A SMALL-SIGNAL HIGH FREQUENCY EQUIVALENT CIRCUIT FOR THE FIELD-EFFECT TRANSISTOR

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ABSTRACT

The field effect transistor is considered as an active, distributed non-uniform transmission line and a differential equation for the small-signal a.c. case is derived. The short-circuit admittance parameters of the device are determined from the solution of the differential equation. A high frequency equivalent circuit for the intrinsic device is then obtained from the first-order approximation of the analysis and the expressions for the elements of this circuit are derived for both the saturated and the non-saturated conditions. The normalized values of these elements are computed as functions of the gate and drain bias voltages and the results of these computations are presented graphically.

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LIST OF PRINCIPAL SYMBOLS

$2a$ = the distance between the two gates

$2b$ = the channel height

E_x = x-component of electric field in the channel

g_m = the transconductance of the F.E.T.

g_{mo} = d.c. or low frequency value of g_m

g_o = output conductance of the F.E.T.

I_d' = total value of channel current

I_d = d.c. value of channel current

i = a.c. value of channel current

L = length of the channel

q = electronic charge

V_g = d.c. potential of the gate referred to the source

V_d = d.c. potential of the drain referred to the source

v = a.c. value of the channel bias

v_g = a.c. potential of the gate referred to the source

v_d = a.c. potential of the drain referred to the source

W = the d.c. value of the channel bias

W' = the total channel bias

W_p = the value of channel bias at pinch-off

y = the ratio (b/a)

y_s = the value of y at the source end of the channel

y_d = the value of y at the drain end of the channel

$y_{1,2,0}$ = the admittances of the equivalent circuit

y_m = the transadmittance of the F.E.T.

Z = the width of the F.E.T.

ϵ = permittivity of the material of the F.E.T.

μ = mobility of the current carriers

ρ_o = charge density of the space-charge regions

ρ_g = charge density of the gate regions

σ_c = conductivity of the channel

ψ = equilibrium barrier potential of the p-n junction

The subscripts s, g, and d refer to the source, gate and drain respectively.

1. INTRODUCTION

1.1 General

Recent advances in the construction techniques of field-effect transistors have enabled devices of greatly improved high frequency performance to be produced. Application of such devices in an optimal fashion to the design of high frequency circuits is facilitated by the use of an equivalent circuit. This thesis is concerned with the theoretical derivation of a high frequency equivalent circuit for the field-effect transistor from a consideration of the basic physical principles involved in the operation of the device.

1.2 Review of the Literature

The field-effect transistor (F.E.T.) is a semiconductor device in which the conductance of the current path is modulated by the application of a transverse electric field. It was first described by Shockley[1] who presented the basic theory of the device. This theory assumed an abrupt p-n junction and a uniformly graded channel along which the potential varied in a gradual way. A brief account of his analysis is given below.

Consider an F.E.T. with an n-type channel and two p-type gates as shown in Fig. 1.1. Suppose that the two gates are shorted to the source and a positive potential V_d is applied to the drain. A current I_d will flow between the source and

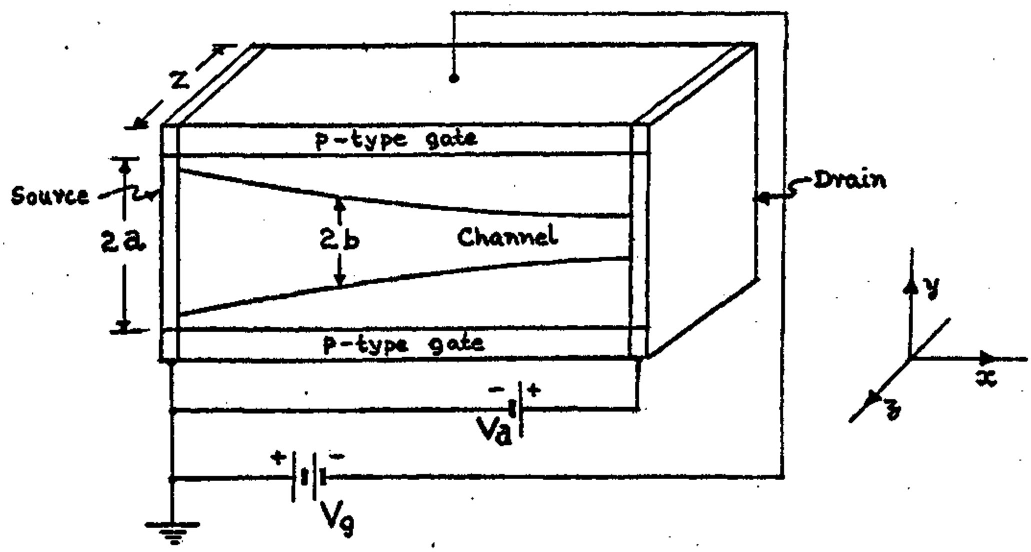

Fig. 1.1 Schematic diagram of an F.E.T.

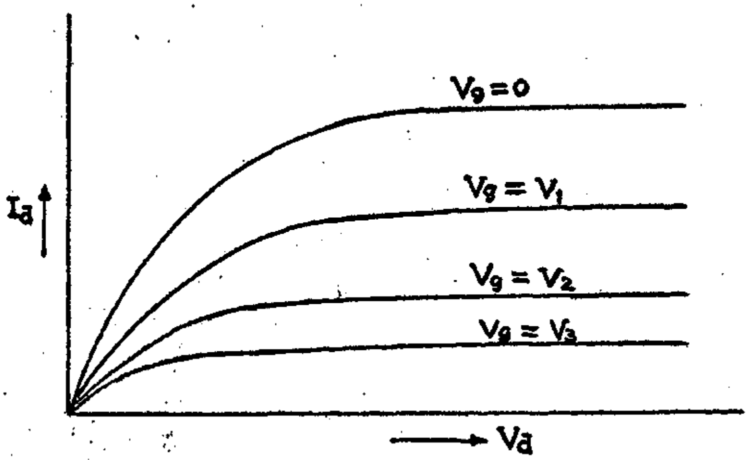

Fig. 1.2 Common source drain voltage-current characteristic of the F.E.T.

the drain causing a voltage drop along the channel. Since the drain is more positive than the source which is directly connected to the gates, the p-n junctions are reverse biased. This reverse bias increases in the positive x direction along the channel and, hence, the space-charge regions will be wider near the drain. As the drain to source voltage is increased the channel becomes narrower until, at a value of channel bias equal to W_p, the two space-charge regions from the opposite gates meet at the drain end. This is called the 'pinch-off' condition and W_p is called the 'pinch-off potential'. The drain current remains essentially saturated for bias voltages higher than W_p as most of the increased voltage appears across the space-charge regions near the drain.

If a negative potential V_g is applied between the gate and the source, the magnitude of the drain voltage V_d required to cut off the channel will be reduced by the amount of gate bias. The drain current will therefore saturate at lower values of drain voltage and current. Thus the common source characteristic (I_d versus V_d) with gate voltage as a parameter is of the form shown in Fig. 1.2.

If W is the potential of the gate with respect to the channel, then according to Shockley,[1]

$$W = W_p(1 - \frac{b}{a})^2 \tag{1.1}$$

where W_p = pinch off potential

$2b$ = channel thickness (Fig. 1)

$2a$ = distance between the two gates

The value of W at the source is given by

$$W = W_s = V_g + \psi$$

and at the drain

$$W = W_d = (V_g - V_d) + \psi.$$

V_g and V_d are the gate and drain voltages referred to the source, respectively. In the above equation, ψ is the equilibrium barrier potential and is a negative quantity for an F.E.T. with an n-type channel.

The x-component of the electric field, E_x, at a distance x from the source along the channel is

$$E_x = \frac{dW}{dx}. \tag{1.2}$$

Hence, the channel current is given by

$$\begin{aligned} I_d &= -2Z\sigma b \cdot E_x \\ &= -2Z\sigma a\left(\frac{b}{a}\right)\frac{dW}{dx} \end{aligned} \tag{1.3}$$

where σ = conductivity of the channel

Z = width of the channel.

Shockley has shown that the channel current is given by

$$I_d = -\frac{2Za\sigma_c W_p}{L}\left[\frac{W_d}{W_p} - \frac{W_s}{W_p} - \frac{2}{3}\left\{\left(\frac{W_d}{W_p}\right)^{\frac{3}{2}} - \left(\frac{W_s}{W_p}\right)^{\frac{3}{2}}\right\}\right] \qquad (1.4)$$

where L is the length of the device. The transconductance, g_{mo}, of the device is therefore given by

$$g_{mo} = \left.\frac{\partial I_d}{\partial V_g}\right|_{V_d=\text{const}} = \frac{2Za\sigma_c}{L}\left[\left(\frac{W_d}{W_p}\right)^{\frac{1}{2}} - \left(\frac{W_s}{W_p}\right)^{\frac{1}{2}}\right] \qquad (1.5)$$

The output conductance, $\frac{1}{R_o}$, of the device becomes

$$\frac{1}{R_o} = \left.\frac{\partial I_d}{\partial V_d}\right|_{V_g=\text{const}} = \frac{2Za\sigma_c}{L}\left[1 - \left(\frac{W_d}{W_p}\right)^{\frac{1}{2}}\right] \qquad (1.6)$$

The equivalent circuit of the F.E.T. for the low frequency case takes the form shown in Fig. 1.3.

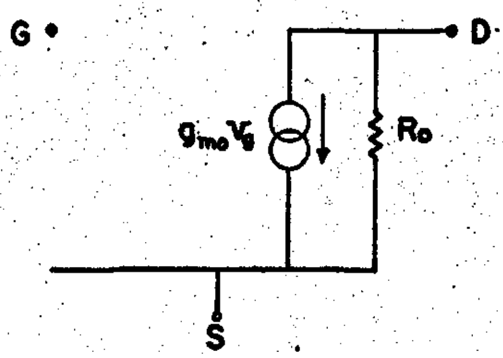

Fig. 1.3 Small-signal low frequency equivalent circuit of the F.E.T.

Shockley's theory has been extended[2-6] to include the effects of changes in the mobility of the carriers with the electric field and non-uniform channel doping.

A number of high frequency equivalent circuits have been proposed[7,8,9,10,11,12] for the F.E.T. In general, these take the form shown in Fig. 1.4. Note that the equivalent circuit of Fig. 1.3 is of this form ($y_1, y_2 = 0$, $y_m = g_{mo}$ and $y_o = \frac{1}{R_o}$). An improvement over the equivalent circuit of Fig. 1.3 has been made by van der Ziel[7] and others[8] by considering charge storage in the space-charge regions of the p-n junction of the device. Analyses of this type lead to the conclusion that y_1 consists of a capacitance C_1 and y_2 consists of a capacitance C_2 and expressions for C_1 and C_2 as functions of the bias voltages have been presented. Olsen[9] has suggested that y_o consists of $\frac{1}{R_o}$ in parallel with a capacitance C_o.

The solid-state devices group at Texas Instruments Inc.[10] has suggested the equivalent circuit shown in Fig. 1.5. This circuit is based on a qualitative analysis of the field-effect transistor operation. The resistors R_3 and R_4 are the d.c. leakage resistances of the reverse biased p-n junctions while R_5 and R_6 are parasitic resistances associated with the source and drain contacts.

Silverthorn's[11,12] equivalent circuit (Fig. 1.6) is a simplification of the Texas Instruments Inc. equivalent circuit valid for the frequency range in which R_3 and R_4 can be

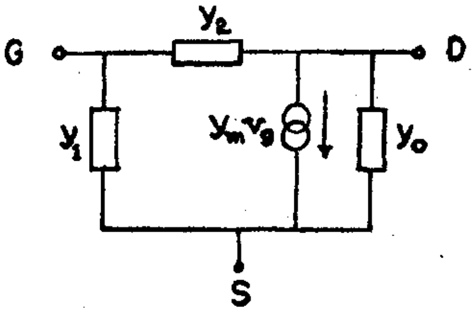

Fig. 1.4 General form of the F.E.T. equivalent circuit.

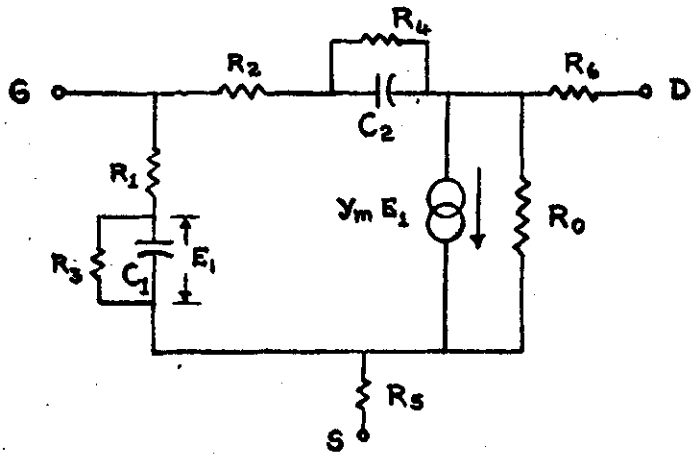

Fig. 1.5 An equivalent circuit for the F.E.T. (suggested by Texas Instruments Inc.)

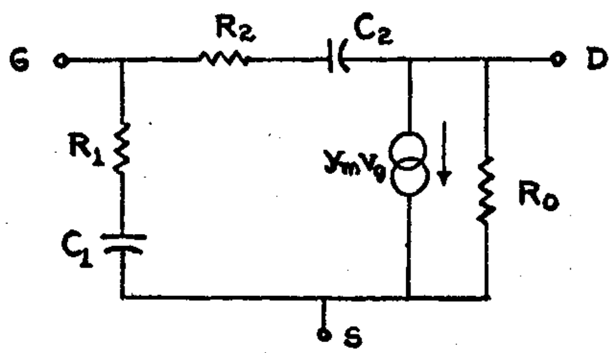

Fig. 1.6 Silverthorn's equivalent circuit for the F.E.T.

neglected. He has shown that R_5 and R_6 can be absorbed into R_1 and R_2 over a wide frequency range and has presented a method for obtaining the equivalent circuit elements from measurements of the short-circuit admittance parameters.

