easy and straight-forward manner. The method of stages can also be used in modelling systems in which the assumption that the up and down times of the components are exponentially distributed is not valid.

The system unavailability in non-Markovian models is very sensitive to the repair and installation time distributions. A range of studies has been conducted using the Monte Carlo simulation method to compare the system unavailability for the sample one spare model as a function of the repair and installation time distributions.

In this chapter, three distributions i.e. Erlang, normal and Weibull are used for the repair and installation processes. The unavailabilities are compared with base values using exponential distributions. The mean values associated with the repair and installation times are held constant in each case. The change in unavailability is therefore entirely due to the change in the shape of distribution.

4.2 Effect On Unavailability Using An Erlang Distribution For Repair And Installation Times

The comparison studies were conducted using the system model shown in Figure 3-1. This model is reproduced as Figure 4-1. The results obtained using exponential distribution are taken to be the base values for comparison.

The assumption of Erlang distribution for the repair and installation times produces distinct changes in the results. The Monte Carlo simulation method described in the previous chapter was used to obtain these values. The assumption of Erlang distribution is applied in steps to the basic model. An Erlang distribution is first introduced for the installation time of the transformer. This is followed by the repair time of the transformer and then both the installation and repair times are included.

It should be noted that the mean state residence times were held constant and that the difference in values is entirely due to changes in shape of the distributions.

The mean of an Erlang distribution = $n\alpha = n/\rho$

where n = shape parameter(also the number of stages)

α = scale parameter(unit of time)

where $\rho = 1/\alpha$ = transition rate

The mean of an exponential distribution = $1/\lambda$

where λ = transition rate

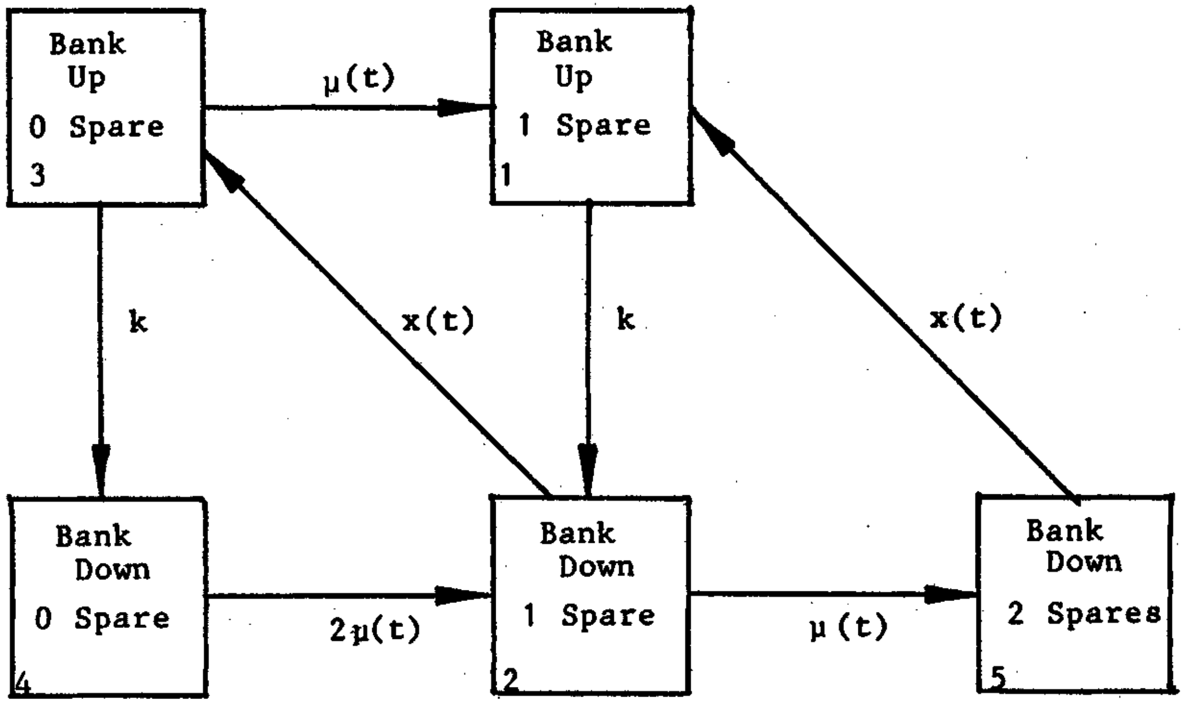

Figure 4-1 : Non-Markovian Model of A Transformer Bank With One Spare.

For comparison, equating means of both distributions gives

$$\frac{n}{\rho} = \frac{1}{\lambda}$$

$$\rho = n\,\lambda$$

In the studies conducted, n has been taken as 3 giving

$$\rho = 3\,\lambda$$

Therefore for the same mean values,

Installation rate using an Erlang distribution

= 3 x installation rate using an exponential distribution

Repair rate using an Erlang distribution

= 3 x repair rate using an exponential distribution

The changes associated with the introduction of an Erlang distribution are discussed in detail in the following section using the following set of data.

Failure rate = 0.25, 1.0, 4.0, 8.0 f/yr

Repair rate = 2, 3, 6, 12, 24 r/yr

Installation rate = 30, 50, 100, 150, 182.5, 365 inst/yr

Shape parameter(number of stages) = 3

Simulation time = 100,000 years for 0.25 and 1.0 f/yr

and 50,000 years for 4.0 and 8.0 f/yr

Case 1: The installation time is Erlang distributed.

The variation in unavailability as a function of the average installation time is shown in Table 4-1.

Table 4-1: The Variation In Unavailability As A Function Of The Average Installation Time When Installation Process Is Erlangian.

Sensitivity Study - Installation time Variable
- Erlang distribution for installation time
- Repair rate = 12 r/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Inst. rate	Exponential	Erlang	Exponential	Erlang
30	0.008414	0.009243	0.034397	0.037316
50	0.005145	0.005454	0.022086	0.023244
100	0.002683	0.002742	0.012698	0.013001
150	0.001860	0.001914	0.009542	0.009662
182.5	0.001567	0.001621	0.008415	0.008437
365	0.000890	0.000901	0.005809	0.005767

	Failure rate =4.0 f/yr		Failure rate = 8.0 f/yr	
Inst rate	Exponential	Erlang	Exponential	Erlang
30	0.140225	0.149417	0.265734	0.278555
50	0.102013	0.105685	0.211062	0.216347
100	0.071669	0.072689	0.166005	0.166951
150	0.061251	0.061574	0.150231	0.150170
182.5	0.057505	0.057552	0.144526	0.144223
365	0.048796	0.048461	0.131196	0.130942

It can be observed from Table 4-1 that the installation time distribution has a significant bearing on the system unavailability for small values of installation rate. As the installation rate increases, the results approach the corresponding base values. The variation in unavailability from the base case values decreases as the installation rate increases i.e. the average installation time becomes small.

Case 2: The repair time is Erlang distributed.

The variation in unavailability as a function of average repair time when the repair process is Erlangian is shown in Table 4-2.

Table 4-2: The Variation In Unavailability As A Function Of The Average Repair Time When The Repair Process Is Erlangian.

Sensitivity Study - Repair time Variable
- Erlang distribution for repair time
- Installation rate = 182.5 inst/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Repair rate	Exponential	Erlang	Exponential	Erlang
2	0.008172	0.008543	0.080806	0.090253
3	0.004503	0.004654	0.044408	0.048724
6	0.002172	0.002230	0.016721	0.017704
12	0.001567	0.001612	0.008415	0.008711
24	0.001415	0.001462	0.006178	0.006268

	Failure rate = 4.0 f/yr	
Repair rate	Exponential	Erlang
2	0.405243	0.446918
3	0.284031	0.319387
6	0.131196	0.148611
12	0.057505	0.062908
24	0.031420	0.033129

It can be observed from Table 4-2 that the unavailabilities with Erlang distributions are higher than the base values. The deviation of the results from the base values increases with decreasing repair rates i.e. increasing mean repair times and increasing failure rates i.e. decreasing mean failure times.

The variation in unavailability as a function of the average installation time when the repair process is Erlangian is shown in Table 4-3.

Table 4-3: The Variation In Unavailability As a Function Of The Average Installation Time When The Repair Process Is Erlangian.

Sensitivity Study - Installation time Variable
- Erlang distribution for repair time
- Repair rate = 12 r/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Inst rate	Expo-nential	Erlang	Expo-nential	Erlang
30	0.008414	0.008513	0.034397	0.034905
50	0.005145	0.005135	0.022086	0.023559
100	0.002683	0.002762	0.012698	0.013120
150	0.001860	0.001890	0.009542	0.009861
182.5	0.001567	0.001612	0.008415	0.008711
365	0.000890	0.000906	0.005809	0.005980

	Failure rate = 4.0 f/yr		Failure rate = 8.0 f/yr	
Inst rate	Expo-nential	Erlang	Expo-nential	Erlang
30	0.140225	0.146904	0.265734	0.282327
50	0.102013	0.109386	0.211062	0.231001
100	0.071669	0.078162	0.166005	0.185866
150	0.061251	0.066963	0.150231	0.169443
182.5	0.057505	0.062908	0.144526	0.163341
365	0.048796	0.053456	0.131196	0.147969

It can be observed from Table 4-3 that if the repair process is Erlangian then, in general, the unavailability is higher than that given by an exponential process. The effect of repair distribution is less evident for lower failure rates i.e. more reliable components. The deviation of the results from the base values is clearly visible for higher failure rates. The repair time distribution dominates at higher installation rates.

Case 3: Both repair and installation times are Erlangian.

