GENERATION OF INTENSE MAGNETIC FIELDS WITH HIGH HOMOGENEITY

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by

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ABSTRACT

The intense magnetic fields with a high degree of field homogeneity have applications in particle accelerators, plasma containers, cyclotrons and other nuclear physics experiments. The problem of satisfying the high homogeneity requirements becomes more severe with an increase in field intensity. Air-core solenoids and iron-core electromagnets are considered as the two basic designs for producing the desired field. In the case of air-core solenoids various coil arrangements are examined. The suitability of a two-coil system and a split-coil design for improving the field homogeneity is illustrated. An analysis based on the method of zonal harmonics is developed. The effect of different coil geometries on field intensity and homogeneity is studied. In the case of the two-coil system, the effect of coil separation on field homogeneity, for various coil geometries, is examined. The method of adding compensating windings to the cylindrical solenoid for improving field homogeneity is investigated in detail. The field homogeneity figures are computed for certain coil arrangements.

The severe problem of cooling high powered air-core solenoids is discussed and the design calculations for the cooling system are presented.

In the iron-core electromagnet design, the effect of pole piece and pole core shape on saturation and gap field homogeneity is discussed. The importance of proper dimensioning of various parts of the magnetic circuit is stressed. A practical design procedure for an iron-core magnet is outlined. Central gap field homogeneity figures are computed for specific pole geometries.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

H_z	Axial Component of Magnetic Field Intensity
$H\rho$	Radial Component of Magnetic Field Intensity
$H_z(0,0)$	Magnetic Field Intensity at the Geometric Centre of the Coil
G	Geometry Factor
V	Scalar Magnetic Potential
a_1	Inner Radius of the Coil
a_2	Outer Radius of the Coil
a_o	Mean Radius of the Coil
$2b$	Axial Width of the Coil
$2d$	Radial depth of the Coil
$2x_1$	Separation Between Two Coils in the Two-Coil System
α	Ratio of Outer Radius to Inner Radius of the Coil
β	Ratio of Coil Width to Inner Diameter
E_n's	Error Coefficients of Main Solenoid
E_n''s	Error Coefficients of Compensating Solenoid
r, θ, ϕ	Spherical System of Coordinates
ρ, ϕ, z	Cylindrical System of Coordinates
i	Current Density in the Coil Winding
W	Electrical Power Input to the Coil
$H_z(r,\theta)$	Axial Component of Magnetic Field Intensity at any Point $P(r,\theta)$
ΔH_z	Degree of Field Homogeneity
v	Volume Factor of the Coil
σ	Surface Density of Magnetic Distribution in a Circular Loop

Ω	Potential Due to a Magnetic Shell
m_s	Specific Mass of Cooling Fluid
C_p	Specific Heat of Cooling Fluid
$2a_{1t}$	Inner Diameter of Tubular Conductor
$2b_{1t}$	Outer Dimension of Tubular Conductor
l	Length of Water Tubing
P	Pressure required to Pump the Fluid
v_f	Velocity of Cooling Fluid
Q	Quantity of Cooling Fluid
Δt	Temperature Rise in Cooling Fluid
Re	Reynold's Number
ν	Coefficient of Viscosity
w_s	Power Dissipated per Unit Cooling Surface
B_o	Magnetic Flux Density at the Geometric Centre in the Air-Gap Region of the Electromagnet
μ_o	Permeability of vacuum
μ_r	Permeability of ferromagnetic Material
Φ	Flux Through the Magnetic Circuit
$2Z_1$	Air-Gap Length
$2r_1$	Pole Face Diameter
$2r_2$	Pole Piece Base Diameter
$2r_3$	Pole Core Base Diameter
θ_1	Pole Piece Taper Angle
θ_1	Pole Core Taper Angle