Ero and van der Ziel[13] have presented an improved small-signal high-frequency analysis of the F.E.T. In this theory, the device is considered to be an active, distributed, non-uniform transmission line and the wave equation for this structure has been solved by an approximate method. The expressions for the short-circuit admittance parameters have been derived for the saturation (pinch-off) case. Expressions for C_1 and R_1 have been obtained and it has been demonstrated that the value of C_1 so computed is in agreement with that obtained from a charge storage analysis. These computations show that $y_2 = 0$ for the pinch-off case in agreement with the previous computations showing that $C_2 = 0$ at pinch-off.

1.3 An Outline of the Scope of This Work

The purpose of this work is to derive an intrinsic high frequency equivalent circuit of the F.E.T. A differential equation for the a.c. case is derived from a knowledge of the physics of the device and a series solution to this equation is obtained. The expressions for short-circuit admittance parameters of the device are derived from the solution. A hybrid-π-network of a general form is assumed to represent the equivalent circuit and the expressions for the circuit elements are determined by comparing the admittance parameters of the hybrid-π -circuit with the derived expressions.

According to van der Ziel's terminology, the d.c. theory of the device, given in Section 1.2, is called the 'zero-order approximation', which means that this theory is for the case of ($j\omega$) approaching zero. The resulting equivalent circuit, shown in Fig. 1.3, has therefore no reactive elements. The 'first-order approximation', then, is taken to mean that only the d.c. and ($j\omega$) terms are retained in the theory and the higher order terms in ($j\omega$) are neglected. The equivalent circuit for this case will have one reactive element in each branch. In this thesis, the equivalent circuit is derived from the 'first-order approximation' of the theory of the device.

The analysis presented in this work is of a more general nature than van der Ziel's analysis because the expressions for the short-circuit admittance parameters and the circuit elements derived here are applicable to both the saturated and the non-saturated conditions.

2. THE DIFFERENTIAL EQUATION AND ITS SOLUTION

2.1 Introduction

The F.E.T. is considered as an active, distributed and non-uniform transmission line and a differential equation of this structure is derived for the small-signal case. It is assumed that the channel is uniformly doped and that the p-n junctions are of the abrupt type. A solution of the differential equation is obtained in the form of a power series which gives the channel current in terms of the channel thickness.

2.2 The Differential Equation

In the small signal case, the d.c. values of channel bias and current have superimposed upon them small sinusoidal a.c. voltage and current variations; hence the expressions for the total channel bias and current can be written as follows:

$$W' = W + ve^{j\omega t} \tag{2.1}$$

$$I_d' = I_d + ie^{j\omega t} \tag{2.2}$$

where v and i are the a.c. components of channel bias and current, respectively.

The statement of Ohm's law as given by equation (1.3) must be augmented by the charge continuity equation in order to derive the differential equation.

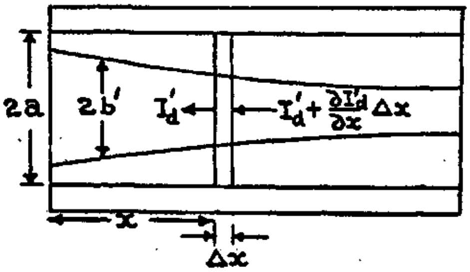

Fig. 2.1 Current flow in a section x of the device.

Consider a volume element of width Δx at a distance x from the source of an F.E.T. with an n-type channel. Under operating conditions, the channel current will flow as shown in Fig. 2.1. The volume of this element is $2aZ\Delta x$ and the net charge ΔQ contained in it is given by

$$\Delta Q = 2(1-\frac{b'}{a})aZ\rho_o\Delta x$$

where $2b'$ is the instantaneous value of the channel height and ρ_o is the charge density of the space charge regions. From equation (1.1) one obtains

$$1 - \frac{b}{a} = 1 - y = (\frac{W}{W_p})^{\frac{1}{2}} \qquad (2.3)$$

where $y = \frac{b}{a}$. Similarly, for the a.c. case,

$$1 - \frac{b'}{a} = 1 - y' = (\frac{W'}{W_p})^{\frac{1}{2}} \qquad (2.4)$$

where y' is the instantaneous value of y. Therefore the expression for ΔQ becomes

$$\Delta Q = 2aZ\,\rho_o\left(\frac{W'}{W_p}\right)^{\frac{1}{2}}\Delta x \tag{2.5}$$

Considerations of charge continuity for this element require that

$$I_d' - \left(I_d' + \frac{\partial I_d'}{\partial x}\Delta x\right) = -\frac{\partial}{\partial t}(\Delta Q)$$

or

$$\frac{\partial I_d'}{\partial x} = \frac{\partial}{\partial t}\left[2aZ\,\rho_o\left(\frac{W'}{W_p}\right)^{\frac{1}{2}}\right]$$

where t is the time.

Since $\frac{\partial I_d}{\partial x} = 0$, use of equations (2.1) and (2.2) yields

$$\frac{\partial i}{\partial x}\,e^{j\omega t} = 2aZ\,\rho_o\,\frac{\partial}{\partial t}\left[\left(\frac{W}{W_p}\right)^{\frac{1}{2}}\left(1 + \frac{v}{W}\,e^{j\omega t}\right)^{\frac{1}{2}}\right]$$

For the small signal case $(\frac{v}{W})$ is a small quantity and hence the second and higher order terms in $(\frac{v}{W})$ can be neglected. Thus the above equation reduces to

$$\frac{\partial i}{\partial x}\,e^{j\omega t} = 2aZ\,\rho_o\left[\left(\frac{W}{W_p}\right)^{\frac{1}{2}}\cdot\ \frac{1}{2}\,\frac{v}{W}\,j\omega e^{j\omega t}\right]$$

With the aid of (2.3), this can be written as

$$\frac{\partial i}{\partial x} = \frac{aZ\,\rho_o}{W_p(1-y)}\,j\omega v \tag{2.6}$$

Equation (1.3) can be written for the a.c. case as follows:

$$I_d' = -A\cdot y'\cdot\frac{dW'}{dx} \tag{2.7}$$

where $A = 2Za\,\sigma_o$.

Use of equations (2.1), (2.2), and (2.4) in (2.7) yields

$$I_d + ie^{j\omega t} = -A\left[1 - \left(\frac{W'}{W_p}\right)^{\frac{1}{2}}\right]\left(\frac{dW}{dx} + \frac{dv}{dx}e^{j\omega t}\right)$$

$$= -A\left[1 - \left(\frac{W}{W_p}\right)^{\frac{1}{2}}\left(1 + \frac{v}{W}e^{j\omega t}\right)^{\frac{1}{2}}\right]\left(\frac{dW}{dx} + \frac{dv}{dx}e^{j\omega t}\right)$$

On neglecting the second and higher order terms in $(\frac{v}{W})$, this reduces, with the aid of (2.3), to

$$I_d + ie^{j\omega t} = -A.y.\frac{dW}{dx} + \frac{A}{2}(1-y)\frac{v}{W}\frac{dW}{dx}e^{j\omega t} - A.y.\frac{dv}{dx}e^{j\omega t} \quad (2.8)$$

The time-independent part of (2.8) along with (2.3) gives

$$\frac{dy}{dx} = \frac{I_d/I_o}{a(1-y)y} \quad (2.9)$$

where $I_o = 4Z\sigma_c W_p$.

Equations (2.3), (2.9), and the time-dependent part of (2.8) can be manipulated to yield

$$\frac{dv}{dy} = -\frac{i(1-y)a}{2Z\sigma_c a(I_d/I_o)} - \frac{v}{y} \quad (2.10)$$

Dividing (2.6) by (2.9), one obtains

$$\frac{di}{dy} = \frac{a^2 Z\rho_o}{W_p} \cdot \frac{j\omega vy}{\left(\frac{I_d}{I_o}\right)} \quad (2.11)$$

The pinch-off potential W_p is given by[6]

$$W_p = -\frac{a^2}{2\epsilon}\rho_o\left[1 - \frac{\rho_o}{\rho_g}\right] \quad (2.12)$$

$$= -\frac{a^2}{2\epsilon}\rho_o.D$$

where ϵ is the permittivity of the channel meterial and ρ_g is the charge density of the gate regions. Use of (2.12) in (2.11) gives

$$\frac{di}{dy} = -\frac{2Z\epsilon j\omega vy}{(I_d/I_o).D} \qquad (2.13)$$

Equations (2.10) and (2.13) can be combined to yield

$$\frac{d^2i}{dy^2} + 2k^2y(1-y)i = 0 \qquad (2.14)$$

where

$$k^2 = -j\omega\frac{\epsilon}{2\sigma(\frac{I_d}{I_o})^2} \qquad (2.15a)$$

and

$$\sigma = \sigma_o.D = \sigma_c(1 - \frac{\rho_o}{\rho_g}) \qquad (2.15b)$$

Equation (2.14) is the required differential equation which expresses the channel current i in terms of the variable parameter y for the a.c. case.

2.3 Solution of the Differential Equation

The differential equation (2.14) is assumed to have a series solution of the following type.

$$i(y) = \sum_{\nu=0}^{\infty} C_\nu\, y^\nu \qquad (2.16)$$

Hence

$$i''(y) = \sum_{\nu=2}^{\infty} \nu(\nu-1)C_\nu y^{\nu-2}$$
$$= 2C_2 + 6C_3 y + \sum_{\nu=4}^{\infty} \nu(\nu-1)C_\nu y^{\nu-2}$$

and

$$2k^2y(1-y) \sum_{\nu=0}^{\infty} C_\nu y^\nu = 2k^2C_oy - 2k^2C_oy^2 + 2k^2C_1y^2 - 2k^2C_1y^3 + 2k^2C_2y^3 \ \ldots\ldots$$
$$= 2k^2C_oy + \sum_{\nu=2}^{\infty} 2k^2(C_{\nu-1}-C_{\nu-2})y^\nu$$

so that the equation (2.14) becomes

$$2C_2 + 6C_3y + \sum_{\nu=4}^{\infty} \nu(\nu-1)C_\nu y^{\nu-2} + 2k^2C_oy + \sum_{\nu=2}^{\infty} 2k^2(C_{\nu-1}-C_{\nu-2})y^\nu = 0$$

This can be written as

$$2C_2 + 6C_3y + 2k^2C_oy + \sum_{\nu=4}^{\infty} \nu(\nu-1)C_\nu y^{\nu-2} + \sum_{\nu=4}^{\infty} 2k^2(C_{\nu-3}-C_{\nu-4})y^{\nu-2} = 0 \qquad (2.17)$$

It is clear that the coefficient of each term should be separately equal to zero.

$$2C_2 = 0 \qquad \text{or} \qquad C_2 = 0$$
$$6C_3 + 2k^2C_o = 0 \qquad \text{or} \qquad C_3 = -\frac{k^2}{3}C_o$$

The last two terms of the equation (2.17) give the recursion formula

$$C_\nu = \frac{2k^2(C_{\nu-4}-C_{\nu-3})}{\nu(\nu-1)} \text{ for } \nu \geq 4\,. \tag{2.18}$$

Hence

$$C_4 = \frac{k^2}{6}(C_o - C_1)$$

$$C_5 = \frac{k^2}{10}(C_1 - C_2) = \frac{k^2}{10}C_1 \quad \text{and so on.}$$

It is therefore obvious that C_ν can be expressed as follows:

$$C_\nu = \alpha_\nu C_o + \beta_\nu C_1, \qquad 0 \leq \nu \leq \infty \tag{2.19}$$

where C_o and C_1 are arbitrary quantities which can be determined by means of boundary conditions.

The most general solution of the equation (2.14) is therefore given by

$$i(y) = C_o \sum_{\nu=0}^{\infty} \alpha_\nu y^\nu + C_1 \sum_{\nu=0}^{\infty} \beta_\nu y^\nu \tag{2.20}$$

where α_ν and β_ν can be determined by the use of the relations (2.18) and (2.19).

Equations (2.18) and (2.19) yield the following values for the coefficients α's and β's.

$$\alpha_0 = 1 \qquad \beta_0 = 0$$

$$\alpha_1 = 0 \qquad \beta_1 = 1$$

$$\alpha_2 = 0 \qquad \beta_2 = 0$$

$$\alpha_3 = -\frac{k^2}{3} \qquad \beta_3 = 0$$

$$\alpha_4 = \frac{k^2}{6} \qquad \beta_4 = -\frac{k^2}{6}$$

$$\alpha_5 = 0 \qquad \beta_5 = \frac{k^2}{10}$$

$$\alpha_6 = \frac{k^4}{45} \qquad \beta_6 = 0$$

$$\alpha_7 = -\frac{k^4}{42} \qquad \beta_7 = \frac{k^4}{126}$$

$$\alpha_8 = \frac{k^4}{168} \qquad \beta_8 = \frac{k^4}{105}$$

$$\alpha_9 = \frac{k^6}{1620} \qquad \beta_9 = \frac{k^4}{360}$$

$$\alpha_{10} = \frac{29}{28350} k^6 \qquad \vdots$$

$$\alpha_{11} = \frac{k^6}{1848}$$

$$\alpha_{12} = \frac{149}{124740} k^6$$

$$\vdots$$

It will be useful to recall the definition of k^2 as given by the relation (2.15)

$$k^2 = -j\omega\left[\frac{\epsilon}{2\sigma(I_d/I_o)^2}\right]$$

It was mentioned in section 1.3 that only the first-order approximation of the theory was going to be used in deriving the equivalent circuit. After carrying out the calculations, it was found that the second order terms in $(j\omega)$ had to be retained in the numerator of the expression for the channel current as they give rise to an additional $(j\omega)$ term. This fact will become clear in chapter 4. Therefore, those values of the coefficients α and β which give rise to $(j\omega)^2$ terms will also be retained in the expression for the channel current. Higher order terms in $(j\omega)$ will be neglected.