The variation in unavailability as a function of average repair time when both repair and installation times are Erlangian is shown in Table 4-4.

Table 4-4: The Variation In Unavailability As A Function Of Average Repair Time When Both Repair And Installation Processes Are Erlangian.

Sensitivity Study - Repair time Variable
- Erlang distribution for both repair and installation times
- Installation rate = 182.5 inst/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Repair rate	Exponential	Erlang	Exponential	Erlang
2	0.008172	0.008825	0.080806	0.089070
3	0.004503	0.004757	0.044408	0.048555
6	0.002172	0.002221	0.016721	0.017638
12	0.001567	0.001589	0.008415	0.008673
24	0.001415	0.001428	0.006178	0.006283

	Failure rate = 4.0 f/yr	
Repair rate	Exponential	Erlang
2	0.405243	0.445686
3	0.284031	0.317906
6	0.131196	0.146170
12	0.057505	0.063060
24	0.031420	0.033300

The variation in unavailability as a function of average installation time when both repair and installation times are Erlangian is shown in Table 4-5.

Table 4-5: The Variation In Unavailability As A Function Of Average Installation Time When Both Repair And Installation Times Are Erlang Distributed.

Sensitivity Study - Installation time Variable
- Erlang distribution for repair and installation times
- Repair rate = 12 r/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Inst rate	Expo-nential	Erlang	Expo-nential	Erlang
30	0.008414	0.009071	0.034397	0.036996
50	0.005145	0.005302	0.022086	0.023091
100	0.002683	0.002697	0.012698	0.013148
150	0.001860	0.001886	0.009542	0.009871
182.5	0.001567	0.001589	0.008415	0.008673
365	0.000890	0.000912	0.005809	0.006024

	Failure rate = 4.0 f/yr		Failure rate = 8.0 f/yr	
Inst rate	Expo-nential	Erlang	Expo-nential	Erlang
30	0.140225	0.153407	0.265734	0.291749
50	0.102013	0.111370	0.211062	0.234403
100	0.071669	0.078722	0.166005	0.186079
150	0.061251	0.067245	0.150231	0.168380
182.5	0.057505	0.063060	0.144526	0.161987
365	0.048796	0.053351	0.131196	0.146665

It can be seen from Tables 4-4 and 4-5 that the unavailabilities with these assumptions are higher than the base values for this case. The difference is clearly seen in the case of higher component failure rates.

In general, the following points can be drawn from all of these studies:

The installation distribution has a significant effect on the system unavailability for lower values of installation rates. As the installation rate increases, the effect of the installation rate on the system unavailability diminishes.

The results of Case 1 are close to the base values for high installation rates (i.e. 182.5 and 365 inst/yr). This indicates that for low installation times, the installation distribution is not particularly important. At low values of installation rate, both repair and installation time distributions affect the determination of system unavailability. The results of Case 2 and Case 3 are quite close to each other for high installation rates. It is therefore obvious that the repair time distribution has a larger impact on the calculated unavailability than the installation time distribution.

The distributional effect is more visible in systems with lower availability. As the system availability increases, the effect of the repair and installation time distributions diminishes.

4.3 Effect Of The Number Of Stages On System Unavailability Of The One Spare Model Shown In Figure 4-1

Table 4-6 shows the results obtained by varying the number of stages. It is assumed that both repair and installation times are Erlang distributed.

Table 4-6: Effect Of The Number Of Stages On System Unavailability.

- Both repair and installation processes are Erlangian
- Repair rate = 12 r/yr
- Installation rate = 182.5 inst/yr

Failure rate	0.25 f/yr	4.0 f/yr
Number of stages	Unavailability	Unavailability
1	0.001567	0.057289
2	0.001575	0.061878
3	0.001589	0.062799
4	0.001605	0.063416

It can be seen that the system unavailability increases as the number of stages increases. The system unavailability is higher in the case of an Erlang distribution. The unavailabilities increase from 0.057289 to 0.063416 for a failure rate of 4.0 f/yr and from 0.001567 to 0.001605 for a failure rate of 0.25 f/yr. It can be observed from Table 4-6 that for a failure rate of 4.0 f/yr the relative variation in unavailability is maximum when the number of stages are increased from 1 to 2 i.e. there is a sudden change in the shape of the distributions from exponential to bell shape. The relative variation is less marked after that. This is not as obvious for lower unavailabilities (i.e. failure rate = 0.25 f/yr).

4.4 Effect On System Unavailability Of Normally Distributed Repair And Installation Times

Normal distributions are quite often used in reliability evaluation. A number of studies are conducted on the model shown in Figure 4-1 to evaluate the effect of normally distributed repair and installation times on system unavailability.

The results using exponential distributions were considered to be the base values. The mean state residence times were held constant and therefore change in the values of unavailabilities were due to the change in the shape of distributions only.

The mean of a normal distribution = m

The mean of an exponential distribution = $1/\lambda$

where λ = transition rate

For equal means of both distributions

$$m = 1/\lambda$$

Therefore,

Mean repair time using a normal distribution =

1/(Repair rate using an exponential distribution)

Mean installation time using a normal distribution =

1/(Installation rate using an exponential distribution)

The following three cases were considered in these studies:

1. The failure and repair processes are exponential, the installation process is normal.
2. The failure and installation processes are exponential, the repair process is normal.
3. The failure process is exponential, the repair and installation processes are normal.

The following set of basic data were utilized for comparison.

Failure rate = 0.25, 1.0, 4.0, 8.0 f/yr

Repair rate = 2, 3, 6, 12, 24 r/yr

Installation rate = 30, 50, 100, 150, 182.5, 365 inst/yr

Standard deviation for the repair time = 0.0005 yrs

Standard deviation for the installation time = 0.0001 yrs

Simulation time = 100,000 years for 0.25 f/yr and 1.0 f/yr
and 50,000 years for 4.0 f/yr and 8.0 f/yr

Case 1: The installation process is normal.

The variation in unavailability as a function of the average installation time is shown in Table 4-7.

Table 4-7: The Variation In Unavailability As A Function Of The Average Installation Time When Installation Time Is Normally Distributed.

Sensitivity Study - Installation time Variable
- Normal distribution for installation time
- Repair rate = 12 r/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Inst rate	Expo-nential	Normal	Expo-nential	Normal
30	0.008414	0.009675	0.034397	0.038742
50	0.005145	0.005653	0.022086	0.023879
100	0.002683	0.002796	0.012698	0.013113
150	0.001860	0.001912	0.009542	0.009782
182.5	0.001567	0.001617	0.008415	0.008715
365	0.000890	0.000909	0.005809	0.005833

	Failure rate = 4.0 f/yr		Failure rate = 8.0 f/yr	
Inst rate	Expo-nential	Normal	Expo-nential	Normal
30	0.140225	0.152836	0.265734	0.284267
50	0.102013	0.107701	0.211062	0.219802
100	0.071669	0.073240	0.166005	0.168540
150	0.061251	0.061962	0.150231	0.151289
182.5	0.057505	0.058484	0.144526	0.145591
365	0.048796	0.049153	0.131196	0.132198

It can be observed from Table 4-7 that the values of unavailability with the normal distribution are higher than the unavailabilities obtained with an exponential distribution for all values of installation rate. The variation in unavailability from the base case decreases as the installation rate increases i.e. the average installation time becomes small. The unavailabilities approach the base values for higher installation rates.

Case 2: The repair time is normally distributed.

The variation in unavailability as a function of average repair time when the repair process is normal is shown in Table 4-8.

Table 4-8: The Variation In Unavailability As A Function Of Average Repair Time When The Repair Process Is Normal.

Sensitivity Study - Repair time Variable
- Normal distribution for repair time
- Installation rate =182.5 inst/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Repair rate	Exponential	Normal	Exponential	Normal
2	0.008172	0.008542	0.080806	0.092625
3	0.004503	0.004620	0.044408	0.049469
6	0.002173	0.002236	0.016721	0.017818
12	0.001567	0.001601	0.008415	0.008703
24	0.001415	0.001414	0.006178	0.006299

Failure rate = 4.0 f/yr

Repair	Exponential	Normal
2	0.405243	0.468646
3	0.284031	0.337441
6	0.131196	0.154531
12	0.057505	0.064267
24	0.031420	0.033520

It can be observed from Table 4-8 that system unavailabilities, in general, are higher than their corresponding base values. The deviation of the results is more evident in the case of higher failure rates. The repair time distribution is the key factor in determining the system unavailability. It has a large impact on the calculated unavailabilities.

The variation in unavailability as a function of the average installation time when the repair process is normal is shown in Table 4-9.

Table 4-9: The Variation In The Unavailability As A Function Of The Average Installation Time When The Repair Process Is Normal.

Sensitivity Study - Installation time Variable
- Normal distribution for repair time
- Repair rate = 12 r/yr

Inst rate	Failure rate = 0.25 f/yr: Expo-nential	Failure rate = 0.25 f/yr: Normal	Failure rate = 1.0 f/yr: Expo-nential	Failure rate = 1.0 f/yr: Normal
30	0.008414	0.008557	0.034397	0.035144
50	0.005145	0.005191	0.022086	0.022745
100	0.002683	0.002694	0.012698	0.013160
150	0.001860	0.001901	0.009542	0.009870
182.5	0.001567	0.001601	0.008415	0.008703
365	0.000890	0.000911	0.005809	0.006005

Inst rate	Failure rate = 4.0 f/yr: Expo-nential	Failure rate = 4.0 f/yr: Normal	Failure rate = 8.0 f/yr: Expo-nential	Failure rate = 8.0 f/yr: Normal
30	0.140225	0.150363	0.265734	0.293643
50	0.102013	0.112633	0.211062	0.242587
100	0.071669	0.079907	0.166005	0.176418
150	0.061251	0.068407	0.150231	0.176418
182.5	0.057505	0.064267	0.144526	0.169978
365	0.048796	0.054624	0.131196	0.154790

It can be seen from Table 4-9 that a normal repair process affects the unavailability results. The values of unavailability are higher than their corresponding base values. The deviation in the results from the base values is clearly visible for relatively high unavailabilities.