1. INTRODUCTION

1.1 General

The generation of high intensity magnetic fields has assumed such great importance during the recent years because a number of groups are building magnet installations comparable in cost and complexity to astronomical telescopes, particle accelerators and nuclear reactors. Numerous investigations in nuclear physics, magneto-optics and very low temperature physics require the use of intense magnetic fields maintained constant for the entire duration of the experiement. The requirement of a high degree of homogeneity at the high field intensity adds to the complexity of the magnet design. The commercial development of nuclear magnetic resonance (NMR) spectroscopy, with continual improvement in sensitivity, stability and resolution, posed a real challenge to the magnet designer to generate a highly homogeneous magnetic field. In order that the NMR technique be useful for many applications, frequency resolution better than 1 cycle/sec is needed, irrespective of the field value. Therefore, the problem of satisfying the high homogeneity and stability requirements becomes more severe with increase in field intensity. Such highly uniform magnetic fields have applications in particle accelerators, plasma containers, study of zeeman effect, cyclotrons, and other nuclear physics applications.

Of the several ways of generating a magnetic field, the choice is dictated by considerations of power, weight, space, duty-cycle, required field intensity, duration, volume, and required field homogeneity. The relative importance of these considerations usually determines the method with little overlapping of choice. The problem at hand states that the field intensity is to be of the order of 20 to 25 kilogauss, direct-current continuous duty, with a field homogeneity of 1 in 10^5 over a central spherical volume of one inch diameter. To meet these specific requirements of field intensity, field volume and field homogeneity, three basic types of magnet design can be considered: (i) superconducting solenoids, (ii) air-core solenoids and (iii) iron core electromagnets. The basic differences, advantages and disadvantages between these three types of magnets are briefly described below.

1.2 Superconducting Solenoids

The development of high quality materials in recent years has tremendously accelerated the development of magnet technology. The availability of superconducting materials which can carry enormous currents has lead to the development of predictable and reliable magnets of all sizes. Superconducting magnets have been applied in nuclear physics for bubble chambers, ionography and deflecting magnets. However, one has to look into the problems associated with the use of superconducting coils and a number of subtleties which distinguish the operation of a superconducting magnet from that of a conventional type. All superconducting magnets share the requirement of a 4.2° K cryogenic environment. This is relatively expensive to maintain, particularly under non-laboratory or special conditions, and specially when these coils have to be operated continuously. More economical and successful solutions than those in current use are called for to accommodate the increasing number of potential applications. It has been recognized since the very first superconducting magnets were built that, superconducting magnets are susceptible to instability problems. This manifests itself in limiting the currents that can be carried in these coils to values well below the critical currents. The trapped moments in the superconducting coil which influence the zero current residual field,and hence the homogeneity, also give rise to a nonlinearity in the current vs. field relationship for the coil[1]. The diamagnetic currents further influence the operation of coils because their local collapse produces flux jump noise. If this is measured in the bore of the magnet, it appears as a spectrum of pulses in the millisecond domain[2]. These are more predominant at lower currents. Because of the very low heat of vaporization of liquid helium the minimization of cryogenic losses is of principal concern in the design of superconducting systems. Certain applications, and requirements of access to the dewar often preclude the use of the most efficient dewar designs.

The successful application of a superconducting magnet invariably involves more than simple consideration of the magnitude of the central field. For many applications one is more concerned with the homogeneity of the field. When the homogeneity in the centre of the solenoid is improved by making the solenoid longer, the volume of superconducting material is, naturally, increased thereby increasing the cost involved. Choosing a greater bore diameter increases the central field homogeneity slightly, but as this increases the volume of superconducting material it is not adopted as a method to improve field homogeneity. In this connection, the technique of adding compensating windings to the main solenoid has been suggested by Garrett[3]. However, in the case of superconductors the situation is complicated by the fact that permanent diamagnetic currents formed within the body of the superconducting wire distort the field. Hence, compensation for these currents has to be done. Apart from this compensation, compensating windings also have to be provided for the main field. The total compensation depends on the strength of the field at which the desired homogeneity is to be achieved.