When the above values of the coefficients are substituted, equation (2.20) becomes

$$i(y) = C_o \cdot T(y) + C_1 \cdot R(y) \tag{2.21}$$

where

$$T(y) = 1 - \frac{1}{3}k^2y^3 + \frac{1}{6}k^2y^4 + \frac{1}{45}k^4y^6 - \frac{1}{42}k^4y^7 + \frac{1}{168}k^4y^8$$
$$- \frac{1}{1620}k^6y^9 + \frac{29}{28350}k^6y^{10} - \frac{1}{1848}k^6y^{11} + \frac{149}{124740}k^6y^{12} \tag{2.22}$$

$$R(y) = y - \frac{1}{6}k^2y^4 + \frac{1}{10}k^2y^5 + \frac{1}{126}k^4y^7 - \frac{1}{105}k^4y^8 + \frac{1}{360}k^4y^9 \tag{2.23}$$

Equation (2.21) is the second-order approximation of the general solution (2.20).

3. EXPRESSIONS FOR ADMITTANCE PARAMETERS

3.1 Introduction

The short-circuit admittance parameters of the F.E.T. are determined in this chapter from the expression for the channel current which was obtained in the previous section. The coefficients C_o and C_1 contained in that expression are evaluated by means of the boundary conditions in both the short-circuited output and the short-circuited input cases.

3.2 Evaluation of C_o and C_1 for the Short-Circuited Output Case

The channel current is given by equation (2.21)

$$i(y) = C_o\, T(y) + C_1\, R(y)$$

Hence

$$\frac{di}{dy} = C_o \cdot T'(y) + C_1\, R'(y)$$

This expression is substituted in equation (2.13) to give

$$v = -\frac{(I_d/I_o)}{(j\omega)2Z\epsilon}\left[C_o\, \frac{T'(y)}{y} + C_1\, \frac{R'(y)}{y}\right] \qquad (3.1)$$

In order to determine the input admittance of the device, the output has to be short-circuited. Therefore, the drain is a.c. short-circuited to the source and this boundary condition gives

$$\left.\begin{aligned} y &= y_s \\ v &= v_g \end{aligned}\right\} \text{ at the source} \tag{3.2a}$$

and

$$\left.\begin{aligned} y &= y_d \\ v &= v_g \end{aligned}\right\} \text{ at the drain} \tag{3.2b}$$

where v_g = a.c. potential of the gate with respect to the source.

When the values (3.2a) are substituted, equation (3.1) becomes (at the source)

$$-\frac{v_g}{B} = C_o\left[\frac{T'(y_s)}{y_s}\right] + C_1\left[\frac{R'(y_s)}{y_s}\right] \tag{3.3}$$

where $B = \dfrac{I_d/I_o}{(j\omega)2Z\epsilon}$,

$$\frac{T'(y_s)}{y_s} = -k^2\Big[y_s - \tfrac{2}{3}y_s^2 - \tfrac{2}{15}k^2y_s^4 + \tfrac{1}{6}k^2y_s^5 - \tfrac{1}{21}k^2y_s^6$$

$$+ \tfrac{1}{18}k^4y_s^7 - \tfrac{29}{2835}k^4y_s^8 + \tfrac{1}{168}k^4y_s^9 - \tfrac{149}{124740}y_s^{10}\Big]$$

$$= -k^2.Y_{s1}$$

and

$$\frac{R'(y_s)}{y_s} = \frac{1}{y_s} - \tfrac{2}{3}k^2y_s^2 + \tfrac{1}{8}k^2y_s^3 + \tfrac{1}{18}k^4y_s^5 - \tfrac{8}{105}k^4y_s^6 + \tfrac{1}{40}k^4y_s^7$$

$$= Y_{s2}$$

Equation (3.3) can therefore be written as

$$-\frac{v_g}{B} = -C_ok^2.Y_{s1} + C_1.Y_{s2} \tag{3.4}$$

Similarly, use of relations (3.2b) in equation (3.1) gives (at the drain)

$$-\frac{v_g}{B} = -C_o k^2 Y_{d1} + C_1 . Y_{d2} \qquad (3.5)$$

where $Y_{d1} = \dfrac{T'(y_d)}{k^2 . y_d}$

and $\quad Y_{d2} = \dfrac{R'(y_d)}{y_d}$

Equations (3.4) and (3.5) yield

$$-\frac{v_g}{B} . Y_{d2} = -C_o k^2 . Y_{s1} . Y_{d2} + C_1 . Y_{s2} . Y_{d2} \qquad (3.6a)$$

$$-\frac{v_g}{B} . Y_{s2} = -C_o k^2 . Y_{d1} . Y_{s2} + C_1 . Y_{d2} . Y_{s2} \qquad (3.6b)$$

C_o is given by subtracting equation (3.6b) from (3.6a).

$$C_o = \frac{\frac{v_g}{B}(Y_{s2} - Y_{d2})}{k^2(Y_{d1} . Y_{s2} - Y_{s1} . Y_{d2})}$$

As explained in section 2.3, $(j\omega)^2$ terms are retained in the numerator of the expression for current, whereas in the denominator they will be neglected as they are not necessary for the first-order approximation of the theory. After simplification, C_o reduces to

$$C_o = \frac{\frac{v_g}{B} . Y_1}{k^2 . \Delta} \qquad (3.7)$$

where
$$Y_1 = \frac{1}{y_s y_d}(u-1) - k^2 y_s\left\{\frac{2}{3}(1-u^2) - \tfrac{1}{2}y_s(1-u^3)\right\}$$
$$+ k^4 y_s^4\left\{\frac{1}{18}(1-u^5) - \frac{8}{105}y_s(1-u^6) + \frac{1}{40}y_s^2(1-u^7)\right\}$$

$$\Delta = \frac{2}{3}\,\frac{1}{u}(1-u^3) - \frac{1}{y_d}(1-u^2) - \frac{2}{3}k^2 y_s^2 u\left\{1 - u - \frac{3}{4}y_s(1-u^2)\right.$$
$$+ \tfrac{1}{2}y_s^2 u(1-u) - \frac{1}{5}\,\frac{1}{u^2}(1-u^5) + \tfrac{1}{4}y_s\frac{1}{u^2}(1-u^6)$$
$$\left. - \frac{1}{14}\,y_s^2\,\frac{1}{u}(1-u^7)\right\} \tag{3.8}$$

and
$$u = \frac{y_d}{y_s} \tag{3.9}$$

The coefficient C_1 is determined in the same way. Equations (3.4) and (3.5) can be written as

$$-\frac{v_g}{B}.Y_{d1} = -C_o k^2.Y_{s1}.Y_{d1} + C_1.Y_{s2}.Y_{d1}$$
$$-\frac{v_g}{B}.Y_{s1} = -C_o k^2 Y_{d1}.Y_{s1} + C_1.Y_{d2}.Y_{s1}$$

These two relations are solved for C_1.

$$C_1 = \frac{\frac{v_g}{B}(Y_{s1} - Y_{d1})}{Y_{s2}.Y_{d1} - Y_{d2}.Y_{s1}}$$

The terms containing k^4 and k^6 are again neglected in the denominator. The expression for C_1 is then simplified and rearranged to give

$$C_1 = \frac{\frac{v_g}{B}.Y_2}{\Delta} \tag{3.10}$$

where

$$Y_2 = 1 - u - \frac{2}{3}y_s(1-u^2) - \frac{2}{15}k^2y_s{}^3(1-u^4) + \frac{1}{6}k^2y_s{}^4(1-u^5)$$

$$- \frac{1}{21}k^2y_s{}^5(1-u^6) + \frac{1}{180}k^4y_s{}^6(1-u^7) - \frac{29}{2835}k^4y_s{}^7(1-u^8)$$

$$+ \frac{1}{168}k^4y_s{}^8(1-u^9) - \frac{149}{124740}k^4y_s{}^9(1-u^{10})$$

Δ and u are given by the equations (3.8) and (3.9), respectively.

3.3 Expression For Channel Current

According to equation (2.15),

$$\frac{-j\omega\epsilon}{2\sigma\left(\frac{I_d}{I_o}\right)^2} = k^2$$

or
$$\frac{j\omega\epsilon}{2\sigma\left(\frac{I_d}{I_o}\right)^2} \cdot 4Z\sigma\left(\frac{I_d}{I_o}\right) = -k^2 \cdot 4Z\sigma\left(\frac{I_d}{I_o}\right)$$

Therefore,

$$\frac{j\omega.2Z\epsilon}{\frac{I_d}{I_o}} = -k^2.4Z\sigma\frac{I_d}{I_o}$$

$$\text{or}\quad \frac{1}{B} = -k^2.4Z\sigma.\frac{I_d}{I_o} \tag{3.11}$$

Equations (3.7), (3.10) and (3.11) are used in (2.25) to get the general expression for the channel current.

$$i(y) = C_o \cdot T(y) + C_1 \cdot R(y)$$

$$= \frac{\frac{v_g}{B} \cdot Y_1}{k^2 \cdot \Delta} \cdot T(y) + \frac{\frac{v_g}{B} \cdot Y_2}{\Delta} \cdot R(y)$$

$$= -k^2 \left(4Z\sigma \frac{I_d}{I_o}\right) \frac{v_g}{\Delta} \left\{ \frac{Y_1}{k^2} \cdot T(y) + Y_2 \cdot R(y) \right\}$$

Third and higher order terms in $(j\omega)$ are again neglected. It can be shown that when simplified and rearranged, the expression for current reduces to

$$i(y) = \frac{4Z\sigma \frac{I_d}{I_o} v_g}{\Delta} \left[\frac{1}{y_s y_d}(1-u) + k^2 y_s \left\{ \frac{2}{3}(1-u^2) - \frac{1}{2} y_s (1-u^3) \right.\right.$$

$$\left. - \frac{1}{3} \frac{y^3}{y_s^2 y_d}(1-u) + \frac{1}{6} \frac{y^4}{y_s^2 y_d}(1-u) - \frac{y}{y_s}(1-u) + \frac{2}{3} y (1-u^2) \right\}$$

$$+ k^4 y_s^4 \left\{ -\frac{1}{18}(1-u^5) + \frac{8}{105} y_s (1-u^6) - \frac{1}{40} y_s^2 (1-u^7) \right.$$

$$- \frac{2}{9} \frac{y^3}{y_s^3}(1-u^2) + \frac{1}{6} \frac{y^3}{y_s^2}(1-u^3) + \frac{1}{9} \frac{y^4}{y_s^3}(1-u^2)$$

$$- \frac{1}{12} \frac{y^4}{y_s^2}(1-u^3) + \frac{1}{45} \frac{y^6}{y_s^5 y_d}(1-u) - \frac{1}{42} \frac{y^7}{y_s^5 y_d}(1-u)$$

$$+ \frac{1}{168} \frac{y^8}{y_s^5 y_d}(1-u) + \frac{2}{15} \frac{y}{y_s}(1-u^4) - \frac{1}{6} y (1-u^5) + \frac{1}{21} y_s y (1-u^6)$$

$$\left.\left. + \frac{1}{6} \frac{y^4}{y_s^4}(1-u) - \frac{1}{9} \frac{y^4}{y_s^3}(1-u^2) - \frac{1}{10} \frac{y^5}{y_s^4}(1-u) + \frac{1}{15} \frac{y^5}{y_s^3}(1-u^2) \right\} \right] \tag{3.12}$$

This is the expression for current at any point along the channel for the case when the drain is a.c. short-circuited to the source.

3.4 The D.C. Case

It is of interest to see what the expression (3.12) for the channel current yields in the d.c. case. Before considering this case, it is necessary to determine $\frac{I_d}{I_o}$ in terms of y. Equation (2.8) gives

$$\frac{1}{a}\frac{I_d}{I_o}\,dx = y(1-y)dy$$

which becomes, on integration

$$\int_0^L \frac{1}{a}\frac{I_d}{I_o}\,dx = \int_{y_s}^{y_d} y(1-y)dy$$

or,

$$\frac{I_d}{I_o}\left(\frac{L}{a}\right) = \frac{y_d^2}{2} - \frac{y_d^3}{3} - \frac{y_s^2}{2} + \frac{y_s^3}{3}$$

$$= -\tfrac{1}{2}(y_s^2 - y_d^2) + \tfrac{1}{3}(y_s^3 - y_d^3)$$

$$= -\frac{y_s^2}{2}\left\{1 - u^2 - \frac{2}{3}\,y_s(1-u^3)\right\}$$

Hence

$$\frac{I_d}{I_o} = -\frac{a}{2L}\cdot y_s^2\left\{1 - u^2 - \frac{2}{3}\,y_s(1-u^3)\right\} \tag{3.13a}$$

As $\omega \rightarrow 0$ (d.c. case) equation (3.12) reduces to

$$\frac{i_d}{v_g} = \frac{i(y_d)}{v_g} = \frac{4Z\sigma.\ \frac{I_d}{I_o}.\frac{1}{y_s y_d}(1-u)}{\frac{2}{3}\,\frac{1}{u}(1-u^3)\ -\ \frac{1}{y_d}(1-u^2)}$$

$$= \frac{-4Z\sigma\frac{I_d}{I_o}\,\frac{1}{y_s}(1-u)(-\frac{1}{y_d})}{\left\{1\ -\ u^2\ -\ \frac{2}{3}\ y_s(1-u^3)\right\}(-\frac{1}{y_d})}$$

On substituting the value of $(\frac{I_d}{I_o})$ from equation (3.13), this expression becomes

$$\frac{i_d}{v_g} = -\ 4Z\sigma\,\frac{1}{y_s}(1-u)(-\ \frac{a}{2L}\ y_s{}^2)$$

$$= \frac{2Z\sigma a}{L}\ (y_s\ -\ y_d)\ =\ g_{mo} \qquad (3.13b)$$

or

$$\frac{i_d}{v_g} = g_{mo} = \frac{2Z\sigma a}{L}\left\{(1-y_d)\ -\ (1-y_s)\right\}$$

$$= \frac{2Z\sigma a}{L}\left\{(\frac{W_d}{W_p})^{\frac{1}{2}}\ -\ (\frac{W_s}{W_p})^{\frac{1}{2}}\right\}$$

which agrees with the equation (1.5)