Case 3: The repair and installation processes are normal.

The variation in unavailability as a function of the average repair time is shown in Table 4-10.

Table 4-10: The Variation In Unavailability As A Function Of Average Repair Time When Both Repair And Installation Processes Are Normal.

Sensitivity Study - Repair time Variable
- Normal distribution for repair and Installation times
- Installation rate = 182.5 inst/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Repair rate	Exponential	Normal	Exponential	Normal
2	0.008172	0.008530	0.080806	0.092609
3	0.004503	0.004606	0.044408	0.049450
6	0.002173	0.002222	0.016721	0.017805
12	0.001567	0.001587	0.008415	0.008690
24	0.001415	0.001419	0.006178	0.006252

Failure rate = 4.0 f/yr

Repair rate	Exponential	Normal
2	0.405243	0.468632
3	0.284031	0.337402
6	0.131196	0.154520
12	0.057505	0.064276
24	0.031420	0.033465

It can be seen from Table 4-10 that normally distributed repair and installation processes result in higher unavailabilities when compared to the base values. On comparing the results of Tables 4-8 and 4-10, it can be seen that the results are quite close. This indicates that for a specified high value of installation rate, the repair time distribution plays a major role in the calculated unavailability. The variation in unavailability as a function of average installation time when both repair and installation processes are normal is shown in Table 4-11.

Table 4-11: The Variation In Unavailability As A Function Of Average Installation Time When Both Repair And Installation Processes Are Normal.

Sensitivity Study - Installation time Variable
- Normal distribution for repair and installation times
- Repair rate = 12 r/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Inst rate	Expo-nential	Normal	Expo-nential	Normal
30	0.008414	0.008486	0.034397	0.035292
50	0.005145	0.005192	0.022086	0.022710
100	0.002683	0.002713	0.012698	0.013097
150	0.001860	0.001884	0.009542	0.009854
182.5	0.001567	0.001587	0.008415	0.008690
365	0.000890	0.000904	0.005809	0.006000

	Failure rate = 4.0 f/yr		Failure rate = 8.0 f/yr	
Inst rate	Expo-nential	Normal	Expo-nential	Normal
30	0.140225	0.152445	0.265734	0.299084
50	0.102013	0.112460	0.211062	0.242675
100	0.071669	0.079845	0.166005	0.194137
150	0.061251	0.068422	0.150231	0.176511
182.5	0.057505	0.064276	0.144526	0.170058
365	0.048796	0.054625	0.131196	0.154827

It is obvious from Table 4-11 that the results show higher unavailabilities as compared to the base values.

In general, the following conclusions can be drawn from this set of studies.

The installation time distribution has a significant effect on the system unavailability for lower values of installation rates. As the installation rate increases, the effect of the installation time distribution diminishes. The results of Case 1 are close to the base values for higher installation rates (i.e. 182.5 and 365 inst/yr). This indicates that for low values of installation time, the installation time distribution is not important. At low values of installation rates, both repair and installation time distributions affect the calculated unavailabilities.

The results of Case 2 and Case 3 are quite close to each other for higher values of installation rate. It therefore emphasizes the fact that the repair time distribution has a larger impact on the calculated results. The effect of the repair time distribution is also seen to be decreasing with a decrease in the average repair times. As the system unavailability increases, the effect of the repair and installation time distributions become more visible.

4.5 Effect On System Unavailability Of Varying Standard Deviation Of A Normally Distributed Repair Time

The variation in system unavailability with standard deviation is shown in Table 4-12. The variation in standard deviation is applied to several repair rates assuming that the repair process is normal. The failure and installation processes are assumed to be exponential.

Table 4-12: The Variation In System Unavailability With Standard Deviation.

Sensitivity Study - Standard deviation and Repair rate Variable
- Repair process is normal
- Installation rate = 182.5 inst/yr

Repair rate	Standard deviation 0.005 yrs	Standard deviation 0.001 yrs	Standard deviation 0.0005 yrs
Failure rate = 1.0 f/yr			
2	0.092545	0.092560	0.092625
3	0.049449	0.049469	0.049481
6	0.017762	0.017785	0.017818
12	0.008691	0.008702	0.008703
24	0.006262	0.006294	0.006299
Failure rate = 4.0 f/yr			
2	0.468503	0.468543	0.468646
3	0.337024	0.337323	0.337441
6	0.154423	0.154476	0.154531
12	0.064167	0.064229	0.064267
24	0.033466	0.033498	0.033520

It can be seen from the Table 4-12 that system unavailabilities, generally, decrease with increasing standard deviations. The effect of varying standard deviation on system unavailability is not very pronounced.

4.6 Effect Of Fixed Installation Time On System Unavailability

The variation in the system unavailability as a function of fixed installation time for a one spare model shown in Figure 4-1 is shown in Table 4-13.

Table 4-13: The Variation In The System Unavailability As A Function Of Fixed Installation Time.

Sensitivity Study - Installation time Variable
- Fixed installation rate
- Repair rate = 12 r/yr

	Failure rate = 0.25 f/yr		Failure rate = 1.0 f/yr	
Inst rate	Expo-nential	Fixed	Expo-nential	Fixed
30	0.008414	0.009660	0.034397	0.038849
50	0.005145	0.005638	0.022086	0.023998
100	0.002683	0.002792	0.012698	0.013213
150	0.001860	0.001888	0.009542	0.009769
182.5	0.001567	0.001567	0.008415	0.008534
365	0.000890	0.000884	0.005809	0.005802

Failure rate = 4.0 f/yr

Inst rate	Exponential	Fixed
30	0.140225	0.153261
50	0.102013	0.107689
100	0.071669	0.073366
150	0.061251	0.061936
182.5	0.057505	0.057891
365	0.048796	0.048880

It can be observed from Table 4-13 that sytem unavailabilities with fixed installation times are generally higher than the base values. The effect is more evident for higher average installation times. The results approach the corresponding base values of the installation time re-establishing the fact that the effect of distributional assumptions diminishes with a decrease in the average installation time.

4.7 Effect Of Fixed Repair Time On System Unavailability

The variation in unavailability as a function of fixed repair time for the one spare model shown in Figure 4-1 is shown in Table 4-14.

Table 4-14: The Variation In Unavailability As A Function Of Fixed Repair Time.

Sensitivity Study - Repair time Variable
- Fixed repair rate
- Installation rate =182.5 inst/yr

Repair rate	Exponential	Normal
Failure rate = 0.25 f/yr		
2	0.008172	0.008719
3	0.004503	0.004712
6	0.002173	0.002238
12	0.001567	0.001574
24	0.001415	0.001453
Failure rate = 4.0 f/yr		
2	0.405243	0.470080
3	0.284031	0.338609
6	0.131196	0.155233
12	0.057505	0.064373
24	0.031420	0.033273

It can be seen from the Table 4-14 that the unavailabilities with fixed repair times are higher than the base values. The deviation is less visible for lower unavailabilities. As system unavailability increases, the impact of a fixed repair time in determining system unavailability increases. In general, the values of unavailability are seen to be higher than the corresponding unavailabilities obtained with normal and Erlang distributions for this set of basic data.

4.8 Effect On System Unavailability Of Weibull Distributed Repair And Installation Times [14]

A Weibull distribution is the most commonly used distribution in reliability studies. Generally, repair and installation times are Weibull distributed. Studies were conducted to determine the effect of Weibull distributions on the steady state unavailabilities. The shape parameter β of the Weibull distribution was varied from 0.5 to 4 with intermediate values of 1 and 2. $\beta = 1$ denotes an exponential distribution and hence analytical values have been used. $\beta = 2$ is a Rayleigh distribution. When $\beta = 4$, the distribution approaches a normal distribution. It has to be noted that the mean values are held constant so that the difference in the unavailabilities is entirely due to the change in the shape of distribution.

Mean Of Weibull distribution $= \alpha\,\Gamma(1 + 1/\beta)$

where α = scale parameter

β = shape parameter

For an exponential distribution

Mean $= 1/\lambda$ where λ = transition rate

For equal means of both distributions

$$\alpha\,\Gamma(1 + 1/\beta) = 1/\lambda$$

When $\beta = 0.5$ (Hyperexponential distribution)

$$\alpha\,\Gamma(1 + 1/0.5) = 1/\lambda$$

$$\alpha = 1/2\lambda$$

When $\beta = 1$ (Exponential distribution)

$$\alpha\,\Gamma(1 + 1) = 1/\lambda$$

$$\alpha = 1/\lambda$$

When $\beta = 2$ (Rayleigh distribution)

$$\alpha\,\Gamma(1 + 1/2) = 1/\lambda$$

$$\alpha\,\Gamma(1.5) = 1/\lambda$$ $\frac{1}{2}\sqrt{\pi}$

$$\alpha = 1/(0.8860756174\,\lambda)$$ 0.88622 69

When $\beta = 4$ (Approximate normal)

$$\alpha\,\Gamma(1 + 1/4) = 1/\lambda$$

$$\alpha\,\Gamma(1.25) = 1/\lambda$$

$$\alpha = 1/(0.9064024771\,\lambda)$$ 0.9064025

The above values of α and β are used to calculate unavailabilities for the one spare model shown in Figure 4-1 for the following data

Failure rate = 0.25, 4.0 f/yr

Repair rate = 2, 3, 6, 12, 24 r/yr

Installation rate = 30, 50, 100, 182.5, 365 inst/yr

β = 0.5, 1, 2, 4

Simulation time = 100,000 years for 0.25 f/yr

and 50,000 years for 4 f/yr

The assumption of Weibull distribution is applied in steps. Firstly, the installation time is assumed to be Weibull distributed. The assumption of a Weibull distribution is then applied to the repair time. Finally, both repair and installation times are assumed to be Weibull distributed.