In many experimental and application situations it is necessary to provide transverse access to the field. This would call for a gap within a pair of coils. By this arrangement one would require larger volume of superconducting material for producing a given field intensity, as compared to the conventional cylindrical solenoid. Although superconducting coils with absolute time stability prove excellent for producing highly homogeneous fields, their high material, installation and maintenance costs do not fully justify their use. for producing a field of strength in the neighbourhood of 20 KG. Hence, superconducting solenoid design is not considered in this thesis.

1.3 Air-Core Solenoids

The art of generating magnetic field by the use of iron-free solenoids is fairly old and well established. Solenoid magnets are generally applied for producing low fields where power is not a consideration, or for very high fields where the introduction of iron would be rendered relatively useless due to saturation effect. In recent years, however, the generation of very intense magnetic fields by the use of air-core solenoids has assumed great importance. The simplest type of solenoid magnet is a cylindrical coil, with straight ends, carrying a uniform current density. Fabry[4], Cockcroft[5], and Bitter[6] have made fundamental contributions to the design of high intensity magnet coils. The advantages of air-core solenoids are:

(1) They can possess a large length and produce quite economically an intense magnetic field.

(2) As the extremities of the coil are free, one can easily observe the bore along its axis and insert a long apparatus.

(3) Very intense magnetic fields even of the order of 150 KG, can be generated[7] and there are no problems of saturation.

(4) Since the magnetic field intensity obtained is proportional to the current density, the field can easily be determined and calibrated. Further, no hysteresis phenomena is manifested due to reversal of direction of current.

(5) Due to the absence of any remanent magnetization, the total suppression of the field is easier and more rapid than in the case of an iron core magnet which must be subjected to a sligh reverse-current to exactly cancel the field in the iron core.

(6) The coils are less heavy and less cumbersome than large electromagnets.

The disadvantages of air-core solenoid magnets are:

(1) Transverse observation is difficult unless two coils separated by a distance are used.

(2) Current intensity must always be stabilized with great care, whereas, in electromagnets with saturated iron the field depends little on current.

(3) The large power consumption required for intense fields demands elaborate and very efficient method of cooling.

(4) At very intense fields tremendous electromagnetic forces act on the conductors.

Various coil constructions are used for the production of high intensity and highly homogeneous magnetic fields. The conflicting requirements of high intensity and high degree of homogeneity pose problems to the magnet designer. The single coil solenoids, though fairly efficient and capable of operating at high power, and hence high field, fail to produce very homogeneous fields. Although it is possible to improve the field uniformity in cylindrical solenoids by lengthening the coil, the limitations of space usually render it impractical. Hak[8] has advocated change in the depth of winding on a cylindrical base according to a simple rule. Bitter[6] and Gaume[9] proposed the use of a non-uniform current distribution, or in other words, the use of different current densities in different parts of a uniformly wound coil, in order to improve the central field homogeneity. Bacon[10] suggested the use of a combination of coils carrying different currents. Either of these schemes has an inherent disadvantage due to the fact that the various sections of the coil have, in general, different thermal behavior affecting the field pattern as the coil warms up.

Helmholtz's two-coil system and Maxwell's three-coil system for producing uniform fields are well known. In the case of the Helmholtz pair with a central gap, not only is the central field intensity reduced, but also the central gap provided for improving the homogeneity is often much too small for radial access. One of the elegant methods for improving field homogeneity is the technique of adding compensating windings to the main solenoid. This is discussed in chapter II. The type of coil arrangement or technique adopted for improving field homogeneity, at a given field intensity and field volume, must strike a compromise between maximum engineering efficiency and maximum uniformity. It is needless to emphasize that no method is practical which adds to the complexity of the cooling problem.

The suitability of air-core solenoids to produce highly homogeneous fields is considered in great detail in chapter II.