3.5 Expression for Δ

It will be useful to reduce the expression for Δ to a convenient form. Equation (3.8) can be written as

$$\Delta\ = \frac{2}{3u}(1-u^3)\ -\ \frac{1}{y_d}(1-u^2)\ -\ \frac{2}{3}\ k^2 y_s{}^2 u(Y_3)$$

where
$$Y_3 = 1\ -\ u\ -\ \frac{3}{4}\ y_s(1-u^2)\ +\ \tfrac{1}{2}y_s{}^2u(1-u)\ -\ \frac{1}{5u^2}(1-u^5)$$

$$+\ \frac{1}{4u^2}\ y_s(1-u^6)\ -\ \frac{1}{14u^2}\ y_s{}^2(1-u^7)$$

and $u = \frac{y_d}{y_s}$

Hence $$\Delta = \left[\frac{2}{3u}(1-u^3) - \frac{1}{y_d}(1-u^2)\right]\left\{1 - \frac{\frac{2}{3}k^2 y_s{}^2 u . Y_3}{\frac{2}{3u}(1-u^3) - \frac{1}{y_d}(1-u^2)}\right\}$$

$$= \left[\frac{2}{3u}(1-u^3) - \frac{1}{y_d}(1-u^2)\right]\left\{1 + j\omega \frac{\epsilon}{2\sigma(\frac{I_d}{I_o})^2} \frac{\frac{2}{3} y_s{}^2 u^2 . Y_3}{(-\frac{1}{y_s})\left[1-u^2-\frac{2}{3}y_s(1-u^3)\right]}\right.$$

It can be shown that this expression reduces, with the aid of (3.13a), to
$$\Delta = \left[\frac{2}{3u}(1-u^3) - \frac{1}{y_d}(1-u^2)\right]\left\{1 + j\omega\tau_1\right\} \tag{3.14a}$$

where $\tau_1 = \frac{4\epsilon L^2}{15\sigma a^2} \cdot \frac{1}{y_s} \frac{Y_4}{Y_o{}^3}$

$$Y_o = 1 - u^2 - \frac{2}{3} y_s(1-u^3) \tag{3.14b}$$

and $$Y_4 = 1 - 5u^2 + 5u^3 - u^5 - \frac{5}{4} y_s(1-3u^2+3u^4-u^6)$$

$$+ \frac{5}{14} y_s{}^2(1-7u^3+7u^4-u^7)$$

3.6 Expression for y_{21}

When $y = y_d$, equation (3.12) yields

$$\frac{i_d}{v_g} = \frac{i(y_d)}{v_g}$$

$$= \frac{4Z\sigma \frac{I_d}{I_o}}{\triangle}\Bigg[\frac{1}{y_s y_d}(1-u) + k^2 y_s\Big\{\tfrac{2}{3}(1-u^2) - \tfrac{1}{2}y_s(1-u^3) - \tfrac{1}{3}u^2(1-u)$$

$$+ \tfrac{1}{6}y_s u^3(1-u) - u(1-u) + \tfrac{2}{3}y_s u(1-u^2)\Big\} + k^4 y_s{}^4\Big\{- \tfrac{1}{18}(1-u^5)$$

$$+ \tfrac{8}{105}y_s(1-u^6) - \tfrac{1}{40}y_s{}^2(1-u^7) - \tfrac{2}{9}u^3(1-u^2) + \tfrac{1}{6}y_s u^3(1-u^3)$$

$$+ \tfrac{1}{9}y_s u^4(1-u^2) - \tfrac{1}{12}y_s{}^2u^4(1-u^3) + \tfrac{1}{45}u^5(1-u)$$

$$- \tfrac{1}{42}y_s u^6(1-u) + \tfrac{1}{168}y_s{}^2u^7(1-u) + \tfrac{2}{15}u(1-u^4) - \tfrac{1}{6}y_s u(1-u^5)$$

$$+ \tfrac{1}{21}y_s{}^2u(1-u^6) + \tfrac{1}{6}u^4(1-u) - \tfrac{1}{9}y_s u^4(1-u^2) - \tfrac{1}{10}y_s u^5(1-u)$$

$$+ \tfrac{1}{15}y_s{}^2u^5(1-u^2)\Big\}\Bigg]$$

which becomes, on simplification,

$$\frac{i_d}{v_g} = \frac{4Z\sigma \frac{I_d}{I_o}(1-u)}{\triangle\, y_s y_d}\Bigg[1 + \tfrac{2}{3}k^2 y_s{}^2 y_d\Big\{1 - \tfrac{1}{2}u - \tfrac{1}{2}u^2 - \tfrac{3}{4}y_s(1 - \tfrac{1}{3}u - \tfrac{1}{3}u^2$$

$$- \tfrac{1}{3}u^3)\Big\} - \tfrac{1}{18}k^4 y_s{}^5 y_d\Big\{1 - \tfrac{7}{5}u - \tfrac{7}{5}u^2 + \tfrac{13}{5}u^3 - \tfrac{2}{5}u^4$$

$$- \tfrac{2}{5}u^5 - \tfrac{48}{35}y_s\big(1 - \tfrac{19}{16}u - \tfrac{19}{16}u^2 + u^3 + u^4 - \tfrac{5}{16}u^5 - \tfrac{5}{16}u^6\big)$$

$$+ \tfrac{9}{20}y_s{}^2\big(1 - \tfrac{19}{21}u - \tfrac{19}{21}u^2 - \tfrac{19}{21}u^3 + \tfrac{51}{21}u^4 - \tfrac{5}{21}u^5 - \tfrac{5}{21}u^6 - \tfrac{5}{21}u^7\big)\Big\}\Bigg]$$

Using equations (2.15a), (3.13a), (3.14a) and (3.13b) it can be shown that this expression takes the form

$$\frac{i_d}{v_g} = y_{21} = \frac{g_{mo}}{(1+j\omega\tau_1)}\Big[1 - j\omega\tau_2(1+j\omega\tau_3)\Big] \qquad (3.15)$$

where $$\tau_2 = \frac{4\epsilon L^2}{3\sigma a^2} \cdot \frac{1}{y_s} \cdot \frac{1}{Y_o^2} \cdot u \cdot Y_5$$

$$\tau_3 = \frac{\epsilon L^2}{6\sigma a^2} \cdot \frac{1}{y_s} \cdot \frac{1}{Y_o^2} \cdot \frac{Y_6}{Y_5}$$

and $$Y_5 = 1 - \tfrac{1}{2}u - \tfrac{1}{2}u^2 - \tfrac{3}{4}y_s(1 - \tfrac{1}{3}u - \tfrac{1}{3}u^2 - \tfrac{1}{3}u^3)$$

$$Y_6 = 1 - \tfrac{7}{5}u - \tfrac{7}{5}u^2 + \tfrac{13}{5}u^3 - \tfrac{2}{5}u^4 - \tfrac{2}{5}u^5$$
$$- \tfrac{48}{35}y_s(1 - \tfrac{19}{16}u - \tfrac{19}{16}u^2 + u^3 + u^4 - \tfrac{5}{16}u^5 - \tfrac{5}{16}u^6)$$
$$+ \tfrac{9}{20}y_s^2(1 - \tfrac{19}{21}u - \tfrac{19}{21}u^2 - \tfrac{19}{21}u^3 + \tfrac{51}{21}u^4 - \tfrac{5}{21}u^5 - \tfrac{5}{21}u^6 - \tfrac{5}{21}u^7)$$

3.7 Expression for y_{11}

When $y = y_s$ equation (3.12) becomes

$$\frac{i(y_s)}{v_g} = \frac{4Z\sigma\frac{I_d}{I_o}}{\Delta}\Bigg[\frac{1}{y_s y_d}(1-u) + k^2 y_s\Big\{\tfrac{2}{3}(1-u^2) - \tfrac{1}{2}y_s(1-u^3) - \tfrac{1}{3u}(1-u)$$
$$+ \tfrac{1}{6u}y_s(1-u) - (1-u) + \tfrac{2}{3}y_s(1-u^2)\Big\} + k^4 y_s^4\Big\{- \tfrac{1}{18}(1-u^5)$$
$$+ \tfrac{8}{105}y_s(1-u^6) - \tfrac{1}{40}y_s^2(1-u^7) - \tfrac{2}{9}(1-u^2) + \tfrac{1}{6}y_s(1-u^3)$$
$$+ \tfrac{1}{9}y_s(1-u^2) - \tfrac{1}{12}y_s^2(1-u^3) + \tfrac{1}{45}\,\tfrac{1}{u}(1-u) - \tfrac{1}{42}\,\tfrac{1}{u}y_s(1-u)$$
$$+ \tfrac{1}{168}\,\tfrac{1}{u}y_s^2(1-u) + \tfrac{2}{15}(1-u^4) - \tfrac{1}{6}y_s(1-u^5) + \tfrac{1}{21}y_s^2(1-u^6)$$
$$+ \tfrac{1}{6}(1-u) - \tfrac{1}{9}y_s(1-u^2) - \tfrac{1}{10}y_s(1-u) + \tfrac{1}{15}y_s^2(1-u^2)\Big\}\Bigg]$$

This is simplified and rearranged to give

$$\frac{i(y_s)}{v_g} = \frac{4Z\sigma\frac{I_d}{I_o}(1-u)}{\triangle\cdot y_s y_d}\left[1 - \tfrac{1}{3}k^2 y_s{}^2 y_d\left\{\frac{1}{u} + 1 - 2u - \tfrac{1}{2}y_s\left(\frac{1}{u} + 1 + u - 3u^2\right)\right\}\right.$$

$$+ k^4 y_s{}^5 y_d\left\{\frac{1}{45}\left(\frac{1}{u} + 1 - \frac{13}{2}u + \frac{7}{2}u^2 + \frac{7}{2}u^3 - \frac{5}{2}u^4\right)\right.$$

$$- \frac{1}{42}y_s\left(\frac{1}{u} + 1 - \frac{16}{5}u - \frac{16}{5}u^2 + \frac{19}{5}u^3 + \frac{19}{5}u^4 - \frac{16}{5}u^5\right)$$

$$\left.\left. + \frac{1}{168}y_s{}^2\left(\frac{1}{u} + 1 + u - \frac{51}{5}u^2 + \frac{19}{5}u^3 + \frac{19}{5}u^4 + \frac{19}{5}u^5 - \frac{21}{5}u^6\right)\right\}\right]$$

Use of equations (2.15a), (3.13a), (3.14a) and (3.13b) as in the previous case yields

$$\frac{i(y_s)}{v_g} = \frac{g_{mo}}{(1+j\omega\tau_1)}\left[1 + j\omega\tau_4(1+j\omega\tau_5)\right] \tag{3.16}$$

where

$$\tau_4 = \frac{2\epsilon L^2}{3\sigma a^2}\cdot\frac{1}{y_s}\cdot\frac{Y_7}{Y_o{}^2}$$

$$\tau_5 = \frac{2\epsilon L^2}{15\sigma a^2}\cdot\frac{1}{y_s}\ \frac{Y_8}{Y_o{}^2 Y_7}$$

and

$$Y_7 = 1 + u - 2u^2 - \tfrac{1}{2}y_s(1 + u + u^2 - 3u^3)$$

$$Y_8 = 1 + u - \frac{13}{2}u^2 + \frac{7}{2}u^3 + \frac{7}{2}u^4 - \frac{5}{2}u^5 - \frac{15}{14}y_s\left\{1 + u - \frac{16}{5}u^2\right.$$

$$\left. - \frac{16}{5}u^3 + \frac{19}{5}u^4 + \frac{19}{5}u^5 - \frac{16}{5}u^6\right\} + \frac{15}{56}y_s{}^2\left\{1 + u\right.$$

$$\left. + u^2 - \frac{51}{5}u^3 + \frac{19}{5}u^4 + \frac{19}{5}u^5 + \frac{19}{5}u^6 - \frac{21}{5}u^7\right\}$$

$$Y_o = 1 - u^2 - \tfrac{2}{3}y_s(1-u^3)$$

Since the net charge in the depletion regions is zero one obtains

$$i_d + i_s + i_g = 0$$

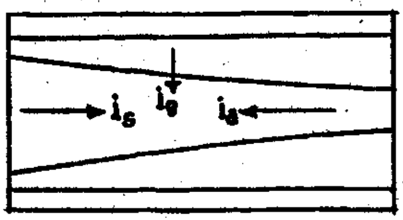

Fig. 3.1 Currents flowing in the device.

The directions of these currents are shown in Fig. 3.1.

Hence

$$\frac{i_g}{v_g} = \frac{-i_s}{v_g} - \frac{i_d}{v_g}$$

$$= \frac{i(y_s)}{v_g} - \frac{i_d}{v_g} \qquad (\text{since } i(y_s) = -i_s)$$

$$= \frac{g_{mo}}{(1+j\omega\tau_1)}\left[1 + j\omega\tau_4(1+j\omega\tau_5) - 1 + j\omega\tau_2(1+j\omega\tau_3)\right]$$

$$= \frac{g_{mo}}{(1+j\omega\tau_1)}\left[j\omega(\tau_4+\tau_2) + (j\omega)^2(\tau_4\tau_5+\tau_2\tau_3)\right]$$

which can be written as

$$\frac{i_g}{v_g} = y_{11} = \frac{g_{mo}}{(1+j\omega\tau_1)}\, j\omega\tau_6(1+j\omega\tau_7) \qquad (3.17)$$

where

$$\tau_6 = \frac{2\epsilon L^2}{3\sigma a^2} \cdot \frac{1}{y_s} \cdot \frac{Y_9}{Y_o^2}$$

$$\tau_7 = \frac{2\epsilon L^2}{15\sigma a^2} \cdot \frac{1}{y_s} \cdot \frac{1}{Y_o^2} \cdot \frac{Y_{10}}{Y_9}$$

and

$$Y_9 = 1 + 3u - 3u^2 - u^3 - \tfrac{1}{2}y_s(1 + 4u - 4u^3 - u^4)$$

$$\begin{aligned} Y_{10} = {} & 1 + \tfrac{7}{2}u - 10u^2 + 10u^4 - \tfrac{7}{2}u^5 - u^6 \\ & - \tfrac{15}{14}y_s(1 + \tfrac{21}{5}u - 7u^2 - 7u^3 + 7u^4 + 7u^5 - \tfrac{21}{5}u^6 - u^7) \\ & + \tfrac{15}{56}y_s^2(1 + \tfrac{26}{5}u - \tfrac{14}{5}u^2 - 14u^3 + 14u^5 + \tfrac{14}{5}u^6 - \tfrac{26}{5}u^7 - u^8) \end{aligned}$$

The input admittance y_{11} of the device is therefore a complex quantity.