Case 1: The installation time is Weibull distributed.

The variation in unavailability as a function of installation time is shown in Table 4-15.

Table 4-15: The Variation In Unavailability As A Function Of Installation Time When Installation Time Is Weibull distributed.

Sensitivity Study - Installation time Variable
\- Weibull distribution for installation time
\- Repair rate 12 r/yr

Installation rate	$\beta = 0.5$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 4.0$
Failure rate = 0.25 f/yr				
30	0.006467	0.008414	0.009325	0.009615
50	0.004177	0.005145	0.005587	0.005601
100	0.002347	0.002683	0.002758	0.002771
150	0.001700	0.001860	0.001880	0.001893
182.5	0.001491	0.001567	0.001589	0.001584
365	0.000865	0.000890	0.000912	0.000907

Installation rate	$\beta = 0.5$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 4.0$
Failure rate = 4.0 f/yr				
30	0.116957	0.140225	0.149277	0.152196
50	0.090390	0.102013	0.106275	0.107134
100	0.067506	0.071669	0.073070	0.073474
150	0.058925	0.061251	0.061970	0.062097
182.5	0.055941	0.057505	0.058167	0.058208
365	0.048285	0.048796	0.048969	0.049299

It can be seen from Table 4-15 that the system unavailability generally increases with an increase in the value of β ($\beta = 1$ is the case of exponential distribution). The unavailabilities for $\beta > 1$ are higher than the base values.

The relative variation in unavailability is greater for lower values of installation rate. The results approach the base values for higher values of installation rate.

Case 2: Repair time is Weibull distributed.

The variation in unavailability as a function of repair rate is shown in Table 4-16.

Table 4-16: The Variation In Unavailability As A Function Of Repair Rate When Repair Time Is Weibull Distributed.

Sensitivity Study - Repair rate Variable
- Weiull distribution for repair time
- Installation rate =182.5 inst/yr

Failure rate = 0.25 f/yr				
Repair rate	β = 0.5	β = 1.0	β = 2.0	β = 4.0
2	0.006336	0.008172	0.008343	0.008525
3	0.004043	0.004503	0.004594	0.004599
6	0.002068	0.002172	0.002184	0.002198
12	0.001564	0.001567	0.001597	0.001584
24	0.001421	0.001415	0.001431	0.001423
Failure rate = 4.0 f/yr				
Repair rate	β = 0.5	β = 1.0	β = 2.0	β = 4.0
2	0.298935	0.405243	0.446141	0.462590
3	0.197366	0.284031	0.319740	0.332367
6	0.130752	0.131196	0.147520	0.152307
12	0.055941	0.057505	0.062850	0.063949
24	0.027388	0.031420	0.032925	0.033358

It can be observed from Table 4-16 that unavailability, generally, increases with an increase in the value of β. The increase is more pronounced for higher failure rates. The deviation of the results from the base values(β = 1) decreases with an increase in the repair rates.

Case 3: Both repair and installation times are Weibull distributed.

The variation in unavailability as a function of repair rate when both repair and installation times are Weibull distributed is shown in Table 4-17.

Table 4-17: The Variation In Unavailability As A Function Of Repair Rate When Repair And Installation Times Are Weibull Distributed.

Sensitivity Study - Repair time Variable
- Weibull distribution for repair and installation times
- Installation rate 182.5 inst/yr

Failure rate = 0.25 f/yr

Repair rate	β = 0.5	β = 1.0	β = 2.0	β = 4.0
2	0.006250	0.008172	0.008306	0.008514
3	0.003622	0.004503	0.004523	0.004589
6	0.001887	0.002172	0.002140	0.002187
12	0.001359	0.001567	0.001580	0.001585
24	0.001187	0.001415	0.001435	0.001420

Failure rate = 4.0 f/yr

Repair rate	β = 0.5	β = 1.0	β = 2.0	β = 4.0
2	0.304379	0.405243	0.446549	0.462582
3	0.200573	0.284031	0.320148	0.332361
6	0.070754	0.131196	0.147973	0.152460
12	0.042057	0.057505	0.062835	0.063728
24	0.024340	0.031420	0.033384	0.033442

It can be seen from Table 4-17 that system unavailability, generally, increases with an increase in the value of the shape parameter β. The system unavailabilities for $\beta < 1$ are lower than the base values. When $\beta > 1$, the system unavailabilities are higher than the base values. The relative variation from the base values decreases as the system becomes more available.

The following points can be drawn from Tables 4-15, 4-16 and 4-17.

The average installation time and the associated distribution has a significant bearing on system unavailability. The system unavailabilities decrease with an increase in the installation rate. The effect of the installation time distribution is more evident for less available systems. The effect of distributional assumptions decrease with an increase in the installation rate. The system unavailability approaches the base value for higher installation rates.

Generally, the results of Case 2 and Case 3 are close to each other for $\beta > 1$. This signifies that the repair time distribution has a larger impact on the unavailability than the installation time distribution for high installation rates. The installation distribution has a significant effect when $\beta < 1$.

4.9 Conclusion

In this chapter, the effect of non-exponential state residence times in a given system was investigated using the Monte Carlo simulation method. The transformer bank with one spare unit was chosen as an example. The non-exponential repair and installation times were assumed to follow Erlang, normal and Weibull distributions. A set of studies were conducted to illustrate the effect of these two parameters on system unavailability as compared to a base case involving exponential distributions.

It can be seen that the shape of the distribution has a significant impact on system unavailability. System unavailabilities in the case of hyper-exponential distributions are basically lower than the base values whereas bell-shaped distributions give relatively higher values of unavailability. The repair time distribution has a larger impact on system unavailability when compared to the installation time distribution. The deviation of the results with non-exponential distributions from the base values decreases as the overall system availability increases.

5. SELECTED STUDIES IN REPAIRABLE SYSTEMS CONTAINING STANDBY COMPONENTS

5.1 Introduction

The previous chapters included various studies conducted on system unavailability with the distribution of state residence time as a varying parameter. Single estimate values (point estimates) were used to provide a comparison of results with different distributions. The accuracy of the results was dependent upon the chosen simulation time. The larger the simulation time, the greater is the accuracy. In such situations, single value estimates are not sufficient indicators of system behaviour. Only through interval estimates(confidence intervals) is the true picture of the underlying system behaviour assessed.

This chapter is concerned with definitions of point and interval estimates and the methodology to determine these estimates. The system unavailability decreases with an increase in the number of spares. In this chapter, the effect of Weibull distributed repair and installation times on system unavailability is evaluated. The frequency of an up or down state is an important parameter in power system reliability evaluation. The Monte Carlo simulation methodology presented earlier is extended in this chapter to evaluate frequency of up and down states of a system.

5.2 Definitions Of Point And Interval Estimates[15]

<u>Point Estimate</u> : A point estimate is a single valued estimate of a population parameter made from a sample. For example, the unavailability 0.001558 is a point estimate of unavailability. There is no probability associated with this type of estimate and the accuracy is not really known. The accuracy increases with an increase in the size of the sample. Point estimates have been used in all studies presented up to this point in the thesis.

<u>Interval Estimates:</u> An interval estimate is a probability statement that a population parameter is between two computed values. For an example, it is 95 percent certain that the unavailability is between 0.001521 and 0.001585. In symbols

$Pr(0.001521 \leq mean \leq 0.001585) = 0.95$

This is a symmetric, two sided or two-tail interval estimate with a 95 percent confidence. The general form of a two sided interval with a $1-\alpha$ confidence level is

$Pr(\text{lower limit} \leq \text{parameter} \leq \text{upper limit}) = 1-\alpha$.

If the estimate is stated as

$$Pr(\text{parameter} \leq \text{upper limit}) = 1-\alpha.$$

it is an upper, or one sided 95 percent confidence interval.

<u>Interval Estimates For The Mean:</u> An interval estimate for the mean is computed using the statistic:

$$t = \frac{\overline{X} - m}{s/\sqrt{n}}$$

where m = mean

n = number of observations

$$\overline{X} = \frac{1}{n}\Sigma X_i \qquad X_i = \text{observation}$$

$$s^2 = \frac{1}{n-1}\Sigma X_i^2 - \frac{n}{n-1}\overline{X}^2$$

It has a Student's t distribution with n-1 degrees of freedom. The confidence that t is between two symmetrically placed t values is 1-α; that is $Pr(-t_\alpha \leqslant t \leqslant +t_\alpha) = 1-\alpha$, where $\pm t_\alpha$ are the two tail values. Substituting the value of t gives:

$$Pr(\overline{X} - \frac{st_\alpha}{\sqrt{n}} \leqslant m \leqslant \overline{X} + \frac{st_\alpha}{\sqrt{n}}) = 1-\alpha$$

The expression in parenthesis is the two sided confidence interval for the mean. A sample of size n is taken and the values of $\overline{X}$ and s are computed to determine confidence limits.