1.4 Iron-Core Electromagnets

The use of iron core electromagnets for producing magnetic fields has been known for many years. The introduction of a ferromagnetic material in an air-core coil diminishes the magnetic resistance of the flux path, enabling the magnetic field generated by a given current in the coil to increase a thousand fold. The limitation in iron core electromagnets is the saturation of ferromagnetic materials which sets an upper limit to the field intensity obtainable. If the field intensity is kept below the saturation level of iron, then the iron core magnets have definite superiority over air-core solenoids and they result in marked saving in power. The design of iron core magnets is, however, not as simple and straightforward as that of air-core solenoids. The design of magnetic circuits becomes extremely complicated due to the non-linear characteristics of iron, whose permeability largely depends on the operating flux density. Hence, even for flux densities in the range of 20 - 25 KG, great care has to be exercised in the shaping of various parts of the magnetic circuit to avoid saturation. Saturation not only renders the ferromagnetic material useless in its purpose as regards to field contribution, but also affects the field homogeneity in the gap. The requirement of high field homogeneity, again, adds to the complexity of the design problem. The pole geometry and the dimensions of various parts of the magnet assume different proportions depending on the degree of homogeneity required. These in turn add to the weight and cost of the magnet as a whole. The demanding requirements of large pole face to gap ratio and wider gap for high field homogeneity, in fact, drastically alter the complete magnetic circuit design. All these factors are examined in detail in chapter IV.

1.5 The Problem

The purpose of this thesis is to investigate the possibility of developing magnetic standards for the production of magnetic fields of intensity in the range of 20 to 25 KG with a homogeneity of one part in 10^5 over a spherical volume of one inch diameter. Air-core solenoids and iron core electromagnets can be considered, as mentioned in the introduction, as the two basic designs for producing the desired field. Although the literature on the production of high intensity fields is quite extensive, very limited information is available on fields of uniformity in the order of 1 in 10^5 at a field intensity of 20 to 25 KG.

In the case of air-core solenoids various coil geometries and coil configurations are considered. The following coil arrangements are analyzed in detail:

(1) Cylindrical solenoid with rectangular cross-section.

(2) Two-coil system.

(3) Three-coil system.

The effect of different coil geometries on field intensity and homogeneity is studied in each of the above coil arrangements. In the case of the two-coil system the effect of coil separation on the field homogeneity is examined for different coil geometries.

A split-coil arrangement with a central gap is studied in detail both as regard to improving the field homogeneity and providing transverse access to the field. The differences between a split-coil design and a two-coil system are also discussed.

The method of adding compensating windings to the cylindrical solenoid to improve field homogeneity is investigated in detail. A field analysis based on zonal harmonics method[12] is given for the compensated solenoid. Different methods of providing compensation to the main solenoid field are discussed. The computed results of field homogeneity are given in the form of tables, including a slotted solenoid.

The production of magnetic fields of intensity in the range of 20 - 25 KG involves operation at a high power level. The cooling problems associated with high power dissipation are severe. A detailed discussion on the serious problem of cooling of air-core solenoids is given in chapter III. Various factors to be taken into consideration for designing a cooling system of an air-core solenoid are discussed and a sample design is worked out.

In the case of the iron core electromagnet, the following aspects are considered:

(1) The effect of pole piece and pole core shape on saturation and field homogeneity.

(2) The importance of proper proportioning of various parts of the magnetic circuit.

(3) The effect of adding shim coils to improve the field homogeneity.