3.8 Evaluation of C_o and C_1 for the Short-Circuited Input Case

The remaining part of this chapter deals with the derivation of expressions for the short-circuit output admittance (y_{22}) and the reverse transfer admittance (y_{12}) of the device. In order to determine the output admittance, the input is a.c. short-circuited and, for this case, the boundary conditions are

$$\left.\begin{aligned} y &= y_s \\ v &= v_g = 0 \end{aligned}\right\} \quad \text{at the source} \qquad (3.18a)$$

$$\text{and} \quad \left.\begin{aligned} y &= y_d \\ v &= -v_d \end{aligned}\right\} \quad \text{at the drain} \qquad (3.18b)$$

These relations are substituted in equation (3.1) to give

$$0 = -C_o k^2 . Y_{s1} + C_1 Y_{s2} \qquad \text{(at the source)} \qquad (3.19)$$

and

$$\frac{v_d}{B} = -C_o k^2 . Y_{d1} + C_1 . Y_{d2} \qquad \text{(at the drain)} \qquad (3.20)$$

B, Y_{s1}, Y_{s2}, Y_{d1} and Y_{d2} have been defined in section (3.2).

The coefficients C_o and C_1 are evaluated for this case by using the equations (3.19) and (3.20).

Equations (3.19) and (3.20) yield

$$0 = -C_o k^2 . Y_{s1} . Y_{d2} + C_1 Y_{s2} . Y_{d2} \qquad (3.21)$$

$$\frac{v_d}{B} . Y_{s2} = -C_o k^2 . Y_{d1} . Y_{s2} + C_1 Y_{d2} . Y_{s2} \qquad (3.22)$$

C_o is determined by subtracting (3.22) from (3.21).

$$C_o = \frac{-\frac{v_d}{B} . Y_{s2}}{k^2 \left[Y_{d1} . Y_{s2} - Y_{s1} . Y_{d2} \right]}$$

This can be rearranged and simplified to give

$$C_o = \frac{-\frac{v_d}{B} . Y_{11}}{k^2 . \Delta} \qquad (3.23)$$

where $Y_{11} = \frac{1}{y_s^2} - \frac{2}{3}k^2 y_s + \frac{1}{2}k^2 y_s^2 + \frac{1}{18}k^4 y_s^4 - \frac{8}{105}k^4 y_s^5 + \frac{1}{40}k^4 y_s^6$

and Δ is given by equation (3.14a).

In order to determine C_1, equations (3.19) and (3.20) are written in the following form.

$$0 = -C_o k^2 \cdot Y_{s1} \cdot Y_{d1} + C_1 \cdot Y_{s2} \cdot Y_{d1} \qquad (3.24)$$

$$\frac{v_d}{B} \cdot Y_{s1} = -C_o k^2 \cdot Y_{d1} \cdot Y_{s1} + C_1 \cdot Y_{d2} \cdot Y_{s1} \qquad (3.25)$$

Equations (3.24) and (3.25) are solved for C_1.

$$C_1 = \frac{-\frac{v_d}{B} \cdot Y_{s1}}{Y_{s2} \cdot Y_{d1} - Y_{d2} \cdot Y_{s1}}$$

It can be shown that this expression reduces to

$$C_1 = \frac{-\frac{v_d}{B} \cdot Y_{12}}{\Delta} \qquad (3.26)$$

where $Y_{12} = 1 - \frac{2}{3}y_s - k^2(\frac{2}{15}y_s^3 - \frac{1}{6}y_s^4 + \frac{1}{21}y_s^5)$

$$+ k^4(\frac{1}{180}y_s^6 - \frac{29}{2835}y_s^7 + \frac{1}{168}y_s^8 - \frac{149}{124740}y_s^9)$$

Substitution of equations (3.23) and (3.26) in (2.21) gives the expression for channel current in this case.

$$i(y) = C_o\, T(y) + C_1\, R(y)$$

$$= -\frac{v_d}{B.\Delta}\left[\frac{Y_{11}}{k^2} . T(y) + Y_{12} . R(y)\right]$$

Use of equations (3.11), (2.22) and (2.23) yields, after neglecting third and higher order terms in $(j\omega)$,

$$i(y) = \frac{4Z\sigma v_d}{\Delta}\frac{I_d}{I_o}\left\{\frac{1}{y_s^2} - k^2 y_s\left[\frac{2}{3} - \frac{1}{2}y_s + \frac{1}{3}\frac{y^3}{y_s^3} - \frac{1}{6}\frac{y^4}{y_s^3}\right.\right.$$

$$\left. - \frac{y}{y_s}(1 - \frac{2}{3}y_s)\right] + k^4 y_s^4\left[\frac{1}{18} - \frac{8}{105}y_s + \frac{1}{40}y_s^2 + \frac{2}{9}\frac{y^3}{y_s^3}\right.$$

$$- \frac{1}{6}\frac{y^3}{y_s^2} - \frac{1}{9}\frac{y^4}{y_s^3} + \frac{1}{12}\frac{y^4}{y_s^2} + \frac{1}{45}\frac{y^6}{y_s^6} - \frac{1}{42}\frac{y^7}{y_s^6}$$

$$+ \frac{1}{168}\frac{y^8}{y_s^6} - \frac{2}{15}\frac{y}{y_s} + \frac{1}{6}y - \frac{1}{21}yy_s - \frac{1}{6}\frac{y^4}{y_s^4} + \frac{1}{9}\frac{y^4}{y_s^3}$$

$$\left.\left. + \frac{1}{10}\frac{y^5}{y_s^4} - \frac{1}{15}\frac{y^5}{y_s^3}\right]\right\} \qquad (3.27)$$

This is the expression for the channel current when the input is a.c. short-circuited.

3.9 Expression for y_{22}

When $y = y_d$, equation (3.27) becomes

$$\frac{i_d}{v_d} = \frac{i(y_d)}{v_d}$$

$$= \frac{4Z\sigma}{\Delta}\frac{I_d}{I_o}\left\{\frac{1}{y_s^2} - \frac{2}{3}k^2 y_s\left[1 - \frac{3}{4}y_s + \frac{1}{2}u^3 - \frac{1}{4}y_s u^4 - \frac{3}{2}u + y_s u\right]\right.$$

$$+ \frac{1}{18}k^4 y_s^4\left[1 - \frac{48}{35}y_s + \frac{9}{20}y_s^2 + 4u^3 - 3y_s u^3 + \frac{3}{2}y_s^2 u^4\right.$$

$$+ \frac{2}{5}u^6 - \frac{3}{7}y_s u^7 + \frac{3}{28}y_s^2 u^8 - \frac{12}{5}u + 3y_s u$$

$$\left.\left. - \frac{6}{7}y_s^2 u - 3u^4 + \frac{9}{5}y_s u^5 - \frac{6}{5}y_s^2 u^5\right]\right\}$$

Equations (2.15) and (3.13) are substituted in this expression to give

$$\frac{i_d}{v_d} = \frac{4Z\sigma}{\Delta}\frac{I_d}{I_o}\frac{1}{y_s^2}\left\{1 + j\omega\left[\frac{4\epsilon L^2}{3\sigma a^2}\cdot\frac{1}{y_s}\cdot\frac{1}{Y_o^2}\left\{1 - \frac{3}{2}u + \frac{1}{2}u^3 - \frac{3}{4}y_s\right.\right.\right.$$

$$\left.\left.(1 - \frac{4}{3}u + \frac{1}{3}u^4)\right\}\right] + (j\omega)^2\left[\frac{2}{9}\frac{\epsilon^2 L^4}{\sigma^2 a^4}\cdot\frac{1}{y_s^2}\cdot\frac{1}{Y_o^4}\left\{1 - \frac{12}{5}u + 4u^3\right.\right.$$

$$- 3u^4 + \frac{2}{5}u^6 - \frac{48}{35}y_s(1 - \frac{35}{16}u + \frac{35}{16}u^3 - \frac{21}{16}u^5 + \frac{5}{16}u^7)$$

$$\left.\left.\left. + \frac{9}{20}y_s^2(1 - \frac{40}{21}u + \frac{10}{3}u^4 - \frac{8}{3}u^5 + \frac{5}{21}u^8)\right\}\right]\right\}$$

which can be written in the form

$$\frac{i_d}{v_d} = y_{22} = \frac{g_{mo}}{(1+j\omega\tau_1)}\left[\frac{u}{(1-u)}\left\{1 + j\omega\tau_8(1+j\omega\tau_9)\right\}\right] \qquad (3.28)$$

where

$$\tau_8 = \frac{4\epsilon L^2}{3\sigma a^2}\cdot\frac{1}{y_s}\cdot\frac{Y_{13}}{Y_o^2}$$

$$\tau_9 = \frac{\epsilon L^2}{6\sigma a^2}\cdot\frac{1}{y_s}\cdot\frac{1}{Y_o^2}\cdot\frac{Y_{14}}{Y_{13}}$$

and

$$Y_{13} = 1 - \frac{3}{2}u + \frac{1}{2}u^3 - \frac{3}{4}y_s(1 - \frac{4}{3}u + \frac{1}{3}u^4)$$

$$Y_{14} = 1 - \frac{12}{5}u + 4u^3 - 3u^4 + \frac{2}{5}u^6$$

$$- \frac{48}{35}y_s(1 - \frac{35}{16}u + \frac{35}{16}u^3 - \frac{21}{16}u^5 + \frac{5}{16}u^7)$$

$$+ \frac{9}{20}y_s^2(1 - \frac{40}{21}u + \frac{10}{3}u^4 - \frac{8}{3}u^5 + \frac{5}{21}u^8)$$

3.10 Expression for y_{12}

When $y = y_s$, equation (3.27) becomes

$$\frac{i(y_s)}{v_d} = \frac{4Z\sigma}{\triangle} \cdot \frac{I_d}{I_o}\left\{\frac{1}{y_s^2} - \frac{2}{3}k^2 y_s(1 - \frac{3}{4}y_s + \frac{1}{2} - \frac{1}{4}y_s - \frac{3}{2} + y_s)\right.$$

$$+ \frac{1}{18}k^4 y_s{}^4(1 - \frac{48}{35}y_s + \frac{9}{20}y_s{}^2 + 4 - 3y_s + \frac{3}{2}y_s{}^2 + \frac{2}{5}$$

$$\left.- \frac{3}{7}y_s + \frac{3}{28}y_s{}^2 - \frac{12}{5} + 3y_s - \frac{6}{7}y_s{}^2 - 3 + \frac{9}{5}y_s - \frac{6}{5}y_s{}^2)\right\}$$

which reduces, with the aid of (2.15a), (3.13a), (3.13b) and (3.14a), to

$$\frac{i(y_s)}{v_d} = \frac{g_{mo}}{(1+j\omega\tau_1)}\,\frac{u}{(1-u)} \qquad (3.29)$$

Since the net charge in the depletion regions is zero,

$$i_d + i_s + i_g = 0$$

Hence

$$i_g = -i_s - i_d$$

$$= i(y_s) - i_d$$

The expression for y_{12} is found by using the relations (3.28) and (3.29).

$$y_{12} = \frac{i_g}{v_d}$$

$$= \frac{i(y_s)}{v_d} - \frac{i_d}{v_d}$$

$$\text{or } y_{12} = -\frac{g_{mo}}{(1+j\omega\tau_1)}\,\frac{u}{(1-u)}\,j\omega\tau_8(1+j\omega\tau_9) \qquad (3.30)$$

The short-circuit admittance parameters of the F.E.T. can therefore be summarized as follows:

$$y_{11} = \frac{i_g}{v_g}\bigg|_{v_d=0} = \frac{g_{mo}}{(1+j\omega\tau_1)}\, j\omega\tau_6(1+j\omega\tau_7)$$

$$y_{12} = \frac{i_g}{v_d}\bigg|_{v_g=0} = \frac{-g_{mo}}{(1+j\omega\tau_1)}\,\frac{u}{1-u}\, j\omega\tau_8(1+j\omega\tau_9)$$

$$y_{21} = \frac{i_d}{v_g}\bigg|_{v_d=0} = \frac{g_{mo}}{(1+j\omega\tau_1)}\left[1 - j\omega\tau_2(1+j\omega\tau_3)\right]$$

$$y_{22} = \frac{i_d}{v_d}\bigg|_{v_g=0} = \frac{g_{mo}}{(1+j\omega\tau_1)}\,\frac{u}{(1-u)}\left[1 + j\omega\tau_8(1+j\omega\tau_9)\right]$$

Under saturated (pinch-off) conditions, $y_d = 0$ and so $u = 0$. Hence

$$y_{12} = 0$$

$$\text{and } y_{22} = 0$$

Also, from equation (3.15), $\tau_2 = 0$. Therefore, y_{21} becomes

$$y_{21} = \frac{g_{mo}}{(1+j\omega\tau_1)}$$

4. THE EQUIVALENT CIRCUIT

4.1 Introduction

In this chapter, an equivalent circuit of the field-effect transistor is formulated by using the expressions for the short-circuit admittance parameters derived in the previous chapter. The elements of the circuit are determined and graphs are drawn to show how their values vary with the drain and ~~source~~ gate voltages.

There are two possible approaches to obtaining equivalent circuits for active devices. The 'circuit or black-box approach' is usually employed when only the terminal characteristics of the device are known. In such cases it is not necessary to know the internal physics of the device. The second method, called the 'device approach', is adopted when the physics of the device is sufficiently well understood. Starting from a knowledge of the physical principles involved, the expressions which adequately describe the operation of the device are derived. Then, an equivalent circuit which satisfies these expressions is found. The advantage of this approach is that it enables each component of the equivalent circuit to be expressed in terms of the fundamental device parameters. The device approach is used here to obtain the equivalent circuit.