5.3 Procedure Used To Set Confidence Interval On System Unavailability

1. The procedure explained in the previous chapter is followed to determine the estimates of system unavailability say X_i. The simulation time is chosen arbitrarily. As seen in chapter 2, estimates improve in statistical accuracy as the simulation time increases.

2. The above procedure is replicated n times to calculate $\overline{X}$. A replication denotes a trial or execution of a simulation model for a given set of parameters including a specified simulation time. A set of independent replications denotes replications with identical inputs except for the random number seeds. On

each replication, the seeds are chosen so that the streams of generated random numbers are independent of the stream on all other replications in a set. This procedure implies that:[16]

1. Estimates of an unavailability computed on each replication are independent and identically distributed.

2. If the simulation time within each replication is sufficiently large, there is a reasonable basis for assuming that the estimates have a normal distribution.

3. The statistical accuracy of the overall average improves as the number of replications increases.

3. The value of the confidence coefficient $(1-\alpha)$ depending upon the level of confidence required can be selected and the corresponding value of t_α for $v=n-1$ degrees of freedom obtained from the Student's t table.

4. The confidence limits of the one sided or two sided confidence level is given by

Two sided $\bar{X} - \frac{st_\alpha}{\sqrt{n}}, \bar{X} + \frac{st_\alpha}{\sqrt{n}}$ Two tail

One sided, upper $-\infty, \bar{X} + \frac{st_\alpha}{\sqrt{n}}$ One tail

One sided, lower $\bar{X} - \frac{st_\alpha}{\sqrt{n}}, +\infty$ One tail

The above procedure has been illustrated using an example of the one spare model of a transformer bank. The flowchart of the simulation procedure for setting confidence interval limits on the system unavailability is shown in Figure 5-1.

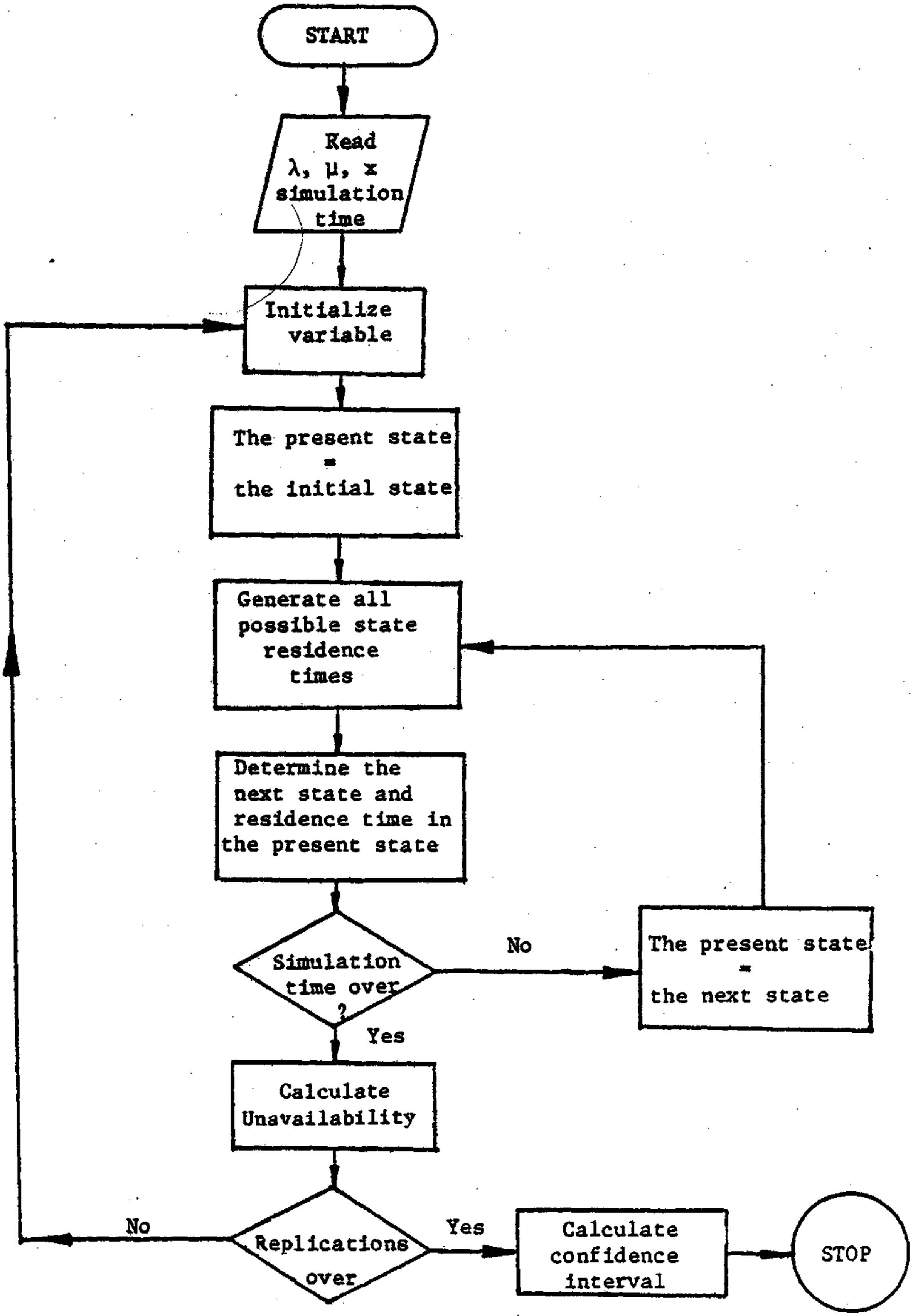

Figure 5-1: Flowchart Of The Simulation Procedure For Setting Confidence Interval Limits On The System Unavailability.

It is assumed that the failure and installation processes are exponential and the repair process is Weibull. The following set of data has been used.

Failure rate = 0.25 f/yr

Repair rate = 2 r/yr

Installation rate = 182.5 inst/yr

β for repair process = 1, 4

Simulation time for one run = 10,000 years

Number of replications n = 10, v = 10-1 = 9

Confidence co-efficient $1-\alpha = 0.95$

Two tail $t_\alpha = 2.262$, One tail $t_\alpha = 1.833$

The following results were obtained using this data

Mean $\bar{X} = 0.007831$, v = 10-1 = 9, s = 0.000617

For $\beta = 1$

Two sided confidence intervals limits are

Upper limit = $\bar{X} + st_\alpha/\sqrt{n}$ = 0.008272

Lower limit = $\bar{X} - st\alpha/\sqrt{n}$ = 0.007390

One sided upper confidence interval limits are

Upper limit = $\bar{X} + st_\alpha/\sqrt{n}$ = 0.008189

Lower limit = $-\infty$

One sided lower confidence interval limits are

Upper limit = $+\infty$

Lower limit = 0.007473

For $\beta = 4$

Two sided confidence limits are

Upper limit = 0.009105, Lower limit = .008255

One sided upper confidence interval limits are

Upper limit = 0.009024, Lower limit = $-\infty$

One sided lower confidence interval limits are

Upper limit = $+\infty$, Lower limit = 0.008336

The confidence limits utilized in the remainder of thesis are all two sided confidence intervals.

5.4 Methods To Estimate Confidence Intervals For A Given Tolerance

Method 1: In this approach, it is assumed that samples are taken from a normal population with a unknown mean m and a known variance σ^2. An estimate of σ can be obtained from either past data or a small trial sample. The sample mean is given by m and variance is given by σ^2/n so that $z = (\bar{X}-m)/(\sigma/\sqrt{n})$ is standard normal. The 100(1-α) percent confidence intervals are given by[17]:

$$(\bar{X} - z_\alpha \sigma/\sqrt{n} \leqslant m \leqslant \bar{X} + z_\alpha \sigma/\sqrt{n})$$

If a tolerance or precision i.e. the amount of deviation from the true value in actual units or percent that is allowed is d, the sample size and in these studies the number of replications in order to produce a symmetrical 100(1-α) percent confidence interval of width 2d is given [17] by

$$n = [\ z_\alpha \sigma/d]^2.$$

Any number of replications which exceeds n will yield more confidence and precision.

Consider the example of the one spare transformer model with the following data to illustrate the above procedure. It is assumed that failure, repair and installation processes are exponential.

Failure rate = 0.25 f/yr

Repair rate = 12 r/yr

Installation rate = 182.5 inst/yr

Simulation time for one run = 10000 years

Precision requirement or tolerance = 0.000016

Confidence coefficient $1-\alpha = 0.95$, $z_{\alpha} = 1.96$

The simulation was carried out for n = 10 using the procedure described in the previous section in order to obtain an estimate of the standard deviation and is estimated as $\sigma = 0.000047$.

Then from the above data

$$n = (1.96 \times 0.000047/0.000016)^2 = 33.1488$$

n = 34 is therefore the required number of replications. The results are X = 0.001550 and $\sigma = 0.000050$. The two sided confidence interval limits on the mean unavailability are

$$\text{Limits for } m = 0.001550 \pm \frac{1.96 \times 0.000050}{\sqrt{34}}$$

$$= 0.001533 \text{ and } 0.001567 \text{ respectively.}$$

The actual precision is $1.96 \times 0.000050/\sqrt{34} = 0.000017$ which exceeds the desired 0.000016. This occurred because σ rose from 0.000047 to 0.000050, when the required $n = 34$ was taken. Had σ remained at 0.000047, $d = 0.000016$ would have resulted. Substitution of $\sigma = 0.000047$ into the limits computation will give $0.001534 \leq m \leq 0.001566$.