2. AIR-CORE SOLENOIDS

2.1 General

In designing a solenoid magnet, one strives to obtain as intense a magnetic field as possible for a given amount of power, under given conditions of working space and homogeneity. Selection of the shape of the coil or a particular coil arrangement is dictated by the requirements to be met by the magnetic field. To date, the design work on air-core solenoids has been concerned either with the production of high intensity fields[6,9] without much emphasis on field homogeneity, or with the generation of relatively weak uniform fields [8,10]. Where the field intensity is of principal concern, one aims to design a highly efficient coil with proper considerations of coil cooling. On the other hand, to realize a high field homogeneity, one has to manipulate the coil dimensions and coil shapes at the expense of coil efficiency. Some of the coil systems such as ellipsoidal construction[11] have just theoretical interest because they are too complicated to put into practice. The coil arrangements like Maxwell's three-coil system[12] and McKeehan's four loop combination[13] are usually not suitable for high power design because of the difficulty of obtaining uniform cooling. On the other hand, the very simple and efficient cylindrical solenoids, which are most suitable for high power construction, fail to produce very homogeneous fields. Hence, other coil arrangements have to be investigated to accomplish the desired degree of field homogeneity without, at the same time, sacrificing too much of coil efficiency.

The production of magnetic fields by cylindrical solenoid is briefly discussed in this chapter. The two-coil system, and the split-coil design are discussed in detail, and the improvements in field homogeneity that can be achieved by these two methods are compared with those of the conventional solenoid. The effects of coil geometries and coil separations on field homogeneity are examined. The method of adding compensating windings to the main solenoid, as a means to improve field homogeneity, is also investigated. An analysis using the method of zonal harmonics is developed. The field homogeneity figures are computed for different methods, and the results are presented in the form of Tables and Graphs.

2.2 Field Analysis

On-axis field calculation is relatively simple and straight forward. Off-axis fields can be calculated by integrating the equation for the field due to a single current loop, using elliptic integrals. This method is illustrated in detail by Garrett[14], and off-axis fields of a semi-infinite solenoidal sheet have been tabulated by Alexander and Downing[15]. Off-axis fields can also be calculated by using scalar magnetic potential whose gradient is the magnetic field. Formulae for the calculation of field and its derivatives, using this method, have been published by Snow[16].

The method of zonal harmonics for the calculation of off-axis magnetic fields has certain distinct advantages. The field can be expressed as a power series with zero and higher order derivative terms. The zero-order term of the zonal harmonic expansion corresponds to the field at the geometric centre of the solenoid, and the higher order terms depend on the geometry of the solenoid. Thus, if one can eliminate or reduce the higher order terms of the zonal harmonic expansion by choosing a proper geometry of coil, one can improve the field homogeneity. Therefore, the method of zonal harmonics serves as an efficient tool for the design of uniform field systems. The applicability of this method for different coil arrangements is illustrated in the subsequent sections.

2.3 Cylindrical Solenoid

The simplest type of magnet coil is cylindrical with straight ends, as shown in Fig. 2.1. The cross-section of the coil is rectangular with axial width 2b, and radial depth 2d. The current density in the coil is assumed to be uniform. It is shown in Appendix A, that the magnetic field intensity H_O, at the geometric centre, O, of the cylindrical solenoid, is given by

$$H_O = G\sqrt{\frac{W\lambda}{\rho a_1}} \qquad (2.1)$$

where W = power input to the coil

λ = space factor = $\dfrac{\text{actual sectional area occupied by the conductors}}{\text{Total cross sectional area of the coil}}$

ρ = specific resistivity of the coil material

a_1 = inner radius of the solenoid

and G is a dimensionless factor called "Fabry factor".

The Fabry factor does not depend on the size, but only on the shape of the coil, and is given by

$$G = \frac{1}{5}\;\frac{(2\pi\beta)^{\frac{1}{2}}}{(\alpha^2-1)^{\frac{1}{2}}}\;\log\frac{\alpha + (\beta^2+\alpha^2)^{\frac{1}{2}}}{1 + (\beta^2+1)^{\frac{1}{2}}} \qquad (2.2)$$