4.2 Assumed Circuit Model

To start with, a hybrid-π network of a general type is assumed to represent the equivalent circuit of the device and the admittance parameters of this circuit model are determined. By comparing these parameters with those derived in the previous chapter, expressions for the different elements of the circuit are derived.

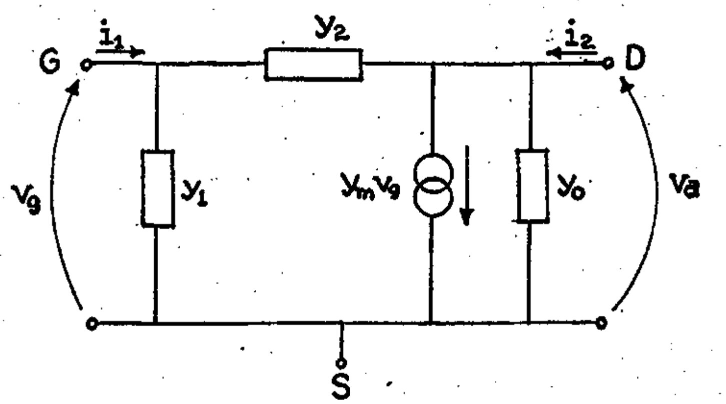

Fig. 4.1 Assumed form of the F.E.T. equivalent circuit.

Fig. 4.1 shows the assumed circuit model of an F.E.T. which is treated as a four terminal device. In this circuit, y_1, y_2 and y_o are the admittance branches and y_mv_g is the current generator; i_1 and i_2 are the input and output currents, respectively. If v_g and v_d are the input and output voltages for this common-source connection, Kirchoff's current relations for the two nodes G and D can be written as follows:

$$i_1 = (y_1+y_2)v_g - y_2v_d$$

$$i_2 = -(y_2-y_m)v_g + (y_2+y_o)v_d$$

Hence

$$y_{11} = y_1 + y_2$$

$$y_{12} = -y_2$$

$$y_{21} = -y_2 + y_m$$

$$y_{22} = y_2 + y_o$$

These relations are solved for y_1, y_2, y_o and y_m.

$$y_1 = y_{11} + y_{12} \tag{4.1}$$

$$y_2 = -y_{12} \tag{4.2}$$

$$y_o = y_{22} + y_{12} \tag{4.3}$$

$$y_m = y_{21} - y_{12} \tag{4.4}$$

The values of y_{11}, y_{12}, y_{21} and y_{22}, derived in chapter 3, are now used to determine y_1, y_2, y_o and y_m.

4.3 Expressions for y_1, C_1 and R_1

By using equations (4.1), (3.17) and (3.30), y_1 is determined.

$$y_1 = y_{11} + y_{12}$$

$$= \frac{g_{mo}}{(1+j\omega\tau_1)}\left[j\omega\tau_6(1+j\omega\tau_7) - \delta\cdot j\omega\tau_8(1+j\omega\tau_9)\right]$$

$$\simeq \frac{g_{mo}}{(1+j\omega\tau_1)}\left[\frac{j\omega\tau_6}{(1-j\omega\tau_7)} - \frac{\delta\cdot j\omega\tau_8}{(1-j\omega\tau_9)}\right]^{*}$$

$$= \frac{g_{mo}}{(1+j\omega\tau_1)}\left[\frac{j\omega(\tau_6-\delta\tau_8) - (j\omega)^2(\tau_6\tau_9-\delta\tau_8\tau_7)}{(1-j\omega\tau_7)(1-j\omega\tau_9)}\right]$$

where $\delta = \frac{u}{1-u}$ (4.5)

As in the previous cases the second order terms in $(j\omega)$ are neglected in the denominator. It can then be shown that the above expression reduces to

$$y_1 = \frac{g_{mo}\cdot j\omega(\tau_6-\delta\tau_8)}{1 + j\omega\left\{\tau_1 + \frac{\tau_6\tau_9-\delta\tau_8\tau_7}{\tau_6-\delta\tau_8} - \tau_7 - \tau_9\right\}} \tag{4.6}$$

which can be written in the form

$$y_1 = \frac{j\omega C_1}{1 + j\omega C_1 R_1} \tag{4.7}$$

where

$$C_1 = g_{mo}(\tau_6 - \delta\tau_8) \tag{4.8}$$

and

$$R_1 = \frac{\tau_1 + \frac{\tau_6\tau_9 - \delta\tau_8\tau_7}{\tau_6 - \delta\tau_8} - \tau_7 - \tau_9}{g_{mo}(\tau_6 - \delta\tau_8)} \tag{4.9}$$

* The validity of this approximation will be discussed in Chapter 5.

Equation (4.7) indicates that y_1 can be conveniently expressed as the admittance of a resistance R_1 in series with a capacitance C_1. As the expressions for g_{mo} and τ's are known, the values of R_1 and C_1 can be determined.

The capacitance C_1 is given by equation (4.8).

$$C_1 = g_{mo}(\tau_6 - \delta\,\tau_8)$$

The values of τ_6 and τ_8 are given by (3.17) and (3.28), respectively, and δ is given by (4.5)

$$(\tau_6 - \delta\tau_8) = \frac{2\epsilon L^2}{3\sigma a^2}\cdot\frac{1}{y_s}\cdot\frac{1}{Y_o^2}\,\frac{1}{(1-u)}\left[1-3u^2+2u^3-\tfrac{1}{2}y_s(1-4u^3+3u^4)\right]$$

Finally, use of equations (3.13b) and (3.14b) yields

$$C_1 = \frac{4Z\epsilon L}{3a}\left\{\frac{1 - 3u^2 + 2u^3 - \frac{1}{2}y_s(1-4u^3+3u^4)}{\left[1 - u^2 - \frac{2}{3}\,y_s(1-u^3)\right]^2}\right\} \qquad (4.10)$$

where $u = \dfrac{y_d}{y_s}$

C_1 is the gate-to-source capacitance and it can be shown that (4.10) agrees with van der Ziel's[8] expression for C_1

In the saturation (pinch-off) case, $y_d = 0$ or $u = 0$, so that (4.10) becomes

$$C_1 = C_{1S} = \frac{4Z\epsilon L}{3a}\left[\frac{1 - \frac{1}{2}y_s}{(1 - \frac{2}{3}y_s)^2}\right] \qquad (4.11)$$

The resistance R_1 is given by equation (4.9). When the values of r's are substituted in (4.9), it becomes too long and cumbersome to write. Therefore, an expression for R_1 in the saturation case is derived.

In the saturation case $y_d = 0$ and $\delta = 0$, so that equation (4.9) becomes

$$R_1 = R_{1S} = \frac{r_1 - r_7}{g_{mo} \cdot r_6}$$

This can be simplified to give

$$R_{1S} = \frac{L}{10\sigma aZ} \cdot \left\{ \frac{1}{y_s} \frac{1 - \frac{37}{21} y_s + \frac{55}{56} y_s^2 - \frac{5}{28} y_s^3}{1 - \frac{5}{3} y_s + \frac{11}{12} y_s^2 - \frac{1}{6} y_s^3} \right\} \qquad (4.12)$$

Since, for the saturation case,

$$g_{mo} = \frac{2Z\sigma a}{L} y_s$$

R_{1S} can also be written as

$$R_{1S} = \frac{1}{5g_{mo}} \left\{ \frac{1 - \frac{37}{21} y_s + \frac{55}{56} y_s^2 - \frac{5}{28} y_s^3}{1 - \frac{5}{3} y_s + \frac{11}{12} y_s^2 - \frac{1}{6} y_s^3} \right\} \qquad (4.13)$$

4.4 Expressions for y_2, C_2 and R_2

Equations (4.2) and (3.30) yield

$$y_2 = -y_{12}$$

$$y_2 = \frac{g_{mo}}{(1+j\omega\tau_1)} \frac{u}{(1-u)} j\omega\tau_8 (1+j\omega\tau_9)$$

$$\approx \frac{g_{mo}}{(1+j\omega\tau_1)} \frac{u}{(1-u)} \frac{j\omega\tau_8}{(1-j\omega\tau_9)}$$

When the second order terms in $(j\omega)$ are neglected, this becomes

$$y_2 = \frac{g_{mo}\left(\frac{u}{1-u}\right) j\omega\tau_8}{1+j\omega(\tau_1-\tau_9)}$$

which can be written in the form

$$y_2 = \frac{j\omega C_2}{1 + j\omega C_2 R_2} \qquad (4.14)$$

where

$$C_2 = g_{mo} \frac{u}{1-u} \tau_8 \qquad (4.15)$$

and

$$R_2 = \frac{\tau_1 - \tau_9}{g_{mo}\left(\frac{u}{1-u}\right)\tau_8} \qquad (4.16)$$

Equation (4.14) indicates that y_2 can be considered as the admittance of a resistance R_2 and a capacity C_2 in series.

Substitution of equations (3.13b) and (3.28) in (4.15) yields

$$C_2 = \frac{8Z\epsilon L}{3a}\left\{ \frac{u\left[1 - \frac{3}{2}u + \frac{1}{2}u^3 - \frac{3}{4}y_s(1-\frac{4}{3}u+\frac{1}{3}u^4)\right]}{\left[1 - u^2 - \frac{2}{3}y_s(1-u^3)\right]^2} \right\} \qquad (4.17)$$

C_2 is the gate-to-drain capacitance. It can be shown that (4.17) agrees with van der Ziel's[8] expression for C_2. In the saturation case u = 0 so that

$$C_2 = C_{2S} = 0$$

After substituting for τ's and g_{mo}, (4.16) becomes

$$R_2 = \frac{L}{10\sigma aZ} \cdot \frac{1}{y_d}\left[\frac{1}{Y_o \cdot Y_{13}}\right]\left[Y_4 - \frac{5}{8}\frac{Y_o}{Y_{13}} \cdot Y_{14}\right] \qquad (4.18)$$

Y_o and Y_4 are defined in section 3.5, and Y_{13} and Y_{14} are defined in section 3.8.

Under saturated conditions, $y_d = 0$ and so (4.18) gives

$$R_2 = R_{2S} = \infty$$

4.5 Expressions for y_o, R_o and L_o

Use of equations (4.3), (3.28) and (3.30) yields the expression for y_o.

$$y_o = y_{22} + y_{12}$$

$$= \frac{g_{mo}}{(1+j\omega\tau_1)}\frac{u}{(1-u)}\left[1 + j\omega\tau_8(1+j\omega\tau_9)\right]$$

$$- \frac{g_{mo}}{(1+j\omega\tau_1)}\frac{u}{(1-u)} j\omega\tau_8(1+j\omega\tau_9)$$

$$= \frac{g_{mo} \cdot \delta}{(1+j\omega\tau_1)} \qquad (4.19)$$

where $\delta = \frac{u}{1-u}$.

Equation (4.19) can be expressed in the form

$$y_o = \frac{g_o}{1 + j\omega g_o L_o} \qquad (4.20)$$

where

$$g_o = g_{mo} \cdot \delta \qquad (4.21)$$

and

$$L_o = \frac{\tau_1}{g_{mo} \cdot \delta} \qquad (4.22)$$

Equation (4.20) shows that the admittance y_o consists of a conductance g_o in series with an inductance L_o whose values are given by the relations (4.21) and (4.22), respectively.

Substituting for g_{mo} and δ , equation (4.21) gives

$$g_o = \frac{2Z\sigma a}{L} \cdot y_s(1-u) \cdot \frac{u}{(1-u)}$$

Since $u = \frac{y_d}{y_s}$, g_o becomes

$$g_o = \frac{2Z\sigma a}{L} \cdot y_d \qquad (4.23)$$

g_o is the output conductance of the device. Recalling that

$$W_d = W_p(1-y_d)^2$$

equation (4.23) can be written as

$$g_o = \frac{2Z\sigma a}{L}\left[1 - \left(\frac{W_d}{W_p}\right)^{\frac{1}{2}}\right]$$

which agrees with the expression (1.6) for output conductance given by the d.c. theory.

An expression for L_o can be derived by the use of equations (4.22) and (3.14a).

$$L_o = \frac{L}{2Z\sigma a} \cdot \frac{1}{y_d} \cdot \frac{4\epsilon L^2}{15\sigma a^2} \cdot \frac{1}{y_s} \cdot \frac{Y_4}{Y_o^3}$$

$$= \frac{2}{15} \frac{\epsilon L^3}{Z\sigma^2 a^3} \cdot \frac{1}{y_d y_s} \cdot \frac{Y_4}{Y_o^3} \qquad (4.24)$$

where

$$Y_4 = 1 - 5u^2 + 5u^3 - u^5 - \frac{5}{4} y_s(1 - 3u^2 + 3u^4 - u^6)$$

$$+ \frac{5}{14} y_s^2(1 - 7u^3 + 7u^4 - u^7)$$

and
$$Y_o = 1 - u^2 - \frac{2}{3} y_s(1 - u^3)$$

Under saturated conditions ($y_d = 0$) equation (4.23) gives

$$g_o = g_{os} = 0$$

or, the output resistance R_o is

$$R_o = R_{os} = \frac{1}{g_{os}} = \infty$$

and the inductance L_o becomes

$$L_o = L_{os} = \infty$$

4.6 Expression for transadmittance y_m

The transadmittance 'y_m' of the device is determined by using equations (4.4), (3.15) and (3.30).

$$y_m = y_{21} - y_{12}$$

$$= \frac{g_{mo}}{(1+j\omega\tau_1)}\left\{1 - j\omega\tau_2(1+j\omega\tau_3)\right\} + \frac{g_{mo}}{1+j\omega\tau_1}\cdot\delta\cdot j\omega\tau_8(1+j\omega\tau_9)$$

$$= \frac{g_{mo}}{(1+j\omega\tau_1)}\left\{1 - j\omega(\tau_2-\delta\tau_8) + (j\omega)^2(-\tau_2\tau_3+\delta\tau_8\tau_9)\right\} \qquad (4.25)$$

At saturation $\delta = 0$ and $\tau_2 = 0$, so that

$$y_m = \frac{g_{mo}}{(1+j\omega\tau_{1S})} \qquad (4.26)$$

where τ_{1S} is the value of τ_1 at saturation. (see Appendix A)

$$\tau_{1S} = \frac{4\epsilon L^2}{15\sigma a^2}\cdot\frac{1}{y_s}\left\{\frac{1 - \frac{5}{4}y_s + \frac{5}{14}y_s^2}{\left(1 - \frac{2}{3}y_s\right)^3}\right\}$$

4.7 The Equivalent Circuit

Having determined all the elements of the assumed circuit model, the equivalent circuit for the F.E.T. can be written as shown in Fig. 4.2.