Two difficulties with the interval estimation procedure as discussed should be noted. First, the assumption that the population is normally distributed does not always hold. The assumption of normality does not pose problem when the sample size or in this case the number of replications is large ($n > 30$). The second difficulty with the formula for confidence interval given above is that it requires knowledge of population variance σ^2. When the number of replications is relatively small and the population variance cannot be calculated, the following method using the Student's t distribution is used to determine confidence intervals.

Method 2: Consider a sequence of independent and identically distributed(i.i.d) random variables with mean m and variance σ^2. To estimate mean m within a tolerance of $\pm d$, where d is a user-specified quantity, the procedure is to collect one observation X_i at a time to form [16]

$$\bar{X} = \frac{1}{n} \Sigma X_i, \quad s^2 = \frac{1}{n-1} \Sigma X_i^2 - \frac{n}{n-1} \bar{X}^2$$

and stop at observation n where

$$n = \min \left[n \; : \; s^2 \leq \frac{nd^2}{t_\alpha^2} \right]$$

where t_α is calculated from Student's t table for n-1 degrees of freedom. The above expression is a stopping rule.

The confidence interval is given by:

$$Pr[\,\overline{X}-d \leqslant m \leqslant \overline{X}+d\,] \geqslant 0.928 \text{ for } 1-\alpha = 0.95$$
$$\geqslant 0.985 \text{ for } 1-\alpha = 0.99.$$

This rule is encouraging for it implies at most, a mild erosion in the confidence level for arbitrary d. If the above expression is used as a stopping rule and observations are collected on two additional replications, the lower bounds increase to 0.9414 for $1-\alpha = 0.95$ and 0.9875 for $1-\alpha = 0.99$.

The above procedure is illustrated in Table 5-1 using selected data to set confidence interval limits on the system unavailabilities. The one spare transformer model is again chosen as an example. The failure, repair and installation processes are assumed to be exponential.

Table 5-1: Procedure To Set Confidence Interval Limits On The System Unavailability For A Given Tolerance.

Failure rate = 0.25 f/yr
Repair rate = 12 r/yr
Installation rate = 182.5 inst/yr
Simulation time for one run = 10,000 years
Required tolerance d = 0.000030

n	X_i	s^2	$nd^2/t_\alpha{}^2$
1	0.001494		
2	0.001504	1.96 E-10	1.1149 E-11
3	0.001519	8.4 E-10	1.4582 E-10
4	0.001527	7.84 E-10	3.5555 E-10
5	0.001534	8.4 E-10	5.8395 E-10
6	0.001539	7.84 E-10	9.0183 E-10

Table 5-1 presents the results using the stopping rule for all $n > 2$. The results in line 6 of Table 5-1 implies that provided X_i is normal

$$Pr(0.001539-0.000030 < m < 0.001539+0.000030)$$

$$= Pr(0.001509 < m < 0.001569) > 0.928.$$

It has been observed that the number of iterations in order to obtain a specific tolerance depends upon the simulation time as well as the number of replications. For example, if the simulation time is increased from 10,000 years to 100,000 years, the number of replications decrease as shown in the following Table 5-2. When simulation time is 100,000 years, the number of replications to obtain the desired tolerance reduce to 3. In this case
$Pr(0.001522 < m < 0.001582) > 0.928$. If the desired tolerance d is 0.000016, quite a few replications have to be carried out

Table 5-2: Simulation Procedure To Set Confidence Interval Limits When Simulation Time Is 100000 Years.

n	X_i	s^2	nd^2/t_α^2
1	0.001558		
2	0.001554	3.6 E-11	1.1149 E-11
3	0.001552	1.6 E-11	1.4582 E-10

when the simulation time is 10,000 years. In the case of a simulation time of 100,000 years and applying the stopping rule, the desired tolerance is achieved at the 3rd replication.

When n=3

$m = 0.001553$, $s^2 = 1.6$ E-11, $nd^2/t_\alpha^2 = 4.1478$ E-11

$Pr(0.001537 < m < 0.001569) > 0.928$.

It is important to note that Student's t and corresponding tabulated critical values are based on the assumption that the sampled population possesses a normal probability distribution as assumed above. It has been observed [3] that the distribution of the t statistic is relatively stable for populations which are non normal but possess a mound shaped distribution. The method can be applied to all problems.

5.5 Effect On System Unavailability Of The Increasing Number Of Spares When The Repair Or Installation Times Are Weibull Distributed

In Chapter 2, the effect on system unavailability of increasing the number of spares was evaluated for a Markovian system. It was observed that the system unavailability decreases with increasing number of spares and approaches a limiting value for infinite spares. In Chapter 3, the assumption of exponentially distributed state residence times was then relaxed to encompass systems with non-Markovian characteristics. The effect of non-exponential distributions on system unavailability was illustrated using a transformer bank with one spare as an example. In this section, one more spare unit has been added to the one spare transformer model and the effect of Weibull distributed state residence times is evaluated. The values of system unavailability with exponentially distributed state residence times ($\beta = 1$) are taken as the base values and are therefore calculated by analytical techniques. The effect of Weibull distributed residence times on system unavailability for the infinite number of spares case is shown later in this chapter. The Weibull distribution with $\beta = 4$ is chosen intentionally because of its bell shaped probability distribution function.

5.5.1 Effect Of Weibull Distributed Repair And Installation Times On System Unavailability Of The Transformer Bank With Two Spares

The state space diagram of the non-Markovian system with two spares is shown in Figure 5-2.

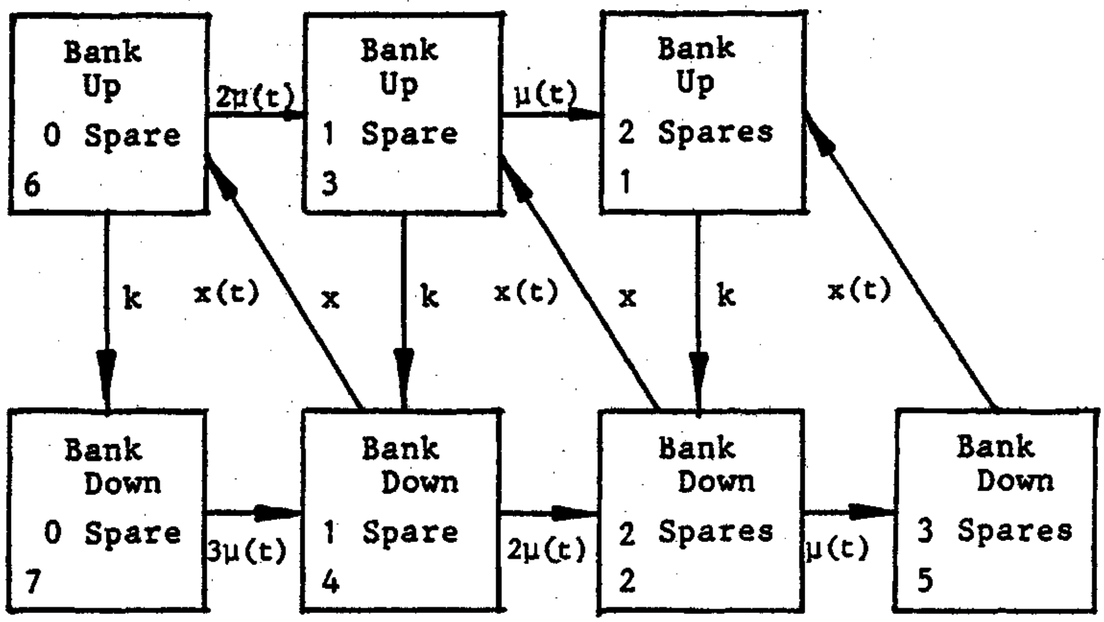

Figure 5-2: Non-Markovian Model Of Transformer Bank With Two Spare Transformers.

In these studies, it is assumed that the failure rate of transformers is constant. The assumption of Weibull distributed repair and installation times is applied in steps. The following data has been used in these investigations.

Failure rate = 4.0, 0.25 f/yr

Repair rate = 2, 3, 6, 12, 24 r/yr

Installation rate = 30, 50, 100, 150, 182.5, 365 inst/yr

Simulation time = 100,000 years for 0.25 f/yr

and 50,000 years for 4.0 f/yr

Shape parameter = 1,4

Case 1: The failure and installation processes are exponential and the repair process is Weibull.

The variation in the system unavailability as a function of the average repair time is shown in Table 5-3.

Table 5-3: The Variation In The Unavailability As A Function Of The Average Repair Time For The Two Spare Transformer Model.

Sensitivity Study - Repair rate Variable
- Repair process Weibull
- Installation rate = 182.5 inst/yr

Repair rate	Exponential	Weibull
Failure rate = 4.0 f/yr		
2	0.219436	0.289930
3	0.122187	0.164223
6	0.043742	0.053536
12	0.024969	0.026429
24	0.021885	0.022070
Failure rate = 0.25 f/yr		
2	0.001645	0.001624
3	0.001452	0.001456
6	0.001378	0.001386
12	0.001369	0.001372
24	0.001369	0.001373

Table 5-3 shows that in the case of the two spare transformer model, the Weibull distributed repair times, generally give higher values of system unavailability as compared to those obtained with exponentially distributed repair times. The effect is more evident in the case of systems with higher unavailabilities.

Case 2: The failure and repair processes are exponential, installation process is Weibull.

The variation in system unavailability as a function of the average installation time is shown in Table 5-4.