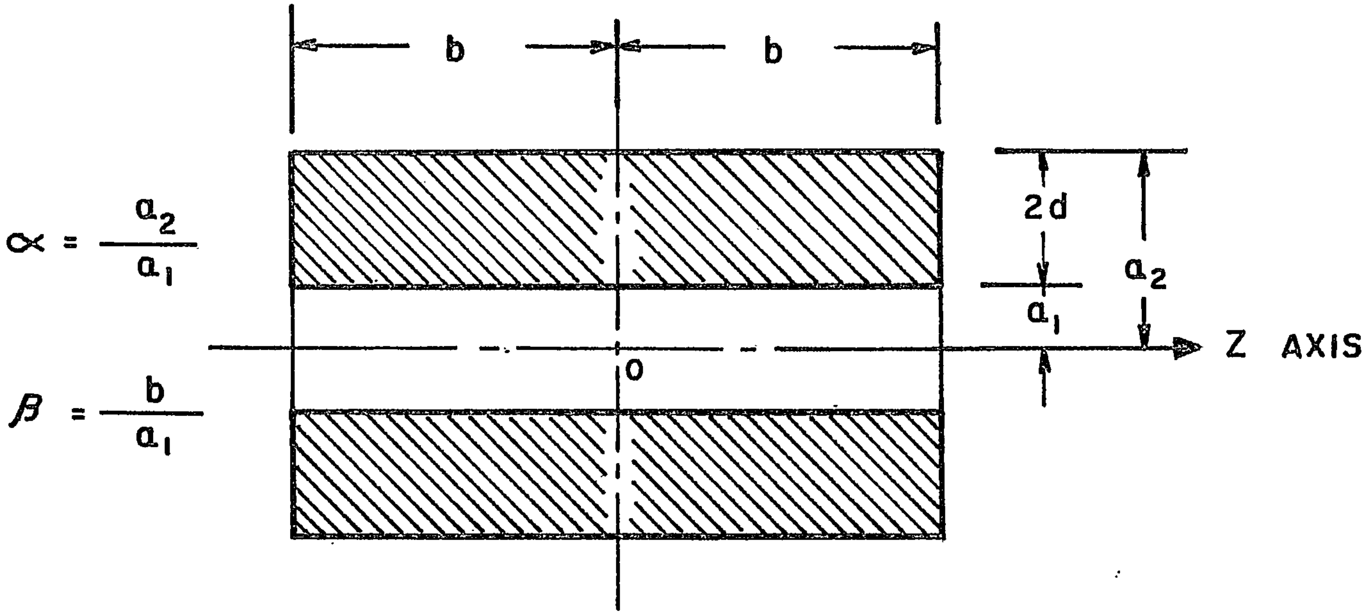

FIGURE 2.1 CYLINDRICAL SOLENOID WITH STRAIGHT ENDS.

where $\alpha = \frac{\text{outer radius } a_2}{\text{inner radius } a_1}$

$\beta = \frac{\text{coil half-length } b}{\text{inner radius } a_1}$

It is to be noted that the value of G given in Eq. 2.2 is for the case of uniform current density in the coil. G will assume different expressions for different current distributions in the winding. Coils with only uniform current density and straight ends are considered in the present discussion.

The Fabry factor G is represented by a family of curves as shown in Fig. 2.2. It has a maximum value of 0.179 for $\alpha = 3$, $\beta = 2$. Larger the value of G, greater is the field intensity H_o, for a given set of values of W, λ, ρ and a_1. Fabry also used a volume factor v defined by

$$v = \frac{\text{Total coil volume}}{{a_1}^3} \qquad (2.3)$$

For the case of a cylindrical coil with straight ends

$$v = 2\pi(\alpha^2 - 1)\beta$$

Curves for various values of v are also shown in Fig. 2.2a. It is to be noted, however, that for many applications a minimum volume factor v is not the only consideration but, the whole configuration produced by the magnet coil has to be taken into account. Selection of the shape for a coil is a compromise dictated by the requirements of homogeneity, volume of homogeneous field, maximum field obtainable for a given homogeneity, and cooling of the coil. A coil geometry which is most efficient from the point of view of field intensity and cooling, may prove totally unsuitable from the