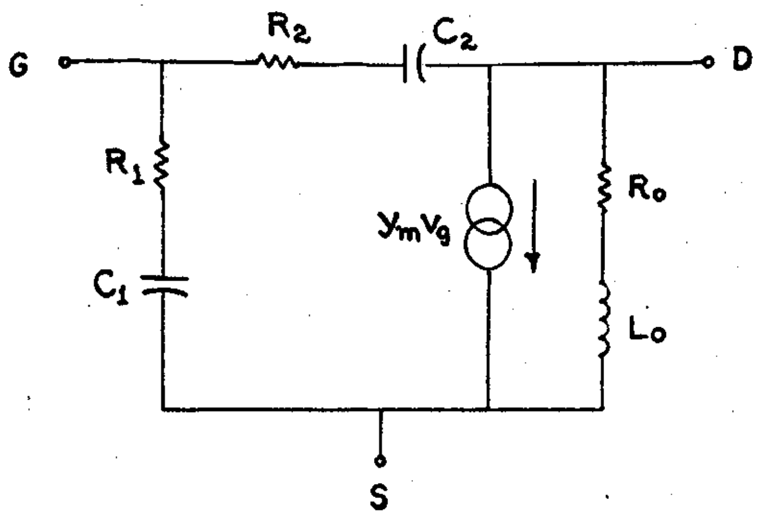

Fig. 4.2 The derived equivalent circuit for the device.

S, D and G are the source, drain and gate terminals, respectively of the device and $y_m v_g$ is the current generator; v_g being the a.c. gate voltage referred to the source. The expressions for R_1, R_2, C_1, C_2 and L_o indicate that the values of these elements vary in a nonlinear manner with the drain and gate voltages.

Normalized values of R_1 and C_1 as given by equations 4.9 and 4.10 have been computed theoretically and are plotted against W_s/W_p for different values of W_d/W_p in Figs. 4.3 and 4.4. It is apparent that when $W_s = W_p$, R_1 goes to infinity for all values of W_d and C_1 goes to zero for all values of W_d except when $W_d = W_p$ (saturation).

It has been shown in the previous section that under saturated conditions R_2 goes to infinity and C_2 reduces to zero. In Figs. 4.5 and 4.6, normalized values of R_2 and C_2 under non-saturated conditions are plotted against W_s/W_p. As W_d/W_p decreases, the value of R_2 reduces and that of C_2 increases.

According to (4.23) and (4.24), both R_o and L_o increase with the drain-to-source voltage and at saturation both go to infinity. It is shown in Chapter 5 that the value of L_o is very small and that it can usually be neglected except at very high frequencies.

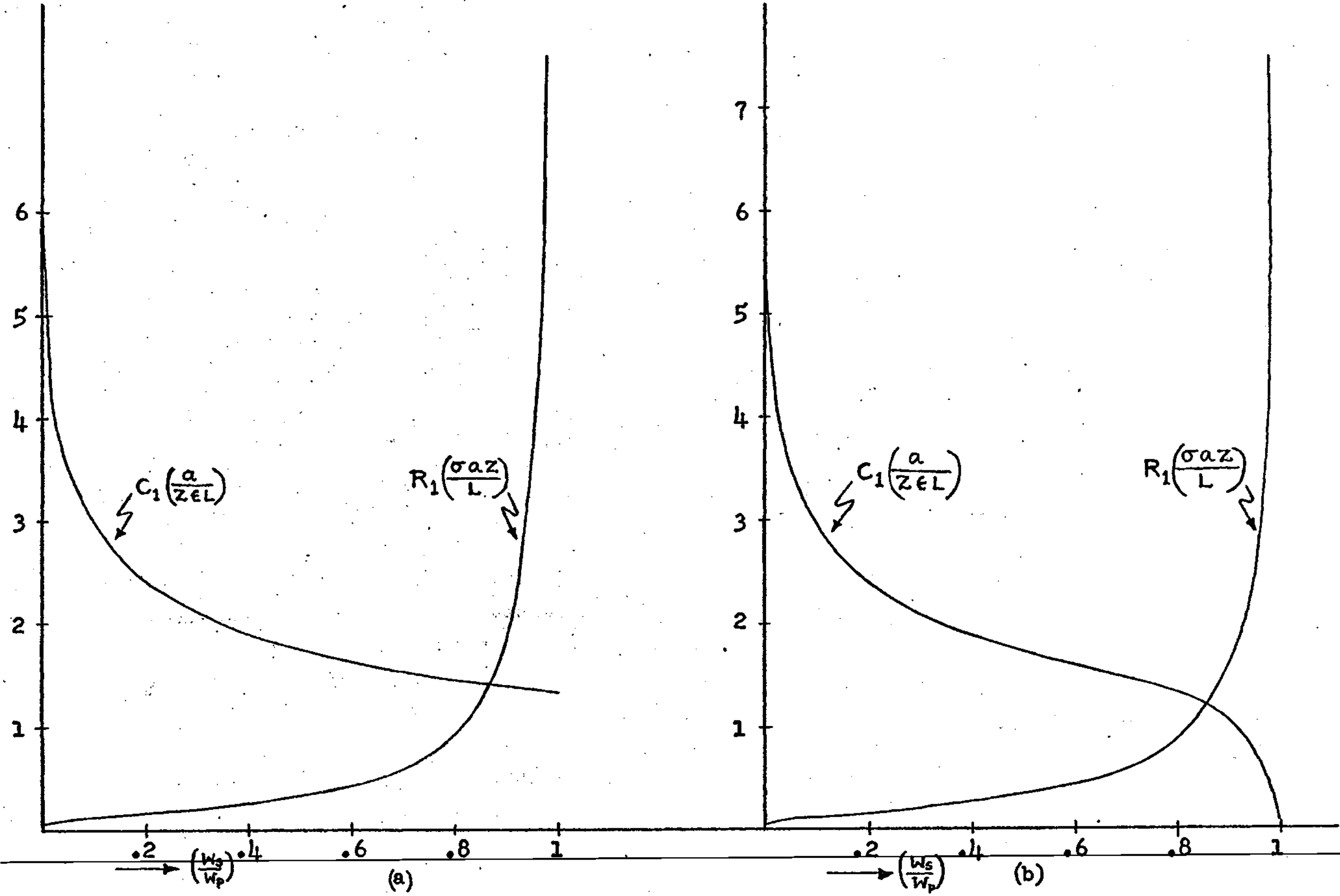

Fig. 4.3 Normalized values of R_1 and C_1 versus normalized gate bias with normalized drain bias as the parameter. (a) $W_a = W_p$ (saturation); (b) $W_a = (0.9)\ W_p$

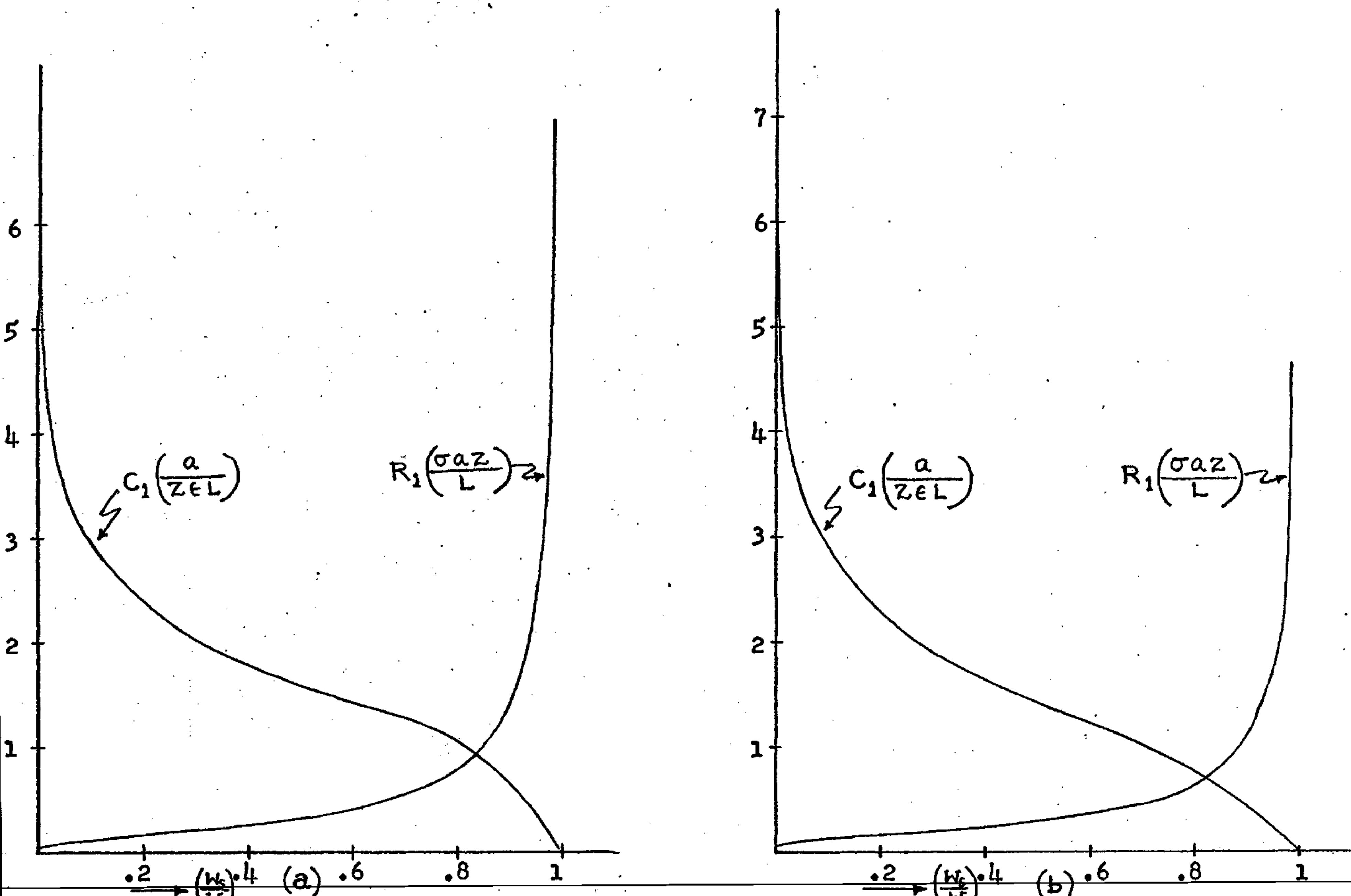

Fig. 4.4 Normalized values of R_1 and C_1 versus normalized gate bias with normalized drain bias as the parameter. (a) $W_d = (0.75)\ W_p$; (b) $W_d = (0.5)\ W_p$.

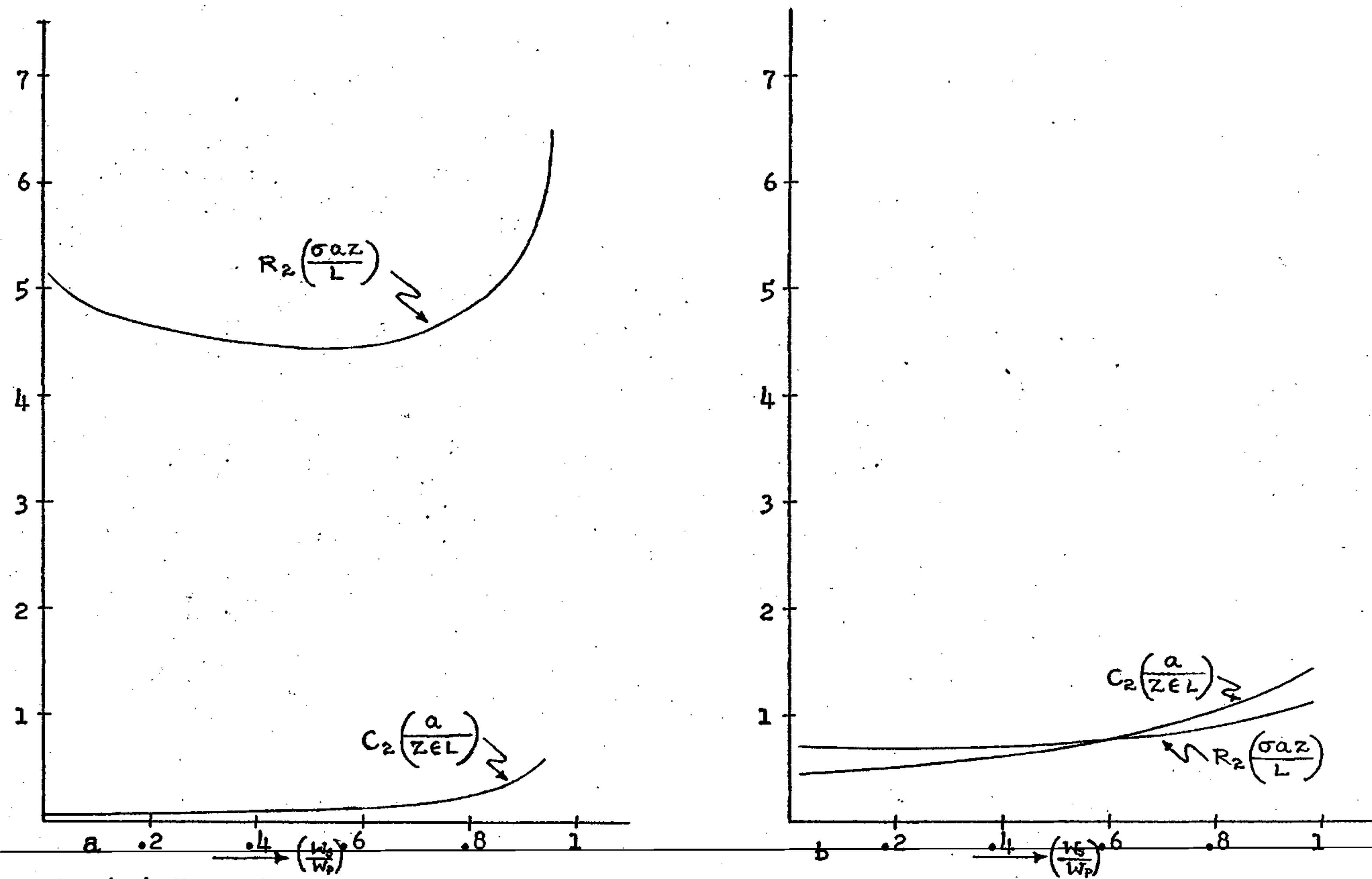

Fig. 4.5 Normalized values of R_2 and C_2 versus normalized gate bias with normalized drain bias as the parameter. (a) $W_d = (0.98)\ W_p$; (b) $W_d = (0.83)\ W_p$.