Table 5-4: The Variation In Unavailability As A Function Of The Average Installation Time For Two Spare Transformer Model

Sensitivity Study - Installation rate Variable
- Installation time Weibull distributed
- Repair rate = 12 r/yr

Failure rate = 4.0 f/yr		
Inst. rate	Exponential	Weibull
30	0.119006	0.136977
50	0.076141	0.084425
100	0.041414	0.043930
150	0.029333	0.030733
182.5	0.024969	0.025866
365	0.014779	0.015021

Failure rate = 0.25 f/yr		
Inst. rate	Exponential	Weibull
30	0.008265	0.009463
50	0.004976	0.005459
100	0.002495	0.002607
150	0.001665	0.001722
182.5	0.001369	0.001405
365	0.000686	0.000692

It can be seen from Table 5-4 that the values of system unavailability with Weibull distributed installation times ($\beta = 4$) are higher than the base values. The relative deviation from the base values decrease with an increase in the installation rates.

5.5.2 Effect Of An Infinite Number Of Spares On The System Unavailability.

As in the case of a Markovian system, the limiting unavailability can be evaluated easily if it is noted that, when there are an infinite number of spares, the repair process becomes irrelevent and can be ignored. The limiting state space diagram reduces to one containing only two states, system up and system down. The transition rate from down to up state i.e. installation rate is now a function of time. The transition from the up state to the down state (failure rate) is constant. The diagram is shown in Figure 5-3.

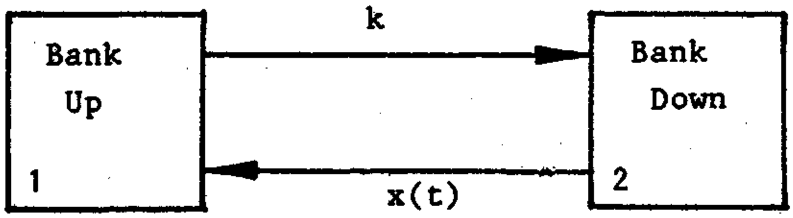

Figure 5-3: Two State Non-Markovian Model Of The Transformer Bank.

Table 5-5 shows the values of system unavailabilities for transformer with infinite number of spares obtained as a function of failure and installation rates for $\beta = 1$ and $\beta = 4$. The simulation times are 10,000 years for 0.25 f/yr

Table 5-5: The Variation In System Unavailabilities As A Function Of Failure And Installation Times.

Failure rate	Inst. rate	Exponential	Weibull
0.25	182.5	0.001366	0.001365
0.50	182.5	0.002732	0.002732
0.75	182.5	0.004092	0.004082
1.00	182.5	0.005450	0.005442
0.25	91.25	0.002732	0.002724
0.25	273.75	0.000912	0.000910
0.25	365.0	0.000684	0.000683
4.0	182.5	0.021447	0.021364

and 50,000 years for 4.0 f/yr. It can be concluded from Table 5-5 that system unavailabilities for $\beta = 1$ and $\beta = 4$ are quite close to each other for an infinite number of spares.

The following points can be drawn from all of the studies conducted in Chapter 4 and Chapter 5.

The system unavailability, generally, decreases with an increase in the number of spare components.

The unavailability values for $\beta = 4$ are, in general, higher than the base values ($\beta = 1$) for a small number of spares. As the number of spares is increased, the unavailability values for $\beta = 4$ approach the base values. When there are an infinite spares, the system unavailabilities for $\beta = 4$ are equal to the base values(within simulation limits).

On comparing the results of Tables 5-5, 2-1, 2-2, and 2-3, it can be seen that in a system with two or more spares, the system unavailabilities are quite close to each other for the following values of failure, repair and installation rates.

Failure rate $\leqslant$ 0.25 f/yr

Repair rate $\geqslant$ 12 r/yr

Installaion rate $\geqslant$ 182.5 inst/yr

In other words, the system unavailabilities approach a constant value for the above basic data and this value is very close to the value given by the expression of calculating unavailability for an infinite spares in the case of Markovian systems.

The most important conclusion that can be drawn from all of these studies is that for a two state model, the distributional assumptions of state residence times do not have any impact on calculated unavailability as long as the average values of these distributions are constant. As the number of outgoing transitions from a state increase, the bell-shaped distributions result in higher unavailabilities. In the case of repairable systems, the distributional impact on unavailability decreases with an increase in the number of spares as in the limit state space diagram for all distributions reduce to a two state model.

5.6 Calculation Of The Frequency Of The Up States And Down States In The Case Of Non-Markovian Systems

The frequency of an up state or down state is an important parameter in power system reliability evaluation studies.

In the case of Markovian systems

Frequency Of encountering a particular state =

(Probability of staying in that state) x
(Rate of departure from the state)

Frequency of encountering cumulated states =

(Sum of frequencies of individual states) -
(Frequency of encounters between these states)

The same basics can be utilized to evaluate frequencies of up and down states in the case of non-Markovian systems. The Monte Carlo simulation method used earlier to evaluate system unavailabilities can be utilized to determine frequencies also with slight modifications.

In this case

$$\text{Frequency of a state} = \frac{\text{Number of transitions in a particular state}}{\text{Total simulation time}}$$

Frequency of cumulative up(down) state =

$$\frac{\text{Sum of frequencies of up(down) states} - \text{Sum of number of transitions among up(down) states}}{\text{Total simulation time}}$$

The above procedure can be illustrated using the one spare model shown in Figure 4-1. In Figure 4-1, State 1 and State 3 represent individual down states. There is one transition from State 3 to State 1 between up States 1 and 3. Similarly, there are two transitions from State 4 to State 2 and from State 2 to State 5 among down states. The values of frequencies of individual states and cumulative up and down states for selected data are shown in Table 5-6. The failure and installation proceses are assumed to be exponential and the repair process is Weibull. In most of the problems encountered, the cumulative values of frequencies are used.

It follows from Table 5-6 that the frequency of an cumulative up state is equivalent to the frequency of the down state. The values of frequency with $\beta = 4$ are slightly lower than the values obtained with $\beta = 1$. The same procedure can be utilized to determine the up state and down state frequencies of a system with two or more spares. The procedure of setting confidence interval limits on unavailability can also be applied to determine a confidence interval for frequency.

5.7 Conclusion

In many availability assessments, the evaluation is performed using a single point availability estimate. This chapter has briefly illustrated how the state histories can be used to obtain unavailabilities at selected levels of confidence. Two methods have been described to set confidence intervals with a given precision and accuracy.

Table 5-6: Frequency Value Of Individual States And Cumulative Up(Down) States For The One Spare Transformer Model Shown In Figure 4-1 When The Repair Process Is Weibull.

Repair rate = 12 r/yr
Repair time is Weibull distributed
Installation rate = 182.5 inst/yr
Simulation time = 100,000 years for 0.25 f/yr
= 50,000 years for 4.0 f/yr

Failure rate = 4.0 f/yr		
State	Exponential	Weibull
1	2.885633	2.703026
2	3.769979	3.743600
3	3.537384	3.742520
4	0.884346	1.040594
5	0.232595	0.001080
Up State (1+3)	3.769979	3.743620
Down State (2+4+5)	3.769979	3.743600

Failure rate = 0.25 f/yr		
State	Exponential	Weibull
1	0.244829	0.243703
2	0.249608	0.248893
3	0.234208	0.248793
4	0.004780	0.005200
5	0.015400	0.000100
Up State (1+3)	0.249608	0.248903
Down State (2+4+5)	0.249608	0.248893

The Monte Carlo simulation methodology utilized in the previous chapter has been used to obtain unavailabilities for non-Markovian systems with two and an infinite number of spares. It was observed that for the two spare transformer model, the values of unavailability obtained with non-exponential state residence times were higher than the base values. The system unavailabilities approach the base values as the number of spares increase. The system unavailabilities with non-exponential state residence times are equal to the base values for infinite spares.

This chapter has also shown how the simulation technique can be extended to provide the additional availability parameter of frequency. This additional index can provide an improved physical appreciation of the reliability performance of a system.

6. GENERAL CONCLUSIONS

The objective of this project was to study the effect of various distributional assumptions on the unavailability associated with repairable systems containing standby components. The study conducted was divided into several parts in which additional objectives apart from the main objective were achieved.

The main elements of the study are given under the different chapter headings and cover the following topics: Markov modelling of repairable systems, non-Markovian modelling of repairable systems, the effect of non-exponential distributions on system unavailability and selected studies in the case of repairable systems with standby components. The initial chapter contains a general introduction. This also includes an overall description of the problem under investigation. Chapter 2 deals with availability modelling techniques of Markovian systems. The various analytical techniques are discussed. The frequency balance approach is used to compute various availability indices for different configurations with an increasing number of spares. In these cases, the closed form expressions were obtained. It was observed that in the case of large and complicated systems, it becomes relatively laborious to write the mathematical expressions. Simulation methods find an increasing application in these circumstances.

Two simulation methodologies based on transition rates and state residence times were presented by illustration. The simulation approach based on transition rates assumes constant transition rates from one state to another. This approach can not be applied to non-Markovian systems. The simulation method based on state residence times is a general technique and can be applied to both Markovian and non-Markovian systems. The simulation method based on transition rates was later used to evaluate unavailabilities of Markovian systems with an increasing number of spares. The results obtained by the simulation method and analytical technique were in close agreement. This illustrates that the simulation method which is quite easy to employ can be used interchangeably with analytical techniques to evaluate availability indices.