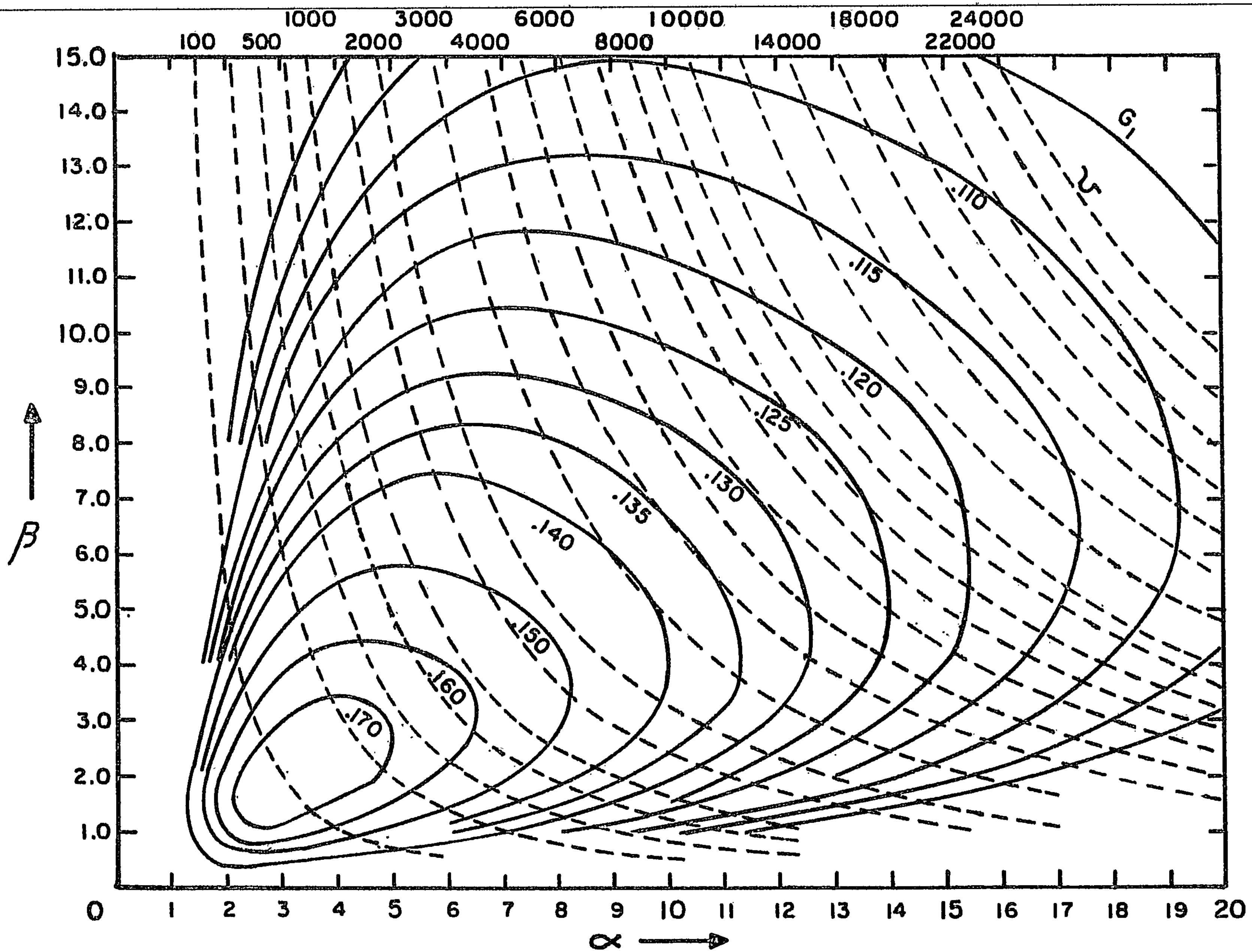

FIGURE 2.2a G-FACTORS FOR UNIFORMLY WOUND COILS — COCKCROFT[5]