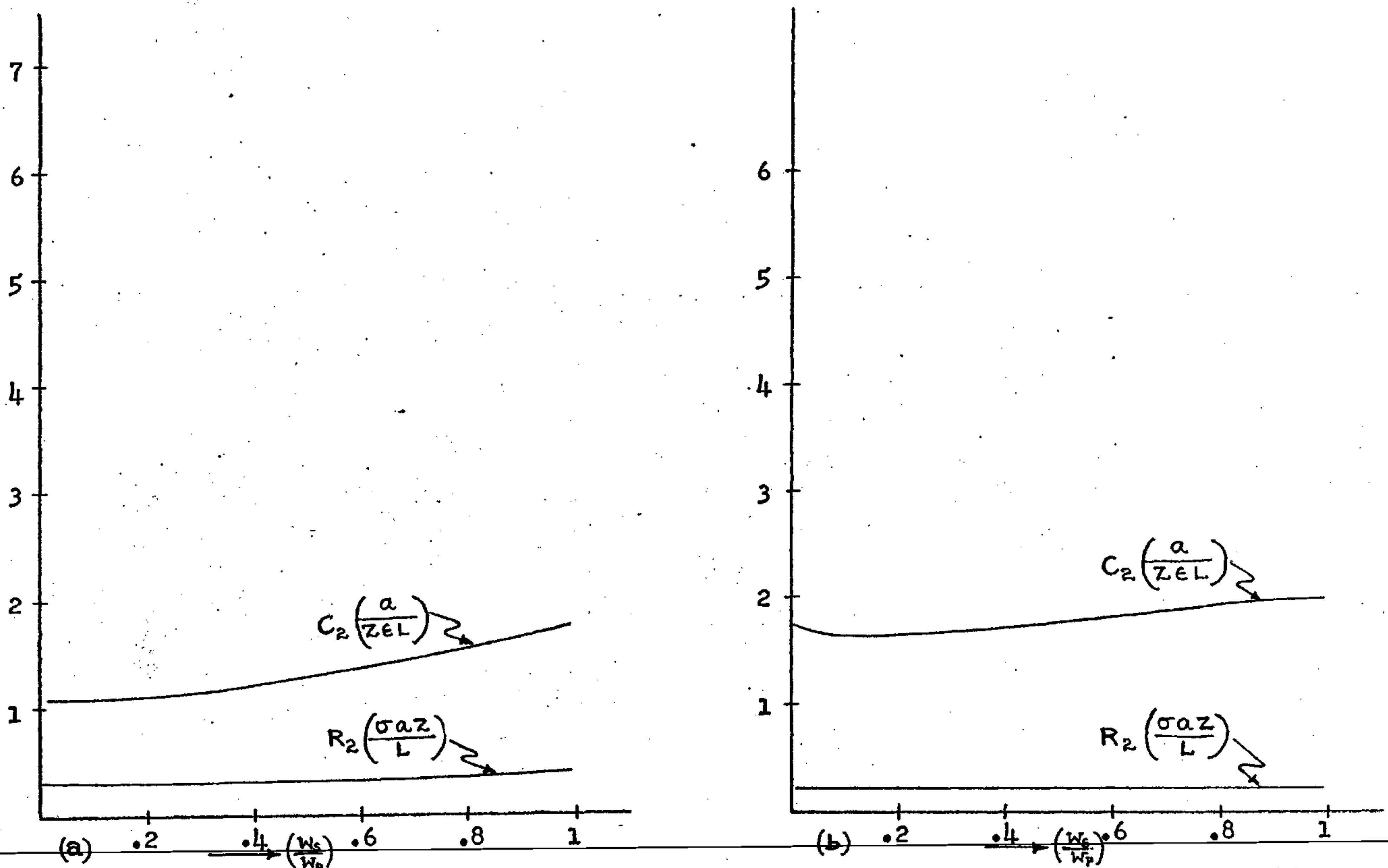

Fig. 4.6 Normalized values of R_2 and C_2 versus normalized gate bias with normalized drain bias as the parameter. (a) $W_d = (0.56\ W_p$; (b) $W_d = (0.35)\ W_p$.

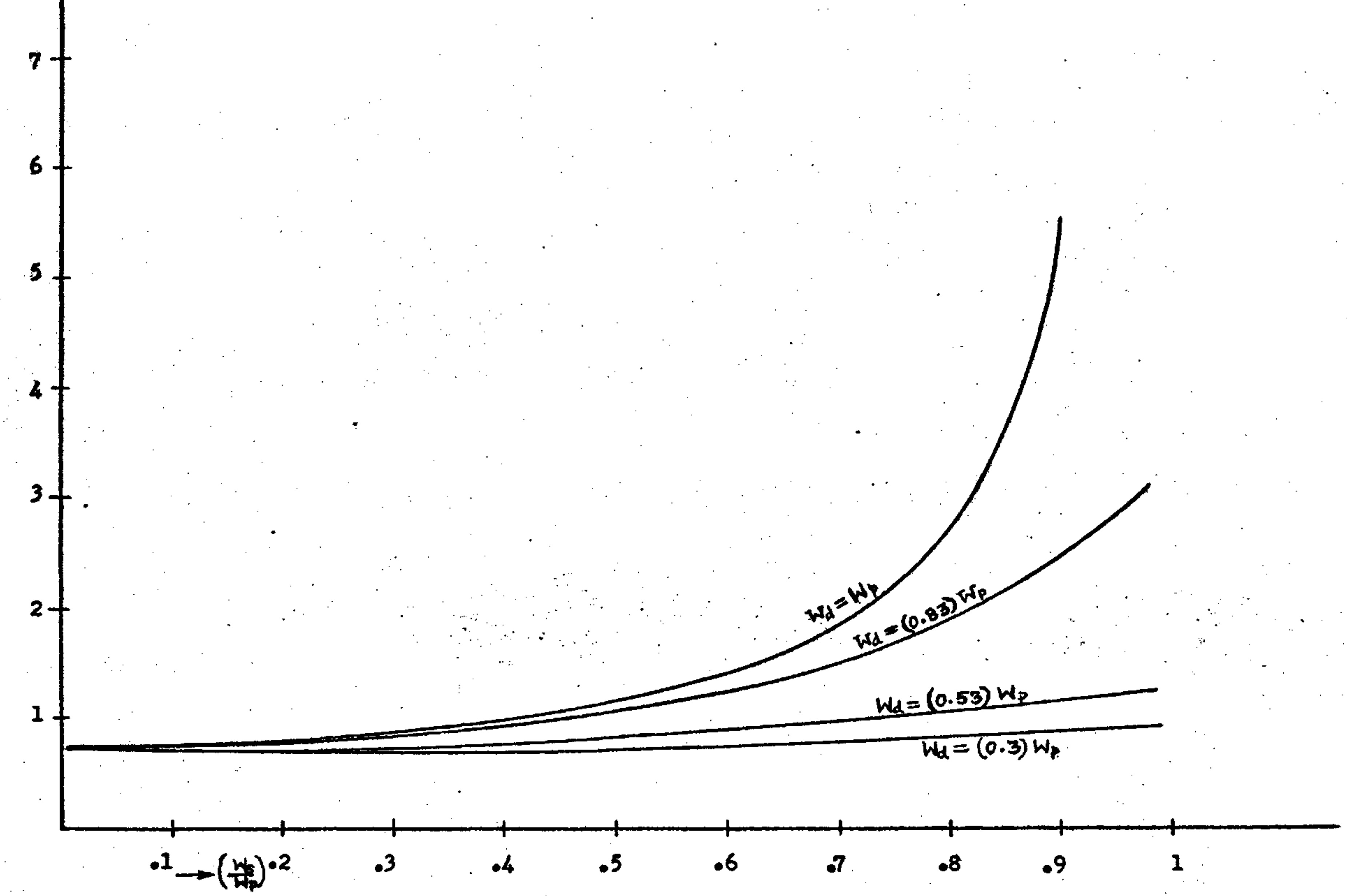

Fig. 4.7 Normalized value of $(\tau_1 + \tau_2 - \delta\,\tau_3)$ versus normalized gate bias with normalized drain bias as the parameter.

5. CONCLUSIONS

Considering the field effect transistor as an active, distributed and non-uniform transmission line, a differential equation for the a.c. case was derived from the physical principles involved in the operation of the device. As these considerations were based on Shockley's basic theory of the device, the analysis presented here is applicable only to those devices for which the assumptions in that theory are valid. In other words, the theory presented in this work is valid for F.E.T.'s with abrupt p-n junctions and uniformly doped channels. Furthermore, since Shockley's theory also assumes a gradual variation of potential along the channel of the device, this analysis is valid as long as the condition $y_s \geqslant y_d$ (i.e. $W_d \geqslant W_s$) is satisfied.

The differential equation was solved and the solution was used to derive the expressions for the short-circuit admittance parameters of the device. An equivalent circuit in the form of a hybrid-π network was then obtained from the first order approximation of the theory. The mathematical approximation made in sections 4.3 and 4.4, while deriving the expressions for the elements of the equivalent circuit, is valid as long as the condition

$$\omega\tau \ll 1$$

is satisfied. The quantity $\frac{\epsilon L^2}{\sigma a^2}$ which determines the order of magnitude of all τ's, depends upon the material, doping and the physical dimensions of the device. For example, it has been shown[14] that for the F.E.T. 2N2498 (manufactured by Texas Instruments Inc.) the value of $\frac{\epsilon L^2}{\sigma a^2} \leqslant 4.0 \times 10^{-9}$. Hence, the approximation is valid up to a frequency of several megacycles.

The Q of the y_o branch can be determined by using this value of $\epsilon L^2/a^2\sigma$. It is given by

$$Q = \frac{\omega L_o}{R_o}$$

Use of (4.23) and (4.24) yields

$$\frac{L_o}{R_o} = \frac{4}{15}\,\frac{\epsilon L^2}{\sigma a^2}\cdot\frac{1}{y_s}\cdot\frac{Y_4}{Y_o^3}$$

Let $y_s = 0.5$ (i.e., $\frac{W_s}{W_p} = 0.25$). Then

$$\frac{L_o}{R_o} \approx 2.4 \times 10^{-9}$$

Therefore the Q of the y_o branch at a frequency of 5 mc is approximately equal to 0.075 which is a very small quantity. The inductance L_o can therefore be neglected except at very high frequencies. When it is omitted, the equivalent circuit of Fig. 4.2 will have the same configuration as Silverthorn's circuit.

Equation (4.26) shows that the transadmittance y_m decreases with frequency. Since the order of magnitude of τ_1 is 10^{-9}

seconds, the variation of y_m with frequency should be very small until a value of about ten megacycles is reached. Measurements made by Silverthorn[11] show this to be correct.

The analysis presented here takes into consideration only the intrinsic device and hence the equivalent circuit obtained is not complete. In an actual F.E.T. structure, there will be some parasitic resistance in series with the source and drain leads, and it has been shown[11,12] that these parasitic resistances can be absorbed into the equivalent circuit of Fig. 4.2 to yield a circuit having the same configuration. Furthermore, the actual capacitances in a practical device will differ somewhat from the values computed in this idealized model because of the non-ideal conditions at the source and drain ends of the channel due to depletion region "end effects".

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APPENDIX A

The values of τ's under saturated condition ($y_d = 0$) are given below.

$$\tau_1 = \frac{4\epsilon L^2}{15\sigma a^2} \cdot \frac{1}{y_s} \left\{ \frac{1 - \frac{5}{4} y_s + \frac{5}{14} y_s^2}{(1 - \frac{2}{3} y_s)^3} \right\}$$

$$\tau_2 = 0$$

$$\tau_3 = \tau_9 = \frac{\epsilon L^2}{6\sigma a^2} \cdot \frac{1}{y_s} \cdot \left\{ \frac{1 - \frac{48}{35} y_s + \frac{9}{20} y_s^2}{(1 - \frac{2}{3} y_s)^2 (1 - \frac{3}{4} y_s)} \right\}$$

$$\tau_4 = \tau_6 = \frac{2}{3} \frac{\epsilon L^2}{\sigma a^2} \cdot \frac{1}{y_s} \left\{ \frac{1 - \frac{1}{2} y_s}{(1 - \frac{2}{3} y_s)^2} \right\}$$

$$\tau_5 = \tau_7 = \frac{2\epsilon L^2}{15\sigma a^2} \cdot \frac{1}{y_s} \left\{ \frac{1 - \frac{15}{14} y_s + \frac{15}{56} y_s^2}{(1 - \frac{2}{3} y_s)^2 (1 - \frac{1}{2} y_s)} \right\}$$

$$\tau_8 = \frac{4\epsilon L^2}{3\sigma a^2} \cdot \frac{1}{y_s} \left\{ \frac{(1 - \frac{3}{4} y_s)}{(1 - \frac{2}{3} y_s)^2} \right\}$$