The state residence times are non-exponentially distributed in the case of non-Markovian systems. The distributions most commonly used in availability studies to represent these residence times are Erlang, normal and Weibull. The brief description of the characteristics of these three distributions is given in Chapter 3 together with an analytical technique, the method of stages, used to compute unavailabilities of non-Markovian systems. The simulation technique based on state residence times is developed to evaluate system unavailabilities of these systems. This method permits the state residence times to follow any distribution. It was observed that the method of stages becomes difficult and cumbersome if not impossible to employ, as the number of required stages and the possible combinations increase. The method of stages is suitable for relatively small models where only a few of the transitions are non-exponential. The simulation method can be applied to all problems concerned with non-Markovian as well as Markovian systems.

Repairable systems with standby components are quite often associated with non-exponential repair and installation times. Chapter 4 has presented a set of studies to illustrate the effect of these two parameters on system unavailability as compared to a base case involving exponential distributions. Practical values of these parameters were selected and sensitivity studies performed with average repair and installation times as varying parameters. The three stages were used to represent Erlang distributed state residence times. The effect of varying the standard deviation was incorporated in the sensitivity studies used to evaluate the effect of normally distributed state residence times. The shape parameter was varied in the case of Weibull distributed repair and installation times. In all of these studies, the one spare transformer model was used as an example. The impact of fixed repair and installation times was also examined.

The shape of the distribution has a significant bearing on the calculated unavailability. Erlangian distributed state residence times result in higher unavailabilities when compared to the base values. As the number of stages increase, system unavailability also increases. Similarly, normally distributed installation and repair times also give higher values of unavailability. The shape parameter of the Weibull distribution can be varied to generate various probability density functions. System unavailabilities in the case of hyper-exponential distributions ($\beta < 1$) are basically lower than the base values whereas bell-shaped distributions ($\beta > 1$) give relatively higher values of unavailability. In all cases, a bell-shaped repair time distribution has a larger impact on system unavailability when compared to the installation time distribution. The system unavailability values approach the base values as the state residence times decrease. The deviation of the results with non-exponential distributions from the base values decreases as the overall system availability increases.

Point estimates of system unavailability may not fulfill the need for a specified accuracy. In Chapter 5, two methods are developed that can be used to set confidence limits on the unavailability indices. The effect of distributional assumptions on system unavailability with two and an infinite number of spares is evaluated. Weibull distributed repair and installation times provide higher values of unavailability when compared to the base values. The unavailabilities with infinite spares were equal to the base values. As the number of spares increase, the system unavailability with all distributions approaches a constant value given by an expression for the unavailability with an infinite number of spares in the case of Markovian systems. The frequencies of encountering up or down states in the case of Weibull distributed residence times are slightly lower than the base values obtained with exponential distributions.

The investigations conducted in this work indicate that for the same mean values the values of system unavailability with non-exponential distributions are equal to the base values for a two state model in which there is only one outgoing transition from each state. As the number of outgoing transitions from a state increase, skewed distributions result in higher unavailabilities. The impact of these distributions diminishes with an increase in overall system availability. The commonly used techniques involving Markov models may be used for highly available systems to evaluate system unavailabilities in certain cases without introducing significant errors.

An important aspect of this work is that the simulation methodologies developed are not confined strictly to the unavailability analysis of repairable systems. These can be applied to any general system which is continous in time and can be represented by a state space diagram.

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APPENDIX A

IMSL ROUTINE - GGEXN

PURPOSE - EXPONENTIAL RANDOM DEVIATE GENERATOR

USAGE - CALL GGEXN (DSEED, XM, N, R)

ARGUMENTS

DSEED - INPUT. AN INTEGER VALUE IN THE EXCLUSIVE RANGE (1, 2147483647). DSEED IS REPLACED BY A NEW DSEED TO BE USED IN SUBSEQUENT CALLS. DSEED MUST BE TYPED DOUBLE PRECISION IN THE CALLING PROGRAM.

XM - INPUT MEAN VALUE.

N - INPUT NUMBER OF DEVIATES TO BE GENERATED.

R - OUTPUT VECTOR OF LENGTH N CONTAINING THE EXPONENTIAL DEVIATES.

REQD. IMSL ROUTINES - GGUBS

<u>Algorithm</u>

GGEXN generates deviates from the exponential distribution with mean m (XM in the calling sequence). The probability density function is

$$f(x) = \begin{cases} (1/m)\, e^{-x/m} & , \quad 0 \leqslant x < \infty,\ 0 < m < \infty \\ 0 & , \quad \text{elsewhere} \end{cases}$$

The corresponding distribution function is

$$F(X) = 1-e^{-x/m}$$

Deviates are generated using the fact that if U has a uniform (0,1) distribution then 1-U also has that distribution. Also the exponential distribution function has an exact form for its inverse. An exponential deviate is produced according to

$$x = -m(\ln(u))$$

where u is a uniform (0,1) deviate produced by IMSL routine GGUBS.

Example

Here, GGEXN is called to generate ten exponential random deviates with mean value 7.9.

INPUT:

```
INTEGER          N
REAL             XM, R(10)
DOUBLE PRECISION DSEED
DSEED =  123457.0D0
XM    =  7.9
N     =  10
CALL GGEXN (DSEED, XM, N, R)
```

OUTPUT:

```
DSEED = .8178780950D09
R(1)  = .27147
R(2)  =  10.620
   .
   .
   .
R(10) =   7.6262
```

APPENDIX B

IMSL ROUTINE - GGNML

PURPOSE - NORMAL OR GAUSSIAN RANDOM DEVIATE GENERATOR

USAGE - CALL GGNML (DSEED, NR, R)

ARGUMENTS

DSEED - INPUT/OUTPUT DOUBLE PRECISION VARIABLE ASSIGNED AN INTEGER VALUE IN THE EXCLUSIVE RANGE (1.D0, 2147483647.D0). DSEED IS REPLACED BY A NEW VALUE TO BE USED IN A SUBSEQUENT CALL.

NR - INPUT NUMBER OF DEVIATES TO BE GENERATED.

R - OUTPUT VECTOR OF LENGTH NR CONTAINING THE NORMAL (0,1) RANDOM NUMBERS.

REQD. IMSL ROUTINES - GGUBS, MDNRIS

Algorithm

GGNML generates pseudo-random normal (0,1) deviates by transforming uniform deviates to normal deviates using the inverse normal routine MDNRIS.

Given DSEED and NR, GGUBS is called to generate NR uniform random numbers in the exclusive range (0,1). Then IMSL routine MDNRIS is called NR times to transform each of the numbers to a normal (0,1) deviate. That is, the uniform deviates generated by GGUBS are transformed to normal (0,1) deviates using the inverse normal probability distribution function MDNRIS.

Random normal (M,S^2) deviates may be obtained by transforming GGNML output according to Y(I)=((R(I))S+M, for I in (1,2,...NR).

Example

In this example, 100 normal random numbers are generated by making one call to GGNML with NR=100 and input DSEED=123457.D0.

Input:

```
INTEGER            NR
REAL               R(100)
DOUBLE PRECISION   DSEED
NR      =   100
DSEED   =   123457.D0
CALL GGNML (DSEED, NR, R)


END
```

OUTPUT:

```
DSEED   =    801129707.D0
R(1)    =   .18279E01
   .
   .
   .
R(100) =  -.32377E00
```

APPENDIX C

IMSL ROUTINE - GGWIB

PURPOSE - WEIBULL RANDOM DEVIATE GENERATOR

USAGE - CALL GGWIB (DSEED, A, N, R)

ARGUMENTS

- DSEED - INPUT. AN INTEGER VALUE IN THE EXCLUSIVE RANGE (1, 2147483647). DSEED IS REPLACED BY A NEW DSEED TO BE USED IN SUBSEQUENT CALLS. DSEED MUST BE TYPED DOUBLE PRECISION IN THE CALLING PROGRAM.
- A - INPUT. SHAPE PARAMETER FOR THE DESIRED WEIBULL FUNCTION. A MUST BE GREATER THAN 0. NO PROGRAM CHECK IS MADE FOR THIS CONDITION.
- N - INPUT NUMBER OF DEVIATES TO BE GENERATED.
- R - OUTPUT VECTOR OF LENGTH N CONTAINING THE WEIBULL DEVIATES.

REQD. IMSL ROUTINES - GGUBS

Algorithm

GGWIB generates pseudo-random deviates from a Weibull distribution, with density

$$f(x) = \begin{cases} Ax^{A-1}e^{-x^A} & x, A > 0 \\ 0 & \text{elsewhere} \end{cases}$$

and distribution function

$$F(x) = 1 - e^{-x^A}$$

By inversion, a Weibull deviate may be generated as

$$x = [-\ln(u)]^{1/A}$$

where u is a pseudo-random deviate from a uniform (0,1) distribution.

Deviates having the more general density function

$$f(x) = \begin{cases} (A/B)[(x-C)/B]^{A-1}\exp-[(x-C)/B]^{A} & x > C,\ A > 0,\ B > 0 \\ 0 & \text{elsewhere} \end{cases}$$

may be generated as follows:

```
  INTEGER          N
  REAL             R(N), A, B, C
  DOUBLE PRECISION  DSEED
     :
     :
  CALL GGWIB(DSEED, A, N, R)
  DO 5 I = 1, N
     R(I) = B*R(I)+C
5 CONTINUE
```

A is generally referred to as the shape parameter. B and C are the scale and location parameters, respectively.

Example

In this example, ten Weibull deviates are generated by calling GGWIB.

Input:

```
INTEGER             N
REAL                R(10), A
DOUBLE PRECISION    DSEED
N     =  10
DSEED =  123457.0D0
A     = .6
   .
   .
   .
CALL GGWIB (DSEED, A, N, R)
   .
   .
   .
END
```

Output:

```
DSEED   =  817878095.0D0
R(1)    = .00363
   .
   .
   .
R(10) =  .94290
```