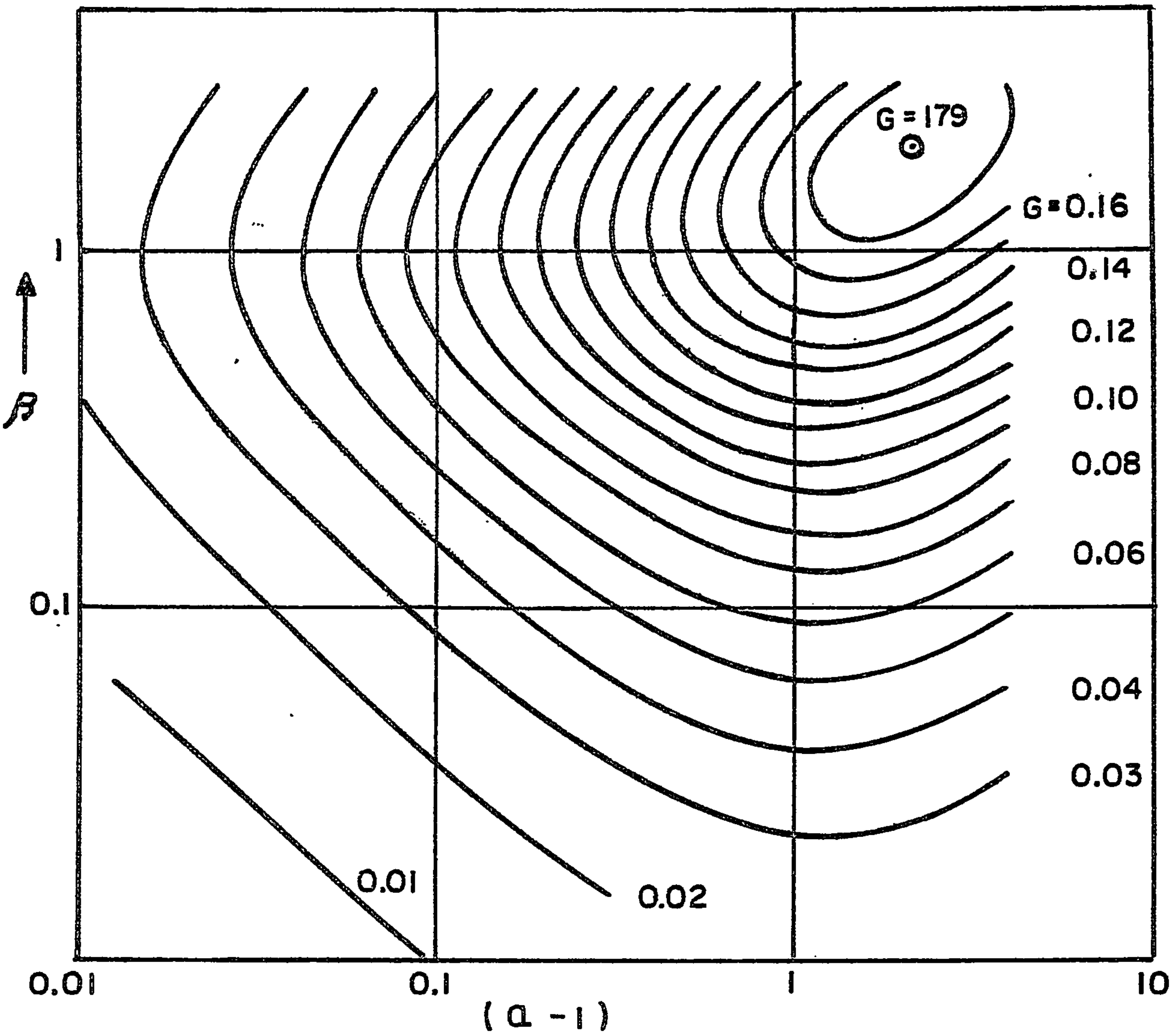

FIGURE 2.2b G-FACTORS FOR UNIFORMLY WOUND COILS WITH SMALL α AND β'S.